Title: A de Sitter Farey Tail

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Abstract: We compute the partition function of quantum Einstein gravity in three dimensional de Sitter space. The Euclidean path integral is formulated as a sum over geometries, including both perturbative loop and non-perturbative instanton corrections coming from geometries with non-trivial topology. These non-trivial geometries have a natural physical interpretation and lead to deviations from the standard thermal behaviour of the de Sitter horizon; this is the de Sitter analog of the celebrated "black hole Farey tail." Perturbative quantum corrections are computed to all orders in perturbation theory and the vacuum partition function, including all instanton and perturbative corrections, is shown to diverge in a way which can not be regulated using standard field theory techniques. Thus the Hartle-Hawking state is not normalizable.

A de Sitter Farey Tail

Holographic Cosmology 2.0, June 21, 2011

Alex Maloney, McGill University

Castro, Lashkari & A. M.

$e \in 2^{\{Castro, Lashkari, A.M., Strominger, \ldots\}}$ in progress

The Problem

Quantum cosmology is confusing:

- How is Unitarity consistent with singularities, inflation, ...?
- Is quantum mechanics modified in cosmological settings?
- What are the appropriate observables for eternal inflation?
- What is the meaning and origin of the entropy of a cosmological horizon?

Similar questions are answered in the context of black hole physics by AdS/CFT.

Let us be bold and apply the same techniques to cosmology.

The Wave-function of the Universe

One lesson of AdS/CFT is that the "wave function of the universe" $|\psi\rangle$ exists and is computable.

The Hartle-Hawking state

$$\langle h|\psi\rangle \sim \int_{g|_{\partial M=h}} \mathcal{D}g\; e^{-S}$$

includes contributions from all geometries. It is the natural "vacuum state" of quantum gravity.

In AdS this is a CFT partition function.

What about in dS?

Goal: Find a theory where we can compute $\langle h|\psi\rangle$.

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e Sitter Space

For three dimensional general relativity with a positive cosmological constant

$$S[g] = \frac{1}{G}\int_M \sqrt{-g}\left(R - \frac{2}{\ell^2}\right)$$

the partition function

$$Z = \int \mathcal{D}g \; e^{-S[g]}$$

can be computed exactly.

We will be inspired by AdS/CFT but we will not use it.

