

Title: Complexity in Fundamental Physics

Date: Jun 23, 2011 05:10 PM

URL: <http://pirsa.org/11060070>

Abstract:

# Outline

## 1 Motivation

## 2 Inspiration

- Spin glass models
- State space structure of the SK model
- Overlap order parameter

## 3 Application to string theory and cosmology

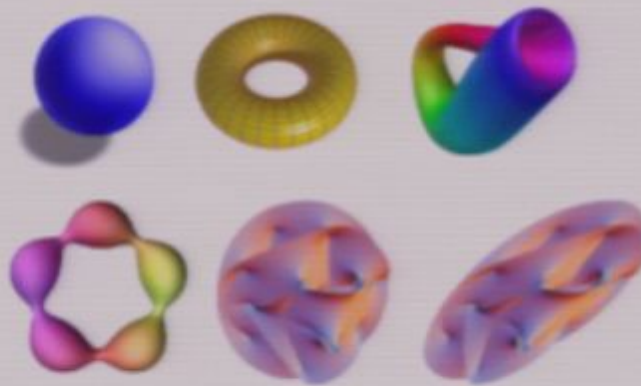
- Generalized overlap order parameters
- Examples of glassy systems in string theory + results
- Cosmology

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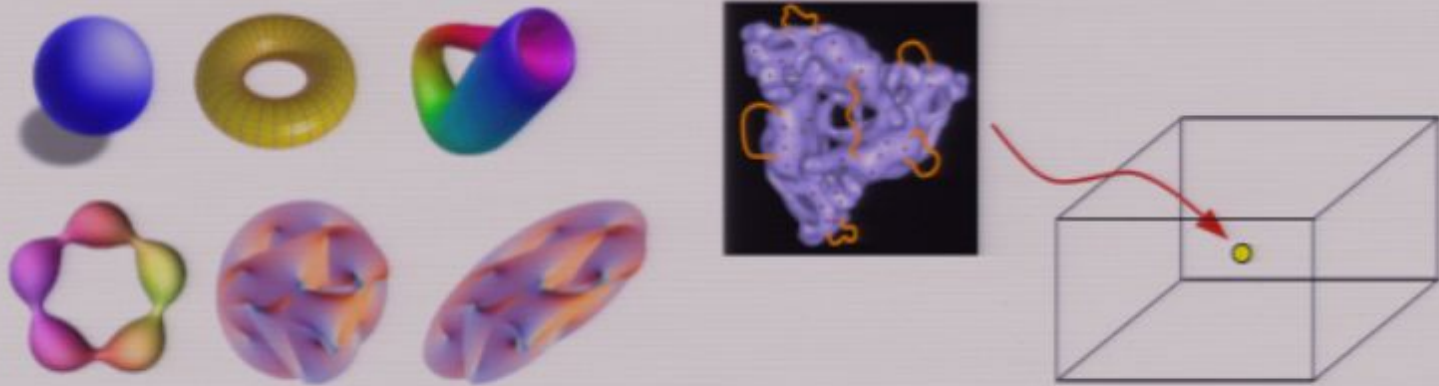
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Quantum gravity (string theory) seems to imply reversal of usual complexity - fundamentalness relation:

- 1 Landscape of compactifications

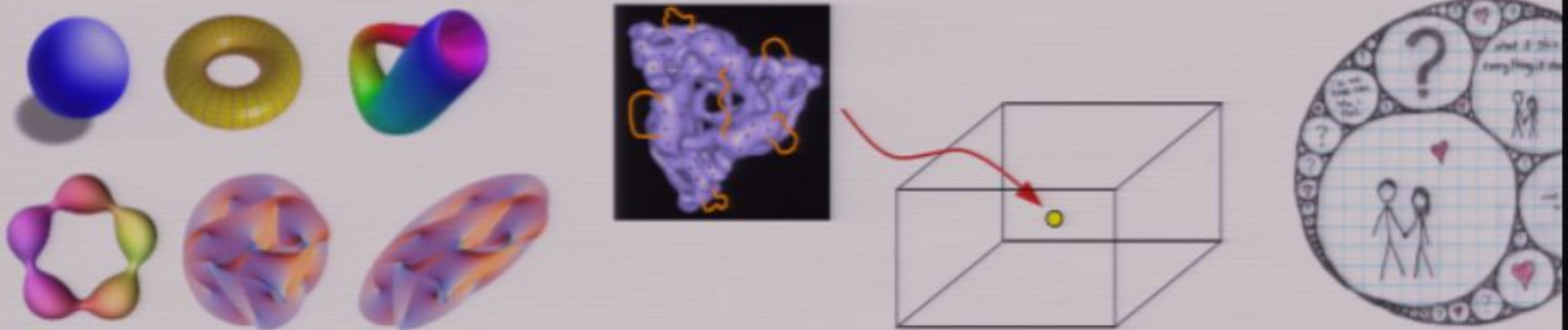
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- 3 Eternal inflation

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- ① Seems unavoidable to understand positive cc in string theory:
  - All string constructions are of high complexity [KKLT, Silverstein, ...]
  - Compactification data not a superselection sector [Coleman-de Luccia, Bousso-Polchinski, ...]
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  - large  $N$  = thermodynamic limit
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- ④ Approach to understand state space geometry quantum cosmology

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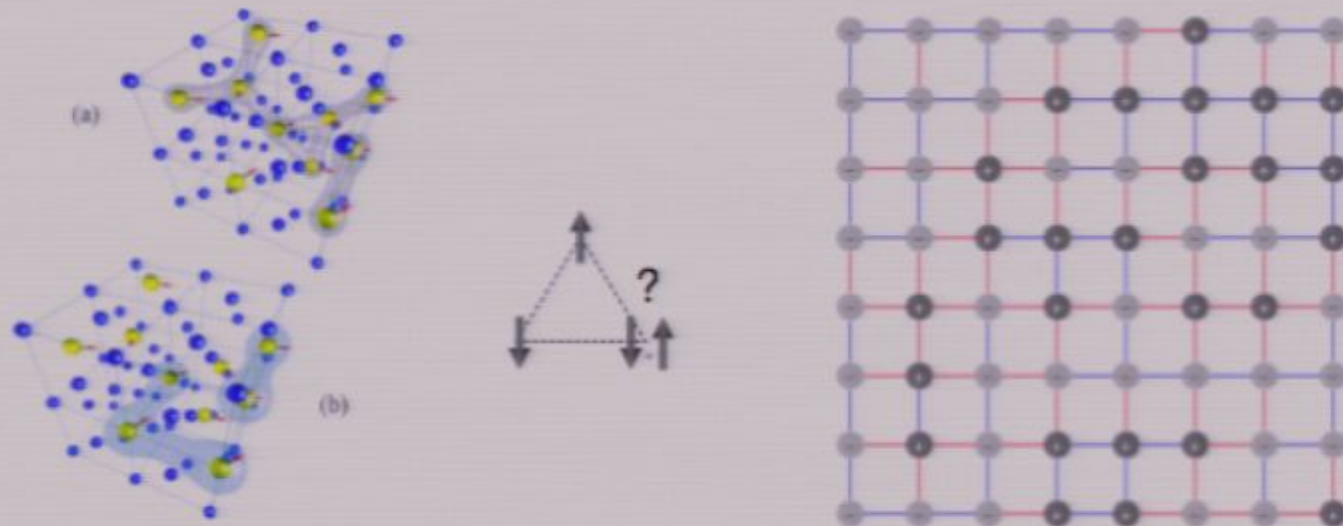
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- Spin glass models
- State space structure of the SK model
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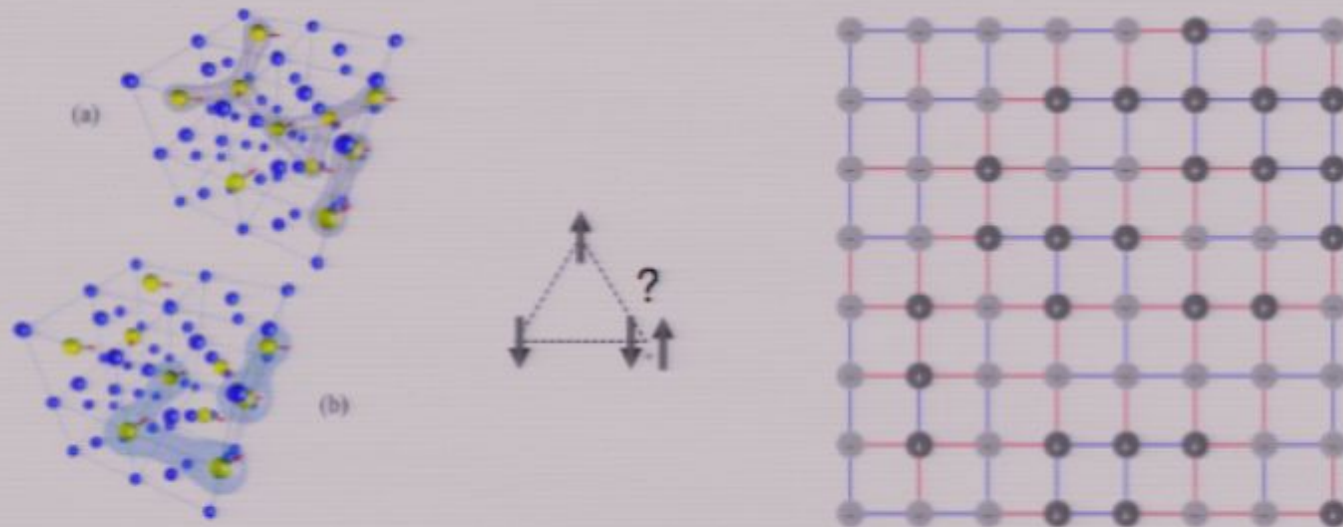
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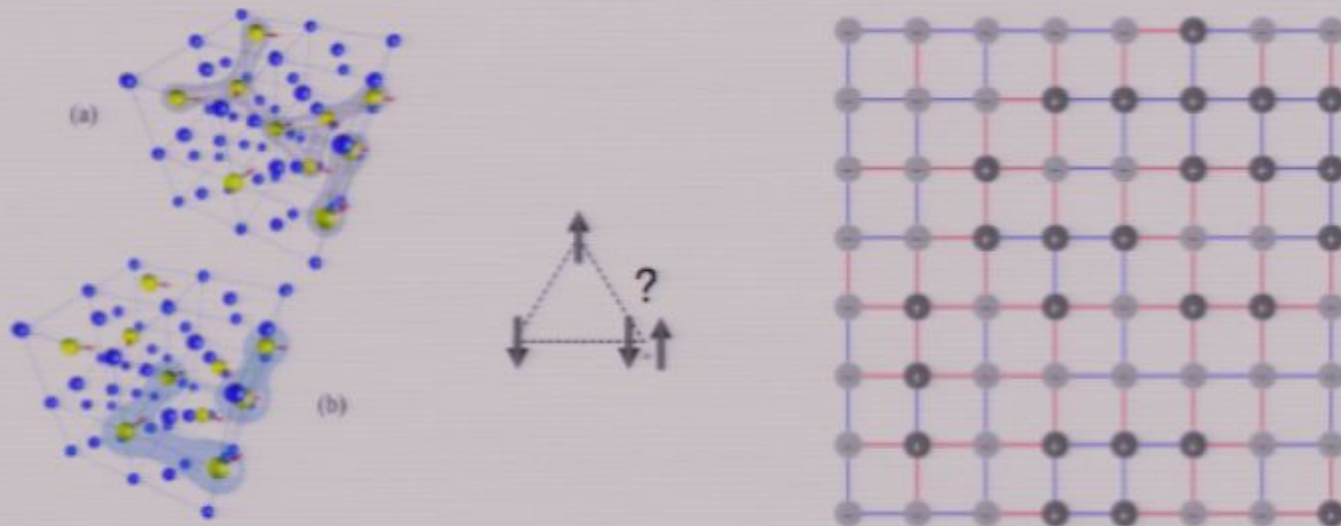


