Title: A Simple Harmonic Universe

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Abstract: We explore simple but novel solutions of general relativity which, classically, approximate cosmologies cycling through an infinite set of ``bounces." These solutions require curvature K=+1, and are supported by a negative cosmological term and matter with -1 < w < -1/3. They can be studied within the regime of validity of general relativity. We argue that quantum mechanically, particle production leads eventually to a departure from the regime of validity of semiclassical general relativity, likely yielding a singular crunch.

A simple harmonic universe

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Work in progress with P. Graham, B. Horn, S. Kachru, and S. Rajendran

Challenges for Early Universe Cosmology

Perimeter Institute, July 2011

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However, there are still important theoretical questions which are not addressed...

Here we focus on the question of the **singularity in the far past.** From cosmological singularity theorems [Penrose, Hawking, ...] **we expect that the universe started from a singularity.**

- Is it possible to evade these singularity theorems?

  Can we make nonsingular eternal universes? Extensions of the singularity theorems?

- Can we construct stable nonsingular cosmologies with infinite cycles of expansion and contraction?

  If so, what's their quantum fate?

√ The search for bouncing cosmologies goes back to Tolman and Lemaitre.

√ More recently: ambitious scenarios like "pre big-bang" and "ekpyrotic/cyclic" universes [Steinhardt/Turok...].

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Our goal: present stable cosmological solutions where

1) the universe is oscillatory with $a_{max} \gg a_{min}$
(bangs and crunches)

2) classical GR is always valid (distances $\gg$ **Planck length)**

3) use matter that satisfies the null energy condition.

1. Evading the singularity theorems

2. A simple harmonic universe

3. Classical and quantum dynamics

I. Evading the singularity theorems

[As far as we know, our analysis and solution are novel. We were also inspired by work by Molina-Paris, Visser, Ellis, Maartens...]

- For $K = 0, -1$ the singularity theorems require the NEC:

$$T_{\mu\nu}k^\mu k^\nu \geq 0 \;\forall\, k^\mu \;\text{ null} \quad\Rightarrow\quad w \geq -1 \quad [p = w\rho]$$

- However, for $K = +1$ they require much stronger assumptions – like the SEC $w \geq 0$.
  This is violated by many physical systems, and is the main point we exploit.

Basic tools: classical GR with FRW metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2\right]$$

...and a fluid $p = w\,\rho$ plus cosmological constant

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi}{3}G_N(1+3w)\rho + \frac{\Lambda}{3} \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi}{3}G_N\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \\ \rho &= \frac{c}{a^{3+3w}}\end{aligned}$$

[We will need a more precise description of the fluid at shorter scales]

We want to find min & max values $a_\pm$ from 3-term structure

$$\dot{a}^2 = -K + \frac{8\pi}{3}\frac{G_N c}{a^{3w+1}} + \frac{\Lambda}{3}a^2 = 0$$

Solution: $K = +1 \;,\; \Lambda < 0 \;,\; -1 < w < -\frac{1}{3}$.

Thus it seems we can evade the inflationary singularity theorems using ordinary and well-understood sources!

- a_- produced by K and ρ, while a_+ produced by Λ and ρ
- we require

$$\frac{|\Lambda|}{3} \ll \left(\frac{8\pi}{3} G_N\, c\right)^{2/(3|w|-1)} \ll M_{Pl}^2 \quad \Rightarrow \quad M_{Pl}^{-1} \ll a_- \ll a_+$$

- sols w/oscillatory behavior and nonsingular bounces and crunches

Let's study this in more detail for the simplest solution $w = -2/3$.

II. A simple harmonic universe (SHU)

When $w = -2/3$ the FRW eqs. describe a harmonic oscillator:

$$\ddot{a} + \frac{|\Lambda|}{3} a = \frac{4\pi}{3} G_N c$$

$$\Rightarrow \quad a(t) \quad = \quad \frac{1}{\sqrt{\gamma}\omega}\left(1 + \sqrt{1-\gamma}\cos(\omega t)\right)$$

$$\omega \quad \equiv \quad \sqrt{\frac{|\Lambda|}{3}} \, , \; \gamma \equiv \frac{3|\Lambda|}{(4\pi G_N c)^2}$$

Focus on the interesting limit

$$\gamma \ll 1 \;\; \Rightarrow \;\; \frac{a_-}{a_+} \approx \gamma$$

• Stability of the simple harmonic universe?

