Title: Observable signatures of anisotropic bubble nucleation

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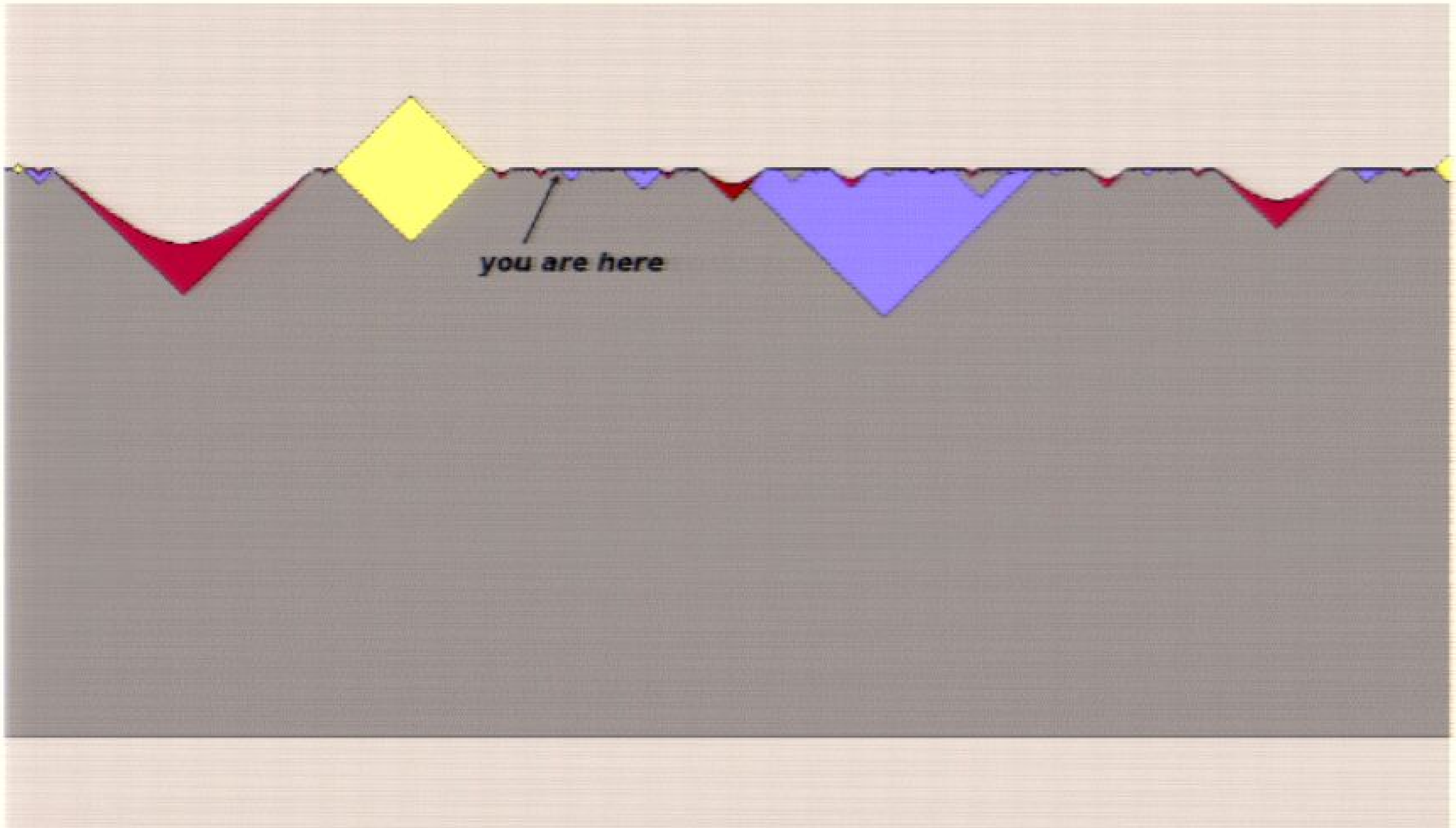
Abstract: Our universe may have formed via bubble nucleation in an eternally-inflating background. Furthermore, the background may have a compact dimension---the modulus of which tunnels out of a metastable minimum during bubble nucleation---which subsequently grows to become one of our three large spatial dimensions. We discuss some potential observational signatures of this scenario.



CHALLENGE: *could there be any crisp observational evidence for this?*

Introduction

- The string landscape is big because of compactification.
- In addition to 4d bubbles, there should be 3d (and other-d) bubbles.
- Do any possibilities lead to observable signatures?

We explore consequences of when our parent vacuum has only two large spatial dimensions; i.e. one of our three large spatial dimensions is compact.

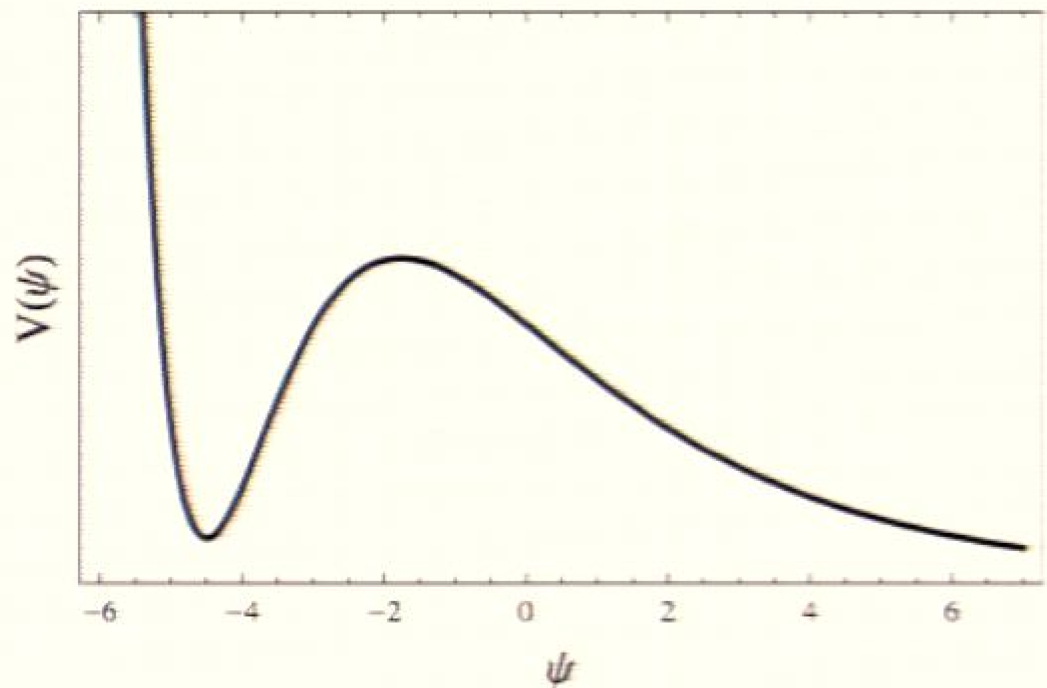
Prior work on a trans-dimensional multiverse:

[Blanco-Pillado, Schwartz-Perlov, and Vilenkin (2009, 2010); Carroll, Johnson, and Randall (2009)]

More recent work:

[Graham, Harnik, Rajendran (2010); Adamek, Campo, Niemeyer (2010)]

Setup: modulus stabilization



We imagine unfurling of spatial dimension z during Coleman–De Luccia barrier penetration involving metastable modulus ψ.

Setup: modulus stabilization

Start with the metric ansatz:

$$ds^2 = e^{-\alpha\psi}\,\overline{g}_{ab}\,dx^a dx^b + L^2 e^{\alpha\psi}\,dz^2\,,$$

where ψ and $\overline{g}_{ab}$ are independent of z, with $0 < z < 2\pi$ and periodic b.c.s.

This gives the effective action:

$$S_{\rm eff} = 2\pi L \int d^3x\,\sqrt{-\overline{g}}\,\left[\frac{1}{16\pi G}\overline{R} - \frac{1}{2}\partial_a\psi\partial^a\psi - \frac{\Lambda}{8\pi G}e^{-\alpha\psi}\right],$$

where Λ is a 4d cosmological constant and $\alpha = \sqrt{16\pi G}$ makes ψ canonical.

As expected ψ simply rolls down the the effective potential to infinity; additional ingredients are needed to create a metastable minimum.

Setup: modulus stabilization

We introduce a complex scalar φ:

$$\mathcal{L}_\varphi = -\frac{1}{2}K(X) - \frac{\lambda}{4}\left(|\varphi|^2 - v^2\right)^2 ,$$

where $X = \partial_\mu \varphi^* \partial^\mu \varphi$. This permits a winding solution

$$\varphi \simeq v e^{inz} \quad \Longrightarrow \quad X \simeq (n^2 v^2 / L^2)\, e^{-\alpha\psi} ,$$

for integer n (assuming $v^2 \gg (n^2/\lambda L^2) K' e^{-\alpha\psi}$).

To generate a metastable minimum of $\overline{V}_{\rm eff}(\psi)$*, we need at least the terms*

$$K(X) = X + \kappa_2 X^2 + \kappa_3 X^3 .$$

(We can choose parameters to satisfy NEC.) [Buniy and Hsu (2005); Babichev (2006)]
This gives:

$$\overline{V}_{\rm eff}(\psi) = \frac{\Lambda}{8\pi G} e^{-\alpha\psi} + \frac{n^2 v^2}{2L^2} e^{-2\alpha\psi} + \frac{\kappa_2\, n^4 v^4}{2L^4} e^{-3\alpha\psi} + \frac{\kappa_3\, n^6 v^6}{2L^6} e^{-4\alpha\psi} .$$

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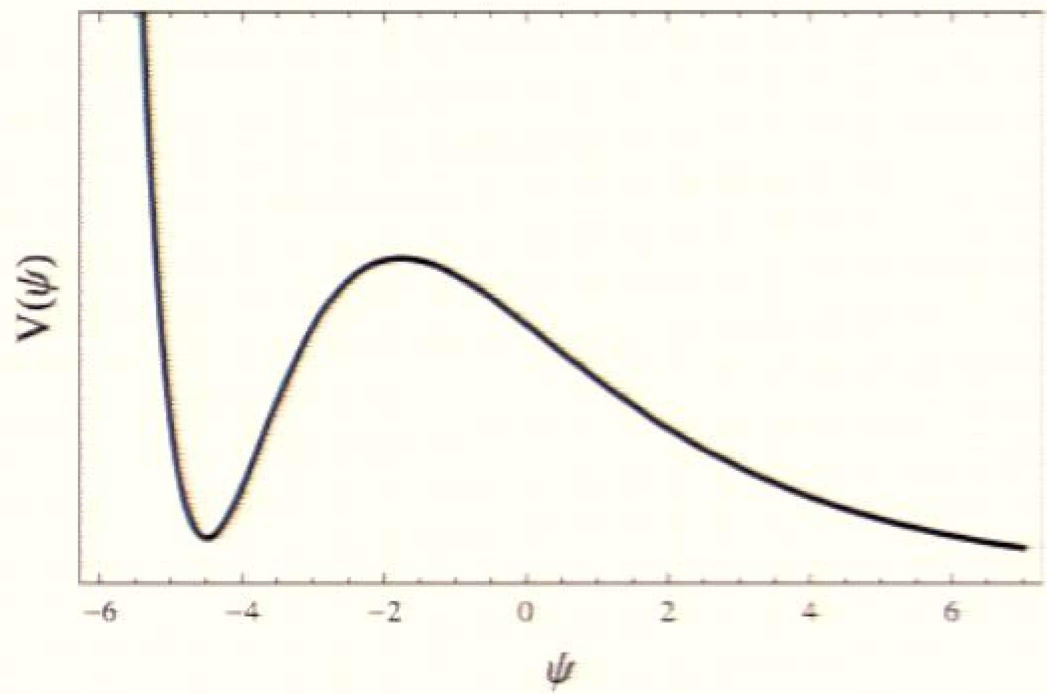