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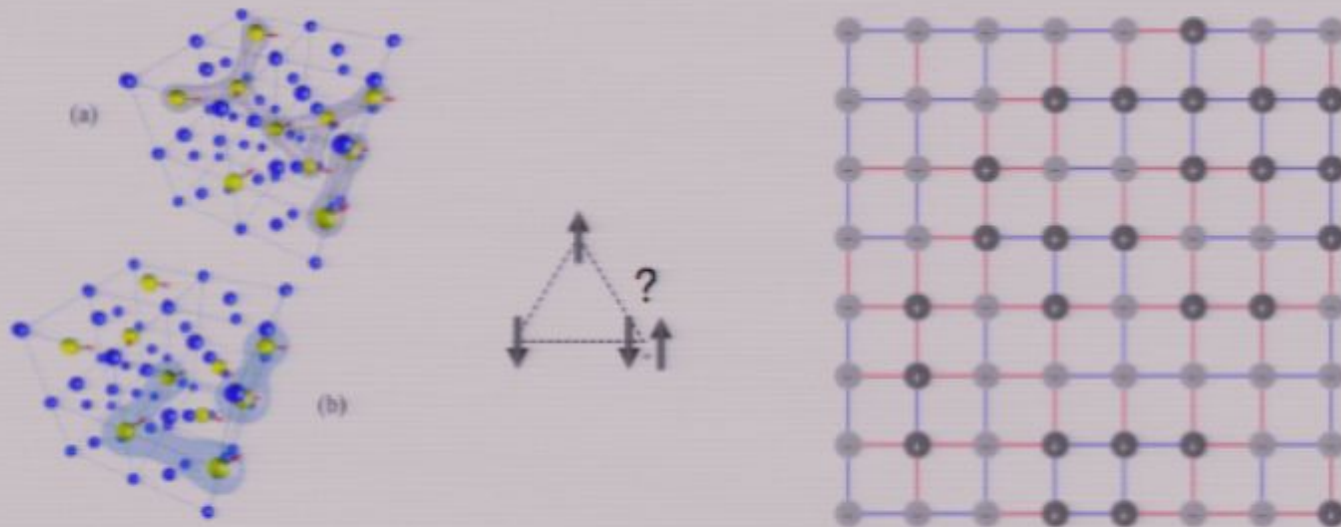
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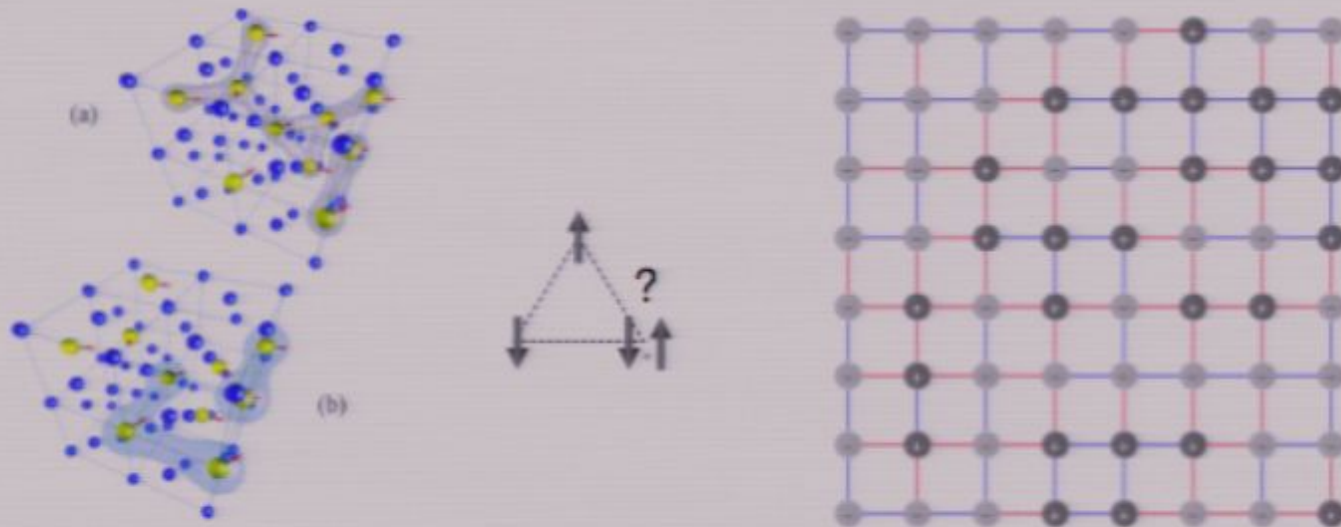
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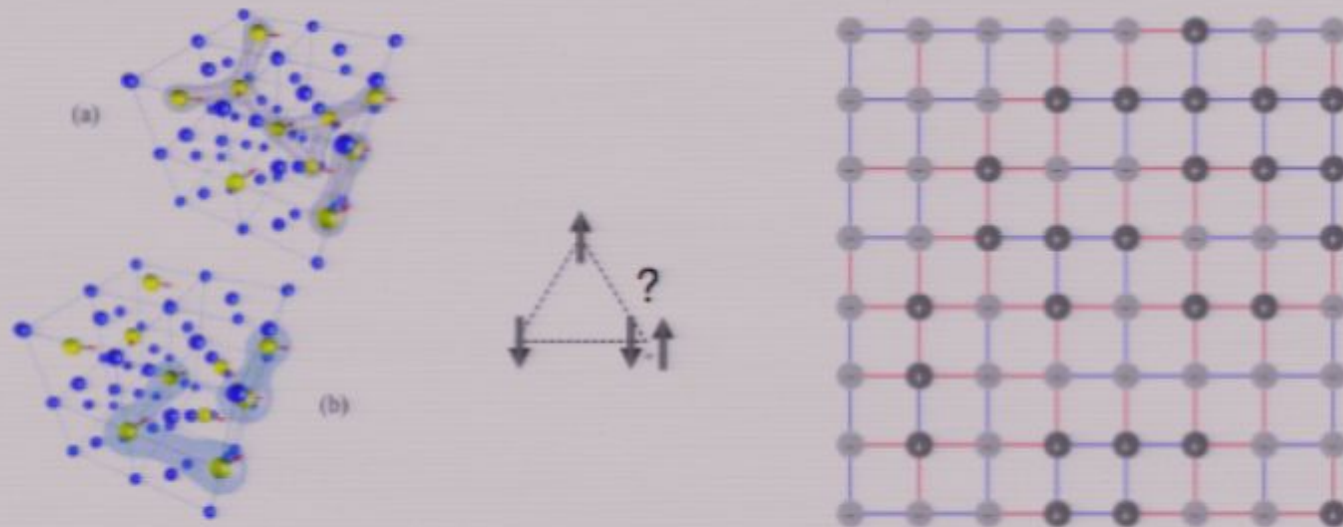
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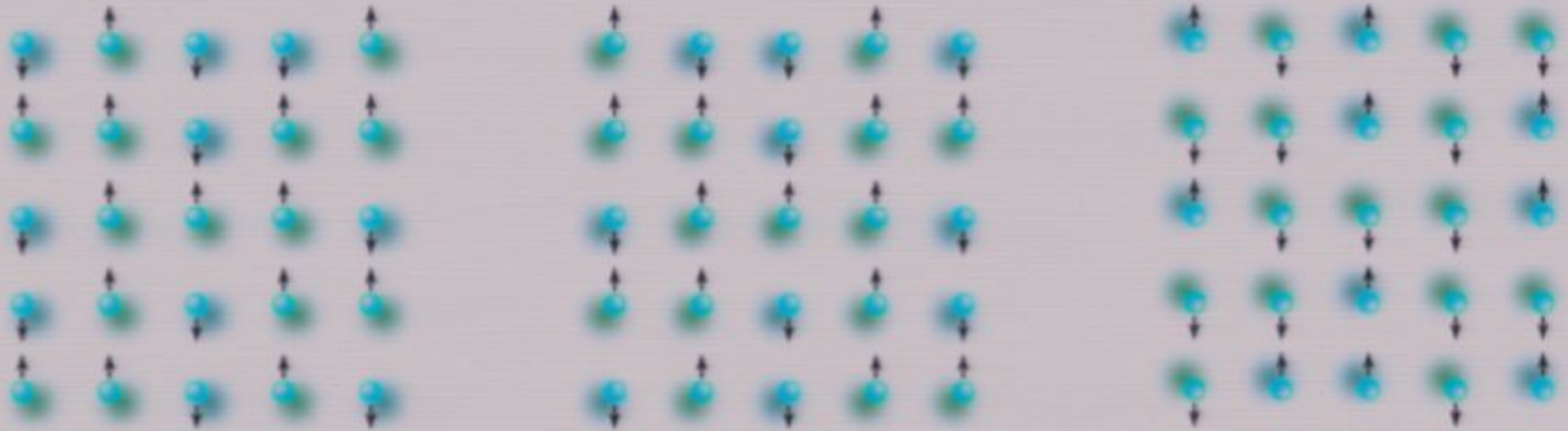
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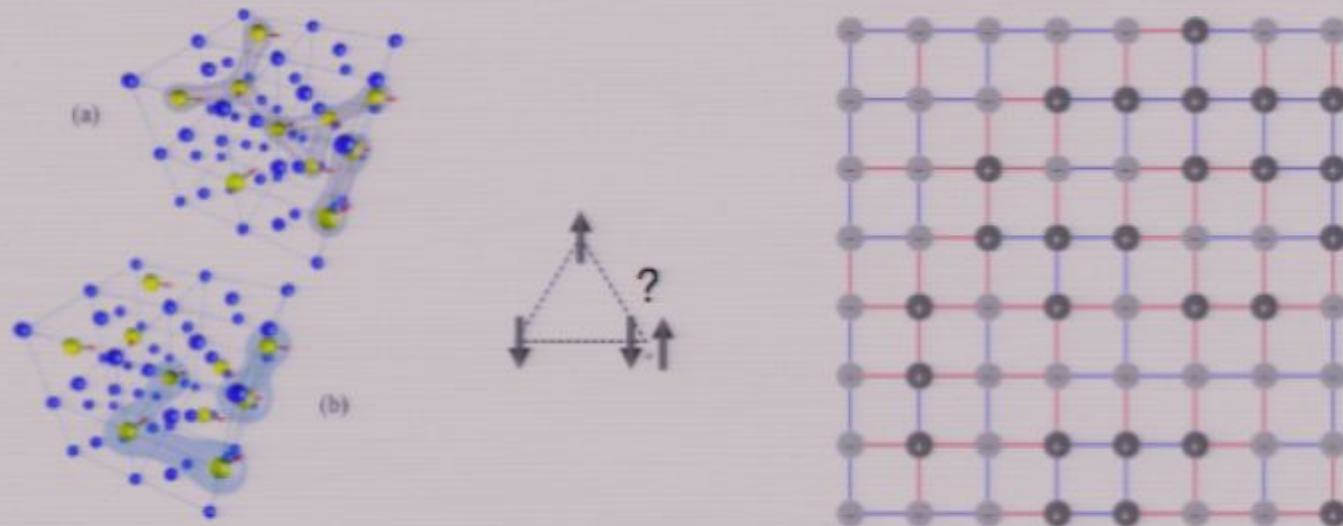
# State space structure of the SK model



- Below critical temperature, spins **freeze** = spin glass phase.
- Different possible freezing patterns possible  $\rightsquigarrow$  **equilibrium states**.
- Local magnetization in state  $\alpha$ :  $m_{i\alpha} \equiv \langle s_i \rangle_\alpha$  (depends on T).
- State **overlap**:

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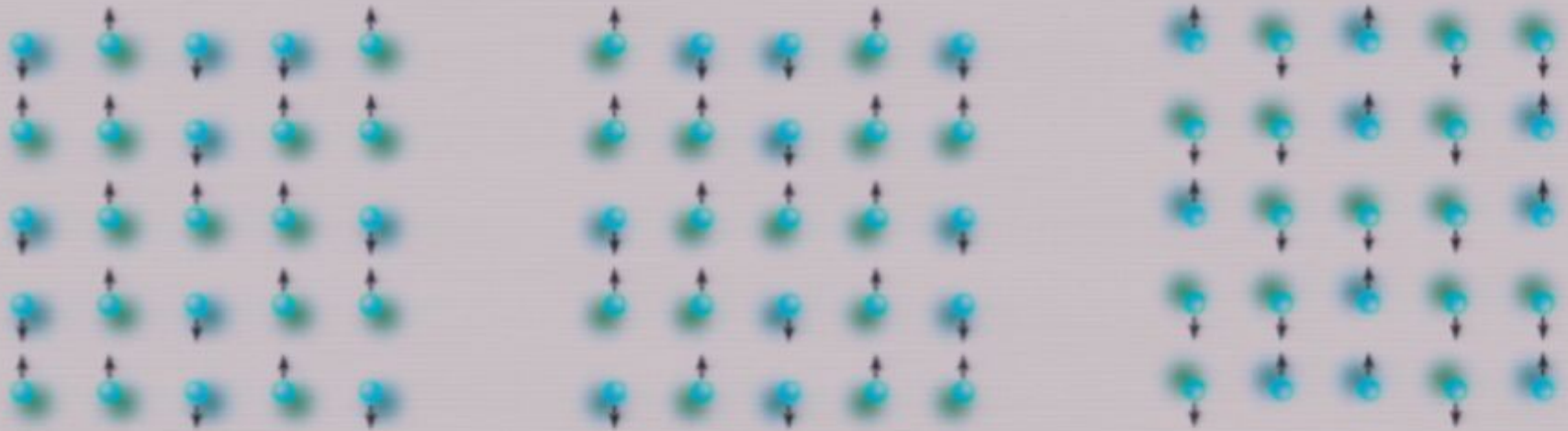


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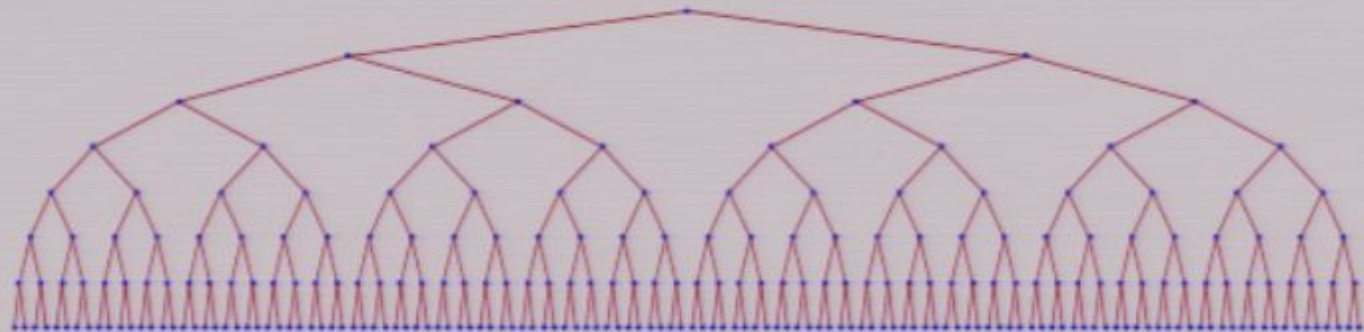


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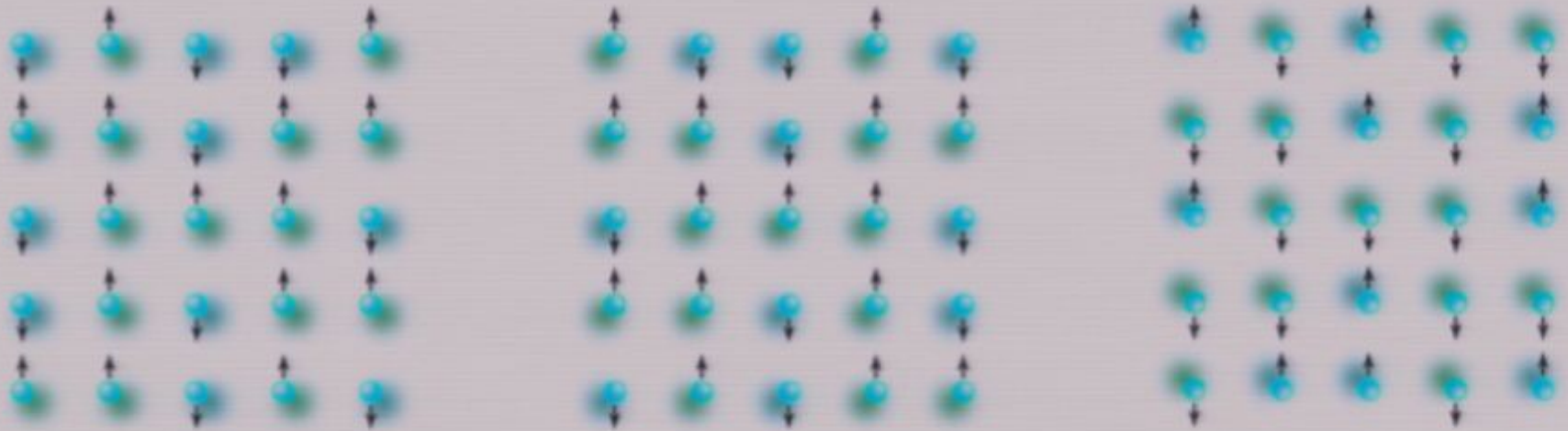
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  - $\Leftrightarrow \forall \alpha, \beta, \gamma : d_{\alpha\beta} \leq \max\{d_{\alpha\gamma}, d_{\beta\gamma}\}$ .
  - $\Leftrightarrow$  All triangles **isosceles**, with unequal side shortest (i.e. largest overlap)
  - $\Leftrightarrow$  States organized as leaves of **tree**.



Analogous to evolution tree: distance = time to common ancestor, equivalently DNA overlap.