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The Idea

The saddle point approximation is

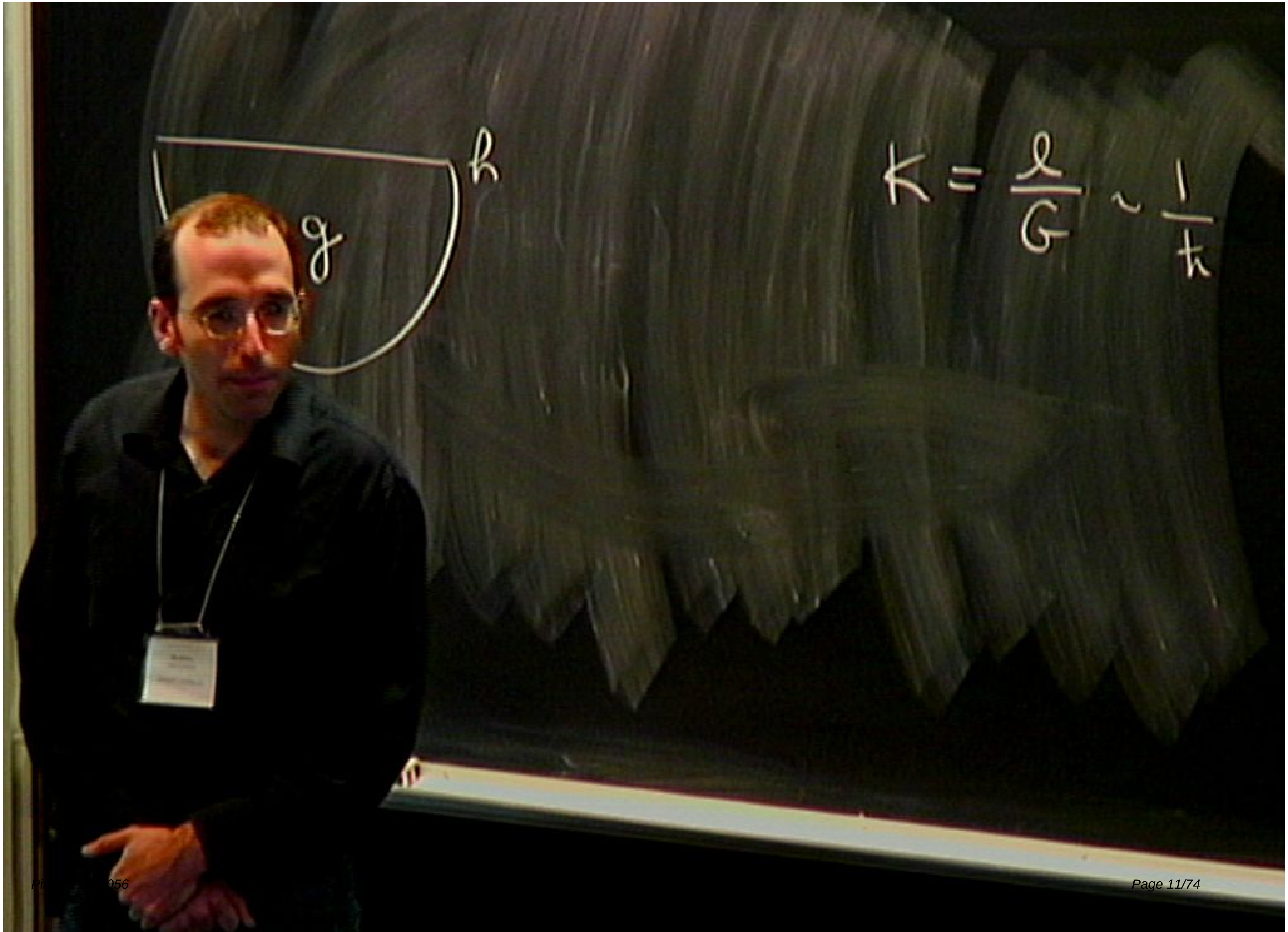
$$Z = \int \mathcal{D}g \; e^{-S[g]} = \sum_{g_0} e^{-kS^0 + S^1 + \frac{1}{k} S^2 + \ldots}$$

where $k = \ell / G$ is the coupling. The approximation becomes exact if we can

- Find all classical saddles
- Compute all perturbative corrections around each saddle

We will do both.

