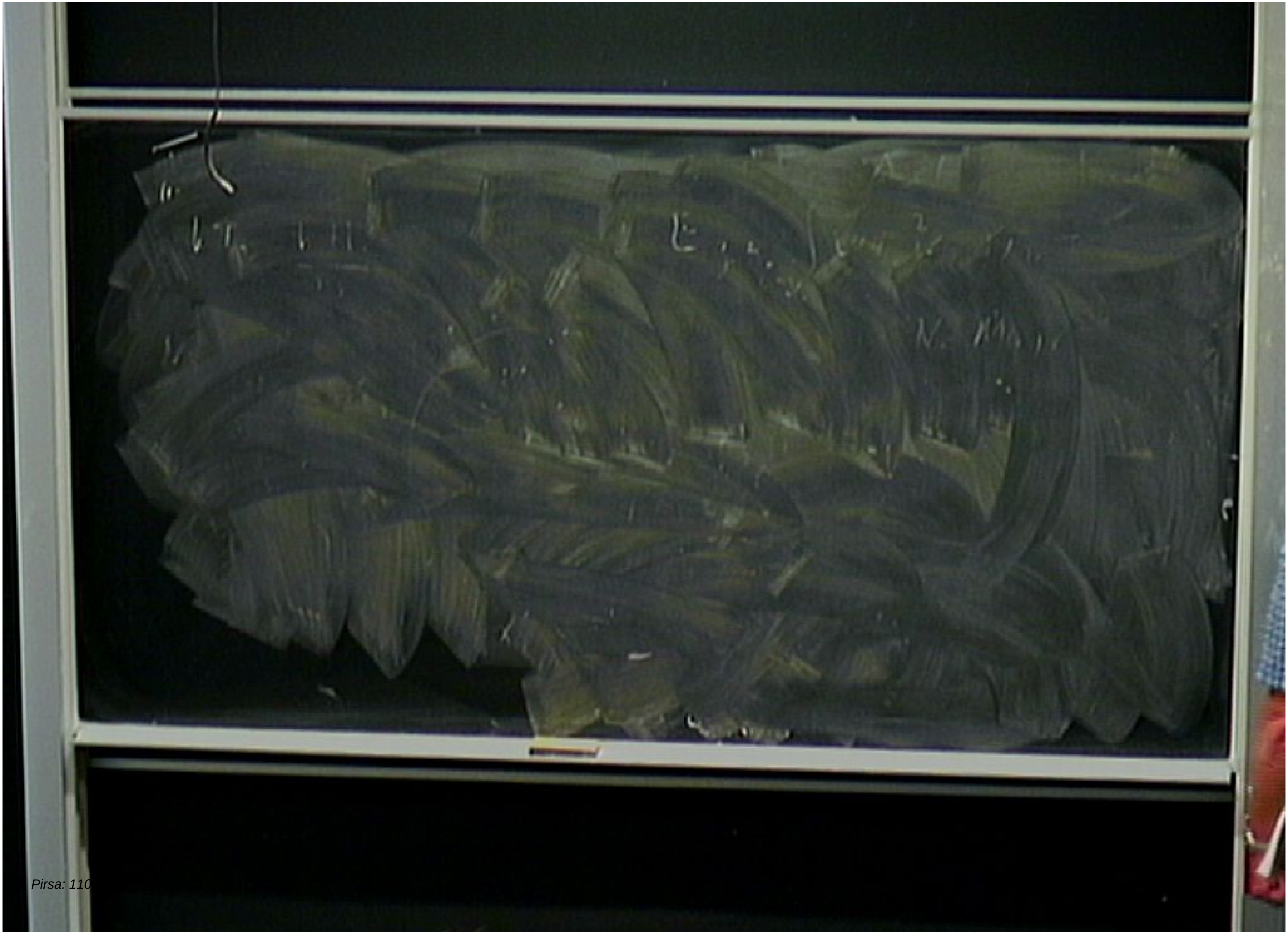
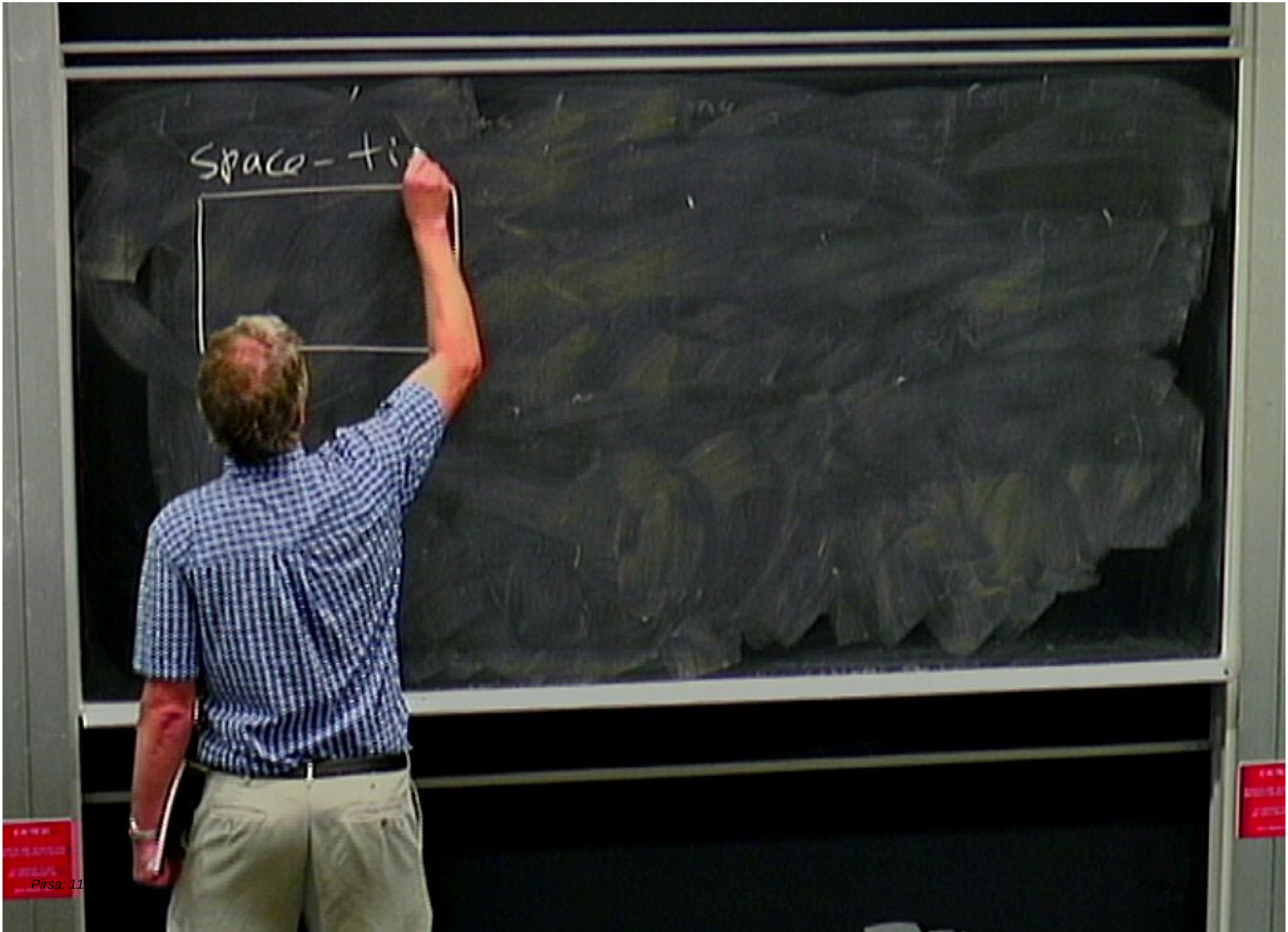
Title: Fuzzy Twistors and Emergent Gravity

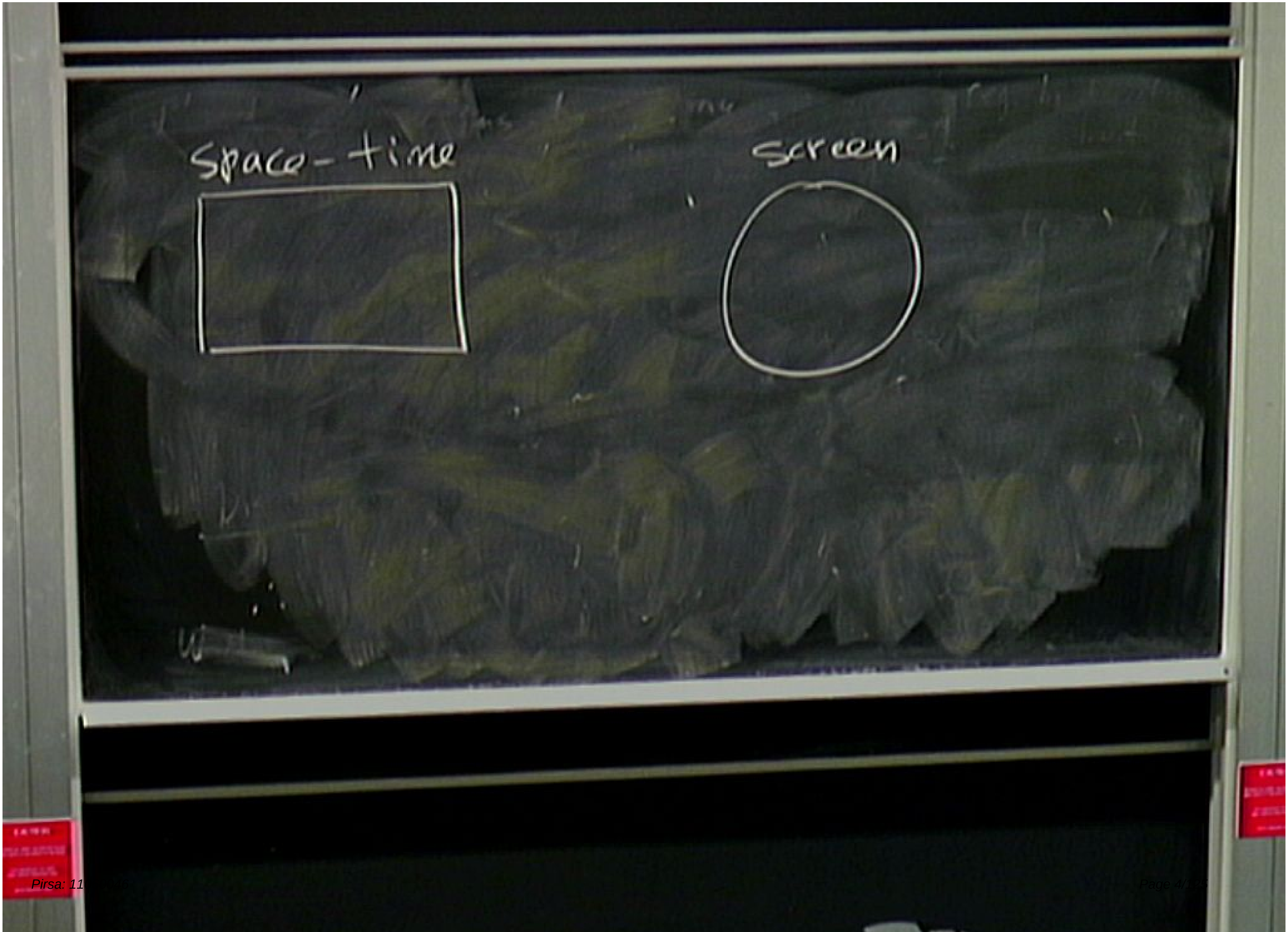
Date: Jun 22, 2011 03:00 PM

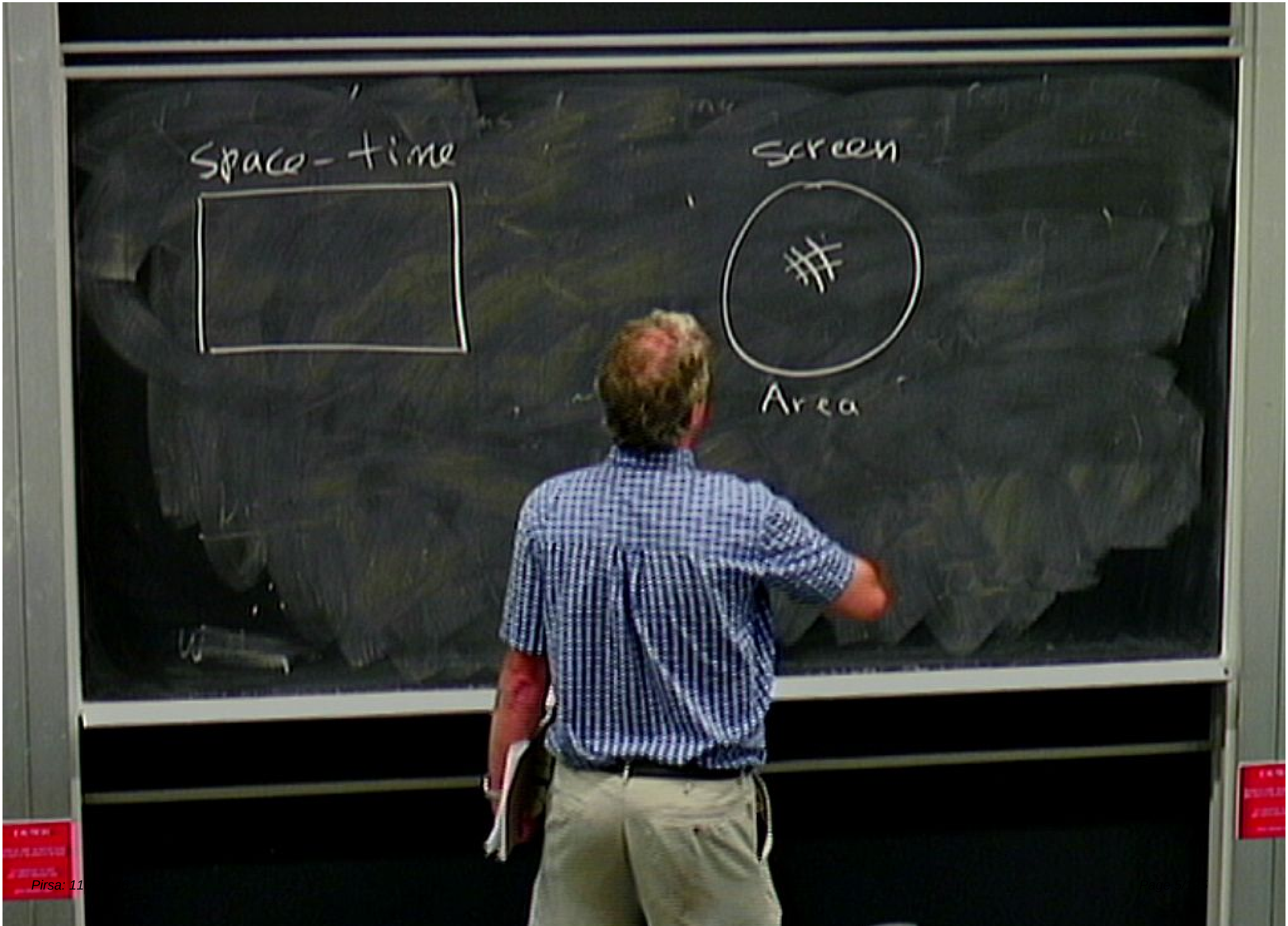
URL: http://pirsa.org/11060046

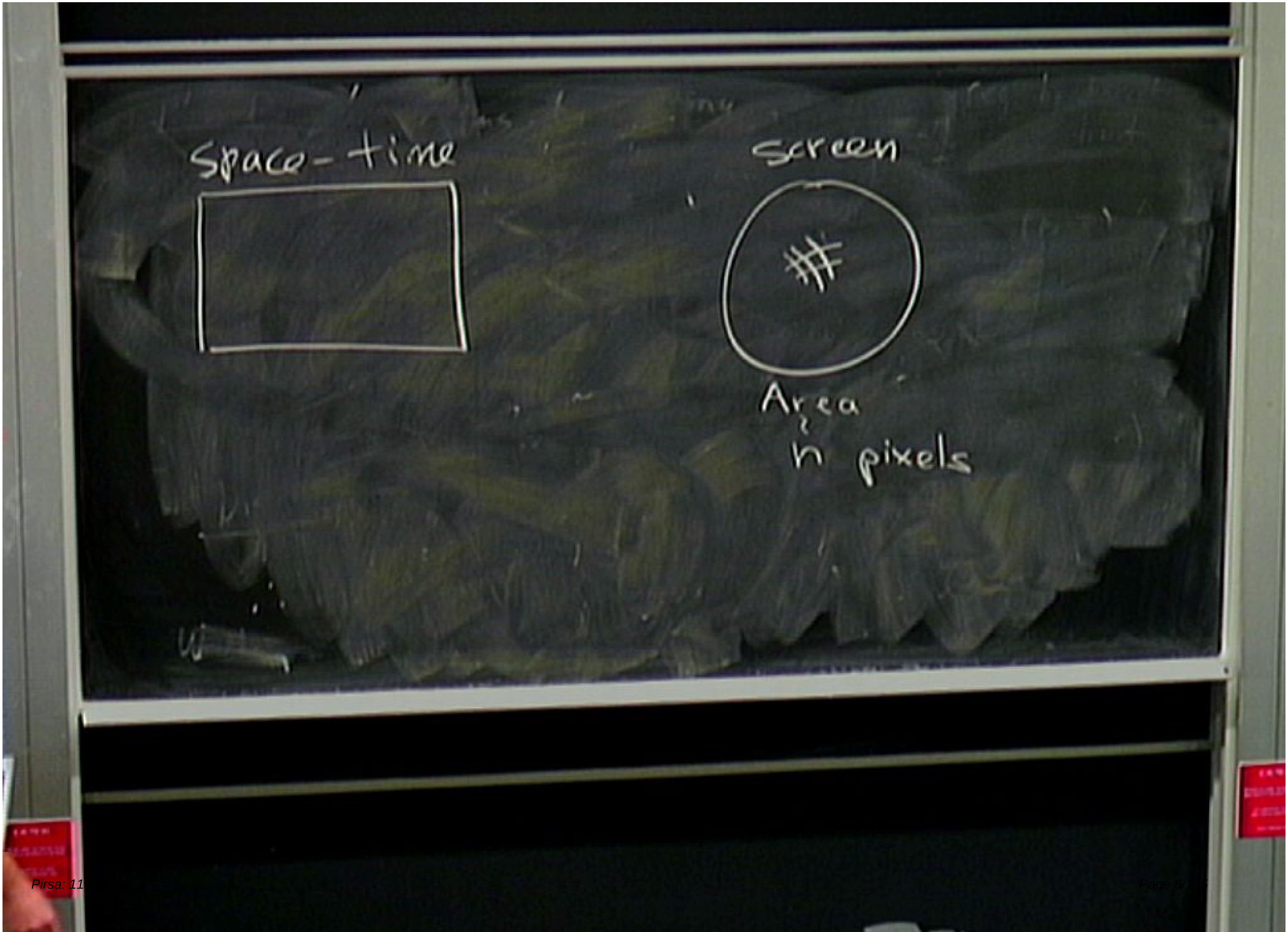
Abstract: In this talk, I explain how twistors can be used to provide a covariant UV cut-off for 4-D gauge theory. I'll then motivate the conjecture that the cut-off gauge theory automatically contains 4-D Einstein gravity. As evidence, I describe how the theory reproduces the gravitational MHV amplitudes.

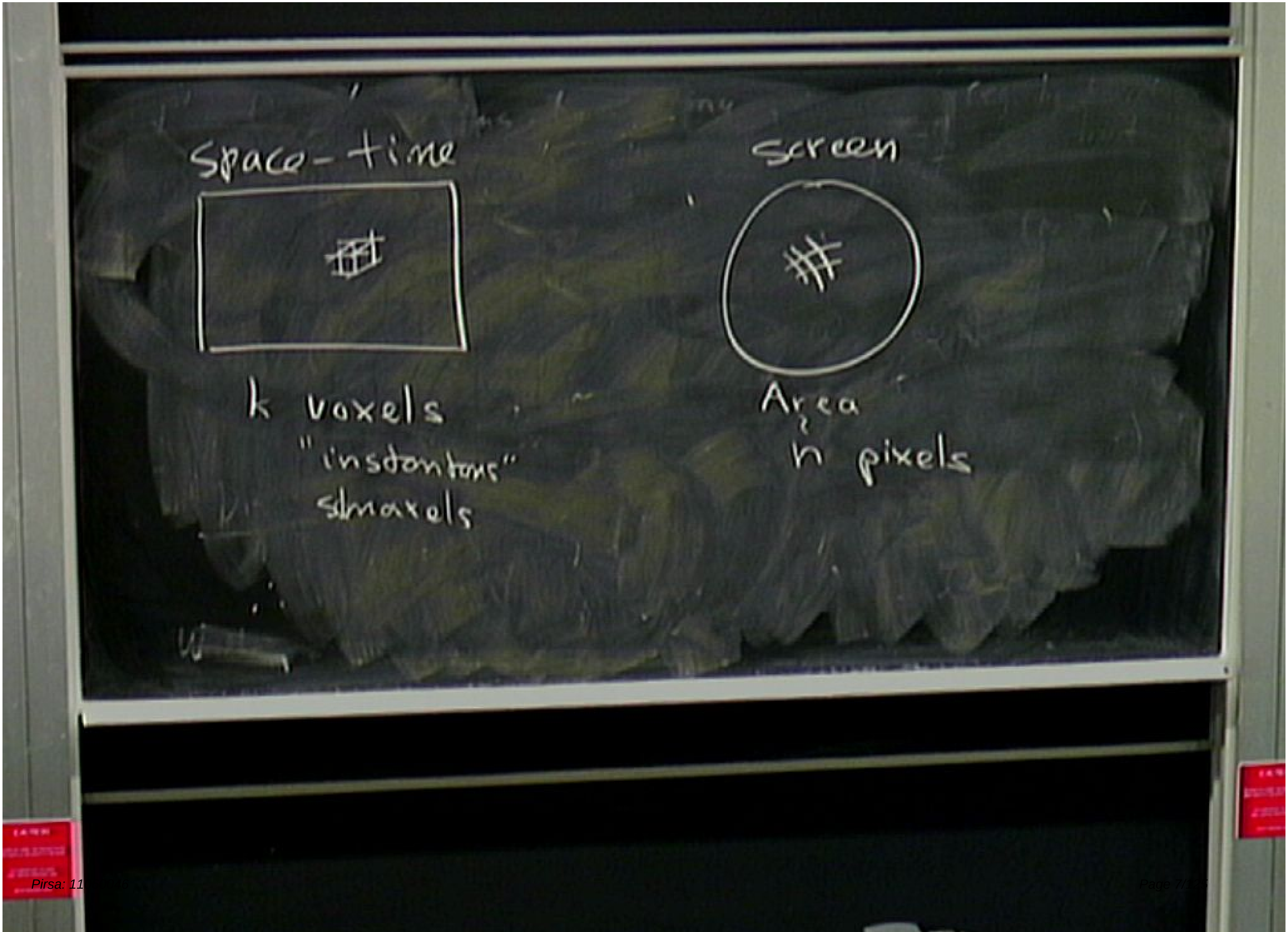


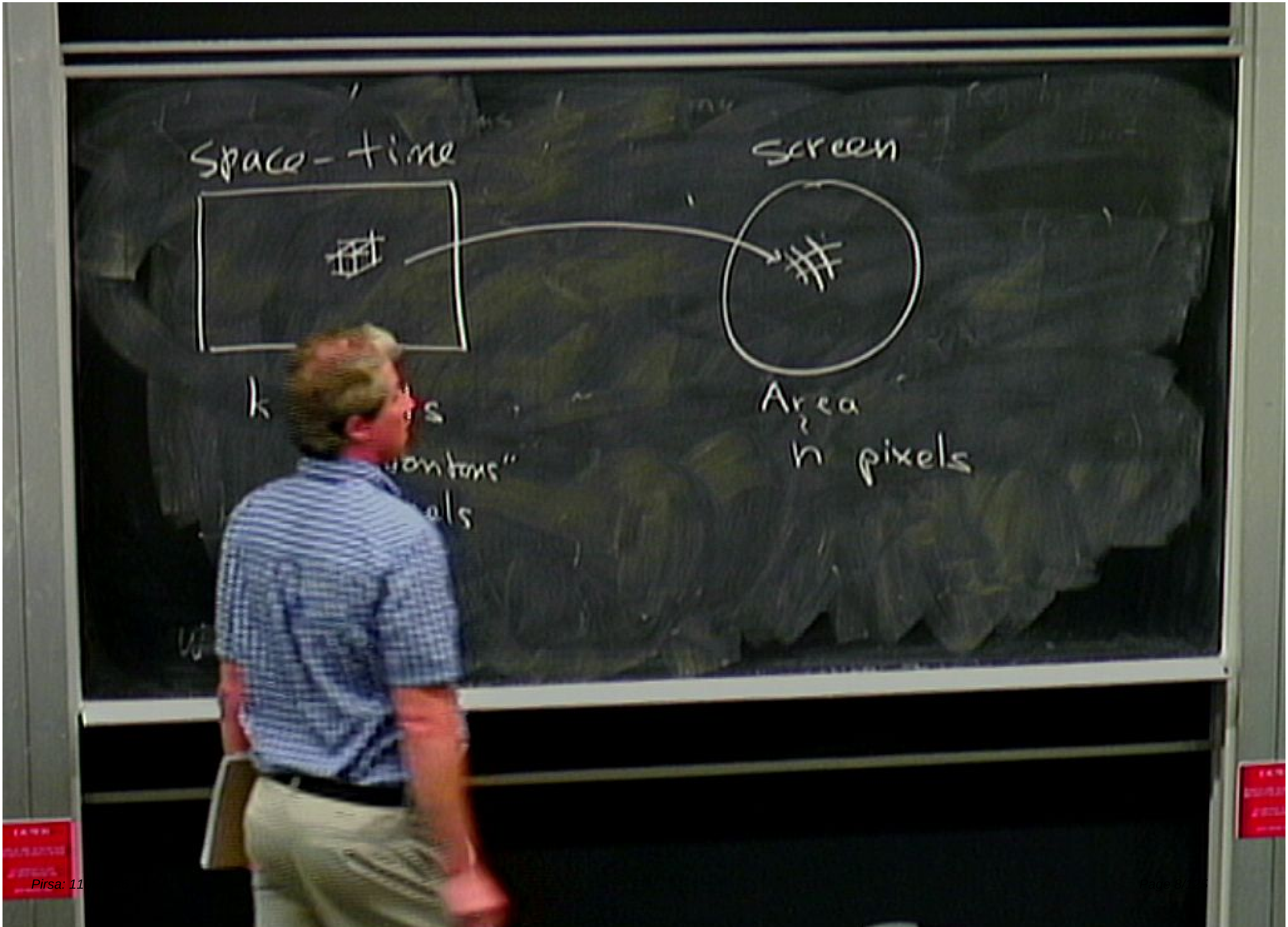

Space-ti


space-time
screen


Space-time
screen
Area


space-time
screen
Area
n pixels


Space-time
k voxels
"instantons"
smaxels
screen
Area
n pixels


Space-time
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Area
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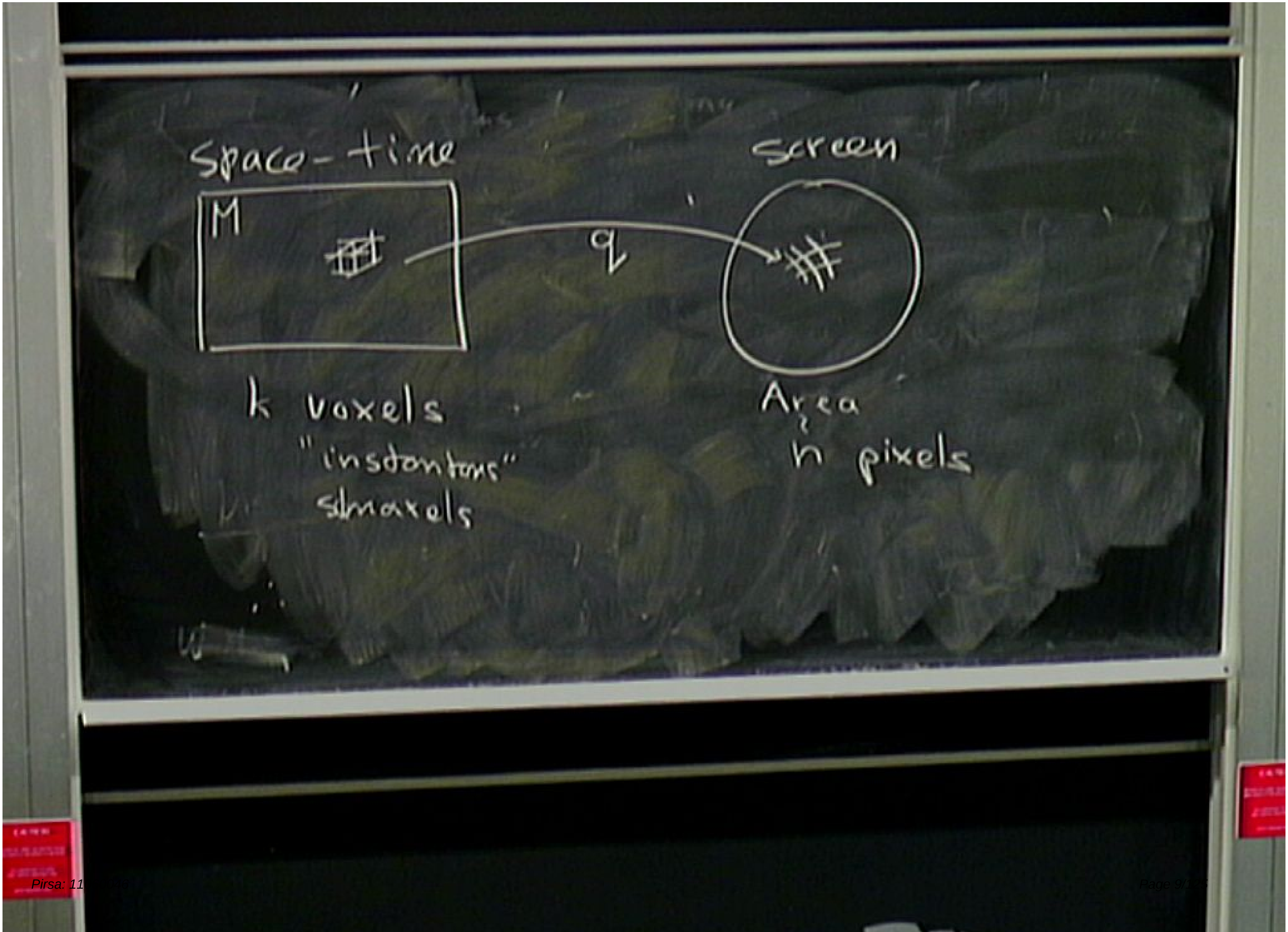
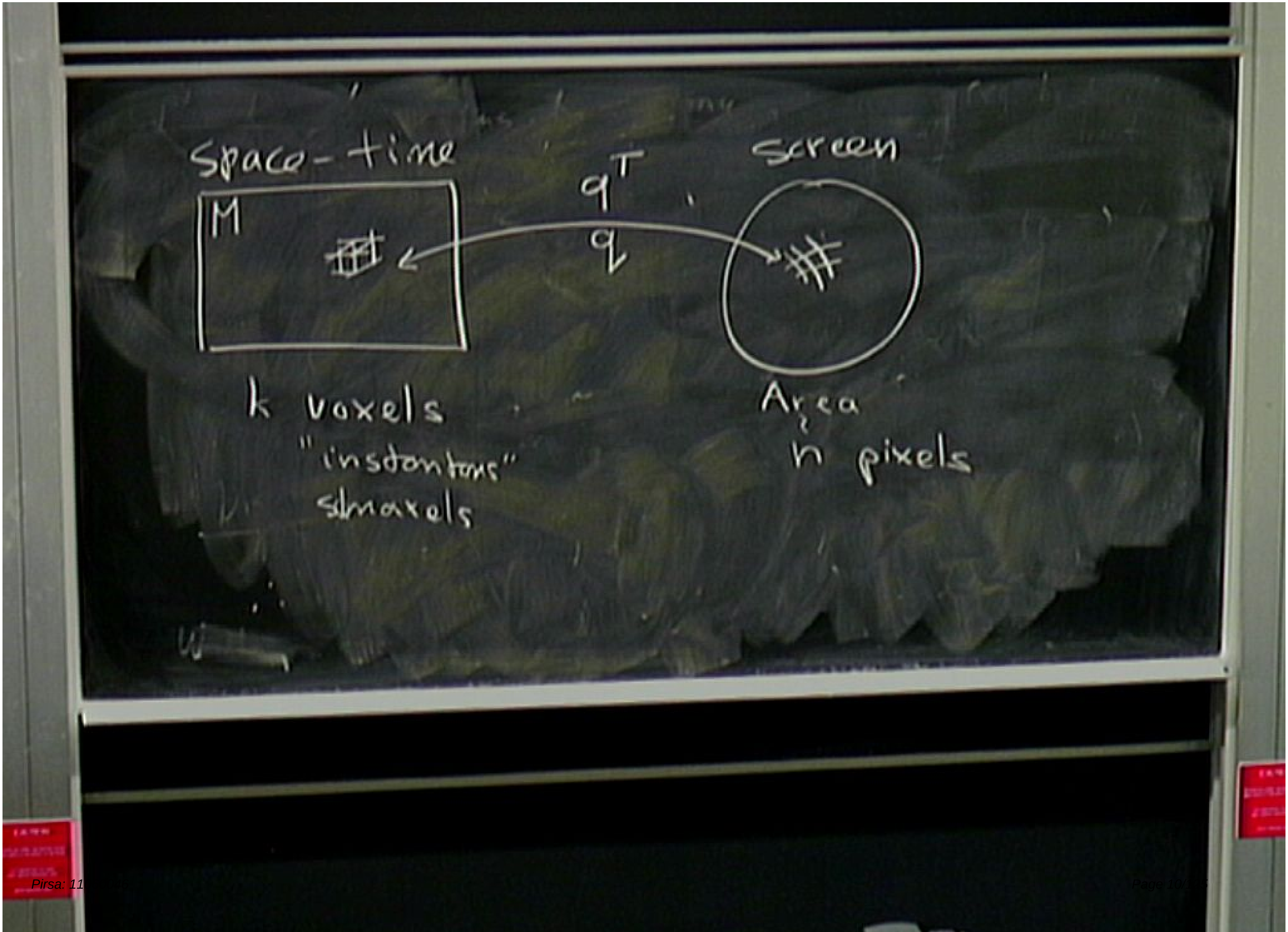
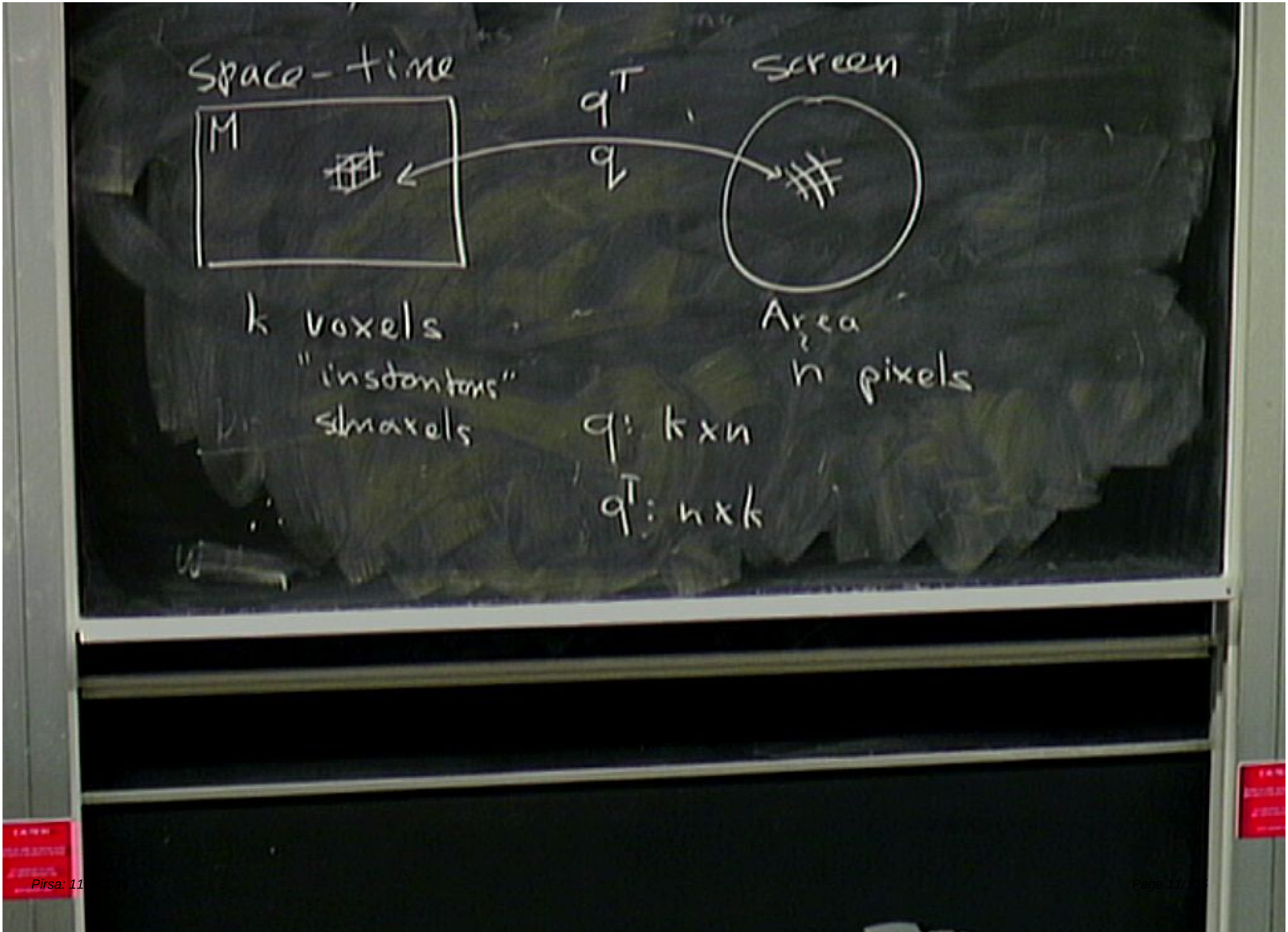
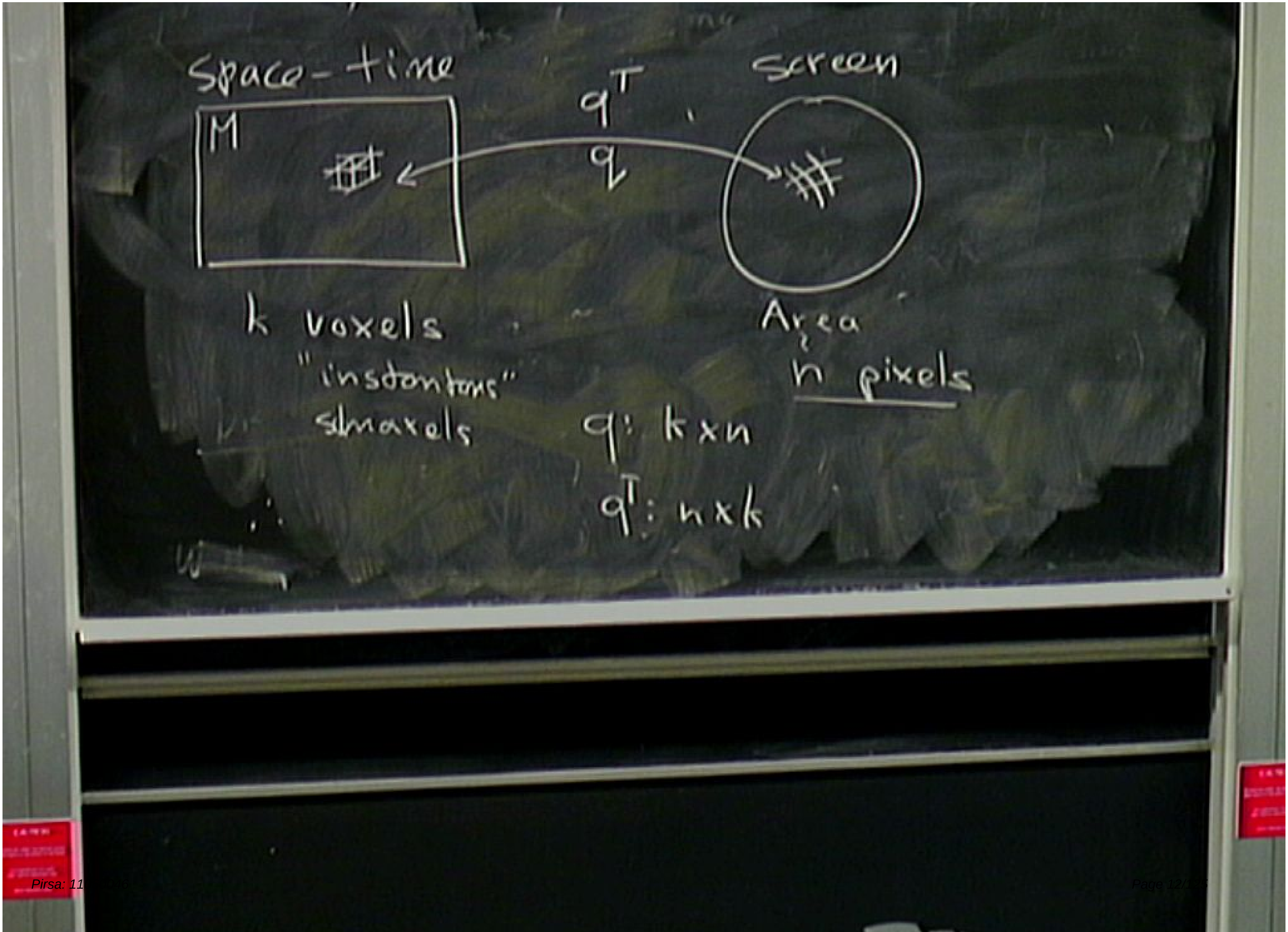

Space-time
M
q
screen
k voxels
"instantons"
smaxels
Area
n pixels


Space-time
M
$q^T$
$q$
screen
k voxels
"instantons"
Area
n pixels


Space-time
M
$q^T$
$q$
screen
k voxels
"instantons"
slmaxels
Area
n pixels
q: k x n
$q^T$: n x k


Space-time
screen
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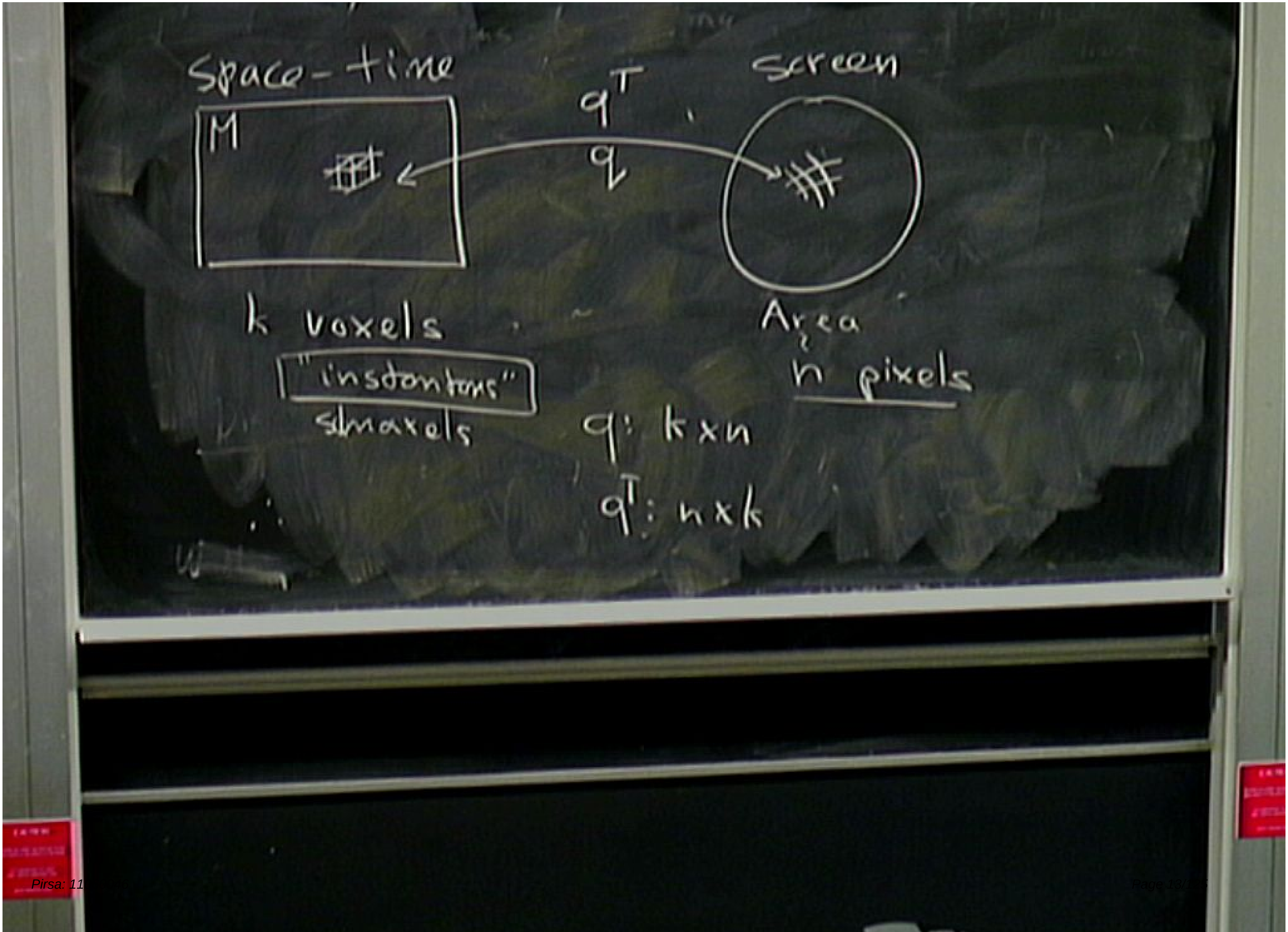
q: k x n
$q^T$: n x k


Space-time
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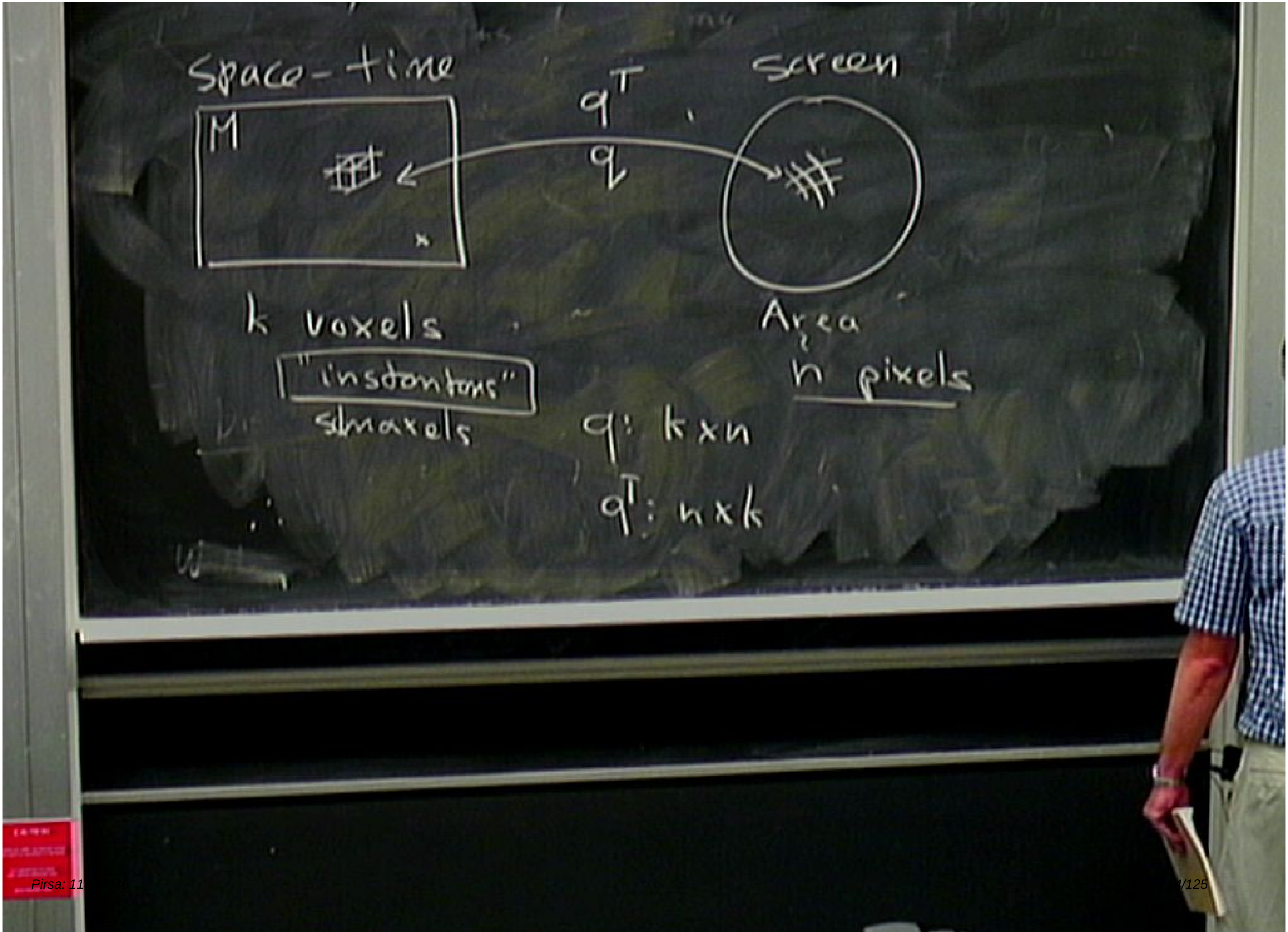
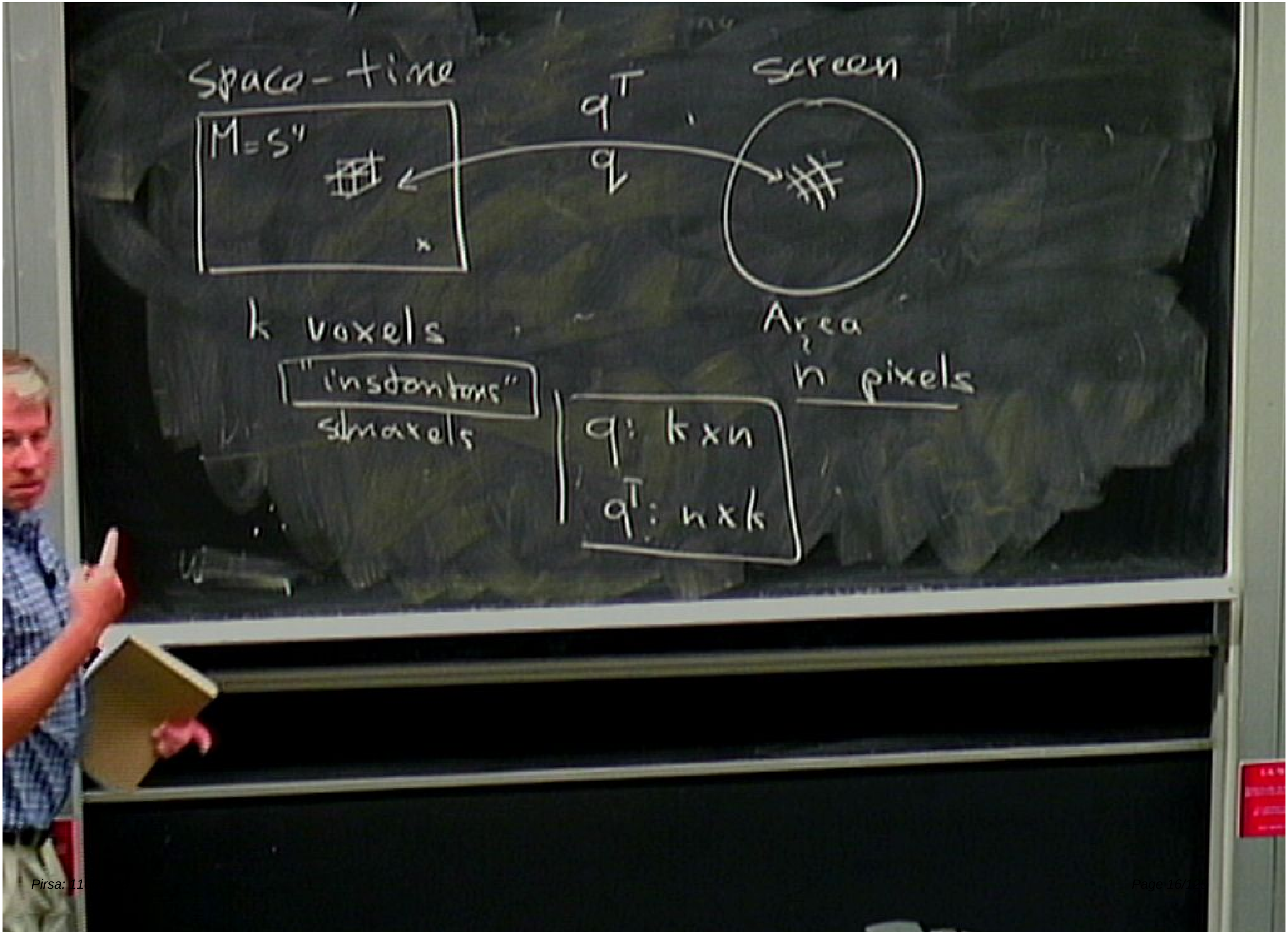
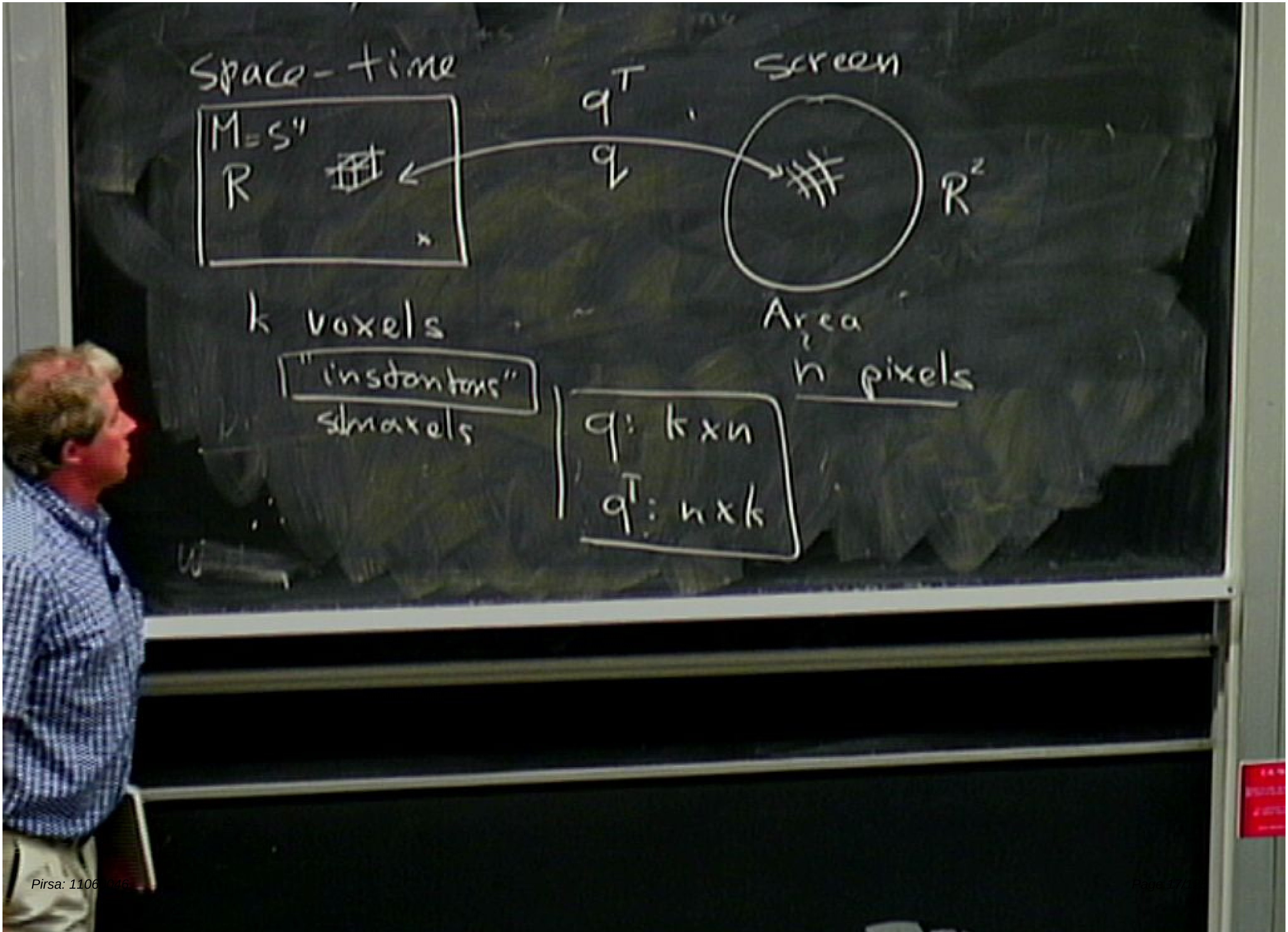
k voxels
"instontons"
smaxels
Area
n pixels
q: k x n
$q^T$: n x k

