Title: Bubble Popper

Date: Jun 24, 2011 03:00 PM

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Abstract: In the context of the AdS/CFT correspondence, I will discuss model-independent properties shared by bulk theories of gravity with consistent dual descriptions. I will then discuss the prospects of extending these ideas to non-conformal theories, in particular to attempts to realize cosmological theories holographically. I will address the status of in-principle falsifiability of various holographic proposals through internal consistency conditions of the boundary theory.

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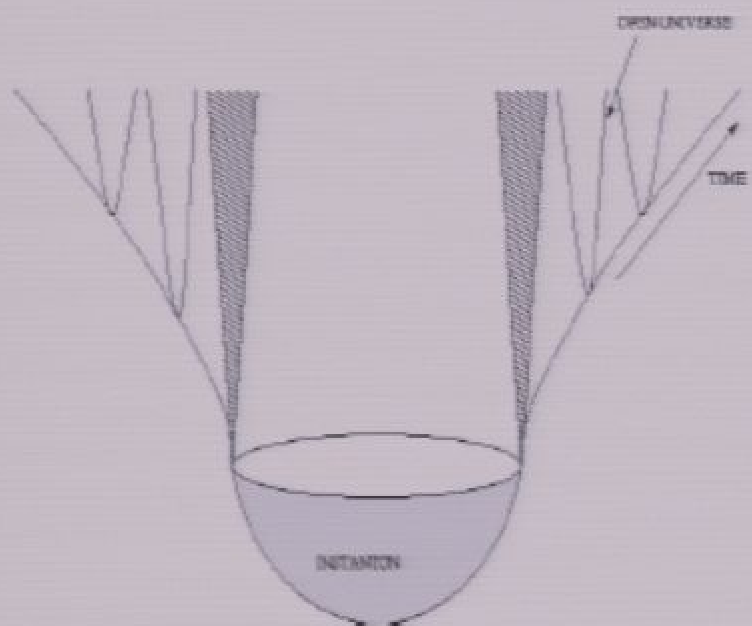


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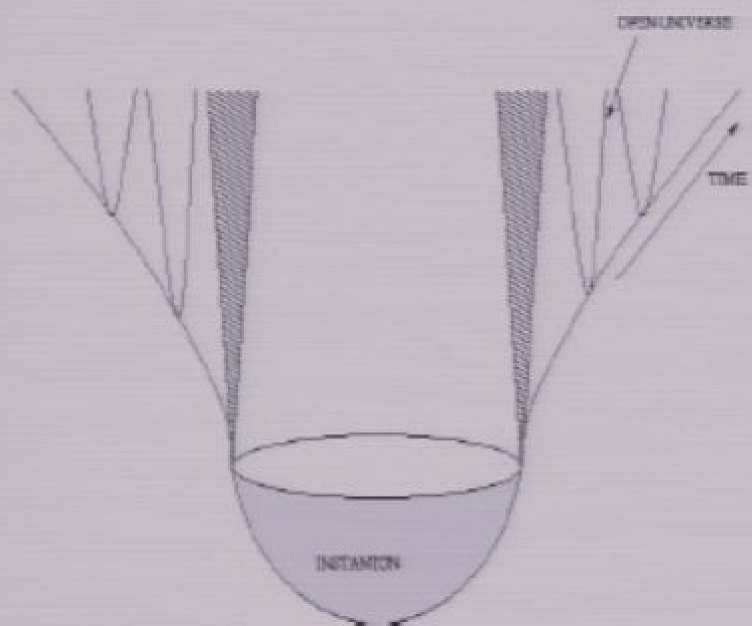


