

Title: Lessons from an exactly solved interacting quantum field theory in de Sitter spacetime

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Abstract: It is argued that the correct quantization of a scalar field theory in de Sitter spacetime involves a de Sitter invariant state which is not the Bunch-Davies vacuum. A novel but natural de Sitter invariant alternative exists and it is suggested that this and is the preferred state for scalar field theories. The argument is based on the exact solution of an interacting scalar field theory.

The spherical constraint (in M^d)

$$M^2 = \Gamma_3 + g_{\beta}^2 [O(\beta) + g_{\beta} \bar{\psi} \psi]$$

$$g_{\beta} = \frac{2\beta H}{6}$$

$$\boxed{G^{-1} : P^2 + M^2}$$

gives a self-consistent eqn for \underline{M} . the mass of Goldstone Particles.

$$\langle \psi^a \rangle = ?$$

$$\langle \psi^a \rangle(H)$$

$$\xrightarrow{H \rightarrow 0} \bar{\varphi}^a$$

const external field H^a

and $\dim H^a \rightarrow 0$

The spherical constraint (in M^d)

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const external field H^a and $\dim H^a \rightarrow 0$

$$\xrightarrow{H \rightarrow 0} \bar{\varphi}^a$$

$$(\square + M^2) \psi^a = H^a$$

$$\langle \square \psi^a + \Gamma_B \psi^a + g_B^2 \psi^2 \psi^a \rangle = H^a$$



The spherical constraint (in M^4)

$$M^2 = \Gamma_B + g_{\frac{1}{2}}^2 [O(M) + g_{\frac{1}{2}} \bar{\psi} \psi]$$

$$g_{\frac{1}{2}} = \frac{2gH}{c}$$

$$\boxed{G^{-1} : P^2 + M^2}$$

gives a self-consistent eqn for \underline{M} . the mass of Goldstone particles

$$\langle \psi \rangle = ?$$

$$\langle \psi^a \rangle (H)$$

const external field \underline{H}^a

and $\dim H^a \rightarrow 0$

$$(\Omega + M^2) \psi^a = H^a \Leftrightarrow \psi^a \neq 0 \quad \langle \Omega \psi^2 + \Gamma_B \psi^2 + g_{\frac{1}{2}}^2 \bar{\psi} \psi \rangle = H^a$$

New phase occurs if $M^2 < 0$.

Large N limit of $O(N)$ vector model.

$\phi^a(x)$ $a=1, \dots, N$
 Look at $\frac{\lambda}{4}(\phi^a \phi^a)^2$ - potential

$$S = N \int d^d x \sqrt{-g} \left[\frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a + \frac{g_0}{2} \phi^a \phi^a - \frac{\lambda_0}{4!} (\phi^a \phi^a)^2 \right]$$

$g_0 \in \mathbb{R}, \lambda_0 N > 0$

$$S[\varphi] = S_2[\varphi] + \lambda S_4[\varphi] \quad \tilde{\varphi}$$

$$= N \left[S_2 \left[\frac{\varphi}{\sqrt{N}} \right] + (N\lambda) S_4 \left[\frac{\varphi}{\sqrt{N}} \right] \right]$$

λN : Fixed 't Hooft coupling

Exactly solvable on Lattice, in any d. and has a proper power diagram.
 2nd order transition
 ≡ Universal Berlin-Mittelman
 Spherical Model.



Berlin Kern Spherical Model

d=3

$$G(x) = \frac{1}{4\pi r} e^{-Mr}$$

$$r = \sqrt{(x_1)^2 + (x_2)^2}$$



d=3

$$G(x) = \frac{1}{4\pi r} e^{-Mr}$$
$$\langle \phi(x) \phi(0) \rangle$$

$$r = \sqrt{(x^i)^2 - (t^i)^2}$$



do3

$$G(x) = \frac{1}{4\pi r} e^{-Mr}$$
$$\langle \phi(x) \phi(0) \rangle \xrightarrow{x \rightarrow 0} \langle \phi^2 \rangle$$

$$r = \sqrt{(x^2 + t^2)}$$

$$\frac{1}{4\pi r_{\min}} \frac{M}{4\pi} + \dots$$

$r_{\min} = \lambda^{-1}$

$$\Gamma_B = -\frac{1}{4\pi r_{\min}} + t$$

$$G(x) = \frac{1}{4\pi r} e^{-\frac{1}{r}}$$

$$\langle \phi(x) \phi(0) \rangle \xrightarrow{x \rightarrow 0} \langle \phi^2 \rangle$$

$$r = \sqrt{(x^2 + t^2)}$$

$$\frac{1}{4\pi r_{\min}} \quad \frac{M}{4\pi}$$

$$r_{\min} = \lambda^{-1}$$

$$\Gamma_B = -\frac{1}{4\pi r_{\min}} + t$$

Spherical constraint becomes.

$$t \approx \frac{g M}{4\pi} = M^2$$

M

$$G(x) = \frac{1}{4\pi r} e^{-\lambda r}$$

$$\langle \phi(x) \phi(0) \rangle$$

$$\lim_{x \rightarrow 0} \langle \phi^2 \rangle$$

$$r = \sqrt{x^2 + t^2}$$

$$\frac{1}{4\pi r_{\min}} = \frac{M}{4\pi} + \dots$$

$$r_{\min} = \lambda^{-1}$$

$$r_B = -\frac{1}{4\pi r_{\min}} + t$$

Spherical constraint becomes.

$$t \approx \frac{g M}{4\pi} = M^2$$

Note: $t=0 \Rightarrow M=0$.

$$M(t) = \frac{g}{8\pi} \left(-1 + \sqrt{1 + \frac{t 8\pi}{g}} \right)$$

$$\underline{b < 0}$$

$$0 + \langle \psi | \psi \rangle$$



231(1)

$$\frac{b < 0}{b + 4\varphi^2 = 0} \quad \sum \varphi$$

$$0 + \langle \varphi \rangle \langle \varphi \rangle$$

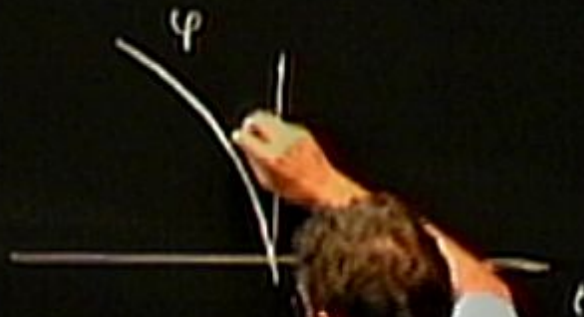


SAFETY
WARNING
FIRE

231(t)

$$\frac{t < 0}{t + 4\psi^2 = 0} \quad \Rightarrow \quad |\psi| = \left(\frac{-t}{4}\right)^{1/2}$$

