Title: A critique of Coleman - De Luccia

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Abstract: In many respects, de Sitter space behaves like a system at finite temperature in finite volume. I will extend this to include the lack of first-order phase transitions. This rules out exponential decay in the de Sitter landscape, which changes the global structure in a significant way.

A critique of Coleman – De Luccia

arXiv : 0911.3142 (hep-th)

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Bottom line:

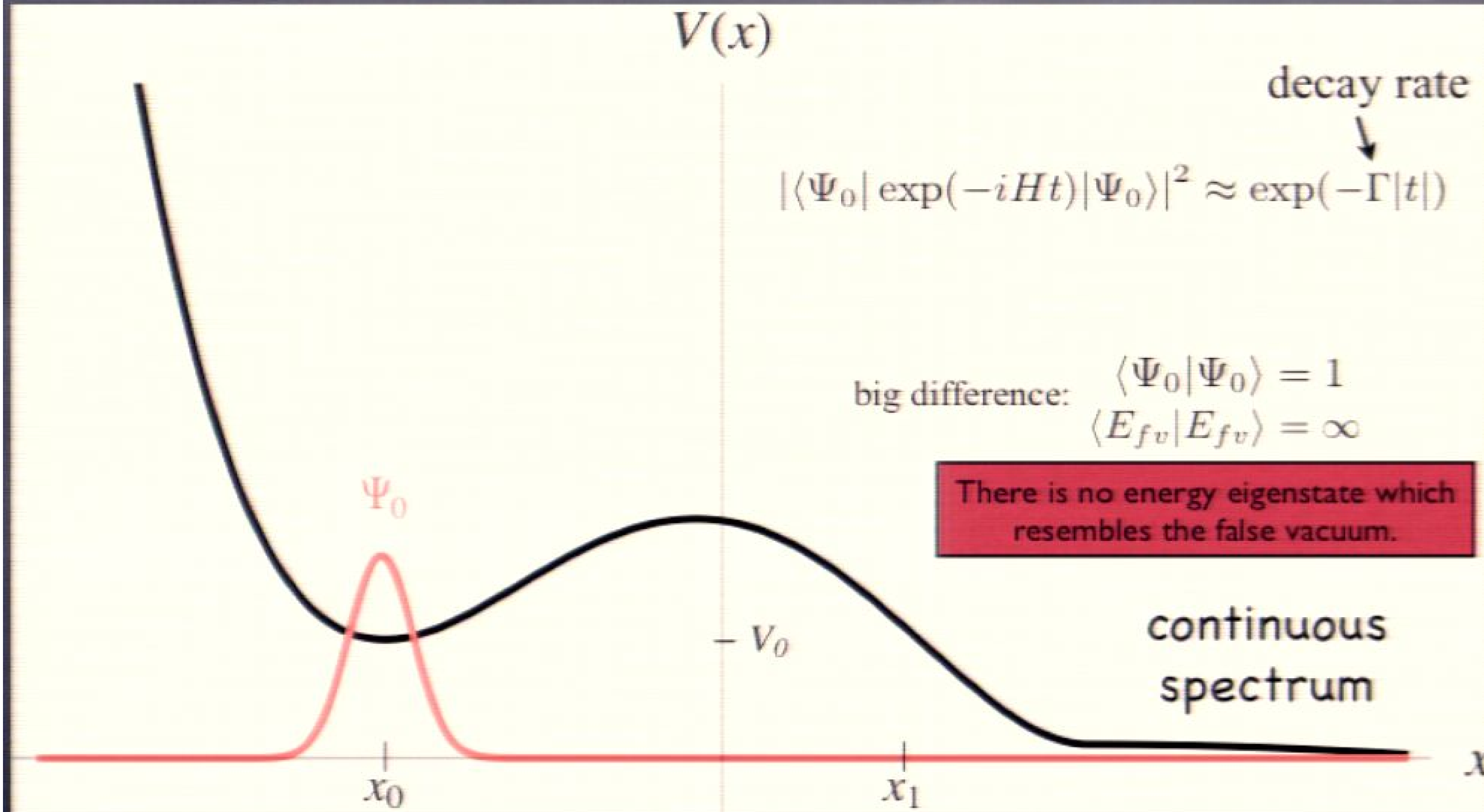
- De Sitter false vacua are stable.
  - no obvious way to populate most of the landscape
  - generic observer never exits a de Sitter false vacuum

This is a big claim, warranting careful scrutiny. Feel free to interrupt and ask questions.

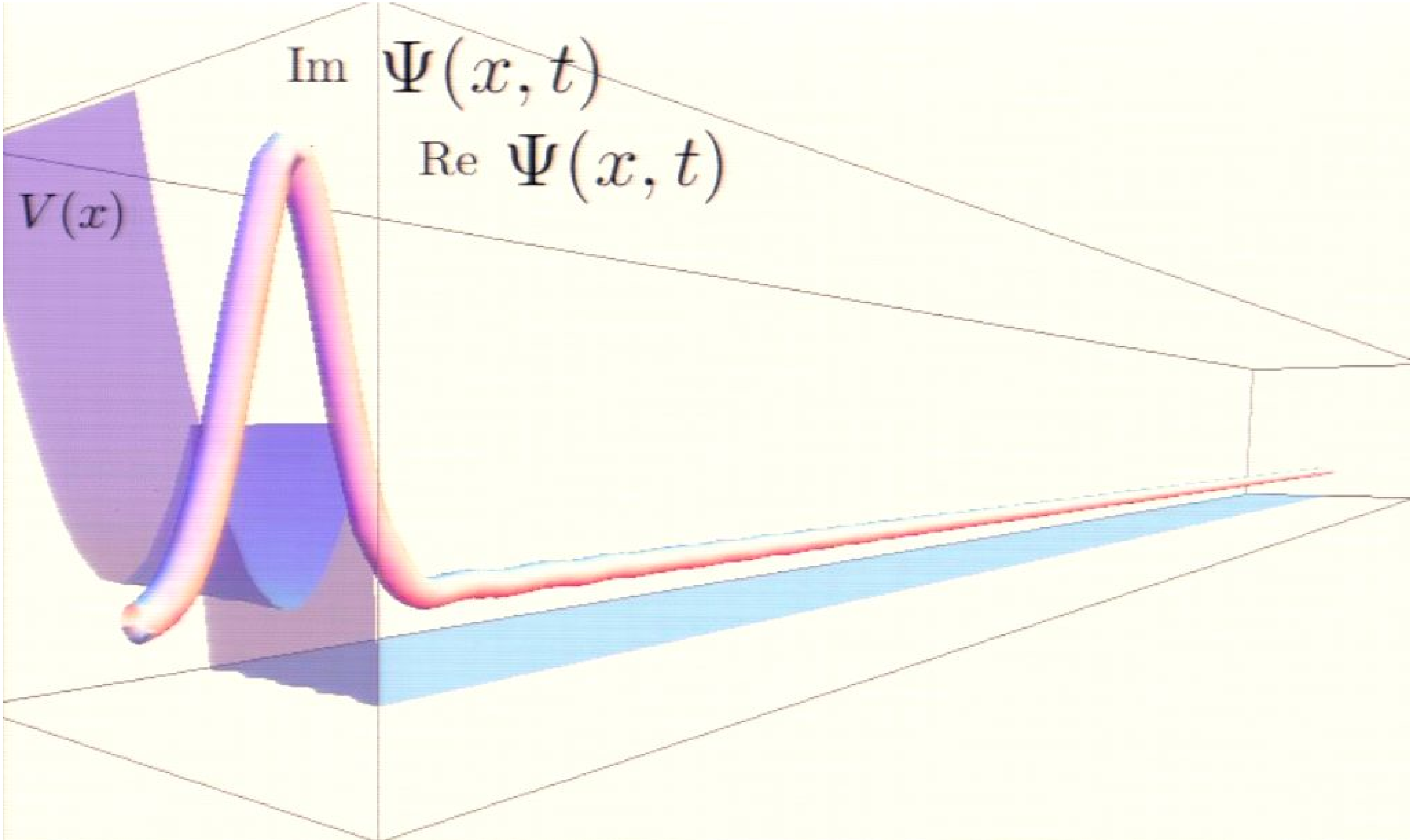
Outline

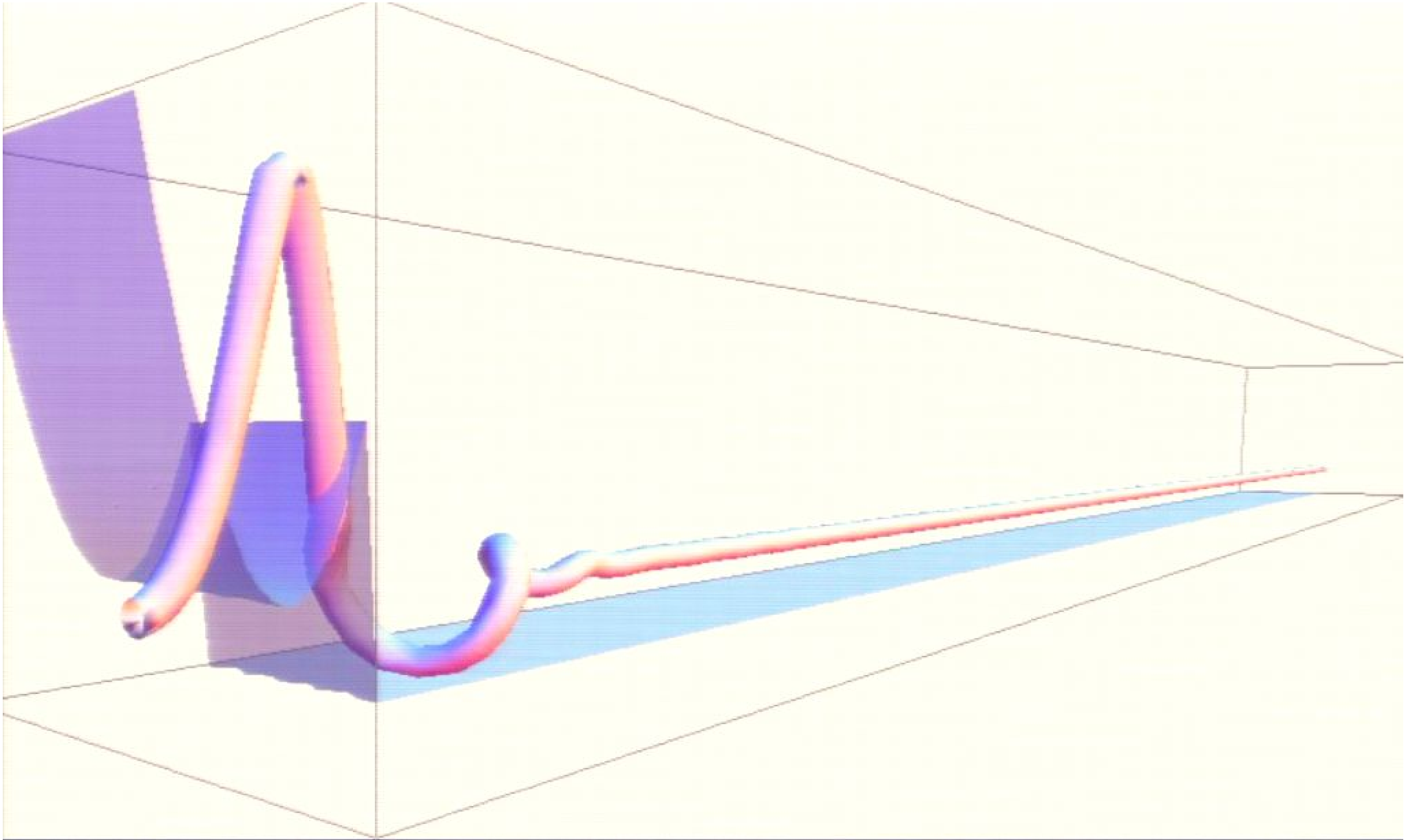
- Exponential decay in QM
  - Euclidean picture (instantons)
  - Density of states picture
- Exponential decay in de Sitter space
  - Euclidean picture (Coleman - De Luccia)
  - Density of states picture
- Conclusions

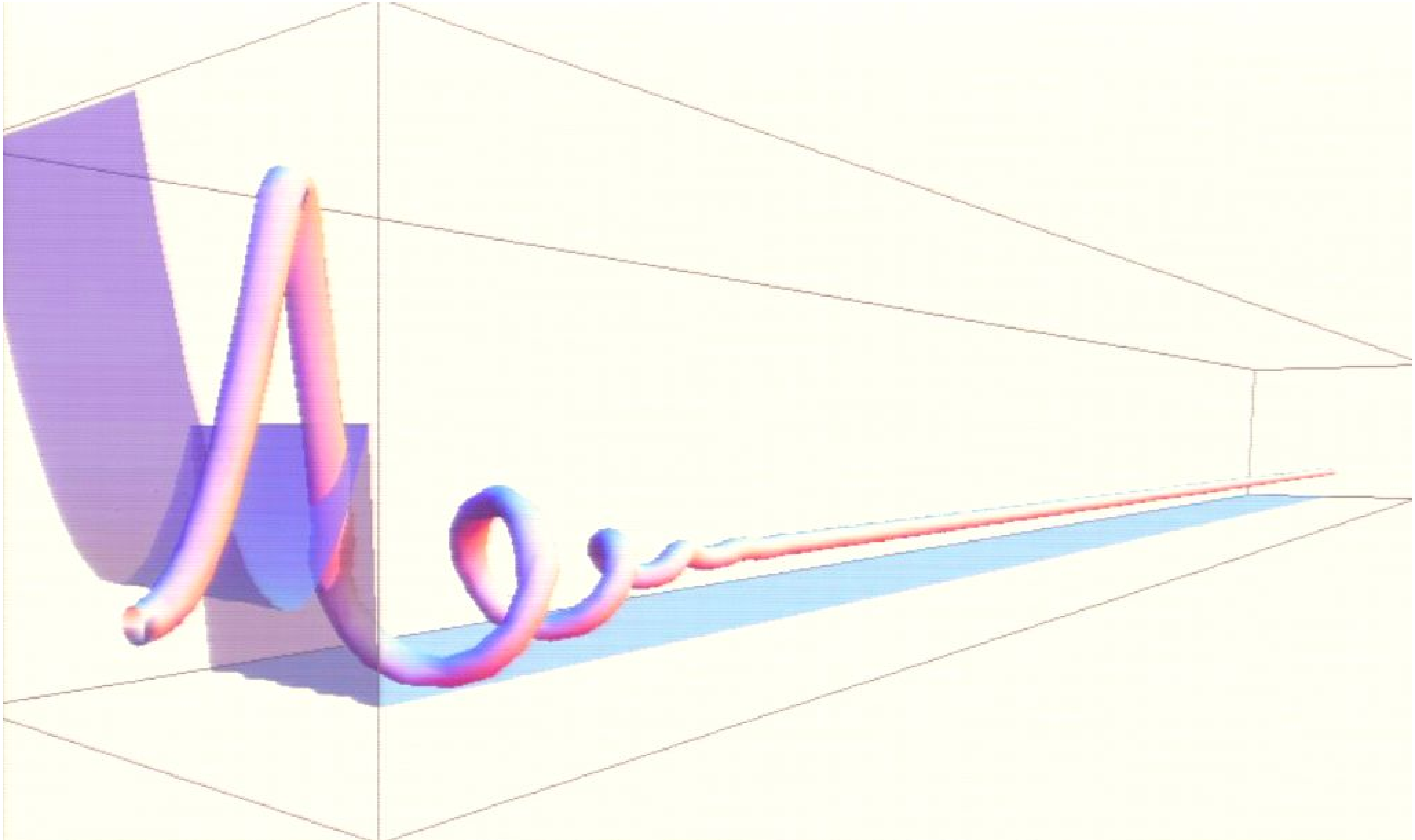
Exponential decay

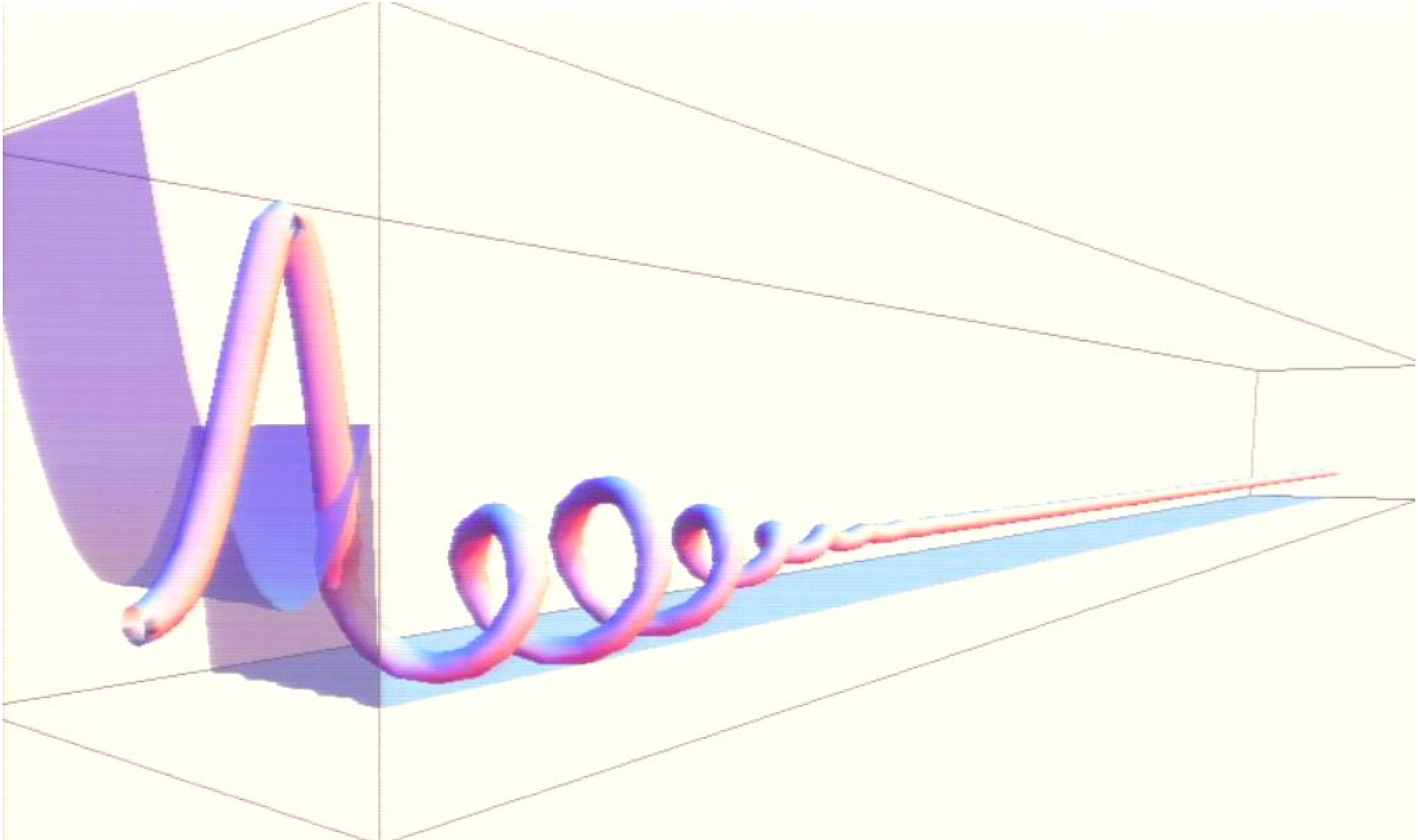


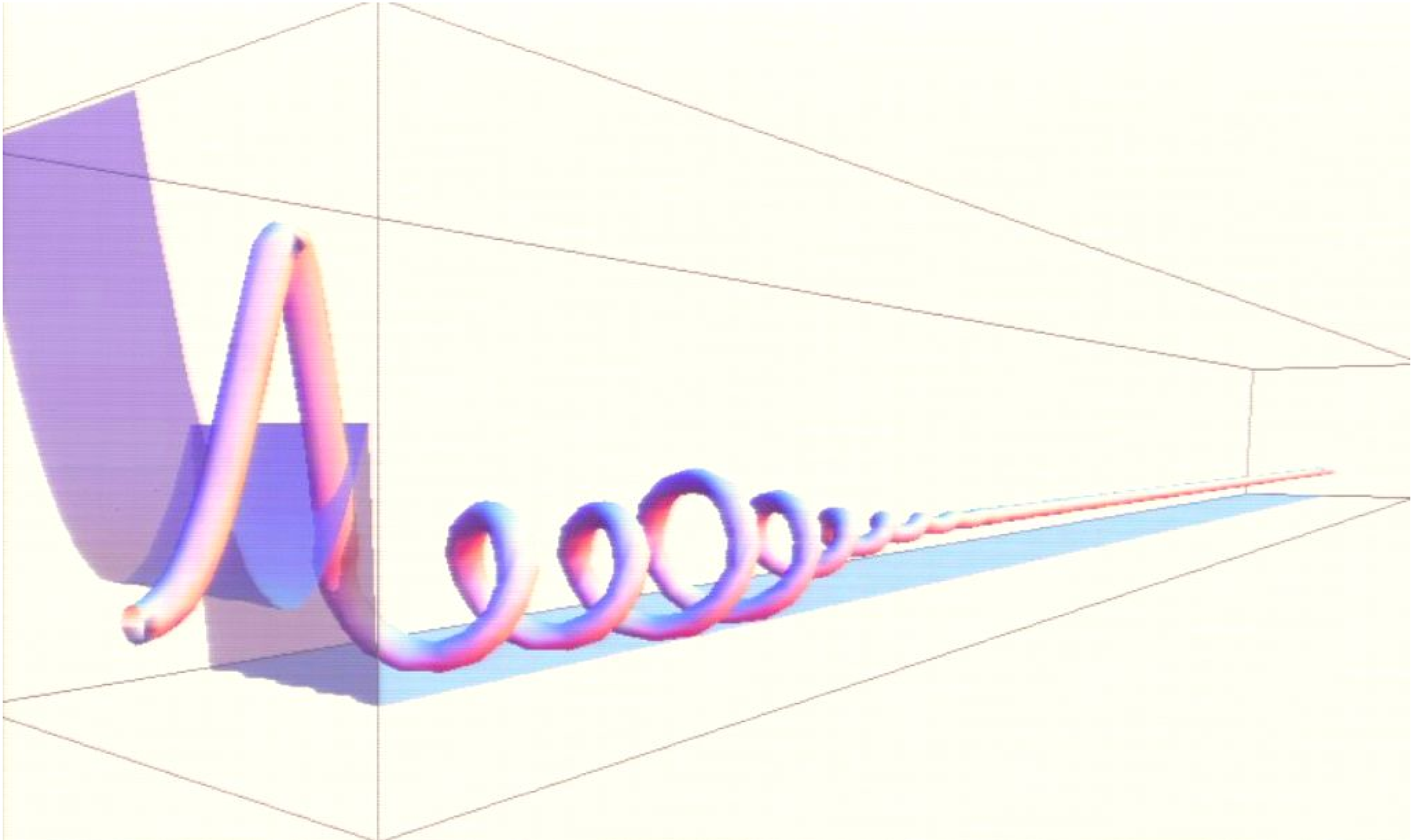
"false vacuum" $|\Psi_0\rangle =$ ground state of *perturbative* Hamiltonian $H_{\rm fv}$ (e.g. Gaussian). $H_{\rm fv}$ is discreet.