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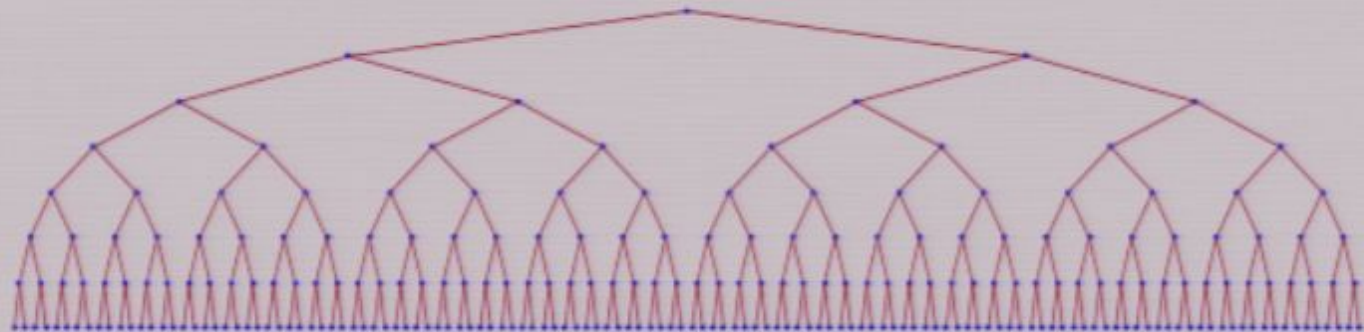
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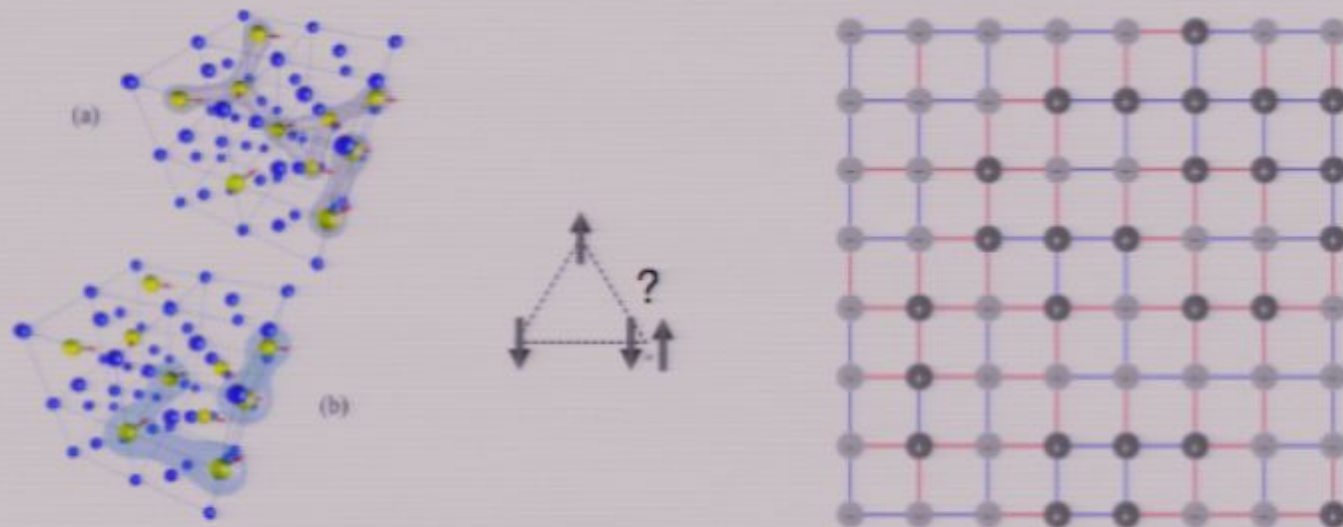
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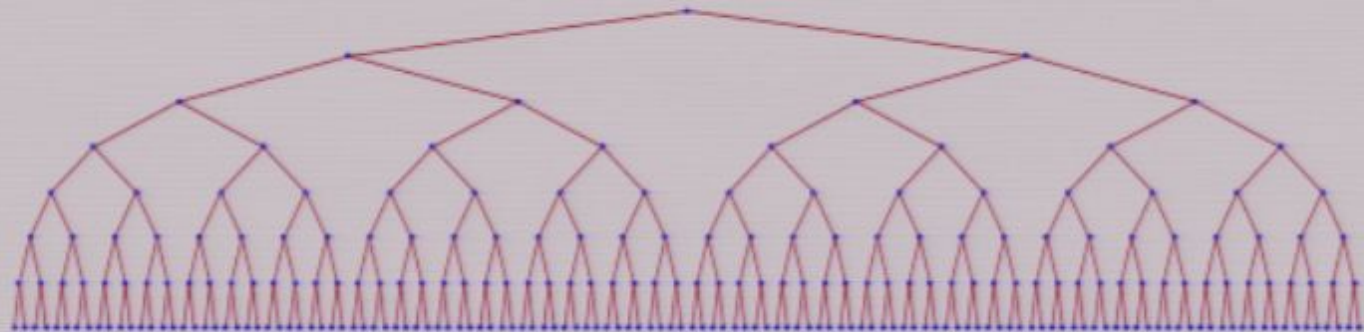
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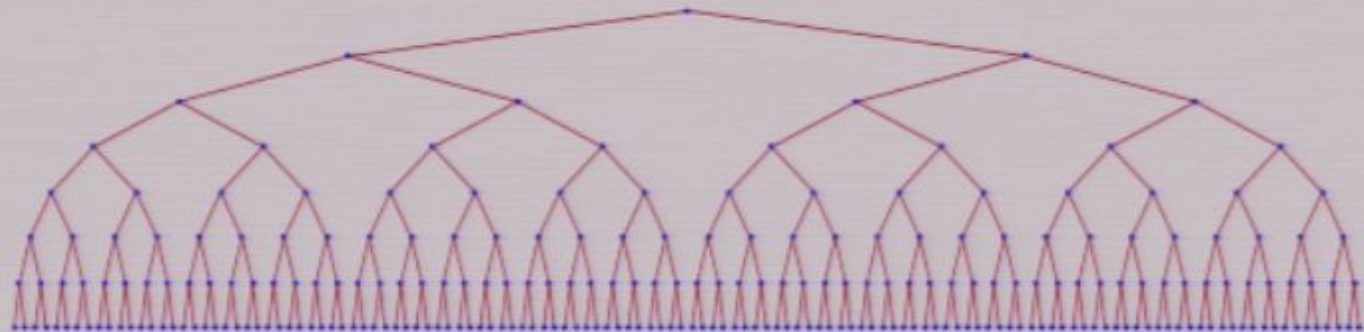
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- Arises purely statically in SK model.



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- Spin glass:  $p_G(s) = \sum_{\alpha} w_{\alpha} p_{\alpha}(s) \rightsquigarrow p_{\alpha}(s) = ?$ .  
 Also:  $M = 0$  for all  $T$ .



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- Definition equilibrium state ("pure state"):  $\rho_\alpha$  such that cluster decomposition holds:  $\lim_{N \rightarrow \infty} \frac{1}{N^2} \sum_{ij} |\langle \mathcal{O}_i \mathcal{O}_j \rangle_\alpha - \langle \mathcal{O}_i \rangle_\alpha \langle \mathcal{O}_j \rangle_\alpha| = 0$

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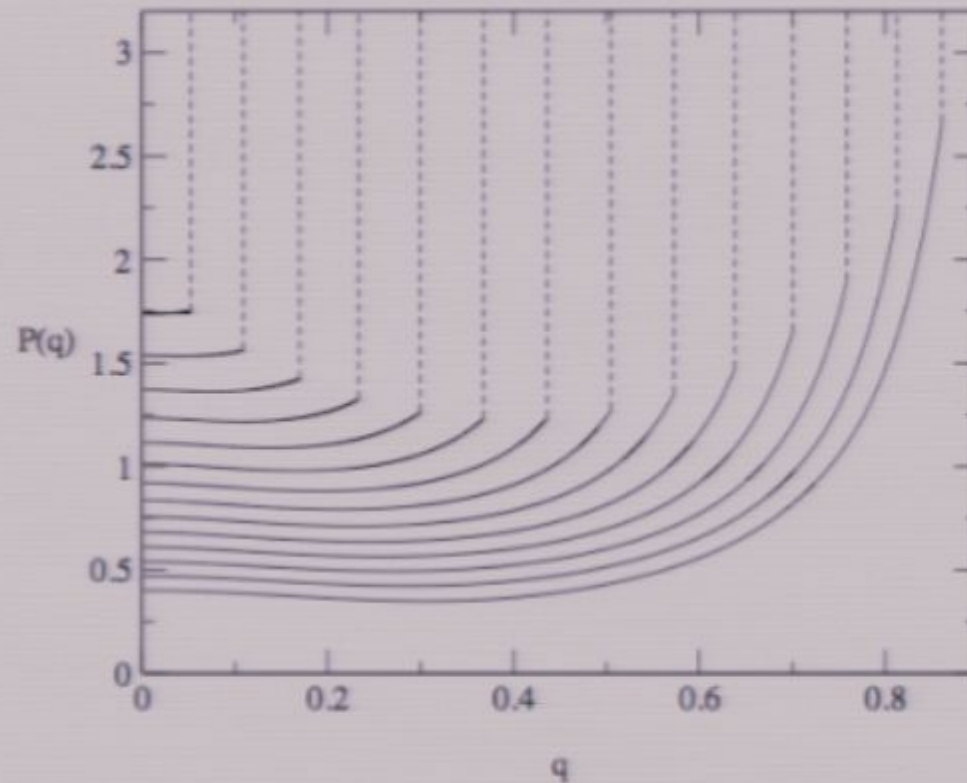
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- Multiple pure states  $\Leftrightarrow P(q)$  nontrivial.



# Result for SK model



$\overline{P(q)}$  for  $T/T_c = 0.95, 0.9, \dots, 0.3$  [Crisanti-Rizzo '02]

# Ultrametricity



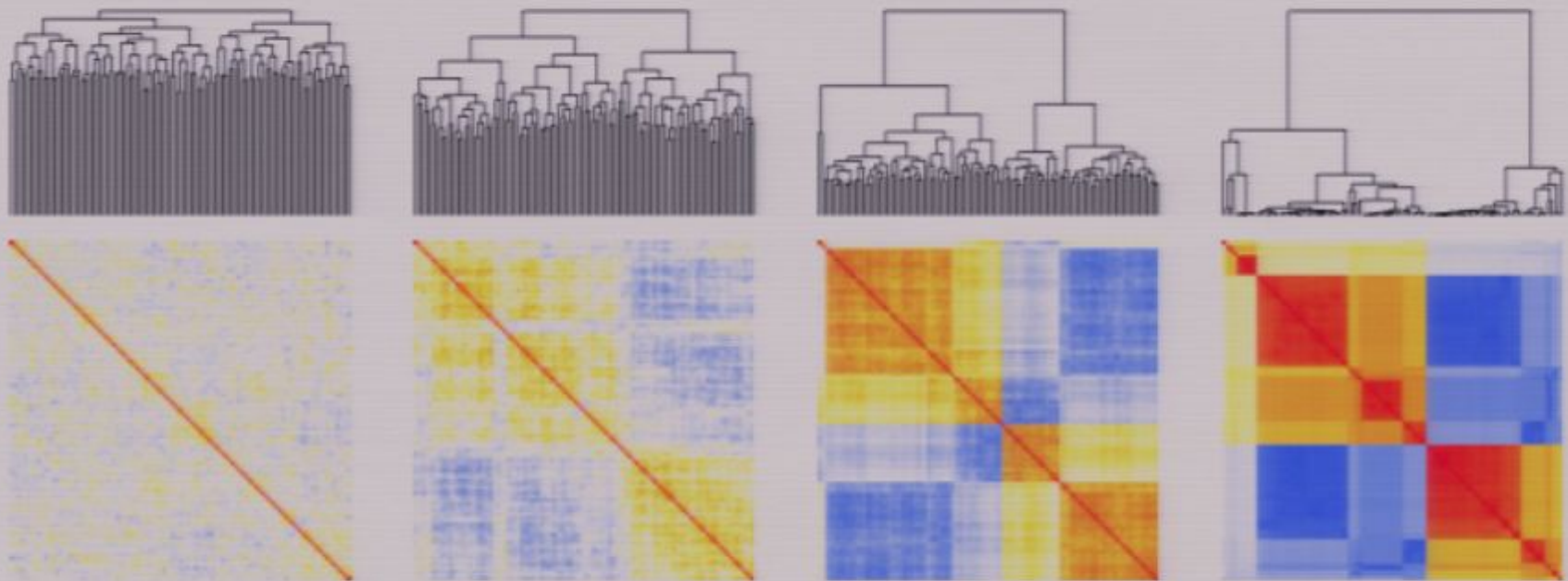
- Triangle distribution:

$$P(q_1, q_2, q_3) \equiv \sum_{\alpha\beta\gamma} w_\alpha w_\beta w_\gamma \delta(q_1 - q_{\beta\gamma}) \delta(q_2 - q_{\gamma\alpha}) \delta(q_3 - q_{\alpha\beta})$$

- Result for SK model,  $q_i \geq 0$ :

$$\begin{aligned} \overline{P(q_1, q_2, q_3)} &= \frac{1}{2} \int_0^{q_1} dq \overline{P(q)} \overline{P(q_1)} \delta(q_1 - q_2) \delta(q_2 - q_3) \\ &\quad + \frac{1}{2} \overline{P(q_1)} \overline{P(q_2)} \Theta(q_1 - q_2) \delta(q_2 - q_3) + \text{perm.} \end{aligned}$$

