

Title: Infrared Challenges for Inflation

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Abstract: I will review some recent work on infrared issues for scalar fields in exact and quasi de Sitter space. Renewed interest in this topic has been driven by the observational potential for a more accurate determination of statistics of the primordial curvature perturbations, especially non-Gaussianity. Interestingly, the resulting questions are not only relevant for mapping inflationary models to observation but also link directly to more fundamental questions about the initial state, eternal inflation, and the long time dynamics of interacting quantum fields in curved space. Infrared questions provide a precisely calculable way to put pressure on inflation as a rigorous and consistent framework, ready to confront future observations.

INFRARED CHALLENGES FOR INFLATION

SARAH SHANDERA
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WITH C.P. BURGESS, R. HOLMAN, L. LEBLOND (0912.1608; 1005.3551)
WITH T. GALVEZ GHERSI, G. GHESHNIZJANI, F. PIAZZA (1103.0783)

REMINDER: DE SITTER IR

Massive field: $\delta = M^2/3H^2$

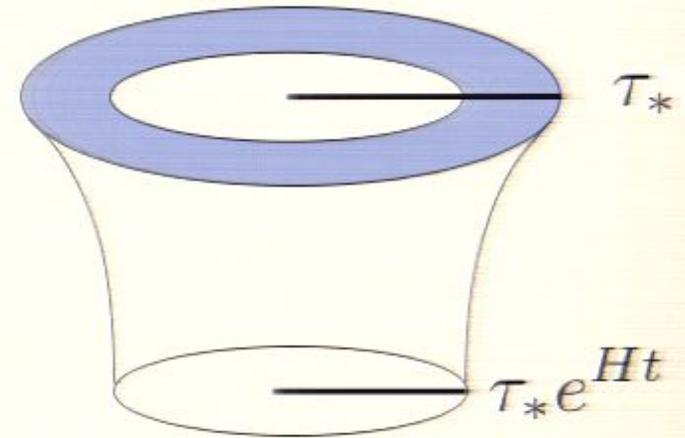
$$\langle \phi(x)^2 \rangle =$$

Massless field:

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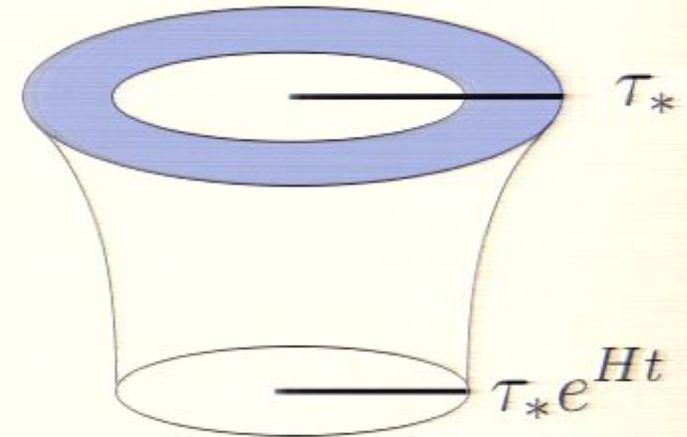
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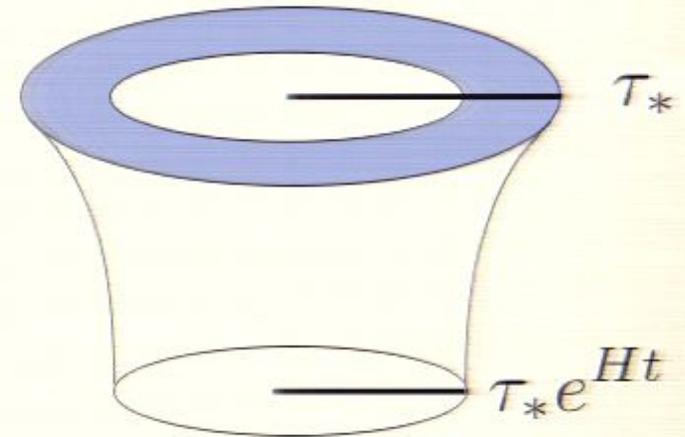
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Massless field:

$$\langle \phi(x)^2 \rangle = \frac{H^3 \Delta t}{4\pi^2} \rightarrow \infty$$



INFRA-RED? AGAIN?

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DATA!

* non-Gaussianity (Planck, Large Scale Structure surveys)

* Interactions: $\langle \zeta^n \rangle \quad n > 2$ (ζ curvature)

- Gravitational
- Everything Else

* Expect patterns

INFRA-RED? AGAIN?

DATA!

* non-Gaussianity (Planck, Large Scale Structure surveys)

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- Everything Else

* Expect patterns

* Matching a scenario \rightarrow Testing the Idea

(MODERN) NATURAL INFLATION

$$\mathcal{L}_0 = -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{4f}\varphi G^a \tilde{G}^a - \frac{\alpha}{4f}\varphi F \tilde{F} + \sum_{n=1}^{\infty} c_n \frac{(\partial\varphi)^{2n+2}}{f^{4n}} + \dots$$
$$+ V_{\text{ex}}(\varphi) + \mu^4 \left[1 - b \cos\left(\frac{\varphi}{f}\right) \right]$$

* Opportunity! Characteristic bispectra;
patterns among moments

* Computational Challenge: Interacting QFT
(Contrast T. Banks' talk)

WHAT MATTERS?

(WHAT'S NEW?)

