Title: Inflationary Cosmology: A Holographic Perspective

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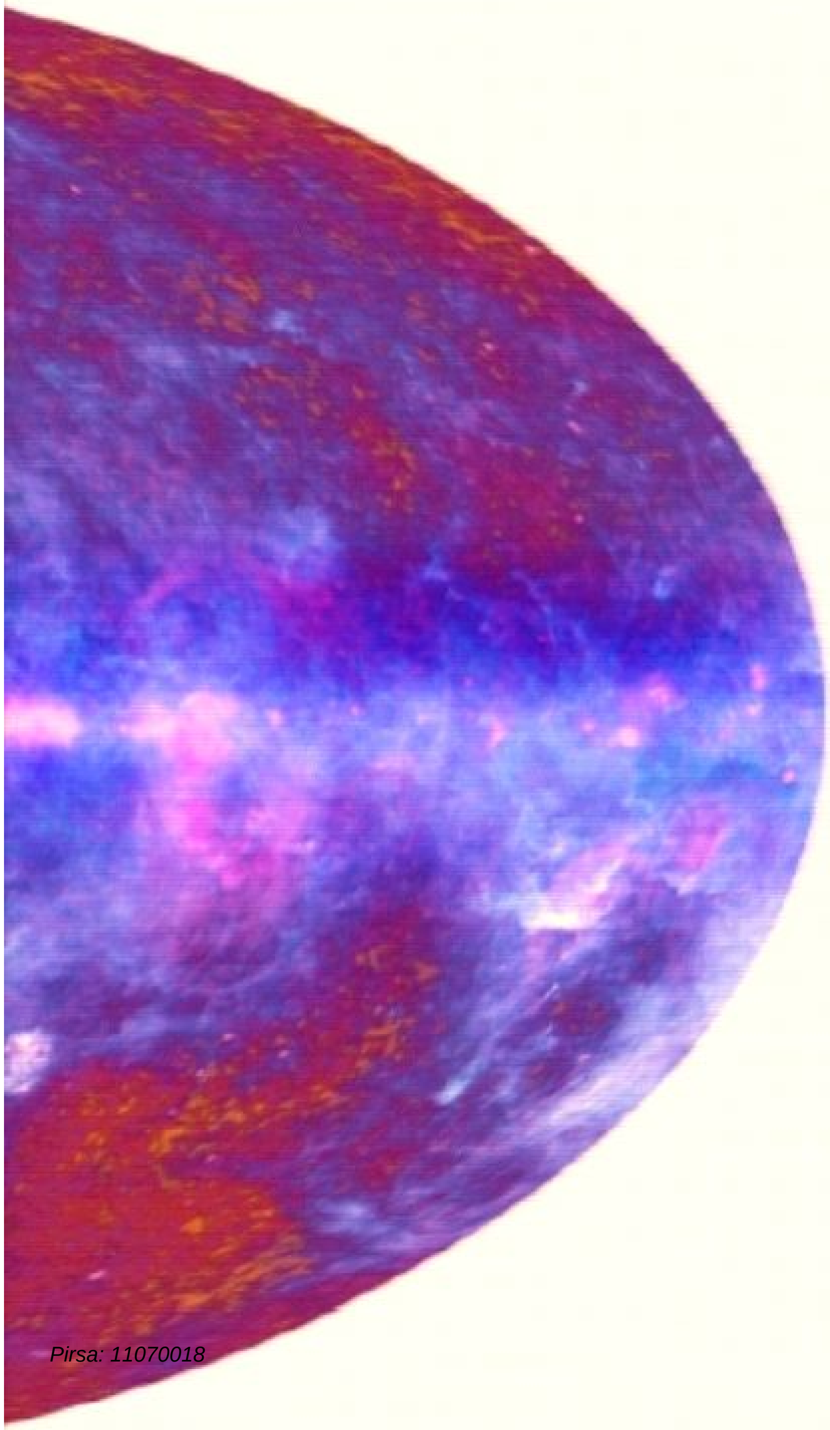
Abstract: We present a holographic framework for inflationary universes, in particular those that are either asymptotically de Sitter or asymptotically power-law. This framework reveals how cosmological observables, including the primordial power spectrum and non-Gaussianities, are encoded in the correlation functions of a three-dimensional non-gravitational quantum field theory. Introducing a simple yet general class of holographic models, we obtain distinctive observational predictions that are compatible with current observational data and may be either confirmed or excluded by Planck.



