Title: A Monte Carlo Method for Radiation Transport

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URL: http://pirsa.org/11060025

Abstract:

# Plan

- Introduction
- Implicit Monte Carlo for photons and neutrinos
- Discrete-diffusion scheme
- Velocity dependence
- Tests

# Neutrino transport in core-collapse supernovae

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n} \cdot \nabla I = \eta - \kappa I$$

# Monte Carlo transport

# Neutrino transport in core-collapse supernovae

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n} \cdot \nabla I = \eta - \kappa I$$

# Monte Carlo transport

# Monte Carlo transport (vs deterministic methods)

- Simpler, and flexible
- Easy multi-D extension
- Parallel scaling
- Expensive (in lower-D?)

# Monte Carlo transport

- Spatial cells and timestep
- MC particle weight
- Sample MC particles in each cell
- Transport MC particles within a timestep
- Update $T$ (and $Y_e$ etc.)

# Some insight from the photon transport community:

Implicit Monte Carlo
[Fleck & Cummings '71]

Discrete-Diffusion Monte Carlo
[Densmore et al. '07]

# Why Implicit Monte Carlo?

$$\Delta t \lesssim \frac{\rho c_v}{a T^3 c \kappa}$$

# Implicit Monte Carlo for photons

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n}\cdot\nabla I = \kappa(B - I)$$

$$\frac{\partial U_m}{\partial t} = \int\int \kappa(B - I)\,d\Omega\,d\epsilon$$

$$U_r(t) \simeq f_n U_{r,n} + \frac{1 - f_n}{c\tilde{\kappa}_n}\int\int \kappa_n I(t)\,d\Omega\,d\varepsilon$$

# Implicit Monte Carlo for photons

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mathbf{n}\cdot\nabla I = \kappa_{ea,n}(B_n - I)$$

$$+\chi_n \int\int \kappa_{es,n} I d\Omega d\varepsilon - \kappa_{es,n} I$$

$$\kappa_{ea,n} = f_n \kappa_n \qquad \kappa_{es,n} = (1 - f_n)\kappa_n$$

$$f_n = \frac{1}{1 + \alpha \Delta t_n \beta_n c \kappa_{a,n}} \qquad 0 \le f_n \le 1$$

# Implicit Monte Carlo for neutrinos

$$\frac{1}{c}\frac{\partial I_i}{\partial t} + \mathbf{n} \cdot \nabla I_i = \kappa_{ai}(B_i - I_i)$$

$$\frac{dU_m}{dt} = \int \int \kappa_{ai}(I_i - B_i) d\Omega d\varepsilon$$

$$\rho N_A \frac{dY_e}{dt} = \Sigma_i s_i \int \int \frac{\kappa_{ai}}{\epsilon}(I_i - B_i) d\Omega d\varepsilon$$

$$f_n = \frac{1}{1 + \alpha c \Delta t_n \gamma_p}$$

# Implicit Monte Carlo at high optical depth

At high optical depth $1/\kappa << \Delta x$ :

$$f_n \to 0$$

$$k_{es,n} = k_a, \quad k_{ea,n} = 0,$$

Effective scattering dominates at high optical depth!

# Treatment of high optical depth

# Gray Discrete-Diffusion
(Densmore et al '07)

$$J = \frac{1}{4\pi}\int I(x,\mu,t)d\Omega$$

$$\frac{1}{c}\frac{\partial J}{\partial t} + \frac{\partial H}{\partial r} = f_n\tilde{\kappa}(B_n - J)$$

$$J_j = \frac{1}{\Delta x_j}\int_{x_{j-1/2}}^{x_{j+1/2}} \phi(x,t)dx \qquad H = -\frac{1}{3\kappa_a}\frac{\partial J}{\partial x}$$

# Treatment of high optical depth

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# Gray Discrete-Diffusion

(Densmore et al '07)

$$\frac{1}{c}\frac{dJ_j}{dt} = f_{n,j}\kappa_{n,j}B_{n,j} - (\kappa_{L,j} + \kappa_{R,j} + f_n\kappa_{a,j})J_j$$

$$+\frac{1}{\Delta x_j}(\kappa_{L,j+1}J_{j+1}\Delta x_{j+1} + \kappa_{R,j-1}J_{j-1}\Delta x_{j-1}$$

Multi group case: “transport” through energy groups.

# Velocity dependence

# Velocity-dependent Monte Carlo: Mixed frame formalism

Emissivities and opacities are calculated in the comoving frame and the transport is performed in the lab frame.

$$\kappa(\mu, \epsilon) = (\epsilon_0/\epsilon)\kappa_0(\epsilon_0)$$

# Velocity-dependent discrete-diffusion

Transport is performed in comoving frame with O(λ*v*/Δ*l*/*c*) accuracy:

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{v}{c}\frac{\partial J_0}{\partial r} + \frac{J_0}{c}\frac{\partial v}{\partial r} + \frac{\varepsilon_0}{3c}\frac{\partial J_0}{\partial \varepsilon_0}\frac{D \ln \rho}{Dt} + \frac{\partial H_0}{\partial x} = \kappa_0(B - J_0$$

Three effects: advection, compression, and Doppler shift.

# Velocity-dependent discrete-diffusion: operator splitting

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{v}{c}\frac{\partial J_0}{\partial r} + \frac{J_0}{c}\frac{\partial v}{\partial r} + \frac{\varepsilon_0}{3c}\frac{\partial J_0}{\partial \varepsilon_0}\frac{D\ln\rho}{Dt} + \frac{\partial H_0}{\partial x} = \kappa_0\left(B - J_0\right)$$

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{\partial H_0}{\partial x} = \kappa_0(B - J_0)$$

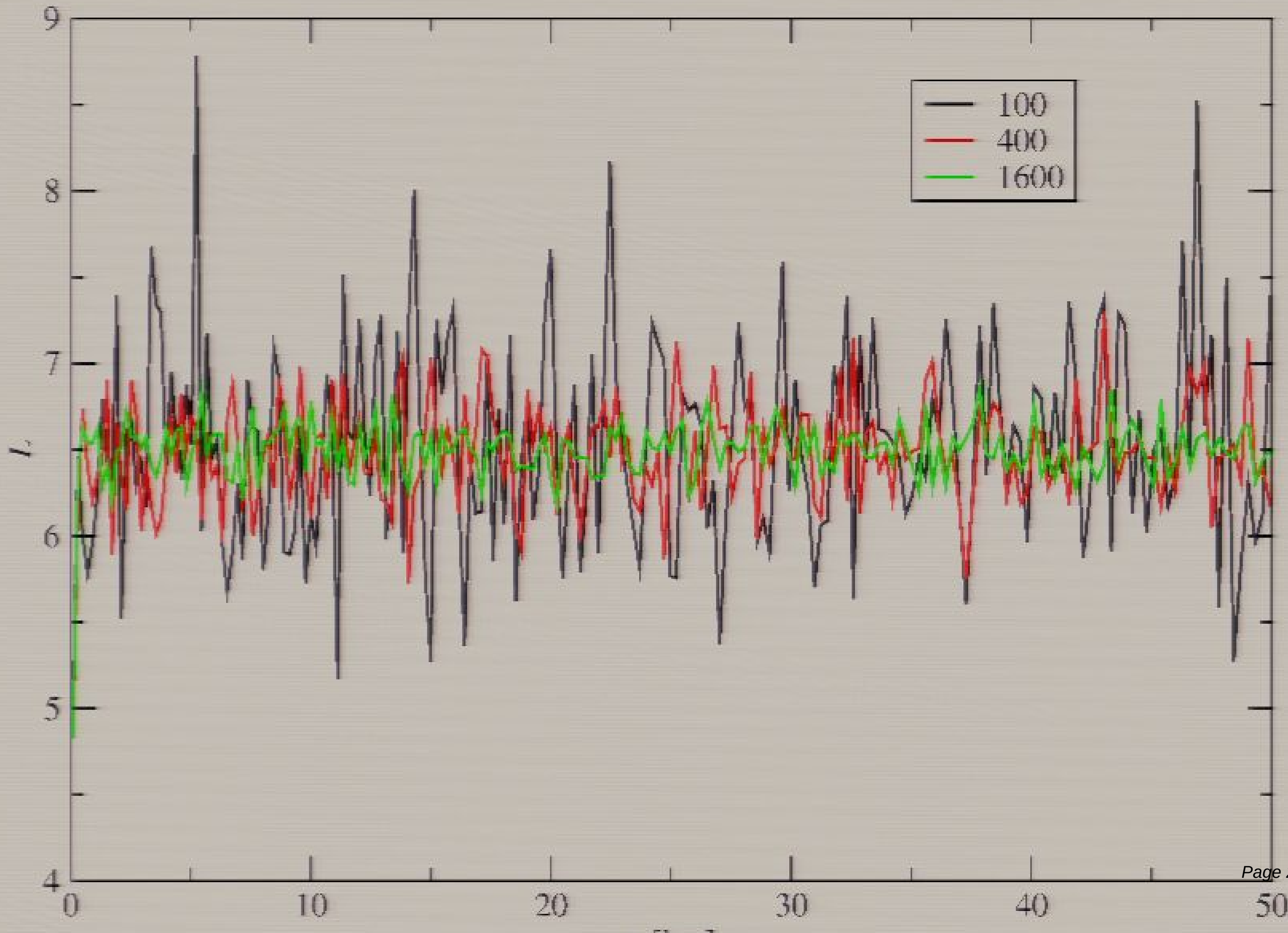
$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{\varepsilon_0}{3c}\frac{\partial J_0}{\partial \varepsilon_0}\frac{D\ln\rho}{Dt} = 0$$

$$\frac{1}{c}\frac{dJ_0}{dt} + \frac{v}{c}\frac{\partial J_0}{\partial r} + \frac{J_0}{c}\frac{\partial v}{\partial r} = 0$$

