

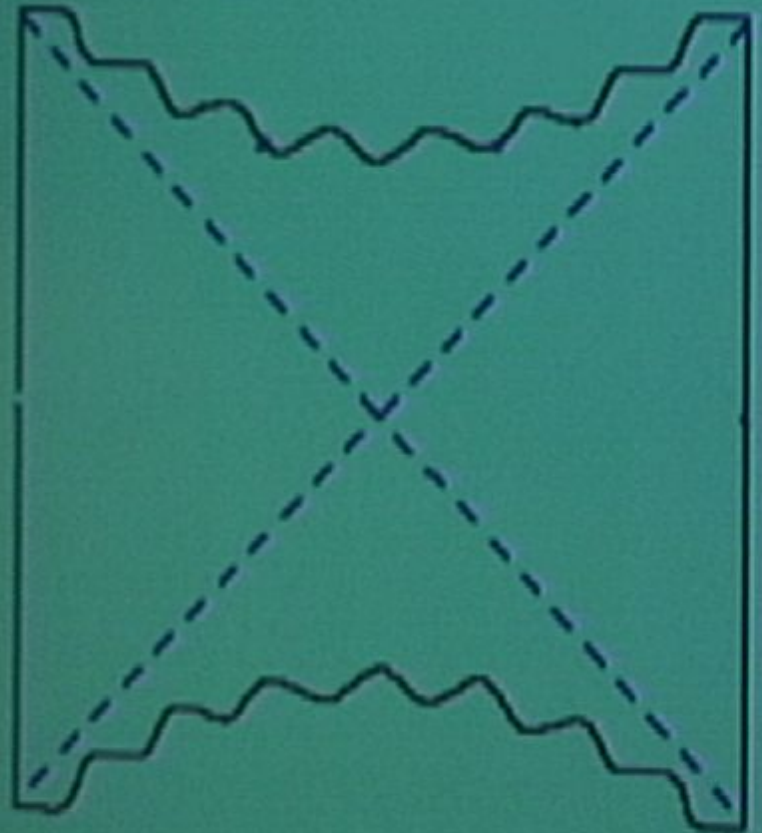
Title: Inside the horizon with holographic Wilsonian RG

Date: Jun 21, 2011 11:00 AM

URL: <http://pirsa.org/11060071>

Abstract:

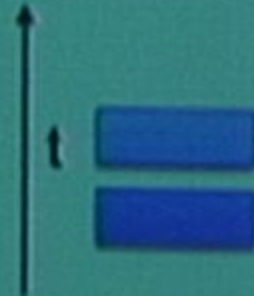
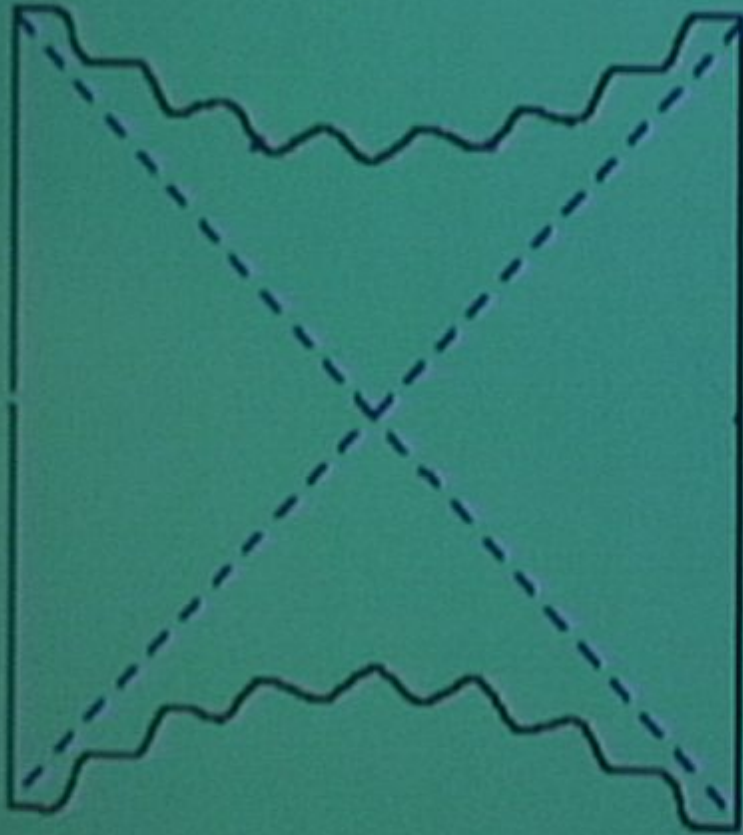
# AdS Schwarzschild black hole



Certain Super Yang-Mills at finite temperature on  $S^3$ .

Example:  $\mathcal{N}=4$  SYM with gauge group  $SU(N)$

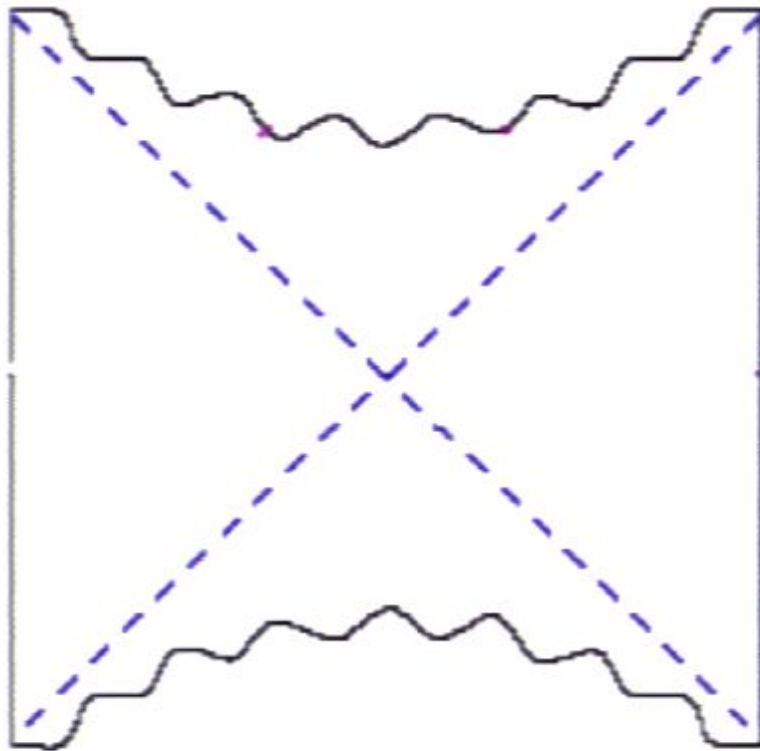
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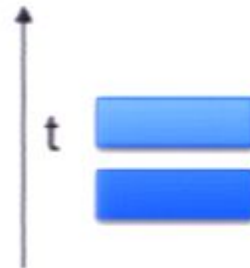
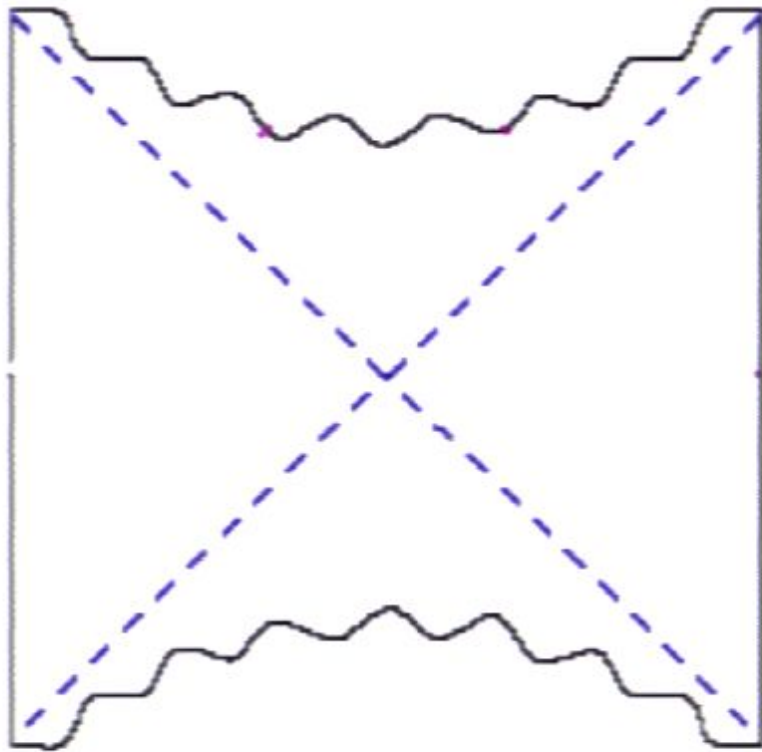


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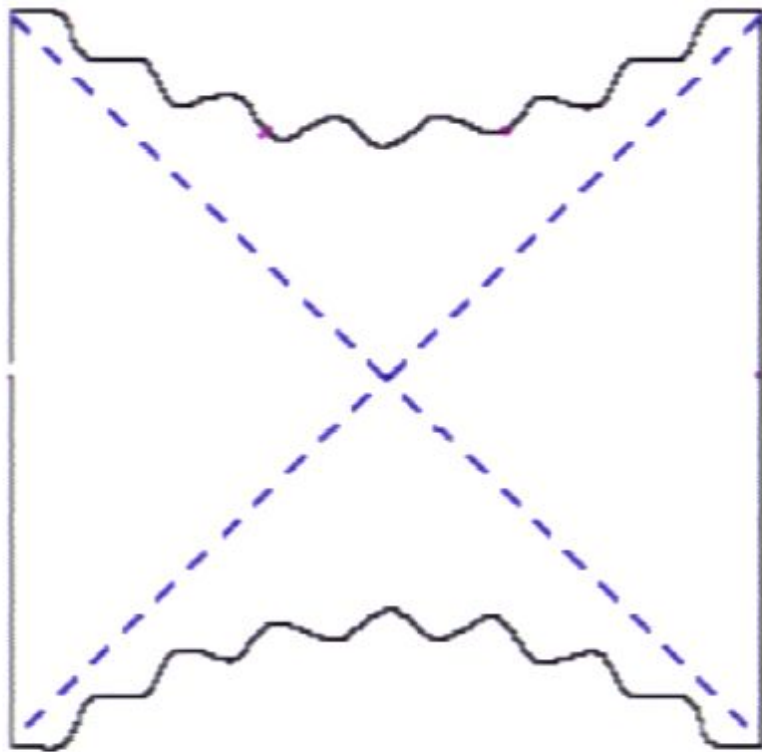


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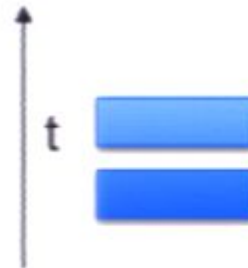
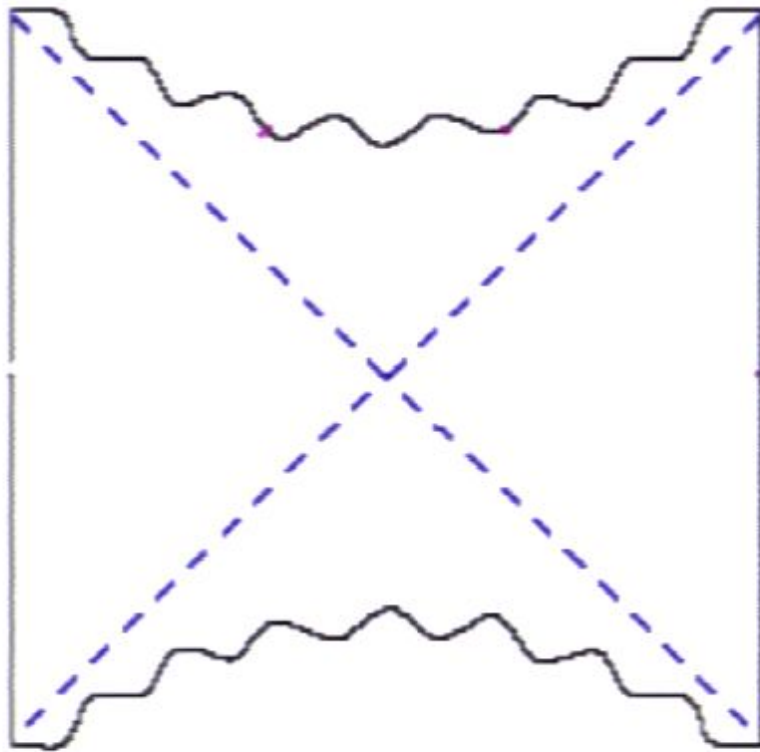


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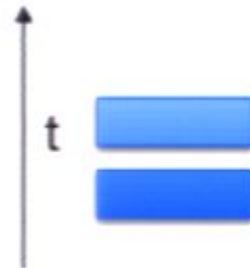
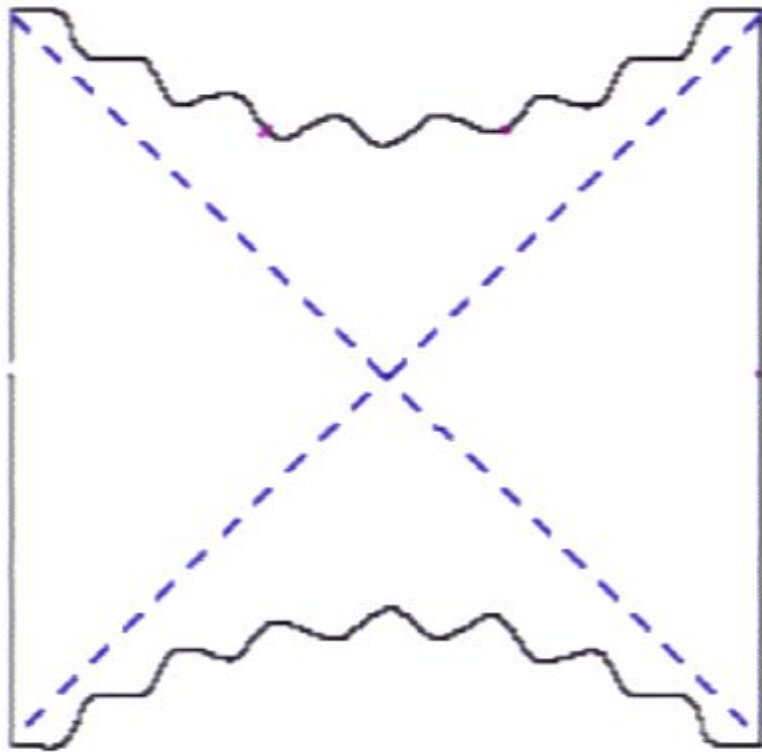


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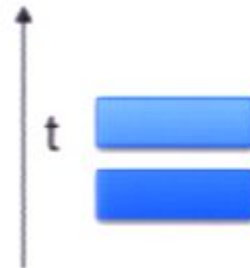
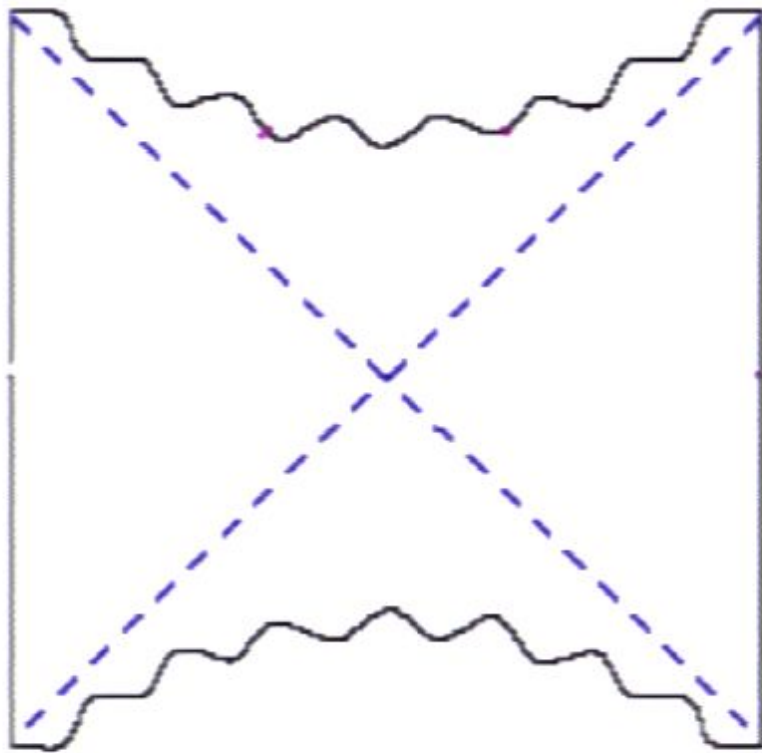
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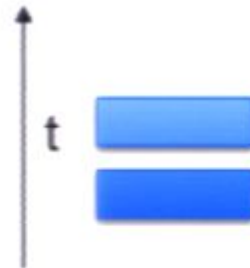
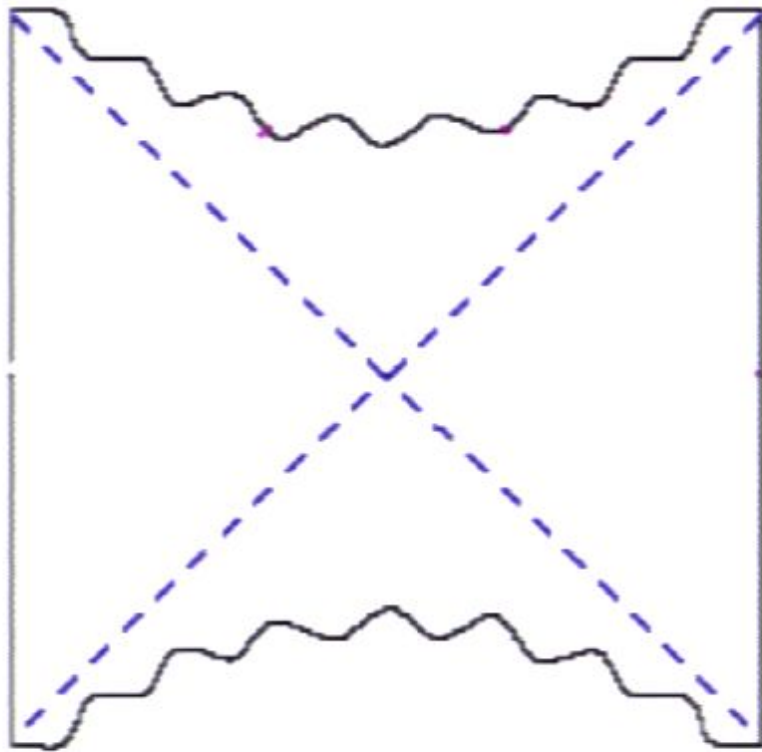


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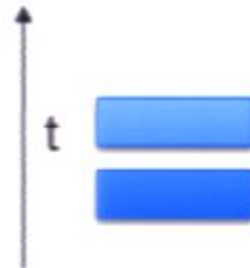
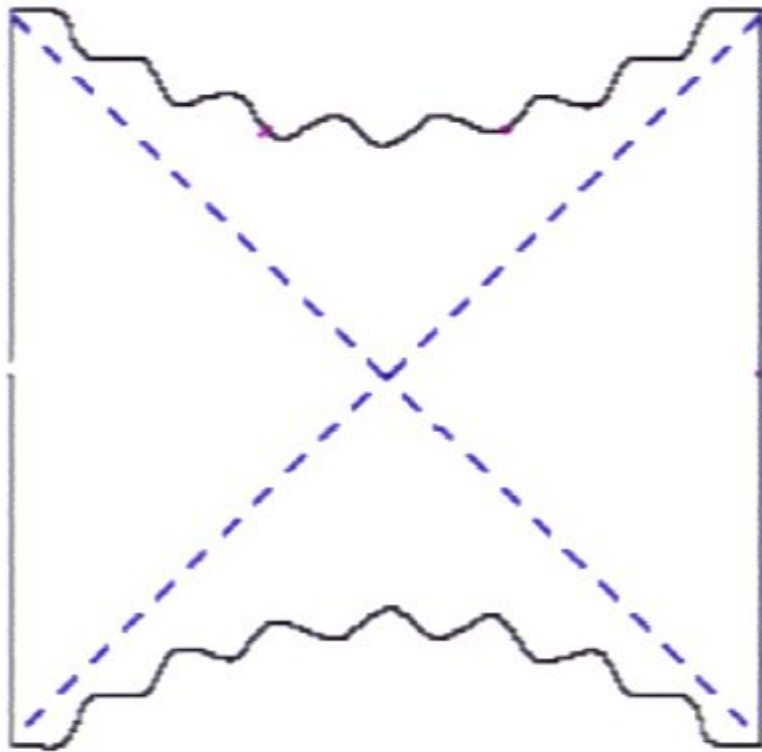


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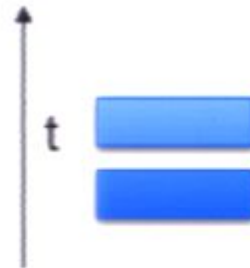
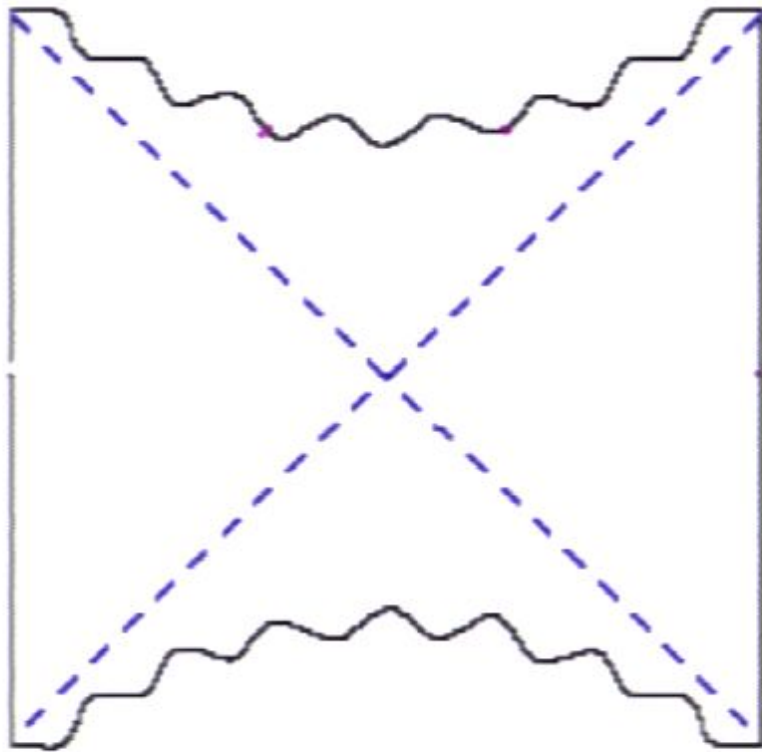
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# Plan

- Holographic Wilsonian RG

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- **Some speculations** on how to describe physics beyond the black hole horizon using AdS/CFT

Festuccia, HL, hep-th/0506202, hep-th/0611098

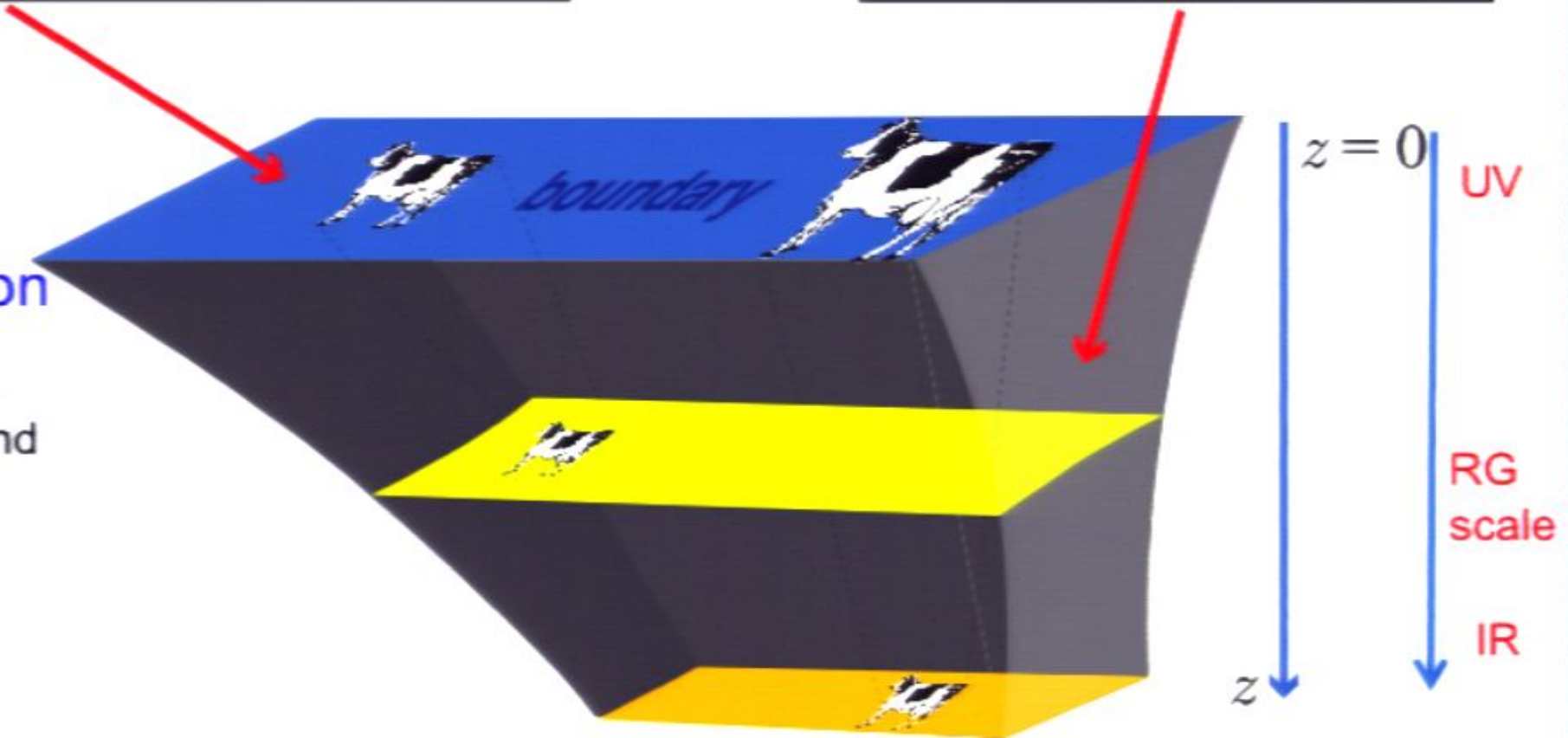
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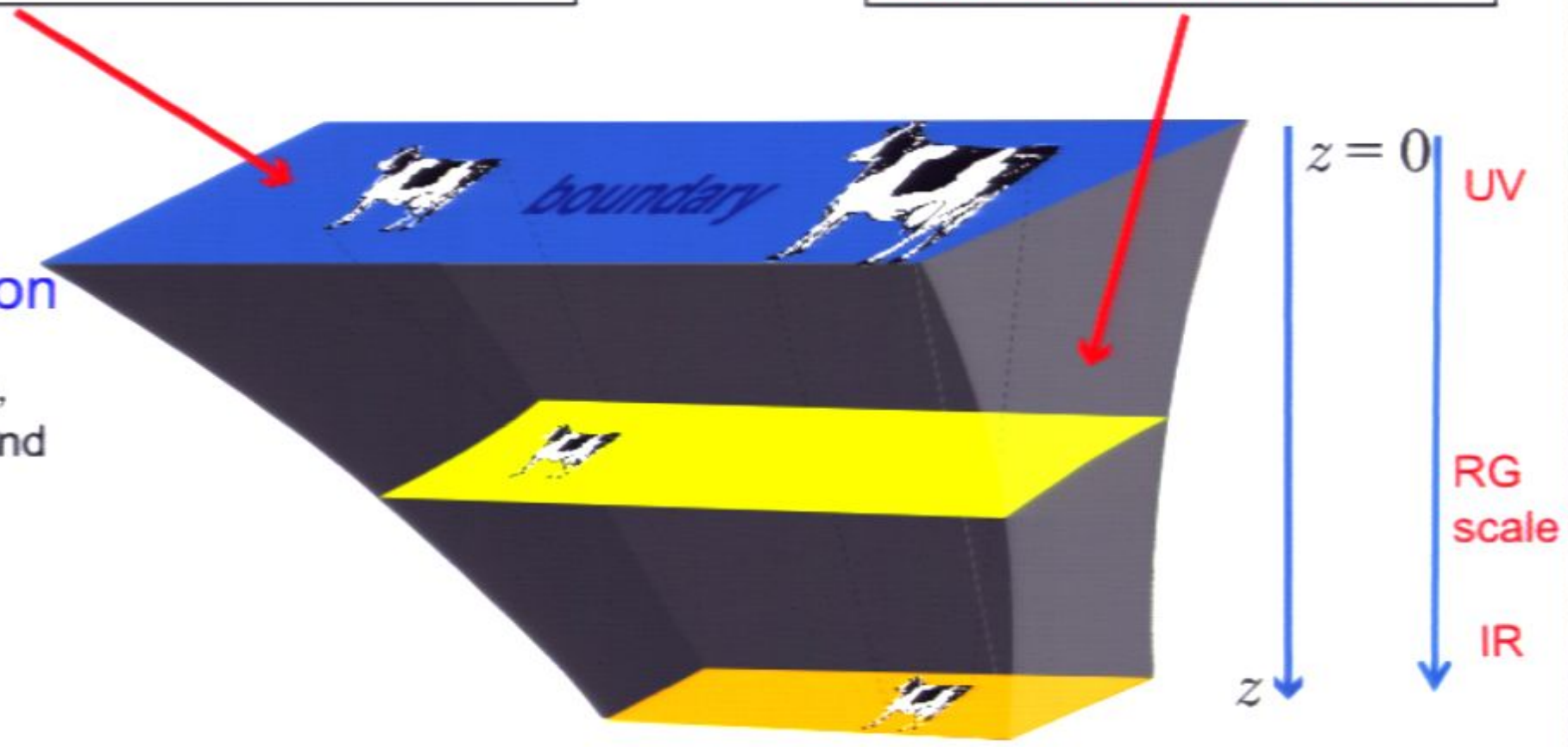
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The extra bulk radial direction: geometrization of the  
renormalization group flow of the boundary system !