Appropriate choice of κ_2 and κ_3 gives $\overline{V}_{\rm eff}(\psi)$ of the desired form.†

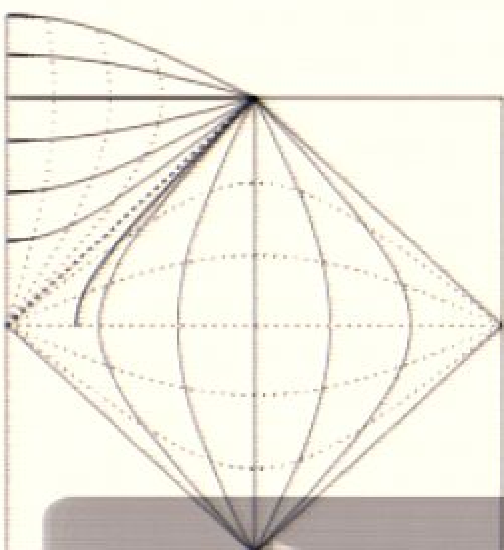
†*This choice of $K(X)$ is ad hoc; but we simply propose it as a toy model proof of principle.*

Setup: anisotropic tunneling instanton

We can now apply Coleman–De Luccia. [Colemand and De Luccia (1980)]

The only difference is the parent vacuum is effectively 3d, *so the instanton only respects* O(3) / SO(2,1) *symmetries*.

Indeed our metric ansatz guarantees the instanton is independent of z.



- 3d CDL, suppresses a circle with radius $\sinh(\xi)$, or $\sin(R)$, $\times$ conformal factor
- 4d geometry adds a circle with fixed radius $\times$ (different) conformal factor
- conformal factor of $z \sim$ constant in parent vacuum, jumps across bubble wall, grows with time in bubble

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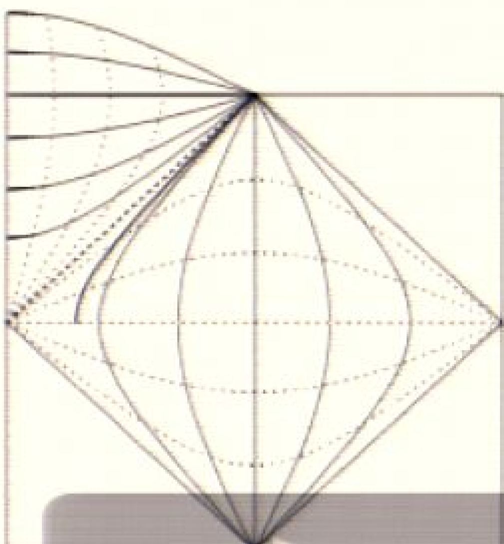

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Meanwhile inside bubble:

$$ds^2 = -d\tau^2 + a^2(\tau)\left[d\chi^2 + \sinh^2(\chi)\,d\phi^2\right] + b^2(\tau)\,dz^2\,.$$

After bubble nucleation the energy density is dominated by cosmological constant ≈ inflaton and a curvature-like term from the modulus potential,

$$\begin{aligned}\overline{V}_{\rm eff}(\psi) \to V(b) &= \frac{\Lambda}{8\pi G} + \frac{n^2 v^2}{2b^2} + \frac{\kappa_2 n^4 v^4}{2b^4} + \frac{\kappa_3 n^6 v^6}{2b^6} \\ &= \rho_{\rm inf} + \frac{1}{8\pi G}\frac{k}{b^2} + \dots\end{aligned}$$

If we take $k \ll 1$ and $\rho_{\rm inf} =$ constant, we obtain a simple analytic solution:

$$a(\tau) = H_{\rm inf}^{-1}\sinh(H_{\rm inf}\tau)\,, \quad b(\tau) = b_{\rm in}\cosh(H_{\rm inf}\tau)\,.$$

Setup: background bubble evolution

During inflation the background anisotropy redshifts away, but it can become significant again during radiation / matter domination.

[Demianski and Doroshkevich (2007); and earlier work]

Define $\Omega_c \equiv 1/(a^2 H^2)$ and $h \equiv \dot{a}/a - \dot{b}/b$, and solve the field equations as an expansion in $\Omega_c, h \ll 1$, assuming matter domination for simplicity.

[Graham, Harnik, and Rajendran (2010)]

After rescaling z to give a and b the same leading-order behavior, this gives

$$a(\tau) \simeq \mathcal{A}\,\tau^{2/3}\left[1 + \frac{1-k}{5}\,\Omega_c(\tau)\right], \quad b(\tau) \simeq \mathcal{A}\,\tau^{2/3}\left[1 - \frac{1-3k}{5}\,\Omega_c(\tau)\right],$$

where $\mathcal{A}$ is an arbitrary constant and $\Omega_c = (9/4)\mathcal{A}^{-2}\tau^{2/3}$.

We can then use these solutions to solve perturbatively for the trajectories of "initially" radial null rays on surface of last scattering.

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Predictions: inflationary perturbations

Incident CMB photon temperature expressed in terms of multipole moments

$$a_{\ell m} = \int d\cos(\theta)\, d\phi\, Y^{\star}_{\ell m}(\theta, \phi)\, \delta_T(\theta, \phi)\,,$$

The temperature of incident photons can be written

$$T_0 = \frac{T_\star}{1 + z_\star}\,, \qquad \delta_T = (1 - \delta_z)\delta_\star - \delta_z\,,$$

where $\delta_\star$ is the temperature contrast on the surface of last scattering, and δ_z is the redshift contrast among CMB photons.

The redshift z has anisotropic ISW effect. Assuming matter domination,

$$\delta_z(\zeta) = -\left\{\frac{1-5k}{15} - \frac{2\,(1-2k)}{15}\left[3\zeta^2 - 1\right]\right\}\Omega^0_c\,.$$

(The homogeneous offset can be absorbed into leading-order component.)