# Monte Carlo simulations



N=800 simulation at  $T/T_c = 1.2, 0.86, 0.55, 0.12$

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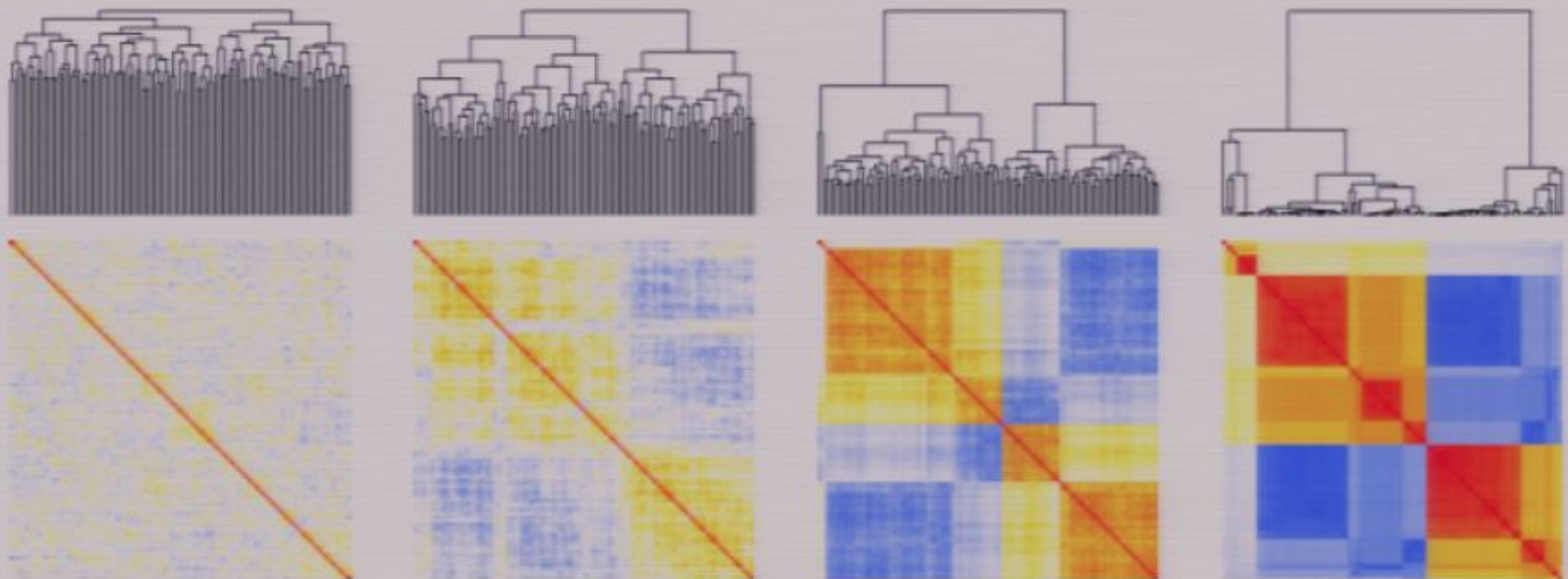
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General definition:

- $\phi_i, i = 1, \dots, N$  subset of d.o.f. of system.
- $\rho = \sum w_\alpha \rho_\alpha, \rho_\alpha$  s.t. cluster decomposition holds.
- $\rho_\alpha^i$  be reduced density matrix for  $\phi_i$  d.o.f.
- Overlap  $q_{\alpha\beta} \equiv \frac{1}{N} \sum_i \text{Tr} \rho_\alpha^i \rho_\beta^i$
- $P(q) = \sum_{\alpha\beta} w_\alpha w_\beta \delta(q - q_{\alpha\beta})$



## Generalized overlap order parameters

Generalization  $P(q)$  from classical Ising spins to general, nonlinear space and quantum systems?

Several possibilities

- ① whatever comes up naturally when integrating out quenched disorder
- ② overlap operators that violate cluster decomposition
- ③ reduced density matrices ←

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Replica formula:  $P(q) = \langle \delta(q - \frac{1}{N} \sum_i \chi_i) \rangle_{n=2}$ , where  $\chi_i$  is the operator exchanging  $\phi_i$  of the two replicas.



# String glasses

(Partial) results for:

# String glasses

(Partial) results for:

- Bousso-Polchinski with quenched random metric on flux lattice
  - analytic: replica symmetric solution
  - numeric: exchange monte carlo

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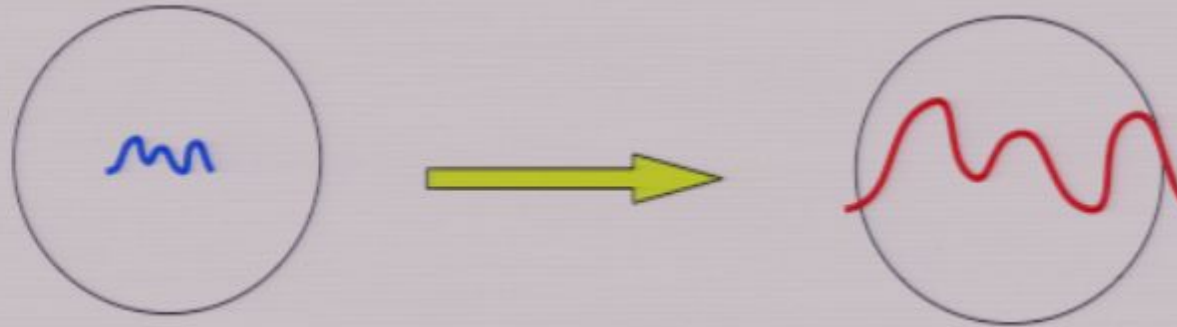
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# Cosmology



- Quantum fluctuations get exponentially stretched to super-Hubble scales and freeze, becoming effectively classical.
- This dynamically generates analog of spin glass “pure states”, in which scalars, metric,... have definite values on large scales and cluster decomposition holds. (Not the case for HH/BD/Euclidean vacuum.)
- Overlap distribution  $\rightarrow$  state space analysis without explicit description of clustering states.
- Useful quantity to compare quantum wave function of universe and “stochastic” measure approaches.

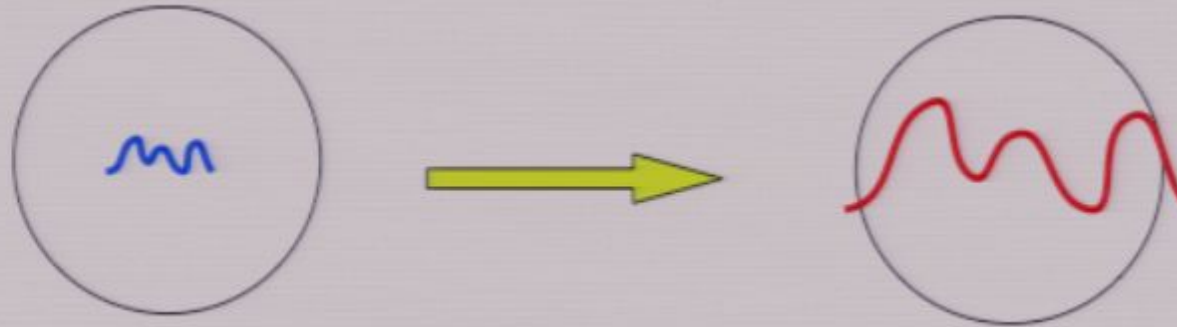


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# Cosmology

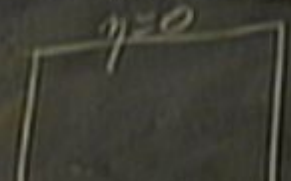


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(MASSLESS SCALAR IN  $dS_2$ )

$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$



$$x \in [0, 2\pi]$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 - \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$\langle \phi(x) \rangle = 0$   
 $\langle \phi(x) \phi(y) \rangle \sim \ln \text{Dis} \left( \frac{x-y}{2} \right)$

- This dynamically generated vacuum (which scalars, metric, cluster decomposition vacuum)
- Overlap distribution → description of clustering
- Useful quantity to compute "stochastic" measure and

Frederik Denef (Harvard, SCGP, Leuven) [Complexity]



(MASSLESS SCALAR IN  $dS_2$ )

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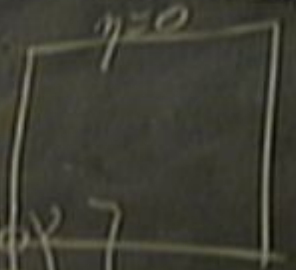
$\eta = 0$   
 $x \in [0, 2\pi]$   
 $\langle \phi(x) \rangle = 0$   
 $\langle \phi(x) \phi(y) \rangle \sim \ln \eta^{-2} \left( \frac{x-y}{2} \right)$

- This dynamically generated vacuum (which scalars, metric, cluster decomposition vacuum.)
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Frederik Denef (Harvard, SCGP, Leuven) [Sample]

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$$\langle \phi(x) \rangle = 0$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \frac{d_S(x,y)}{L^2}$$

- o This dyn... which se... cluster de... vacuum.)
- o Overlap of... description
- o Useful qu... "stochasti"

Frederik Denef (Harvard, 5)

$$d\vec{r}^2 = \frac{dy^2 + dx^2}{\eta^2}$$

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$



$$x \in [0, 2\pi]$$

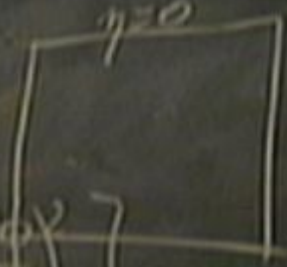
$$\langle \phi(x) \rangle = 0$$

$$\langle \phi(x) \phi(x') \rangle \sim \ln \frac{1}{|x-x'|}$$



(MASSLESS SCALAR IN  $ds_2$ )

$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$



$$x \in [0, 2\pi]$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$\langle \phi(x) \rangle = 0$   
 $\langle \phi(x) \phi(y) \rangle \sim \ln \sin\left(\frac{x-y}{2}\right)$

(MASSLESS SCALAR IN  $ds_2$ )

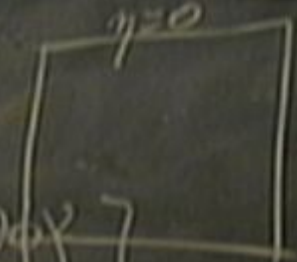
$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$\eta=0$   
 $x \in [0, 2\pi]$   
 $\langle \phi(x) \rangle = 0$   
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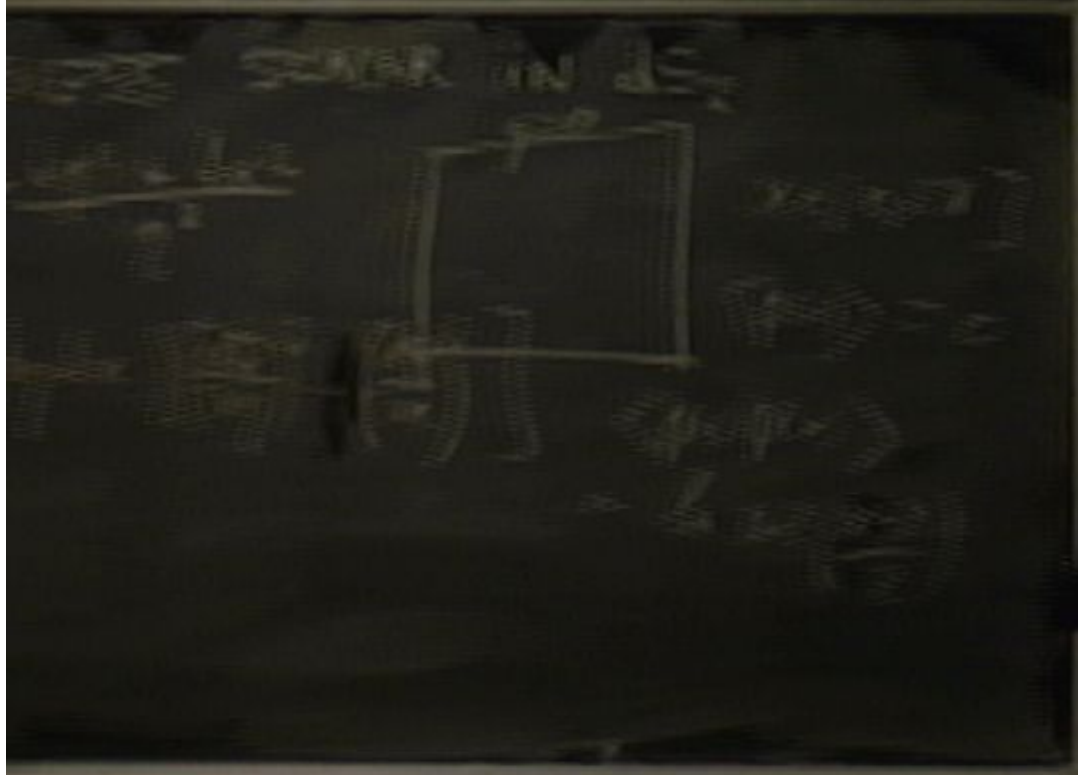
- Q
- sc
- T
- wh
- ch
- va
- Ov
- des
- Use
- 'st

Frederik Denef



scales and freeze, becoming effectively classical.

- This dynamically generates analog of spin glass "pure states", in which scalars, metric, ... have definite values on large scales and cluster decomposition holds. (Not the case for HH/BD/Euclidean vacuum.)
- Overlap distribution  $\rightarrow$  state space analysis without explicit description of clustering states.
- Useful quantity to compare quantum wave function of universe and "stochastic" measure approaches.



- Quantum fluctuations get exponential scales and freeze, becoming effective
- This dynamically generates analogies in which scalars, metric... have de Sitter cluster decomposition holds. (No vacuum...)
- Overlap distribution  $\rightarrow$  state space description of clustering states.
- Useful quantity to compare quantum "stochastic" measure approaches

# MASSLESS SCALAR IN $dS_2$

$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$

$$Z = \int d\eta dx \left[ \int_{\eta=0}^{\infty} \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$x \in [0, 2\pi]$   
 $\langle \phi(x) \rangle = 0$   
 $\langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{2} \frac{x-y}{2}$

- Quantum fluctuations get ex... scales and freeze, becoming...
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# MASSLESS SCALAR IN $dS_2$

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$$= \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

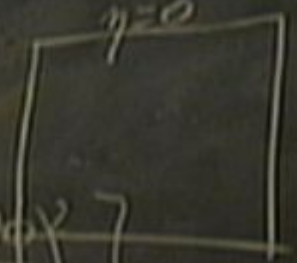
$x \in [0, 2\pi]$   
 $\langle \phi(x) \rangle = 0$   
 $\langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$

- Quantum fluctuations get extremely large at small scales and freeze, becoming classical (at large scales)
- This dynamically generates a state in which scalars, metric, ... have a cluster decomposition (cluster decomposition holds in vacuum.)
- Overlap distribution  $\rightarrow$  state description of clustering state
- Useful quantity to compare quantum and "stochastic" measure approaches

Frederik Denef (Harvard, SCGP, Leuven) Complexity and ...

(MASSLESS SCALAR IN  $dS_2$ )

$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$



$$x \in [0, 2\pi]$$

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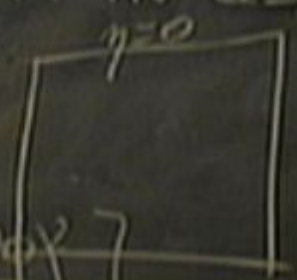
$$\langle \phi(x) \phi(y) \rangle \sim \ln \frac{2}{\eta^2} \left( \frac{x-y}{2} \right)$$

- Quantum fluctuations at small scales and from vacuum.
- This dynamical system which scalars cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

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$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$



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$$\langle \phi(x) \phi(y) \rangle \sim \ln \text{Li}_2 \left( \frac{x-y}{2} \right)$$

$\phi(x)$

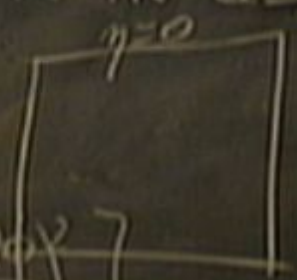
- Quantum fluctuations at small scales and from the vacuum.
- This dynamical system, which scalars cluster decorrelate from the vacuum.)
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- Useful quantities are "stochastic" processes.