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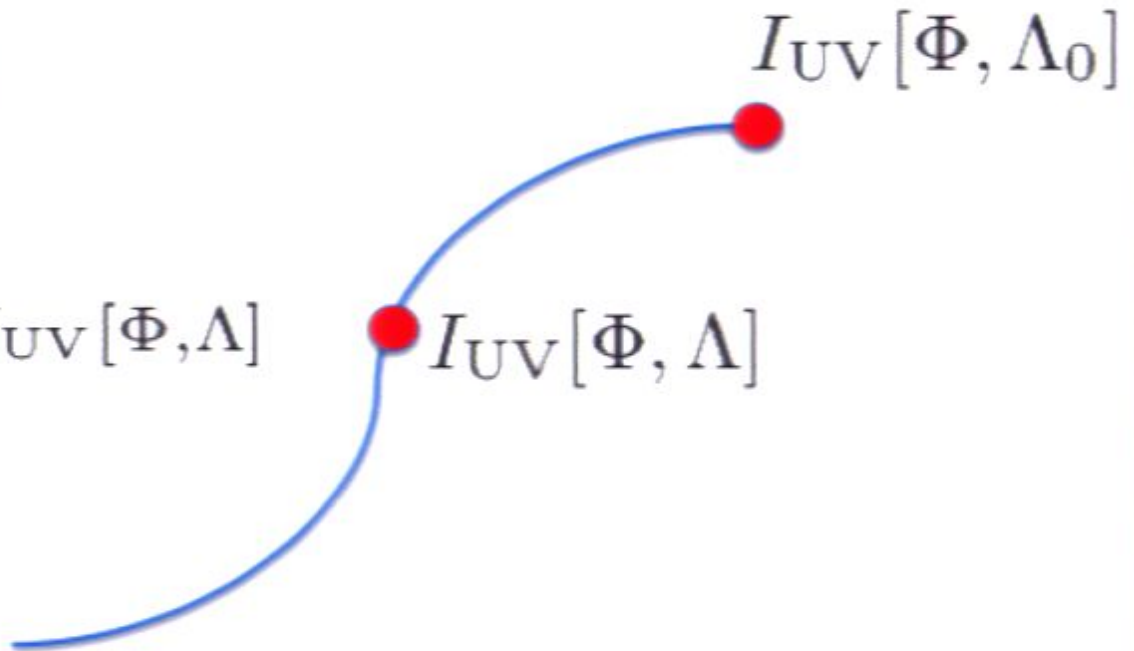
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The diagram shows a blue curved arrow pointing from the lower energy scale  $\Lambda$  to the higher energy scale  $\Lambda_0$ . Two red dots mark the points  $I_{UV}[\Phi, \Lambda]$  and  $I_{UV}[\Phi, \Lambda_0]$  on the curve, representing the UV effective action at different scales.

Flow is reversible if keeping  
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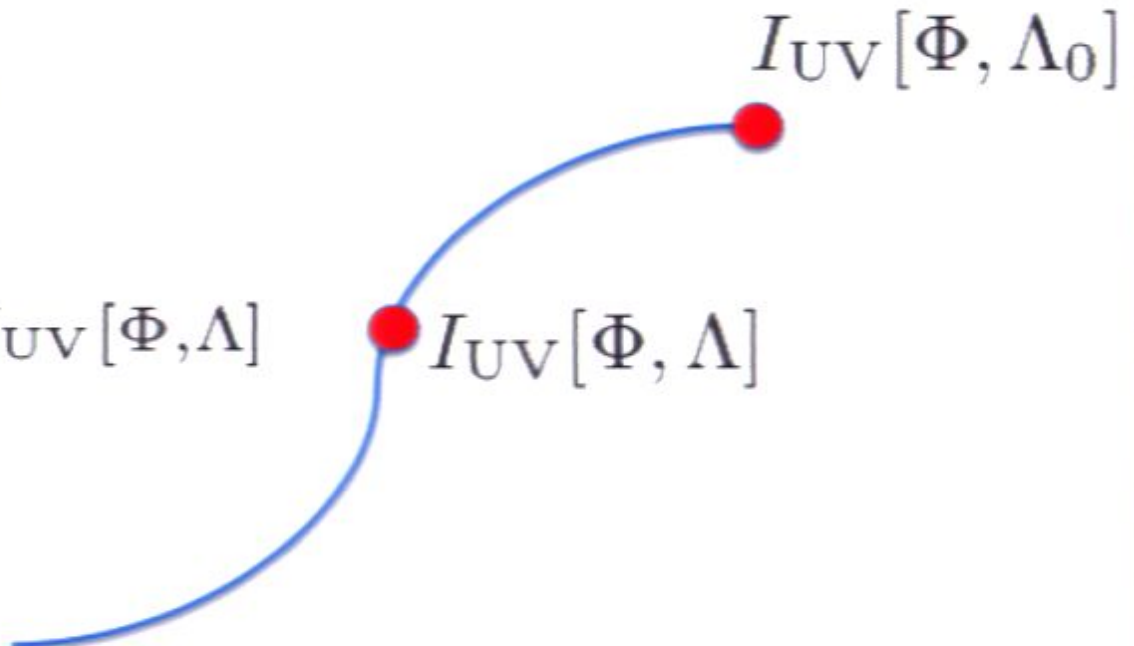
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**Will work in the large  $N$  limit, i.e. classical bulk gravity.**

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# Gravity setup



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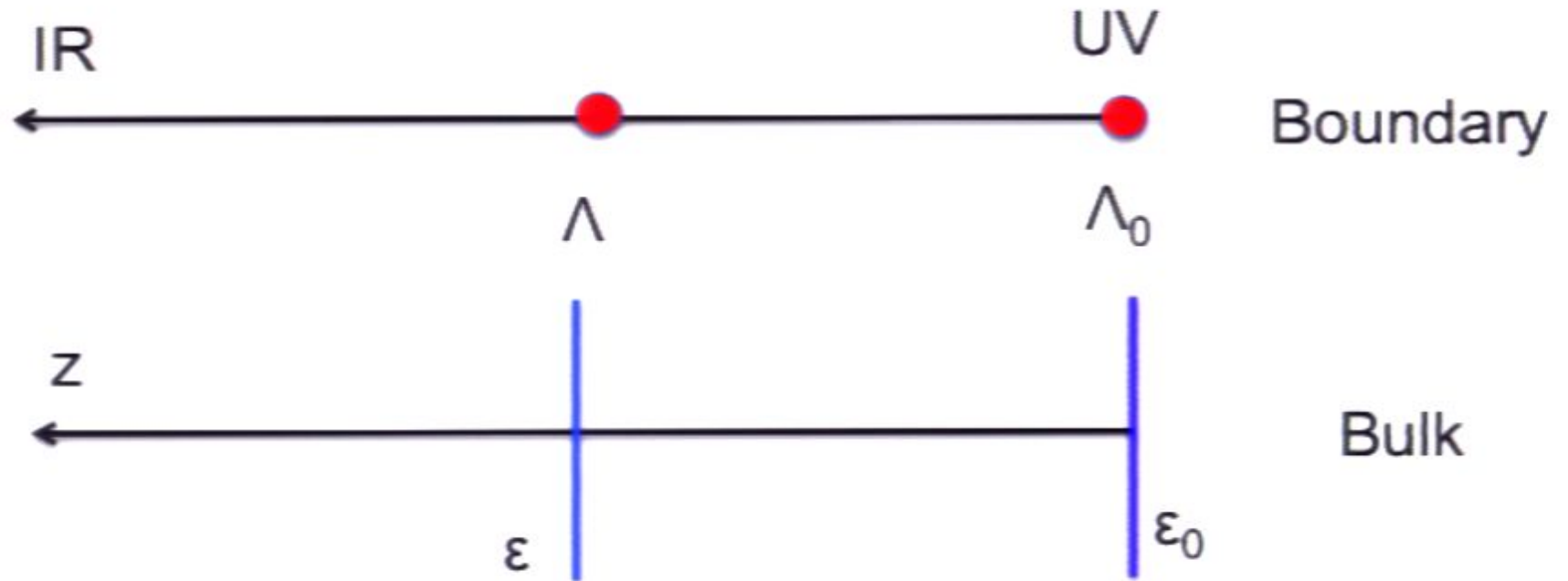
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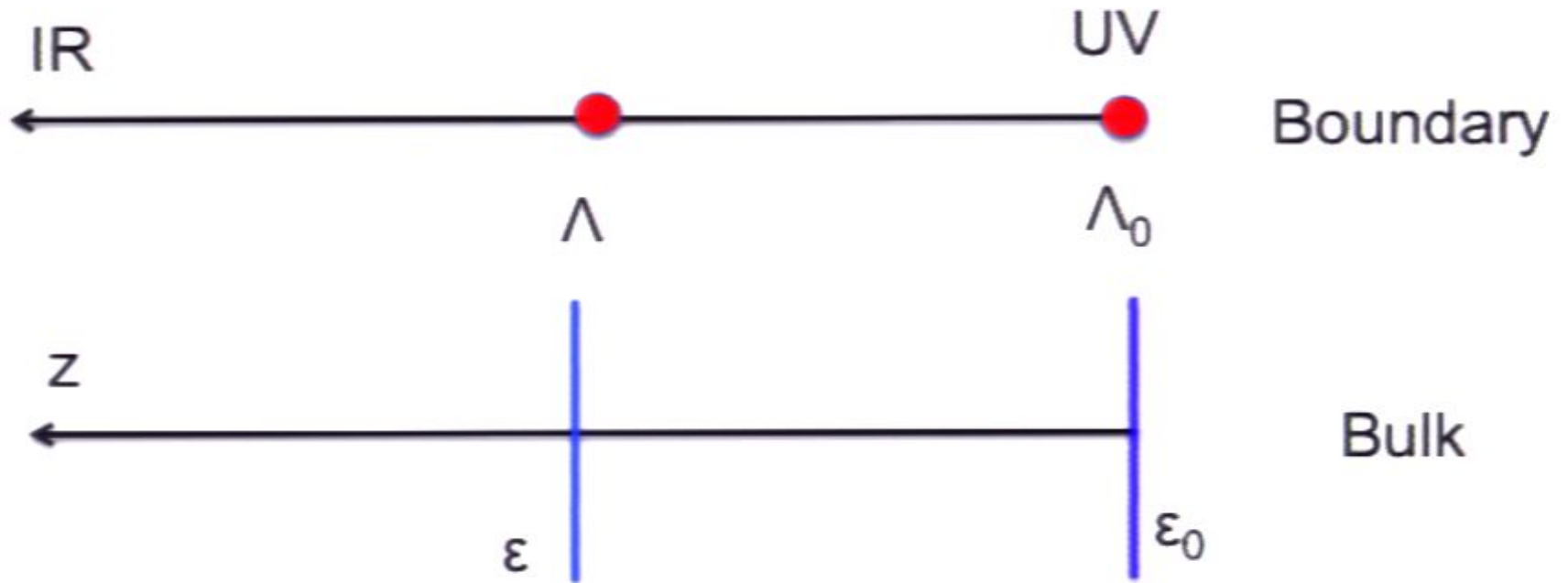
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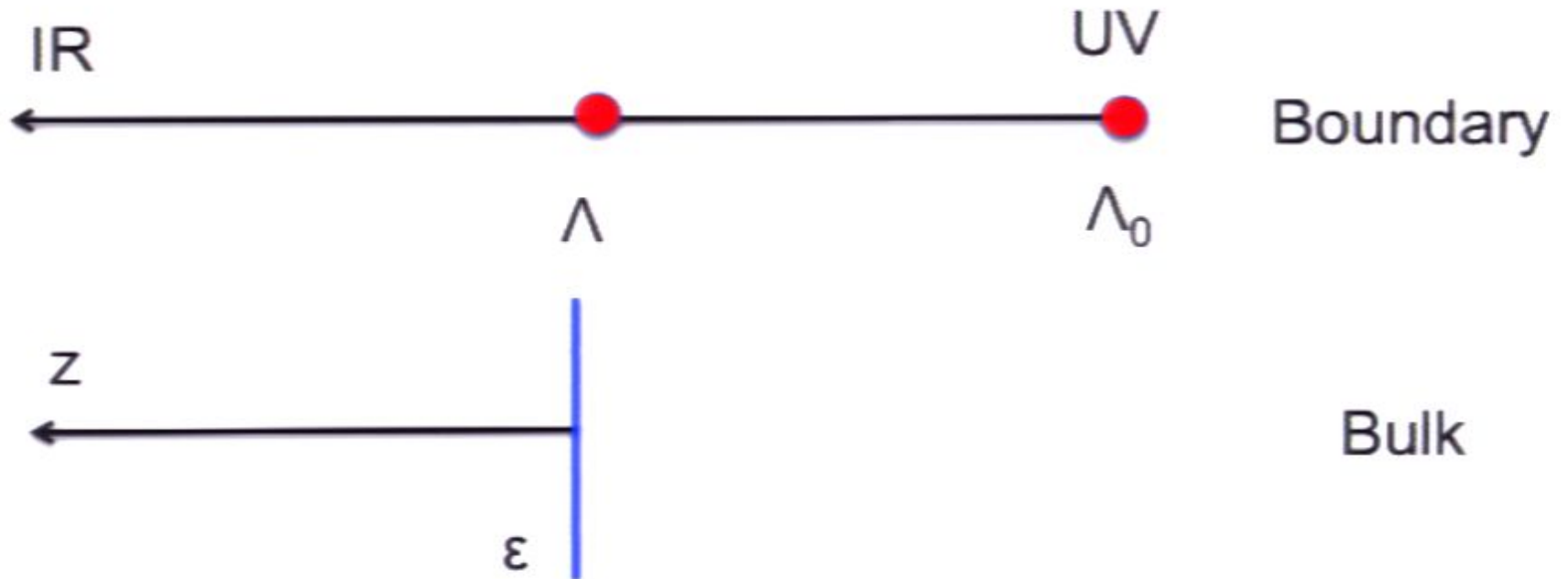


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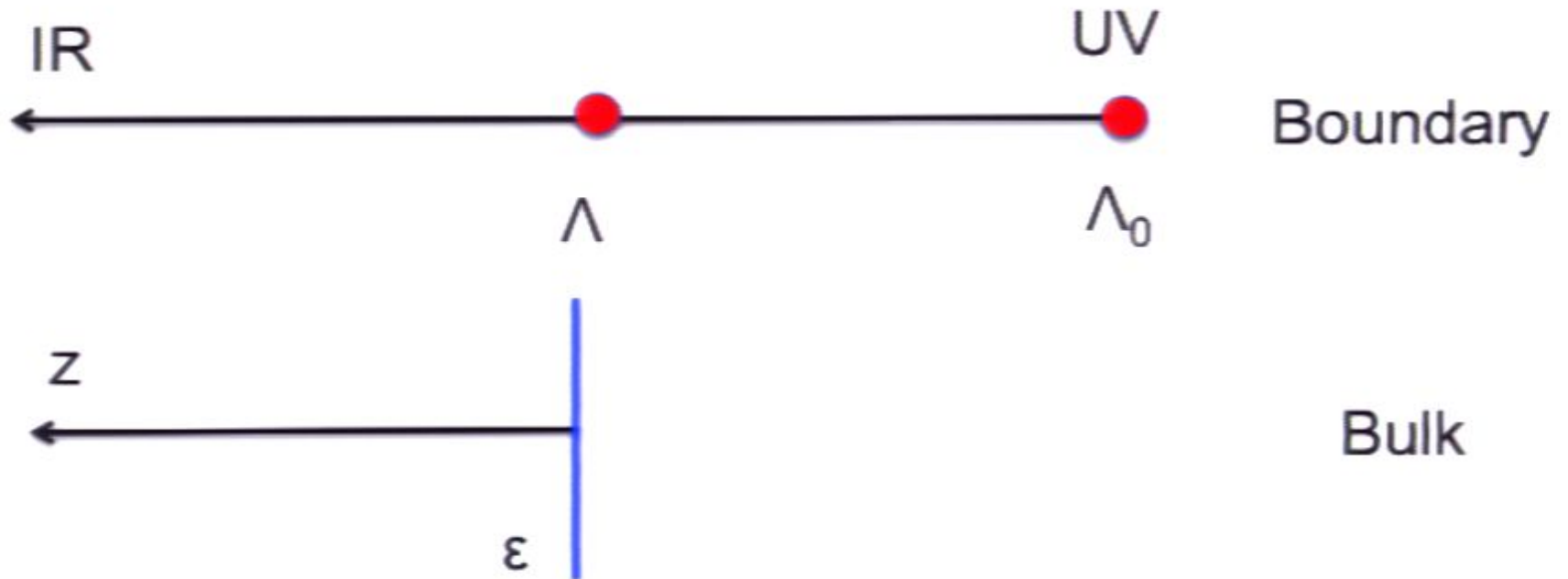
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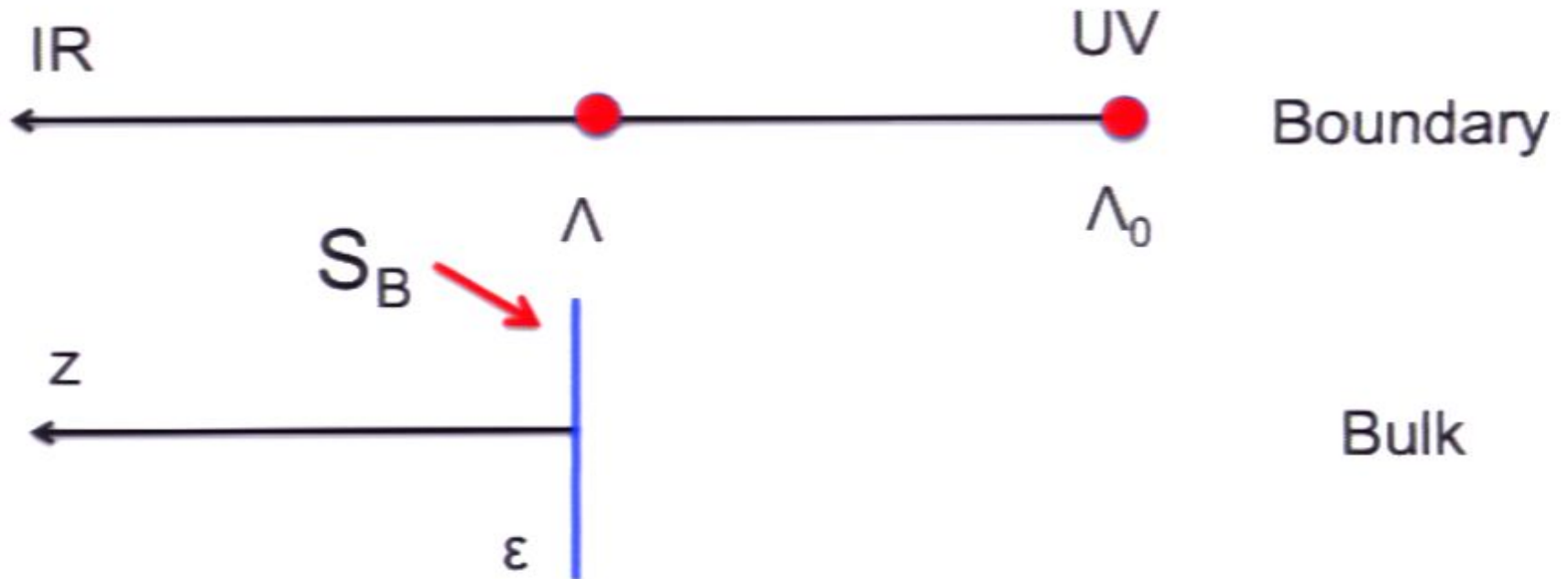
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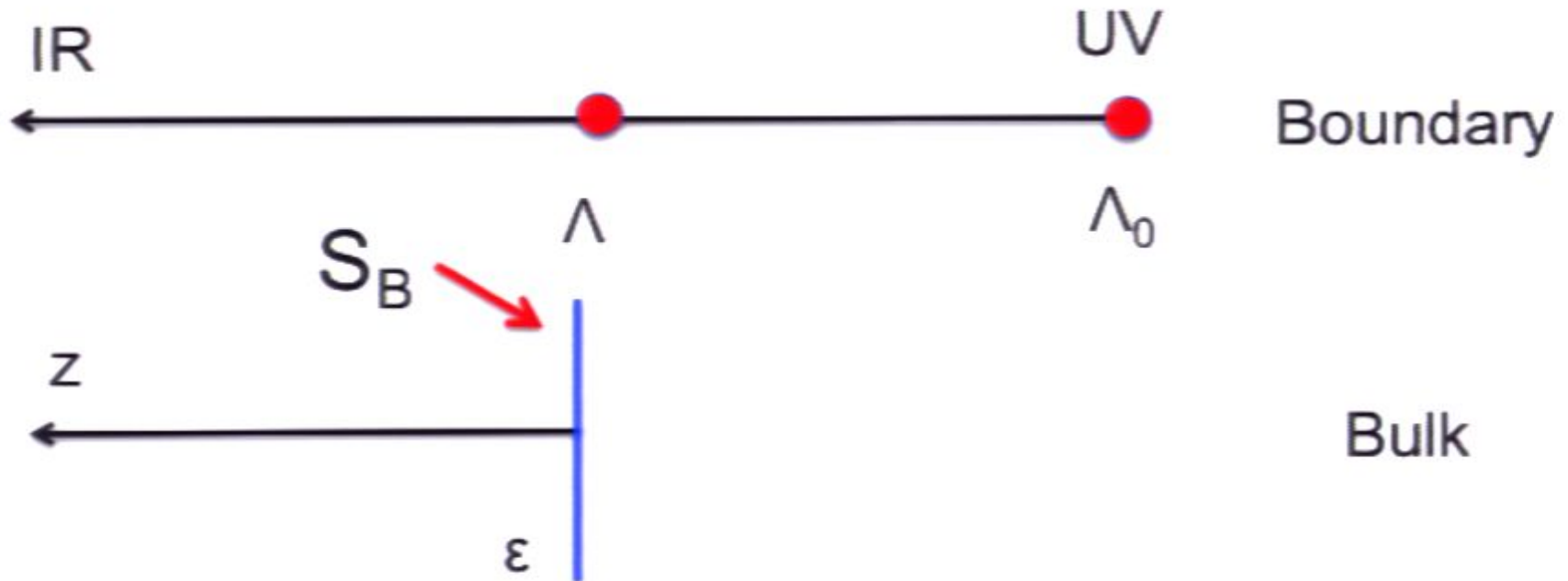
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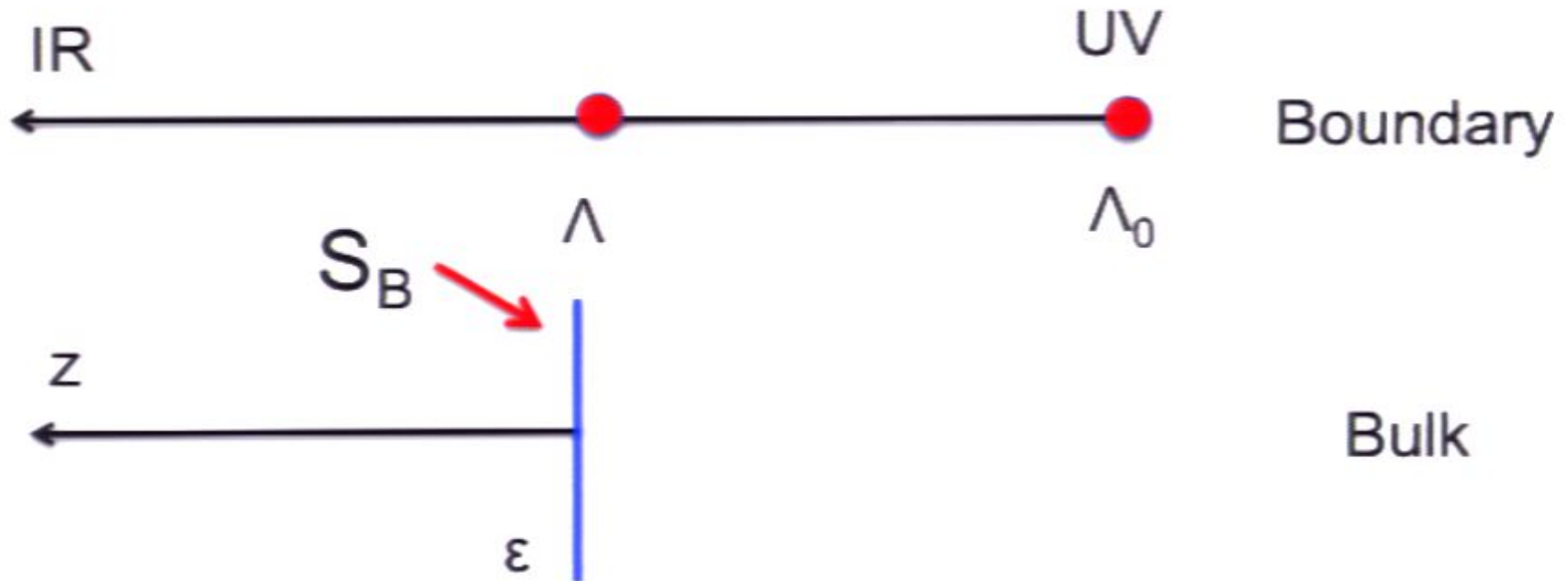


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Consistent with field theory expectations.

