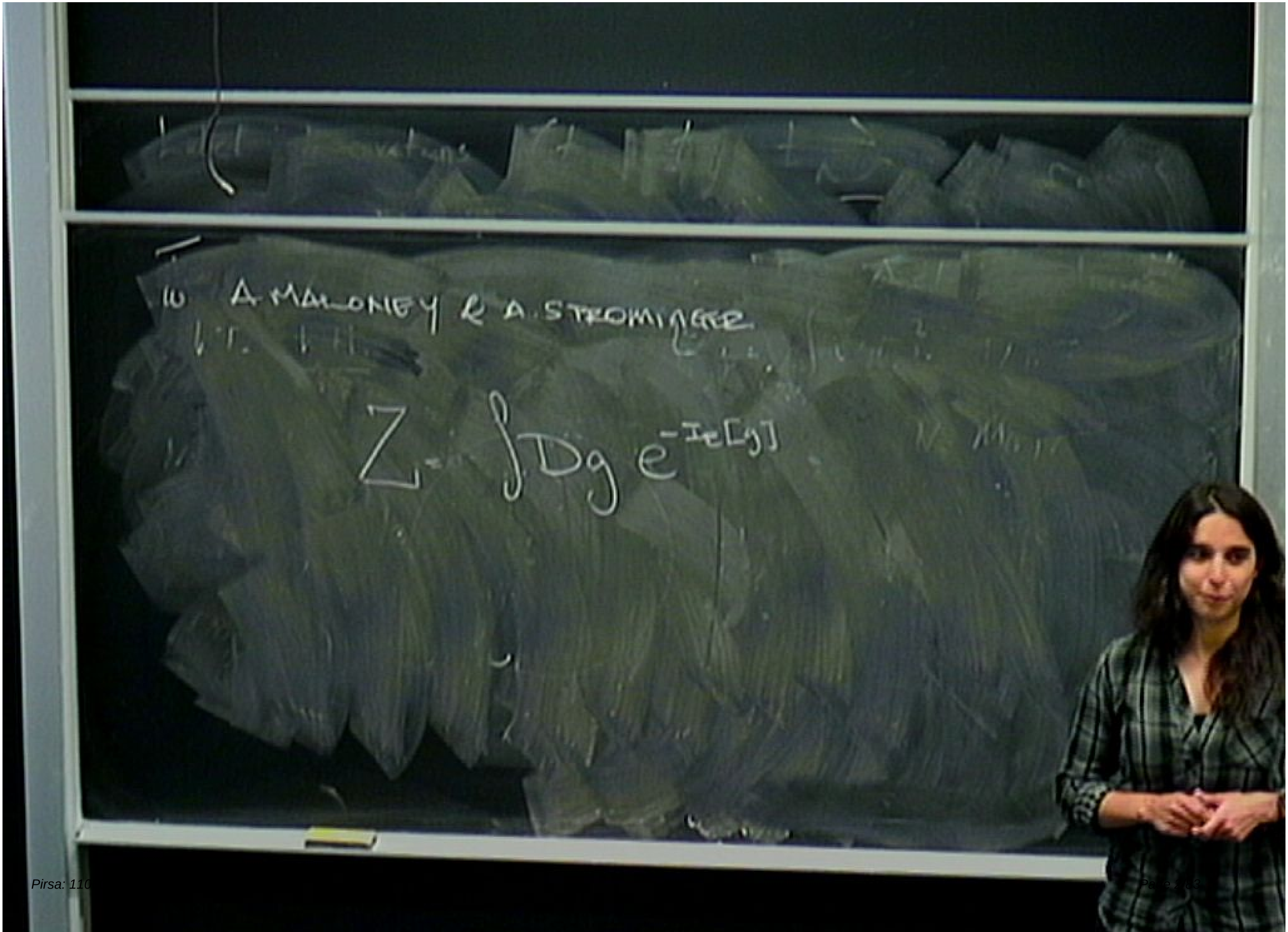
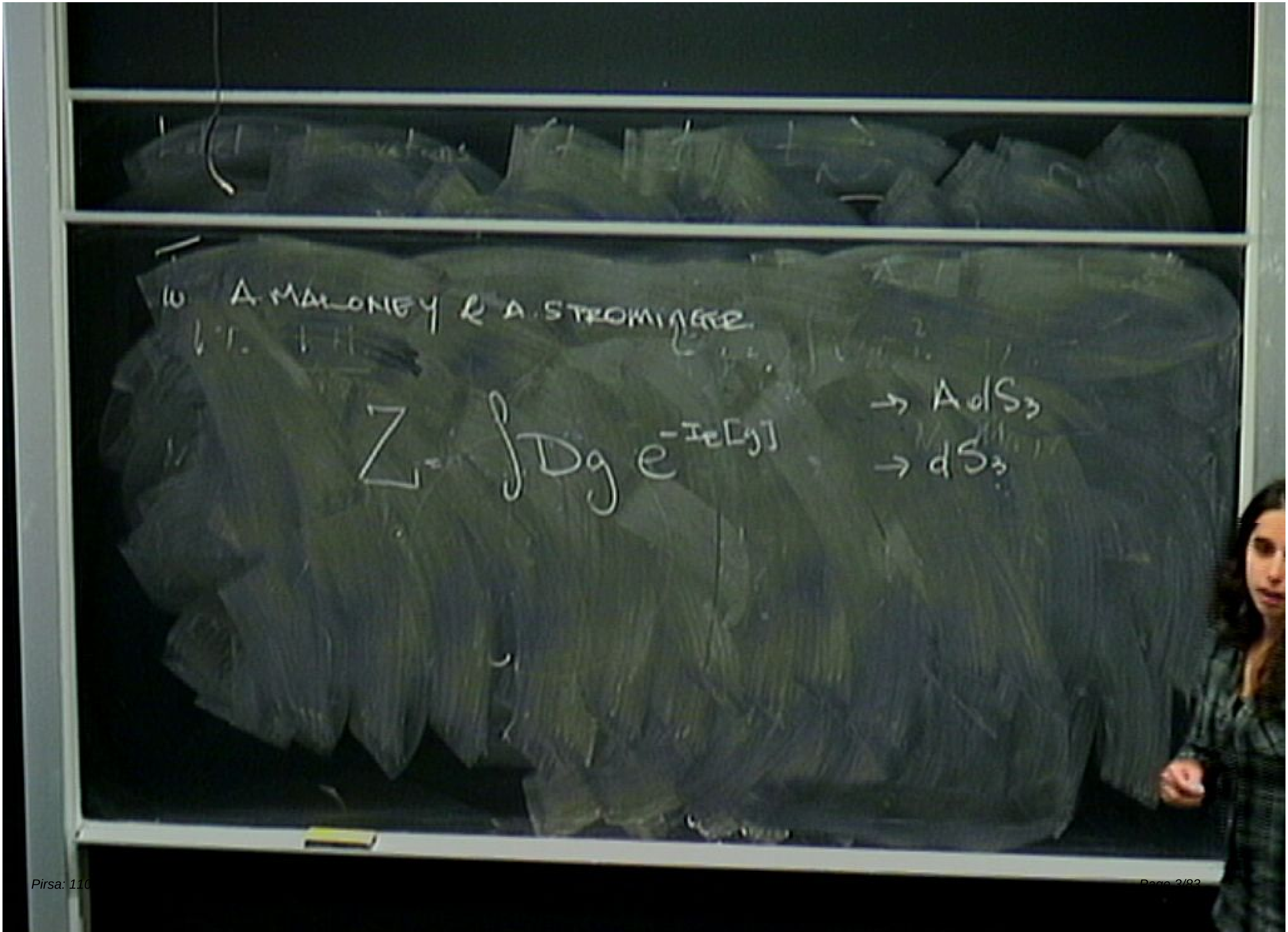
Title: The partition function of quantum de Sitter

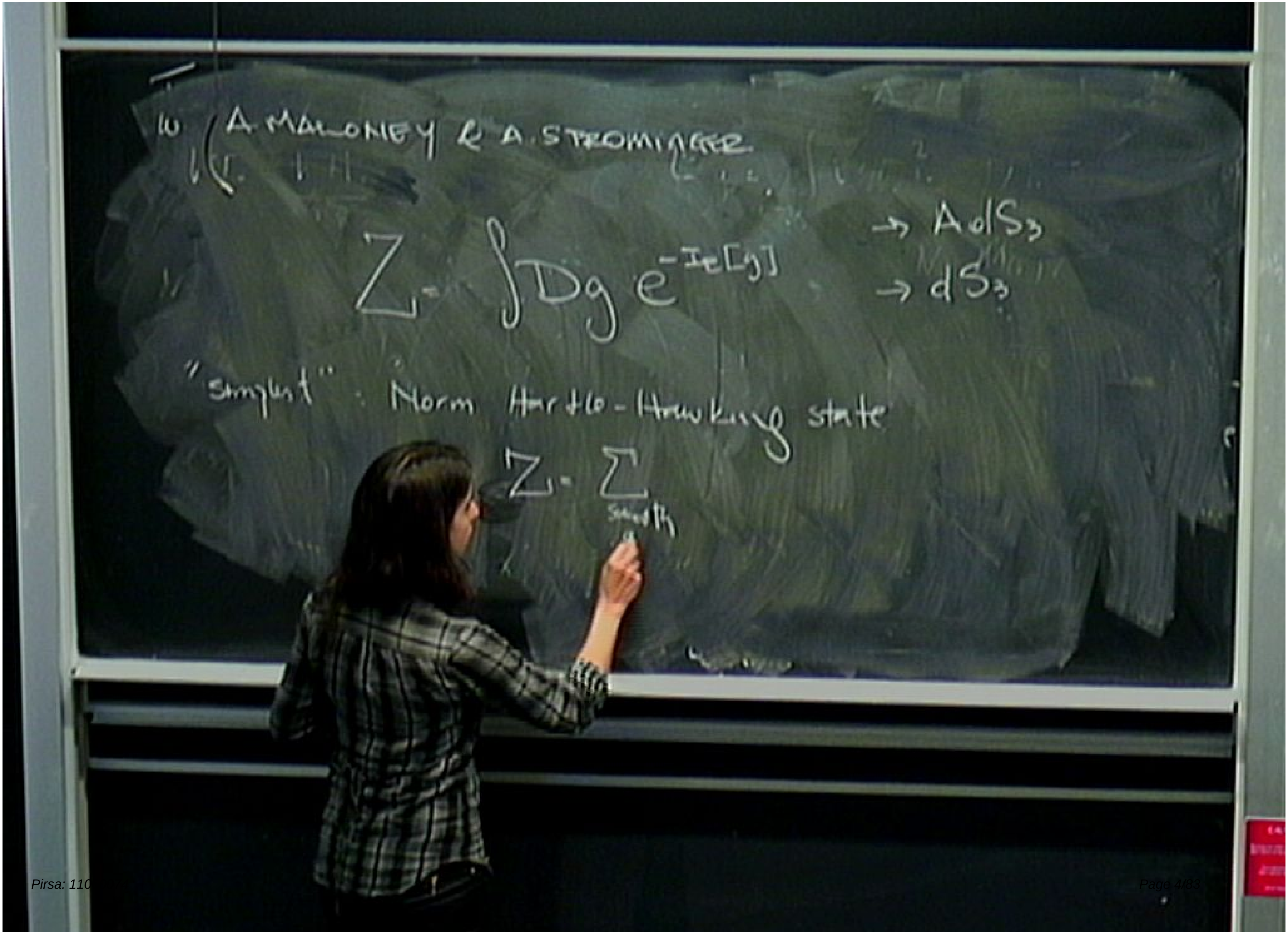
Date: Jun 22, 2011 11:55 AM

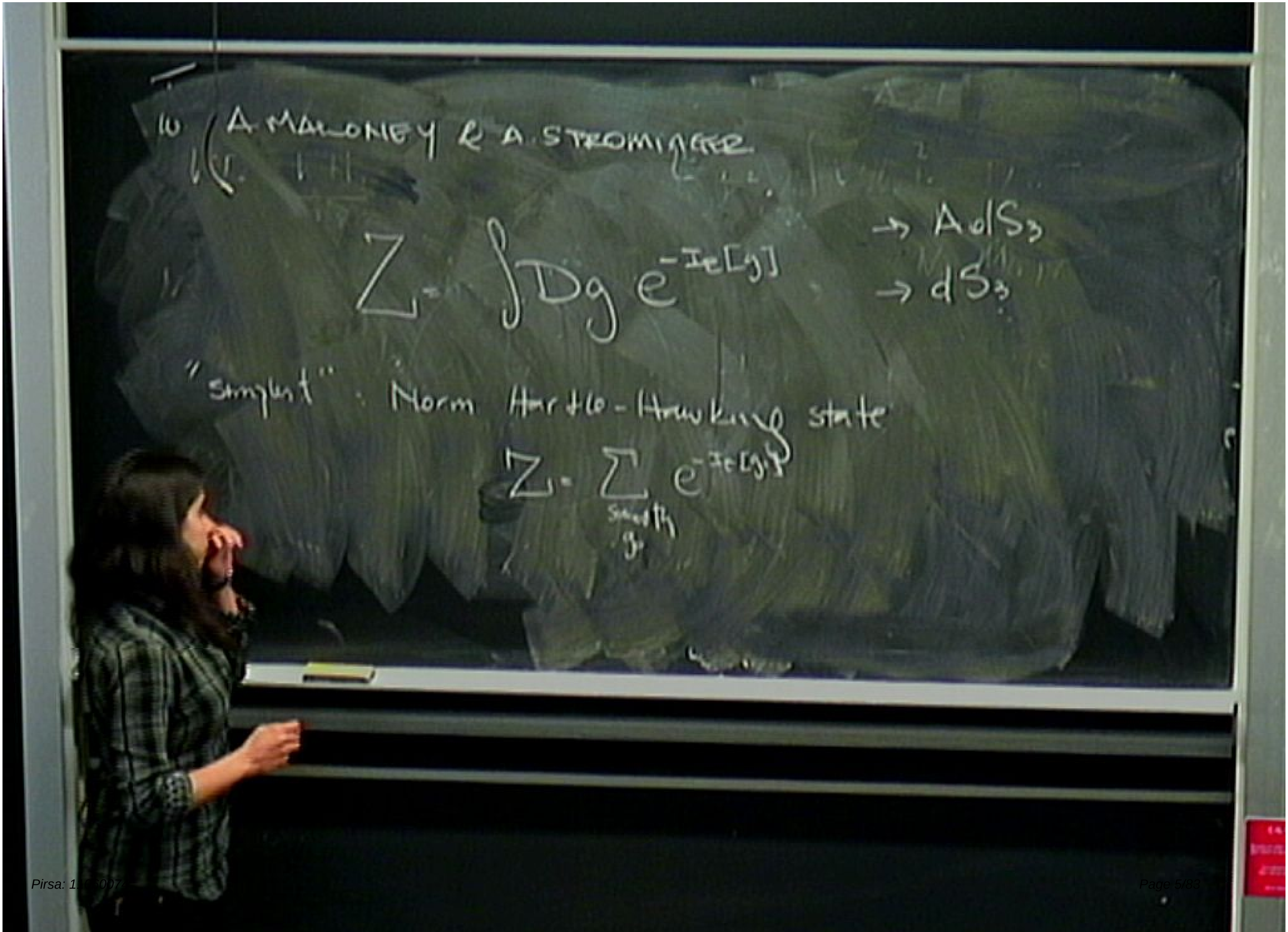
URL: http://pirsa.org/11060074

Abstract: We propose that quantum gravity in de Sitter space should be de


(ii) A. MALONEY & A. STROMINGER
Z = ∫Dg e^{-I_E[g]}


w A. MALONEY & A. STROMINGER
$Z = \int Dg\, e^{-I_E[g]}$
→ AdS_3
→ dS_3


A. MALONEY & A. STROMINGER
$Z = \int Dg \, e^{-I_E[g]}$
→ AdS_3
→ dS_3
"Simplest": Norm Hartle-Hawking state
$Z = \sum_{\text{smooth}}$


A. MALONEY & A. STROMINGER
$Z = \int Dg \, e^{-I_E[g]}$
$\to AdS_3$
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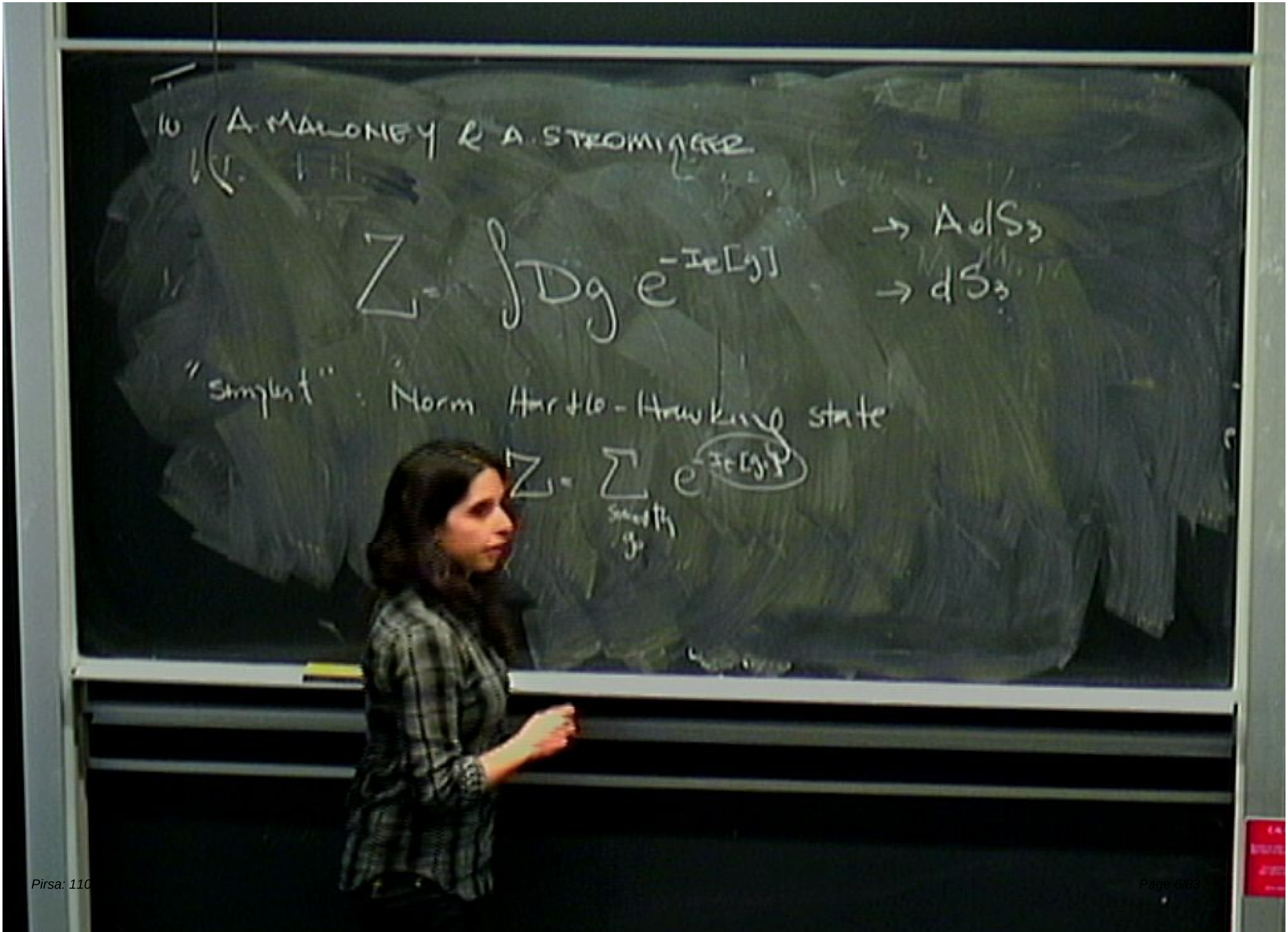
"Simplest": Norm Hartle-Hawking state
$Z = \sum_{\text{smooth } g} e^{-I_E[g]}$


A. MALONEY & A. STROMINGER
$Z = \int Dg\, e^{-I_E[g]}$
→ AdS$_3$
→ dS$_3$
"Simplest": Norm Hartle-Hawking state
$Z = \sum_{\text{smooth } g_i} e^{-I_E[g_i]}$

w A. MALONEY & A. STROMINGER

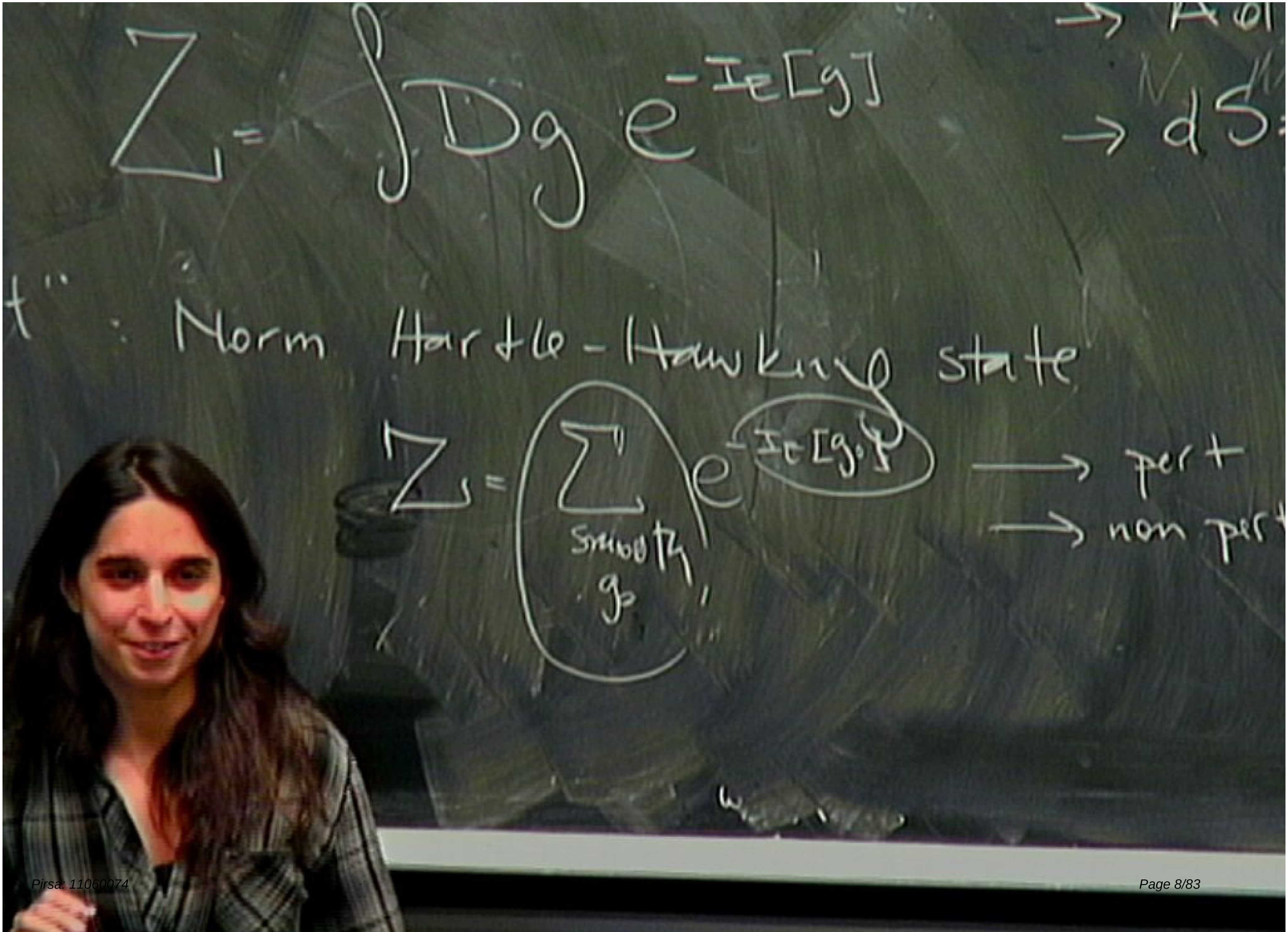
$$Z = \int Dg\, e^{-I_E[g]}$$

$\rightarrow AdS_3$

$\rightarrow dS_3$

"simplest": Norm Hartle-Hawking state

$$Z = \sum_{\substack{\text{smooth} \\ g_c}} e^{-I_E[g_c]}$$


Z = ∫ Dg e^{-I_E[g]}
→ dS
Norm Hartle-Hawking state
Z = Σ' e^{-I_E[g_0]}
smooth g_0
→ pert
→ non pert