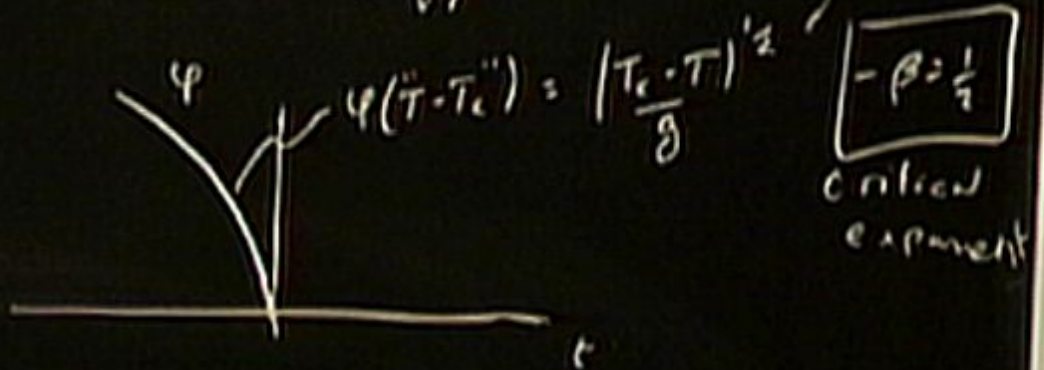
Phase diagram.



$$\langle \psi \rangle + \langle \psi \rangle \langle \psi \rangle$$

$$\frac{b < 0}{t + \beta\varphi^2 = 0} \quad \Rightarrow \quad |\varphi| = \left(\frac{-t}{\beta}\right)^{1/2}$$

Phase diagram .



r_{min}

$g \rightarrow \infty$

a universal
answer

$$M(t) = \frac{g}{8\pi} \left(-1 + \sqrt{1 + \frac{t^2 8\pi}{g}} \right)$$

Parameter value for a massless
particle.

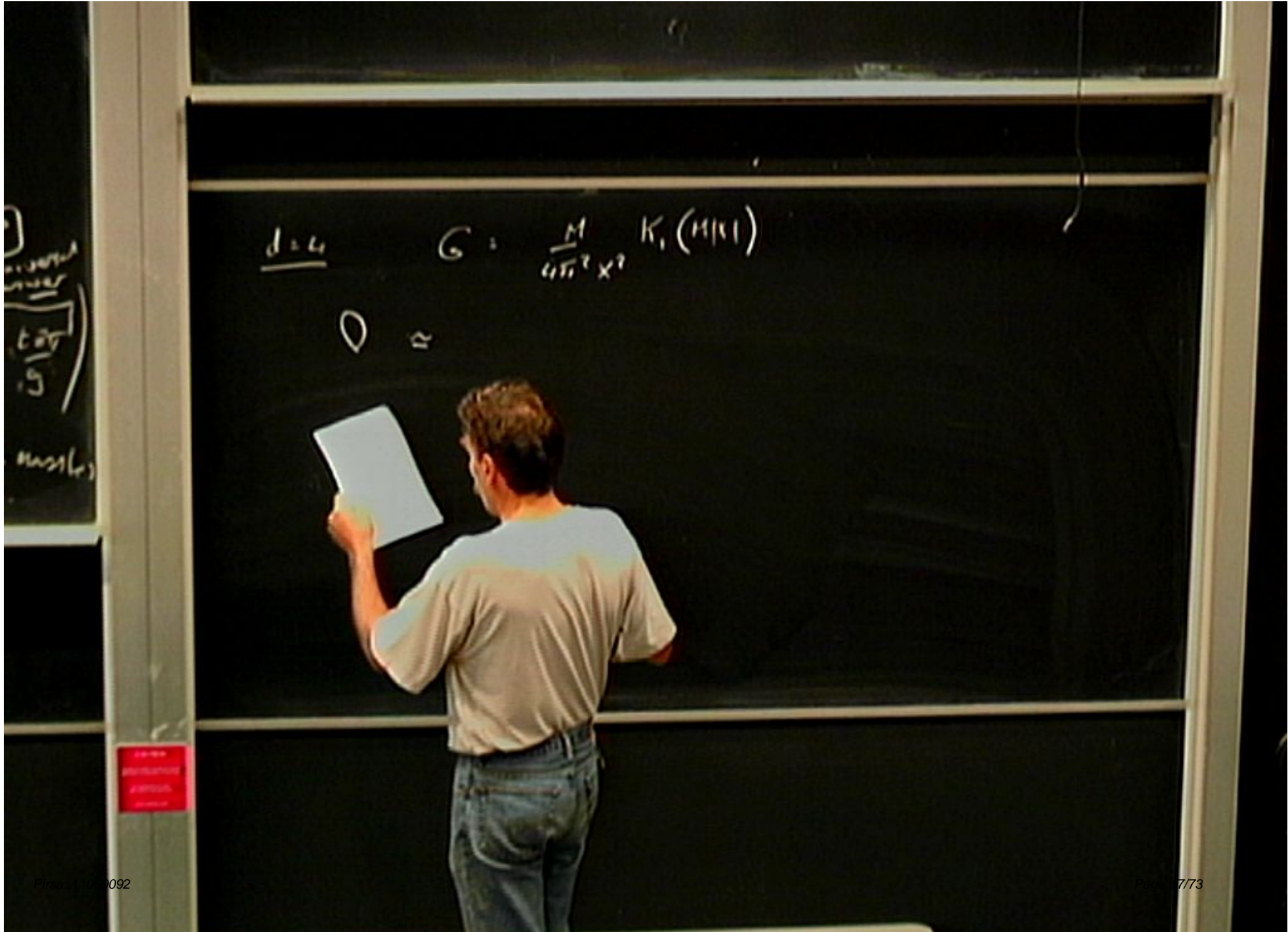
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Parameter value for a massless
particle.



$$\underline{d=4}$$

$$G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$$

\mathcal{O} \mathbb{R}

divergent
power
 $\frac{1}{s}$
massless

SAFETY
INSTRUMENTS
CORPORATION

d = 4 $G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$

$0 \approx \frac{1}{4\pi^2 x_{min}^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right)$

divergent
power
 $\left(\frac{d-2}{2} \right)$
massless



(1) $\frac{d}{dt} \ln \frac{1}{g}$
 (2) $\frac{d}{dt} \ln \frac{1}{g}$

$\frac{d}{dt} \ln \frac{1}{g} = \frac{1}{g} \frac{dg}{dt}$
 $G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$

$\frac{d}{dt} \ln \frac{1}{g} = \frac{1}{4\pi^2 x_{min}^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2 x_{min}^2}{16\pi^2} + 2\gamma - 1 \right)$

