Title: Scale Invariance from Spontaneous Breaking of Conformal Symmetry

Date: Jul 14, 2011 09:50 AM

URL: http://pirsa.org/11070017

Abstract: I will discuss a novel framework of the very early universe which addresses the traditional horizon and flatness problems of big bang cosmology and predicts a scale invariant spectrum of perturbations. Unlike

inflation, this scenario requires no exponential superluminal expansion of

space-time. Instead, the early universe is described by a conformal field theory minimally coupled to gravity. The conformal fields develop a time-dependent expectation value which breaks the flat space so(4,2) conformal symmetry down to so(4,1), the symmetries of de Sitter, giving perturbations a scale invariant spectrum. The solution is an attractor, at

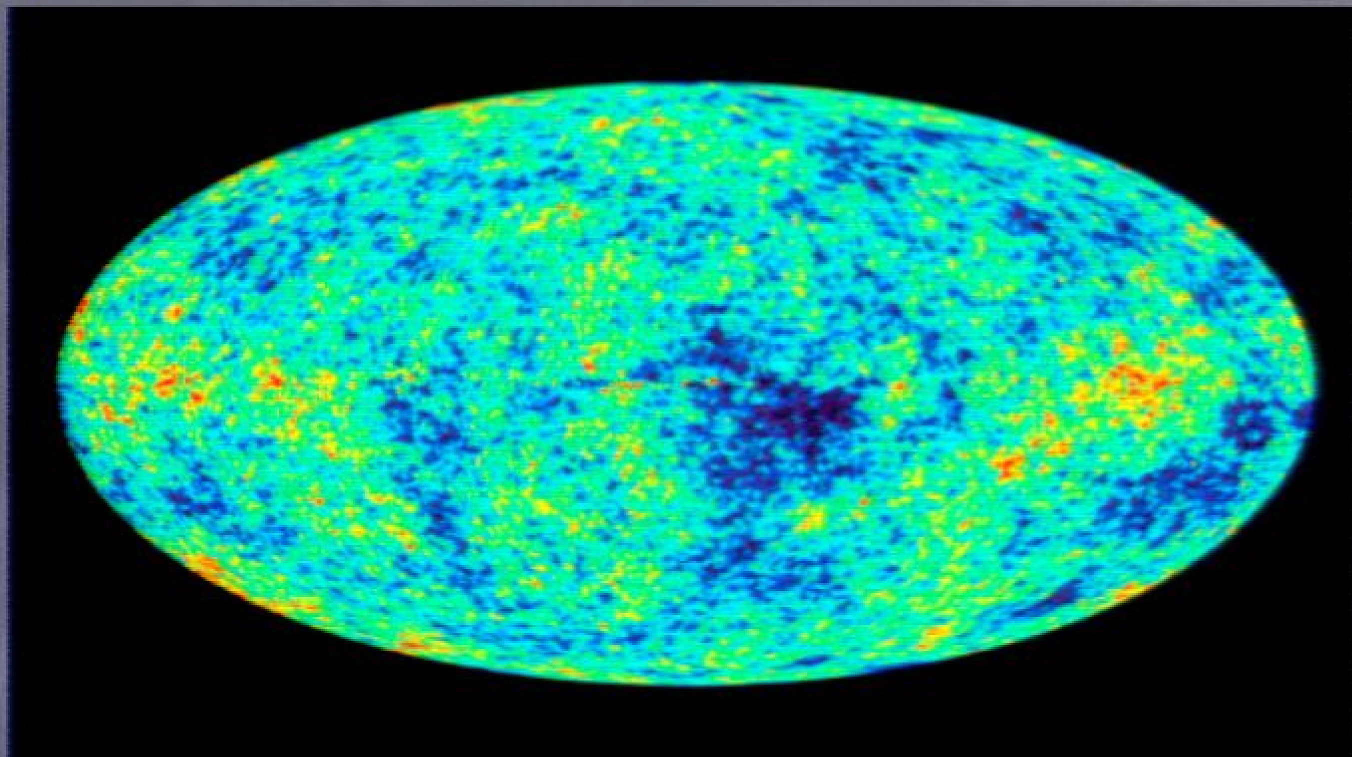
least in the case of a single time-dependent field. Meanwhile, the metric background remains approximately flat but slowly contracts, which makes the universe increasingly flat, homogeneous and isotropic. The essential features of the scenario depend only on the symmetry breaking pattern and not on the details of the underlying lagrangian.

The Pseudo-Conformal Universe: Scale Invariance from Conformal Invariance

Justin Khoury (UPenn)

with K. Hinterbichler, arXiv:1106.1428 [hep-th]

Work in progress with K. Hinterbichler, A. Joyce, G. Miller, V. Balasubramanian, Z. Saleem, and J. Stokes



Primordial perturbations:

- Gaussian
- Linear
- Adiabatic
- Scale-invariant

Theorem

Khoury and Miller, 1012.0846; Khoury and Joyce, to appear;
Baumann, Senatore & Zaldarriaga, 1101.3320; Geshnizjani, Kinney & Dizg
1107.1241.

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Inflation is unique mechanism, with single field and attractor background, capable of generating scale invariant and gaussian perturbations over a broad range of scales.

- With $c_s = 1$, scale inv. + attractor requires

Inflation

$$a(\tau) \sim 1/(-\tau) \quad w \simeq \text{const.}$$

Adiabatic ekpyrosis

$$a(\tau) \approx \text{const.}; \quad w \simeq 1/\tau^2$$

Khoury and Steinhardt, PRL (201

$\Longrightarrow$ Identical 2-point function

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- The argument generalizes to arbitrary $c_s(t)$, including models with $c_s \gg 1$

Armendariz-Picon (2006); Magueijo (2008); Bessada, Kinney, Stojkovic & Wang (2010).

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$\Longrightarrow$ much more freedom

e.g. $$S = \int \mathrm{d}^4x\sqrt{-g}\, f(\phi)(\partial\chi)^2$$

In practice, not so trivial:

- Non-inflationary multi-field mechanisms are generally unstable

  e.g. Lehners, McFadden, Turok & Steinhardt, hep-th/0702153
  Buchbinder, Khoury & Ovrut, hep-th/0702154
  Creminelli & Senatore, hep-th/0702165
  Tolley & Wesley, hep-th/0703101

- Symmetry principle?

