Title: Entropy in the Universe

Date: Jul 16, 2011 09:50 AM

URL: http://pirsa.org/11070006

Abstract: "A positive cosmological constant allows arbitrarily many different quantum states, but apparently only if there can be big bangs and/or big crunches. Without any big bang or big crunch, the entropy may be limited by the Gibbons-Hawking entropy of pure deSitter, and the matter entropy might even more limited by a value roughly the three-fourths power of the Gibbons-Hawking entropy. A classical analogue of an upper limit on the entropy is the finite canonical measure for nonsingular cosmologies.

Inflation by itself does not explain the arrow of time and seems to require that not only a small region have suitable initial conditions. A special quantum state, such as the Symmetric-Bounce one, with a suitable measure, such as volume averaging and Agnesi weighting, appears possible to explain the arrow of time and other observations, such as high order that would not be expected for Boltzmann brain observations, distant stars that would not be expected if we were fluctuations of empty de Sitter spacetime, and positive cosmological constant that appears not to dominate many other measures."

Entropy in the Universe

P. C. W. Davies, "Inflation and Time Asymmetry in the Universe," *Nature* **301**, 398-400 (1983):

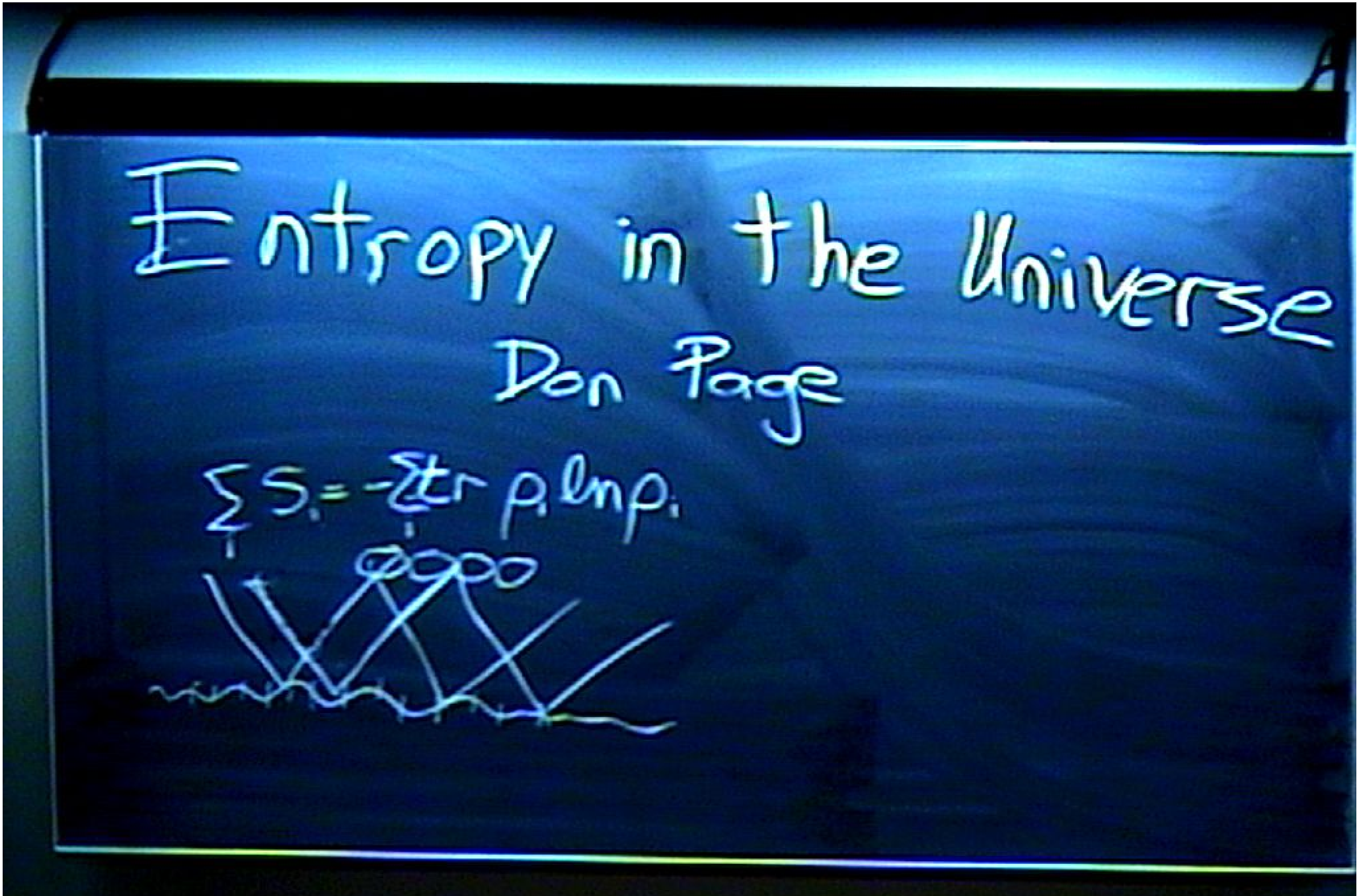
"I argue here that it [the recently proposed inflationary Universe scenario] also provides a natural explanation for the origin of time asymmetry ('time's arrow') in the Universe."

"In conclusion, it is now possible, using the inflationary scenario, to postulate a Universe which begins in a arbitrary, equilibrium state, both as far as matter and gravity are concerned but which, because of the linkage between the quantum state of the matter and the gravitational behaviour through Λ, first gets wound up gravitationally by inflation, then gets wound up in the nuclear sense at a much later epoch by the conventional expansion. The remaining history of the Universe is the subsequent attempt to unwind by gravitational clumping . . . and nucleosynthesis . . ."

D. N. Page, "Inflation Does Not Explain Time Asymmetry," *Nature* **304**, 39-41 (1983):

"Davies has argued that the inflationary cosmological scenario provides a natural explanation for the time asymmetry of the Universe. Here I dispute this argument by noting that the inflationary scenario implicitly invokes time asymmetry with the assumption of an absence of initial spatial correlations. No scenario based on charge-parity-time (CPT)-invariant dynamical laws can explain the time asymmetry apart from postulating or explaining these special initial conditions, as Penrose has emphasized."

If n regions each with Hilbert-space dimension D, joint system has density matrix with $D^{2n}-1$ real parameters, roughly $10^{10^{122.377}}$ for $n = 10^{122.597}$, $D = 2$, whereas an uncorrelated joint density matrix has only $n(D^2-1)$ parameters, roughly $10^{10^{2.09017}}$, far, far less, so such an uncorrelated initial state that can give the second law is very, very special.


Entropy in the Universe
Don Page
$\sum_i S_i = -\sum_i \mathrm{tr}\, \rho_i \ln \rho_i$

D. N. Page, "Inflation Does Not Explain Time Asymmetry," *Nature* **304**, 39-41 (1983):

"Davies has argued that the inflationary cosmological scenario provides a natural explanation for the time asymmetry of the Universe. Here I dispute this argument by noting that the inflationary scenario implicitly invokes time asymmetry with the assumption of an absence of initial spatial correlations. No scenario based on charge-parity-time (CPT)-invariant dynamical laws can explain the time asymmetry apart from postulating or explaining these special initial conditions, as Penrose has emphasized."

If n regions each with Hilbert-space dimension D, joint system has density matrix with $D^{2n}-1$ real parameters, roughly $10^{10^{122.377}}$ for $n=10^{122.597}$, $D=2$, whereas an uncorrelated joint density matrix has only $n(D^2-1)$ parameters, roughly $10^{10^{2.09017}}$, far, far less, so such an uncorrelated initial state that can give the second law is very, very special.

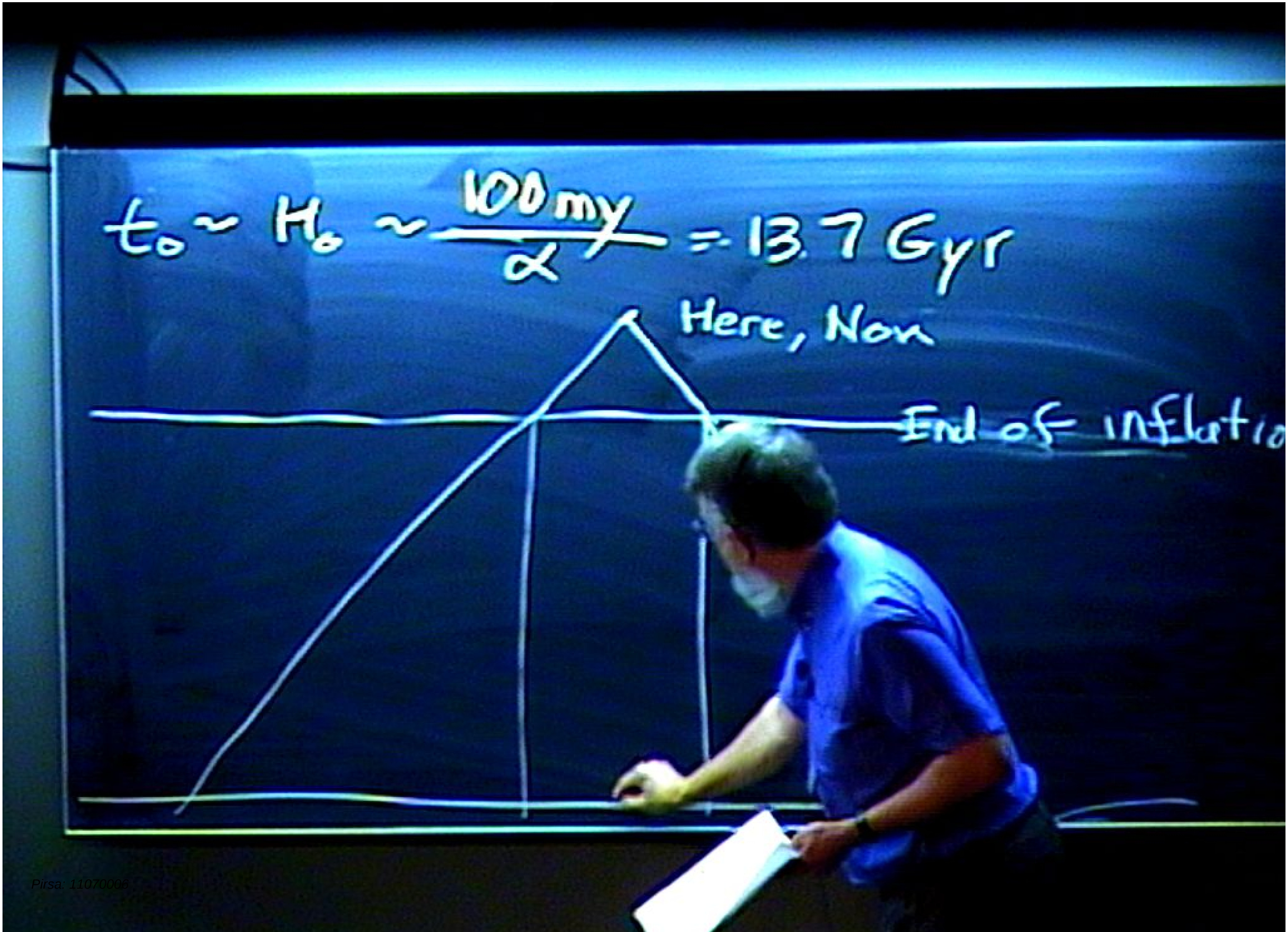
Does Inflation Require Only a Small Initial Region to Have Suitable Initial Conditions?

Often it is said that if a small region has suitable initial conditions, it will lead to inflation.

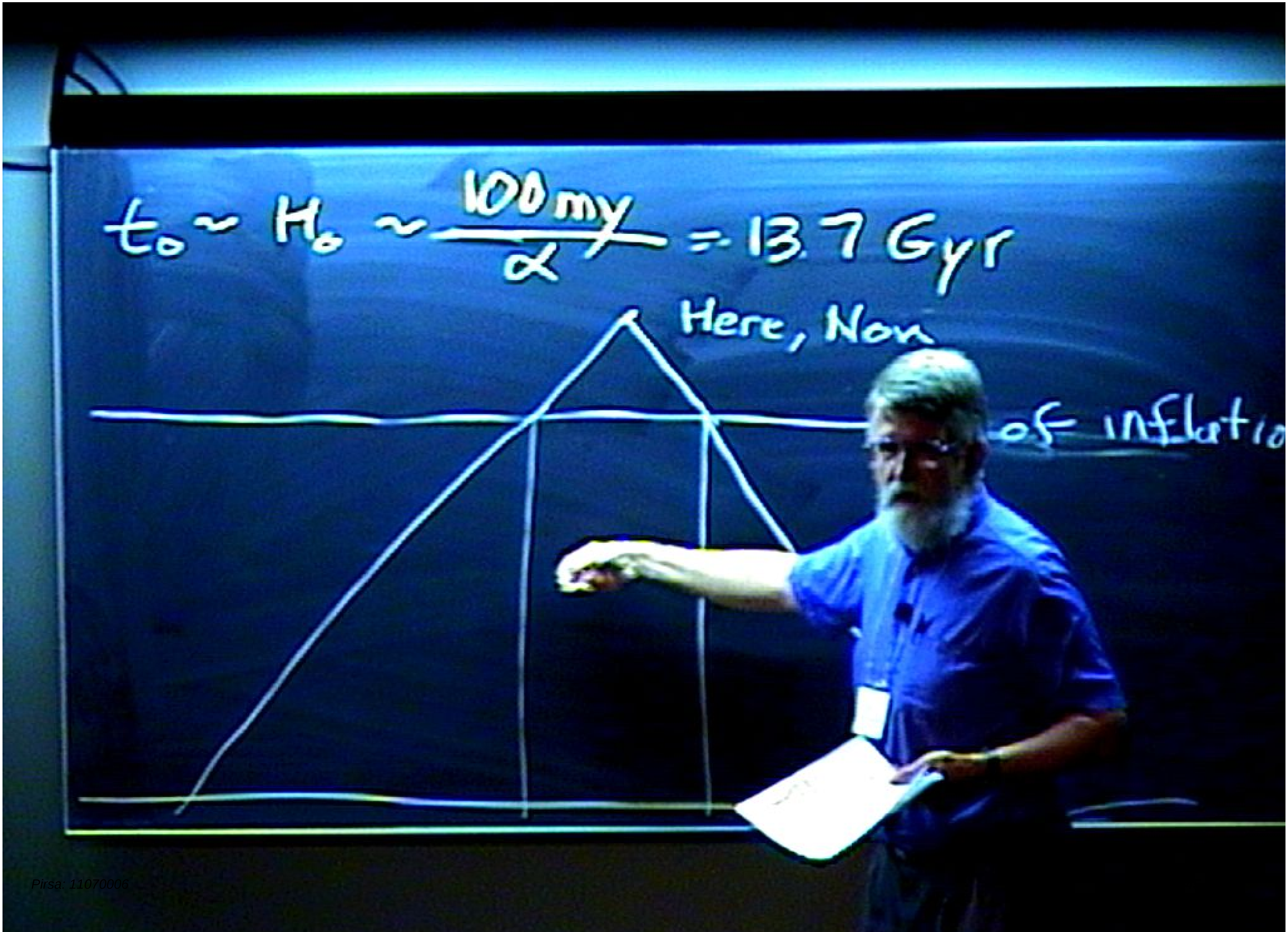
An implicit assumption is that that region evolves approximately autonomously.