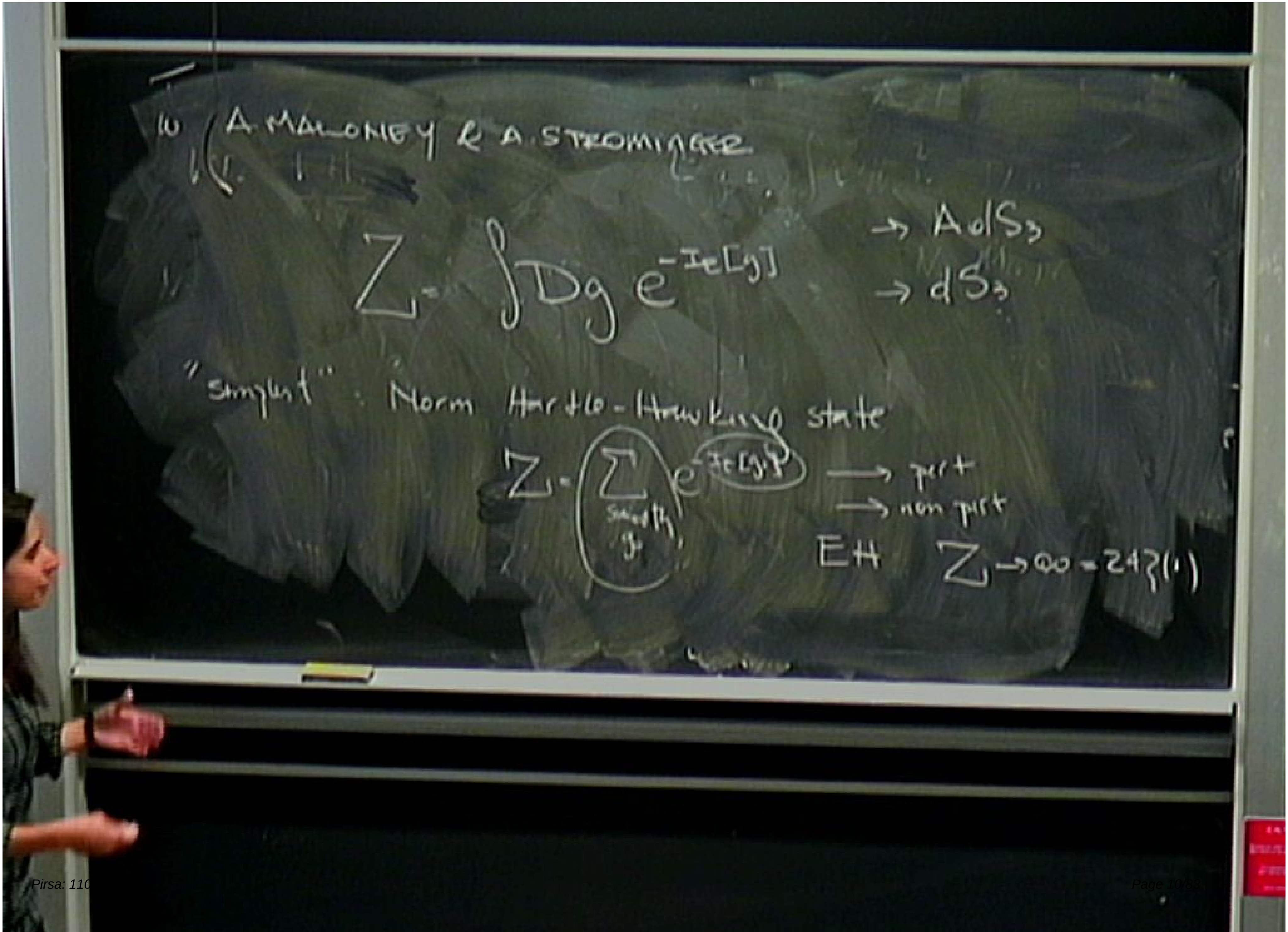
MALONEY & A. STROMINGER

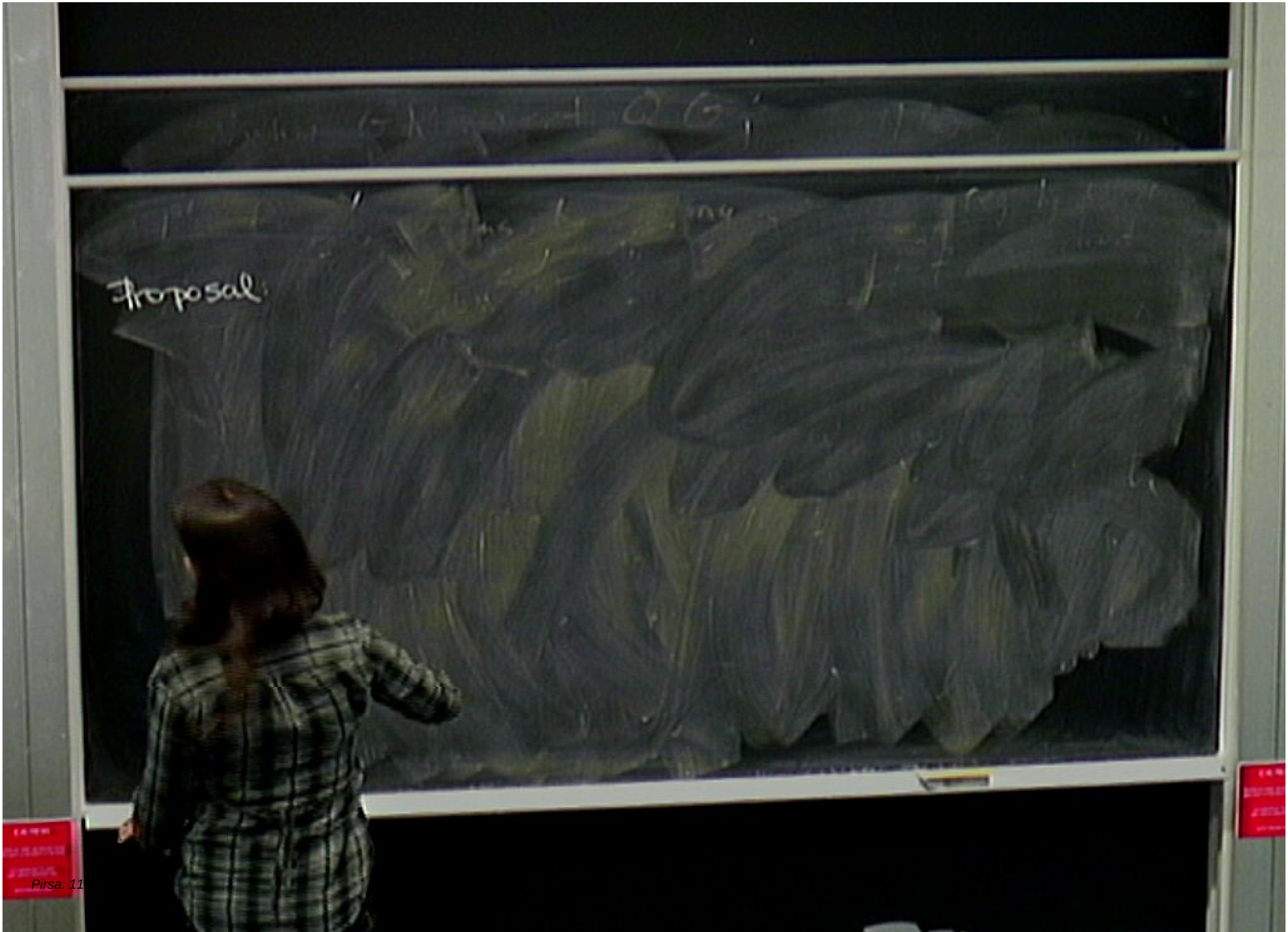
$$Z = \int Dg\, e^{-I_E[g]} \quad \begin{array}{l} \to AdS_3 \\ \to dS_3 \end{array}$$

"...plest" : Norm Hartle-Hawking state

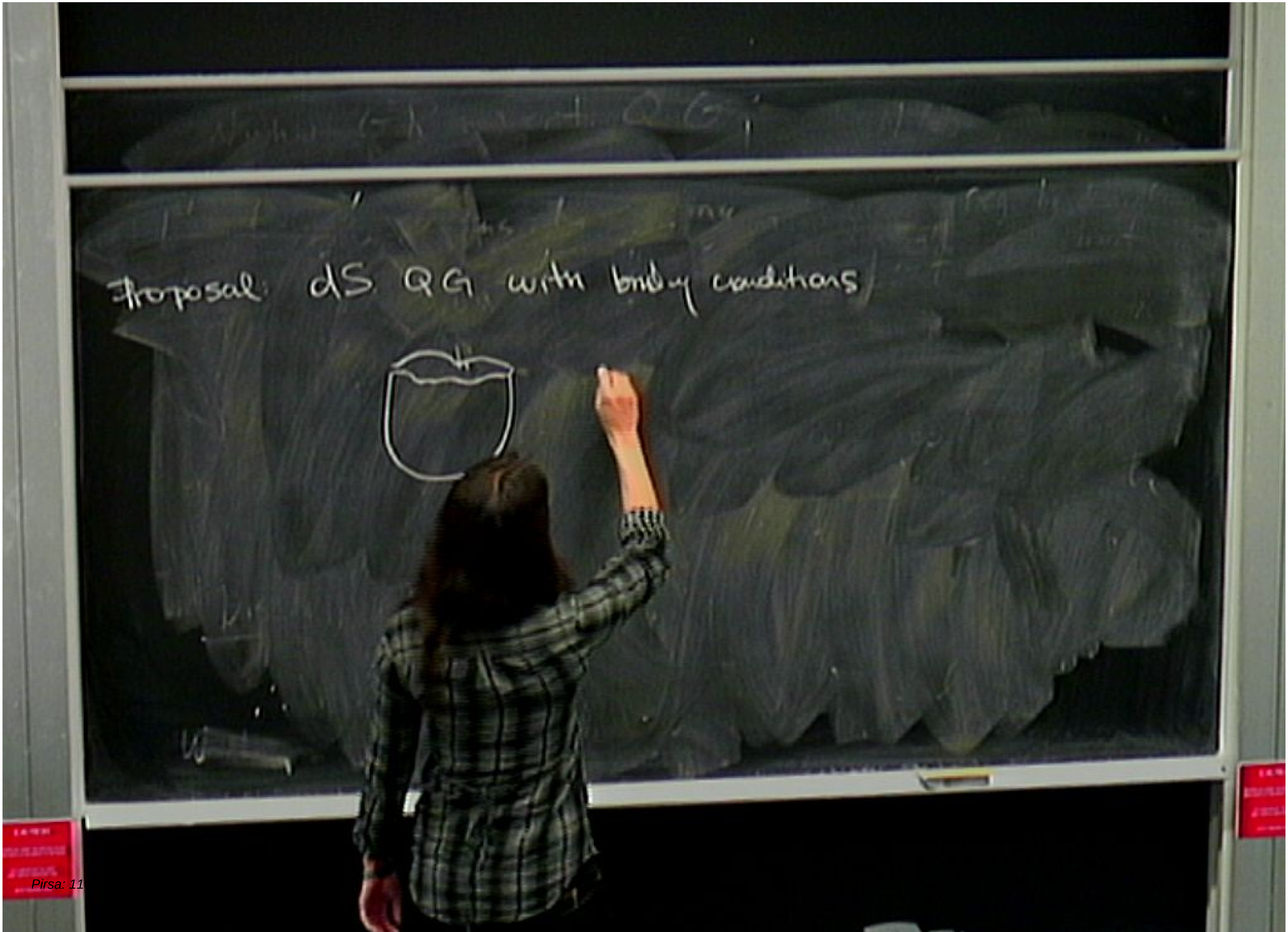
$$Z = \sum_{\substack{\text{smooth} \\ g_0}} e^{-I_E[g, g_0]} \quad \begin{array}{l} \longrightarrow \text{pert} \\ \longrightarrow \text{non pert} \end{array}$$

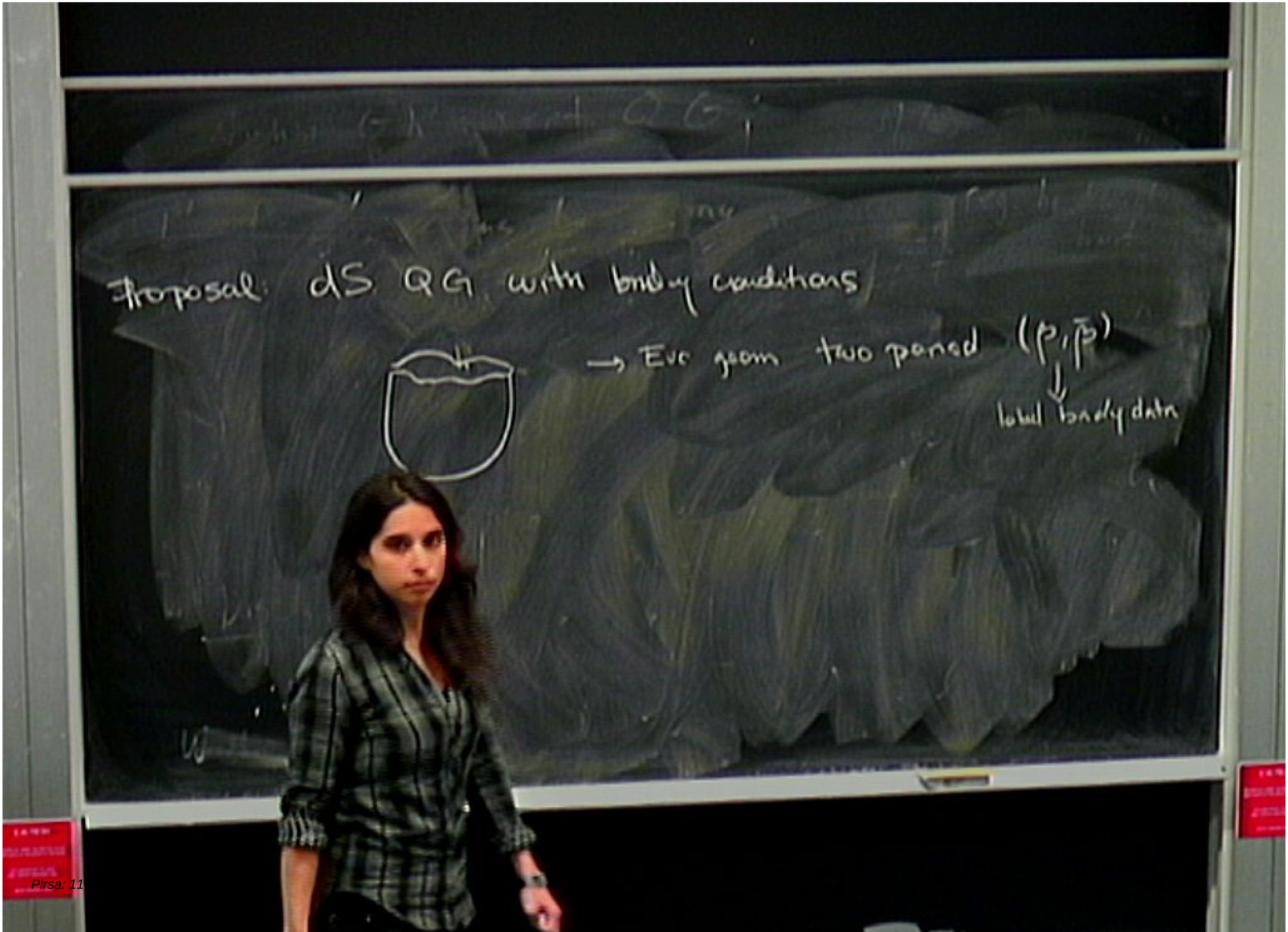
EH $\quad Z \to \infty = 24\zeta(-1)$


w/ A. MALONEY & A. STROMINGER
Z = ∫Dg e^{-I_E[g]}
→ AdS_3
→ dS_3
"simplest": Norm Hartle-Hawking state
Z = Σ_{smooth g} e^{-I_E[g]}
→ pert
→ non pert
EH Z → ∞ = 24ζ(-1)


Proposal:

Proposal: dS QG with bndry conditions


Proposal: dS QG with bdry conditions


Proposal: dS QG with bndy conditions
→ Euc geom two pointed (p, p̄)
label bndy data

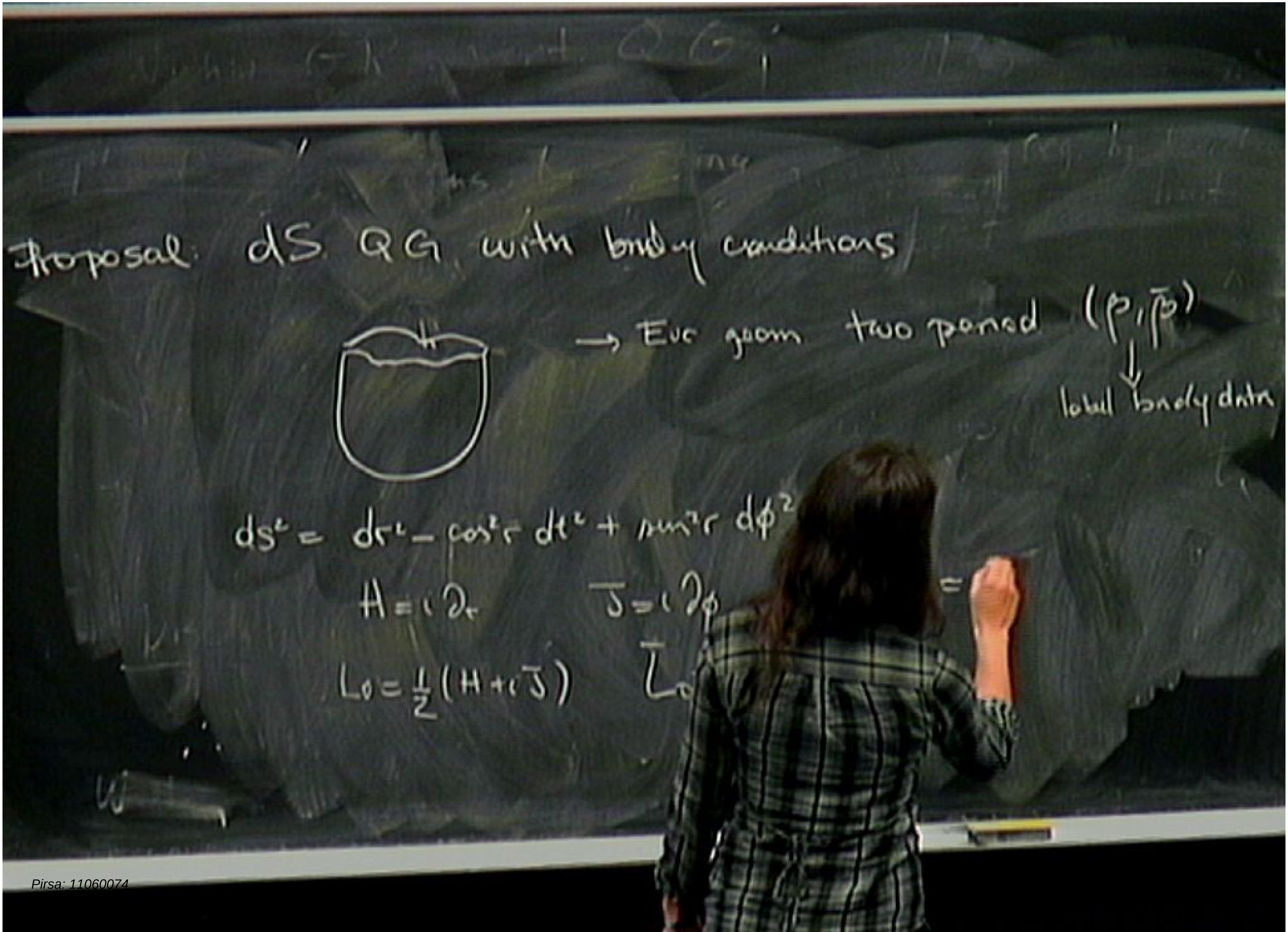
Proposal: dS QG with bndy conditions

$\rightarrow$ Euc geom two period $(\rho, \bar{\rho})$

$\downarrow$

label bndy data

$$ds^2 = dr^2 - \cos^2 r \, dt^2 + \sin^2 r \, d\phi^2$$


Proposal: dS QG with bndy conditions
→ Euc geom two period $(\rho, \bar{\rho})$
label bndy data
$ds^2 = dr^2 - \cos^2 r\, dt^2 + \sin^2 r\, d\phi^2$
$H = i\partial_t$
$J = i\partial_\phi$
$L_0 = \frac{1}{2}(H + iJ)$
$\bar{L}_0$

Proposal: dS QG with bndy conditions

$\rightarrow$ Euc geom two period $(\rho, \bar{\rho})$

label bndy data

$$ds^2 = dr^2 - \cos^2 r \, dt^2 + \sin^2 r \, d\phi^2$$

$H = i\partial_t$ $\qquad J = i\partial_\phi$ $\qquad \rho = e^{(2\pi i\beta L_0 - 2\pi i\bar{\beta}\bar{L}_0)}$

$L_0 = \frac{1}{2}(H + iJ)$ $\qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$

Proposal: dS QG with bndry conditions

$\to$ Euc geom two period $(\beta, \bar{\beta})$

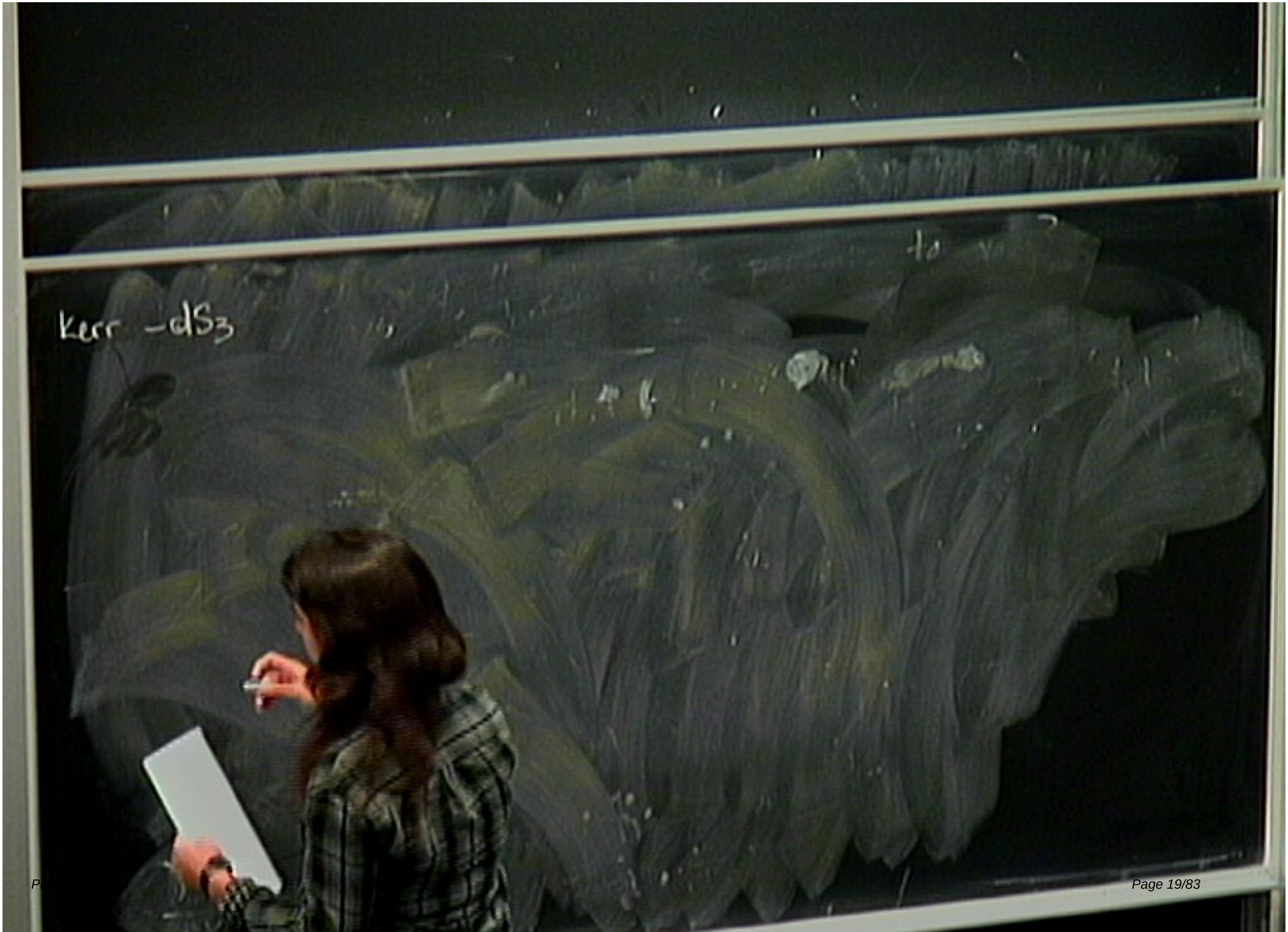
$\downarrow$

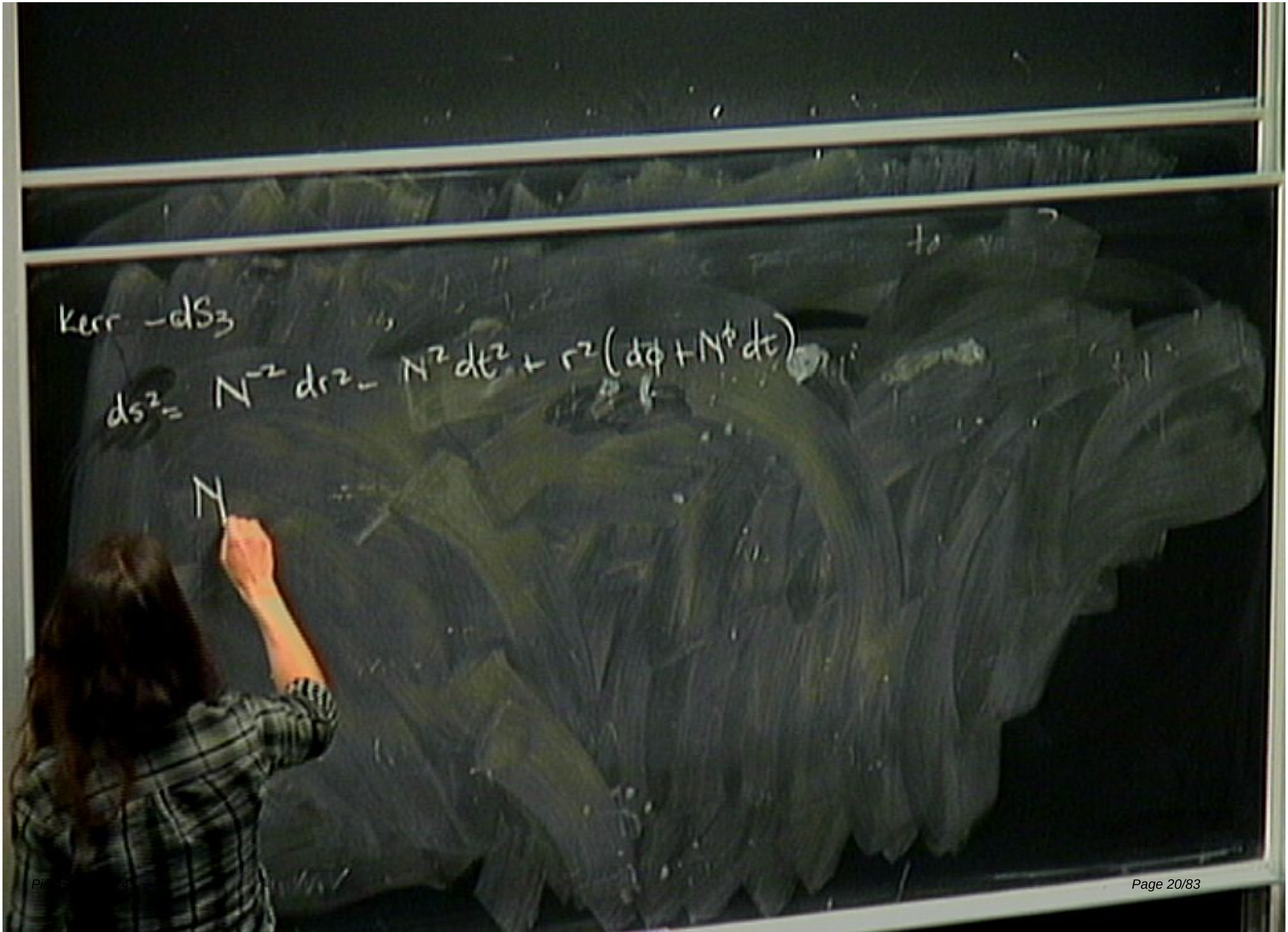
local bndry data

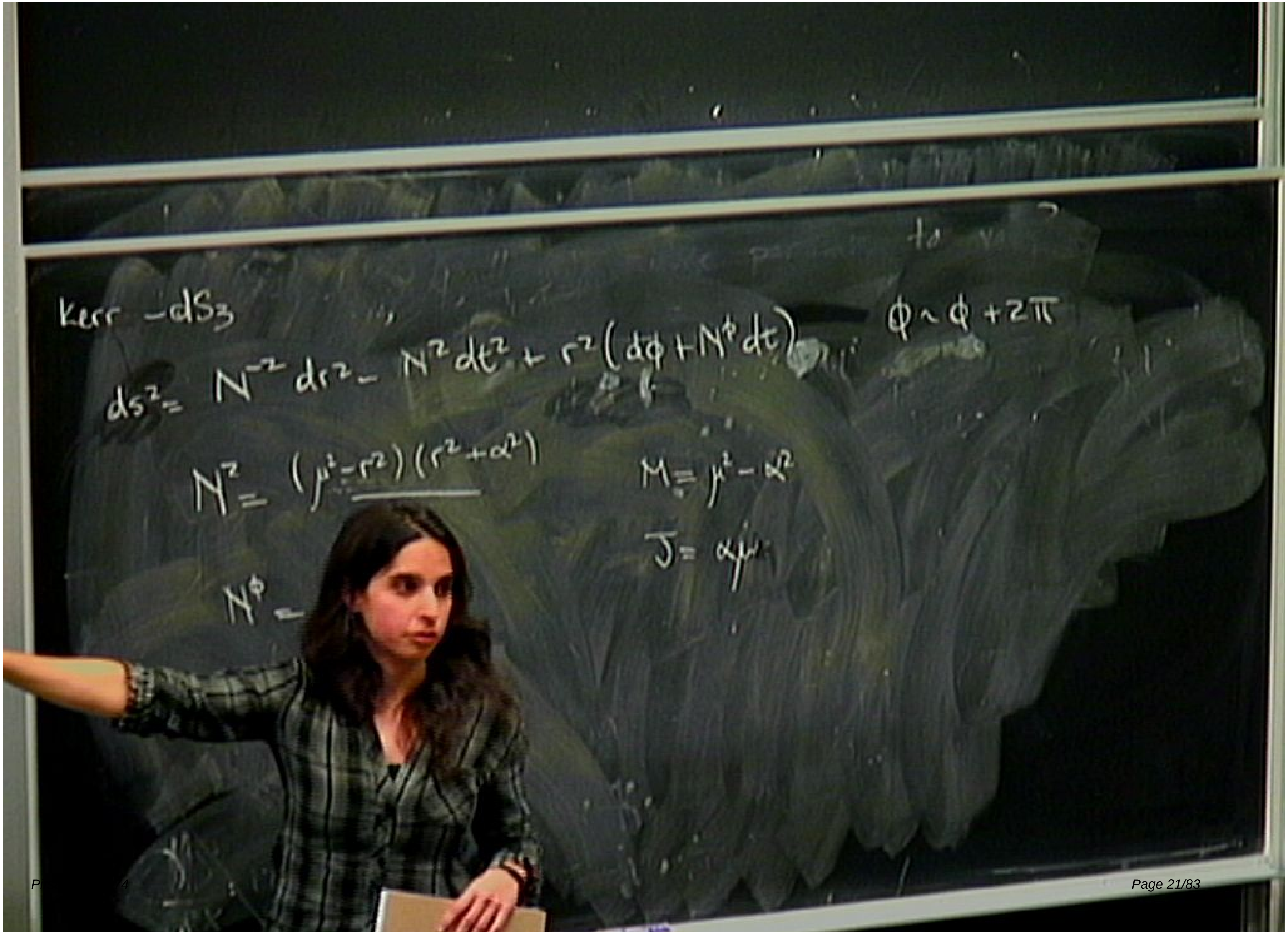
$$ds^2 = dr^2 - \cos^2 r \, dt^2 + \sin^2 r \, d\phi^2$$