The new classical saddles have a straightforward physical interpretation and lead to quantum gravitational effects for de Sitter observers.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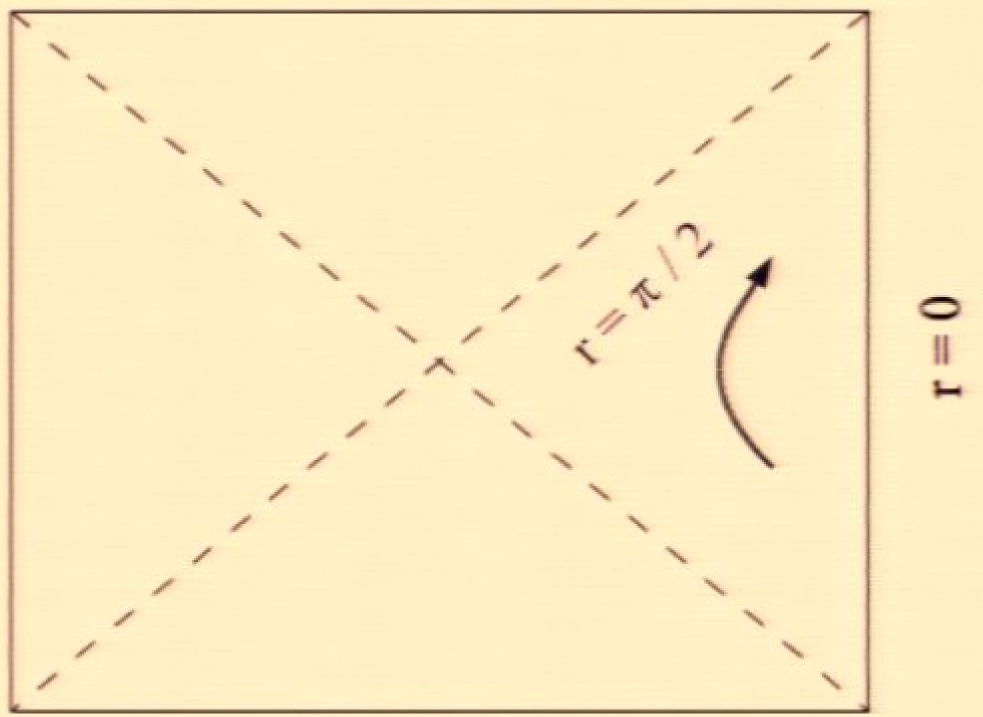
The Plan for Today:

- Find the Classical Solutions
- Compute the Partition Function
- Discuss the Result

The Causal Patch

A single observer can access only a small portion of the de Sitter geometry. The metric in this patch is

$$ds^2 = dr^2 - \cos^2 r\; dt^2 + \sin^2 r\; d\phi^2$$



Quantum Field Theory in de Sitter

There is no global definition of energy in dS, but there is a notion of energy associated with an observer.

The two killing vectors

- $H = \partial_t$ generates time translations
- $J = \partial_\phi$ generates rotations.

States are labelled by energy H and angular momentum J.

The operators H and J can be constructed explicitly for free QFT.

The definition of the QFT vacuum state requires some care. It is typically constructed by Wick rotation $t = it_E$.

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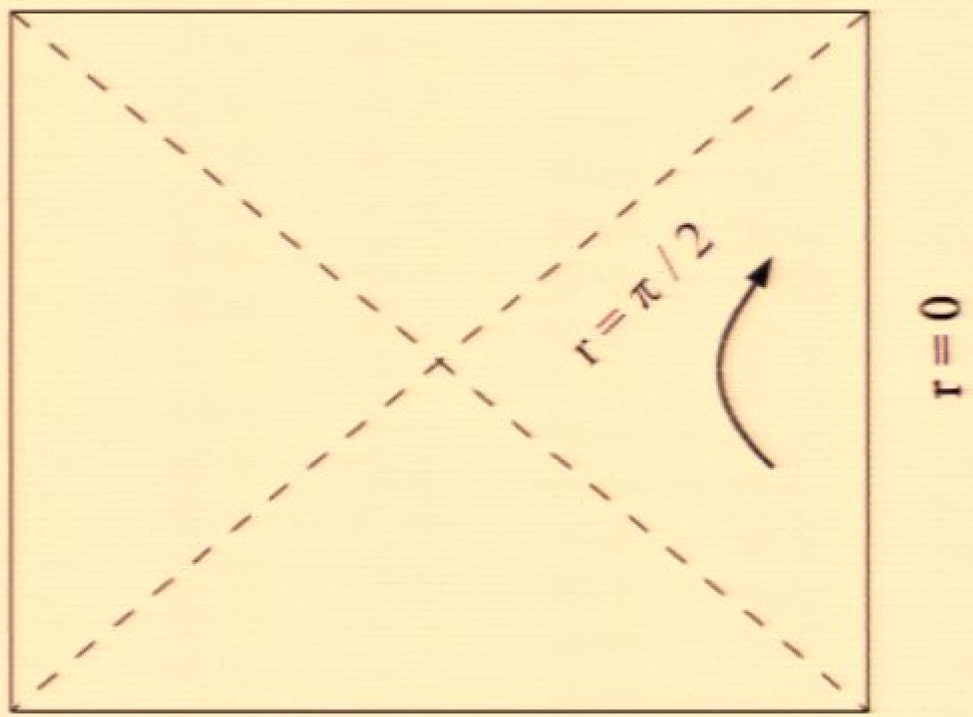

