Title: Lessons from an exactly solved interacting quantum field theory in de Sitter spacetime

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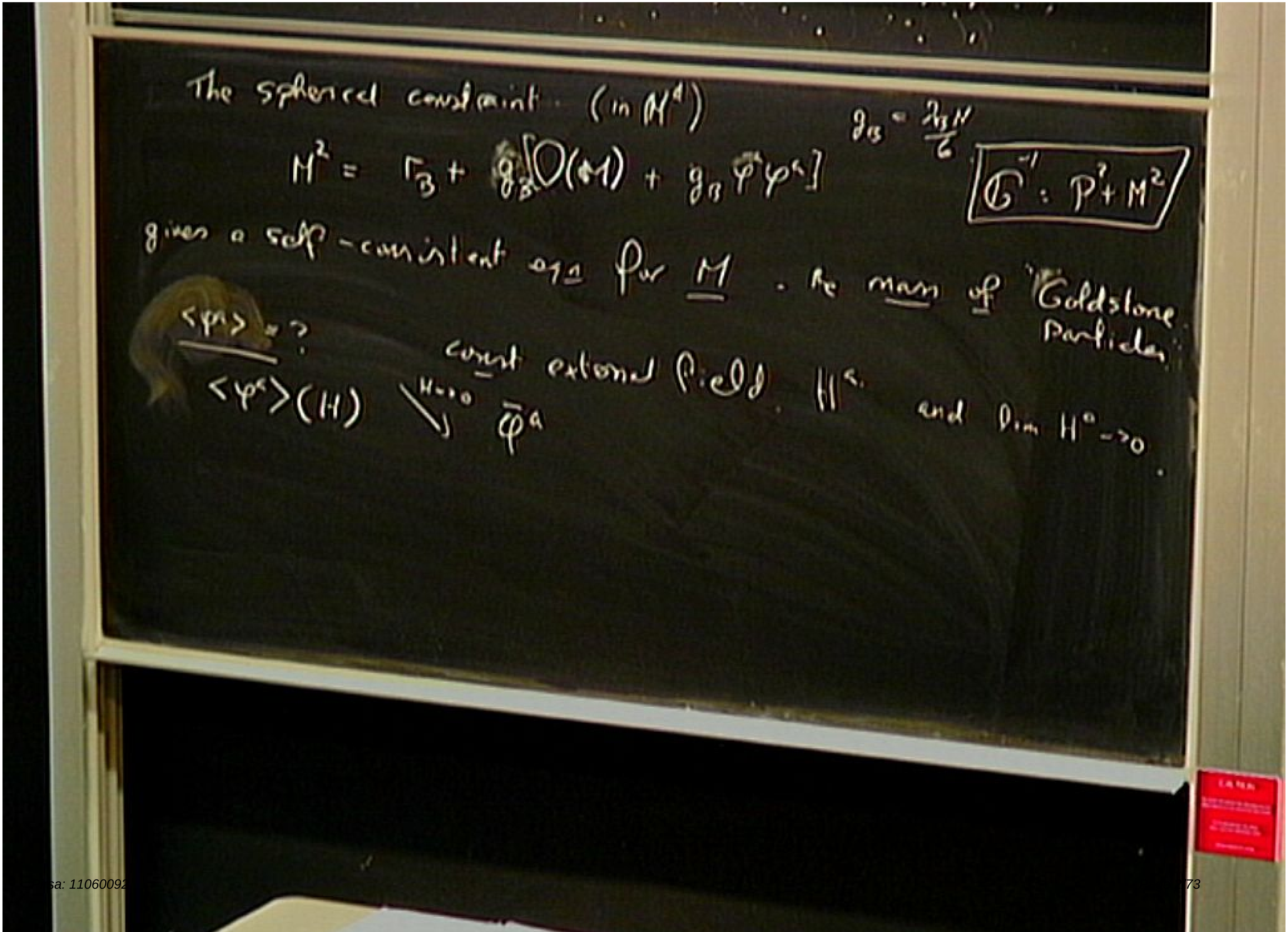
Abstract: It is argued that the correct quantization of a scalar field theory in de Sitter spacetime involves a de Sitter invariant state which is not the Bunch-Davies vacuum. A novel but natural de Sitter invariant alternative exists and it is suggested that this and is the prefered state for scalar field theories. The argument is based on the exact solution of an interacting scalar field theory.


The spherical constraint (in N^d)
g_B = g_B N / 6
M^2 = r_B + g_B [O(M) + g_B φ^a φ^a]
G^-1 = p^2 + M^2
gives a self-consistent eqn for M . the mass of Goldstone Particles
<φ^a> = ?
const external field H^a
<φ^a>(H) H→0 φ̄^a
and lim H^a → 0

The spherical constraint (in M^4)

$g_B = \frac{\lambda_B N}{6}$

$$M^2 = r_B + g_B[O(M) + g_B \varphi^a\varphi^a]$$

$$G^{-1} = p^2 + M^2$$

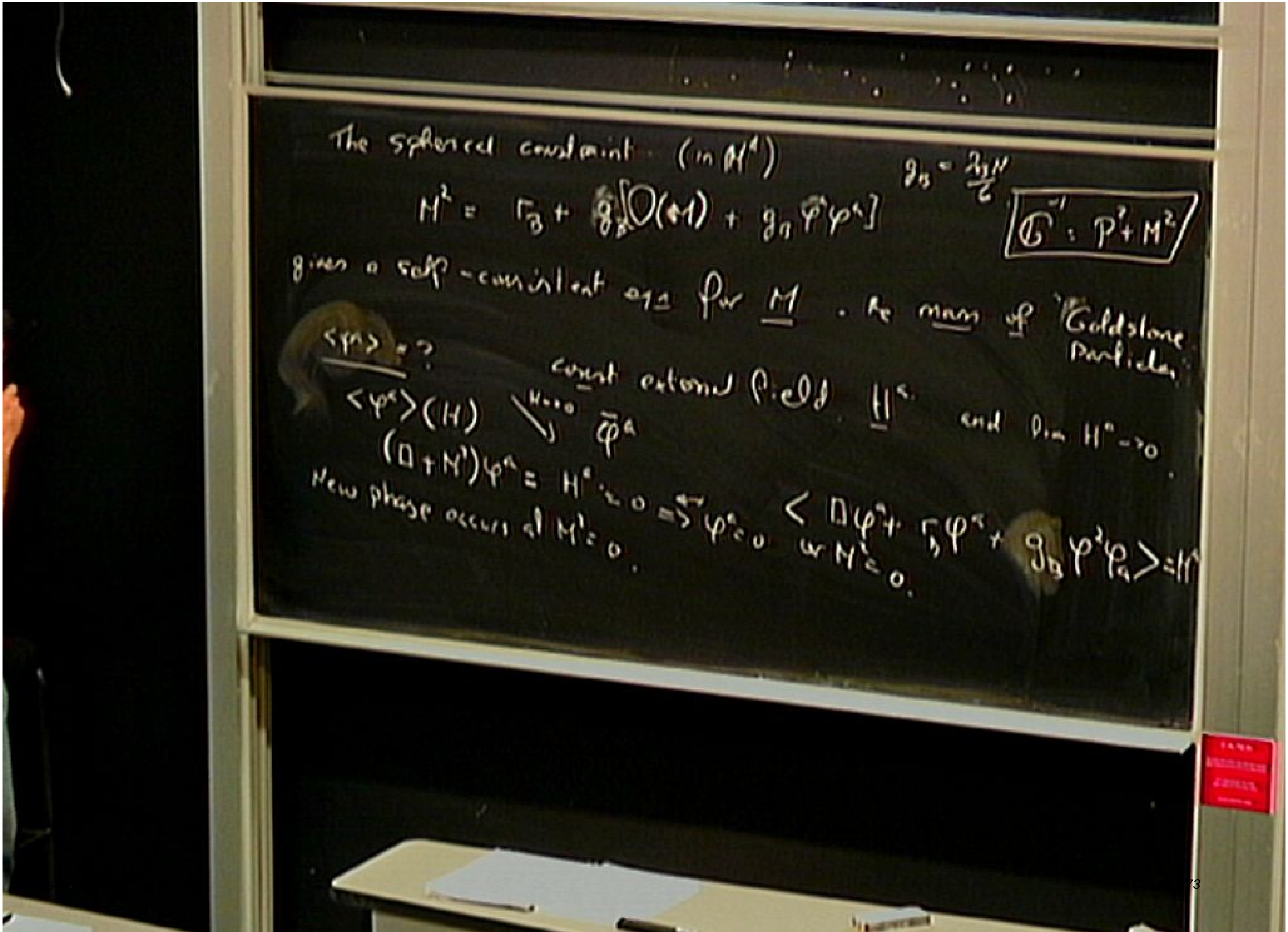
gives a self-consistent eqn for $\underline{M}$. the $\underline{\text{mass}}$ of Goldstone Particles.

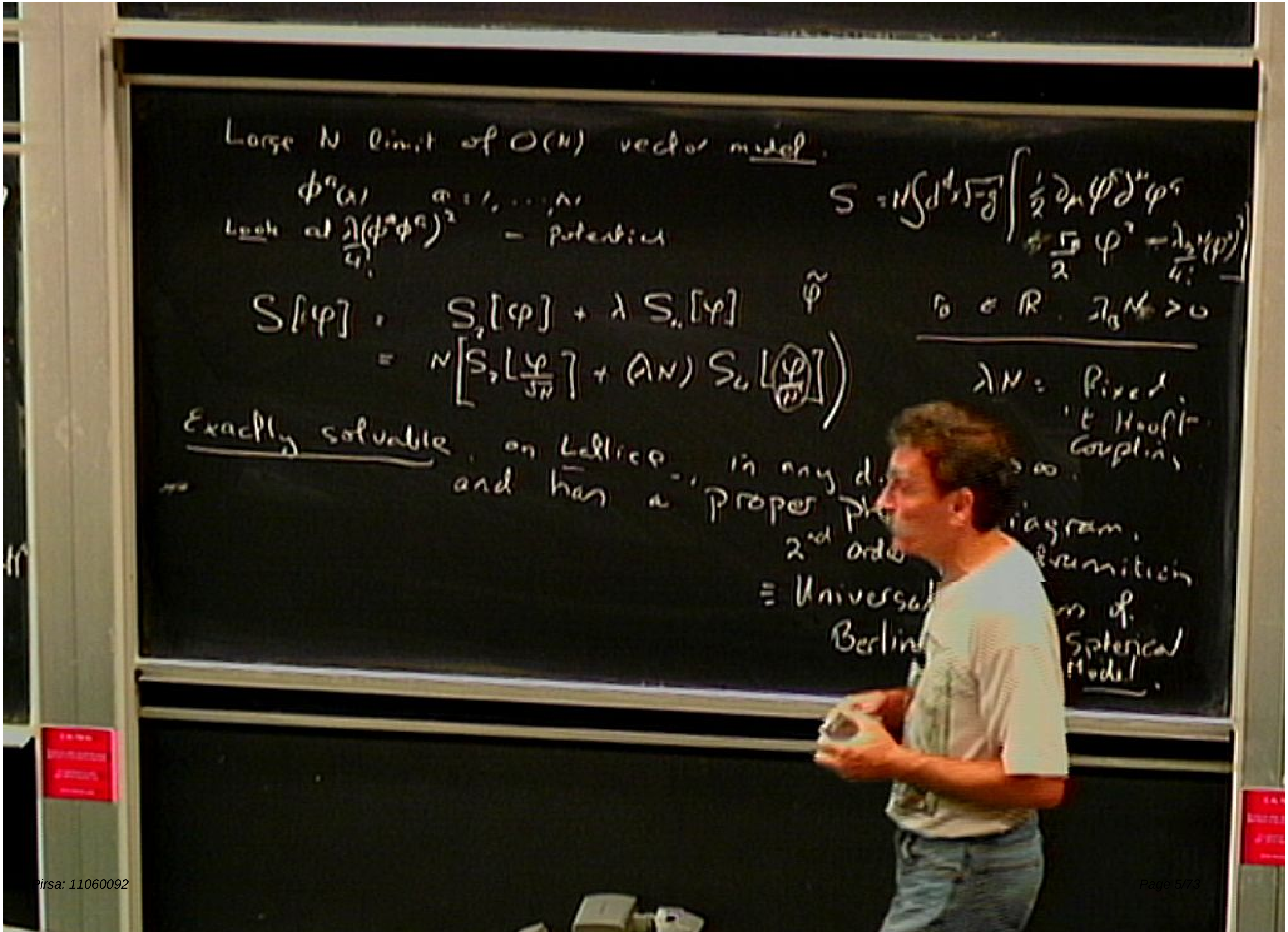
$\langle\varphi^a\rangle = ?$

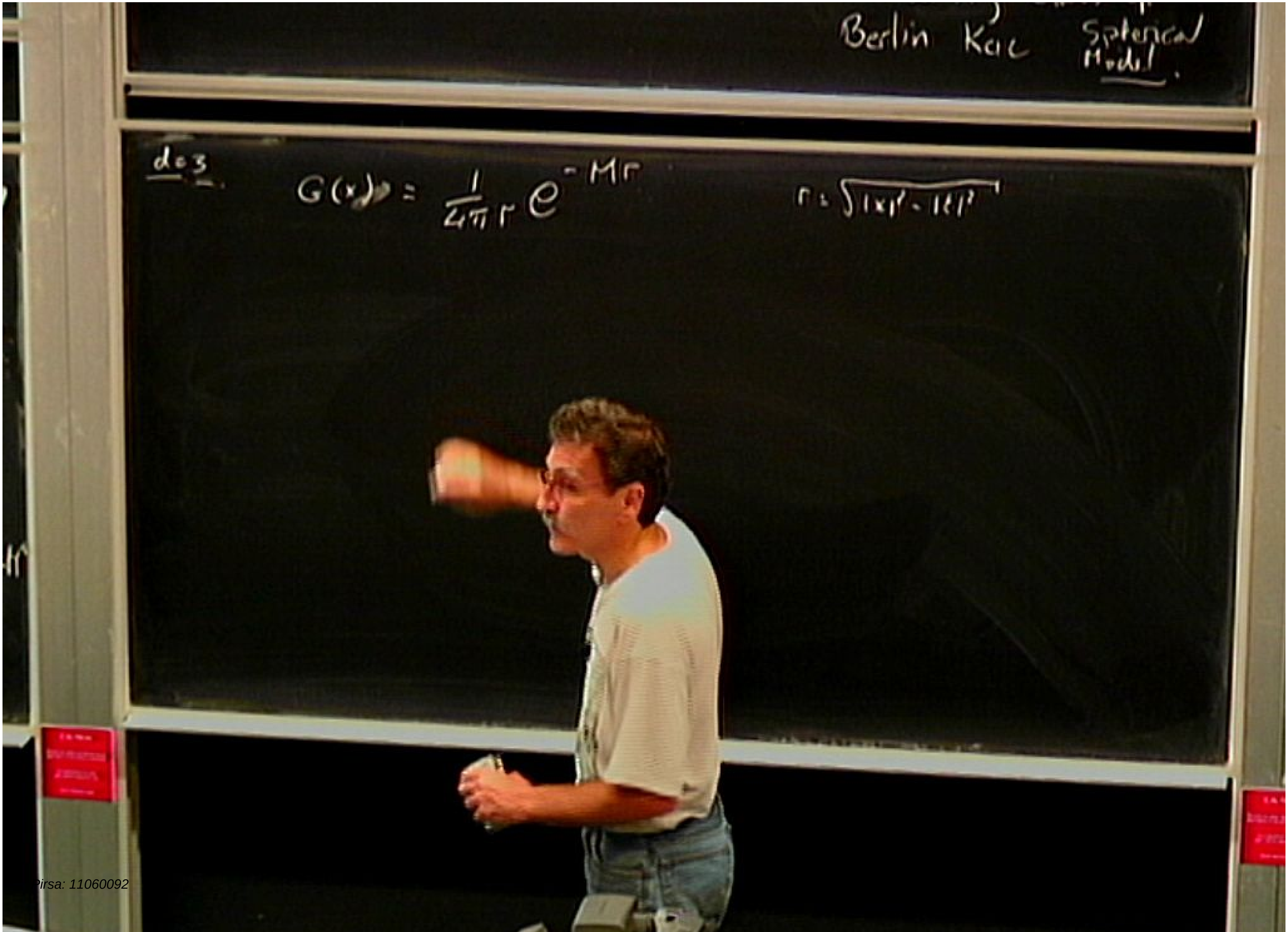
const external field $\underline{H}^a$ and lim $H^a \to 0$.

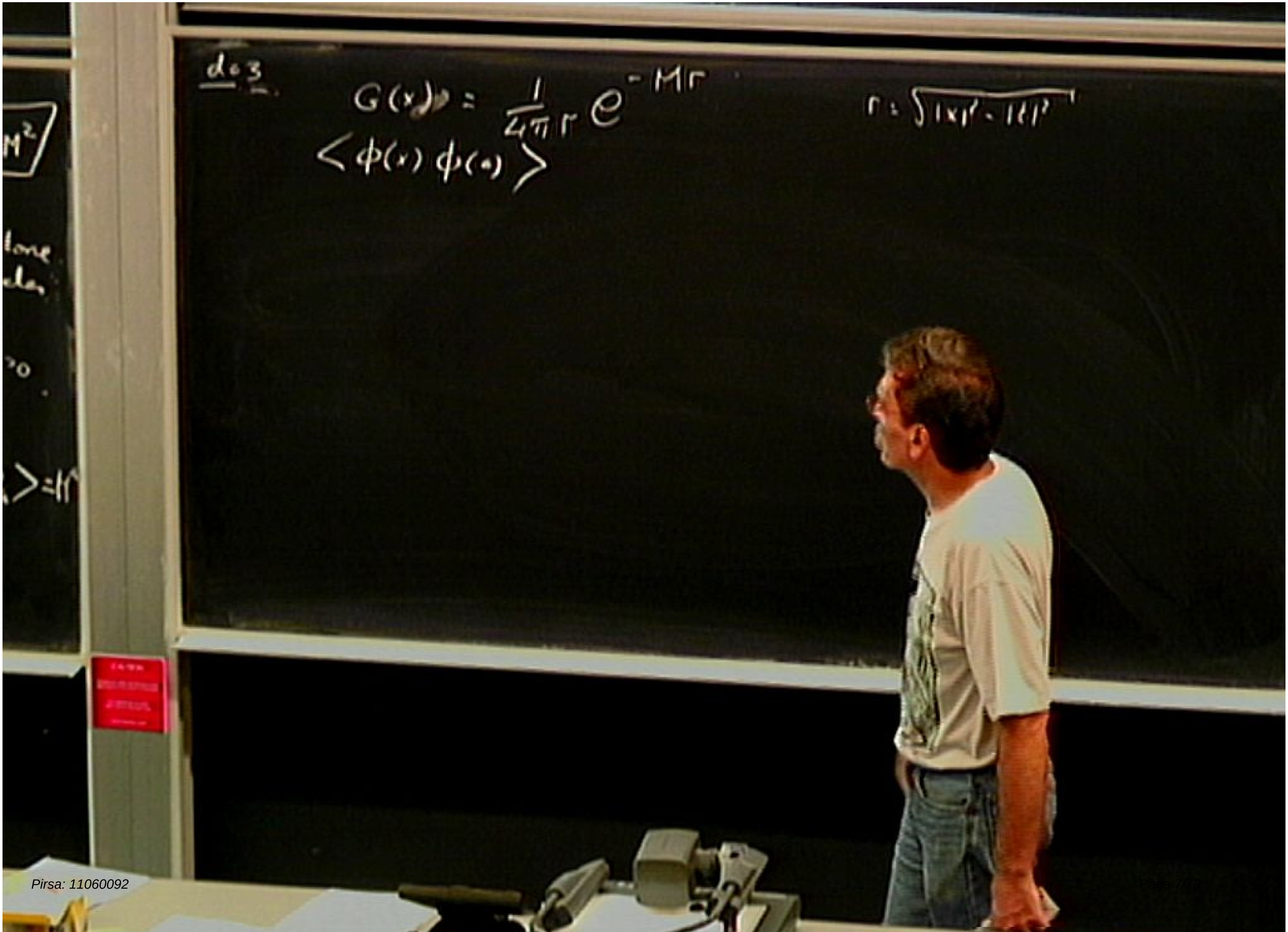
$$\langle\varphi^a\rangle(H) \xrightarrow{H\to 0} \bar{\varphi}^a$$

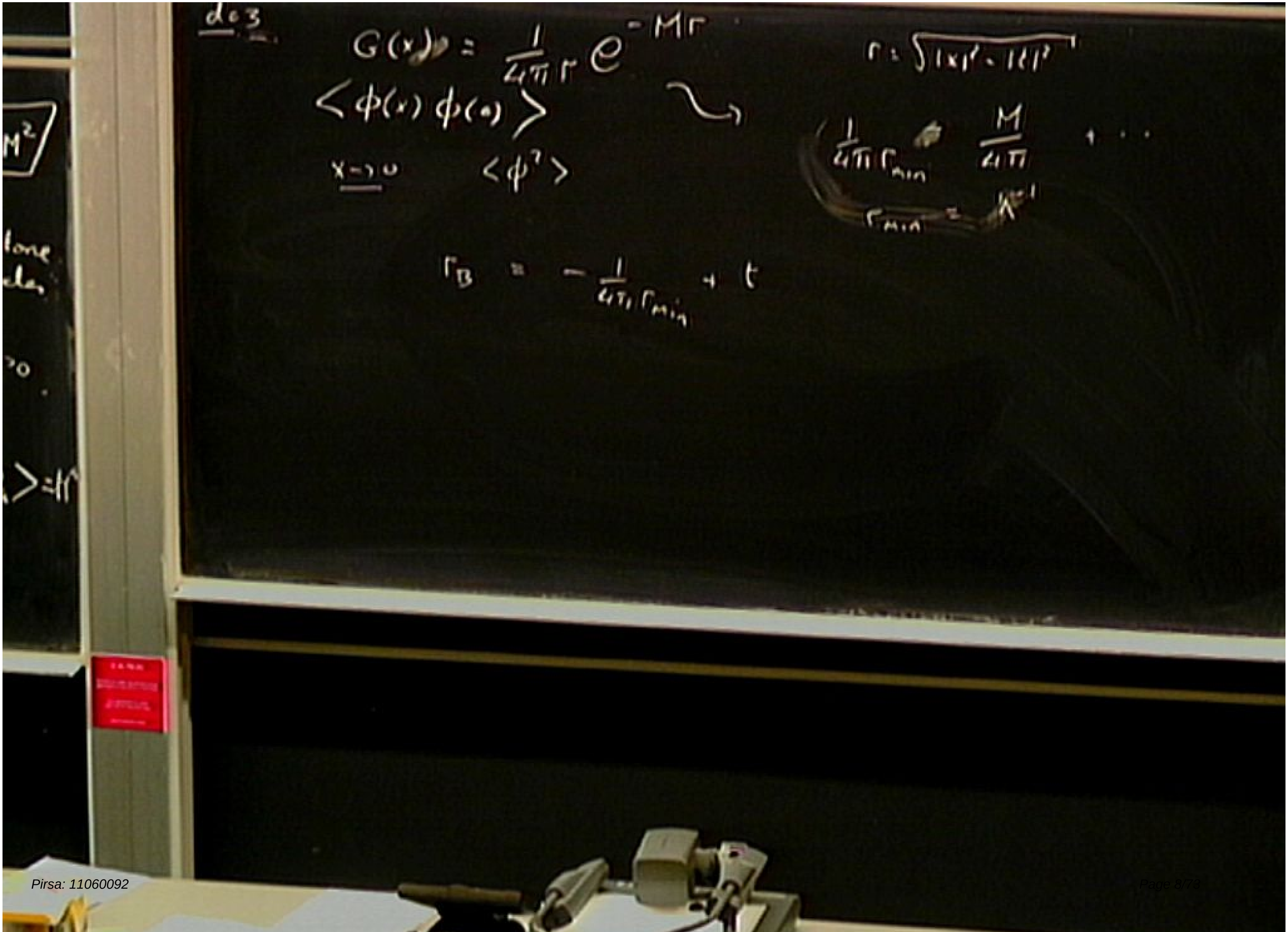
$$(\Box + M^2)\varphi^a = H^a . \quad \Leftrightarrow \quad \langle \Box\varphi^a + r_B\varphi^a + g_B\varphi^2\varphi_a\rangle = H^a$$

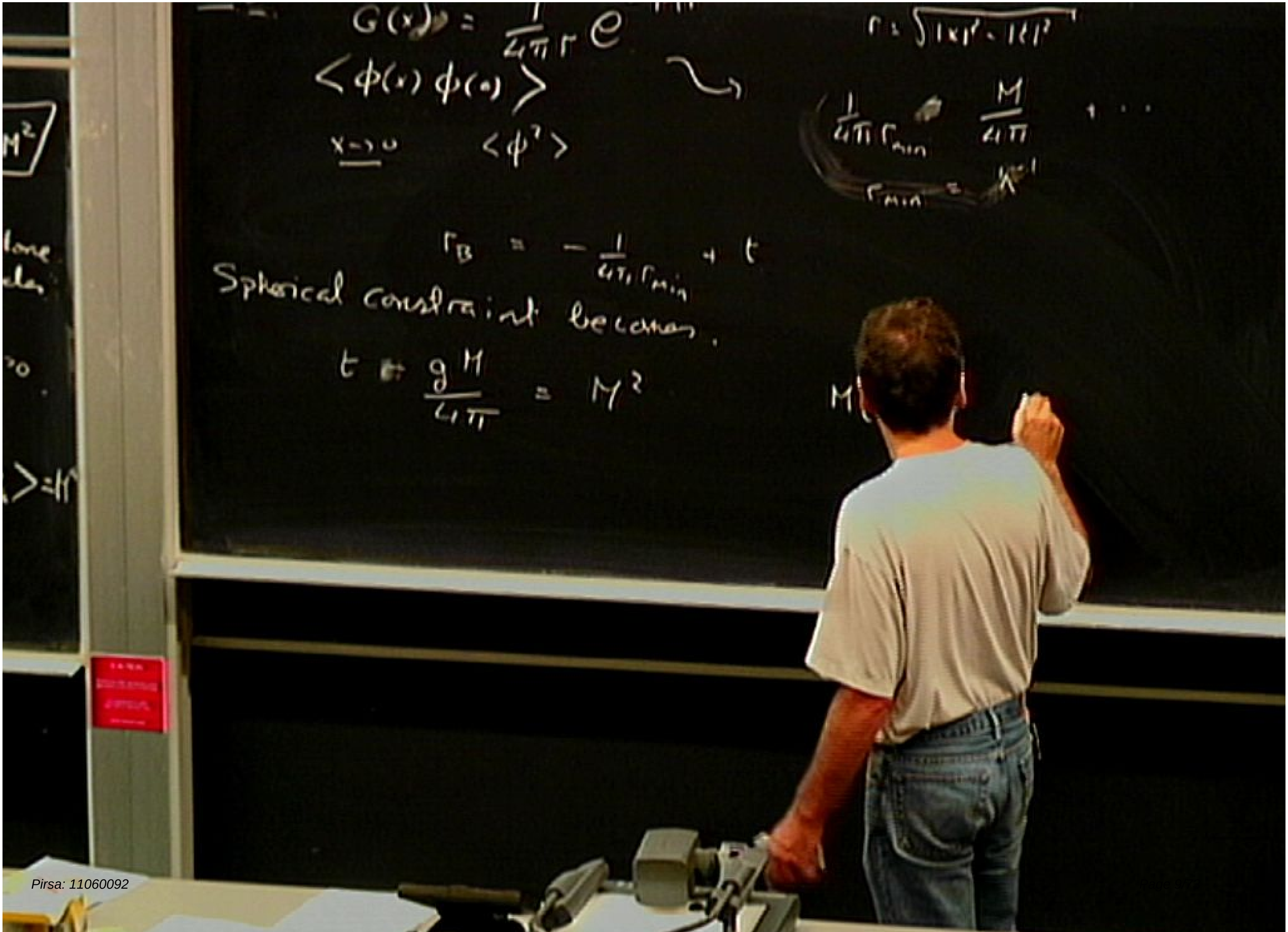

The spherical constraint (in R^4)
M^2 = r_B + g_B [Ω(M) + g_R φ^a φ^a]
G^{-1} = p^2 + M^2
gives a self-consistent eqn for M . the mass of Goldstone particles
<φ^a> = ?
const external field H^a and lim H^a -> 0
<φ^a>(H) -> H->0 φ̄^a
(□ + M^2) φ^a = H^a = 0 => φ^a = 0 or M^2 = 0.
New phase occurs at M^2 = 0.
< □φ^a + r_B φ^a + g_B φ^2 φ_a > = H^a

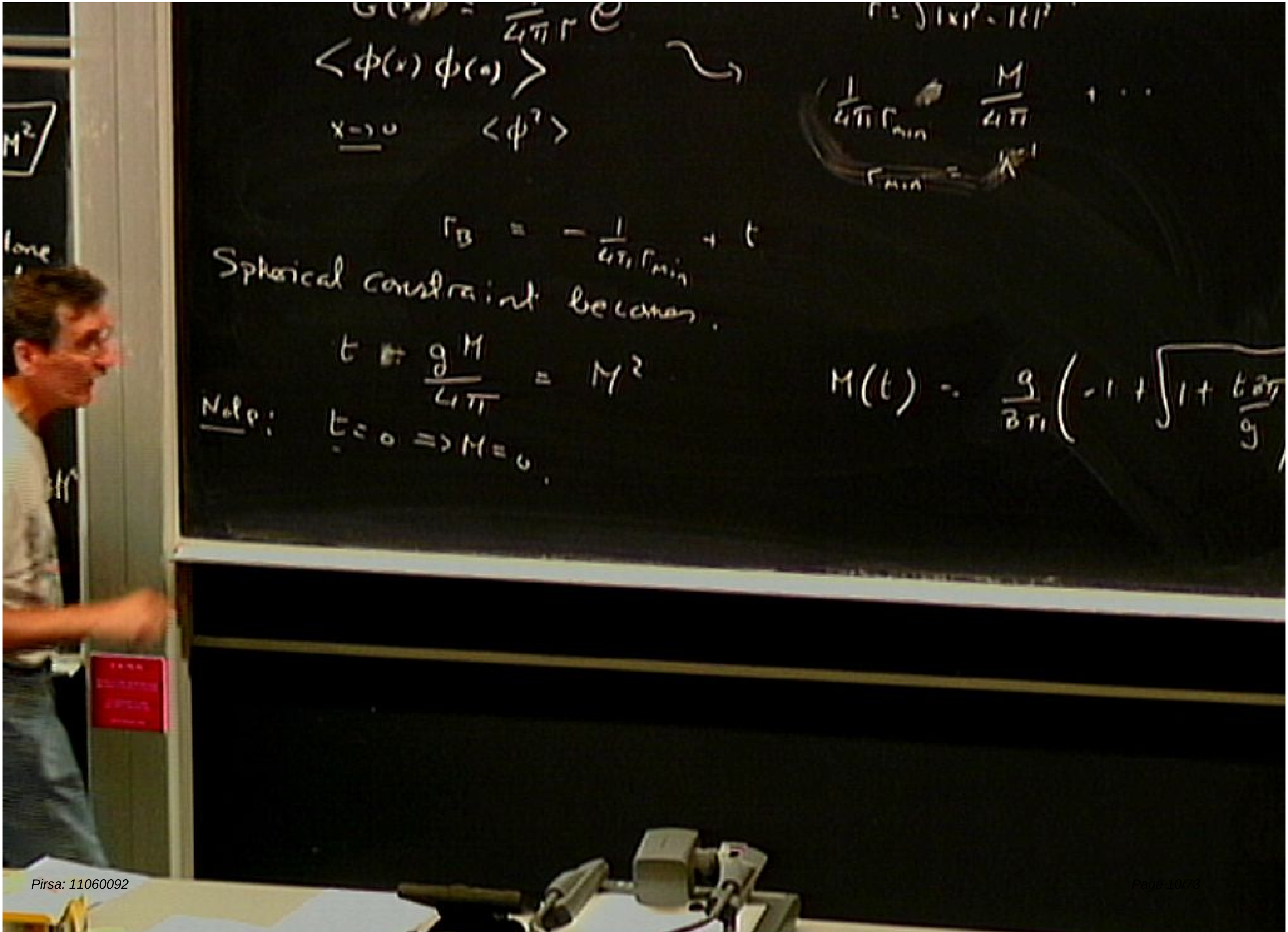

Large N limit of O(N) vector model.
$\phi^a(x)$ $a = 1, \ldots, N$
Look at $\frac{\lambda}{4!}(\phi^a\phi^a)^2$ — Potential
$S = N\int d^dx \sqrt{-g} \left[\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi \right.$
$\left. + \frac{r_0}{2}\varphi^2 + \frac{\lambda_0 N}{4!}(\varphi^2)^2 \right]$
$S[\varphi] = S_2[\varphi] + \lambda S_4[\varphi]$
$= N\left[S_2\left[\frac{\varphi}{\sqrt{N}}\right] + (\lambda N)\, S_4\left[\frac{\varphi}{\sqrt{N}}\right] \right]$
$\tilde{\varphi}$
$r_0 \in \mathbb{R}$, $\lambda_0 N > 0$
λN = fixed
't Hooft Coupling
Exactly solvable on Lattice, in any d
and has a proper
diagram.
2^{nd} order
transition
≡ Universal
of
Berlin
Spherical
Model.