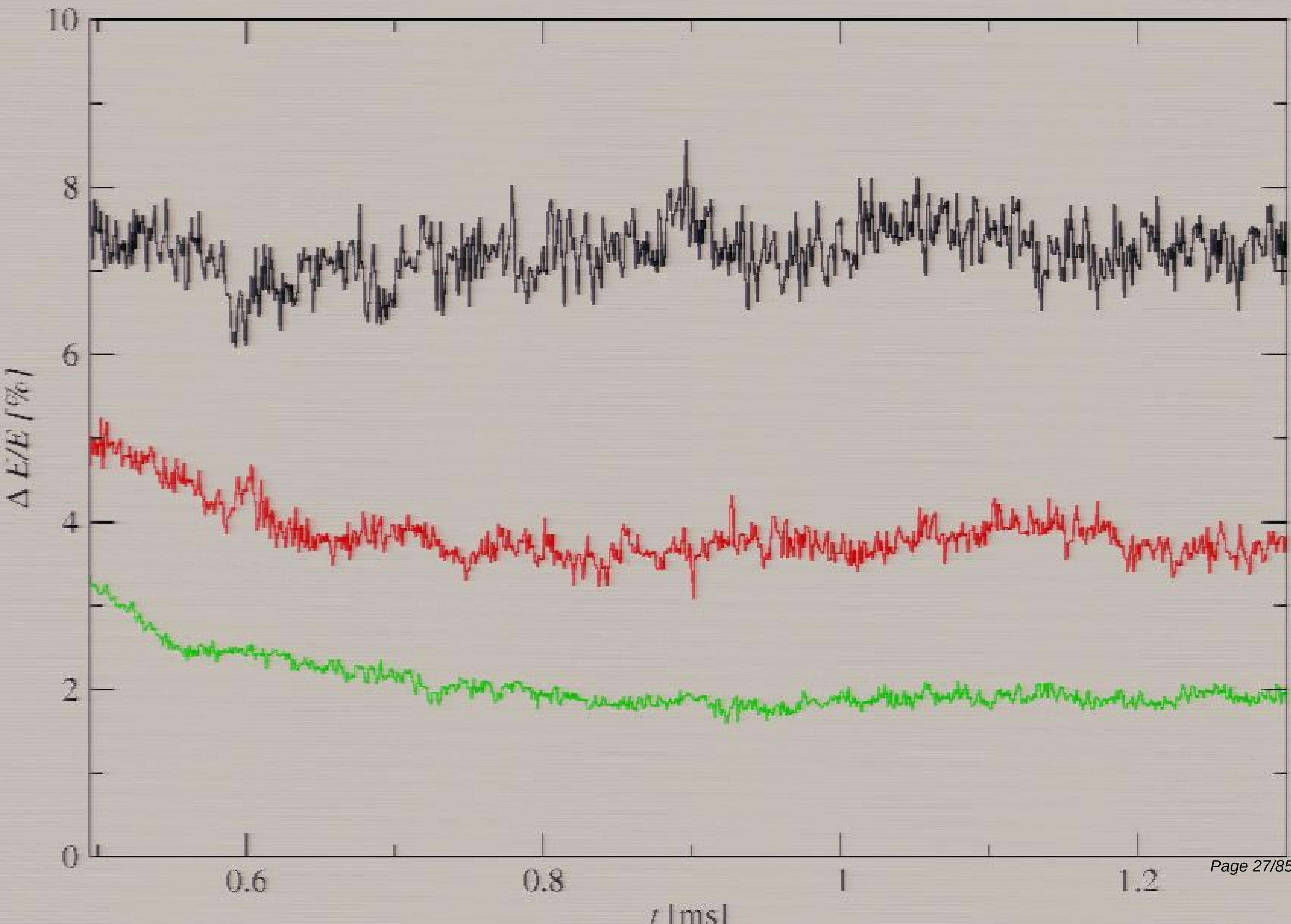
# 1D spherically symmetric code

# Test I
# Static scattering atmosphere

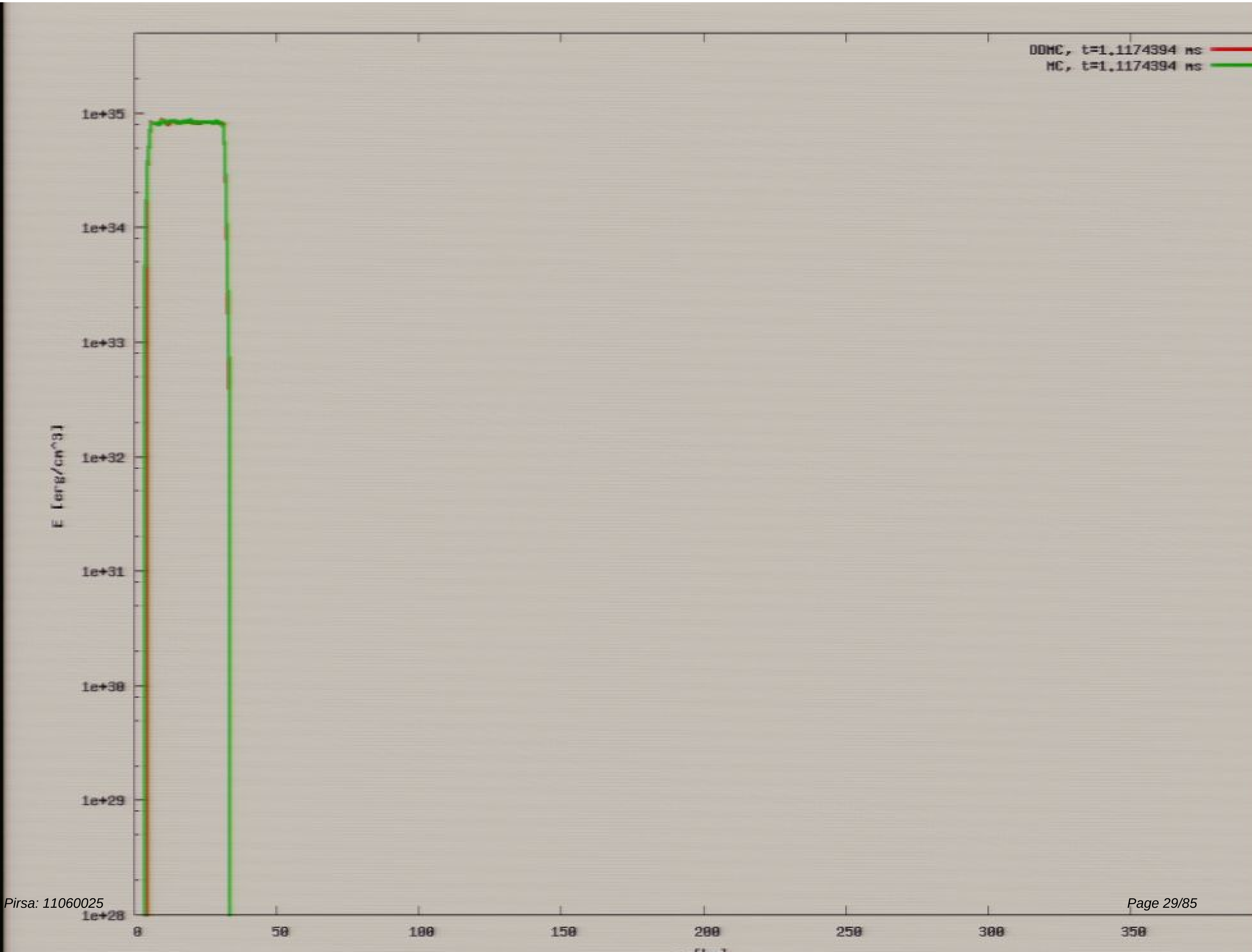
# Tests I: Static scattering atmosphere



# Test I: Static scattering atmosphere



# Test II
# Homologously expanding shell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MC, t=1.1174394 ms
E [erg/cm^3]
1e+35
1e+34
1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
0
50
100
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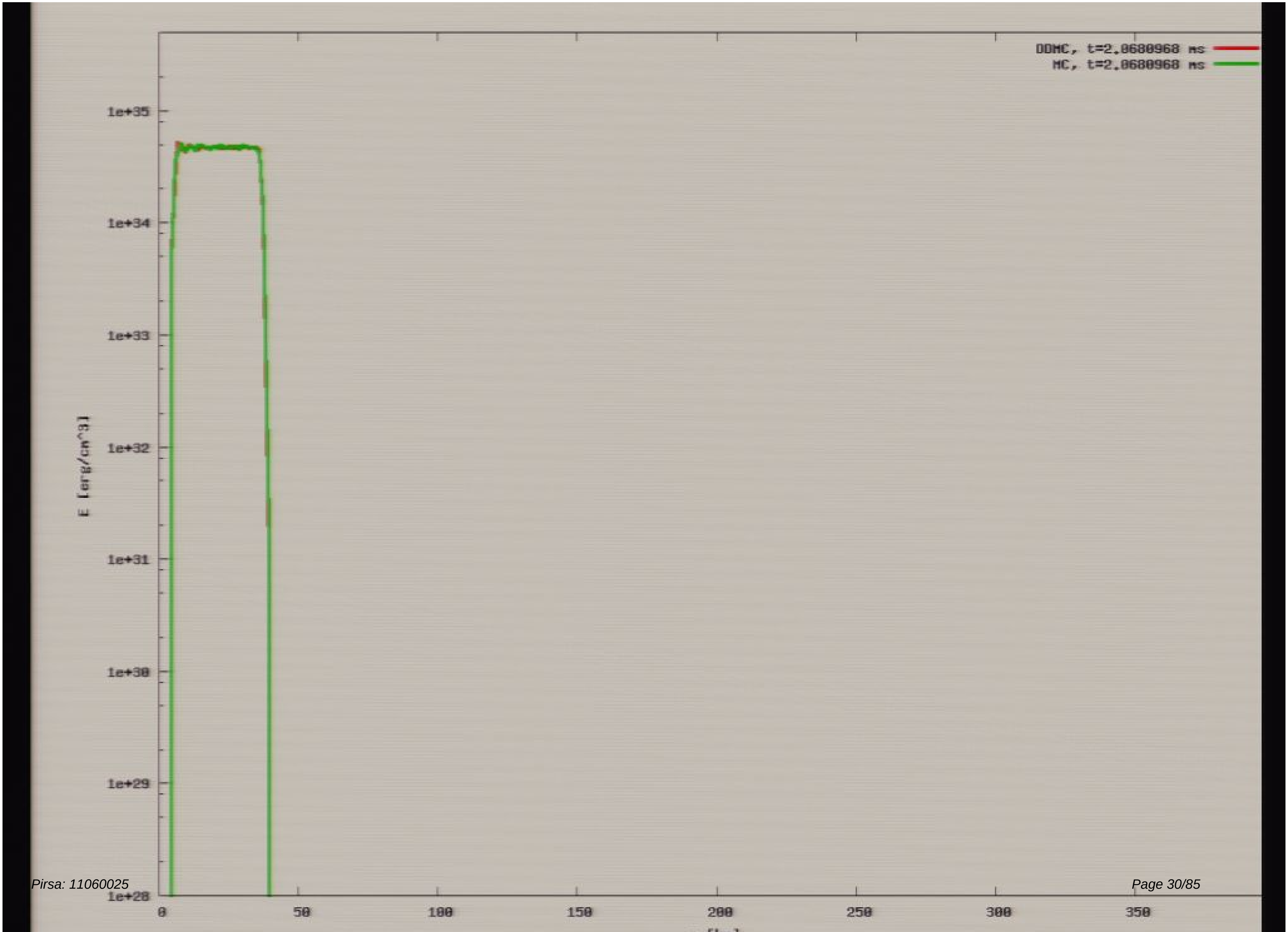
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1e+34
1e+33
1e+32
1e+31
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1e+29
1e+28
E [erg/cm^3]
0
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100
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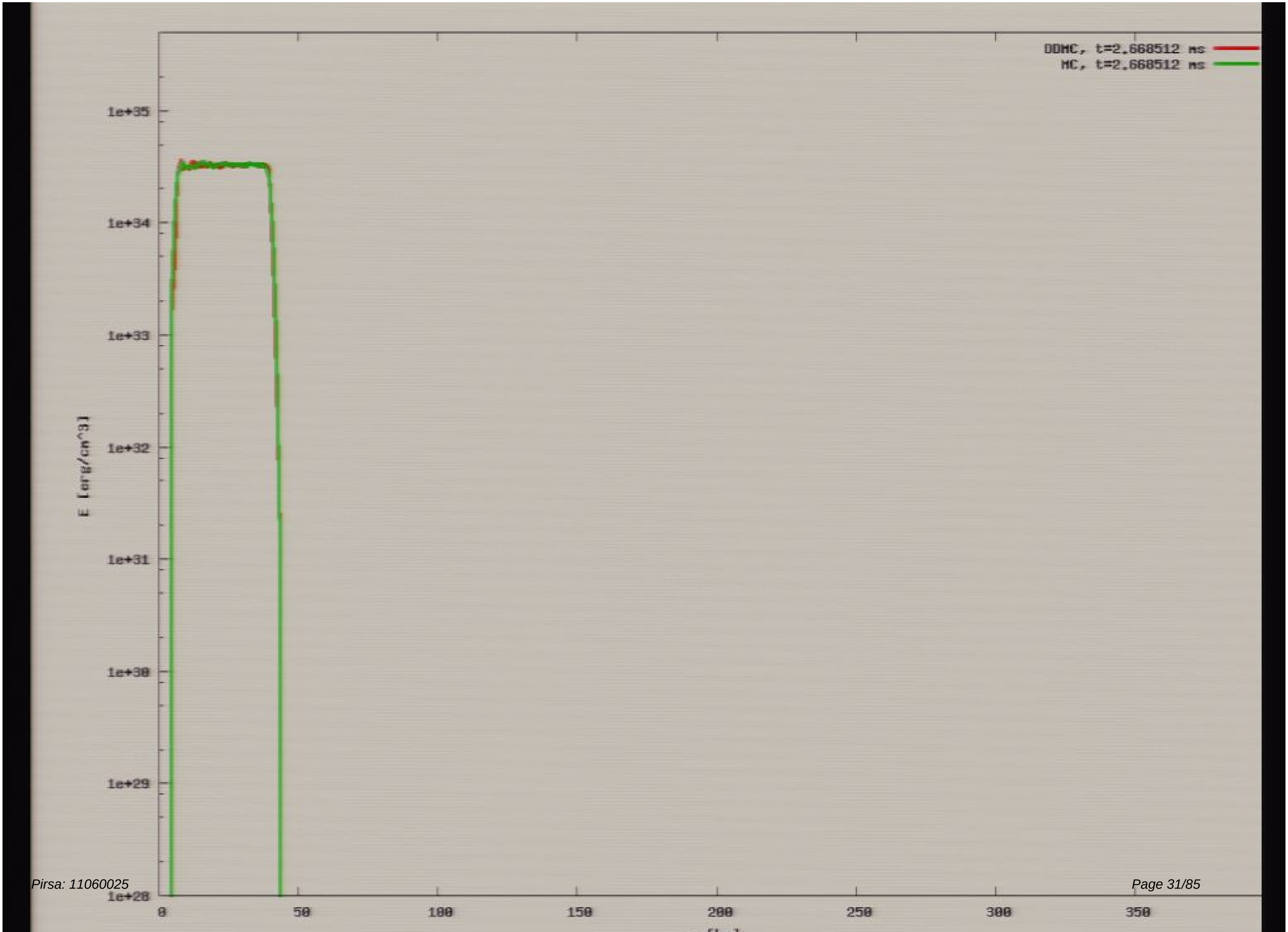
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MC, t=2.668512 ms
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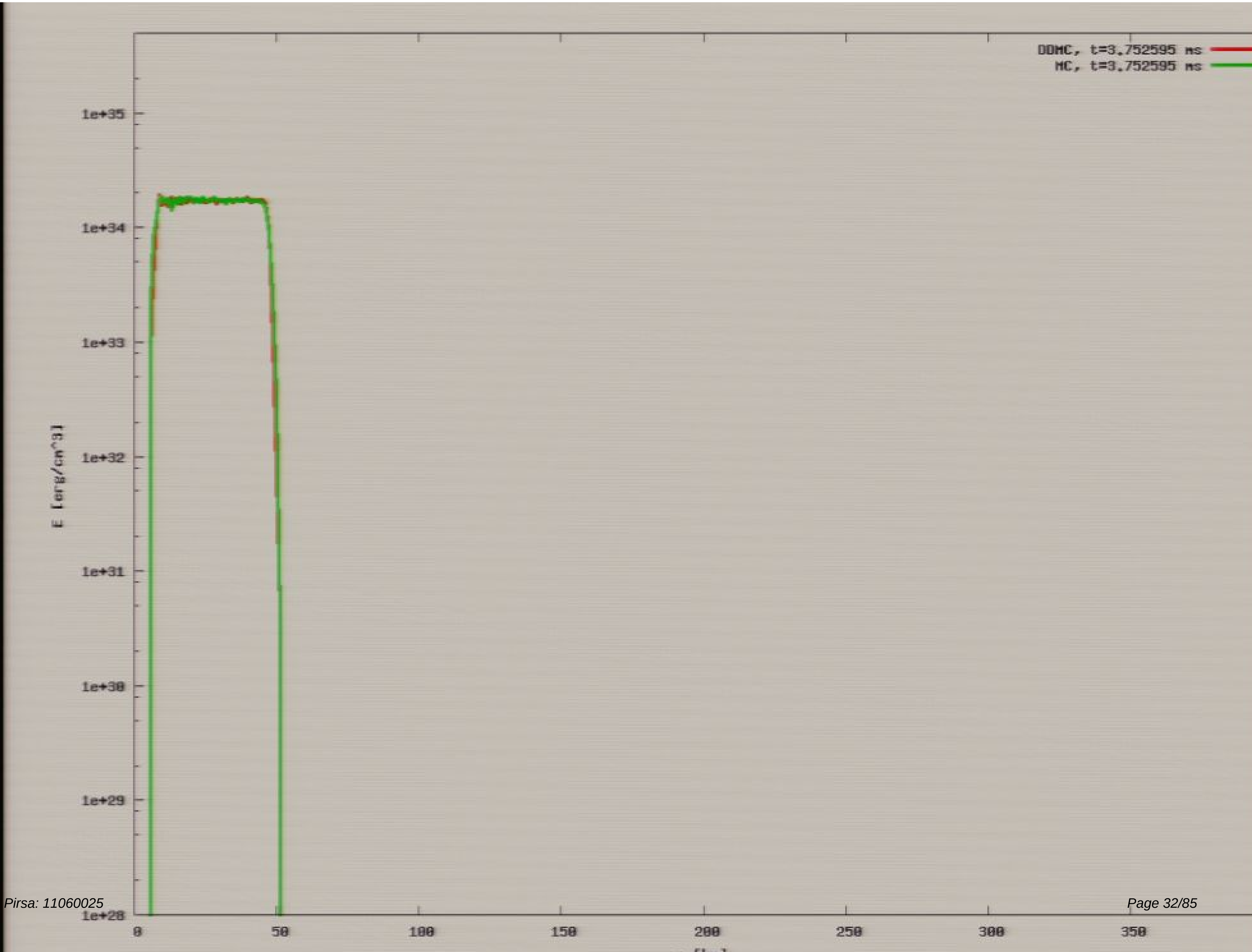