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Euclidean Continuation

The Euclidean metric

$$ds^2 = dr^2 + \cos^2 r \; dt_E^2 + \sin^2 r \; d\phi^2 \; .$$

is smooth only if we identify $t_E \sim t_E + 2\pi n$. This is the sphere S^3.

The identification is generated by the operator

$$\rho = e^{-\beta H}, \qquad \beta = 2\pi$$

so field theory correlators obtained by analytic continuation are in canonical ensemble at finite temperature.

The horizon emits a bath of thermal radiation at the Hawking temperature β^{-1}.

Euclidean Continuation II

But the metric is also smooth if we identify

$$(t_E, \phi) \sim (t_E, \phi) + 2\pi \left(\frac{1}{p} n, m + \frac{q}{p} n \right)$$

for any $(p, q) = 1$. This is the lens space $L(p, q) = S^3/\mathbb{Z}_p$.

The identification is generated by the operator

$$\rho = e^{-\beta H + \theta J}, \qquad \beta = \frac{2\pi}{p}, \quad \theta = 2\pi i \frac{q}{p}$$

so field theory correlators are in grand canonical ensemble at finite temperature and chemical potential.

The Hawking radiation now contains correlations.

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VGA-1

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Quantum Gravity

At the level of QFT in a fixed background these geometries are all equally good "Euclidean continuations of de Sitter."

Which one should we use? All of them!

The full partition function of quantum gravity includes a sum over geometries. Each geometry contributes

$$Z \sim e^{-k \cdot Vol} \sim \exp\left\{-\frac{\ell}{Gp}\right\}$$

In the semi-classical $G \to 0$ limit the sphere dominates and the state is approximately thermal.

The other geometries lead to calculable quantum gravity deviations from the standard thermal behaviour of the Bunch-Davies vacuum.

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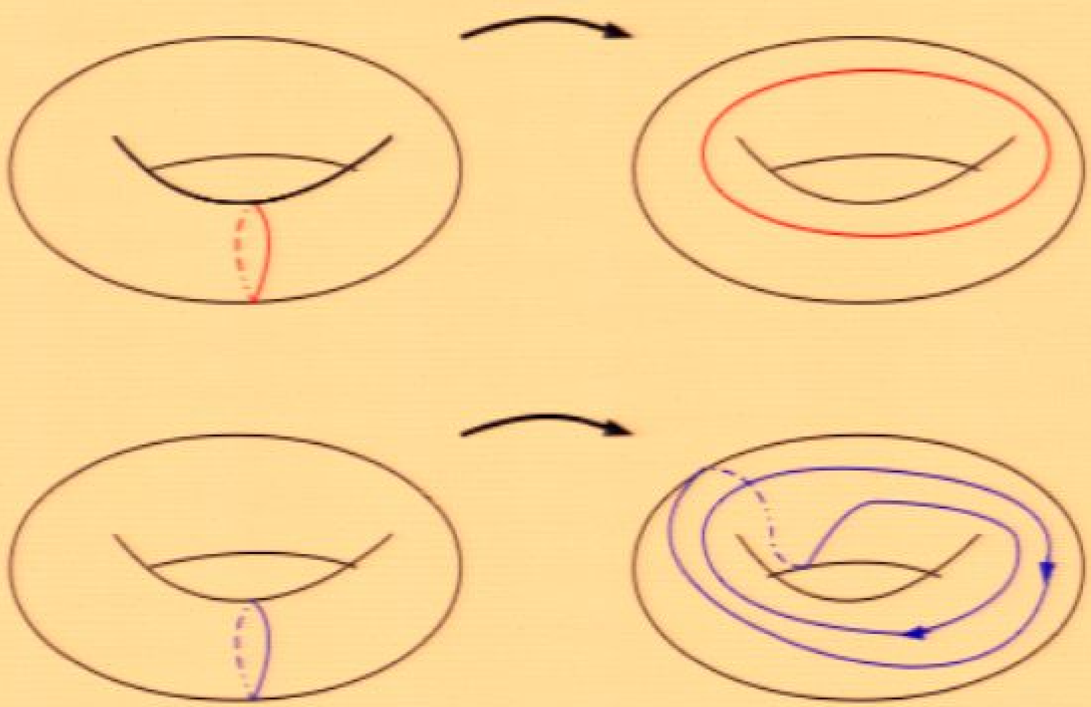

