Title: Complexity in Fundamental Physics

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Abstract:

Outline

1. Motivation
2. Inspiration
   - Spin glass models
   - State space structure of the SK model
   - Overlap order parameter
3. Application to string theory and cosmology
   - Generalized overlap order parameters
   - Examples of glassy systems in string theory + results
   - Cosmology

Complexity in fundamental physics

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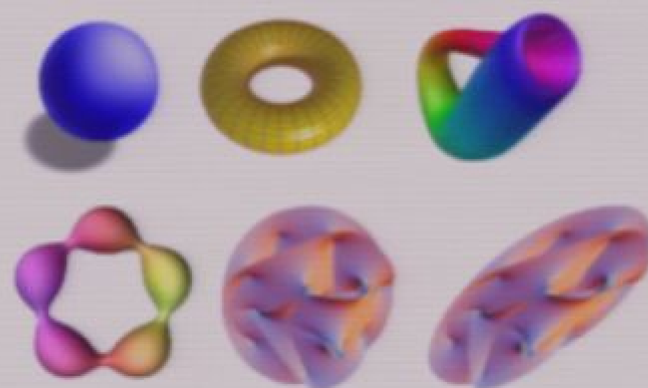
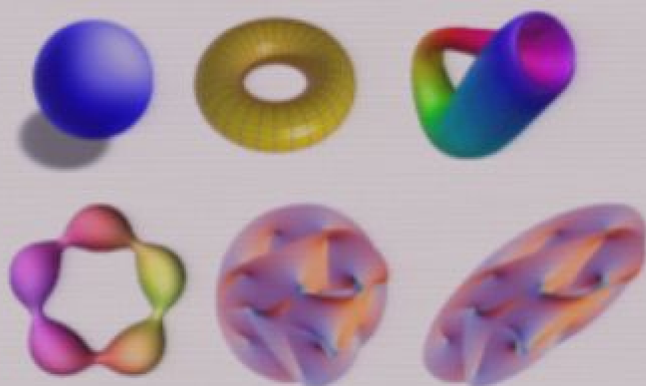
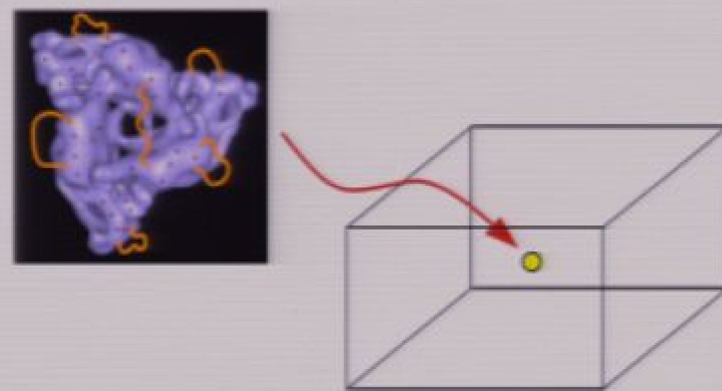


Quantum gravity (string theory) seems to imply reversal of usual complexity - fundamentalness relation:

1. Landscape of compactifications

Complexity in fundamental physics





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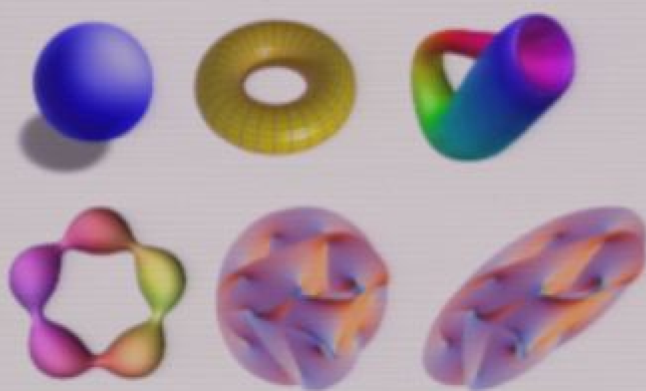

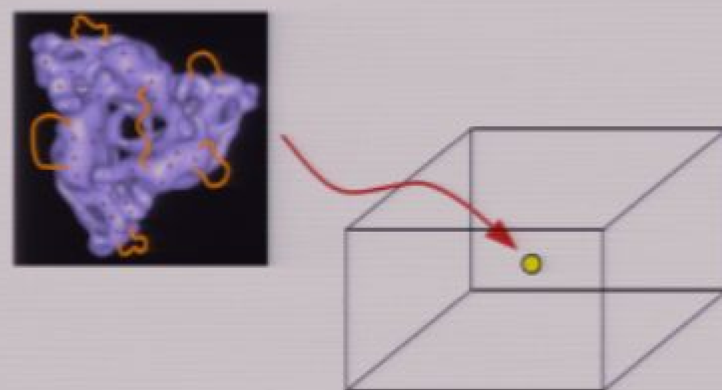

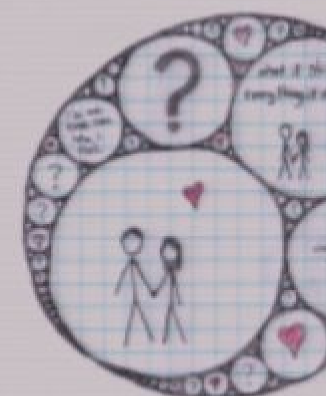

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   - All string constructions are of high complexity [KKLT,Silverstein,...]
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4. Approach to understand state space geometry quantum cosmology

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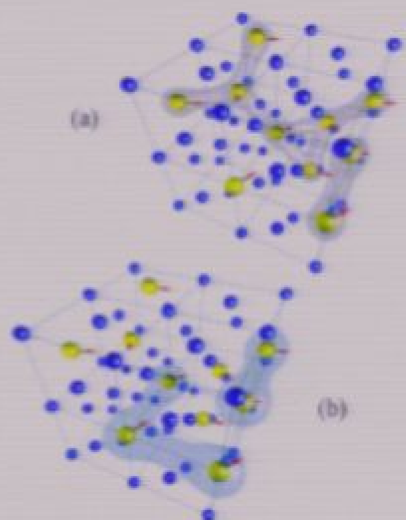

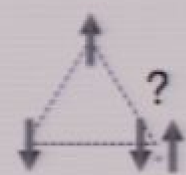

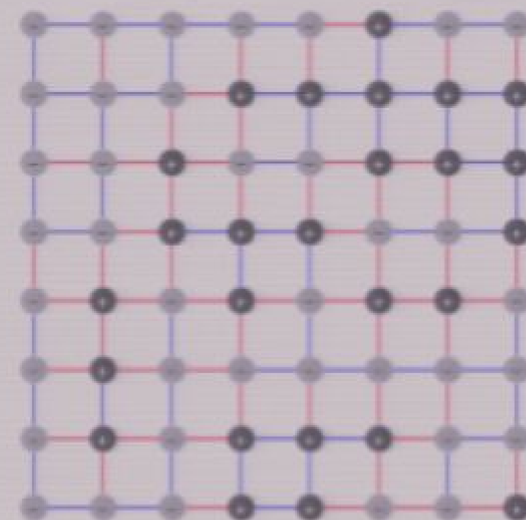

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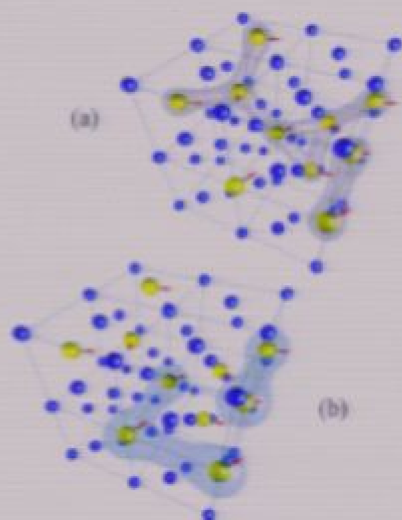
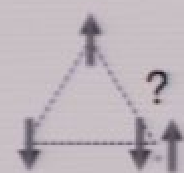
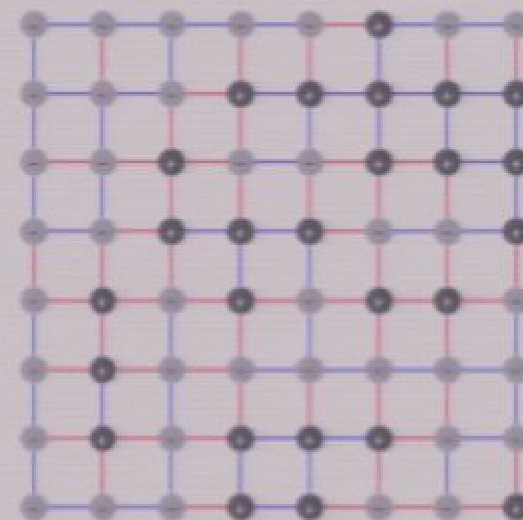
Spin glasses







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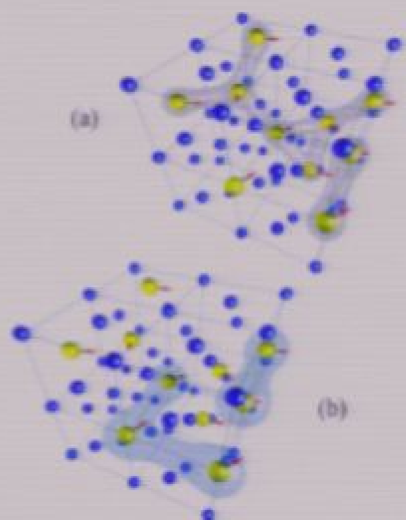

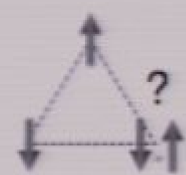

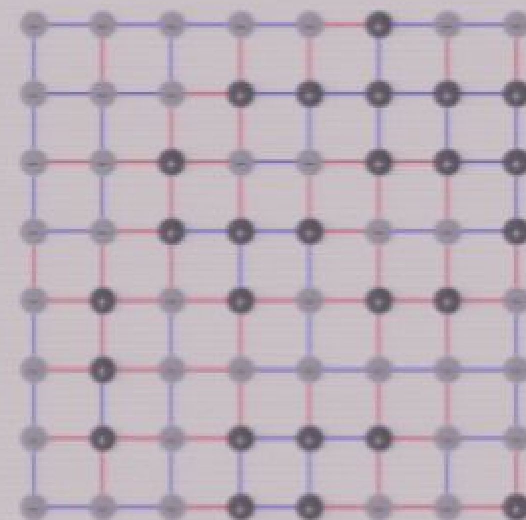

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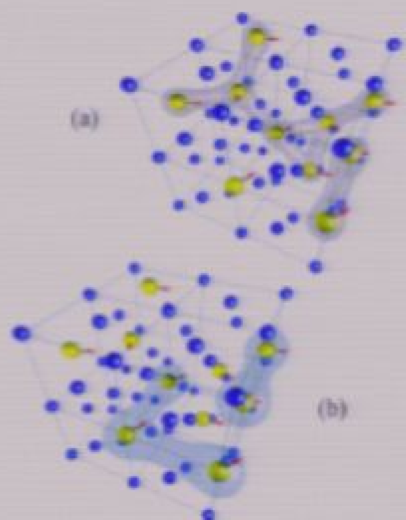

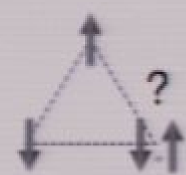

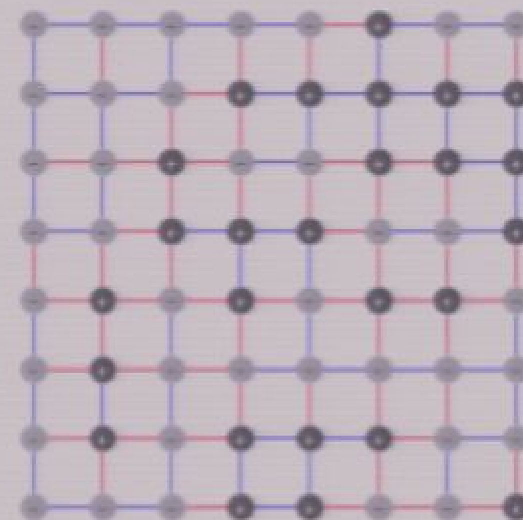

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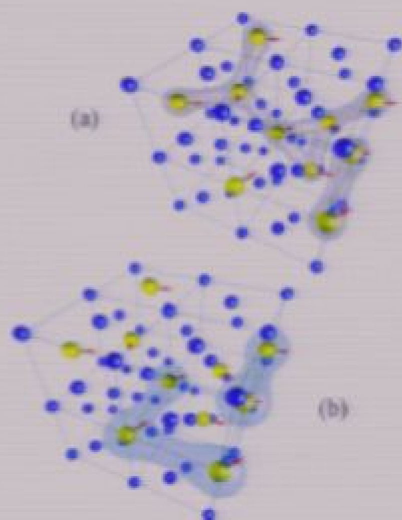

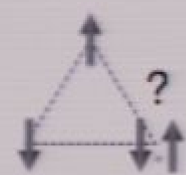

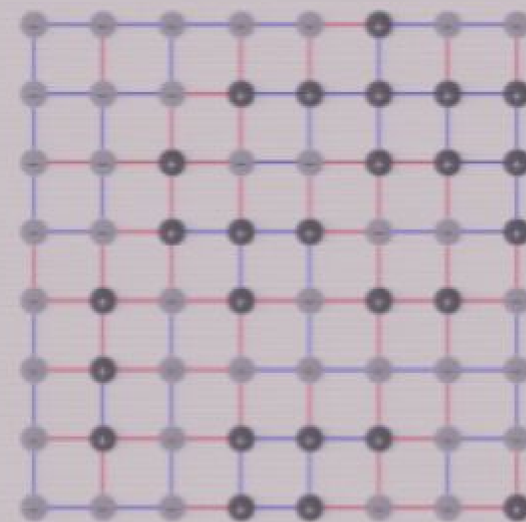

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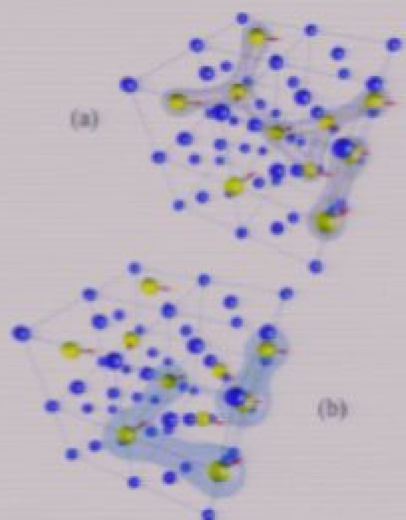

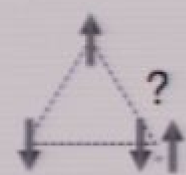

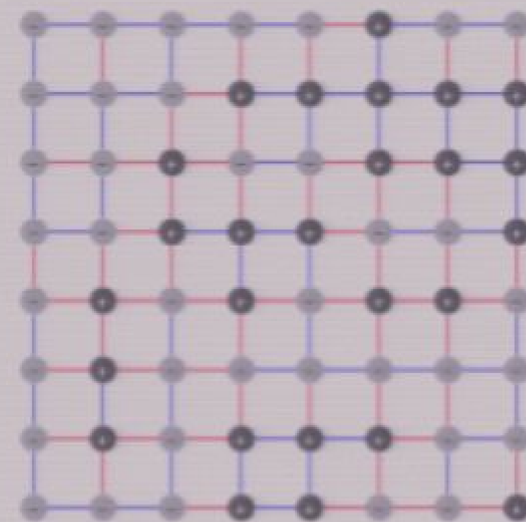

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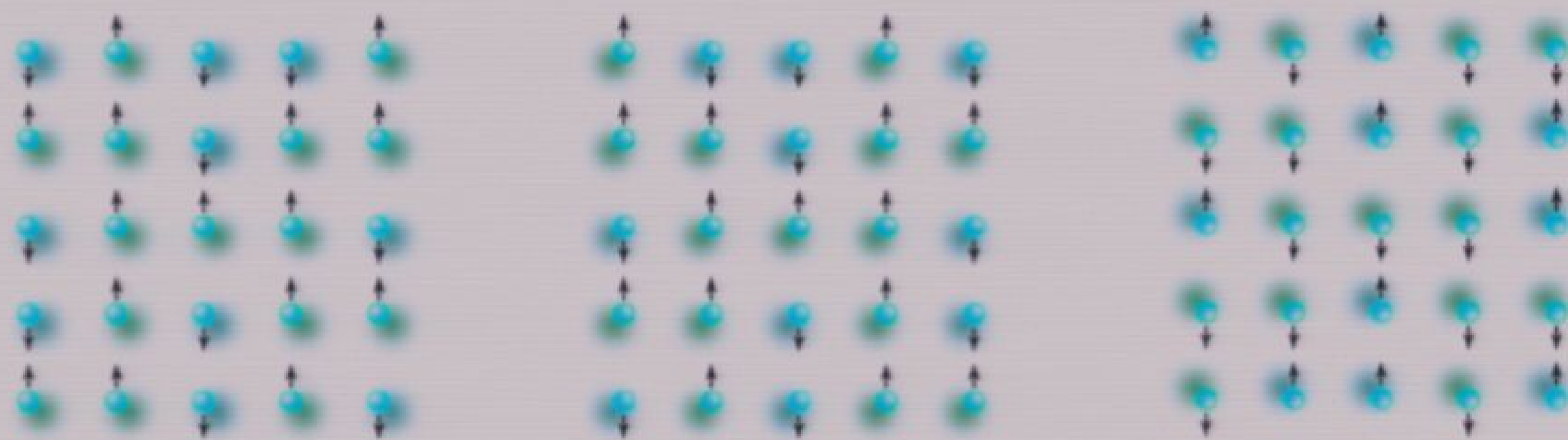
State space structure of the SK model



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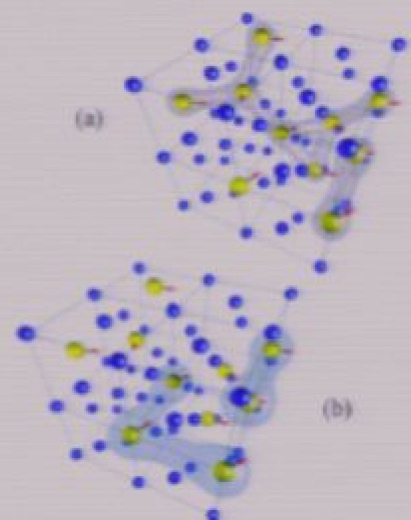

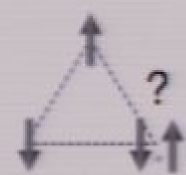

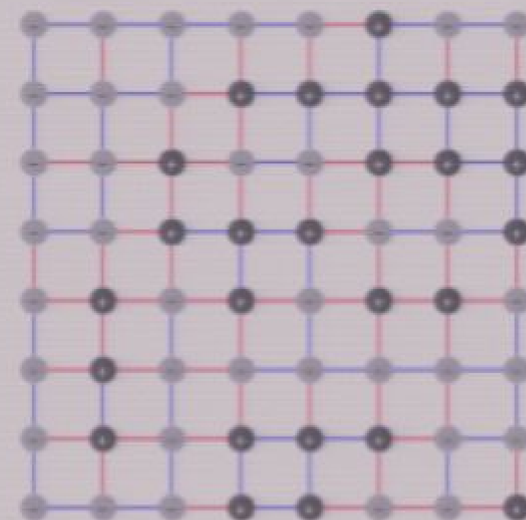

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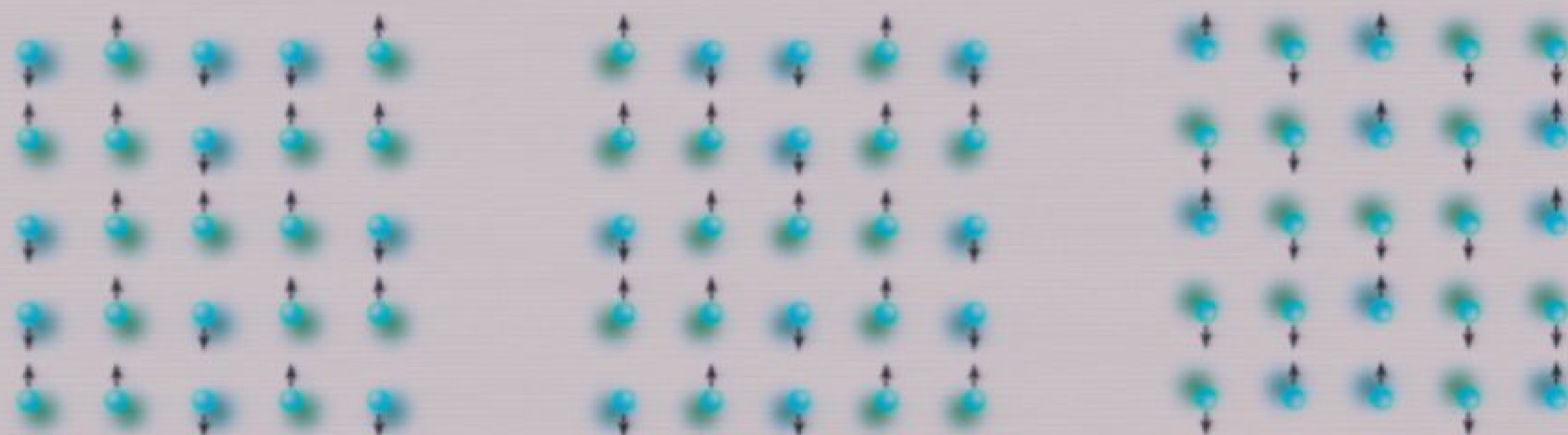

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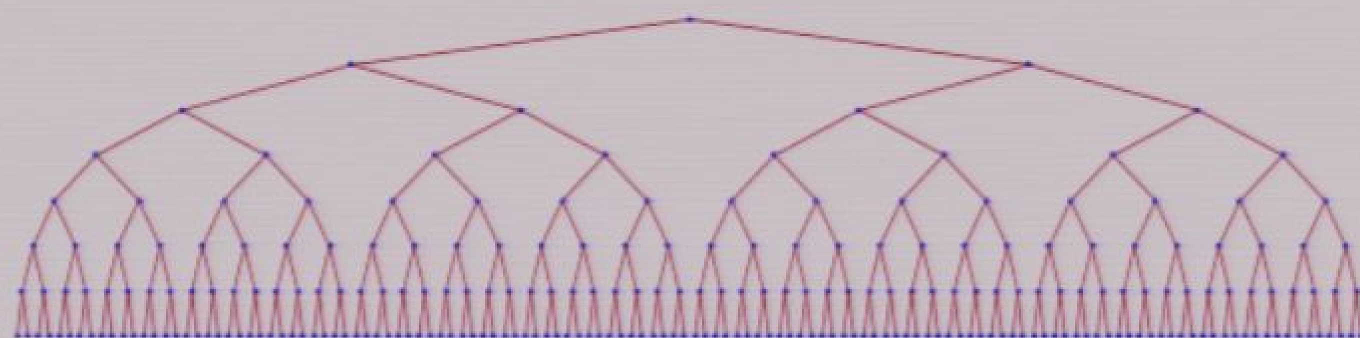

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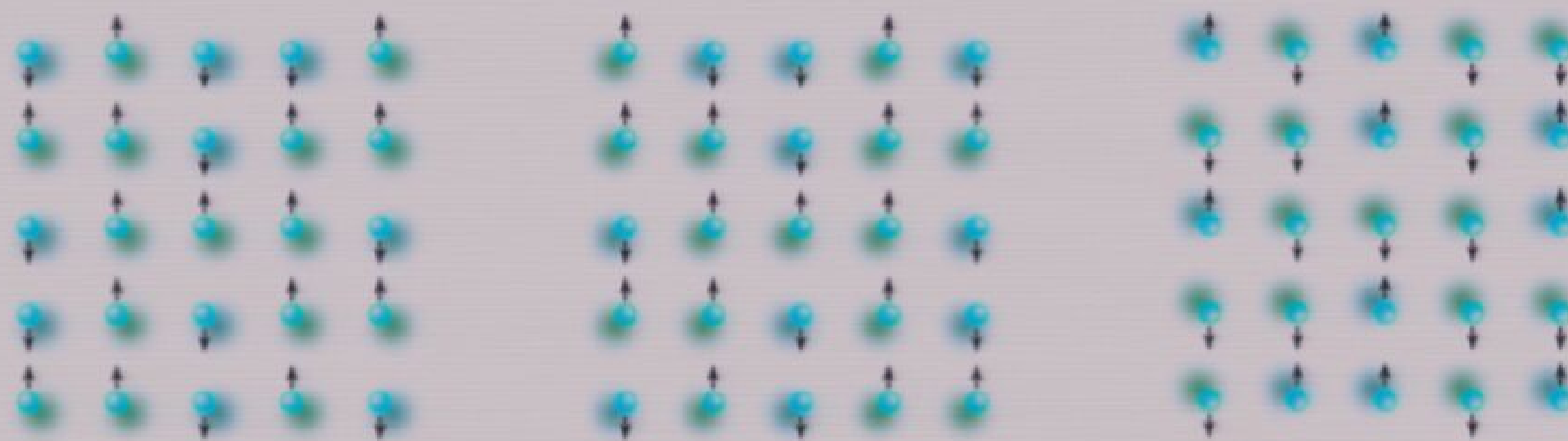

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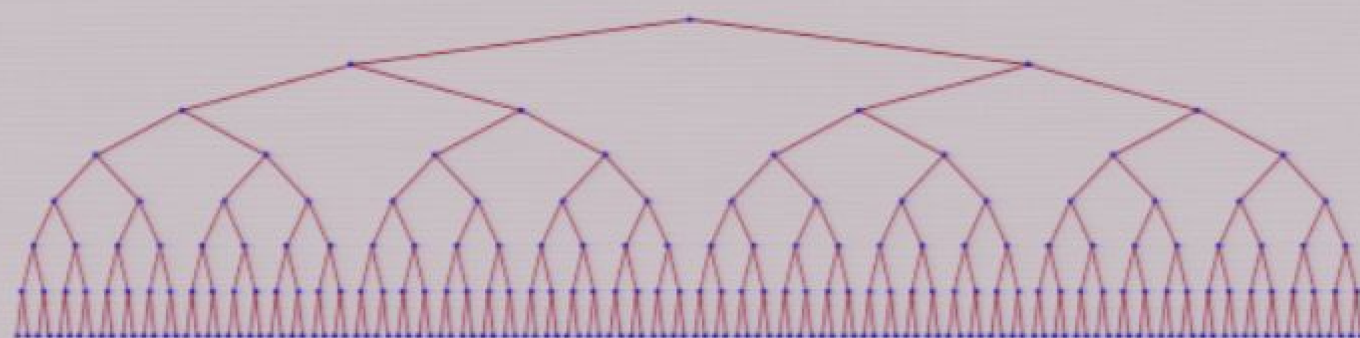

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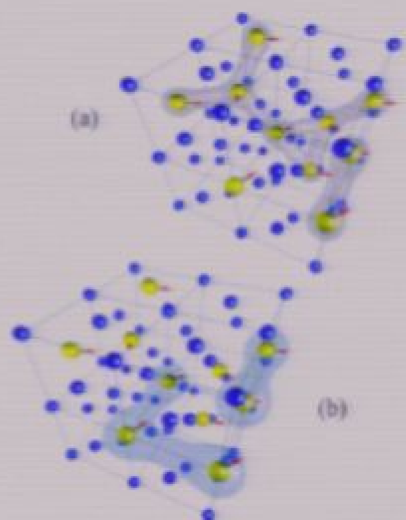

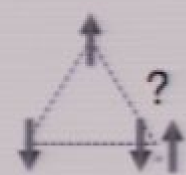

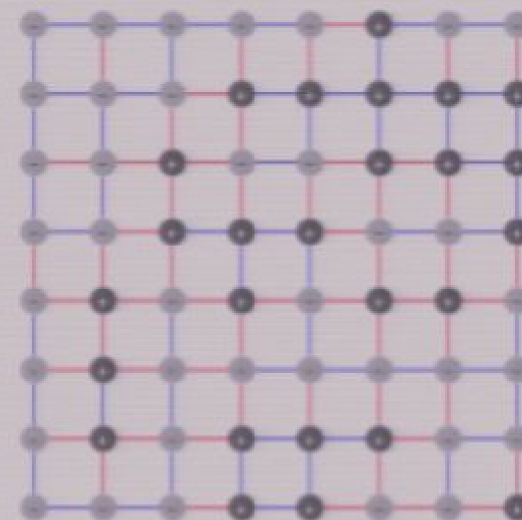

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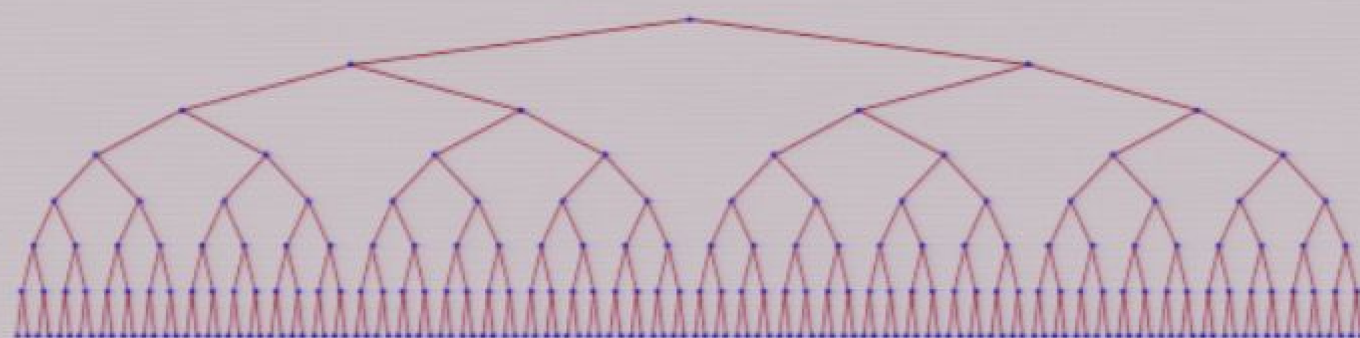

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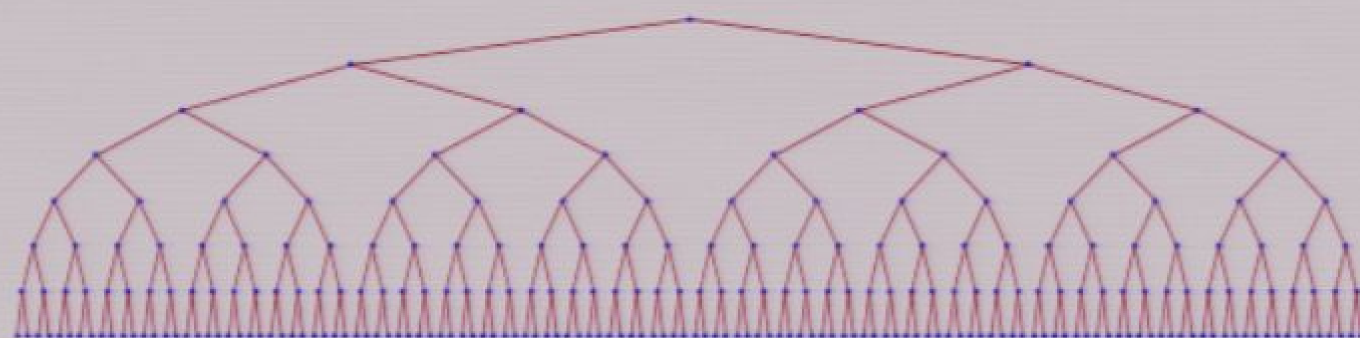

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- Spin glass: $p_G(s) = \sum_\alpha w_\alpha \, p_\alpha(s) \rightsquigarrow p_\alpha(s) =?$.
  Also: $M = 0$ for all T.

Parisi's overlap "order" "parameter"

- Definition equilibrium state ("pure state"): p_α such that cluster decomposition holds: $\lim_{N\to\infty}\frac{1}{N^2}\sum_{ij}|\langle\mathcal{O}_i\mathcal{O}_j\rangle_\alpha - \langle\mathcal{O}_i\rangle_\alpha\langle\mathcal{O}_j\rangle_\alpha| = 0$

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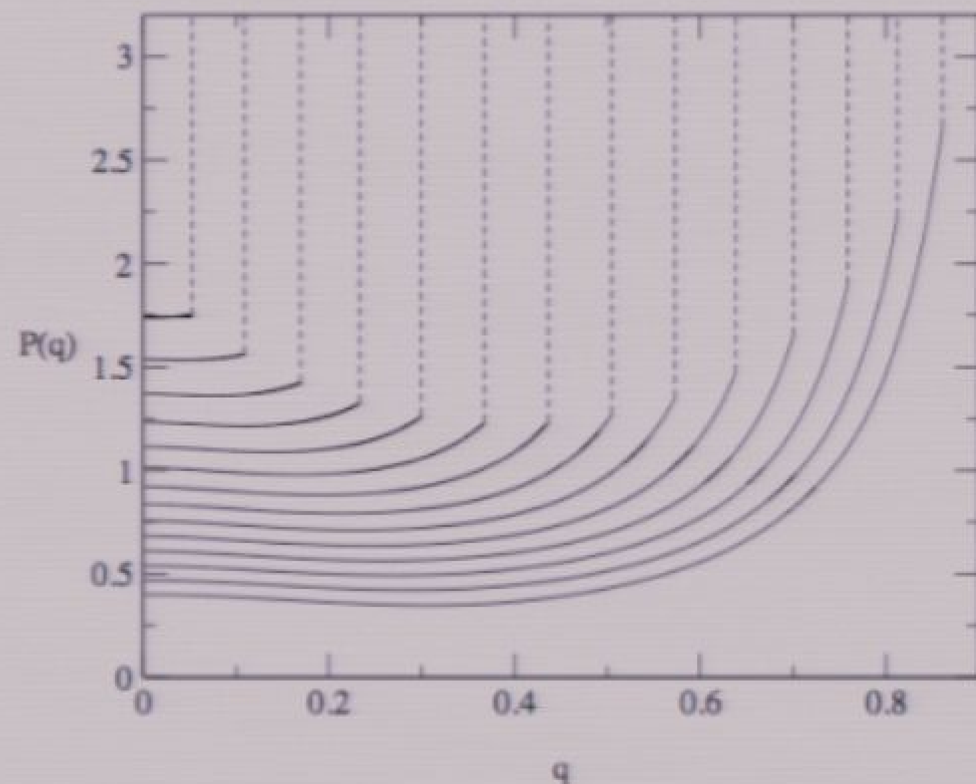
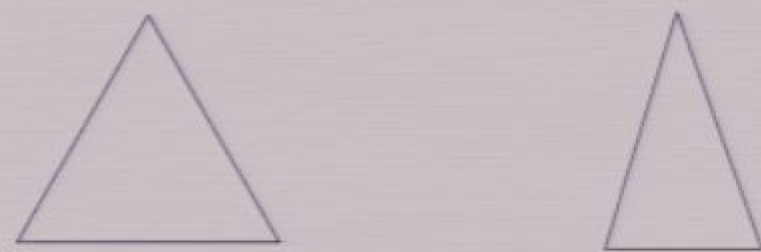
- Multiple pure states $\Leftrightarrow P(q)$ nontrivial.

Result for SK model



$\overline{P(q)}$ for $T/T_c = 0.95, 0.9, \ldots, 0.3$ [Crisanti-Rizzo '02]

Ultrametricity



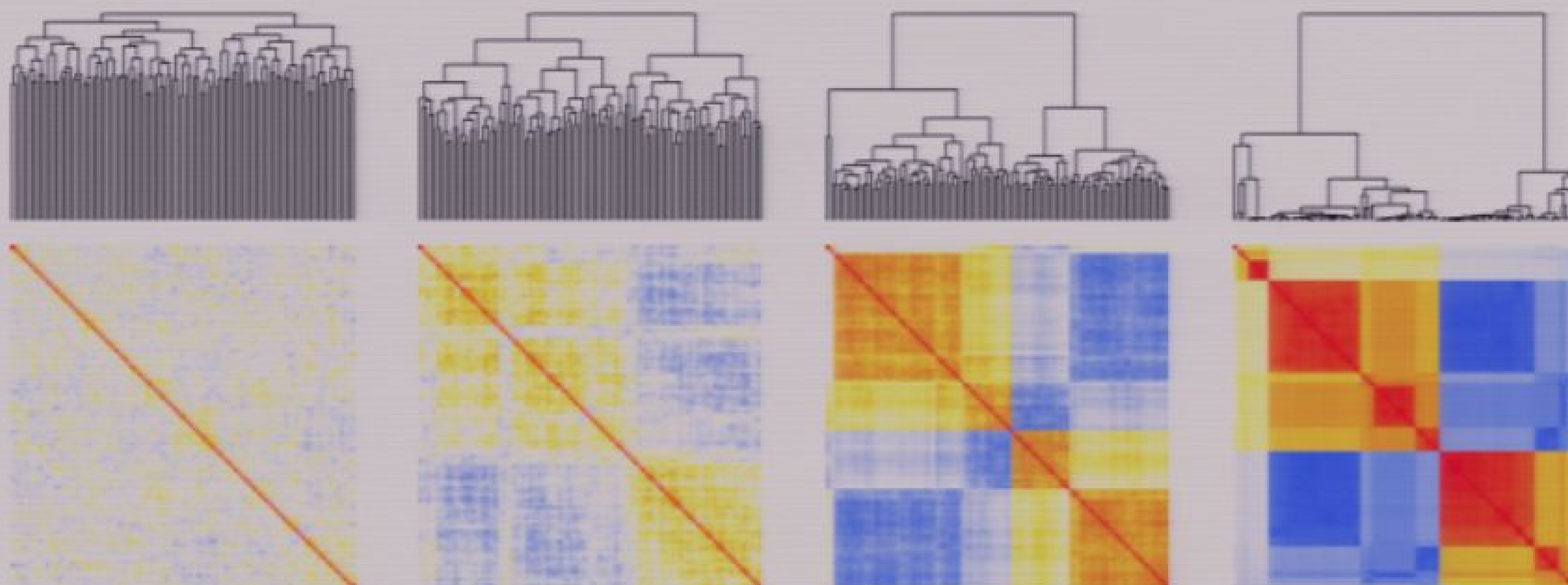
- Triangle distribution:

$$P(q_1, q_2, q_3) \equiv \sum_{\alpha\beta\gamma} w_\alpha w_\beta w_\gamma \, \delta(q_1 - q_{\beta\gamma})\delta(q_2 - q_{\gamma\alpha})\,\delta(q_3 - q_{\alpha\beta})$$

- Result for SK model, $q_i \geq 0$:

$$\overline{P(q_1, q_2, q_3)} = \frac{1}{2}\int_0^{q_1} dq\, \overline{P(q)}\,\overline{P(q_1)}\,\delta(q_1 - q_2)\,\delta(q_2 - q_3)$$
$$+\frac{1}{2}\overline{P(q_1)}\,\overline{P(q_2)}\,\Theta(q_1 - q_2)\,\delta(q_2 - q_3) + \text{perm.}$$

Monte Carlo simulations



N=800 simulation at $T/T_c = 1.2, 0.86, 0.55, 0.12$

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2. Inspiration
   - Spin glass models
   - State space structure of the SK model
   - Overlap order parameter

3. Application to string theory and cosmology
   - Generalized overlap order parameters
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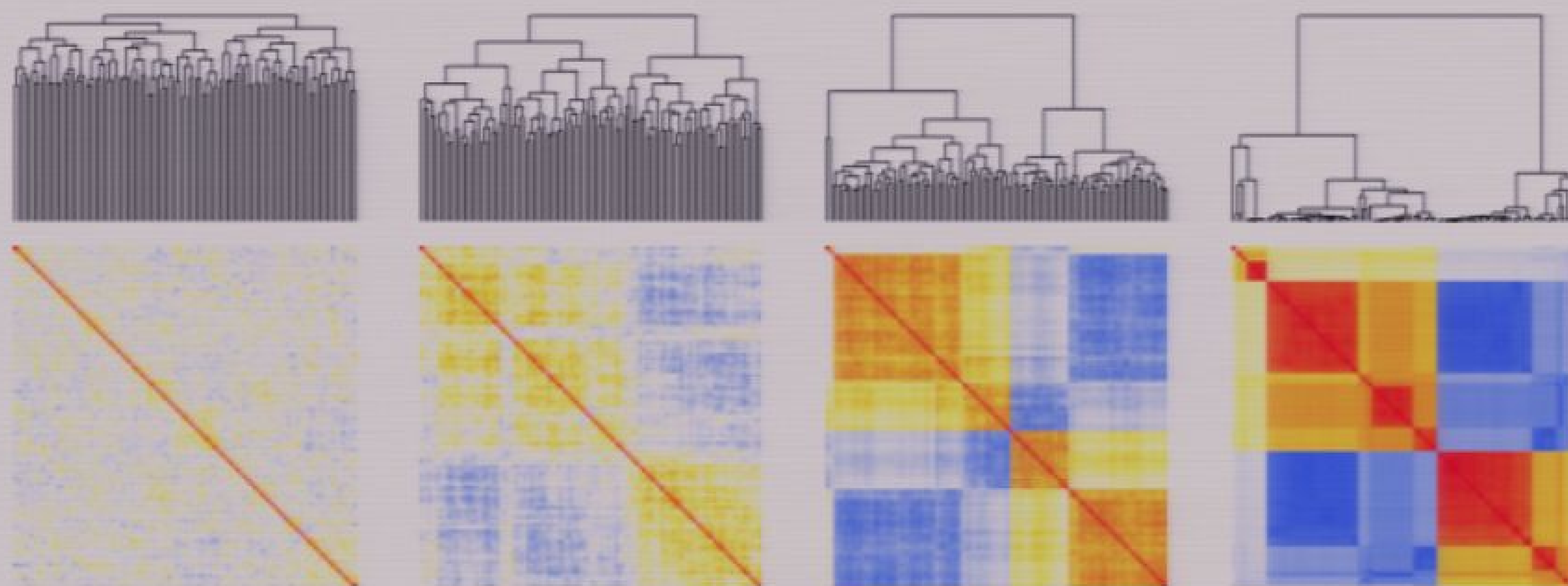

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Generalization $P(q)$ from classical Ising spins to general, nonlinear spac and quantum systems?

Several possibilities

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- ϕ_i, $i = 1, \cdots, N$ subset of d.o.f. of system.
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Replica formula: $P(q) = \langle \delta(q - \frac{1}{N} \sum_i \chi_i) \rangle_{n=2}$, where χ_i is the operator
exchanging ϕ_i of the two replicas.

String glasses

(Partial) results for:

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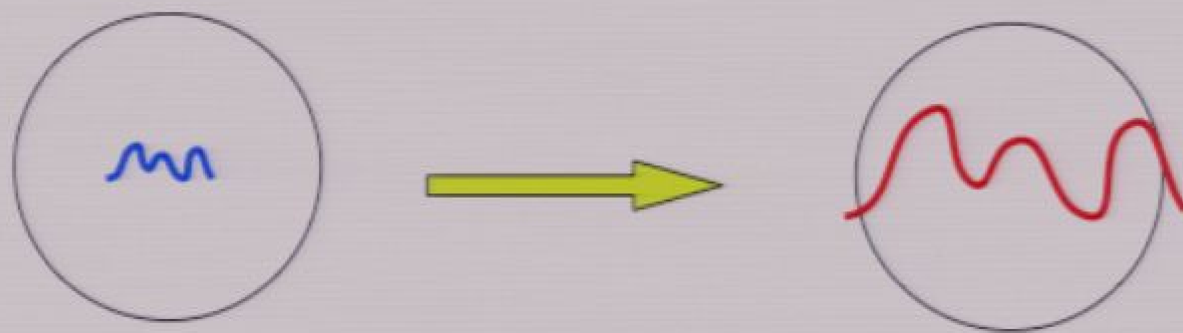
Cosmology



- Quantum fluctuations get exponentially stretched to super-Hubble scales and freeze, becoming effectively classical.
- This dynamically generates analog of spin glass "pure states", in which scalars, metric,... have definite values on large scales and cluster decomposition holds. (Not the case for HH/BD/Euclidean vacuum.)
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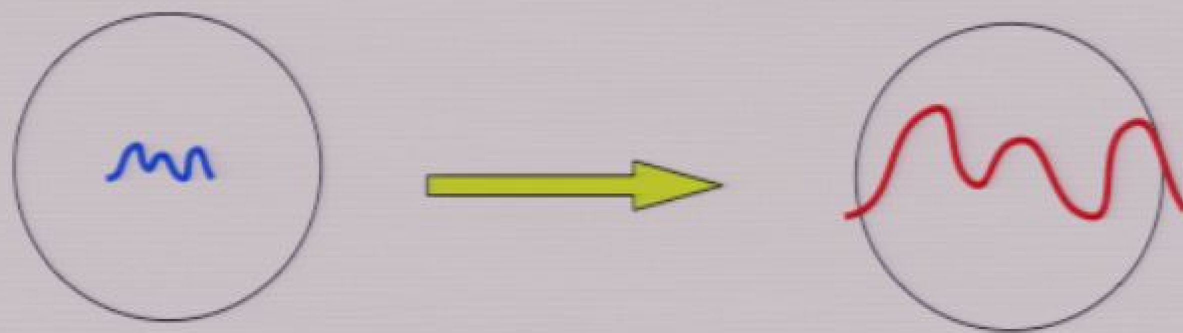

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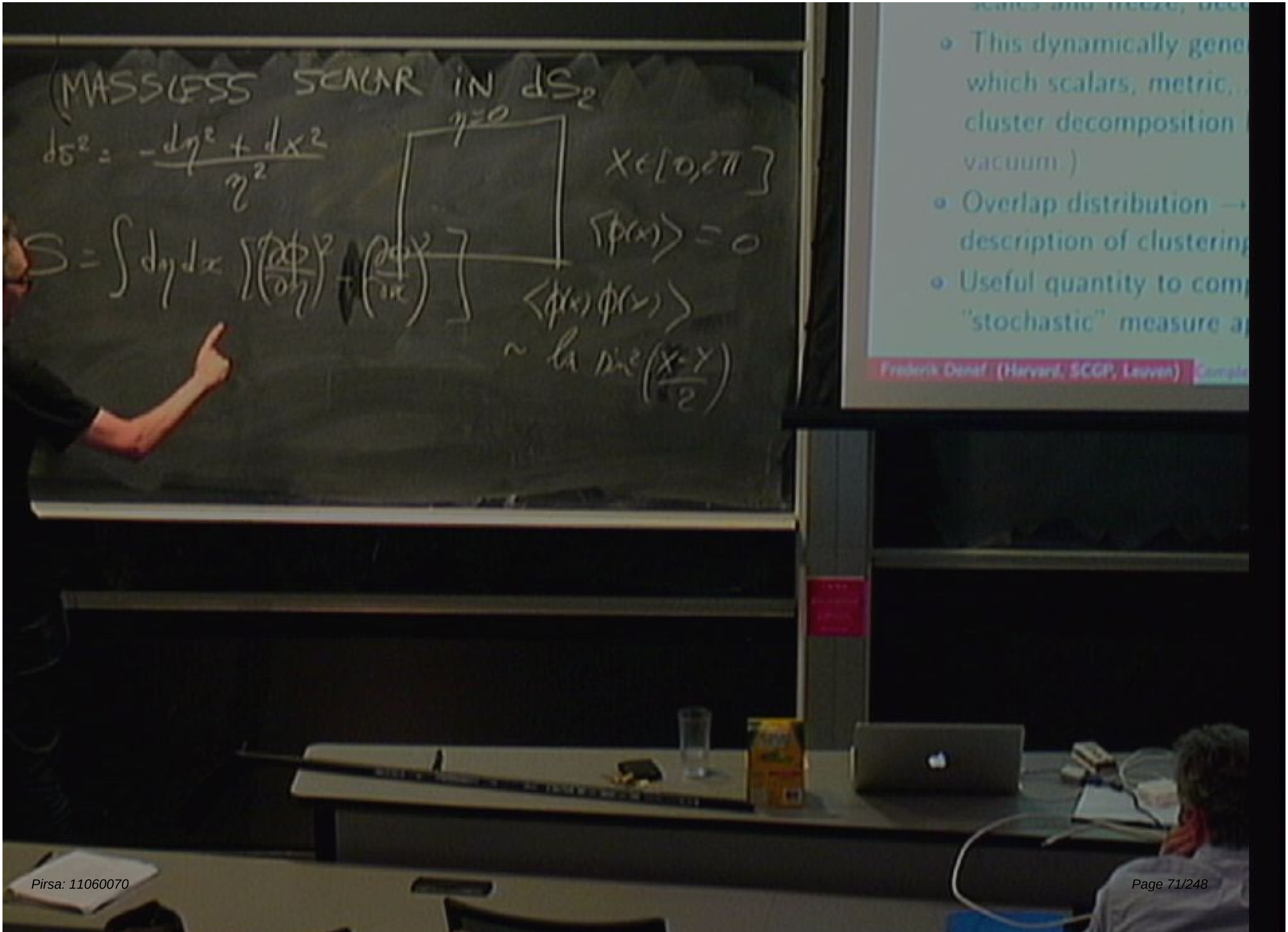
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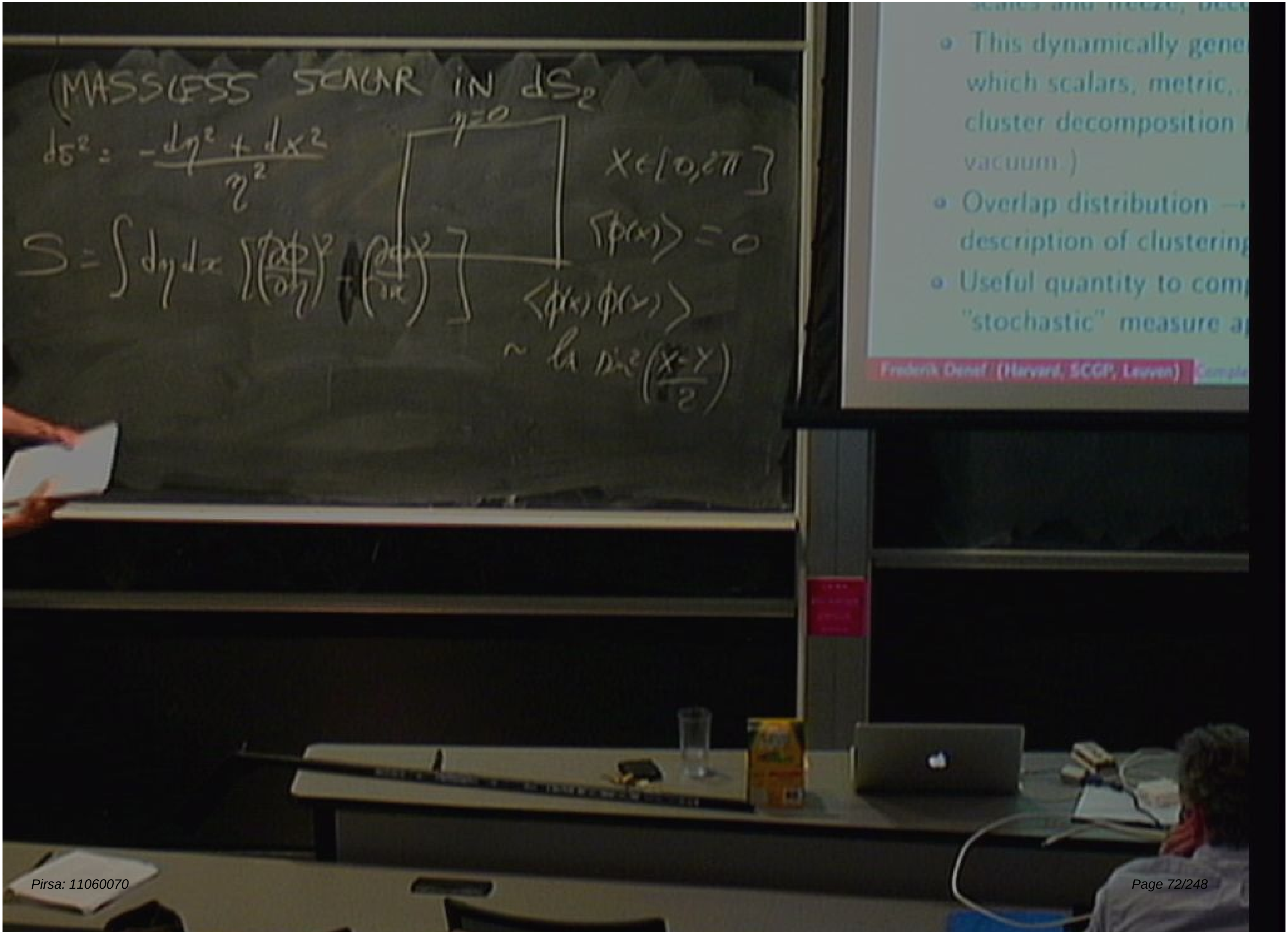
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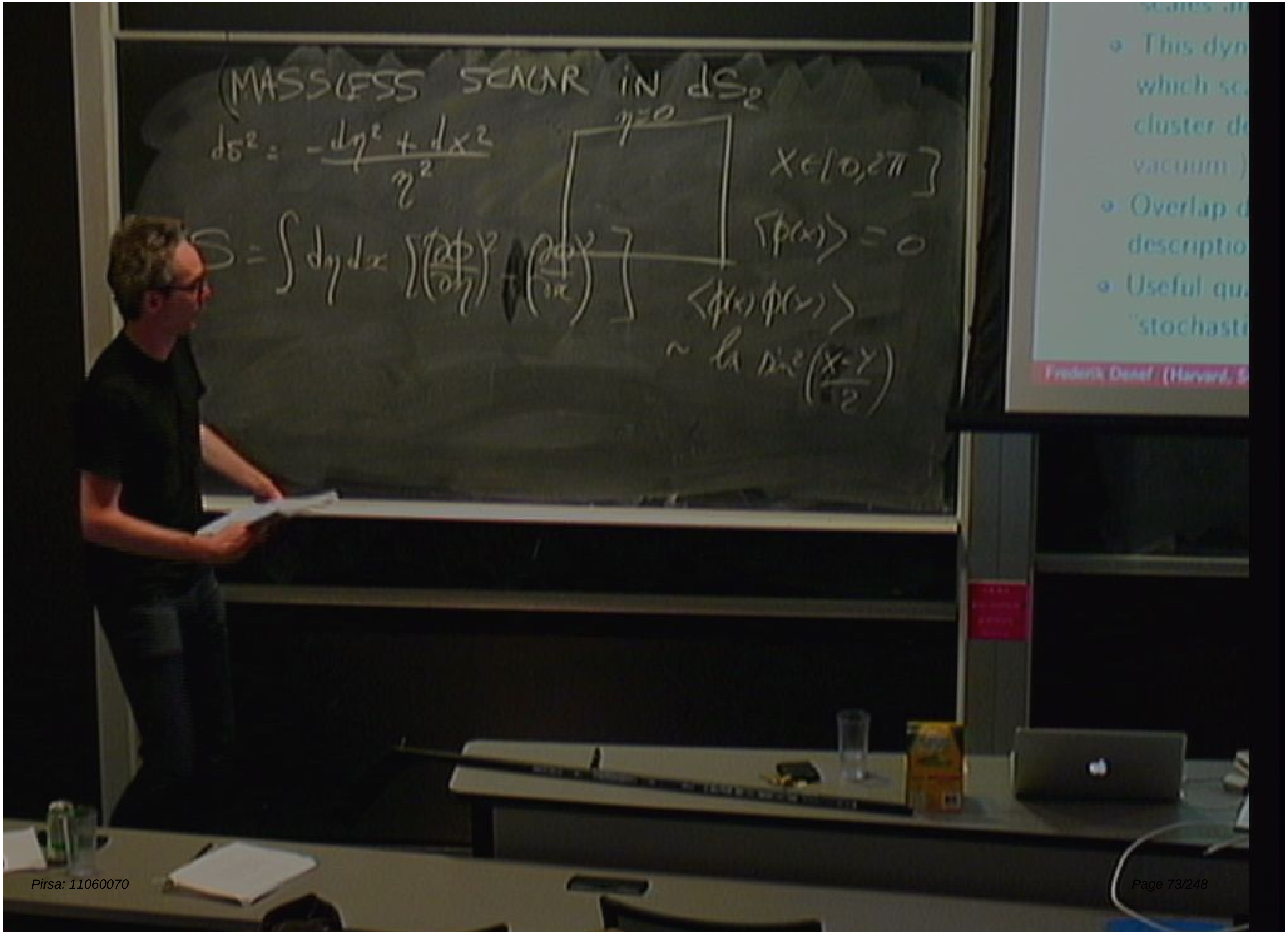
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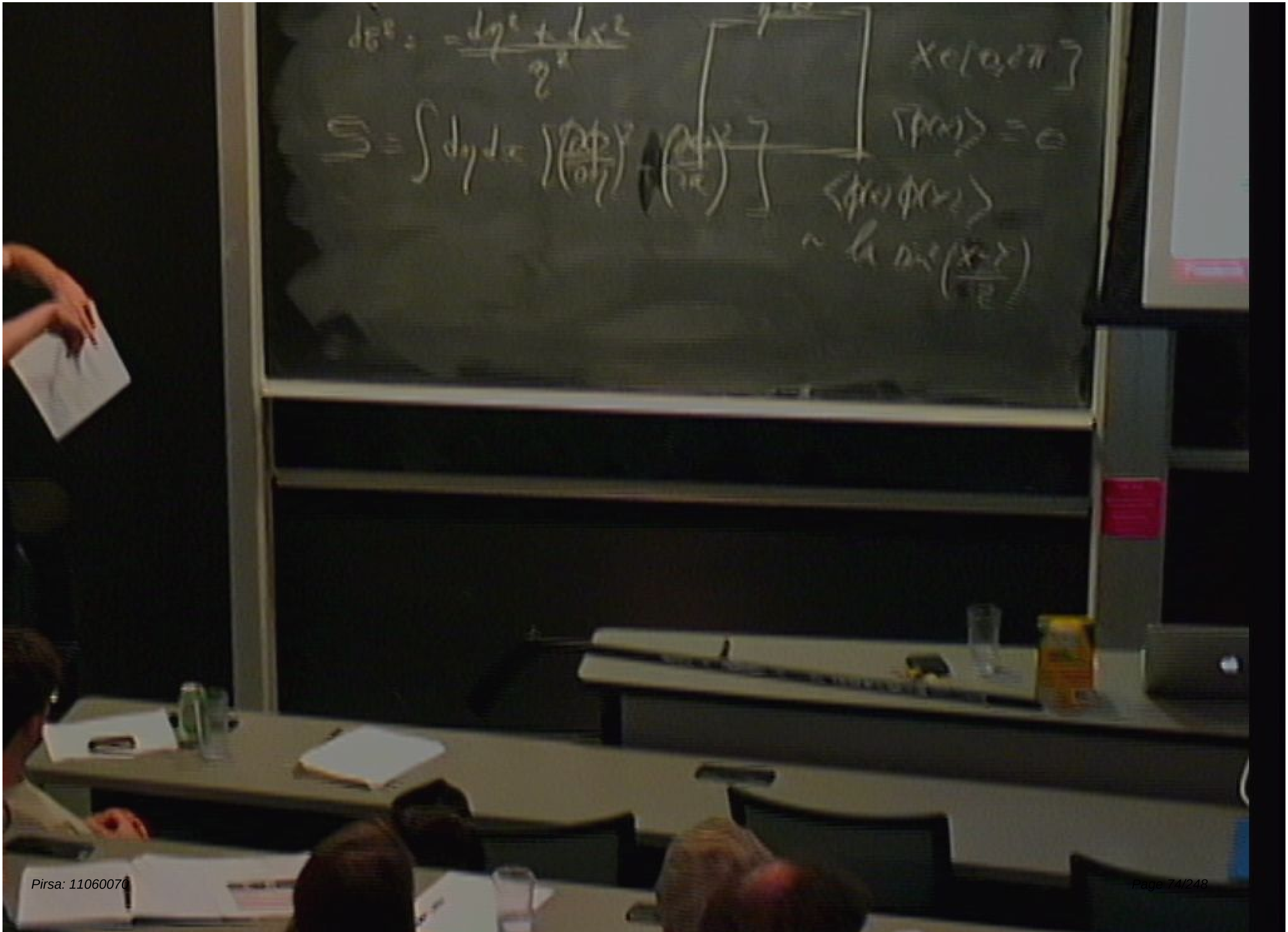

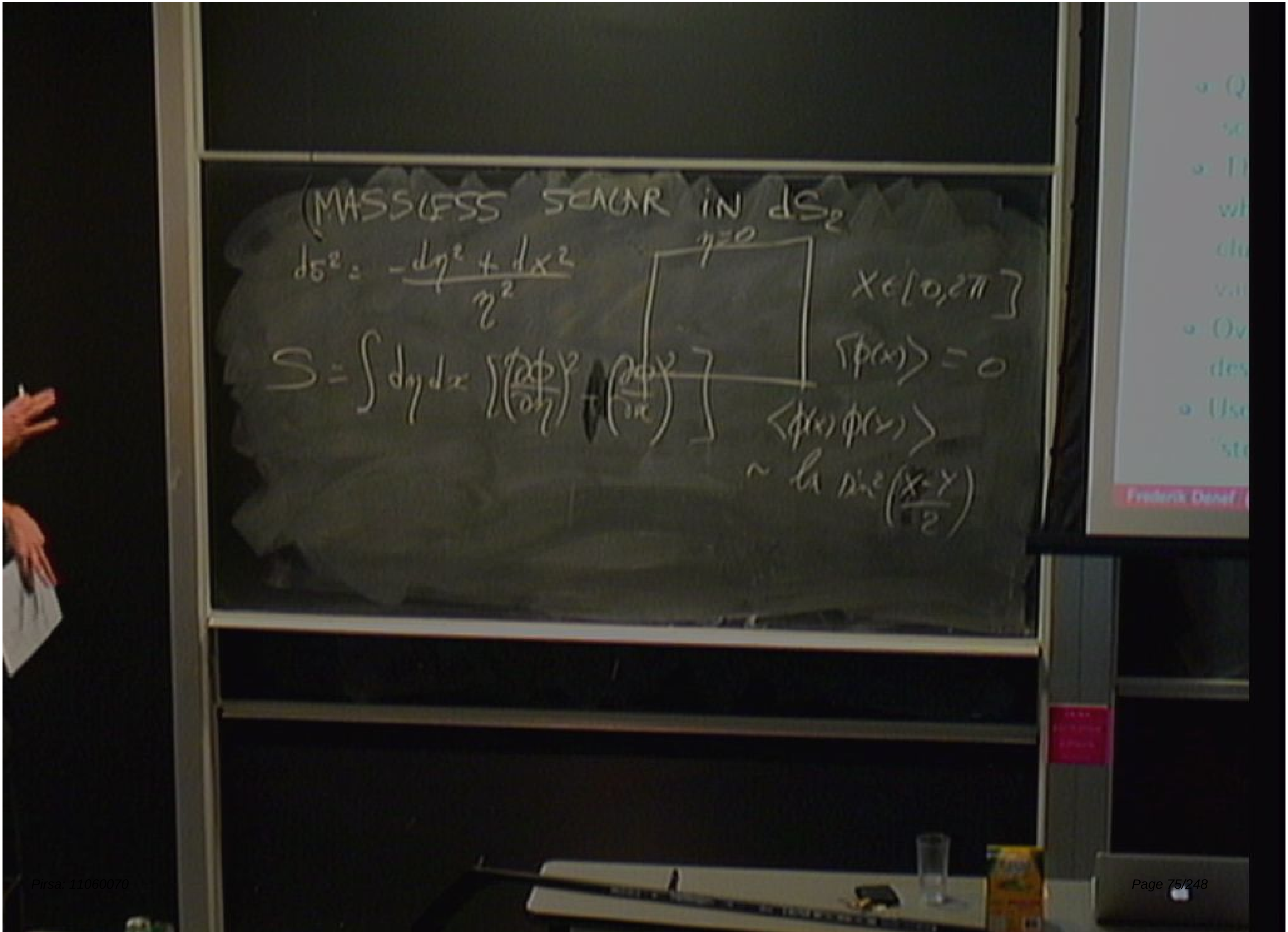
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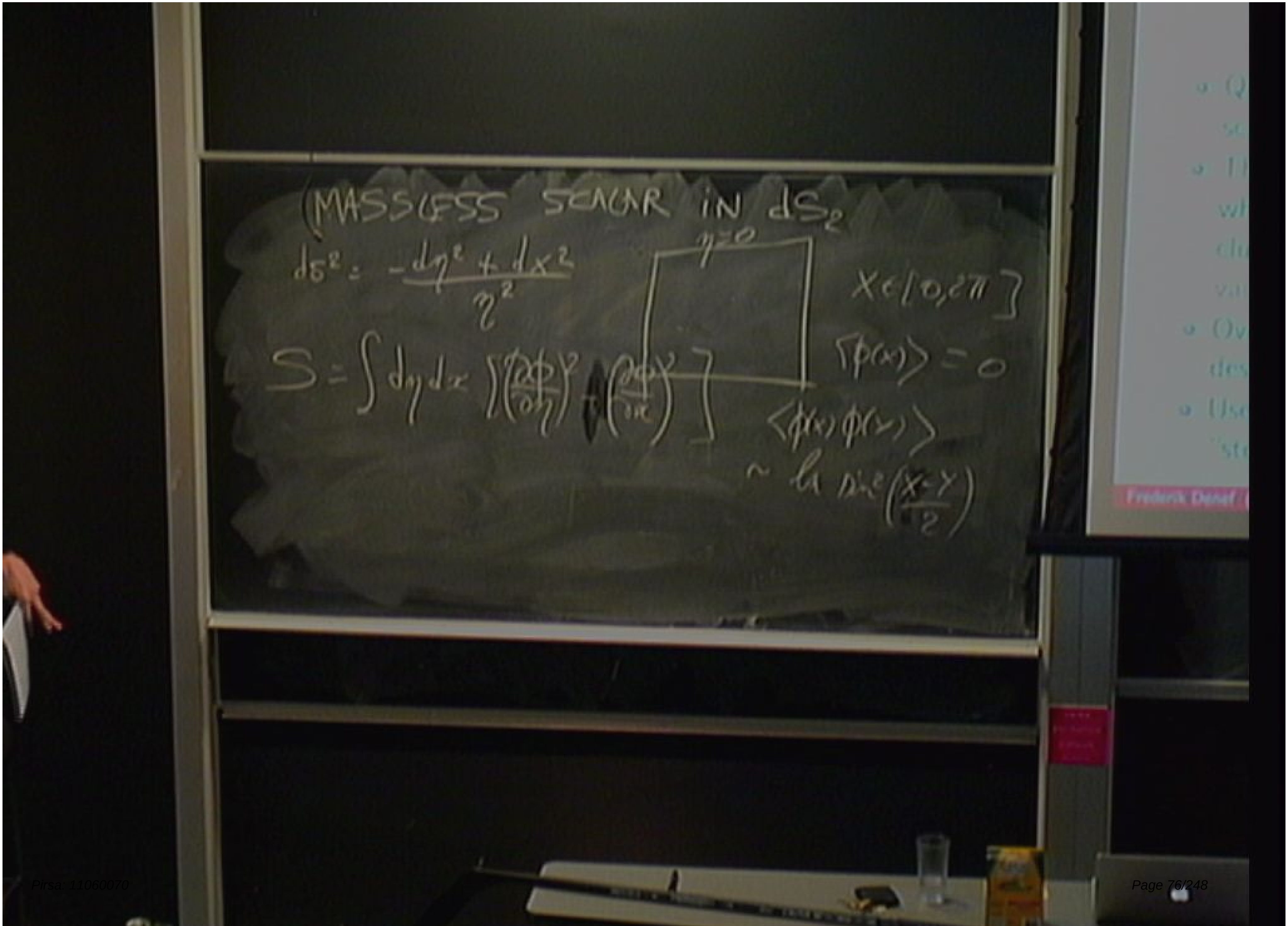
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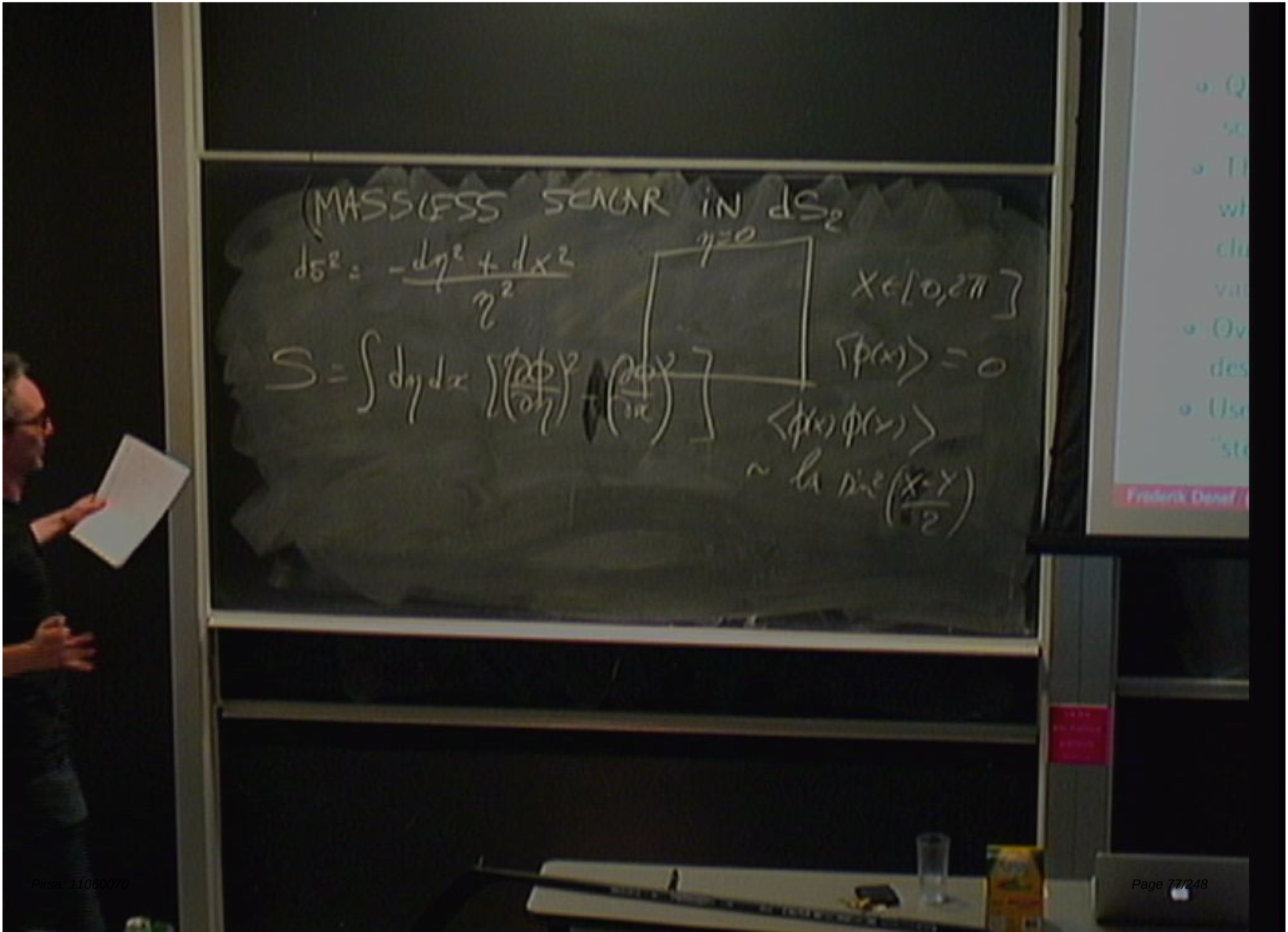
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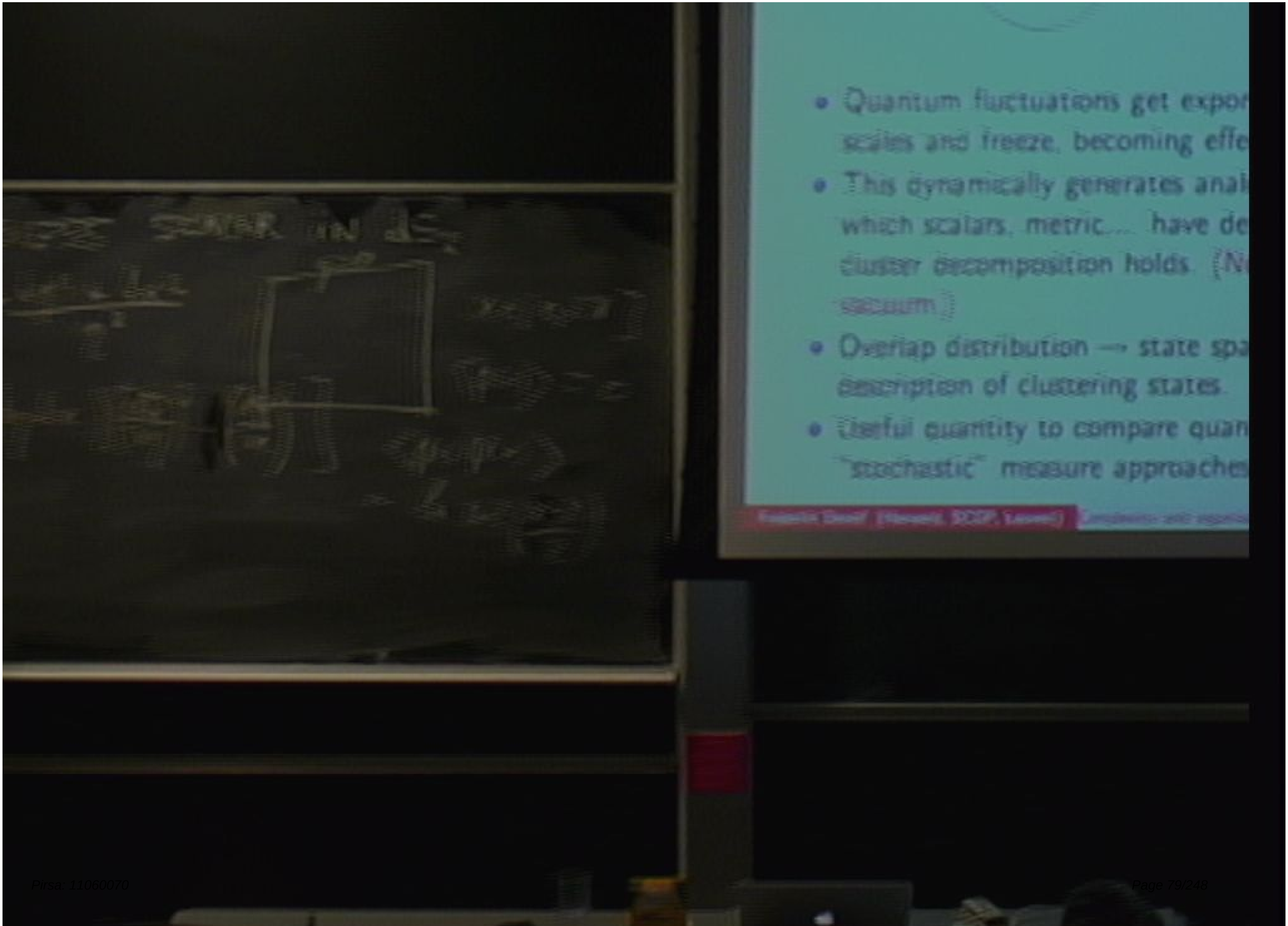
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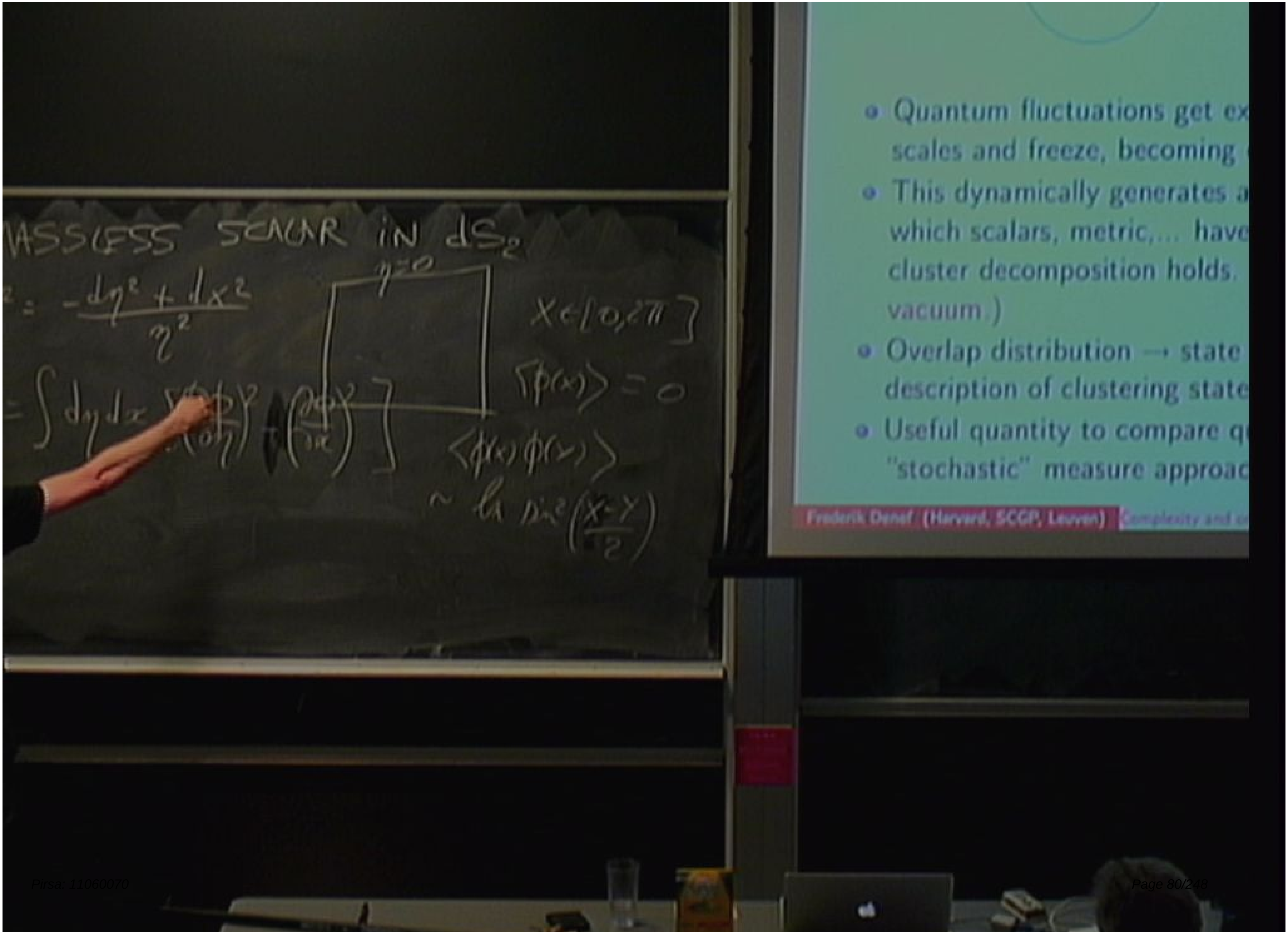
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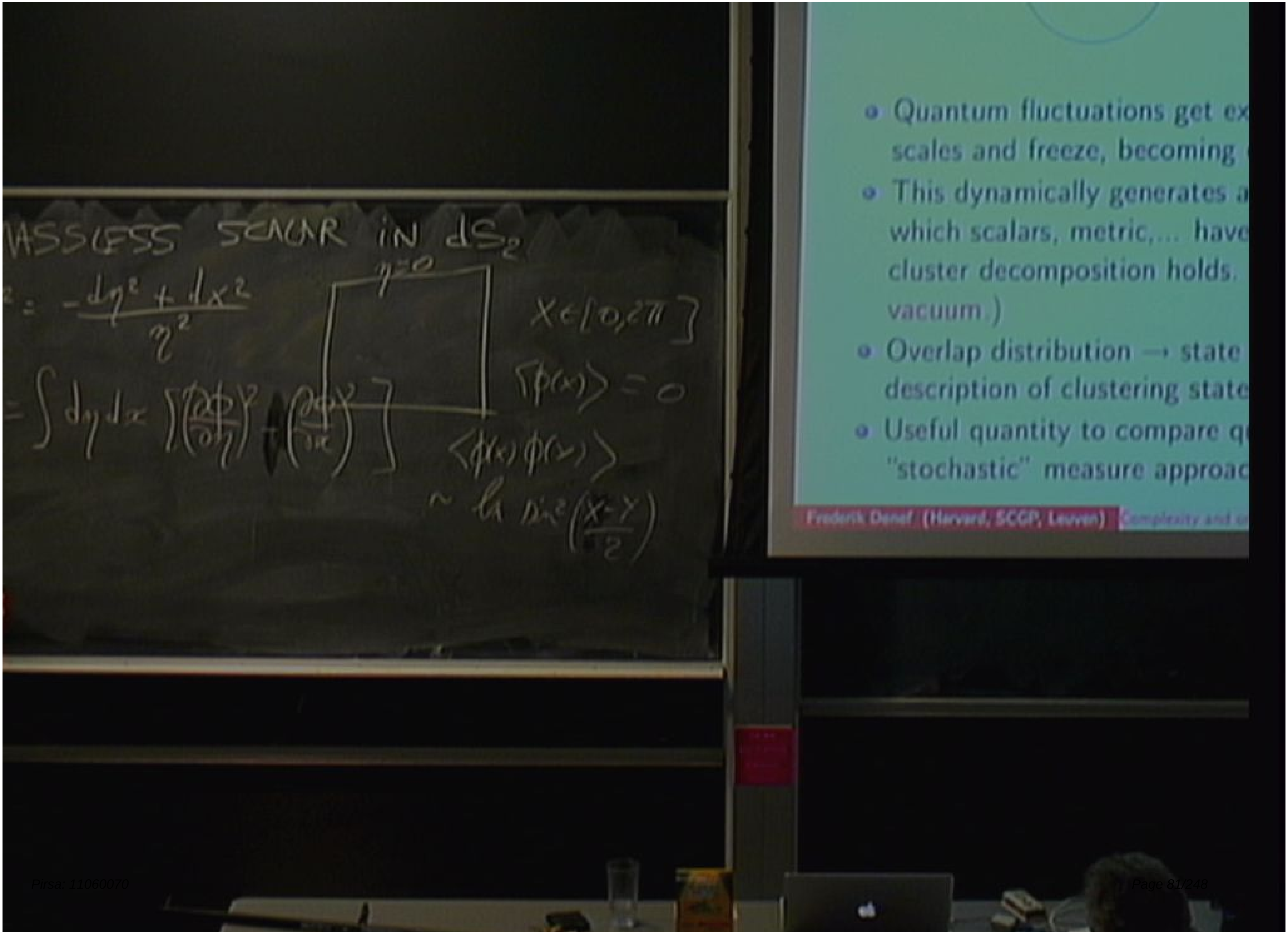
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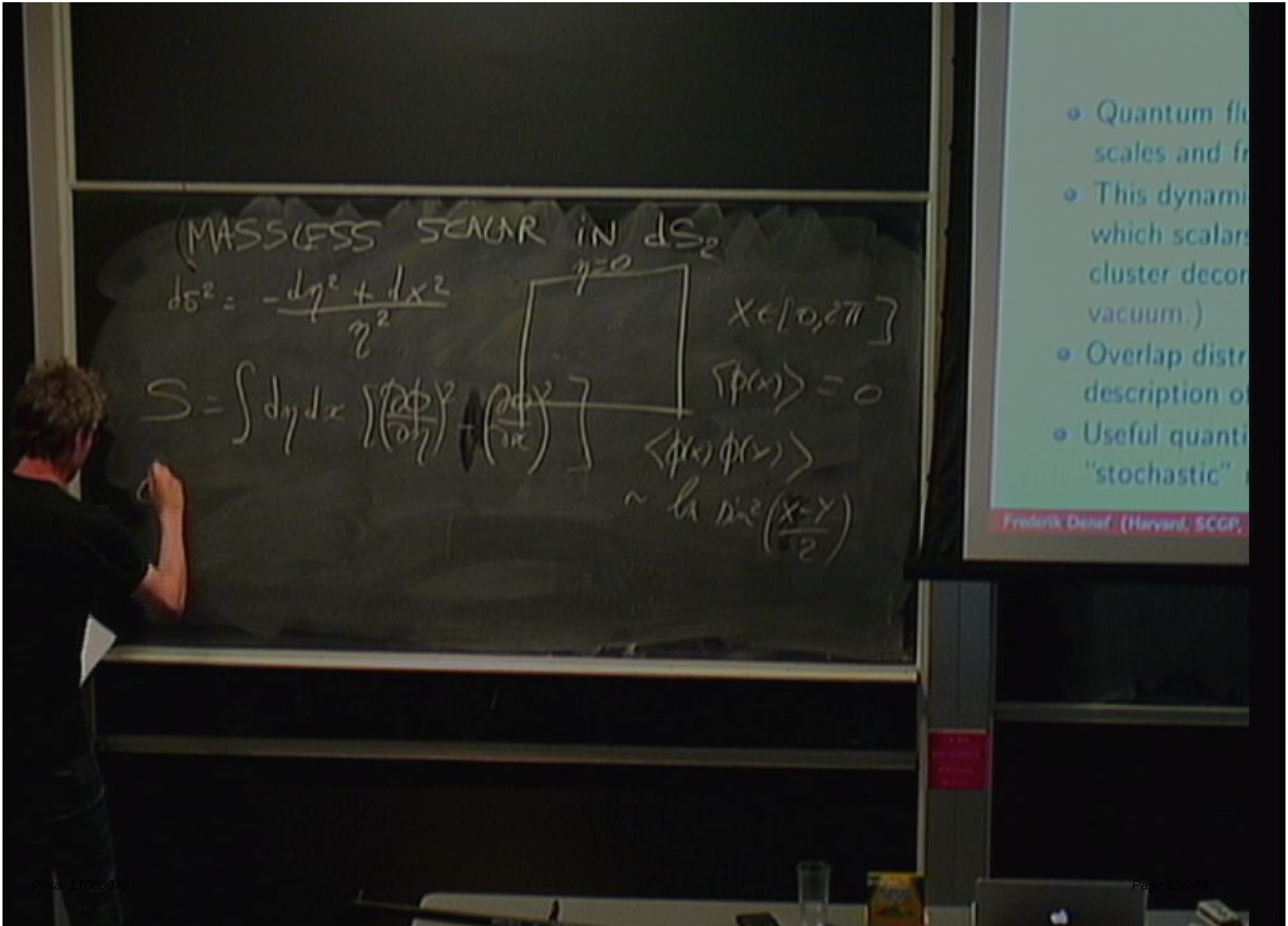
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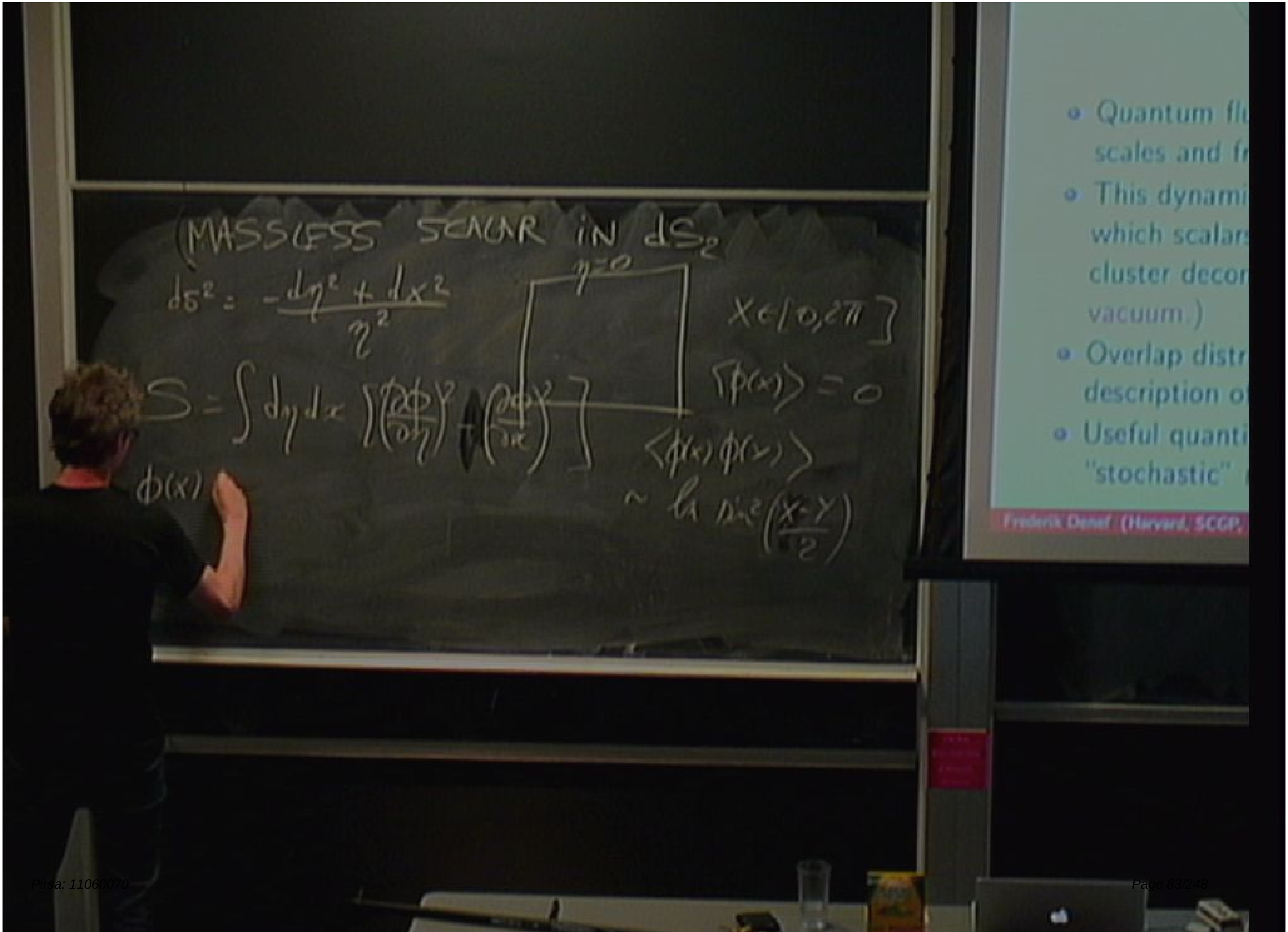
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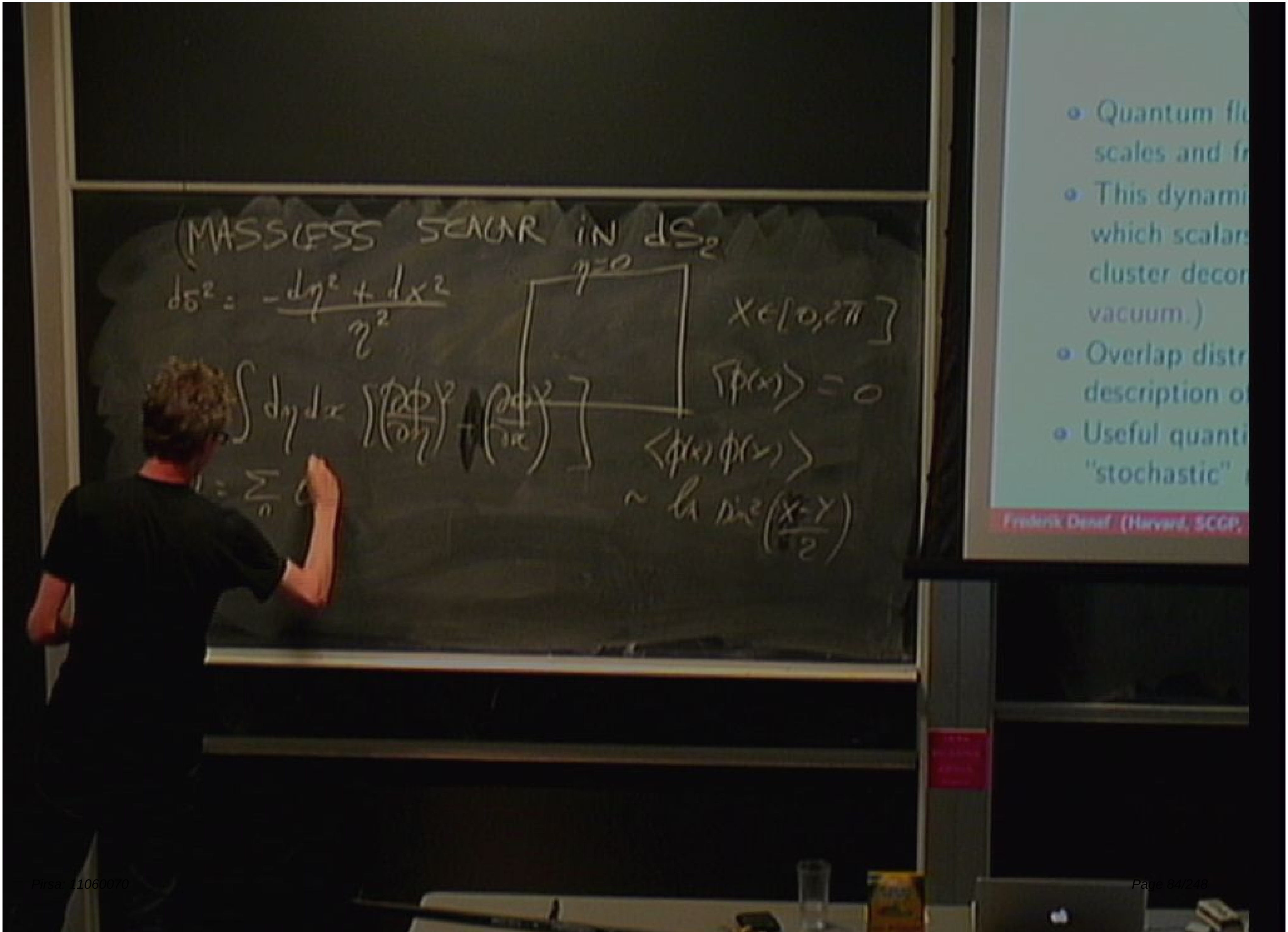
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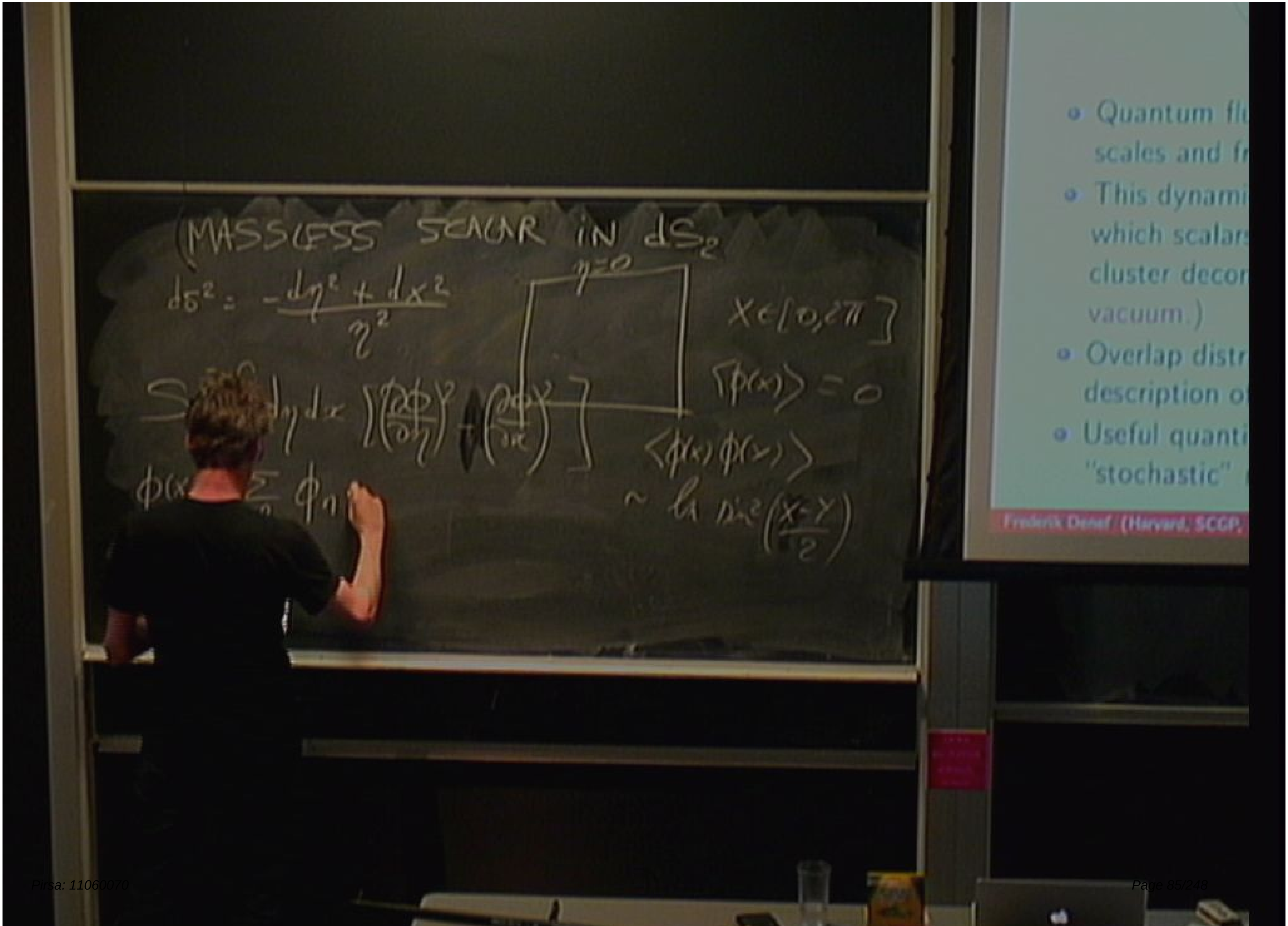

