Title: Correlation functions in FRW/CFT duality

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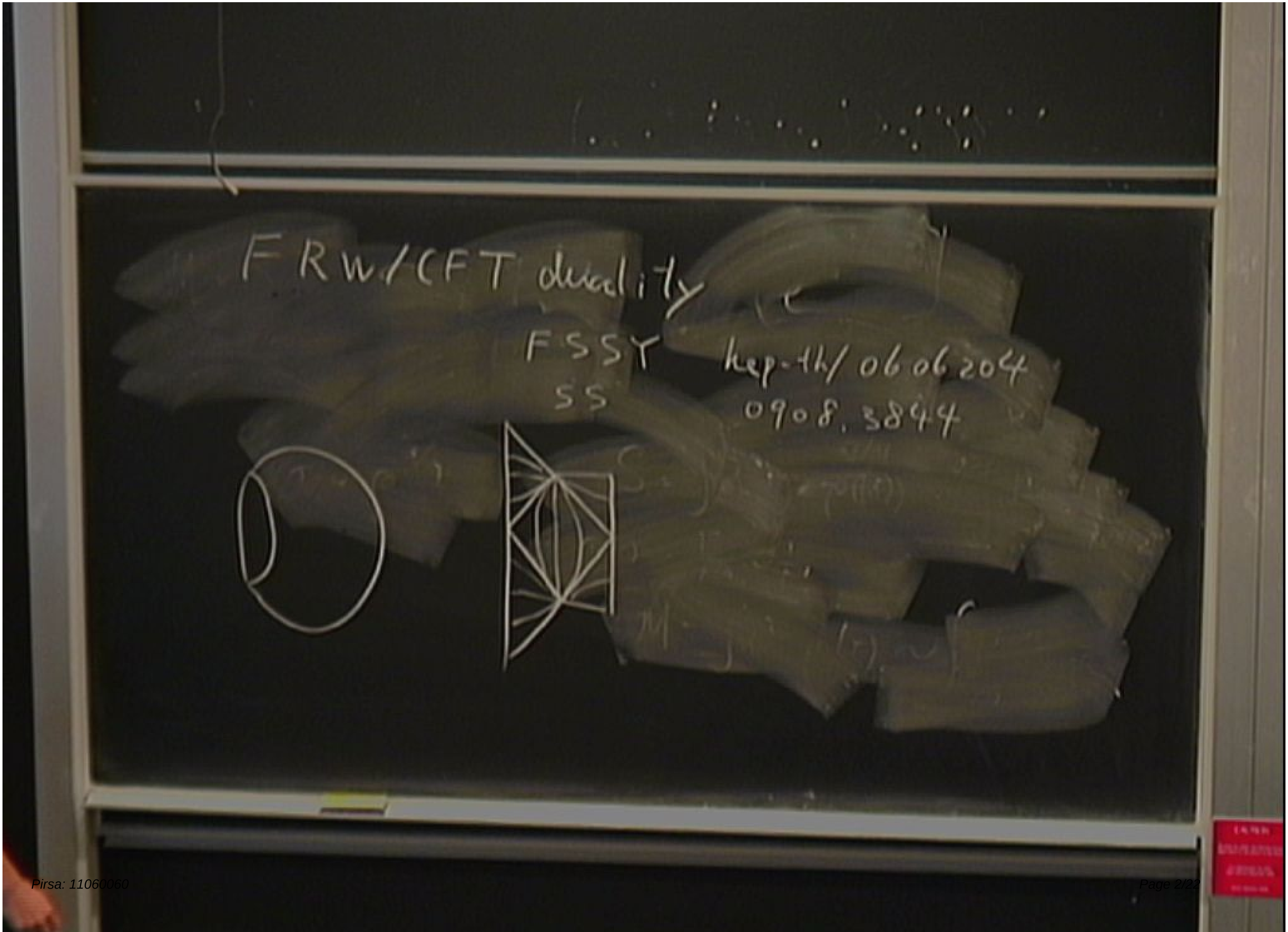
URL: http://pirsa.org/11060060

Abstract: FRW/CFT duality is a proposal for a holographic dual description for universe created by bubble nucleation. For (3+1) dimensional universe, the dual theory is defined on 2-sphere at the boundary of open universe. I will study correlation functions and explain essential features of FRW/CFT duality: One bulk field corresponds to a tower of CFT operators The boundary theory contains 2D gravity, and the Liouville field plays the role of time. Energy-momentum tensor has dimension 2, as required from the 2D conformal symmetry.


