Title: A Canonical Measure for Inflation

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Abstract:

A Canonical Measure for Inflation

Neil Turok, Perimeter Institute

Question: Is there a natural measure on the space of cosmological solutions?

What is the likelihood of a universe like ours in a given physical model? eg inflation, cyclic,

Two key ingredients in this talk

I: Penrose critique of inflation - Hamiltonian evolution almost never turns a generic state into an unusual state. Canonical measure is invariant.

II: Counting of states in gravity should be done in an asymptotic region where global properties of spacetime become sharp

In a specific setup, we shall obtain a precise, canonical measure and show a universe like ours is extremely unlikely in slow-roll inflationary models

- standard slow-roll inflation

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = \frac{1}{3}\rho_\phi - \frac{k}{a^2}; \quad \rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi)$$

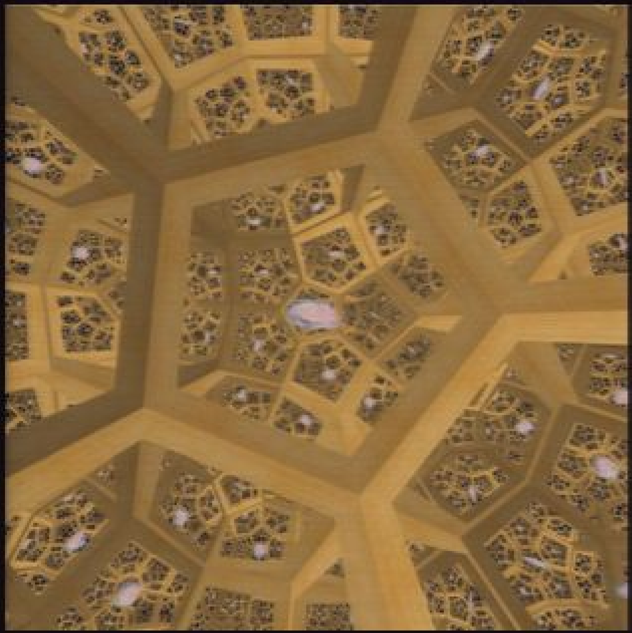
$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi = -V_{,\phi}$$

$$\Rightarrow \dot H = -\frac{1}{2}\dot\phi^2 + \frac{k}{a^2}$$

- Hamiltonian and time reversal invariant

(For now, I'll focus on FRW spacetimes - thi
is of course generous to inflation - and
assume V(φ) is monotonic away from its min)

I'll focus on k=-1 (so a and H are monotonic) and compactify the spatial slices



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* (but in fact has been advocated as a very natural setup fo chaotic inflation eg by Linde)

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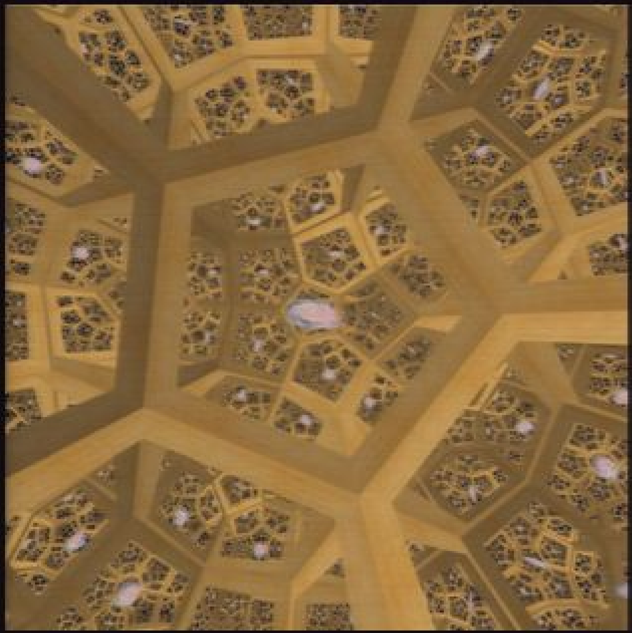

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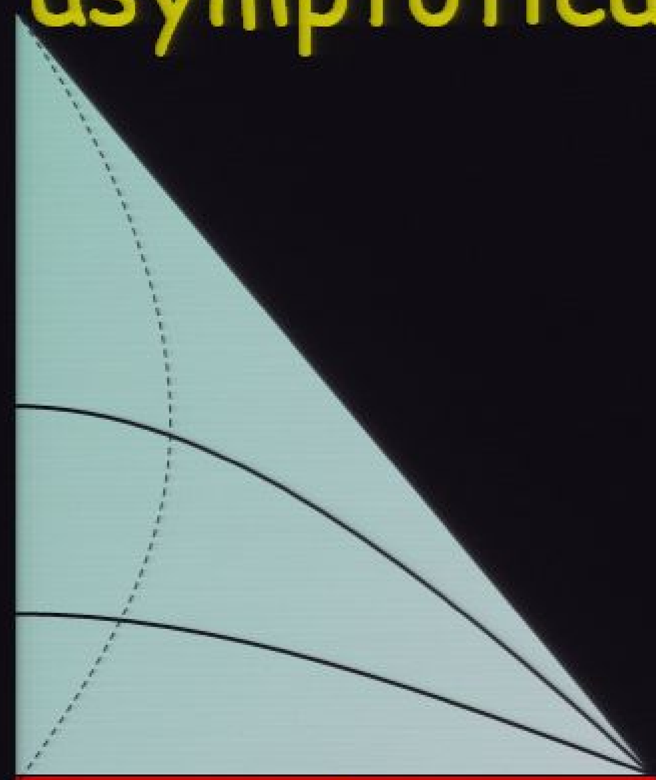
2-parameter family of solutions with an initial singularity:



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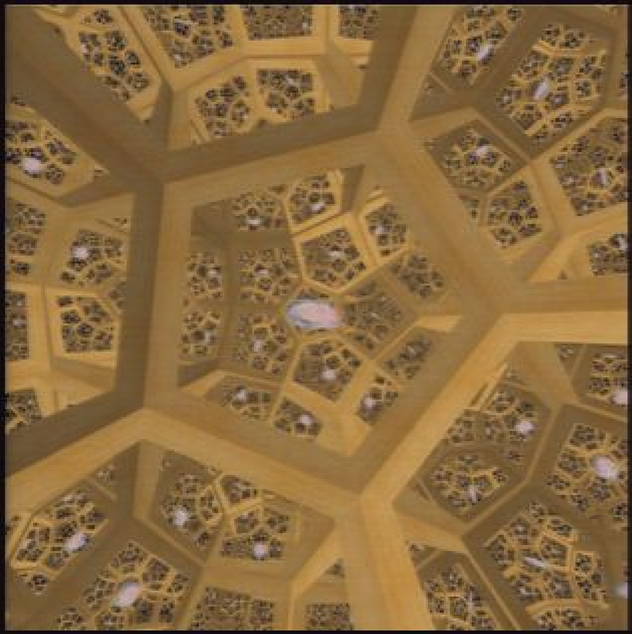

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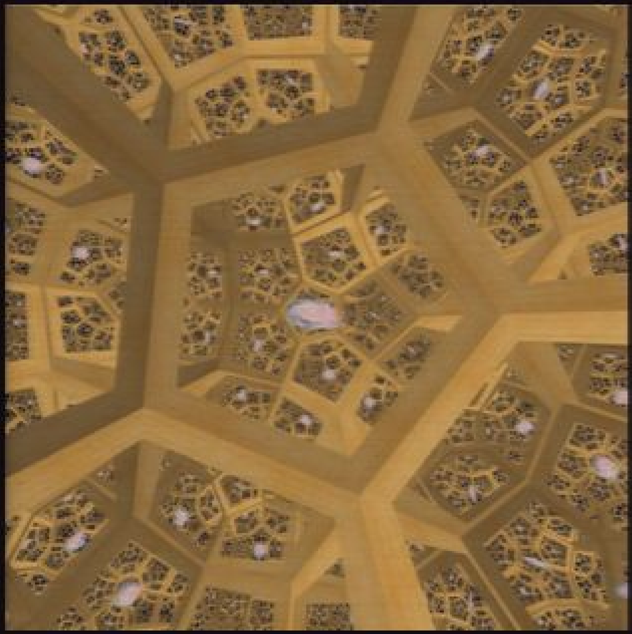