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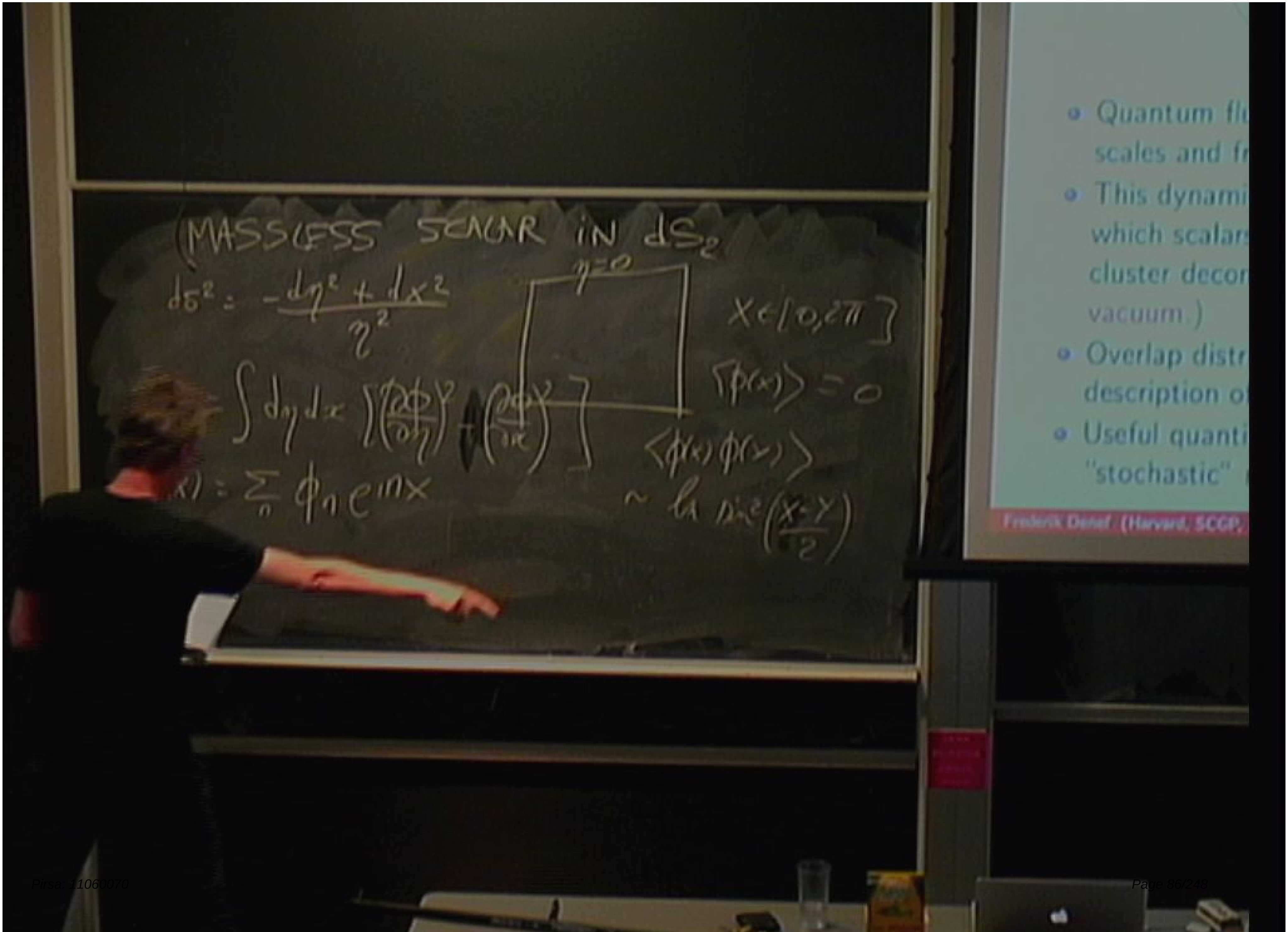
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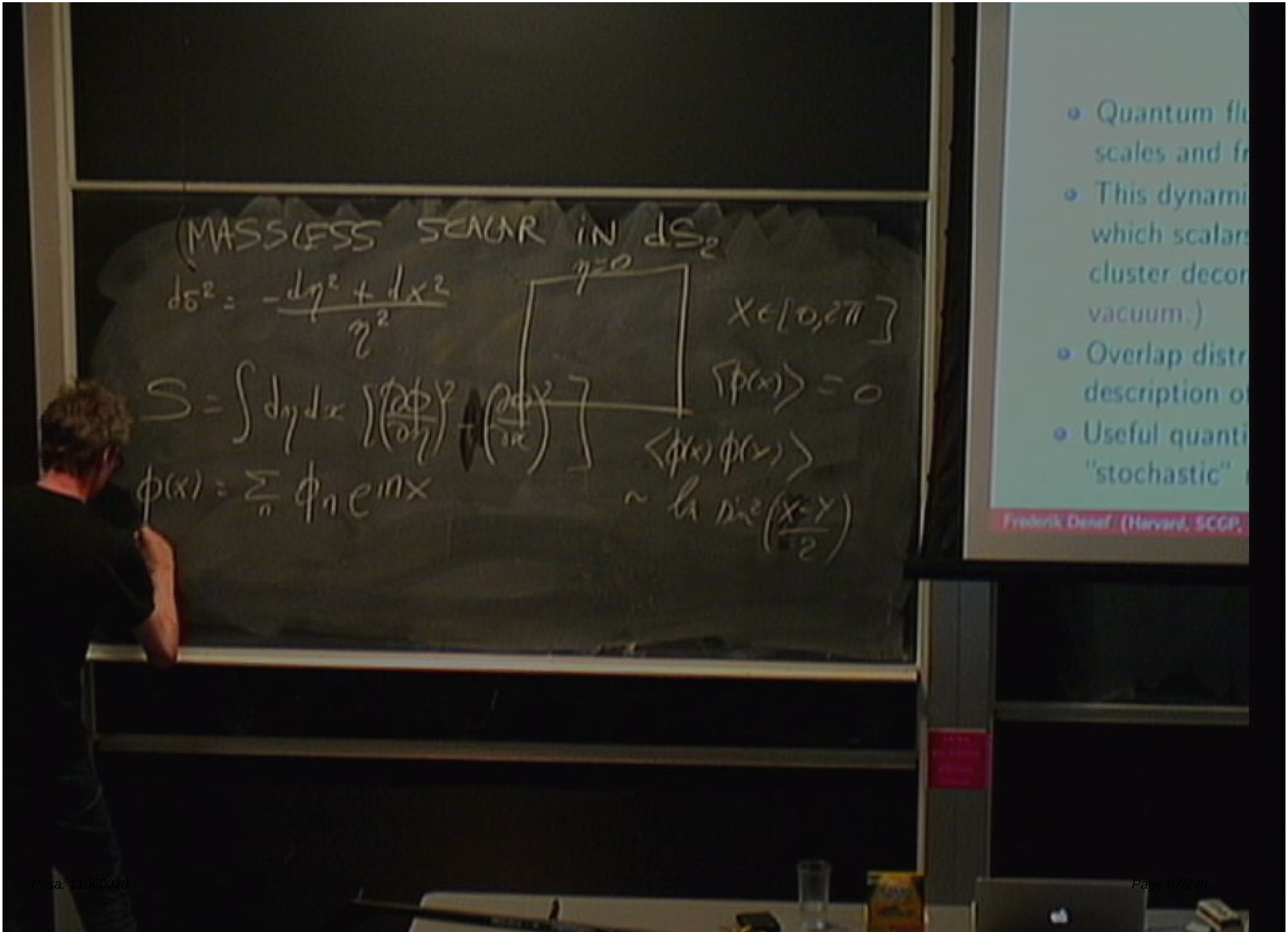
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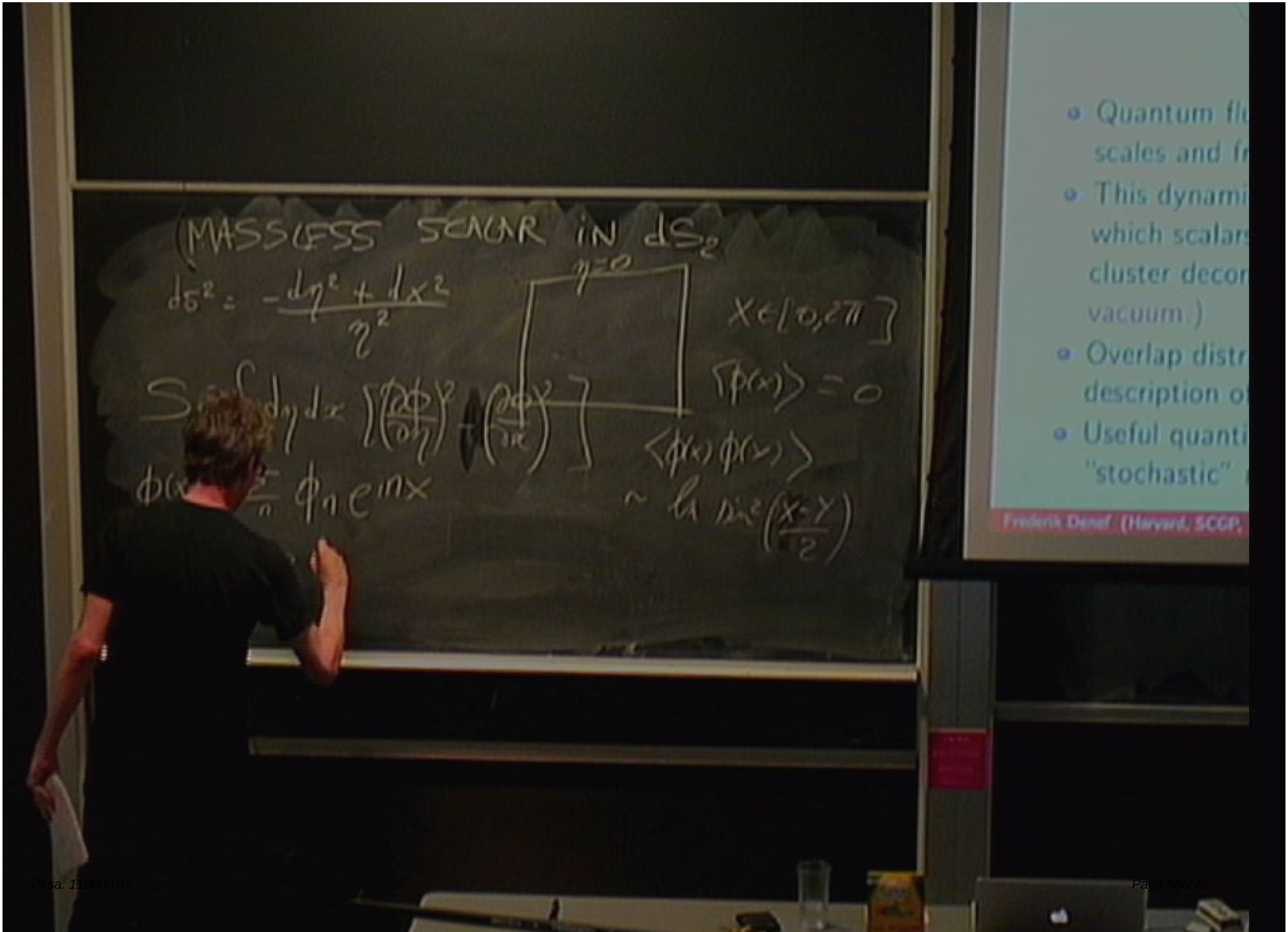
MASSLESS SCALAR IN dS2
η=0
X∈[0,2π]
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⟨φ(x) φ(y)⟩
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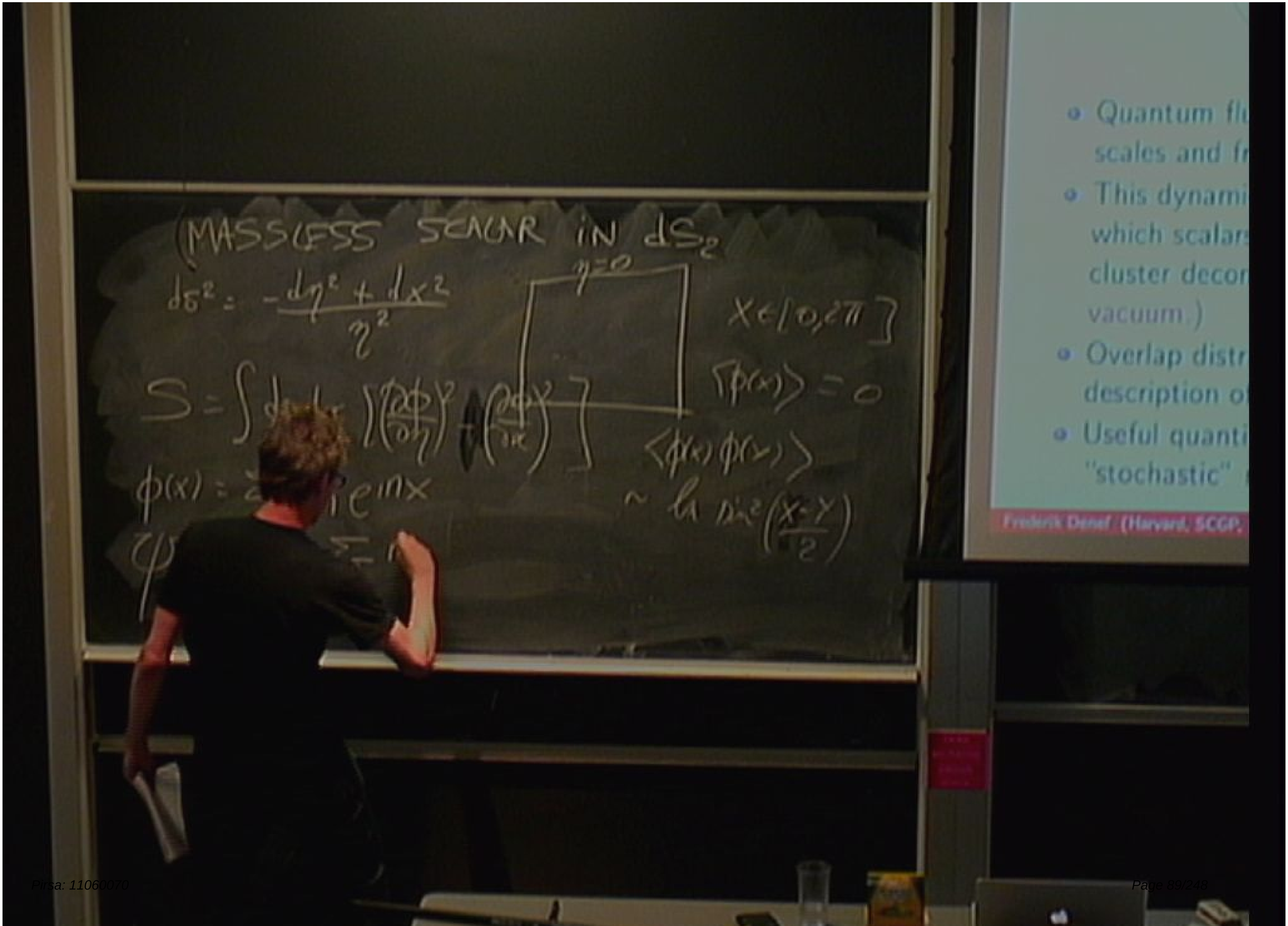
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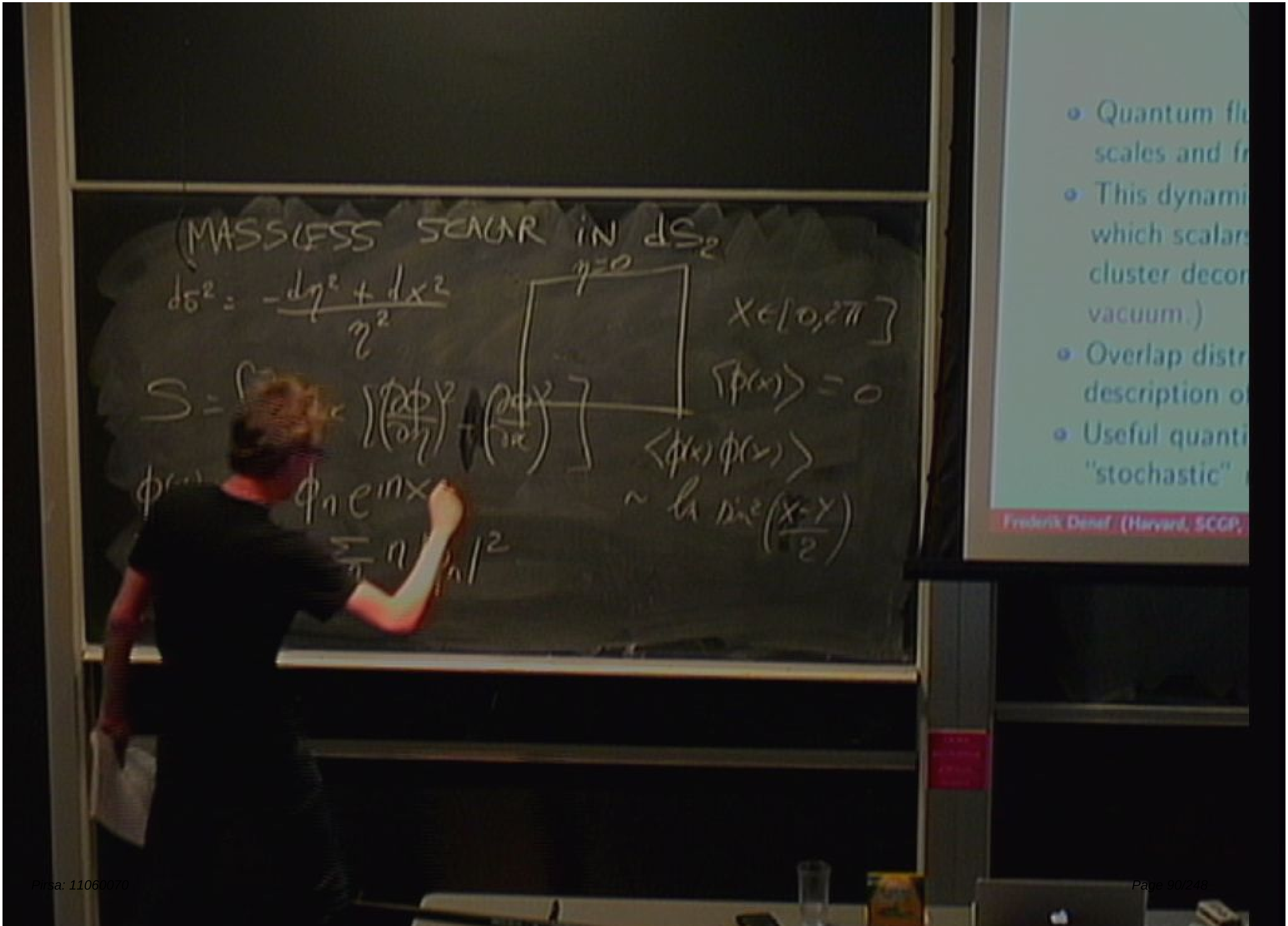
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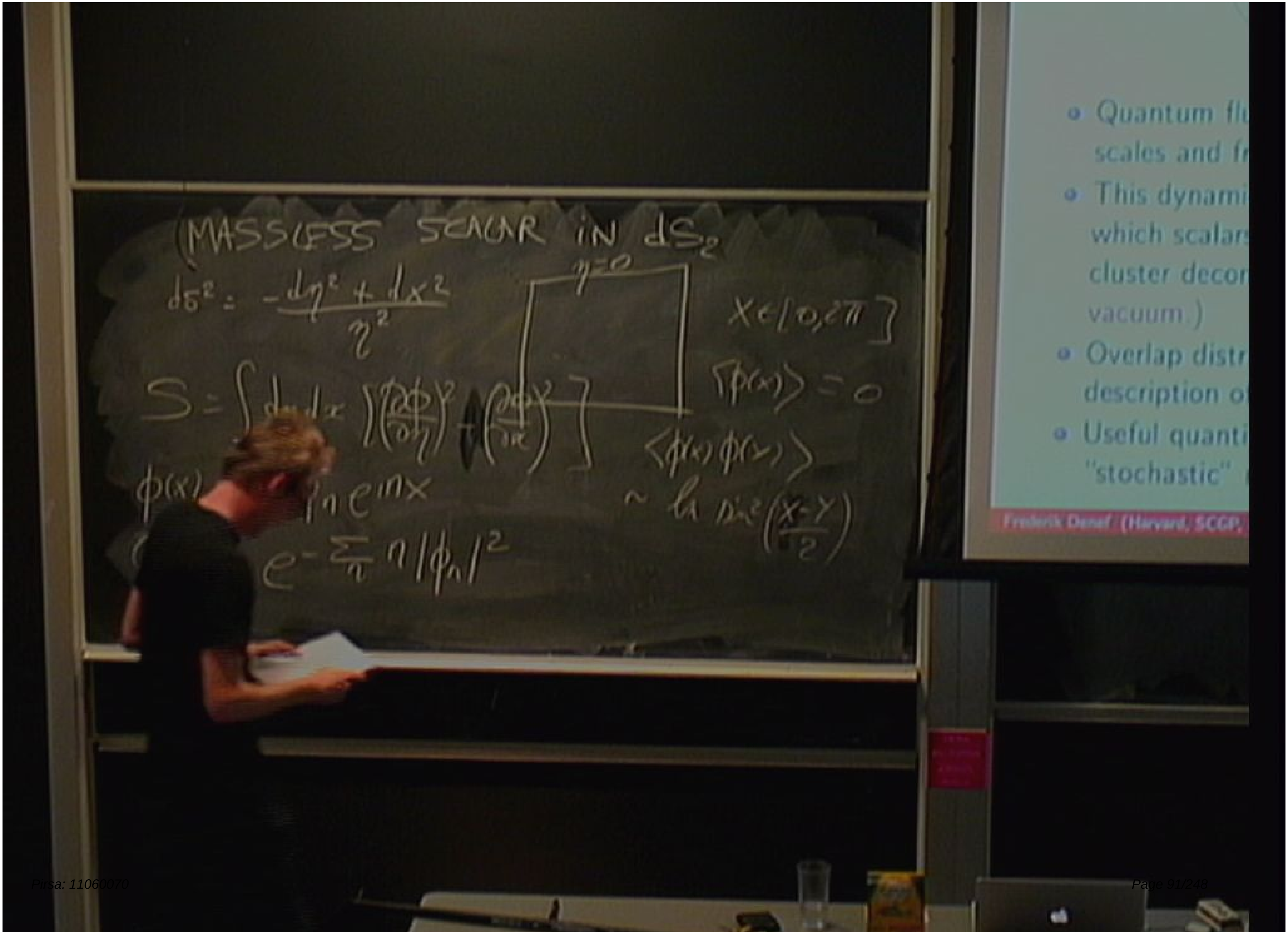
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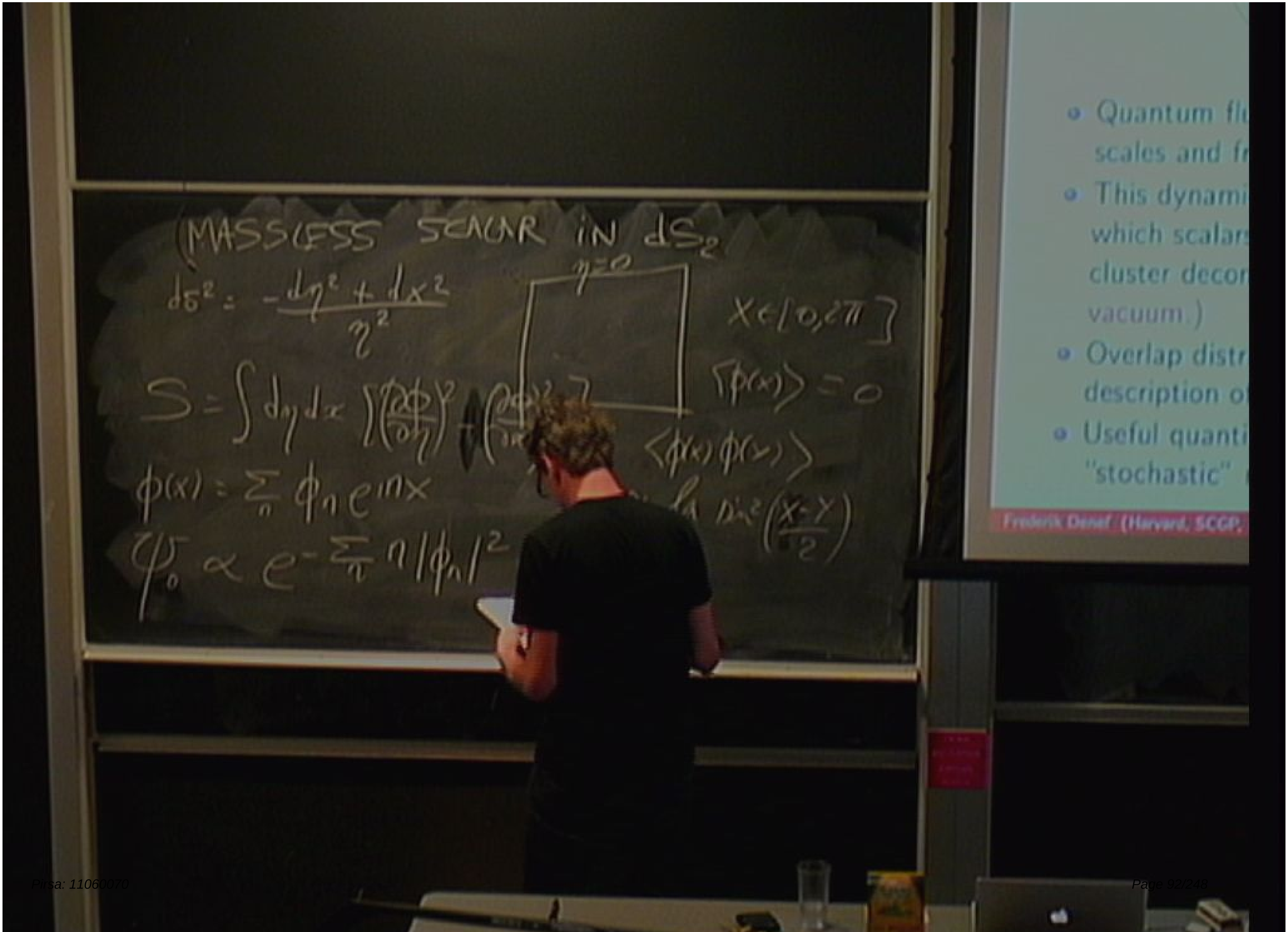
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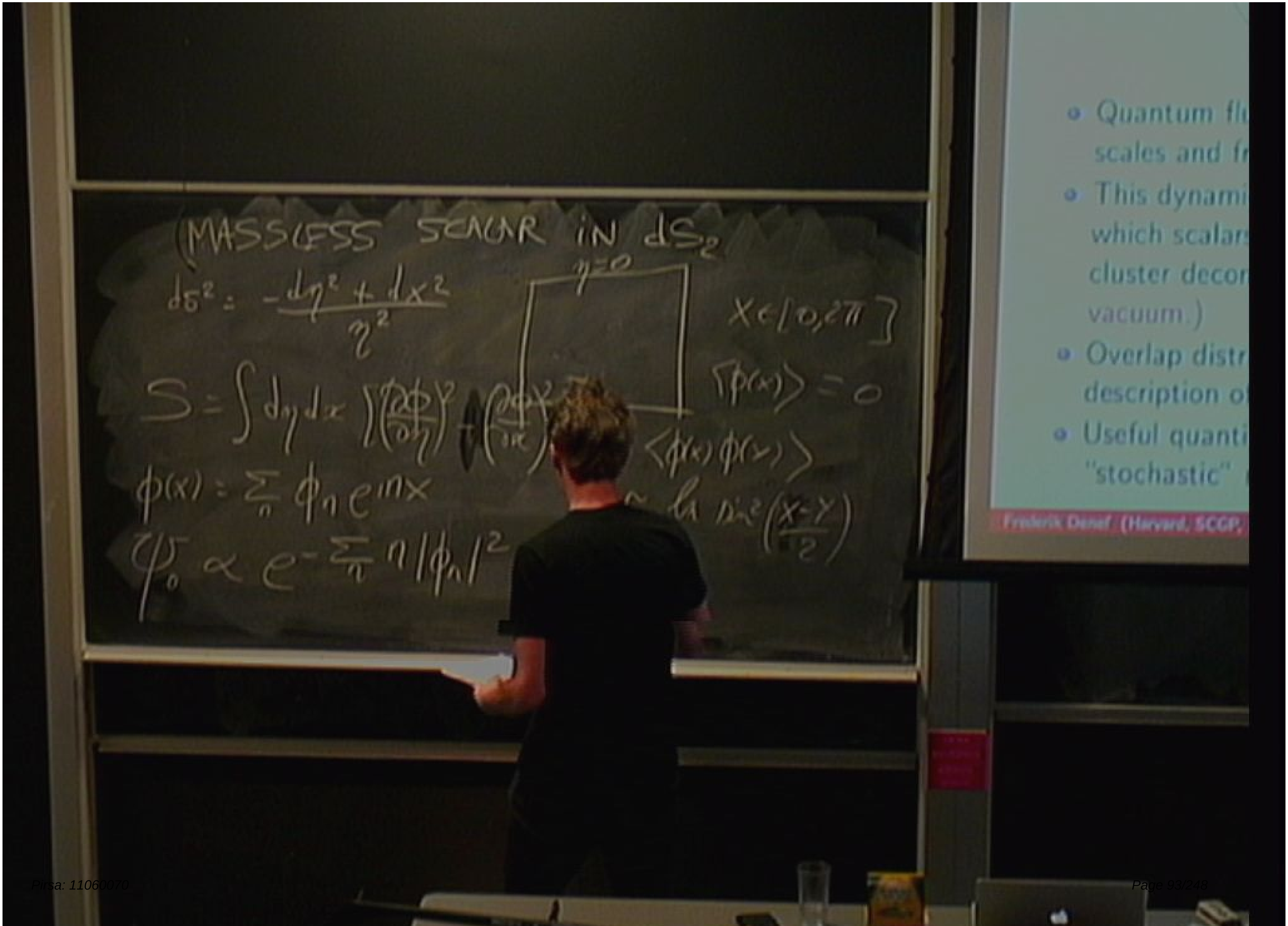
MASSLESS SCALAR IN dS2
ds² = (−dη² + dx²)/η²
η=0
x ∈ [0, 2π]
S = ∫ dη dx ((∂φ/∂η)² − (∂φ/∂x)²)
⟨φ(x)⟩ = 0
⟨φ(x) φ(y)⟩
φ(x) = Σn φn e^{inx}
ψ0 ∝ e^{−Σn n|φn|²}
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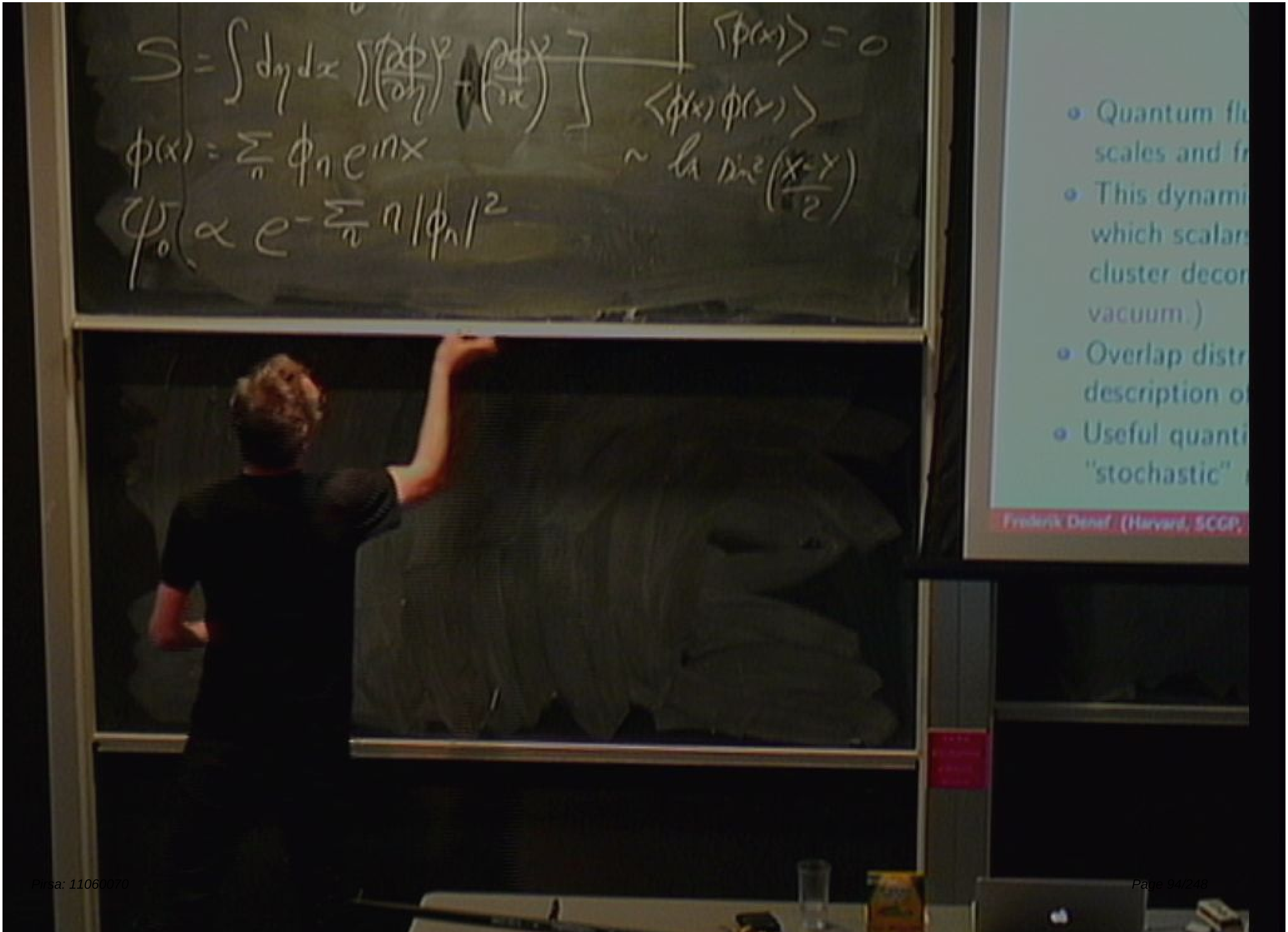
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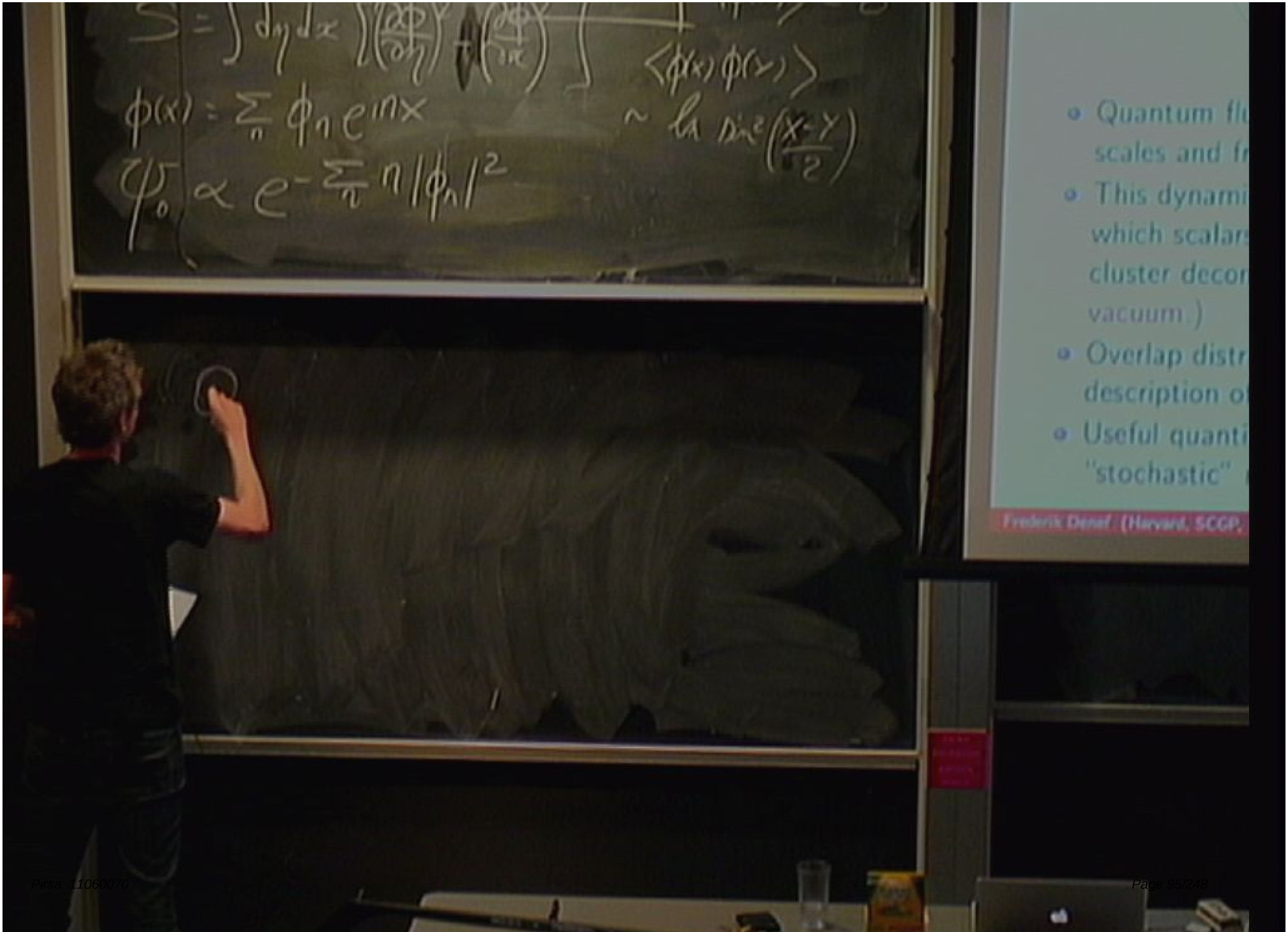
$\langle\phi(x)\phi(y)\rangle$
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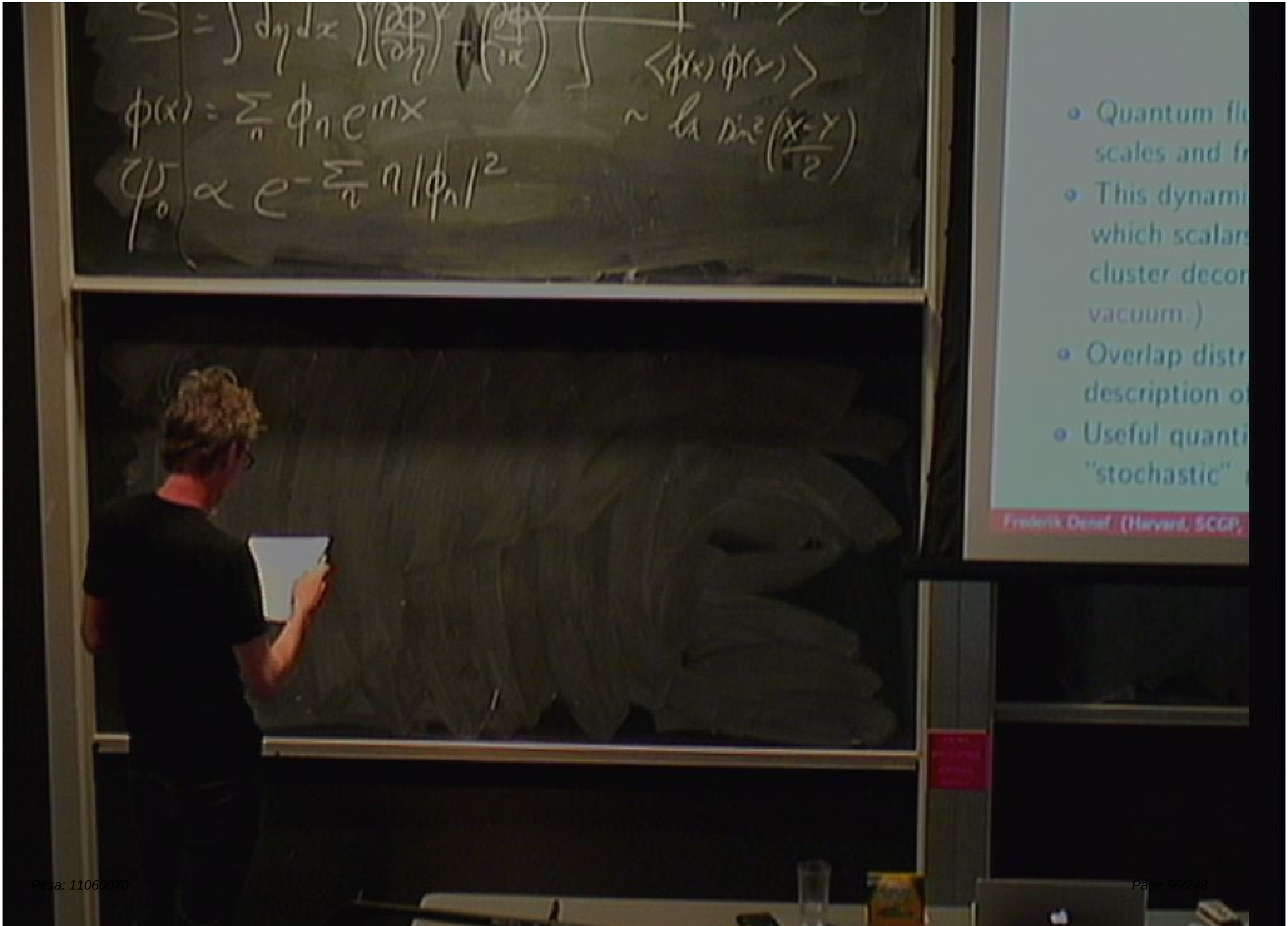
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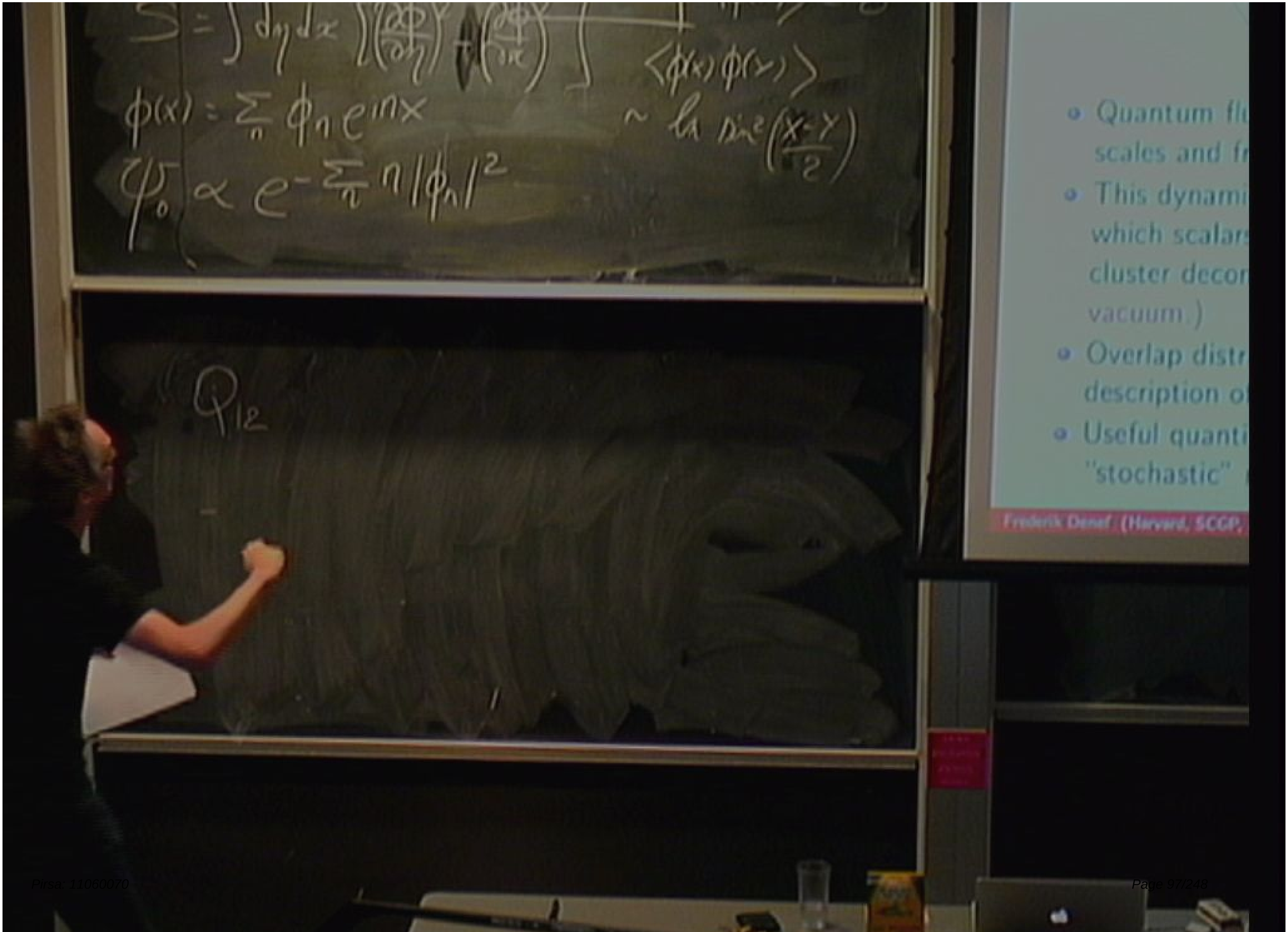
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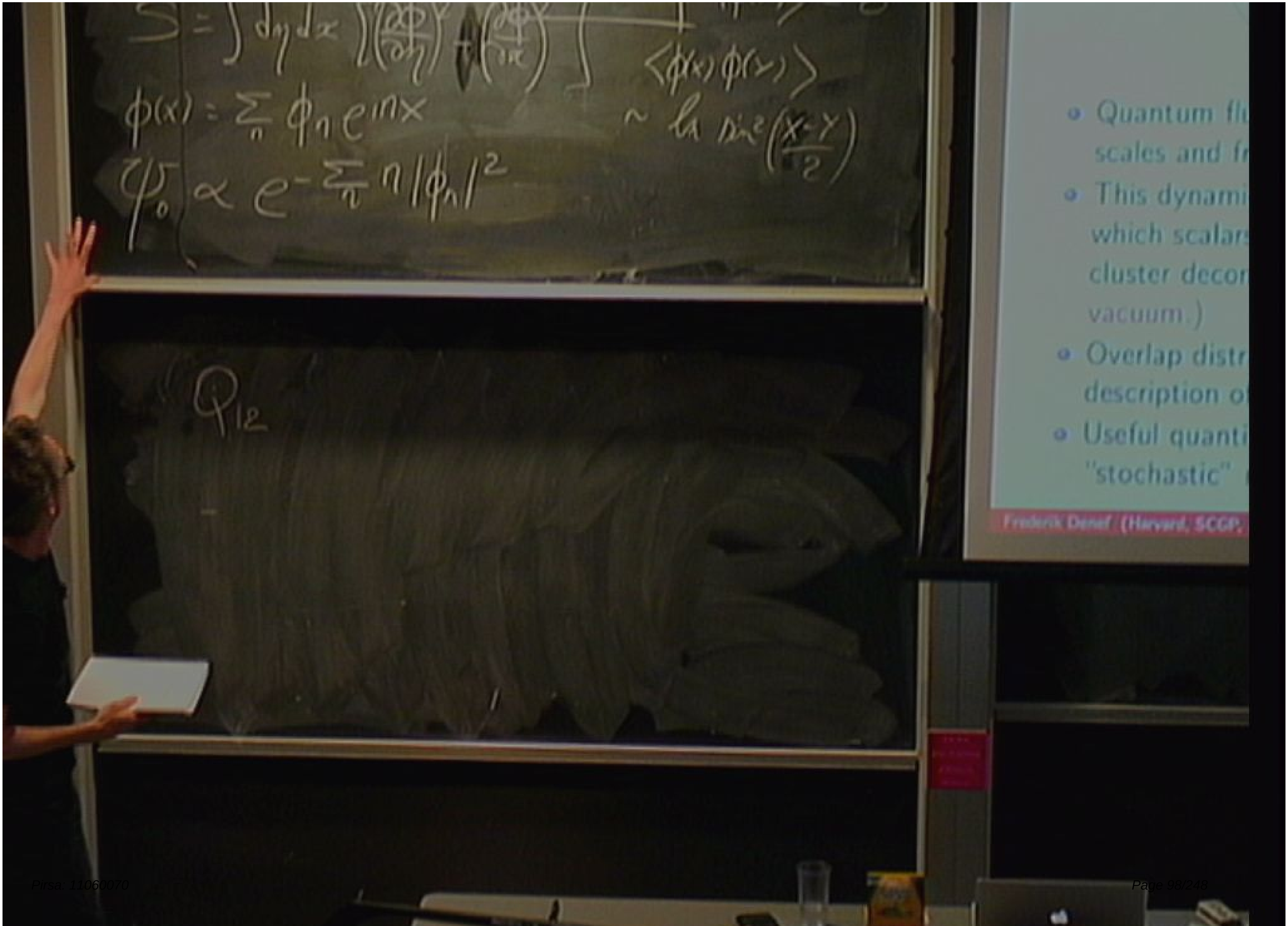
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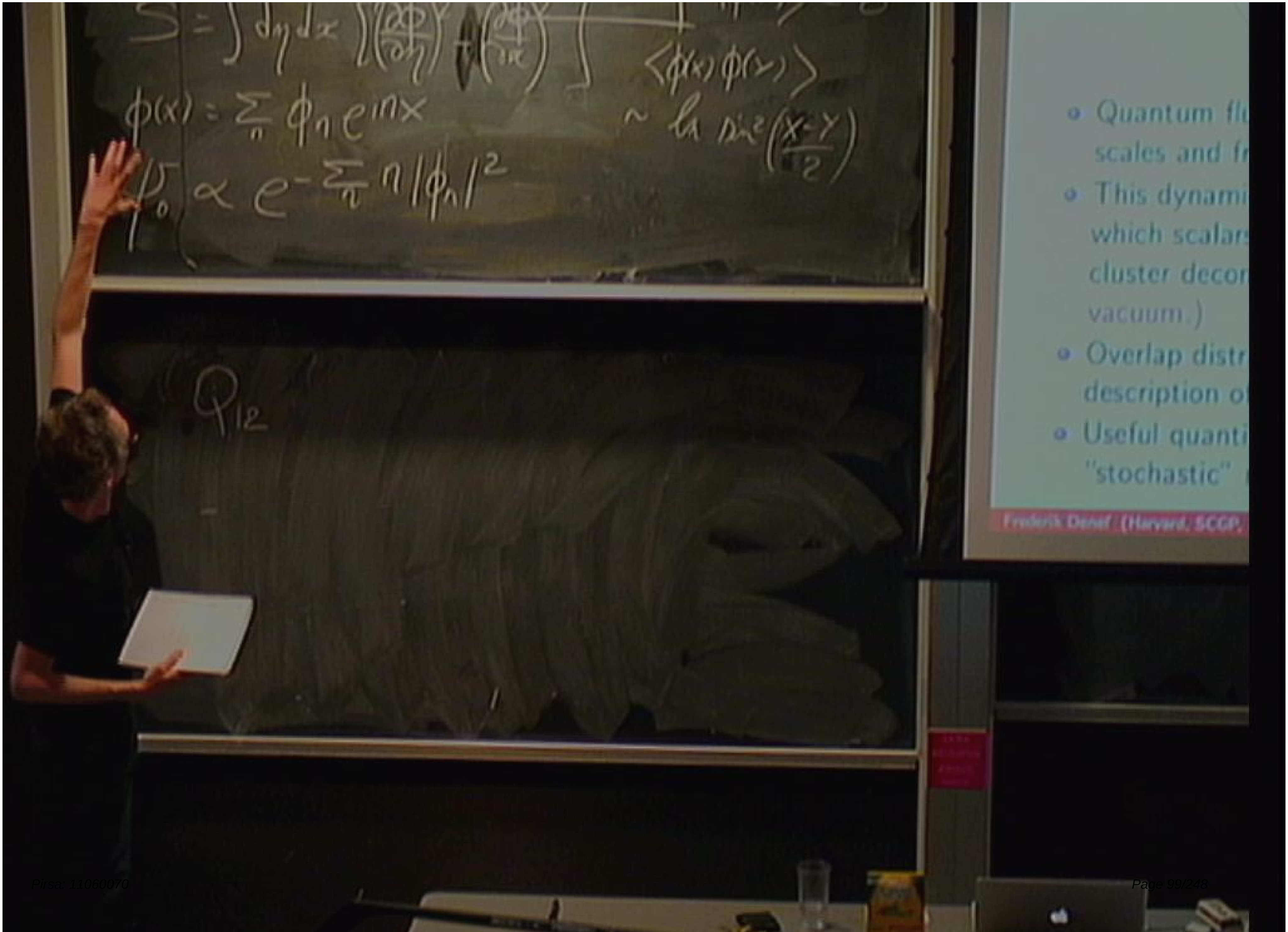
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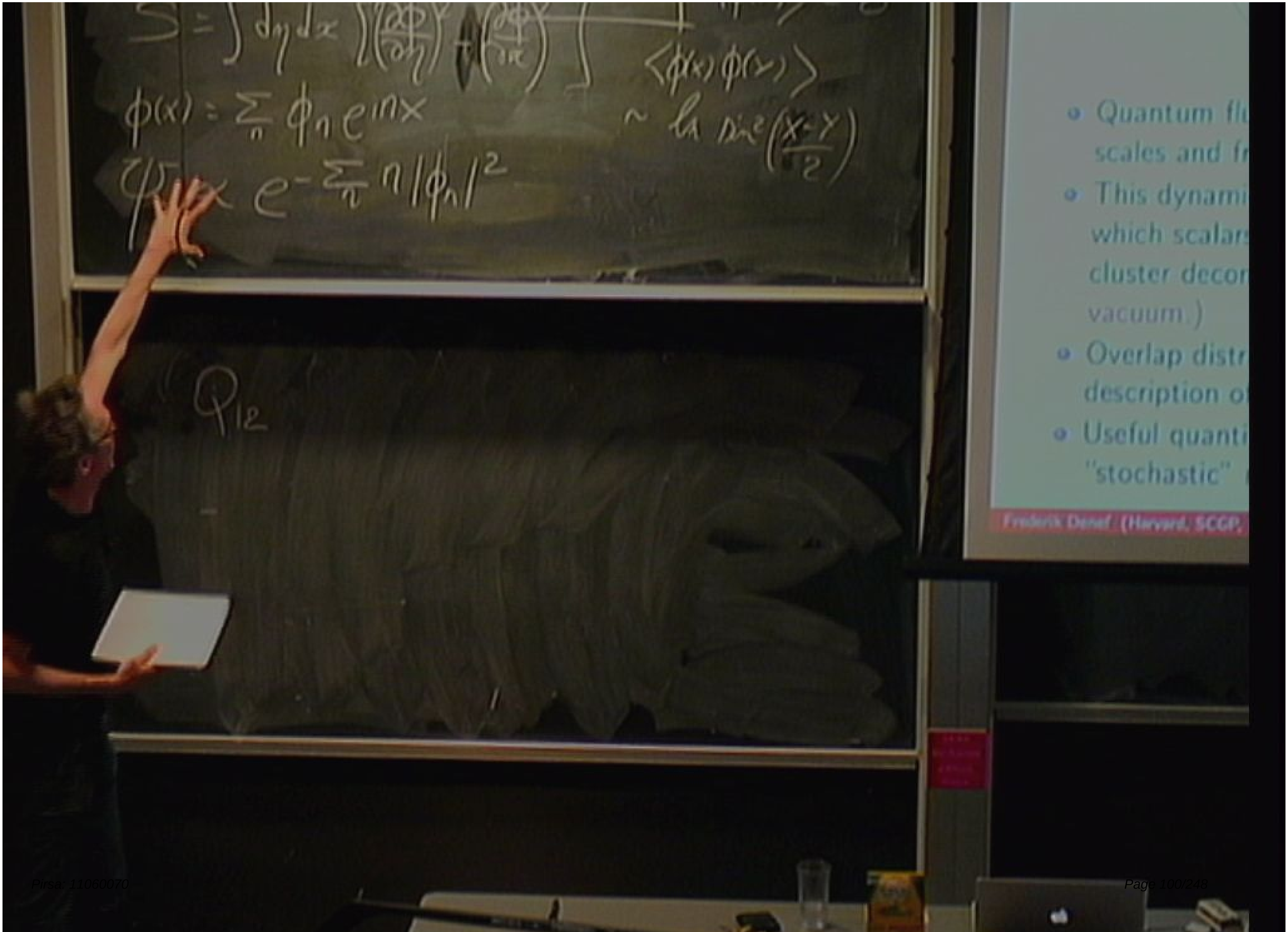
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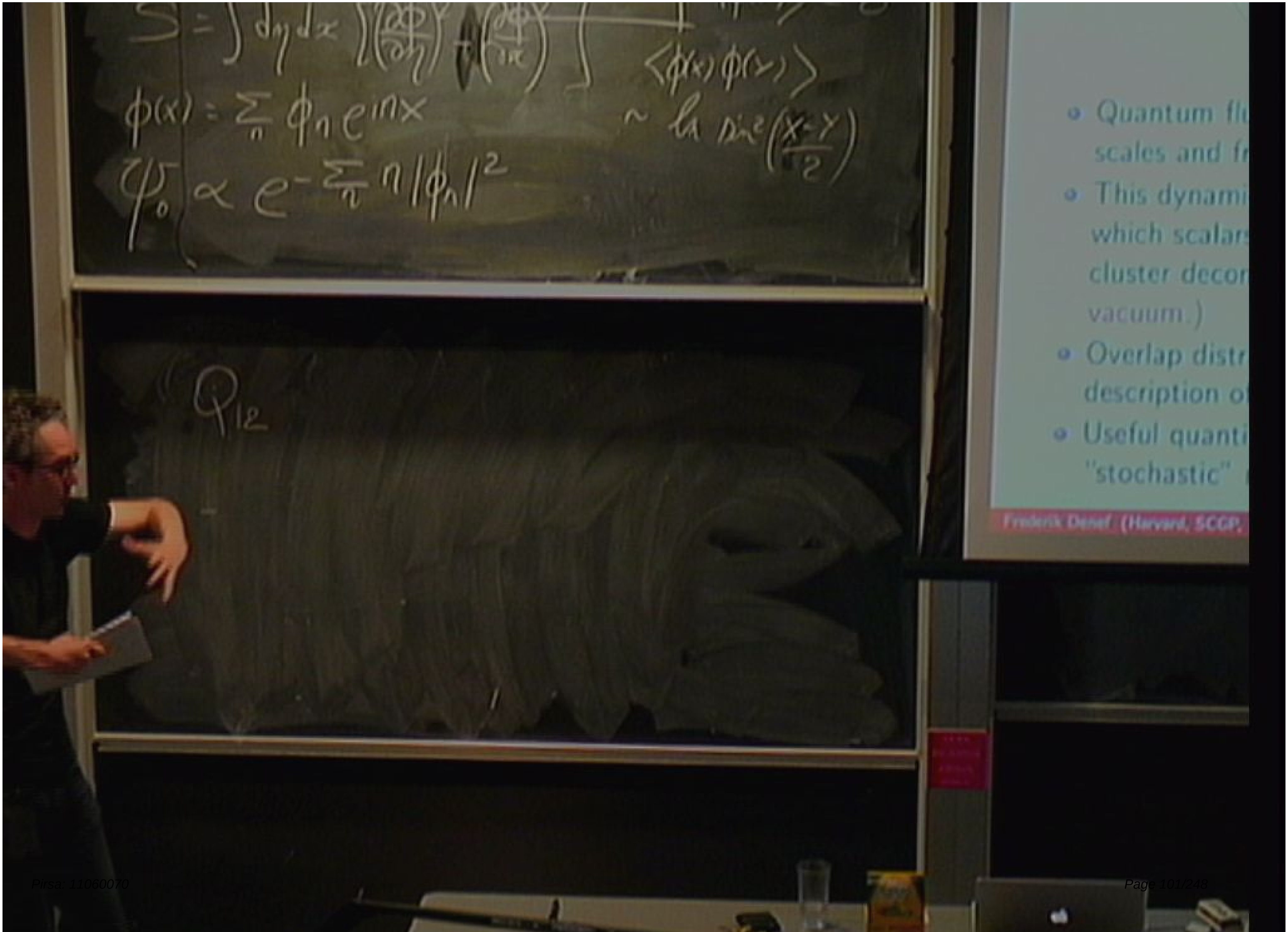
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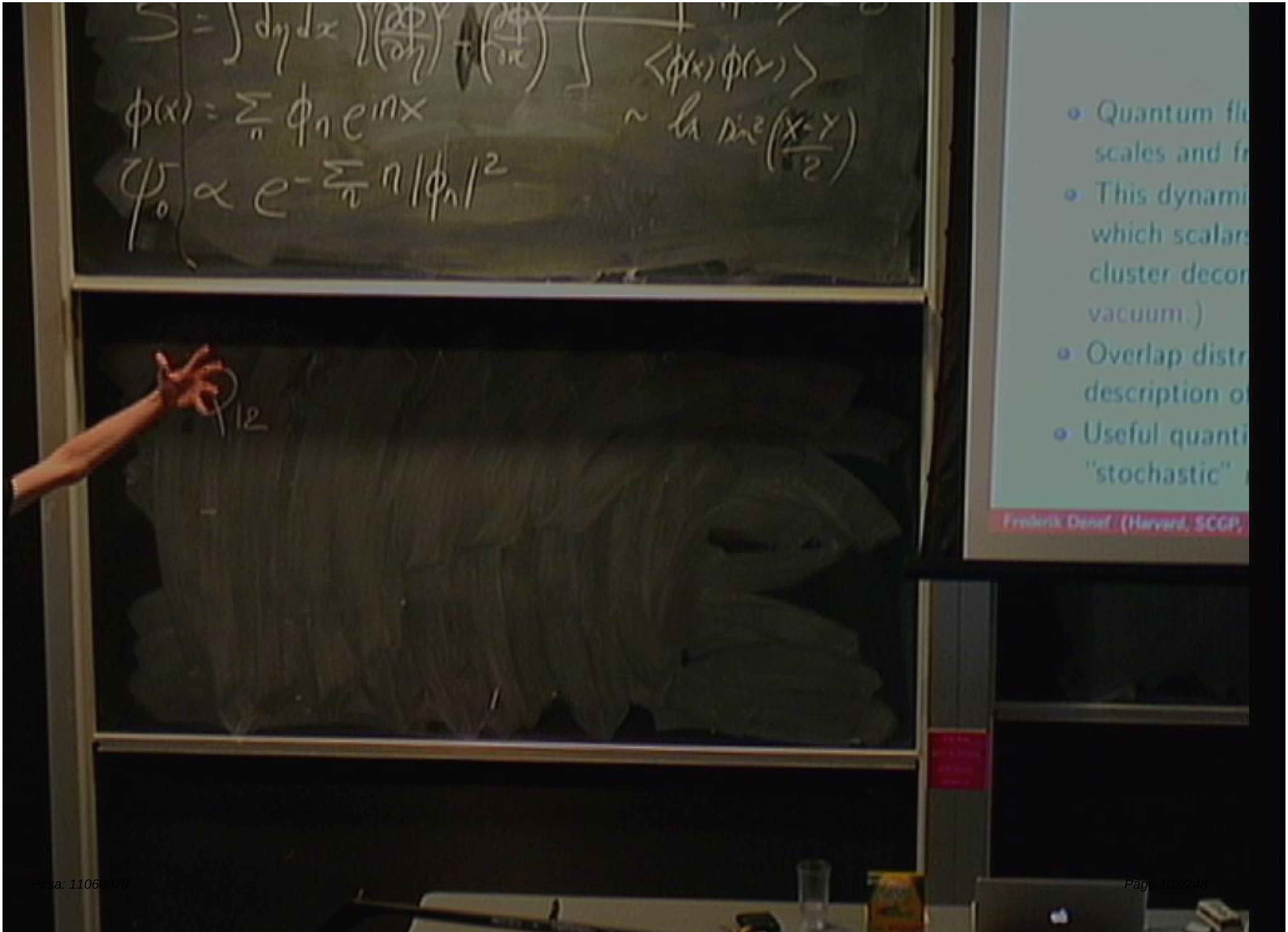
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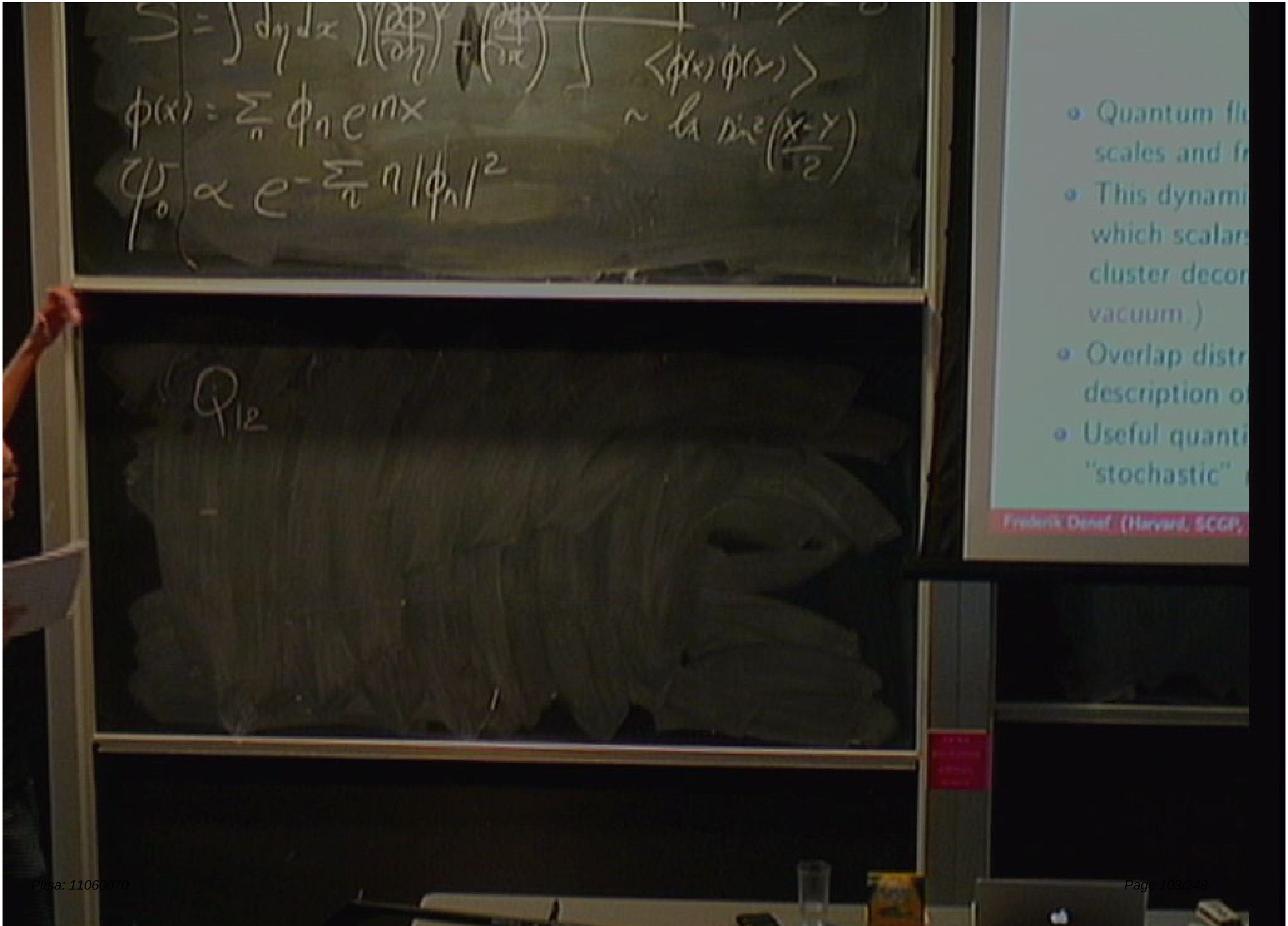
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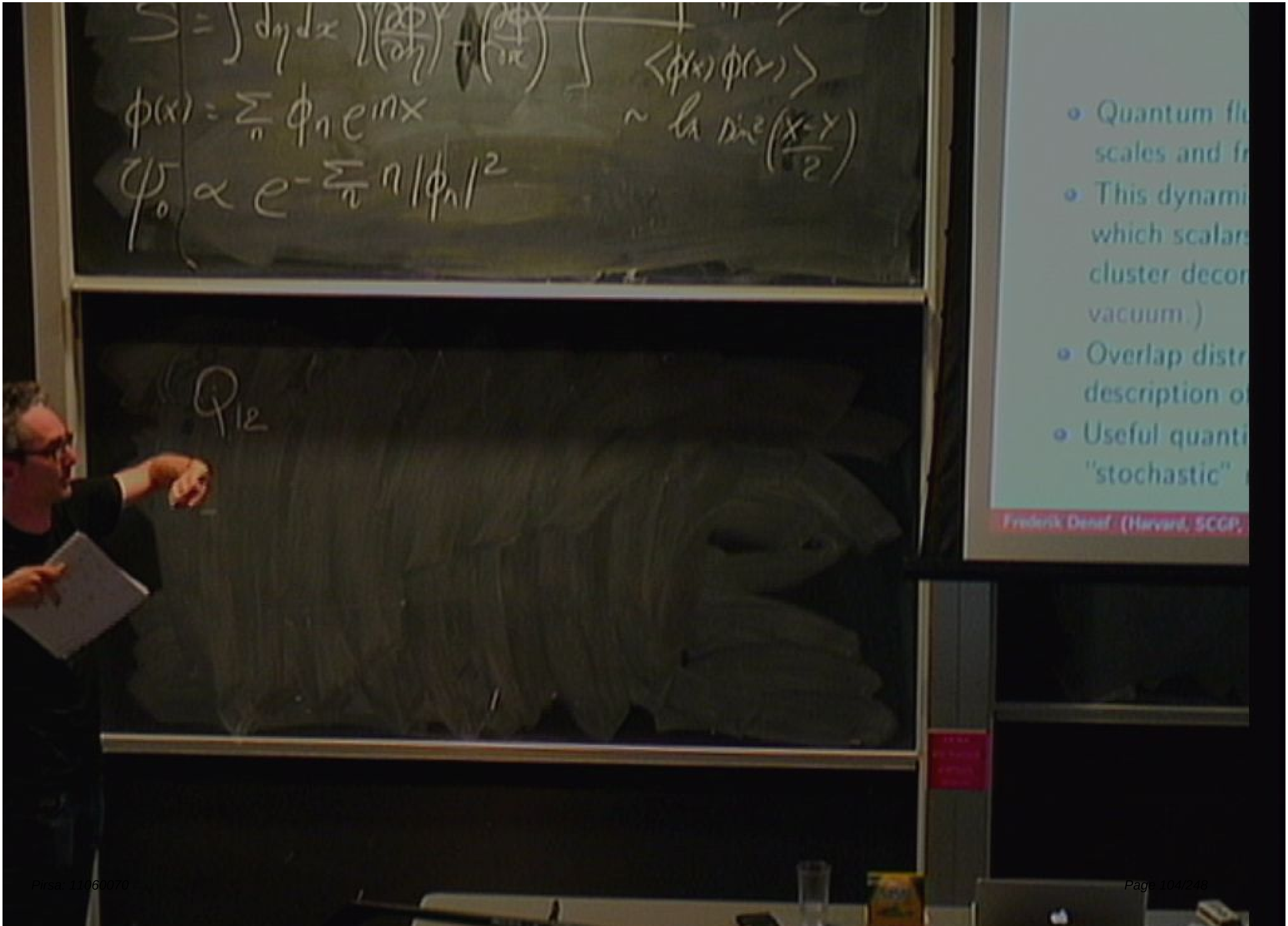
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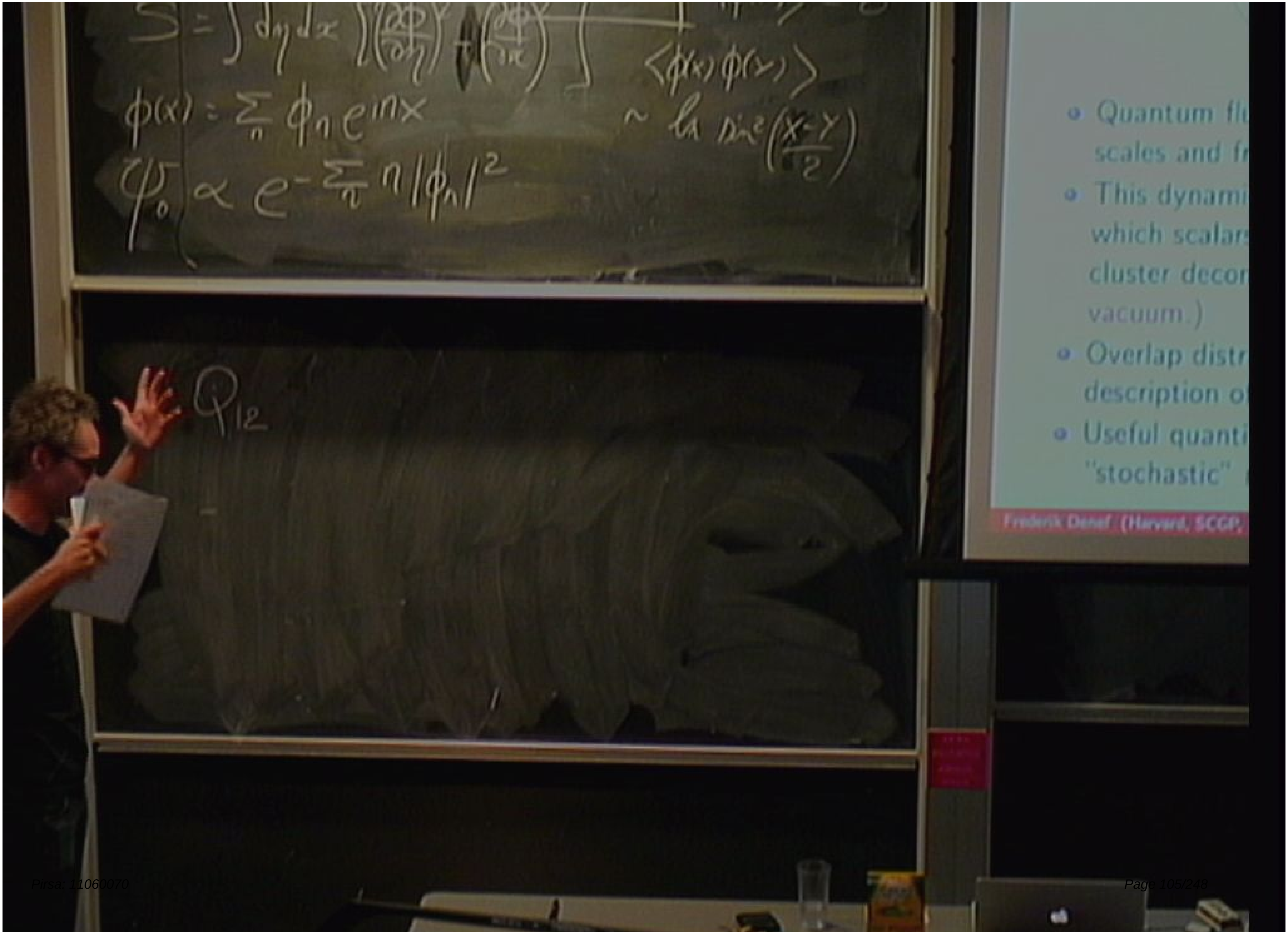
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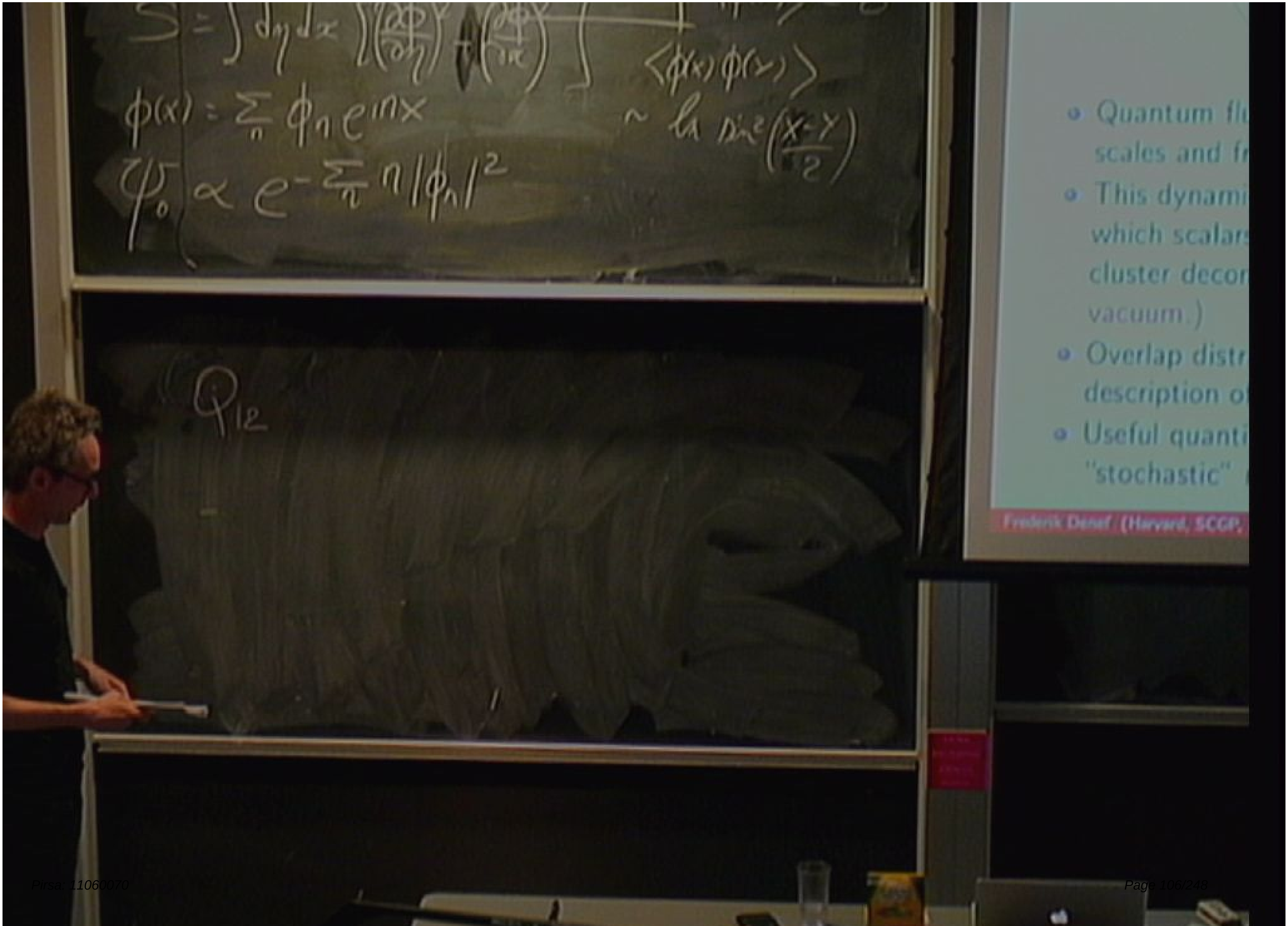