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- Realizations
  - Rubakov's U(1) model
    V. Rubakov, 0906.3693; 1007.3417; 1007.4949; 1105.6230
  - Galilean Genesis
    Creminelli, Nicolis & Trincherini, 1007.0027

General CFT on 4D Minkowski space, described by scalar ops:

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Theory invariant under 15 conformal symmetries:

$$\begin{aligned} \delta_{P_\mu}\phi_I &= -\partial_\mu\phi_I\,. & \delta_{J^{\mu\nu}}\phi_I &= (x^\mu\partial^\nu - x^\nu\partial^\mu)\phi_I\,. \\ \delta_D\phi_I &= -(d_I + x^\mu\partial_\mu)\phi_I\,. & \delta_{K_\mu}\phi_I &= \left(-2x_\mu d_I - 2x_\mu x^\nu\partial_\nu + x^2\partial_\mu\right)\phi_I\,. \end{aligned}$$

Define: $\delta_{J_{-2,-1}} = \delta_D\,,\ \delta_{J_{-2,\mu}} = \frac{1}{2}(\delta_{P^\mu} - \delta_{K^\mu})\,,\ \delta_{J_{-1,\mu}} = \frac{1}{2}(\delta_{P^\mu} + \delta_{K}$

$$\Longrightarrow\quad [\delta_{J_{AB}}, \delta_{J_{CD}}] = \eta_{AC}\delta_{J_{BD}} - \eta_{BC}\delta_{J_{AD}} + \eta_{BD}\delta_{J_{AC}} - \eta_{AD}\delta_{J_{BC}}$$

$$\eta_{AB} = \mathrm{diag}(-1, 1, \eta_{\mu\nu})$$

Commutation relations of so(4,2)

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$$\Longrightarrow \quad [\delta_{J_{ab}}, \delta_{J_{cd}}] = \eta_{ac}\delta_{J_{bd}} - \eta_{bc}\delta_{J_{ad}} + \eta_{bd}\delta_{J_{ac}} - \eta_{ad}\delta_{J_{bc}}$$

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Commutation relations of so(4,1)

$$\therefore \quad so(4,2) \to so(4,1)$$

As usual in spontaneous symmetry breaking, much of the physics derives from symmetry breaking pattern,

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Phenomenological Lagrangian

Use symmetries to fix quadratic action for the perturbations,

$$\varphi_I = \phi_I - \bar{\phi}_I$$

- Unbroken so(4,1) subalgebra acts linearly:

$$\begin{aligned} \delta_{P_i}\varphi_I &= -\partial_i\varphi_I\,, & \delta_{J^{ij}}\varphi_I &= \left(x^i\partial^j - x^j\partial^i\right)\varphi_I\,. \\ \delta_D\varphi_I &= -\left(d_I + x^\mu\partial_\mu\right)\varphi_I\,, & \delta_{K_i}\varphi_I &= \left(-2x_i d_I - 2x_i x^\nu\partial_\nu + x^2\partial_i\right)\varphi_I \end{aligned}$$

- 5 broken symmetries act nonlinearly:

$$\begin{aligned} \delta_{P_0}\varphi_I &= -\frac{d_I}{t}\bar{\phi}_I - \dot{\varphi}_I\,, \qquad \delta_{J^{0i}}\varphi_I = \frac{d_I x^i}{t}\bar{\phi}_I + \left(t\partial_i + x^i\partial_t\right)\varphi_I \\ \delta_{K_0}\varphi_I &= \frac{d_I x^2}{t}\bar{\phi}_I + \left(2td_I + 2tx^\nu\partial_\nu + x^2\partial_t\right)\varphi_I\,. \end{aligned}$$

Phenomenological Lagrangian (cont'd)

General 2-derivative quadratic lagrangian :

$$\mathcal{L}_{\rm quad} = \frac{1}{2} M_1^{IJ}(t)\dot{\varphi}_I\dot{\varphi}_J - \frac{1}{2} M_2^{IJ}(t)\vec{\nabla}\varphi_I\cdot\vec{\nabla}\varphi_J - \frac{1}{2} M_3^{IJ}(t)\varphi_I\varphi_J$$

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Imposing linearly and non-linearly realized symmetries:

$$\mathcal{L}_{\rm quad} \sim \sum_{\rm blocks}\left(-\frac{1}{2}(-t)^{2(d-1)}\eta^{\mu\nu}\partial_\mu\varphi_I\partial_\nu\varphi^I - \frac{1}{2}(-t)^{2(d-2)}M^{IJ}\varphi_I\varphi_J\right)$$

where $d\ M^{IJ}c_J = d(d+1)(d-4)\ c^I$

(recall $\bar\phi_I = c_I/(-t)^{d_I}$)

- Quadratic action fixed by symmetries
- Fields of different conformal dimensions do not mix
- Exactly luminal propagation (because of so(4,1))

Phenomenological Lagrangian (cont'd)

Hinterbichler, Joyce & Khoury, in progress

- Coset construction: non-linear realization of so(4,2), with linearly realized so(4,1) subgroup.

  cf. Coleman, Wess & Zumino (1969); Salam & Strathdee (1969); Low & Manohar (2002).

- More on systematic approach later...

