

Title: Antigravity Predicted Between Crunch and Bang in a Cyclic Universe

Date: Jul 14, 2011 04:30 PM

URL: <http://pirsa.org/11070013>

Abstract: Einstein's theory of General Relativity and its couplings to matter in 3+1 dimensions can be slightly enlarged with the requirement of a local scale (conformal) symmetry and the corresponding gauge degrees of freedom. This form of the theory is a prediction from 2T-gravity in 4+2 dimensions. It has no dimensionful constants, not even the gravitational constant, and requires all scalar fields to be conformally coupled to gravity and to the rest of matter. The theory can be gauge fixed to the usual gravity theory in the Einstein frame, thus generating the gravitational constant. Other physically equivalent forms of gauge fixing lead to the complete set of exact analytic solutions of the usual Friedmann equations, including radiation, curvature, anisotropy and a special potential for a scalar field coupled minimally to gravity. These analytic cosmological solutions, which are geodesically complete at singularities, reveal many surprising properties that are not noticeable with approximate cosmological solutions. Some aspects of the exact solutions will be reviewed in this lecture. In particular, it is predicted that the universe is cyclic and furthermore is has a period of antigravity between every big crunch and the following big bang.

Antigravity Between Crunch and Bang in a Geodesically Complete Cyclic Universe

Itzhak Bars

University of Southern California

Talk @

Challenges for Early Universe Cosmology
Perimeter Institute, July 2011

- 1) I.B. and S.H. Chen, **1004.0752**
- 2) I.B., and S.H. Chen and Neil Turok, **1105.3606**
- 3) I.B. + Chen + Turok + Steinhardt., in preparation (several papers)

Antigravity Between Crunch and Bang in a Geodesically Complete Cyclic Universe

Itzhak Bars

University of Southern California

Talk @
Challenges for Early Universe Cosmology
Perimeter Institute, July 2011

- 1) I.B. and S.H. Chen, **1004.0752**
- 2) I.B., and S.H. Chen and Neil Turok, **1105.3606**
- 3) I.B. + Chen + Turok + Steinhardt., in preparation (several papers)

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

FRW

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

Friedmann equations

$$\frac{\dot{a}_E^2}{a_E^4} = \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4}$$

$$\frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} = -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4}$$

$$\frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) = 0.$$

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

FRW

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

Friedmann equations

$$\begin{aligned} \frac{\dot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4} \\ \frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} &= -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4} \\ \frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) &= 0. \end{aligned}$$

6 parameters
 4=b,c,K, ρ_0
 2=(4-2) initial values

Analytically solved with this V:

found ALL solutions

I.B. and S.H. Chen, 1004.0752

I.B. + Chen + Turok, 1105.3606

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa} \right)^4 \left[b \cosh^4 \left(\frac{\kappa\sigma}{\sqrt{6}} \right) + c \sinh^4 \left(\frac{\kappa\sigma}{\sqrt{6}} \right) \right]$$

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

FRW

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

Friedmann equations

$$\begin{aligned} \frac{\dot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4} \\ \frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} &= -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4} \\ \frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) &= 0. \end{aligned}$$

6 parameters
 4=b,c,K, ρ_0
 2=(4-2) initial values

Analytically solved with this V:

found ALL solutions

I.B. and S.H. Chen, 1004.0752

I.B. + Chen + Turok, 1105.3606

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa} \right)^4 \left[b \cosh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) + c \sinh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) \right]$$

Generic solution is geodesically incomplete in Einstein gravity. There is a subset of geodesically complete solutions only with conditions on initial values and parameters of the model.

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

FRW

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

Friedmann equations

$$\begin{aligned} \frac{\dot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4} \\ \frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} &= -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4} \\ \frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) &= 0. \end{aligned}$$

6 parameters
 4=b,c,K, ρ_0
 2=(4-2) initial values

Analytically solved with this V:

found ALL solutions

I.B. and S.H. Chen, 1004.0752

I.B. + Chen + Turok, 1105.3606

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa} \right)^4 \left[b \cosh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) + c \sinh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) \right]$$

Generic solution is geodesically incomplete in Einstein gravity. There is a subset of geodesically complete solutions only with conditions on initial values and parameters of the model.

Cosmology with a scalar coupled to gravity

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

+radiation + matter

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

FRW

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

Friedmann equations

Also anisotropic metrics:
Kasner, Bianchi IX.

Two more fields in metric
important only near BB

Analytically solved with this V:

found ALL solutions

I.B. and S.H. Chen, 1004.0752

I.B. + Chen + Turok, 1105.3606

$$\begin{aligned} \frac{\dot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4} \\ \frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} &= -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4} \\ \frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) &= 0. \end{aligned}$$

6 parameters
4=b,c,K, ρ_0
2=(4-2) initial
values

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa} \right)^4 \left[b \cosh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) + c \sinh^4 \left(\frac{\kappa \sigma}{\sqrt{6}} \right) \right]$$

Generic solution is geodesically incomplete in Einstein gravity. There is a subset of
geodesically complete solutions only with conditions on initial values and parameters of the model.

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom) 3/14

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} \text{gravitational parameter} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom)

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

A prediction of **2T-gravity in 4+2 dims.** Also motivated by colliding branes scenario.

Fundamental: Gauge symmetry in phase space
I.B. 0804.1585, I.B.-Chen 0811.2510

Steinhardt + Turok

McFadden + Turok 0409122

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom) 3/14

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

A prediction of **2T-gravity in 4+2 dims.** Also motivated by colliding branes scenario.

Fundamental: Gauge symmetry in phase space
I.B. 0804.1585, I.B.+Chen 0811.2510

Steinhardt + Turok

McFadden + Turok 0409122

Weyl symmetry can be gauge fixed in several forms.

Einstein gauge $\frac{1}{12} (\phi_E^2 - s_E^2) = \frac{1}{2\kappa^2}$ $\phi_E(x), s_E(x), g_E^{\mu\nu}(x)$

$$\phi_E(x) = \pm \frac{\sqrt{6}}{\kappa} \cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right), \quad s_E(x) = \frac{\sqrt{6}}{\kappa} \sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g_E) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom) 3/14

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

A prediction of **2T-gravity in 4+2 dims.** Also motivated by colliding branes scenario.

Fundamental: Gauge symmetry in phase space
I.B. 0804.1585, I.B.+Chen 0811.2510

Steinhardt + Turok

McFadden + Turok 0409122

Weyl symmetry can be gauge fixed in several forms.

Einstein gauge $\frac{1}{12} (\phi_E^2 - s_E^2) = \frac{1}{2\kappa^2}$ $\phi_E(x), s_E(x), g_E^{\mu\nu}(x)$

$$\phi_E(x) = \pm \frac{\sqrt{6}}{\kappa} \cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right), \quad s_E(x) = \frac{\sqrt{6}}{\kappa} \sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g_E) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

This is not the whole story: Einstein gauge is valid only when the *gauge invariant* quantity $[1 - s^2(x^\mu) / \phi^2(x^\mu)]$ is positive.

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom) 3/14

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

A prediction of **2T-gravity in 4+2 dims.** Also motivated by colliding branes scenario.

Fundamental: Gauge symmetry in phase space
I.B. 0804.1585, I.B.+Chen 0811.2510

Steinhardt + Turok

McFadden + Turok 0409122

Weyl symmetry can be gauge fixed in several forms.

Einstein gauge $\frac{1}{12} (\phi_E^2 - s_E^2) = \frac{1}{2\kappa^2}$ $\phi_E(x), s_E(x), g_E^{\mu\nu}(x)$

$$\phi_E(x) = \pm \frac{\sqrt{6}}{\kappa} \cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right), \quad s_E(x) = \frac{\sqrt{6}}{\kappa} \sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g_E) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

This is not the whole story: Einstein gauge is valid only when the *gauge invariant* quantity $[1 - s^2(x^\mu) / \phi^2(x^\mu)]$ is positive.

For geodesic completeness: a slight extension of Einstein gravity (gauge degrees of freedom) 3/14

Local scaling symmetry (Weyl): allows only conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions, more conformal scalars, in complete Weyl invariant theory.)

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\nu} \partial_\mu s \partial_\nu s + \frac{1}{12} (\phi^2 - s^2) R(g) - \phi^4 f\left(\frac{s}{\phi}\right) \right)$$

A prediction of **2T-gravity in 4+2 dims.** Also motivated by colliding branes scenario.

Fundamental: Gauge symmetry in phase space
I.B. 0804.1585, I.B.+Chen 0811.2510

Steinhardt + Turok

McFadden + Turok 0409122

Weyl symmetry can be gauge fixed in several forms.

Einstein gauge $\frac{1}{12} (\phi_E^2 - s_E^2) = \frac{1}{2\kappa^2}$ $\phi_E(x), s_E(x), g_E^{\mu\nu}(x)$

$$\phi_E(x) = \pm \frac{\sqrt{6}}{\kappa} \cosh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right), \quad s_E(x) = \frac{\sqrt{6}}{\kappa} \sinh\left(\frac{\kappa\sigma(x)}{\sqrt{6}}\right)$$

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g_E) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\}$$

This is not the whole story: Einstein gauge is valid only when the *gauge invariant* quantity $[1 - s^2(x^\mu) / \phi^2(x^\mu)]$ is positive.

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

case of only time dependent fields

$$ds_\gamma^2 = -d\tau^2 + \frac{dr^2}{1 - kr^2/r_0^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{FRW}_\gamma$$

$$R(g_\gamma) = 6K, \text{ with } K \equiv \frac{k}{r_0^2}, \quad k = 0, \pm 1.$$

$$L = \frac{1}{2}(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2) - \frac{K}{2}(-\phi_\gamma^2 + s_\gamma^2) - \phi^4 f\left(\frac{s}{\phi}\right)$$

Plus the energy constraint: $H=0$ This is equivalent to the 00 Einstein eq. $G_{00}=T_{00}$ which compensates for the ghost.