Two possible sources:

1) Homogeneous perturbations

Recall instability of Einstein's static universe:

- The dust energy density and size of the universe have to be tuned to match the c.c.
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Recall instability of Einstein's static universe:

- The dust energy density and size of the universe have to be tuned to match the c.c.
- Any deviation causes a large instability.

** However, we have a parameter space of solutions. Small homogeneous fluctuations are expected to lead to qualitatively similar oscillating universes.

Concretely, the most general homogeneous perturbation

$$ds^2 = -dt^2 + \sum_{i=1}^{3} a_i(t)^2 \sigma_i^2 \ , \quad a_i(t) = a(t) + \delta a_i(t)$$

leads to small oscillatory δa_i sols.

2) Inhomogeneous perturbations

e.g. scalar modes $\delta\rho$, δp and

$$ds^2 = -(1 + 2\Psi)\, dt^2 + a^2(t)\,(1 - 2\Psi)\, d\Omega_3^2$$

$$\ddot{\Psi} + \frac{\dot{a}}{a}(4 + 3c_s^2)\dot{\Psi} + \left(2\frac{\ddot{a}}{a} + (1 + 3c_s^2)\left(\frac{\dot{a}^2}{a^2} - \frac{K}{a^2}\right)\right)\Psi - \frac{c_s^2}{a^2}\nabla_{S^3}^2\Psi = 0$$

where $\delta p = c_s^2\, \delta\rho$.

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If matter were really a perfect fluid, since we need negative pressure,

$$c_s^2 = w < -1/3$$

Sound wave perturbations would have an imaginary speed!
$\rightsquigarrow$ **Disastrous high momentum instabilities!**

⋆ **Solution:** consider a matter source that behaves like a fluid with $w < -1/3$ at long distances, but under inhomogeneous perturbations it behaves like a solid.

Based on [Bucher, Spergel: Solid dark matter]

This adds extra elastic resistance to deformations, producing a positive speed of sound and stabilizing the inhomogeneous modes.

Concrete example: frustrated network of domain walls, with $w = -2/3$. $c_s^2 > 0$.

III. Classical and quantum dynamics

Dynamics of massless scalars (e.g. graviton) in the SHU:

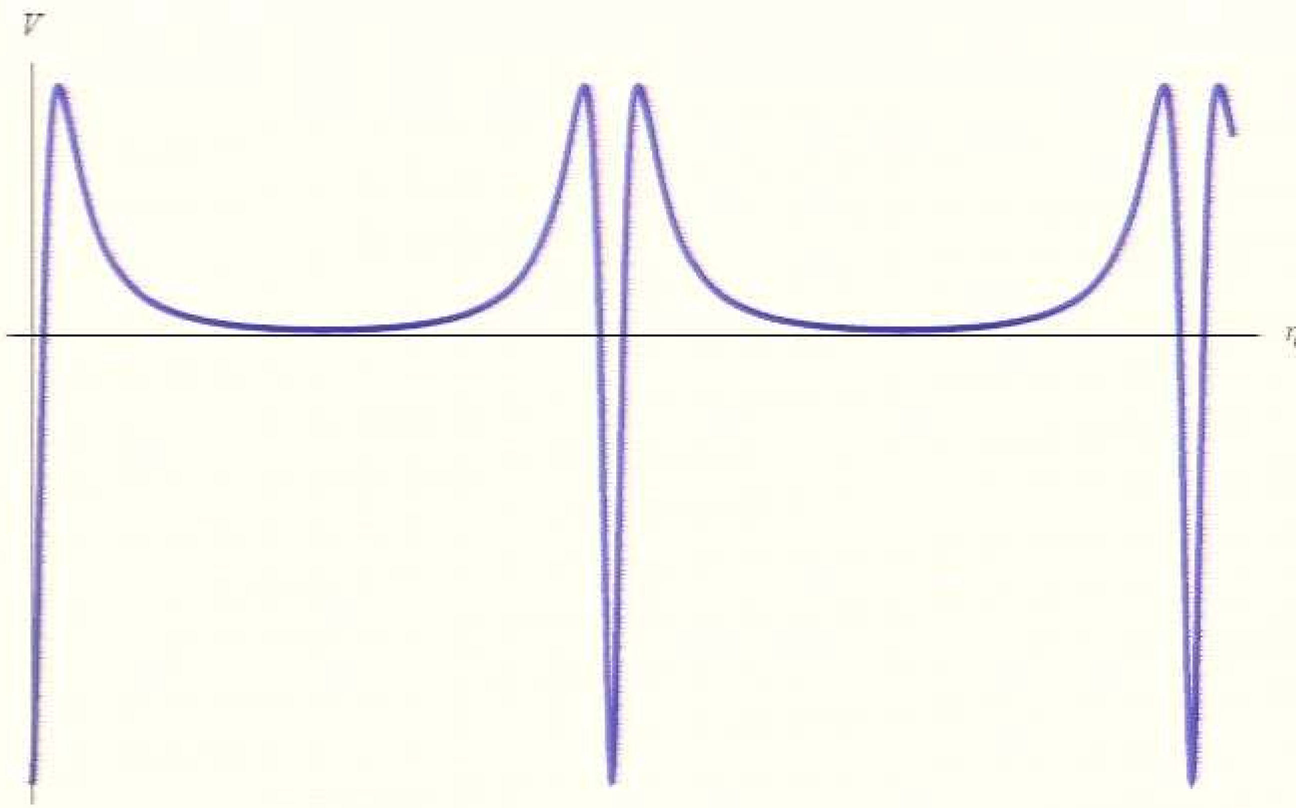
$$S = \int d^4x \sqrt{g^{(3)}}\, a^2(\eta) \left[(\partial_\eta \phi)^2 - (\nabla_{S^3}\phi)^2\right]$$

Normalizing $\chi = a(\eta)\phi$ and expanding in spherical harmonics,

$$\chi_k'' + \left(k(k+2) - \frac{a''}{a}\right)\chi_k = 0$$

$\Rightarrow$ Schrödinger problem for a particle in a periodic potential,

$$-\chi'' + V(\eta)\,\chi = k(k+2)\,\chi \quad , \quad V(\eta) \equiv \frac{a''}{a}$$



Three regimes:

- $k = 0$, homogeneous mode
- intermediate momenta $0 < k < 1/\sqrt{\gamma}$
- high momenta $k \gg 1/\sqrt{\gamma}$ $\rightsquigarrow$ flat space modes

Dynamics of particles in the harmonic universe
$\Leftrightarrow$ electrons in a Bloch potential

$\surd$ Instead of periodic b.c., we want to give initial conditions for χ

$\surd$ Set of decoupled harmonic oscillators in each cycle

$\surd$ Patch sols across 'barriers' and 'wells' of the potential

► The homogeneous mode exhibits linear growth,

$$\phi(\eta) \sim \frac{\eta}{\gamma^2}$$

► For intermediate momenta $k < 1/\sqrt{\gamma}$,

$$\phi(\eta) \sim \exp\left[\sqrt{1 - \frac{k^2}{k_c^2}}\,\eta\right] \quad , \quad k_c^2 \sim 1/\gamma$$

► At high momenta $k > 1/\sqrt{\gamma}$, modulated Minkowski modes,

$$\phi(\eta) \sim (\sin \eta)\, e^{ik\eta}$$

Backreaction from the scalar field

Compute the energy density including modes up to $k_c \sim 1/\sqrt{\gamma}$ and compare to the background.

- Classically, initial amplitude ϕ_0 arbitrary. Backreaction important after a number of cycles

$$N_* \sim \log \frac{M_{Pl}^2}{\phi_0^2}$$

- Quantum-mechanically, $\phi_0 \sim \sqrt{\gamma}\,\omega$. Particle production important when

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Physically, too much radiation will lead to a singular crunch.

