Title: On the fast scrambling conjecture

Date: Jun 23, 2011 03:00 PM

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Abstract: Motivated by the consistency of black hole complementarity, Sekino and Susskind have conjectured that no physical system can "scramble" its internal degrees of freedom in time faster than (1/T) log S, where T is temperature and S the system's entropy. By considering a number of toy examples and general Lieb-Robinson-type causality bounds, I'll explore the range of validity of the conjecture. Some of these examples suggest that nonlocal Hamiltonians can delocalize information at rates exceeding the fast scrambling bound, but the physical relevance of these examples is unclear. Joint work with Nima Lashkari and Douglas Stanford.

Scrambling

- Minimum time for "localized" information to become inaccessible without measuring fraction O(1) of the whole system

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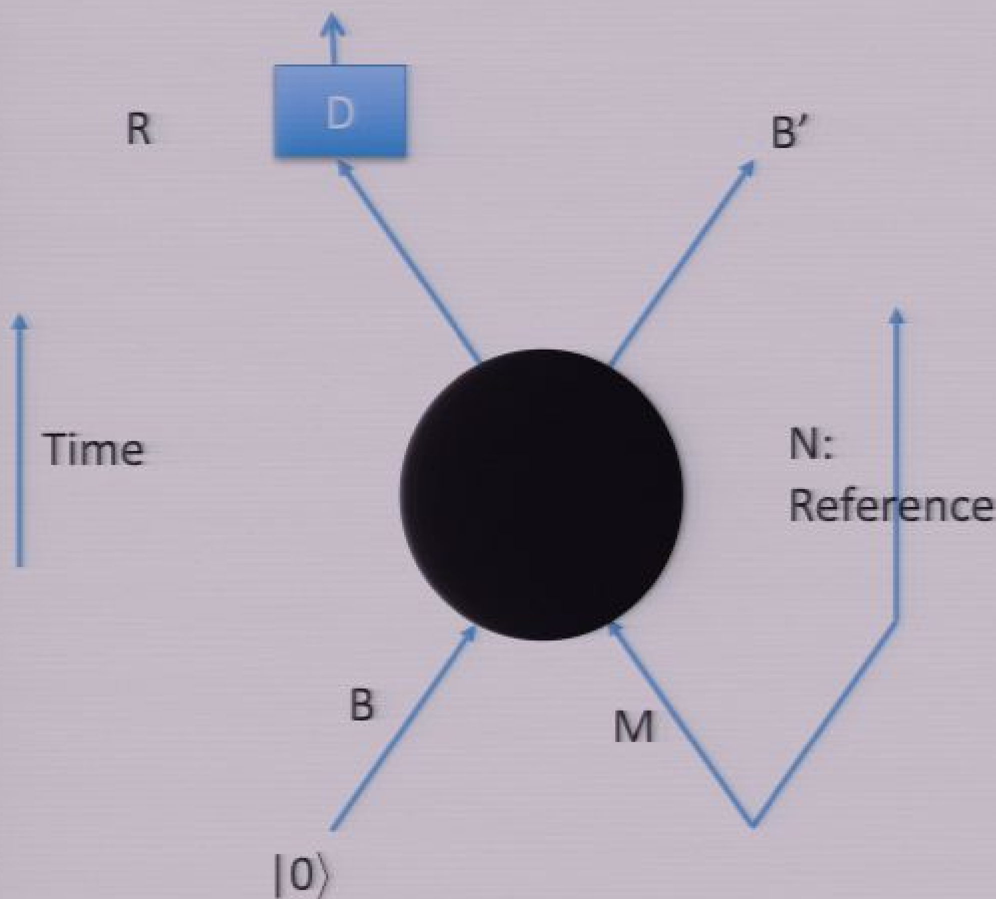
- Normal systems: geometrical locality
  - $Tt^* \sim L^{const} \sim S^{const/d}$

Temperature
Linear size
Scrambling time
Entropy

- Sch. Black holes, thermal cell AdS BH, static patch de Sitter:
  - $Tt^* \sim \hbar \log S$ (estimate based on charge spreading)
- Conjecture: no system can scramble its degrees of freedom faster [Sekino-Susskind'08, Susskind'11]
  - Motivation: black hole complementarity principle [H-Preskill'07]

Decoupling and information



Sending arbitrary states from M to R is *equivalent* to establishing entanglement between N and R

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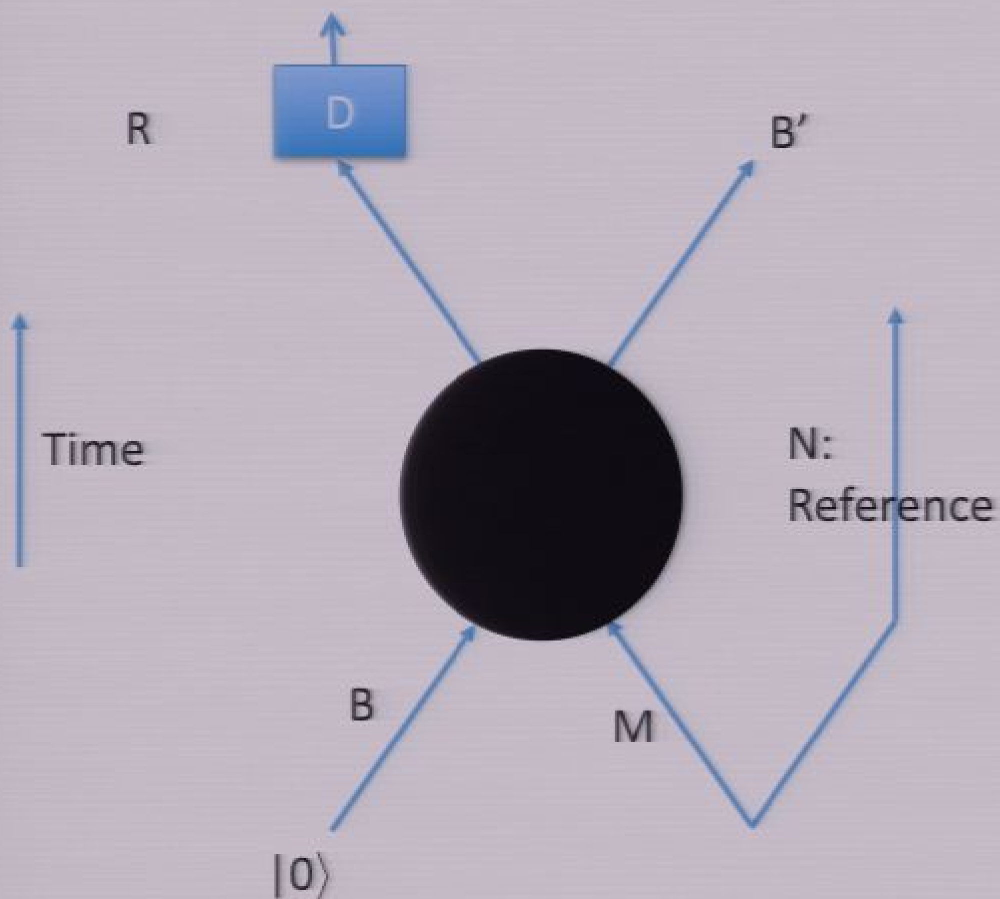

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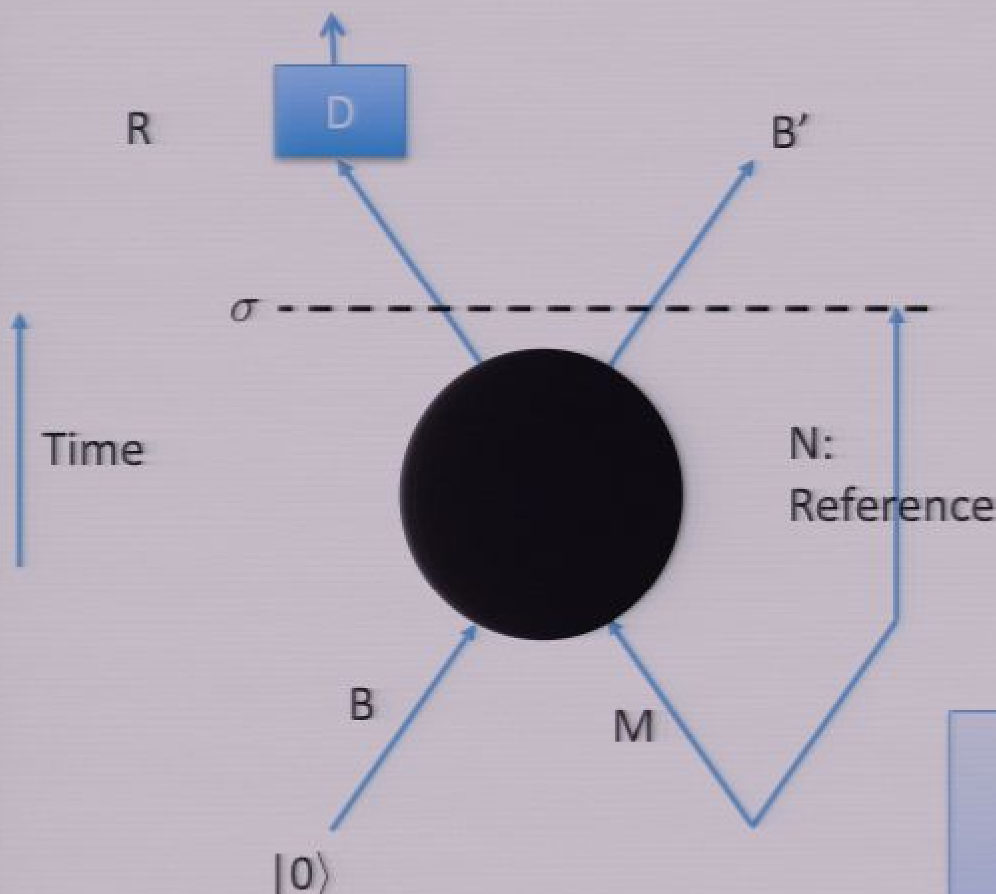

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Establishing entanglement between N and R is *equivalent* to eliminating all correlations between N and B′

$$\mathrm{Tr}_R\, \sigma_{NB'R} = \tau_N \otimes \xi_{B'}$$

$$\Rightarrow$$

$$|\sigma_{NB'R}\rangle = (\mathrm{id}_{NB'} \otimes U_R)\,|\phi_{NR1}\rangle\,|\psi_{B'R2}\rangle$$

Big picture versus toy examples

String theory descriptions of black holes couple degrees of freedom nonlocally.

e.g. BFSS Matrix theory: $$L = \sum_a \mathrm{tr}\, \dot{M}^a \dot{M}^a - \sum_{ab} \mathrm{tr}[M^a, M^b]^2$$

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HARD

Goal of this talk is more modest:

1) Find examples of toy systems that scramble quickly
2) Prove general lower bounds on scrambling times

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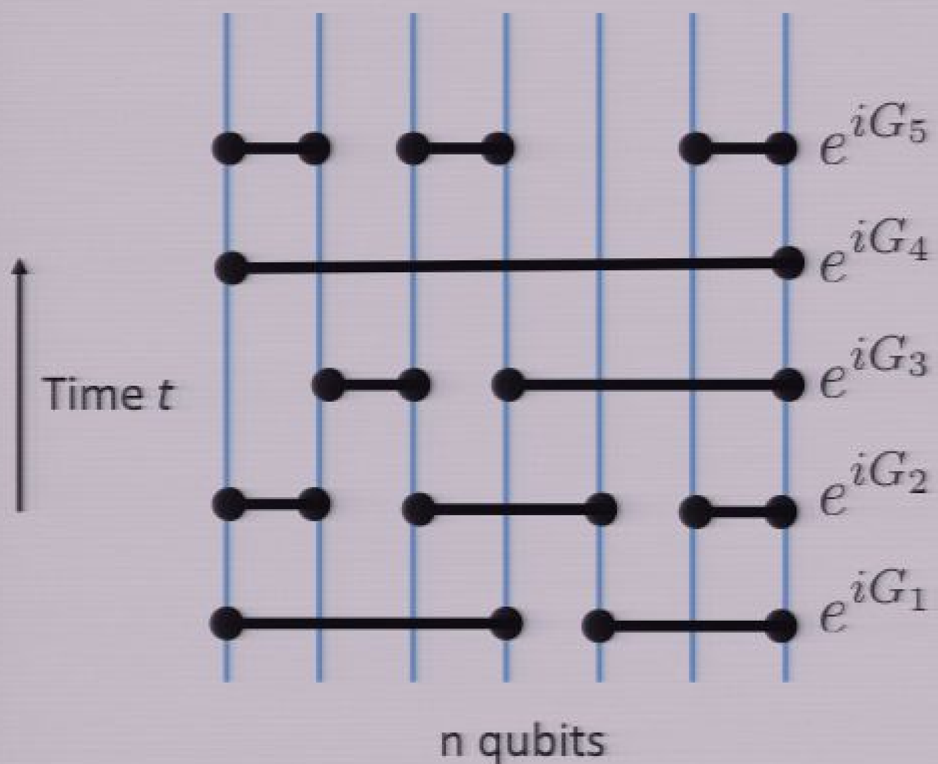
Test: 1) That fast scrambling is even possible
2) Whether scrambling faster than the conjecture might be possible

Outline

- Brownian quantum circuits
- Ising interaction on random graphs
- Feynman's circuit simulator
- Lieb-Robinson bounds for nonlocal interactions*