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

case of only time dependent fields

$$ds_\gamma^2 = -d\tau^2 + \frac{dr^2}{1 - kr^2/r_0^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{FRW}_\gamma$$

$$R(g_\gamma) = 6K, \text{ with } K \equiv \frac{k}{r_0^2}, \quad k = 0, \pm 1.$$

$$L = \frac{1}{2}(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2) - \frac{K}{2}(-\phi_\gamma^2 + s_\gamma^2) - \phi^4 f\left(\frac{s}{\phi}\right)$$

Plus the energy constraint: $H=0$ This is equivalent to the 00 Einstein eq. $G_{00}=T_{00}$ which compensates for the ghost.

connection between the γ -gauge and the Einstein gauge

BCRT transform
Bars Chen
Steindhardt Turak

$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right) \quad \begin{array}{l} \text{Positive} \\ \text{region} \end{array}$$

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

case of only time dependent fields

$$ds_\gamma^2 = -d\tau^2 + \frac{dr^2}{1 - kr^2/r_0^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad \text{FRW}_\gamma$$

$$R(g_\gamma) = 6K, \text{ with } K \equiv \frac{k}{r_0^2}, k = 0, \pm 1.$$

$$L = \frac{1}{2}(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2) - \frac{K}{2}(-\phi_\gamma^2 + s_\gamma^2) - \phi^4 f\left(\frac{s}{\phi}\right)$$

Nothing singular in γ -gauge

Plus the energy constraint: $H=0$ This is equivalent to the 00 Einstein eq. $G_{00}=T_{00}$ which compensates for the ghost.

connection between the γ -gauge and the Einstein gauge

BCRT transform
Bars Chen
Steindhardt Turak

Pirsa: 11070013

$$\phi^2(1-s^2/\phi^2)$$

$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

Positive region

Page 19/44

BB singularity in E-gauge: $a_E=0$: when gauge invariant factor vanishes in γ -gauge, or any gauge!!

Analytic solutions – all of them!!

$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} \left(-\phi_\gamma^2 + s_\gamma^2 \right) - \phi^4 f \left(\frac{s}{\phi} \right)$$

Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

Analytic solutions – all of them!!

$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} \left(-\phi_\gamma^2 + s_\gamma^2 \right) - \phi^4 f \left(\frac{s}{\phi} \right)$$

Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

BCST transform
Friedmann equations
become:

$$0 = \ddot{\phi}_\gamma - 4b\phi_\gamma^3 + K\phi_\gamma,$$

$$0 = \ddot{s}_\gamma + 4cs_\gamma^3 + Ks_\gamma,$$

$$0 = - \left(\frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{1}{2}K\phi_\gamma^2 \right) + \left(\frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{1}{2}Ks_\gamma^2 \right) + \rho_0$$

Completely decoupled equations,
except for the zero energy condition.
Solutions are **Jacobi elliptic functions**,
with various boundary conditions.

Analytic solutions – all of them!!

$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} \left(-\phi_\gamma^2 + s_\gamma^2 \right) - \phi^4 f \left(\frac{s}{\phi} \right)$$

Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

BCST transform
Friedmann equations become:

$$0 = \ddot{\phi}_\gamma - 4b\phi_\gamma^3 + K\phi_\gamma,$$

$$0 = \ddot{s}_\gamma + 4cs_\gamma^3 + Ks_\gamma,$$

$$0 = - \left(\frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{1}{2}K\phi_\gamma^2 \right) + \left(\frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{1}{2}Ks_\gamma^2 \right) + \rho_0$$

Completely decoupled equations, except for the zero energy condition. Solutions are **Jacobi elliptic functions**, with various boundary conditions.

First integral

$$\frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{K}{2}s_\gamma^2 = E_s \quad \frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{K}{2}\phi_\gamma^2 = E_\phi, \quad E_s \equiv E, \quad E_\phi = E + \rho_0$$

Particle in a potential problem, intuitively solved by looking at the plot of the potential.

$$H(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4,$$

$$H(s) = \frac{1}{2}\dot{s}^2 + V(s) \quad V(s) = \frac{1}{2}Ks^2 + cs^4$$

Analytic solutions – all of them!!

$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} \left(-\phi_\gamma^2 + s_\gamma^2 \right) - \phi^4 f \left(\frac{s}{\phi} \right)$$

Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

BCST transform
Friedmann equations become:

$$0 = \ddot{\phi}_\gamma - 4b\phi_\gamma^3 + K\phi_\gamma.$$

$$0 = \ddot{s}_\gamma + 4cs_\gamma^3 + Ks_\gamma.$$

$$0 = - \left(\frac{1}{2}\ddot{\phi}_\gamma - b\phi_\gamma^4 + \frac{1}{2}K\phi_\gamma^2 \right) + \left(\frac{1}{2}\ddot{s}_\gamma^2 + cs_\gamma^4 + \frac{1}{2}Ks_\gamma^2 \right) + \rho_0$$

First integral $\frac{1}{2}\dot{s}_\gamma^2 + cs_\gamma^4 + \frac{K}{2}s_\gamma^2 = E_s \quad \frac{1}{2}\dot{\phi}_\gamma^2 - b\phi_\gamma^4 + \frac{K}{2}\phi_\gamma^2 = E_\phi, \quad E_s \equiv E, \quad E_\phi = E + \rho_0$

Particle in a potential problem, intuitively solved by looking at the plot of the potential.

$$H(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4,$$

$$H(s) = \frac{1}{2}\dot{s}^2 + V(s) \quad V(s) = \frac{1}{2}Ks^2 + cs^4$$

$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4.$$

K=0 case
Quartic
potentials

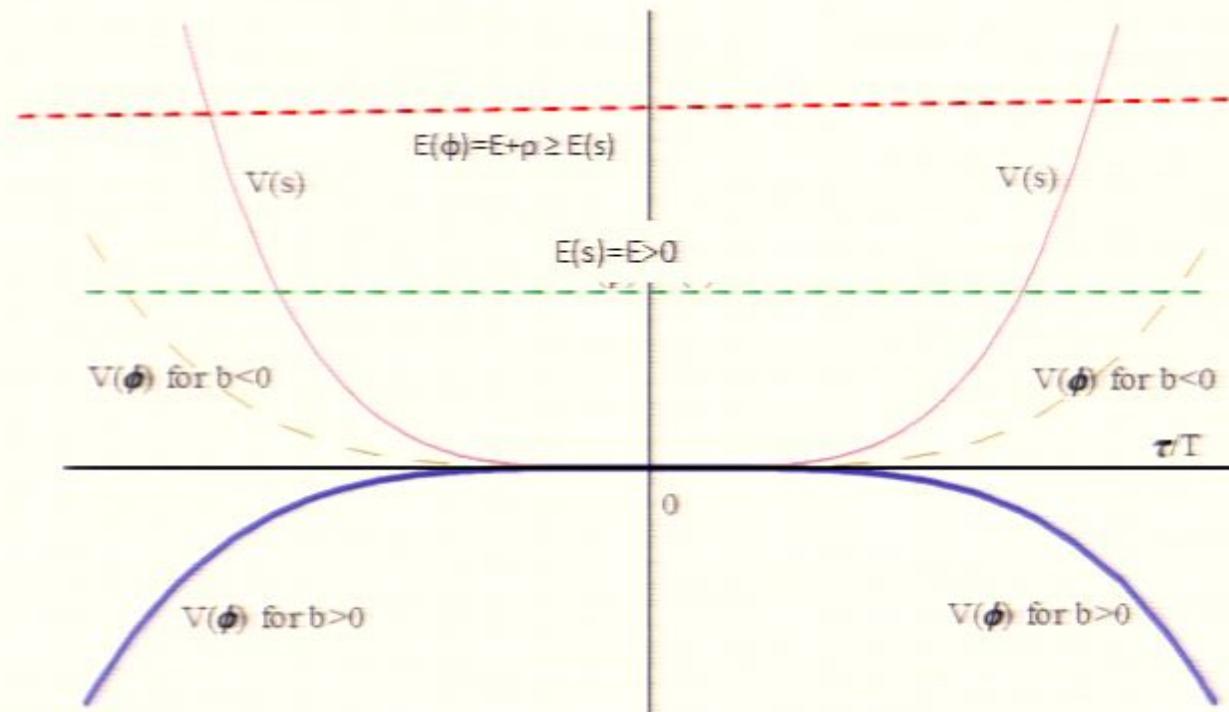


FIG. 1: The flat FRW universe, $k = 0$.