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MC, t=3.752595 ms
E [erg/cm^3]
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1e+31
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1e+29
1e+28
0
50
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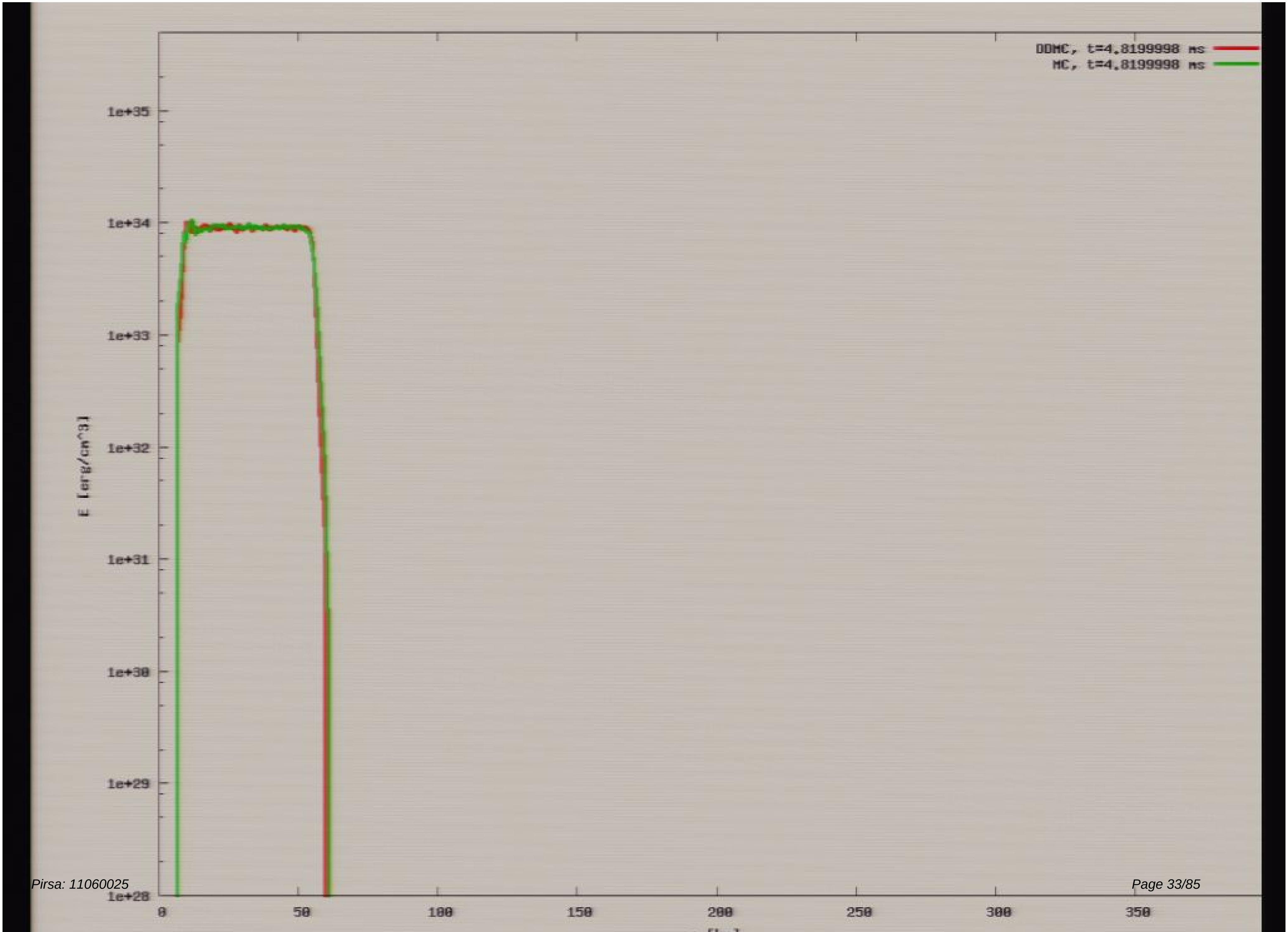
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MC, t=4.8199998 ms
1e+35
1e+34
1e+33
1e+32
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1e+29
1e+28
E [erg/cm^3]
0
50
100
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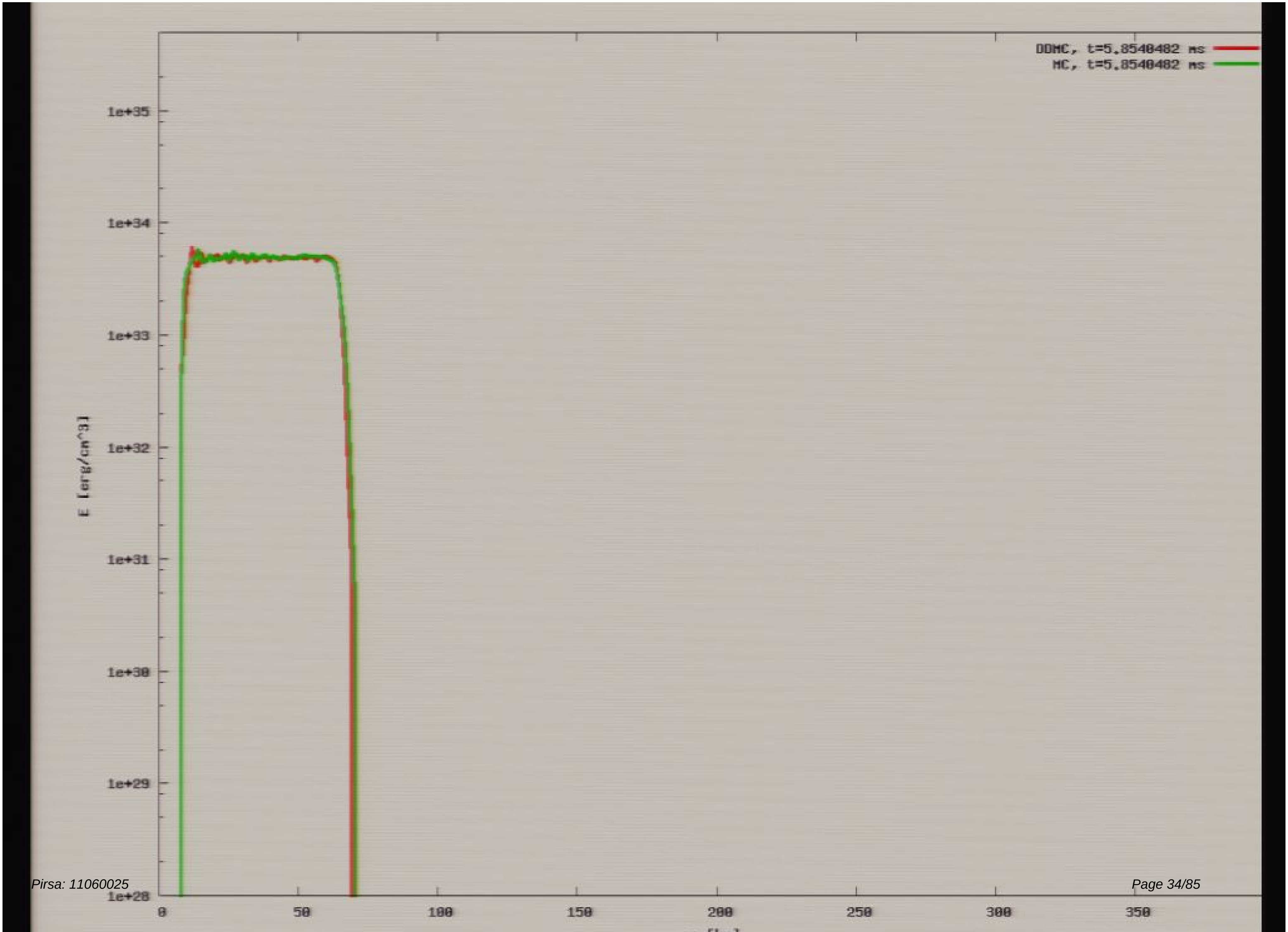
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1e+34
1e+33
1e+32
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1e+29
1e+28
E [erg/cm^3]
0
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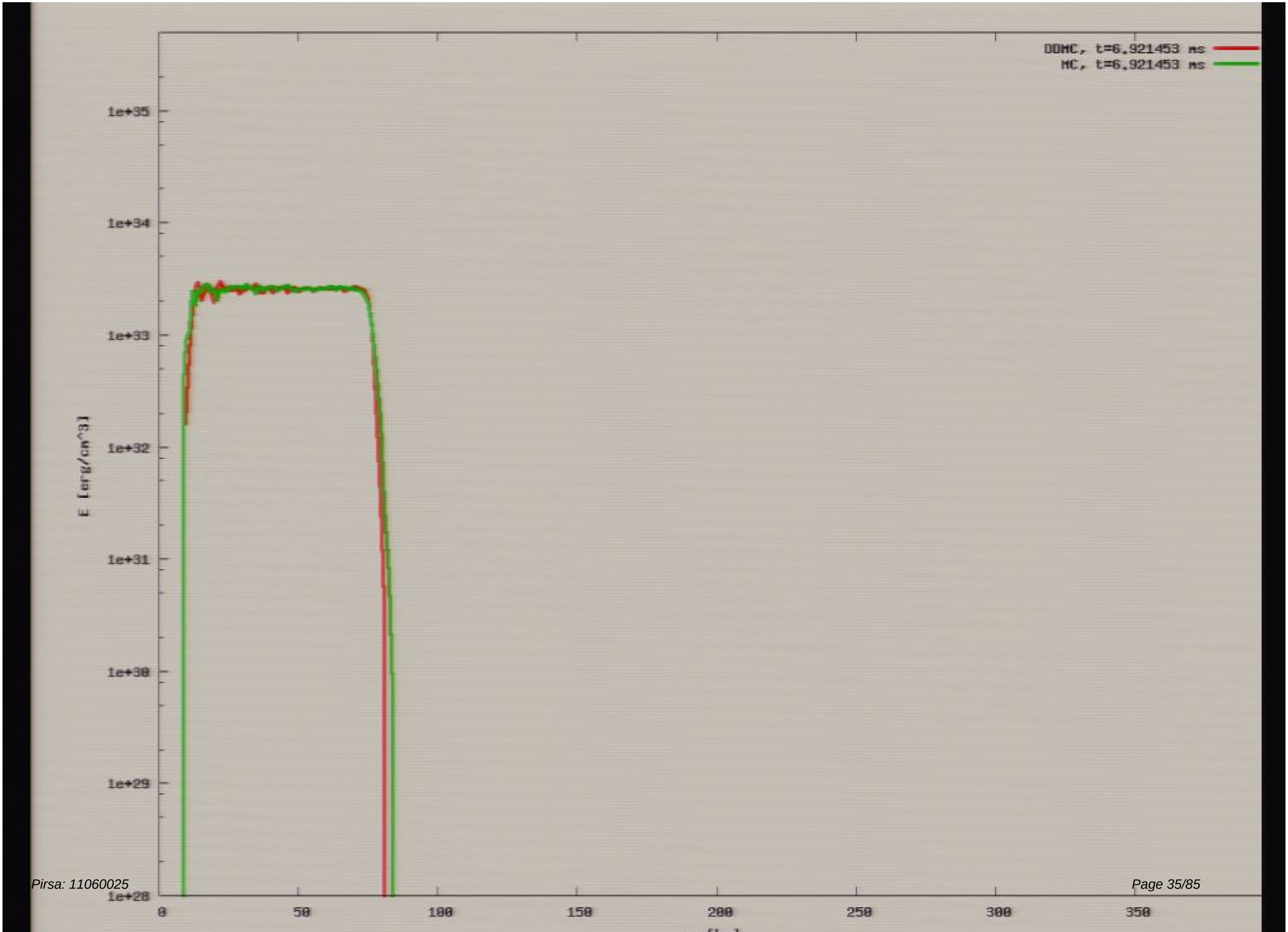
