Title: What is the state of the early Universe

Date: Jul 08, 2011 11:00 AM

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Abstract: String theory should give a well-defined answer to the following question: What is the state of matter in the limit of infinite energy density? We use results obtained from the understanding of black hole entropy to conjecture this equation of state, noting that the maximum entropy state in string theory has vastly more entropy than the states used in traditional approaches to early Universe Cosmology. The evolution of the Universe with this equation of state can be obtained in closed form.

What is the state of the Early Universe ?

Samir D. Mathur

Perimeter Institute, 2011

(Work in collaboration with Borun Chowdhury)
(see also papers by Kalyan Rama)

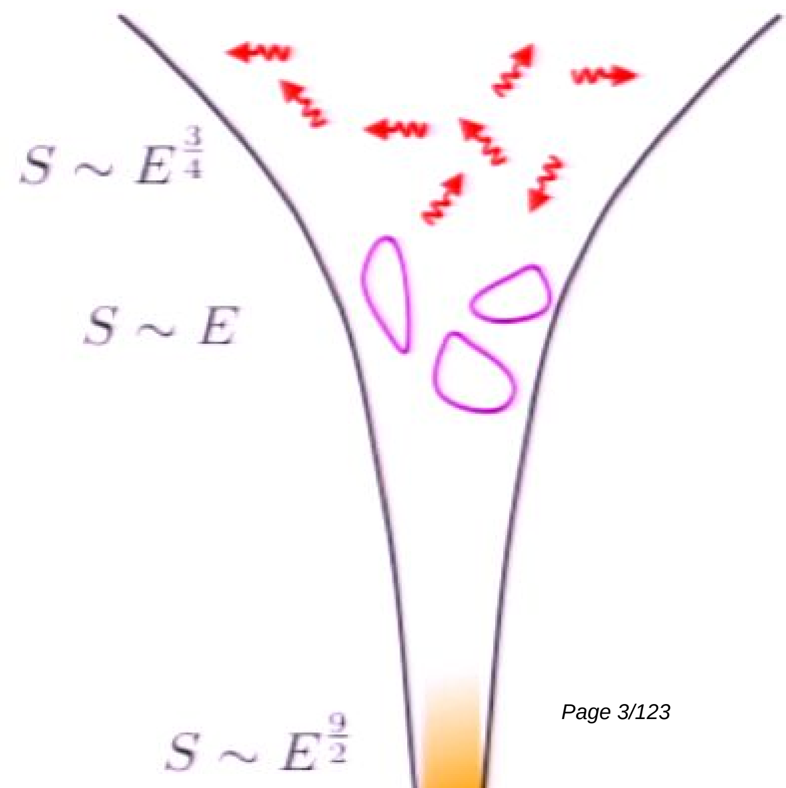
If we follow the Universe back to early times, it is possible that the density goes to infinity

String theory should give us a well defined answer to the question:

What is the equation of state in the limit of infinite density ?

In this talk we will set up this problem in string theory and suggest an answer:

The fractional brane gas



Let the Universe be a box of volume V

(This toroidal geometry is taken just for convenience, should not matter at the end)

In this box, put energy E

Question: What is the state of maximal entropy, and how much is S(E,V) ?

S(E,V) ??

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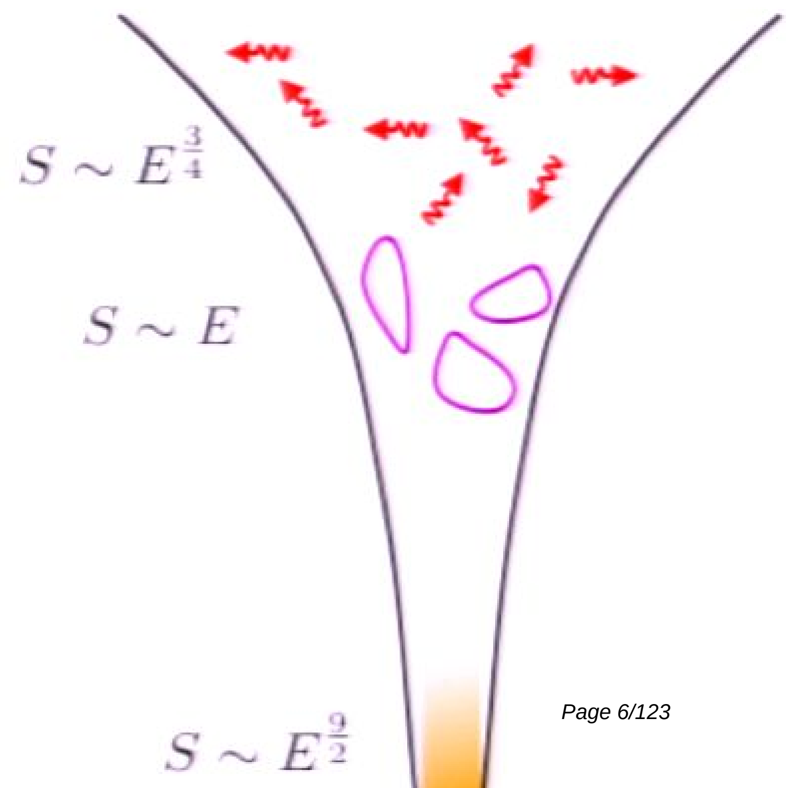

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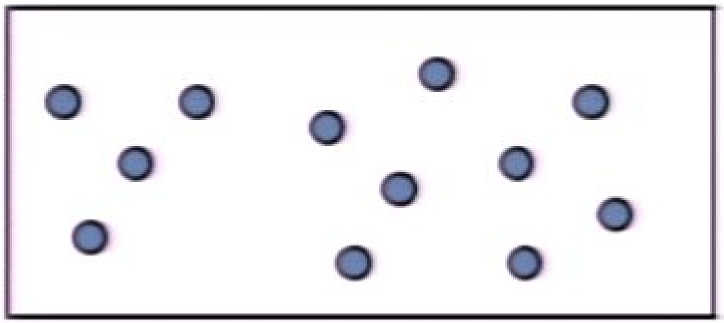
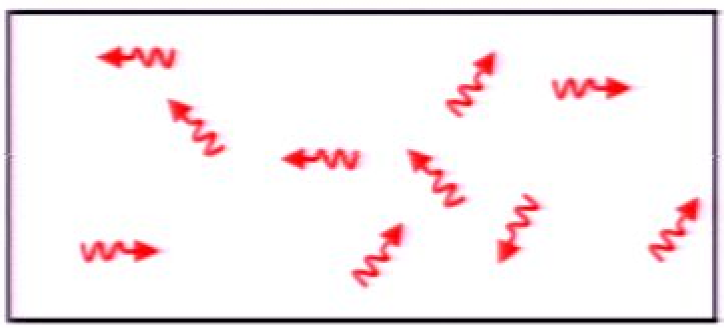


Dust

$S \sim 0$



Radiation

$S \sim E^{\frac{D-1}{D}}$

In the early Universe, radiation has more entropy, so we choose the state to be radiation

This question is quite well-defined

$S(E,V)$??

General relativity allows us any value of E for a given V

Putting more E will give a larger expansion rate dV/dt, but there is no constraint on E itself

We will assume that dV/dt is still small enough that we can have a definition of entropy, since defining entropy needs some approximate equilibrium

Our question is: What does string theory give for $S(E,V)$?

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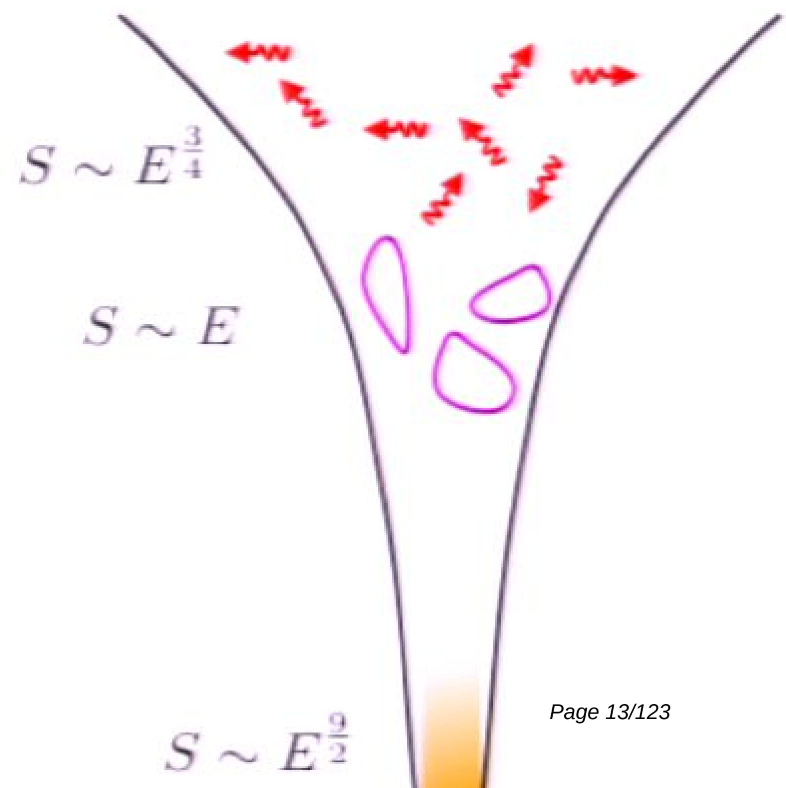

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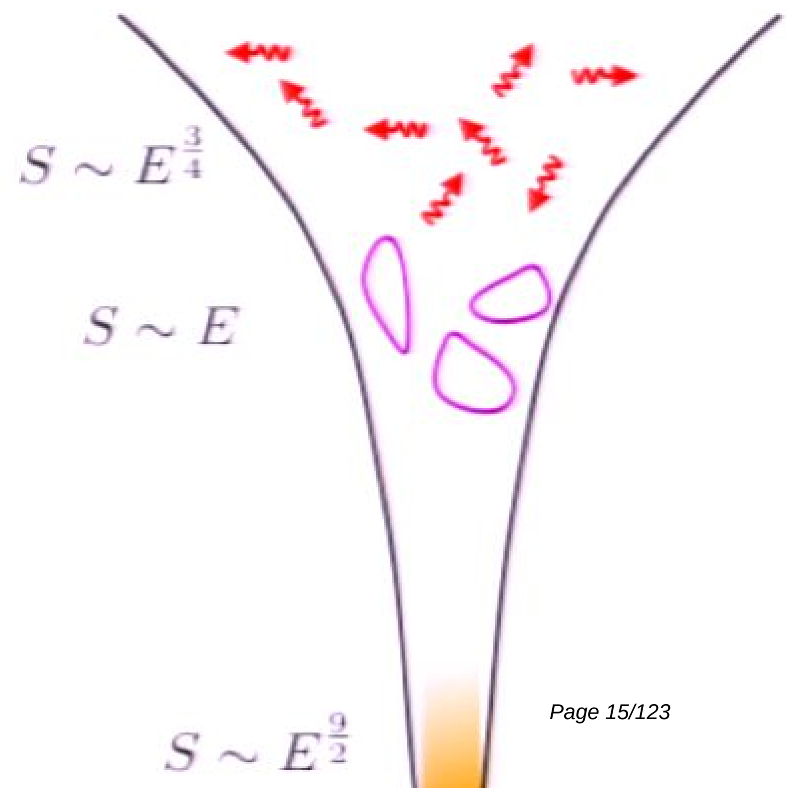

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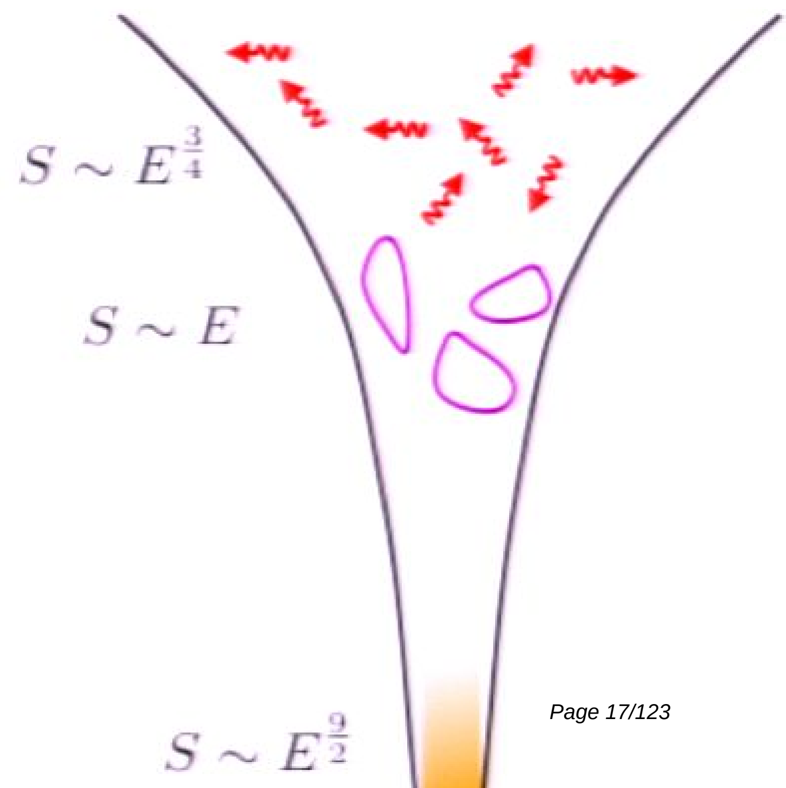

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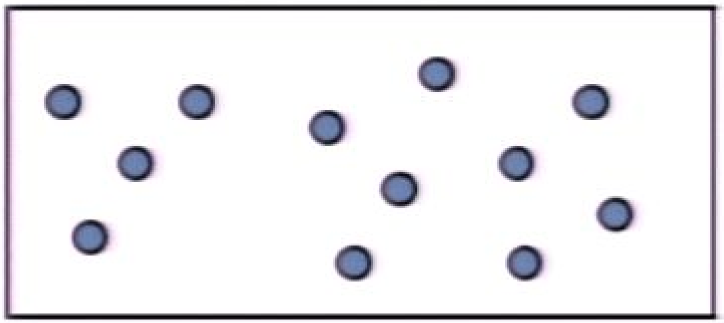

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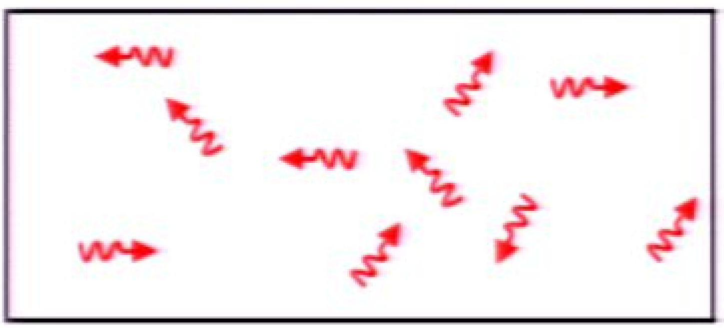

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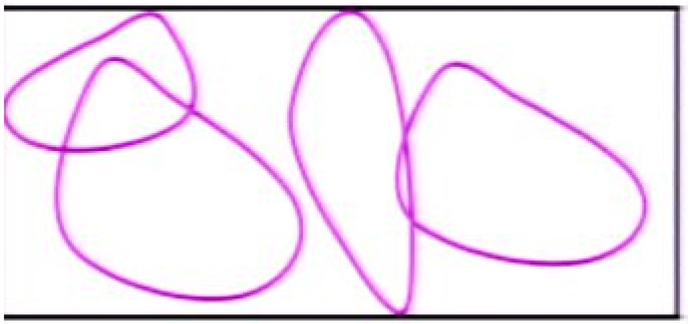
In the early Universe, radiation has more entropy, so we choose the state to be radiation

When we learnt about strings, we looked for a string gas phase for very early times



String gas
(Hagedorn phase)

$$S \sim E \sim \sqrt{E}\sqrt{E}$$

(Brandenberger+Vafa)

Question: Can we get an even higher power of E from string theory ?

Black holes in 3+1: $$S = \frac{A}{4} \sim M^2 \sim E^2$$

Brane configurations in string theory can reproduce this entropy ...

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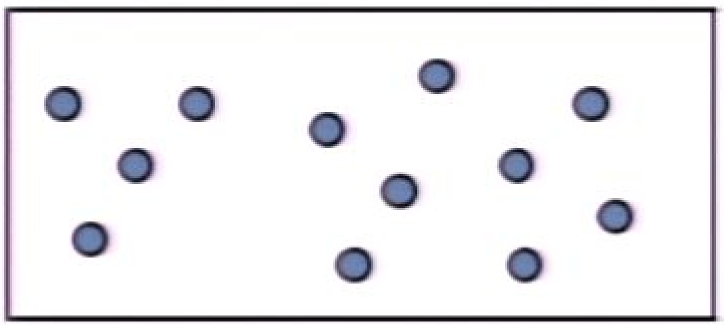

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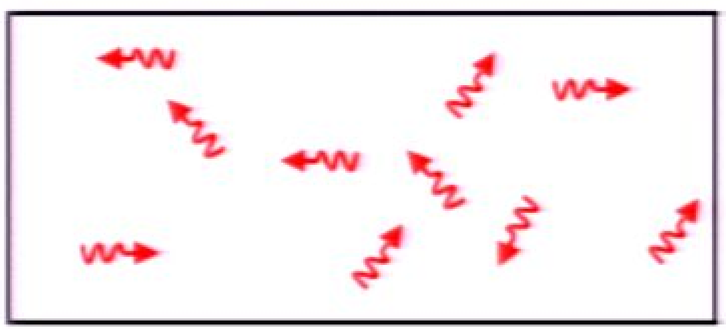

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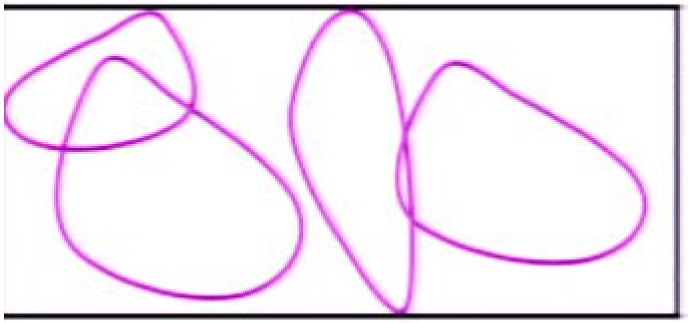

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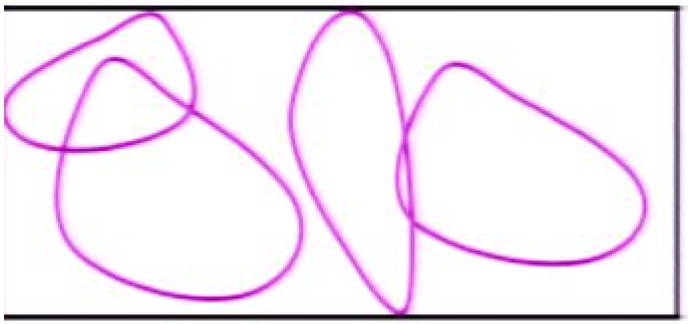