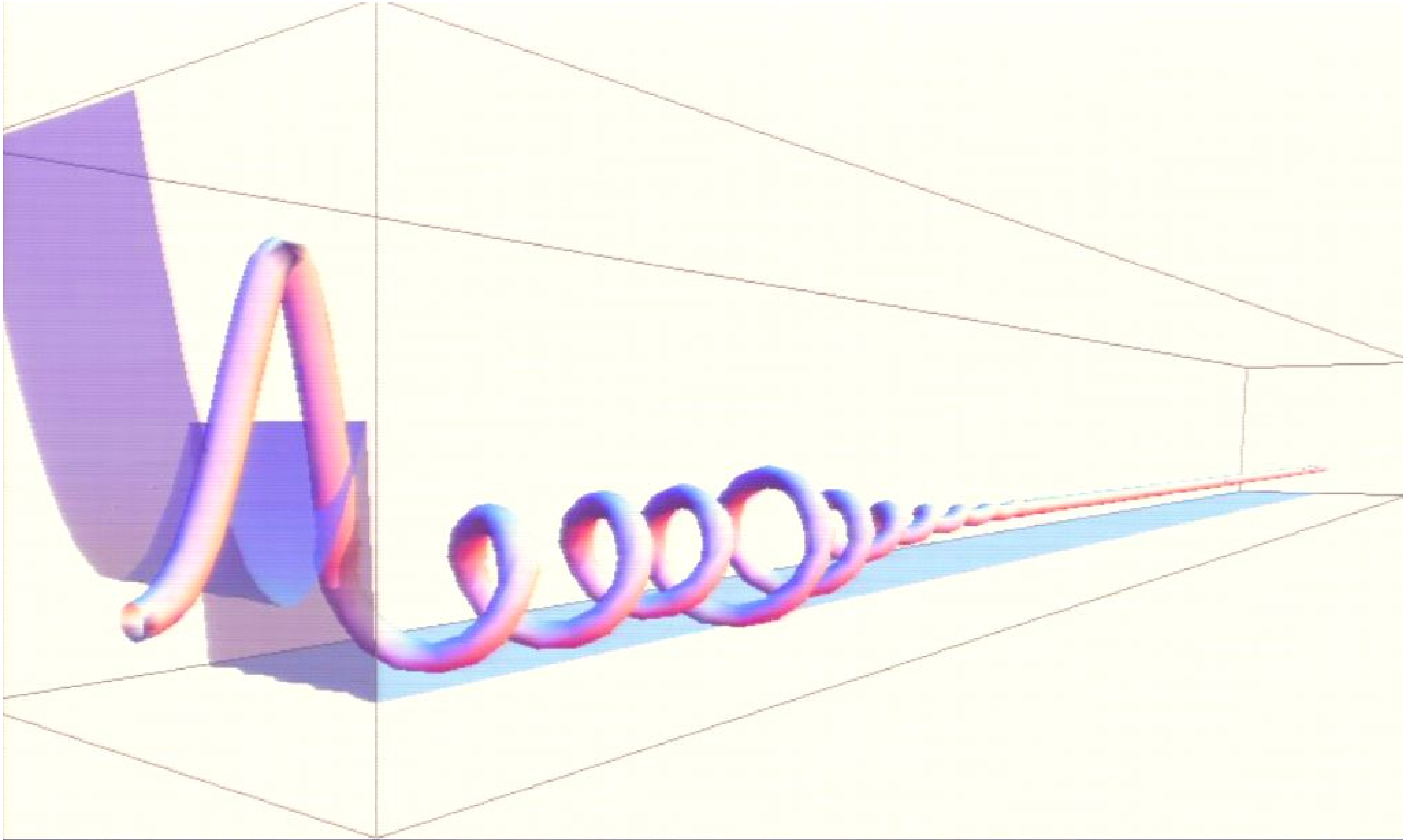

$\mathrm{Im}\ \Psi(x,t)$
$\mathrm{Re}\ \Psi(x,t)$
$V(x)$

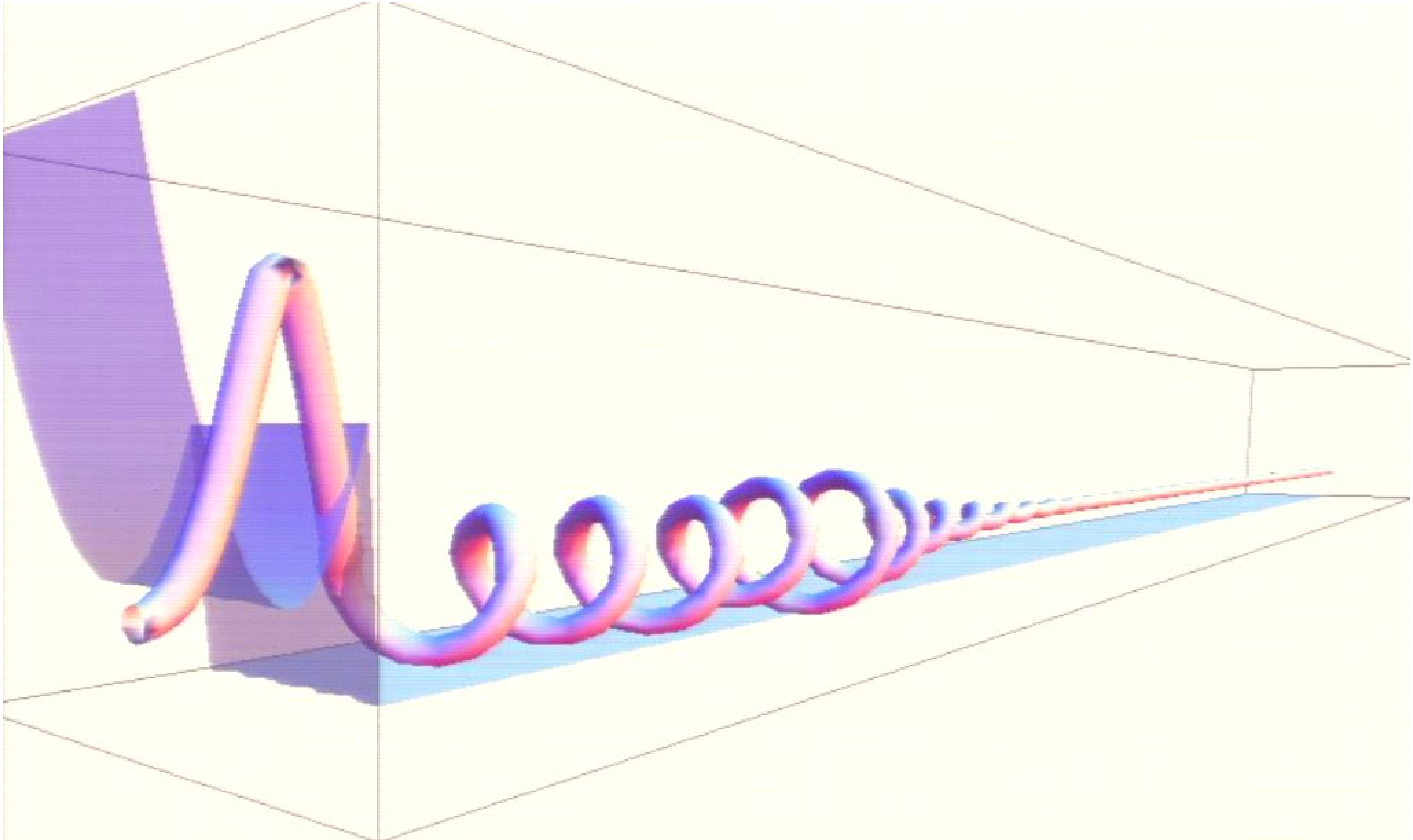


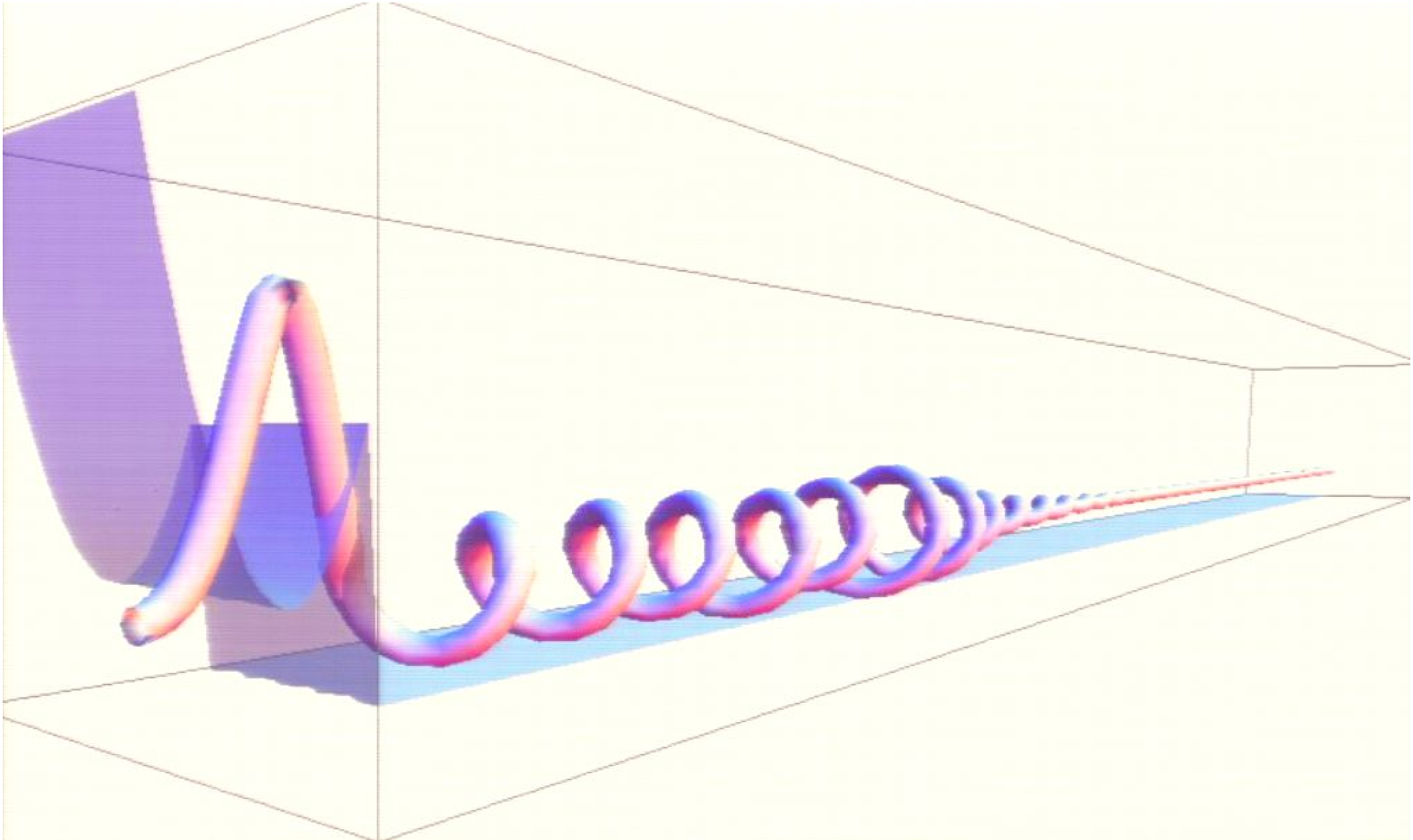


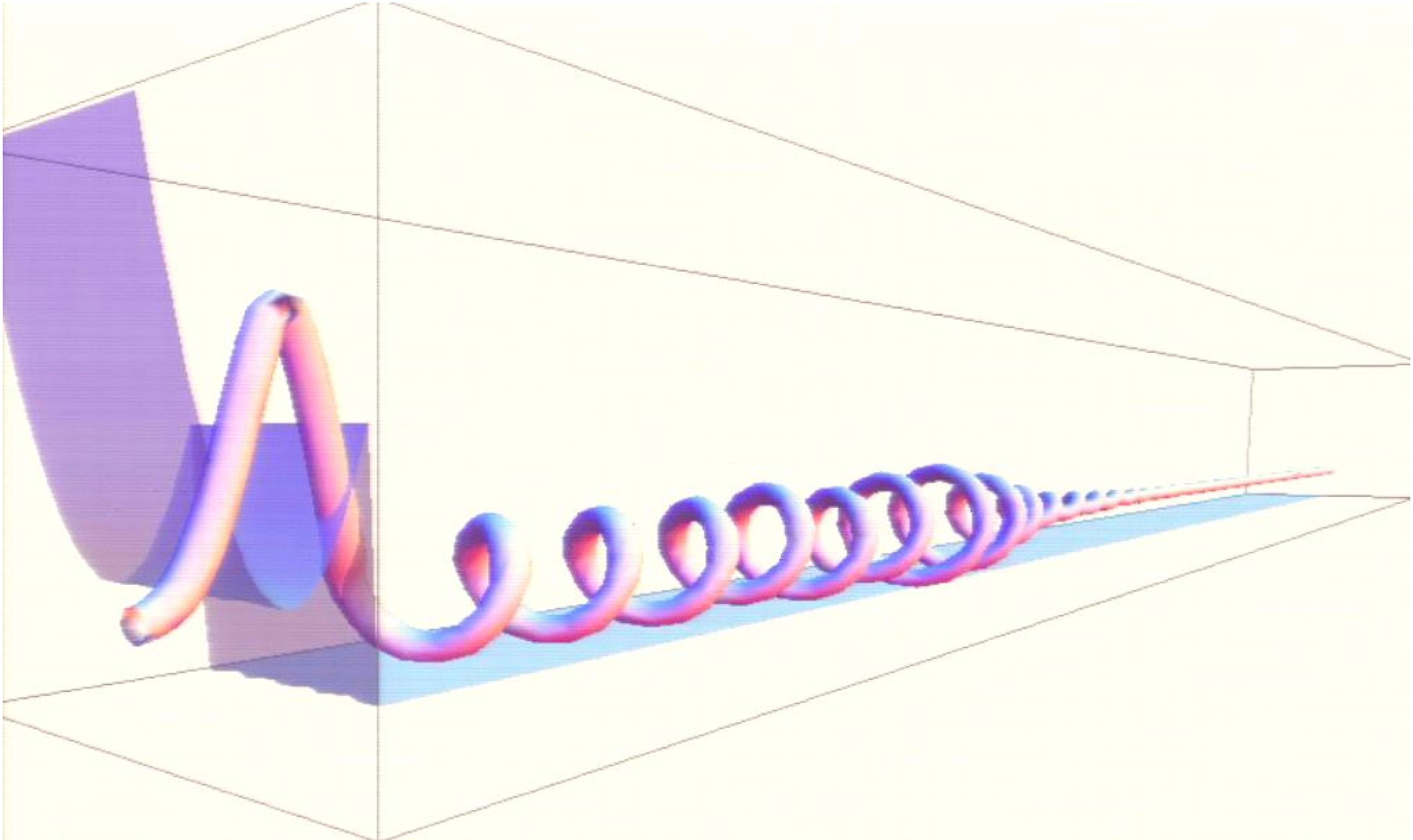


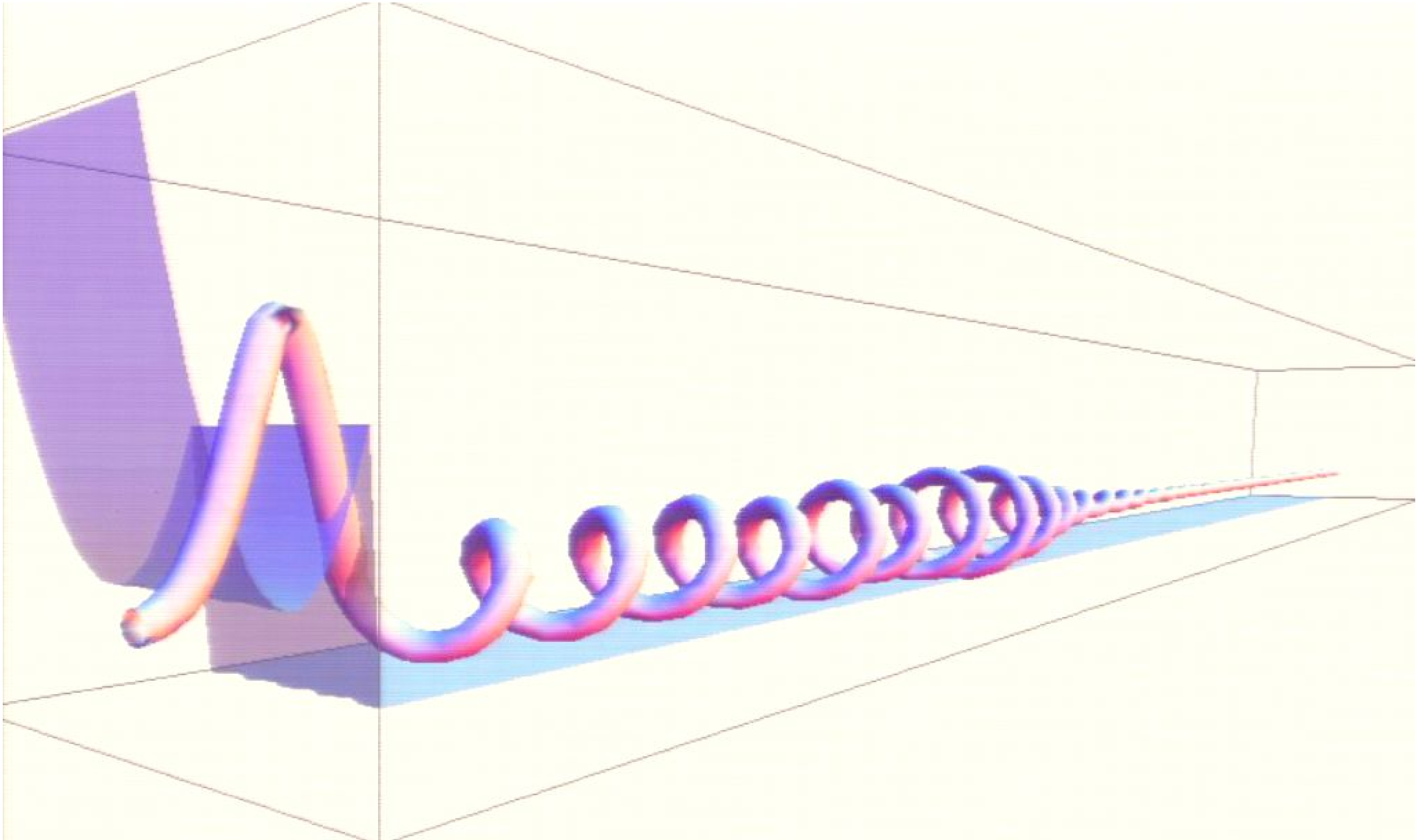


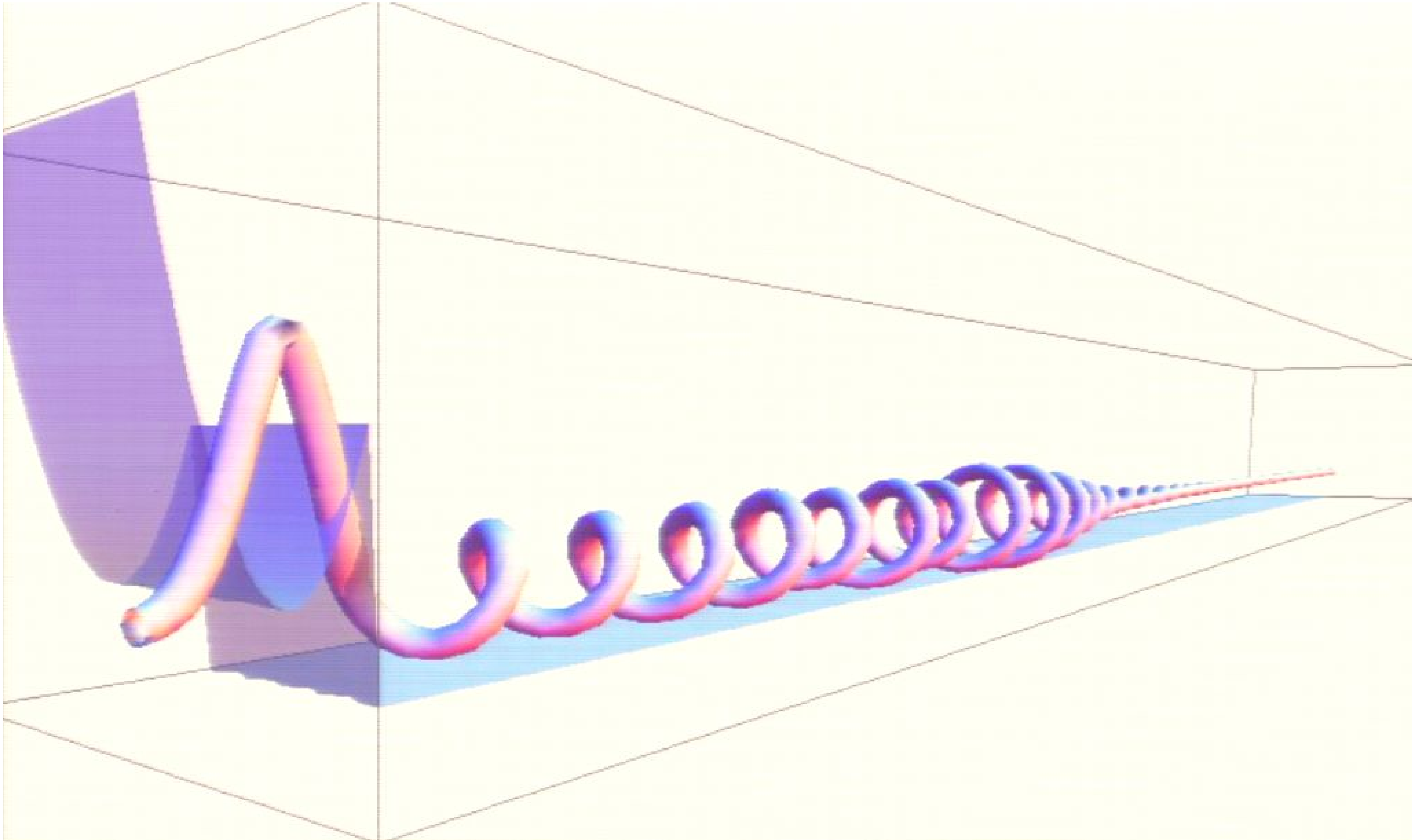


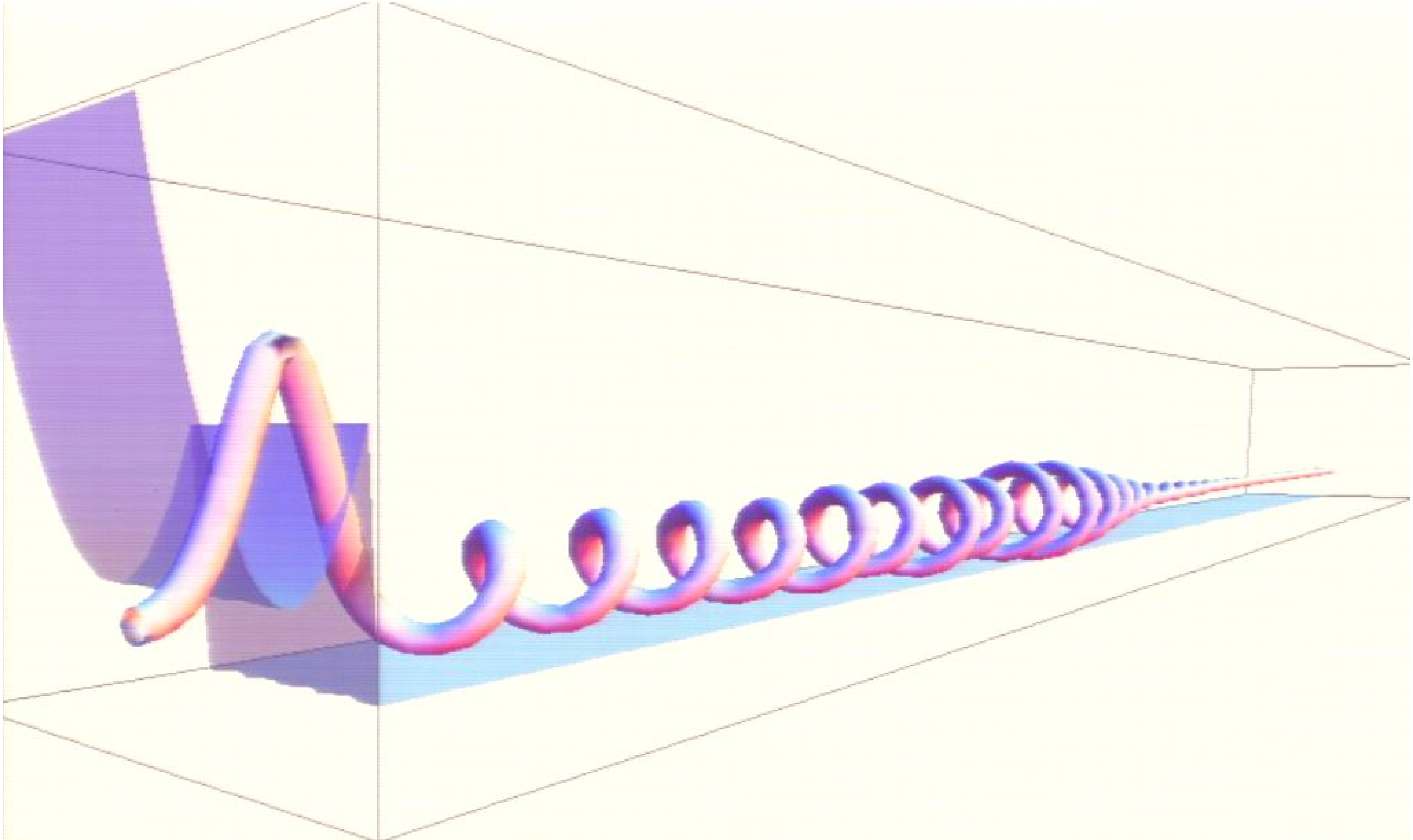


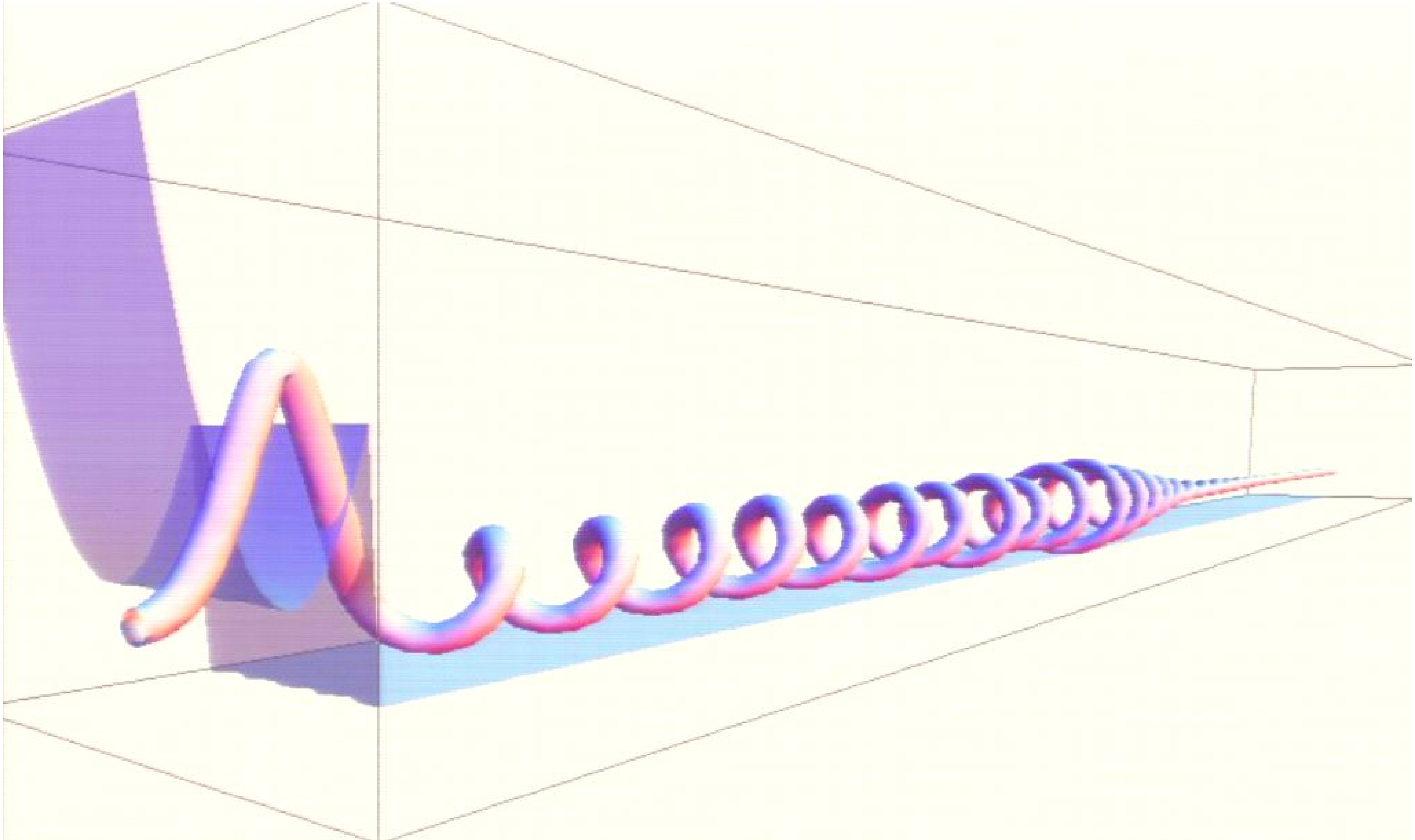


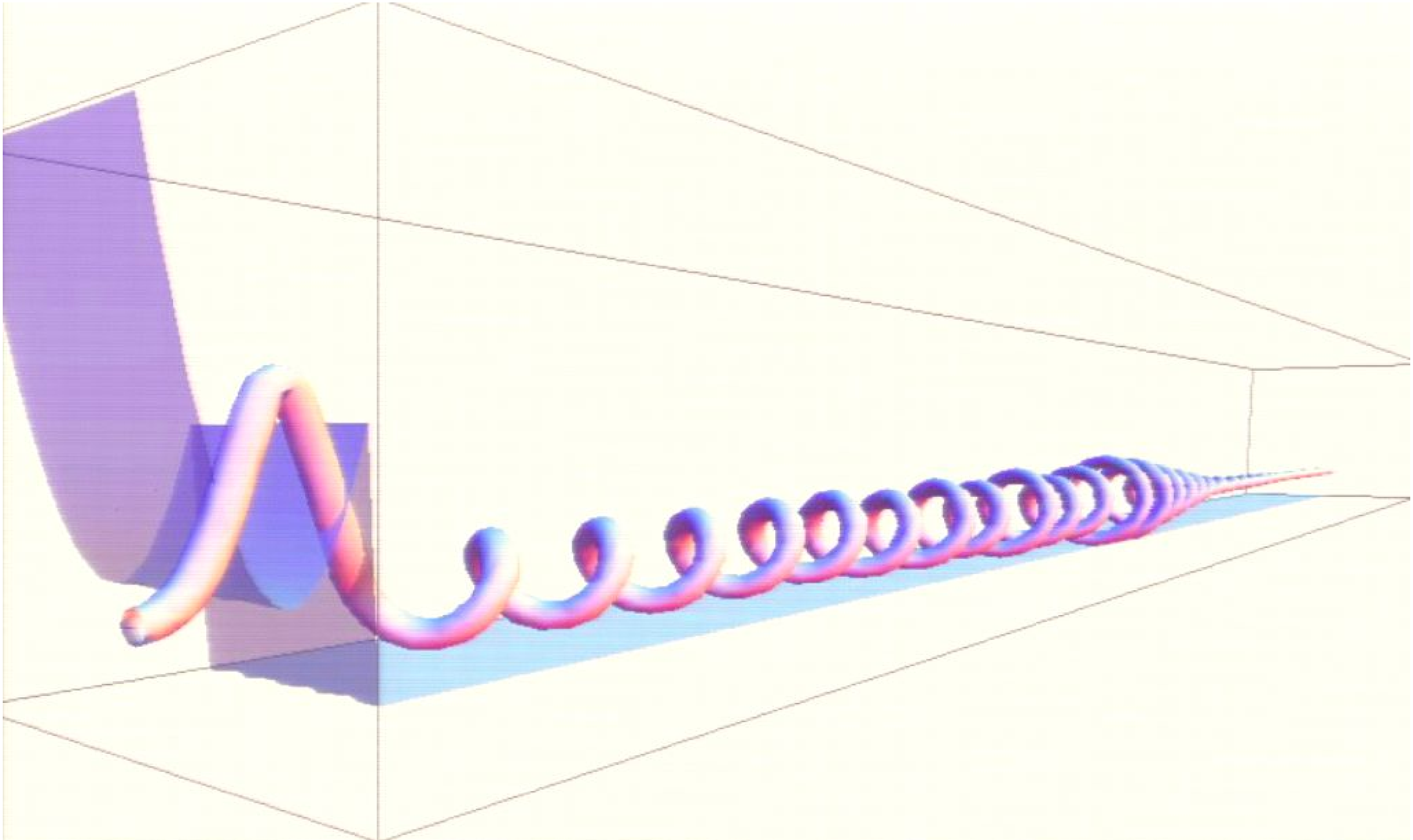


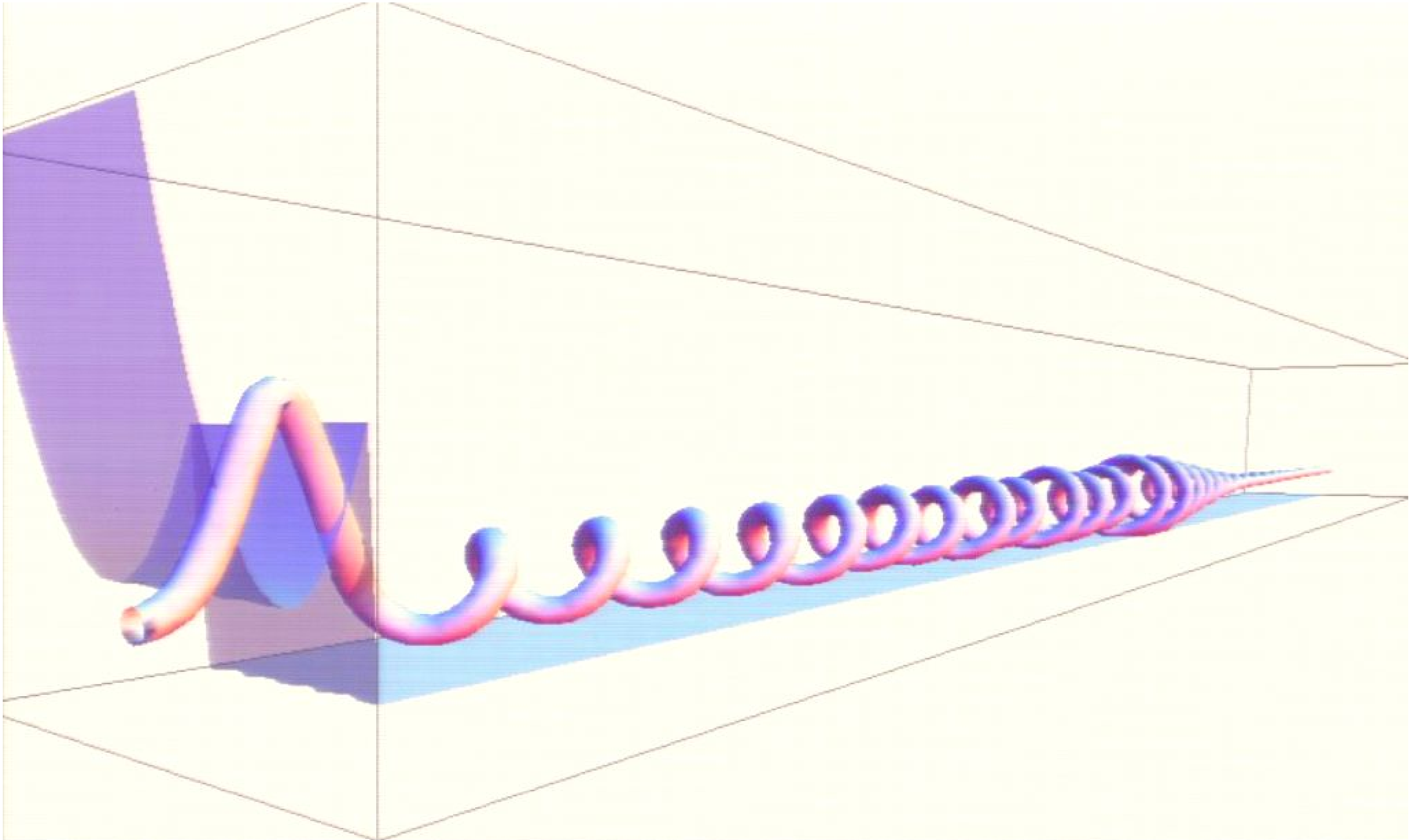


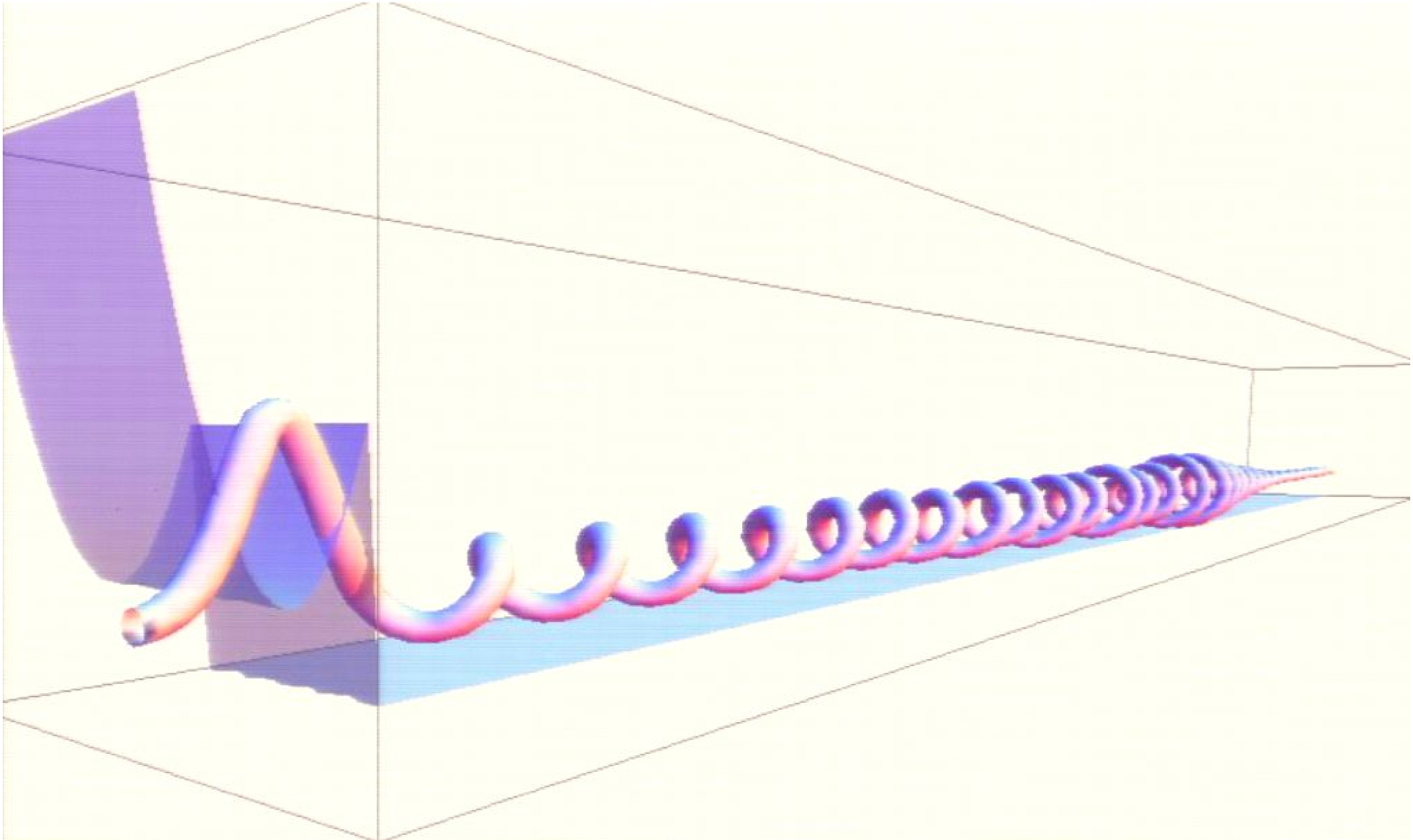


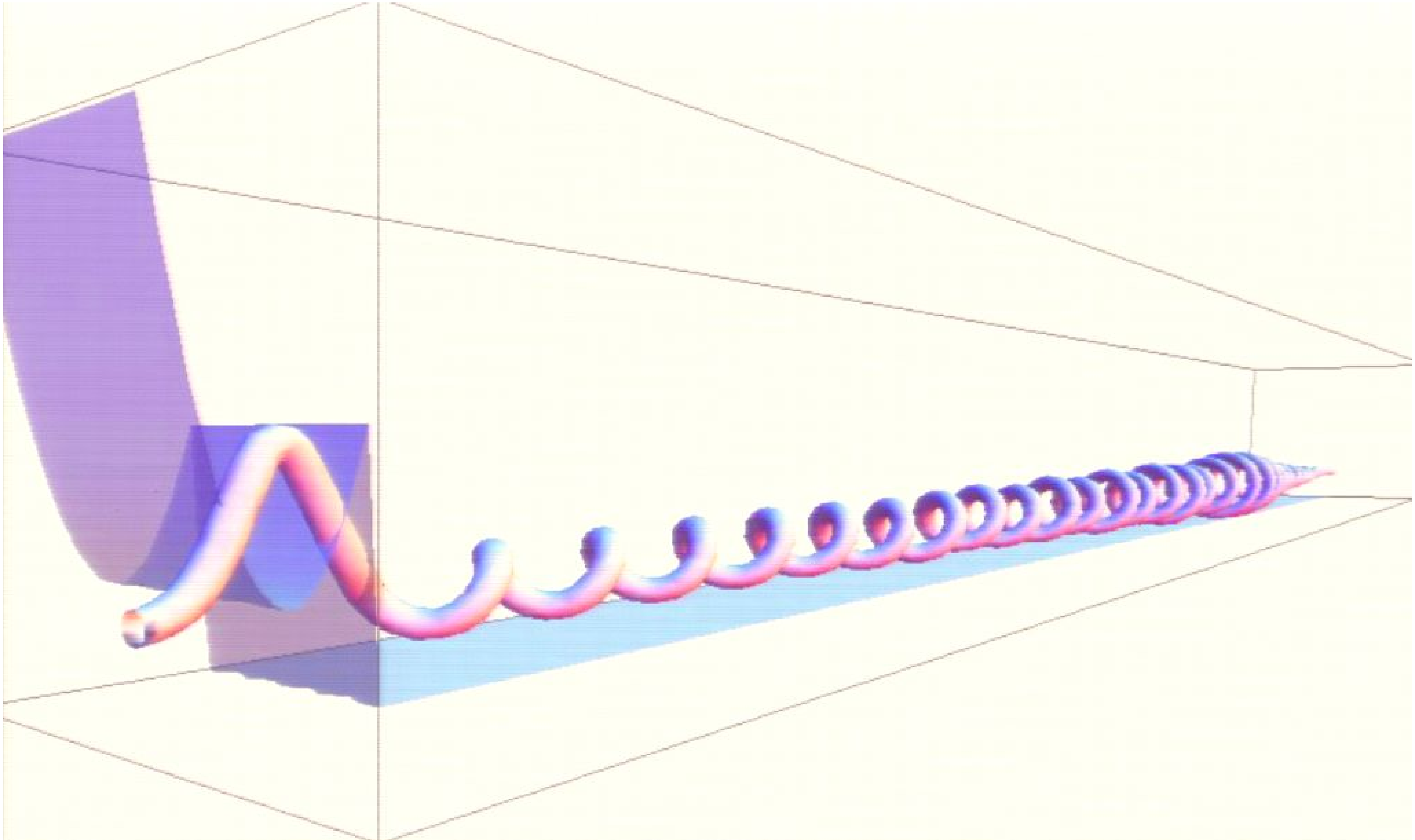


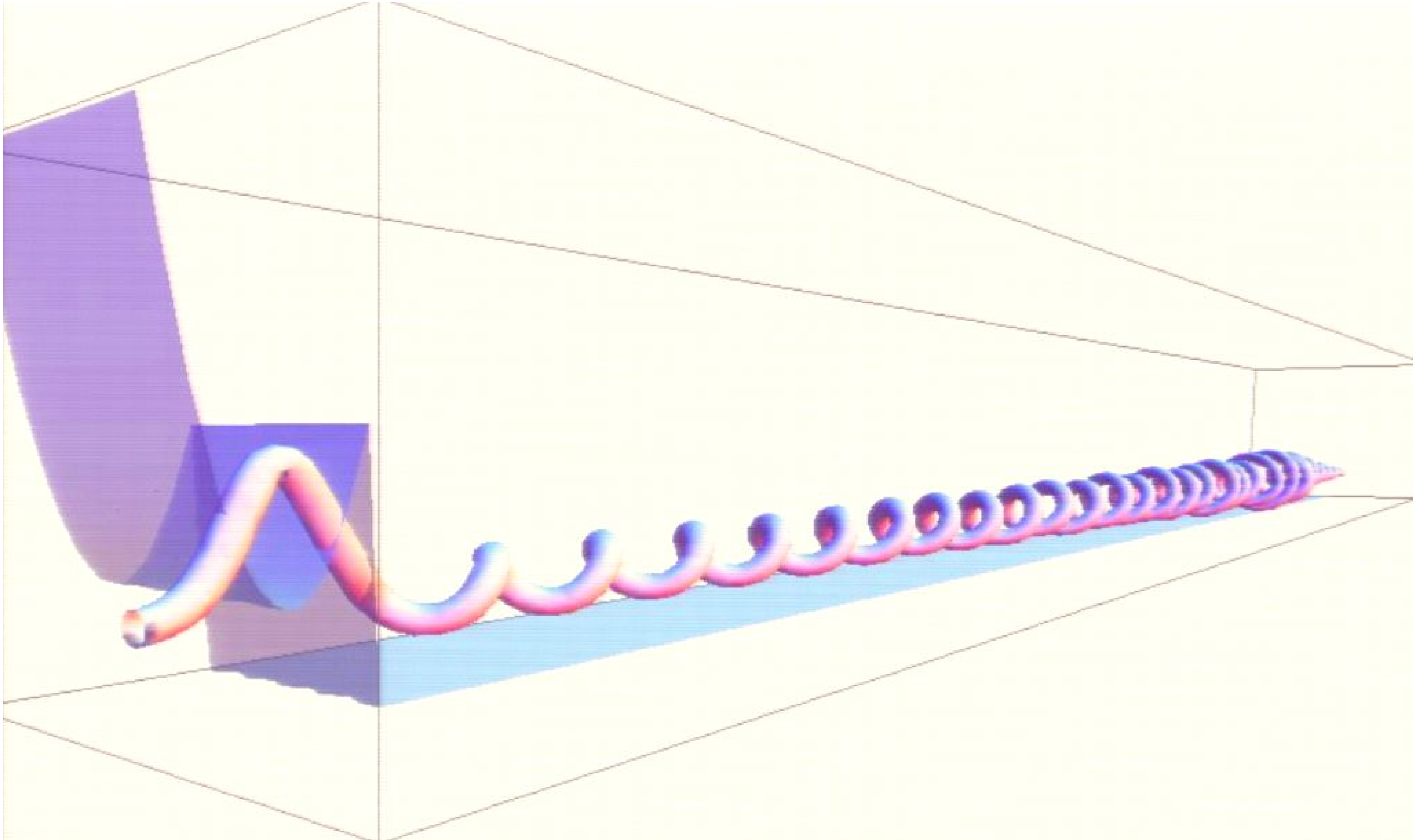












Instanton method

S. Coleman
"The uses of instantons"

$$|\langle\Psi_0|\exp(-iHt)|\Psi_0\rangle|^2 \approx \exp(-\Gamma|t|)$$

$$\langle x_f|e^{-iHt}|x_i\rangle = \int [dx] e^{iS[x]}$$

$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau \qquad V(x) \to -V(x)$$

$t \to -it = -T$ This is the *definition* of the path integral

$$\langle x_0|e^{-HT}|x_0\rangle = \int [dx]\, e^{-S_E[x]}$$

On the left: Large T picks out the lowest energy states which overlap $|x_0\rangle$.

On the right: We will use method of steepest descent to calculate the late time behavior of the low energy states.

$$H = H_{pert.} + \Delta$$

$$\boxed{\langle x_0 | e^{-HT} | x_0 \rangle} = \sum_n \langle x_0 | \, e^{-(H_{pert.}+\Delta)T} \, | n \rangle \langle n | x_0 \rangle$$

$$\overset{\text{large } T}{\longrightarrow} e^{-(E_{fv}+\delta_{fv})T} \, |\langle x_0 | \Psi_0 \rangle|^2$$

If we treat Ψ_0 like an eigenstate of H, its energy has an imaginary part.