Berlin Kac
Spherical Model
d=3
G(x) = 1/(4πr) e^{-Mr}
r = √(|x|² - |t|²)

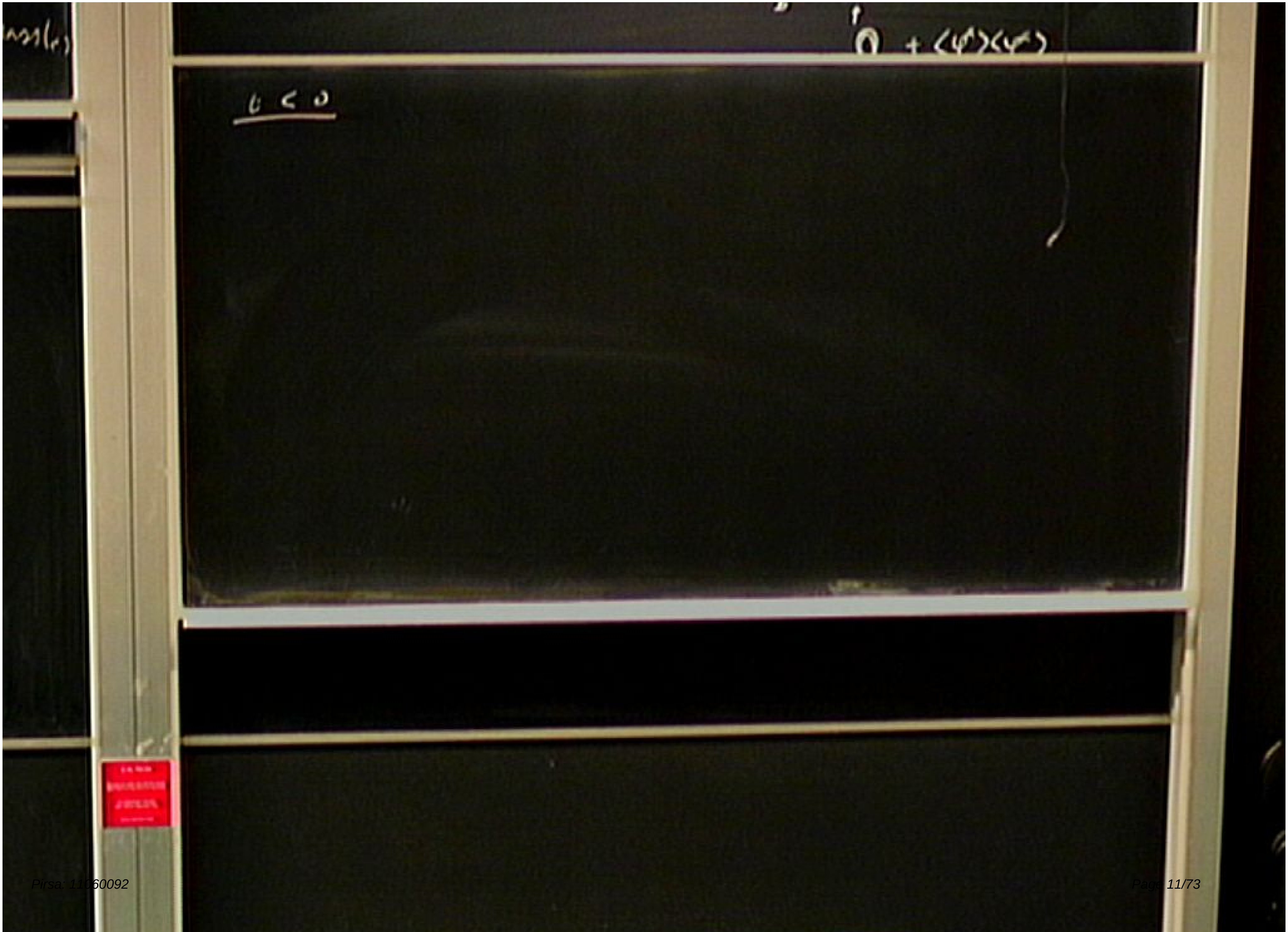

d=3
G(x) = 1/(4π r) e^{-Mr}
r = √(|x|² - |t|²)
<φ(x) φ(0)>

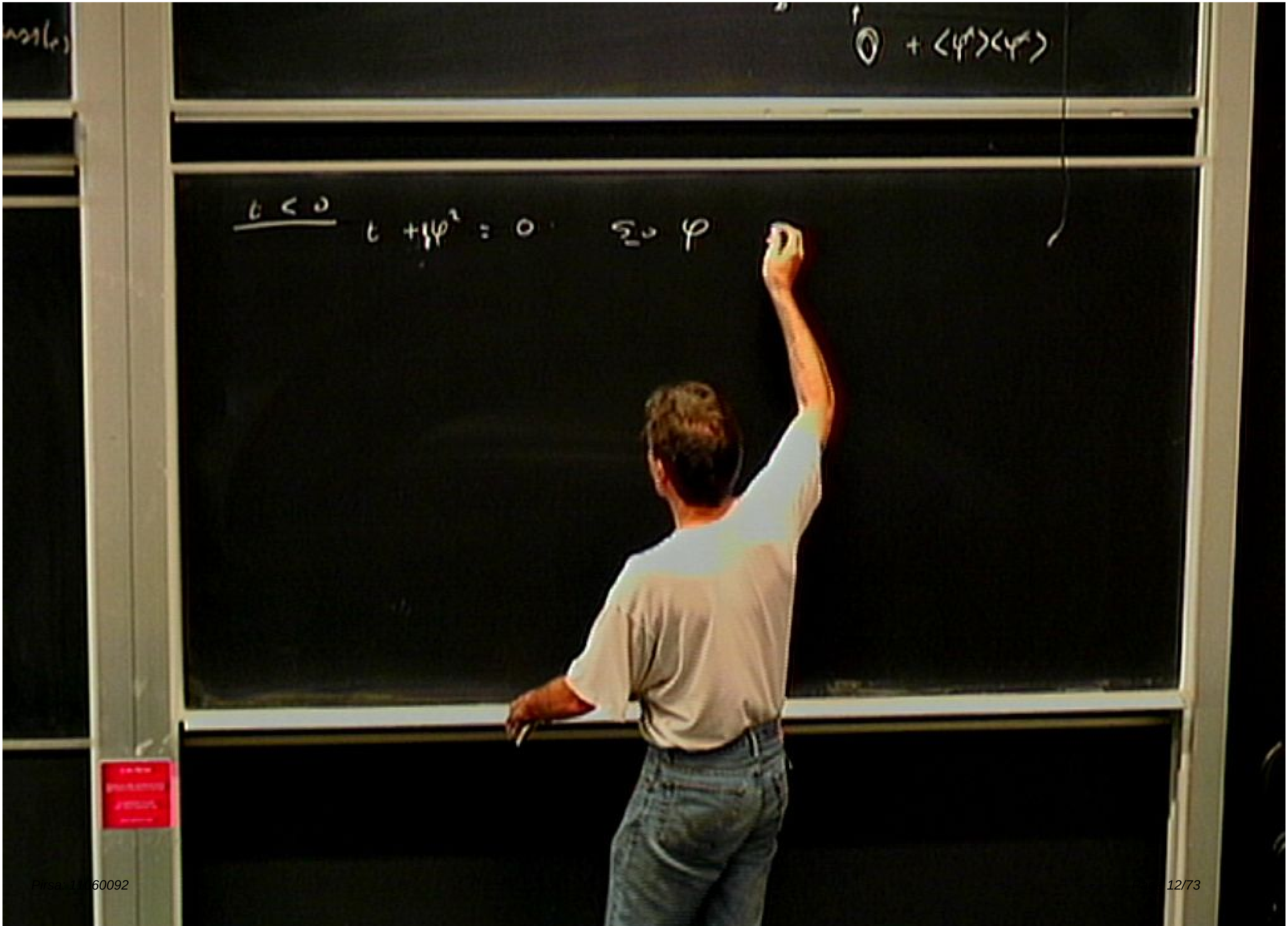

G(x) = 1/(4π r) e^{-Mr}
r = √(|x|² + |t|²)
⟨φ(x) φ(0)⟩
x → 0 ⟨φ²⟩
1/(4π r_min) + M/(4π) + . . .
r_min = Λ^{-1}
r_B = − 1/(4π r_min) + t

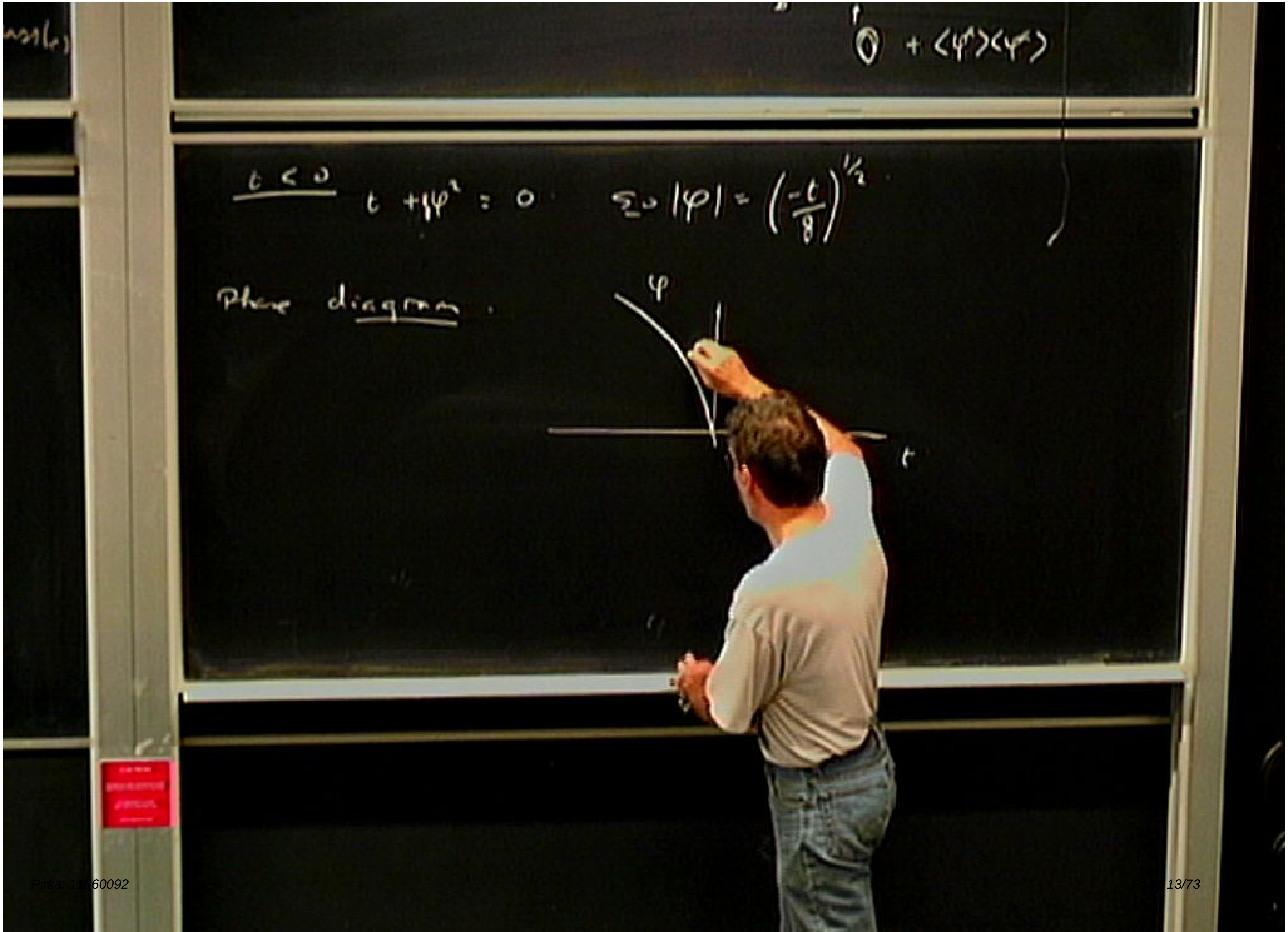

Spherical constraint becomes.

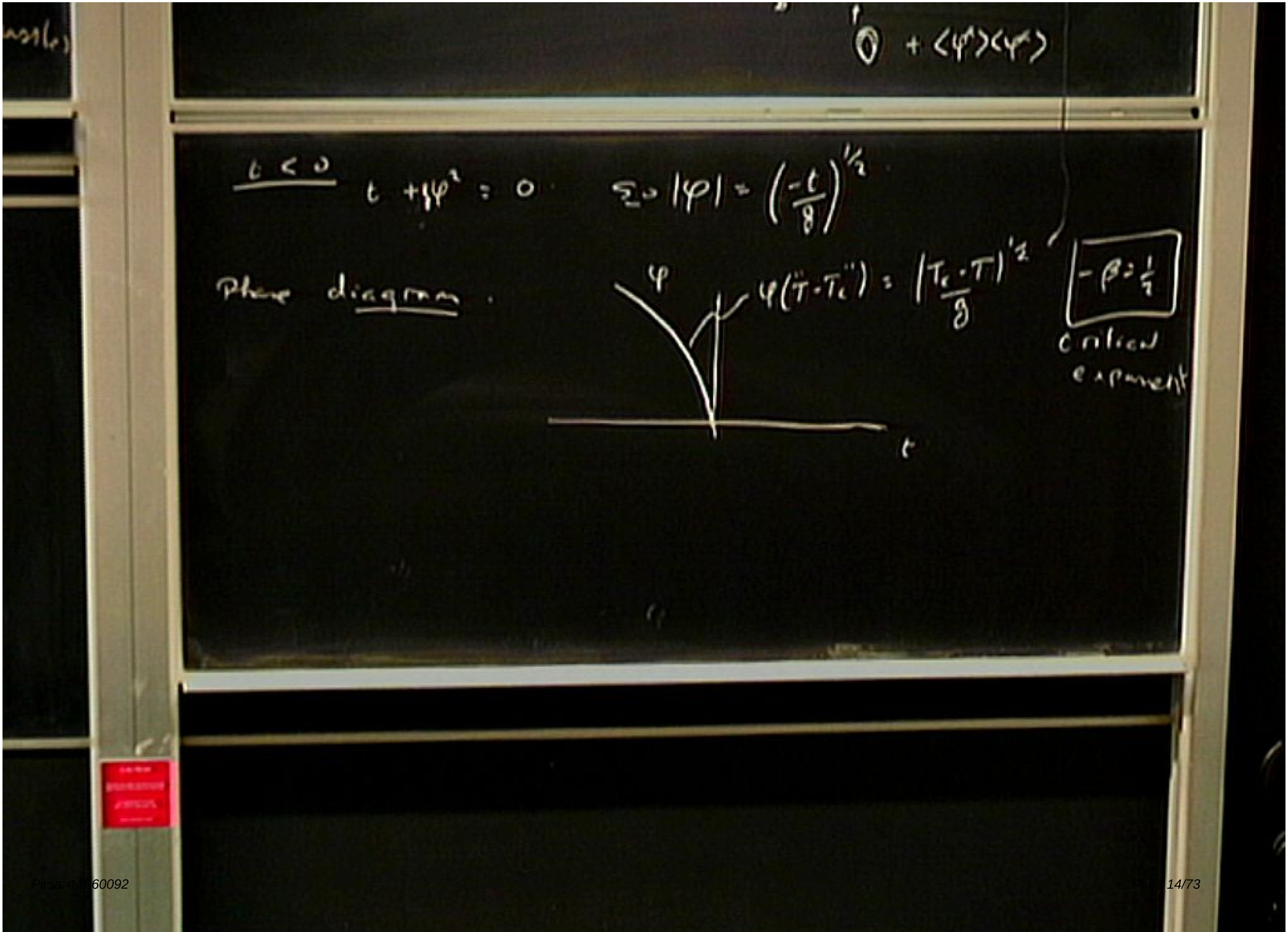


<φ(x) φ(0)>
x → 0 <φ²>
r_B = − 1/(4π r_min) + t
Spherical constraint becomes.
t = gM/4π = M²
Note: t = 0 ⇒ M = 0.
M(t) = g/8π (−1 + √(1 + t8π/g


t < 0


t < 0


t < 0
t + gφ² = 0
Phase diagram
φ
t


+ ⟨φ²⟩⟨φ²⟩
t < 0
t + gφ² = 0
|φ| = (−t/g)^{1/2}
Phase diagram.
φ
φ(T = T_c) = ((T_c − T)/g)^{1/2}
β = 1/2
critical exponent
t

$$g \to \infty$$

a universal answer

$$M(t) = \frac{g}{8\pi}\left(-1 + \sqrt{1 + \frac{t\,8\pi}{g}}\right)$$

Parameter value for a mass(r_0) Particle.

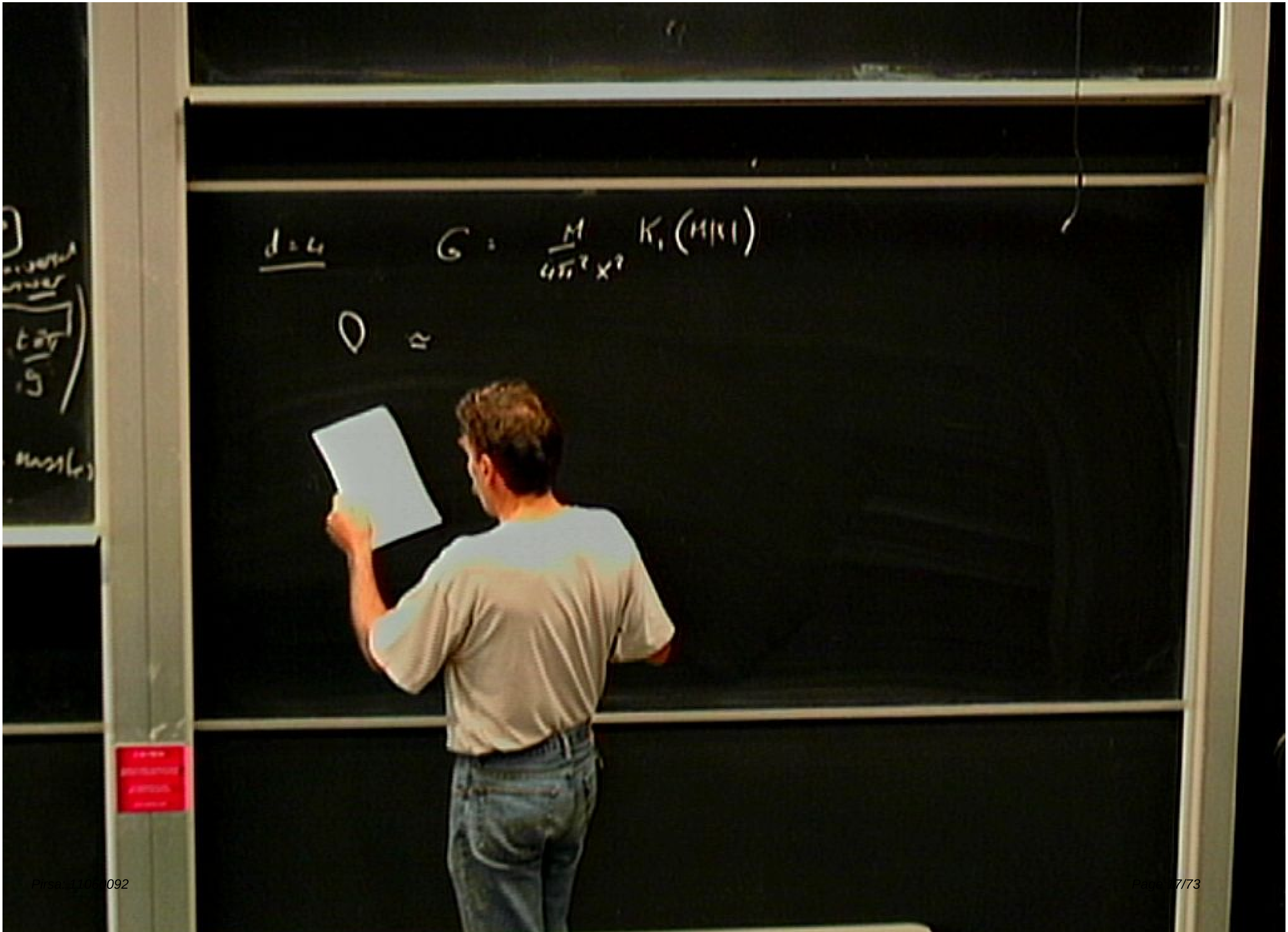
min

$$g \to \infty$$

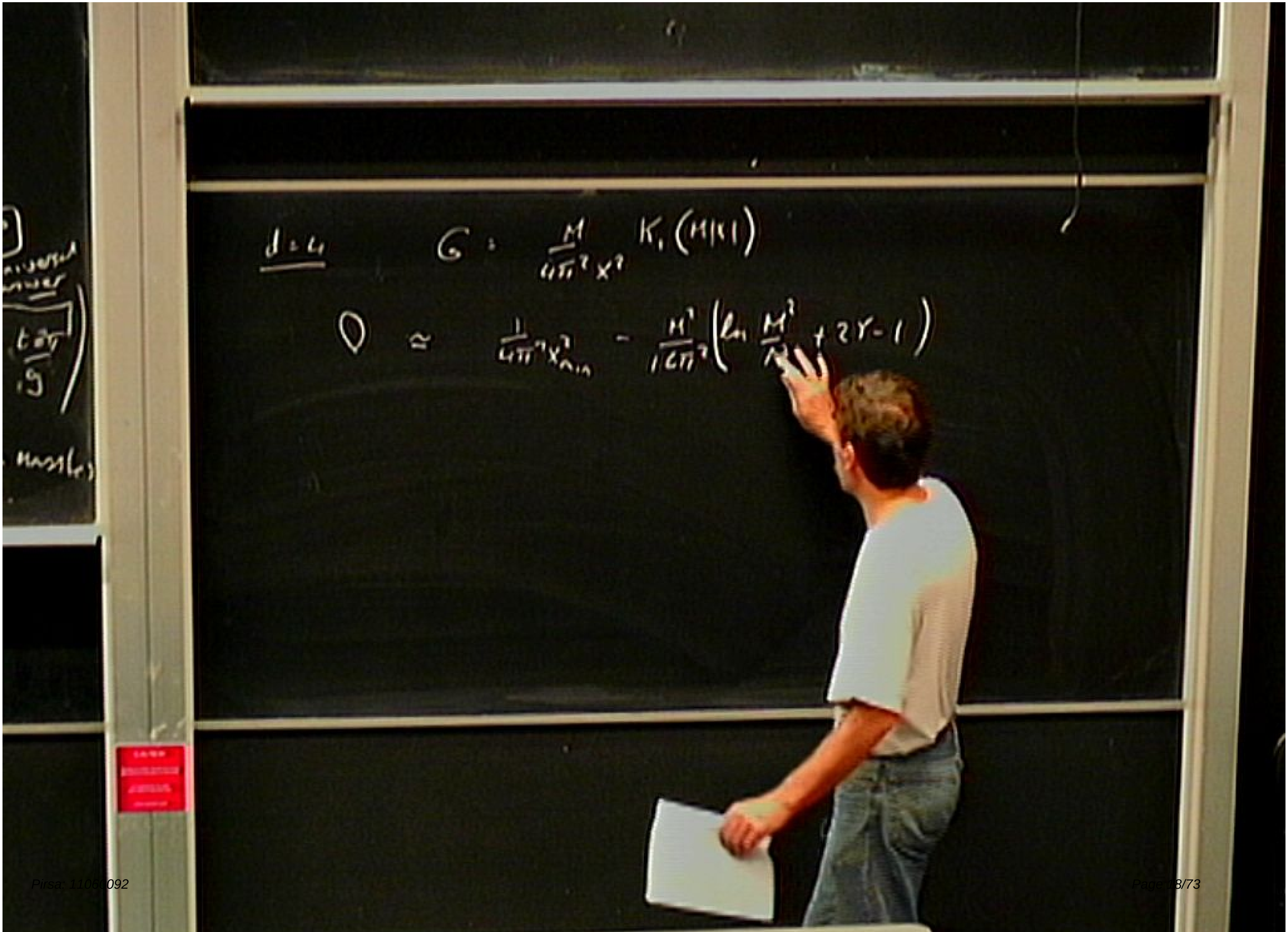
a universal answer

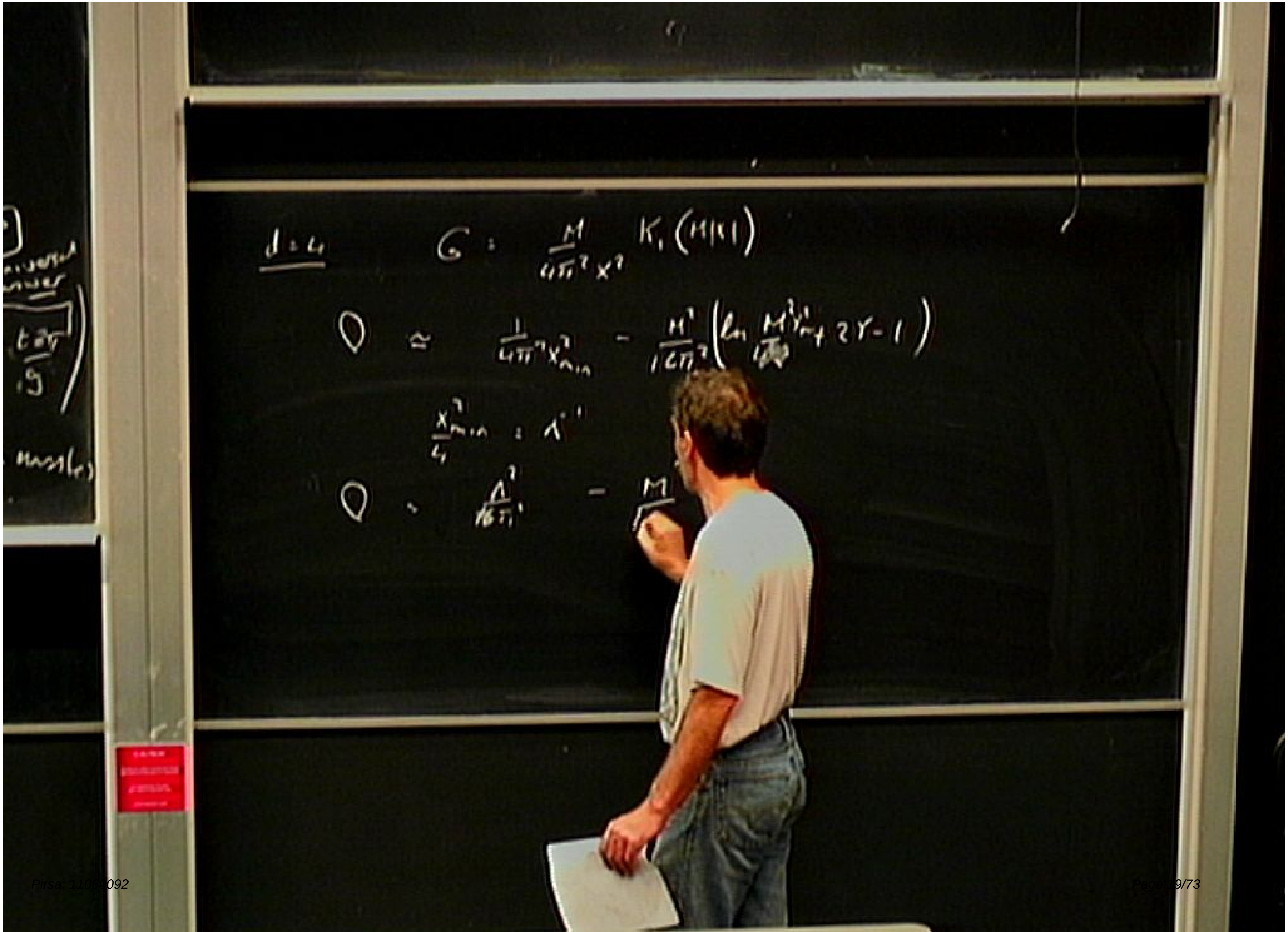
$$M(t) = \frac{g}{8\pi}\left(-1 + \sqrt{1 + \frac{t\,8\pi}{g}}\right)$$

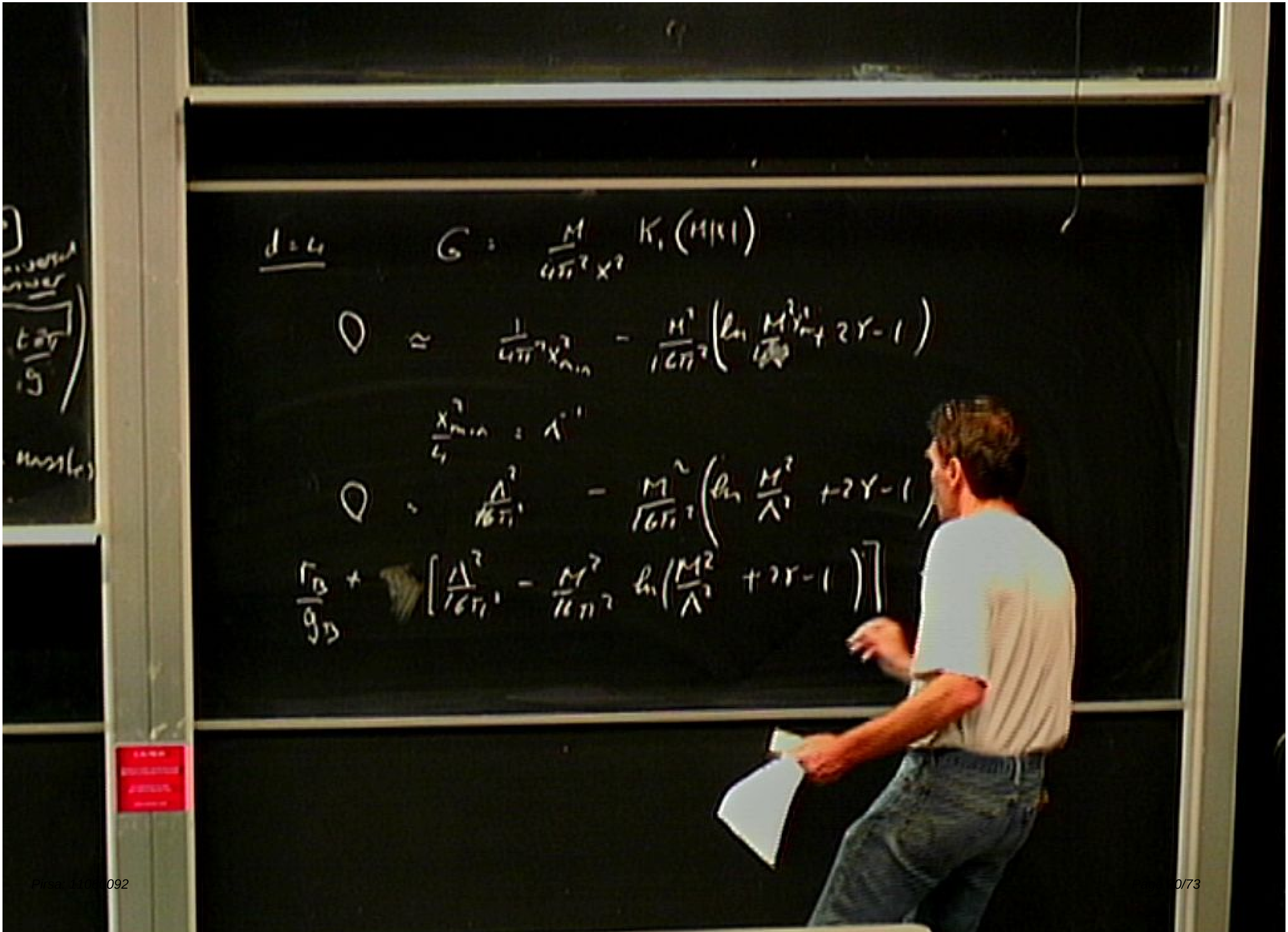
e Parameter value for a mass(+) particle.