Lesson #1: Dynamical Attractor

If there is only a single field ϕ with $d \neq 0$, then

$$\mathcal{L}_{\text{quad}} \sim -\frac{1}{2}(-t)^{2(d-1)}\eta^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}(-t)^{2(d-2)}(d+1)(d-4)\varphi^2$$

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$$\Longrightarrow \qquad \varphi_k \sim \frac{1}{(-t)^{d+1}} \quad \text{and} \quad \varphi_k \sim (-t)^{4-d}$$

growing decaying

But growing mode is just a time shift:

$$\bar{\phi}(t+\varepsilon) = \bar{\phi}(t) + \varepsilon\dot{\bar{\phi}}(t) \sim \frac{1}{(-t)^d}\left(1 - \frac{d\varepsilon}{t}\right)$$

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Quantum fluctuations: assuming adiabatic vacuum,

$$\varphi_k \sim \frac{1}{(-t)^{d-1/2}} H^{(1)}_{5/2}(-kt) \to \frac{1}{k^{5/2}(-t)^{d+1}}$$

red spectrum

Lesson #2: Scale Invariance

Scale invariant perturbations originate from conformal dimension-0 fields, χ_I :

$$\mathcal{L}_{\rm quad}^{(d=0)} = -\frac{1}{2}t^{-2}\eta^{\mu\nu}\partial_\mu\chi_I\partial_\nu\chi^I - \frac{1}{2}M^{IJ}t^{-4}\chi_I\chi_J$$

$\Longrightarrow$ Action of scalars on de Sitter space $g_{\mu\nu}^{\rm eff} \sim \frac{1}{t^2}\eta_{\mu\nu}$

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Assuming mass matrix negligible,

$$\Longrightarrow \quad k^{3/2}|\chi_k| \simeq \text{constant}$$

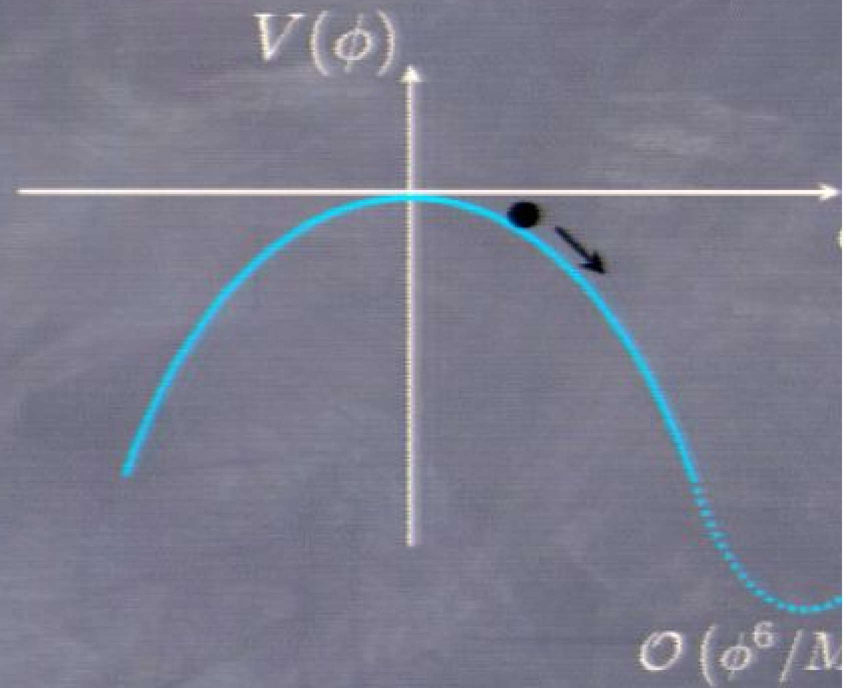
- 2nd field only amplifies to a constant (attractor)
- Contrast with other 2-field mechanisms...
- No special tuning necessary

e.g. Lehners, McFadden, Turok & Steinhardt, hep-th/0702153
Buchbinder, Khoury & Ovrut, hep-th/0702154
Creminelli & Senatore, hep-th/0702165

An Example: Negative Quartic

Rubakov, JCAP 0909, 30 (2009)

Hinterbichler & Khoury, arXiv:1106.1428



$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4$$

$\lambda > 0 \implies$ asymptotically free

Assuming homogeneous evolution,

$$\ddot{\phi} = \lambda\phi^3 \implies \phi(t) = \frac{\sqrt{2}}{\sqrt{\lambda}(-t)}$$

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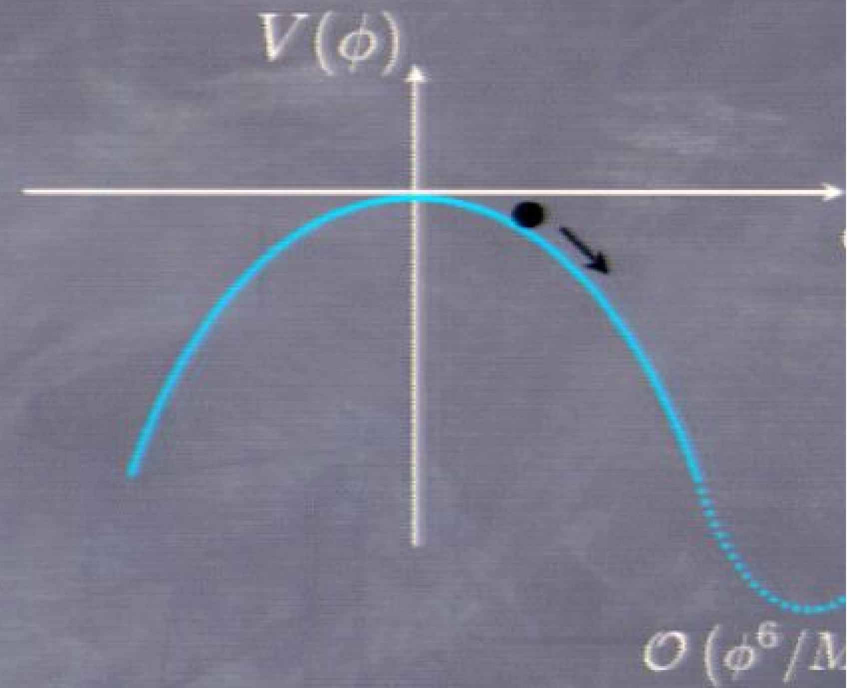

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Conformal dimension 0 field χ :

$$\mathcal{L}_\chi = \underbrace{-\frac{1}{2}\phi^2(\partial\chi)^2}_{\sim t^{-2}(\partial\chi)^2} \underbrace{- \frac{\kappa}{2}\lambda\phi^4\chi^2 + \frac{\xi}{2}\phi\Box\phi\chi^2}_{\sim t^{-4}\chi^2} + \mathcal{O}(\chi^3)$$