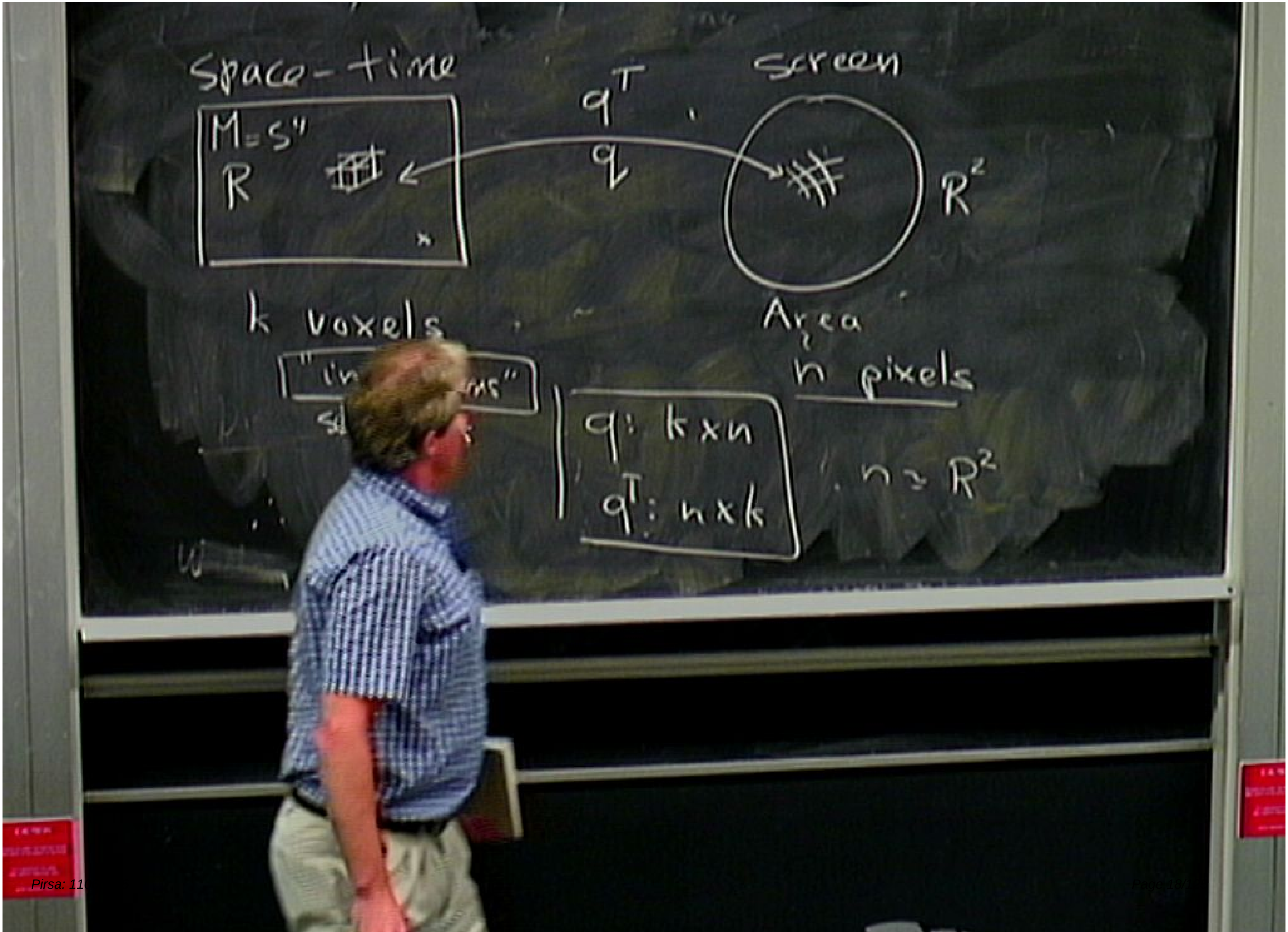

Space-time
screen
M
q^T
q
k voxels
Area
"instantons"
n pixels
smaxels
q: k x n
q^T: n x k

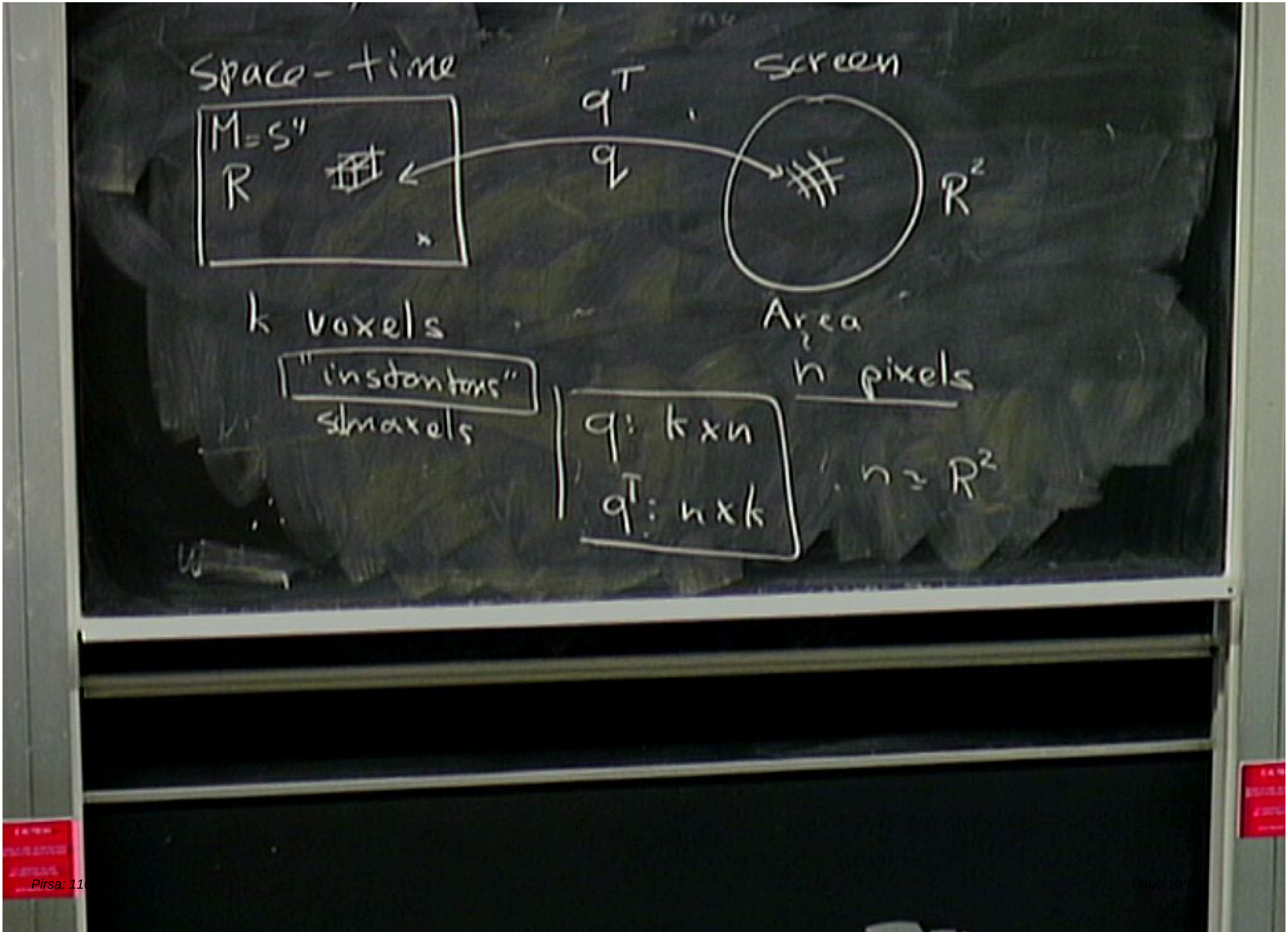
$$\underset{n\times n}{A} = \langle q^T | d_x | q \rangle$$

$k$ = instanton number

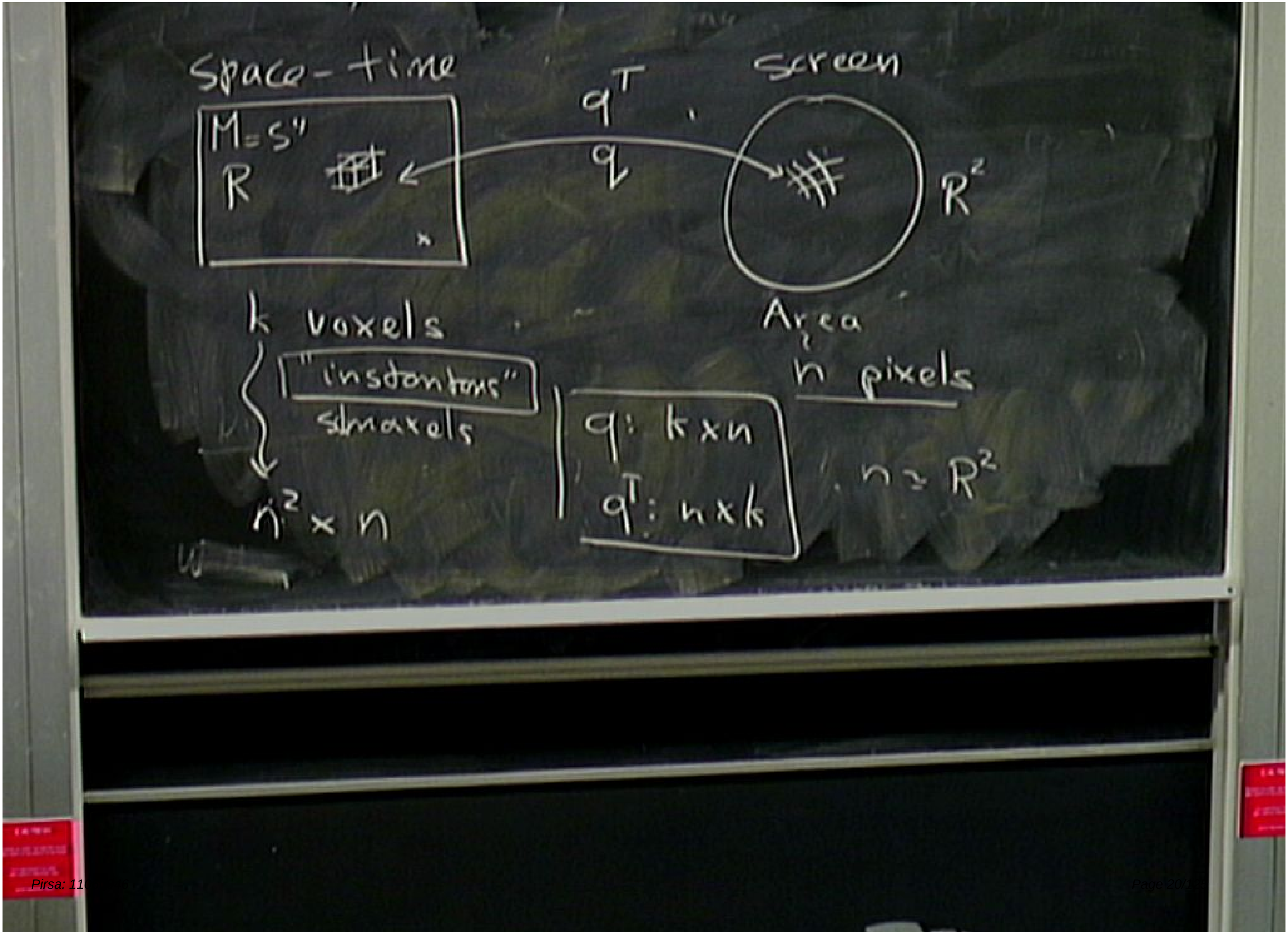

space-time
screen
$q^T$
$q$
M=S"
k voxels
"instantons"
smaxels
Area
n pixels
q: k x n
$q^T$: n x k


Space-time
screen
$q^T$
$q$
R
$R^2$
k voxels
"instantons"
Area
n pixels
q: k x n
$q^T$: n x k

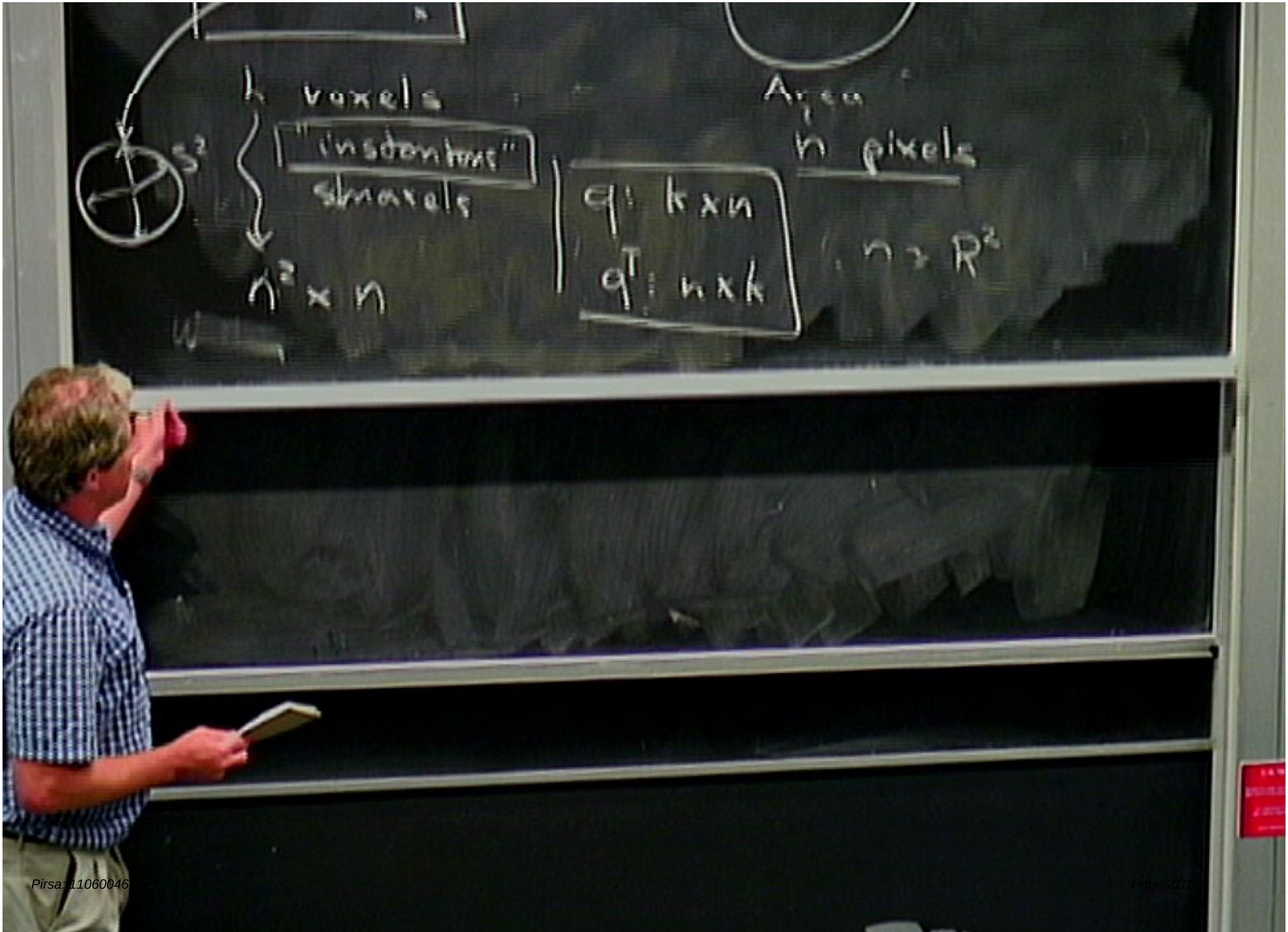

Space-time
screen
R
q
R
k voxels
Area
n pixels
q: k x n
n ≈ R



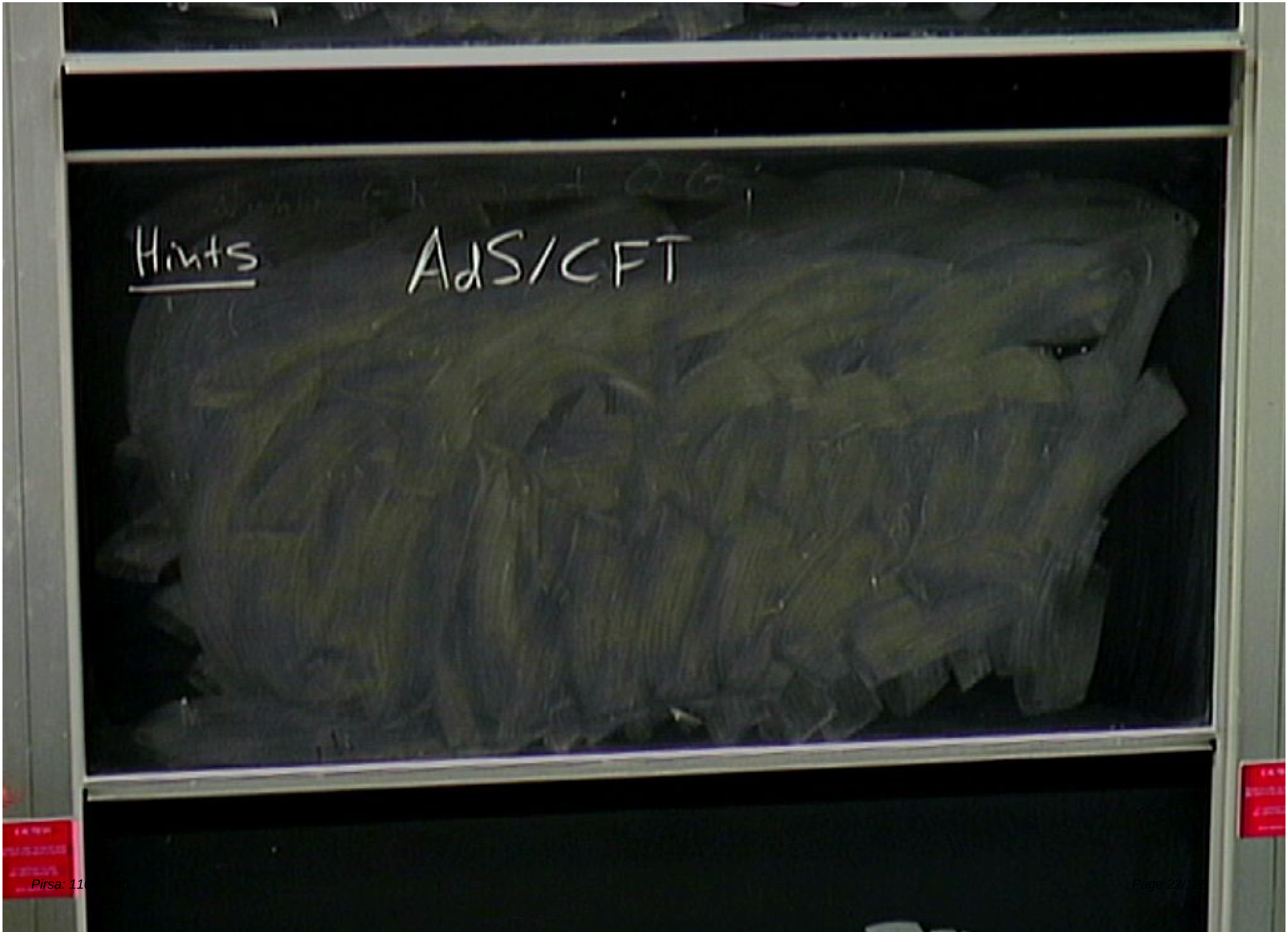
Space-time
screen
$q^T$
$q$
M = S"
R
$R^2$
k voxels
"instantons"
Area
n pixels
q: k x n
$q^T$: n x k
n ≈ $R^2$



space-time
screen
$q^T$
$q$
R
$R^2$
k voxels
"instantons"
Area
n pixels
q: k x n
$q^T$: n x k
$n \simeq R^2$
$n^2 \times n$



k voxels
$S^2$
$n^2 \times n$
q: k x n
$q^T$: n x k
Area
n pixels
$n \gtrsim R^2$

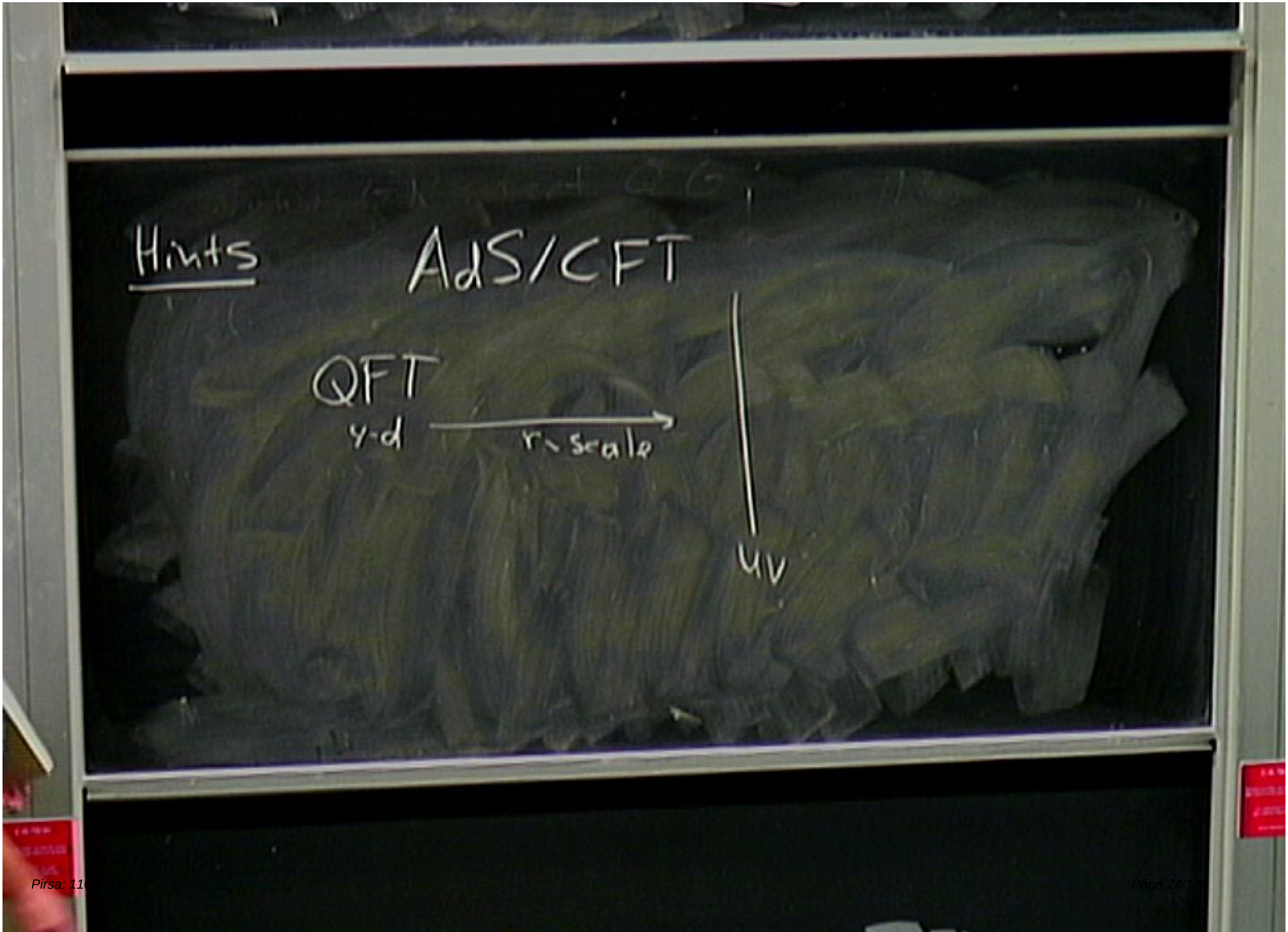

Hints
AdS/CFT

Hints

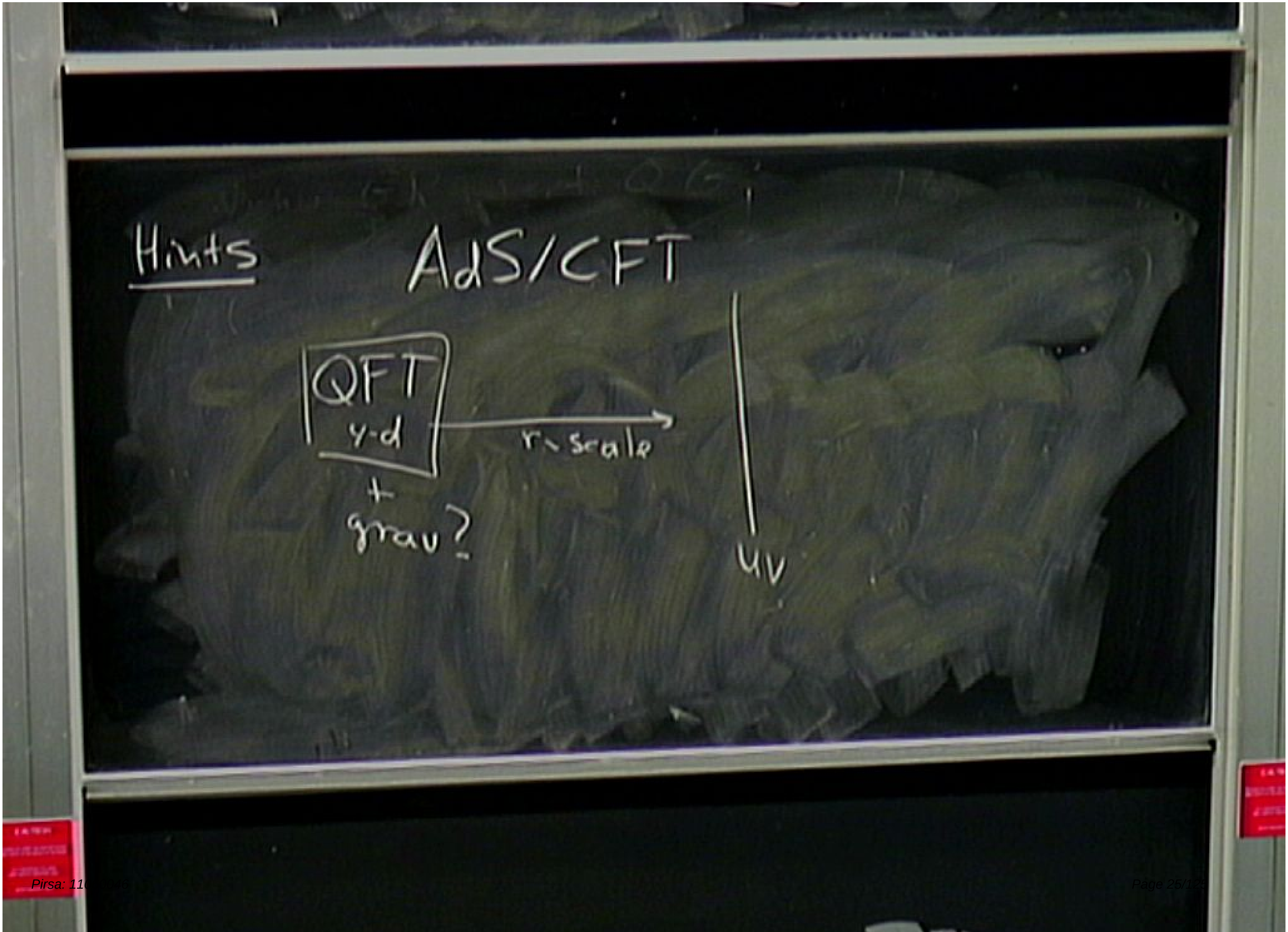
AdS/CFT

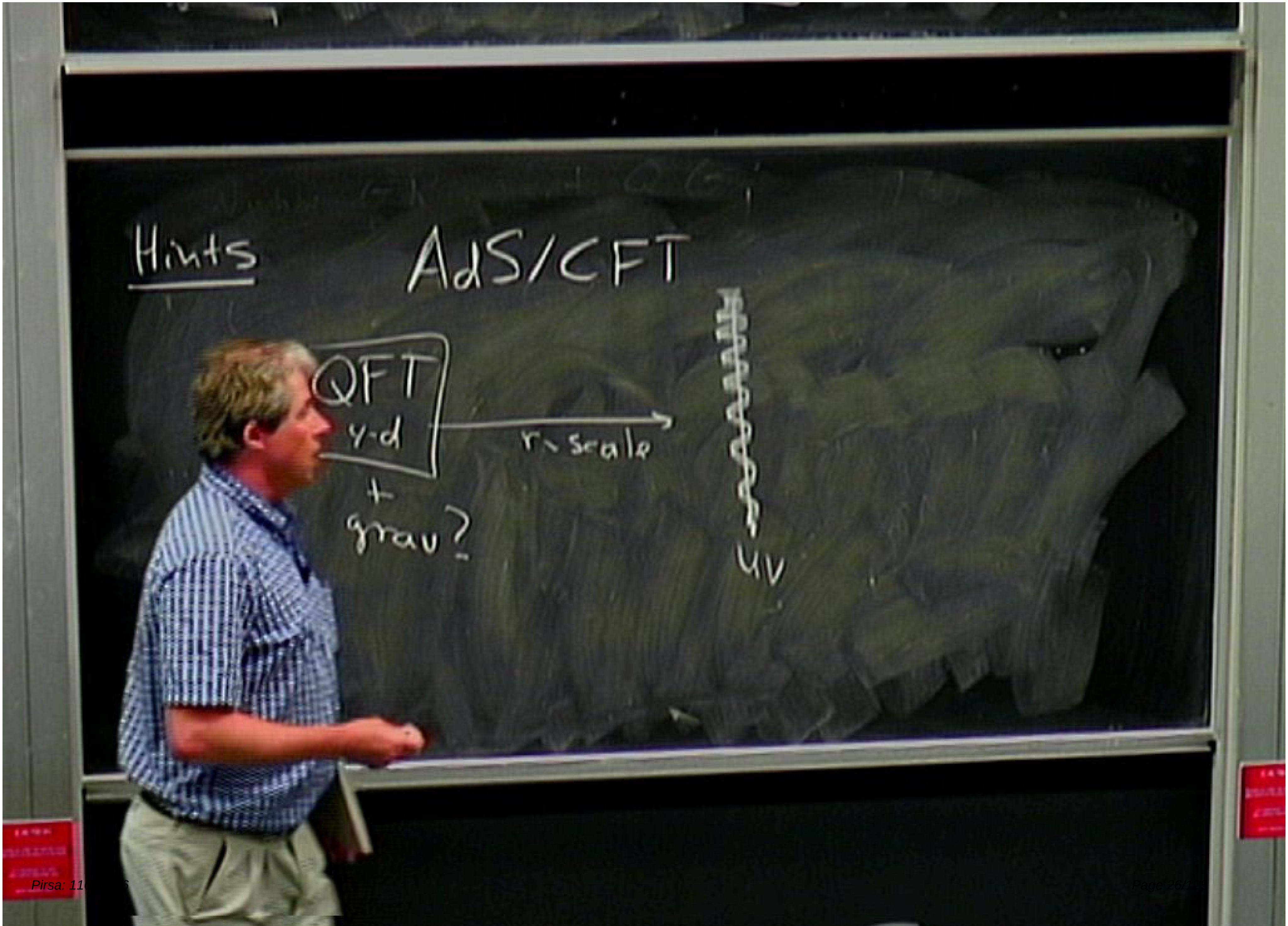
QFT

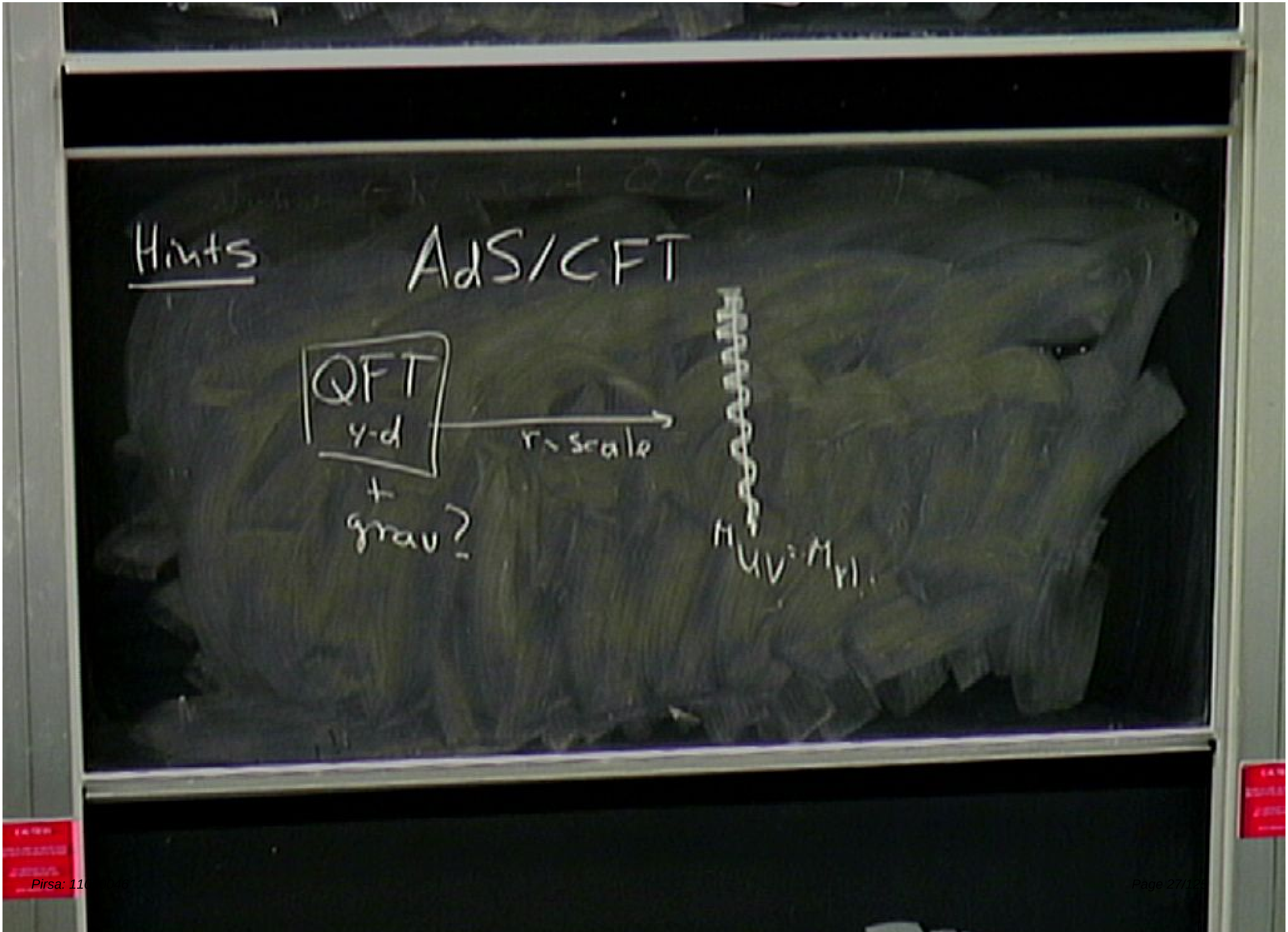
4-d → r

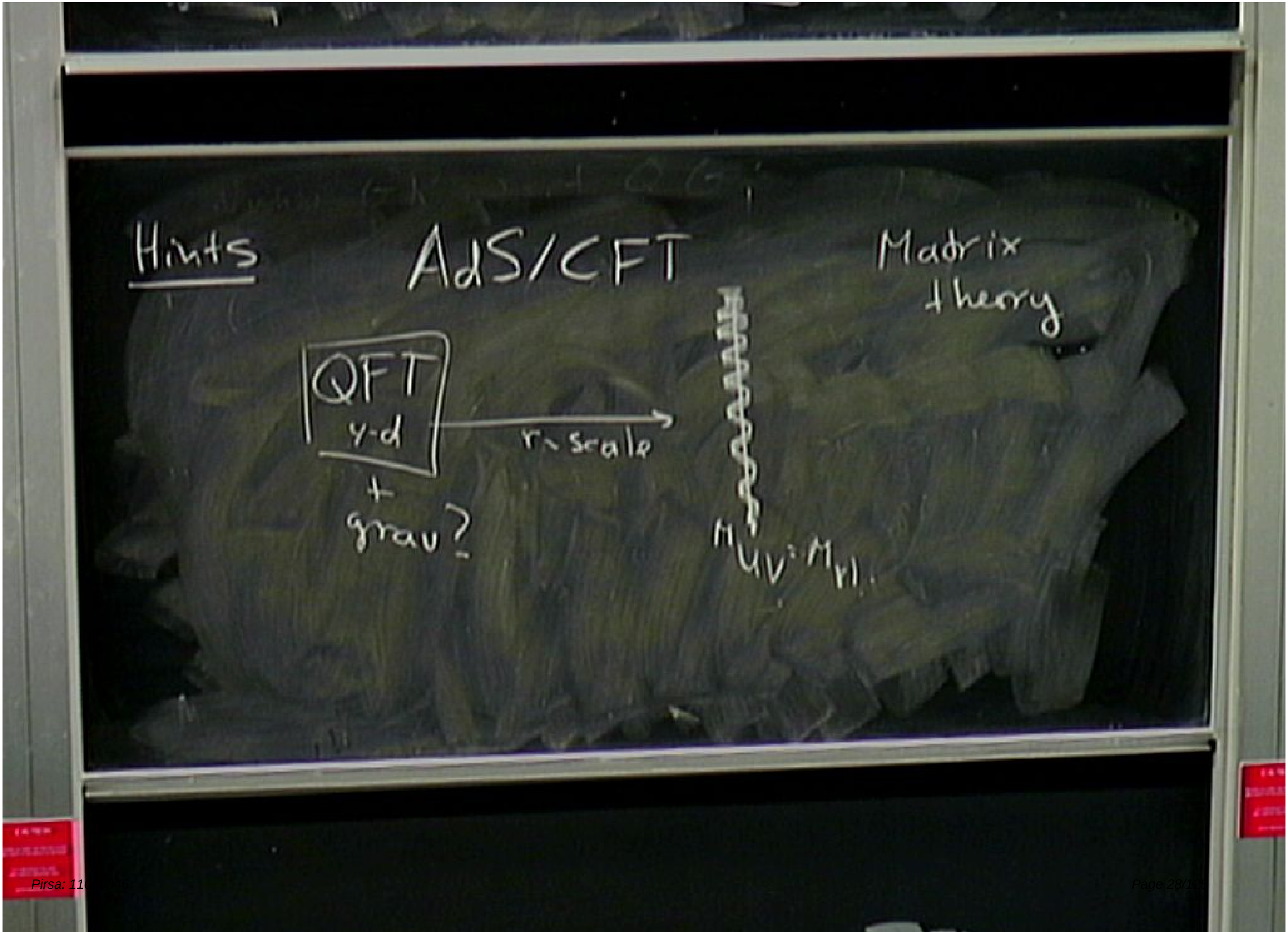


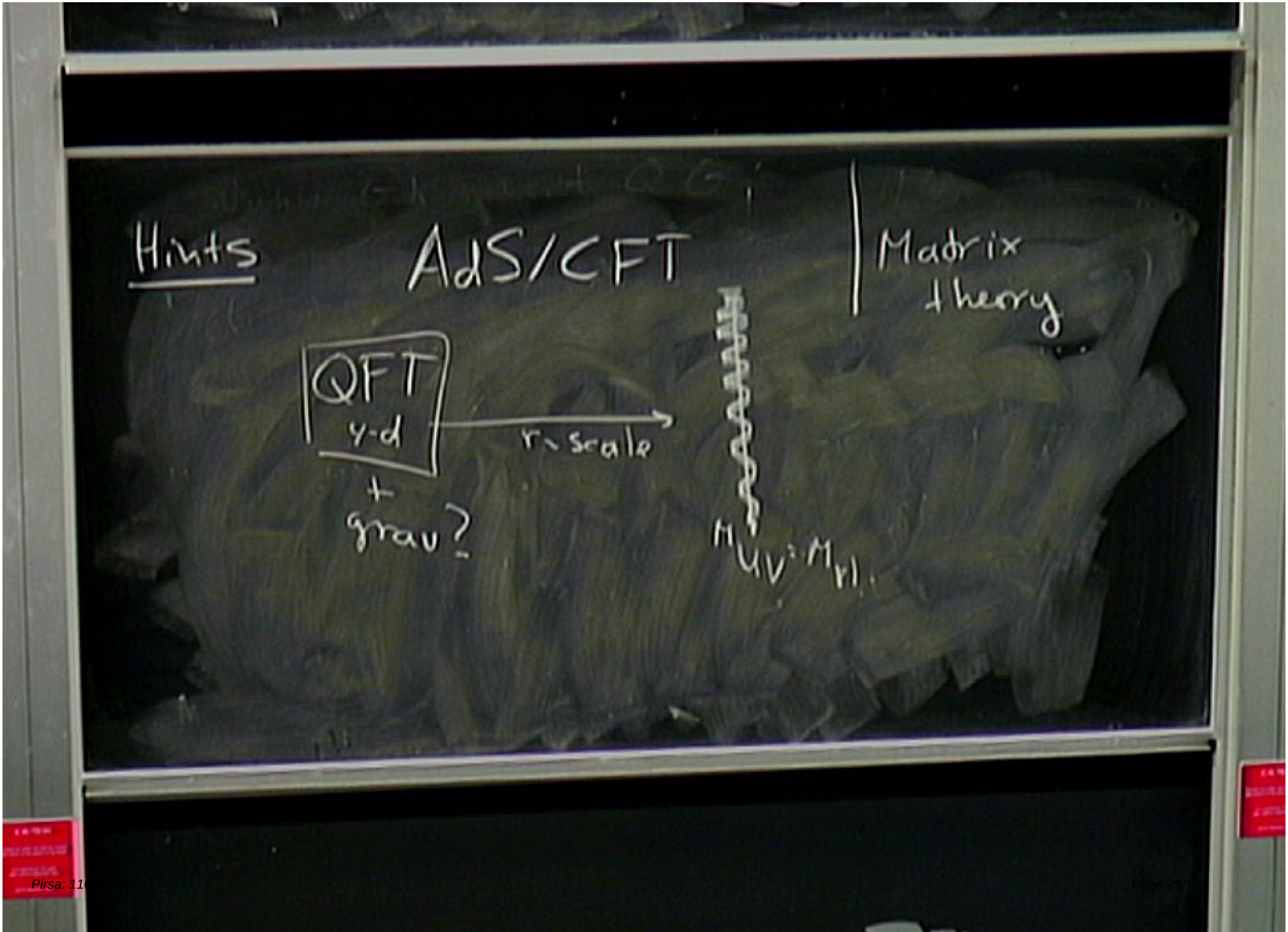
Hints
AdS/CFT
QFT
4-d
r ~ scale
UV

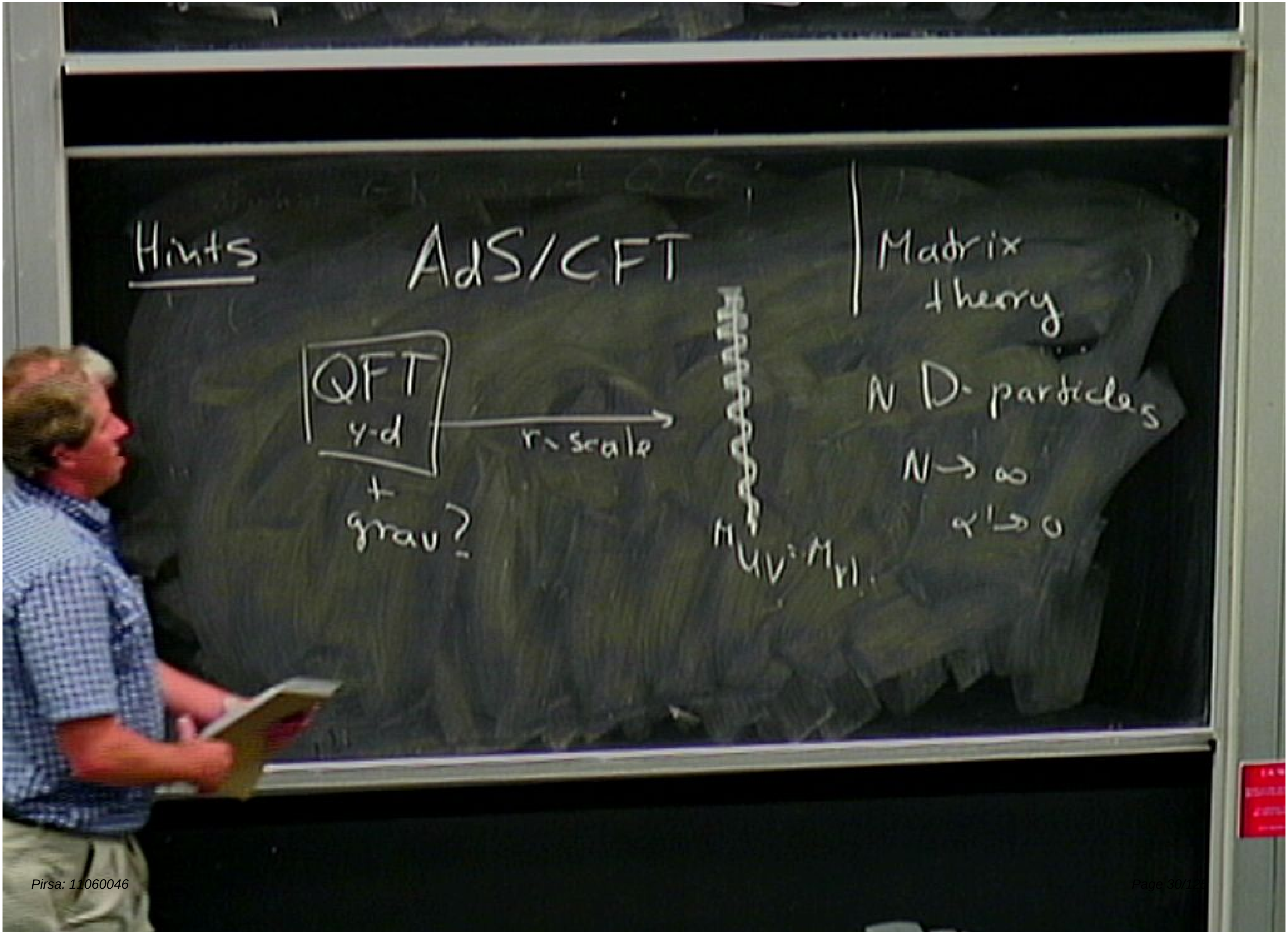

Hints
AdS/CFT
QFT
4-d
r ~ scale
+
grav?
UV

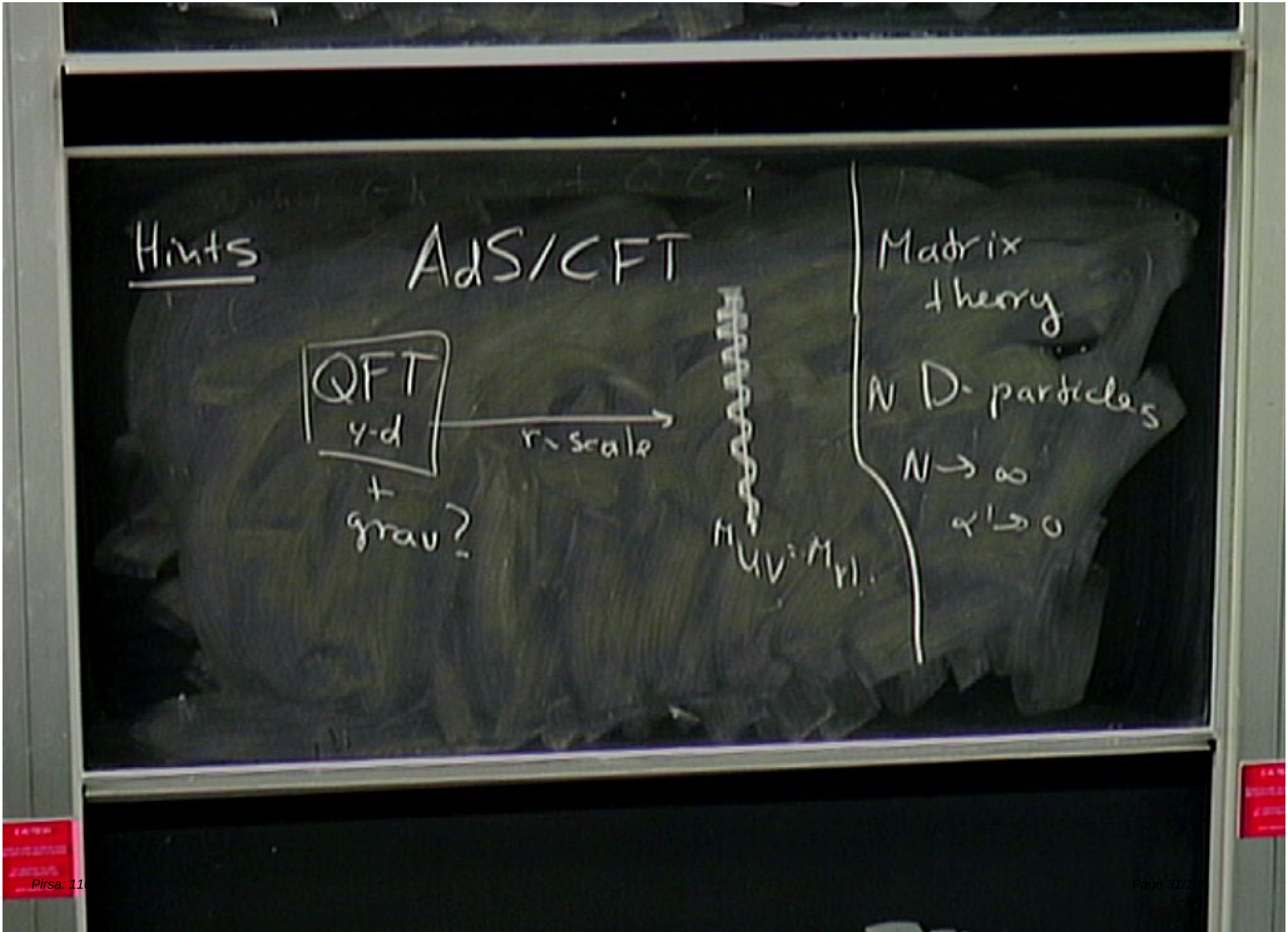

Hints
AdS/CFT
QFT
4-d
+
grav?
r ~ scale
UV

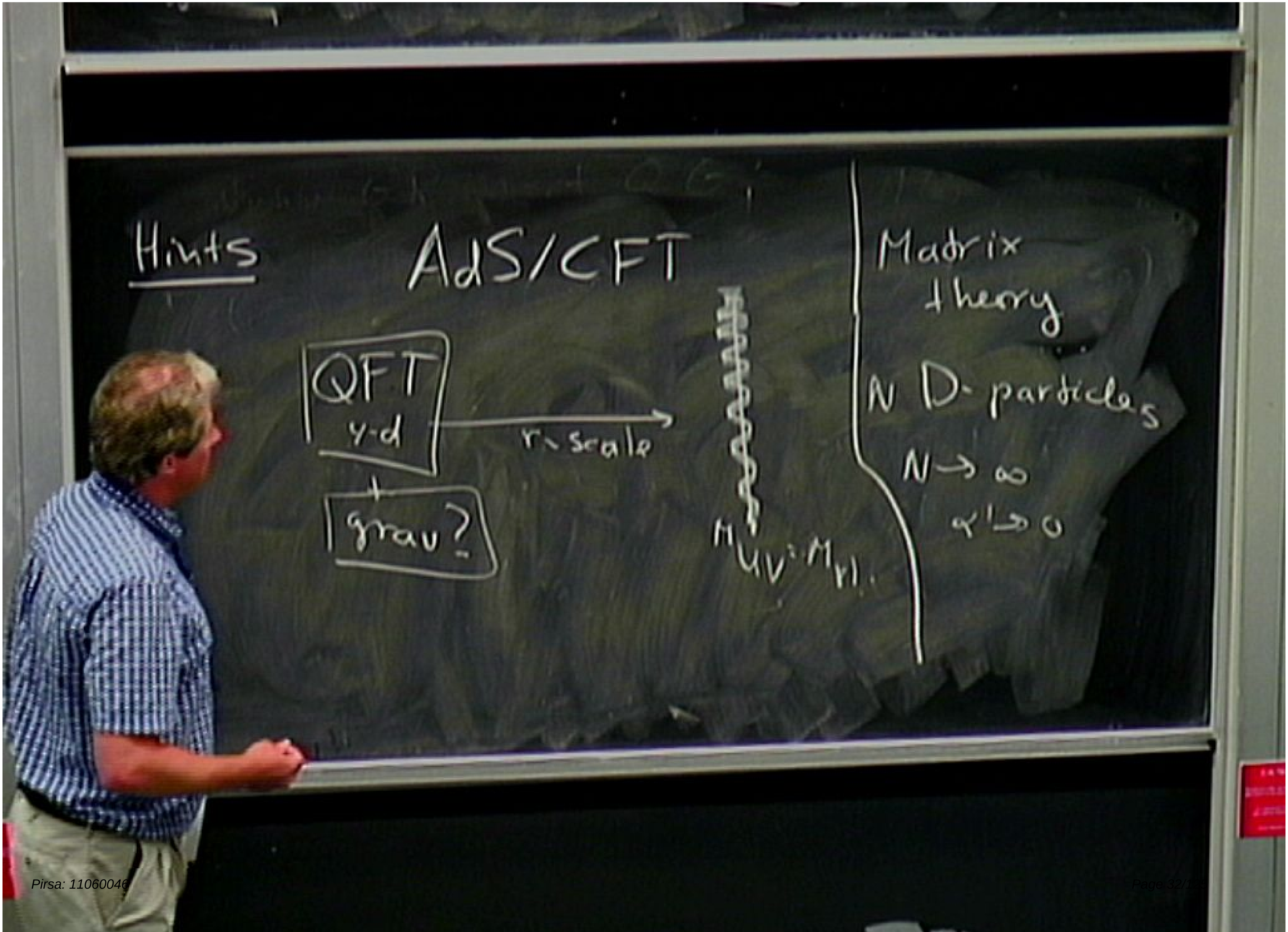

Hints
AdS/CFT
QFT
4-d
+
grav?
r ~ scale
M_UV = M_Pl.

