Title: The semiclassical wavefunction of the universe near de Sitter space

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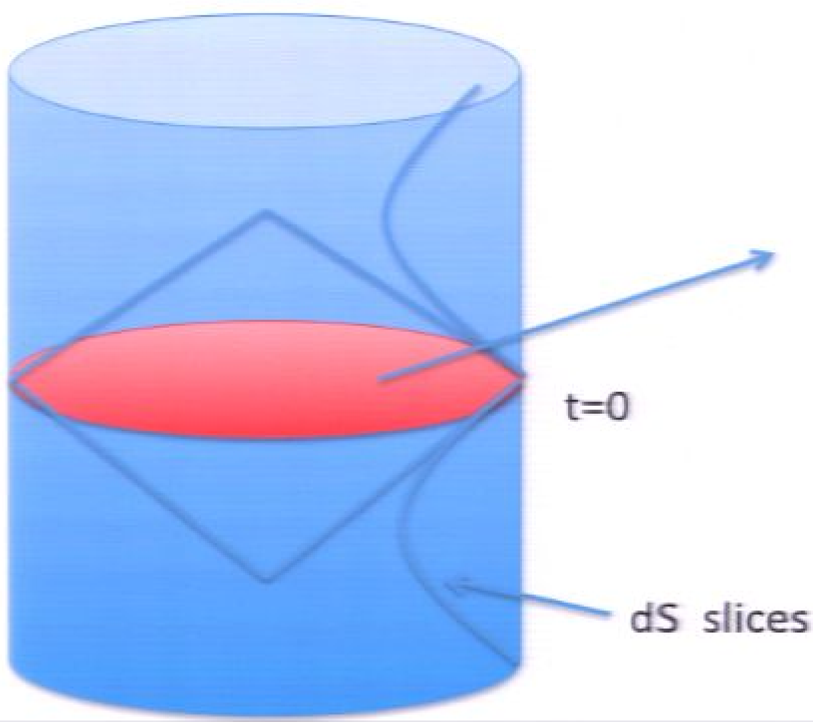
Abstract: We make some remarks about the semiclassical wavefunction of the universe around de-Sitter space. In five dimensional gravity with a positive cosmological constant it is possible to compute the full semiclassical measure for arbitrary geometries at superhorizon scales. In four dimensions, the same computation can be reformulated as a problem in conformal gravity.

AdS crunches

$$ds^2 = -dt^2 + \cos^2 t\, ds^2_{H_d}$$ Crunching slicing

$$ds^2 = d\rho^2 + \sinh^2 \rho\, ds^2_{dS_d}$$ De Sitter slicing



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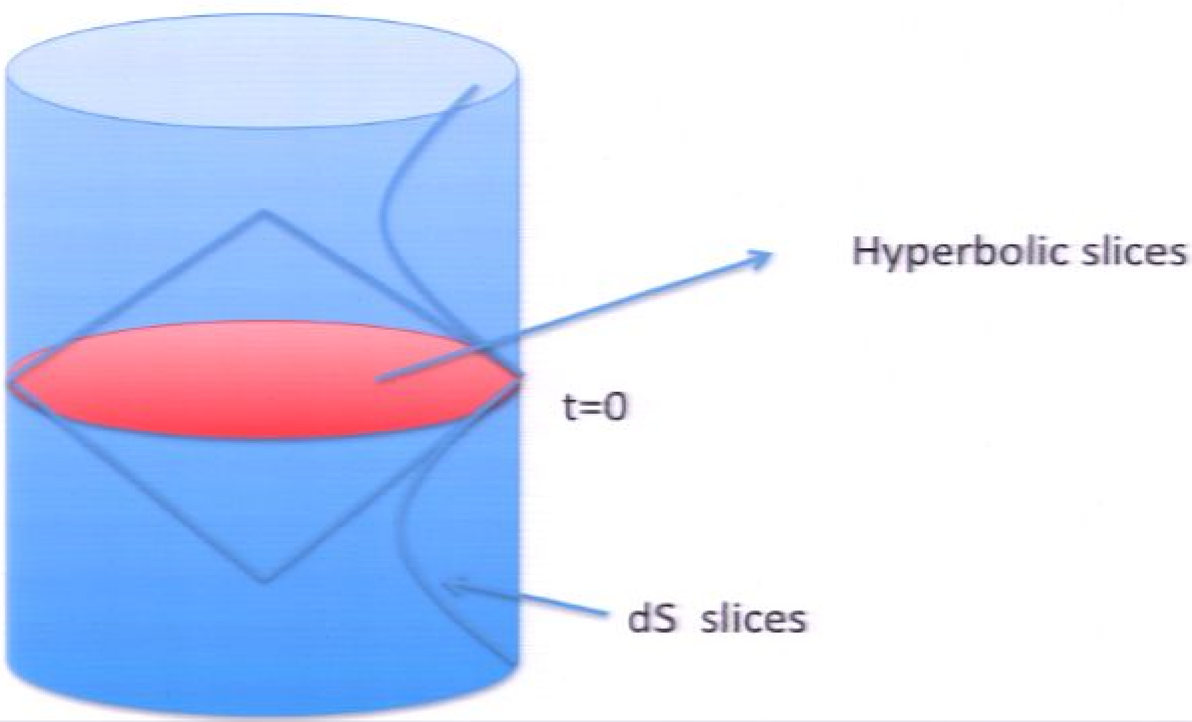