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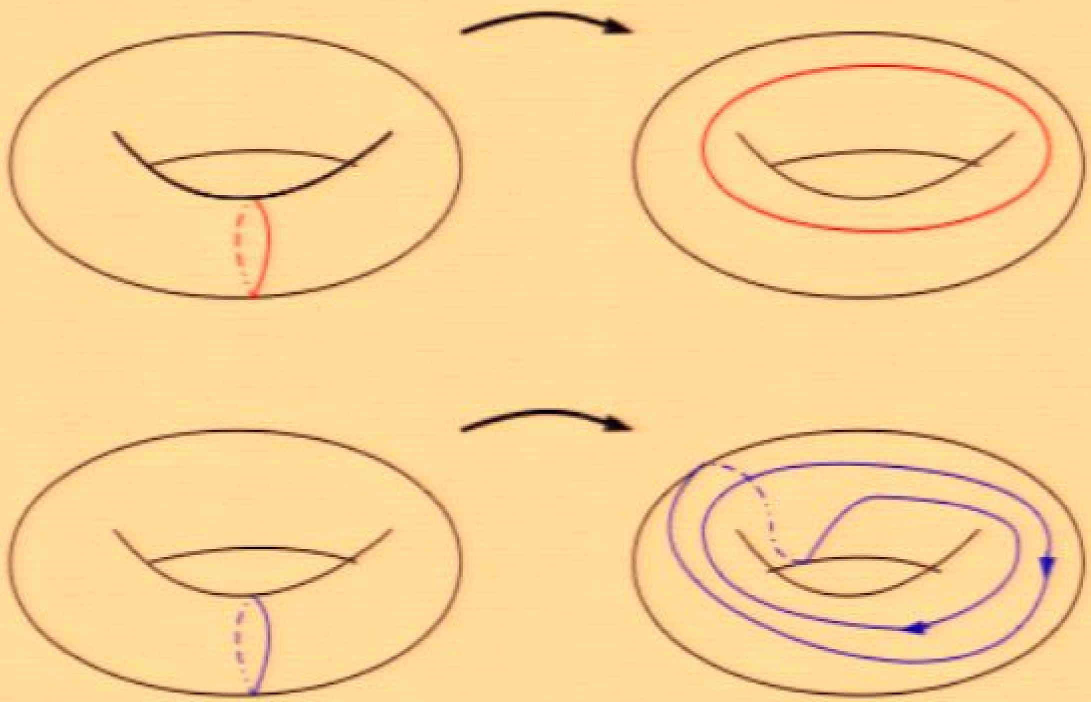