$\frac{d}{dt} \ln \frac{1}{g} = \frac{A^2}{16\pi^2} - \frac{M^2}{16\pi^2}$



10/10/092

1) $\frac{d}{dx}$
 2) $\frac{d}{dx}$
 3) $\frac{d}{dx}$

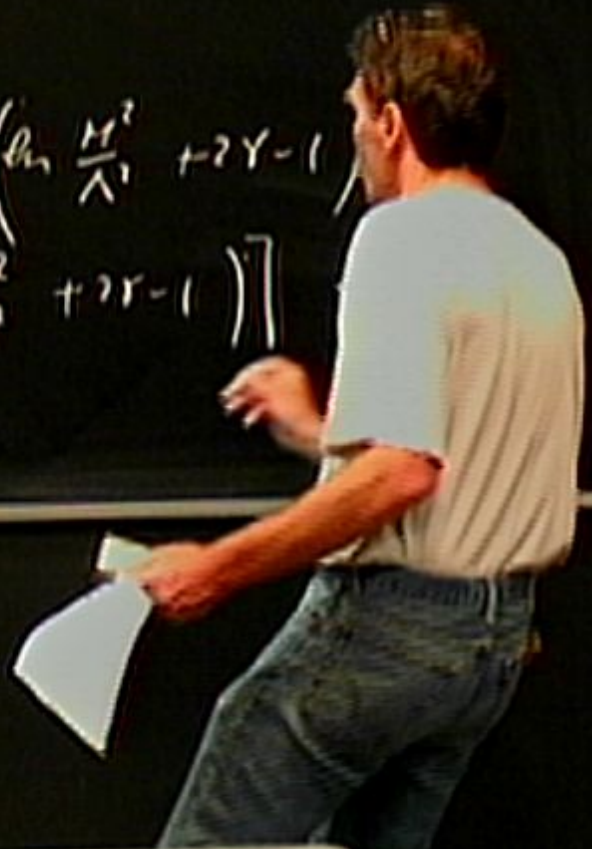
$$\frac{d}{dx} \quad G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$$

$$0 \approx \frac{1}{4\pi^2 x_{min}^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2 x_{min}^2}{\Lambda^2} + 2\gamma - 1 \right)$$

$$\frac{x_{min}^2}{\Lambda^2} = \Lambda^{-2}$$

$$0 = \frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right)$$

$$\frac{d}{dx} \Rightarrow \left[\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \ln \left(\frac{M^2}{\Lambda^2} + 2\gamma - 1 \right) \right]$$



$\frac{d}{dt}$
 $\frac{d}{dt}$
 $\frac{d}{dt}$

$$\frac{d}{dt} \quad G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$$

$$0 \approx \frac{1}{4\pi^2 x_{min}^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2 x_{min}^2}{\Lambda^2} + 2\gamma - 1 \right)$$

$$\frac{1}{4\pi^2 x_{min}^2} = \frac{M^2}{16\pi^2}$$

$$0 \approx \frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right)$$

$$\frac{d}{dt} \Rightarrow \left[\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right) \right] = \frac{M^2}{5}$$



1)

$$\frac{1}{g_B} = \frac{1}{g(\mu)} - \frac{M^2 \mu^2}{16\pi^2 \Lambda^2}$$

$$\ln \frac{M^2 \mu^2}{\Lambda^2} + 2\gamma - 1$$

$$\frac{\mu_B}{g_B} = \ln \mu$$

$$\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1$$

$d = 4$ $G = \frac{M}{4\pi^2 x^2} K_1(M|x|)$

\circ \circ $\frac{1}{4\pi^2 x_{min}^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2 x_{min}^2}{\Lambda^2} + 2\gamma - 1 \right)$

$\frac{x_{min}^2}{4} = \Lambda^{-2}$

\circ $\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right)$

$\left[\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \ln \left(\frac{M^2}{\Lambda^2} + 2\gamma - 1 \right) \right] = \frac{M^2}{S_B}$

$\frac{L}{S_B} = 2$

$\frac{1}{g_B} = \frac{1}{g(\mu)} - \frac{M^2}{16\pi^2} \frac{L}{\Lambda^2}$

$\frac{L}{g_B} = \ln \nu$

$\beta(g) = \frac{g^2}{8\pi^2}$



Final temp:

Finite temp : $\mathbb{R}^{d-1} \times S^1$

Finite temp: $\mathbb{R}^{d-1} \times S^1$

d=3

$g \rightarrow \infty$

$$z = \frac{M}{4\pi} - \ln(1 - e^{-M/\pi})$$

Finite temp: $R^{d-1} \times S^1$

$d=3$ $g \rightarrow \infty$

$$Z = \frac{M}{4\pi} - \ln(1 - e^{-M/\pi})$$

$M \rightarrow 0$ for $\tau \rightarrow -\infty$

No phase transition

Finite temp: $\mathbb{R}^{d-1} \times S^1$

$d=3$

$g \rightarrow \infty$

$$Z = \frac{M}{4\pi} - \ln(1 - e^{-M/\tau})$$

$M \rightarrow 0$ for $\tau \rightarrow \infty$

- Goldstone modes prevent the transition

No phase transition

Finite temp: $\mathbb{R}^{d-1} \times S^1$

$d=3$ $g \rightarrow \infty$

$$Z = \frac{M}{4\pi} - \ln(1 - e^{-M/T})$$

$M \rightarrow 0$ for $T \rightarrow -\infty$

- Goldstone modes prevent the transition
 $3 < d < 4$

No phase transition

$$Z \neq \frac{6d}{T^{d-2}} = \frac{10}{T^{d-2}} (M/T)$$

Finite temp: $\mathbb{R}^{d-1} \times S^1$

$d \geq 3$

$g \rightarrow \infty$

$$z = \frac{M}{4\pi} - \ln(1 - e^{-M/T})$$

$M \rightarrow 0$ for $T \rightarrow \infty$

- Goldstone modes present. The transition

No phase transition

$3 < d < 4$

$z \approx \frac{6d}{T^{d-2}} = \frac{10}{T^{d-2}} (M/T) \sim$ interpolates between $d=2$ and $d=3$