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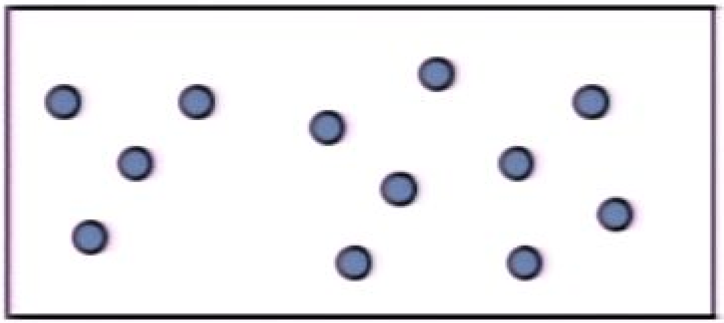

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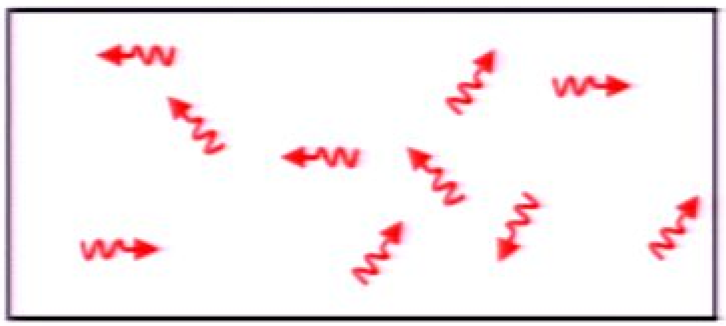

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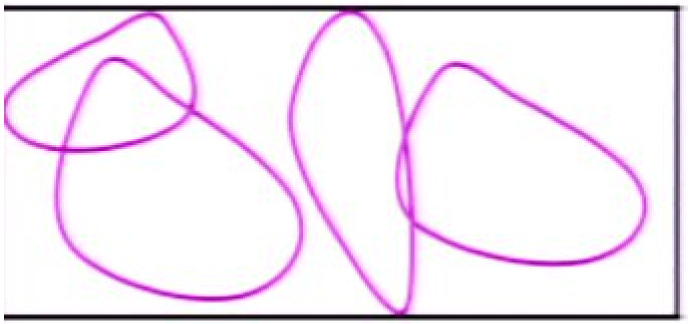

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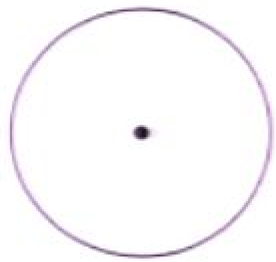
Bekenstein showed by Gedanken experiments that a black hole must have entropy

$$S_{bek} = \frac{A}{4}$$



Counting states in string theory reproduces this entropy

We will review how this entropy is found, since we will then extend the underlying ideas to the Universe

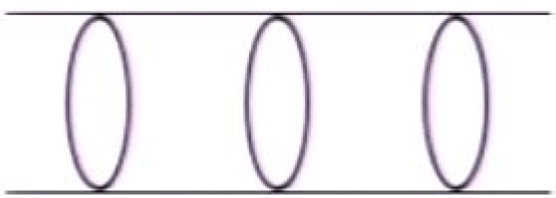
Strings wound around a circle give objects with winding charge

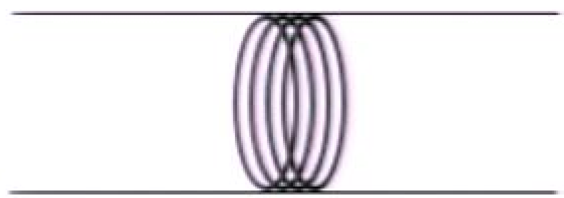
Gravitons running around compact directions give objects carrying momentum charge

1-charge

String theory has a set of S, T dualities that can map any charge to any other charge

Winding charge: Strings wrapped on a circle

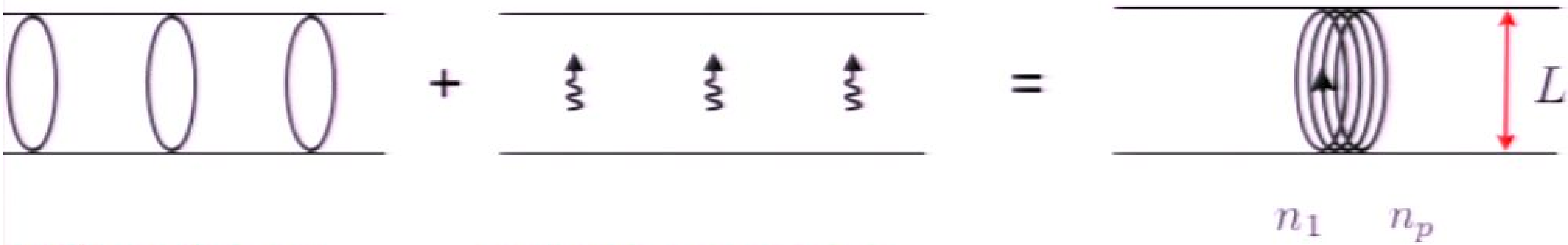




Bound state of many strings has a unique configuration:
The multi-wound string

Thus $S = \ln 1 = 0$

2-charges



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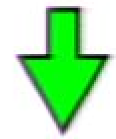

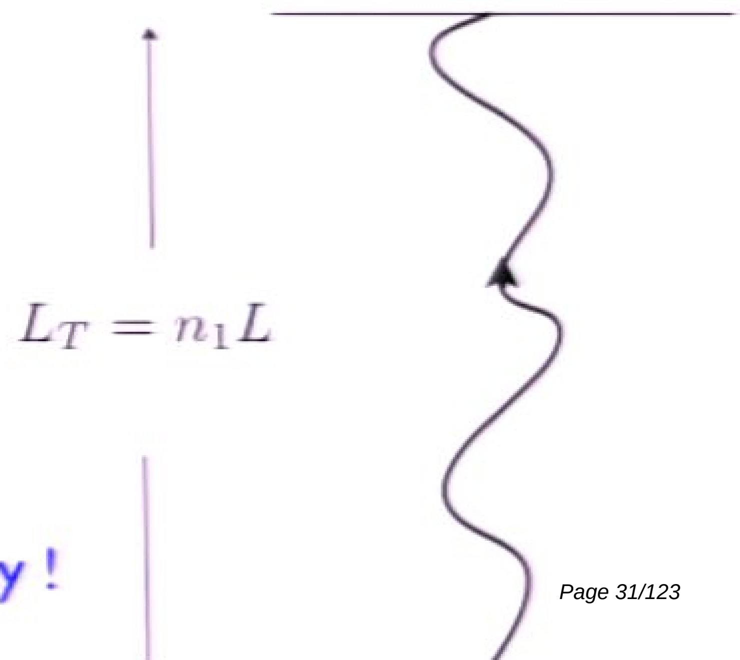

Open up string to its 'covering space'
We have transverse vibrations
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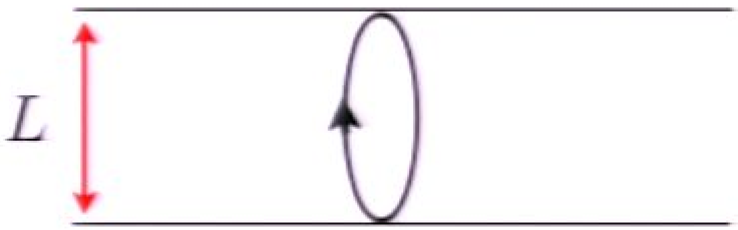
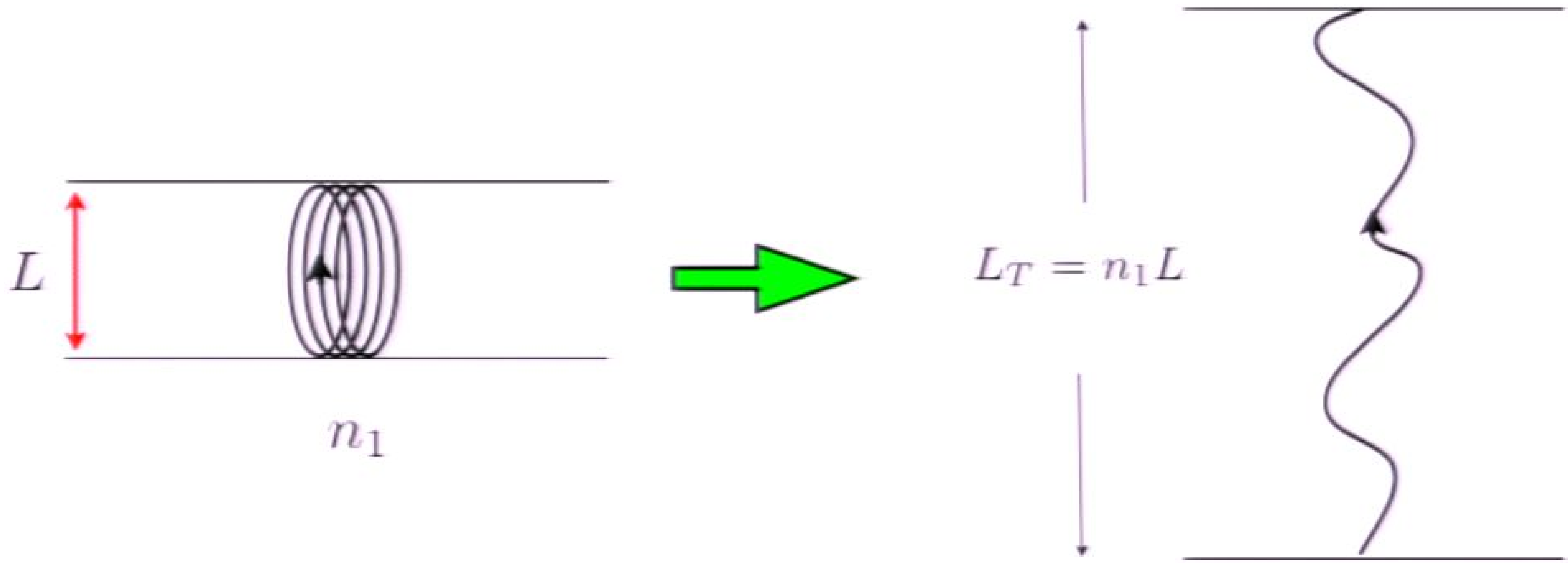
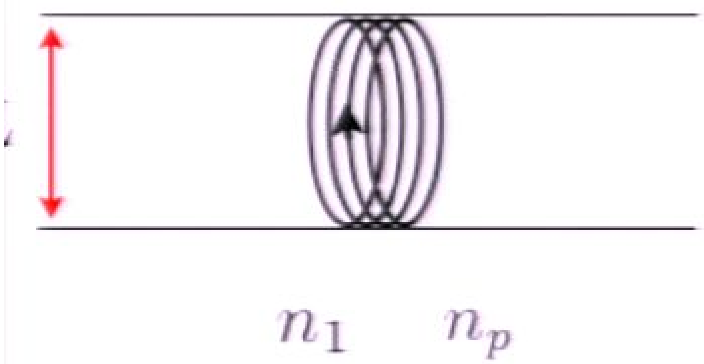


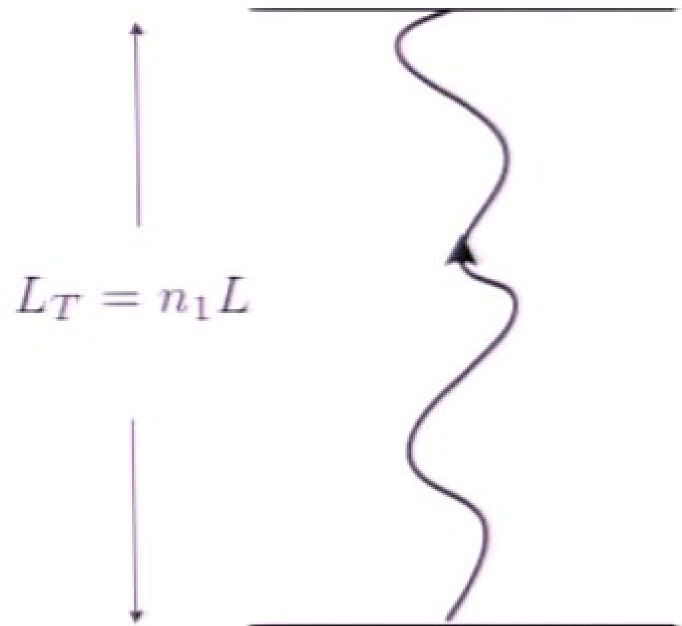
Momentum comes in units of $\frac{2\pi}{L}$



Momentum excitations come in units of $\frac{2\pi}{L_T} = \frac{2\pi}{n_1 L}$ (fractionation)

But total momentum must still be in units of $\frac{2\pi}{}$





Each quantum of harmonic k carries momentum $\frac{2\pi k}{L_T}$

Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi (n_1 n_p)}{L_T}$$

Let there be n_k units of excitations in harmonic k

Then

$$\sum k \, n_k = n_1 n_p$$

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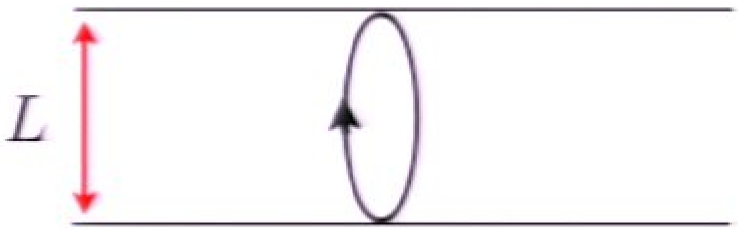

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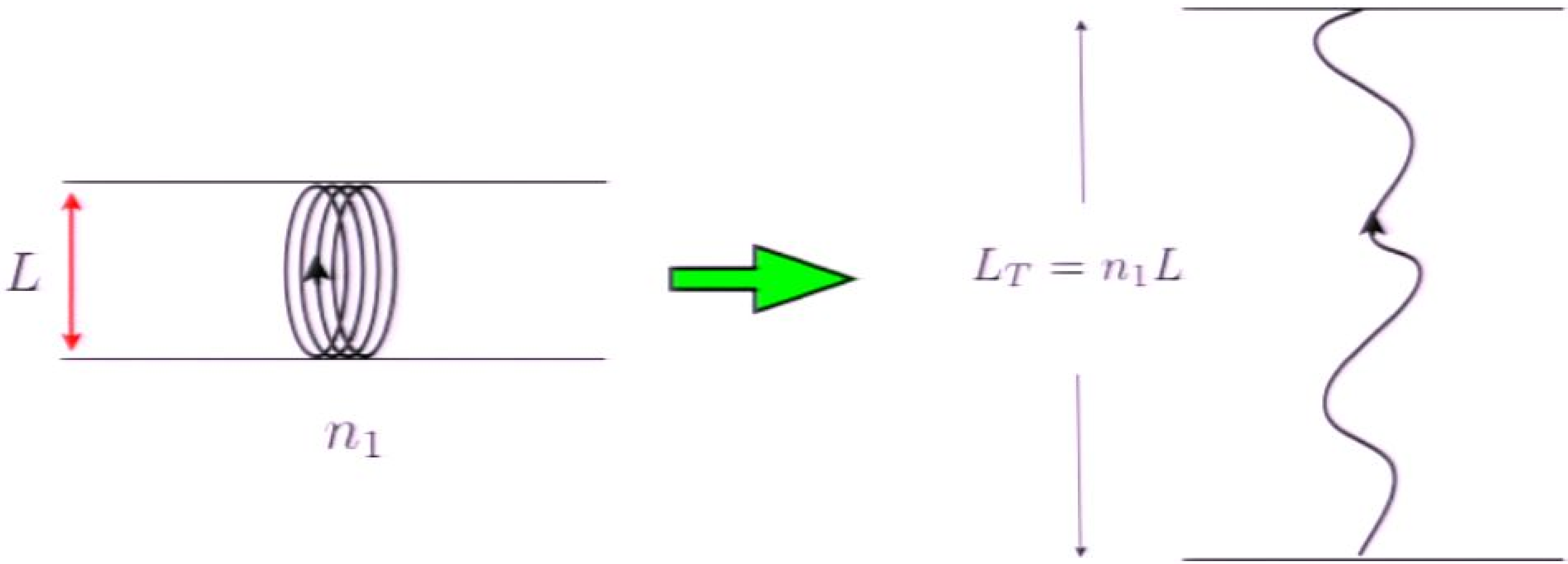

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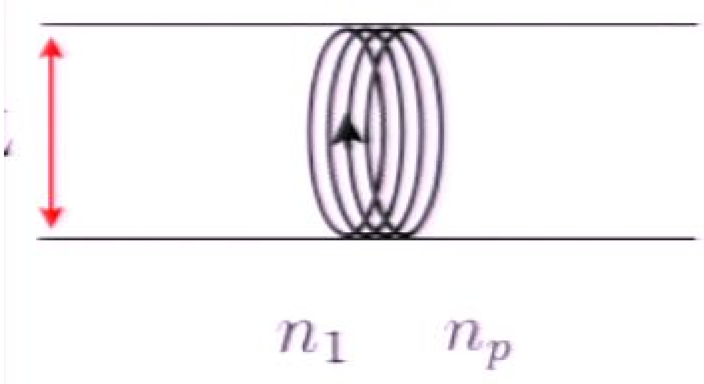