Holographic cosmology –

# de Sitter etc.

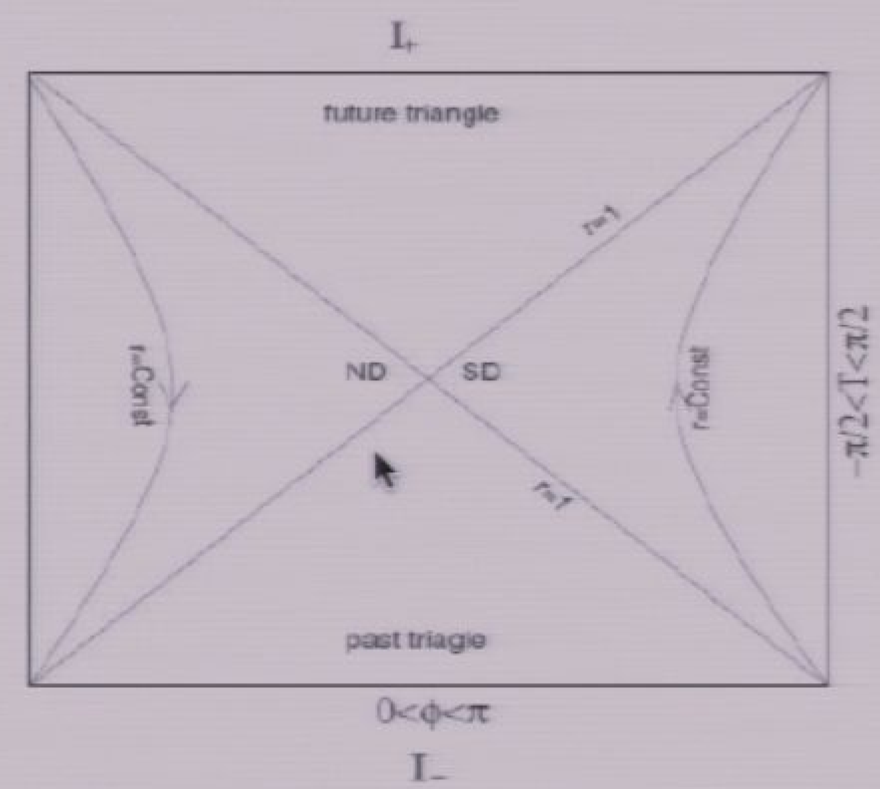


Figure: The Universe

Our best phenomenological model of cosmology indicates that our present Universe has a positive vacuum energy and will asymptote to de Sitter space.