Miao Li (2000)

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Semi-classical limit: **Hamilton-Jacobi equation**

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Physics should not depend on where we choose  $z=\epsilon$  surface



Flow equation for  $S_B[\epsilon]$

Semi-classical limit: **Hamilton-Jacobi equation**

$$\partial_\epsilon S_B[\phi, \epsilon] = - \int_{z=\epsilon} d^d x H \left( \phi, \Pi = \frac{\delta S_B}{\delta \phi} \right)$$

**H: bulk Hamiltonian corresponding to z-foliation.**

The above equation should be treated as a **functional equation**:

$$S_B[\phi, \epsilon] = \int \sum_i g_i(\epsilon) \phi^n \quad \longrightarrow \quad \partial_\epsilon g_i = \beta_i(\{g\})$$

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$$S_B[\phi, \epsilon] = \int \sum_i g_i(\epsilon) \phi^n \quad \longrightarrow \quad \partial_\epsilon g_i = \beta_i(\{g\})$$

# Flow equation for $S_B$

Physics should not depend on where we choose  $z=\epsilon$  surface



Flow equation for  $S_B[\epsilon]$

Semi-classical limit: **Hamilton-Jacobi equation**

$$\partial_\epsilon S_B[\phi, \epsilon] = - \int_{z=\epsilon} d^d x H \left( \phi, \Pi = \frac{\delta S_B}{\delta \phi} \right)$$

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$$S_B[\epsilon, \phi] = \Lambda(\epsilon) + \int d^d x \sqrt{-\gamma} J(x, \epsilon) \phi(x) \\ - \frac{1}{2} \int d^d x \sqrt{-\gamma} f(\epsilon) \phi^2(x) + \dots$$



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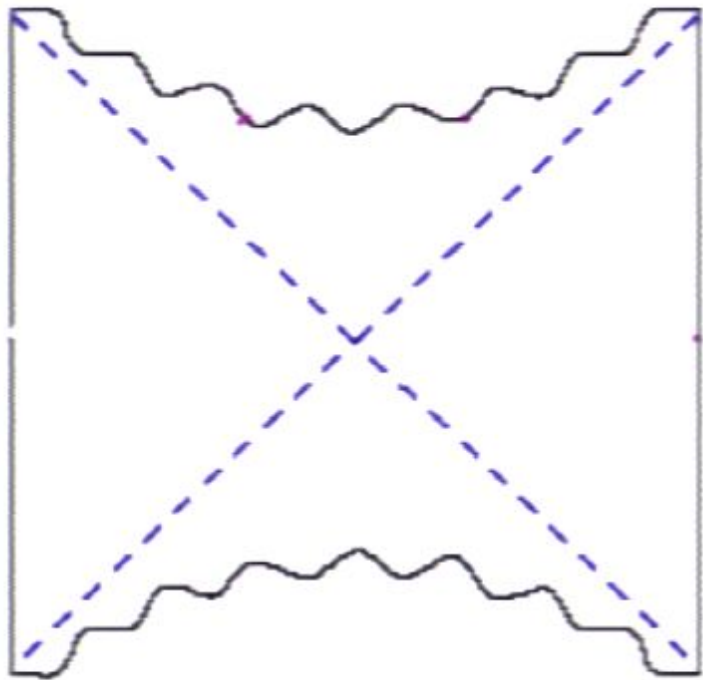
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Both  $\log Z_{\text{bulk}}[\epsilon_0]$  and  $S_B$  **satisfy Hamilton-Jacobi equation,**  
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# Schwarzschild black hole in AdS

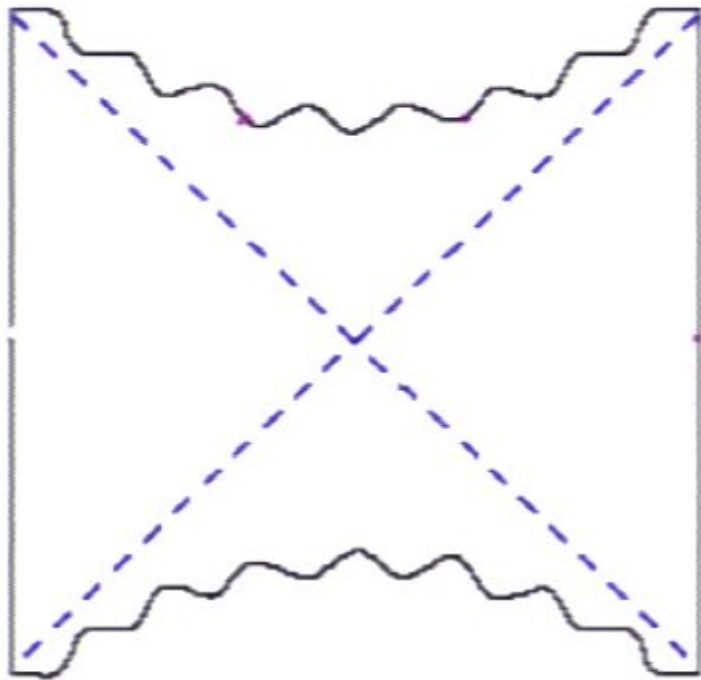
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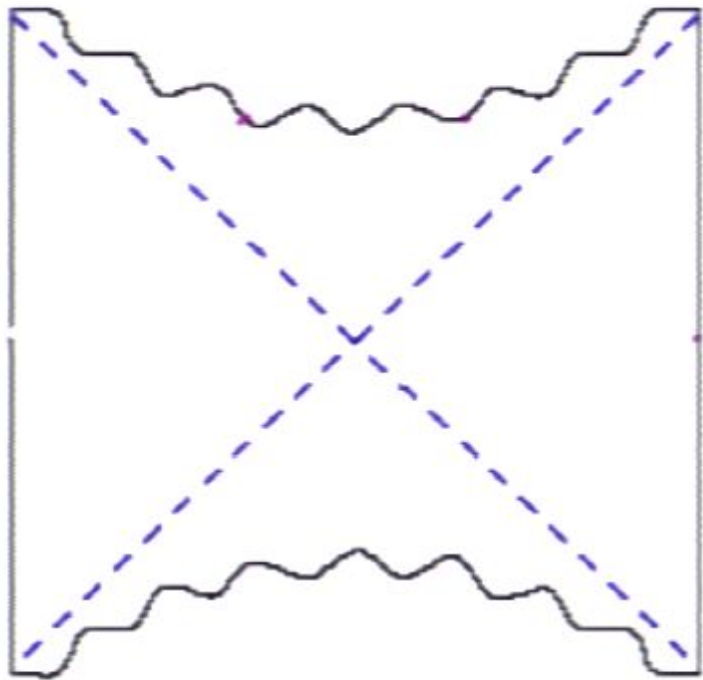


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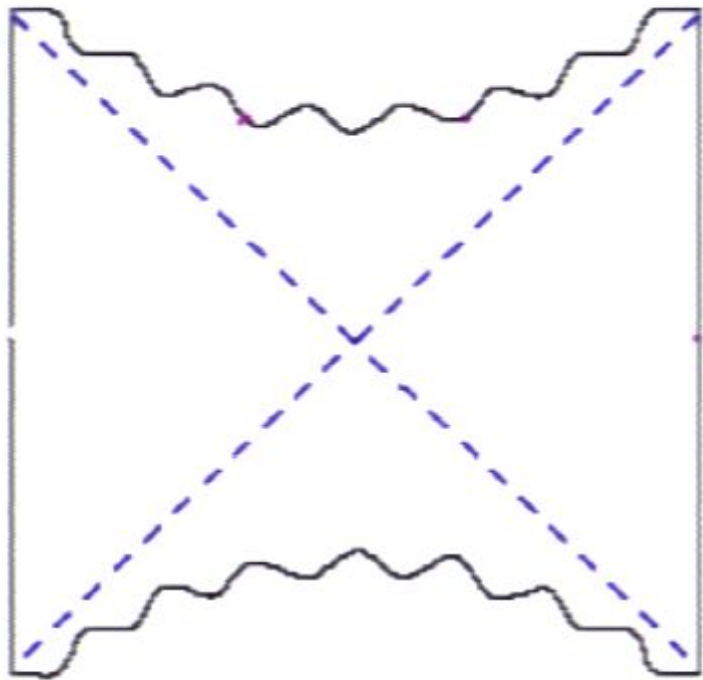
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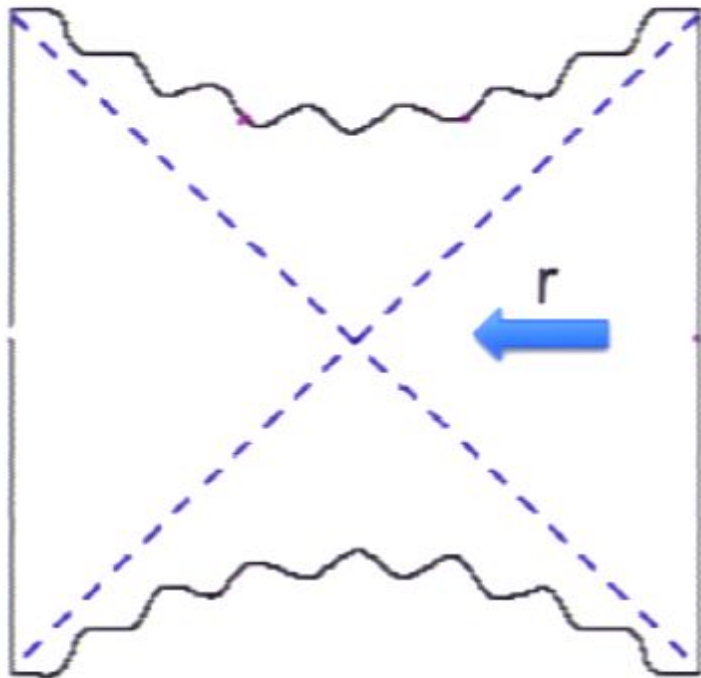
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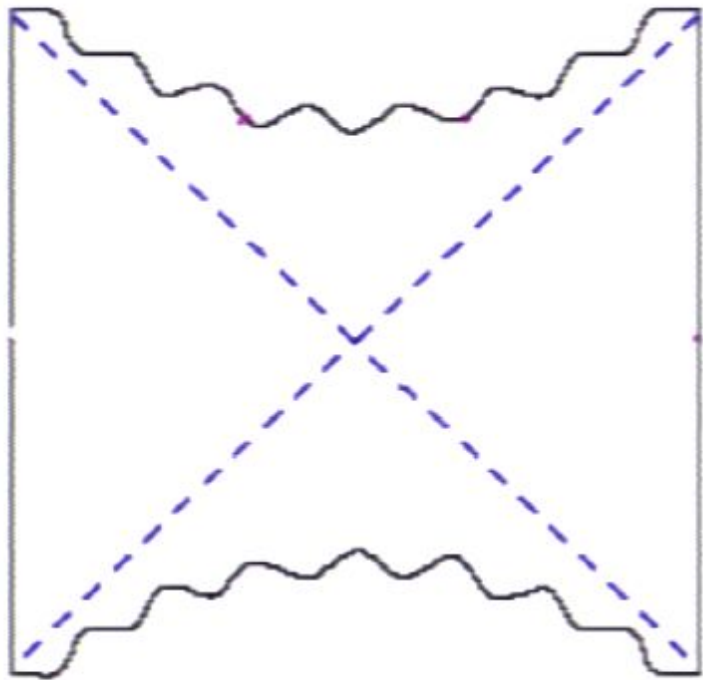
Boundary:  $f(r \rightarrow \infty) = +\infty$

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# Schwarzschild black hole in AdS



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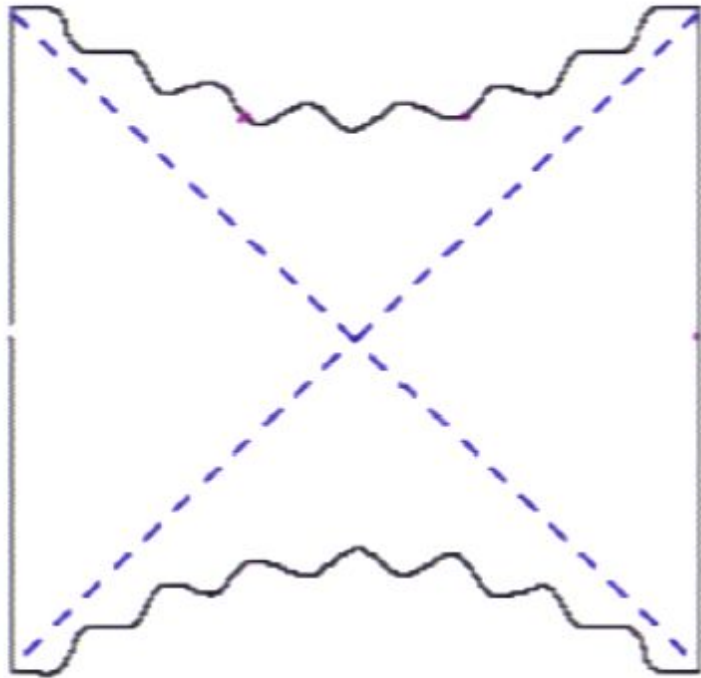
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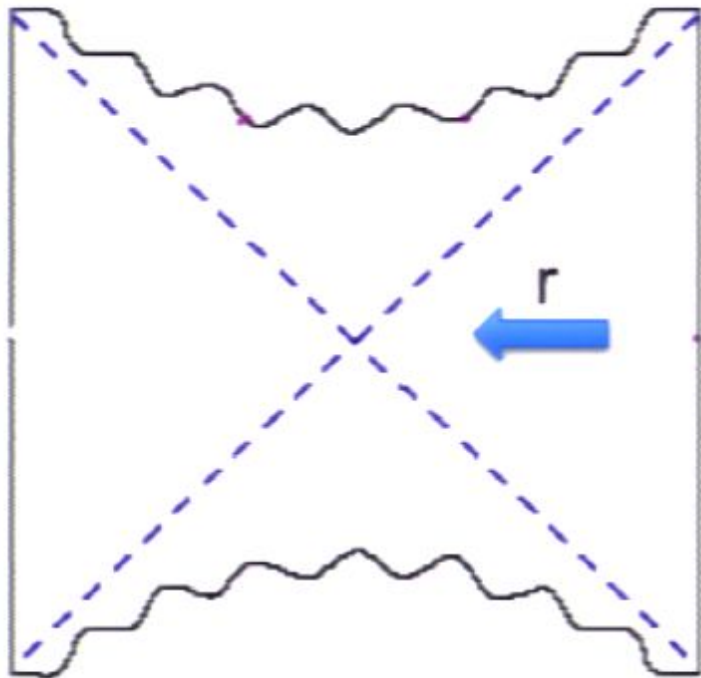
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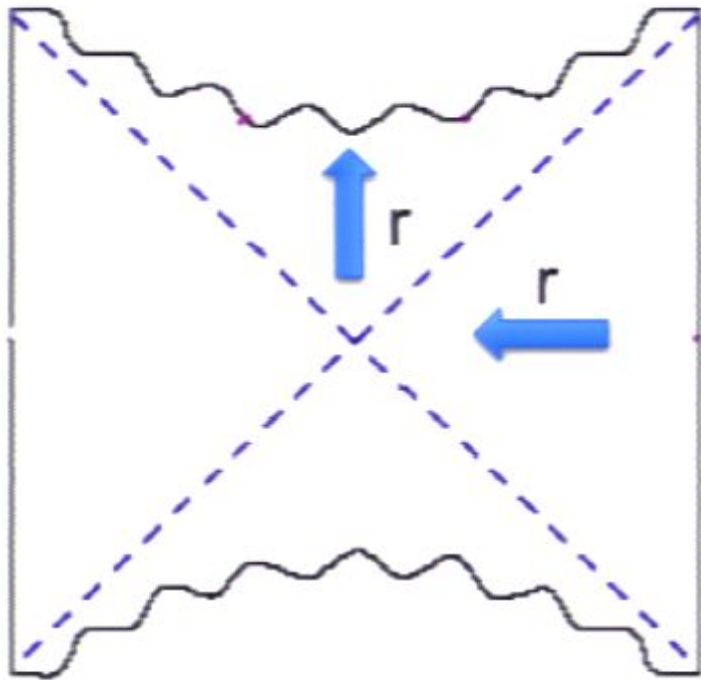
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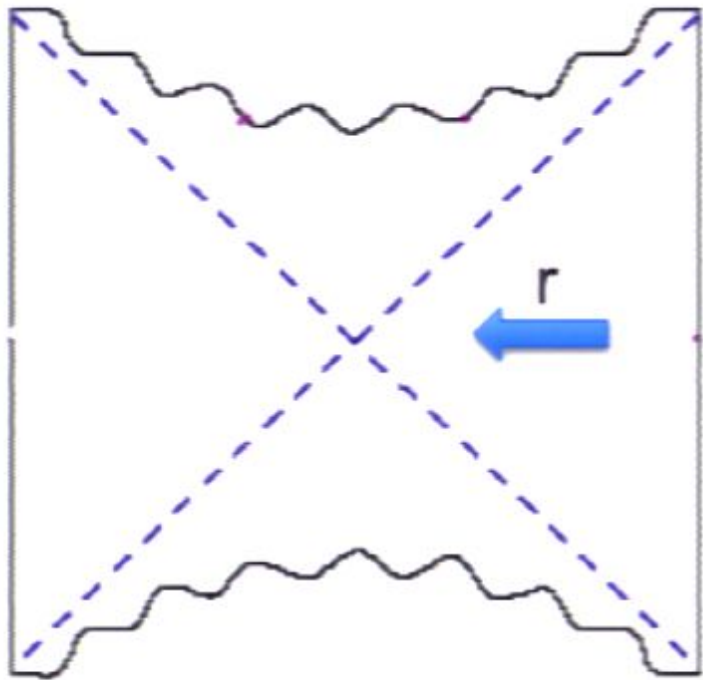
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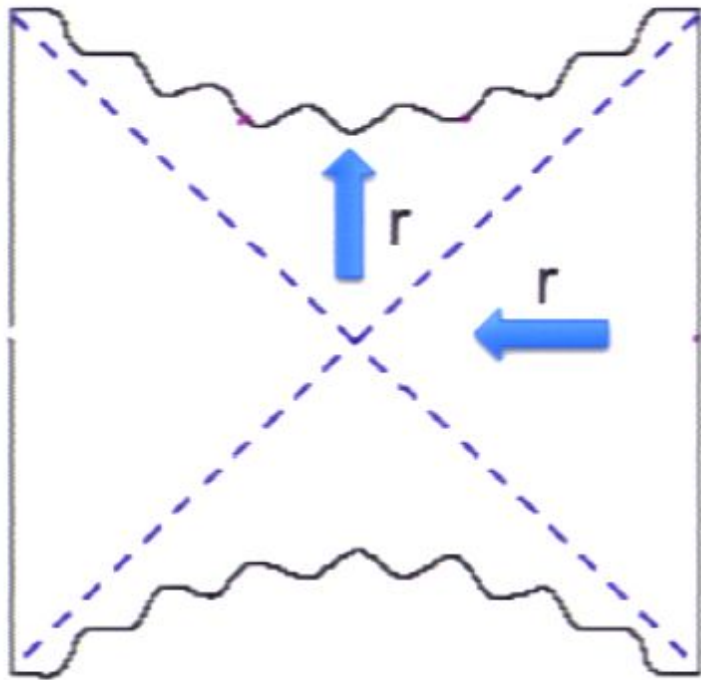
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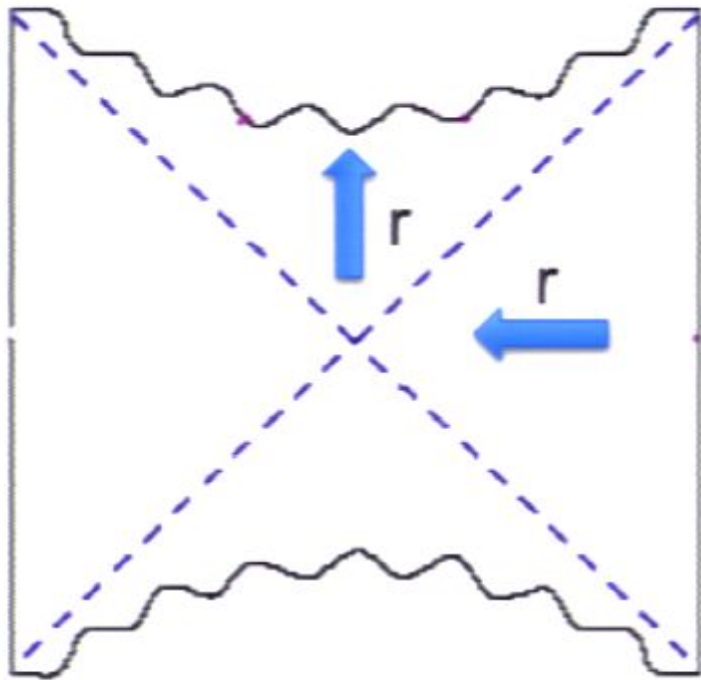
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- Each geodesic is characterized by a **turning point.**

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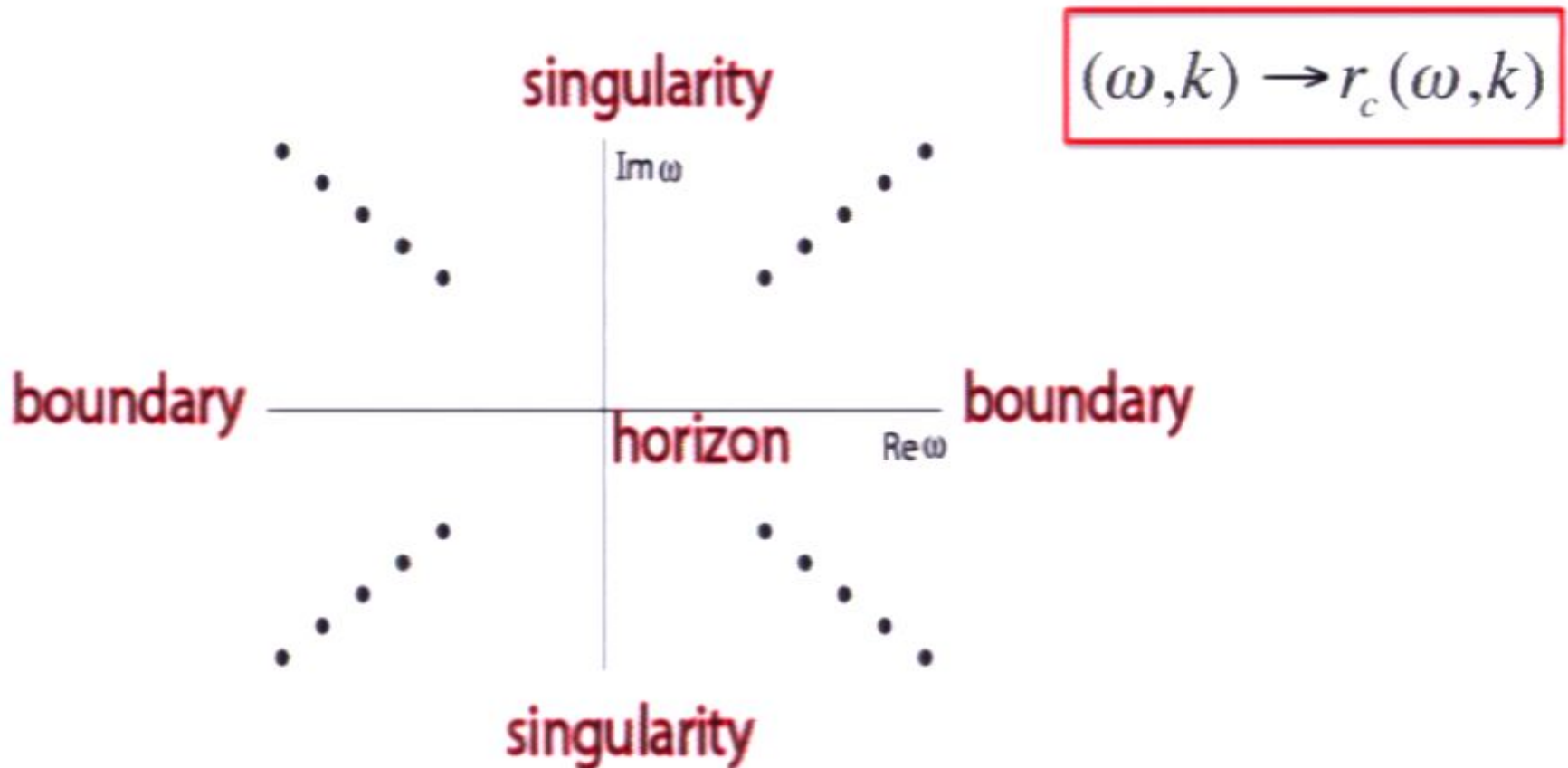
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# Frequencies versus turning points

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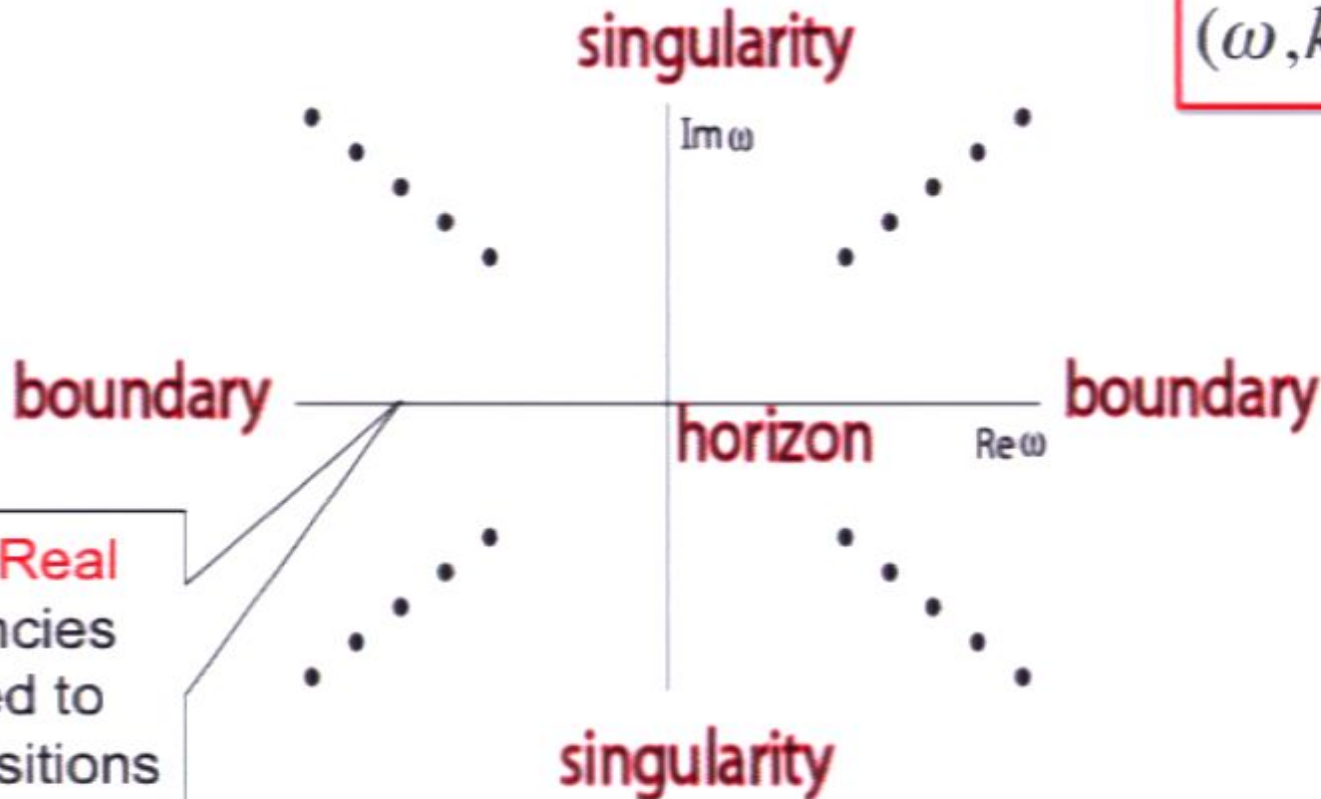


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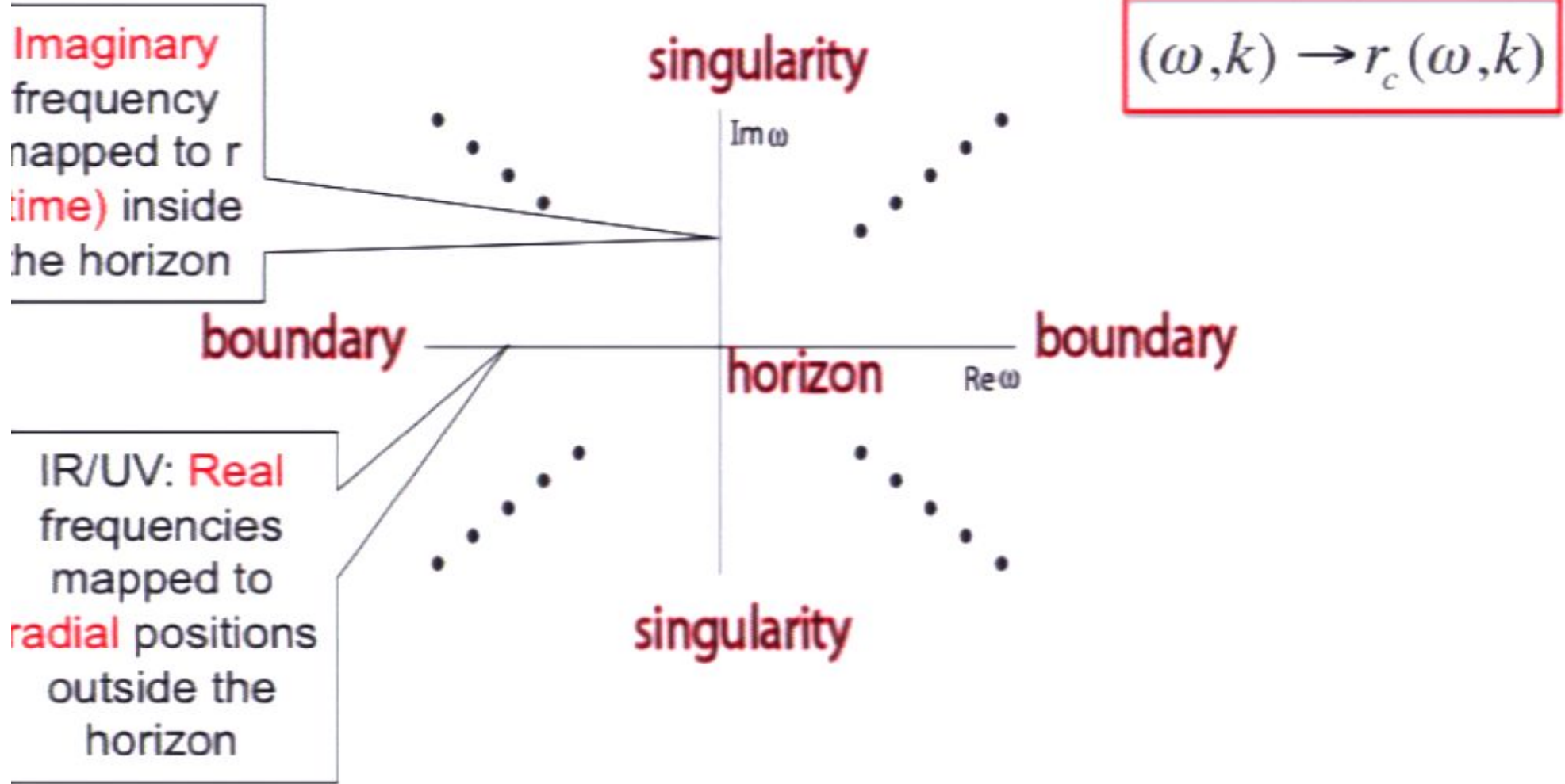
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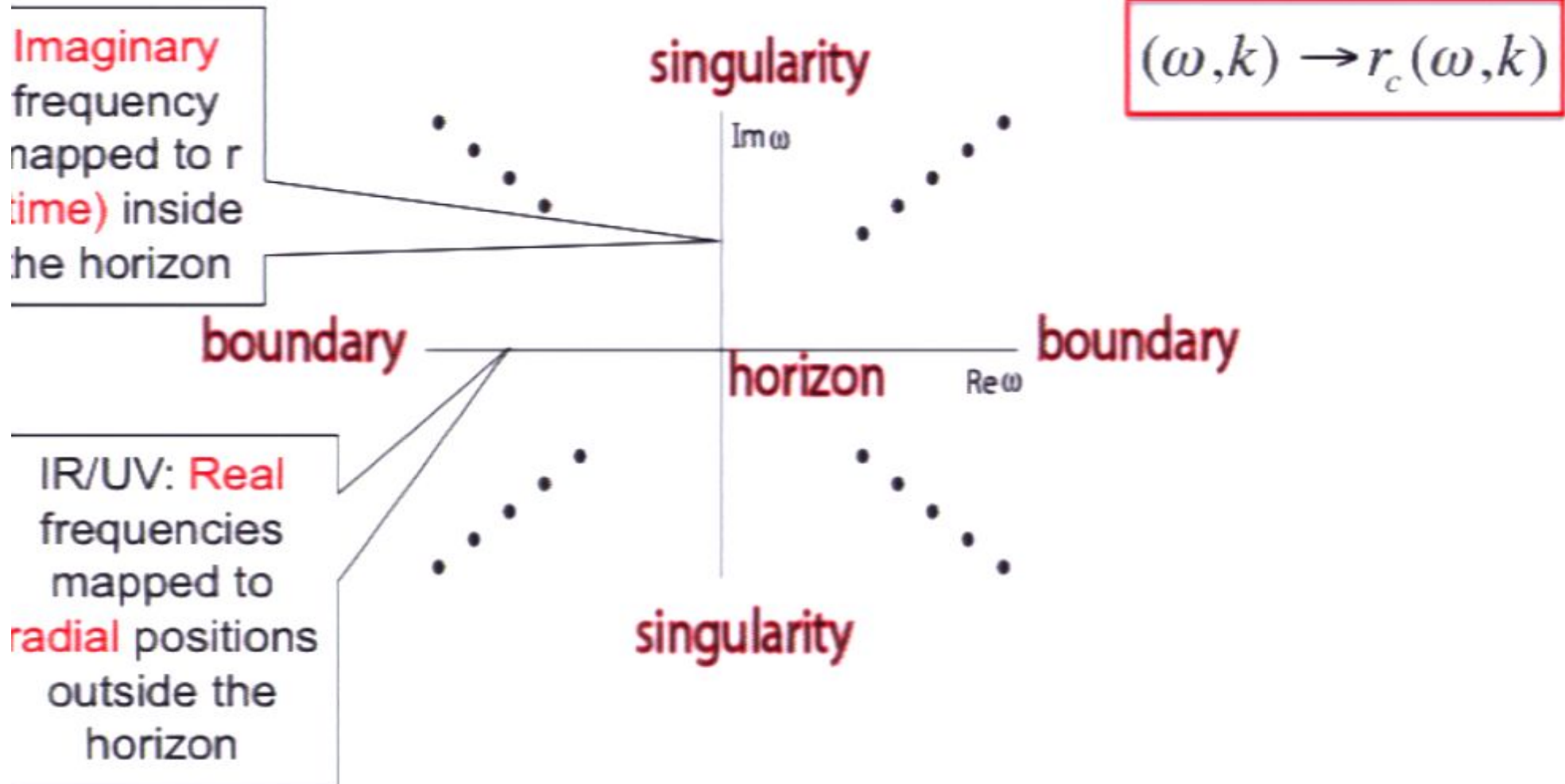