Finite temp: $\mathbb{R}^{d-1} \times S^1$

$d=3$ $g \rightarrow \infty$

$$z = \frac{M}{4\pi} - \ln(1 - e^{-M/T})$$

$M \rightarrow 0$ for $T \rightarrow \infty$

- Goldstone modes present
 $3 < d < 4$

The transition

No phase transition

$$z \neq \frac{6d}{T^{d-2}} = M \sqrt{\frac{d}{4\pi T}}$$

interpolates between $\frac{1}{4\pi T}$ and $\frac{6d}{T^{d-2}}$

$2^{d-1} \times S'$

$$T_c = \left(\frac{-\zeta}{b_d} \right)^{\frac{1}{d-2}}$$

$$\ln(1 - e^{-M/T})$$

for $\zeta \rightarrow -\infty$

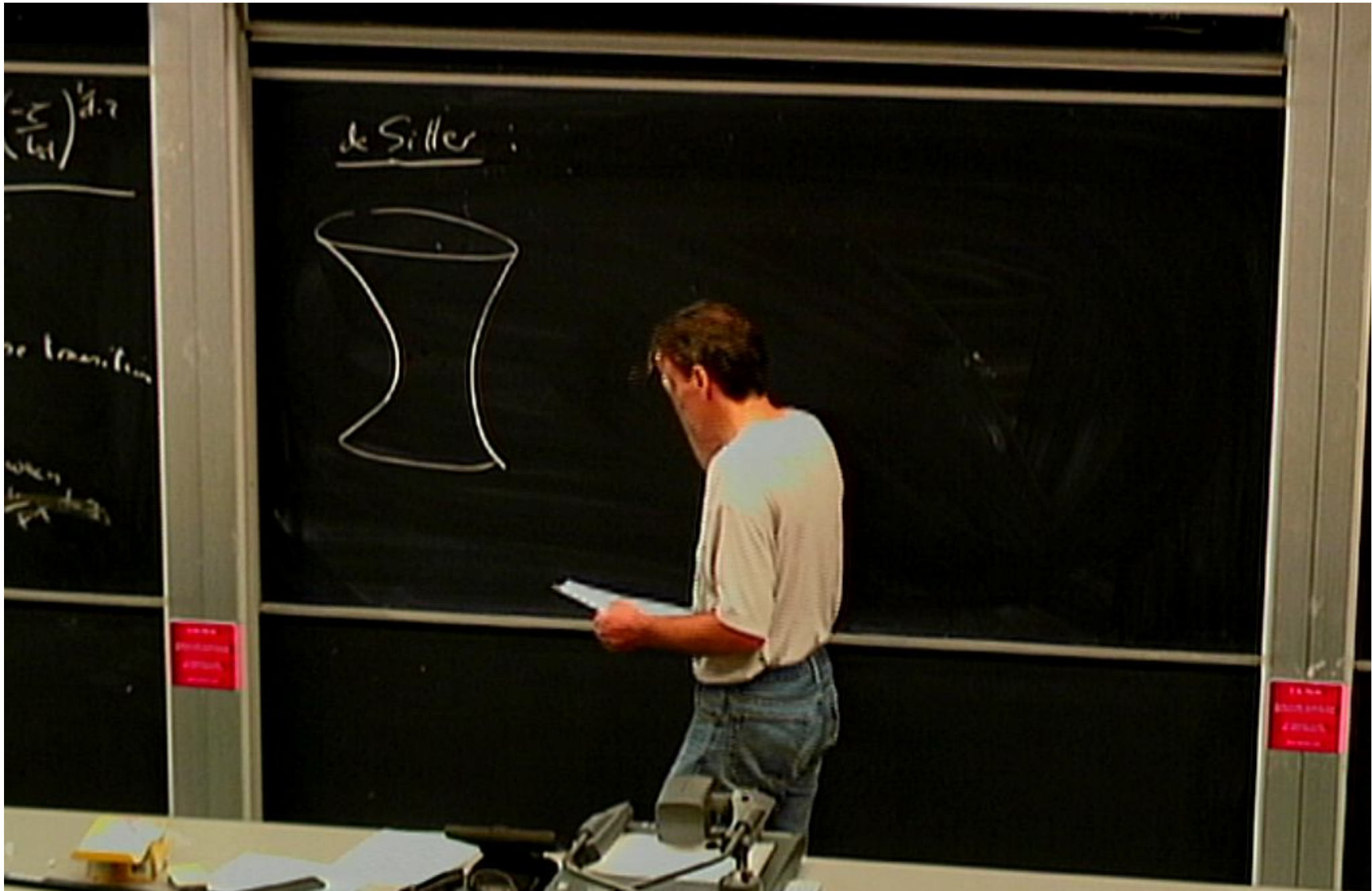
No phase transition

The transition

$$M^{d/2} f(M/T)$$

interpolates between

1 and $\frac{1}{d-2}$

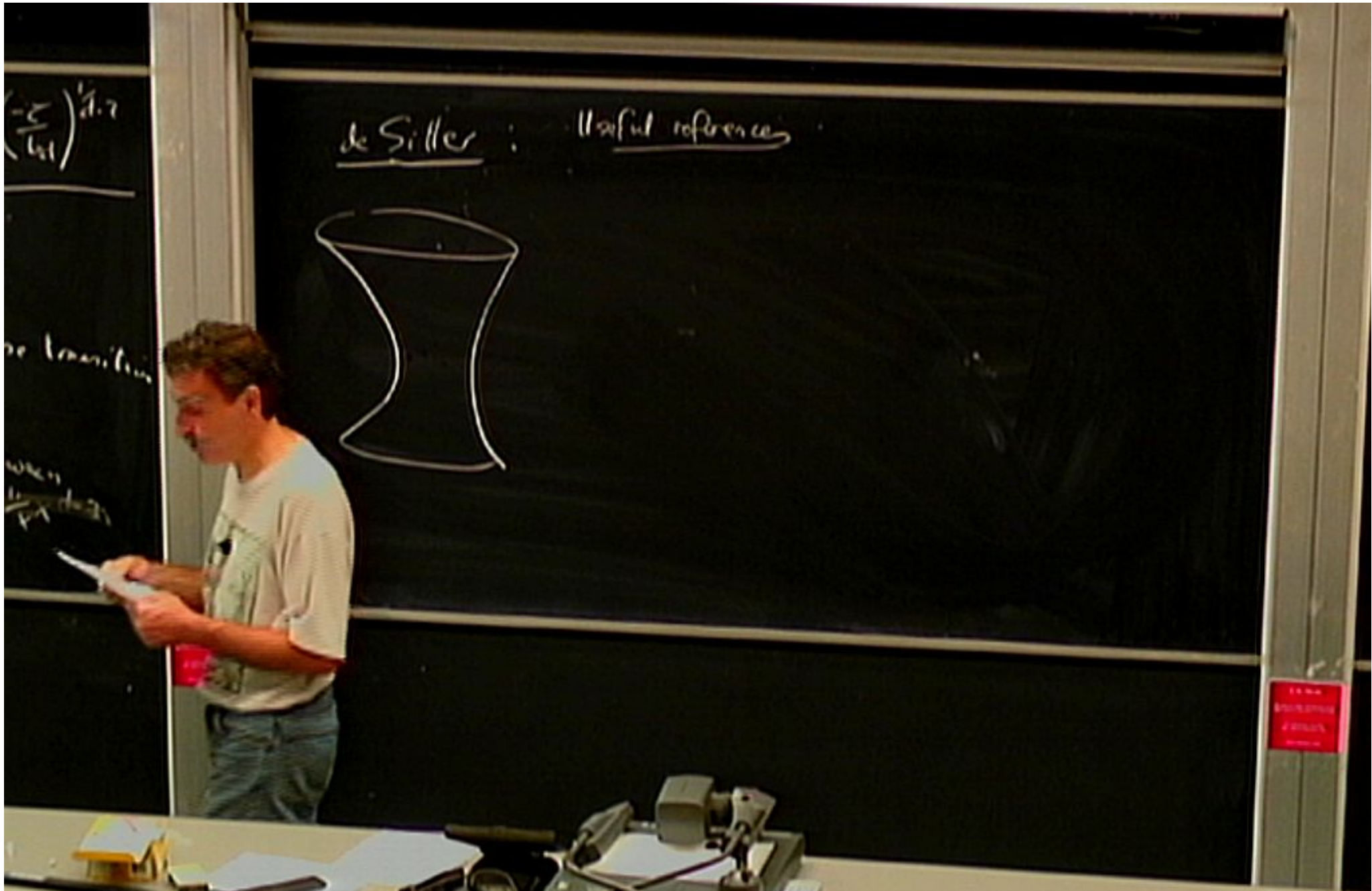


$$\left(\frac{-\epsilon}{4t}\right)^{1/d-2}$$

transition

de Sitter :





$$\left(-\frac{\epsilon}{4t}\right)^{1/d-2}$$

de Sitter : Useful reference



no transition

when

$$\left(\frac{-\sigma}{4\pi}\right)^{1/2}$$

no transition

over
the

de Sitter :