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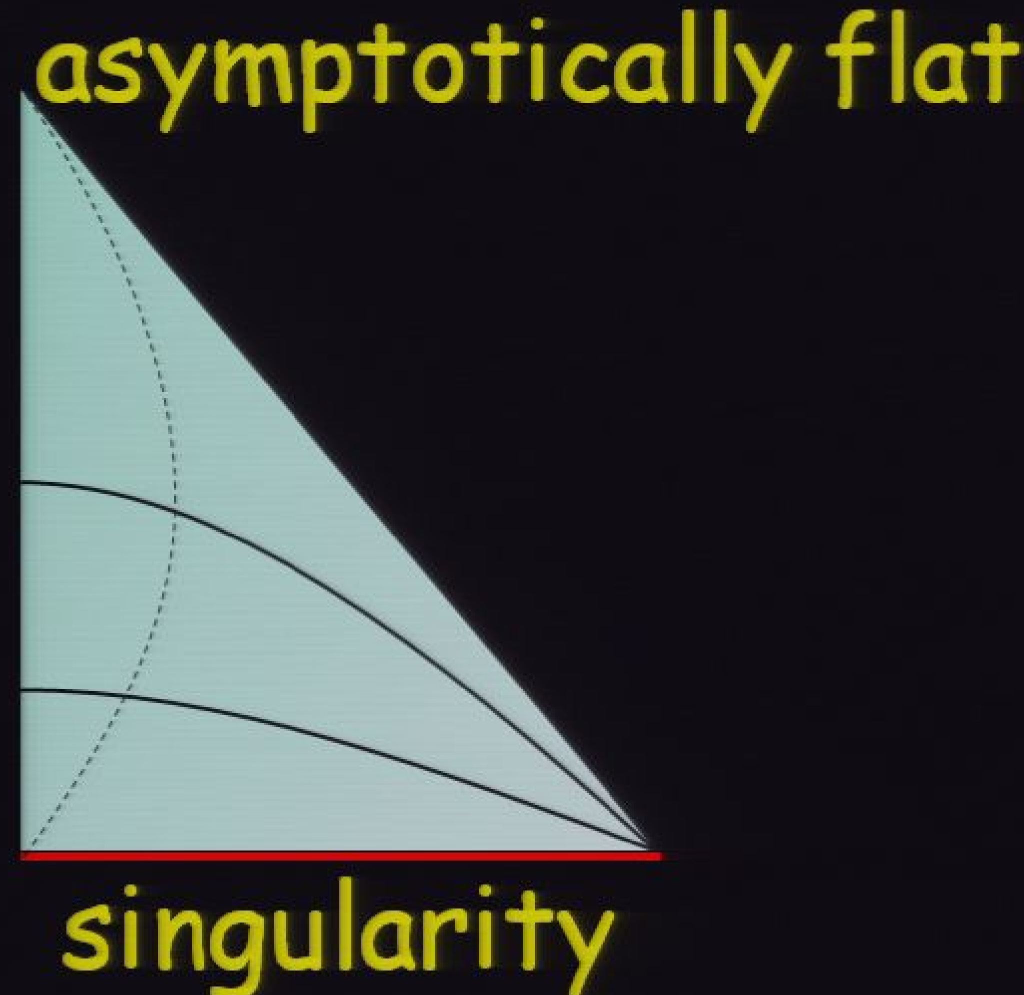


Criteria for a Measure

(i) Positive, normalisable
(ii) Independent of slicing or coordinates on either space-time or field space
(iii) No ad hoc external structures eg comoving observers, volume factors ...
(iv) Natural extension of canonical quantum measur for fluctuations to the background (why use for former but not for latter???)

Canonical measure on space of solutions

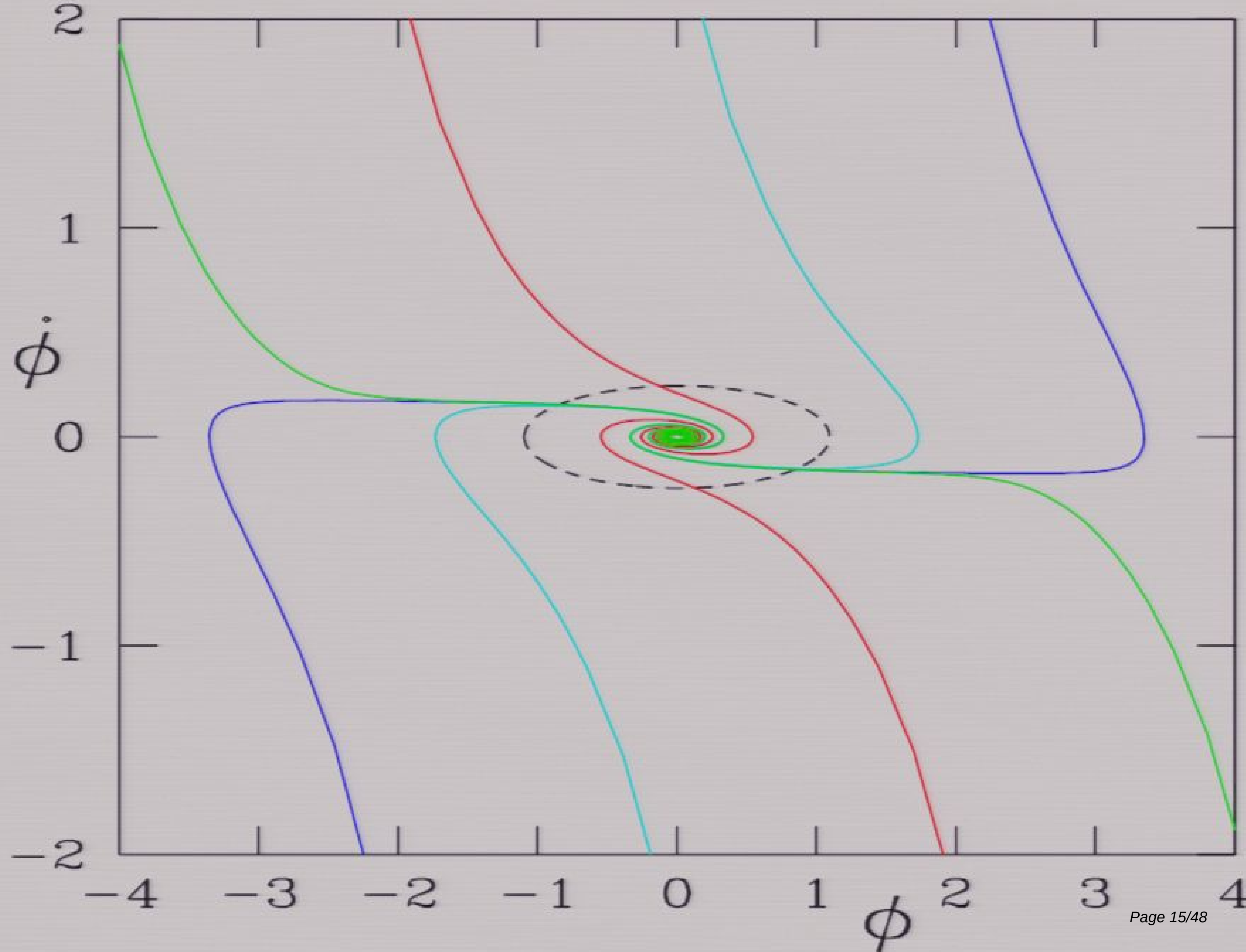
$$\omega_c = dp_a \wedge da + dp_\phi \wedge d\phi$$