IR/UV: **Real** frequencies mapped to **radial** positions outside the horizon

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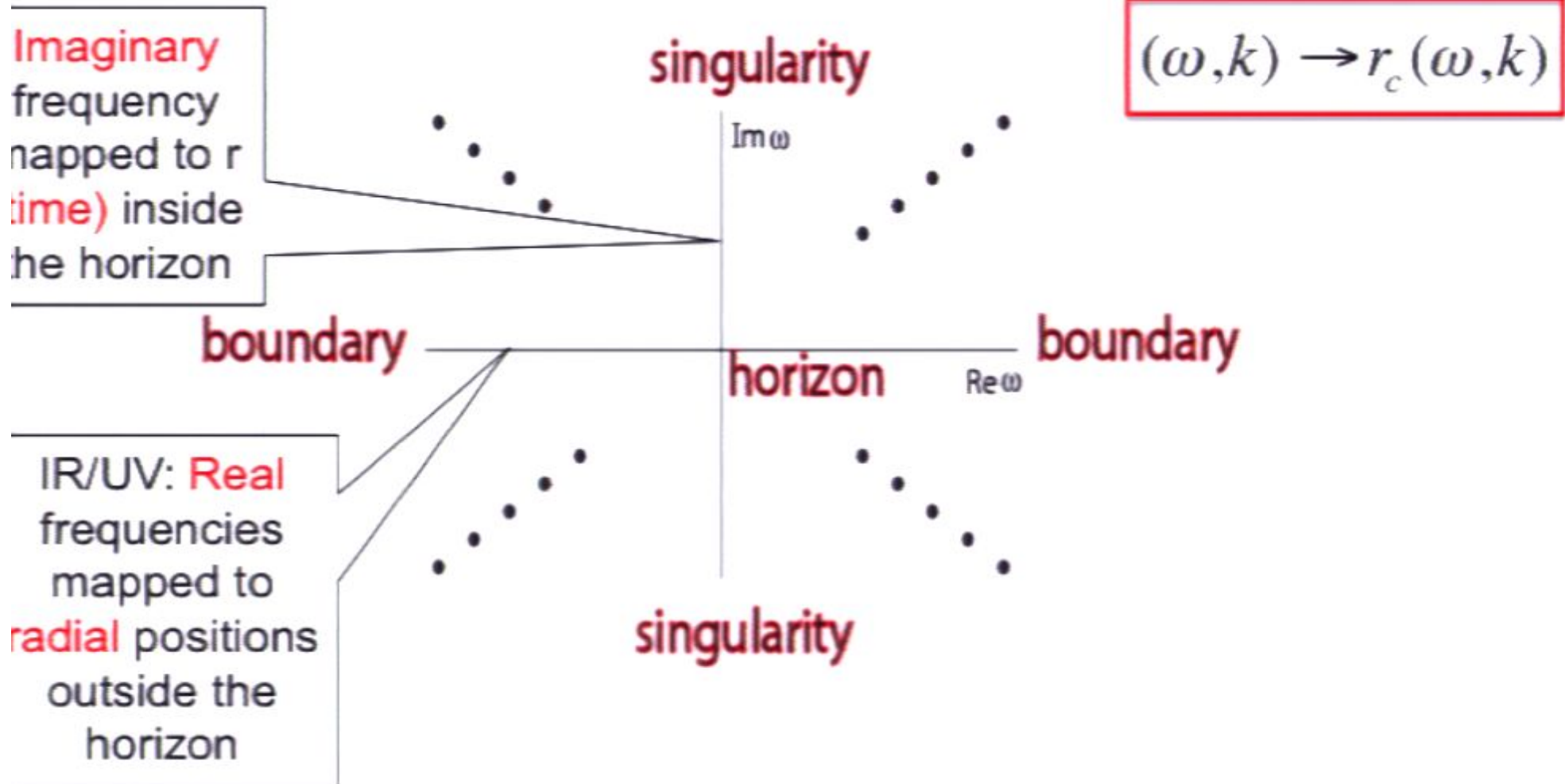
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Consistent with picture obtained from the heuristic “redshift” analysis.

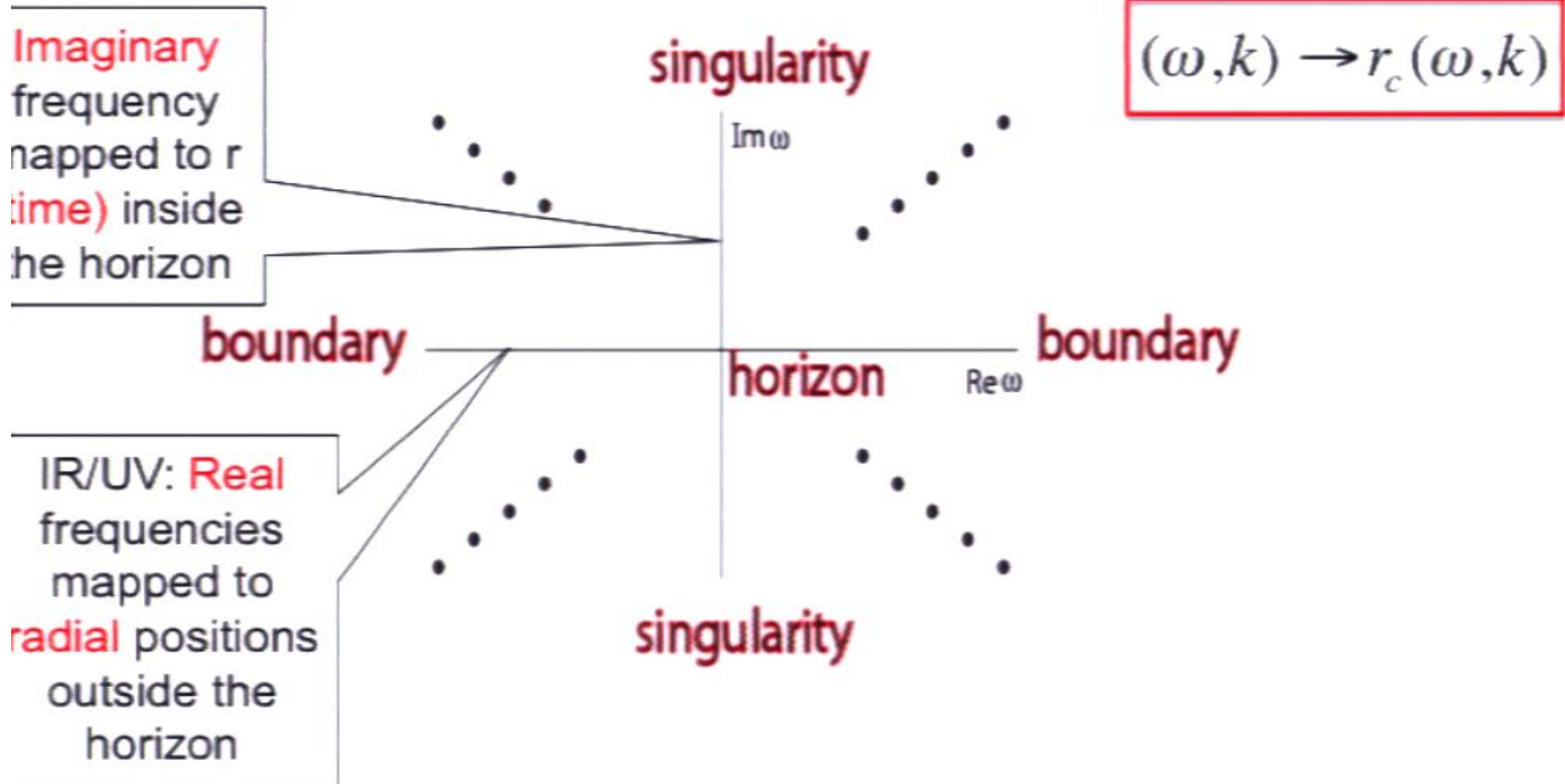


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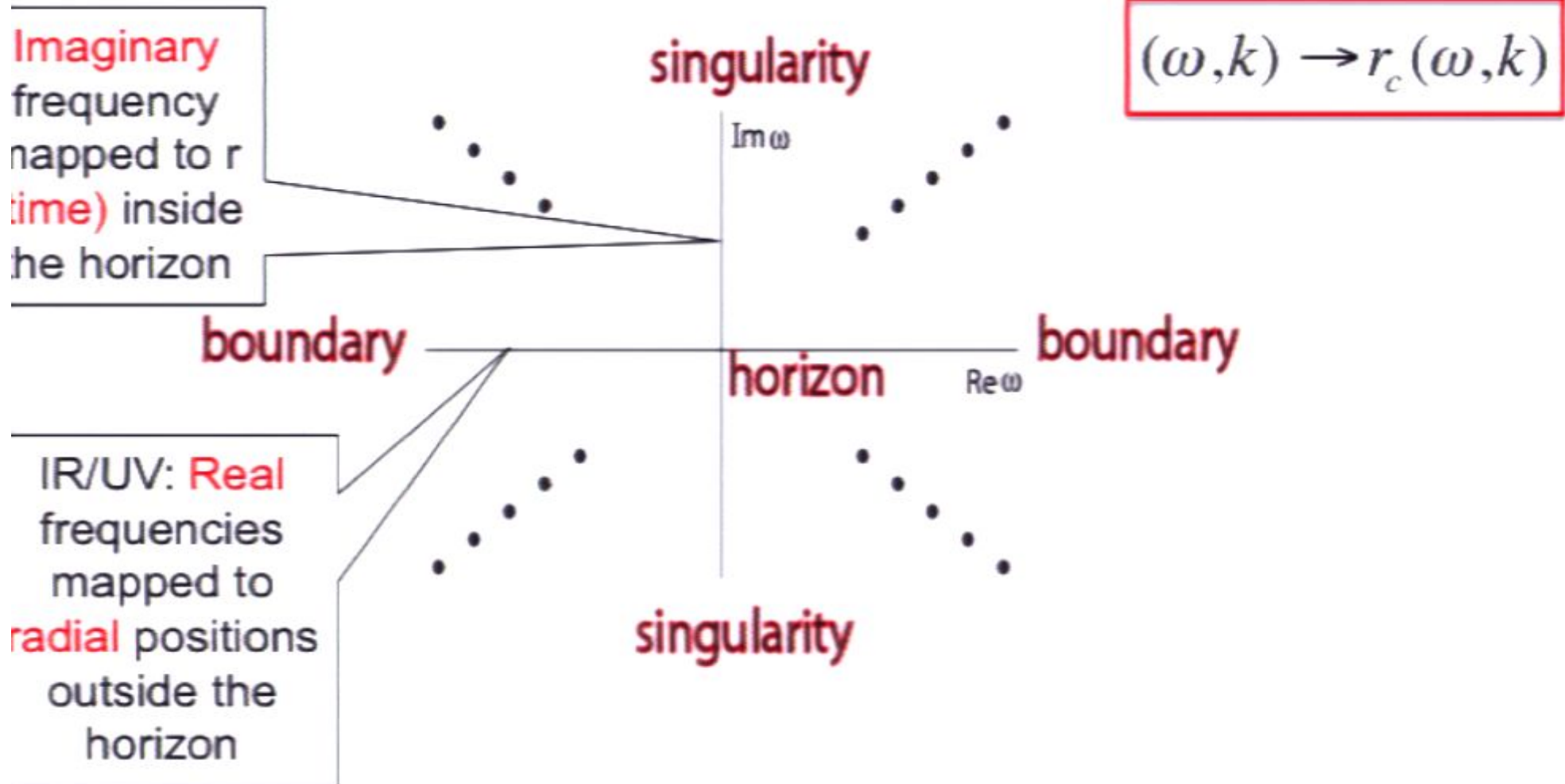
# Inside the horizon: a dual “Euclidean” theory?

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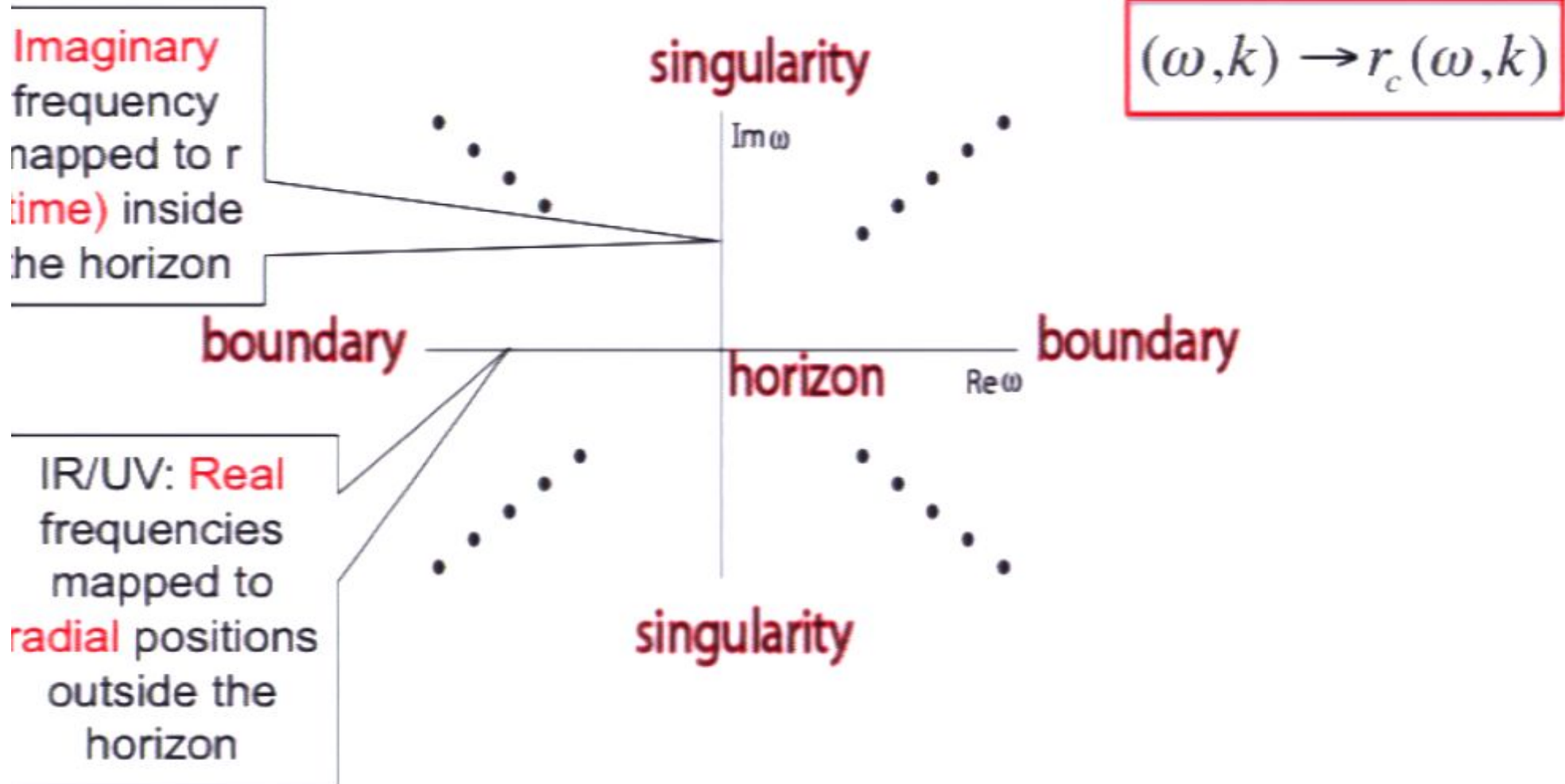


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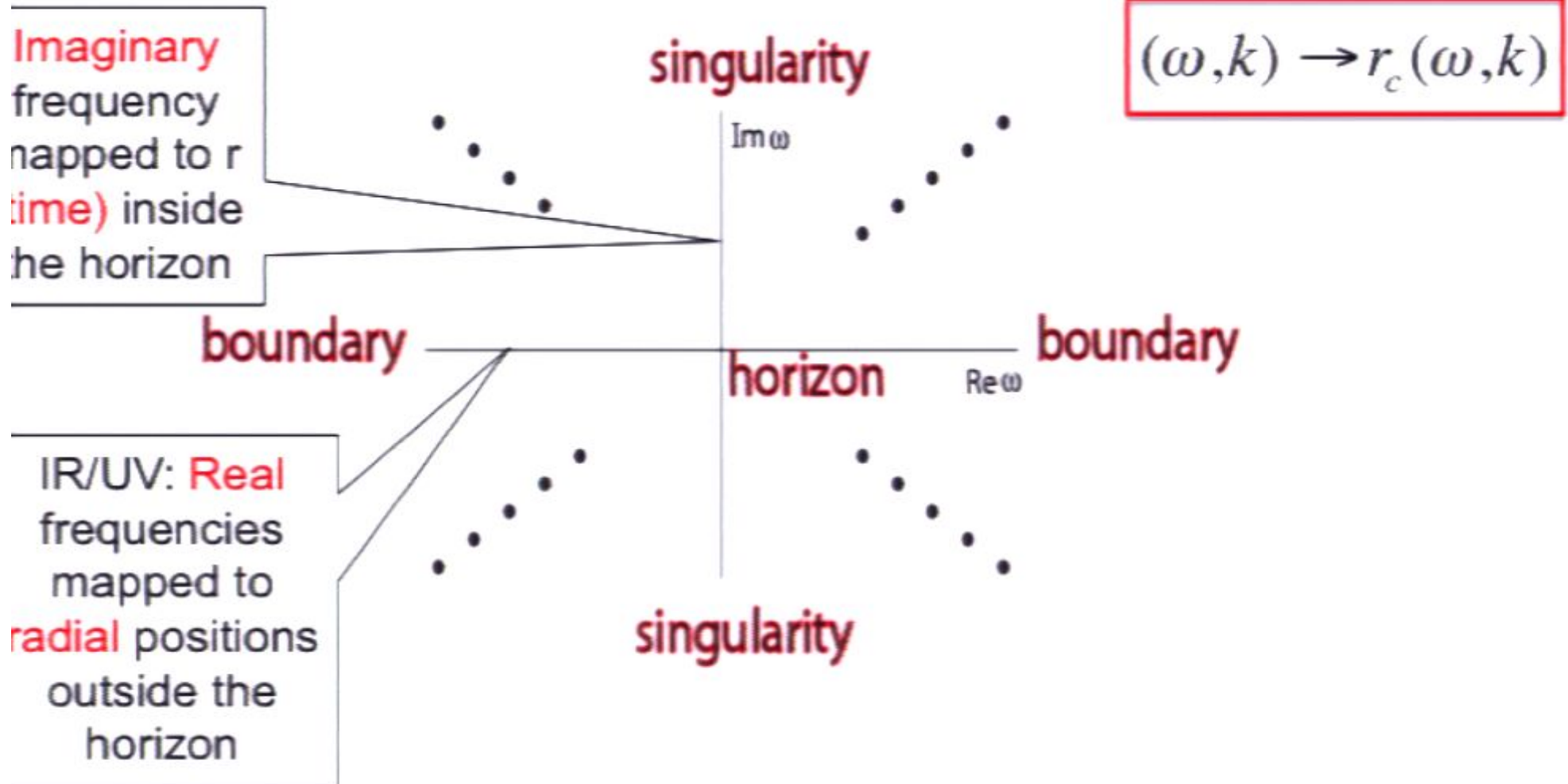
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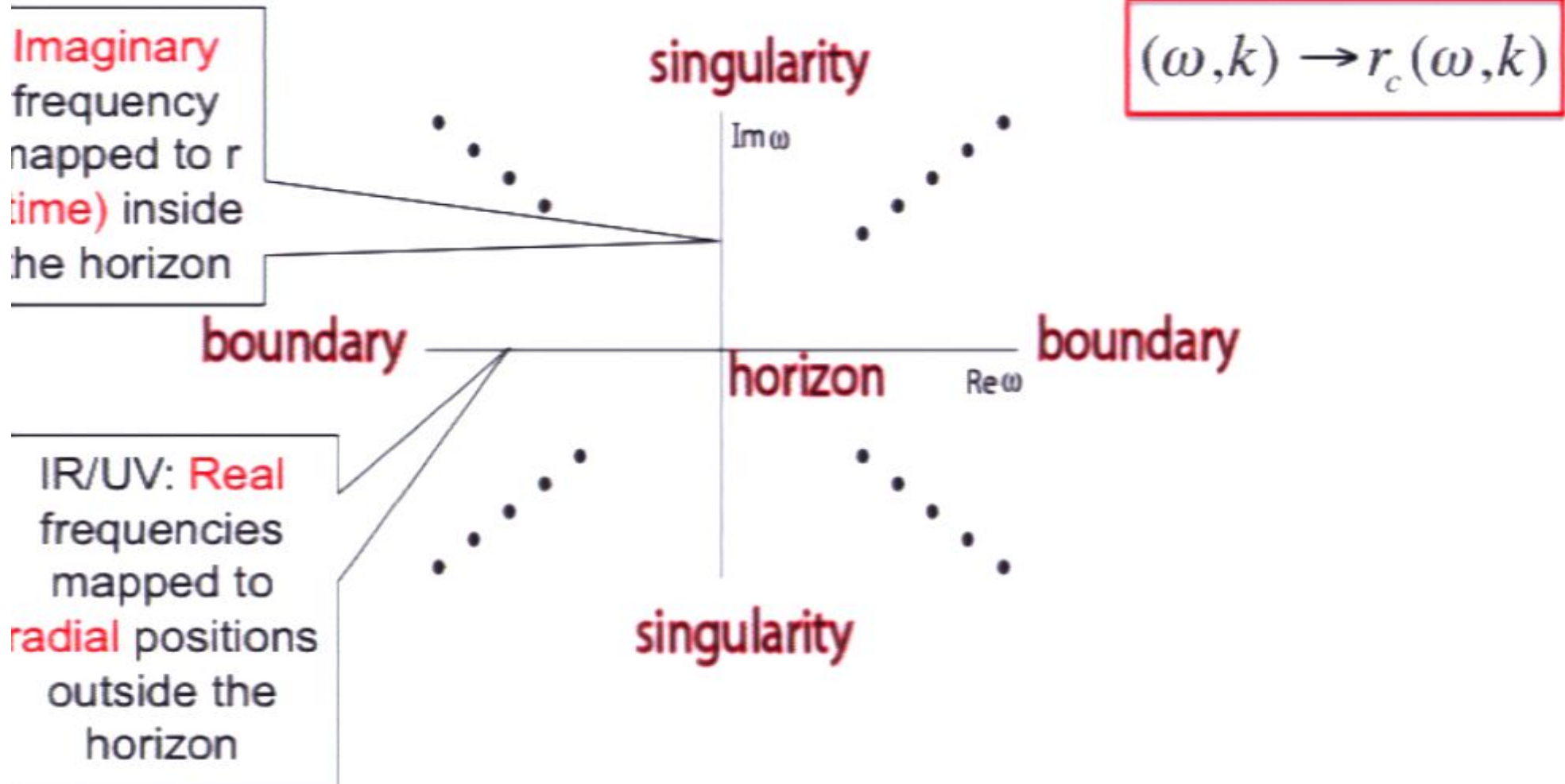
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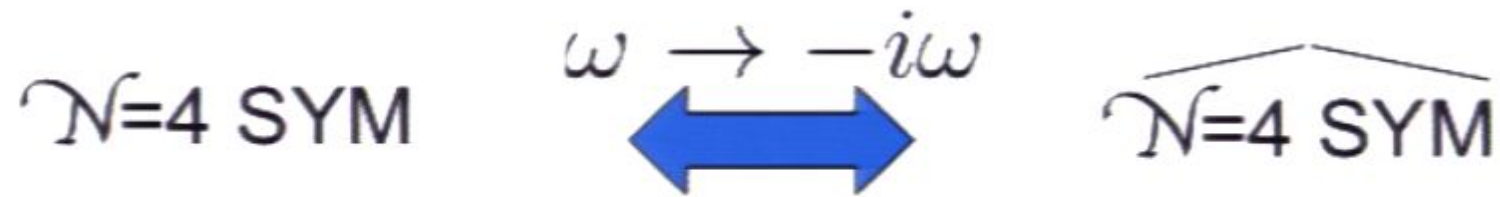
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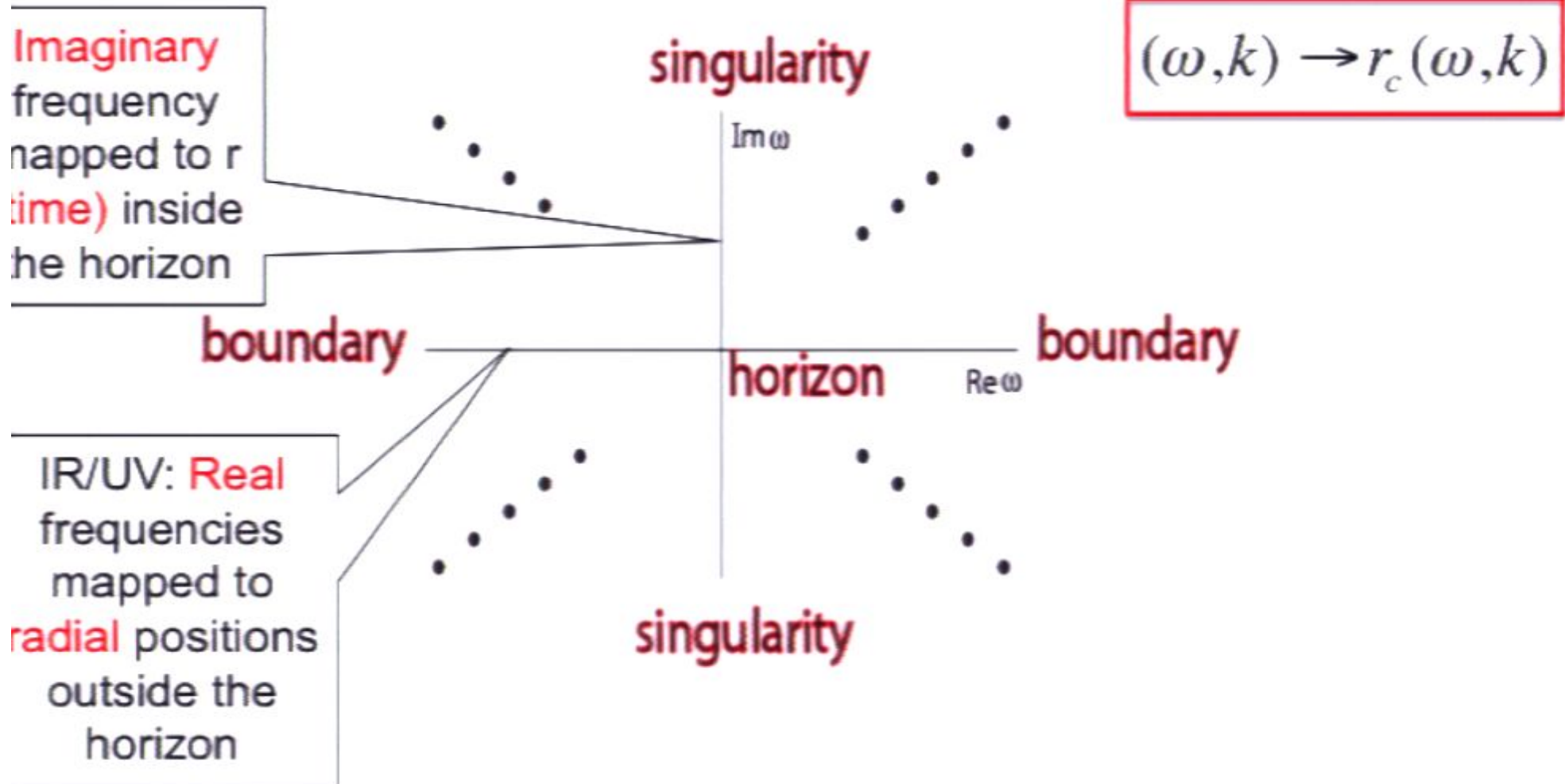
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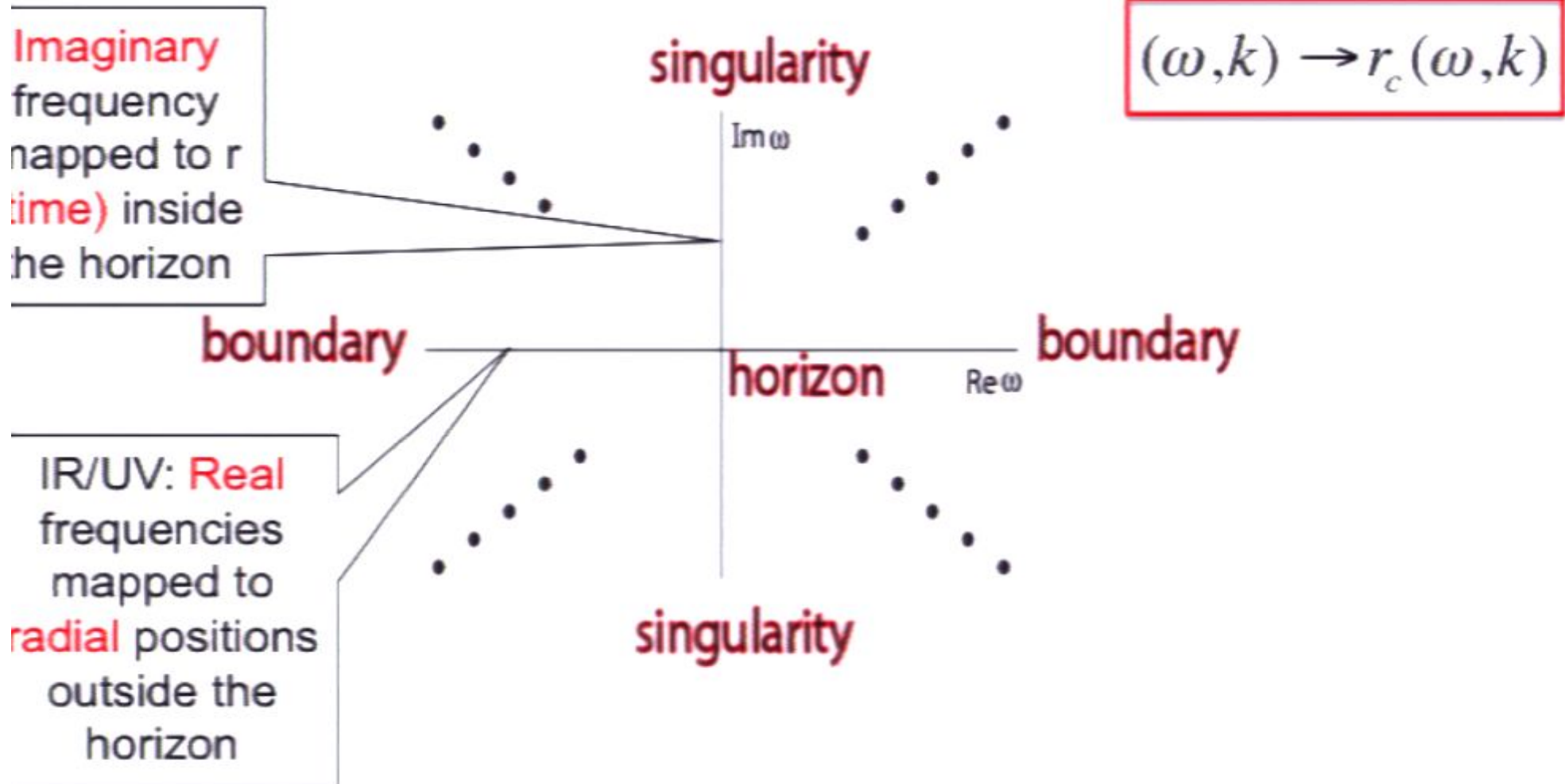
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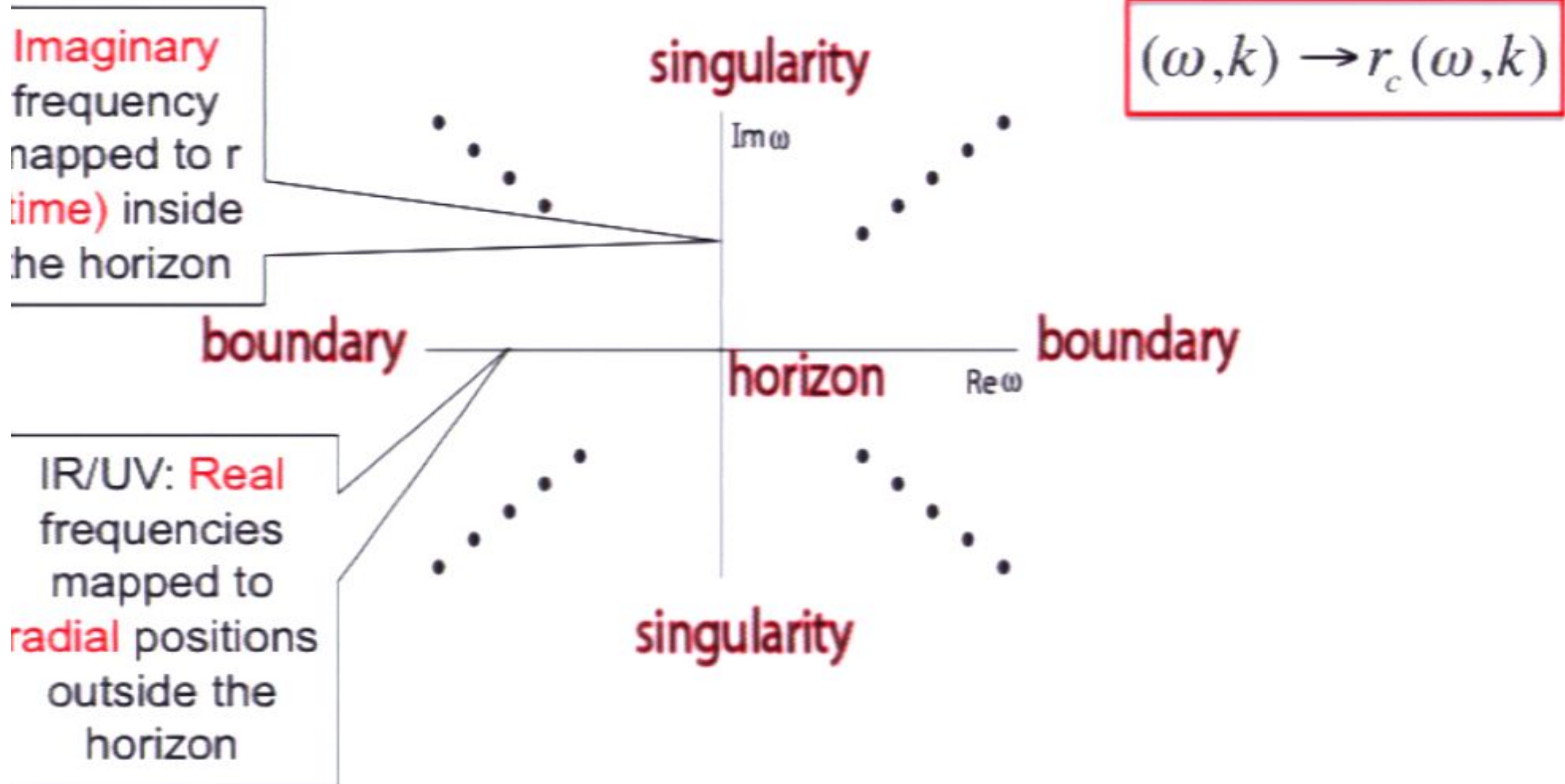