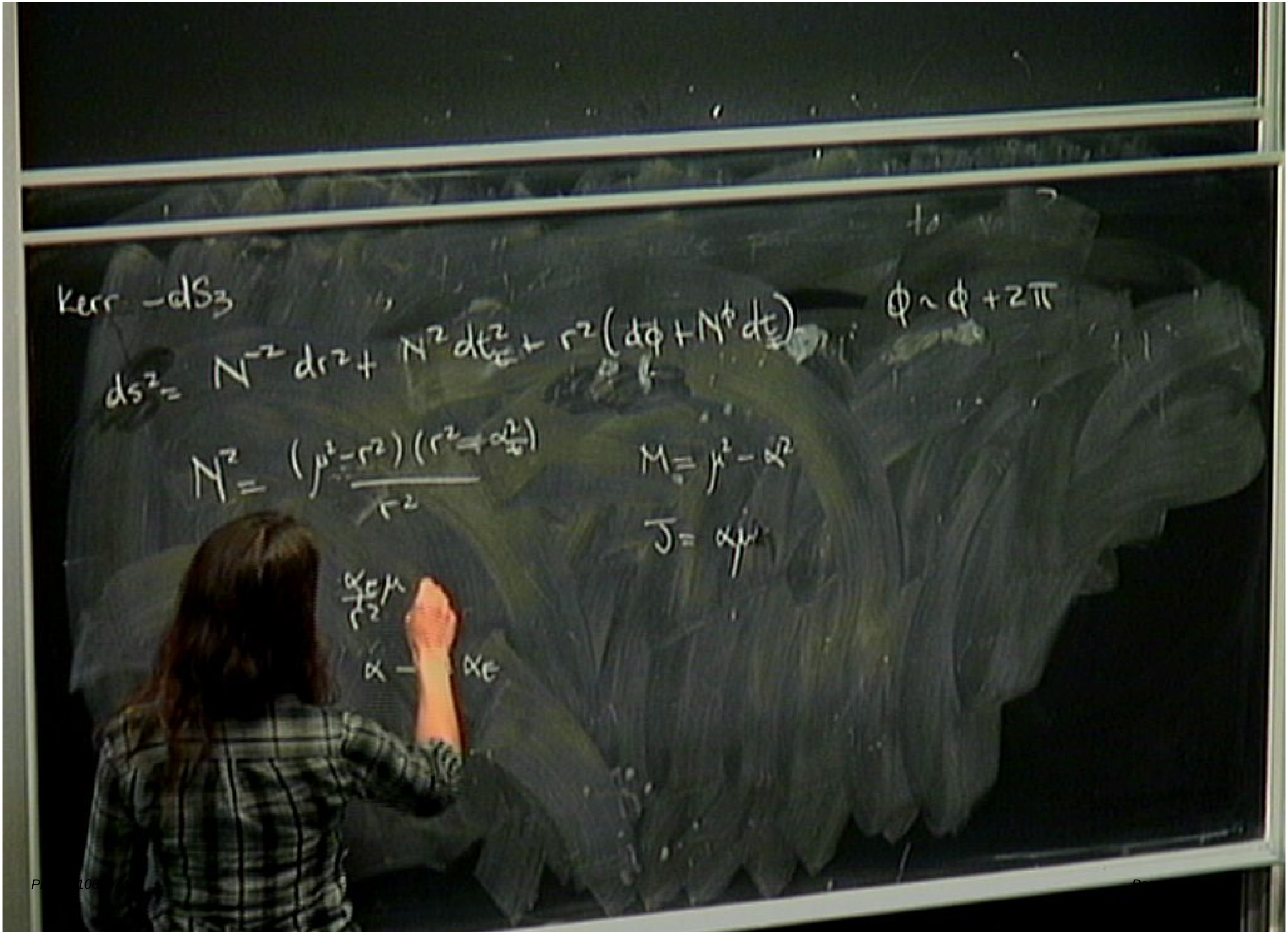
$$H = i\partial_t \qquad J = i\partial_\phi \qquad \rho = e^{(2\pi i \beta L_0 - 2\pi i \bar{\beta} \bar{L}_0)}$$

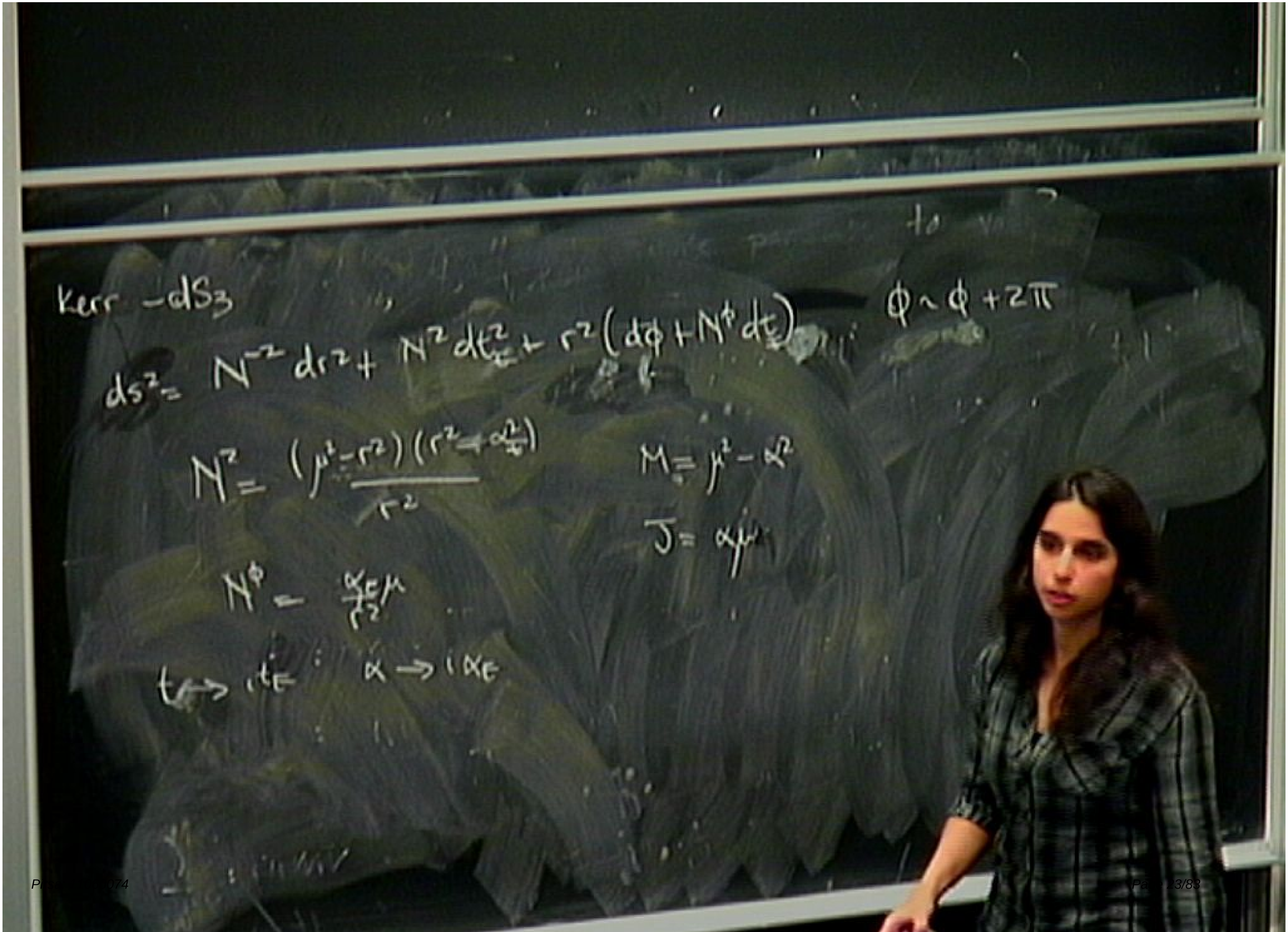
$$L_0 = \frac{1}{2}(H + iJ) \qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$$


Kerr -dS3


Kerr - dS_3
$ds^2 = N^{-2} dr^2 - N^2 dt^2 + r^2 (d\phi + N^\phi dt)$
N


Kerr -dS3
ds2 = N-2 dr2 - N2 dt2 + r2(dφ + Nφ dt)2
φ ~ φ + 2π
N2 = (μ2 - r2)(r2 + α2)
M = μ2 - α2
J = αμ
Nφ =


Kerr - dS3
$\phi \sim \phi + 2\pi$
$ds^2 = N^{-2} dr^2 + N^2 dt^2 + r^2 (d\phi + N^\phi dt)^2$
$M = \mu^2 - \alpha^2$
$J = \alpha\mu$


Kerr - dS3
ds² = N⁻² dr² + N² dt_E² + r² (dφ + N^φ dt_E)
φ ~ φ + 2π
N² = (μ² - r²)(r² + α_E²) / r²
M = μ² - α²
J = αμ
N^φ = α_E μ / r²
t → i t_E ; α → i α_E

Kerr $-dS_3$

$$ds^2 = N^{-2} dr^2 + N^2 dt_E^2 + r^2 (d\phi + N^\phi dt_E)^2 \qquad \phi \sim \phi + 2\pi$$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$$t \to i t_E \quad : \quad \alpha \to i \alpha_E$$

mmr =

$$\tilde{t}_E = \mu_1 t_E + \alpha_E \phi$$

$$\tilde{\phi}_E = \mu_1 \phi + \alpha_E t_E$$

Proposal: dS QG with bndy conditions

→ Euc geom two period $(\beta, \bar{\beta})$

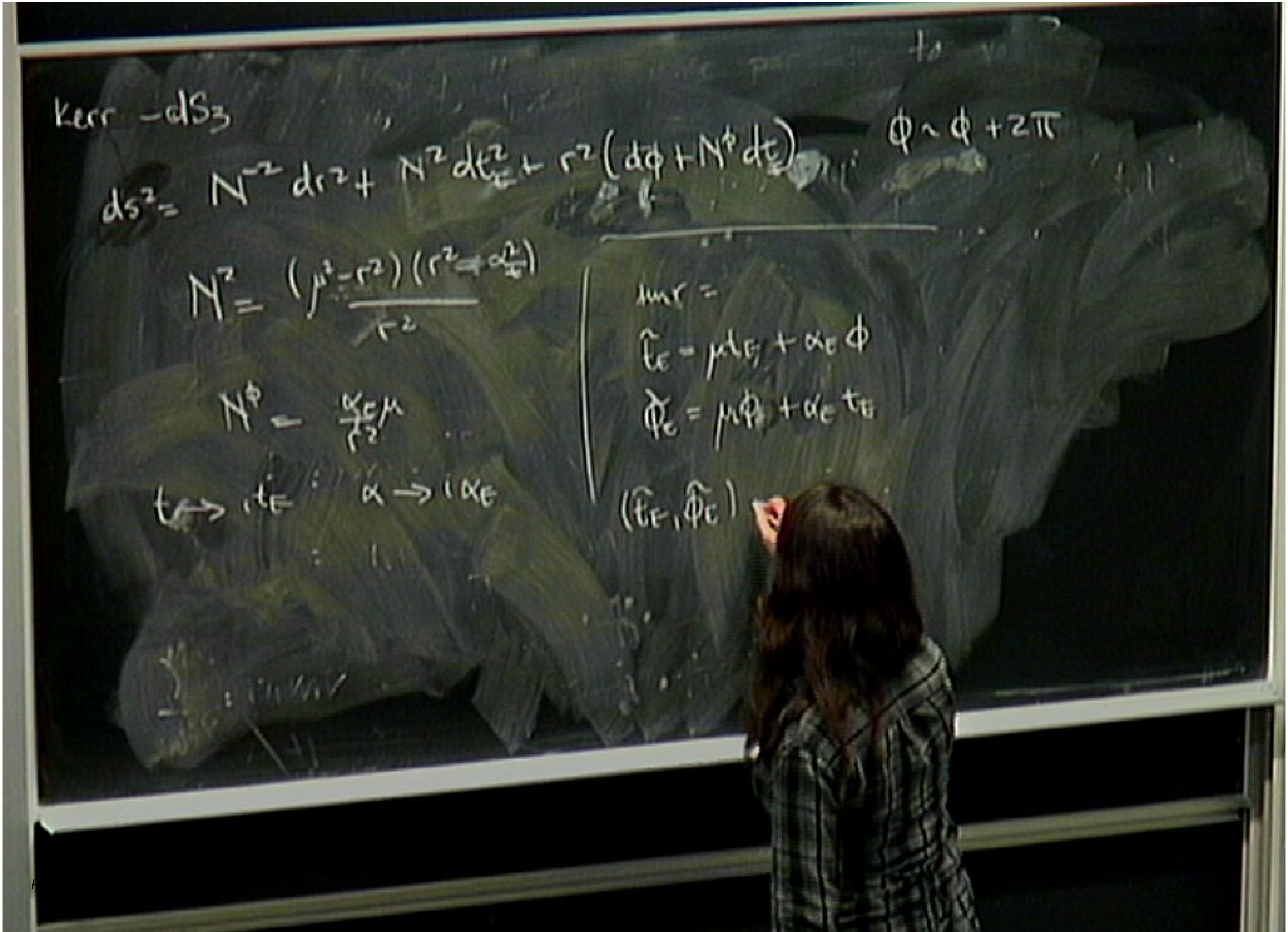
↓

label bndy data

$$ds^2 = dr^2 + \cos^2 r\, d\tilde{t}_E^2 + \sin^2 r\, d\tilde{\phi}^2$$

$H = i\partial_t$ $\qquad J = i\partial_\phi$ $\qquad \rho = e^{(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0)}$

$L_0 = \frac{1}{2}(H + iJ)$ $\qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$


Kerr – dS_3
$\phi \sim \phi + 2\pi$
$ds^2 = N^{-2} dr^2 + N^2 dt_E^2 + r^2 (d\phi + N^\phi dt_E)$
$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$
$N^\phi = \frac{\alpha_E \mu}{r^2}$
$t \to i t_E \quad \alpha \to i \alpha_E$
$\hat{t}_E = \mu t_E + \alpha_E \phi$
$\hat{\phi}_E = \mu \phi + \alpha_E t_E$
$(\hat{t}_E, \hat{\phi}_E)$

$-dS_3$

$\phi \sim \phi + 2\pi$

$$ds^2 = N^{-2}dr^2 + N^2 dt_E^2 + r^2(d\phi + N^\phi dt_E)$$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

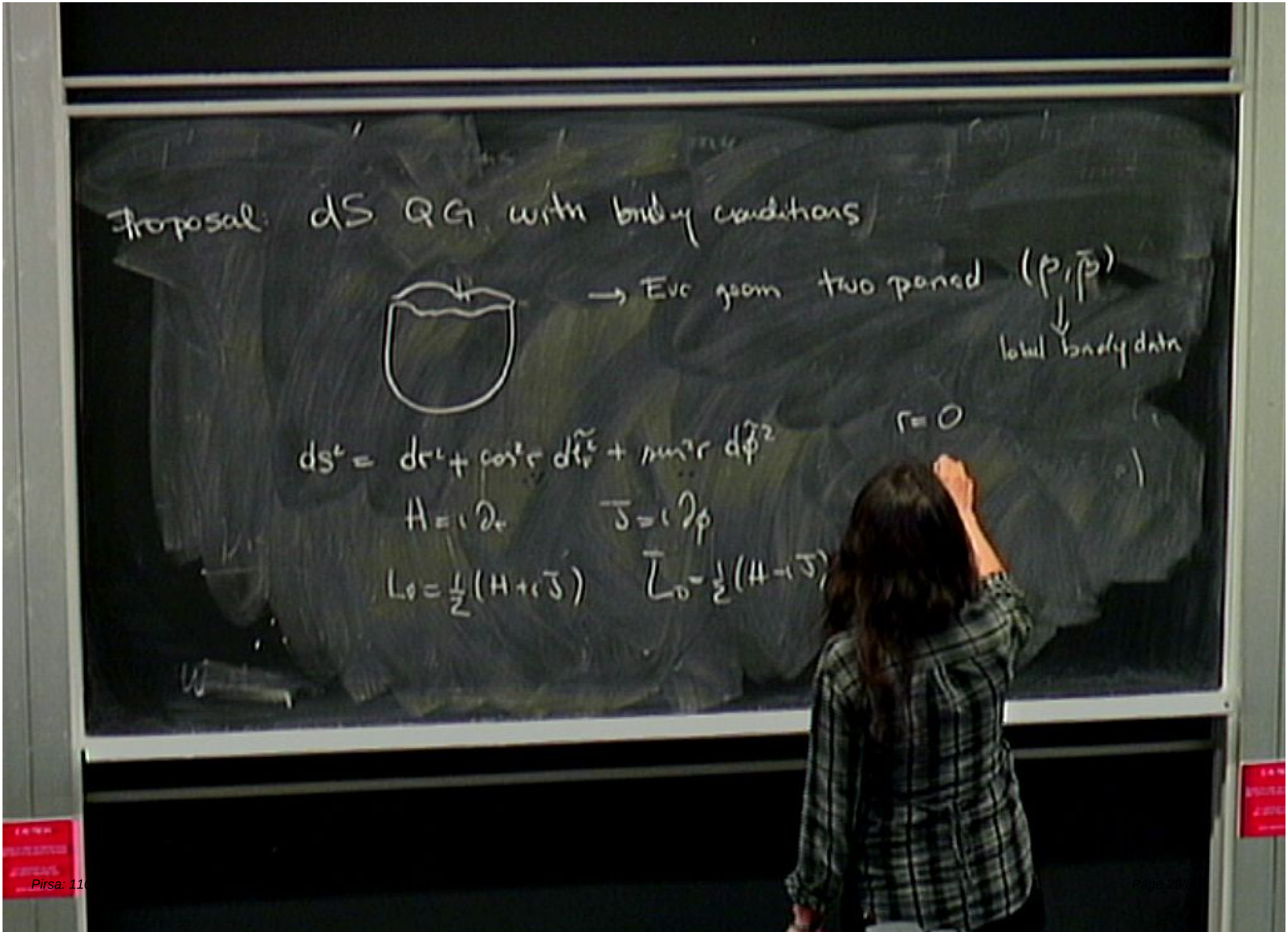
$t \to i t_E$: $\alpha \to i\alpha_E$

hmr =

$$\hat{t}_E = \mu t_E + \alpha_E \phi$$

$$\hat{\phi}_E = \mu \phi + \alpha_E t_E$$

$$(\hat{t}_E, \hat{\phi}_E) \sim (\hat{t}_E, \hat{\phi}_E) + 2\pi m(\alpha_E, \mu)$$


Proposal: dS QG with bndy conditions
→ Euc geom two pointed $(\rho, \bar{\rho})$
local bndy data
$r = 0$
$ds^2 = dr^2 + \cos^2 r \, d\tilde{\tau}^2 + \sin^2 r \, d\tilde{\phi}^2$
$H = i\partial_\tau$ $J = i\partial_\phi$
$L_0 = \frac{1}{2}(H + iJ)$ $\bar{L}_0 = \frac{1}{2}(H - iJ)$

Proposal: dS QG with bdry conditions

$\longrightarrow$ Euc geom two period $(\beta, \bar{\beta})$

$\downarrow$

local bdry data

$$ds^2 = dr^2 + \cos^2 r \, d\tilde{t}_E^2 + \sin^2 r \, d\tilde{\phi}^2$$

$r = 0$

$r = \pi/2$

$H = i\partial_t \qquad J = i\partial_\phi$

$L_0 = \frac{1}{2}(H + iJ) \qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$

Proposal: dS QG with bdry conditions

 $\longrightarrow$ Euc geom two period $(\rho, \bar{\rho})$

$\downarrow$

local bndy data

$r = 0 \rightarrow$ dS lives

$r = \pi/2$

$$ds^2 = dr^2 + \cos^2 r \, d\tilde{t}^2 + \sin^2 r \, d\tilde{\phi}^2$$

$H = i\partial_t \qquad J = i\partial_\phi$

$L_0 = \frac{1}{2}(H + iJ) \qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$$t \to i t_E \quad : \quad \alpha \to i \alpha_E$$

$\lim r =$

$$\tilde{t}_E = \mu t_E + \alpha_E \phi$$

$$\tilde{\phi}_E = \mu \phi_E + \alpha_E t_E$$

$$(\tilde{t}_E, \tilde{\phi}_E) \sim (\tilde{t}_E, \tilde{\phi}_E) + 2\pi m(\alpha_E, \mu)$$

Proposal: dS QG with bndy conditions

$\longrightarrow$ Euc geom two pared $(\rho, \bar{\rho})$

$\downarrow$

label bndy data

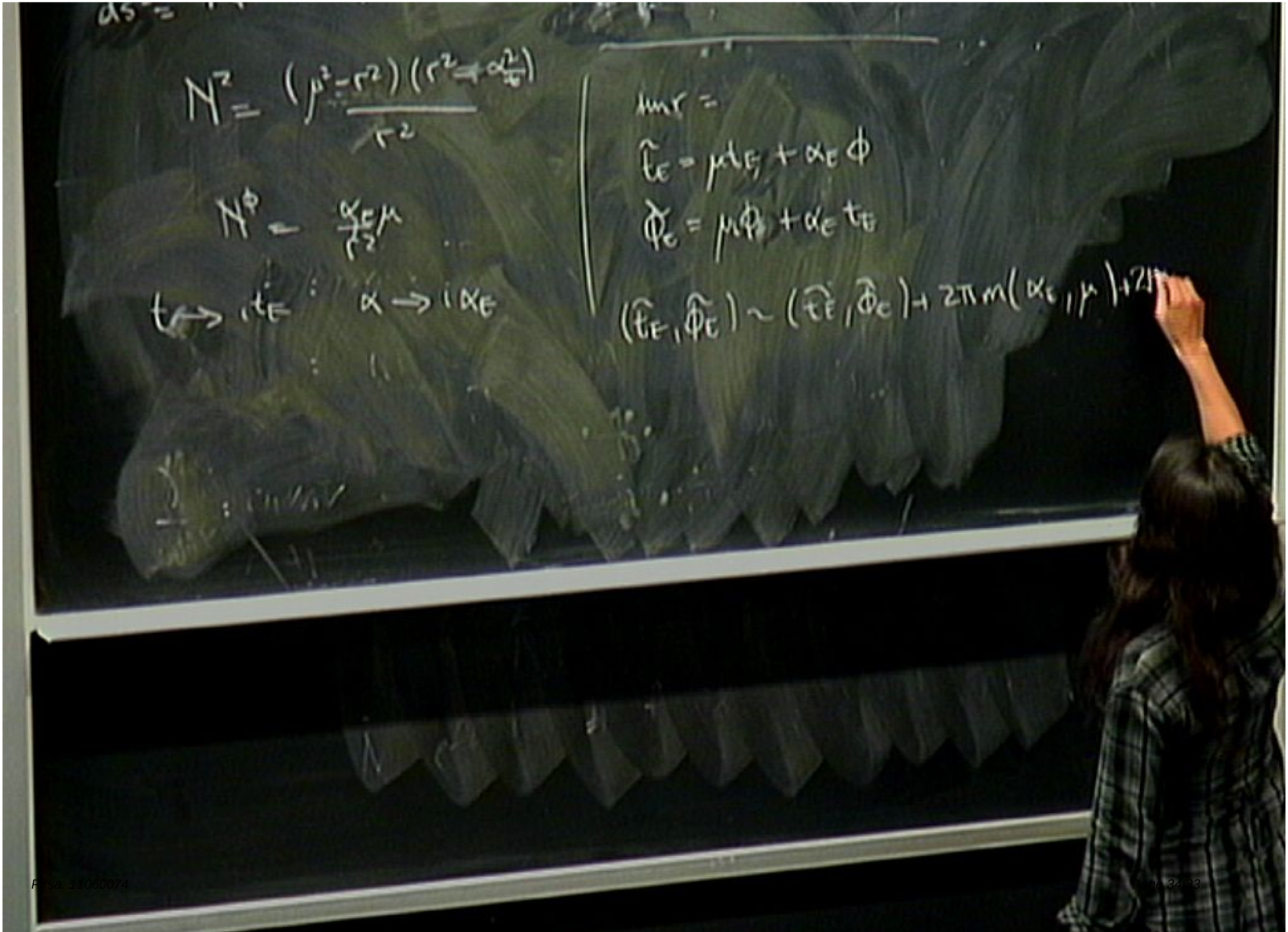
$r = 0 \rightarrow$ dS lines

$r = \pi/2$)

$$ds^2 = dr^2 + \cos^2 r \, d\tilde{t}_E^2 + \sin^2 r \, d\phi^2$$

$H = i\partial_t \qquad J = i\partial_\phi$

$L_0 = \frac{1}{2}(H + iJ) \qquad \bar{L}_0 = \frac{1}{2}(H - iJ)$


$N^\phi = \frac{\alpha_E \mu}{r^2}$
$t \to i t_E \quad \alpha \to i \alpha_E$
$\tilde{t}_E = \mu t_E + \alpha_E \phi$
$\tilde{\phi}_E = \mu \phi_E + \alpha_E t_E$
$(\tilde{t}_E, \tilde{\phi}_E) \sim (\tilde{t}_E, \tilde{\phi}_E) + 2\pi m(\alpha_E, \mu)$