# de Sitter etc.

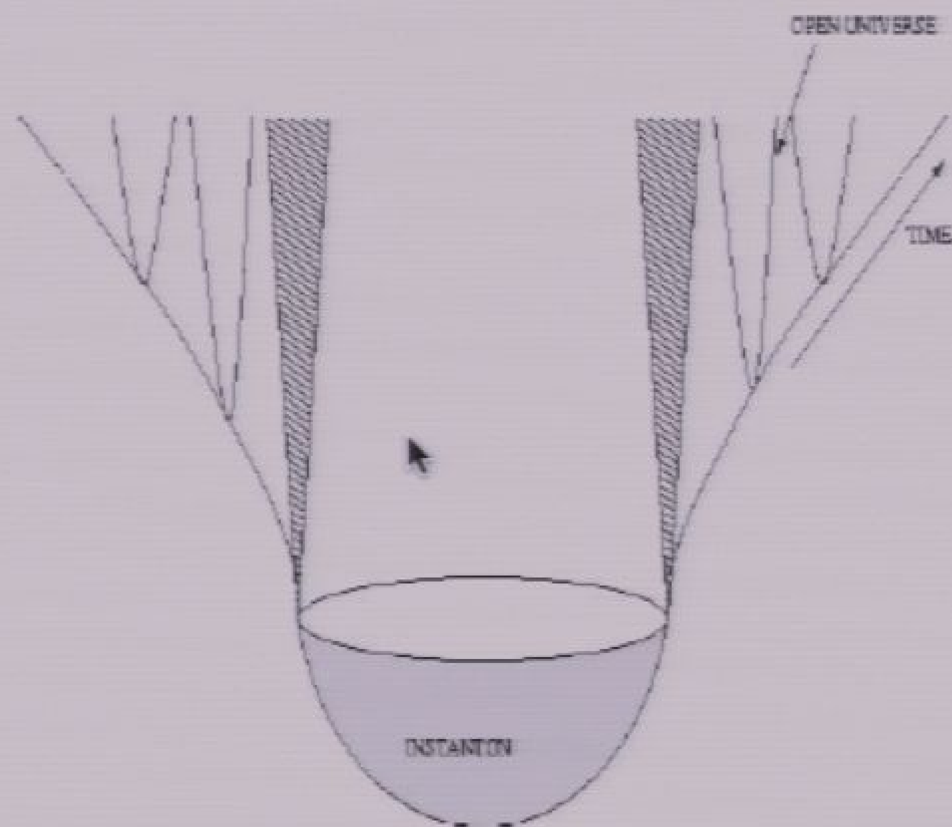


Figure: The Universe

Eternal inflation suggests our observable Universe was born from a Coleman-de-Luccia bubble.

# Falsifiability

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Figure: The Opposite Of Constipated

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- To amplify the second point, by observable or falsifiable, we can be referring to sectors of the landscape apparently disjoint from our own. Because someday a census taker will be able to look back from a terminal SUSY vacuum and see all of them.
- In fact, generating a list of metastable de Sitter solutions represents one observable that's accessible in principle to a terminal observer and should be calculable.
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  - For a given gauge group, what is the highrest possible mass of the lightest charged state? (NOT fuzzy.)
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- All these questions have AdS analogs.
- What does it mean to solve the dS bootstrap?

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- Let's consider a model analog calculation for AdS to understand how in principle we'd want to proceed and what tools may or may not be missing for the dS case.

# BTW

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If I say your theory is unfasifiable – don't take it personally –

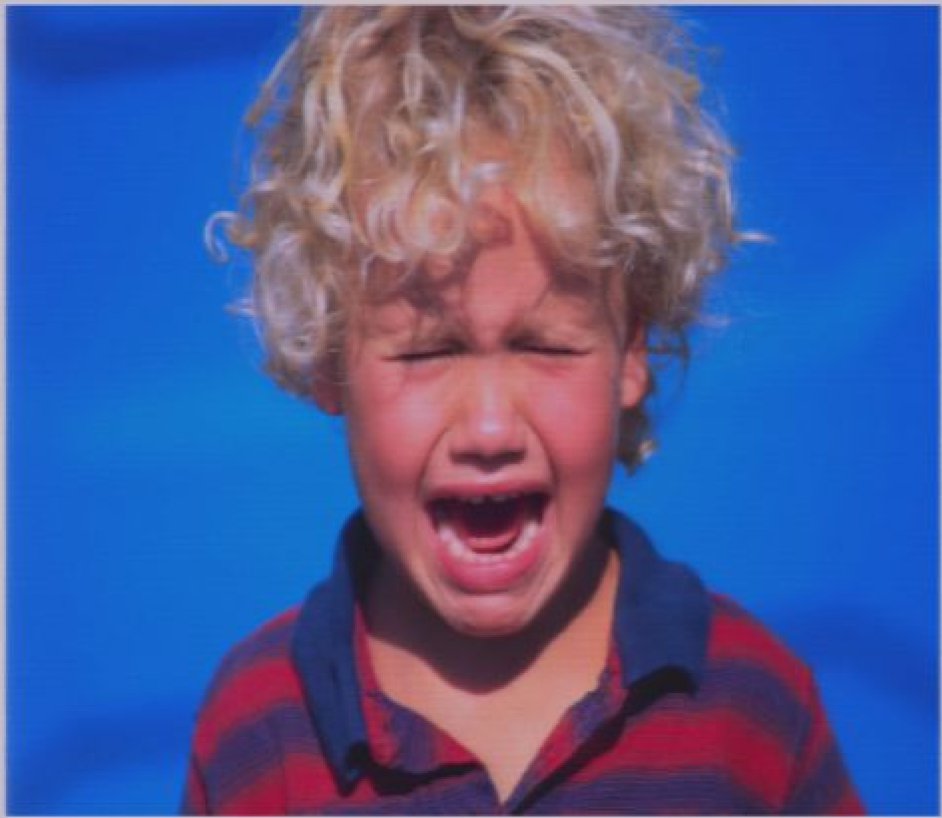

Figure: Don't cry!