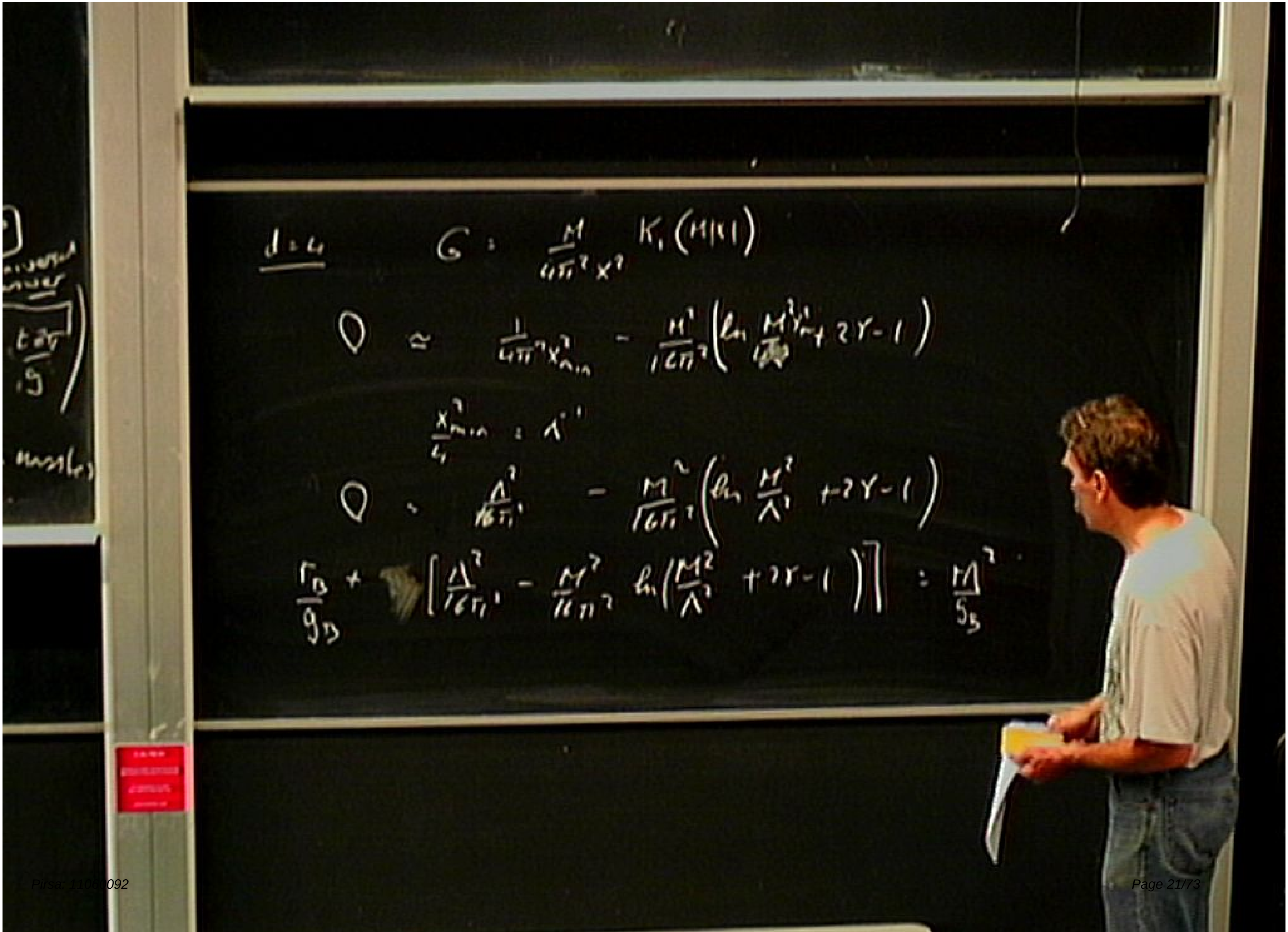


d = 4
G = M/(4π² x²) K₁(M|x|)


d = 4
G = M / (4π² x²) K₁(M|x|)
≃ 1 / (4π² x²_min) − M² / (16π²) (ln M²/Λ² + 2γ − 1)


d = 4
G = M/(4π² x²) K₁(M|x|)
x²min/4 = Λ⁻²


d = 4


d = 4
G = M/(4π² x²) K₁(M|x|)

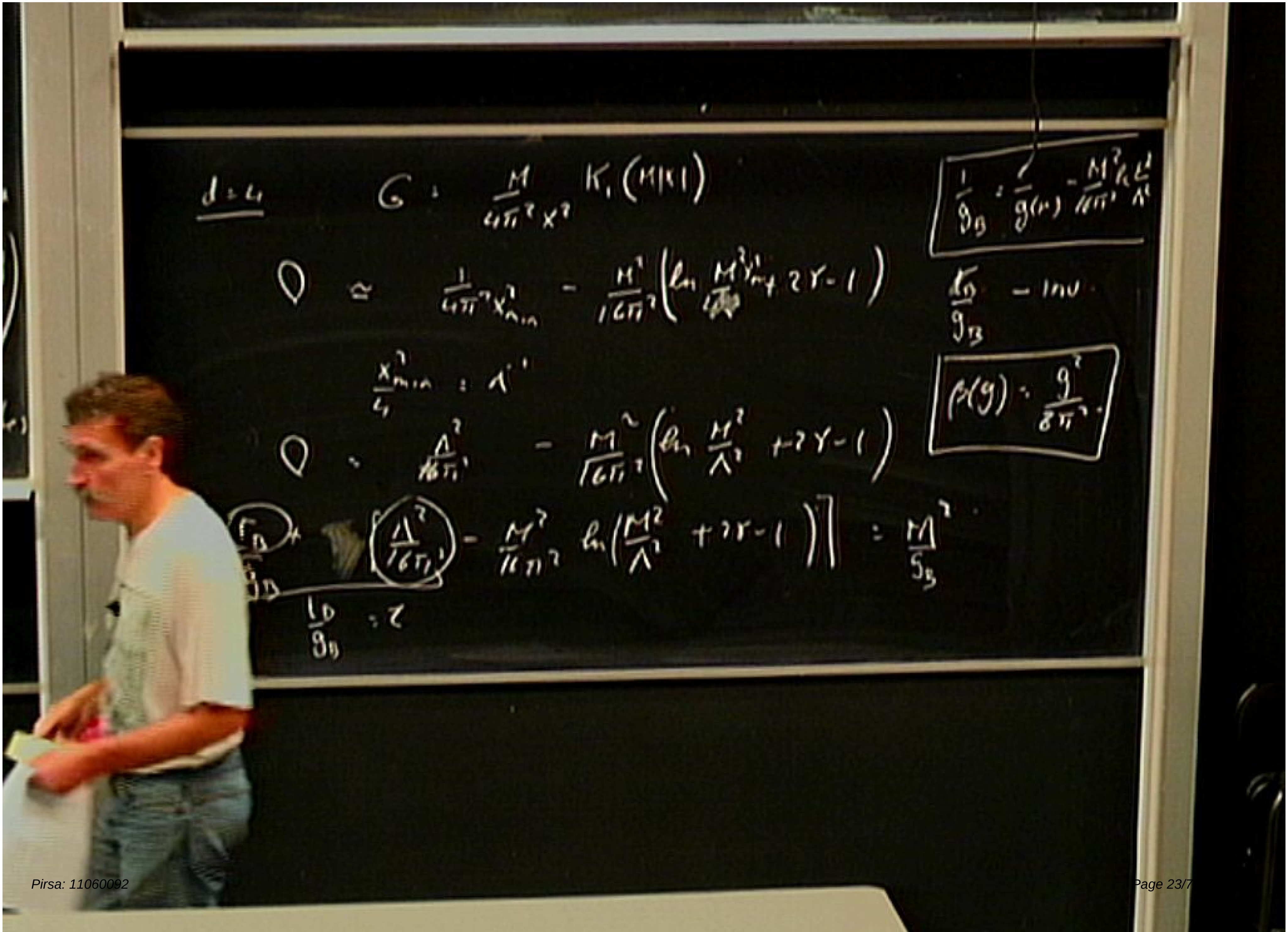
$1)$

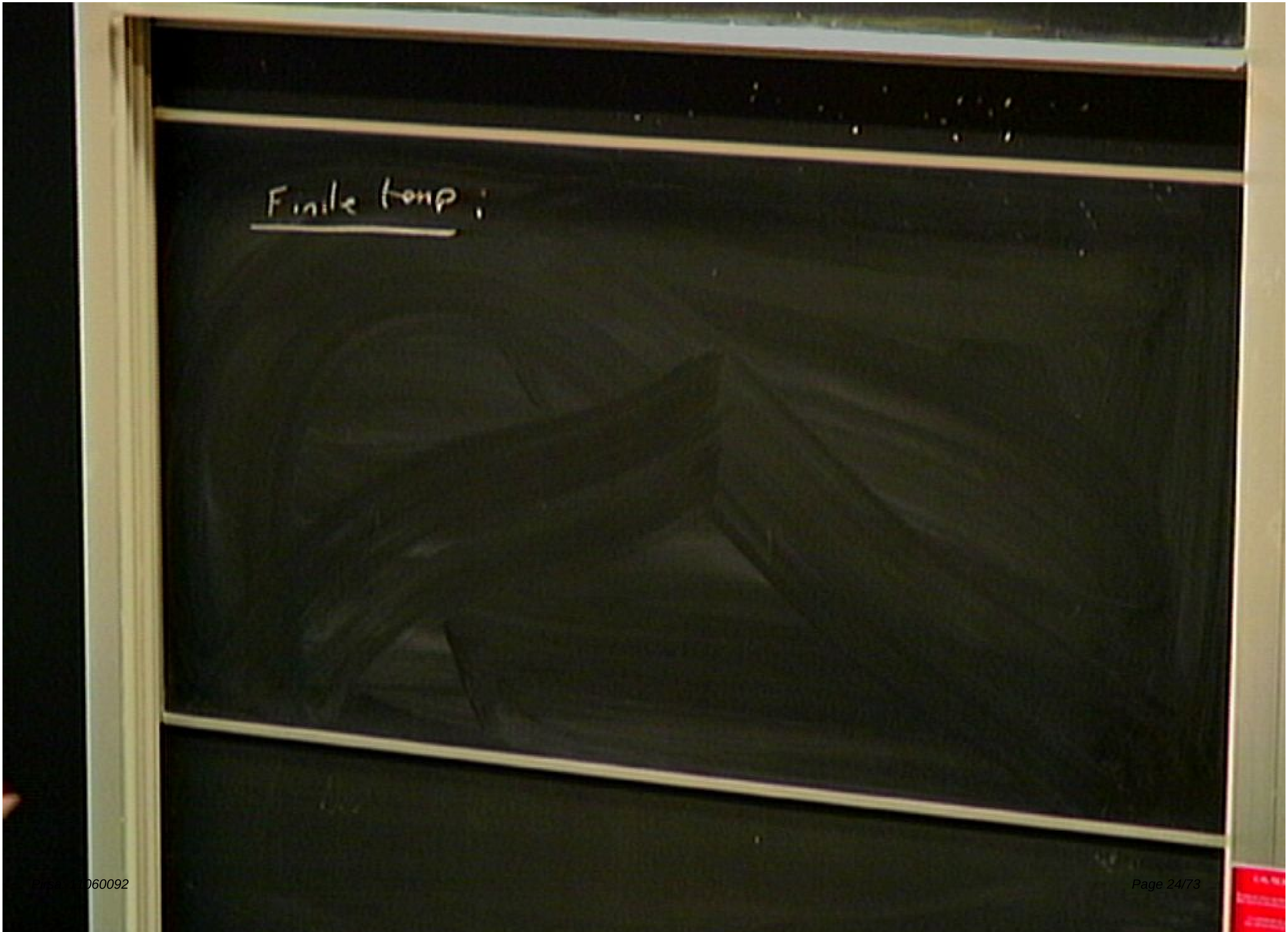
$$\frac{1}{g_B} = \frac{1}{g(\mu)} - \frac{M^2}{16\pi^2} \ln \frac{\mu^2}{\Lambda^2}$$

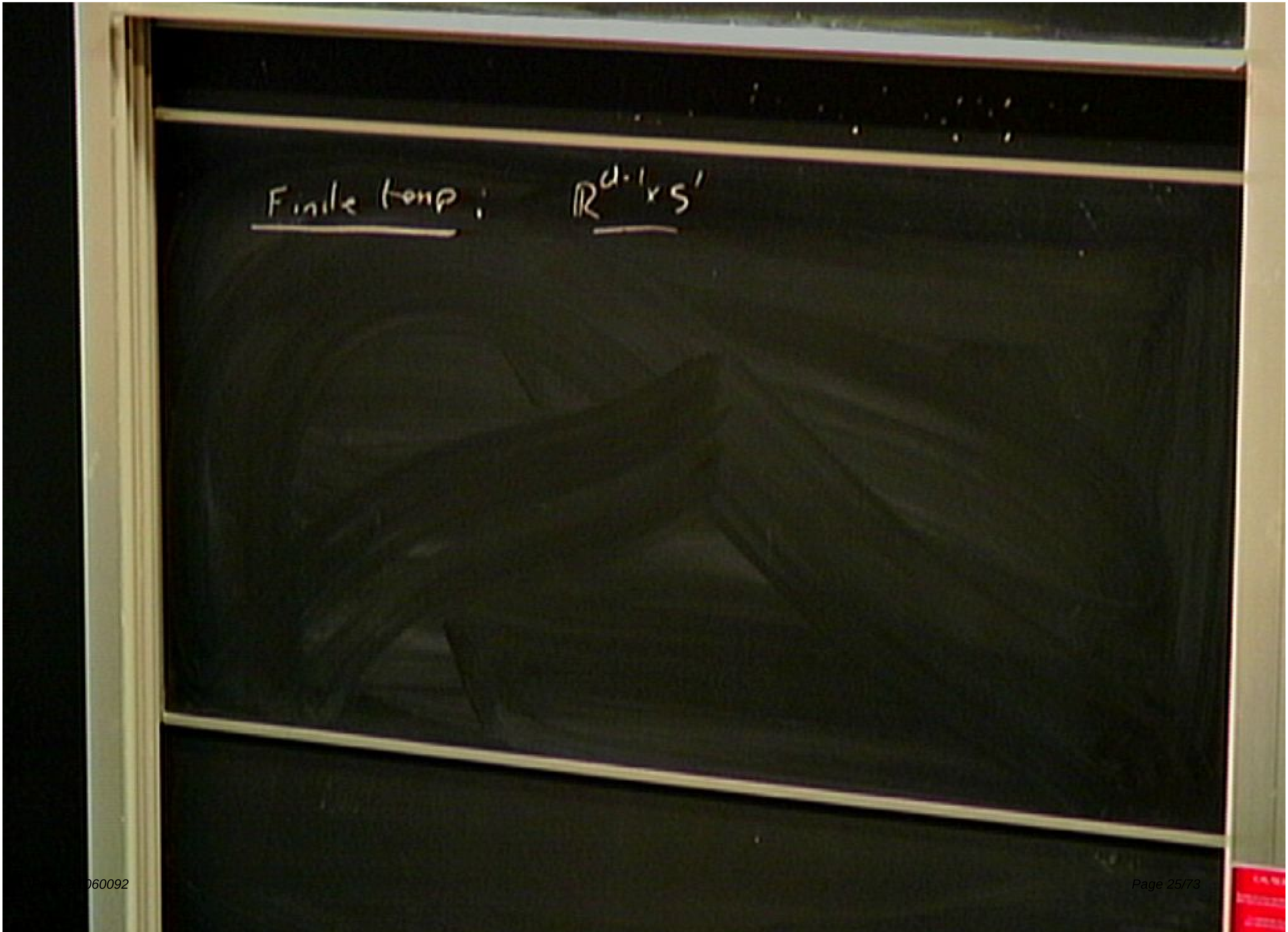
$\ln \frac{M^2 x_{min}^2}{4} + 2\gamma - 1)$

$\frac{t_B}{g_B} \quad - \text{inv}$

$\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1)$


d = 4
G = M/(4π² x²) K₁(M|x|)
β(g) = g²/(8π²)
x²min/4 = Λ⁻²

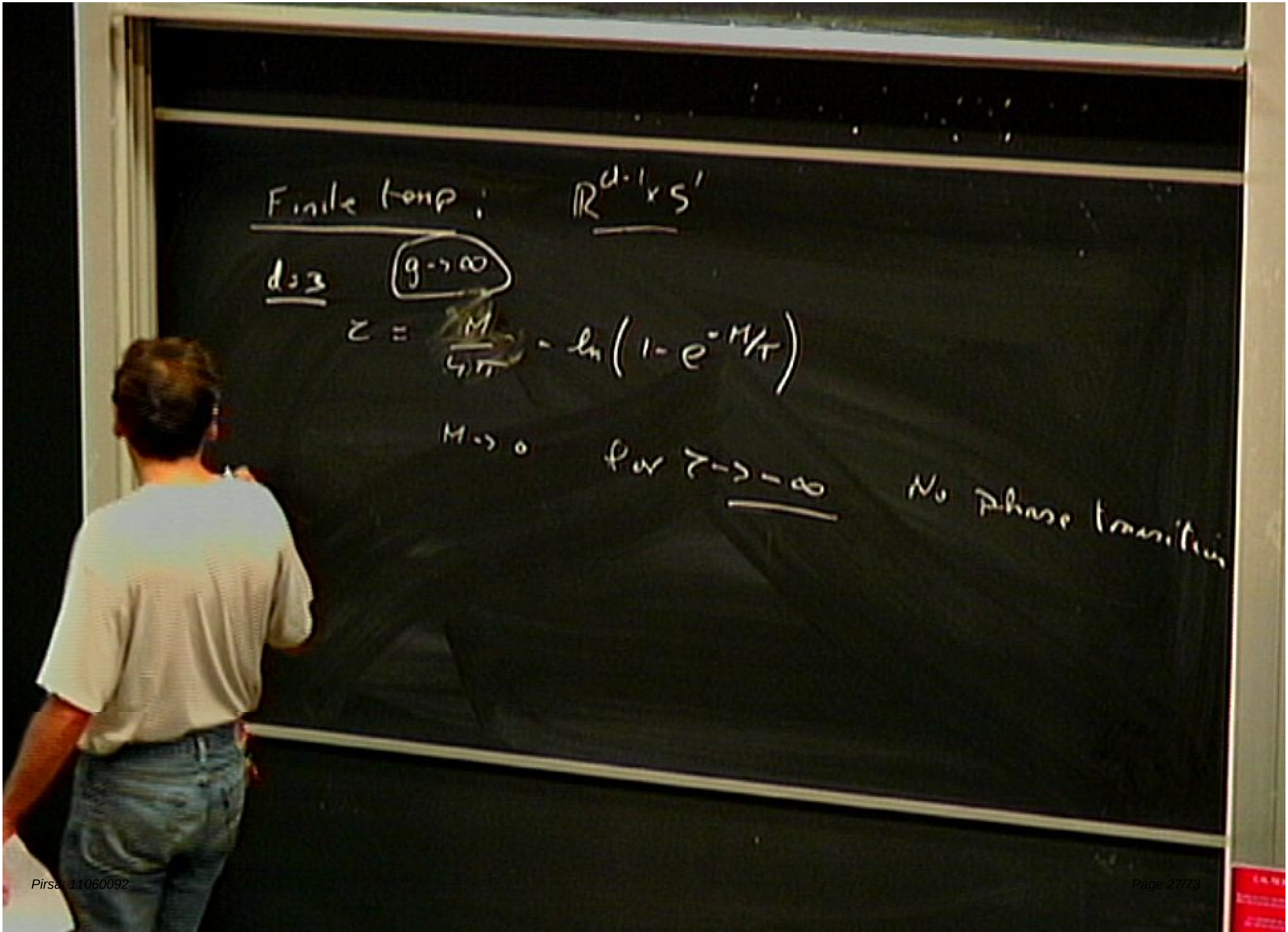

Finite temp :


Finite temp :
$\mathbb{R}^{d-1} \times S^1$

Finite temp : $\mathbb{R}^{d-1} \times S^1$

d = 3 $g \to \infty$

$$Z = \frac{M}{4\pi} - \ln\left(1 - e^{-M/T}\right)$$


Finite temp: R^{d-1} × S^1
d=3
g→∞
z = 2M/4π − ln(1 − e^{-M/T})
M→0 for z→−∞
No phase transition

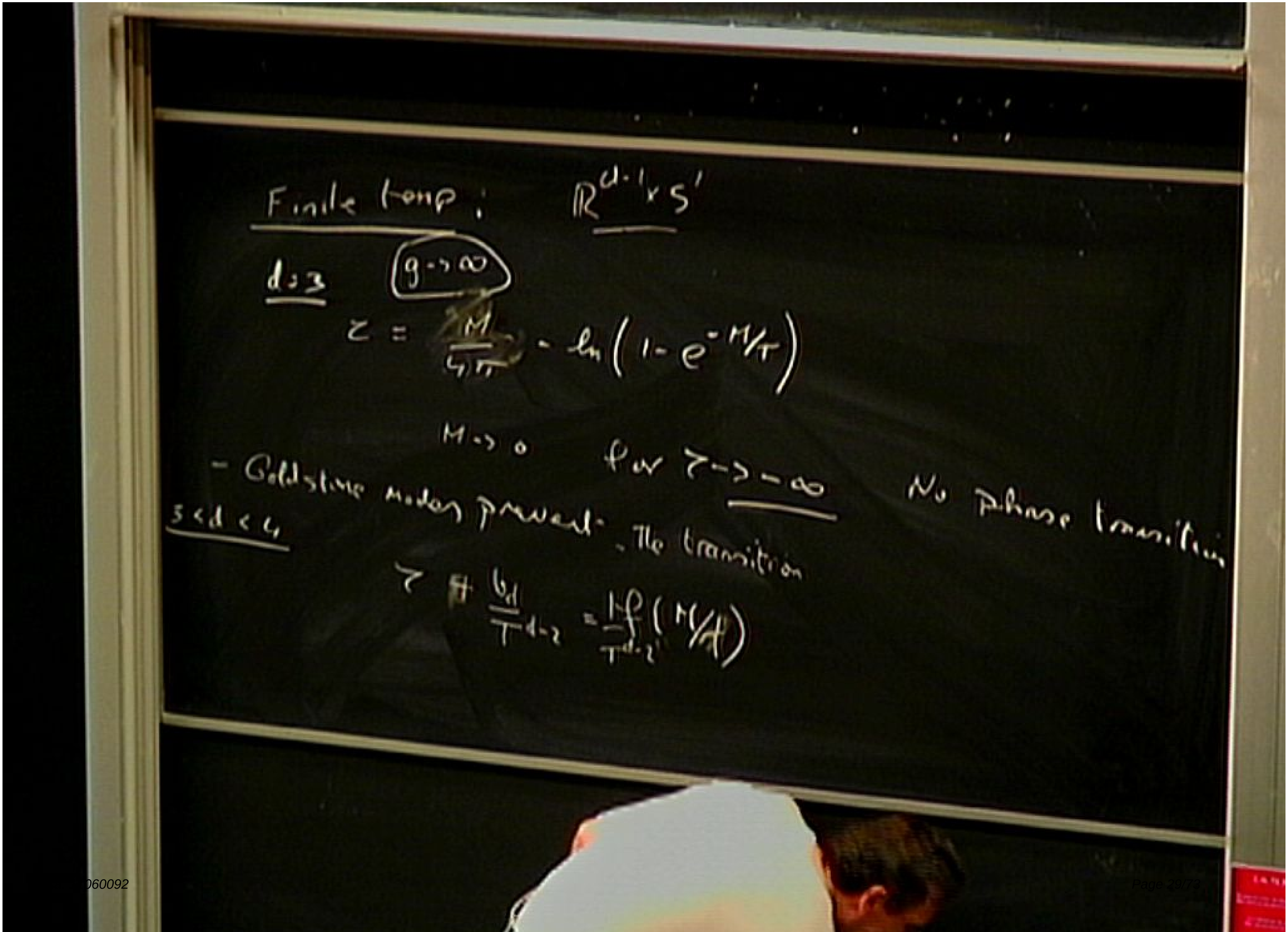
Finite temp: $\mathbb{R}^{d-1}\times S^1$

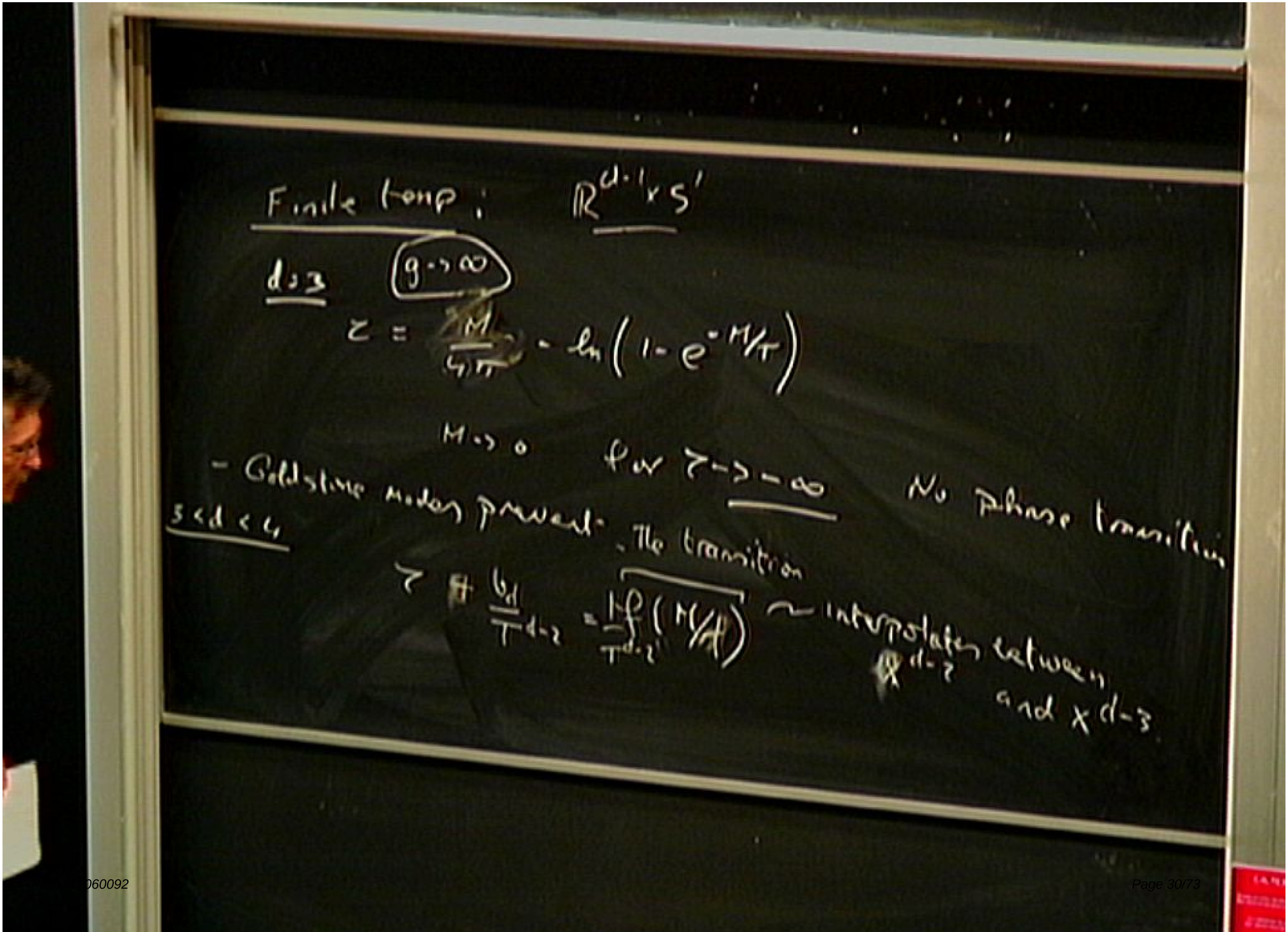
$d=3$ $\quad g\to\infty$

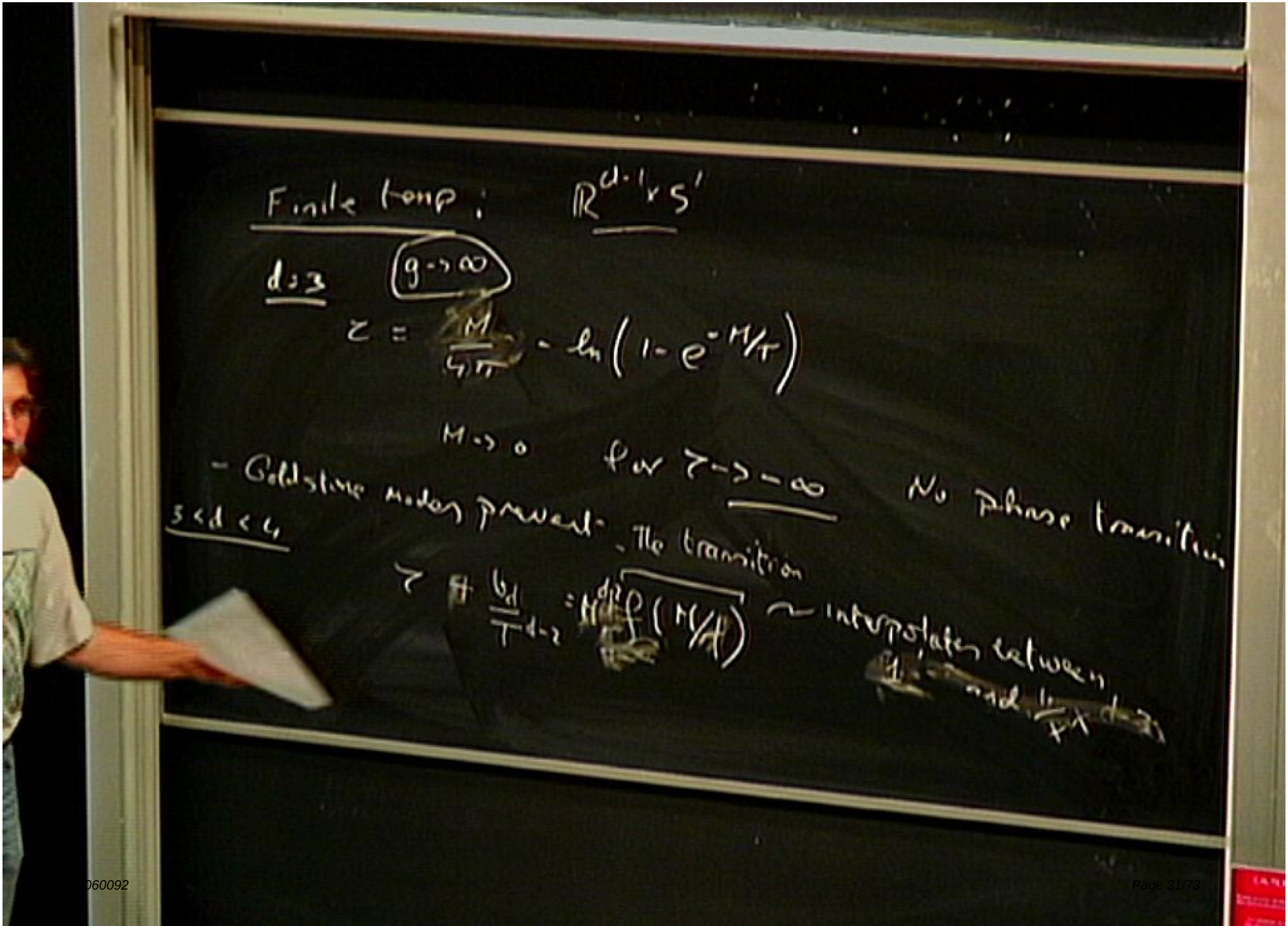
$$Z = \frac{M}{4\pi} - \ln\left(1-e^{-M/T}\right)$$