Inflationary cosmology: a holographic perspective

Paul McFadden

Universiteit van Amsterdam

Challenges for Early Universe Cosmology

Perimeter Institute 14.7.11

References

Based on work with Kostas Skenderis:

- Cosmological 3-point correlators from holography [1104.3894]
- Holographic non-Gaussianity [1011.0452]
- Observational signatures of holographic models of inflation [1010.0244]
- The holographic universe [1007.2007]
- Holography for cosmology [0907.5542]

Work with Richard Easther, Raphael Flauger and KS:

- Constraining holographic inflation with WMAP [1104.2040]

And new work to appear shortly with Adam Bzowski and KS.

Holography

Black hole physics suggests that any gravitational theory should be *holographic*, i.e., should admit a dual description in terms of a non-gravitational theory in one dimension less.

The best understood examples originate from string theory via decoupling limit of branes:

- D3, M5, etc. $\Rightarrow$ asymptotically AdS spacetimes $\Rightarrow$ dual to QFTs that become conformal in the UV.
- D2, D4, etc. $\Rightarrow$ asymptotically power-law spacetimes $\Rightarrow$ dual to QFTs with a generalised conformal structure.

Holography for cosmology

Here I will describe a holographic framework for inflationary spacetimes. Specifically, those that either:

1. approach de Sitter spacetime at late times

$$\mathrm{d}s^2 \to -\mathrm{d}t^2 + e^{2t}\mathrm{d}x_i\mathrm{d}x^i, \qquad \text{as} \quad t \to \infty$$

2. approach power-law scaling solutions at late times

$$\mathrm{d}s^2 \to -\mathrm{d}t^2 + t^{2n}\mathrm{d}x_i\mathrm{d}x^i \quad (n > 1), \qquad \text{as} \quad t \to \infty.$$

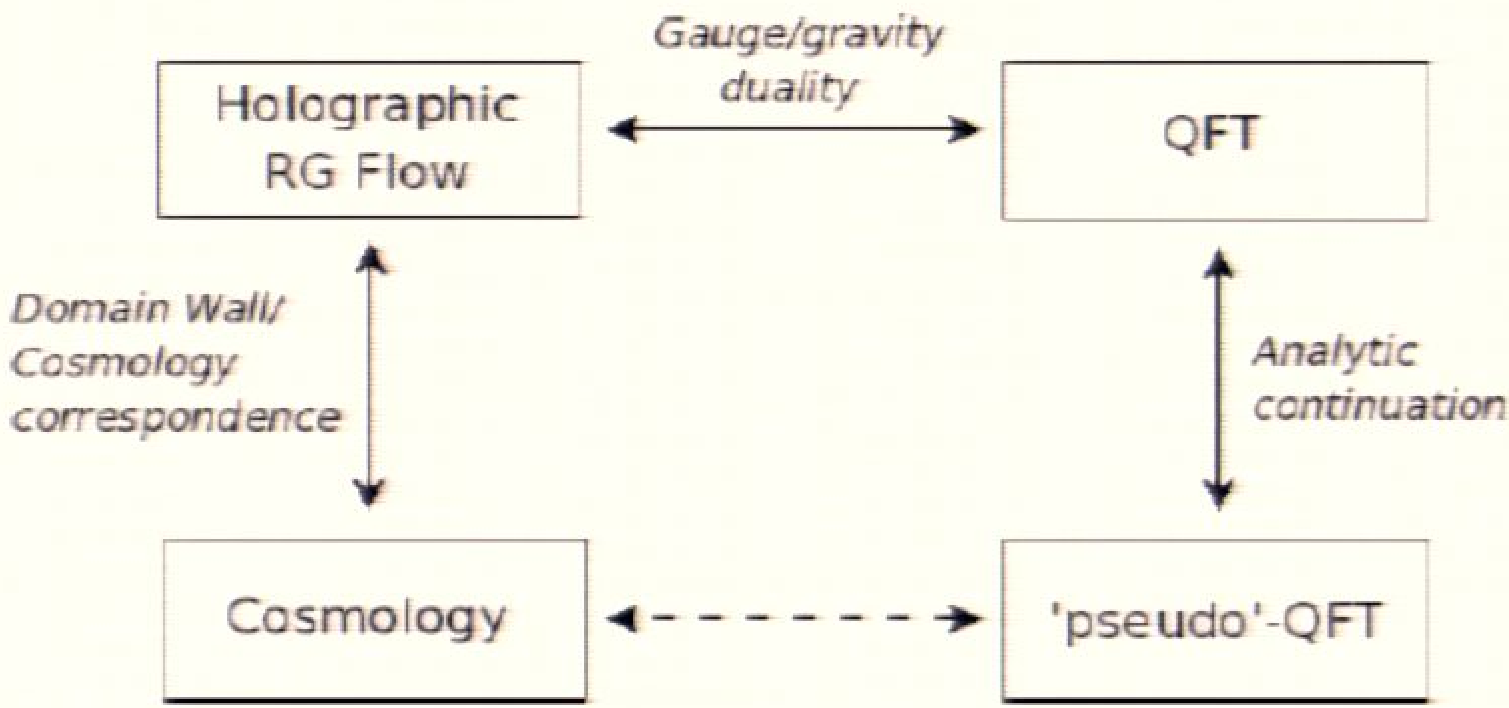
These spacetimes are directly related via analytic continuation to spacetimes with a well-defined holographic interpretation, namely asymptotically AdS or asymptotically power-law holographic RG flows.