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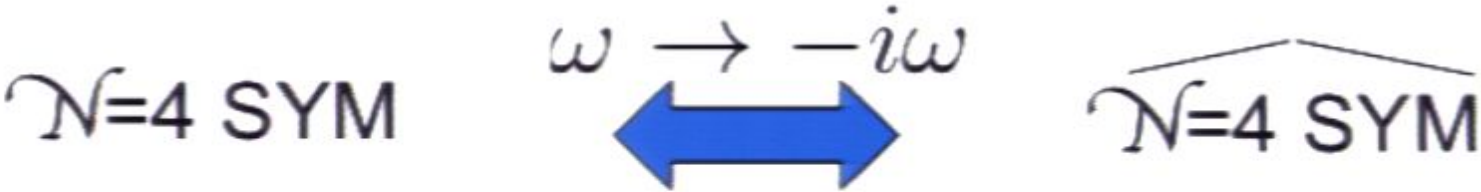


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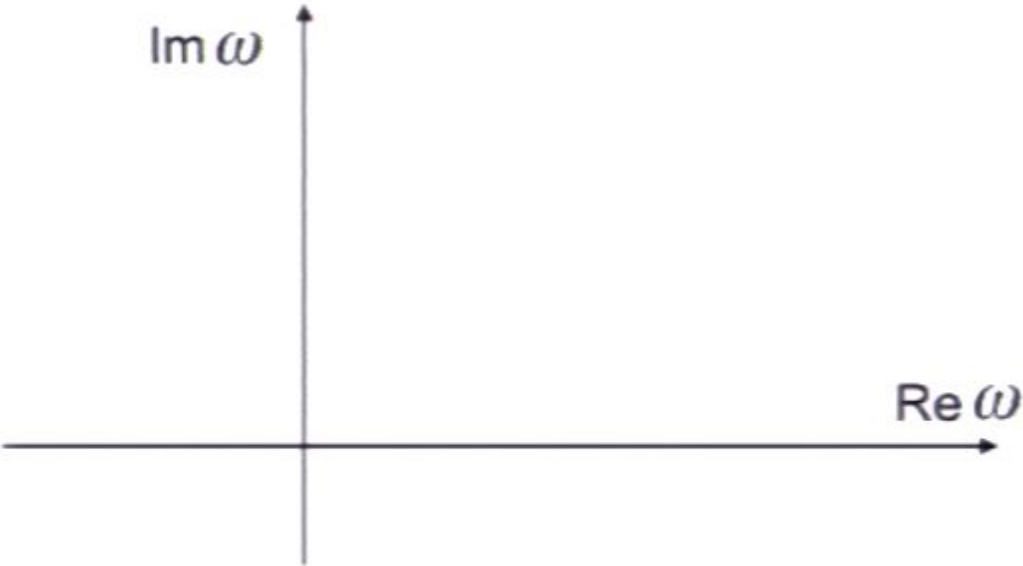
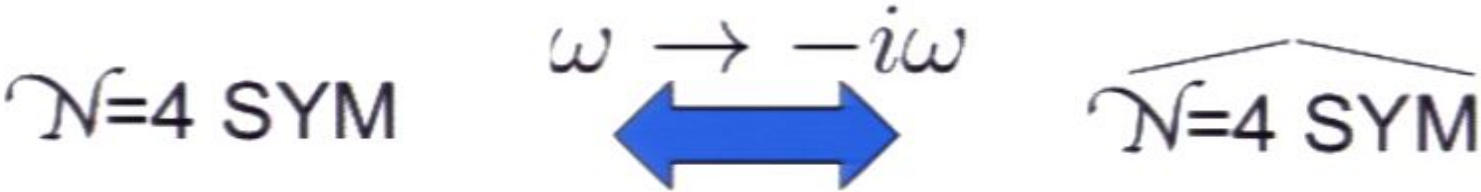
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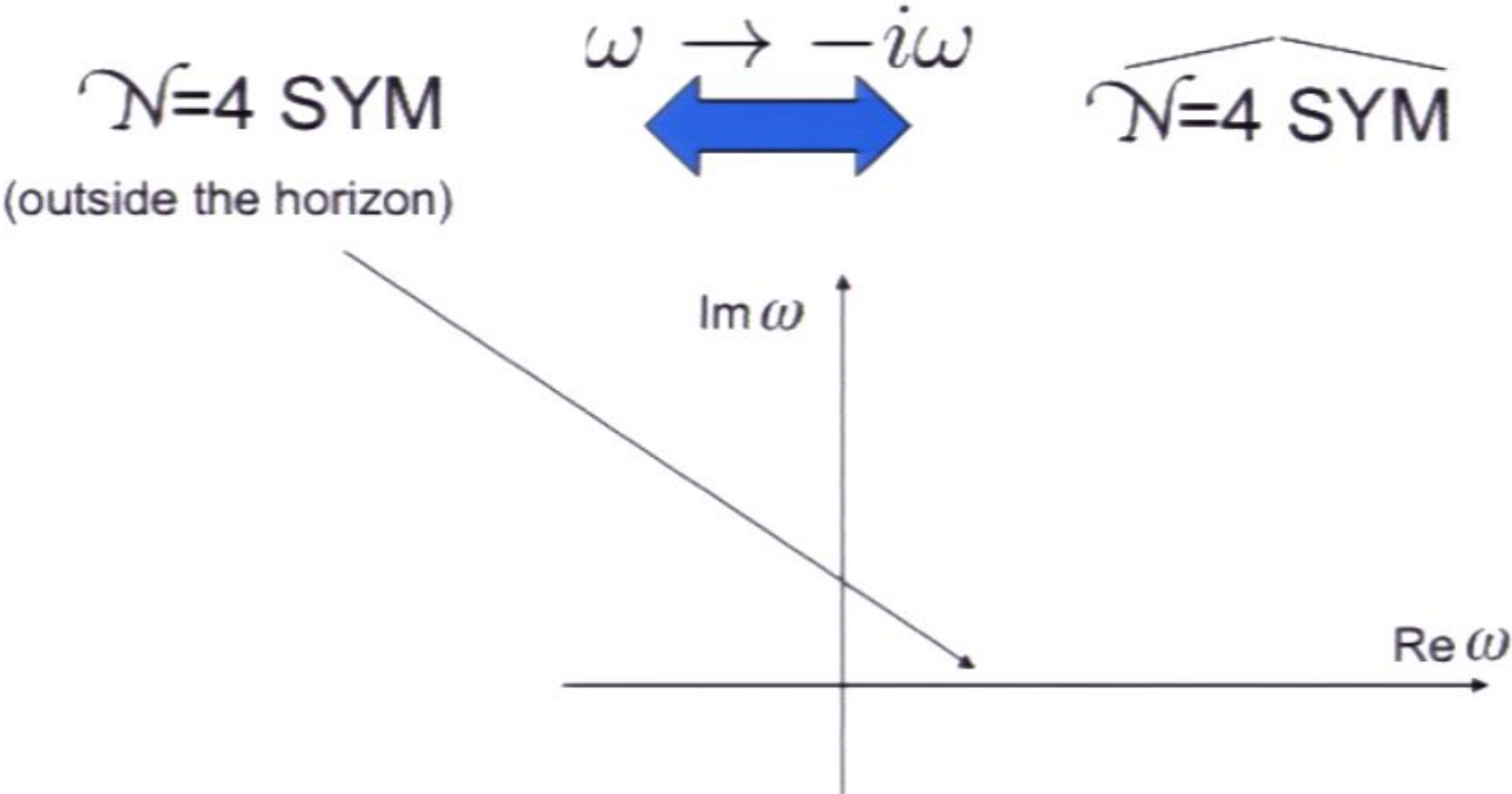




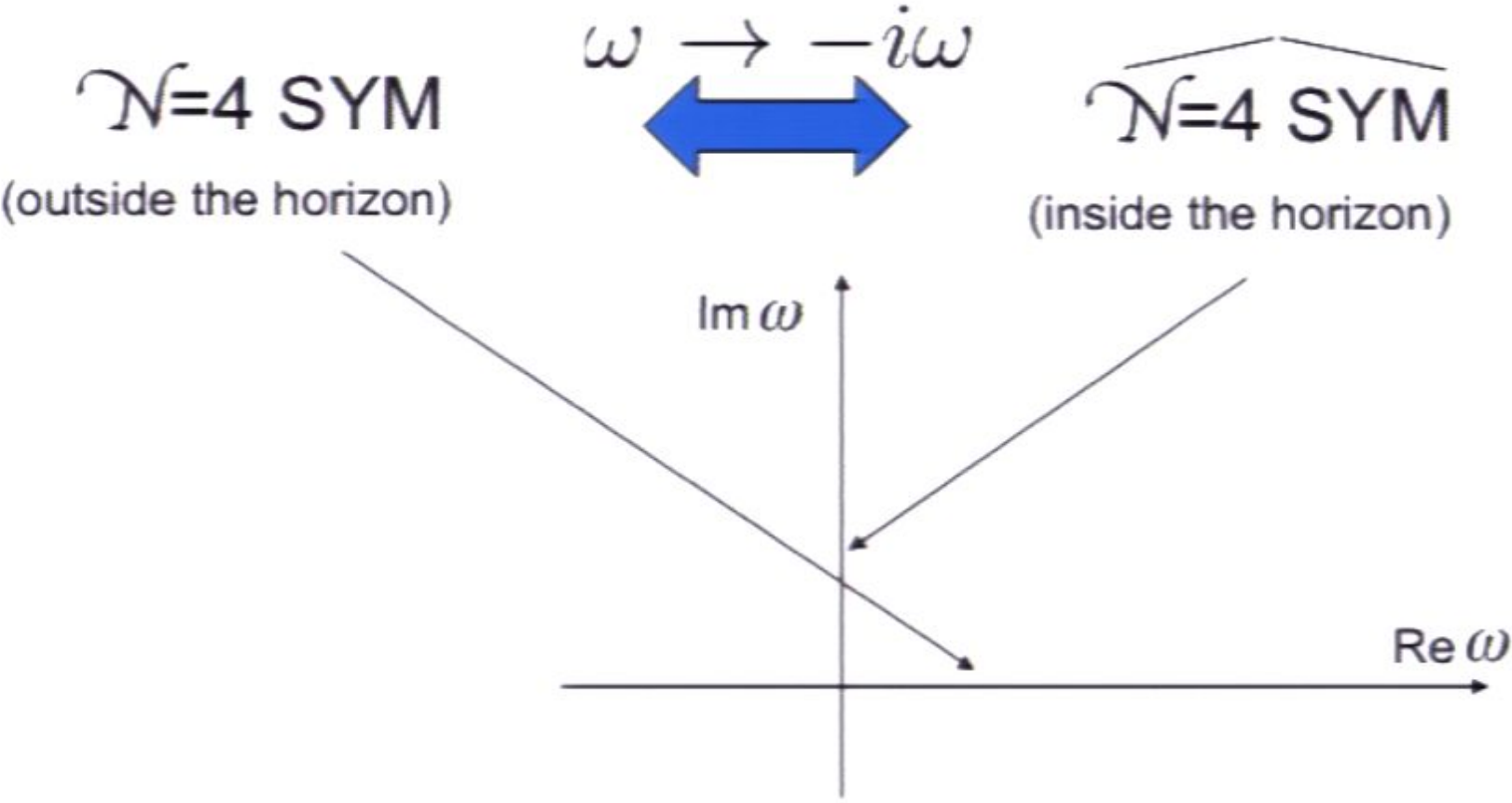
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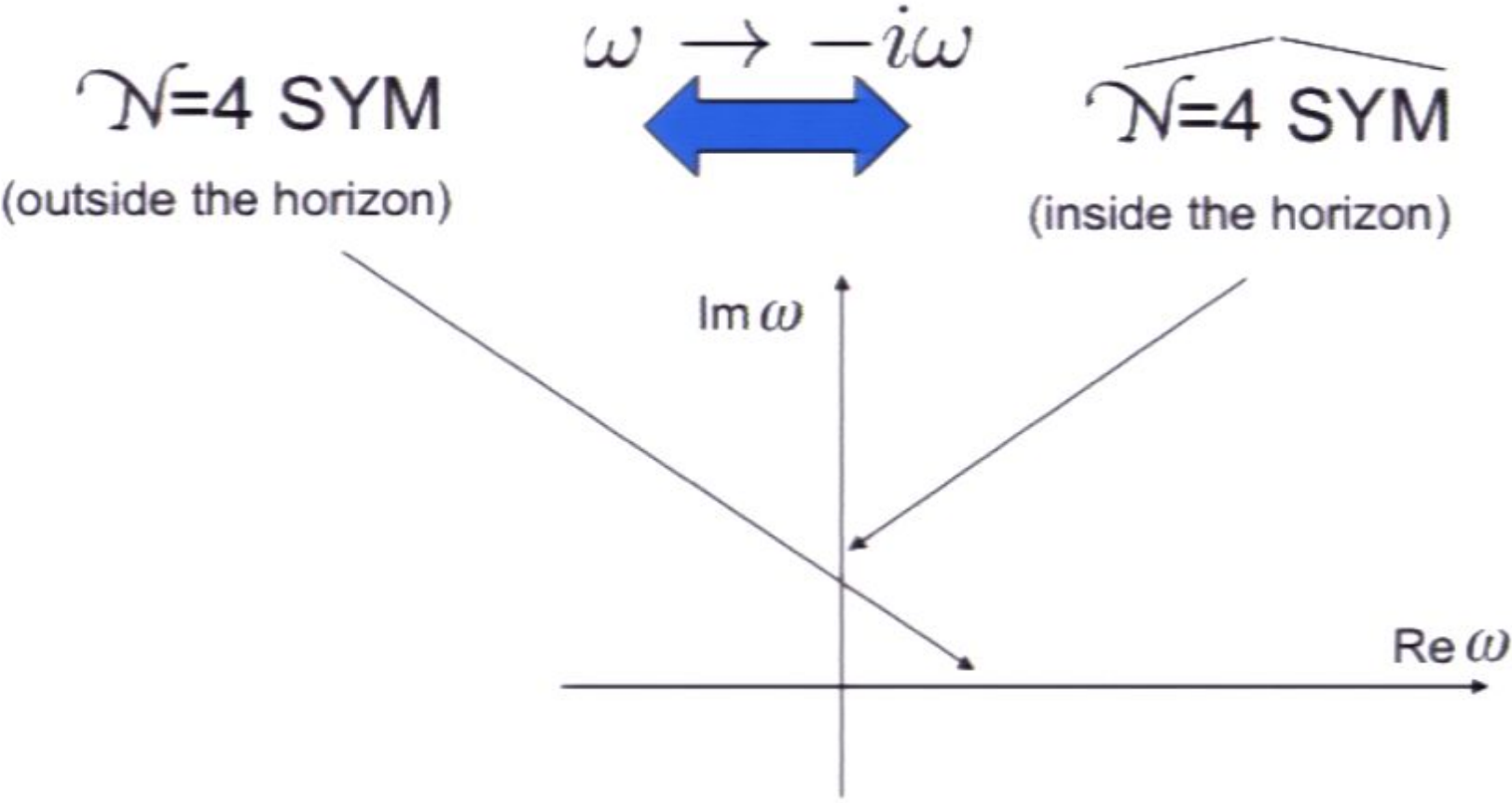
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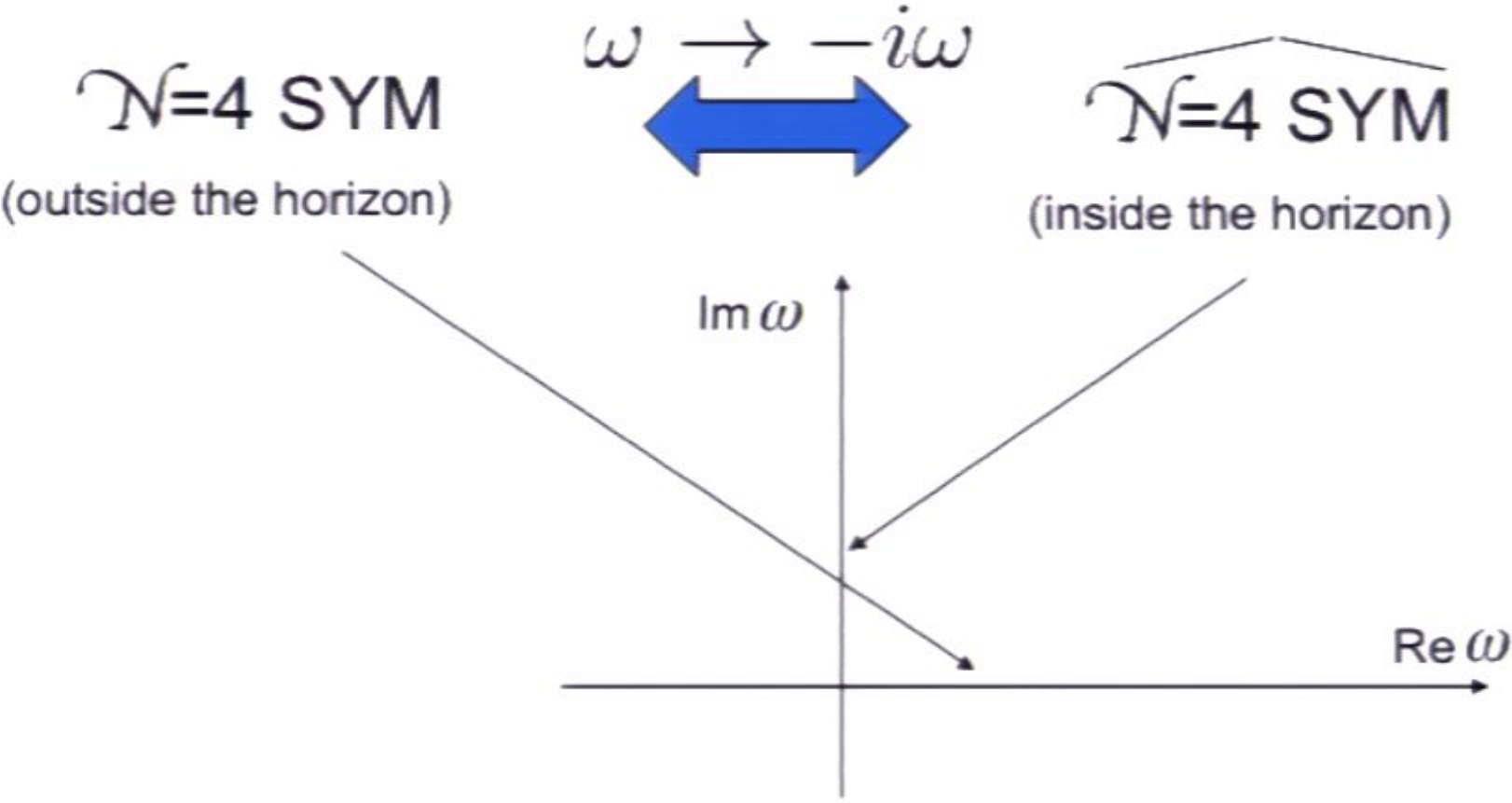
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Different from the standard Euclidean analytic continuation!