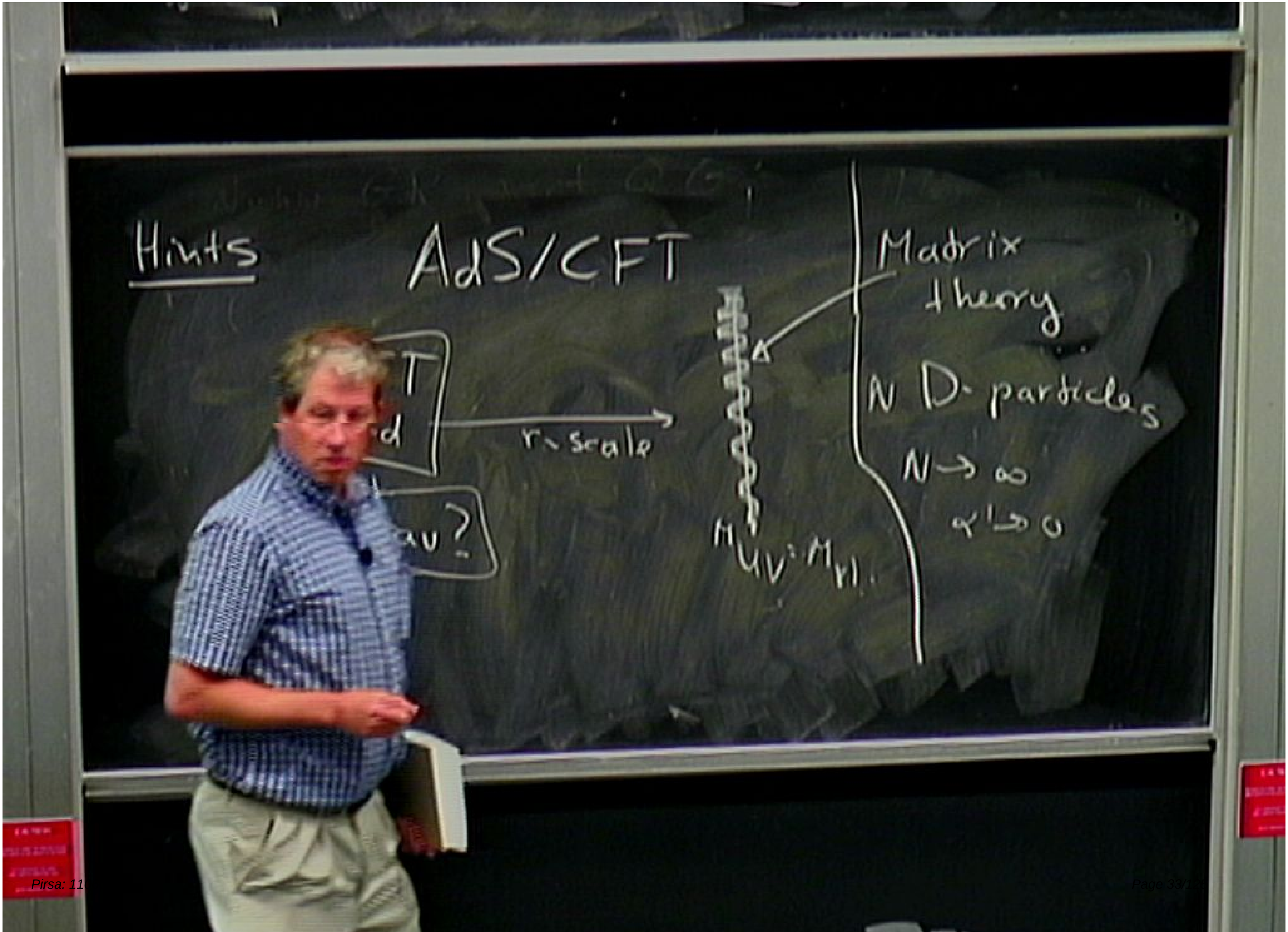

Hints
AdS/CFT
Matrix theory
QFT
4-d
+ grav?
r ~ scale
M_UV = M_Pl

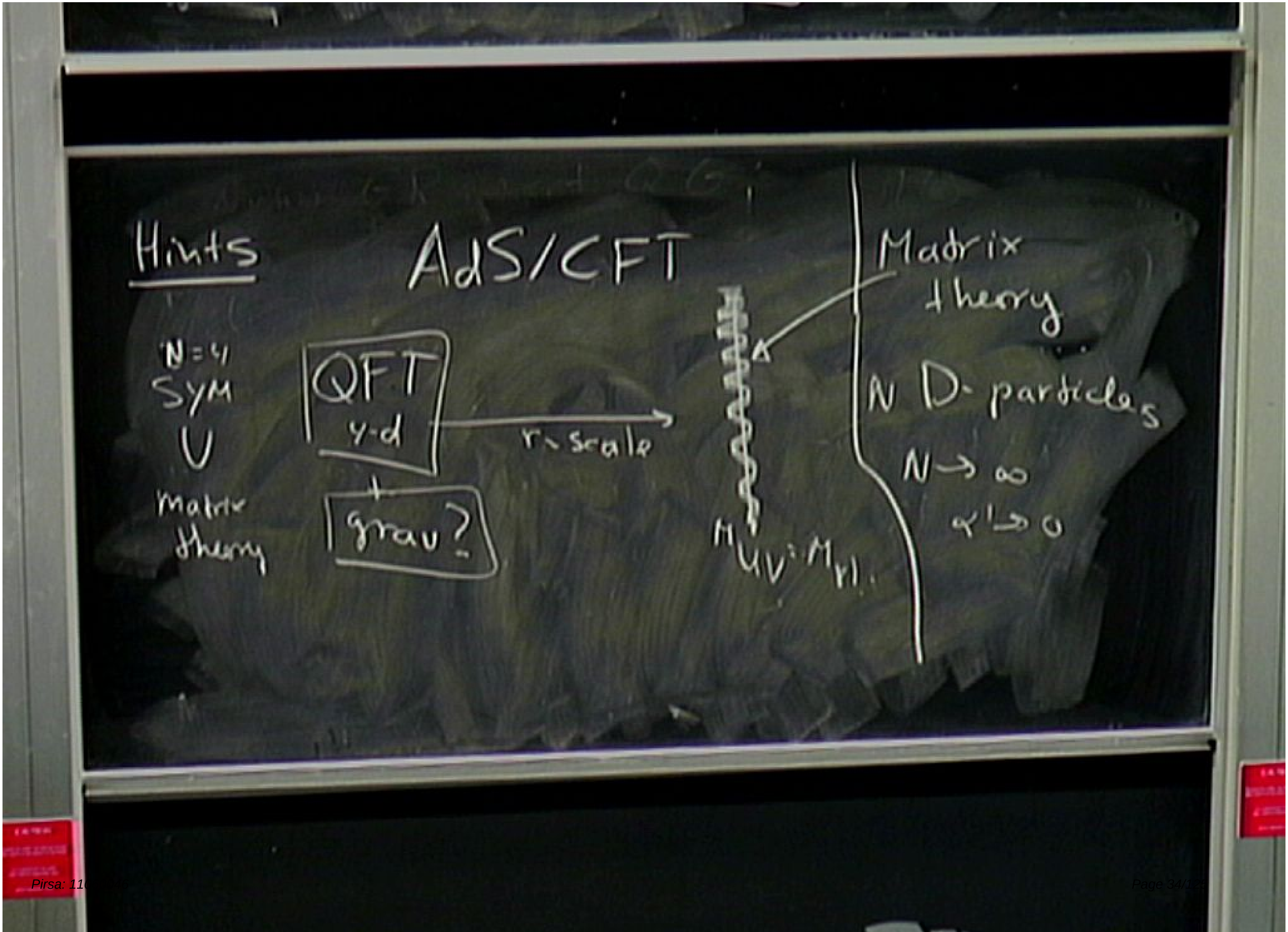

Hints
AdS/CFT
Matrix theory
QFT
4-d
+ grav?
r ~ scale
M_UV = M_Pl.

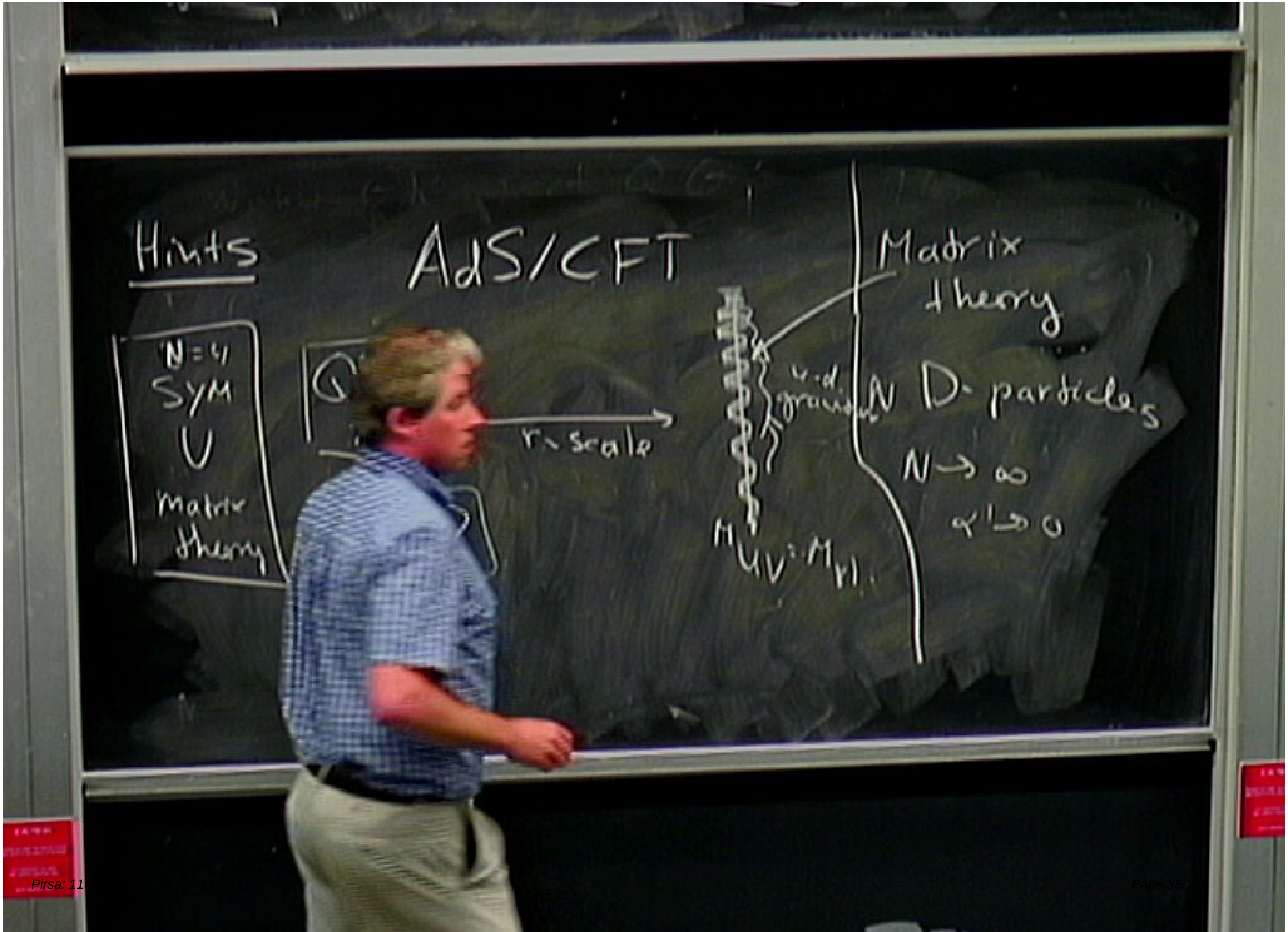

Hints
AdS/CFT
Matrix theory
QFT
4-d
+
grav?
r ~ scale
M_UV = M_pl
N D-particles
N → ∞
α' → 0

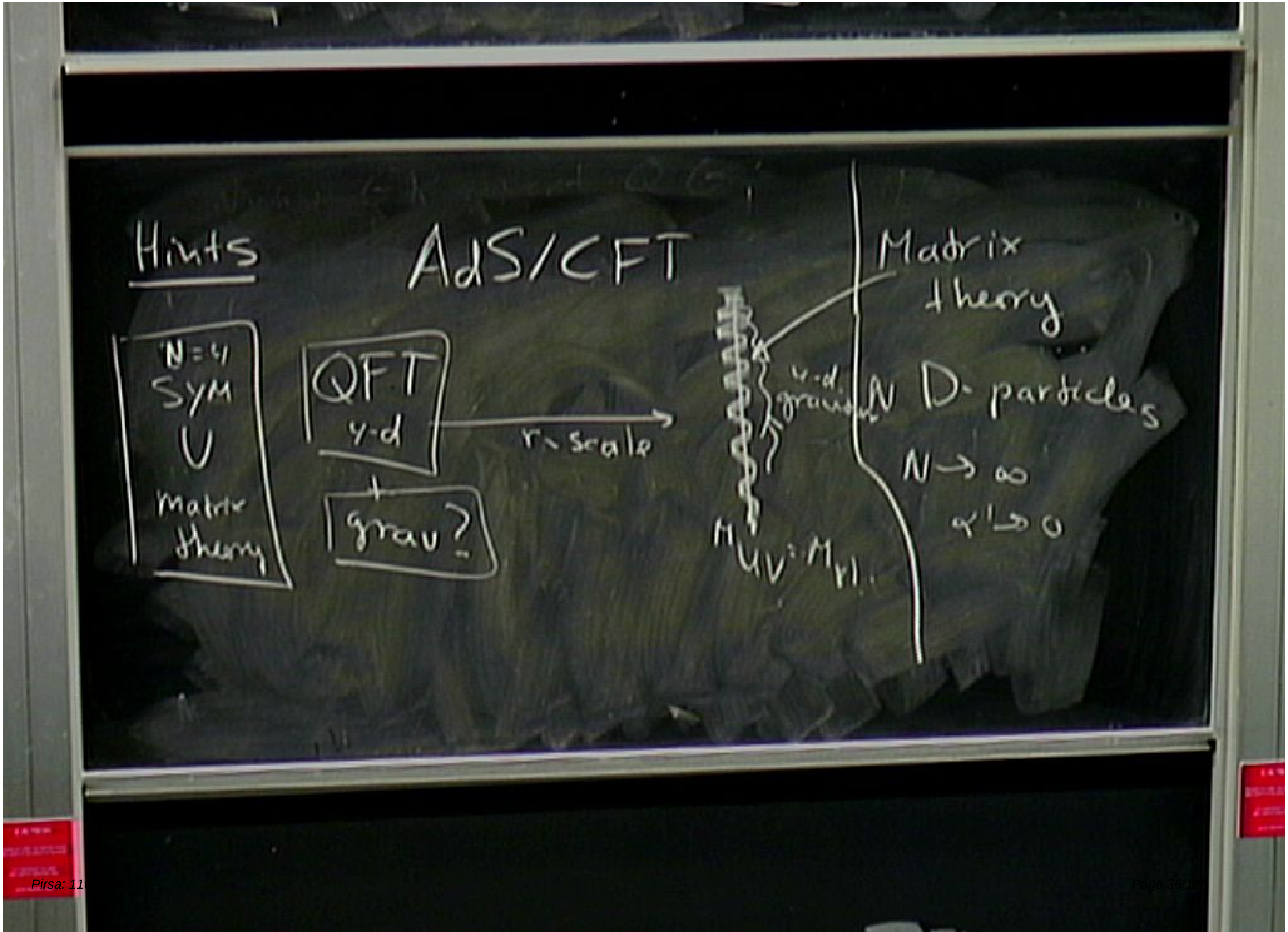

Hints
AdS/CFT
QFT
4-d
+
grav?
r ~ scale
$M_{UV} = M_{Pl}$
Matrix
theory
N D-particles
$N \to \infty$
$\alpha' \to 0$

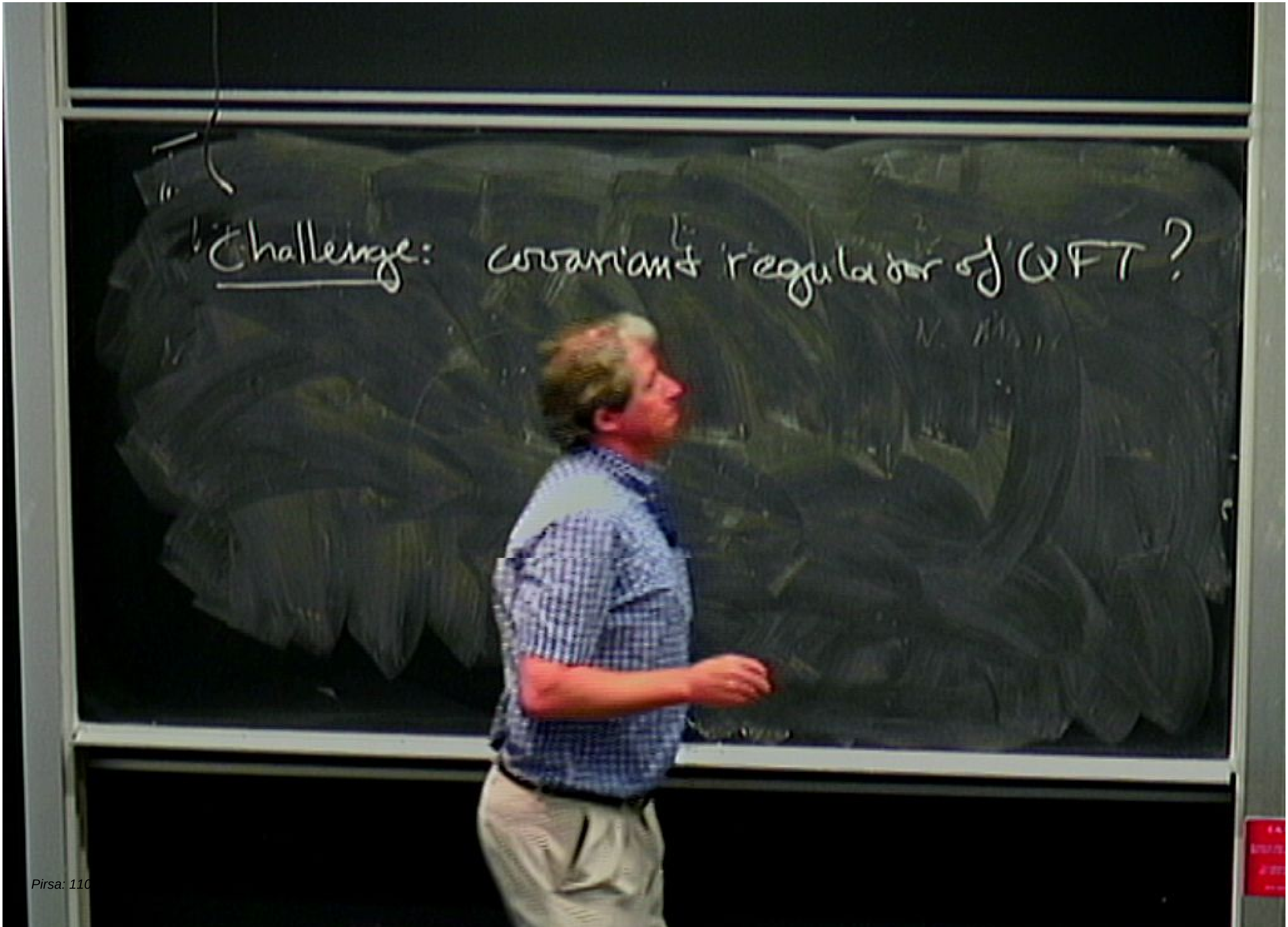

Hints
AdS/CFT
QFT
4-d
+
grav?
r ~ scale
M_UV ~ M_pl.
Matrix theory
N D-particles
N → ∞
α' → 0

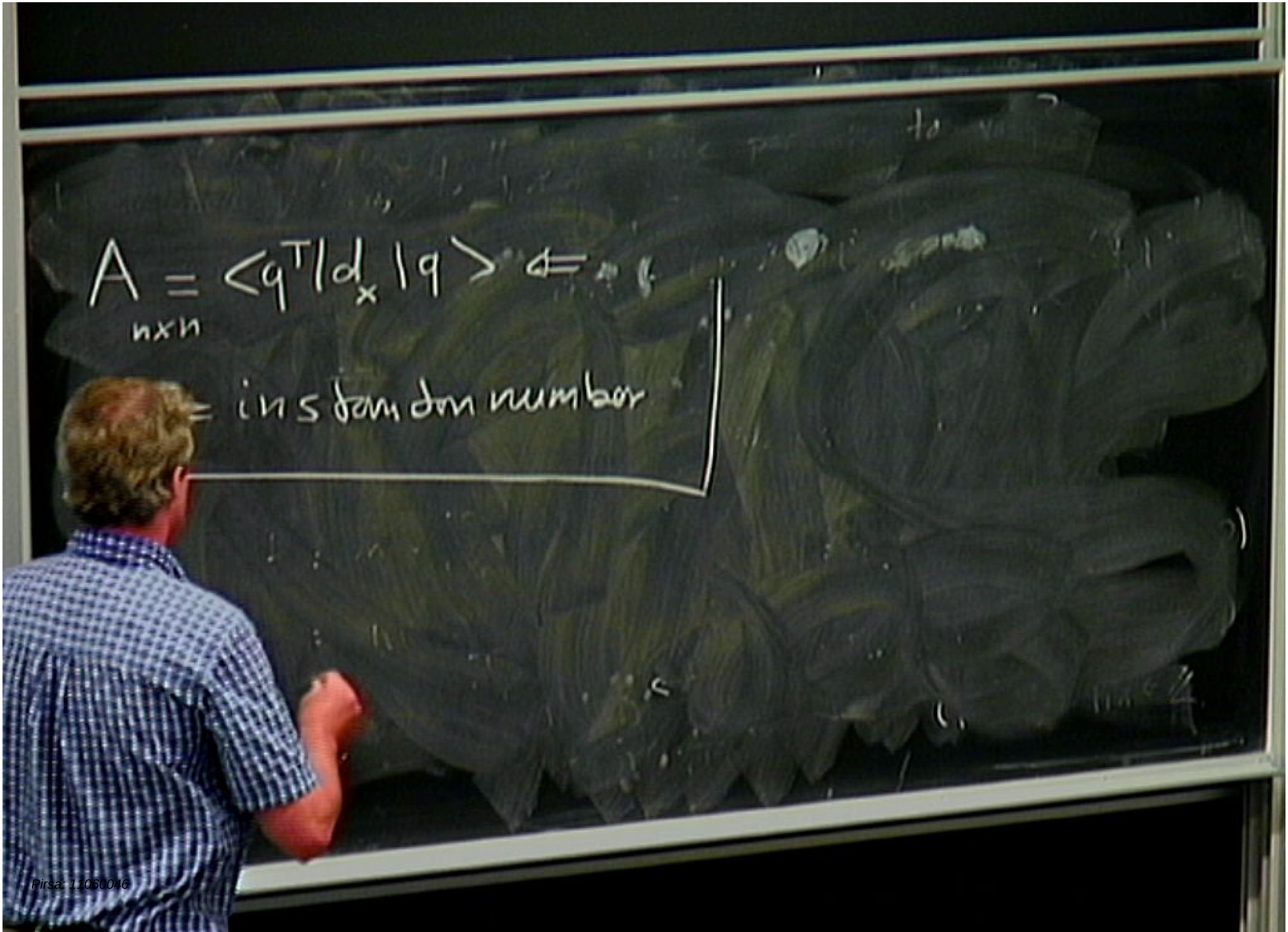

Hints
AdS/CFT
Matrix theory
N D-particles
N → ∞
α' → 0
r ~ scale
M_UV = M_Pl


Hints
AdS/CFT
Matrix theory
N=4 SYM
matrix theory
QFT 4-d
grav?
r ~ scale
N D-particles
N → ∞
α' → 0


Hints
AdS/CFT
Matrix theory
N=4
SYM
U
matrix theory
r ~ scale
4-d graviton
N D-particles
N → ∞
α' → 0
M_UV = M_Pl.


Hints
AdS/CFT
N=4
SYM
∪
matrix
theory
QFT
4-d
+
grav?
r ~ scale
4-d
gravity
N
M_UV = M_pl.
Matrix
theory
D-particles
N → ∞
α' → 0

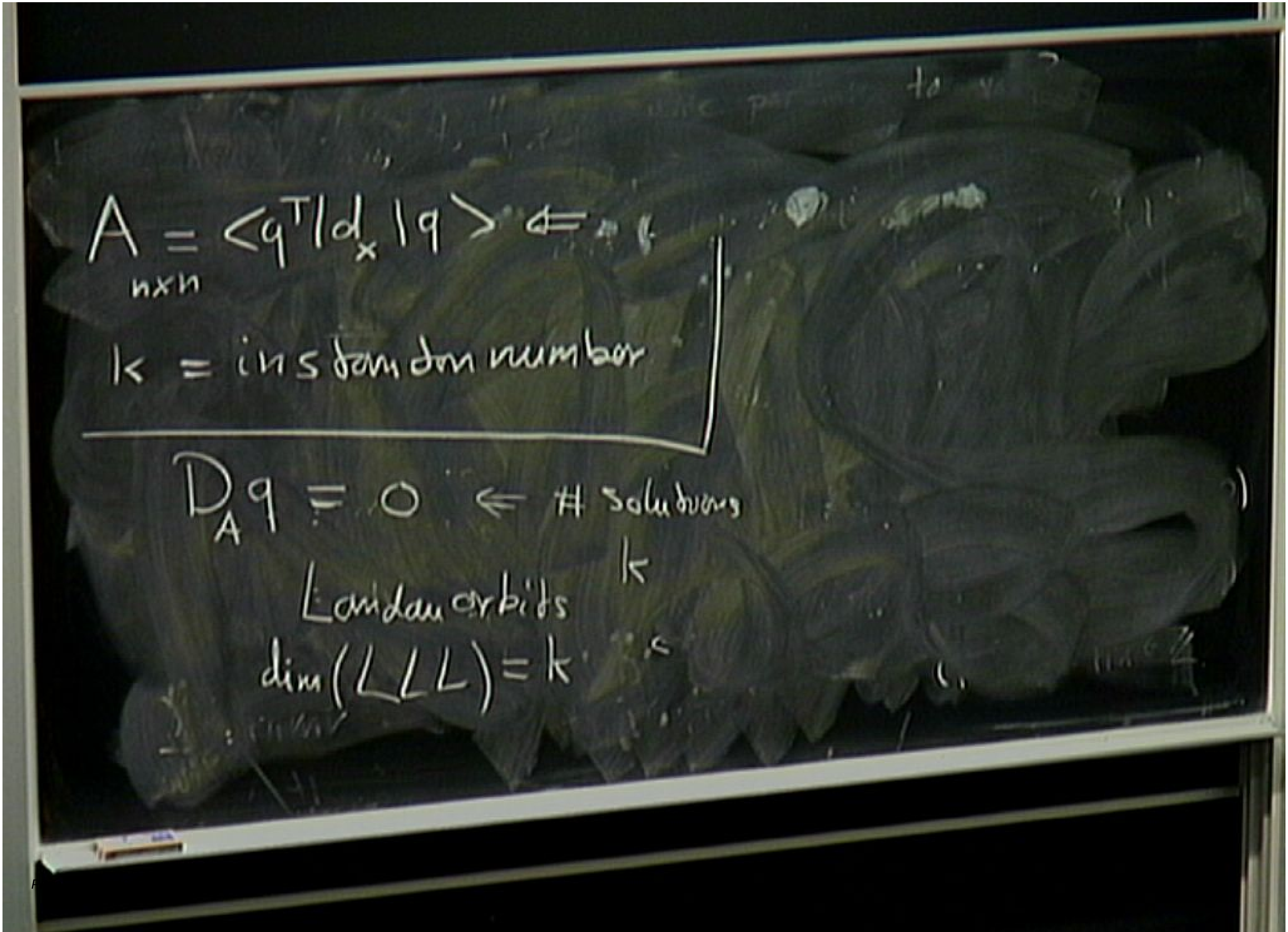

Challenge: covariant regulator of QFT ?

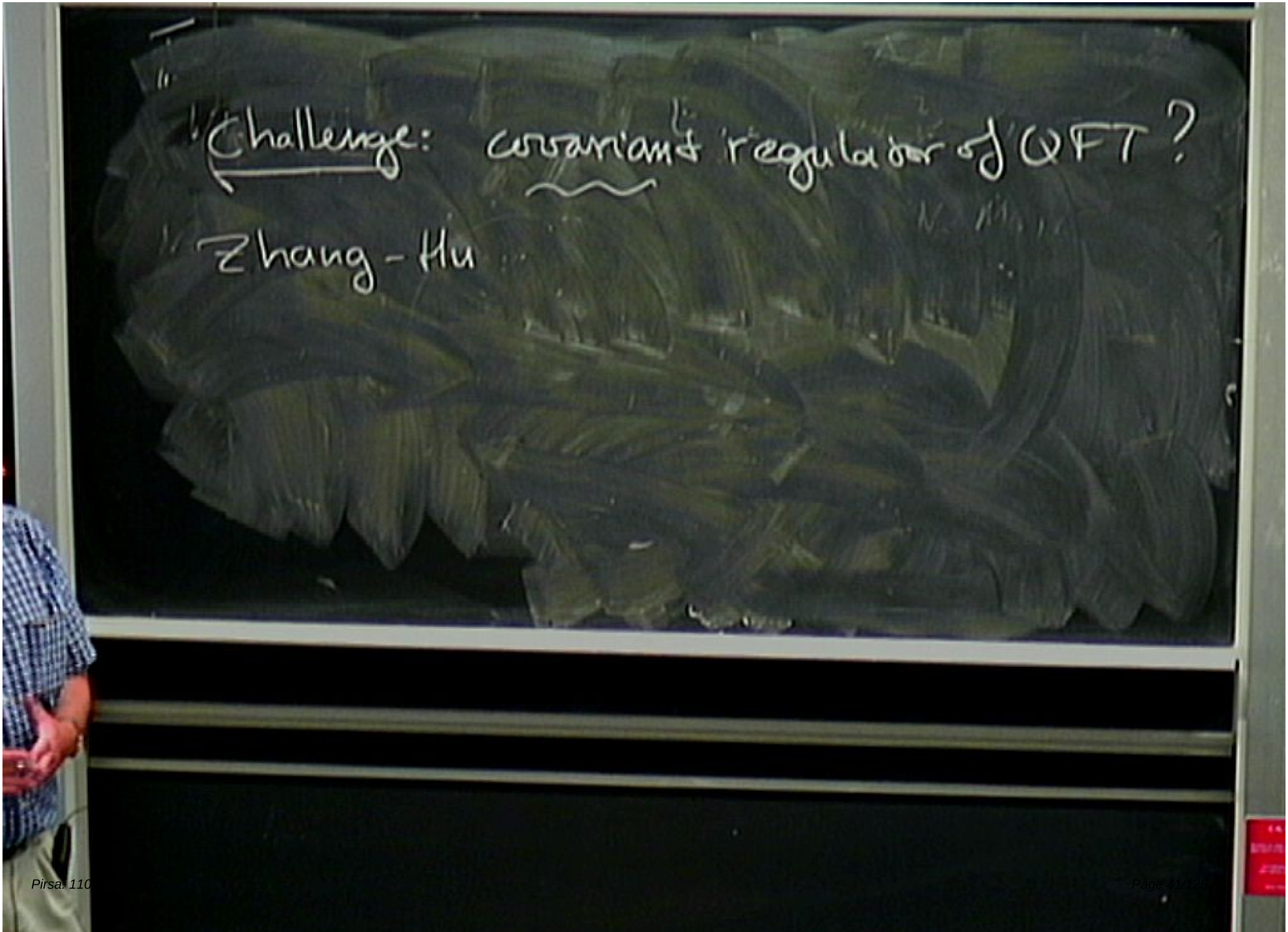

$A_{n\times n} = \langle q^T | d_x | q \rangle \Leftarrow$
= instanton number

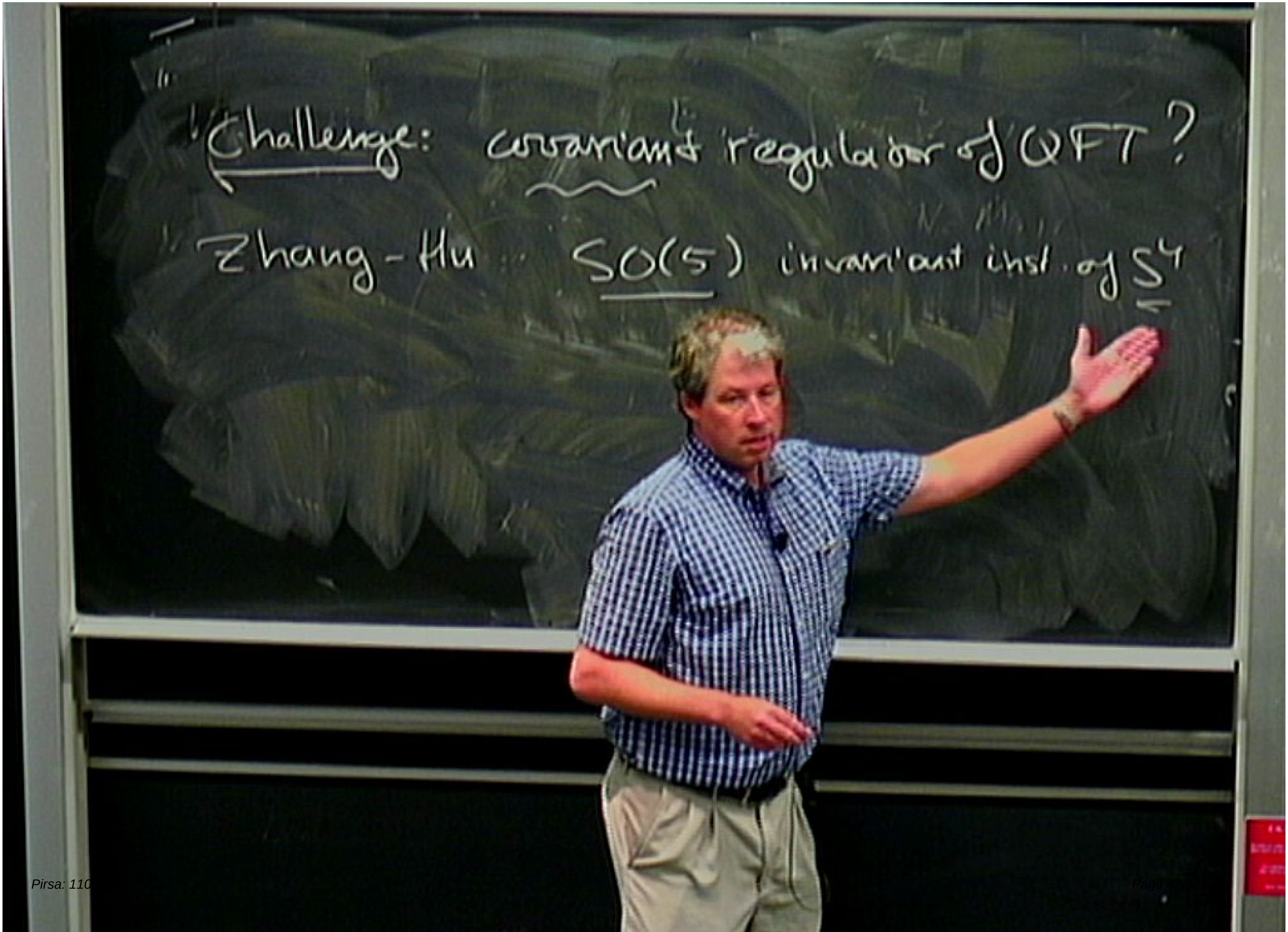
$$A_{n\times n} = \langle q^T | d_x | q \rangle \Leftarrow$$

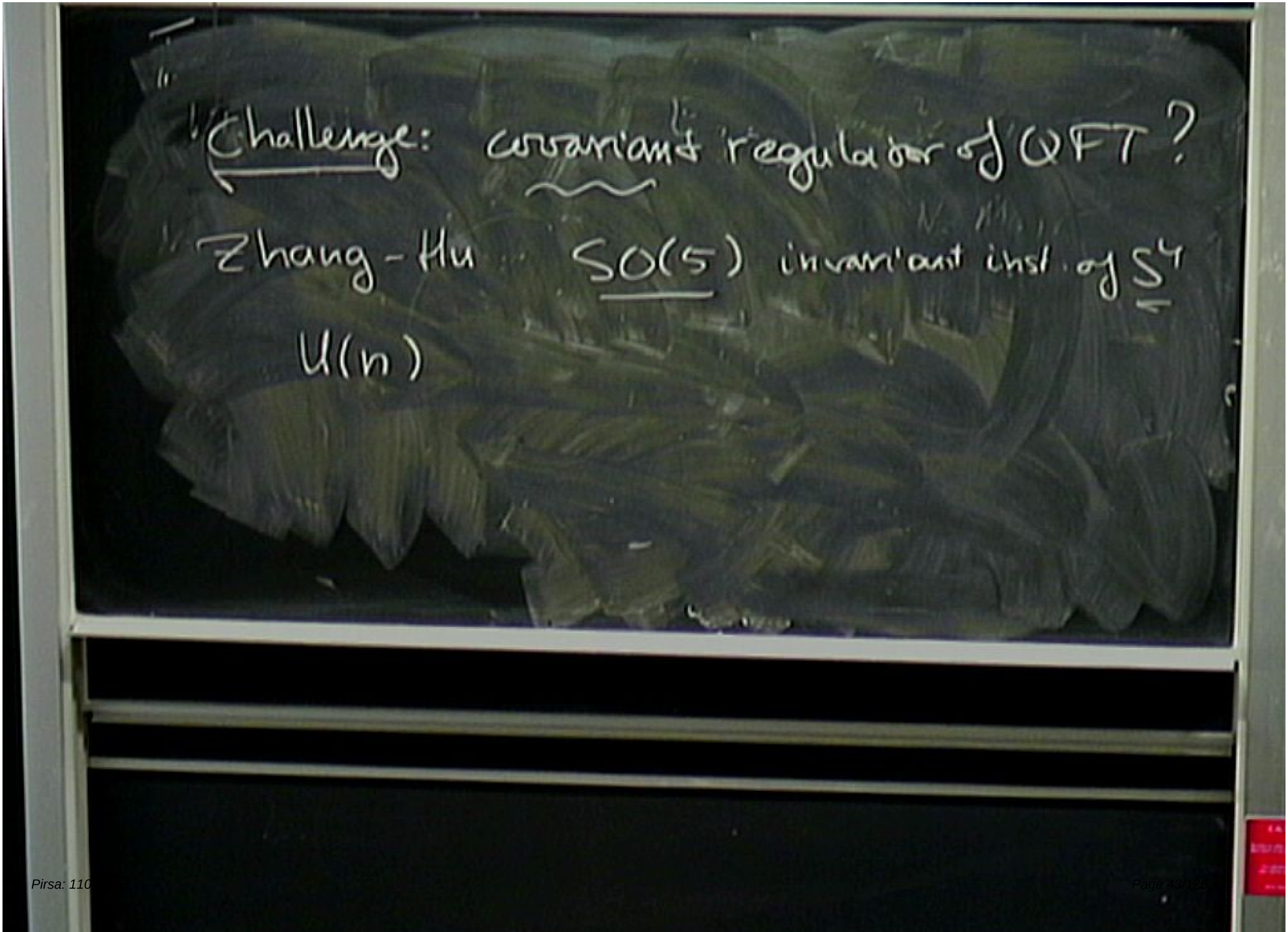
$k$ = instanton number

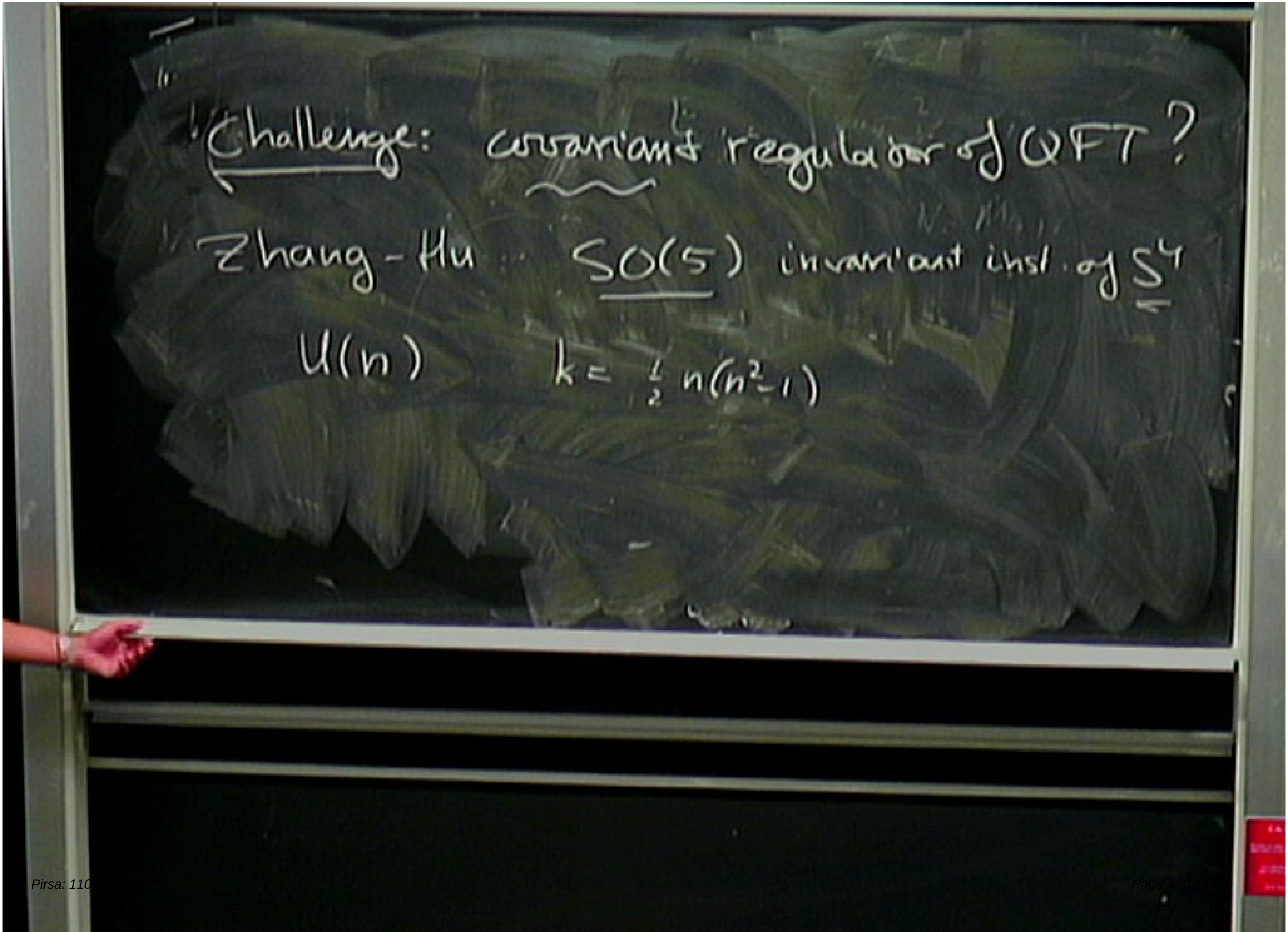
$$D_A q = 0$$


A_{n×n} = <q^T|d_x|q> ⇐
k = instanton number
D_A q = 0 ⇐ # solutions
k
Landau orbits
dim(LLL) = k


Challenge: covariant regulator of QFT ?
Zhang-Hu


Challenge: covariant regulator of QFT ?
Zhang-Hu SO(5) invariant inst. of S4

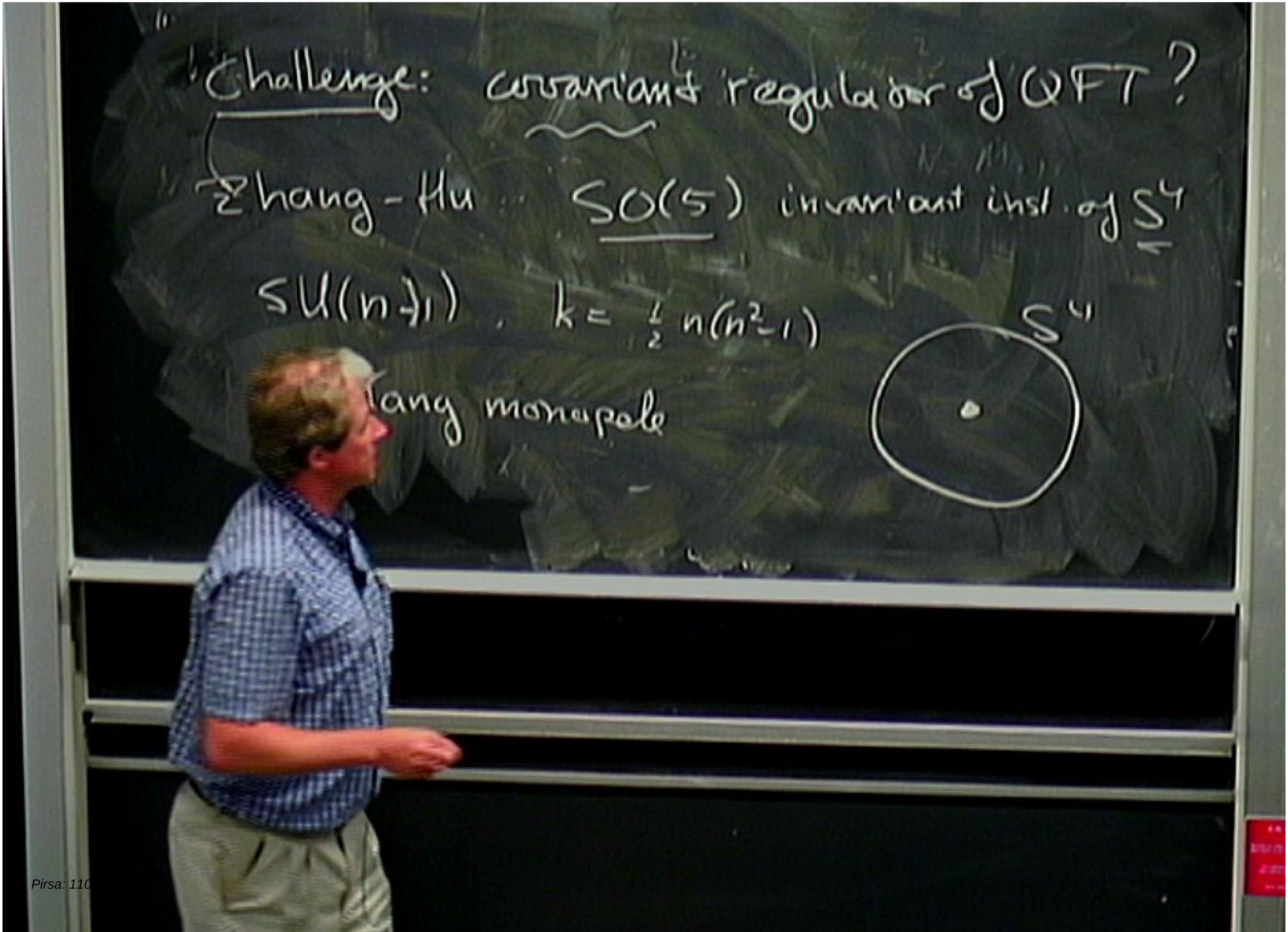

"Challenge: covariant regulator of QFT?
Zhang-Hu SO(5) invariant inst. of $S^4$
U(n)


Challenge: covariant regulator of QFT?
Zhang-Hu SO(5) invariant inst. of $S^4$
U(n) $k = \frac{1}{2} n(n^2-1)$

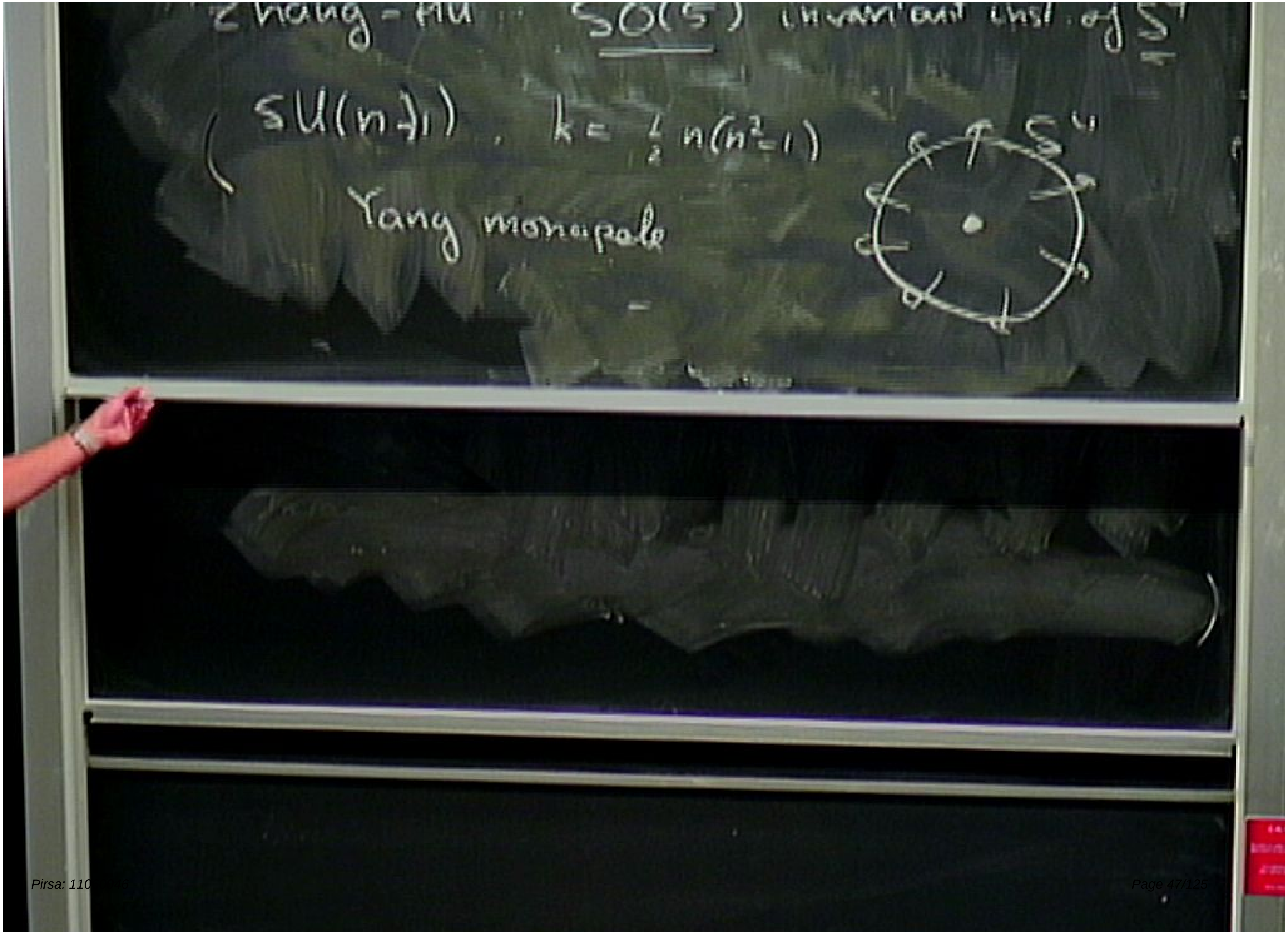
Challenge: covariant regulator of QFT?