Useful references

Allen Phys Rev D 32 (1985) 3136

Spalding, Strominger and Volovich
arXiv hep-th/0008147



$$\left(\frac{-\epsilon}{4\pi}\right)^{1/2}$$

no transition

de Sitter

Useful references

Allen Phys Rev D 32 (1985) 3136

Spalding, Strominger and Volovich

arkiv hep-th/0112007

Mitoff & Morrison 1010.5327

Hollands 1010.5367



REDACTED

$$\left(\frac{-\epsilon}{4t}\right)^{1/2}$$

no transition

over

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1010.5327

Hollands

1010.5367

← Hyperboloid
in M^4



REDACTED

REDACTED

$$\left(\frac{-\epsilon}{4\pi}\right)^{1/2}$$

no transition

when

de Sitter : Useful references



Allen Phys Rev D 32 (1985) 3136

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Holland's 1010.5367

paraboloid
in M^4
 ρ^2

$$x^i = sh \tau$$
$$x^j = wch \tau$$

$$\left(\frac{-\epsilon}{4\pi}\right)^{1/2} d.7$$

no transition

de Sitter :

Useful references

Allen Phys Rev D 32 (1985) 3136

Spalding, Strominger na Volovich

arkiv hep-th/011007

Mitoff & Morrison

1010.5327

Hollands

1010.5367



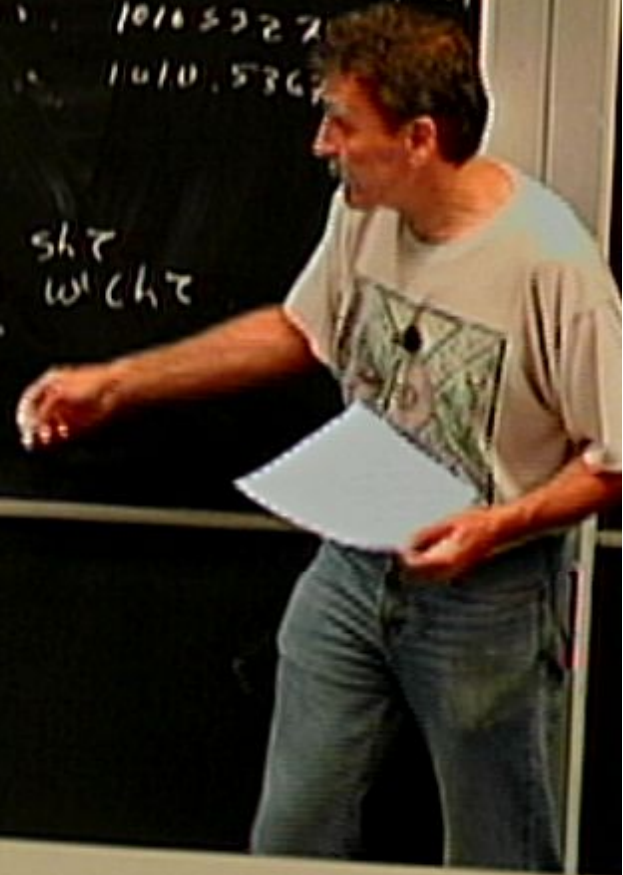
Hyperboloid
in $M^{1,3}$

$$-x_0^2 + x_1^2 = \rho^2$$

$$x^i = \rho \chi^i$$

$$x^i = \rho \chi^i$$

$$ds^2 = -dt^2 + \rho^2 d\Omega_{2,1}^2$$



Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



The spherical constraint - (in M^4)

$$M^2 = r_3 + g_3 [O(M)] + g_0 \bar{\psi} \psi$$

$$g_3 = \frac{2\lambda M}{\epsilon}$$

$$\boxed{G^{-1} = P^2 + M^2}$$

gives a self-consistent eqn for M - the mass of Goldstone

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



The spherical constraint (in M^4)

$$M^2 = r_3^2 + g_3^2 [O(M) + g_0 \bar{\psi} \psi]$$

$$g_3 = \frac{2g_0 M}{\epsilon}$$

$$\boxed{G^{-1} = P^2 + M^2}$$

gives a self-consistent eqn for \underline{M} . The mass of Goldstone

$$\langle \psi \bar{\psi} \rangle = ?$$

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime.



The SFT

gives a sol

$$\langle \varphi \rangle = ?$$

The transition

(in M^4)

$$[(-1) + g_0 \varphi^2]$$

$$g_0 = \frac{2gM}{\epsilon}$$

$$\boxed{G^{-1} : P^2 + M^2}$$

for M . the mass of Goldstone

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



... EFT version is S^d



... the same order prevent. The transition

... fixed constant - (in M^d)

$$= \Gamma_3 + g_3 [O(M) + g_{11} \bar{\psi} \psi] \quad g_3 = \frac{2g_1 M}{\epsilon}$$

$$\boxed{G^{-1} : P^2 + M^2}$$

... consistent eqs for M . the mass of Goldstone

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



... EFT version is S^d

<

The transition to the spherical constraint (in M^d)

$$M^2 = r_3 + g_3^2 [O(M) + g_0 \bar{\psi} \psi]$$

$$g_3 = \frac{2g_0 M}{\epsilon}$$

$$\boxed{G^{-1} = P^2 + M^2}$$

gives a self-consistent eqn for M . The mass of Goldstone

$\langle \psi \rangle = ?$

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



... EFT version is S^d

$\langle \varphi \rangle = \underline{\text{const}}$

The spherical constraint (in M^d)

$$M^2 = r_3 + g_3 [O(M) + g_0 \varphi^2]$$

$$g_3 = \frac{2g_0 M}{\epsilon}$$

$$\boxed{G^{-1} : P^2 + M^2}$$

gives a self-consistent eqn for M . The mass of Goldstone

$\langle \varphi \rangle = ?$

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



Euclidean version is S^d

$$\langle \varphi \rangle = \text{loop} + \langle \varphi \rangle^2 \langle \varphi \rangle$$

