

Title: A Simple Harmonic Universe

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Abstract: We explore simple but novel solutions of general relativity which, classically, approximate cosmologies cycling through an infinite set of ``bounces." These solutions require curvature $K=+1$, and are supported by a negative cosmological term and matter with $-1 < w < -1/3$. They can be studied within the regime of validity of general relativity. We argue that quantum mechanically, particle production leads eventually to a departure from the regime of validity of semiclassical general relativity, likely yielding a singular crunch.

A simple harmonic universe

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Work in progress with P. Graham, B. Horn, S. Kachru, and S. Rajendran

Challenges for Early Universe Cosmology

Perimeter Institute, July 2011

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Here we focus on the question of the **singularity in the far past**. From cosmological singularity theorems [Penrose, Hawking, ...] **we expect that the universe started from a singularity.**

- Is it possible to evade these singularity theorems?

Can we make nonsingular eternal universes? Extensions of the singularity theorems?

- Can we construct stable nonsingular cosmologies with infinite cycles of expansion and contraction?

If so, what's their quantum fate?

✓ The search for bouncing cosmologies goes back to Tolman and Lemaitre.

✓ More recently: ambitious scenarios like “pre big-bang” and “ekpyrotic/cyclic” universes [Steinhardt/Turok...].

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Our goal: present stable cosmological solutions where

**1) the universe is oscillatory with $a_{max} \gg a_{min}$
(bangs and crunches)**

2) classical GR is always valid (distances \gg Planck length)

3) use matter that satisfies the null energy condition.

1. **Evading the singularity theorems**
2. **A simple harmonic universe**
3. **Classical and quantum dynamics**

I. Evading the singularity theorems

[As far as we know, our analysis and solution are novel. We were also inspired by work by Molina-Paris, Visser, Ellis, Maartens...]

- For $K = 0, -1$ the singularity theorems require the NEC:

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0 \quad \forall k^{\mu} \text{ null} \Rightarrow w \geq -1 \quad [\rho = w\rho]$$

- However, for $K = +1$ they require much stronger assumptions – like the SEC $w \geq 0$.
This is violated by many physical systems, and is the main point we exploit.

Basic tools: classical GR with FRW metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 d\Omega_2^2 \right]$$

...and a fluid $p = w \rho$ plus cosmological constant

$$\begin{aligned}\frac{\ddot{a}}{a} &= -\frac{4\pi}{3}G_N(1+3w)\rho + \frac{\Lambda}{3} \\ \left(\frac{\dot{a}}{a}\right)^2 &= \frac{8\pi}{3}G_N\rho - \frac{K}{a^2} + \frac{\Lambda}{3} \\ \rho &= \frac{c}{a^{3+3w}}\end{aligned}$$

[We will need a more precise description of the fluid at shorter scales]

We want to find min & max values a_{\pm} from 3-term structure

$$\dot{a}^2 = -K + \frac{8\pi}{3}\frac{G_N c}{a^{3w+1}} + \frac{\Lambda}{3}a^2 = 0$$

Solution: $K = +1$, $\Lambda < 0$, $-1 < w < -\frac{1}{3}$.

Thus it seems we can evade the inflationary singularity theorems using ordinary and well-understood sources!

- ▶ a_- produced by K and ρ , while a_+ produced by Λ and ρ
- ▶ we require

$$\frac{|\Lambda|}{3} \ll \left(\frac{8\pi}{3} G_N c \right)^{2/(3|w|-1)} \ll M_{Pl}^2 \Rightarrow M_{Pl}^{-1} \ll a_- \ll a_+$$

- ▶ sols w/oscillatory behavior and nonsingular bounces and crunches

Let's study this in more detail for the simplest solution $w = -2/3$.

II. A simple harmonic universe (SHU)

When $w = -2/3$ the FRW eqs. describe a harmonic oscillator:

$$\ddot{a} + \frac{|\Lambda|}{3} a = \frac{4\pi}{3} G_N c$$

$$\Rightarrow a(t) = \frac{1}{\sqrt{\gamma}\omega} \left(1 + \sqrt{1 - \gamma} \cos(\omega t) \right)$$

$$\omega \equiv \sqrt{\frac{|\Lambda|}{3}}, \quad \gamma \equiv \frac{3|\Lambda|}{(4\pi G_N c)^2}$$

Focus on the interesting limit

$$\gamma \ll 1 \Rightarrow \frac{a_-}{a_+} \approx \gamma$$

- **Stability of the simple harmonic universe?**

Two possible sources:

- 1) Homogeneous perturbations**

Recall instability of Einstein's static universe:

- The dust energy density and size of the universe have to be tuned to match the c.c.
- Any deviation causes a large instability.