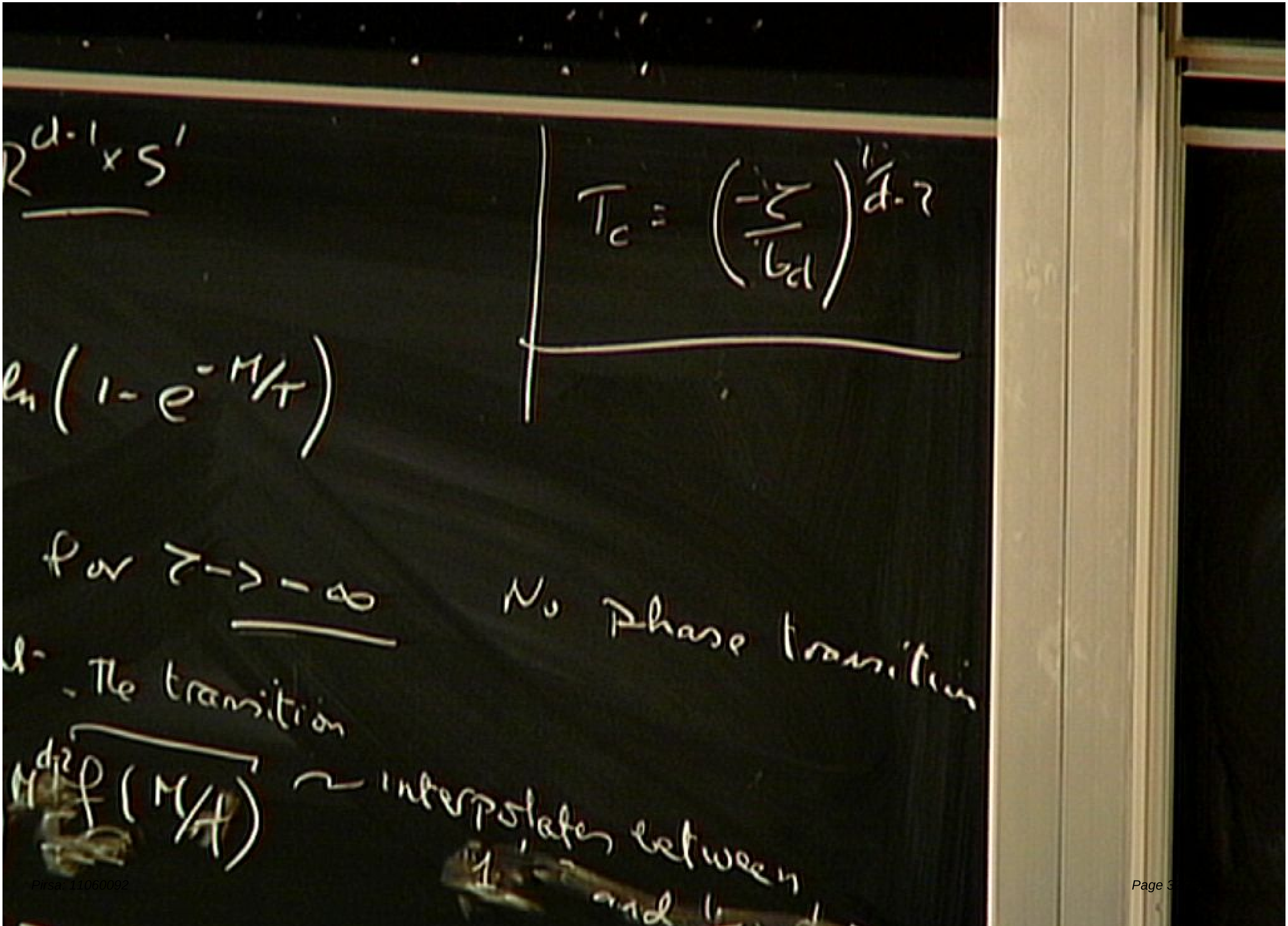
$M\to 0$ for $Z\to-\infty$ No phase transition

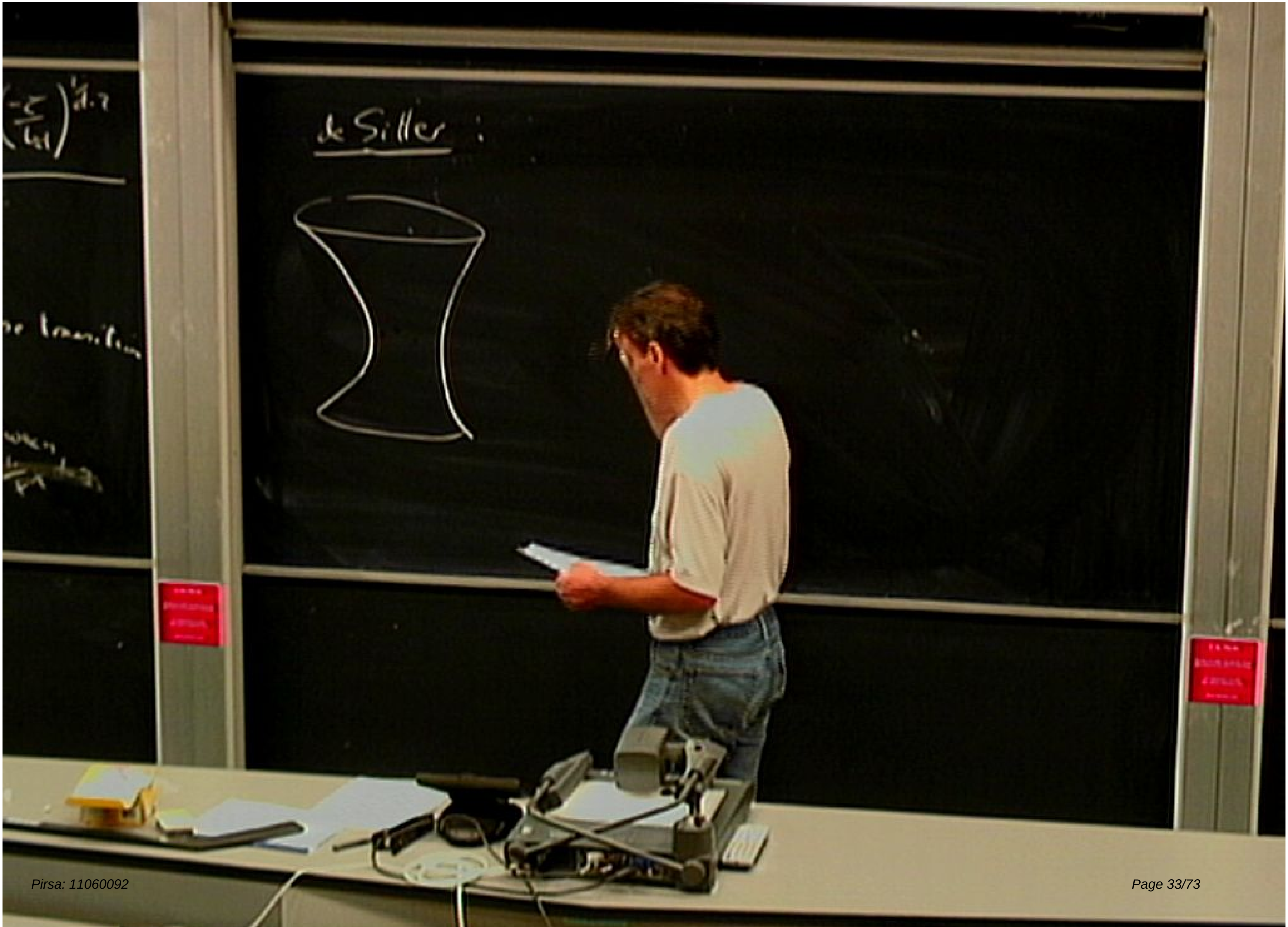
- Goldstone modes prevent the transition


Finite temp: $\mathbb{R}^{d-1}\times S^1$
d=3
g→∞
$\tau = \frac{M}{4\pi} - \ln(1-e^{-M/T})$
M→0 for τ→−∞
No phase transition
− Goldstone modes prevent the transition
3<d<4
$\tau \# \frac{b_d}{T^{d-2}} = \frac{M}{T^{d-2}} f(M/T)$

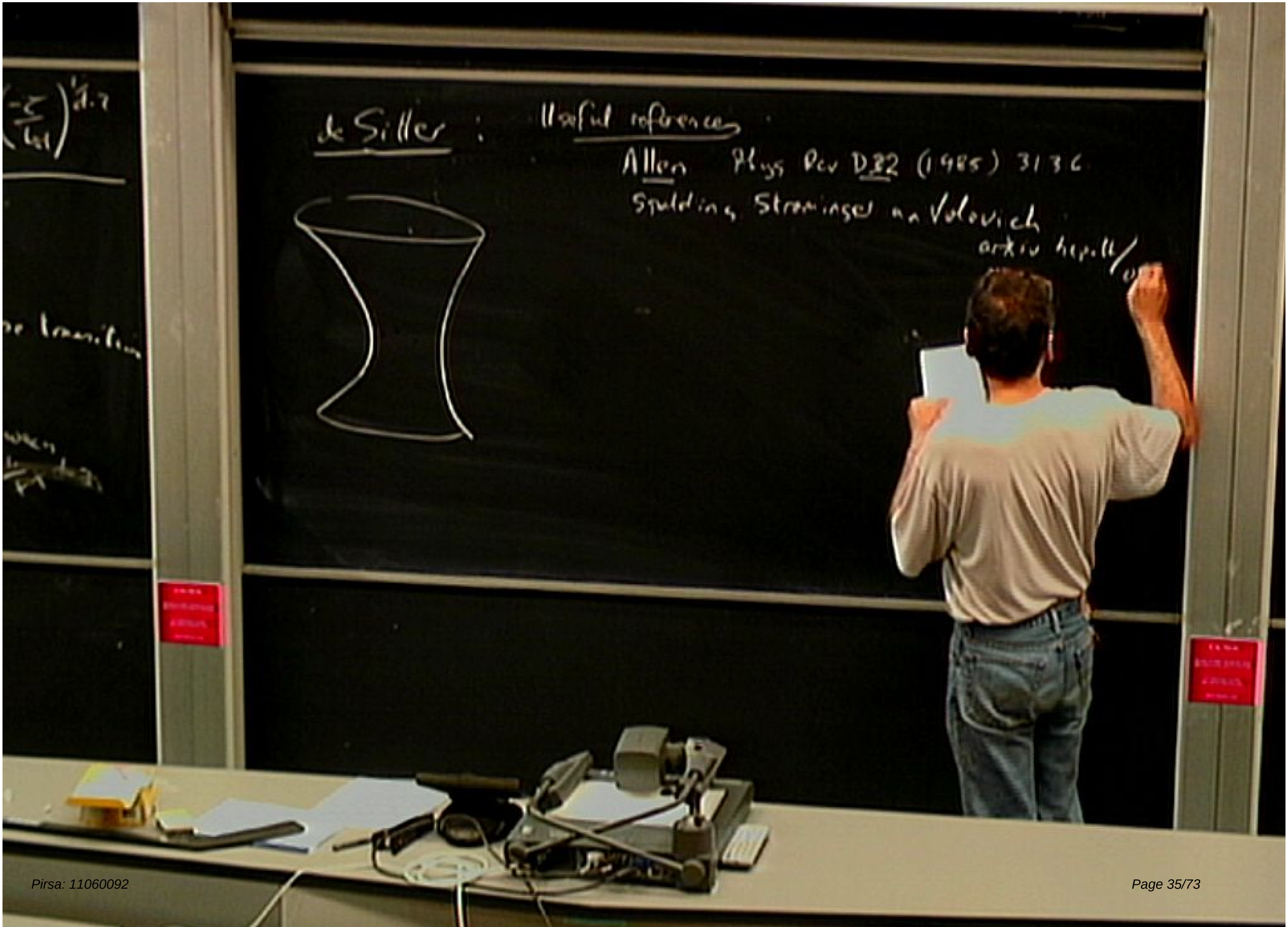

Finite temp: R^{d-1} × S^1
d=3
g→∞
M→0
No phase transition
- Goldstone modes present. The transition
3<d<4
interpolates between
and

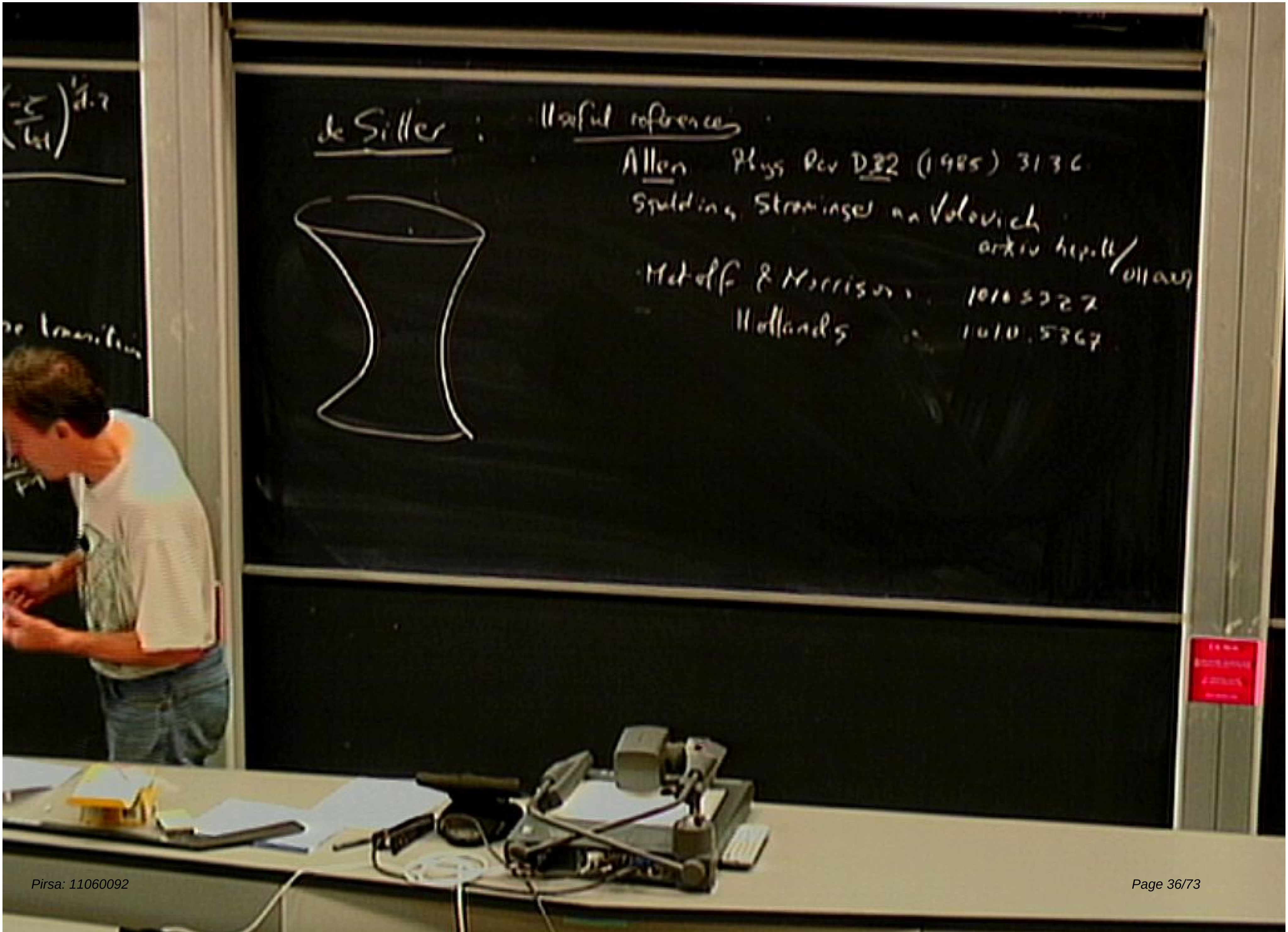

Finite temp: R^{d-1} x S^1
d=3
g→∞
M→0 for z→-∞
- Goldstone modes present
No phase transition
3<d<4
The transition
interpolates between


$R^{d-1} \times S^1$
$T_c = \left(\frac{-\zeta}{b_d}\right)^{1/d-2}$
$\ln(1 - e^{-M/T})$
for $\zeta \to -\infty$
No phase transition
The transition
interpolates between
and


de Sitter :