The sphere

$$M^2$$

gives a solp

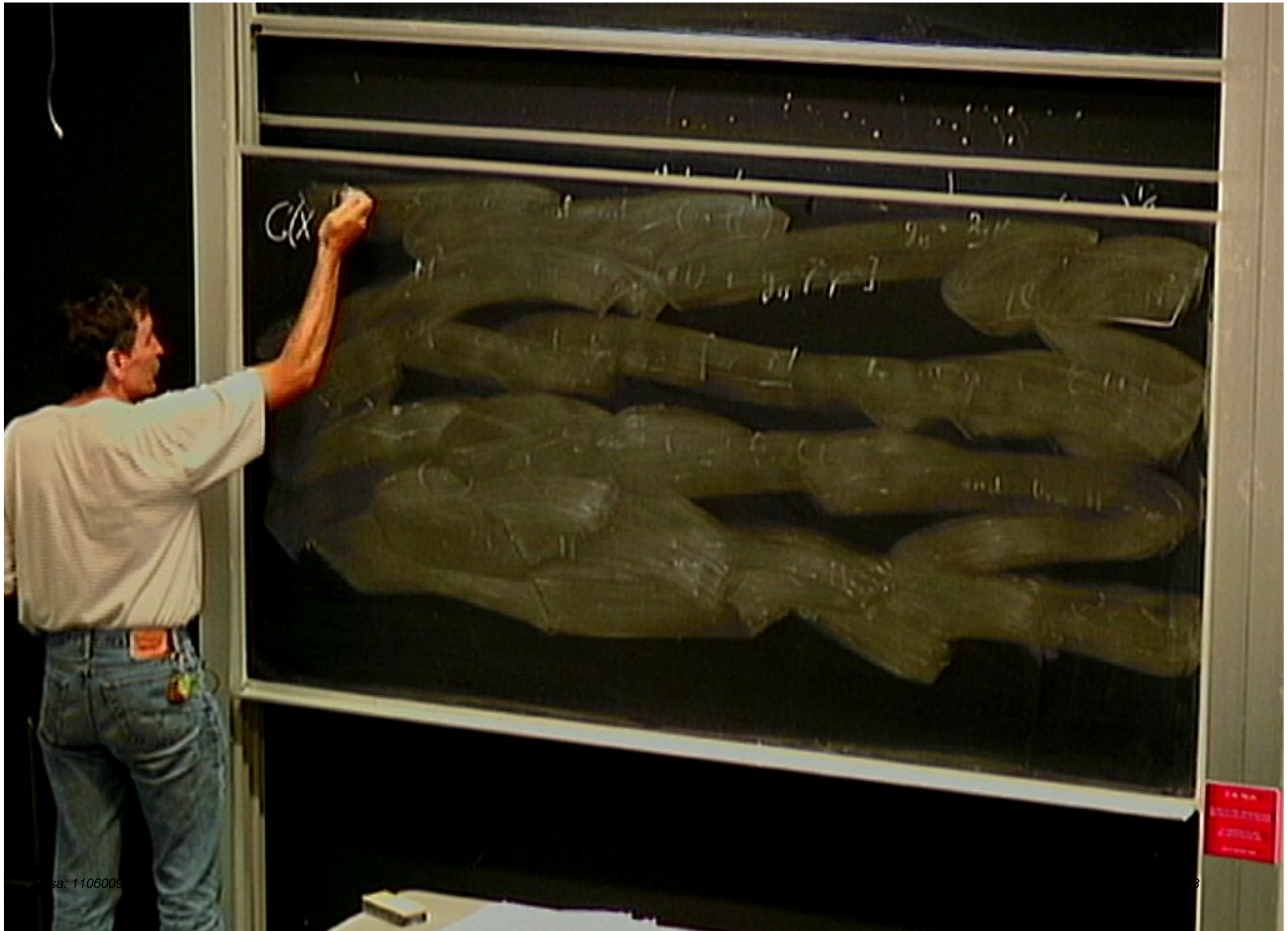
$$V(\varphi) + g_0 \varphi^2 \varphi^2$$

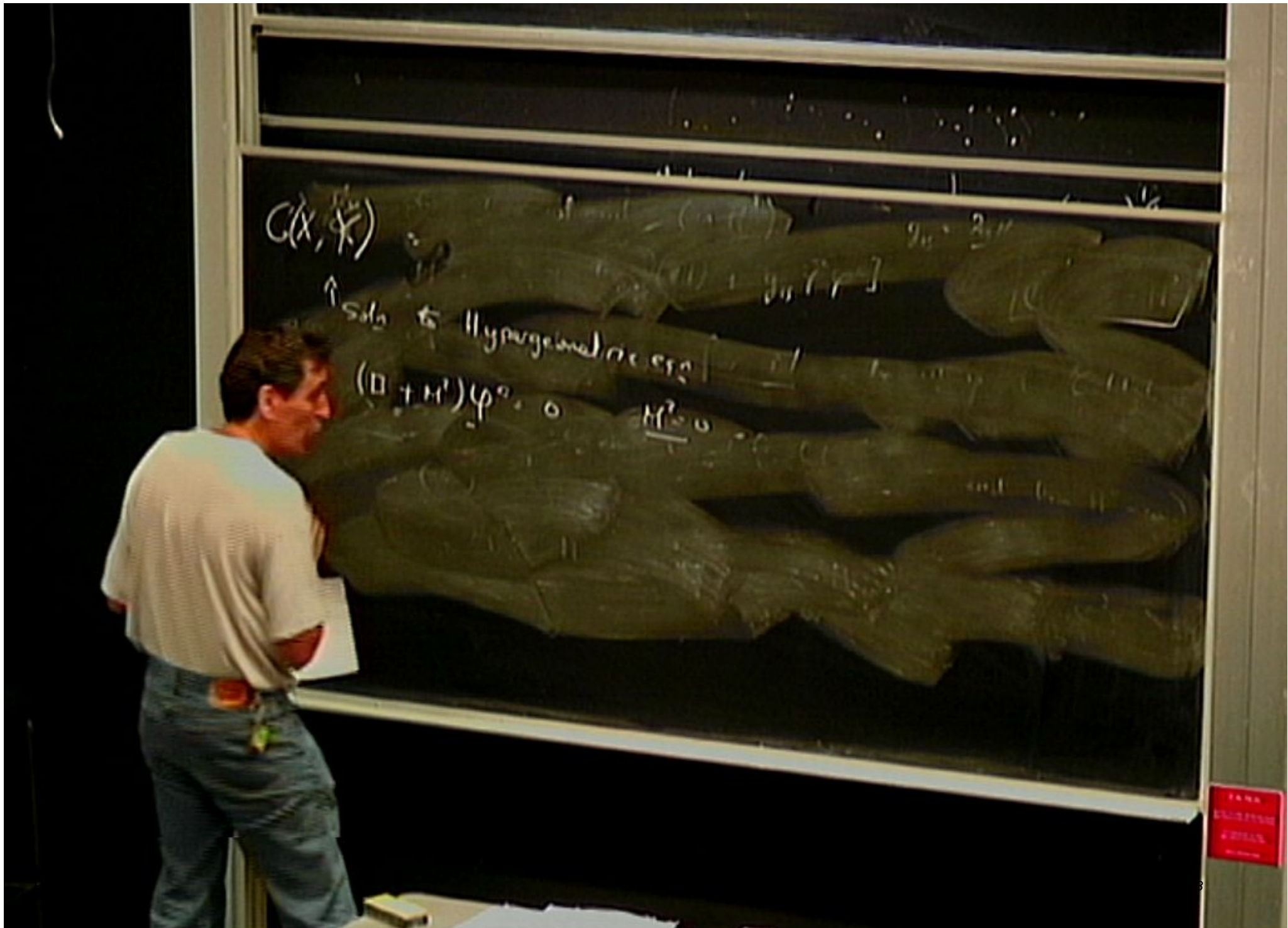
$$g_0 = \frac{2gM}{\epsilon}$$

$$\boxed{G^{-1} = P^2 + M^2}$$

for M the mass of Goldstone

$$\langle \varphi \rangle = ?$$





$$C(x, \lambda)$$

↑ Soln to hypergeometric eqn

$$(\partial + M^2)\varphi^2 = 0 \quad \underline{M^2 = 0}$$

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$G(x, \kappa)$

↑ Soln to Hypergeometric

$$(D + M')\varphi = 0$$

$$\left[(1-p^2) \frac{d^2}{dp^2} - d P \frac{d}{dp} \right]$$

$P \rightarrow -P$ a Symmetry

1000
1000000
1000000000

$G(x, \kappa)$

↑ Soln to hypergeometric eqn

$G(P)$

$$\left[(1-P^2) \frac{d^2}{dP^2} - d P \frac{d}{dP} + M^2 e^2 \right] G = 0$$

$P \rightarrow -P$ a Symmetry

$-P$ - the antipole to P .