$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4.$$

$K=0$ case

Quartic
potentials

$A \times F[sn(z|m),$
 $cn(z|m),$
 $dn(z|m)]$

$z=(\tau-\tau_0)/T$

A, m, T
 depend on
 b, c, K, ρ, E

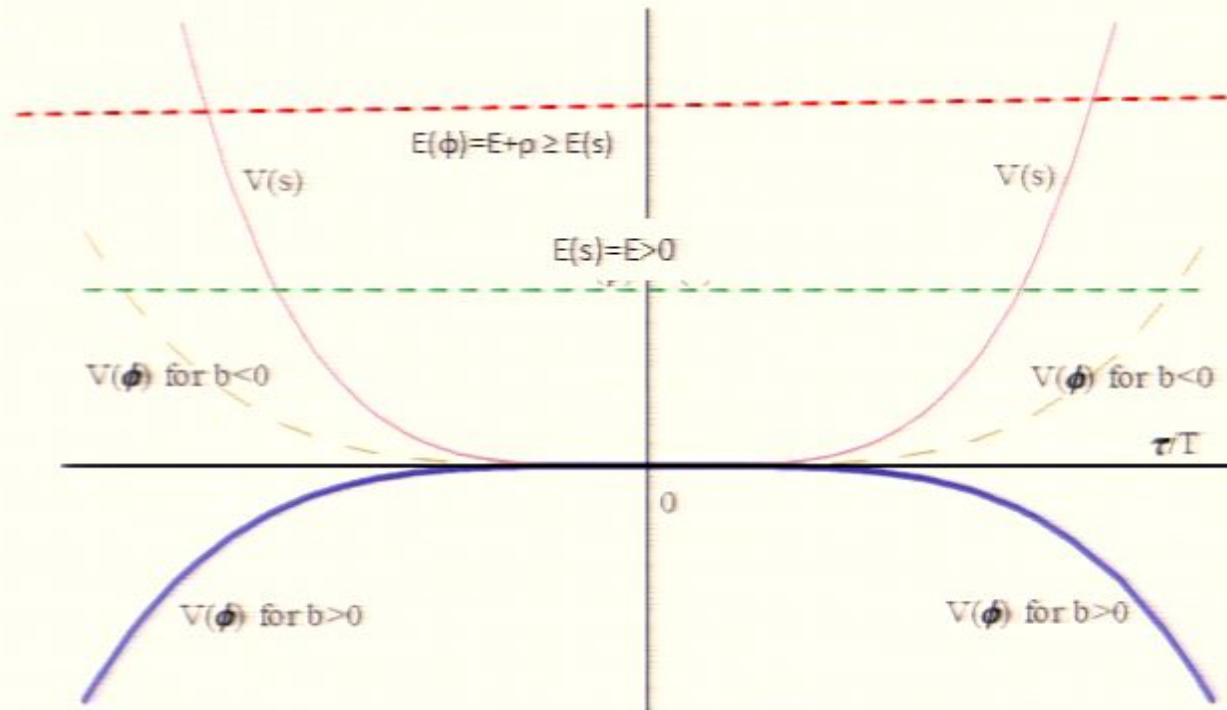


FIG. 1: The flat FRW universe, $k = 0$.

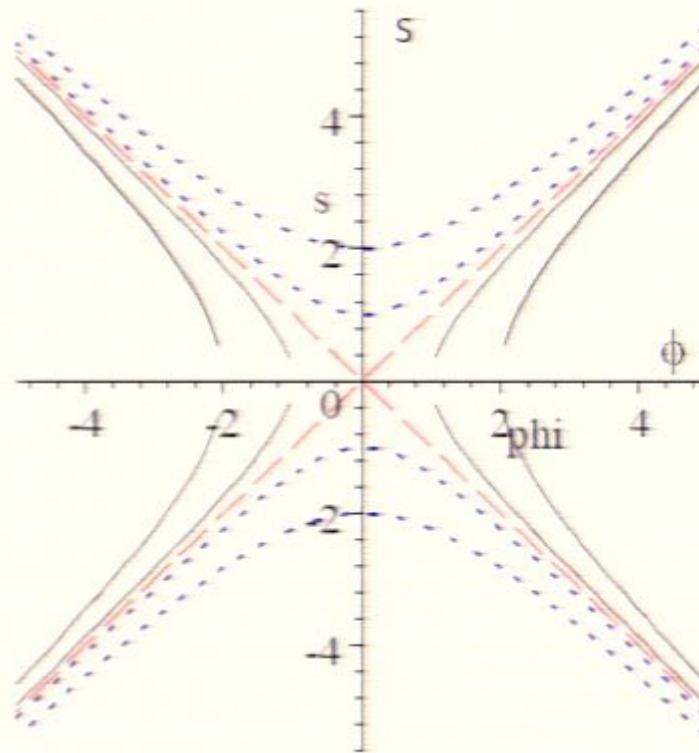
$\phi(\tau), s(\tau)$ perform independent oscillations

For generic initial conditions, the sign of $(\phi^2 - s^2)(\tau)$ changes over time.

So the generic solution is geodesically incomplete in the Einstein gauge.

There are special solutions that are geodesically complete, but must constrain parameter space.

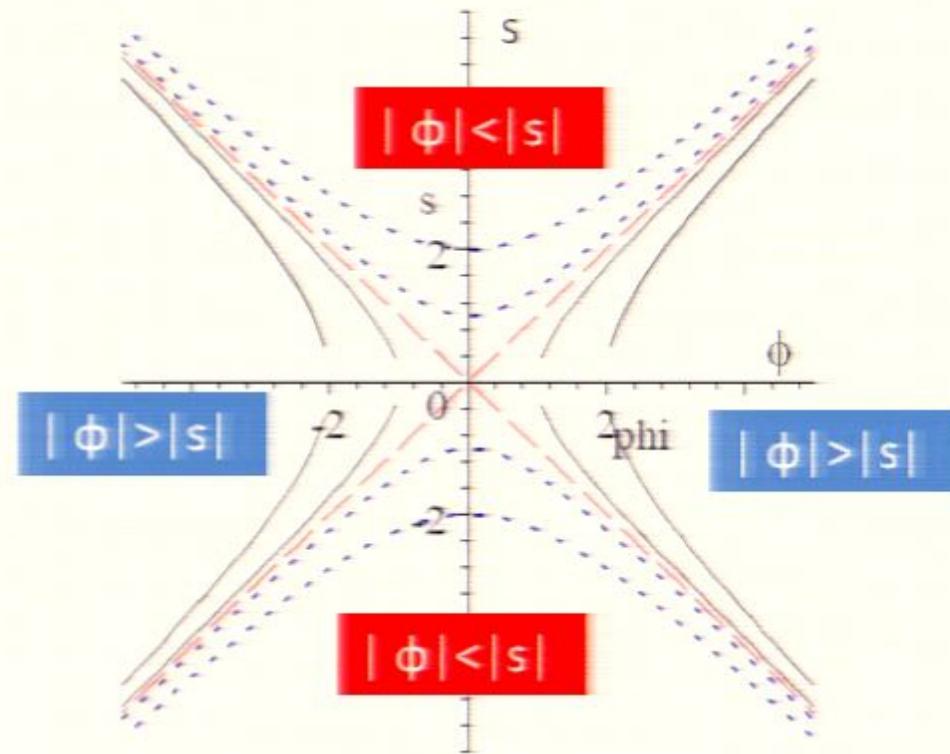
Geodesically complete larger space: ϕ_γ, s_γ plane



$$a_E^2 = z \theta(z), \quad z \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2), \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

$$\phi_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \cosh \left(\frac{\kappa \sigma}{\sqrt{6}} \right), \quad s_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \sinh \left(\frac{\kappa \sigma}{\sqrt{6}} \right).$$

Geodesically complete larger space: ϕ_γ, s_γ plane

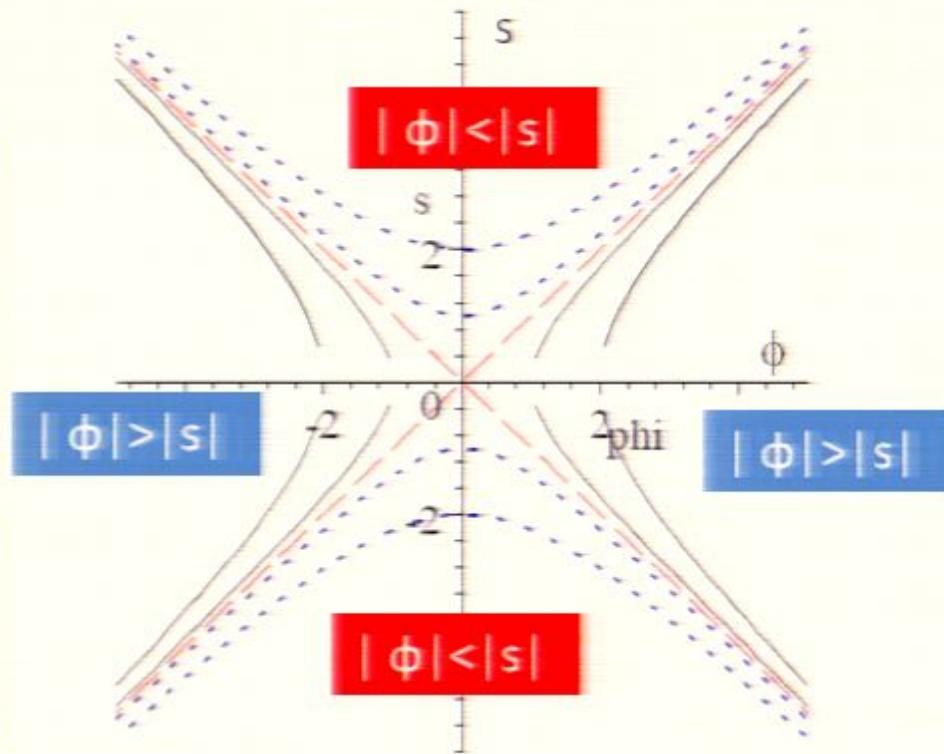


$$a_E^2 = z \theta(z), \quad z \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2), \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

$$\phi_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \cosh \left(\frac{\kappa \sigma}{\sqrt{6}} \right), \quad s_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \sinh \left(\frac{\kappa \sigma}{\sqrt{6}} \right).$$

Geodesically complete larger space: ϕ_γ, s_γ plane

Generic solution:
 $(\phi(\tau), s(\tau)$ periodic)
 is a smooth curve
 that spans the
 various quadrants.
 Closed curve if
 periods relatively
 quantized.
 (parametric plot
 using Mathematica)