Kerr $-dS_3$

$$ds^2 = N^{-2} dr^2 + N^2 dt_E^2 + r^2(d\phi + N^\phi dt_E)$$

$$\phi \sim \phi + 2\pi$$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$$t \to i t_E \; : \; \alpha \to i\alpha_E$$

$$\tilde{t}_E = \mu t_E + \alpha_E \phi$$

$$\tilde{\phi}_E = \mu \phi_E + \alpha_E t_E$$

$$(\hat{t}_E, \hat{\phi}_E) \sim (\hat{t}_E, \hat{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1, 0)$$

Kerr $-dS_3$

$$ds^2 = N^{-2}\,dr^2 + N^2\,dt_E^2 + r^2\left(d\phi + N^\phi\,dt_E\right)$$

$$\phi \sim \phi + 2\pi$$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$$t \to i t_E \; : \quad \alpha \to i\alpha_E$$

$\mathrm{Im}\, r =$

$$\tilde{t}_E = \mu t_E + \alpha_E \phi$$

$$\tilde{\phi}_E = \mu \phi_E + \alpha_E t_E$$

$$(\tilde{t}_E, \tilde{\phi}_E) \sim (\tilde{t}_E, \tilde{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1,0)$$

$$+ 2\pi(m + \quad)(\alpha_E, \mu) + \frac{2\pi n}{p}(1,0)$$

Kerr $-dS_3$

$$ds^2 = N^{-2}dr^2 + N^2 dt_E^2 + r^2(d\phi + N^\phi dt_E)$$

$$\phi \sim \phi + 2\pi$$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 + \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$$t \to i t_E \; : \; \alpha \to i\alpha_E$$

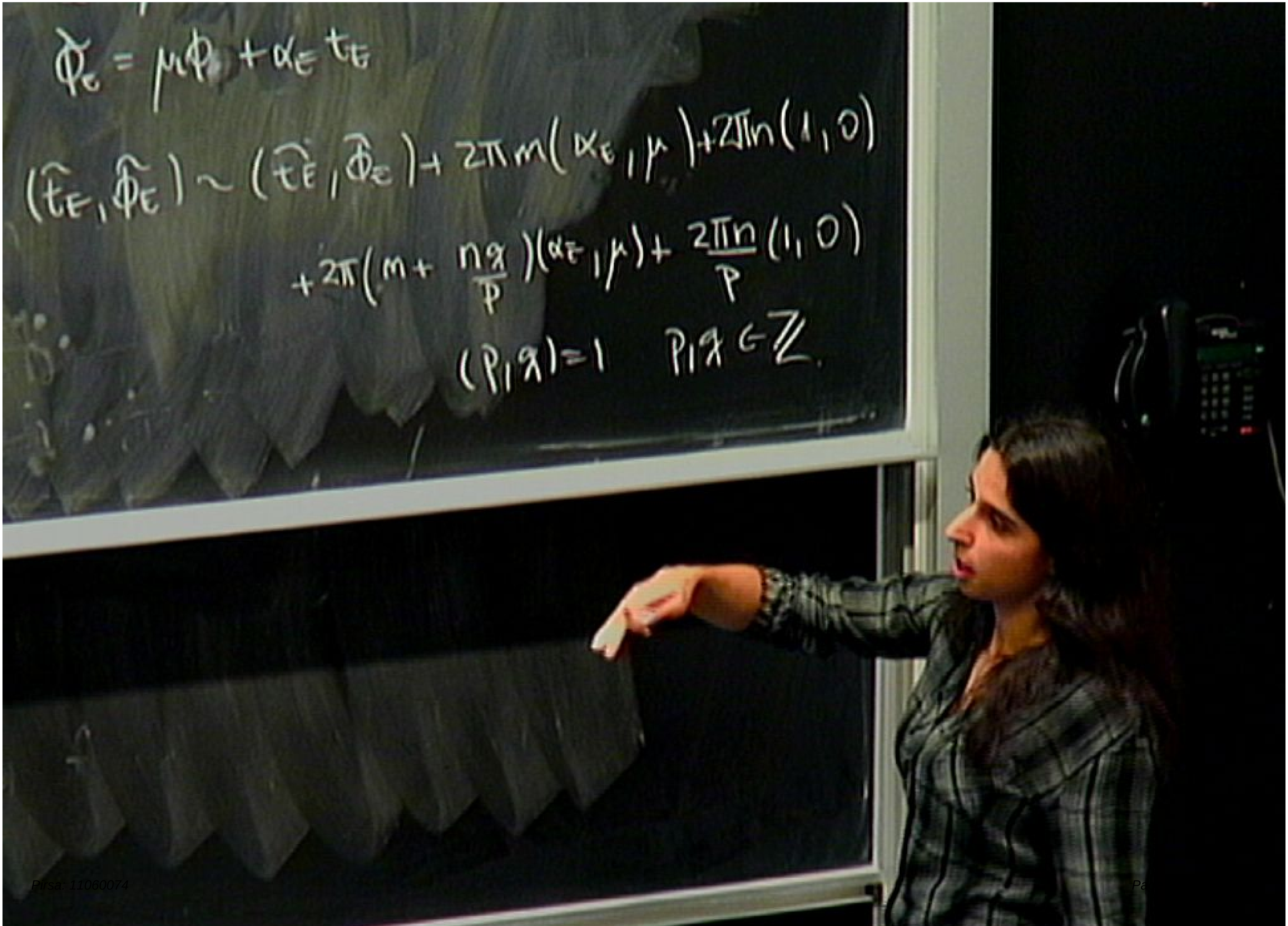
tmr =

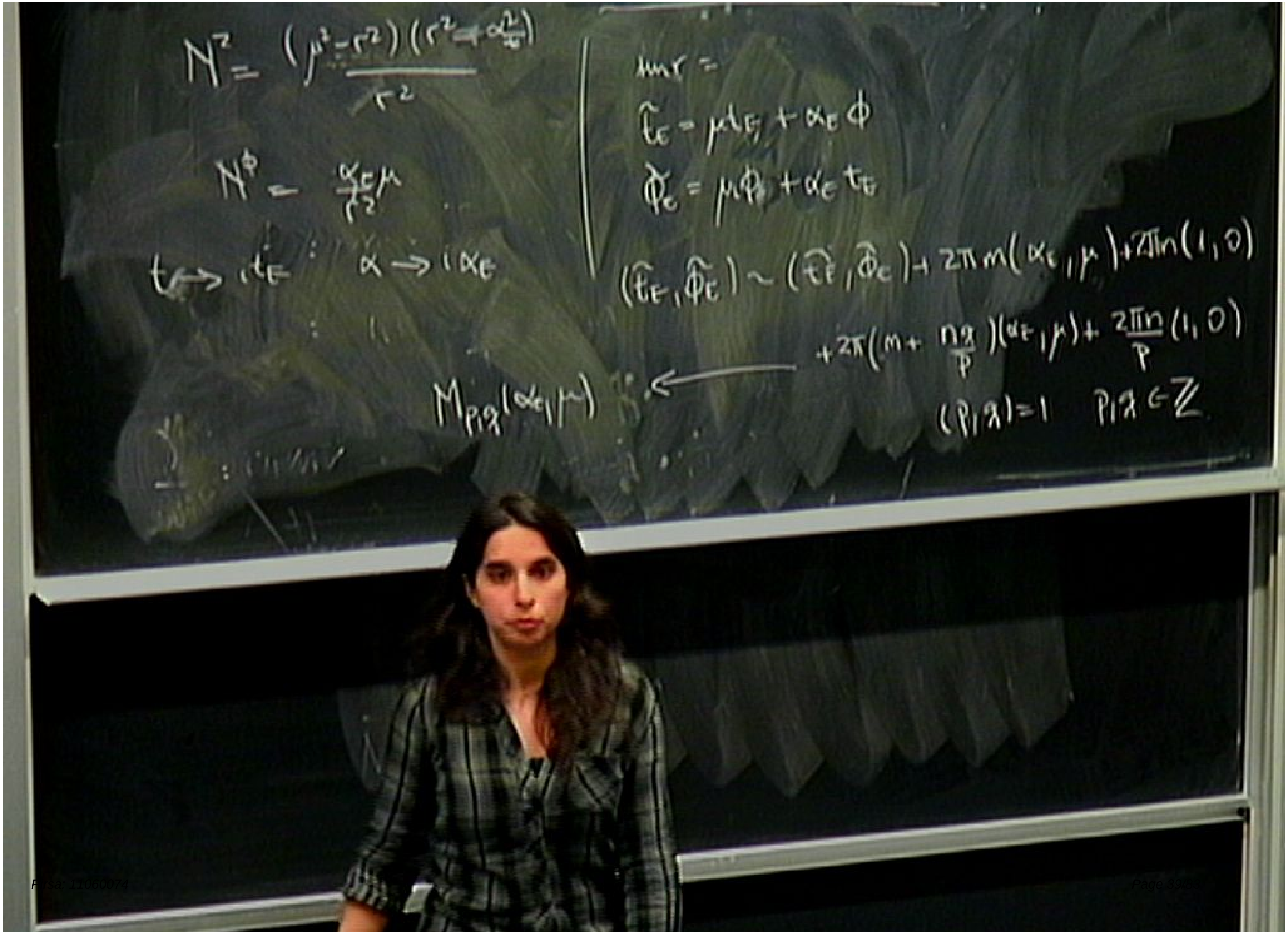
$$\hat{t}_E = \mu t_E + \alpha_E \phi$$

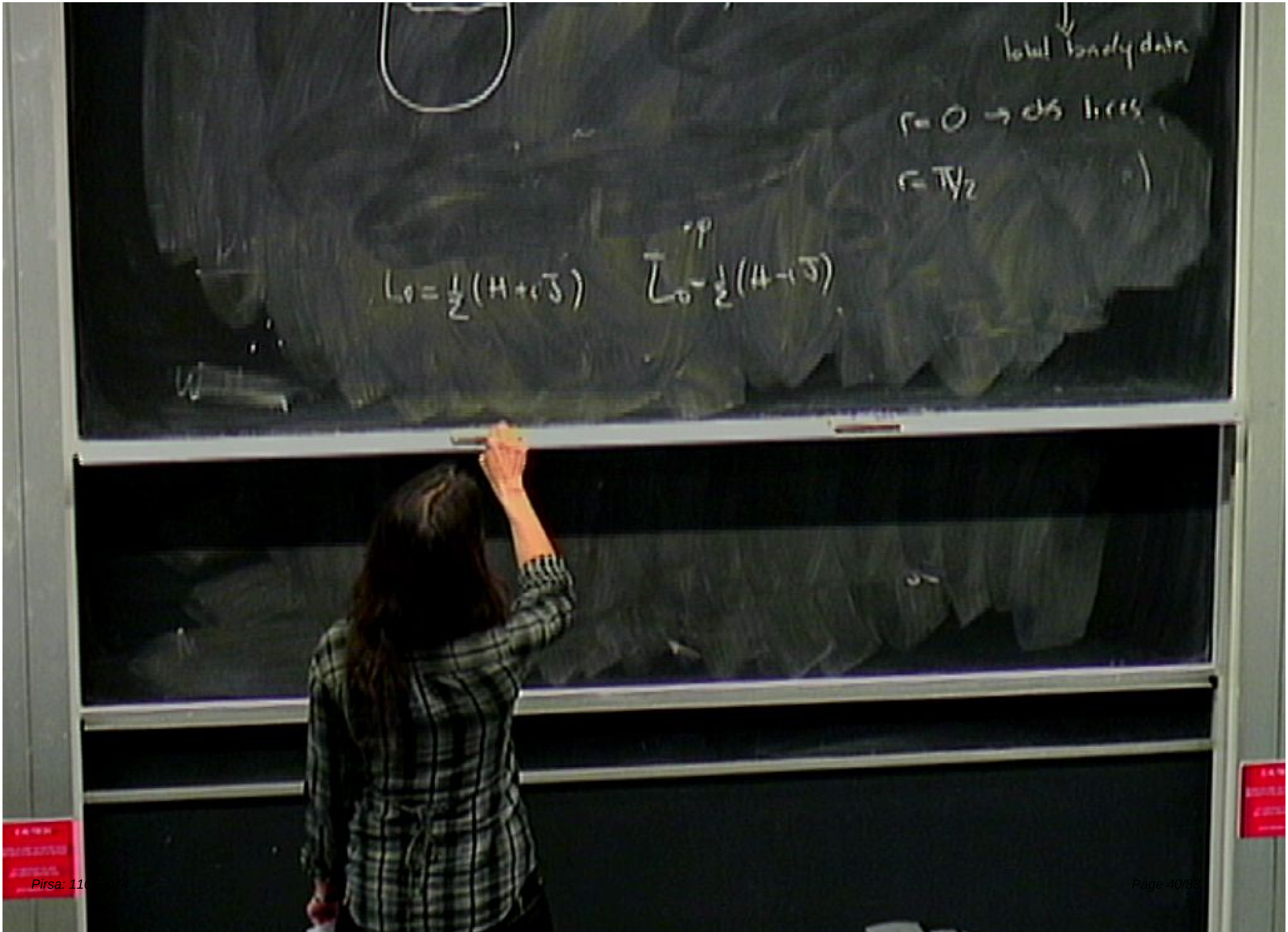
$$\hat{\phi}_E = \mu \phi_E + \alpha_E t_E$$

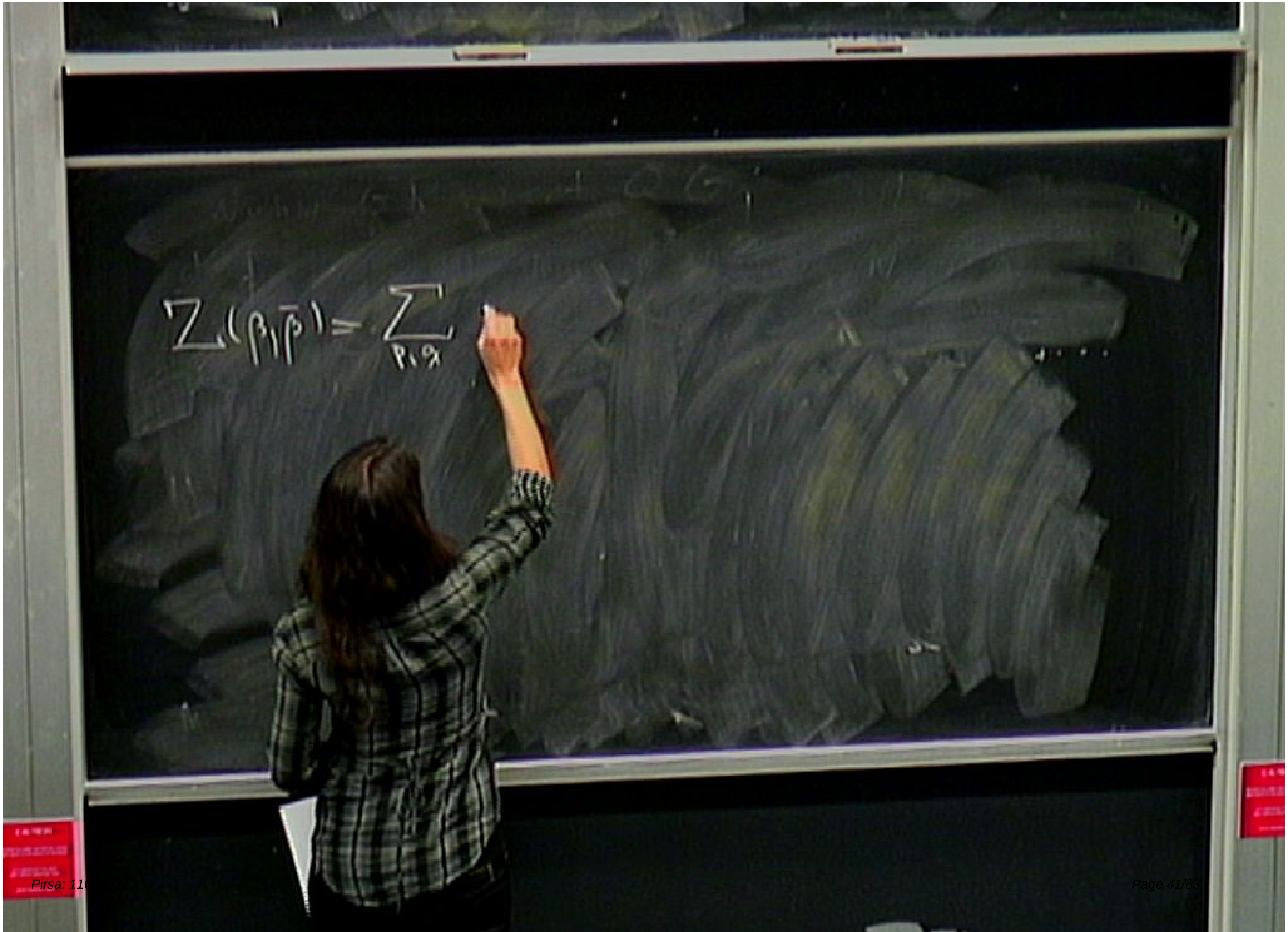
$$(\hat{t}_E, \hat{\phi}_E) \sim (\hat{t}_E, \hat{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1,0)$$

$$+ 2\pi\left(m + \frac{nq}{p}\right)(\alpha_E, \mu) + \frac{2\pi n}{p}(1,0)$$


$\tilde{\phi}_E = \mu\phi + \alpha_E t_E$
$(\hat{t}_E, \hat{\phi}_E) \sim (\hat{t}_E, \hat{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1,0)$
$+ 2\pi(m + \frac{nq}{p})(\alpha_E, \mu) + \frac{2\pi n}{p}(1,0)$
$(p,q)=1 \quad p,q \in \mathbb{Z}$


$N^\phi = \frac{\alpha_E \mu}{r^2}$
$\hat{t}_E = \mu t_E + \alpha_E \phi$
$\hat{\phi}_E = \mu \phi_E + \alpha_E t_E$
$(\hat{t}_E, \hat{\phi}_E) \sim (\hat{t}_E, \hat{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1,0)$
$+ 2\pi(m + \frac{nq}{p})(\alpha_E, \mu) + \frac{2\pi n}{p}(1,0)$
$M_{p,q}(\alpha_E, \mu)$
$(p,q)=1 \quad p,q \in \mathbb{Z}$


local bndy data
r = 0 → dS lines
r = π/2
L_0 = 1/2 (H + iJ)
L̄_0 = 1/2 (H − iJ)


$Z(\beta, \bar{\beta}) = \sum_{p,q}$

$$Z(\beta, \bar{\beta}) = \sum_{p,q} \exp(-S[M_{pq}(x_E, r)]$$

$$\rho \sim \exp($$

$$N^2 = \frac{(\mu^2 - r^2)(r^2 - \alpha_E^2)}{r^2}$$

$$N^\phi = \frac{\alpha_E \mu}{r^2}$$

$t \to i t_E \quad ; \quad \alpha \to i\alpha_E$

$\lim r =$

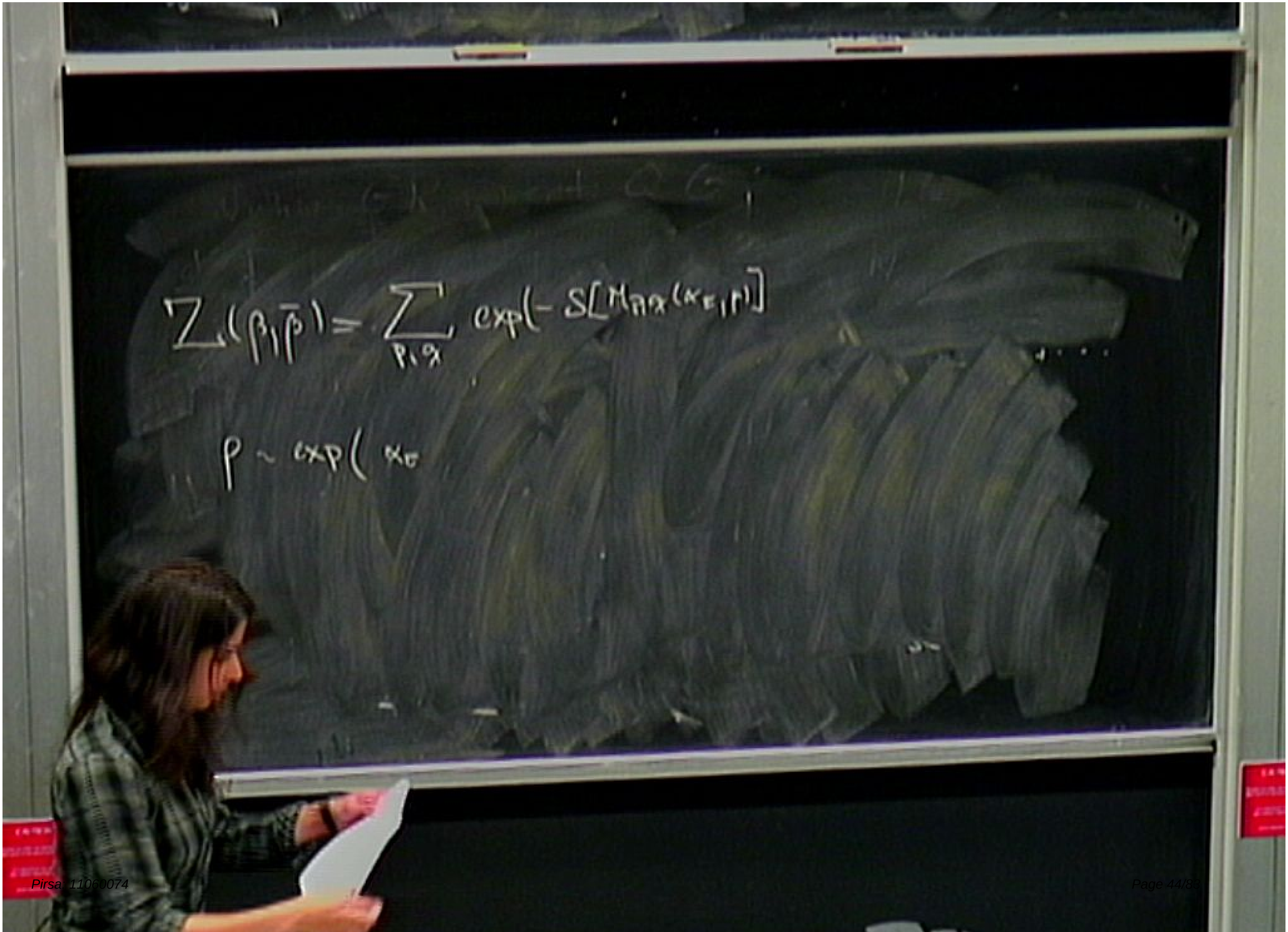
$\tilde{t}_E = \mu t_E + \alpha_E \phi$

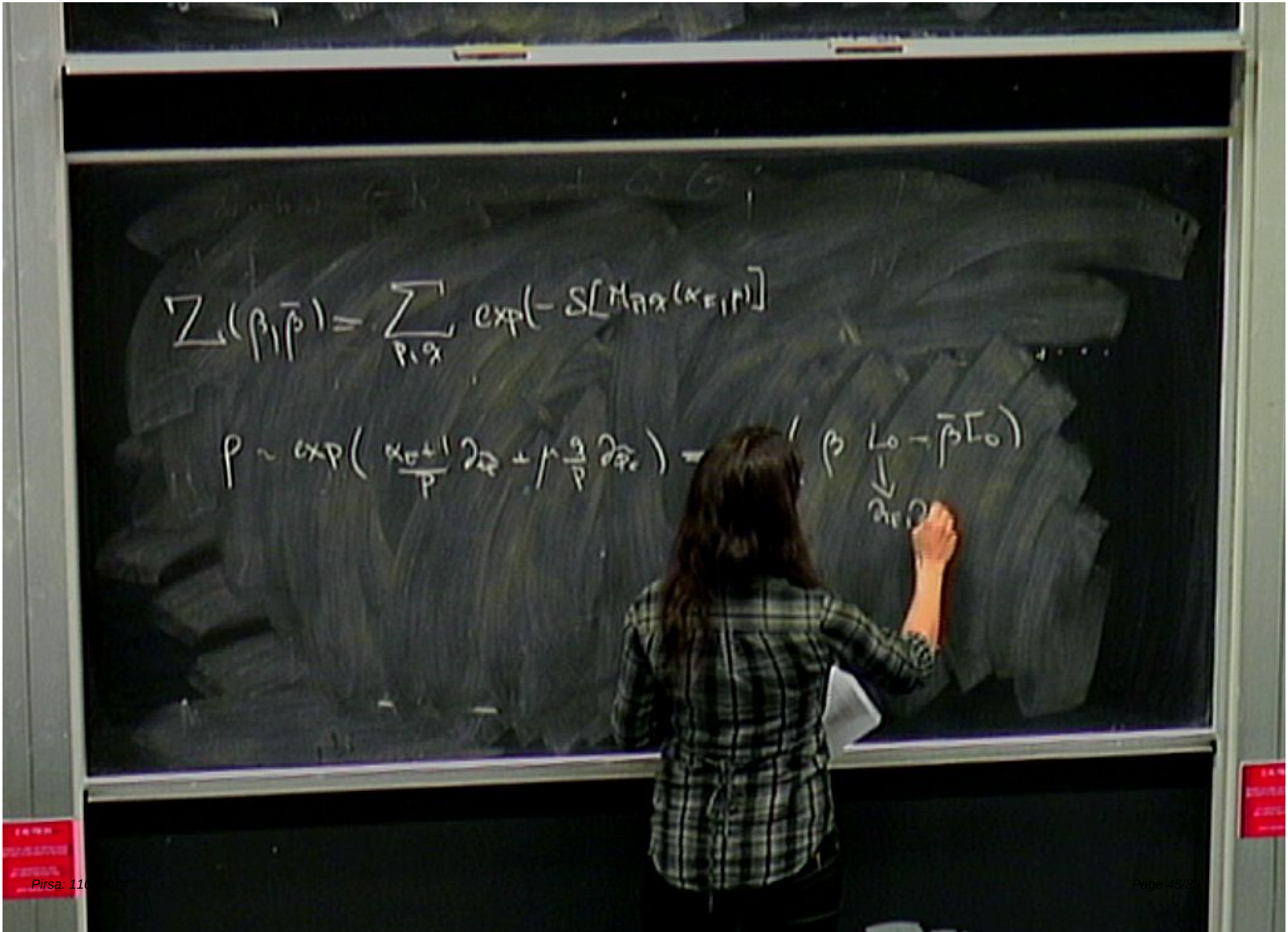
$\tilde{\phi}_E = \mu \phi_E + \alpha_E t_E$

$$(\tilde{t}_E, \tilde{\phi}_E) \sim (\tilde{t}_E, \tilde{\phi}_E) + 2\pi m(\alpha_E, \mu) + 2\pi n(1,0)$$