Actually, Δ has off-diagonal terms which preserve unitarity.

$$\delta_{fv} = -\frac{i}{2}\Gamma$$

non-perturbative correction to false-vacuum "energy"

$$|\langle \Psi_0 | \exp(-iHt) | \Psi_0 \rangle|^2 \approx \exp(-\Gamma |t|)$$

Method of steepest descent

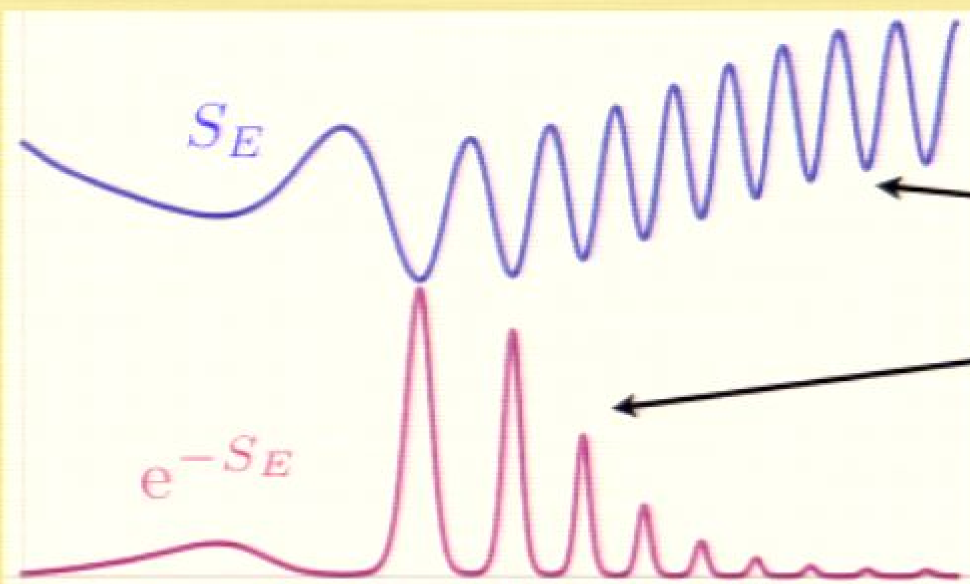
$$S_E[x] \approx \sum_{x_{cl}} S_E[x_{cl}] + \frac{1}{2}\delta x \left. \frac{\delta^2 S_E}{\delta x^2}\right|_{x_{cl}} \delta x$$

$$\left.\frac{\delta^2 S_E}{\delta x^2}\right|_{x_{cl}} = -\partial_\tau^2 + V''(x_{cl}(\tau))$$

Fluctuations = measure

expand about the local **minima** of $S_E[x]$

$$\int [dx] \mathrm{e}^{-S_E[x]} \approx \sum_{x_{cl}} \mathrm{e}^{-S_E[x_{cl}]} \sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$

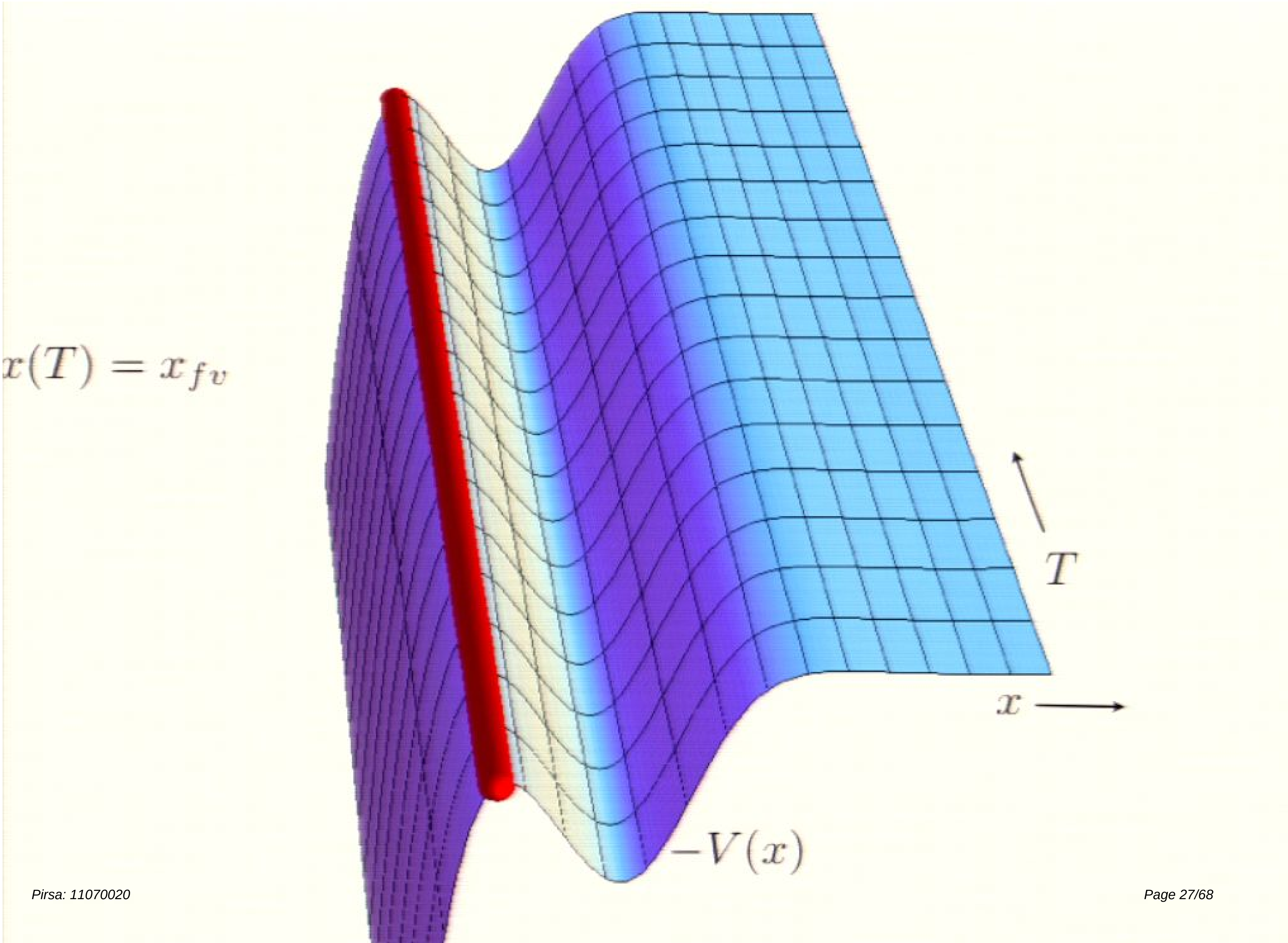


many minima (x_{cl})

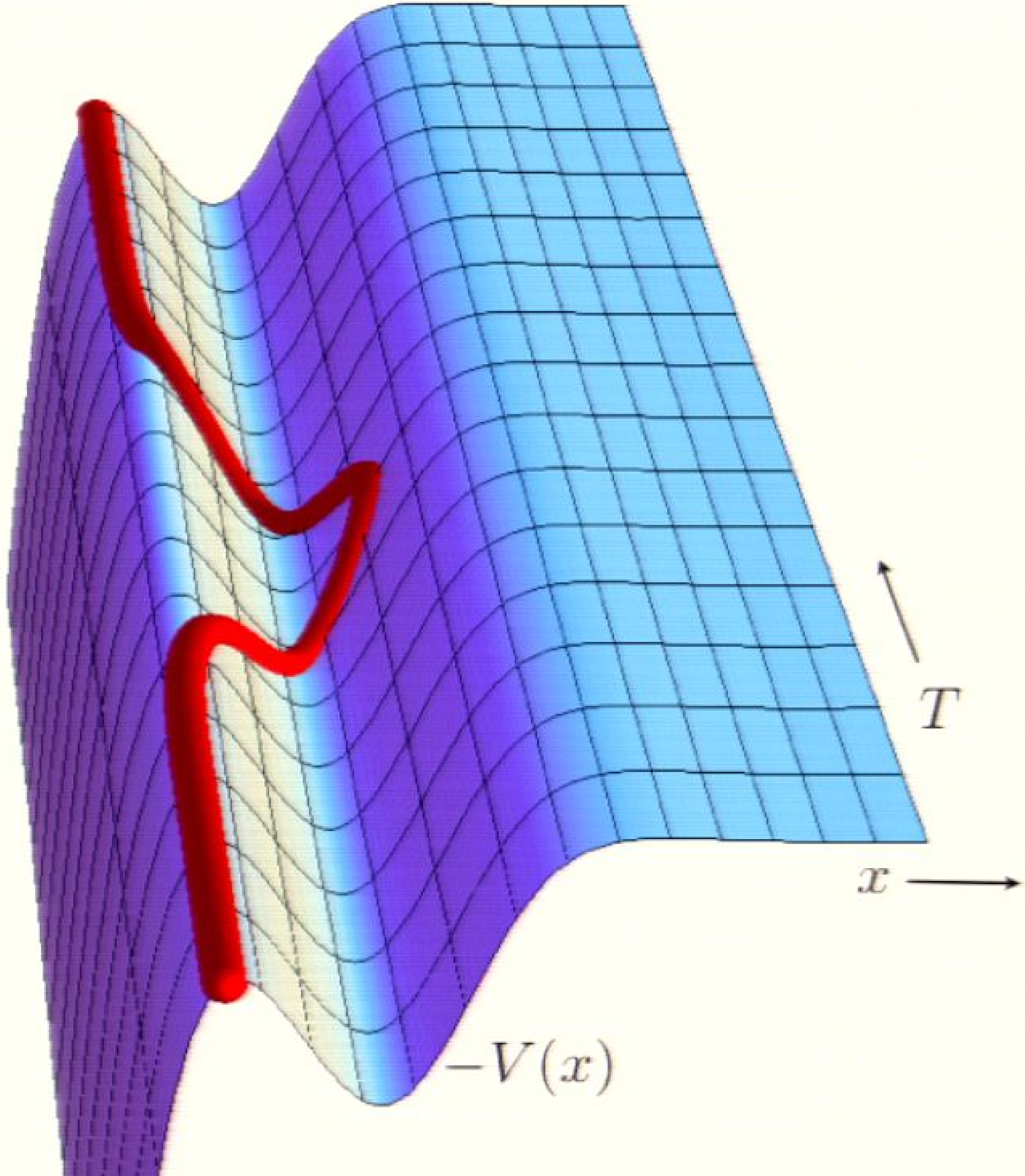
many Gaussians

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{\pi}} \mathrm{e}^{-\lambda x^2} = \frac{1}{\sqrt{\lambda}}$$

What are the x_{cl}?


$x(T) = x_{fv}$
T
x
$-V(x)$

$(T) = x_B(T)$



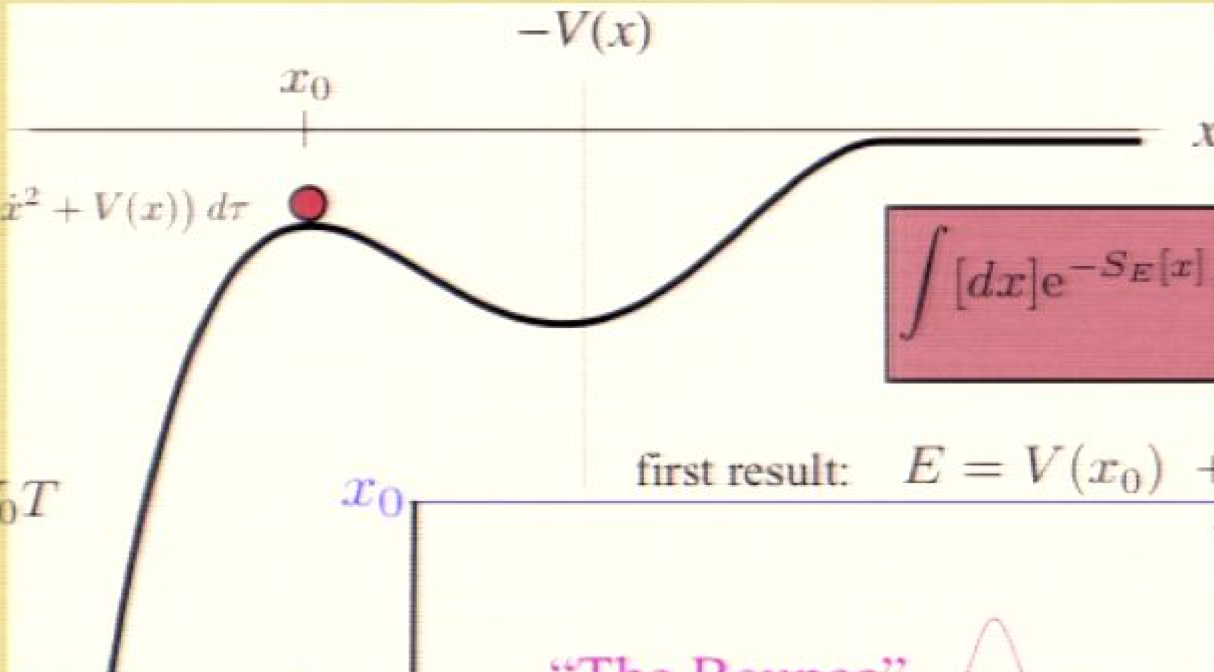
'he instanton is
symptotic to the
false vacuum

What are the x_{cl}?

any number of bounces ≥ 0

"instantons"

$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau$$



$$\int [dx] e^{-S_E[x]} \approx \sum_{x_{cl}} e^{-S_E[x_{cl}]} \sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$

first result: $E = V(x_0) + \frac{\hbar}{2}\sqrt{V''(x_0)} + \mathcal{O}(\hbar^2)$

$S_E[x_0] = V_0 T$

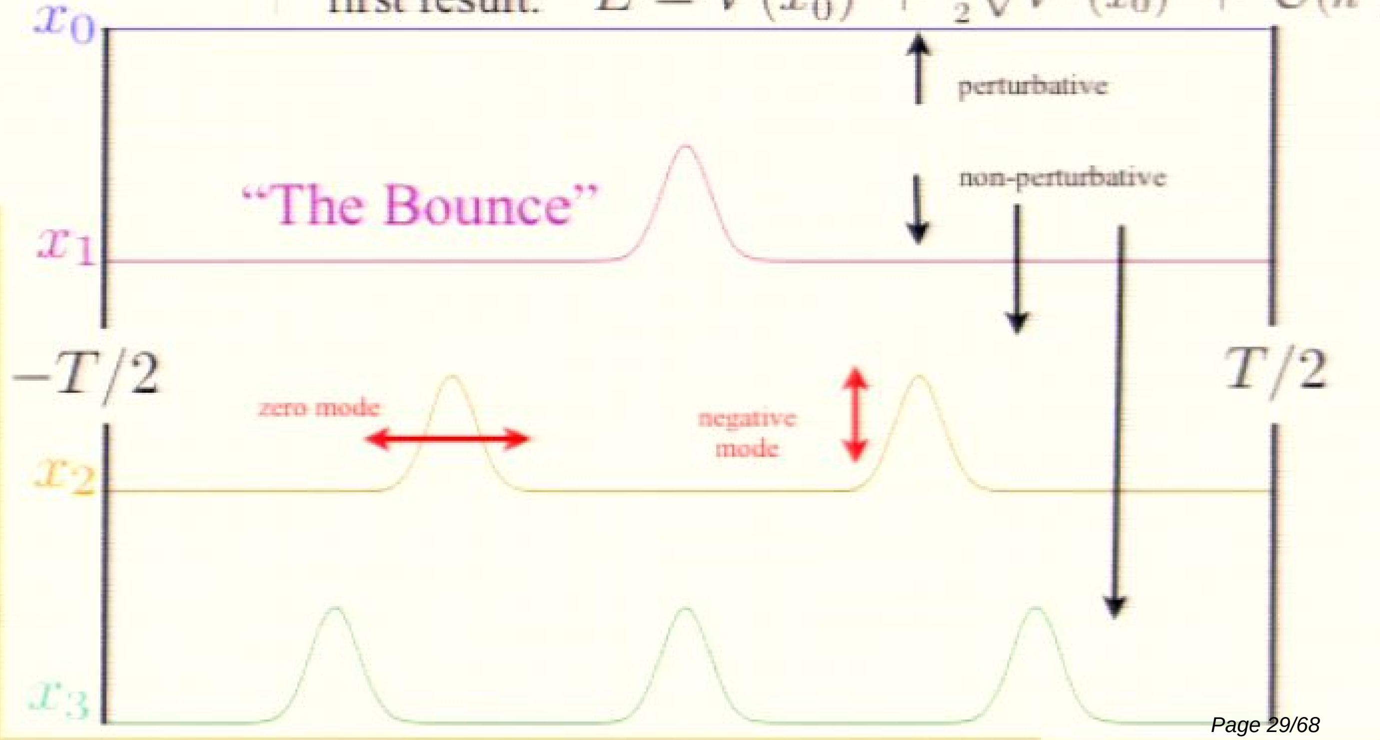
$S_E[x_1] = V_0 T + S_B$

$S_E[x_n] = V_0 T + n S_B$

n zero modes!

n negative modes!

$$S_B = 2\int \sqrt{2(V(x) - V_0)}\,dx$$

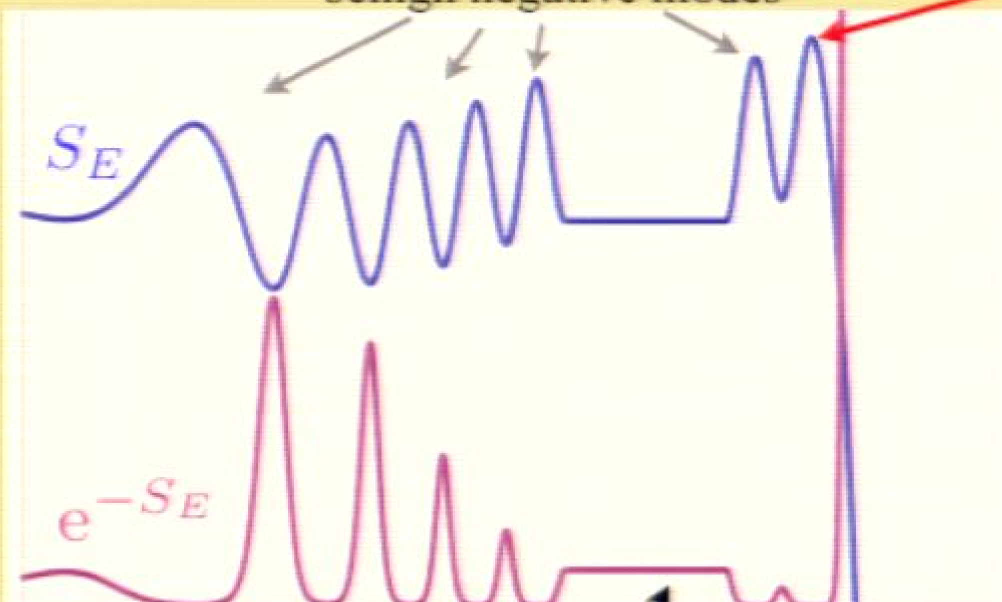


Each "dangerous" negative mode contributes a *tiny* imaginary factor.

$$\int_{-\infty}^{\infty} e^{+|\lambda|x^2} dx = \frac{i}{2}\sqrt{\frac{\pi}{|\lambda|}}$$

Sagredo forgot this factor of $1/2$.

benign negative modes



Dangerous negative modes allow for exponential decay

usual formula

$$\int_{-\infty}^{\infty} e^{-\lambda z^2} dz = \sqrt{\frac{\pi}{\lambda}}$$

Each zero mode contributes a large, but finite factor T.

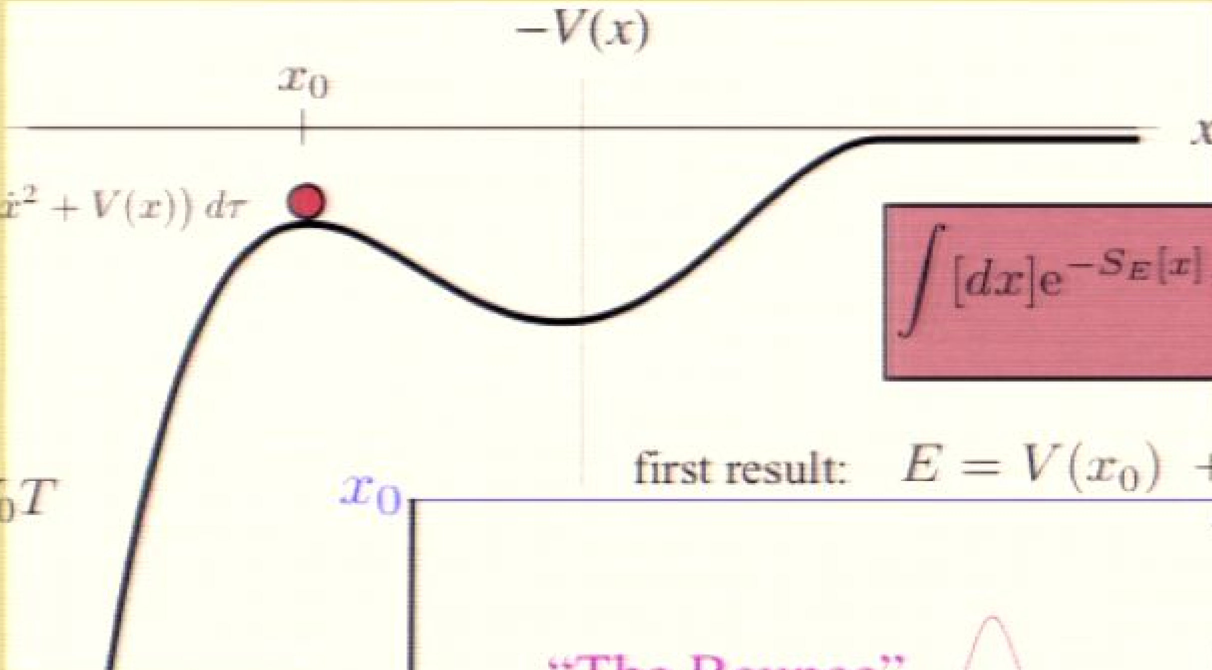
Zero modes make a small effect observable at macroscopic time (extensive).

$$\int_{-T/2}^{T/2} e^{-0\tau^2} d\tau = T$$

What are the x_{cl}?

any number of bounces ≥ 0

"instantons"



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$S_E[x_1] = V_0 T + S_B$

$S_E[x_n] = V_0 T + n S_B$

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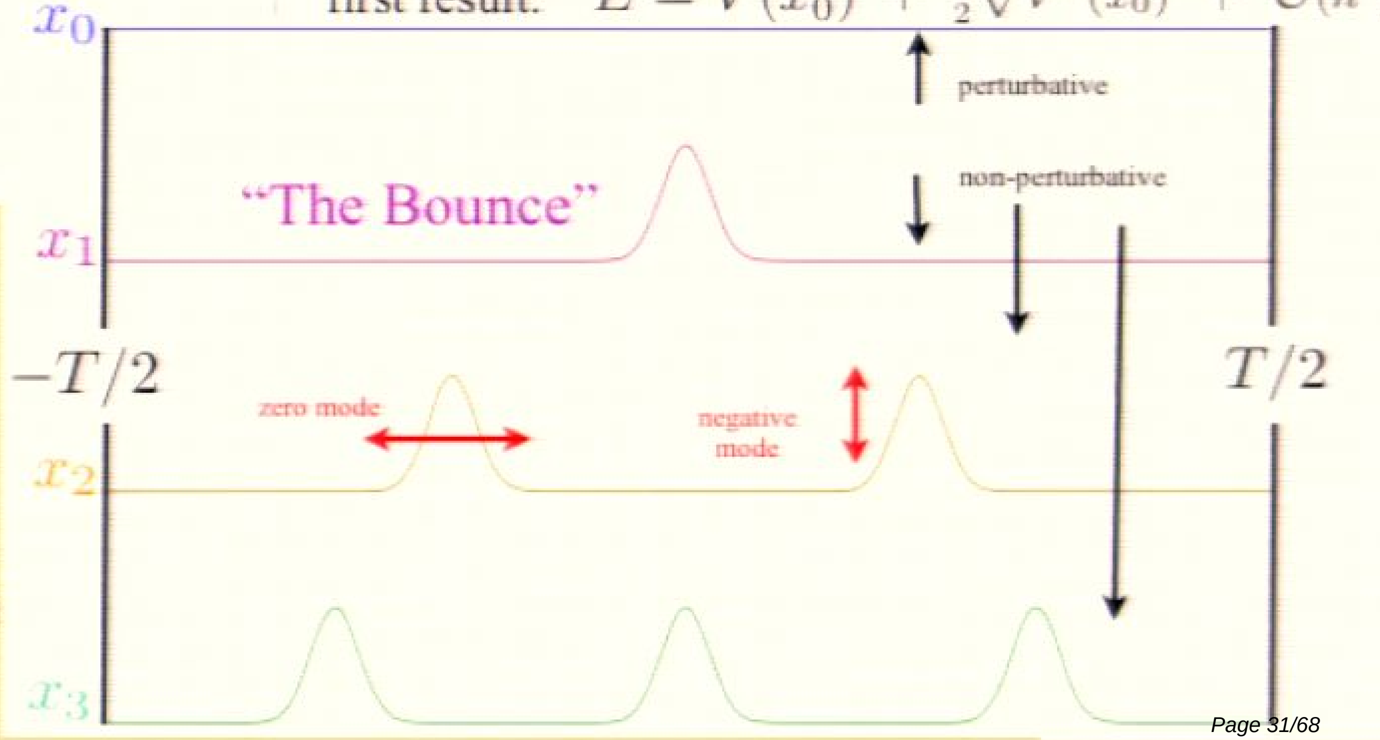

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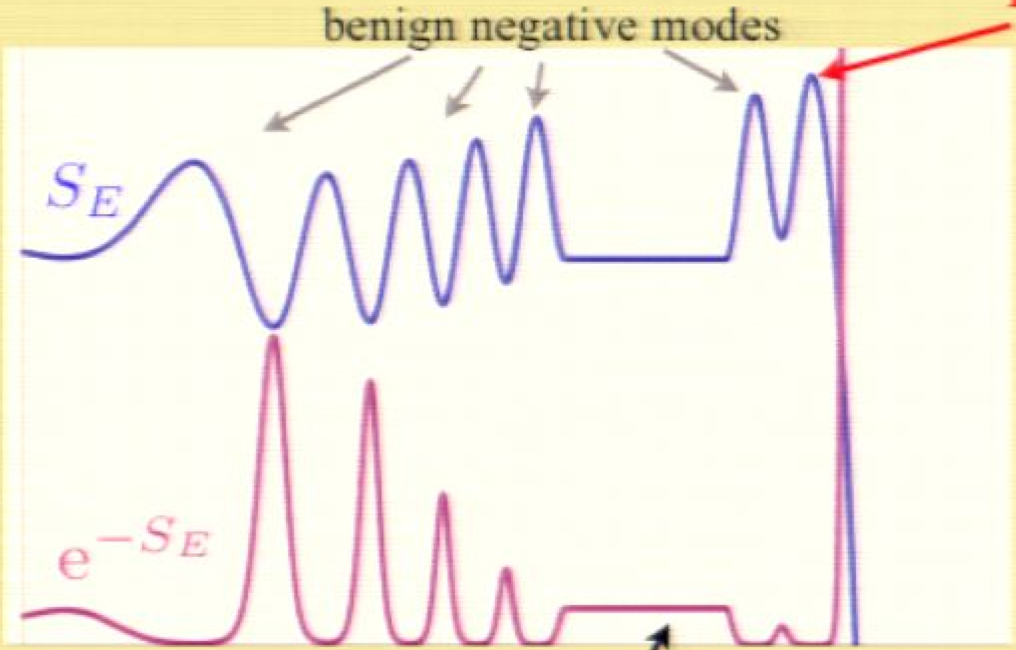