- Matter fields are *not spectators* (semi-classical)
- Matter fields *interact*
- Background is *not exact, eternal dS*
- Initial state may *not* be *Bunch Davies* vacuum
- Match/predict *observables* post inflation

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 *Techniques should be able to handle all this*

WHY **IR** ISSUES ARE **INTERACTION** ISSUES

$$\langle \zeta(k)^2 \rangle \sim \frac{H^2}{k^3} \frac{1}{\epsilon M_p^2}$$

Observer's two-point function:

WHY IR ISSUES ARE INTERACTION ISSUES

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Observer's two-point function:

$$\begin{aligned} \langle \zeta(x)\zeta(y) \rangle &\sim \int_{L_{IR}^{-1}} \frac{dk}{k} \frac{\text{Sin}[k|\vec{x} - \vec{y}|]}{k|\vec{x} - \vec{y}|} e^{-kR} \\ &\sim \text{Log} \left[\frac{|\vec{x} - \vec{y}|}{L_{IR}} \right] \end{aligned}$$

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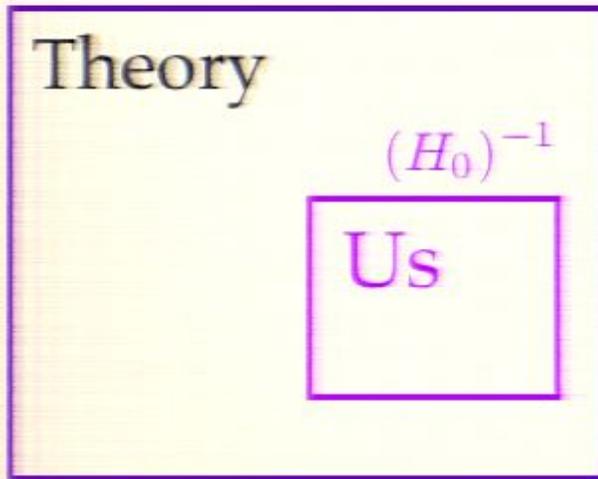
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* No IR scale except horizon size (local background sets the "0")

“CURVATURE IN A BOX”

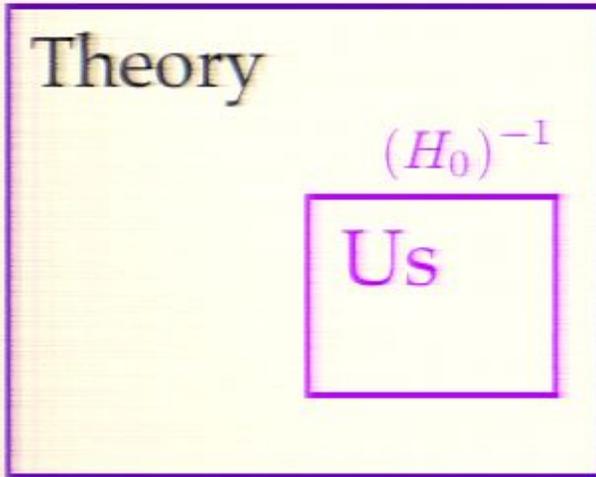
L_{IR}



Lyth; Enqvist et al

“CURVATURE IN A BOX”

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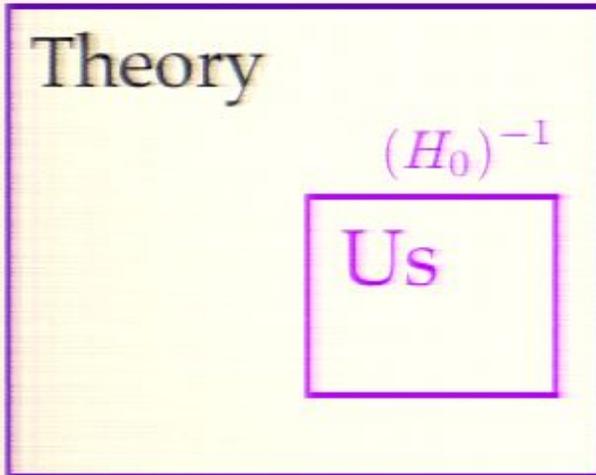


Lyth; Enqvist et al

* Add interactions
(modes are correlated)

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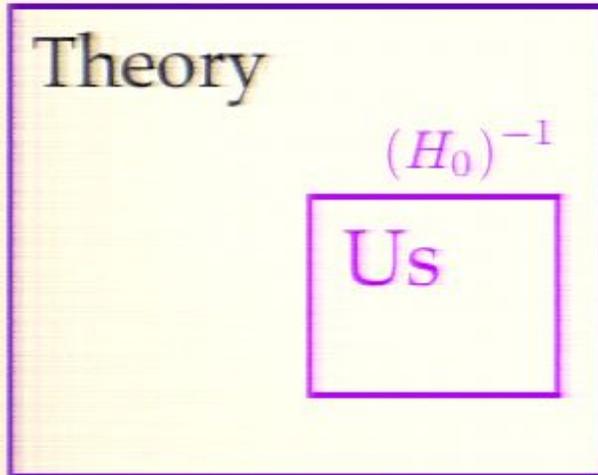
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$$\langle \zeta_k^4 \rangle_c \neq 0$$

$$\langle \zeta_k^2 \rangle_c \sim \frac{H^2}{k^3} \frac{1}{\epsilon M_p} \left[1 + A \int \frac{dk}{k} + \dots \right]$$

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- * Is $\langle \zeta_k^2 \rangle_0$ actually divergent?
- * What regulates the IR?
- * Are obs. sensitive to loops?
- * Are obs. sensitive to regulator?
- * What lessons?