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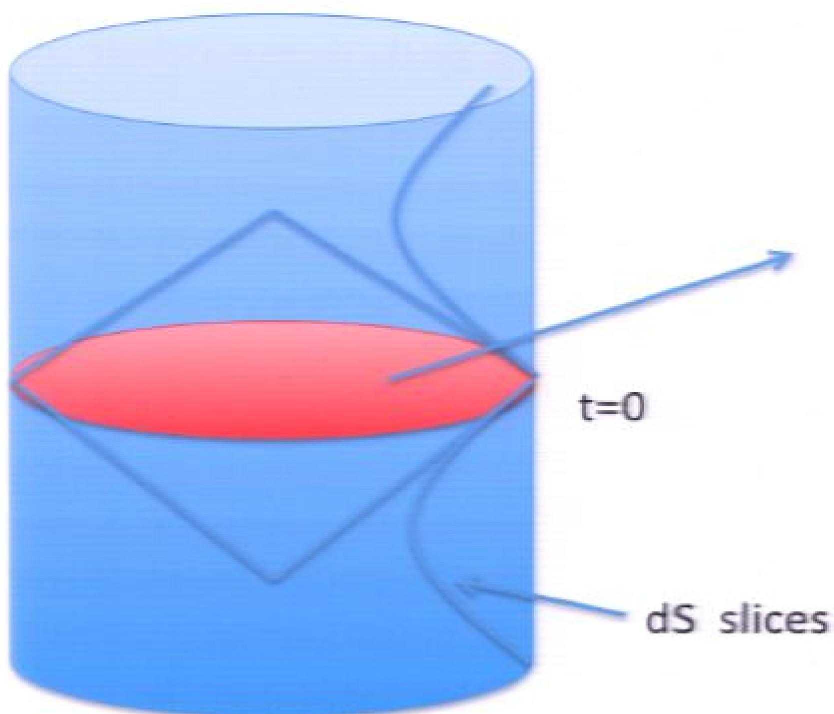

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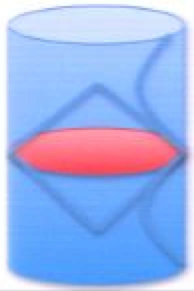
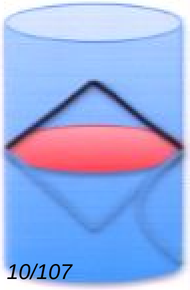
- No crunch
- Boltzman brains in the de Sitter picture, just the vacuum in the cylinder.
- Conformaly coupled scalars have large superhorizon fluctuations in de Sitter → just collapse smoothly, if undisturbed (ie. Nothing mesured). They decohere and cohere again!.



Non-conformal field theories in de Sitter

Buchel, Langfelder, Walcher, Aharony, Fabinger, Horowitz, Silverstein Balasubramanian, Ross, Cai, Titchener, Alishahiha, Karch, Tong, , Larjo, Simon, Hirayama, He, Rozali, Hutsaioti, Kumar, Rafferty, Marlolf, Rangamani, Van Raamsdonk



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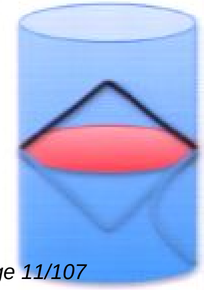

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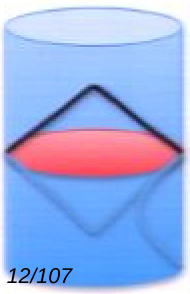

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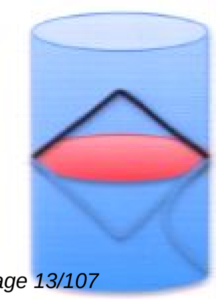

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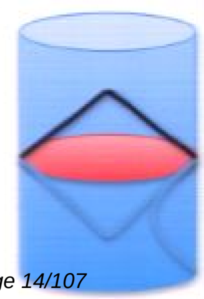

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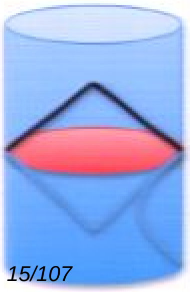
- Continuing beyond infinity? Seems to require more data, if at all possible.



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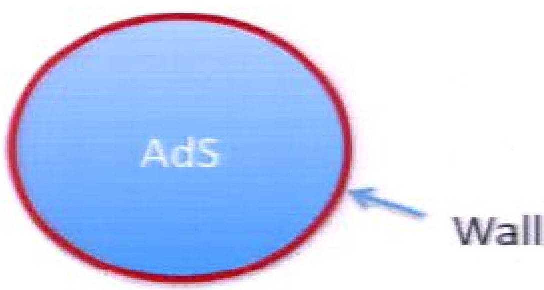

Hertog, Horowitz, Barbon, Rabinovici

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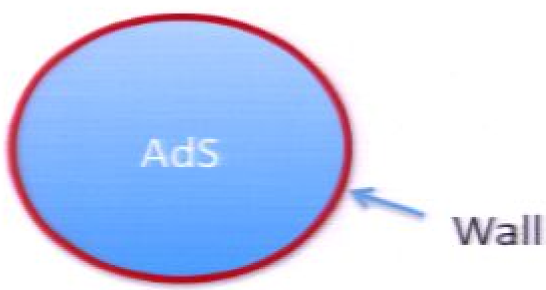

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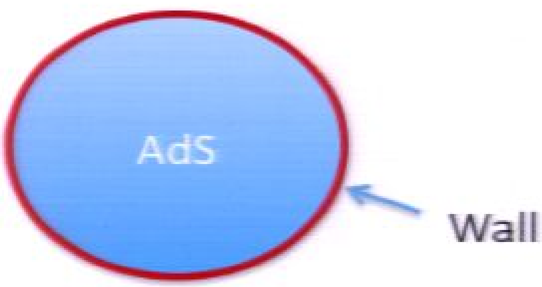

- Then we can approximately replace the AdS region by a field theory living on the wall. It is a CFT plus an irrelevant perturbation, dual to the massive field in AdS. The field theory has a UV cutoff related to the radial position where the wall is sitting. With this field theory description, there is no singularity (or interior).