Zhang-Hu   SO(5) invariant inst. of $S^4$

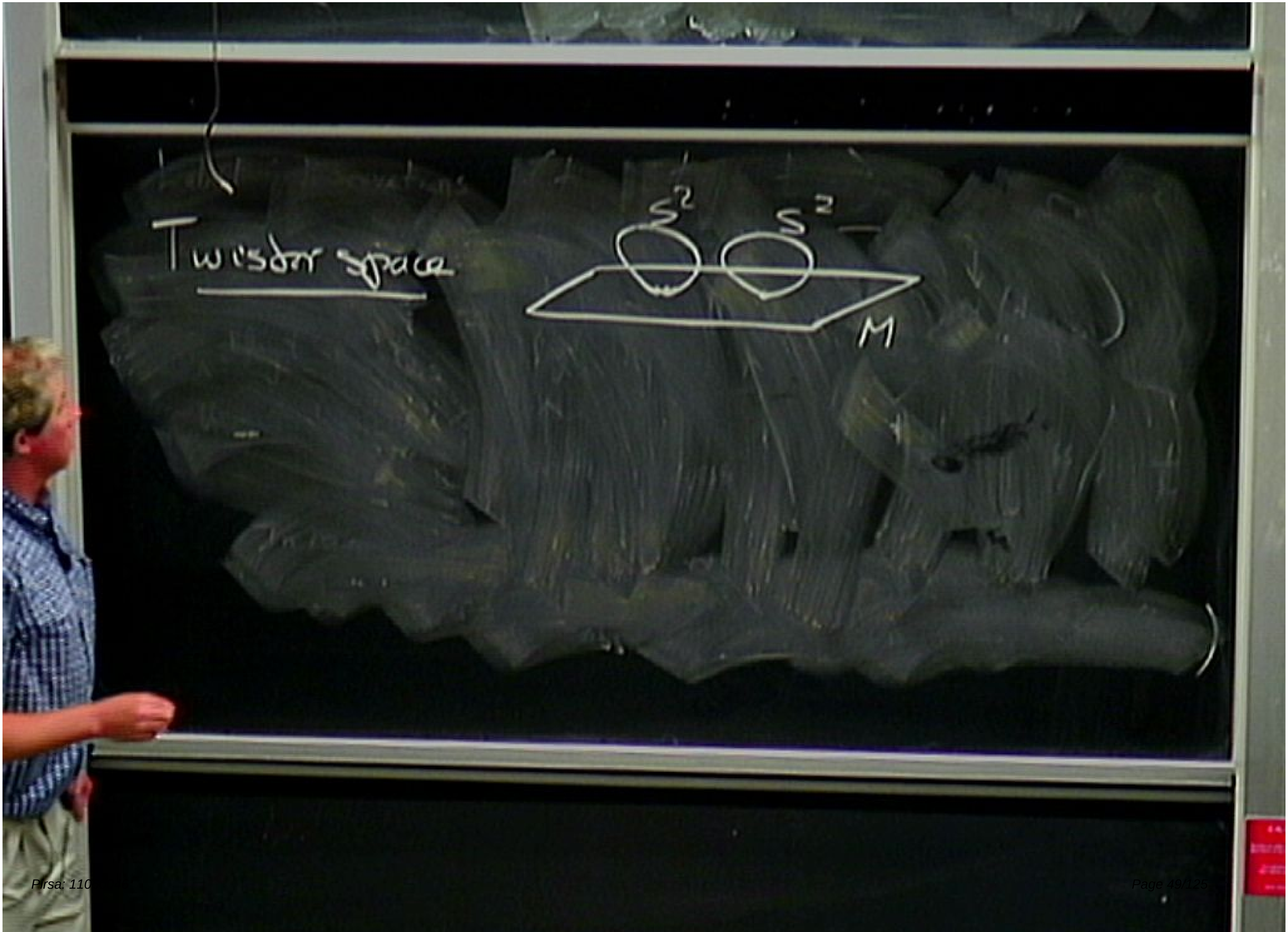
$U(n+1)$   $k = \frac{1}{2} n(n^2-1)$

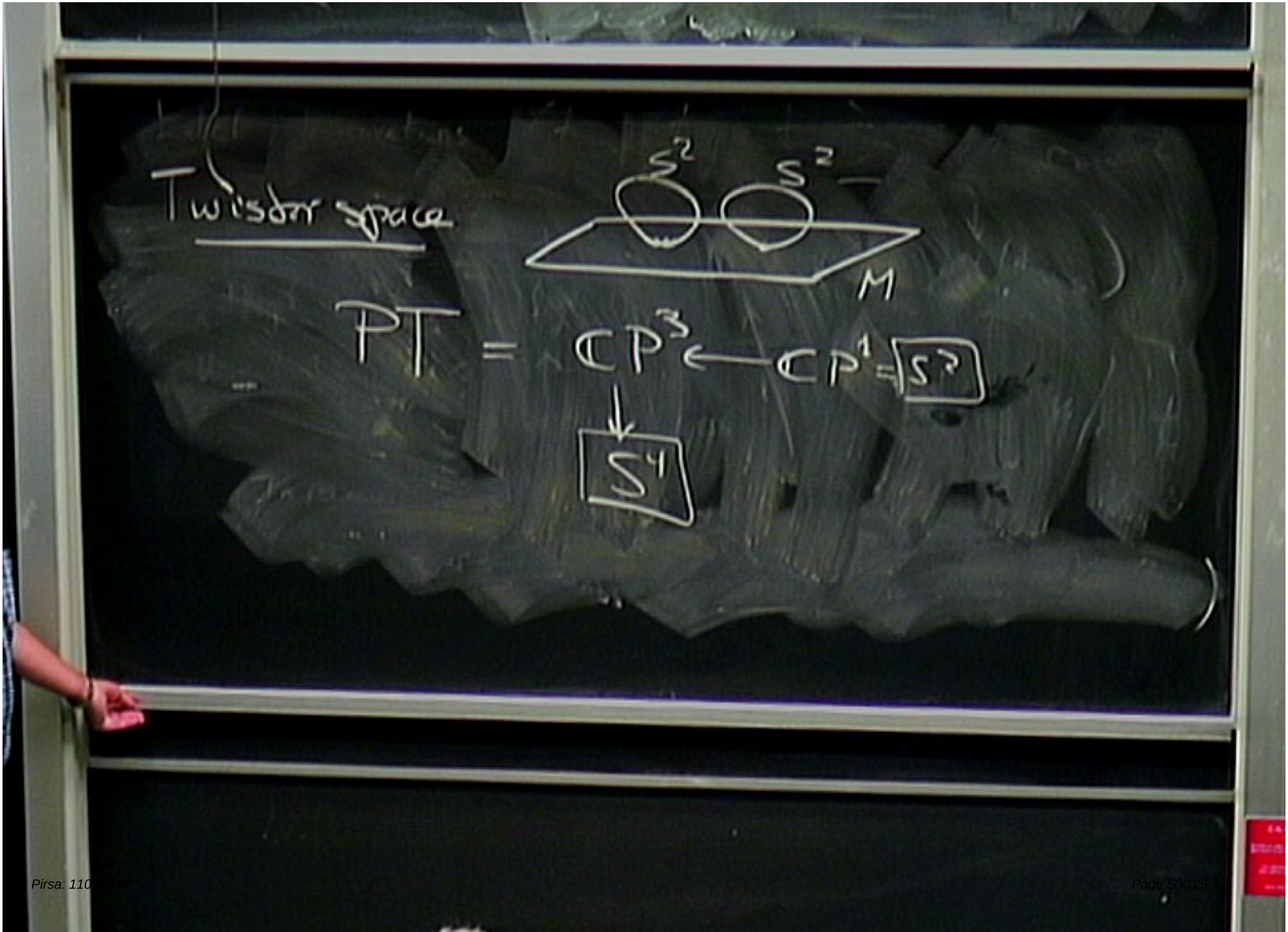


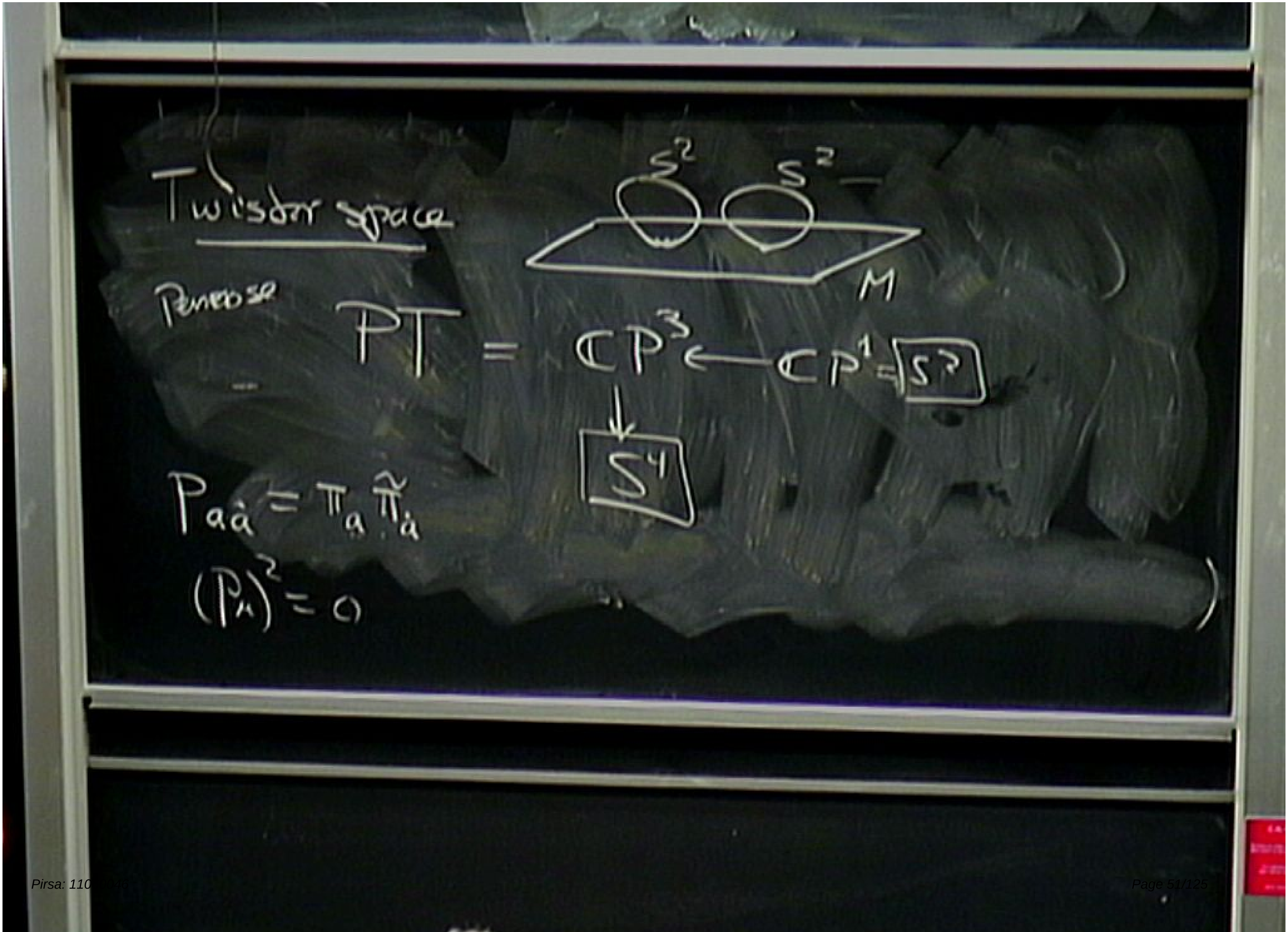
Challenge: covariant regulator of QFT ?
Zhang-Hu SO(5) invariant inst. of S4
SU(n+1), k = 1/2 n(n2-1)
ang monopole
S4

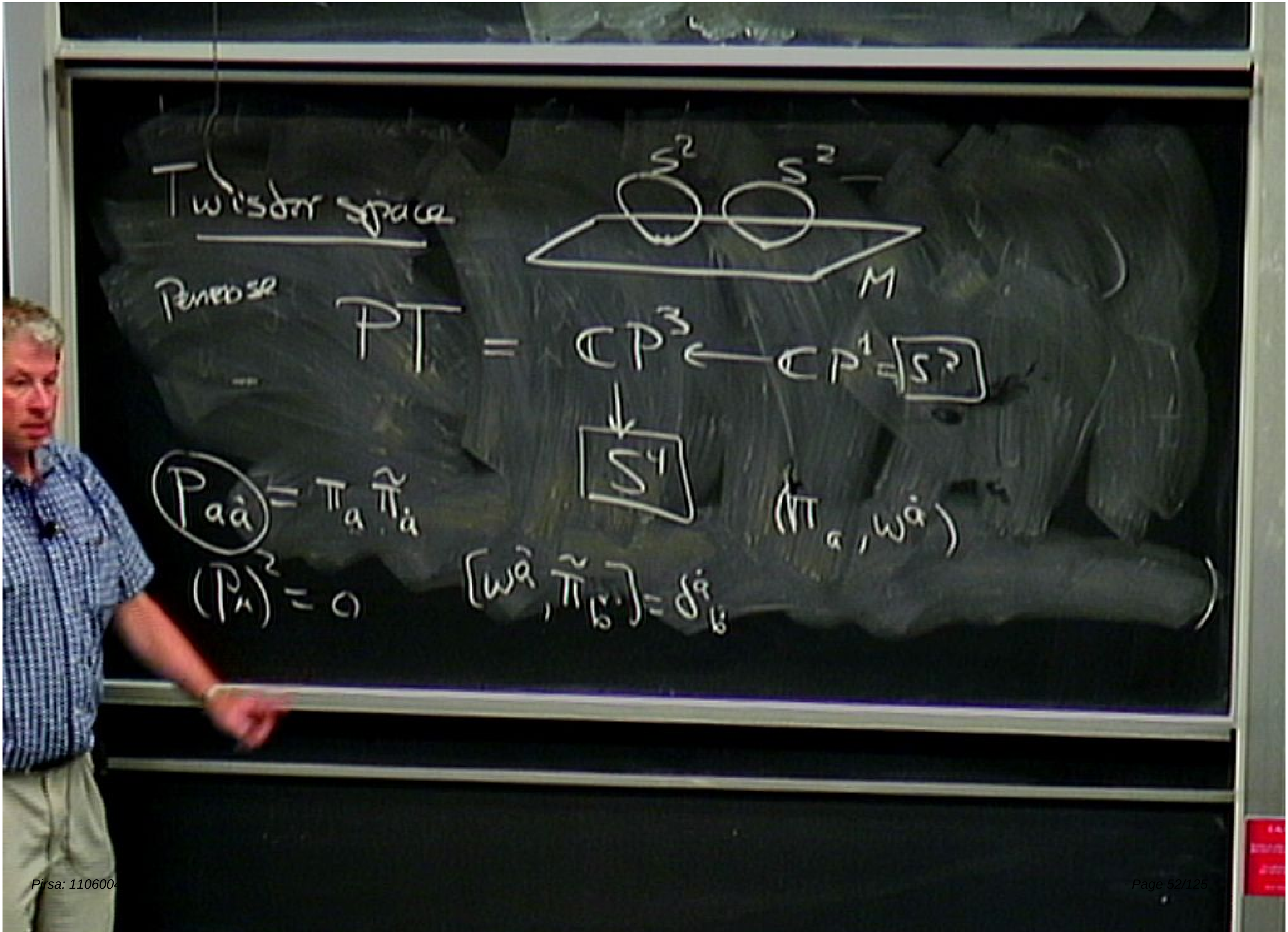

Zhang-Hu SO(5) invariant inst. of $S^4$
SU(n+1)
Yang monopole
$S^4$

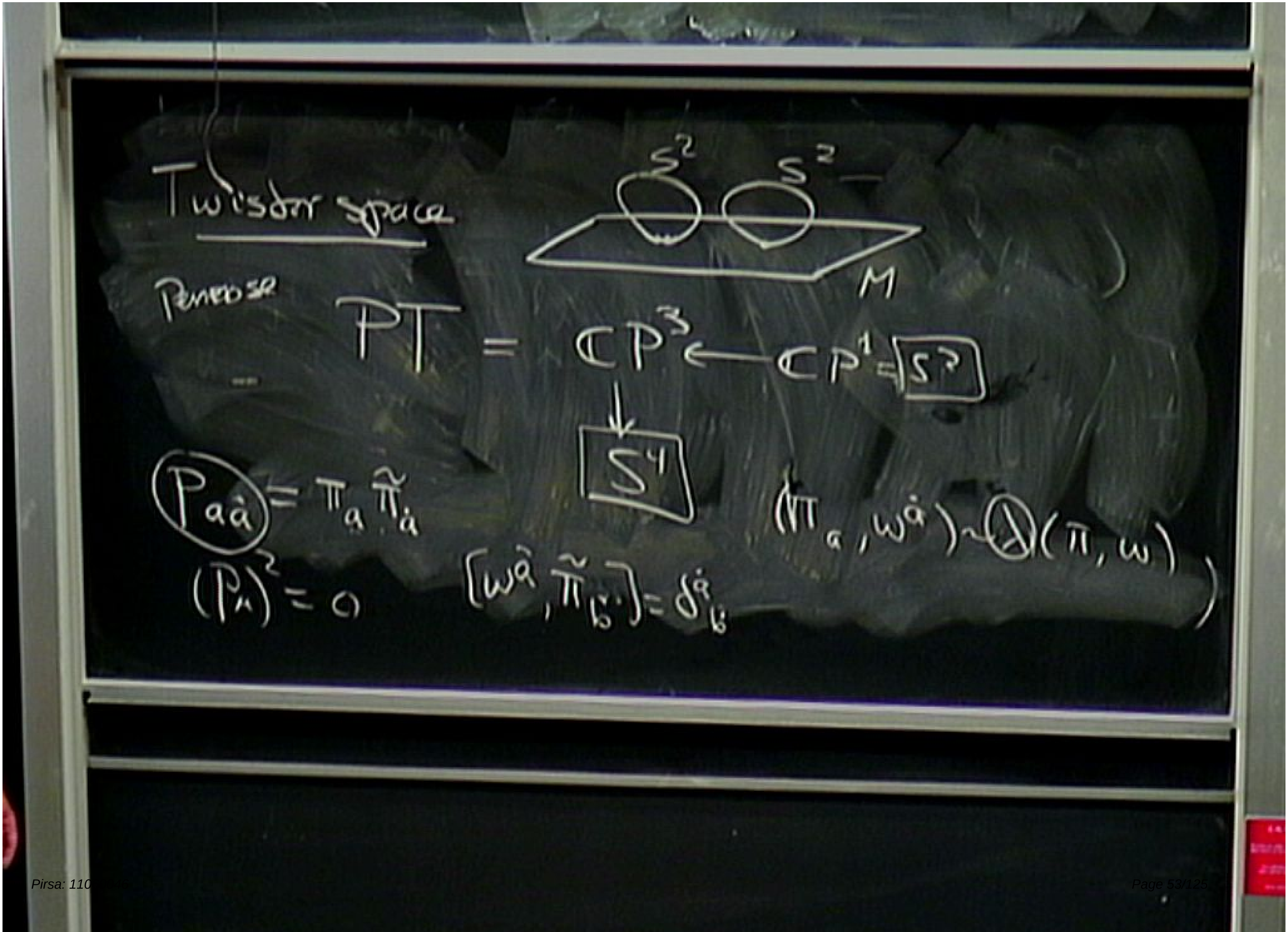

Twistor space

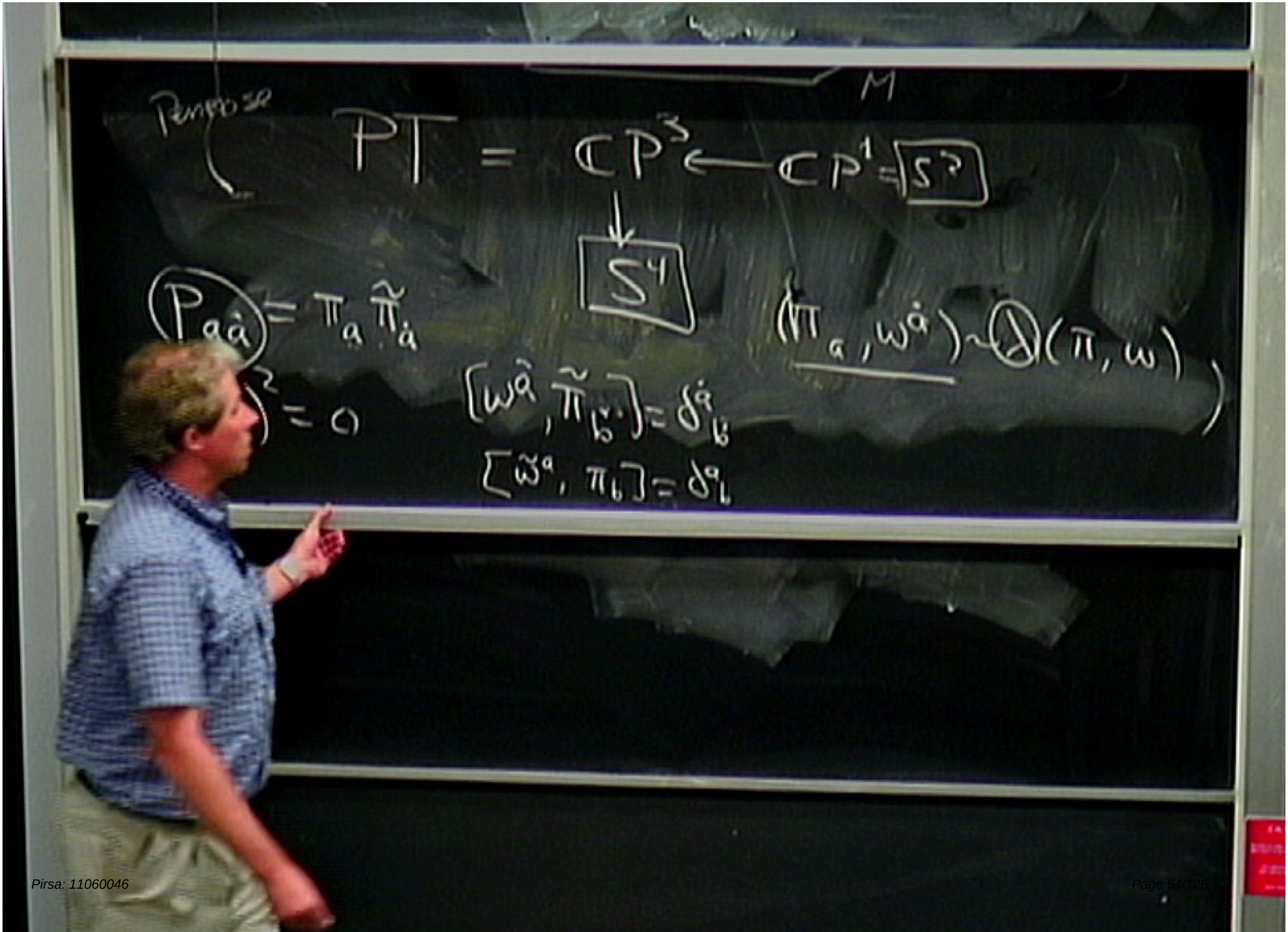

Twistor space
S2
S2
M

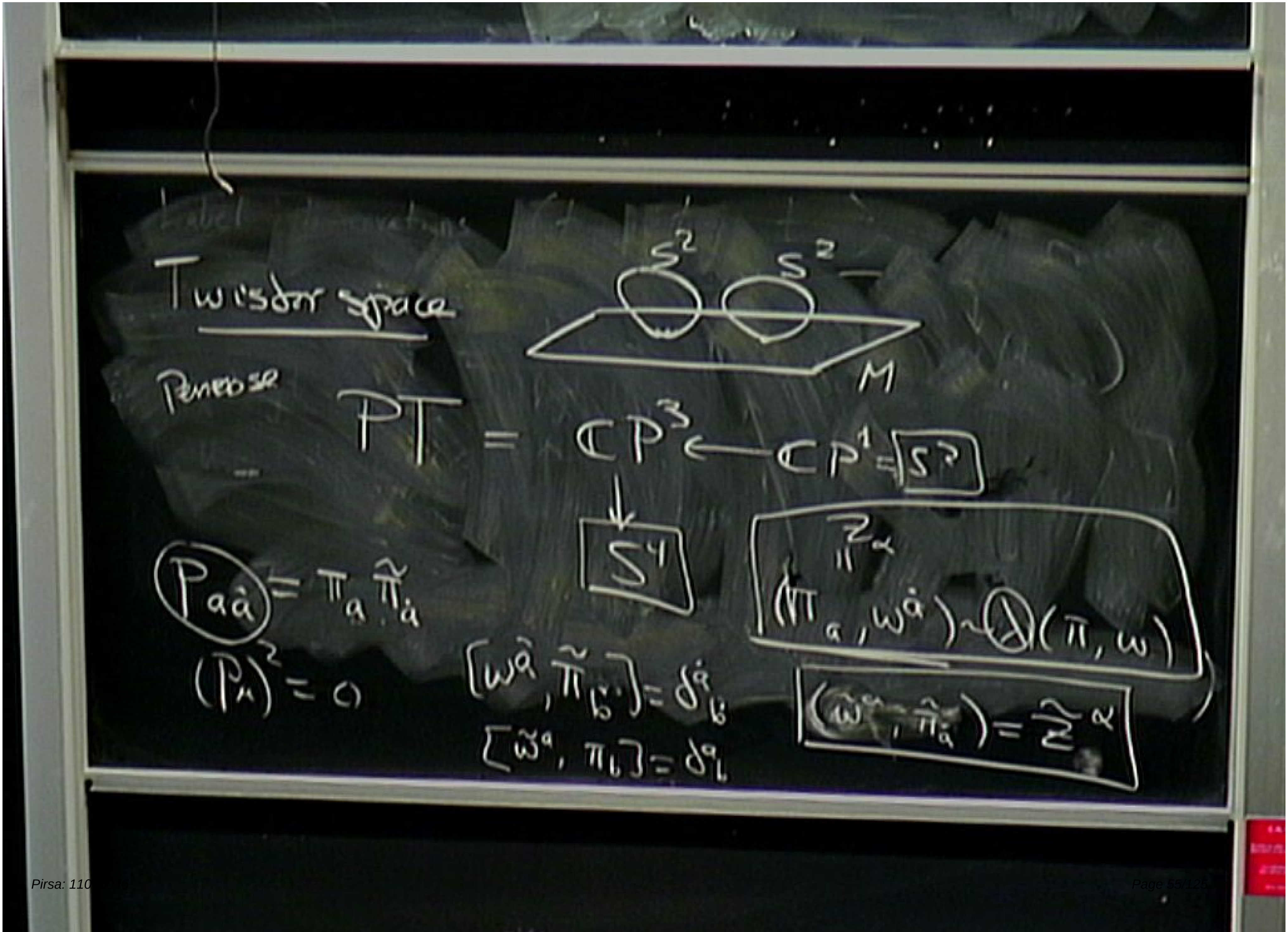

Twistor space
S²
S²
M
PT = CP³ ← CP¹ = S²
S⁴

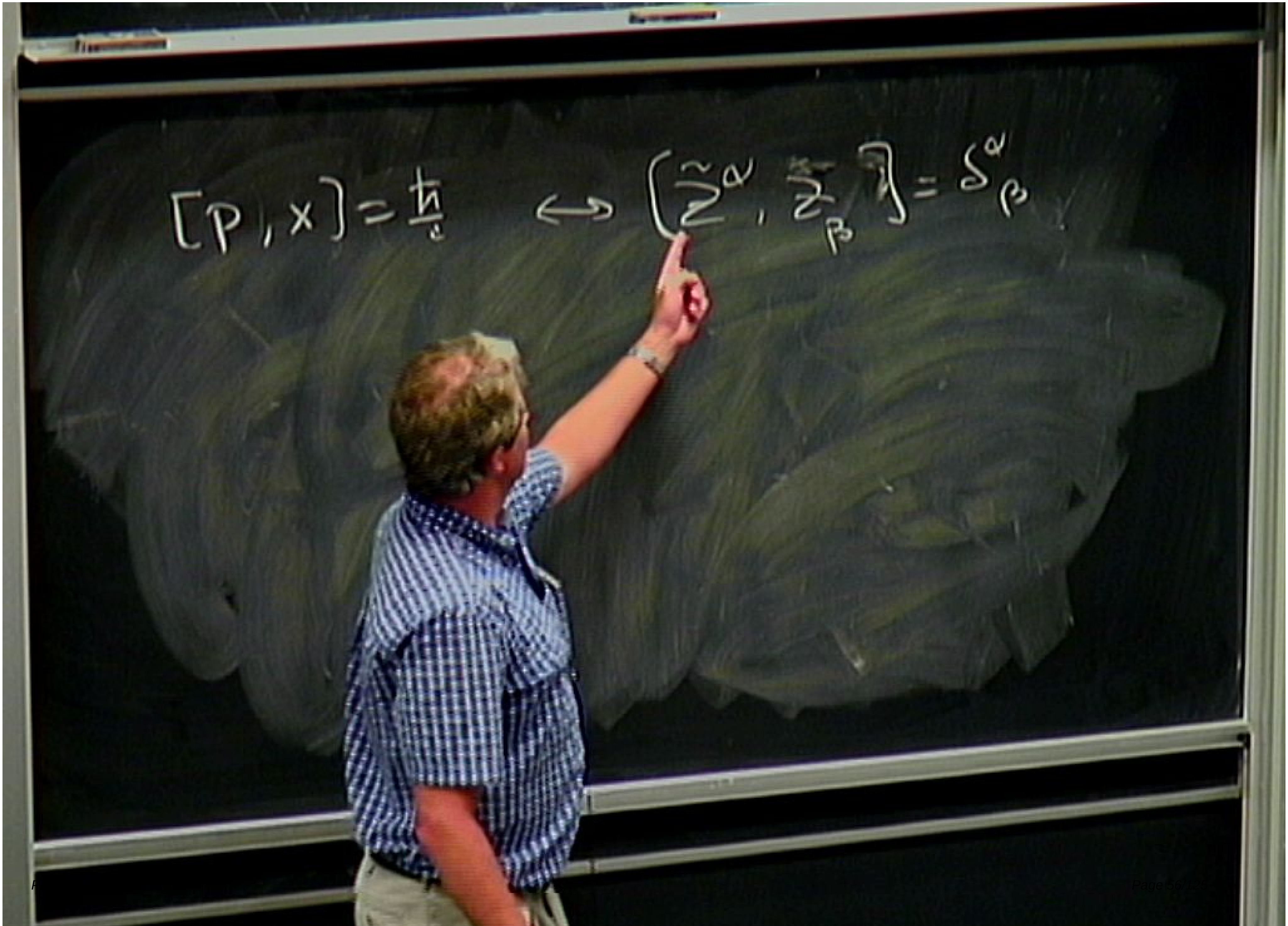

Twistor space
Penrose
$S^2$
$S^2$
$M$
$PT = \mathbb{C}P^3 \leftarrow \mathbb{C}P^1 = S^2$
$S^4$
$P_{a\dot{a}} = \pi_a \tilde{\pi}_{\dot{a}}$
$(P_\mu)^2 = 0$

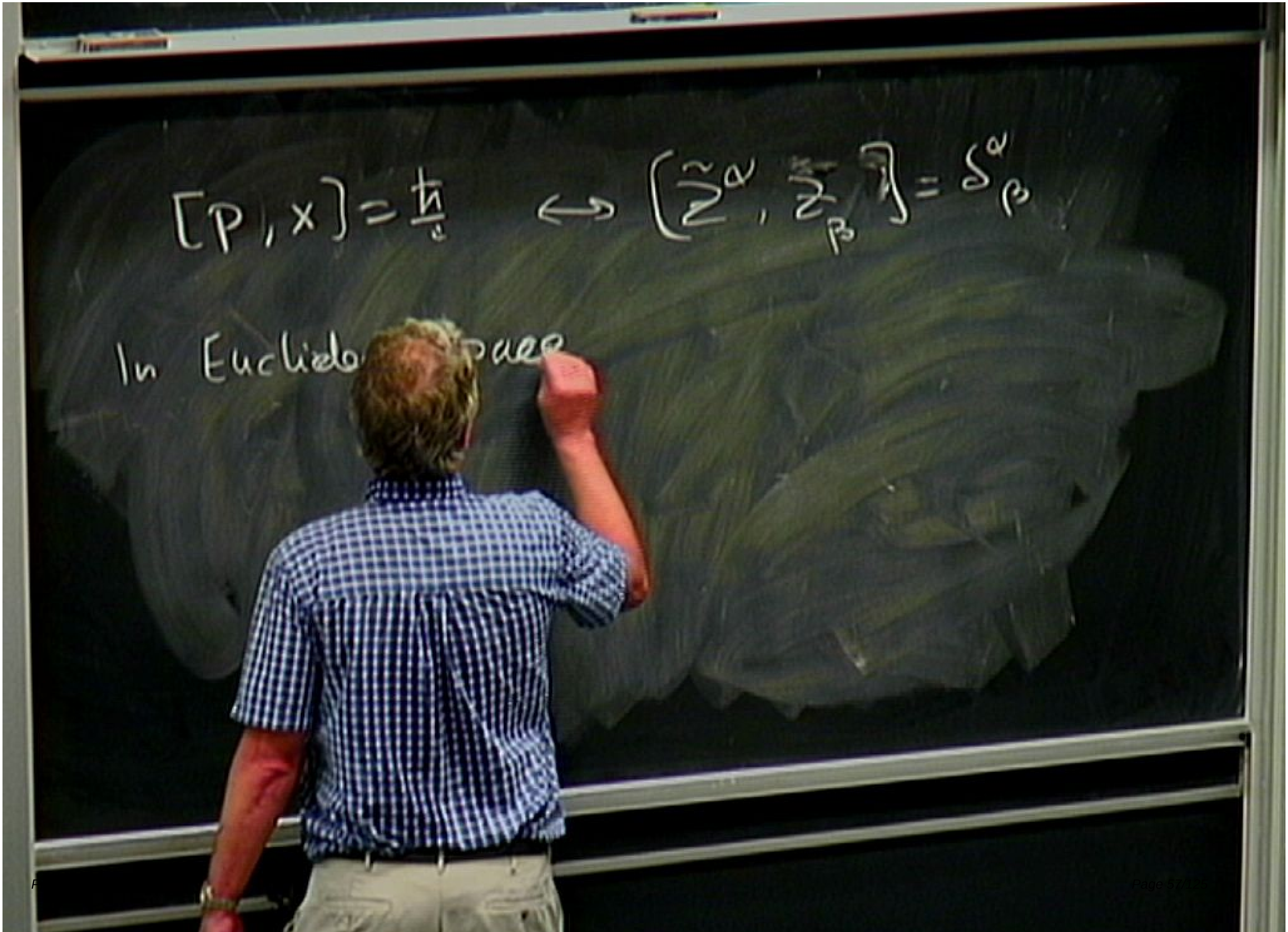

Twistor space
S^2
S^2
M
Penrose
PT = CP^3 ← CP^1 = S^2
S^4
P_{aȧ} = π_a π̃_ȧ
(π_a , w^ȧ)
(P_A)^2 = 0
[w^ȧ , π̃_ḃ] = δ^ȧ_ḃ

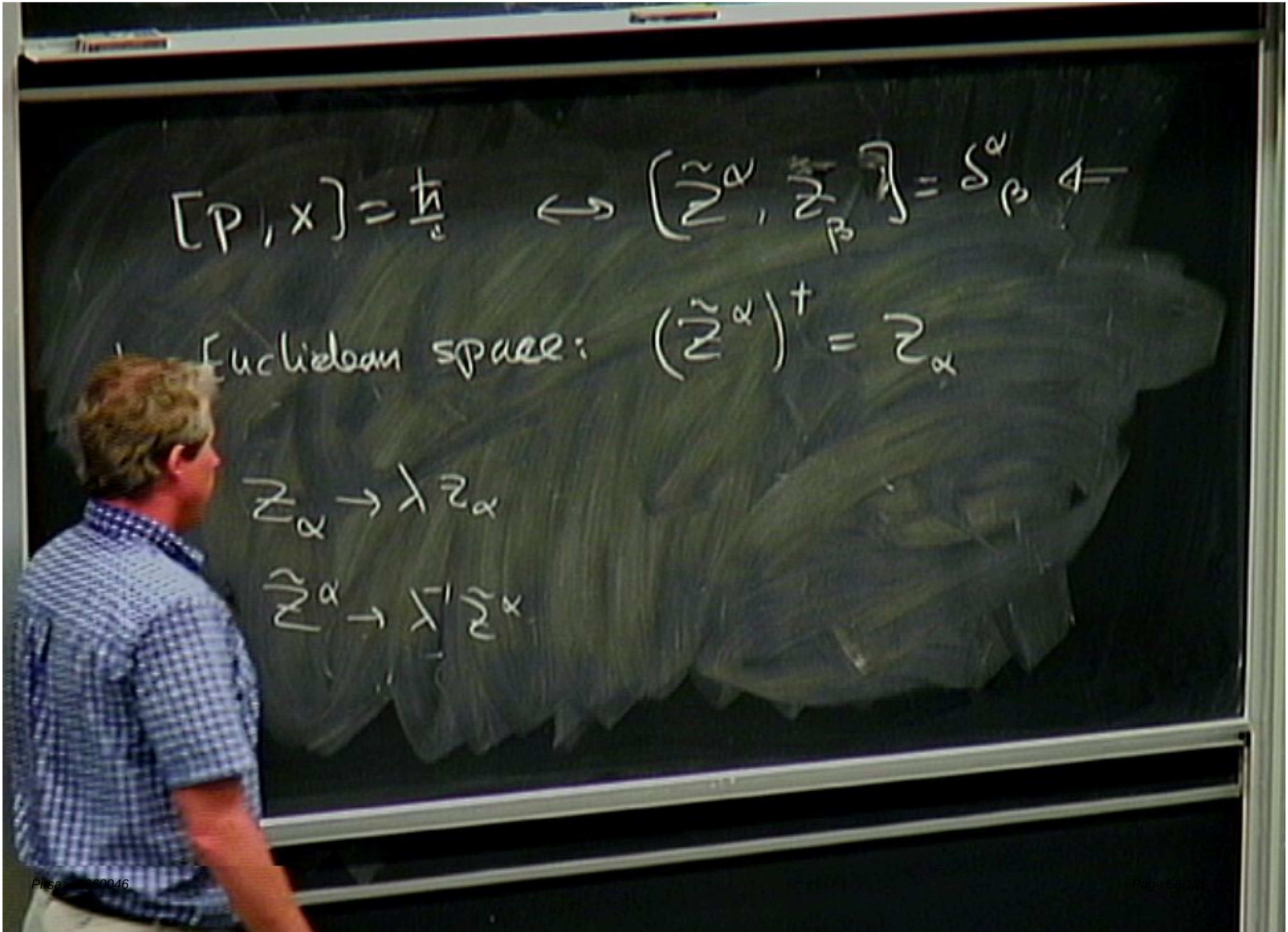

Twistor space
Penrose
S^2 S^2
M
PT = CP^3 ← CP^1 = S^2
S^4
P_{aȧ} = π_a π̃_ȧ
(P_μ)^2 = 0
[ω^ȧ, π̃_ḃ] = δ^ȧ_ḃ
(π_a, ω^ȧ) ~ λ(π, ω)


Penrose
M
PT = CP^3 ← CP^1 = S^2
S^4
P_{aȧ} = π_a π̃_ȧ
(π_a, ω^ȧ) ~ λ(π, ω)
[ω^ȧ, π̃_ḃ] = δ^ȧ_ḃ
[ω̃^a, π_b] = δ^a_b


Twistor space
Penrose
$S^2$ $S^2$
$M$
$PT = \mathbb{C}P^3 \leftarrow \mathbb{C}P^1 = S^2$
$S^4$
$Z^\alpha$
$(\pi_a, \omega^{\dot a}) \sim \lambda(\pi, \omega)$
$(\tilde\omega^a, \tilde\pi_{\dot a}) = \tilde Z^\alpha$
$P_{a\dot a} = \pi_a \tilde\pi_{\dot a}$
$(P_\mu)^2 = 0$
$[\omega^{\dot a}, \tilde\pi_{\dot b}] = \delta^{\dot a}_{\dot b}$
$[\tilde\omega^a, \pi_b] = \delta^a_b$


$[p, x] = \frac{\hbar}{i} \leftrightarrow [\tilde{z}^\alpha, \tilde{z}_\beta] = \delta^\alpha_\beta$


[p , x] = ħ/i ⟷ [z̃^α , z̃_β] = δ^α_β
In Euclid


$[p, x] = \frac{\hbar}{i} \longleftrightarrow [\tilde{Z}^\alpha, \bar{Z}_\beta] = \delta^\alpha_\beta$
Euclidean space: $(\tilde{Z}^\alpha)^\dagger = Z_\alpha$
$Z_\alpha \to \lambda Z_\alpha$
$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$

$$[p, x] = \frac{\hbar}{i} \quad \longleftrightarrow \quad [\tilde{Z}^\alpha, \tilde{Z}_\beta] = \delta^\alpha_\beta$$

In Euclidean space: $(\tilde{Z}^\alpha)^\dagger = Z_\alpha$

$Z_\alpha \to \lambda Z_\alpha$ conserved charge $Q = \tilde{Z}^\alpha Z_\alpha$

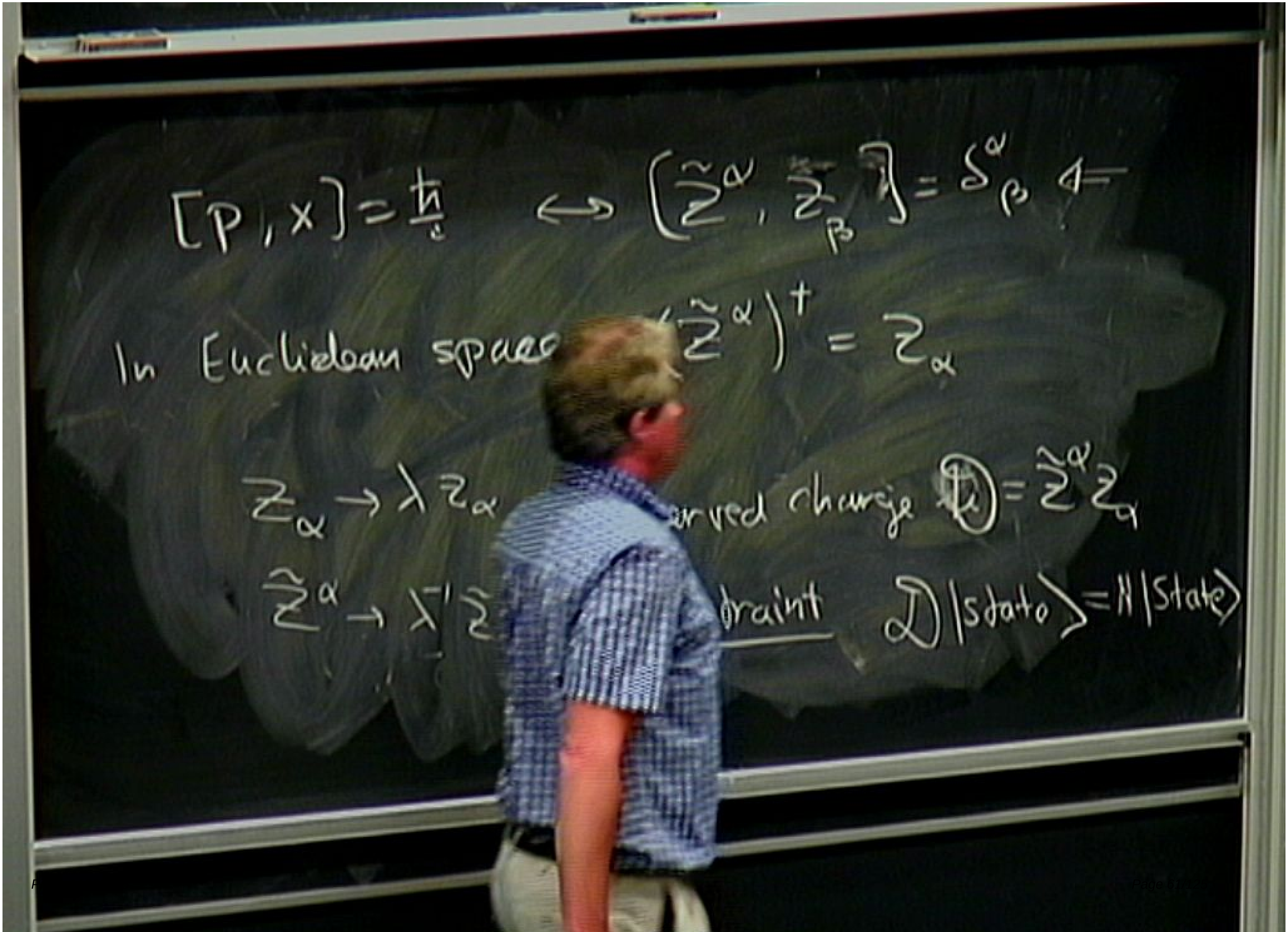
$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$

$$[p, x] = \frac{\hbar}{i} \quad \longleftrightarrow \quad [\tilde{Z}^\alpha, Z_\beta] = \delta^\alpha_\beta \;\leftarrow$$

In Euclidean space: $(\tilde{Z}^\alpha)^\dagger = Z_\alpha$

$Z_\alpha \to \lambda Z_\alpha$ conserved charge $Q = \tilde{Z}^\alpha Z_\alpha$

$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$ constraint

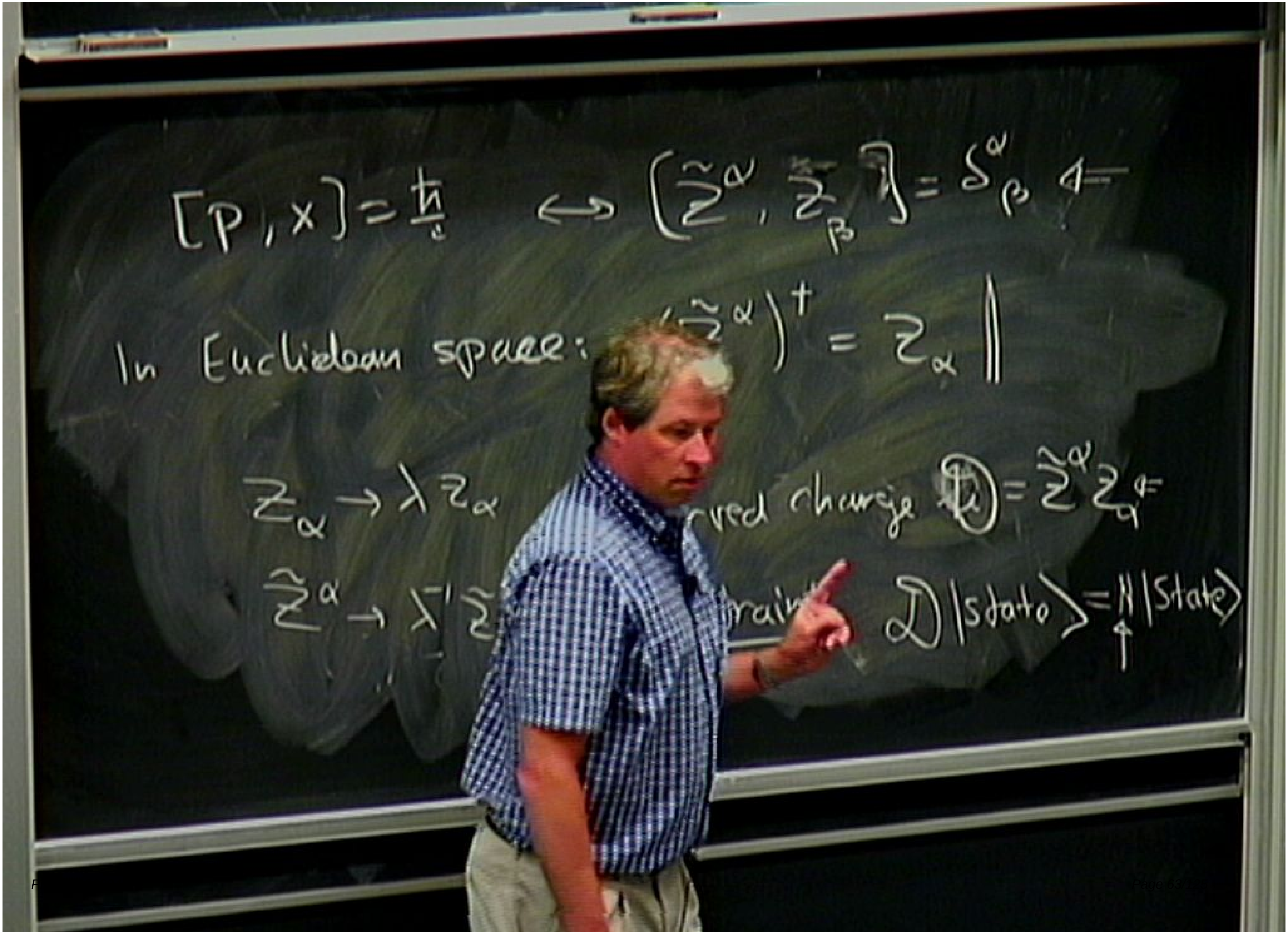

[p, x] = ħ/i ⟷ [Z̃^α, Z_β] = δ^α_β
In Euclidean space (Z̃^α)† = Z_α
Z_α → λ Z_α
Z̃^α → λ⁻¹ Z̃
rved charge D = Z̃^α Z_α
traint
D |state⟩ = N |State⟩

$$[p, x] = \frac{\hbar}{i} \quad \longleftrightarrow \quad [\tilde{Z}^\alpha, Z_\beta] = \delta^\alpha_\beta$$

In Euclidean space: $(\tilde{Z}^\alpha)^\dagger = Z_\alpha$

$Z_\alpha \to \lambda Z_\alpha$ conserved charge $D = \tilde{Z}^\alpha Z_\alpha$

$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$ Constraint $D|\text{state}\rangle = N|\text{state}\rangle$


[p, x] = ħ/i
In Euclidean space:
charge
|state⟩ = N |state⟩

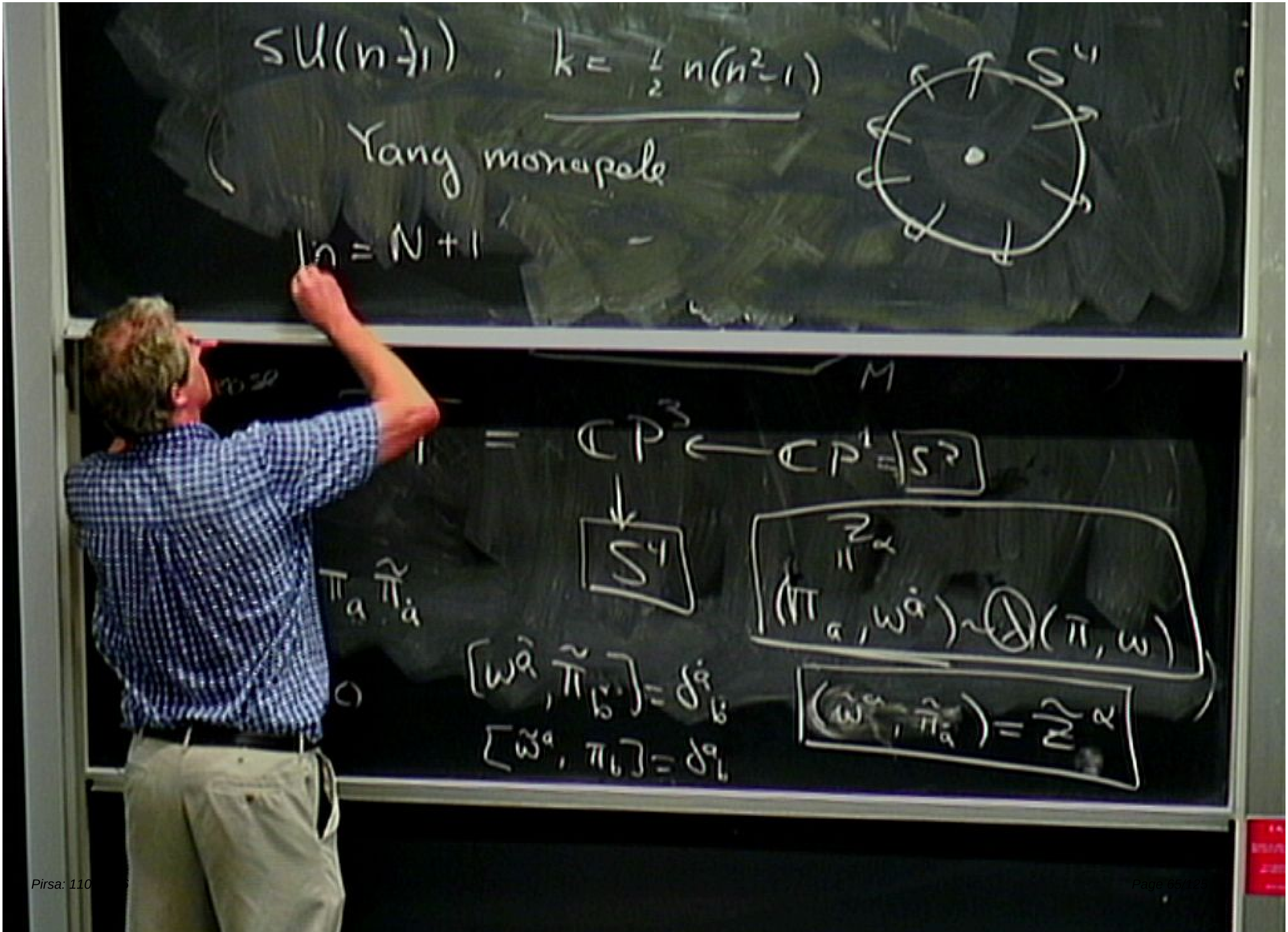
$$[p, x] = \frac{\hbar}{i} \quad \longleftrightarrow \quad [\tilde{Z}^\alpha, Z_\beta] = \delta^\alpha_\beta \leftarrow$$

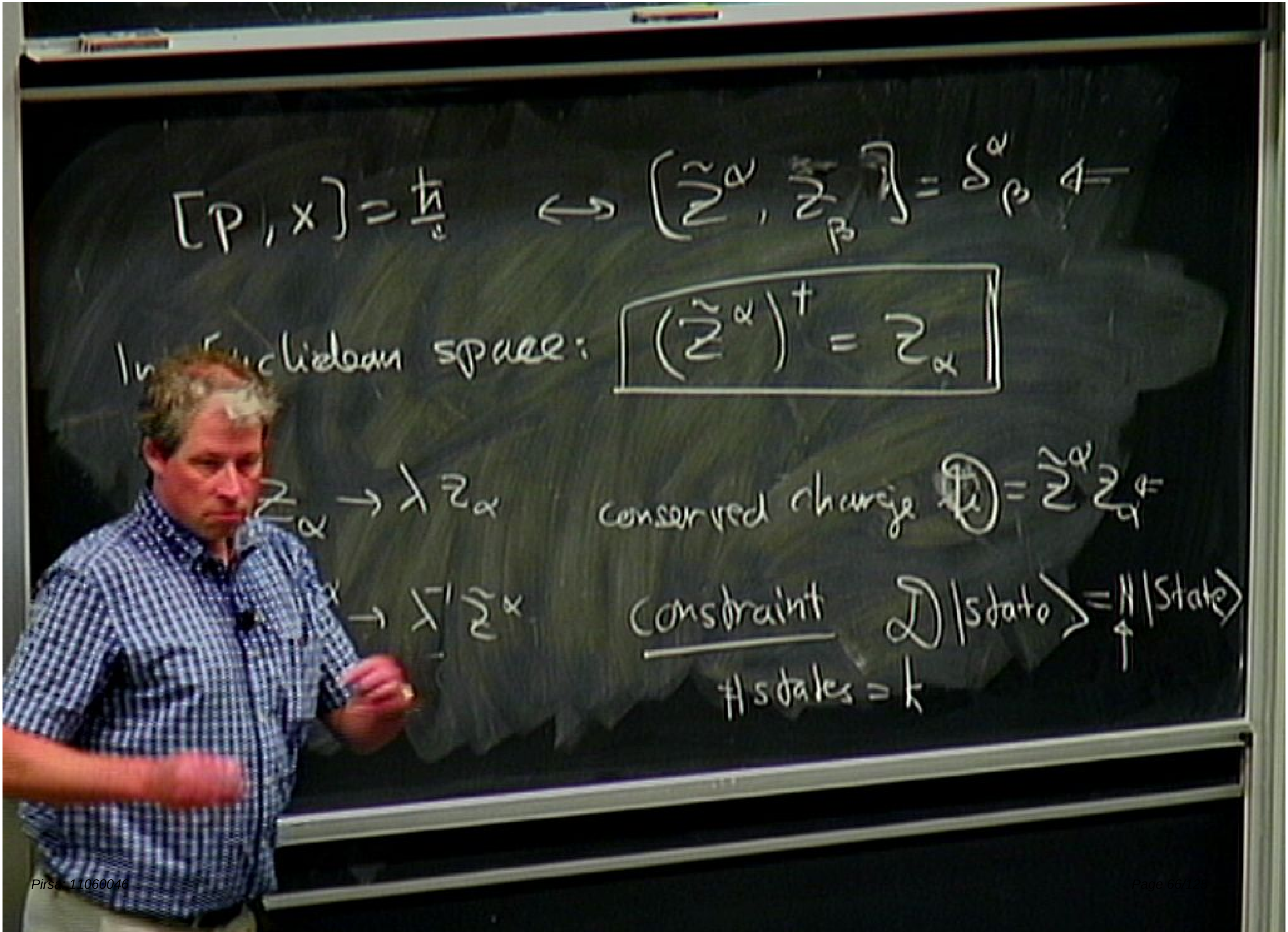
In Euclidean space: $(\tilde{Z}^\alpha)^\dagger = Z_\alpha$