Frederik Denef (Harvard, SCGP)



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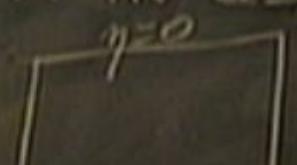
$$= \sum_n \dots$$

- Quantum fluctuations at small scales and from vacuum.)
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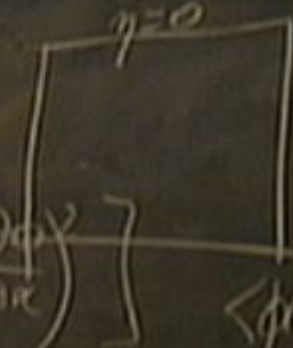
$$\phi(x) = \sum_n \phi_n$$

- Quantum fluctuations at small scales and frequencies
- This dynamical system exhibits a phase transition (in which scalars cluster decorrelate from the vacuum.)
- Overlap distribution is a useful description of the system
- Useful quantities are "stochastic" processes

Frederik Denef (Harvard, SCGP)

(MASSLESS SCALAR IN  $dS_2$ )

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$$\int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\langle \phi(x) \rangle = 0$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \text{Li} e^{\frac{x-y}{2}}$$

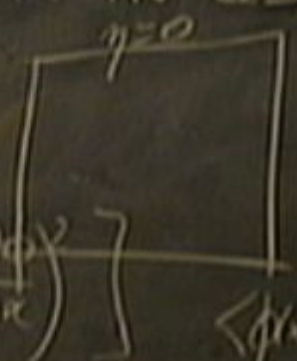
- Quantum fluctuations at small scales and from vacuum.)
- This dynamical system (in which scalars cluster decorrelate from vacuum.)
- Overlap distribution of states (description of entanglement)
- Useful quantities (e.g. "stochastic" processes)

Frederik Denef (Harvard, SCGP)



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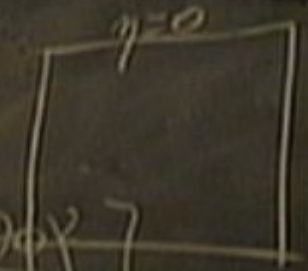
$$\langle \phi(x) \phi(y) \rangle \sim \ln \sin\left(\frac{x-y}{2}\right)$$

- Quantum fluctuations at small scales and from the vacuum.)
- This dynamical system, which scalars cluster decorrelate vacuum.)
- Overlap distribution as a description of the vacuum state.
- Useful quantities are "stochastic" processes.

Frederik Denef (Harvard, SCGP)

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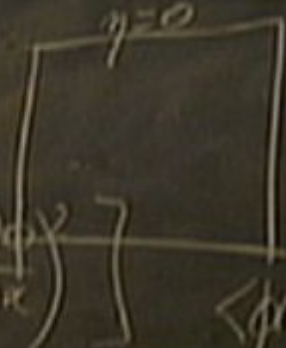
$$\langle \phi(x) \phi(y) \rangle \sim \ln \sin\left(\frac{x-y}{2}\right)$$

- Quantum fluctuations on small scales and from the vacuum.
- This dynamical system, which scalars cluster decorrelate from the vacuum.)
- Overlap distribution of the vacuum description of the system.
- Useful quantities are "stochastic" processes.

Frederik Denef (Harvard, SCGP)

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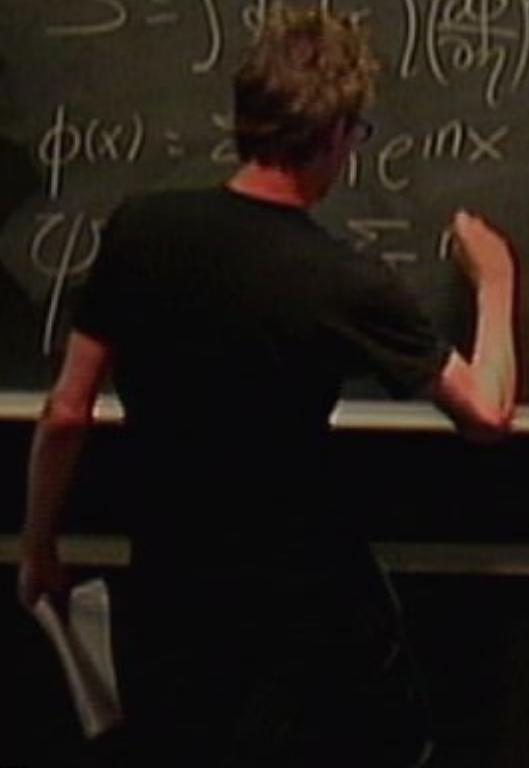


$$x \in [0, 2\pi]$$

$$S = \int d\eta dx \left[ \frac{1}{2} \left( \frac{\partial \phi}{\partial \eta} \right)^2 - \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \quad \langle \phi(x) \rangle = 0$$

$$\phi(x) = \sum_n a_n e^{inx}$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$



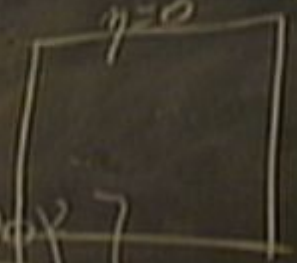
- Quantum fluctuations at small scales and from the vacuum.
- This dynamical system exhibits a phase transition which scalars cluster decorrelate from the vacuum.)
- Overlap distribution of the ground state provides a description of the geometry of the vacuum.
- Useful quantities are "stochastic" processes.

Frederik Denef (Harvard, SCGP)



(MASSLESS SCALAR IN  $dS_2$ )

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$$S = \int \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right]$$

$$\langle \phi(x) \rangle = 0$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \text{sinc} \left( \frac{x-y}{2} \right)$$

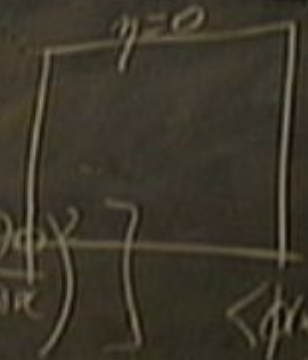
$$\phi(x) = \sum_n \frac{1}{\sqrt{2\pi}} \eta^{1/2} e^{inx}$$

- Quantum fluctuations scales and fractal
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

(MASSLESS SCALAR IN  $dS_2$ )

$$ds^2 = -\frac{d\eta^2 + dx^2}{\eta^2}$$



$$x \in [0, 2\pi]$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \quad \langle \phi(x) \rangle = 0$$

$$\phi(x) = \int \eta e^{inx}$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \sin\left(\frac{x-y}{2}\right)$$

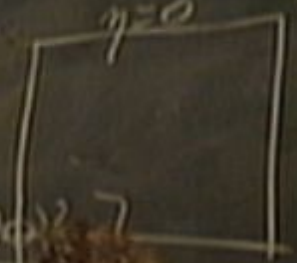
$$e^{-\sum_n \eta |\phi_n|^2}$$

- Quantum fluctuations scales and fr...
- This dynamical which scalars cluster decor vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

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$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$\langle \phi(x) \phi(y) \rangle \propto \ln \Delta_{dS_2} \left( \frac{x-y}{2} \right)$$

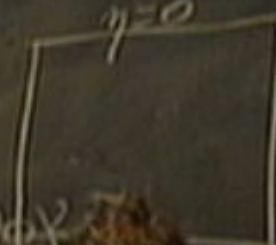
- Quantum fluctuations at small scales and frequencies.
- This dynamical system exhibits a phase transition (in which scalars cluster decorrelate from the vacuum.)
- Overlap distribution is a useful description of the "stochastic" nature of the system.

Frederik Denef (Harvard, SCGP)



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$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

- Quantum fluctuations at small scales and frequencies
- This dynamical system exhibits a phase transition (in which scalars cluster decorrelate from the vacuum.)
- Overlap distribution is a useful description of the system
- Useful quantities are "stochastic" processes

Frederik Denef (Harvard, SCGP)

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$$\phi(x) = \sum_n \phi_n e^{inx} \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta d\alpha \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial \alpha} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate vacuum.)
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$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
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Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$\psi_{12}$

- Quantum fluctuations scales and fr
- This dynamical which scalars cluster decor vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int dy dx \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \sim \ln \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$\phi_{12}$

- Quantum fluctuations scales and frequencies
- This dynamical system which scalars cluster decomposition (vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



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$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \frac{1}{|x-y|}$$

$$P_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$Q_{12}$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution description of
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Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \text{dis} \left( \frac{x-y}{2} \right)$$

$$\psi(x) = e^{-\sum_n n |\phi_n|^2}$$

$\mathcal{Q}_{12}$

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Frederik Denef (Harvard, SCGP)

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$Q_{12}$

- Quantum fluctuations scales and frequencies
- This dynamical system which scalar fields cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



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$\psi_0$

- Quantum fluctuations scales and fr...
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- Frederik Denef (Harvard, SCGP)

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$Q_{12}$

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- This dynamical system (which scalar fields cluster decorrelate vacuum.)
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Frederik Denef (Harvard, SCGP)

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$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$\phi_{12}$

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Frederik Denef (Harvard, SCGP)



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$\psi_0$

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- This dynamical system (which scalars cluster decorrelate vacuum.)
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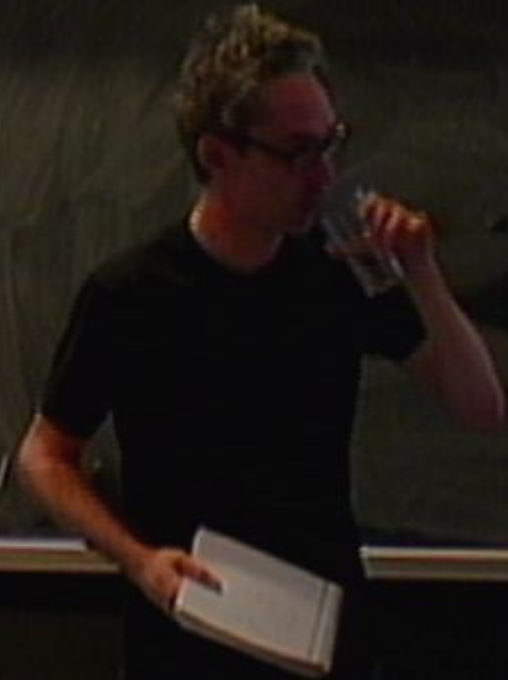


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$Q_{12}$



- Quantum fluctuations (scales and frequencies)
  - This dynamical system (which scalars cluster decorrelate vacuum.)
  - Overlap distribution (description of)
  - Useful quantities ("stochastic")
- Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

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$Q_{12}$

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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 - \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

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$$\langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

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$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$



$\phi_{12}$

- Quantum fluctuations at small scales and frequencies.
- This dynamical system is characterized by a set of scalar fields which cluster decorrelate in the vacuum.
- Overlap distribution is a useful description of the system.
- Useful quantities are "stochastic" variables.

Frederik Denef (Harvard, SCGP)



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$Q_{12}$

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$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$Q_{12}$

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$$\phi(x) = \sum_n \phi_n e^{inx} \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$Q_{12}$

- Quantum fluctuations at small scales and frequencies.
- This dynamical system exhibits a phase transition which separates a phase with scalar cluster decomposition from a phase with a vacuum state.
- Overlap distribution function provides a description of the phase transition.
- Useful quantities are "stochastic" variables.

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \text{dist} \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$\rho_{12}$

- Quantum fluctuations scales and fr
- This dynamical which scalars cluster decor vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{2} e^{\frac{x-y}{2}}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
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- Useful quantities "stochastic"

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$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} \int d\eta$$

- Quantum fluctuations scales and fr...
- This dynamical description which scalars cluster decor vacuum.)
- Overlap distribution description of...
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int \phi''(x)$$

- Quantum fluctuations at small scales and frequencies
- This dynamical system exhibits a phase transition (which scalars cluster decorrelate in the vacuum.)
- Overlap distribution provides a description of the ground state
- Useful quantities are "stochastic" variables

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left[ \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi(x) \phi(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
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$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \frac{dx}{2}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system which scalars cluster decomposition (vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

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$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

- Quantum fluctuations scales and fr...
  - This dynamical system (with scalar fields cluster decomposition vacuum.)
  - Overlap distribution description of...
  - Useful quantities "stochastic" ...
- Frederik Denef (Harvard, SCGP)

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$



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$$C_2 = \int dx \phi^{(n)}(x) \phi^{(n)}(x)$$

- Quantum fluctuations scales and fr...
  - This dynamical system (which scalar fields cluster decorrelate in vacuum.)
  - Overlap distribution description of...
  - Useful quantities "stochastic"
- Frederik Denef (Harvard, SCGP)



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$$Q = \int dx \phi^{(n)}(x) \phi^{(n')}(x)$$

- Quantum fluctuations at small scales and frequencies.
- This dynamical system is characterized by a set of scalar fields which cluster decorrelate in the vacuum.)
- Overlap distribution provides a description of the ground state.
- Useful quantities are "stochastic" variables.