Holography for cosmology

Via this correspondence, which also holds when we include perturbations, one can directly relate cosmological observables to the correlation functions of a dual 3d non-gravitational QFT.

$\Rightarrow$ 'Holographic formulae' for cosmological power spectra & non-Gaussianities.

Framework


Holographic RG Flow
Gauge/gravity duality
QFT
Domain Wall/ Cosmology correspondence
Analytic continuation
Cosmology
'pseudo'-QFT

Analytic continuation

Consider a single scalar field minimally coupled to gravity, with a potential V.

- On the bulk side: (barred = hologr. RG flow, unbarred = cosmo)

$$\bar{\kappa}^2\bar{V} = -\kappa^2 V, \qquad \bar{q} = -iq, \qquad q = \sqrt{\vec{q}^2}.$$

Bunch-Davies vacuum $\leftrightarrow$ regularity in interior

Valid at nonlinear order in perturbation theory: works for non-Gaussianities.

- On the QFT side, this translates to:

$$\bar{N} = -iN, \qquad \bar{q} = -iq,$$

where $\bar{N}$ is the rank of the gauge group of the dual QFT.

Note $g^2_{\text{eff}} = g^2_{\text{YM}} N/q$ invariant.

Analytic continuation

- In practice, to compute tree-level cosmological correlators, we only need apply this continuation to the large-N correlators of the regular QFT dual to the holographic RG flow.

- For example, in a class of theories we will meet later,

$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\rm eff}^2).$$

Under the continuation, $g_{\rm eff}^2 \to g_{\rm eff}^2$, $N^2 \to -N^2$, $q \to -iq$ hence

$$\langle T(q)T(-q)\rangle \to -iN^2 q^3 f(g_{\rm eff}^2).$$

Holographic formulae

One can now derive holographic formulae expressing cosmological 2-point functions in terms of (the analytic continuation of) the 2-point function of the dual stress tensor:

$$\langle\zeta(q)\zeta(-q)\rangle = \frac{-1}{8\mathrm{Im}[B(-iq)]}, \qquad \langle\gamma^{(s)}(q)\gamma^{(s')}(-q)\rangle = \frac{-\delta^{ss'}}{\mathrm{Im}[A(-iq)]},$$

[0907.5542]

where

$$\langle T_{ij}(q)T_{kl}(-q)\rangle = A(q)\Pi_{ijkl} + B(q)\pi_{ij}\pi_{kl}.$$

Here the projectors $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{jl} + \pi_{il}\pi_{jk} - \pi_{ij}\pi_{kl})$, $\quad \pi_{ij} = \delta_{ij} - q_i q_j/q^2$.

Note one takes the imaginary part after continuing N and q.

Holographic formulae

Similarly, one can show

$$\langle\zeta(q_1)\zeta(q_2)\zeta(q_3)\rangle = -\frac{1}{256}\frac{\mathrm{Im}\big[\langle T(-iq_1)T(-iq_2)T(-iq_3)\rangle + (\text{semilocal terms})\big]}{\prod_i \mathrm{Im}[B(-iq_i)]}$$

[1011.0452]

Here, the semilocal terms correspond to contributions where two of the three insertion points are coincident: their form is important and contributes to 'local'-type non-Gaussianity. We compute them precisely.