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Figure: make-up gift

# CFT FAQ

Not everyone knows what a CFT is.
A CFT is an object defined by a set of local operators $\mathcal{O}_i(z)$ and an operator product expansion.

$$\mathcal{O}_i(z_1) \cdot \mathcal{O}_j(z_2) = \sum_k f_{ij}{}^k(z_1, z_2)\mathcal{O}_k(z_2) \ ,$$

including the identity $\mathcal{O}_0 = 1$.

These local operators define a set of expectation values such that the OPE is satisfied inside the expectation value.

$$\langle \mathcal{O}_{i_1}(z_1)\mathcal{O}_{i_2}(z_2) \cdot (\text{other operators})\rangle = \sum_j f_{i_1 i_2}{}^j(z_1, z_2) \langle \mathcal{O}_j(z_3) \cdot (\text{other}$$

# CFT FAQ

This product is taken to be associative, and the expansion is convergent, for $z_1$ sufficiently close to $z_2$.

One of these operators is taken to be the stress tensor $T_{ab}$.

Furthermore the theory is taken to be defined on an arbitrary* manifold $M$ with an arbitrary * background geometry $g_{ab}$.

# CFT FAQ

Finally the stress tensor is given by the variation of the theory with resepect to $g_{ab}$:

$$\left\langle T^{ab}(z) \cdot (\text{operators}) \right\rangle = \frac{\delta}{\delta g_{ab}} \langle (\text{operators}) \rangle$$

For a theory depending only on the conformal structure, and not on the local scale, the stress tensor must be traceless: $T_a{}^a = 0$.

In particular, our expectation values depend only on the intrinsic geometry and topology and not on the coordinate system.

The invariance under infinitesimal coordinate transformations is equivalent to the condition that $\nabla^b T_{ab} = 0$.

The invariance under coordinate transformations not connected to the identity is referred to as modular invariance.

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That is, we know a great deal about special classes of CFT –(SUSY, holomorphically factorized, integrable,$\cdots$) – but not characteristics of the entire landscape of CFT.

# $AdS_3/CFT_2$

Both of these difficulties are easier to deal with in two dimensional CFT. So we will try to learn the maximum mass gap for a theory of quantum gravity with $\Lambda < 0$ in three dimensions.

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$$E_n = \frac{|n|}{L} \qquad L \equiv \sqrt{-\Lambda}\,.$$

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When the Hilbert space completely factorizes as a product of left- and right-moving states, then it is possible to prove the following bound:

$$h_1 \leq \frac{c}{24} + 1$$
$$\tilde{h}_1 \leq \frac{\tilde{c}}{24} + 1$$

Furthermore, this is the best possible bound for holomorphically factorized CFT * (not necessarily true).

The "extremal CFT"

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$$Z(\tau,\bar{\tau}) \neq Z_{\mathrm{RIGHT}}(\tau)\cdot Z_{\mathrm{LEFT}}(\bar{\tau})$$

in general!

We would like to extract the underlying principle and generalize the bound to the non-factorized

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More generally, at higher odd order $p$, the condition for modular invariance is

$$\left\langle f_p(E) \right\rangle \Big|_{\beta=2\pi} = 0 \, ,$$

where $f_p(E)$ is a polynomial defined by

$f_p(E)$

# Modular Invariance

The first order condition of moular invariance at medium temperature is that the average energy is exactly equal to zero when $\beta = 2\pi$ :

$$\left\langle E \right\rangle \Big|_{\beta=2\pi} = 0 \, .$$

More generally, at higher odd order $p$, the condition for modular invariance is

$$\left\langle f_p(E) \right\rangle \Big|_{\beta=2\pi} = 0 \, ,$$

where $f_p(E)$ is a polynomial defined by

$$f_p(E) \equiv (-1)^p \cdot \exp(+2\pi E) \cdot \left( \beta \partial_\beta \right)^p$$

# Modular Invariance

The first two odd polynomialsare

# Modular Invariance

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we can write

$$\left( \beta\partial_\beta \right)^3 Z^{(ex)} \Big|_{\beta=2\pi} = - \left( \beta\partial_\beta \right)$$