Brownian circuits



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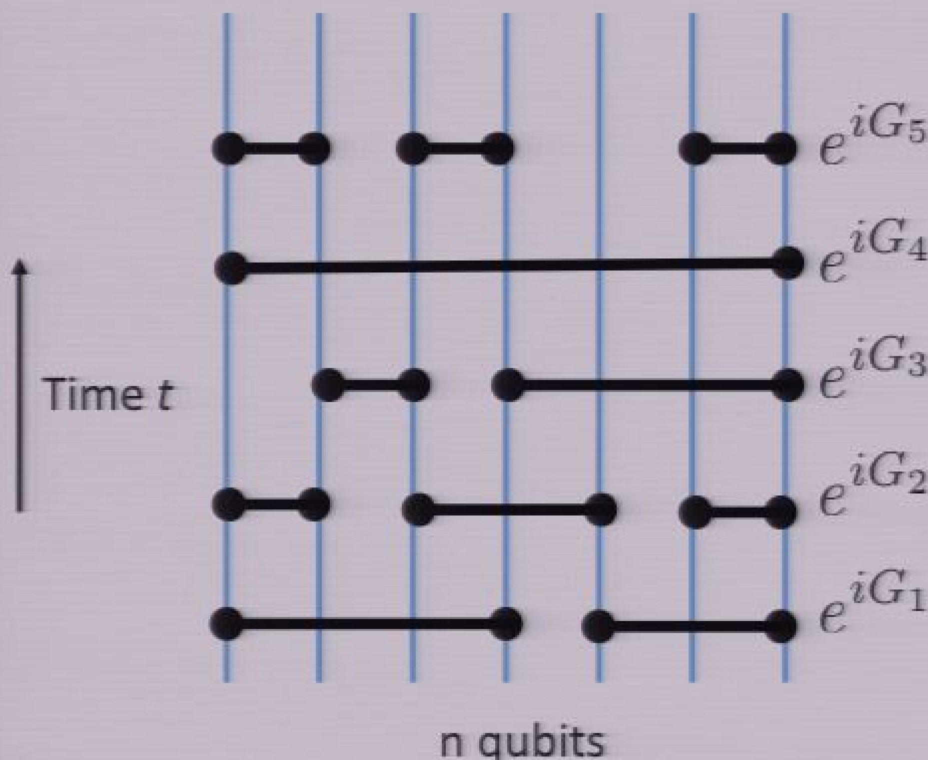
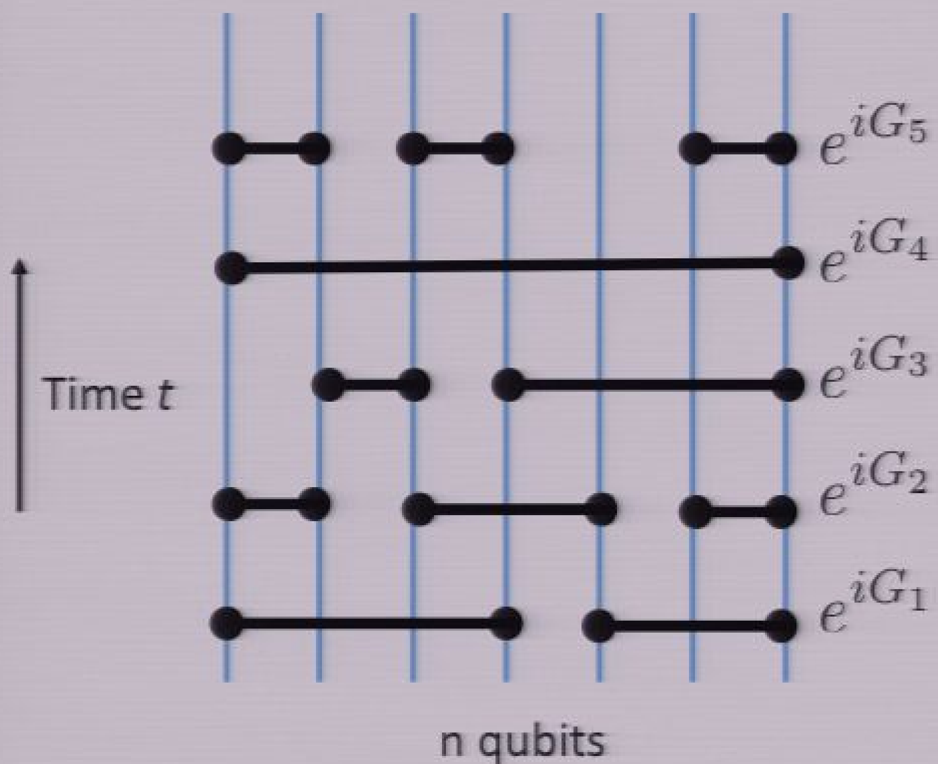


Dankert et al.: Construction of circuit scrambling in time O(log n)

Brownian circuits



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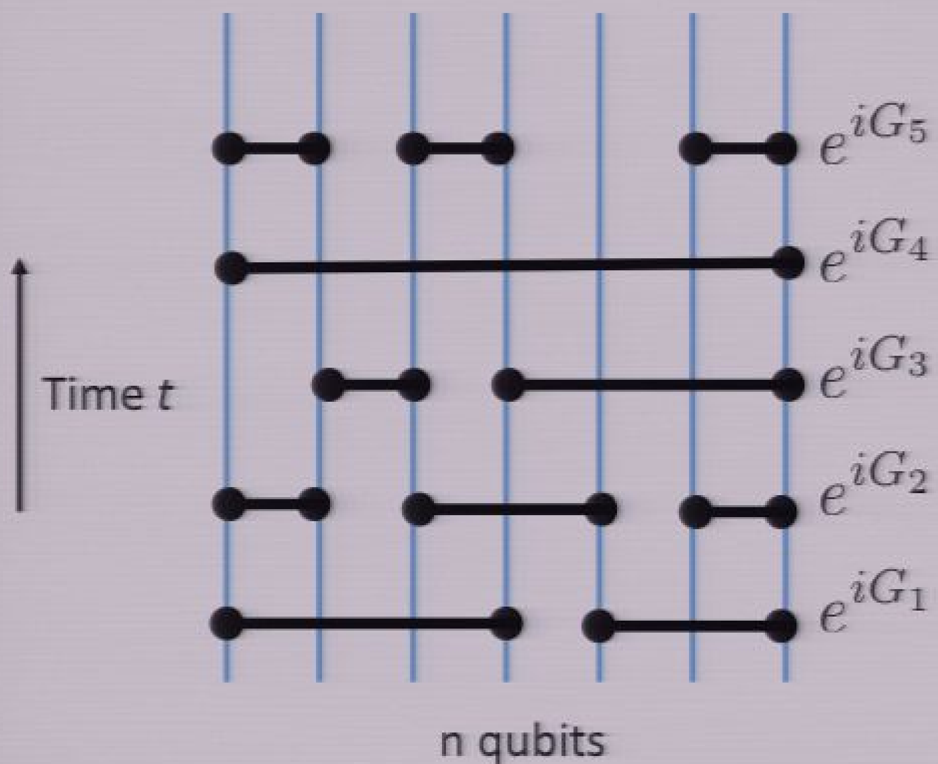

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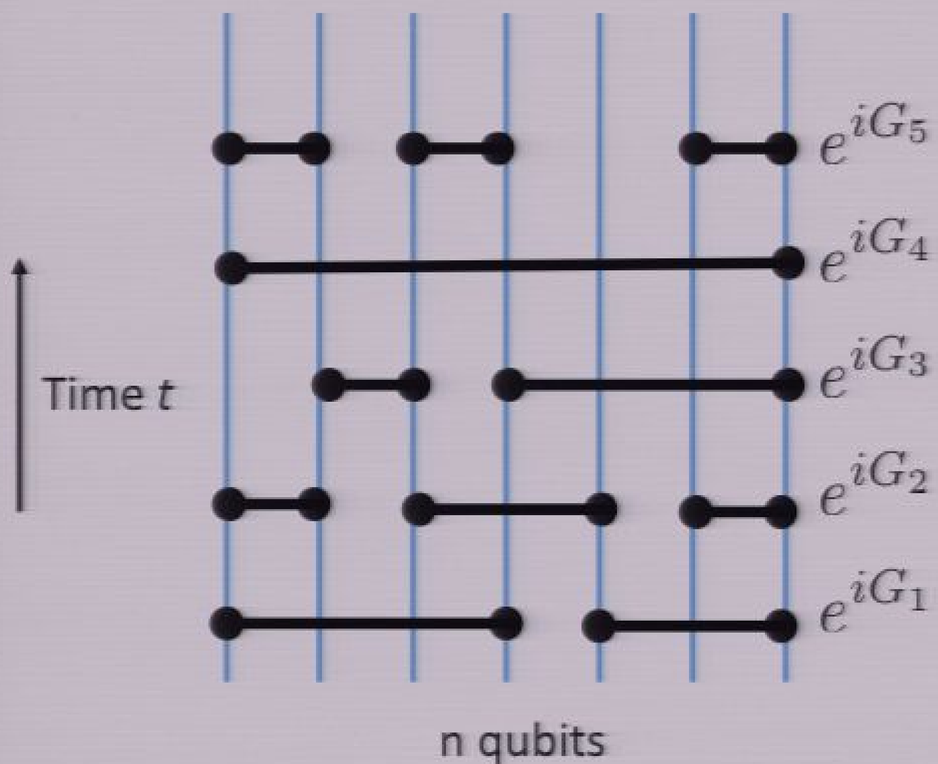
Brownian circuits

G_i random pairwise interaction



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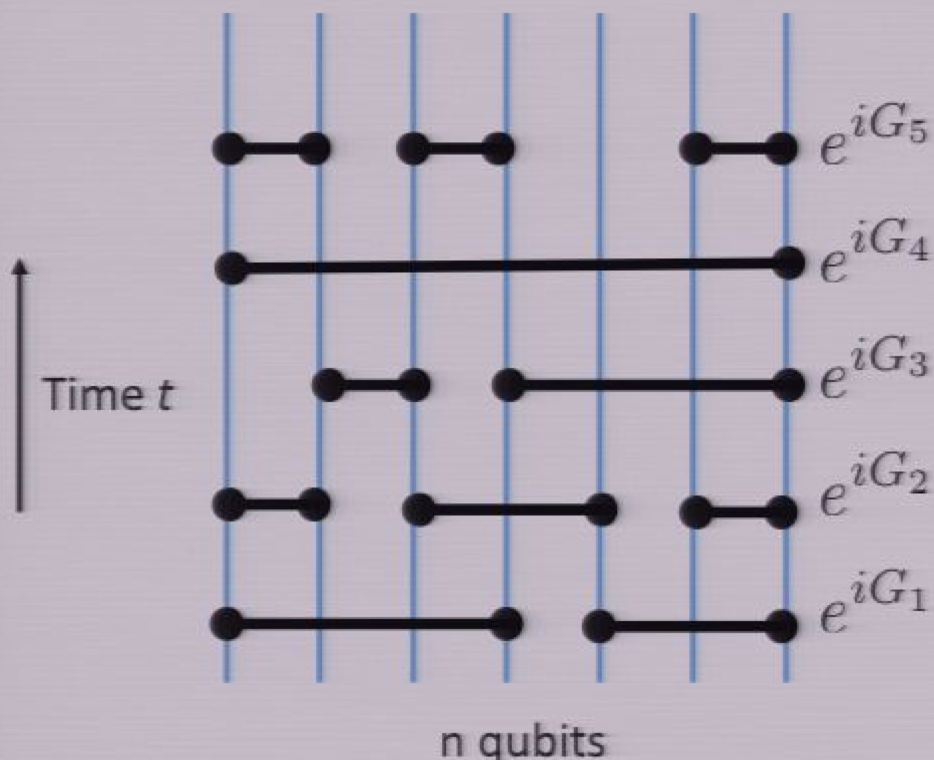

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$$G_i = \sum_{<j,k>} \sum_{\alpha_j,\alpha_k} \sigma_{\alpha_j}^{(j)} \otimes \sigma_{\alpha_k}^{(k)} g_{ijk\alpha_j\alpha_k}$$

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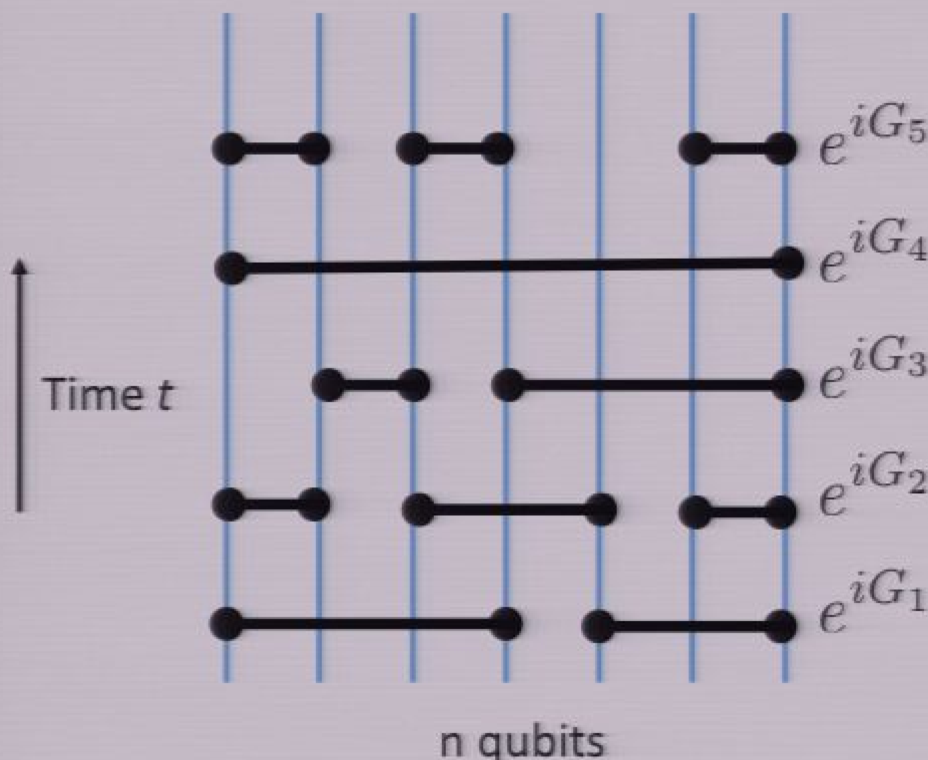

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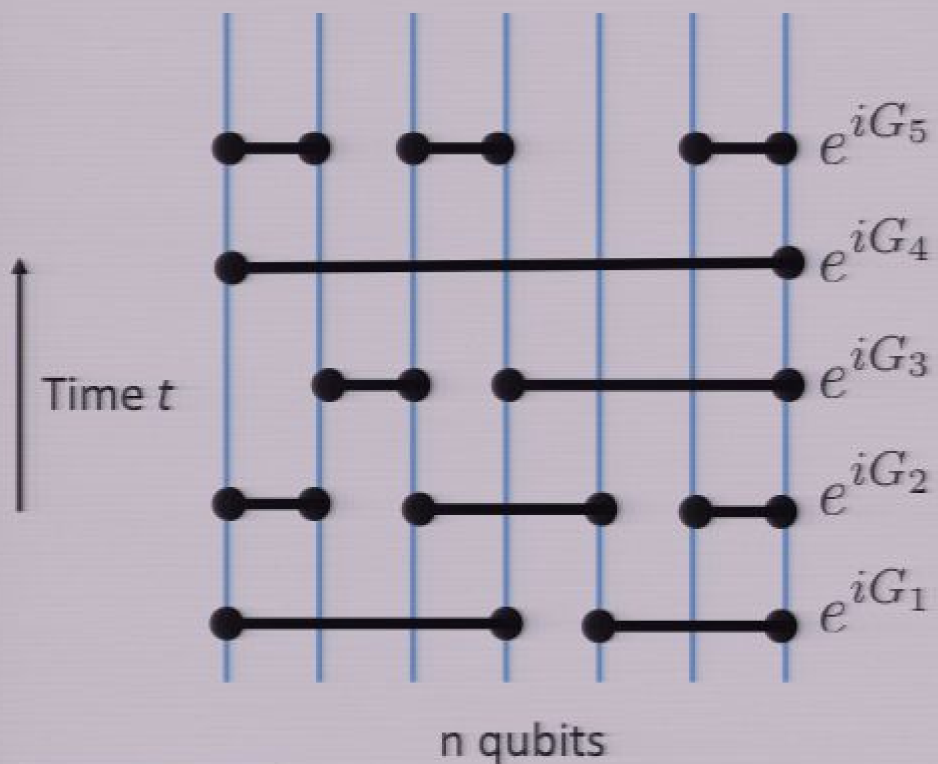