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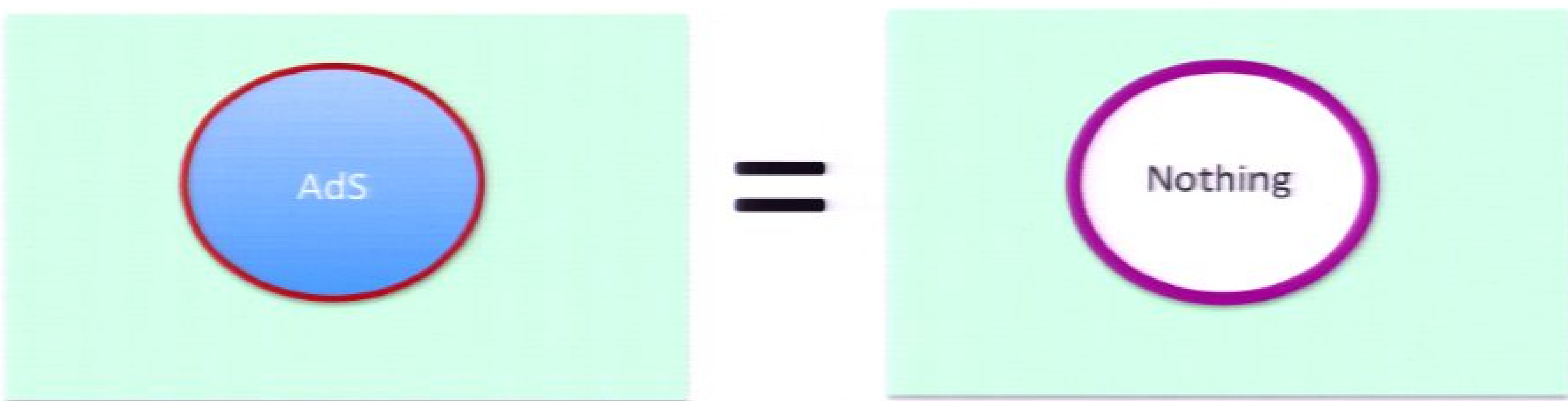
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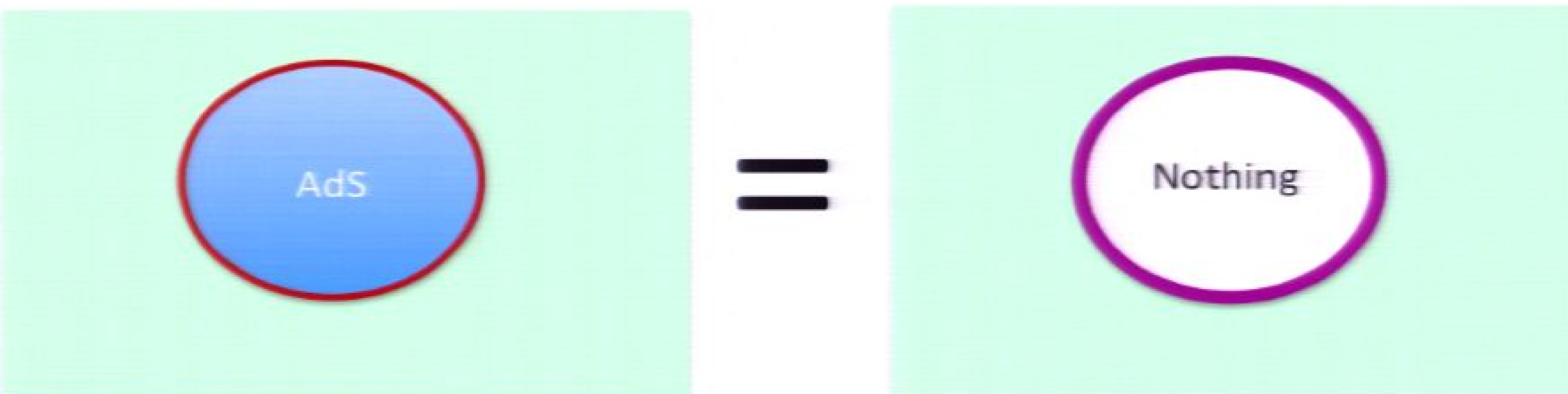

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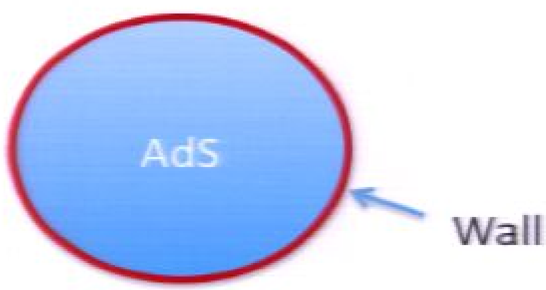

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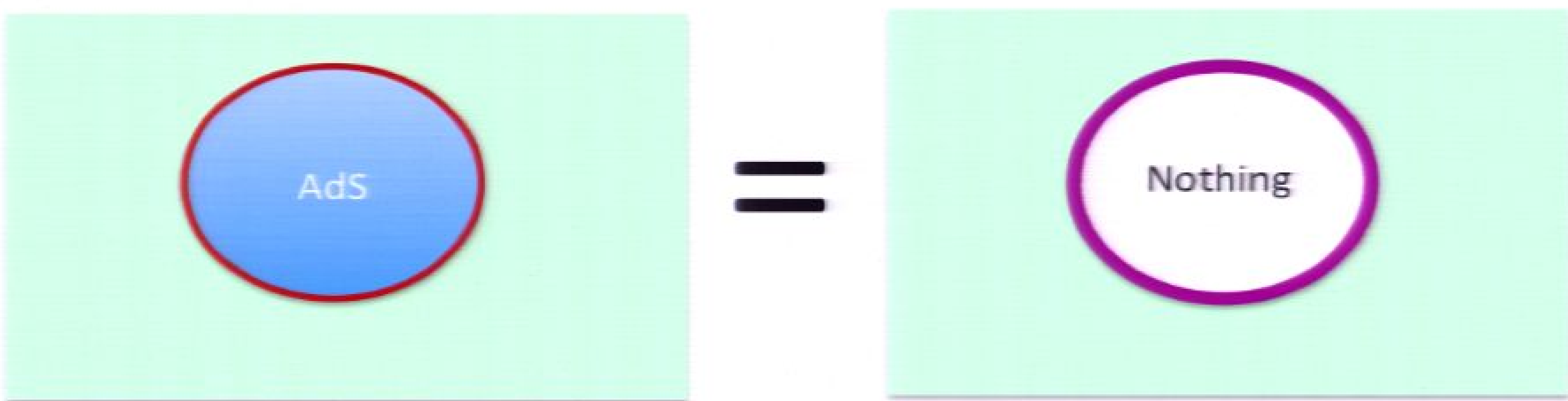
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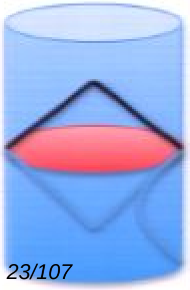