However, if the surroundings are in a generic chaotic state, it is not obvious why large perturbations from them will not enter the region and destroy its inflationary evolution.

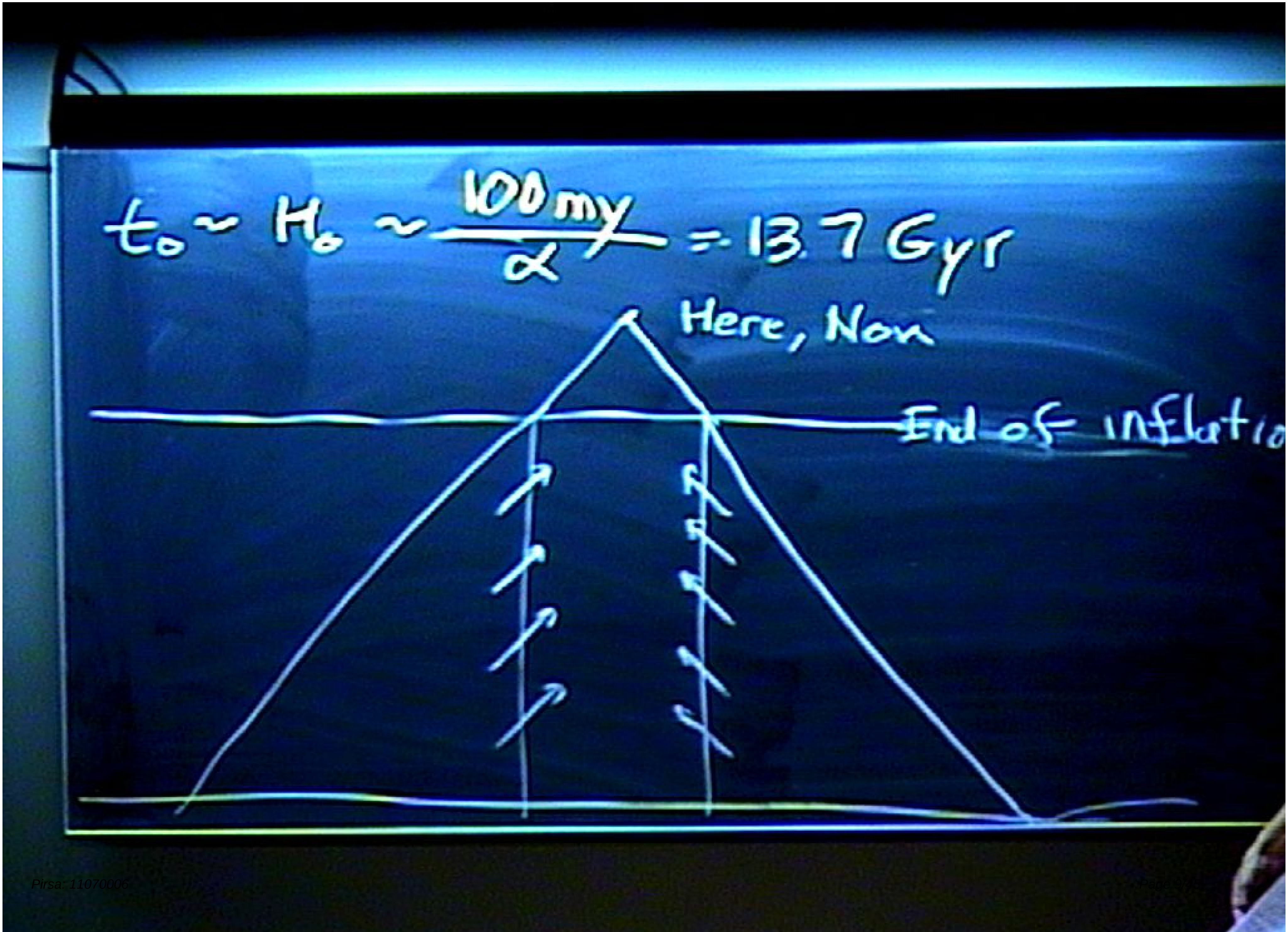
Assuming that this does not happen seems rather analogous to assuming a very special initial state over a large region, so again it seems that one needs a highly restricted quantum state to get the observed arrow of time.



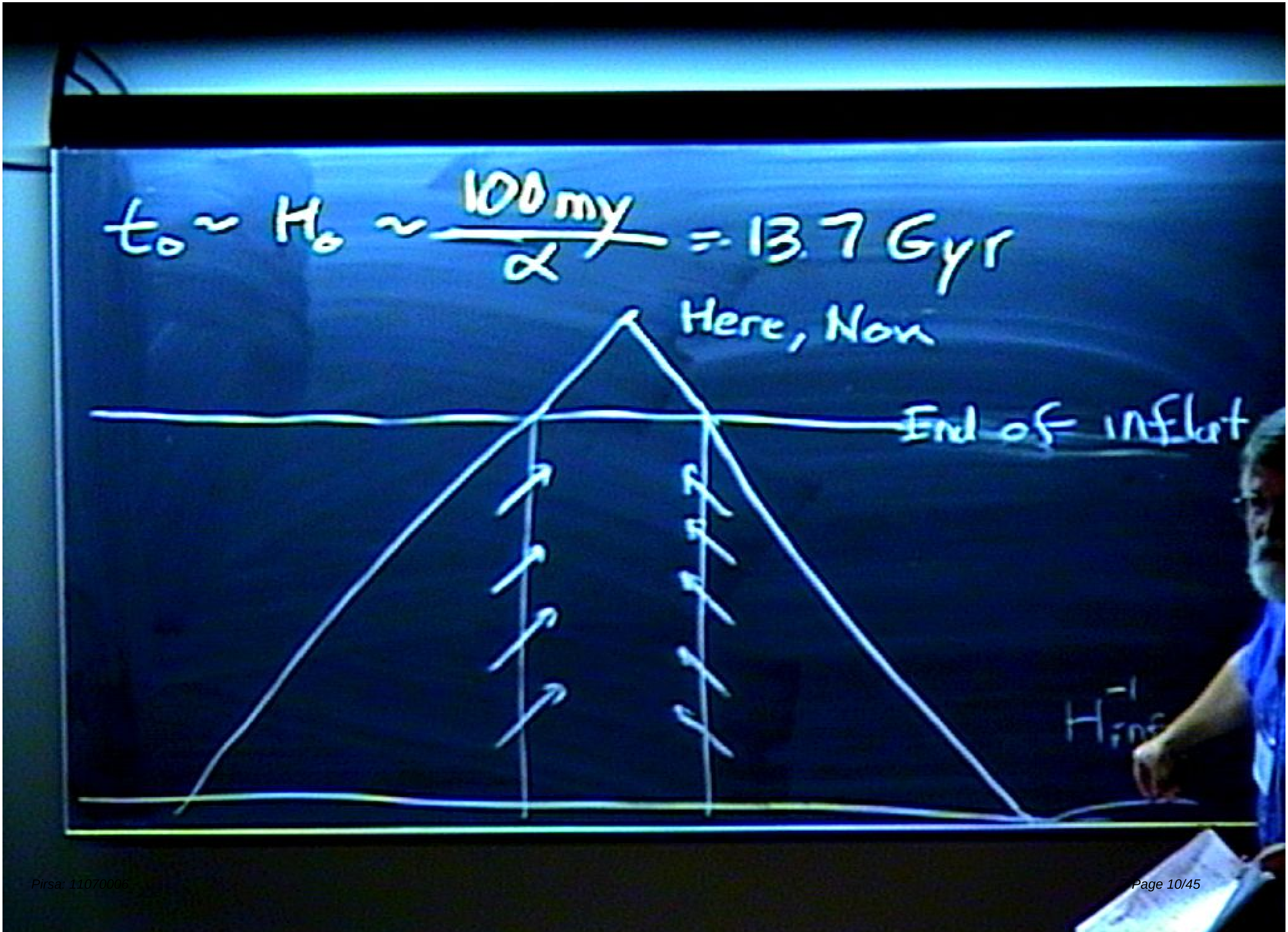
$t_0 \sim H_0 \sim \frac{100\,my}{\alpha} = 13.7\ Gyr$
Here, Now
End of inflatio

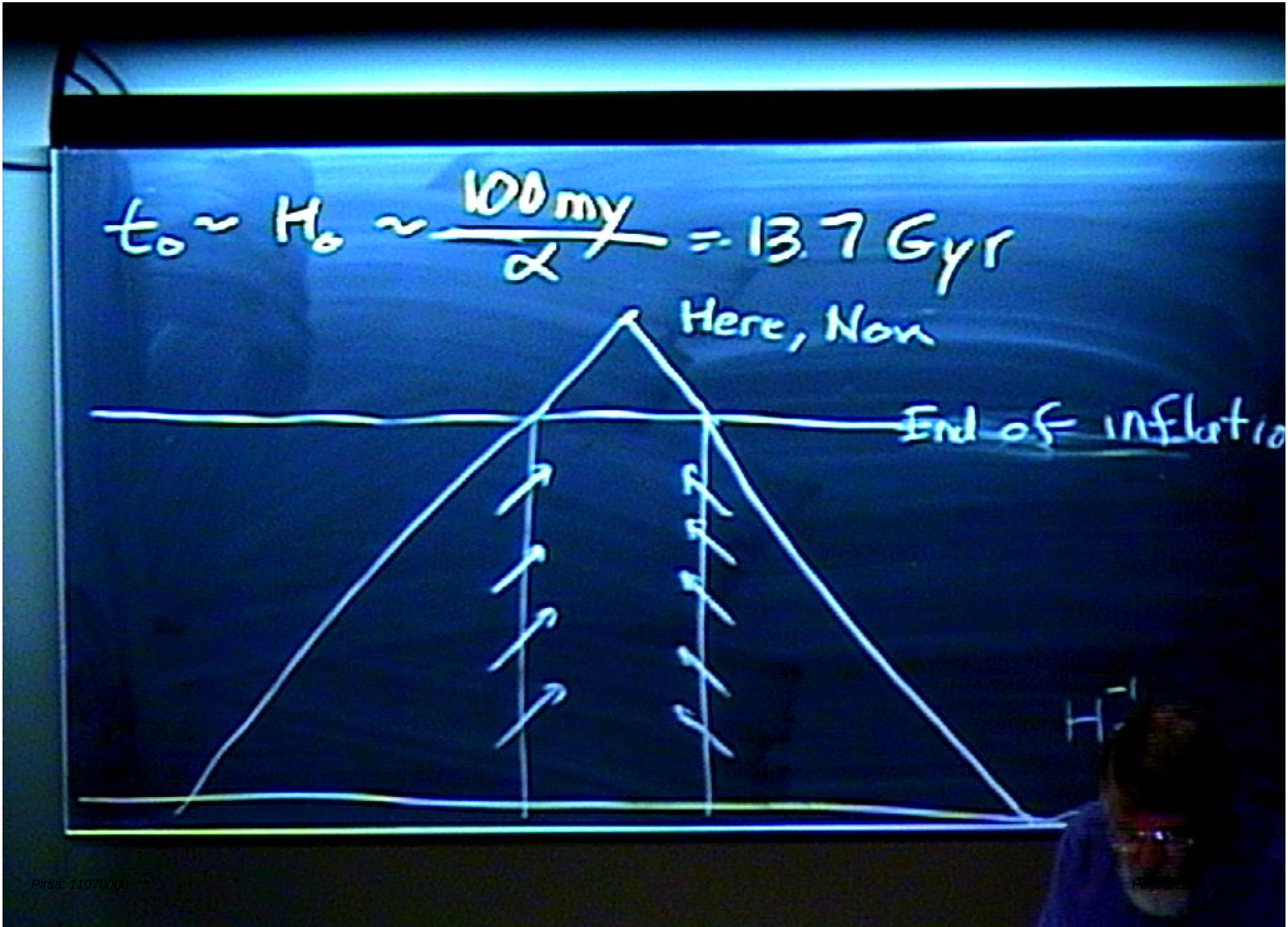


$t_0 \sim H_0 \sim \frac{100\,my}{\alpha} = 13.7\,Gyr$
Here, Now
of inflatio



$t_0 \sim H_0 \sim \frac{100\,my}{\alpha} = 13.7\,Gyr$
Here, Now
End of inflatio


$t_0 \sim H_0 \sim \frac{100\,my}{\alpha} = 13.7$ Gyr
Here, Now
End of inflat
H_{inf}^{-1}


$t_0 \sim H_0 \sim \frac{100\,my}{\alpha} = 13.7$ Gyr
Here, Now
End of inflatio
H^{-1}

Eternal Inflation

Eternal inflation with tunneling between different bubbles suggests that the universe may consist of an infinite branching sequence of bubble universes.

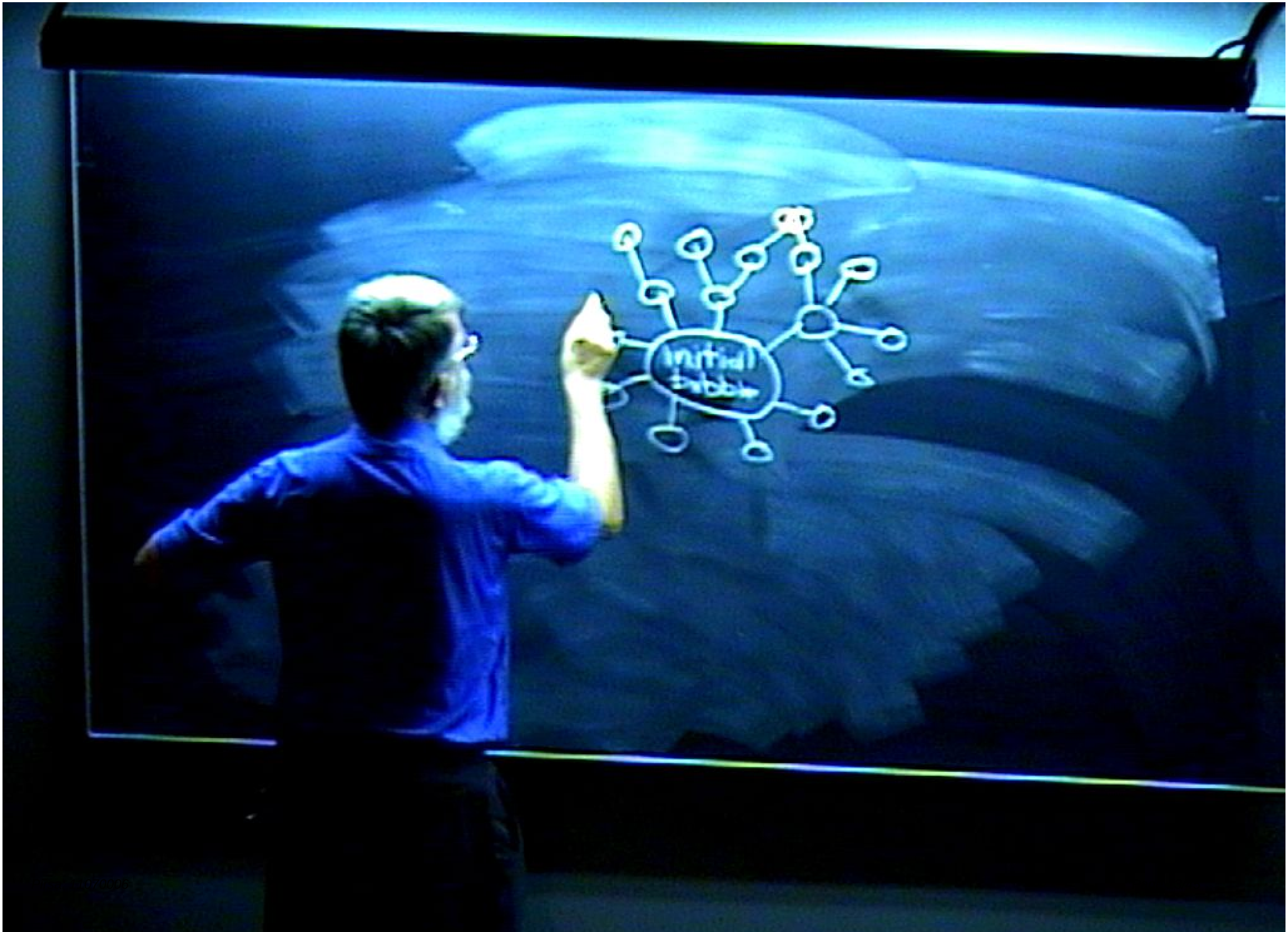
Are we near the beginning of a sequence or far down a chain?