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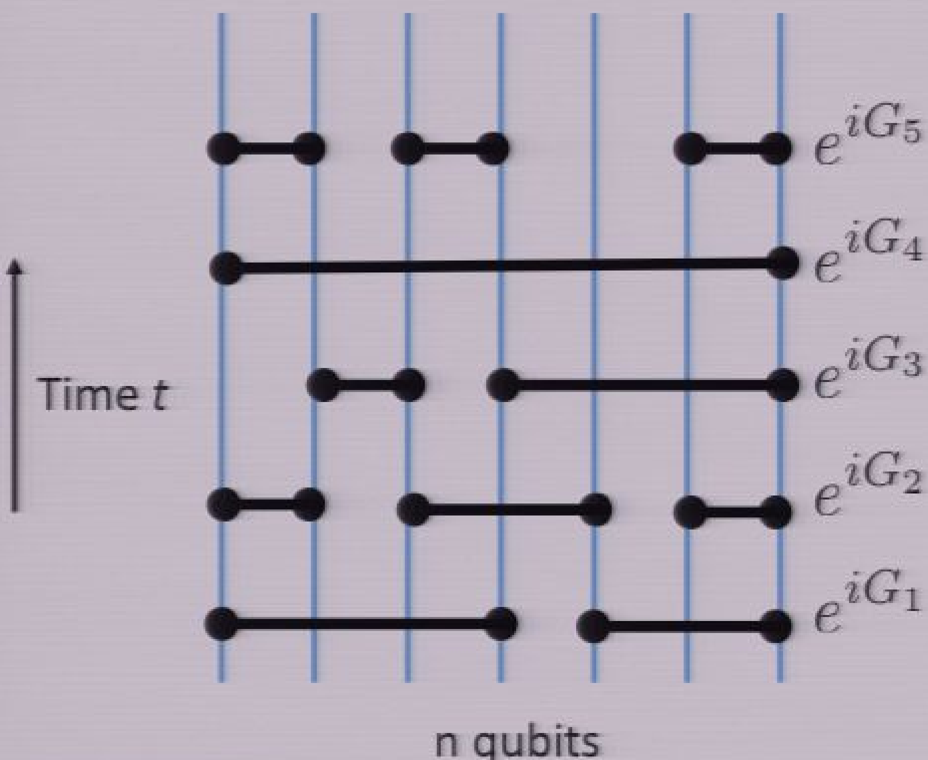

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Limit of infinitesimal ε:

$$U(t) = \prod_{j=1}^{t/\epsilon} e^{iG_j}$$

$$dU(t) = \sum_{<j,k>} \sum_{\alpha_j,\alpha_k} \sigma^{(j)}_{\alpha_j} \otimes \sigma^{(k)}_{\alpha_k} U(t) dW_{jk\alpha_j\alpha_k}(t) - \frac{1}{2}U(t)$$

Weiner process

Entanglement production

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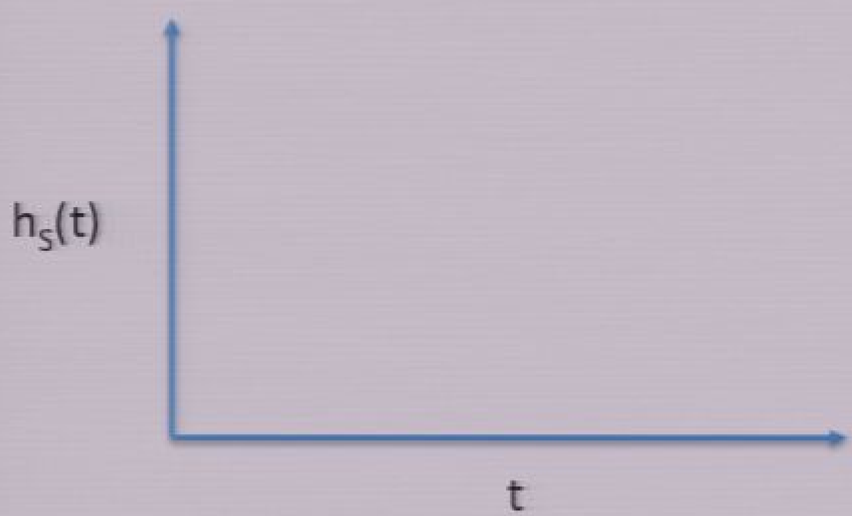

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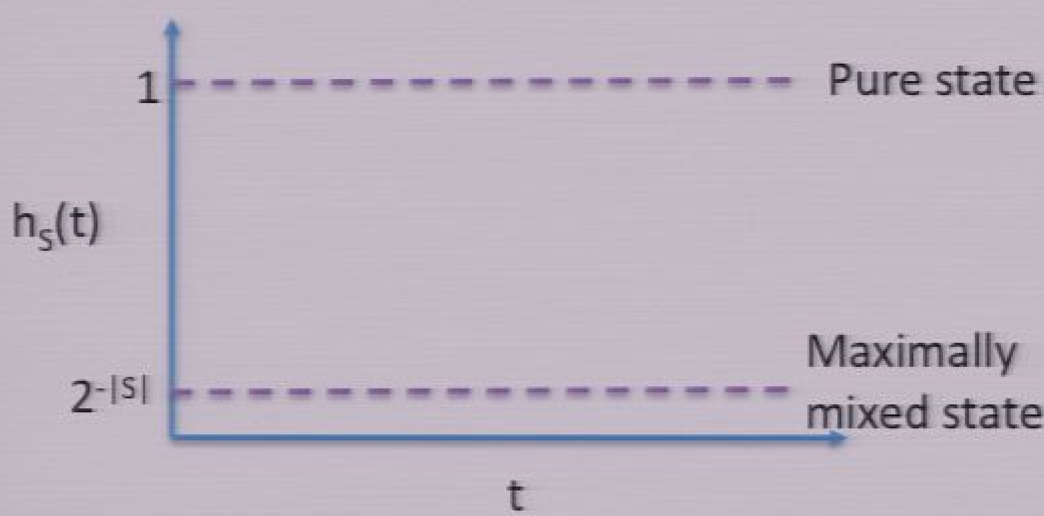

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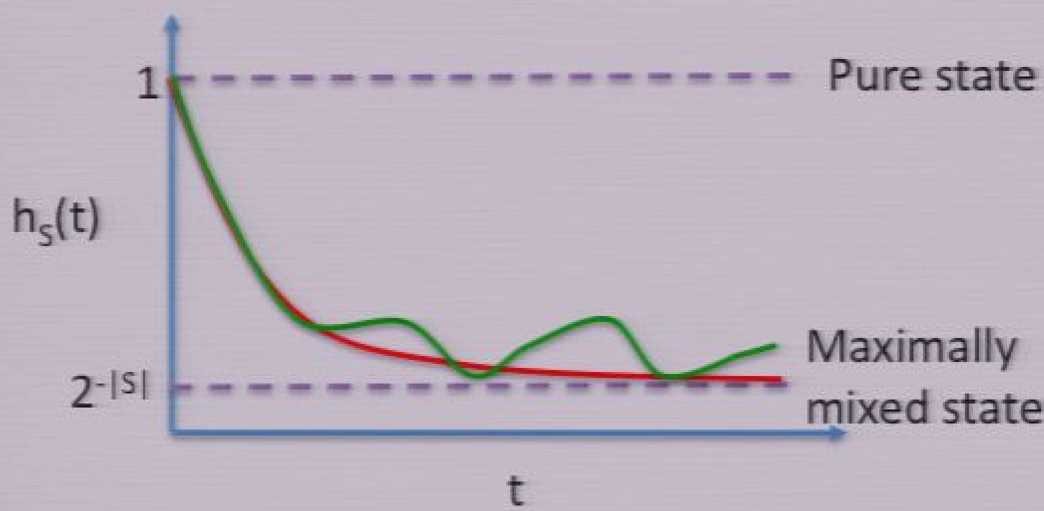

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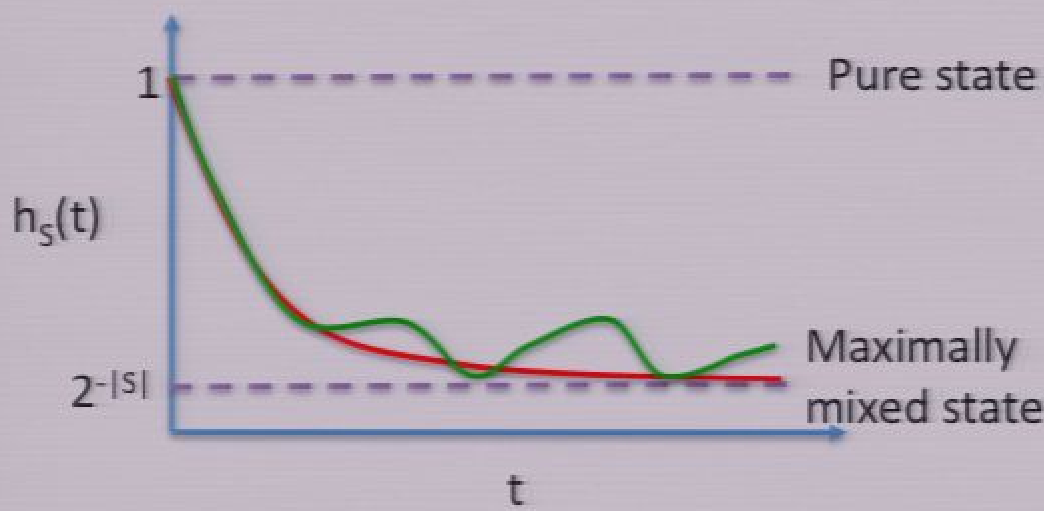

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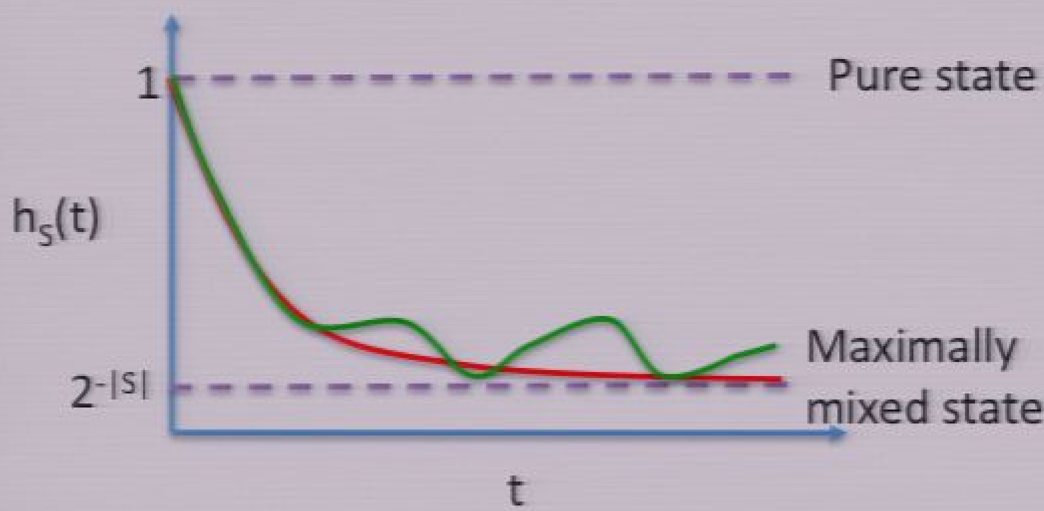

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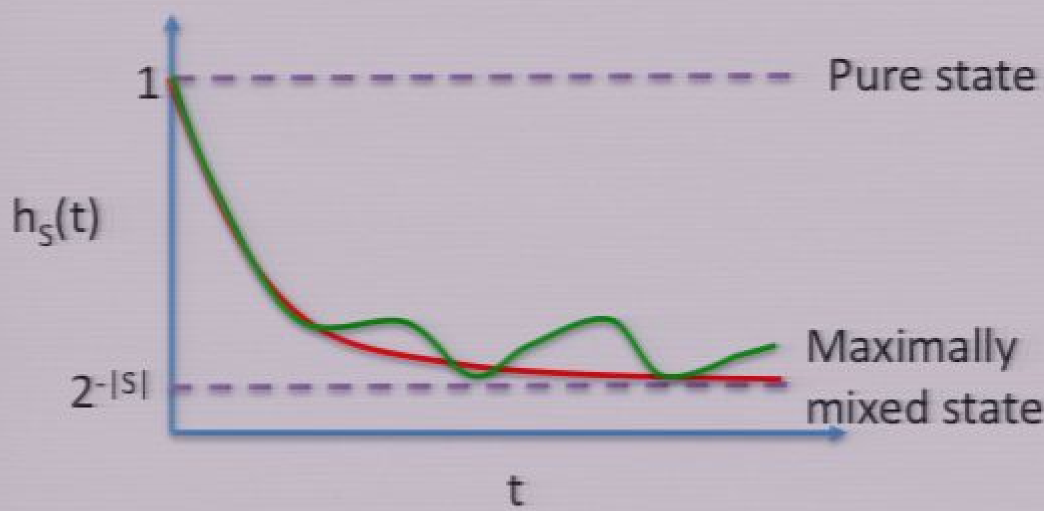

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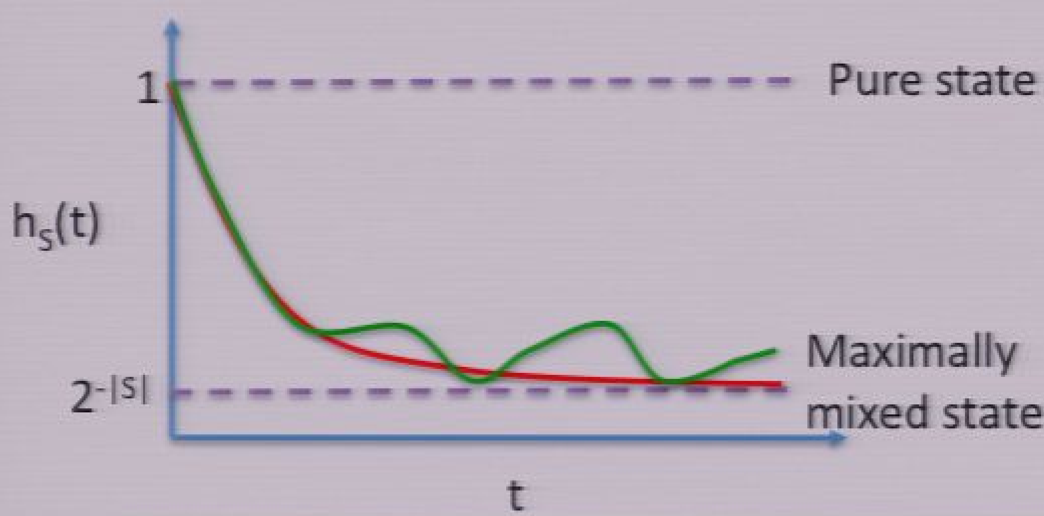