TOY PROBLEM:

Spectator scalar with quartic interaction

$$S = - \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V_0 + \frac{m_0^2}{2} \phi^2 + \frac{\lambda_0}{4!} \phi^4 \right]$$

C.P. Burgess, R. Holman, L. Leblond, S. Shandera
(0912.1608, 1005.3551)

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Other work: Starobinsky, Yokoyama ('94); Riotto, Sloth;
van der Meulen, Smit; Petri; Marolf,
Morrison; Garbrecht, Rigopoulos

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MECHANICS FOR EVALUATING CORRELATORS

$$\langle \mathcal{O}(t) \rangle = \left\langle \text{in} \left| \left[\bar{\mathbb{T}} \exp \left(i \int_{t_{\text{in}}}^t dt' \mathcal{H}(t') \right) \right] \mathcal{O}(t) \left[\mathbb{T} \exp \left(-i \int_{t_{\text{in}}}^t dt' \mathcal{H}(t') \right) \right] \right| \text{in} \right\rangle$$

Double the fields: $\phi \rightarrow \{\phi^+, \phi^-\}$

$$G_c = -\frac{i}{2} (G^{-+} + G^{+-})$$

$$G_R^0(k, \tau_1, \tau_2) = \theta(\tau_1 - \tau_2) (G^{-+} - G^{+-})$$

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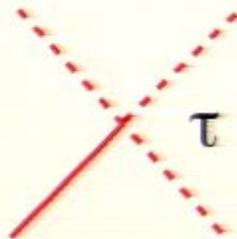
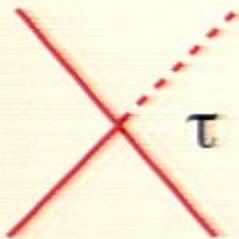
EVALUATING DIAGRAMS

$$k\tau \ll 1$$

$$G_C^0(k, \tau, \tau) \simeq \frac{H^2}{2k^3} (k\tau)^{2\delta}$$

$$G_R^0(k, \tau_1, \tau_2) \simeq \theta(\tau_1 - \tau_2) \frac{H^2}{3} (\tau_1^{3-\delta} \tau_2^\delta - \tau_1^\delta \tau_2^{3-\delta})$$

Interactions



Pirsa: 11070019

$$ia^4(\tau)\lambda$$

$$-ia^4(\tau)\frac{\lambda}{4}$$

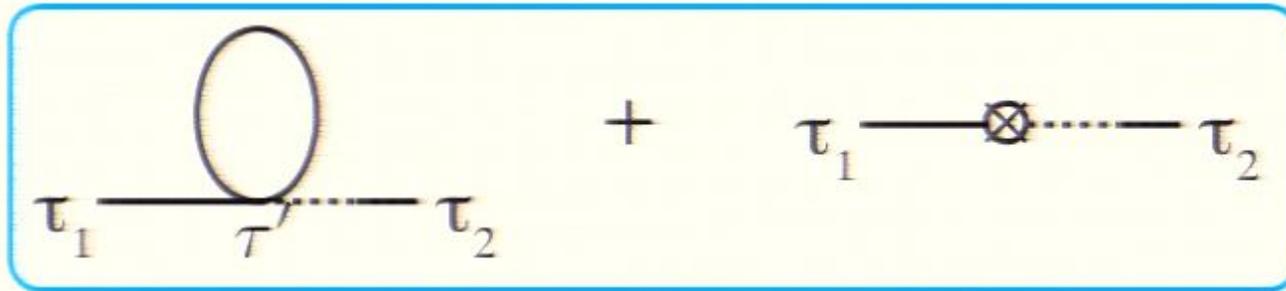
Use:

Flat slicing
Bunch Davies

$$\delta = M^2/3H^2$$

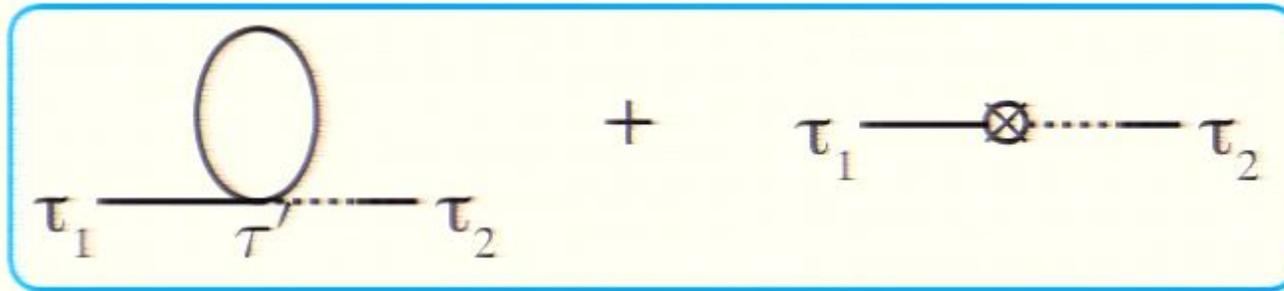
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1-LOOP



* Momentum integral, Time integral

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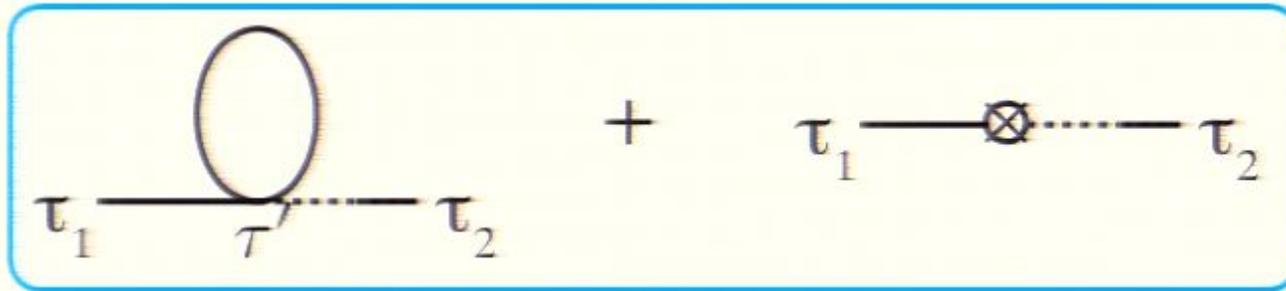