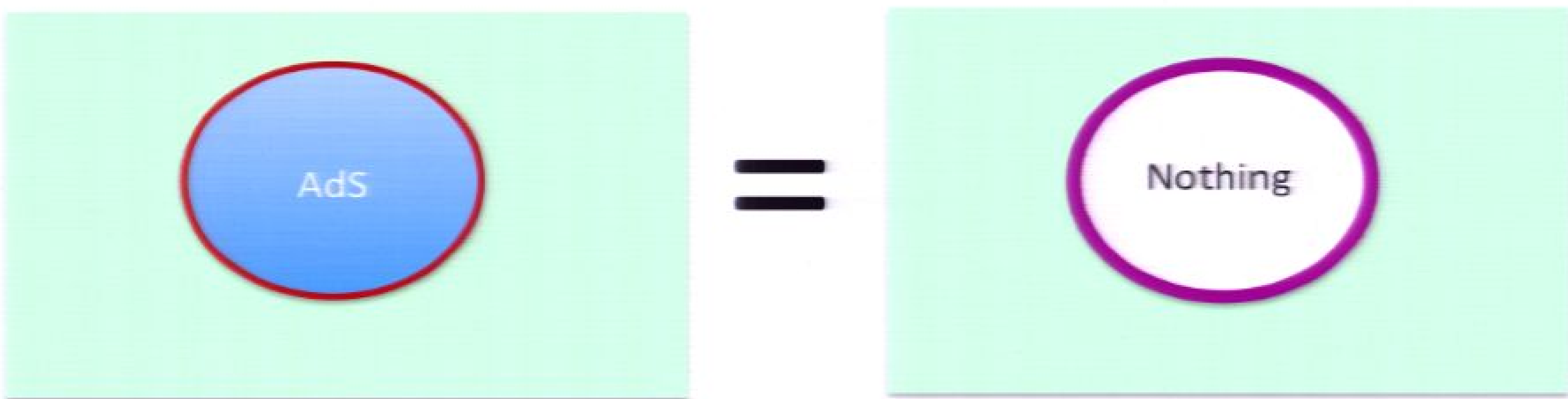
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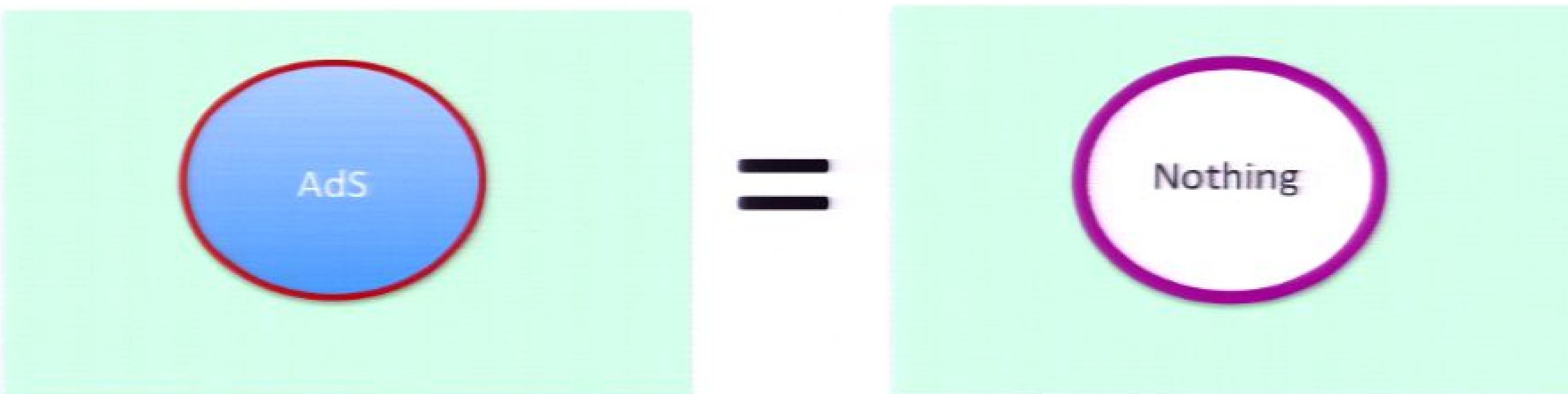