Another Example: Galileon Genesis

Creminelli, Nicolis & Trincherini, JCAP 1011, 021 (2010)

$$\mathcal{L}_{\rm gal} = c_2\mathcal{L}_2 + c_3\mathcal{L}_3 + c_4\mathcal{L}_4 + c_5\mathcal{L}_5$$

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2 ,$$

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$$\mathcal{L}_4 = \frac{(\partial\phi)^2}{\phi^4}\left[-\frac{1}{2}\frac{(\Box\phi)^2}{\phi^2} + \frac{1}{2}\frac{\phi^{;\mu\nu}\phi_{;\mu\nu}}{\phi^2} + \frac{6}{5}\frac{(\partial\phi)^2\Box\phi}{\phi^3} - \frac{6}{5}\frac{\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}}{\phi^3} - \frac{3}{20}\frac{(\partial\phi)^4}{\phi^4}\right] .$$

$$\mathcal{L}_5 = \frac{(\partial\phi)^2}{\phi^6}\left[-\frac{1}{2}\frac{(\Box\phi)^3}{\phi^3} - \frac{\phi^{;\mu\nu}\phi_{;\nu\rho}\phi^{;\rho}{}_{\mu}}{\phi^3} + \frac{3}{2}\frac{\Box\phi\,\phi^{;\mu\nu}\phi_{;\mu\nu}}{\phi^3} + 3\frac{\phi^{;\mu\nu}\phi_{;\nu\rho}\phi^{;\rho}\phi_{;\mu}}{\phi^4} - 3\frac{\Box\phi\,\phi^{;\mu\nu}\phi_{;\mu}\phi}{\phi^4}\right.$$

$$\left. - 3\frac{(\partial\phi)^2\phi^{;\mu\nu}\phi_{;\mu\nu}}{\phi^4} + 3\frac{(\Box\phi)^2(\partial\phi)^2}{\phi^4} - \frac{36}{7}\frac{(\partial\phi)^4\Box\phi}{\phi^5} + \frac{36}{7}\frac{(\partial\phi)^2\phi^{;\mu}\phi^{;\nu}\phi_{;\mu\nu}}{\phi^5} - \frac{3}{56}\frac{(\partial\phi)^6}{\phi^6}\right]$$

$$\Longrightarrow \boxed{\bar\phi(t) = \frac{\alpha}{(-t)}}$$ with $\alpha c_2 - \frac{3}{2\alpha}c_3 + \frac{3}{2\alpha^3}c_4 - \frac{3}{4\alpha^5}c_5 = 0$

- Can violate Null Energy Condition
- Superluminal propagation

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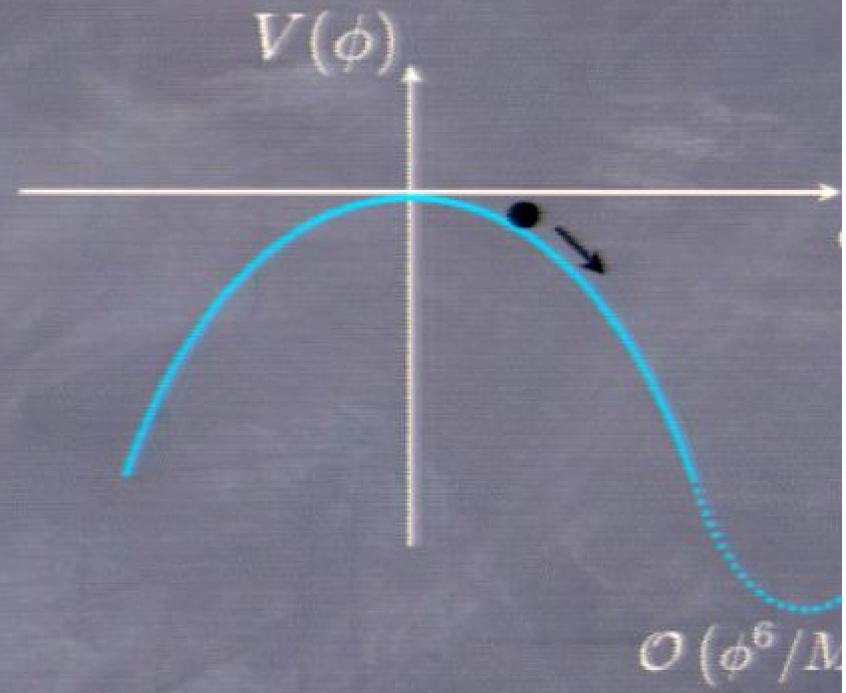

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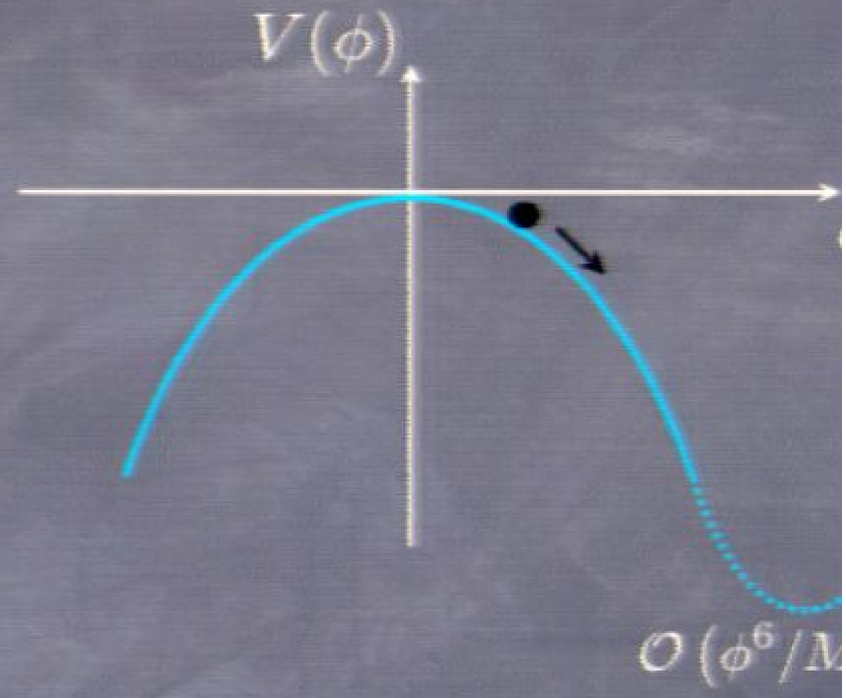