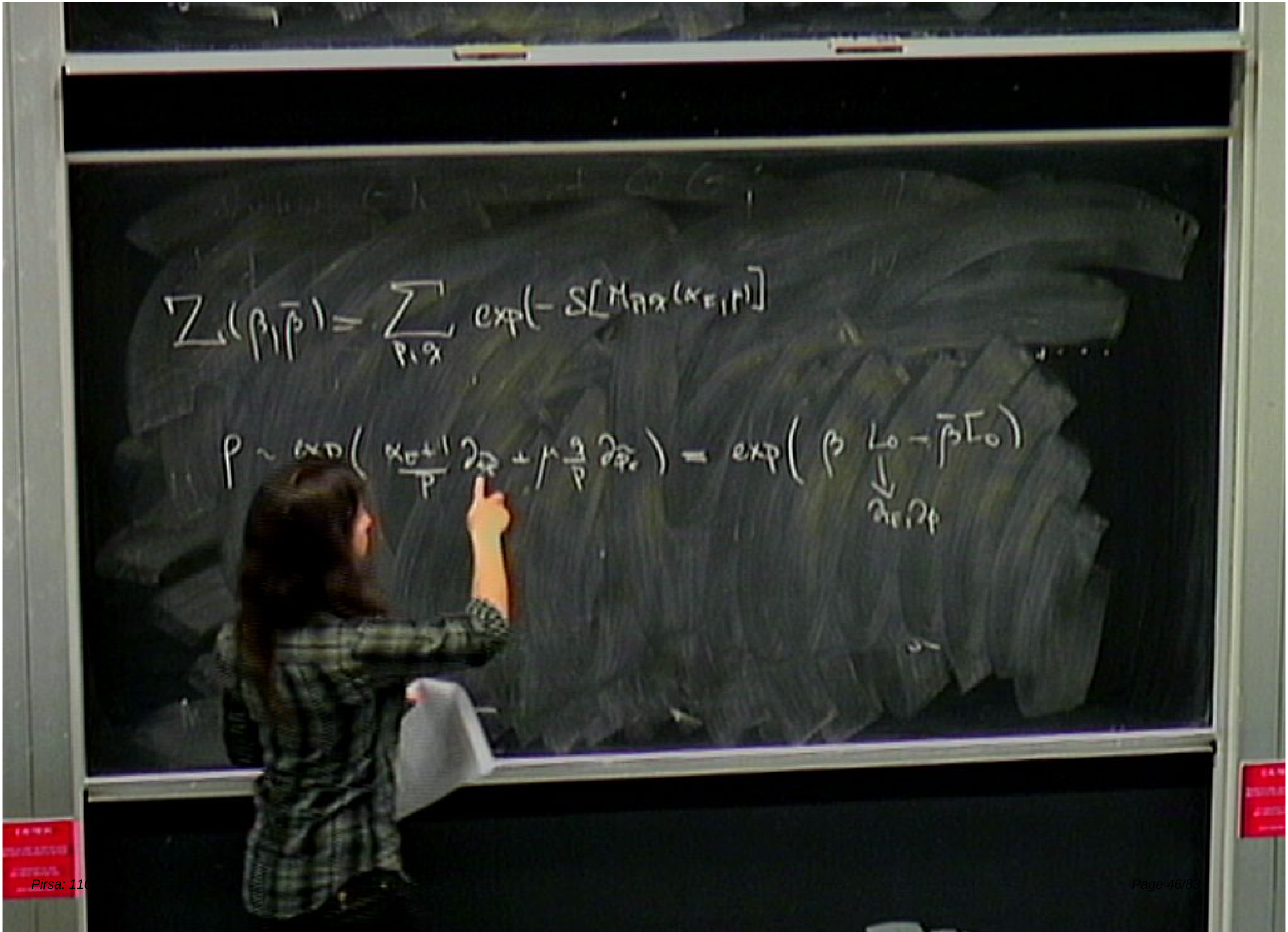
$$M_{p,q}(\alpha_E, \mu) \leftarrow + 2\pi\left(m + \frac{nq}{p}\right)(\alpha_E, \mu) + \frac{2\pi n}{p}(1,0)$$

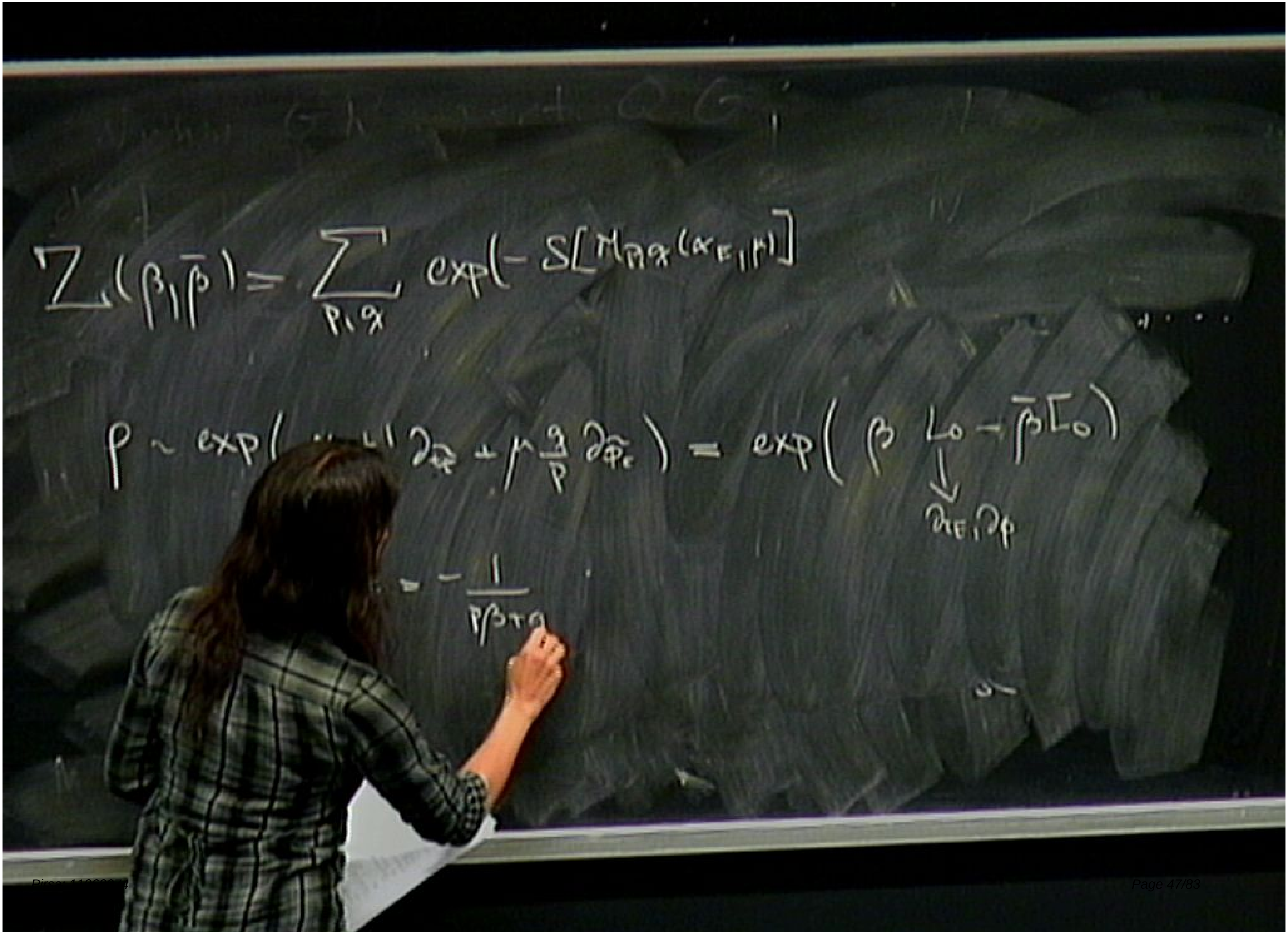
$(p,q) = 1 \quad p,q \in \mathbb{Z}$


$Z(\beta,\bar{\beta}) = \sum_{p,\alpha} \exp(-S[M_{p\alpha}(x_E, r)]$
$\rho \sim \exp(x_\sigma$



$Z(\beta,\bar\beta) = \sum_{p,\alpha} \exp(-S[\mathrm{Max}(\alpha_E, p)]$


$Z(\beta,\bar\beta) = \sum_{p,x} \exp(-S[\mathrm{Map}(x_E,p)]$
$= \exp(\beta L_0 - \bar\beta \bar L_0)$



$Z(\beta, \bar{\beta}) = \sum_{p,q} \exp(-S[M_{p,q}(x_E, \rho)]$
$\rho \sim \exp(\quad \partial_{\tilde{\varphi}} + \mu \frac{q}{p} \partial_{\tilde{\varphi}_E}) = \exp(\beta L_0 - \bar{\beta} \bar{L}_0)$
$\partial_{t_E}, \partial_\rho$
$= -\frac{1}{p\beta + q}$

$$Z(\beta, \bar{\beta}) = \sum_{p,q} \exp\left(- S[M_{p,q}(\alpha_E, \mu)]\right) = \sum_{p,q} Z_{p,q}($$

$$\rho \sim \exp\left(\frac{\alpha_E + 1}{p} \partial_{\tilde{\tau}} + \mu \frac{q}{p} \partial_{\tilde{\phi}_E} \right) = \exp\left(\left(\;\; L_0 + \bar{\beta} \bar{L}_0 \right)\right.$$

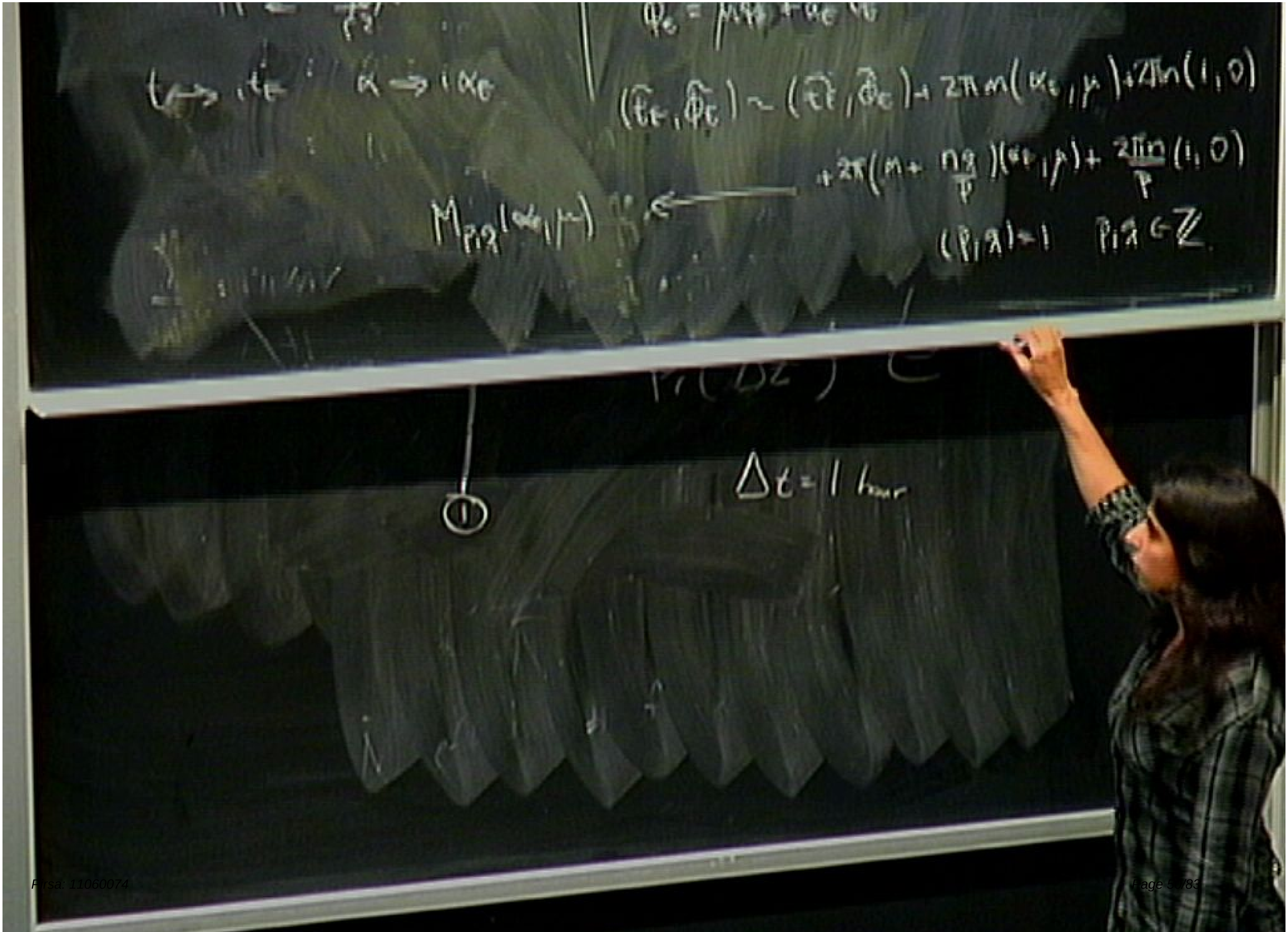
$$\mu + \alpha_E = -\frac{1}{p\beta + q} \qquad \mu - \alpha_E = \frac{1}{p\bar{\beta} + q}$$

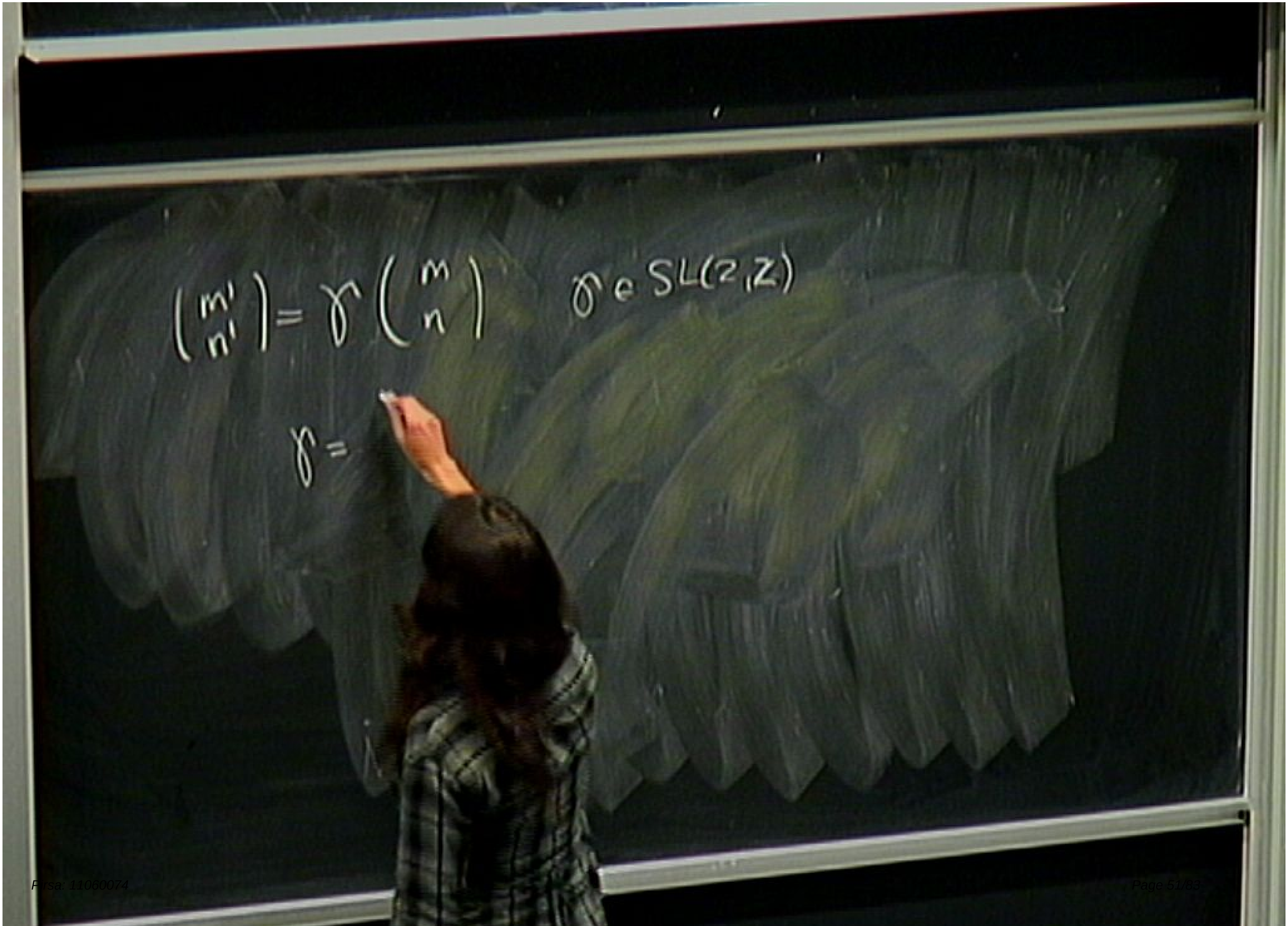
$$Z(\beta,\bar{\beta}) = \sum_{p,q} \exp\left(-S[M_{p,q}(\alpha_E, \mu)]\right) = \sum_{p,q} Z_m\left(p\beta+q,\, p\bar{\beta}+q\right)$$

$$\rho \sim \exp\left(\frac{\alpha_E+1}{p}\partial_{t_E} + \mu\frac{q}{p}\partial_{\phi_E}\right) = \exp\left(\beta L_0 - \bar{\beta}\bar{L}_0\right)$$

$$\partial_{t_E}, \partial_\phi$$

$$\mu+\alpha_E = -\frac{1}{p\beta+q} \qquad \mu-\alpha_E = \frac{1}{p\bar{\beta}+q}$$


(p,q)=1 p,q ∈ Z
Δt = 1 hour


$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix}$ $\gamma \in SL(2,\mathbb{Z})$
$\gamma =$

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

$$(\hat{\theta}_e, \hat{\phi}_e) \sim (\hat{\theta}_e, \hat{\phi}_e) + 2\pi m' (\alpha_e', \beta') + 2\pi n' (1, 0)$$

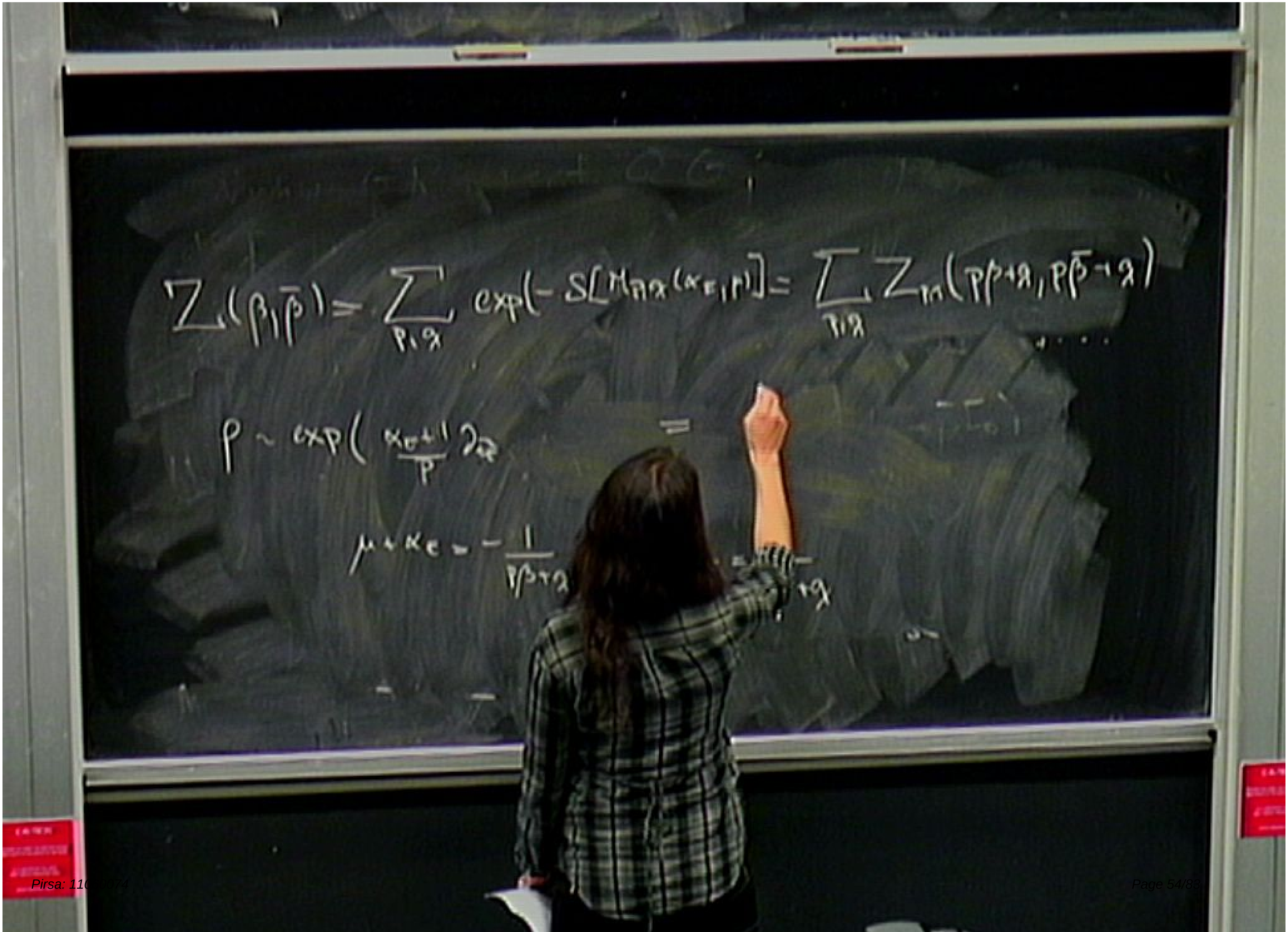
$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

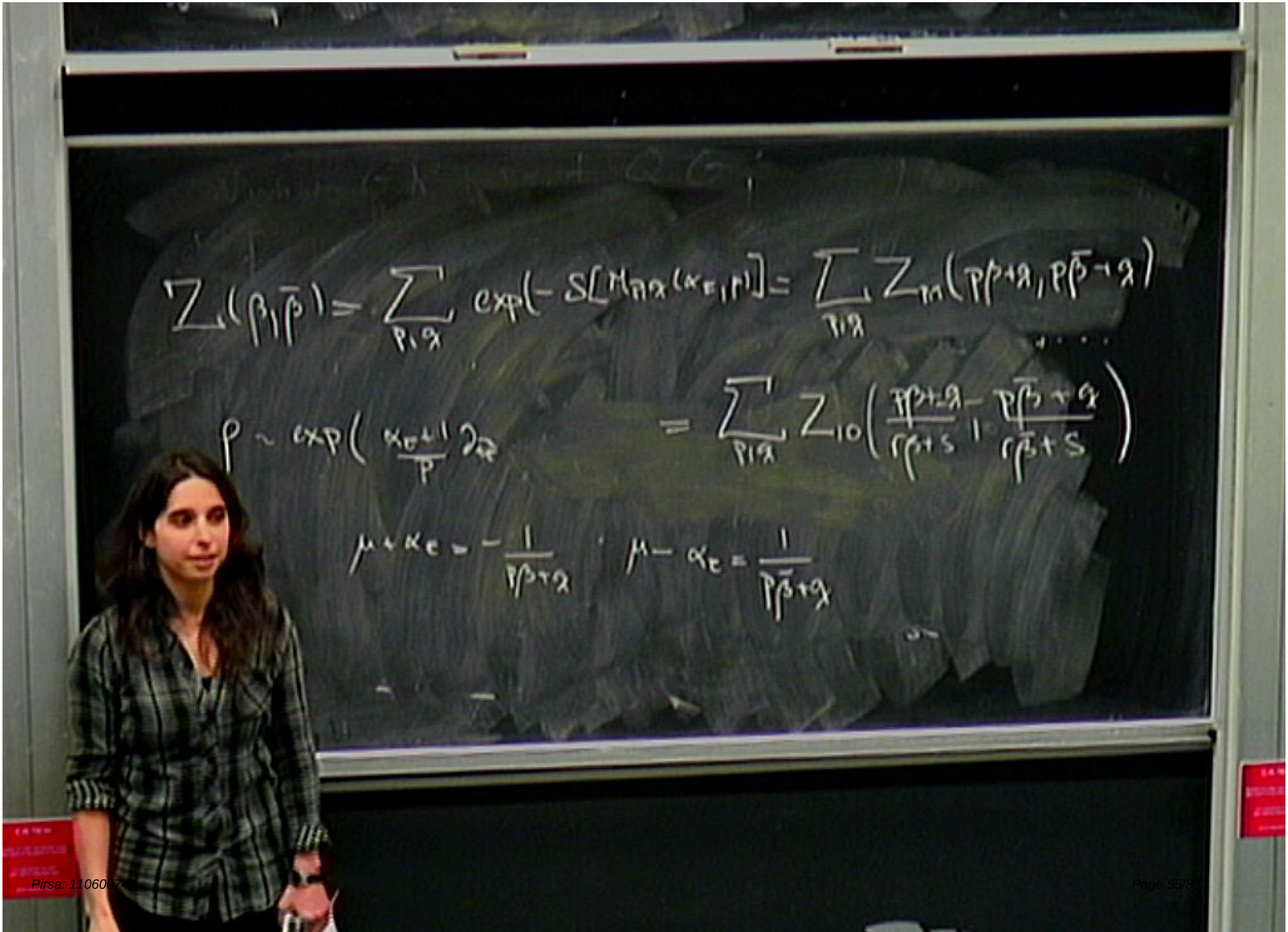
$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$$