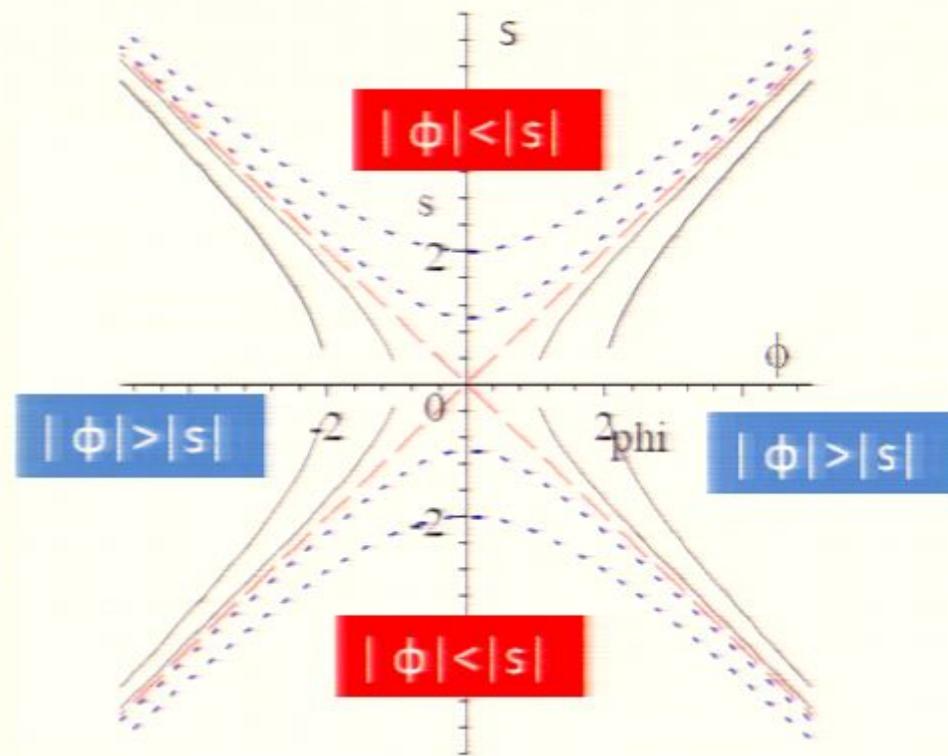


$$a_E^2 = z \theta(z), \quad z \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2), \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

$$\phi_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \cosh \left(\frac{\kappa \sigma}{\sqrt{6}} \right), \quad s_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \sinh \left(\frac{\kappa \sigma}{\sqrt{6}} \right).$$

Geodesically complete larger space: ϕ_γ, s_γ plane

Generic solution:
 $(\phi(\tau), s(\tau))$ periodic
 is a smooth curve
 that spans the
 various quadrants.
 Closed curve if
 periods relatively
 quantized.
 (parametric plot
 using Mathematica)



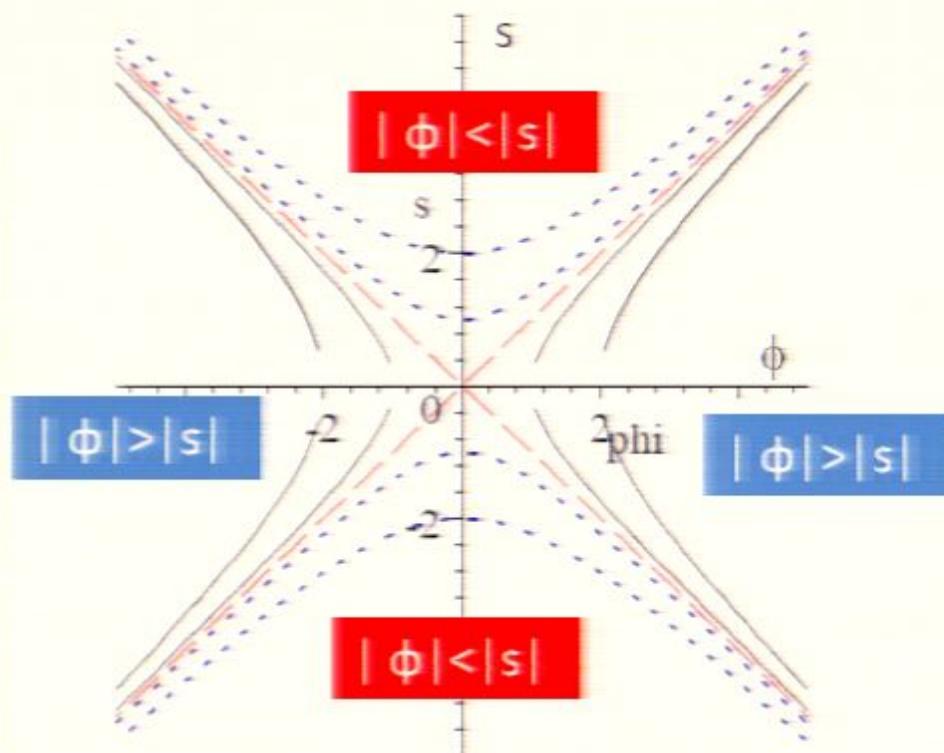
**big bangs or
 big crunches
 at the lightcone.**

$$a_E^2 = z \theta(z), \quad z \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2), \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

$$\phi_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \cosh \left(\frac{\kappa \sigma}{\sqrt{6}} \right), \quad s_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \sinh \left(\frac{\kappa \sigma}{\sqrt{6}} \right).$$

Geodesically complete larger space: ϕ_γ, s_γ plane

Generic solution:
 $(\phi(\tau), s(\tau))$ periodic
 is a smooth curve
 that spans the
 various quadrants.
 Closed curve if
 periods relatively
 quantized.
 (parametric plot
 using Mathematica)



**big bangs or
 big crunches
 at the lightcone.**

**Generic solution is
 a cyclic universe
 with antigravity
 stuck between
 crunch and bang!**

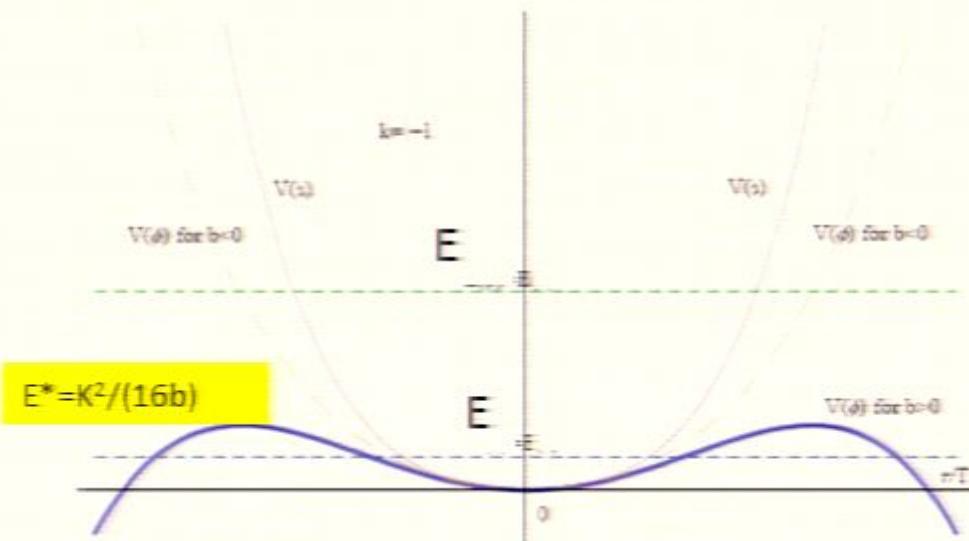
$$a_E^2 = z \theta(z), \quad z \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2), \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

$$\phi_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \cosh \left(\frac{\kappa \sigma}{\sqrt{6}} \right), \quad s_\gamma = \pm \frac{\sqrt{6}}{\kappa} \sqrt{|z|} \sinh \left(\frac{\kappa \sigma}{\sqrt{6}} \right).$$

$$V(s) = \frac{1}{2}K\phi^2 + cs^4$$

$$V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4.$$

$K > 0$ case

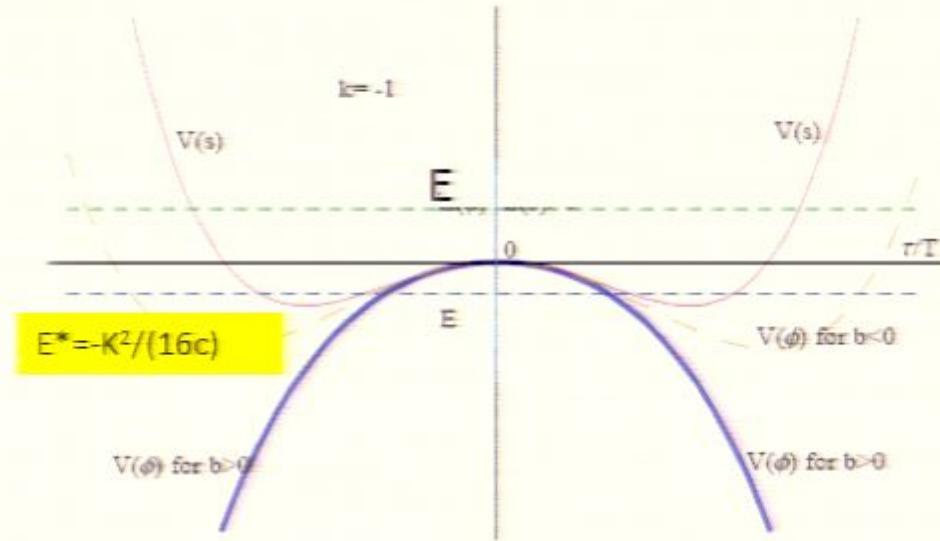


Higher level $E > E^*$, with $E_s = E$, $E_\phi = E + \rho$
Similar behavior to $K=0$ case.

Lower level $E, E_\phi < E^*$, with $E_s = E$, $E_\phi = E + \rho$
s oscillates in the V_s well, while
 ϕ oscillates outside the V_ϕ hill.