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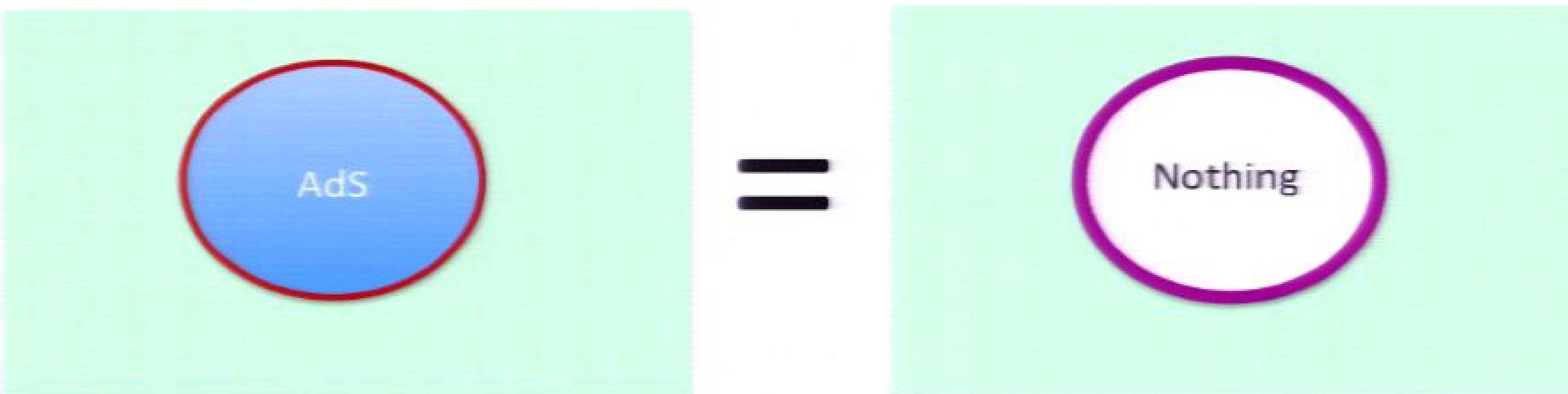

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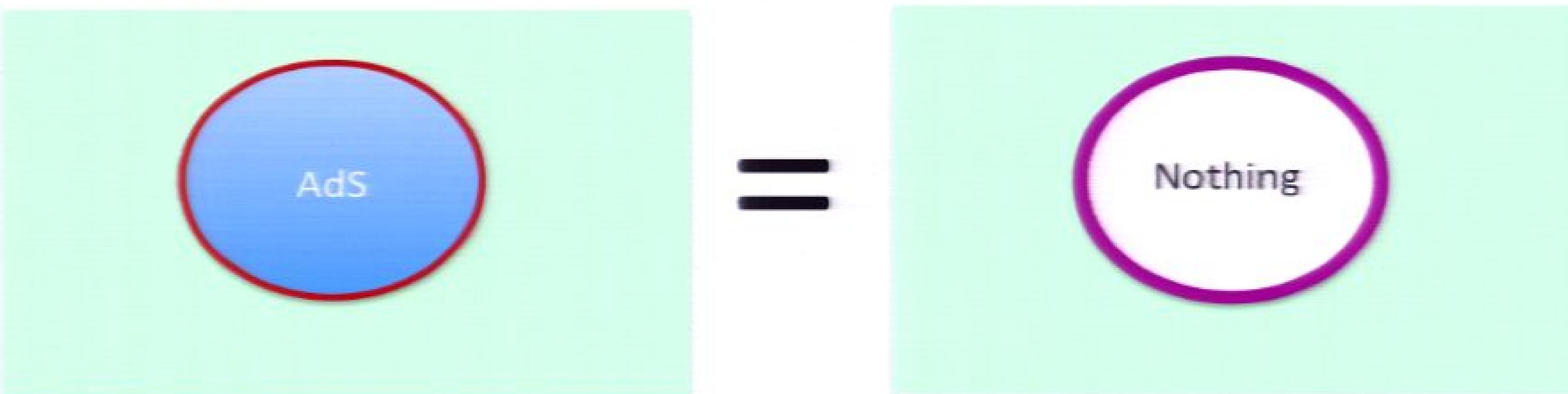

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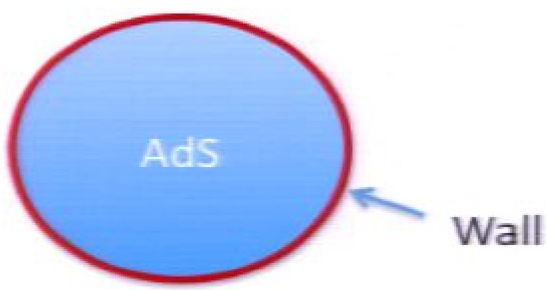

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2^{nd} part

Solution of the tree level 5d measure problem in pure 5d gravity. Finding the probability for different shapes for the spatial sections.

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- In quantum theories probabilities are given by the square of the wavefunction
- If you have IR divergencies in a physical question → it was not a good question, you need to modify the question.

Perturbation theory around dS

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- This perturbative gravity (and scalar field) has made successful predictions for our universe!. (CMB anisotropies, etc..).

5d pure gravity in de Sitter

- Gravity with positive cosmological constant
- Consider the BD vacuum in the weakly coupled regime, $\frac{R^3}{G_N} \gg 1$

$$ds^2 = \frac{-d\eta^2 + g_{ij}dx^i dx^j}{\eta^2} \qquad g_{ij} = \delta_{ij} + h_{ij}$$

$\Psi(g_{ij})$ Wavefunction of the universe

- The wavefunction of the universe is well defined perturbatively. It is a perturbative solution of the measure problem. Here we just discuss the tree level part.

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$$\Psi_R[\Omega^2(x)g_{ij}] = \Psi_R[g_{ij}]$$

Up to the conformal anomaly

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$$z \to -i\eta \,, R_{AdS} = -iR_{dS} \;,$$

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The boundary conditions also transform properly:

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In de Sitter → continuation from EAdS.

Starobinski,
Fefferman-Graham
Heningson-Skenderis

In Euclidean space, we have a real answer:

$$\Psi(\frac{1}{\epsilon^2}g_{ij}) = e^{c_{AdS}\left[\frac{1}{\epsilon^4}\int\sqrt{g}+\frac{1}{\epsilon^2}\int\sqrt{g}R+\log\epsilon\int W^2-E+\mathrm{Finite}(g)\right]}$$

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All terms become purely imaginary, including the finite term. The only real part arises via

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(Depends on the metric of the four dimensional slice)

Action of conformal gravity

Gives a topological term, the Euler number.