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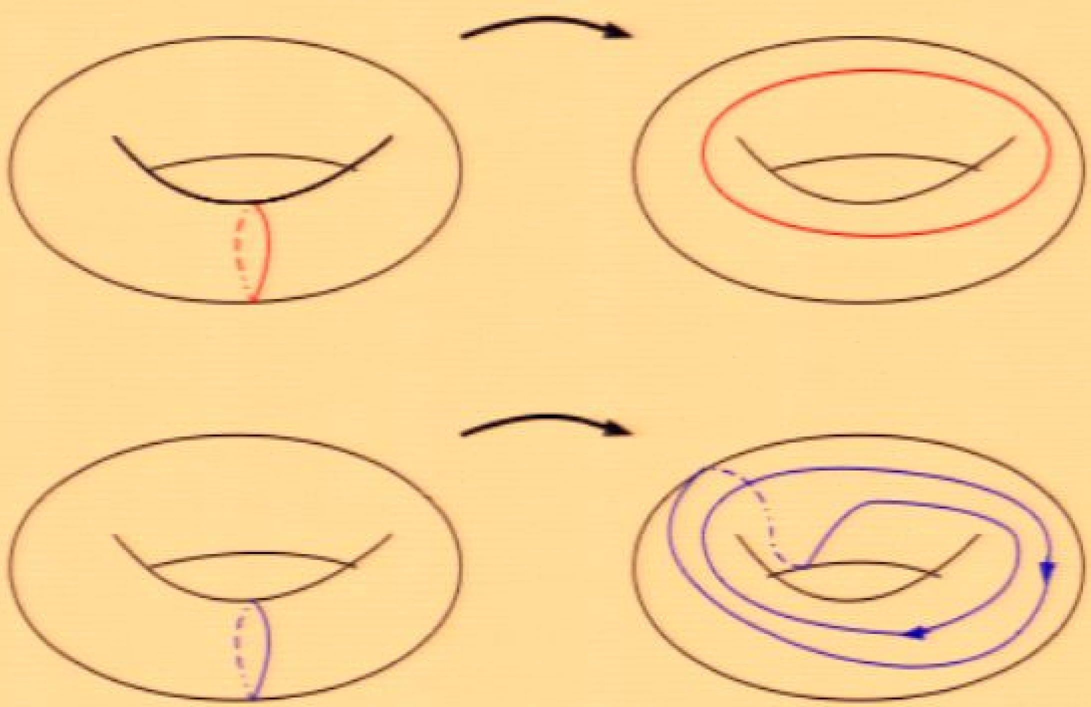

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The Partition Function

The norm of the Hartle-Hawking state is the partition function

$$Z = \langle \psi | \psi \rangle = \int \mathcal{D}g \; e^{-S[g]} = \sum_{g_0} e^{-kS^0 + S^1 + \frac{1}{k} S^2 + \dots}$$

Although 3D gravity has no local degrees of freedom there are "global" degrees of freedom which give quantum corrections.

For example, the one loop determinants of the ghost and linearized metric fluctuations do not quite cancel

$$e^{S^1} = \frac{\det \Delta_{ghost}}{\det \Delta_{graviton}} \neq 1$$

To determine the S^i we can either compute the all-loop Feynman diagrams or we can cheat. Let's cheat!

Chern-Simons Theory

Classical 3D gravity is an $SU(2) \times SU(2)$ Chern-Simons theory with imaginary levels $k_{\pm} = \pm ik$