For analogous formulae for 3-point functions involving tensors, see [1104.3894].

The dual QFT

The two classes of asymptotic behaviours with a well-understood holographic description correspond to two classes of dual QFTs:

1. Asymptotically de Sitter $\rightarrow$ QFT is a deformation of a CFT.
2. Asymptotically power-law $\rightarrow$ QFT has generalised conformal structure.

A detailed analysis of this first class of QFTs is currently in progress. In the remainder of this talk, we'll focus instead on the latter class.

The dual QFT

We require that the theory has the following properties:

1. It admits a large N limit.
2. All fields are massless.
3. It has a dimensionful coupling constant.
4. All terms in the Lagrangian have the same scaling dimension, which is different to three.

Properties ❷ to ❹ imply the theory admits a generalised conformal structure: the theory would be conformal if the coupling constant is promoted to a background field transforming non-trivially under conformal transformations.

[Jevicki, Kazama, Yoneya (1998)]
[Kanitscheider, Skenderis, Taylor (2008)]

The dual QFT

A class of models exhibiting these properties is given by the following super-renormalisable theory:

$$S = \frac{1}{g_{\rm YM}^2} \int d^3x \, \mathrm{tr}\Big[\frac{1}{2}F^I_{ij}F^{Iij} + \frac{1}{2}(D\phi^J)^2 + \frac{1}{2}(D\chi^K)^2 + \bar{\psi}^L \not{D}\psi^L + \lambda_{M_1M_2M_3M_4}\Phi^{M_1}\Phi^{M_2}\Phi^{M_3}\Phi^{M_4} + \mu^{\alpha\beta}_{ML_1L_2}\Phi^M\psi^{L_1}_\alpha\psi^{L_2}_\beta\Big]$$

i.e. gauge fields, minimal and conformal scalars, fermions, all in adjoint representation. Parameters are N, field content, and single dimensionful coupling $g^2_{\rm YM}$.

New holographic models

- Conventional inflationary models are described by strongly coupled QFT.
- New models arise when we consider the dual QFT at weak coupling, but still at large N.
- In these models, the very early universe is in a non-geometric phase. This phase should have a string theory description in terms of a strongly coupled sigma model. Here we use holography to describe it.
- The end of this phase is the beginning of conventional hot big bang cosmology.

➡ To extract predictions, we use our holographic formulae.

Holographic power spectrum

At large N,

$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\rm eff}^2) \quad \Rightarrow \quad \Delta_{\mathcal{R}}^2(q) = \frac{1}{4\pi^2 N^2}\frac{1}{f(g_{\rm eff}^2)},$$

and for $g_{\rm eff}^2 \ll 1$

$$f(g_{\rm eff}^2) = f_0(1 - f_1 g_{\rm eff}^2 \ln g_{\rm eff}^2 + f_2 g_{\rm eff}^2 + O(g_{\rm eff}^4)).$$

- We determined f_0 via a 1-loop calculation in perturbation theory.
- f_1 is determined at 2-loop order (though not yet computed).
- f_2 is related to a physical scale generated by infrared effects $q_{\rm IR} \sim g_{\rm YM}^2$.
  [Jackiw, Templeton (1981); Appelquist, Pisarski (1981)].

Provided one probes the theory at scales large compared to the IR scale this term is negligible.

Holographic power spectrum

Setting $f_1 g_{\mathrm{eff}}^2 = gq_*/q$, where q_* is an arbitrary pivot scale, the holographic power spectrum takes the form:

$$\Delta_{\mathcal{R}}^2(q) = \Delta_{\mathcal{R}}^2 \frac{1}{1 + (gq_*/q)\ln|q/gq_*|}.$$

- $\Delta_{\mathcal{R}}^2 = 1/(4\pi^2 N^2 f_0)$ so small amplitude $\Delta_{\mathcal{R}}^2 \sim 10^{-9}$ consistent with large $N \sim 10^4$.
- Perturbative approach requires $(gq_*/q) \ll 1$ hence spectrum is near scale invariant.