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Notice that in the wavefunction, reparametrization symmetry translates into relatively simple equations, or conditions on the answer.

Of course, when we want to define gauge invariant projection operators, the problem of finding gauge invariant observables will come back again.

Conclusions, 5d

- In five dimensional de-Sitter there is a huge simplification if we compute the wavefunction.
- We simply get the action of conformal gravity in 4d. This is the 4d spatial slice of the 5d geometry at superhorizon distances.

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Heningson-Skenderis

$$\Psi(\frac{1}{\epsilon^2} g_{ij}) = e^{c_{AdS}\left[\frac{1}{\epsilon^4}\int\sqrt{g} + \frac{1}{\epsilon^2}\int\sqrt{g}R + \log\epsilon\int W^2 - E + \mathrm{Finite}(g)\right]}$$

$$c_{AdS} = \frac{R_{AdS}^3}{G_N} \to i\frac{R_{dS}^3}{G_N}$$

All terms become purely imaginary, including the finite term. The only real part arises via

$$\log\epsilon \to \log|\eta_0| + i\frac{\pi}{2}$$

$$|\Psi|^2 = e^{-c_{dS}\pi\int d^4x\sqrt{g}(W^2 - E)}$$

(Depends on the metric of the four dimensional slice)

Action of conformal gravity

Gives a topological term, the Euler number.

4 dimensional de Sitter gravity

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- The same problem in four dimensions is more complicated.
- The answer is non-local. It is simply the same as the non-local answer in the EAdS case (up to an overall sign).
- Curious observation: This non-local, and finite part, can be computed using conformal gravity.

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- Renormalized action for ordinary Einstein gravity → Equal to the action of ordinary gravity.

- A simple boundary condition on the fields of conformal gravity selects the Einstein gravity solutions.
- Conformal gravity equations: 4th order. 2 boundary conditions in the past from Bunch Davies. Two in the future:

$$g_{ij}(\eta = 0) = g^b_{ij} \ , \qquad \partial_\eta g_{ij}(\eta = 0) = 0$$

$$ds^2 = \frac{-d\eta^2 + (g^0 + \eta^2 g^2 + \eta^3 g^3 + \cdots) dx dx}{\eta^2}$$

Einstein solutions.

No time derivative

Useful identity:

$$\int Euler = \int W^2 + 2\int Ricci^2 - \frac{1}{3}R^2$$

Equations of motion of Weyl gravity → Involves Ricci tensor. For Einstein spaces, Ricci is proportional to the metric. So the equations of motion are proportional to the metric, but the equations of motion (Bach tensor) are traceless → must be zero

Evaluating the Einstein action on an Einstein space → Same as evaluating the 4 volume. The volume arises from the Ricci terms.

$$S_E \propto \int \sqrt{g} \propto \int (W^2 - E)$$

$$S_{\mathrm{E,Renormalized}} = \int d^4x\sqrt{g} - \mathrm{Boundary} = \int d^4x\sqrt{g}W^2 - (\mathrm{Euler\ Number})$$

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No time derivative

$$\Psi_{Conformal}[h, h' = 0] = \Psi_{Einstein,\mathrm{Re}normalized}[h]$$

$$e^{c\int W^2} = e^{S_{E,\mathrm{Re}normalized}} \qquad c = \frac{M^2}{H^2}$$

-We get the ``right'' sign for the conformal gravity action for dS and the ``wrong'' one for AdS

-The overall constant is simply the ``central'' charge, or the de Sitter entropy, which is given by M^2/H^2

-This is also the only dimensionless coupling constant for pure gravity in dS (or AdS) ...(at tree level).

Ordinary de-Sitter wavefunctions:

$$h = (1 - ik\eta)e^{ik\eta}$$

Can be viewed as the combination of conformal gravity wavefunctions obeying the Neumann boundary condition.

We can use the propagators of conformal gravity with a Neumann condition + the vertices of conformal gravity

Or

The usual propagators of Einstein gravity

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- All we did, was to evaluate the ghost wavefunctions at zero values for the ghost fields.
- A quartic action + conformal couplings to background curvature → to an action in dS or AdS, which is the sum of two quadratic fields, one with positive norm one with negative norm. We are simply putting zero boundary conditions for the negative norm one.

Quartic Scalar field

$$S = \frac{1}{2}\int d^4x\sqrt{g}\left[(\nabla^2 C)^2 - 2(R_{\mu\nu} - \frac{1}{3}g_{\mu\nu}R)\partial_\mu C\partial_\nu C\right]$$

$$S = -\frac{1}{2}\int_{AdS_4}\sqrt{g}\left[(\nabla^2 C)^2 - 2(\nabla C)^2\right]$$

$$S = \int_{AdS_4}\sqrt{g}\left\{[\nabla(C+\varphi)]^2 - [(\nabla\varphi)^2 - 2\varphi^2]\right\}$$

Massive (tachyonic in AdS) field

Massless field

(setting this to zero at the boundary)

Quantum Questions

- Some versions of N=4 conformal sugra appear to be finite. Fradkin Tseytlin
- (one of these appears from the twistor string theory) Berkovits Witten
- Can this truncation be extended to the N=4 theory? Do we get an ordinary O(4) gauged sugra? (suggested by Berkovits)

$$\int e^C (W^2 + C\nabla^4 C)$$

In N=4 conformal supergravity, the coupling Constant is the vev of a field → sets the ratio of the Planck scale to the cosmological constant scale We can get large hierarchies from a not so large C.

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- Can the quantum theory with a Neumann boundary condition be interpreted as the result of a Unitary bulk theory ?
  - Note that we would only get the wavefunction at one time. Only superhorizon wavefunction.