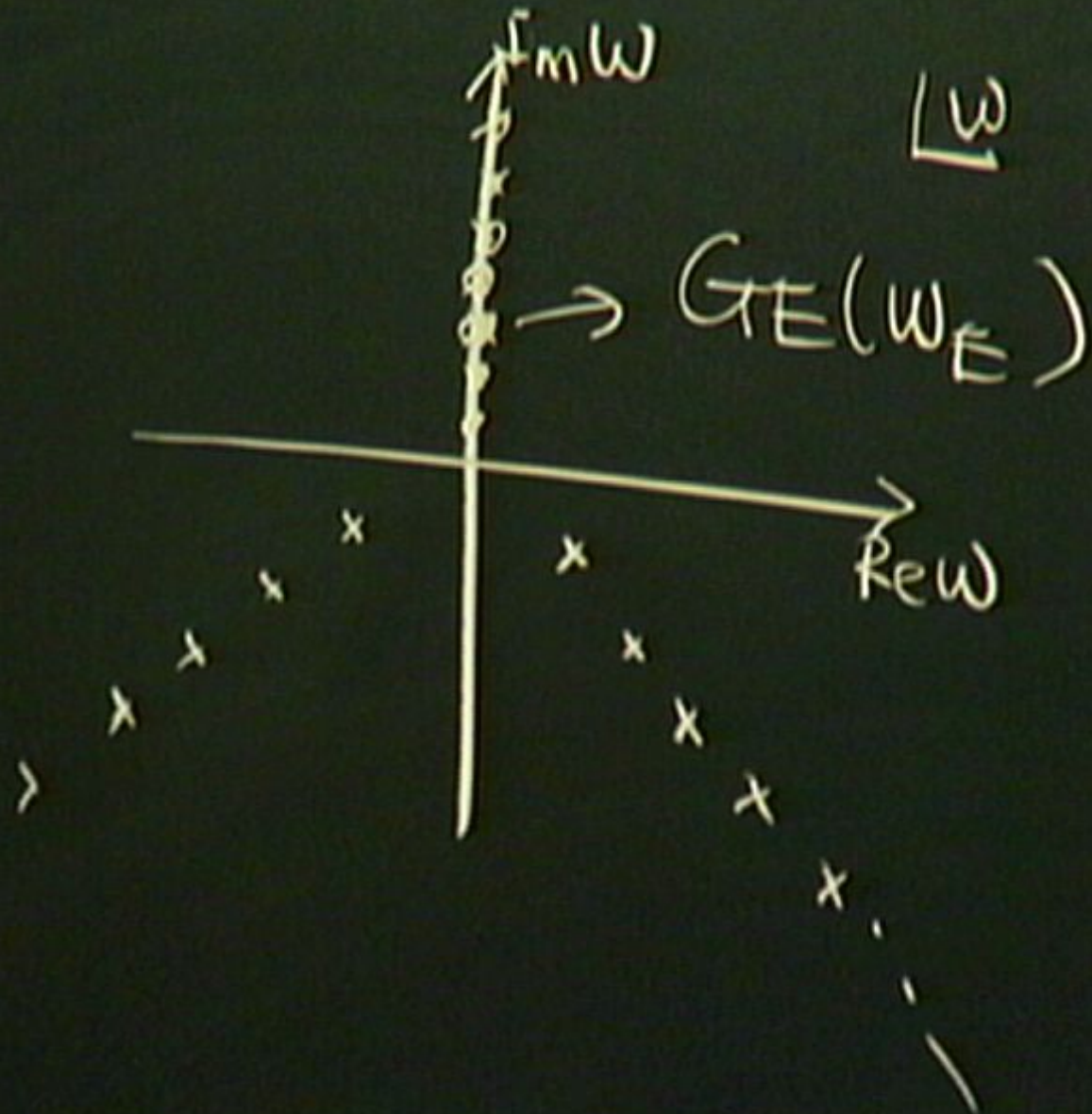


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GR

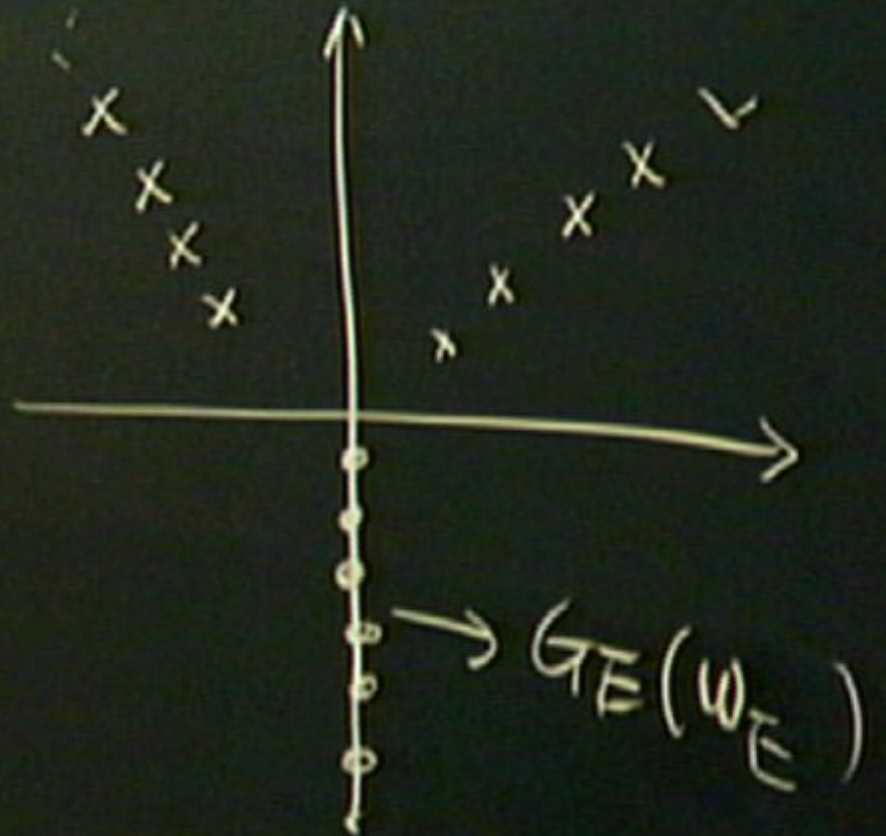
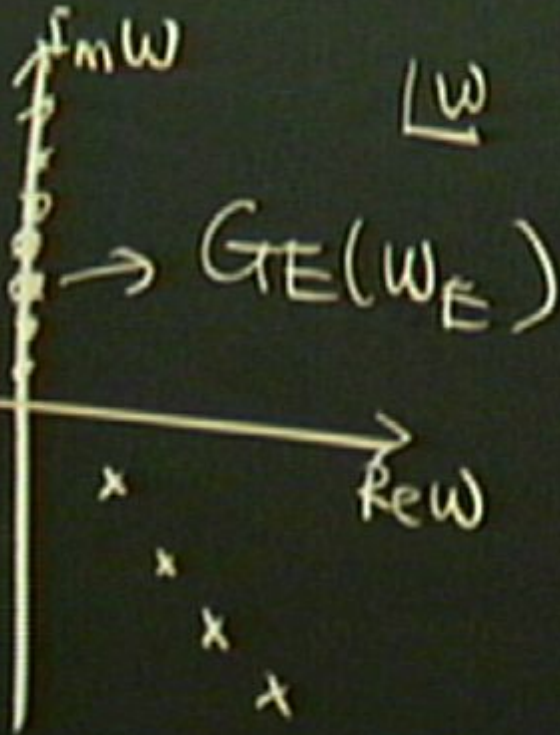




$$G_T = G_{TR} - G_{TA}$$

$G_{TR}$

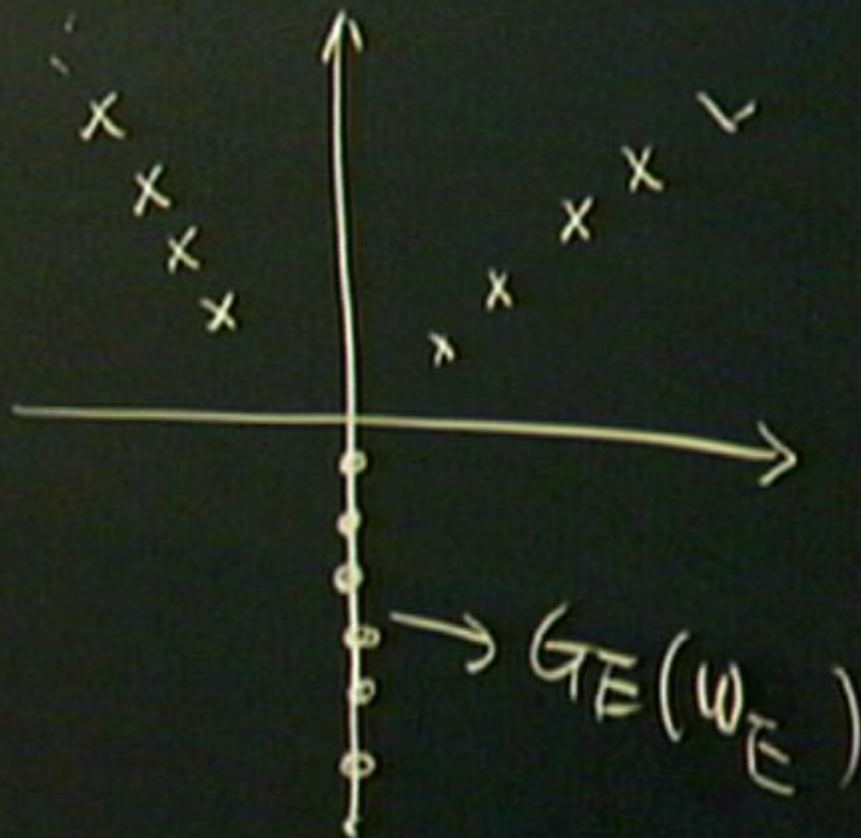
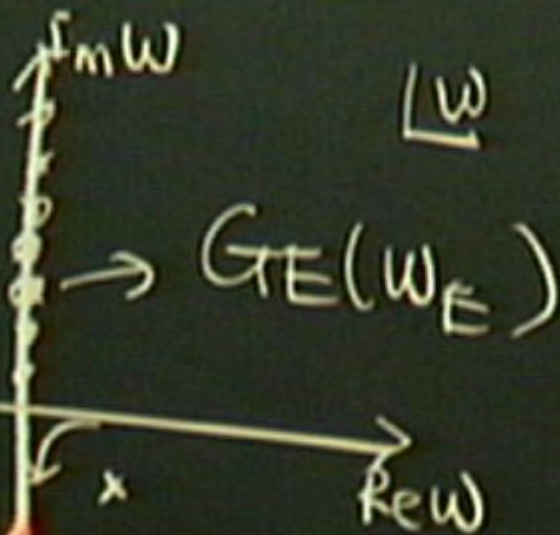
$G_{TA}$



$G_R$

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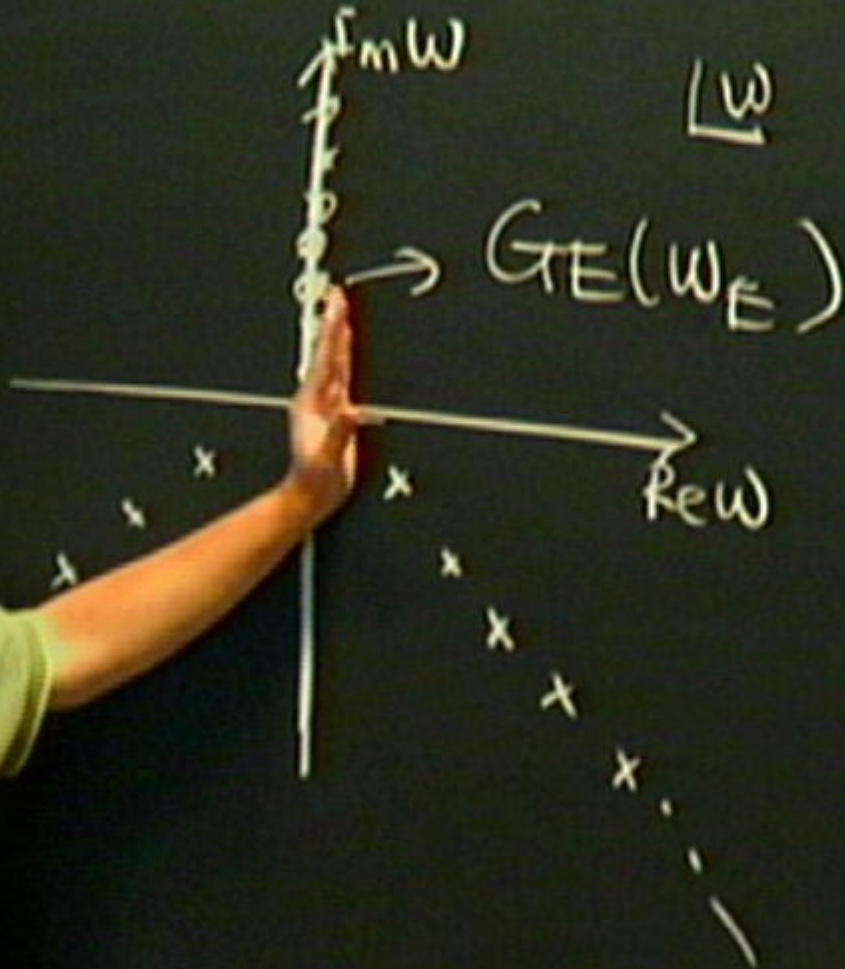
$G_A$



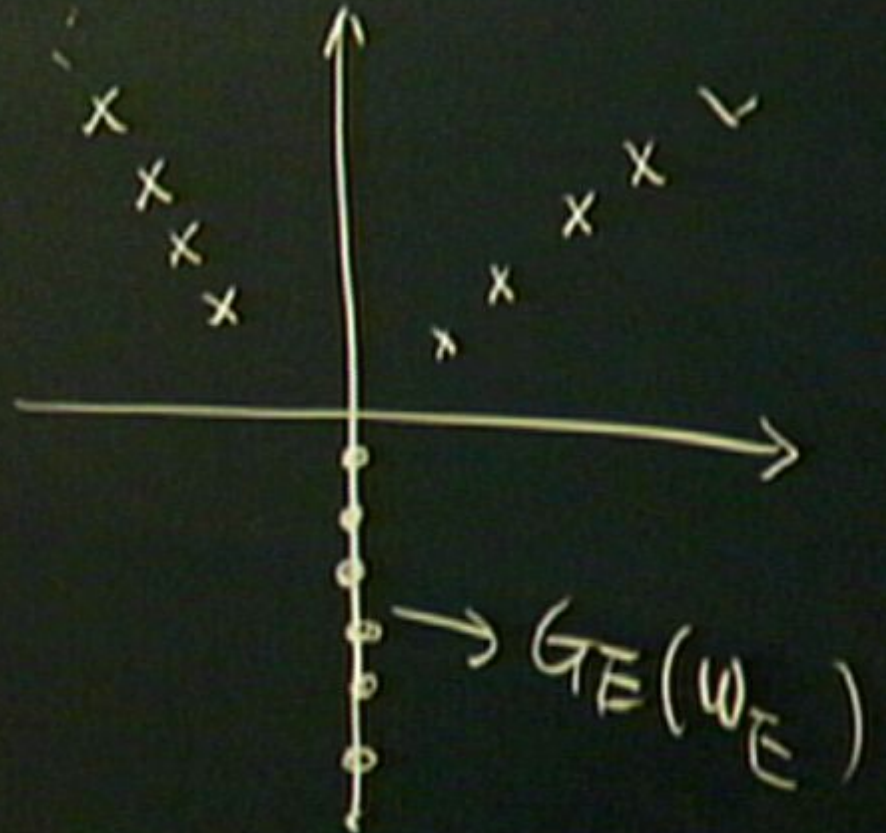


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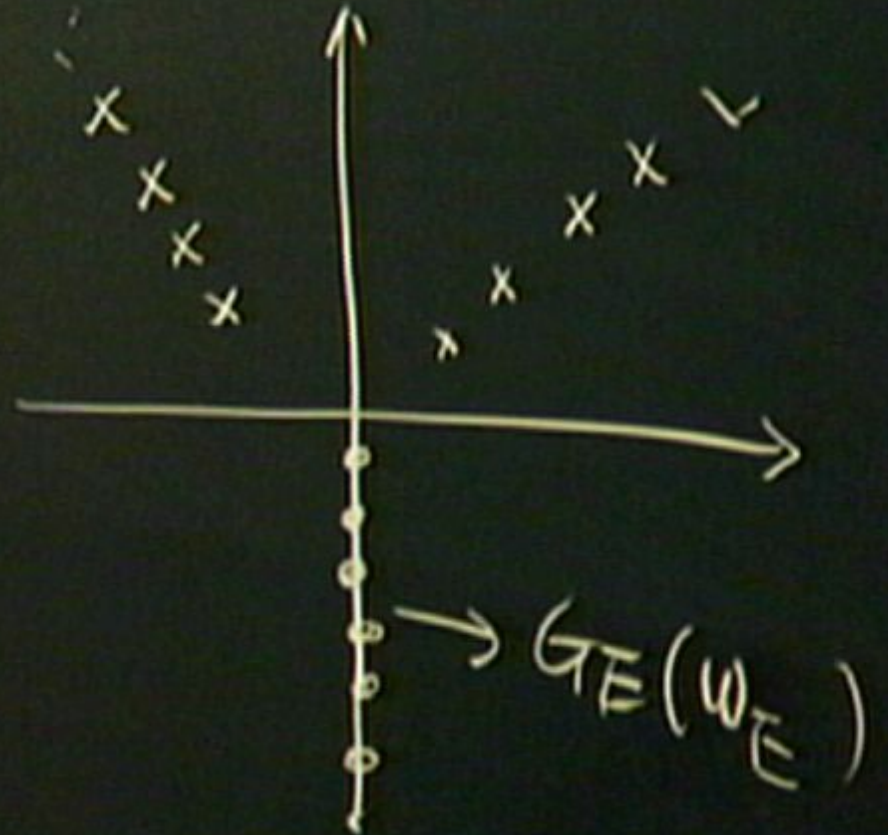
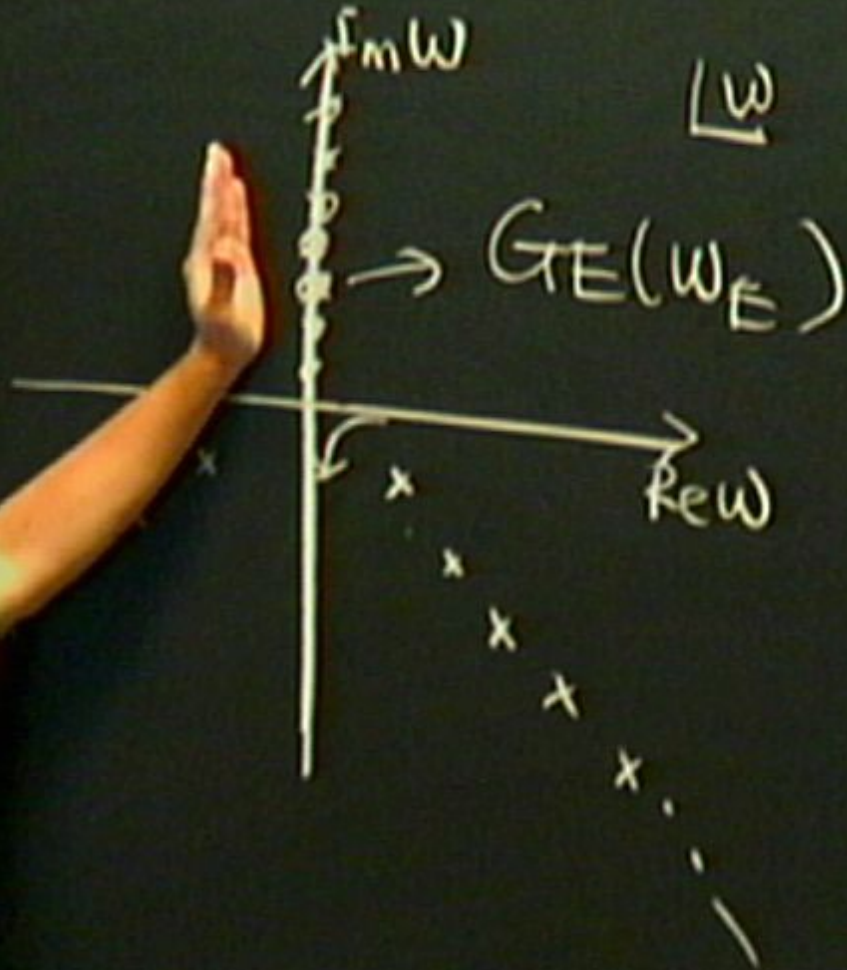
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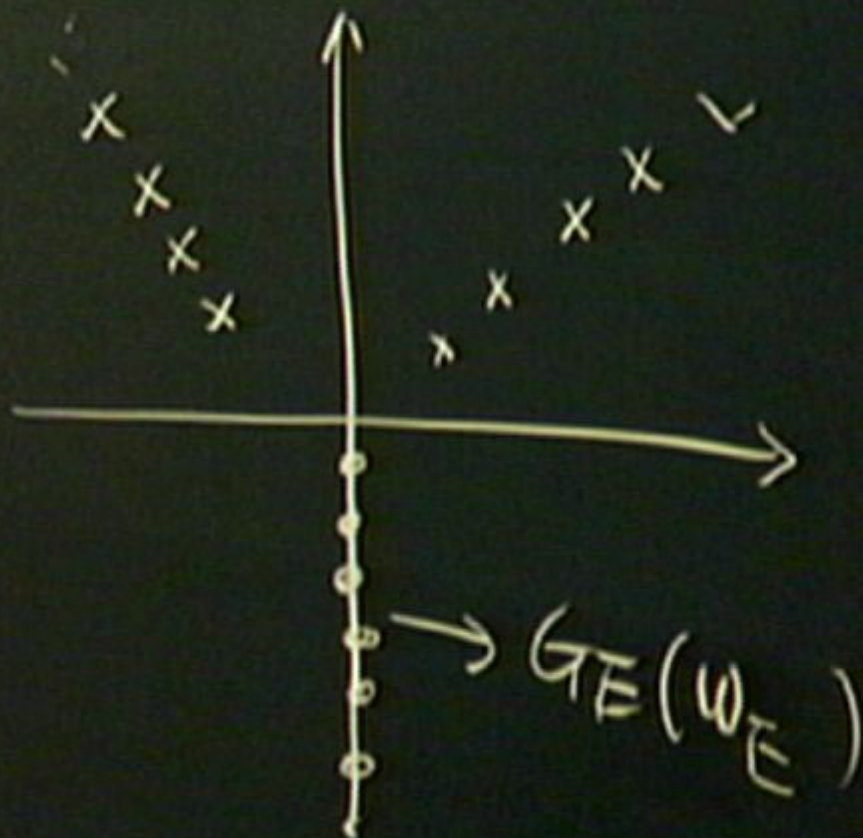
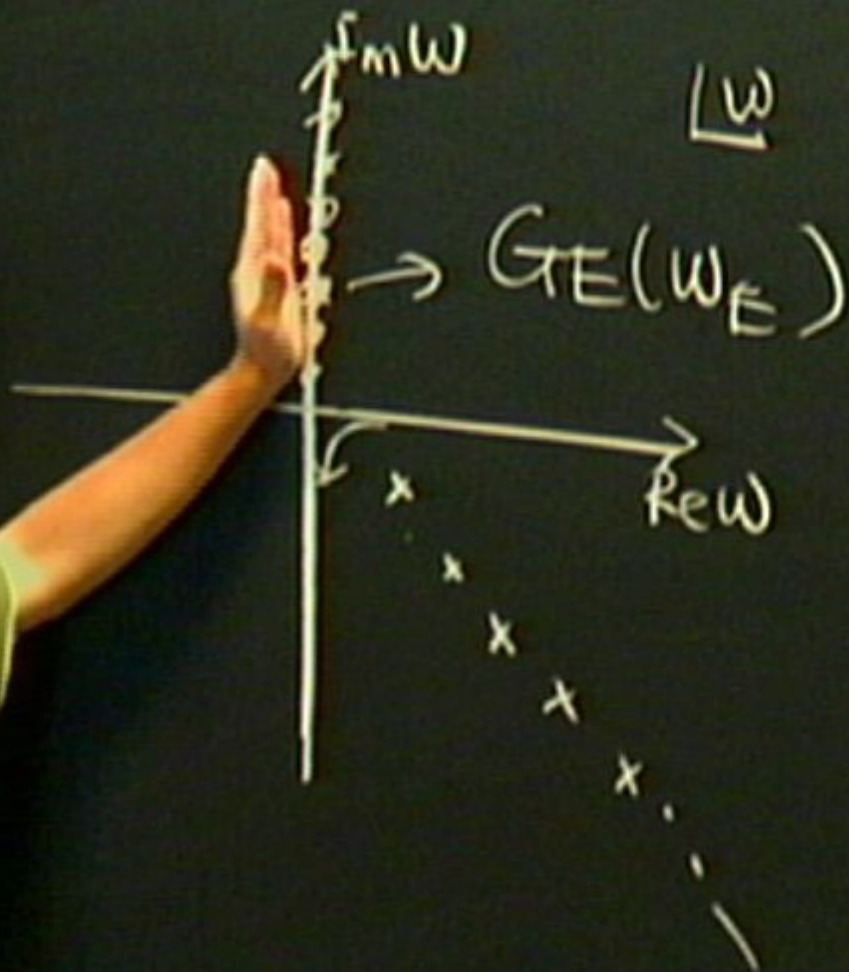




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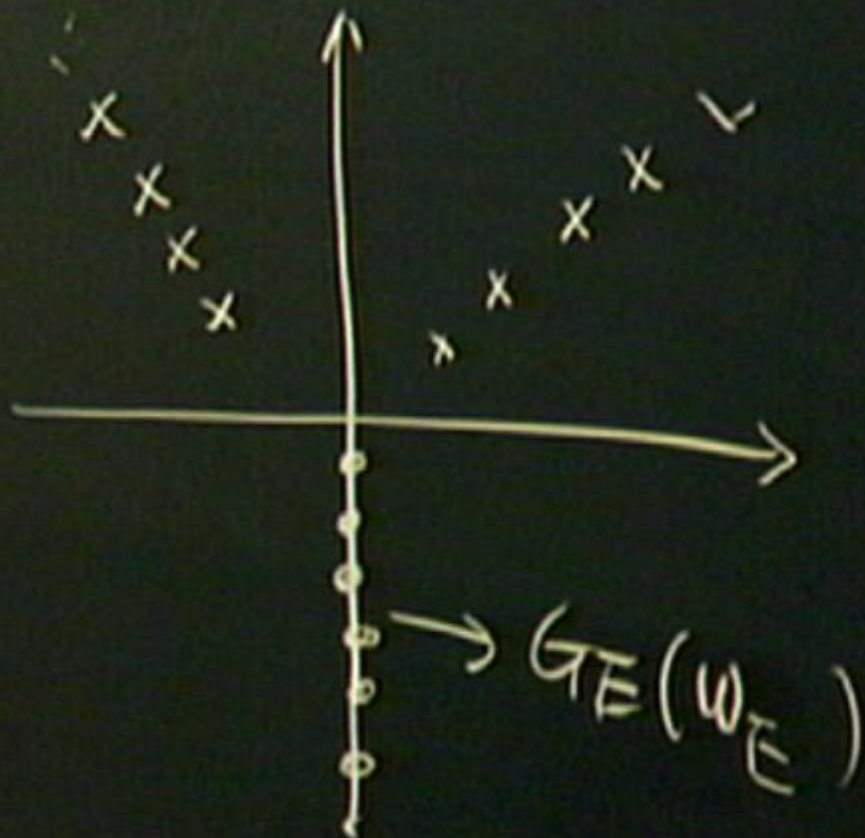
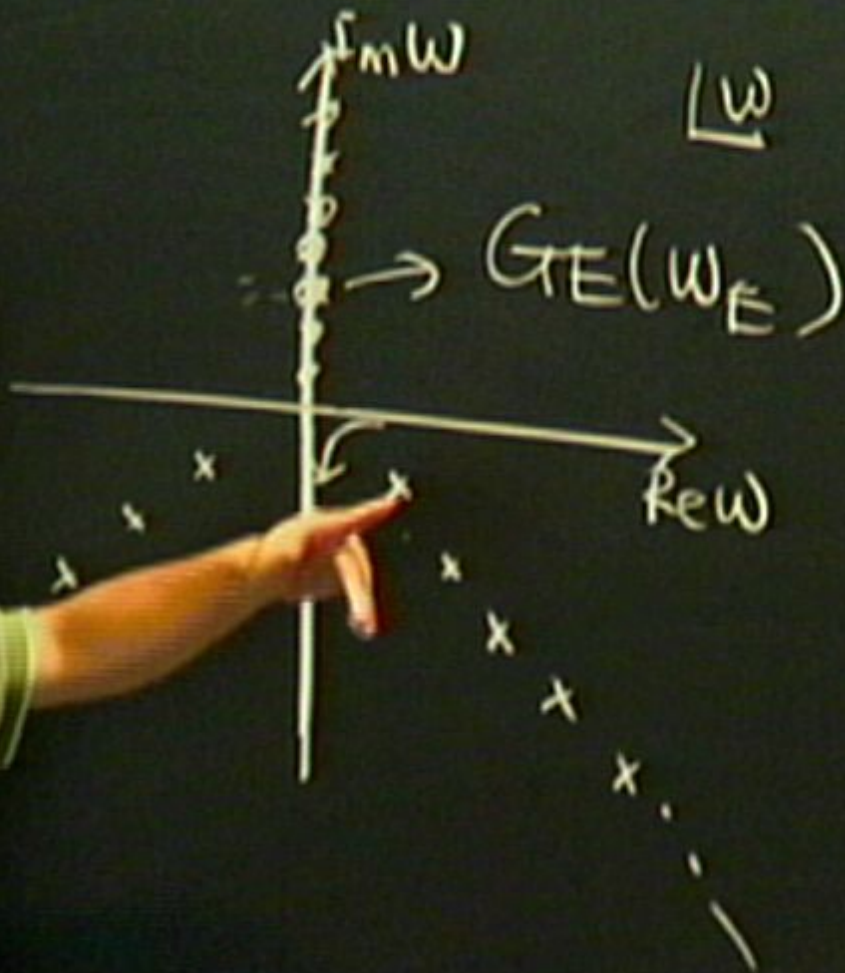
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$$G_T = G_{TR} - G_{TA}$$