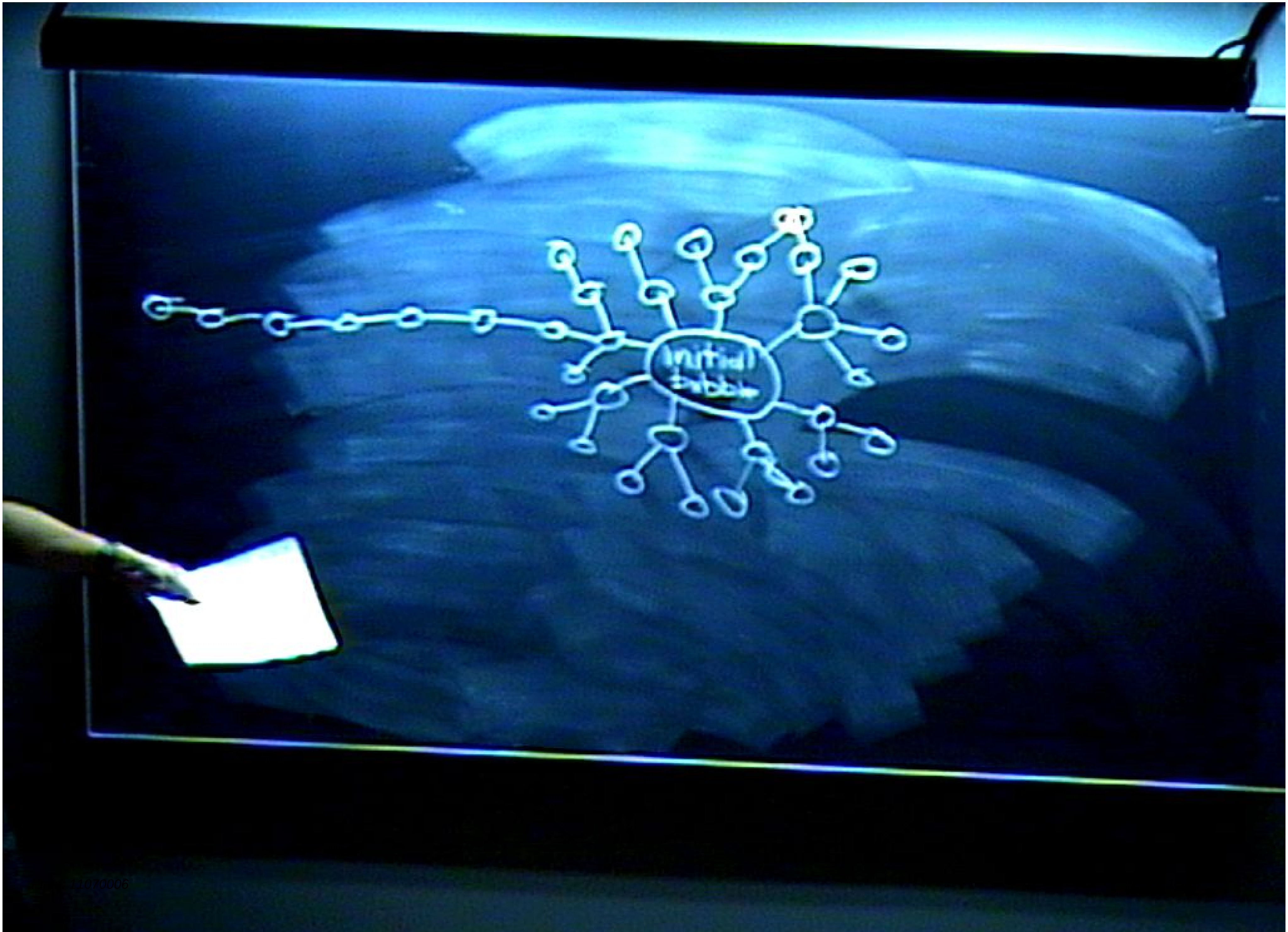
If we are far down, is there a finite or infinite number of predecessors?

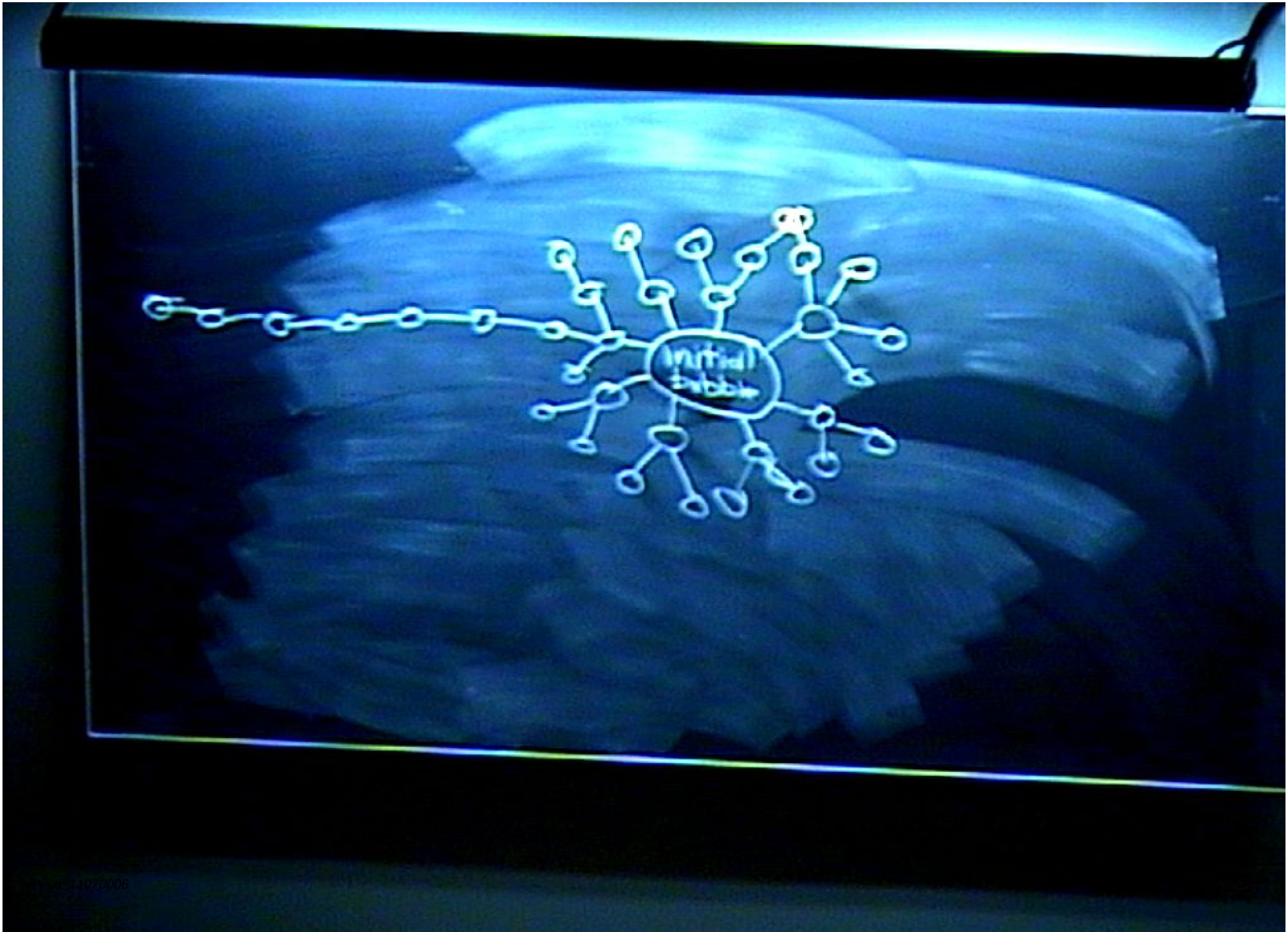
If it is a finite number, what sets its order of magnitude?

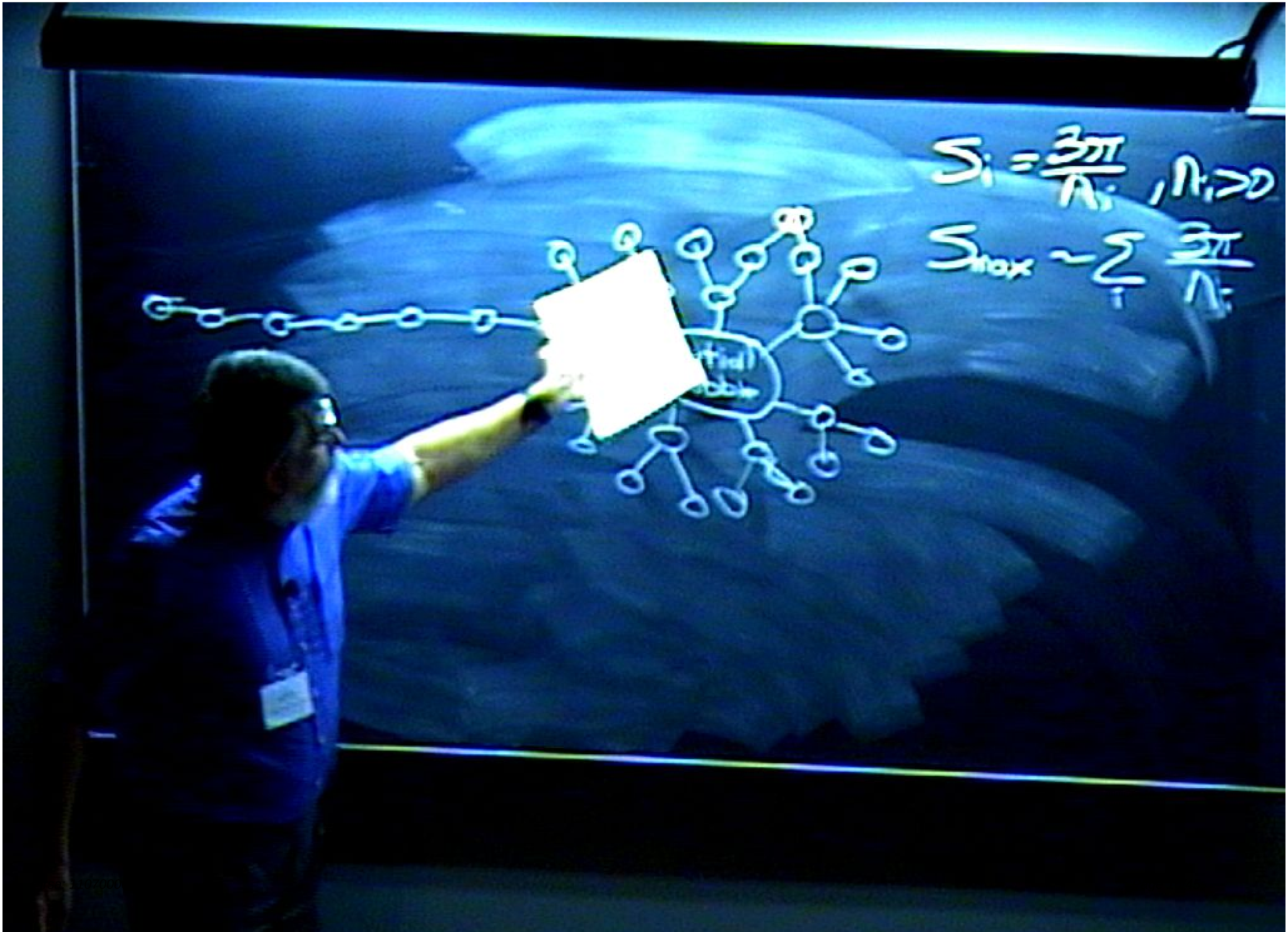
If it is an infinite number, why hasn't the universe thermalized?

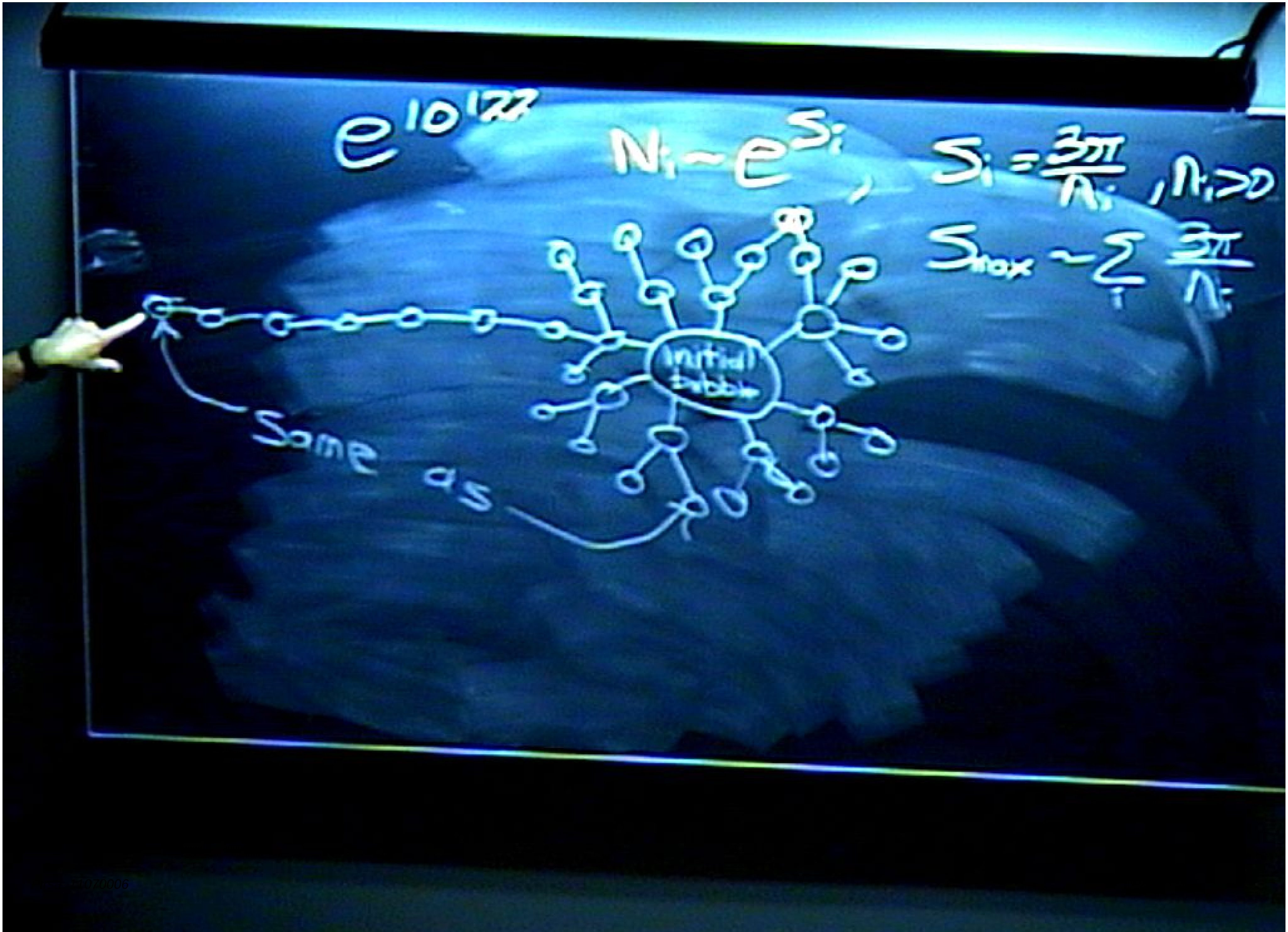
Is the state space infinite in such a way that even though an infinite part is filled, it does not thermalize?