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Recall instability of Einstein's static universe:

- The dust energy density and size of the universe have to be tuned to match the c.c.
- Any deviation causes a large instability.

** However, we have a parameter space of solutions. Small homogeneous fluctuations are expected to lead to qualitatively similar oscillating universes.

Concretely, the most general homogeneous perturbation

$$ds^2 = -dt^2 + \sum_{i=1}^3 a_i(t)^2 \sigma_i^2, \quad a_i(t) = a(t) + \delta a_i(t)$$

leads to small oscillatory δa_i sols.

2) Inhomogeneous perturbations

e.g. scalar modes $\delta\rho$, δp and

$$ds^2 = -(1 + 2\Psi) dt^2 + a^2(t) (1 - 2\Psi) d\Omega_3^2$$

$$\ddot{\Psi} + \frac{\dot{a}}{a}(4 + 3c_s^2)\dot{\Psi} + \left(2\frac{\ddot{a}}{a} + (1 + 3c_s^2)\left(\frac{\dot{a}^2}{a^2} - \frac{K}{a^2}\right)\right)\Psi - \frac{c_s^2}{a^2}\nabla_{S^3}^2\Psi = 0$$

where $\delta p = c_s^2 \delta\rho$.

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If matter were really a perfect fluid, since we need negative pressure,

$$c_s^2 = w < -1/3$$

Sound wave perturbations would have an imaginary speed!

→ **Disastrous high momentum instabilities!**

★ **Solution:** consider a matter source that behaves like a fluid with $w < -1/3$ at long distances, but under inhomogeneous perturbations it behaves like a solid.

Based on [Bucher, Spergel: Solid dark matter]

This adds extra elastic resistance to deformations, producing a positive speed of sound and stabilizing the inhomogeneous modes.

Concrete example: frustrated network of domain walls, with $w = -2/3$. $c_s^2 > 0$.

III. Classical and quantum dynamics

Dynamics of massless scalars (e.g. graviton) in the SHU:

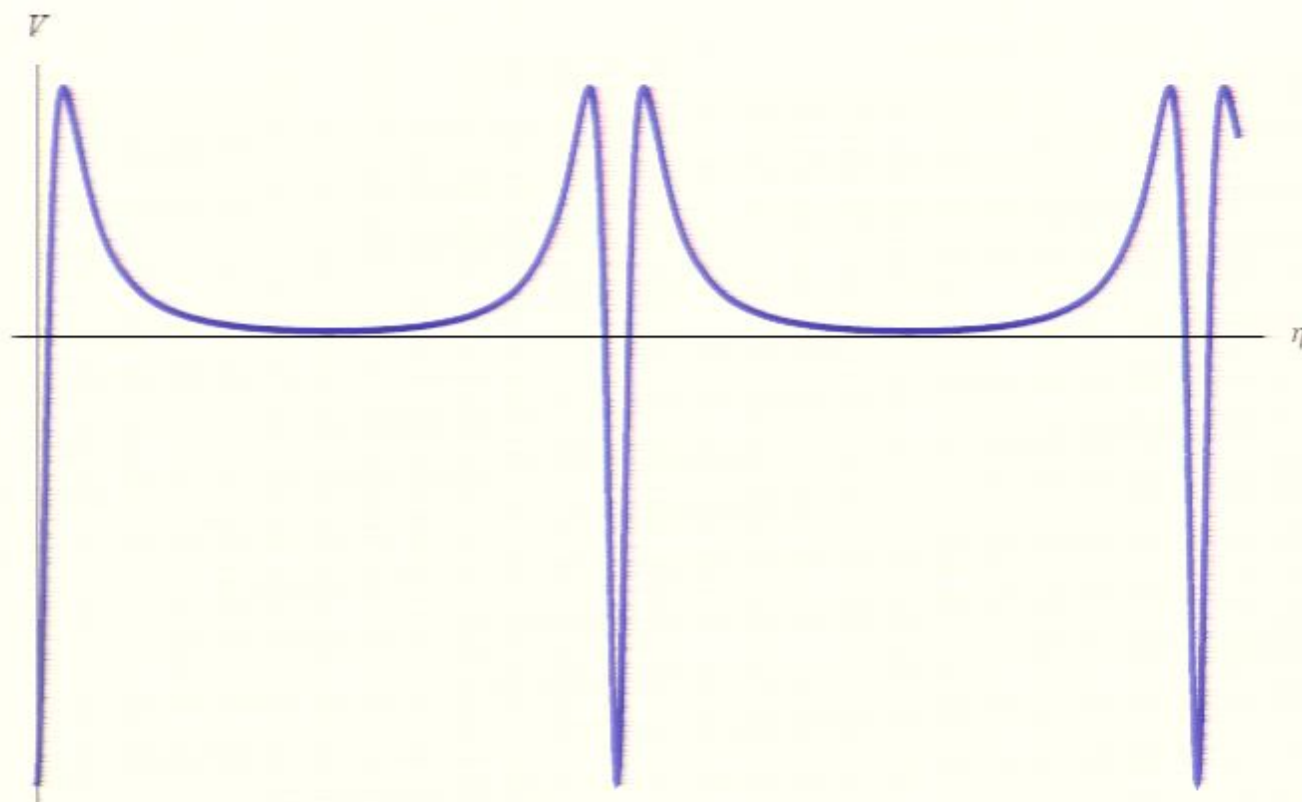
$$S = \int d^4x \sqrt{g^{(3)}} a^2(\eta) [(\partial_\eta \phi)^2 - (\nabla_{S^3} \phi)^2]$$