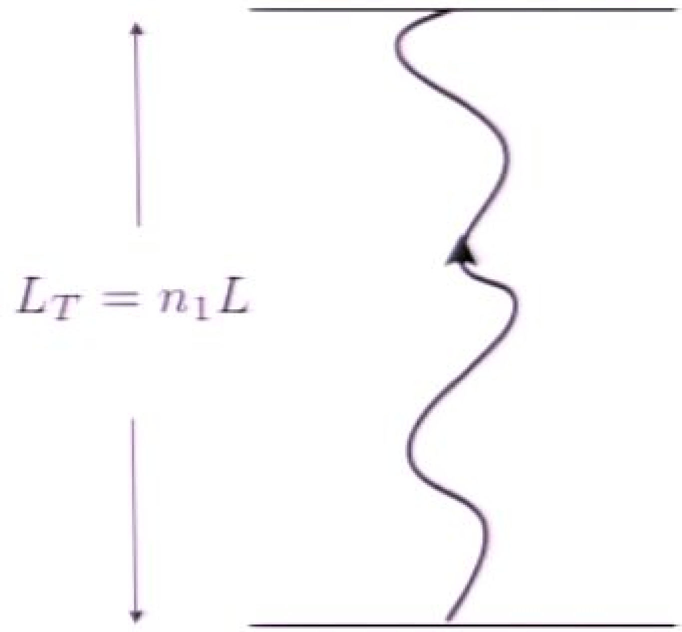

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Count 'partitions' of

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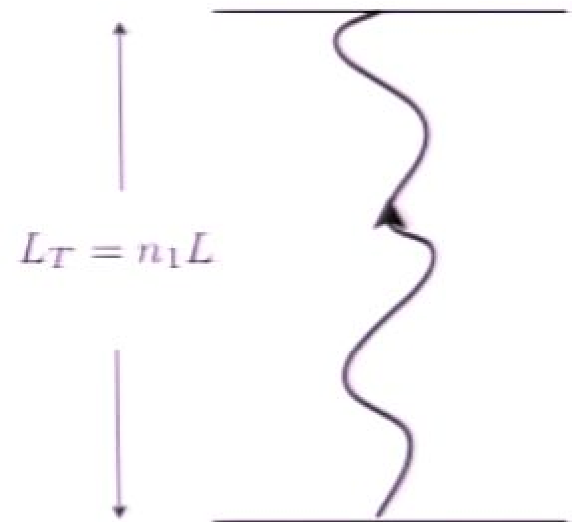


bosonic + 8 fermionic degrees of freedom

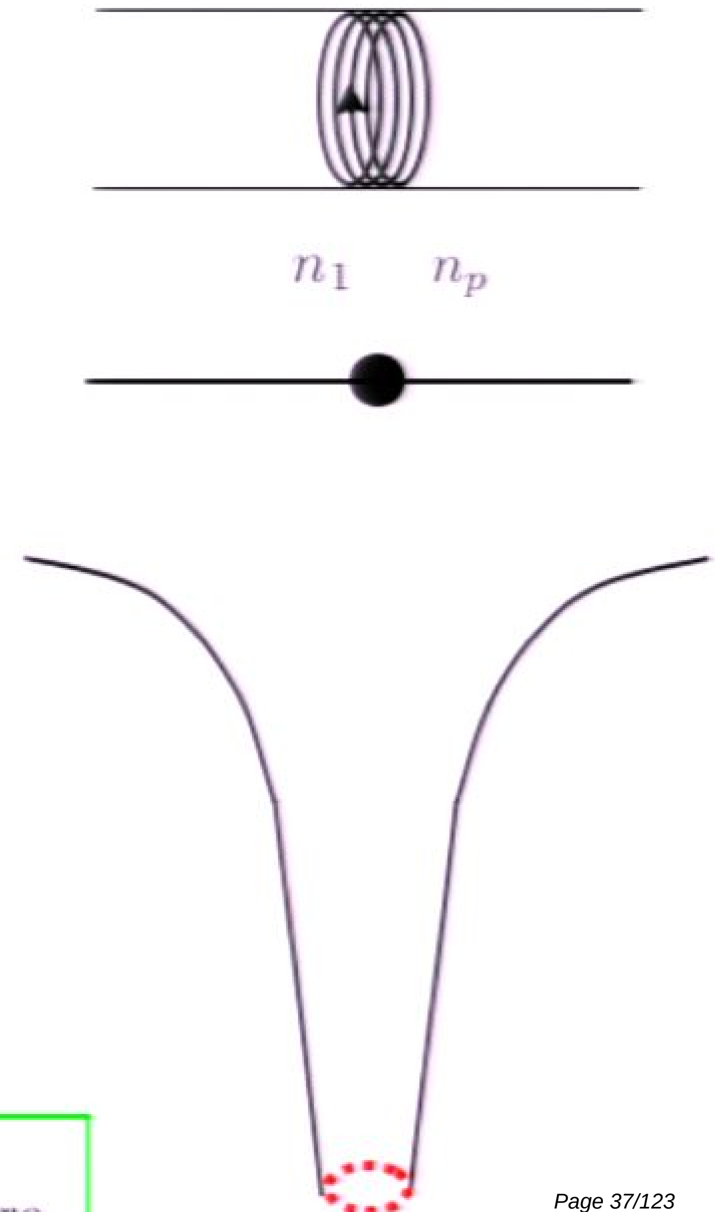
$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}}$ states

$$S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p} \qquad T^4 \times S^1$$

$$S_{micro} = 4\pi\sqrt{n_1 n_p} \qquad K3 \times S^1$$

(Susskind '93, Sen '94)

Now let us make a black hole with both these charges ...



For $K3 \times S^1$ compactification, geometry gives a Bekenstein - Wald entropy

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p}$$

(Dabholkar '04)

Thus we see that

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$

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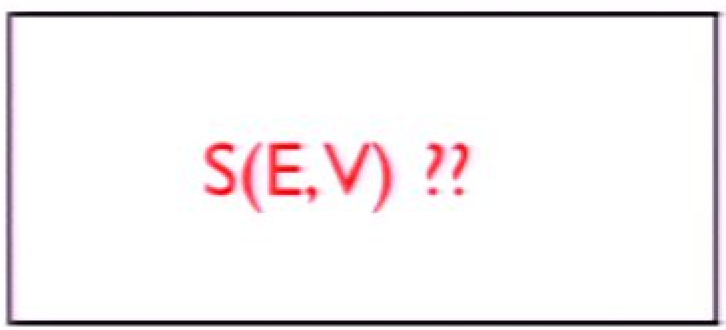

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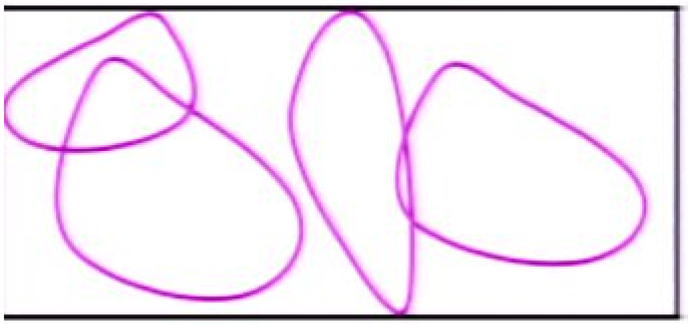

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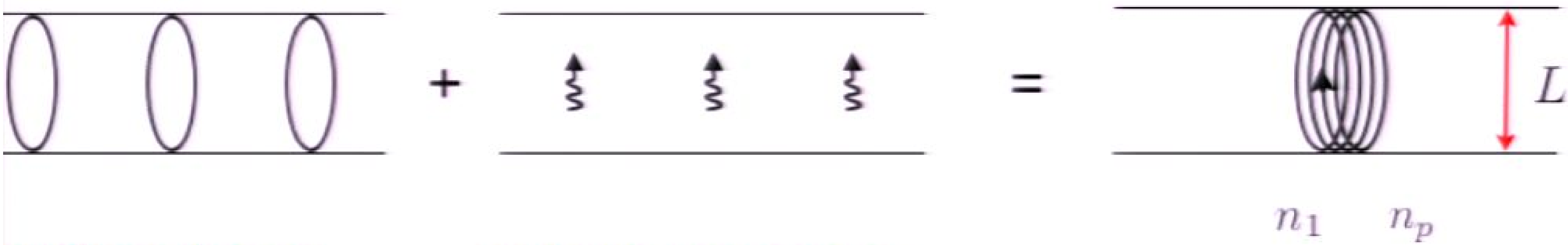

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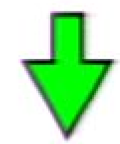

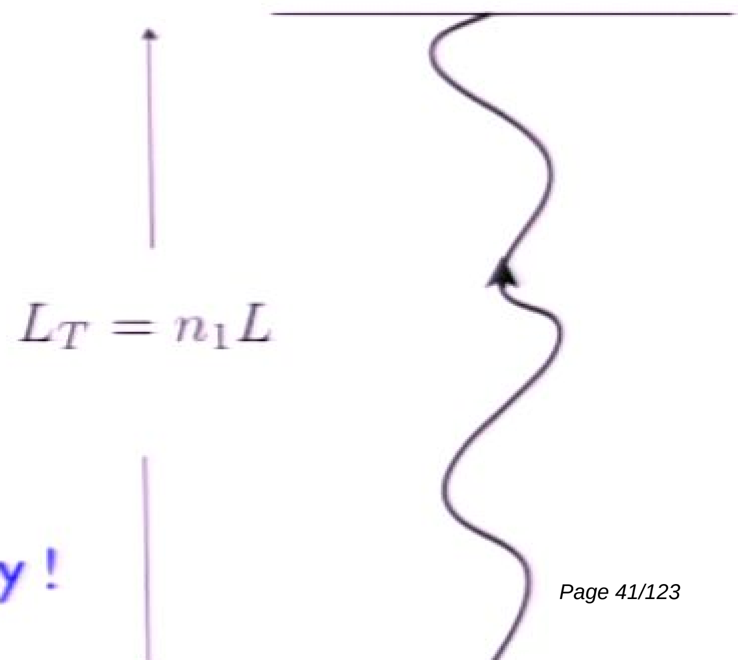

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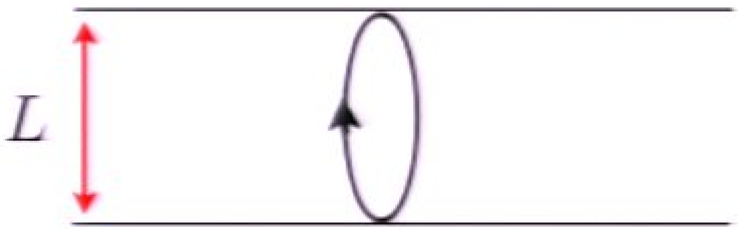

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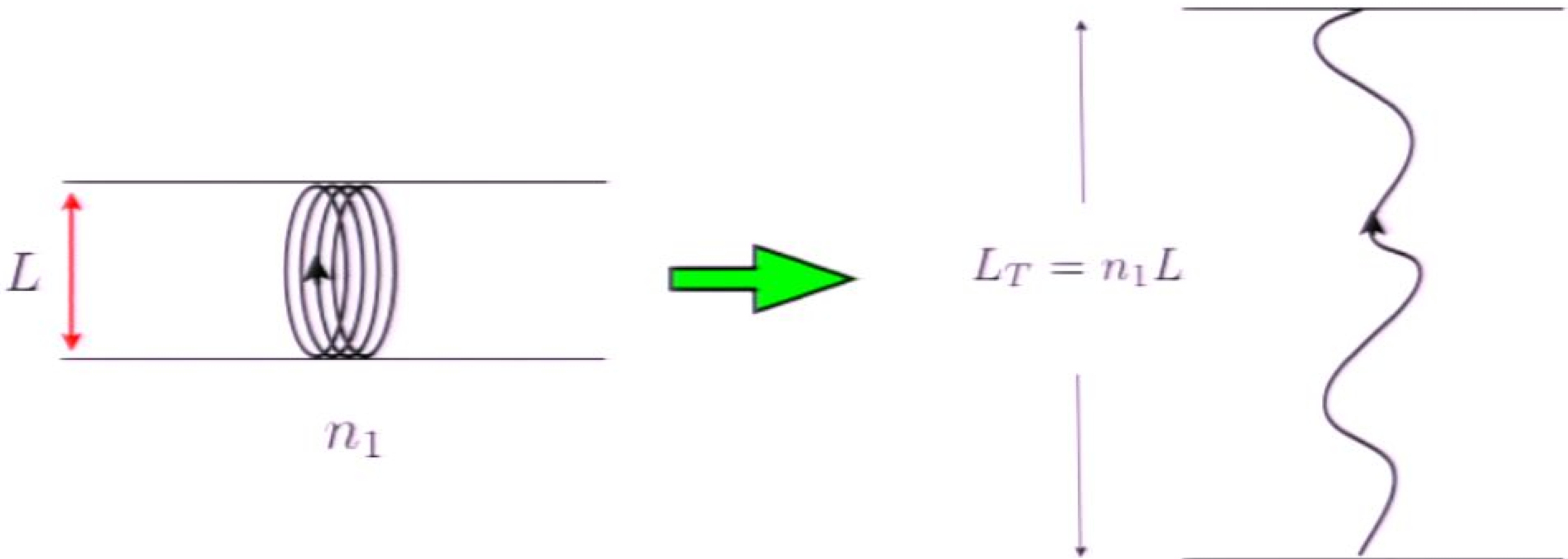

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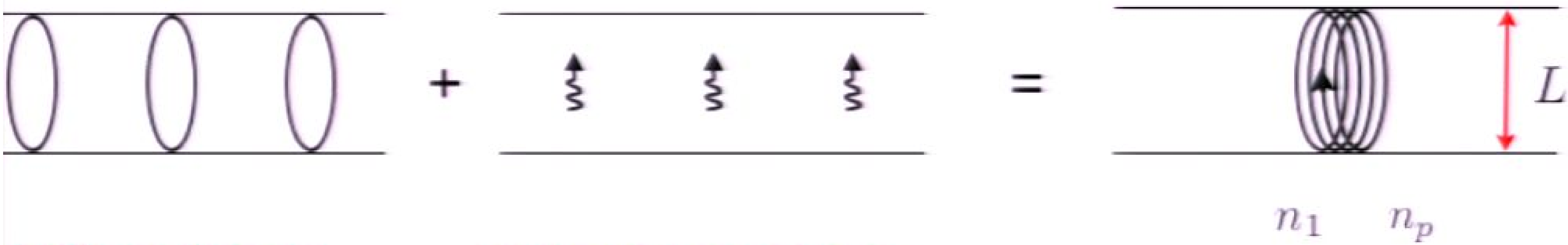

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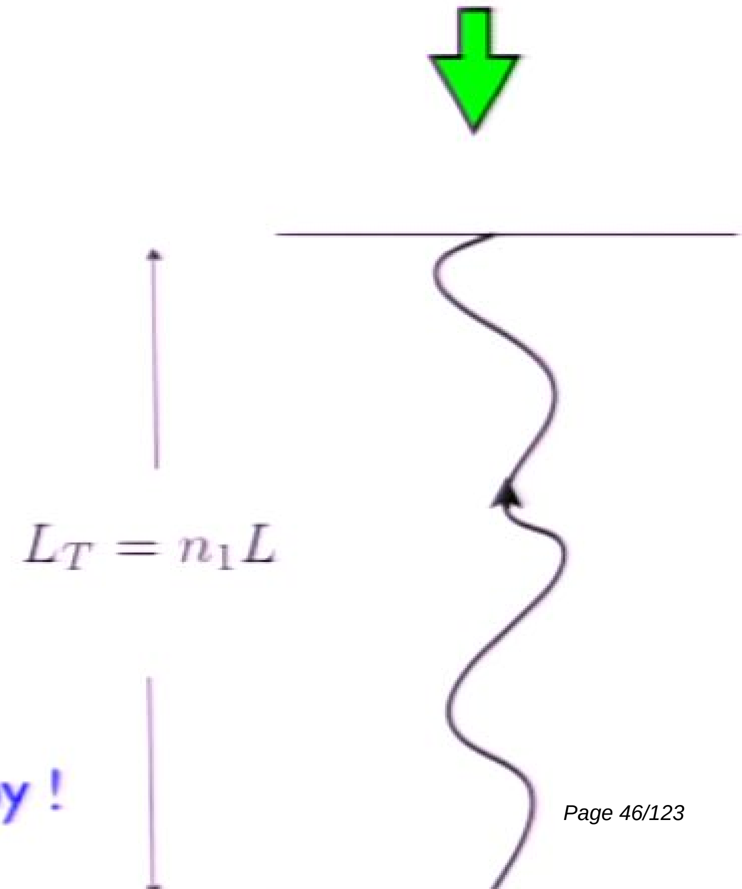

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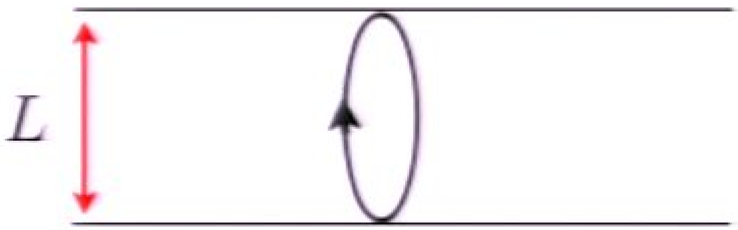

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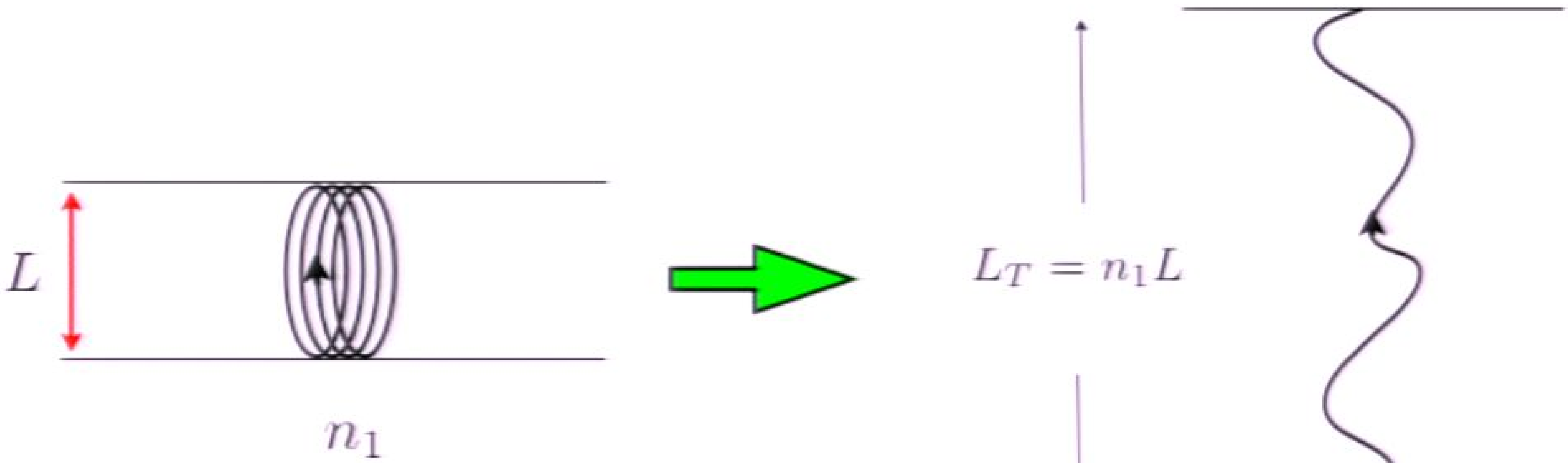
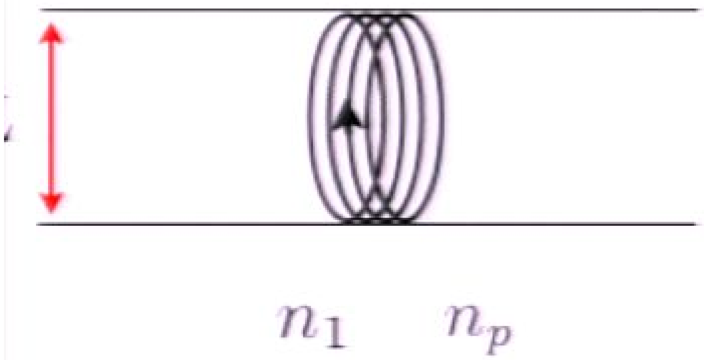
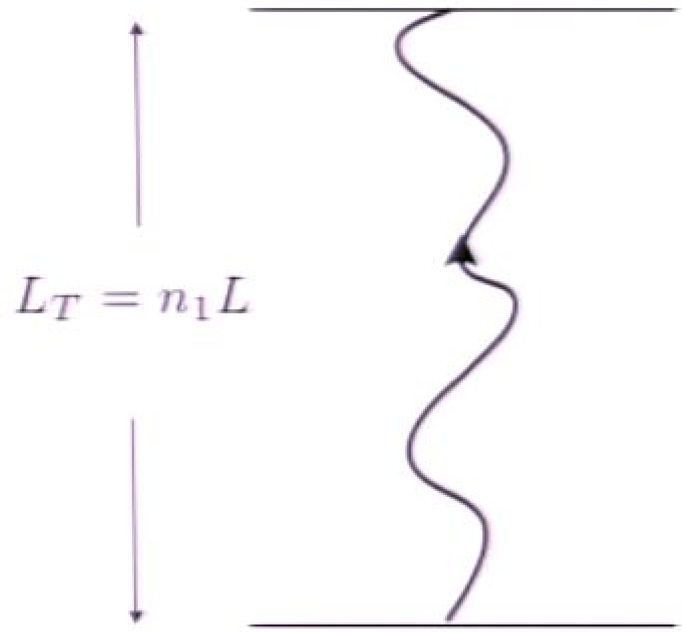


Momentum excitations come in units of $\frac{2\pi}{L_T} = \frac{2\pi}{n_1 L}$ (fractionation)

But total momentum must still be in units of $\frac{2\pi}{L}$





Each quantum of harmonic k carries momentum $\dfrac{2\pi k}{L_T}$

Total momentum

$$P = \frac{2\pi n_p}{L} = \frac{2\pi(n_1 n_p)}{L_T}$$

Let there be n_k units of excitations in harmonic k

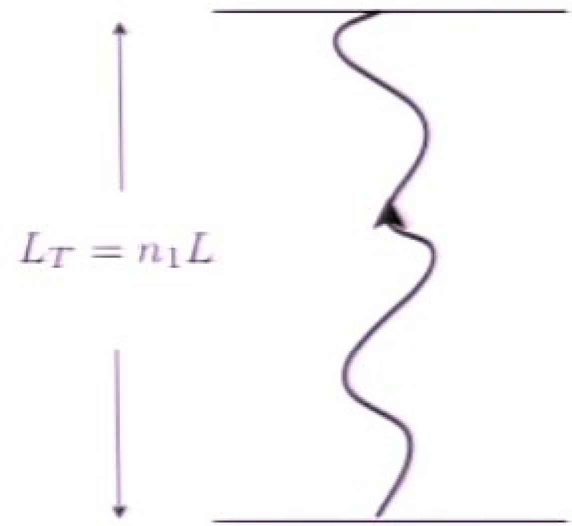
Then

$$\sum k \; n_k = n_1 n_p$$

$$\sum k\ n_k = n_1 n_p$$

Count 'partitions' of

$n_1 n_p$



| bosonic + 8 fermionic degrees of freedom

$e^{2\pi\sqrt{2}\sqrt{n_1 n_p}}$ states

$$S_{micro} = 2\pi\sqrt{2}\sqrt{n_1 n_p} \qquad T^4 \times S^1$$

$$S_{micro} = 4\pi\sqrt{n_1 n_p} \qquad K3 \times S^1$$

(Susskind '93, Sen '94)

Now let us make a black hole with both these charges ...