$G_{TR}$

$G_{TA}$

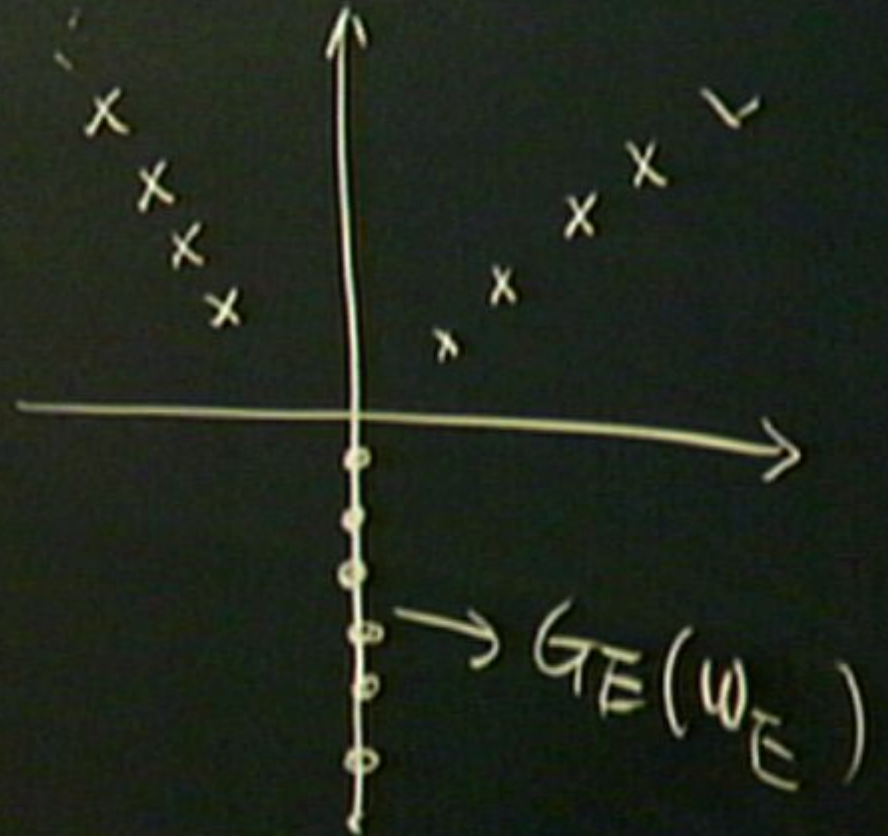
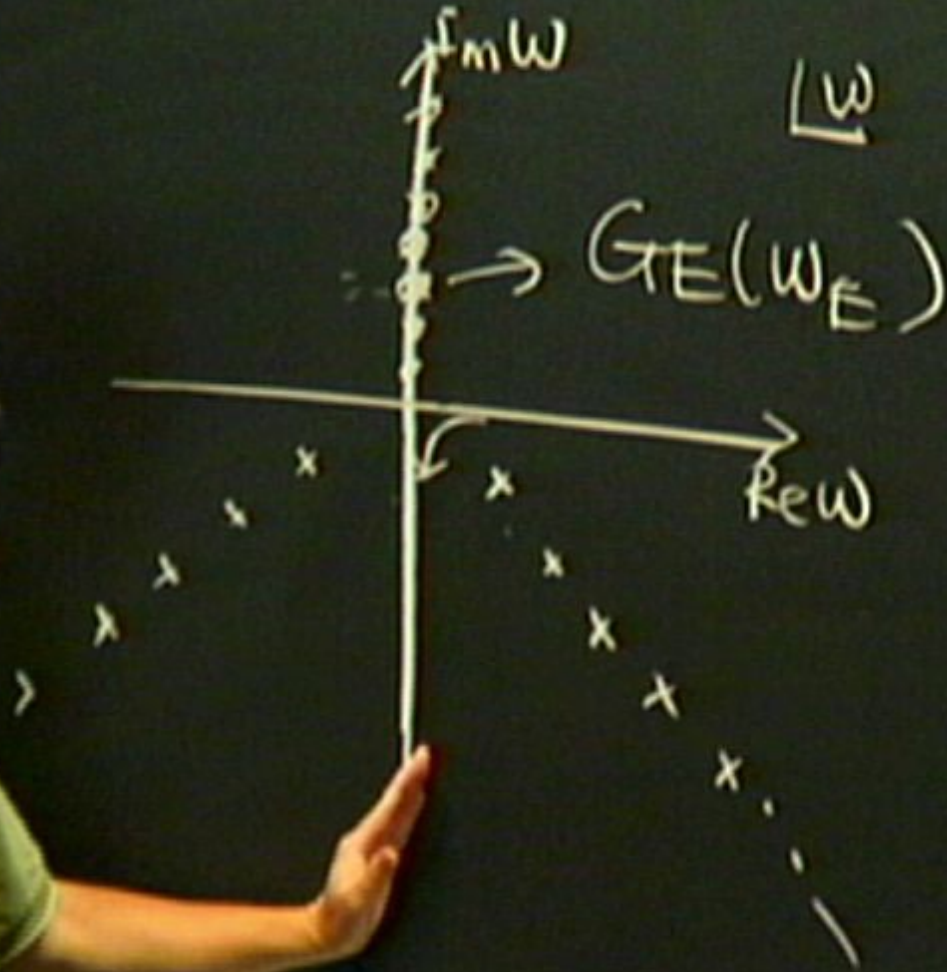




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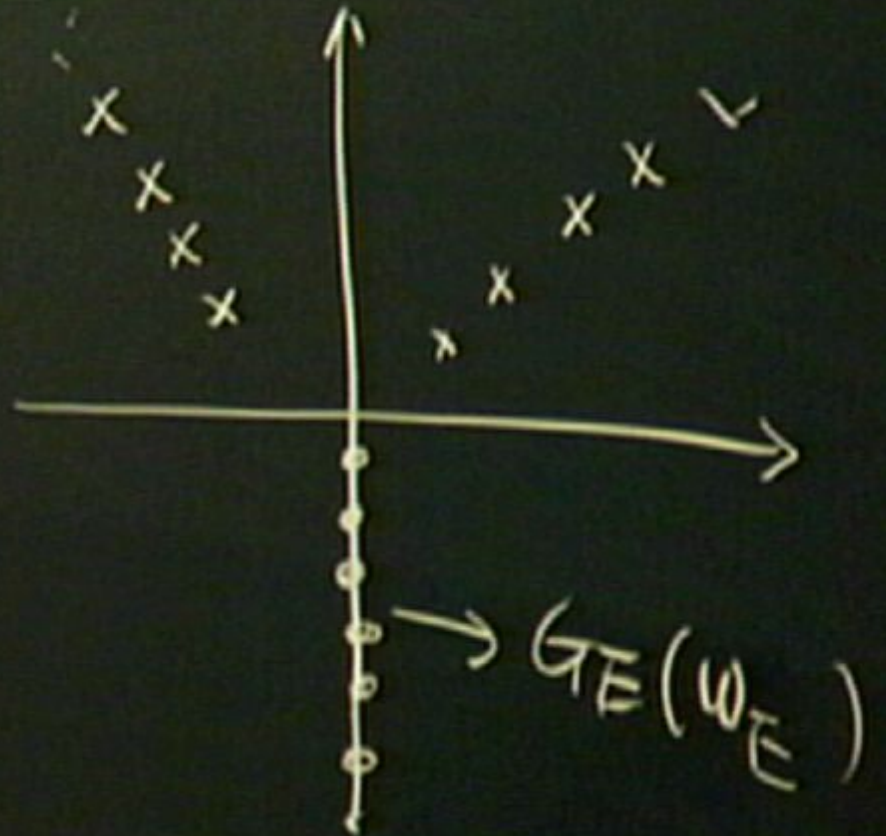
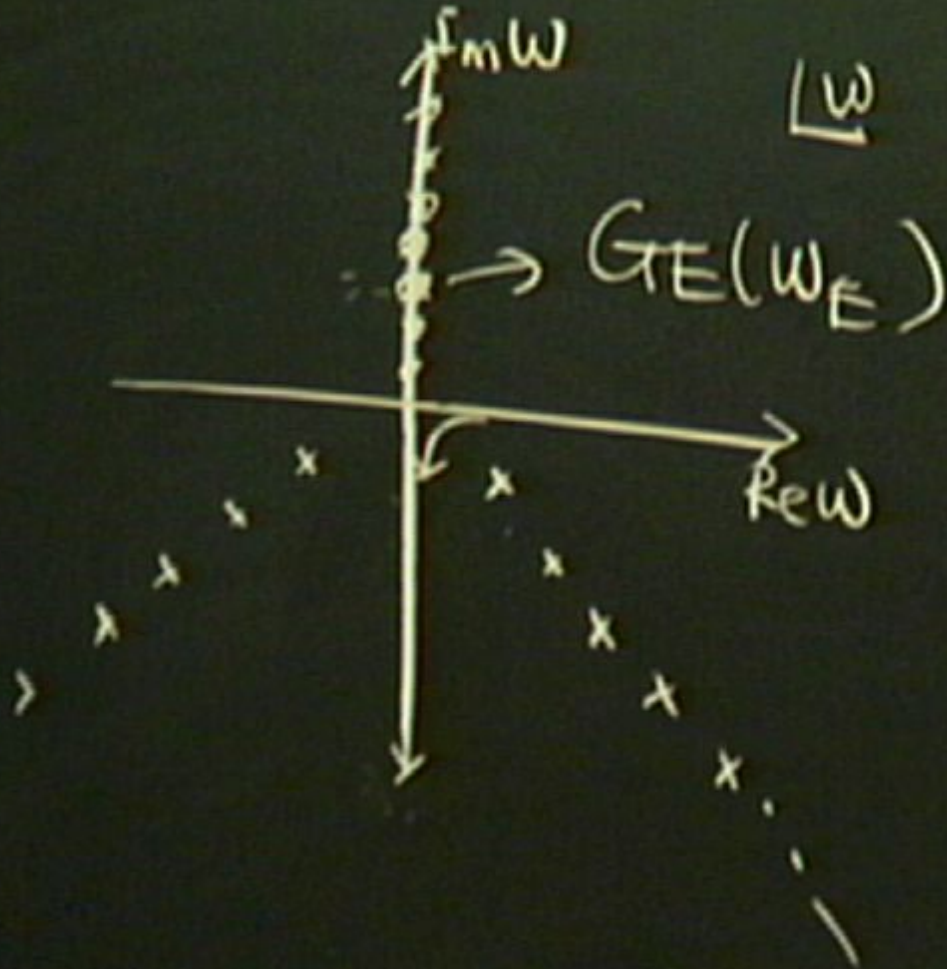
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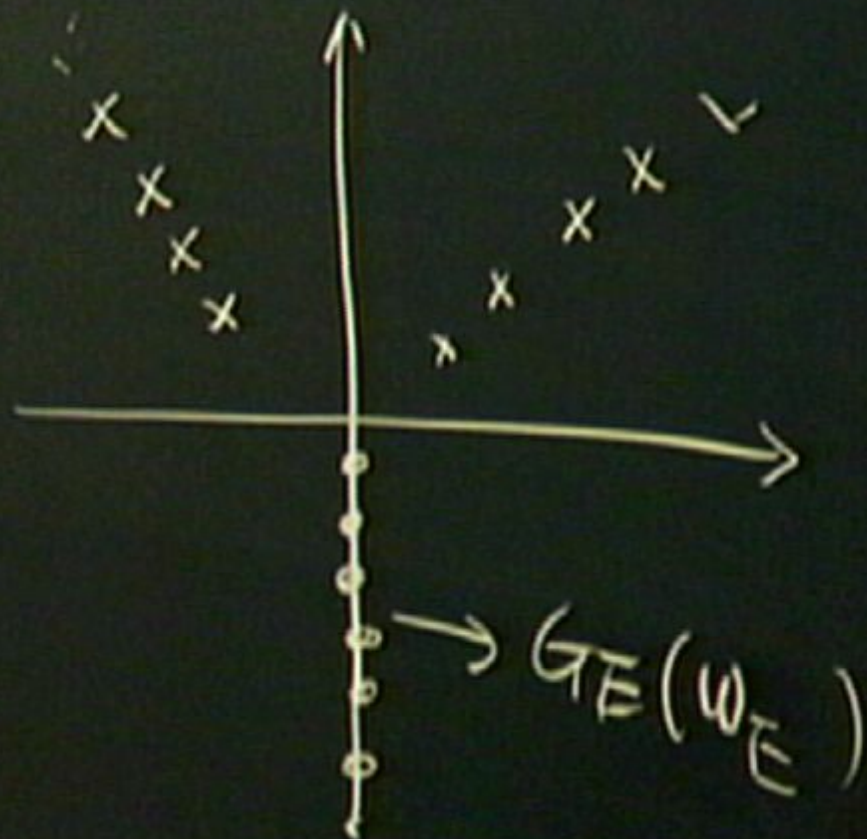
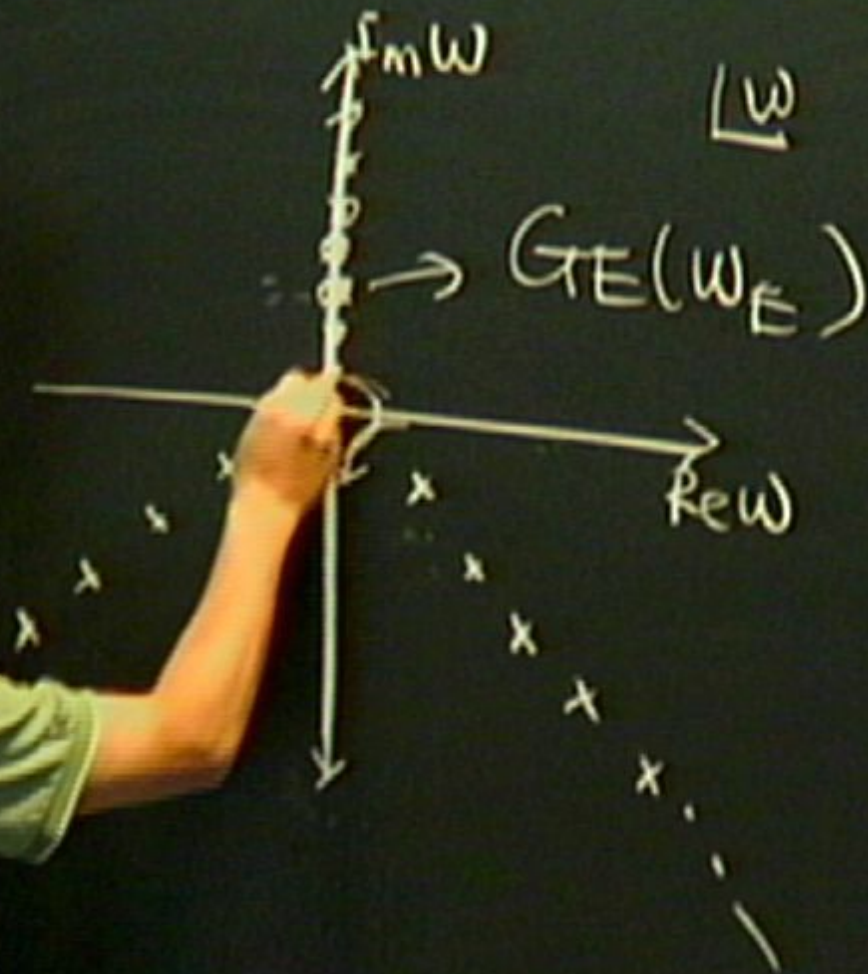




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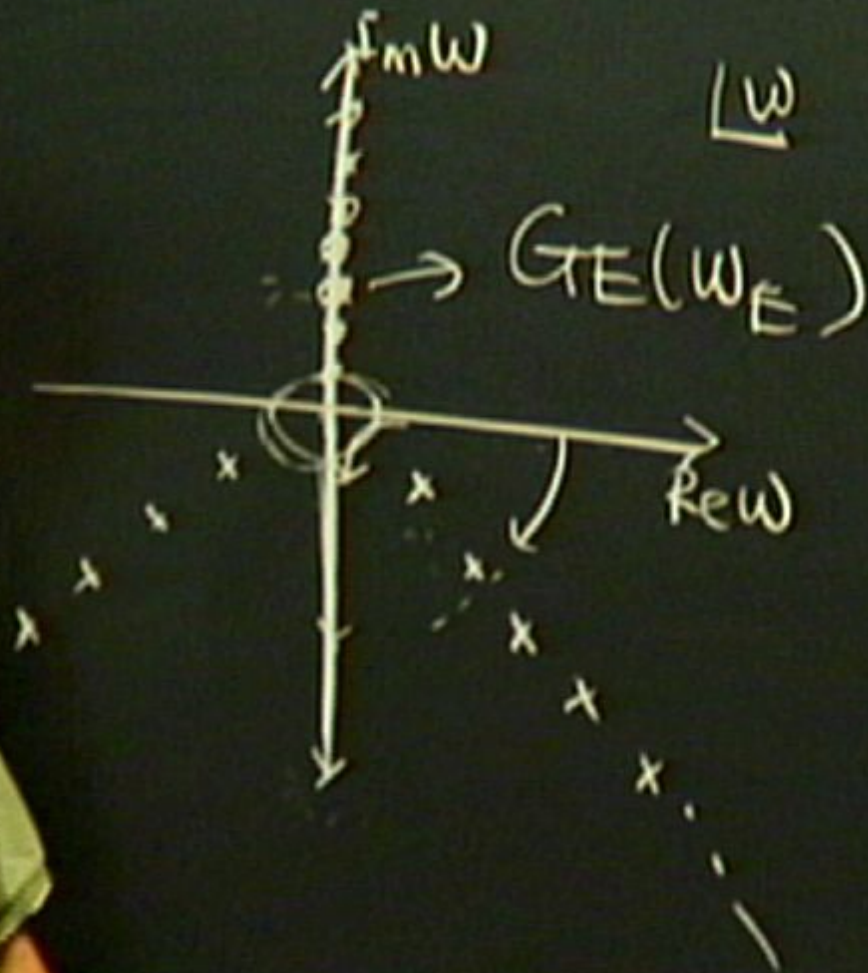
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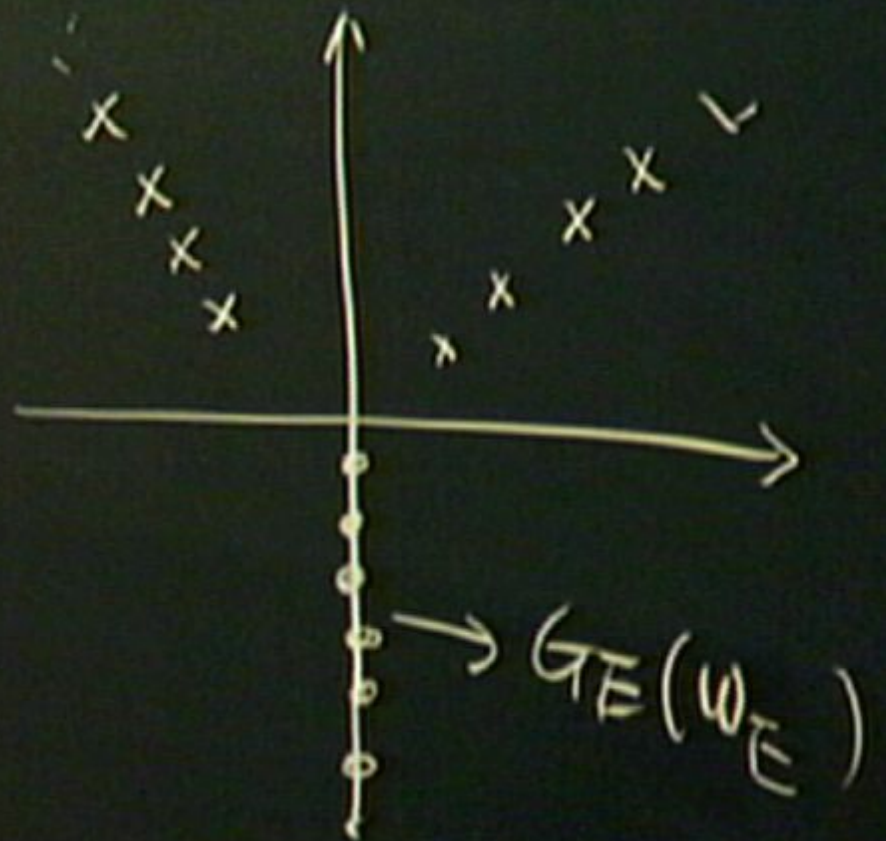


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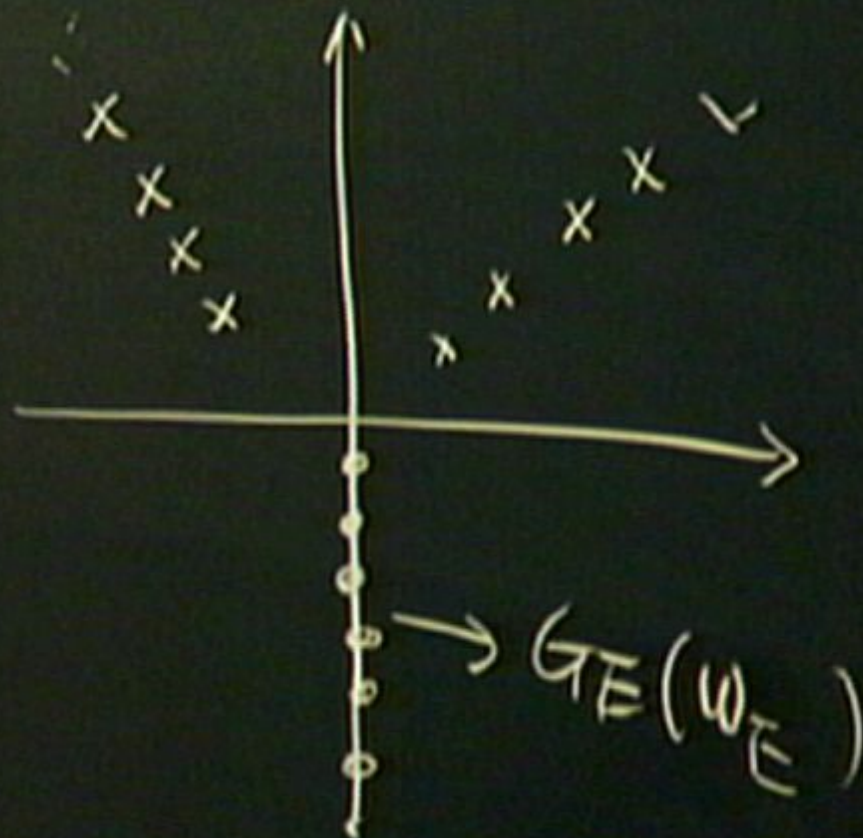
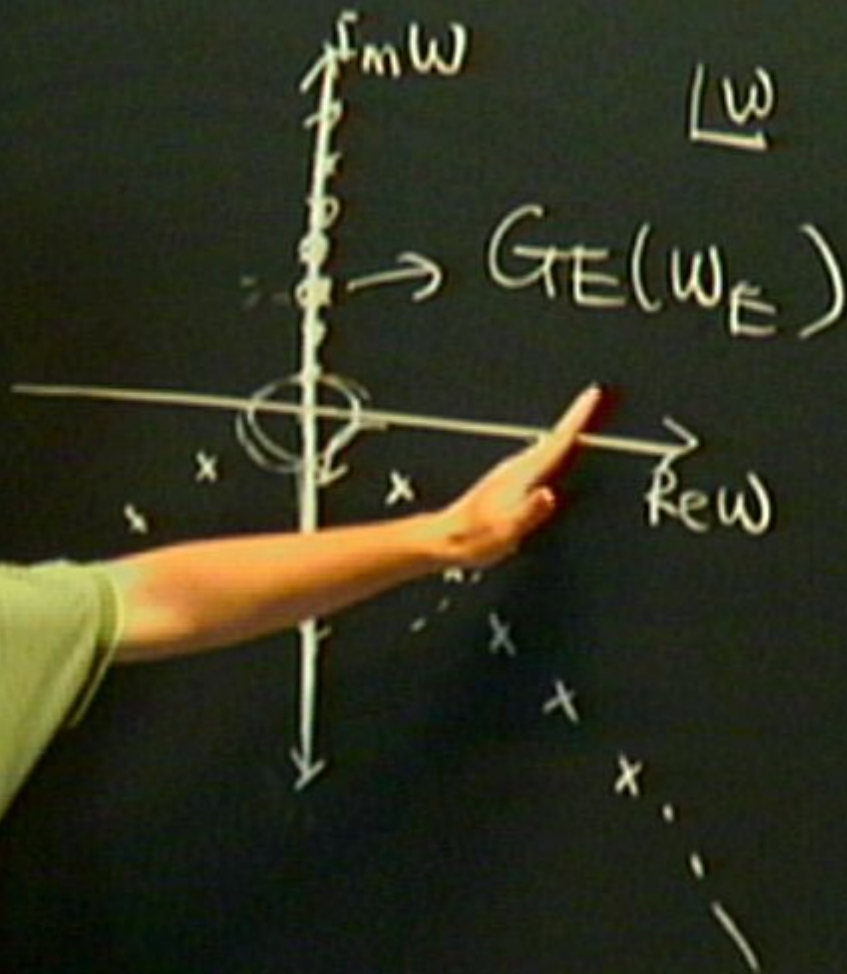




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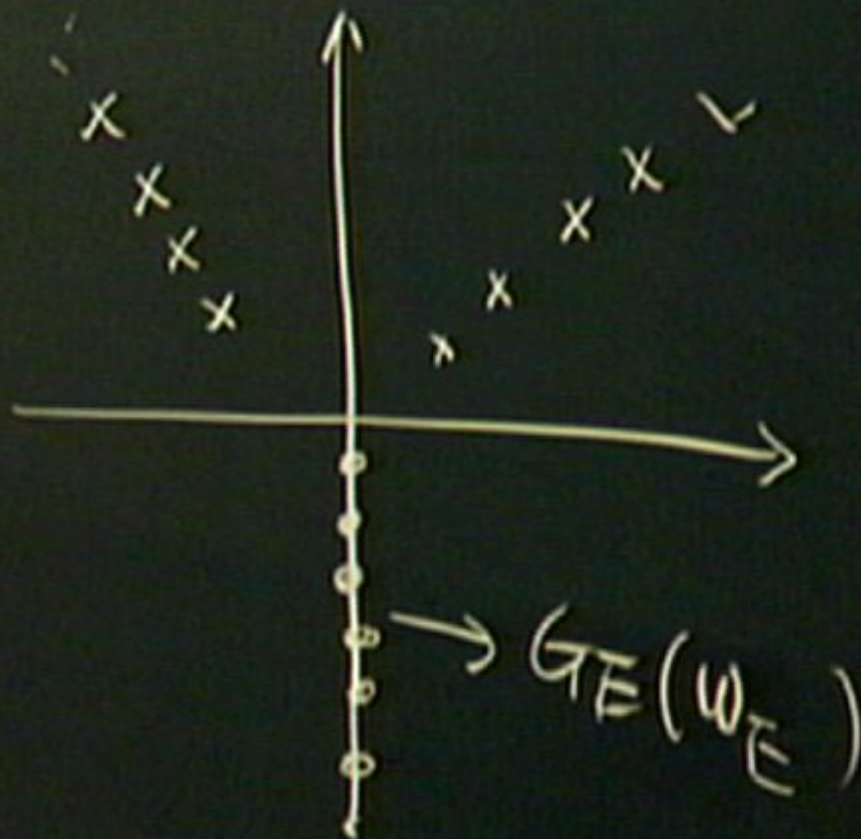
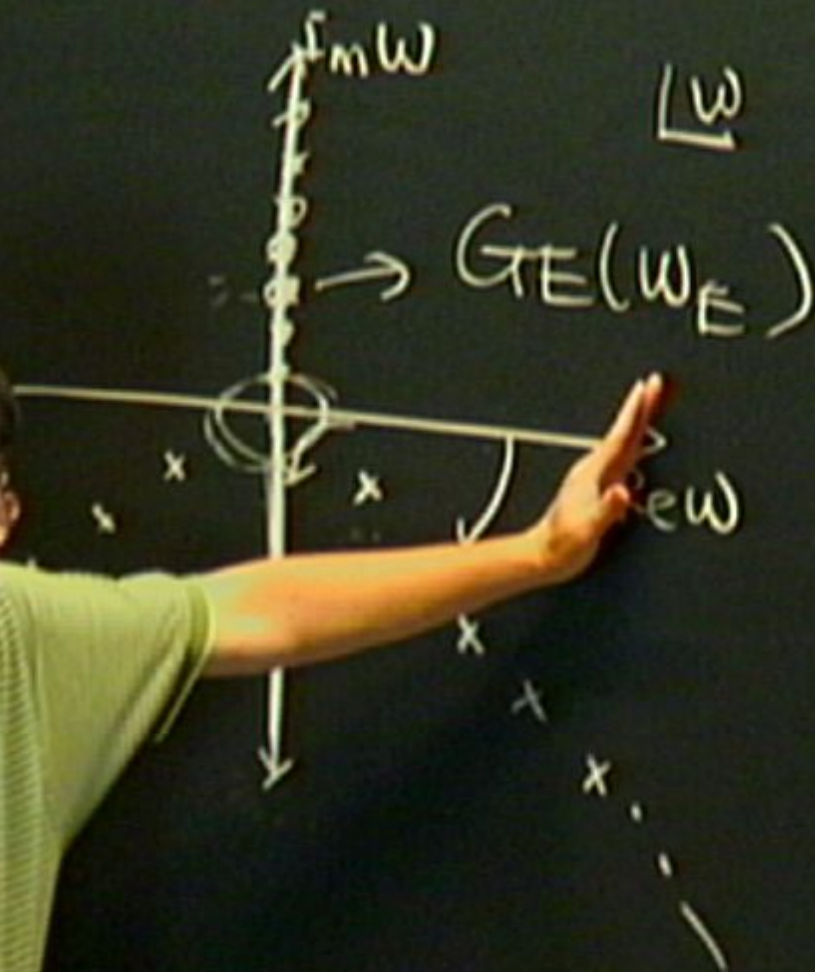