$$S_{GR} = kI_{cs}[A_+] - kI_{cs}[A_-]$$

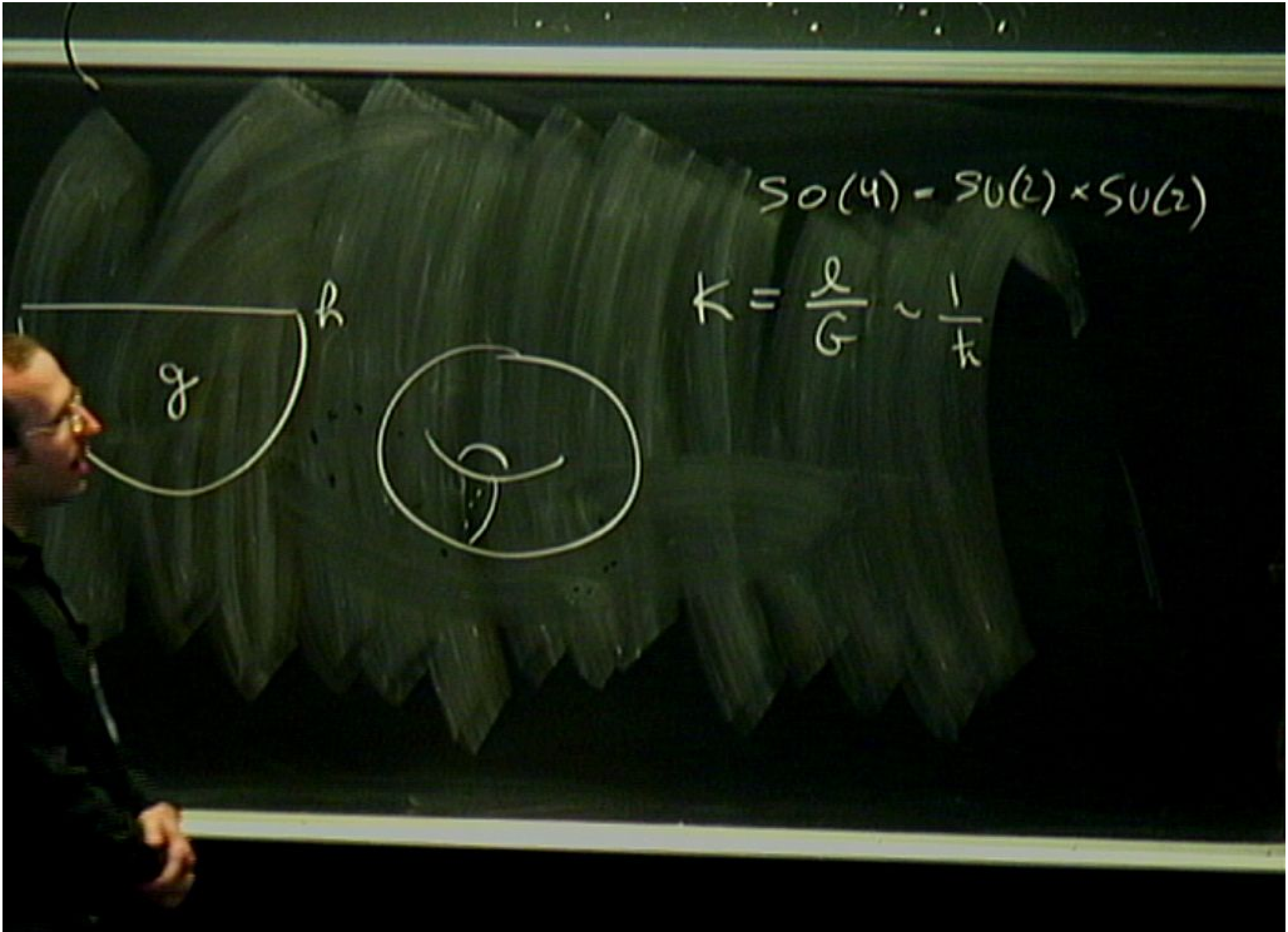
where I_{cs} is the $SU(2)$ Chern-Simons invariant

$$I_{cs}[A] = \mathrm{Tr} \int \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$

For Minkowski and AdS gravity subtleties arise because the gauge group and space-time are not compact.

But we are now studying CS theory with a compact gauge group on a compact manifold.

The CS partition function can be computed exactly using TQFT methods.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Chern-Simons Theory $\neq$ Gravity

The two theories are not equivalent at the quantum level.

The CS path integral is a sum over all flat connections on a manifold of fixed topology. The gravity path integral is a sum over topologies, with a specific flat connection (a metric) for each topology.

But they are equivalent at all orders in perturbation theory around a given classical saddle.

To compute the gravity perturbative corrections we must isolate the contribution to the CS path integral which comes from the flat connection of gravity.