Large number of bounces for $|\Lambda|/M_{Pl}^2 \ll 1$.

IV. Conclusions

- We presented new GR solutions of cosmologies that cycle through an infinite set of nonsingular bounces
- The solutions have $K = +1$, negative c.c. and matter with $-1 < w < -1/3$
- Singularity theorems evaded for $\gamma > 1/4$, while for smaller γ classical perturbations become important after a large number of cycles.
- QM: particle production eventually becomes important, and may lead to a singular crunch. Quantum extension of sing. theorems.
- Next step, combine our solution with other components. Inflation, radiation/matter domination, ...

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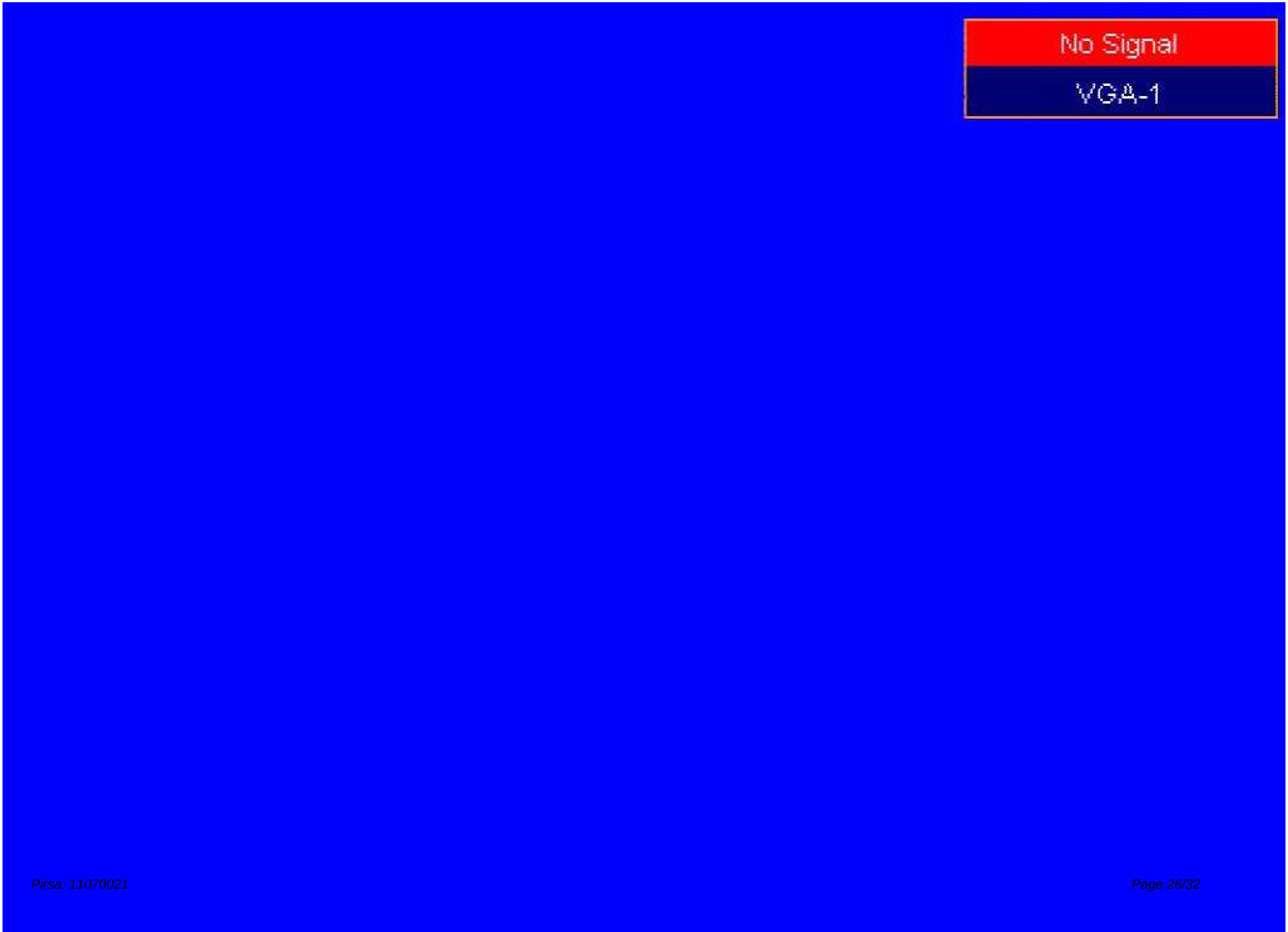
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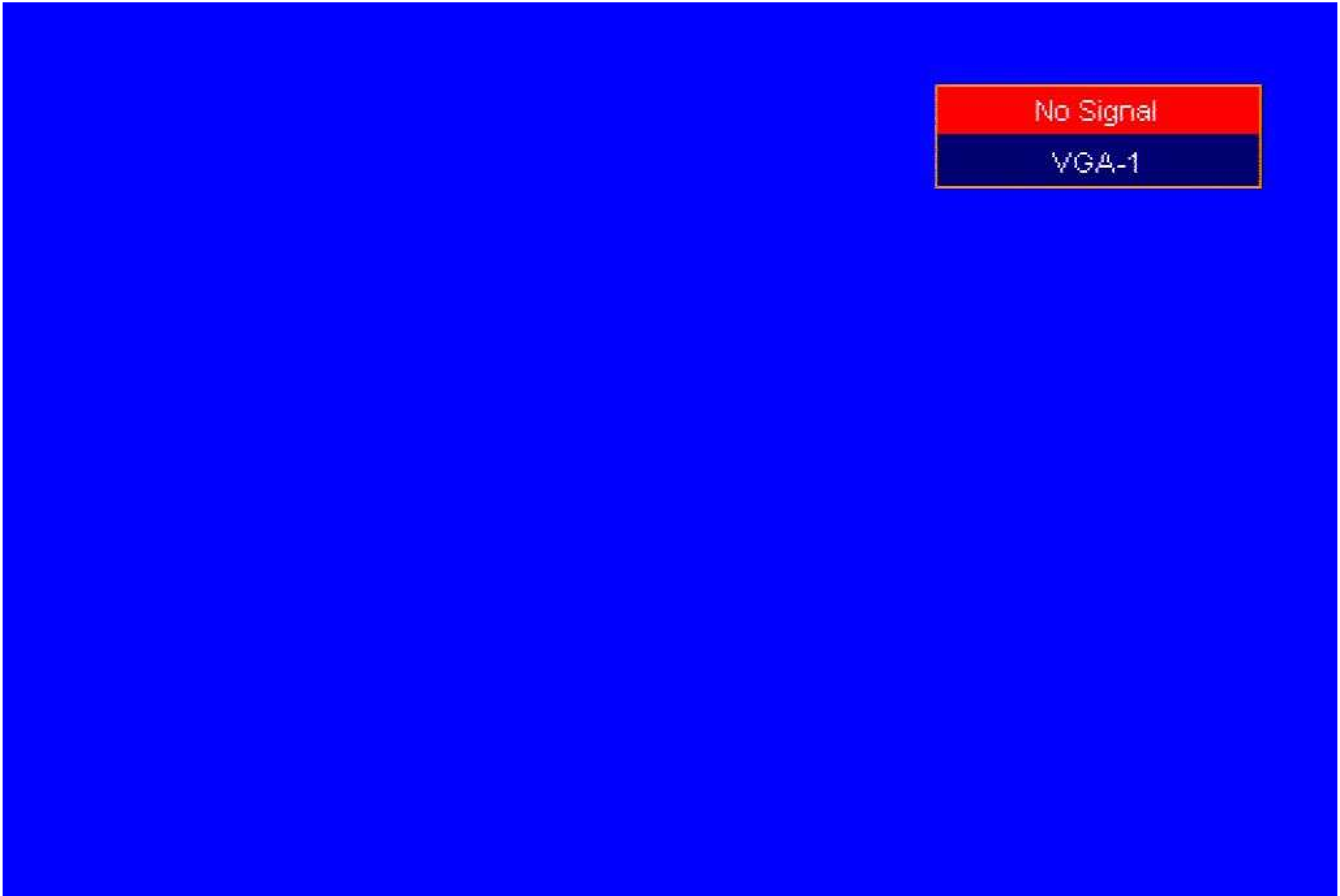