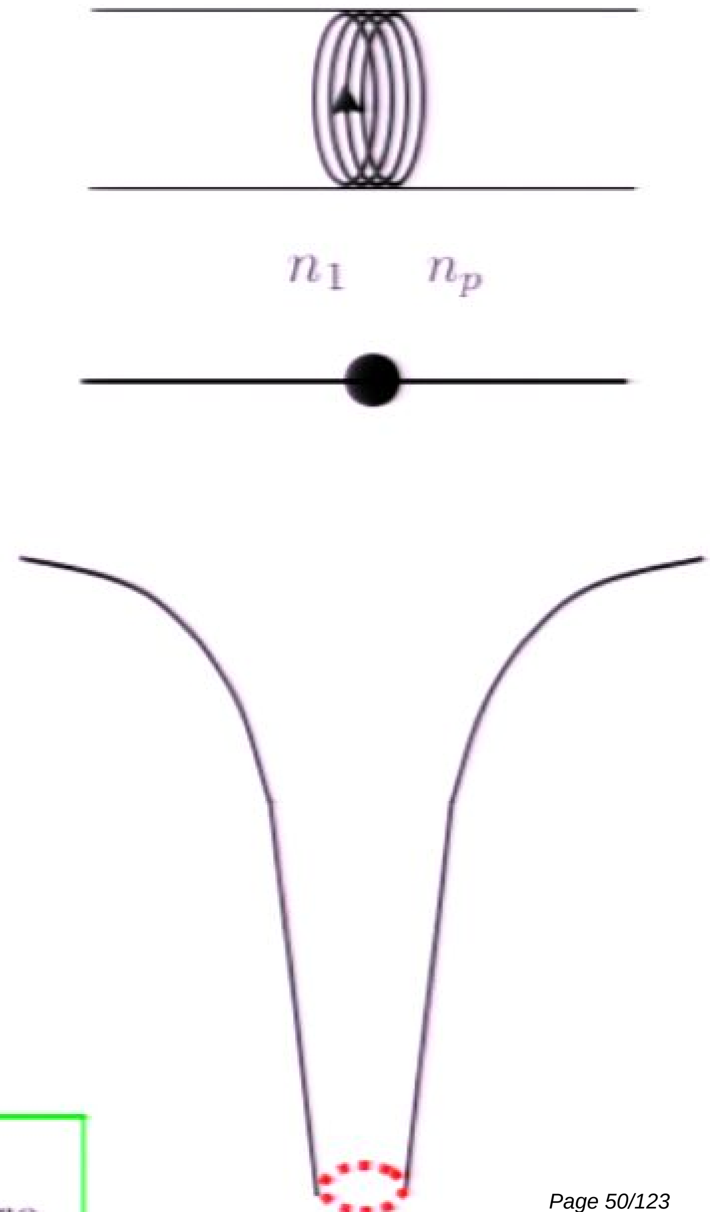
For $K3 \times S^1$ compactification, geometry gives a Bekenstein - Wald entropy

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p}$$

(Dabholkar '04)

Thus we see that

$$S_{bek} = \frac{A}{2G} = 4\pi\sqrt{n_1 n_p} = S_{micro}$$



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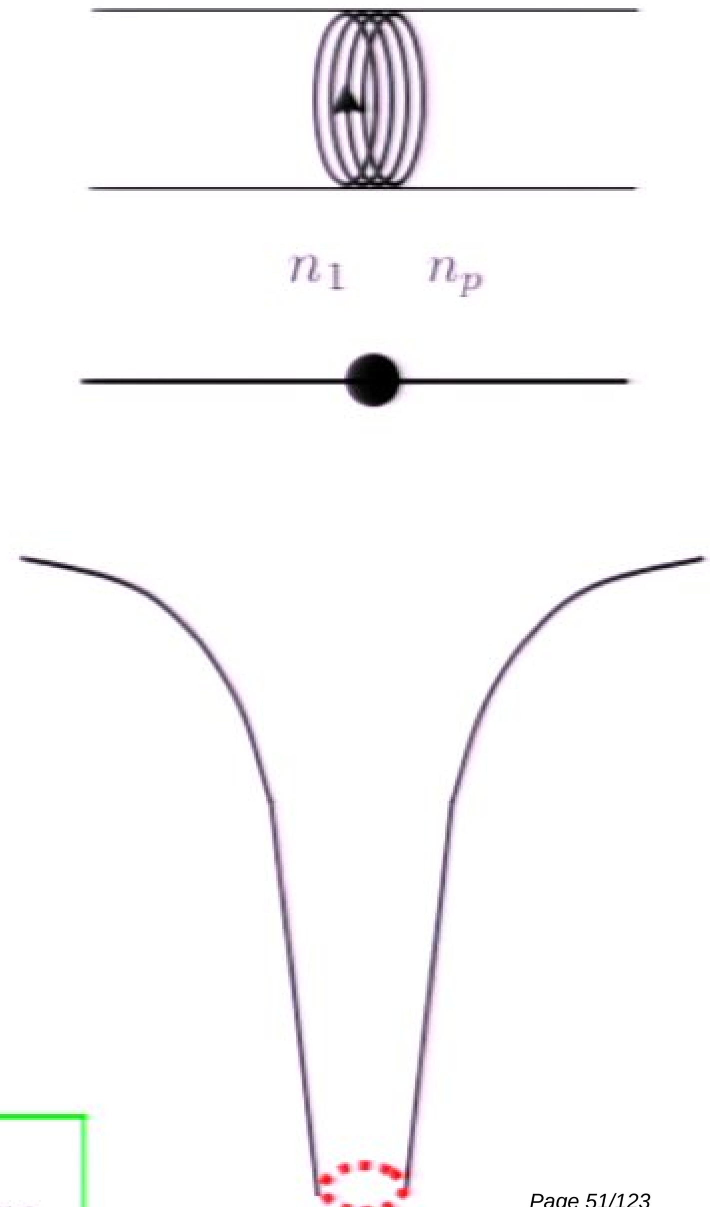

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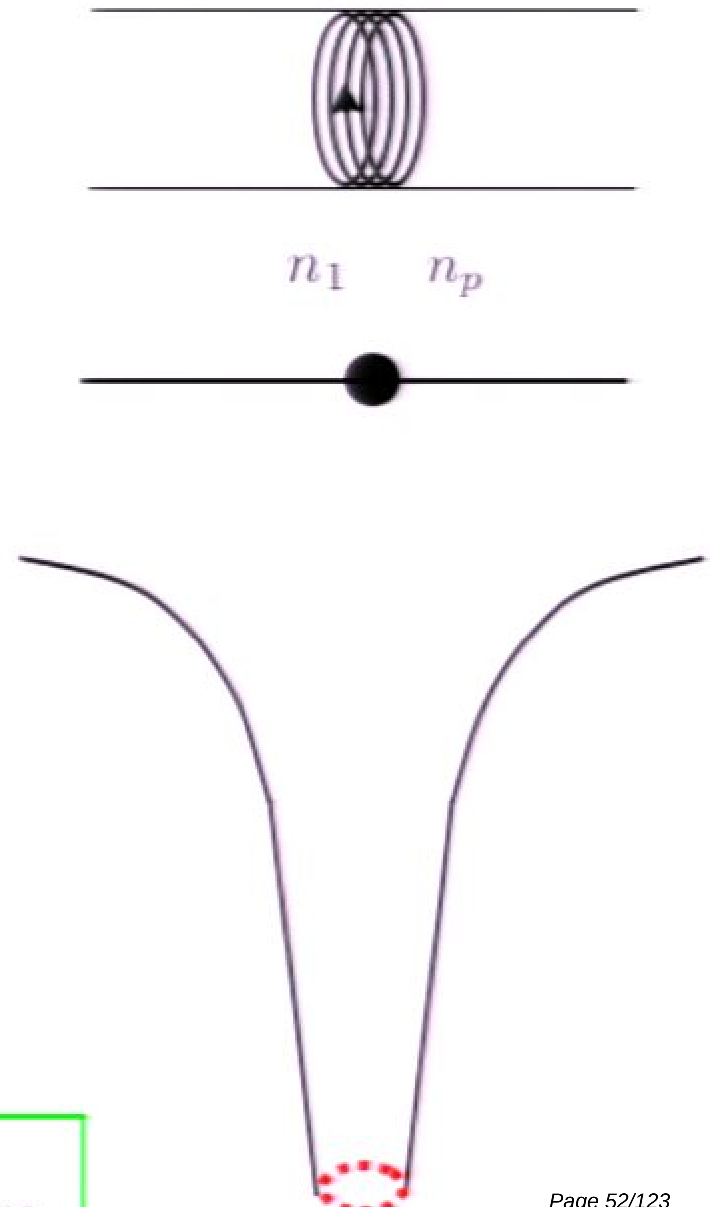

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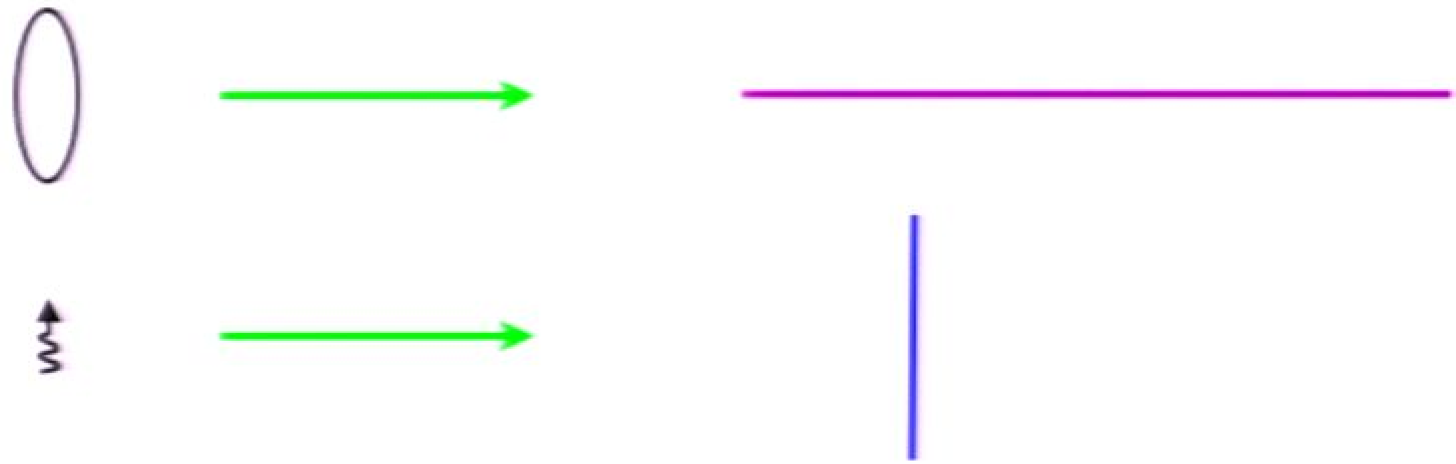
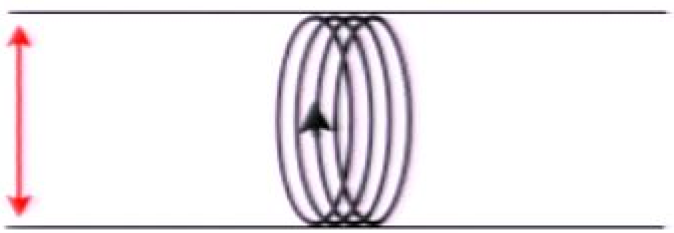
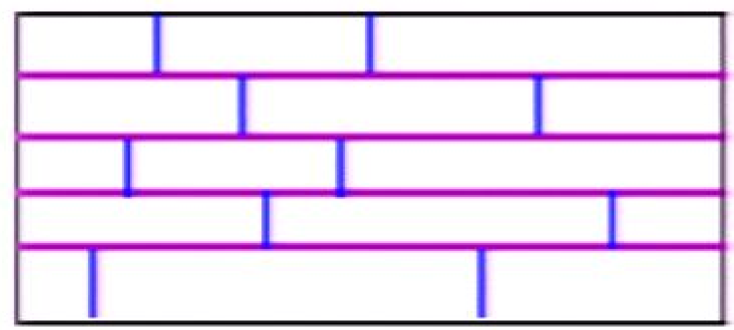
ractionation in other descriptions





n_1



Momentum excitations come in units of $\dfrac{2\pi}{L_T} = \dfrac{2\pi}{n_1 L}$

One set of branes comes in fractional units when bound to another set of branes

$$S = 2\pi\sqrt{2}\sqrt{n_1 n_2}$$

ore entropy ?

2 kinds of charges: $S = 2\pi\sqrt{2}\sqrt{n_1 n_2}$

Suppose our total energy is E. Let us put half in excitations of the first kind and half in the second. Then

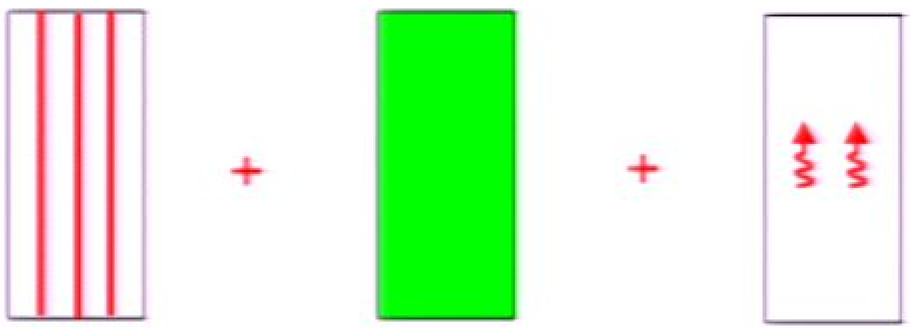
$$S \sim \sqrt{\frac{E}{2}\frac{E}{2}} \sim E$$

Question: Can we mix 3 kinds of charges and get

$$S \sim \sqrt{n_1 n_2 n_3} \sim E^{\frac{3}{2}}$$??

Yes ! This was done in the Strominger-Vafa black hole

3-charge extremal $M_{9,1} \to M_{4,1} \times S^1 \times T^4$

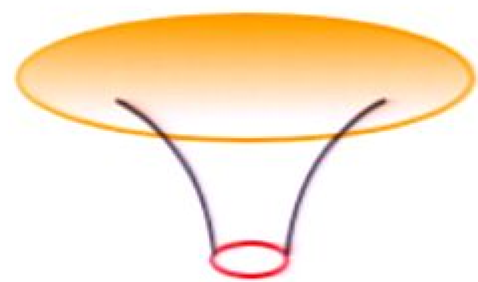


n_1	n_5	n_p
D1 branes	D5 branes	momentum

Count states:

$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

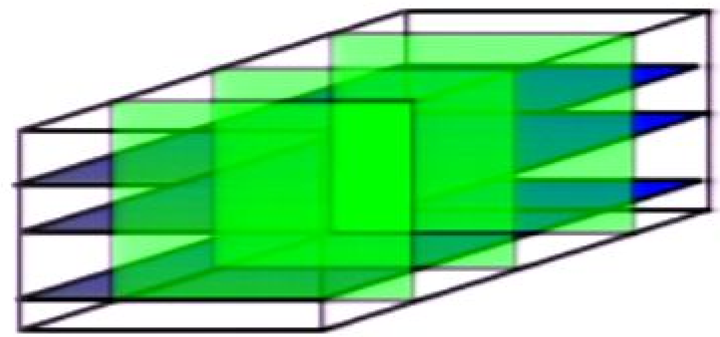
Make an extremal black hole with same charges



$$S_{bek} = \frac{A}{4G} = S_{micro}$$

(Strominger + Vafa 96)

3-charges: $S \sim \sqrt{n_1 n_2 n_3} \sim E^{\frac{3}{2}}$



Entropy comes from different ways to group the $n_1 n_2 n_3$ intersection points

The 4-charge extremal case works the same way (3+1 d black holes)

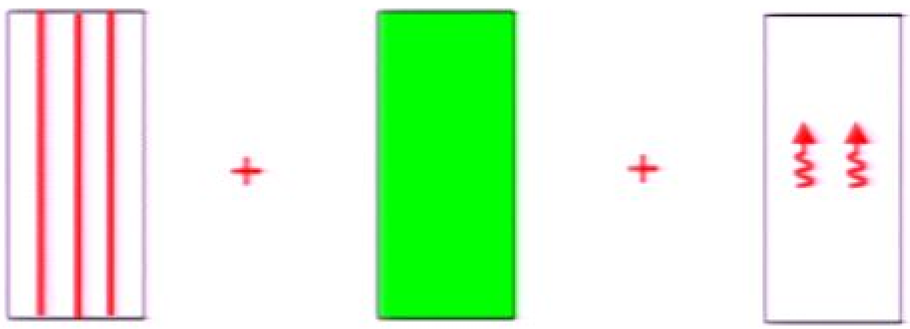
$$S_{micro} = 2\pi\sqrt{n_1 n_2 n_3 n_4} = S_{bek}$$