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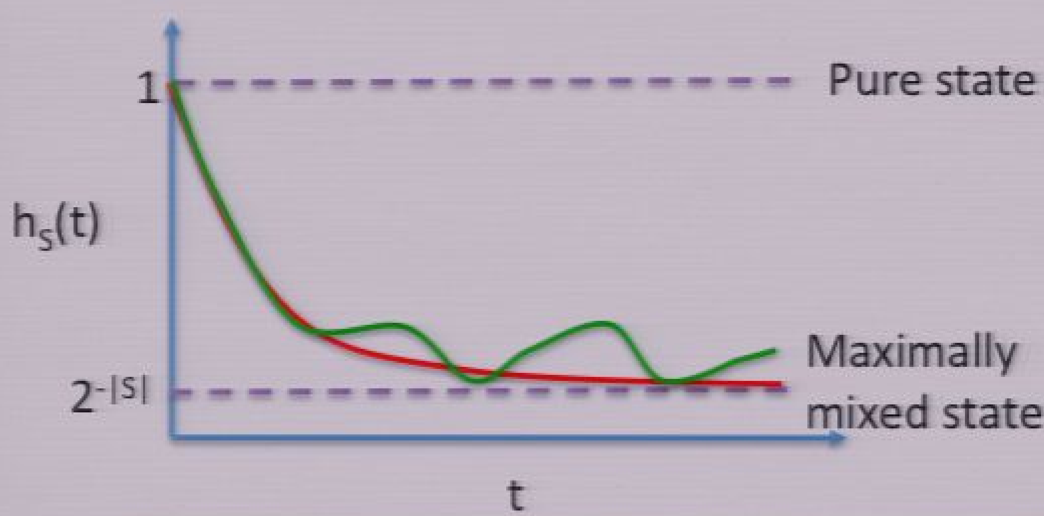

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Analysis of ODE

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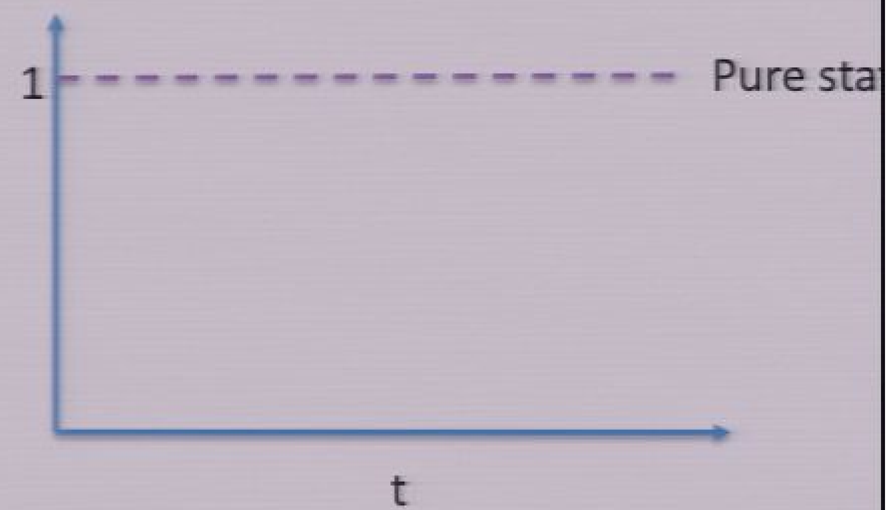

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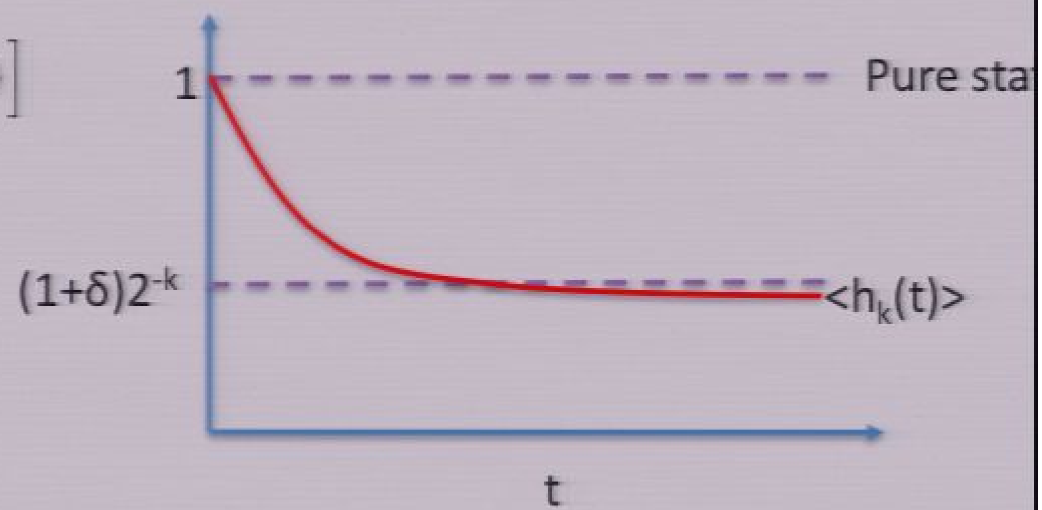

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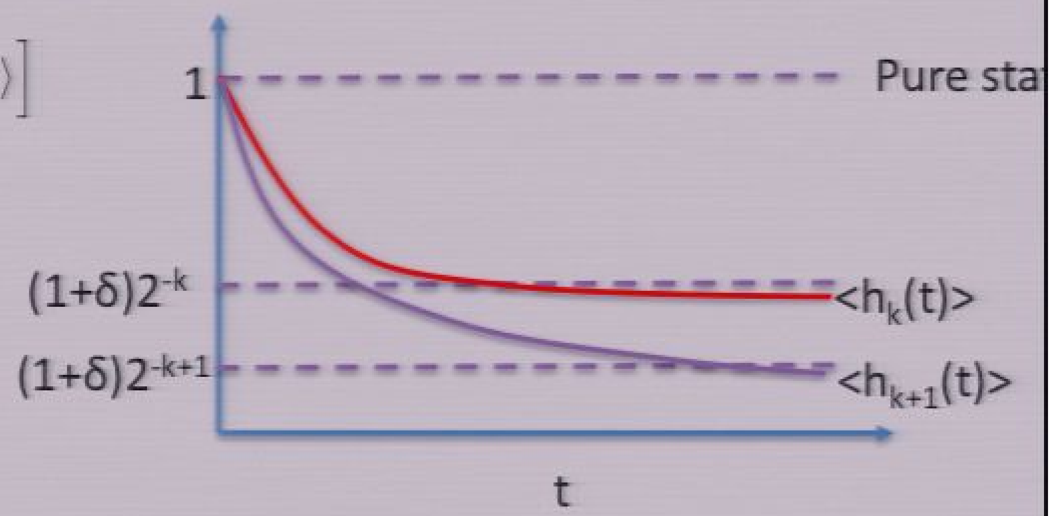

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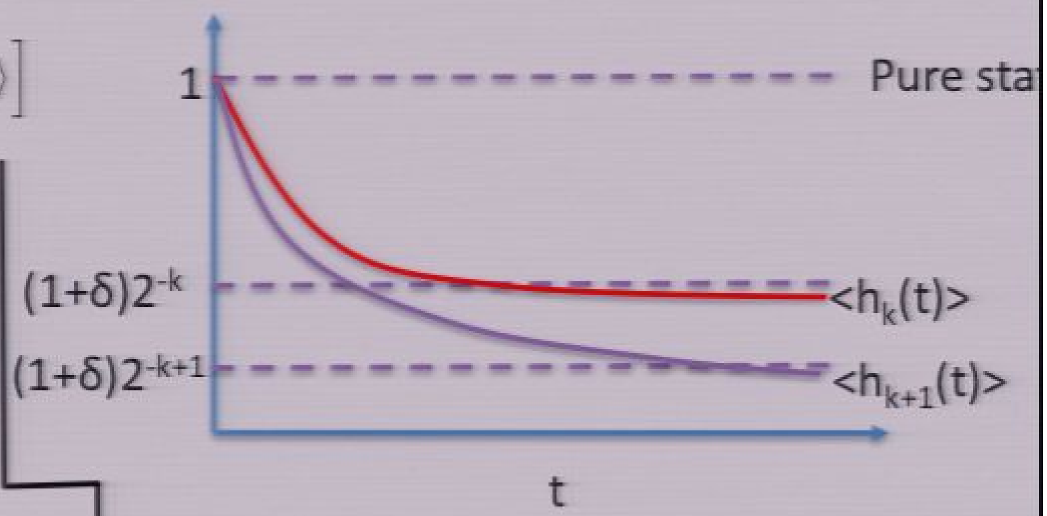

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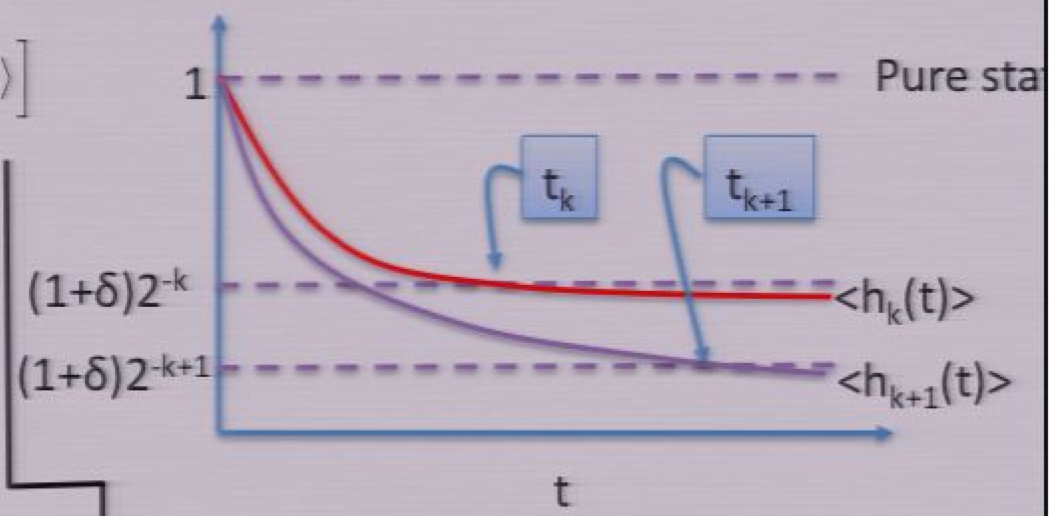

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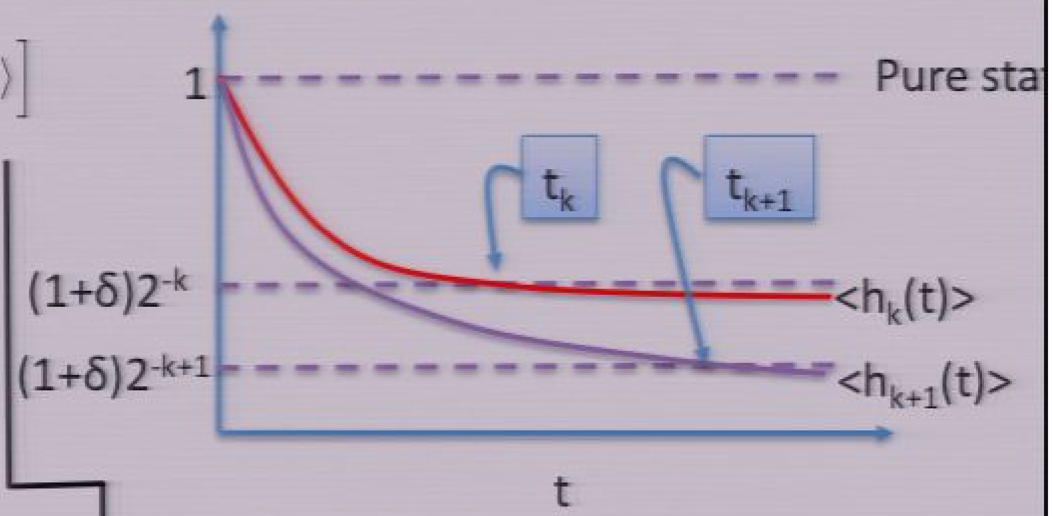

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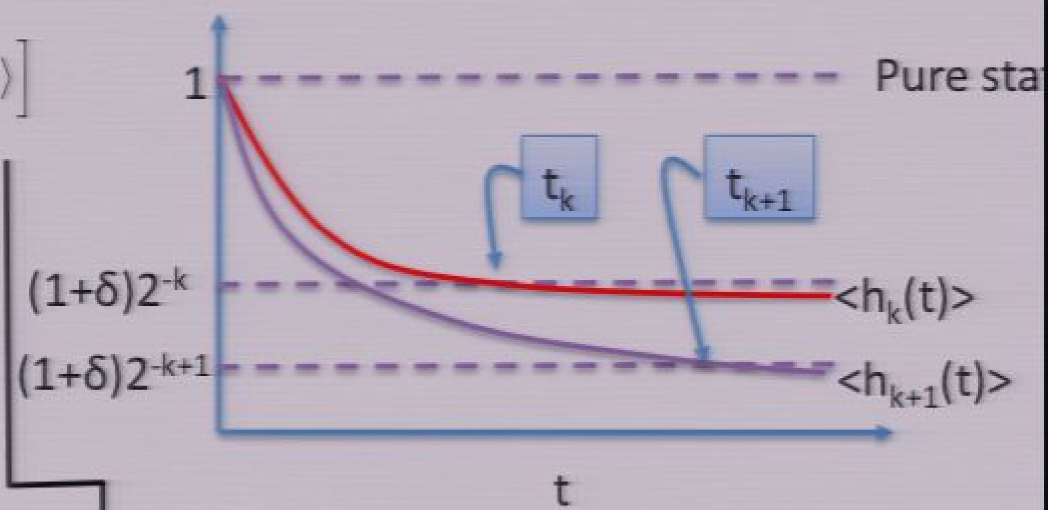

