Title: Holographic Cosmology Part 1

Date: Jul 12, 2011 12:10 PM

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Abstract: We will describe how a quantum mechanical description of a flat FRW with equation of state pressure =energy density, emerges.

Holographic Cosmology
Part 1

"Challenges for Early Universe Cosmology"
Perimeter Institute, July 2011

The thermodynamics of the equation of state: $p = \rho$

$$dE = TdS - pdV$$

locally:

$$p + \rho = T\sigma$$

$$d\rho = Td\sigma$$

$$\sigma \sim \rho^{1/2}$$

This is the statistical mechanics of a 1+1 dim. CFT

(not a homogeneous scalar field)

$$dE = T\,dS - p\,dV$$

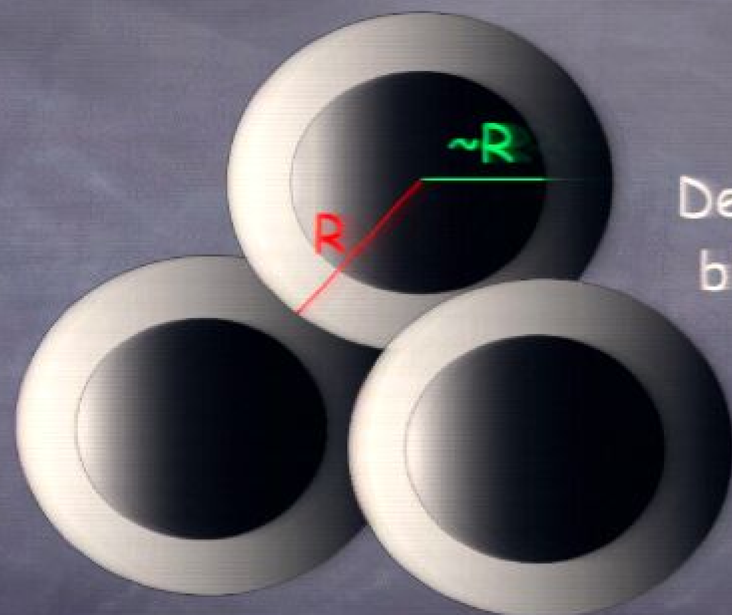
$$\boxed{p = \tfrac{1}{3}\rho}$$

$$E = \rho V$$

$$S = sV$$

$$\sigma \sim \sqrt{\rho} \longrightarrow$$

Mechanical picture of $p = \rho$



Dense gas of black holes

$$\sigma \sim \frac{R^{D-2}}{R^{D-1}} \sim R^{-1} \qquad \rho \sim \frac{R^{D-3}}{R^{D-1}} \sim R^{-2}$$

$$\sigma \sim \rho^{1/2}$$

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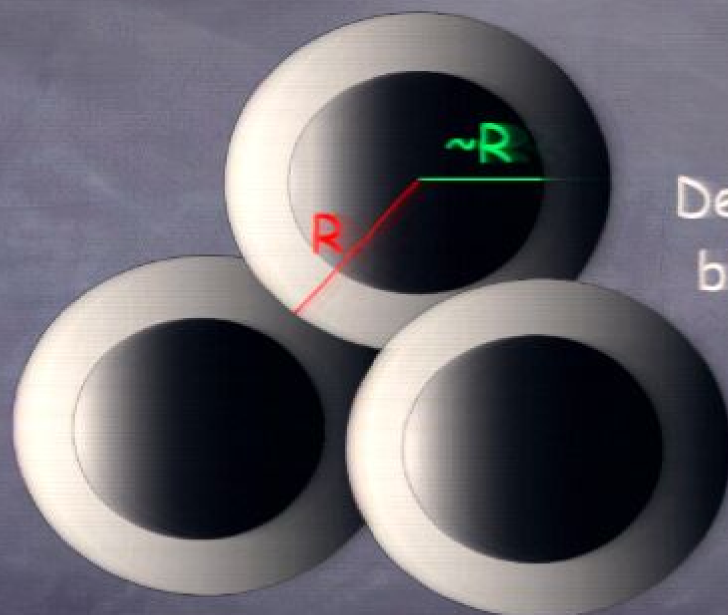

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The storage of information requires space

The merger of black holes leads to bigger black holes

$\longrightarrow$ pressure in the dense gas of black holes

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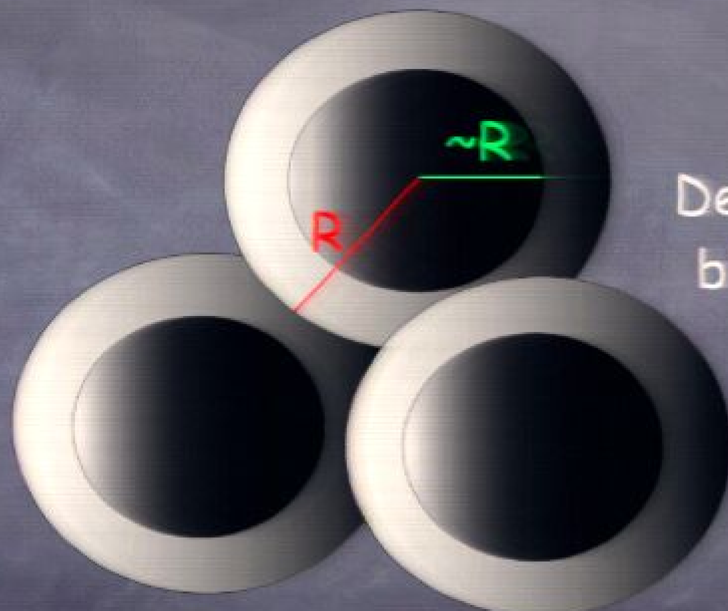

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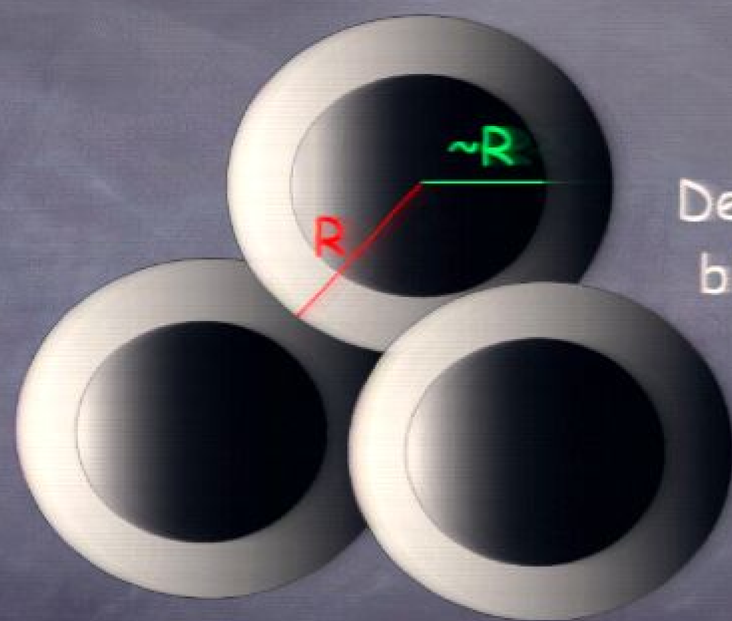

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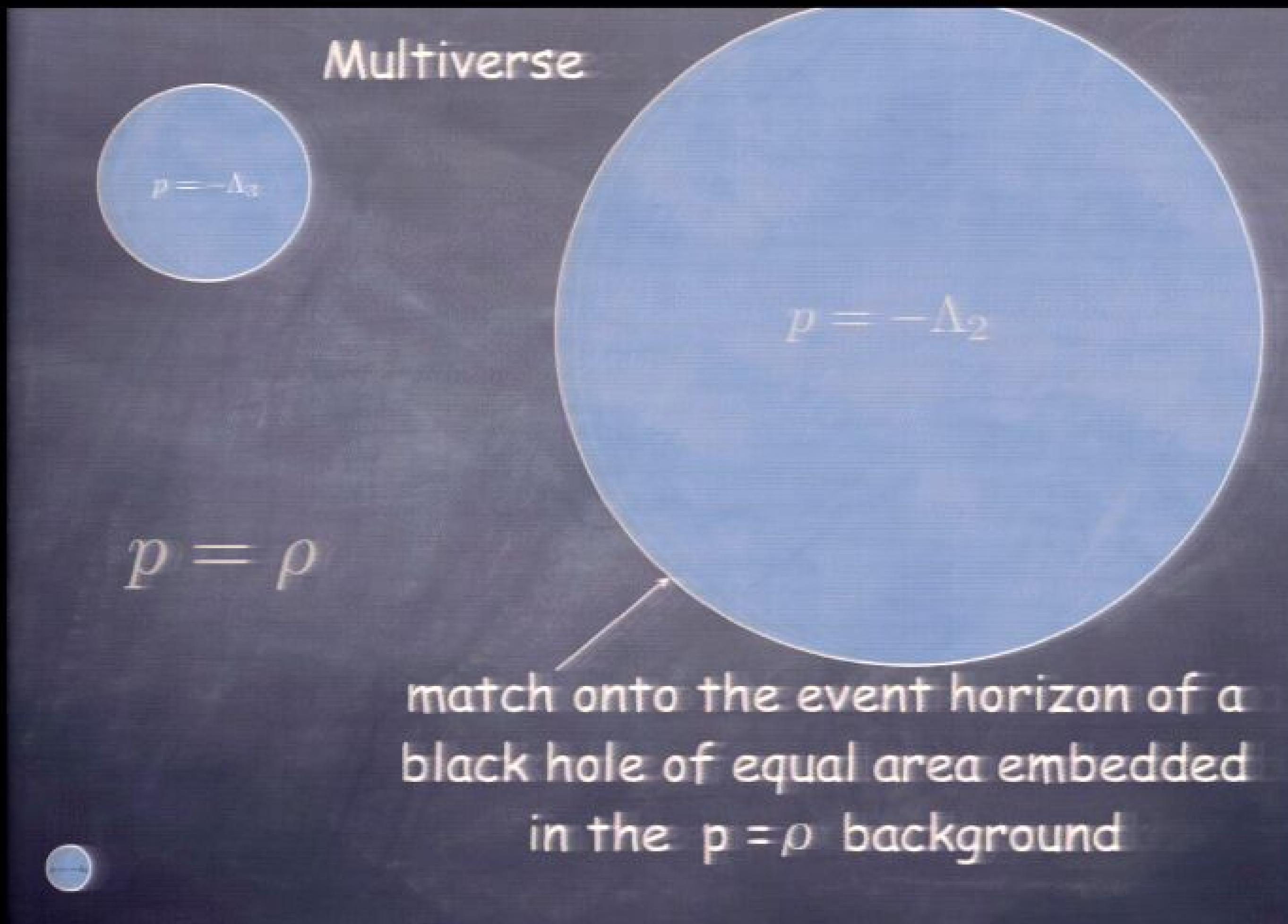
Synchronize times:
equal area slicing