Predictions: inflationary perturbations

We approximate the density perturbations as the dS fluctuations of a free massless scalar σ, ignoring processing within the horizon

$$\delta_\star(\zeta,\,\phi) \propto \sigma(\tau_\star,\,\mathbf{x}_\star(\zeta,\,\phi))\,.$$

LS coords $\mathbf{x}_\star$ are related to sky coords (ζ,ϕ) using radial, null rays.

Perform mode expansion

$$\hat{\sigma}(\tau,\,\mathbf{x}) = \int dq \sum_{p,\,r,\,s} \left[\frac{\Upsilon_{pqs}(\tau)}{\sqrt{a(\tau)b(\tau)}}\, U_{qrs}(\mathbf{x})\, \hat{a}_{pqrs} + \text{h.c.} \right]\,.$$

"Fourier" modes U_{qrs} (q real, r and s integer) anisotropic.

The mode functions Υ_{pqs} are computed in the Bunch–Davies vacuum.
(Reminder: we treat parent and inflation in bubble as pure dS, take $k \ll 1$.)

[Yamamoto, Sasaki, Tanaka (1995,1995,1996); Bucher, Goldhaber, Turok (1995,1995); Garriga, Montes, Sasaki, Tanaka (1998,1999).]

Predictions: inflationary perturbations

The multipole correlator $C_{\ell\ell' mm'} \equiv \langle \hat{a}_{\ell m} \hat{a}^{\dagger}_{\ell' m'} \rangle$ contains a quadrupole:

$$C^k_{2200} = \frac{64\pi}{1125}(1-2k)^2(\Omega_c^0)^2 .$$

(Note that $64\pi/1125 \approx 0.42^2$.)

The induced quadrupole implies $\Omega_c^0 \lesssim 10^{-5}$*, unless* $k \approx 1/2$.

We focus on $k \approx 1/2$ and $k \ll 1$.

- replace the sum over s with an integral
- use $P_{qs} \propto (q^2+\mu^2)^{-3/2}$, where $\mu = s/b_{\rm in}H_{\rm inf}$

To see the deviations from isotropy, we define

$$\delta C_{\ell\ell' mm'} = \frac{C_{\ell\ell' mm'} - C^{(0)}_{\ell\ell' mm'}}{\max\{C^{(0)}_{\ell\ell mm}, C^{(0)}_{\ell'\ell' m'm'}\}} ,$$

where $C^{(0)}_{\ell\ell mm}$ is the correlator in the limit $\Omega_c^0 \to 0$.

Predictions: inflationary perturbations

One feature of this model is off-diagonal elements of $C_{\ell\ell' mm'}$*:*

	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	$\ell = 7$	$\ell = 8$	$\ell = 9$
$\ell' = \ell$	-1.0^*	-0.88	-0.84	-0.80	-0.80	-0.79	-0.79	-0.79
$\ell' = \ell \pm 1$	0	0	0	0	0	0	0	0
$\ell' = \ell \pm 2$	0.78	0.84	0.88	0.91	0.93	0.96	0.98	0.99

Table 1: The multipole correlator contrast $\delta C_{\ell\ell' mm'}$, in units of Ω_c^0, for several values of ℓ and ℓ', $m = m' = 0$ and $k \ll 1$ (data computed using $\Omega_c^0 = 10^{-5}$).

*Does not include induced quadrupole.

	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	$\ell = 7$	$\ell = 8$	$\ell = 9$
$\ell' = \ell$	-1.2	-1.1	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
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$\ell' = \ell \pm 2$	0.33	0.35	0.36	0.37	0.38	0.40	0.40	0.40

Table 2: The multipole correlator contrast $\delta C_{\ell\ell' mm'}$, in units of Ω_c^0, for several values of ℓ and ℓ', $m = m' = 0$ and $k \approx 1/2$ (data computed using $\Omega_c^0 = 10^{-5}$).

The spectrum is not unlike: [Ackerman, Carroll, and Wise (2007).]

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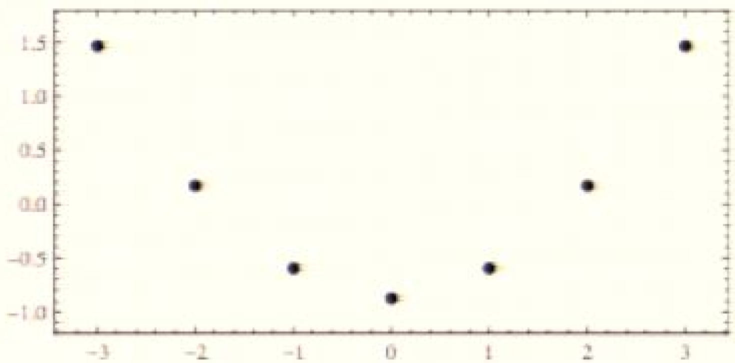

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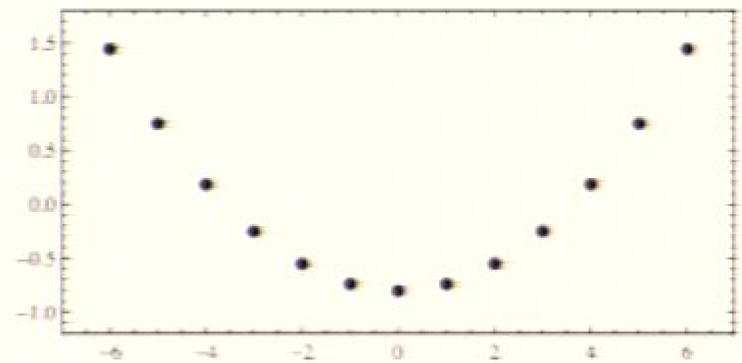