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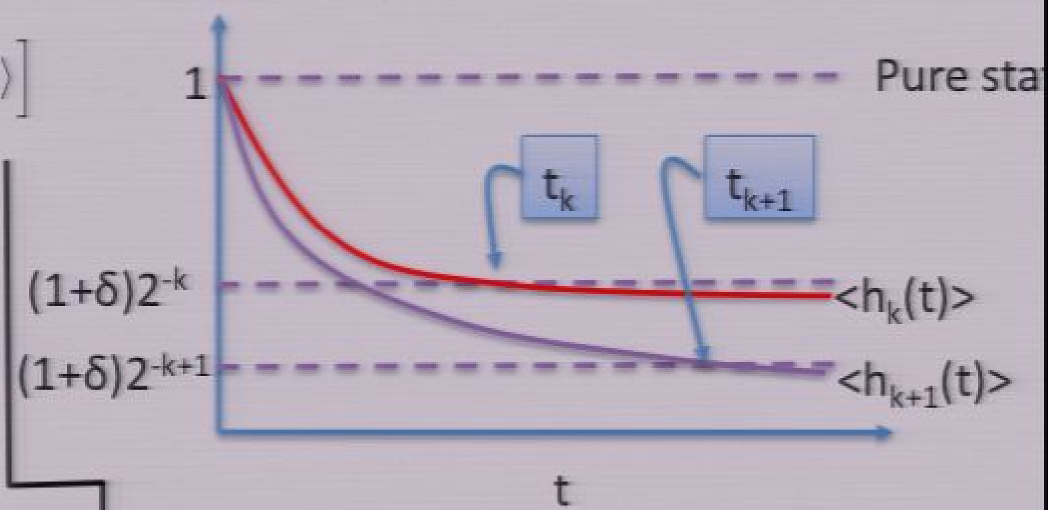

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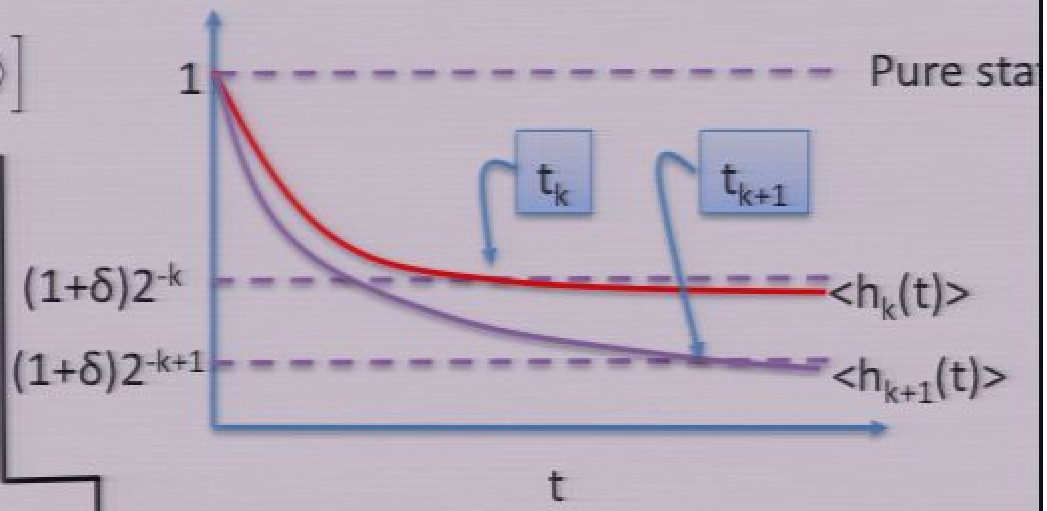


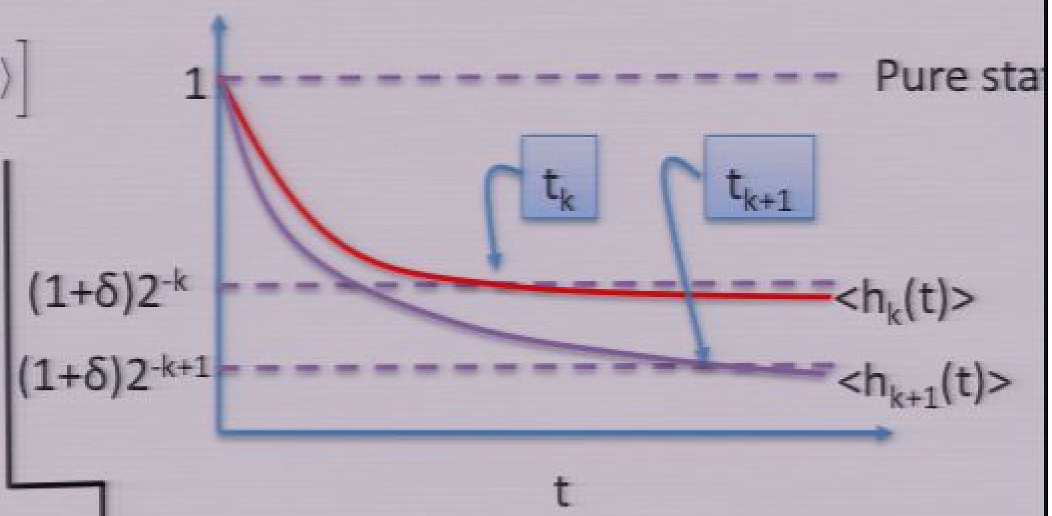
Careful analysis

Analysis of ODE

$|\psi(0)\rangle = |\psi_1\rangle|\psi_2\rangle \ldots |\psi_n\rangle$

Purity $h_k(t) = \mathrm{tr}\, \psi_{|S|=k}(t)^2$

$$\frac{d\langle h_k\rangle}{dt} = k(n-k)\Big[2\langle h_{k-1}\rangle - 5\langle h_k\rangle + 2\langle h_{k+1}\rangle\Big]$$



Quick and dirty analysis

Let t_k be time at which $<h_k(t)> =(1+\delta)2^{-k}$

For $t>t_{k-1}$

$$\begin{aligned}\frac{d\langle h_k\rangle}{dt} \;&\sim\leq\; kn\Big[2\frac{1+\delta}{2^{k-1}} - 5\langle h_k\rangle + 2\langle h_{k+1}\rangle\Big] \\ &\leq\; kn\Big[2\frac{1+\delta}{2^{k-1}} - 3\langle h_k\rangle\Big]\end{aligned}$$

Exponential decay with rate proportional to k.

So $t_k - t_{k-1} \leq O(1/k)$

$$t_k \sim \sum_{j=1}^{k} \frac{1}{k} \sim \log(k)$$

Careful analysis

Solve using Gauss hypergeometric functior

Brownian circuits: take-home

- Scramble very effectively: subsystems of size smaller than half become almost maximally entangled
- Scramble quickly: $t^*/t_1 = O(\log n)$

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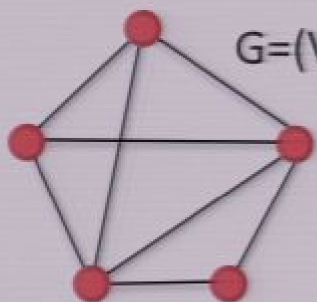
- But:
  - Time-dependent
  - Not very physical
  - Lots of randomness

Ising model: surprisingly instructive



G=(V,E)

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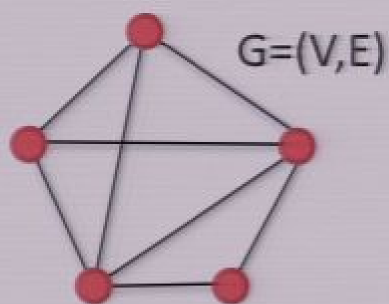

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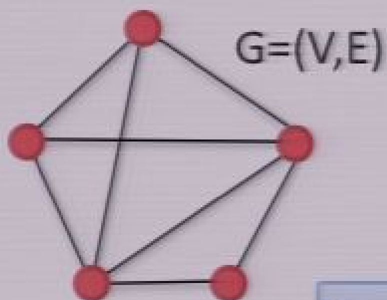

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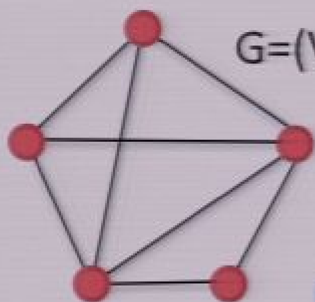

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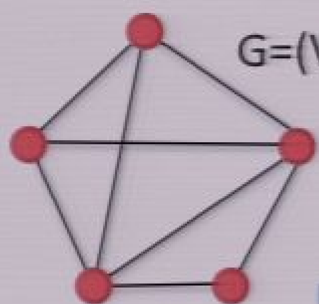

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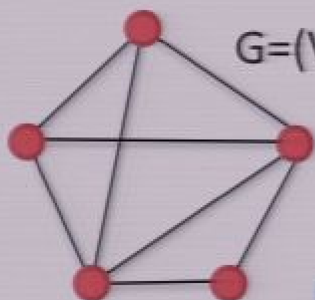

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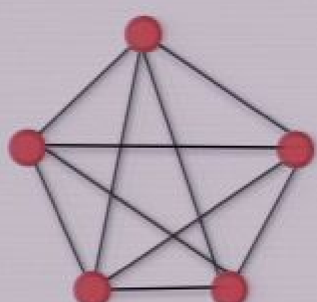

Complete graph

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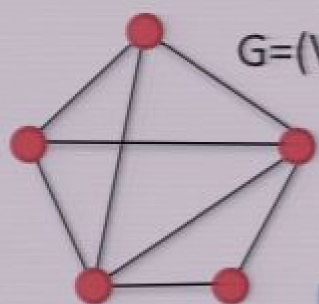

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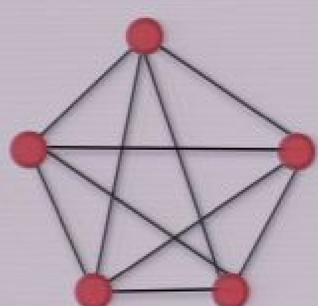

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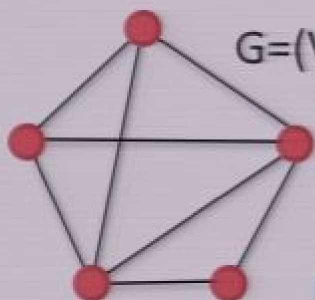

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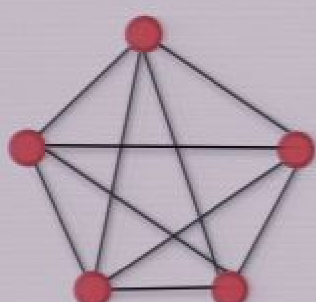

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|Ψ(t*)> locally unitarily equivalent to |00000>+|11111>

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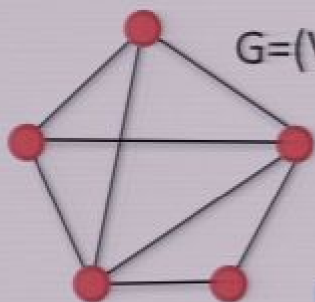

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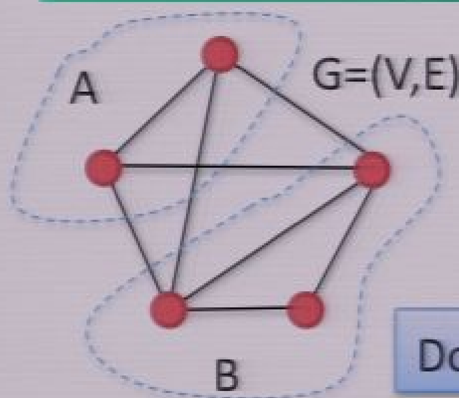

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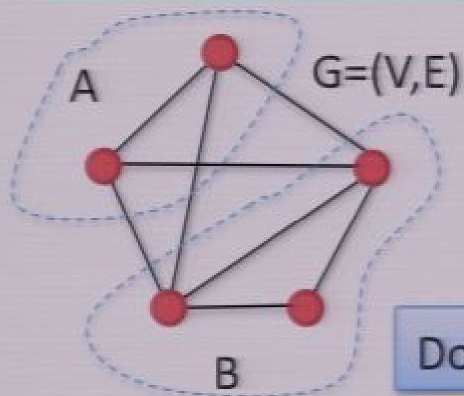

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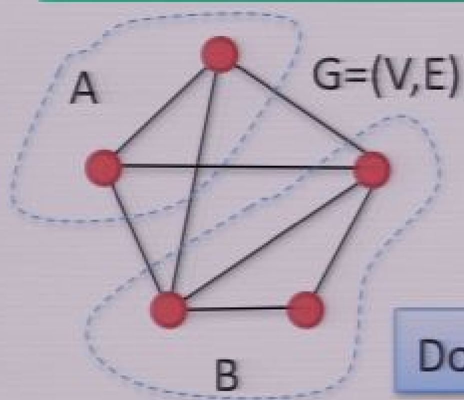

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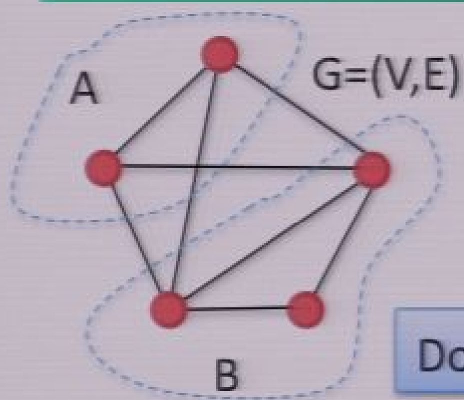

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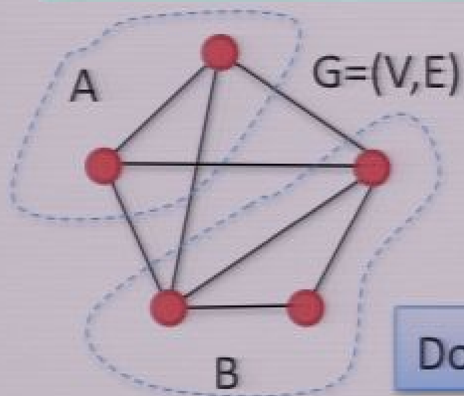

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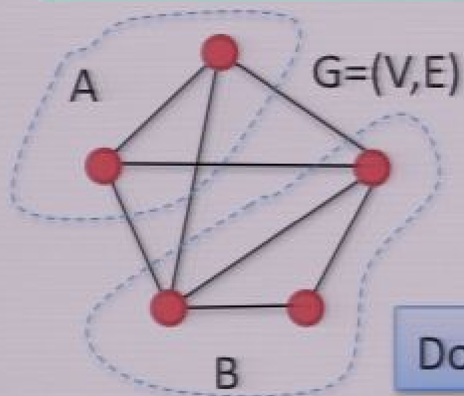

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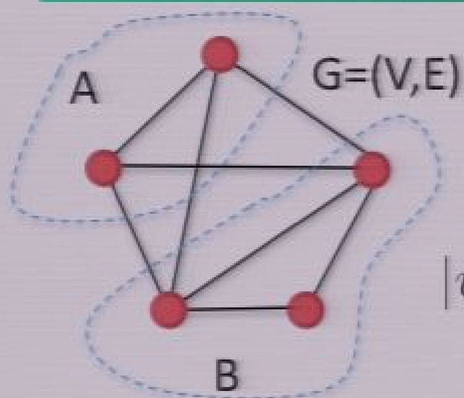