Strings and Rigid strings

(Pointed out by Polyakov)

$$S_{usual} = \int \sqrt{g}$$

Polyakov

$$S_{rigid} = \int \sqrt{g} \hat{K}^i_{ab} \hat{K}^{i,ab}$$

↑ Induced metric

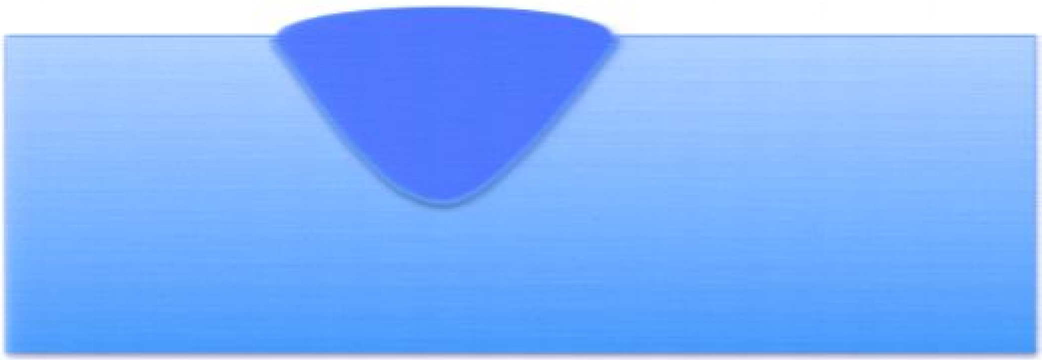
Weyl invariant in target space.

The problem of computing a Wilson loop in AdS is equivalent to computing a Wilson loop in flat space with the rigid string action, with an extra Neuman boundary condition on the fields. $X^\mu(\sigma = 0) = f^\mu(\tau)$, $\partial_\sigma X^\mu(\sigma = 0) = 0$

Alexakis

Value of the Wilson loop = Value of the rigid string action.

Membranes in dS and rigid strings



Membrane (domain wall in 4d) is created in the probe approximation. (Or connecting same energy vacua). Its dS boundary is a two dimensional surface. The probability that this surface has a given shape → Given by the rigid string action.

$$|\Psi(X)|^2 = e^{-R^3 T\pi(S_{rigid}-Euler)}$$

Same argument using the conformal anomaly for the membrane action

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Conclusions

- Conformal gravity with Neumann boundary conditions is equivalent (at tree level) to ordinary gravity on superhorizon distances.
- In AdS: The partition function of conformal gravity with Neumann boundary conditions is the same as that of ordinary gravity
- Gives a different way to compute AdS gravity correlation functions. Connections with Twistor string?
- This is non-linear, but classical (or semiclassical) relation
- It would be interesting to see what happens in the quantum case. One probably needs to do it for N=4 conformal sugra, which is finite.

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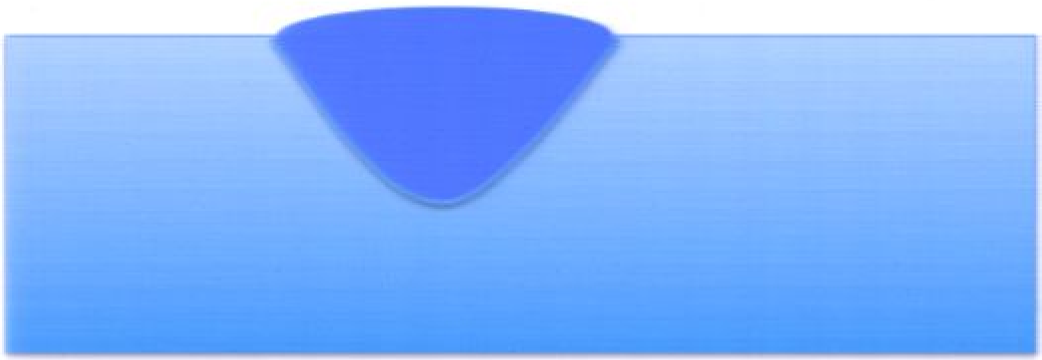