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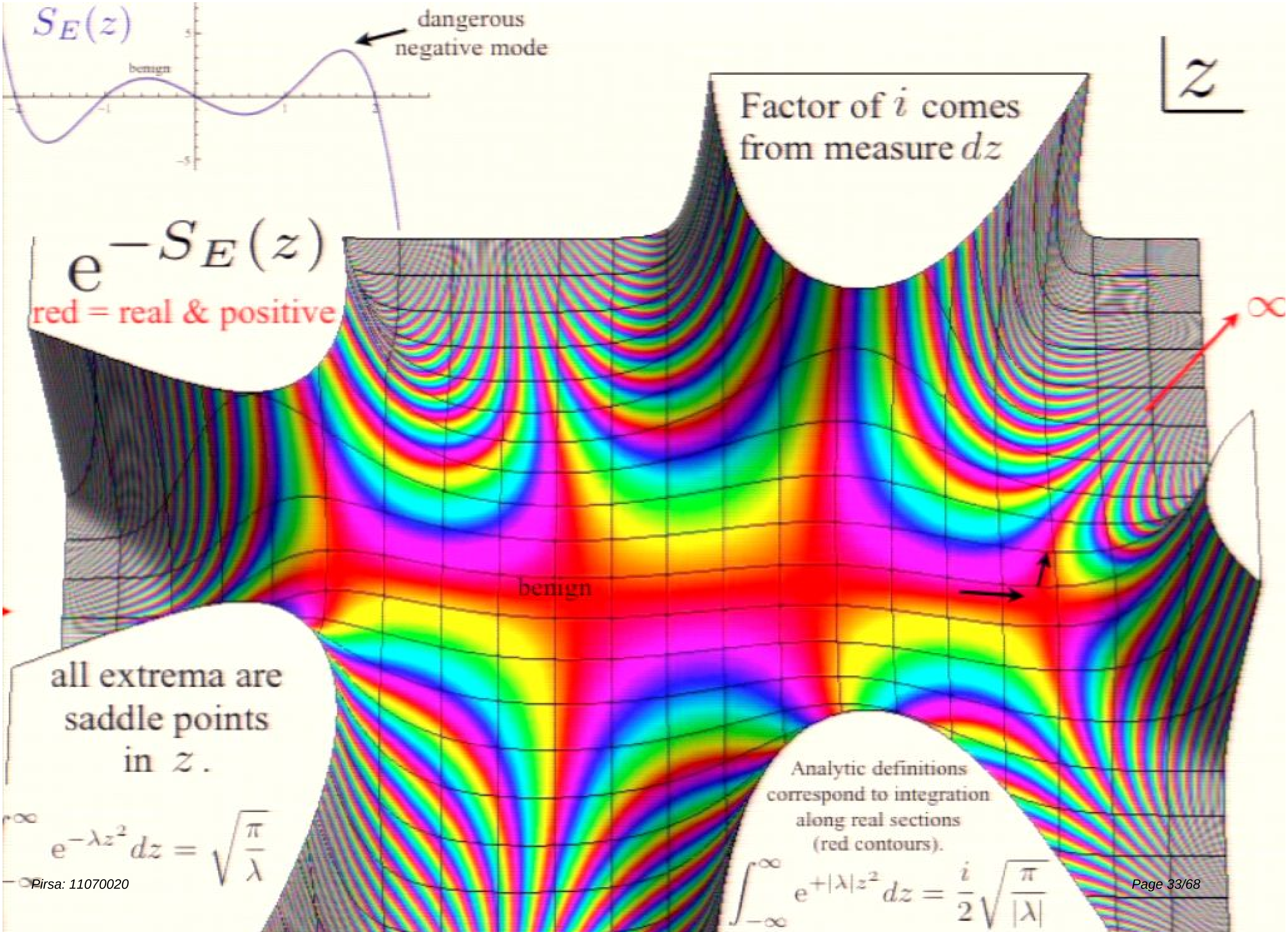

$S_E(z)$
dangerous negative mode
benign
z
Factor of i comes from measure dz
$e^{-S_E(z)}$
red = real & positive
∞
benign
all extrema are saddle points in z.
Analytic definitions correspond to integration along real sections (red contours).
$\int_{-\infty}^{\infty} e^{-\lambda z^2} dz = \sqrt{\frac{\pi}{\lambda}}$
$\int_{-\infty}^{\infty} e^{+|\lambda| z^2} dz = \frac{i}{2}\sqrt{\frac{\pi}{|\lambda|}}$

$$x_n \;\equiv\; n \qquad \text{widely separated bounces}$$

$$S_E[x_n] = S_E[x_0] + nS_B \qquad S_B = 2\int \sqrt{2(V(x)-V_0)}dx$$

$$\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_n}^{-\frac{1}{2}} = \frac{T^n}{n!}\left(\frac{-i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_n}^{-\frac{1}{2}} = \frac{T^n}{n!}\left(\frac{-i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{\frac{n}{2}} \det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_0}^{-\frac{1}{2}}$$

$$\sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]_{x_0}}} = e^{-\frac{1}{2}\omega T + \mathcal{O}(\hbar^2)T} \qquad \omega = \sqrt{V''(x_0)}$$

$$\int [dx]e^{-S_E[x]} \approx \sum_{x_{cl}} e^{-S_E[x_{cl}]}\sqrt{\frac{1}{\det\left[\frac{\delta^2 S_E}{\delta x^2}\right]}}$$

$$= \sum_{n=0}^{\infty} e^{-\left(V_0+\frac{\omega}{2}+\mathcal{O}(\hbar^2)\right)T}\left(\frac{T^n}{n!}\left(\frac{i}{2}\right)^n \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{\frac{n}{2}} e^{-nS_B}\right)$$

$$= e^{-\left(V_0+\frac{\omega}{2}-\frac{i}{2}\det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} e^{-S_B}\right)T}$$

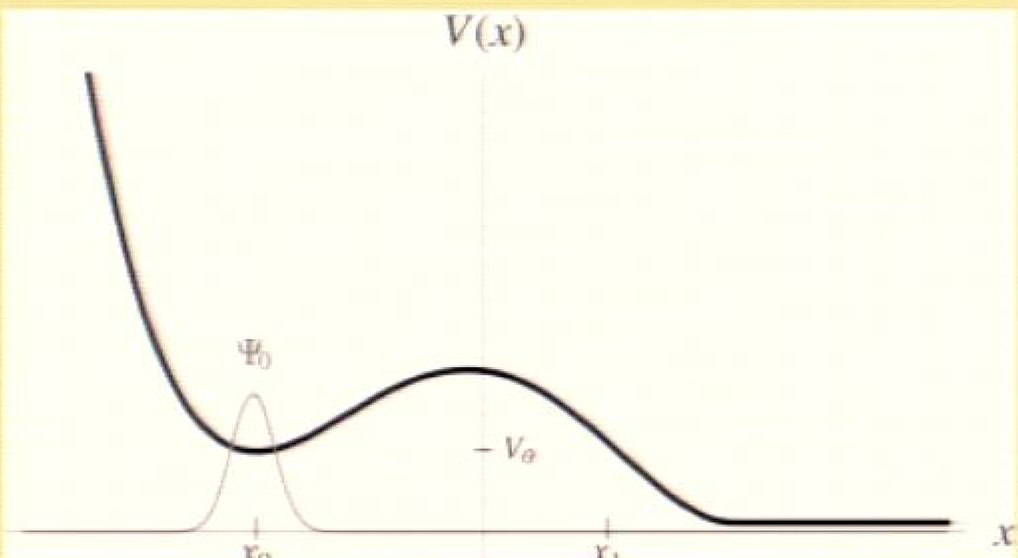
At large T

$$e^{-(E_{fv}+\delta_{fv})T} = e^{-\left(V_0+\frac{\omega}{2}-\frac{i}{2}\det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} e^{-S_B}\right)T}$$

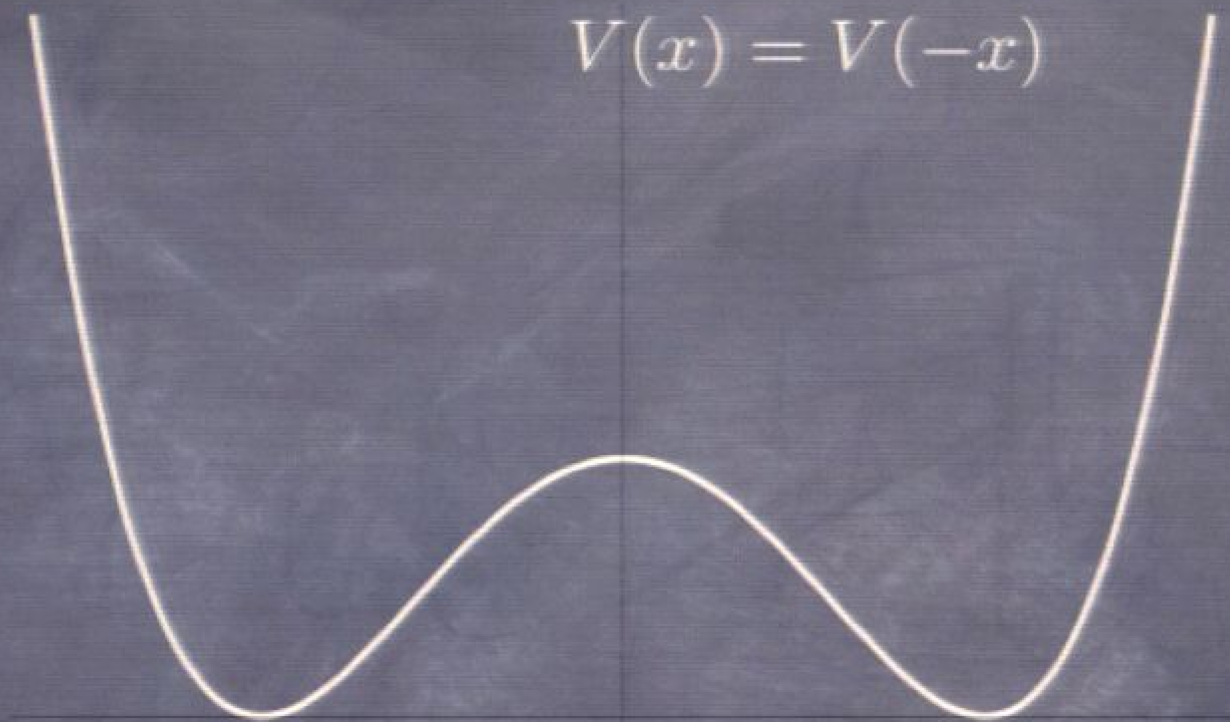
$$\delta_{fv} = -\frac{i}{2}\Gamma$$

$$\Gamma = \det{}'\left[\left|\frac{\delta^2 S_E}{\delta x^2}\right|\right]_{x_B}^{-\frac{1}{2}} e^{-S_B}$$

$$|\langle\Psi_0|\exp(-iHt)|\Psi_0\rangle|^2 \approx \exp(-\Gamma|t|)$$



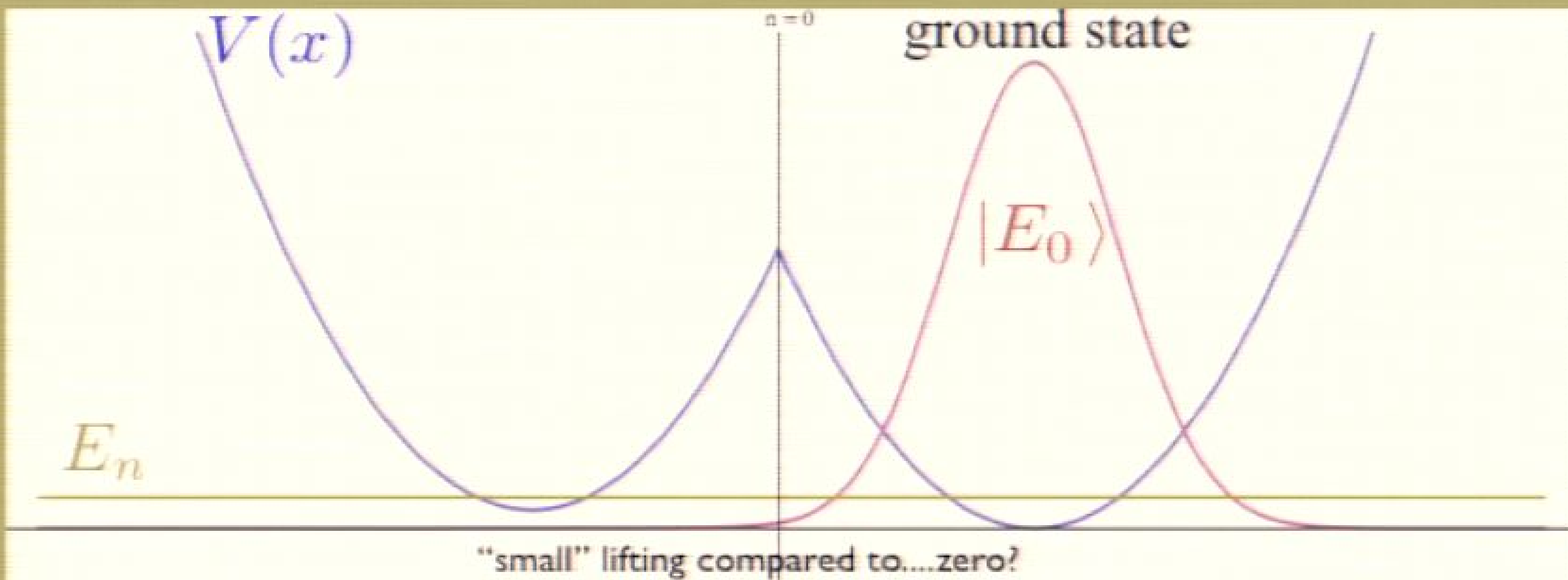
The symmetric double well



he spectra of H_{l} and H_{r} are identical to all orders in perturbation theory due to a Z_2 symmetry.

But instantons lift the degeneracy.

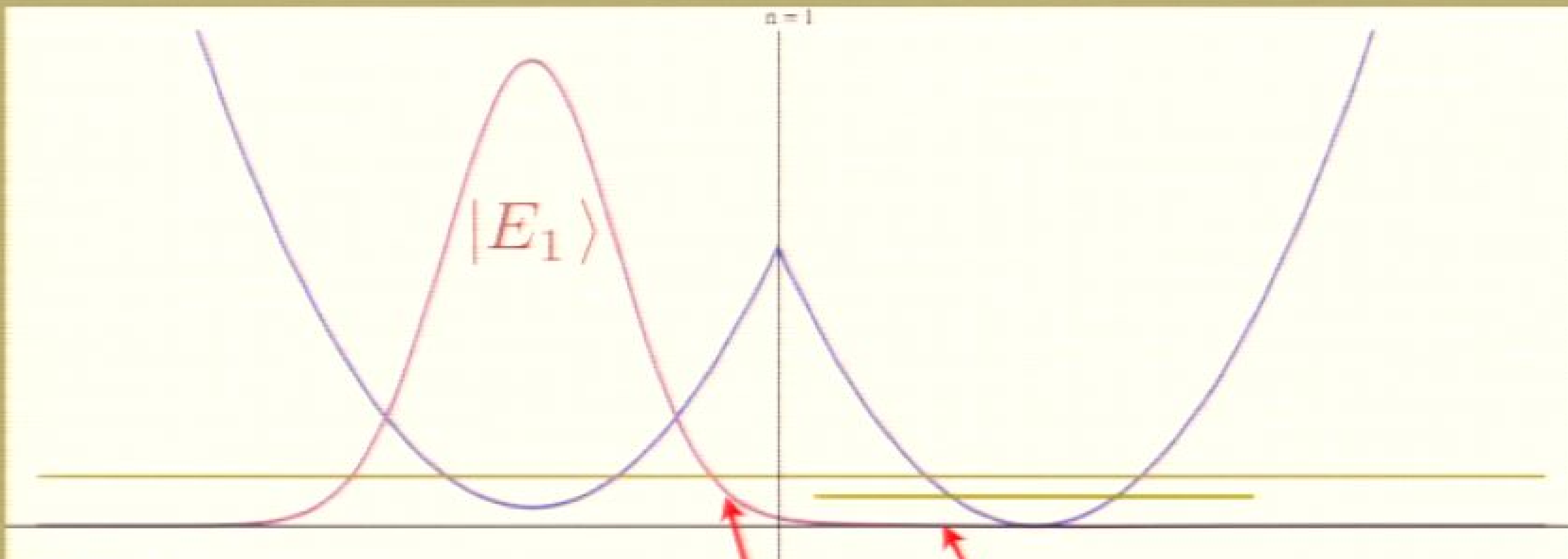
$\Longrightarrow$ *resonance*



The asymmetric double well

not a perturbation of the symmetric double well!

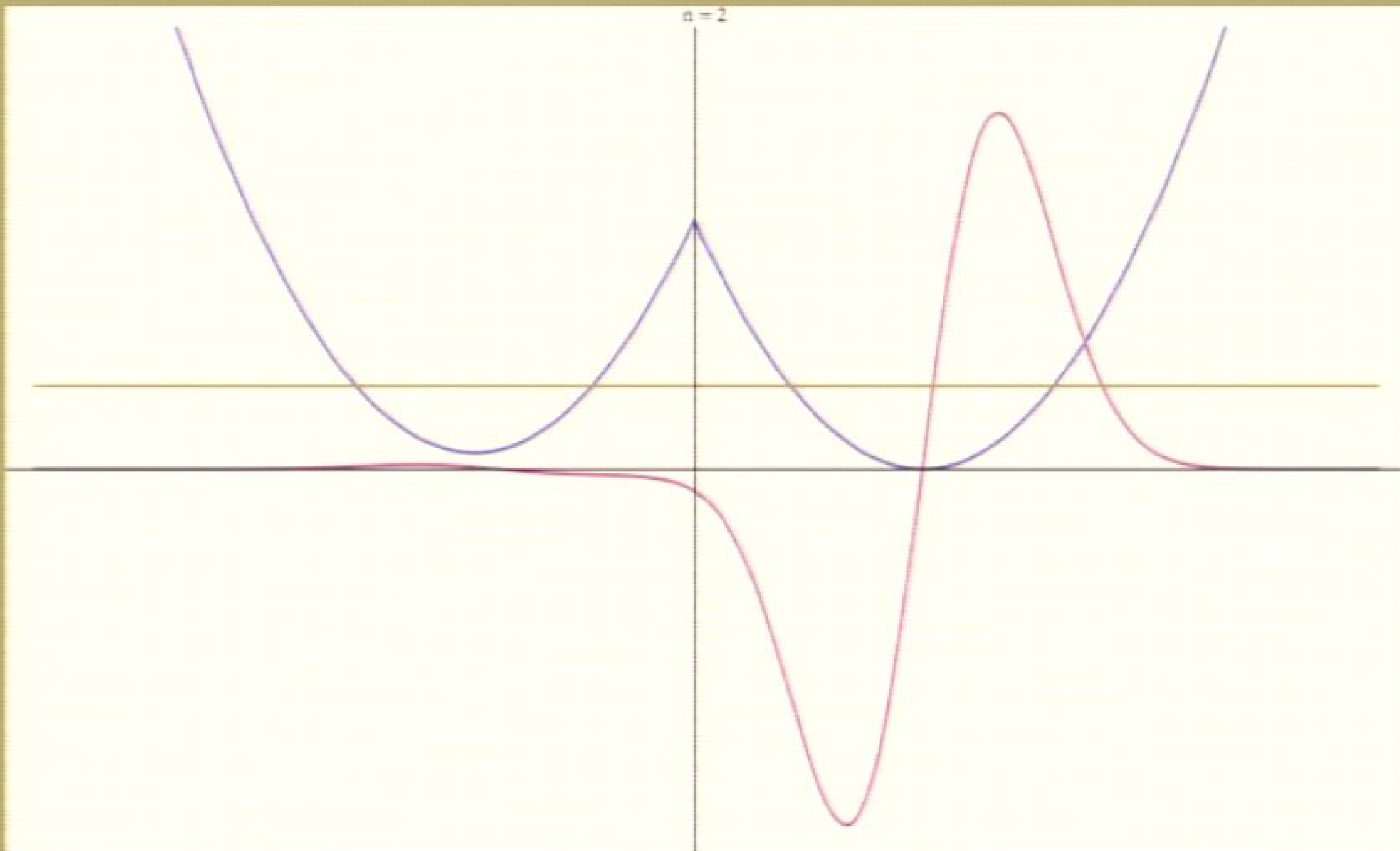
A Stable False Vacuum



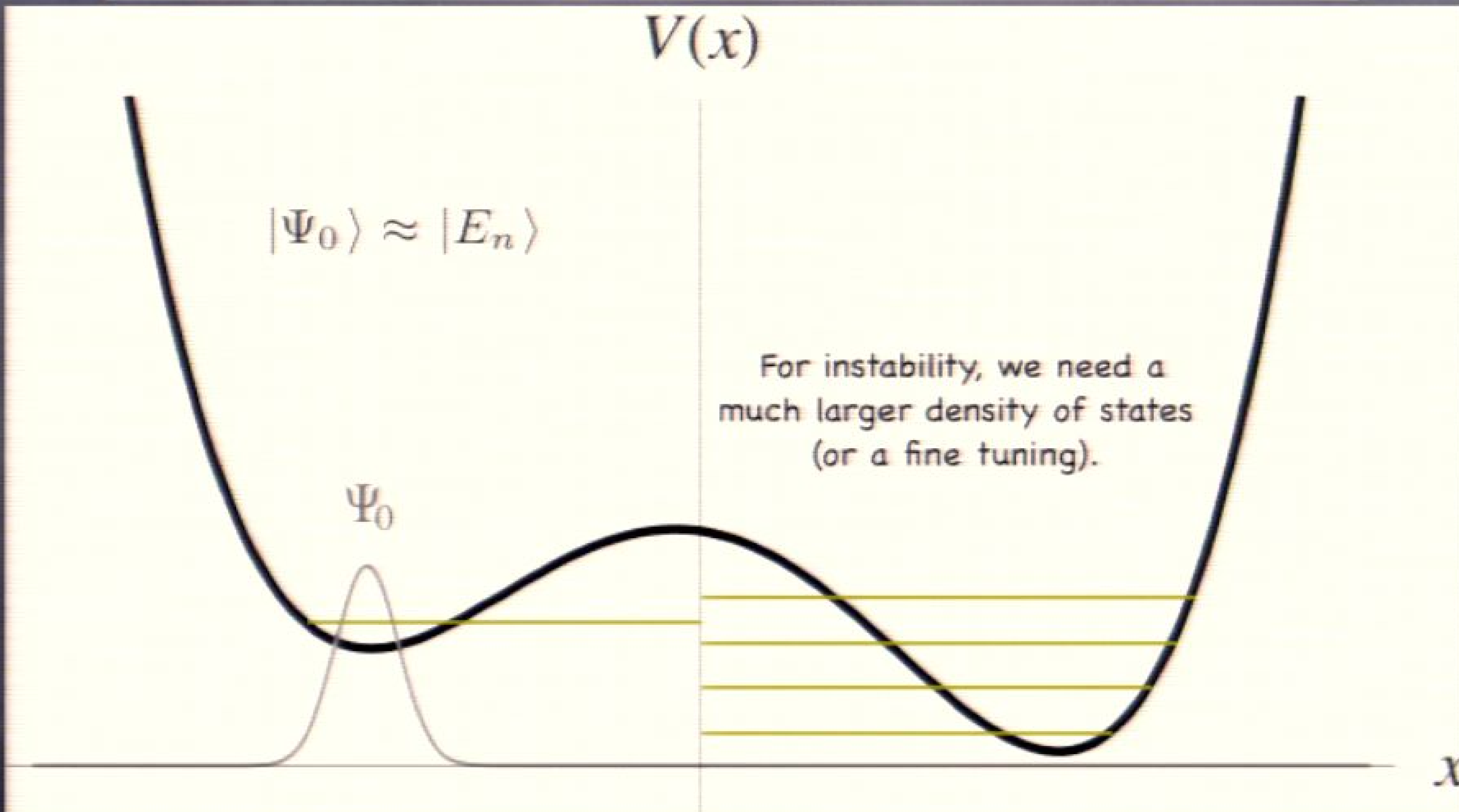
1. Fix 2 boundary conditions here

2. Choose energy for convergence here

3. We are left with no free parameters. All wave functions will have a growing and a decaying barrier mode with $\mathcal{O}(1)$ coefficients. One of them will thus always dominate, and all solutions are primarily supported on the left, or the right.