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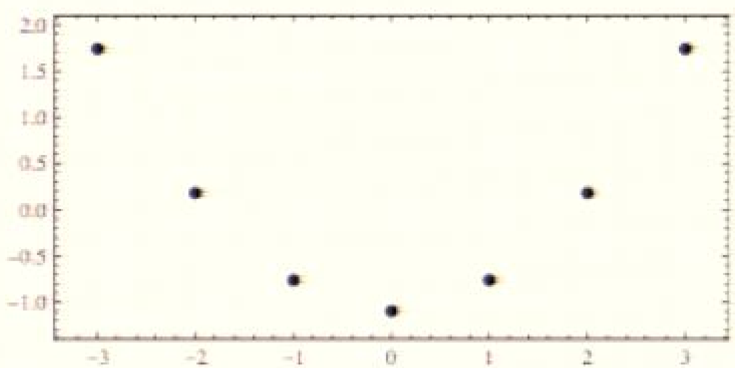

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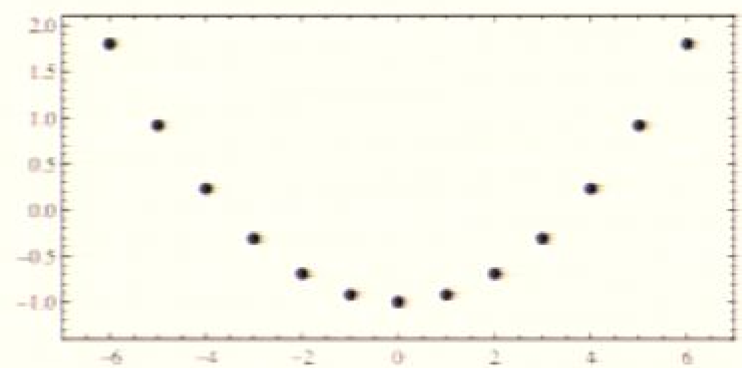

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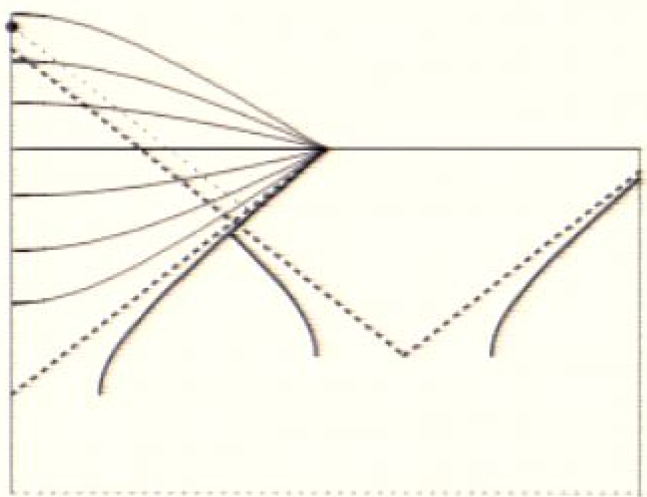
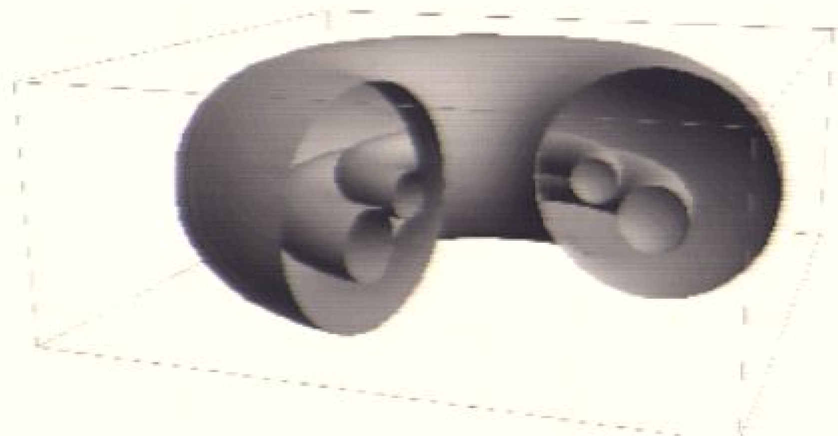
Predictions: bubble collisions

Finally, consider bubble collisions. [Aguirre[4], Chang[2], Czeck, Garriga, Guth, Freivogel, Johnson[4], Kleban[4], Larjo, Levi[3], Nicolis, Shomer, Sigurdson[2], Tysanner, Vilenkin]

With respect to a "radial" × time slice of spacetime, (2+1)-dimensional bubble collision looks the same as in (3+1) dimensions:





Instead of each point representing a two-sphere, each point represents a torus.

Predictions: bubble collisions

The calculation proceeds in analogy to in the 4d case (recall MK's talk).

EXCEPT, the affected region in sky is

$$\cos(\phi - \phi_{\rm nuc})\sin(\theta) \geq \text{constants}\,.$$

As in 4d the colliding bubble can nucleate at any azimuthal angle $\phi_{\rm nuc}$, but in this model it must always be centered at $\theta = \pi/2$.

We remind that the effects of collisions can be observable even if Ω_c^0 is not. [Chang, Kleban, Levi (2008,2009).]

However, in present case

$$N \sim \frac{H_{\rm p}}{H_{\rm inf}}\frac{\Gamma}{H_{\rm p}^3}\sqrt{\Omega_k}\,.$$

Conclusions

- The parent to our bubble universe might have had only two large spatial dimensions, our third becoming large after bubble nucleation.
- If inflation within our bubble did not last too long, this creates statistical anisotropy among CMB perturbations, and generates a quadrupole in standard-candle luminosities at fixed redshift.
- Absent fine-tuning / fortuitous cancellations, both signals appear at the margin of observability, due to an induced quadrupole $\sim \Omega_c^0$.
- There will also be bubble collisions, the likelihood of which to observe depends on microscopic model parameters.
- Circumstances surrounding observation appear fortuitous; on the other hand they give hope to substantiate landscape / multiverse hypothesis.
- *It happens somewhere in the multiverse!!!*