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta d\alpha \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial \alpha} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

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$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

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- Quantum fluctuations (scales and frequencies)
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution (description of)
- Useful quantities ("stochastic")

Frederik Denef (Harvard, SCGP)



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- Quantum fluctuations at small scales and frequencies
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- Quantum fluctuations (scales and frequencies)
- This dynamical system (which scalars cluster decorrelates vacuum.)
- Overlap distribution (description of)
- Useful quantities ("stochastic")

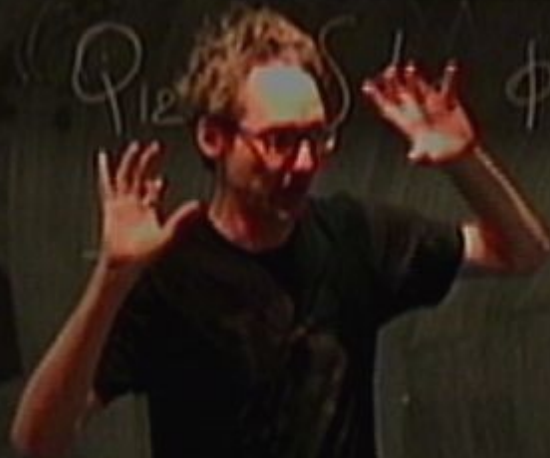
Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|} \left( \frac{x-y}{2} \right)$$

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$$Q_{12} \phi^{(1)}(x) \phi^{(2)}(x)$$



- Quantum fluctuations scales and fractal dimensions
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution description of criticality
- Useful quantities "stochastic" growth

Frederik Denef (Harvard, SCGP)

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- Quantum fluctuations scales and frequencies
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- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard/SCGP)



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- Quantum fluctuations at small scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate in vacuum.)
- Overlap distribution as a description of the vacuum state
- Useful quantities are "stochastic" variables

Frederik Denef (Harvard, SCGP)



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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
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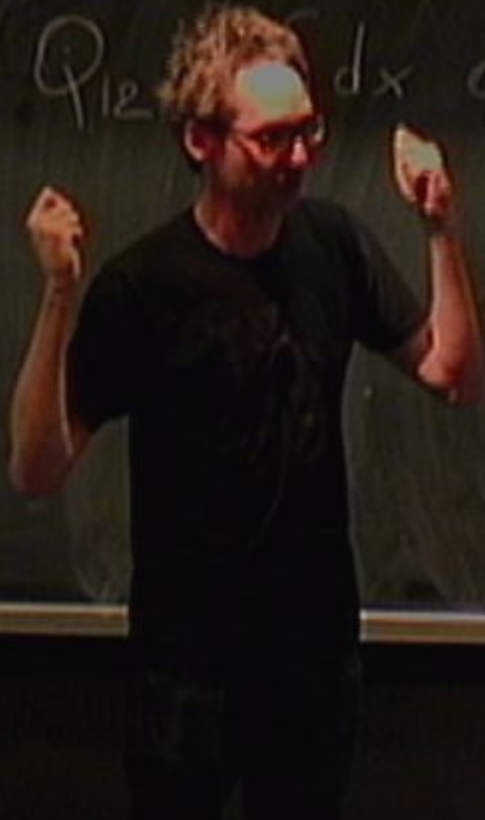
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$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$



- Quantum fluctuations (scales and frequencies)
- This dynamical system (which scalar fields cluster decorrelate in vacuum.)
- Overlap distribution (description of the ground state)
- Useful quantities ("stochastic")

Frederik Denef (Harvard, SCGP)

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- Quantum fluctuations scales and fr
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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
- Overlap distribution description of
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- This dynamical system (which scalars cluster decorrelate in vacuum.)
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- Quantum fluctuations at small scales and frequencies
- This dynamical system (which scalars cluster decorrelate in vacuum.)
- Overlap distribution description of the ground state
- Useful quantities are "stochastic" variables

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- Quantum fluctuations scales and fr...
- This dynamical description which scalars cluster decor...
- Overlap distribution description of...
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- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
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$$\langle \phi^m(x) \phi^{m'}(x) \rangle = \int dx \phi^m(x) \phi^{m'}(x)$$



- Quantum fluctuations scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate in vacuum.)
- Overlap distribution description of the ground state
- Useful quantities "stochastic" process

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- Quantum fluctuations at small scales and frequencies
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$$\langle \phi^n \rangle = \int dx \phi^n(x) \phi^{(n)}(x)$$

- Quantum fluctuations at small scales and frequencies
- This dynamical system (with scalar fields) exhibits cluster decomposition in the vacuum.
- Overlap distribution provides a description of the vacuum state.
- Useful quantities are "stochastic" variables.

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- Quantum fluctuations scales and frequencies
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$$= \int \phi^{(n)}(x) \phi^{(n')}(x)$$

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$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and fractal dimensions
- This dynamical system (which scalar fields cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic" process

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \text{Dis} \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$= \int dx \phi^{(n)}(x) \phi^{(n')}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta d\alpha \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial \alpha} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int d\phi \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations (scales and frequencies)
- This dynamical system (which scalars cluster decomposition (vacuum).)
- Overlap distribution (description of)
- Useful quantities ("stochastic")

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la sine}\left(\frac{x-\gamma}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalars cluster decomposition vacuum.)
- Overlap distribution description of
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Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{in x} \quad \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations scales and fr
- This dynamical which scalars cluster decor vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

- Quantum fluctuations (scales and frequencies)
- This dynamical system (which scalars cluster decorrelate in vacuum.)
- Overlap distribution (description of)
- Useful quantities ("stochastic")

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{in x} \quad \sim \ln \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n$$

- Quantum fluctuations scales and fr
- This dynamical which scalars cluster decor vacuum.)
- Overlap distribution description of
- Useful quantities "stochastic"

Frederik Denef (Harvard, SCGP)

$$S = \int d\eta dx \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} c_n^{(2)}$$

- Quantum fluctuations scales and fr...
- This dynamical system (which scalars cluster decor...
- Overlap distribution description of...
- Useful quantities "stochastic" ...

Frederik Denef (Harvard, SCGP)



$$S = \int d\eta dx \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int \phi_n^{(1)} \phi_{-n}^{(2)}$$

- Quantum fluctuations scales and frequencies
- This dynamical system (which scalar fields cluster decorrelate vacuum.)
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Frederik Denef (Harvard, SCGP)

$$S = \int dy dx \left( \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{in x} \quad \sim \text{la} \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$



$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$\sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$



$$S = \int dy dx \left( \left( \frac{\partial \phi}{\partial t} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$(x)$

$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{La sine} \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{iQ} \rangle$$

$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \ln \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q} \rangle$$



$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \quad | \langle \phi(x) \rangle = 0$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$\langle \phi^{(1)} \phi^{(2)} \rangle = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$\sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$|\phi\rangle$

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \int \phi(x) \phi(x) - 0$$

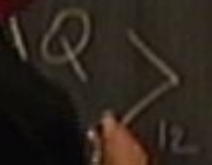
$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$



$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\sim \frac{1}{\sqrt{2}} \exp\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$\langle Q_{12} \rangle$



$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \int \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la } \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{1 \leq 2}$$

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + V(\phi) \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{in x} \quad \sim \text{la } \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle$$

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + V(\phi) \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{in x} \quad \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$\langle \phi^{(1)} \phi^{(2)} \rangle = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int dx \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\phi} \rangle$$



$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + V(\phi) \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la } \text{la } \text{la } e^{i(x-y)/2}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int \phi_n^{(1)} \phi_{-n}^{(2)}$$

$\langle e^{i\phi} \rangle$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 + V(\phi) \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{like } \text{se}^{-\frac{|x-y|}{2}}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2}$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 - V(\phi) \right] \quad \langle \phi(x) \phi(y) \rangle$$

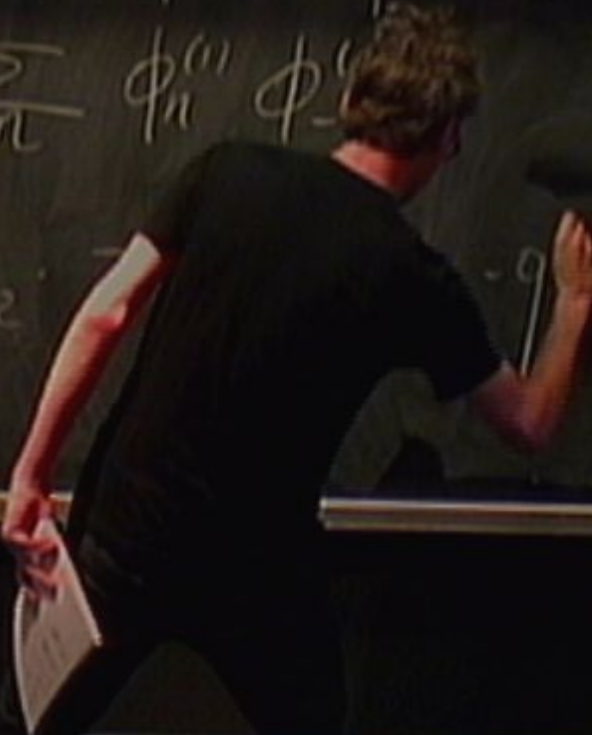
$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \frac{1}{|x-y|}$$

$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2}$$





$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \int dx \langle \phi(x) \phi(x) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \frac{1}{|x-y|}$$

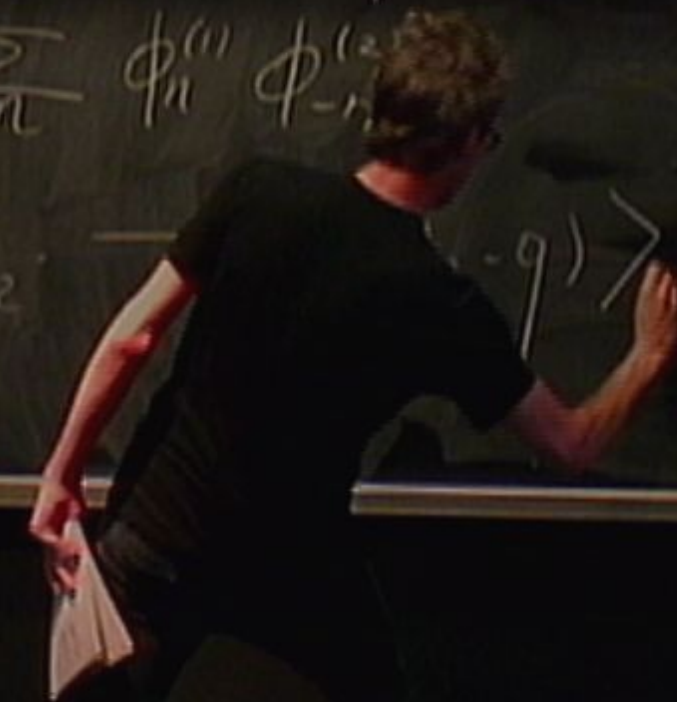
$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2}$$

$$| -q \rangle \rangle$$



$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \operatorname{disc} \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \frac{1}{|x-y|}$$

$$\Psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$



$$S = \int dy dx \left[ \left( \frac{\partial \Phi}{\partial y} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right] \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Y_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{iY_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int dy dx \left( \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \ln \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$= \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle \int dx Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$



$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle \delta(Q_{12}) \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right] \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la sine} \left( \frac{x-y}{2} \right)$$

$$\Psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$



$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la sine} \left( \frac{x-y}{2} \right)$$

$$\Psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

No Signal  
VGA 1

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\Psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle \delta(Q_{12} - q) \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

No Sign  
V04.1

$$S = \int dy dx \left( \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

No Signal  
VGA 1



$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle \sim \ln \sin\left(\frac{x-y}{2}\right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle Q_{12} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

in Sign  
VSA.1

$$S = \int d\eta dx \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

No Signal

VGA-1

$$S = \int dy dx \left( \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

to be  
V04.1



$$S = \int d\eta dx \left[ \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle \sim \ln \sin^2 \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

10/10/16

V04.1

$$S = \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] - \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle \delta(Q_{12} - q) \rangle_{n=2}$$



$$S = \int d\eta dx \left[ \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle \xrightarrow{n \rightarrow 2} \langle \delta(\phi) \rangle$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1/2}$$

No Signal

USA 1

$$S = \int dy dx \left[ \left( \frac{\partial \phi}{\partial t} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right] \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1/2}$$

No Signal  
VSA1

$$\int \int d\eta dx \left( \left( \frac{\partial \Phi}{\partial \eta} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{tr} e^{\frac{(x-y)}{2}}$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1/2}$$



$$S = \int dy dx \left( \left( \frac{\partial \Phi}{\partial y} \right)^2 + \left( \frac{\partial \Phi}{\partial x} \right)^2 \right) \langle \phi(x) \phi(y) \rangle \sim \ln \operatorname{sinc} \left( \frac{x-y}{2} \right)$$

$$\phi(x) = \sum_n \phi_n e^{inx}$$

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$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

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$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle e^{-i\lambda Q_{12}} \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]$$

$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin^2\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

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$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

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$$S = \int d\eta dx \left( \left( \frac{\partial \phi}{\partial \eta} \right)^2 + \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \quad \langle \phi(x) \phi(y) \rangle$$

$$\phi(x) = \sum_n \phi_n e^{inx} \quad \sim \text{la} \sin\left(\frac{x-y}{2}\right)$$

$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \delta(Q_{12} - q) \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1/2}$$



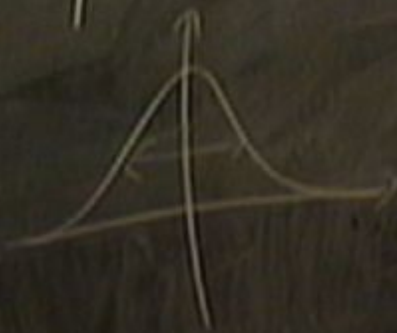
$$P(q) \propto \frac{1}{\cosh^2 q}$$



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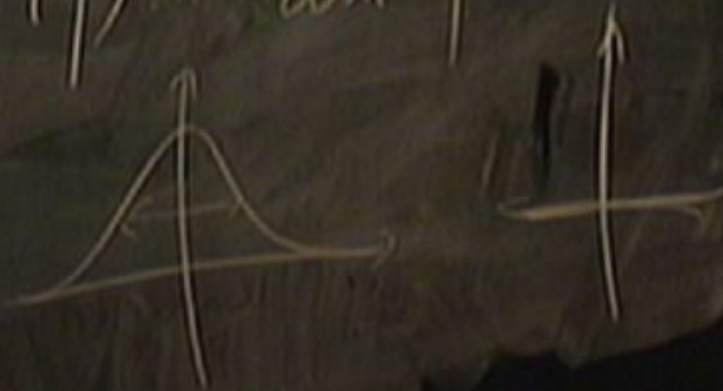