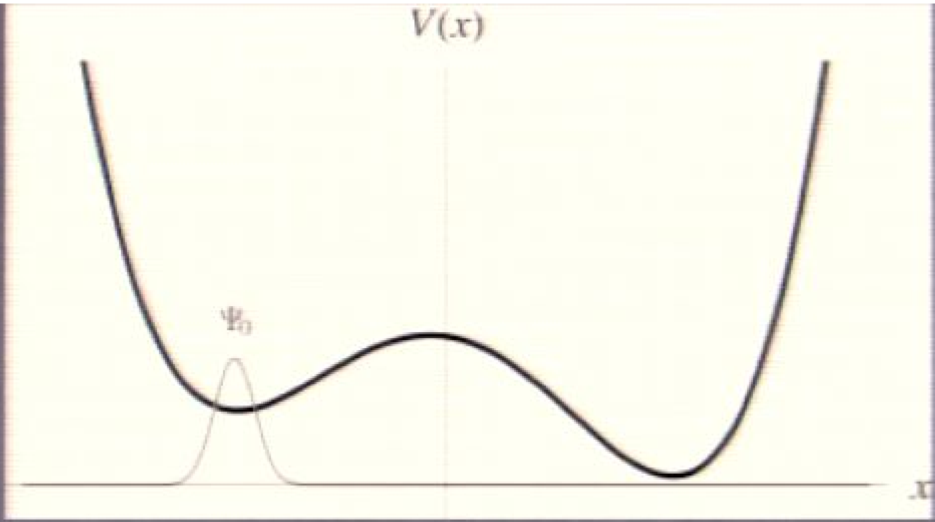

n = 2

The asymmetric double well has a stable false vacuum.



The (LHS) perturbative vacuum **is** an approximate energy eigenstate ($S_B \gg 1$).

This approximate energy eigenstate is stable despite these two facts:

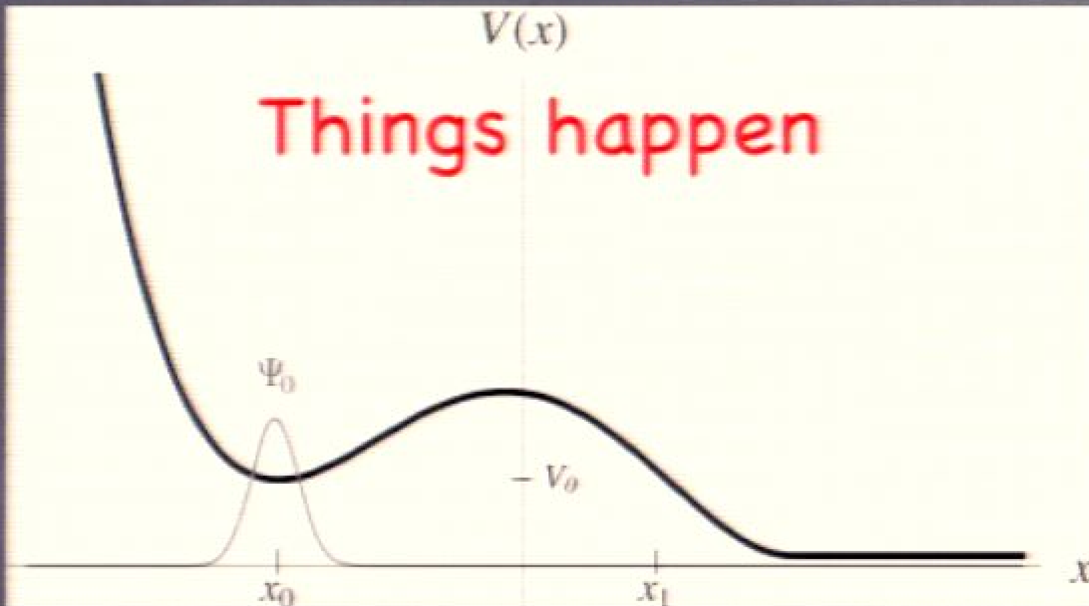


1. There is a non-zero probability for an observer to find the particle on the right, in the true vacuum). (single freak observer)

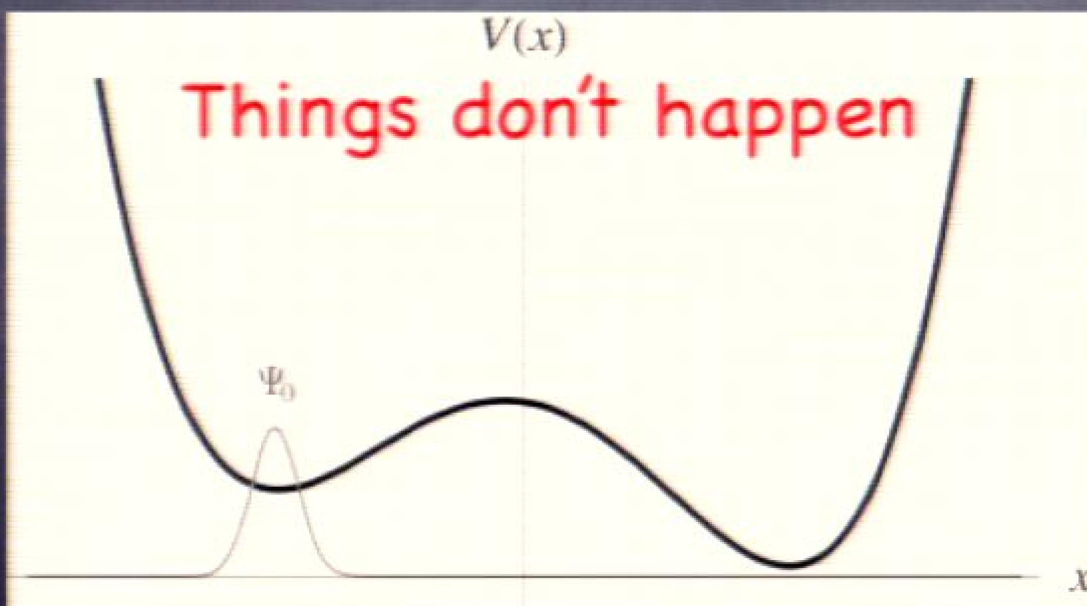
By this definition, everything is unstable.

2. By coupling the particle to a big enough detector (e.g. the electromagnetic continuum), the particle will irreversibly and certainly end up in the true vacuum. (perpetual macroscopic observer)

different Hamiltonian, not the problem we are considering



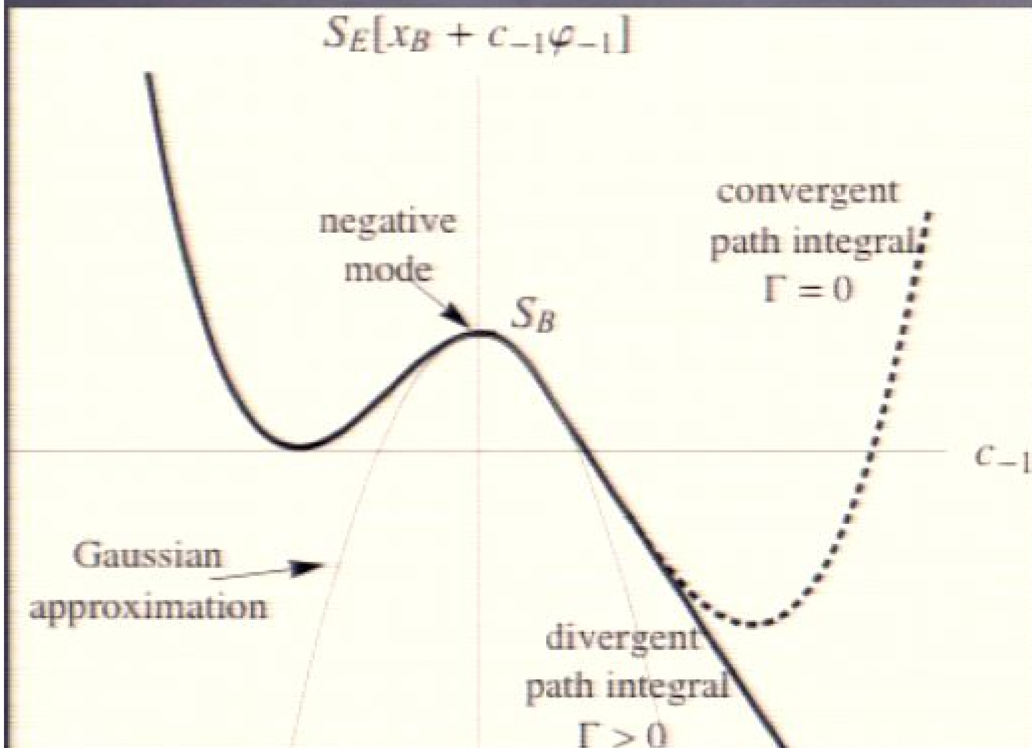
$$P_{\text{survival}} = \mathrm{e}^{-\Gamma t}$$



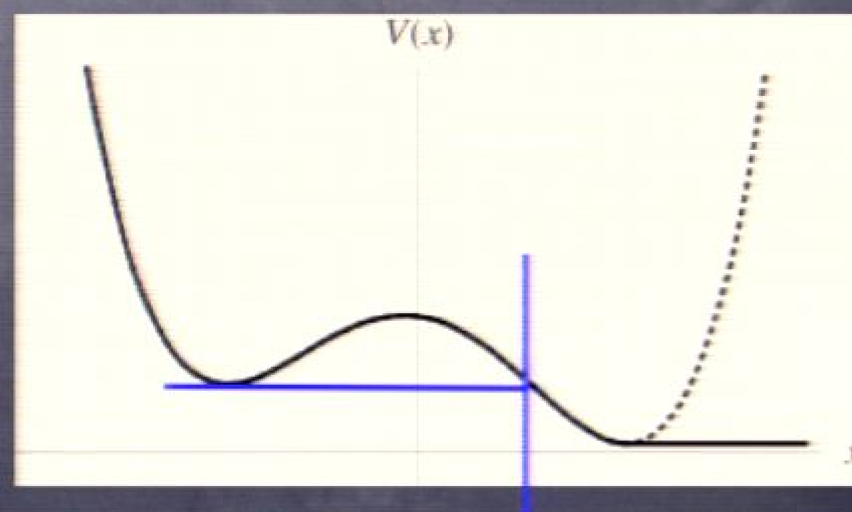
$$P_{\text{survival}} = \text{const.} \approx 1$$

But what about the bounce?

The instanton formalism appears to predict exponential decay. The resolution is that the single negative mode is rendered benign by "compactifying" the true vacuum.

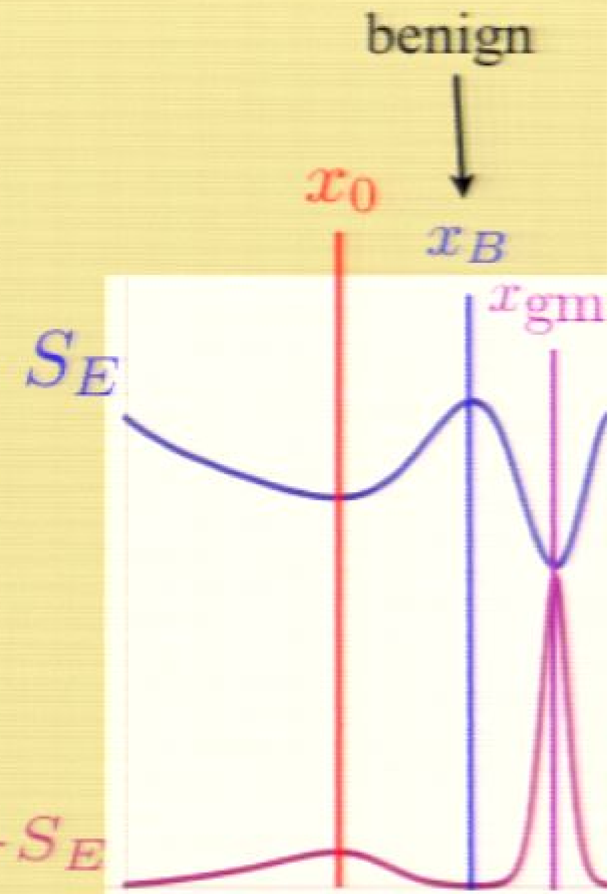


$$S_E = \int_{-T/2}^{T/2} \left(\tfrac{1}{2}\dot{x}^2 + V(x)\right) d\tau$$

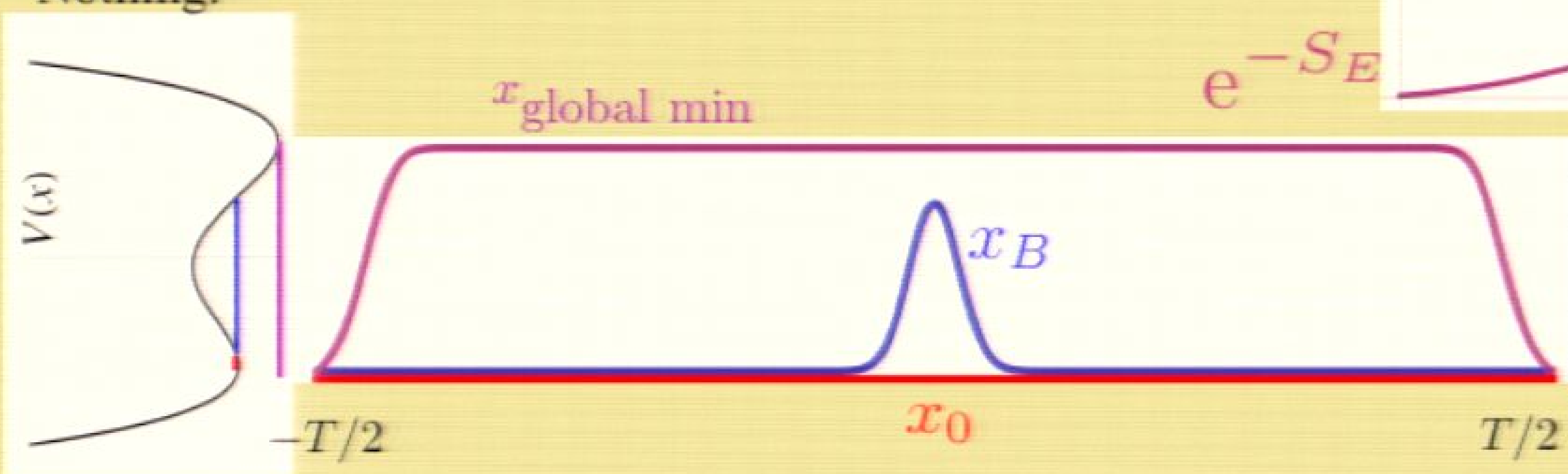


The double well has Euclidean action bounded below!

For the asymmetric double well, "the bounce" has a *benign* negative mode. The lower action solutions to either side of "the bounce" are both finite and much more important. The contribution to the partition function from "the bounce" is negligible.

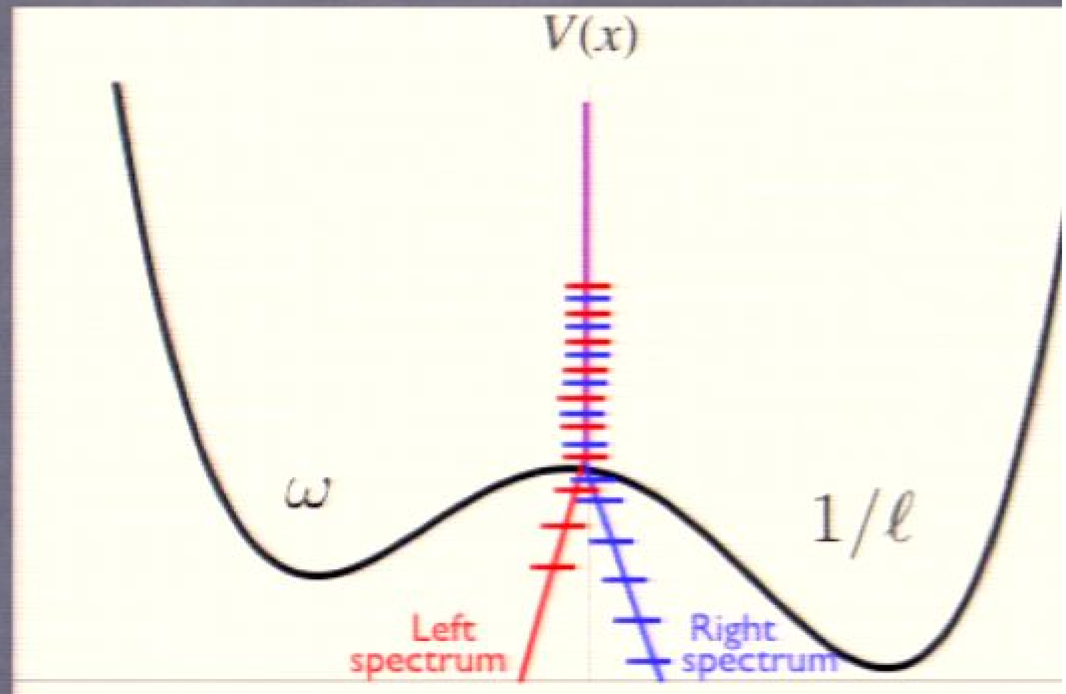


Then what is the significance of the bounce?
Nothing.



ne false vacuum cannot decay because there are no excited true vacuum states with overlapping energy. This is found experimentally in cavity QED.

The true vacuum volume ℓ must be *very* large for instability to be generic.



$$\nu(E) := \frac{1}{\Delta E}$$

$$\nu_{\rm tv}(E) \sim \ell\sqrt{\frac{m}{E}}$$

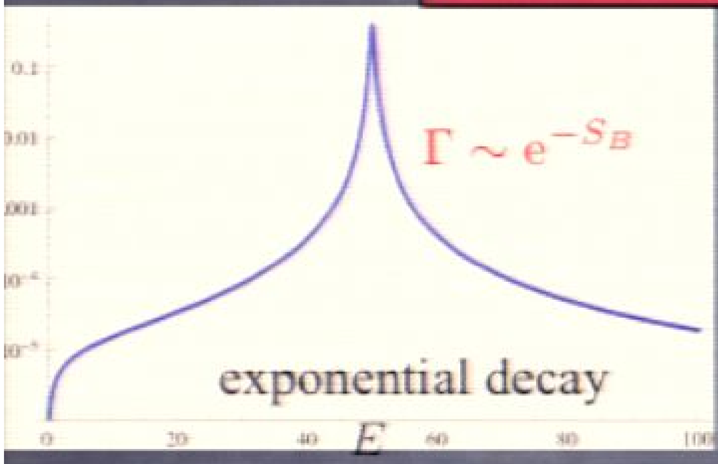
$$\nu_{\rm fv}(E) \sim 1/\omega$$

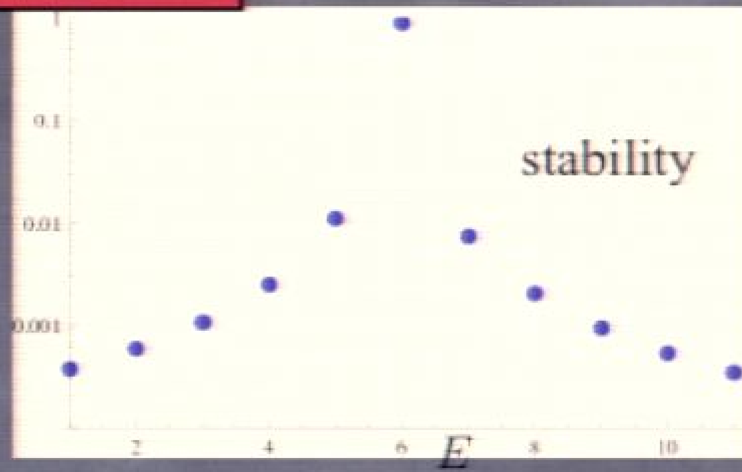
$$\delta E_{\rm resonance} \sim {\rm e}^{-S_B/2}$$

tability for $\frac{\nu_{\rm tv}(E_{\rm fv})}{\nu_{\rm fv}(E_{\rm fv})} \lesssim {\rm e}^{S_B/2} \quad \Longrightarrow \quad \ell \lesssim {\rm e}^{S_B/2}\sqrt{\frac{E_{\rm fv}}{\omega^2 m}}$

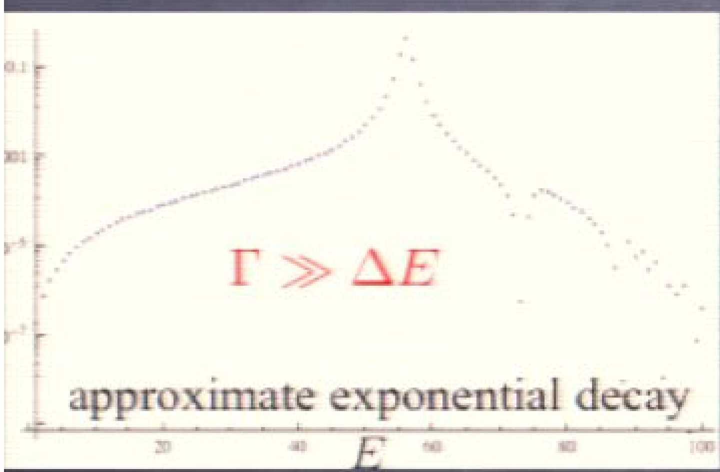
At larger ℓ, messy crossover behavior, not exponential decay.

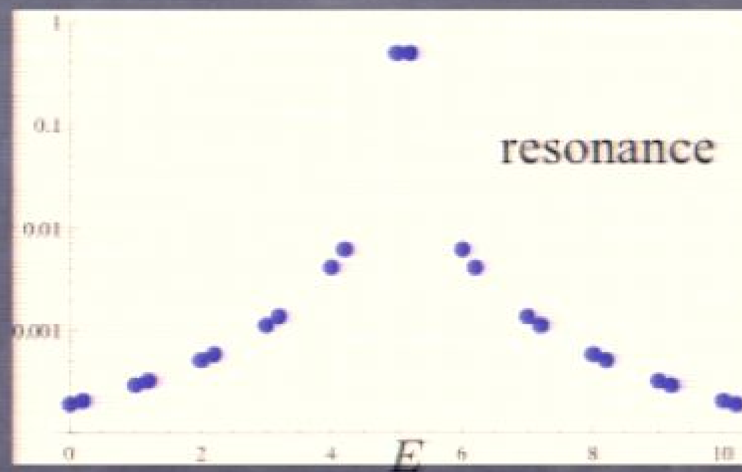
$|\langle E|\Psi_0(t)\rangle|^2 \leftarrow$ time independent





density of states set by volume





peak width Γ set by e^{-S_B}

finite volume QM

$\nu(E) \sim E^{d-1}\ell^d$

finite volume QFT

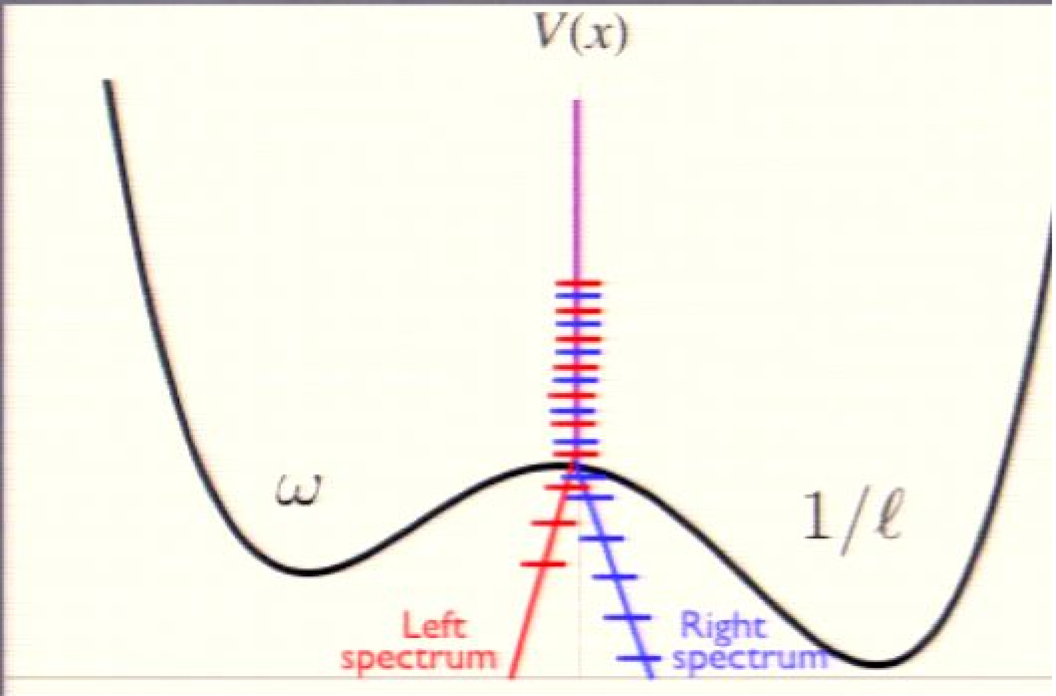
$\nu(E) \sim \exp[(E\ell)^{\frac{d-1}{d}}]$

static patch of de Sitter space QFT

$\nu(E) = \infty$

ne false vacuum cannot decay because there are no excited
true vacuum states with overlapping energy. This is found
experimentally in cavity QED.