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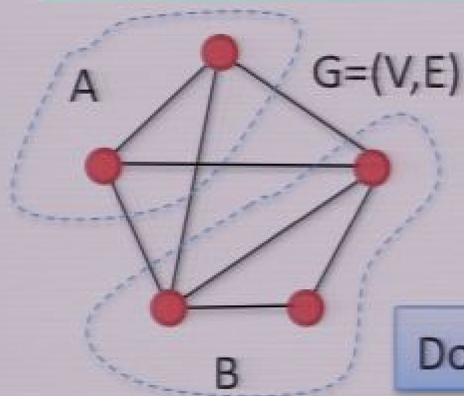

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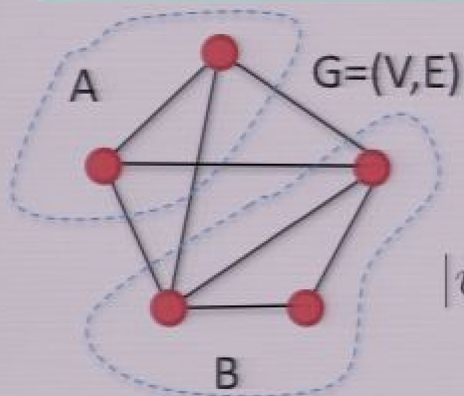

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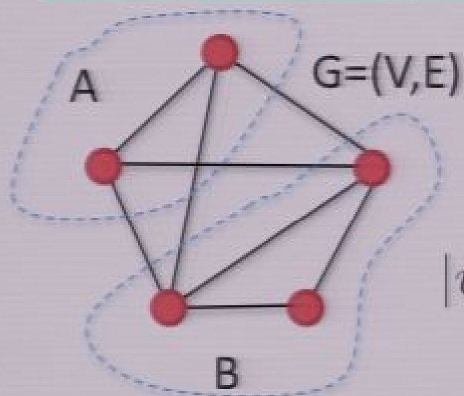

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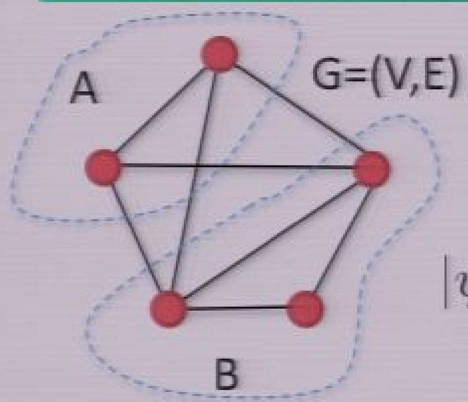

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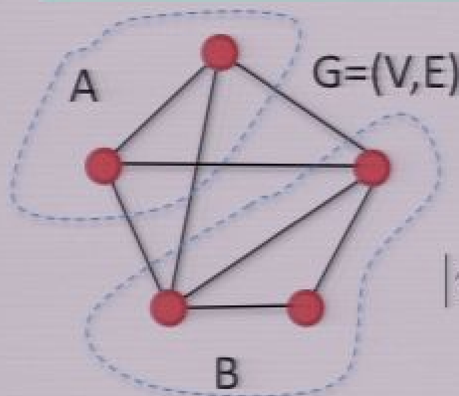

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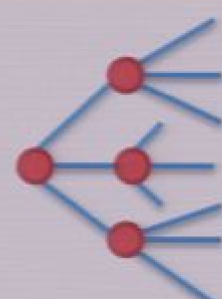

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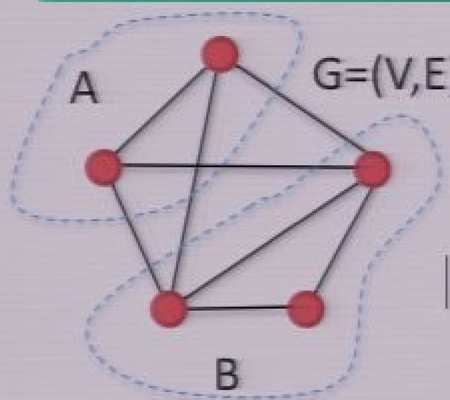

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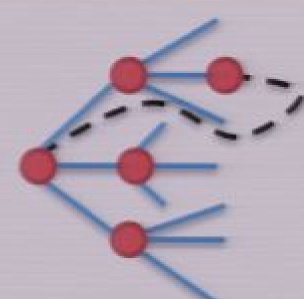
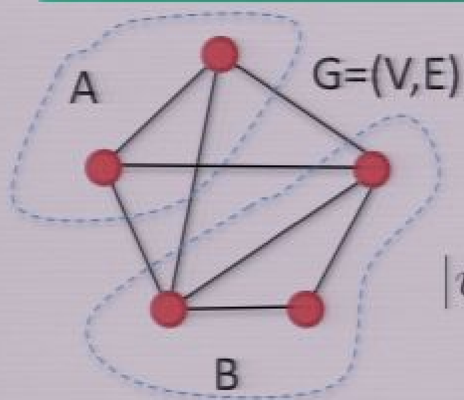


Solve Ising on tree: single site scrambles in time $t_1 \sim (\log n)^{1/2}$

Ising model on a random graph



$$H = \frac{|V|}{|E|} \sum_{(i,j)\in E} \frac{1}{4}(I+\sigma_z^{(i)}) \otimes (I+\sigma_z^{(j)})$$

$$|\psi(0)\rangle = \otimes_{v\in V}(|0\rangle + |1\rangle) \quad |\psi(t_*)\rangle = \left\{|\varphi\rangle|\sigma_z^{(v)} \prod_{u\in \mathrm{nhd}(v)} \sigma_x^{(u)}\right\}$$

$$S(\psi(t_*)_A) = \mathrm{rank}_{\mathbb{Z}_2} \mathrm{Adj}_{A,B}$$

Maximal entanglement: $\mathrm{Adj}_{A,B}$ full rank

Make G a random graph: Pr((u,v) is an edge) = p

Sparsest graph for which this is true: p ~ log n / n

$$t_* = \pi \log n/2$$

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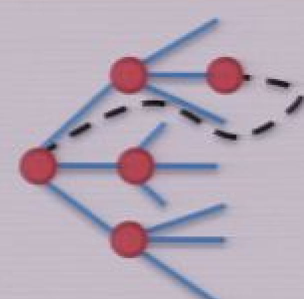


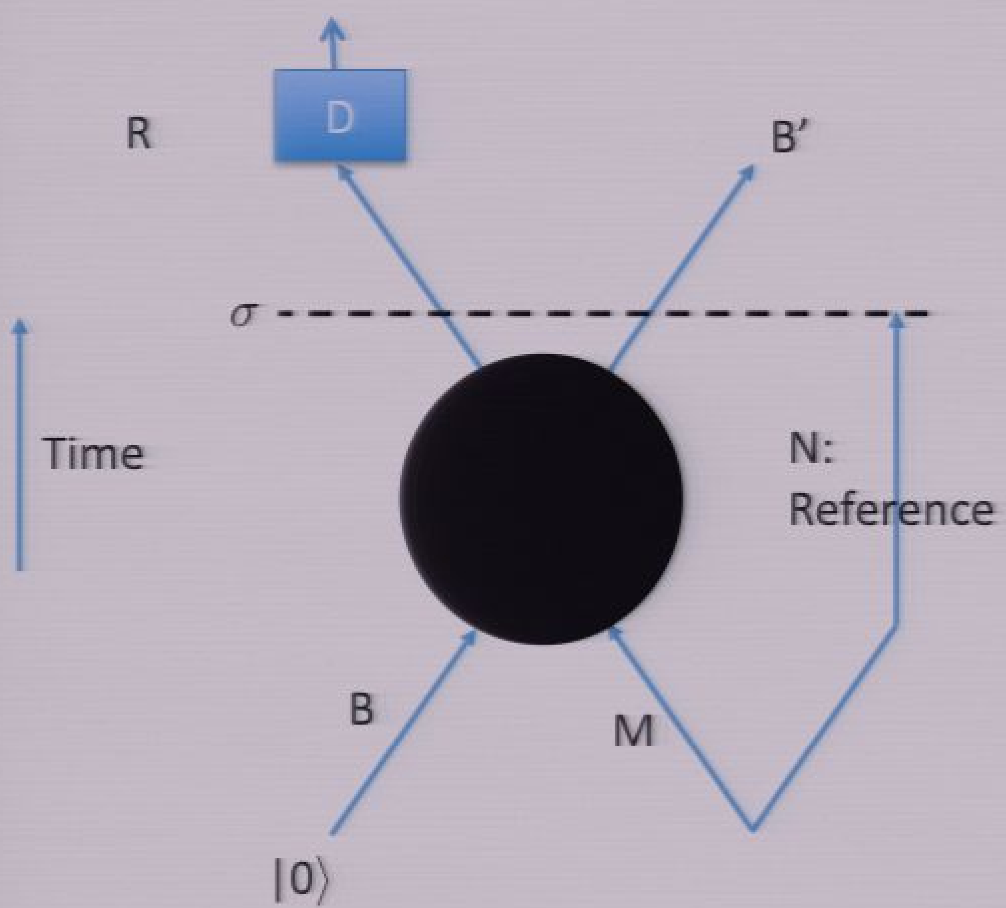
Find $t_*/t_1 \sim (\log n)^{1/2}$:
Faster than scrambling conjecture?

Solve Ising on tree: single site scrambles in time $t_1 \sim (\log n)^{1/2}$

Ising model: take-home

- Generates entanglement very quickly:
  - Vertex degree normalization of time $t^* = O(\log n)$
  - Very fast scaling: $t^*/t_1 = O((\log n)^{1/2})$
- Doesn't scramble completely
  - Relevance to information leakage?

Decoherence and information



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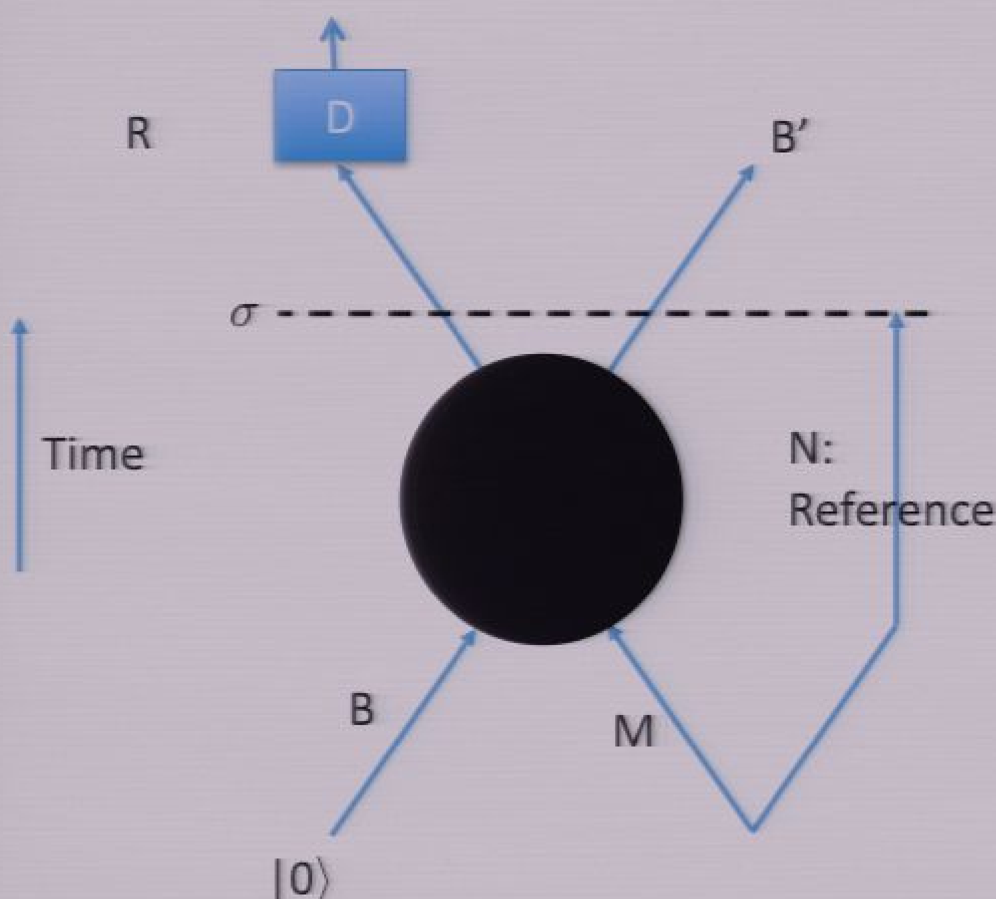

Sending classical message from M to R is *equivalent* to establishing classical correlation between N and R

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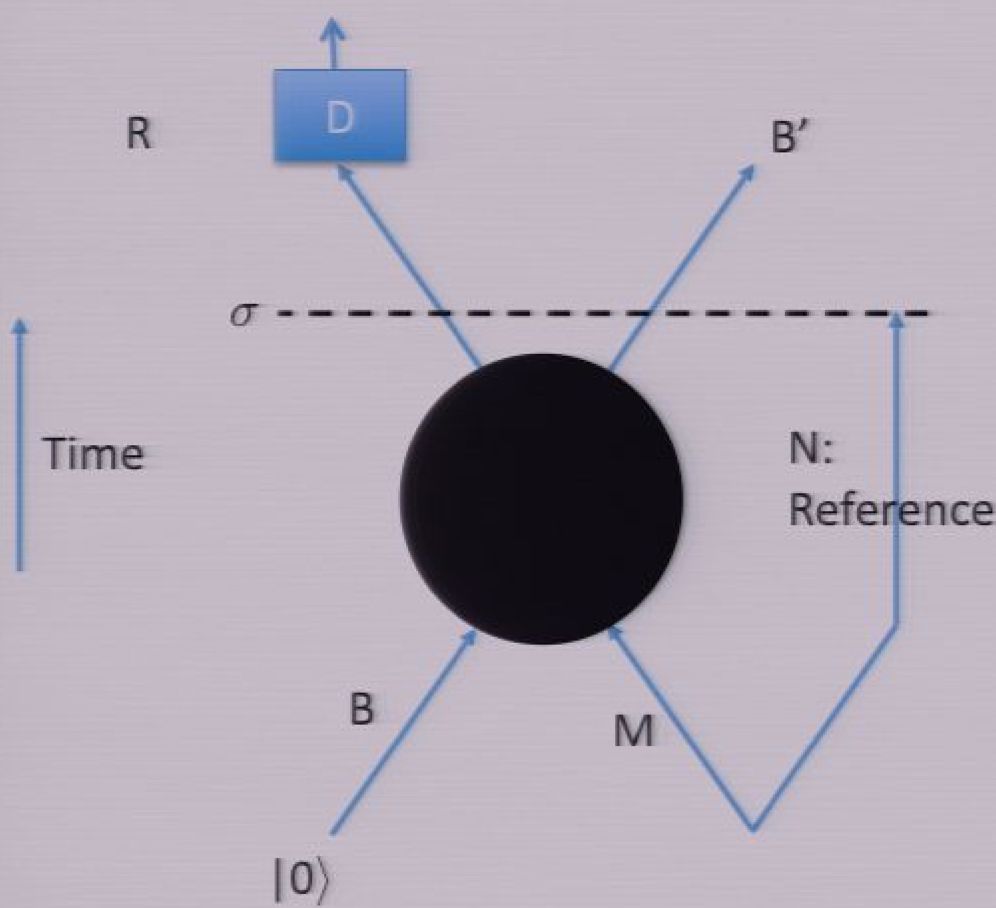