Charges can be taken as D1 D5 P KK, or D3 D3 D3 D3

Note that $S \sim \sqrt{n_1 n_2 n_3 n_4} \sim E^2$

(Johnson, Khuri, Myers 96, Horowitz, Lowe, Maldacena 96)

3-charge extremal $M_{9,1} \to M_{4,1} \times S^1 \times T^4$

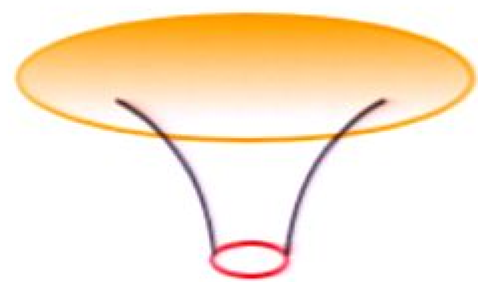


n_1 D1 branes + n_5 D5 branes + n_p momentum

Count states:

$$S_{micro} = 2\pi\sqrt{n_1 n_5 n_p}$$

Make an extremal black hole with same charges



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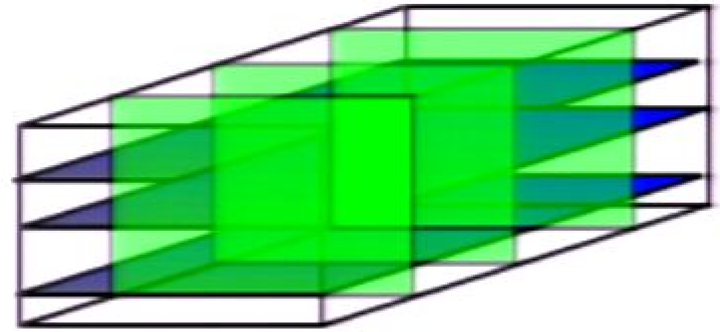
3-charges: $S \sim \sqrt{n_1 n_2 n_3} \sim E^{\frac{3}{2}}$



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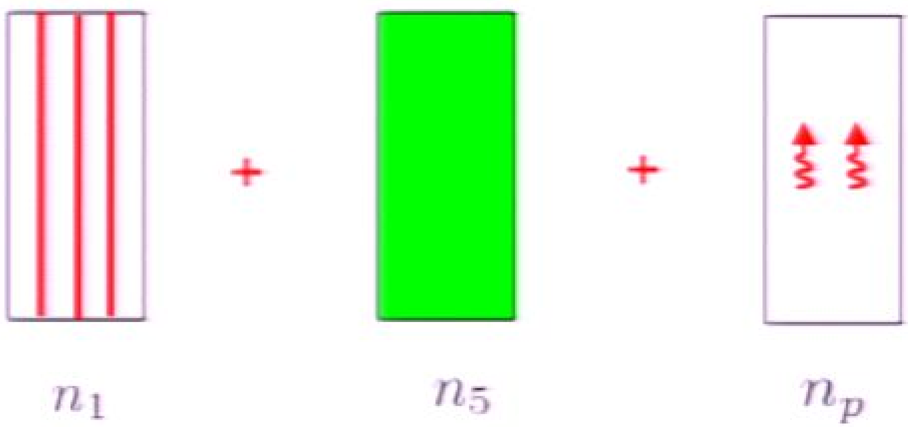
Note that $S \sim \sqrt{n_1 n_2 n_3 n_4} \sim E^2$

(Johnson,Khuri,Myers 96,
Horowitz,Lowe,Maldacena 96)

Non-extremality

The systems we have studied so far are extremal: e.g. 3-charge extremal



D1 charge n_1

D5 charge n_5

P charge n_p

Thus we just add the mass of separate constituents

Energy $E = n_1 m_1 + n_5 m_5 + n_p m_p$

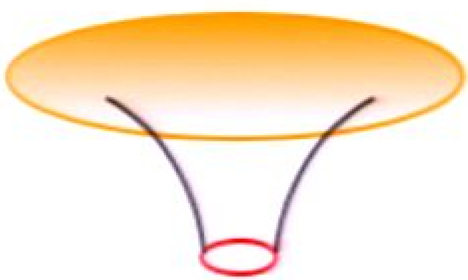
But the Universe is overall neutral (no place for flux to escape)

Thus we have to study non-extremal brane bound states

We will find a very similar story here ....

Let us go down from 3 charges to 2 charges, but make the system a little nonextremal

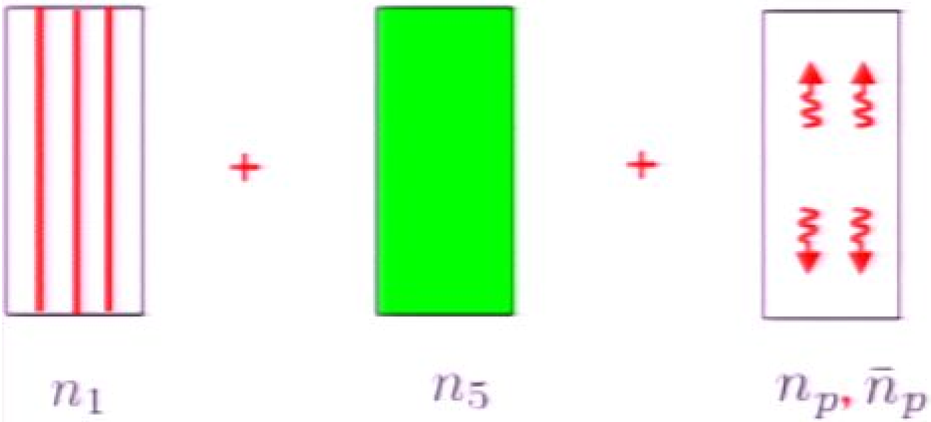
Thus we have large charges D1, D5, and a little extra energy over and above the mass of the D1, D5 branes



We can find the Bekenstein entropy of such a hole ...

Can we reproduce this by a microscopic count ? Yes !!

ear-extremal 2-charge:



D1 charge $\hat{n}_1 = n_1$

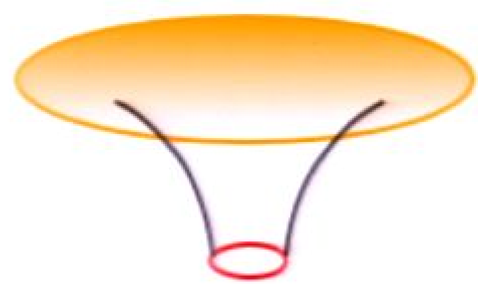
D5 charge $\hat{n}_5 = n_5$

P charge $\hat{n}_p = n_p - \bar{n}_p$

Let us try to simply add the energies of all constituents, and also extend the entropy expression in the most natural way

Energy $E = n_1 m_1 + n_5 m_5 + (n_p + \bar{n}_p) m_p$

Entropy $S_{micro} = 2\pi\sqrt{n_1 n_5}(\sqrt{n_p} + \sqrt{\bar{n}_p})$



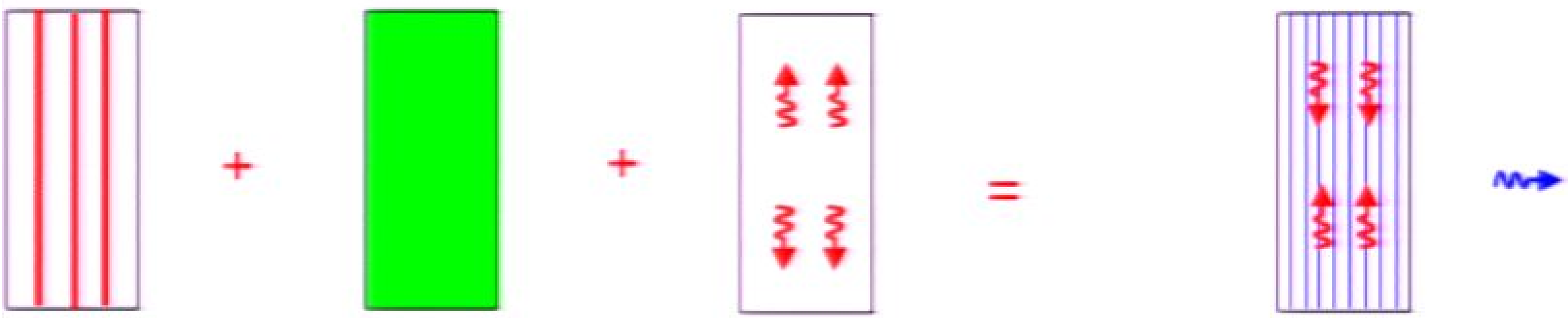
Then we find

$$S_{micro} = S_{bh}$$

(Callan,Maldacena 96)

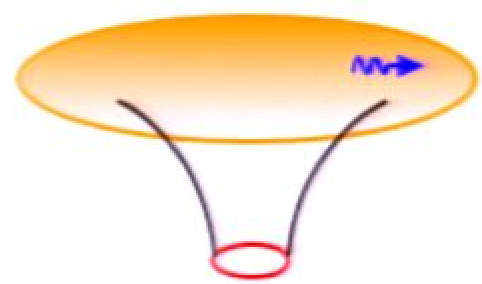
ls this agreement of entropies a coincidence ?

No ! Since the dynamics also agrees ...



nnihilation of momentum and
nti-momentum modes gives radiation

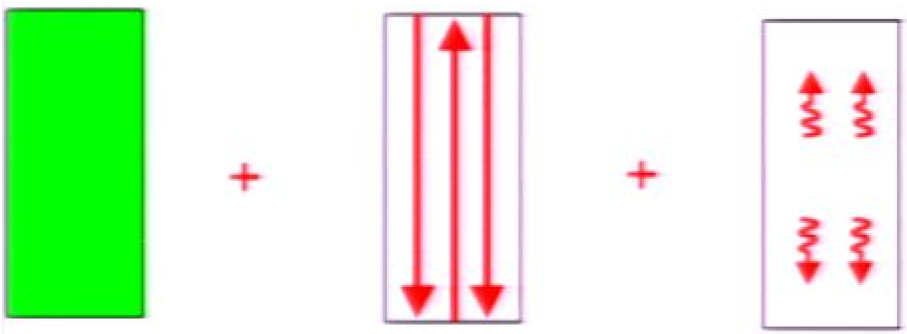
xact agreement with Hawking radiation
pectrum of black holes !



$$\Gamma_{micro} = \Gamma_{hawking}$$

Das+SDM 96, Maldacna+Strominger 96)

ear-extremal 1-charge:



n_5 $\qquad$ $n_1, \bar{n}_1$ $\qquad$ $n_p, \bar{n}_p$

D1 charge $\quad \hat{n}_1 = n_1 - \bar{n}_1$

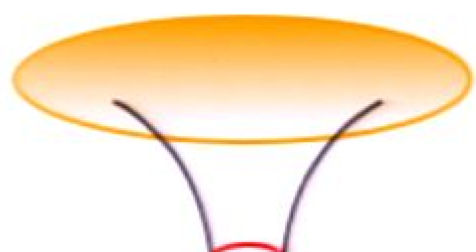
D5 charge $\quad \hat{n}_5 = n_5$

P charge $\quad \hat{n}_p = n_p - \bar{n}_p$

$$E = (n_1 + \bar{n}_1)m_1 + n_5 m_5 + (n_p + \bar{n}_p)m_p$$

$$S_{micro} = 2\pi\sqrt{n_5}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

Maximize S_{micro} subject to the total charge, total energy constraints



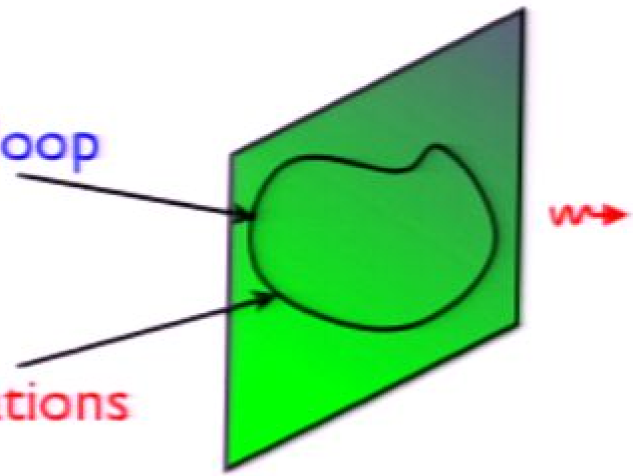
Near-extremal 5-brane

Then we find $\quad S_{micro} = S_{bek}$

(Maldacena 96)

Again, the dynamics agrees

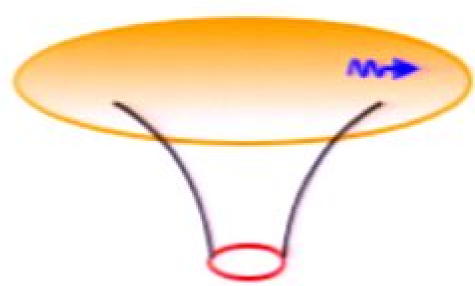
D1 and anti-D1 make a loop



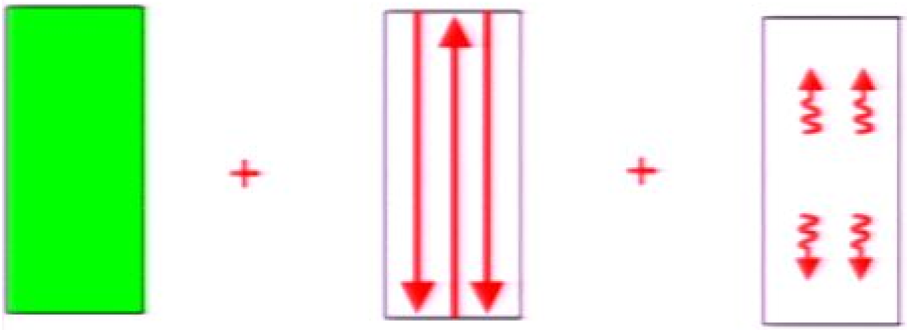
P and anti-P create vibrations of loop

(Klebanov and SDM 97)

Low energy radiation from vibrating loop agrees exactly with low energy radiation from near-extremal D5 brane geometry



ear-extremal 1-charge:



n_5 $n_1, \bar{n}_1$ $n_p, \bar{n}_p$

D1 charge $\hat{n}_1 = n_1 - \bar{n}_1$

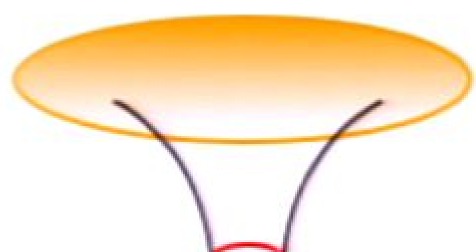
D5 charge $\hat{n}_5 = n_5$

P charge $\hat{n}_p = n_p - \bar{n}_p$

$$E = (n_1 + \bar{n}_1)m_1 + n_5 m_5 + (n_p + \bar{n}_p)m_p$$

$$S_{micro} = 2\pi\sqrt{n_5}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

Maximize S_{micro} subject to the total charge, total energy constraints



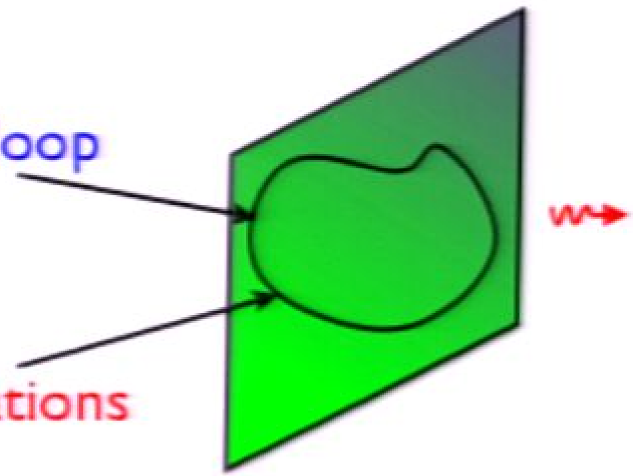
Near-extremal 5-brane

Then we find $S_{micro} = S_{bek}$

(Maldacena 96)

Again, the dynamics agrees

D1 and anti-D1 make a loop



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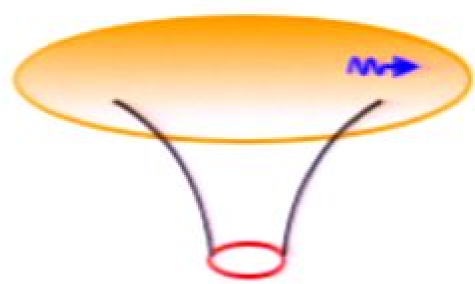
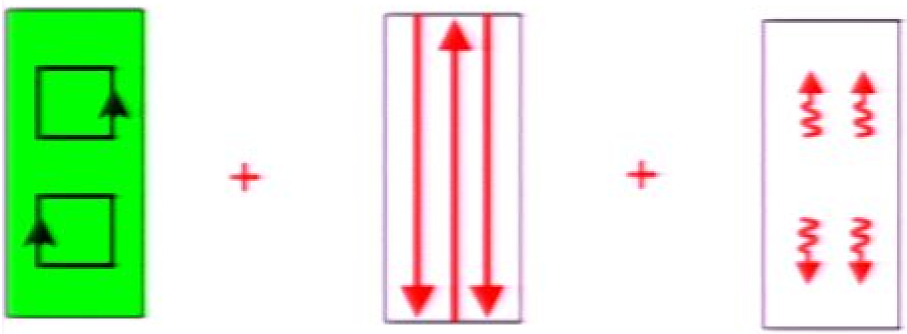


eneral system (including neutral)



$n_5, \bar{n}_5$ $n_1, \bar{n}_1$ $n_p, \bar{n}_p$

D1 charge $\hat{n}_1 = n_1 - \bar{n}_1$

D5 charge $\hat{n}_5 = n_5 - \bar{n}_5$

P charge $\hat{n}_p = n_p - \bar{n}_p$

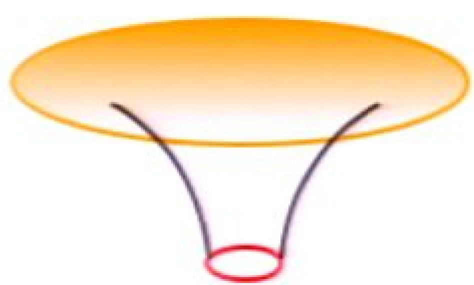
$$E = (n_1 + \bar{n}_1)m_1 + (n_5 + \bar{n}_5)m_5 + (n_p + \bar{n}_p)m_p$$

$$S_{micro} = 2\pi(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p})$$

Maximize S_{micro} subject to the total charge, total energy constraints

Then we again find !!

$$S_{micro} = S_{bek}$$



All black holes, including Schwarzschild

(Horowitz, Maldacena, Strominger 96)

The 4-charge hole works in the same way:

$$S_{micro} = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_2} + \sqrt{\bar{n}_2})(\sqrt{n_3} + \sqrt{\bar{n}_3})(\sqrt{n_4} + \sqrt{\bar{n}_4}) = S_{bek}$$

(Horowitz,Lowe,Maldacena 96)

Thus we arrive at the following conjecture for the entropy of states in string theory :

$$S = C \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i})$$

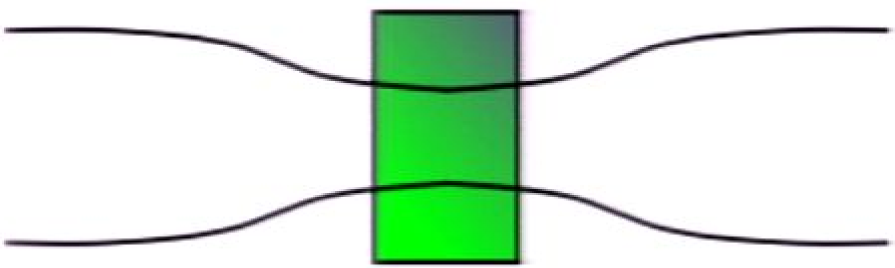
where A is a constant of order unity, and the other parameters are determined as follows

There is just one more relation that we need from black hole physics ...