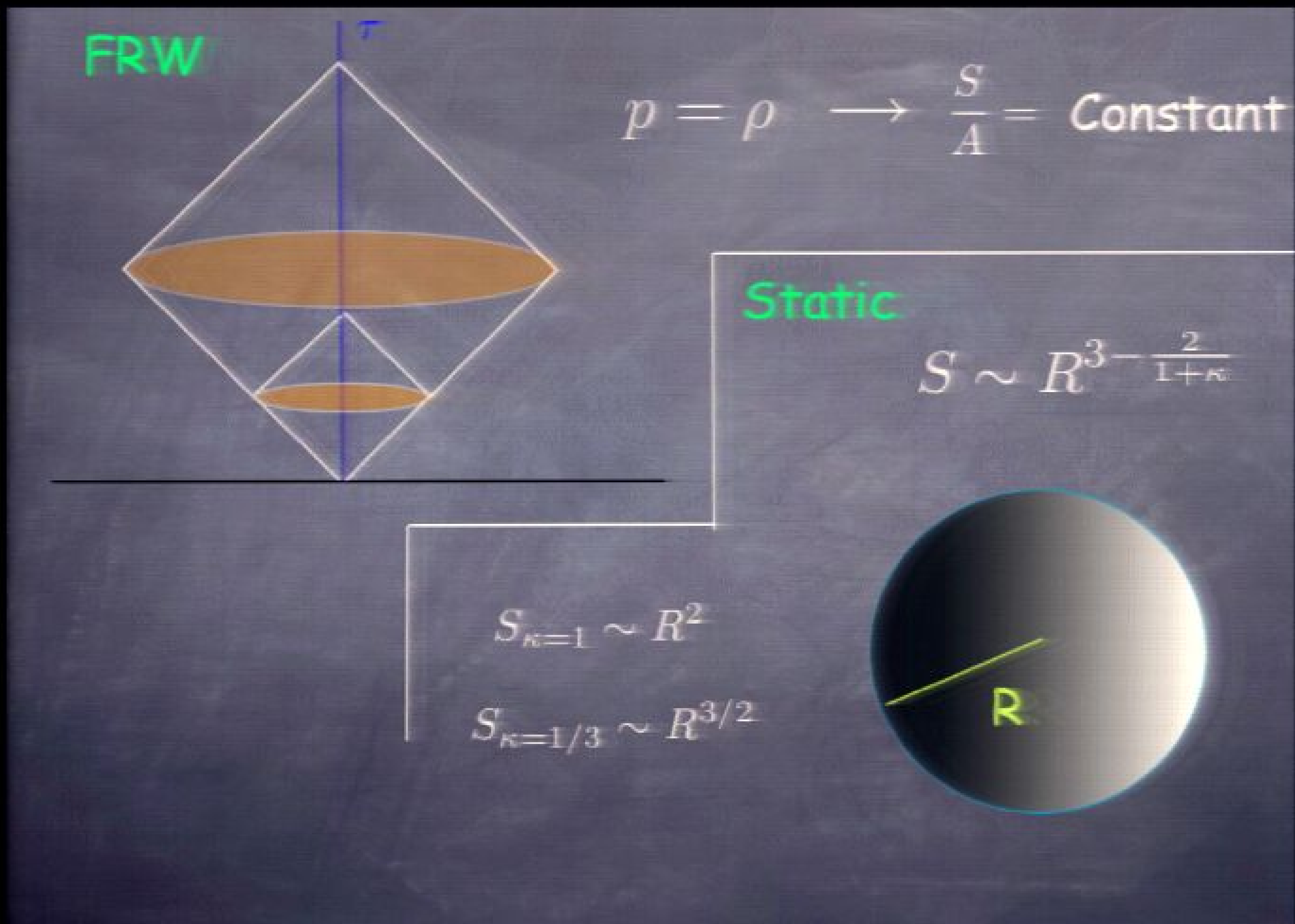
Continuity of the geometry:

$$tR^2(t) = t^{2/3}L^2$$

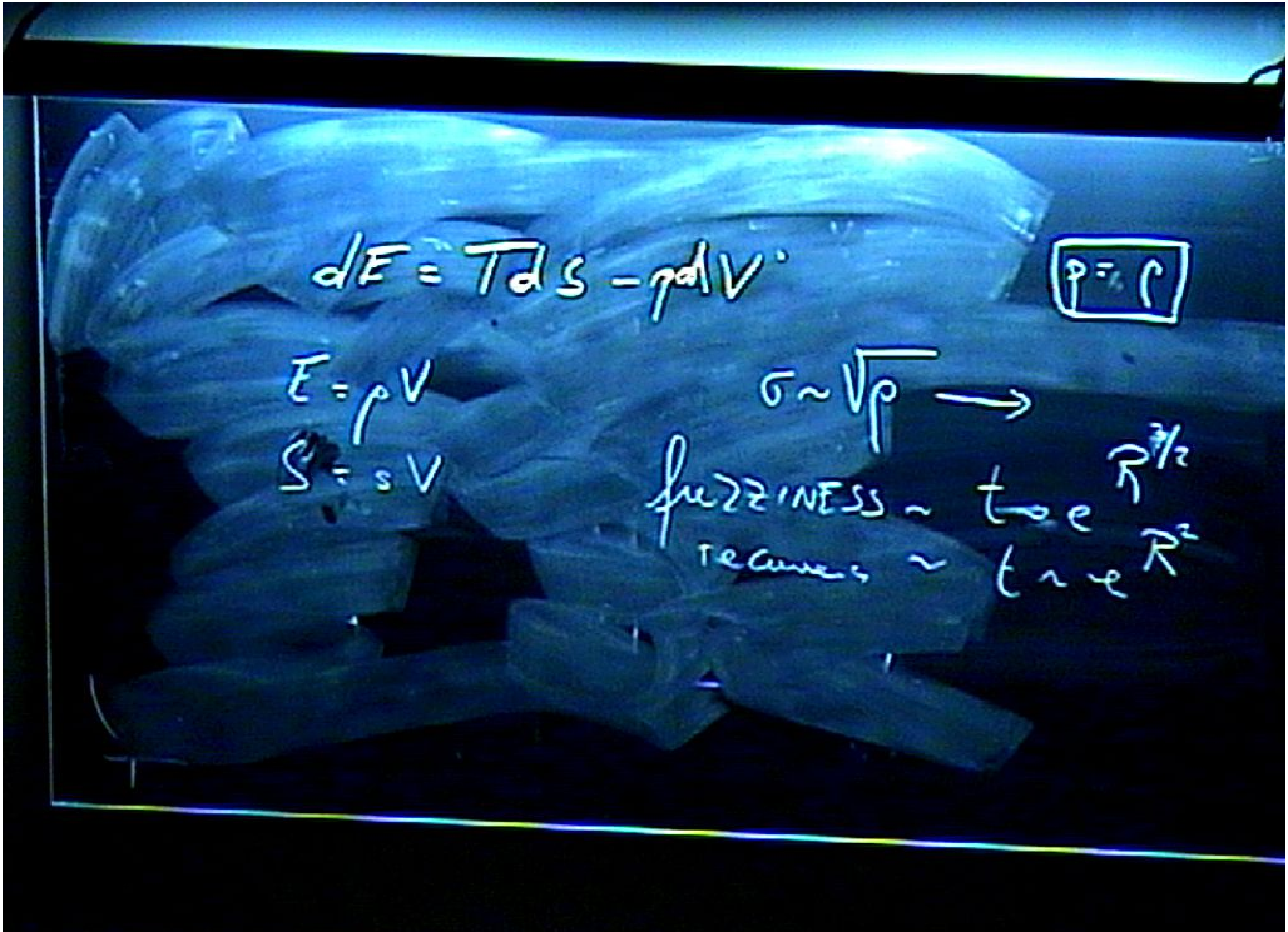
$$R(t) \sim t^{-1/6}$$

"normal" region shrinks

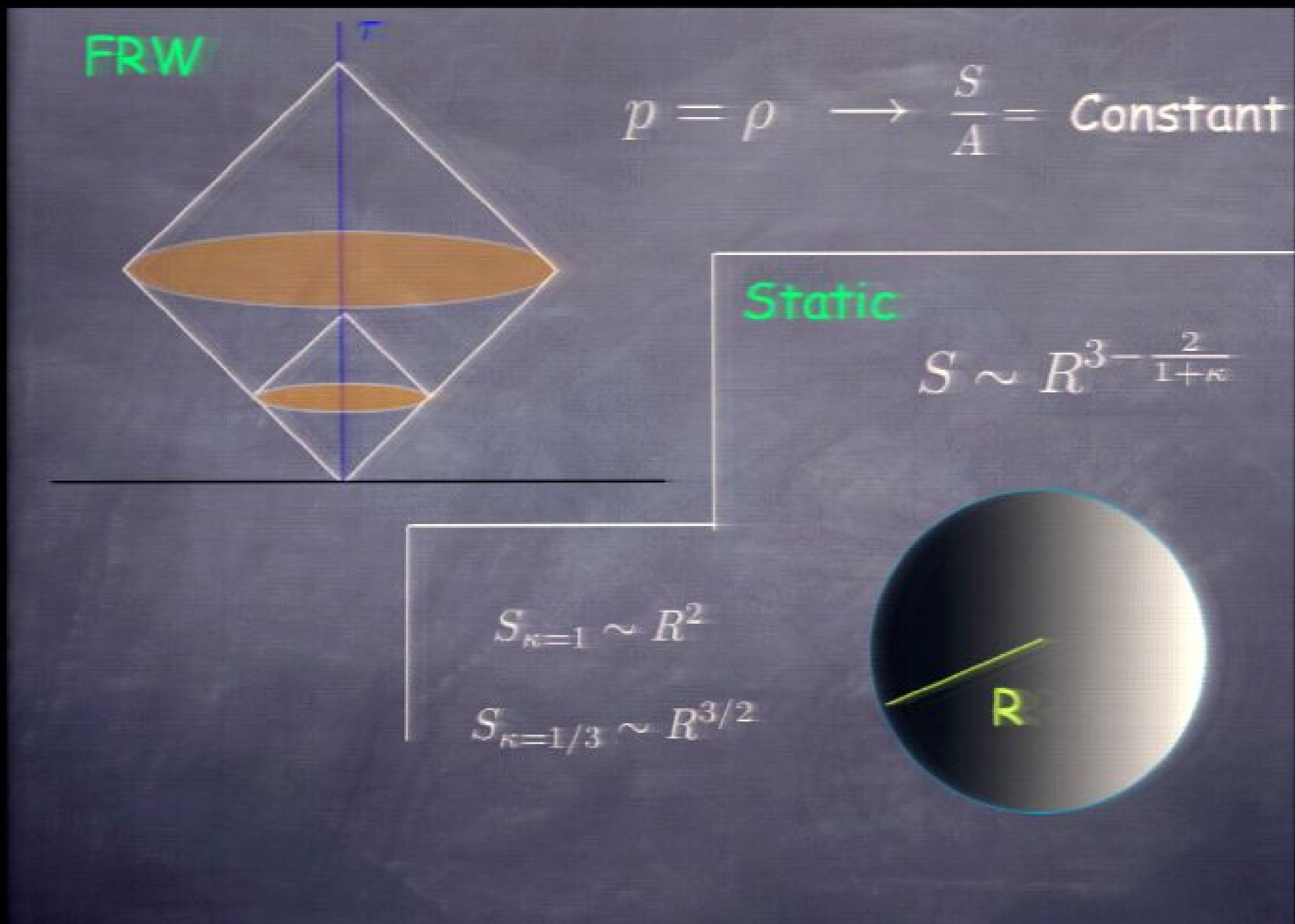

Multiverse
$p = -\Lambda_3$
$p = -\Lambda_2$
$p = \rho$
match onto the event horizon of a
black hole of equal area embedded
in the $p = \rho$ background


FRW
τ
$p = \rho \longrightarrow \frac{S}{A} =$ Constant
Static
$S \sim R^{3-\frac{2}{1+\kappa}}$
$S_{\kappa=1} \sim R^2$
$S_{\kappa=1/3} \sim R^{3/2}$
R

[redacted]



dE = Tds − pdV
E = ρV
S = sV
σ ~ √ρ ⟶
FUZZINESS ~ t ~ e^{R^{3/2}}
~ t ~ e^{R^2}


FRW
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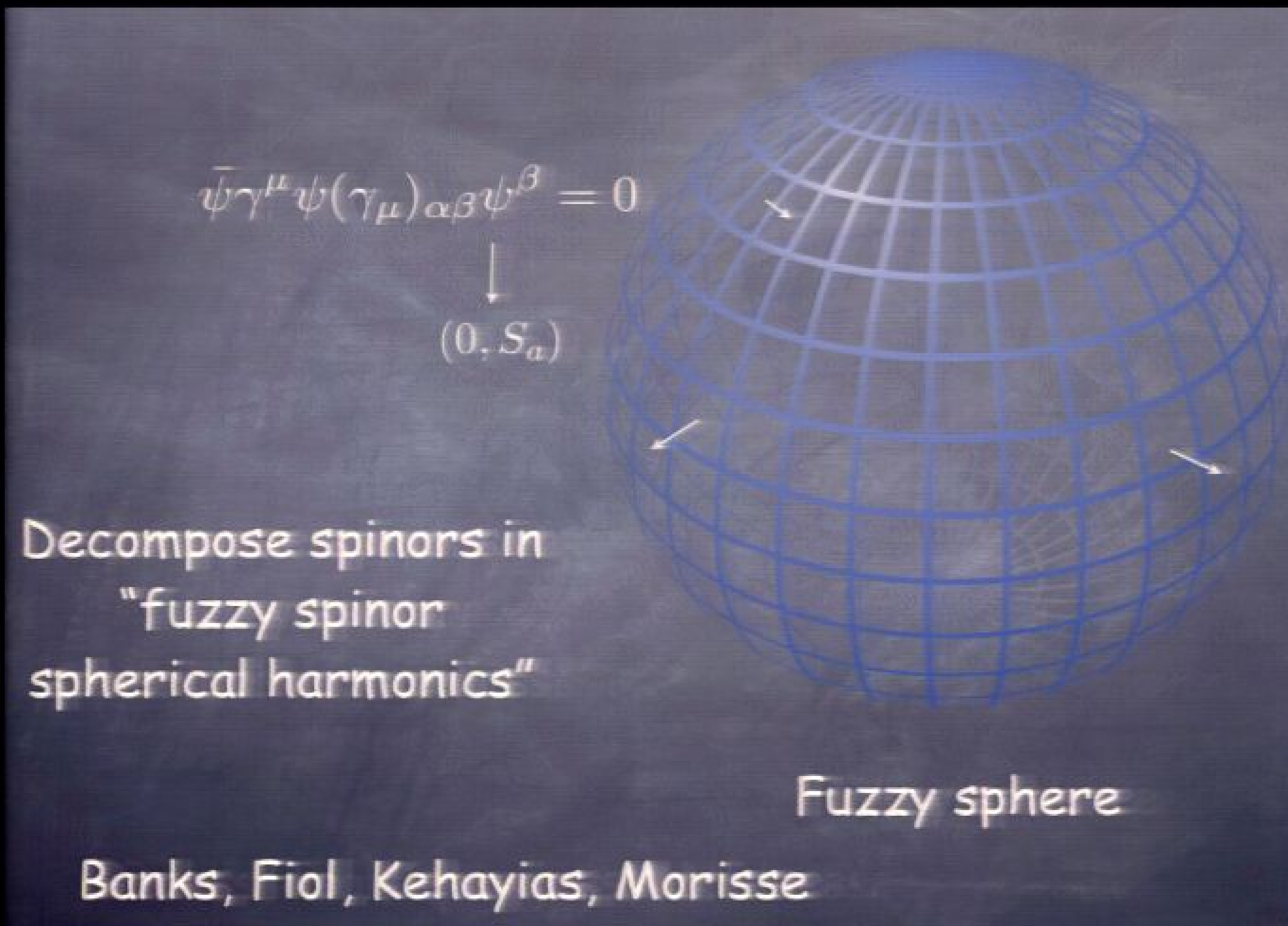
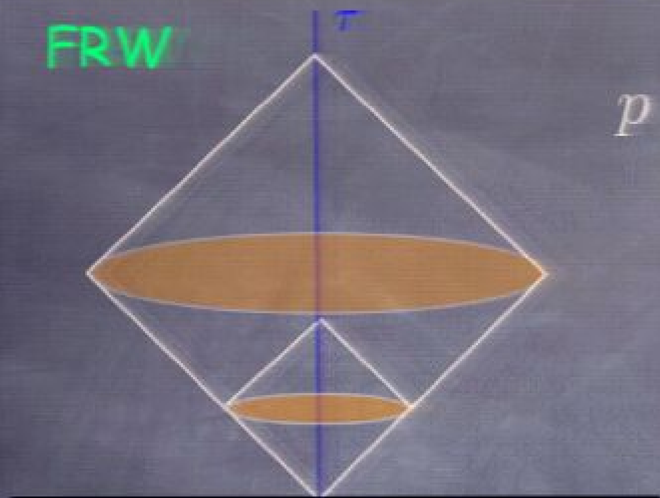
R