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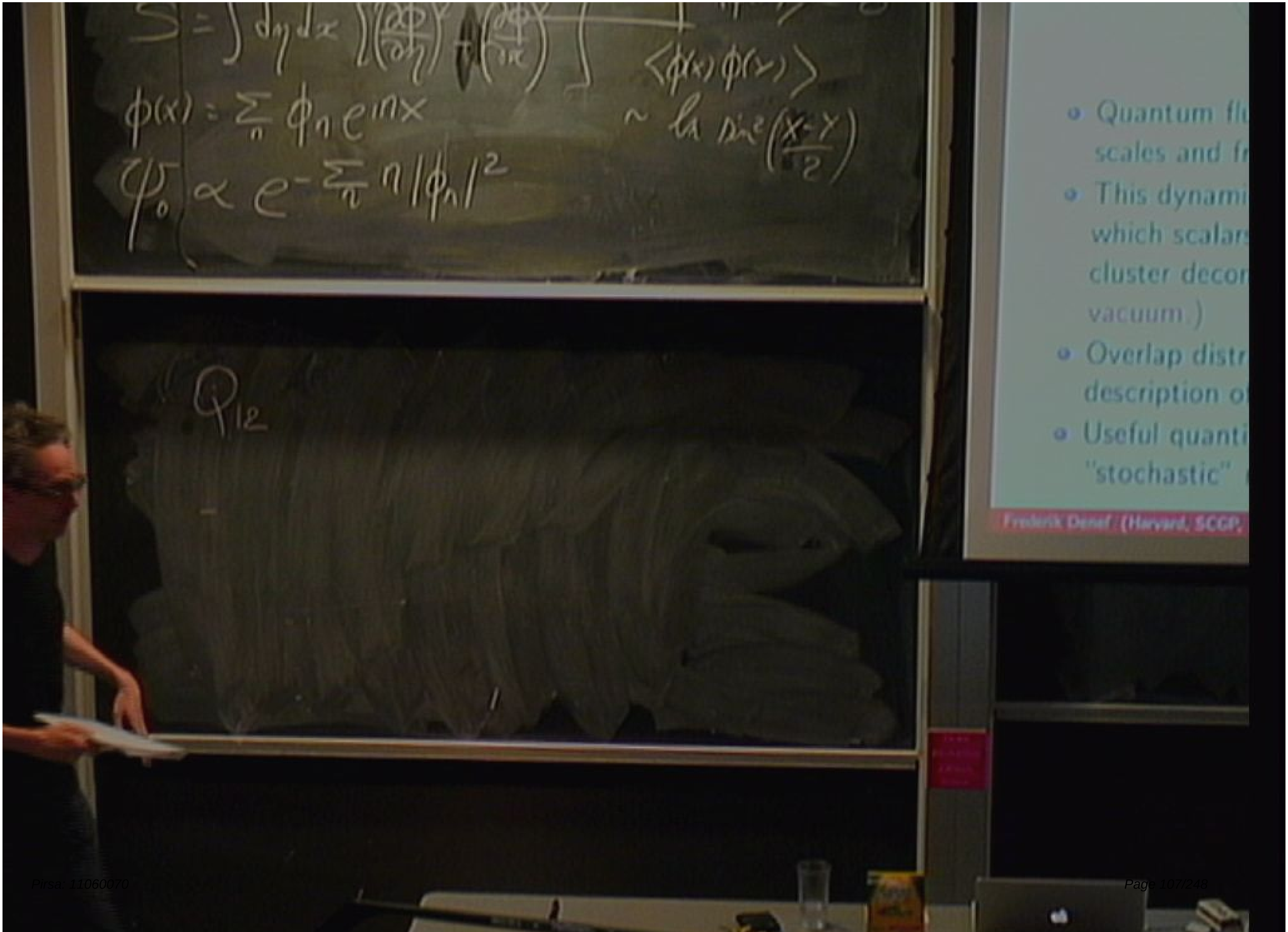
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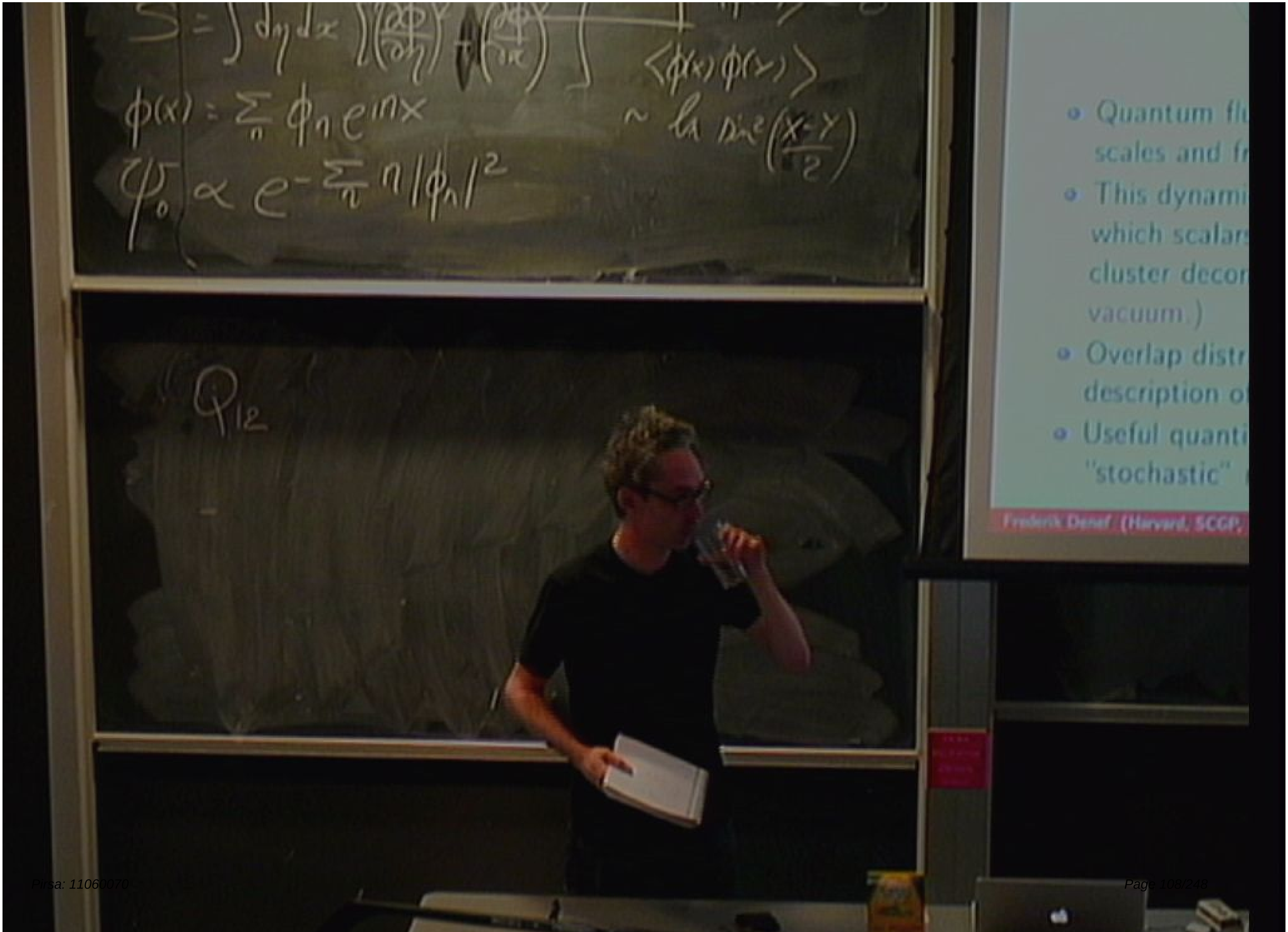

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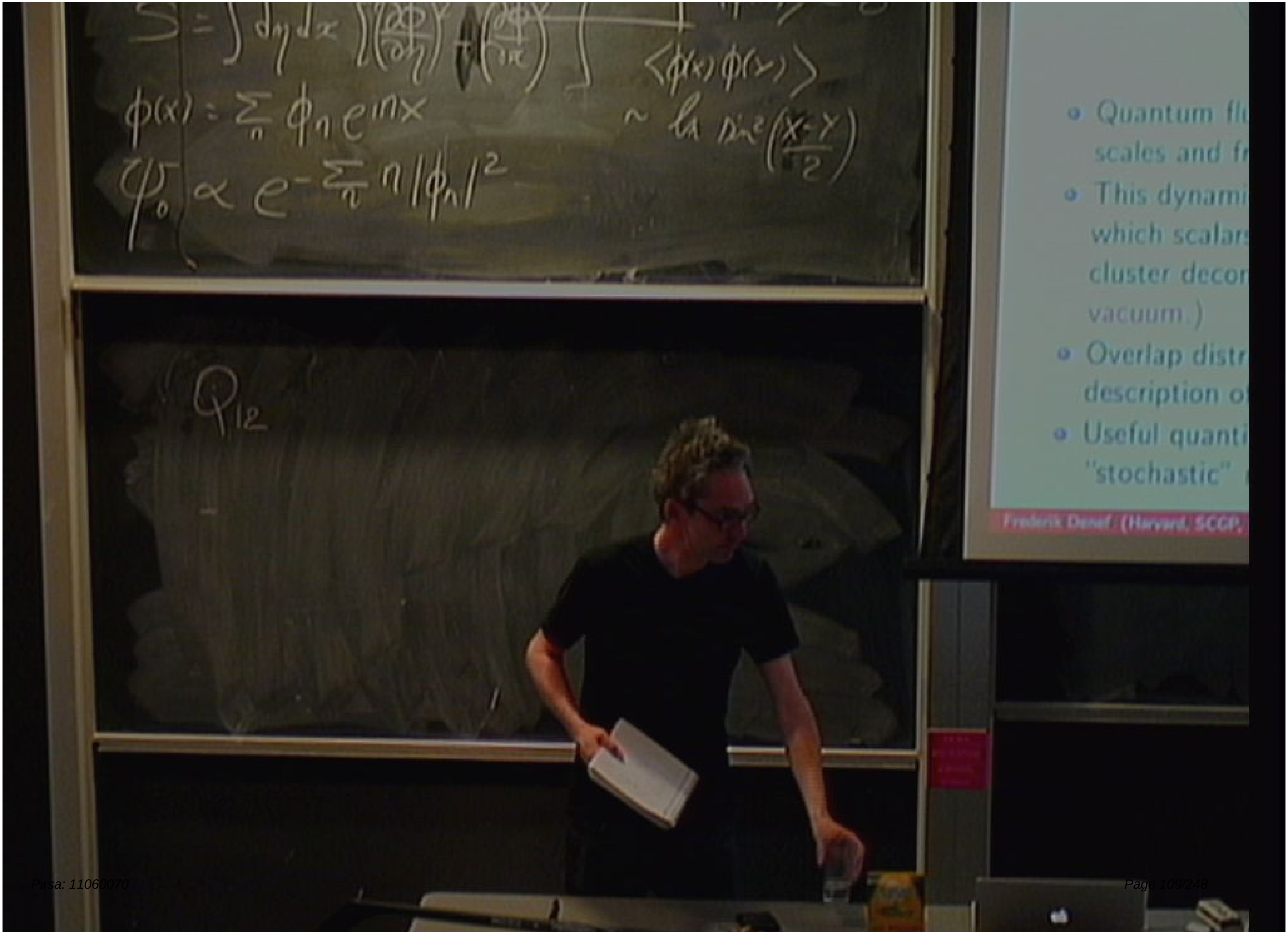
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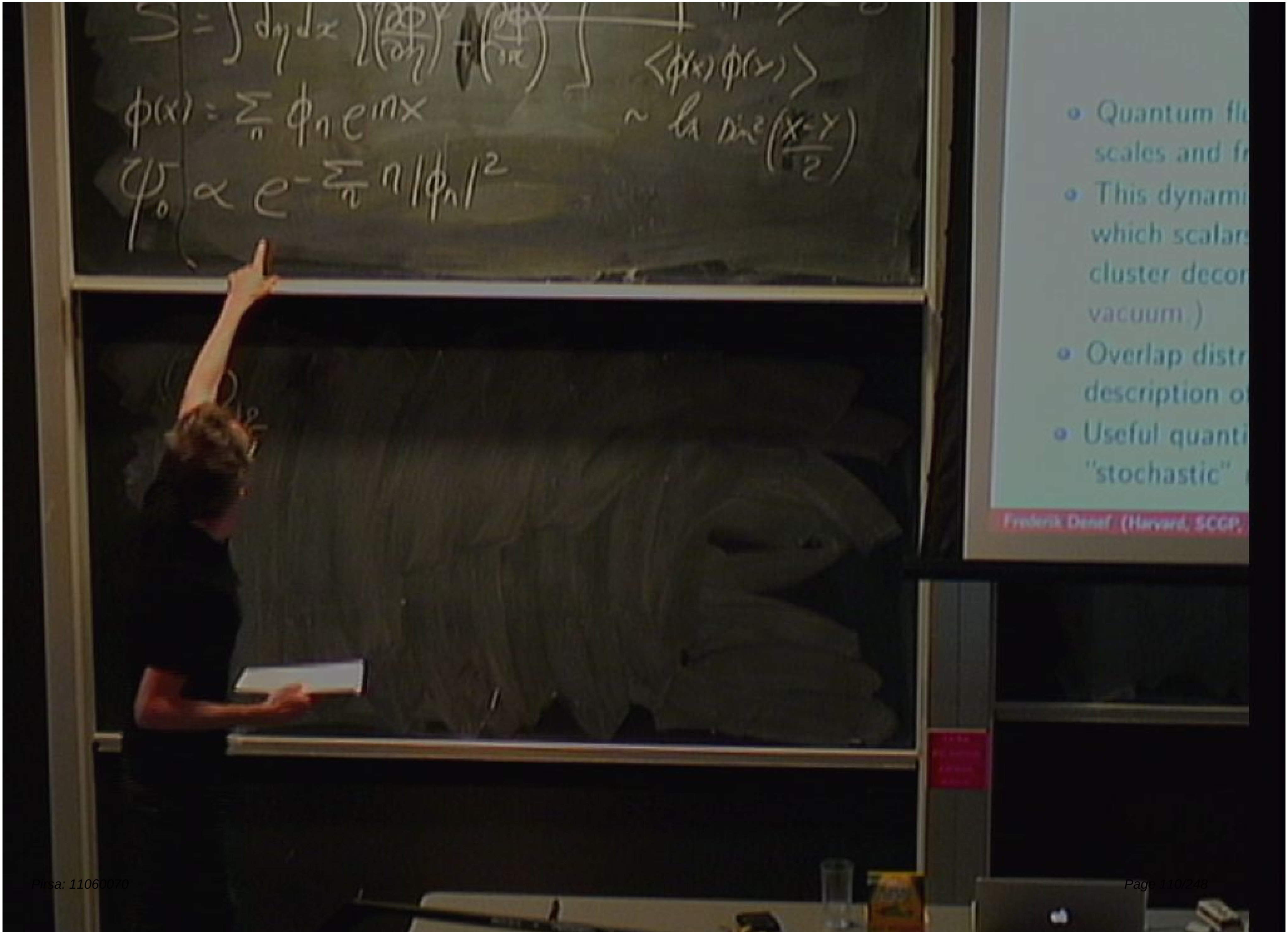
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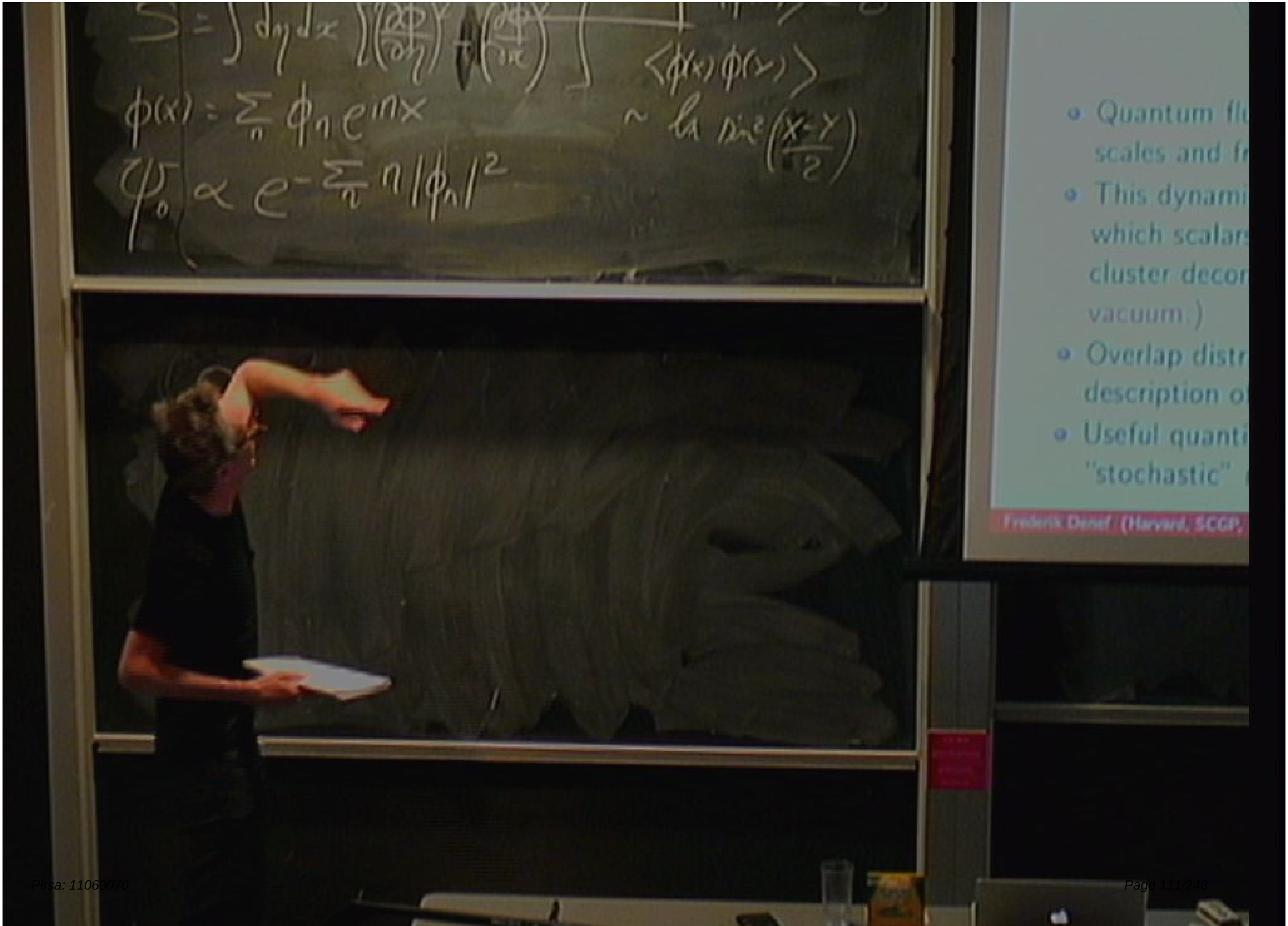

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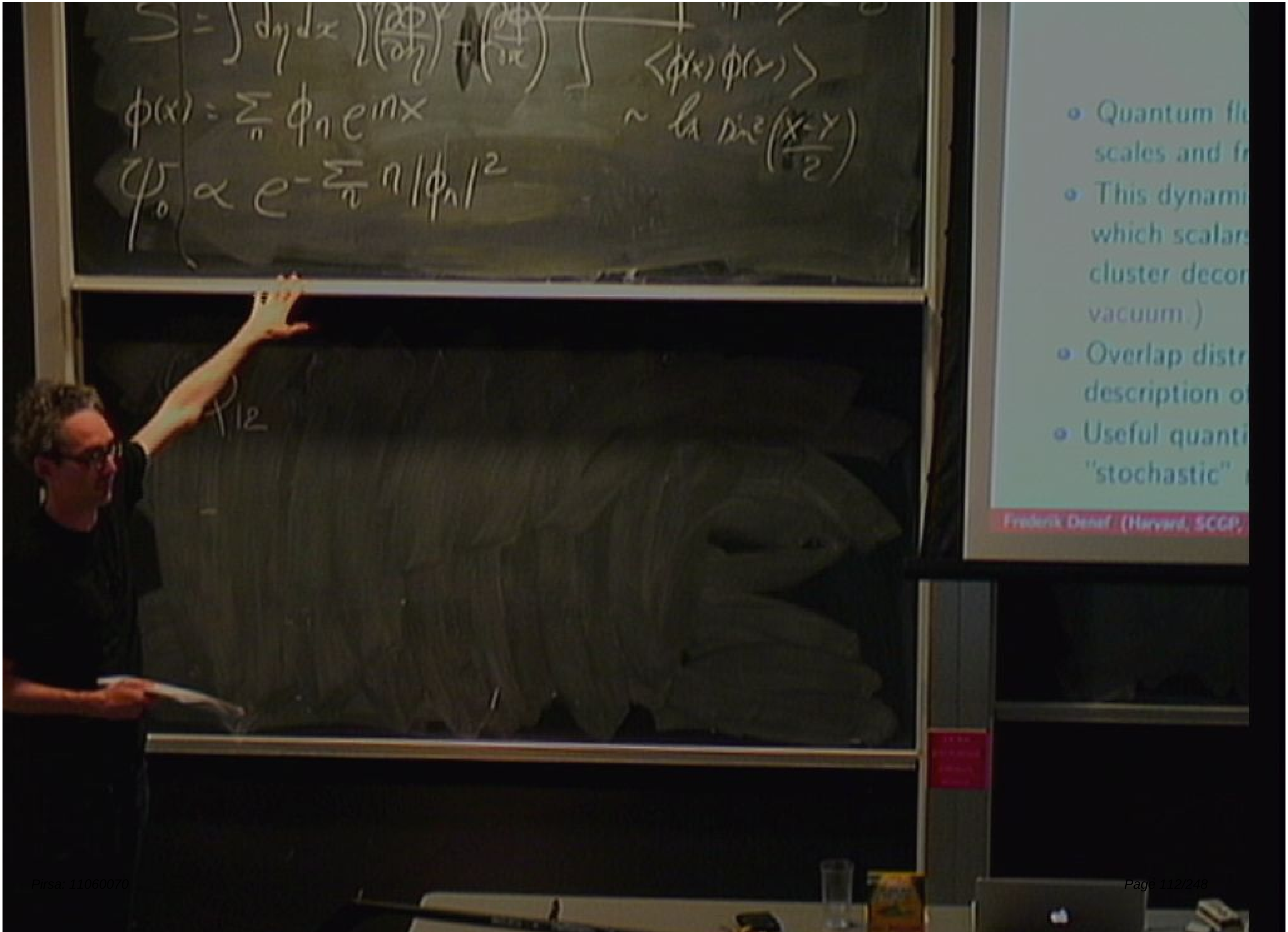
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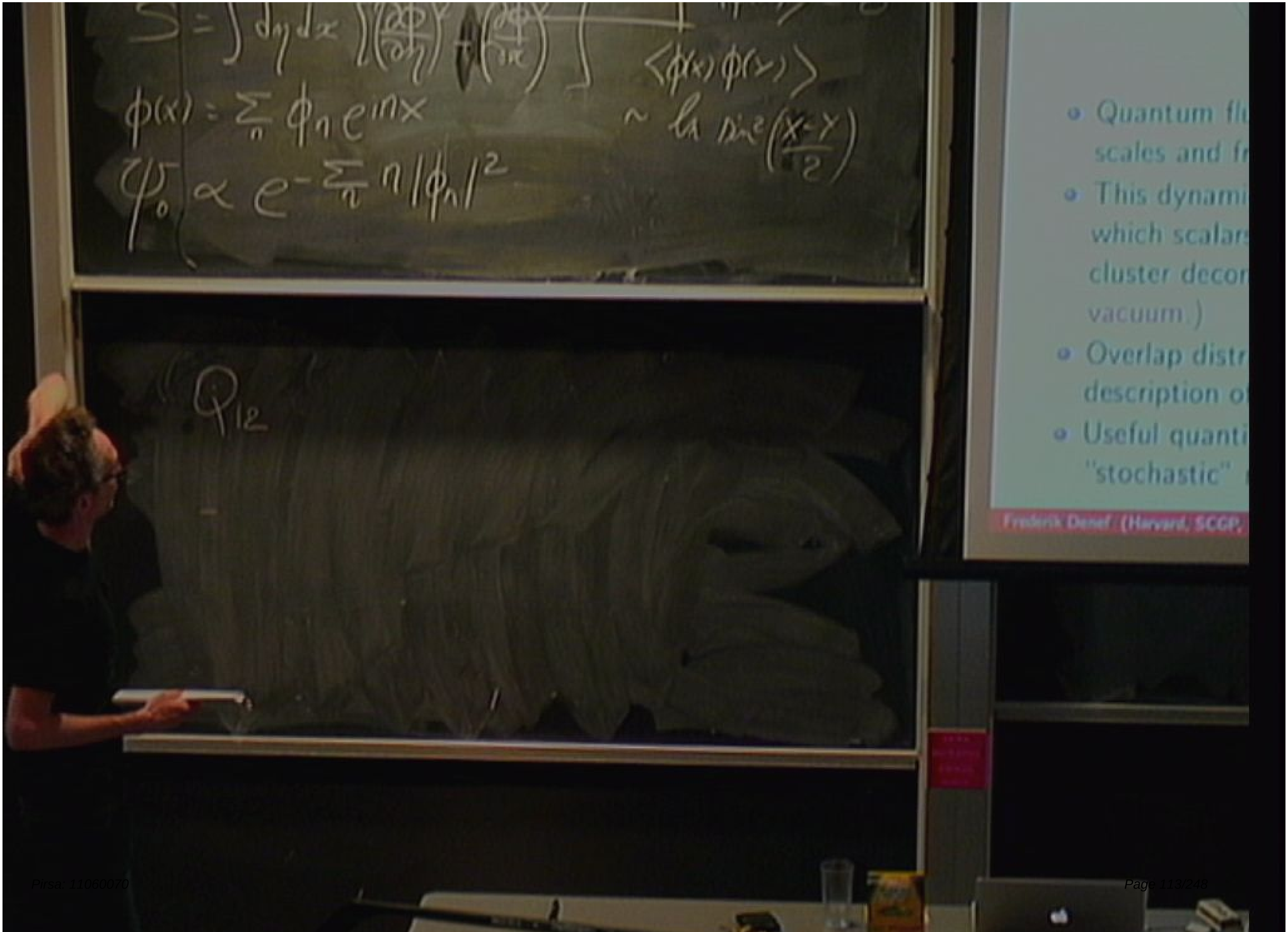
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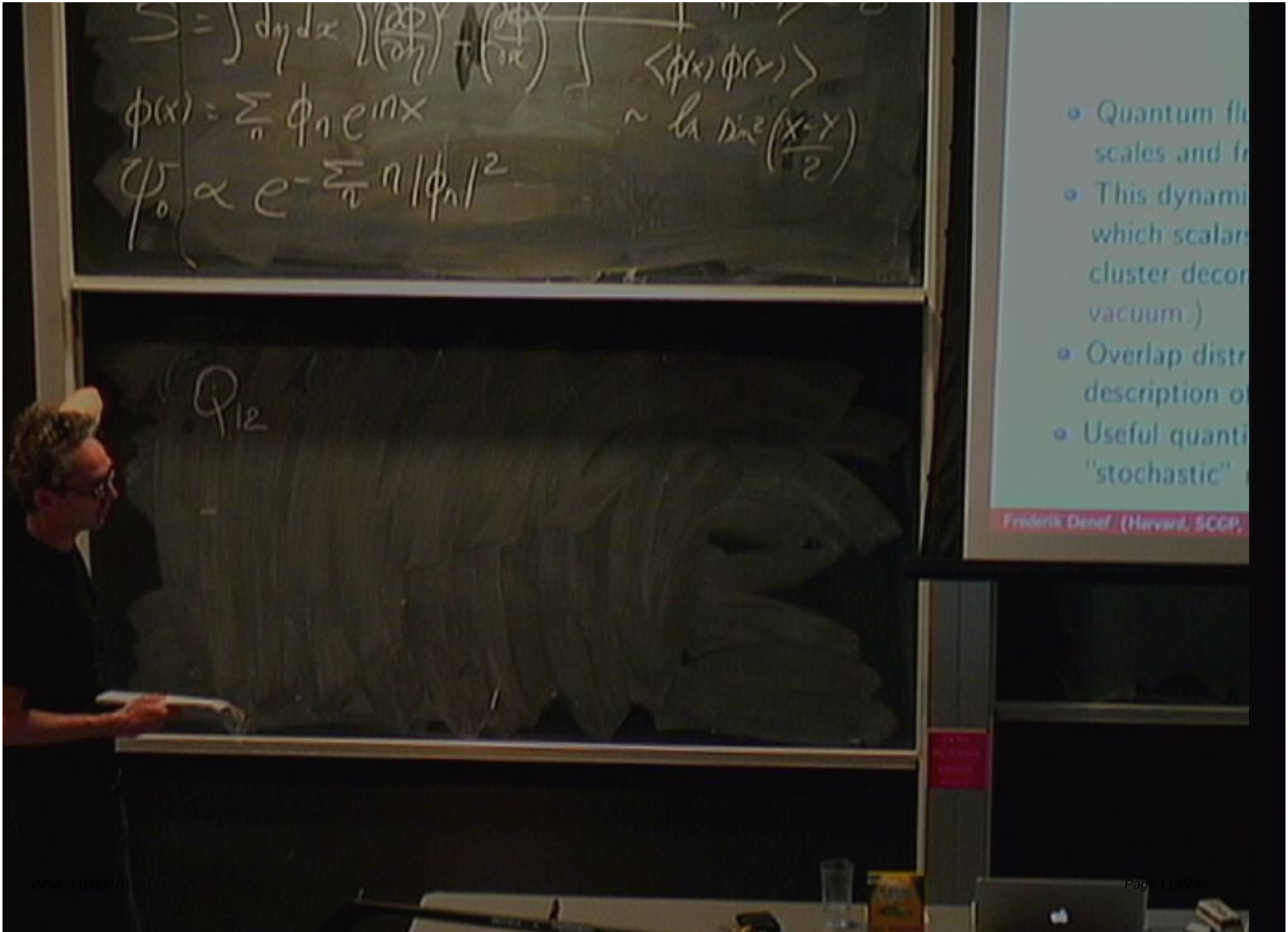
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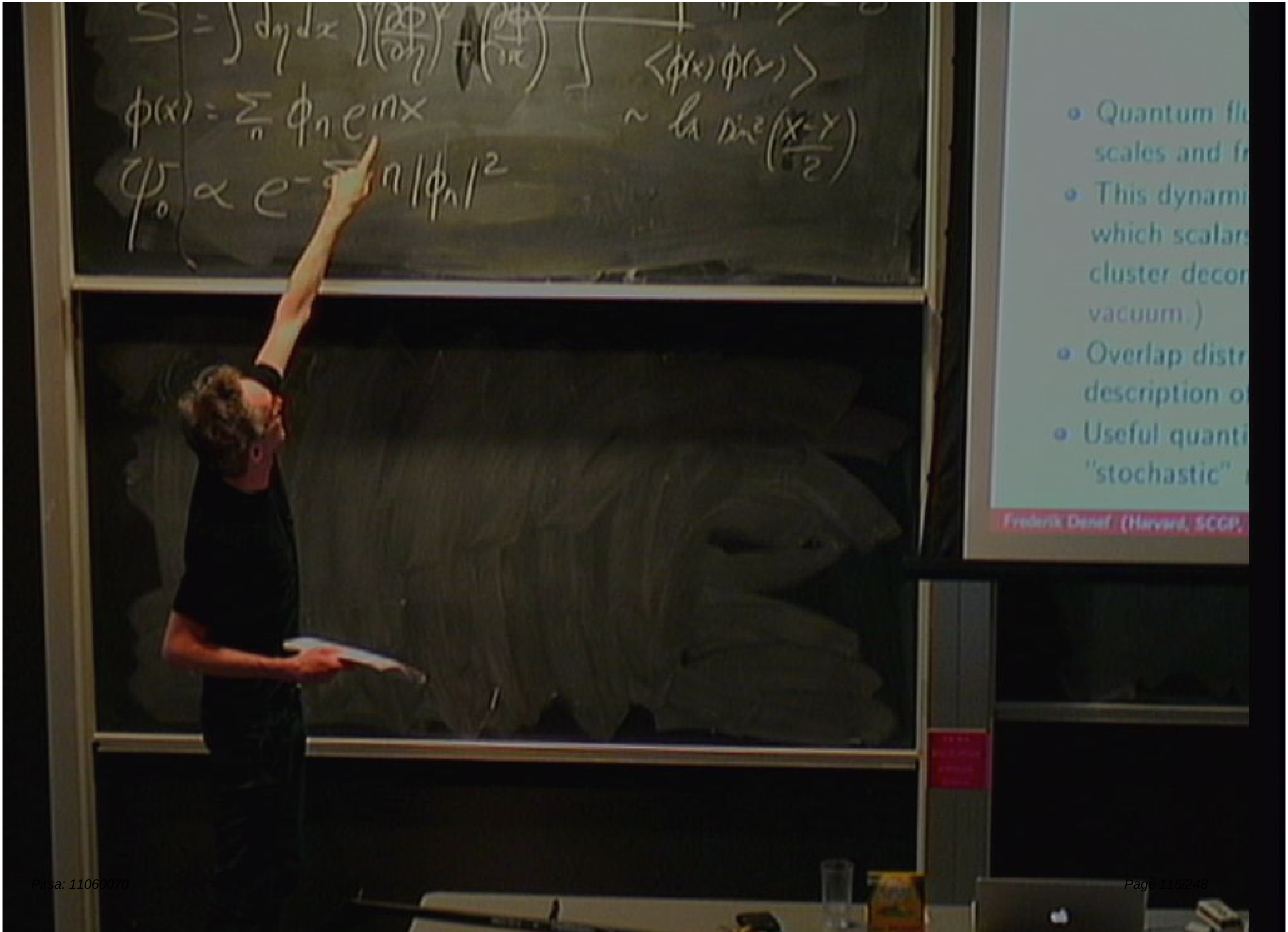
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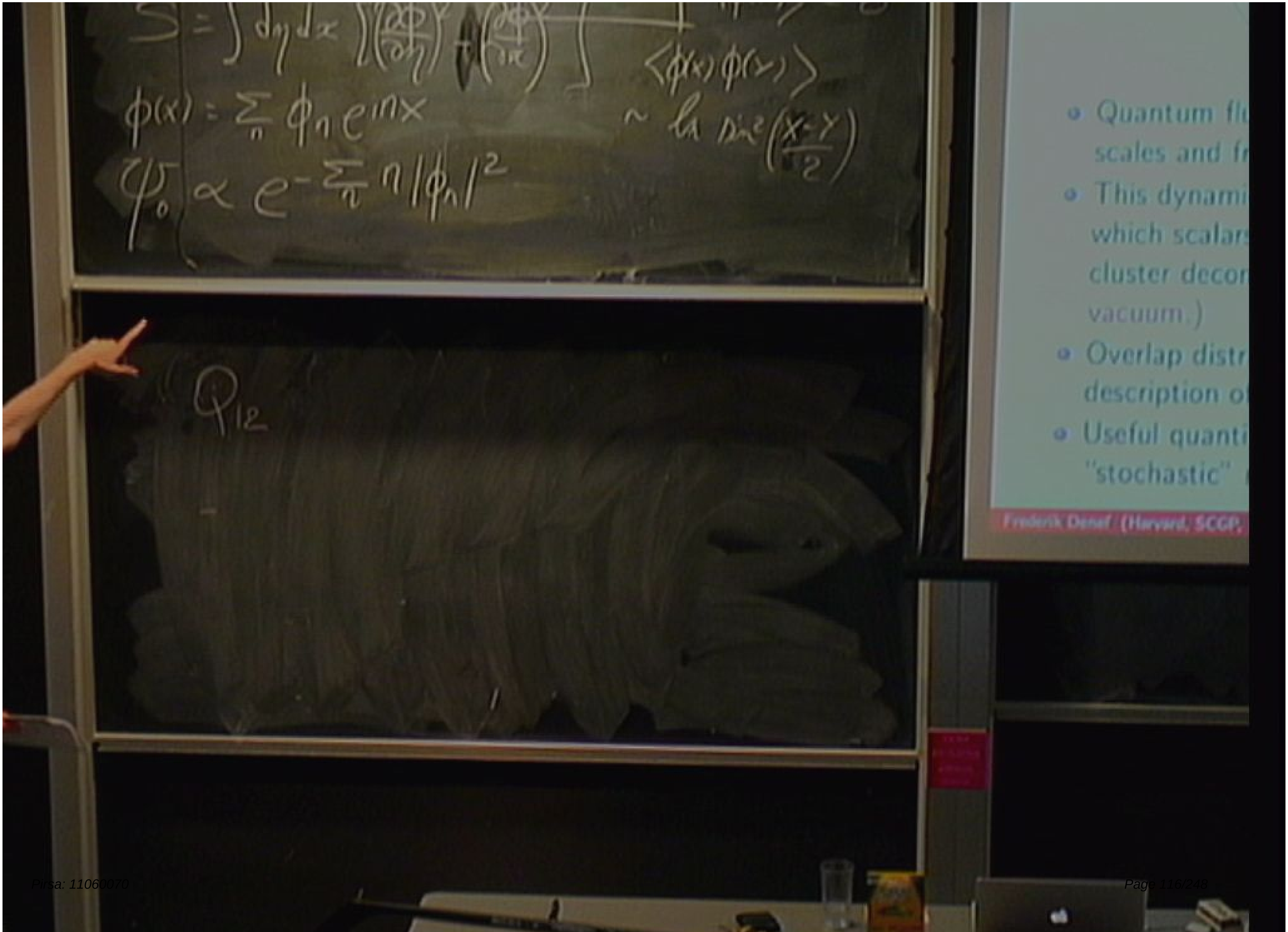
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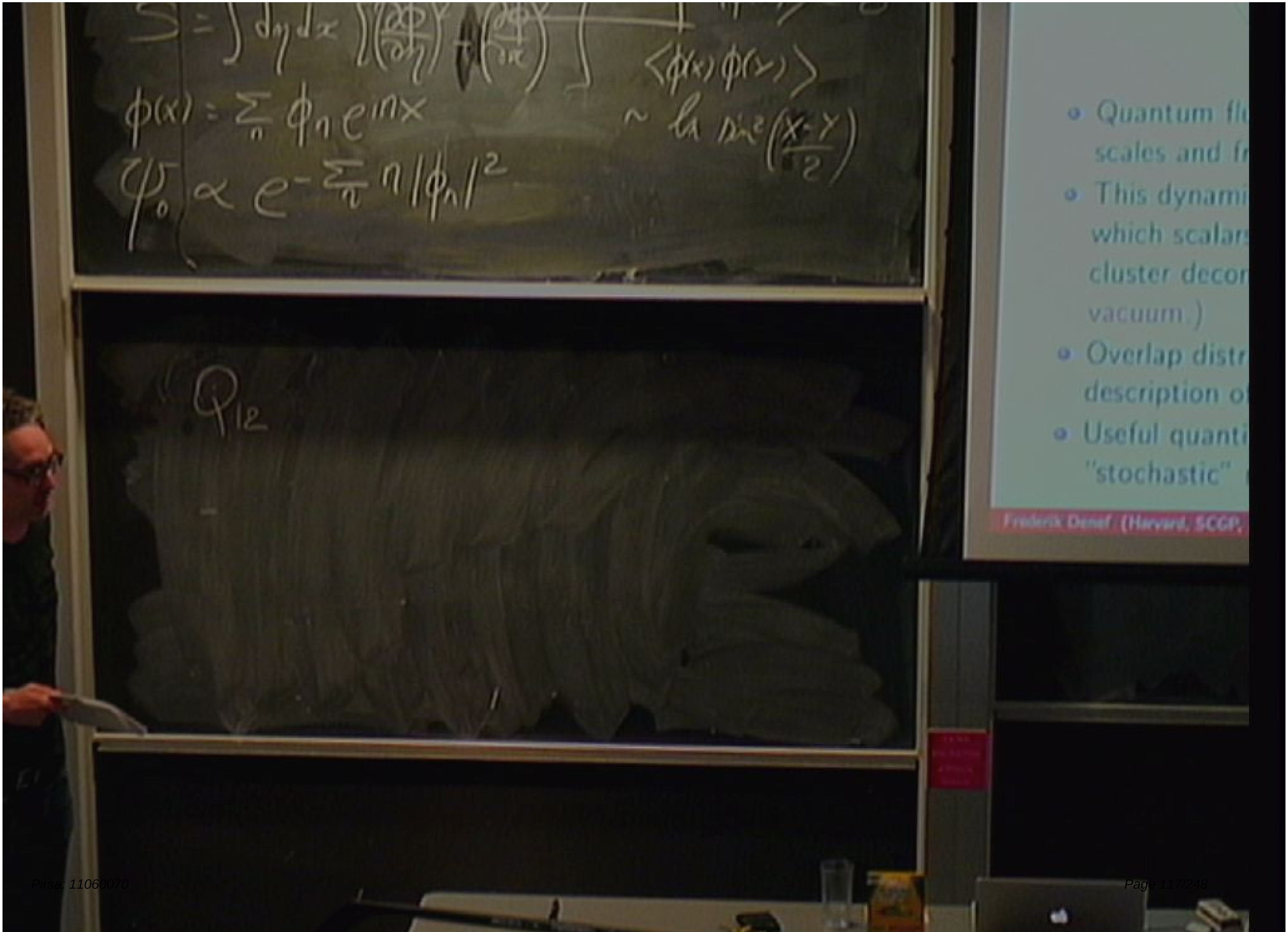
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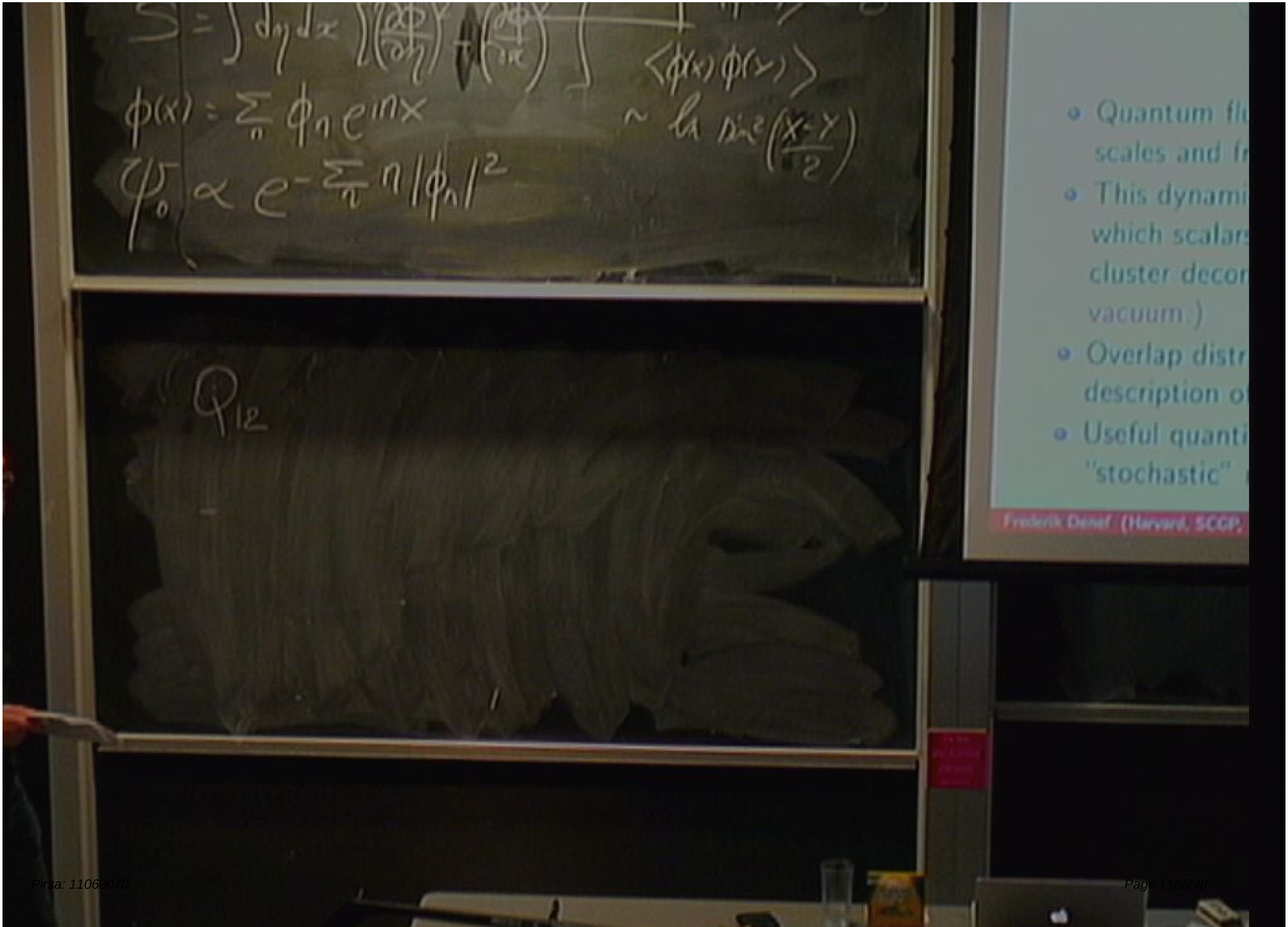
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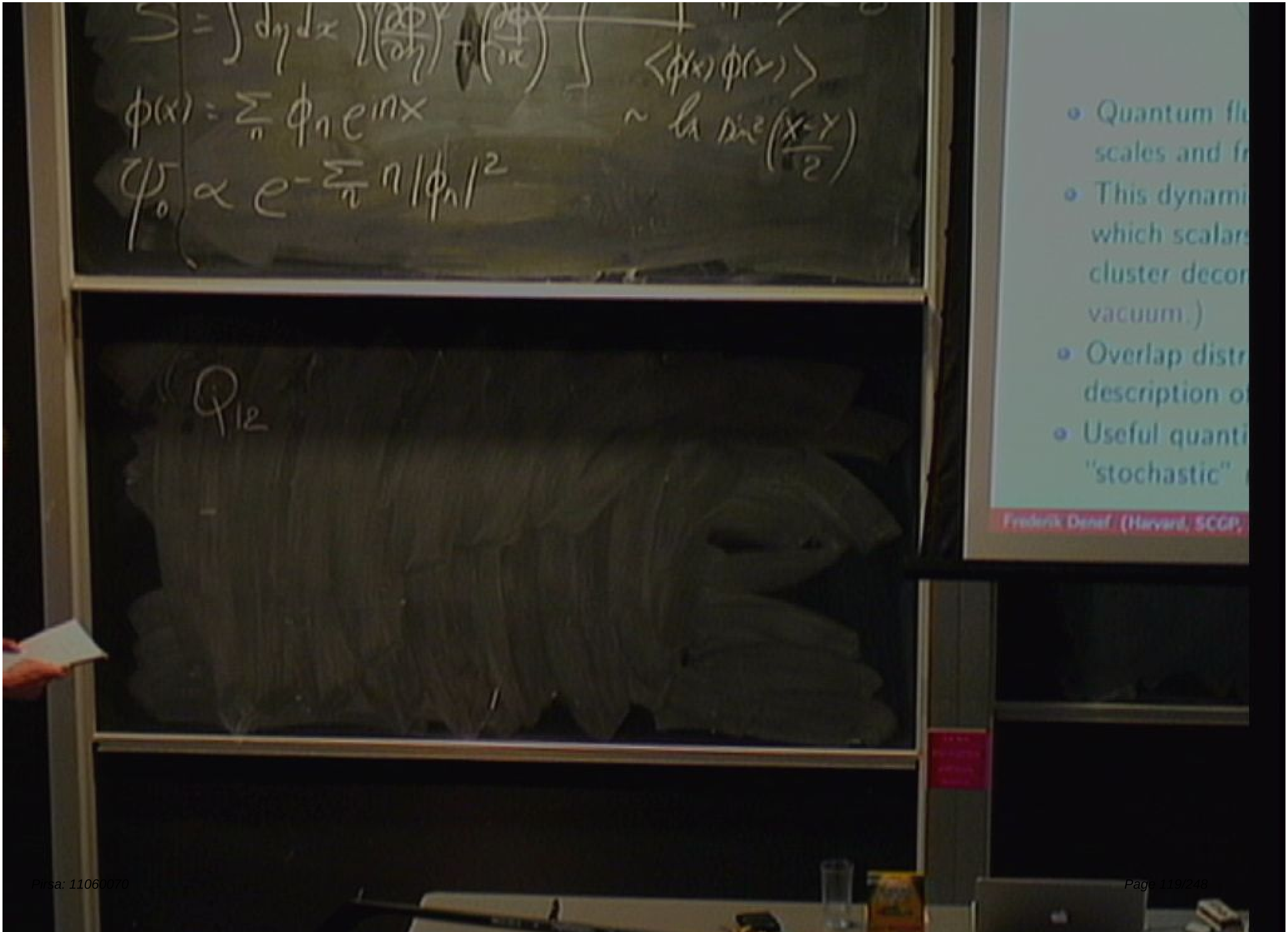
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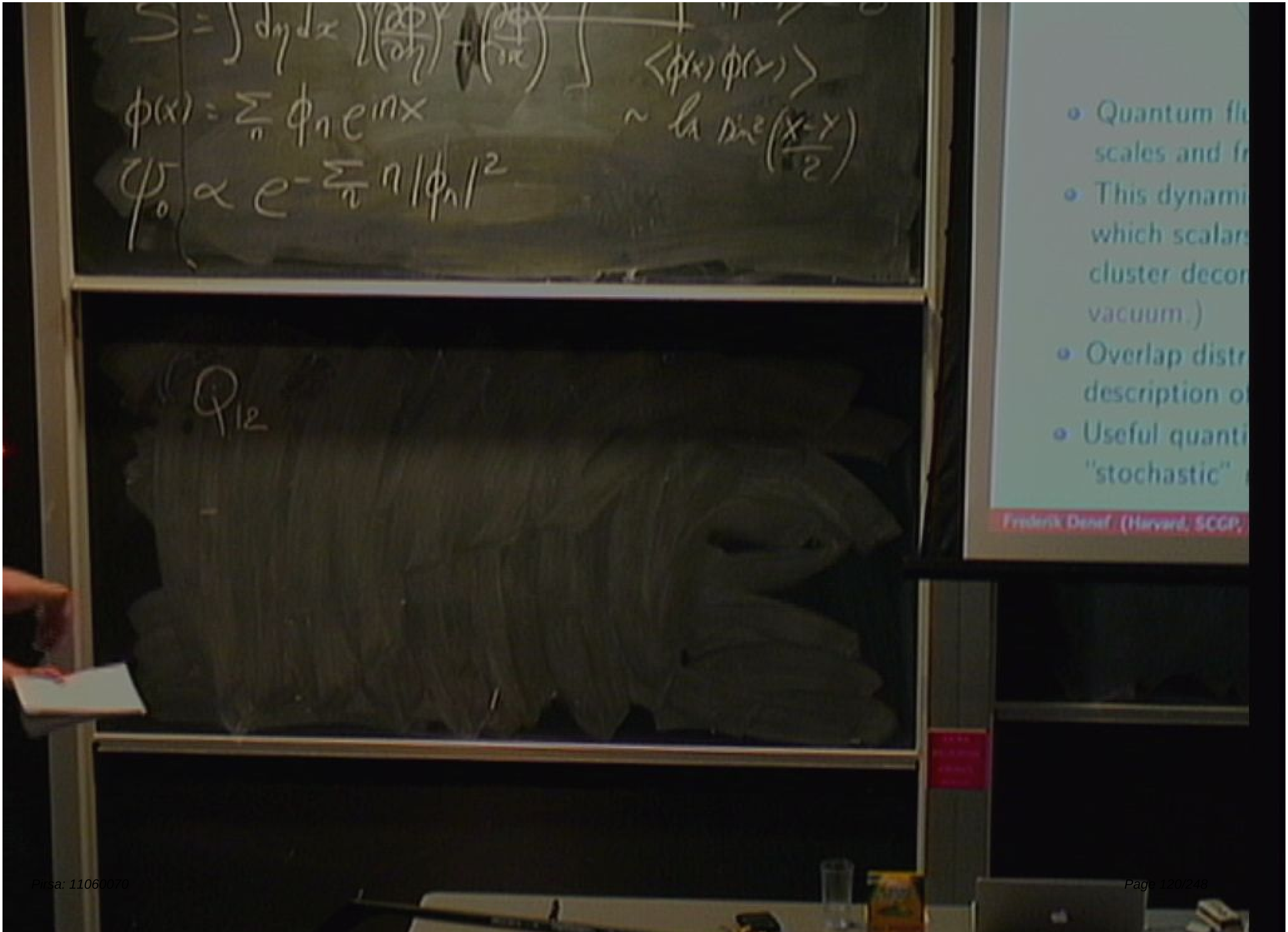
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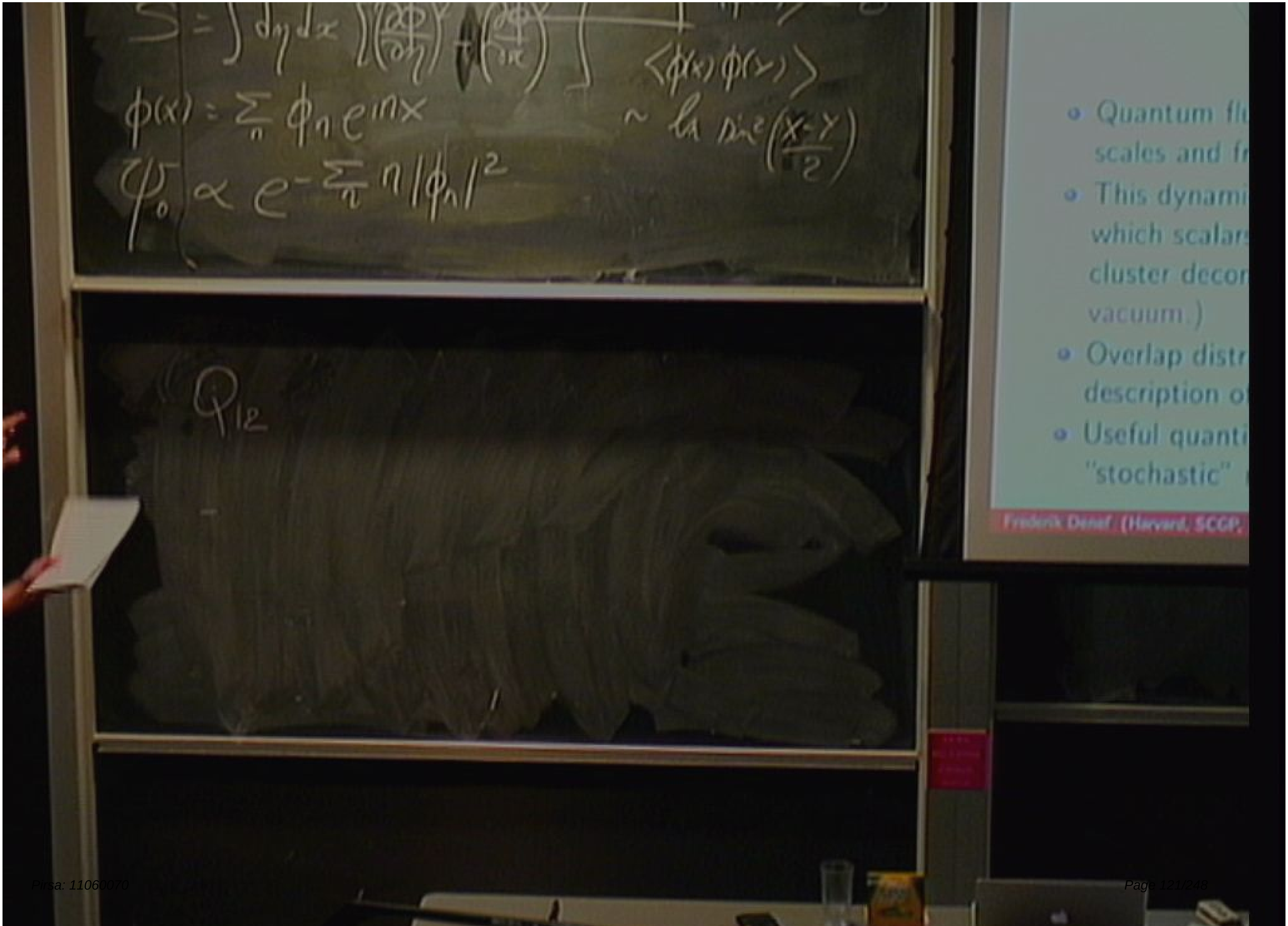
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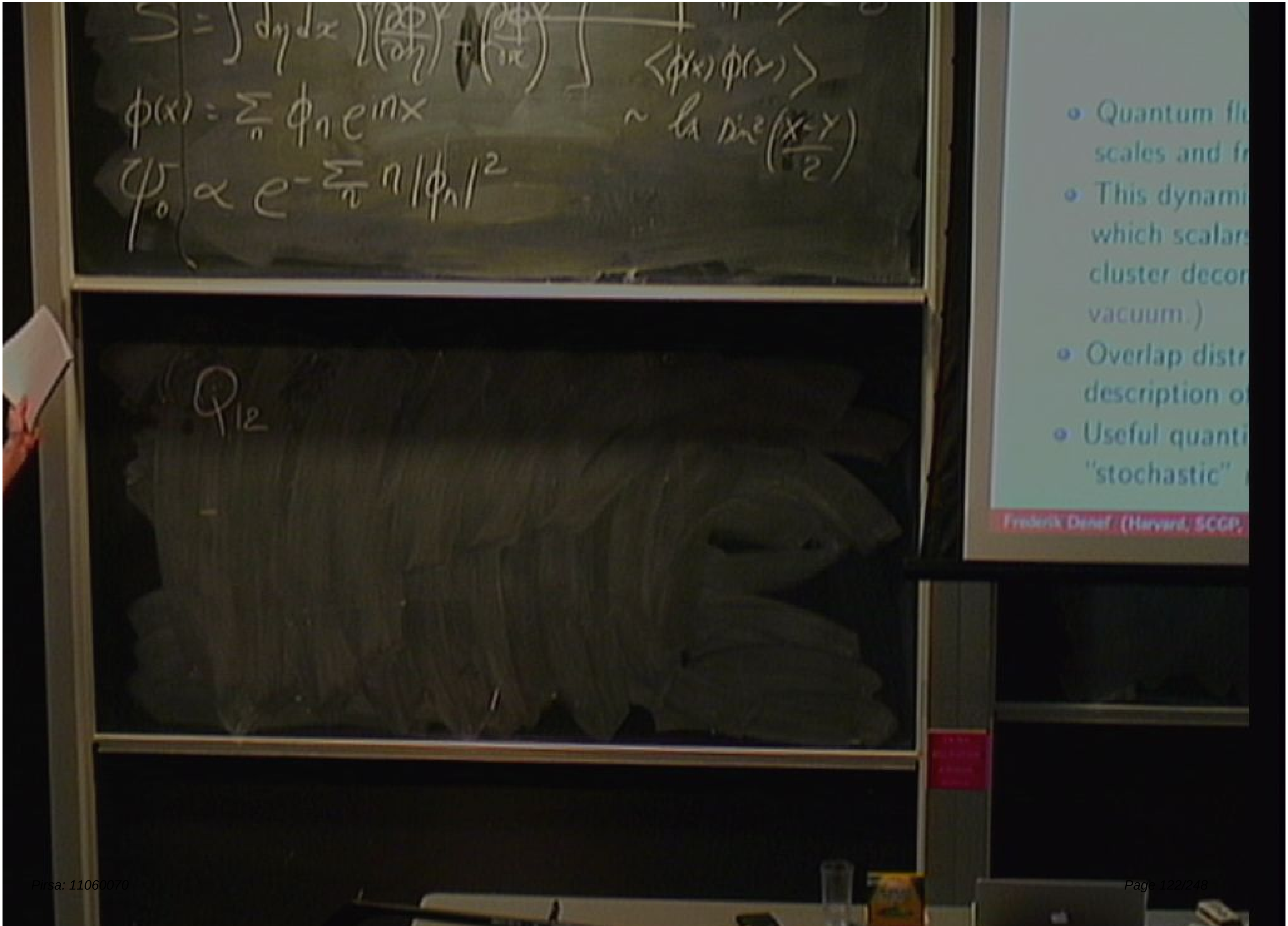
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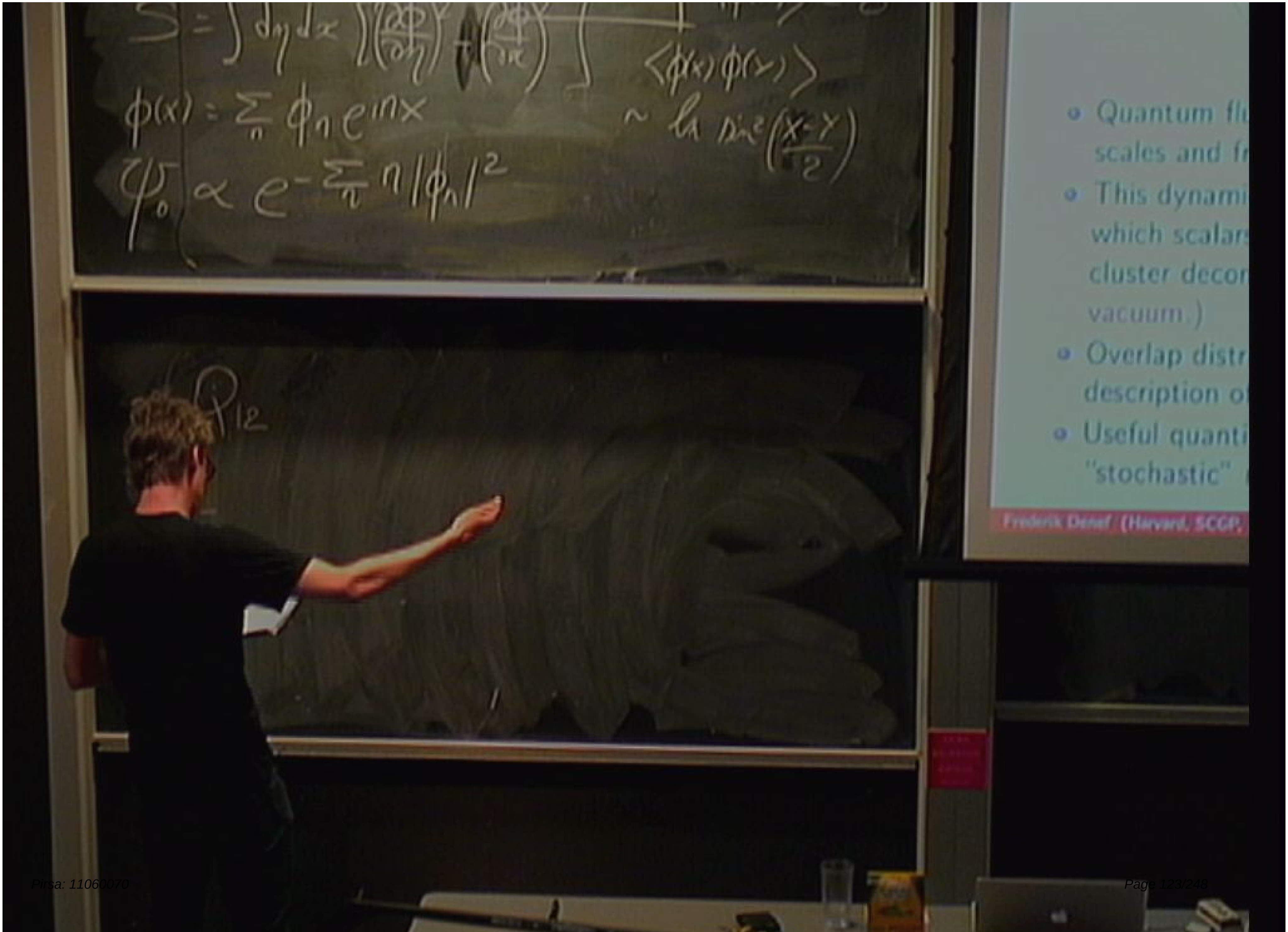
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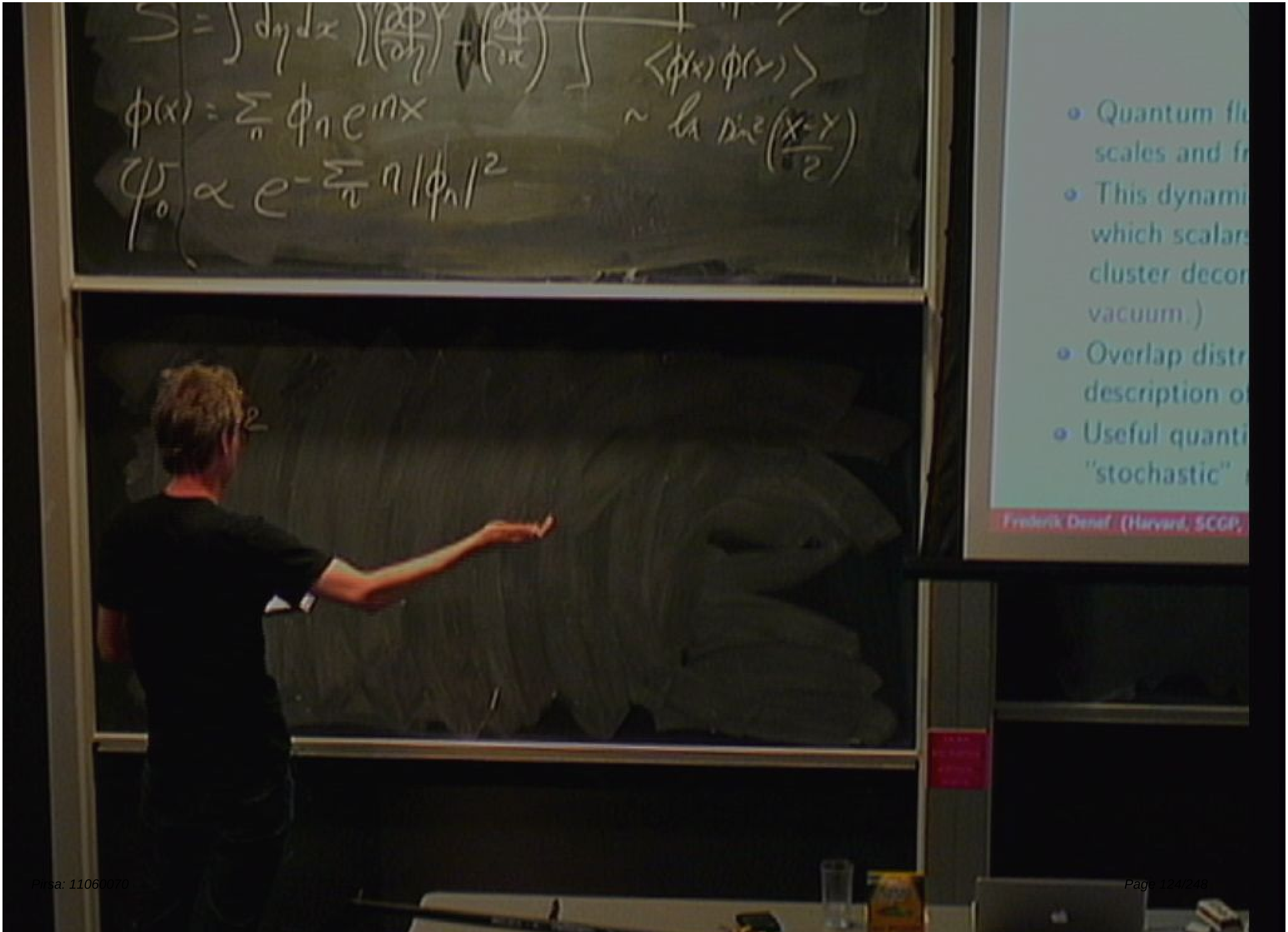
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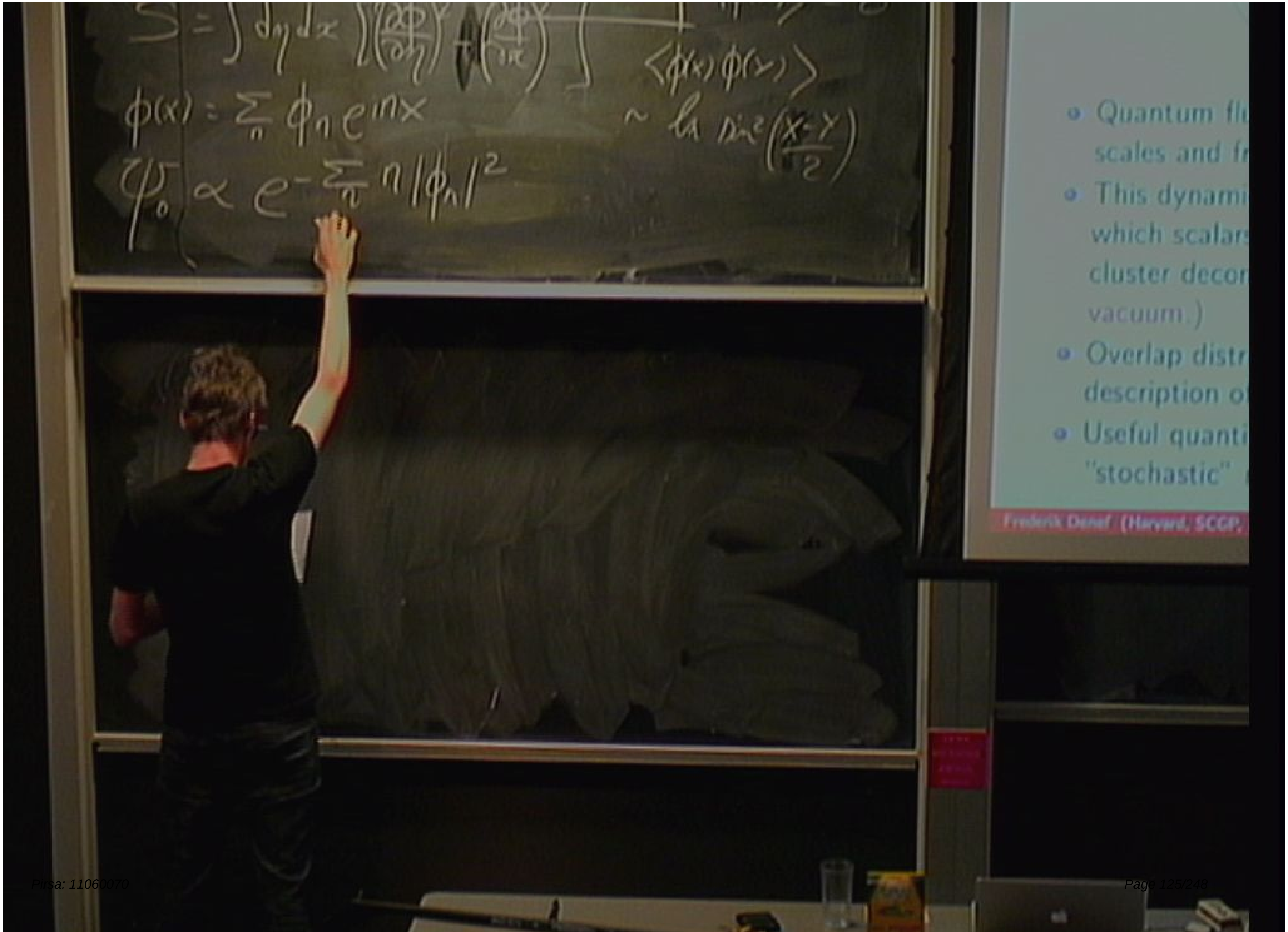
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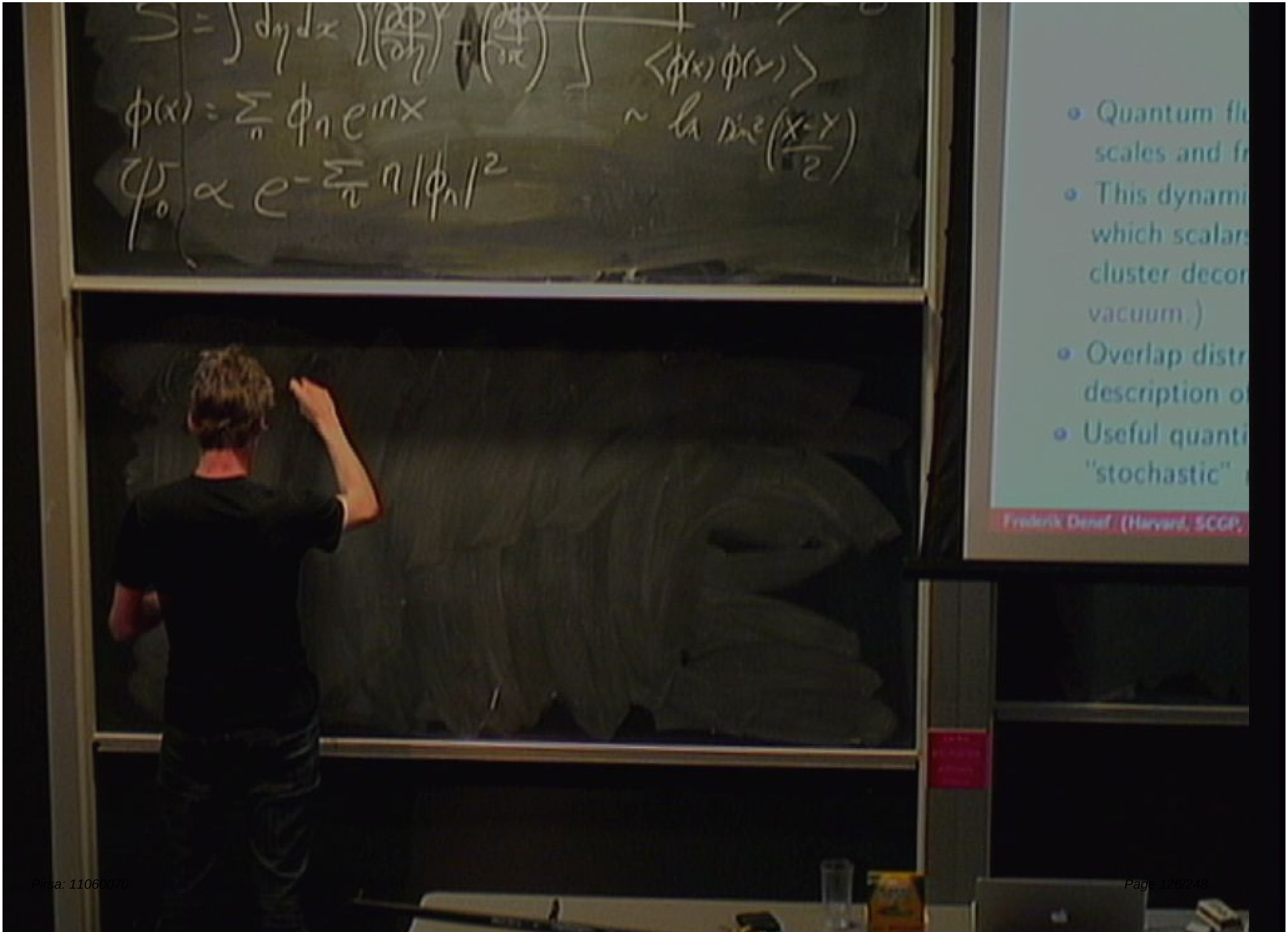
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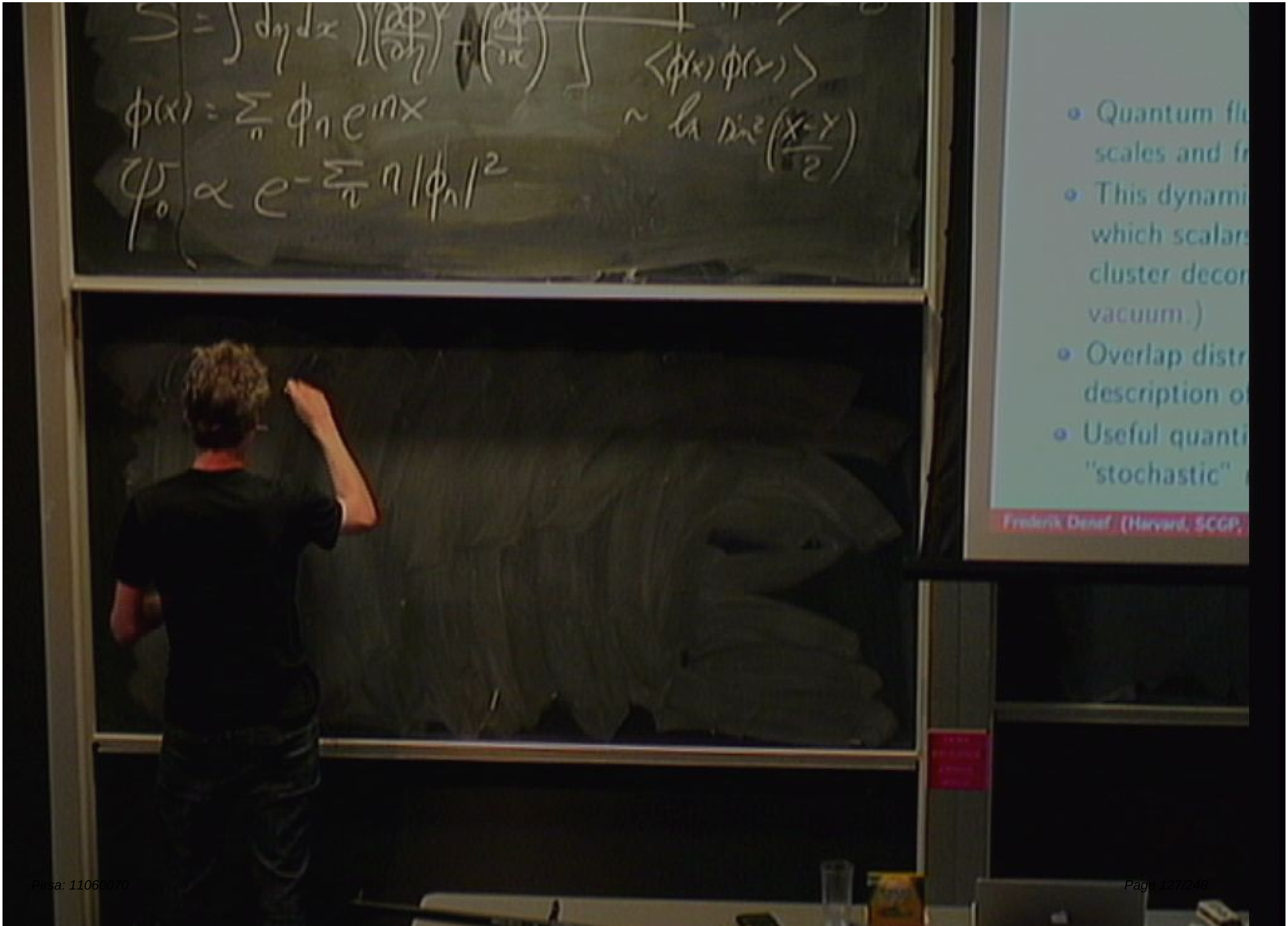
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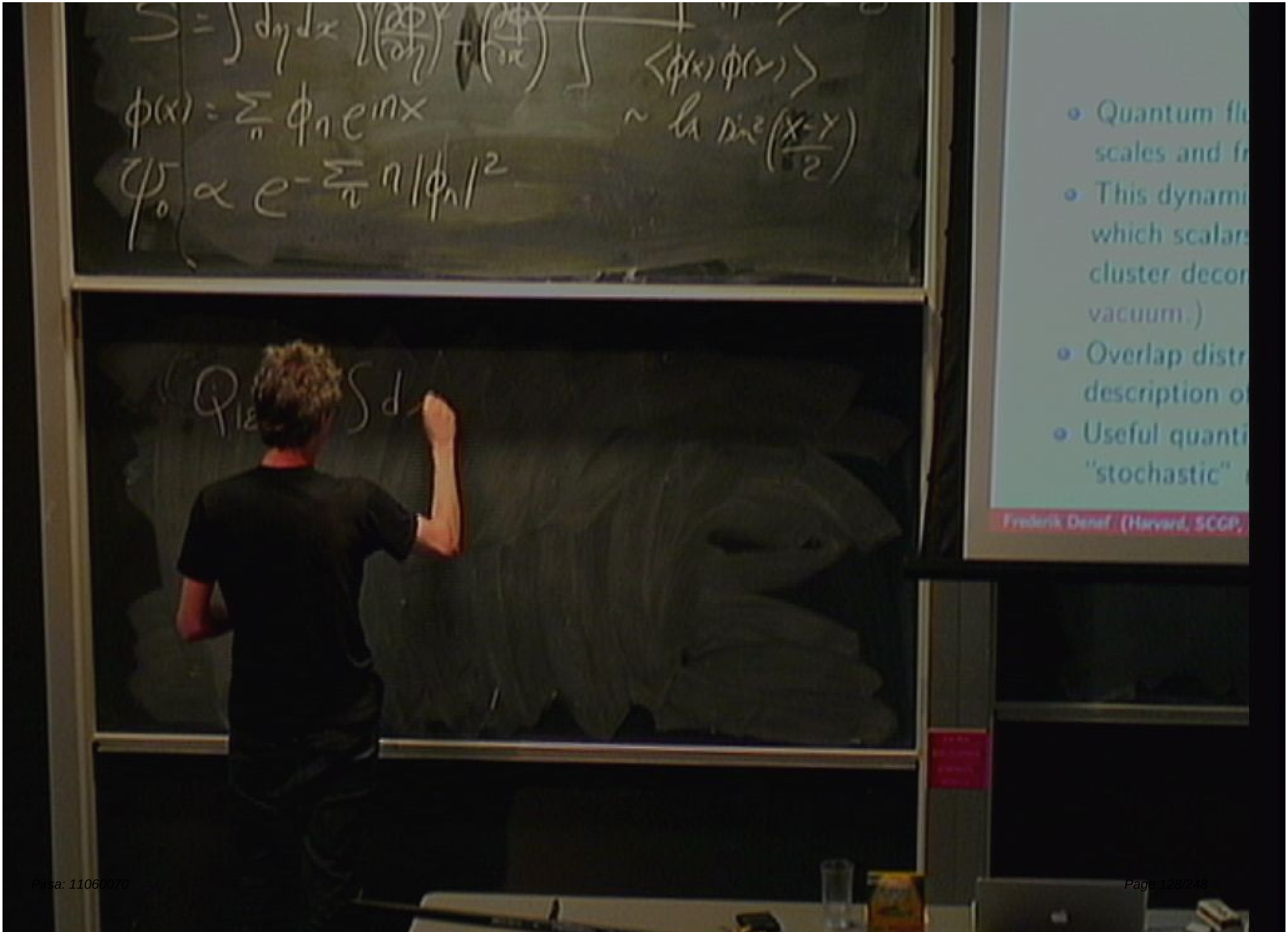
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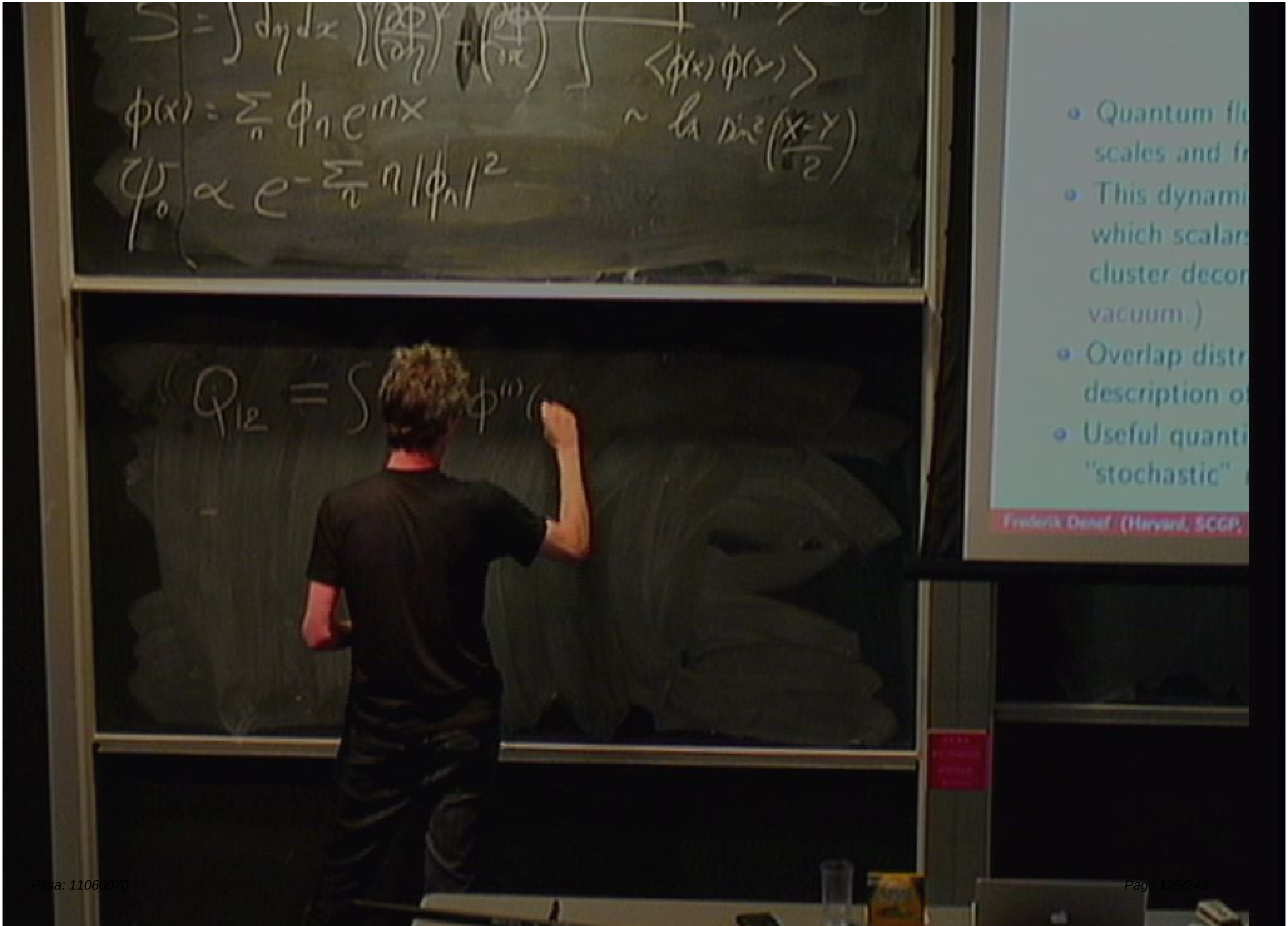
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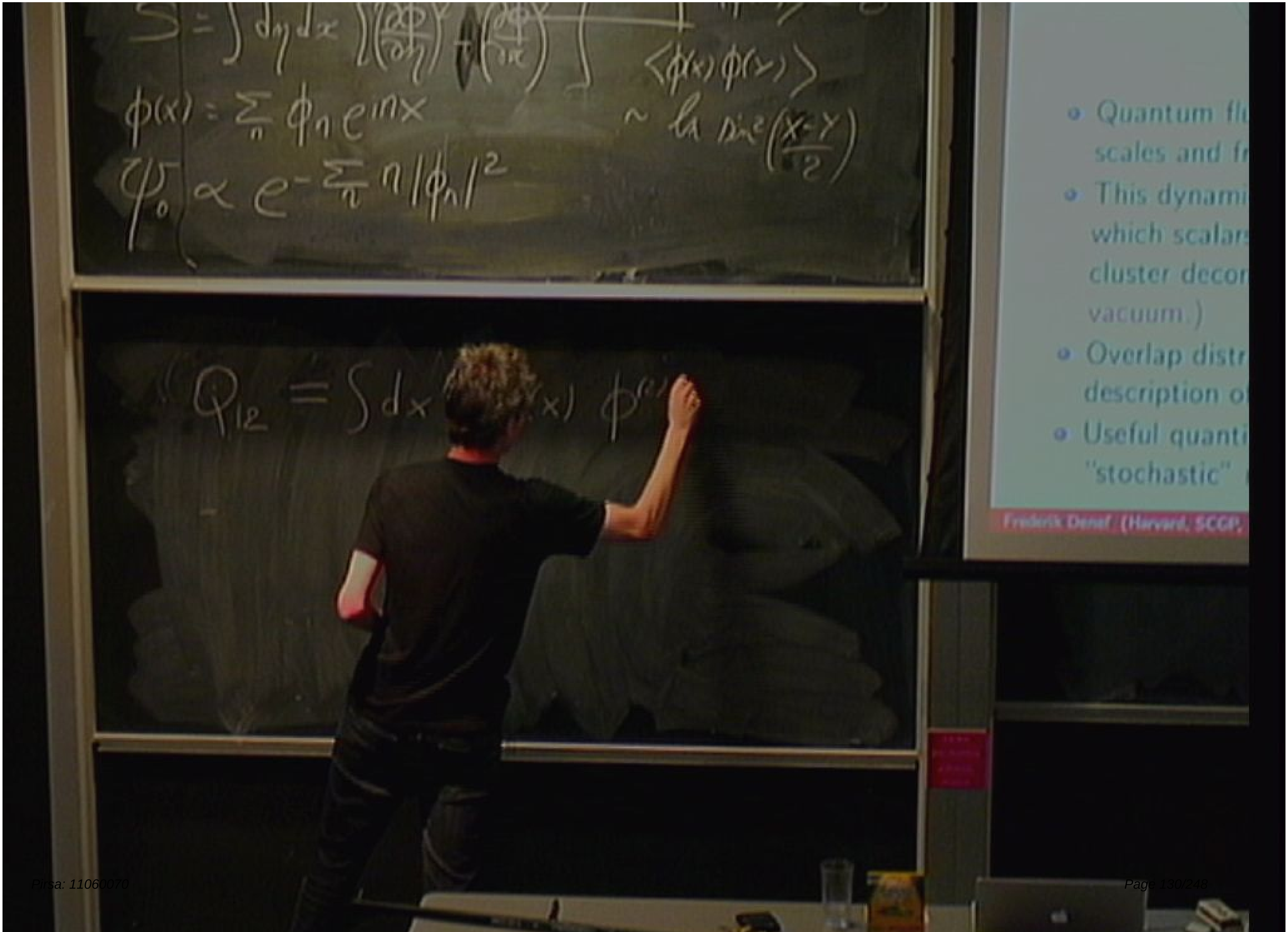
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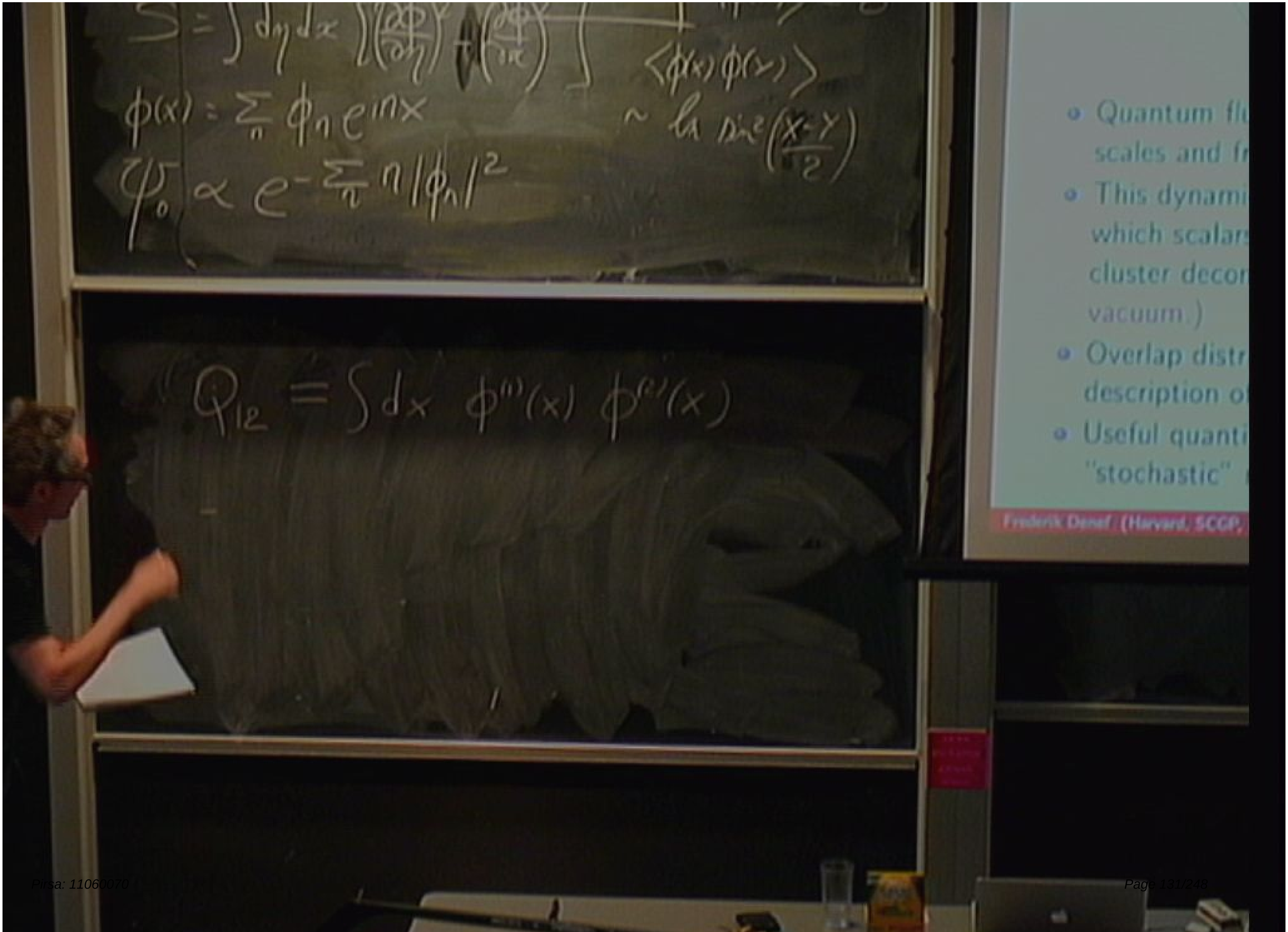
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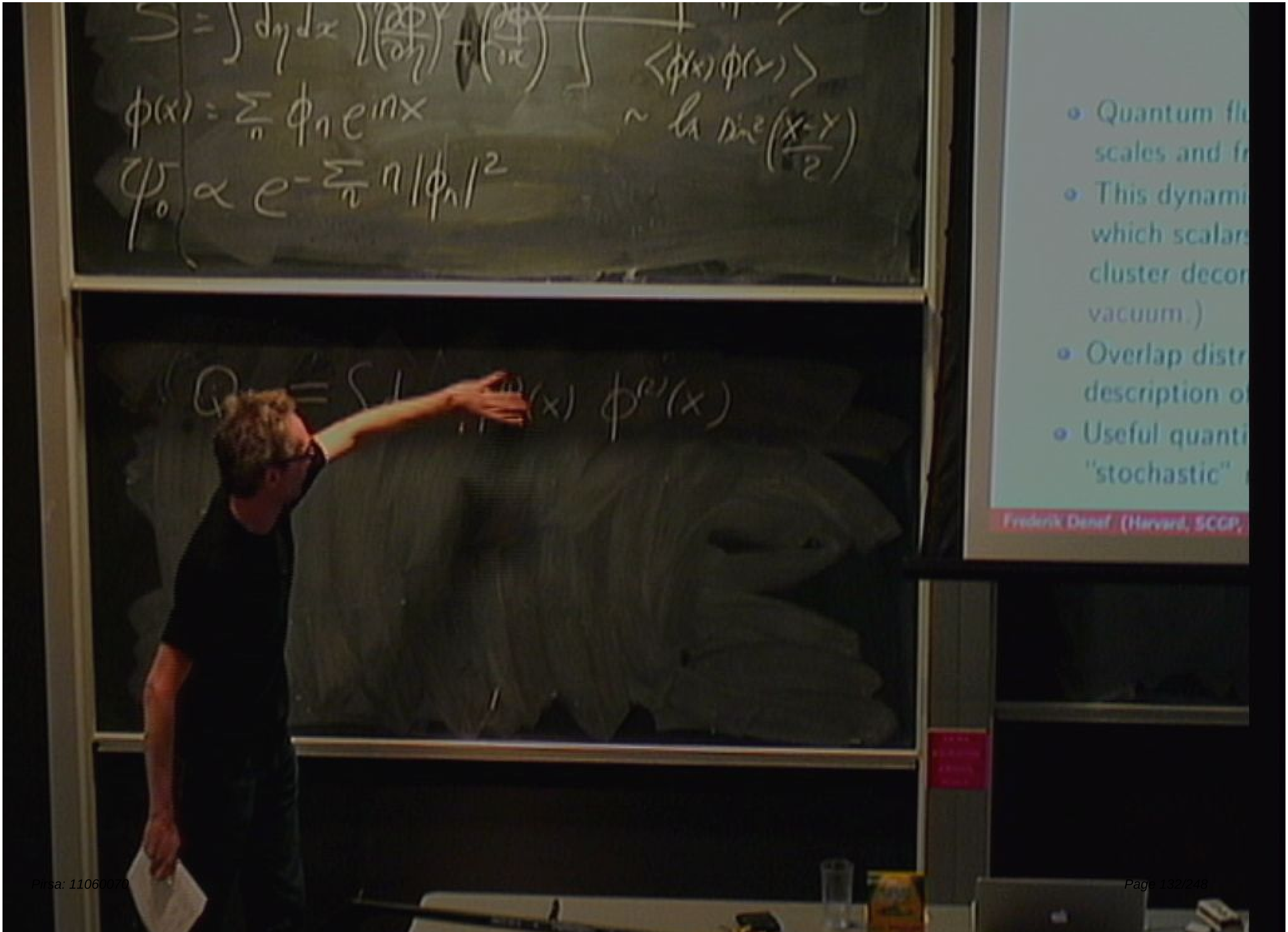
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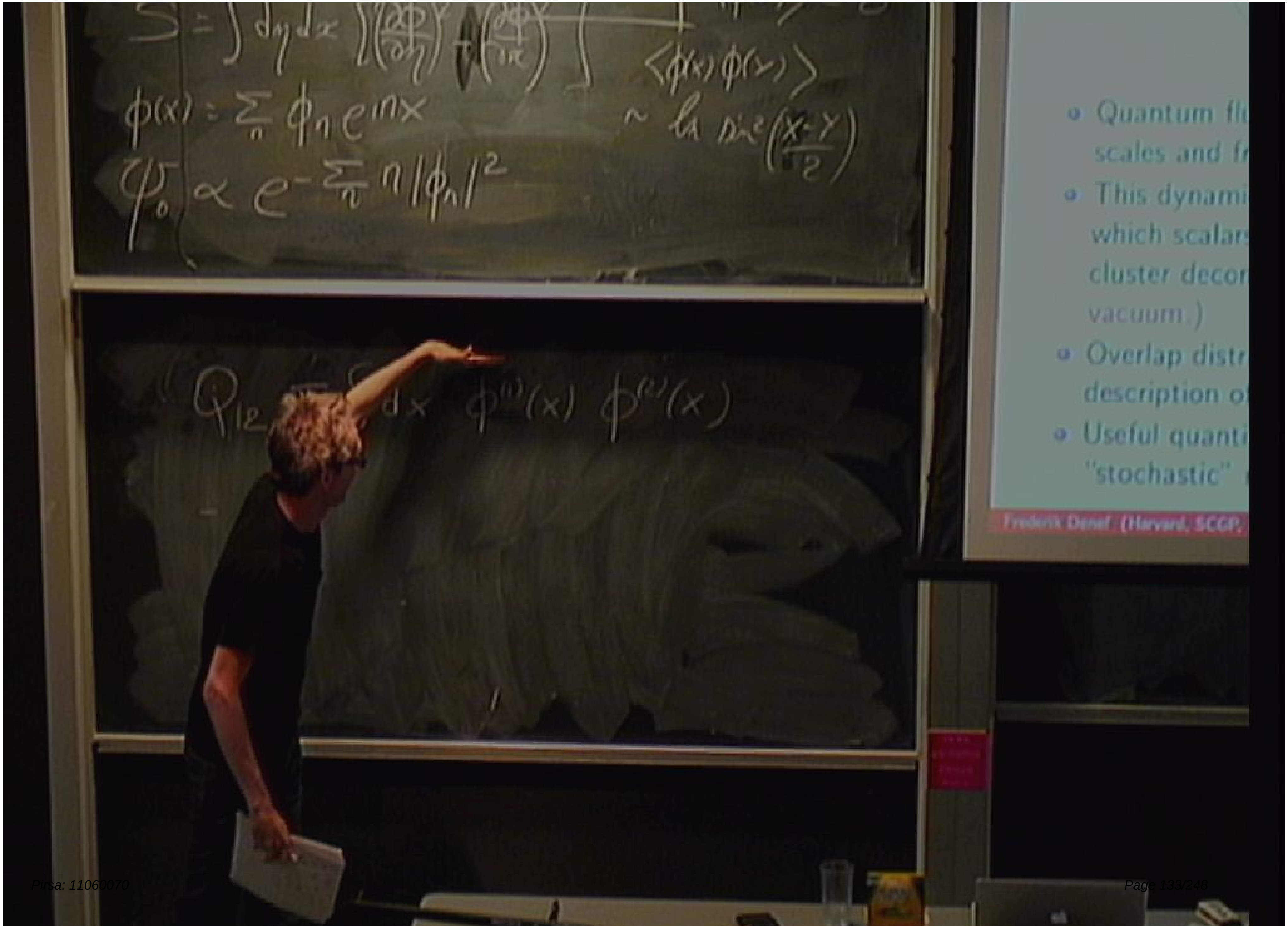