Then for any initial values there is a finite bounce at size $a_e \neq 0$. NO antigravity.

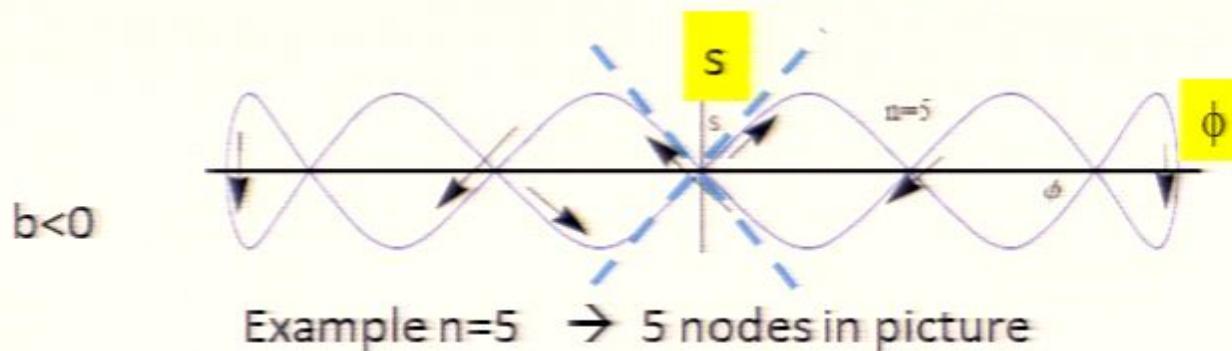
$K < 0$ case



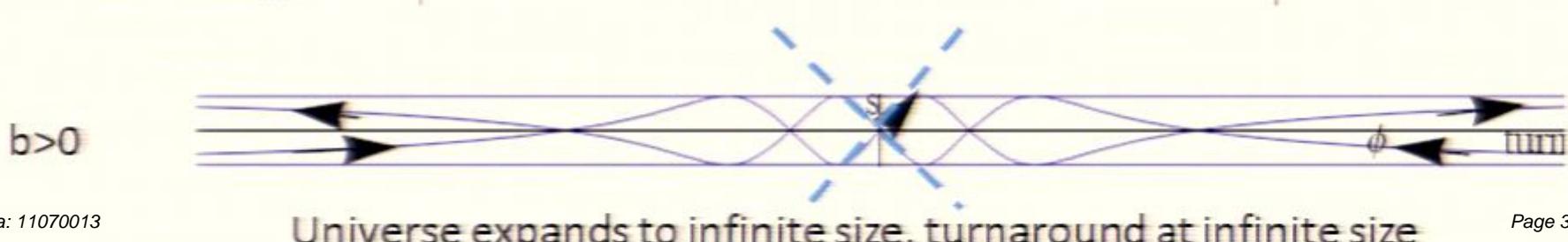
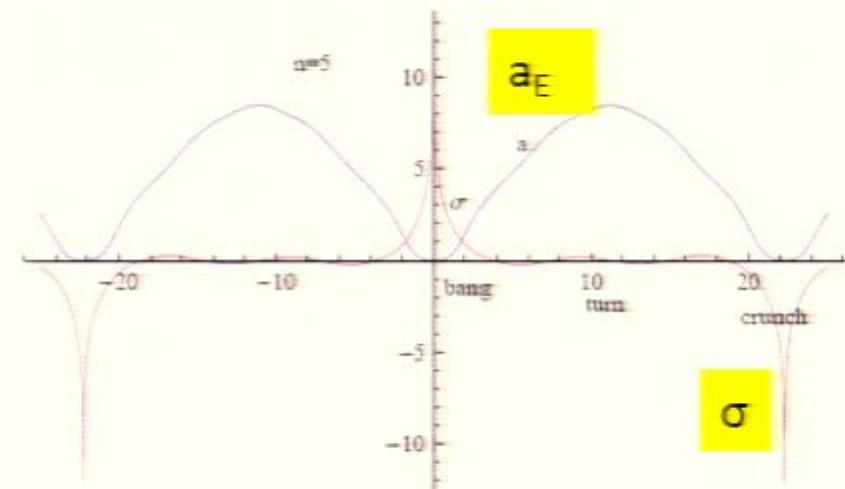
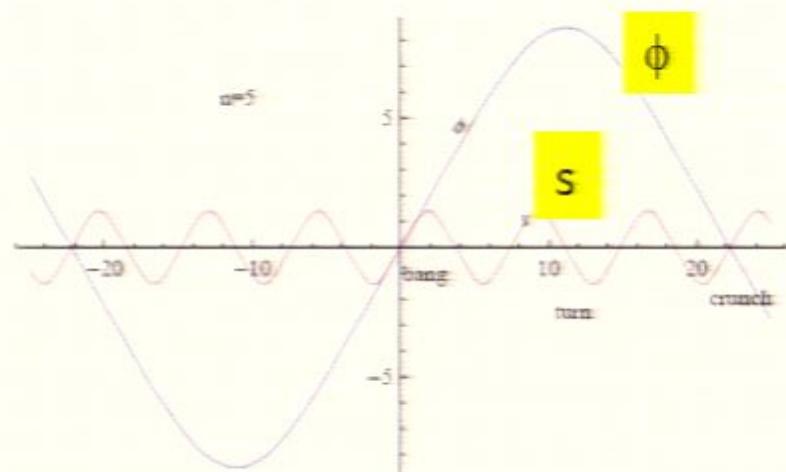
Higher level $E > 0$, with $E_s = E$, $E_\phi = E + \rho$
Similar behavior to $K=0$ case.

Lower level $E^* < E < 0$,
with $E_s = E$, $E_\phi = E + \rho$
All solutions are geodesically incomplete in
the Einstein gauge. **There is no way to
avoid antigravity.**

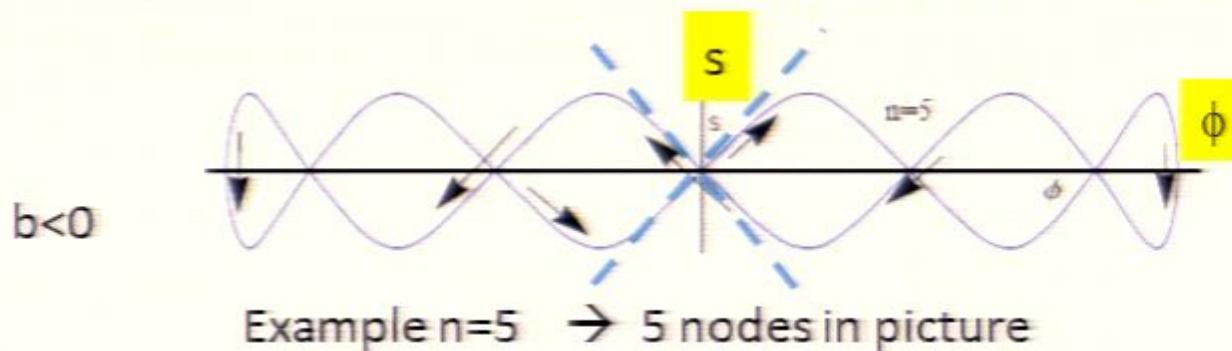
Geodesically complete solutions in the Einstein gauge, without antigravity



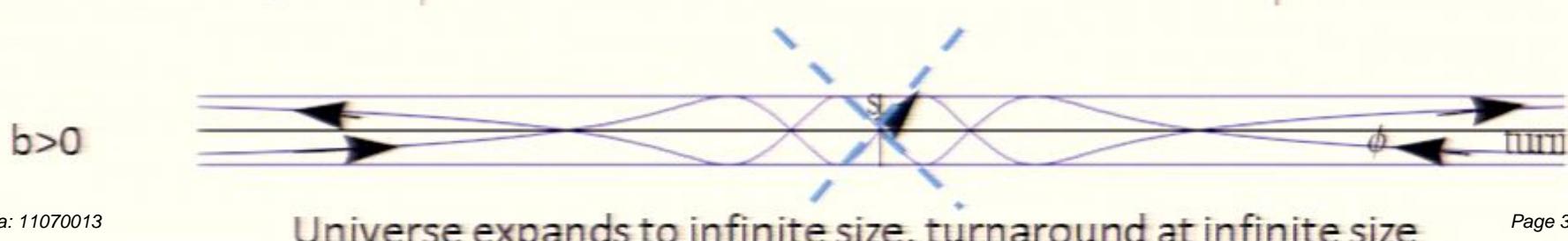
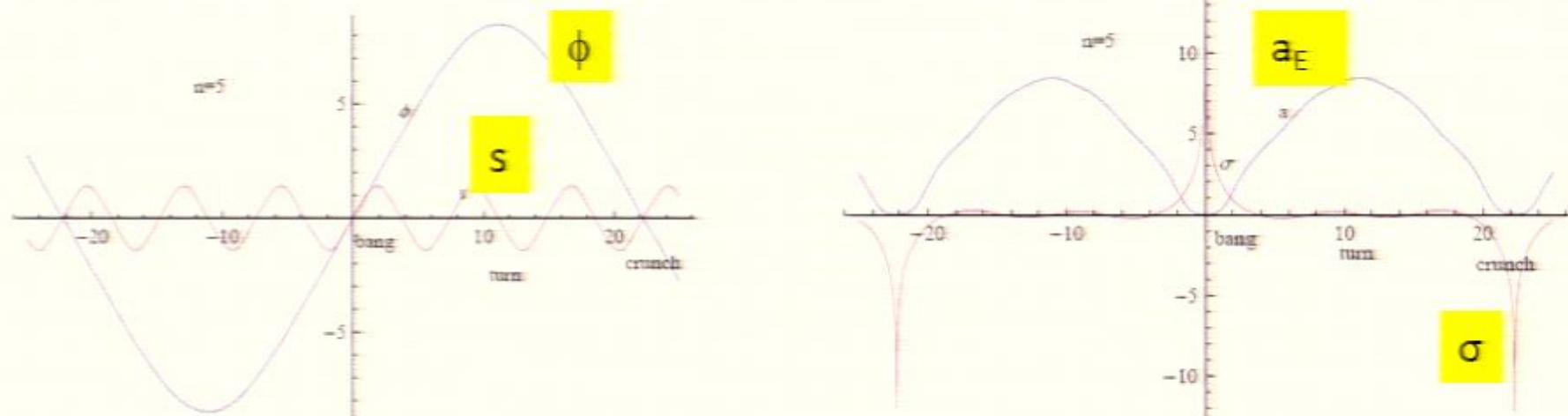
Conditions on 6 parameter space:
 (1) Synchronized initial values
 $\phi(0)=s(0)=0$
 (2) Relative quantization of periods
 $P_\phi=nP_s$