$$P(q) \propto \frac{1}{\cosh^2 q}$$

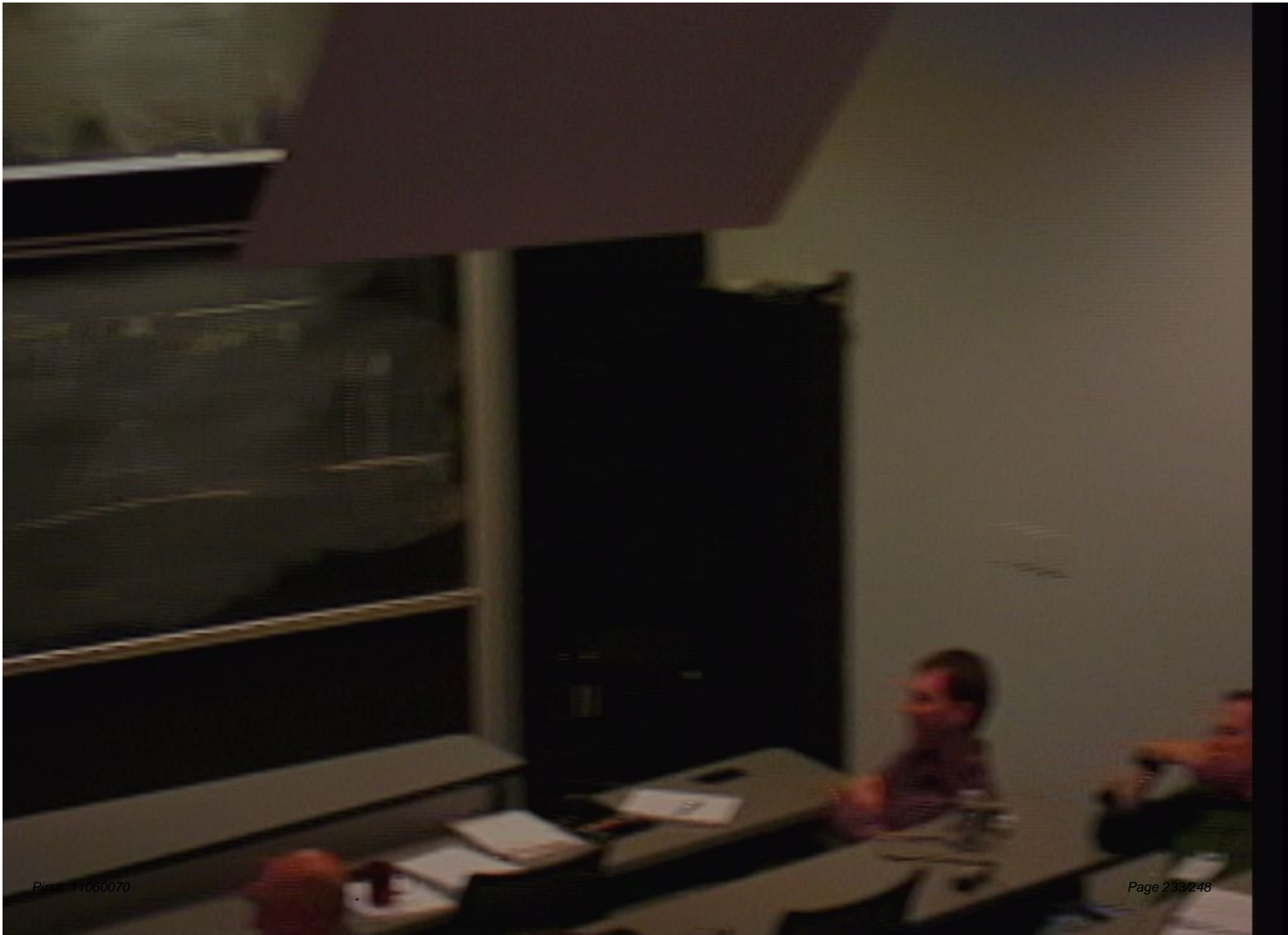




$$P(q) \propto \frac{1}{\cosh^2 q}$$









$$\phi(x) = \sum_n \phi_n e^{inx} \quad \langle \phi(x) \phi(y) \rangle \sim \ln \frac{1}{|x-y|}$$

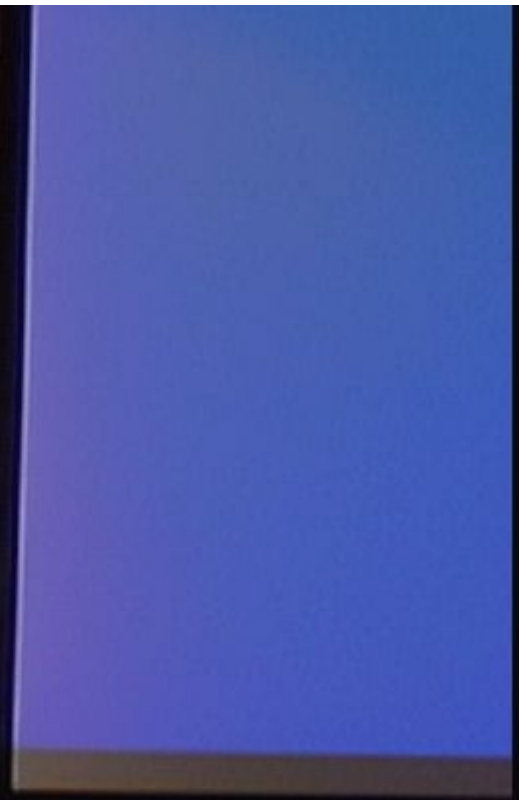
$$\psi_0 \propto e^{-\sum_n \eta |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)} \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{g=2} = \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right)$$

$$\langle (Q_{12} - g) \rangle_{g=2}$$



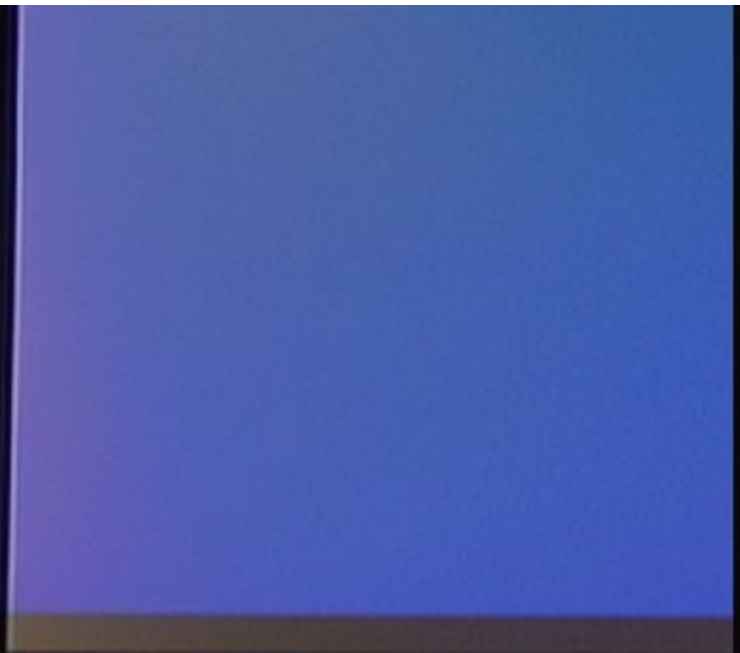
$$\psi_0 \propto e^{-\sum_n a |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1}$$



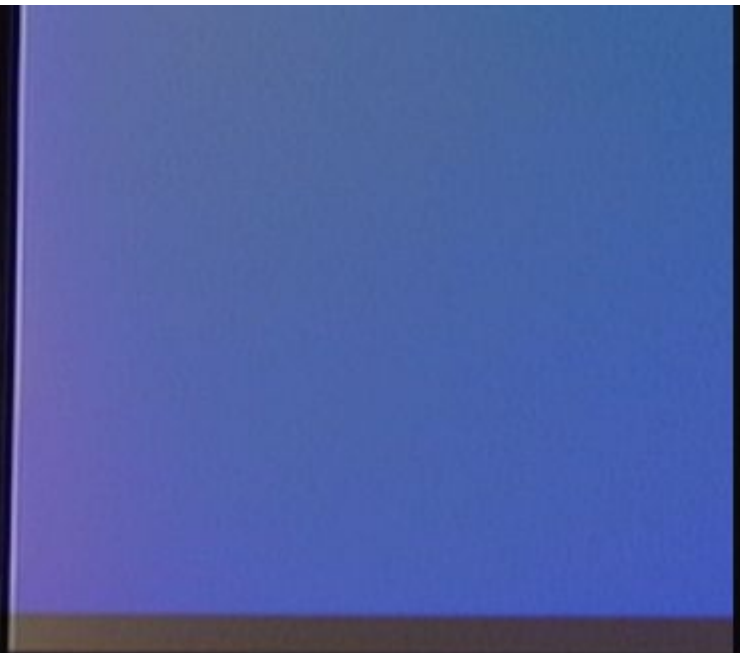
$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle_{n=2} \rightarrow \langle \dots \rangle_{n=2}$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]^{-1}$$





$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

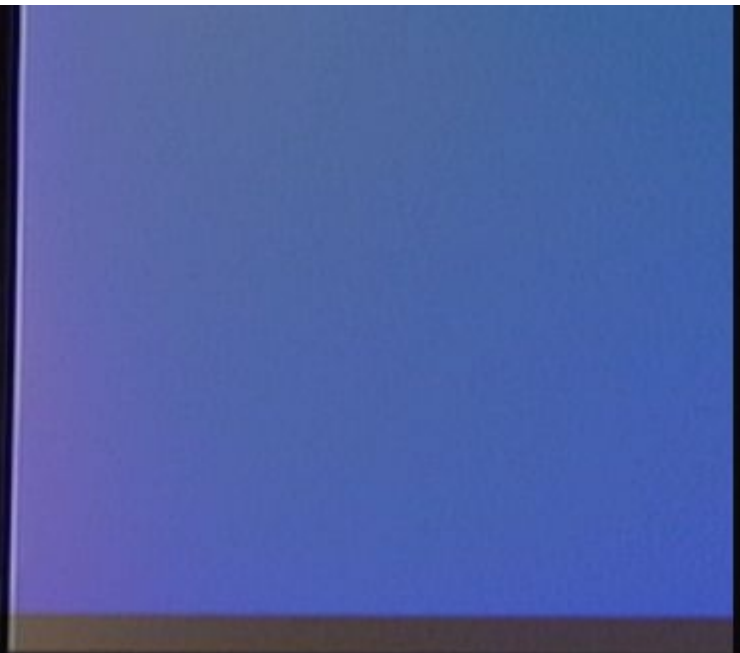
$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle$$

$$= \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]$$

$$\delta(Q_{12} - q) \Big|_{n=2}$$

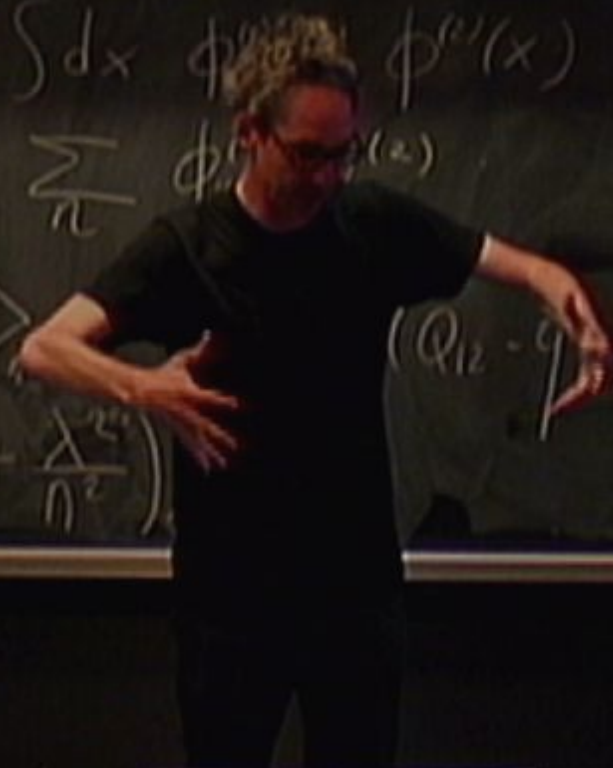
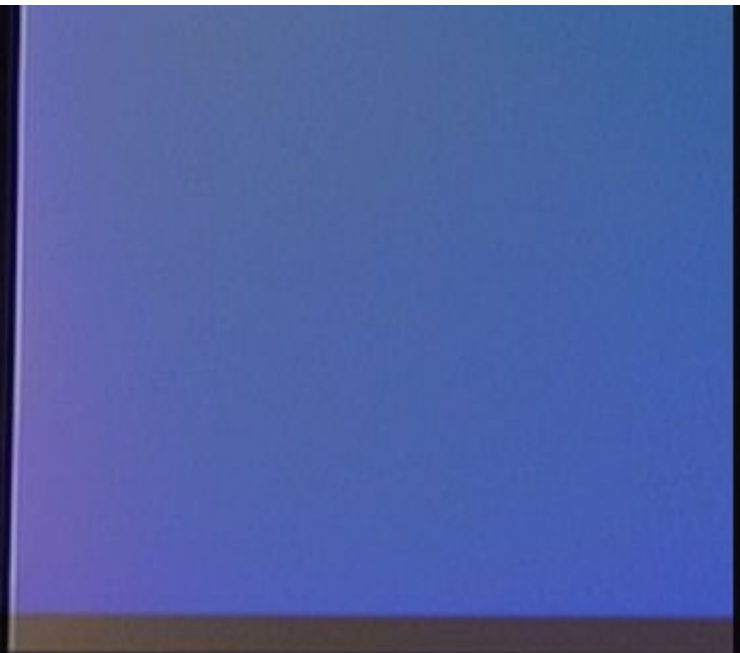


$$\psi_0 \propto e^{-\sum_n a |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle = \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]$$



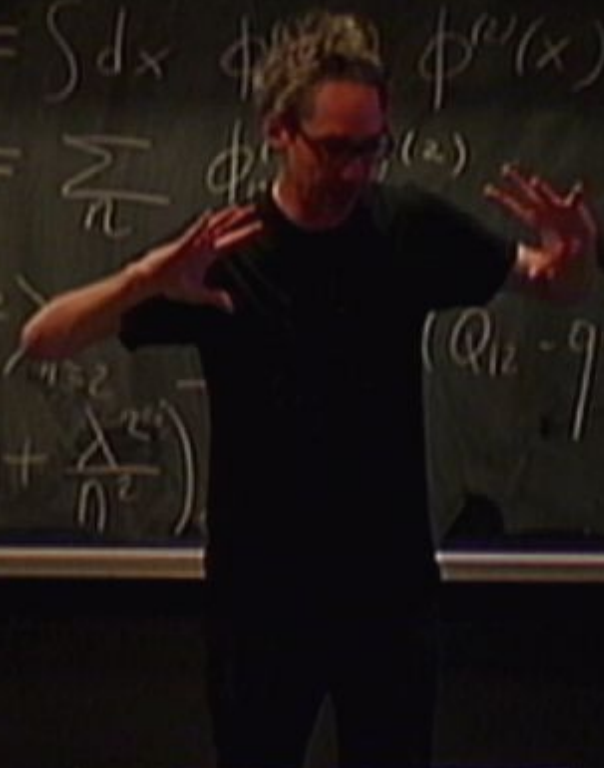
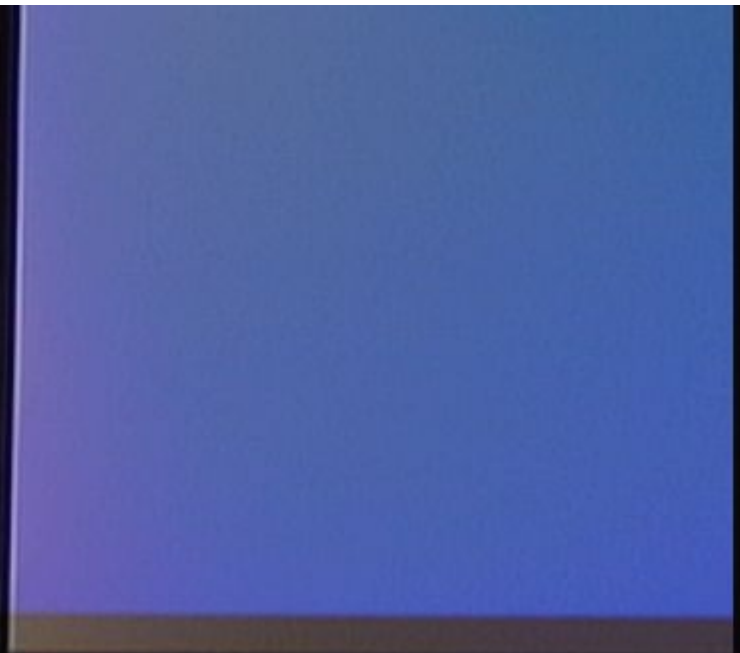
$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)} \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle = \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right)$$