de Sitter :
Useful references


de Sitter :
Useful references
Allen Phys Rev D32 (1985) 3136
Spradlin, Strominger and Volovich
arXiv hep-th/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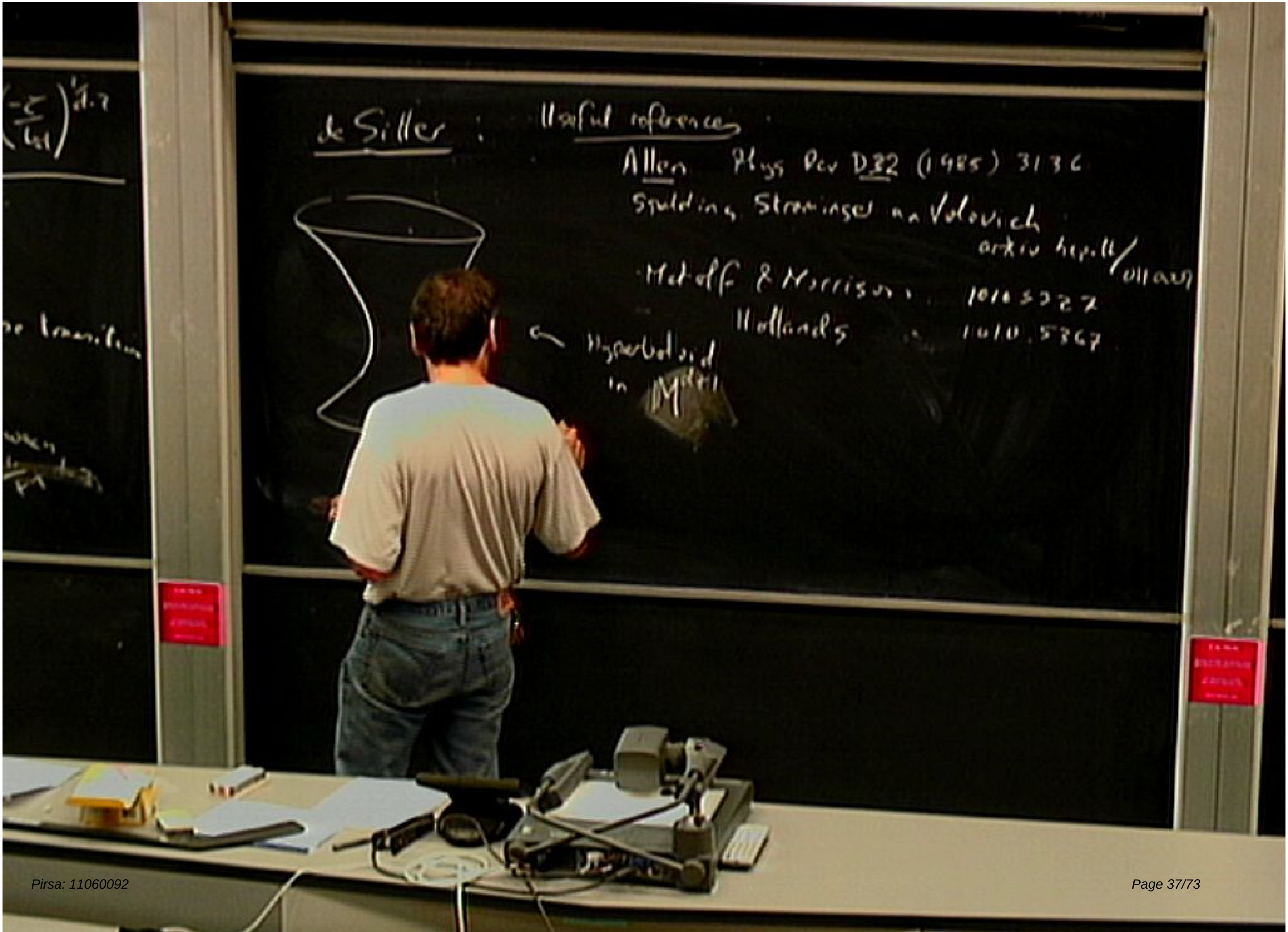
Spradlin, Strominger and Volovich
arXiv hep-th/0110007
Metcalf & Morrison : 1010.5327
Hollands : 1010.5367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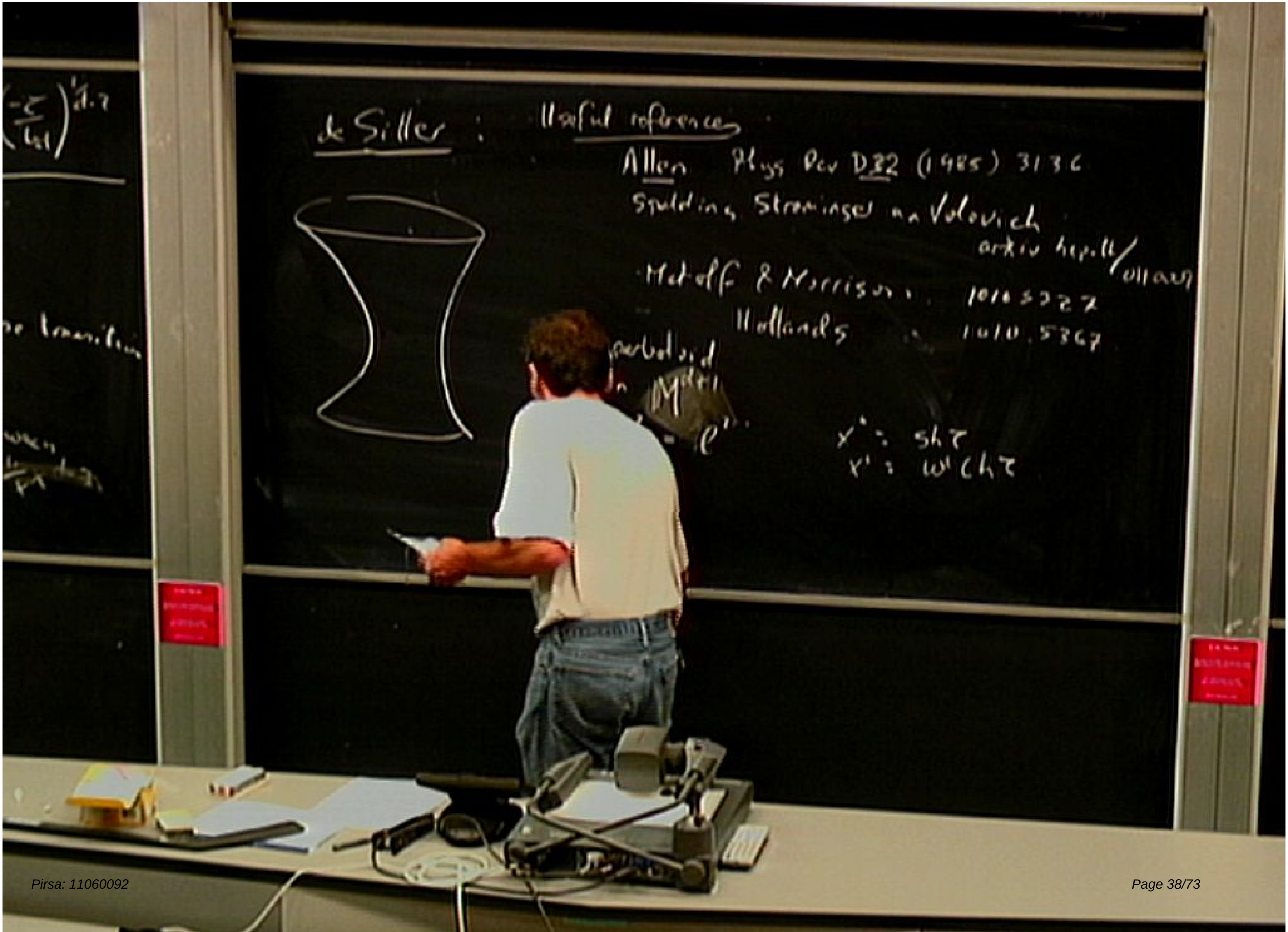
Hyperboloid
in M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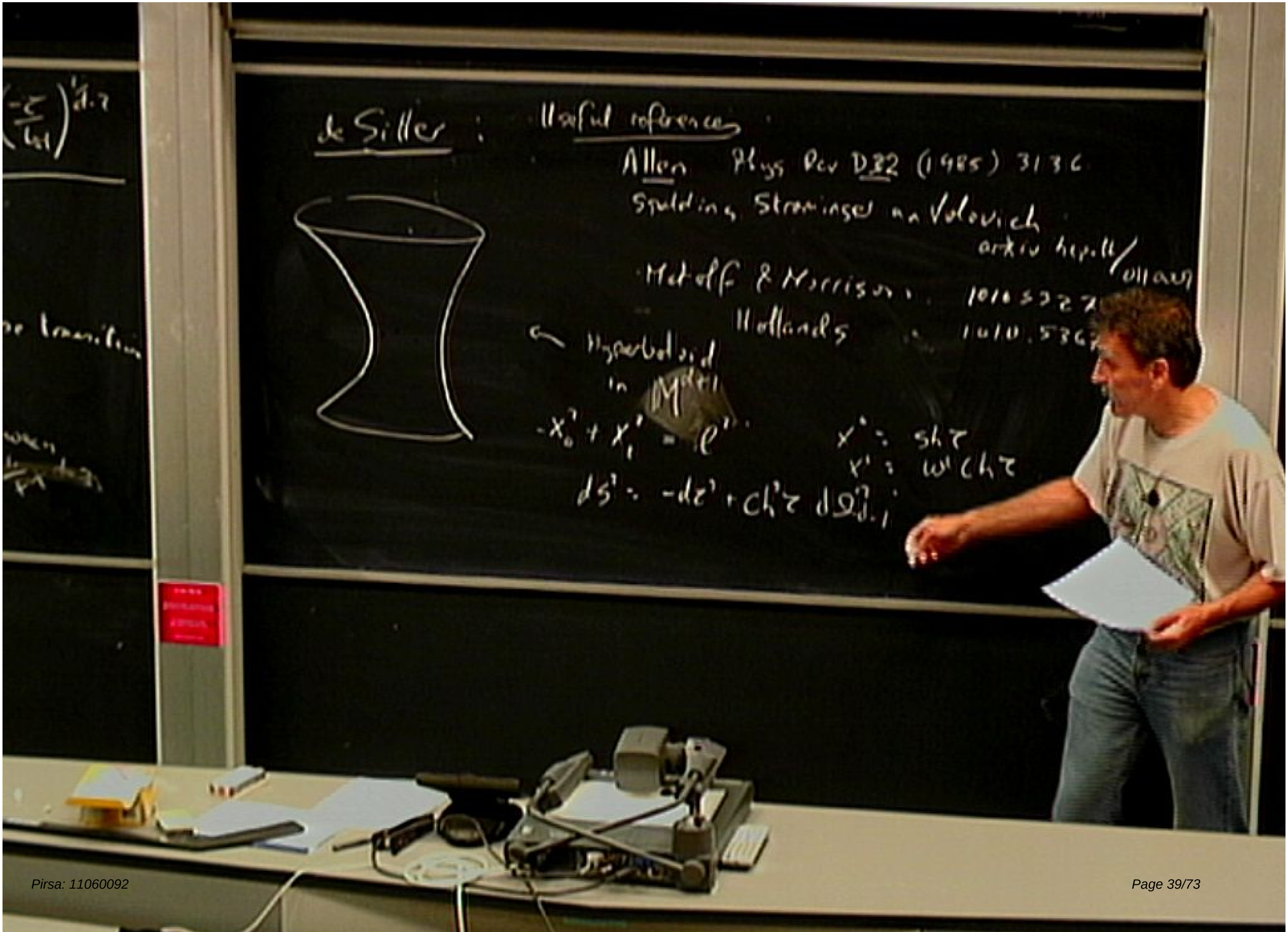
Hollands 1010.5367
hyperboloid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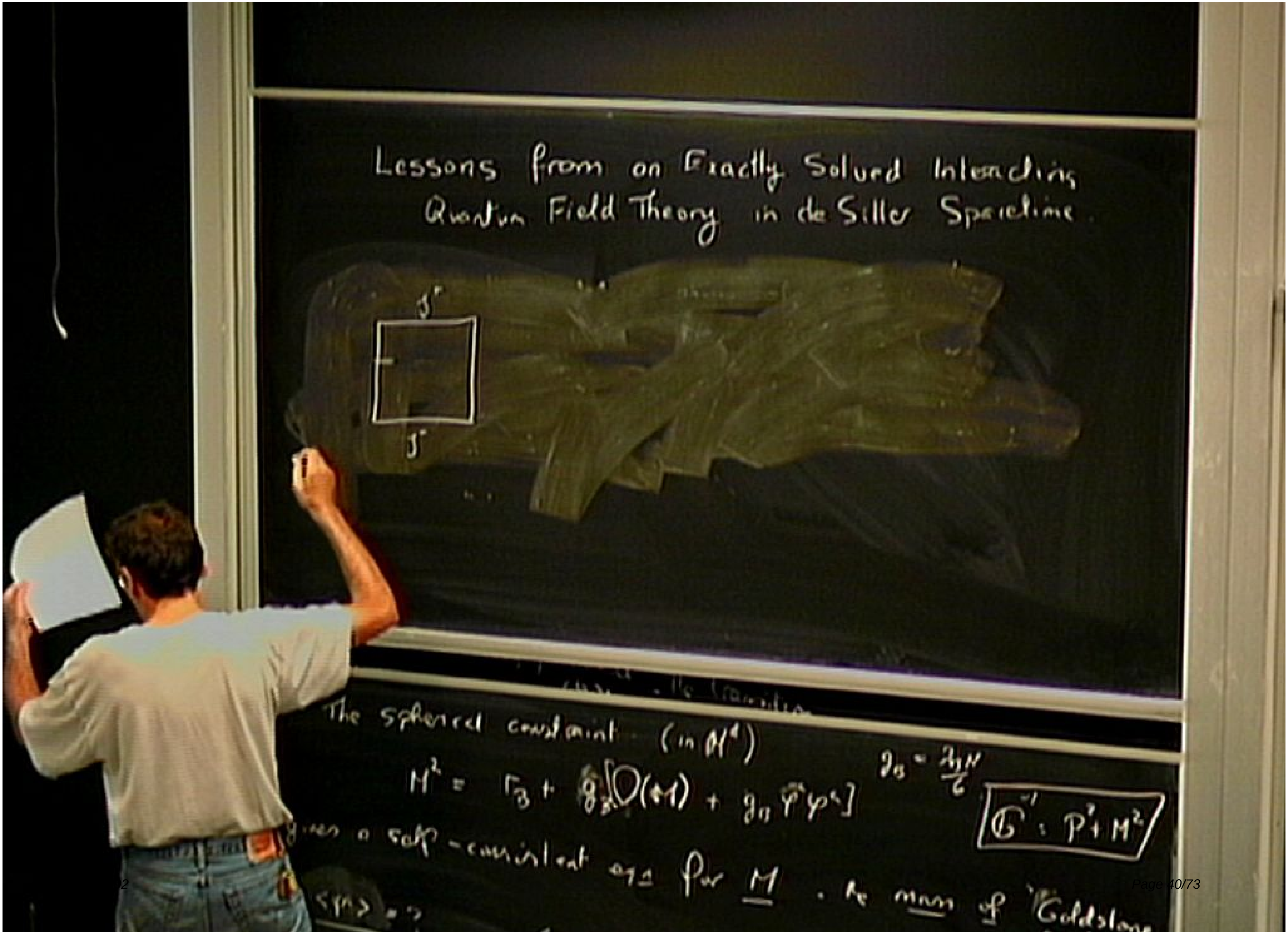
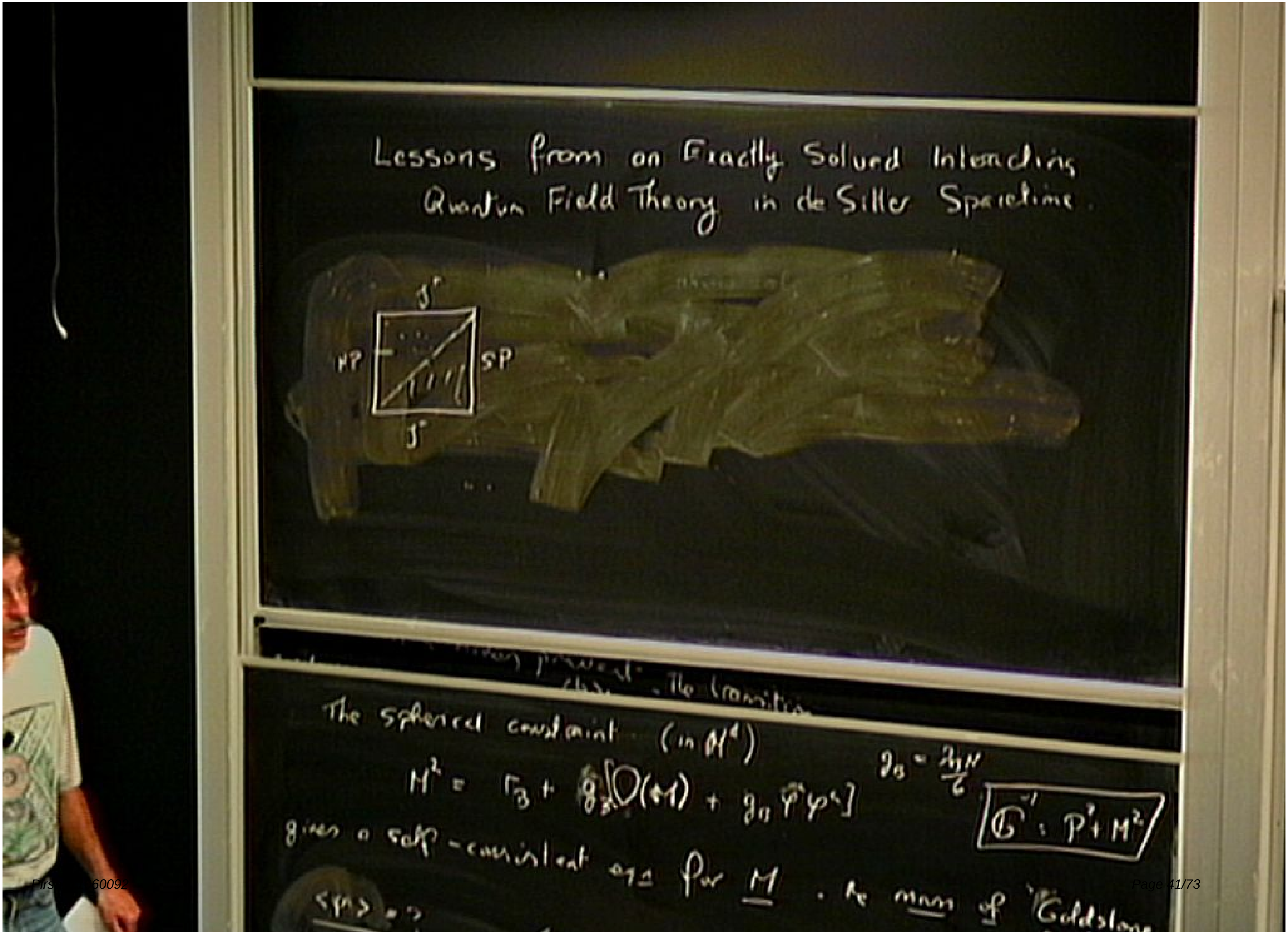
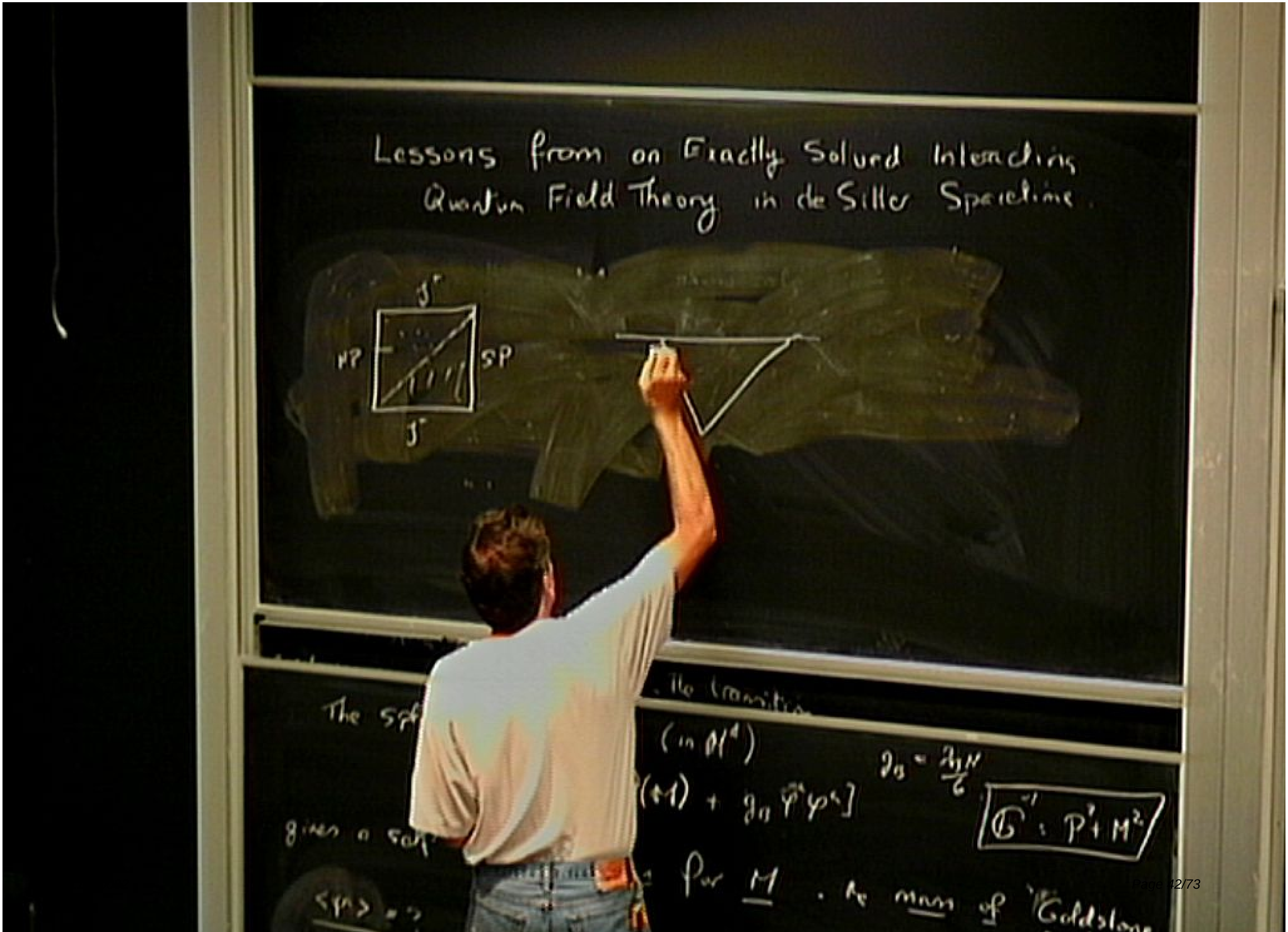
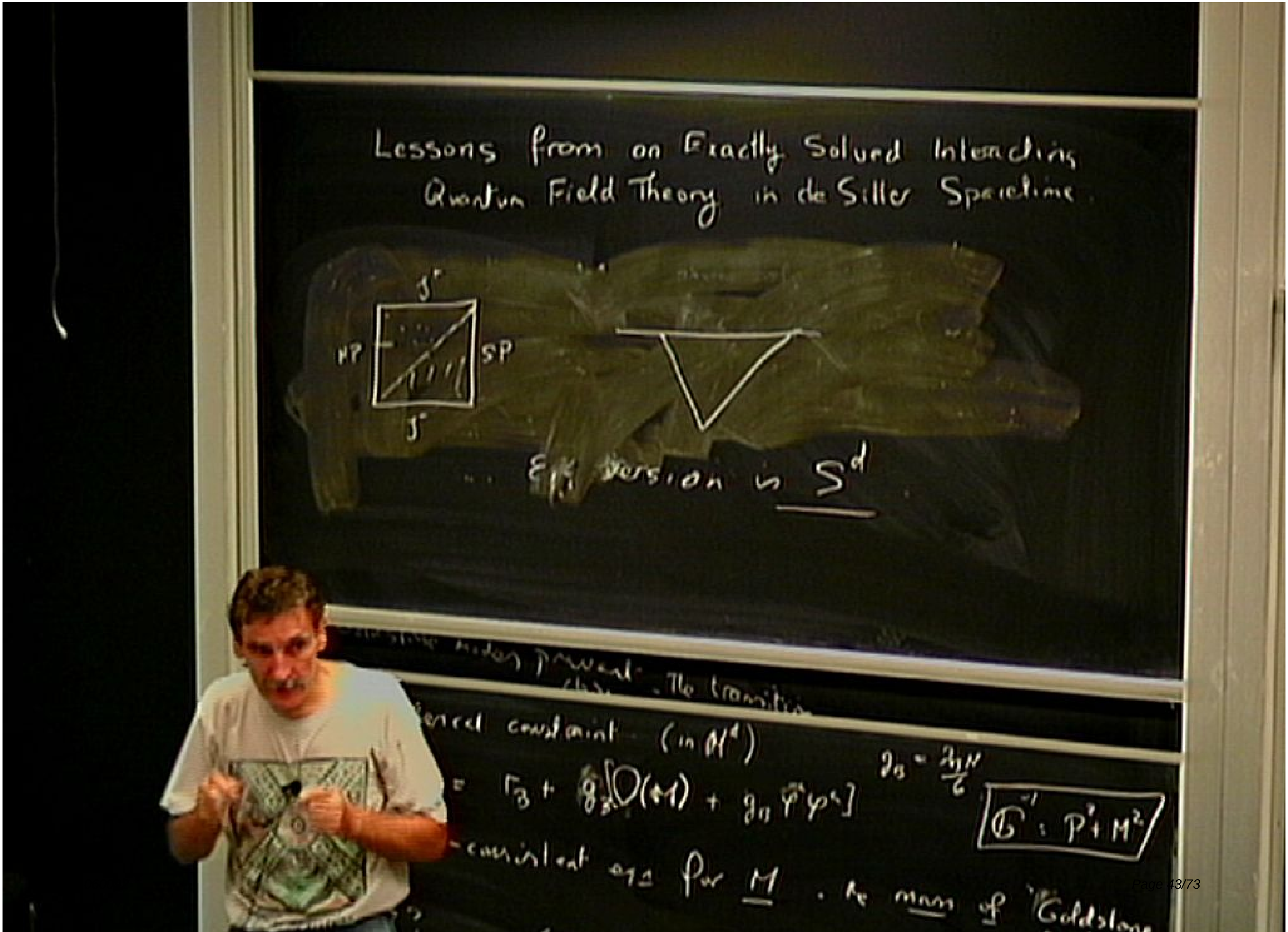
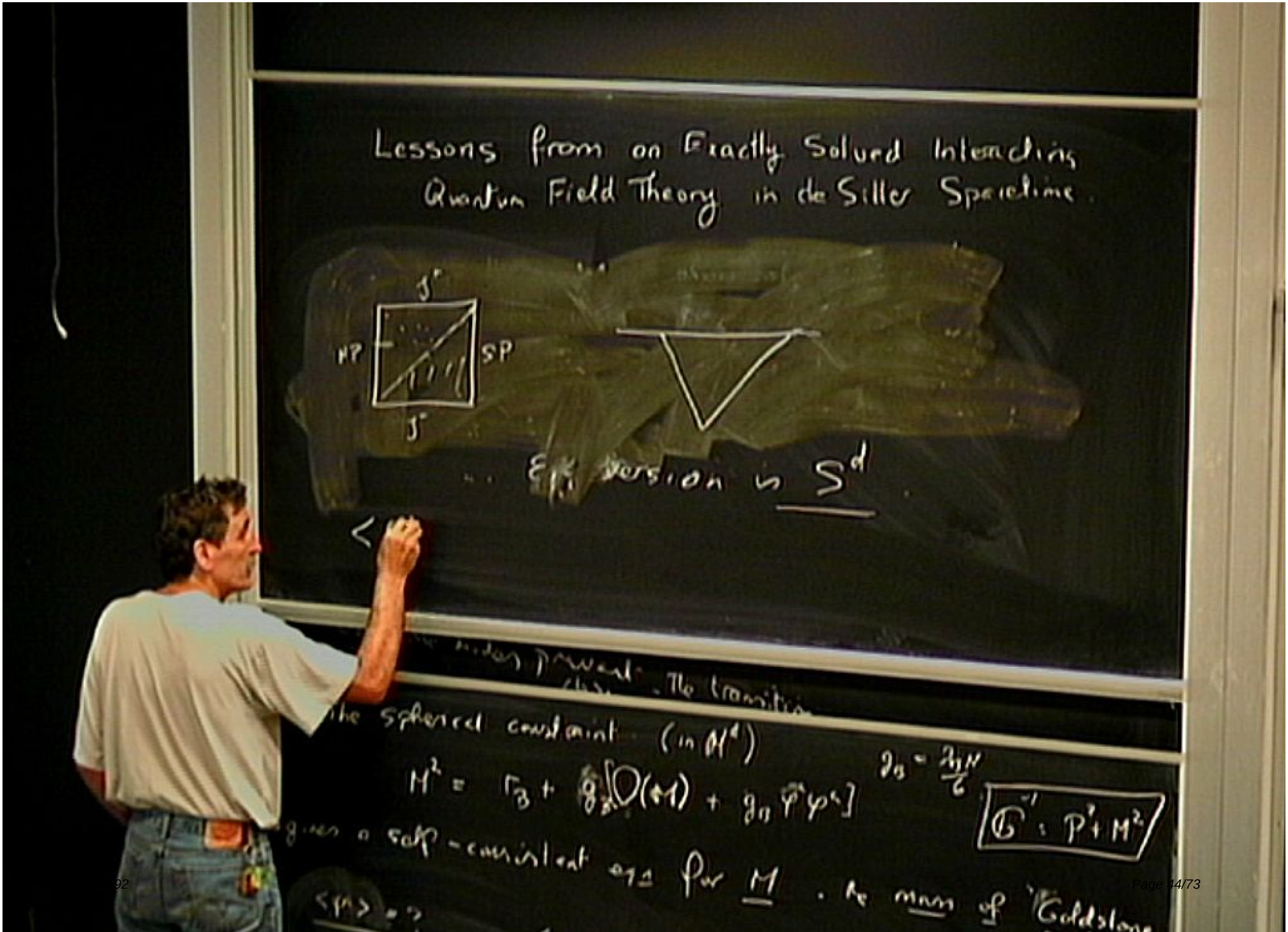
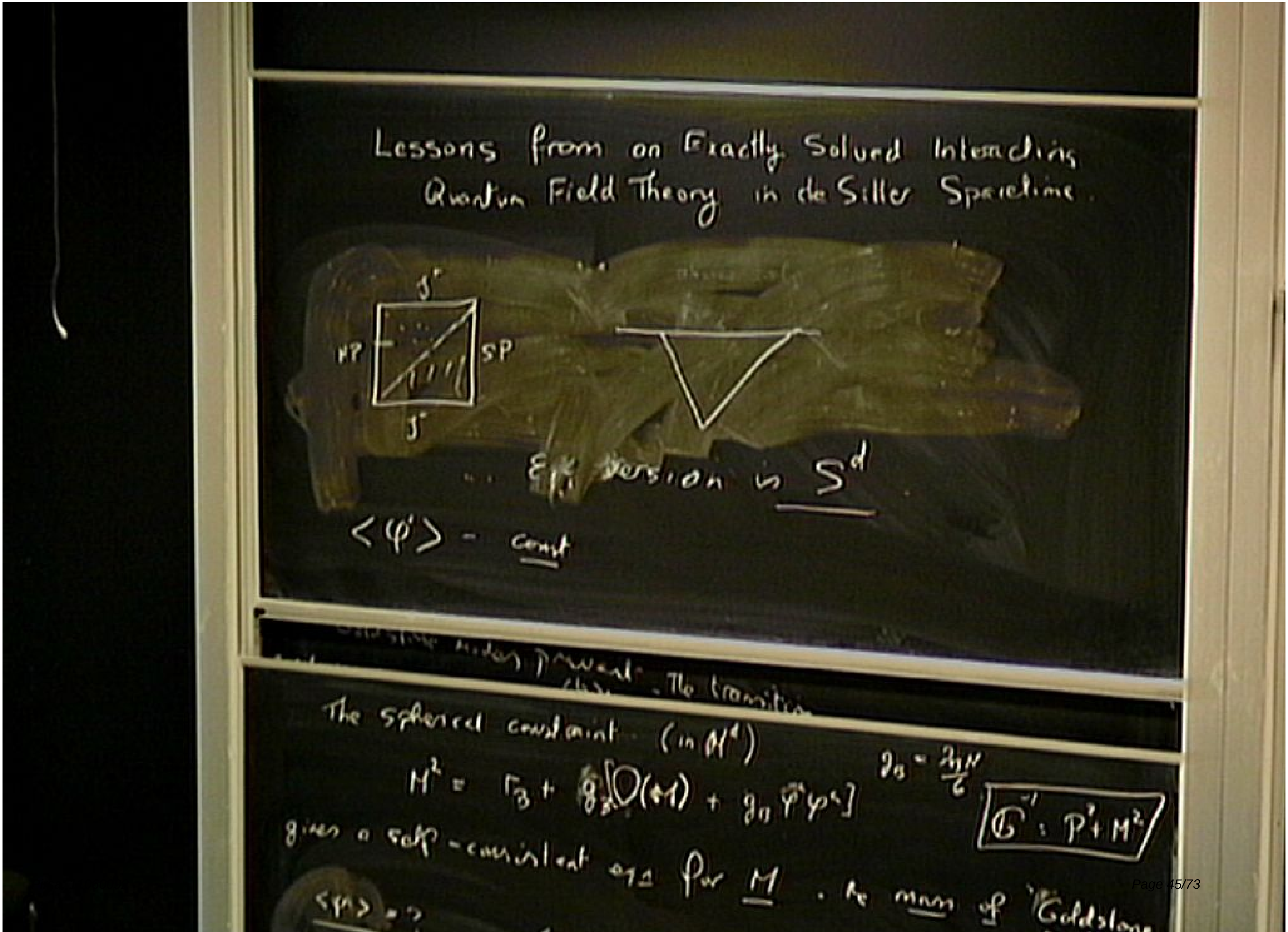

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
The spherical constraint
gives a self-consistent eqn for M

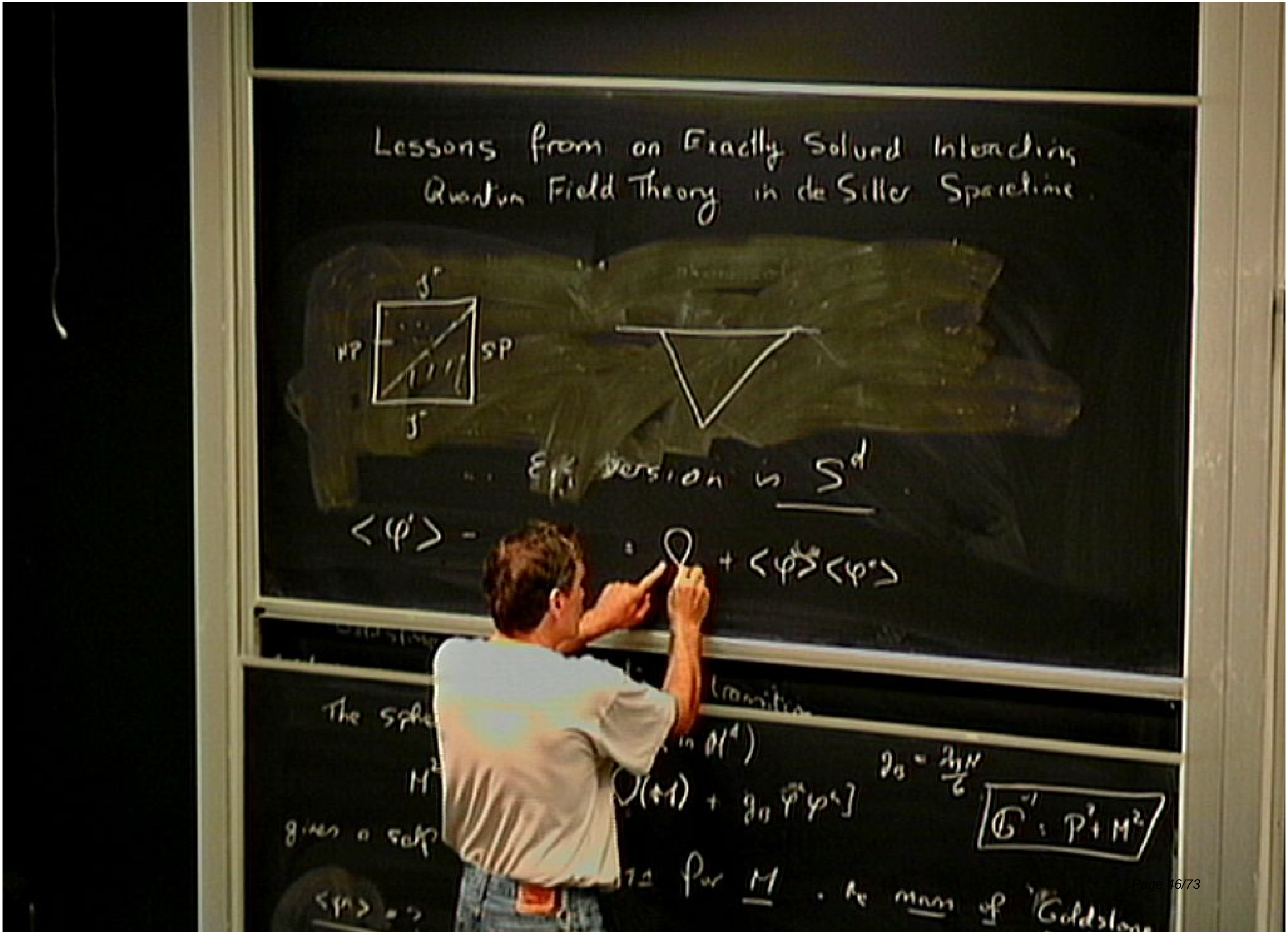

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime.
J+
NP
SP
J-
The spherical constraint (in M^d)
gives a self-consistent eqn for M

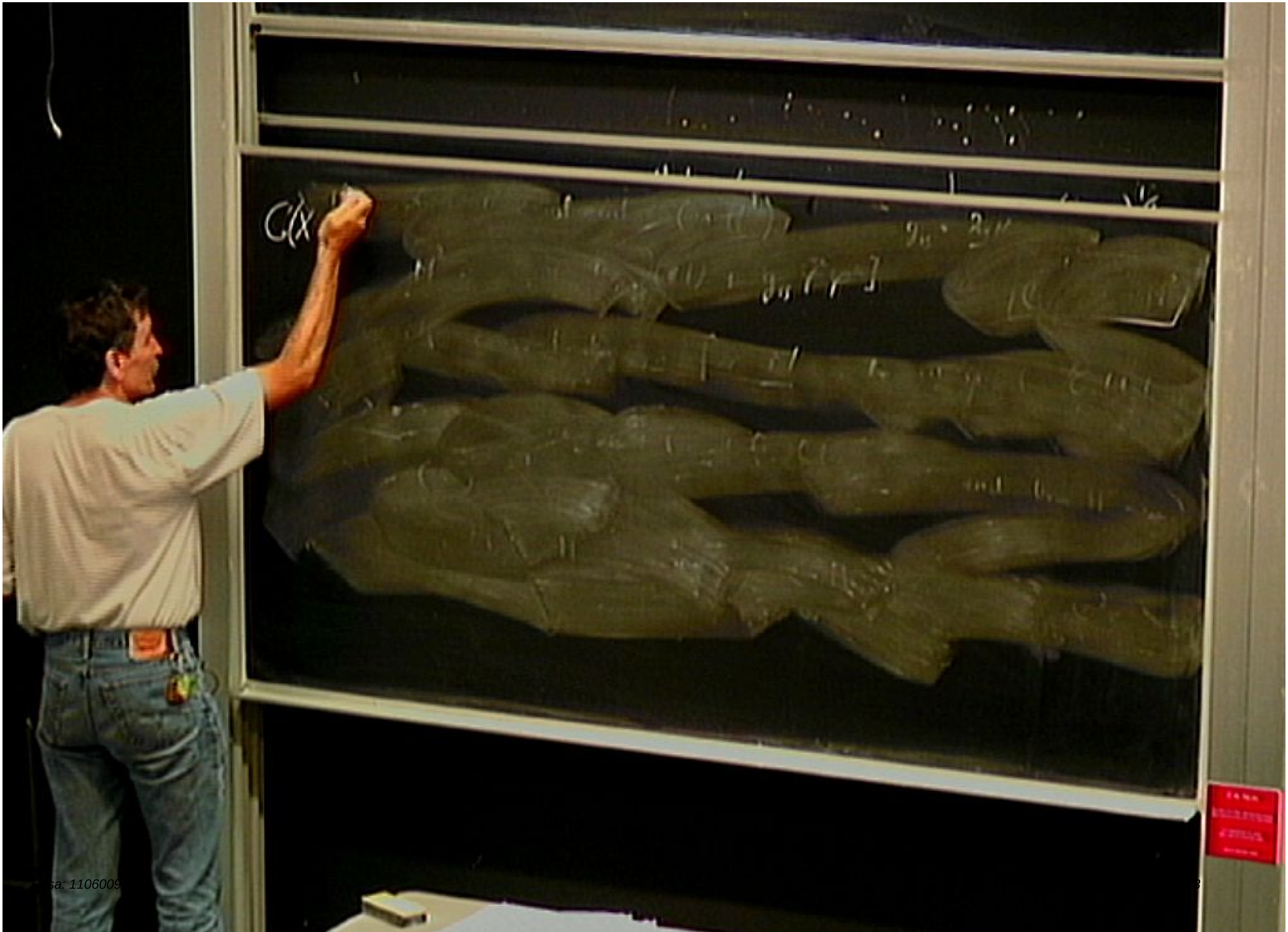

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
NP
SP
The
gives a
$G^{-1} = P^2 + M^2$

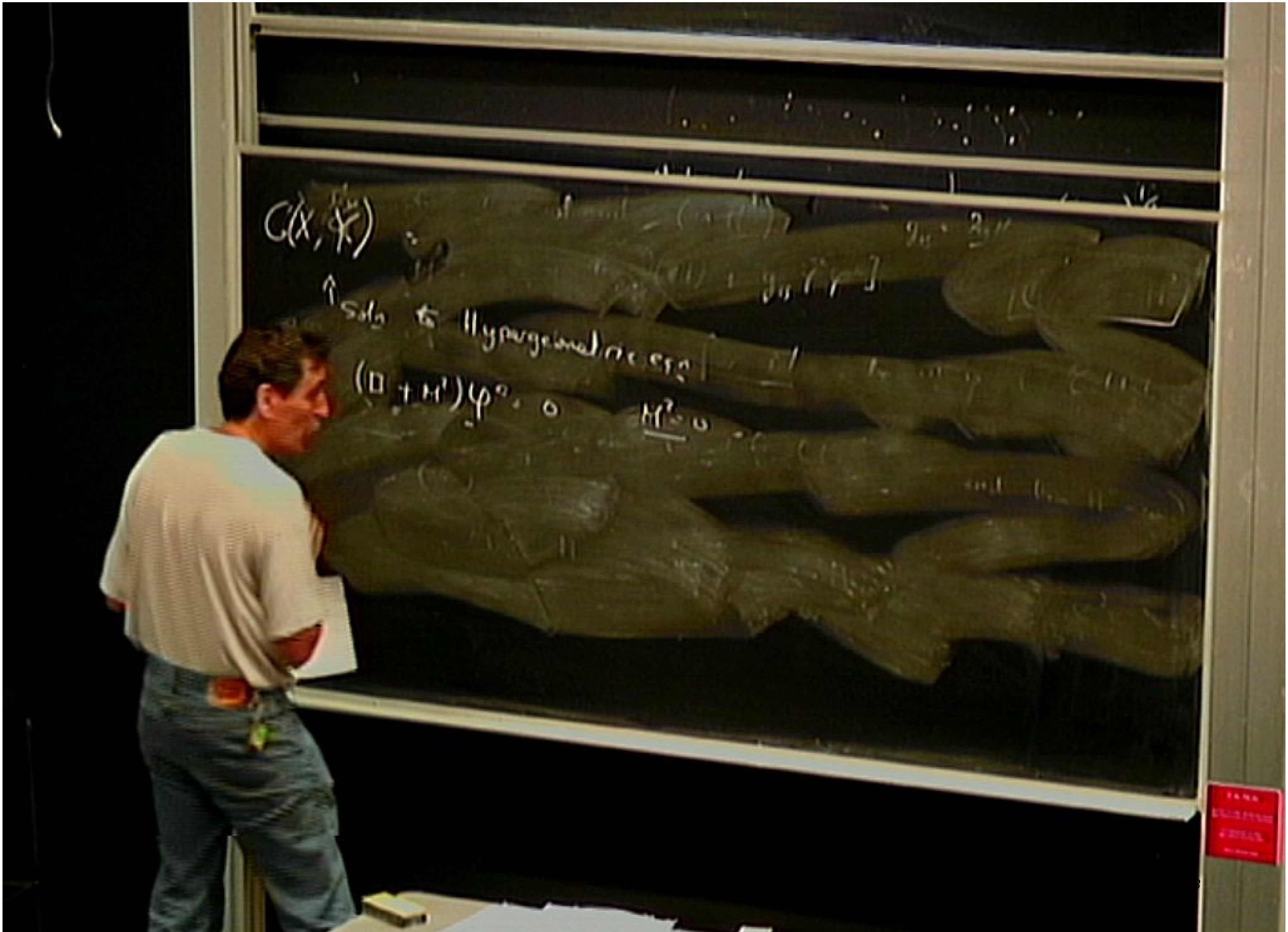

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
NP
SP
Eucl. version in S^d
constraint (in M^d)
G^-1 = P^2 + M^2
eqs for M
mass of Goldstone

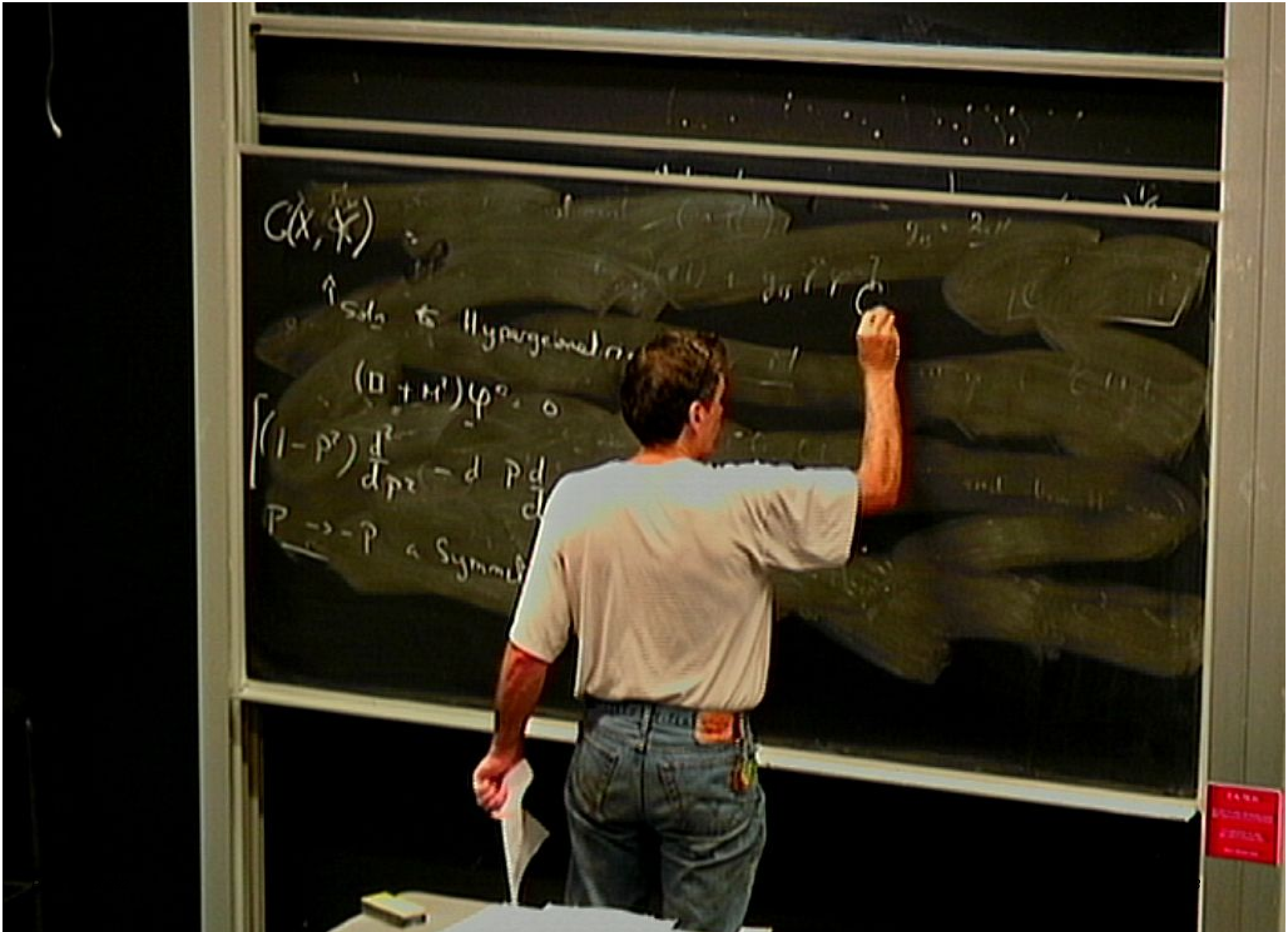

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime.