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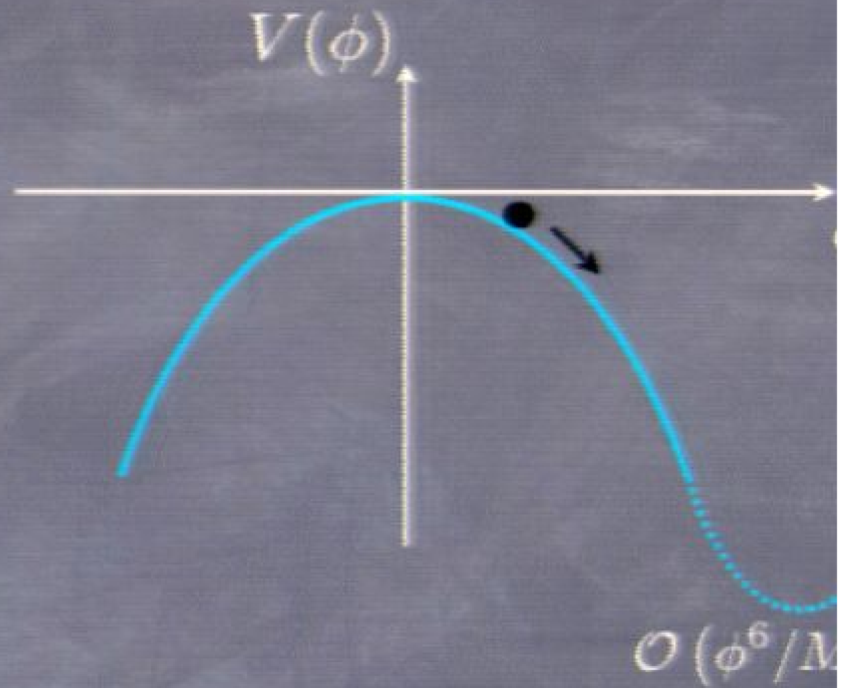
$$\Longrightarrow \boxed{\bar{\phi}(t) = \frac{\alpha}{(-t)}}$$ with $\alpha c_2 - \frac{3}{2\alpha}c_3 + \frac{3}{2\alpha^3}c_4 - \frac{3}{4\alpha^5}c_5 = 0$

- Can violate Null Energy Condition
- Superluminal propagation

An Example: Negative Quartic

Rubakov, JCAP 0909, 30 (2009)
Hinterbichler & Khoury, arXiv:1106.1428

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 + \frac{\lambda}{4}\phi^4$$



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Assuming homogeneous evolution,

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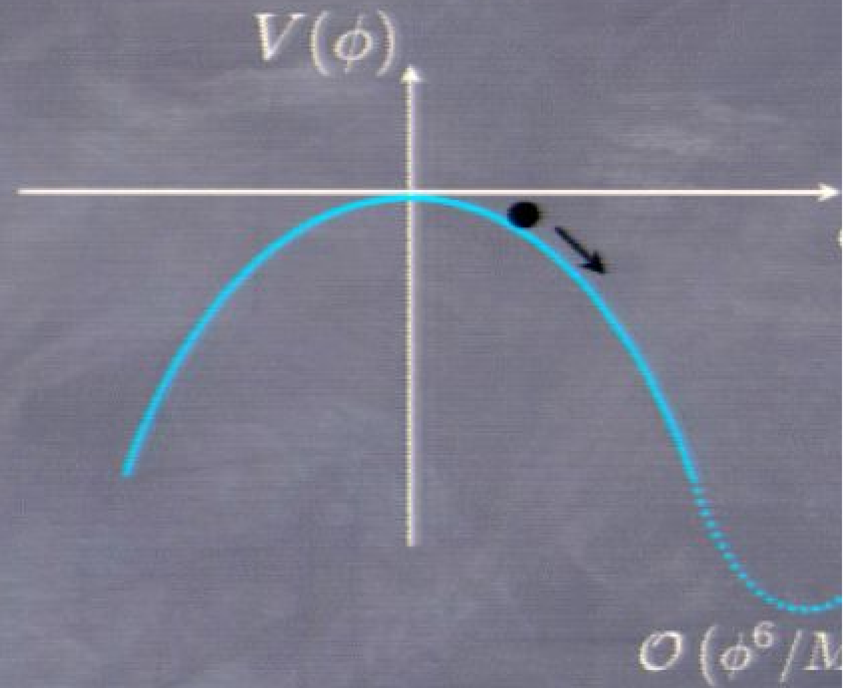

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Conformal dimension 0 field χ :

$$\mathcal{L}_\chi = \underbrace{-\frac{1}{2}\phi^2(\partial\chi)^2}_{\sim t^{-2}(\partial\chi)^2} \underbrace{-\frac{\kappa}{2}\lambda\phi^4\chi^2 + \frac{\xi}{2}\phi\Box\phi\chi^2}_{\sim t^{-4}\chi^2} + \mathcal{O}(\chi^3)$$

Another Example: Galileon Genesis

Creminelli, Nicolis & Trincherini, JCAP 1011, 021 (2010)

$$\mathcal{L}_{\rm gal} = c_2\mathcal{L}_2 + c_3\mathcal{L}_3 + c_4\mathcal{L}_4 + c_5\mathcal{L}_5$$

$$\mathcal{L}_2 = -\frac{1}{2}(\partial\phi)^2\,.$$

$$\mathcal{L}_3 = -\frac{1}{2}\frac{(\partial\phi)^2\Box\phi}{\phi^3} + \frac{1}{4}\frac{(\partial\phi)^4}{\phi^4}\,.$$