FRW/CFT duality
FSSY hep-th/0606204
SS 0908.3844

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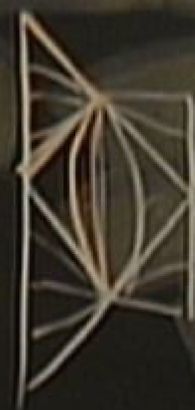


$$ds^2 = a^2(T)\left(-dT^2 + \overbrace{dR^2 + \sinh^2 R \, d\Omega_2^2}^{H^3}\right)$$

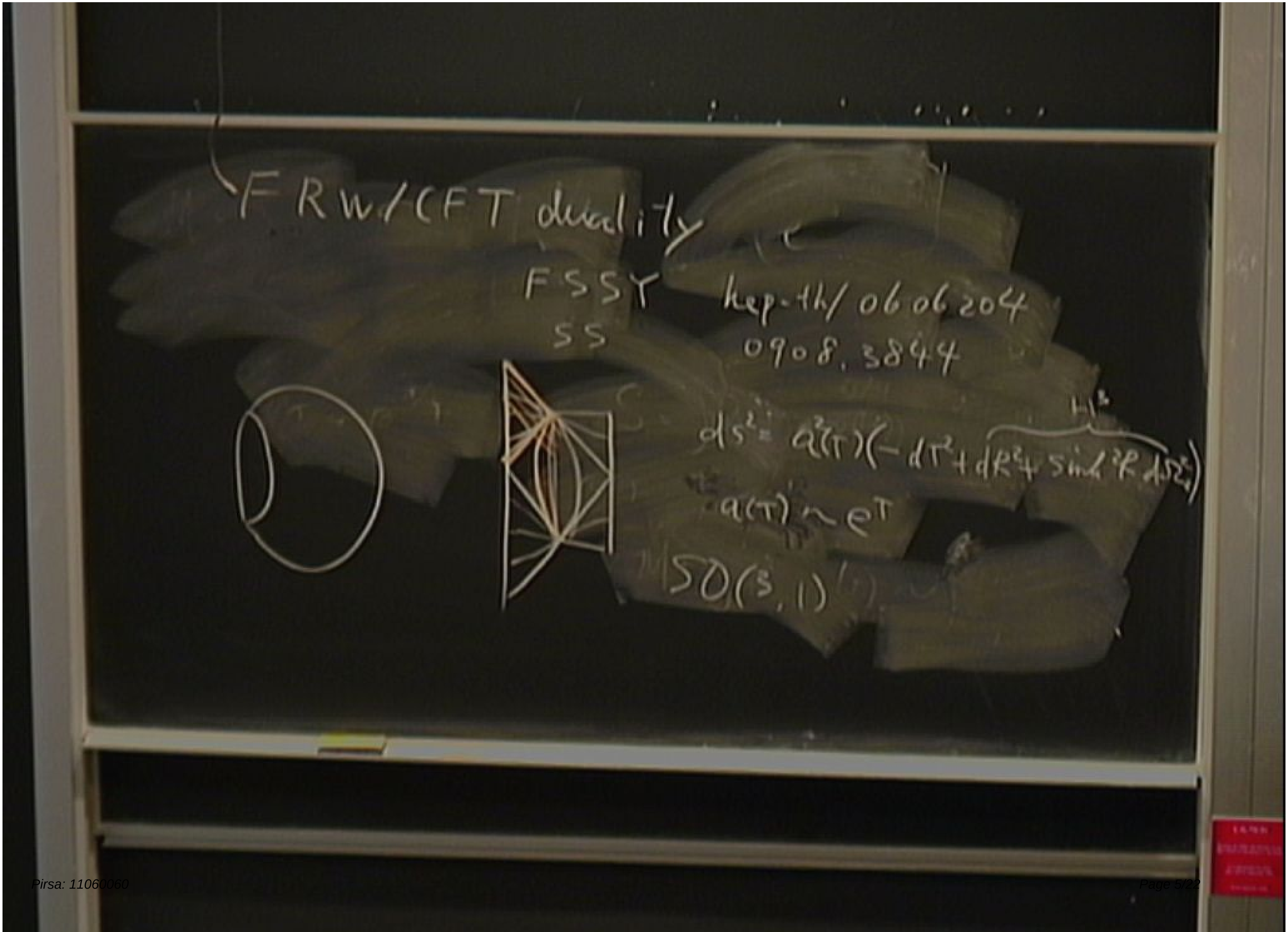
$$a(T) \sim e^T$$

$$SO(3,1)$$

Dual theory:

2D CFT on S^2
at the bdry H^3

- contains gravity (Liouville)
- Liouville plays the role of time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$ds^2 = a^2(T)(-dT^2 + \underbrace{dR^2 + \sinh^2 R \, d\Omega_2^2}_{H^3})$