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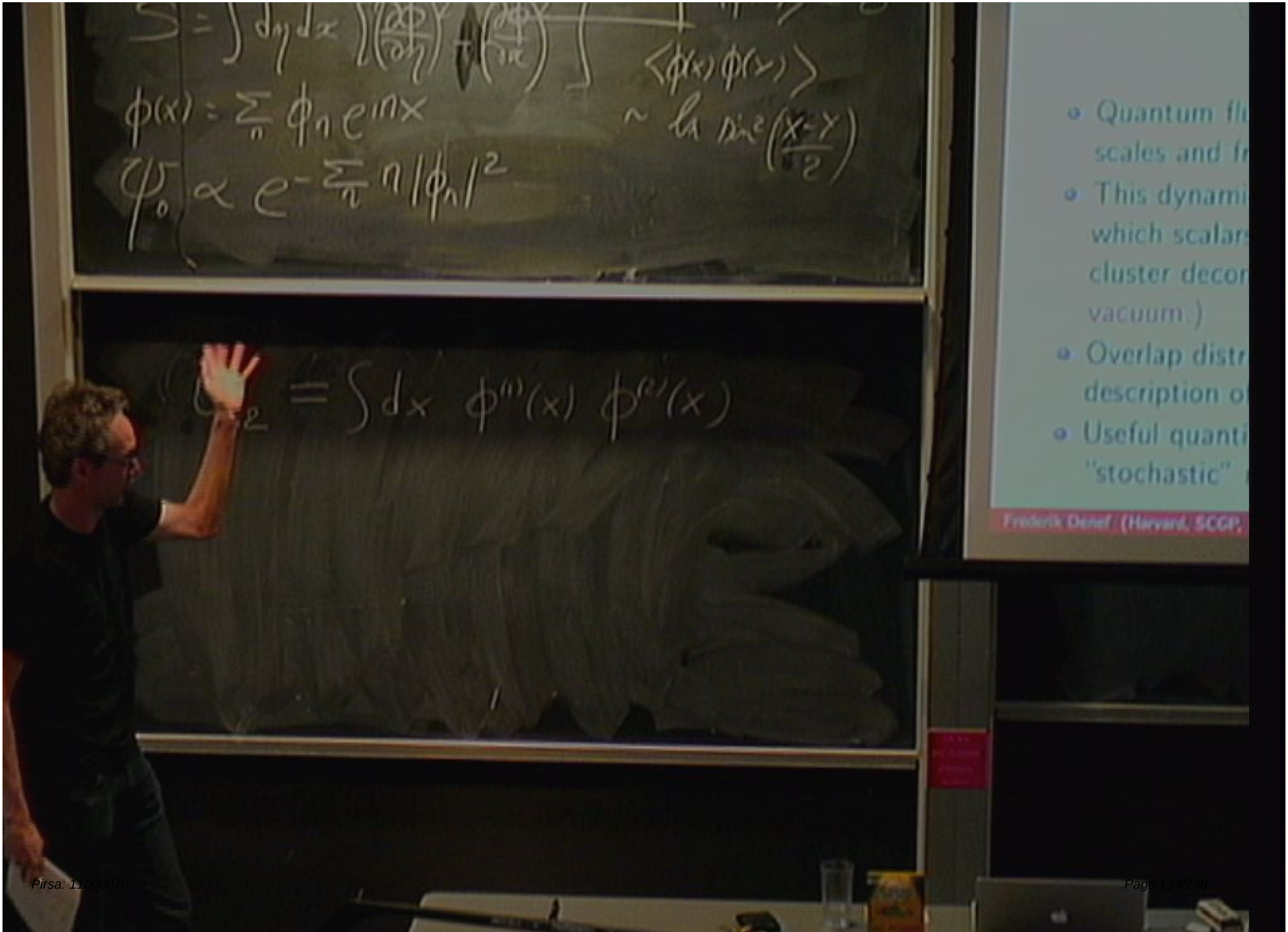

$\langle \phi(x)\phi(y) \rangle$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$
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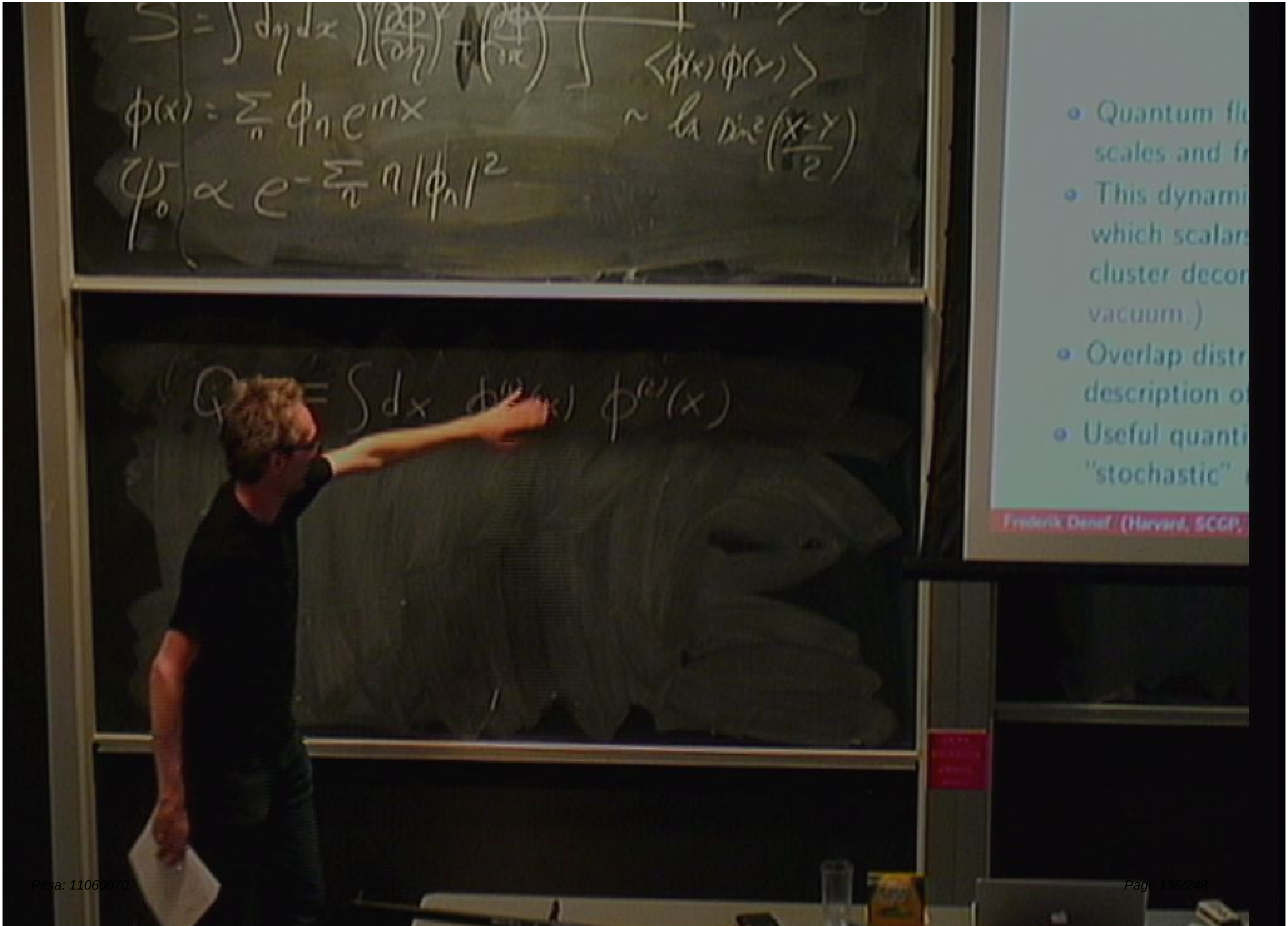
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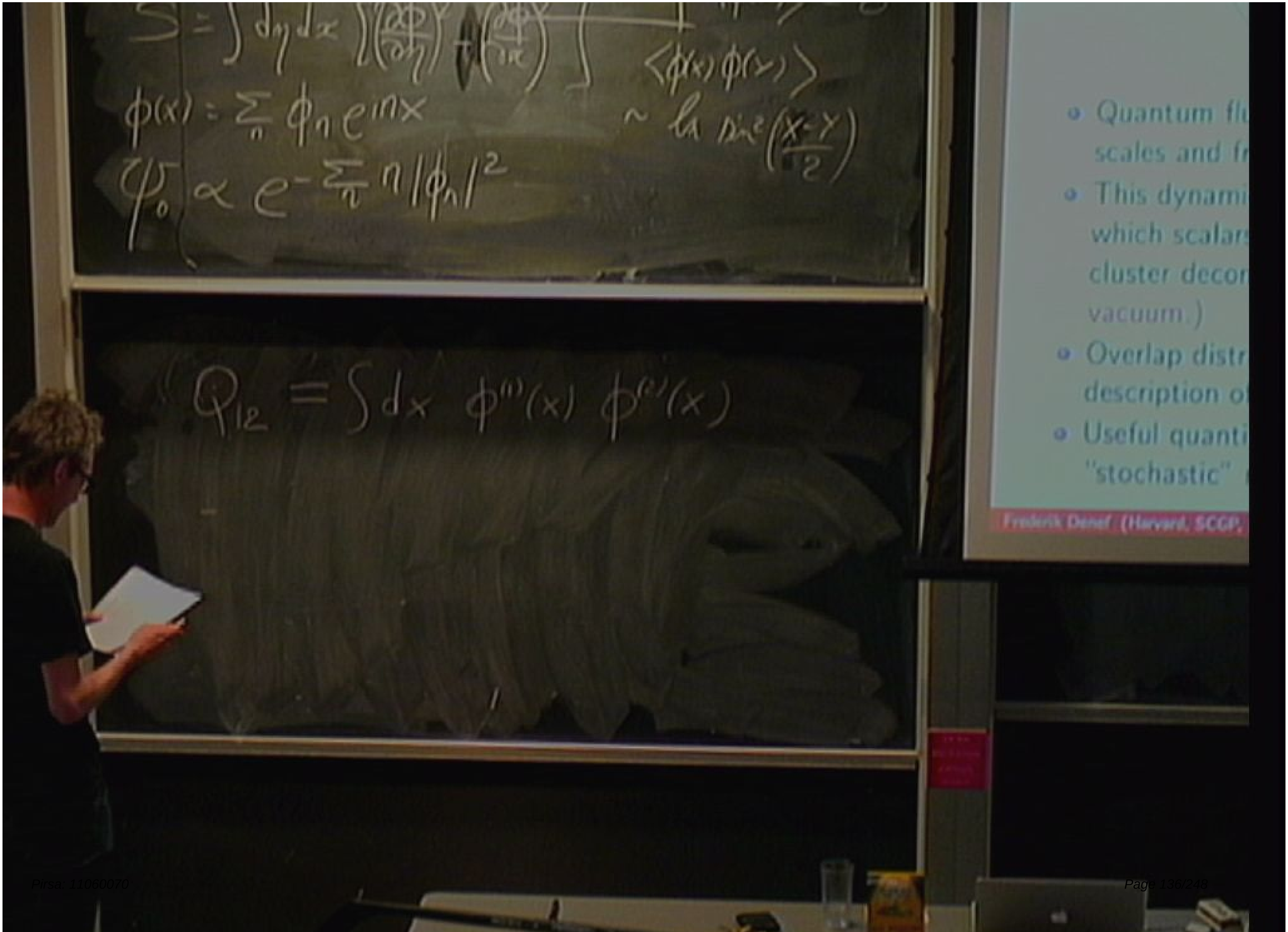

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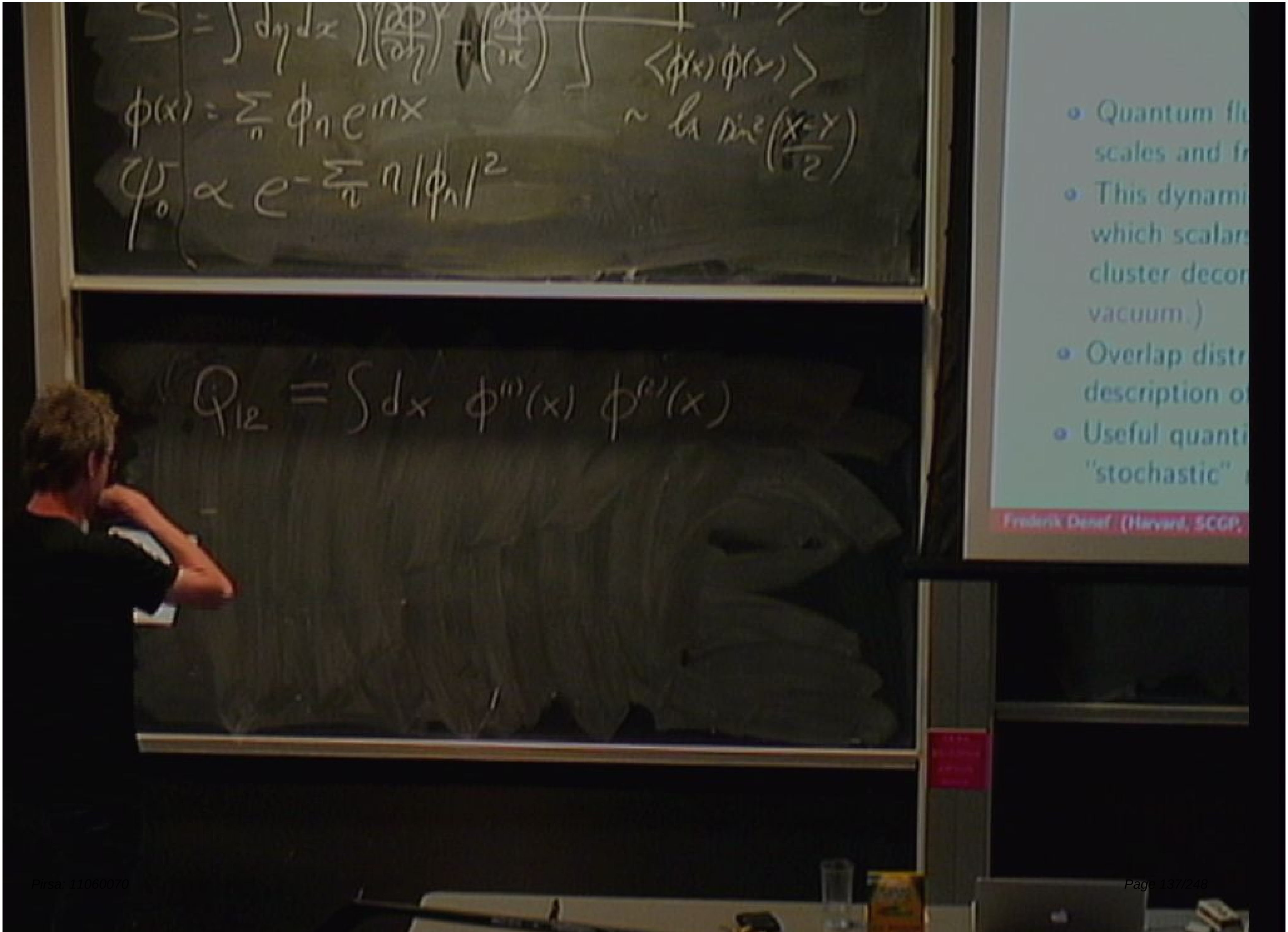

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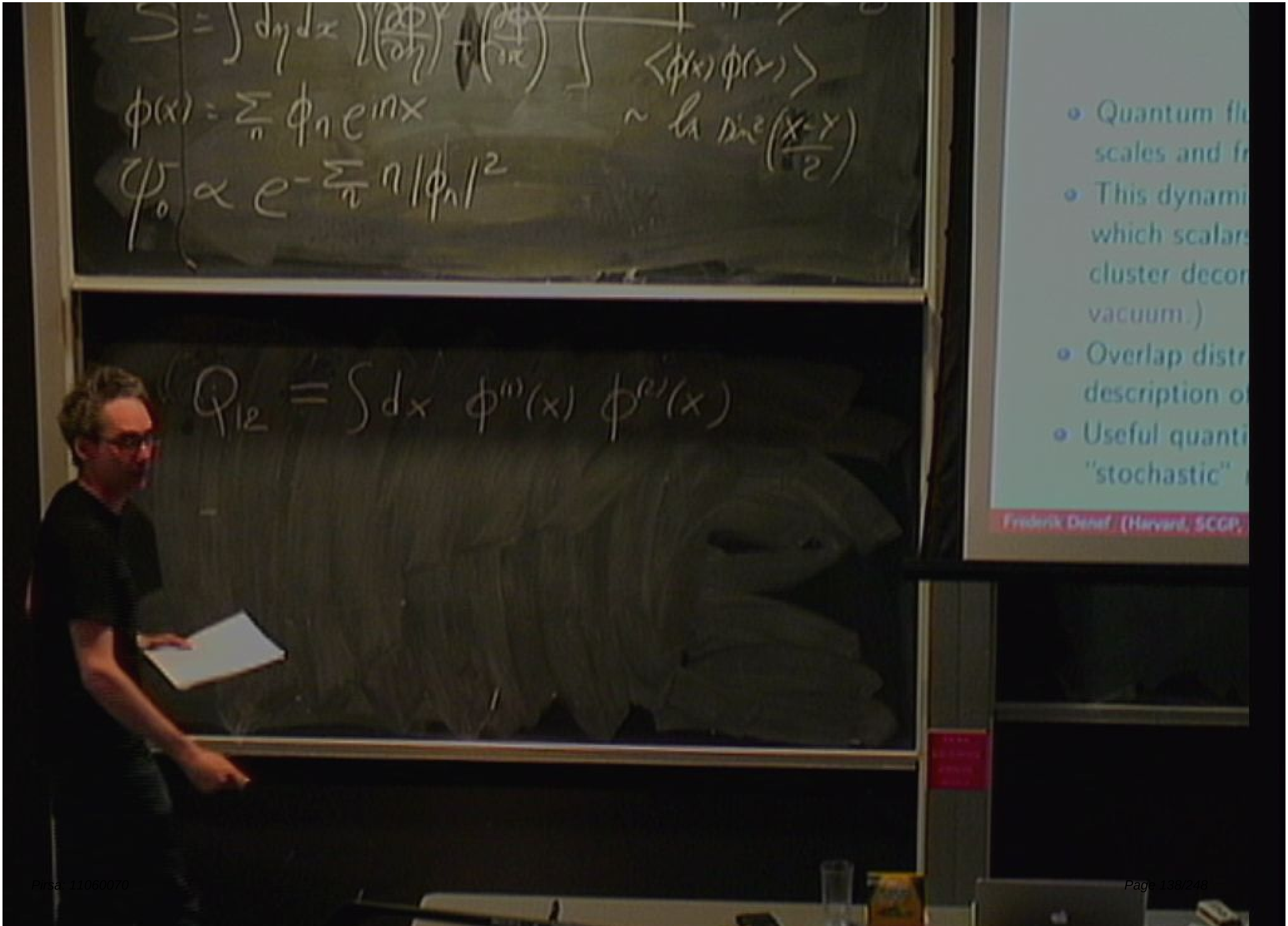
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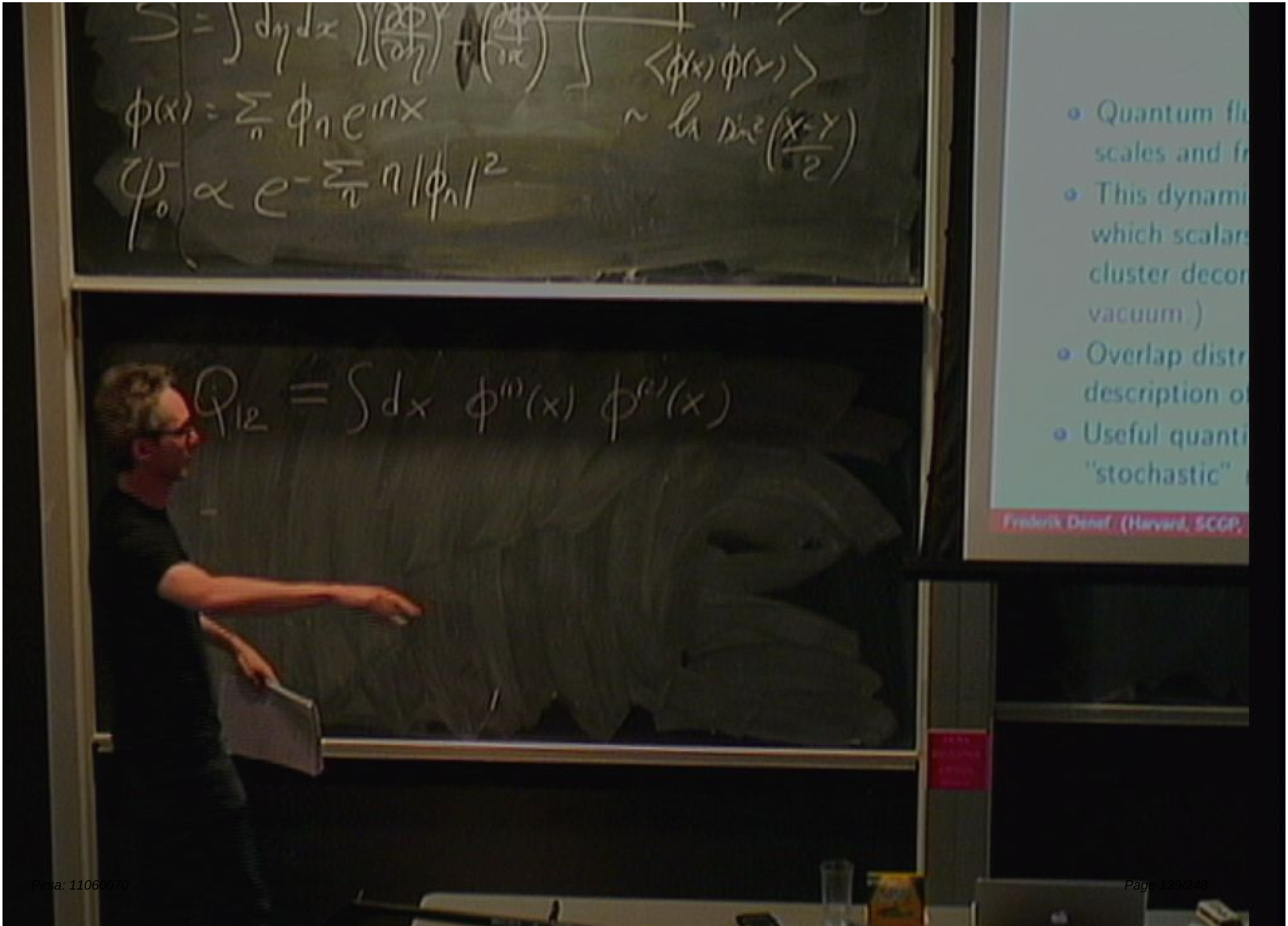
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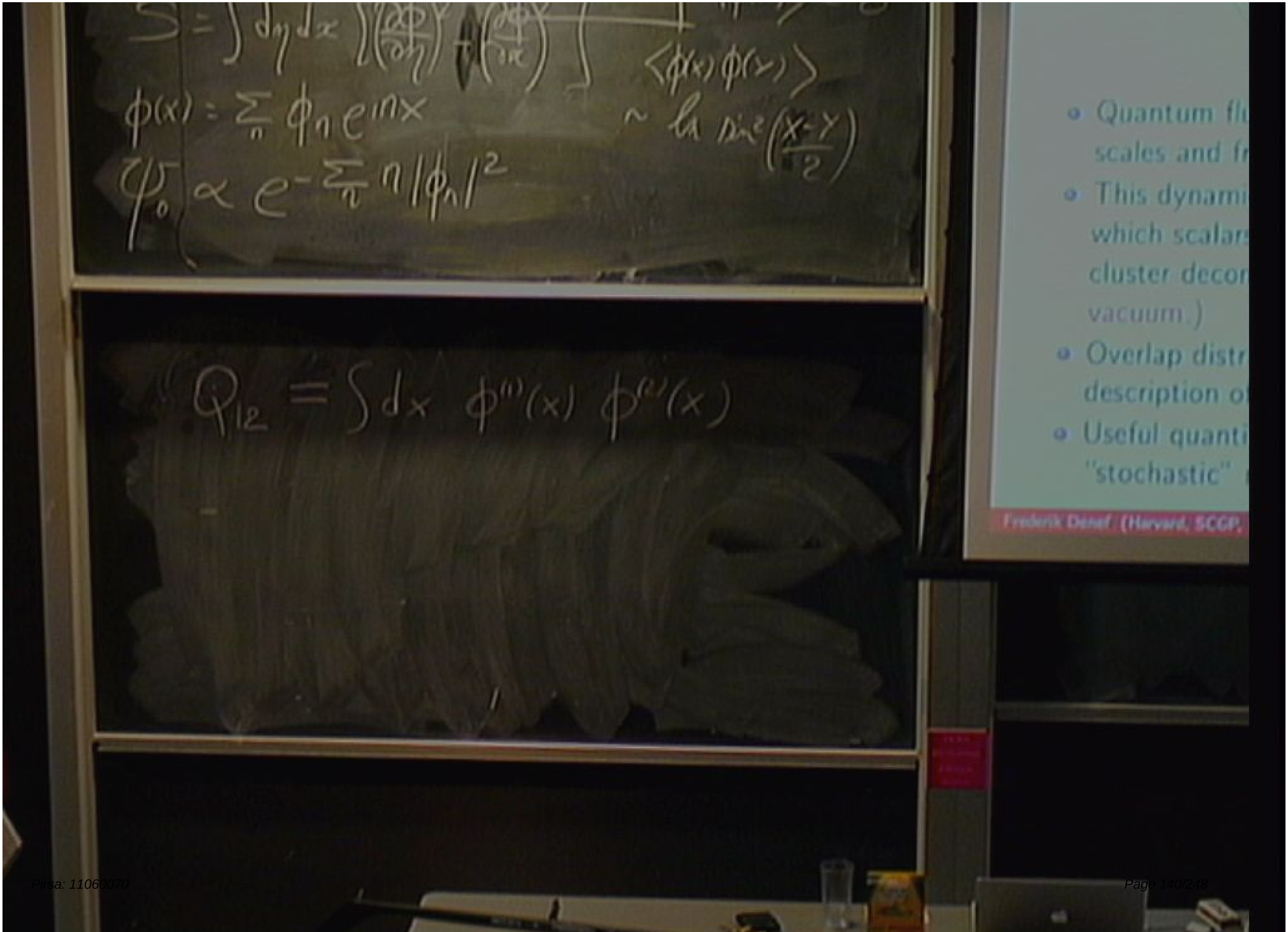

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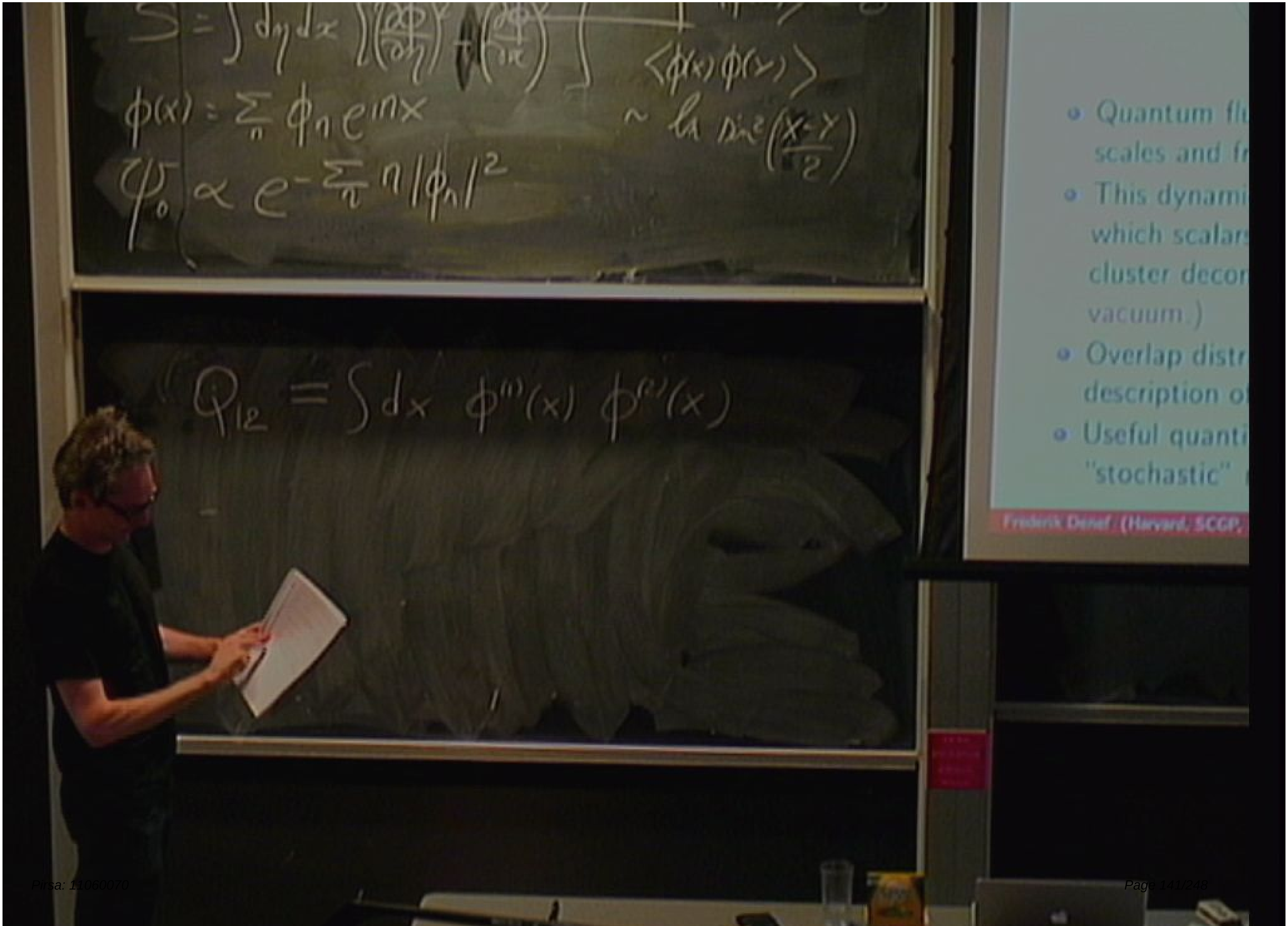
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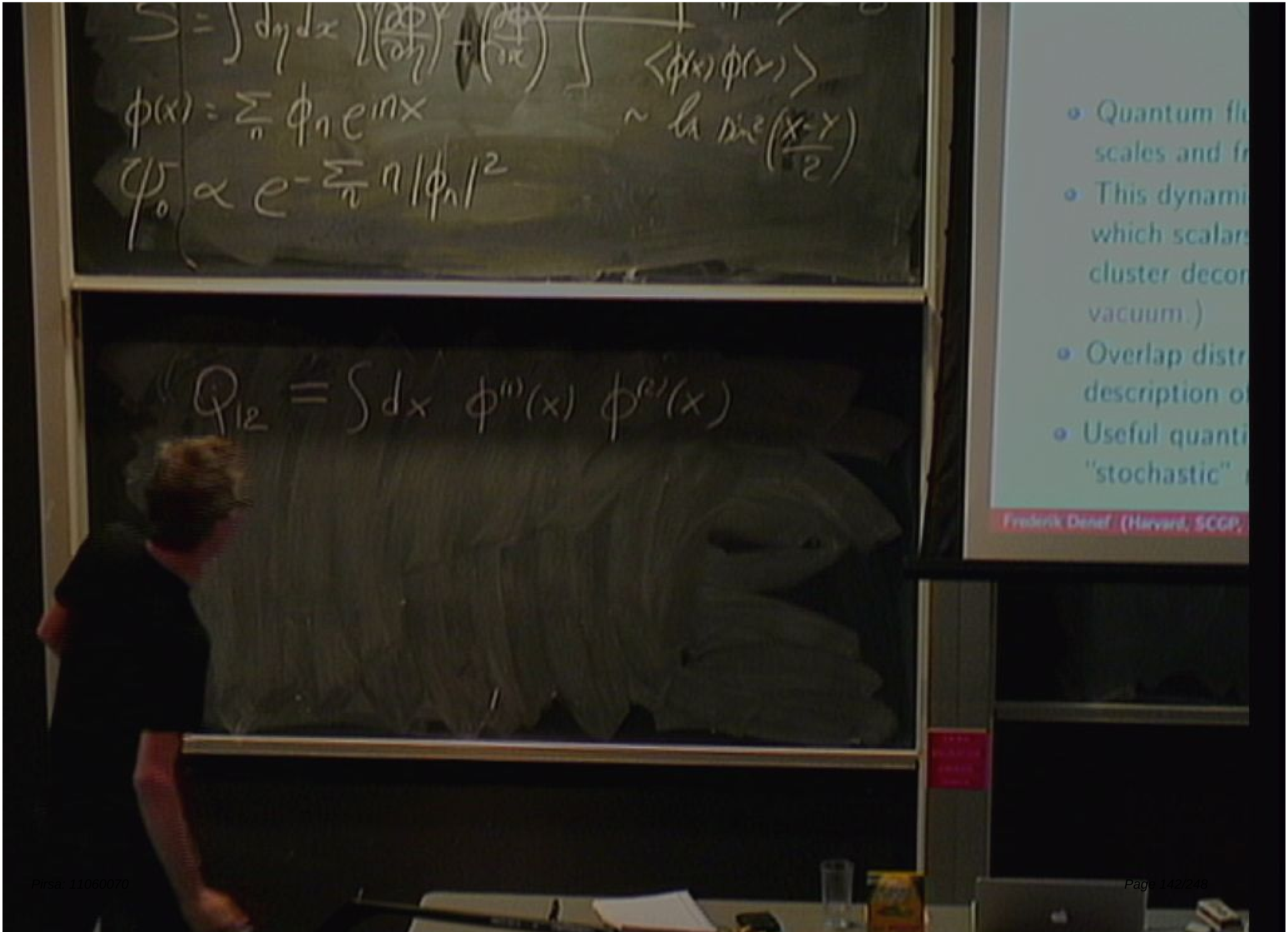
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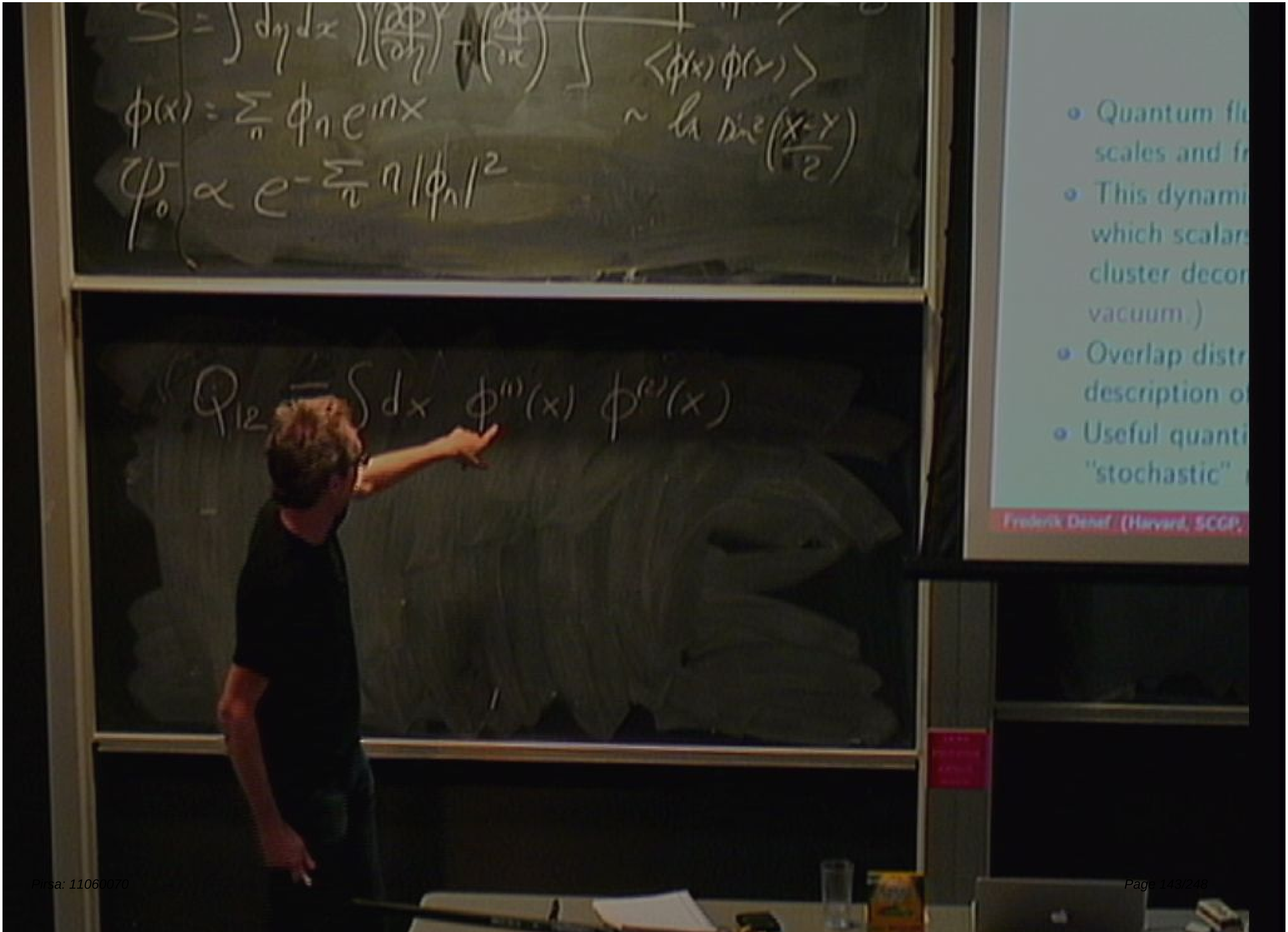
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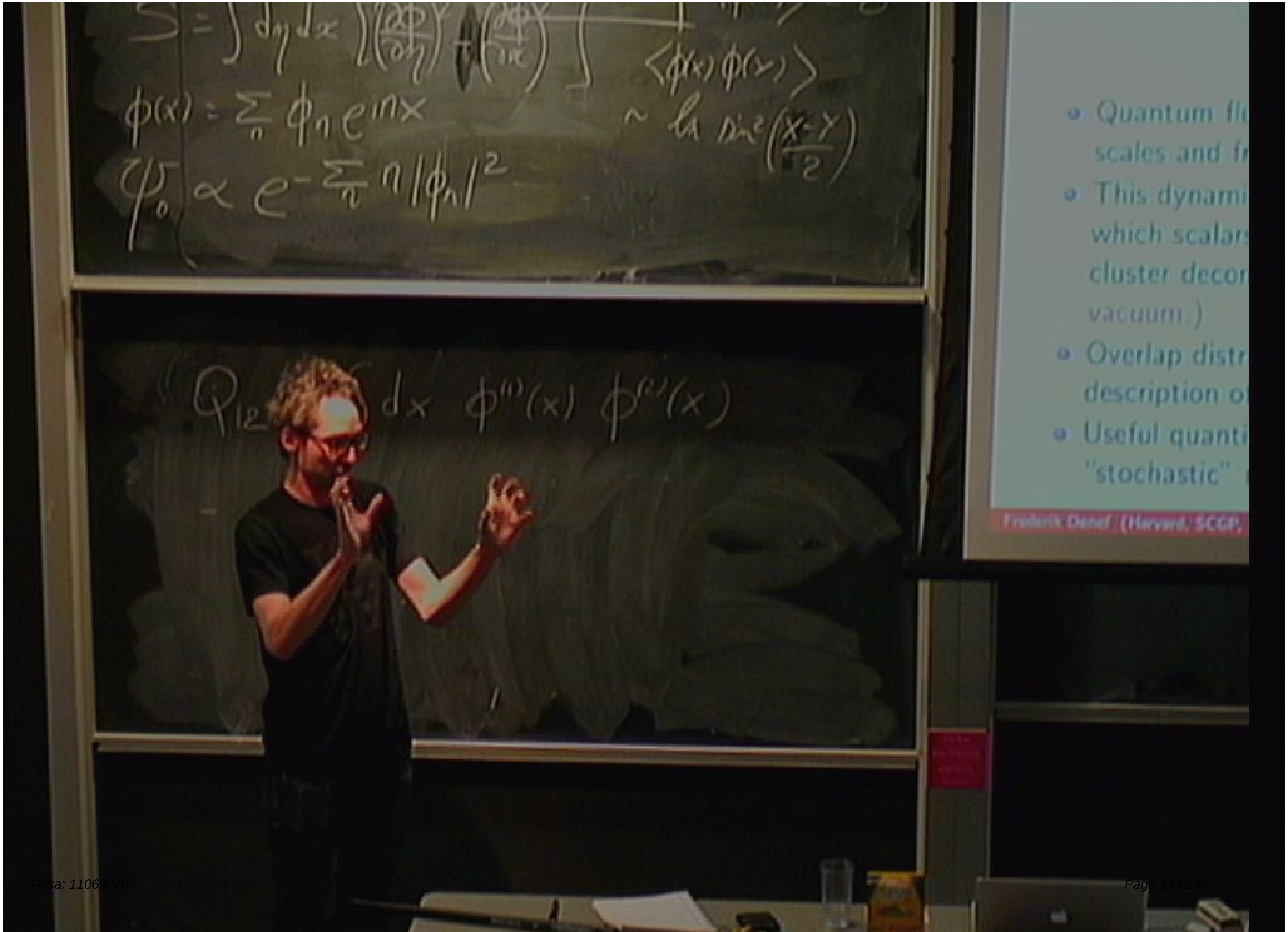
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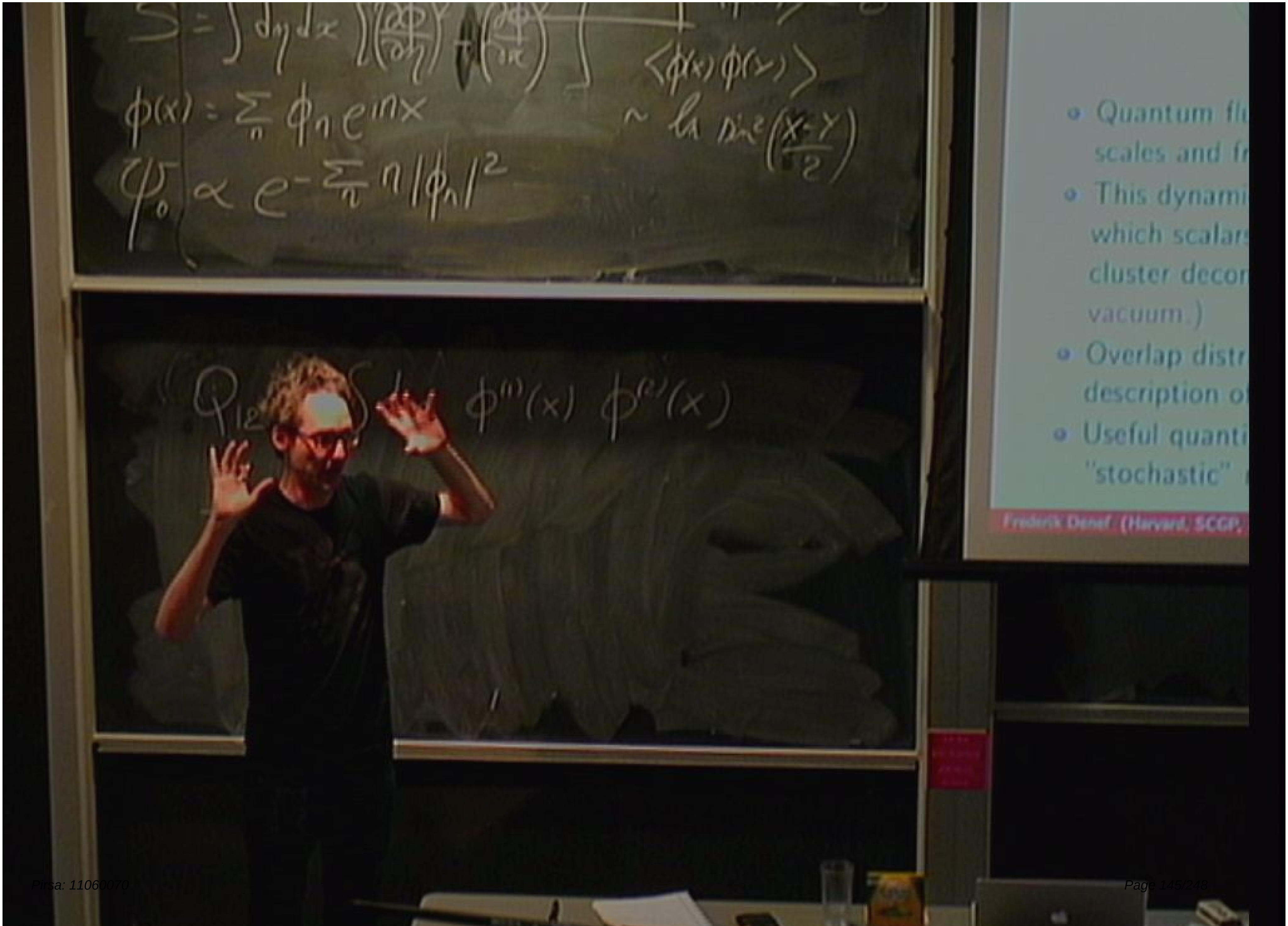
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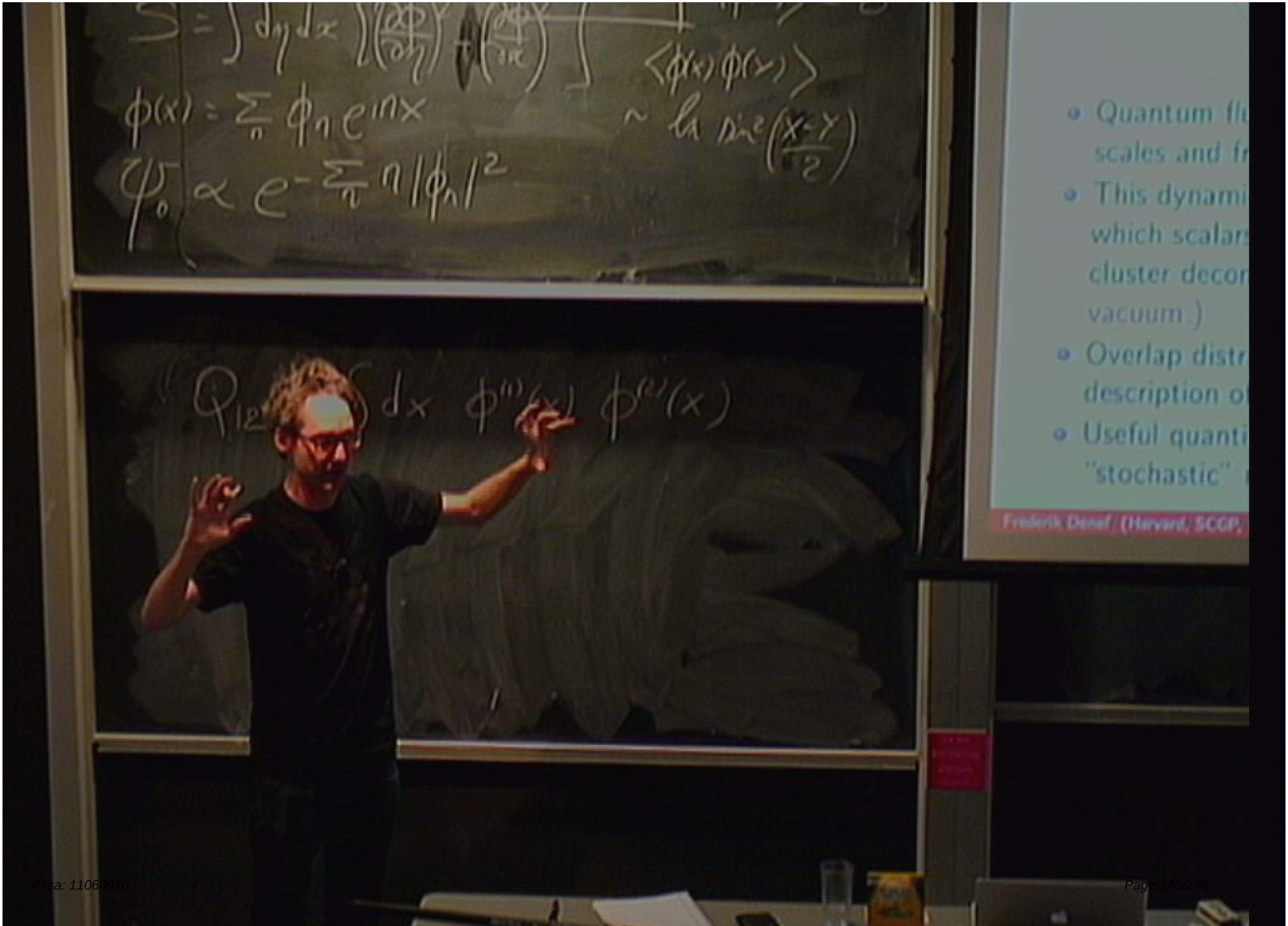
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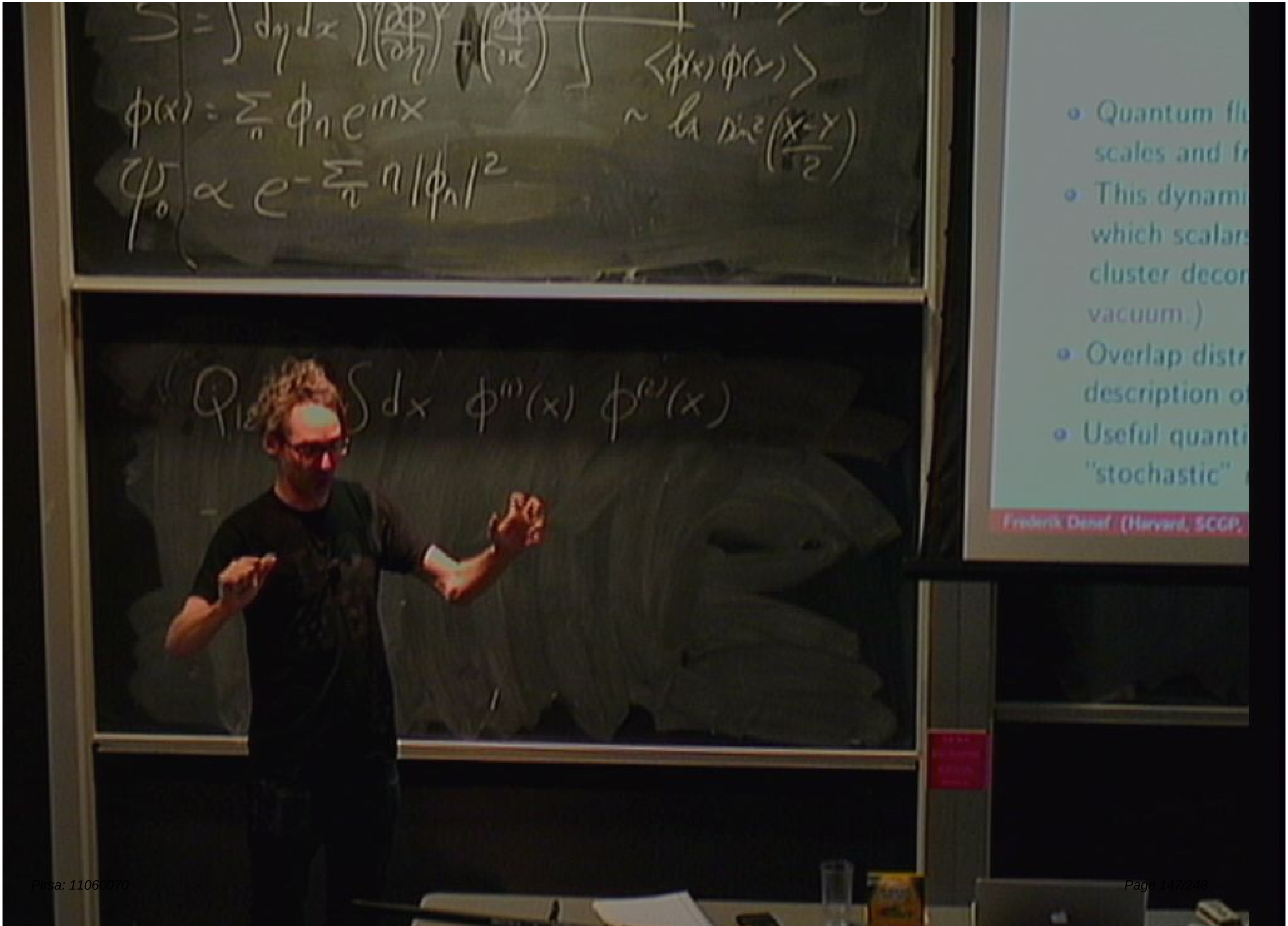
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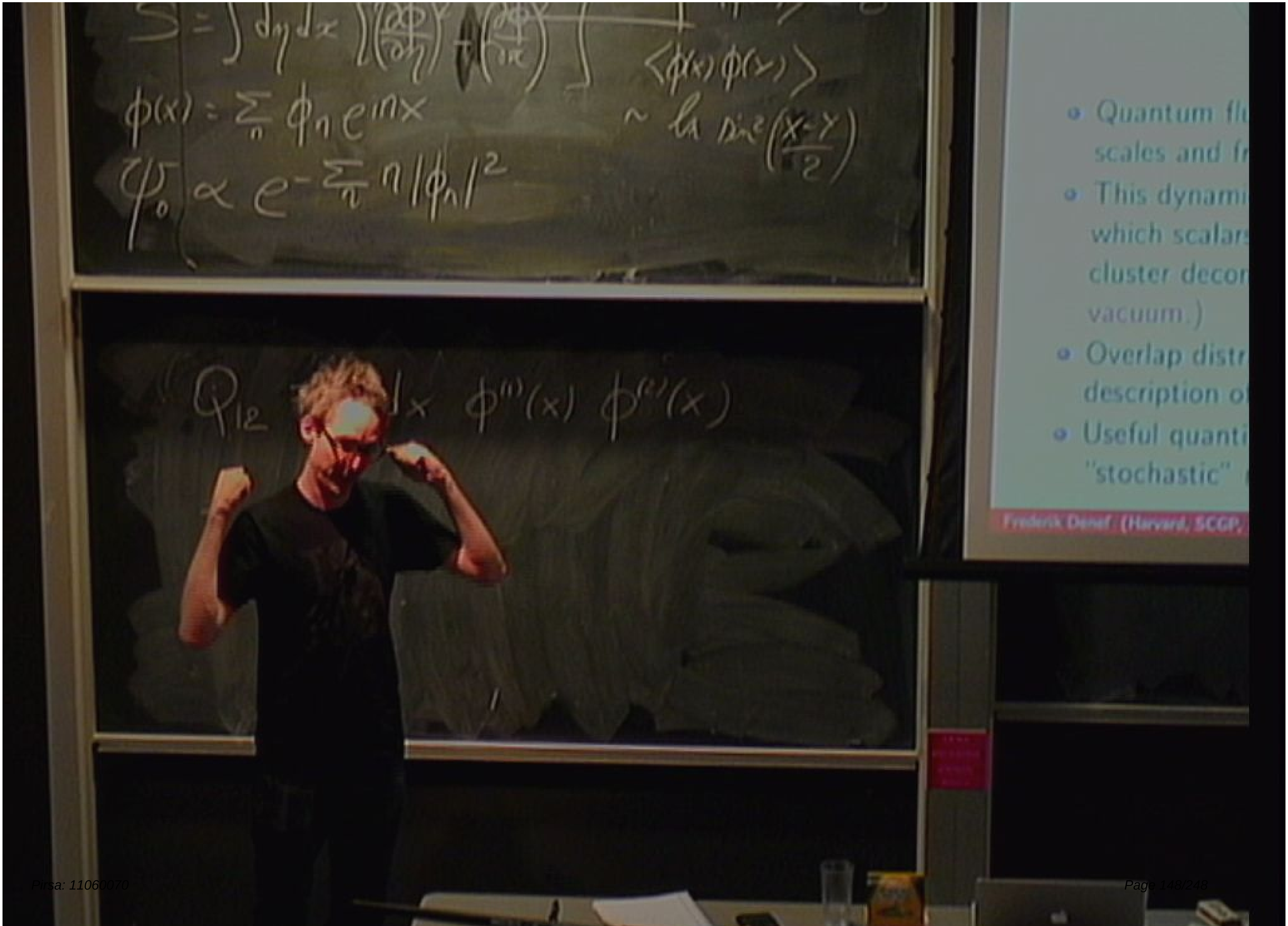

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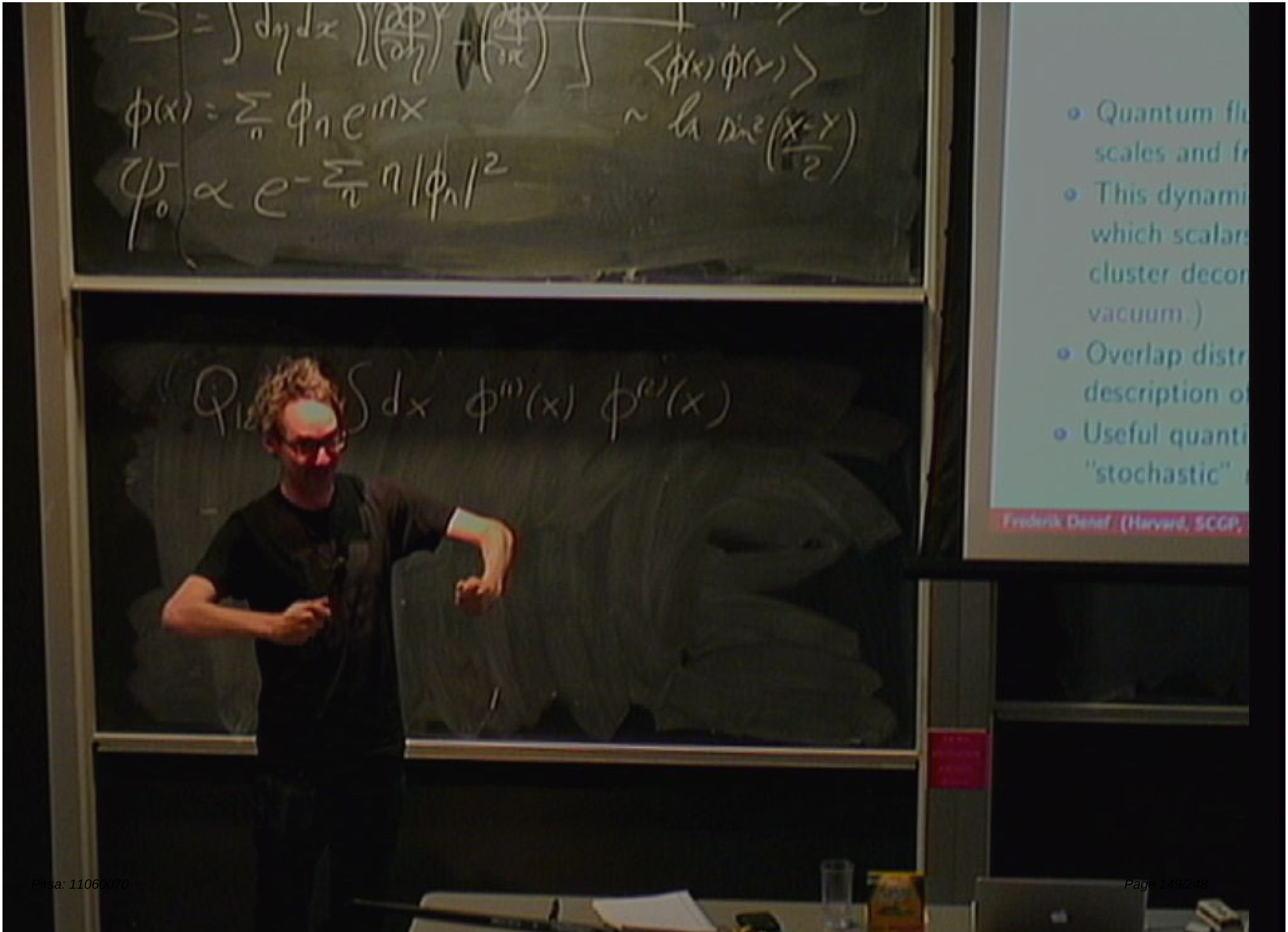
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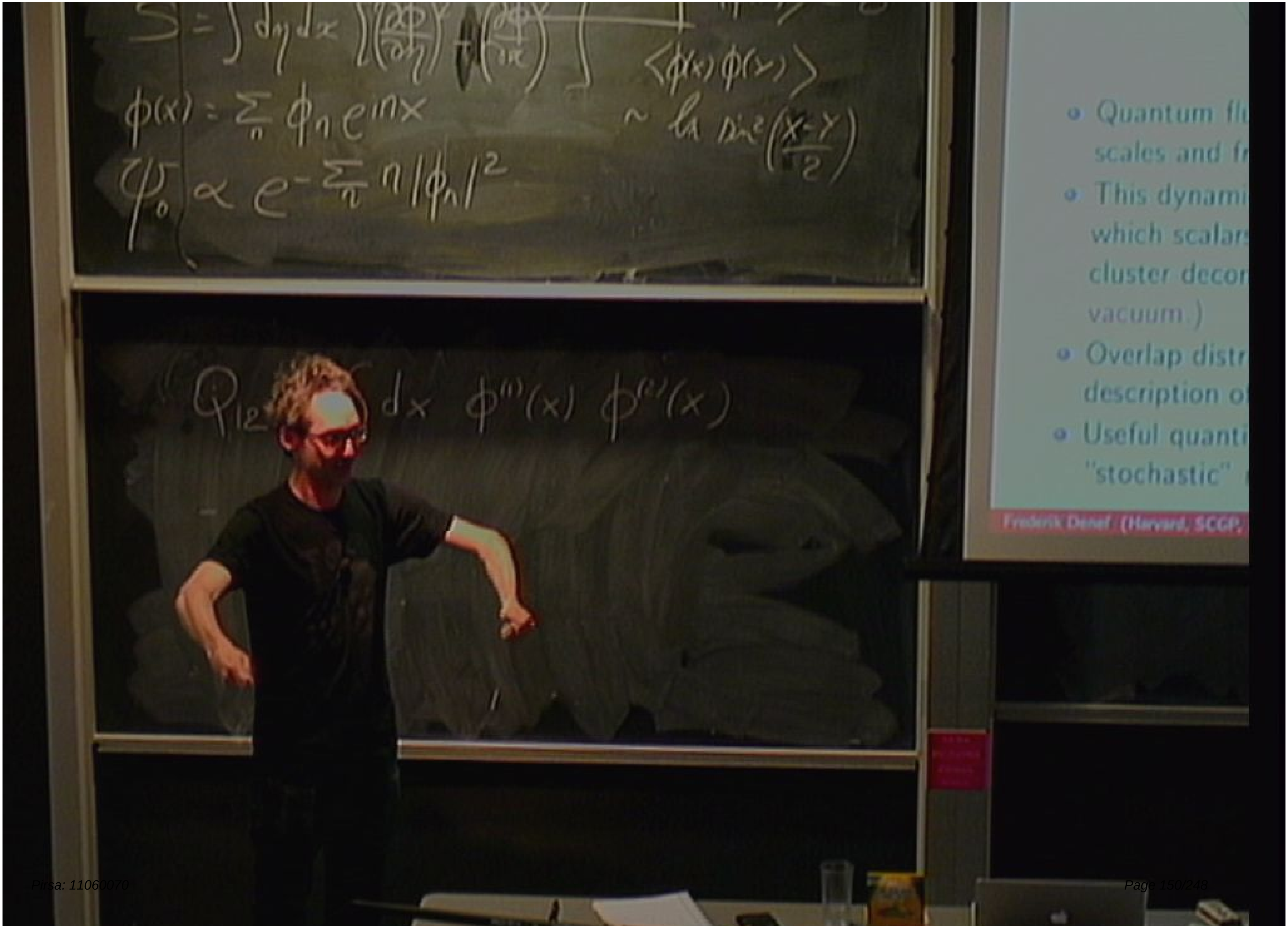
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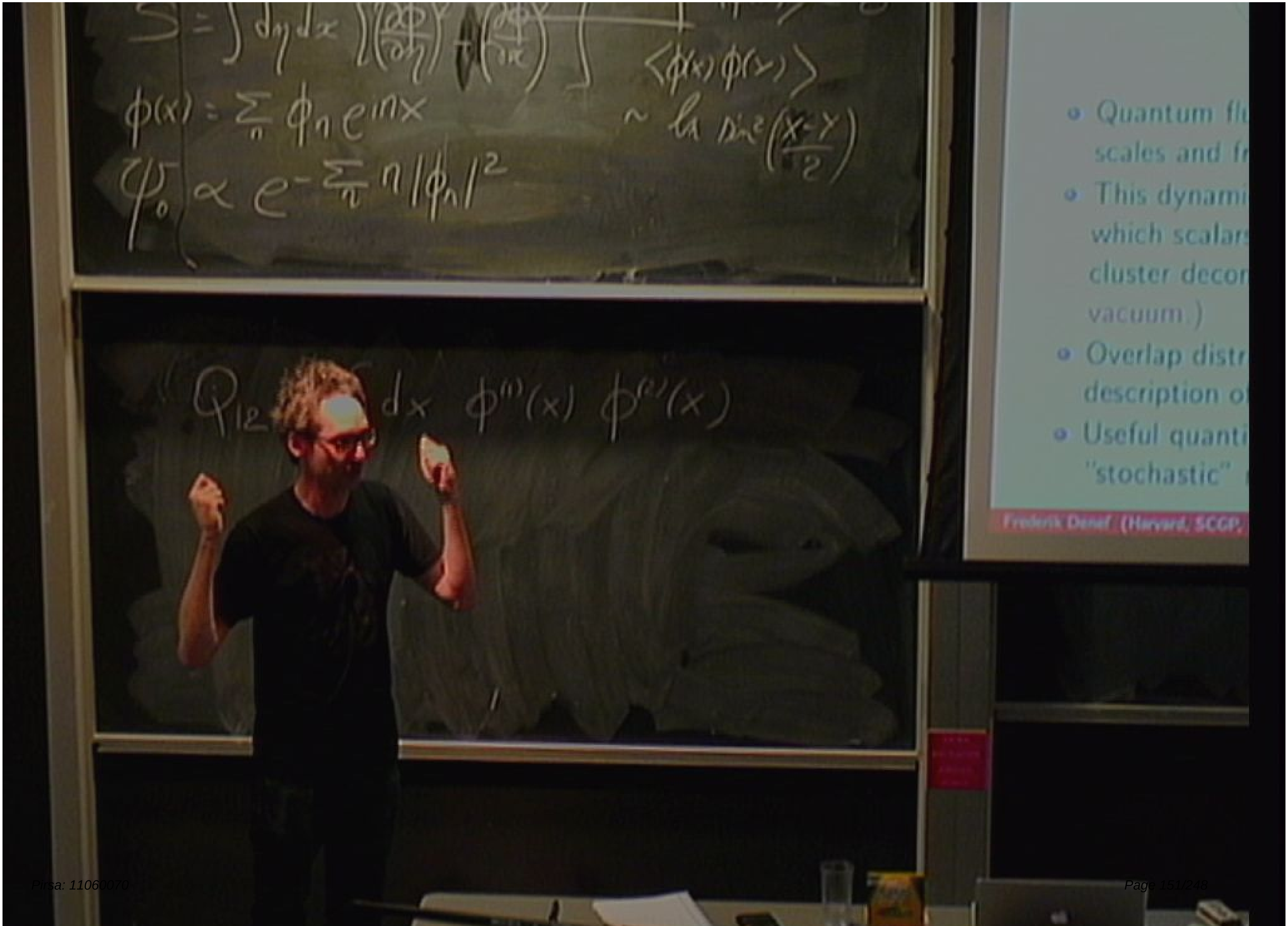
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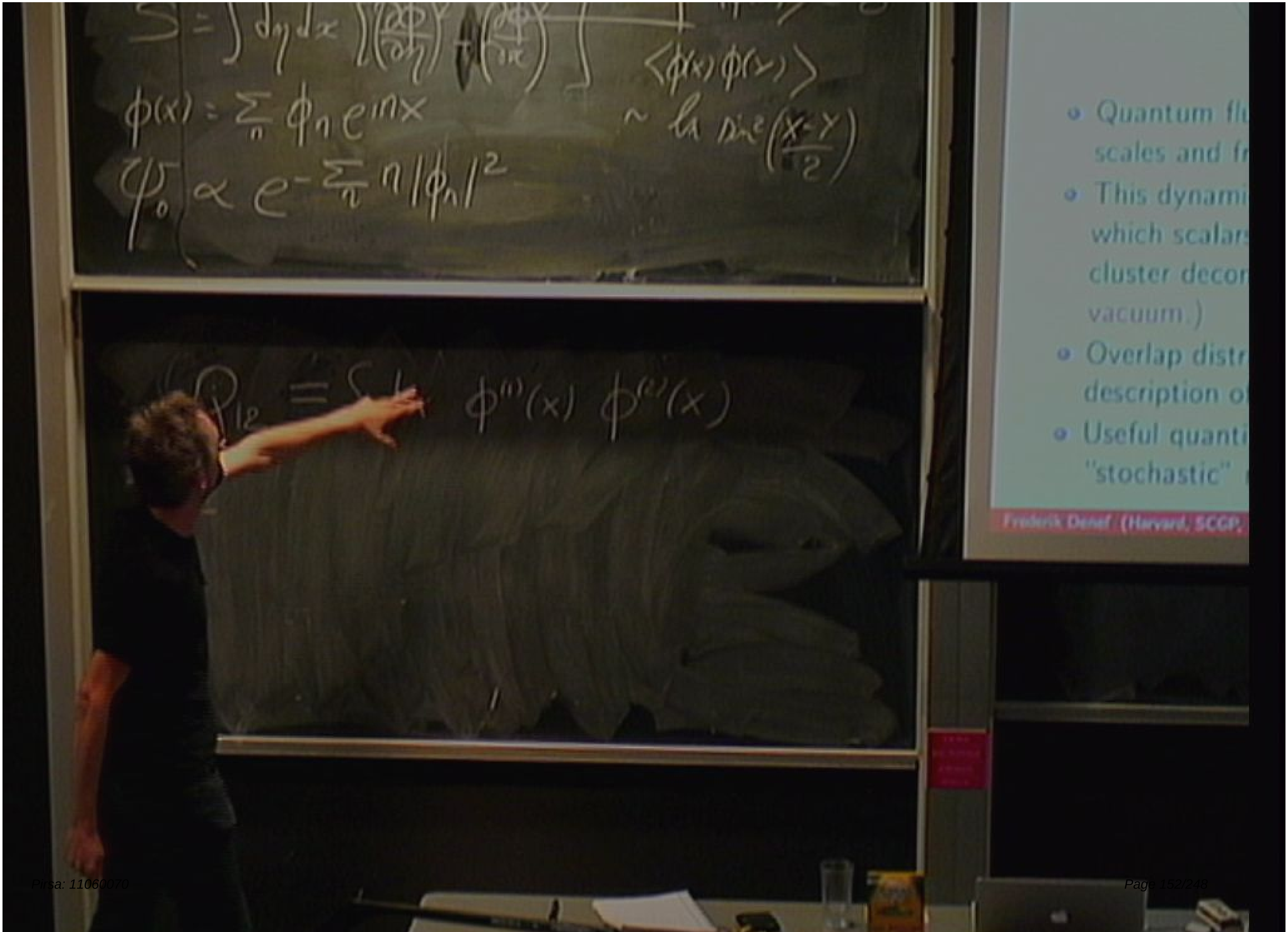
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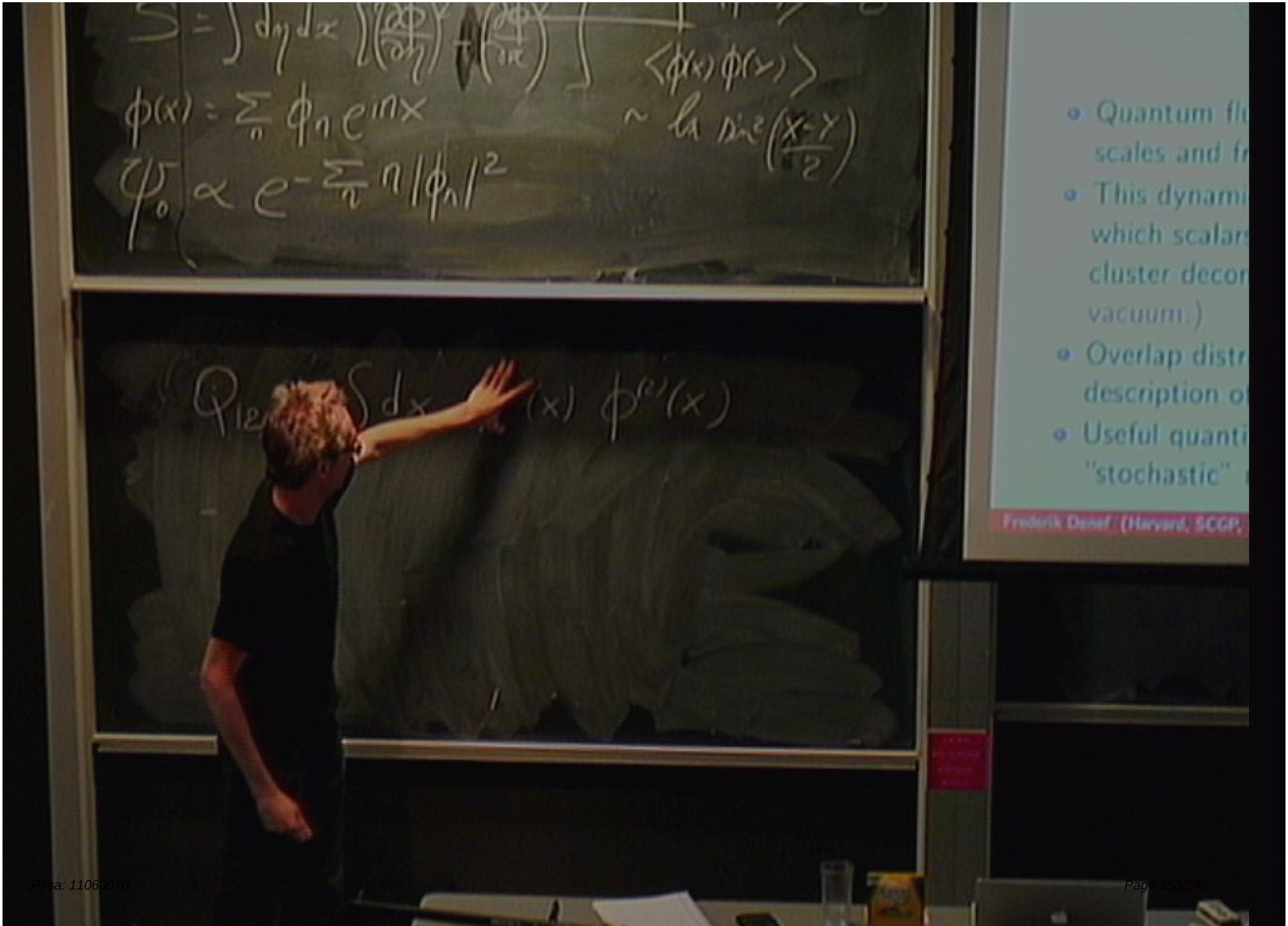
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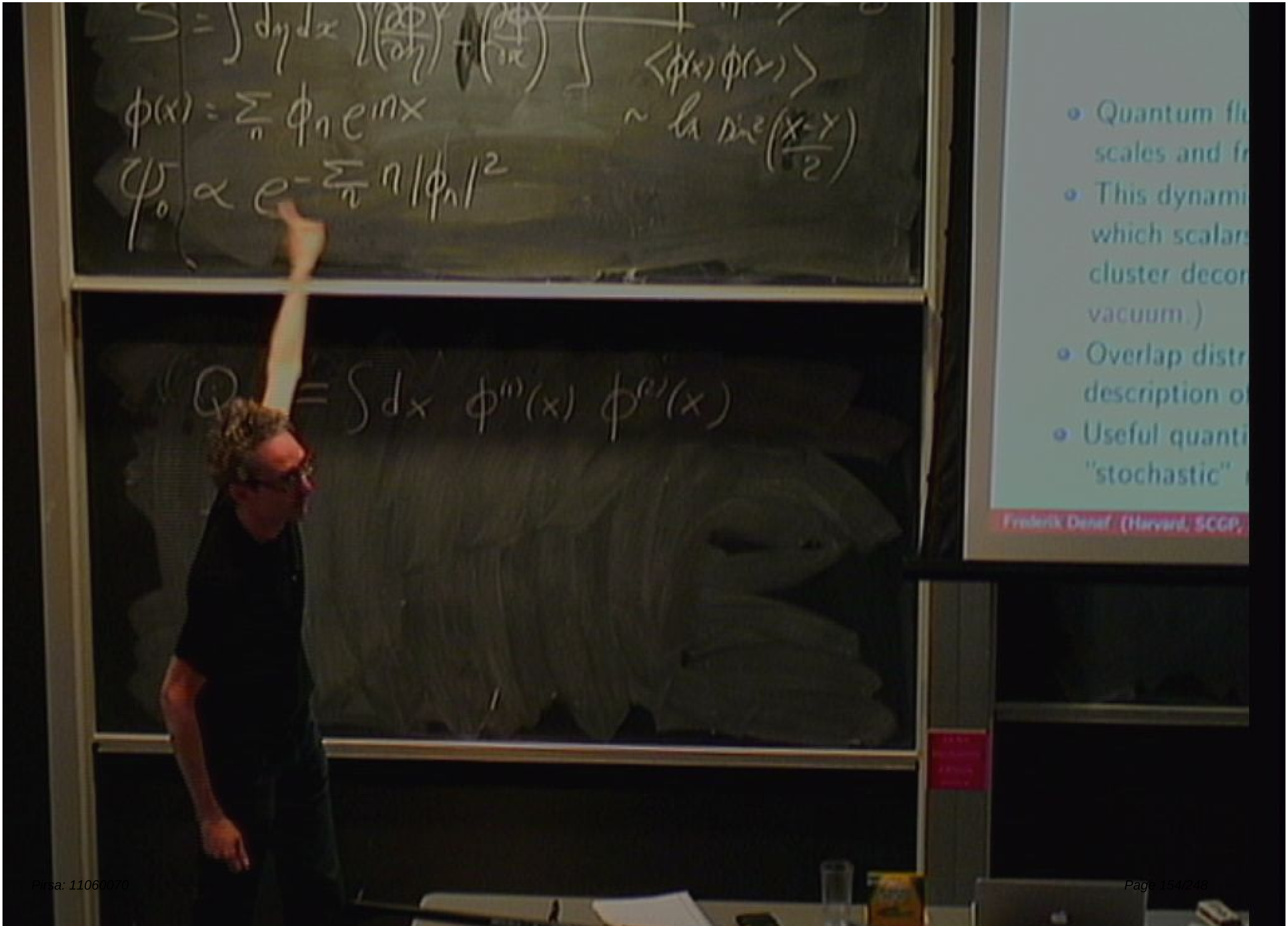
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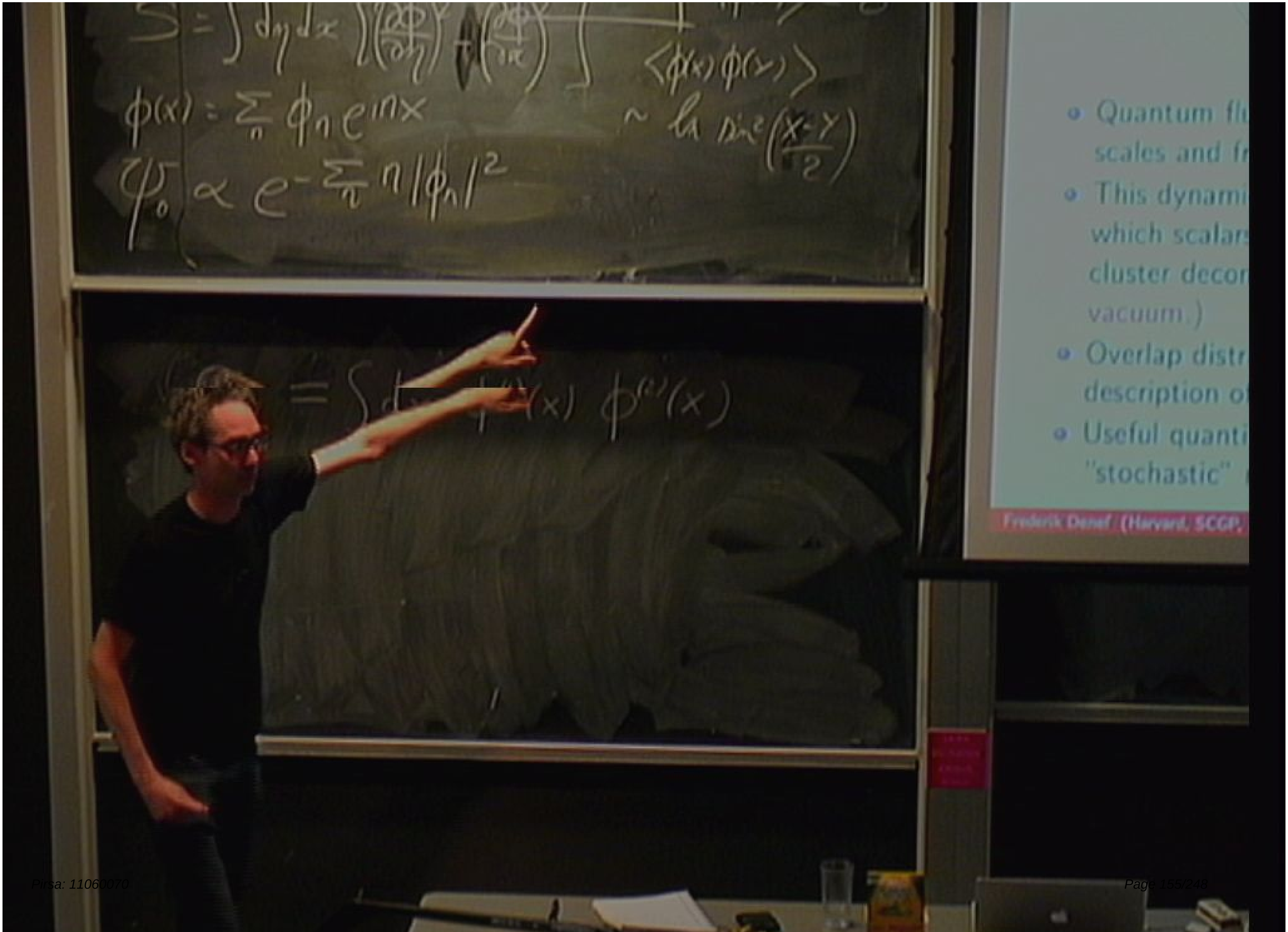
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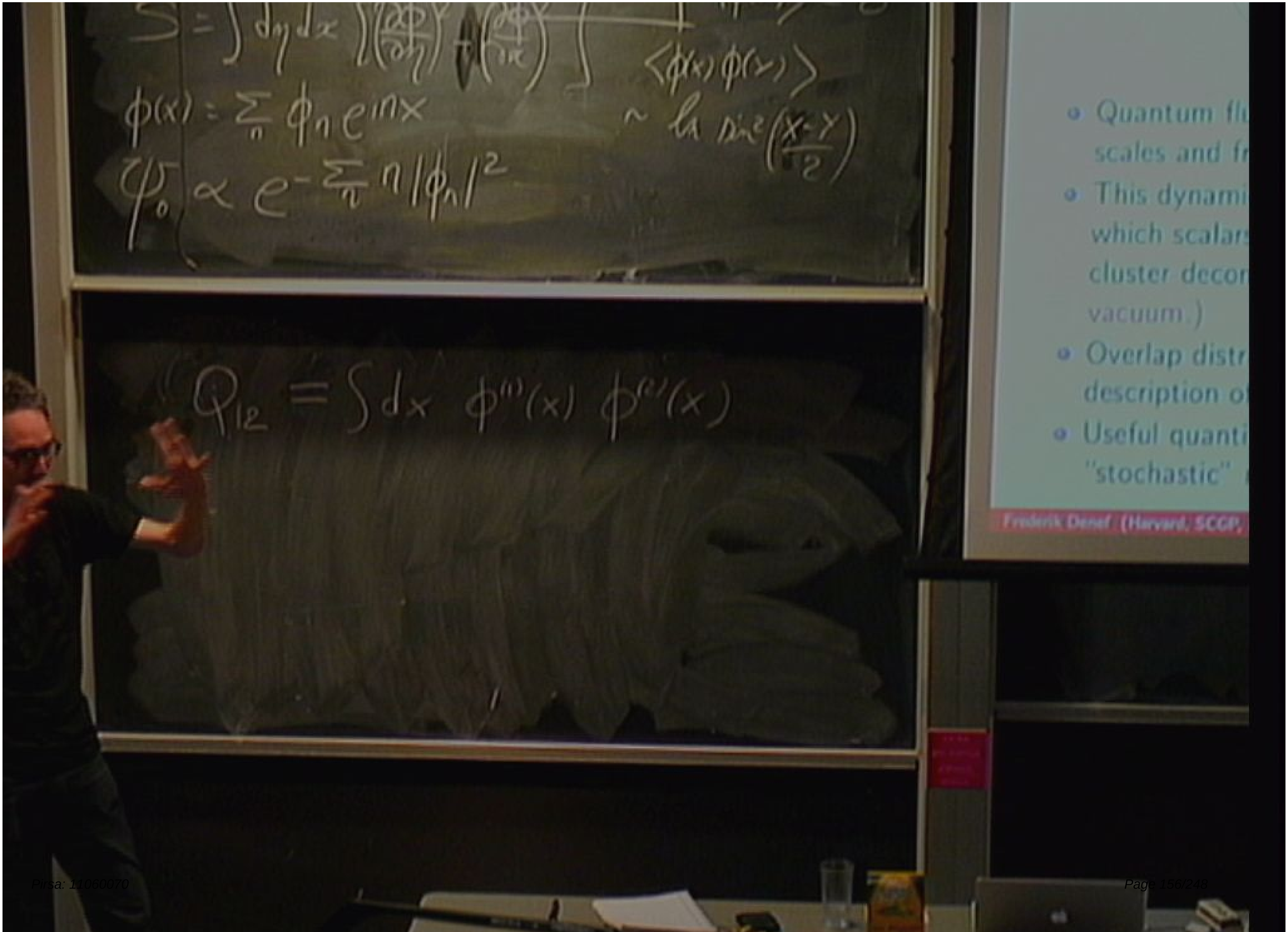