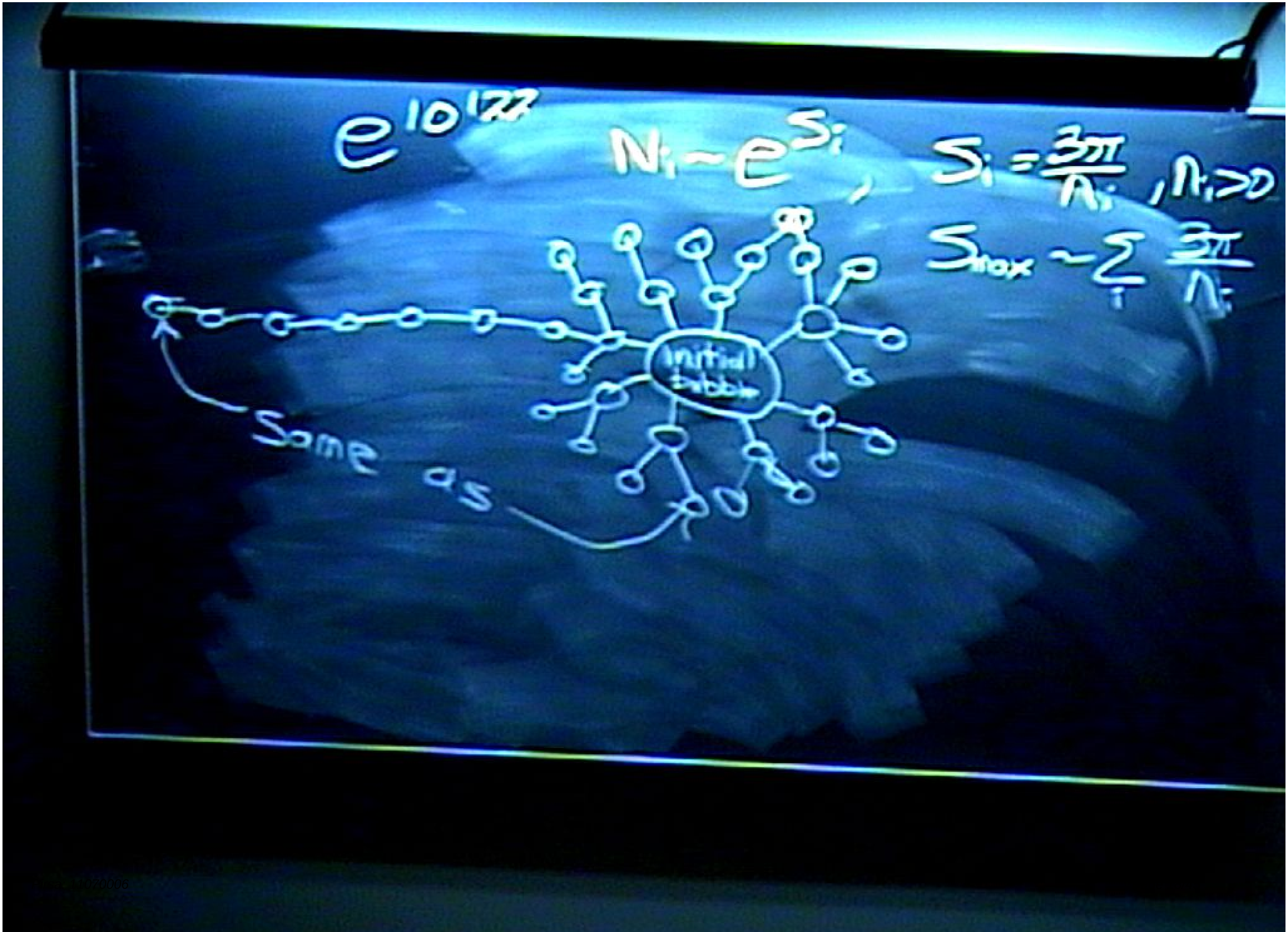

initial


Initial


Initial


$S_i = \frac{3\pi}{\Lambda_i}, \Lambda_i > 0$
$S_{max} \sim \sum_i \frac{3\pi}{\Lambda_i}$


$e^{10^{122}}$
$N_i \sim e^{S_i}$,
$S_i = \frac{3\pi}{\Lambda_i}$, $\Lambda_i > 0$
$S_{max} \sim \sum_i \frac{3\pi}{\Lambda_i}$
Initial bubble
Same as

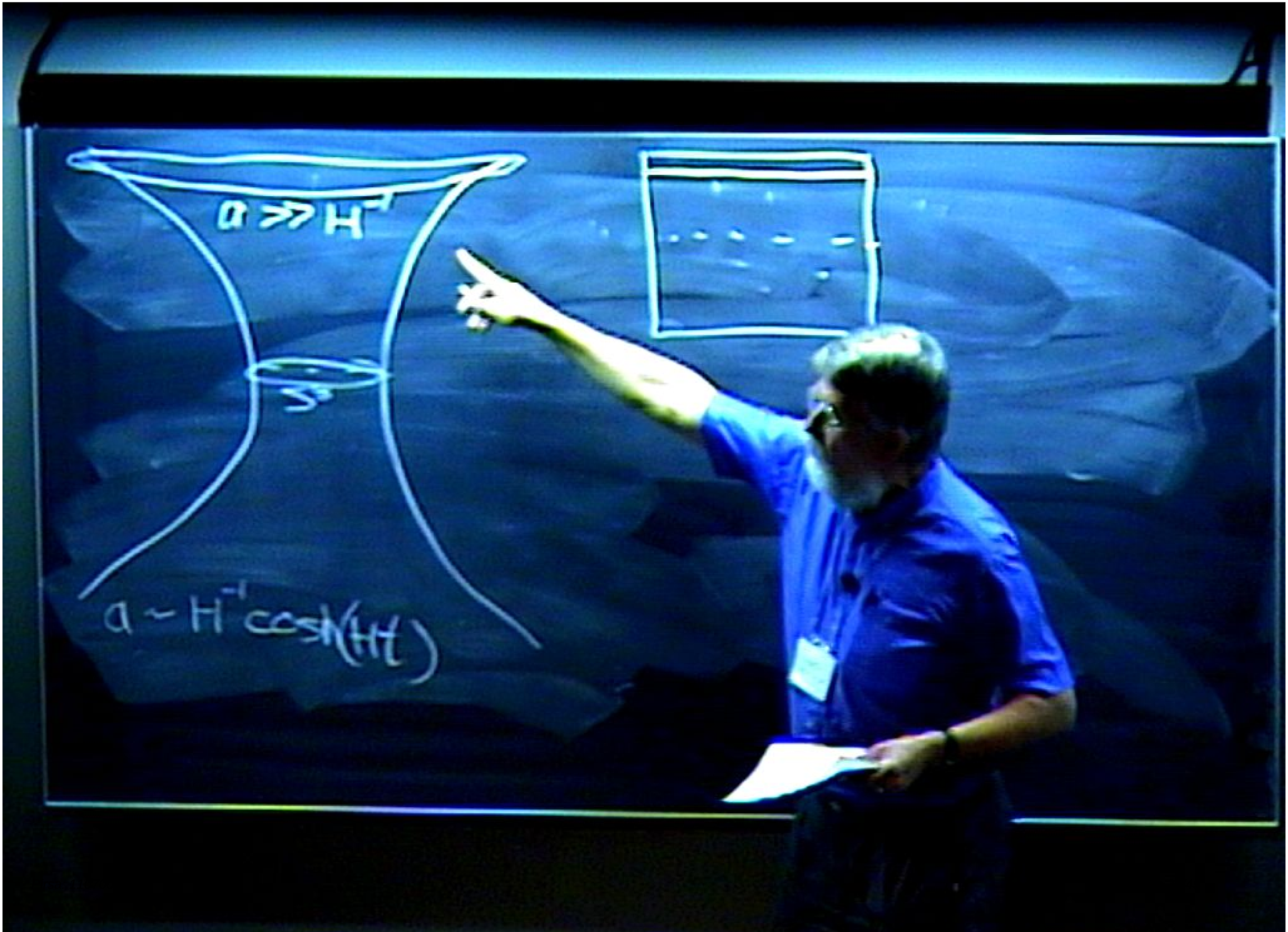

$e^{10^{122}}$
$N_i \sim e^{S_i}$, $S_i = \frac{3\pi}{\Lambda_i}$, $\Lambda_i > 0$
$S_{max} \sim \sum_i \frac{3\pi}{\Lambda_i}$
Initial bubble
Same as

Does a Positive Cosmological Constant Imply Finite Entropy?

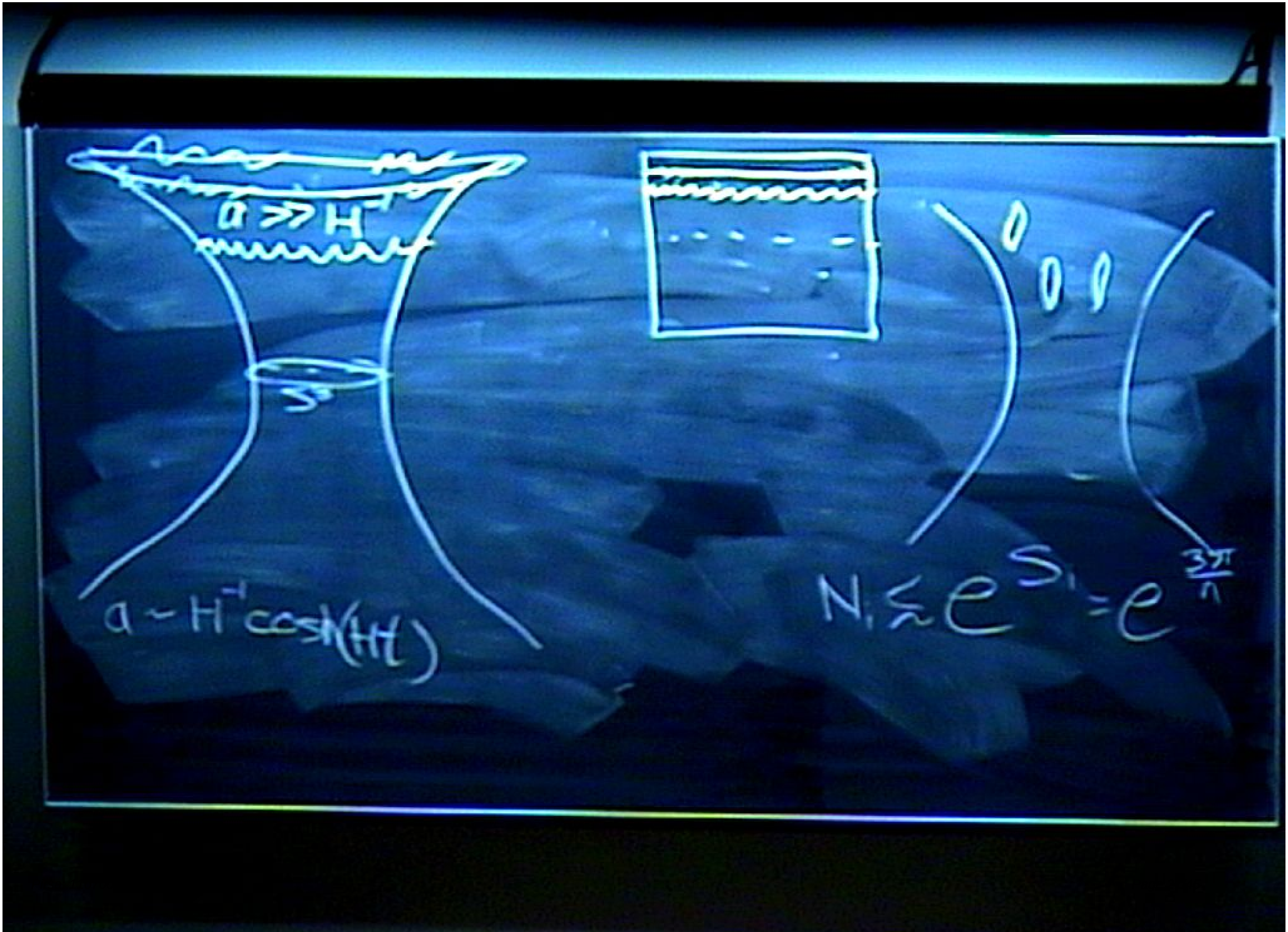
DeSitter spacetime has a Gibbons-Hawking entropy of $S = 3\pi/\Lambda$. Often that is thought to be the upper limit for fixed $\Lambda > 0$. However, one can take an arbitrarily large $k = +1$ hypersurface at late times in deSitter and perturb it with radiation having arbitrarily large entropy.

A more plausible conjecture would be that a nonsingular spacetime with fixed $\Lambda > 0$ has $S \leq 3\pi/\Lambda$.