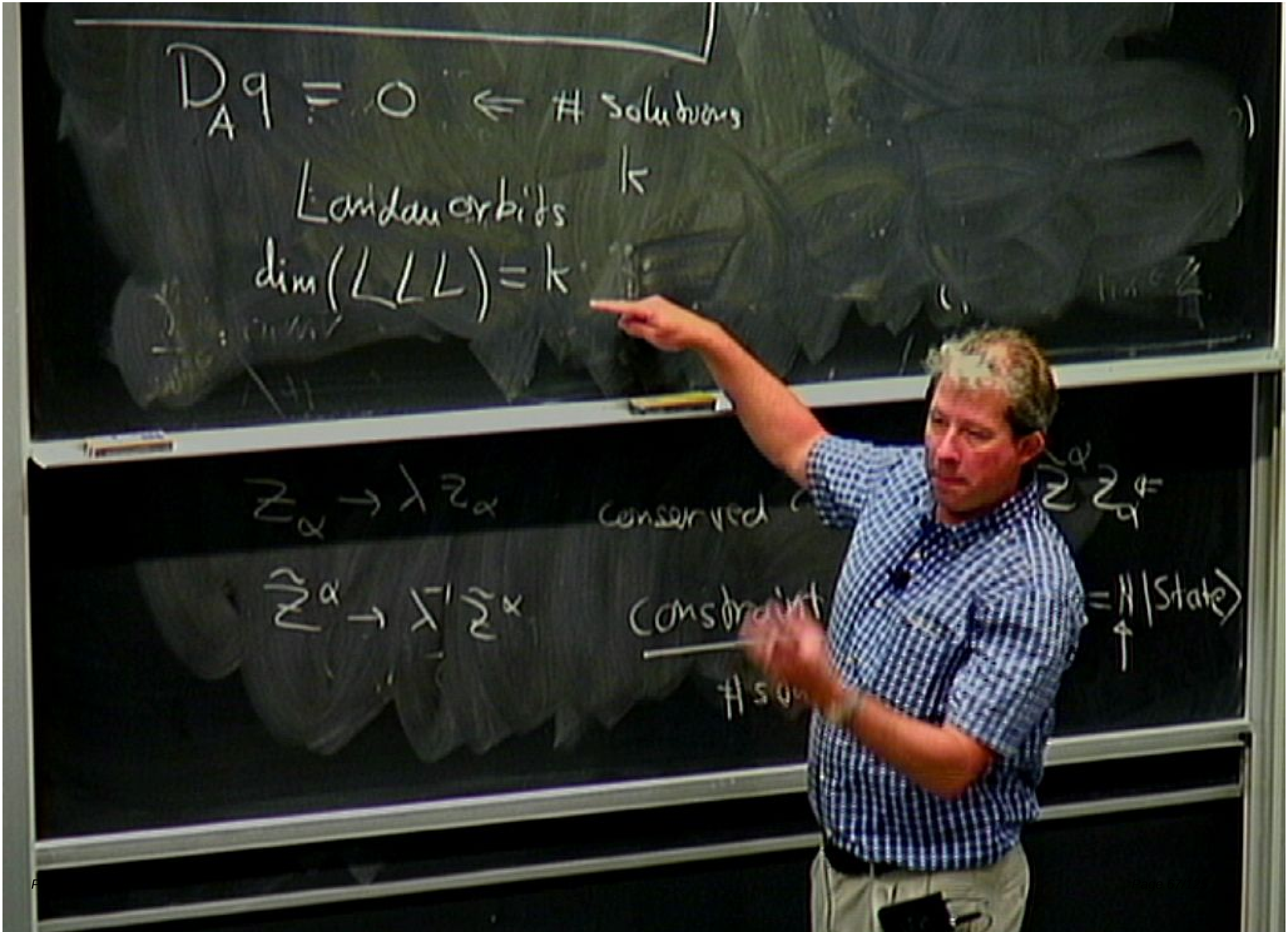
$Z_\alpha \to \lambda Z_\alpha$ conserved charge $\mathcal{D} = \tilde{Z}^\alpha Z_\alpha \leftarrow$

$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$ Constraint $\mathcal{D}|\text{state}\rangle = N|\text{state}\rangle$

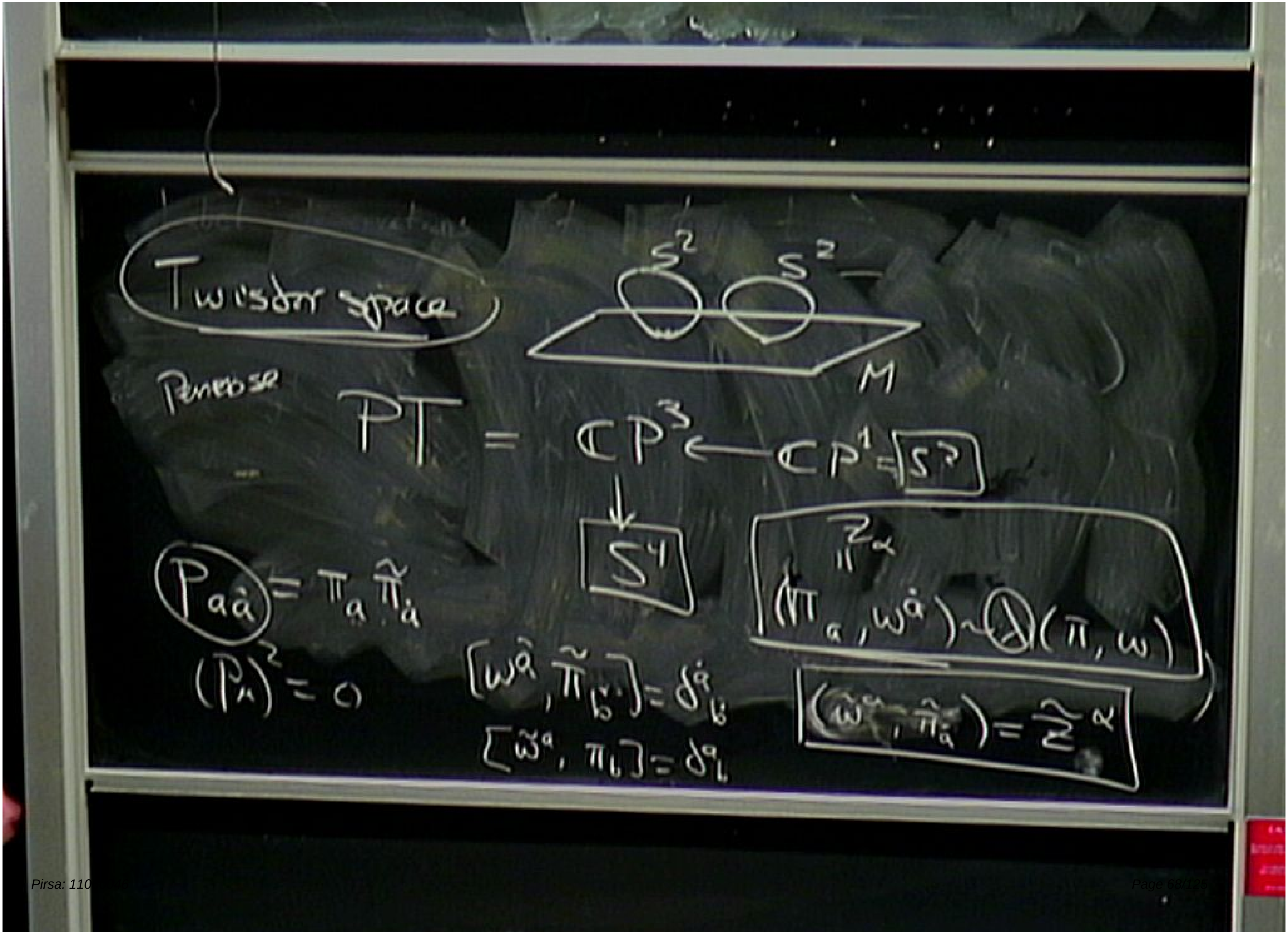
# states = $k$

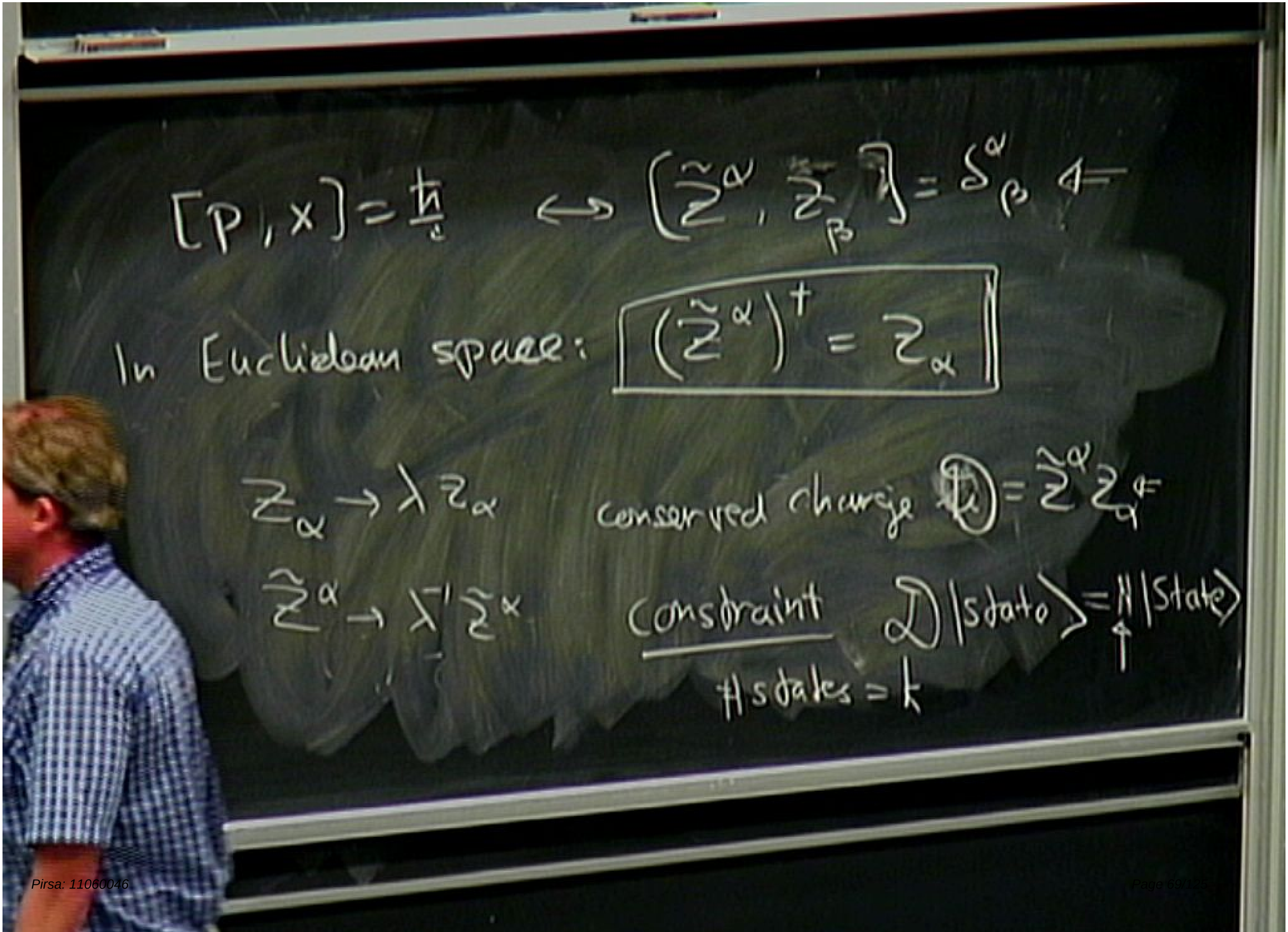

SU(n+1)
k = ½ n(n²-1)
Yang monopole
n = N+1
S⁴
M
CP³ ← CP¹ = S²
S⁴
[ω, π̃] = δ
[ω̃, π] = δ


[p, x] = ħ/i ⟷ [Z̃^α, Z_β] = δ^α_β
space: (Z̃^α)† = Z_α
Z_α → λ Z_α
conserved charge = Z̃^α Z_α
→ λ^-1 Z̃^α
Constraint
|state⟩ = N |state⟩
# states = k

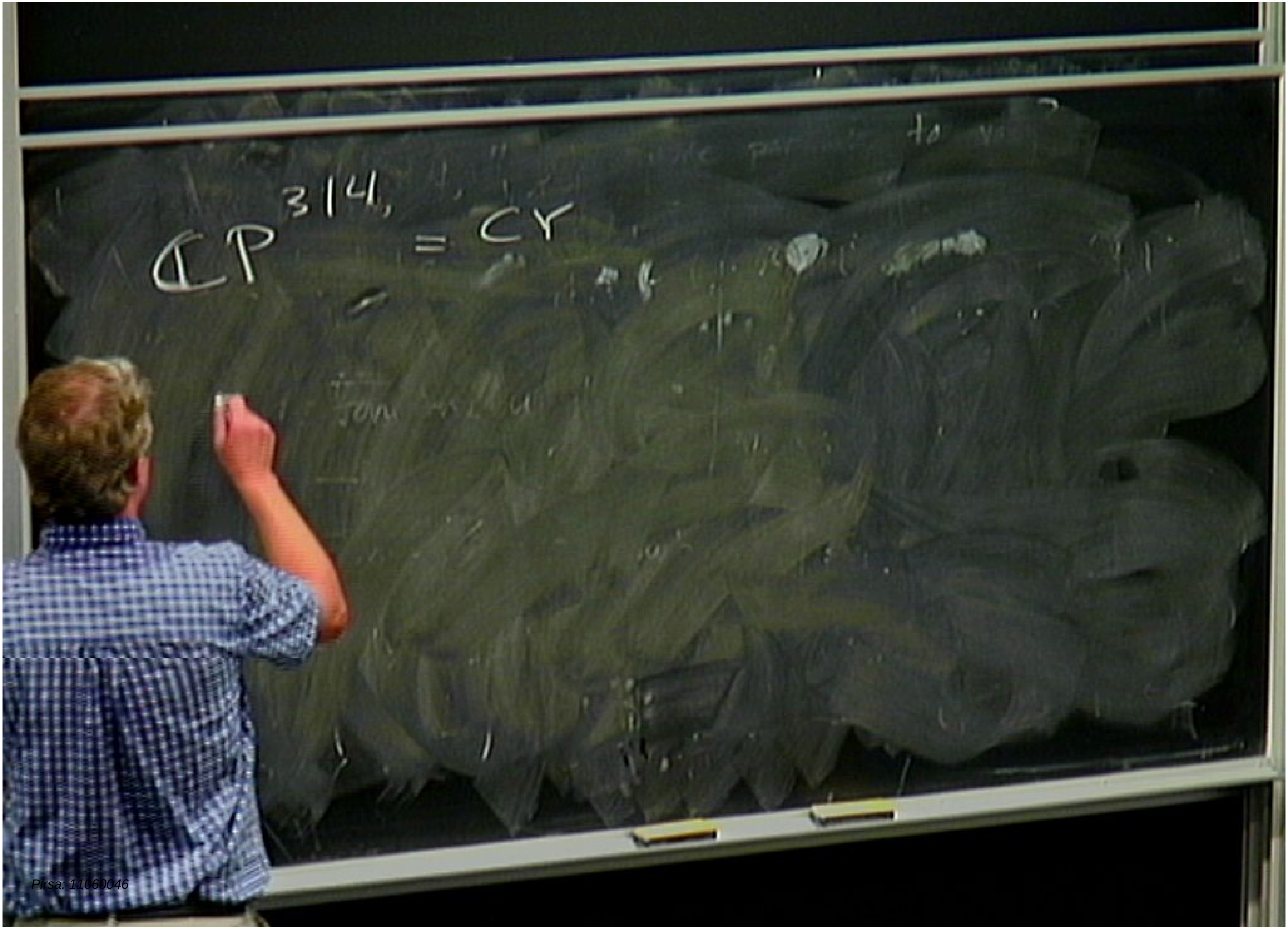


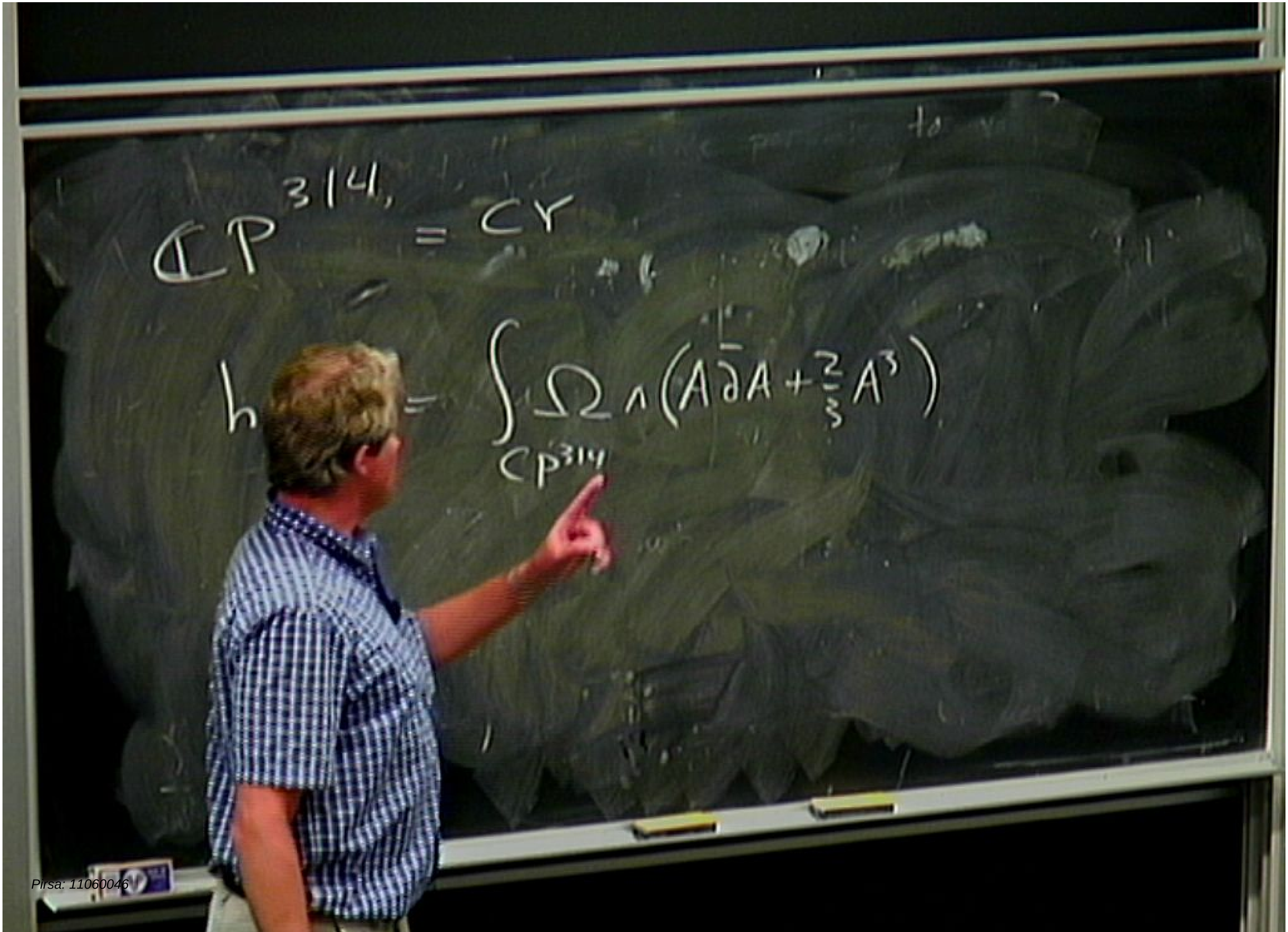
D_A q = 0 ⇐ # solutions
k
Landau orbits
dim(LLL) = k
z_α → λ z_α
conserved
z̃^α → λ^{-1} z̃^α
constraint
= N |state⟩


Twistor space
Penrose
S^2 S^2
M
PT = CP^3 ← CP^1 = S^2
S^4
Z_α
(π_a, ω^ȧ) ~ λ(π, ω)
P_{aȧ} = π_a π̃_ȧ
(P_A)^2 = 0
[ω^ȧ, π̃_ḃ] = δ^ȧ_ḃ
[ω̃^a, π_b] = δ^a_b
(ω̃^a, π̃_ȧ) = Z̃^α


[p, x] = ħ/i ⟷ [Z̃^α, Z_β] = δ^α_β
In Euclidean space: (Z̃^α)† = Z_α
Z_α → λ Z_α
Z̃^α → λ⁻¹ Z̃^α
conserved charge D = Z̃^α Z_α
Constraint D |state⟩ = N |state⟩
# states = k

$\mathbb{CP}^{3|4}$,

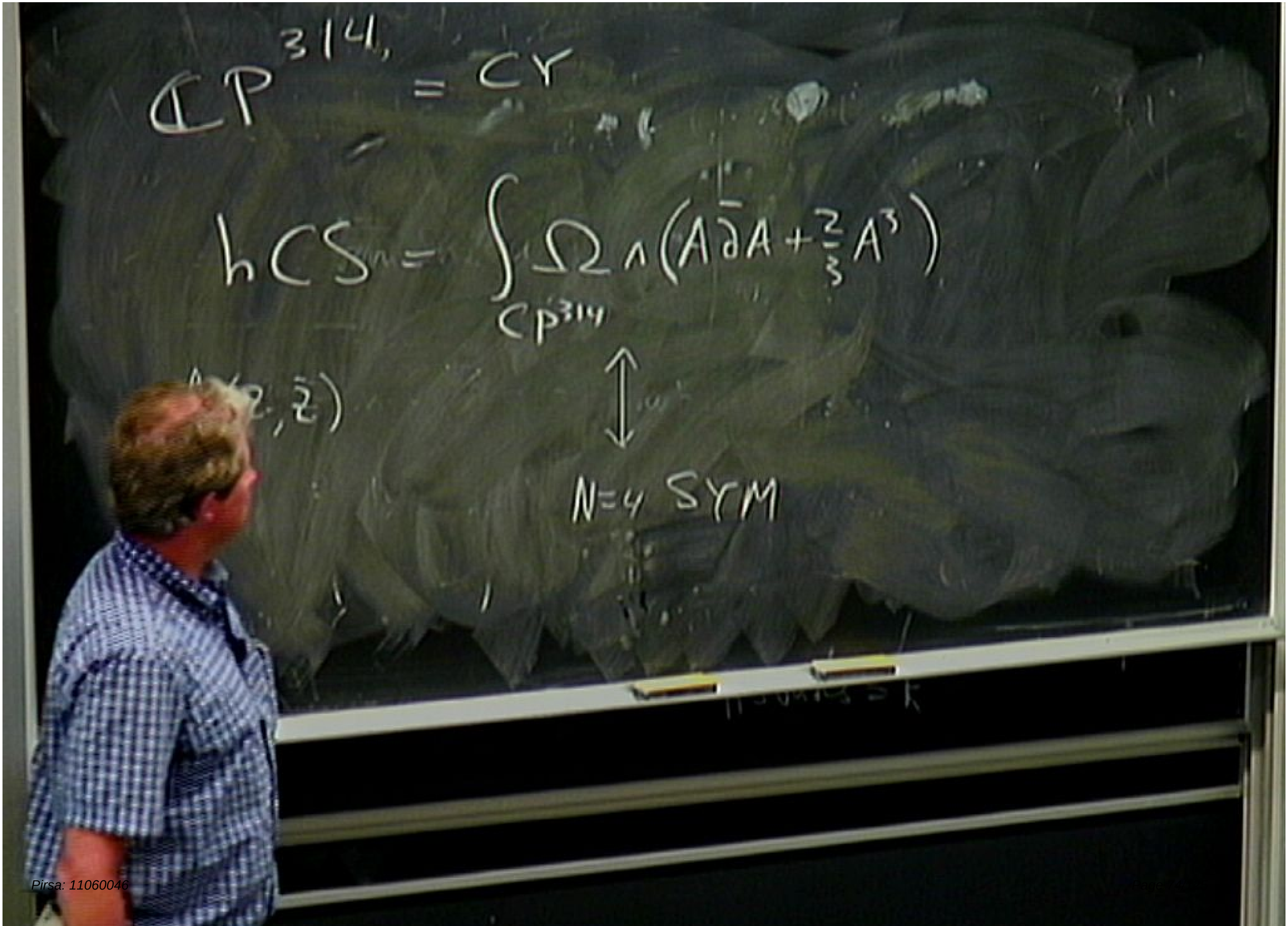

ℂP3|4, = CY

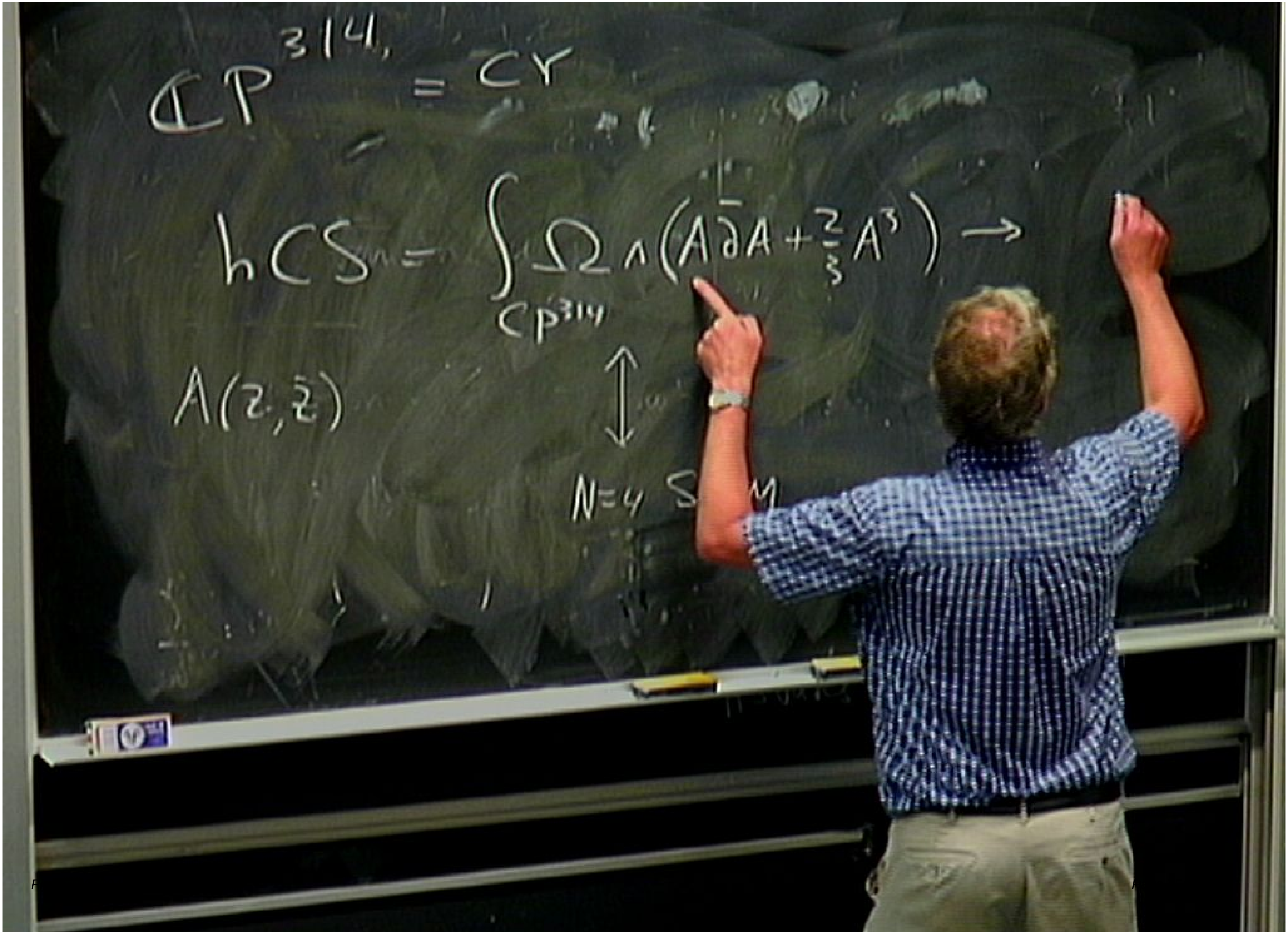

$\mathbb{C}P^{3|4} = CY$
$\int_{CP^{3|4}} \Omega \wedge (A \partial A + \frac{2}{3} A^3)$

$$\mathbb{CP}^{3|4} = CY$$

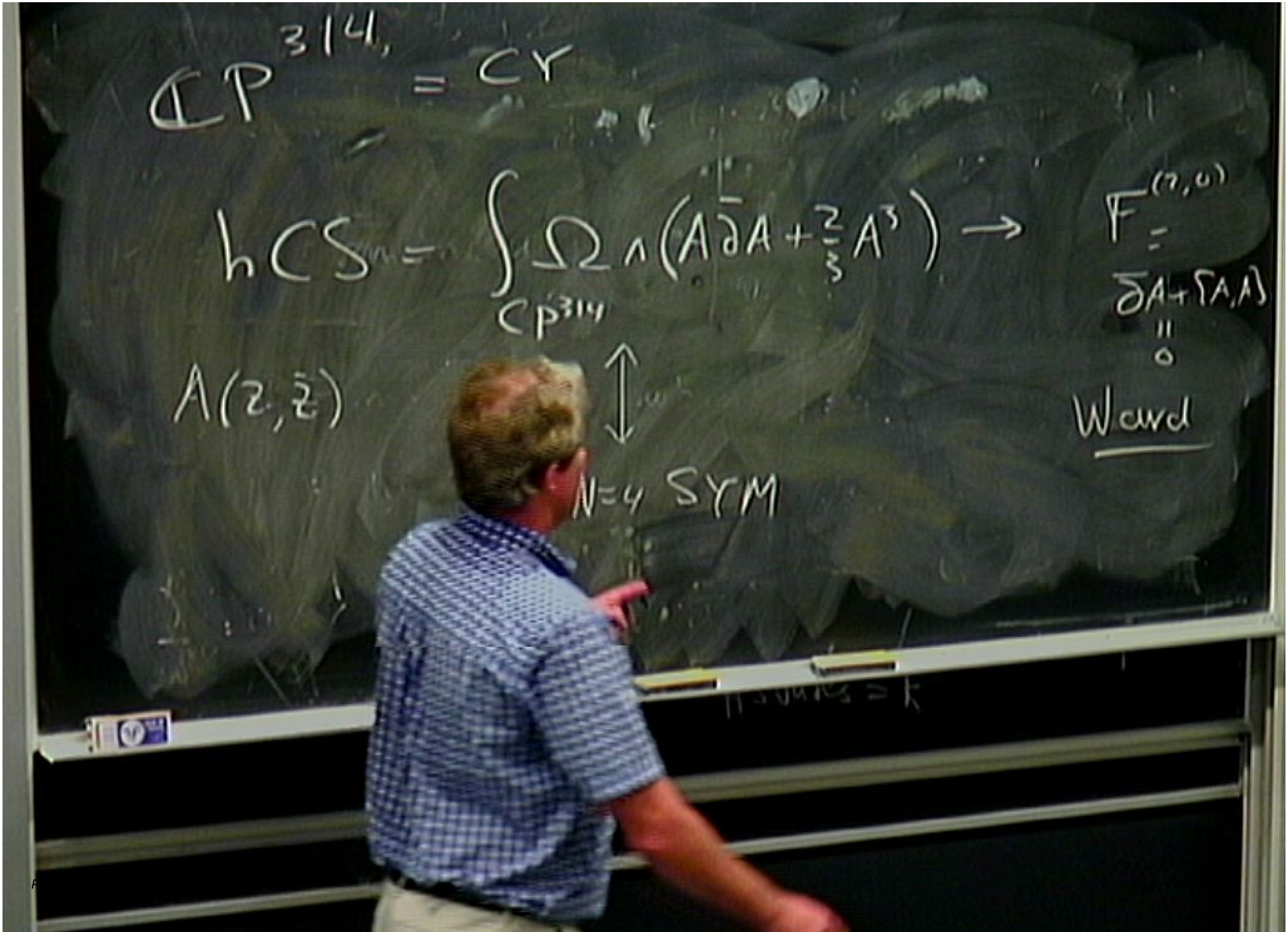
$$hCS = \int_{CP^{3|4}} \Omega \wedge \left( A \bar{\partial} A + \frac{2}{3} A^3 \right)$$

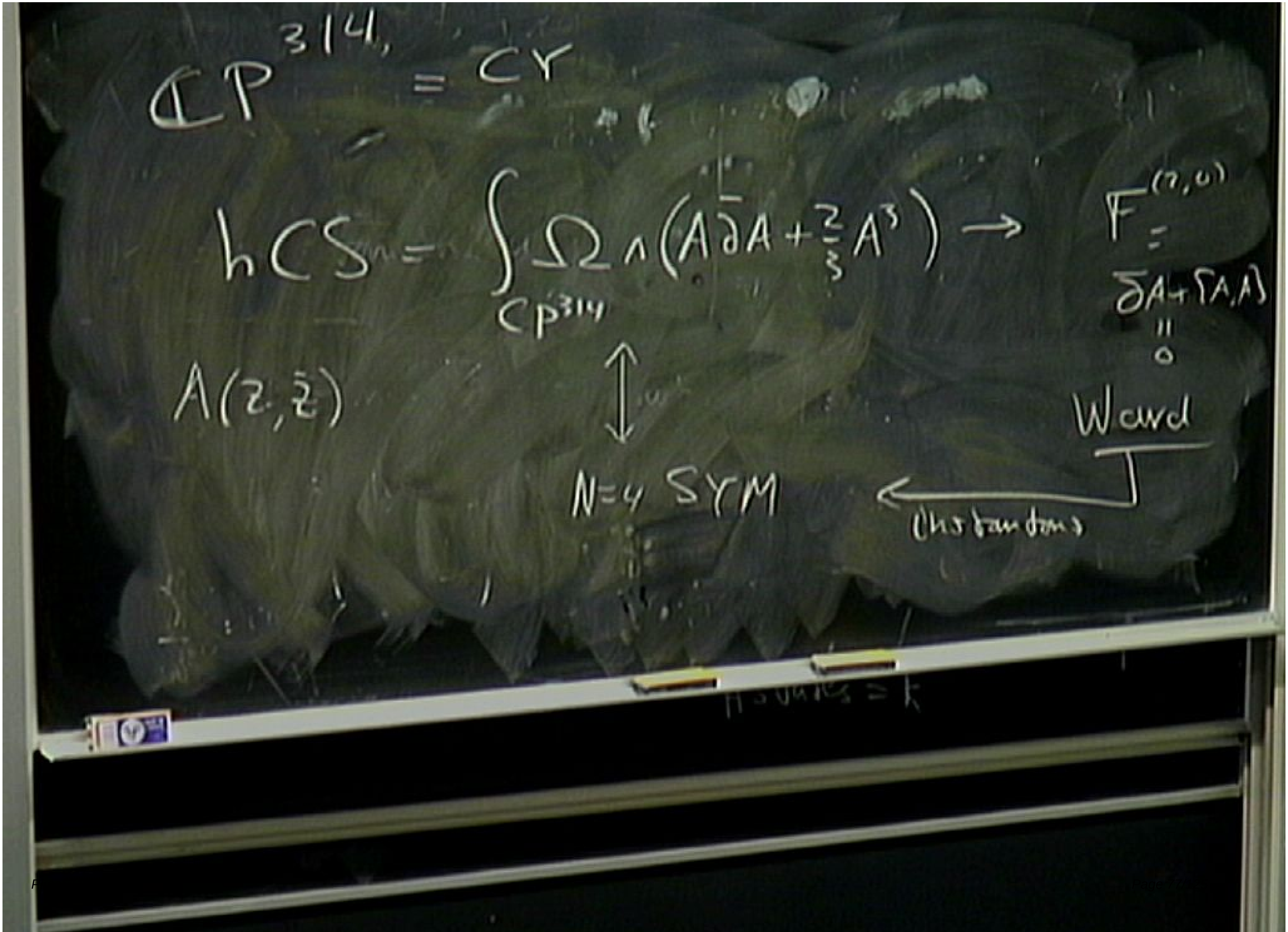
$$A(z, \bar{z})$$

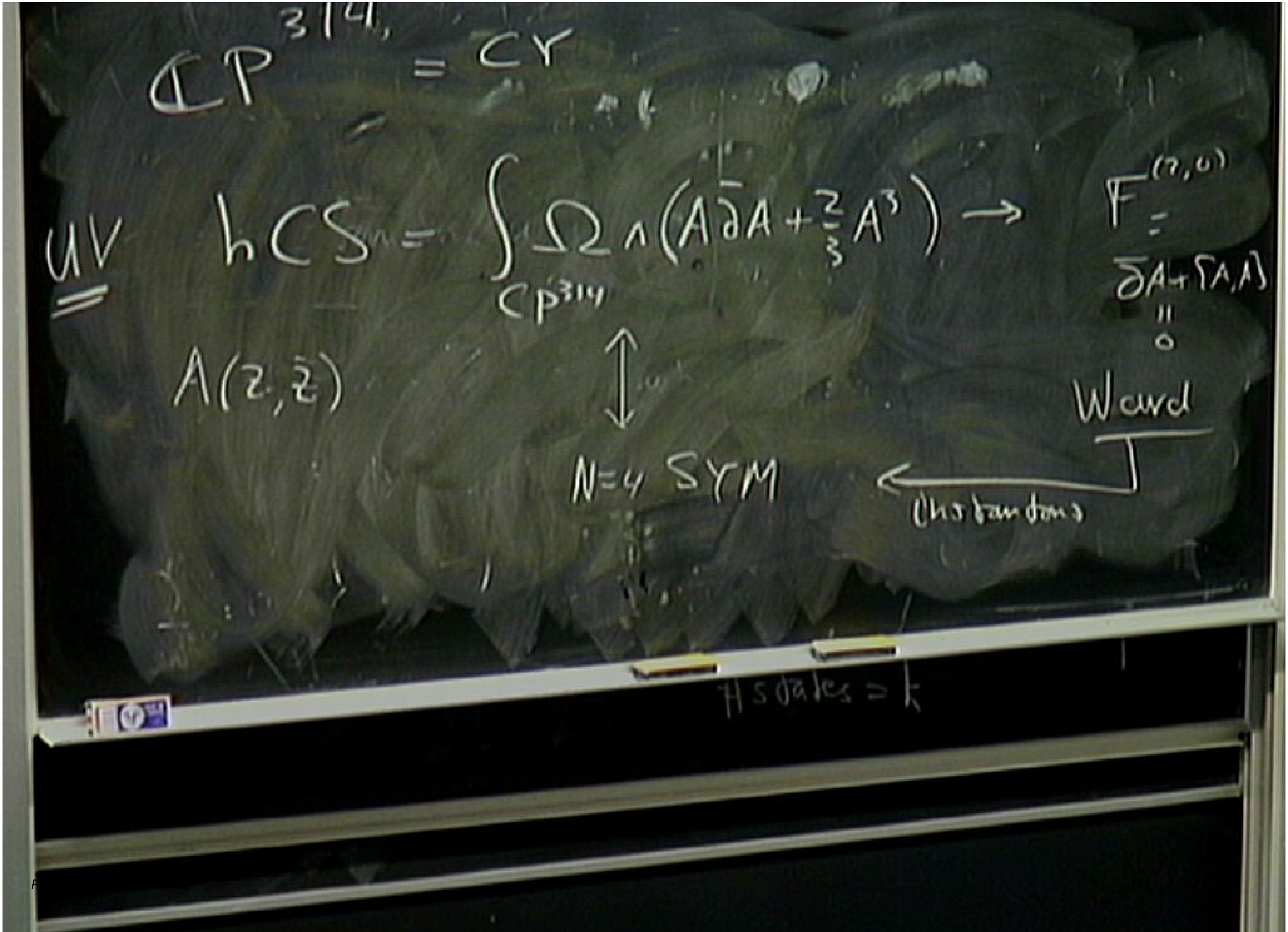

CP3|4 = CY
hCS = ∫ Ω ∧ (A∂̄A + 2/3 A3)
CP3|4
N=4 SYM



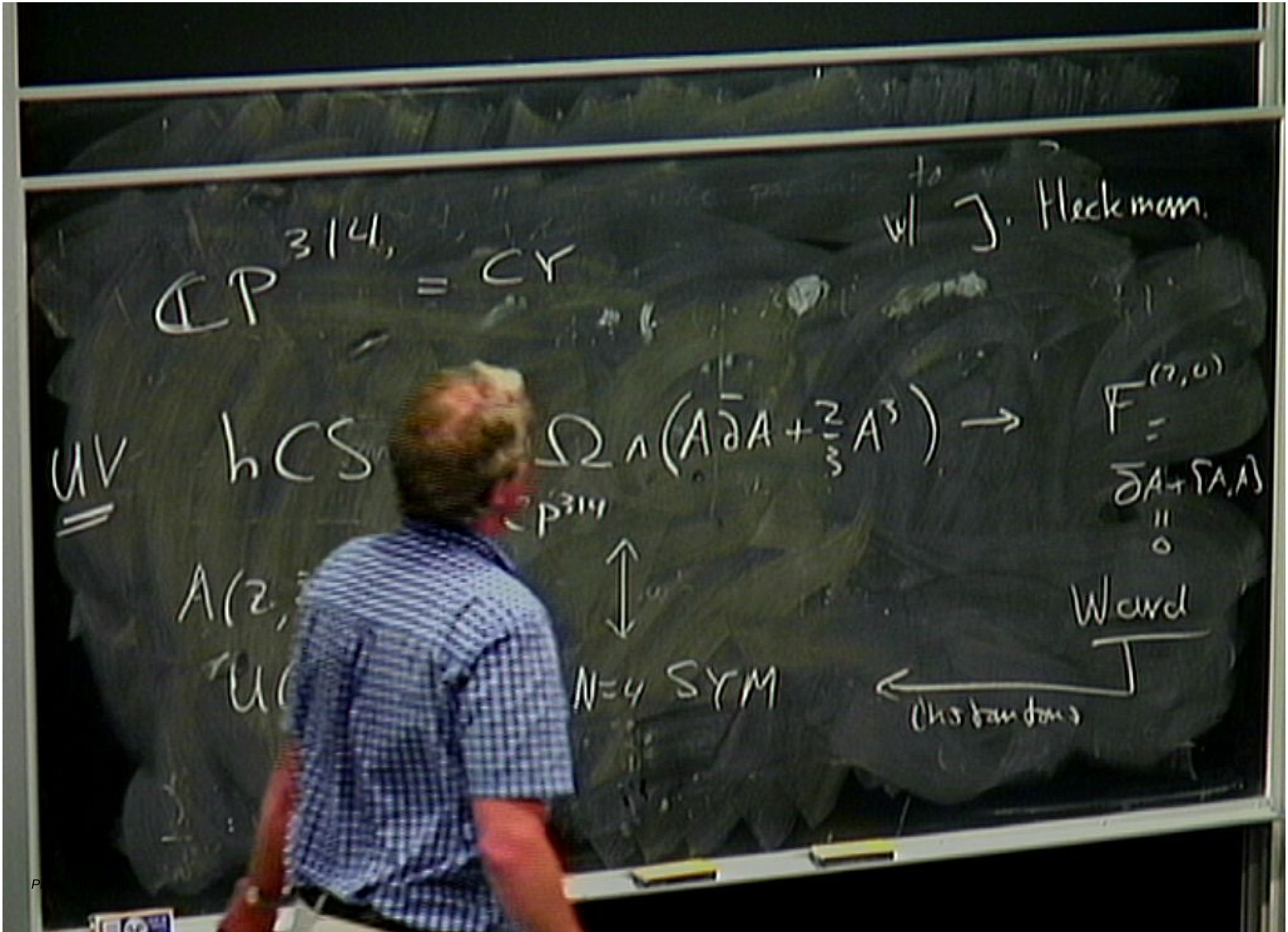
$\mathbb{C}P^{3|4} = CY$
$hCS = \int_{CP^{3|4}} \Omega \wedge (A\bar{\partial}A + \frac{2}{3}A^3) \rightarrow$
$A(z, \bar{z})$
N=4

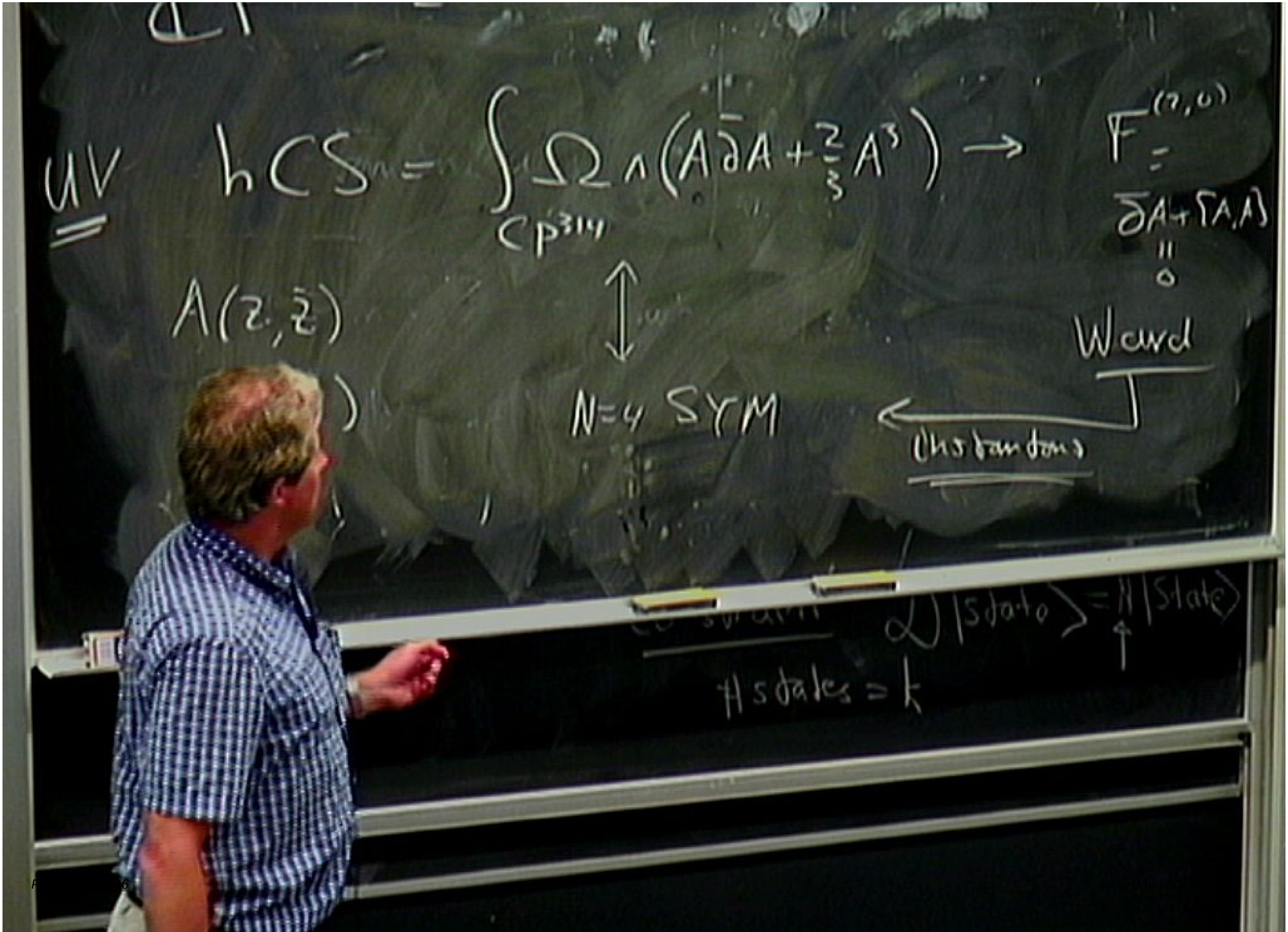

CP^{3|4} = CY
hCS = ∫_{CP^{3|4}} Ω ∧ (A∂̄A + 2/3 A^3) → F^{(2,0)} = ∂̄A + [A,A] = 0
A(z, z̄)
N=4 SYM
Ward

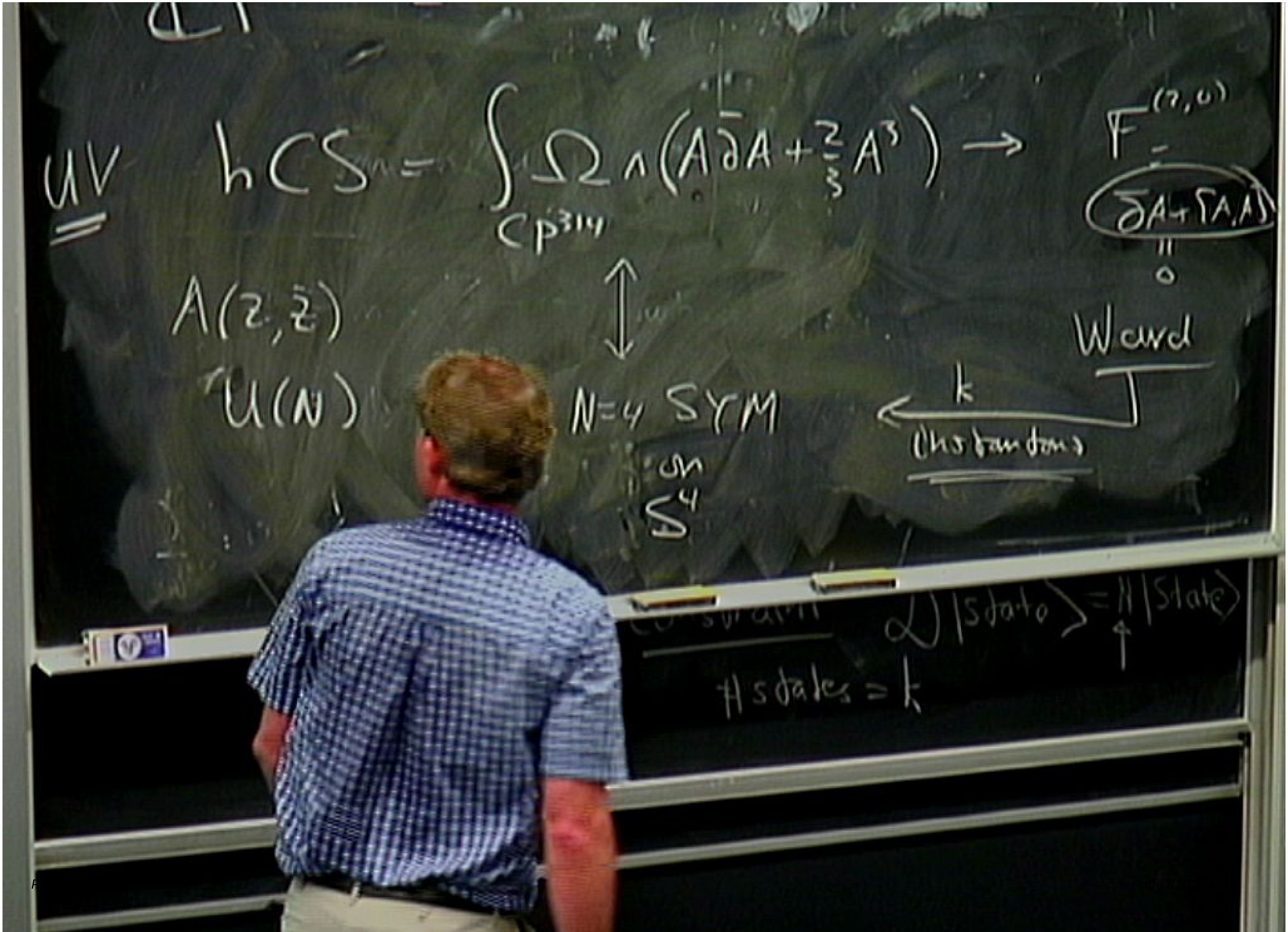

CP^{3|4} = CY
hCS = ∫_{CP^{3|4}} Ω ∧ (A∂̄A + 2/3 A^3) → F^{(2,0)} = ∂̄A + [A,A] = 0
A(z, z̄)
N=4 SYM
Ward
(instantons)