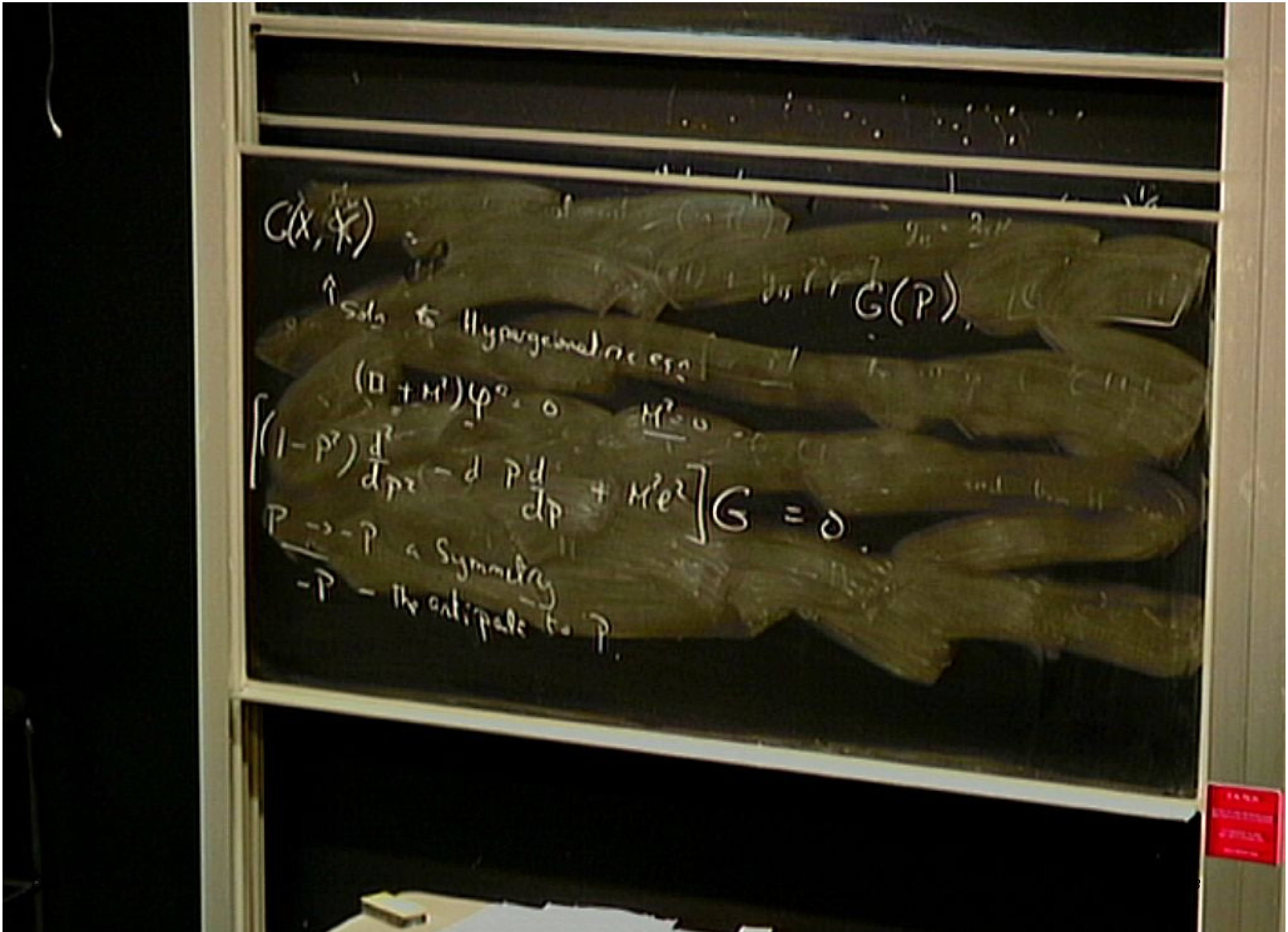

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime.
NP
SP
version in S^d
<φ'> = const
The spherical constraint (in M^d)
gives a self-consistent eqn for M . the mass of Goldstone
G^-1 = p^2 + M^2


Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
NP
SP
version in S^d
The sphe
given a self
<φ> = ?

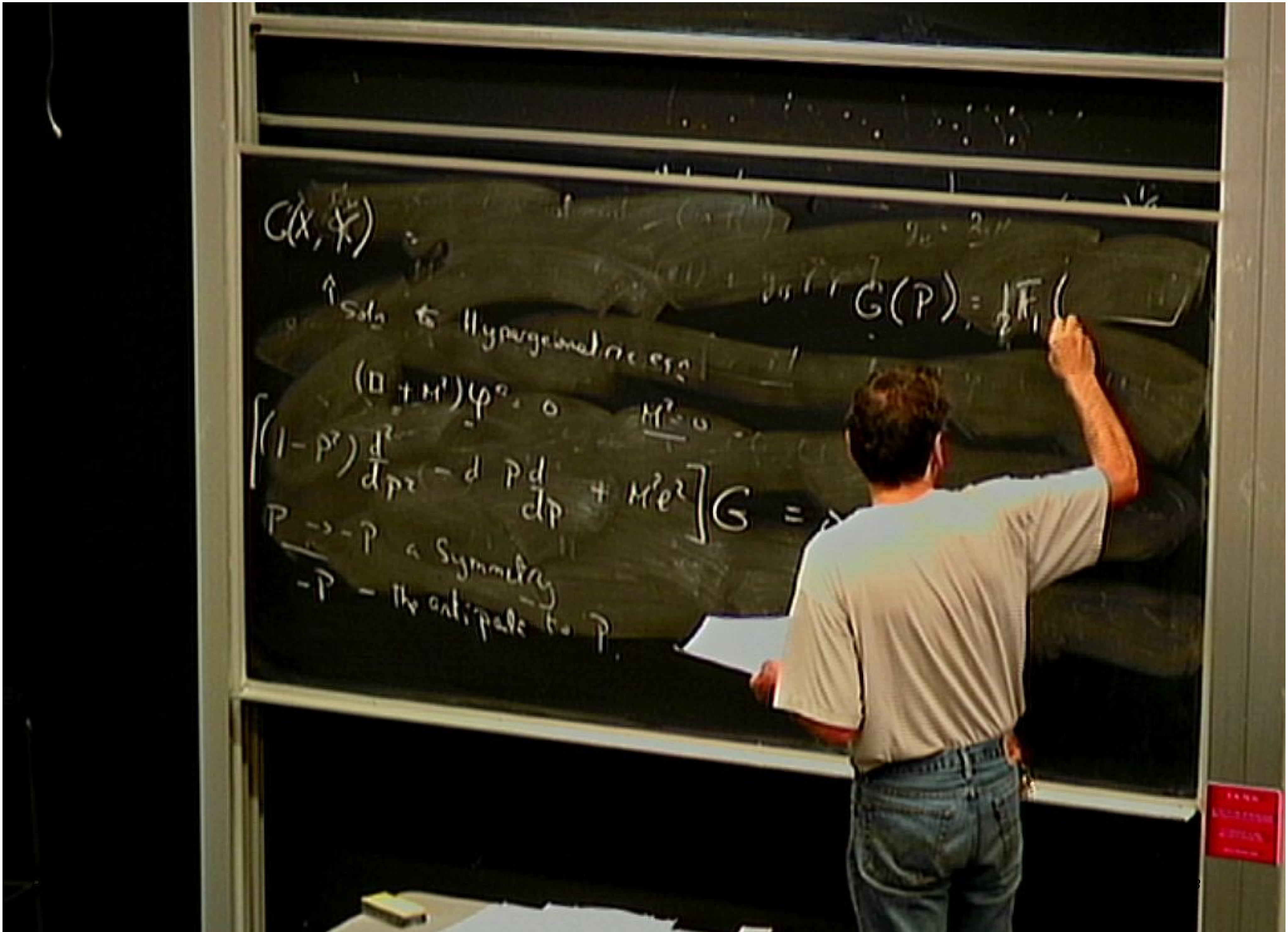

C(x

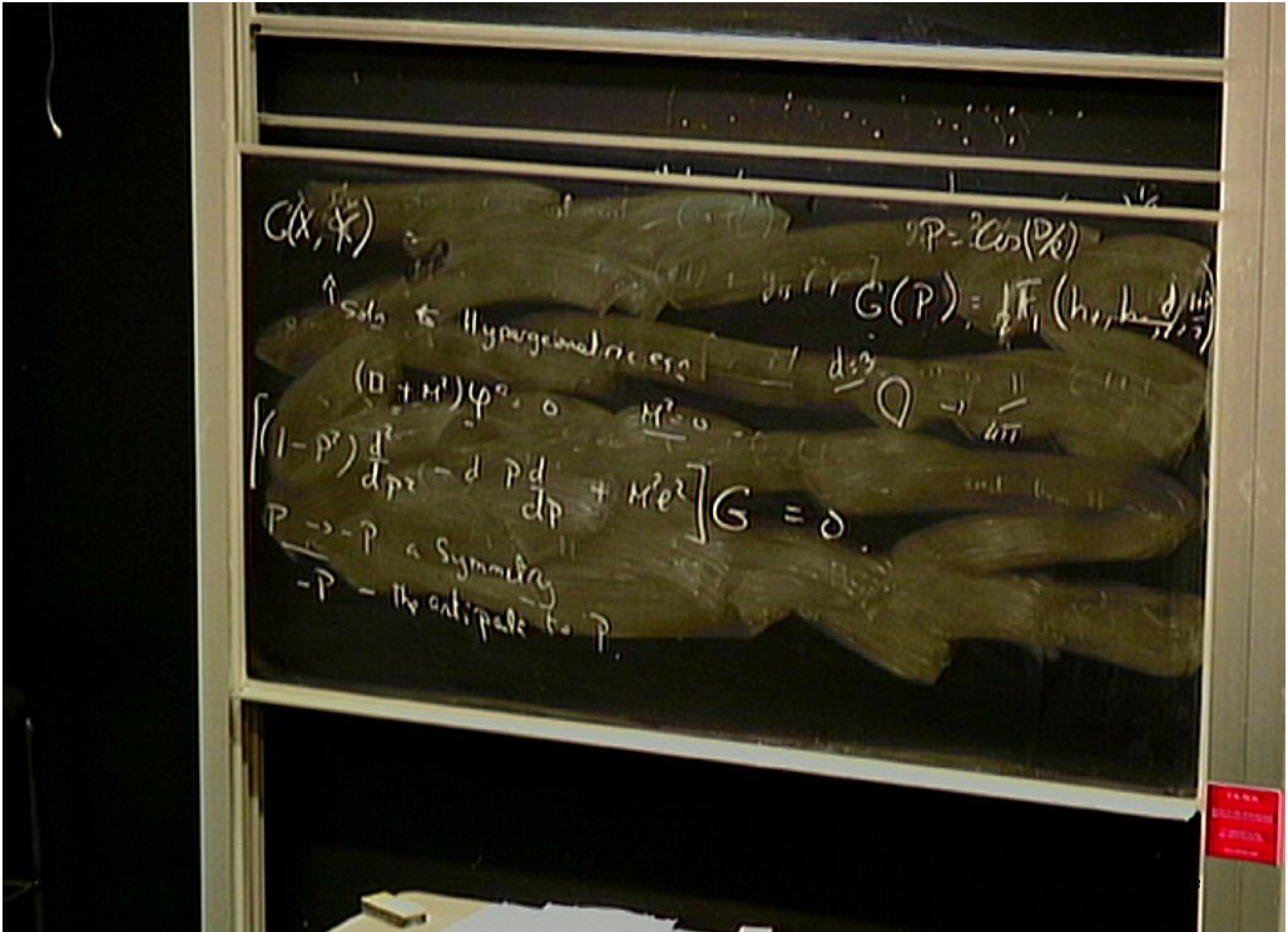




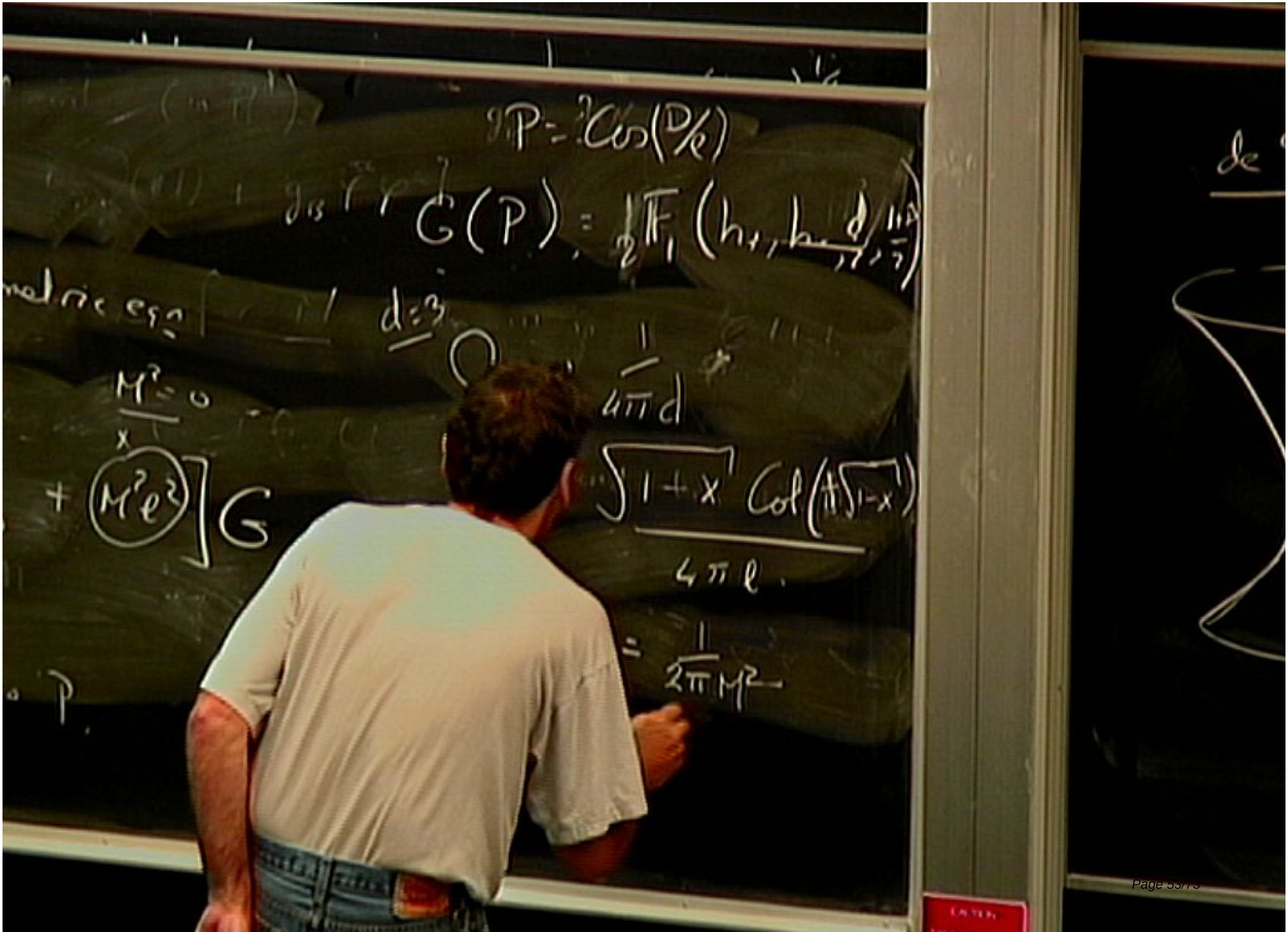


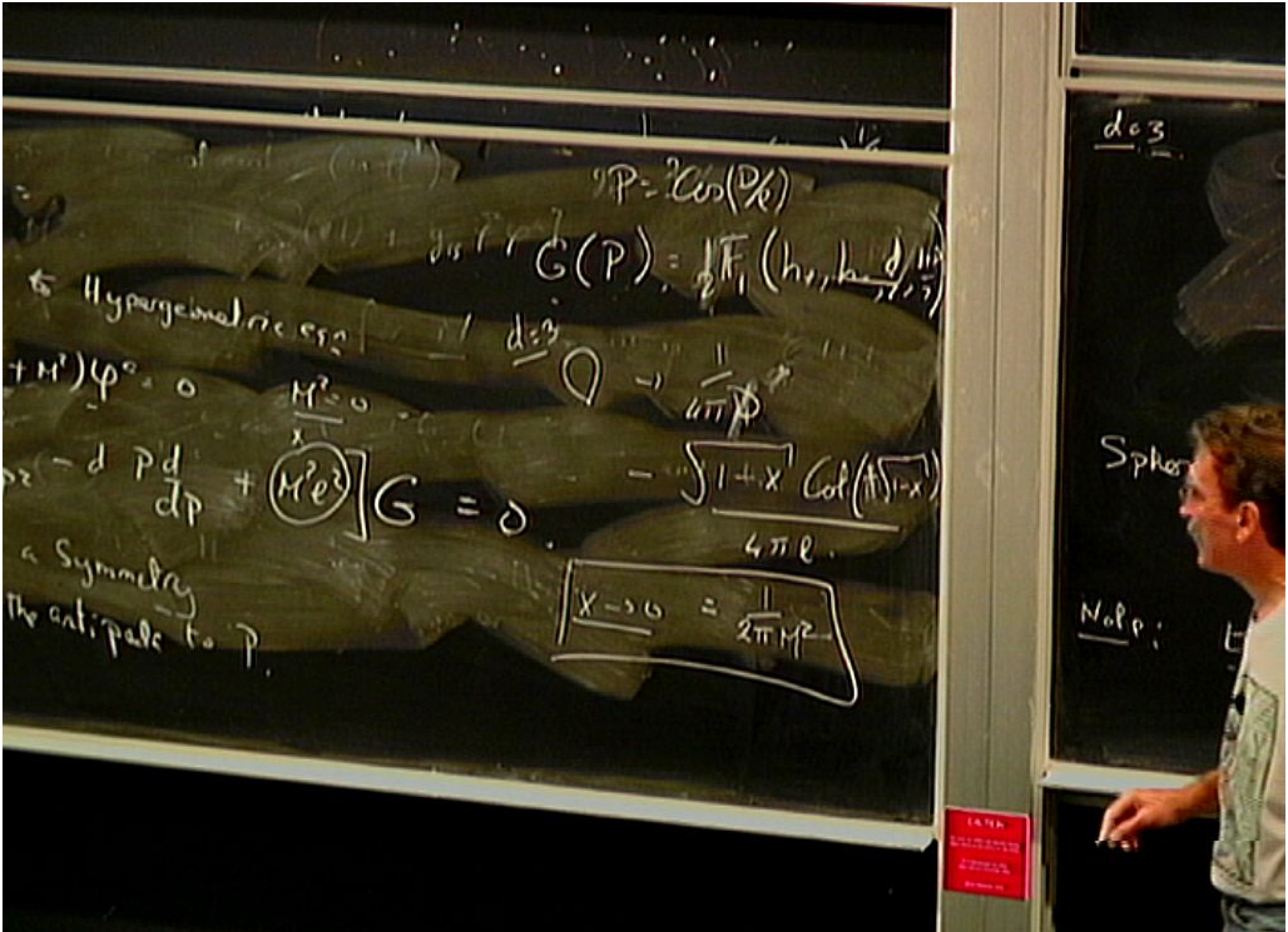
G(x, x')
G(P),
↑ Soln to Hypergeometric eq
(□ + M²)φ° = 0   M² = 0
[(1 − P²) d²/dP² − d P d/dP + M²e²] G = 0.
P → −P a symmetry
−P − the antipode to P.


Soln to Hypergeometric eq
P → −P a Symmetry
−P – the antipode to P

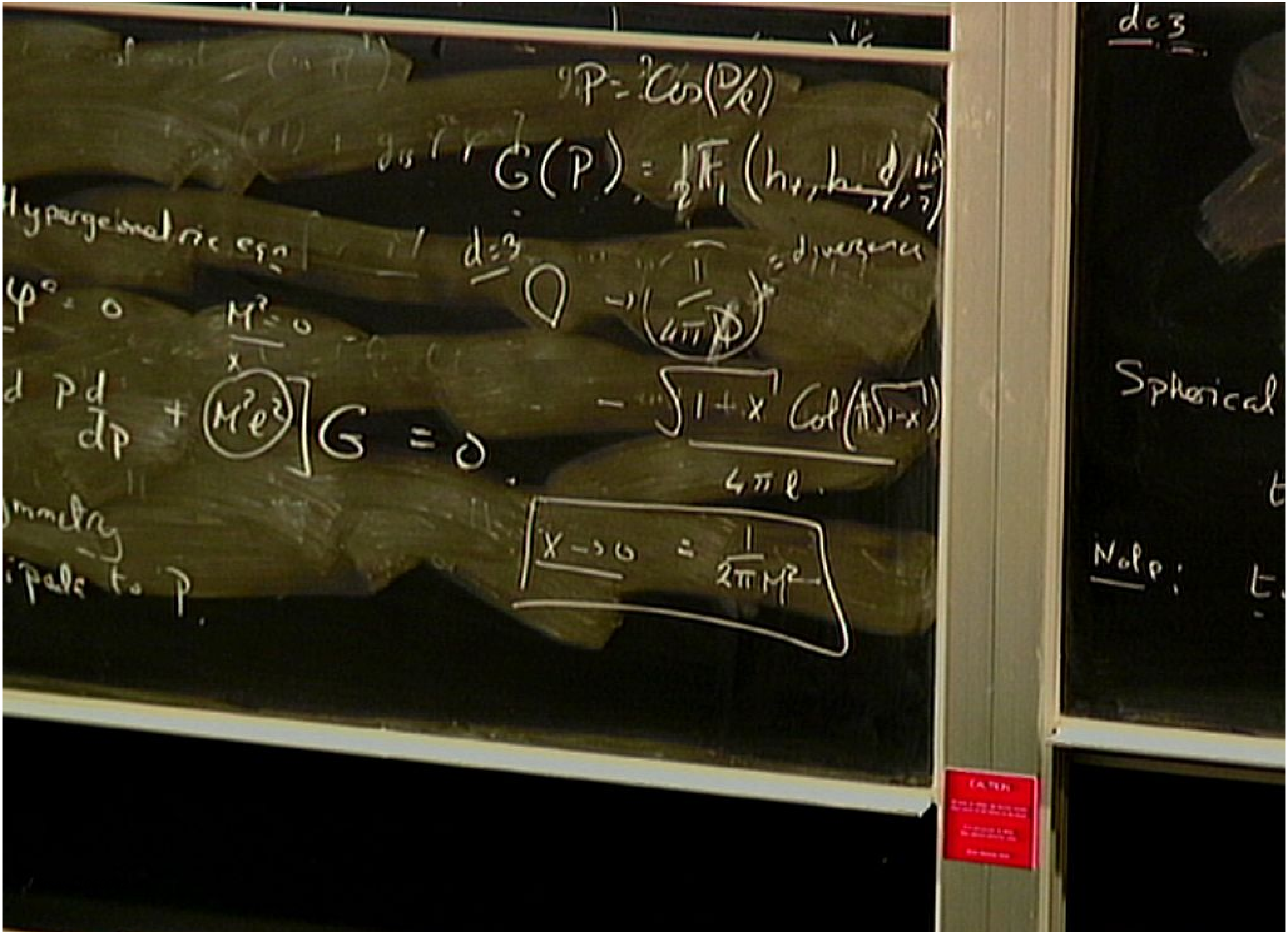


G(x, y)
P = cos(D/e)
G(P) = F (h, h, d/2
Soln to Hypergeometric eqn
(□ + M²)φ = 0
M² = 0
d=3
1/4π
[(1-P²) d²/dP² - d P d/dP + M²e²] G = 0.
P → -P a Symmetry
-P - the antipode to P.