$\bar{\psi}\gamma^\mu\psi(\gamma_\mu)_{\alpha\beta}\psi^\beta = 0$
$(0, S_a)$
Decompose spinors in "fuzzy spinor spherical harmonics"
Fuzzy sphere
Banks, Fiol, Kehayias, Morisse

FRW



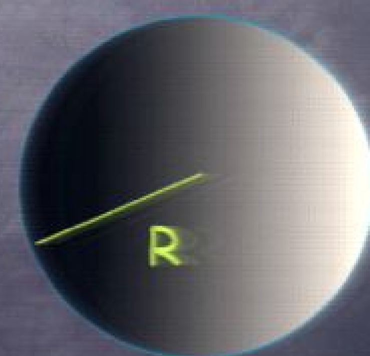
$$p = \rho \longrightarrow \frac{S}{A} = \text{Constant}$$

Static

$$S \sim R^{3-\frac{2}{1+\kappa}}$$

$$S_{\kappa=1} \sim R^2$$

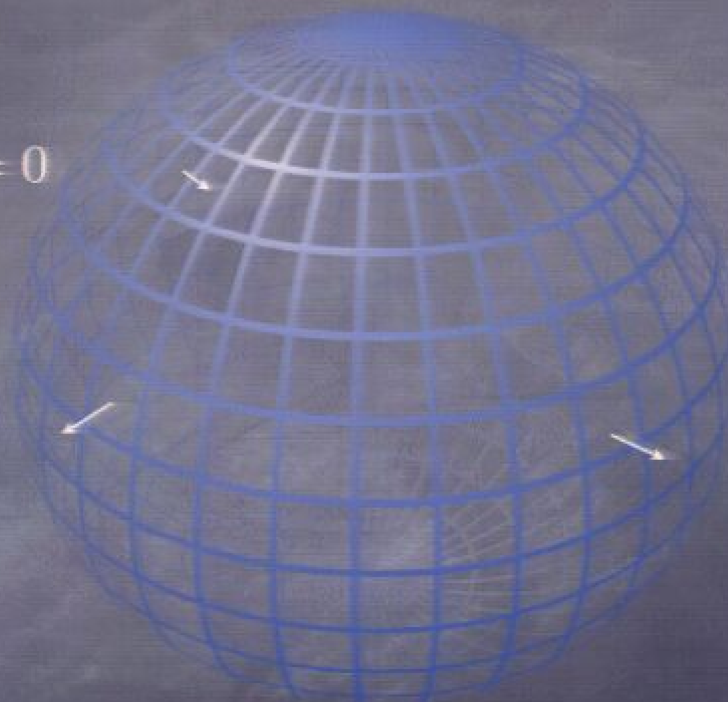
$$S_{\kappa=1/3} \sim R^{3/2}$$



$$\bar{\psi}\gamma^{\mu}\psi(\gamma_{\mu})_{\alpha\beta}\psi^{\beta} = 0$$

$$\downarrow$$

$$(0, S_a)$$



Decompose spinors in "fuzzy spinor spherical harmonics"

Fuzzy sphere

Banks, Fiol, Kehayias, Morisse

Reducible representation of SU(2) up to spin N-1

$$M_{A,B} = \begin{pmatrix} m_{1,1} & m_{1,2} & \cdots & m_{1,N} \\ m_{2,1} & m_{2,2} & \cdots & m_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ m_{N,1} & m_{N,2} & \cdots & m_{N,N} \end{pmatrix}$$

Reducible representation of SU(2) up to spin N-1/2

$$S_{A,i} = \begin{pmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,N} \\ s_{2,1} & s_{2,2} & \cdots & s_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ s_{N+1,1} & s_{N+1,2} & \cdots & s_{N+1,N} \end{pmatrix}$$

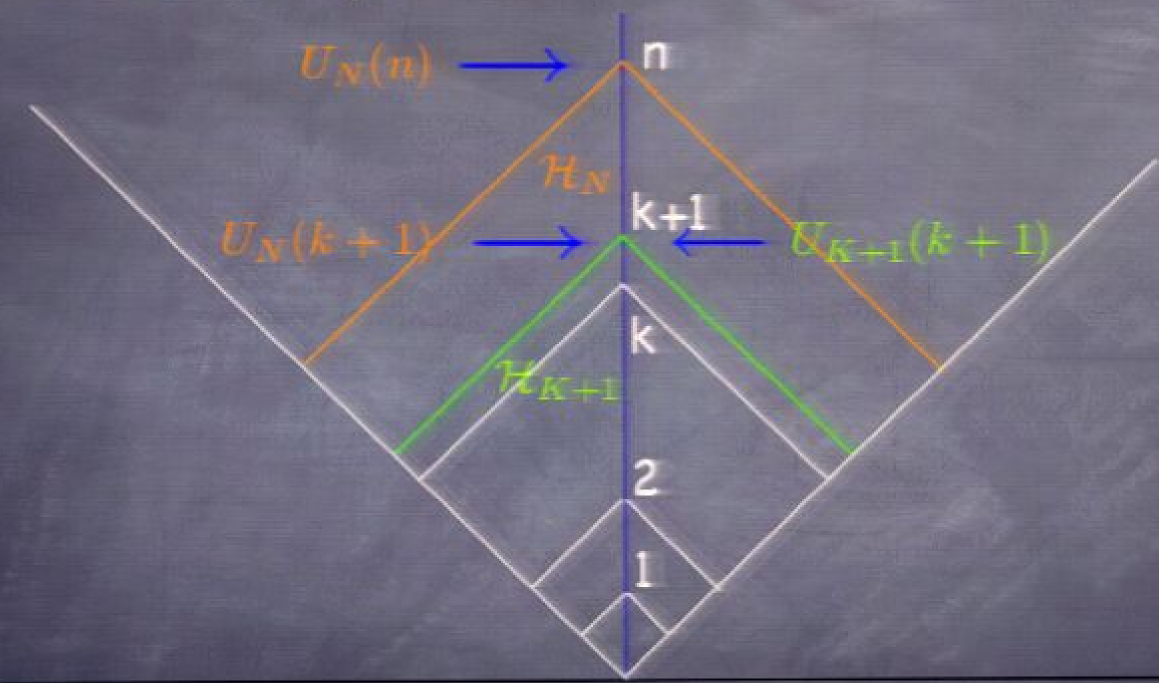
SU(2) covariant quantization

$$[S^A{}_i, S\dagger^j{}_B]_+ = \delta^j{}_i \delta^A{}_B$$

Hamiltonian

$$H \sim S\dagger^A_i(m) h_{mn} S^i_A(n)$$

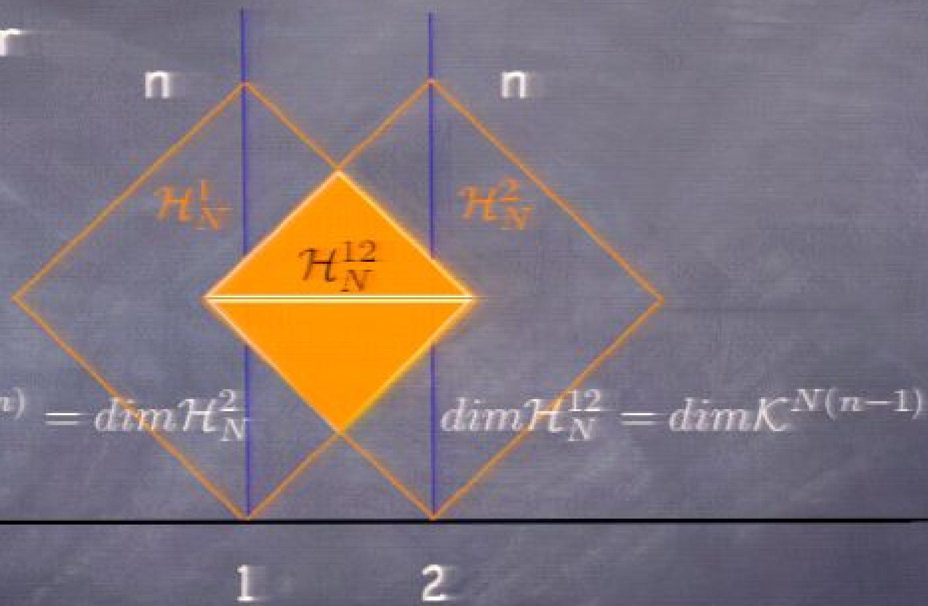
Consistency requirement:



$$U_N(k+1) = U_{K+1}(k+1) \otimes V_{NK+1}(k+1)$$

$$H_N(k+1) = H_{K+1}(k+1) \otimes I + I \otimes V_N(k+1)$$

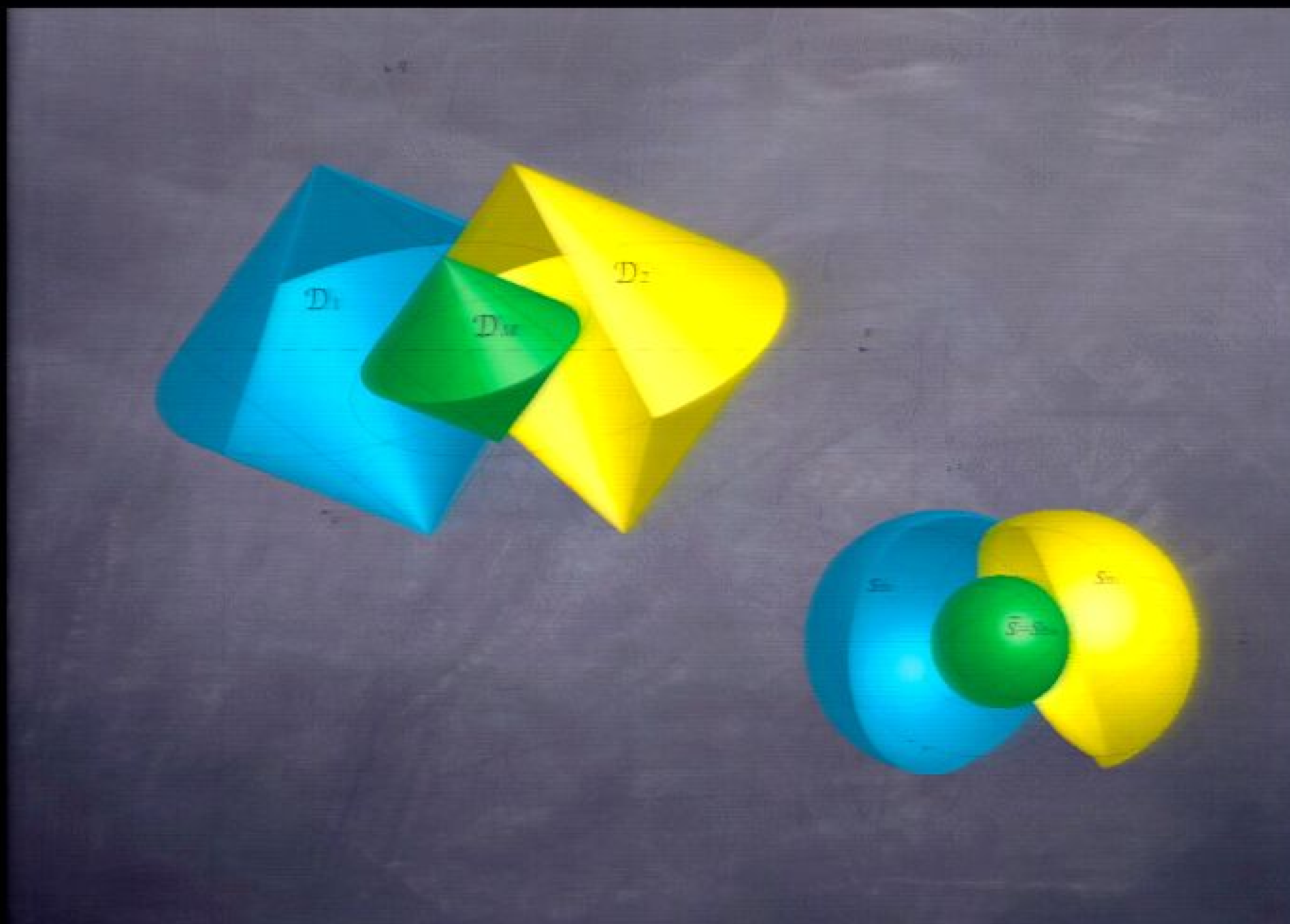
Nearest neighbor

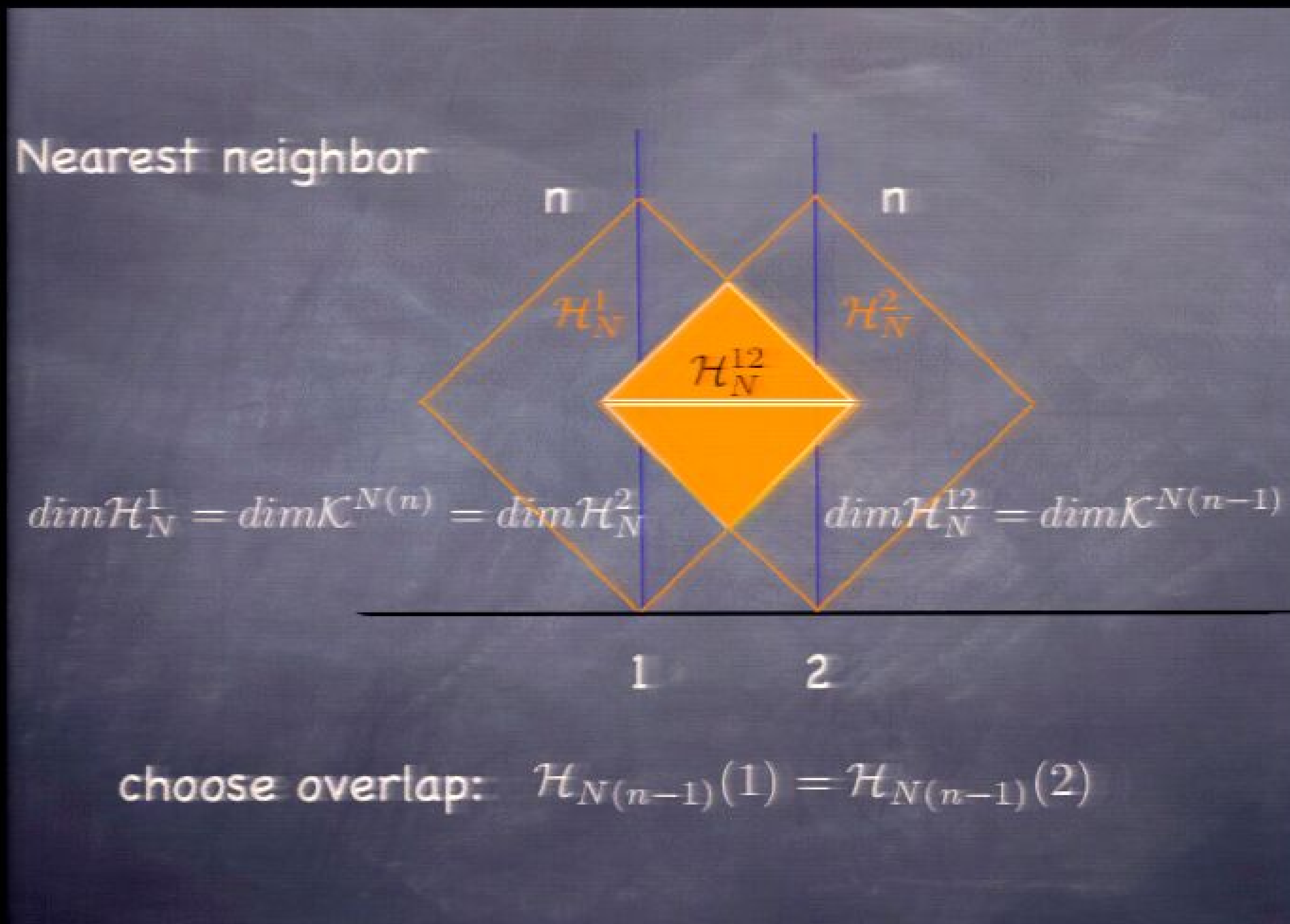


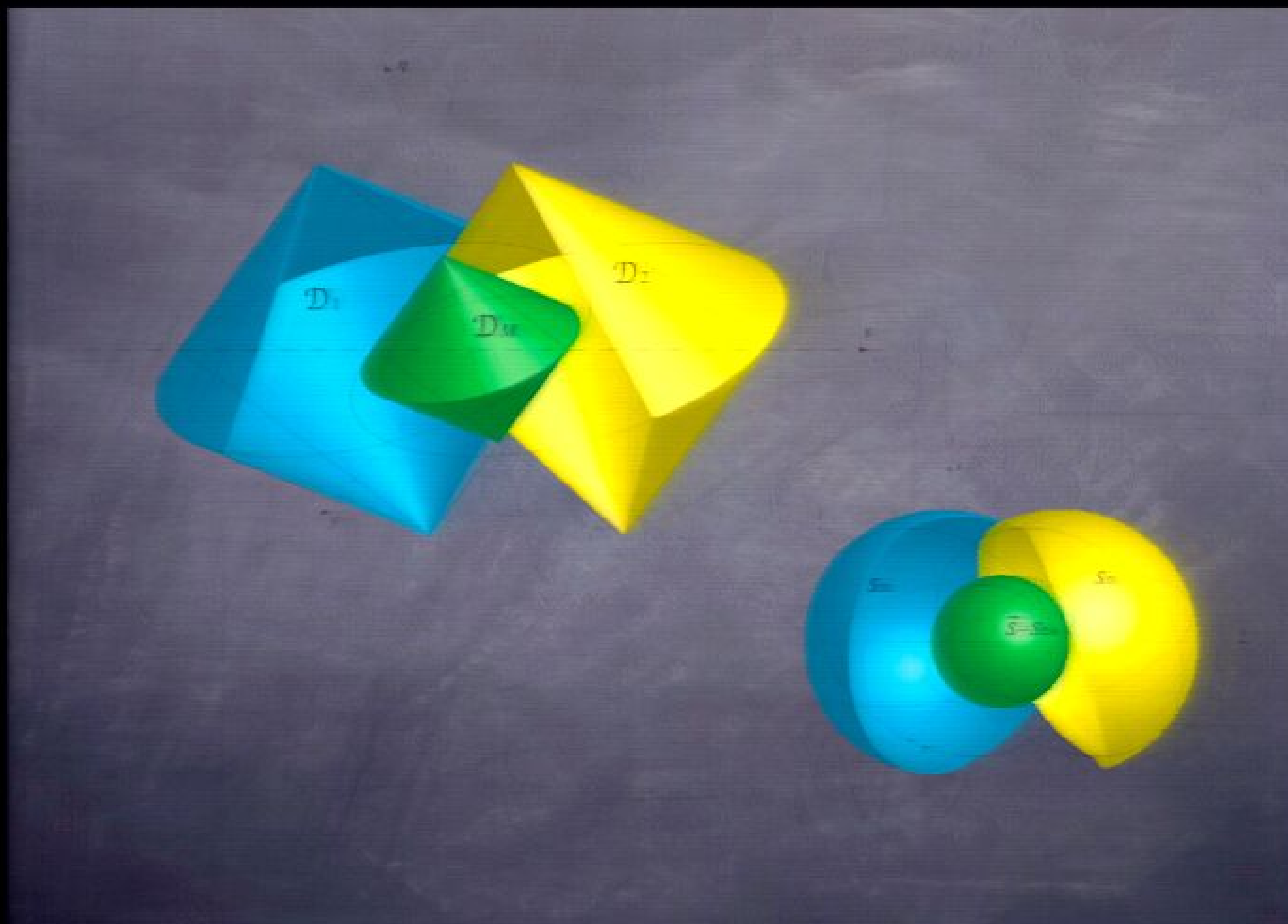