$$\int_\Sigma \omega_c \Big|_{\mathbb{H}=0}$$

Liouville
Gibbons, Hawking, Stewa
Hawking, Page
Hollands Wald
Kofman, Linde, Mukhanov
Gibbons, NT
Carroll, Tan

with Σ pierced once by every trajectory
e.g. a=const or H=const

Satisfies all of conditions (i)-(iv)
except normalisability, because Σ is not compact (because $\mathbb{H}$ isn't positive)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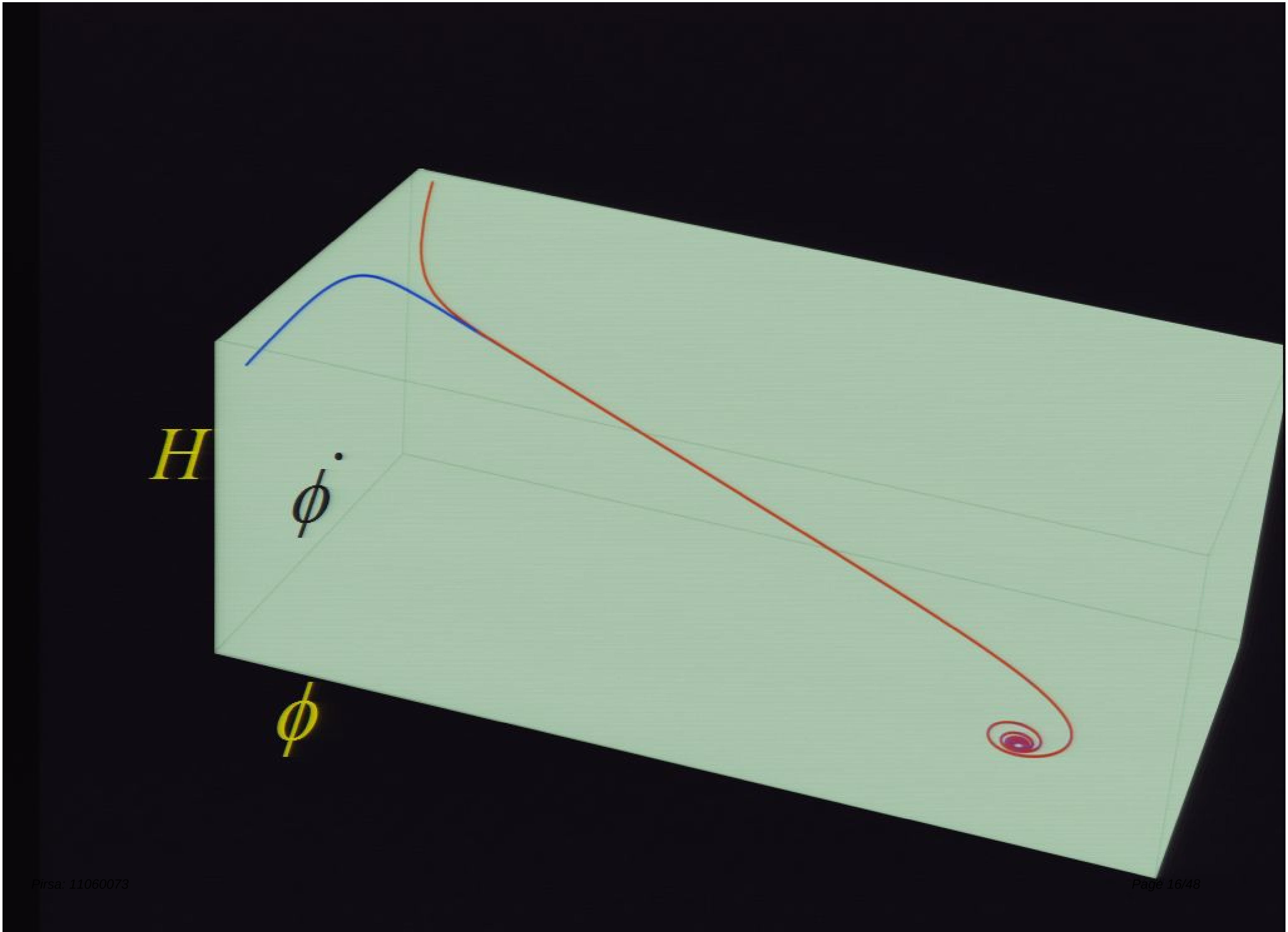
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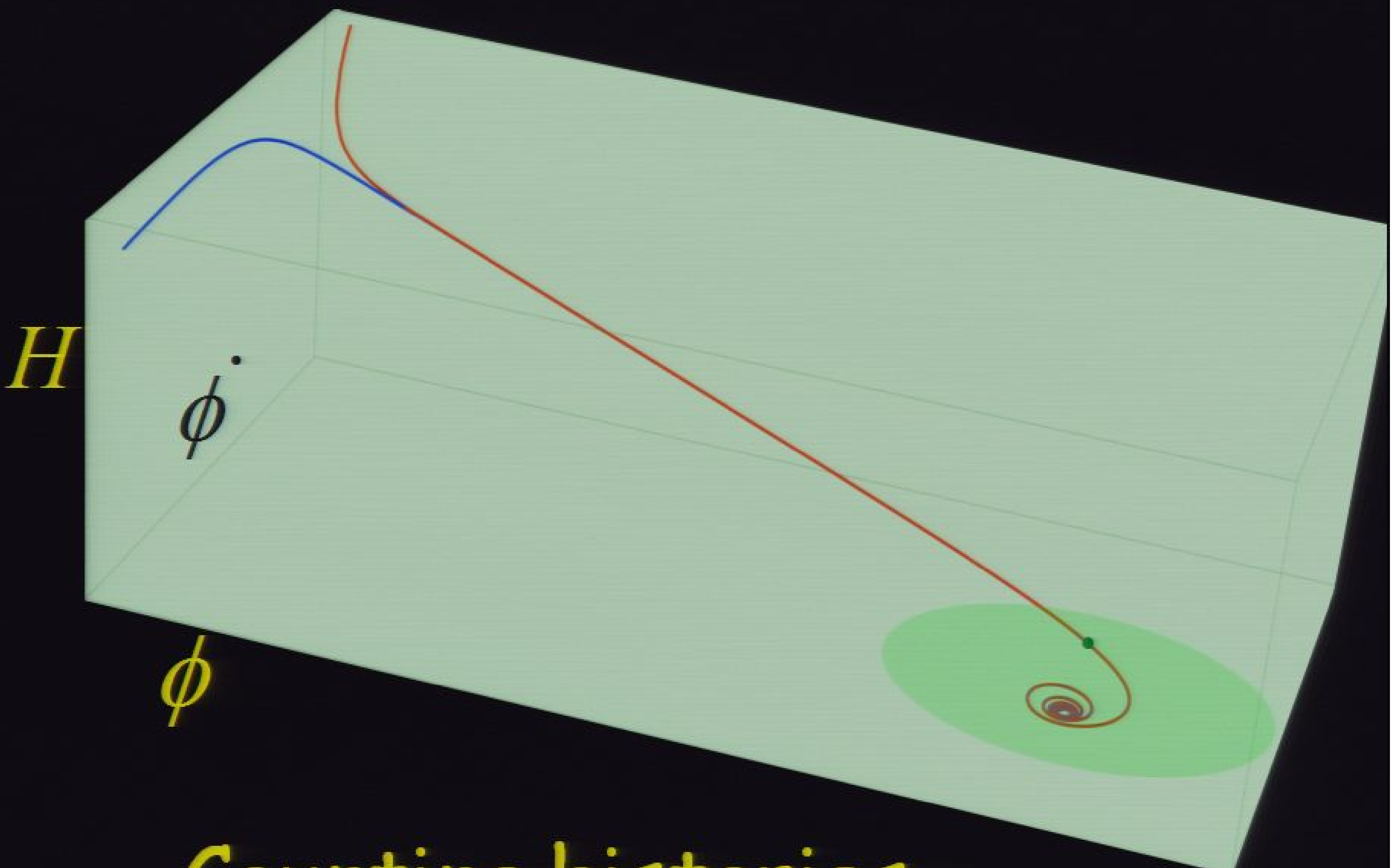


Counting histories

Flat space canonical (Gibbs) ensemble.
Cannot just integrate over Liouville: instead, we maximise entropy $S = -\sum p_i \ln p_i$

subject to constraint $E = \sum p_i E_i$

Note: in information theory approach, max ent principle is very general, can even be applied to non-equilibrium situations (see e.g. beautiful papers of E.T. Jaynes)

But in GR, $\mathbb{H}=0$ on all physical states, so we cannot constrain its expectation value to make measure finite

What do we do?