$a(T) \sim e^T$
SO(3,1)

harmonics on H^3:

$$\nabla_H^2 W_{(k\ell m)} = -(k^2+1) W_{(k\ell m)}$$

$$W_{(k\ell m)} = N \frac{\sin kR}{\sinh R}$$

k : real   normalizable $\sim e^{-R}$

Non-normalizable modes

$k = i$

$$\nabla_H^2 W_{(k=i,\ell m)} = 0$$

- const.
- finite as $R \to \infty$ w/ any (ℓ, m)

Important for massless fields

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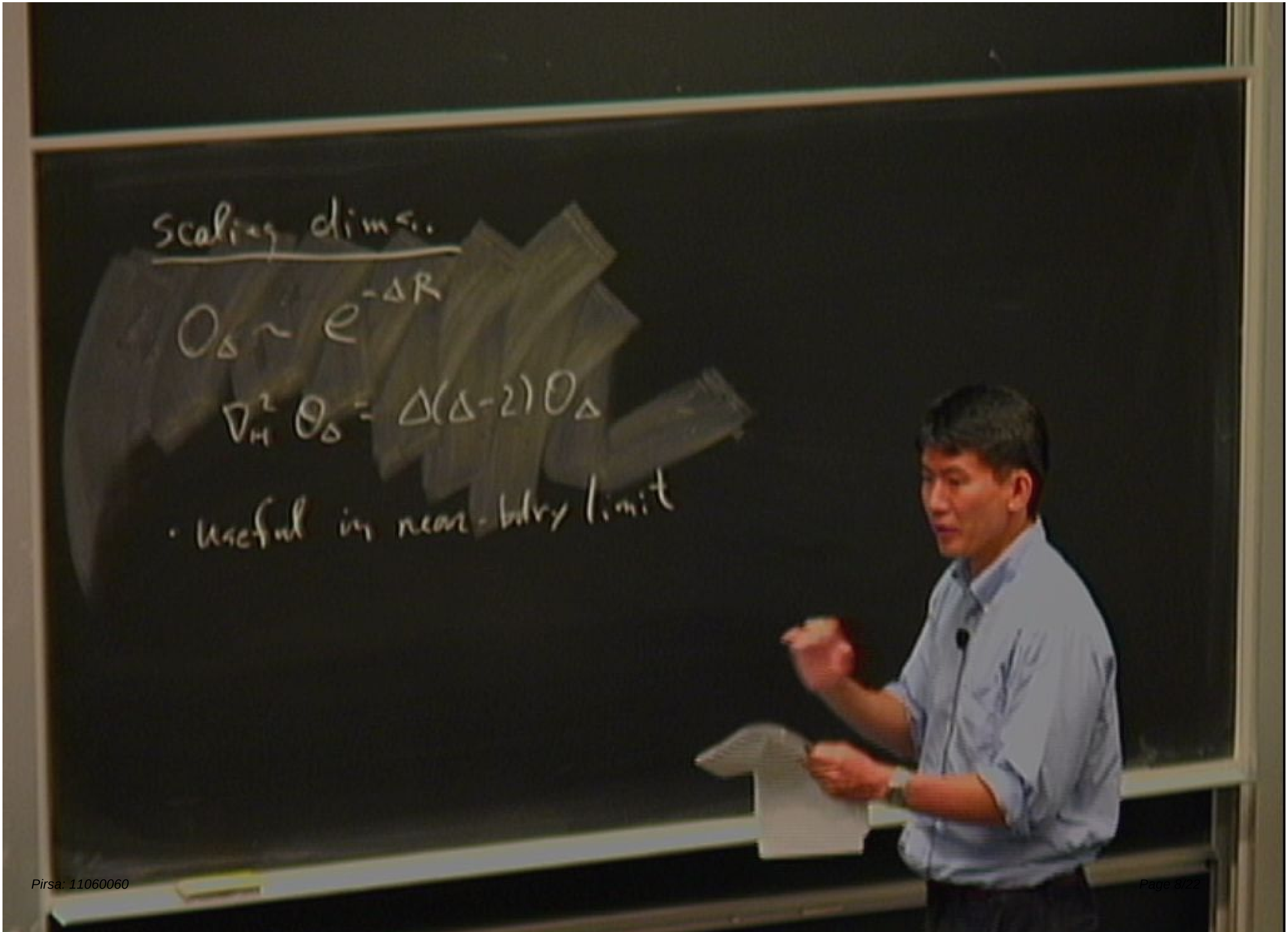
scaling dim's
$O_\Delta \sim e^{-\Delta R}$
$\nabla_H^2 O_\Delta = \Delta(\Delta-2) O_\Delta$
· useful in near-bdry limit