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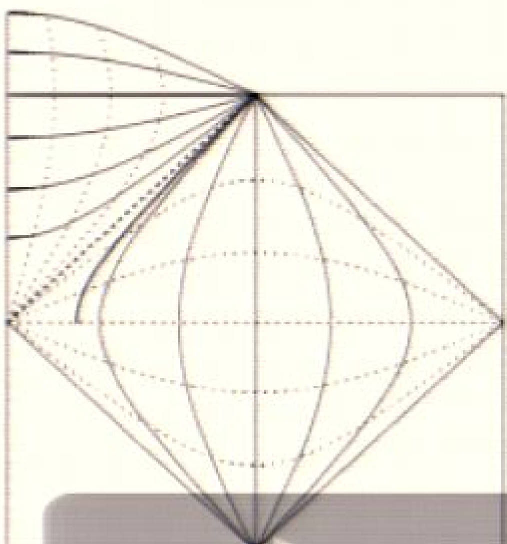

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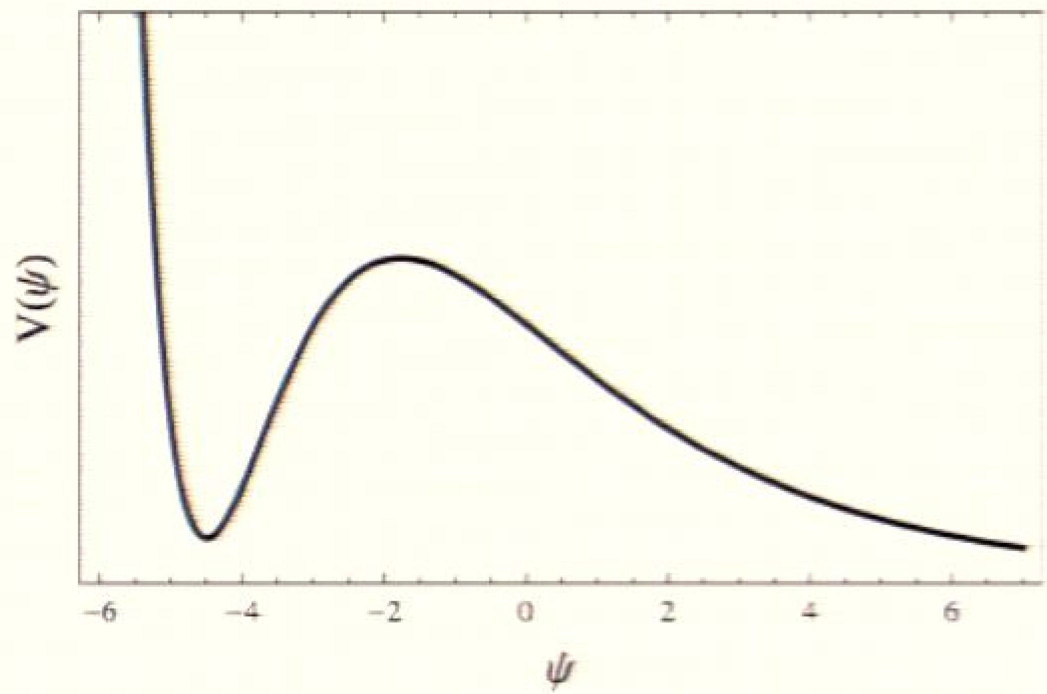

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†This choice of $K(X)$ is ad hoc; but we simply propose it as a toy model proof of principle.

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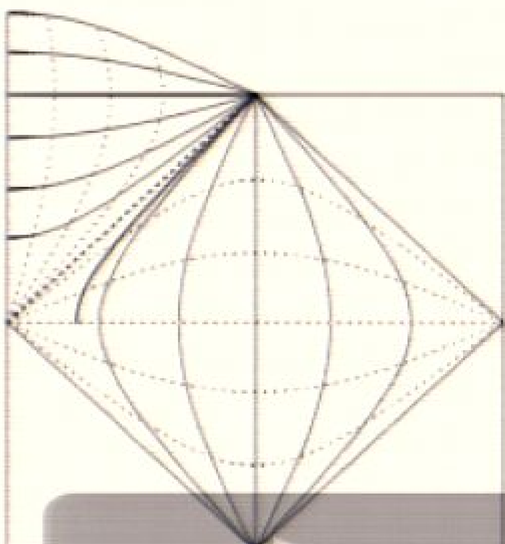
We can now apply Coleman–De Luccia. [Colemand and De Luccia (1980)]

The only difference is the parent vacuum is effectively 3d, *so the instanton only respects* O(3) / SO(2,1) *symmetries.*

Indeed our metric ansatz guarantees the instanton is independent of z.



- 3d CDL, suppresses a circle with radius $\sinh(\xi)$, or $\sin(R)$, $\times$ conformal factor
- 4d geometry adds a circle with fixed radius $\times$ (different) conformal factor
- conformal factor of $z \sim$ constant in parent vacuum, jumps across bubble wall, grows with time in bubble

Setup: modulus stabilization

We introduce a complex scalar φ:

$$\mathcal{L}_\varphi = -\frac{1}{2}K(X) - \frac{\lambda}{4}\left(|\varphi|^2 - v^2\right)^2 ,$$

where $X = \partial_\mu\varphi^*\partial^\mu\varphi$. This permits a winding solution

$$\varphi \simeq ve^{inz} \quad \Longrightarrow \quad X \simeq (n^2v^2/L^2)\, e^{-\alpha\psi} ,$$

for integer n (assuming $v^2 \gg (n^2/\lambda L^2)K' e^{-\alpha\psi}$).