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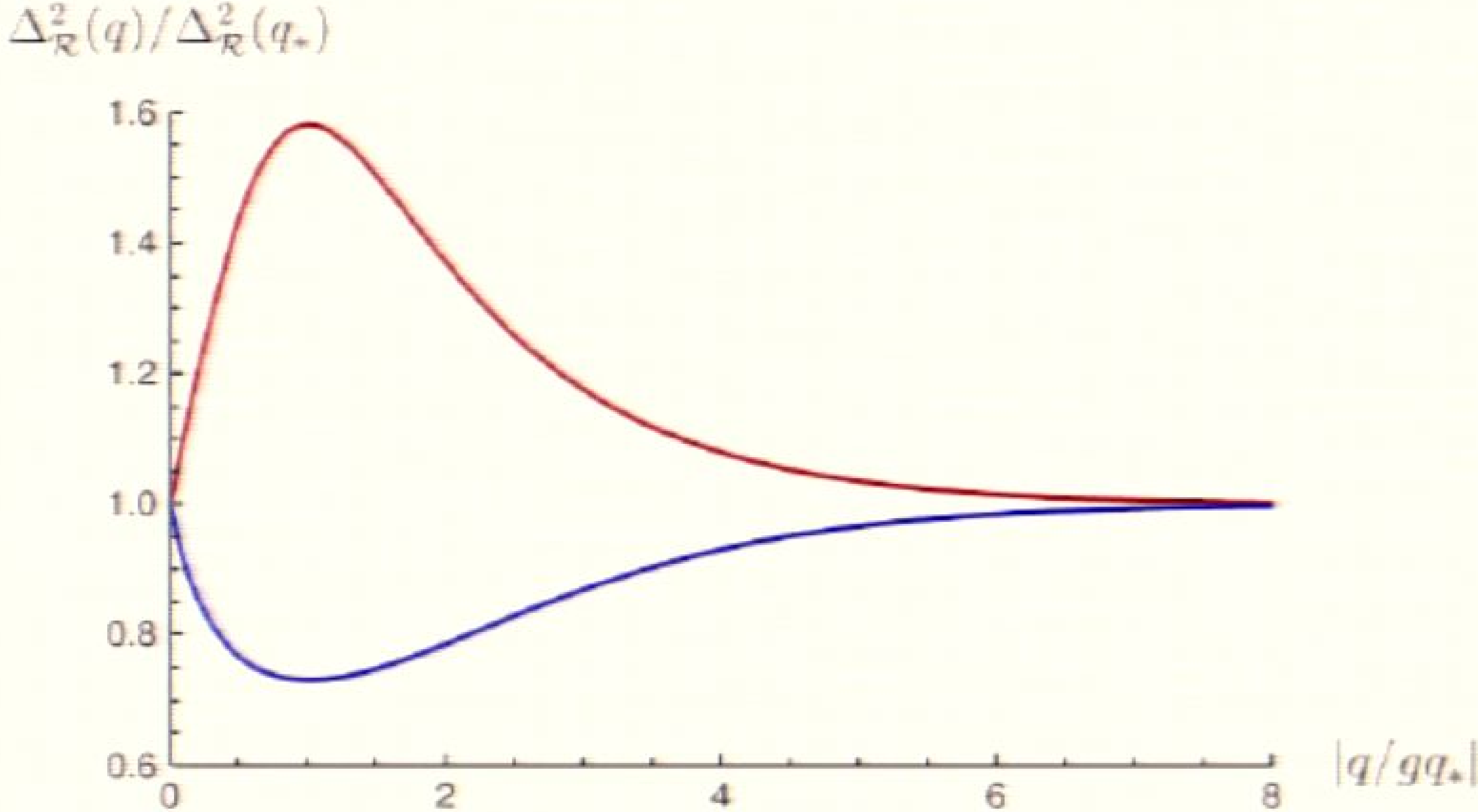

Red curve $g < 0$, blue curve $g > 0$. Perturbative calculation only reliable for large momenta $q/gq_* \gg 1$ far from peak/trough. At very high momenta spectrum rapidly becomes scale invariant (asymptotic freedom).