$$dim\mathcal{H}_N^1 = dim\mathcal{K}^{N(n)} = dim\mathcal{H}_N^2$$

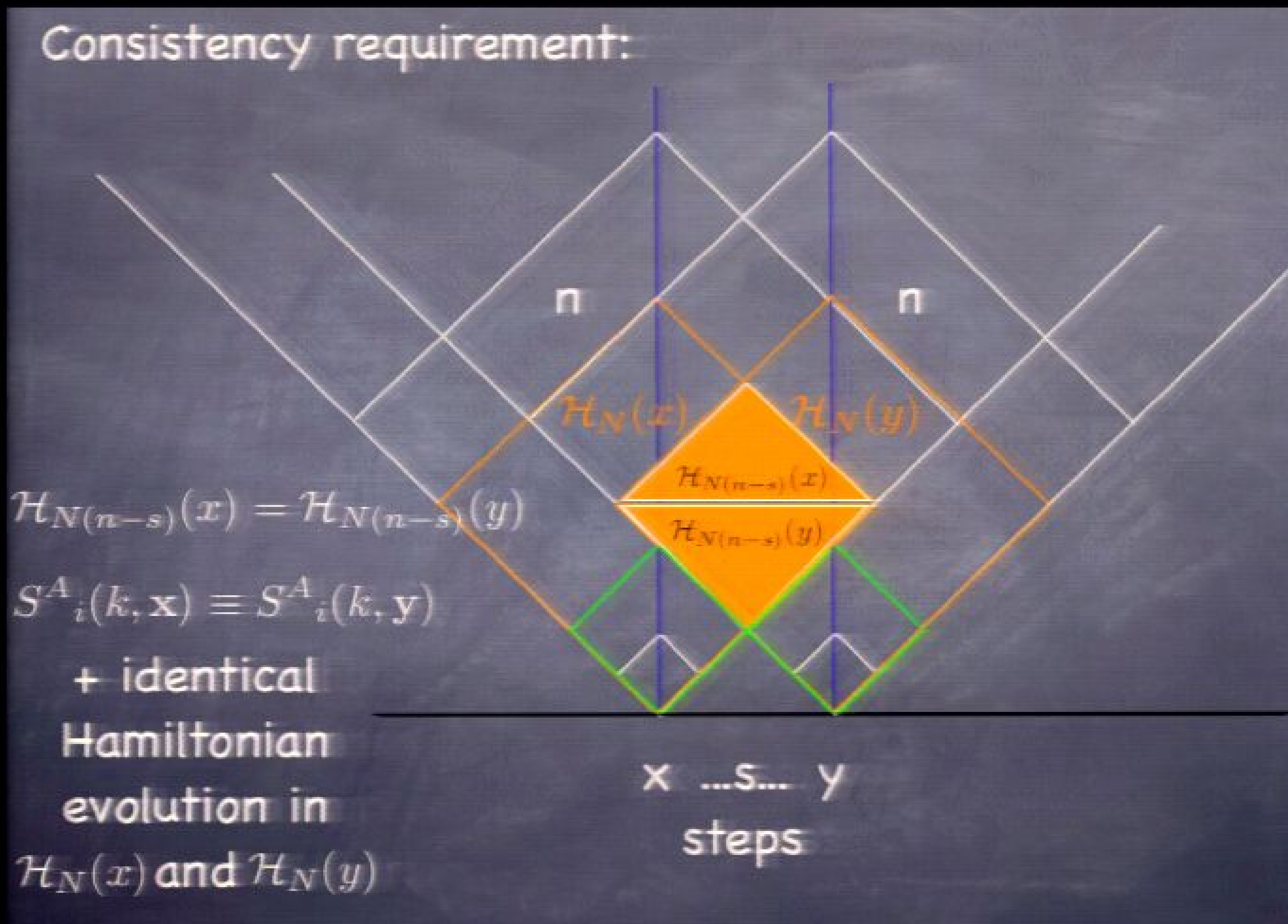
$$dim\mathcal{H}_N^{12} = dim\mathcal{K}^{N(n-1)}$$

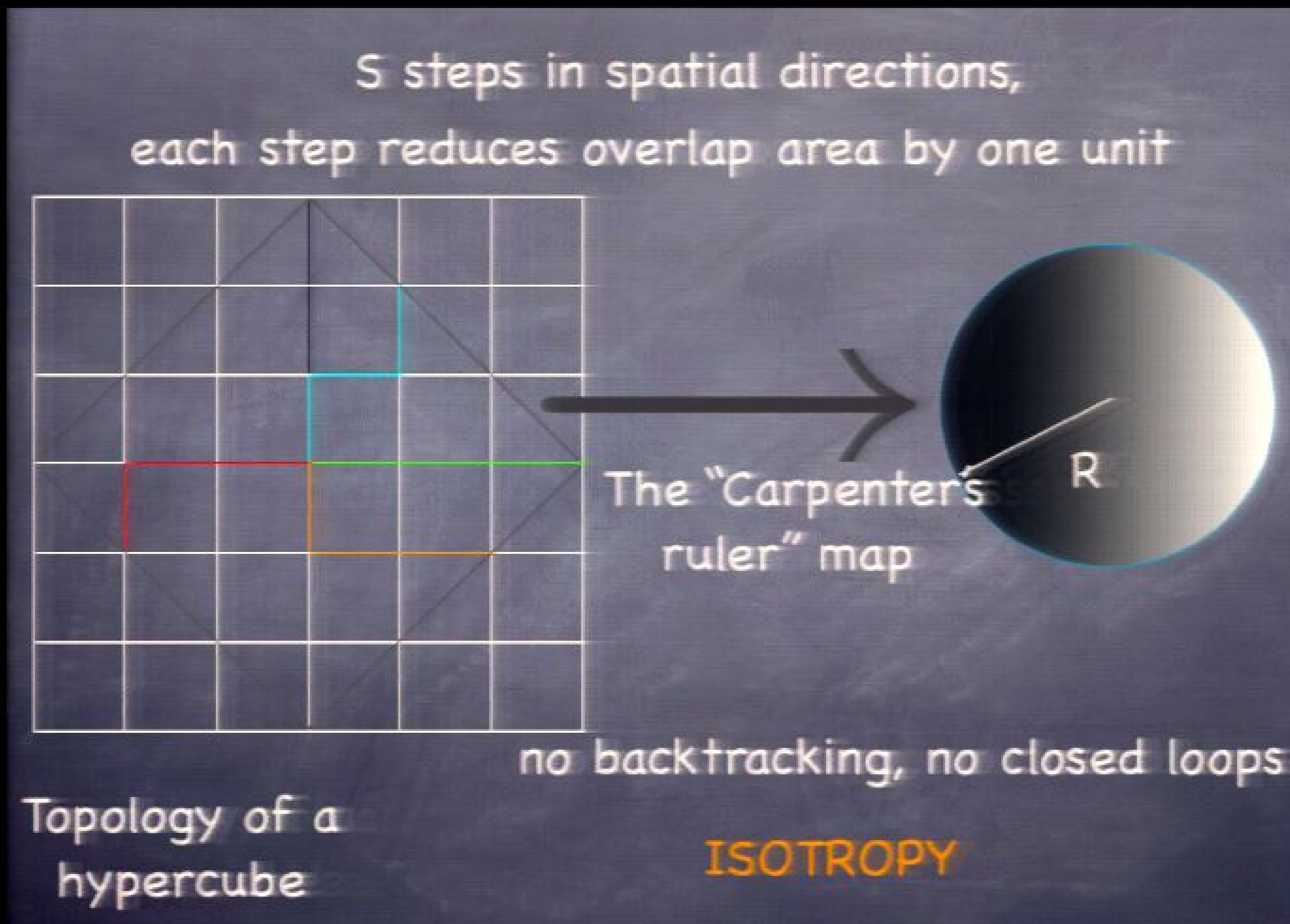
choose overlap: $\mathcal{H}_{N(n-1)}(1) = \mathcal{H}_{N(n-1)}(2)$