Geodesically complete solutions in the Einstein gauge, without antigravity



Conditions on 6 parameter space:
 (1) Synchronized initial values
 $\phi(0)=s(0)=0$
 (2) Relative quantization of periods
 $P_\phi = n P_s$



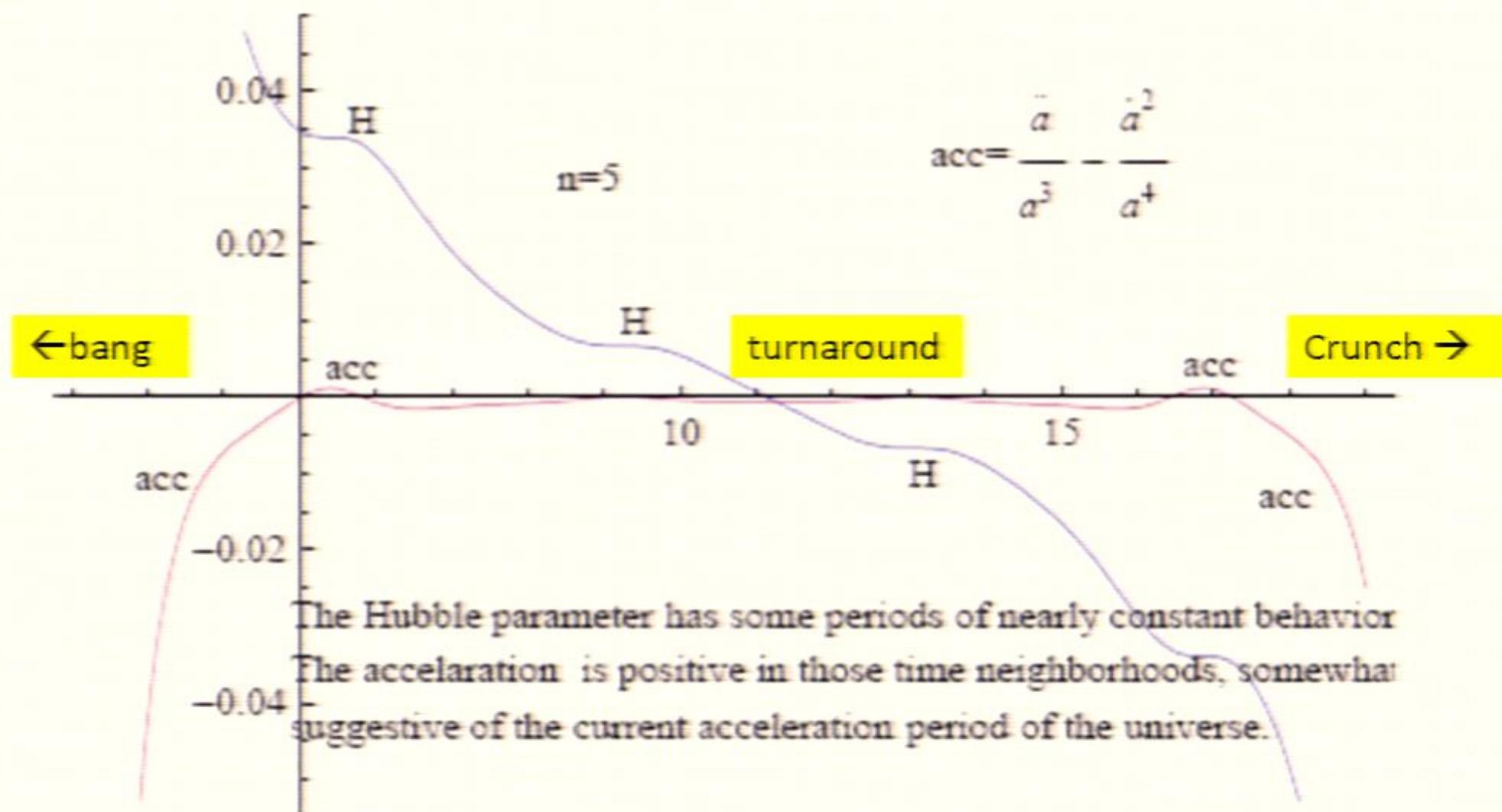


FIG. 12: Temporary inflation periods.

Anisotropy

$$ds^2 = a^2(\tau) (-d\tau^2 + ds_3^2) \quad \text{If } K \neq 0, \text{ ds}_3 = \text{Bianchi IX (Misner)}$$

$$K \rightarrow 0 \quad (ds_3^2)_{Kasner} = e^{2\alpha_1(\tau) + 2\sqrt{3}\alpha_2(\tau)} dx^2 + e^{2\alpha_1(\tau) - 2\sqrt{3}\alpha_2(\tau)} dy^2 + e^{-4\alpha_1(\tau)} dz^2$$

Anisotropy

$$ds^2 = a^2(\tau) (-d\tau^2 + ds_3^2) \quad \text{If } K \neq 0, \text{ ds}_3 = \text{Bianchi IX (Misner)}$$

$$K \rightarrow 0 \quad (ds_3^2)_{\text{Kasner}} = e^{2\alpha_1(\tau) + 2\sqrt{3}\alpha_2(\tau)} dx^2 + e^{2\alpha_1(\tau) - 2\sqrt{3}\alpha_2(\tau)} dy^2 + e^{-4\alpha_1(\tau)} dz^2$$

In Friedmann equations, 2 more fields $\alpha_1(\tau), \alpha_2(\tau)$, just like the $\sigma(\tau)$

Friedman Eqs: kinetic terms for α_1, α_2 just like σ , plus anisotropy potential if $K \neq 0$

$$V(\alpha_1, \alpha_2) = \frac{K}{\kappa^2 a^2} \left(e^{-8\alpha_1} + 4e^{\alpha_1} \sinh^2(2\sqrt{3}\alpha_2) - 4e^{-2\alpha_1} \cosh(2\sqrt{3}\alpha_2) - 3 \right)$$

Anisotropy

$$ds^2 = a^2(\tau) (-d\tau^2 + ds_3^2) \quad \text{If } K \neq 0, \text{ ds}_3 = \text{Bianchi IX (Misner)}$$

$$K \rightarrow 0 \quad (ds_3^2)_{\text{Kasner}} = e^{2\alpha_1(\tau) + 2\sqrt{3}\alpha_2(\tau)} dx^2 + e^{2\alpha_1(\tau) - 2\sqrt{3}\alpha_2(\tau)} dy^2 + e^{-4\alpha_1(\tau)} dz^2$$

In Friedmann equations, 2 more fields $\alpha_1(\tau), \alpha_2(\tau)$, just like the $\sigma(\tau)$

Friedman Eqs: kinetic terms for α_1, α_2 just like σ , plus anisotropy potential if $K \neq 0$

$$V(\alpha_1, \alpha_2) = \frac{K}{\kappa^2 a^2} \left(e^{-8\alpha_1} + 4e^{\alpha_1} \sinh^2(2\sqrt{3}\alpha_2) - 4e^{-2\alpha_1} \cosh(2\sqrt{3}\alpha_2) - 3 \right)$$

Free scalars if $K=0$, then canonical conjugate momenta p_1, p_2 are constants of motion.

Near singularity, kinetic terms dominate, so all potentials, including $V(\sigma)$ negligible.

Then σ momentum q is also conserved near the singularity.

For a range of q, p_1, p_2 mixmaster universe is avoided when σ is present (agree with BKL, etc.)

$$p_1 = (a_E)^2 \partial_\tau \alpha_1 \text{ etc.}$$

Without potentials can find all solutions analytically

for any (initial) anisotropy momenta p_1, p_2 ; or σ momentum q
including the parameters K, ρ_0 .

q = σ momentum

p = anisotropy momentum.