Dual theory:

2D CFT on S^2
at the bdry H^3

- contains gravity (Liouville)
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$$g = e^{2\varphi} \hat{g}$$

Dual theory:

2D CFT on S^2

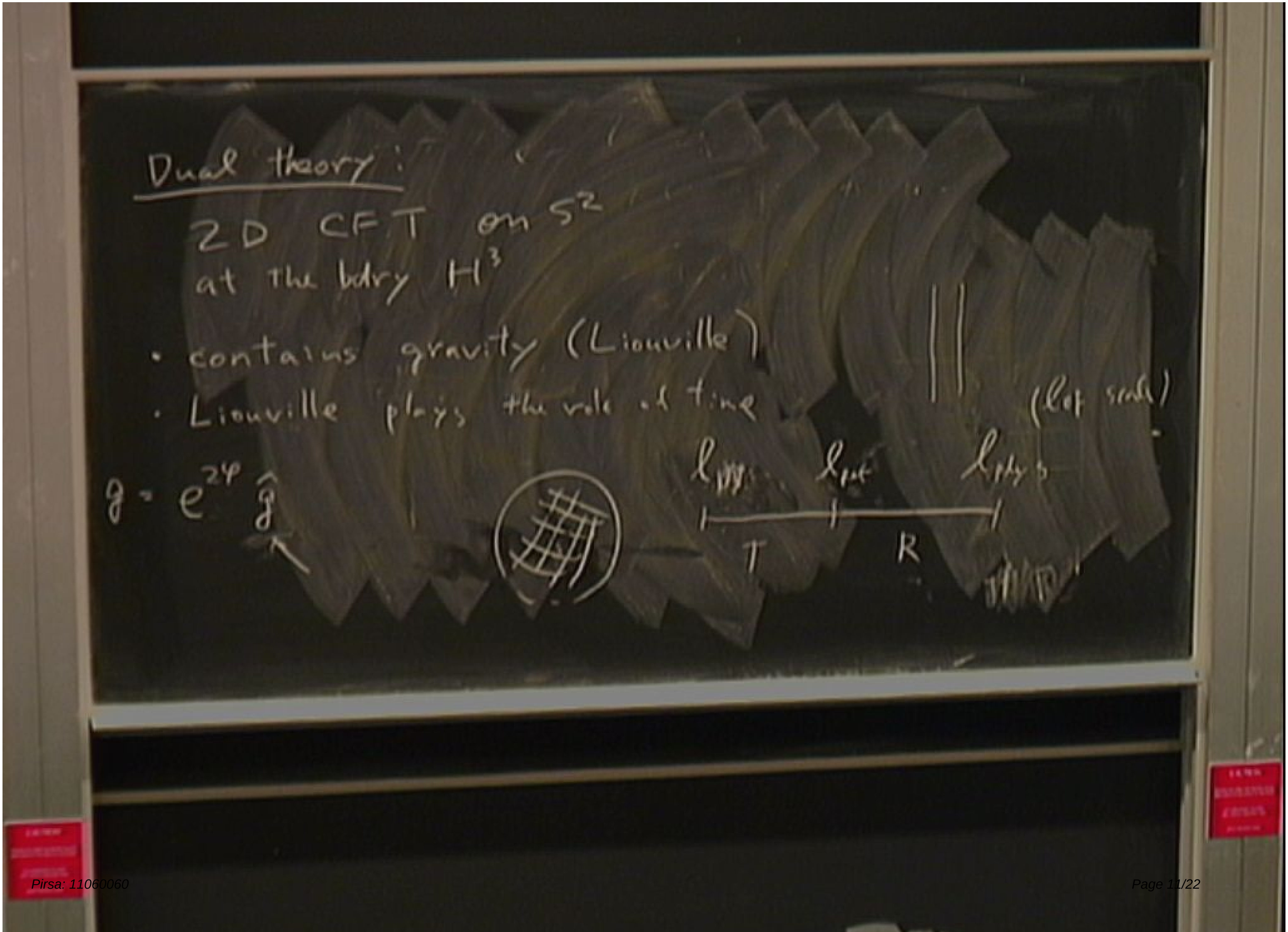
at the bdry H^3

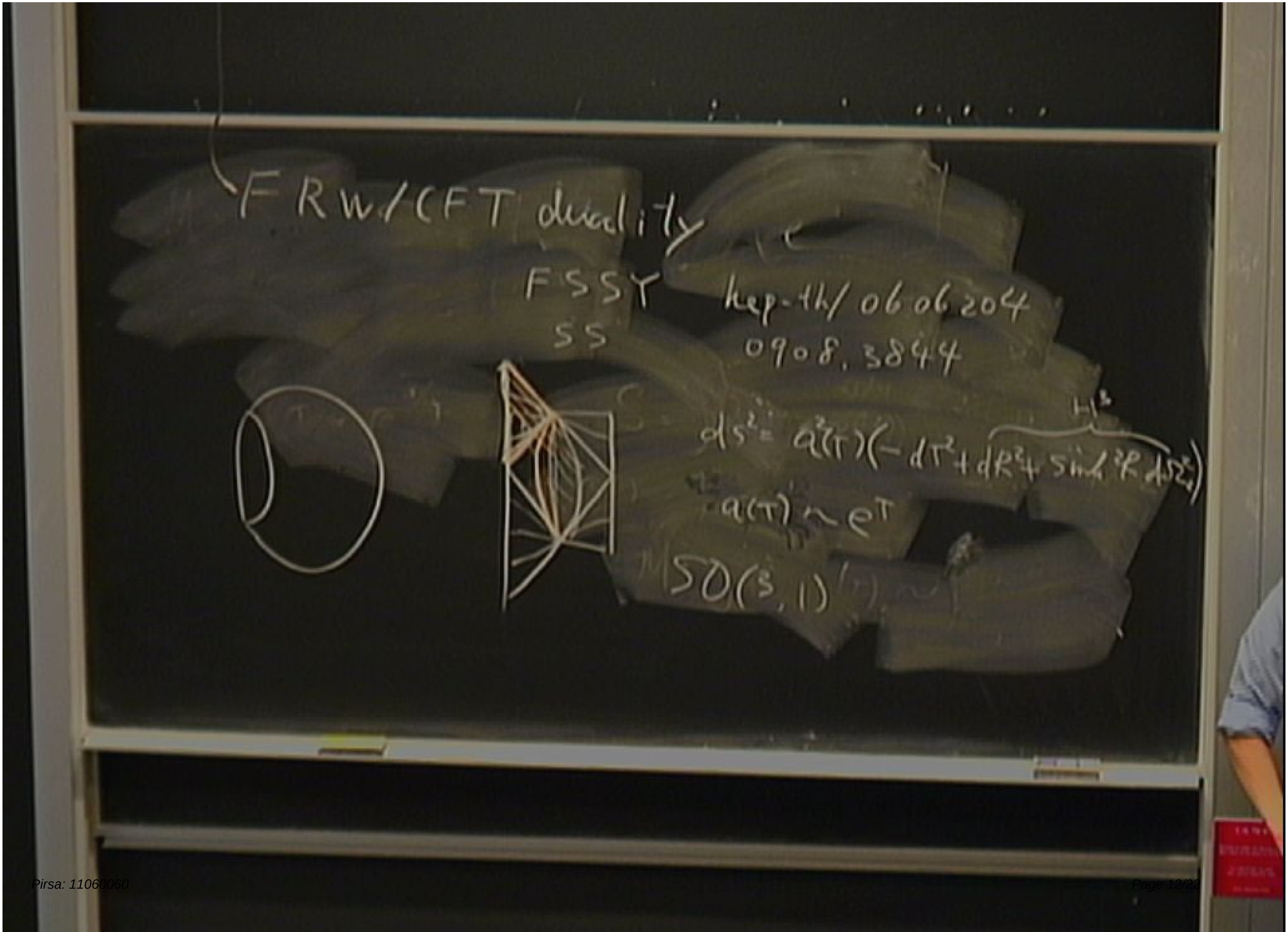
- contains gravity (Liouville)
- Liouville plays the role of time (log scale)