D1
D2
DM


Nearest neighbor
n
n
$\mathcal{H}_N^1$
$\mathcal{H}_N^2$
$\mathcal{H}_N^{12}$
$dim\mathcal{H}_N^1 = dim\mathcal{K}^{N(n)} = dim\mathcal{H}_N^2$
$dim\mathcal{H}_N^{12} = dim\mathcal{K}^{N(n-1)}$
1
2
choose overlap: $\mathcal{H}_{N(n-1)}(1) = \mathcal{H}_{N(n-1)}(2)$


$\mathfrak{D}_1$
$\mathfrak{D}_2$
$\mathfrak{D}_M$


Consistency requirement:
n
n
$\mathcal{H}_N(x)$
$\mathcal{H}_N(y)$
$\mathcal{H}_{N(n-s)}(x)$
$\mathcal{H}_{N(n-s)}(y)$
$\mathcal{H}_{N(n-s)}(x) = \mathcal{H}_{N(n-s)}(y)$
$S^A{}_i(k, \mathbf{x}) \equiv S^A{}_i(k, \mathbf{y})$
+ identical Hamiltonian evolution in $\mathcal{H}_N(x)$ and $\mathcal{H}_N(y)$
x ...s... y
steps


S steps in spatial directions,
each step reduces overlap area by one unit
R
The "Carpenter's ruler" map
no backtracking, no closed loops
Topology of a hypercube
ISOTROPY

$$U_N(N) = e^{iH_N(N)}$$

$$H_N(N) = S^{\dagger A}{}_i(m) h_{mn} S^i{}_A(n) \equiv \frac{1}{N} H_{FT}$$

The matrix h is chosen from a random gaussian ensemble

$$P(h) = e^{-NTrh^2}$$

This ensures that whatever the initial state of the system is, it will explore, in the N-evolution, all of the Hilbert space

In the large N limit: $d(\lambda) = \sqrt{1-\lambda^2}$

(Wigner's semi-circle)

The spectrum is linear near the origin, $\lambda_i \sim \frac{1}{N}$
with cut-off$\sim$1

The large N thermodynamics of $H_{FT} \equiv N H_N(N)$

1 + 1 dimensional fermion system with UV
cutoff 1 and spatial length N

$$\sigma \sim \sqrt{\rho}$$

The thermodynamics is dominated by the IR of a 1+1 CFT

This will be unchanged by a large class of perturbations of H_{FT}

Only the four Fermi interaction can be marginally relevant (the mass term = 0 because of the random ansatz)

There is the additional restriction on the class of allowed operators in H_{FT} :

locality in the IR

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locality in the IR

FLATNESS

– The model saturates the entropy bound

At large N, excited states are generic states obtained by the action of a sequence of random Hamiltonians

Flat $p = \rho$ saturates the entropy bound

– The spectrum is that of a $1 + 1$ CFT, the corresponding FRW universe must have a conformal isometry

Every flat FRW with a single component eq. of state

(spatial curvature violates the scale invariance)

The $p = \rho$ FRW Universe

has a conformal Killing symmetry:

$$ds^2 = dt^2 - t^{2/3} dx_i dx^i$$

$$t \rightarrow \lambda t$$

$$x_i \rightarrow \lambda^{2/3} x_i$$

TIME DEPENDENCE

Flat FRW: $A \sim t^{d-2} \leftrightarrow N(k) \sim k^{d-2} \sim t^{d-2}$

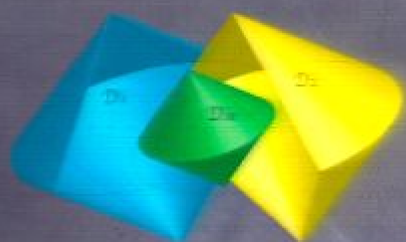
$$-iH(t)\Delta t \sim -iH_N \Delta N$$

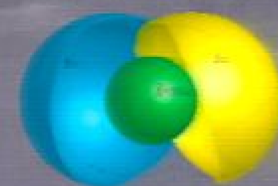
$$\boxed{H(t) \sim N^{\frac{d-3}{d-2}} H_N \sim N^{\frac{d-3}{d-2}}} \longleftrightarrow \boxed{M_{BH} \sim S_{BH}^{\frac{d-3}{d-2}}}$$

$H_N \sim \rho_{1+1} \sim 1$ $S \sim N$

$$\boxed{\rho \sim \frac{N^{\frac{d-3}{d-2}}}{N^{\frac{d-1}{d-2}}} \sim \frac{1}{t^2}} \qquad \boxed{\sigma \sim \frac{N}{N^{\frac{d-1}{d-2}}} \sim \frac{1}{t}}$$

$$\sigma \sim \sqrt{\rho}$$

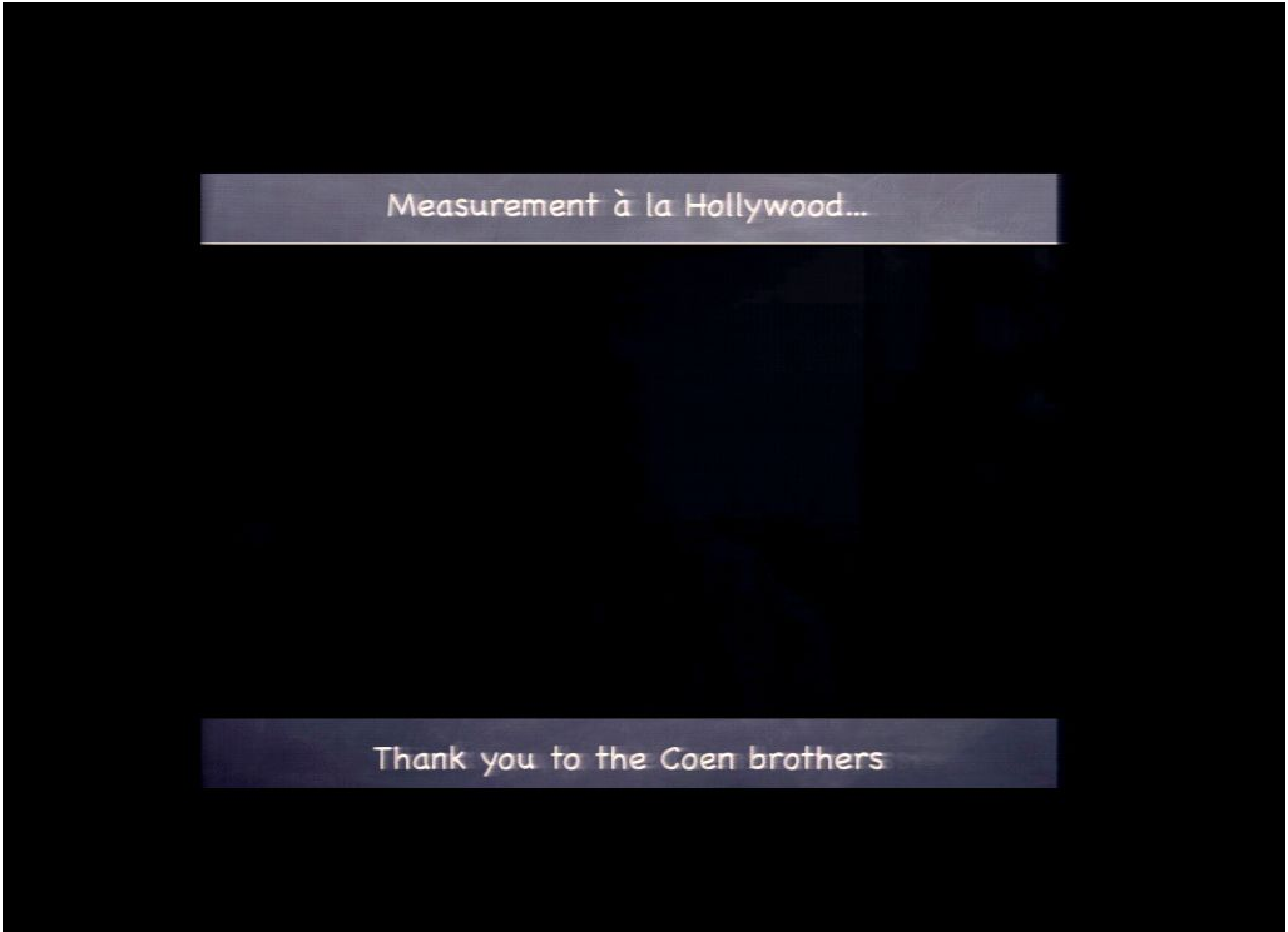


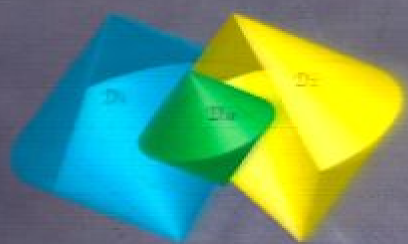


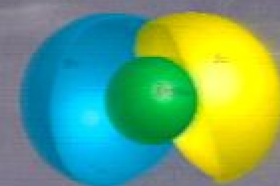
Area of overlaps comparison between the $p = \rho$ geometry and the overlap rules.