Antigravity Loop (K=0 case)

$$\phi + s = \sqrt{\tau} \left(\sqrt{p^2 + q^2} + \rho \tau \right) \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

$$\phi - s = -2 \frac{\tau}{\sqrt{\tau}} \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{-\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

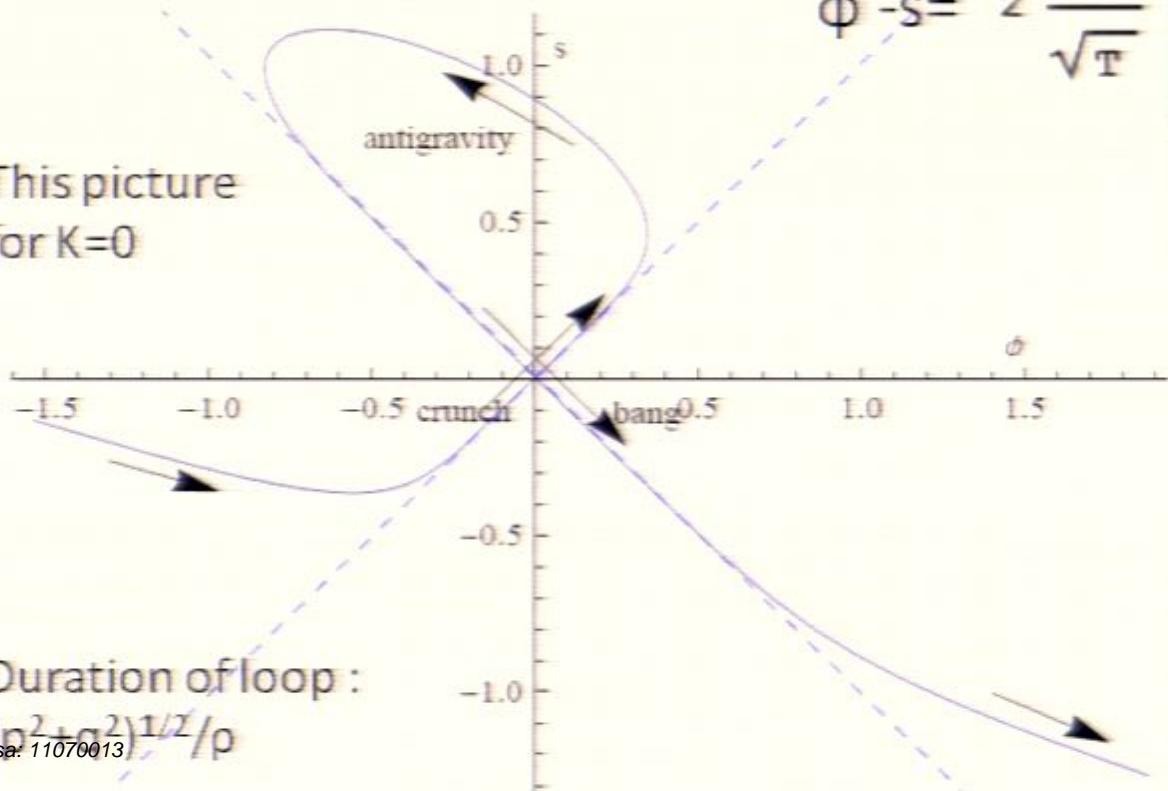
$q = \sigma$ momentum
 $p = \text{anisotropy momentum.}$

Antigravity Loop ($K=0$ case)

$$\phi + s = \sqrt{\tau} \left(\sqrt{p^2 + q^2} + \rho \tau \right) \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

$$\phi - s = 2 \frac{\tau}{\sqrt{\tau}} \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{-\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

This picture
for $K=0$



Duration of loop:
 $(p^2 + q^2)^{1/2}/\rho$

Print: 11070013

if p_1 or p_2 is not 0 :
 FOR ALL INITIAL CONDITIONS
 both $\phi, s \rightarrow 0$ at the big bang
 or crunch singularity,
 FOCUSING !!

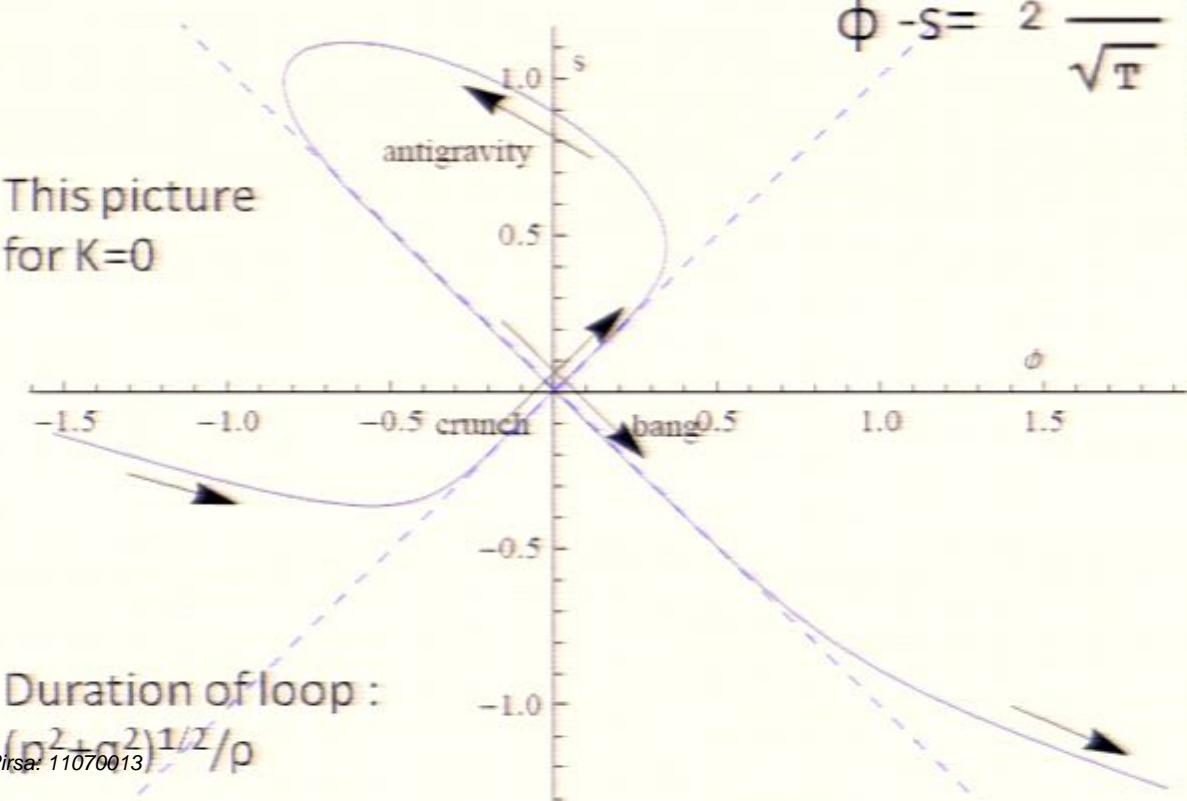
ALWAYS
 a period of antigravity
 sandwiched between
 crunch and bang

Page 39/44

$q = \sigma$ momentum
 $p = \text{anisotropy momentum.}$

$K=0: p_1, p_2$ are conserved throughout motion.

q changes during the loop because of $V(\sigma)$.
If small loop, \approx no change.



Antigravity Loop ($K=0$ case)

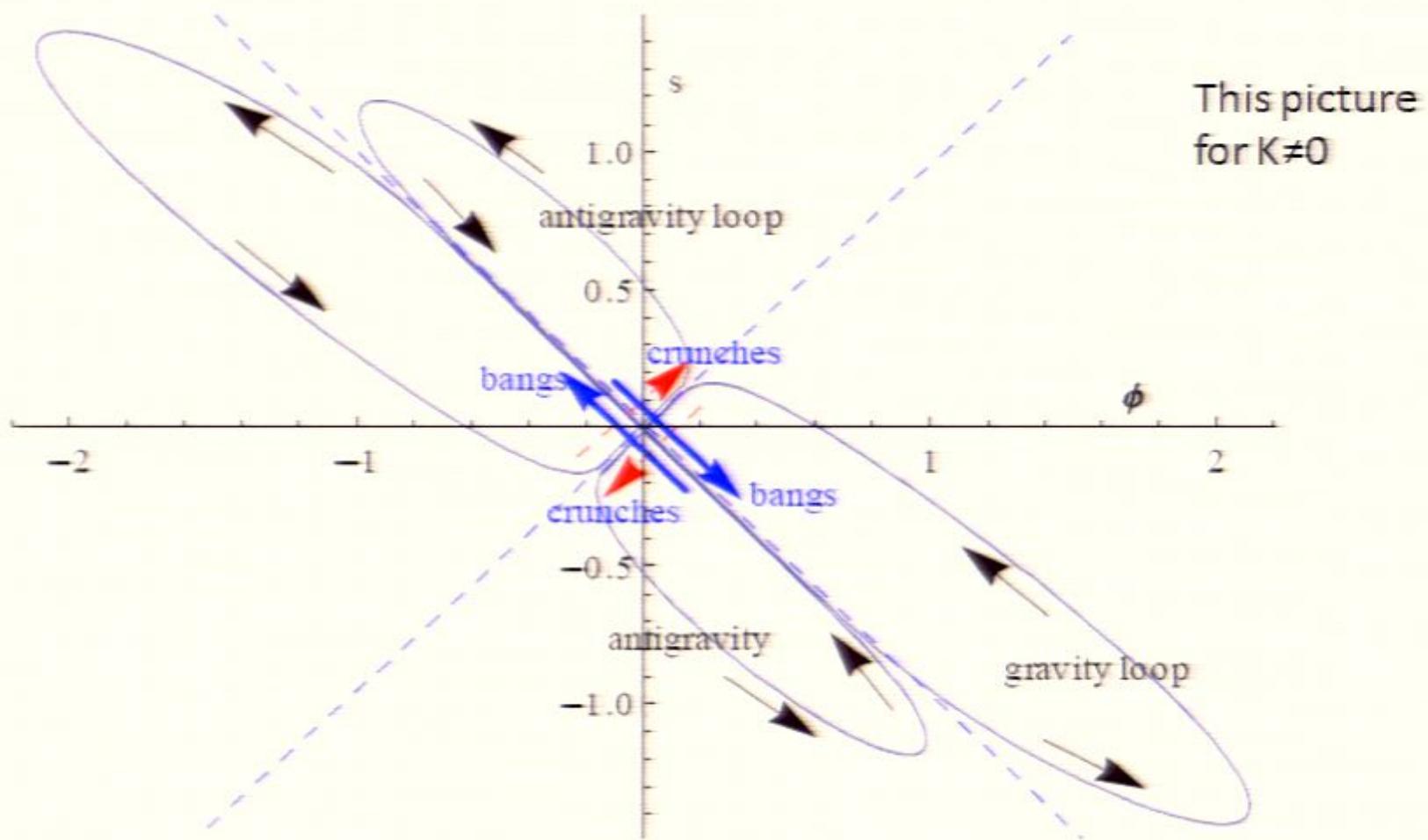
$$\phi + s = \sqrt{\tau} \left(\sqrt{p^2 + q^2} + \rho \tau \right) \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

$$\phi - s = 2 \frac{\tau}{\sqrt{\tau}} \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{-\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

if p_1 or p_2 is not 0 :
FOR ALL INITIAL CONDITIONS
both $\phi, s \rightarrow 0$ at the big bang
or crunch singularity,
FOCUSING !!