One might extend this conjecture to allow black holes to form and evaporate, so long as asymptotically there is no big bang or big crunch singularity at the beginning or end of the universe.



a >> H^-1
S^3
a ~ H^-1 cosh(Ht)


a >> H⁻¹
S³
a ~ H⁻¹ cosh(Ht)
N₁ ≲ e^S₁ = e^(3π/Λ)

"No-Bang Quantum State of the Cosmos," *Classical and Quantum Gravity* **25**, 154011 (2008):

For a homogeneous S^3 neck of radius R with inflaton ϕ and thermal radiation of temperature T with radiation constant a, the maximum entropy of the radiation will come from having the maximum value of the temperature T consistent with $\ddot{R} \geq 0$ at $\dot{\phi} = 0$, which gives

$$T = (2a)^{-1/4}\left(m^2\phi^2 + \frac{\Lambda}{4\pi G}\right)^{1/4},$$

$$R = \left(\frac{8\pi G}{3}\right)^{-1/2}\left(m^2\phi^2 + \frac{\Lambda}{4\pi G}\right)^{-1/2},$$

$$\begin{aligned} S &= \left(\frac{9\pi^2 a}{512}\right)^{1/4}\left(G^2 m^2\phi^2 + \frac{G\Lambda}{4\pi}\right)^{-3/4} \\ &\leq S_{\text{no-bang}} = \left(\frac{9\pi^5 a}{8G^3\Lambda^3}\right)^{\frac{1}{4}}. \end{aligned}$$

Another way to get arbitrarily large entropy for fixed $\Lambda > 0$ is to add radiation to perturb the neck of the Kottler metric,

$$ds^2_{\text{neck}} = dx^2 + r^2(x)(d\theta^2 + \sin^2\theta d\varphi^2),$$

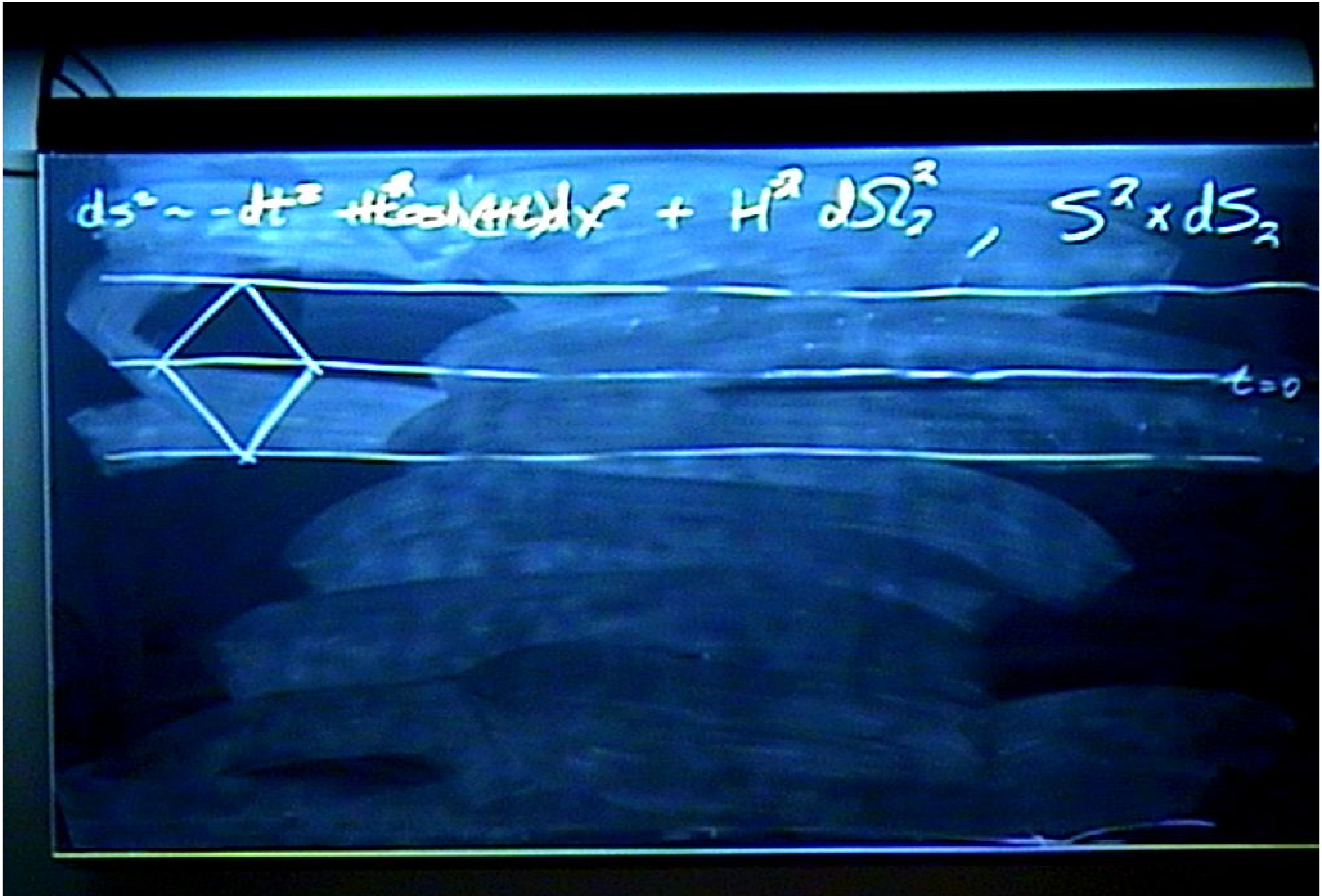
where the Einstein constraint equation gives

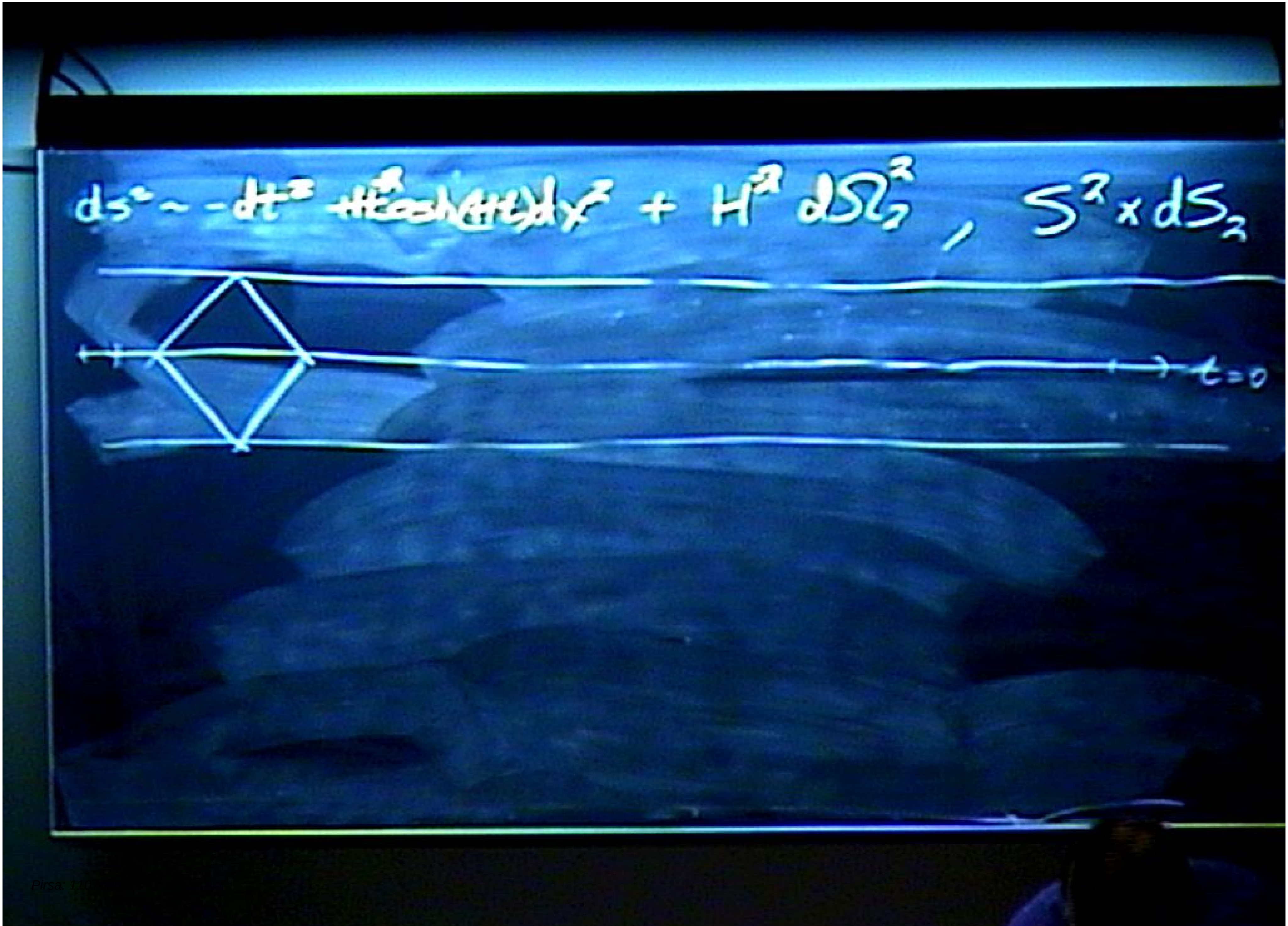
$$\frac{1 - r'^2 - 2rr''}{r^2} = -G^0_0 = \Lambda + 4\pi G(m^2\phi^2 + 2aT^4),$$

with a homogeneous solution

$$r = [\Lambda + 4\pi G(m^2\phi^2 + 2aT^4)]^{-1/2}.$$

If x has length X, then the total 3-volume of this homogeneous neck is $4\pi r^2 X$, and the entropy of the radiation is $S = (16\pi/3)aT^3r^2X$. But since the length X can be made arbitrarily large, the entropy of the neck is unbounded above.


ds² ~ -dt² + H⁻² cosh²(Ht) dx² + H⁻² dΩ₂² , S² × dS₂
t=0


ds² ~ -dt² + H² cosh²(Ht) dx² + H² dΩ₂² , S² x dS₂
t=0

Another way to get arbitrarily large entropy for fixed $\Lambda > 0$ is to add radiation to perturb the neck of the Kottler metric,

$$ds^2_{\text{neck}} = dx^2 + r^2(x)(d\theta^2 + \sin^2\theta d\varphi^2),$$

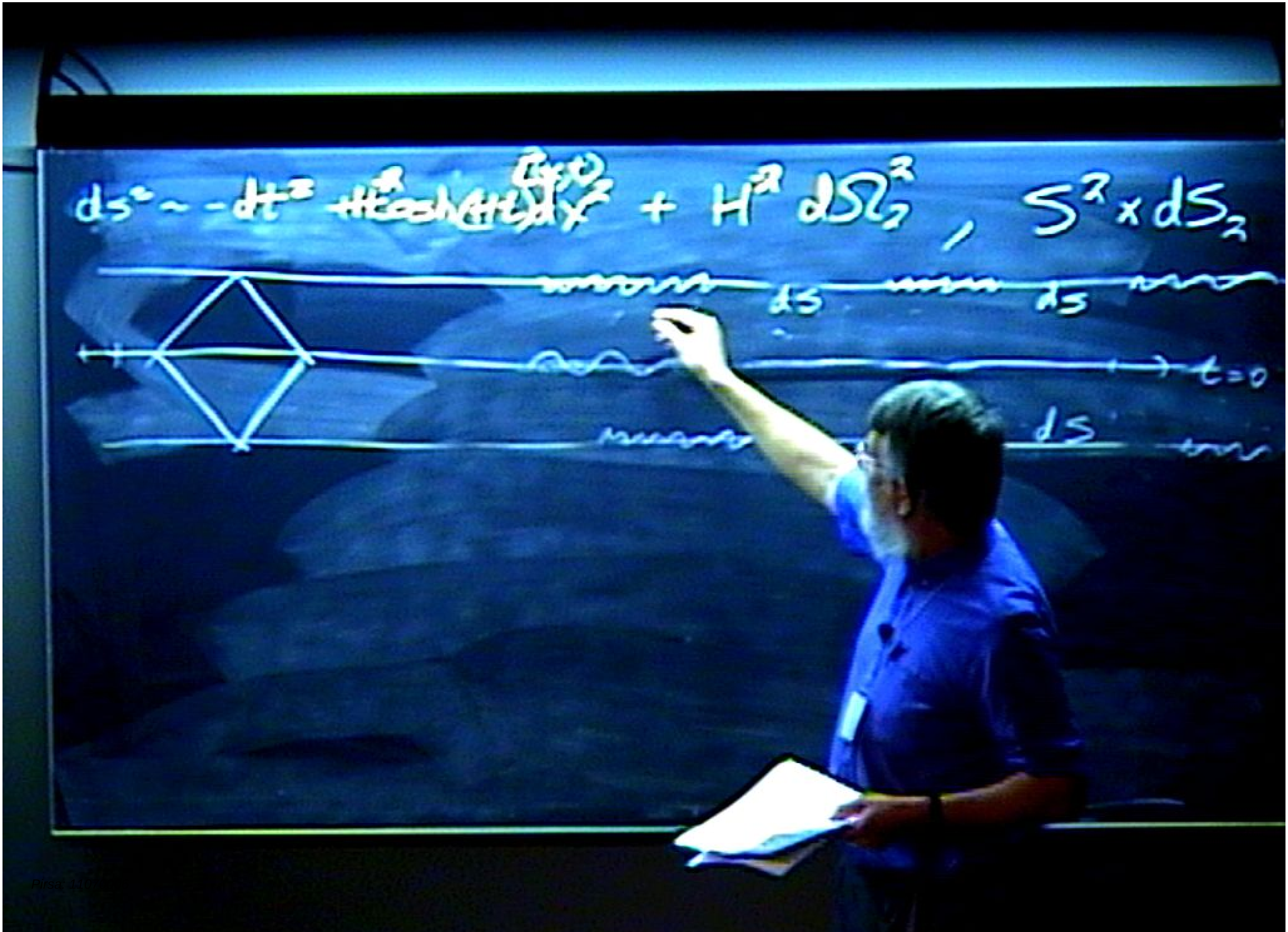
where the Einstein constraint equation gives

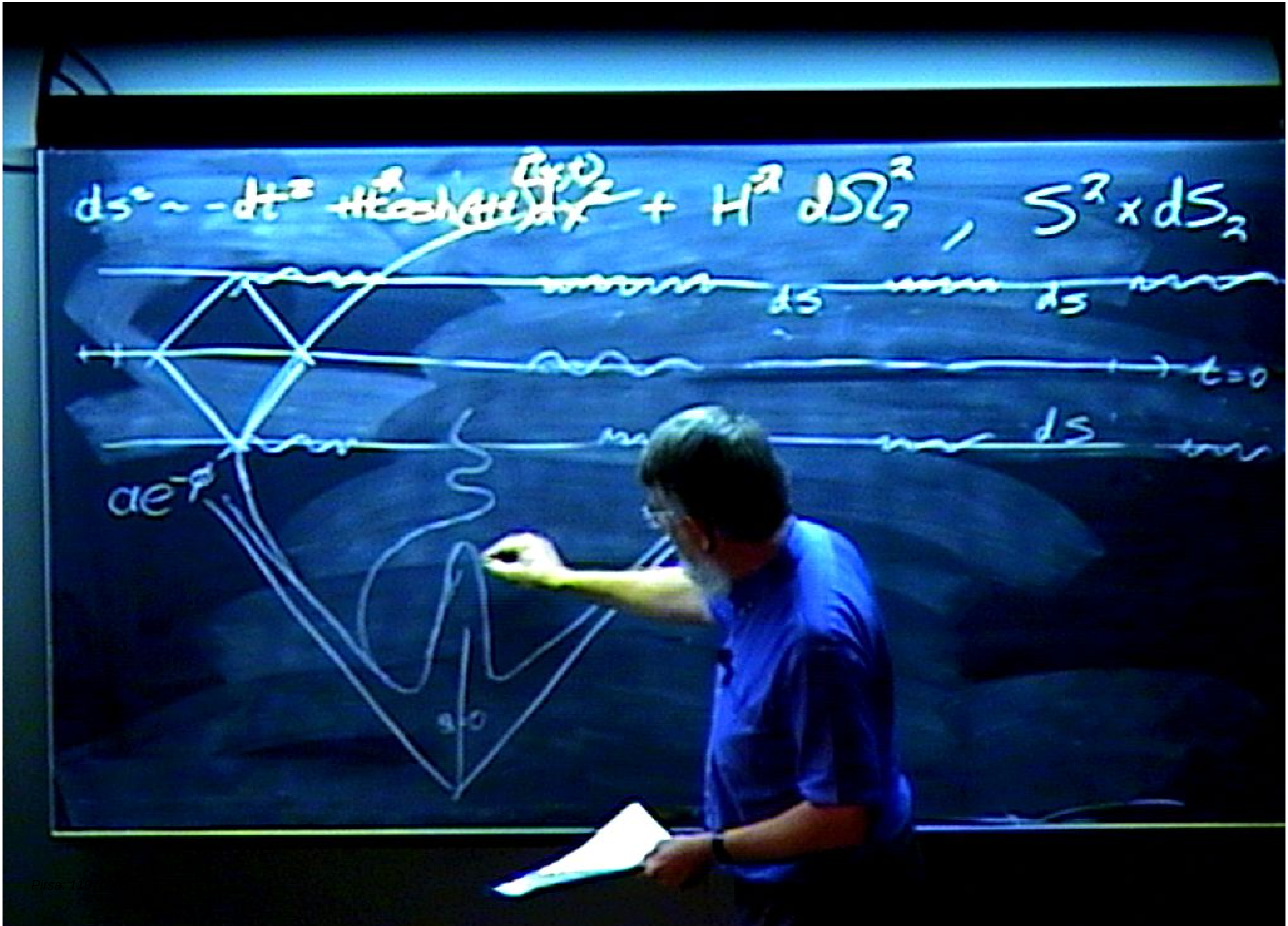
$$\frac{1 - r'^2 - 2rr''}{r^2} = -G^0_0 = \Lambda + 4\pi G(m^2\phi^2 + 2aT^4),$$