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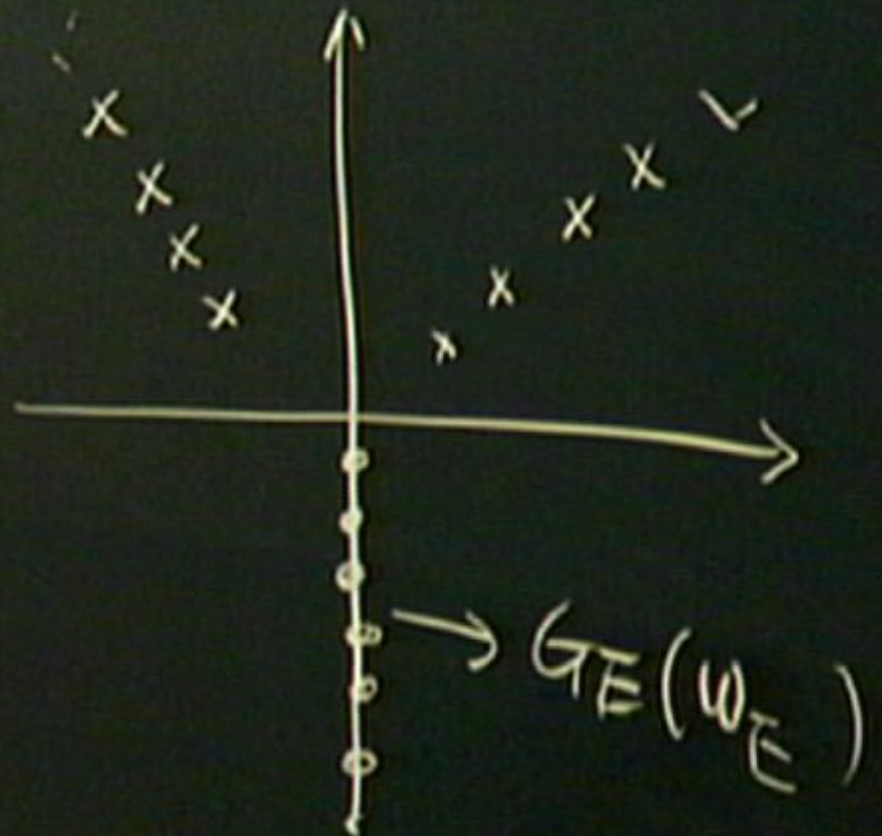
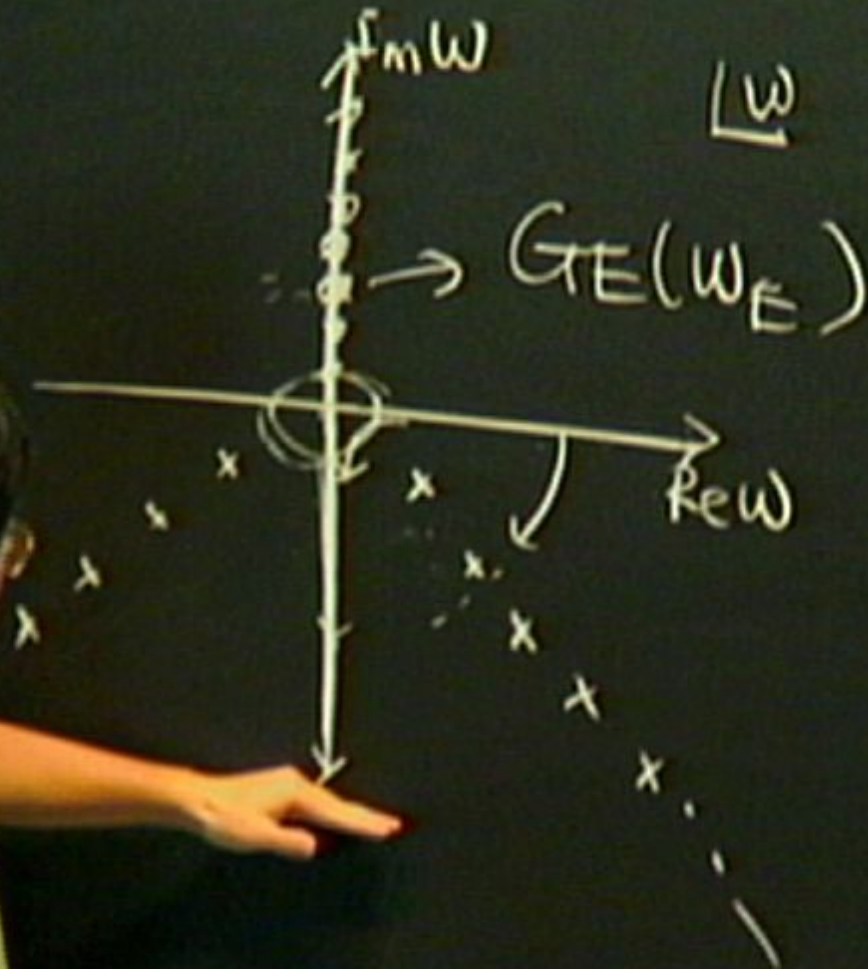




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$G_{TA}$



# Singularity as a UV fixed point?

UV of  $\mathcal{N}=4$  SYM at finite T



Low energies



Analytic continuation to  
 $\mathcal{N}=4$  SYM



UV of  $\mathcal{N}=4$  SYM

Boundary of AdS black hole



Horizon (from outside)



Horizon (from inside)



singularity



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Pirsa: 11060071  
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Boundary of AdS black hole



Horizon (from outside)



Horizon (from inside)



singularity

# Singularity as a UV fixed point?

UV of  $\mathcal{N}=4$  SYM at finite T



Low energies



Analytic continuation to  
 $\mathcal{N}=4$  SYM



UV of  $\mathcal{N}=4$  SYM

Boundary of AdS black hole



Horizon (from outside)



Horizon (from inside)



singularity

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The story we just described is at the large  $N$  limit.

What happens at finite  $N$ ?

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# Correlation functions at finite N

$\mathcal{N}=4$  SYM theory on  $S^3$  at finite N:



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- Discrete energy spectrum.

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- Discrete energy spectrum.
- Wightman functions have simple analytic structure:

$$G_+(\omega, l) = 2\pi \sum_{m,n} e^{-\beta E_m} \rho_{mn} \delta(\omega - E_n + E_m)$$

A discrete sum of delta functions

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But the Wightman functions we obtained from gravity have a **continuous spectrum**, due to presence of horizon.

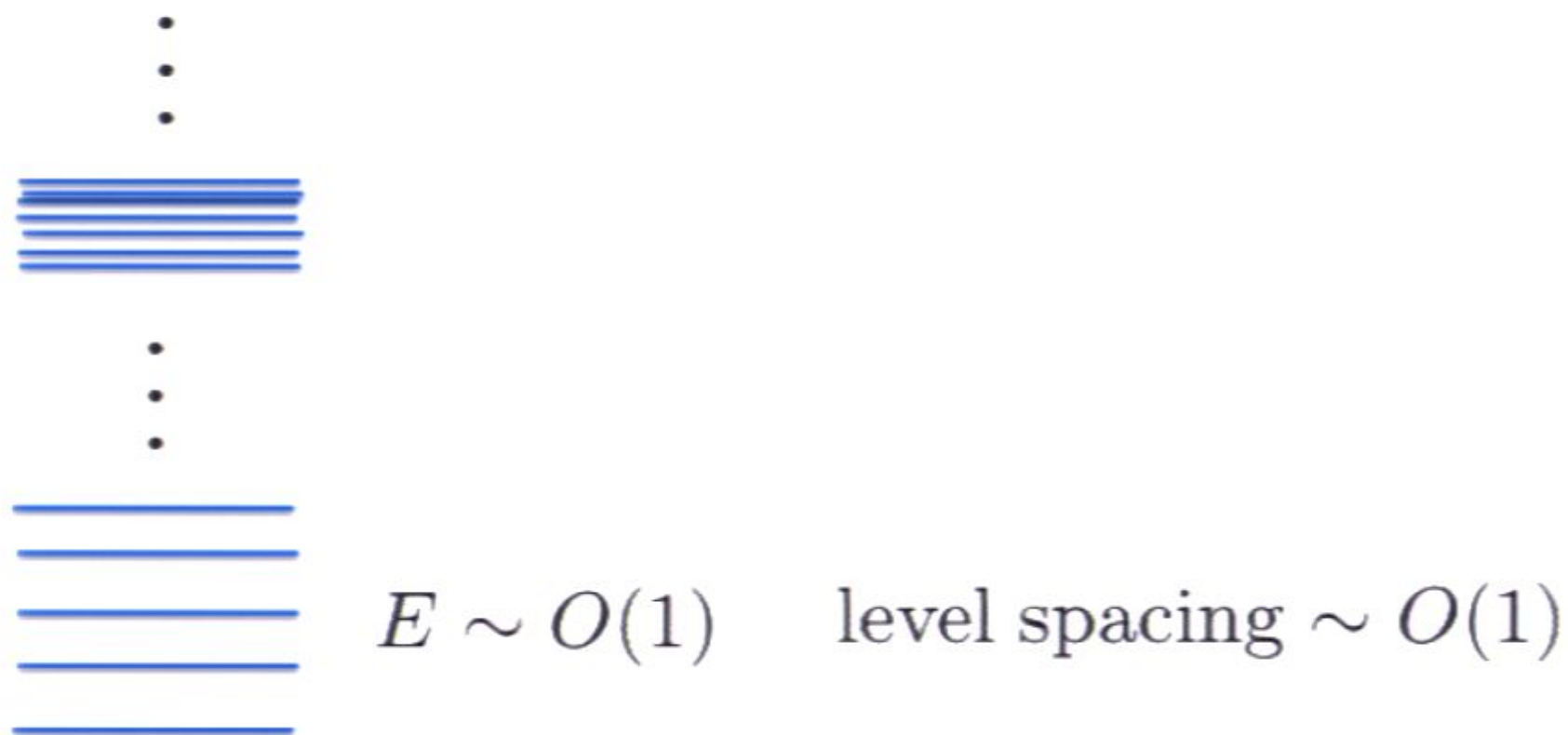
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Energy spectrum of  
SYM on a compact space

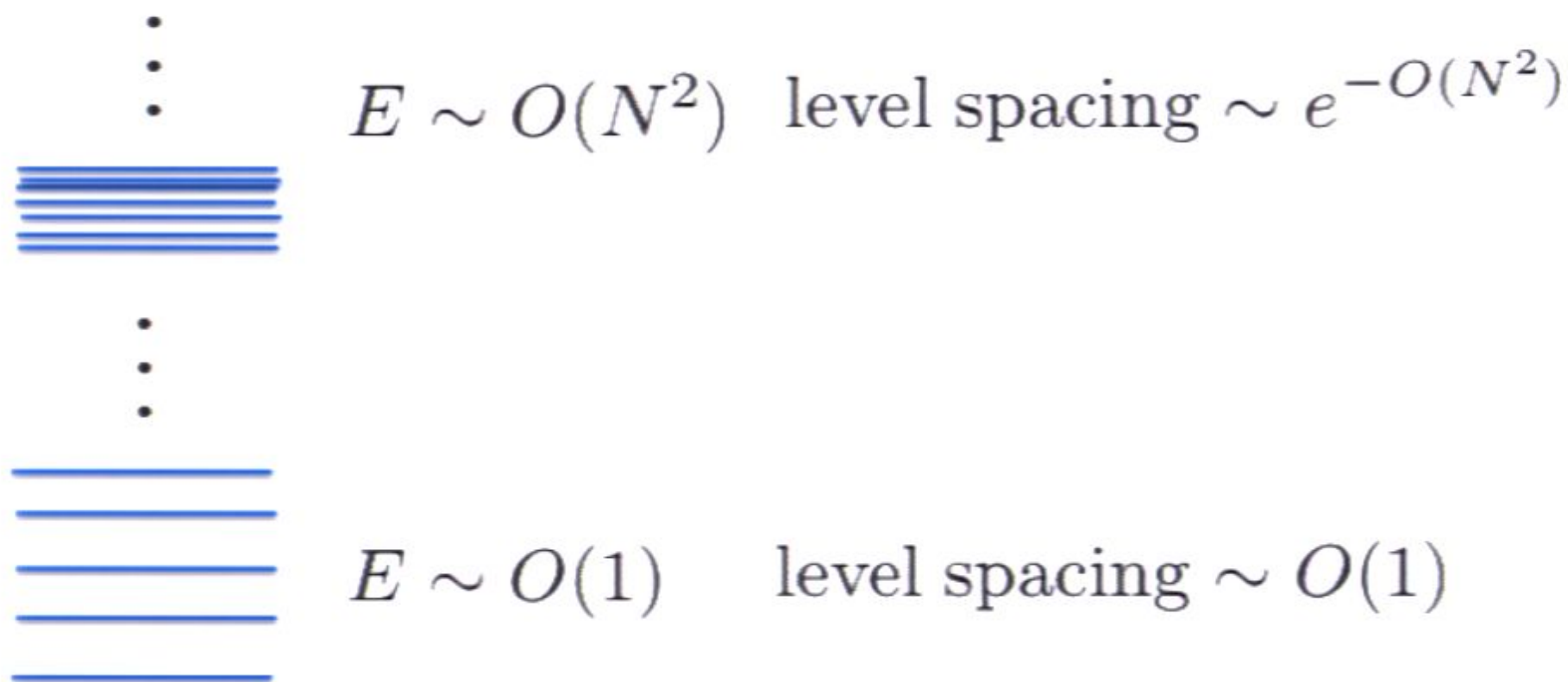


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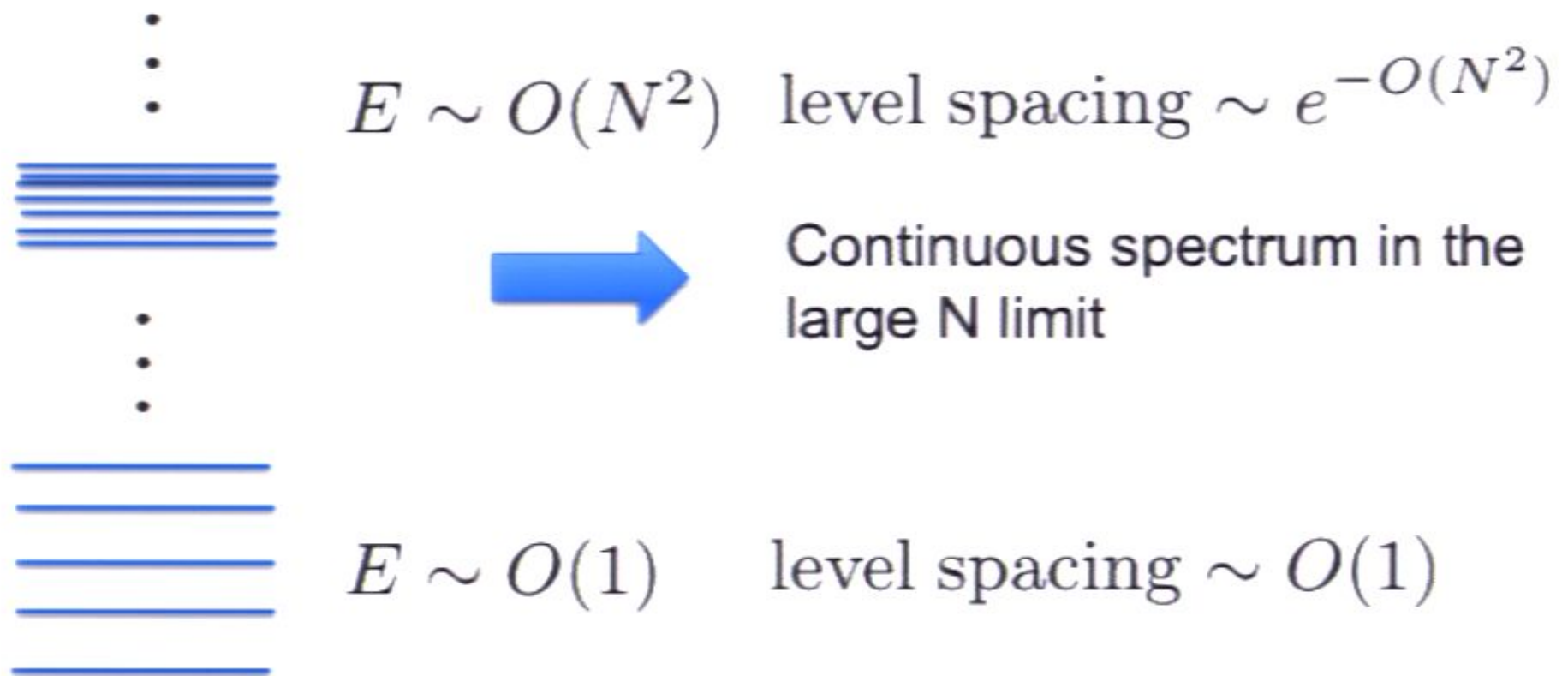
Energy spectrum of  
SYM on a compact space

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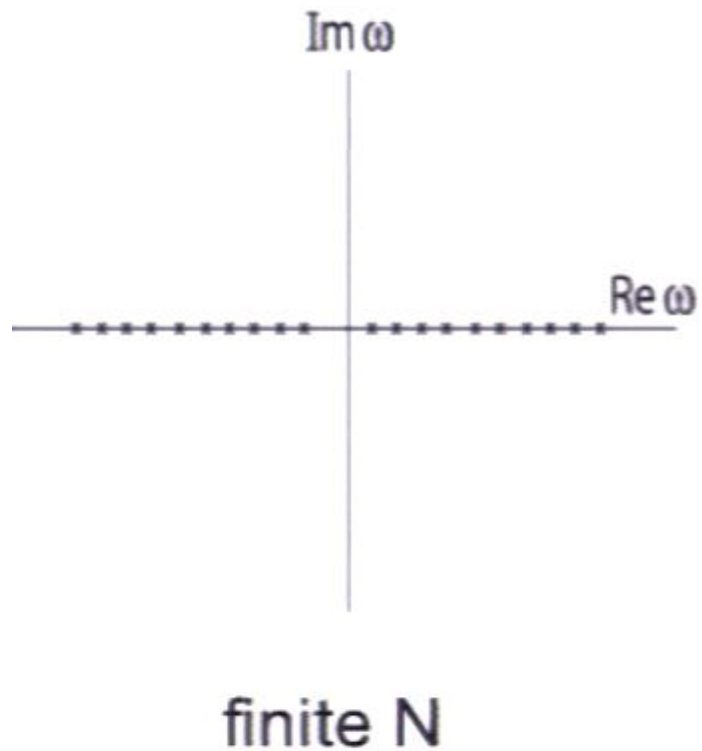
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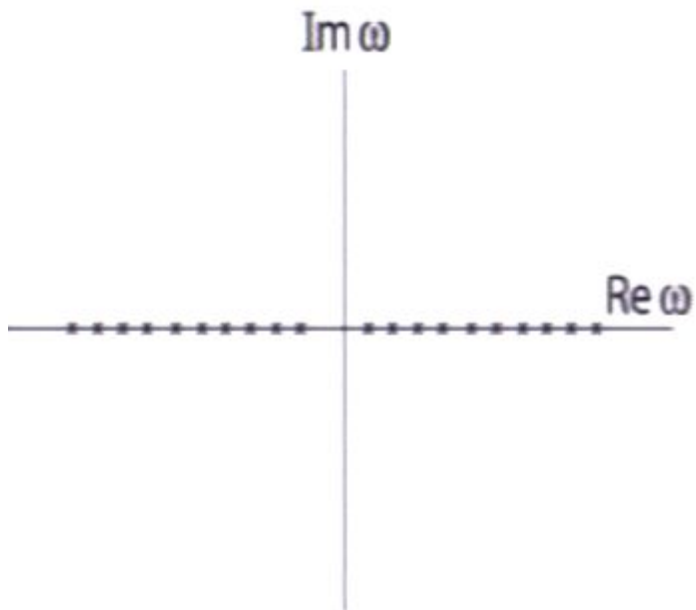
# Finite versus infinite $\mathbb{N}$



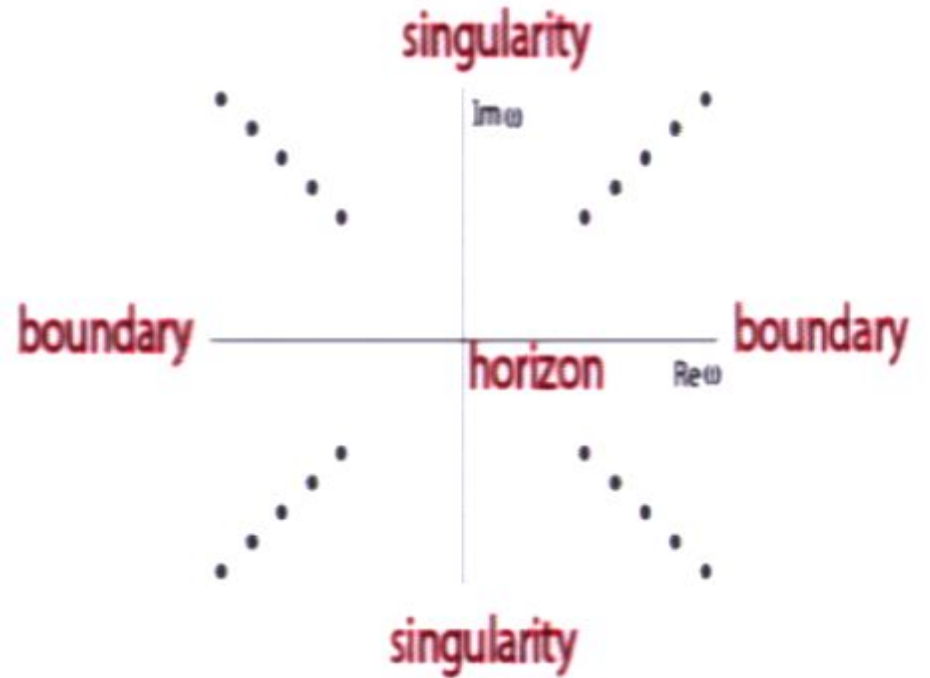
# Finite versus infinite N



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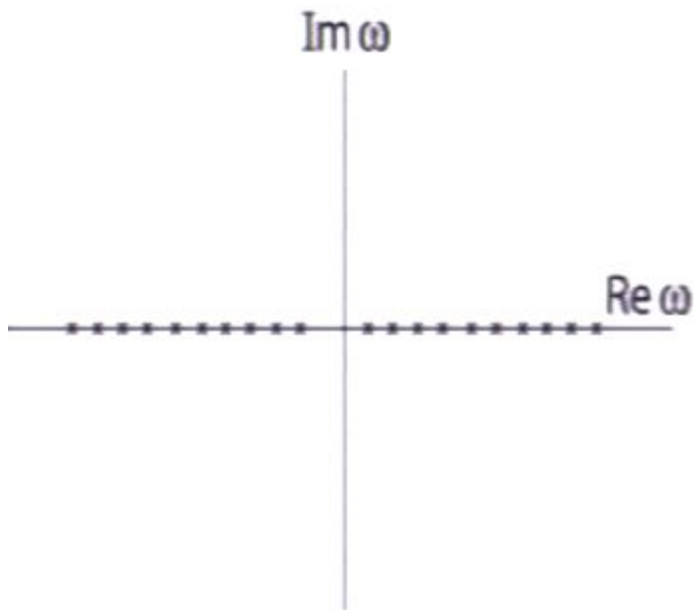


finite N



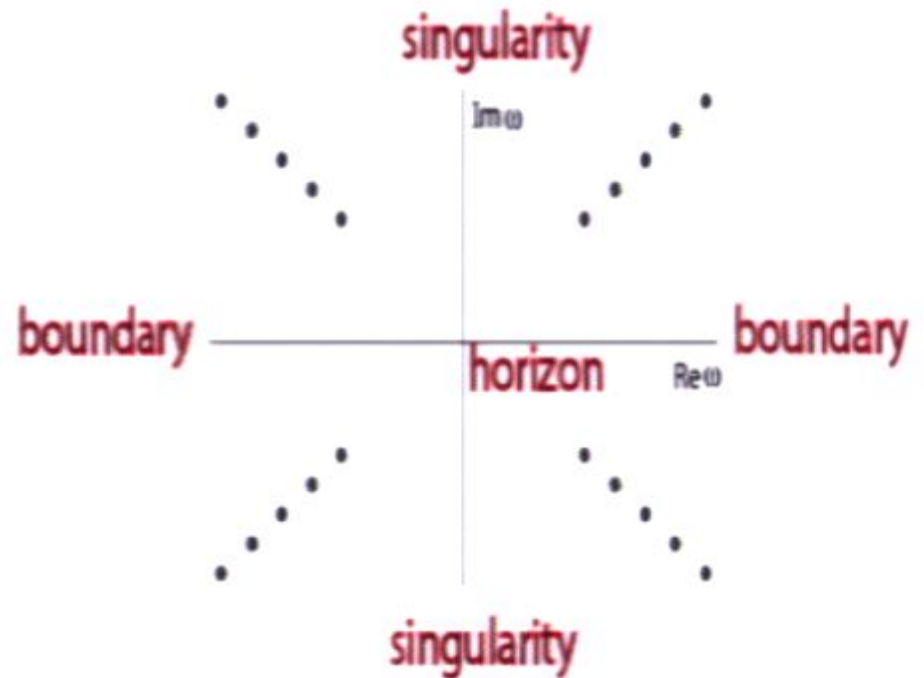
Infinite N

# Finite versus infinite N



finite N

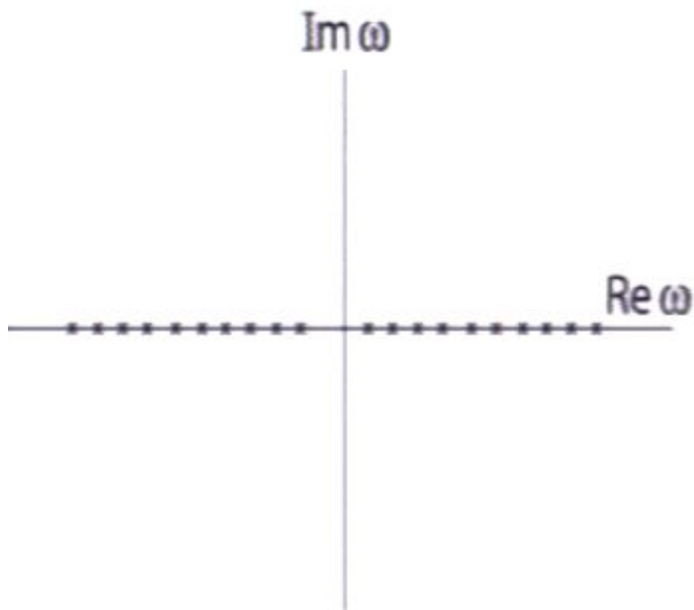
$\mathcal{N}=4$  SYM



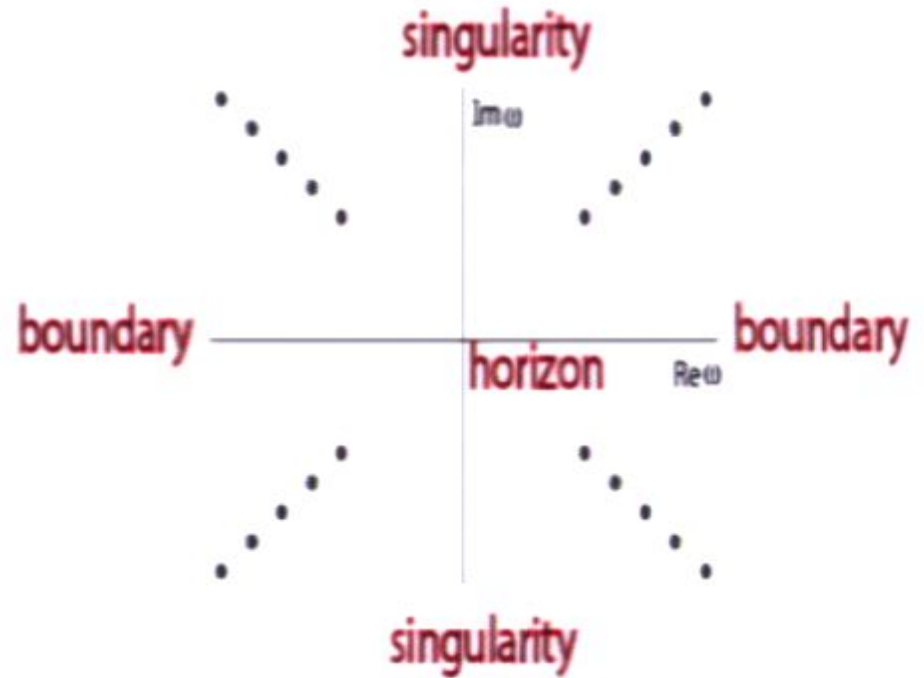
Infinite N

spacetime inside the horizon  
including singularities:

# Finite versus infinite N



finite N



Infinite N

$\mathcal{N}=4$  SYM

spacetime inside the horizon  
including singularities:

approximate concept  
appearing only  
in the large N limit





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No Signal

VGA-1

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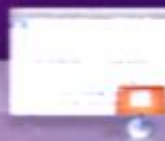




Macintosh HD



PJ talk

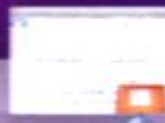




Macintosh HD



PI talk







Macintosh HD



PJ talk