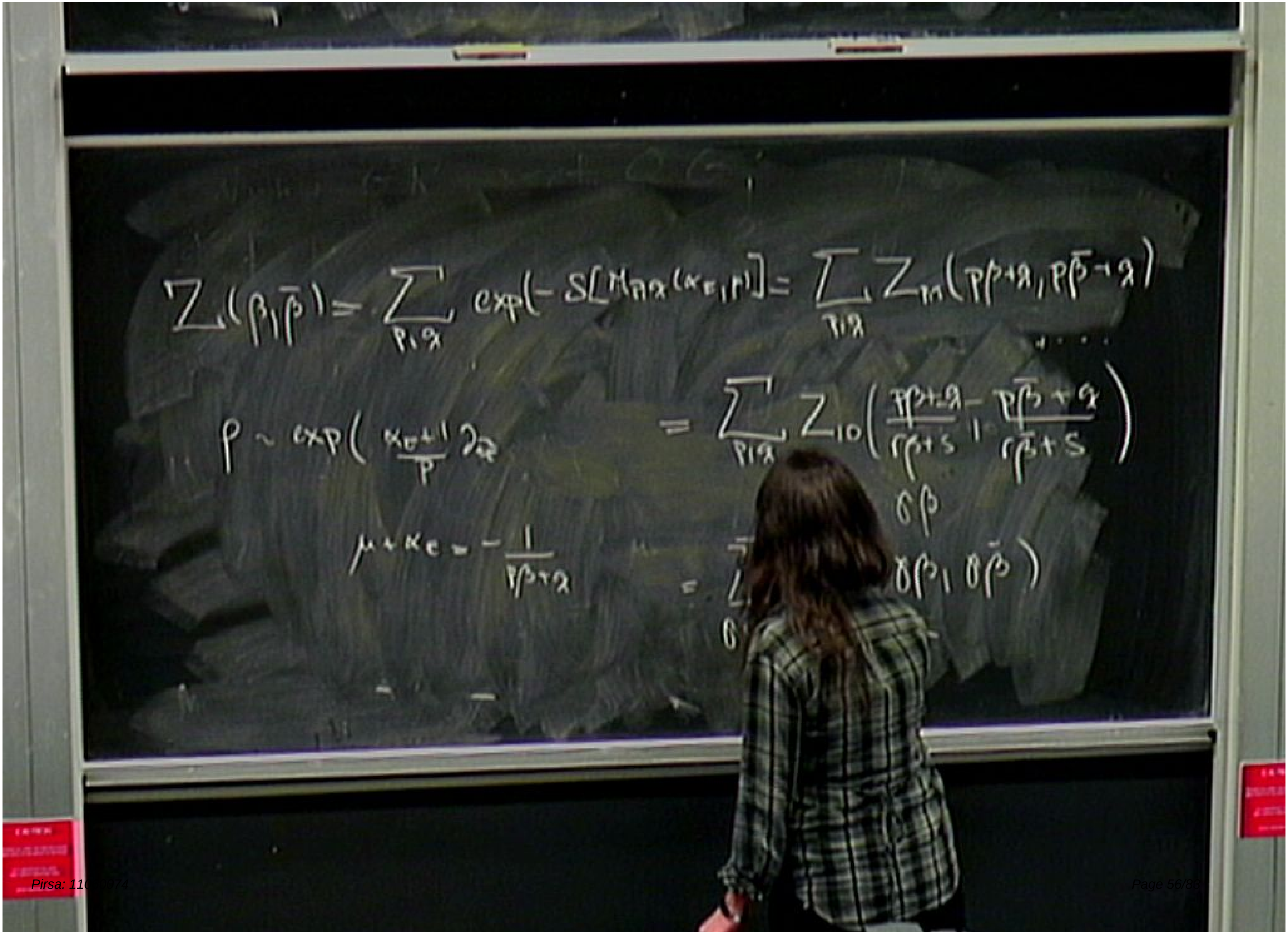
$$M_{p,q}(\alpha_E, \mu) = M_{1,0}(\alpha_E', \mu')$$

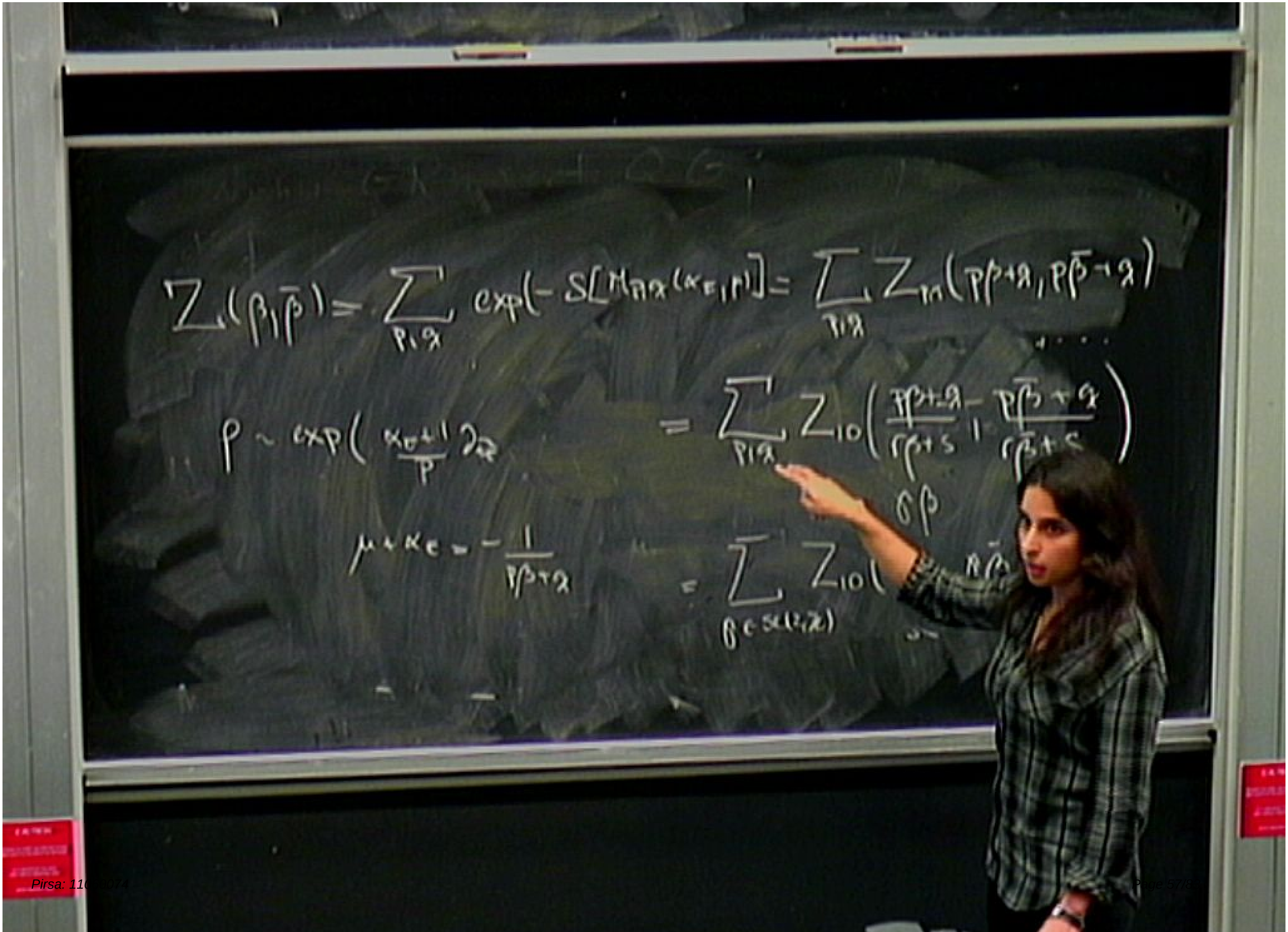
$$(\tilde{t}_E, \tilde{\phi}_E) \sim (\tilde{t}_E, \tilde{\phi}_E) + 2\pi m' (\alpha_E', \mu') + 2\pi n'(1,0)$$

$$\alpha_E' = \frac{\alpha_E - r}{p} \qquad \mu' = \frac{\mu}{p}$$


Z(β,β̄) = Σ_{p,q} exp(-S[M_{p,q}(α_E, r)]) = Σ_{p,q} Z_{cl}(pβ+q, pβ̄+q)
ρ ~ exp((α_E+1)/p ∂_α
μ + κ_E = − 1/(pβ+q)


$Z(\beta,\bar\beta) = \sum_{p,q} \exp(-S[M_{p,q}(\alpha_E, r)]) = \sum_{p,q} Z_{M}(p\beta+q, p\bar\beta+q)$
$= \sum_{p,q} Z_{1,0}\left(\frac{p\beta+q}{r\beta+s}, \frac{p\bar\beta+q}{r\bar\beta+s}\right)$
$\rho \sim \exp\left(\frac{\alpha_E+1}{p}\partial_{\bar\beta}\right.$
$\mu+\alpha_E = -\frac{1}{p\beta+q}$
$\mu-\alpha_E = \frac{1}{p\bar\beta+q}$


$Z(\beta,\bar\beta) = \sum_{p,q} \exp(-S[M_{p,q}(\kappa_E, r)]) = \sum_{p,q} Z_M(p\beta+q, p\bar\beta+q)$
$\rho \sim \exp\left(\frac{\kappa_E+1}{p} \partial_z \right.$
$= \sum_{p,q} Z_{10}\left(\frac{p\beta+q}{r\beta+s}, \frac{p\bar\beta+q}{r\bar\beta+s} \right)$
$\mu + \kappa_E = -\frac{1}{p\beta+q}$
$\gamma\beta$
$= \sum \; (\gamma\beta, \gamma\bar\beta)$



$Z(\beta,\bar\beta) = \sum_{p,q} \exp(-S[M_{p,q}(\alpha_E, r)]) = \sum_{p,q} Z_{10}(p\beta+q, p\bar\beta+q)$
$\rho \sim \exp\left(\frac{\alpha_E+1}{p}\partial_{\bar z}\right)$
$= \sum_{p,q} Z_{10}\left(\frac{p\beta+q}{r\beta+s}, \frac{p\bar\beta+q}{r\bar\beta+s}\right)$
$\mu+\kappa_E = -\frac{1}{p\beta+q}$
$= \sum_{\beta\in SL(2,\mathbb{Z})} Z_{10}($

$$Z(\beta,\bar\beta) = \sum_{p,q} \exp(-S[M_{p,q}(\alpha_E, \rho)]) = \sum_{p,q} Z_{10}(p\beta+q, p\bar\beta+q)$$

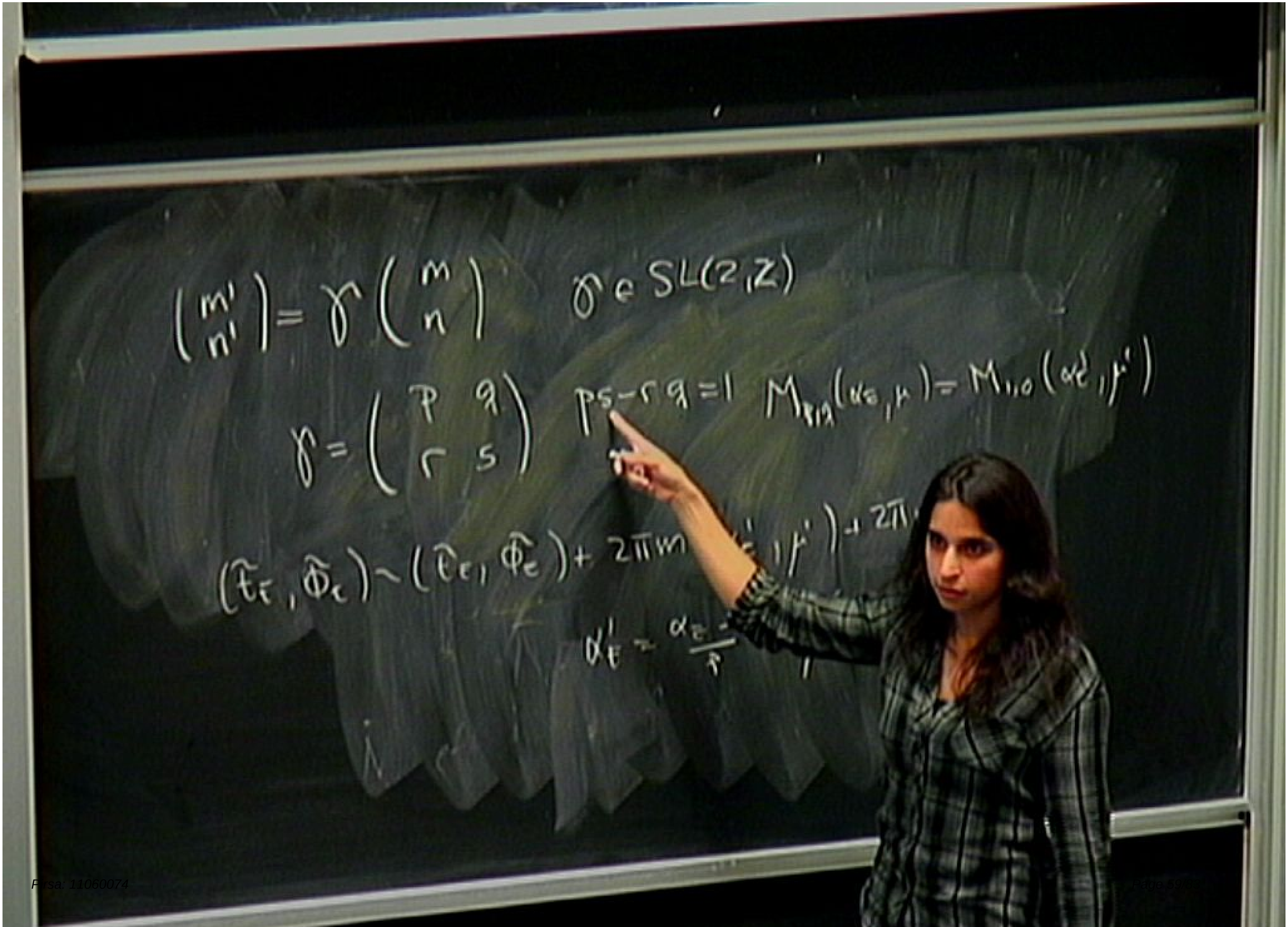
$$\rho \sim \exp\left(\frac{\alpha_E+1}{p} \partial_q \right.$$

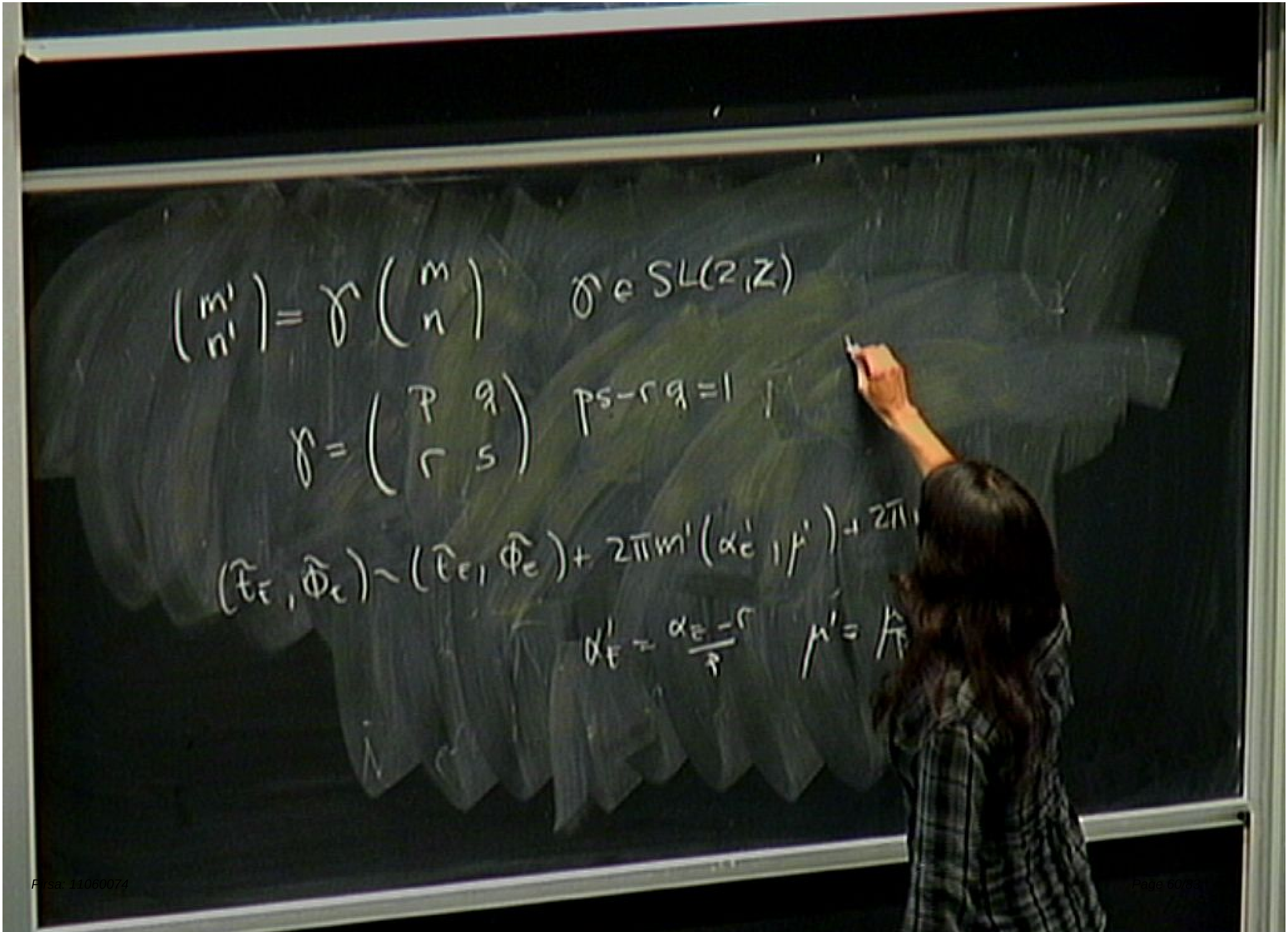
$$= \sum_{p,q} Z_{10}\left(\frac{p\beta+q}{r\beta+s}, \frac{p\bar\beta+q}{r\bar\beta+s} \right)$$

$$\gamma\beta$$

$$\mu + \alpha_E = -\frac{1}{p\beta+q}$$

$$= \sum_{\gamma \in SL(2,\mathbb{Z})} Z_{10}(\gamma\beta, \gamma\bar\beta)$$


$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix}$ $\gamma \in SL(2,Z)$
$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ $ps - rq = 1$ $M_{p,q}(\alpha_E, \mu) = M_{1,0}(\alpha'_E, \mu')$


$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix}$ $\gamma \in SL(2,\mathbb{Z})$
$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ $ps - rq = 1$
$(\hat{t}_E, \hat{\Phi}_e) \sim (\hat{t}_E, \hat{\Phi}_e) + 2\pi m'(\alpha'_E, \mu') + 2\pi$
$\alpha'_E = \frac{\alpha_E - r}{p}$ $\mu' =$

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1 \qquad \begin{matrix} r \to r + p \\ s \to s + q \end{matrix}$$

$$(\tilde{\tau}_E, \tilde{\Phi}_E) \sim (\tilde{\tau}_E, \tilde{\Phi}_E) + 2\pi m' (\alpha_E', \mu') + 2\pi n' (1, 0)$$

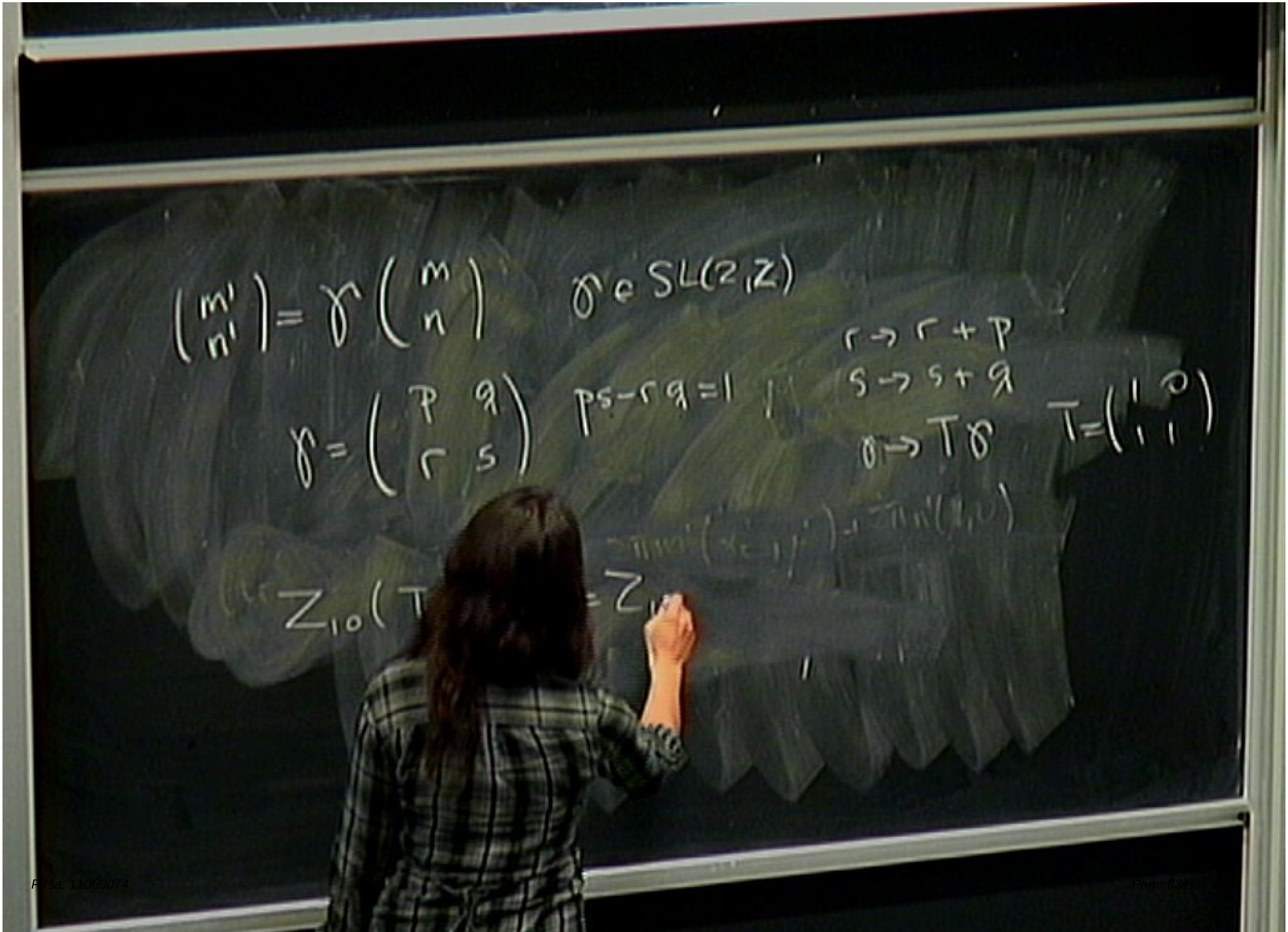
$$\alpha_E' = \frac{\alpha_E - r}{p} \qquad \mu' = \frac{\mu}{p}$$

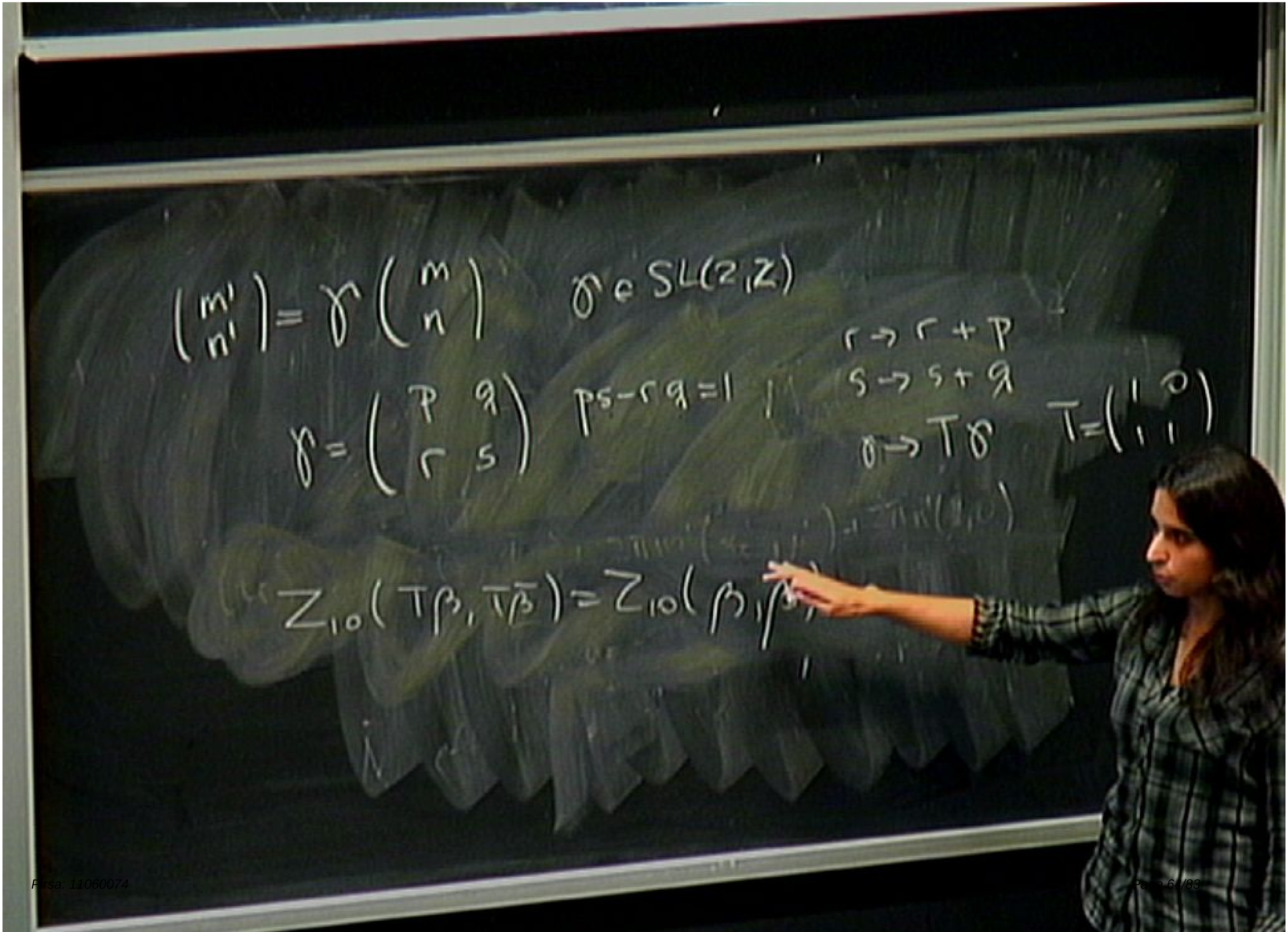
$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1$$

$$r \to r + p$$
$$s \to s + q$$
$$\gamma \to T\gamma \qquad T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\mu' = \frac{\mu}{p}$$


(m' n') = γ (m n)
γ ∈ SL(2,Z)
γ = (p q r s)
ps − rq = 1
r → r + p
s → s + q
γ → Tγ
T = (1 0 1 1)
Z₁,₀(T