with a homogeneous solution

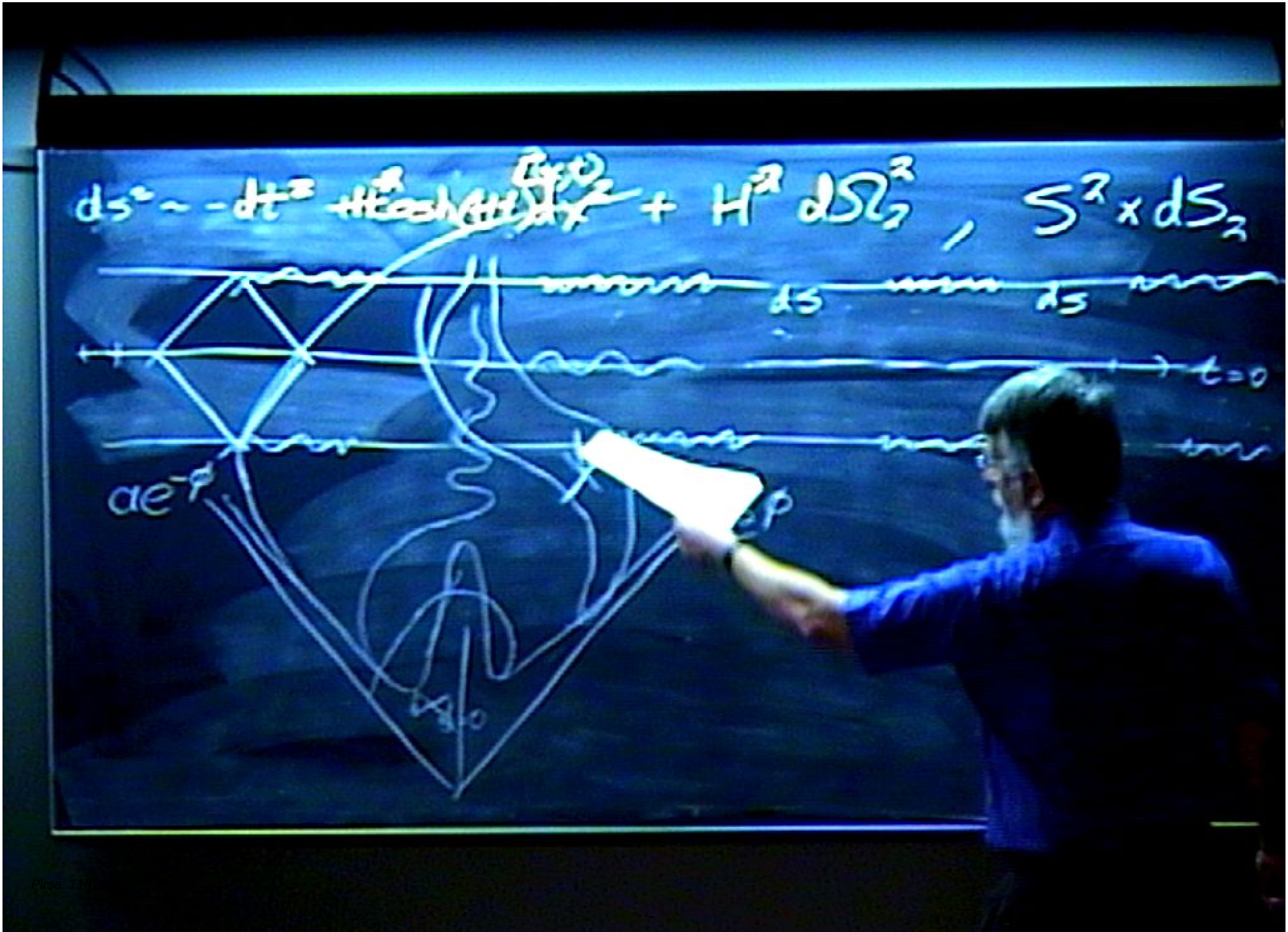
$$r = [\Lambda + 4\pi G(m^2\phi^2 + 2aT^4)]^{-1/2}.$$

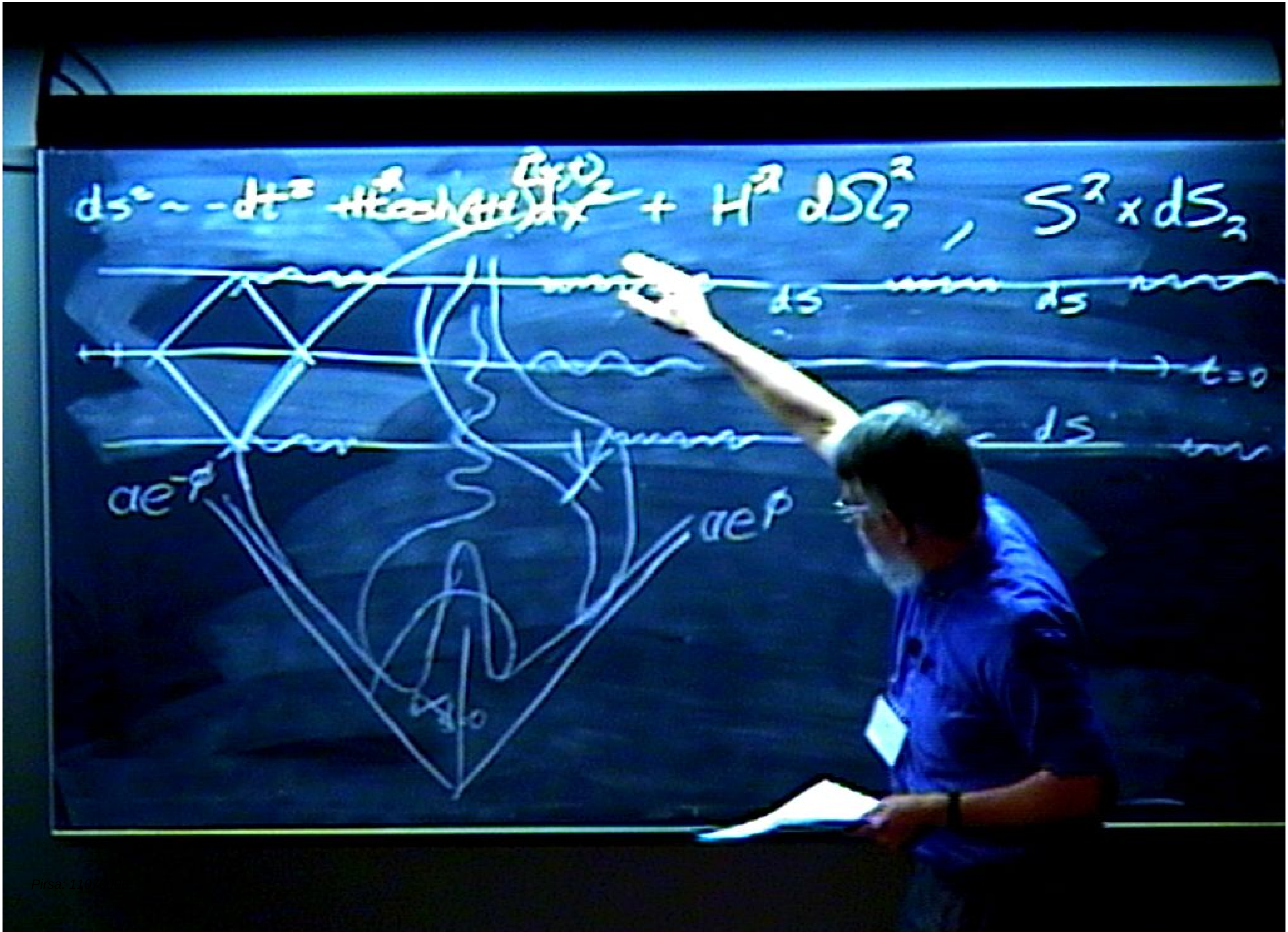
If x has length X, then the total 3-volume of this homogeneous neck is $4\pi r^2 X$, and the entropy of the radiation is $S = (16\pi/3)aT^3 r^2 X$. But since the length X can be made arbitrarily large, the entropy of the neck is unbounded above.