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MC, t=6.921453 ms
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1e+34
1e+33
1e+32
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1e+30
1e+29
1e+28
E [erg/cm^3]
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150
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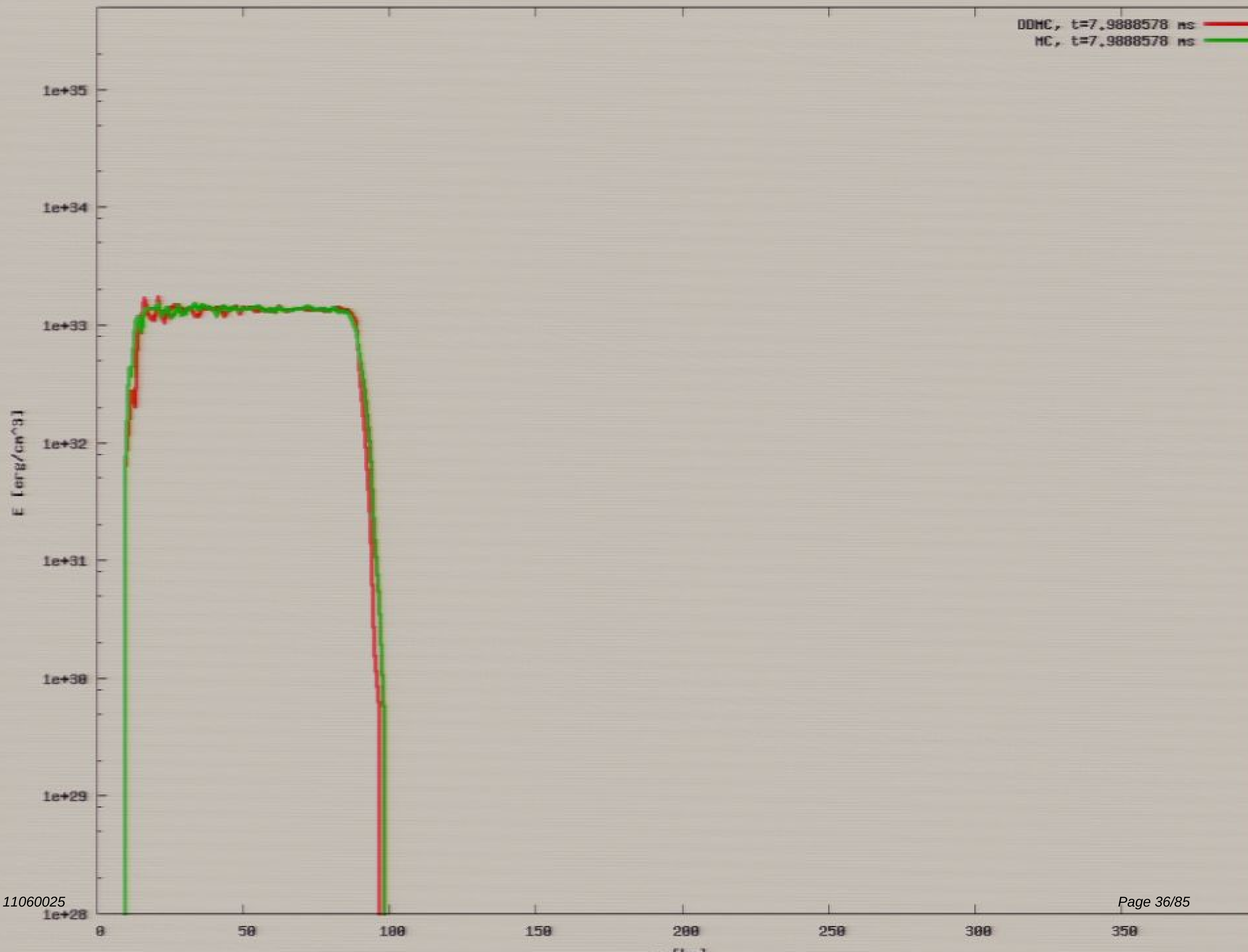
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MC, t=7.9888578 ms
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1e+34
1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
E [erg/cm^3]
0
50
100
150
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250
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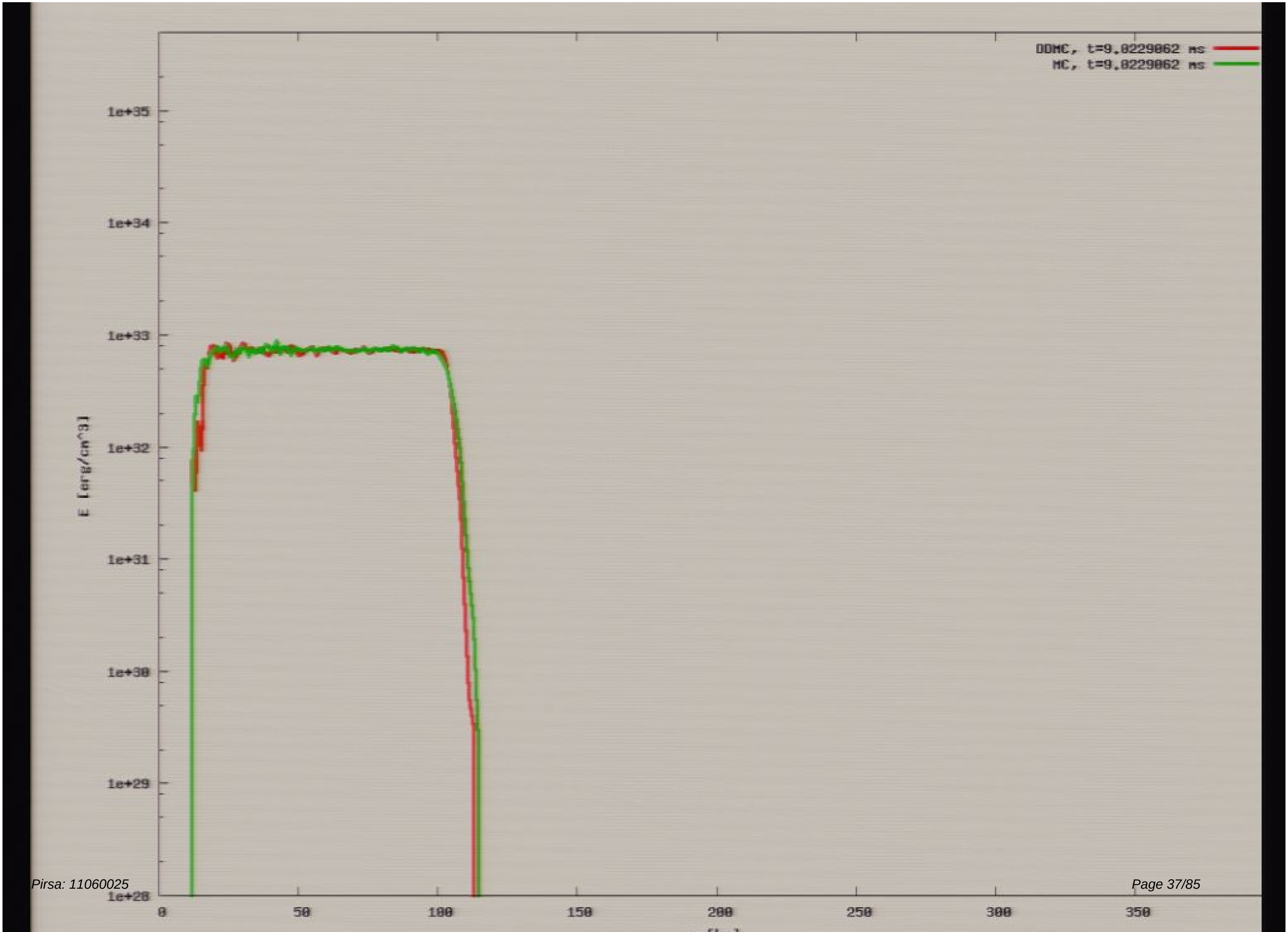