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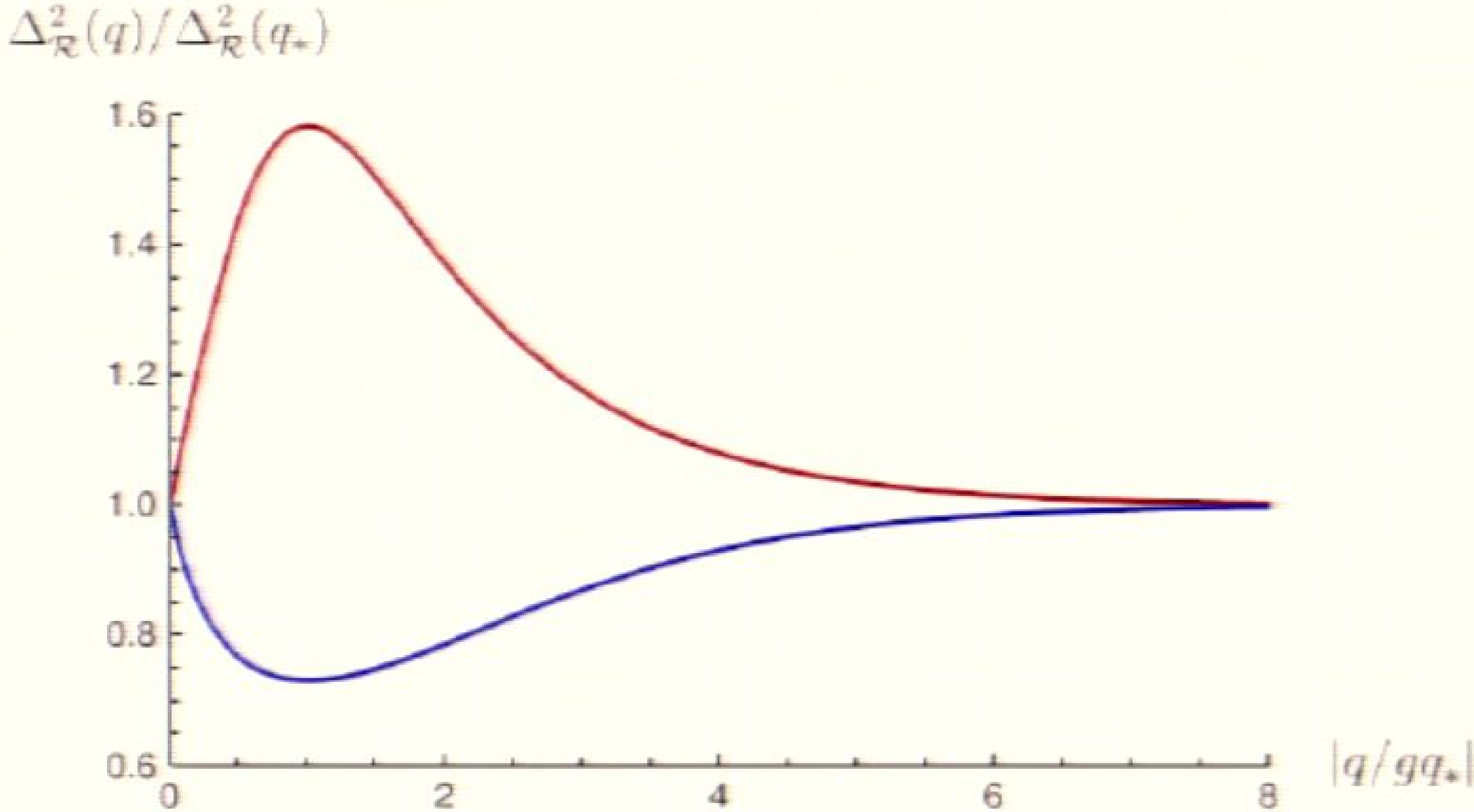

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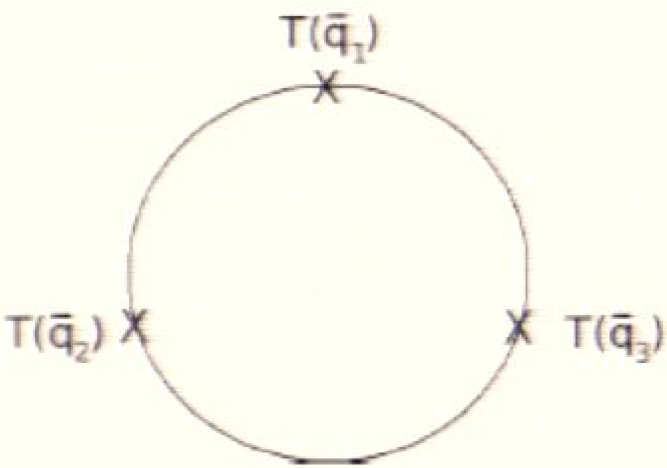
Non-Gaussianity



Evaluating the QFT 3-pt function, our holographic formula predicts a scalar bispectrum of exactly the equilateral form with

$$f_{NL}^{equil} = 5/36.$$

[1011.0452]

- This result is independent of all details of the theory, such as QFT field content.
- Too small for direct detection by Planck, but observation of larger f_{NL} would exclude model.
- For tensor bispectra see [1104.3894] and work to appear with Bzowski, Skenderis.