$$g = e^{2\varphi}\,\hat{g}$$

$l_{phys} \quad l_{pl} \quad l_{phys}$

T R


Dual theory:
2D CFT on S^2
at the bdry H^3
• contains gravity (Liouville)
• Liouville plays the role of time
$g = e^{2\varphi}\hat{g}$
(lat scale)
l_{phys} l_{lat} l_{phys}
T R


FRW/CFT duality
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H³
ds² = a²(T)(−dT² + dR² + sinh²R dΩ₂²)
a(T) ~ e^T
SO(3,1)

$$ds^2 = a^2(x)\left(d\chi^2 + d\theta^2 + \sin^2\theta\, d\Omega_2^2\right)$$

$$ds^2 = a^2(x)\left(dx^2 + d\theta^2 + \sin^2\theta \, d\Omega_2^2\right)$$

e.g. free massless scalar

$$\left[-\partial_x^2 - \nabla_S^2 + \frac{a''}{a}\right]\left(a(x)\, a(x') \langle \phi(x,\theta)\, \phi(x',0) \rangle\right) = \delta(x-x') \, \delta^{(3)}(\theta)$$

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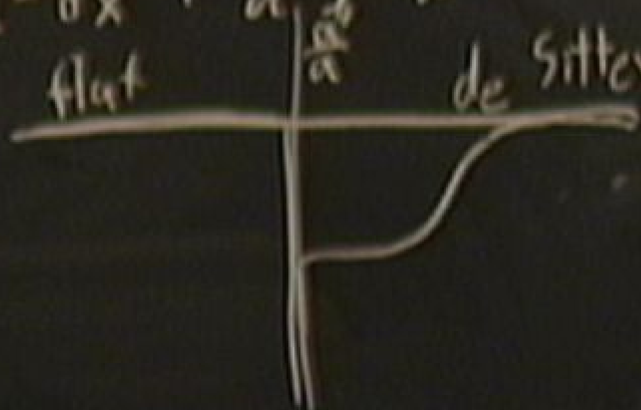
$$\left[-\partial_x^2 + \frac{a''}{a}\right] u_k(x) = (k^2+1)\, u_k(x)$$



$$ds^2 = a^2(x)\left(dx^2 + d\theta^2 + \sin^2\theta \, d\Omega_2^2\right)$$

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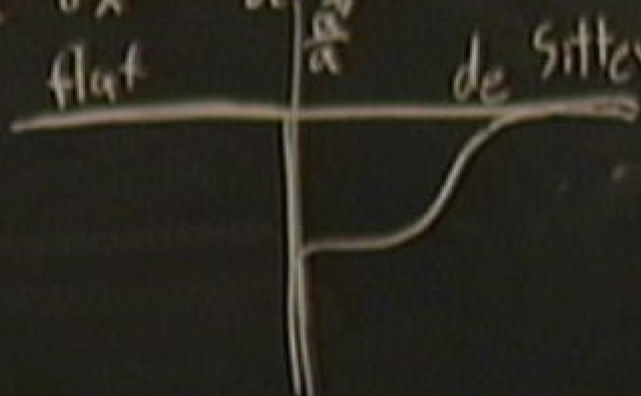