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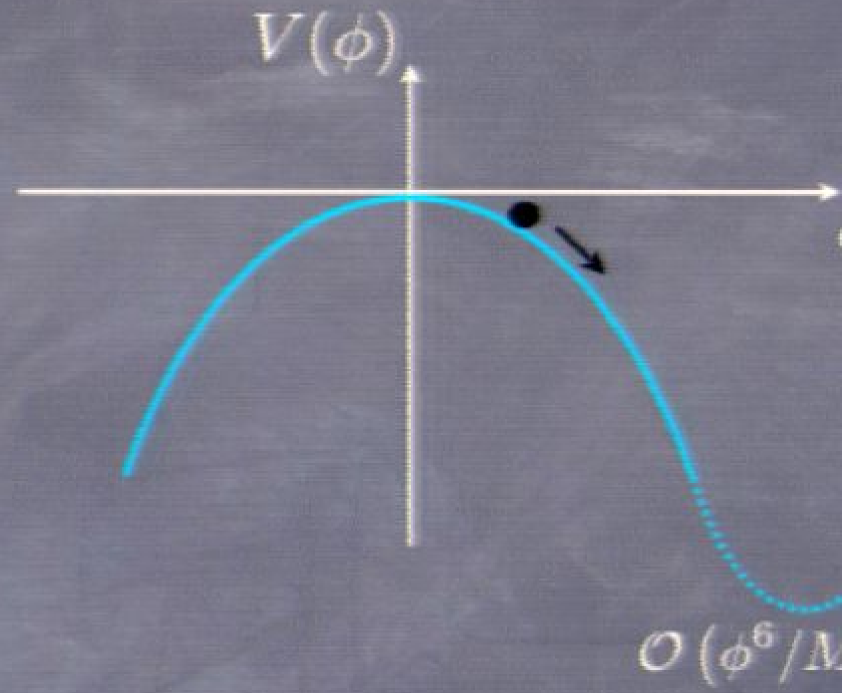

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Turning on Gravity

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}R \ + \ \mathrm{CFT}[g_{\mu\nu}]$$

$\Longrightarrow$ conformal invariance broken explicitly at $1/M_{\rm Pl}$

Gravity can be neglected at early times: $\bar{\phi}_I(t) \simeq \dfrac{c_I}{(-t)^{d_I}}$

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e.g. $\beta = 2/\lambda > 0$ for $V(\phi) = -\lambda\phi^4/4$

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$$H(t) \approx \frac{\beta}{6t^3 M_{\rm Pl}^2} \quad \Longrightarrow \quad \boxed{a(t) \simeq 1 - \frac{\beta}{12 t^2 M_{\rm Pl}^2}}$$

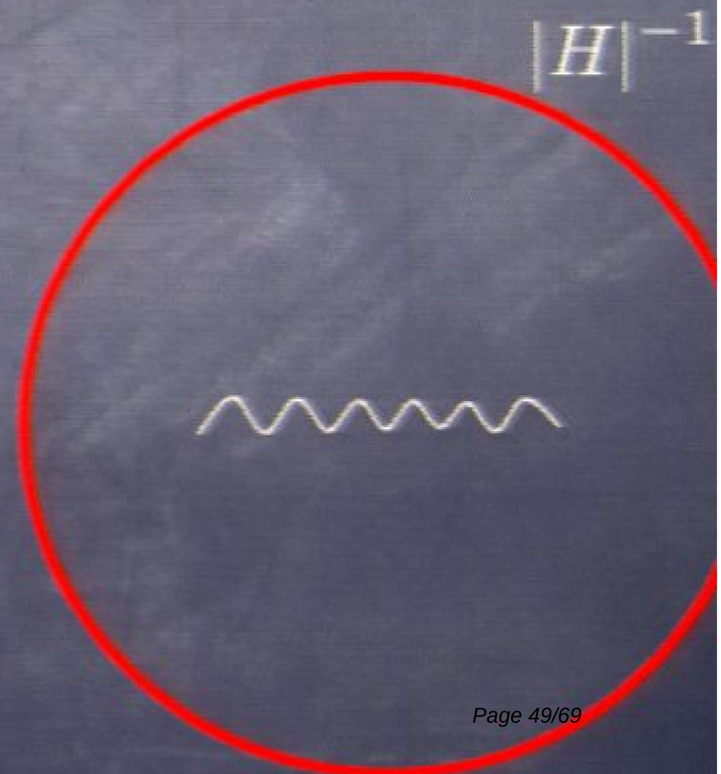
$\therefore$ Universe nearly static for: $t \lesssim t_{\rm end} \equiv -\frac{\sqrt{\beta}}{M_{\rm Pl}}$

- $\bar{\phi}_I(t_{\rm end}) \sim \frac{1}{(-t_{\rm end})^{d_I}} \sim M_{\rm Pl}^{d_I}$

- All scale inv. modes are super-Hubble:

$$\frac{k}{|H(t_{\rm end})|} < \frac{k_{\rm end}}{|H(t_{\rm end})|} \sim \frac{t_{\rm end}^2 M_{\rm Pl}^2}{\beta} \sim \mathcal{O}(1)$$

$k_{\rm end} \sim |t_{\rm end}|^{-1}$



Flatness and Homogeneity

$$H(t) \approx \frac{\beta}{6t^3 M_{\rm Pl}^2}$$

$$a(t) \simeq 1 - \frac{\beta}{12t^2 M_{\rm Pl}^2}$$

Universe becomes increasingly flat and homogeneous:

$$3H^2 M_{\rm Pl}^2 = -\frac{3K}{a^2} + \frac{C_{\rm mat}}{a^3} + \frac{C_{\rm rad}}{a^4} + \frac{C_{\rm aniso}}{a^6} + \ldots + \rho_{\rm CFT}$$

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$$\rho_{\rm CFT} \sim \frac{1}{t^6 M_{\rm Pl}^2}$$

Akin to ekpyrotic cosmologies (contracting universe with w >> 1)

Gratton, Khoury, Steinhardt & Turok (2003);
Erickson, Wesley, Steinhardt & Turok (2004).