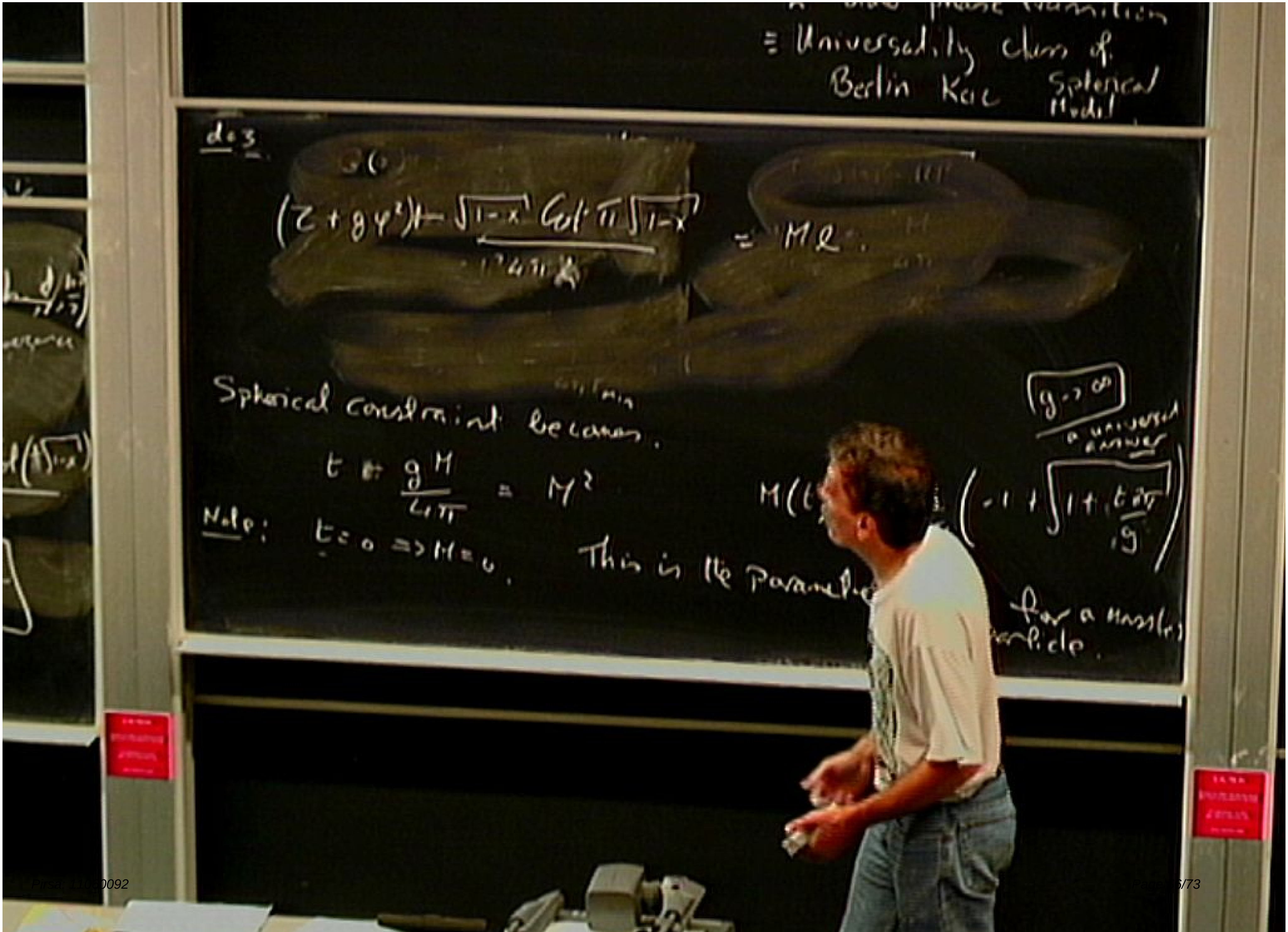

P = Cos(D/ℓ)
G(P) = ₂F₁(h₊, h₋, d/2 ...
d=3
1/4πd
√(1+x) Cot(...)
4πℓ
1/2πM²

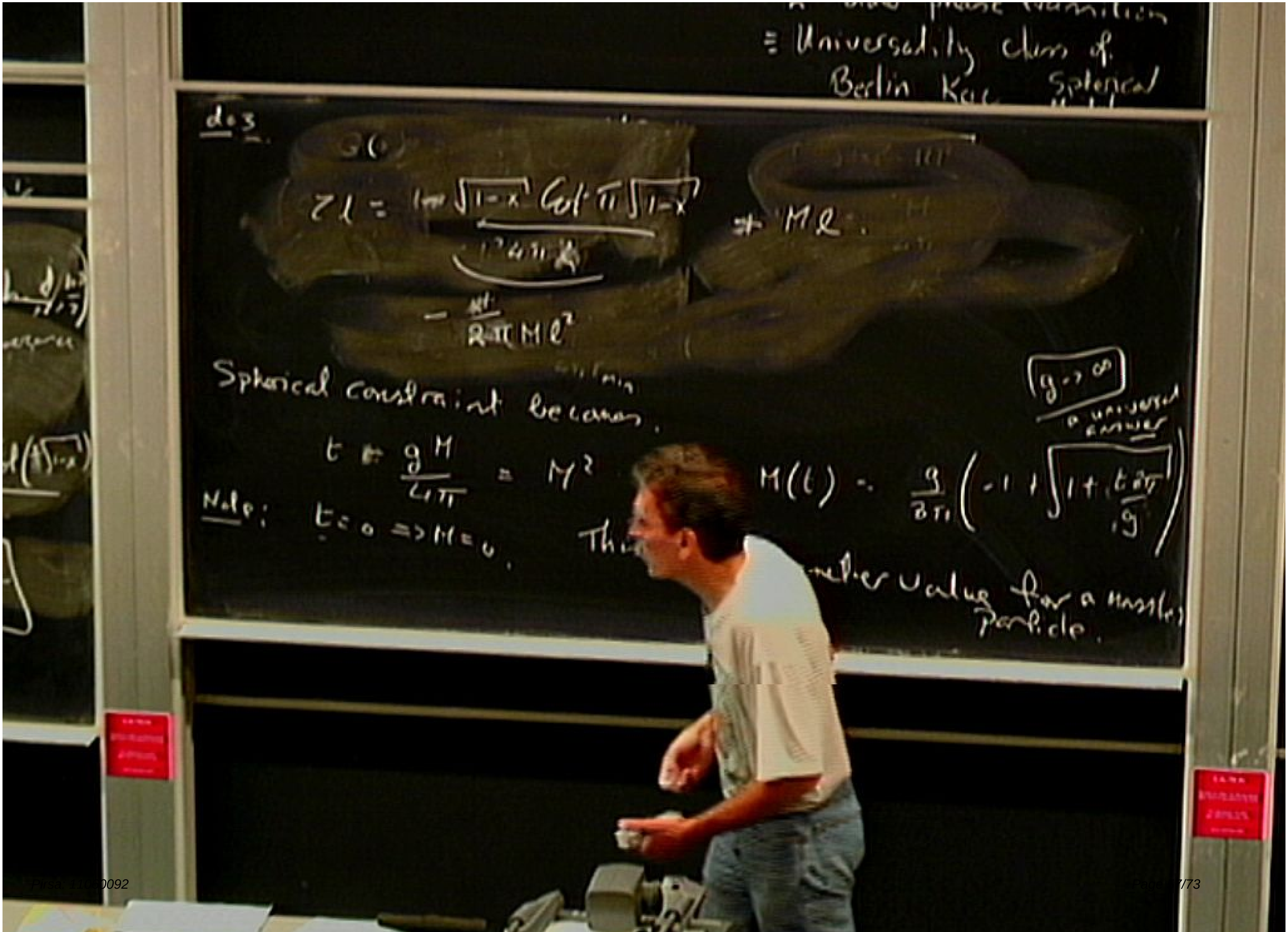


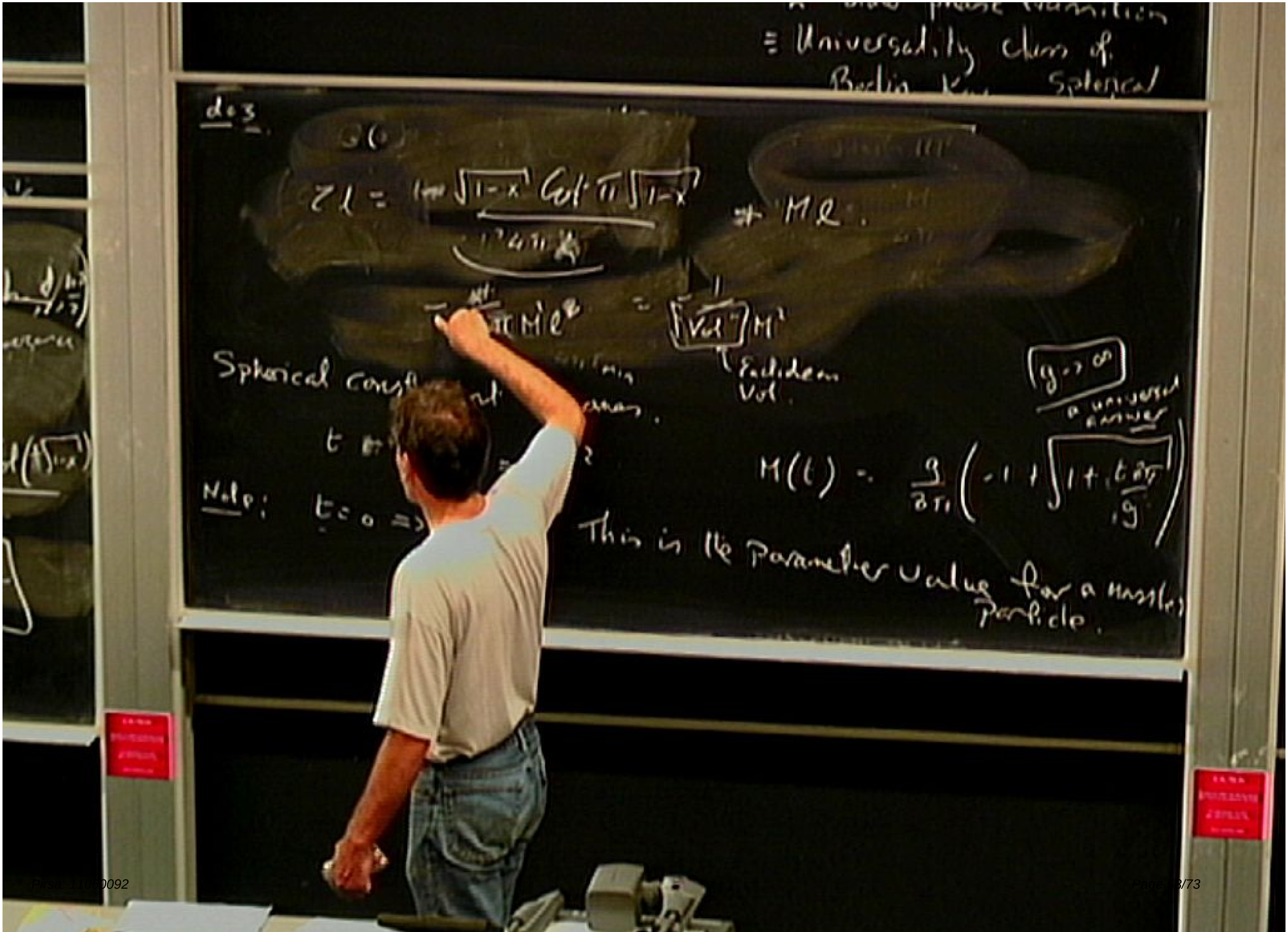
Hypergeometric eqn
d=3
G = 0
4πℓ
a Symmetry
the antipode to P
d=3
Spher
Note:

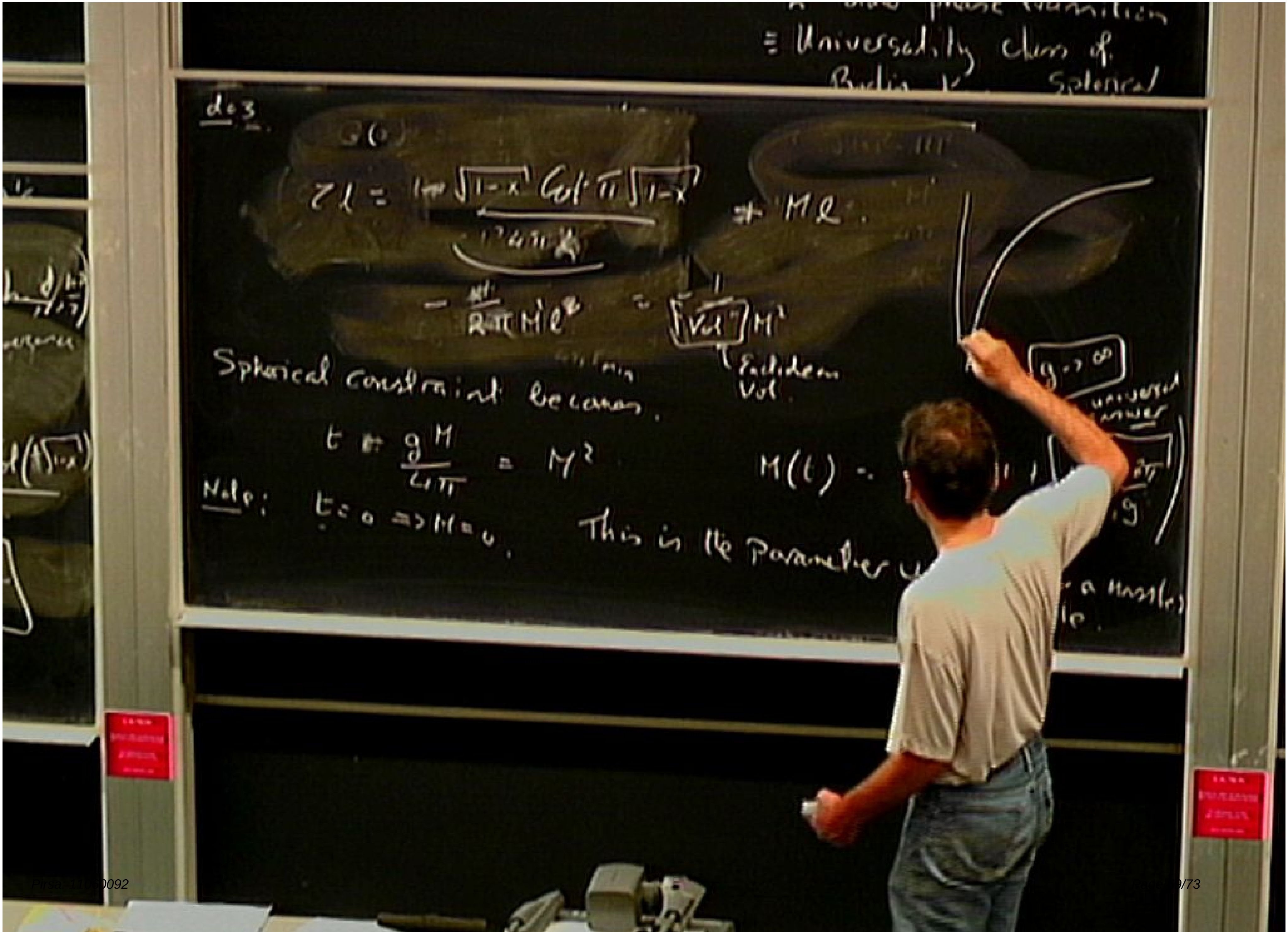


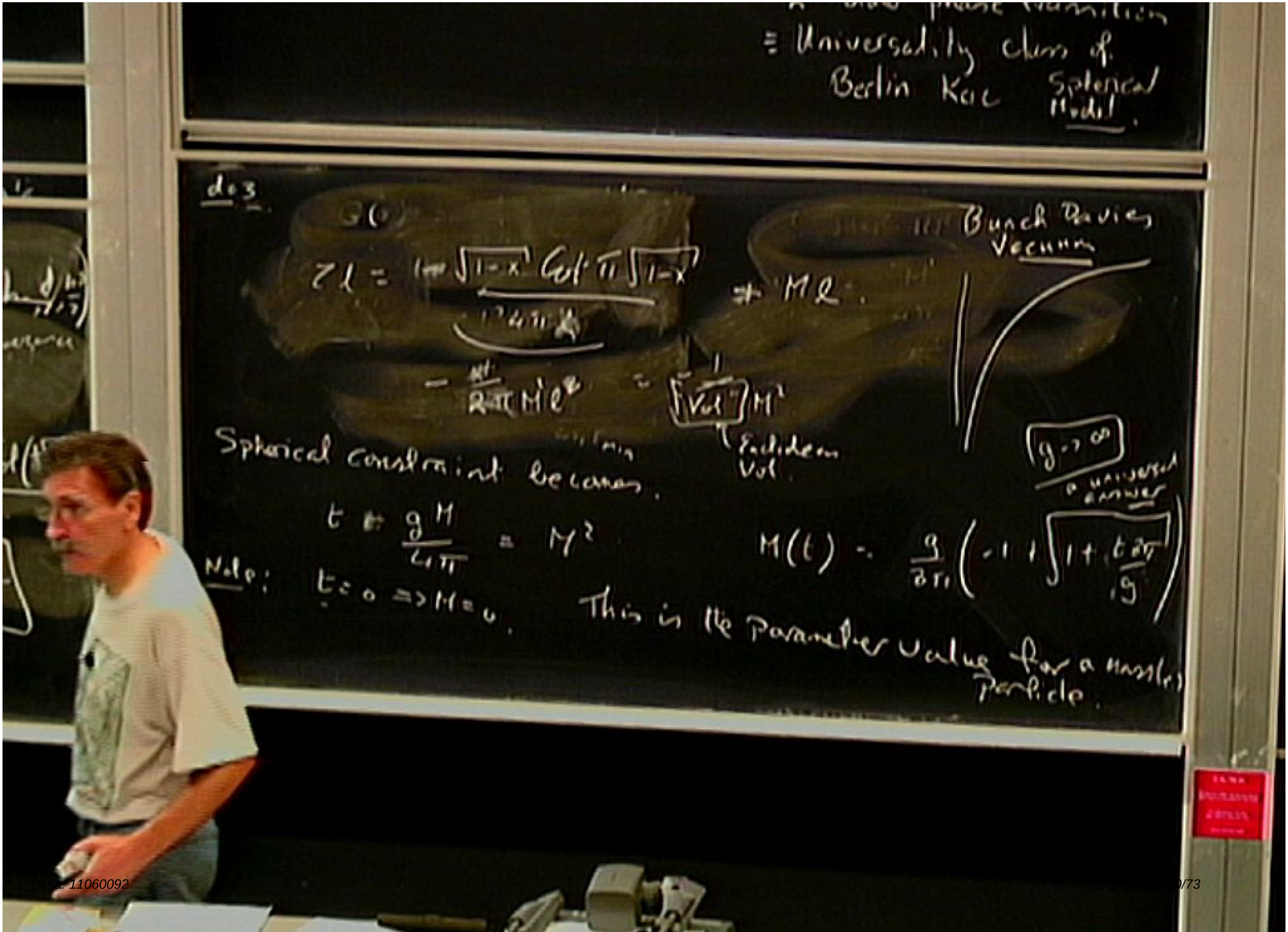
d=3
G(P) = ₂F₁(h₊, h₋, d/2
Hypergeometric eqn
d=3
divergence
4πℓ
Spherical
Note:

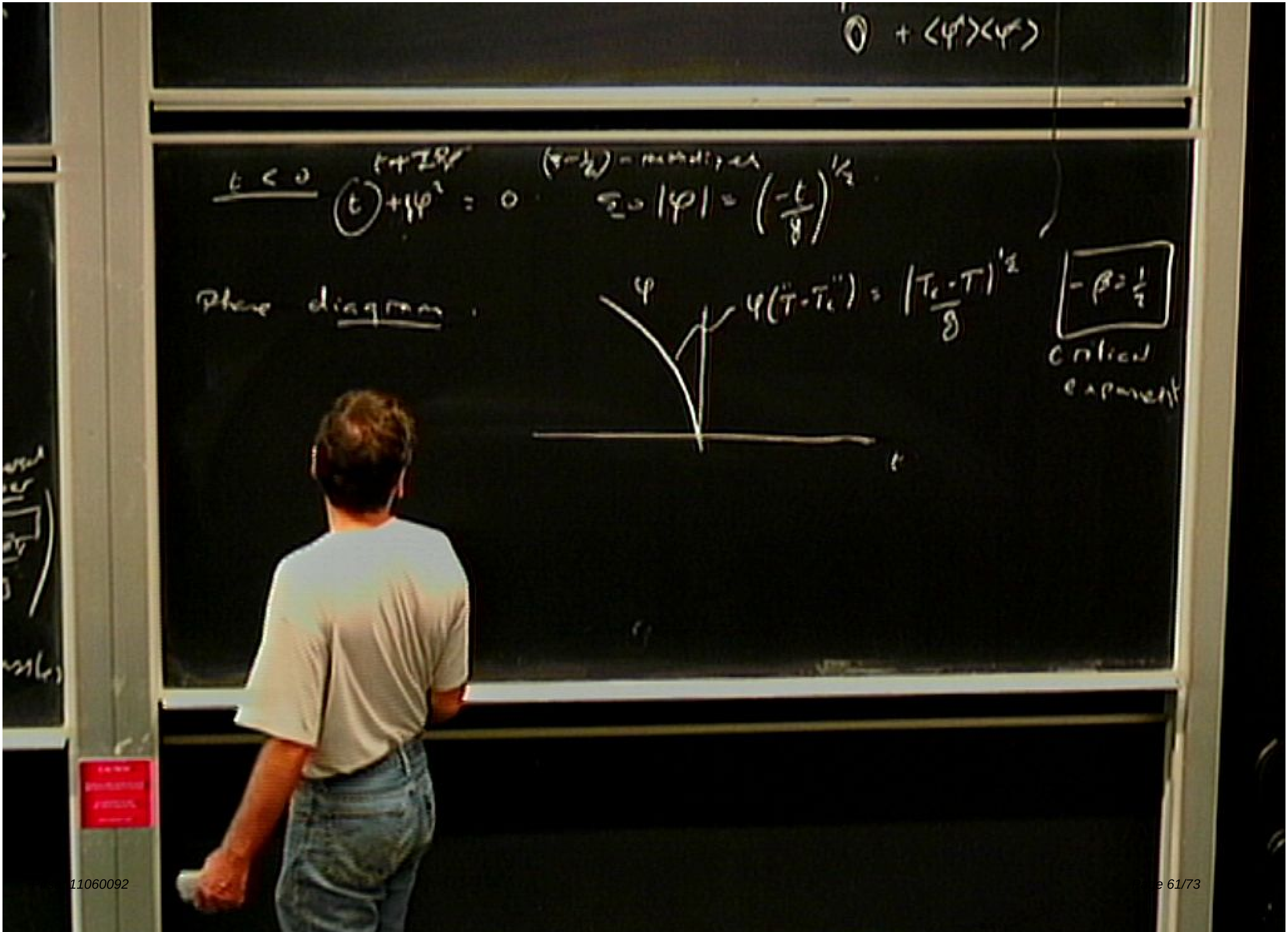

≡ Universality class of
Berlin Kac Spherical Model
d=3
Spherical constraint becomes.
Note:
This is the parameter
for a massless
particle.
a universal answer

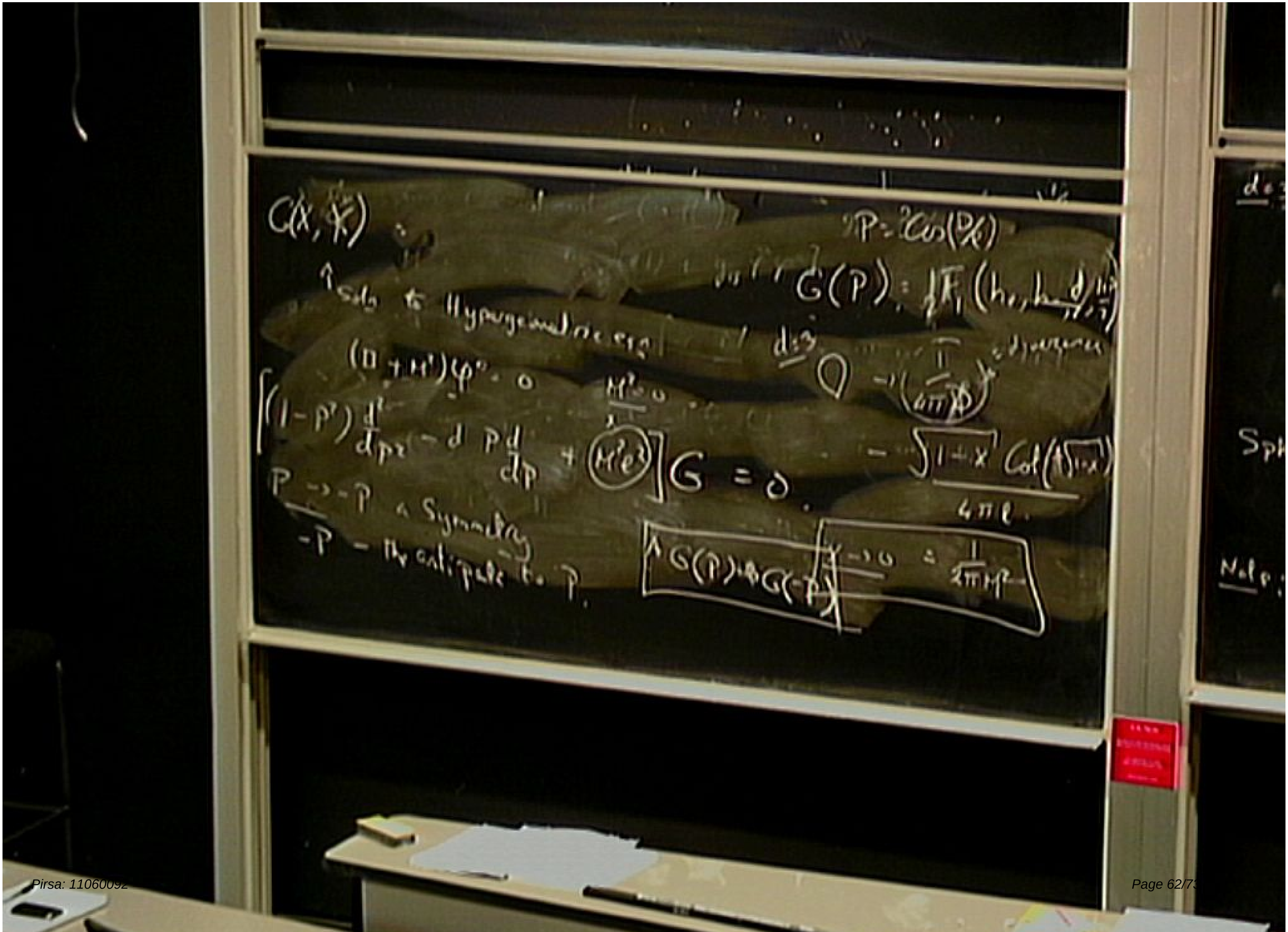

Universality class of
Berlin Kac Spherical
d=3
Spherical constraint becomes
Note: t=0 => M=0.
value for a massless particle.

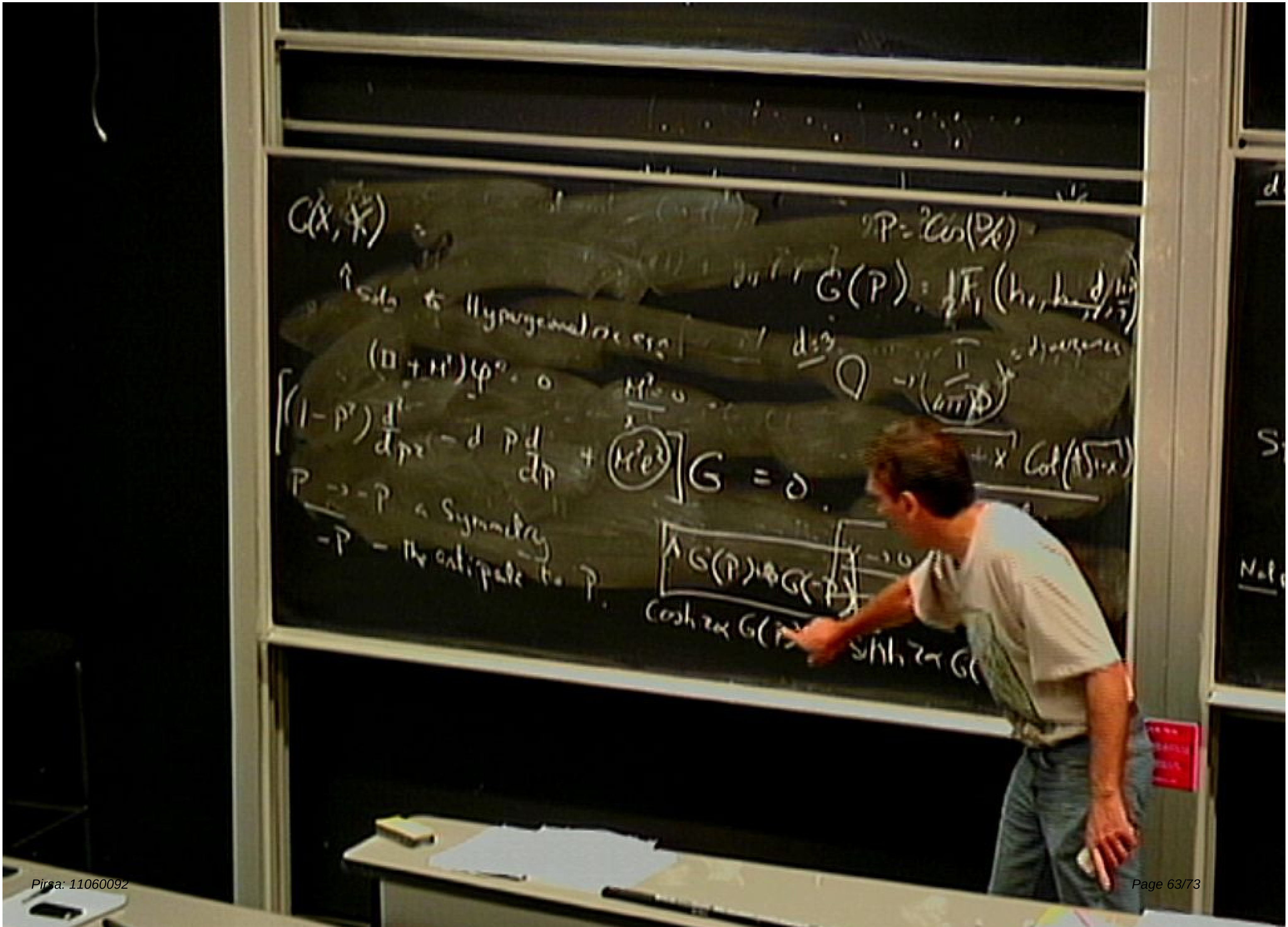

≡ Universality class of
Spherical
Spherical config
Euclidean Vol.
a universal answer
This is the parameter value for a massless particle.

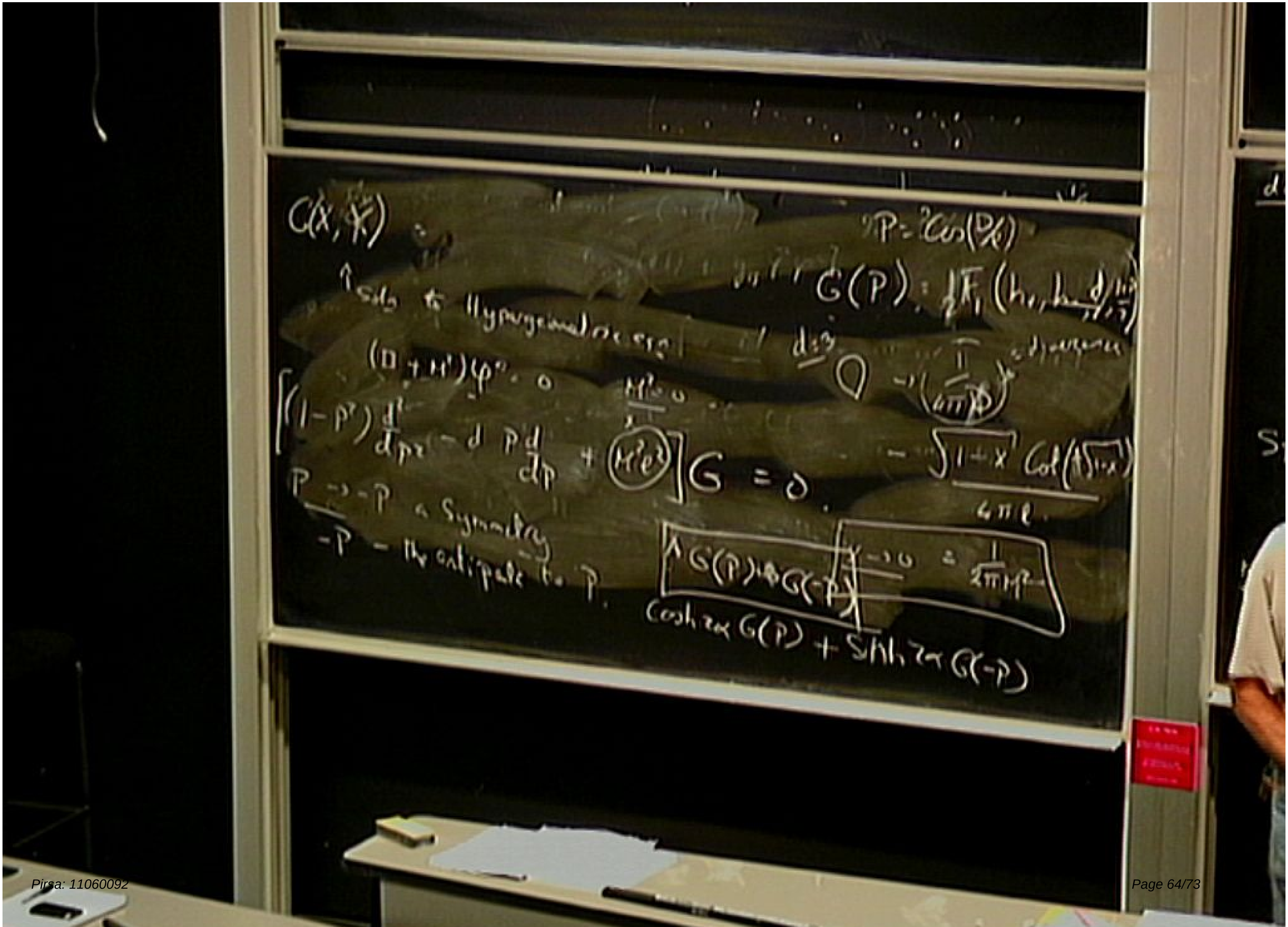

≡ Universality class of
d=3
Spherical constraint becomes
Euclidean Vol
Note: t=0 ⇒ M=0.
This is the parameter
g→∞
universal answer


≡ Universality class of
Berlin Kac Spherical Model.
d=3
Bunch Davies Vacuum
Spherical constraint becomes.
Euclidean Vol.
a universal answer
Note: t=0 => M=0. This is the parameter value for a massless particle.