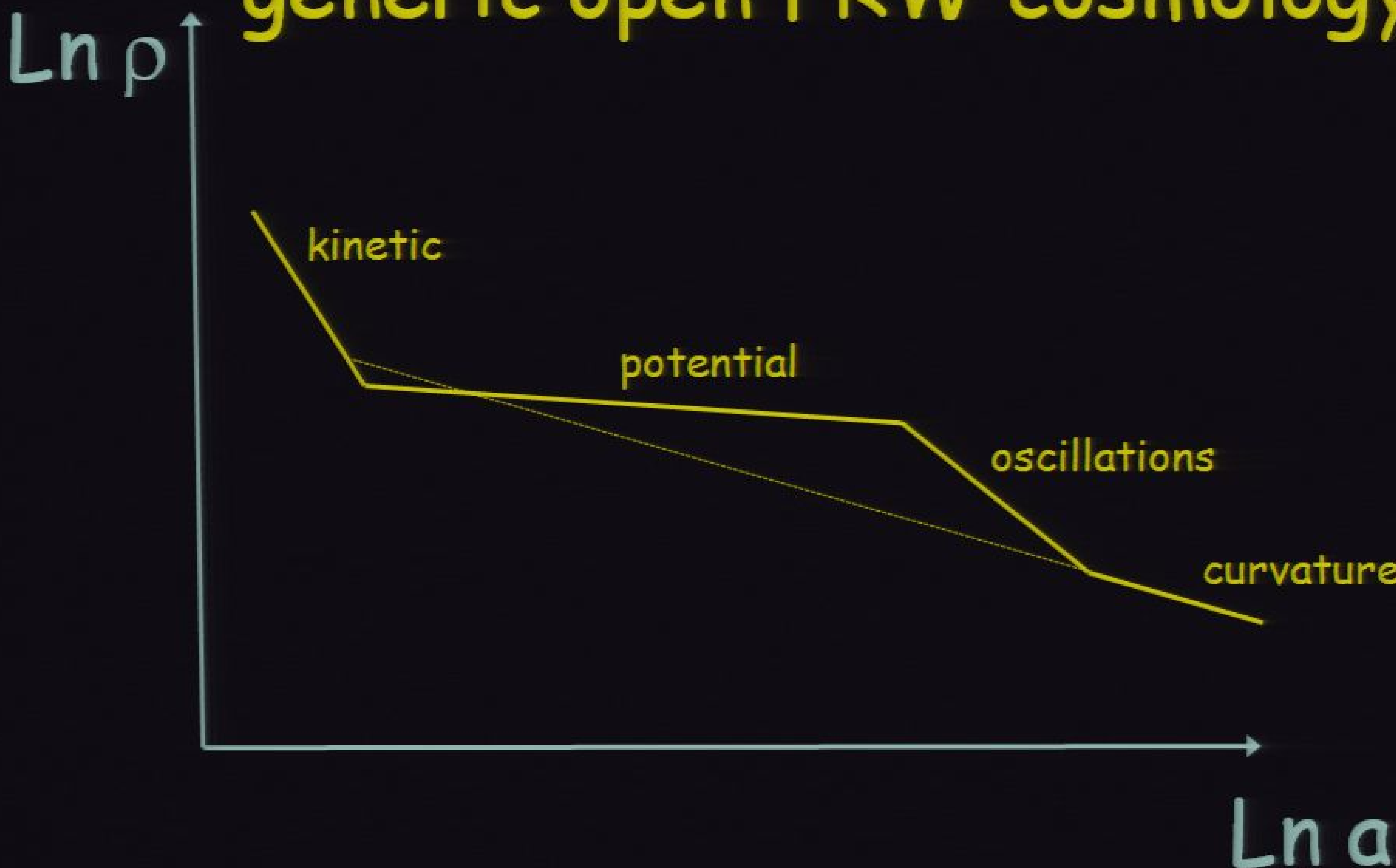
In k=+1, zero Λ cosmologies, matter density is diluted away at large a -> gravity becomes negligible, expansion of 'box' is adiabatic

-> entropy reduces to that of the matter (inc grav waves), and is an adiabatic invariant