The true vacuum volume ℓ must be *very* large for instability to be generic.



$$\nu(E) := \frac{1}{\Delta E}$$

$$\nu_{\rm tv}(E) \sim \ell\sqrt{\frac{m}{E}}$$

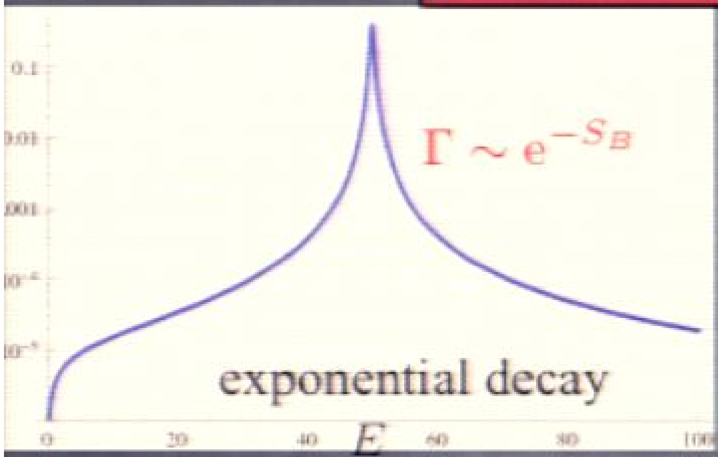
$$\nu_{\rm fv}(E) \sim 1/\omega$$

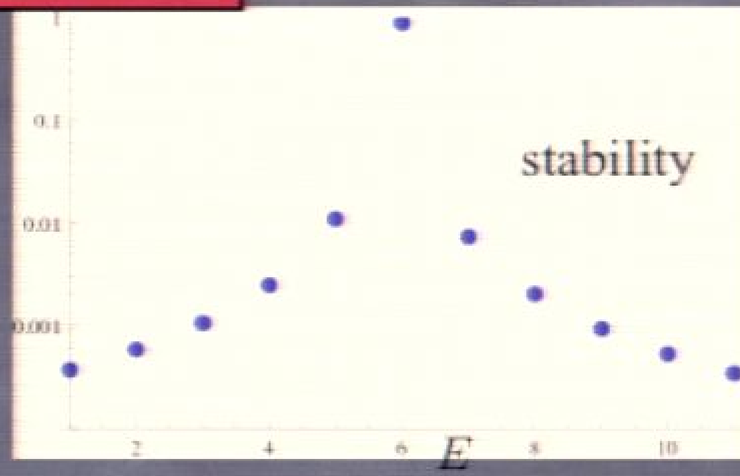
$$\delta E_{\rm resonance} \sim {\rm e}^{-S_B/2}$$

tability for $\frac{\nu_{\rm tv}(E_{\rm fv})}{\nu_{\rm fv}(E_{\rm fv})} \lesssim {\rm e}^{S_B/2} \implies \ell \lesssim {\rm e}^{S_B/2}\sqrt{\frac{E_{\rm fv}}{\omega^2 m}}$

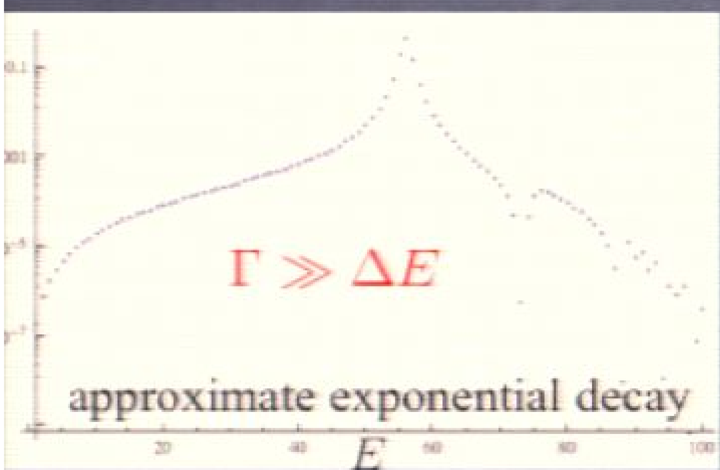
At larger ℓ, messy crossover behavior, not exponential decay.

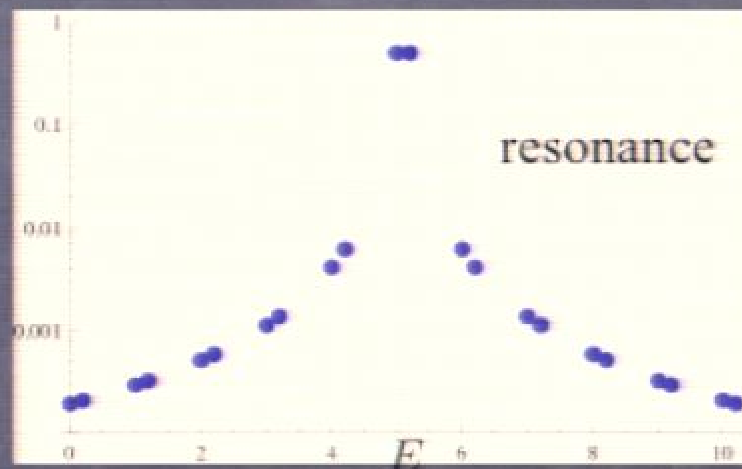
$$|\langle E|\Psi_0(t)\rangle|^2$$ ← time independent





density of states set by volume





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static patch of de Sitter space QFT

$$\nu(E) = \infty$$

Examples of false vacuum decay

Quantum Mechanics: Unstable state evolves into a spherical wave of decay products.

1+1 d QFT: Unstable electric field nucleates a quark - antiquark pair with less electric field between them. This pair expands and eventually collides with other similarly nucleated particles.

3+1 d QFT: Unstable vacuum nucleates a bubble containing true vacuum. This bubble expands and eventually collides with other similarly nucleated bubbles.

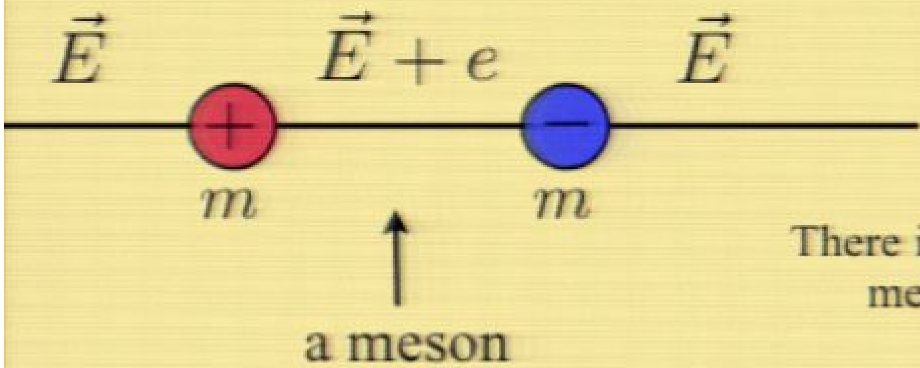
The Schwinger model

(Massive, Bosonic)

vacuum electric field $= \frac{\theta e}{2\pi}$.

$$S = -\int \left(\tfrac{1}{4}F^2 + \tfrac{1}{2}\overline{D_\mu \Phi} D^\mu \Phi + \tfrac{m^2}{2}\bar{\Phi}\Phi\right) d^2x + \tfrac{e}{2\pi}\theta \int F$$

$\vec{E} = *F$ (a scalar)



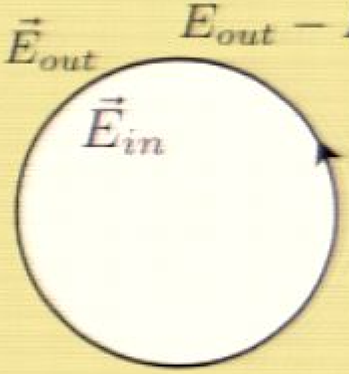
There is no photon: low energy excited mesons are stable if $\left|\vec{E}\right| < e/2$.

Confinement: All quarks undergo piecewise uniform acceleration

Pair production (vacuum instability)

$$S_E = \oint_{\partial\Sigma} m - \int_{\Sigma} \epsilon$$

Exponential decay of the false vacuum is calculated precisely as it was in quantum mechanics:



$\vec{E}_{out} - \vec{E}_{in} = \pm e$

$$\mathbb{R}^{1,1} \to \mathbb{R}^2$$

$$\epsilon = \frac{1}{2}\left(\vec{E}^2_{out} - \vec{E}^2_{in}\right)$$

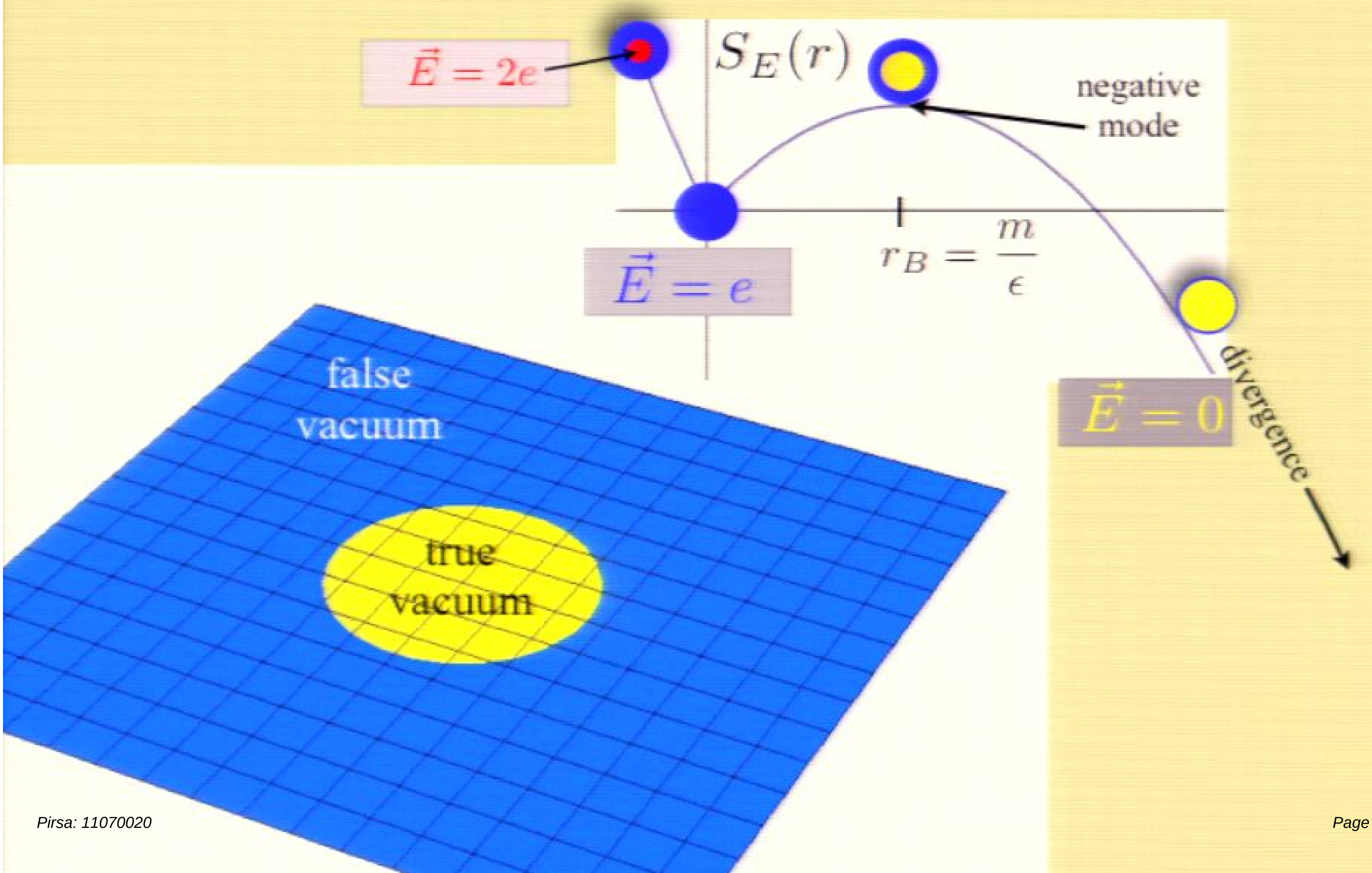
"The Bounce" = S^1 domain wall in $\mathbb{R}^2$

2 zero modes (translations in $\mathbb{R}^2$)

Single negative mode corresponds to dilatations of the bubble

Euclidean action unbounded below

$$S_E = 2\pi r m - \pi r^2 \epsilon$$



See Appendix of Leblond, B.S., Siemens

Nucleation rate per unit length

$$\left\langle \vec{E} \left| e^{-\mathcal{H}XT} \right| \vec{E} \right\rangle = \int [d\Sigma] e^{-S_E[\Sigma]}$$

$$S_E = m \oint_{\partial\Sigma} ds + \frac{\vec{E}^2}{2} \int_\Sigma d^2x$$

$$\vec{E}_{in} - \vec{E}_{out} = \pm e$$

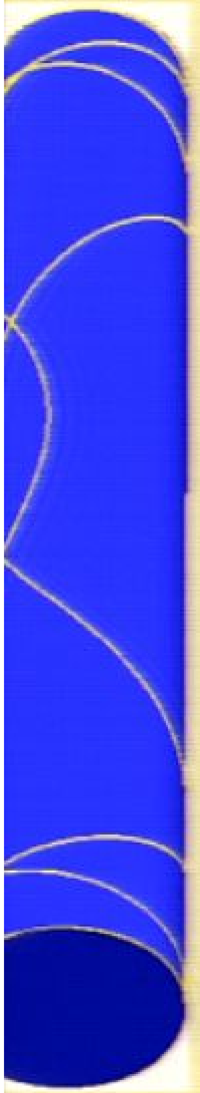
$$\langle \mathcal{H} \rangle = \frac{1}{2}\vec{E}^2 - i\frac{e\vec{E}}{4\pi} \exp\left(-\pi m^2/e\vec{E}\right)$$

decay rate per unit volume

$$\Gamma = \frac{1}{2} \det{}' \left[\left| \frac{\delta^2 S_E}{\delta \partial \Sigma^2} \right| \right]^{-\frac{1}{2}} e^{-S_B} = \frac{e\vec{E}}{2\pi} \exp\left(-\pi m^2/e\vec{E}\right)$$

Sagredo's missing factor

What happens if we compactify the Schwinger model on a circle?



As we dilatate the instanton on the cylinder, it will overlap itself.

In the probe approximation, this is irrelevant:

$$S_E = 2\pi r m - \pi r^2 \epsilon$$

$$\Rightarrow \Gamma > 0$$

The negative mode is still "dangerous"

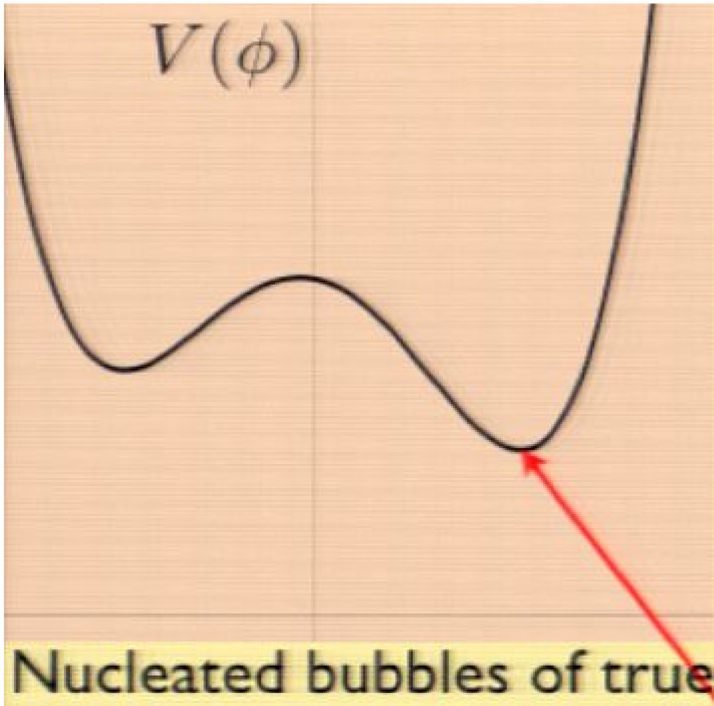
This can be confirmed by the method of Bogolyubov coefficients.

But if we include the back-reaction on the electric field, the action is bounded below.

The negative mode is actually "benign"

$$\Rightarrow \Gamma = 0$$

This can be confirmed by explicit computation of the discrete spectrum when $\Gamma\ell \ll 1/\ell$



In a typical QFT, the density of states grows exponentially with the volume. One should thus expect a rapid transition from stable false vacua to meta-stable false vacua as the volume is increased.

Nucleated bubbles of true-vacuum will expand and (self) collide, exciting the scalar quanta of the true vacuum.

/ithout bubble collisions, the accessible density of states is made up of the bubble wall phase space. This does not depend exponentially on volume.

3+1 d QFT in flat space

$$S_E = \int_{\mathbb{R}^4} \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) d^4x$$

Integrate out the UV:
nly domain walls remain

$$S_E \;=\; \mu \int_{S^3} d^3x \;-\; \epsilon \int_{B^4} d^4x$$

a.k.a. Brown - Teitelboim

xponential decay of the false vacuum is calculated precisely as it was in the Schwinger model:

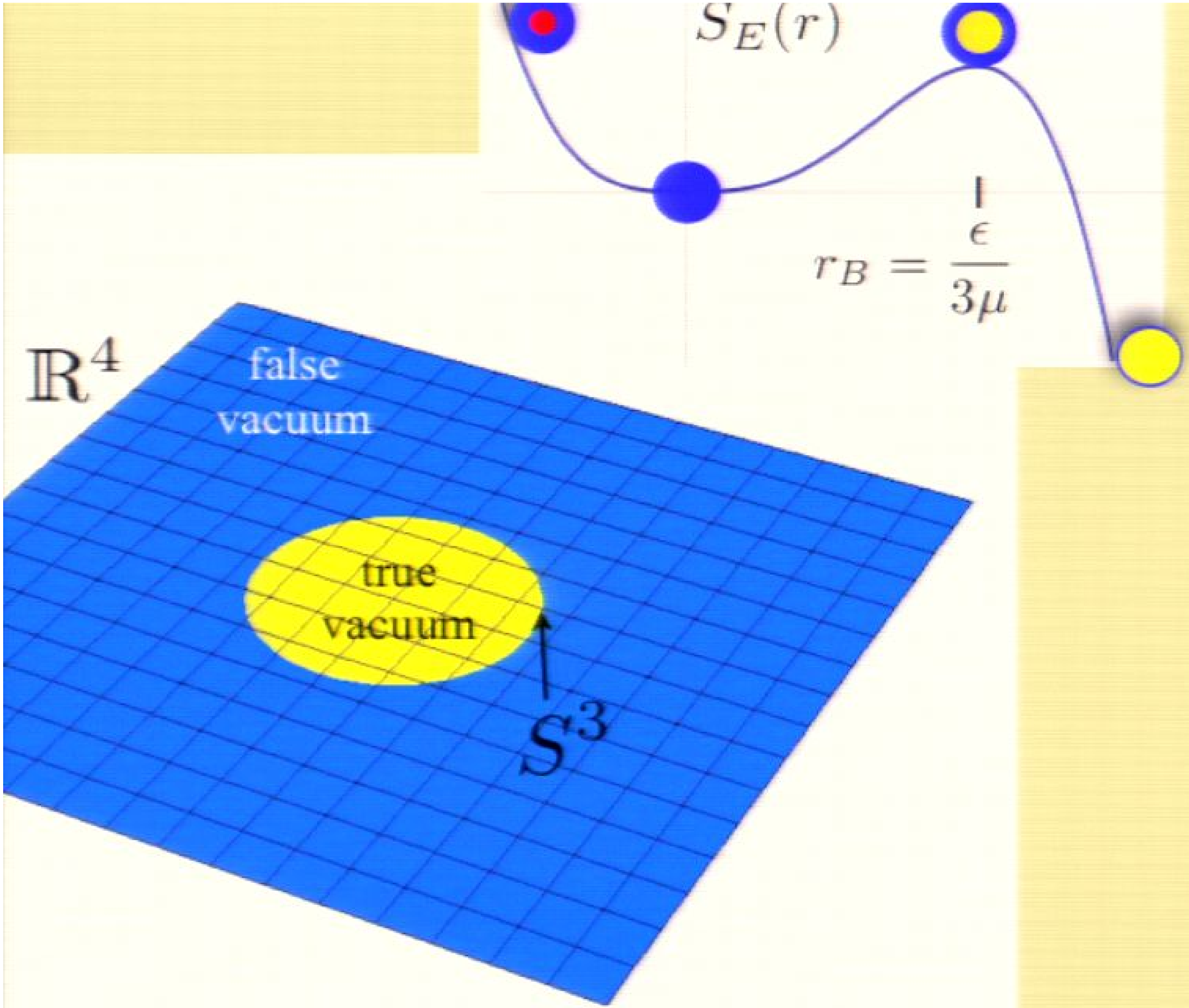
$$\mathbb{R}^{1,3} \longrightarrow \mathbb{R}^4 \qquad ds^2 \to d\tau^2 + dx^2 + dy^2 + dz^2$$

"The Bounce" = S^3 domain wall in $\mathbb{R}^4$

4 zero modes (translations in $\mathbb{R}^4$)

Negative mode corresponds to dilatations of the bubble
It is "dangerous."