* Momentum integral, Time integral

$$\Lambda(\tau) \simeq \frac{H^2}{(2\pi)^2} \int_{a\Lambda_{IR}}^{a\Lambda_{UV}} \frac{dp}{p} (-p\tau)^{2\delta} + \text{c.t.} \simeq \frac{H^2}{(2\pi)^2} \int_{a\Lambda_{IR}}^{a\mu} \frac{dp}{p} (-p\tau)^{2\delta}$$

$$\simeq \frac{1}{2\delta} \left(\frac{H}{2\pi} \right)^2 \left[\left(\frac{\mu}{H} \right)^{2\delta} - \left(\frac{\Lambda_{IR}}{H} \right)^{2\delta} \right]$$

1-LOOP



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Physical scale; *Comoving* scale would introduce $a(\tau')$

1-LOOP, CONT'D

* Momentum integral, Time integral

$$G_c^1(k, \tau, \tau) = -\lambda \left[\frac{1}{2\delta} \left(\frac{H}{2\pi} \right)^2 \left(\frac{\mu}{H} \right)^{2\delta} \right] \frac{1}{6k^3} \int_{-1/k}^{\tau} \frac{d\tau'}{\tau'} \left[\left(\frac{\tau}{\tau'} \right)^{3-\delta} - \left(\frac{\tau}{\tau'} \right)^{\delta} \right] (k^2 \tau' \tau)^{\delta}$$

1-LOOP, CONT'D

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* So the full answer:

$$G_C(k, \tau) \simeq \frac{H^2}{2k^3} (-k\tau)^{2\delta} \left[1 + \frac{\lambda}{6(2\pi)^2 \delta} \left(\frac{\mu}{H} \right)^{2\delta} \ln(-k\tau) + \dots \right]$$

1-LOOP, CONT'D

* Momentum integral, **Time integral**

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renormalization scale secular growing term

“IR” scale (physical)

SUMMING UP THE LOGS

* Apply “Dynamical Renormalization Group”:
(Boyanovsky, de Vega)

$$G_C(k, \tau) \simeq \frac{H^2}{2k^3} (-k\tau)^{2\delta + \Delta_m}$$

$$\Delta_m = \frac{\lambda}{6(2\pi)^2 \delta} \left(\frac{\mu}{H} \right)^{2\delta}$$

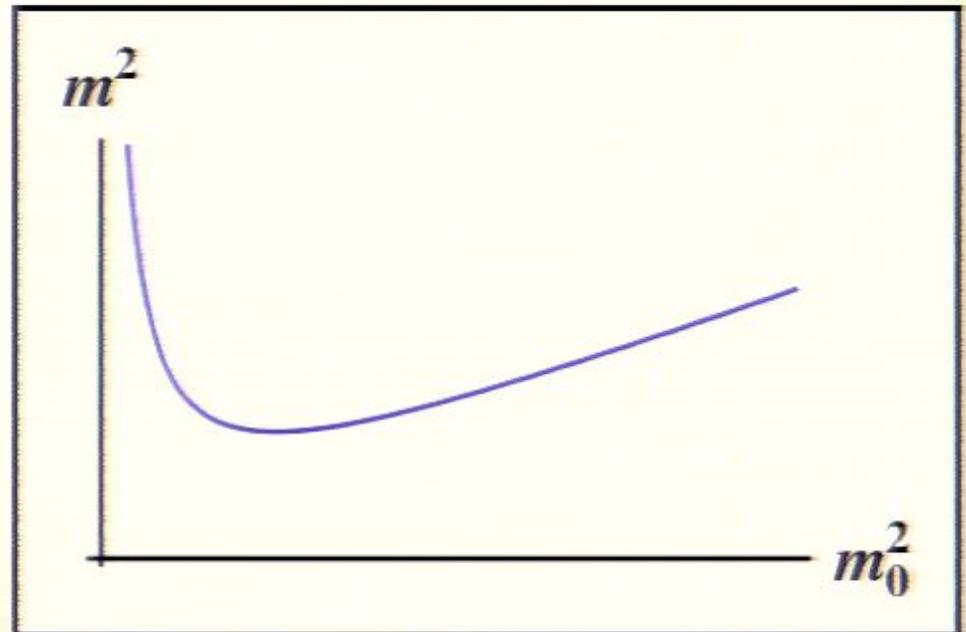
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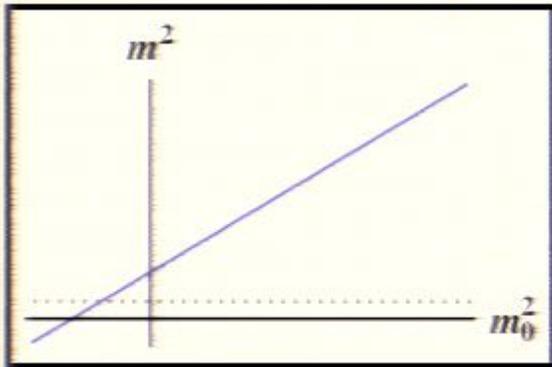
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$$m^2 = m_0^2 + \lambda \frac{H^4}{m_0^2}$$

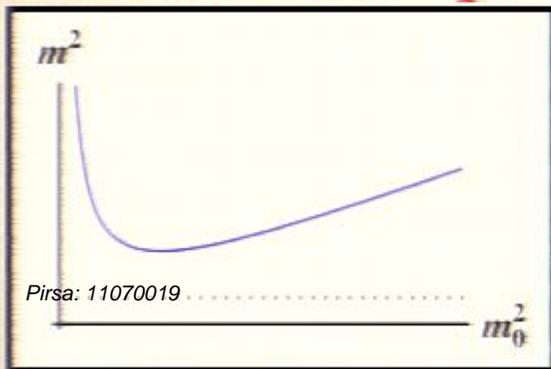


SIMILAR TO THERMAL FT

Thermal:
(Div. at 2 loops)



dS:
(Div. at 1 loop)