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MC, t=9.0229062 ms
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1e+34
1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
E [erg/cm^3]
0
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100
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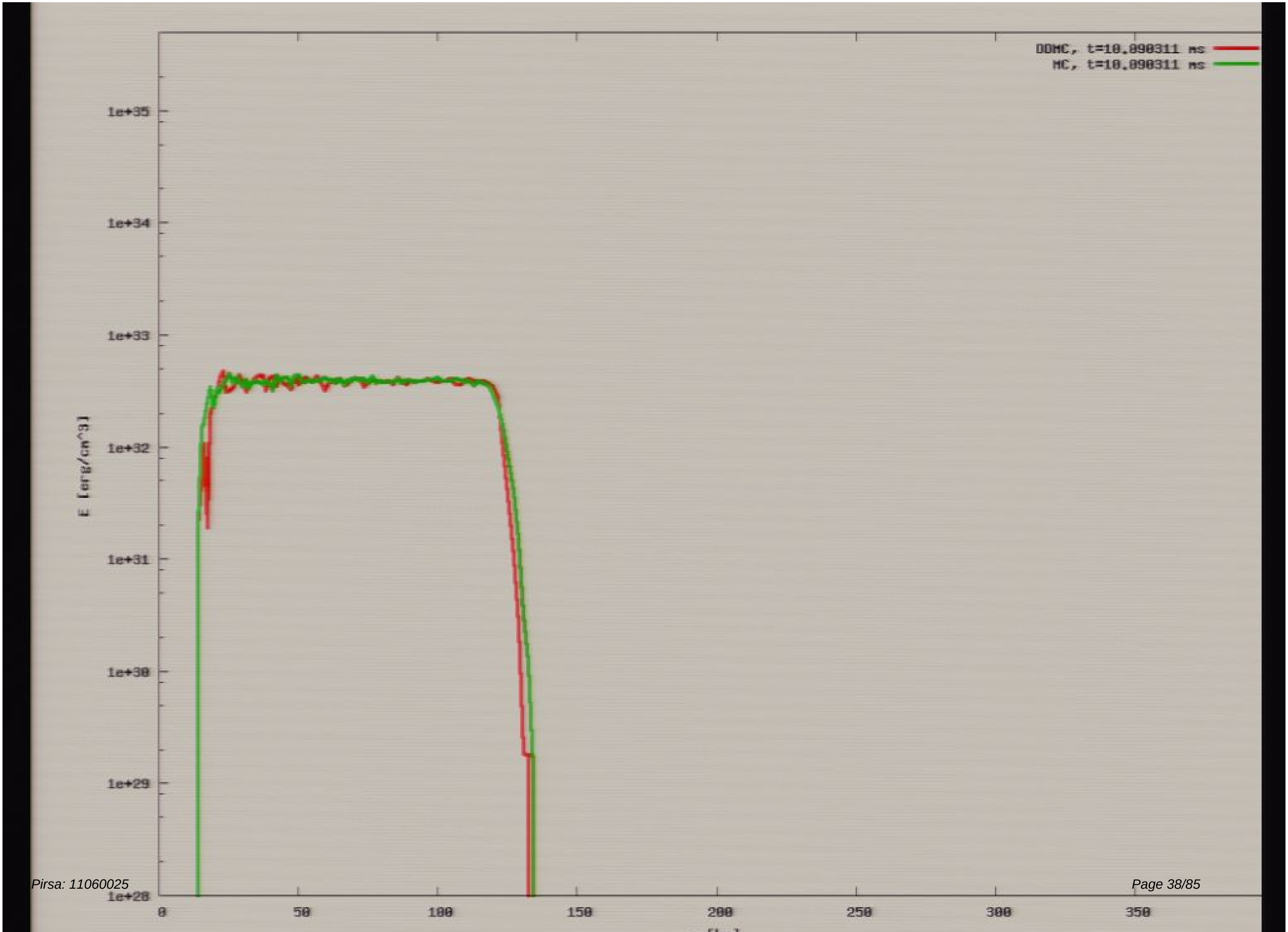
DDMC, t=10.090311 ms
MC, t=10.090311 ms
E [erg/cm^3]
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1e+34
1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
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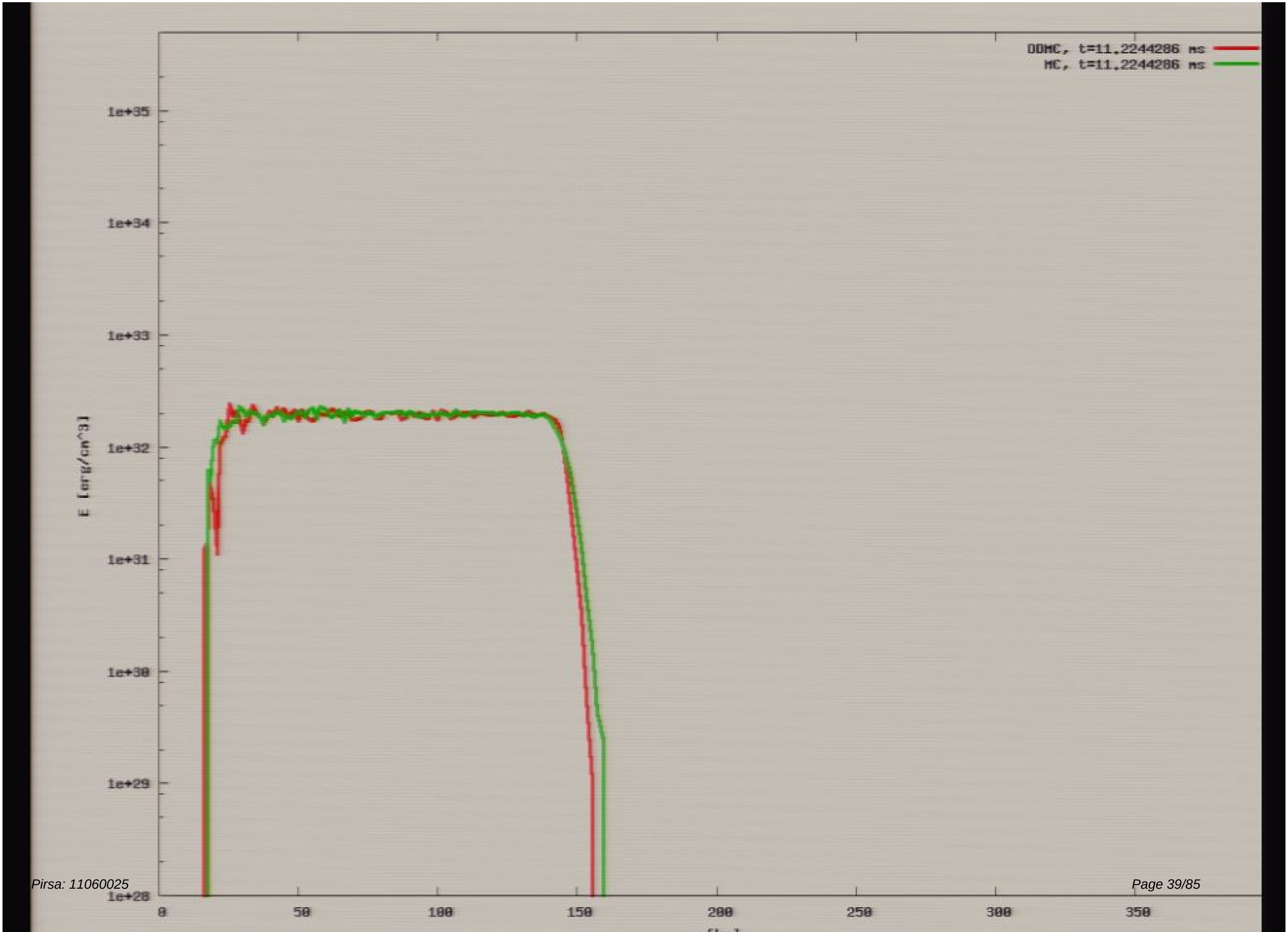
DDMC, t=11.2244286 ms
MC, t=11.2244286 ms
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1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
E [erg/cm^3]
0
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100
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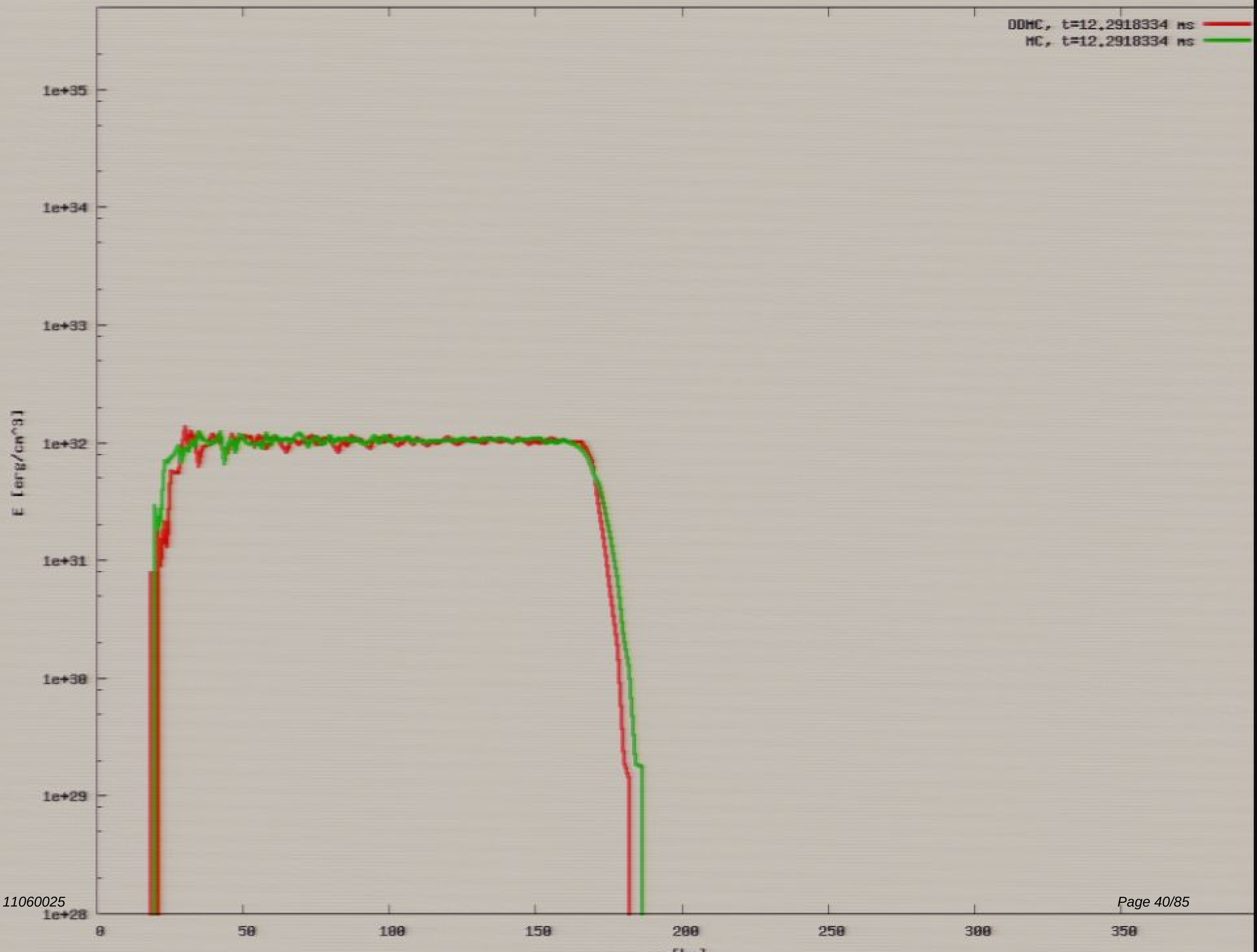