# Modular Invariance

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we can write

$$\left(\beta\partial_\beta\right)^3 Z^{(ex)}\Big|_{\beta=2\pi} = -\left(\beta\partial_\beta\right)^3 Z^{(vac)}\Big|_{\beta=2\pi}$$

$$\left(\beta\partial_\beta\right)^1 Z^{(ex)}\Big|_{\beta=2\pi} = -\Big($$

# Modular Invariance

The first two

$f_1(E) = \ 2\pi E$

Using these p
equations for
$E_1$ is too high

$Z^{(vac)} \equiv$

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# Modular Invariance

$$\left( \beta\partial_\beta \right)^3 Z^{(\mathrm{ex})} \Big|_{\beta=2\pi} = -\left( \beta\partial_\beta \right)^3 Z^{(\mathrm{vac})} \Big|_{\beta=2\pi}$$

$$\left( \beta\partial_\beta \right)^1 Z^{(\mathrm{ex})} \Big|_{\beta=2\pi} = -\left( \beta\partial_\beta \right)^1 Z^{(\mathrm{vac})} \Big|_{\beta=2\pi}$$

# Modular Invariance

$$\left( \beta\partial_\beta \right)^3 Z^{(ex)} \Big|_{\beta=2\pi} \quad - \left( \beta\partial_\beta \right)^3 Z^{(vac)} \Big|_{\beta=2\pi}$$

$$=$$

$$\left( \beta\partial_\beta \right)^1 Z^{(ex)} \Big|_{\beta=2\pi} \quad - \left( \beta\partial_\beta \right)^1 Z^{(vac)} \Big|_{\beta=2\pi}$$

# Modular Invariance

$$\frac{\left(\beta\partial_\beta\right)^3 Z^{(\mathrm{ex})}\Big|_{\beta=2\pi}}{\left(\beta\partial_\beta\right)^1 Z^{(\mathrm{ex})}\Big|_{\beta=2\pi}} = \frac{+ (}{+ (}$$

$$E)^2 - 3(2\pi E) + 1 \quad \equiv \quad I_{31}(E_0)\ .$$

# Modular Invariance

$$\frac{\left(\ \beta\partial_\beta\ \right)^3 Z^{(ex)}\ \Big|_{\beta=2\pi}}{\left(\ \beta\partial_\beta\ \right)^1 Z^{(ex)}\ \Big|_{\beta=2\pi}} = \sum_{m=1}^{\infty} f_3(E_m)\exp(\qquad$$

$$I_{31}(E_0)\ .$$

# Modular Invariance

$$\frac{\left(\beta\partial_\beta\right)^3 Z^{(ex)}\Big|_{\beta=2\pi}}{\left(\beta\partial_\beta\right)^1 Z^{(ex)}\Big|_{\beta=2\pi}} = \frac{\sum_{m=1}^{\infty} f_3(E_m)\exp(-2\pi E_m)}{\sum_{n=1}^{\infty} f_1(E_n)\exp(-}$$

$I_{31}(E_0)$ .

# Modular Invariance

$$= \frac{\sum_{m=1}^{\infty} \quad \frac{f_3(E_m)}{f_1(E_m)} \cdot f_1(E_m) \quad \exp\left(-2\pi E_m\right)}{\sum_{n=1}^{\infty} f_1(E_n) \exp\left(-2\pi E_n\right)}$$

$$= I_{31}(E_0) \ .$$

# Modular Invariance

$$\frac{\sum_{m=1}^{\infty} \quad I_{31}(E_m) \cdot f_1(E_m) \quad \exp(-2\pi E_m)}{\sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)} \quad - \quad I_{31}(E_0) = \quad 0$$

Now subtract $I(E_0)$ from both sides.