Every trajectory ends up on an adiabat curve $S_m(E_m,a)$= const

Natural to label an ensemble of spacetimes by the asymptotic entropy $S=S_m$

generic open FRW cosmology



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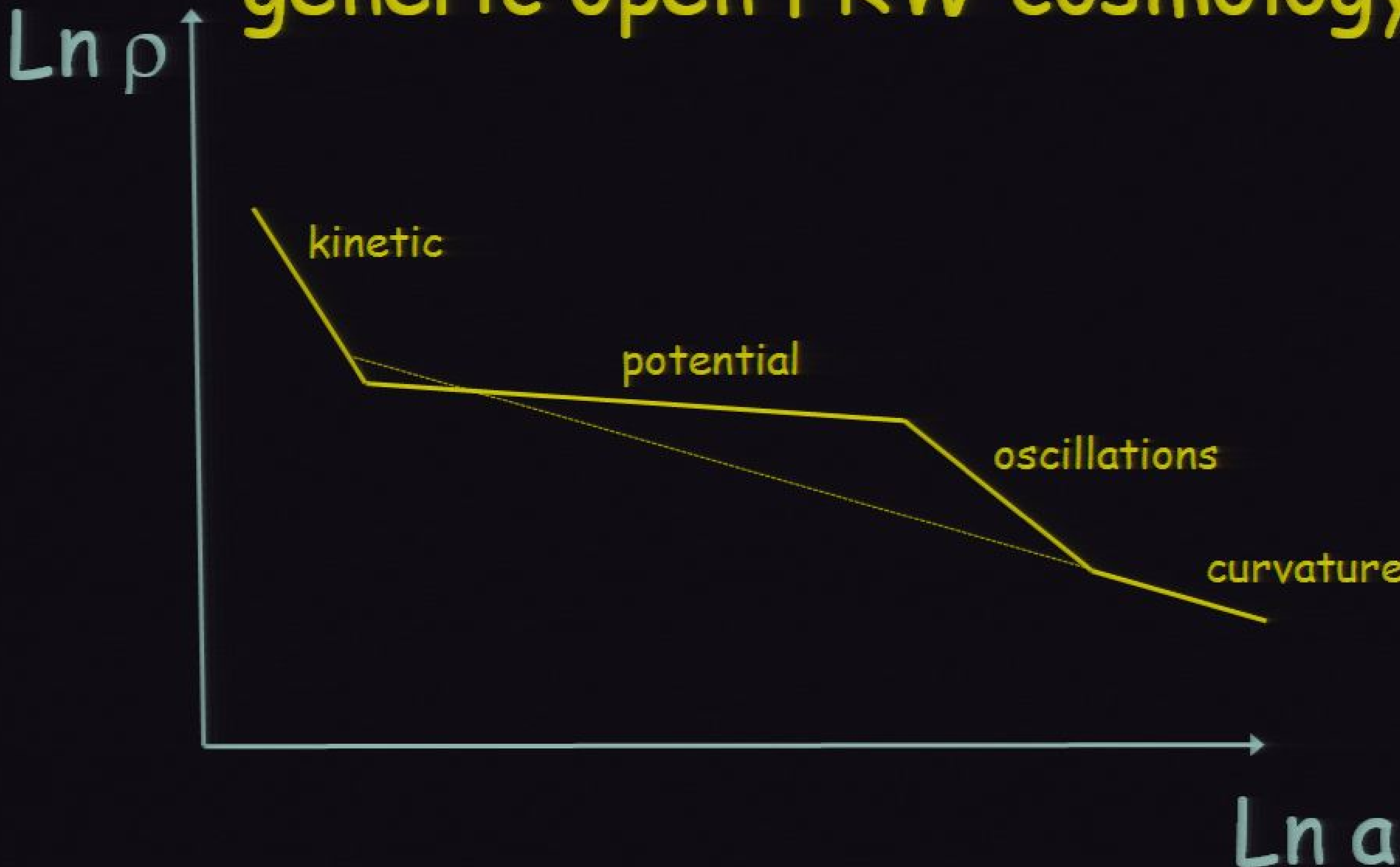
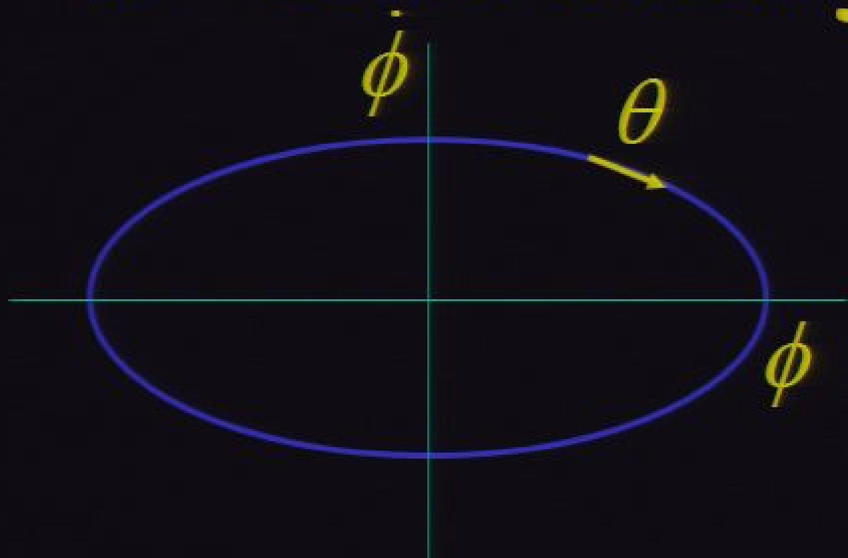


a =const slicing

$$\omega_c = a^3 d\dot{\phi}\, d\phi$$



$$\rho = \tfrac{1}{2}(\dot{\phi}^2 + m^2\phi^2)$$

$$\sim C/a^3,\ \ a \to \infty,$$

ρa^3 adiabatically conserved

$$ds^2 = \frac{da^2}{1 + 8\pi G \rho a^2/3} + a^2 dH_3^2 \approx dM_4^2 - \frac{8\pi G}{3}\frac{C}{a}da^2$$

(C is analogous to the AdM mass)

in large a limit, effect of matter on background spacetime (i.e. gravity) becomes negligible

we just have flat spacetime, and an adiabatically expanding box filled with matter

statistical ensemble: minisuperspace

$$\mathbb{H}_{\rm m}(p_\phi,\phi)=\tfrac{1}{2}\left(\frac{p_\phi^2}{Ua^3}+Ua^3V(\phi)\right)$$

$$\langle \mathbb{H}_{\rm m}\rangle=\frac{\int dp_\phi d\phi e^{-\beta\mathbb{H}_{\rm m}}\mathbb{H}_{\rm m}}{\int dp_\phi d\phi e^{-\beta\mathbb{H}_{\rm m}}}=E(a,\beta)$$

entropy $S=S_m=\ln(\frac{Ua^3\rho_\phi}{m})=$ adiabatic invarian

constant entropy=fixed C

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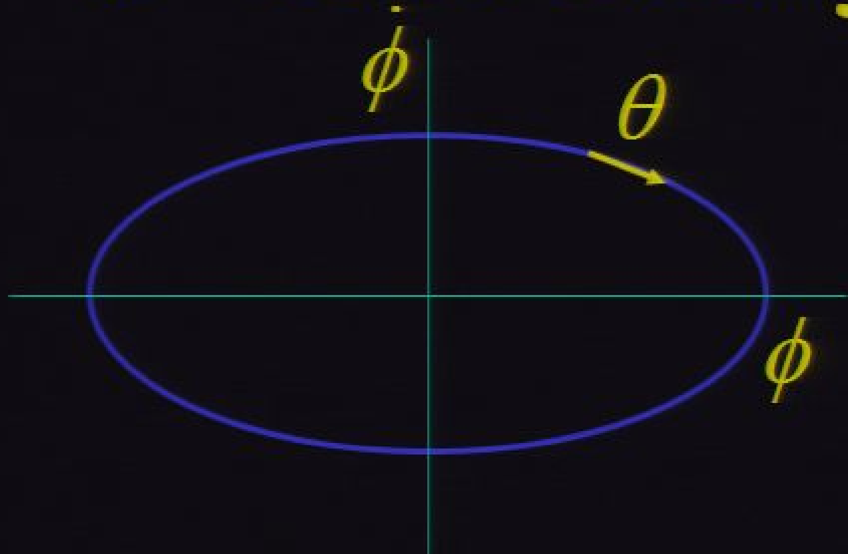

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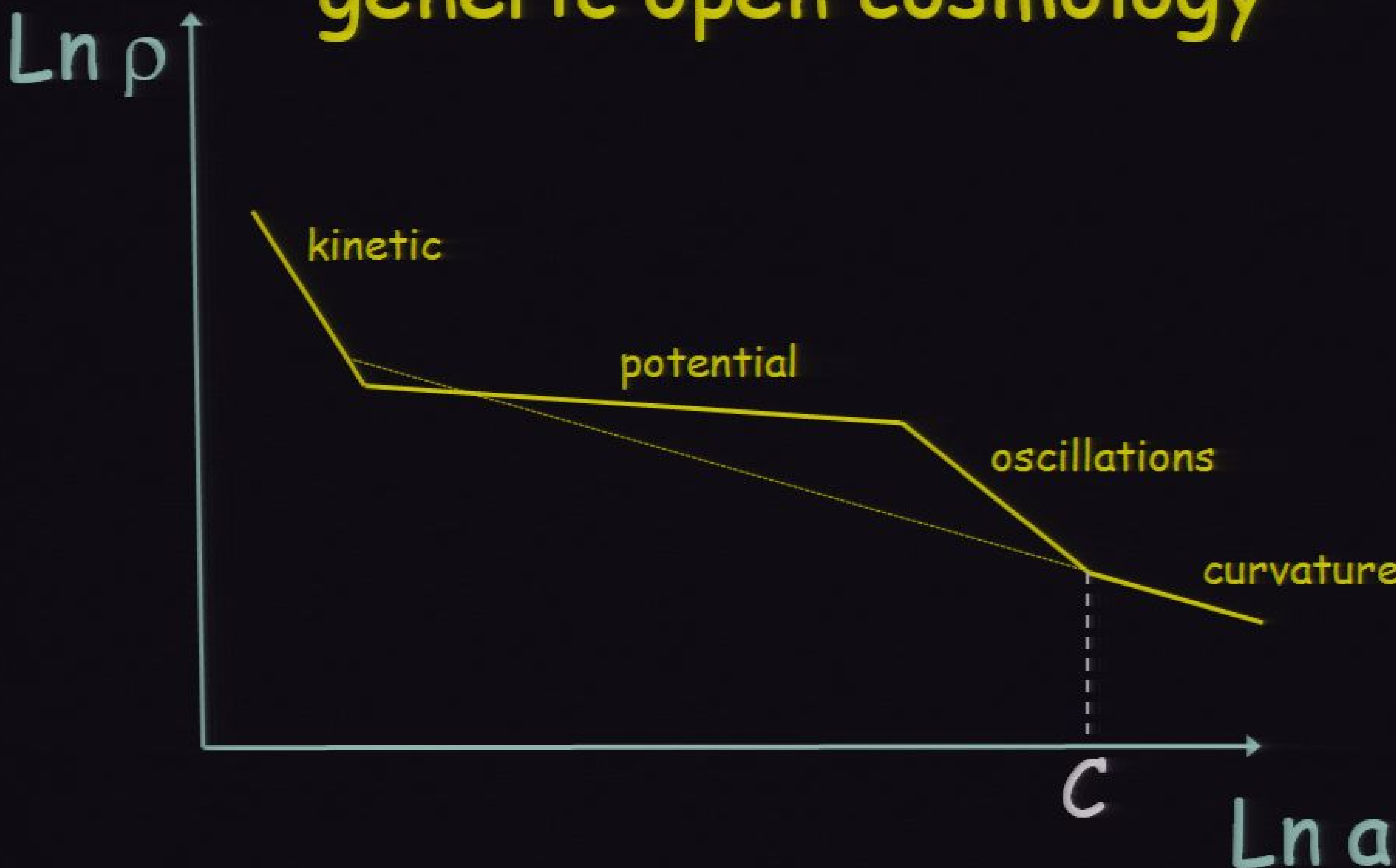