$G(x, \kappa)$

↑ Soln to hypergeometric eqn

$(\square + M^2)\varphi^2 = 0$ $M^2 = 0$

$$\left[(1-p^2) \frac{d^2}{dp^2} - d \, p \frac{d}{dp} + M^2 e^2 \right] G = 0$$

$p \rightarrow -p$ a Symmetry

$-p$ - the antipole to p .

$G(p) = \frac{1}{2} F_1(\dots)$

$G(x, \kappa)$

$P = \cos(D/k)$

↑ Soln to hypergeometric eqn

$$G(P) = {}_2F_1\left(h, h, \frac{d}{1-d}\right)$$

$$(D + M^2)\varphi^2 = 0$$

$$M^2 = 0$$

$$Q = \frac{1}{4\pi}$$

$$\left[(1-P^2) \frac{d^2}{dP^2} - d P \frac{d}{dP} + M^2 e^2 \right] G = 0$$

$P \rightarrow -P$ a Symmetry

$-P$ - the antipole to P .

$$g_P = \cos(D/k)$$

$$G(P) = \frac{1}{2} F_1 \left(h_1, h_2, \frac{d}{112} \right)$$

metric eqn

$d=3$

$$\left[\begin{array}{l} M^2 = 0 \\ + M^2 e^2 \end{array} \right] G$$

$$\frac{1}{4\pi d}$$

$$\frac{\int \sqrt{1+x} \operatorname{Col}(\sqrt{1-x})}{4\pi l}$$

$$= \frac{1}{2\pi M^2}$$

$$P = \cos(D/k)$$

$$G(P) = \frac{1}{2} F_1 \left(h, k, \frac{d}{2}, \frac{1}{2} \right)$$

Hypergeometric eqn

$$+ M^2 \varphi^2 = 0$$

$$M^2 = 0$$

$$\frac{1}{4\pi P}$$

$$\left[-d P \frac{d}{dP} + M^2 \right] G = 0$$

$$- \frac{\int \sqrt{1+x} \cos(\sqrt{1-x})}{4\pi l}$$

Symmetry
the antipole to P.

$$\boxed{\frac{x \rightarrow 0}{2\pi M^2}}$$

d=3

Sphere

Note:

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$$\rho = \cos(\theta/k)$$

$$G(P) = \int \frac{F_1(h_1, k_1, d, \frac{1}{2})}{r^2}$$

Hypergeometric eqn

$$\varphi'' = 0$$

$$M^2 = 0$$

$d=3$

∇

$$\rightarrow \left(\frac{1}{4\pi r} \right) = \text{divergence}$$

$$\left[\frac{d}{dx} + \frac{x}{M^2} \right] G = 0$$

$$= \frac{\int \sqrt{1+x} \cot(\pi \sqrt{1-x})}{4\pi l}$$

symmetry
applies to P

$$\boxed{\lim_{x \rightarrow 0} = \frac{1}{2\pi M^2}}$$

$d=3$

Spherical

Notes:

CAUTION
DO NOT TOUCH THE BOARD
OR THE CHALK

... phase transition
 = Universality class of
 Berlin Katz Spherical
 Model

de3

36

$$(z + g\gamma^2) - \frac{\sqrt{1-x} \cot \pi \sqrt{1-x}}{4\pi g} = M^2$$

Spherical constraint becomes

$$t \equiv \frac{gM}{4\pi} = M^2$$

Note: $t=0 \Rightarrow M=0$

This is the parameter

$g \rightarrow \infty$
 a universal answer

$$M(t) = \left(-1 + \sqrt{1 + \frac{t \cot \pi}{g}} \right)$$

for a massive particle



... phase transition
 = Universality class of
 Berlin-Kac Spherical
 Model

do 3

$$Z_1 = \int_{-1}^1 \sqrt{1-x} \underbrace{\int_0^{2\pi} d\phi \pi \sqrt{1-x}}_{\text{circumference}} + M R$$

$$= \frac{4\pi}{R} M R^2$$

Spherical constraint becomes

$$t \approx \frac{g M}{4\pi} = M^2$$

Note: $t=0 \Rightarrow M=0$

This is the critical value for a massive particle.

$$M(t) = \frac{g}{8\pi} \left(-1 + \sqrt{1 + \frac{t \cdot 8\pi}{g}} \right)$$

$g \rightarrow \infty$
 a universal answer



... phase transition
 = Universality class of
 Berlin-Kim Spherical

do3

$$Z_1 = \frac{1}{(2\pi)^{3/2}} \int_0^1 \sqrt{1-x^2} \int_0^{2\pi} \int_0^\pi \sqrt{1-x^2} \sin\theta d\theta d\phi dx$$

+ MR

$$\frac{4\pi}{3} M^3 = \int_{Vol} M^3 = \text{Euclidean Vol.}$$

Spherical constraint

$g \rightarrow \infty$
 a universal answer

$$M(t) = \frac{g}{2\pi} \left(-1 + \sqrt{1 + \frac{t \cdot 2\pi}{g}} \right)$$

Note: $t=0 \Rightarrow$

This is the parameter value for a massive particle.

... phase transition
 = Universality class of
 Berlin & Spherical

d=3

$$Z_1 = \int_{\text{Vol}} \sqrt{1-x} \text{Vol} \pi \sqrt{1-x} \quad \neq M^2$$

$$- \frac{M^2}{2\pi M^2} = \boxed{\text{Vol}} M^2$$

Euclidean Vol.

Spherical constraint becomes

$$t \neq \frac{g M}{4\pi} = M^2$$

Note: $t=0 \Rightarrow M=0$

This is the parameter u



... phase transition
 = Universality class of
 Berlin Katz Spherical Model

d=3

$$Z_1 = \frac{1}{(2\pi)^3} \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 \dots$$

$\neq M^2$

$$- \frac{M^2}{2\pi M^2} = \frac{1}{\text{Vol}} M^2$$

Euclidean Vol.

Spherical constraint becomes

$$t \neq \frac{g M}{4\pi} = M^2$$

Note: $t=0 \Rightarrow M=0$

Bunch Davies vacuum

$g \rightarrow \infty$
 a universal answer

$$M(t) = \frac{g}{2\pi} \left(-1 + \sqrt{1 + \frac{t 2\pi}{g}} \right)$$

This is the parameter value for a massive particle.