We can check that

- Tree level: Einstein-Hilbert action $\leftrightarrow$ CS invariant
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But they are equivalent at all orders in perturbation theory around a given classical saddle.

To compute the gravity perturbative corrections we must isolate the contribution to the CS path integral which comes from the flat connection of gravity.

We can check that

- Tree level: Einstein-Hilbert action $\leftrightarrow$ CS invariant
- One-loop level: GR determinant $\leftrightarrow$ CS determinant (hard!)

Chern-Simons Theory $\neq$ Gravity

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Results & Discussion

The Answer

We now compute the path integral, including all perturbative (loop) and non-perturbative (instanton) corrections coming from lens spaces.

For 3D Einstein gravity the answer is divergent due to the sum over geometries with small volume

$$Z = 24\zeta(1) + \dots$$

This divergence cannot be regulated using standard field theory techniques.

Conclusion: the Hartle–Hawking state of Einstein gravity is not normalizable. It does not live in a finite dimensional Hilbert space.

nterpretation

What does this mean?

Perhaps:

- de Sitter gravity does not exist. (Unlikely)
- de Sitter gravity exists but we have not done the path integral correctly. (Unlikely)
- de Sitter gravity exists but we are computing the wrong thing. (Possible)
- de Sitter gravity exists and the wave function is normalizable only if we include more interesting degrees of freedom. (Likely)

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TMG in dS

For example, if we modify the theory to include a gravitational Chern-Simons term

$$S = \frac{1}{G}\int\left(R - \frac{2}{\ell^2} + \mu^{-1} I_{cs}\right)$$

the theory now has a local degree of freedom.

The Euclidean saddles are the same but the action is different. For certain values of the coupling

$$\frac{\ell}{G} \in (\mu\ell)\,\mathbb{Z}$$

phases in the sum conspire to make the divergent piece cancel. The Hartle–Hawking state has finite norm.

Interpretation: "Pure" quantum gravity does not exist. Additional degrees of freedom are required.

S/CFT

Near $\mathcal{I}^+$ the Hartle-Hawking wave function is conjectured to become a CFT partition function times local counterterms

$$\langle h|\psi\rangle \to Z_{CFT}(h)$$

Modular invariance

$$Z(\tau) = Z(\gamma\tau) \qquad \gamma \in SL(2,\mathbb{Z})$$

allows us to determine the high energy density of states.

With certain (strong) assumptions, Cardy's formula reproduces the de Sitter entropy:

$$S = \frac{A}{4G}$$

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e Sitter Entropy

Where did this modular invariance come from?

In the presence of a boundary, our saddle points are related by a (large) coordinate transformation. The group of such coordinate transformations is $SL(2, \mathbb{Z})$.

This explains the modular invariance of the dS/CFT partition function

$$\textit{Modular invariance} \Leftrightarrow \textit{General Coordinate Invariance}$$

and at least partly explains the applicability of Cardy's formula.

Interpretation: de Sitter entropy is a feature of any consistent, diffeomorphism invariant theory of quantum gravity in de Sitter space.

c.f. Castro's Talk Tomorrow

peculation: Peaks of the Wave Function

Near $\mathcal{I}^+$ the wave function $\langle h|\psi\rangle$ approaches a CFT partition function, regarded as a function of the conformal structure of the spatial slice.

It is a CFT with negative central charge, so will have the reverse of the usual properties

- It vanishes when the spatial slice is very inhomogeneous
- It is peaked when the spatial slice has automorphisms

Speculation: The wave function of the universe prefers geometries with symmetry.

Speculation: Cosmological Constant

We can not explain the small value of the cosmological constant.

The partition function converges only when the coupling constant of the theory obeys certain (number theoretic) equation. For Einstein gravity this equation had no solutions.

For topologically massive gravity this required the cosmological constant to be integer quantized.

Speculation: For more realistic theories of gravity the cosmological constant solves a more complicated number theoretic equation. This may explain its small value.