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MC, t=12.2918334 ms
E [erg/cm^3]
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1e+34
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1e+31
1e+30
1e+29
1e+28
0
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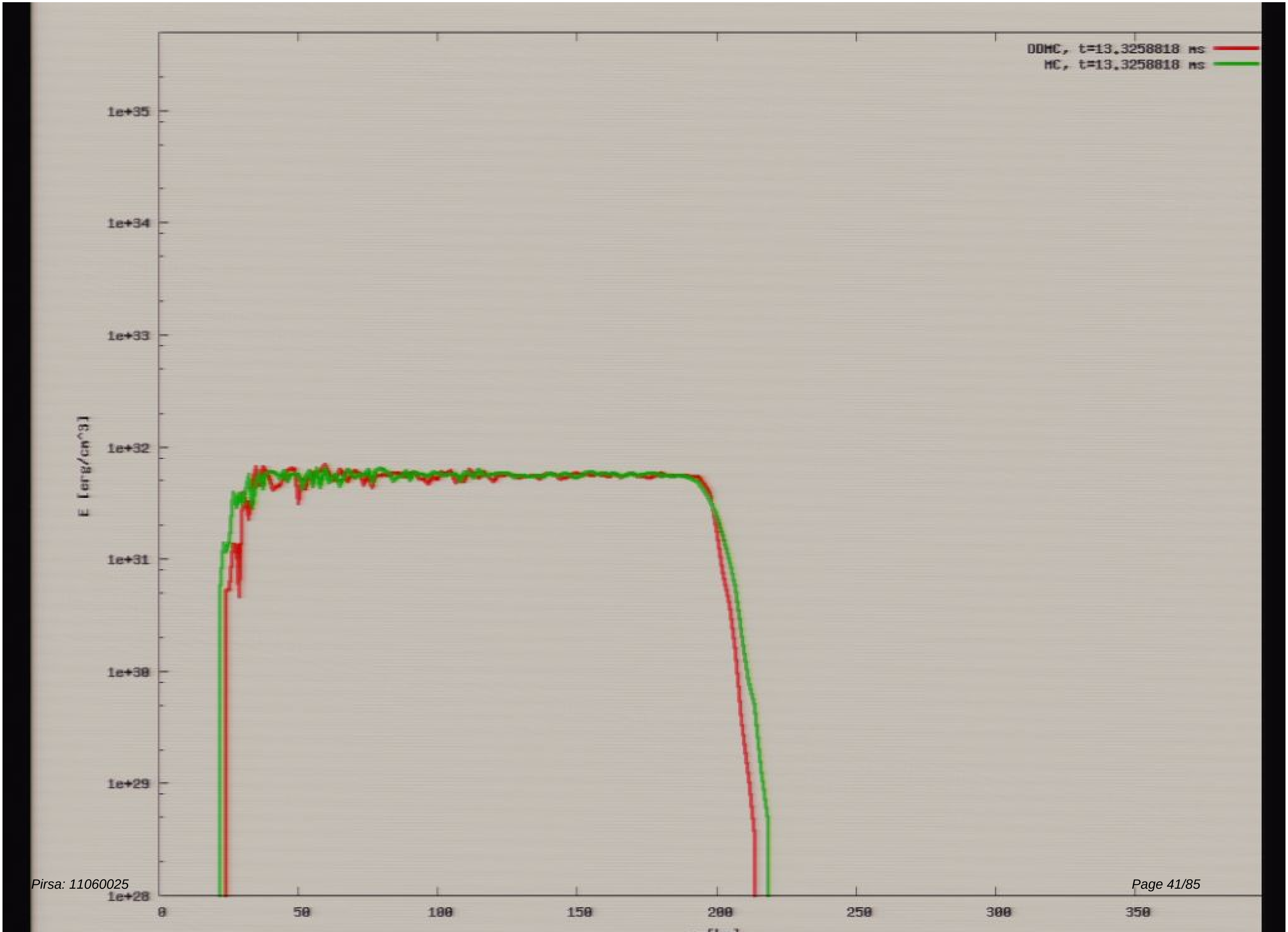