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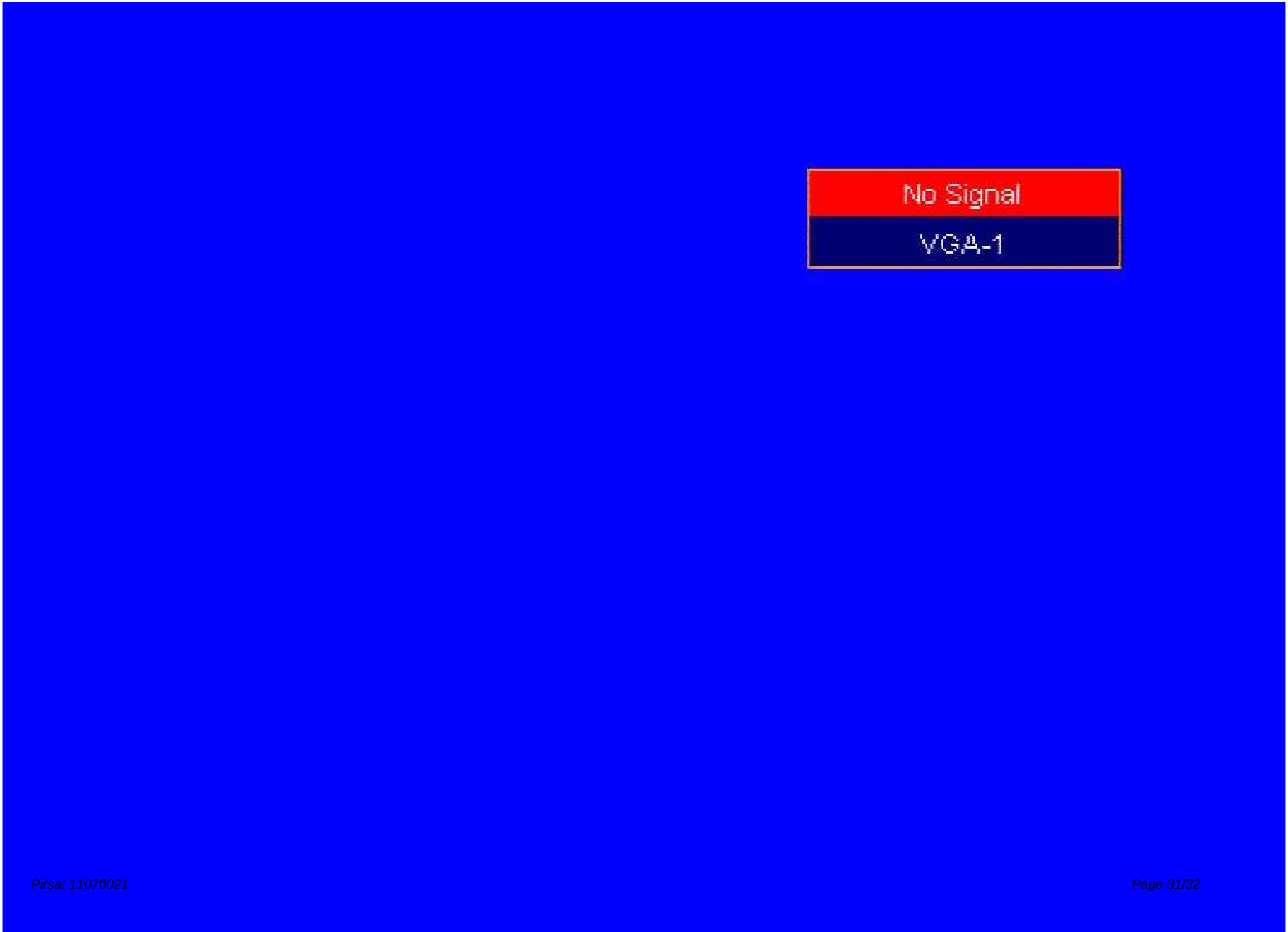
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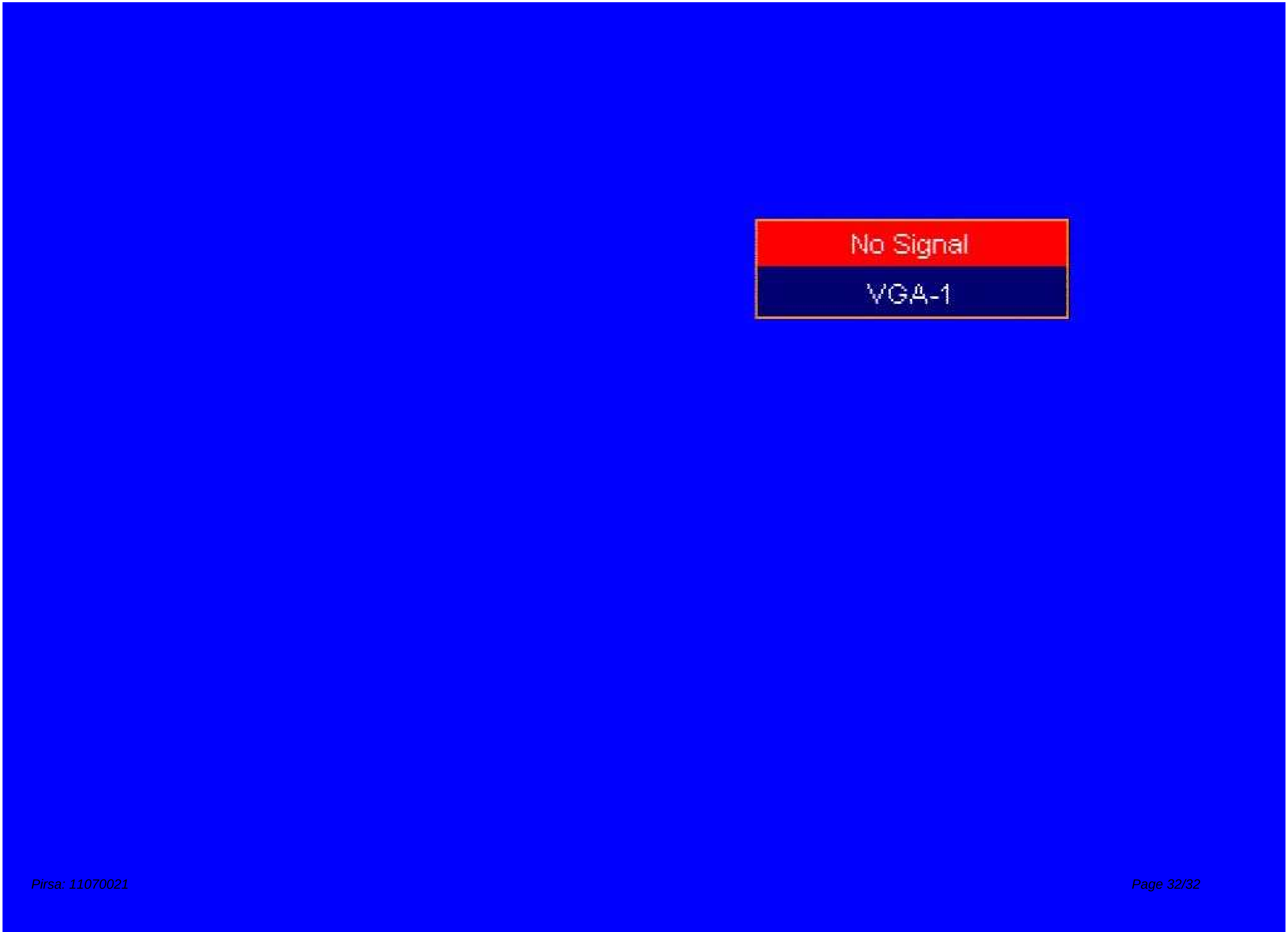
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