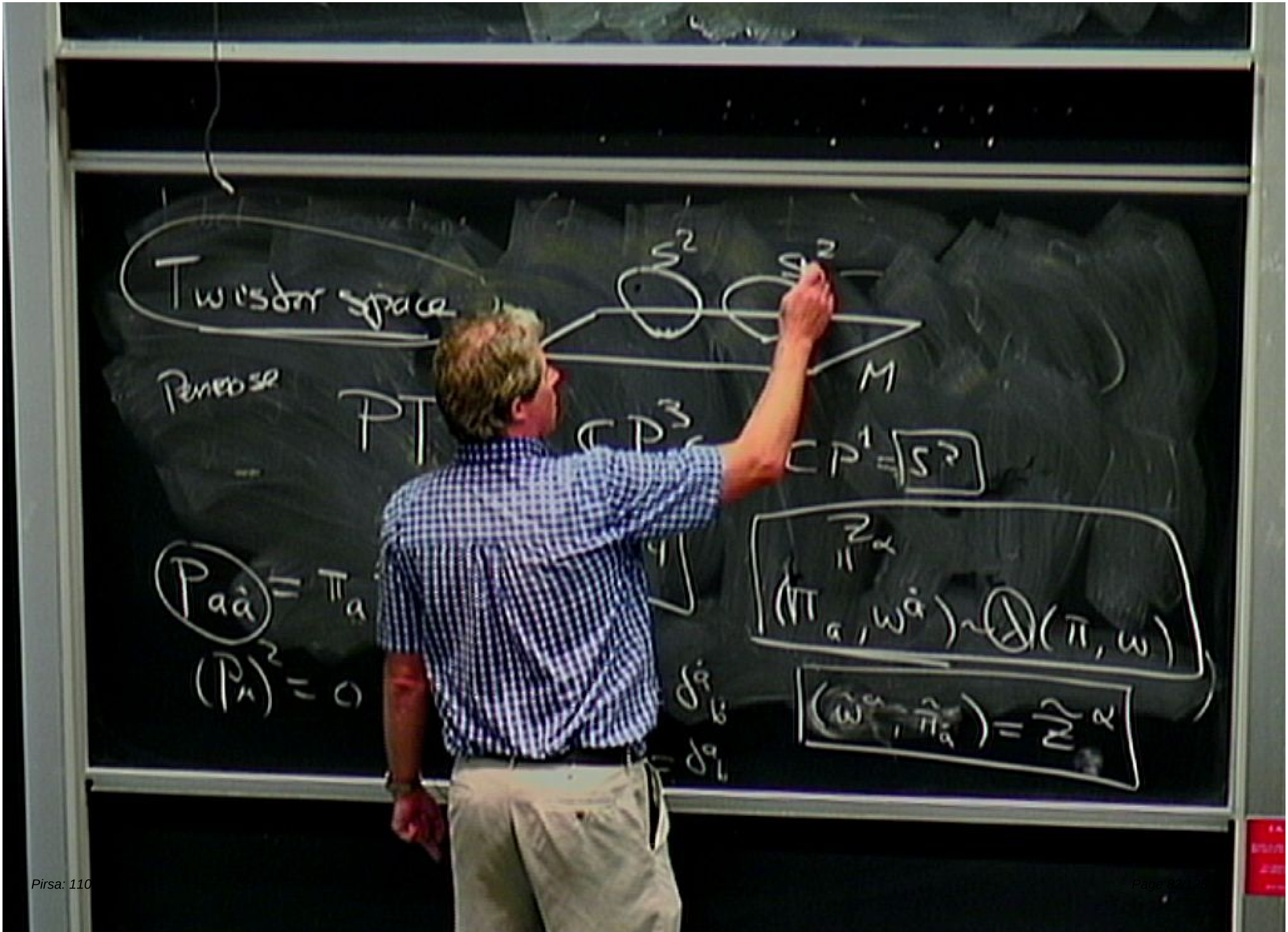


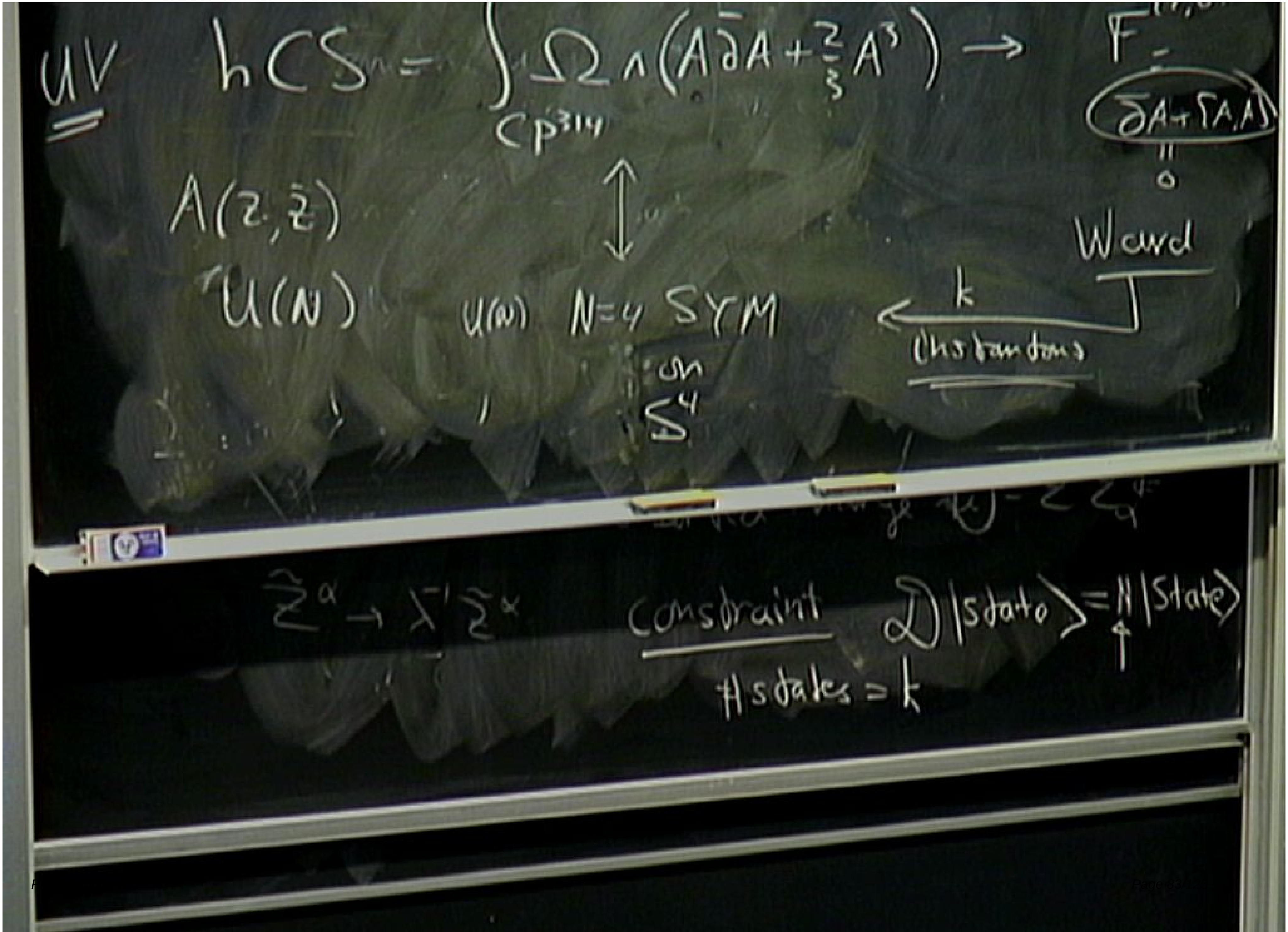
CP^{3|4} = CY
UV
hCS = ∫_{CP^{3|4}} Ω ∧ (A∂̄A + 2/3 A^3) → F^{(2,0)} = ∂̄A + [A,A] = 0
A(z, z̄)
N=4 SYM
Ward

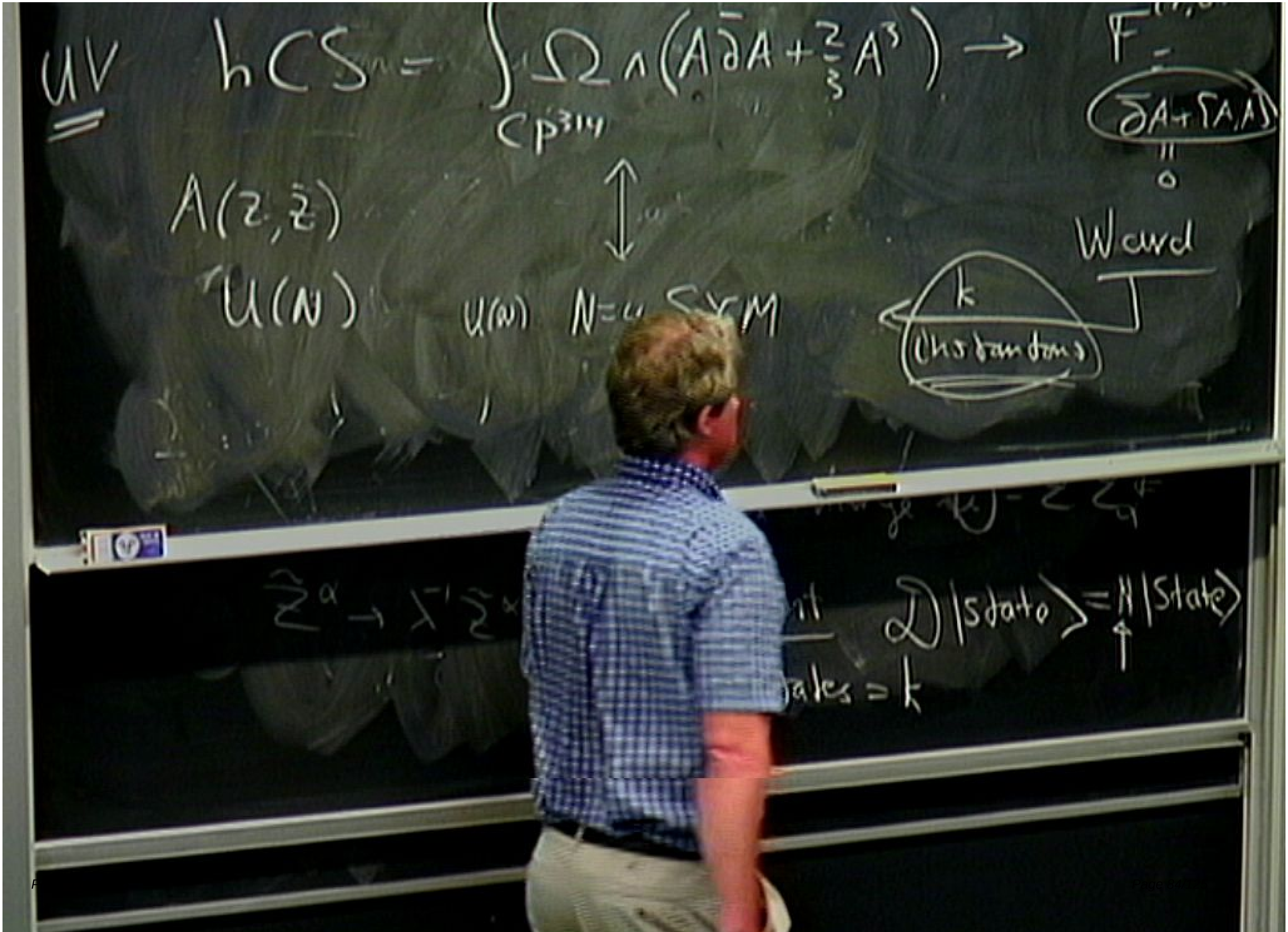

w/ J. Heckman.
ℂP^{3|4} = CY
UV
hCS
Ω ∧ (A∂̄A + 2/3 A³) →
P^{3|4}
F^{(2,0)} =
∂̄A + [A,A]
= 0
A(z,
U(
N=4 SYM
Ward

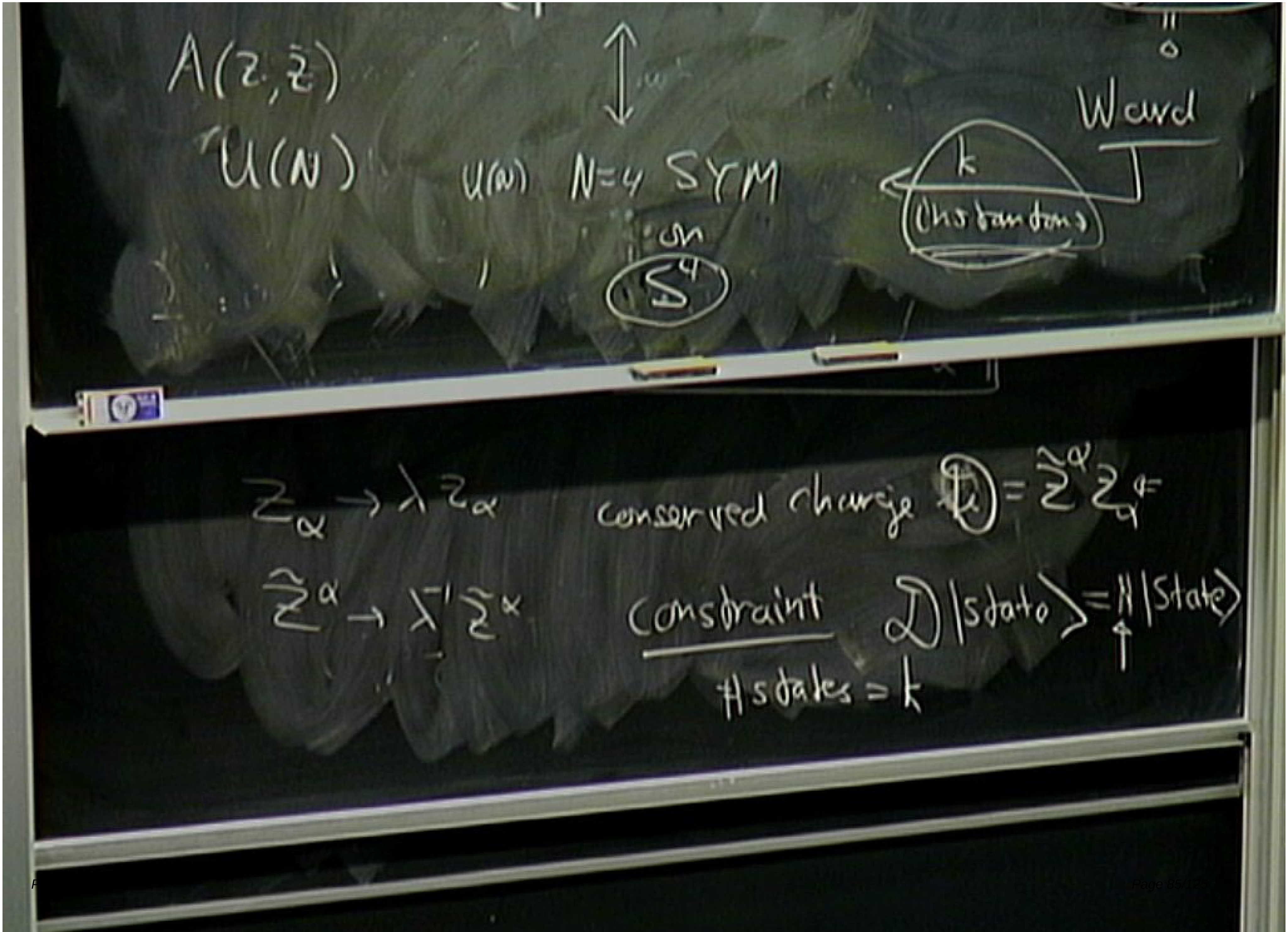

UV
hCS = ∫ Ω ∧ (A∂̄A + 2/3 A³) → F^(2,0) = ∂̄A + [A,A] = 0
CP^3|4
A(z, z̄)
N=4 SYM
Ward
(instantons)
#states = k


UV
hCS
CP3|4
A(z, z̄)
U(N)
N=4 SYM
on
S4
Ward
k
#states = k


Twistor space
Penrose
PT
M


UV
$hCS_m = \int_{CP^{3|4}} \Omega \wedge (A\bar{\partial}A + \frac{2}{3}A^3) \to F^{(0,2)}$
$\bar{\partial}A + [A,A] = 0$
$A(z,\bar{z})$
$U(N)$
$U(N)$ $N=4$ SYM on $S^4$
Ward
k
(Instantons)
$\tilde{z}^a \to \lambda^{-1}\tilde{z}^a$
Constraint
$\mathcal{D}|state\rangle = N|state\rangle$
#states = k


UV
hCS = ∫ Ω ∧ (A∂̄A + 2/3 A³) →
Cp³|⁴
A(z, z̄)
U(N)
U(N) N=4 SYM
Ward
k
(instantons)
|state⟩ = N |state⟩


A(z, z̄)
U(N)
U(N) N=4 SYM
on
S^4
k
(instantons)
Ward
z_α → λ z_α
conserved charge = z̃^α z_α
z̃^α → λ^{-1} z̃^α
Constraint
|state⟩ = N |state⟩
# states = k

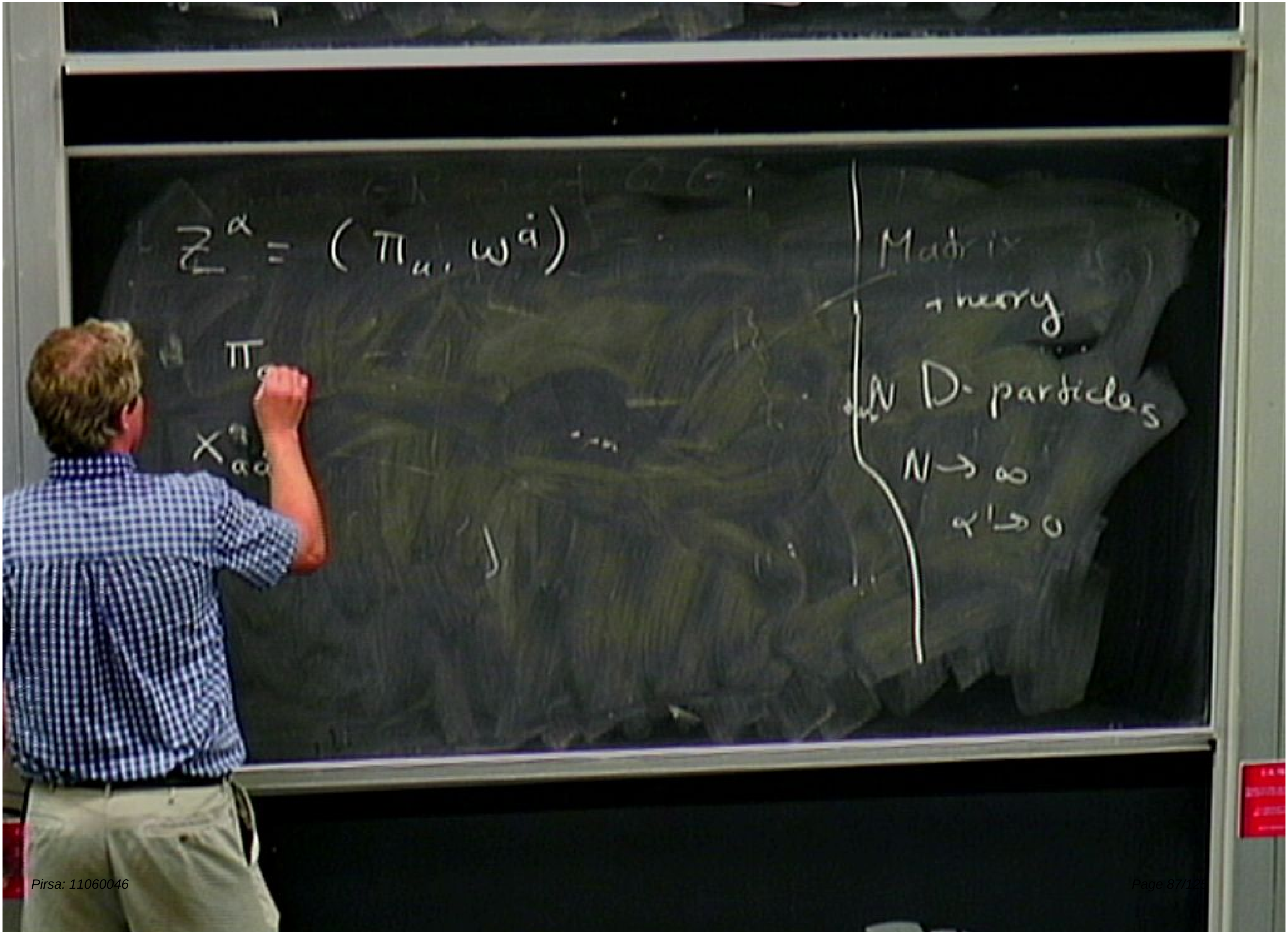
$$[p, x] = \frac{\hbar}{i} \quad \leftrightarrow \quad [\tilde{Z}^\alpha, Z_\beta] = \delta^\alpha_\beta \leftarrow$$

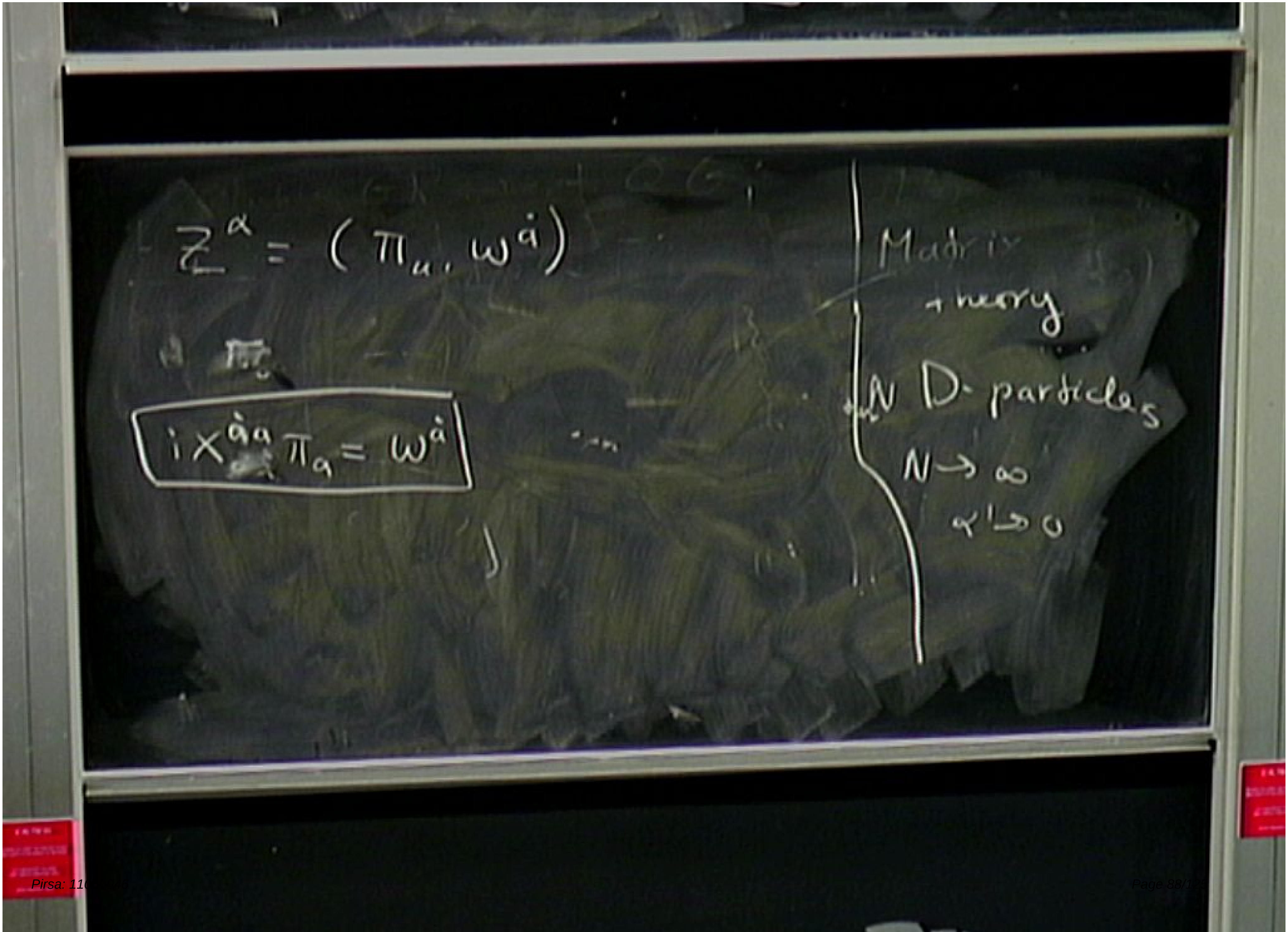
In Euclidean space: $\boxed{(\tilde{Z}^\alpha)^\dagger = Z_\alpha}$

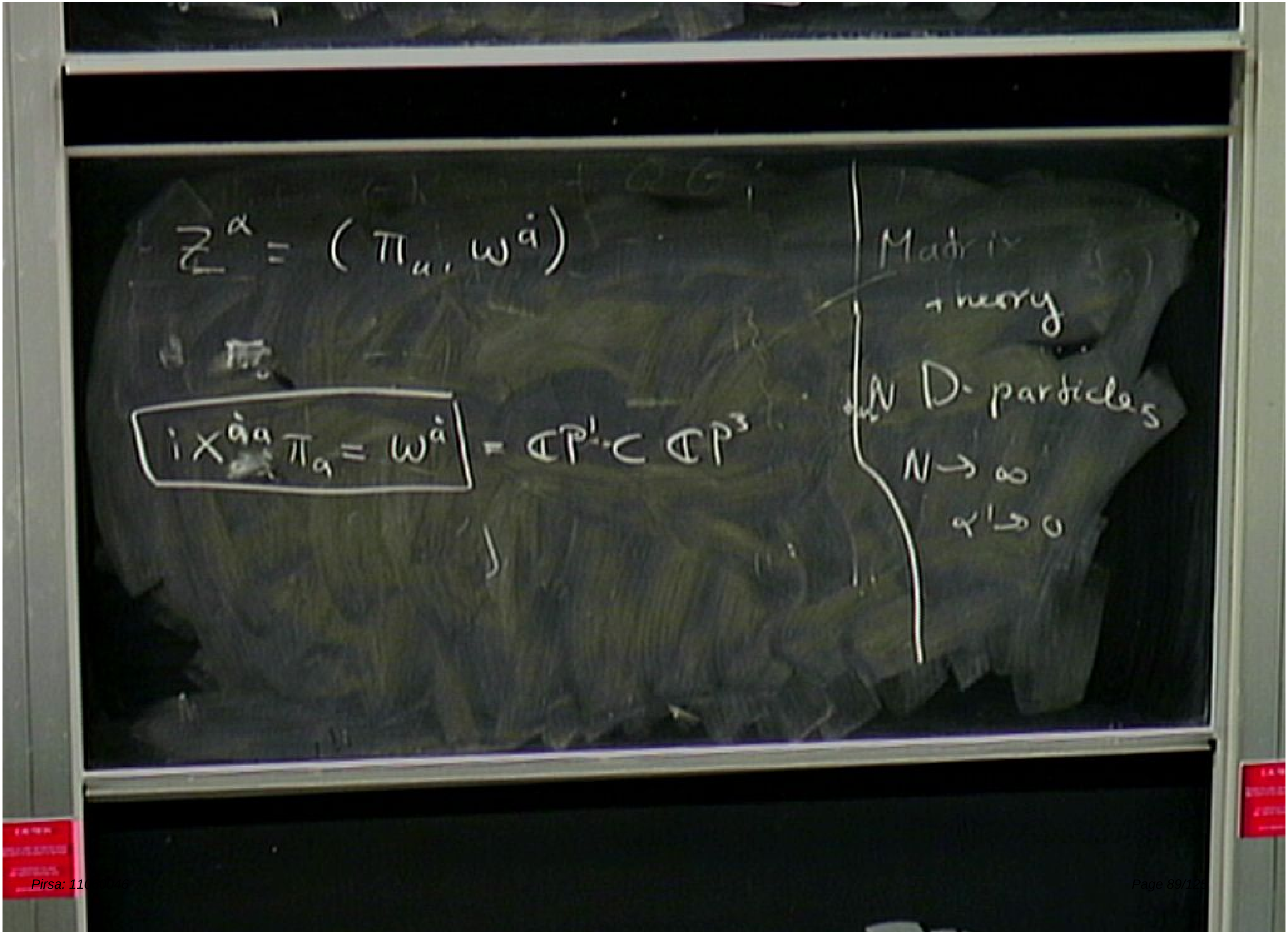
$Z_\alpha \to \lambda Z_\alpha$ conserved charge $(D) = \tilde{Z}^\alpha Z_\alpha \leftarrow$

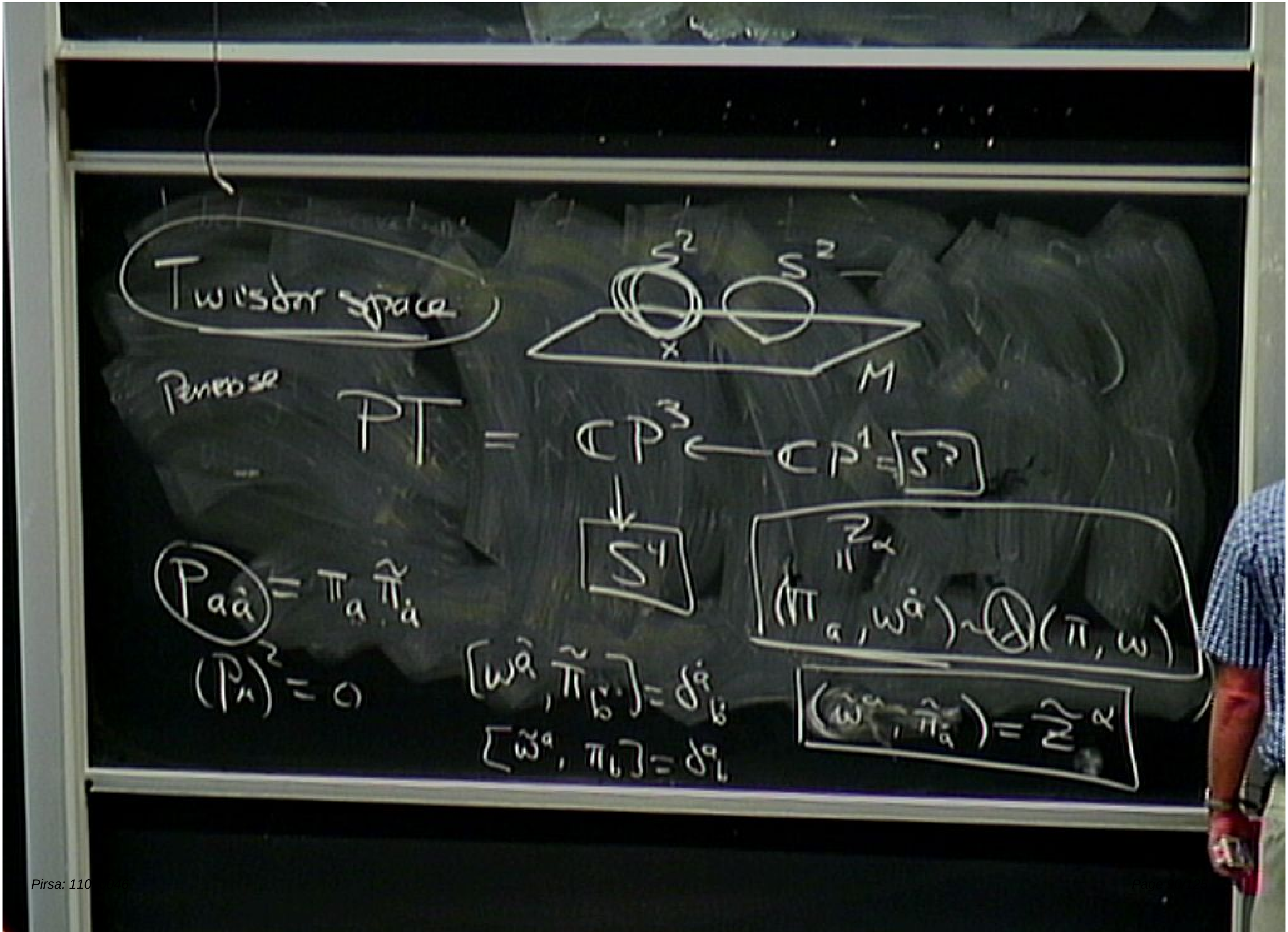
$\tilde{Z}^\alpha \to \lambda^{-1} \tilde{Z}^\alpha$ <u>Constraint</u> $D|\text{state}\rangle = N|\text{state}\rangle$

$\#\text{states} = k$


Z^α = (π_a, ω^ȧ)
π_a
X^a_{aȧ}
Matrix
theory
N D-particles
N → ∞
α' → 0


$Z^\alpha = (\pi_a, \omega^{\dot a})$
$i x^{\dot a a} \pi_a = \omega^{\dot a}$
Matrix theory
N D-particles
$N \to \infty$
$\alpha' \to 0$


$Z^\alpha = (\pi_a, \omega^{\dot{a}})$
$i x^{\dot{a}a} \pi_a = \omega^{\dot{a}}$ = $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Matrix theory
N D-particles
$N \to \infty$
$\alpha' \to 0$


Twistor space
$S^2$
$S^2$
x
M
Penrose
$PT = \mathbb{C}P^3 \leftarrow \mathbb{C}P^1 = S^2$
$S^4$
$Z^\alpha$
$(\pi_a, \omega^{\dot a}) \sim \lambda(\pi, \omega)$
$P_{a\dot a} = \pi_a \tilde\pi_{\dot a}$
$(P_A)^2 = 0$
$[\omega^{\dot a}, \tilde\pi_{\dot b}] = \delta^{\dot a}_{\dot b}$
$[\tilde\omega^a, \pi_b] = \delta^a_b$
$(\tilde\omega^a, \tilde\pi_{\dot a}) = \tilde Z^\alpha$

$$Z^\alpha = (\pi_a, \omega^{\dot a})$$

twistor line

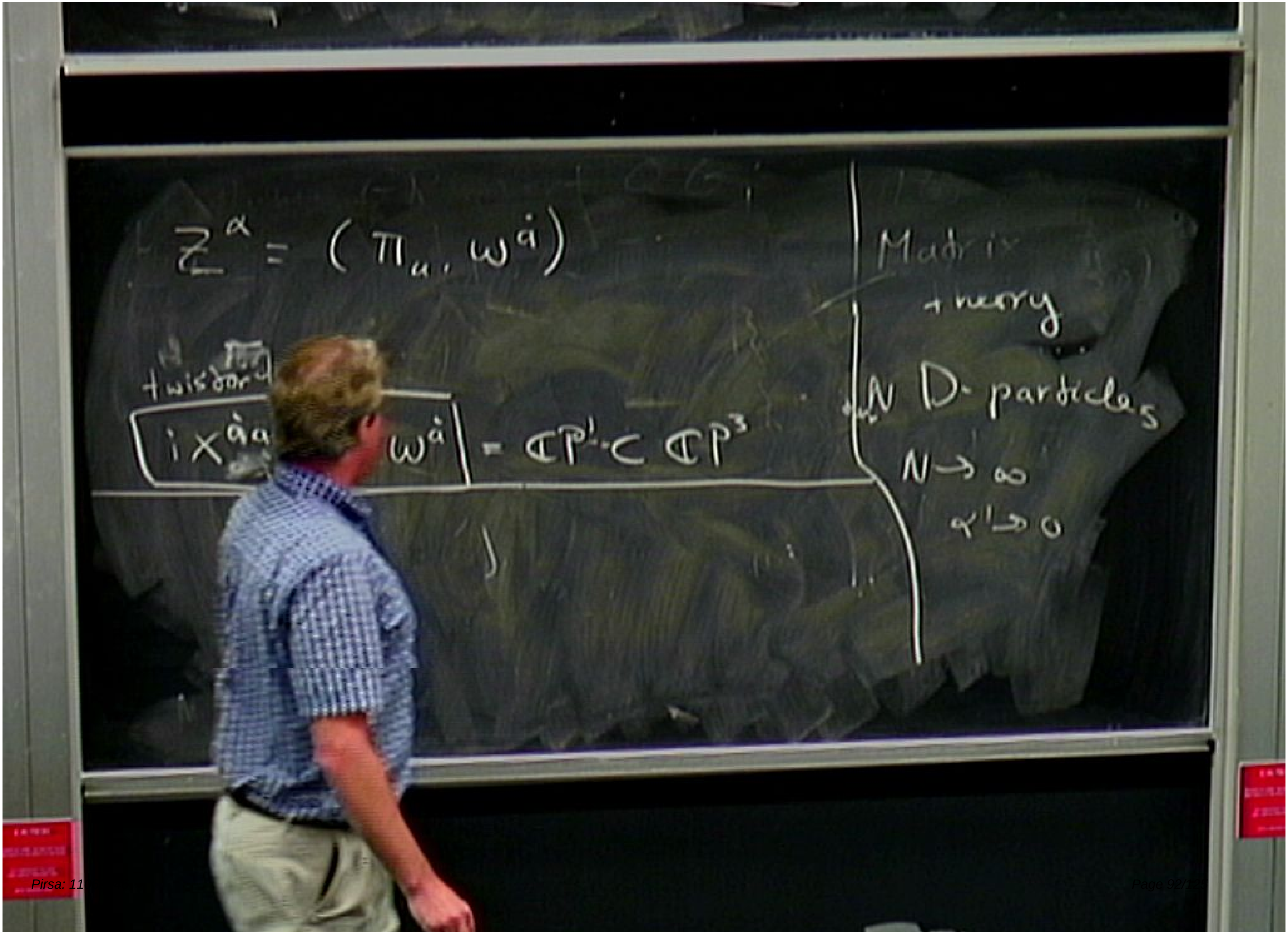
$$\boxed{i x^{\dot a a} \pi_a = \omega^{\dot a}} = \mathbb{CP}^1 \subset \mathbb{CP}^3$$

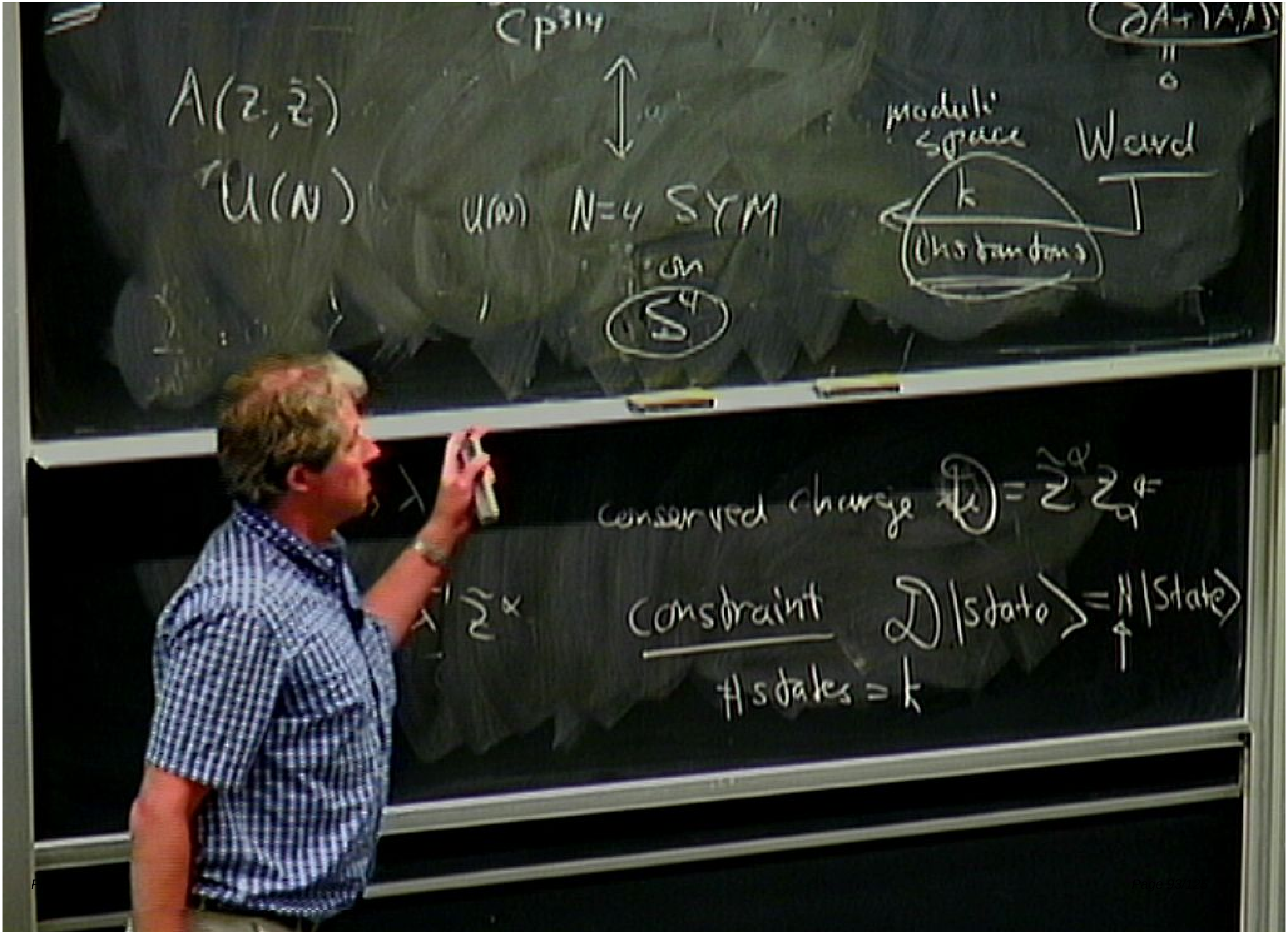
Matrix theory

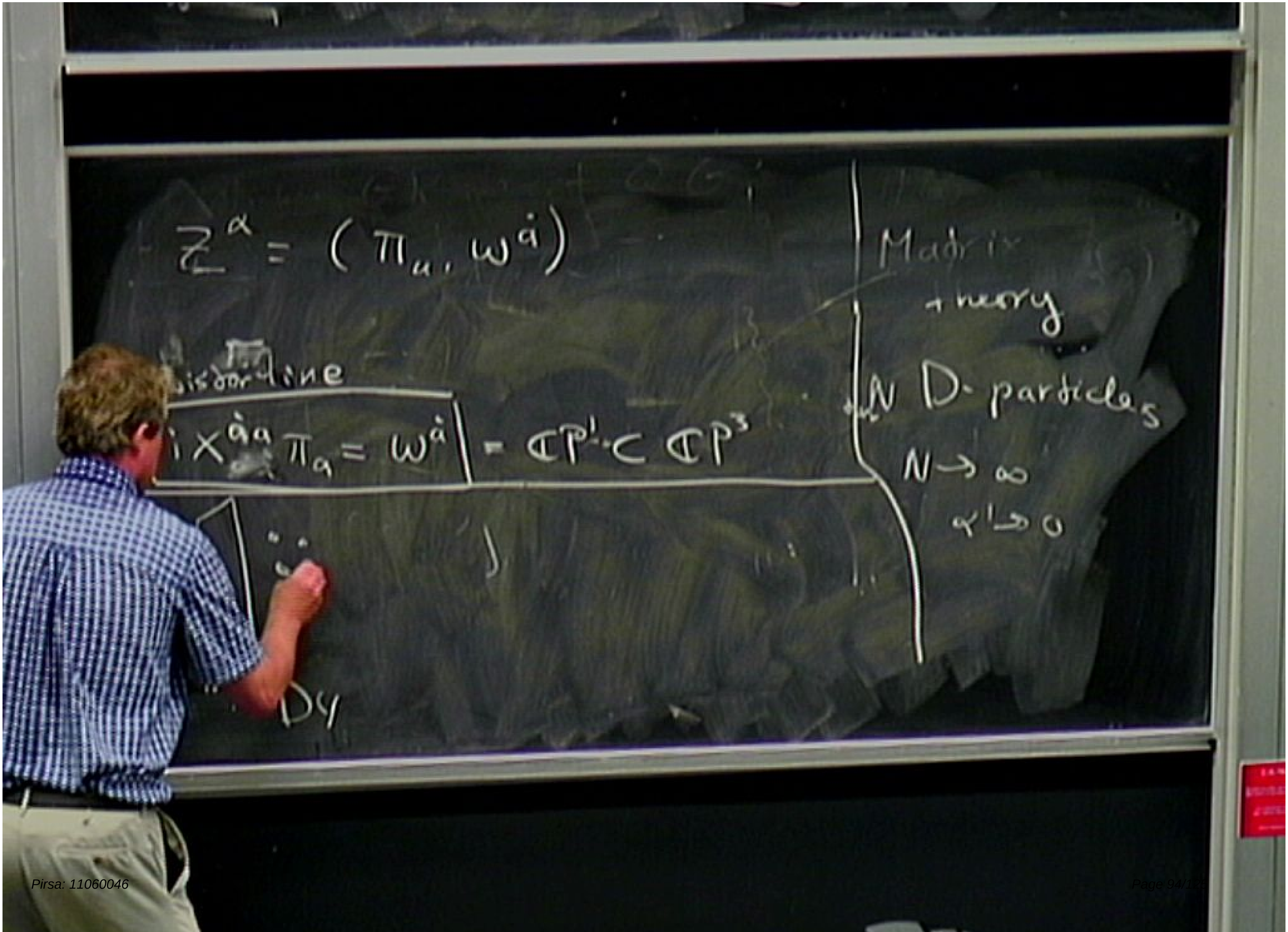
N D-particles

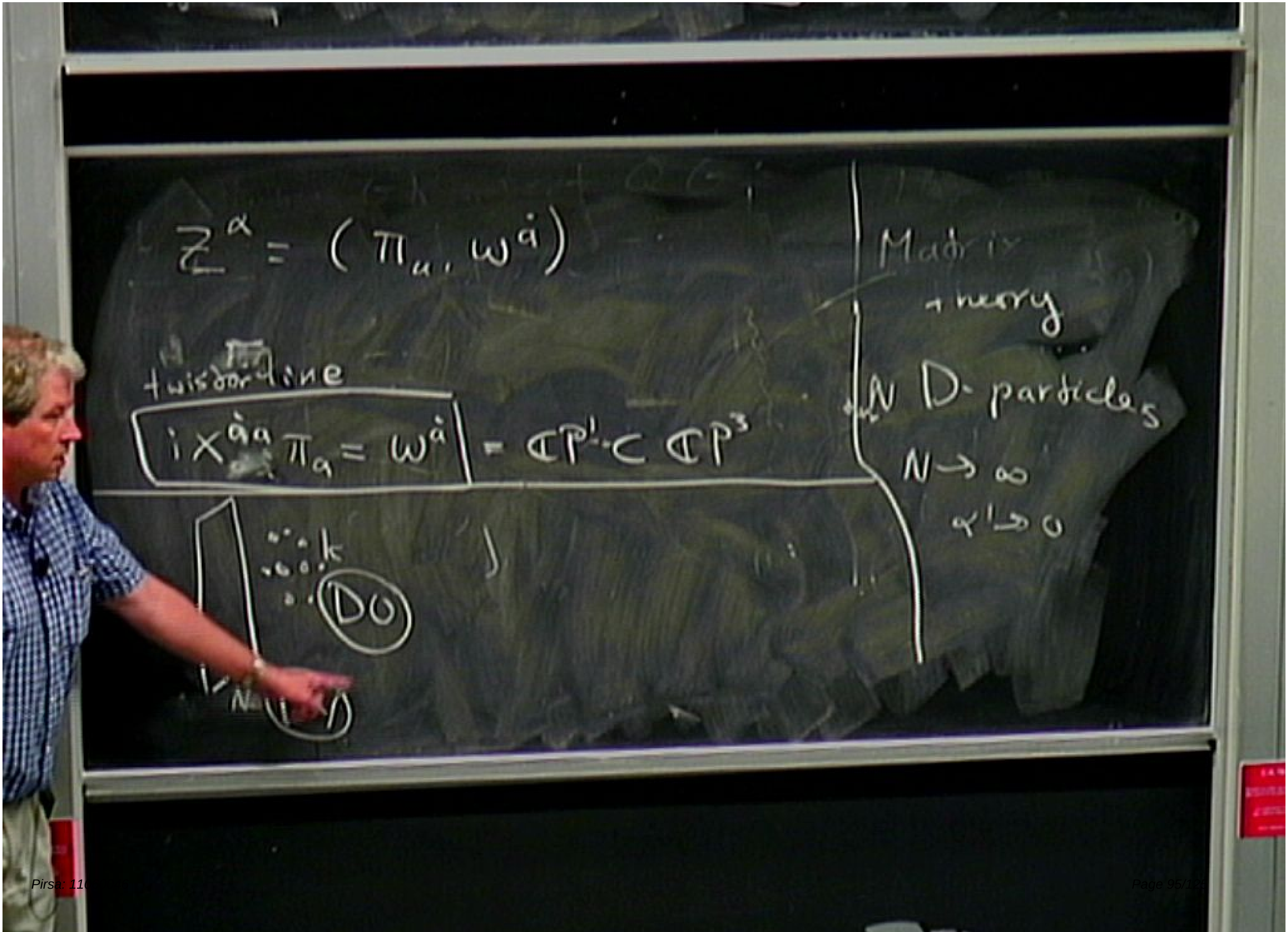
$N \to \infty$

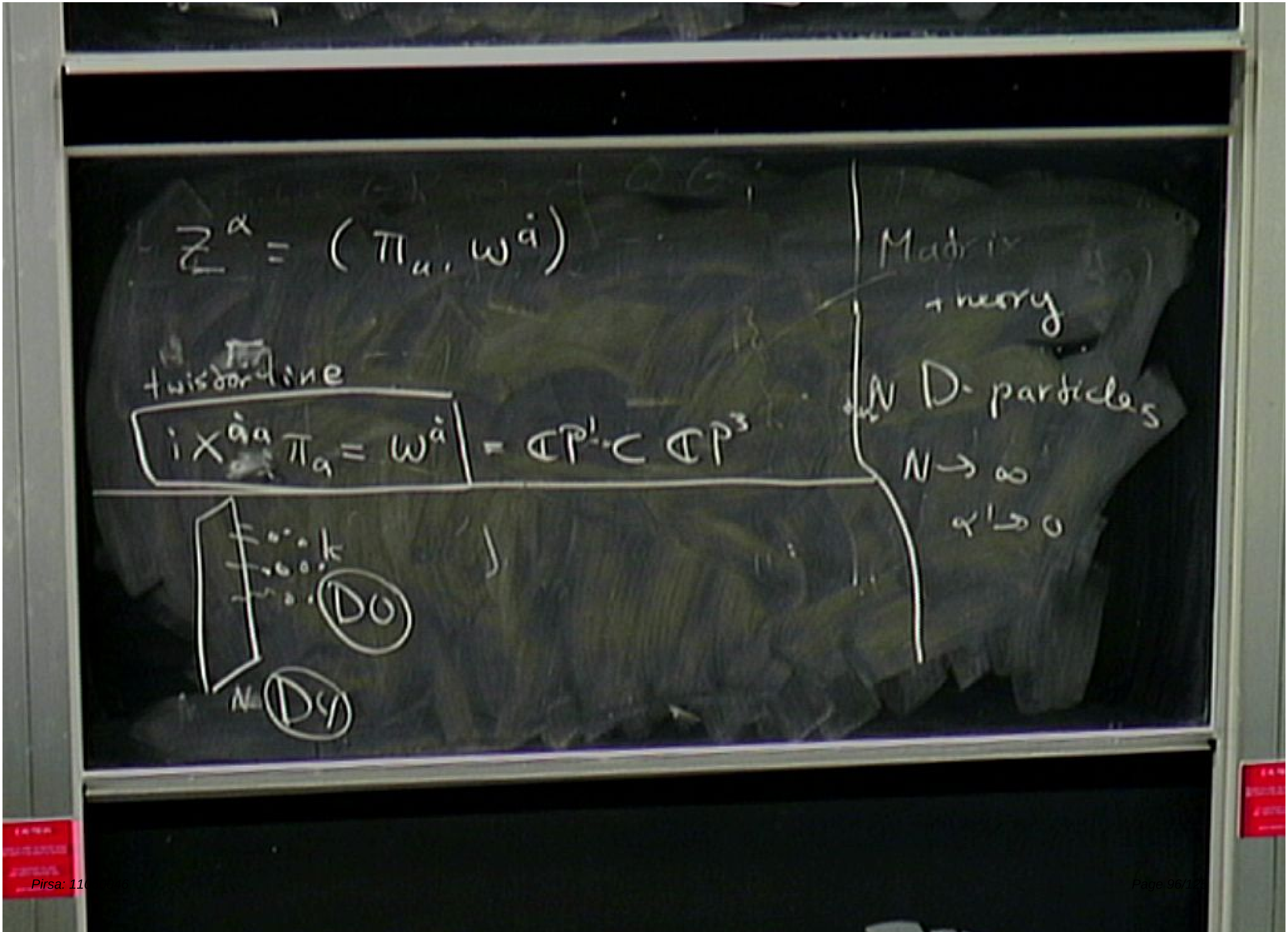
$\alpha' \to 0$

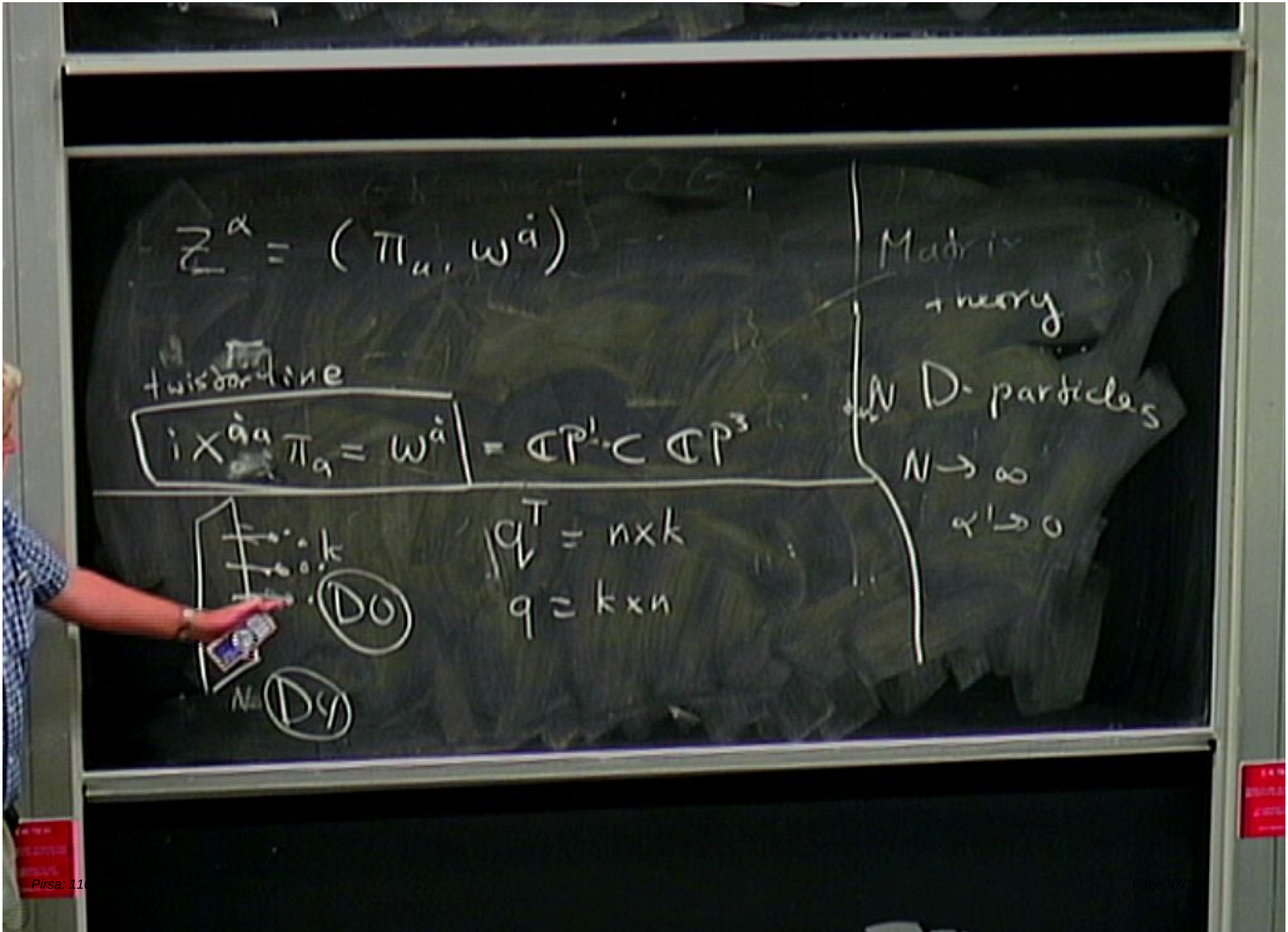

Z^α = (π_α, ω^α̇)
twistor
ix^α̇α ... ω^α̇ = CP^1 ⊂ CP^3
Matrix theory
N D-particles
N → ∞
α' → 0