DDMC, t=13.3258818 ms
MC, t=13.3258818 ms
E [erg/cm^3]
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1e+34
1e+33
1e+32
1e+31
1e+30
1e+29
1e+28
0
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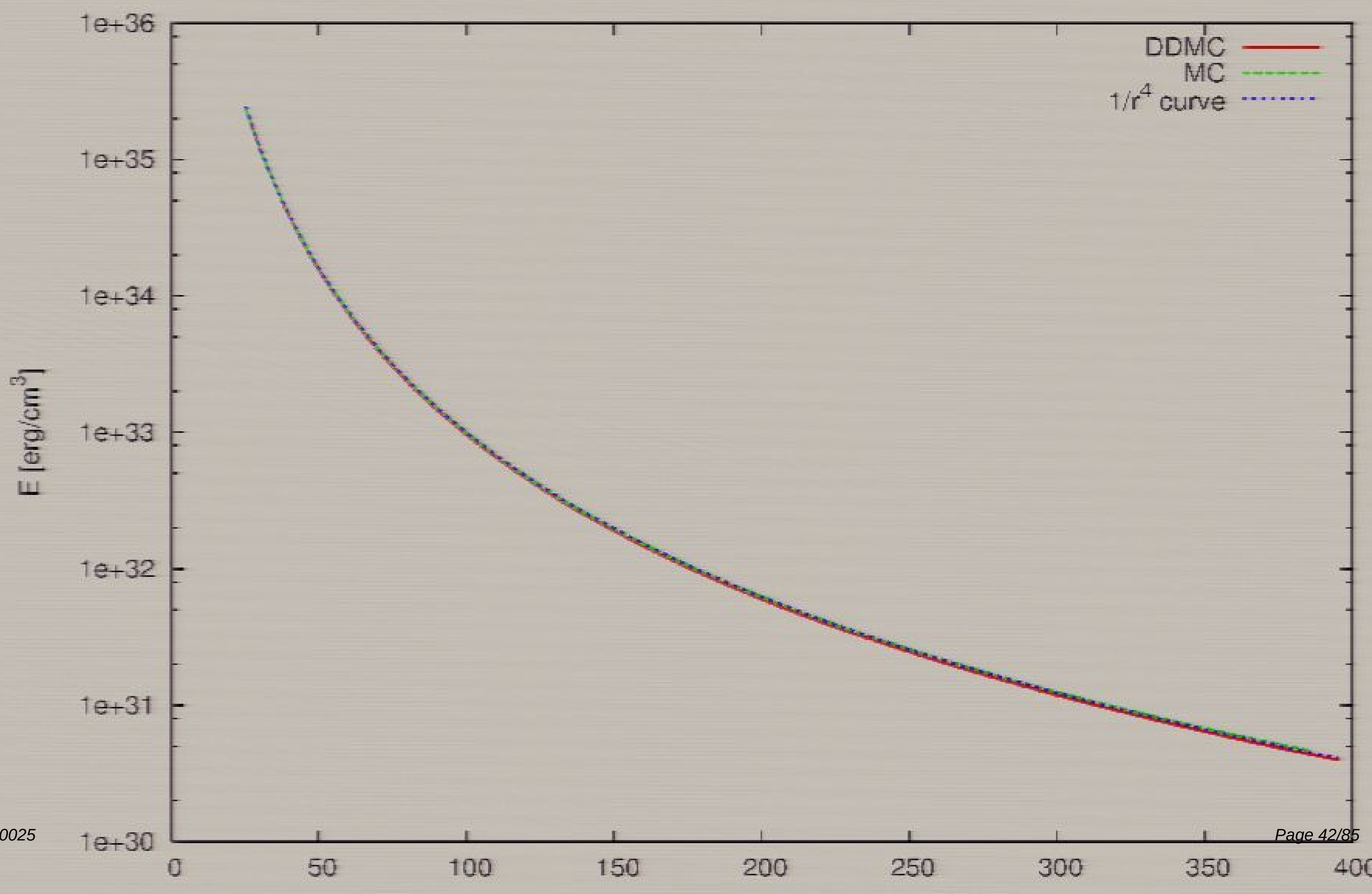
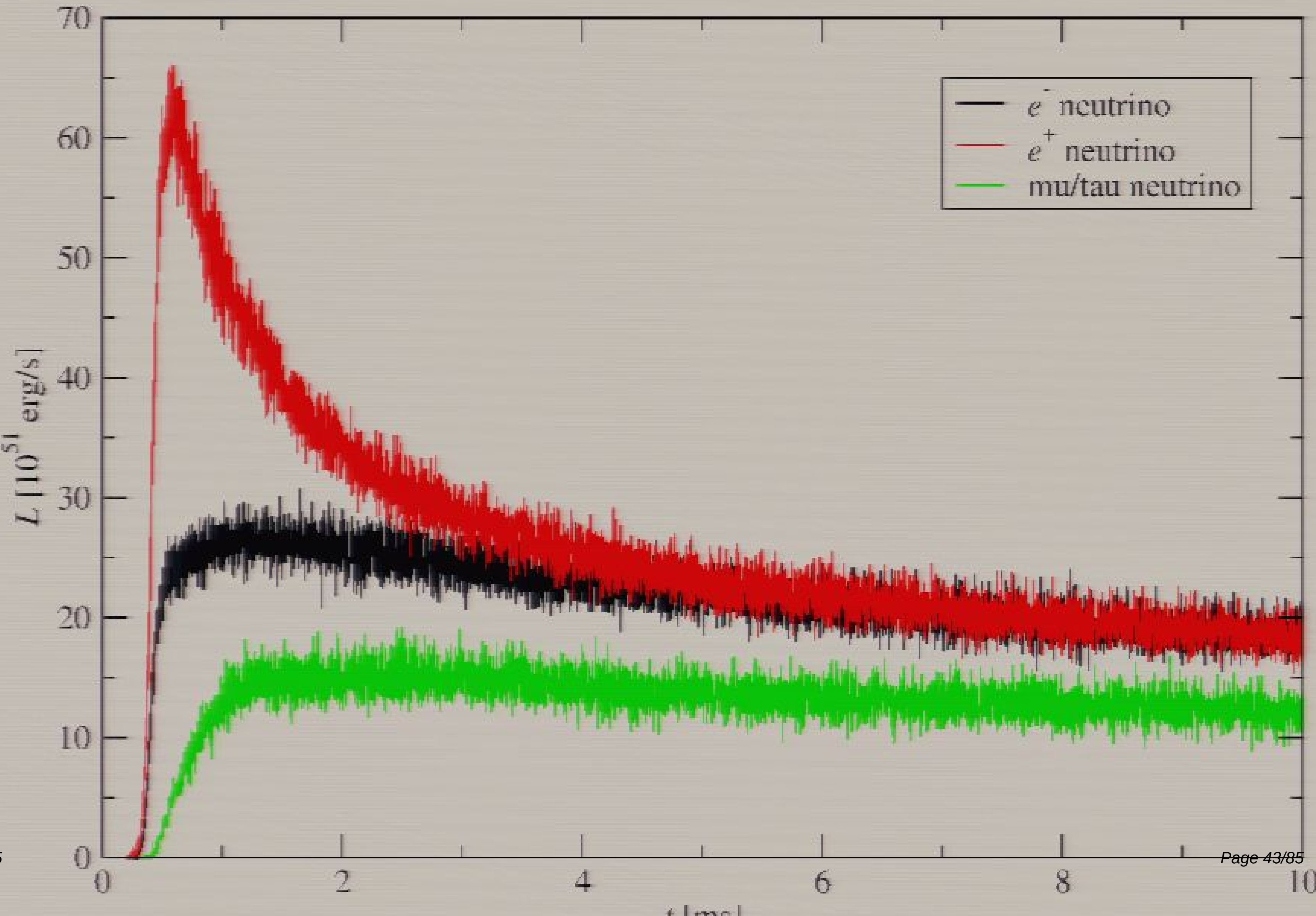
100
150
200
250
300
350