• Green's fn. on S^3

$$\left[-\nabla_3^2 + (k^2+1)\right] G_k(\theta) = \delta^3(0)$$

$$G_k(\theta) = \frac{\sinh k(\pi-\theta)}{\sinh k\pi \, \sin\theta}$$

$SO(3,1)$

$$\langle \phi \phi \rangle = \frac{1}{a(x)a(x')} \int dk\, u_k^*(x) u_k(x') G_k(\theta)$$

$$= e^{-(x+x')} \int dk \left(e^{ik(x-x')} + R(k) e^{ik(x+x')} \right) G_k(\theta)$$

$=$

$SO(3,1)$

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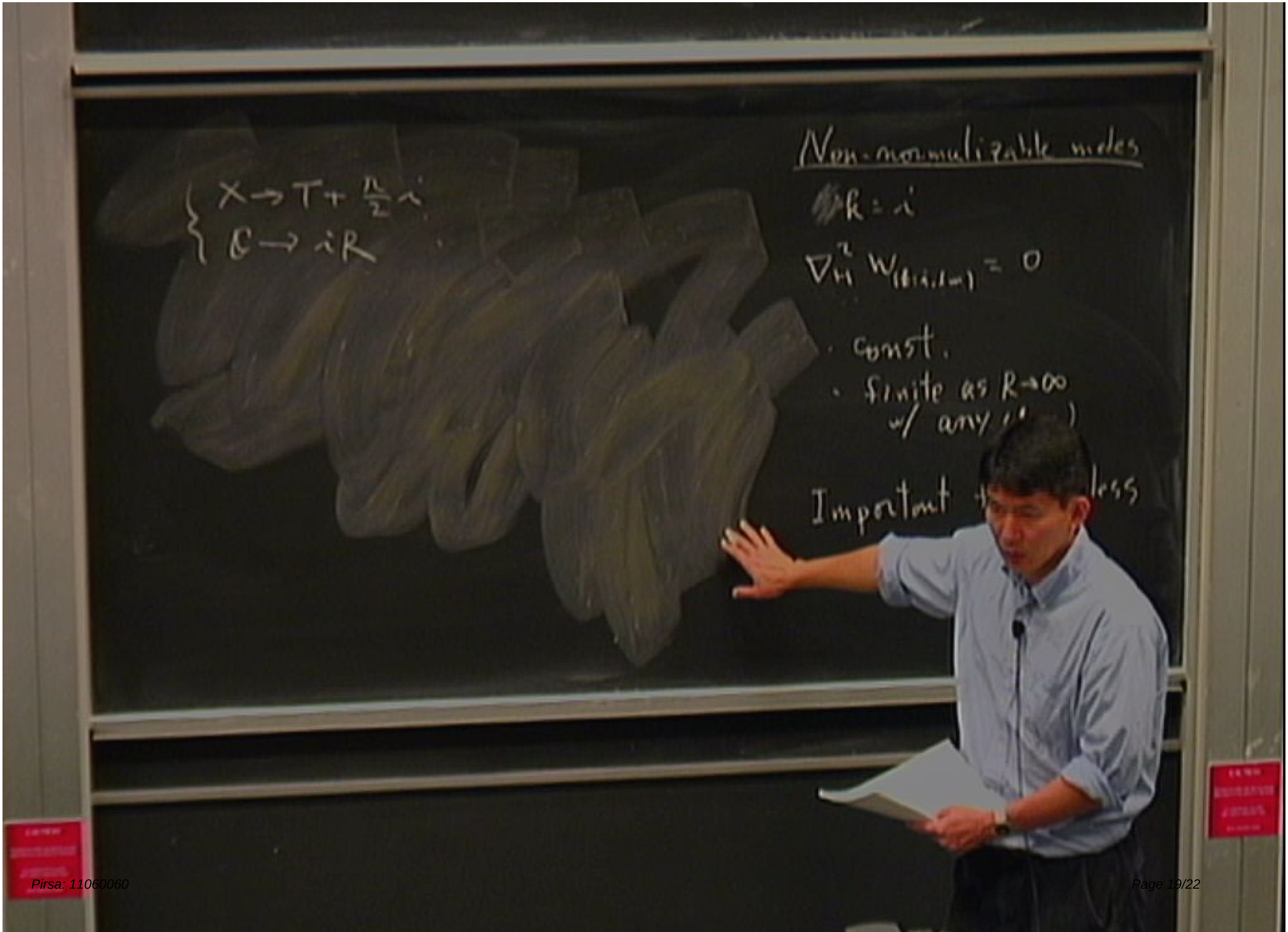
$$+ \text{(bound states)}$$