To generate a metastable minimum of $\overline{V}_{\rm eff}(\psi)$, we need at least the terms

$$K(X) = X + \kappa_2 X^2 + \kappa_3 X^3 .$$

(We can choose parameters to satisfy NEC.) [Buniy and Hsu (2005); Babichev (2006)]
This gives:

$$\overline{V}_{\rm eff}(\psi) = \frac{\Lambda}{8\pi G}e^{-\alpha\psi} + \frac{n^2v^2}{2L^2}e^{-2\alpha\psi} + \frac{\kappa_2\, n^4v^4}{2L^4}e^{-3\alpha\psi} + \frac{\kappa_3\, n^6v^6}{2L^6}e^{-4\alpha\psi} .$$

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Meanwhile inside bubble:

$$ds^2 = -d\tau^2 + a^2(\tau)\left[d\chi^2 + \sinh^2(\chi)\, d\phi^2\right] + b^2(\tau)\, dz^2 \,.$$

After bubble nucleation the energy density is dominated by cosmological constant ≈ inflaton and a curvature-like term from the modulus potential,

$$\begin{aligned} \overline{V}_{\rm eff}(\psi) \to V(b) &= \frac{\Lambda}{8\pi G} + \frac{n^2 v^2}{2b^2} + \frac{\kappa_2 n^4 v^4}{2b^4} + \frac{\kappa_3 n^6 v^6}{2b^6} \\ &= \rho_{\rm inf} + \frac{1}{8\pi G}\frac{k}{b^2} + \dots \end{aligned}$$

If we take $k \ll 1$ and $\rho_{\rm inf} =$ constant, we obtain a simple analytic solution:

$$a(\tau) = H_{\rm inf}^{-1} \sinh(H_{\rm inf}\tau)\,, \quad b(\tau) = b_{\rm in} \cosh(H_{\rm inf}\tau)\,.$$

Setup: modulus stabilization

Start with the metric ansatz:

$$ds^2 = e^{-\alpha\psi}\,\bar{g}_{ab}\,dx^a dx^b + L^2 e^{\alpha\psi}\,dz^2\,,$$

where ψ and $\bar{g}_{ab}$ are independent of z, with $0 < z < 2\pi$ and periodic b.c.s.

This gives the effective action:

$$S_{\rm eff} = 2\pi L \int d^3x\,\sqrt{-\bar{g}}\,\left[\frac{1}{16\pi G}\bar{R} - \frac{1}{2}\partial_a\psi\partial^a\psi - \frac{\Lambda}{8\pi G}e^{-\alpha\psi}\right],$$

where Λ is a 4d cosmological constant and $\alpha = \sqrt{16\pi G}$ makes ψ canonical.

As expected ψ simply rolls down the the effective potential to infinity; additional ingredients are needed to create a metastable minimum.

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Neglecting back-reaction of bubble wall, parent vacuum:

$$ds^2 = H_{\rm p}^{-2}\mathrm{sech}^2(\overline{\eta})\left[d\overline{\eta}^2 - d\overline{\chi}^2 + \cosh^2(\overline{\chi})\,d\phi^2\right] + b_{\rm in}^2\,dz^2\,.$$

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Setup: background bubble evolution

During inflation the background anisotropy redshifts away, but it can become significant again during radiation / matter domination.

[Demianski and Doroshkevich (2007); and earlier work]

Define $\Omega_c \equiv 1/(a^2 H^2)$ and $h \equiv \dot{a}/a - \dot{b}/b$, and solve the field equations as an expansion in $\Omega_c, h \ll 1$, assuming matter domination for simplicity.

[Graham, Harnik, and Rajendran (2010)]

After rescaling z to give a and b the same leading-order behavior, this gives

$$a(\tau) \simeq \mathcal{A}\tau^{2/3}\left[1 + \frac{1-k}{5}\,\Omega_c(\tau)\right], \quad b(\tau) \simeq \mathcal{A}\tau^{2/3}\left[1 - \frac{1-3k}{5}\,\Omega_c(\tau)\right],$$

where $\mathcal{A}$ is an arbitrary constant and $\Omega_c = (9/4)\mathcal{A}^{-2}\tau^{2/3}$.

We can then use these solutions to solve perturbatively for the trajectories of "initially" radial null rays on surface of last scattering.

Predictions: inflationary perturbations

One feature of this model is off-diagonal elements of $C_{\ell\ell' mm'}$:

	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	$\ell = 7$	$\ell = 8$	$\ell = 9$
$\ell' = \ell$	-1.0^*	-0.88	-0.84	-0.80	-0.80	-0.79	-0.79	-0.79
$\ell' = \ell \pm 1$	0	0	0	0	0	0	0	0
$\ell' = \ell \pm 2$	0.78	0.84	0.88	0.91	0.93	0.96	0.98	0.99

Table 1: The multipole correlator contrast $\delta C_{\ell\ell' mm'}$, in units of Ω_c^0, for several values of ℓ and ℓ', $m = m' = 0$ and $k \ll 1$ (data computed using $\Omega_c^0 = 10^{-5}$).

*Does not include induced quadrupole.

	$\ell = 2$	$\ell = 3$	$\ell = 4$	$\ell = 5$	$\ell = 6$	$\ell = 7$	$\ell = 8$	$\ell = 9$
$\ell' = \ell$	-1.2	-1.1	-1.0	-1.0	-1.0	-1.0	-1.0	-1.0
$\ell' = \ell \pm 1$	0	0	0	0	0	0	0	0
$\ell' = \ell \pm 2$	0.33	0.35	0.36	0.37	0.38	0.40	0.40	0.40

Table 2: The multipole correlator contrast $\delta C_{\ell\ell' mm'}$, in units of Ω_c^0, for several values of ℓ and ℓ', $m = m' = 0$ and $k \approx 1/2$ (data computed using $\Omega_c^0 = 10^{-5}$).

The spectrum is not unlike: [Ackerman, Carroll, and Wise (2007).]

Predictions: inflationary perturbations

The multipole correlator $C_{\ell\ell' mm'} \equiv \langle \hat{a}_{\ell m} \hat{a}^{\dagger}_{\ell' m'} \rangle$ contains a quadrupole:

$$C^k_{2200} = \frac{64\pi}{1125}(1-2k)^2(\Omega^0_c)^2 \, .$$

(Note that $64\pi/1125 \approx 0.42^2$.)