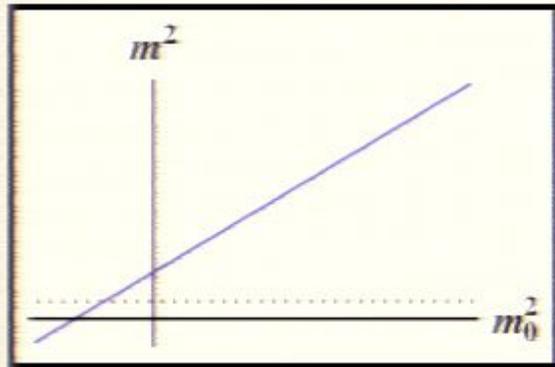


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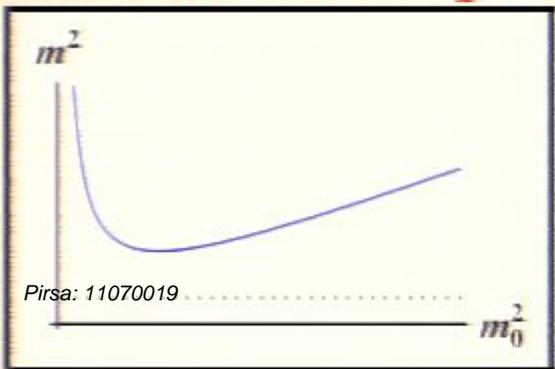
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Loop size	$\mathcal{O}(1)$	$\mathcal{O}(\sqrt{\lambda})$	$\mathcal{O}(\lambda)$
	m_C	m_{dyn}	m_{max}
<u>Thermal</u>	λT	$\sqrt{\lambda} T$	T
<u>de Sitter</u>	$\sqrt{\lambda} H$	$\lambda^{1/4} H$	H

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LESSONS?

1. Similar, not identical, to *stochastic* result. (Why?)

$$m_{\text{dyn}} = A \lambda^{1/4} H$$

* What range of applicability for stochastic (eg, with sound horizon?)

2. How to count relevant modes (*measure?*)

BACK TO A MASSIVE SCALAR:

* How many super-horizon modes matter?

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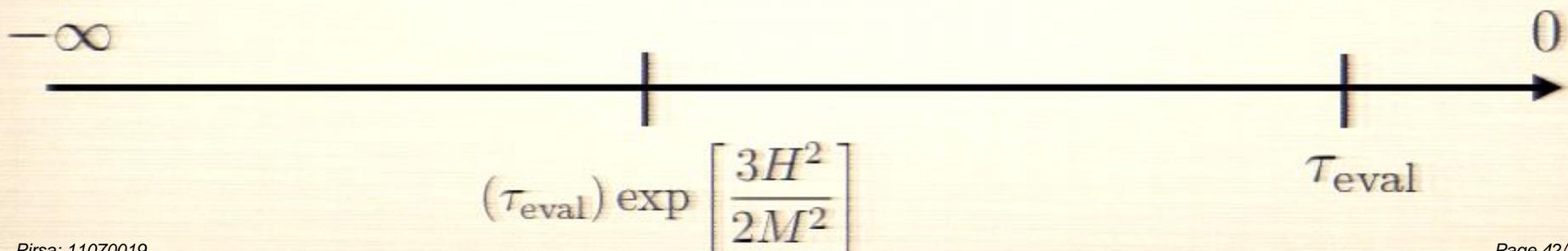
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* How many super-horizon modes matter?

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INFLATIONARY OBSERVABLES

- Use the 'resummed' story to define the background in the Hubble patch...
- what about fluctuations?

$$\langle \zeta_k^2 \rangle = \langle \zeta_k^2 \rangle_0 [1 + \alpha_1 \langle \zeta(x)^2 \rangle_* + \alpha_2 \langle h(x)^2 \rangle_* + \dots]$$

$$\langle h_k^2 \rangle = \langle h_k^2 \rangle_0 [1 + \alpha_3 \langle \zeta(x)^2 \rangle_* + \alpha_4 \langle h(x)^2 \rangle_* + \dots]$$

$$\alpha_1, \alpha_3 \propto n_s - 1$$

$$\alpha_2, \alpha_4 \propto n_t$$

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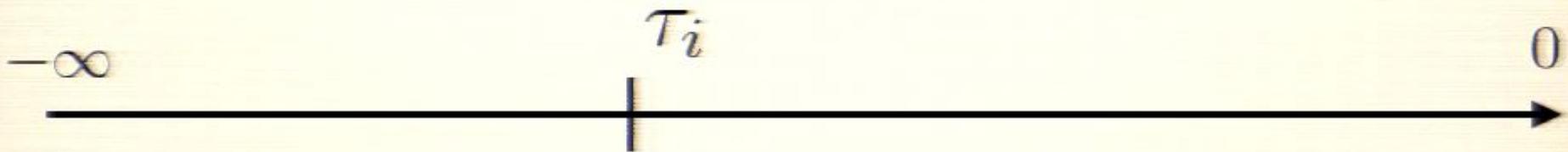
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INFLATIONARY OBSERVABLES, CONT'D

$$\langle \zeta(x)^2 \rangle_* \approx \int_{a_i H}^{a_* H} \frac{dk}{k} \frac{H^2}{4\pi^2} \frac{1}{2\epsilon M_p^2} \sim H^3 t$$