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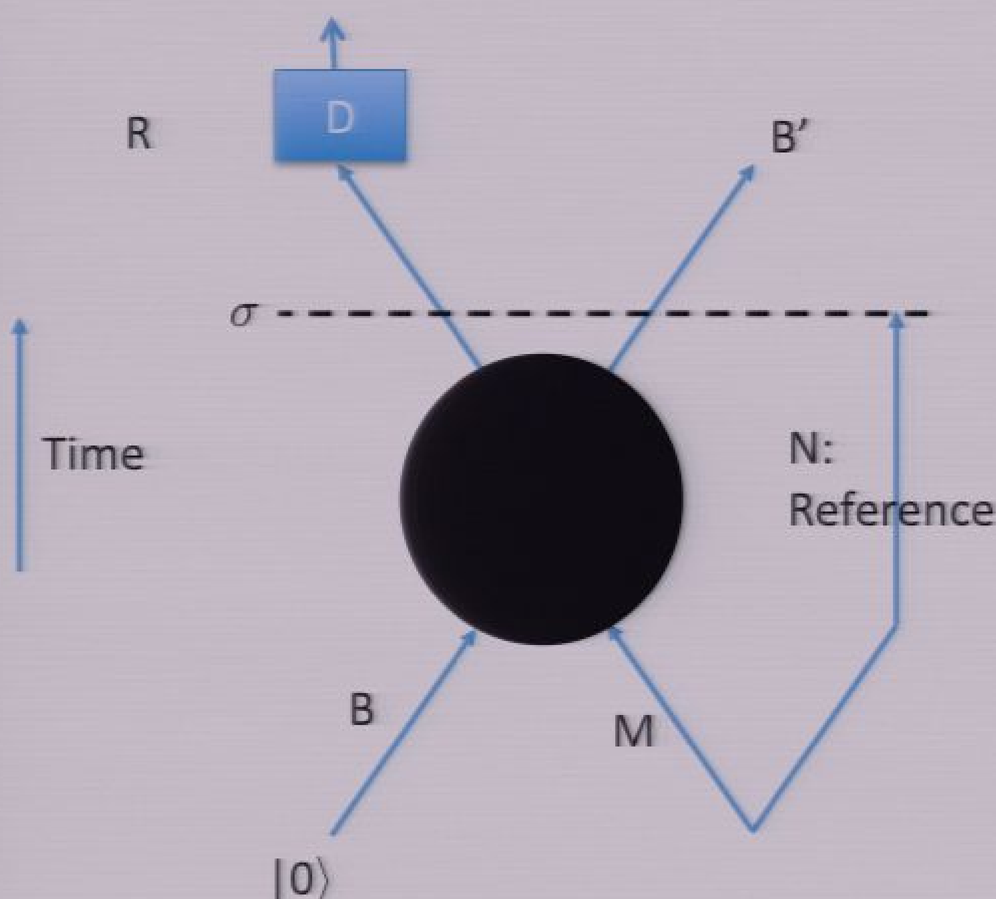
Establishing classical correlation between N and R is *equivalent* to decohering N B′

Decoherence and information



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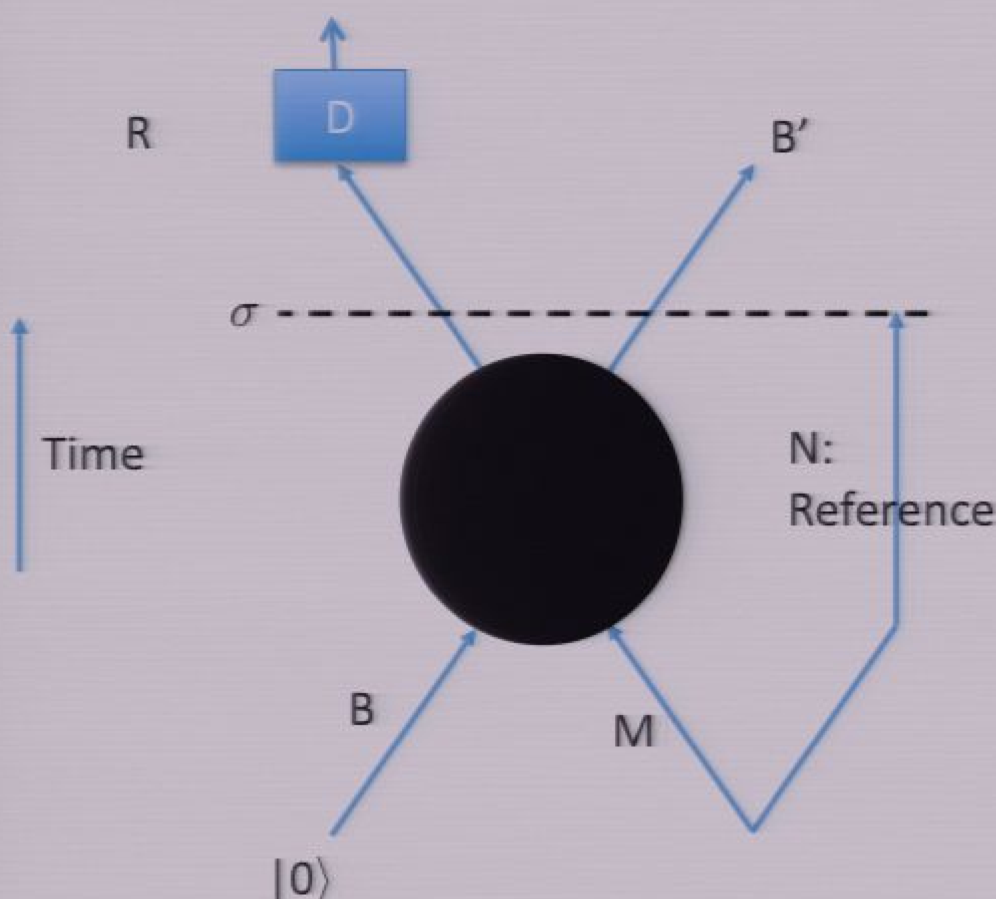
$$\mathrm{tr}_R\, \sigma_{NB'R} = \sum_j p_j |j,j\rangle\langle j,j|_{NB'}$$

$$\Rightarrow$$

$$|\sigma\rangle_{NB'R} = \sum_j \sqrt{p_j}\, |j,j\rangle_{NB'} |e_j\rangle_R$$

Decoherence and information



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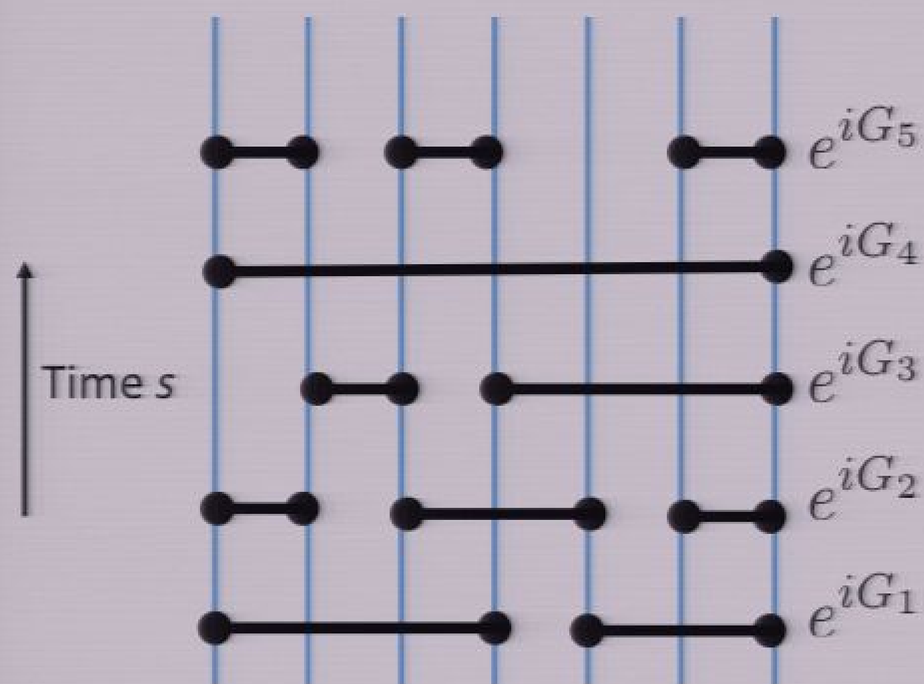
So decoherence as in Ising implies information transmission, but no cloning issues

Feynman's simulator

How to make a time-dependent quantum circuit into a time-independent Hamiltonian?



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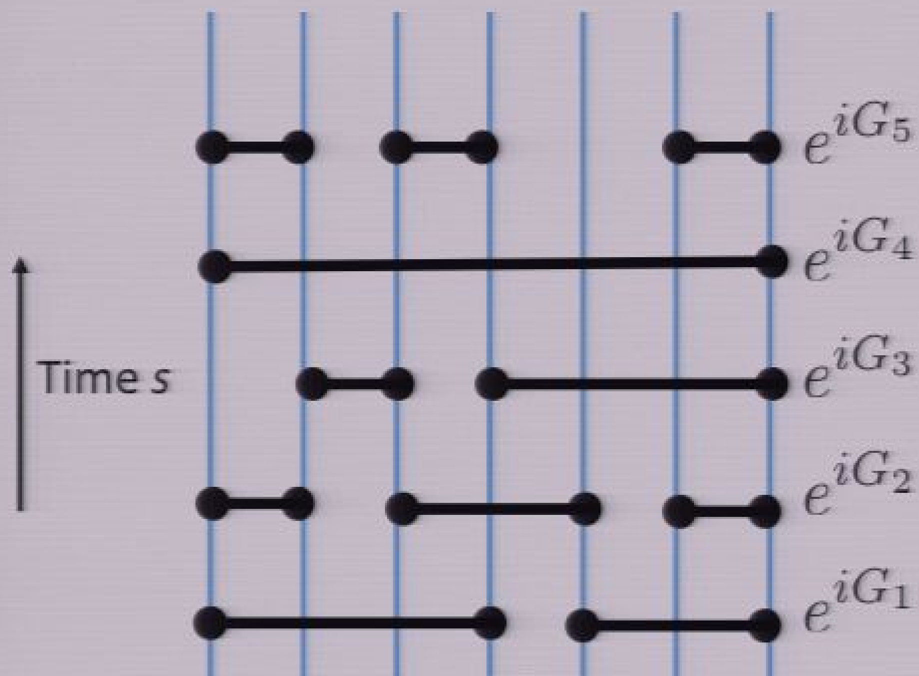

$$H = \sum_{s=1}^{S} \left[e^{iG_s} \otimes |s\rangle\langle s-1| + e^{-iG_s} \otimes |s-1\rangle\langle \right.$$

“Spin” register

“Clock” register

$$\begin{aligned} e^{iHt}|\varphi\rangle|0\rangle \quad = \quad & \alpha_0(t)|\varphi\rangle|0\rangle \\ & +\alpha_1(t)e^{iG_1}|\varphi\rangle|1\rangle \\ & +\alpha_2(t)e^{iG_2}e^{iG_1}|\varphi\rangle|2\rangle + \end{aligned}$$

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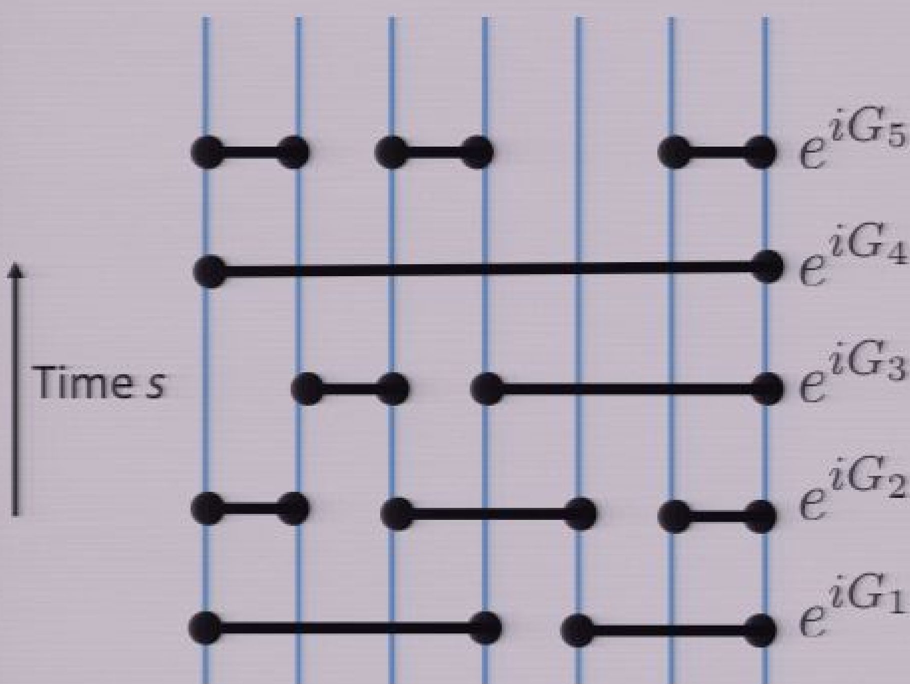

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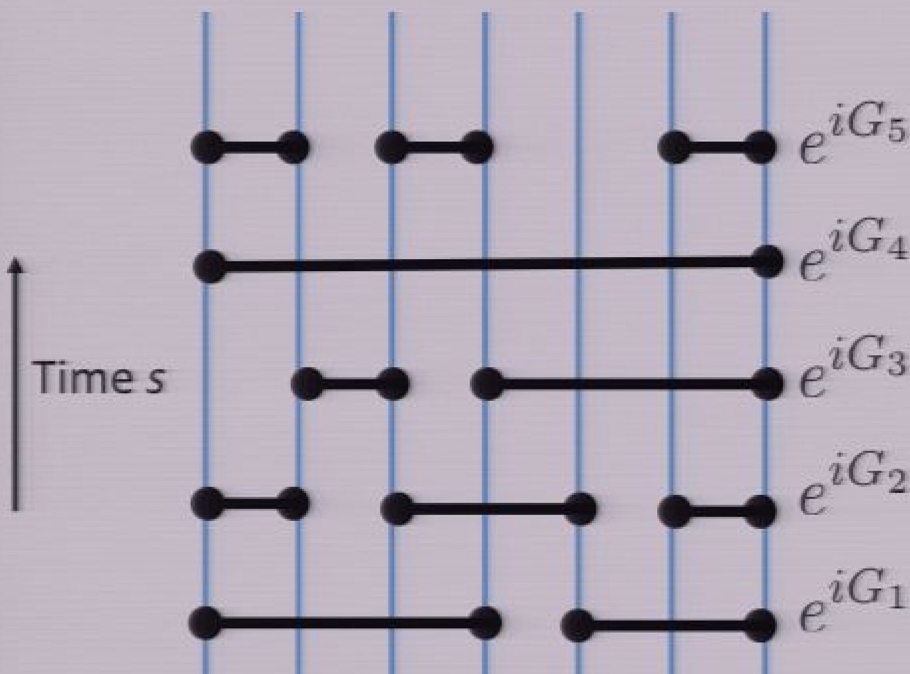

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k-body: how big is k?