Holographic model vs ΛCDM

We undertook a custom fit of the holographic model to the current cosmological data, using the empirical ΛCDM model to provide a comparison.

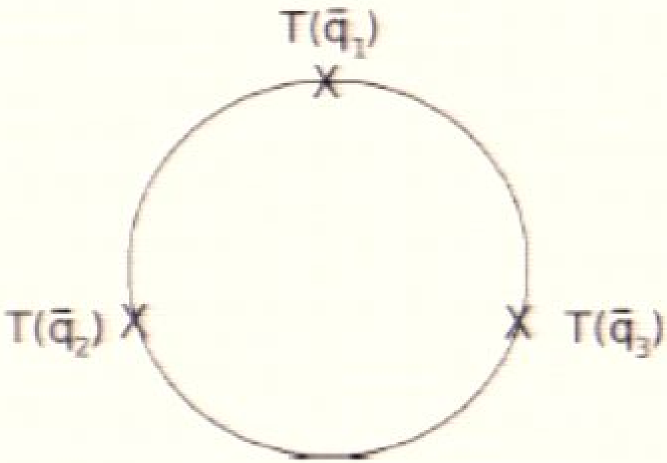
[Easther, Flauger, PM, Skenderis (2011)]
See also [Dias (2011)]

Both models have six parameters: five of these ($\Omega_b h^2$, $\Omega_c h^2$, h, $\Delta^2_{\mathcal{R}}$) are common to both and were found to lie within one standard deviation of each other.

The sixth parameter is the tilt n_s (ΛCDM) or the coupling g (holographic model). The WMAP7 data favour a slightly red spectrum with a best-fit:

$$g = (-1.27 \pm 0.93) \times 10^{-3}, \qquad q_* = 0.05\,\mathrm{Mpc}^{-1}.$$

Non-Gaussianity



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Holographic model vs ΛCDM

The difference in best-fit log likelihoods are:

	Holographic Model	ΛCDM	$\Delta \ln \mathcal{L}_{\text{best}}$
WMAP7	3735.5	3734.3	1.2
WMAP+BAO+H_0	3737.3	3735.7	1.6
WMAP+CMB	3815.0	3812.5	2.5

ΛCDM therefore provides a somewhat better fit, i.e. probability of the data given the optimal choice of model parameters.

To perform model comparison one should compute the Bayesian evidence, i.e. the probability of the model given the data and the prior probability distribution for the model's parameters.

$$E = \int \mathrm{d}\alpha_M P(\alpha_M) \mathcal{L}(\alpha_M)$$

Bayesian Evidence

The specification of priors is important:

- For the holographic model we restricted $|g| < |g_{\max}|$, i.e. couplings for which perturbation theory valid over entire CMB range.
- Difficult to fairly assign prior for tilt n_s in ΛCDM: we tried two choices

$$\text{(i) } 0.92 < n_s < 1.0, \qquad \text{(ii) } 0.9 < n_s < 1.1.$$

The first is near optimal for ΛCDM; the second is symmetric about $n_s = 1$ (since we don't tell the holographic model the sign of the tilt).

For choice (i) we find weak evidence in favour of ΛCDM ($\Delta \ln E \sim 1.2$ to 1.8).

For choice (ii) the evidence is inconclusive ($\Delta \ln E \lesssim 1$).

We conclude that more data is required (Planck), as well as a better theoretical understanding of 2-loop and IR effects, to permit more scale dependence.

Conclusions

1. Standard inflation is holographic: observables such as power spectra and non-Gaussianities can be expressed in terms of analytic continuations of correlation functions of a strongly coupled dual QFT.
2. There are new holographic models based on perturbative QFT. These describe a universe that started in a non-geometric strongly coupled phase.
3. A class of such models based on a super-renormalisable QFT was custom-fit to the data and are compatible with current data. Planck should allow a definitive test through predicted spectral running.