infinite N_I slow-roll solution

$$H^2 = \frac{1}{3}V + \frac{2}{3}\left(\frac{dH}{d\phi}\right)^2$$

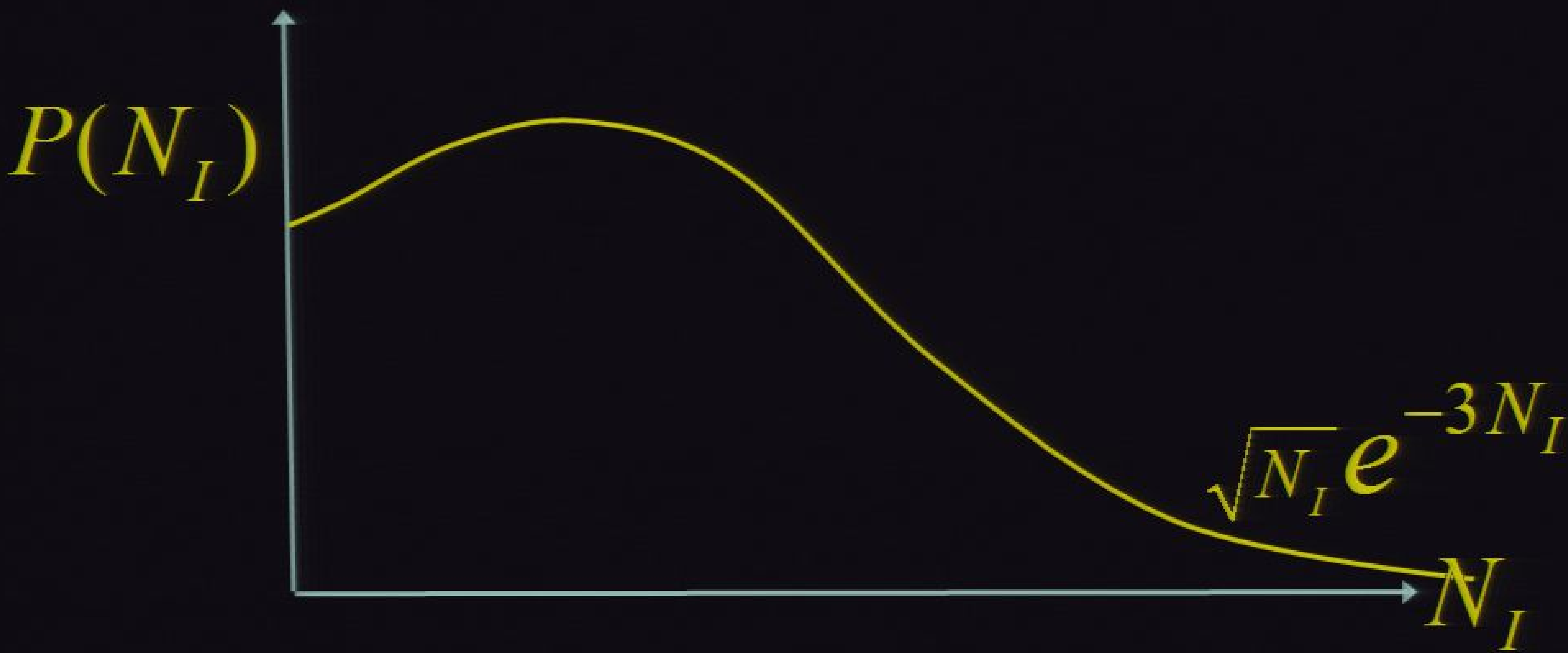
$$H_{SR}(\phi) \approx \sqrt{\frac{V(\phi)}{3}}\left(1 + \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2 ..\right)$$

Deviations (going back in N):