Phenomenological Lagrangian (revisited)

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Tractor Calculus

Bailey, Eastwood & Grover (1994); Eastwood & Rice (1987); T.Y. Thomas (1926).

Tractors are to Weyl invariance what tensors are to diffeomorphism invariance.

Riemannian geometry: $g_{\mu\nu}\,,\;\; \nabla_\mu\,,\;\; \phi,\;\; A_\mu\;\ldots$

$$\Longrightarrow\; S=\int \mathrm{d}^4x\sqrt{-g}\,\mathcal{L}\,(g_{\mu\nu},\nabla_\mu,\phi,A_\mu\ldots)$$

Conformal geometry: $[g_{\mu\nu}]=\left[\Omega^2 g_{\mu\nu}\right]$

However, connection is naturally defined in 4+2 dimensions.

Tractor: $\boxed{T^M=(T_+,T^\mu,T_-)}$

Tractor Calculus

Dimension w tractor: $$T^M = (T_+, T^\mu, T_-)$$

Under $\hat{g}_{ab} = \Omega^2 g_{ab}$, tractors transform nicely: $$\hat{T}^M = \Omega^w U^M_{\ N} T^N$$

where $$U^M_{\ N} = \begin{pmatrix} \Omega & 0 & 0 \\ \Upsilon^\mu & \delta^\mu_\nu & 0 \\ -\frac{1}{2}\Omega^{-1}\Upsilon^\kappa\Upsilon_\kappa & -\Omega^{-1}\Upsilon_\nu & \Omega^{-1} \end{pmatrix}.$$

with $\Upsilon_\mu = \Omega^{-1}\partial_\mu \Omega$

Can contract indices with metric: $$\eta_{MN} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \eta_{\mu\nu} & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

which is invariant $$\eta_{AB} = U^M_{\ A} U^N_{\ B} \eta_{MN}$$

Tractor Derivatives

Tractor connection: $\mathcal{D}_\mu T^M = \mathcal{D}_\mu \begin{pmatrix} T_+ \\ T^\mu \\ T_- \end{pmatrix} = \begin{pmatrix} \partial_\mu T_+ - T_\mu \\ \partial_\mu T^\nu + \delta^\nu_\mu T_- \\ \partial_\mu T_- \end{pmatrix}$.

- Nice transformation properties: $\hat{\mathcal{D}}_\mu = U \mathcal{D}_\mu U^{-1}$
- Metric compatible: $\mathcal{D}_\mu \eta_{MN} = 0$

Unfortunately, does not transform as tractor.

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Unfortunately, does not transform as tractor.

Define Thomas D operator:

$$D^M = \begin{pmatrix} w(d+2w-2) \\ (d+2w-2)\mathcal{D}^\mu \\ -\mathcal{D}^2 \end{pmatrix}$$

- Maps dim'n w tractors to dim'n $w-1$ tractors.
- $D_M D^M = 0$
- Integration by parts: $\int \mathrm{d}^d x \, V_M D^M \varphi = \int \mathrm{d}^d x \, \varphi D^M V_M$

The power of tractors

Hinterbichler, Joyce & Khoury, in progress

Consider $w = 1$ field φ. Generic dilatation invariant terms are:

$$\varphi^{-d+2n-m}\partial^{2n}\varphi^{m}$$

Great. But which linear combinations are conformally invariant?

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Consider $w=1$ field φ. Generic dilatation invariant terms are:

$$\varphi^{-d+2n-m}\partial^{2n}\varphi^{m}$$

Great. But which linear combinations are conformally invariant?

- 0 derivatives: Only 1 possible term, φ^{-d}

$$\Longrightarrow \quad S=\int \mathrm{d}^d x\, \varphi^{-d}$$

- 2 derivatives: Now 2 possible terms, $\varphi^{-d}(\partial\varphi)^2\,,\ \ \varphi^{1-d}\Box\varphi\,,$ but related by total derivative

$$\Longrightarrow S=\int \mathrm{d}^d x\, \varphi^{-d} D_M\varphi D^M\varphi \longrightarrow -d(d-2)\int \mathrm{d}^d x\, \varphi^{-d}(\partial\varphi)^2$$

- 4 derivatives: 7 possible terms,

$$\frac{\Box^2\varphi}{\varphi^{d-3}}, \quad \frac{\partial^\mu\varphi\,\partial_\mu\Box\varphi}{\varphi^{d-2}}, \quad \frac{(\Box\varphi)^2}{\varphi^{d-2}}, \quad \frac{(\partial_\mu\partial_\nu\varphi)^2}{\varphi^{d-2}}, \quad \frac{(\partial\varphi)^2\Box\varphi}{\varphi^{d-1}}, \quad \frac{\partial^\mu\varphi\,\partial^\nu\varphi\,\partial_\mu\partial_\nu\varphi}{\varphi^{d-1}}, \quad \frac{(\partial\varphi)^4}{\varphi^d}$$

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- Of remaining 3, can form 2 conformally-invariant combinations.

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$$\begin{aligned} S &= \int \mathrm{d}^d x\, \varphi^{-d}(D\varphi)^4 \\ &\longrightarrow d^2 \int \mathrm{d}^d x \Big[d^2\varphi^{-d}(\partial\varphi)^4 - 4d\varphi^{1-d}\Box\varphi(\partial\varphi)^2 + 4\varphi^{2-d}(\Box\varphi)^2\Big] \end{aligned}$$

$$\begin{aligned} S &= \int \mathrm{d}^d x\, \varphi^{2-d} D_M D_N \varphi D^M D^N \varphi \\ &\longrightarrow d(d-2)^2 \int \mathrm{d}^d x \Big[\frac{d(2-d)(1-d)}{2}\varphi^{-d}(\partial\varphi)^4 + \frac{3d(2-d)}{2}\varphi^{1-d}\Box\varphi(\partial\varphi \\ &\qquad + (d-1)\varphi^{2-d}(\Box\varphi)^2\Big] \end{aligned}$$

- 6 derivatives: 30 possible terms, with 20 total derivative combinations! And then must find which are conformally-invariant combinations...