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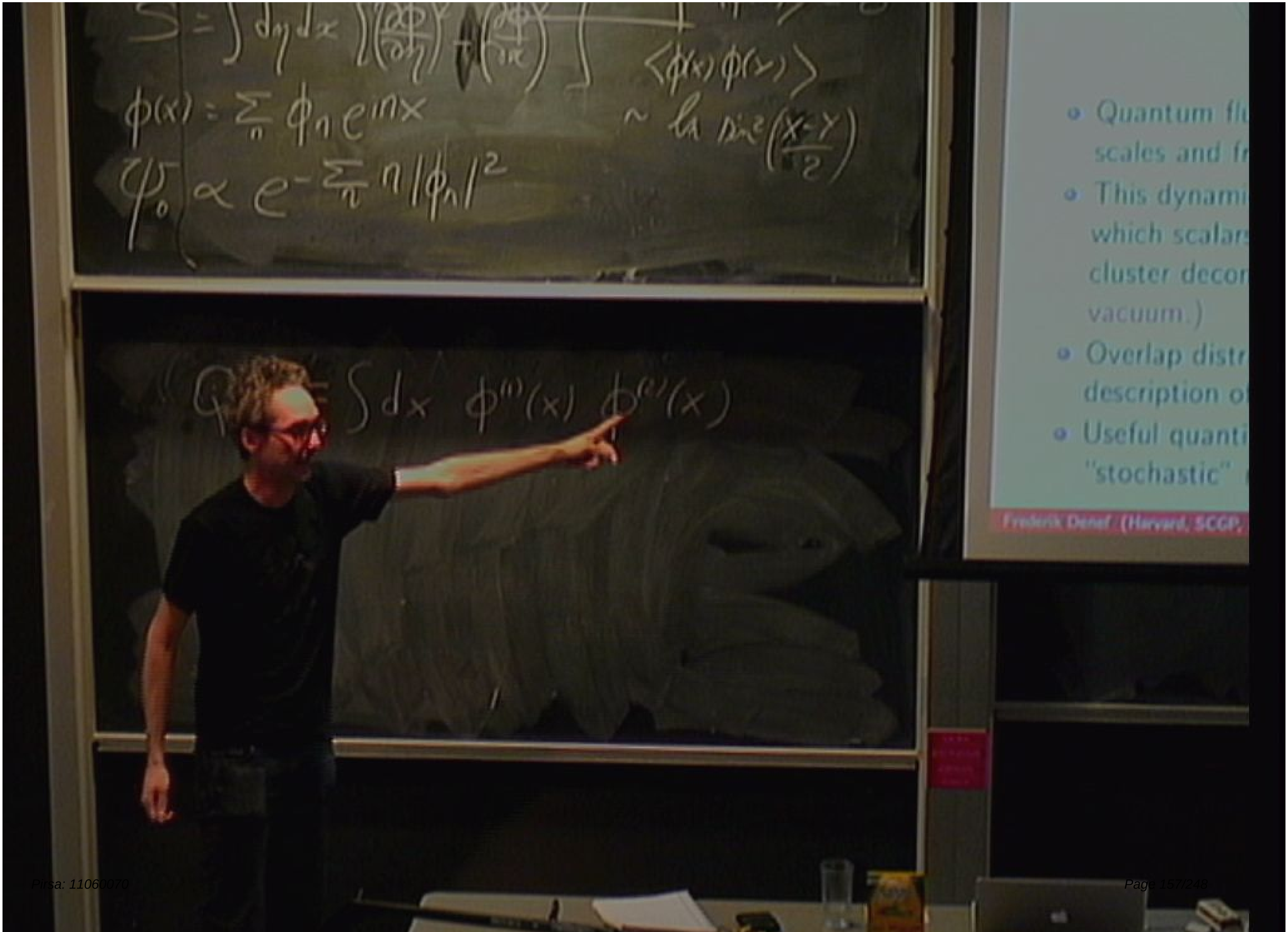

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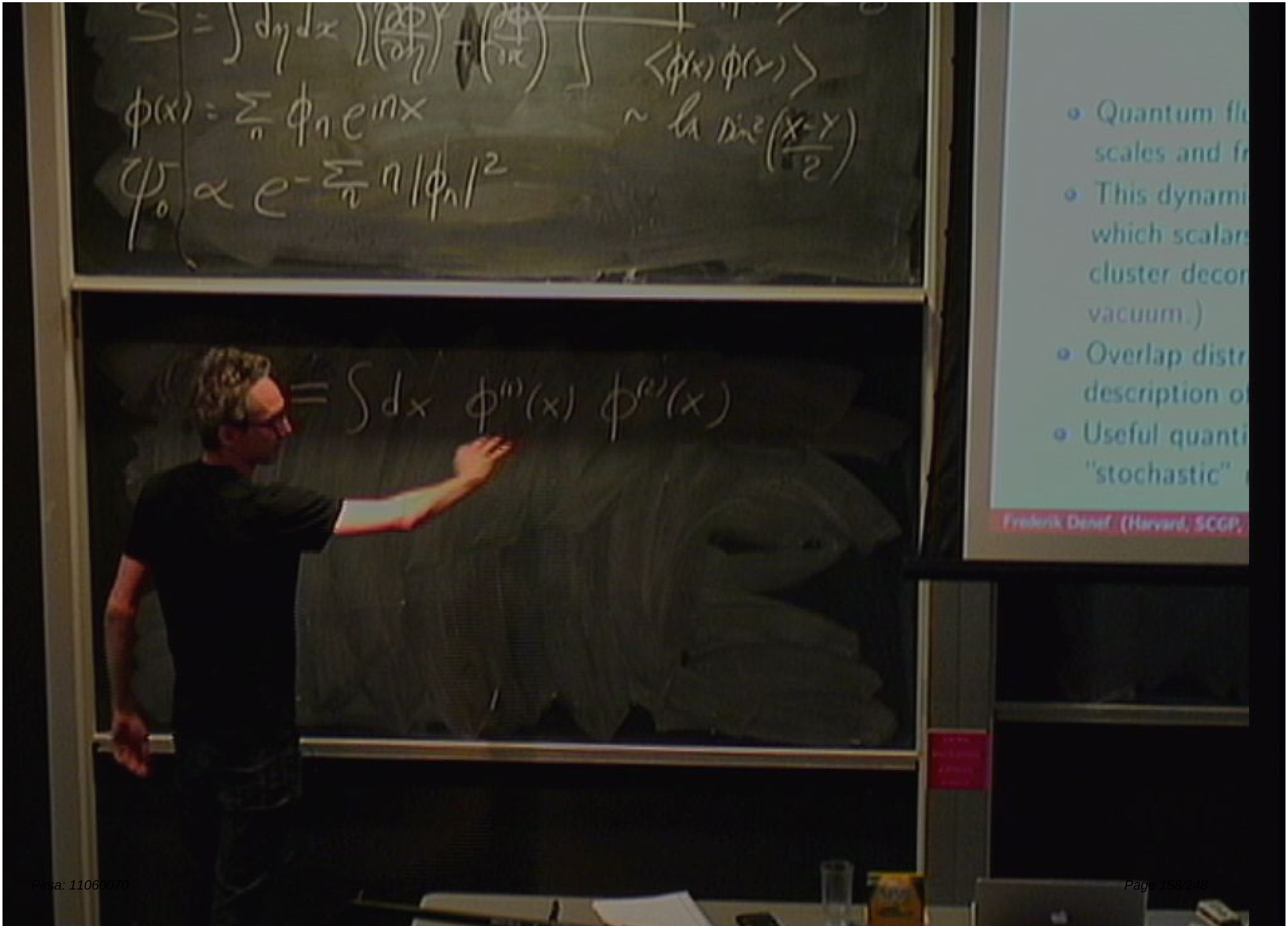
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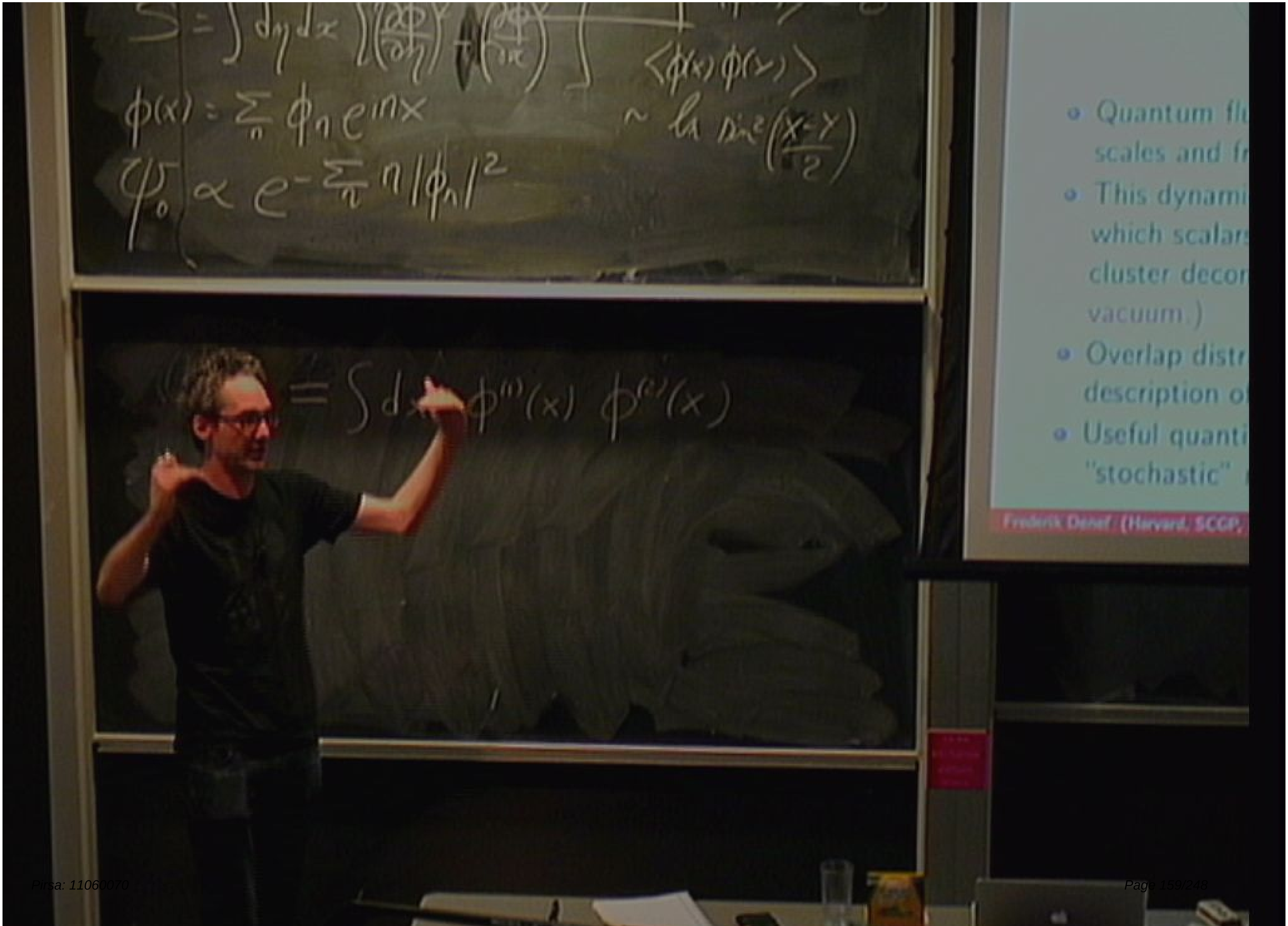
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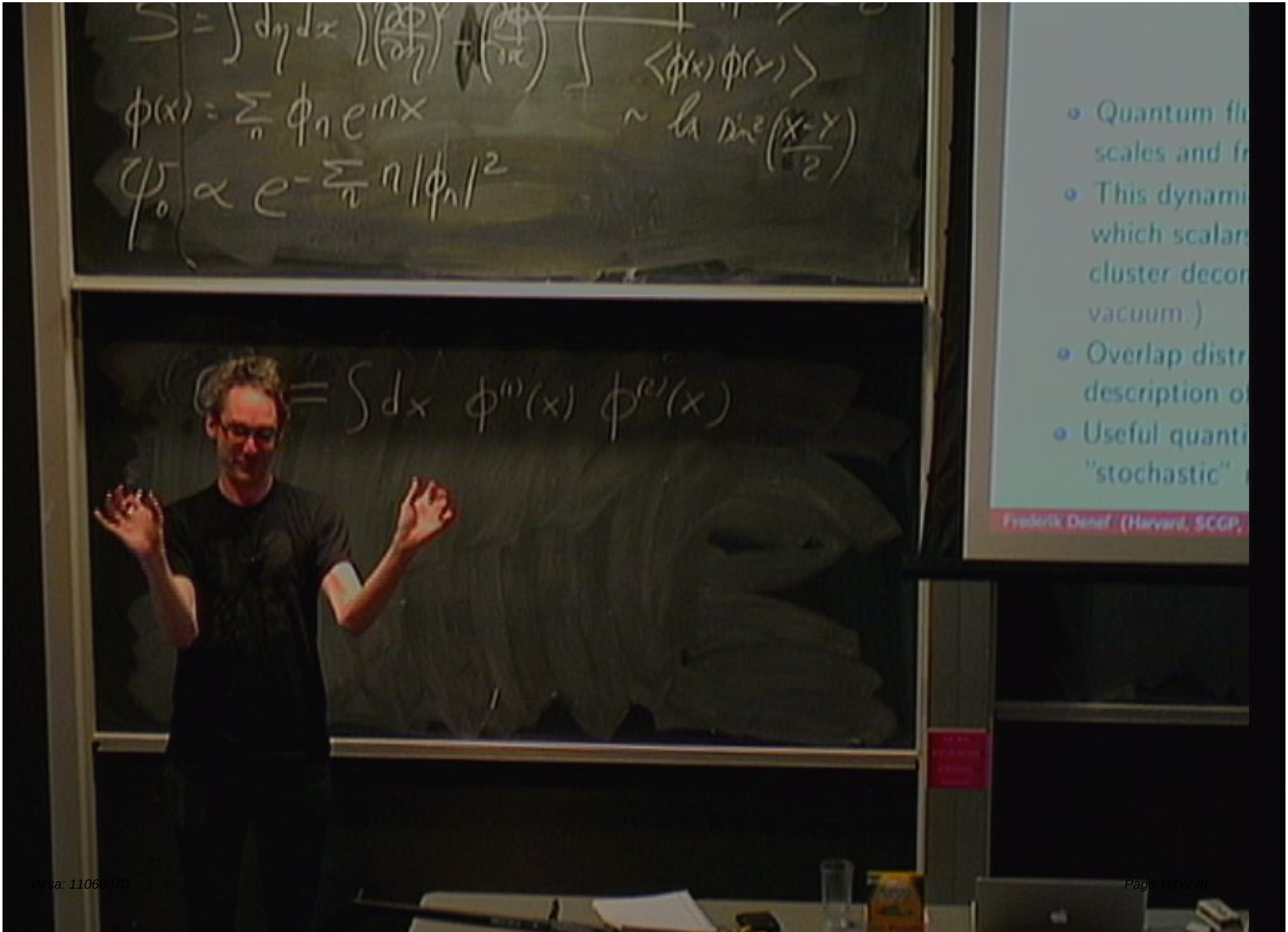
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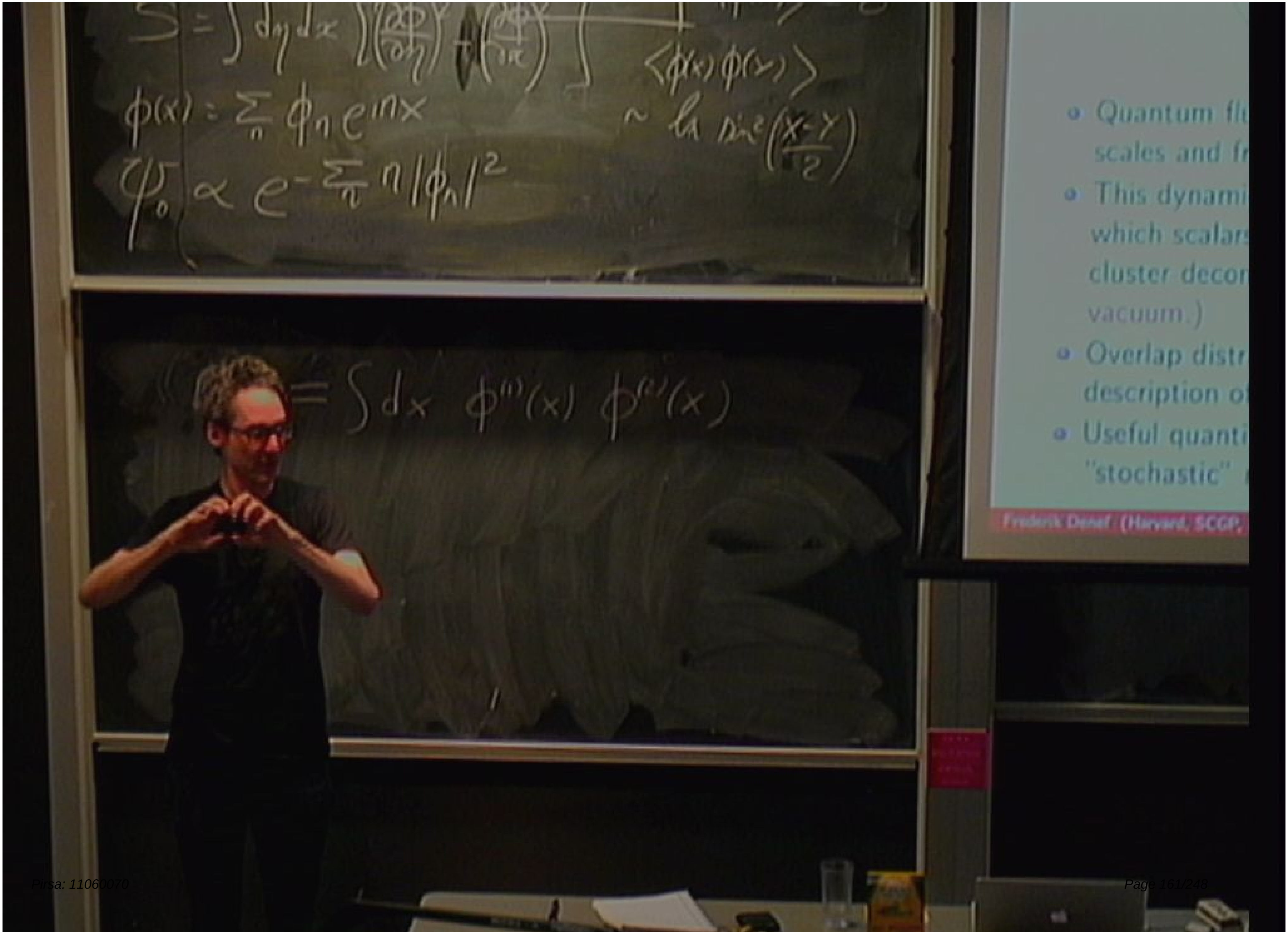
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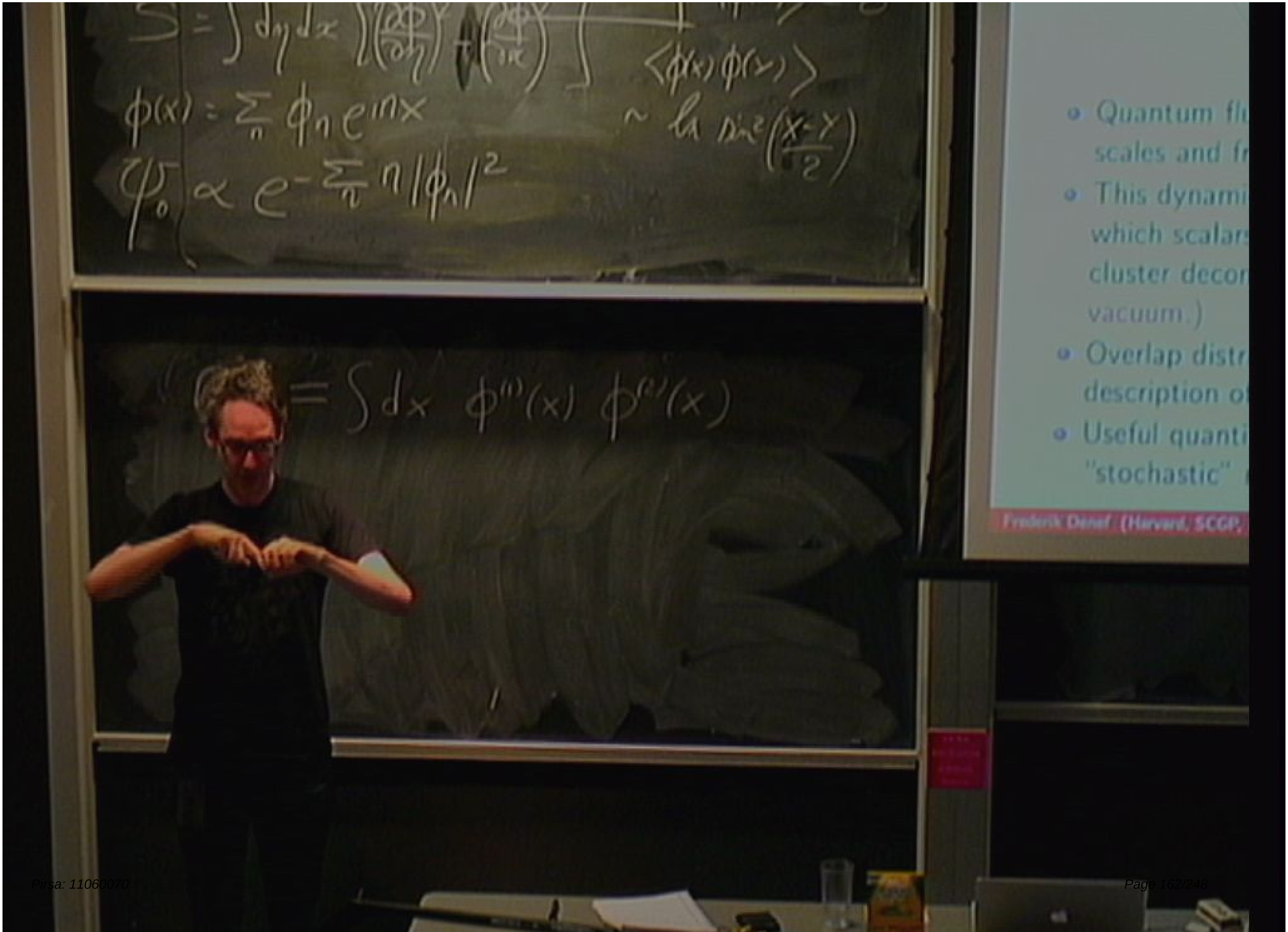
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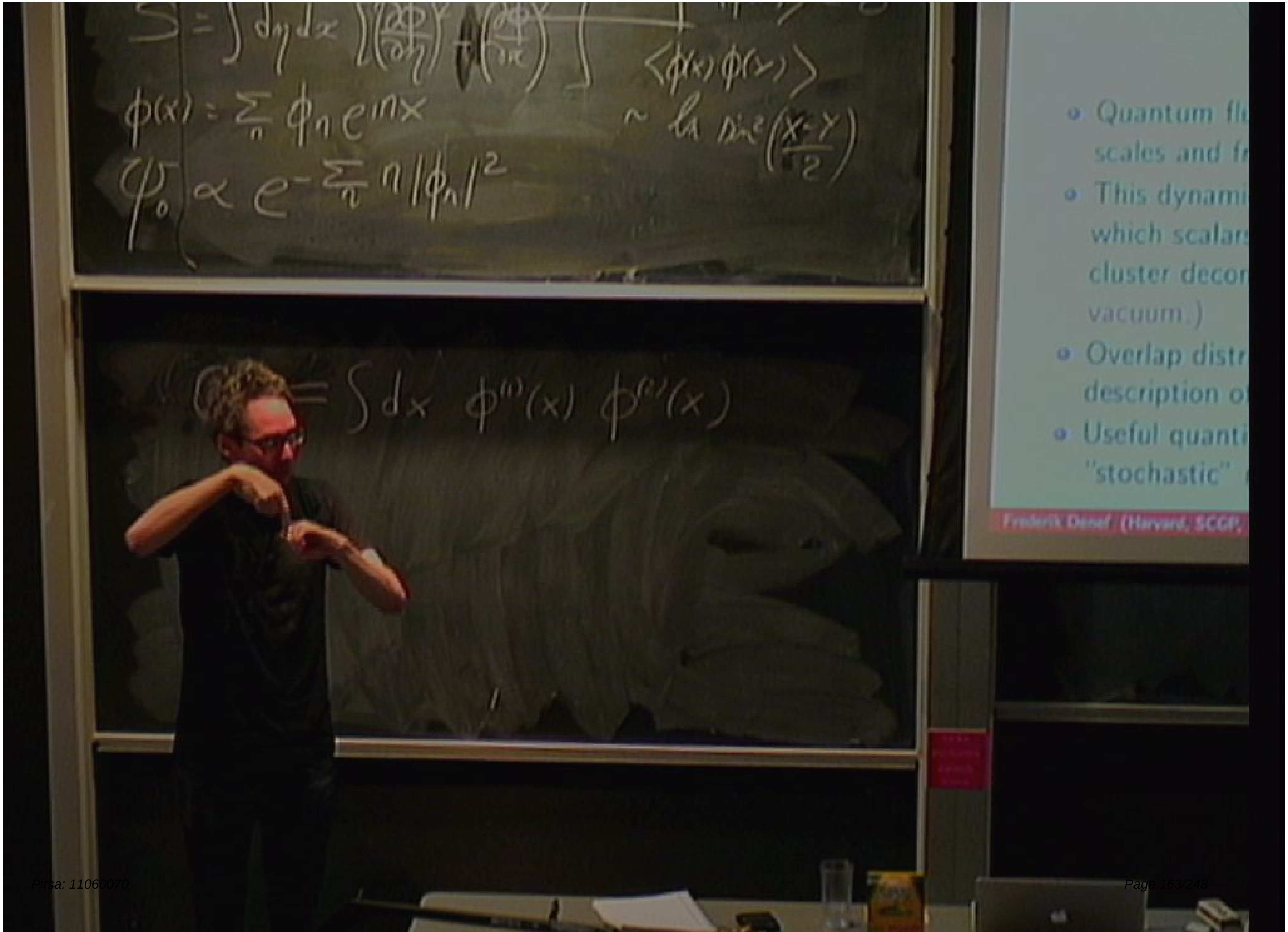
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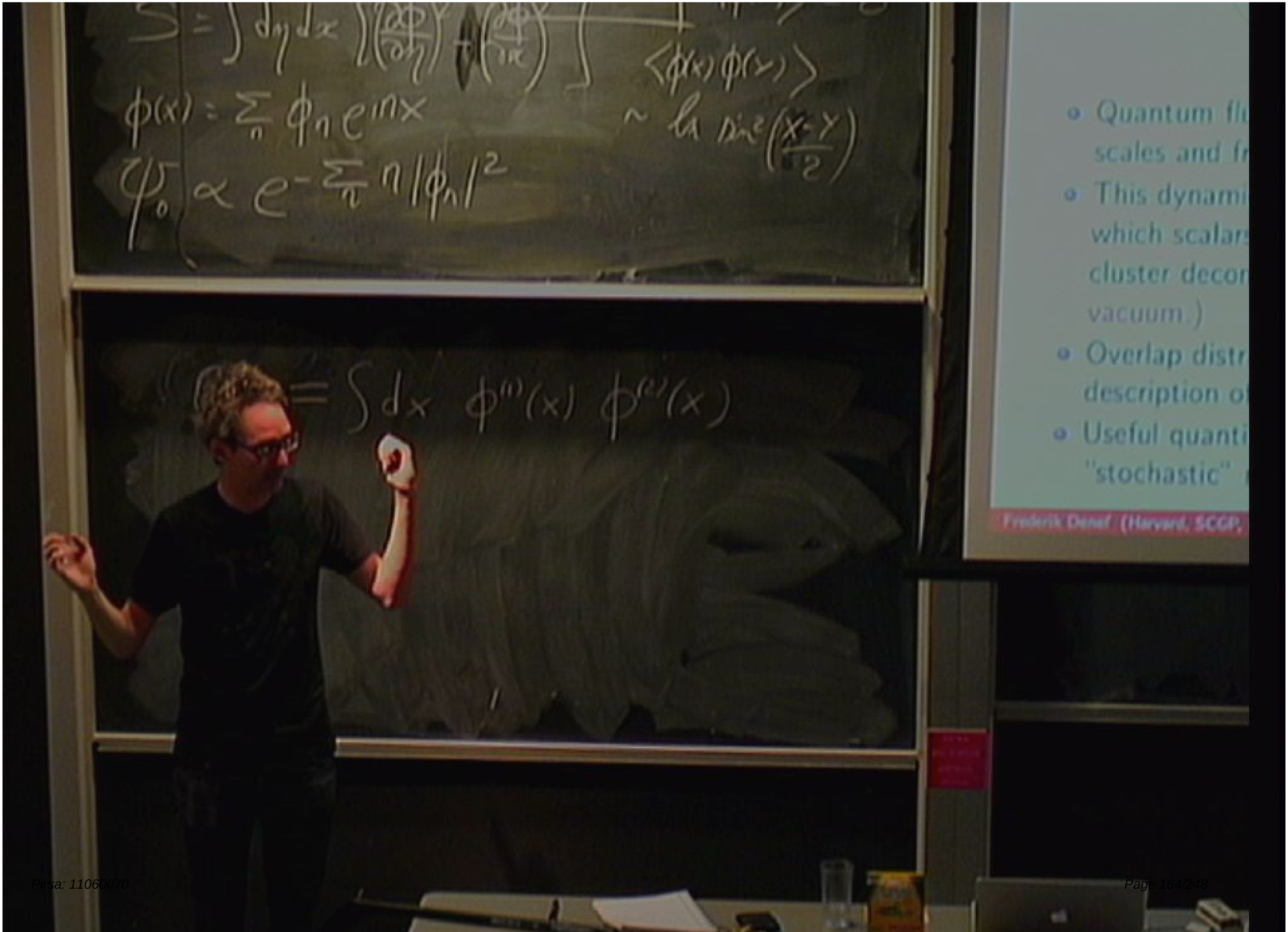

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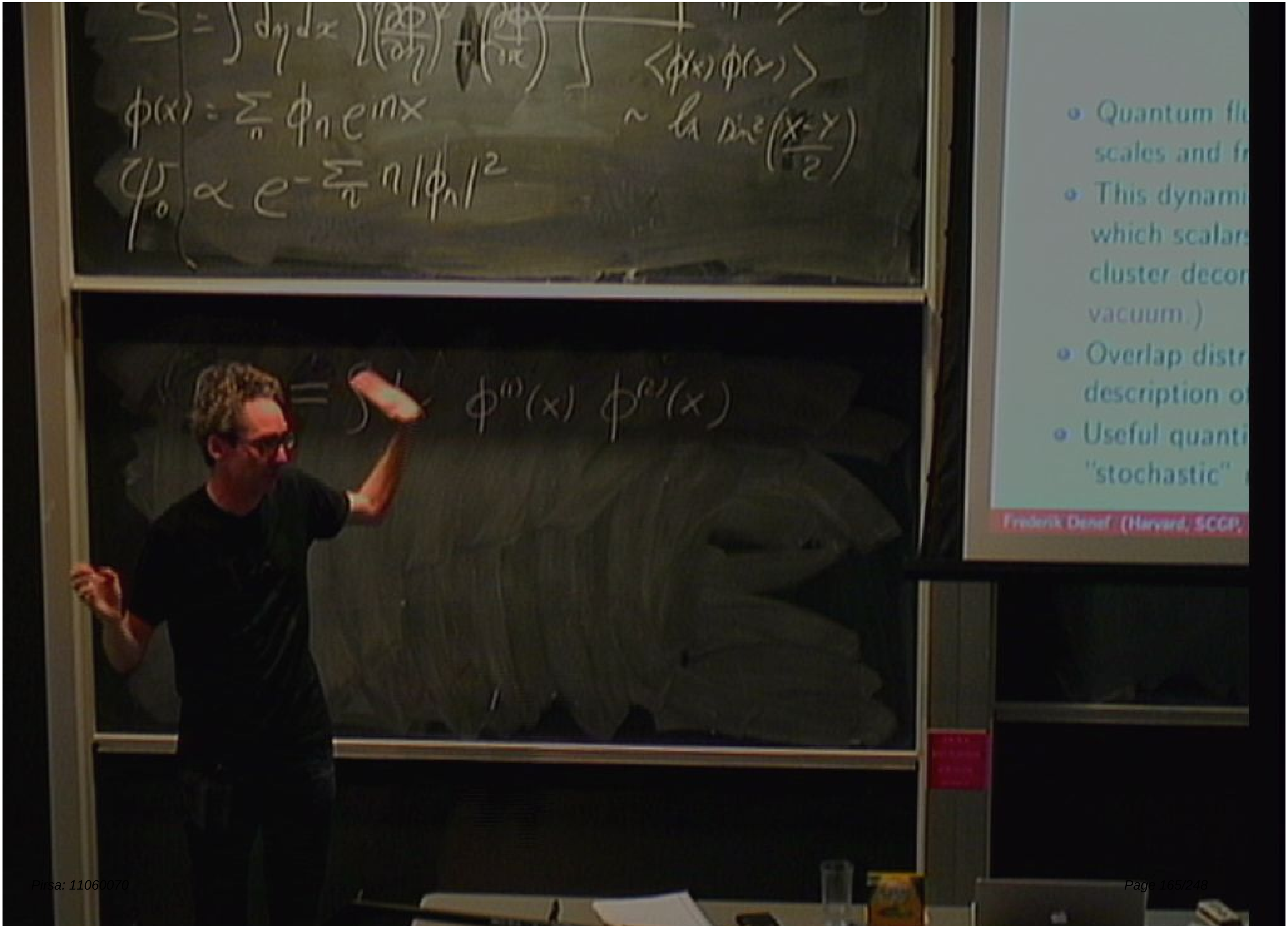
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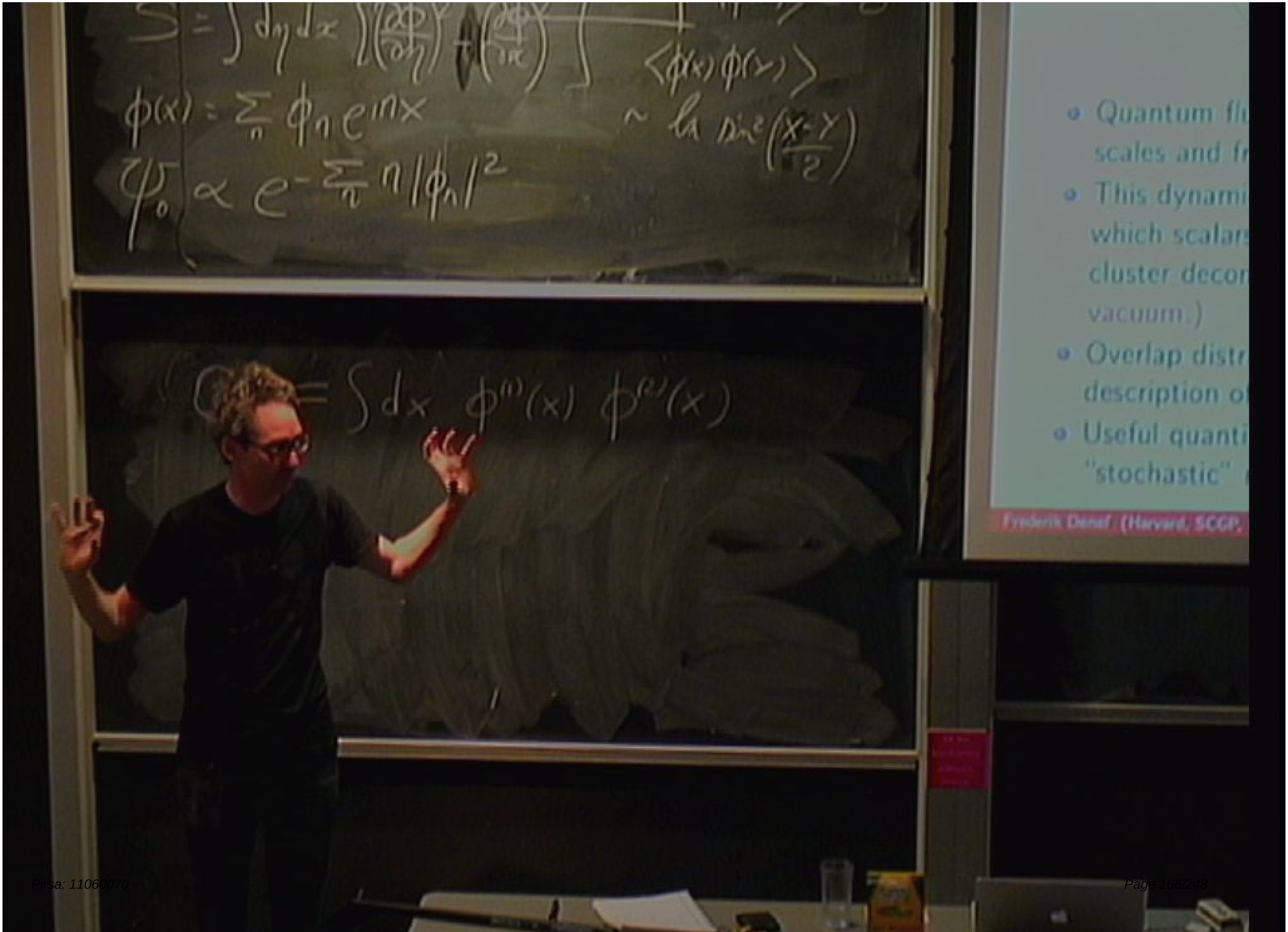
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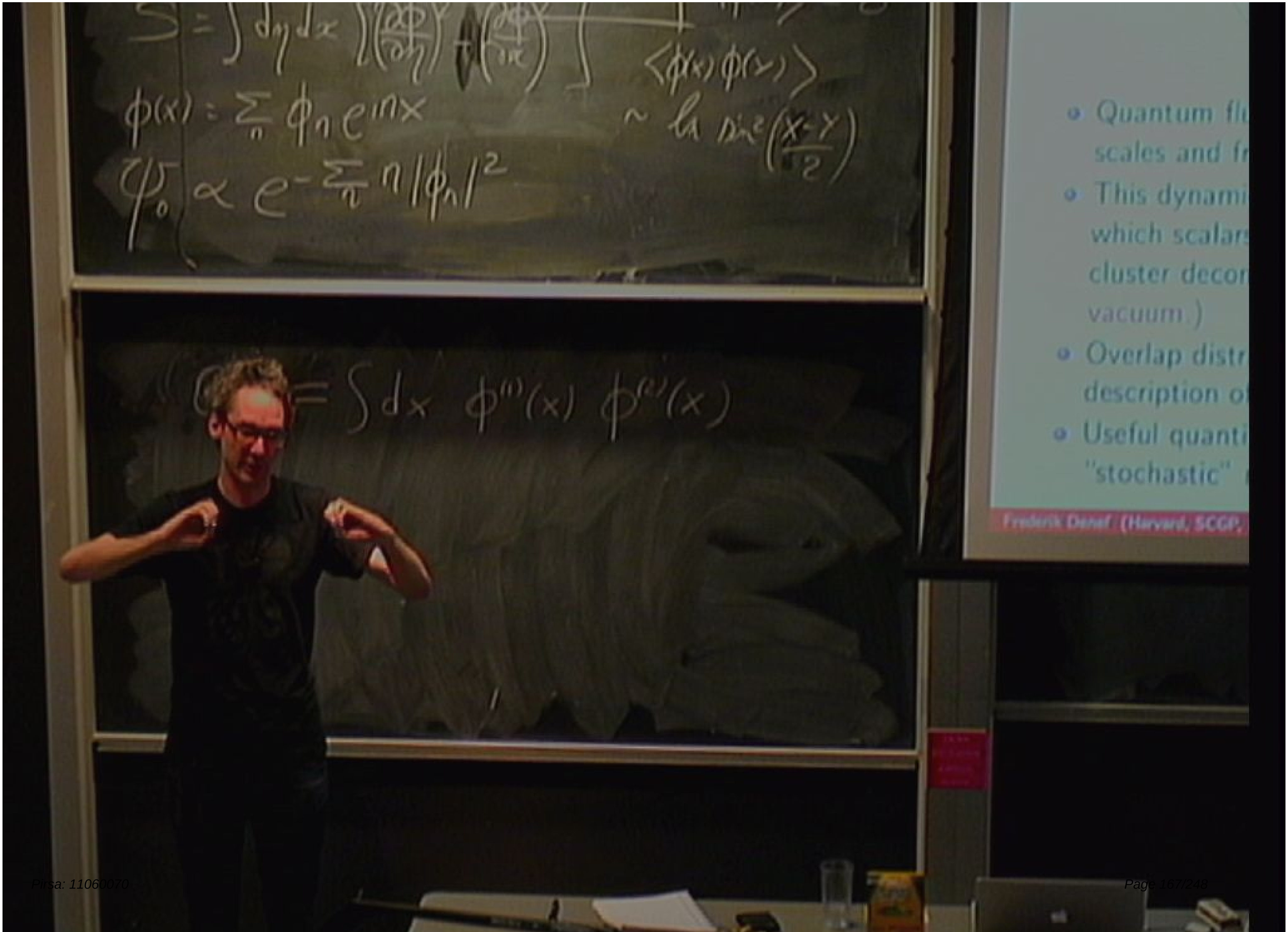
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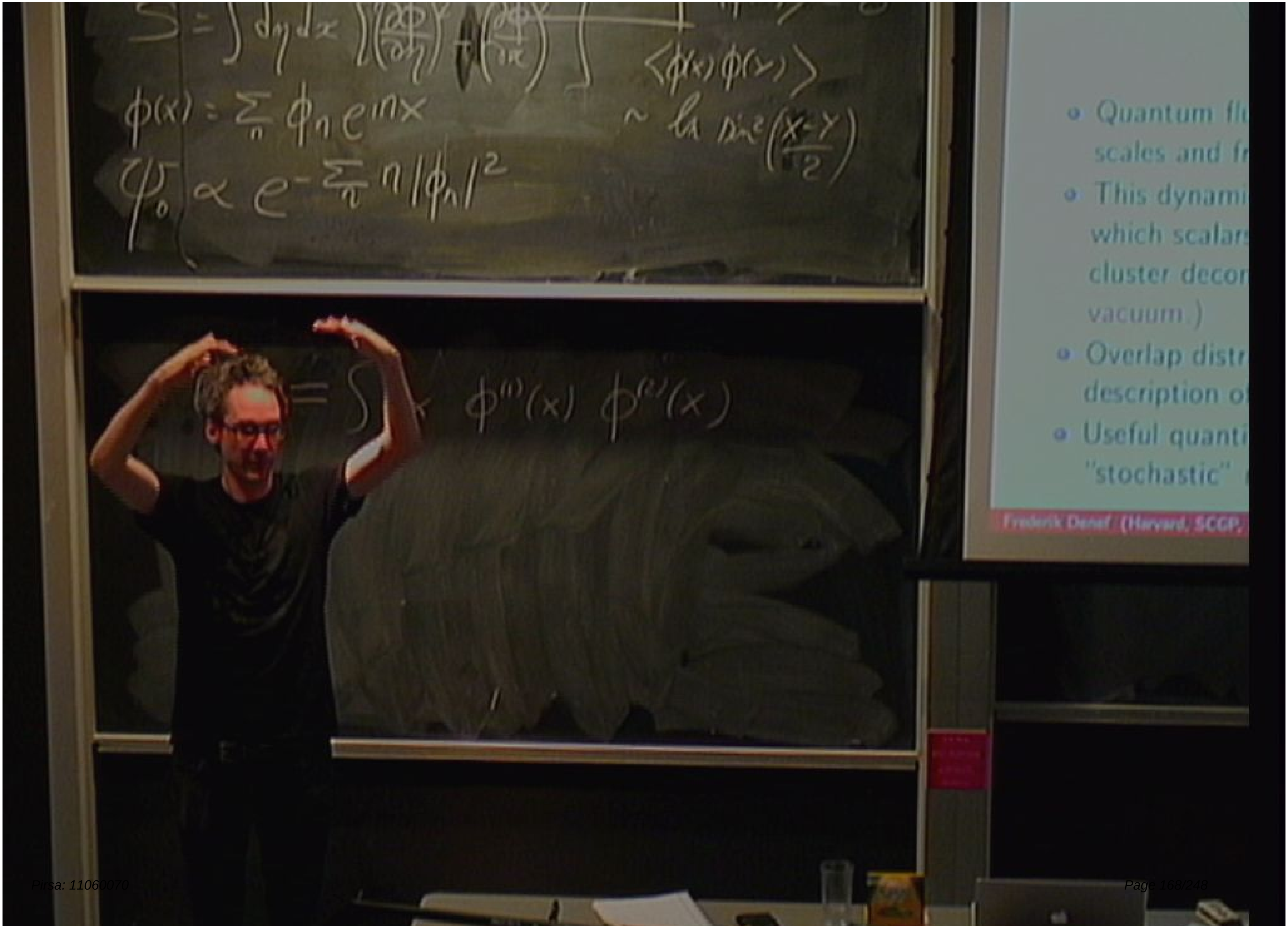
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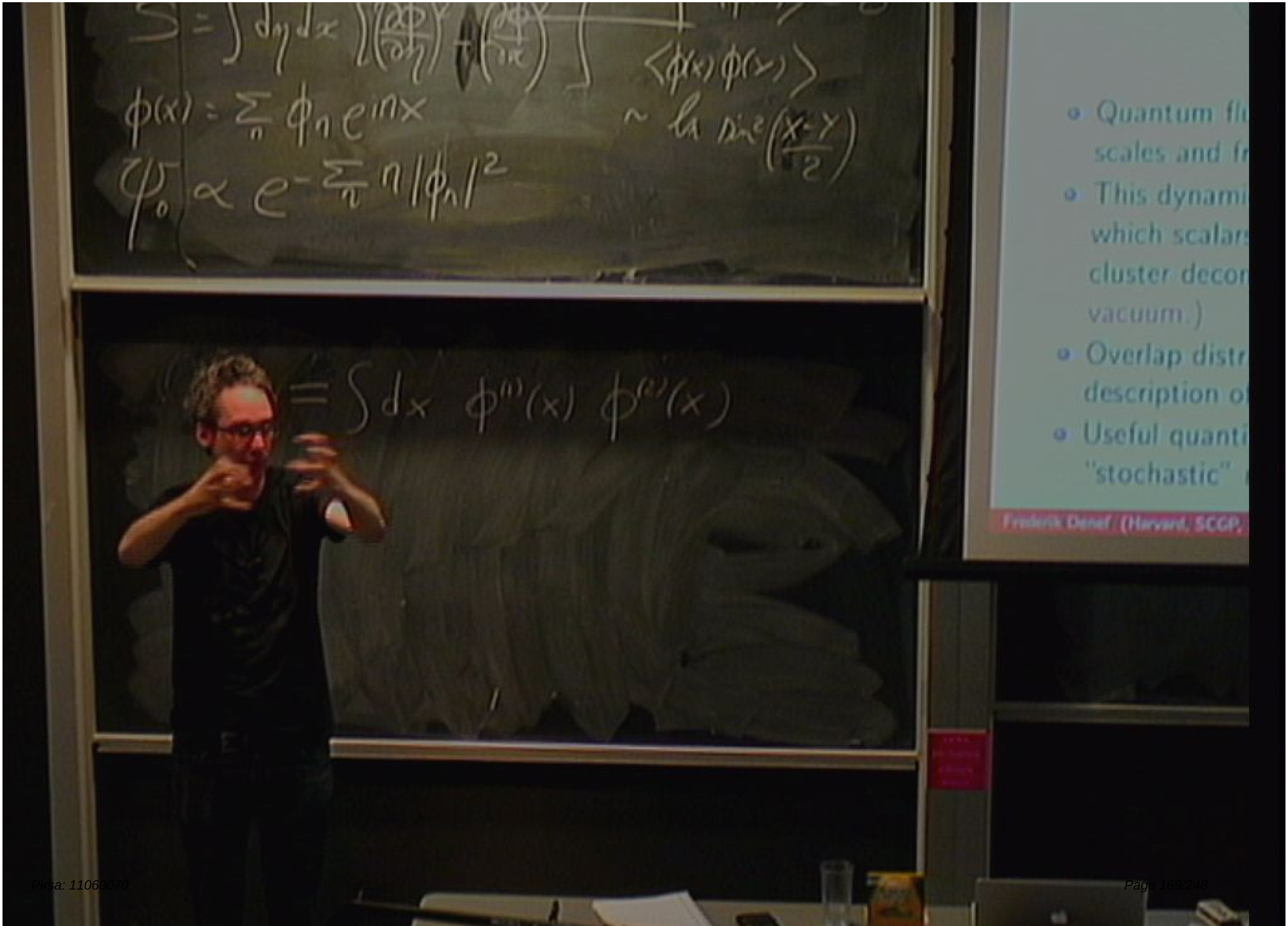
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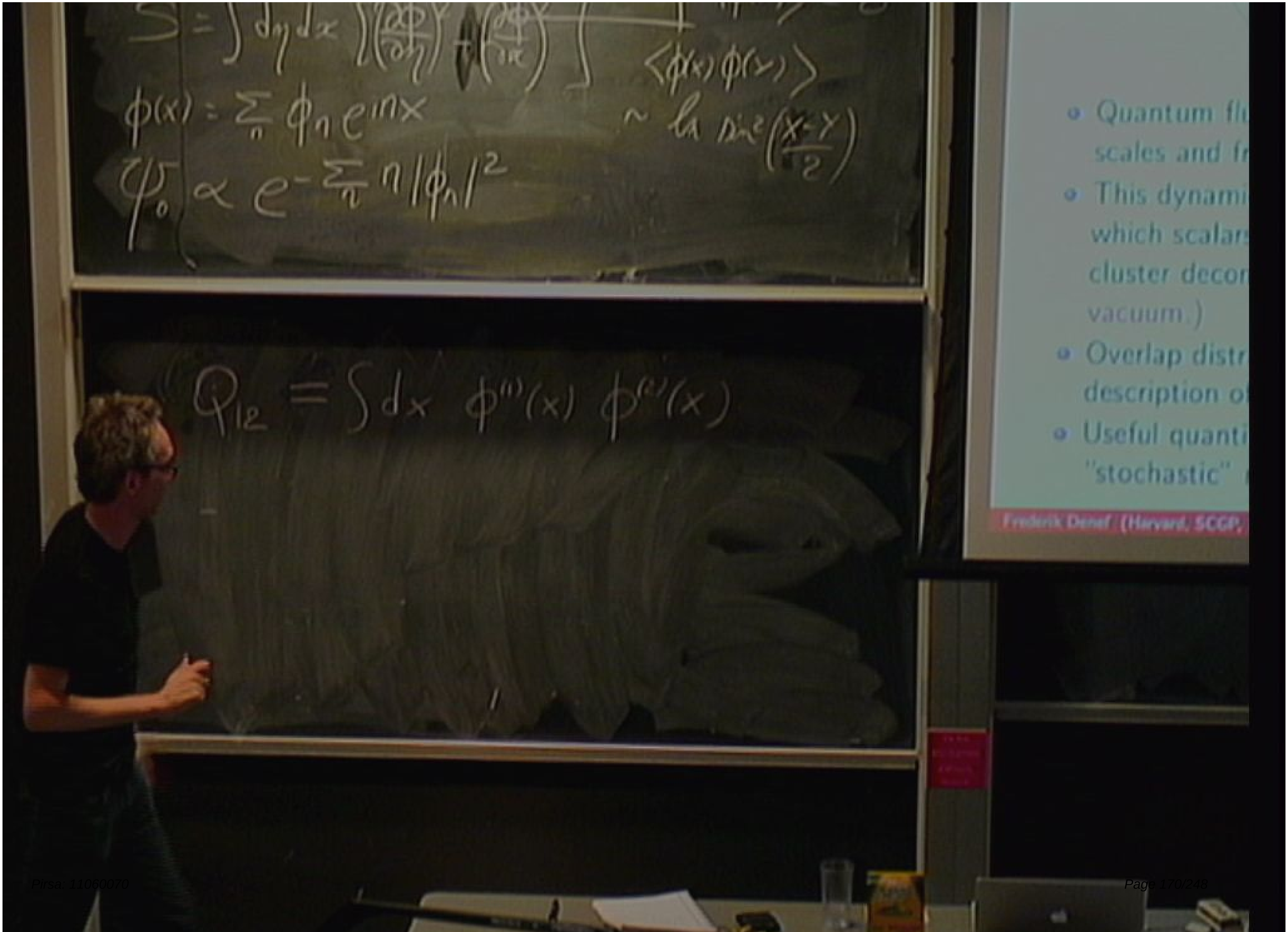
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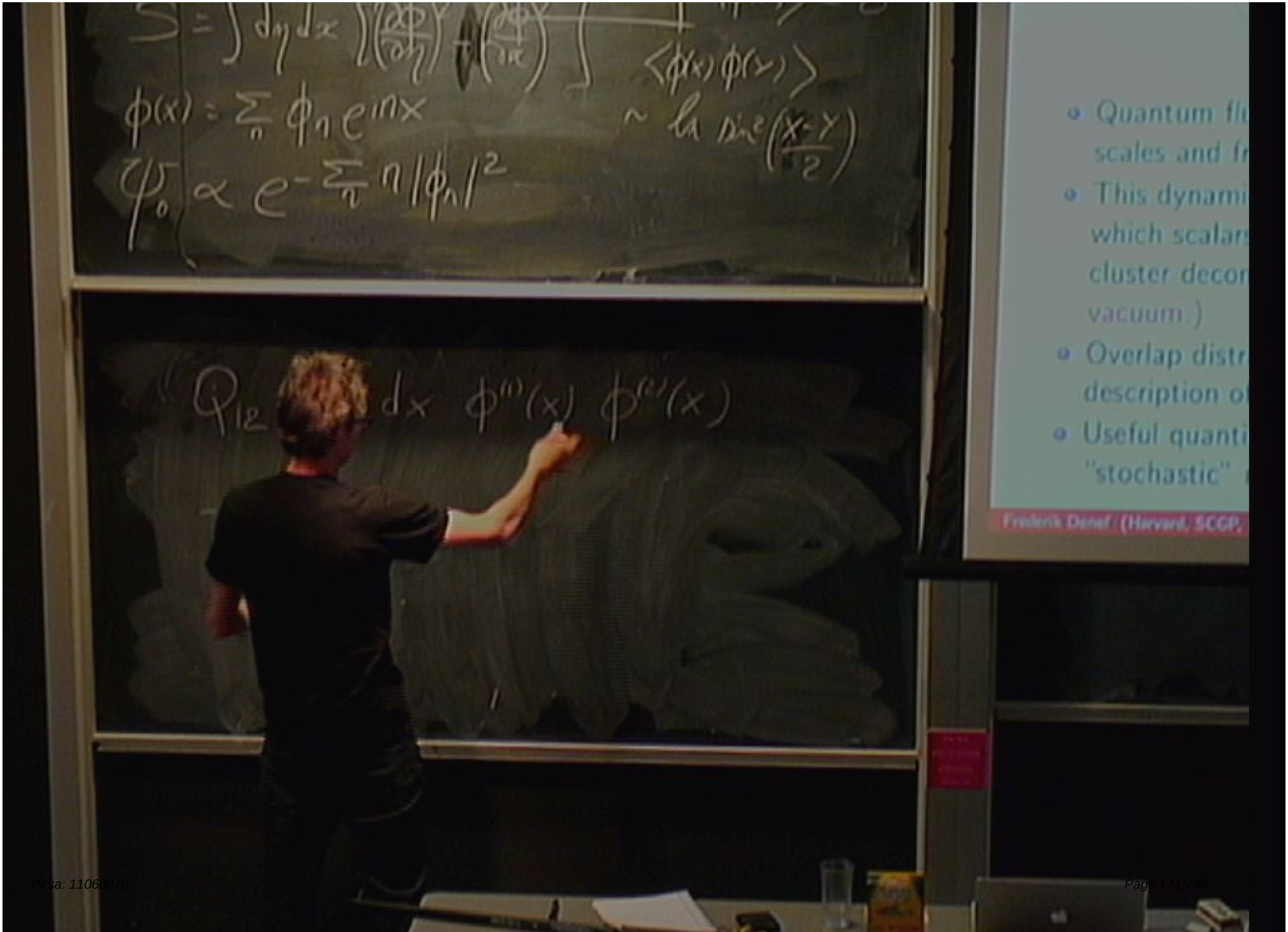
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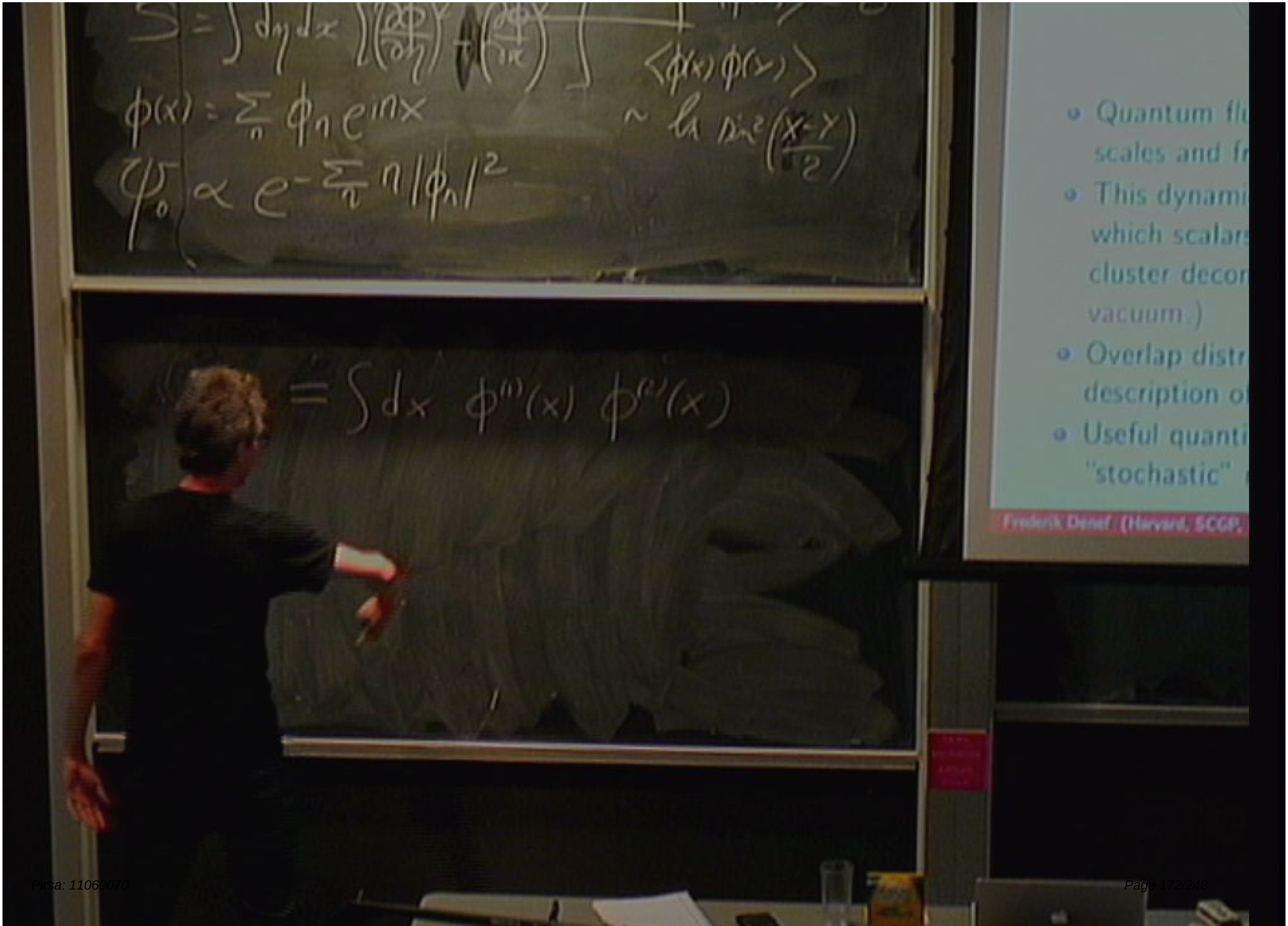
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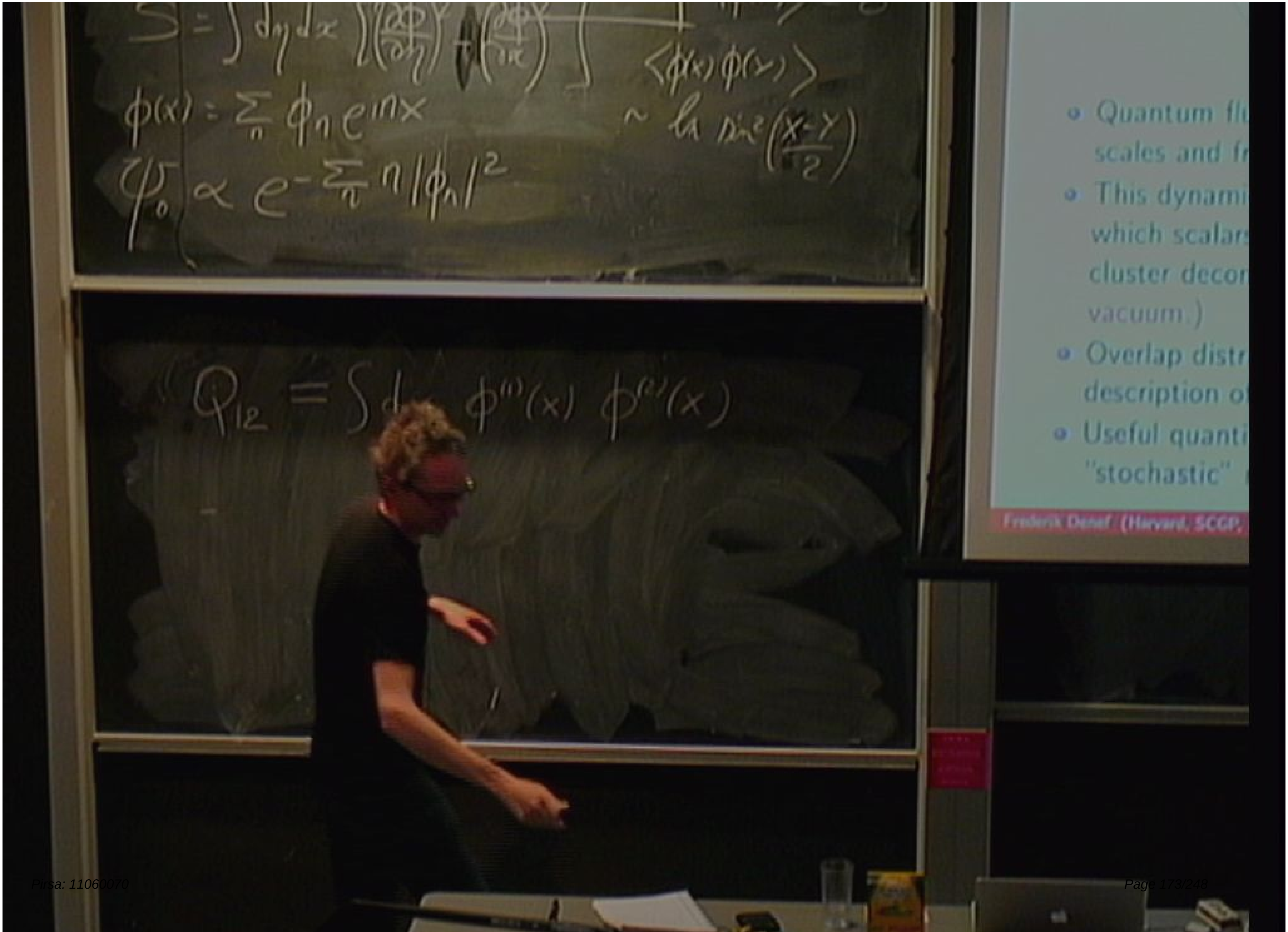
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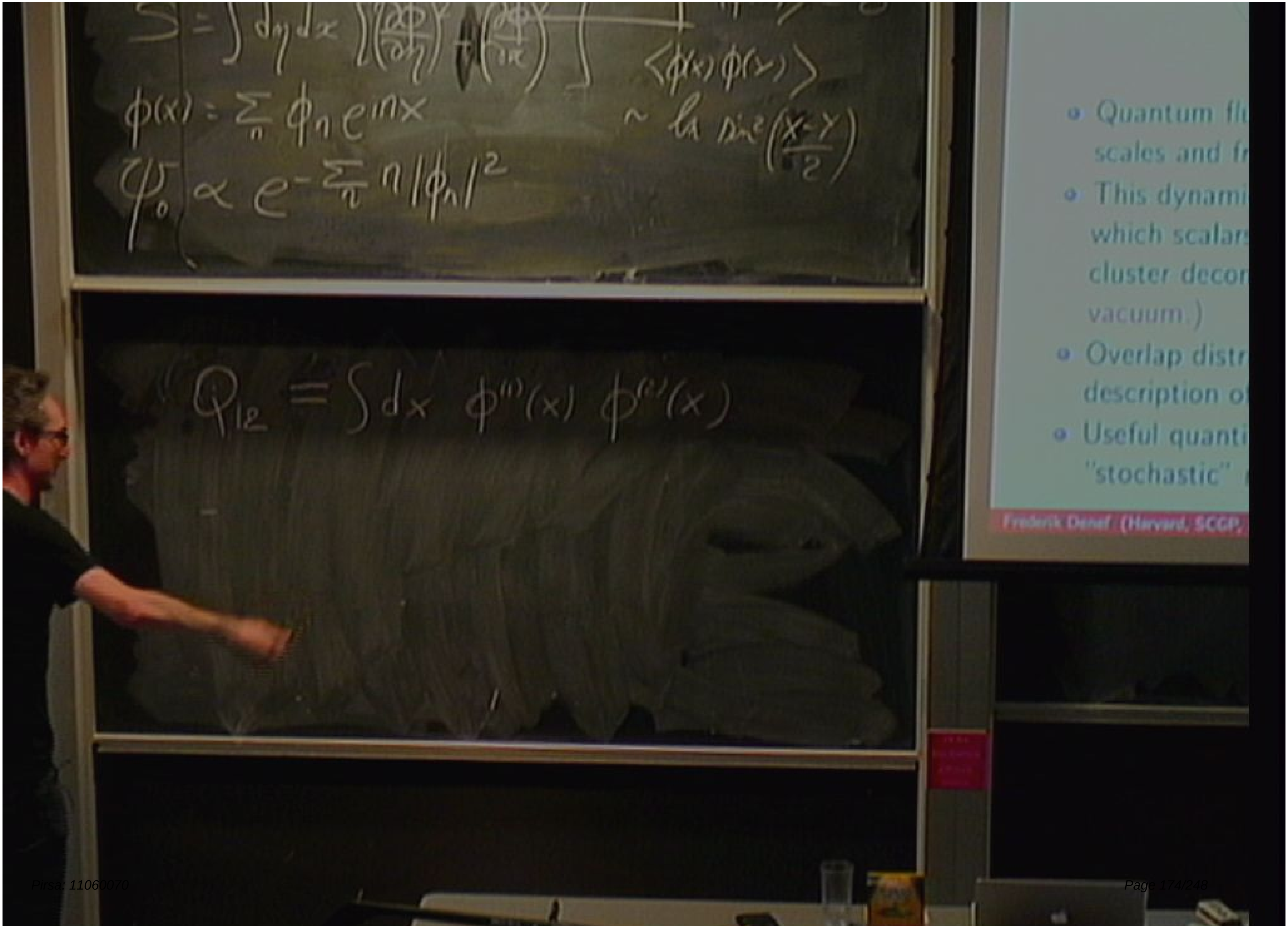
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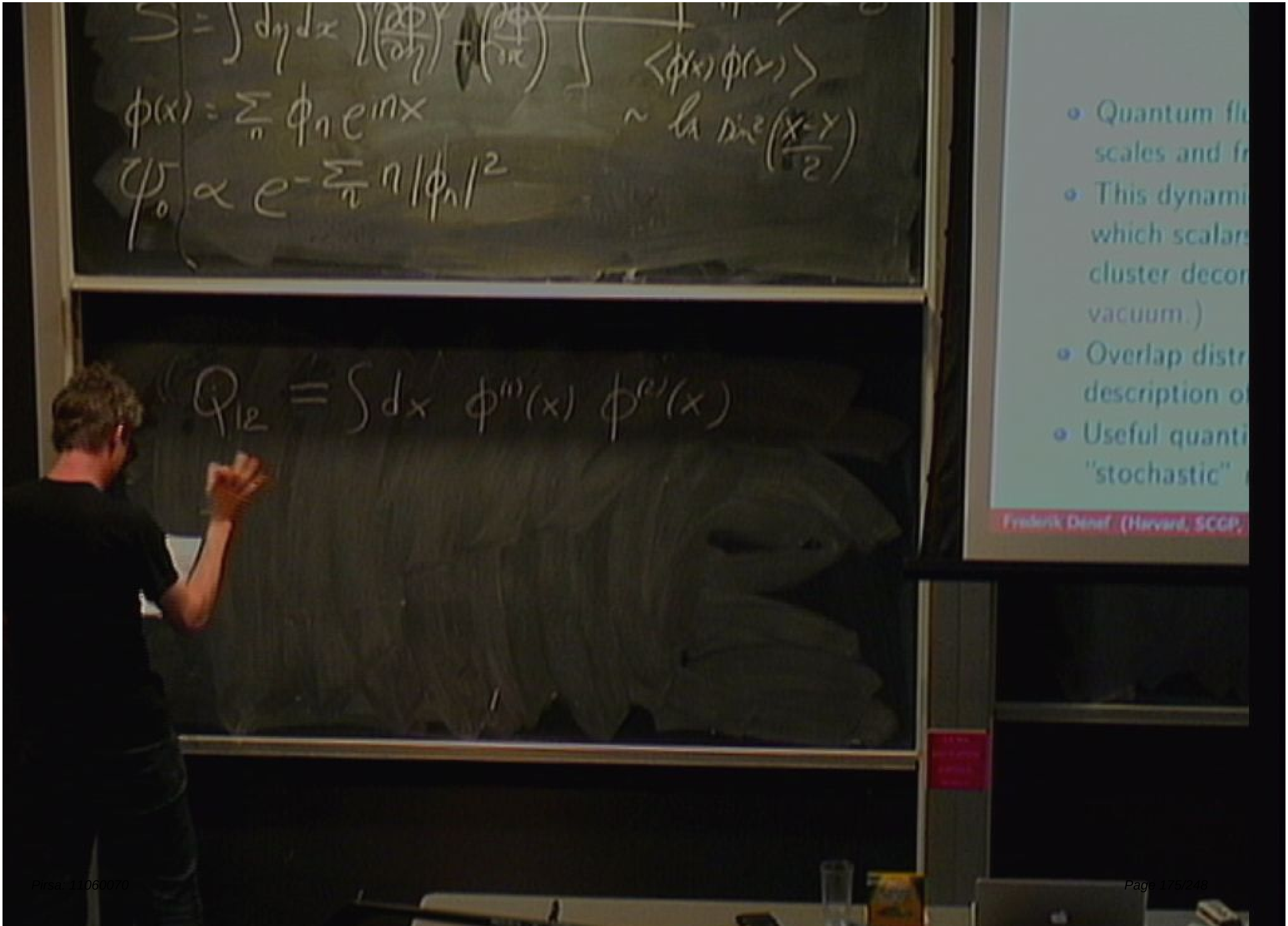
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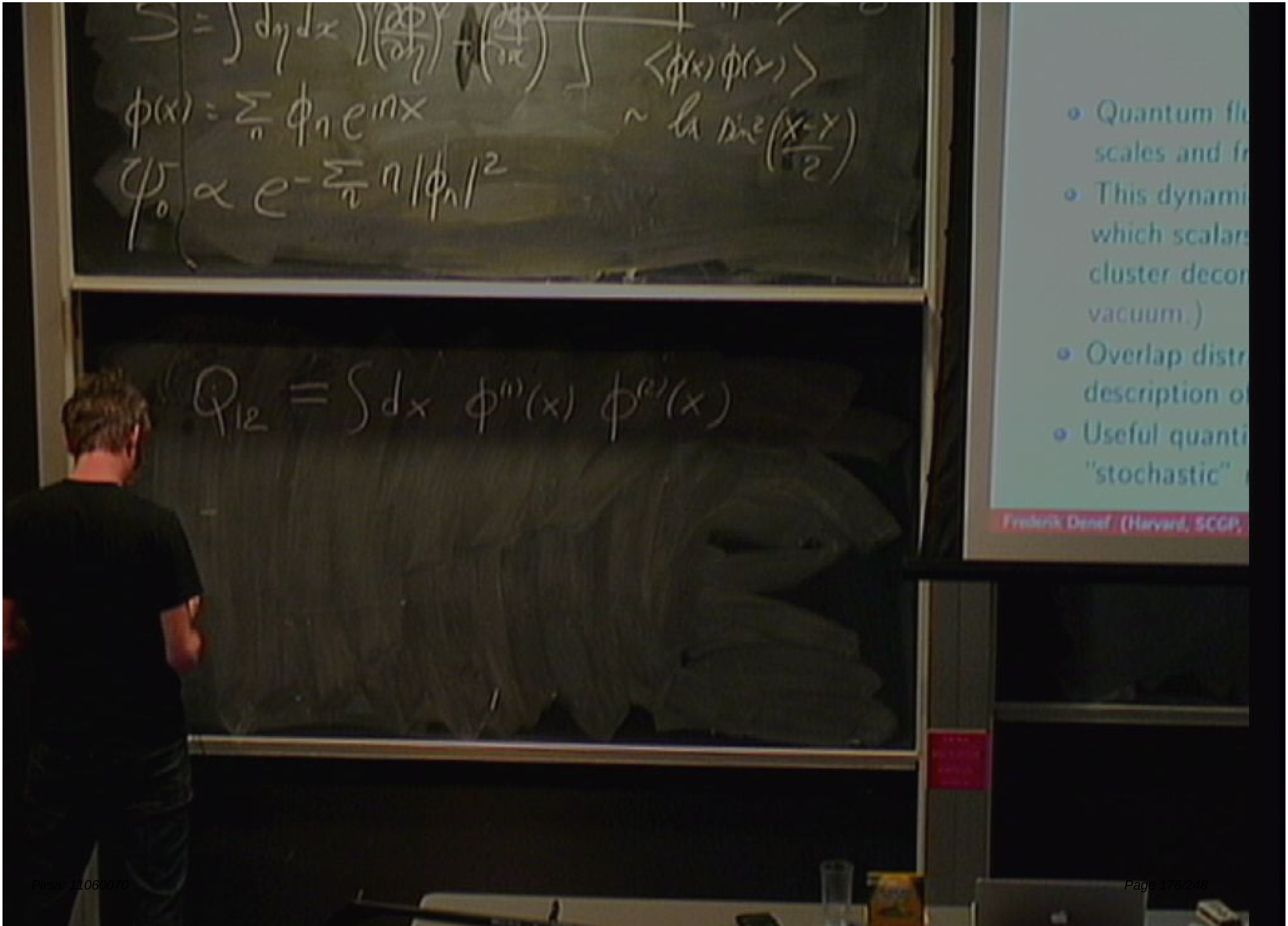
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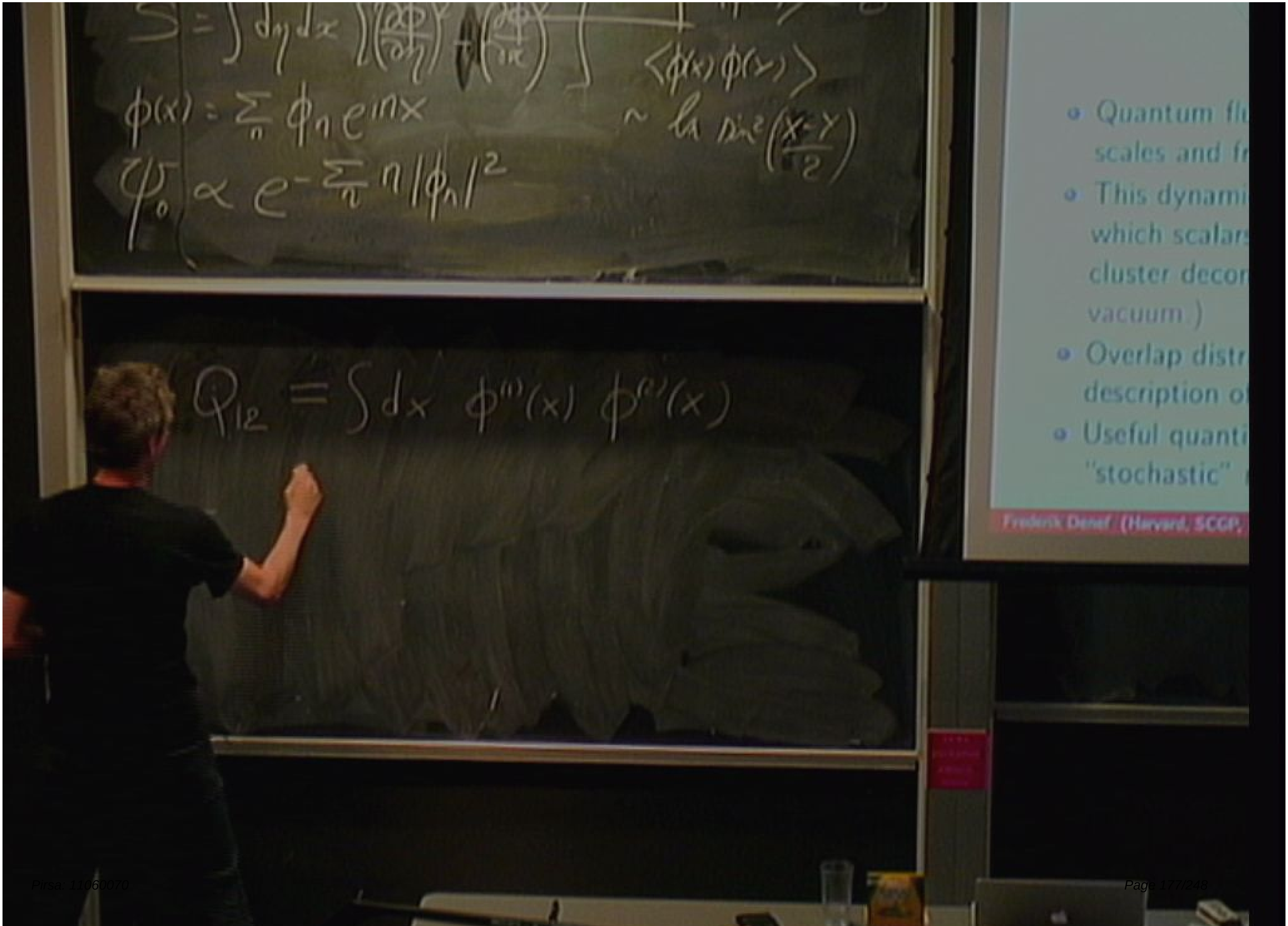