# Modular Invariance

$$\frac{\sum_{m=1}^{\infty} \quad I_{31}(E_m) \cdot f_1(E_m) \quad \exp(-2\pi E_m)}{\sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)} \quad - \quad I_{31}(E_0) = \quad 0$$

Multiply through

# Modular Invariance

$$\frac{\sum_{m=1}^{\infty} \quad I_{31}(E_m) \cdot f_1(E_m) \quad \exp(-2\pi E_m)}{\sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)} = I_{31}(E_0)$$

Now subtract

# Modular Invariance

$$\frac{\sum_{m=1}^{\infty} \quad I_{31}(E_m) \cdot f_1(E_m) \quad \exp(-2\pi E_m)}{\sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)}$$

$$= I_{31}(E_0)$$

# Modular Invariance

$$\frac{\sum_{m=1}^{\infty} \quad l_{31}(E_m) \cdot f_1(E_m) \quad \exp(\cdot}{\sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)}$$

$$\Rightarrow$$

$$\sum_{m=1}^{\infty} l_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - l_{31}(E_0) \cdot \sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n) = 0$$

# Modular Invariance

$$\Rightarrow$$

$$\sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp\left(-2\pi E_m\right) - I_{31}(E_0) \cdot \sum_{n=1}^{\infty} f_1(E_n) \exp\left(-2\pi E_n\right) = 0$$

# Modular Invariance

$$\sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - I_{31}(E_0) \cdot \sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)$$

$$= \sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - \sum_{n=1}^{\infty} I_{31}(E_0)\, f_1(E_n) \exp(-2\pi E_n) = 0$$

Now bring $I(E_0)$ inside the sum –

# Modular Invariance

$$\sum_{m=1}^{\infty} l_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - l_{31}(E_0) \cdot \sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)$$

$$= \sum_{m=1}^{\infty} l_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - \sum_{n=1}^{\infty} l_{31}(E_0)\, f_1(E_n) \exp(-2\pi E_n) = 0$$

– change dummy indices –

# Modular Invariance

$$\sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - I_{31}(E_0) \cdot \sum_{n=1}^{\infty} f_1(E_n) \exp(-2\pi E_n)$$

$$= \sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp(-2\pi E_m) - \sum_{m=1}^{\infty} I_{31}(E_0)\, f_1(E_m) \exp(-2\pi E_m) =$$

– and group

# Modular Invariance

$$\sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp\left(-2\pi E_m\right) - I_{31}(E_0)$$

$$= \sum_{m=1}^{\infty} I_{31}(E_m)\, f_1(E_m) \exp\left(-2\pi E_m\right) - \sum_{m=1}^{\infty} I_{31}($$

$$= \sum_{m=1}^{\infty} \Big( I_{31}(E_m) - I_{31}(E_0) \Big)\, f_1(E_m) \exp\left(-2\pi E_m\right) = 0$$

# Modular Invariance

$$\sum_{m=1}^{\infty} \Big( I_{31}(E_m) - I_{31}(E_0) \Big) f_1(E_m) \exp(-2\pi E_m) = 0$$

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# Modular Invariance

$$\sum_{m=1}^{\infty}\Big( I_{31}(E_m) - I_{31}(E_0) \Big) f_1(E_m) \exp(-2\pi E_m) = 0$$

We will derive a universal inequality from this identity

# Modular Invariance

$$\sum_{m=1}^{\infty} \left( I_{31}(E_m) - I_{31}(E_0) \right) f_1(E_m) \exp(-2\pi E_m) = 0$$

The ratio $I_{31}(E)$ is given by $I_{31}(E) =$

# Modular Invariance

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The ratio $I_{31}(E)$ is given by $I_{31}(E) = (2\pi E)^2 - 3(2\pi E) + 1$.
Fixing

# Modular Invariance

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Fixing $E_0$, the roots of the equation $I_{31}(E)$

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Fixing $E_0$, the roots of the equation $I_{31}(E) = I_{31}(E_0)$ are $E =$

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Fixing $E_0$, the roots of the equation $I_{31}(E) = I_{31}(E_0)$ are $E = E_0$, and $E = E_+$, with

$$E_+ \equiv$$

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Note that $E_+$ is positive: we are assuming unitarity

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If a an energy $E$ is greater than

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If a an energy $E$ is greater than $E_+$, then $I_{31}(E) - I_{31}(E_0)$ and $f_1(E)$