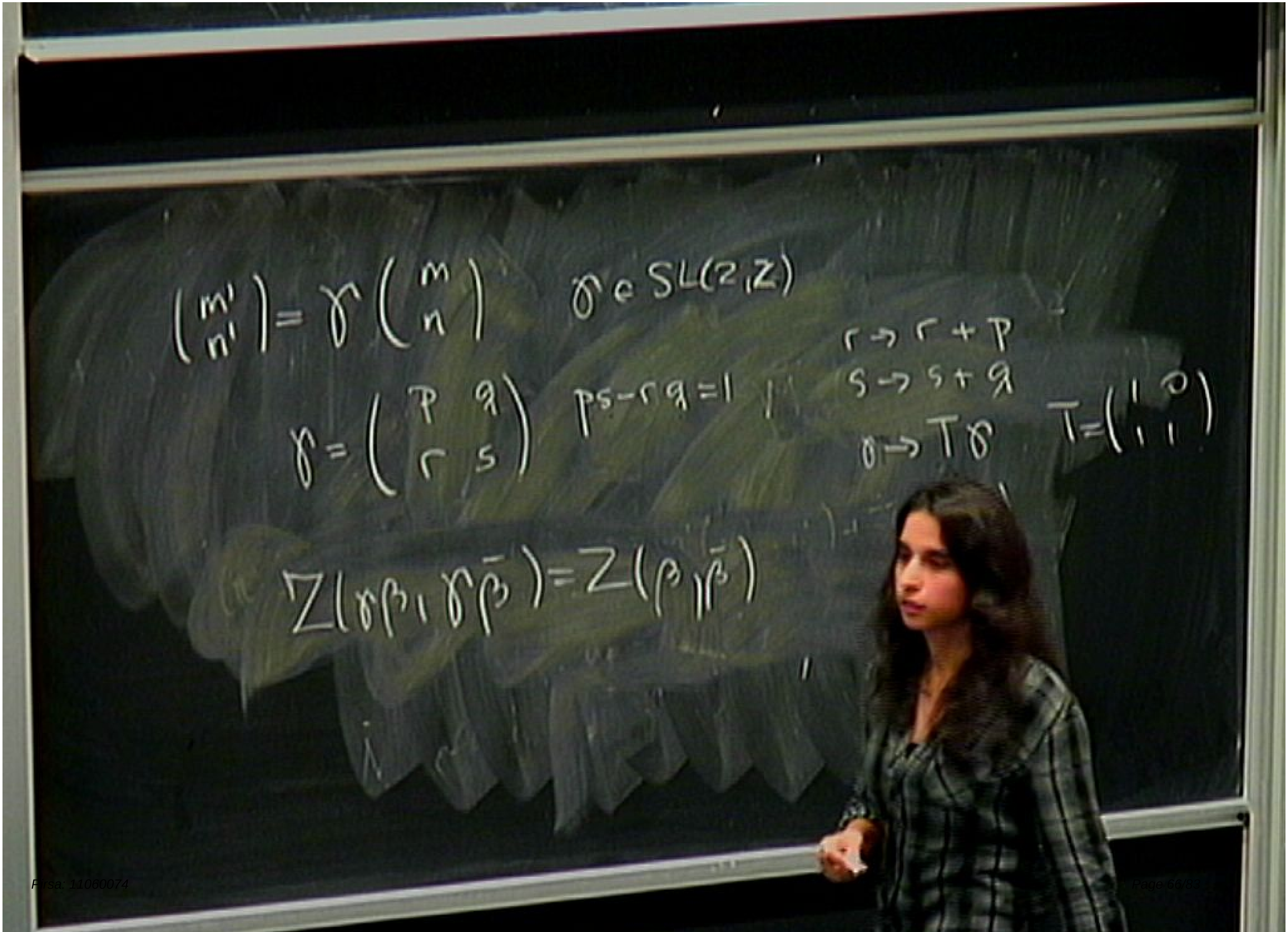

(m' n') = γ (m n)
γ ∈ SL(2,Z)
γ = (p q r s)
ps − rq = 1
r → r + p
s → s + q
γ → Tγ
T = (1 0 1 1)
Z10(Tβ, Tβ̄) = Z10(β, β̄)

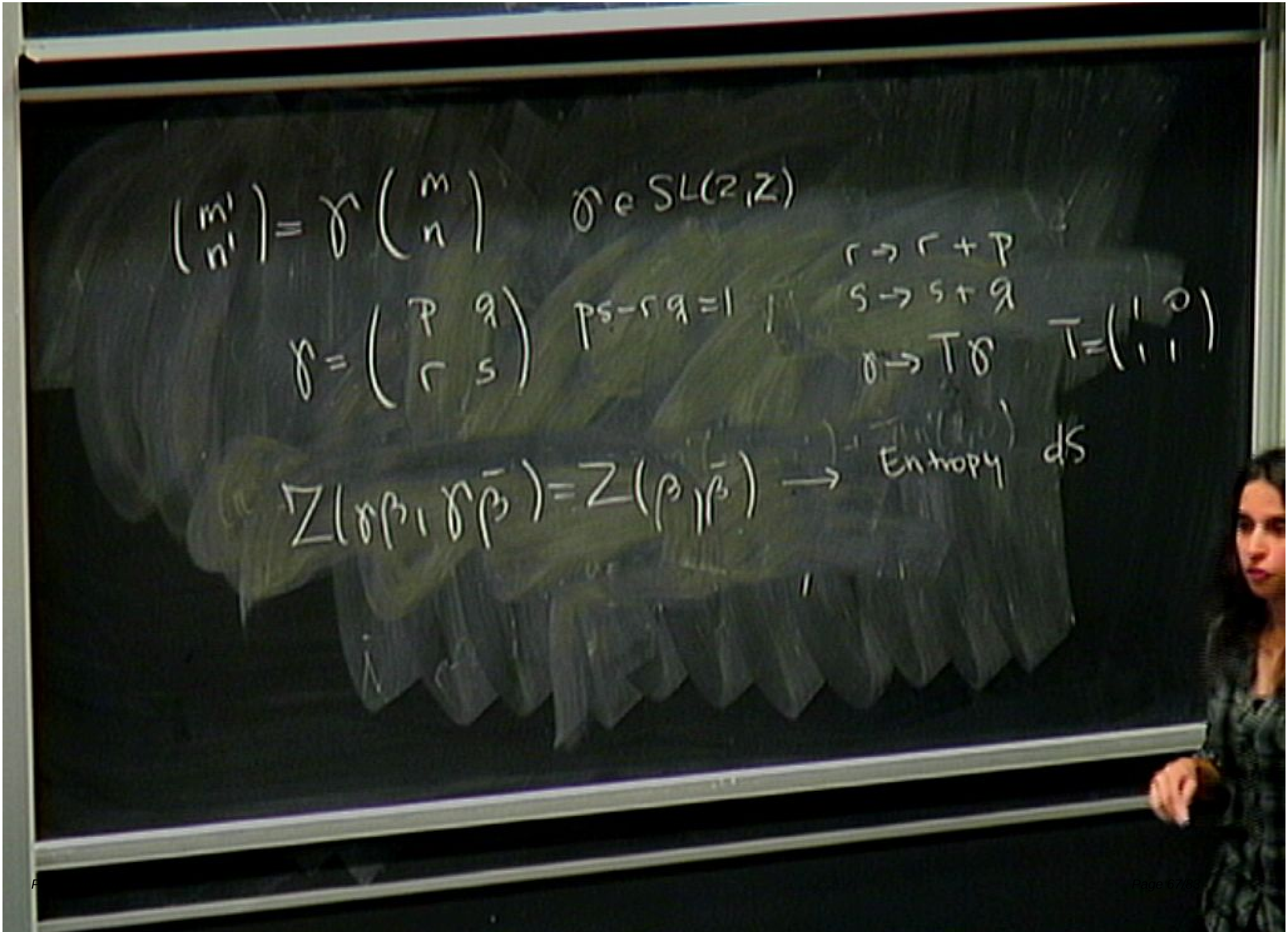
$$Z(\beta,\bar\beta) = \sum_{p,q} \exp\left(-S[M_{p,q}(\alpha_E, r)]\right) = \sum_{p,q} Z_{10}(p\beta+q,\, p\bar\beta+q)$$

$$\rho \sim \exp\left(\frac{\alpha_E+i}{p}\,\partial_x\right) \qquad = \sum_{p,q} Z_{10}\left(\frac{p\beta+q}{r\beta+s},\, \frac{p\bar\beta+q}{r\bar\beta+s}\right)$$

$$\gamma\beta$$

$$\mu + \kappa_E = -\frac{1}{p\beta+q} \qquad = \sum_{\gamma\in SL(2,\mathbb{Z})/\mathbb{Z}} Z_{10}(\gamma\beta,\, \gamma\bar\beta)$$


$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix}$ $\gamma \in SL(2,Z)$
$r \to r + p$
$s \to s + q$
$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ $ps - rq = 1$
$\gamma \to T\gamma$ $T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$
$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta})$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$\gamma \to T\gamma$ $T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$
$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) \longrightarrow$ Entropy dS

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1$$

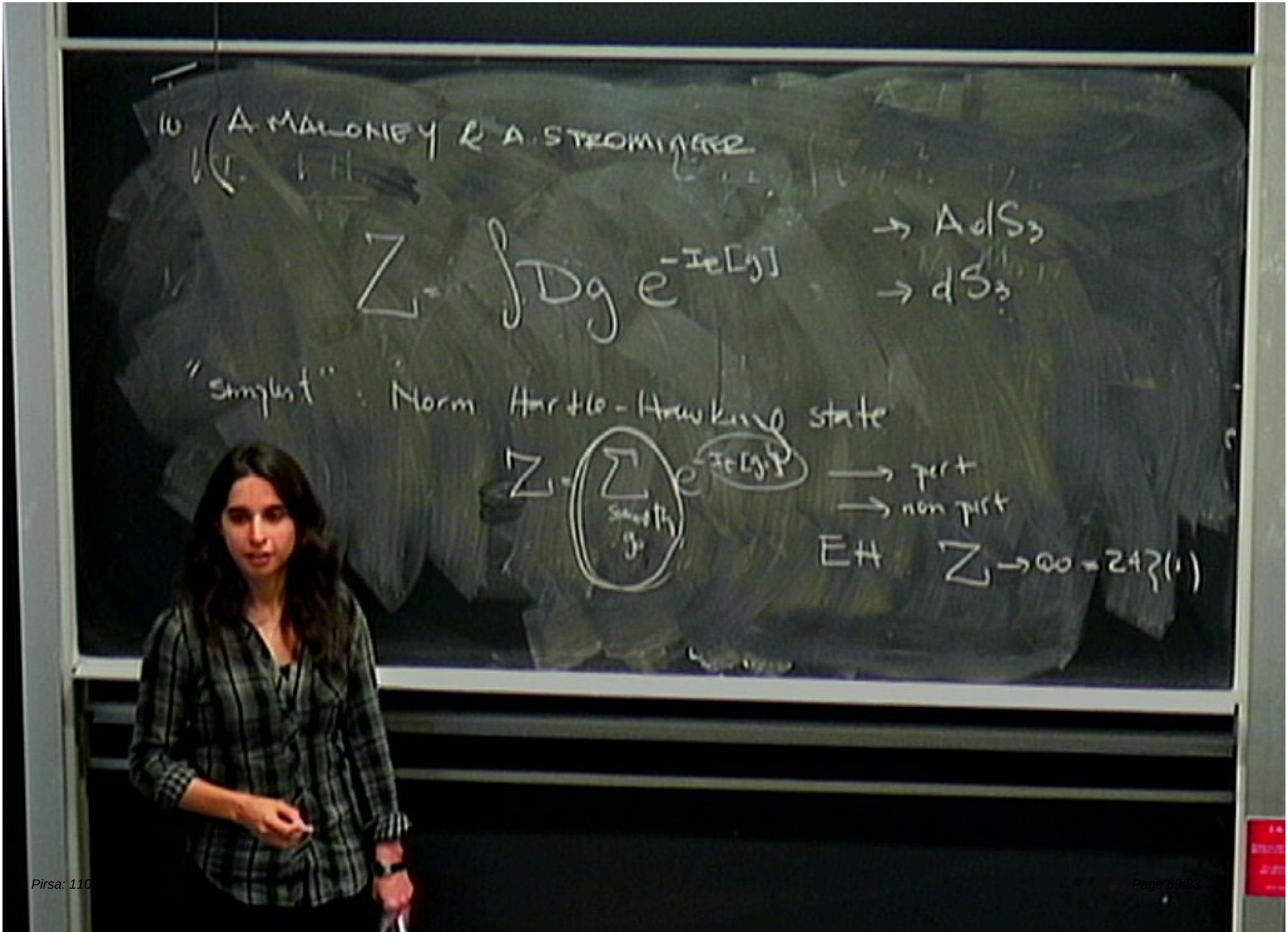
$$r \to r + p$$

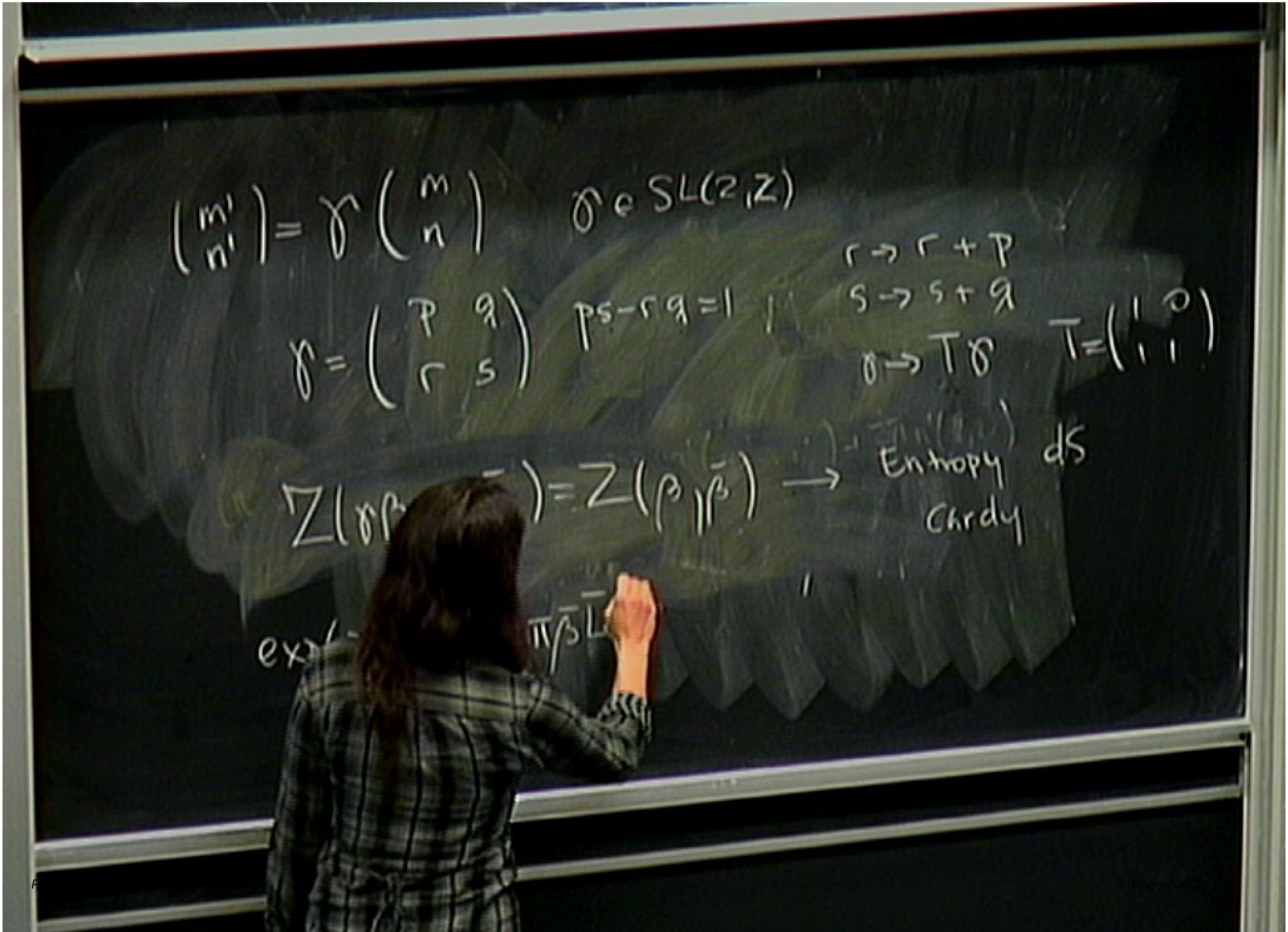
$$s \to s + q$$

$$\gamma \to T\gamma \qquad T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) \longrightarrow \text{Entropy } dS$$

Cardy


A. MALONEY & A. STROMINGER
Z = ∫Dg e^{-I_E[g]}
→ AdS3
→ dS3
"Simplest": Norm Hartle-Hawking state
→ pert
→ non pert
EH


γ ∈ SL(2,Z)
ps − rq = 1
r → r + p
s → s + q
γ → Tγ
Entropy dS
Cardy

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1$$

$r \to r + p$

$s \to s + q$

$\gamma \to T\gamma \qquad T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) \longrightarrow \text{Entropy} \quad dS$$

Cardy

$$\exp(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0)$$

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1$$

$r \to r + p$

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$$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) \longrightarrow \text{Entropy} \quad dS$$

Cardy

$$\exp(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0) \qquad L_0 = \tfrac{1}{2}(H + iJ)$$

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

$$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix} \qquad ps - rq = 1$$

$$r \to r + p$$

$$s \to s + q$$

$$\gamma \to T\gamma \qquad T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) =$$

$$\exp(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0) \qquad L_0 = \tfrac{1}{2}(H + i$$

$$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix} \qquad \gamma \in SL(2,\mathbb{Z})$$

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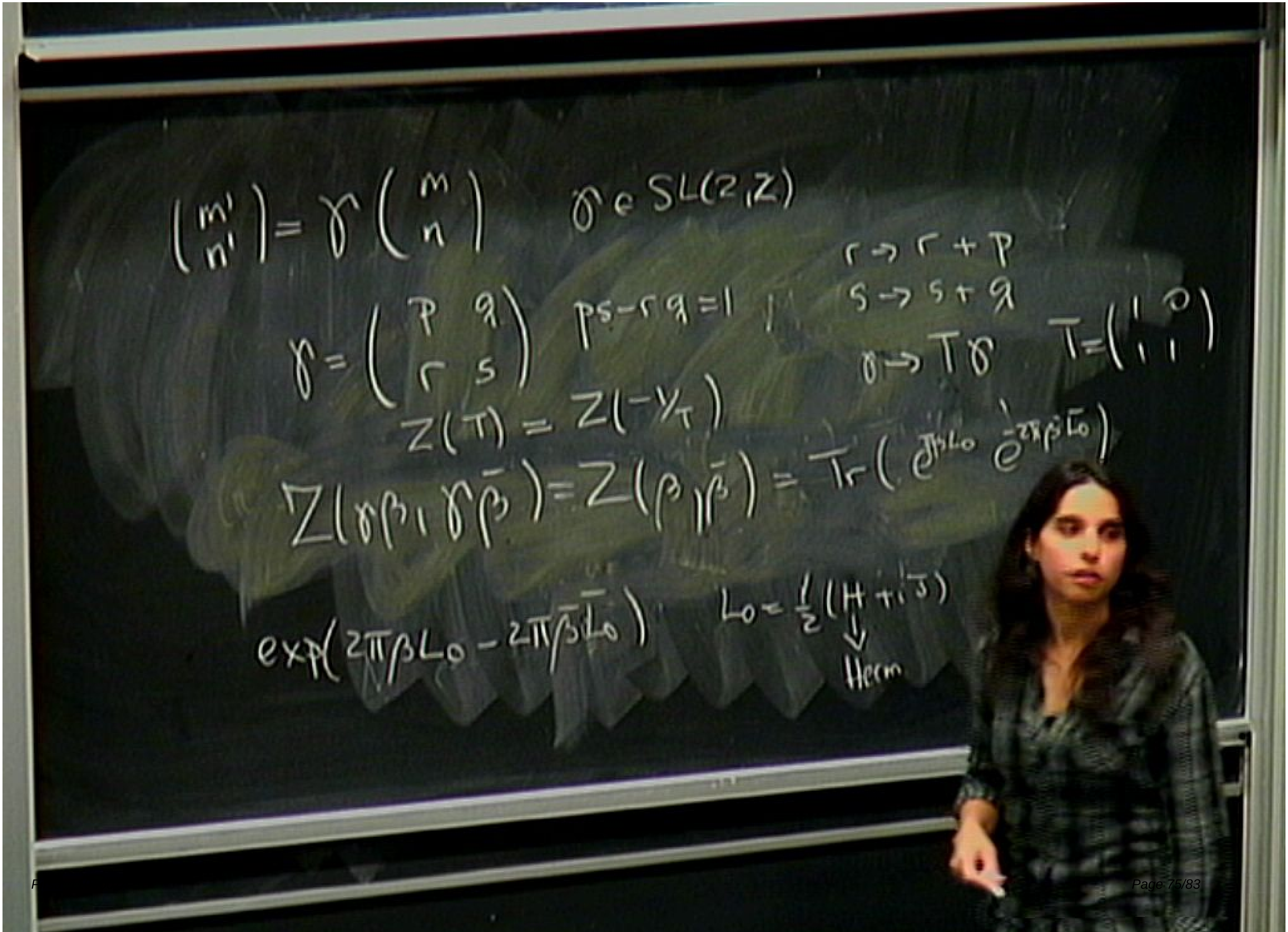
$$\gamma \to T\gamma \qquad T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) = \mathrm{Tr}\left(e^{2\pi\beta L_0}\, e^{-2\pi\bar{\beta}\bar{L}_0} \right)$$

$$\exp\left(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0\right) \qquad L_0 = \tfrac{1}{2}(H + iJ)$$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$r \to r + p$
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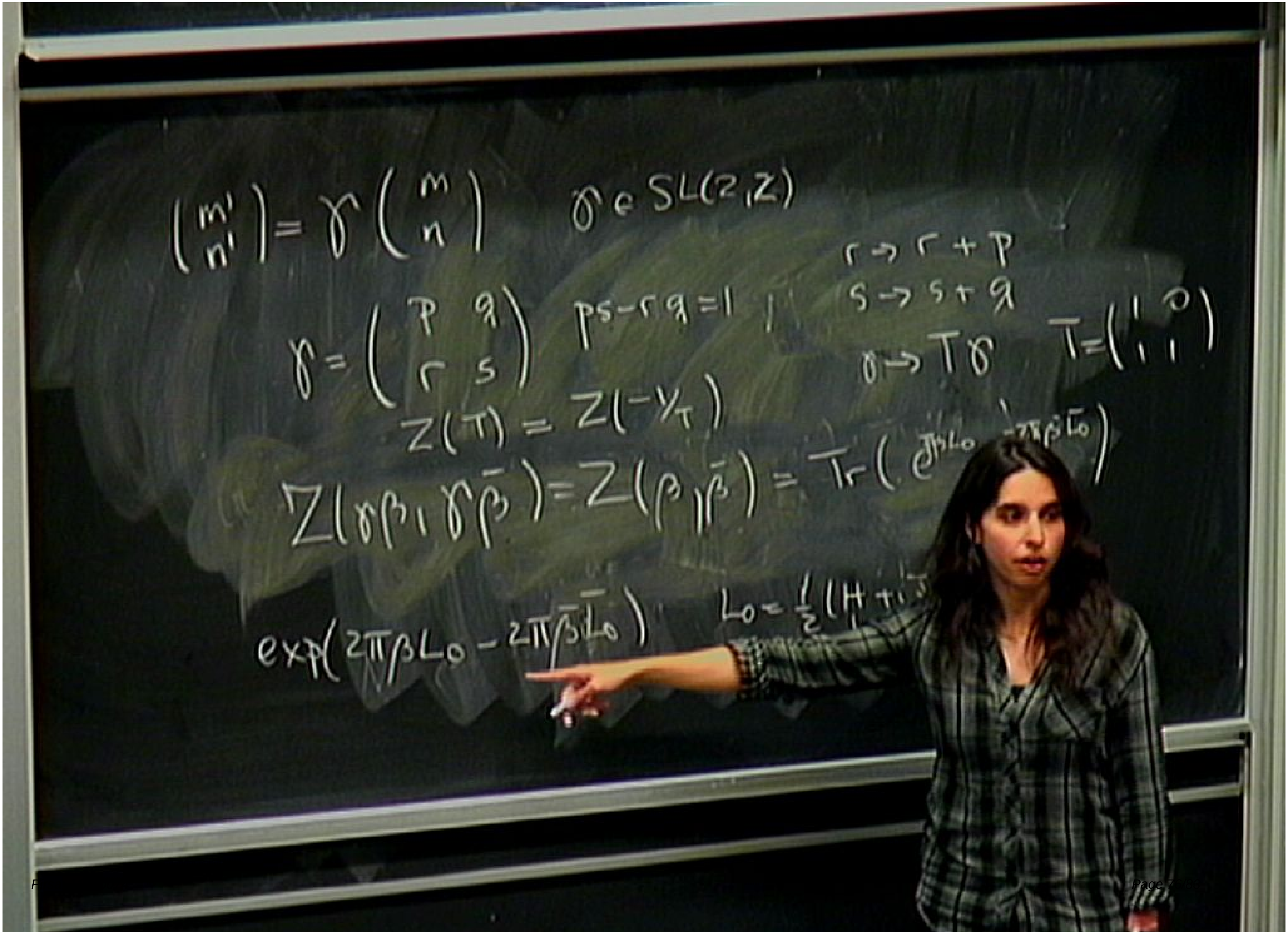
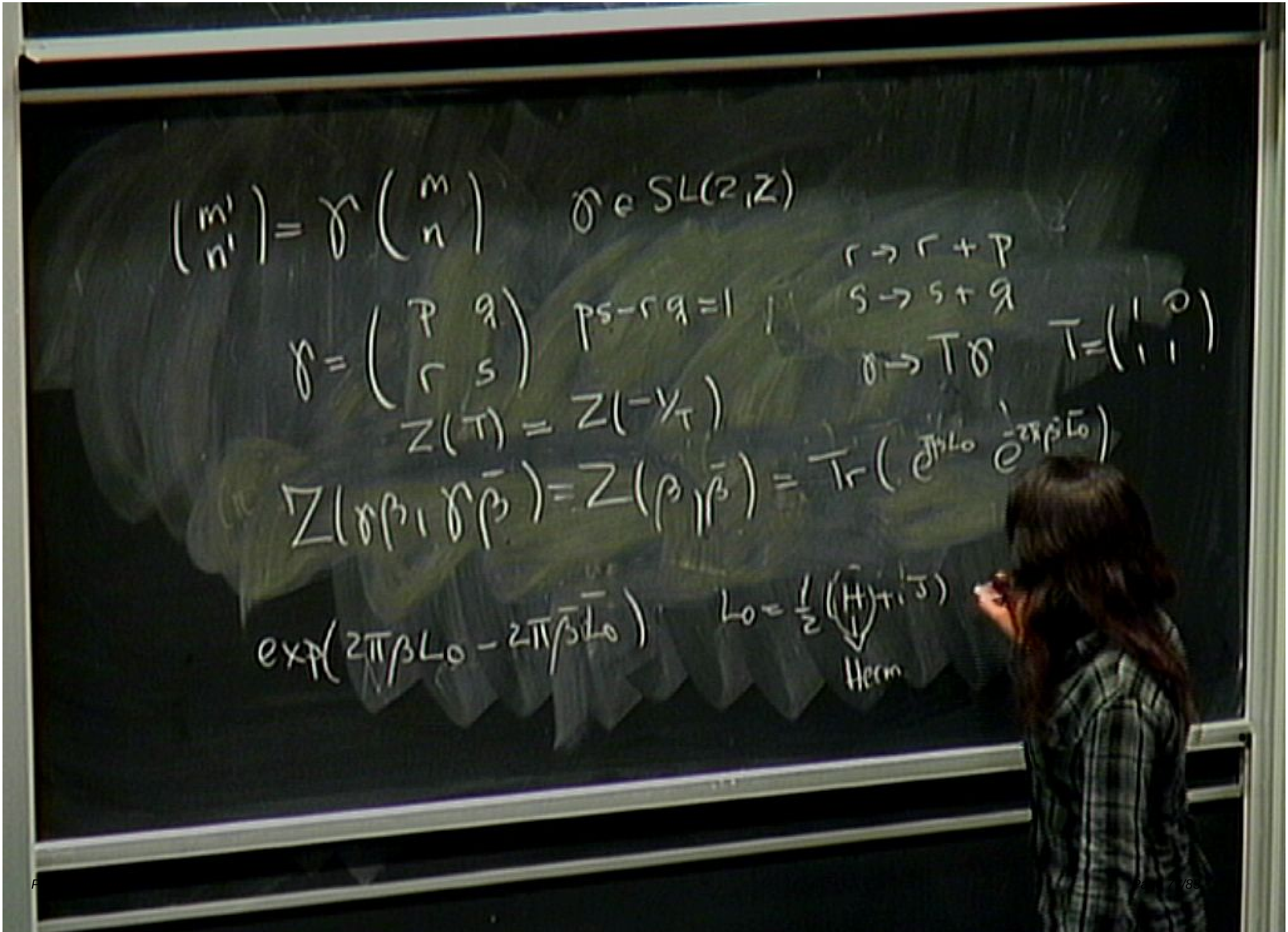
$\gamma = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ $ps - rq = 1$
$\gamma \to T\gamma$
$Z(\tau) = Z(-1/\tau)$
$Z(\gamma\beta, \gamma\bar{\beta}) = Z(\beta, \bar{\beta}) = \mathrm{Tr}(e^{2\pi i\beta L_0} e^{-2\pi i\bar{\beta}\bar{L}_0})$
$\exp(2\pi\beta L_0 - 2\pi\bar{\beta}\bar{L}_0)$
$L_0 = \frac{1}{2}(H + iJ)$
Herm