$Q_{12} = \int dx\ \phi^{(1)}(x)\ \phi^{(2)}(x)$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\langle \phi(x)\phi(y) \rangle$
Quantum fl
scales and fr
This dynami
which scalars
cluster decon
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Overlap distr
description of
Useful quanti
"stochastic"
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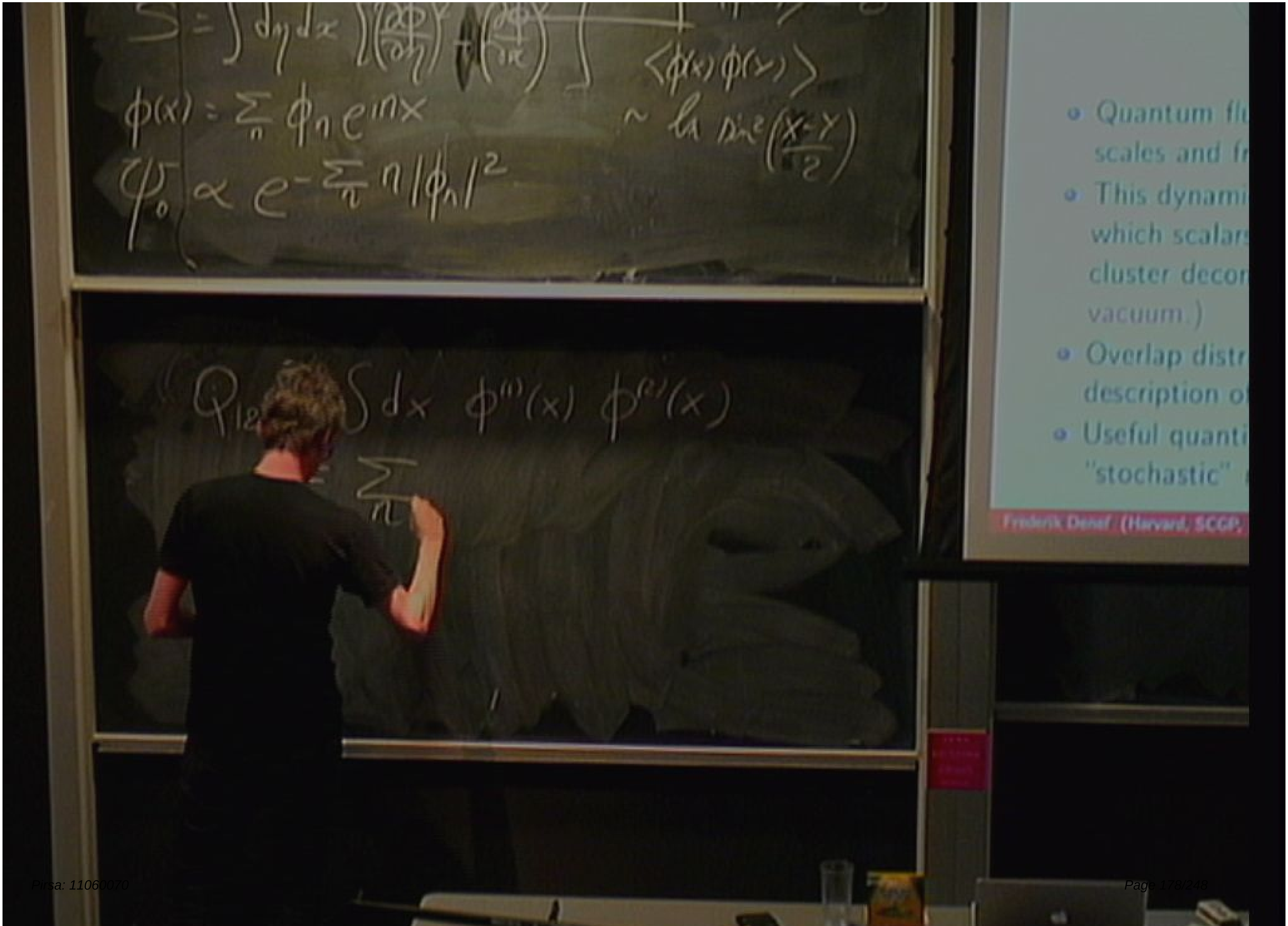
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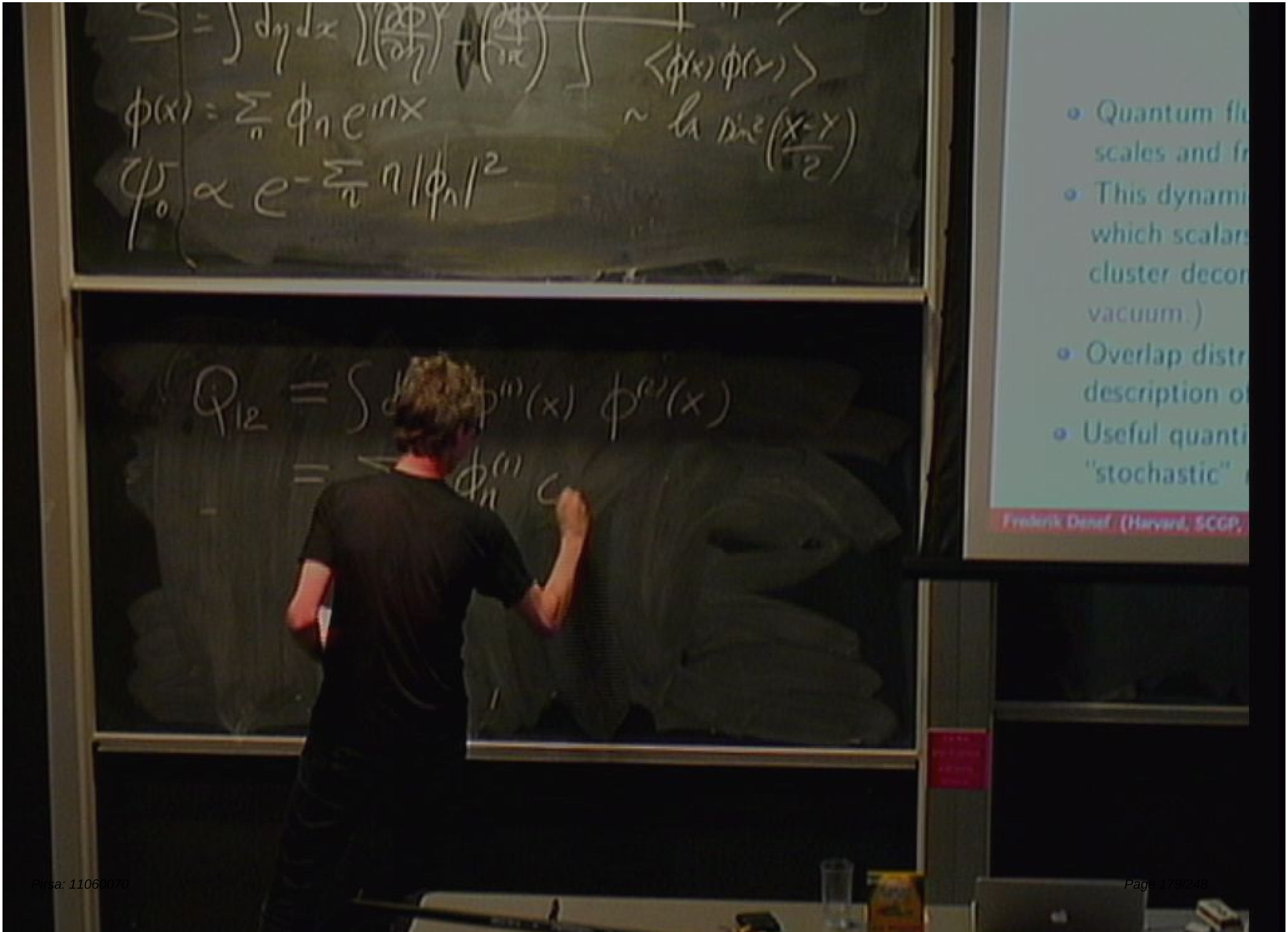
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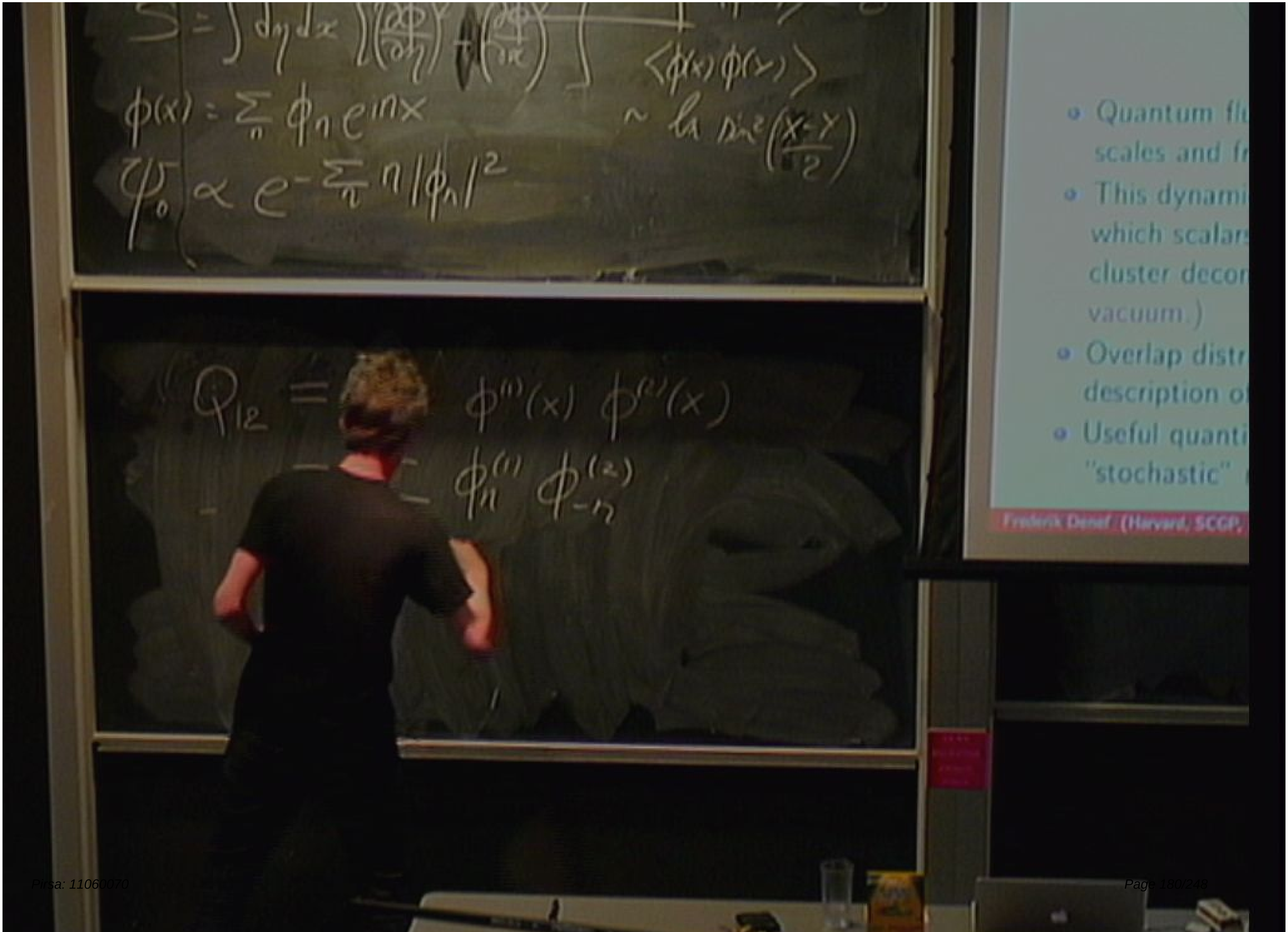
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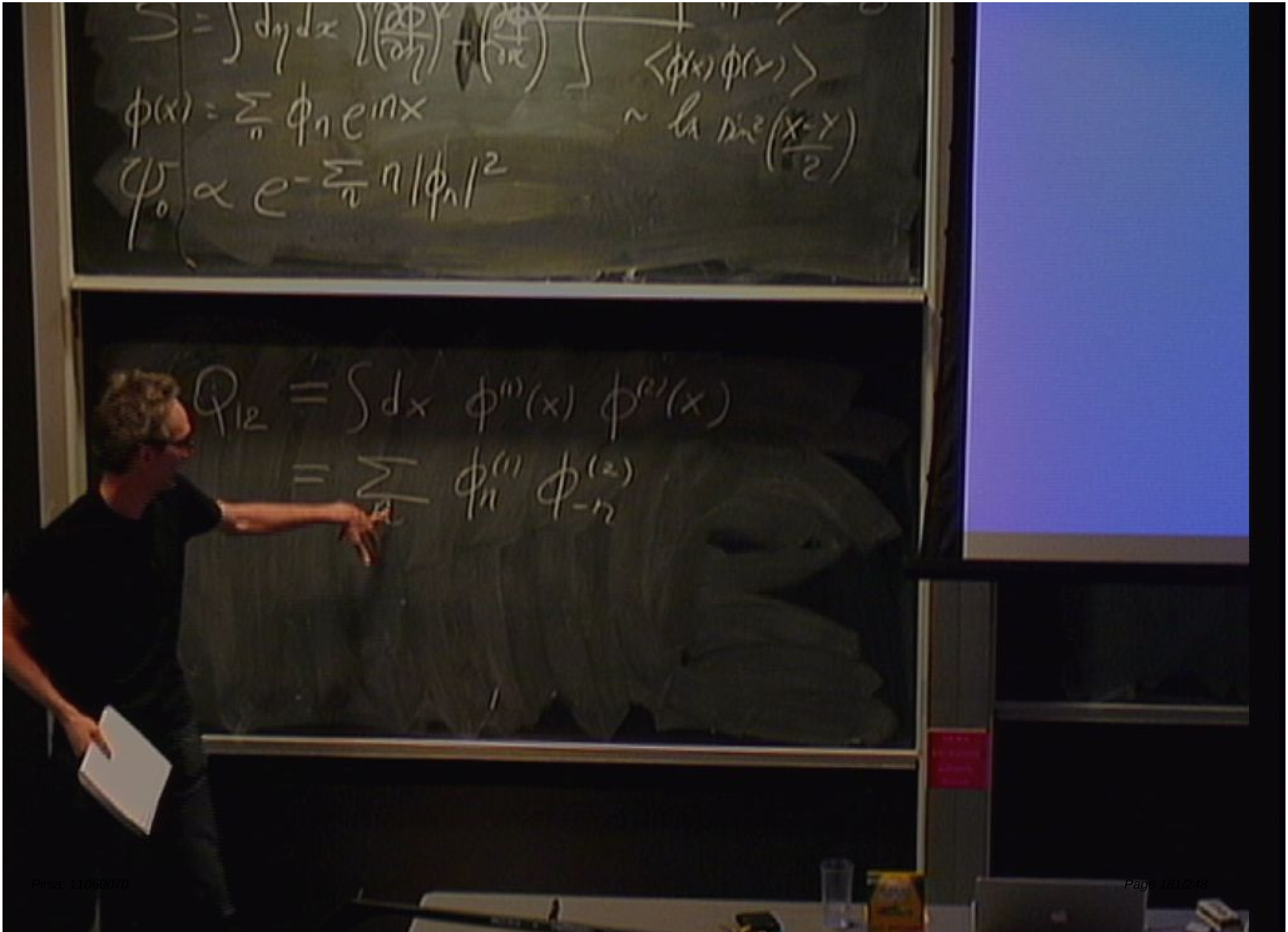
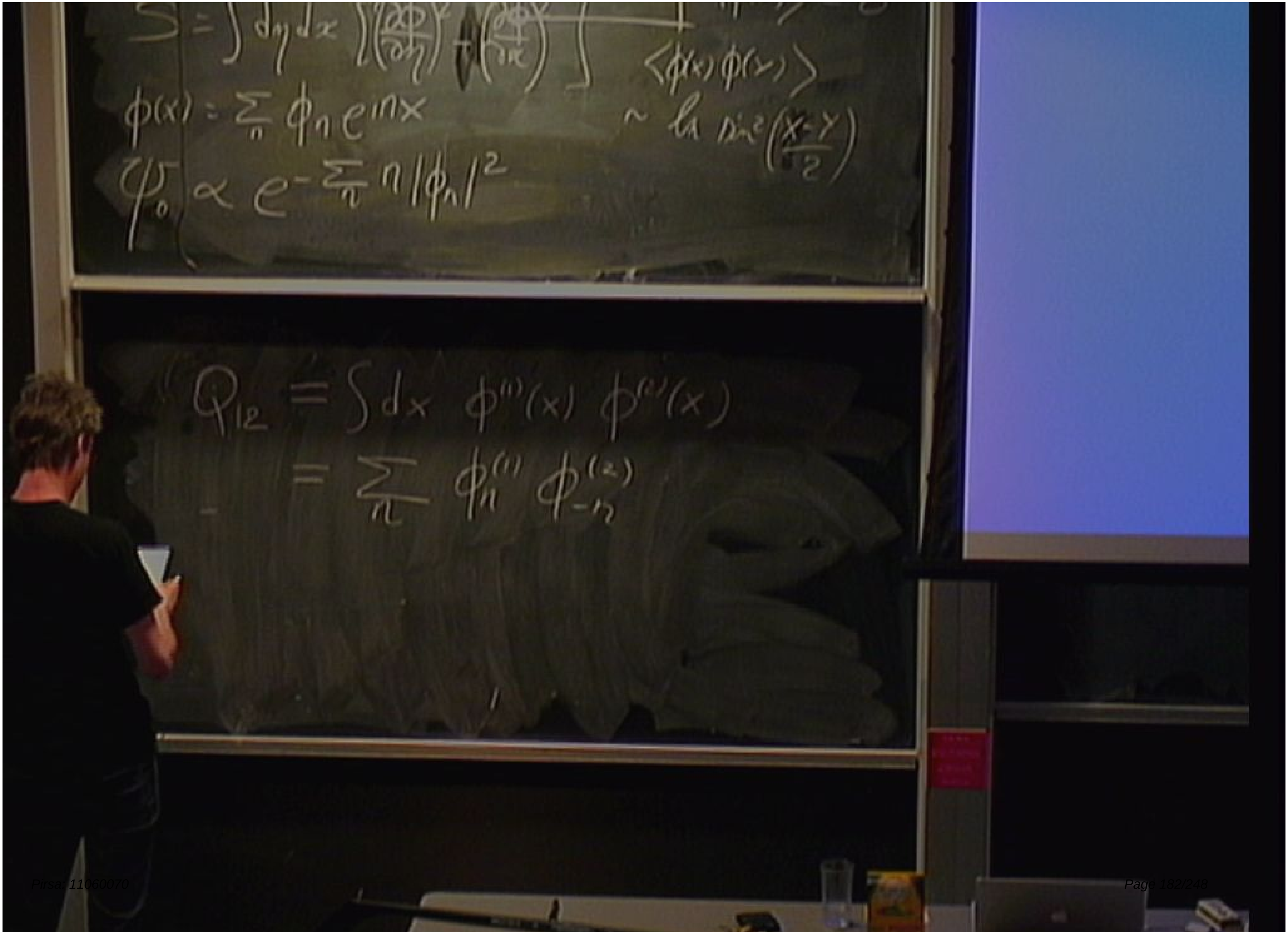
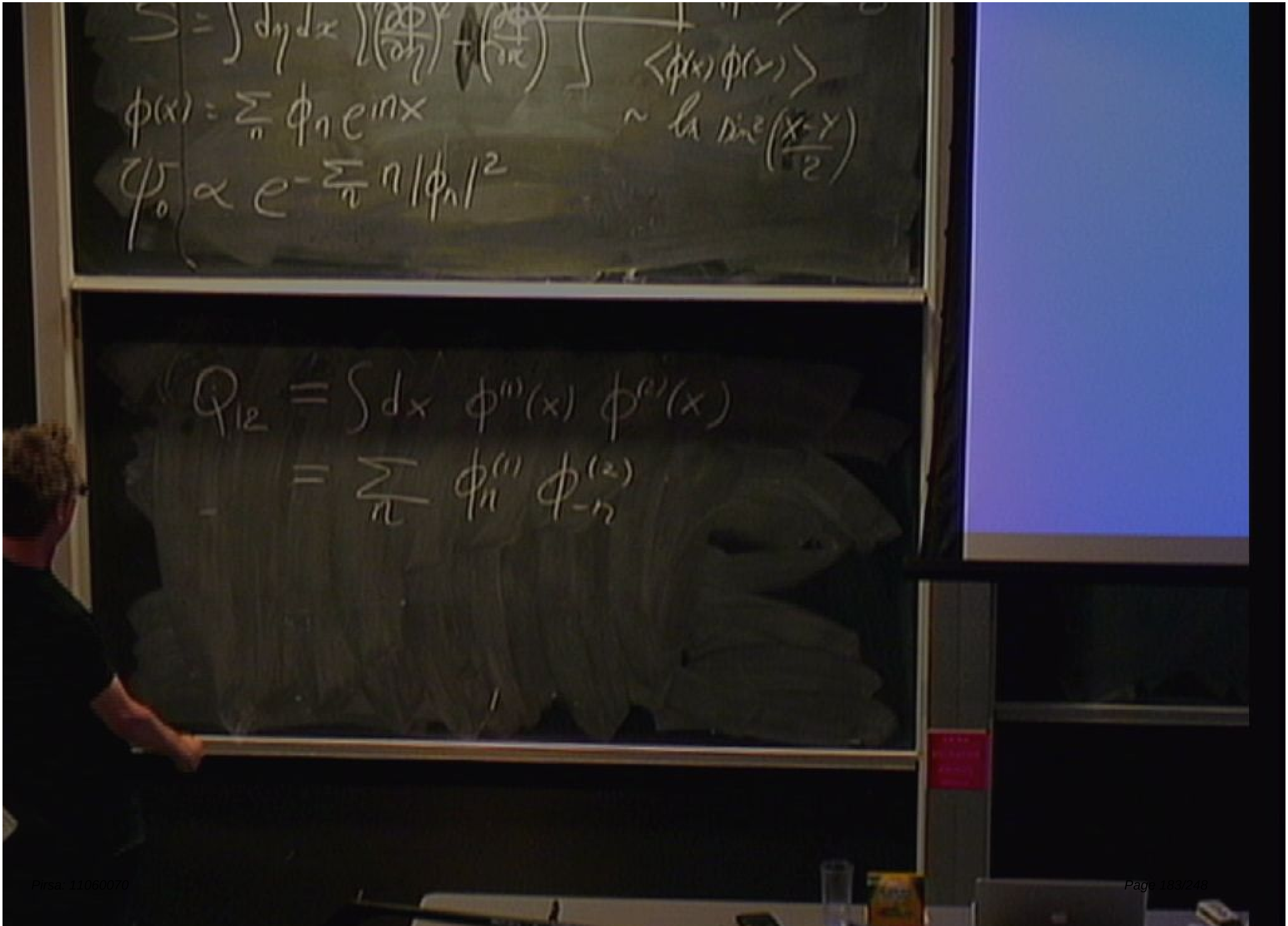


φ(x) = Σ_n φ_n e^{inx}
⟨φ(x) φ(y)⟩
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{-n}^(2)


⟨φ(x) φ(y)⟩
~ ln sin²((x−y)/2)
φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{−Σ_n n|φ_n|²}
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{−n}^(2)



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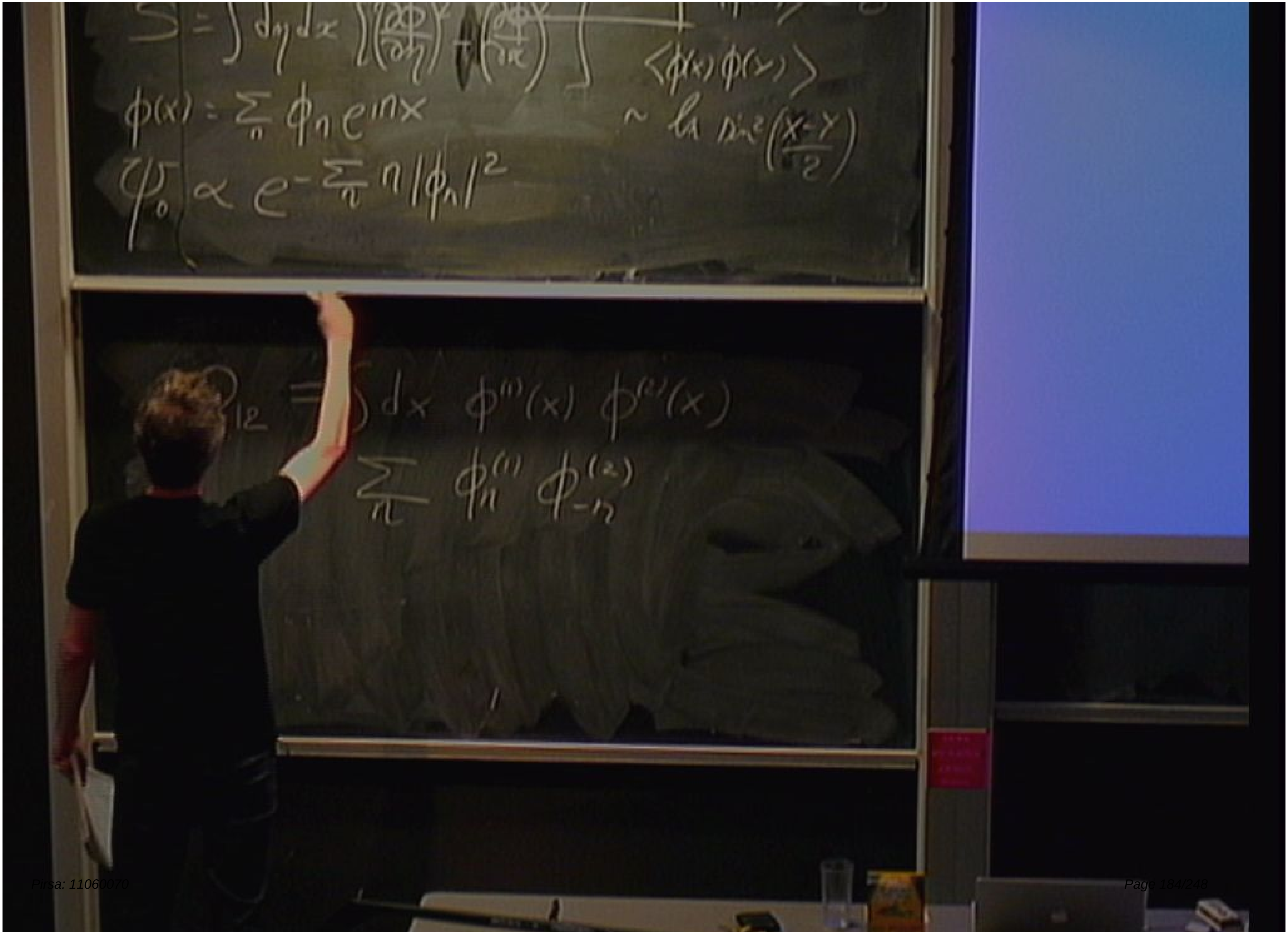
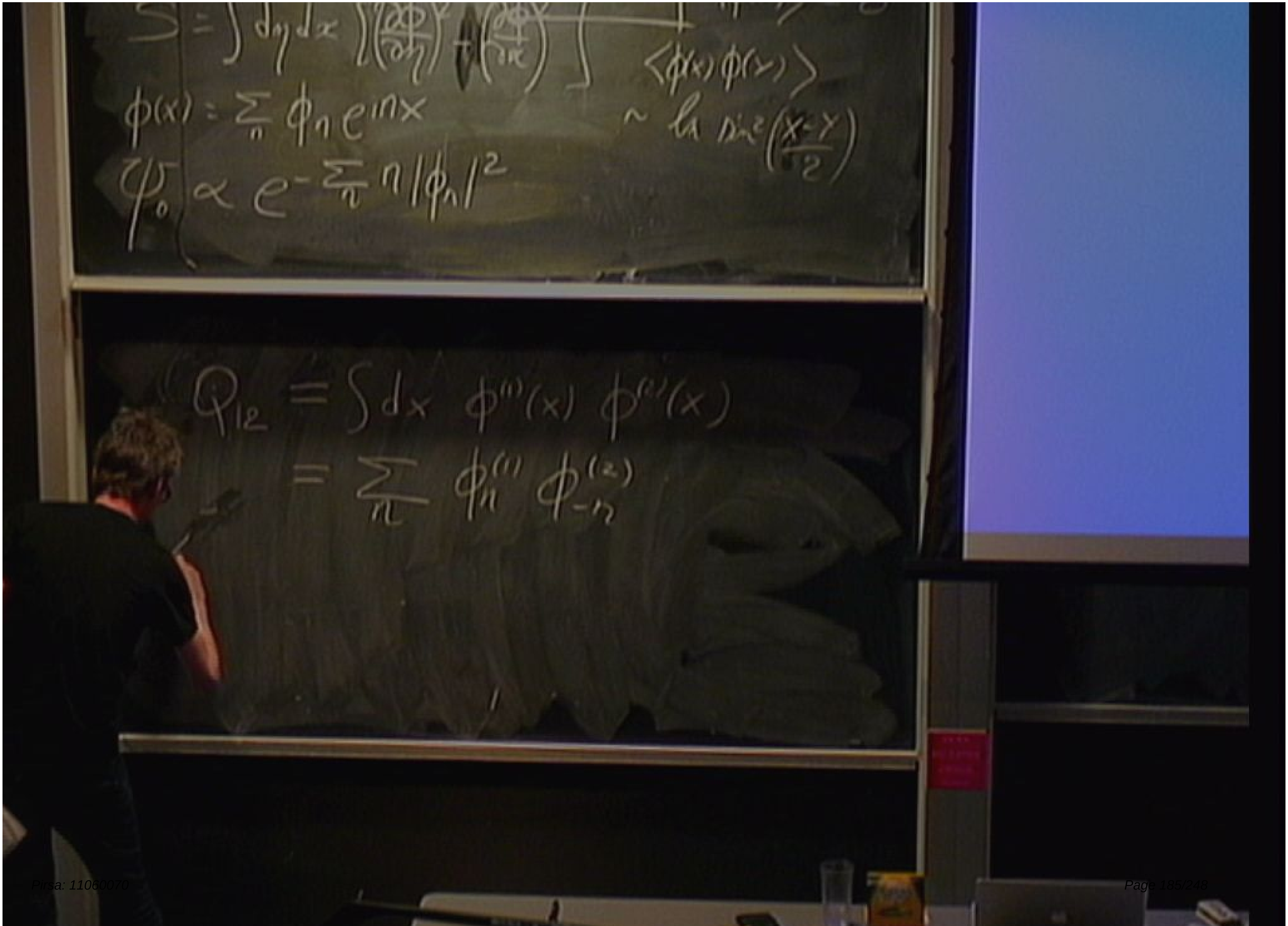

⟨φ(x) φ(y)⟩
φ(x) = Σn φn e^inx
Ψ0 ∝ e^(−Σn n|φn|²)
∫ dx φ^(1)(x) φ^(2)(x)
Σn φn^(1) φ−n^(2)


⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
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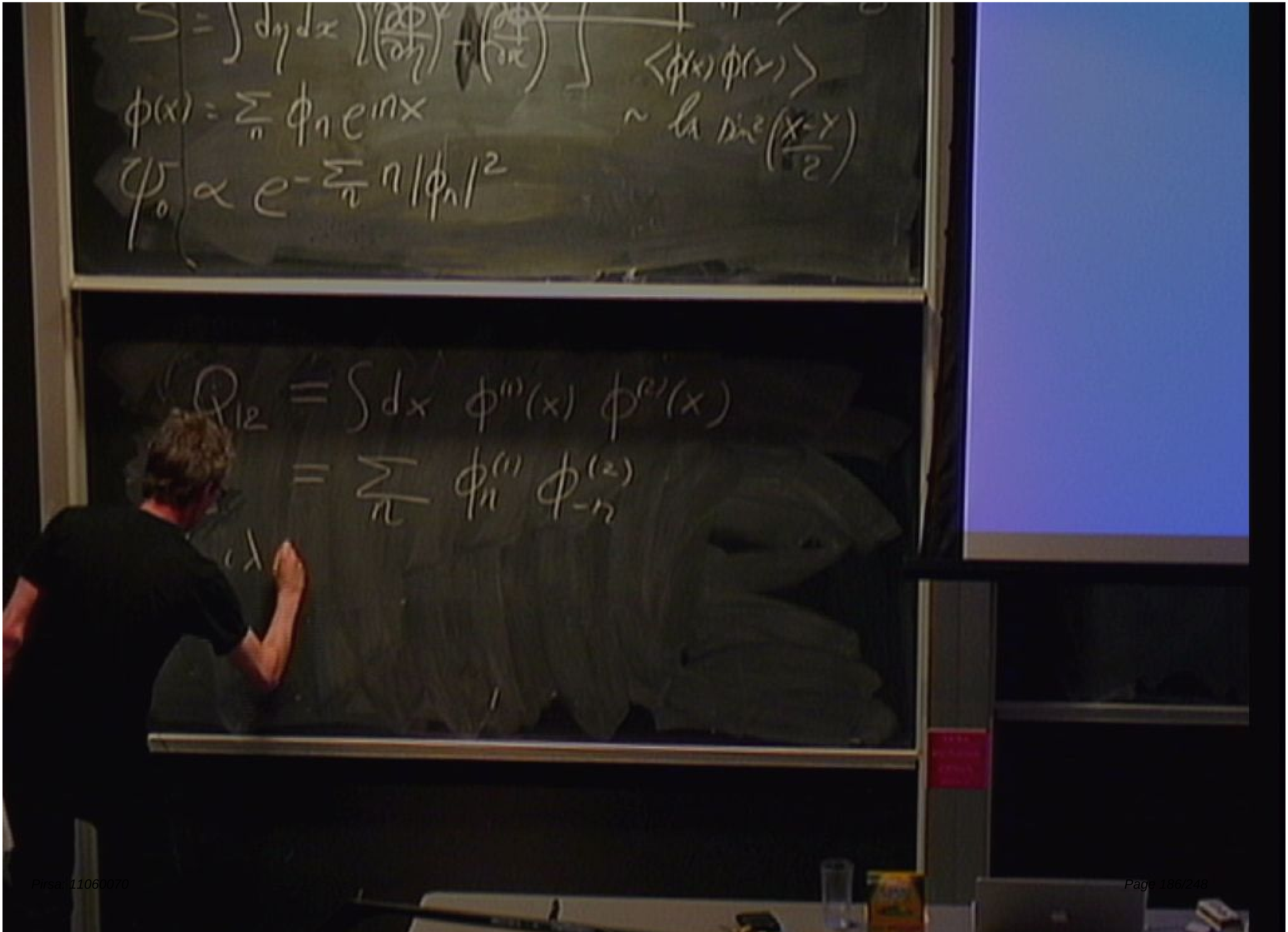
⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
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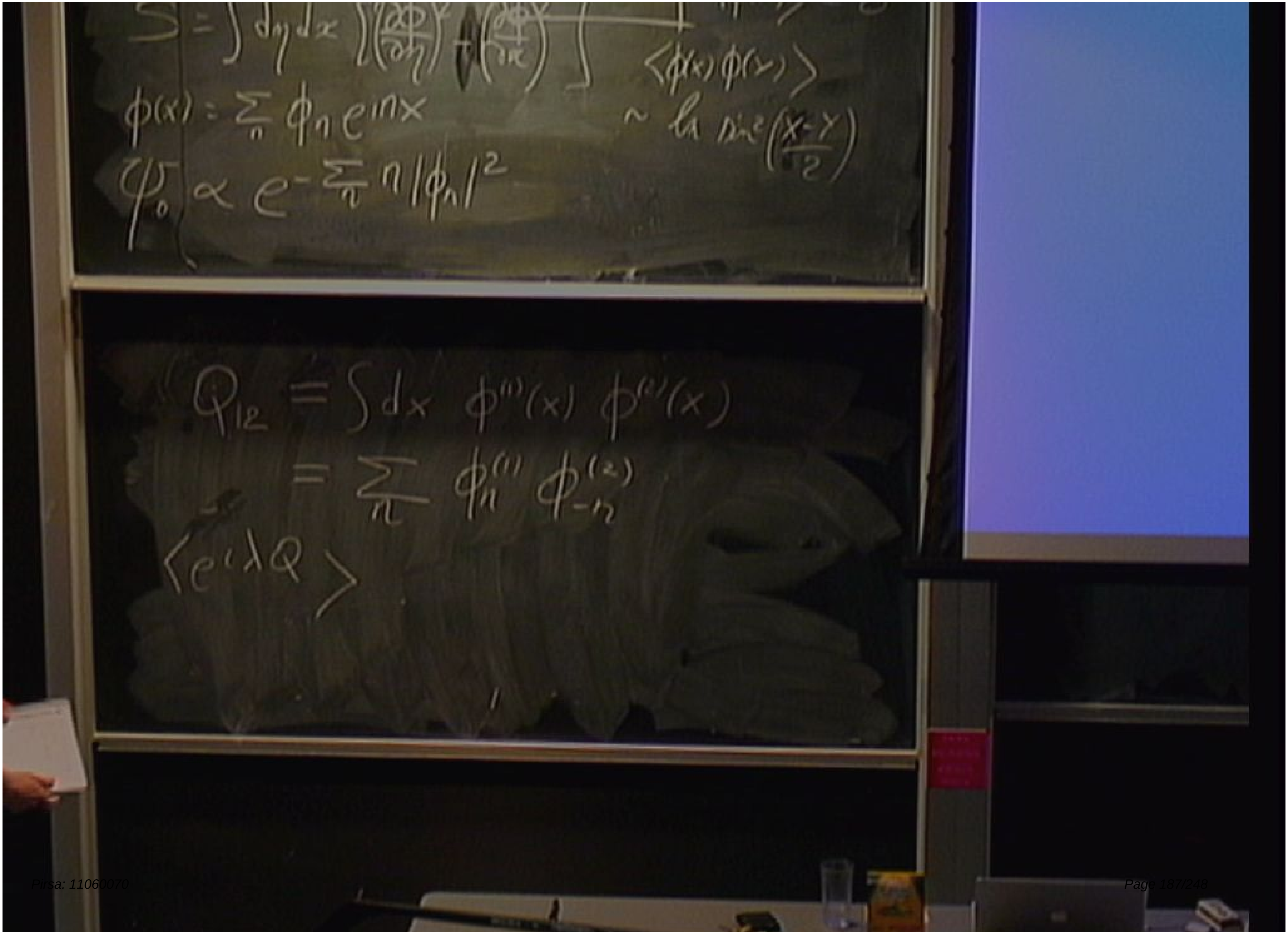
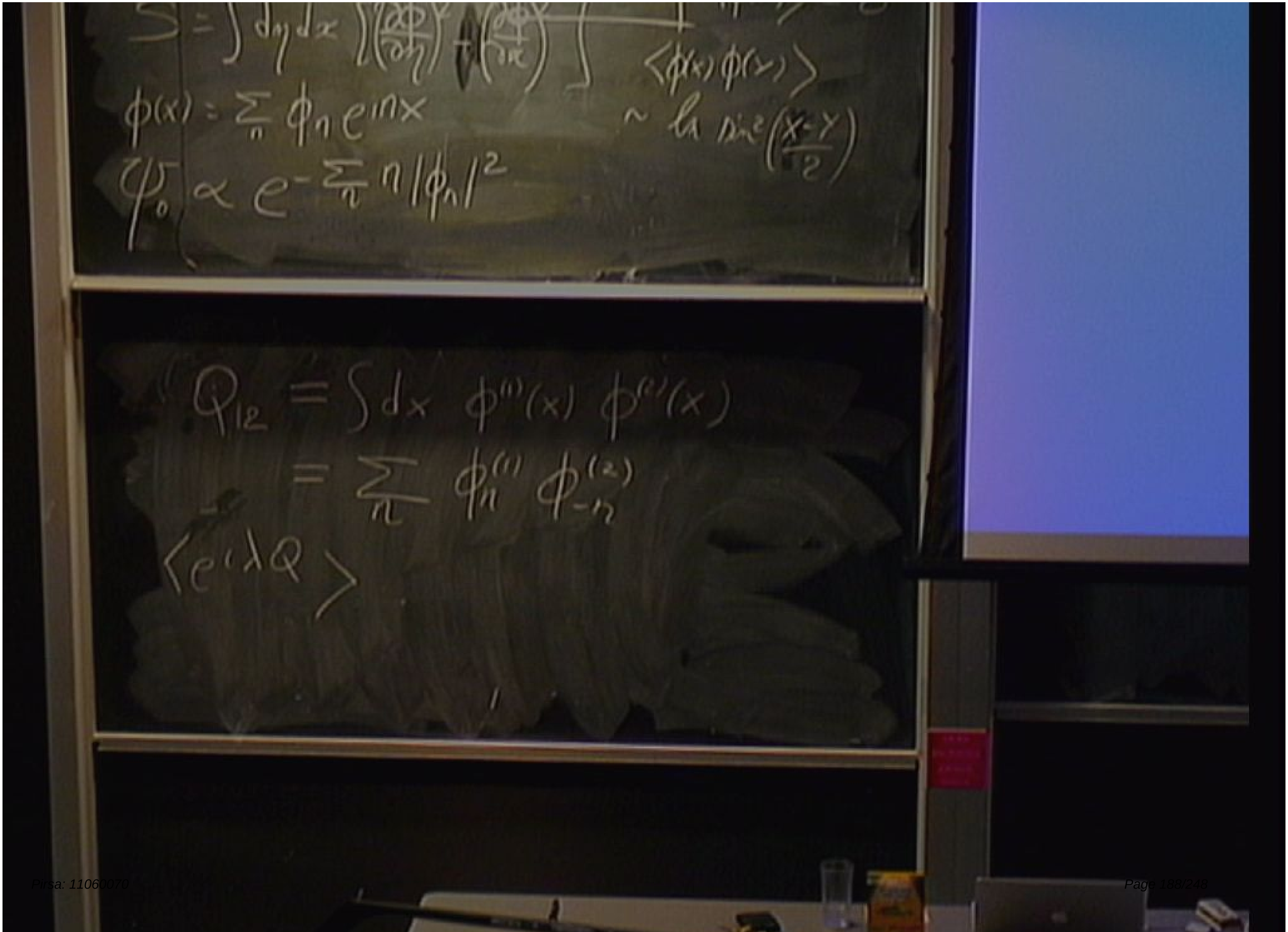

$\langle \phi(x)\phi(y) \rangle$
$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx\ \phi^{(1)}(x)\ \phi^{(2)}(x)$
$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$
$\langle e^{i\lambda Q} \rangle$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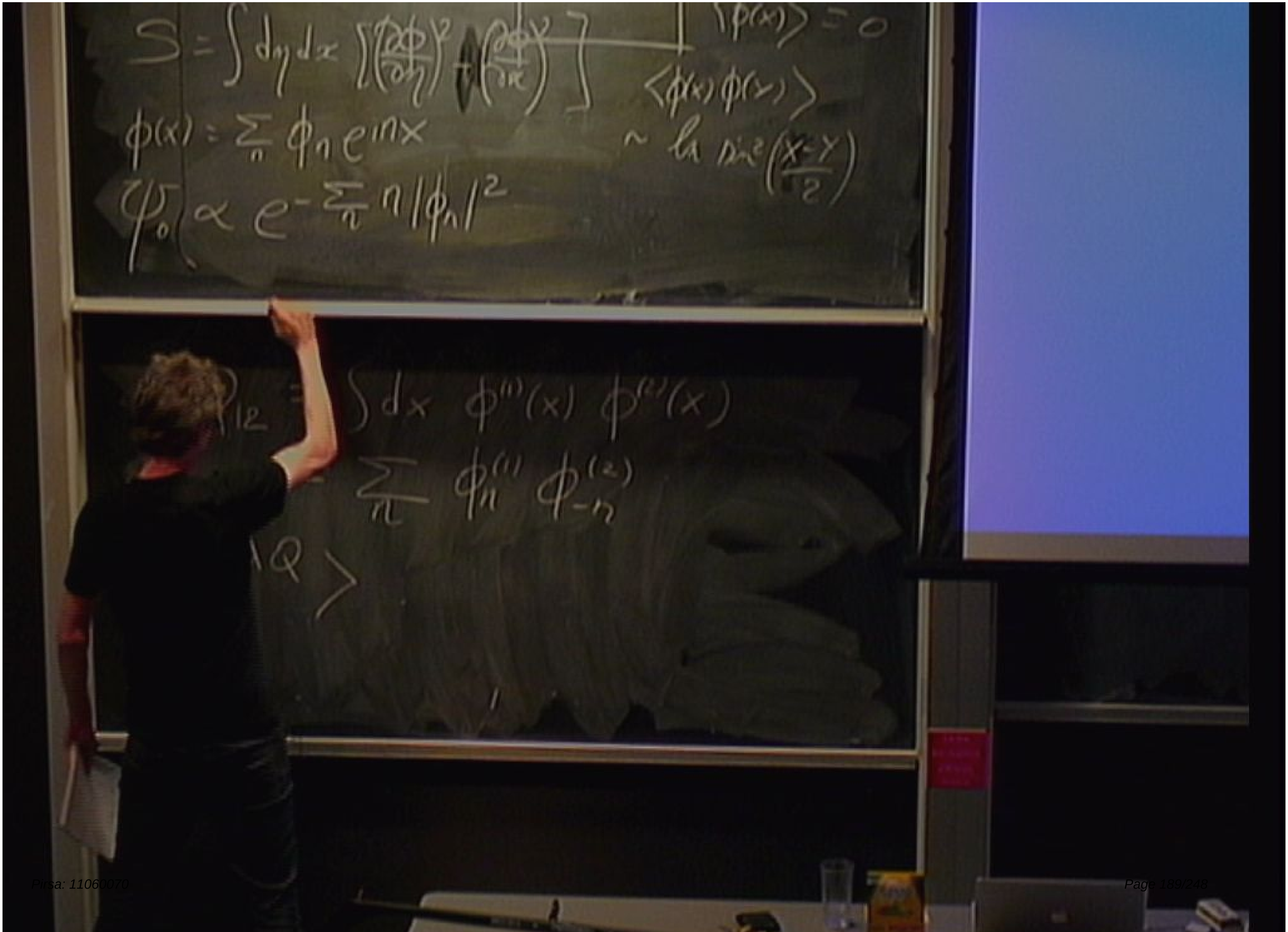
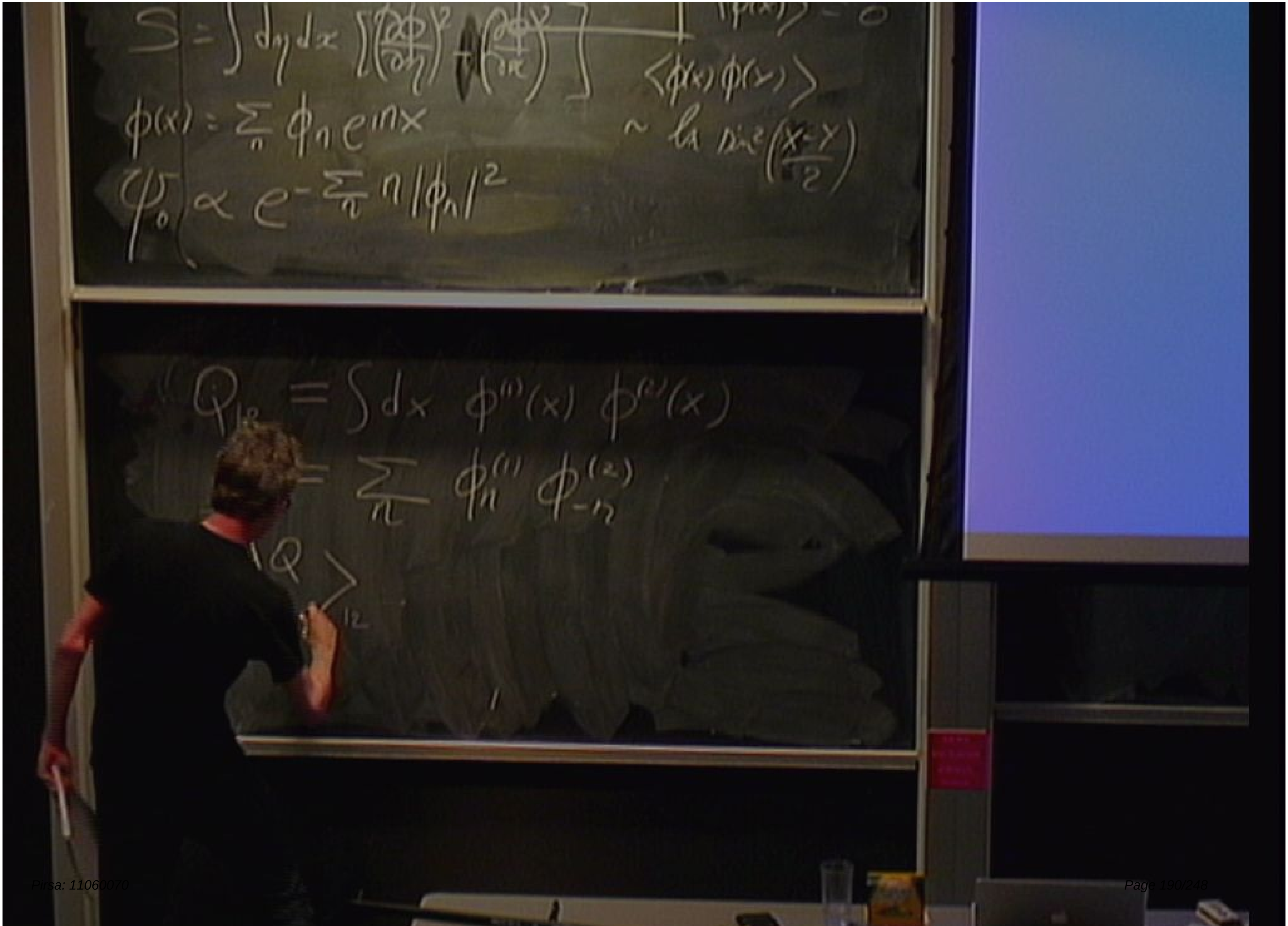


S = ∫ dη dx [(∂φ/∂η)² + (∂φ/∂x)²]
⟨φ(x)⟩ = 0
⟨φ(x) φ(y)⟩
~ ln sin²((x−y)/2)
φ(x) = Σn φn e^inx
Ψ0 ∝ e^(−Σn n|φn|²)
12 = ∫ dx φ^(1)(x) φ^(2)(x)
= Σn φn^(1) φ−n^(2)


φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
⟨φ(x) φ(y)⟩
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
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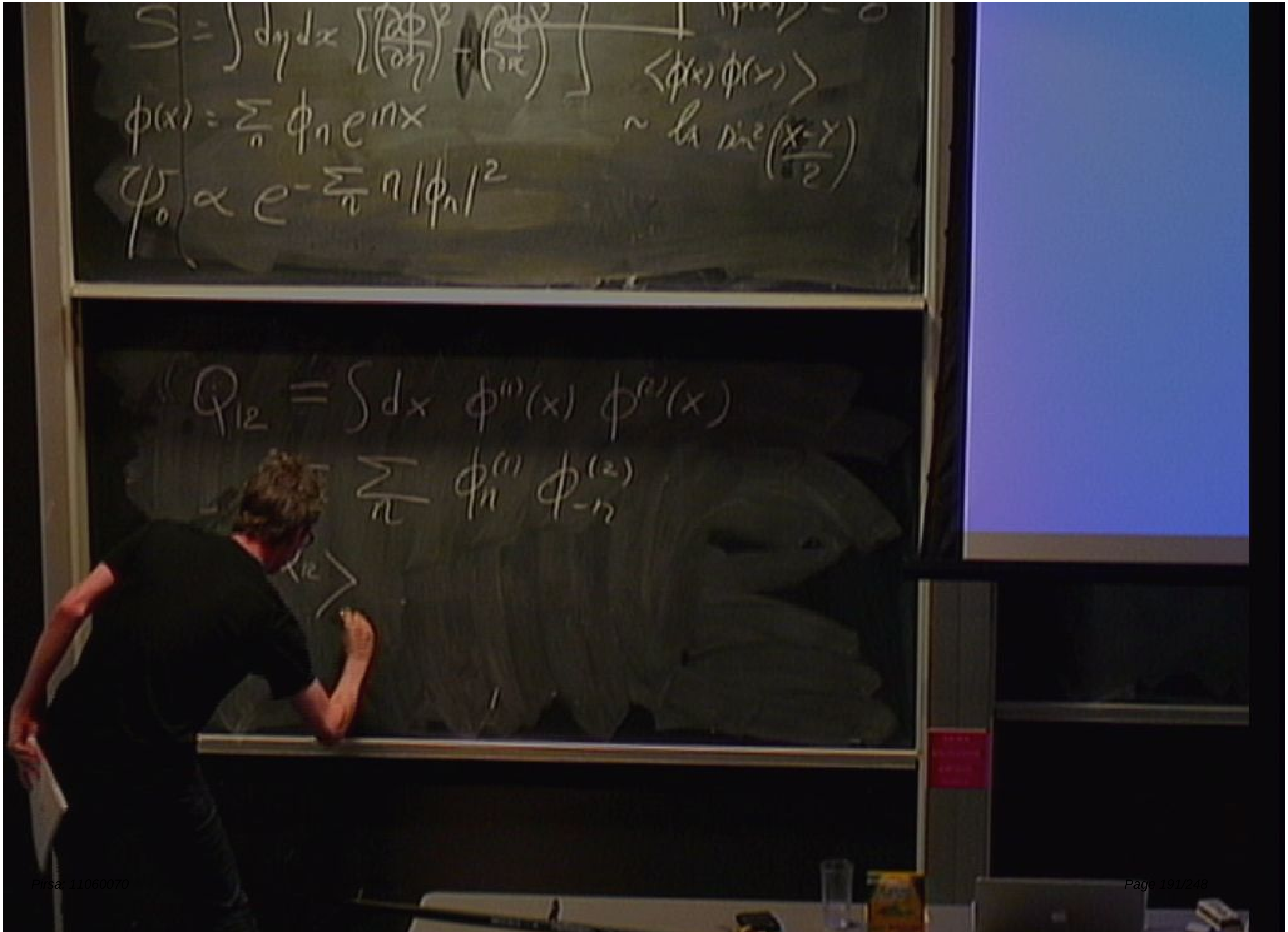
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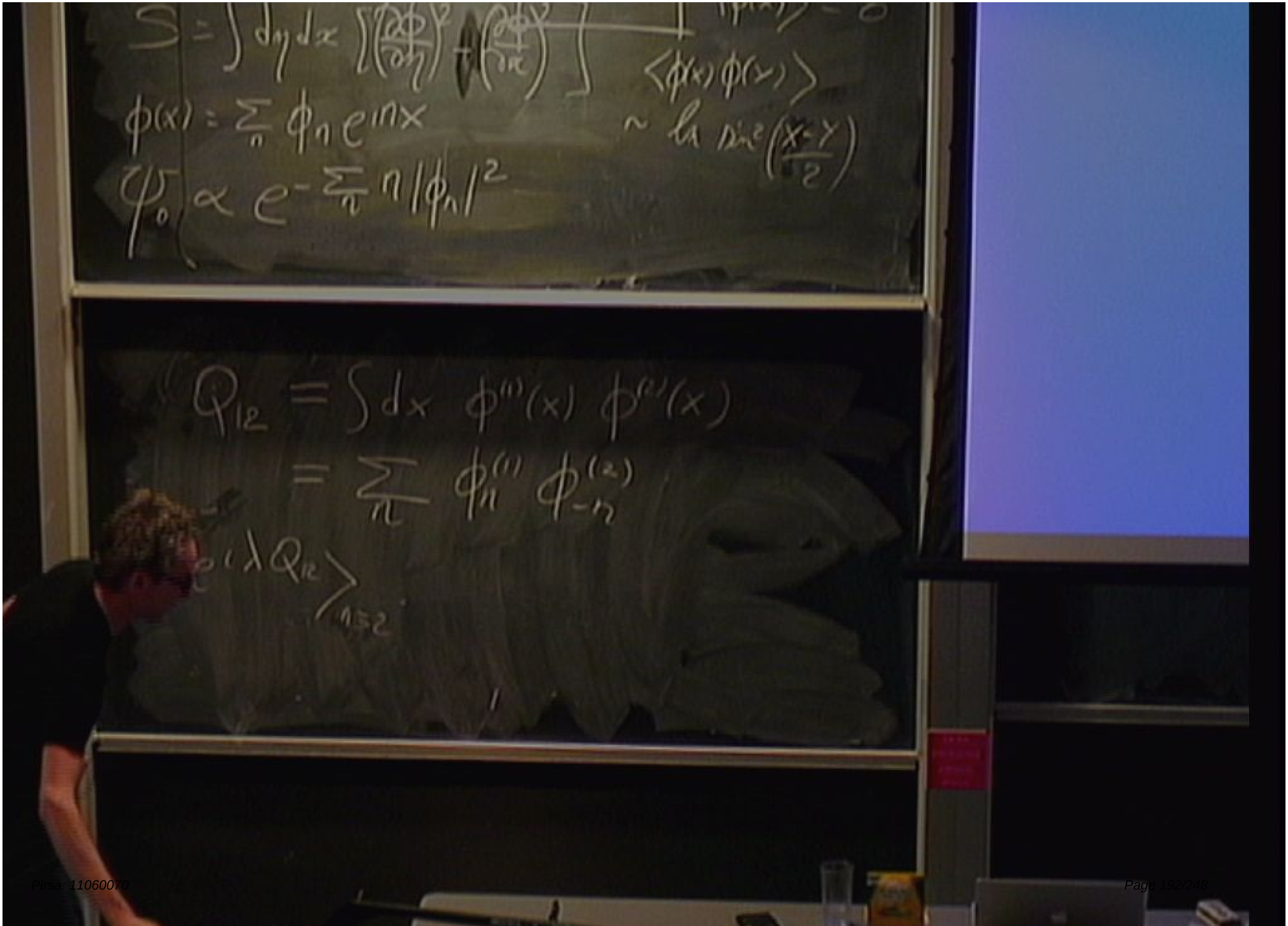
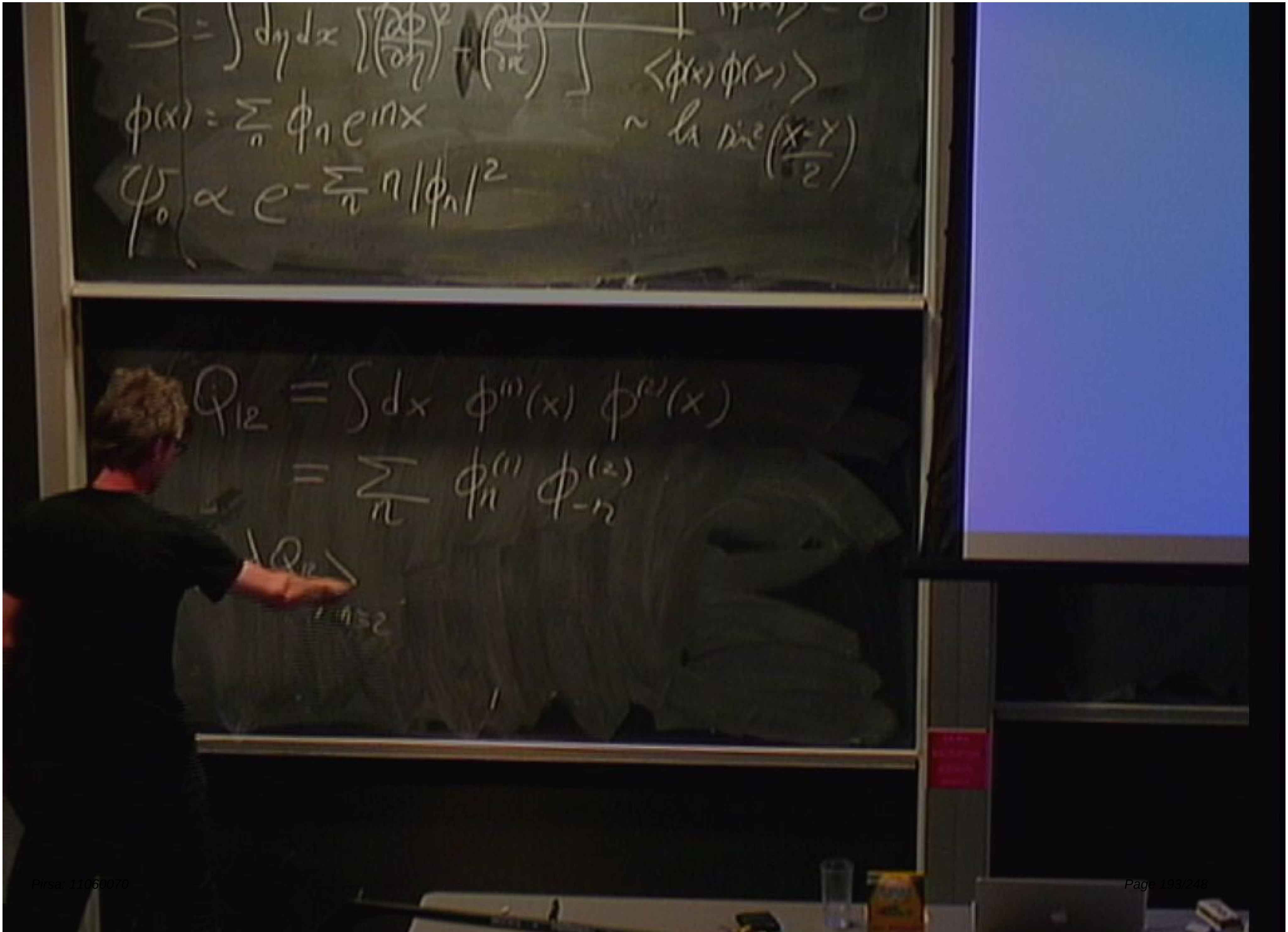
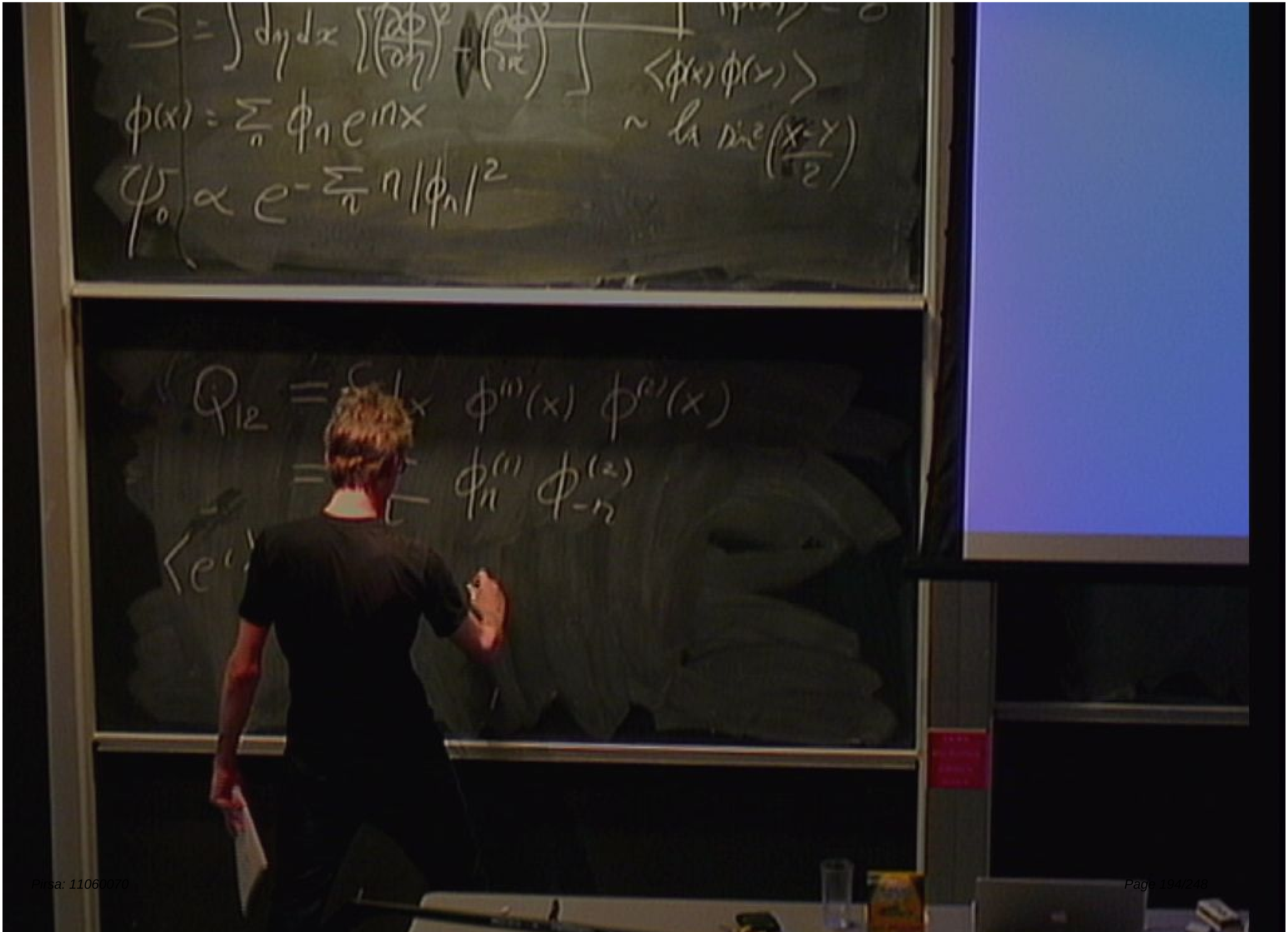
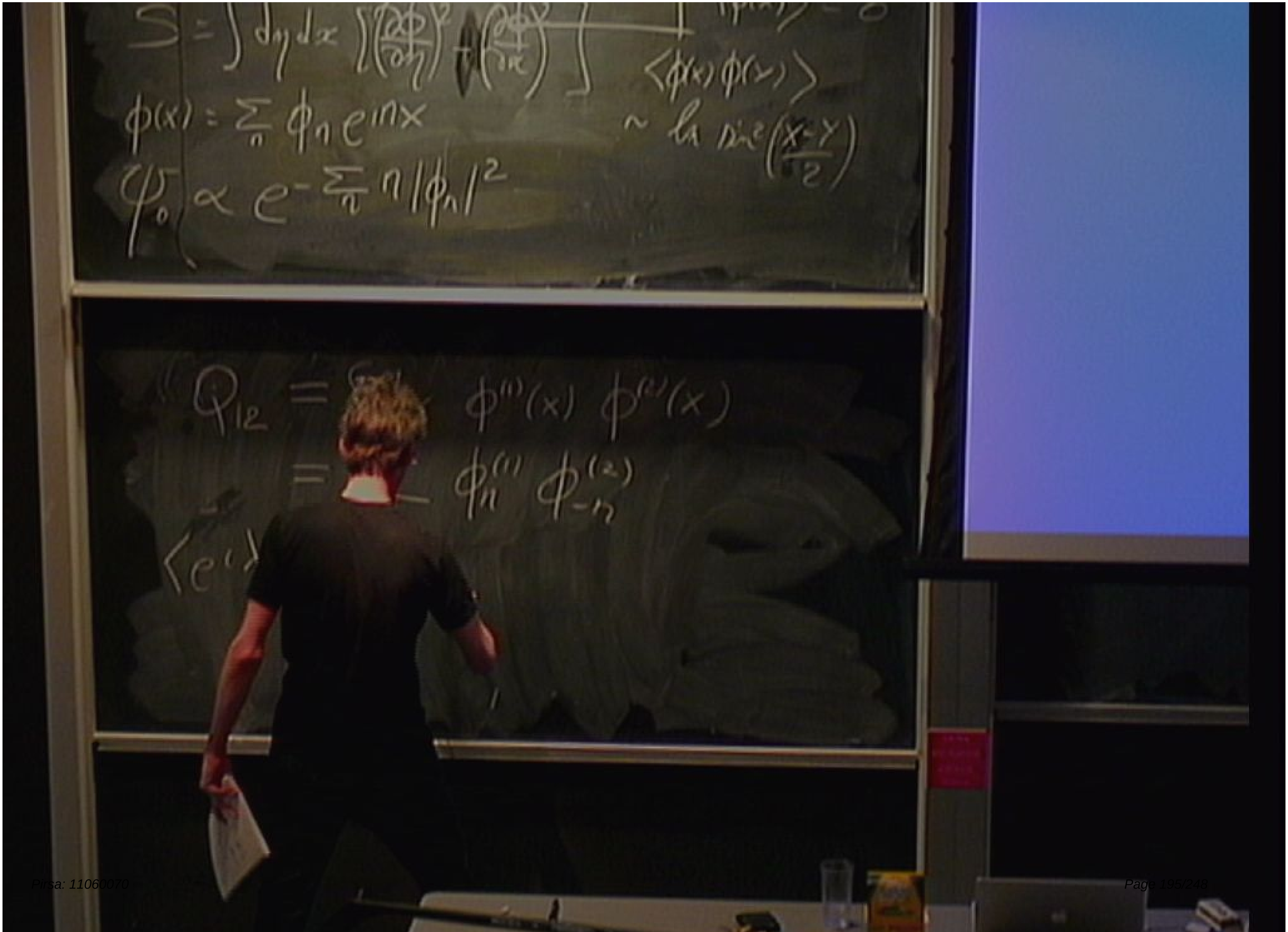
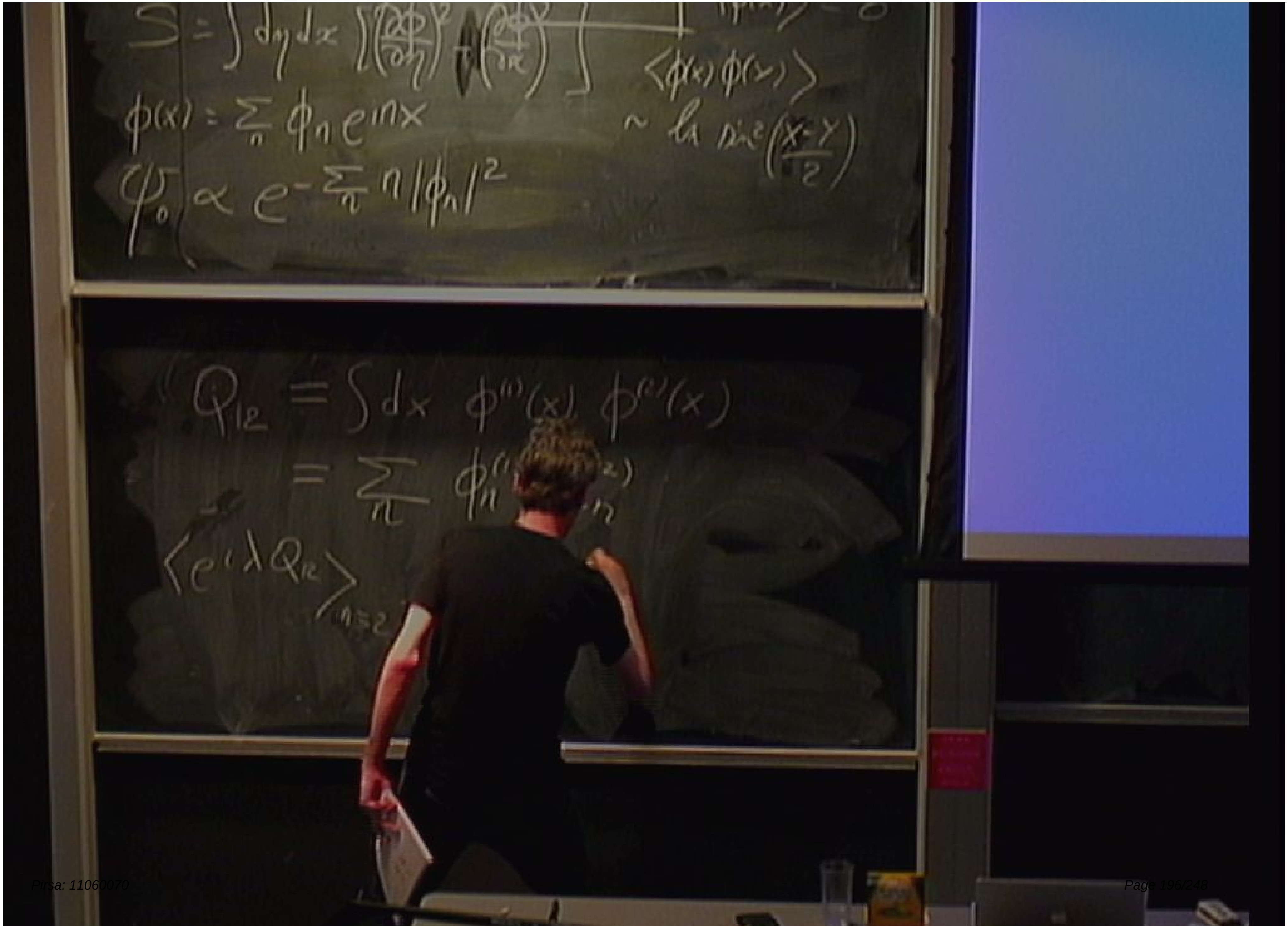



$S = \int d\eta\, dx \left[\left(\frac{\partial\phi}{\partial\eta}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$
$\langle \phi(x)\phi(y) \rangle$
$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx\, \phi^{(1)}(x)\, \phi^{(2)}(x)$
$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$
$\langle Q_{12} \rangle$



φ(x) = Σ_n φ_n e^{inx}
⟨φ(x) φ(y)⟩





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⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
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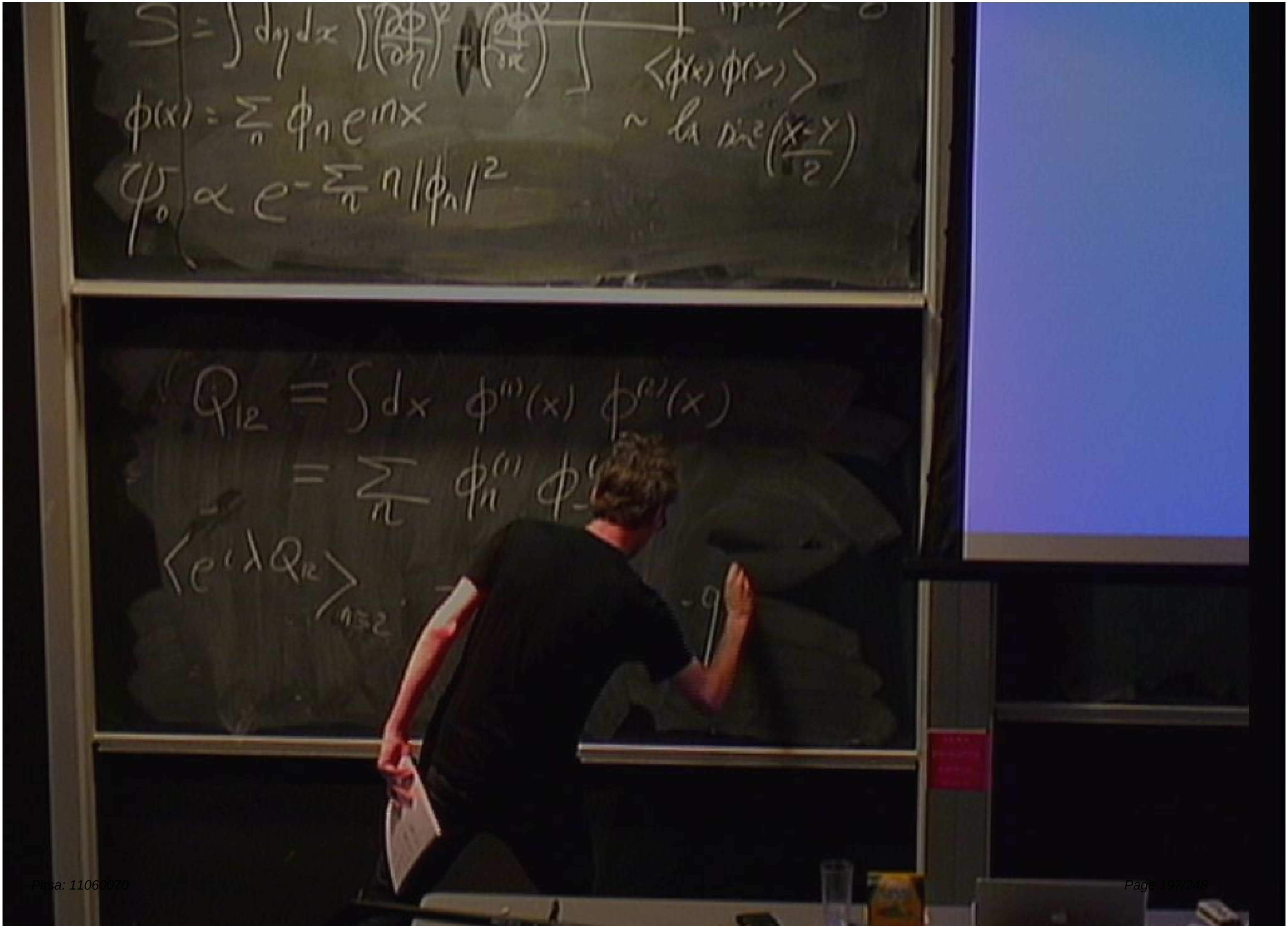
ψ_0 ∝ e^{−Σ_n n|φ_n|²}
Q_12 = ∫ dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{−n}^(2)
⟨e^{iλQ_12}⟩_{n≥2}


φ(x) = Σ_n φ_n e^{inx}
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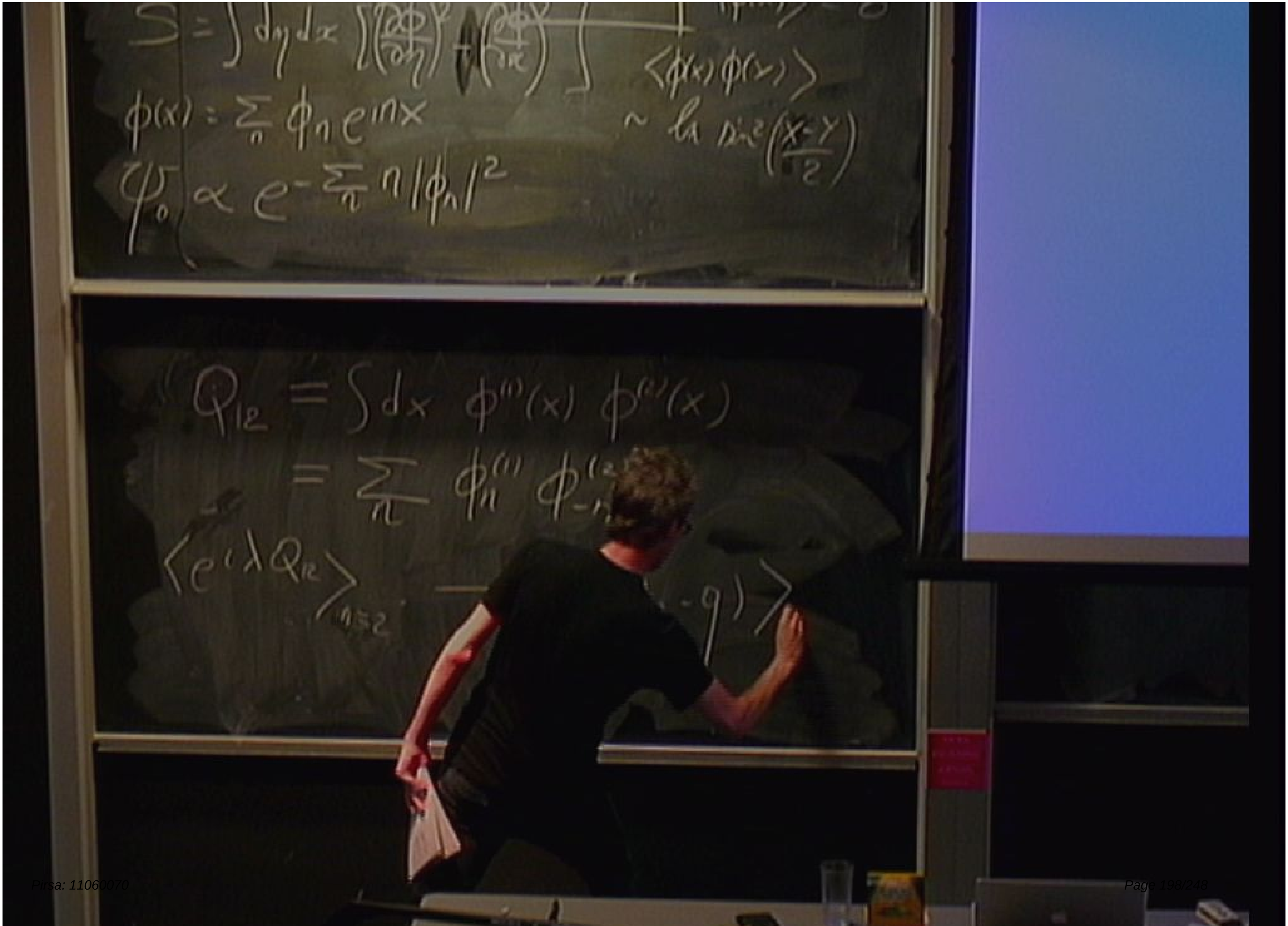
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
⟨e^{iλQ_12}⟩


φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
⟨φ(x) φ(y)⟩
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)

$$S = \int d\eta\, dx \left[\left(\frac{\partial\phi}{\partial\eta}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$$

$$\langle \phi(x)\phi(y) \rangle \sim \ln \sin^2\left(\frac{x-y}{2}\right)$$

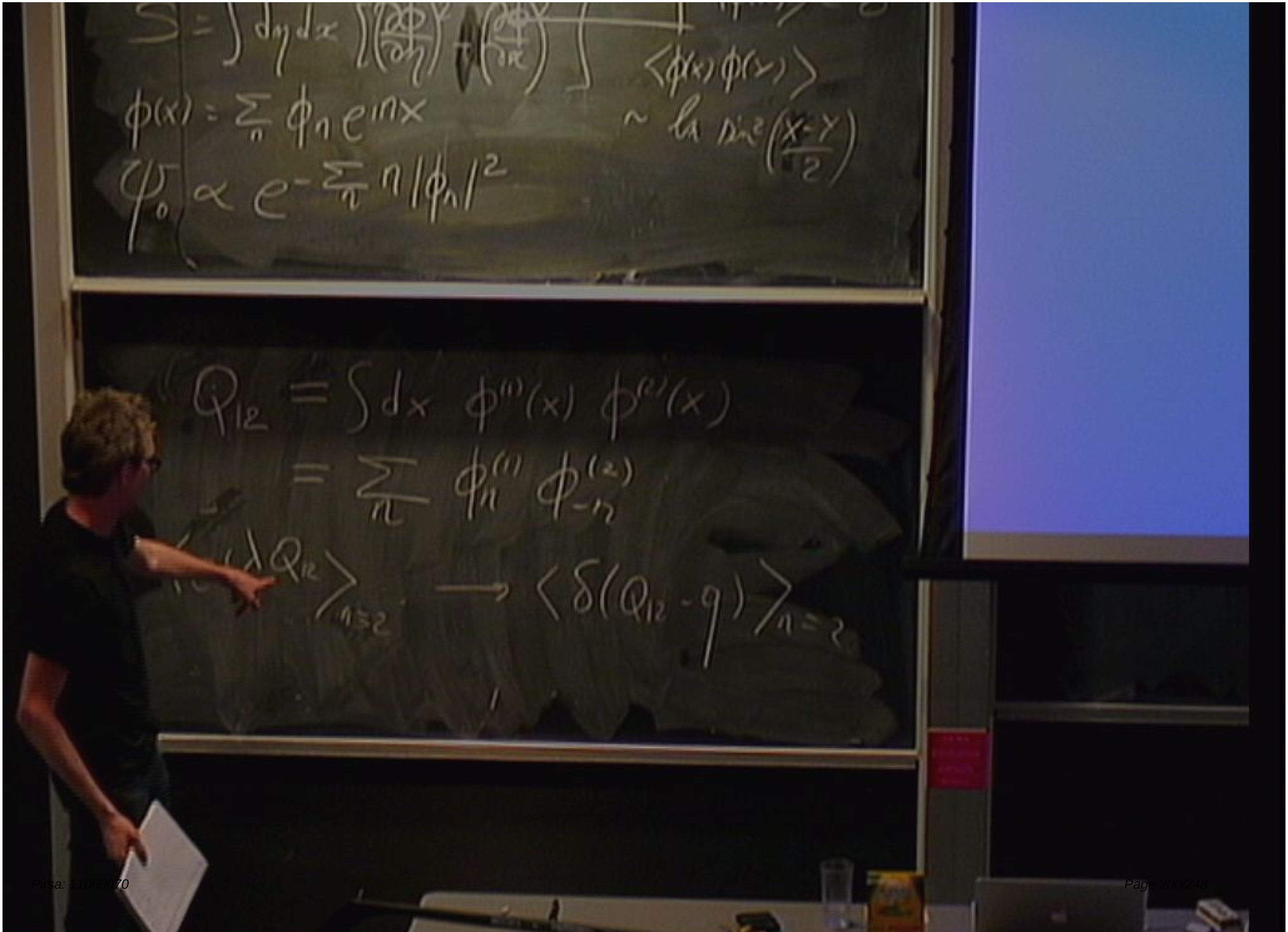
$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$$

$$Q_{12} = \int dx\, \phi^{(1)}(x)\, \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$


φ(x) = Σ_n φ_n e^{inx}
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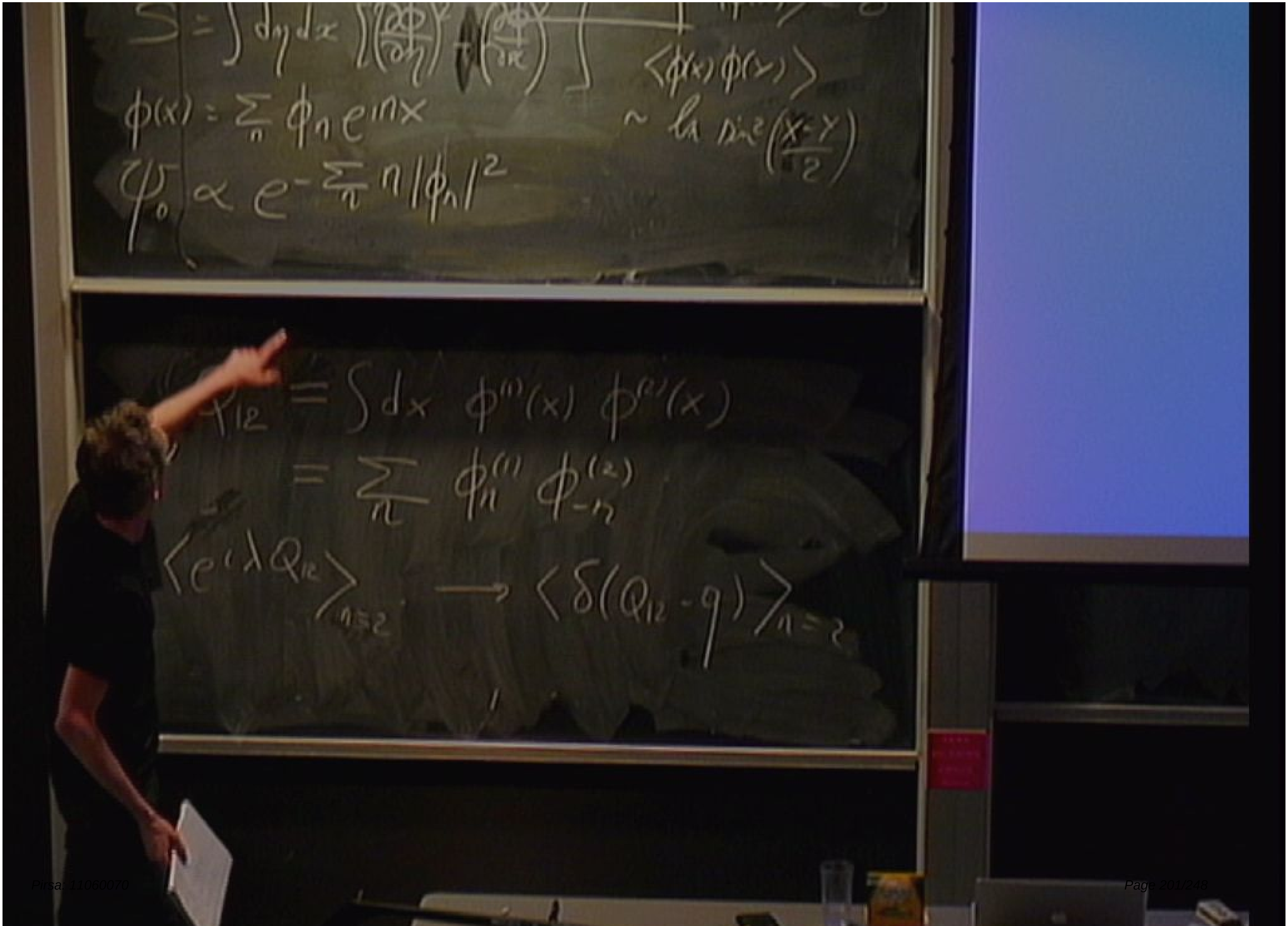
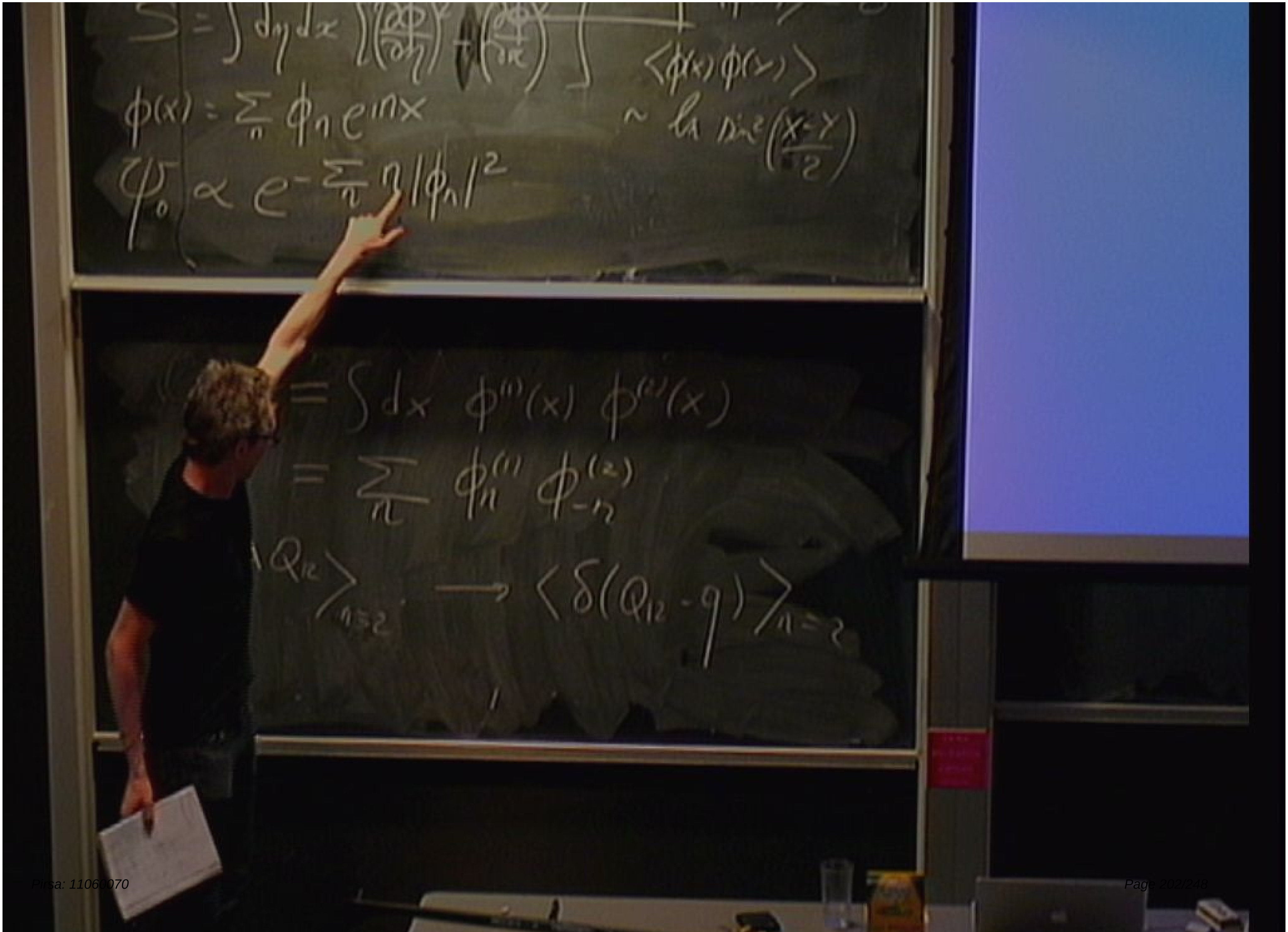
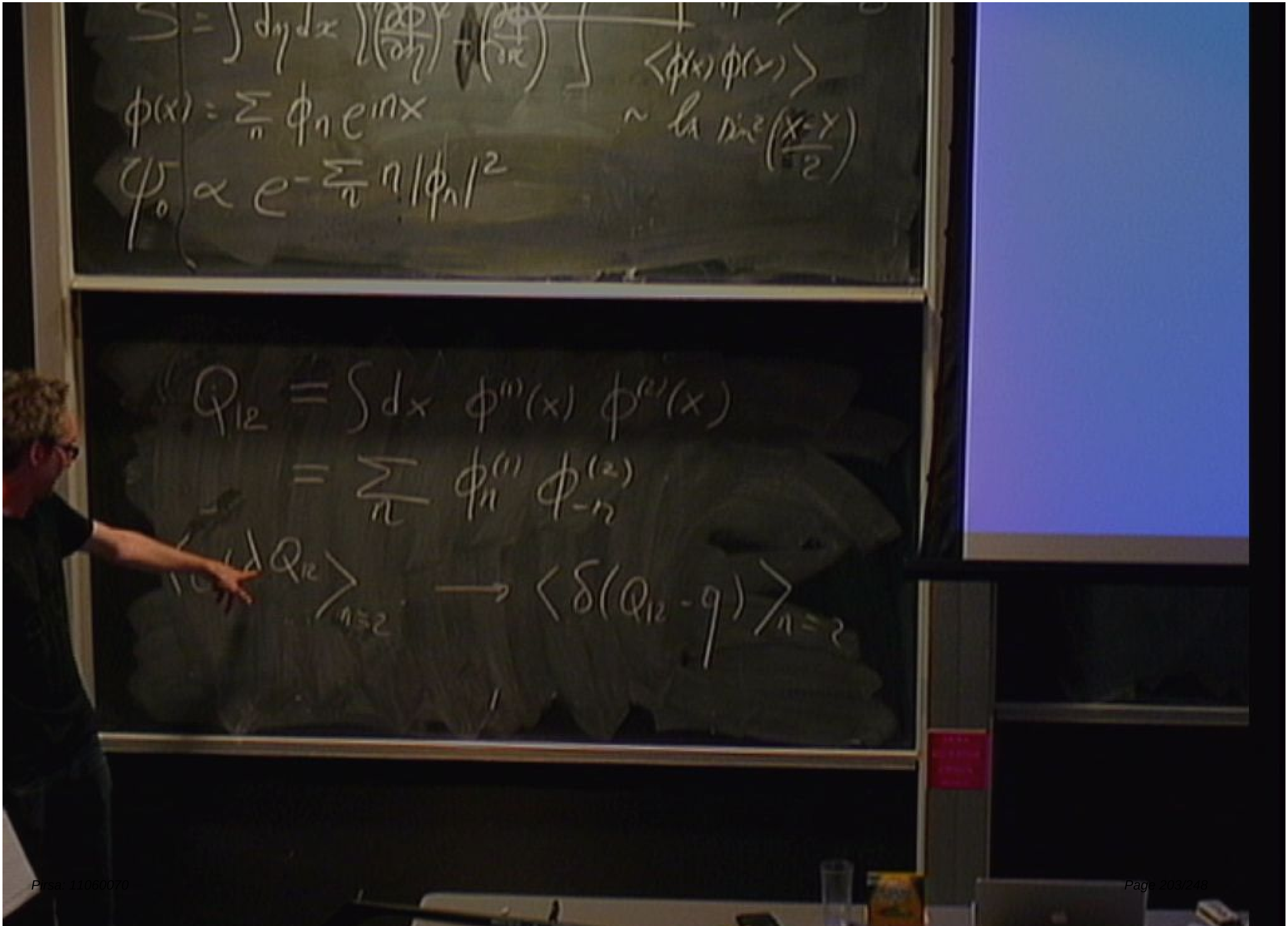
~ ln sin²((x-y)/2)
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{-n}^(2)
⟶ ⟨δ(Q_12 - q)⟩_{n=2}


⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
= ∫dx φ^{(1)}(x) φ^{(2)}(x)
= Σ_n φ_n^{(1)} φ_{-n}^{(2)}
⟨e^{iλQ_{12}}⟩_{n=2} ⟶ ⟨δ(Q_{12} - q)⟩_{n=2}


φ(x) = Σn φn e^inx
= ∫dx φ(1)(x) φ(2)(x)
⟨φ(x) φ(y)⟩


φ(x) = Σ_n φ_n e^{inx}
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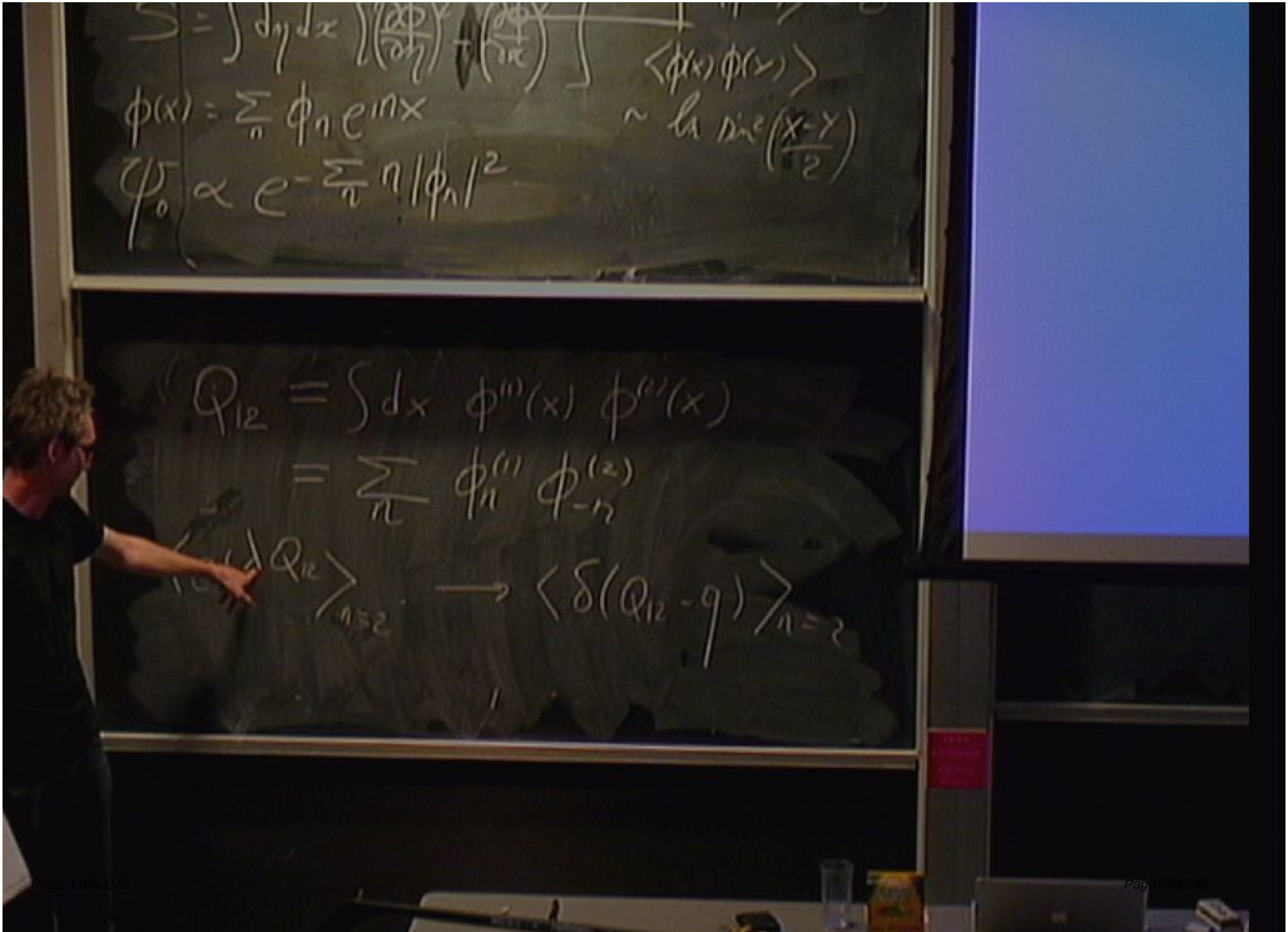
⟶ ⟨δ(Q_12 - q)⟩_{n=2}


⟨φ(x) φ(y)⟩
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Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
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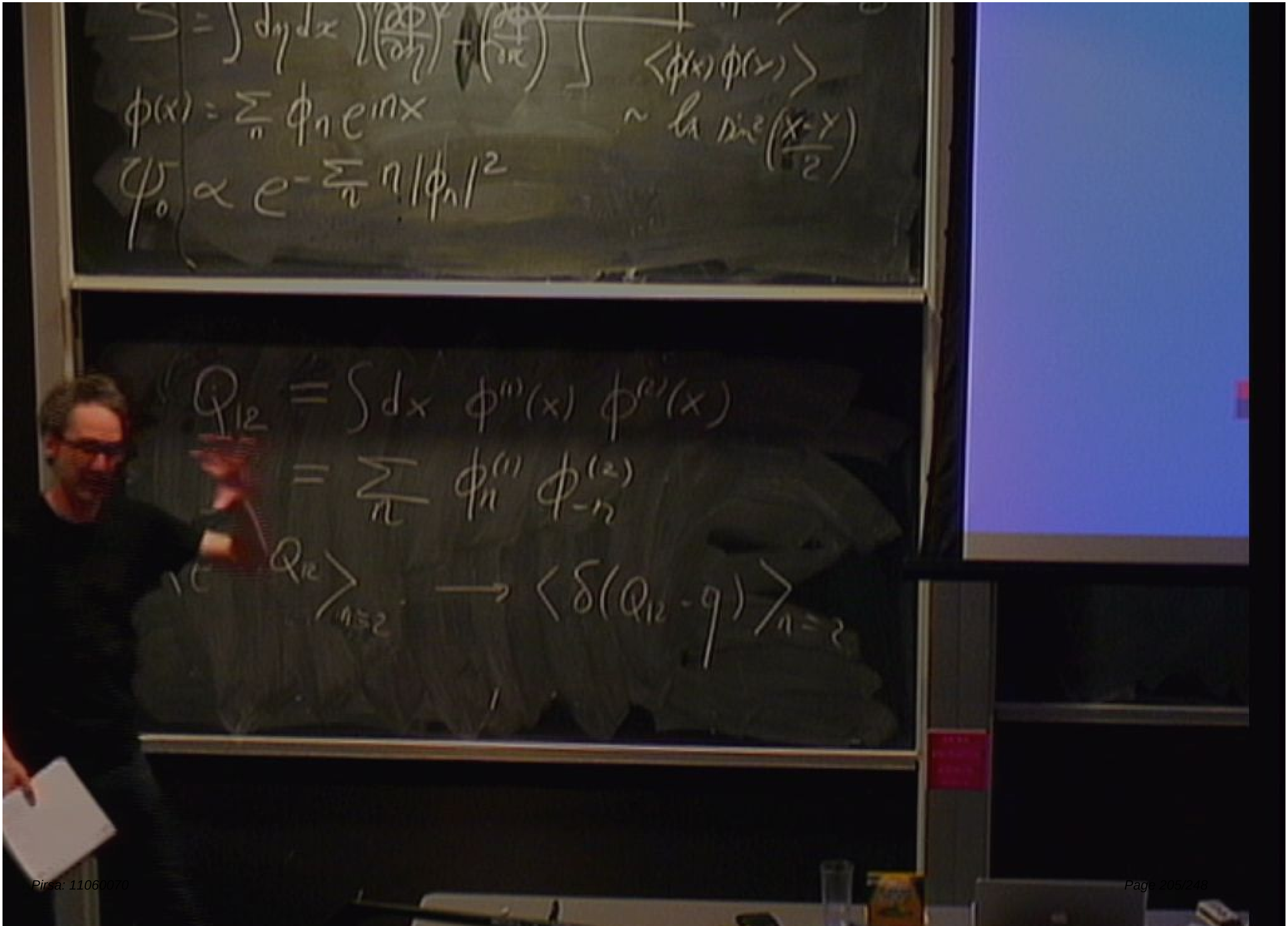
⟨δ(Q_12 − q)⟩_{n=2}



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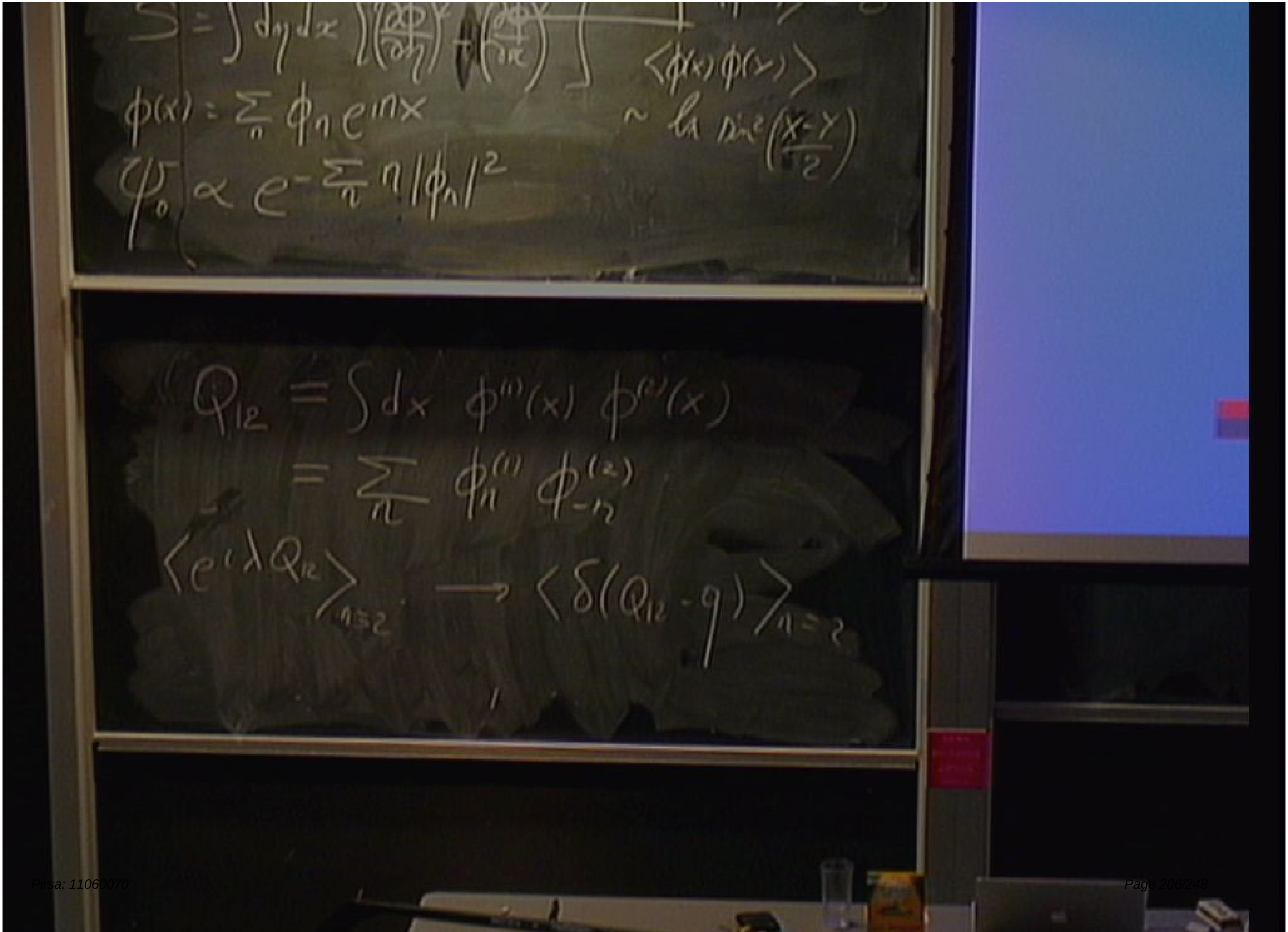
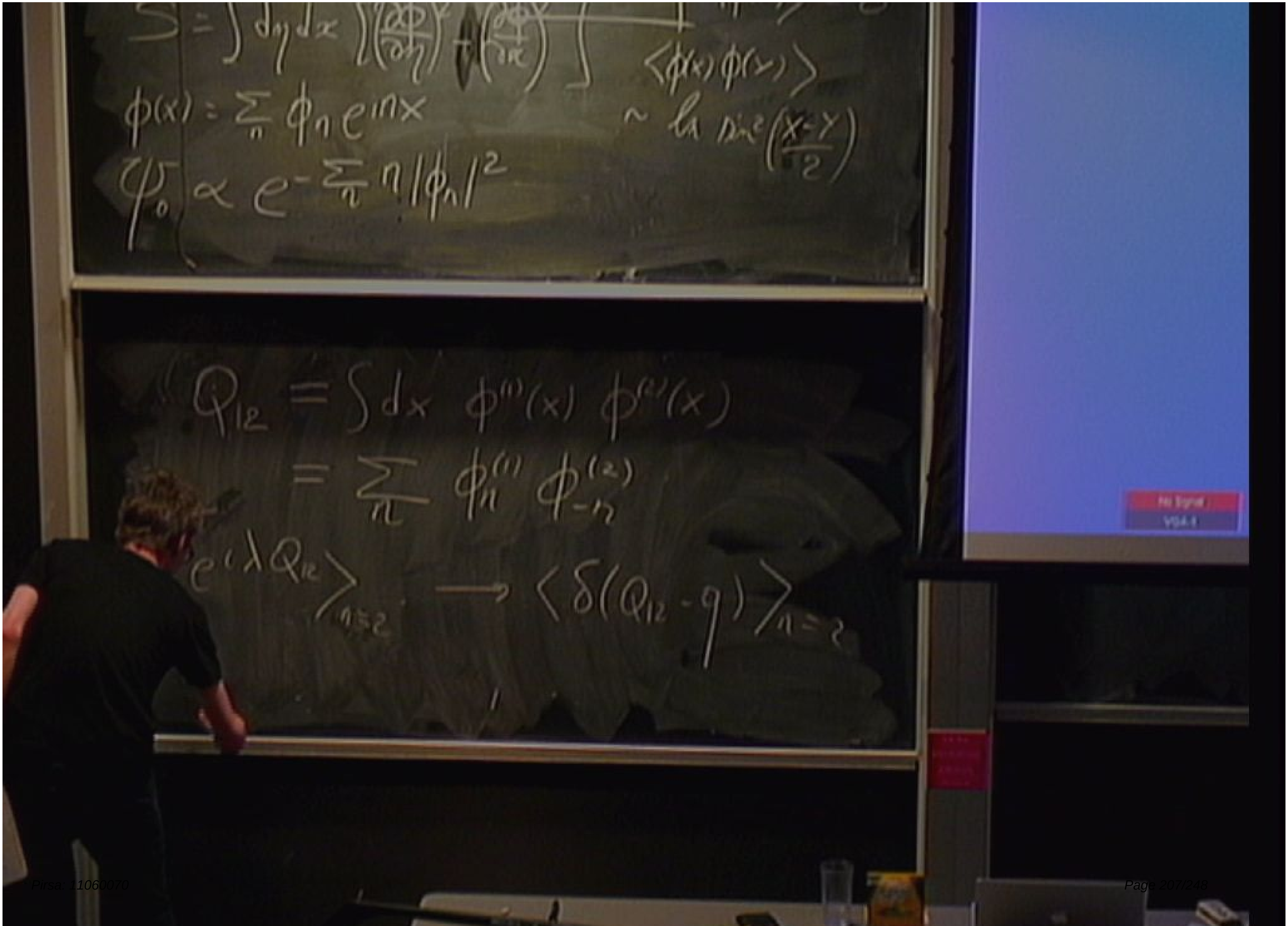
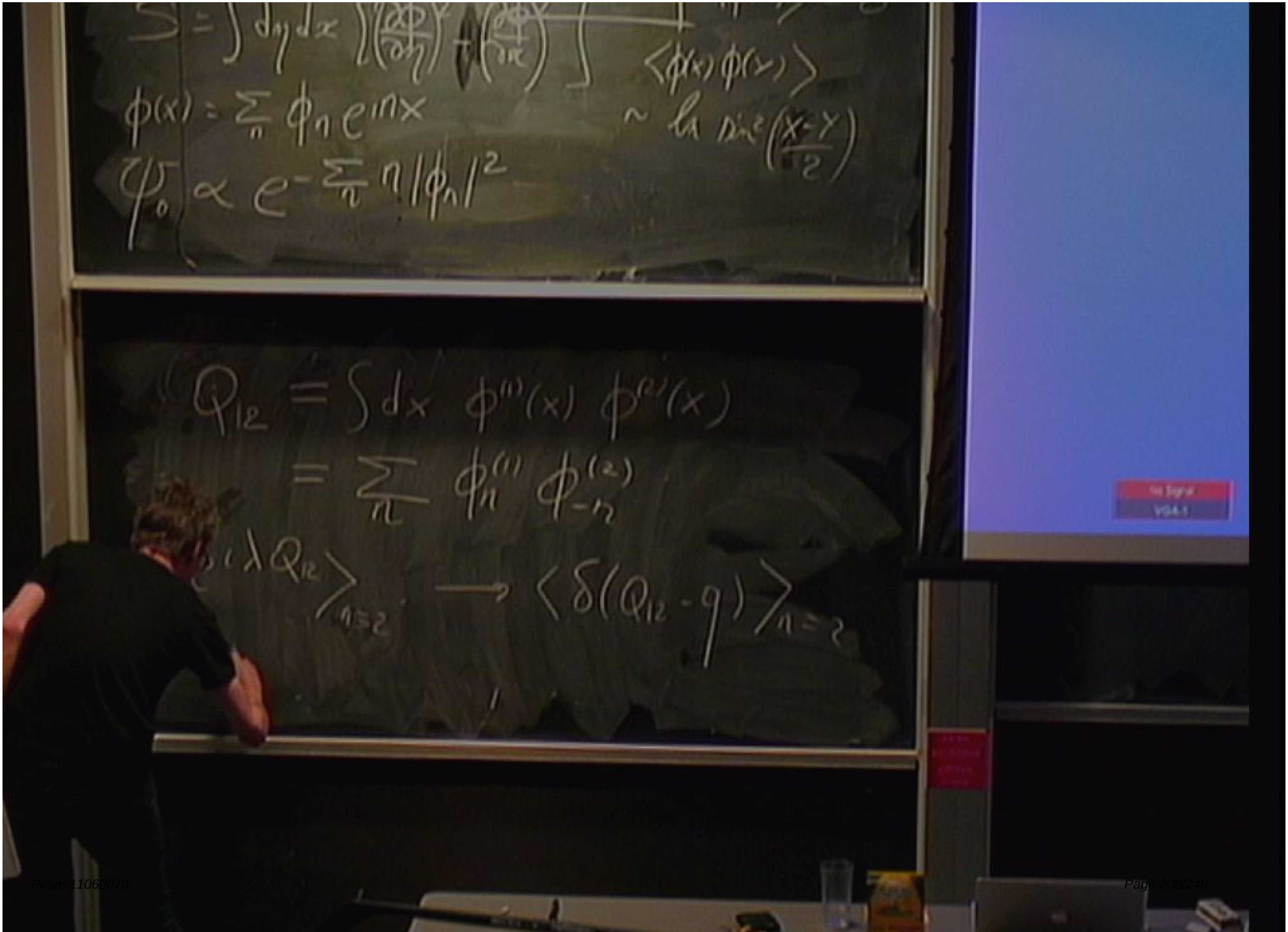
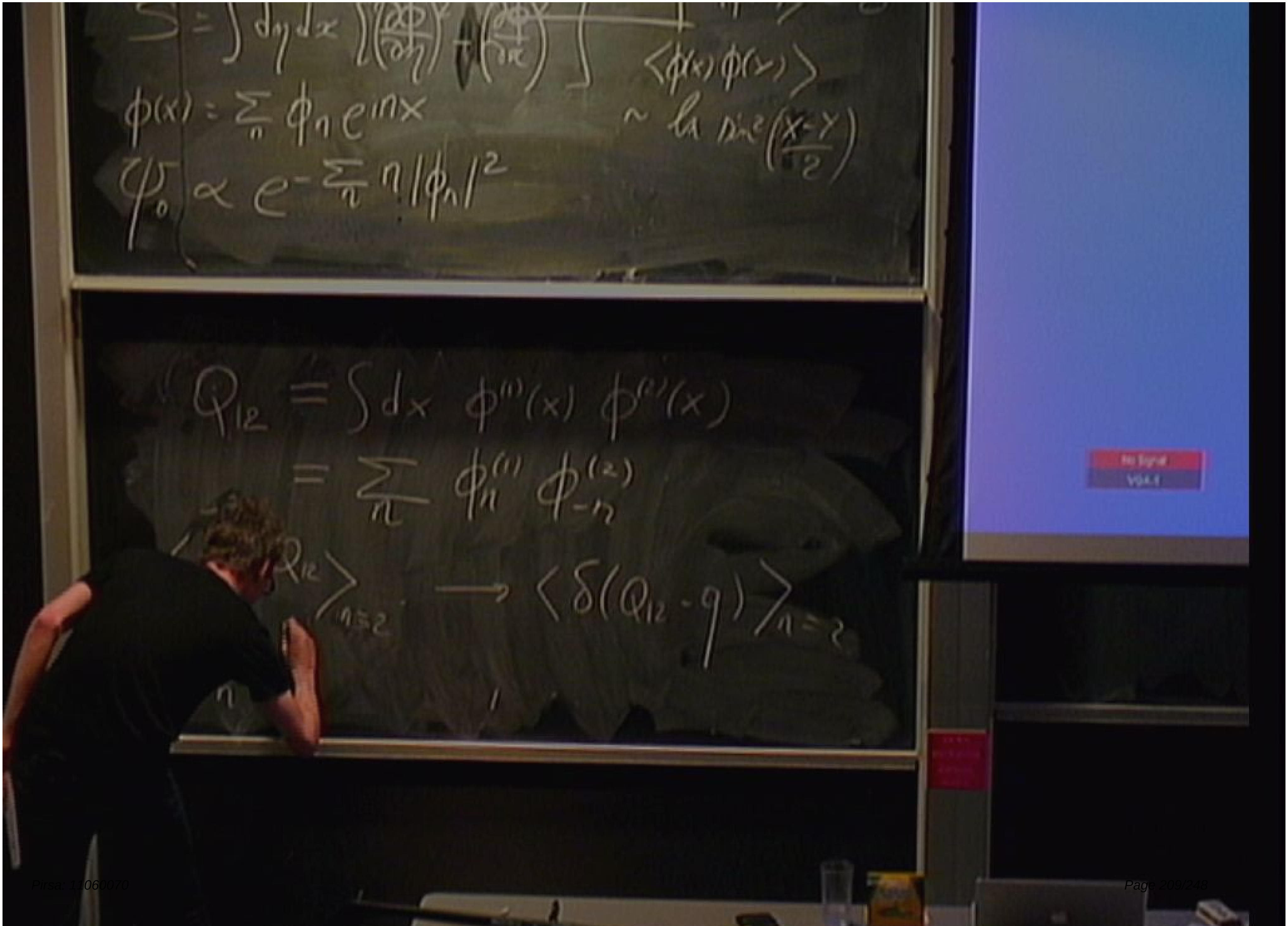
⟨δ(Q_12 - q)⟩_{n=2}

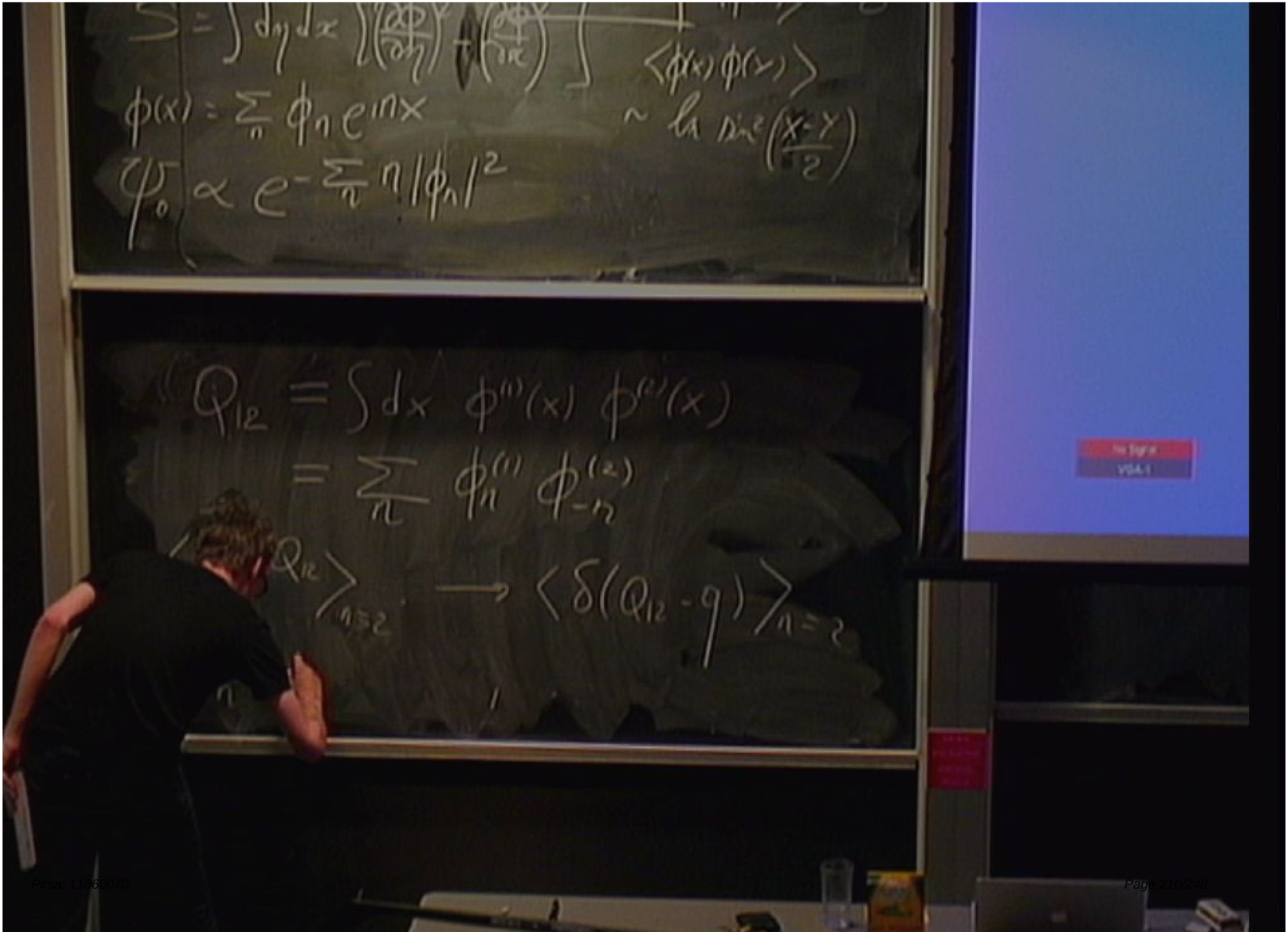


$\langle \phi(x)\phi(y) \rangle$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx\, \phi^{(1)}(x)\, \phi^{(2)}(x)$
$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$
$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$


φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
⟨φ(x) φ(y)⟩ ~ ln sin^2((x-y)/2)
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{-n}^(2)
⟨e^{iλQ_12}⟩_{n=2} → ⟨δ(Q_12 - q)⟩_{n=2}


⟨φ(x) φ(y)⟩
φ(x) = Σn φn e^{inx}
Ψ0 ∝ e^{-Σn n|φn|^2}
Q12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σn φn^(1) φ-n^(2)
→ ⟨δ(Q12 - q)⟩n=2


$\langle \phi(x)\phi(y) \rangle$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx \; \phi^{(1)}(x) \; \phi^{(2)}(x)$
$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$
$\longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$
No Signal
VGA 1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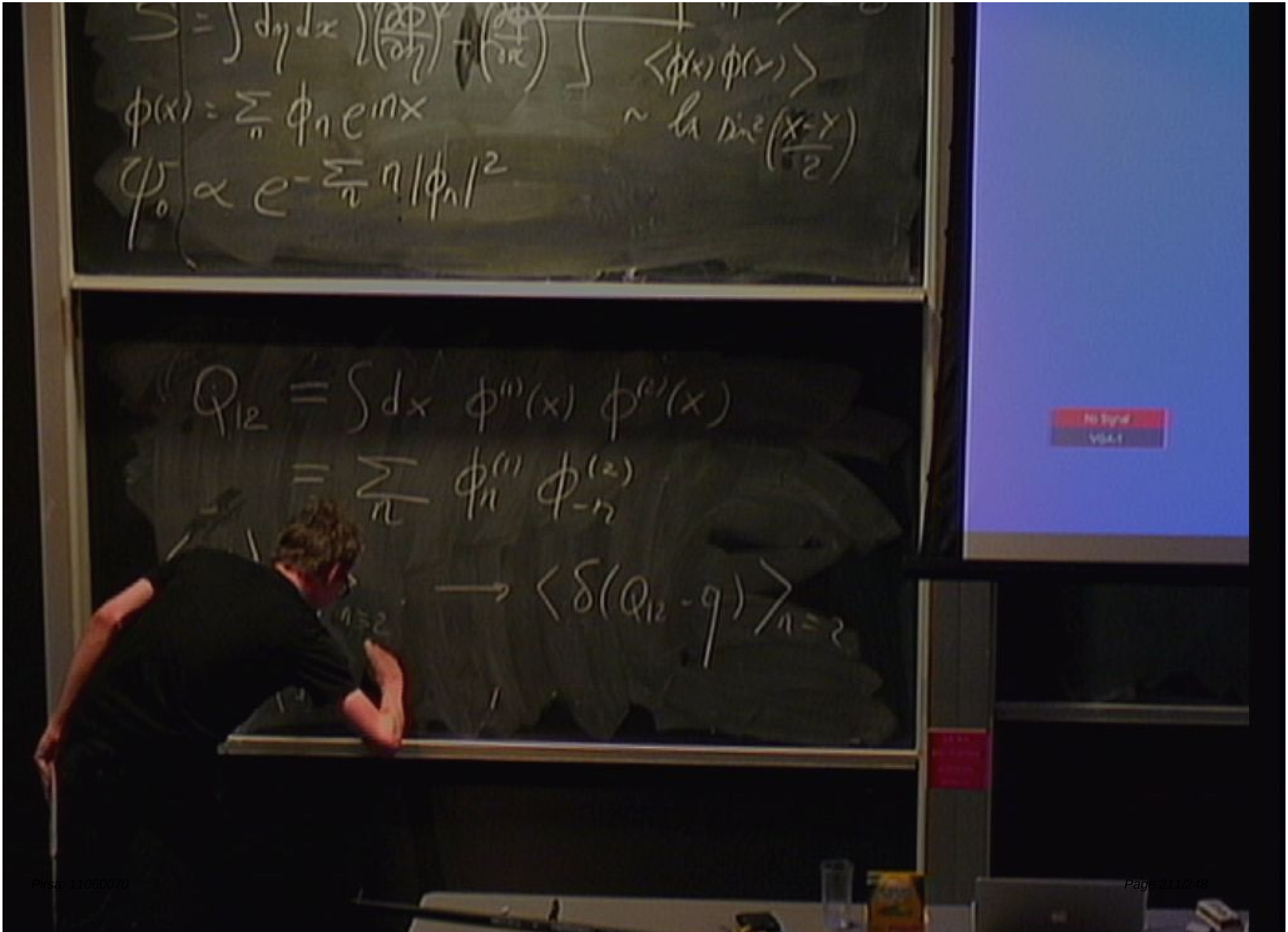
Ψ_0 ∝ e^{-Σ_n n|φ_n|^2}
⟨φ(x) φ(y)⟩
~ ln sin²((x-y)/2)
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{-n}^(2)
⟨Q_12⟩_{n=2} → ⟨δ(Q_12 - q)⟩_{n=2}


⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
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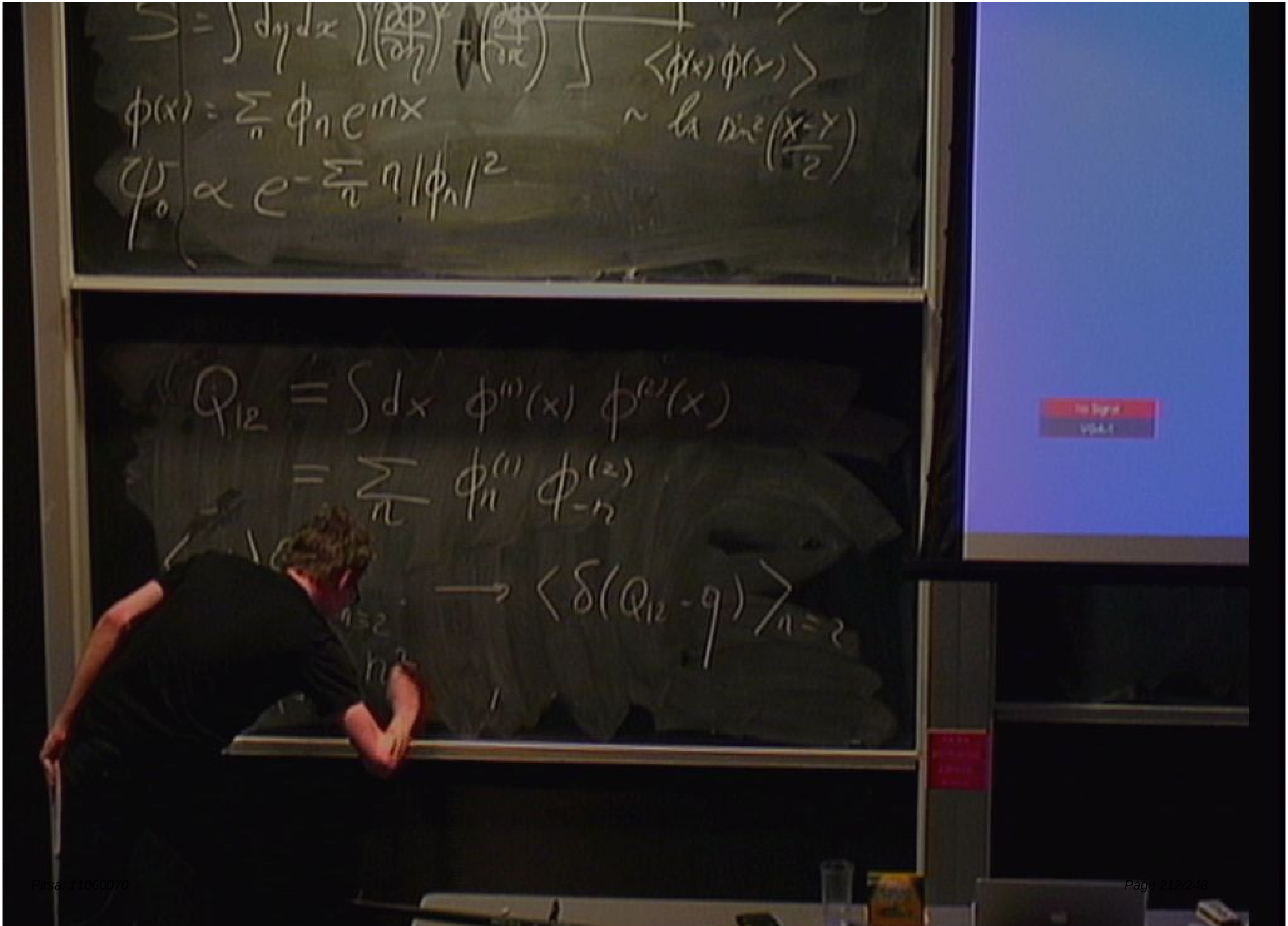
= Σ_n φ_n^(1) φ_{−n}^(2)
→ ⟨δ(Q_12 − q)⟩_{n=2}


$\phi(x) = \sum_n \phi_n e^{inx}$
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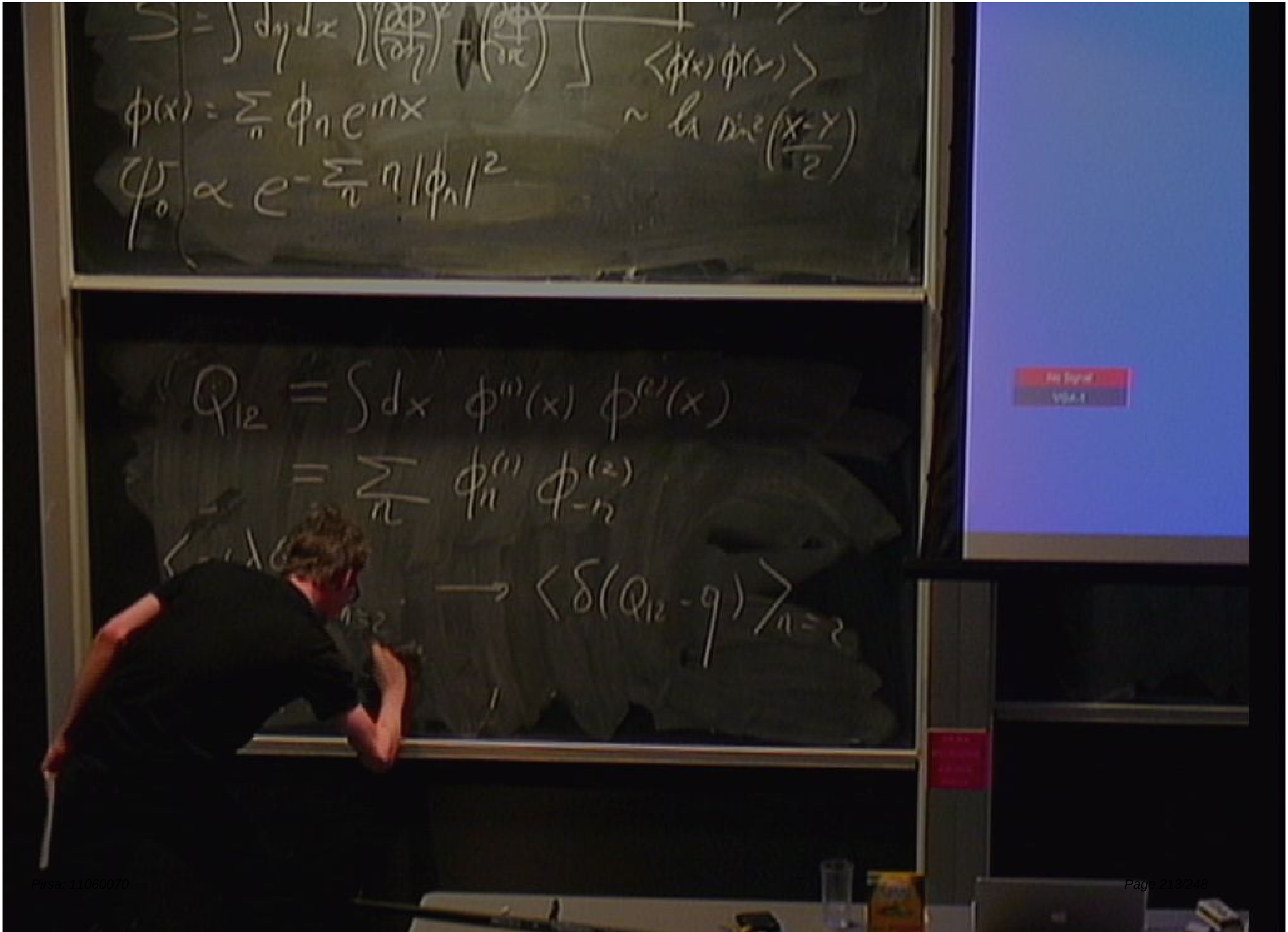
$\langle \phi(x)\phi(y) \rangle$
$Q_{12} = \int dx\ \phi^{(1)}(x)\ \phi^{(2)}(x)$
$= \sum_n \phi^{(1)}_n \phi^{(2)}_{-n}$
$\longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$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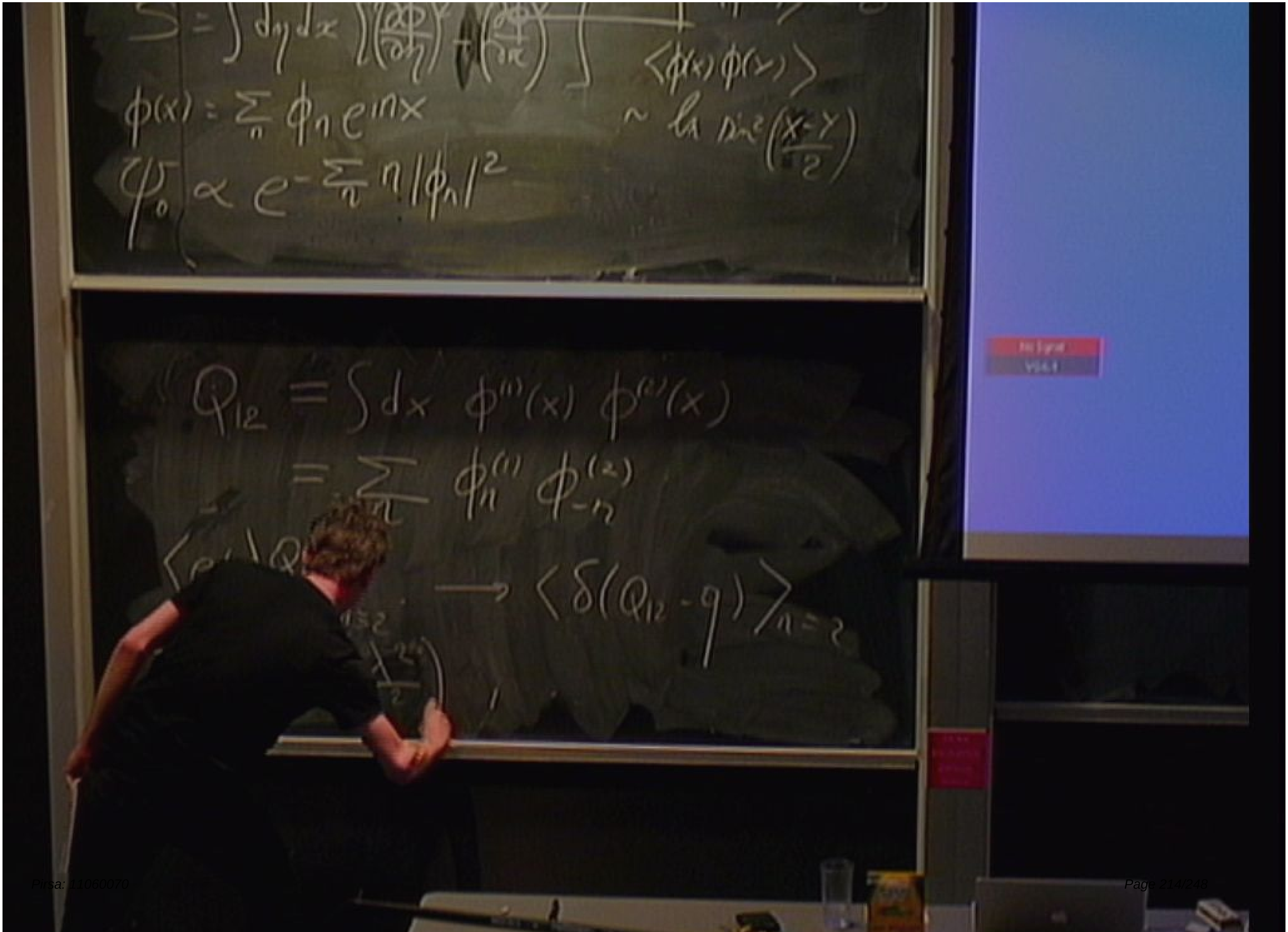
→ ⟨δ(Q_12 − q)⟩_{n=2}
No Signal
VGA 1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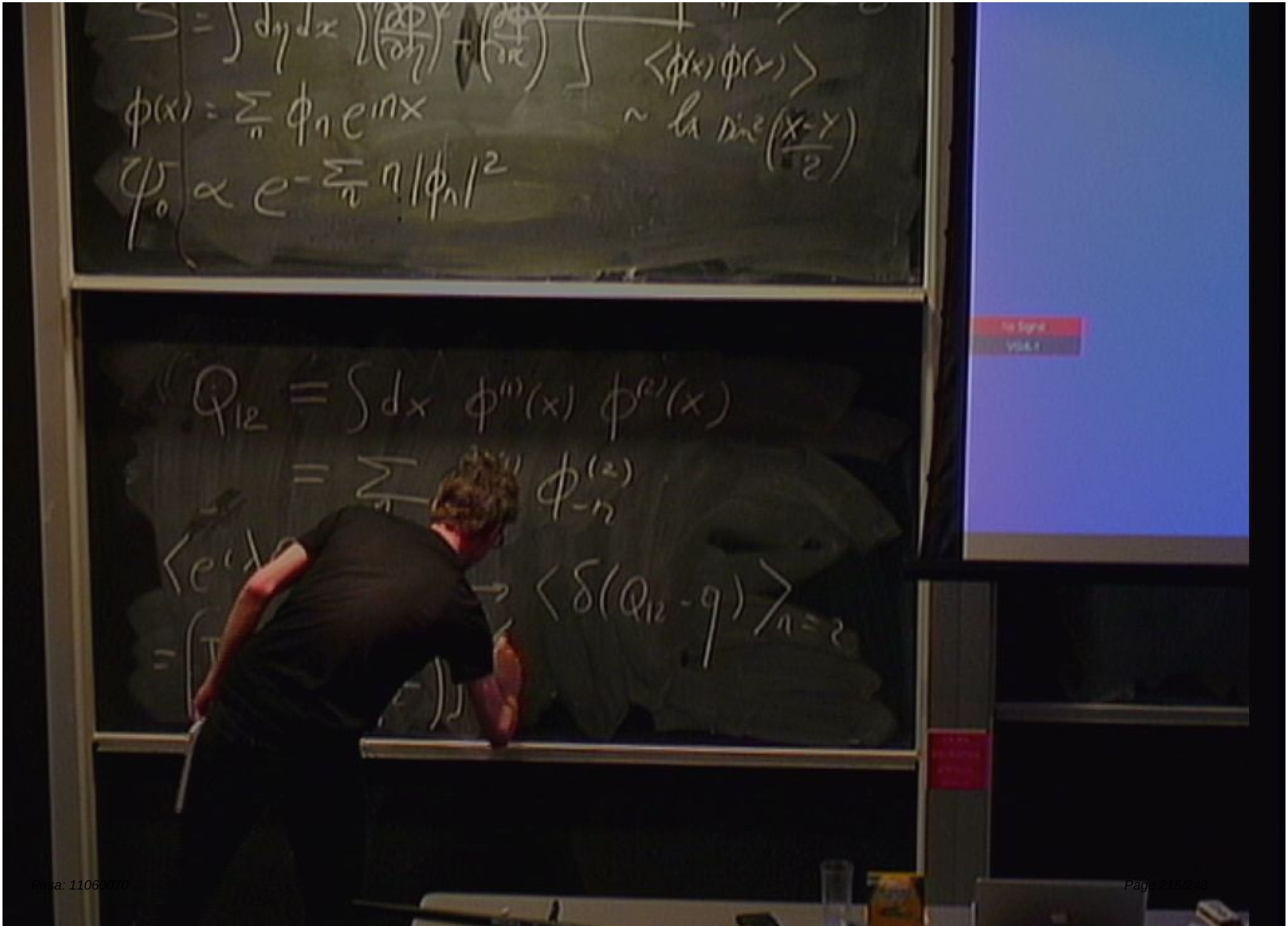
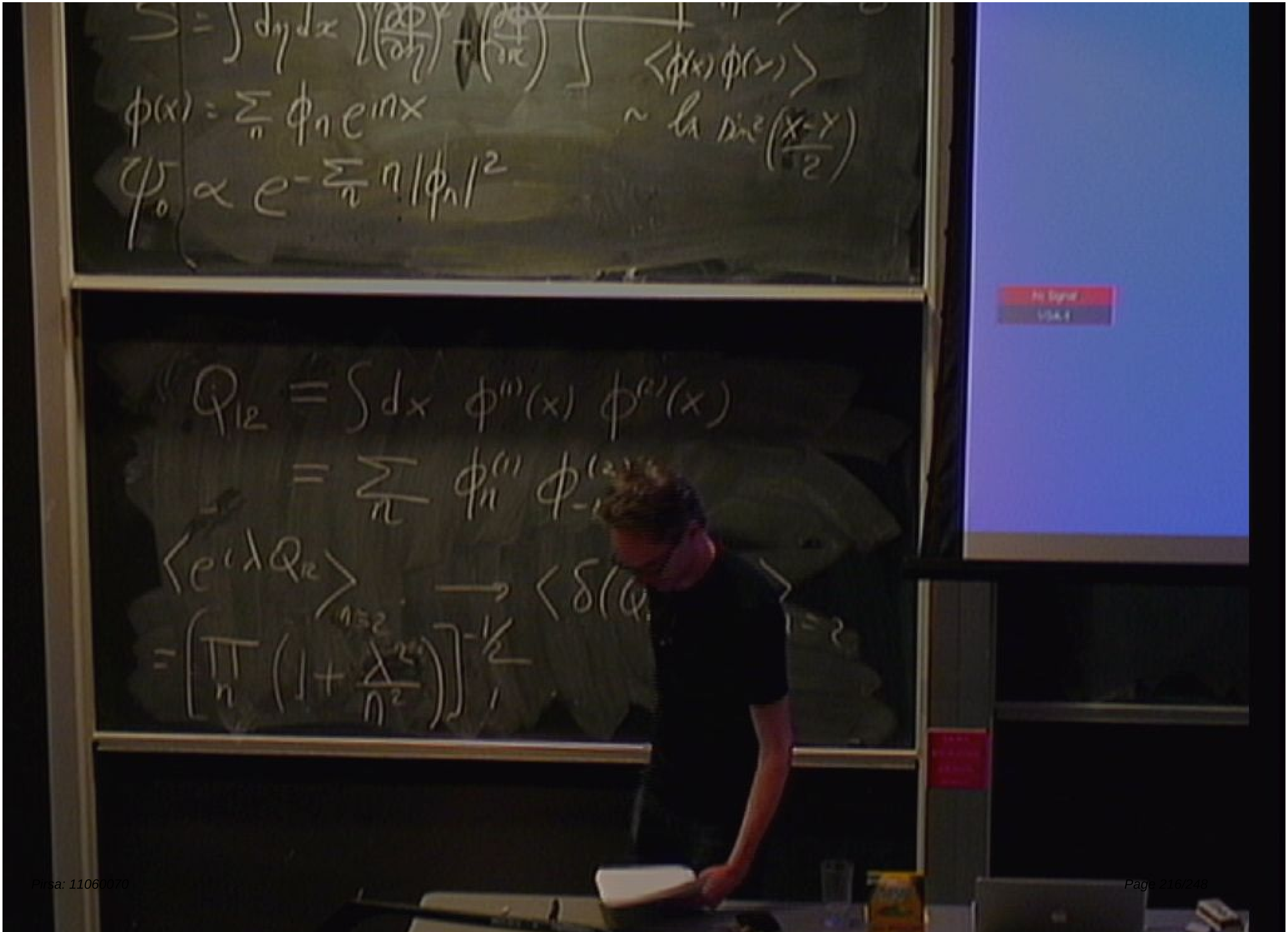
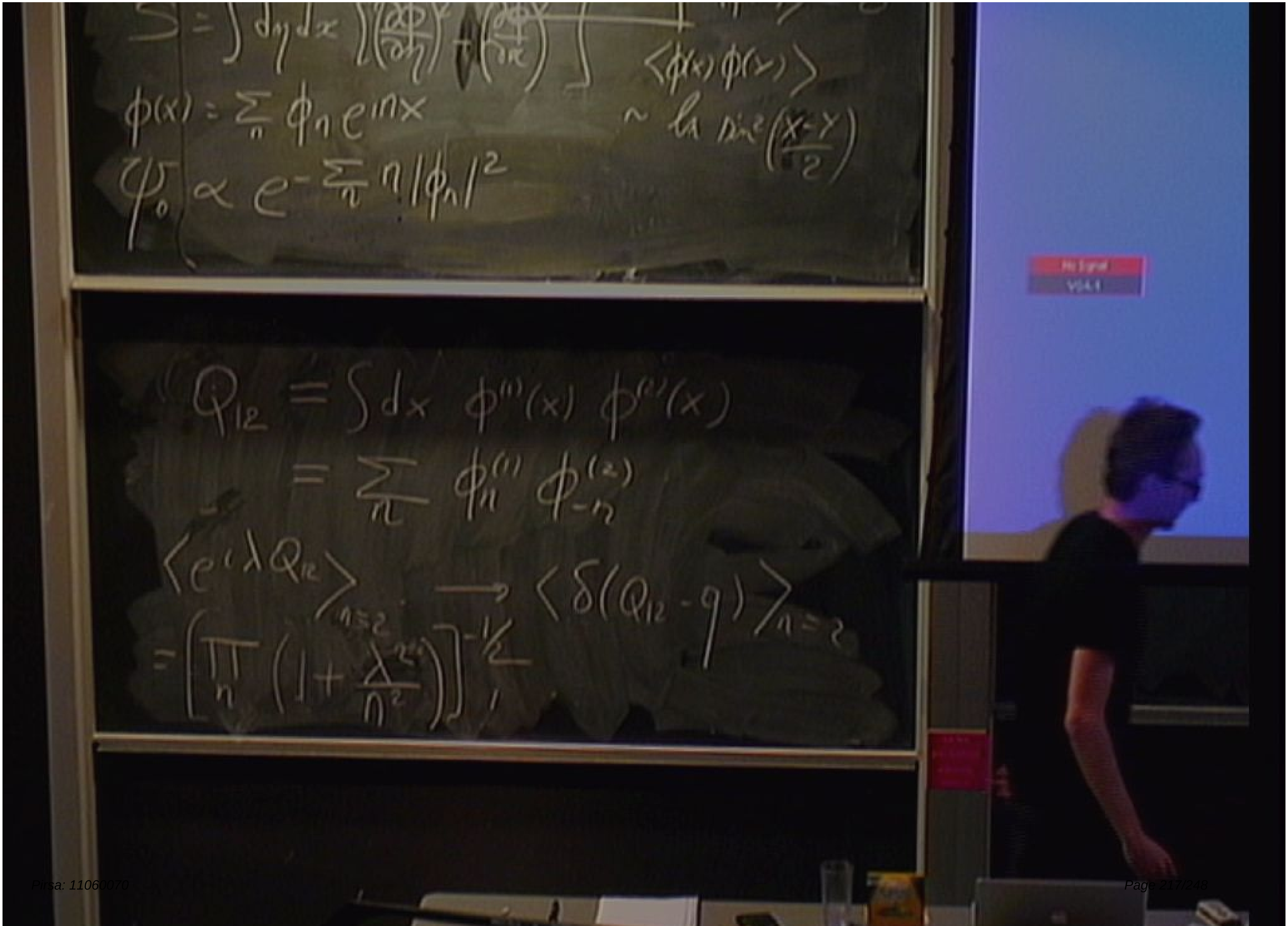
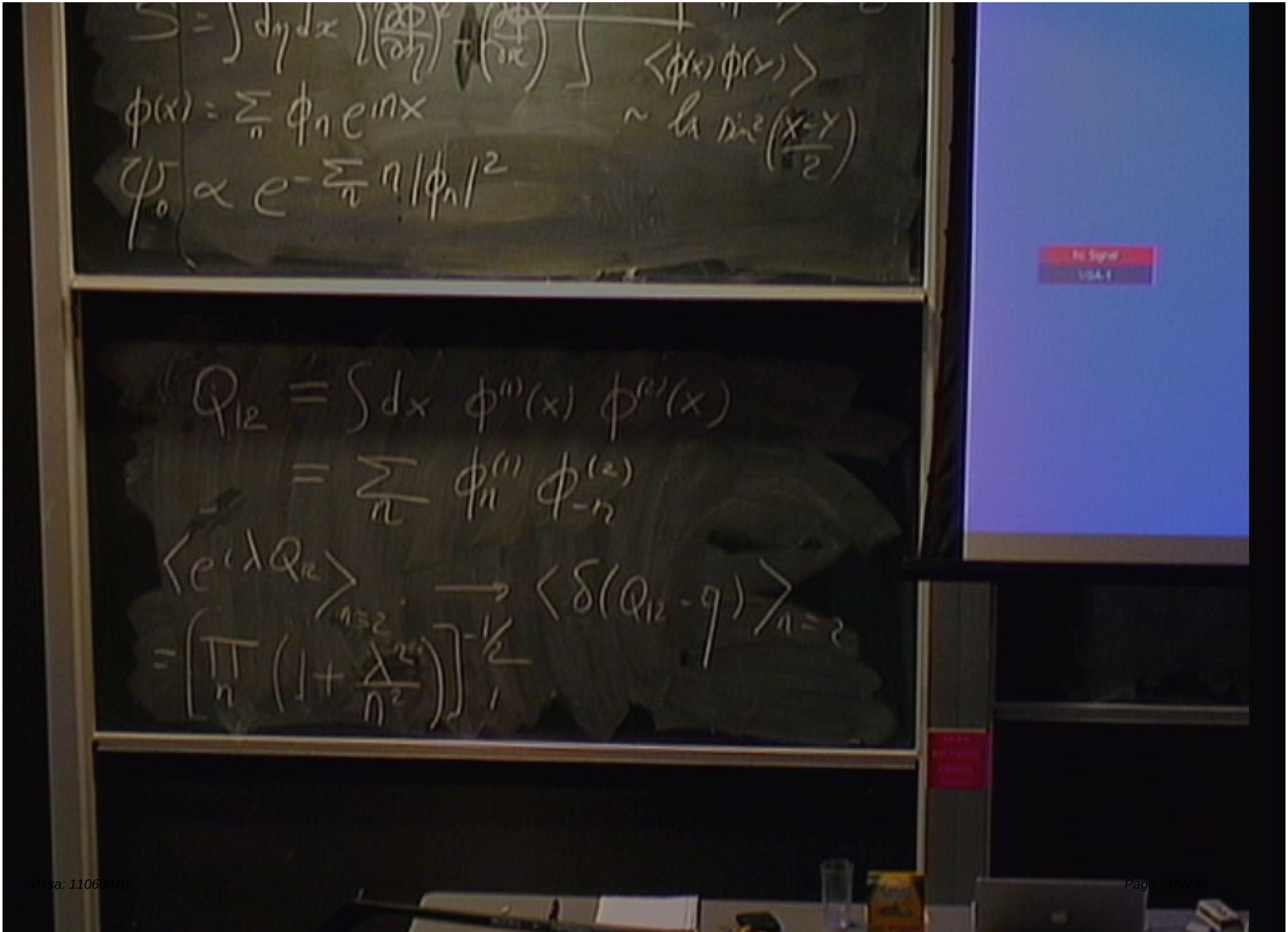
→ ⟨δ(Q_12 - q)⟩_{n=2}

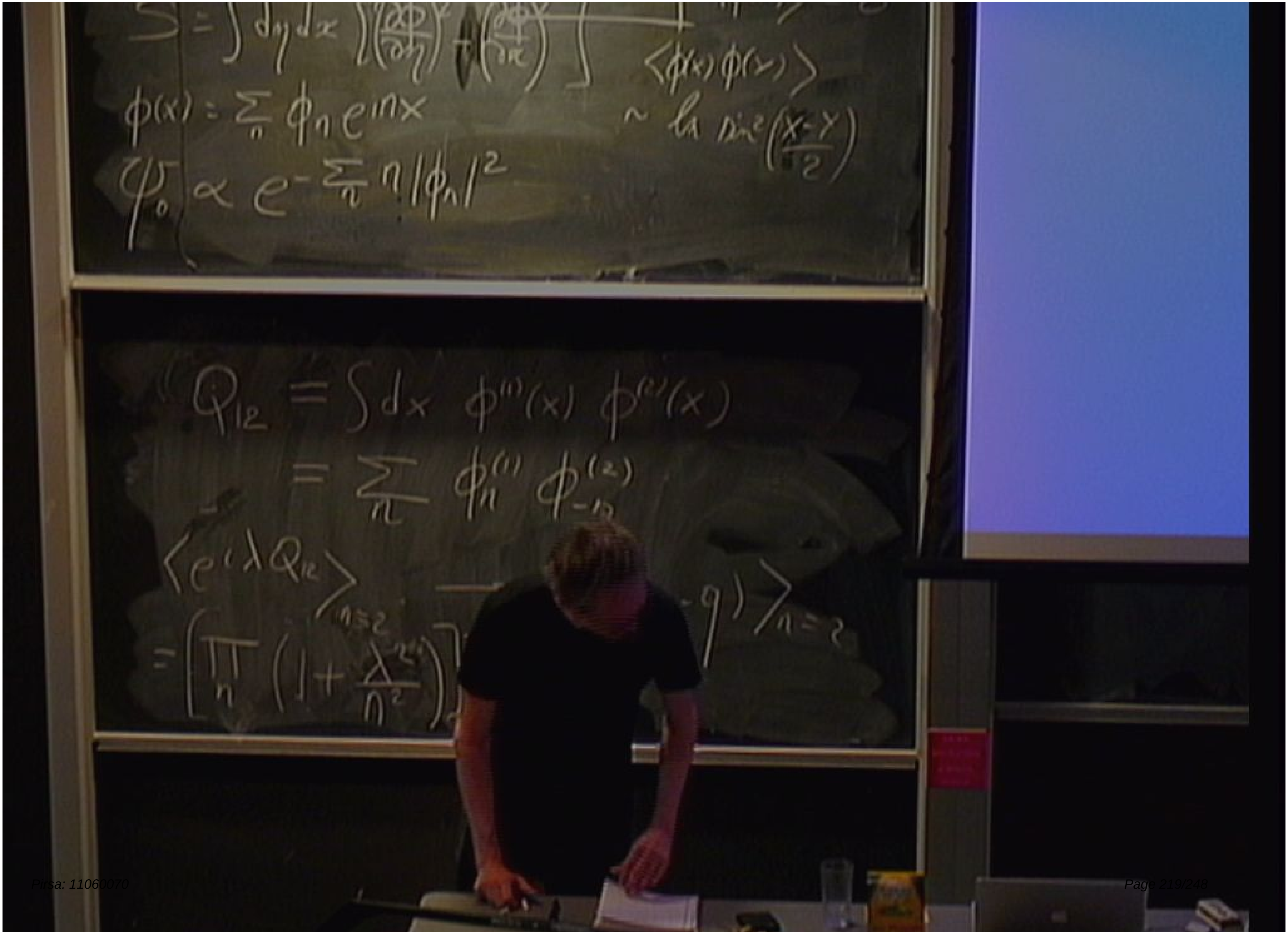

$\langle \phi(x)\phi(y) \rangle$
$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx\ \phi^{(1)}(x)\ \phi^{(2)}(x)$
$\langle \delta(Q_{12} - q) \rangle_{n=2}$

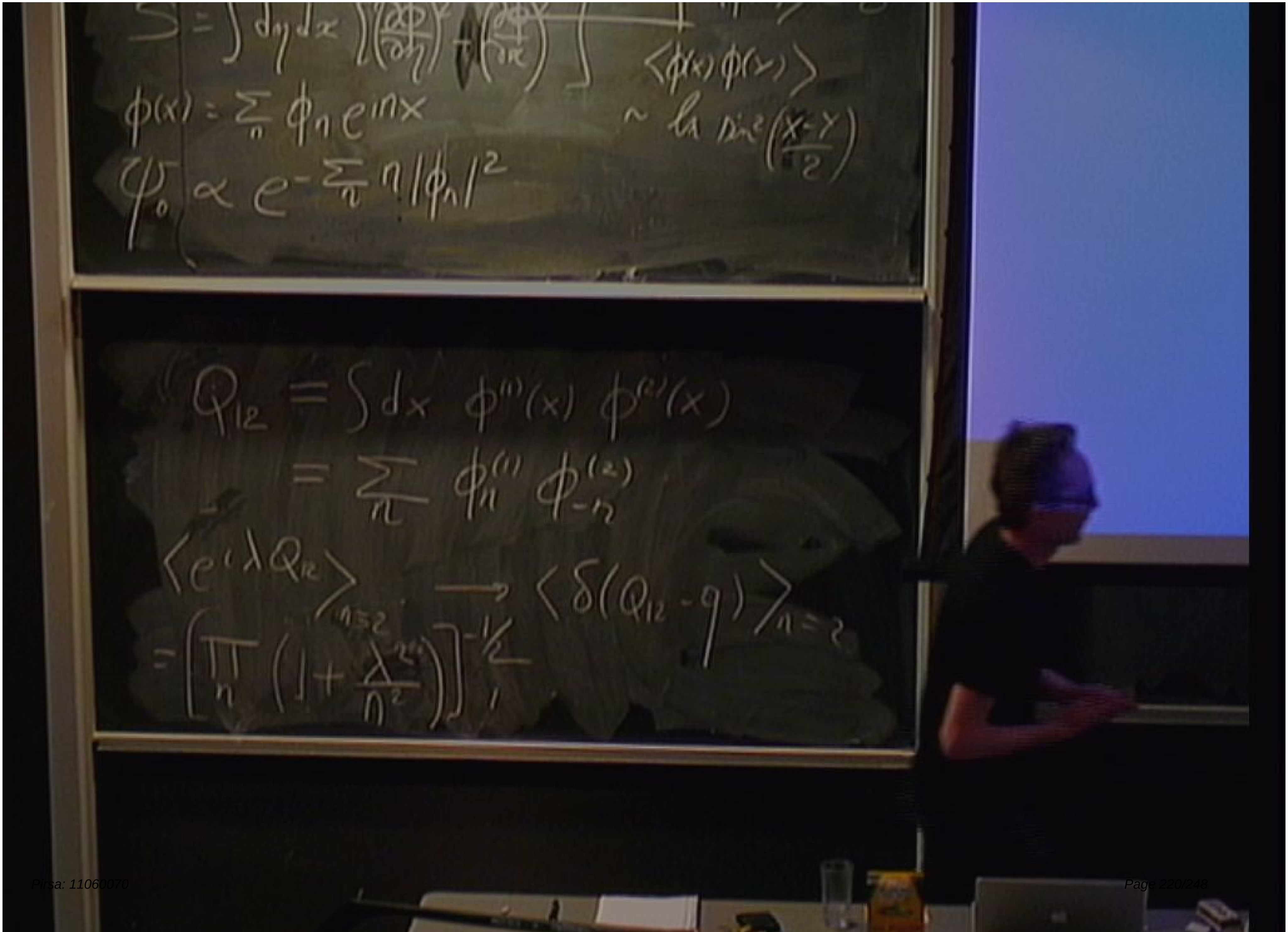

S = ∫ dη dx [(∂φ/∂η)² + (∂φ/∂x)²]
⟨φ(x) φ(y)⟩
φ(x) = Σ_n φ_n e^{inx}
~ ln sin²((x−y)/2)
Ψ_0 ∝ e^{−Σ_n n|φ_n|²}
Q_12 = ∫ dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{−n}^(2)
⟨e^{iλQ_12}⟩_{n=2} → ⟨δ(Q_12)⟩_{n=2}
= [Π_n (1 + λ²/n²)]^{−1/2}



⟨φ(x) φ(y)⟩ ~ ln sin²((x−y)/2)
φ(x) = Σ_n φ_n e^{inx}
Ψ_0 ∝ e^{−Σ_n n|φ_n|²}
Q_12 = ∫dx φ^(1)(x) φ^(2)(x)
= Σ_n φ_n^(1) φ_{−n}^(2)
⟨e^{iλQ_12}⟩_{n=2} → ⟨δ(Q_12 − q)⟩_{n=2}
= [Π_n (1 + λ²/n²)]^{−1/2}


$S = \int d\eta\, dx \left[\left(\frac{\partial\phi}{\partial\eta}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$
$\phi(x) = \sum_n \phi_n e^{inx}$
$\Psi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$\langle \phi(x)\phi(y) \rangle \sim \ln \sin^2\left(\frac{x-y}{2}\right)$
$Q_{12} = \int dx\, \phi^{(1)}(x)\, \phi^{(2)}(x) = \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$
$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$
$= \left[\prod_n \left(1 + \frac{\lambda^2}{n^2}\right)\right]^{-1/2}$


φ(x) = Σn φn e^inx
Ψ0 ∝ e^-Σn n|φn|²
⟨φ(x) φ(y)⟩
~ ln sin²((x-y)/2)
Q12 = ∫dx φ(1)(x) φ(2)(x)
= Σn φn(1) φ-n(2)


$S = \int d\eta\, dx \left[\left(\frac{\partial\phi}{\partial\eta}\right)^2 + \left(\frac{\partial\phi}{\partial x}\right)^2 \right]$
$\langle \phi(x)\phi(y) \rangle$
$\phi(x) = \sum_n \phi_n e^{inx}$
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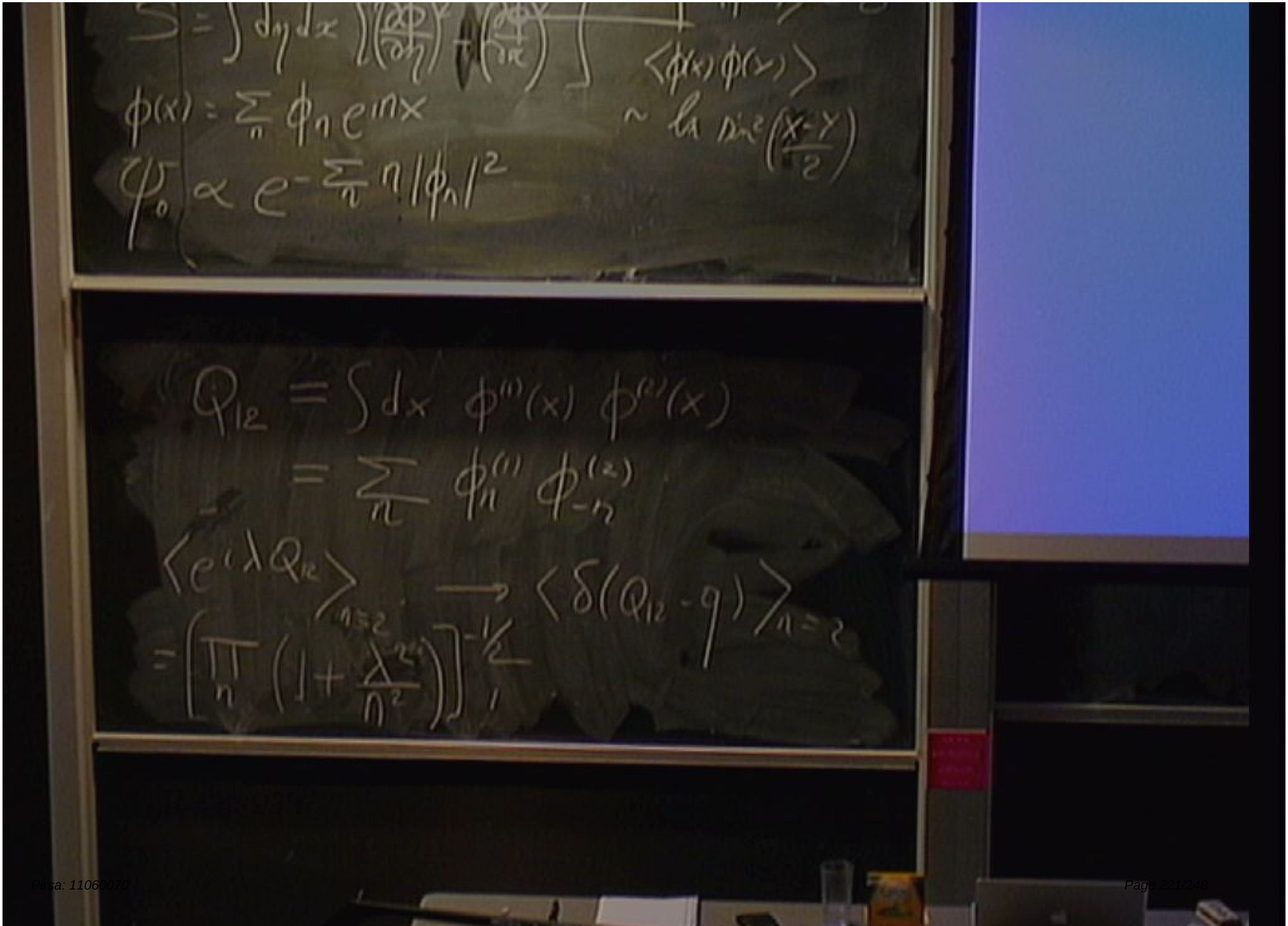
$= \left[\prod_n \left(1 + \frac{\lambda^2}{n^2} \right) \right]^{-1/2}$


$\langle \phi(x)\phi(y) \rangle$
$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$
$\phi(x) = \sum_n \phi_n e^{inx}$
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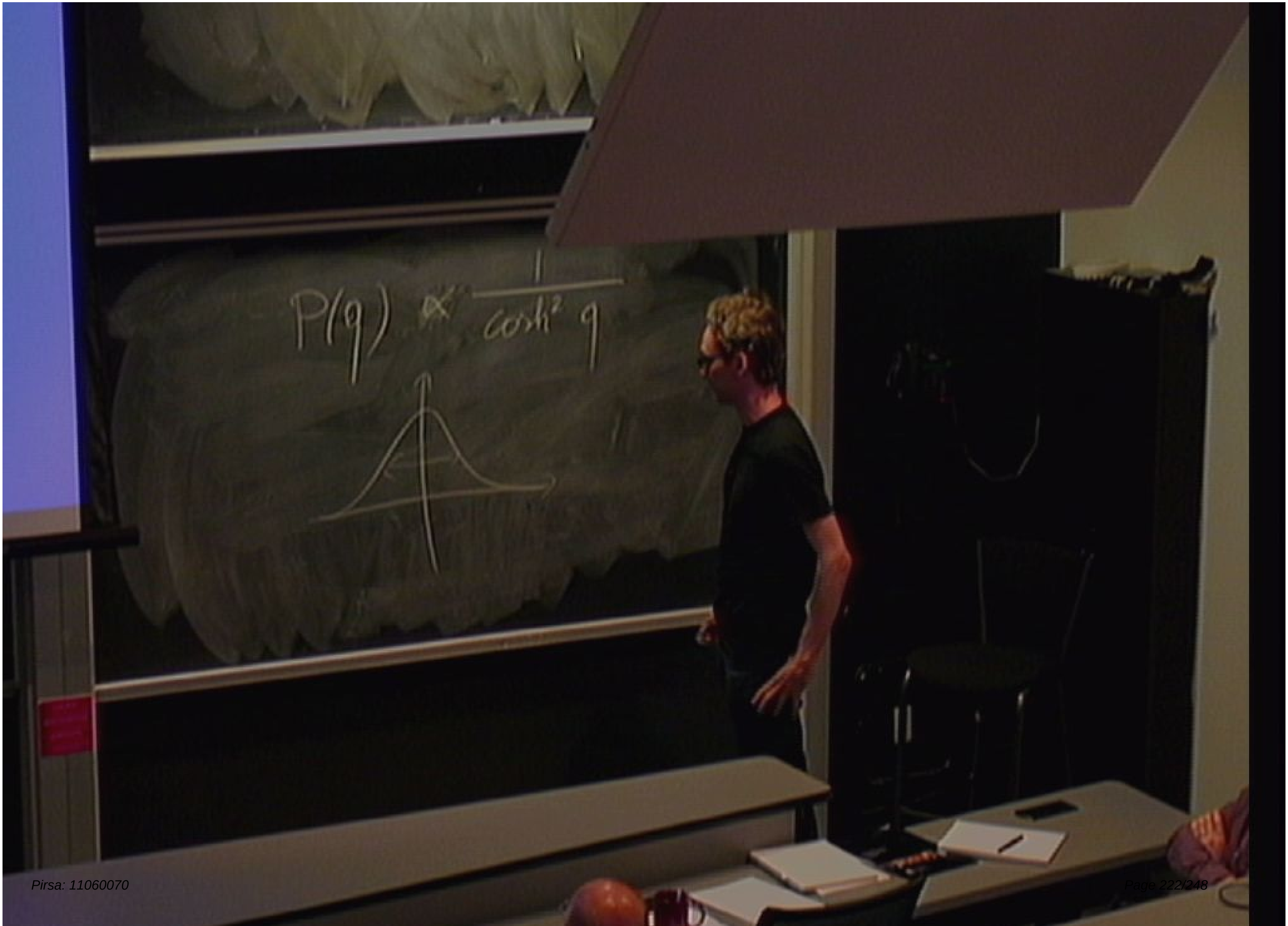
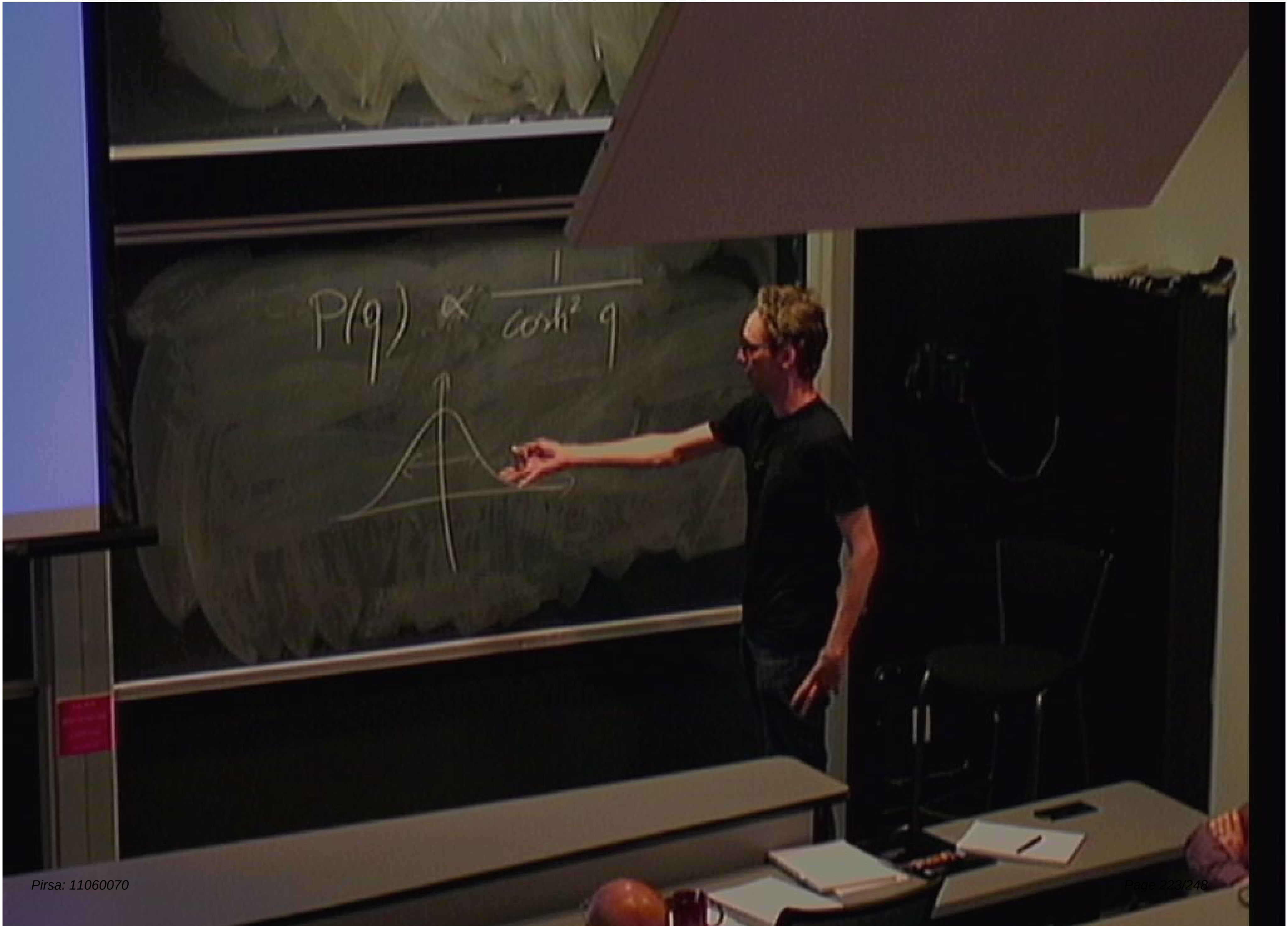
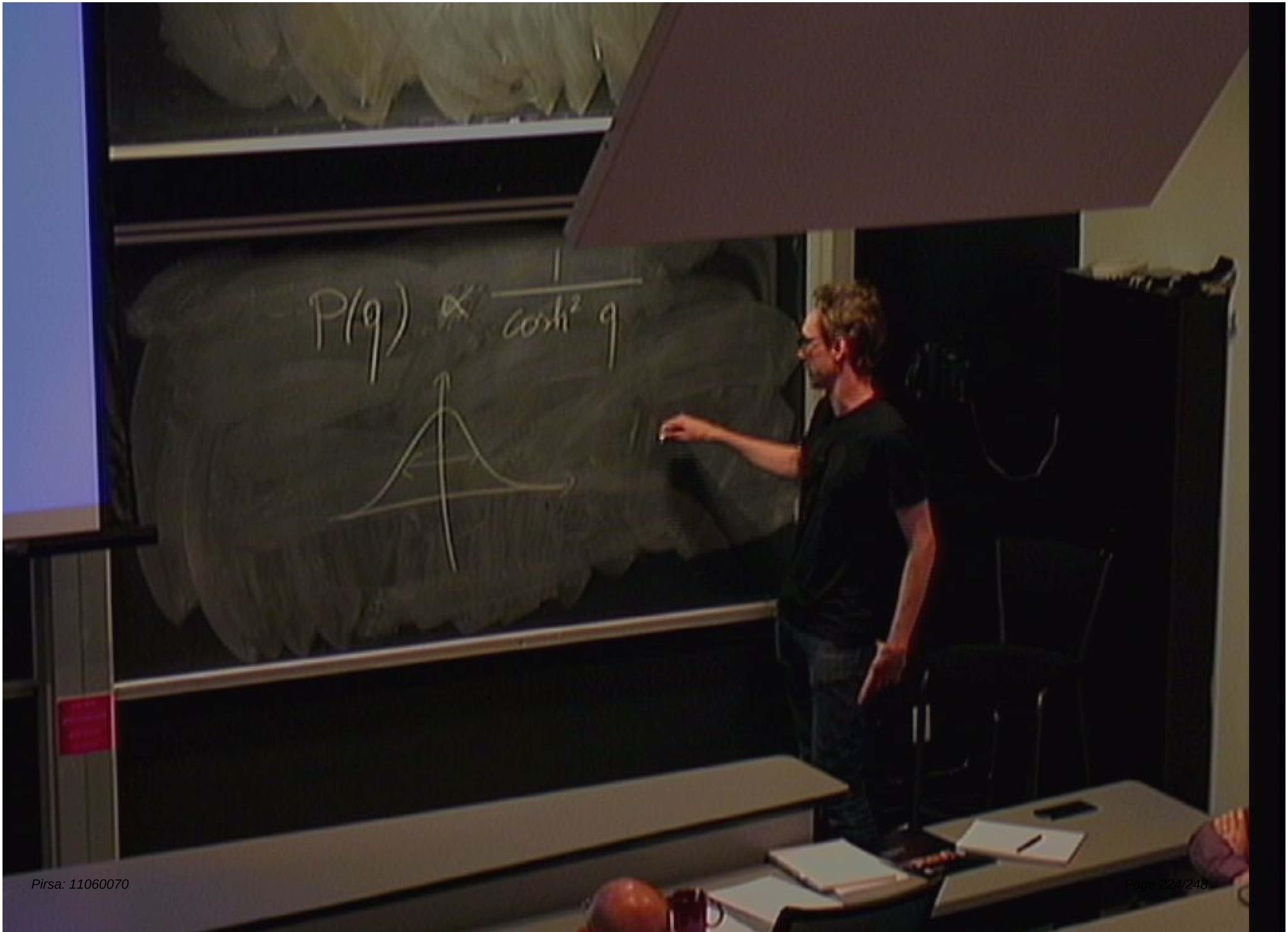
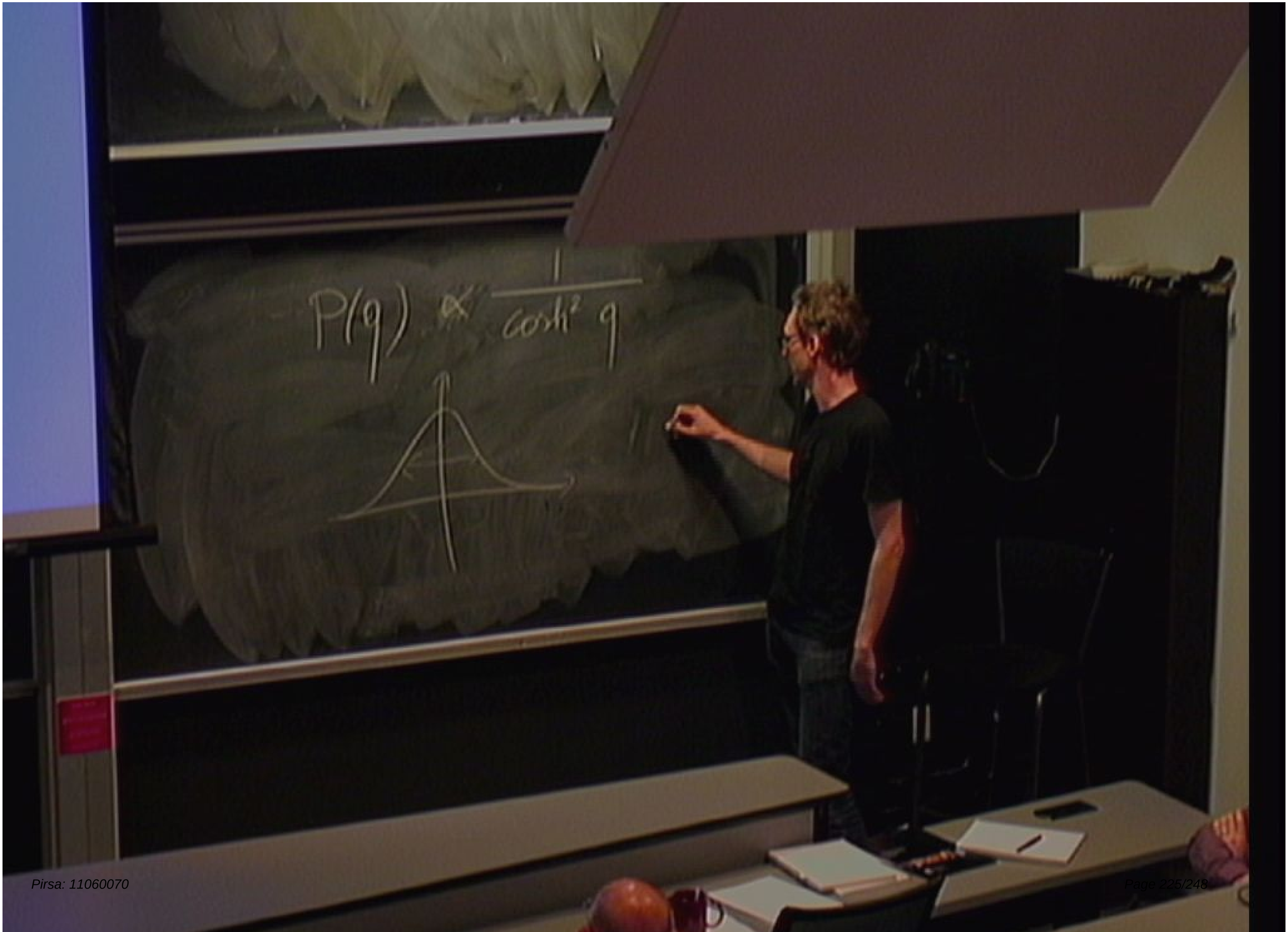
$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \longrightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$