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Note that $E_+$ is positive: we are assuming unitarity so $E_0 < 0$.
If a an energy $E$ is greater than $E_+$, then $I_{31}(E) - I_{31}(E_0)$ and $f_1(E) = 2\pi E$ are both positive:

$$f_1(E)\ ,\ \Big(\ I_{31}(E) -$$

# Modular Invariance

$$\sum_{m=1}^{\infty} \Big( I_{31}(E_m) - I_{31}(E_0) \Big) f_1(E_m) \exp(-2\pi E_m) = 0$$

$$E_+ \equiv \frac{3}{2\pi} - E_0 .$$

$$f_1(E) \,,\ \Big( I_{31}(E) - I_{31}(E_0) \Big) > 0 \qquad \text{for} \qquad E > E_+ .$$

# Modular Invariance

$$\sum_{m=1}^{\infty} \Big( I_{31}(E_m) - I_{31}(E_0) \Big) f_1(E_m) \exp(-2\pi E_m) = 0$$

$$E_+ \equiv \frac{3}{2\pi} - E_0 \; .$$

$$f_1(E) \; , \; \Big( I_{31}(E) - I_{31}(E_0) \Big) > 0 \qquad \text{for} \qquad E > E_+ \; .$$

If every excited level $E_n$, $n \geq 1$is greater than $E_+$, then the left-hand side of our medium-temperature modular identity is strictly positive. Therefore some level $E_n$, $n \geq 1$ must be

# Modular Invariance

$$\sum_{m=1}^{\infty} \left( I_{31}(E_m) - I_{31}(E_0) \right) f_1(E_m) \exp(-2\pi E$$

$$E_+ \equiv \frac{3}{2\pi} - E_0 \; .$$

$$f_1(E) \; , \; \left( I_{31}(E) - I_{31}(E_0) \right) > 0 \qquad \text{for}$$

$$E_1 < E_+ \; .$$

# Universal inequality

$$E_+ \equiv \frac{3}{2\pi} - E_0 \ .$$
$$E_1 < E_+ \ .$$

This is a univeral inequality for unitary, modular-invariant CFT with discrete spectrum.

# Universal inequality

Written in terms of operator dimensions $\Delta \equiv E - E_0$, we have

$$\Delta_1 \; <$$

# Universal inequality

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$$\begin{aligned} \Delta_1 &< \Delta_+ \, , \\ \Delta_+ &\equiv \frac{3}{2\pi} + \frac{c_{\text{tot}}}{12} \\ &= \end{aligned}$$

# Universal inequality

Written in terms of operator dimensions $\Delta \equiv E - E_0$, we have

$$\begin{aligned} \Delta_1 &< \Delta_+ \,, \\ \Delta_+ &\equiv \frac{3}{2\pi} + \frac{c_{\rm tot}}{12} \\ &= 0.477465 + \frac{c_{\rm tot}}{12} \,. \end{aligned}$$

For low central charge

# Universal inequality

$$\Delta_+ \equiv \frac{3}{2\pi} + \frac{c_{\text{tot}}}{12} \; .$$
$$\Delta_1 < \Delta_+ \; .$$

For $c_{\text{tot}} \geq$

# Universal inequality

$$\Delta_+ \equiv \frac{3}{2\pi} + \frac{c_{\text{tot}}}{12} \, .$$
$$\Delta_1 < \Delta_+ \, .$$

For $c_{\text{tot}} \geq 24 - \frac{18}{\pi} \simeq 18.2704$, the bound is uninformative, since $\Delta_+ \geq 2$ in this range. (There is always a stress tensor in a CFTanyway, with $\Delta = 2$).
But we can adapt

# Universal inequality for primary operators

The first few polynomials are :

$$f_0(E) = 1 \,,$$

$$f_1(E)$$

# Universal inequality for primary operators

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$$
\begin{aligned}
f_0(E) &= 1\,, \\
f_1(E) &= 2\pi\,E - \frac{1}{2}\,, \\
f_2(E) &= (\,2\pi\,E\,)^2 - 2\,(\,2\pi\,E\,) + \Big(
\end{aligned}
$$

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$$f_3(E) = (2\pi$$

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f_3(E) &= (2\pi\, E)^3 - \frac{9}{2}\,(2\pi\, E)^2 \\
&+ \left( \frac{41}{8} + 6\, r_{20} \right)
\end{aligned}
$$