(m' n') = γ (m n)
γ ∈ SL(2,Z)
r → r + p
s → s + q
γ = (p q r s)
ps − rq = 1
γ → Tγ
Z(τ) = Z(−1/τ)
Z(γβ, γβ̄) = Z(β, β̄) = Tr(
exp(2πβL0 − 2πβ̄L̄0)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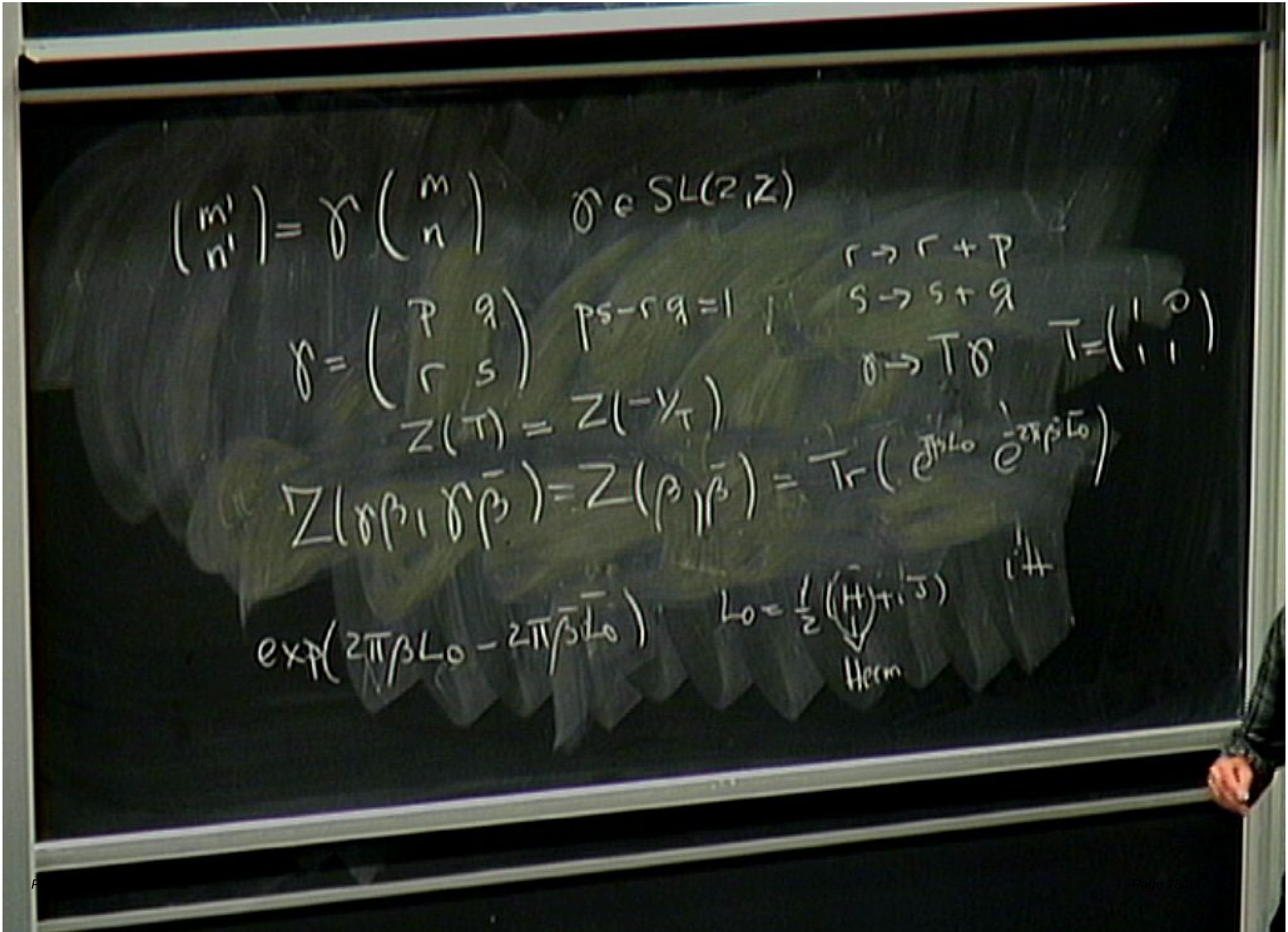
$exp(2\pi\beta L_0 - 2\pi\bar\beta \bar L_0)$
$L_0 = \frac{1}{2}(H + iJ)$
Herm


$\begin{pmatrix} m' \\ n' \end{pmatrix} = \gamma \begin{pmatrix} m \\ n \end{pmatrix}$ $\gamma \in SL(2,Z)$
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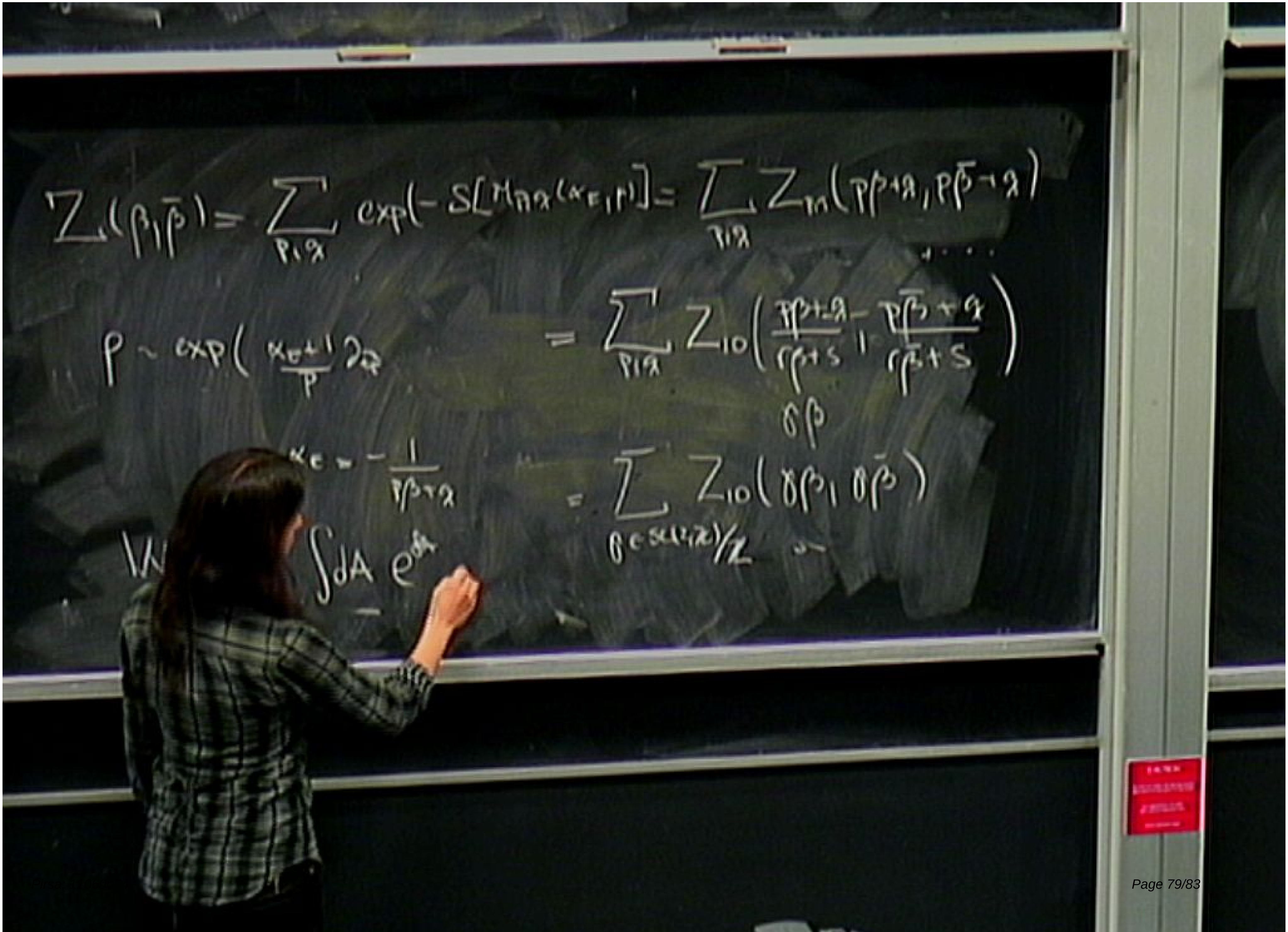
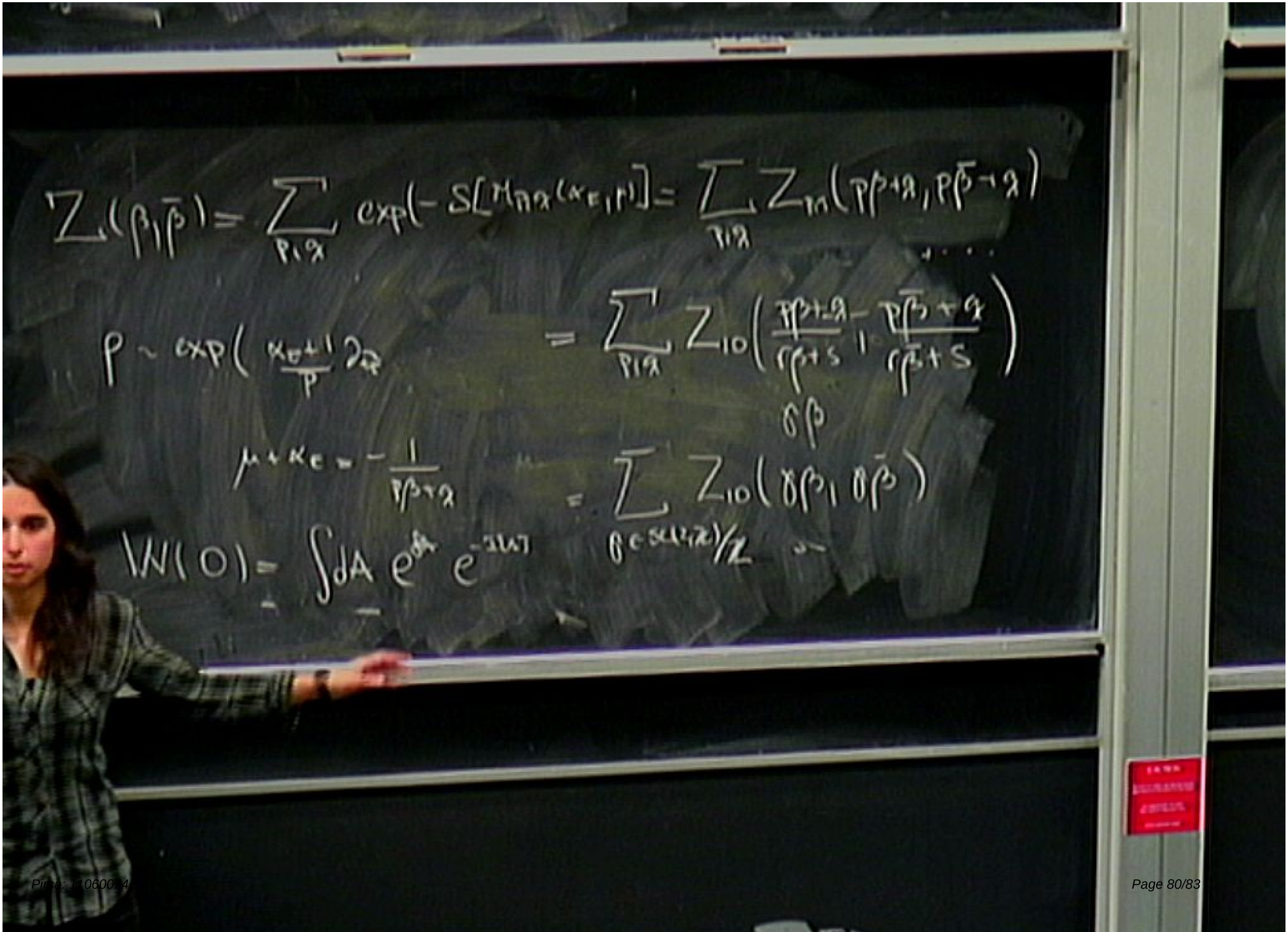
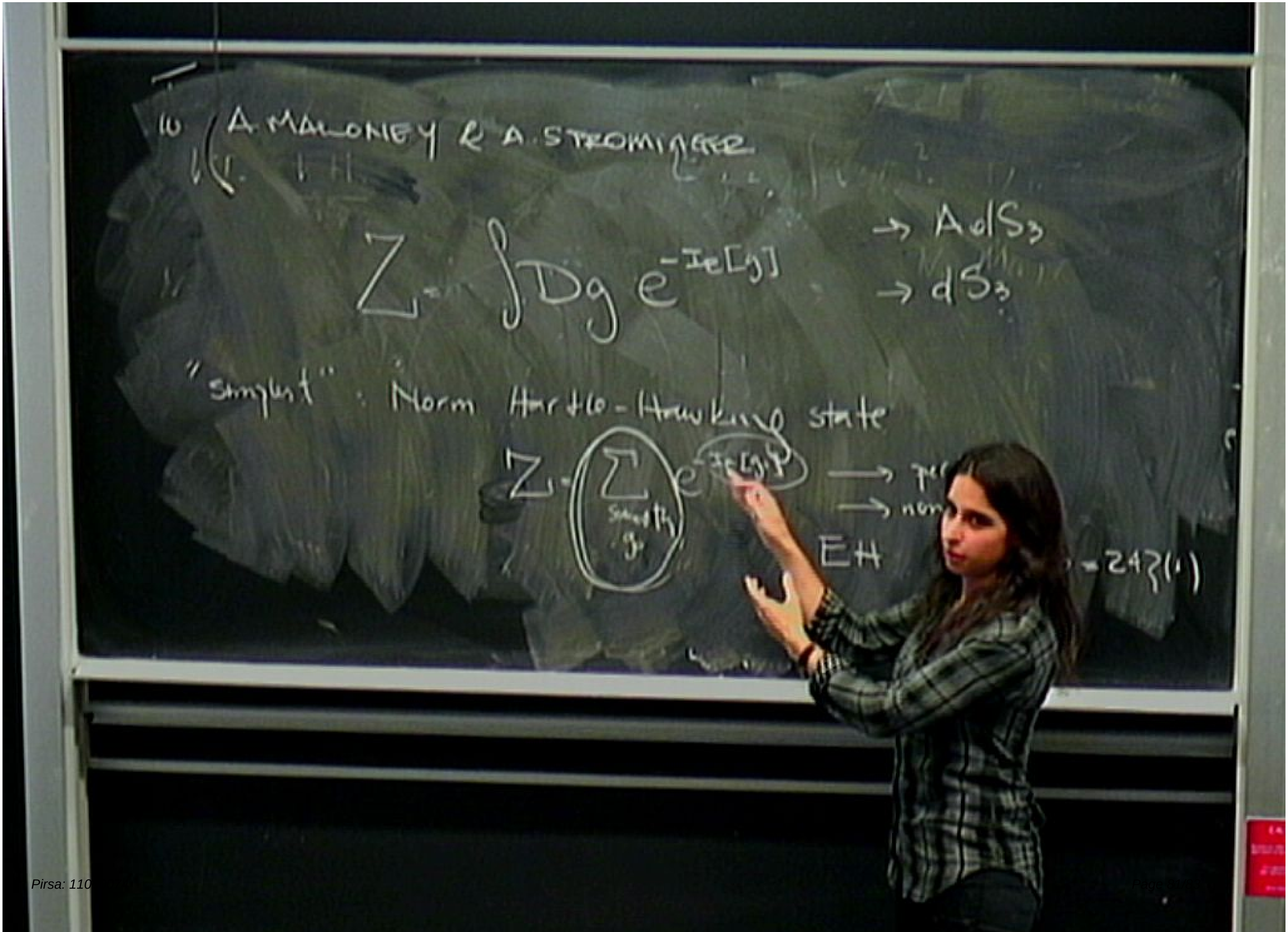
$\exp(2\pi\beta L_0 - 2\pi\bar\beta\bar L_0)$ $L_0 = \frac{1}{2}(H + iJ)$ iH
Herm



$Z(\beta, \bar\beta) = \sum_{p,q} \exp(-S[M_{p,q}(\alpha_E, r)]) = \sum_{p,q} Z_{p,q}(p\beta+q, p\bar\beta+q)$
$\rho \sim \exp\left(\frac{\alpha_E+1}{p}\right)$
$= \sum_{p,q} Z_{1,0}\left(\frac{p\beta+q}{r\beta+s}, \frac{p\bar\beta+q}{r\bar\beta+s}\right)$
$\gamma\beta$
$\alpha_E = -\frac{1}{p\beta+q}$
$= \sum Z_{1,0}(\gamma\beta, \gamma\bar\beta)$
$\int dA\, e^{\alpha A}$


Z(β, β̄) = Σ_{p,q} exp(− S[M_{p,q}(κ_E, μ)]) = Σ_{p,q} Z_{p,q}(pβ+q, pβ̄+q)
ρ ~ exp((κ_E+1)/p ∂_?
= Σ_{p,q} Z_{1,0}((pβ+q)/(rβ+s), (pβ̄+q)/(rβ̄+s))
γβ
μ + κ_E = − 1/(pβ+q)
= Σ_{γ∈SL(2,Z)/Z} Z_{1,0}(γβ, γβ̄)
W(0) = ∫dA e^{...} e^{−S[A]}


A. MALONEY & A. STROMINGER
Z = ∫Dg e^{-I_E[g]}
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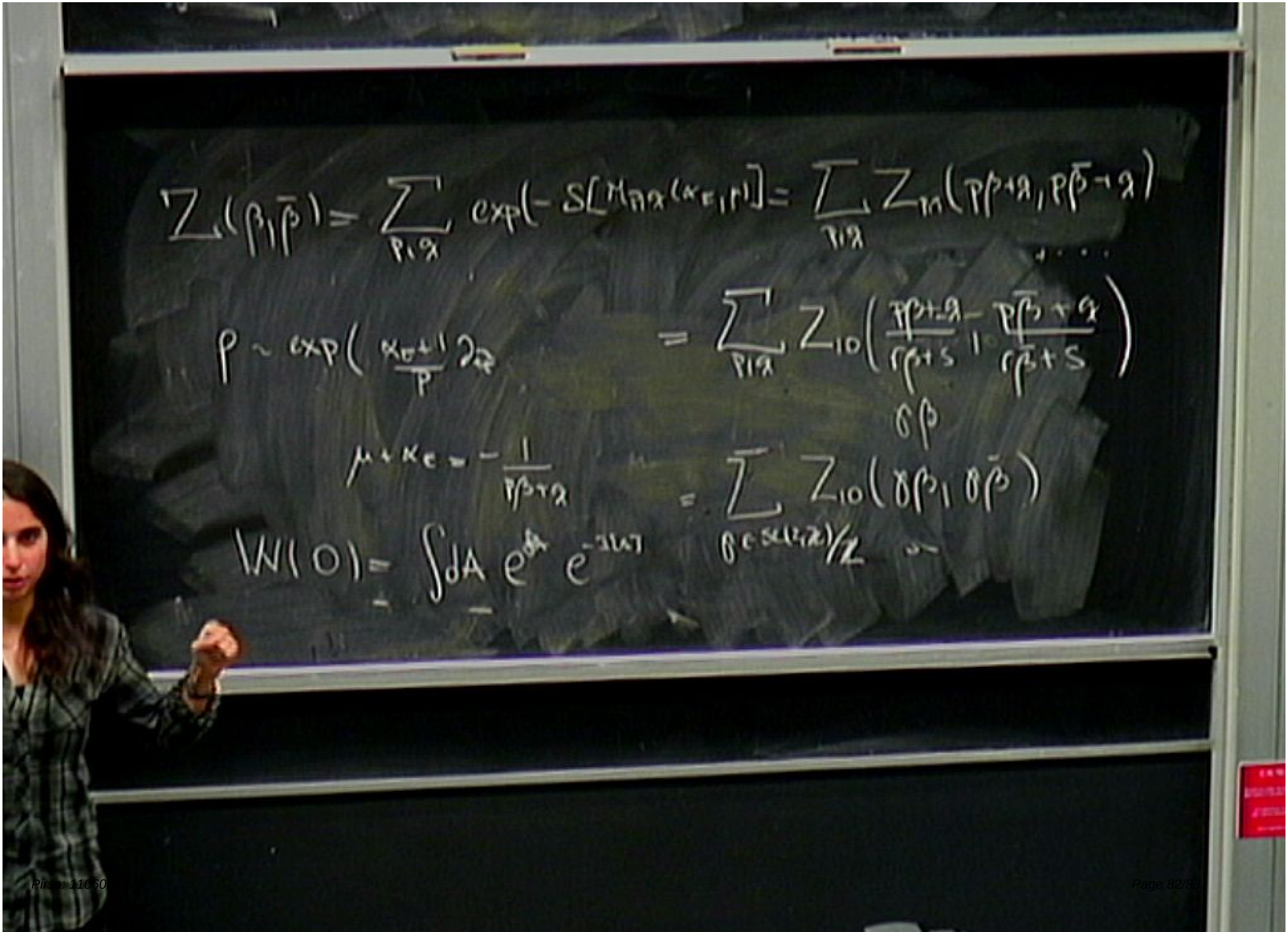
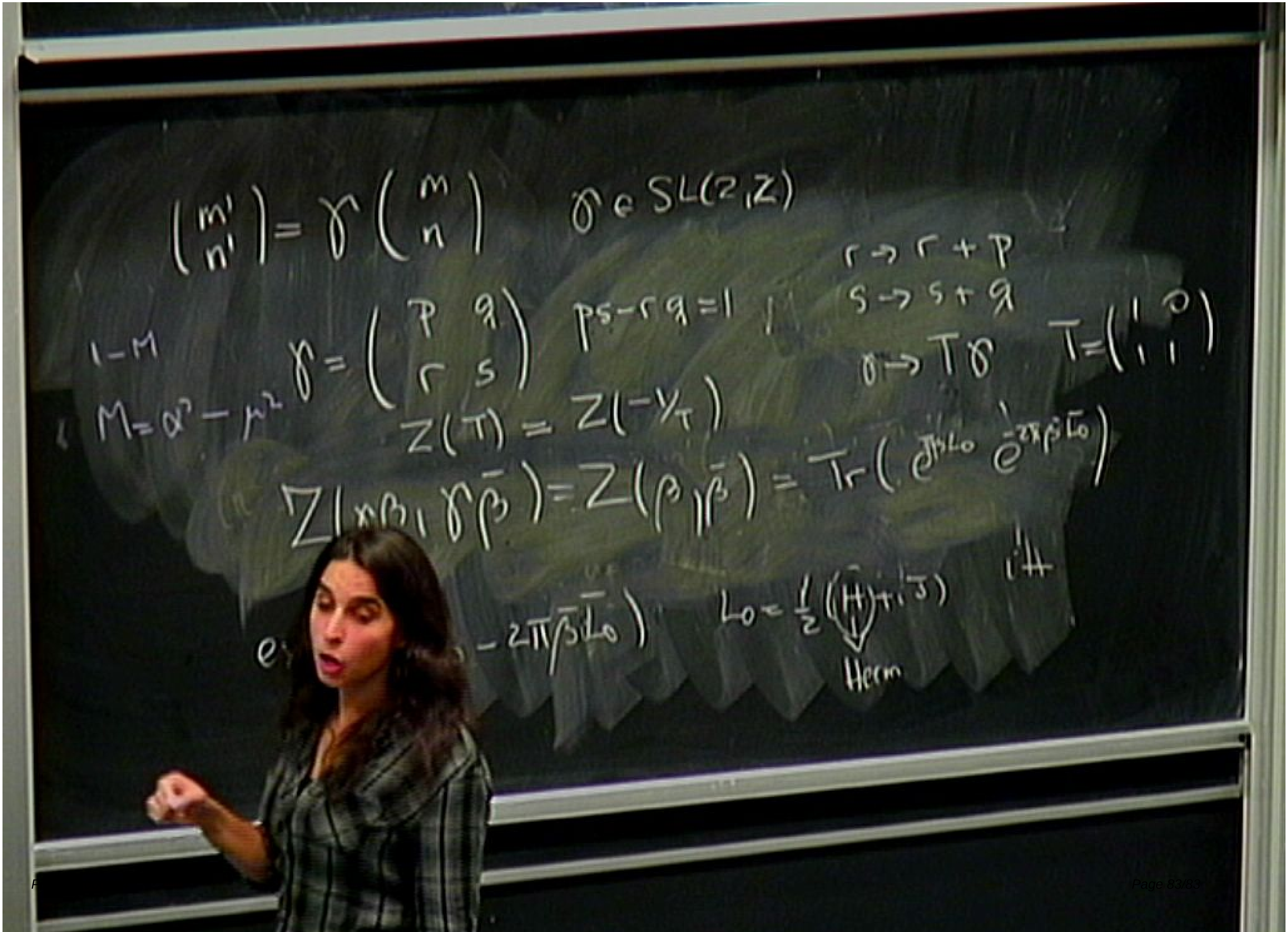

W(0) = ∫dA



(m' n') = γ (m n)   γ ∈ SL(2,Z)
r → r + p
s → s + q
γ = (p q / r s)   ps − rq = 1
γ → Tγ   T = (1 0 / 1 1)
1 − M
M = α² − μ²
Z(τ) = Z(−1/τ)
Z(γβ, γβ̄) = Z(β, β̄) = Tr(e^{βL₀} e^{2πβ̄L̄₀})
L₀ = ½(H + iJ)
Herm