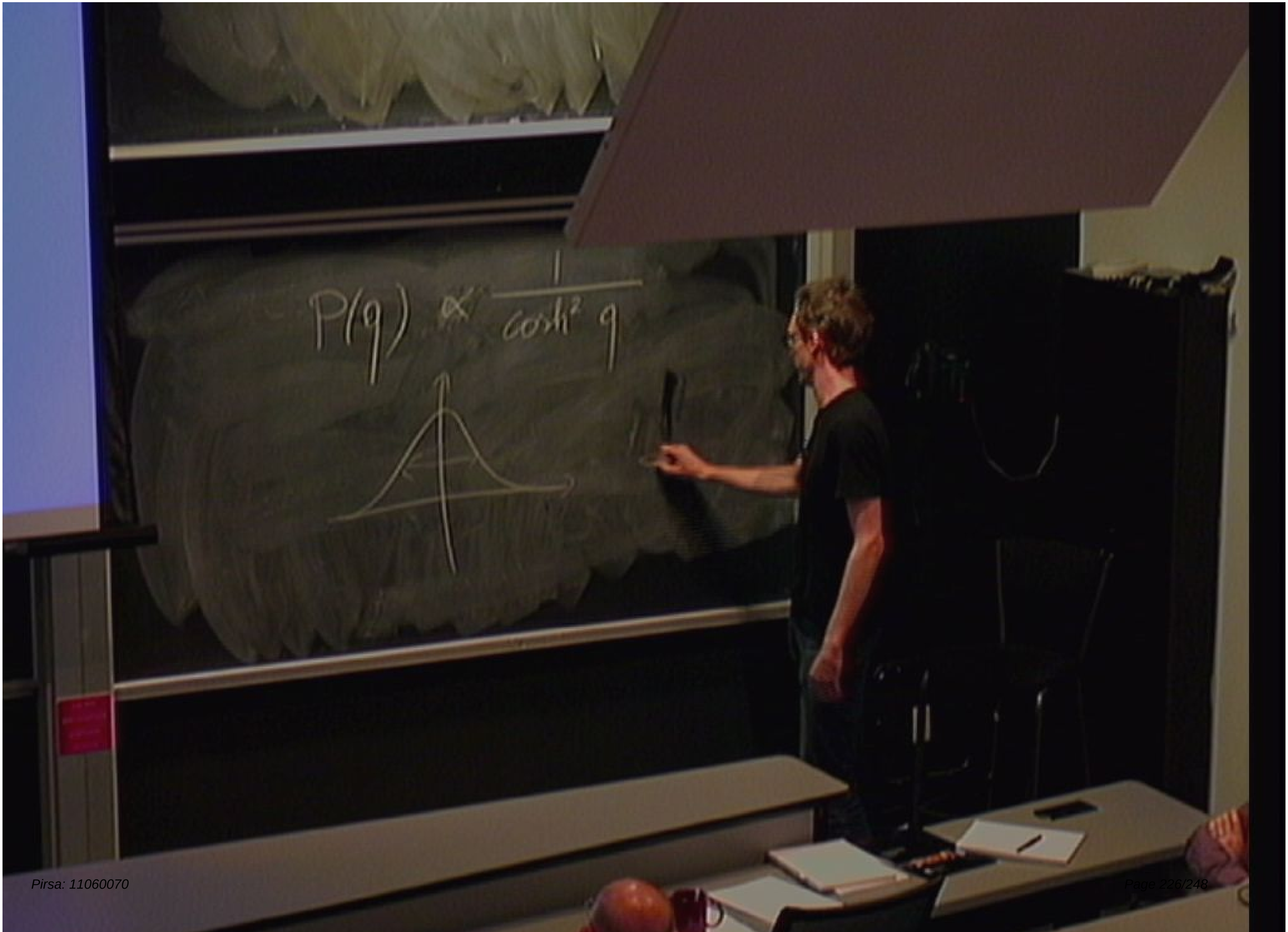

P(q) ∝ 1/cosh² q

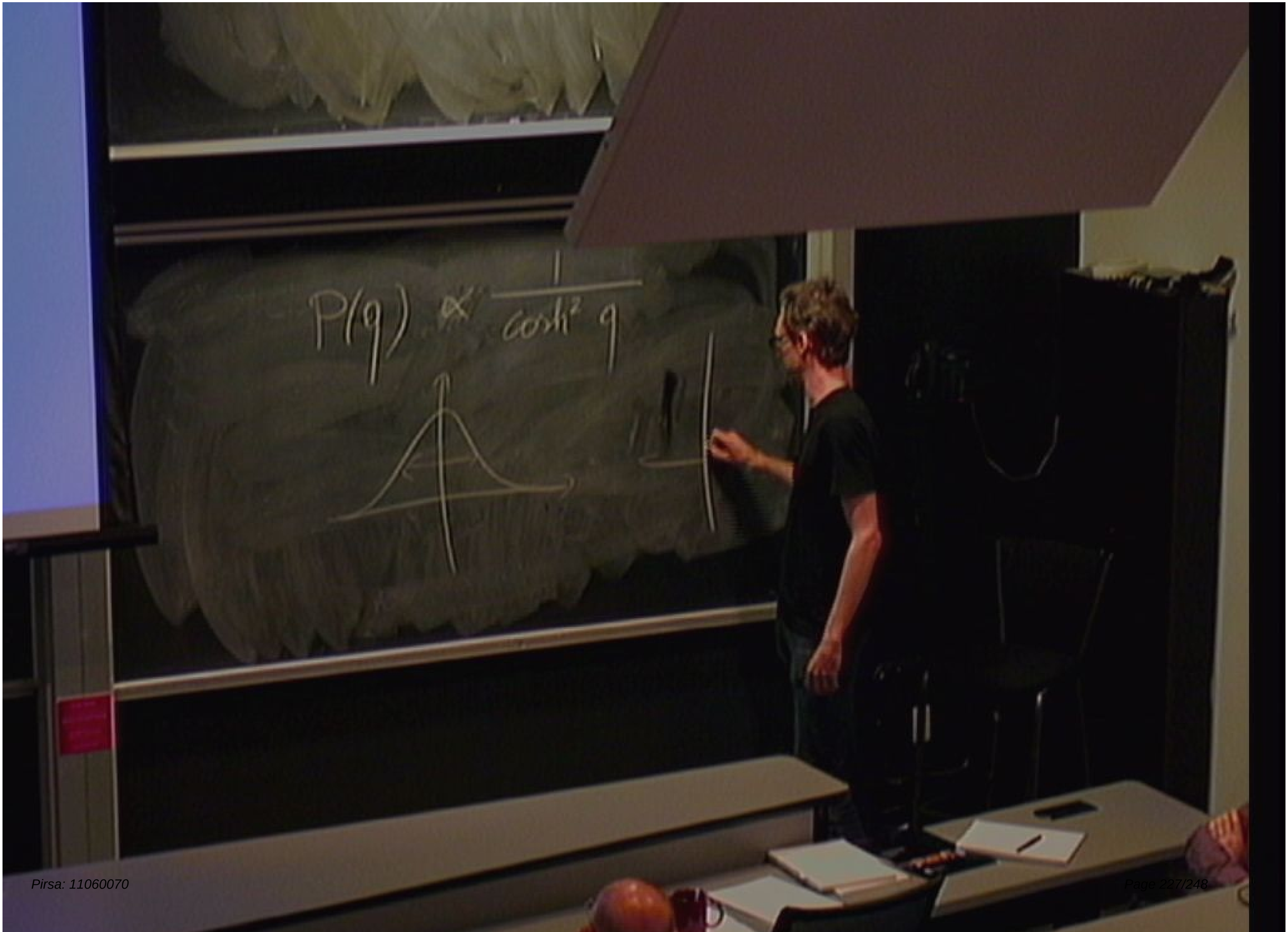

$P(q) \propto \frac{1}{\cosh^2 q}$

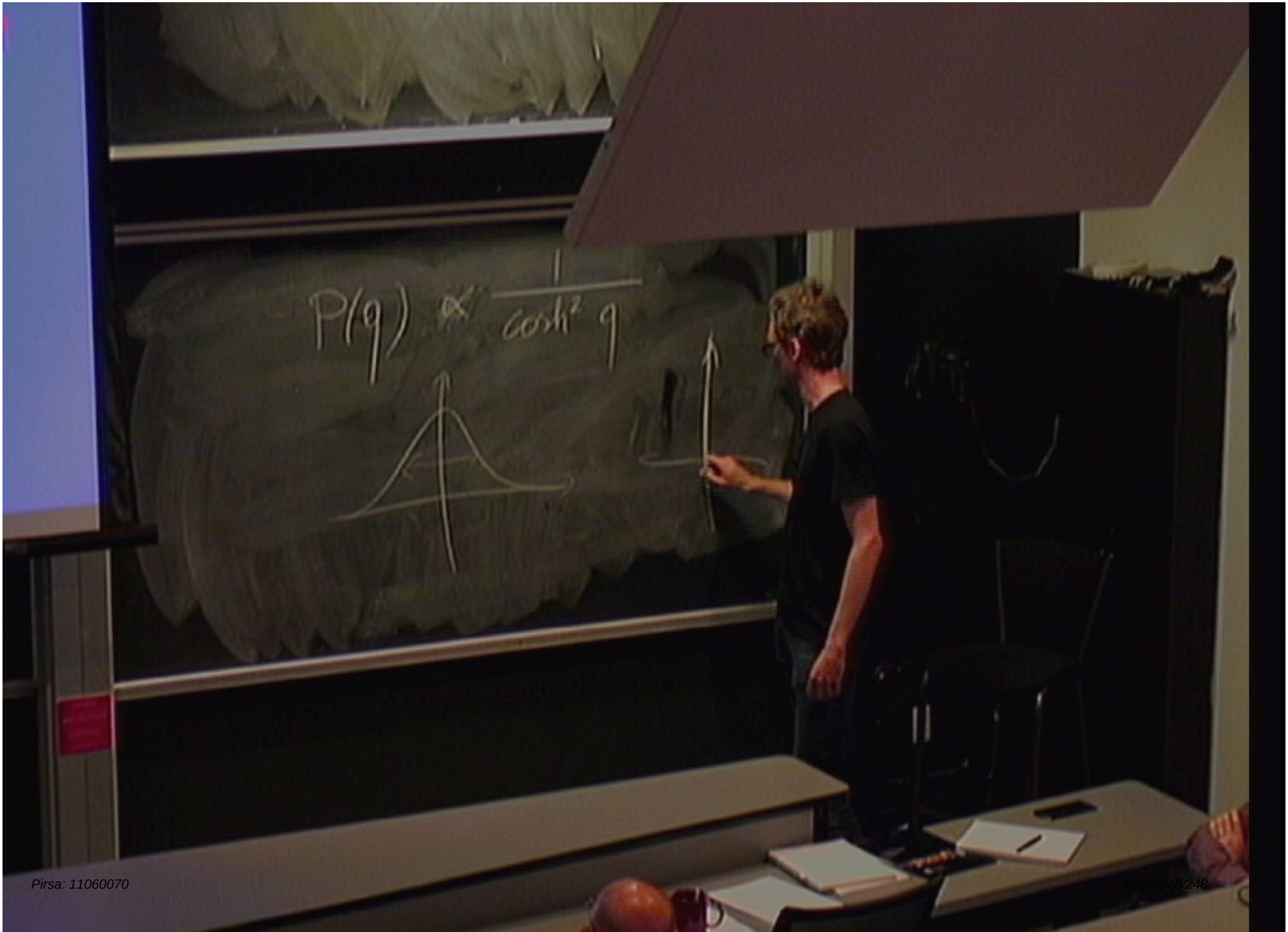


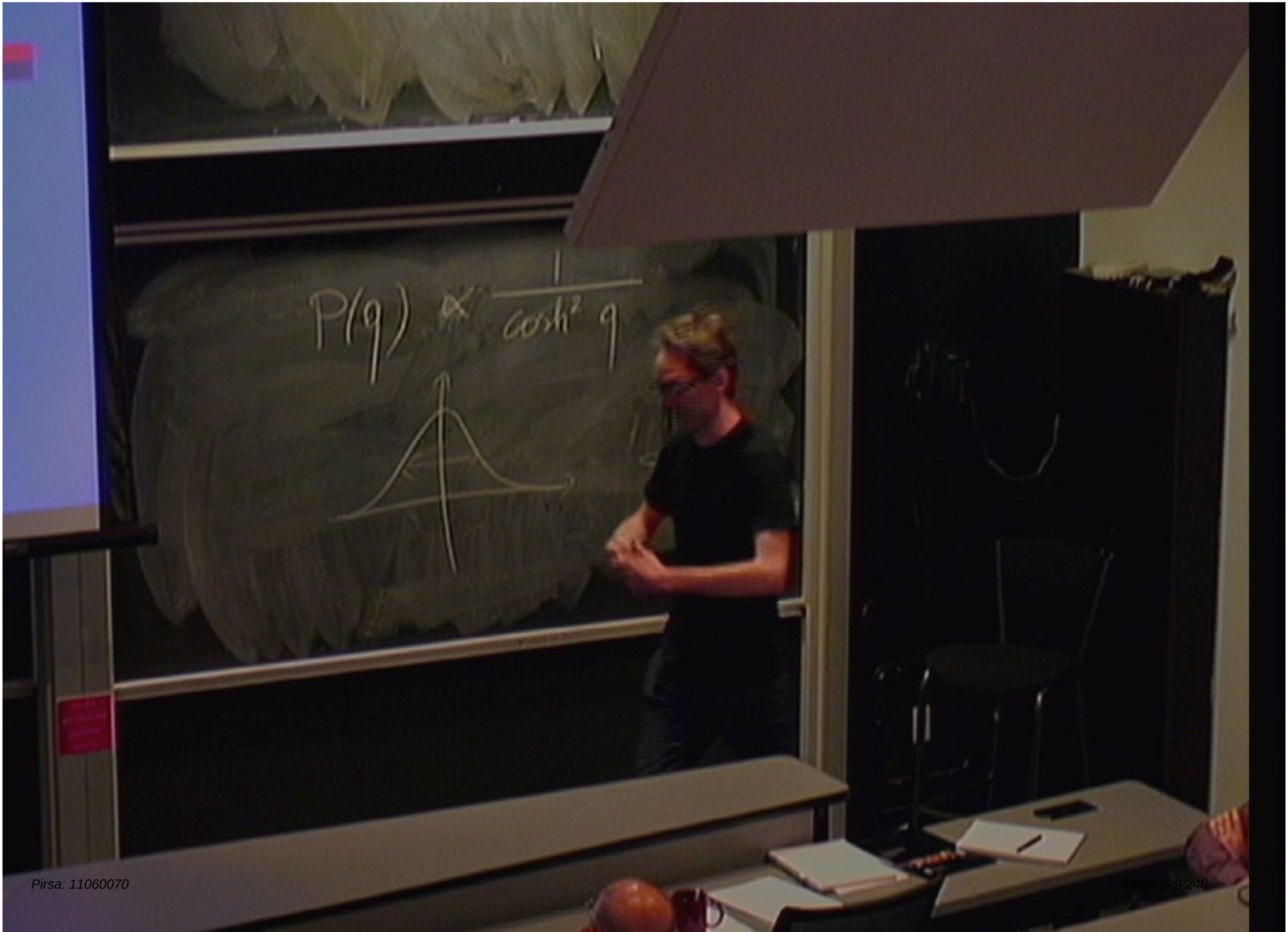
$P(q) \propto \frac{1}{\cosh^2 q}$

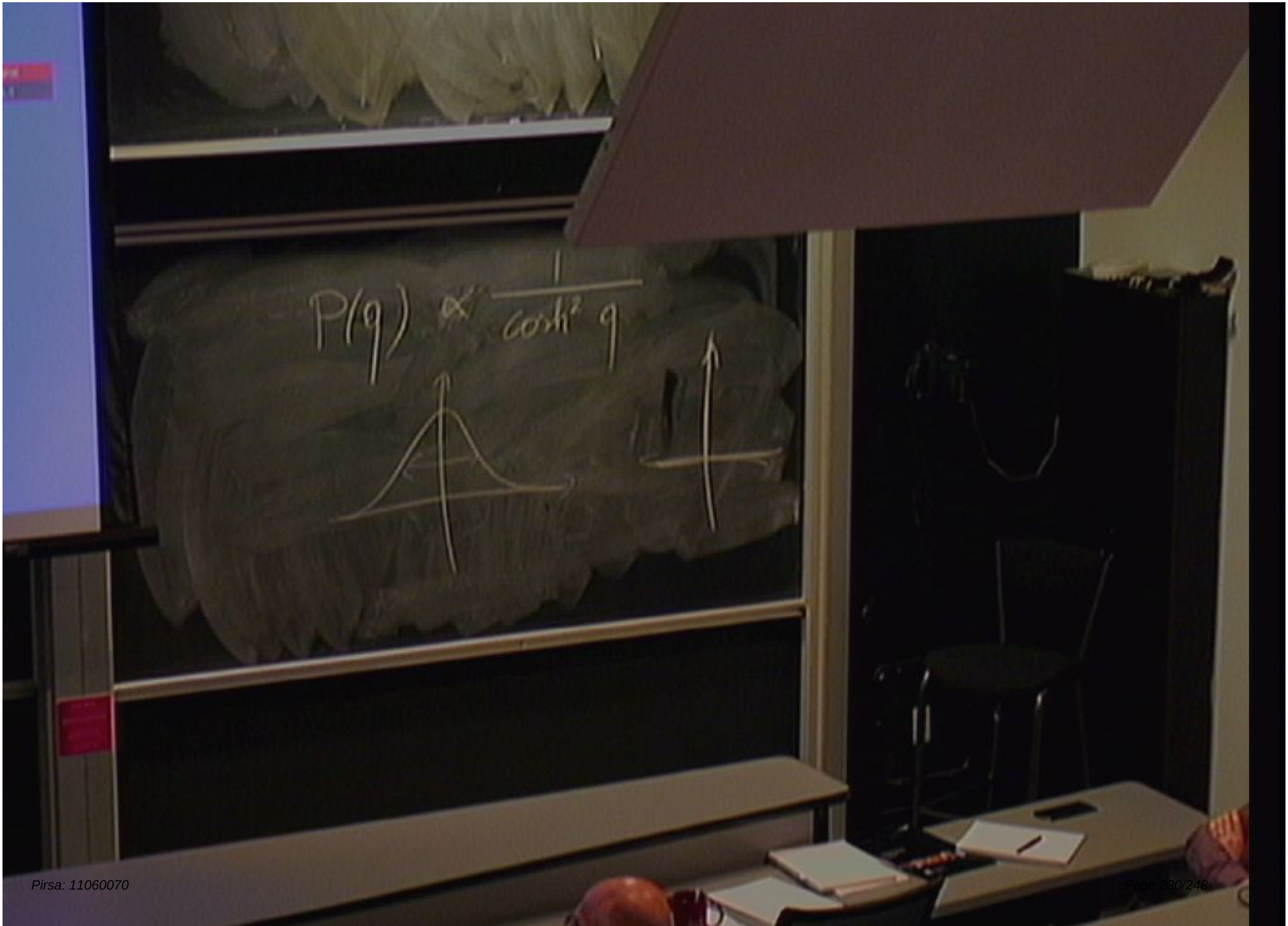

P(q) ∝ 1 / cosh² q

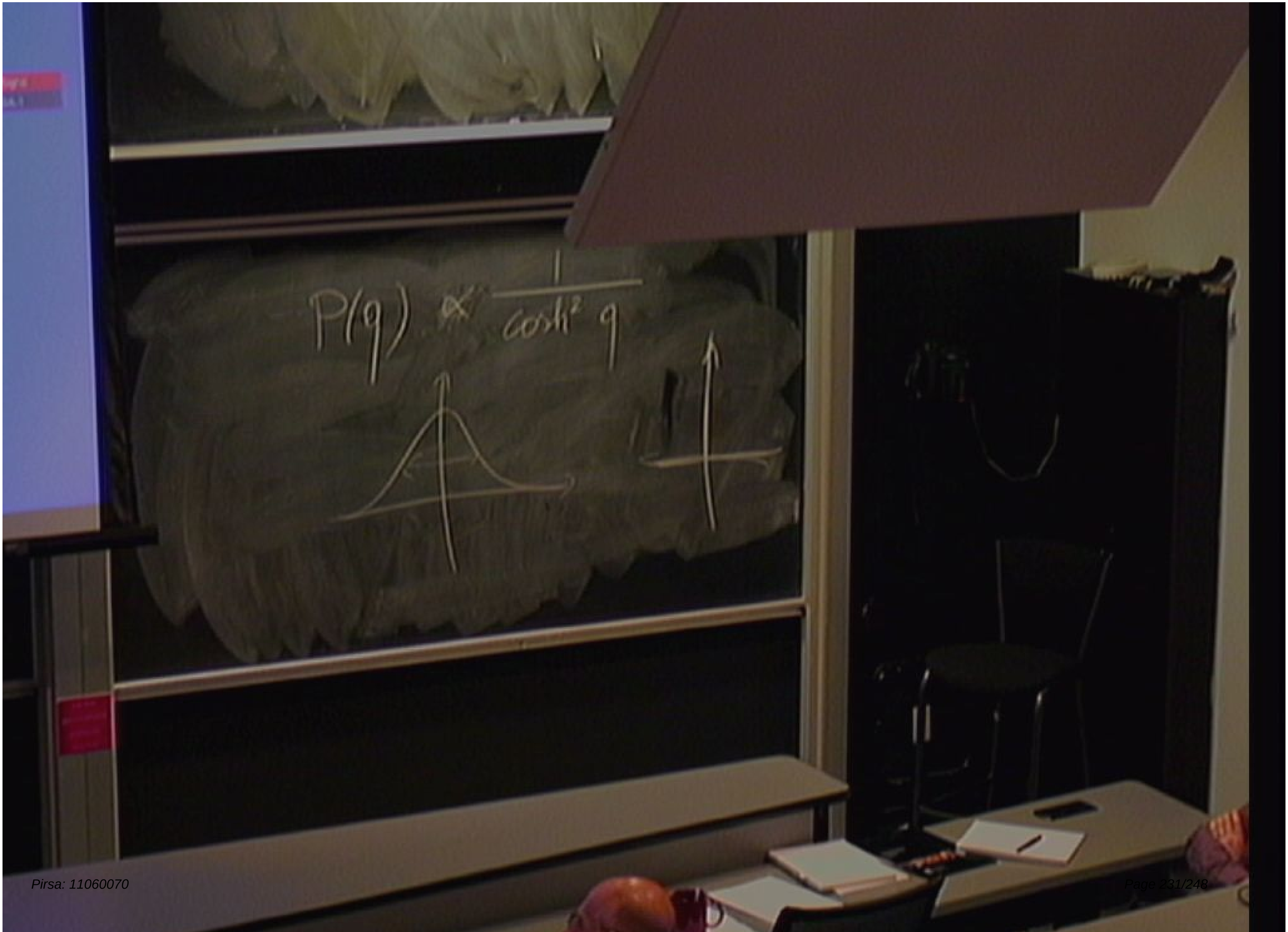

$P(q) \propto \frac{1}{\cosh^2 q}$

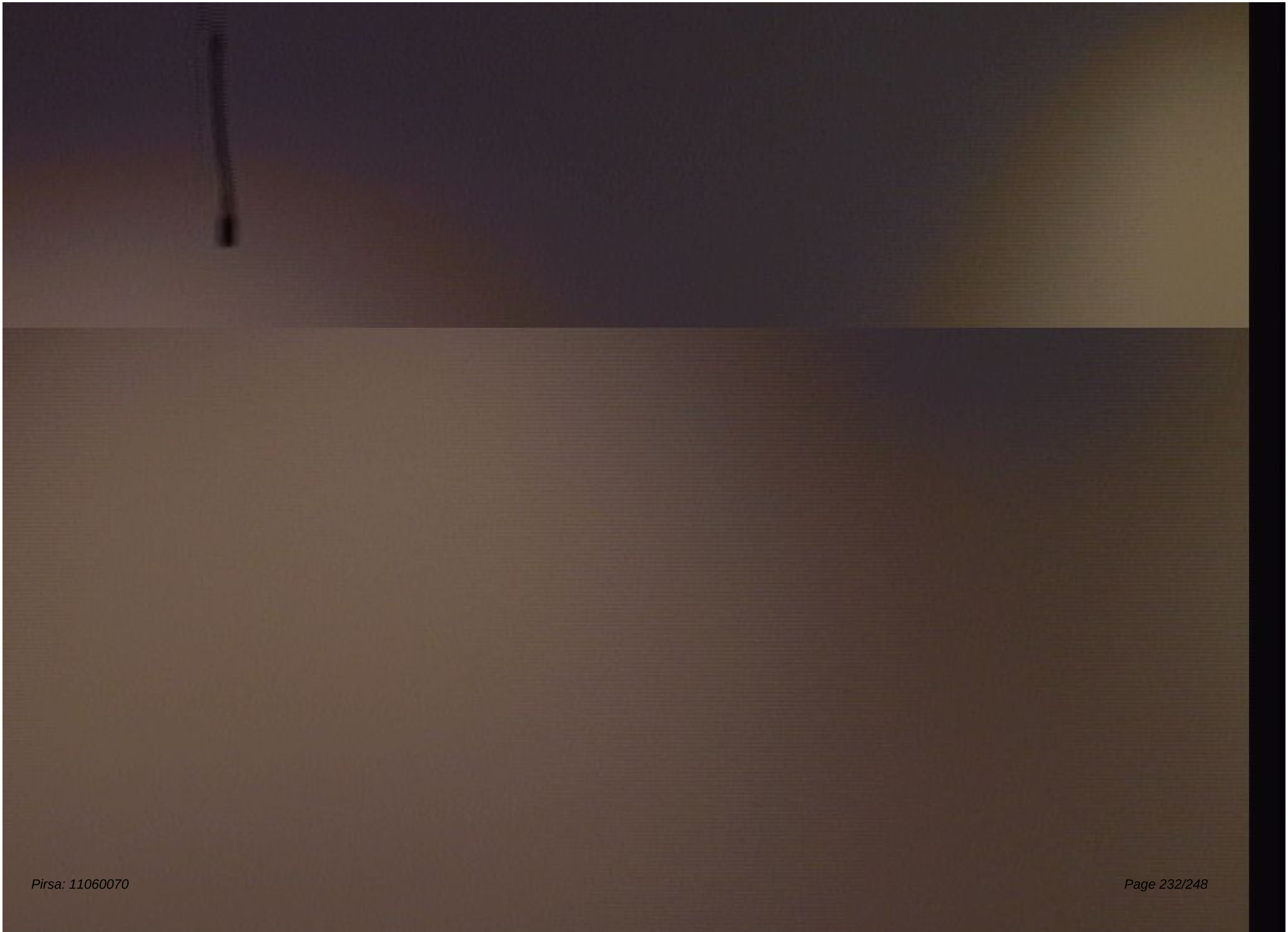

P(q) ∝ 1 / cosh² q

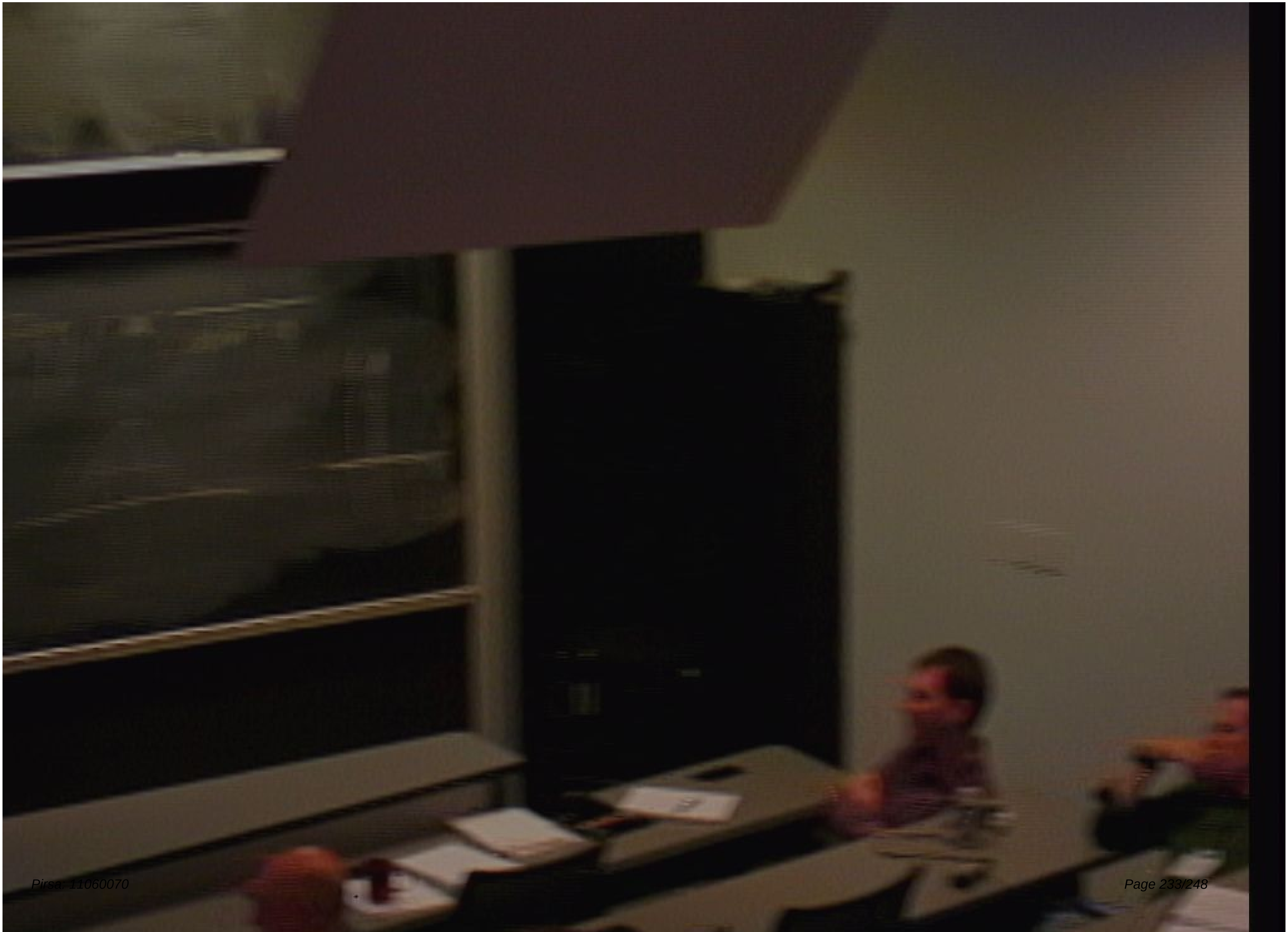

P(q) ∝ 1/cosh² q

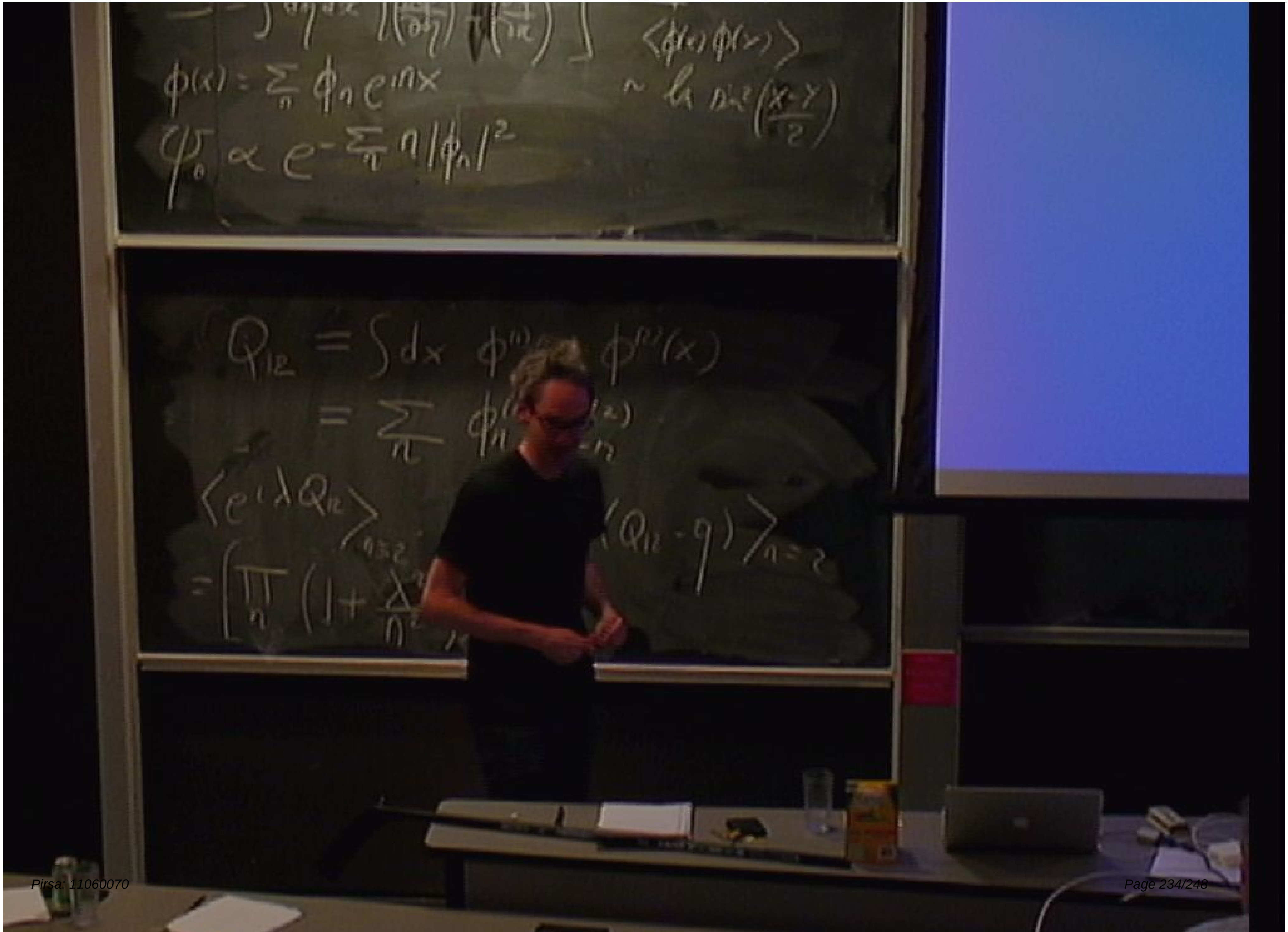

P(q) ∝ 1/cosh² q

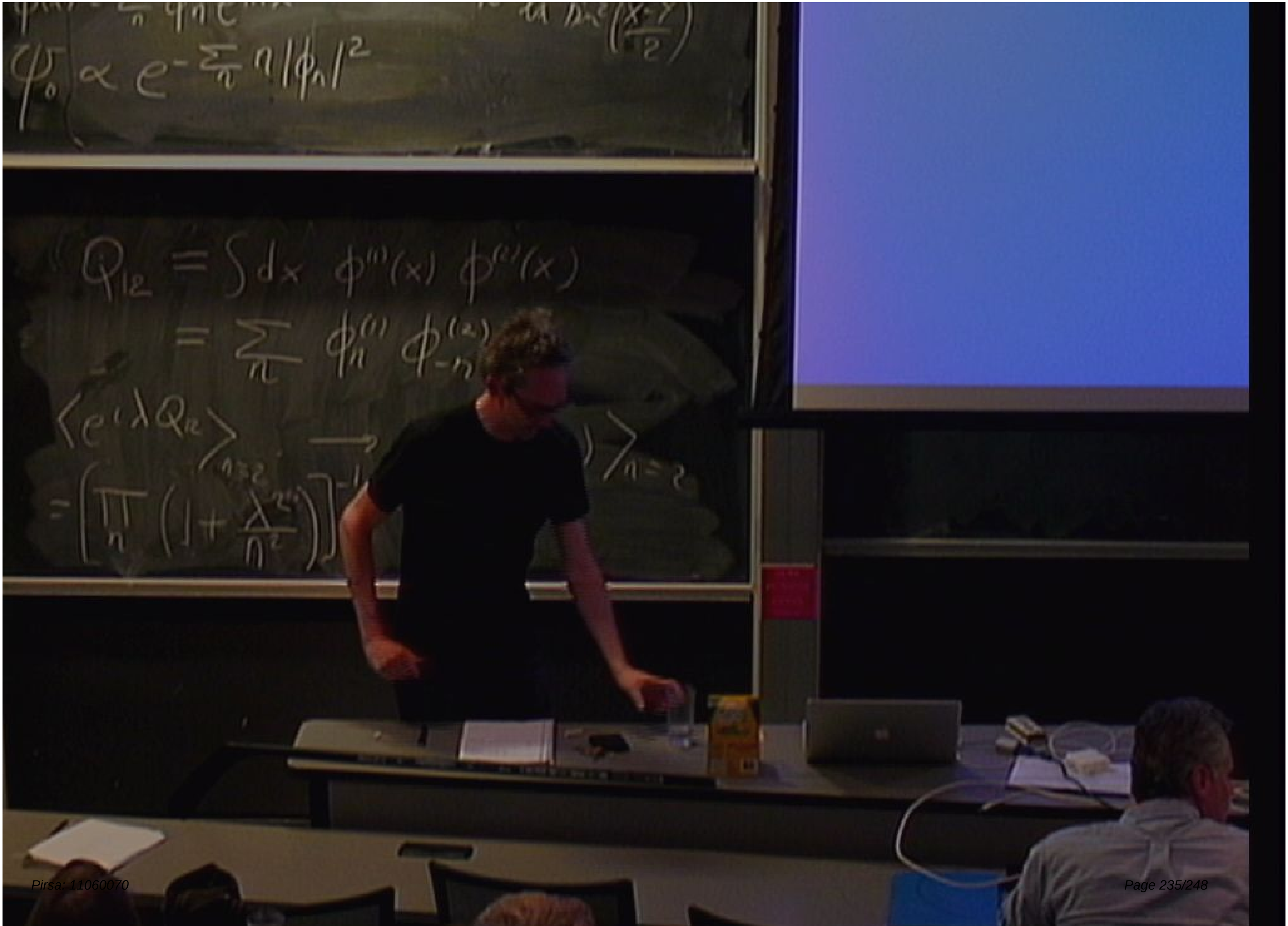

P(q) ∝ 1/cosh² q

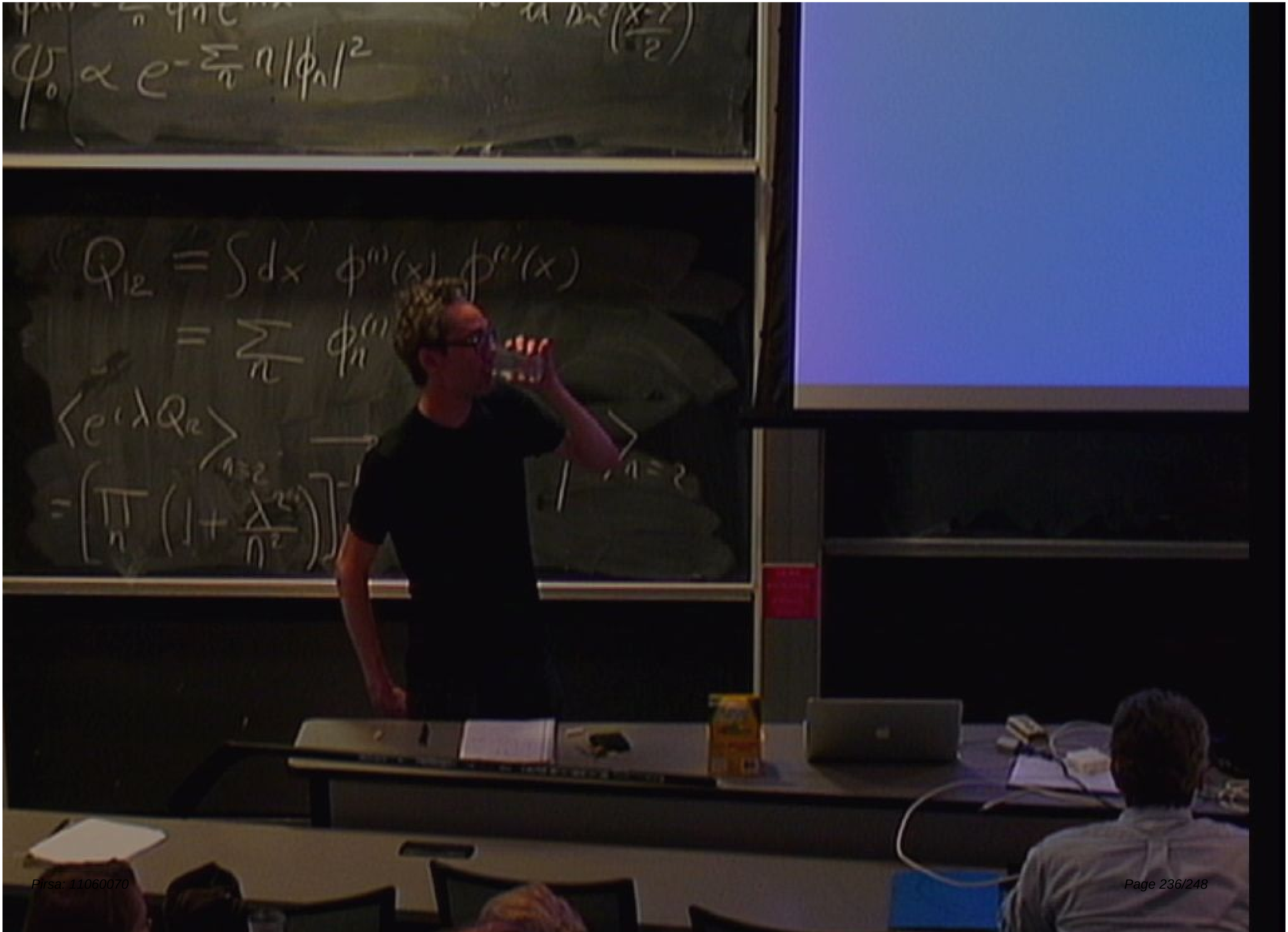

P(q) ∝ 1/cosh² q



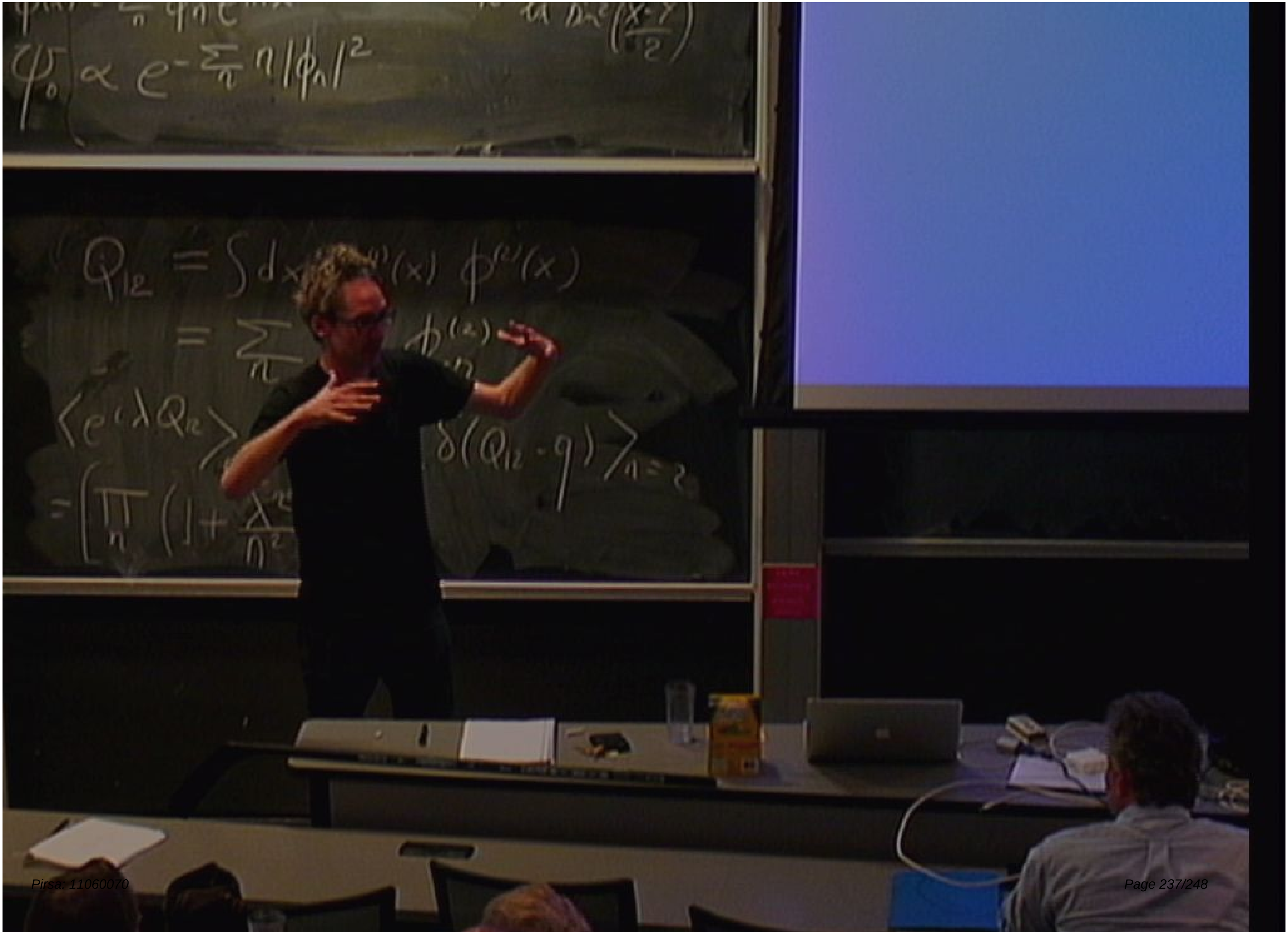


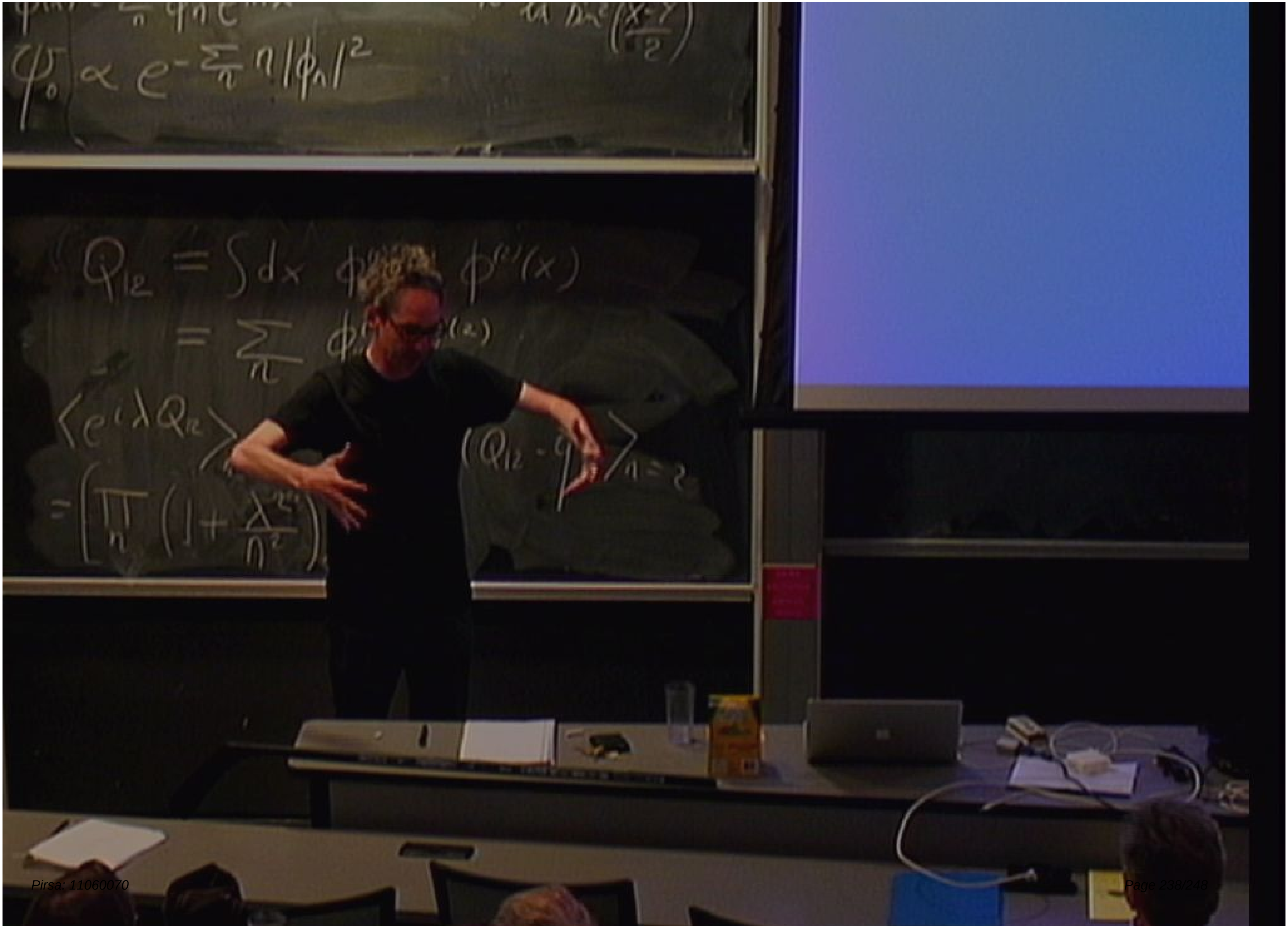


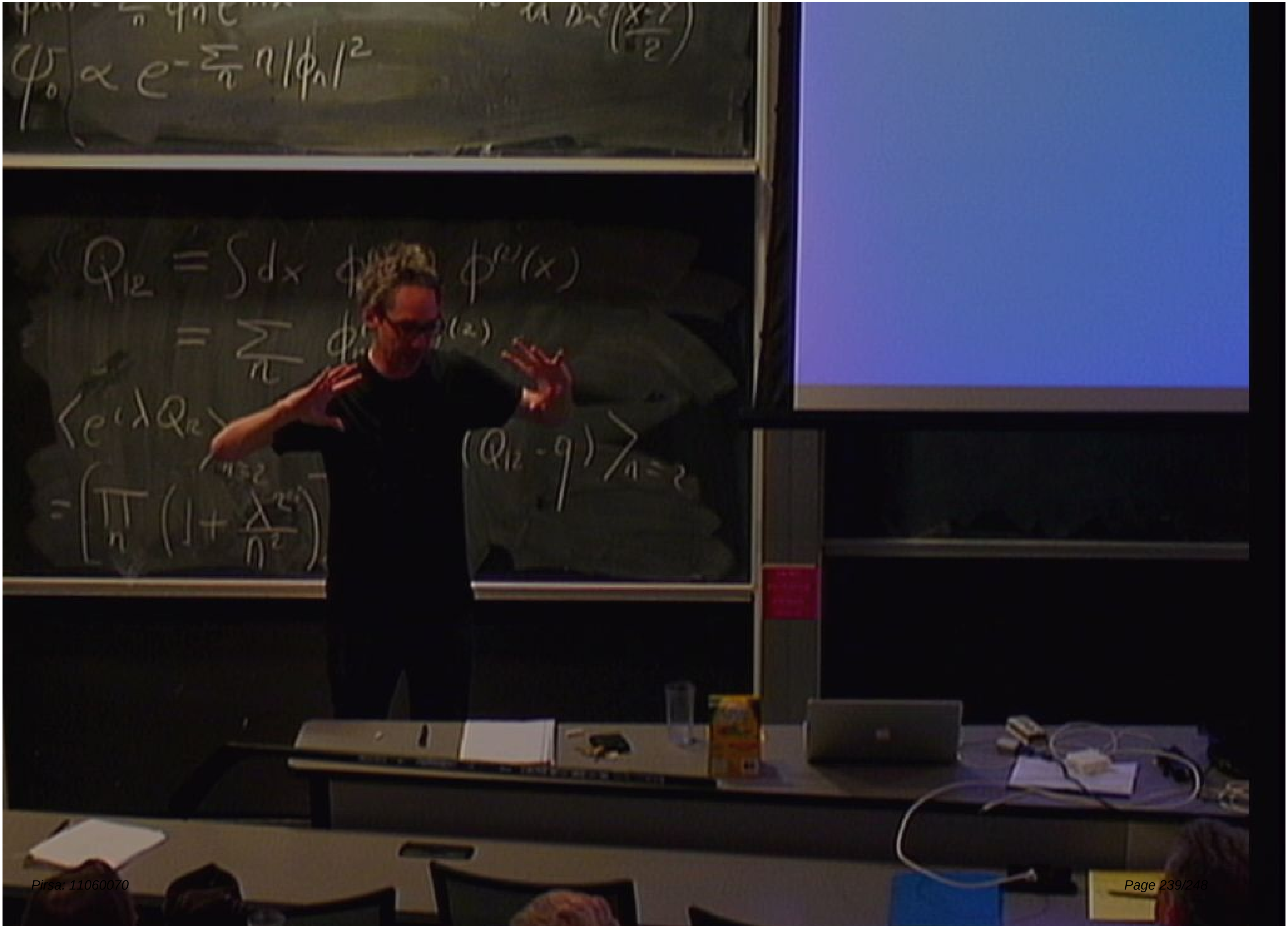

$\phi_0 \propto e^{-\sum_n n|\phi_n|^2}$
$Q_{12} = \int dx\ \phi^{(1)}(x)\ \phi^{(2)}(x)$
$= \sum_n \phi_n^{(1)}\ \phi_{-n}^{(2)}$

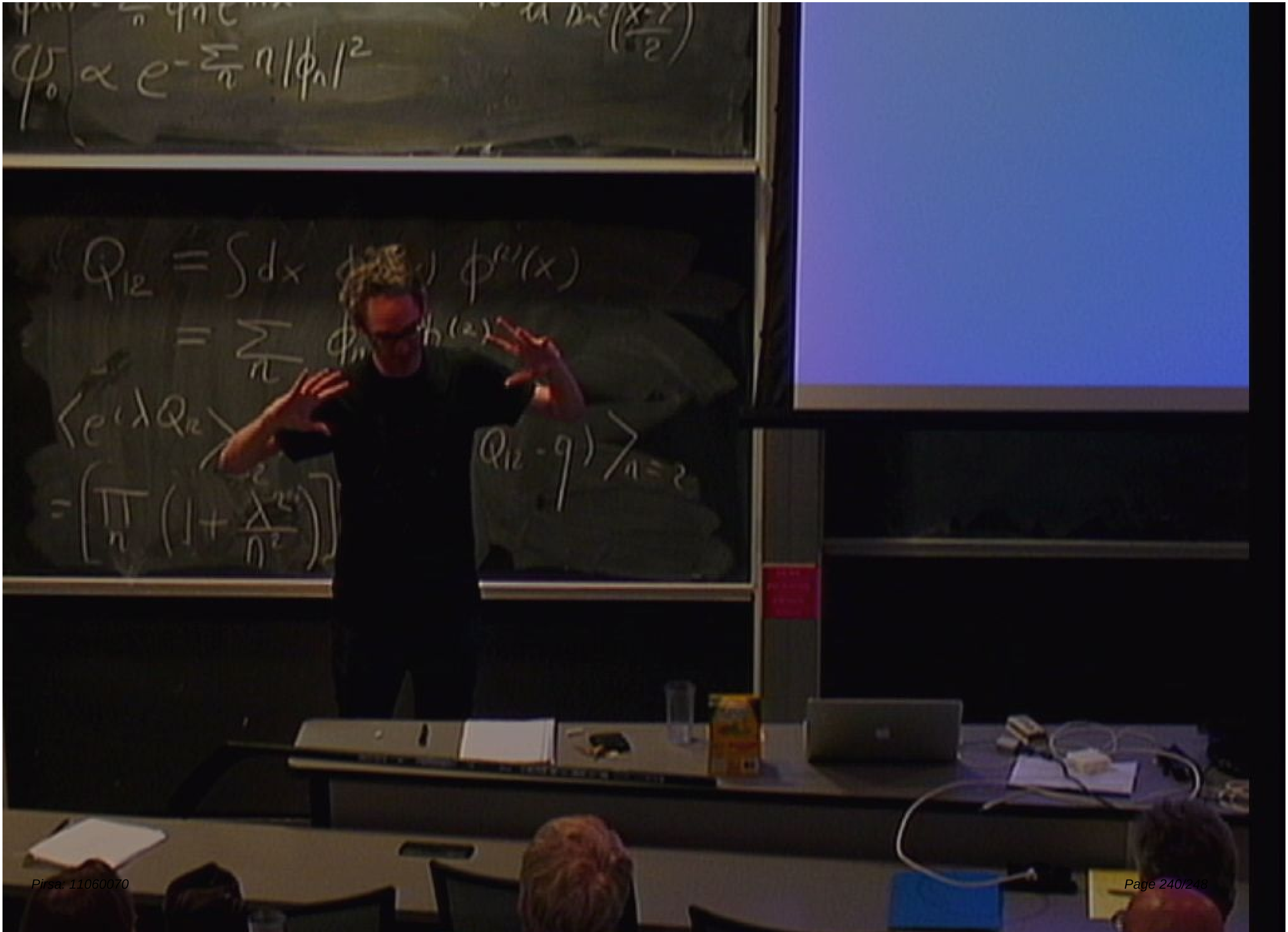


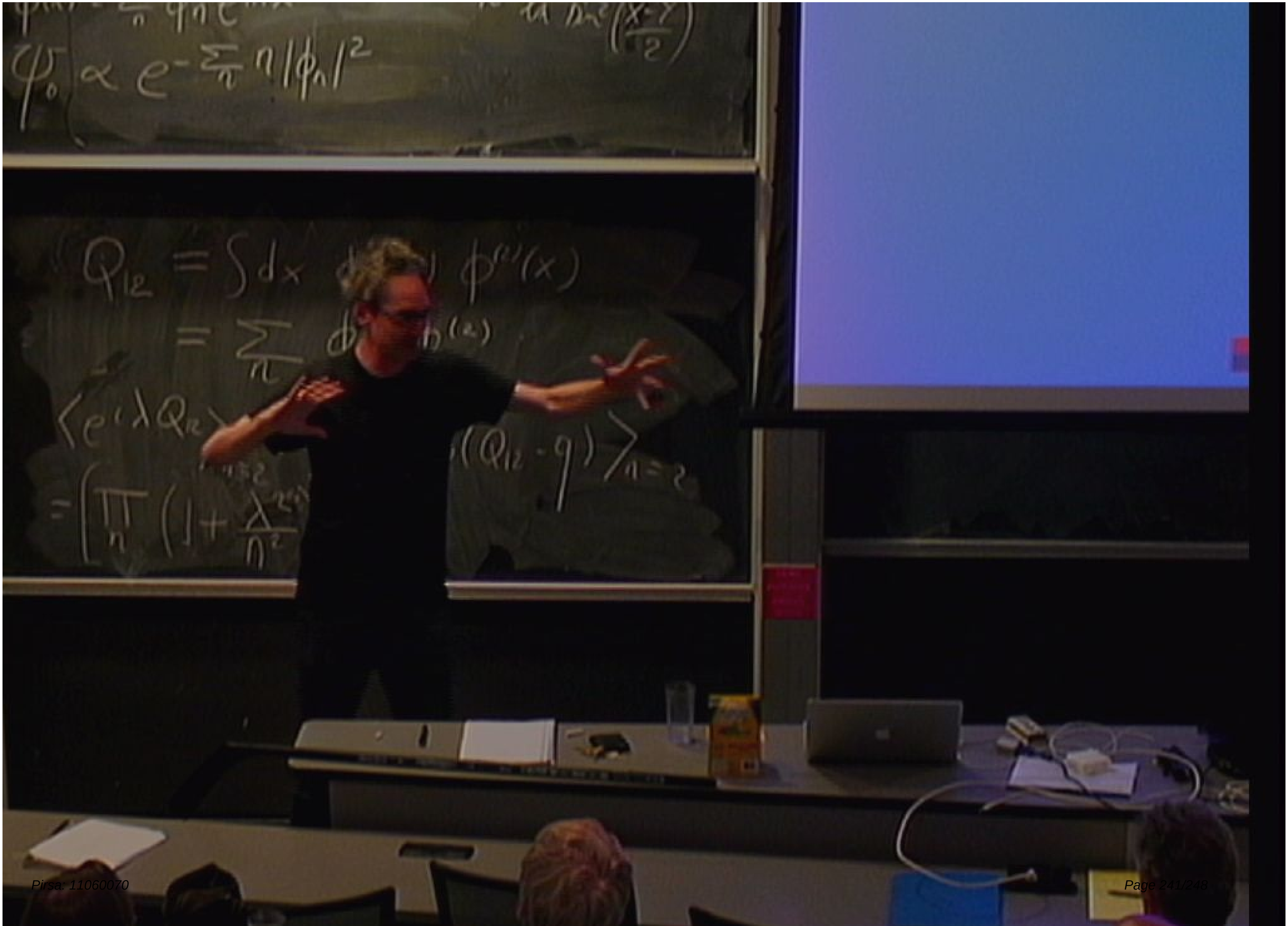
φ₀ ∝ e^{-Σ_n n|φ_n|²}
Q₁₂ = ∫dx φ^{(1)}(x) φ^{(2)}(x)

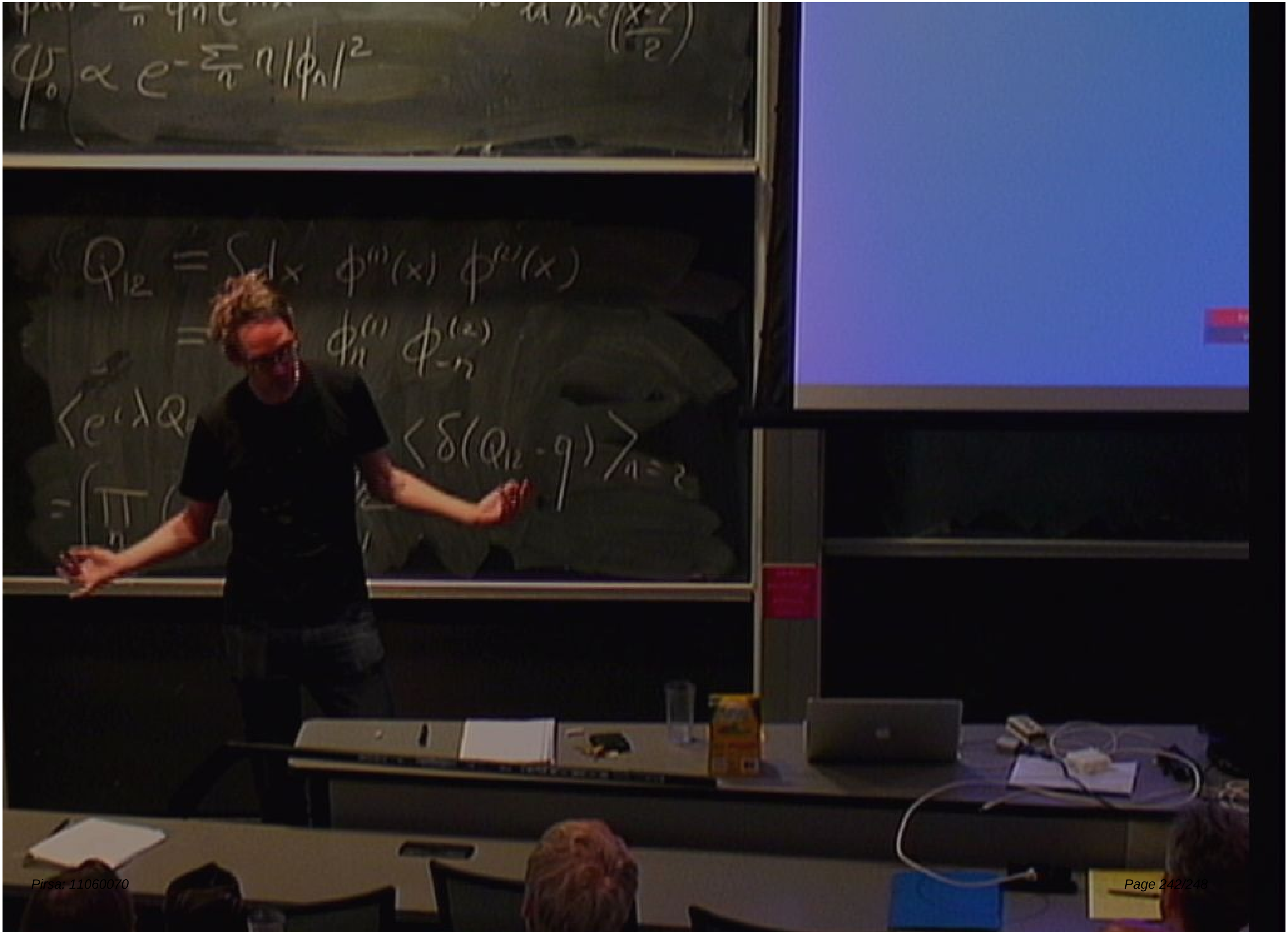


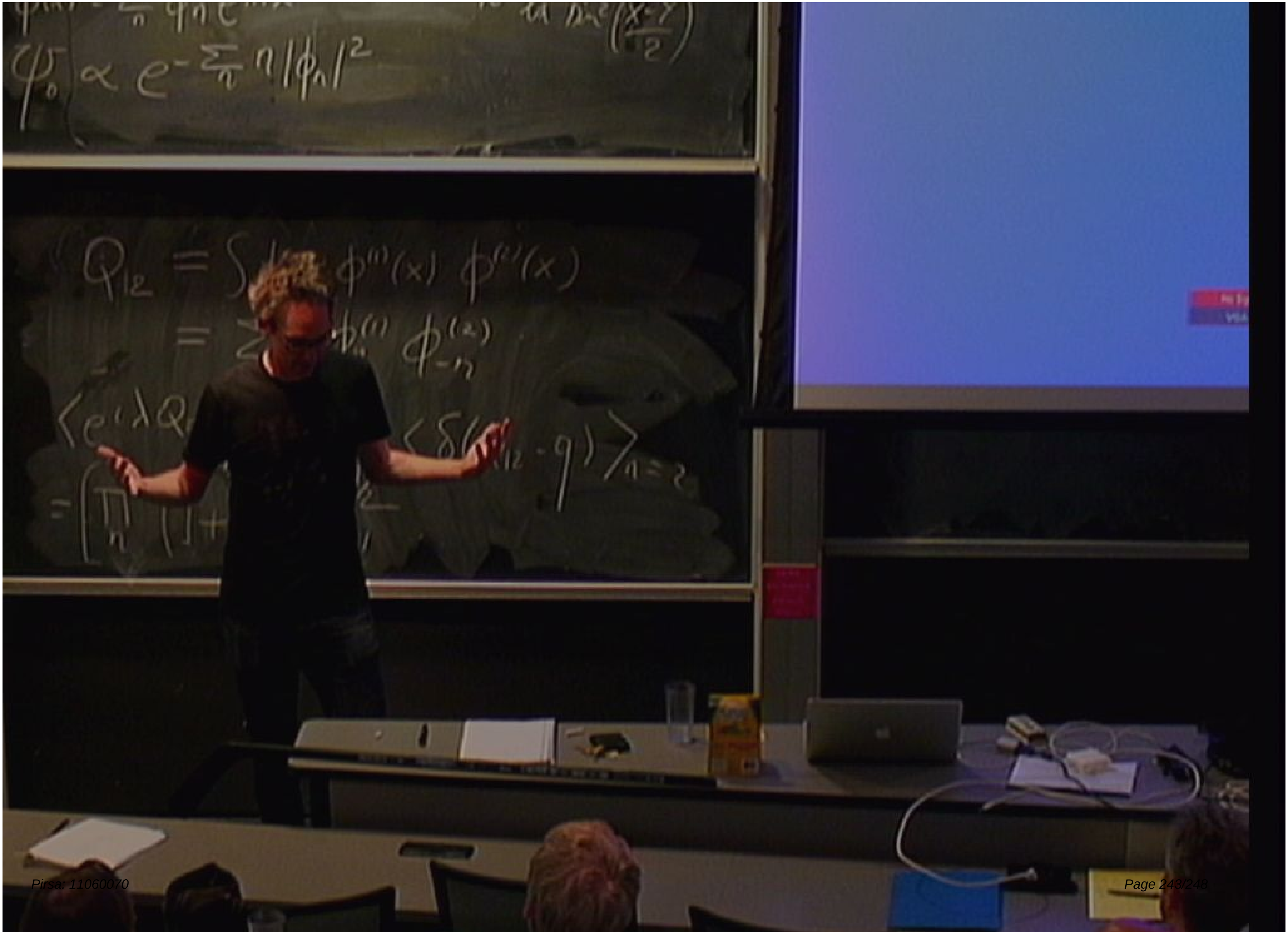


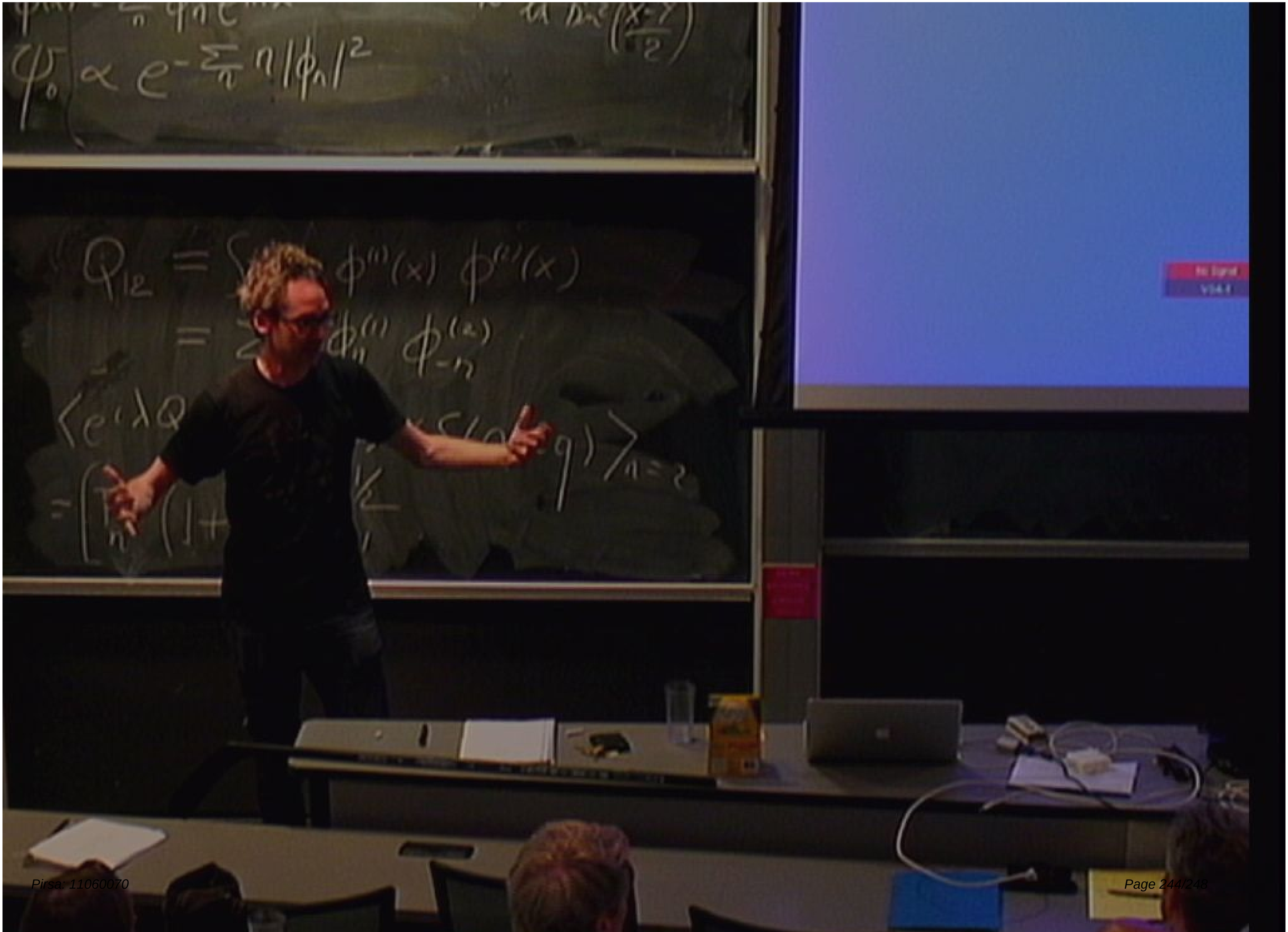


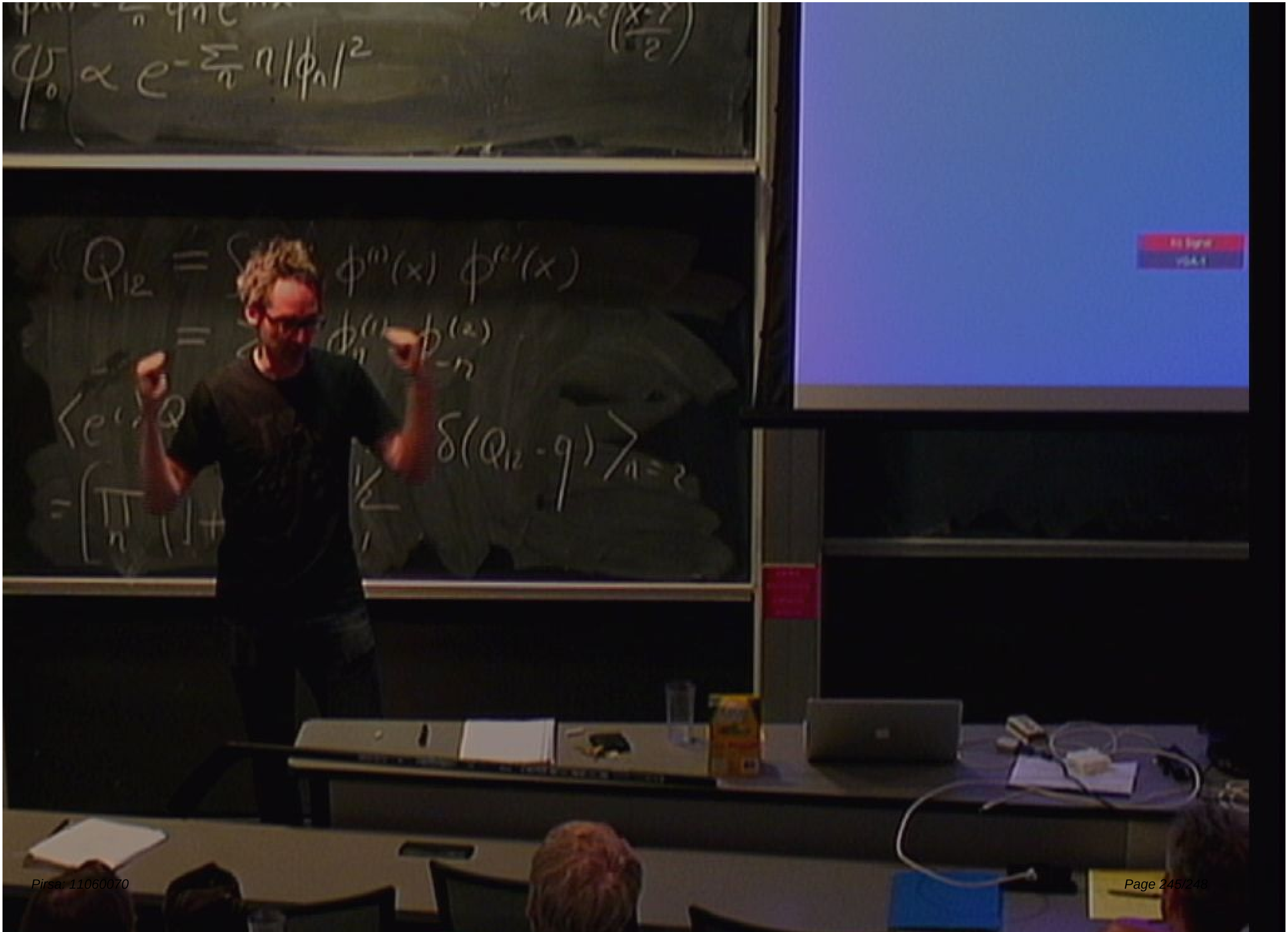


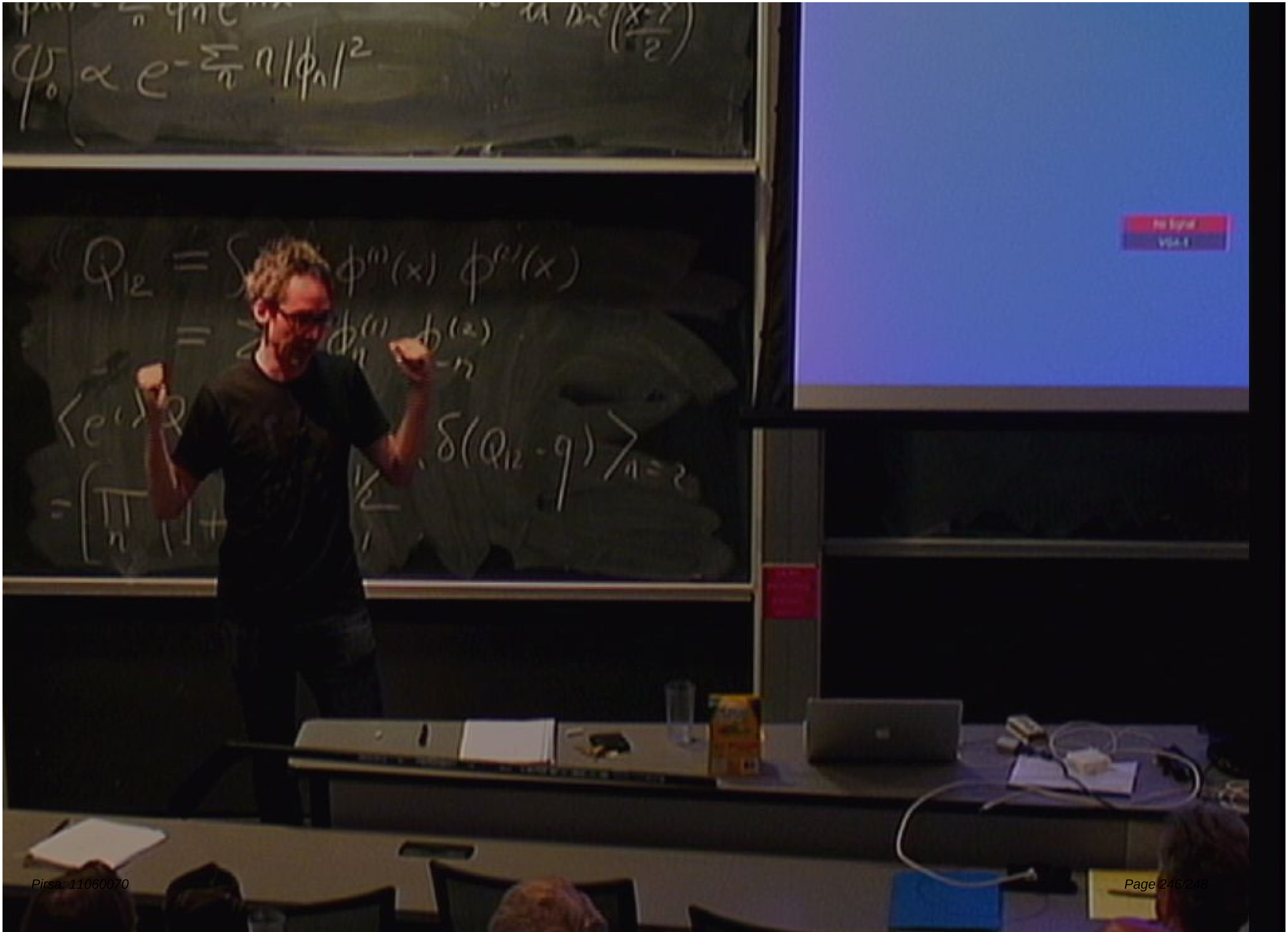


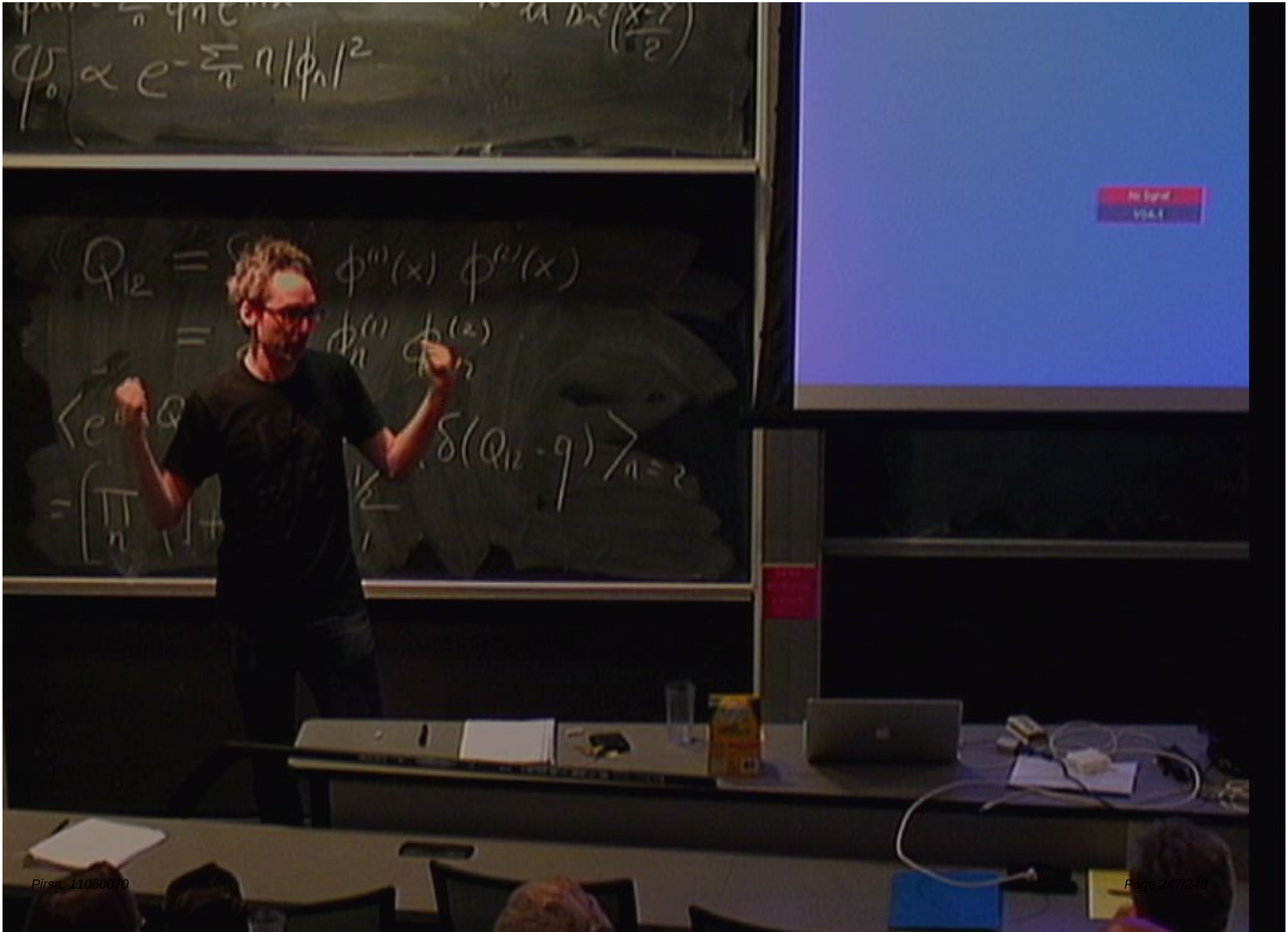


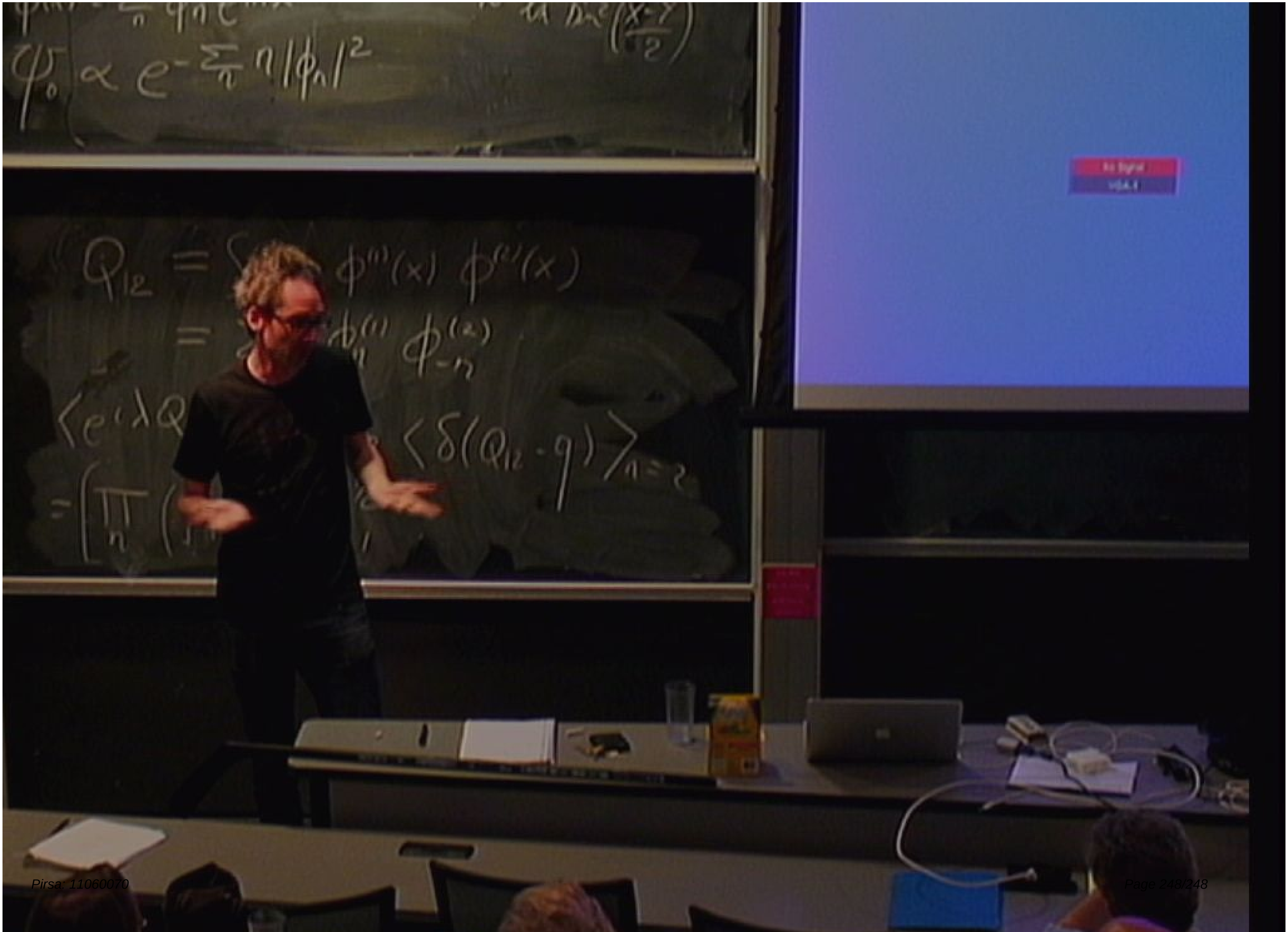

$\phi_0 \propto e^{-\sum_n n|\phi_n|^2}$




$\phi_0 \propto e^{-\sum_n n|\phi_n|^2}$



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$Q_{12} =$
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