$$\frac{d\delta H}{dN} = 3\delta H \Rightarrow \delta H \propto e^{3N}$$

$$\delta\theta \sim \sqrt{N_I}\, e^{3(6-N_I)} \sim P(N \geq N_I)$$

Canonical measure for inflation



Finite C result is always lower than $C = \infty$ result at large N_I

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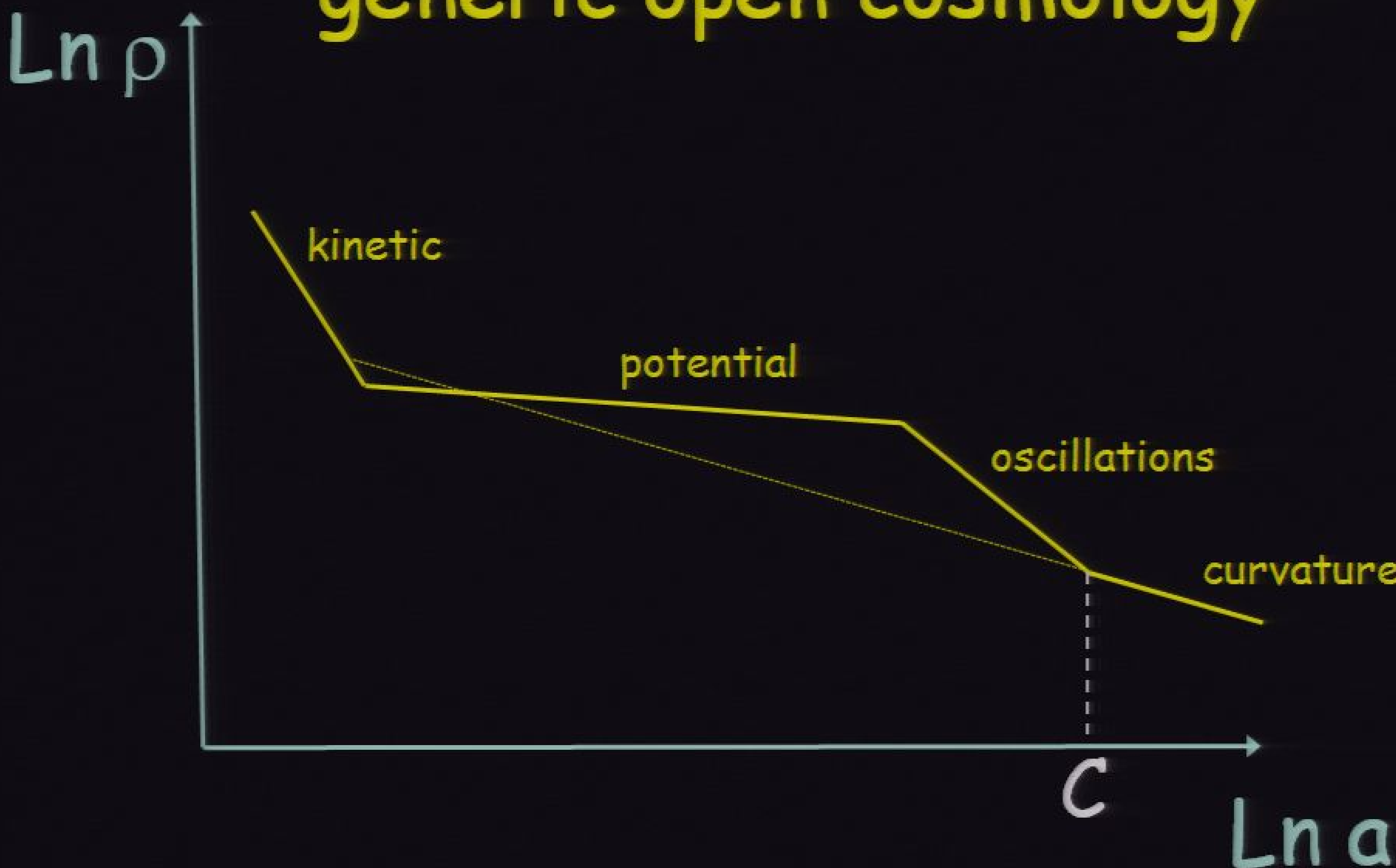

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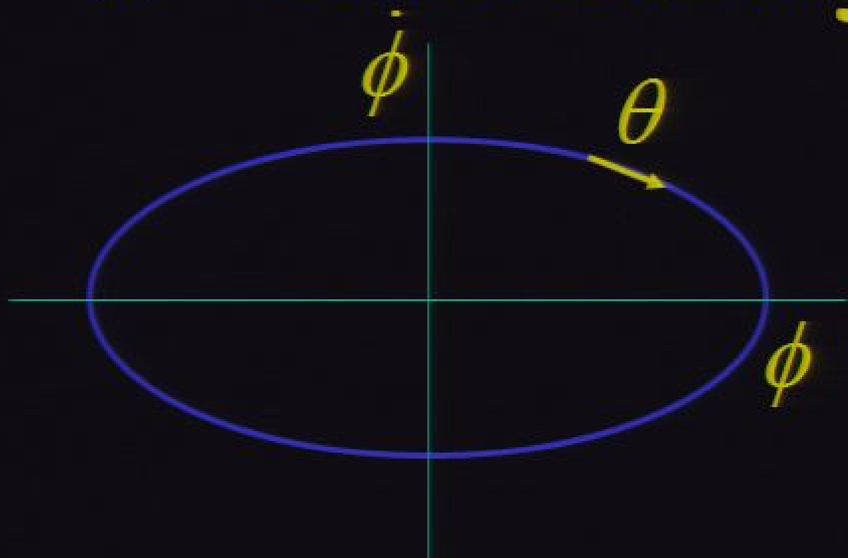

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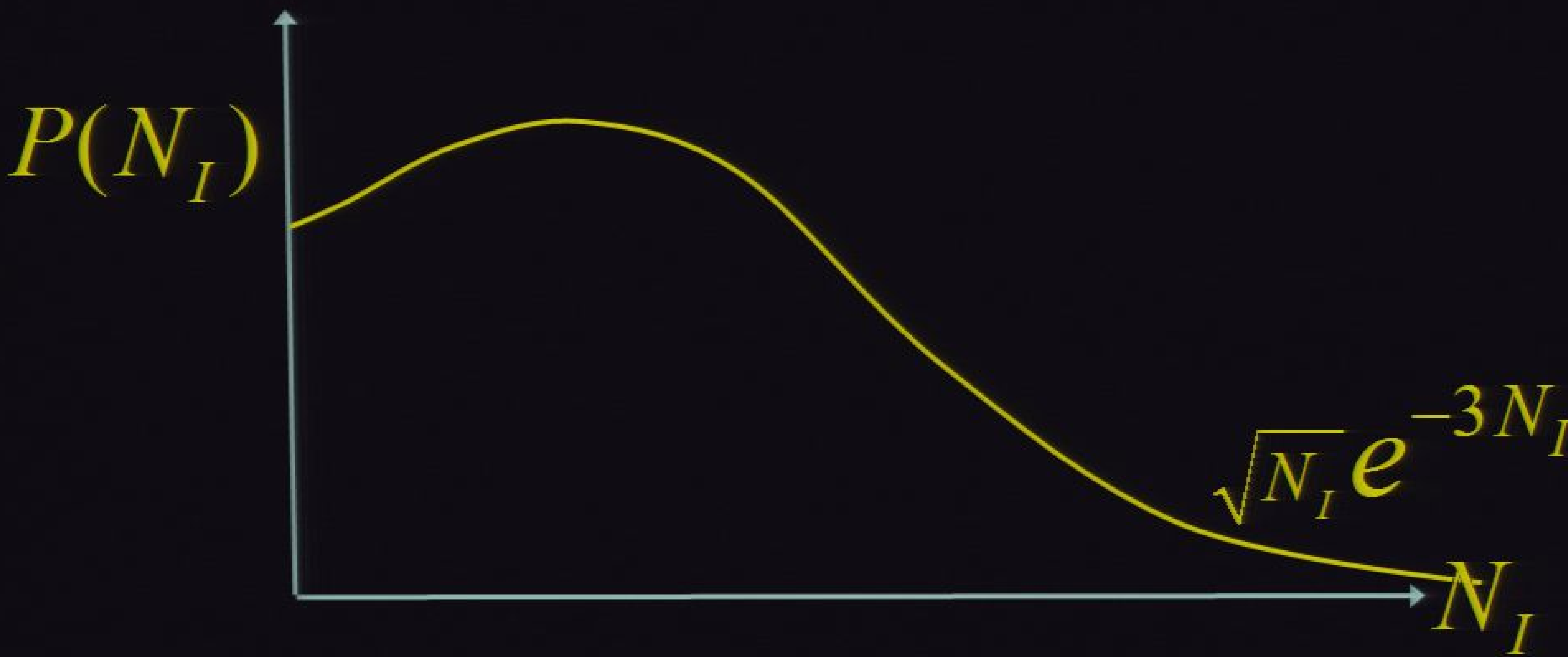

Finite C result is always lower than $C=\infty$ result at large N_I

* Note: "attractor" becomes "repeller" because statistical ensemble defined in asymptotic region where gravity becomes unimportant: the future

* more inflaton fields makes problem worse

$$P_{>}(N_I) \propto (\delta\theta)^m \sim e^{-3N_I m}$$ cf N-flation

* this analysis makes precise a problem identified by Penrose long ago (Annals NYAS, 1989)

* with this canonical measure, inflation cannot be considered a viable explanation for the observed state of the cosmos.

What could be wrong?