ALWAYS
a period of antigravity
sandwiched between
crunch and bang

Antigravity Loop

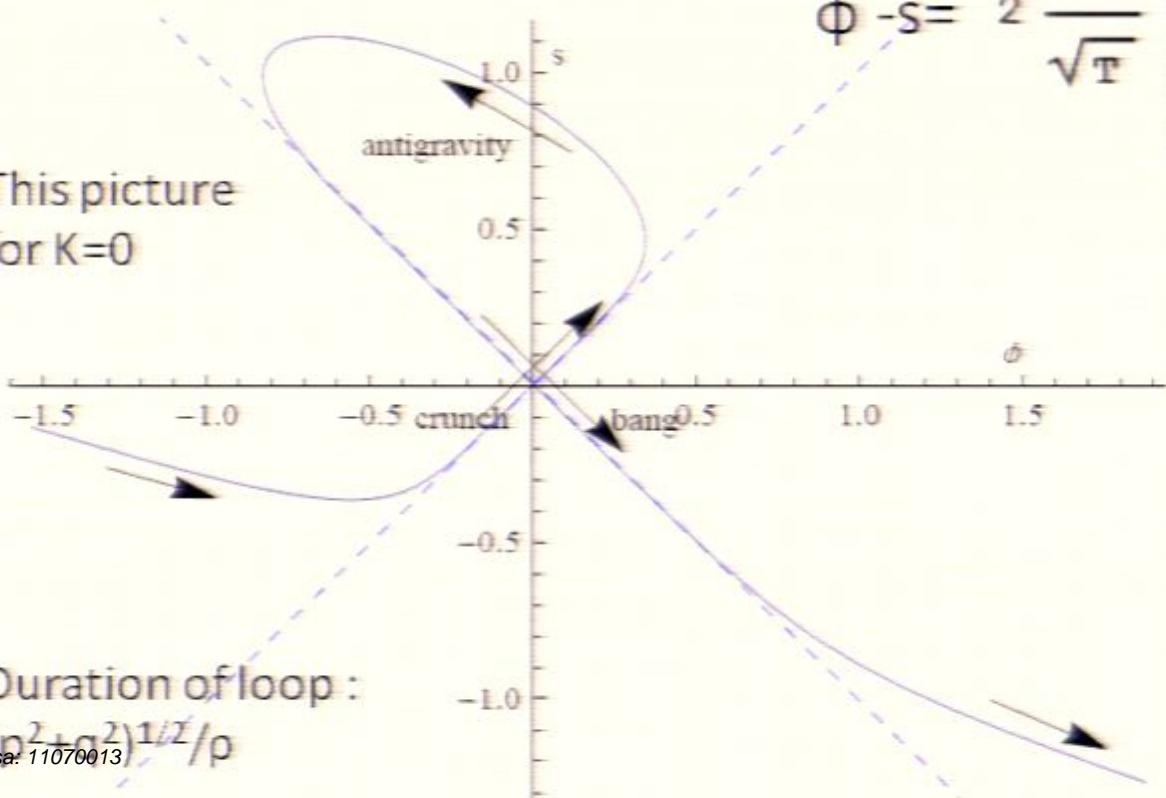


q, p_1, p_2 change during each loop (if loop is large) because of the potentials, but trajectory always returns to the origin and connects gravity \leftrightarrow antigravity regions

$q = \sigma$ momentum
 $p = \text{anisotropy momentum.}$

$K=0: p_1, p_2$ are conserved throughout motion.

q changes during the loop because of $V(\sigma)$.
If small loop, \approx no change.



Antigravity Loop ($K=0$ case)

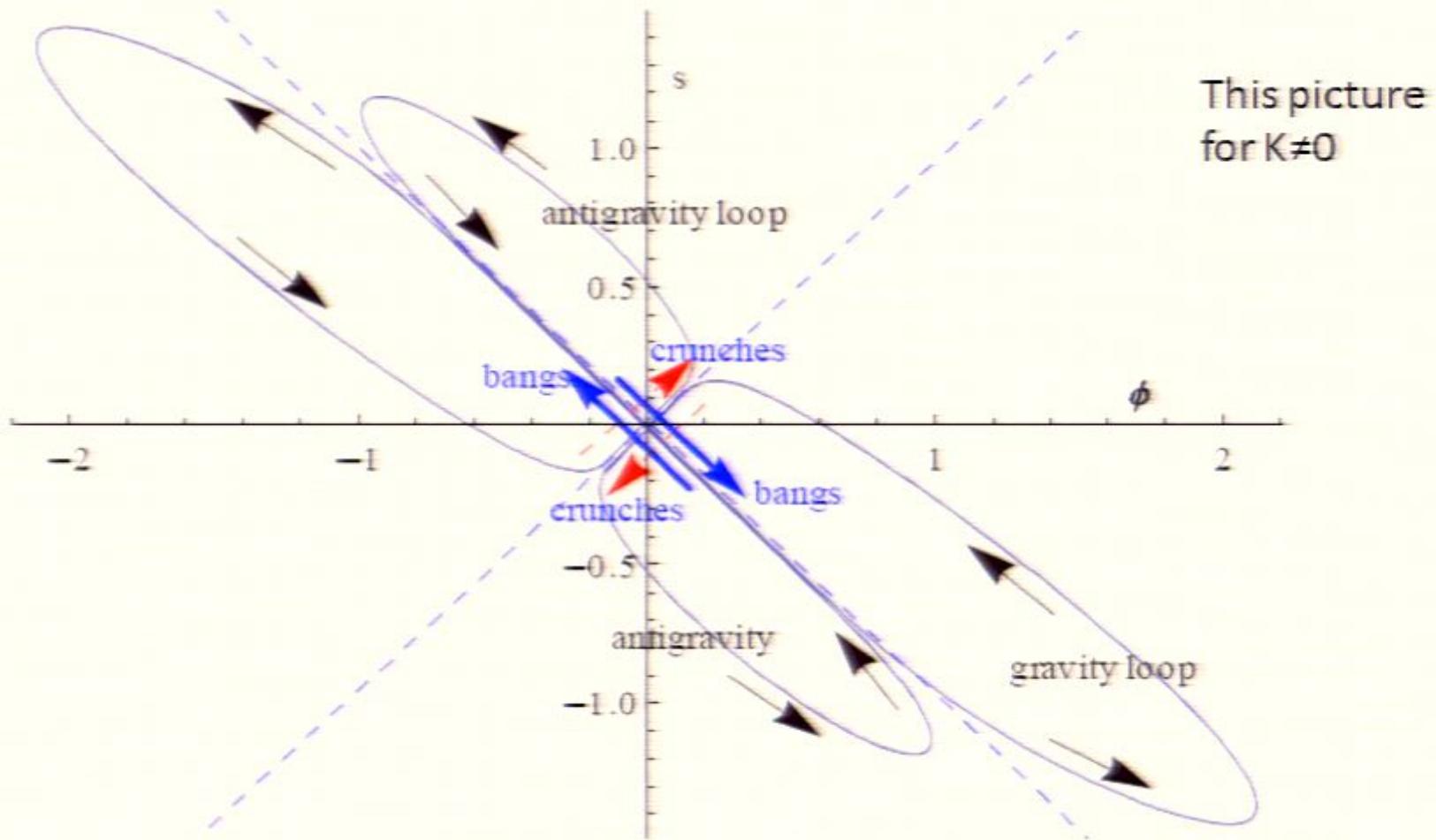
$$\phi + s = \sqrt{\tau} \left(\sqrt{p^2 + q^2} + \rho \tau \right) \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

$$\phi - s = 2 \frac{\tau}{\sqrt{\tau}} \left(\frac{\left(\frac{\tau}{\tau} \right)^2}{\left(\rho * \tau + \sqrt{p^2 + q^2} \right)^2} \right)^{-\frac{1}{4}} \left(1 + \frac{q}{\sqrt{p^2 + q^2}} \right)$$

if p_1 or p_2 is not 0 :
FOR ALL INITIAL CONDITIONS
both $\phi, s \rightarrow 0$ at the big bang
or crunch singularity,
FOCUSING !!

ALWAYS
a period of antigravity
sandwiched between
crunch and bang

Antigravity Loop



q, p_1, p_2 change during each loop (if loop is large) because of the potentials, but trajectory always returns to the origin and connects gravity \leftrightarrow antigravity regions

What have we learned?

- 1) Have found new techniques to solve cosmological equations analytically. Found all solutions for several special potentials.
- 2) Antigravity is very hard to avoid. Anisotropy requires it.
- 3) Have studied Wheeler-deWitt equation for the same system, can solve some cases exactly, others semiclassically. Same conclusions with quantum fuzziness.
- 4) Open: Are there observational effects today of a past antigravity period? This is an important future project. Study of small fluctuations and fitting to current observations.
- 5) Will this new insight survive the effects of a full quantum theory. How does it affect the study of string theory ?
- 6) These phenomena are direct predictions of 2T-physics in 4+2 dimensions (although they can be stated in 1T-physics, 1T gravity does not demand ALL scalars to be conformally coupled, 2T does).