Normalizing $\chi = a(\eta)\phi$ and expanding in spherical harmonics,

$$\chi_k'' + \left(k(k+2) - \frac{a''}{a} \right) \chi_k = 0$$

\Rightarrow Schrödinger problem for a particle in a periodic potential,

$$-\chi'' + V(\eta)\chi = k(k+2)\chi, \quad V(\eta) \equiv \frac{a''}{a}$$



Three regimes:

- ▶ $k = 0$, homogeneous mode
- ▶ intermediate momenta $0 < k < 1/\sqrt{\gamma}$
- ▶ high momenta $k \gg 1/\sqrt{\gamma} \rightsquigarrow$ flat space modes

Dynamics of particles in the harmonic universe
 \Leftrightarrow **electrons in a Bloch potential**

- ✓ Instead of periodic b.c., we want to give initial conditions for χ
- ✓ Set of decoupled harmonic oscillators in each cycle
- ✓ Patch sols across 'barriers' and 'wells' of the potential

- The homogeneous mode exhibits linear growth,

$$\phi(\eta) \sim \frac{\eta}{\gamma^2}$$

- For intermediate momenta $k < 1/\sqrt{\gamma}$,

$$\phi(\eta) \sim \exp \left[\sqrt{1 - \frac{k^2}{k_c^2}} \eta \right], \quad k_c^2 \sim 1/\gamma$$

- At high momenta $k > 1/\sqrt{\gamma}$, modulated Minkowski modes,

$$\phi(\eta) \sim (\sin \eta) e^{ik\eta}$$

Backreaction from the scalar field

Compute the energy density including modes up to $k_c \sim 1/\sqrt{\gamma}$ and compare to the background.

- ▶ **Classically**, initial amplitude ϕ_0 arbitrary. Backreaction important after a number of cycles

$$N_* \sim \log \frac{M_{Pl}^2}{\phi_0^2}$$

- ▶ **Quantum-mechanically**, $\phi_0 \sim \sqrt{\gamma} \omega$. Particle production important when

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Physically, too much radiation will lead to a singular crunch.

Large number of bounces for $|\Lambda|/M_{Pl}^2 \ll 1$.

IV. Conclusions

- ▶ We presented new GR solutions of cosmologies that cycle through an infinite set of nonsingular bounces
- ▶ The solutions have $K = +1$, negative c.c. and matter with $-1 < w < -1/3$
- ▶ Singularity theorems evaded for $\gamma > 1/4$, while for smaller γ classical perturbations become important after a large number of cycles.
- ▶ QM: particle production eventually becomes important, and may lead to a singular crunch. Quantum extension of sing. theorems.
- ▶ Next step, combine our solution with other components. Inflation, radiation/matter domination, ...

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Bookmarks

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