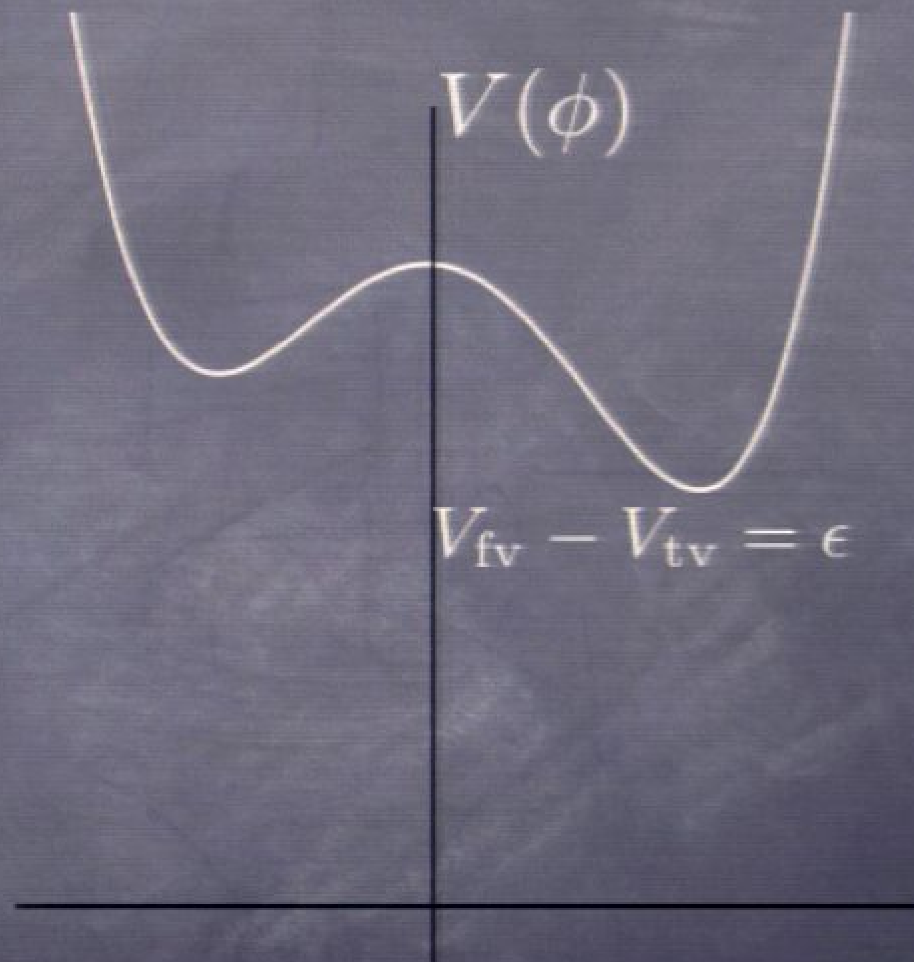
$$\Gamma \sim e^{-S_B} \qquad S_B = \frac{27\pi^2\mu^4}{2\epsilon^3}$$



$S_E(r)$
$r_B = \frac{\epsilon}{3\mu}$
$\mathbb{R}^4$
false
vacuum
true
vacuum
S^3

QFT in de Sitter space

Consider a scalar field with potential $V(\phi)$



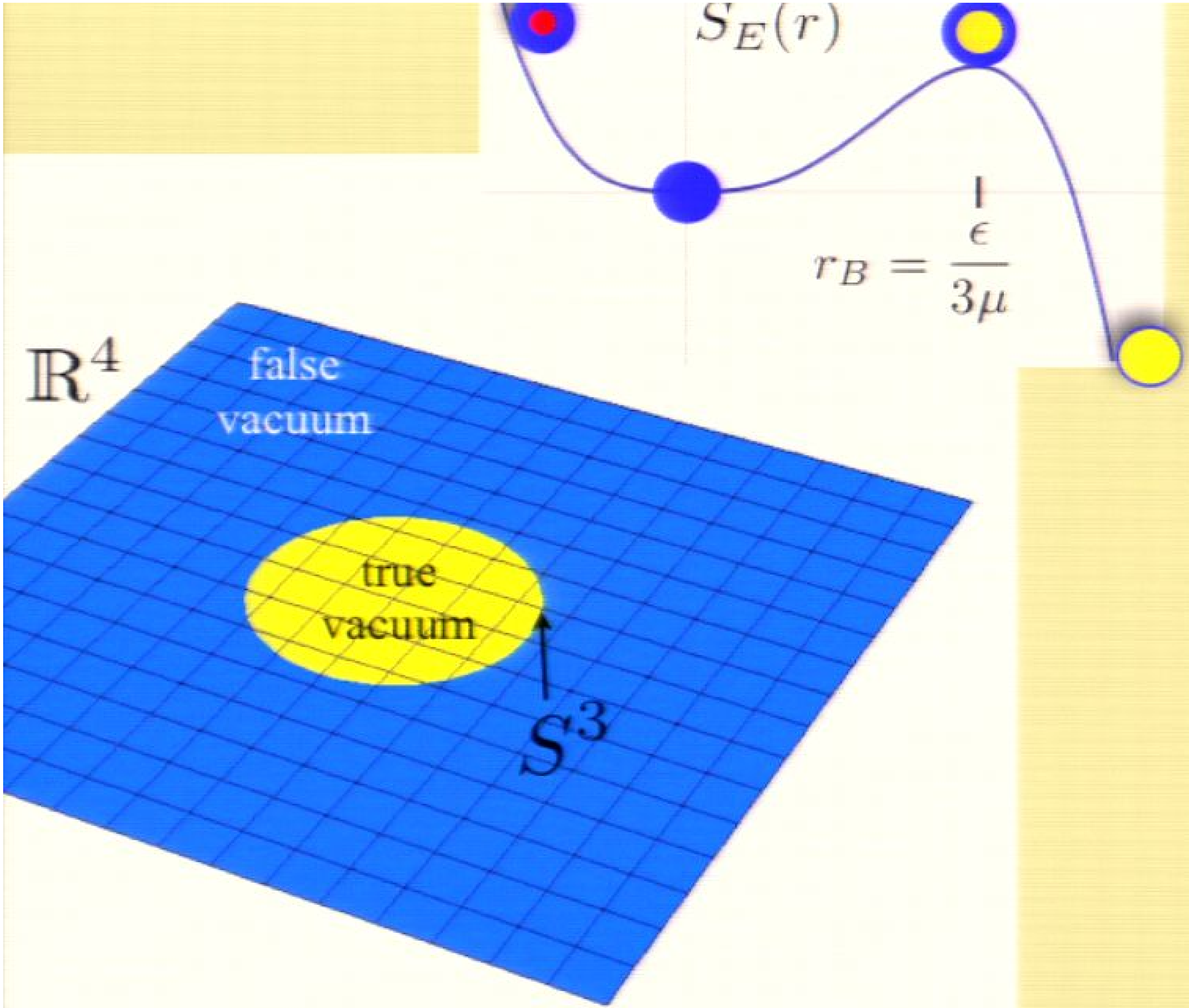
de Sitter radius $\ell \sim \sqrt{\frac{M_{\rm P}^{d-2}}{V}}$

$S_{\rm GH} \sim (M_{\rm P}\ell)^{d-2}$

semi-classical limit ϵ, ℓ fixed

$V_{\rm tv} \to \infty$
$M_{\rm P} \to \infty$

$\Delta S_{\rm GH} \to \epsilon \ell^d$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false vacuum
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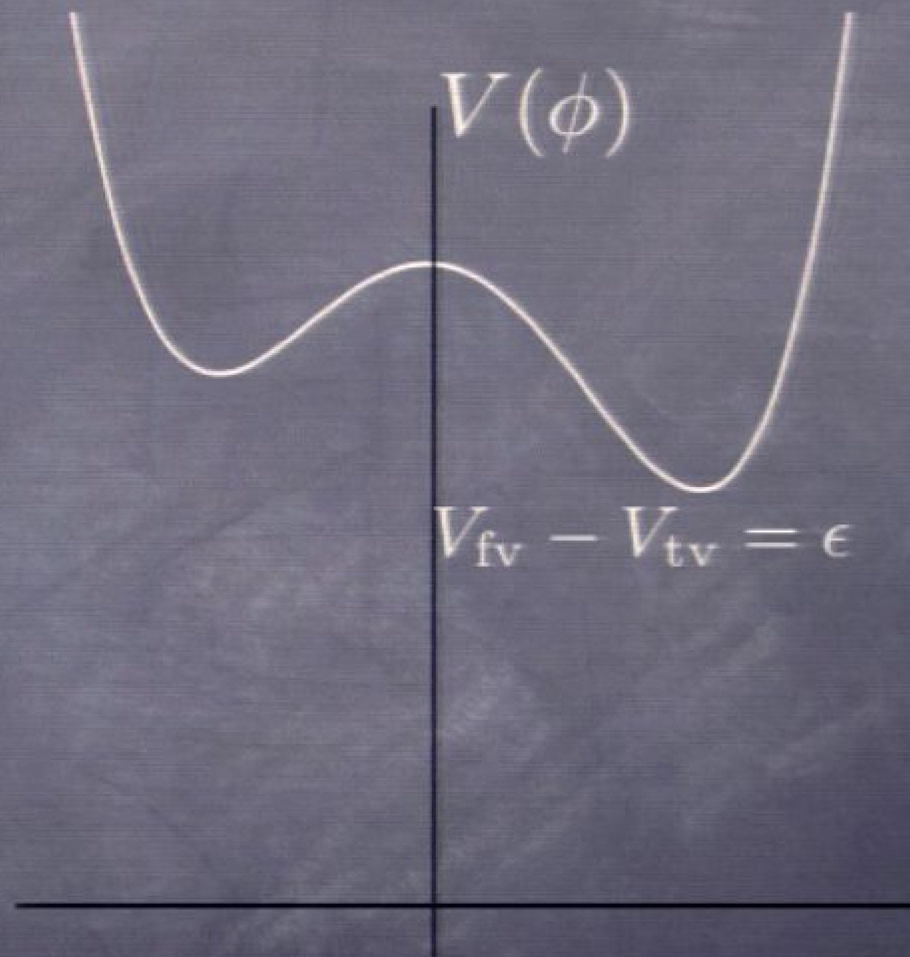

de Sitter radius $\ell \sim \sqrt{\dfrac{M_{\rm P}^{d-2}}{V}}$

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semi-classical limit ϵ, ℓ fixed

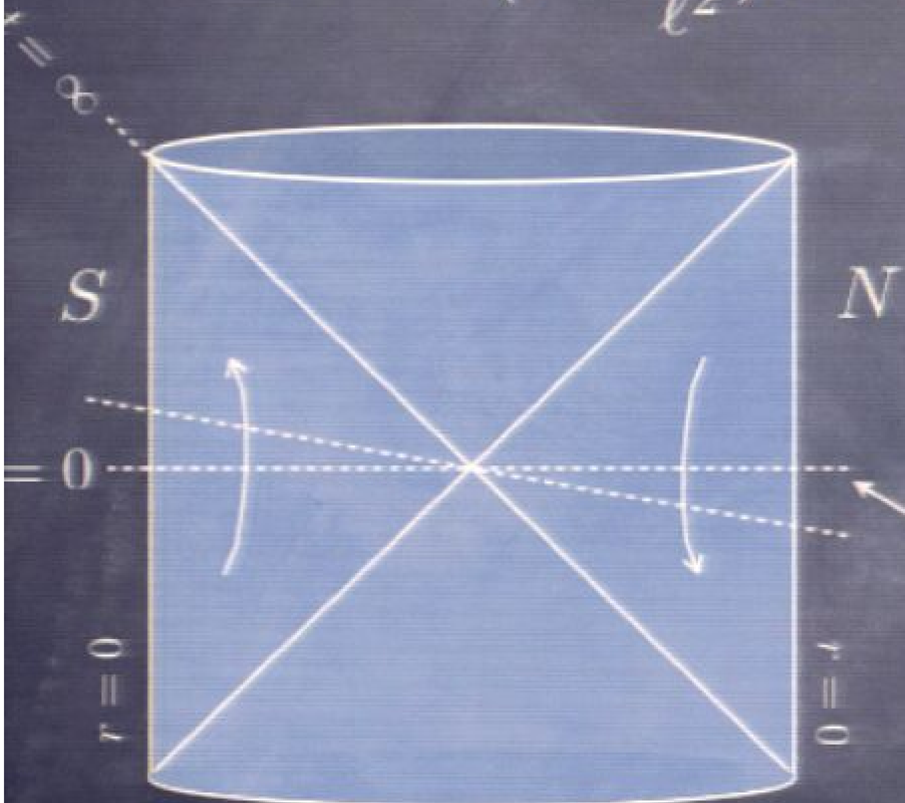
$V_{\rm tv} \to \infty$
$M_{\rm P} \to \infty$

$\Delta S_{\rm GH} \to \epsilon \ell^d$

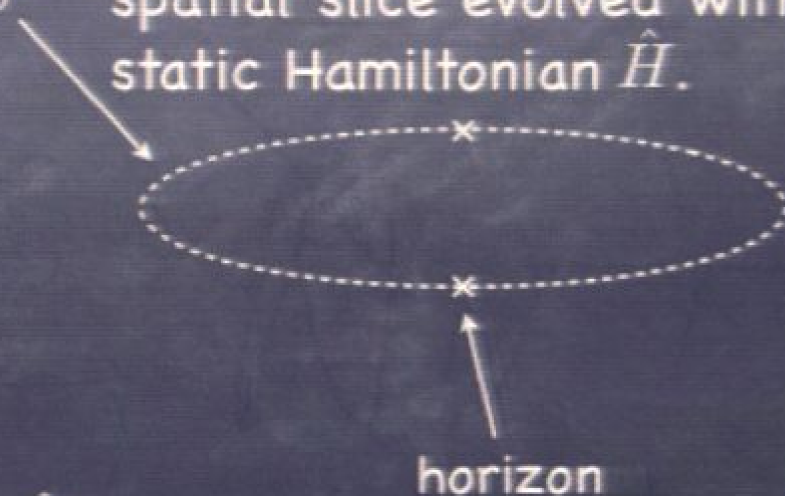
de Sitter space

$$ds^2 = -(1 - \frac{r^2}{\ell^2})dt^2 + (1 - \frac{r^2}{\ell^2})^{-1}dr^2 + r^2 d\Omega^2_{d-2}$$

S^{d-2} at every point (except r=0)



S^{d-1} spatial slice evolved with static Hamiltonian $\hat{H}$.



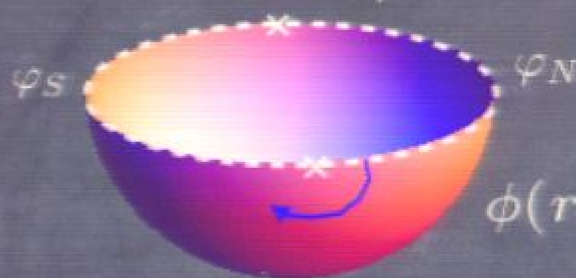
$$i\frac{\partial}{\partial t}\Psi[t,\varphi(r)] = \hat{H}_{\text{full}}\Psi[t,\varphi(r)]$$

$$\hat{H}_{\text{full}} = \hat{H}_S - \hat{H}_N$$

The Hartle-Hawking state $\quad t \to -it$

$$\varphi_S\varphi_N|\mathrm{HH}\rangle = \int_{S^d/2} [d\phi]\mathrm{e}^{-S_E[\phi]} = \langle\varphi_S|\mathrm{e}^{-\pi\ell\hat{H}}|\varphi_N$$

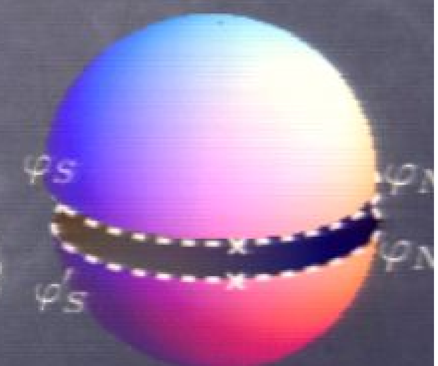
$$\hat{H} = \hat{H}_S = \hat{H}_N$$



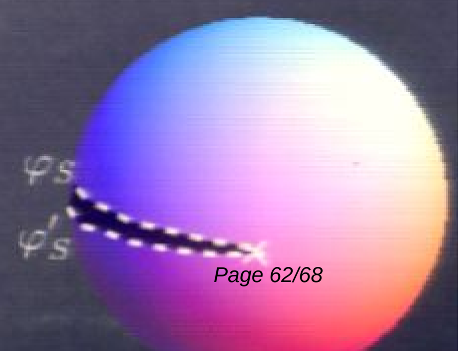
$$\phi(r,t)\Big|_{t=0} = \begin{cases} \varphi_S(r) & r \in S \\ \varphi_N(r) & r \in N \end{cases}$$

$$\rho_S = \mathrm{Tr}_N\, |\mathrm{HH}\rangle\langle\mathrm{HH}|$$

$$\varphi'_S|\rho_S|\varphi_S\rangle = \int_{S^{d-1}/2} [d\varphi_N]\langle\varphi'_S\varphi_N|\mathrm{HH}\rangle\langle\mathrm{HH}|\varphi_S\varphi_N\rangle$$



$$= \int_{S^d} [d\phi]\mathrm{e}^{-S_E[\phi]} = \langle\varphi'_S|\mathrm{e}^{-2\pi\ell\hat{H}}|\varphi_S\rangle$$



$$1/T = 2\pi\ell$$

"nothing happens" in the exact vacuum state

$$\rho_S = \frac{1}{Z}\int_0^\infty dE\, \mathrm{e}^{-2\pi\ell E}|E\rangle\langle E|$$

$$Z = \int_0^\infty dE\, \nu(E)\mathrm{e}^{-2\pi\ell E}$$

$$|\mathrm{HH}\rangle \sim |\mathrm{tv}\rangle \;+\; |\mathrm{fv}\rangle \;+\; |\mathrm{np}\rangle$$

$$\mathcal{O}(1) \qquad \mathcal{O}(\mathrm{e}^{-S_{\mathrm{fv}}/2}) \qquad \mathcal{O}(\mathrm{e}^{-S_{\mathrm{CDL}}/2})$$

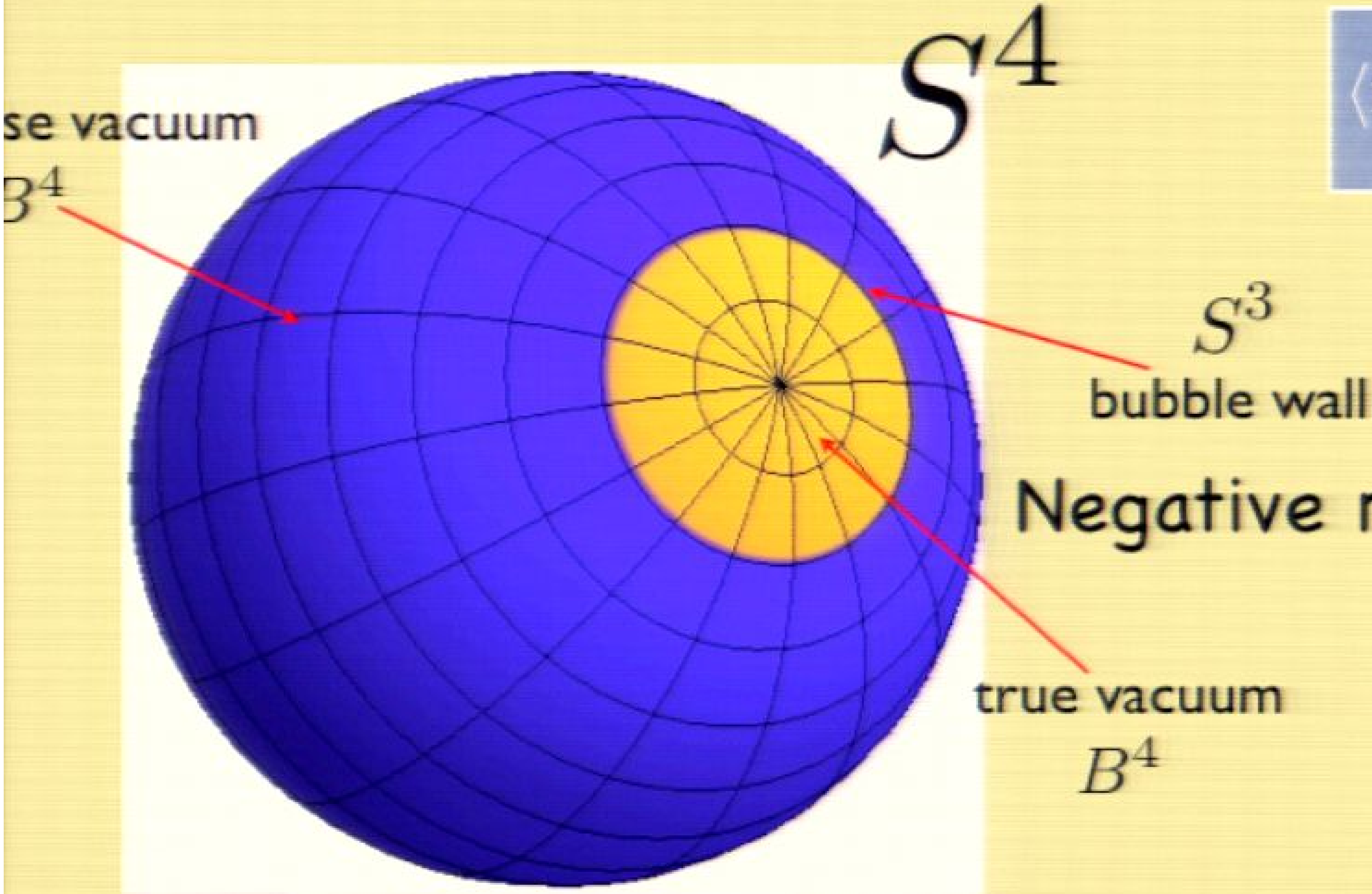
$$S_{\mathrm{fv}} = \beta E_{\mathrm{fv}} \sim \ell^d \epsilon \qquad S_{\mathrm{CDL}} \sim S_{\mathrm{fv}} + \frac{S_1^d}{\epsilon^{d-1} + S_1^{d-1}/\ell^{d-1}}$$

$$S_{\mathrm{fv}} \approx \Delta S_{\mathrm{GH}}$$

Notice this is also the difference in Gibbons-Hawking entropy

what about the perturbative false vacuum $|\mathrm{fv}\rangle$?

QFT in de Sitter space

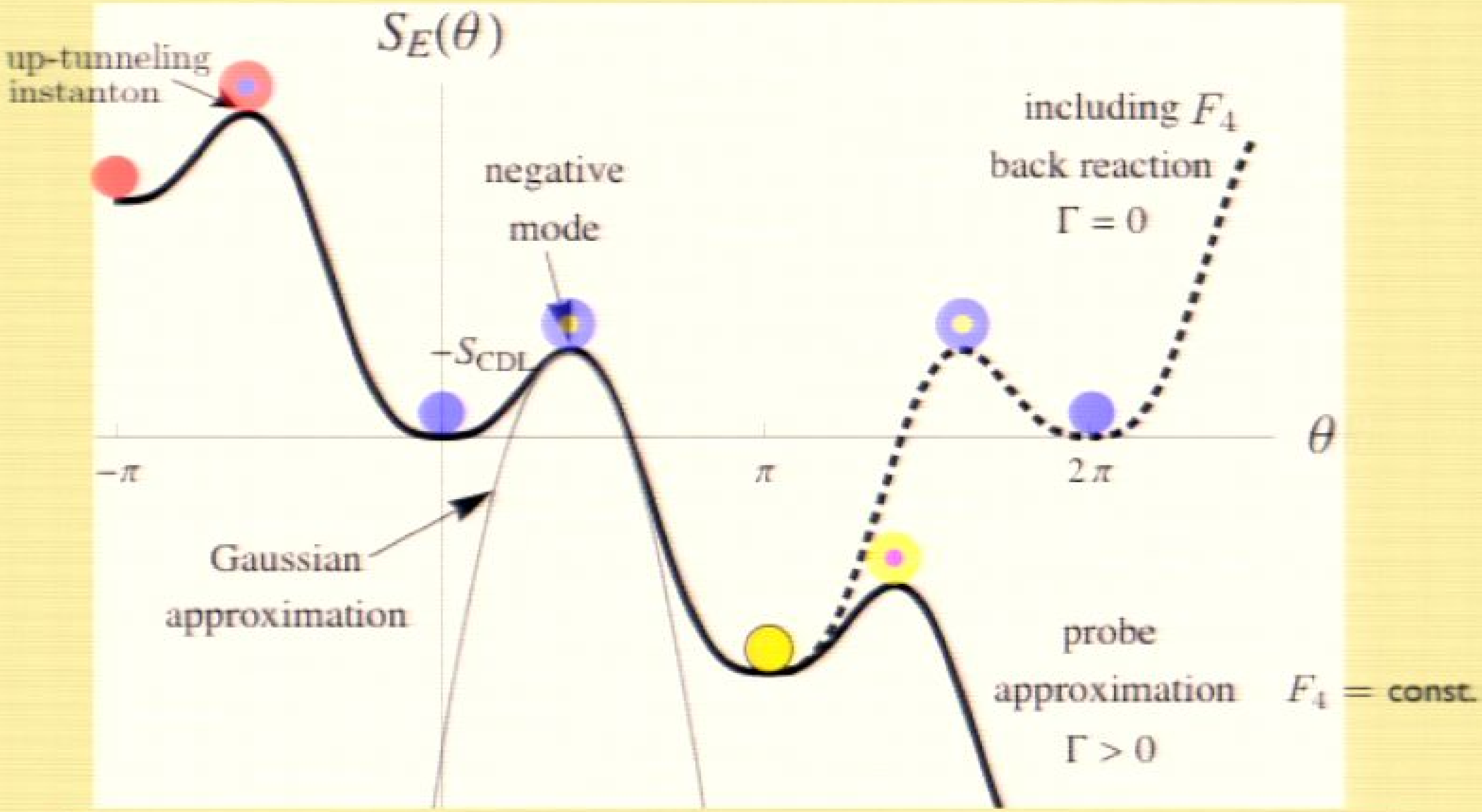


$\langle \text{fv}|e^{-i\hat{H}t}|\text{fv}\rangle$

Negative mode is benign

Quantum correction to false vacuum energy density is small and real.

Coleman - De Luccia action as a function of Euclidean bubble radius



The Coleman - De Luccia negative mode is *benign* in de Sitter space.

The (static patch) false vacuum energy density does not receive any imaginary part from quantum effects.

density of states in the de Sitter static patch

ecall from before:

finite volume QM	finite volume QFT	de Sitter space QFT
$\nu(E) \sim E^{d-1}\ell^d$	$\nu(E) \sim \exp[(E\ell)^{\frac{d-1}{d}}]$	$\nu(E) = \infty$

regulate this with physical cutoff $E_{\rm loc} < M_{\rm P}$

number of modes of energy < E: $g(E) \sim E\ell^{d-1}M_{\rm P}^{d-2}$

$$g(1/\beta) \sim S_{\rm GH}$$

$$\nu(1/\beta) \sim {\rm e}^{S_{\rm GH}}$$

$$\nu(E) = \delta(E - \hat{H})$$

$$\nu(E)_{\rm eff} = \sum_s \delta(E - E_s)$$

The accessible density of states:

modulo conserved charges

see e.g. Randall - Schwartz - Sanz

Rotational invariance of the false vacuum: $d_{\text{eff}} = 2$

$$S_{\text{GH}} \sim \ell^{d-2} M_{\text{P}}^{d-2} \qquad d > 2$$

$$S_{\text{GH}} \sim \log(\ell M_{\text{P}}) \qquad d = 2$$

$$\nu(E)_{\text{eff}} = \sum_s \delta(E - E_s)$$

QFT in d-dimensional static patch behaves like infinite volume, but of dimension d-2.

$$g(E)_{\text{eff}} = \log(2M_{\text{P}}/E)$$

$$\Delta S_{\text{eff}} \sim \log(\ell)$$

Hence

$$\frac{\nu_{\text{tv}}(E_{\text{fv}})}{\nu_{\text{fv}}(E_{\text{fv}})} \sim \ell^{d-1}$$

the density of **accessible** states is power-law, like QM in a box!

$\Longrightarrow$ Stable false vacua when $\ell \lesssim \mathrm{e}^{S_B/2}$

At larger de Sitter radius, messy crossover behavior, not bubble nucleation.

Conclusions

- A negative mode is either:
  - dangerous -> there is an out of equilibrium state
  - benign -> this instanton is a subdominant configuration
- Both the Euclidean- and density-of-states-pictures agree: quantum corrections to classical vacua are zero-dimensional effects on scales >> ℓ (for a way out, see last week's talk by O'connor)
- There are no first-order phase transitions in de Sitter space.