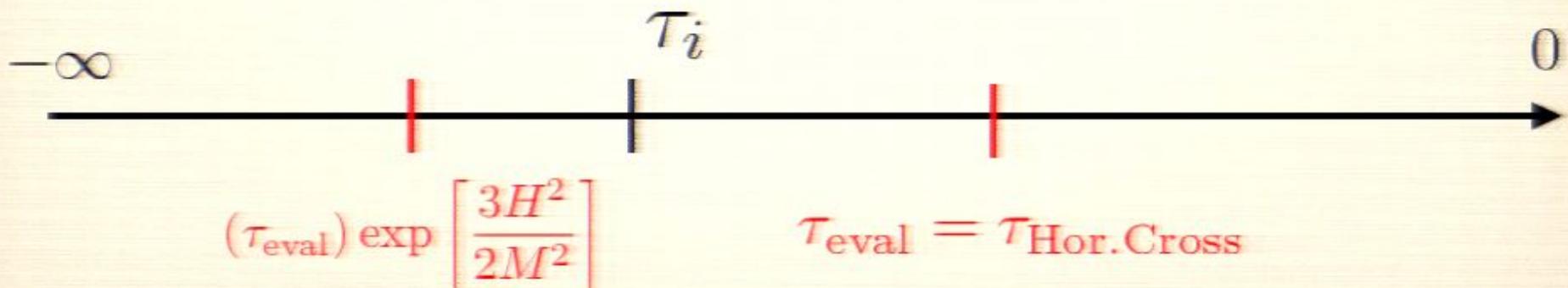
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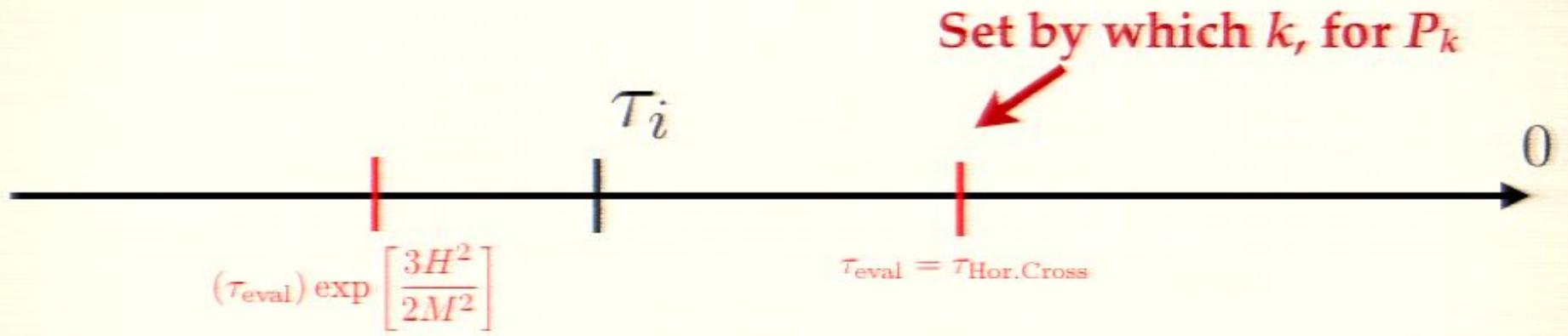


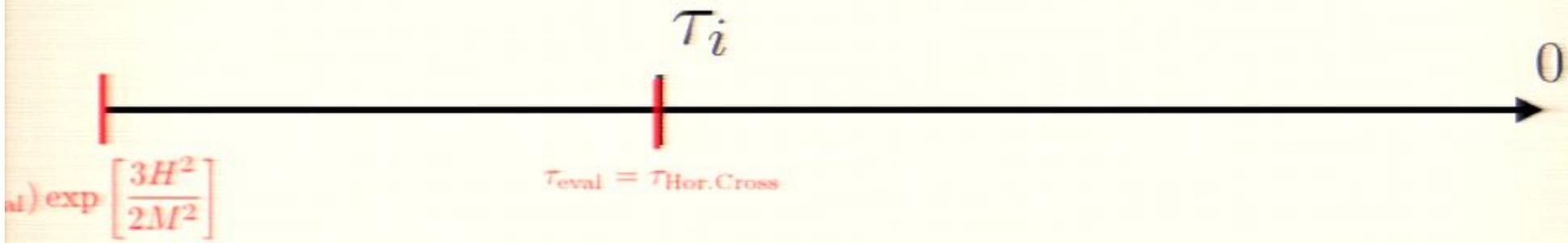
INFLATIONARY OBSERVABLES, CONT'D

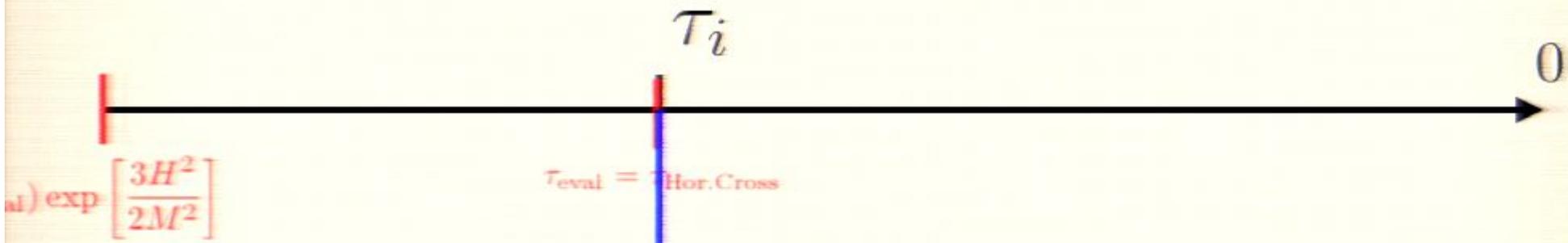
$$\langle \zeta(x)^2 \rangle_* \approx \int_{a_i H}^{a_* H} \frac{dk}{k} \frac{H^2}{4\pi^2} \frac{1}{2\epsilon M_p^2} \sim H^3 t$$

$$\langle h(x)^2 \rangle_* \approx \int_{a_i H}^{a_* H} \frac{dk}{k} \frac{H^2}{4\pi^2} \sim H^3 t$$