If branes wrap a compact direction, they exert a pressure on that direction

This leads to a behavior $L - \frac{\alpha}{r}$

for the radius of the compact direction, and the pressure can be read off from α



From the black hole metric, we find that $P_a = \sum_i (n_i + \bar{n}_i)\ p_a^i$

Thus pressure is just the simple sum of the pressures exerted by the individual branes and antibranes wrapping that compact direction loral from all that we know about black holes in string theory:

hings simplify dramatically at high density

ı the correct variables (fractional branes) everything is free :

lacroscopic quantities are given by a simple sum over
contributions from individual branes and antibranes

Charges are given by $\hat{n}_i = n_i - \bar{n}_i$

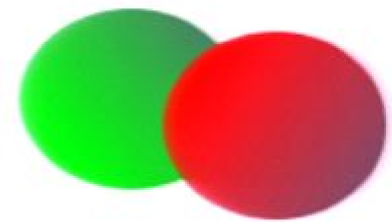
Energy is given by $E = \sum_i (n_i + \bar{n}_i)\, m_i$

Pressures are given by $P_a = \sum_i (n_i + \bar{n}_i)\, p_a^i$

Entropy is given by

$$S = C \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i})$$

\nalogy: Hadron physics is very complicated ....



3ut at high energies the physics is free if we use the
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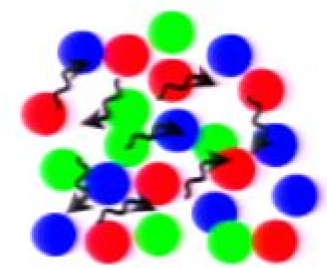
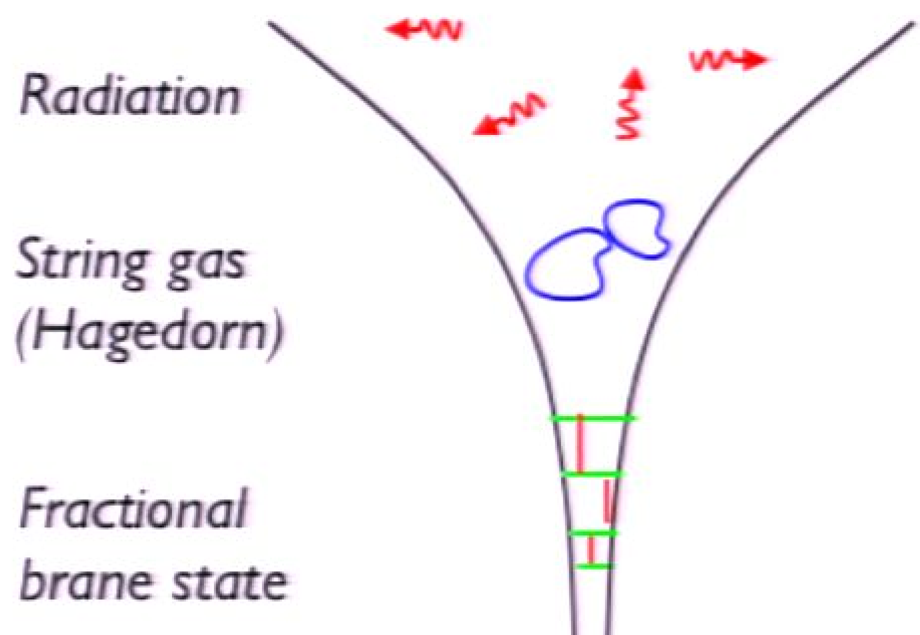


Now we will apply these results to the Cosmology of the Early Universe



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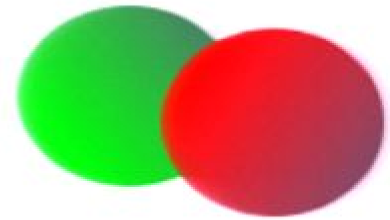
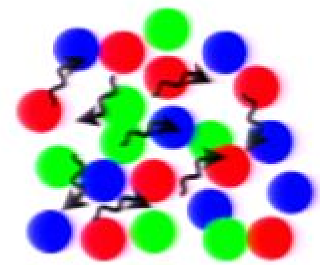


But at high energies the physics is free if we use the correct variables: quarks and gluons



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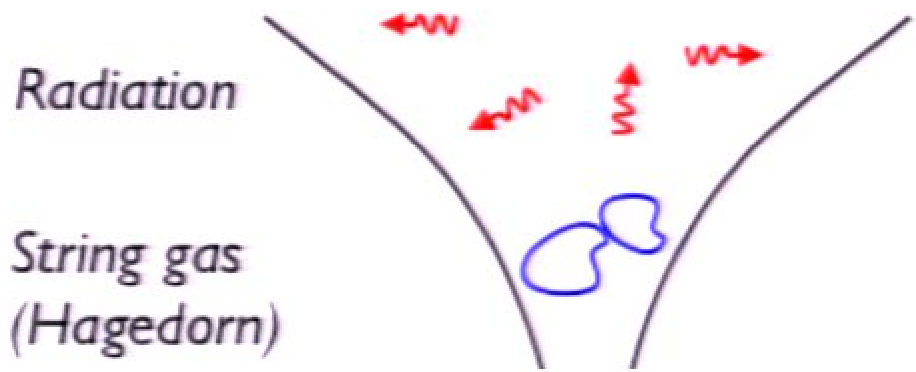
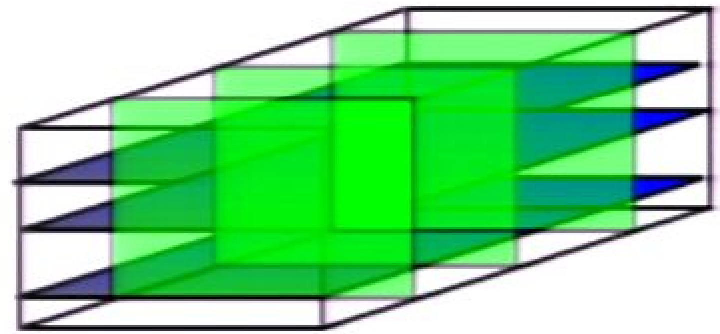


(A) Take M theory, which is the parent of all string theories

It lives in 10+1 spacetime dimensions

Compactify all 10 space directions into a torus



Choose appropriate sets of directions to wrap different types of branes (and their antibranes)

The mass of a brane is given by

$$m_i = T_p \prod_j L_j$$

The Universe is neutral, so set $n_1 = \bar{n}_i$

$$S = A \prod \sqrt{n_i}$$

(B) Maximize entropy S for given total energy E

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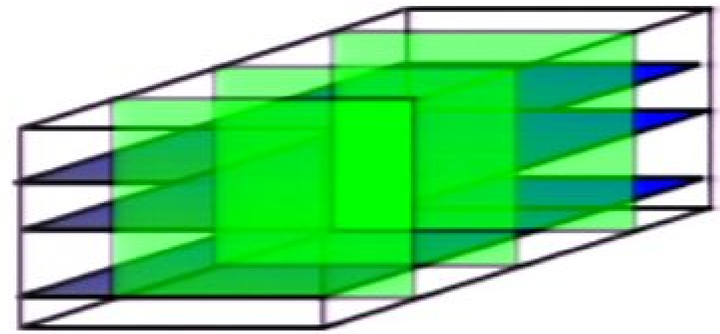
This tells us that energy is equipartitioned among different types of branes

$$E_k = n_k m_k = \frac{E}{2N}$$

(A) Take M theory, which is the parent of all string theories

It lives in 10+1 spacetime dimensions

Compactify all 10 space directions
into a torus



Choose appropriate sets of directions to wrap different types of branes (and their antibranes)

The mass of a brane is given by

$$m_i = T_p \prod_j L_j$$

The Universe is neutral, so set

$$n_1 = \bar{n}_i$$

$$S = A \prod \sqrt{n_i}$$

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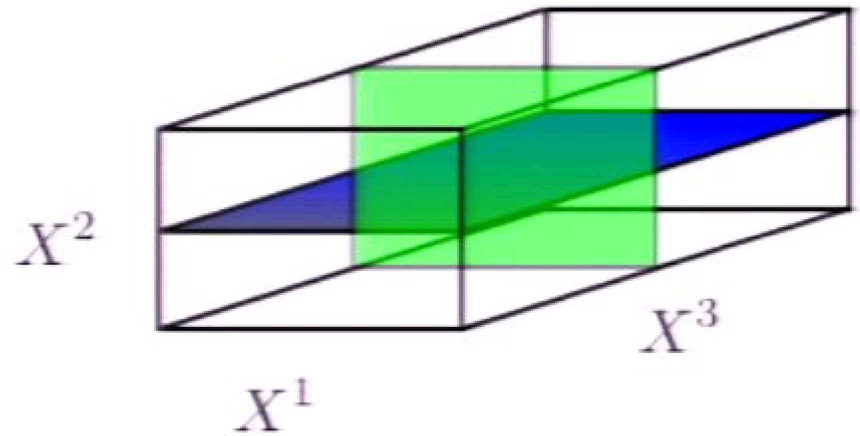
Let N_i of these types extend along the direction X^i

Define $$w_i \equiv \frac{N_i}{N}$$



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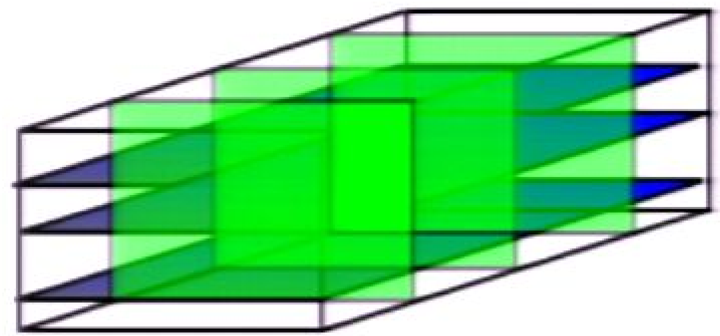
$$p_i = w_i \rho$$

Where ρ is the energy density of the Universe

(E) Solving Einstein's equations :

Take a Kasner-type metric ansatz

$$ds^2 = -dt^2 + \sum_{i=1}^{D-1} a_i^2(t) dx_i^2$$



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Interestingly, the problem can be solved in closed form ...

(Chowdhury + SDM 06)

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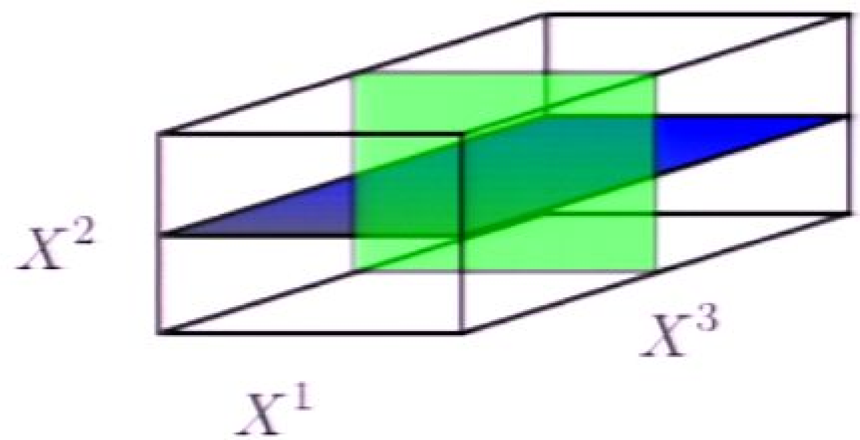

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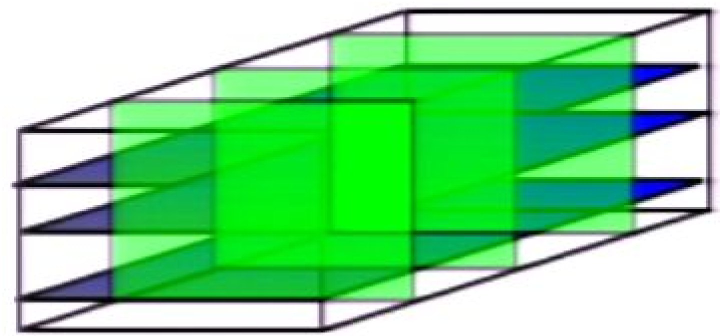

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(F) The solution :

Define the constants

$$W \equiv \sum_i w_i, \qquad U \equiv \sum_i w_i^2$$

(Recall that $w_i \equiv \frac{N_i}{N}$)

Compute the constants

$$\begin{aligned} K_1 &= \frac{(D-1-W)}{2(D-2)} \\ K_2 &= -\frac{1}{2}[\frac{1-W}{D-2}W + U] \end{aligned}$$

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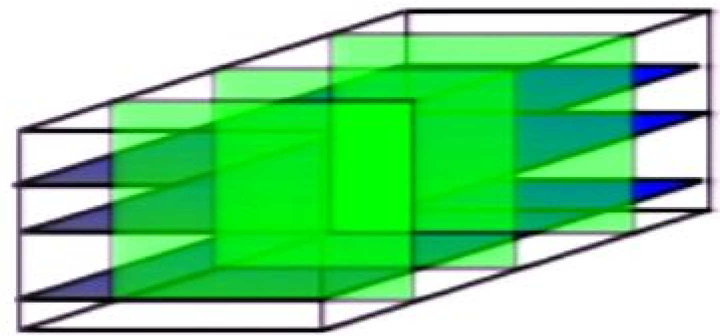

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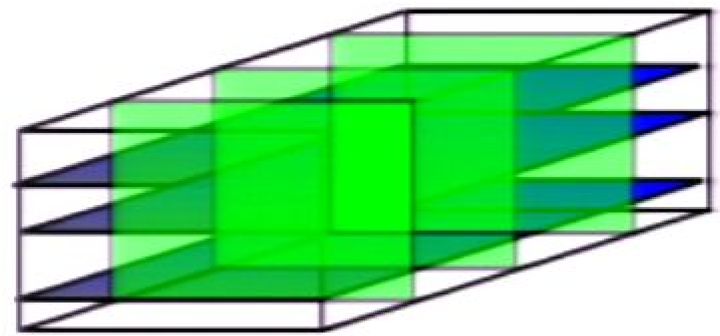

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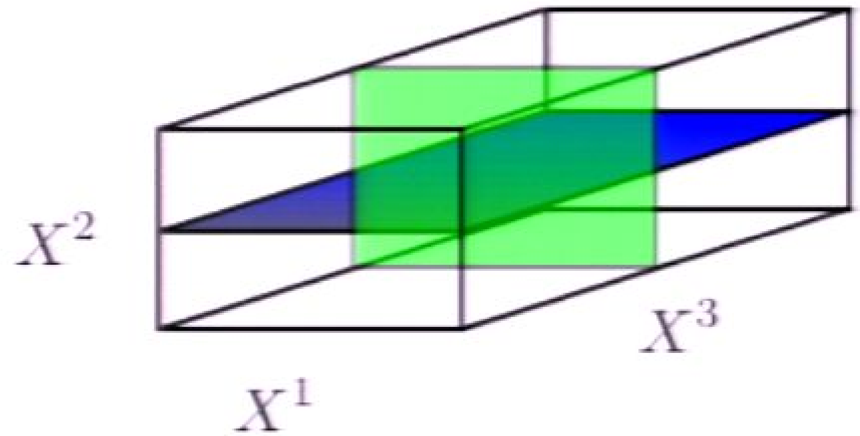

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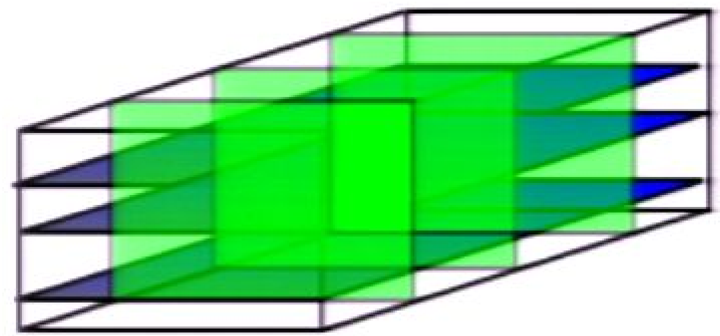

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Then

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where τ is an auxiliary time parameter defined by

$$(t - t_0) = \frac{1}{A_4} \int_0^{\tau} (\tau' - r_1)^{\frac{2(-r_1 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} (\tau' - r_2)^{-\frac{2(-r_2 K_2 + A_2)}{(K_1+K_2)(r_1-r_2)}} d\tau'$$

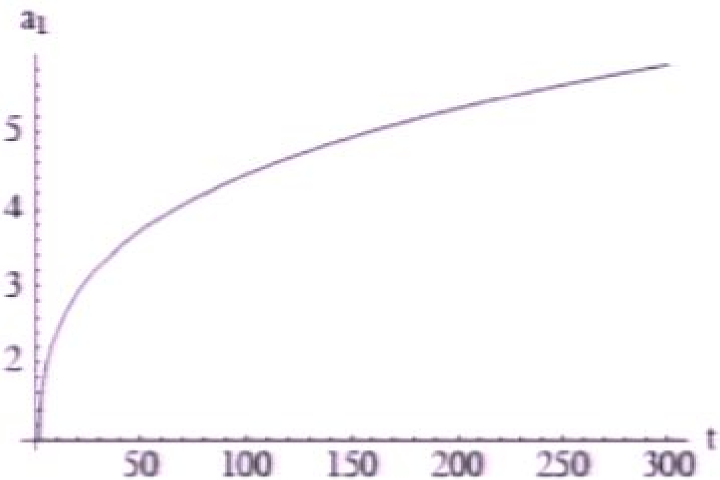
Recall that this integral is just the incomplete Beta function

$$B_x(p,q) = \int_0^x s^{p-1}(1-s)^{q-1} ds$$

At late times the evolution becomes power law ...