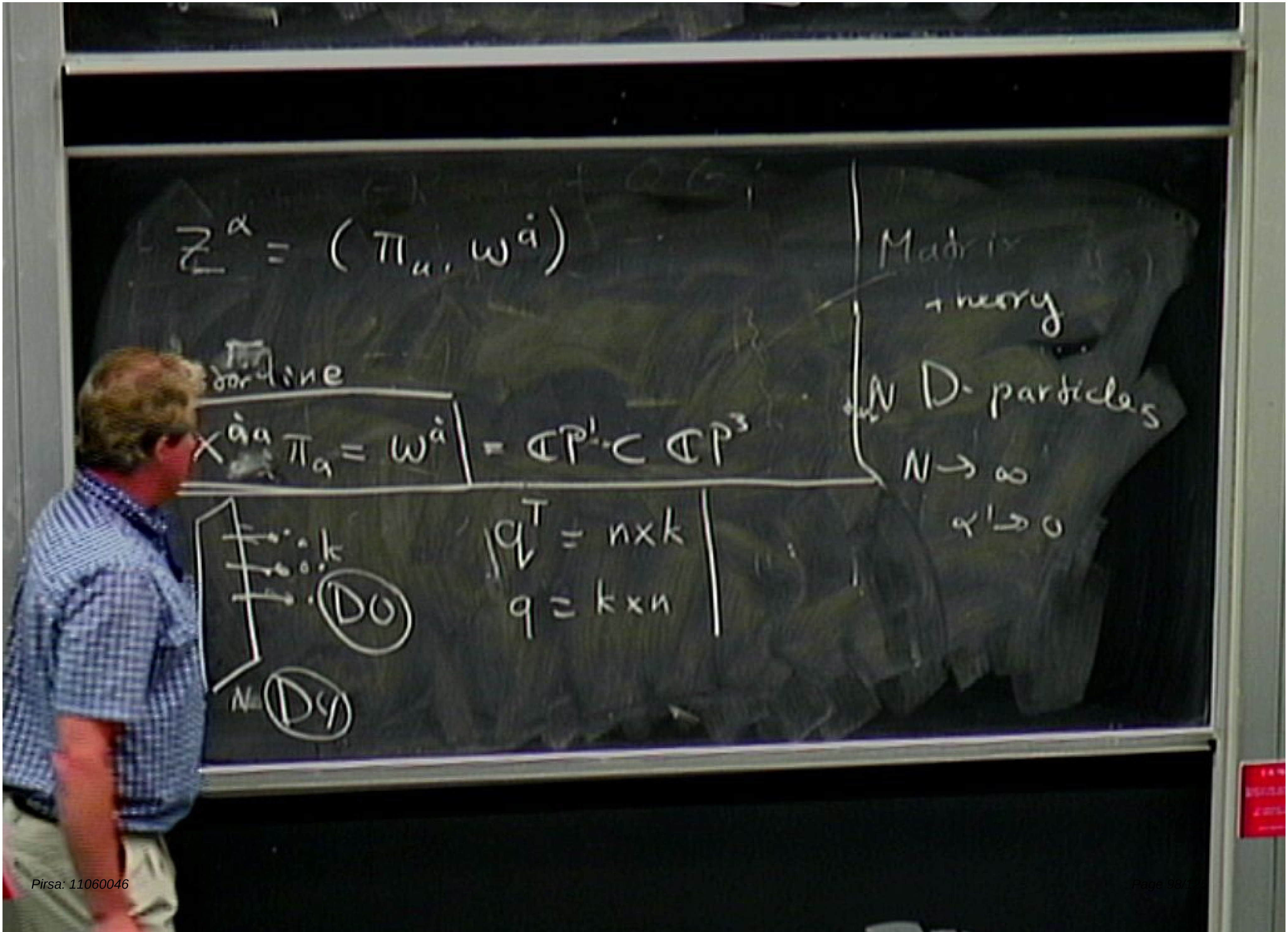

CP^{3|4}
A(z, z̄)
U(N)
U(N) N=4 SYM
on
S^4
moduli space
Ward
k
(instantons)
conserved charge
constraint
|state⟩ = N |state⟩
#states = k

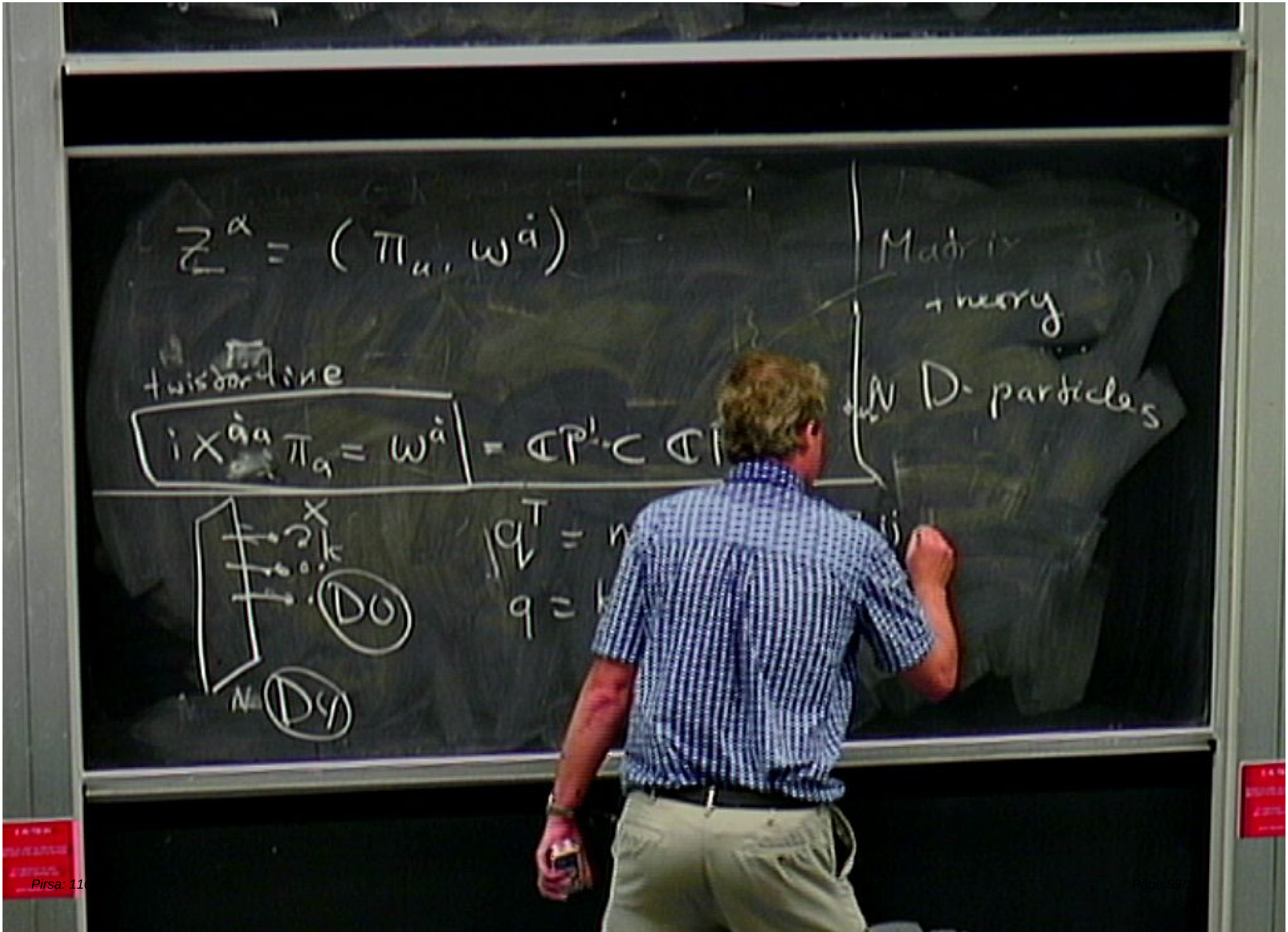

Z^α = (π_a, ω^ȧ)
x^{ȧa} π_a = ω^ȧ = CP^1 ⊂ CP^3
Matrix theory
N D-particles
N → ∞
α' → 0


Z^α = (π_a, ω^ȧ)
twistor line
i x^{ȧa} π_a = ω^ȧ = CP^1 ⊂ CP^3
Matrix theory
N D-particles
N → ∞
α' → 0
k
D0

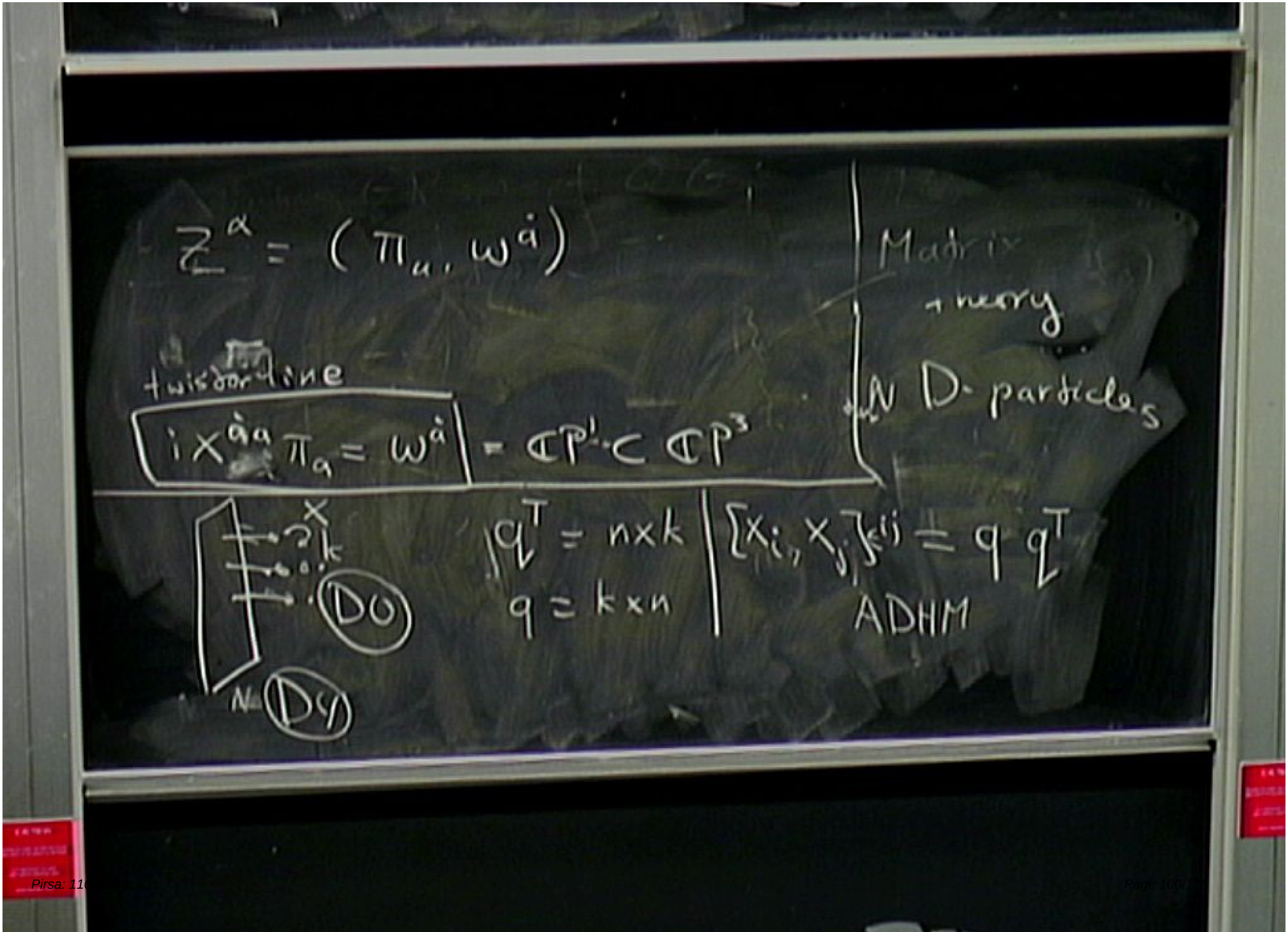

Z^α = (π_a, ω^ȧ)
twistor line
i x^{ȧa} π_a = ω^ȧ
CP^1 ⊂ CP^3
Matrix theory
N D-particles
N → ∞
α' → 0
k
D0
N D4

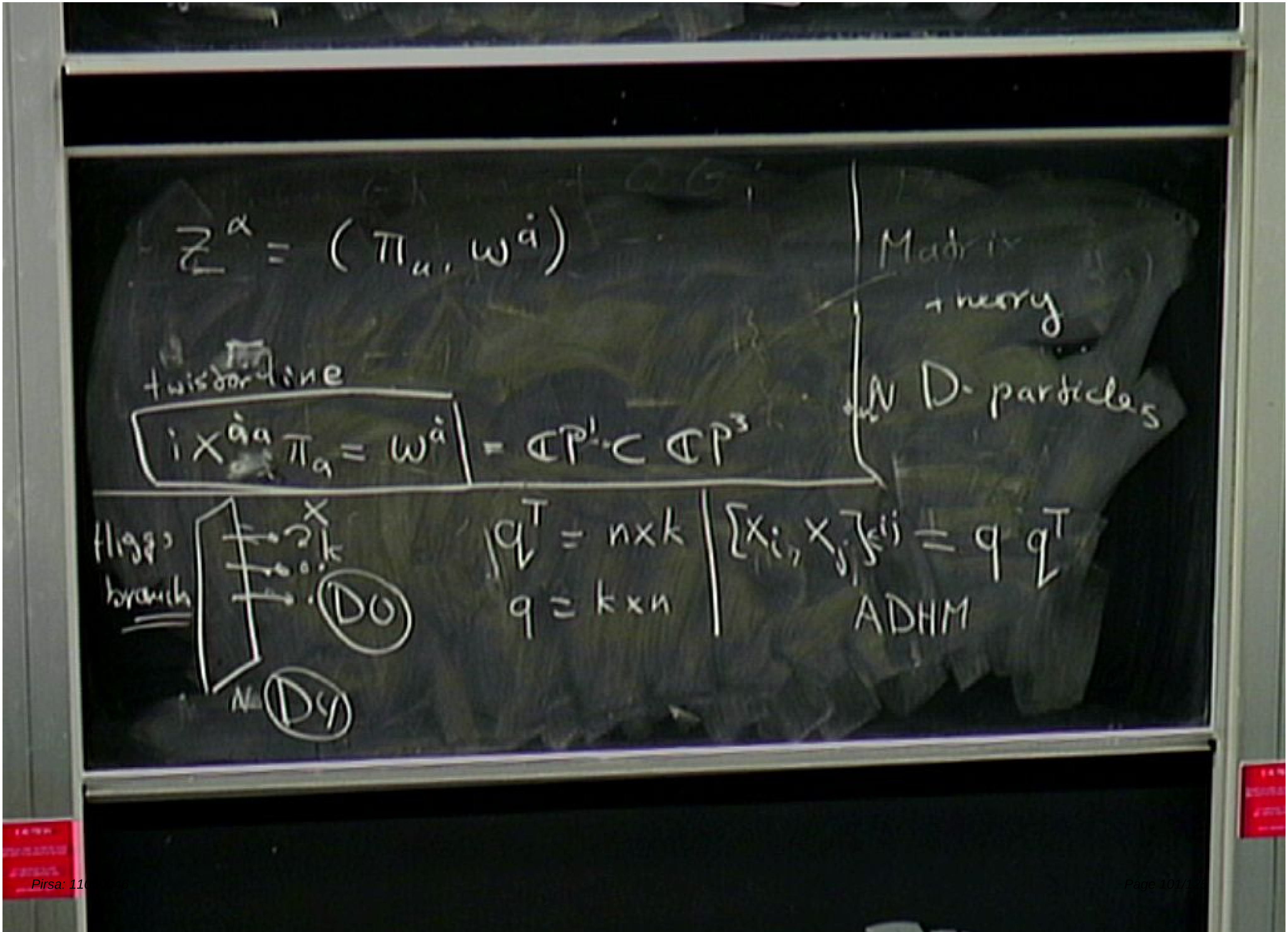

Z^α = (π_a, ω^ȧ)
twistor line
i x^{ȧa} π_a = ω^ȧ = CP^1 ⊂ CP^3
q^T = n×k
q = k×n
D0
D4
Matrix theory
N D-particles
N → ∞
α' → 0

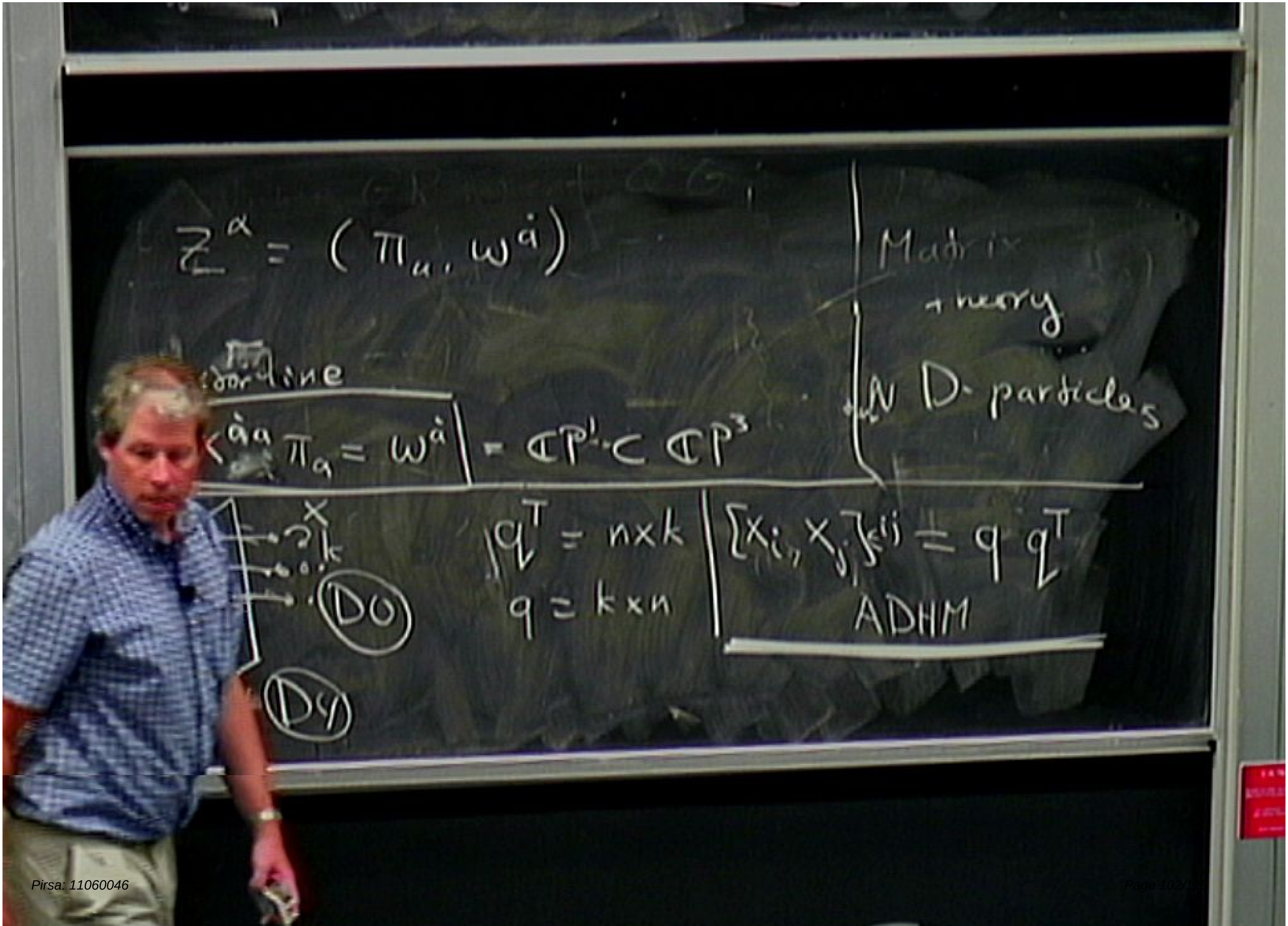

Z^α = (π_a, ω^ȧ)
Matrix theory
N D-particles
N → ∞
α' → 0
x^ȧa π_a = ω^ȧ
= CP^1 ⊂ CP^3
q^T = n×k
q = k×n
D0
D4

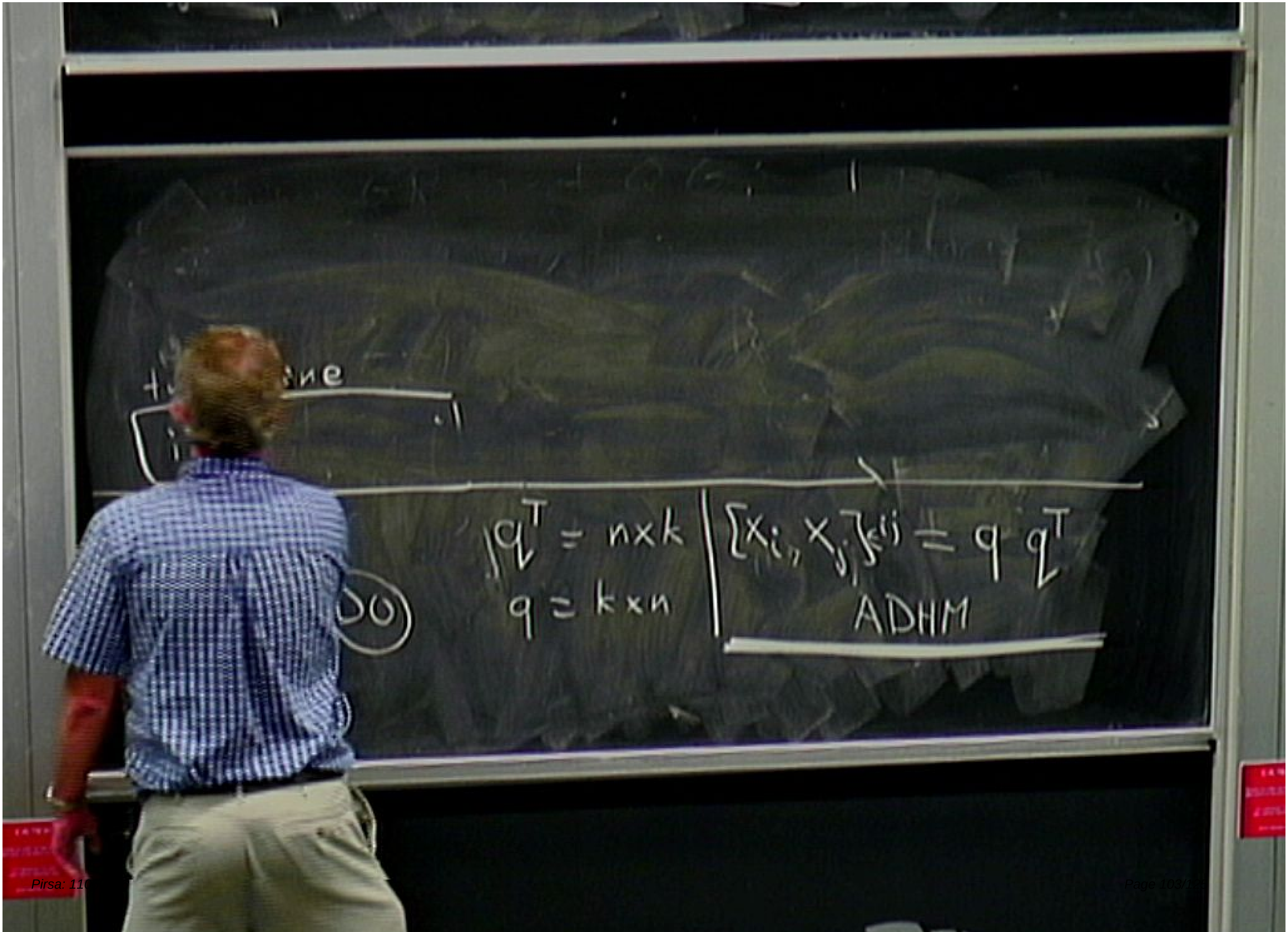


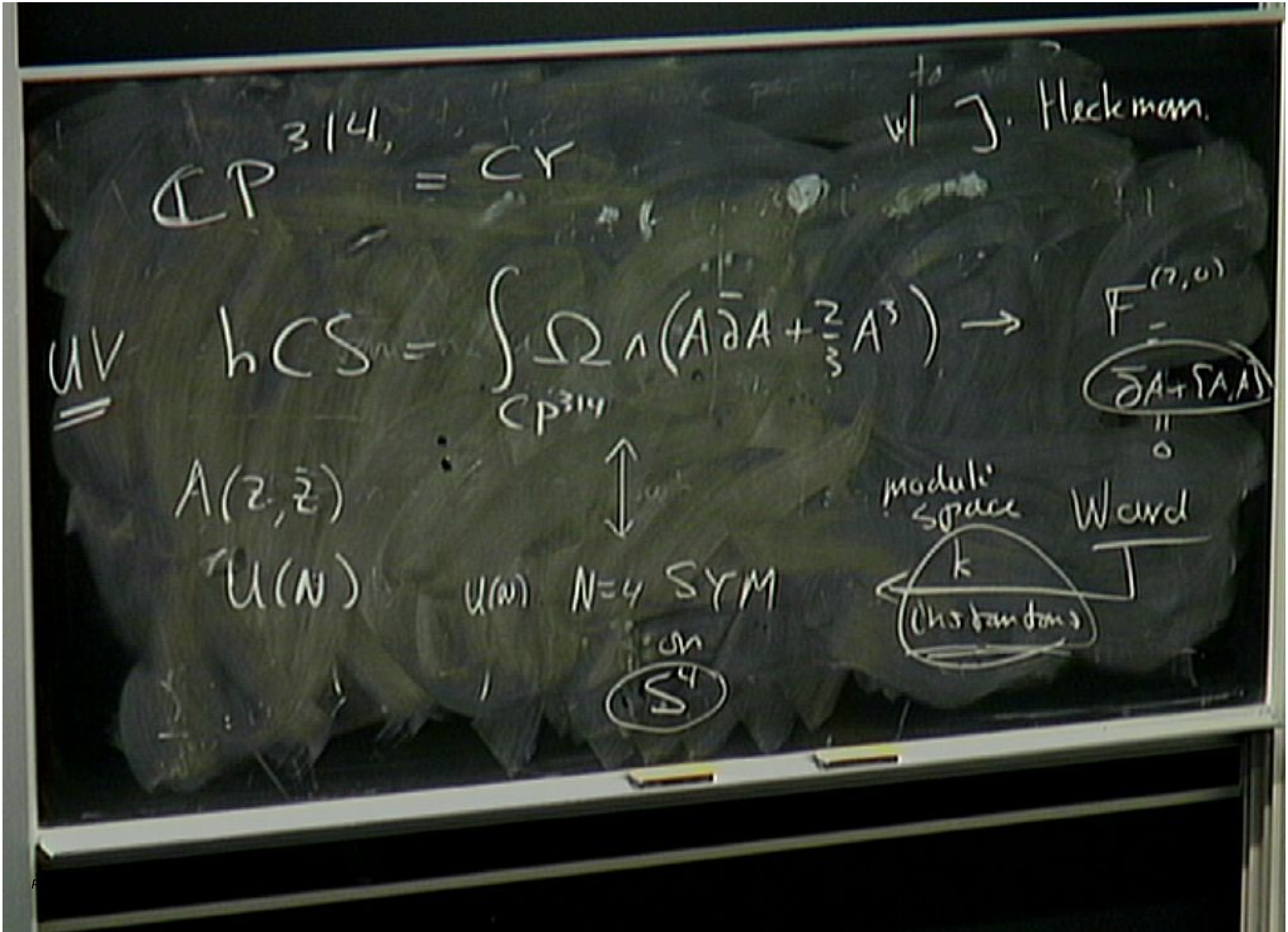
Z^α = (π_a, ω^ȧ)
twistor line
i x^{ȧa} π_a = ω^ȧ
= CP^1 ⊂ CP
D0
D4
Matrix theory
N D-particles

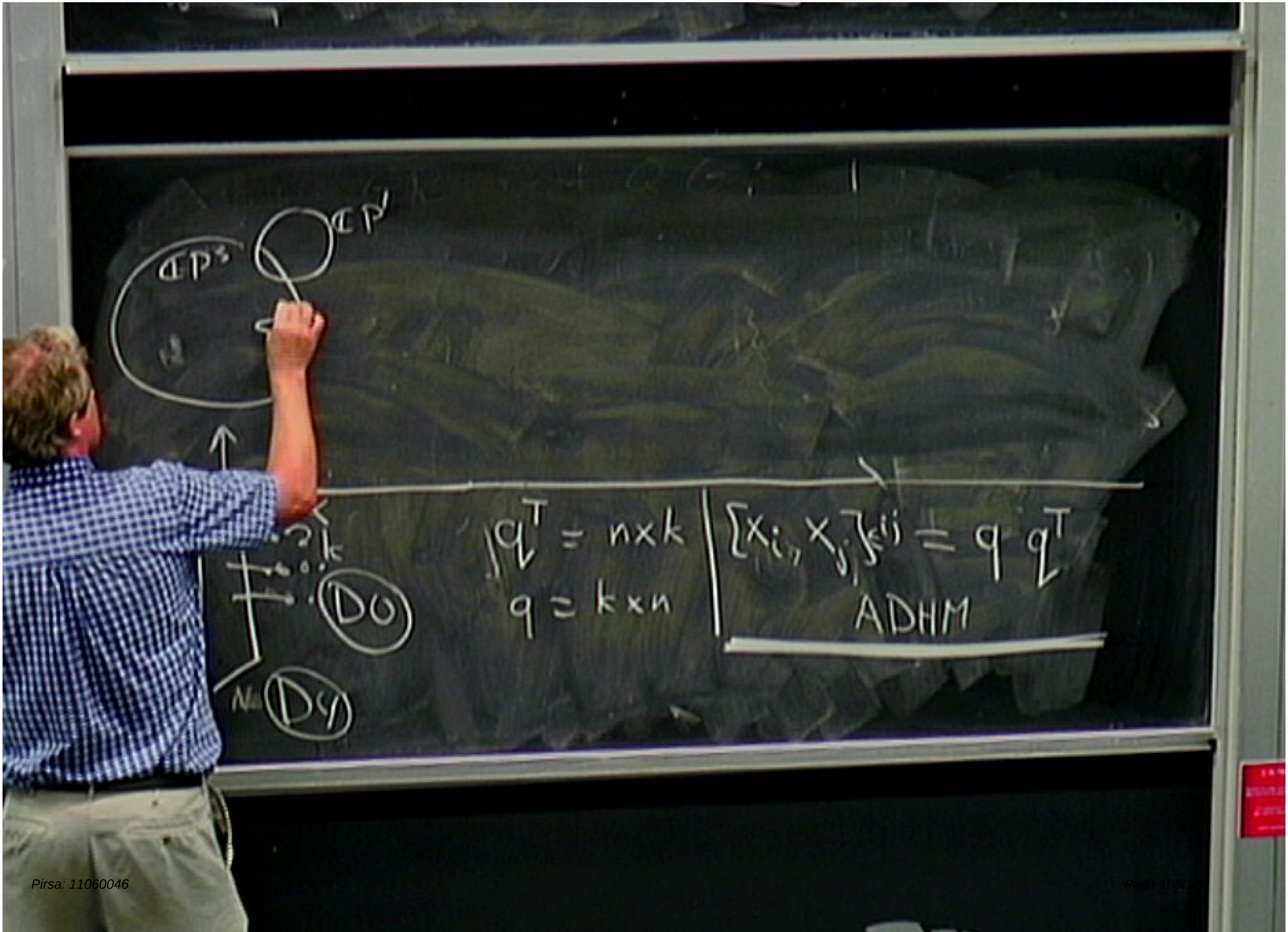

Z^α = (π_a, ω^ȧ)
twistor line
i x^{ȧa} π_a = ω^ȧ = CP¹ ⊂ CP³
Matrix theory
N D-particles
X
k
DO
N
D4
q^T = n×k
q = k×n
[X_i, X_j]ε^{ij} = q q^T
ADHM

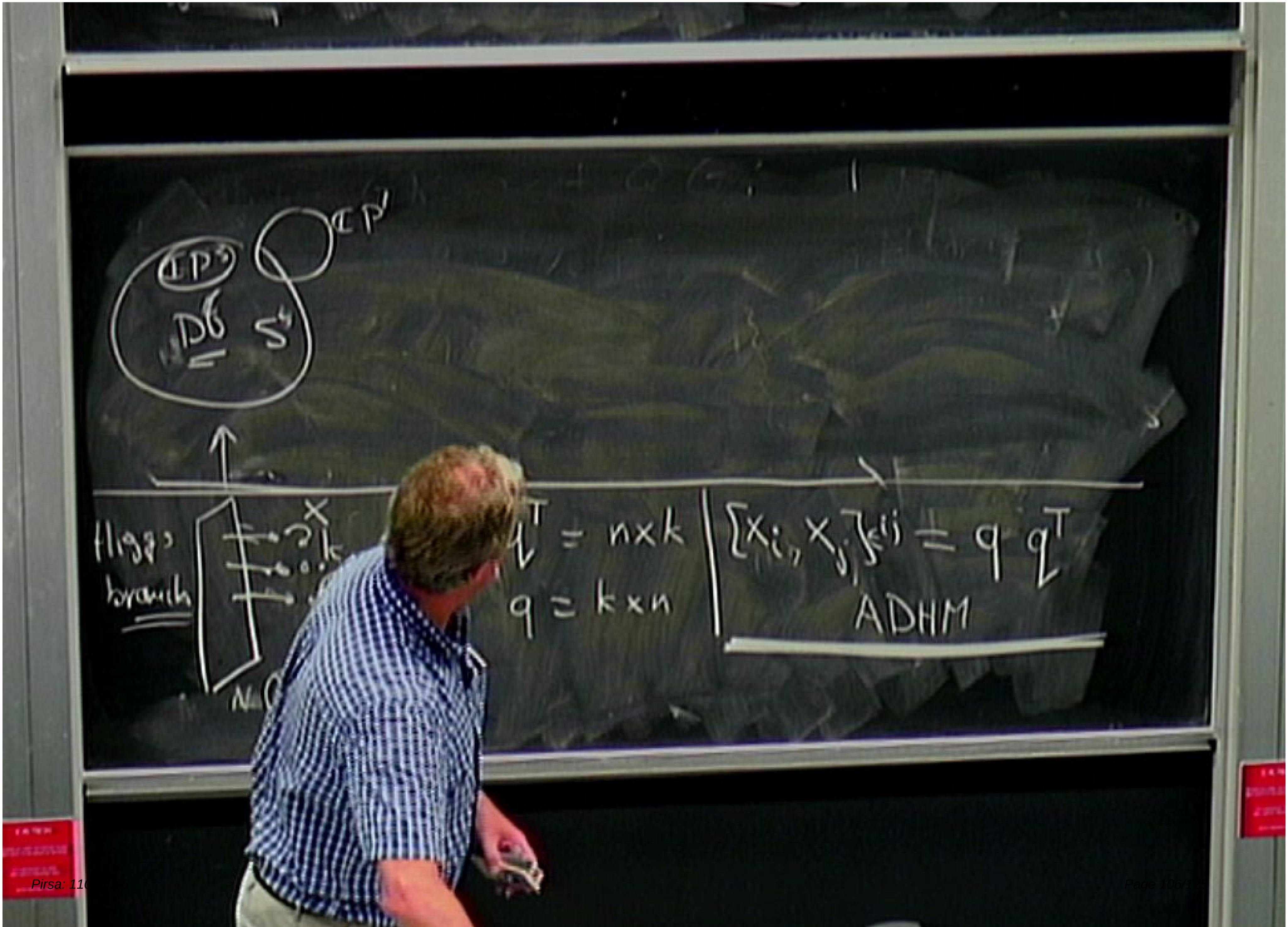

$Z^{\alpha} = (\pi_{a}, \omega^{\dot{a}})$
twistor line
$i x^{\dot{a}a} \pi_a = \omega^{\dot{a}}$ = $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Matrix theory
N D-particles
Higgs branch
X
k
D0
N D4
$q^T = n \times k$
$q = k \times n$
$[X_i, X_j]\epsilon^{ij} = q\, q^T$
ADHM

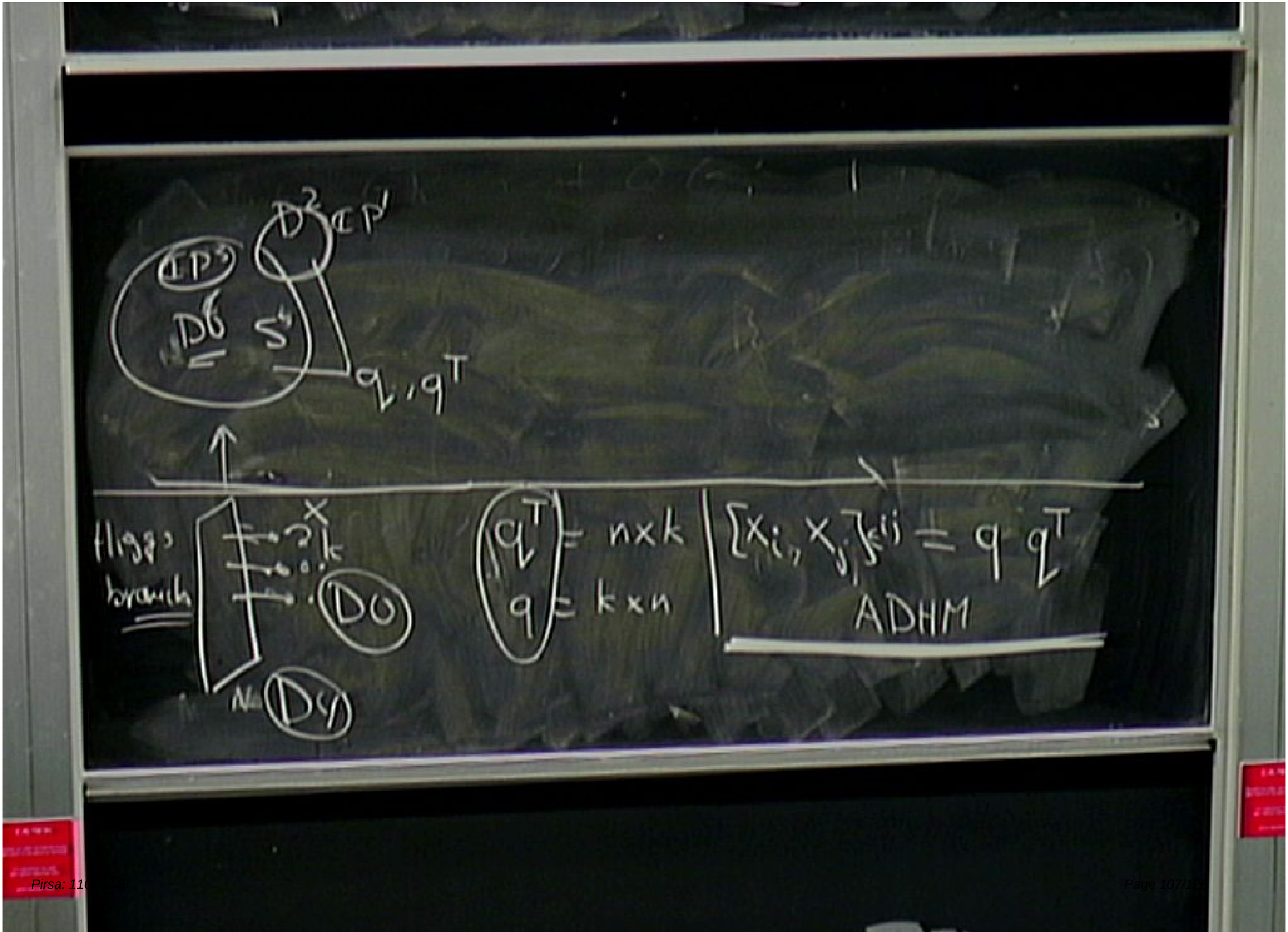

Z^α = (π_a, ω^ȧ)
Matrix theory
N D-particles
x^{ȧa} π_a = ω^ȧ
CP^1 ⊂ CP^3
q^T = n×k
q = k×n
[X_i, X_j]ε^{ij} = q q^T
ADHM
D0
D4

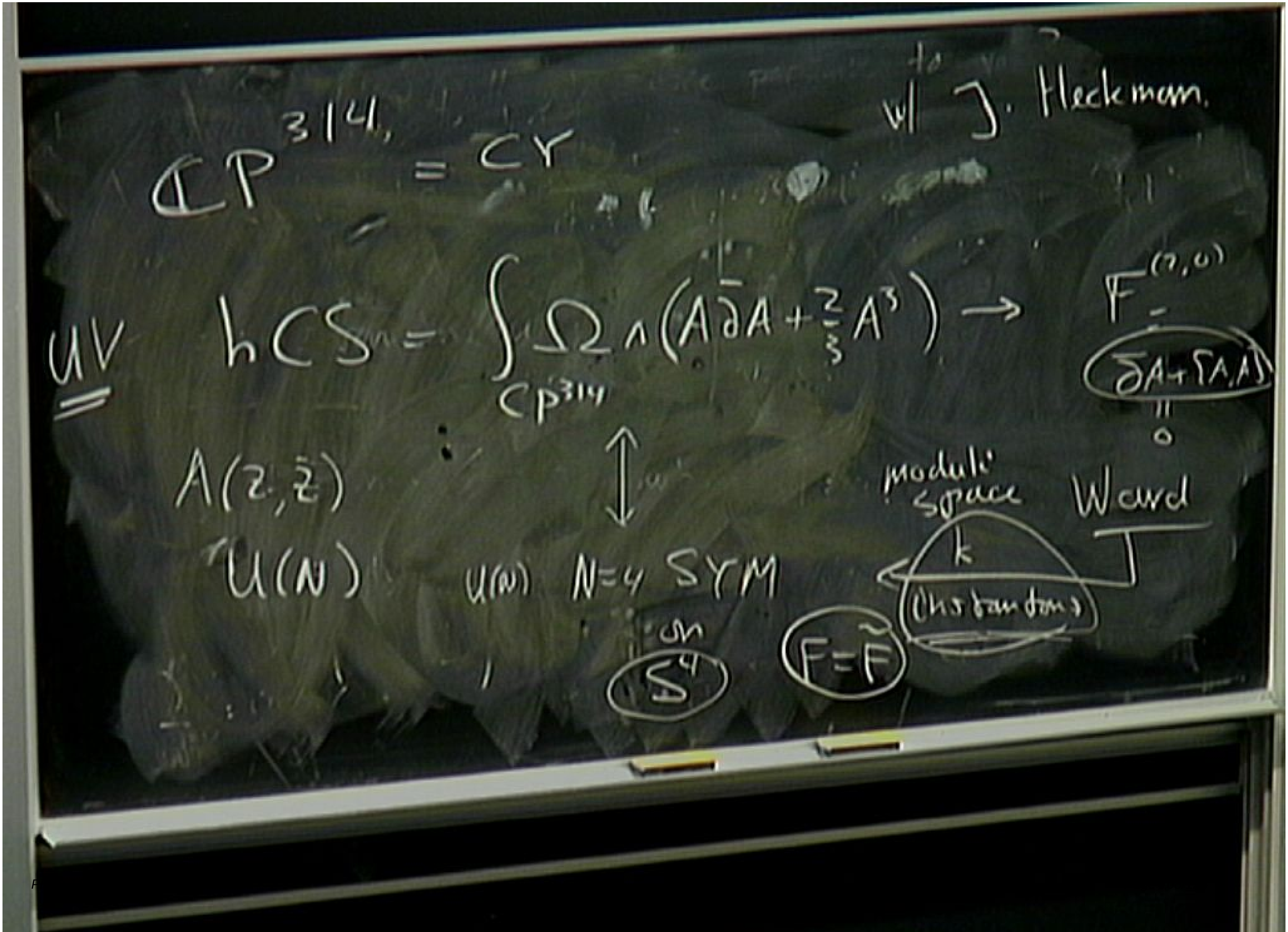

q^T = n×k
q = k×n
[X_i, X_j]ε^{ij} = q q^T
ADHM

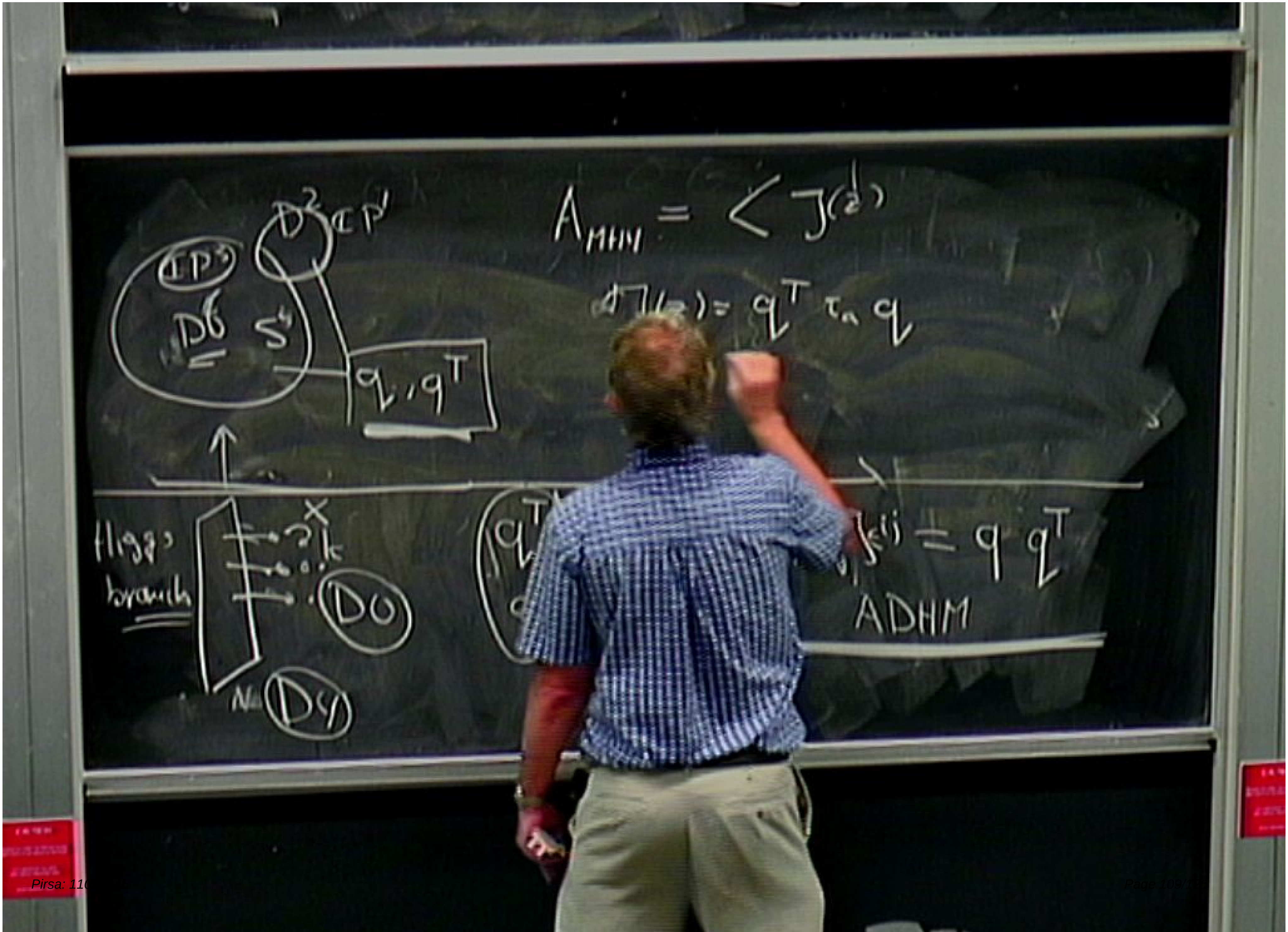

w/ J. Heckman
$\mathbb{C}P^{3|4}$ = CY
UV
hCS = $\int_{\mathbb{C}P^{3|4}} \Omega \wedge (A\bar{\partial}A + \frac{2}{3}A^3) \rightarrow F^{(2,0)}$
$\bar{\partial}A + [A,A]$ = 0
$A(z,\bar{z})$
U(N)
moduli space
Ward
k
instantons
U(N) N=4 SYM
on
$S^4$

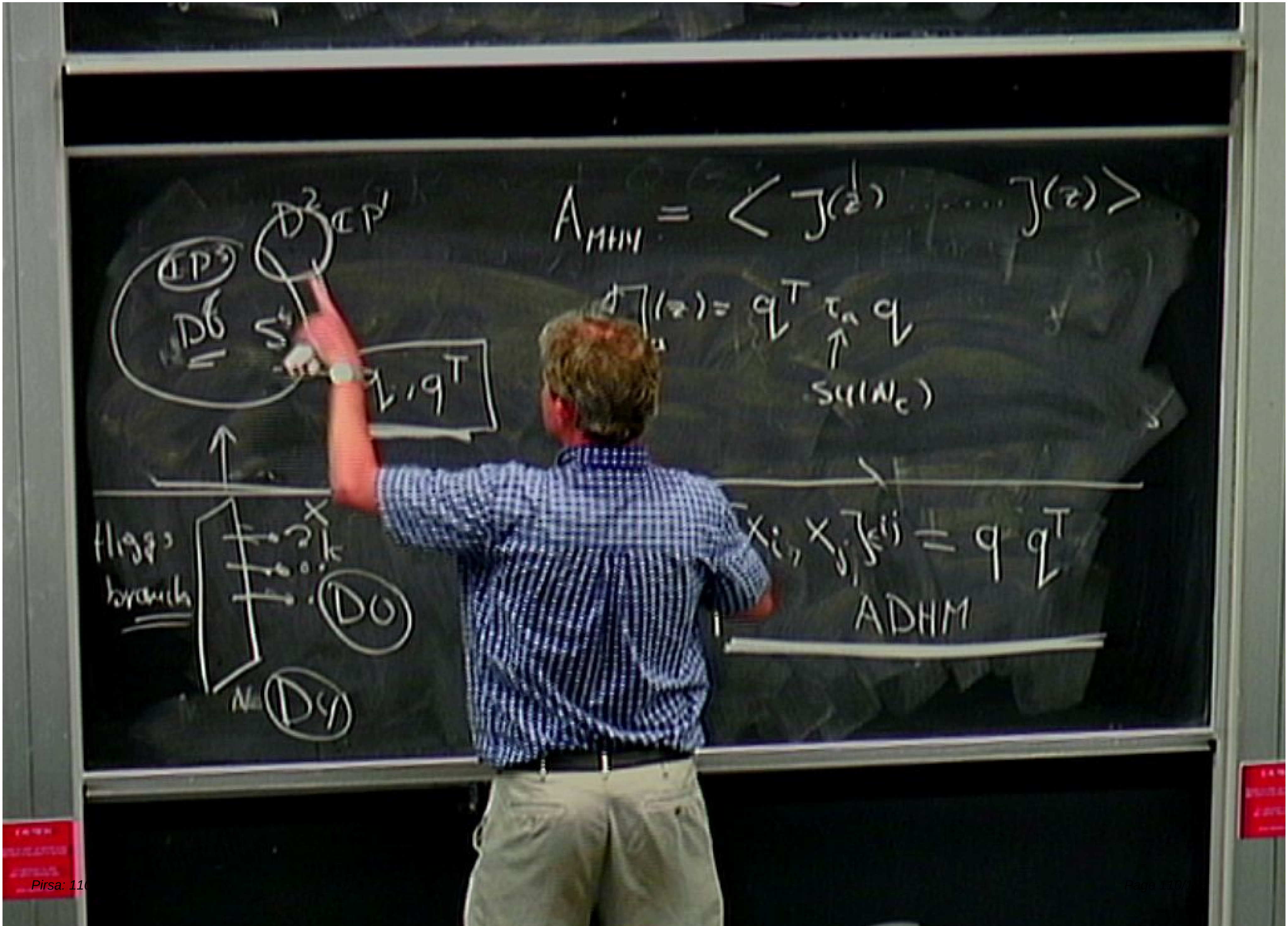

CP³
CP¹
k
D0
D4
qᵀ = n×k
q = k×n
[X_i, X_j]ε^ij = q qᵀ
ADHM

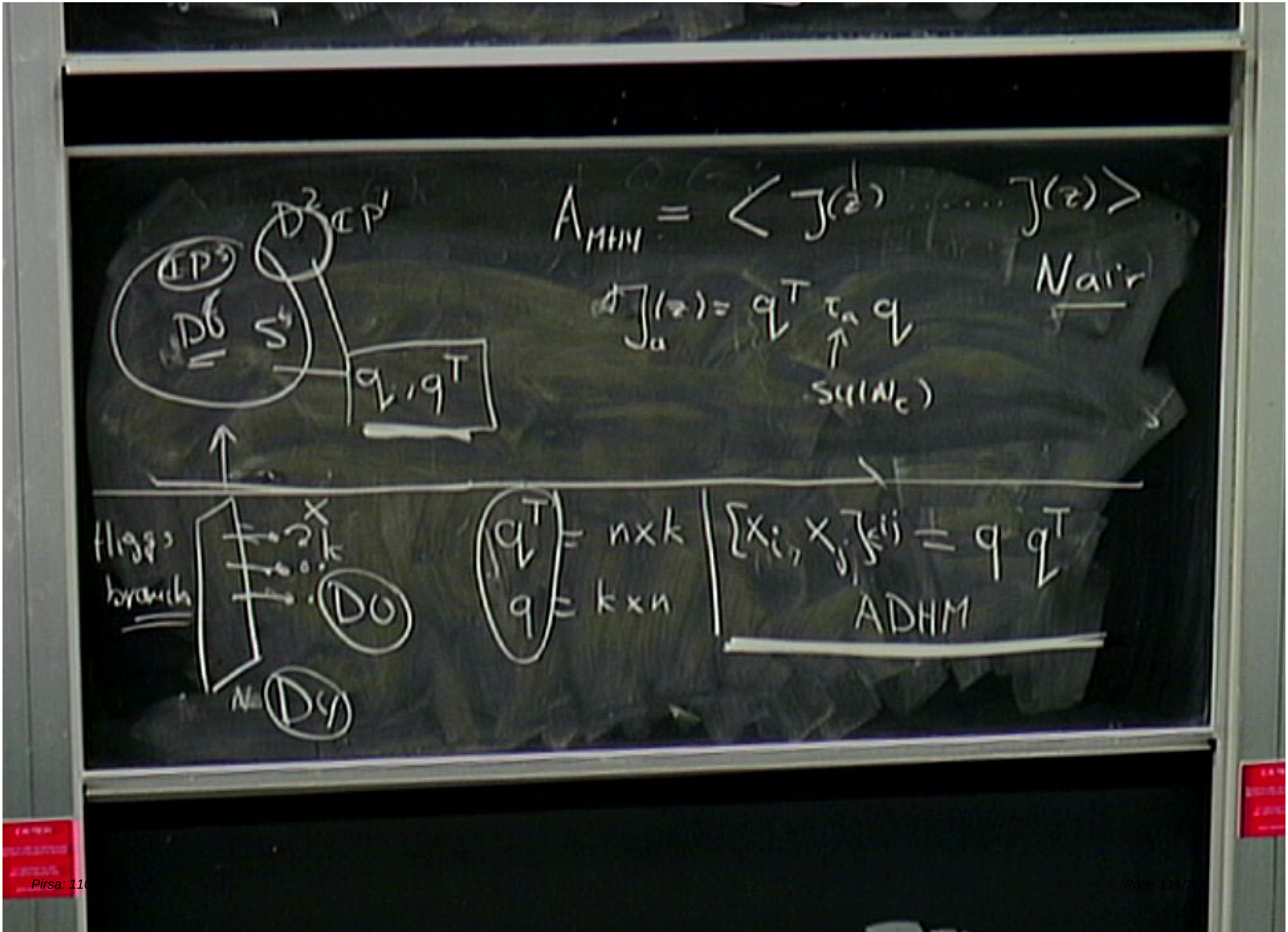

CP^3
CP^1
D6
S^4
Higgs branch
= n×k
q = k×n
[X_i, X_j]ε^{ij} = q q^T
ADHM


D2 ⊂ CP1
CP3
D6
S4
q, qT
Higgs branch
k
D0
N D4
qT = n×k
q = k×n
[Xi, Xj] εij = q qT
ADHM