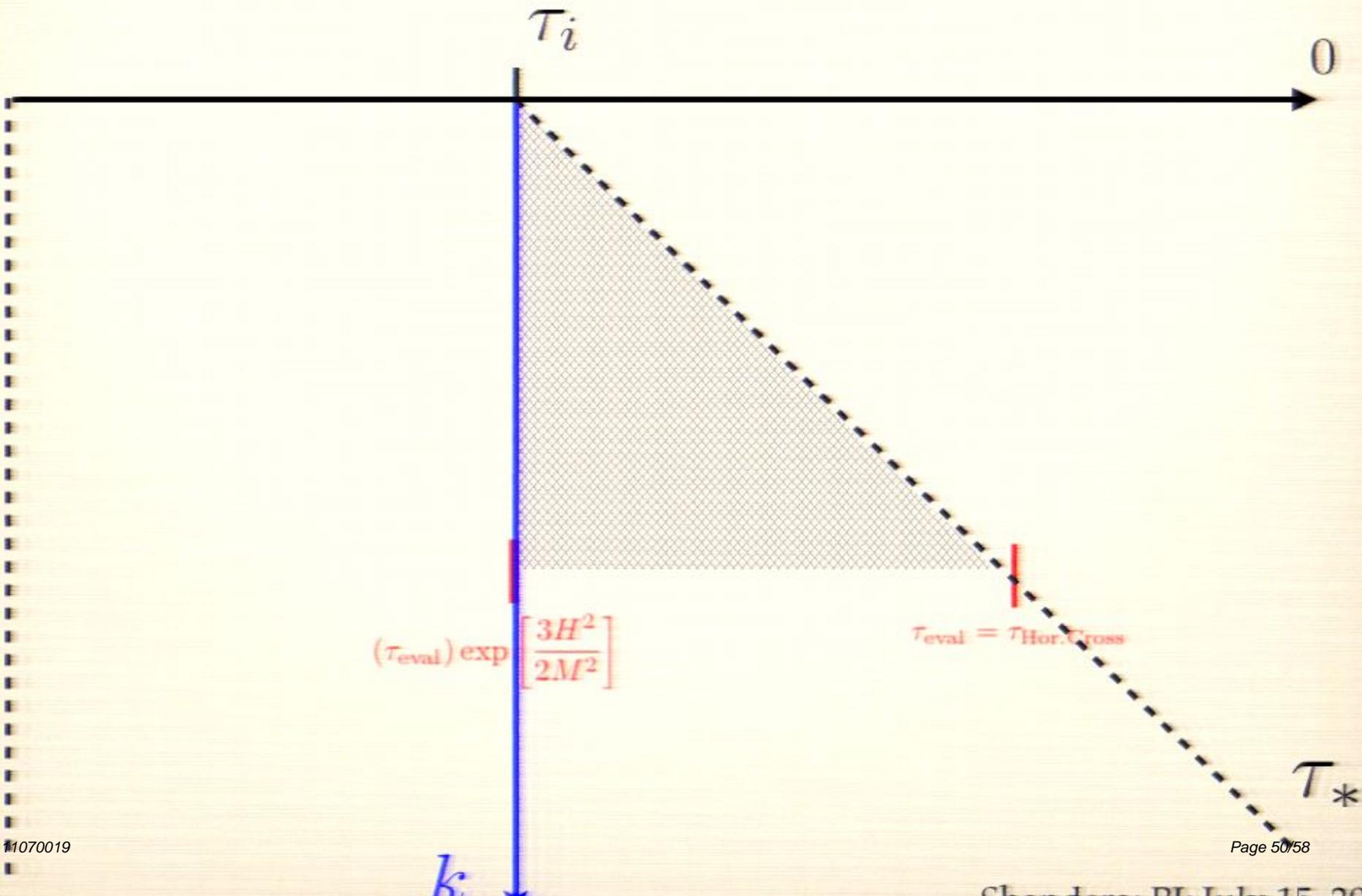


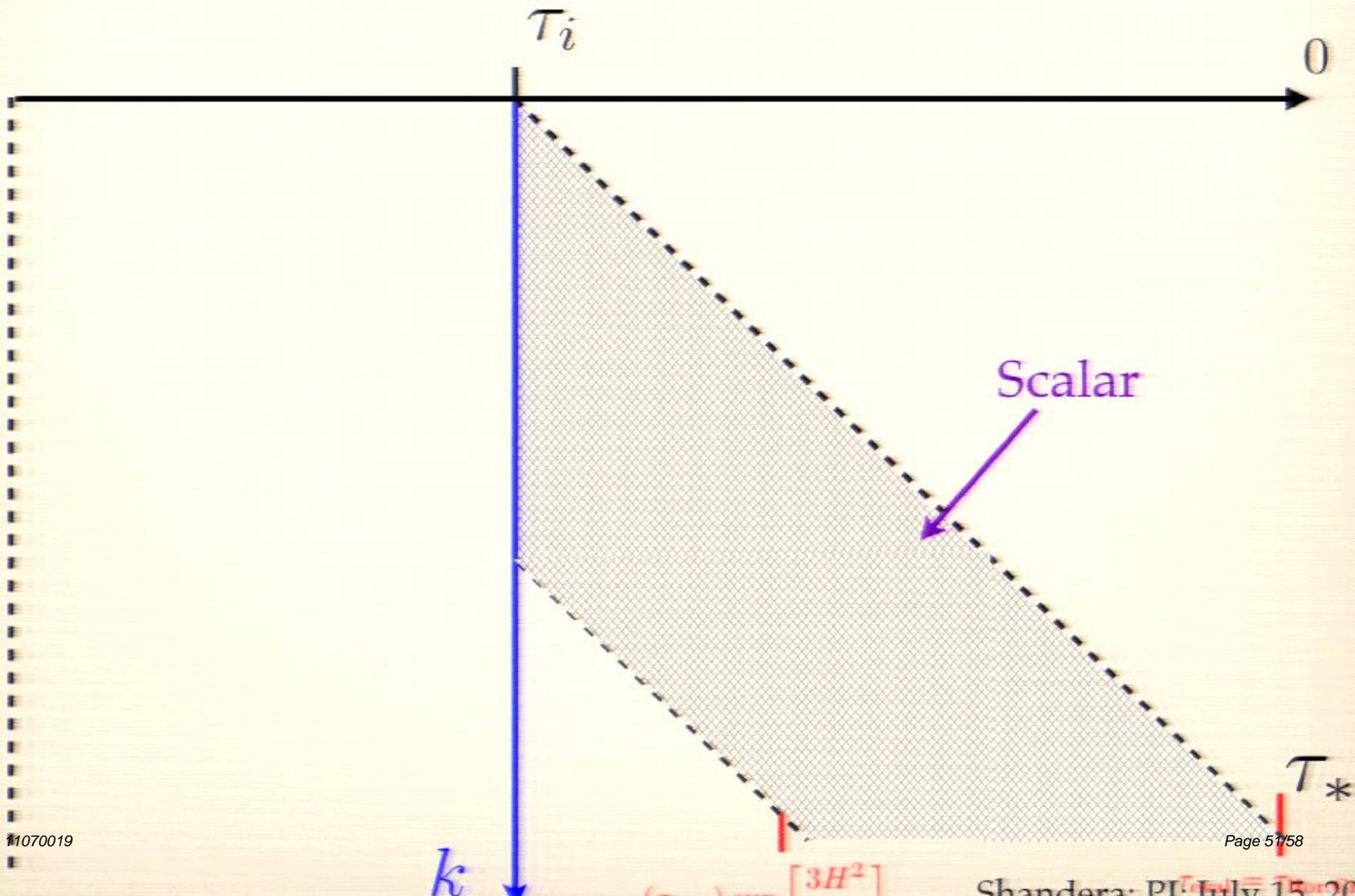






$\text{al) exp} \left[\frac{3H^2}{2M^2} \right]$





FUTURE DIRECTIONS I: INFRARED PUZZLE?

$$\langle \zeta_k^2 \rangle = \langle \zeta_k^2 \rangle_0 [1 + \alpha_1 \langle \zeta(x)^2 \rangle_* + \alpha_2 \langle h(x)^2 \rangle_* + \dots]$$

$$\underbrace{\hspace{10em}}_{H^3 t}$$

Perturbative breakdown?

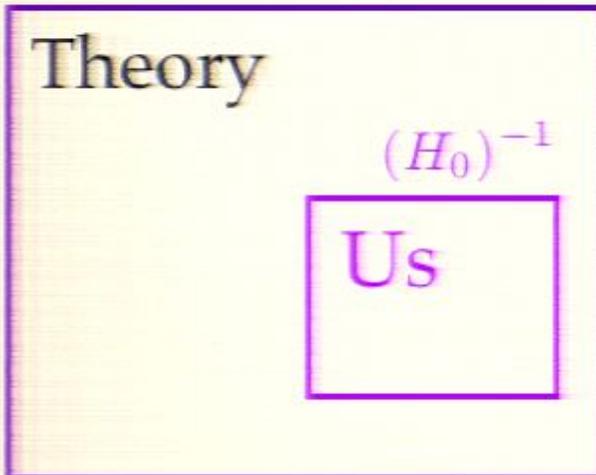
$$t > \frac{M_p^2}{H^3} \sim H^{-1} \times \frac{H^{-2}}{G} \sim RS$$

- * Stochastic eternal inflation...must have quasi dS
- * A sharp analogy to BH information?

FUTURE DIRECTIONS II: MATCHING & SIMULATING