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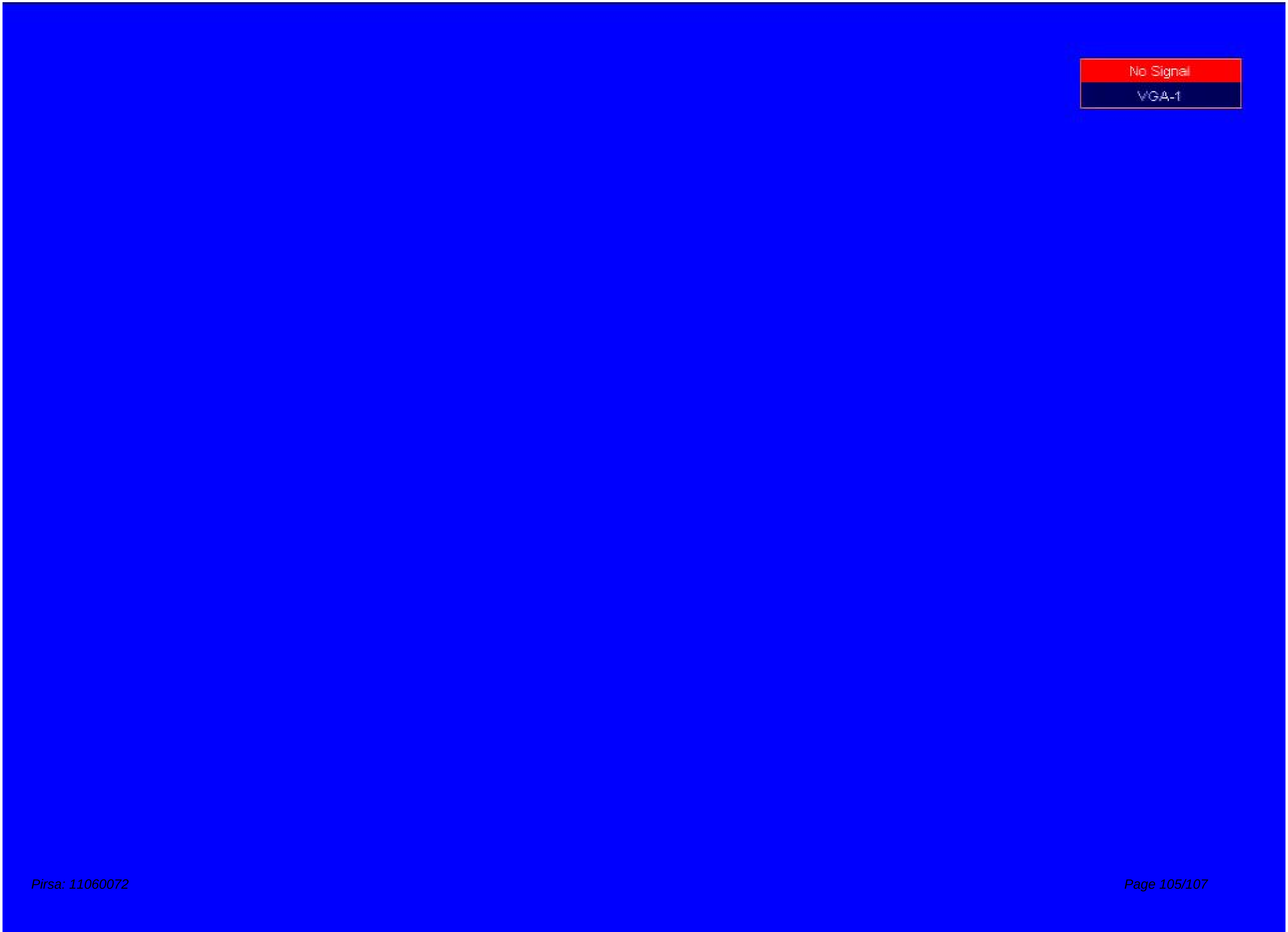
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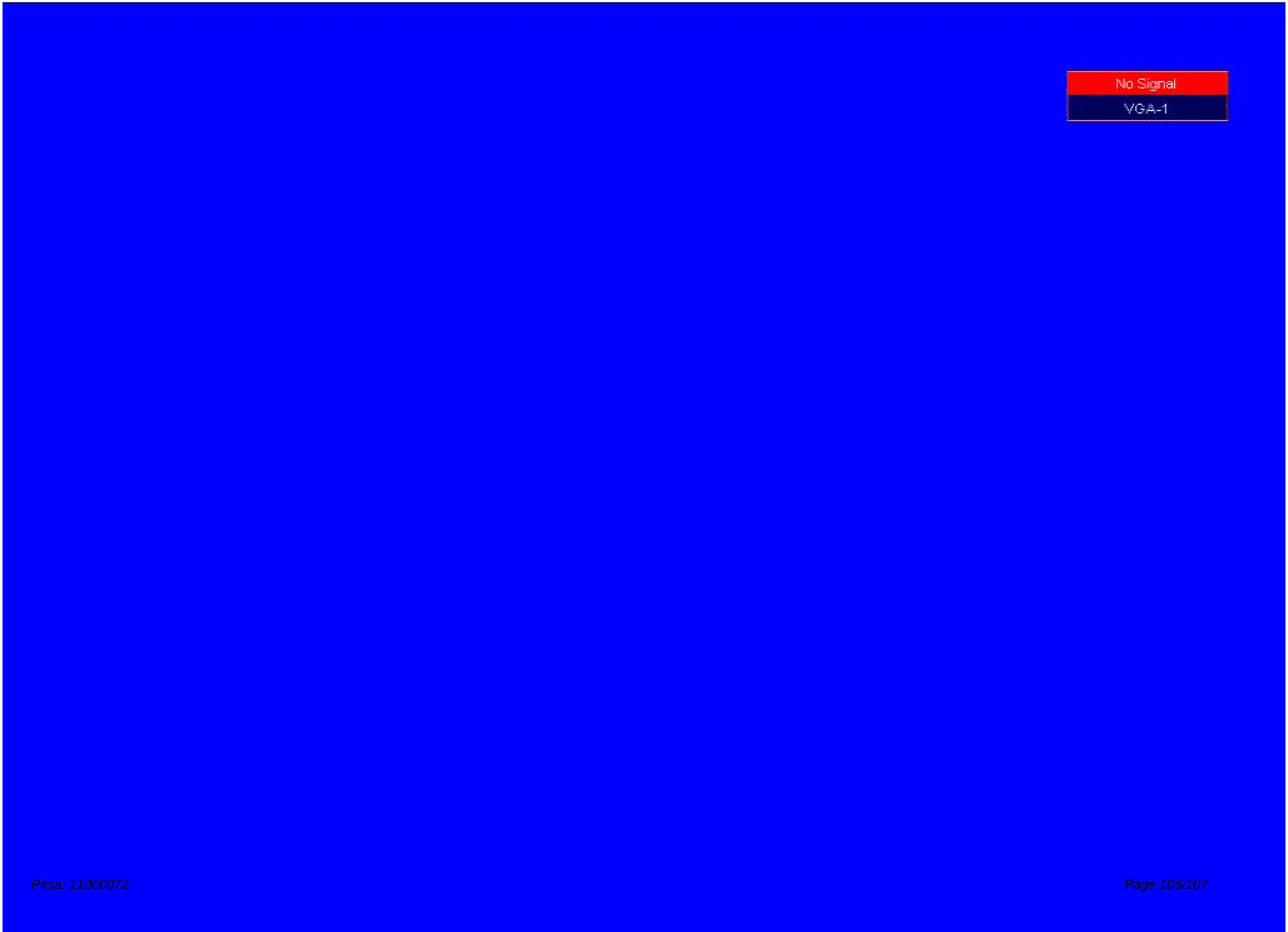
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