The induced quadrupole implies $\Omega^0_c \lesssim 10^{-5}$, *unless* $k \approx 1/2$.

We focus on $k \approx 1/2$ and $k \ll 1$.

- replace the sum over s with an integral
- use $P_{qs} \propto (q^2 + \mu^2)^{-3/2}$, where $\mu = s/b_{\rm in} H_{\rm inf}$

To see the deviations from isotropy, we define

$$\delta C_{\ell\ell' mm'} = \frac{C_{\ell\ell' mm'} - C^{(0)}_{\ell\ell' mm'}}{\max\{C^{(0)}_{\ell\ell mm}, C^{(0)}_{\ell'\ell' m'm'}\}} \, ,$$

where $C^{(0)}_{\ell\ell mm}$ is the correlator in the limit $\Omega^0_c \to 0$.

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Predictions: inflationary perturbations

We approximate the density perturbations as the dS fluctuations of a free massless scalar σ, ignoring processing within the horizon

$$\delta_\star(\zeta,\,\phi) \propto \sigma(\tau_\star,\,\mathbf{x}_\star(\zeta,\,\phi))\,.$$

LS coords $\mathbf{x}_\star$ are related to sky coords (ζ, ϕ) using radial, null rays.

Perform mode expansion

$$\hat{\sigma}(\tau,\,\mathbf{x}) = \int dq \sum_{p,\,r,\,s} \left[\frac{\Upsilon_{pqs}(\tau)}{\sqrt{a(\tau)b(\tau)}}\, U_{qrs}(\mathbf{x})\, \hat{a}_{pqrs} + \text{h.c.} \right].$$

"Fourier" modes U_{qrs} (q real, r and s integer) anisotropic.

The mode functions Υ_{pqs} are computed in the Bunch–Davies vacuum.
(Reminder: we treat parent and inflation in bubble as pure dS, take $k \ll 1$.)

[Yamamoto, Sasaki, Tanaka (1995,1995,1996); Bucher, Goldhaber, Turok (1995,1995); Garriga, Montes, Sasaki, Tanaka (1998,1999).]

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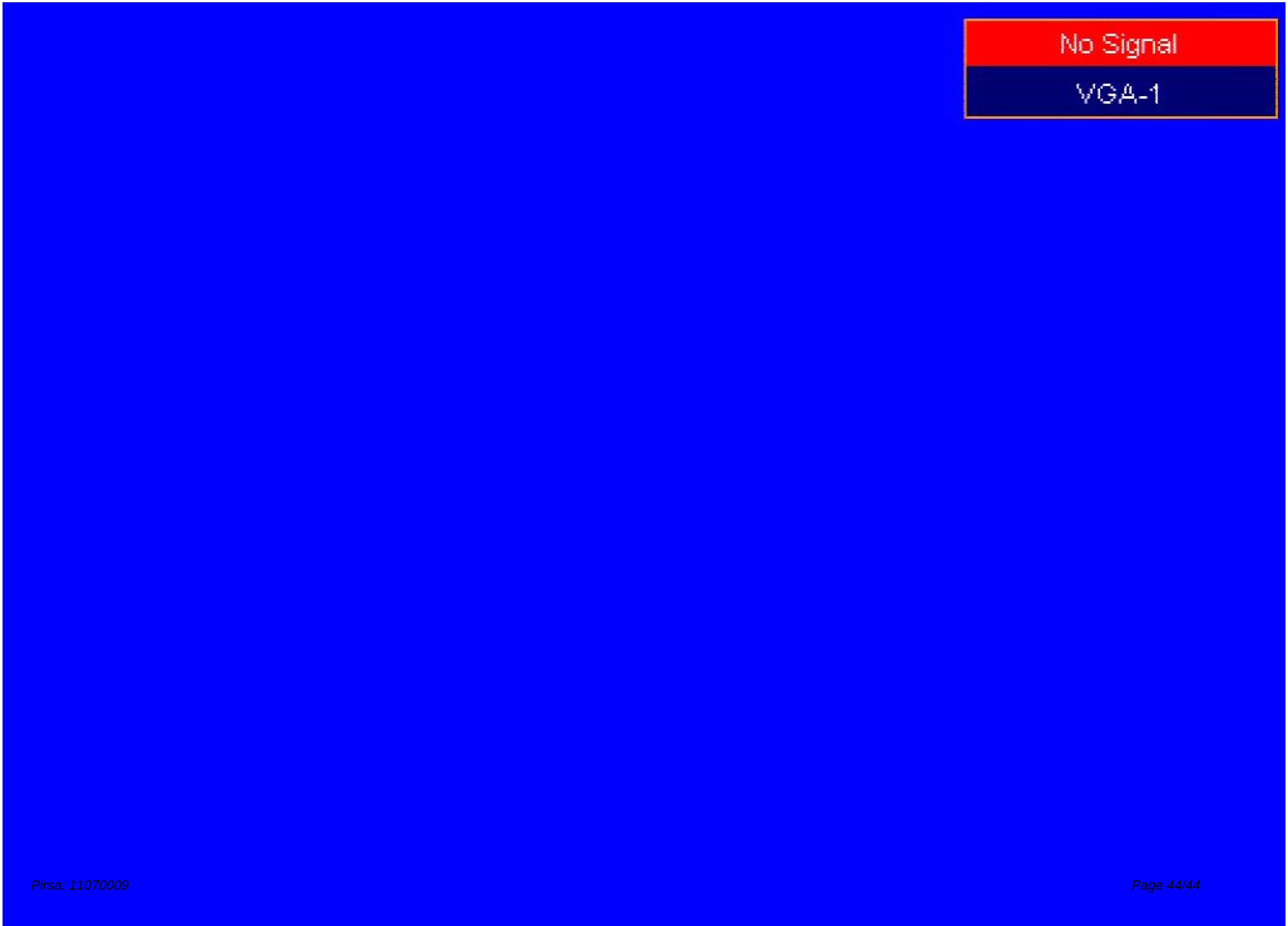
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