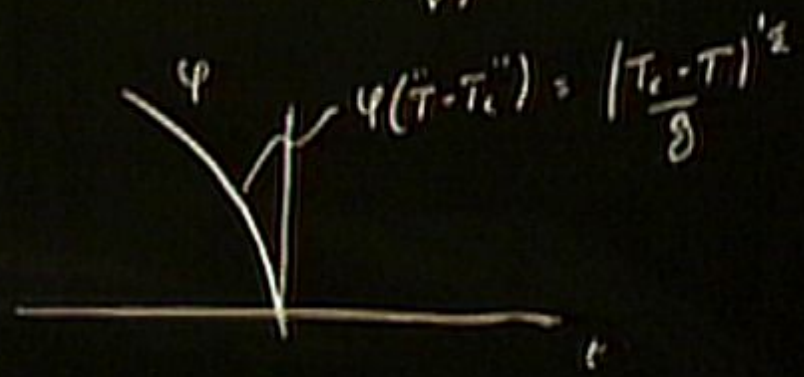


$$\langle \psi \rangle + \langle \psi \rangle \langle \psi \rangle$$

$t < 0$ $t + 2\beta\psi$ $(\psi - \frac{1}{2}) = \text{modified}$

$$\langle \psi \rangle + \psi^2 = 0 \quad \Rightarrow |\psi| = \left(\frac{-t}{8} \right)^{1/2}$$

Phase diagram



$-\beta = \frac{1}{2}$
critical exponent

$G(x, y)$

↑ Soln to Hypergeometric eqn

$P = \cos(\theta/k)$

$G(P) = {}_2F_1\left(\begin{matrix} h, h \\ h+1 \end{matrix} \middle| \frac{d}{4\pi l}\right)$

$(D + M^2)G = 0$

$\left[(1-P^2) \frac{d^2}{dP^2} - d P \frac{d}{dP} + M^2 \right] G = 0$

$Q \rightarrow \left(\frac{1}{4\pi l}\right)^2$

$-\frac{\sqrt{1-x^2} \cot(\sqrt{1-x^2})}{4\pi l}$

$P \rightarrow -P$ a Symmetry
 $-P$ - the antipode to P

$G(P) = G(-P) = \frac{1}{2\pi i P}$



$G(x, y)$

↑ leads to hypergeometric eq.

$$(D + M)G = 0$$

$$\left[(1-P^2) \frac{d^2}{dP^2} - d P \frac{d}{dP} + M^2 e^2 \right] G = 0$$

$P \rightarrow -P$ - Symmetry

$-P$ - the antiparticle to P .

$$P = \cos(\theta)$$

$$G(P) = \int F_1 \left(h, h, \frac{d}{1-P^2} \right)$$

$$Q \rightarrow \left(\frac{1}{4MP} \right)^2$$

$$+ x \cot(\sqrt{1-x^2})$$

$$\begin{matrix} \boxed{G(P) \leftrightarrow G(-P)} \\ \text{Cosh } 2\alpha G(P) \\ \text{sinh } 2\alpha G(P) \end{matrix}$$

$G(x, y)$

↑ leads to Hypergeometric eq

$$(D + M)(p^n) = 0$$

$$\left[(1-p^2) \frac{d^2}{dp^2} - d P \frac{d}{dp} + M^2 e^2 \right] G = 0$$

$P \rightarrow -P$ a Symmetry

$-P$ - the antiparticle to P

$$P = \cos(\theta)$$

$$G(P) = \int F_1 \left(h, h, \frac{d}{1-p^2} \right)$$

$$Q = \left(\frac{1}{4\pi P} \right)^2$$

$$= \frac{\sqrt{1-x} \cot(\sqrt{1-x})}{4\pi l}$$

$$\boxed{G(P) \leftrightarrow G(-P)} \quad \frac{1}{2\pi i P}$$

$$\cosh 2\alpha G(P) + \sinh 2\alpha G(-P)$$

$$\left[\frac{d}{dp} + \frac{M^2 e^2}{x} \right] G = 0$$

$$-\frac{\int \sqrt{1+x} \operatorname{Col}(\pm \sqrt{1-x})}{4\pi l}$$

$$\frac{A G(p) + B G(-p)}{\cosh 2\alpha G(p) + \sinh 2\alpha G(-p)} \Big|_{x \rightarrow 0} = \frac{1}{2\pi M^2}$$

CAUTION
 It is dangerous to use
 this device in the
 presence of fire.

$\frac{d}{dp} + \left(\frac{M^2 e^2}{x} \right) G = 0$

$-\frac{\sqrt{1+x} \operatorname{Col}(\pm \sqrt{1-x})}{4\pi l}$

$\lim_{x \rightarrow 0} \frac{A G(p) + B G(-p)}{\cosh 2\alpha G(p) + \sinh 2\alpha G(-p)} = \frac{1}{2\pi M^2}$

CAUTION
 Do not touch the blackboard
 when it is being used
 for teaching purposes only
 2018-2019

$$G(x, \kappa)$$

↑ Soln to Hypergeometric eqn

$$P = \cos(D/\ell)$$

$$G(P) \approx \frac{1}{2} F_1 \left(h_+, h_-, \frac{d}{\ell} \right)$$

$$(\square + M^2) \psi = 0$$

$$\left[(1-P^2) \frac{d^2}{dP^2} - d P \frac{d}{dP} + M^2 e^2 \right] G$$

$$M^2 = 0$$

$$\frac{1}{4\pi P} = \text{divergence}$$

$$= \frac{\int \sqrt{1+x} \text{Col}(A) \sqrt{1-x}}{4\pi \ell}$$

$P \rightarrow -P$ a Symmetry
 $-P$ - the antipole to P

$$\frac{1}{2\pi M^2}$$

$$G(-P)$$



$$G = G_0(P) - G(-P) \quad \checkmark$$



$$\frac{x}{\lambda} = \lambda^{-1}$$

$$\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right)$$

$$\left[\frac{\Lambda^2}{16\pi^2} - \frac{M^2}{16\pi^2} \left(\ln \frac{M^2}{\Lambda^2} + 2\gamma - 1 \right) \right] = \frac{M^2}{S_B}$$

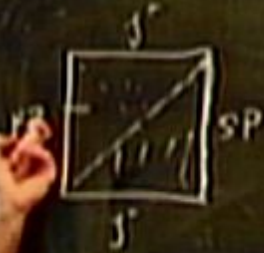
$\frac{L_D}{S_B} = 2$

$$\frac{1}{g_B} = \frac{1}{g(r)} - \frac{M^2}{16\pi^2} \frac{L^2}{\Lambda^2}$$

$$\frac{L_D}{S_B} = 1 \text{ nu}$$

$$\rho(g) = \frac{g^2}{8\pi^2}$$

Lessons from an Exactly Solved Interacting Quantum Field Theory in de Sitter Spacetime



EFT version is S^d

$$\langle \varphi \rangle = \text{const} = \varphi_0 + \langle \varphi^* \varphi \rangle$$



NO
SMOKING
HERE

Lessons from an Exactly Solvable Interacting
Quantum Field Theory in de Sitter Spacetime

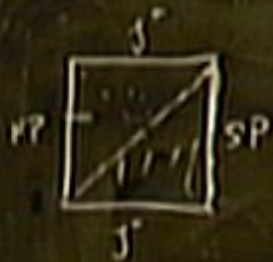


Euclidean version is S^d

$$= \mathcal{Q}_1 + \langle \varphi \rangle \langle \varphi^* \rangle$$

DO NOT
touch
without
permission

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime



$\langle \varphi \rangle = \text{const}$

S^d
 $\langle \varphi \rangle$

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime



Euclidean version is S^d

$$\langle \varphi \rangle = \text{const} = \varphi_1 + \langle \varphi \rangle^* \langle \varphi \rangle$$

Lessons from an Exactly Solved Interacting
Quantum Field Theory in de Sitter Spacetime



$\langle \psi \rangle = \text{const}$

$\langle \psi \rangle = \langle \psi \rangle$