- bound state at $k = i$
- "kinematical" poles $k = in \quad (n \in \mathbb{Z})$


X → T + π/2 i
θ → iR
Non-normalizable modes
k = i
∇²_H W(k,l,m) = 0
· const.
· finite as R→∞
w/ any
Important

Non-normalizable modes

$$\begin{cases} X \to T + \frac{\pi}{2} i \\ \theta \to iR \end{cases}$$

$$R = R_1 + R_2 + \ln(1-\cos\theta)$$

$$\langle O_\Delta O_\Delta \rangle = \frac{1}{(1-\cos\theta)^\Delta}$$

$$\langle \phi(T_1, R_1, \Omega)\, \phi(T_2, R_2, 0) \rangle$$

$$= \sum_{\Delta_2} q_{\Delta_2} e^{-\Delta_1(R_1+T_1) - \Delta_2(R_2+T_2)} \langle O_{\Delta_1} O_{\Delta_2} \rangle$$

$$+ \sum_{\Delta'_1} q_{\Delta'_1} e^{-\Delta_1 R_1 + (\Delta_1 - d) T_1 - \Delta_1 R_2 + (\Delta_1 - d) T_2} \langle O_{\Delta'} O_{\Delta'} \rangle$$

Non-normalizable modes

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$$+ \sum_{\Delta'_2} g_{\Delta'_2} e^{-\Delta_2 R_1 + (\Delta_1 - 2) T_1 - \Delta_2 R_2 + (\Delta_2 - 2) T_2} \langle \mathcal{O}_{\Delta'} \mathcal{O}_{\Delta'} \rangle$$

$\mathcal{O}_{\Delta_1}$: defined at UV scale

$\mathcal{O}_{\Delta_2}$: defined at ref. scale

$$\cancel{\partial^\alpha h_{\alpha i}} + \partial^j h_{ji} = 0$$

$$ds^2 = a^2(x)\left(dx^2 + d\theta^2 + \sin^2\theta\, d\Omega_2^2\right) \qquad \langle \sqrt{g}^{(2)} R^{(2)} \sqrt{g}^{(1)} R^{(1)} \rangle$$

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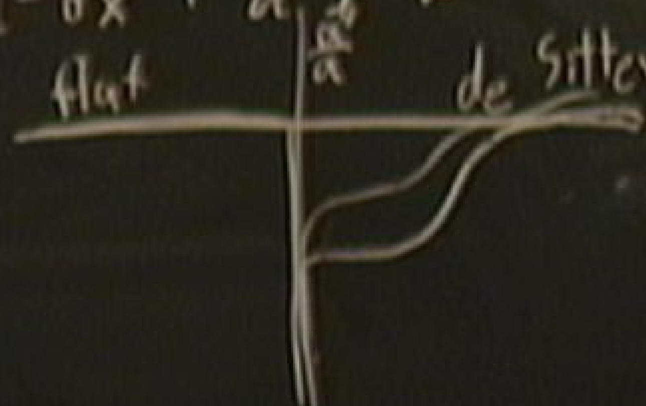

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