$w_i = -.2$ for all i

All a_i equal



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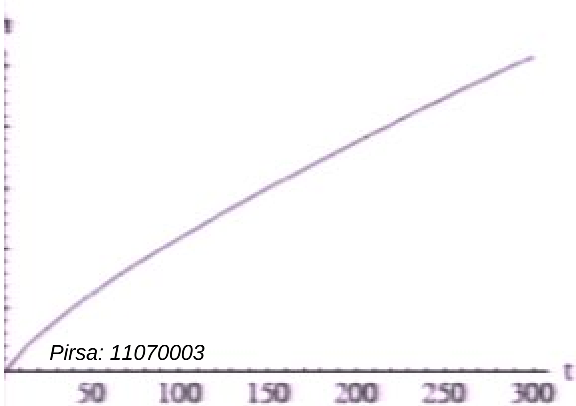

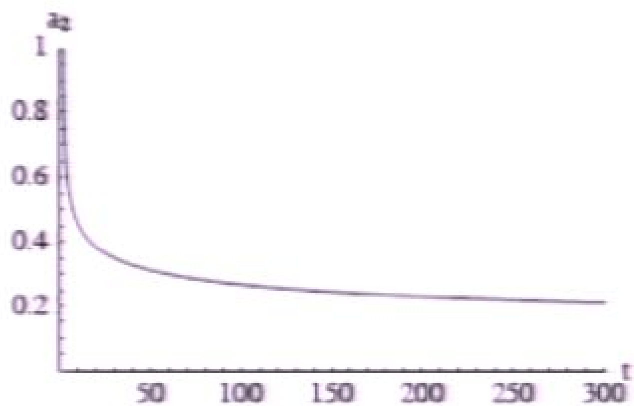

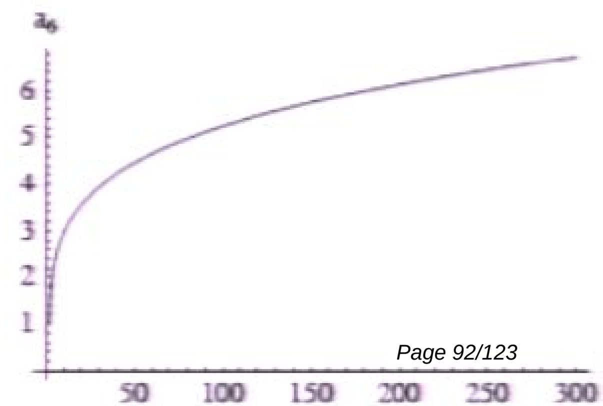

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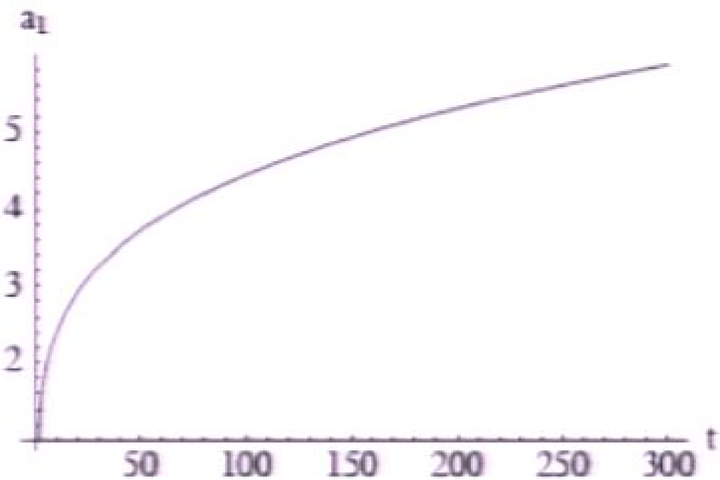

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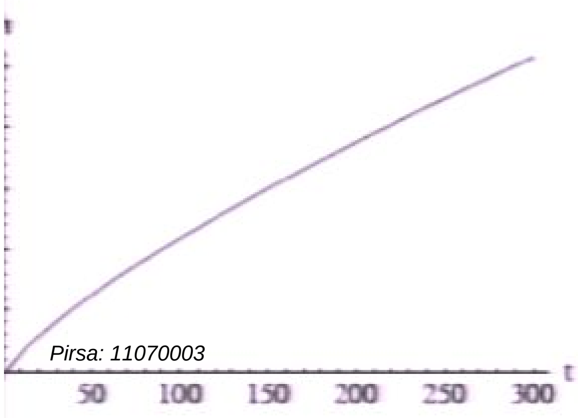

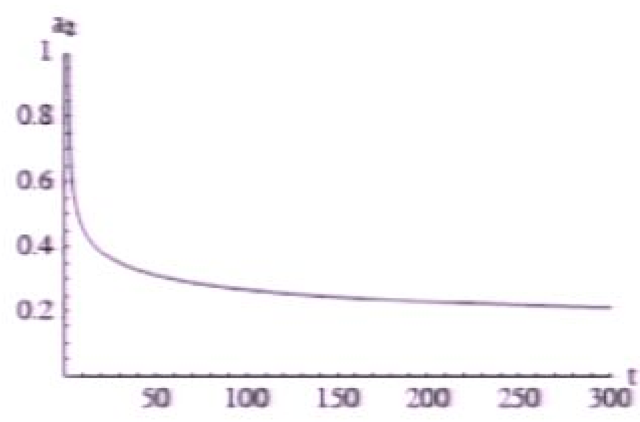

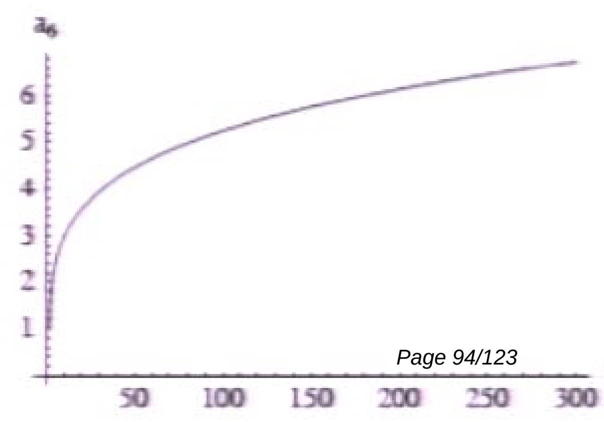

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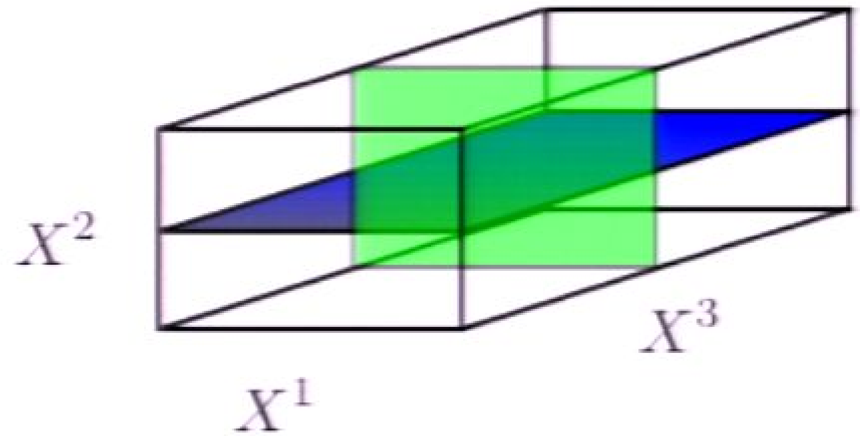

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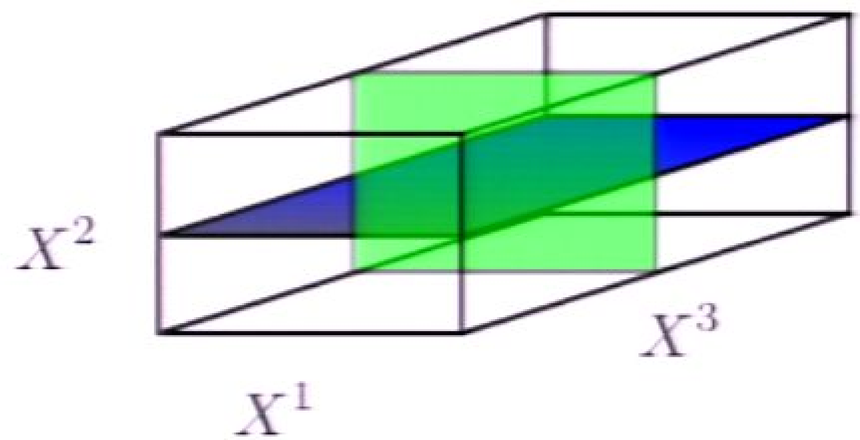

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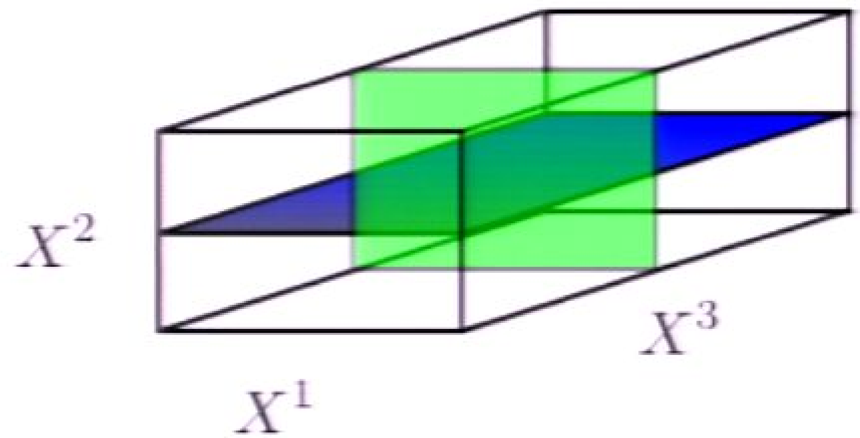

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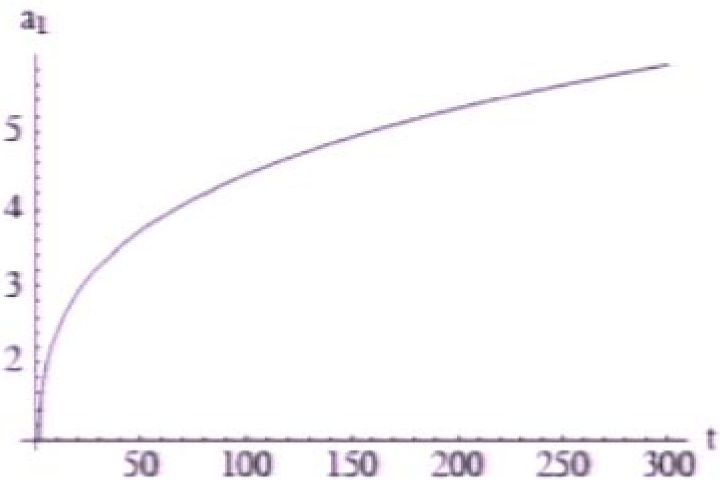

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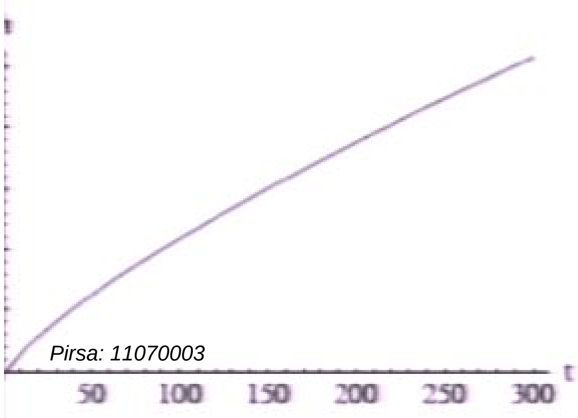
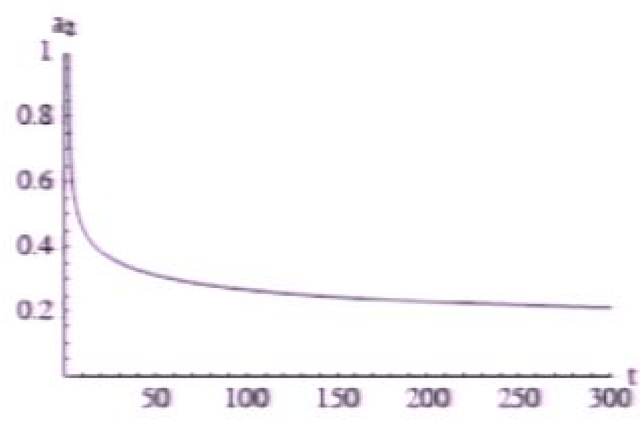
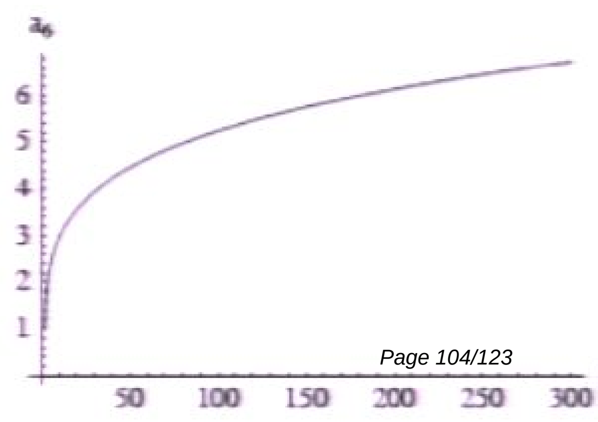




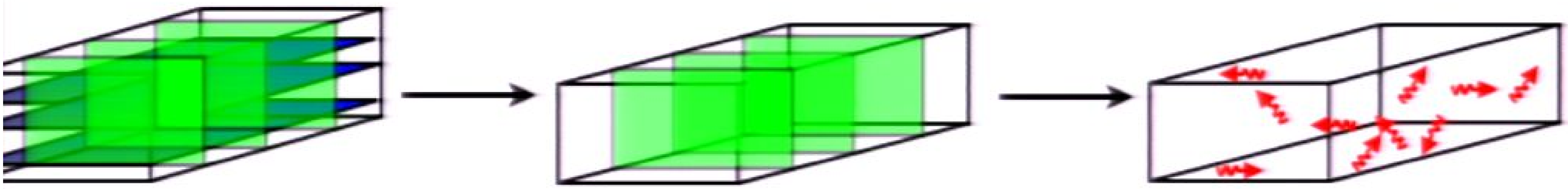


Conjectures

A) At very early times we can get upto 9 kinds of branes.

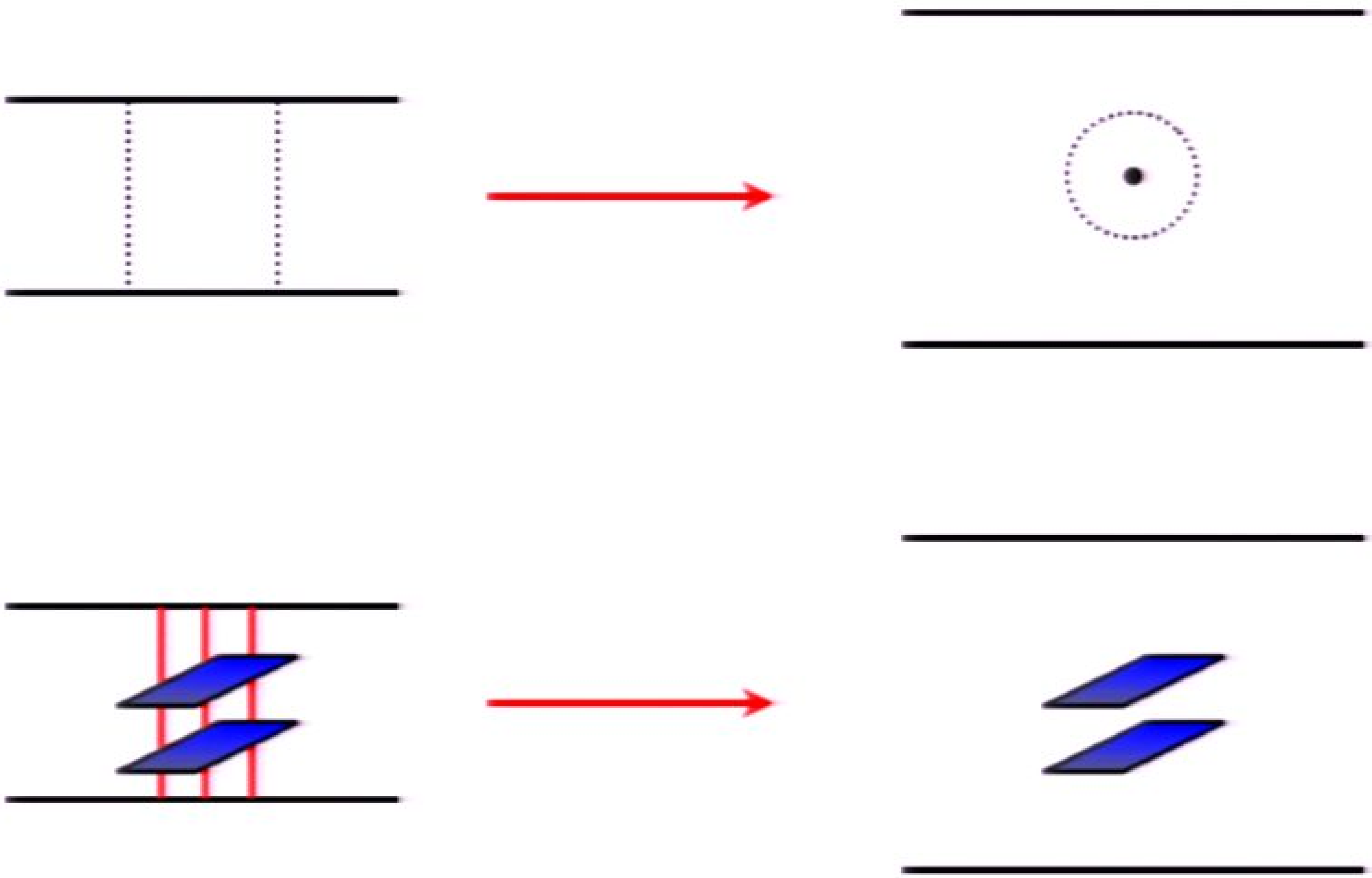
As the Universe expands, it becomes advantageous to have less kinds of branes 9 , 8, 7, ...

Finally we might get down to just radiation ...



We can compute the details becuase we have a microscopic model of the Gregory - Laflamme transition ...