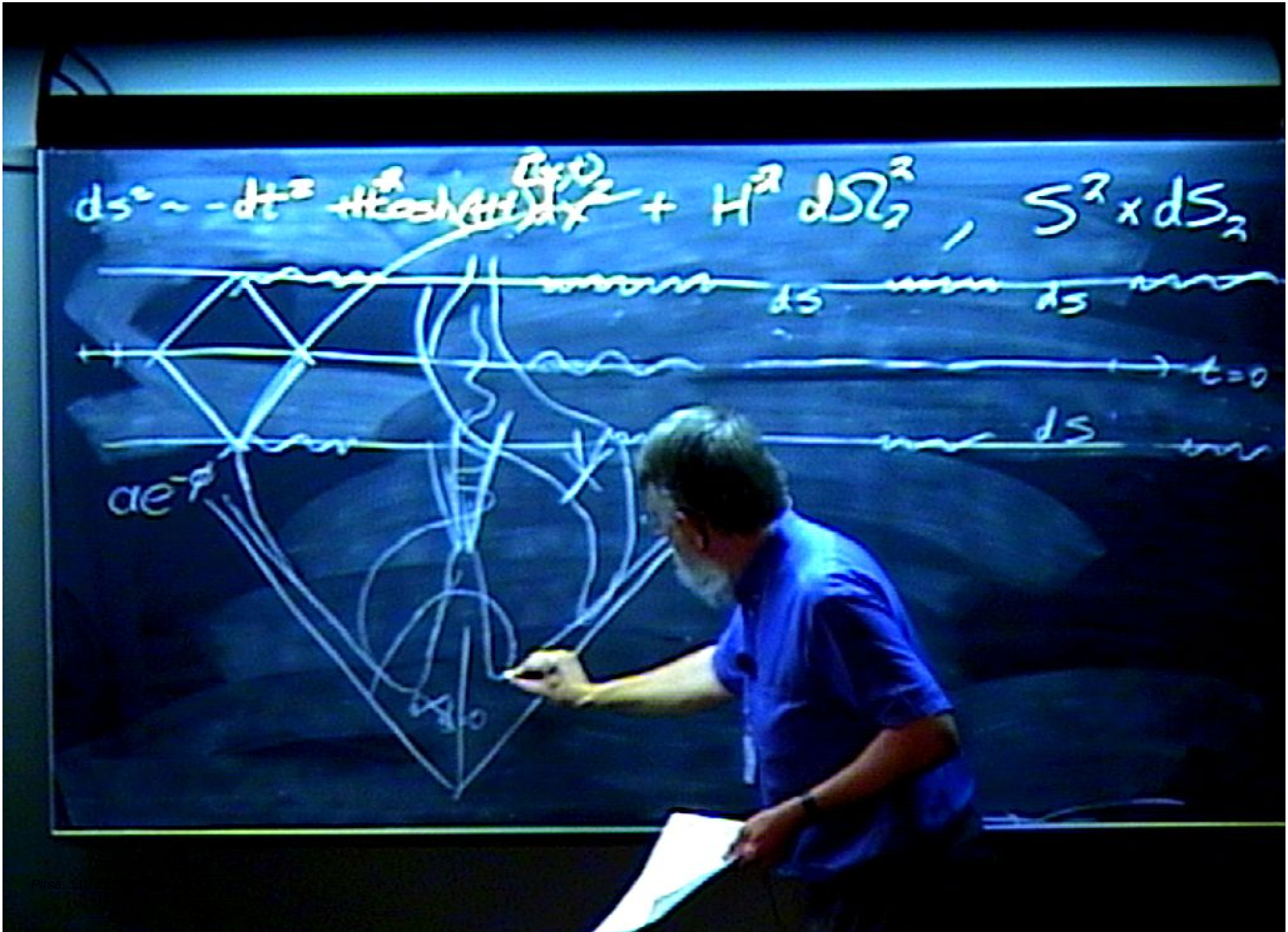

S² x dS₂
dS
dS
t=0
dS

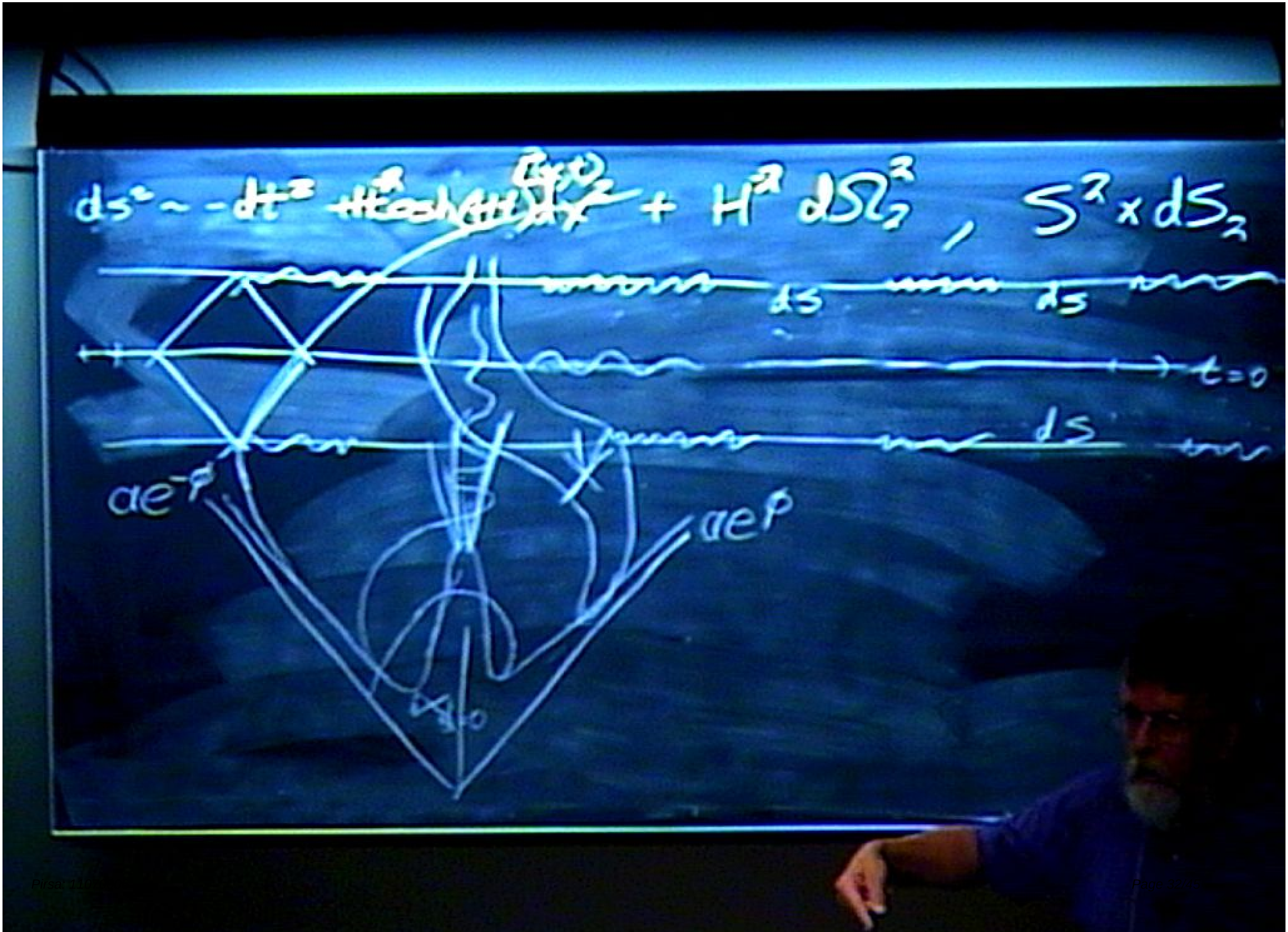


S² x dS₂
dS
dS
t=0
dS


S² x dS₂
dS
dS
t=0
ae^{-φ}


$S^2 \times dS_2$
dS
dS
t=0
dS
$ae^{-\rho}$
ae^{ρ}


dS
dS
t=0
dS



dS
dS
t=0
dS
ae^{-ρ}
ae^{ρ}
S^2 x dS_2

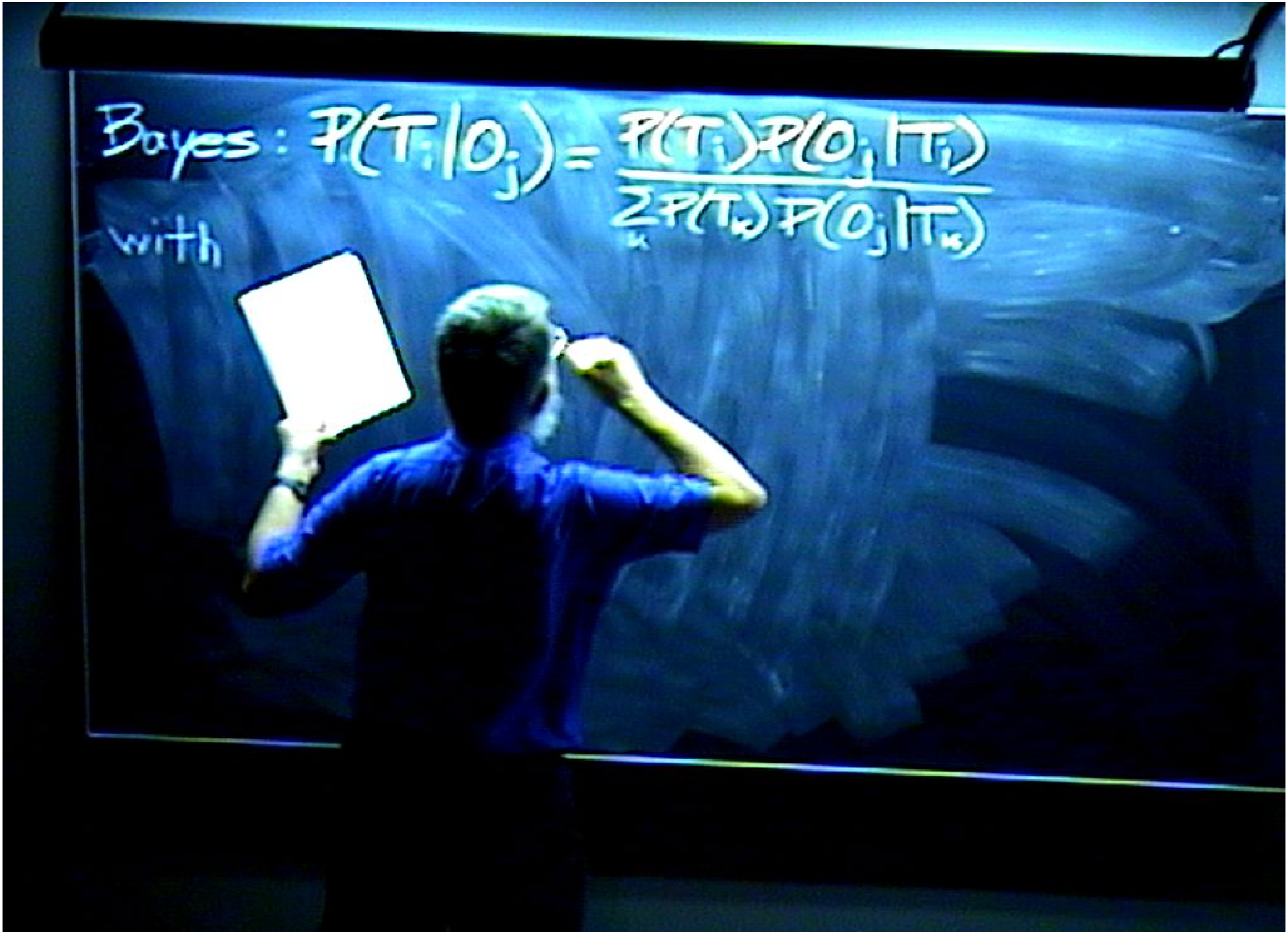
Explaining the Arrow of Time

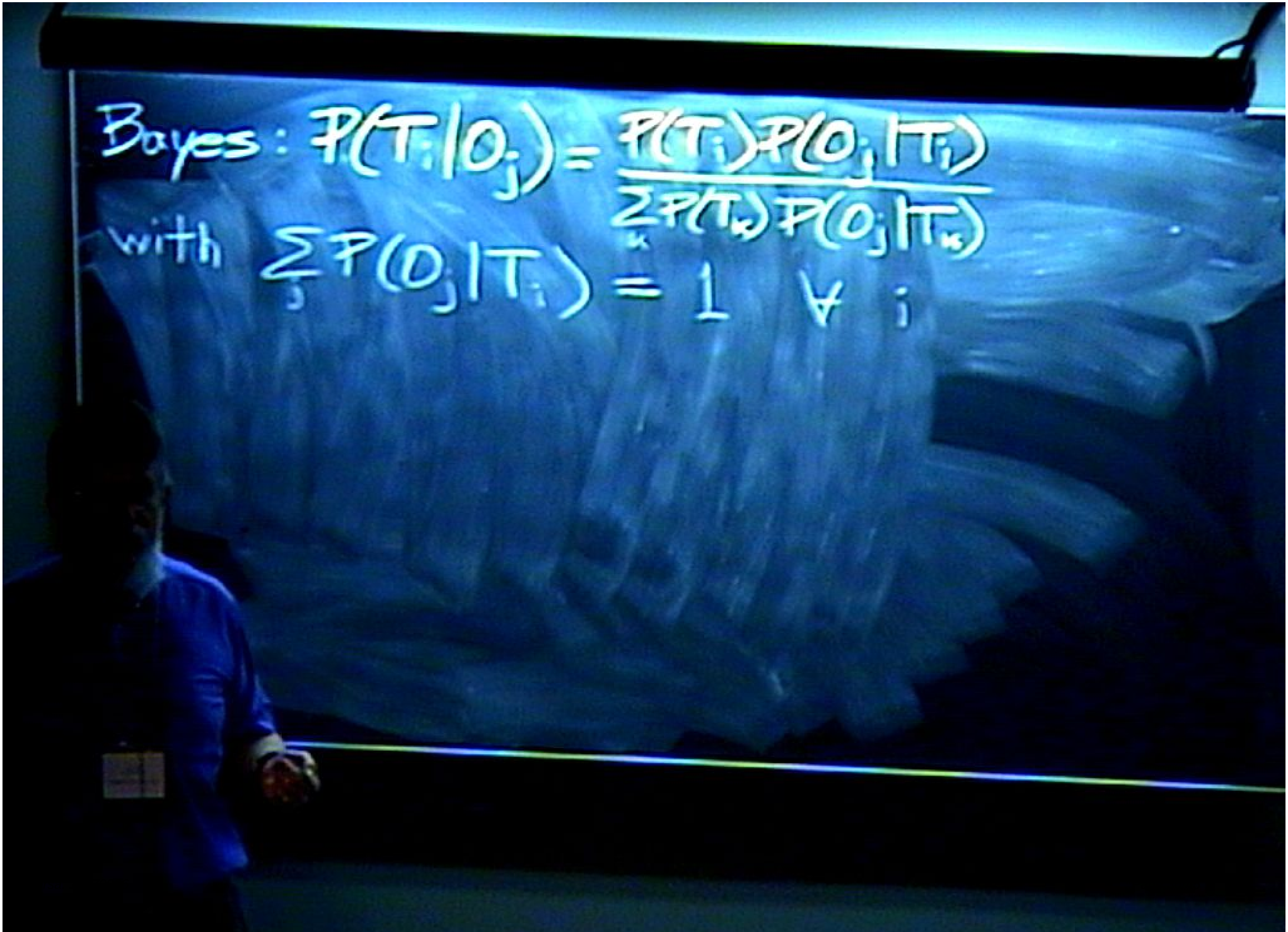
To explain our observations, it may not be enough just to have finite entropy. We may need a special quantum state that has small coarse-grained entropy when the universe is small but growing entropy when the universe expands.

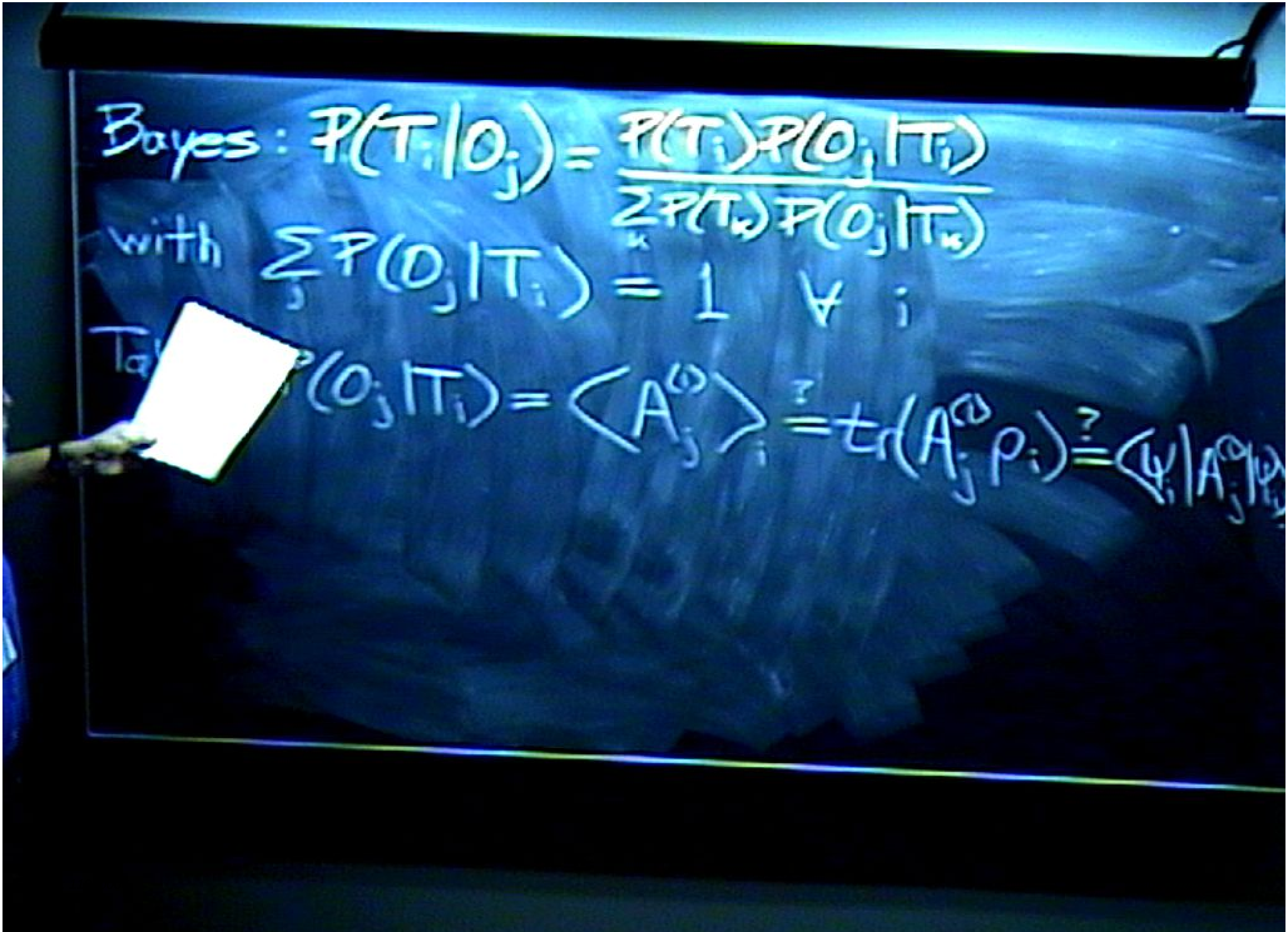
I used to favor the Hartle-Hawking no-boundary proposal, but in "Susskind's Challenge to the Hartle-Hawking No-Boundary Proposal and Possible Resolutions," *JCAP* **0701**, 004 (2007), I fleshed out Susskind's argument that the HH state is dominated by empty de Sitter.

Then I favored my "No-Bang Quantum State of the Cosmos," *Classical and Quantum Gravity* **25**, 154011 (2008), but it seems to have Boltzmann brain problems.

Now I favor my "Symmetric-Bounce Quantum State of the Universe," *JCAP* **0909**, 026 (2009), which is a pure state that is macroscopically time symmetric about a homogeneous, isotropic bounce with inhomogeneities and anisotropies then in their ground state. When combined with the volume averaging of "Cosmological Measures without Volume Weighting," *JCAP* **0810**, 025 (2008) (which takes the spatial density of observations rather than the total number at each moment of time), and "Agnesi Weighting for the Measure Problem of Cosmology," *JCAP* **1103**, 031 (2011) (which uses the simple weighting factor $dt/(1+t^2)$ to make the time integral of the volume averaging converge), it seems to fit observations and avoid problems other states and measures have with vacua of zero or negative cosmological constant. The measure it gives for at least N e-folds of inflation is $\propto N^{-1/4}$, which is consistent with observations.