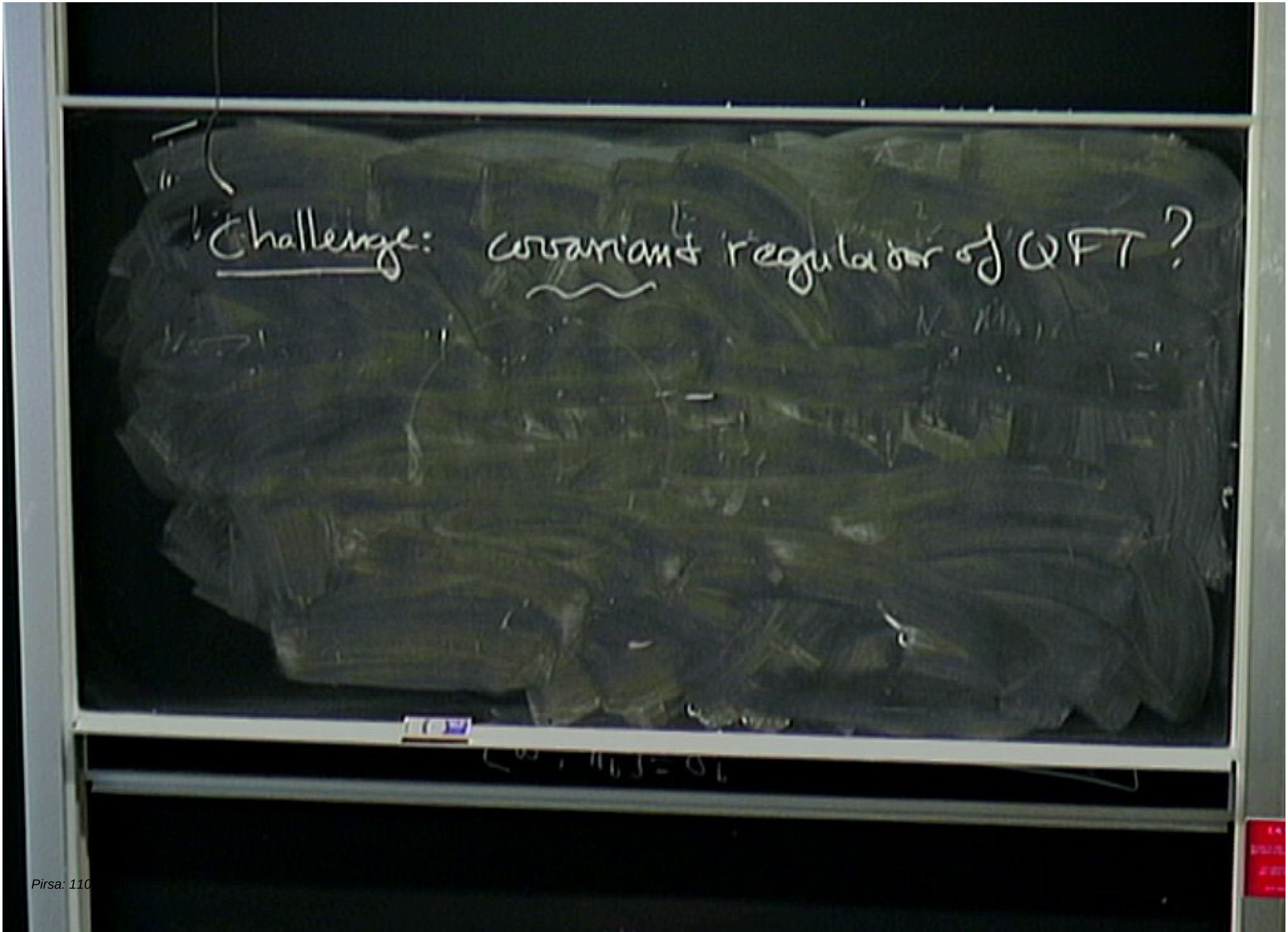

w/ J. Heckman.
$\mathbb{C}P^{3|4} = CY$
UV
$hCS = \int_{CP^{3|4}} \Omega \wedge (A\bar{\partial}A + \frac{2}{3}A^3) \to F^{(2,0)}$
$\bar{\partial}A + [A,A] = 0$
$A(z, \bar{z})$
U(N)
U(N) N=4 SYM
on $S^4$
$F = \tilde{F}$
moduli space
Ward
k
instantons

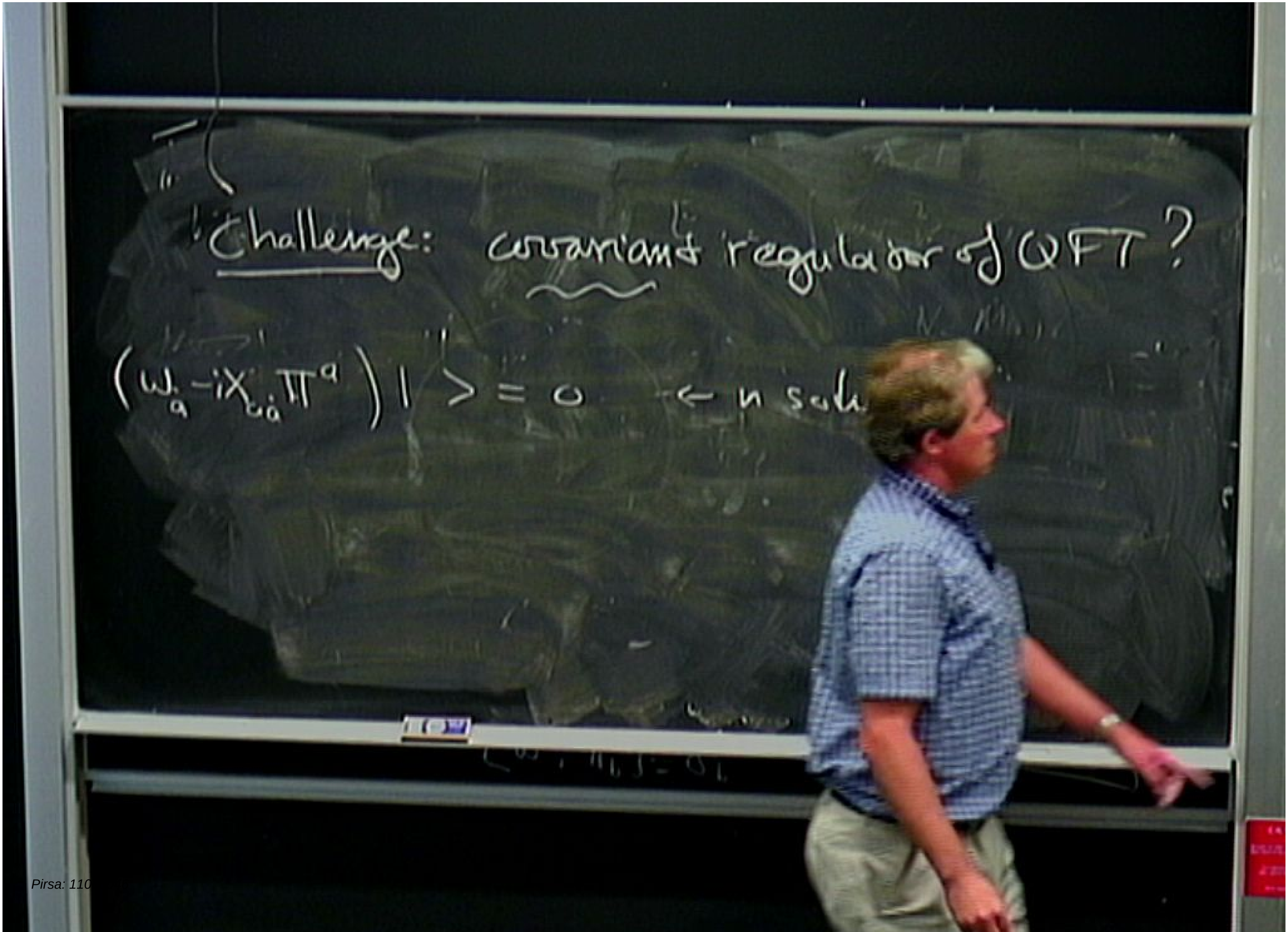

$A_{MHM} = \langle J(z)$
$q^T \tau_n q$
$q, q^T$
$CP^3$
$D^2 \; CP^1$
$D6$
$S^4$
Higgs branch
D0
D4
ADHM
$= q \, q^T$


$A_{MHM} = \langle J(z) \ldots\ldots J(z) \rangle$
$D^2 \mathbb{CP}^1$
$\mathbb{CP}^3$
$D6$
$S^4$
$q, q^T$
$J(z) = q^T \tau_a q$
$SU(N_c)$
$[X_i, X_j]\epsilon^{ij} = q\, q^T$
ADHM
Higgs branch
D0
D4



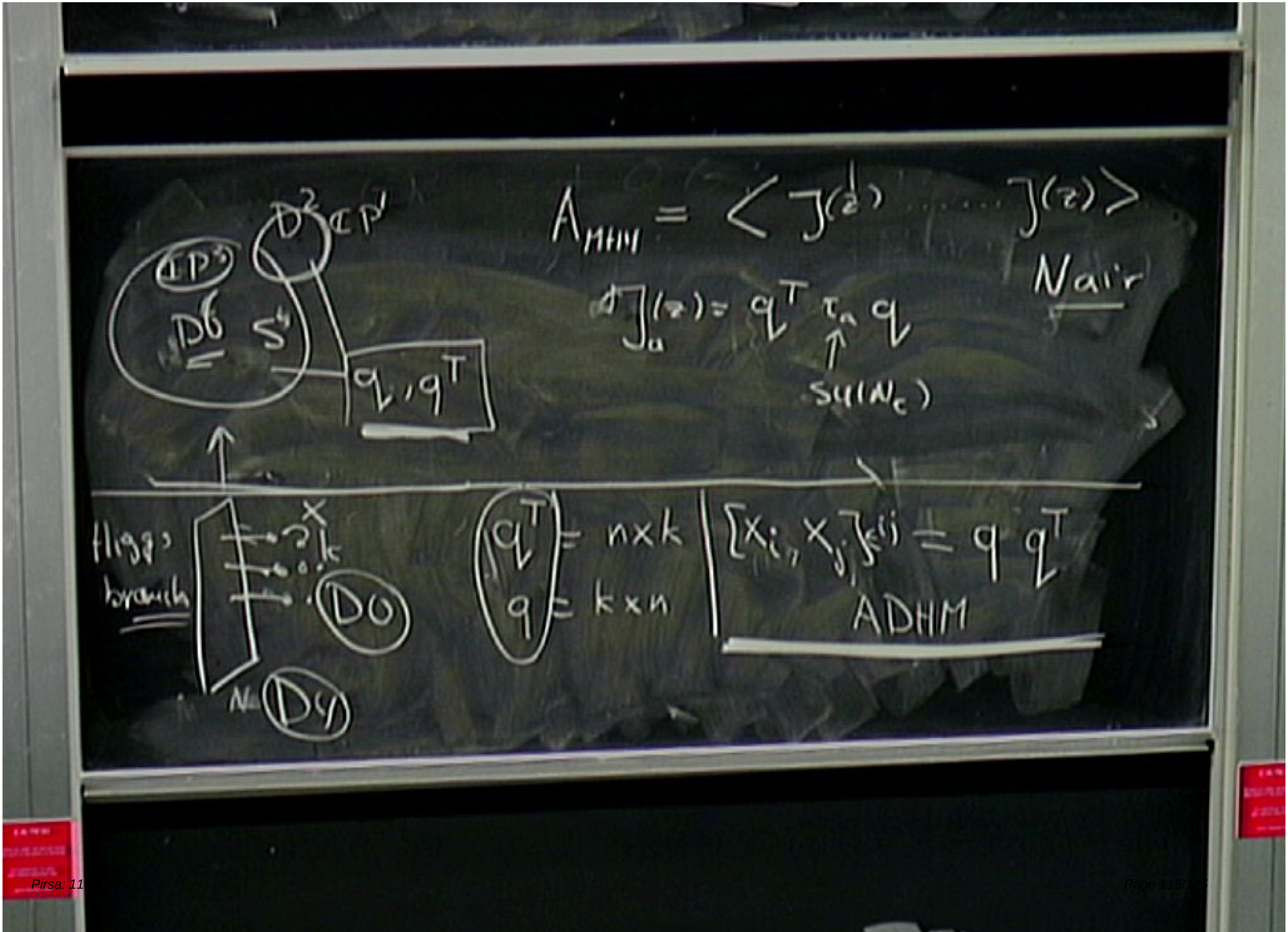
A_MHV = < J(z) ...... J(z) >
Nair
J_a(z) = q^T τ_a q
SU(N_c)
D^2
CP^1
CP^3
D6
S^4
q, q^T
X
k
Higgs branch
D0
N
D4
q^T = n×k
q = k×n
[X_i, X_j]ε^{ij} = q q^T
ADHM


Challenge: covariant regulator of QFT?


Challenge: covariant regulator of QFT ?
$(\omega_a - iX_{a\dot{a}}\Pi^a)\,|\;\rangle = 0$ ← n sol

Challenge: covariant regulator of QFT?

$$\left(\omega_{a} - i X_{a\dot{a}} \Pi^{\dot{a}}\right) |\ \rangle = 0 \quad \leftarrow n \text{ solutions.}$$



A_MHV = < J(z) ...... J(z) >
Nair
J_a(z) = q^T τ_a q
SU(N_c)
CP^3
D^2 CP^1
D6
S^4
q, q^T
X
Higgs branch
k
D0
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q^T = n x k
q = k x n
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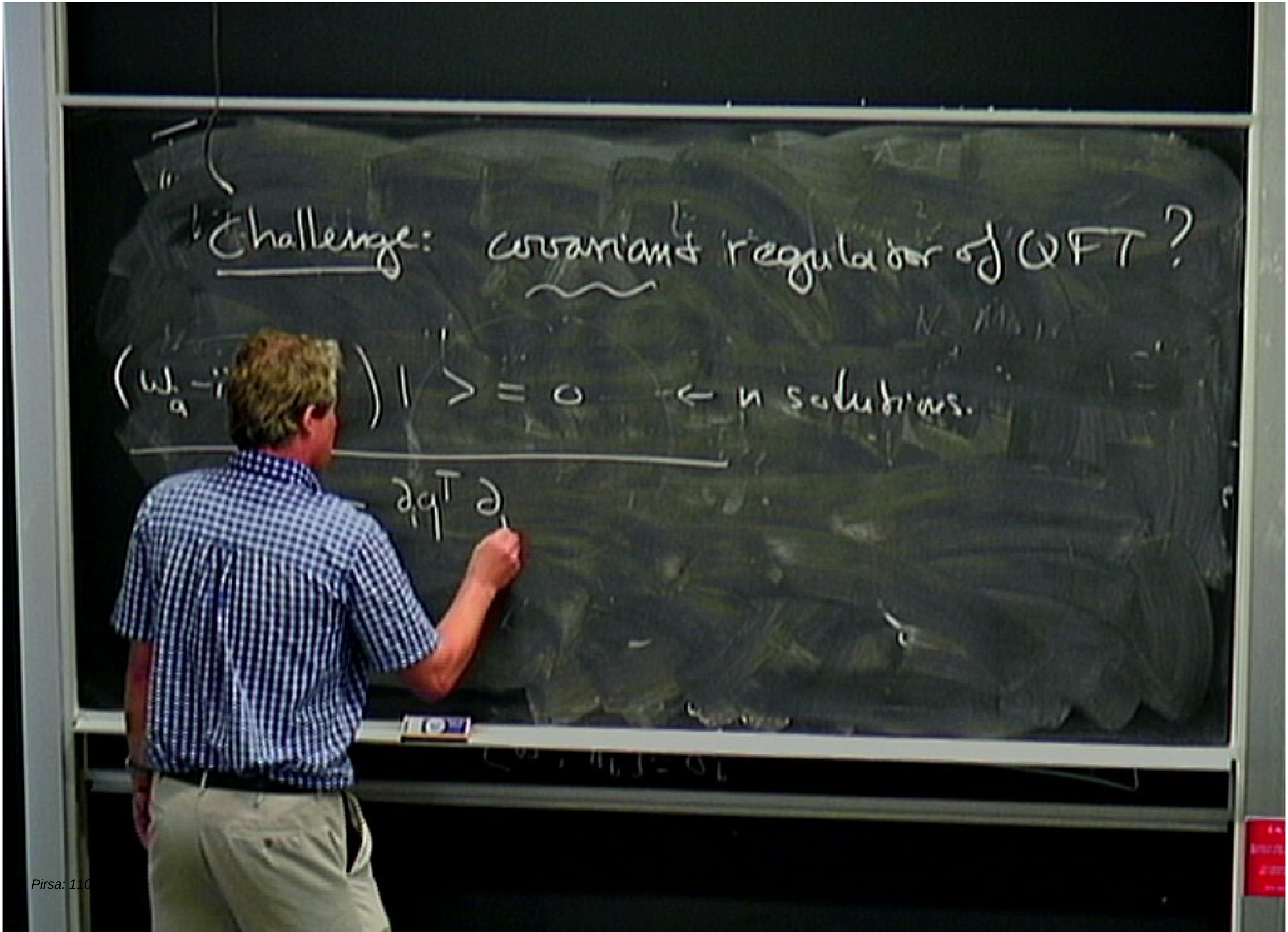
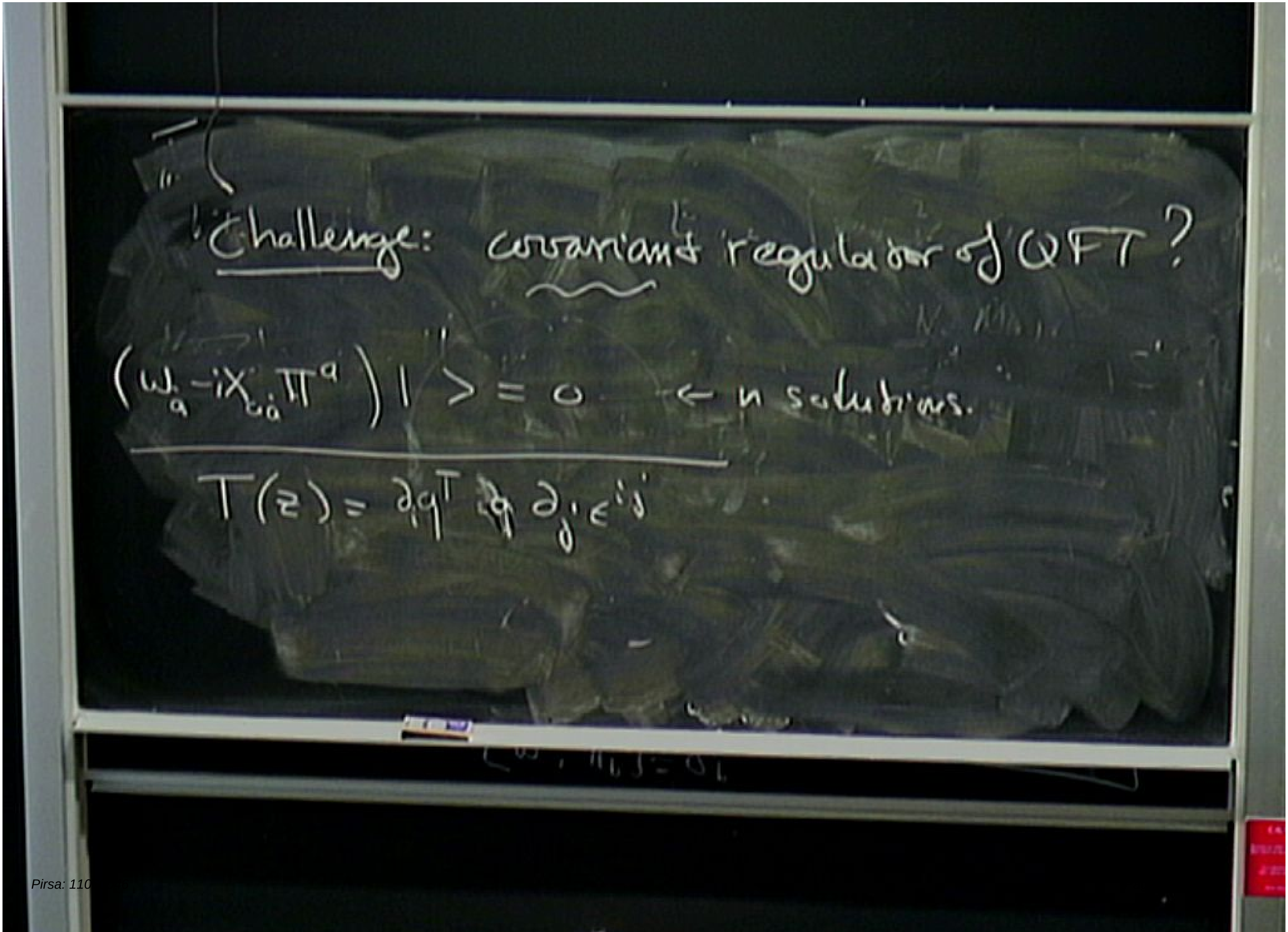
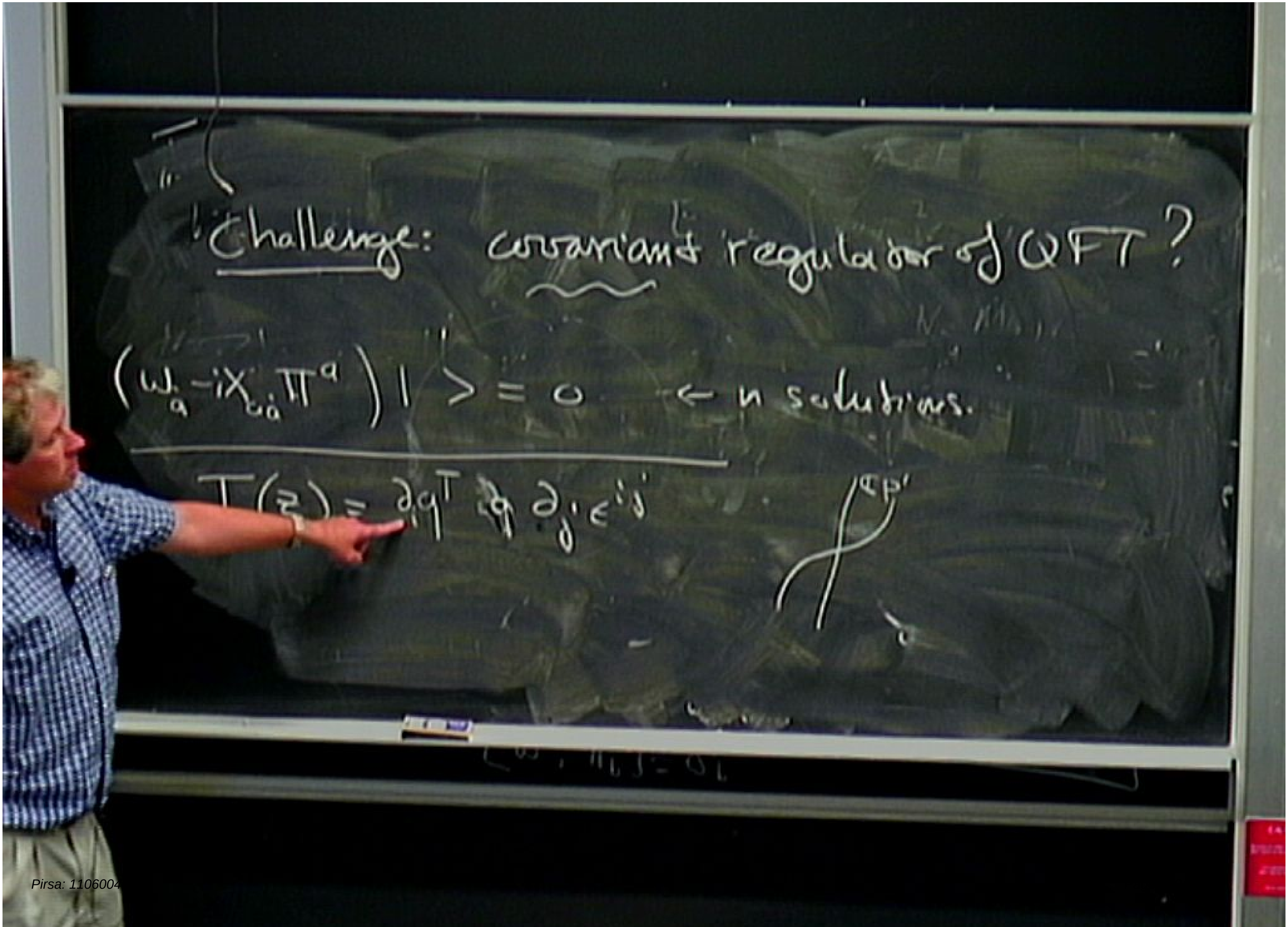
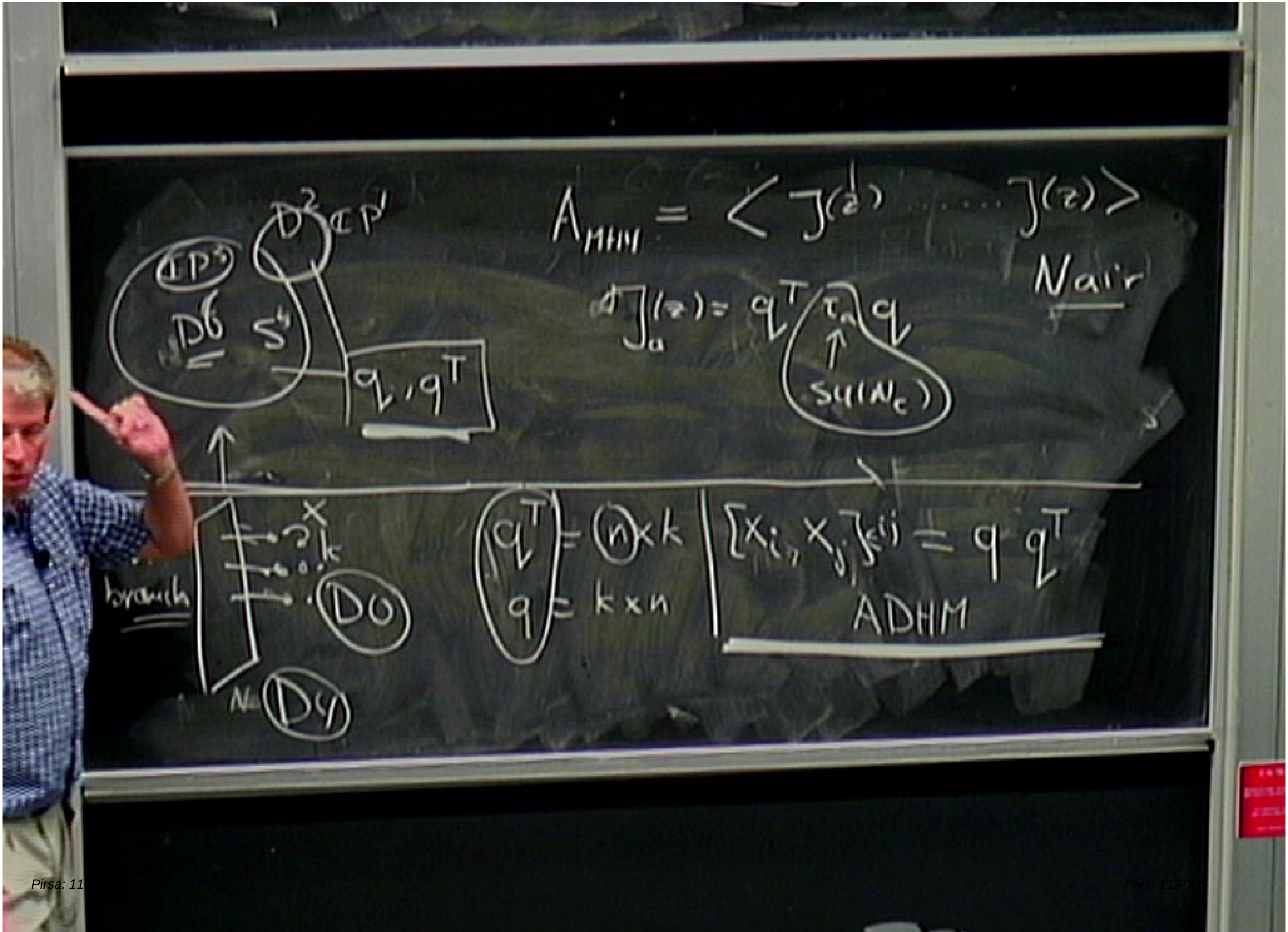
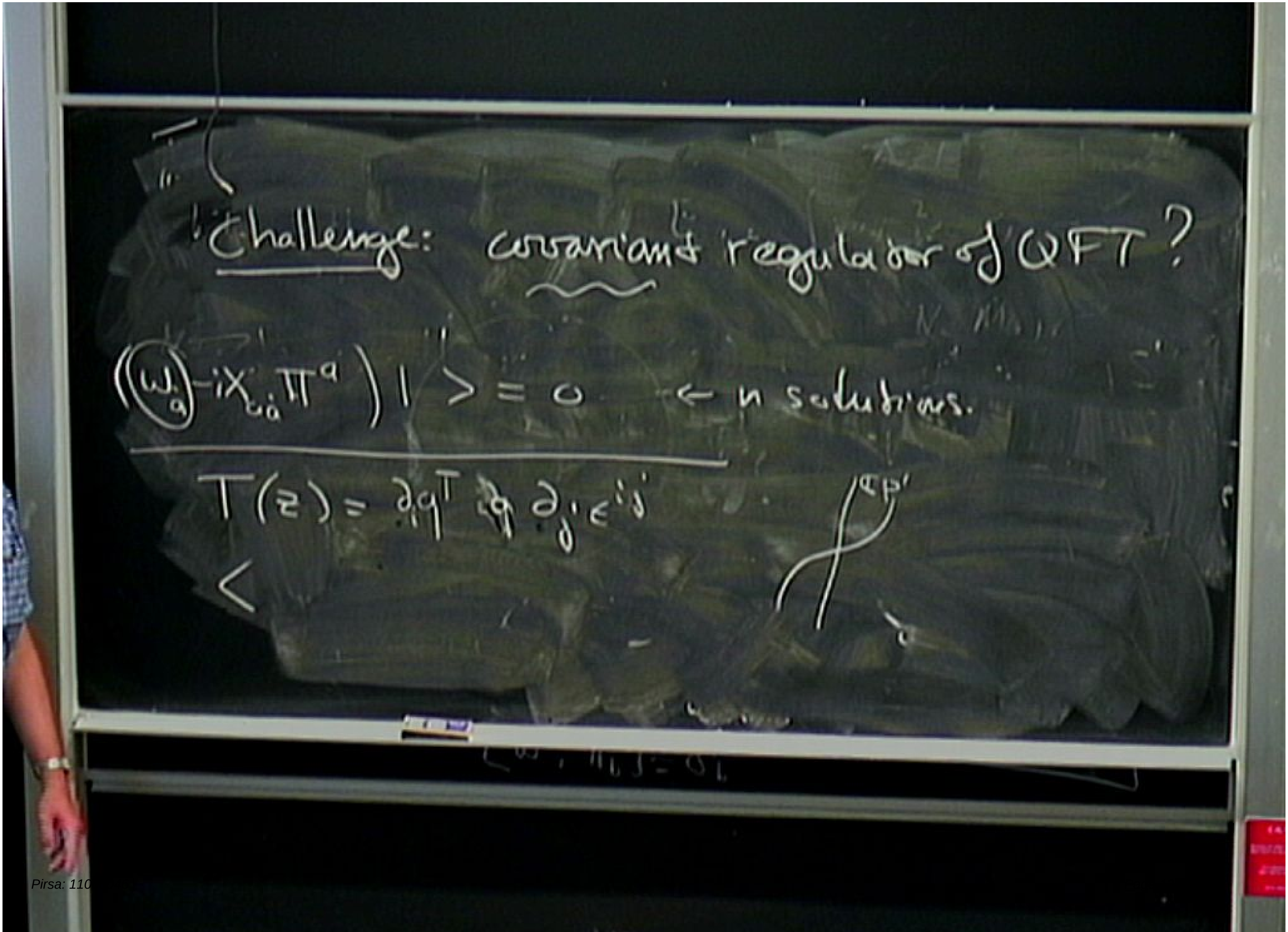
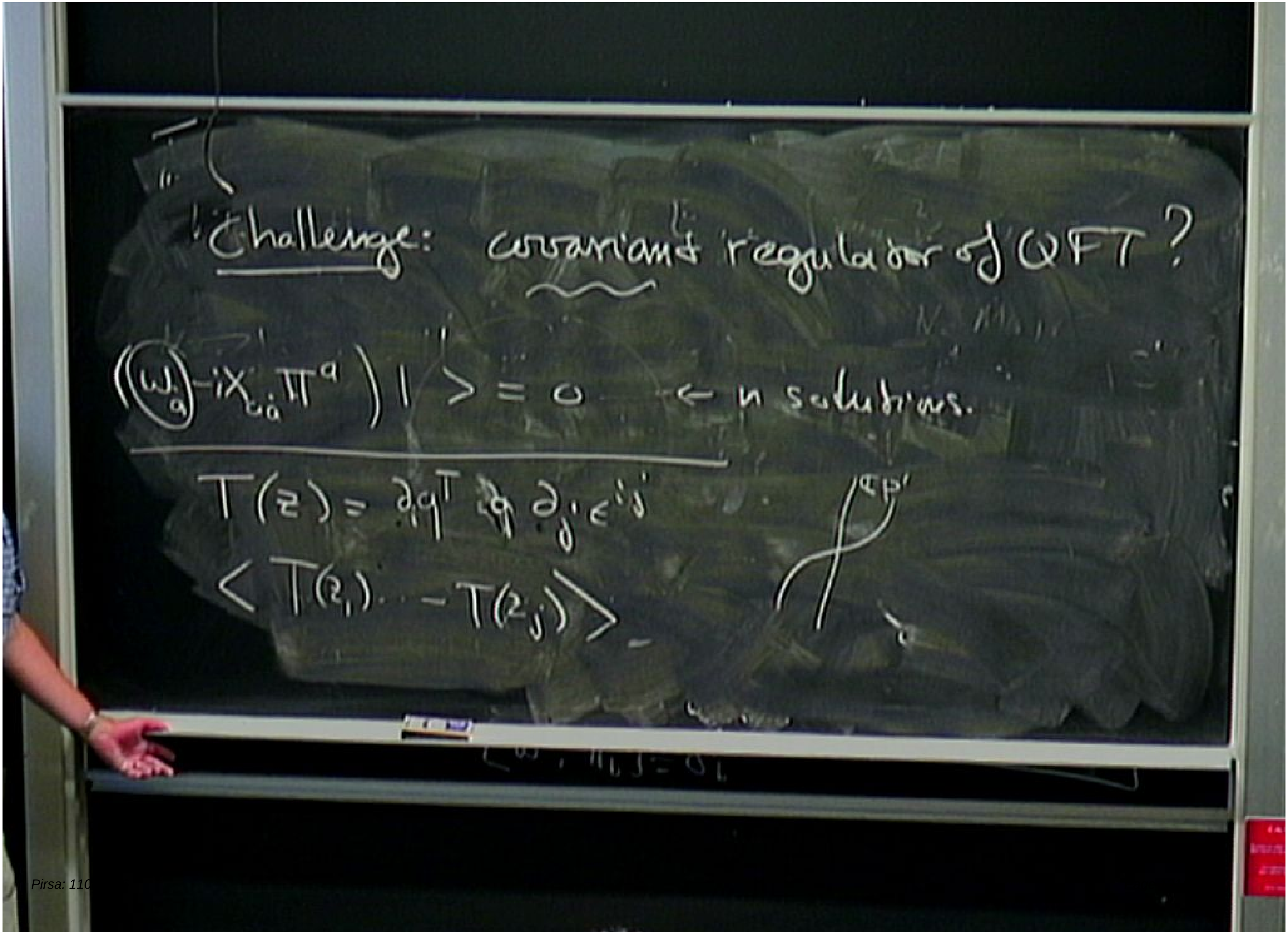

Challenge: covariant regulator of QFT ?
← n solutions.

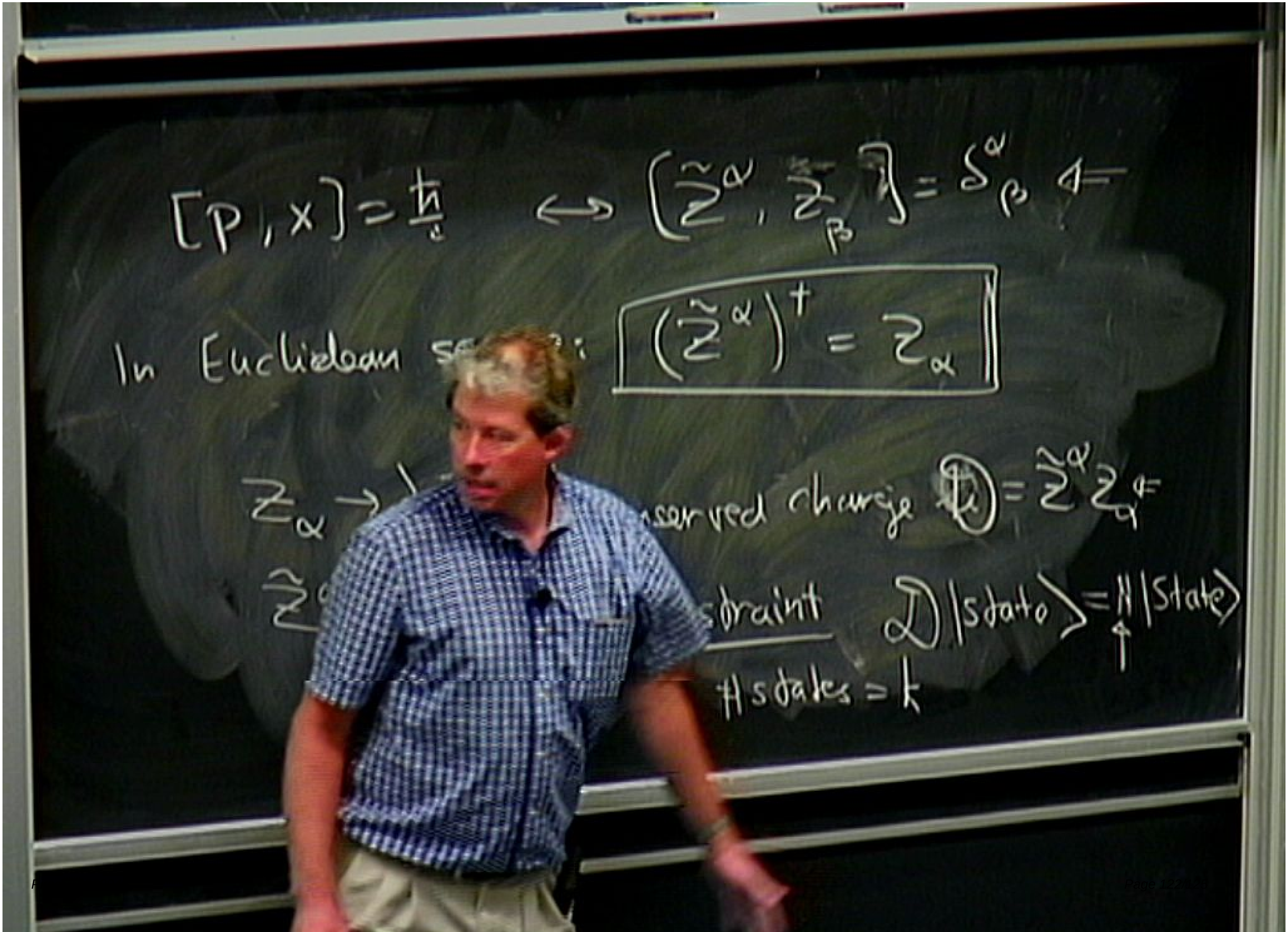

Challenge: covariant regulator of QFT ?
(ω_a − iX_aa Π^a) | > = 0 ← n solutions.
T(z) = ∂_i q^T q ∂_j ε^ij

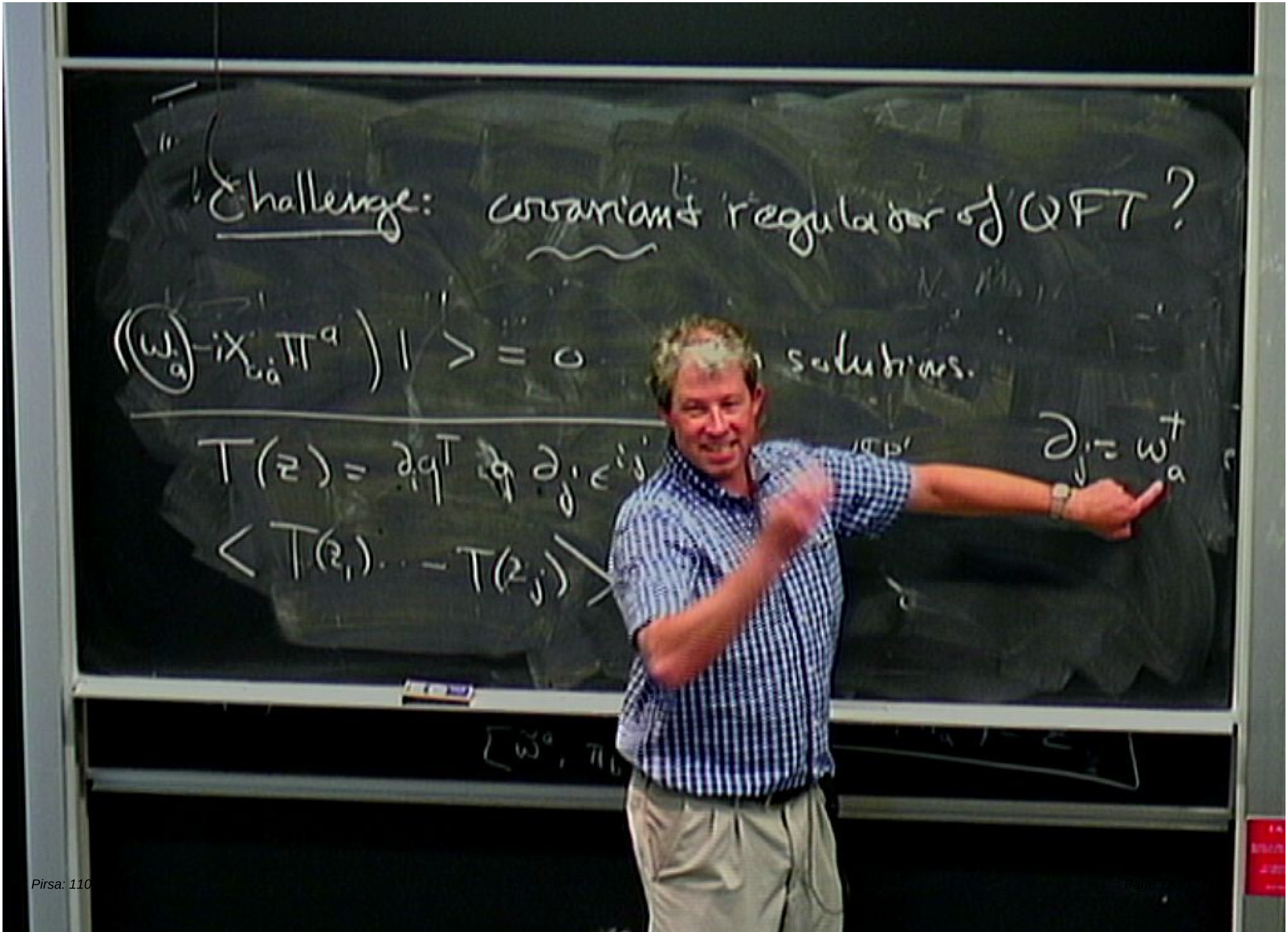

Challenge: covariant regulator of QFT ?
$(\omega_a - i X_{a\dot{a}} \Pi^a) | \rangle = 0$ ← n solutions.
$T(z) = \partial_i q^T$
Pirsa: 1106004

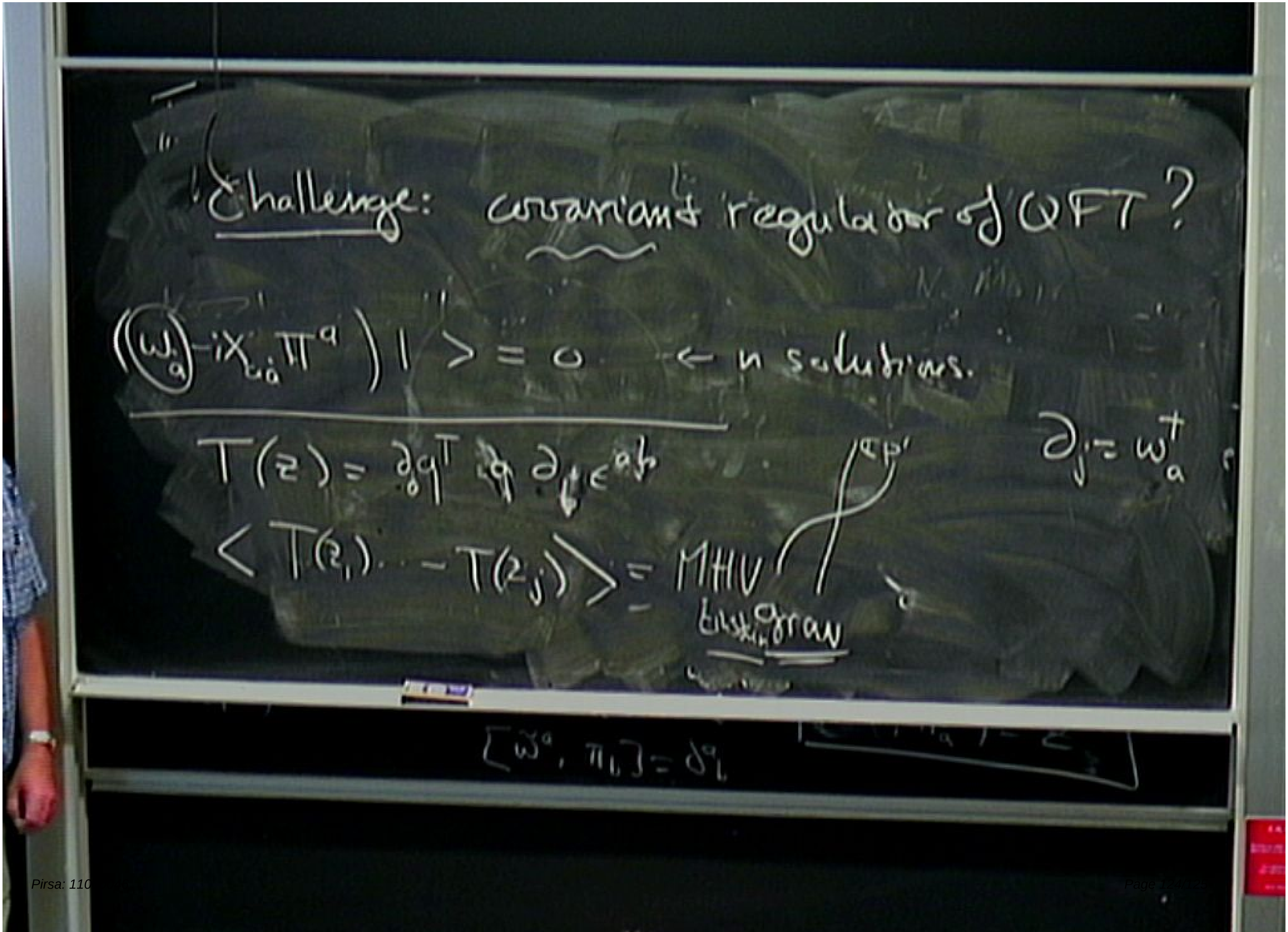

A_MHM = < J(z) ...... J(z) >
Nair
J_a(z) = q^T τ_a q
SU(N_c)
D2
CP^1
CP^3
D6
S^4
q, q^T
q^T = n×k
q = k×n
[X_i, X_j] ε^{ij} = q q^T
ADHM
branch
D0
D4

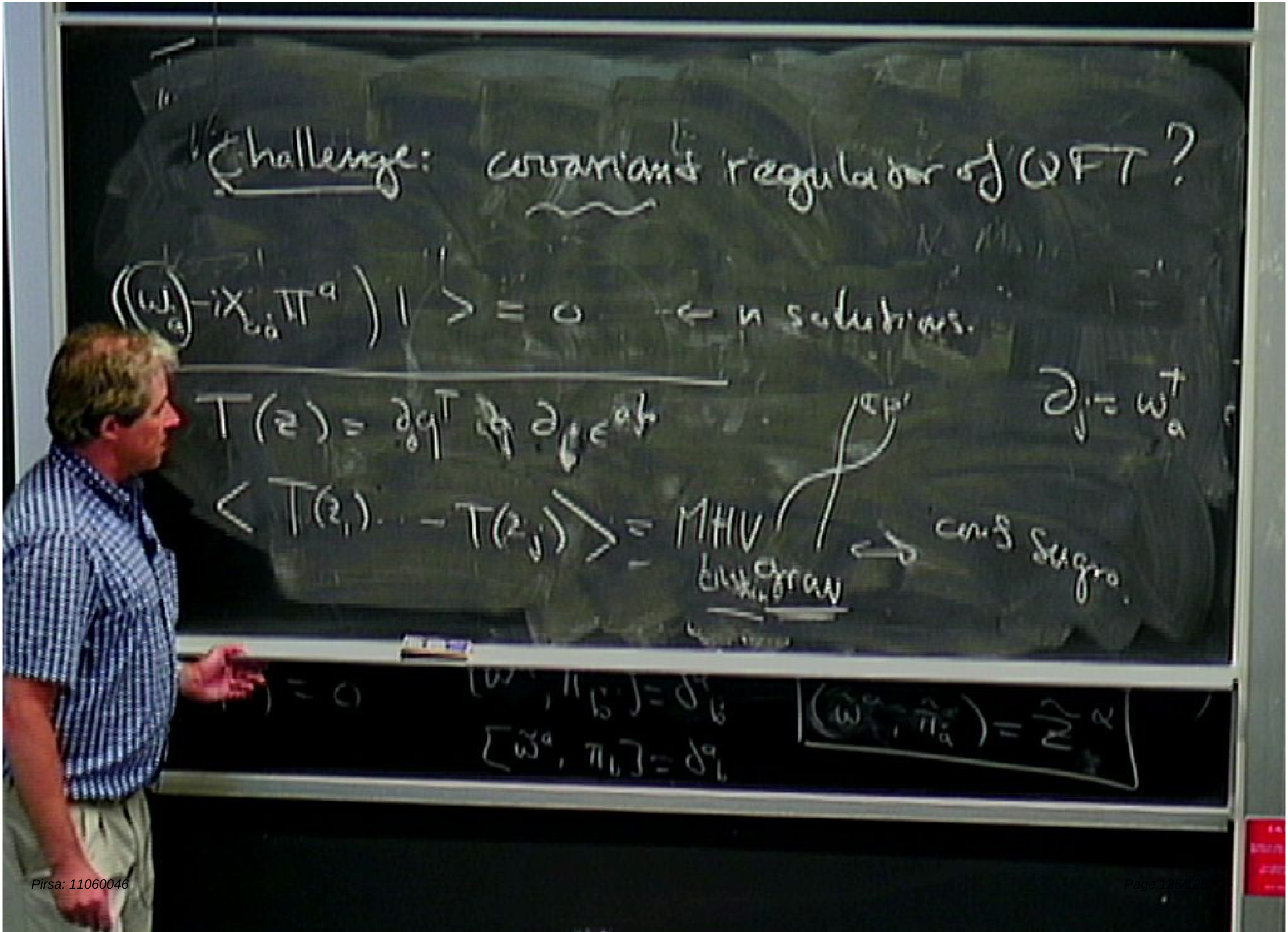

Challenge: covariant regulator of QFT ?
((ω_a) − iX_{aa} Π^a ) | > = 0 ← n solutions.
T(z) = ∂_i q^T ∂_q ∂_j ε^{ij}


Challenge: covariant regulator of QFT ?
$((\omega_a) - iX_{aa}\Pi^a)\,|\,\rangle = 0$ ← n solutions.
$T(z) = \partial_i q^T q\, \partial_j \epsilon^{ij}$
$\langle T(z_1) \ldots T(z_j) \rangle$


[p, x] = ħ/i
In Euclidean
served charge
straint
|state⟩ = N|state⟩
#states = k


Challenge: covariant regulator of QFT ?
solutions.


Challenge: covariant regulator of QFT?
$((\omega_a) - i X_{\dot{a}a} \pi^a) | \rangle = 0$ ← n solutions.
$T(z) = \partial q^T q\, \partial_b \epsilon^{ab}$
$\partial_j = \omega^\dagger_a$
$\langle T(z_1) \cdots T(z_j) \rangle = \text{MHV}$
$[\omega^a, \pi_b] = \delta^a_b$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MHV