(Chowdhury, Giusto, SDM 06)

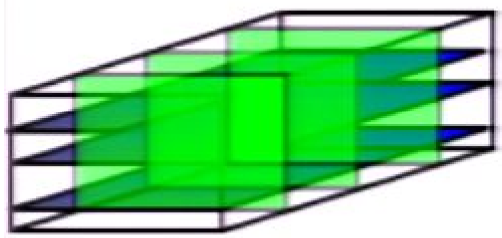


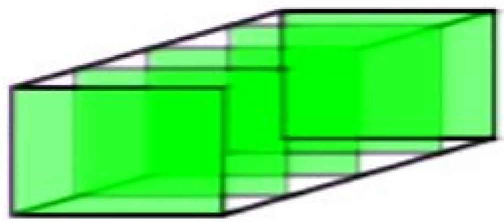
Using the same microscopic entropy formulae as before, we reproduce the main features of the tension-energy phase diagram

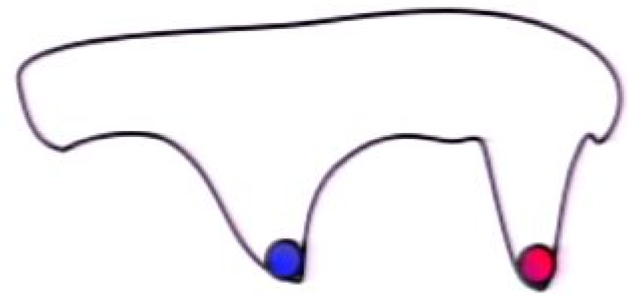
→ Brane ‘snapping’

B) Brane annihilation is a slow process, since it gives exactly the rate of Hawking radiation

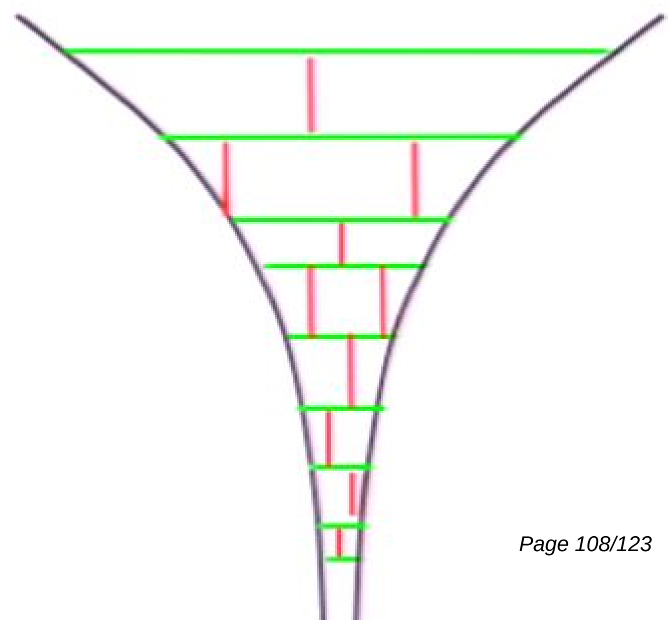
Rate of annihilation is $\sim \dfrac{1}{N^k} \sim \hbar^\alpha$







Some branes/antibranes may not annnihilate, may be stuck in a KKLT phase, which would give inflation ...

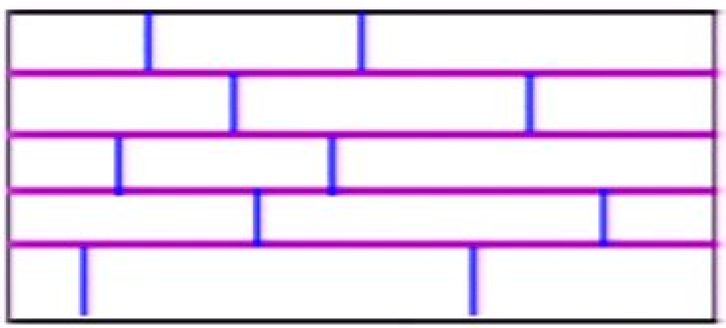


C) Fractional branes may be left over today, and may form some part of dark matter ...

(D) We may have to re-think what the horizon problem is ....

Usually quantum nonlocality extends over distances $\sim l_p$

But the fractional branes are correlated across the entire size of the Universe, so maybe we should think of the nonlocality scale as $\sim N^{\alpha} l_p$



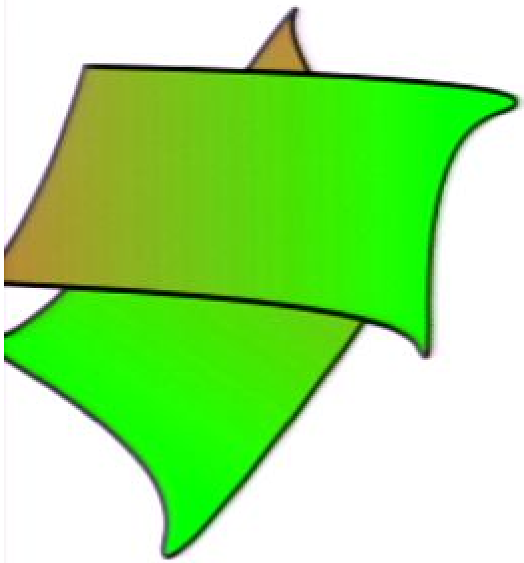
(E) Flatness problem: We should examine what is the entropy of the fractiona brane state on a curved space as compared to a flat space ...

If flat space maximises the entropy, then flat spatial slices will be preferred strongly ...

Observations

A) The fractional brane gas is very different from the usual brane gas

(Brandenberger et al, Greene et al ...)



Brane gas: Density of branes is low; occasional interactions

$$S \sim A \sim E$$

Hagedorn behavior: entropy of thermal vibrations cancels energy cost of brane area

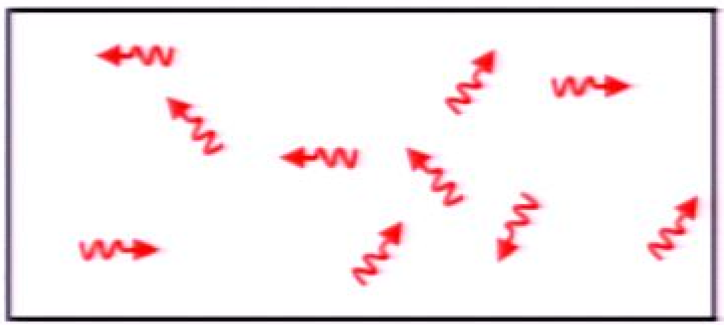
Fractional brane gas:

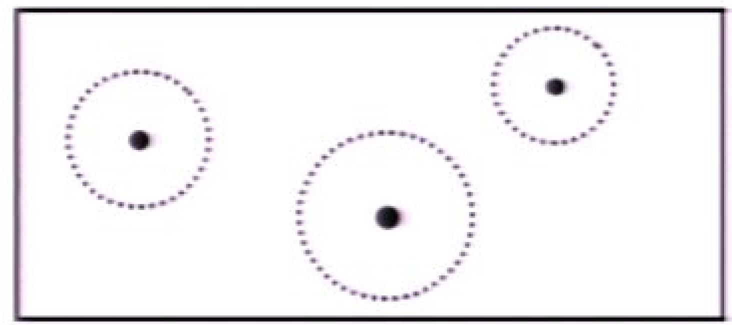
Branes are crushed together to planck distances

They form a bound state where branes break up on other branes

$$S \sim E^{\frac{N}{2}}, \quad N \leq 9$$

(B) What about back holes in the Early Universe ?





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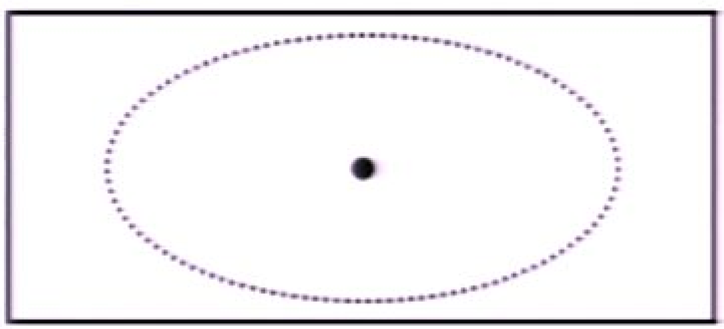

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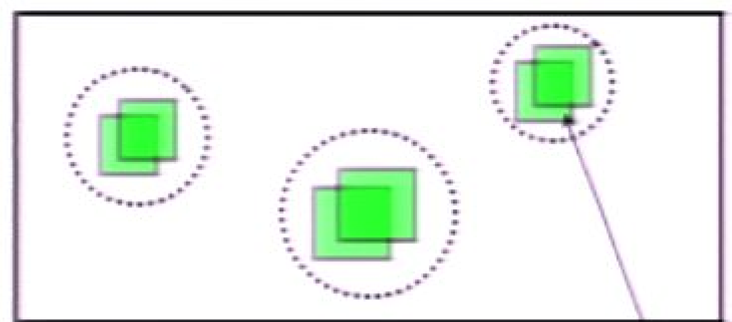
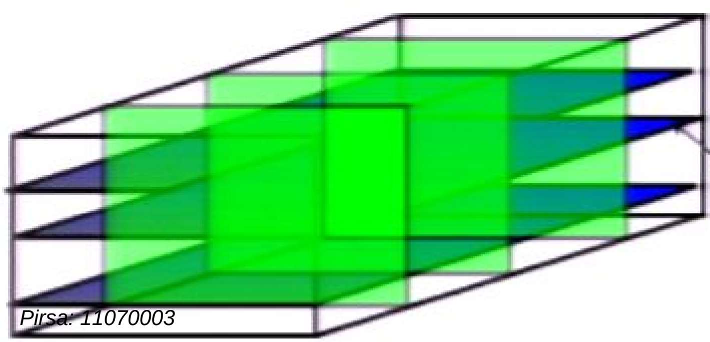


black holes
as branes



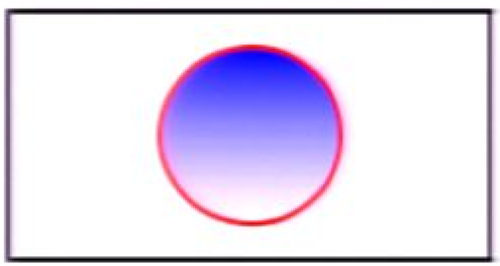
branes wrap
new cycles

Can keep adding more
energy E !!

Suggests that fundamental entropy law is not $S = \frac{A}{4}$

But rather $S = C \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i})$

Analogy: Put a balloon in a box, and blow air in it



At first, mass of air M determines area A, pressure P

After balloon fills the box, M can be further raised, but area A does not rise ... pressure P keeps rising

$PV = NkT$

Question

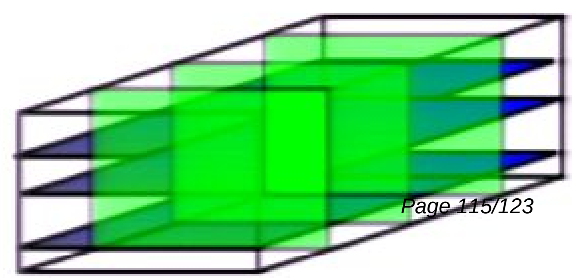
Results on black holes in string theory are solid, well understood

Suggest $$S = C\prod_{i=1}^{N}(\sqrt{n_i}+\sqrt{\bar{n}_i})$$

For Cosmology, we do not know how to set initial conditions ...

Should we look for states with large entropy, or states with a high expansion rate, or states found by some other principle ?

Question: What, if any, is the role of the fractional brane state in Cosmology ?



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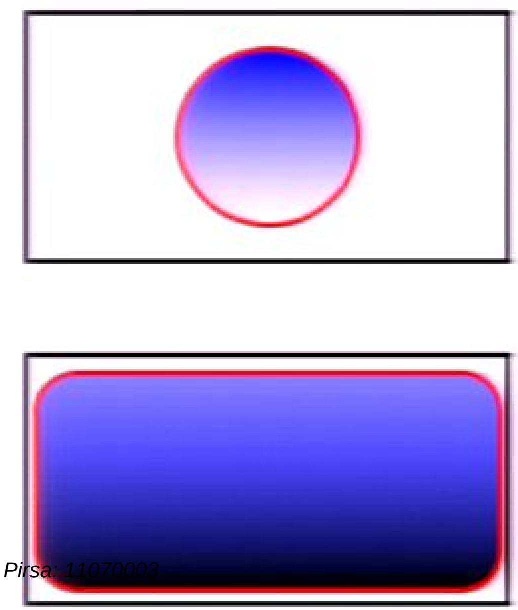

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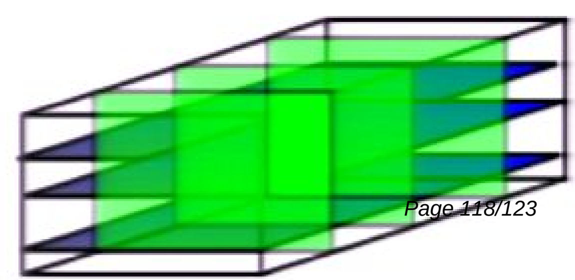

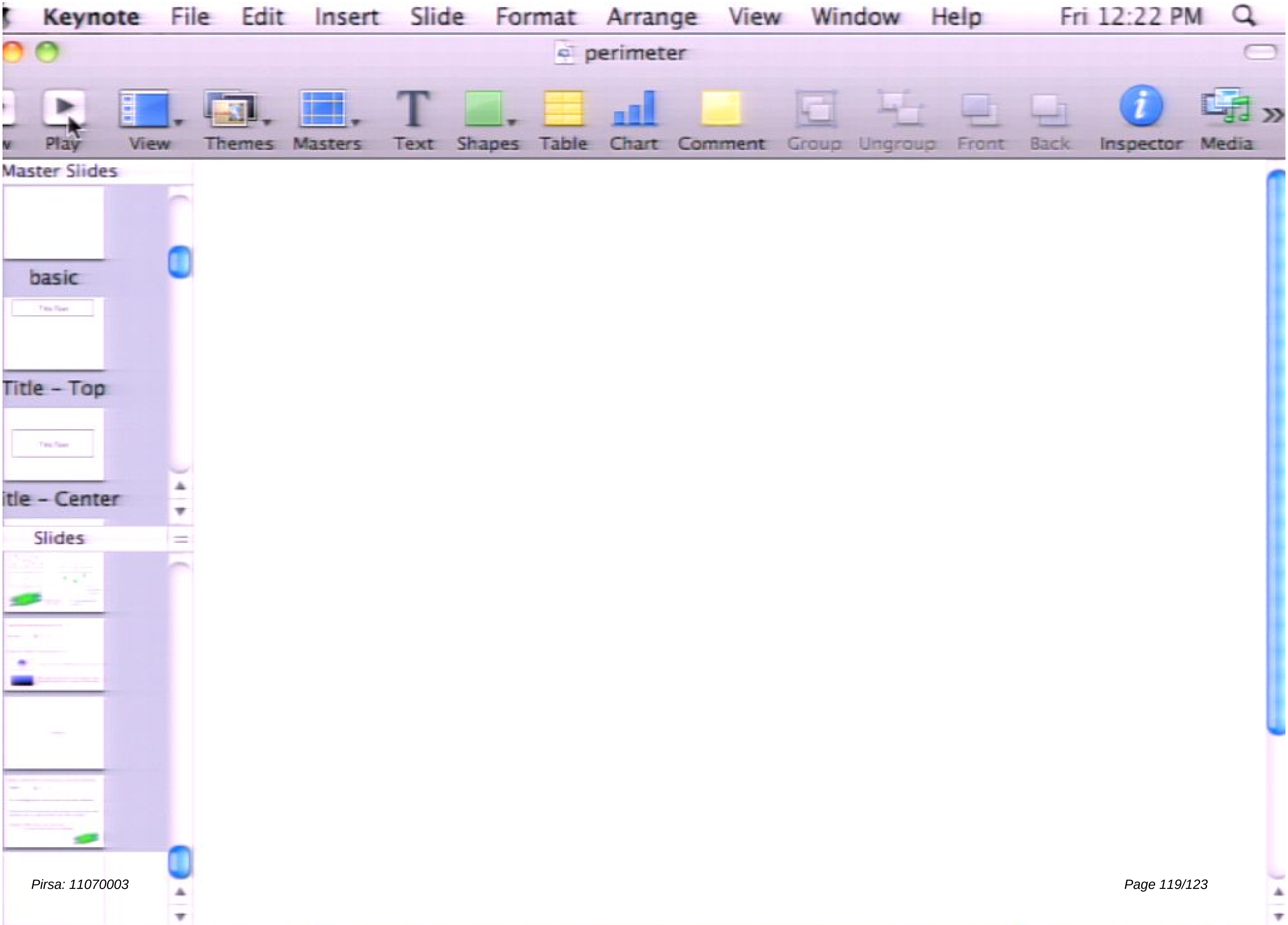
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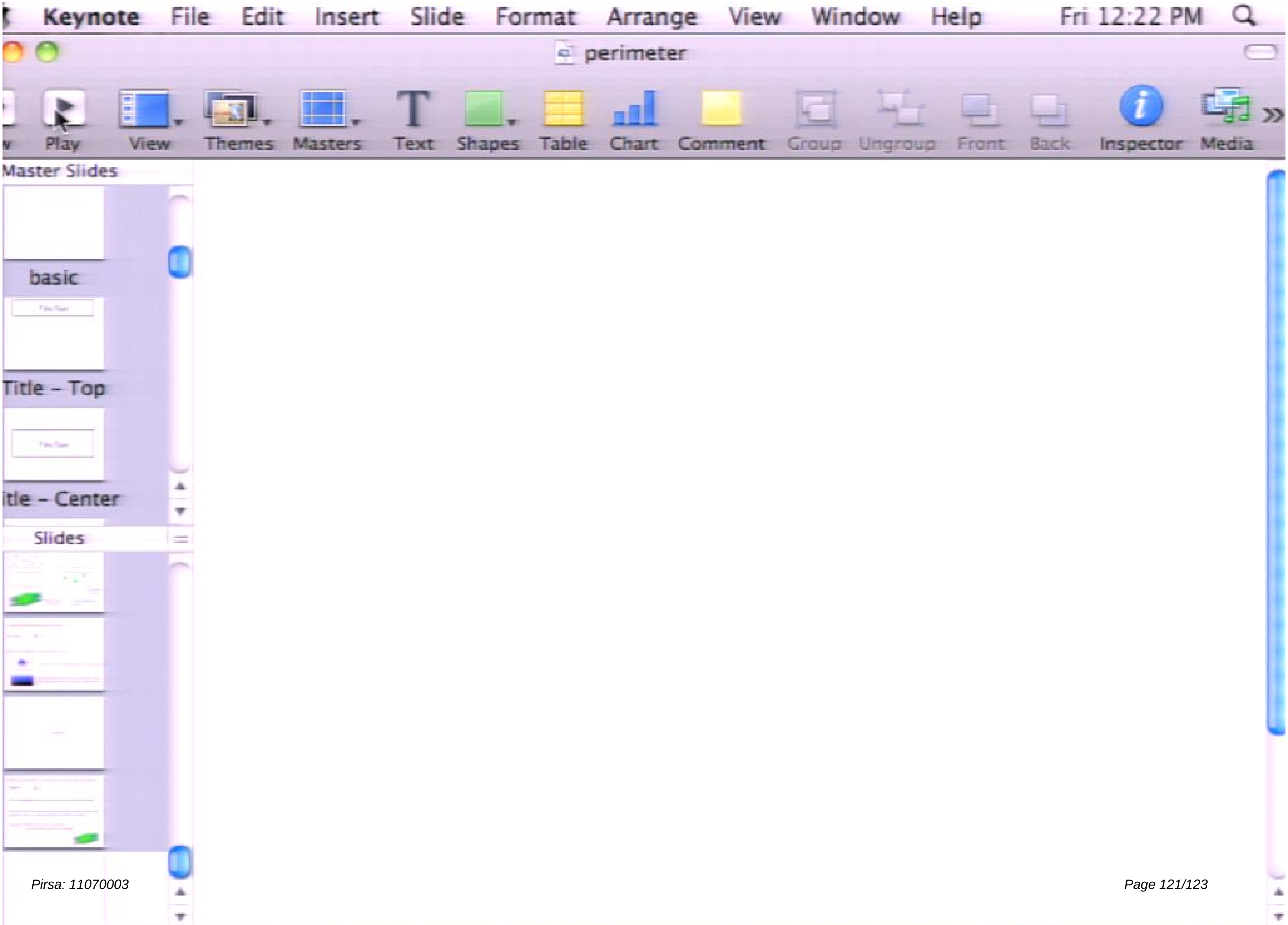
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