$$(Q_{12} - q) \Big|_{n=2}$$





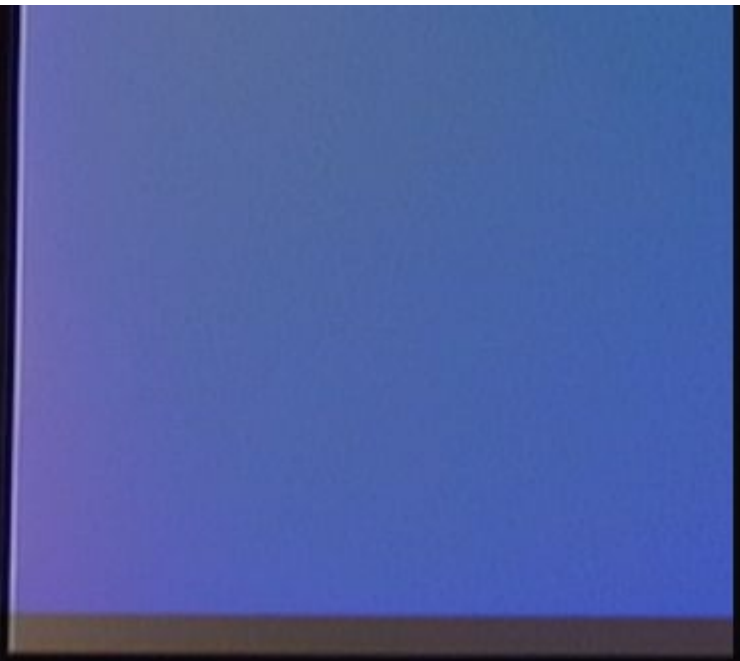
$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle = \left[ \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right) \right]$$

$$Q_{12} = \langle \dots \rangle_{n=2}$$



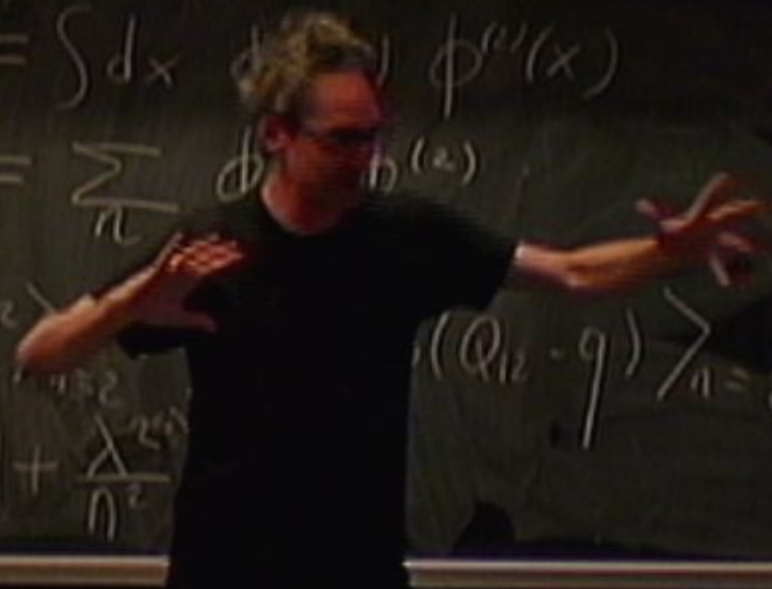
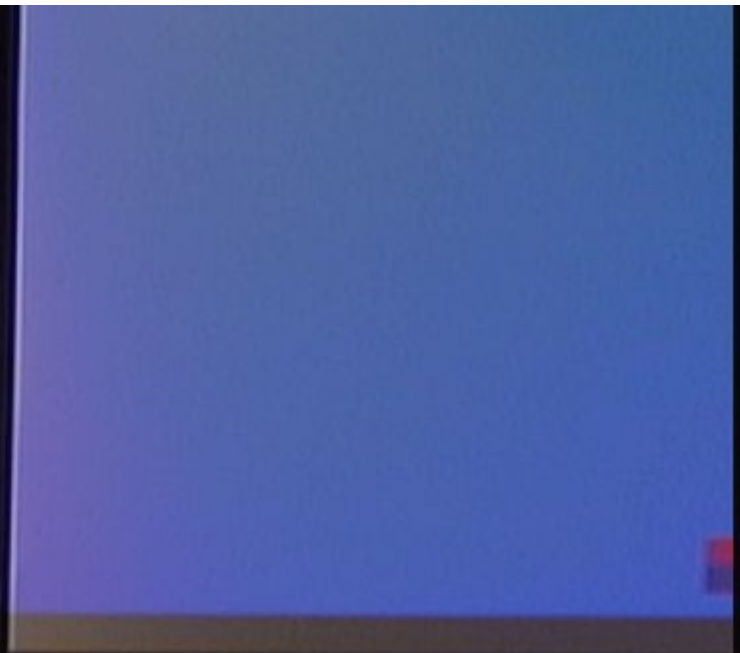
$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q_{12}} \rangle = \prod_n \left( 1 + \frac{\lambda^2}{n^2} \right)$$

$$\langle (Q_{12} - q) \rangle_{n=2}$$

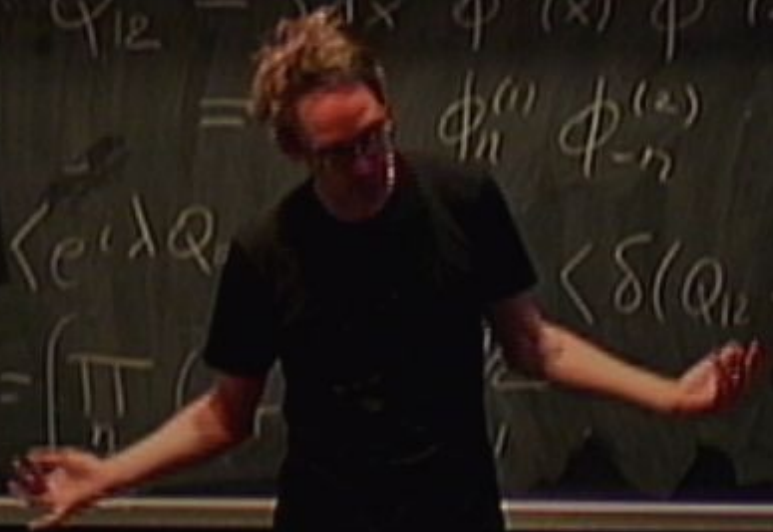
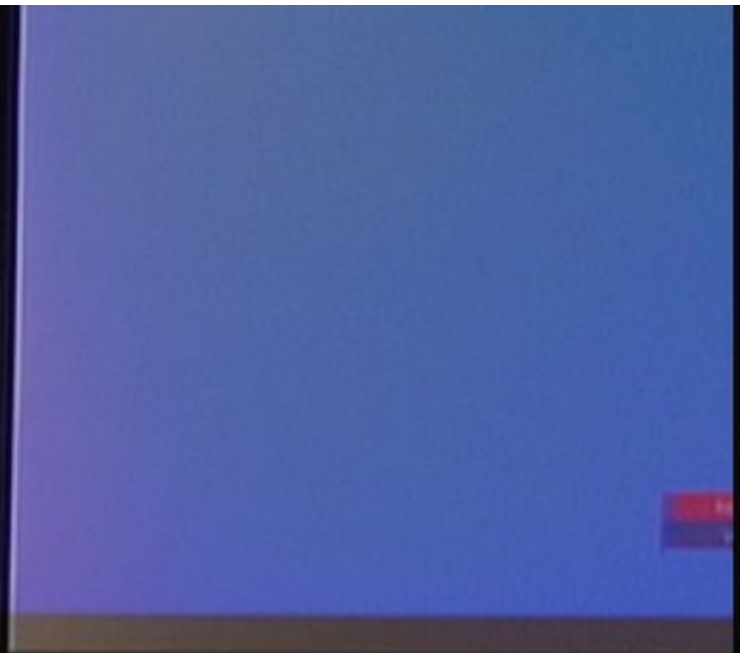


$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int dx \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int dx \phi_n^{(1)} \phi_{-n}^{(2)}$$

$$\langle \delta(Q_{12} - q) \rangle_{n=2}$$



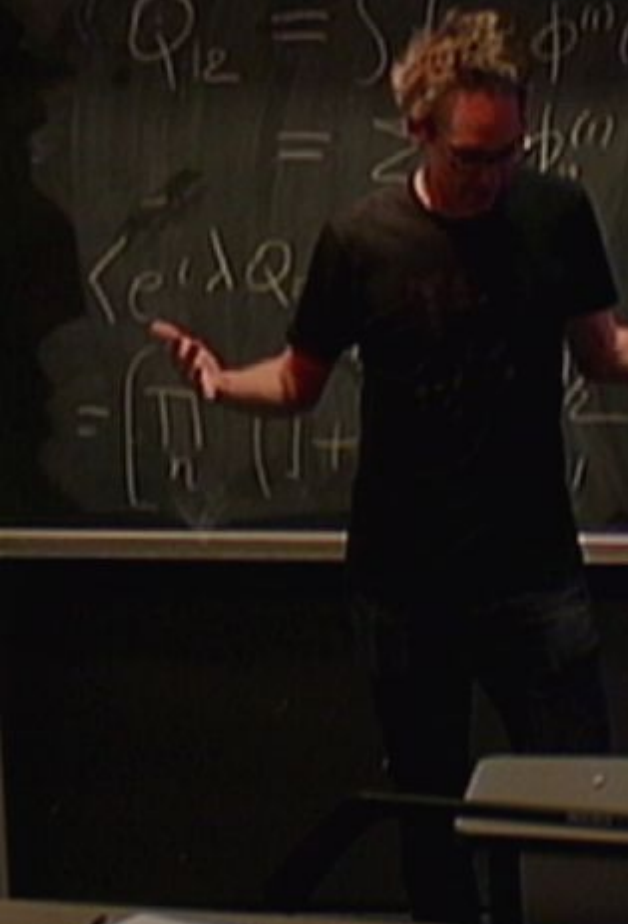
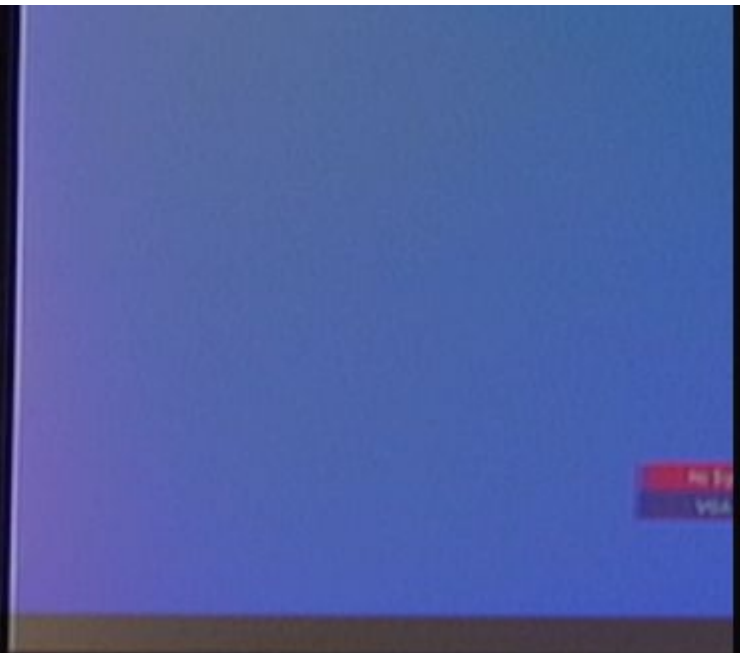


$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int \prod \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum \phi^{(1)} \phi^{(2)}$$

$$\langle e^{i\lambda Q} \rangle = \int \prod \phi^{(1)} \phi^{(2)} \langle \delta(\phi^{(1)} - \phi^{(2)}) \rangle_{n=2}$$

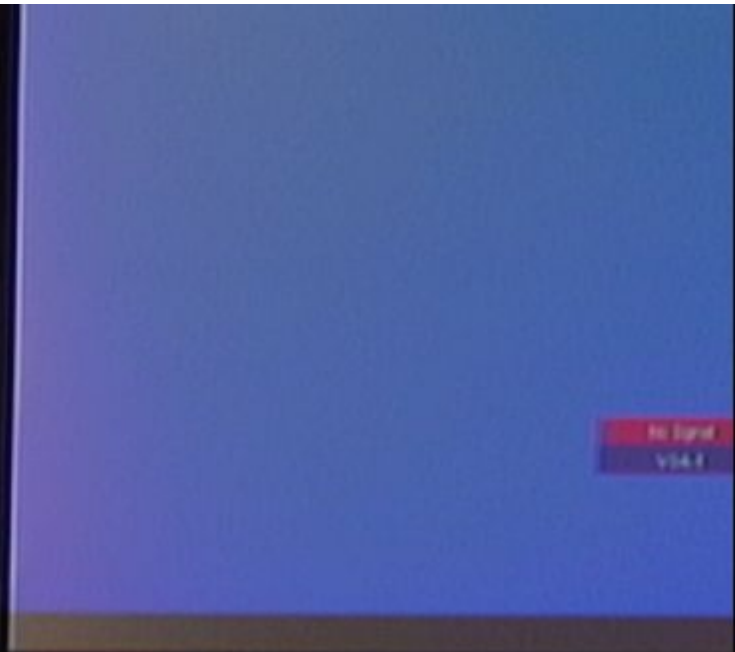


$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q} \rangle = \int \prod_n (1 + \dots)$$



$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{iQ} \rangle = \int \prod_n (1 + \dots)$$

$$\delta(Q_{12} - q) \int_{n=2}^{\infty}$$





$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \sum_n \phi_n^{(1)} \phi_n^{(2)}$$

$$\delta(Q_{12} - q) \int_{n=2}$$



$$\psi_0 \propto e^{-\sum_n n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{iQ} \rangle = \int \prod_n \dots \delta(Q_{12} - q) \int_{n=2}$$



$$\psi_0 \propto e^{-\sum_n a_n |\phi_n|^2}$$

$$Q_{12} = \int \phi^{(1)}(x) \phi^{(2)}(x)$$

$$= \int \phi_n^{(1)} \phi_n^{(2)}$$

$$\langle e^{i\lambda Q} \rangle = \int \prod_n \left( \dots \right)$$

$$\langle \delta(Q_{12} - q) \rangle_{n=2}$$