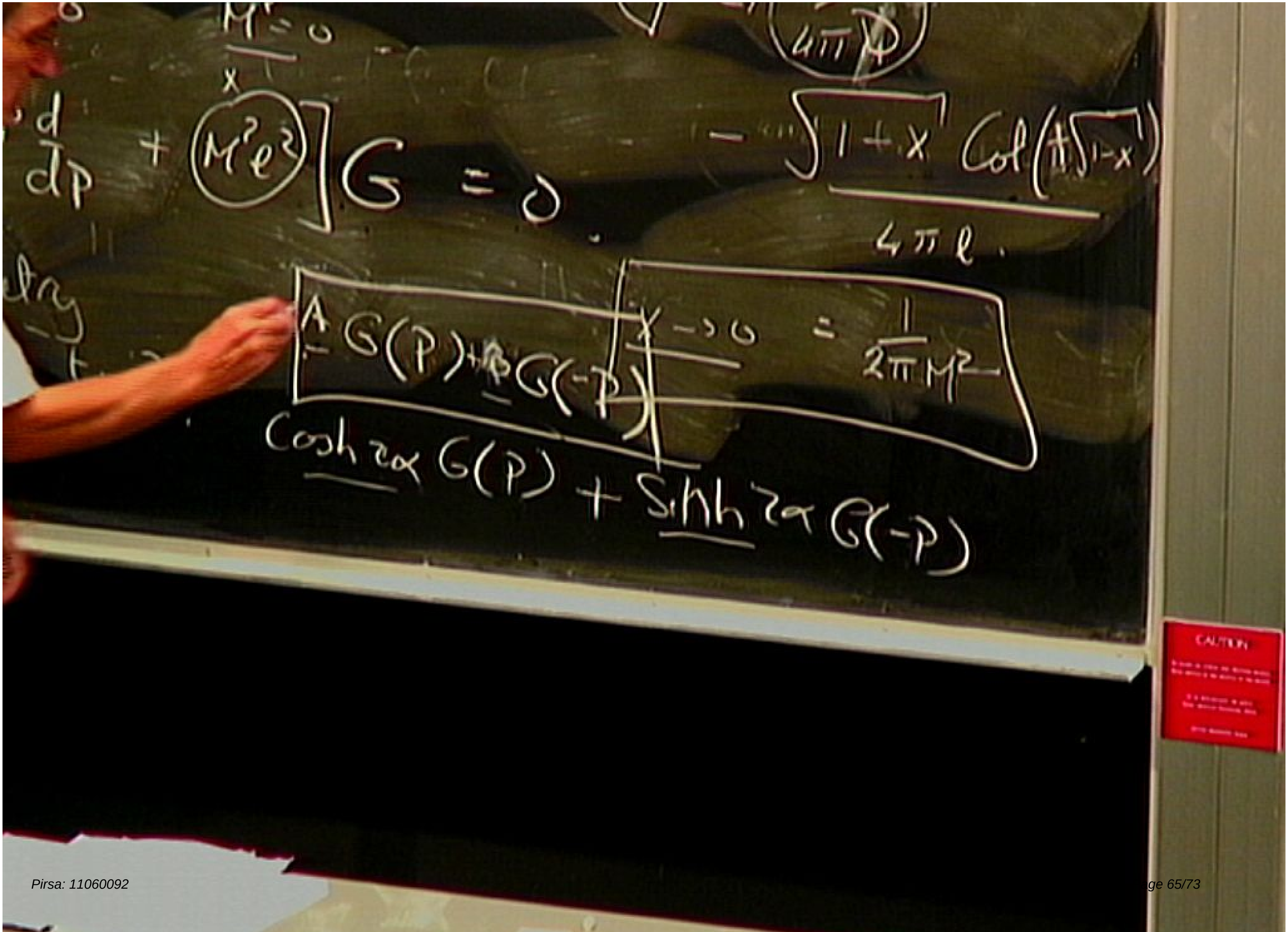

t < 0
Phase diagram
critical
exponent

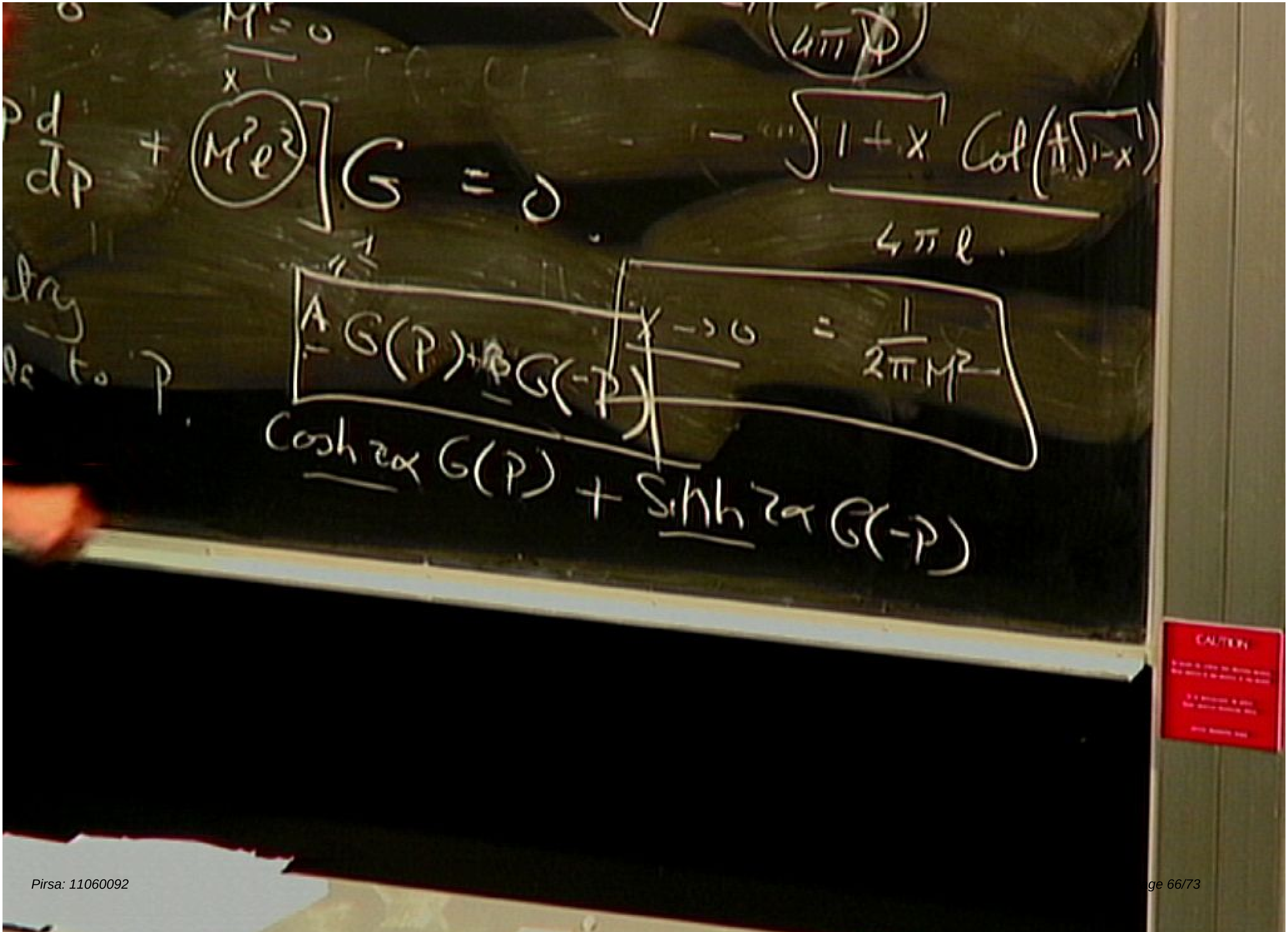

G(x, y)
Soln to Hypergeometric eqn
G(P) = 2F1
Symmetry
G = 0.
4πℓ

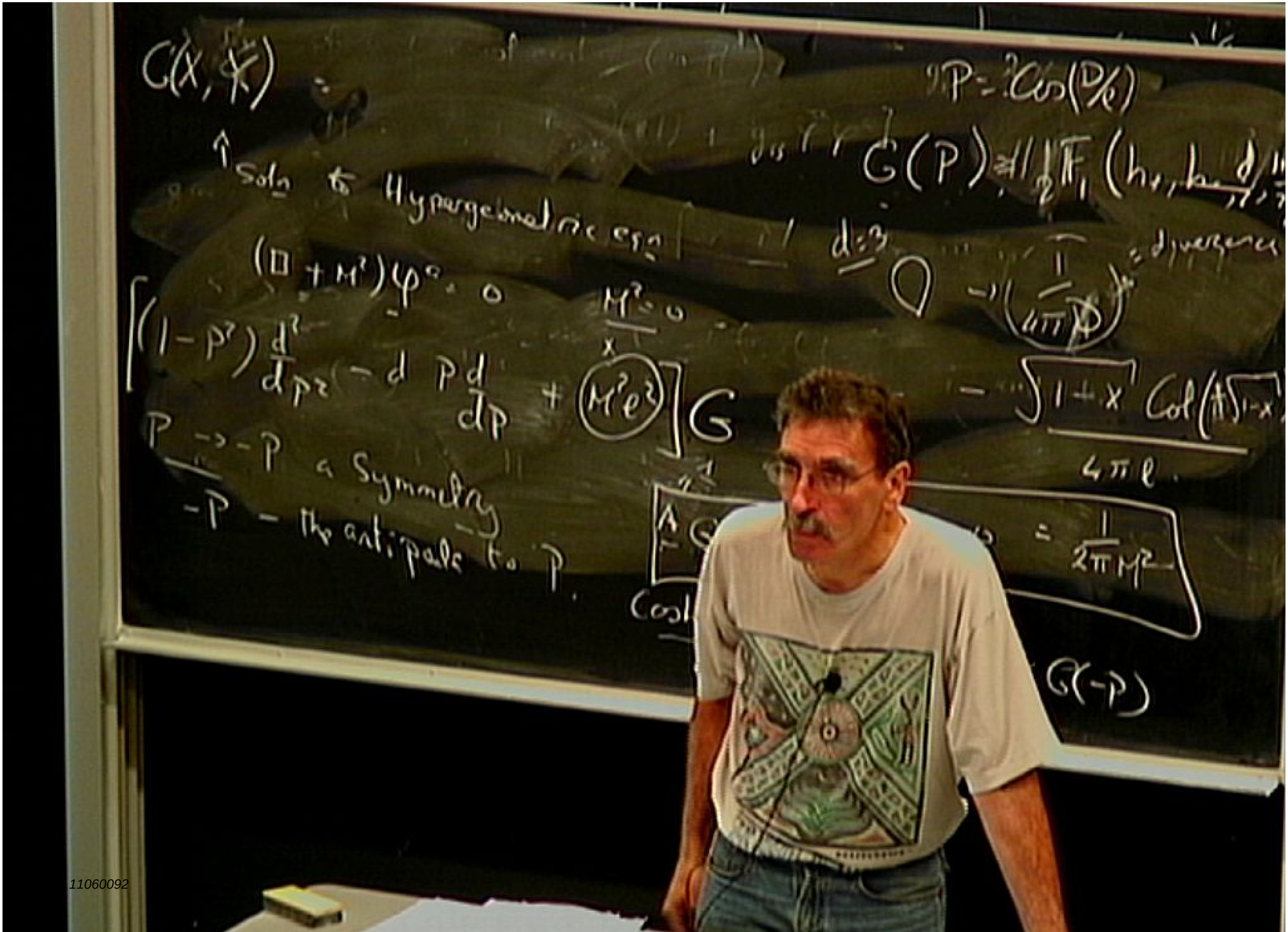

P → -P ← Symmetry
-P ← the antipode to P.



G(P)
sols to Hypergeometric eqs
G = 0
P → -P a Symmetry
-P = the antipode to P
Cosh 2α G(P) + Sinh 2α G(-P)


d/dp + M²ℓ²] G = 0
4πℓ
A G(p) + B G(−p)
x → 0 = 1/(2πM²)
Cosh 2α G(p) + Sinh 2α G(−p)
CAUTION


G = 0
4πℓ
x → 0
Cosh 2α G(P) + Sinh 2α G(-P)


Soln to Hypergeometric eqn
divergence
P -> -P a Symmetry
-P - the antipode to P

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$$G = G_0(P) - G'(-P) \checkmark$$

$$\boxed{\frac{1}{g_B} = \frac{1}{g(\mu)} - \frac{M^2}{16\pi^2} \ln\frac{\mu^2}{\Lambda^2}}$$

$$\frac{\Gamma_B}{g_B} - \text{inv}$$

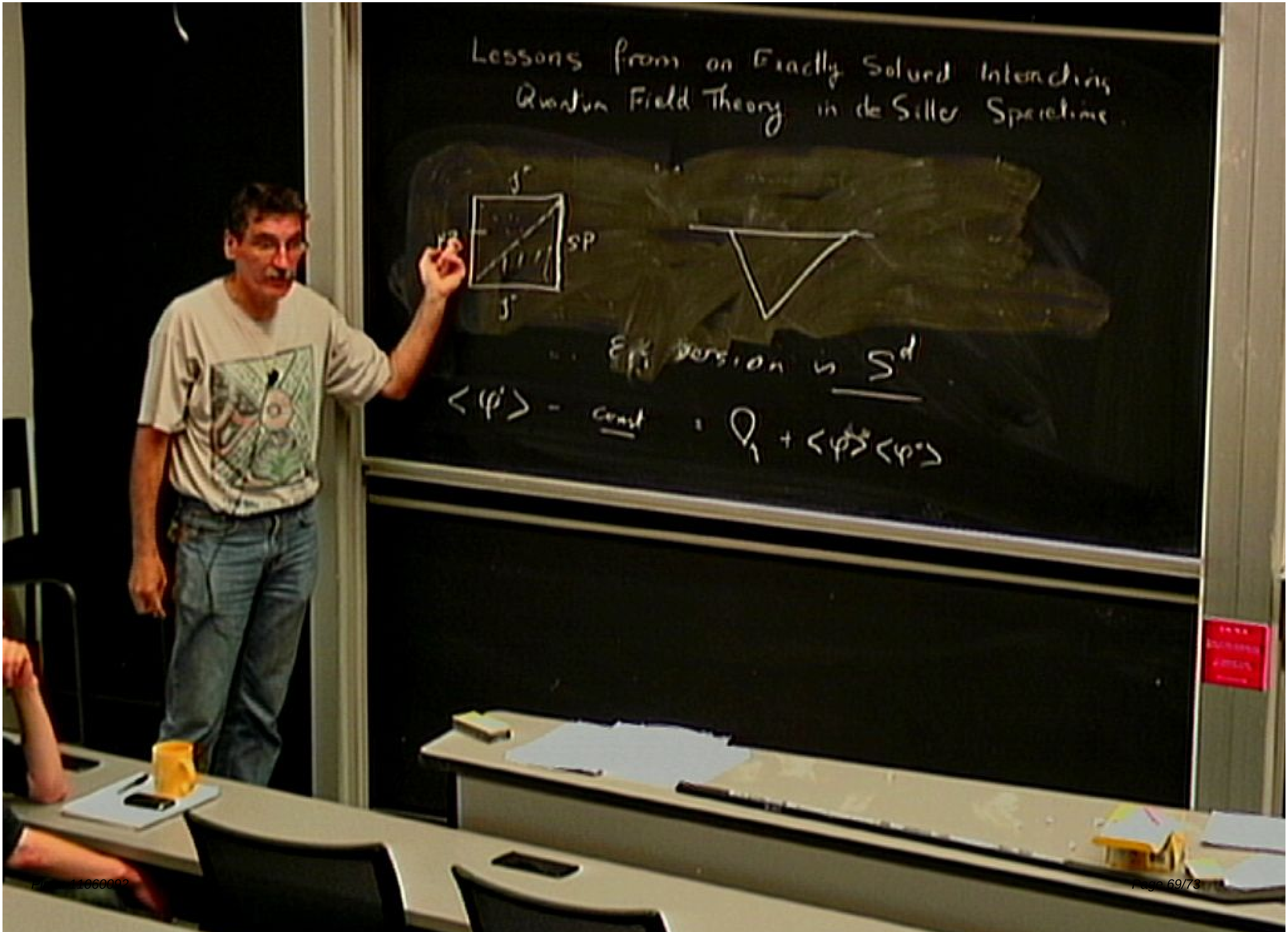
$$\boxed{\beta(g) = \frac{g^2}{8\pi^2}}$$

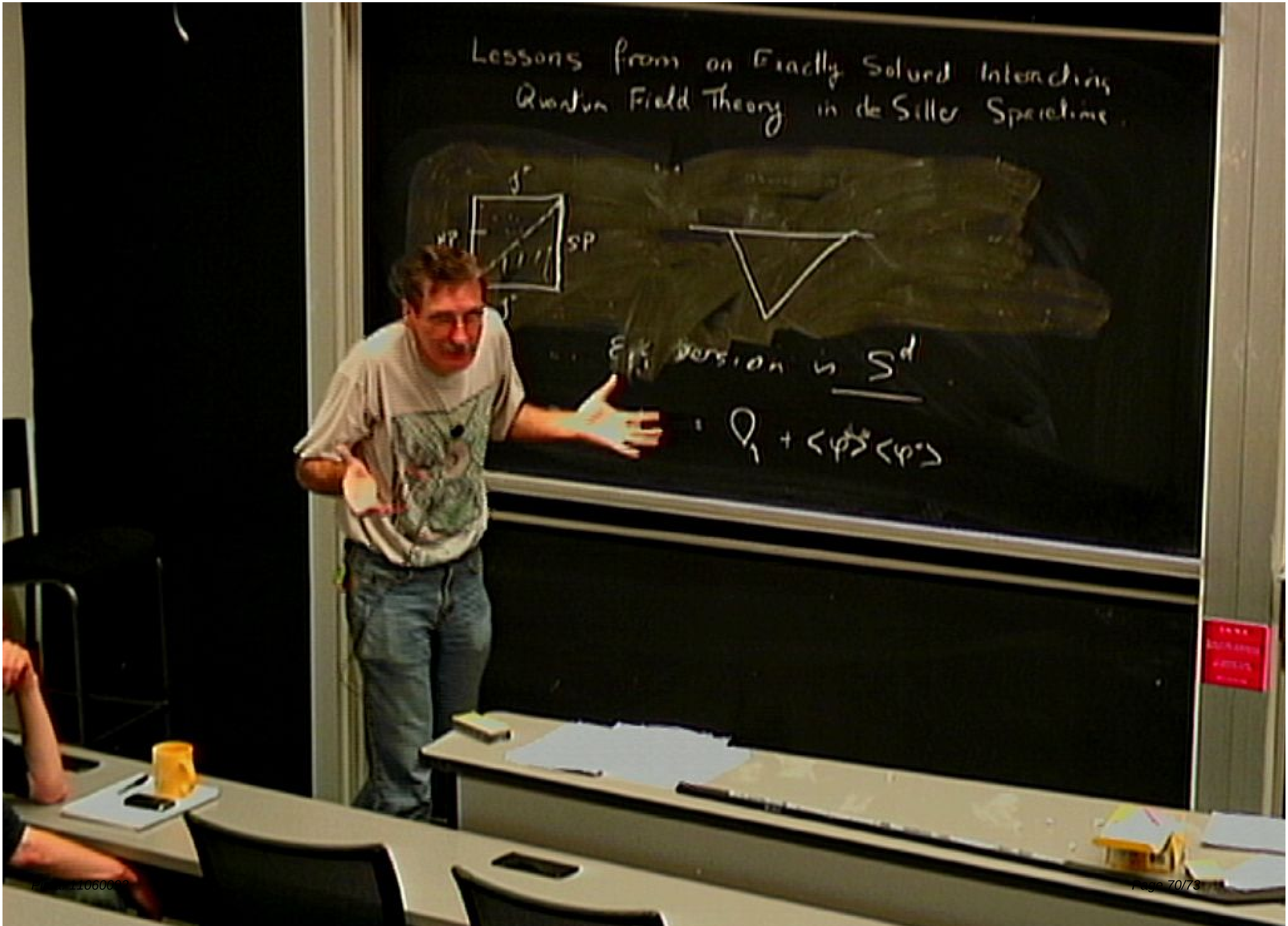
$$\frac{x_{min}^2}{4} = \Lambda^{-2}$$

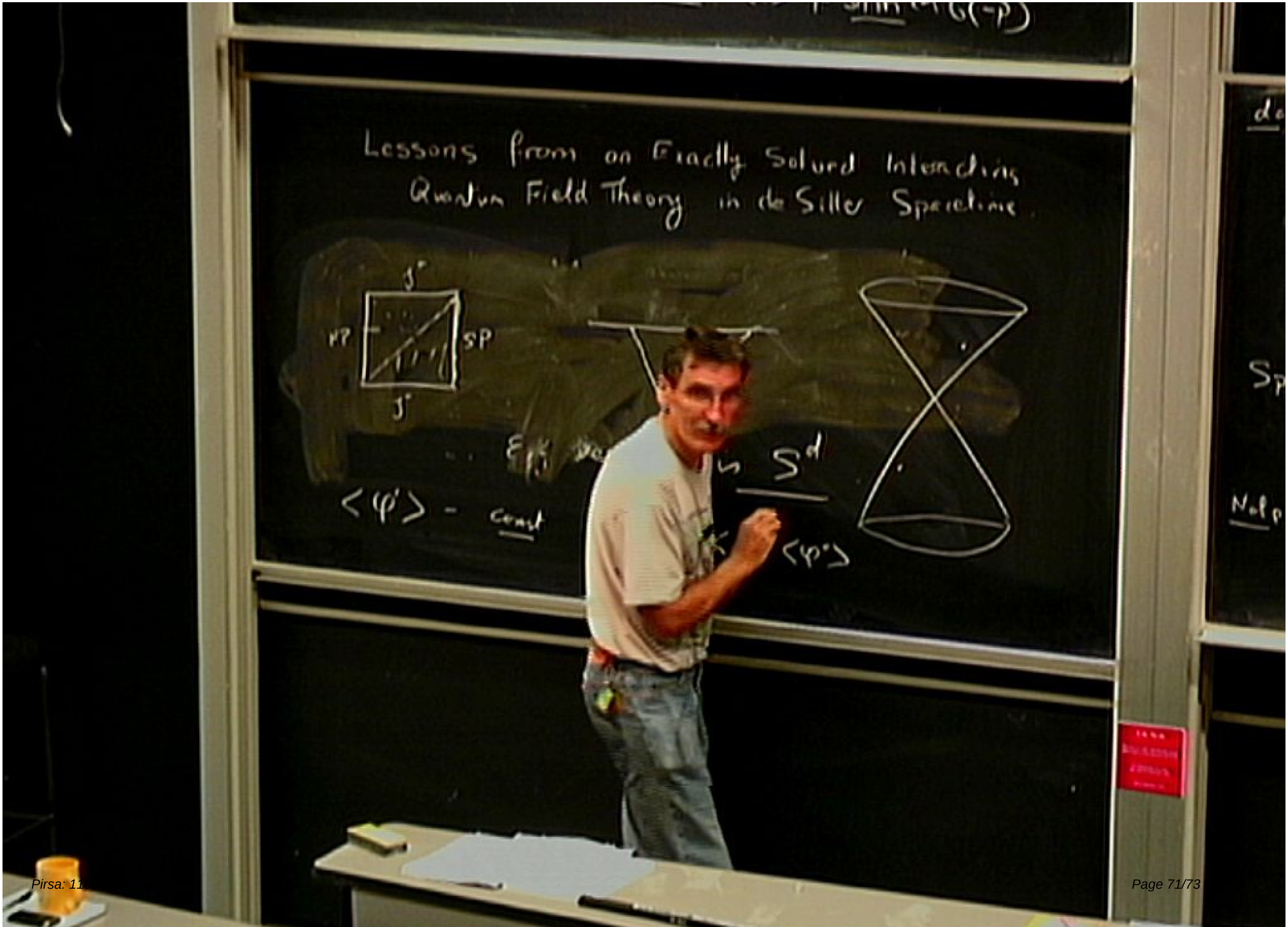
$$Q = \frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2}\left(\ln\frac{M^2}{\Lambda^2} + 2\gamma - 1\right)$$

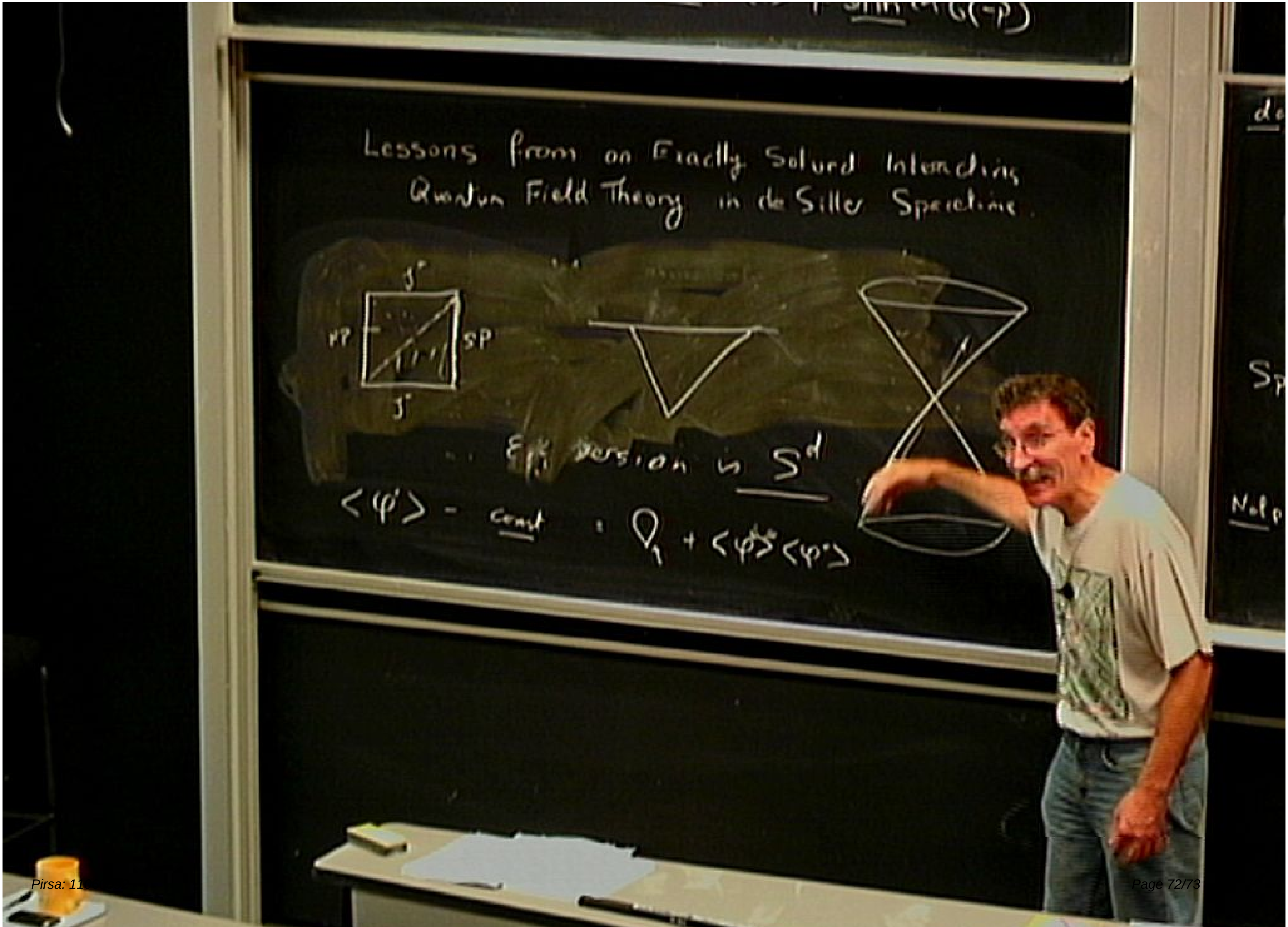
$$\frac{\Gamma_B}{g_B} + \left[\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2}\ln\left(\frac{M^2}{\Lambda^2} + 2\gamma - 1\right)\right] = \frac{M^2}{g_B}$$

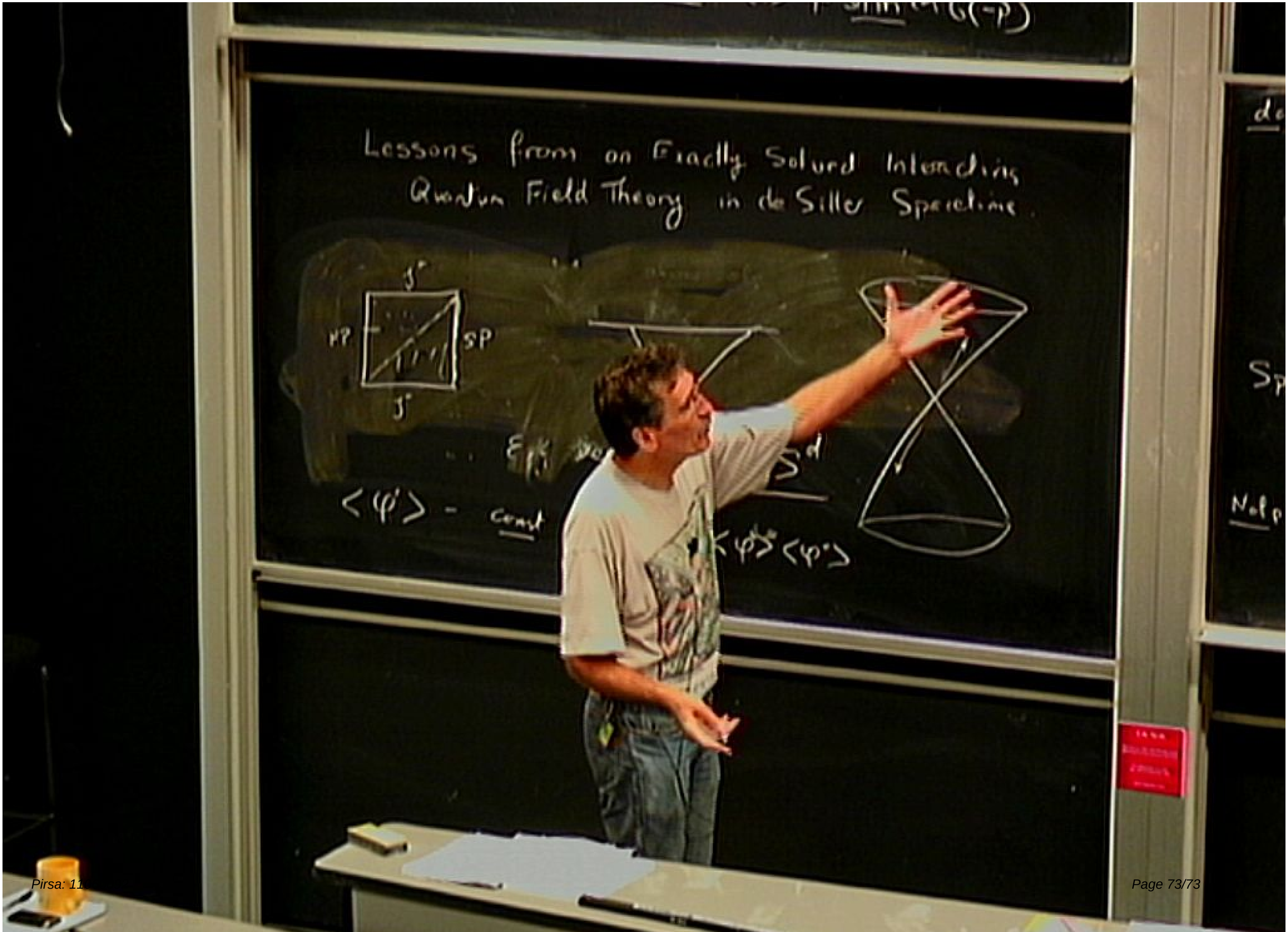
$$\frac{\Gamma_B}{g_B} = \tau$$


Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime


Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime


Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
SP
$\langle \varphi^2 \rangle$ - const
S^d


Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
SP



Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime
NP
SP