L_{IR}

* Two-Point:



$$\langle \zeta_k^2 \rangle \propto A_0 \left(\frac{k}{k_0} \right)^{n_s - 1}$$

Loop-corrections (re)normalize

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L_{IR}

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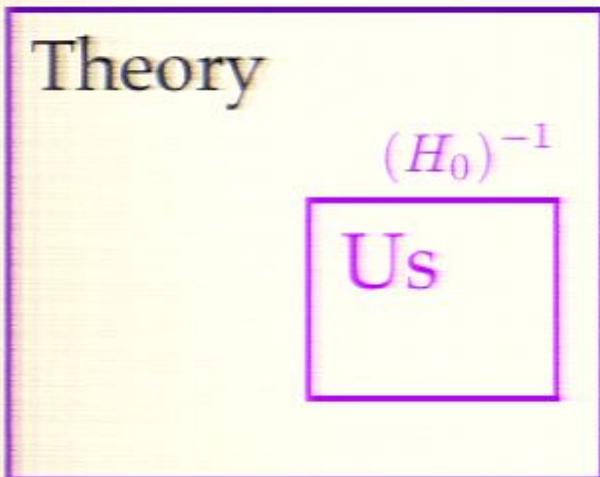
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Loop-corrections (re)normalize

* Three-Point:

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_{12}} \rangle \propto AB(k_1, k_2, k_{12})$$

Loop-corrections: change the shape?



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Present Day Dark Energy