If each G_s only involves a single pair: k = 2 + 1 = 3 body.

By using Dankert et al. results, get a full scrambler with $t^*/t_- = O(\log n)$

Scrambling and derandomization

Quality of scrambling:

* Size of set of perturbations - "Adversariality"
* Lack of extra randomness
* Distinguishability of scrambled perturbations

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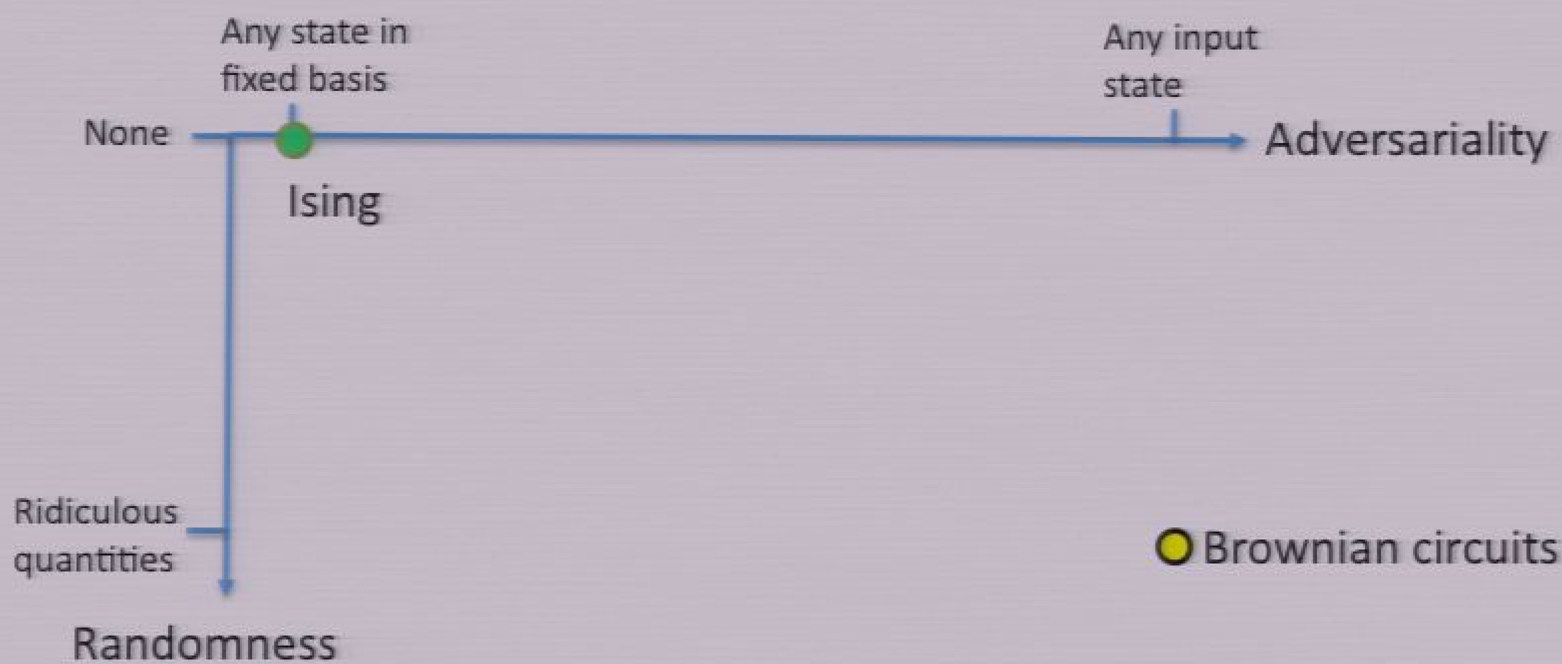

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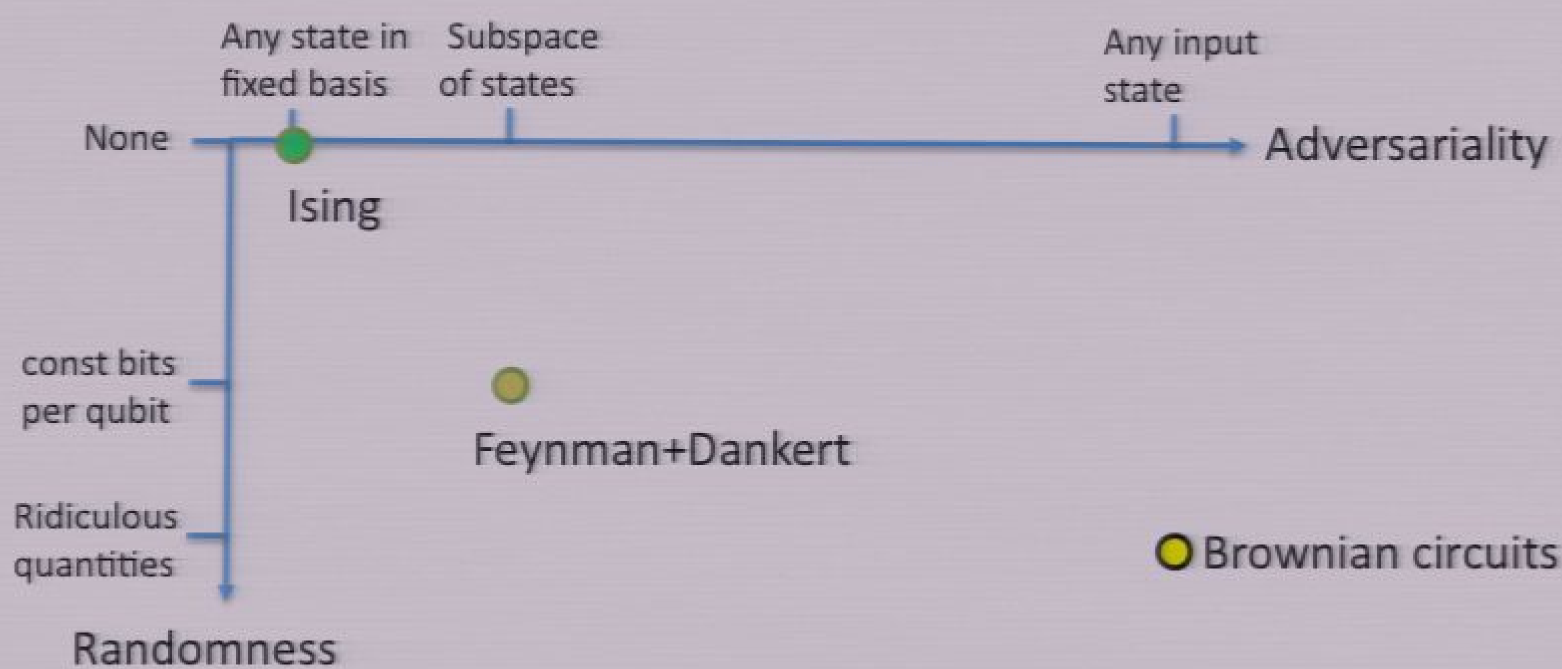

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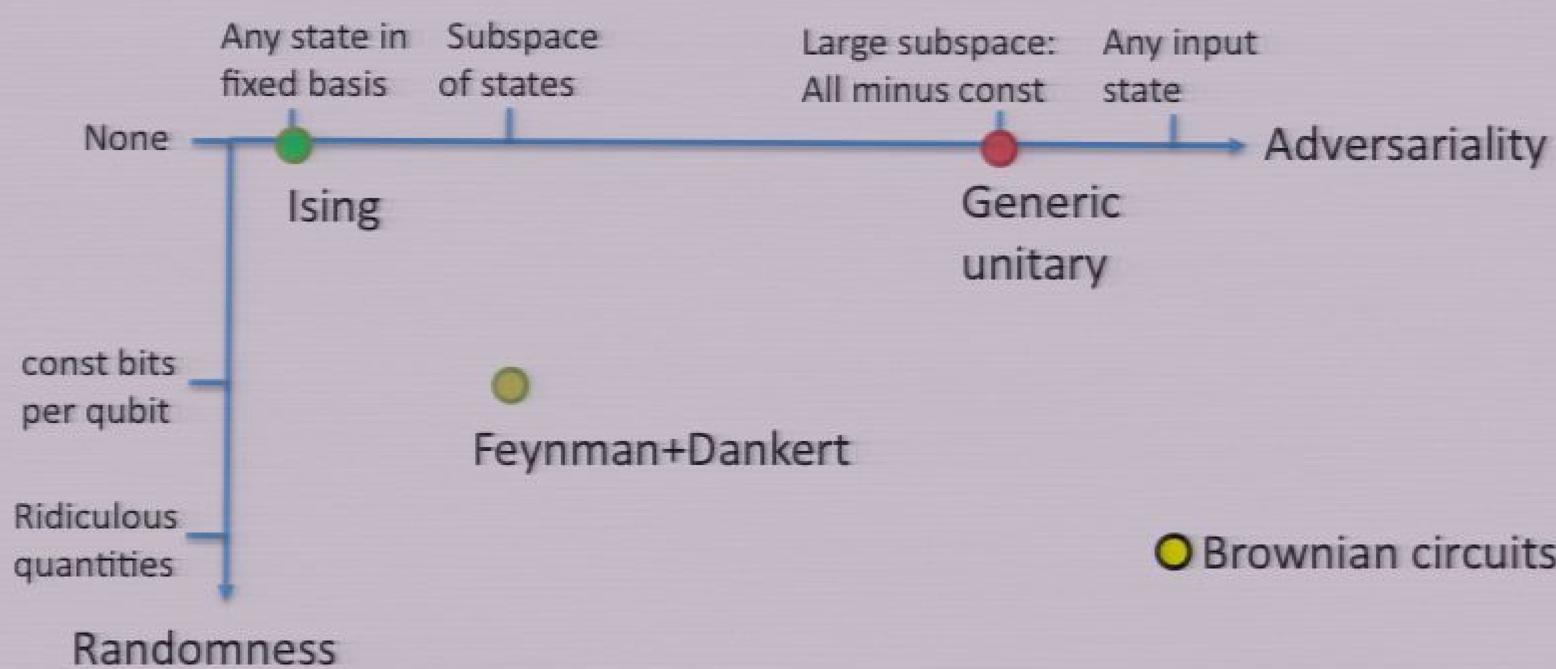
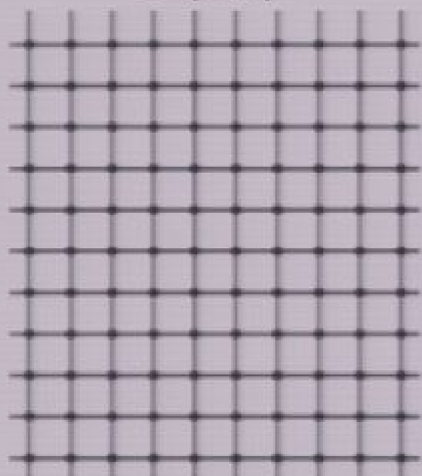


Lessons from Lieb-Robinson

G=(V,E)



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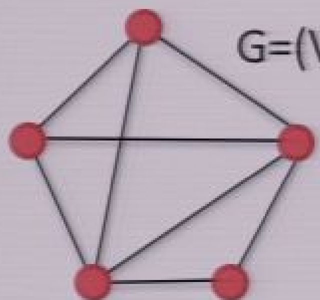

$$H = \sum_{e \in E} H_e$$

Hilbert space for every vertex

$$\forall u \in V : \sum_{v \in \mathrm{nhd}(u)} \|H_{(u,v)}\| \leq \mathrm{const}$$

Observables X_u and Y_v localized at u and v.

$$\|[X_u(t), Y_v]\| \sim\leq \frac{1}{D} \exp(\mathrm{const}\, t)$$

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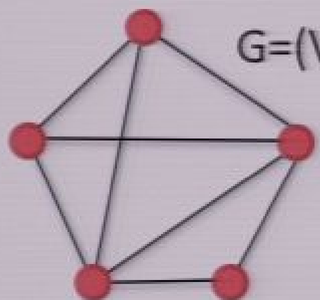

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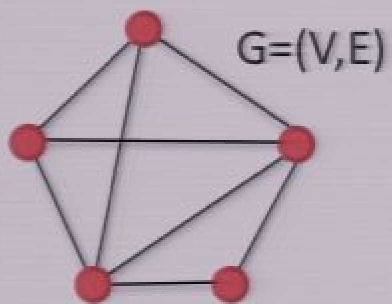

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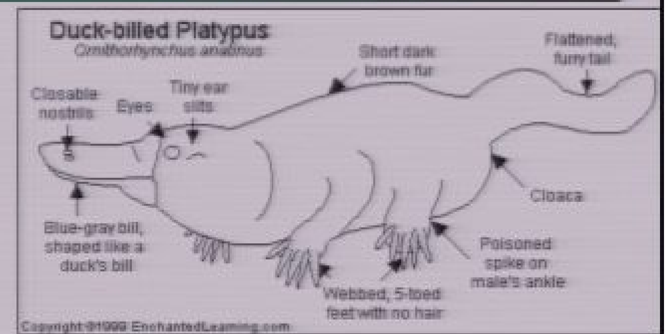
$$\delta \leq \|[X_u(t), Y_v]\| \sim\leq \exp(vt - \mathrm{dist}(u,v))$$

Consider max possible vertices M(k) at distance ≤k from a given vertex

$$M(k) \leq D^k$$

Conclusions



- Brownian circuits
  - Good: full scrambling $t^*/t_1 = O(\log n)$
  - Bad: not time independent
- Ising model on a sparse random graph
  - Good: time independent, generates lots of entanglement, $t^* = O(\log n)$
  - Bad: not a full scrambler, $t^*/t_1 = O((\log n)^{1/2})$
- Feynman's simulator
  - Good: time independent, full scrambling, $t^*/t_1 = O(\log n)$
  - Bad: not really that fast, ad hoc