(Geodesic distance)^2 ↔ log dim (overlap Hilbert space)

This works.


Measurement à la Hollywood...
Thank you to the Coen brothers





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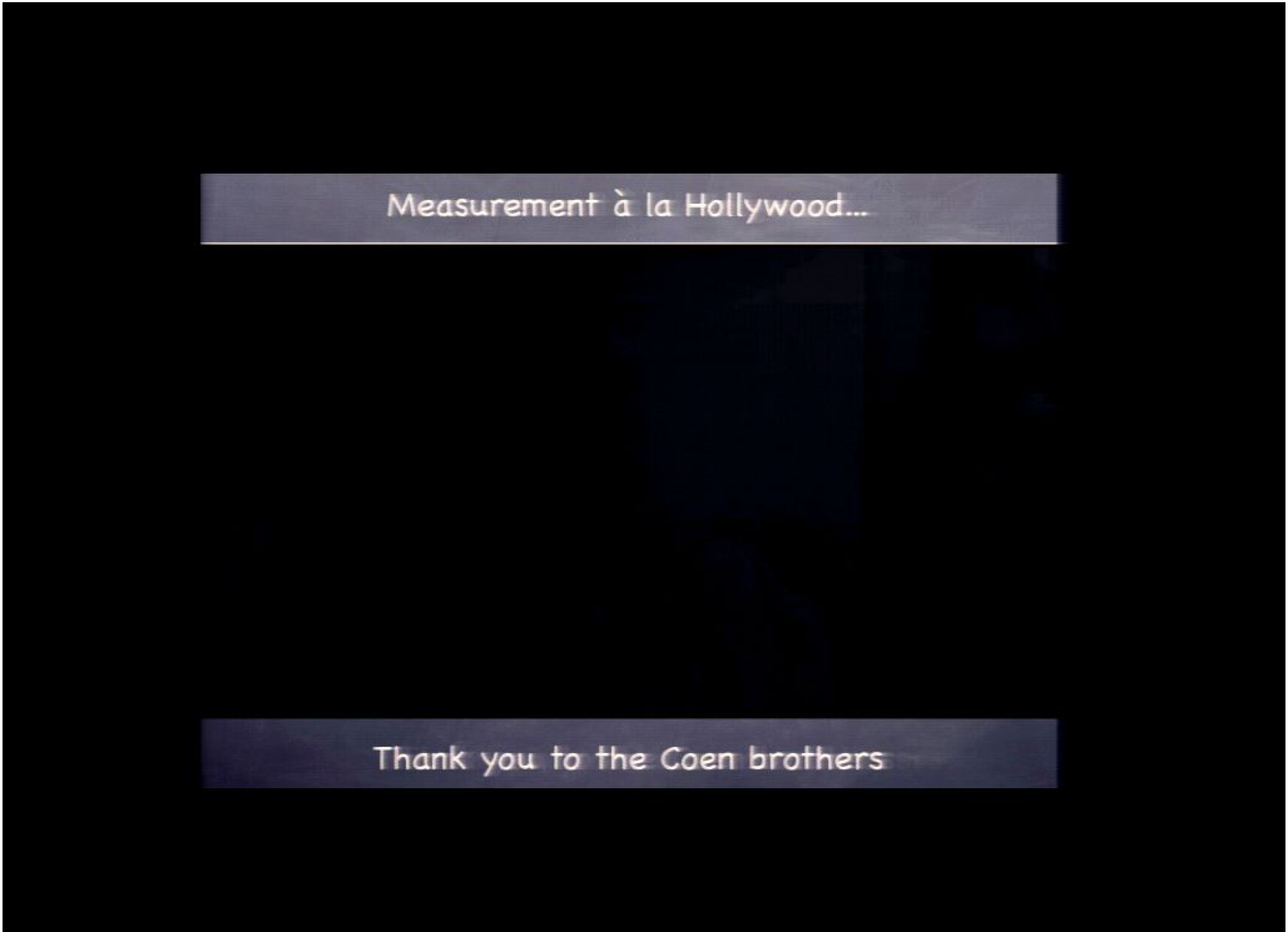
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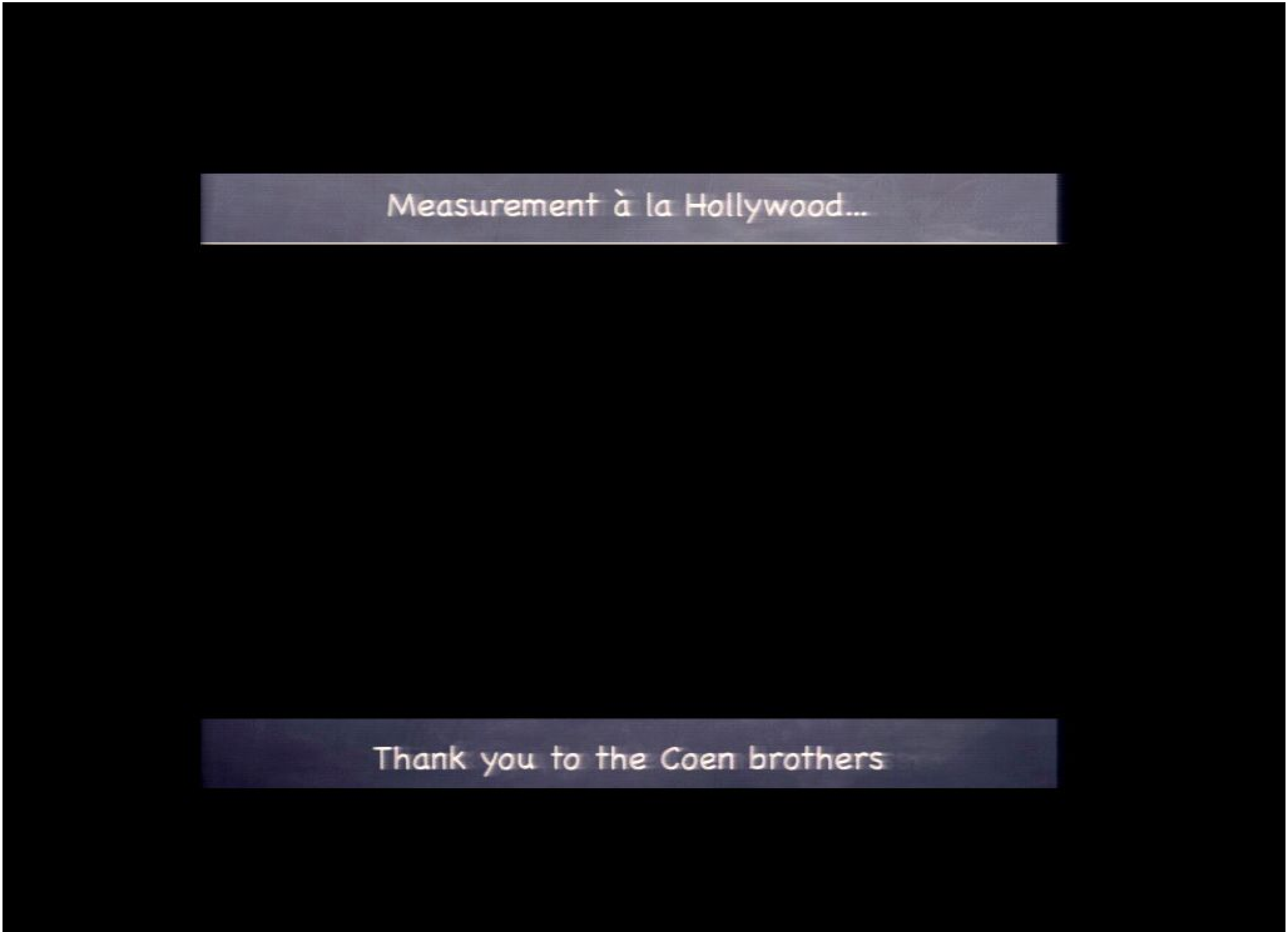
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