* canonical measure?
* neglect of: entropy production?
  no: reheating is unitary, cannot alter proportion of states $P(N \geq N_I)$
  inhomogeneities?
  quantum fluctuations?
* inflation?

cf "cyclic/ekpyrotic" theory, where gravity is unimportant in the past, and according to the analogous canonical measure, every trajectory undergoes near-maximal ekpyrosis (w/P. Steinhard in progress)

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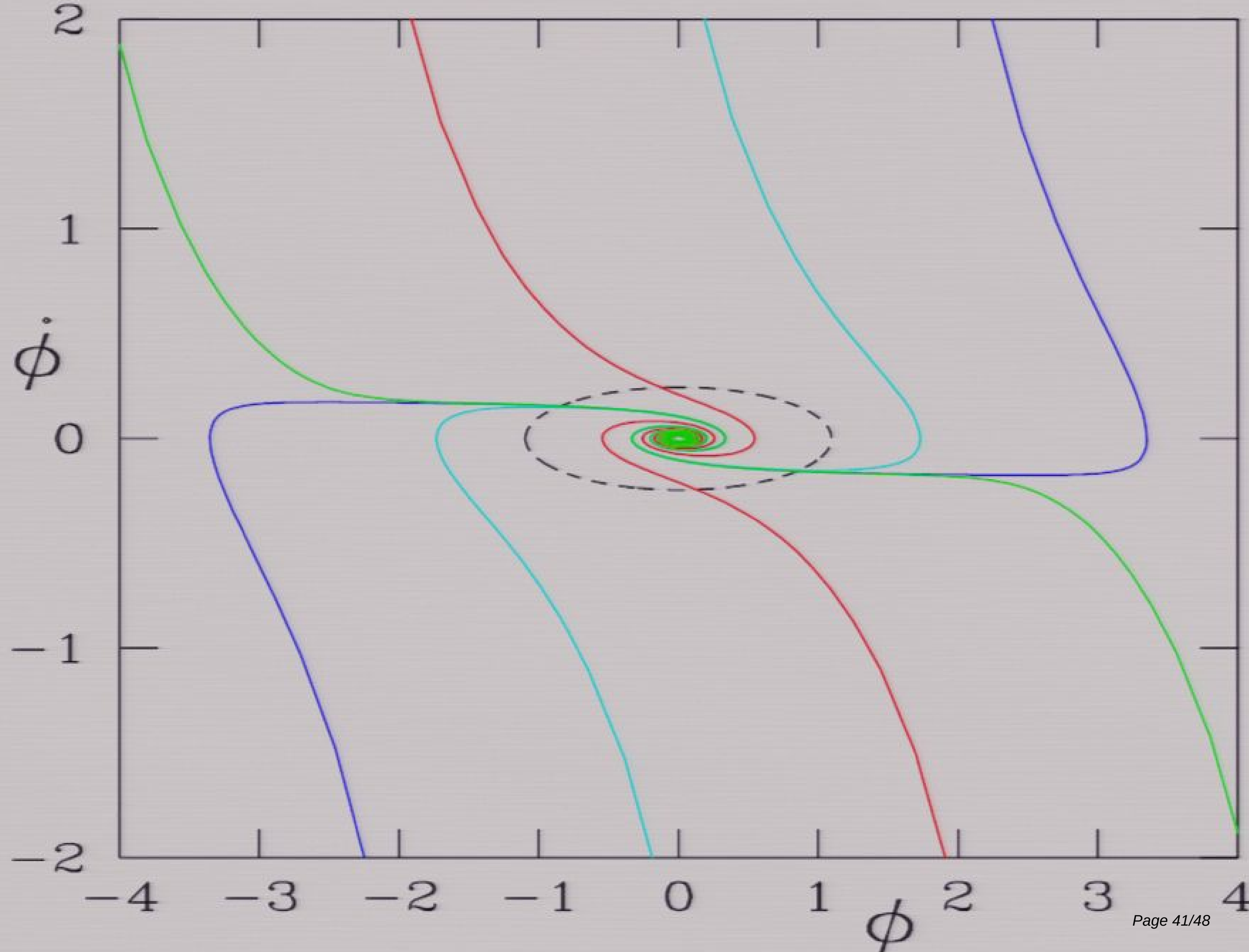
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Future:

semiclassical theory including quantum jumps and tunneling -> "eternal" inflation

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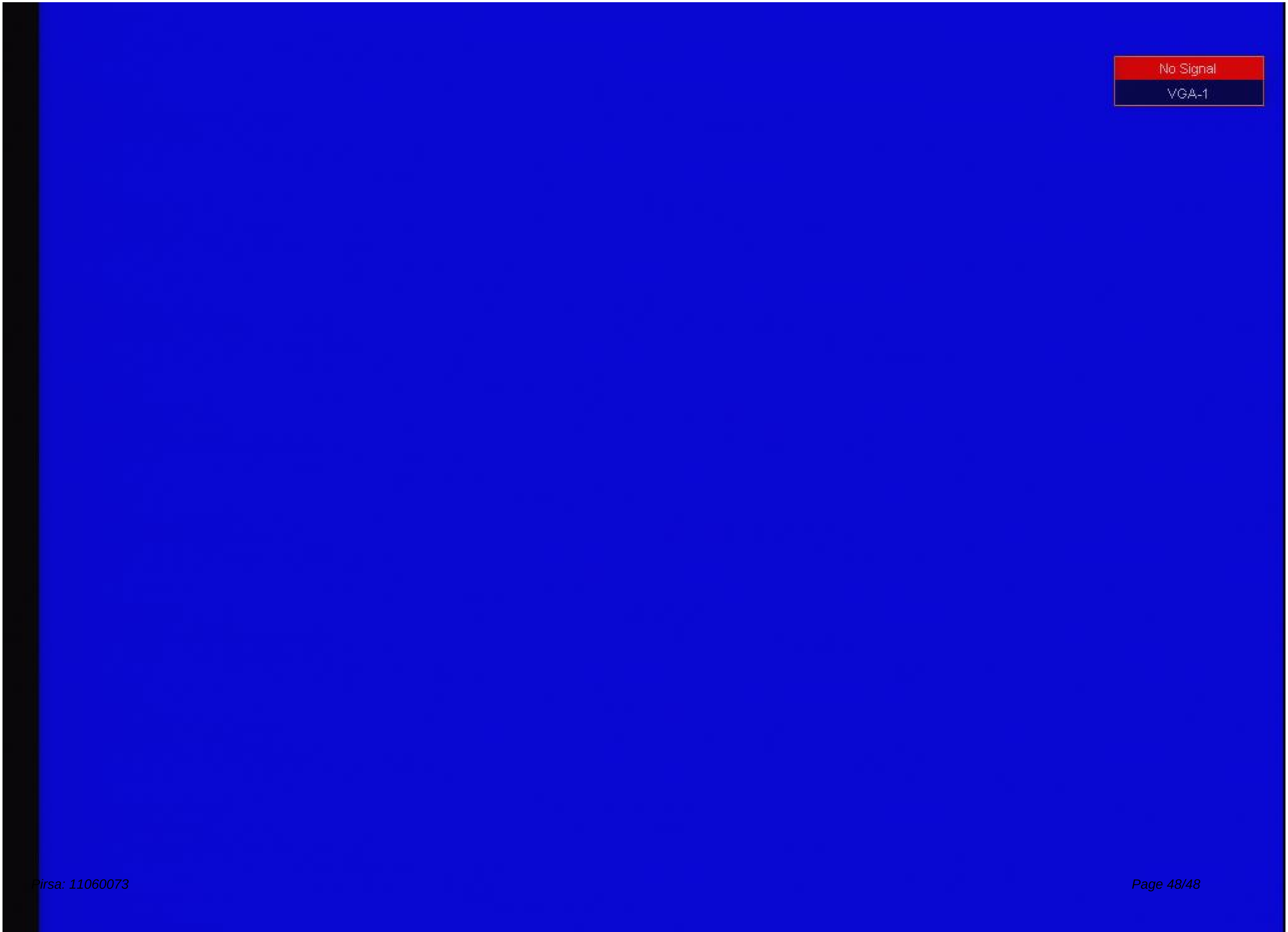
No Signal
VGA-1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