With tractors, life is easy:

$$D_M D_N D_K \varphi D^M D^N D^K \varphi$$

$$(D\varphi)^6$$

$$(D\varphi)^2 (D_M D_N \varphi)^2$$

$$D_M D_N \varphi D^M D_K \varphi D^K D^N \varphi$$

Tractor calculus offers an elegant and powerful framework for writing down general effective action for our mechanism

AdS/CFT connection

Balasubramanian, Hinterbichler, Khoury, Saleem & Stokes, in progress

For strongly coupled CFT, can look for AdS_5 gravity dual.

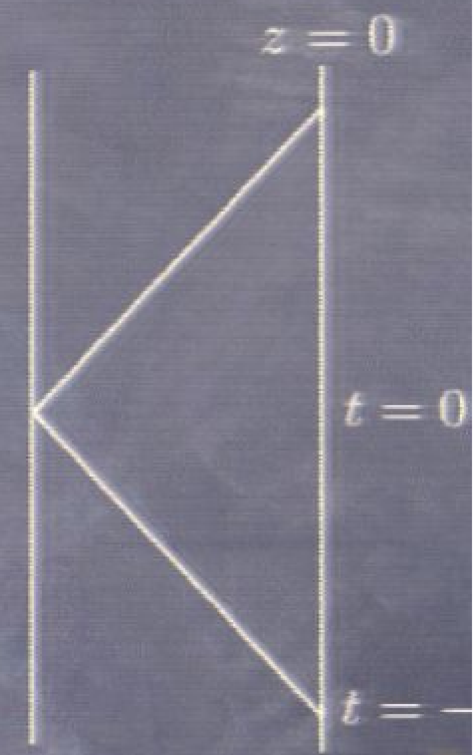
On CFT side, need: $$\langle \mathcal{O} \rangle \sim \frac{1}{(-t)^{\Delta}}$$ $\Delta \equiv$ conformal dim'n

AdS/CFT connection

Balasubramanian, Hinterbichler, Khoury, Saleem & Stoke
in progress

Poincare coords: $\mathrm{d}s^2 = \frac{1}{z^2}\left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}z^2\right)$

Massive scalar: $\Box\phi = m^2\phi$



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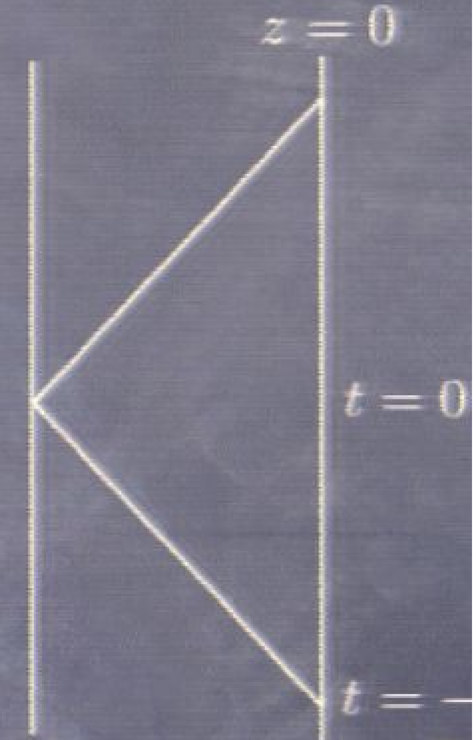

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Look for $\phi = \phi(x)$, where $x \equiv z/(-t)$

$$x^2(1-x^2)\phi''(x) - x(3+2x^2)\phi'(x) = m^2\phi(x)$$

Near bdy $(x=0)$,

$$\phi(x) \sim \underbrace{C_+ x^{-\lambda_+}}_{\text{non-norm.}} + \underbrace{C_- x^{-\lambda_-}}_{\text{norm.}}$$

$$\lambda_\pm = -2 \pm \sqrt{4+m^2}$$

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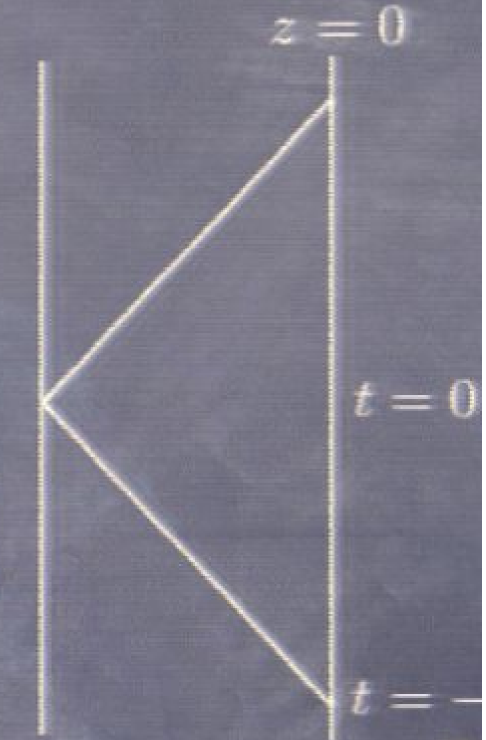

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- Dual to $\mathcal{O}$ with dim'n $\Delta = 2 + \sqrt{4+m^2}$

$$\Longrightarrow \quad \langle \mathcal{O} \rangle = \frac{C_-}{(-t)^{-\lambda_-}} = \frac{C_-}{(-t)^{\Delta}}$$

Conclusions

- Spontaneous conformal symmetry breaking: $so(4,2) \to so(4,1)$
- For scalar ϕ_I of conformal dim'n d_I, this is realized by

$$\phi_I \sim t^{-d_I}$$

- Effective action for pertns fixed by symmetry breaking pattern
  - coset construction
  - tractor calculus — Hinterbichler, Joyce & Khoury, in progress
- Gravity unimportant, universe driven to flatness and homogeneity
- Predictions:
  - significant non-gaussianity
  - negligible gravity waves
- AdS/CFT connection: 5D gravity dual in AdS_5?

Balasubramanian, Hinterbichler, JK, Saleem & Stokes, in progress

[redacted]