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f_3(E) &= (2\pi\, E)^3 - \frac{9}{2}\,(2\pi\, E)^2 \\
&\quad + \left(\frac{41}{8} + 6\, r_{20}\right)(2\pi\, E) - \left(\frac{17}{16} + 3\, r_{20}\right)\,,
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where the numerical constant $r_{20}$ is defined as:

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\end{aligned}
$$

where the numerical constant $r_{20}$ is defined as:

$$
r_{20} \equiv \frac{\eta''(i)}{\eta(i)} = -\frac{1}{16} + \sum_{n=1}^{\infty} \frac{\pi^2 n^2}{\sinh^2(\pi n)} = 0
$$

# The gravitational interpretation

So our universal inequality for primary operators is:

$$\Delta_+ \leq$$

# Universal inequality for primary operators

Proceeding in parallel with our warm-up proof, we derive the inequality

$$\Delta_1 \leq \Delta_+ \ ,$$

where $\Delta_+$ is defined as the largest root $\Delta$ of the cubic equation

$$f_3(\Delta + \hat{E}_0) - \frac{b_3(\hat{E}_0)}{b_1(\hat{E}_0)} f_1(\Delta + \hat{E}_0) = 0$$

The function $\Delta_+$ is well-defined for all values of $c_{tot}$. At large $c_{tot}$ it can be expanded as

$$\Delta_+ = \frac{c_{tot}}{12} + \delta_0 + o\left( c_{tot}^{-1} \right) \ ,$$

$$\delta_0 = \frac{(12 - \pi) + (13\pi - 12)\exp(-2\pi)}{6\pi\ (1 - \exp(-2\pi))} \simeq 0.473695 + o\left( 10^{-7} \right)$$

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It is also possible to prove that $\Delta_+$ is uniformly bounded above by

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So our universal inequality for primary operators is:

$$\Delta_+ \leq \frac{c_{\text{tot}}}{12} + \delta_0 \ ,$$

$$\delta_0 \equiv 0.473695 + o\Big(\ 10$$

# The gravitational interpretation

- We have seen that there is a universal upper limit on the energy to which a theory of quantum gravity and matter can EVER be extended.
- The bound can be proved rigorously with no use of perturbation theory or semiclassical methods.
- As $\Lambda \to 0$ the bound is independent of the boundary condition, and makes a universal statement about local bulk physics.
- It is similar in spirit to the weak gravity conjecture.

# The gravitational interpretation

So we use the $AdS_3/CFT_2$ dictionary:

$$c_{\text{tot}} = \frac{3\,L_{AdS}}{G_N} \qquad\qquad \Delta = L_{AdS}\,M\ ,$$

where $M$ is the mass of a state in the bulk.
Using this translation, we obtain:

$$M_1 \leq \frac{1}{4\,G_N} + \frac{\delta_0}{L_{AdS}}\ .$$

This inequality is universal for all theories of gravity and matter in 3 dimensions with negative cosmological constant. It is exact at finite AdS radius, and approaches a finite limit when the AdS radius goes to infinity.

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- We have seen that there is a universal upper limit on the energy to which a theory of quantum gravity and matter can EVER be extended.
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- As $\Lambda \to 0$ the bound is independent of the boundary condition, and makes a universal statement about local bulk physics.

# The gravitational interpretation

So we use the $AdS_3/CFT_2$ dictionary:

$$c_{\rm tot} = \frac{3\,L_{AdS}}{G_N} \qquad\qquad \Delta = L_{AdS}\,M\ ,$$

where $M$ is the mass of a state in the bulk.
Using this translation, we obtain:

$$M_1 \leq \frac{1}{4\,G_N} + \frac{\delta_0}{L_{AdS}}\ .$$

This inequality is universal for all theories of gravity and matter in 3 dimensions with negative cosmological constant. It is exact at finite AdS radius, and approaches a finite limit when the AdS radius goes to infinity.

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- Seems dS/CFT can never do more than describe effective field theory in the bulk.


Simeon Hellerman’s MacBook Air
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