Bayes : $P(T_i|O_j) = \frac{P(T_i)P(O_j|T_i)}{\sum_k P(T_k)P(O_j|T_k)}$
with


Bayes: $P(T_i|O_j) = \frac{P(T_i)P(O_j|T_i)}{\sum_k P(T_k)P(O_j|T_k)}$
with $\sum_j P(O_j|T_i) = 1 \quad \forall i$

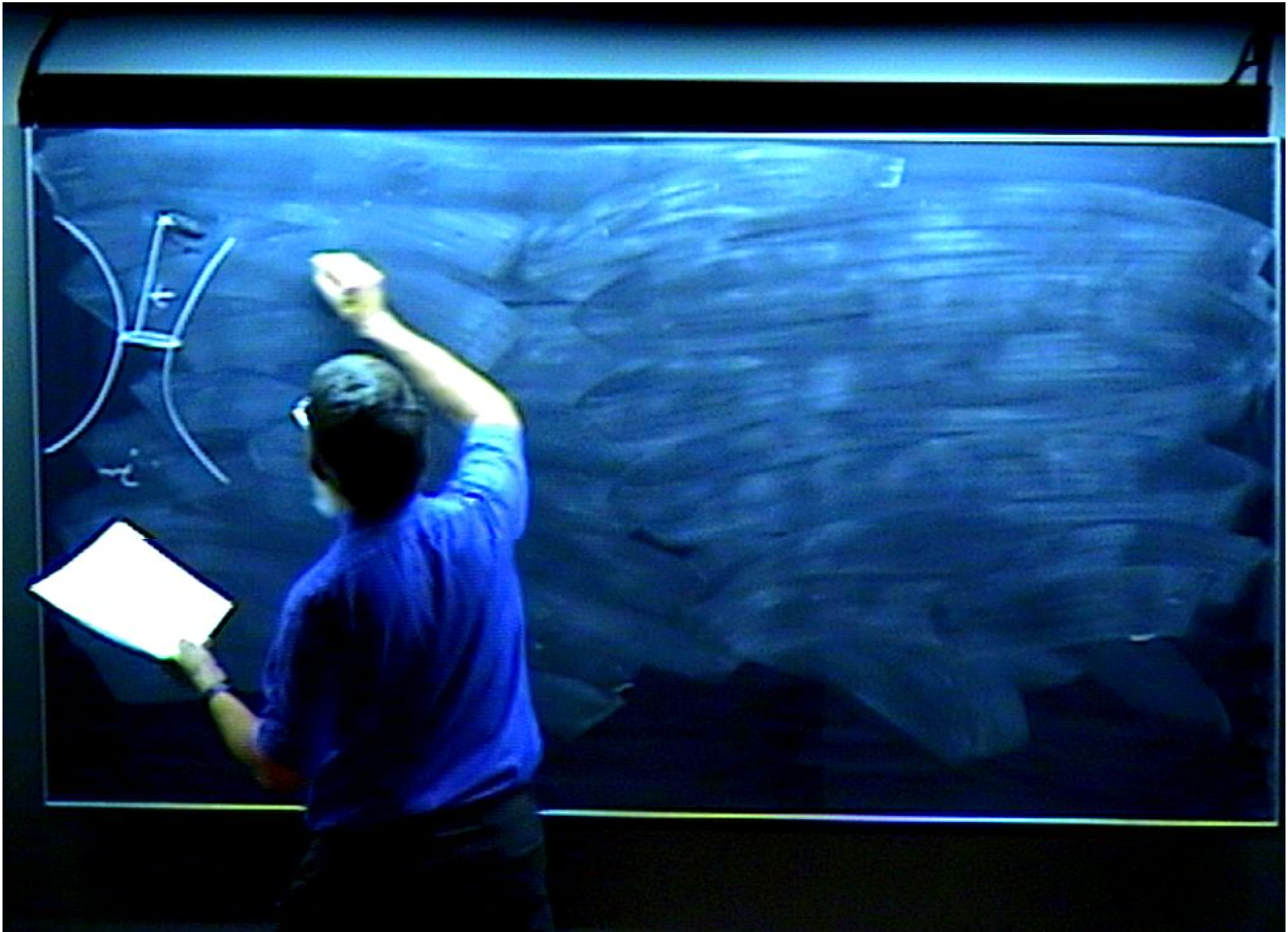


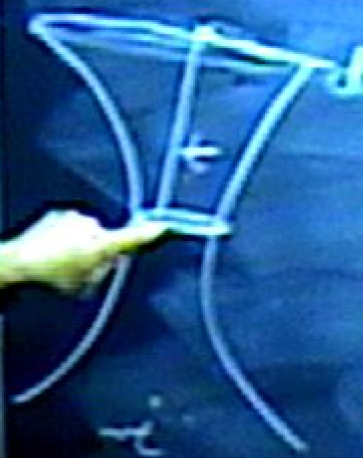
Bayes: $P(T_i|O_j) = \frac{P(T_i)P(O_j|T_i)}{\sum_k P(T_k)P(O_j|T_k)}$
with $\sum_j P(O_j|T_i) = 1 \ \forall \ i$
Ta
$P(O_j|T_i) = \langle A_j^{(i)} \rangle_i \overset{?}{=} tr(A_j^{(i)} \rho_i) \overset{?}{=} \langle \psi_i | A_j^{(i)} | \psi_i \rangle$

Bayes: $P(T_i|O_j) = \dfrac{P(T_i)P(O_j|T_i)}{\sum_k P(T_k)P(O_j|T_k)}$

with $\sum_j P(O_j|T_i) = 1 \quad \forall \; i$

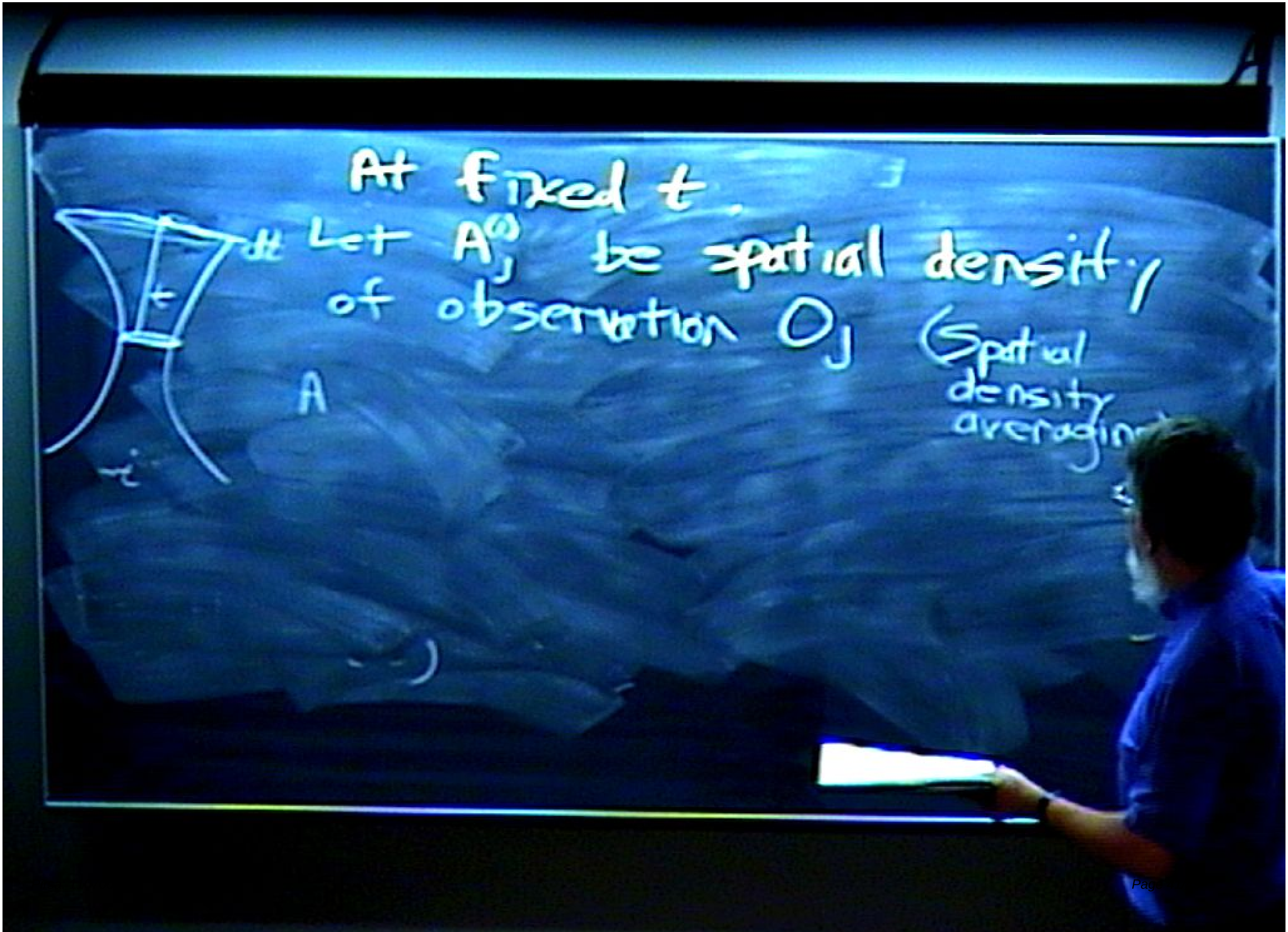
Take $P(O_j|T_i) = \langle A_j^{(i)} \rangle_i \overset{?}{=} tr(A_j^{(i)} \rho_i) \overset{?}{=} \langle \psi_i | A_j^{(i)} | \psi_i \rangle$

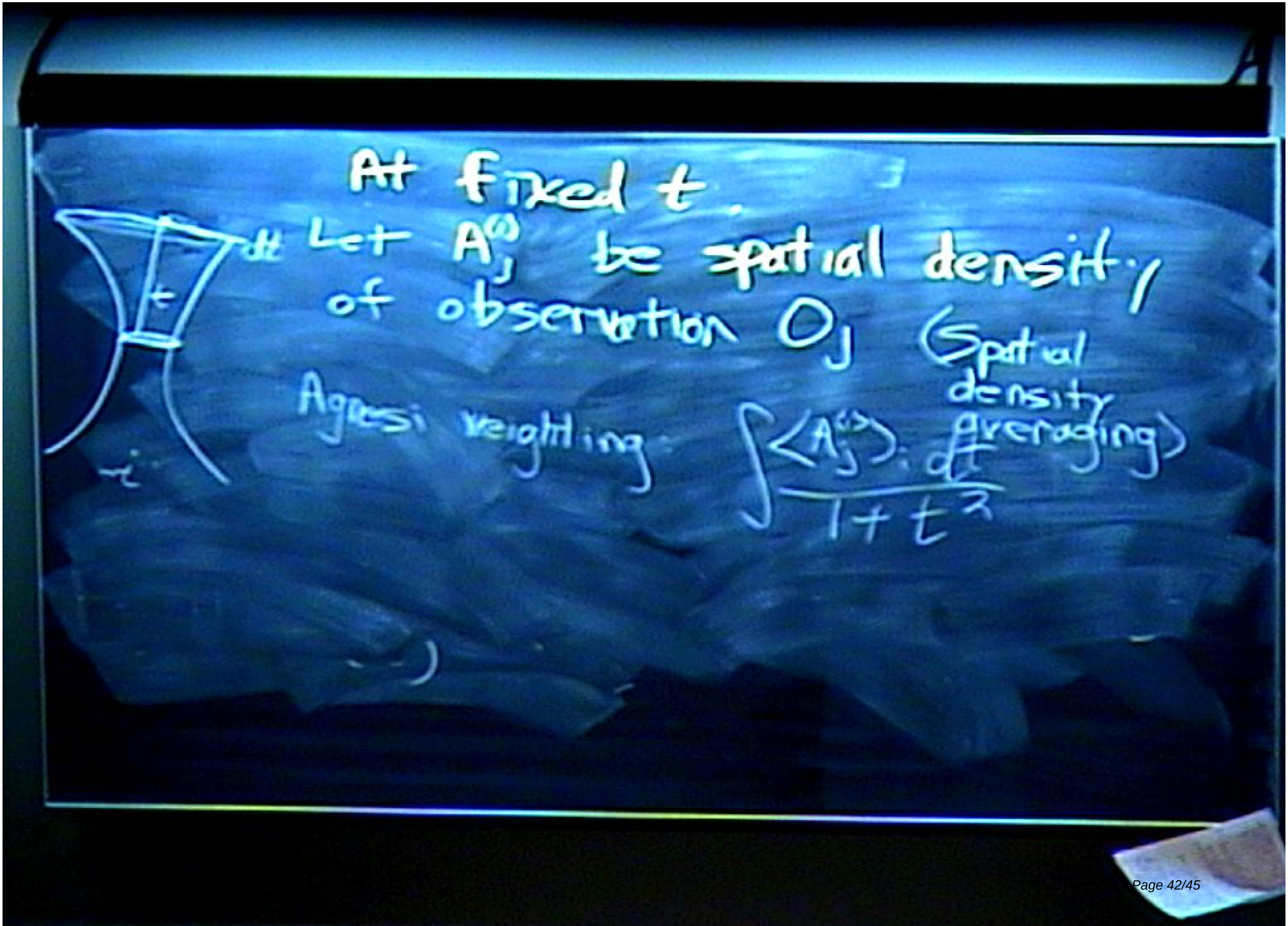




At fixed t

Let $A_J^{(s)}$ be spatial density of observation O_J


At fixed t
Let A_J^(i) be spatial density
of observation O_J (Spatial
density
averaging)
A


At fixed t
Let $A_j^{(t)}$ be spatial density of observation O_j (Spatial density averaging)
Agnesi weighting: $\int \frac{\langle A_j^{(t)} \rangle \, dt}{1+t^2}$

At fixed t

Let $A_J^{(t)}$ be spatial density of observation O_J (Spatial density averaging)

Agnesi weighting: $\int \frac{\langle A_J^{(t)} \rangle \, dt}{1+t^2}$

Bayes: $P(T_i|O_j) = \frac{P(T_i)P(O_j|T_i)}{\sum_k P(T_k)P(O_j|T_k)}$

with $\sum_j P(O_j|T_i) = 1 \quad \forall\ i$

Take $P(O_j|T_i) = \langle A_j^{(i)} \rangle_i \stackrel{?}{=} \mathrm{tr}(A_j^{(i)} \rho_i) \stackrel{?}{=} \langle \psi_i | A_j^{(i)} | \psi_i \rangle$

Now I favor my "Symmetric-Bounce Quantum State of the Universe," *JCAP* **0909**, 026 (2009), which is a pure state that is macroscopically time symmetric about a homogeneous, isotropic bounce with inhomogeneities and anisotropies then in their ground state. When combined with the volume averaging of "Cosmological Measures without Volume Weighting," *JCAP* **0810**, 025 (2008) (which takes the spatial density of observations rather than the total number at each moment of time), and "Agnesi Weighting for the Measure Problem of Cosmology," *JCAP* **1103**, 031 (2011) (which uses the simple weighting factor $dt/(1+t^2)$ to make the time integral of the volume averaging converge), it seems to fit observations and avoid problems other states and measures have with vacua of zero or negative cosmological constant. The measure it gives for at least N e-folds of inflation is $\propto N^{-1/4}$, which is consistent with observations.