# Homologously expanding shell: total energy scaling



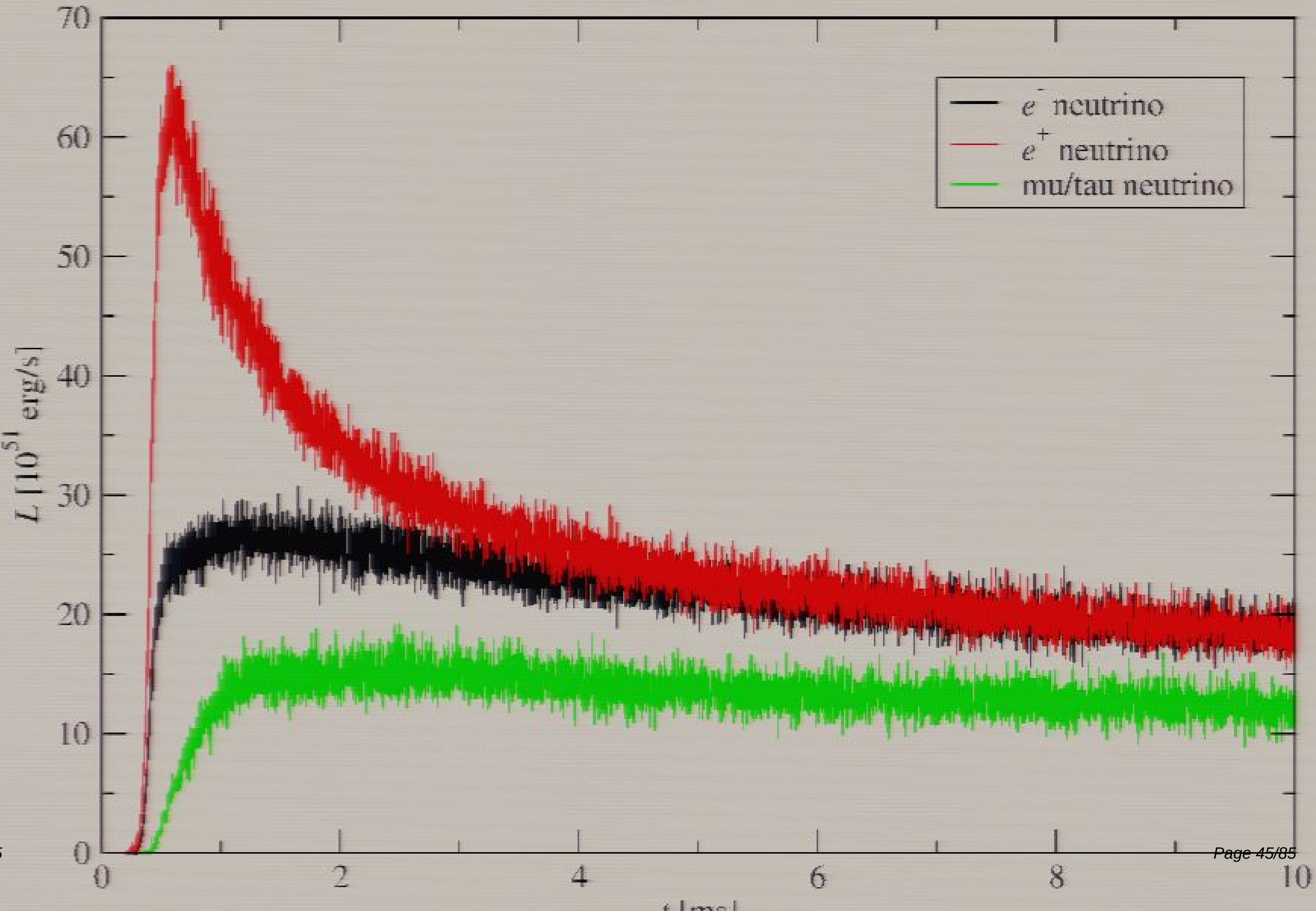
# Neutrino emission by proto-NS



# Summary

- Implicit MC for neutrino transport
- Multi-group discrete-diffusion
- Velocity-dependent MC and DDMC
- Applicable to both neutrino and photon transport

# Neutrino emission by proto-NS



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VGA-1

No Signal

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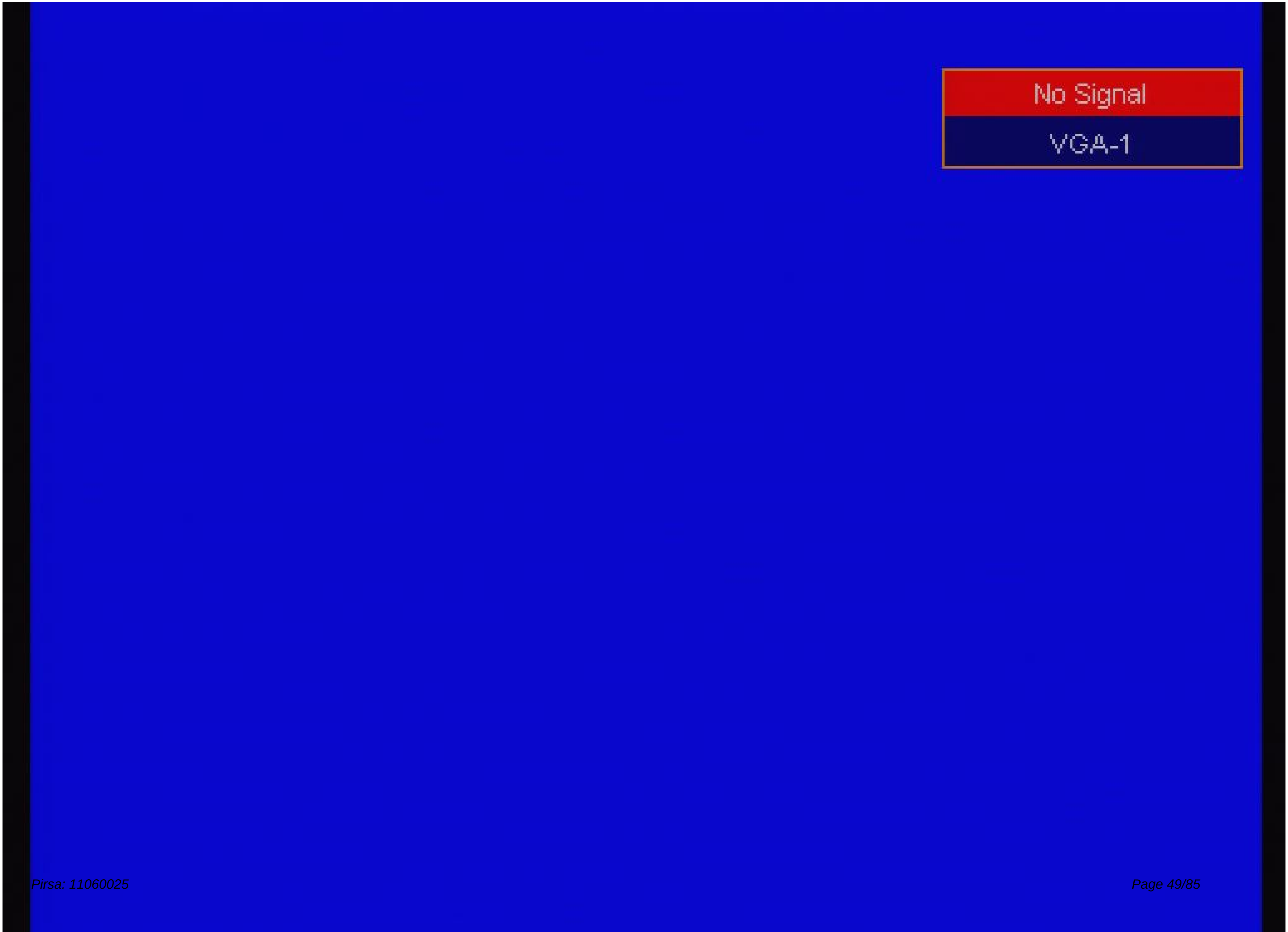
No Signal
VGA-1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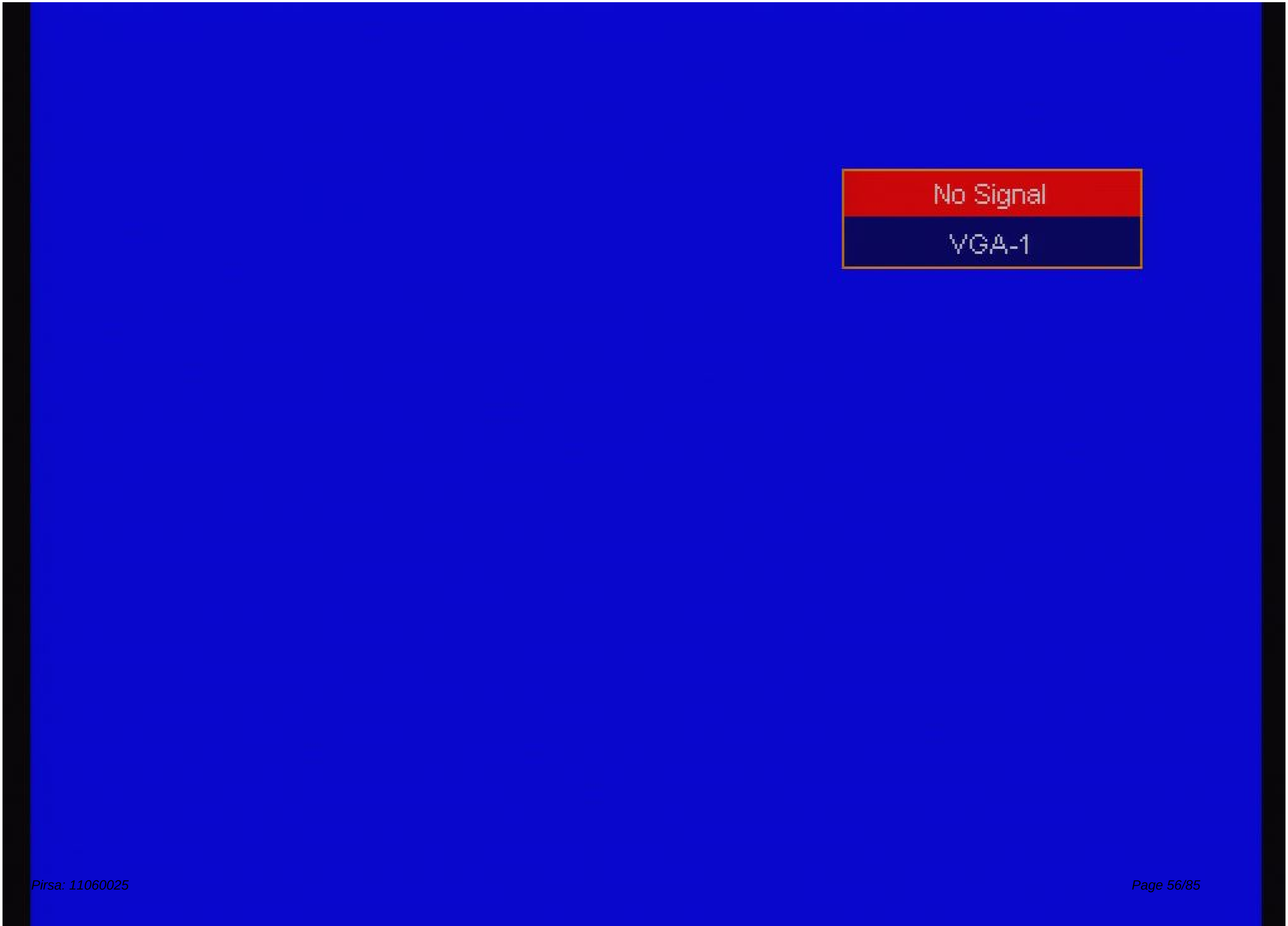
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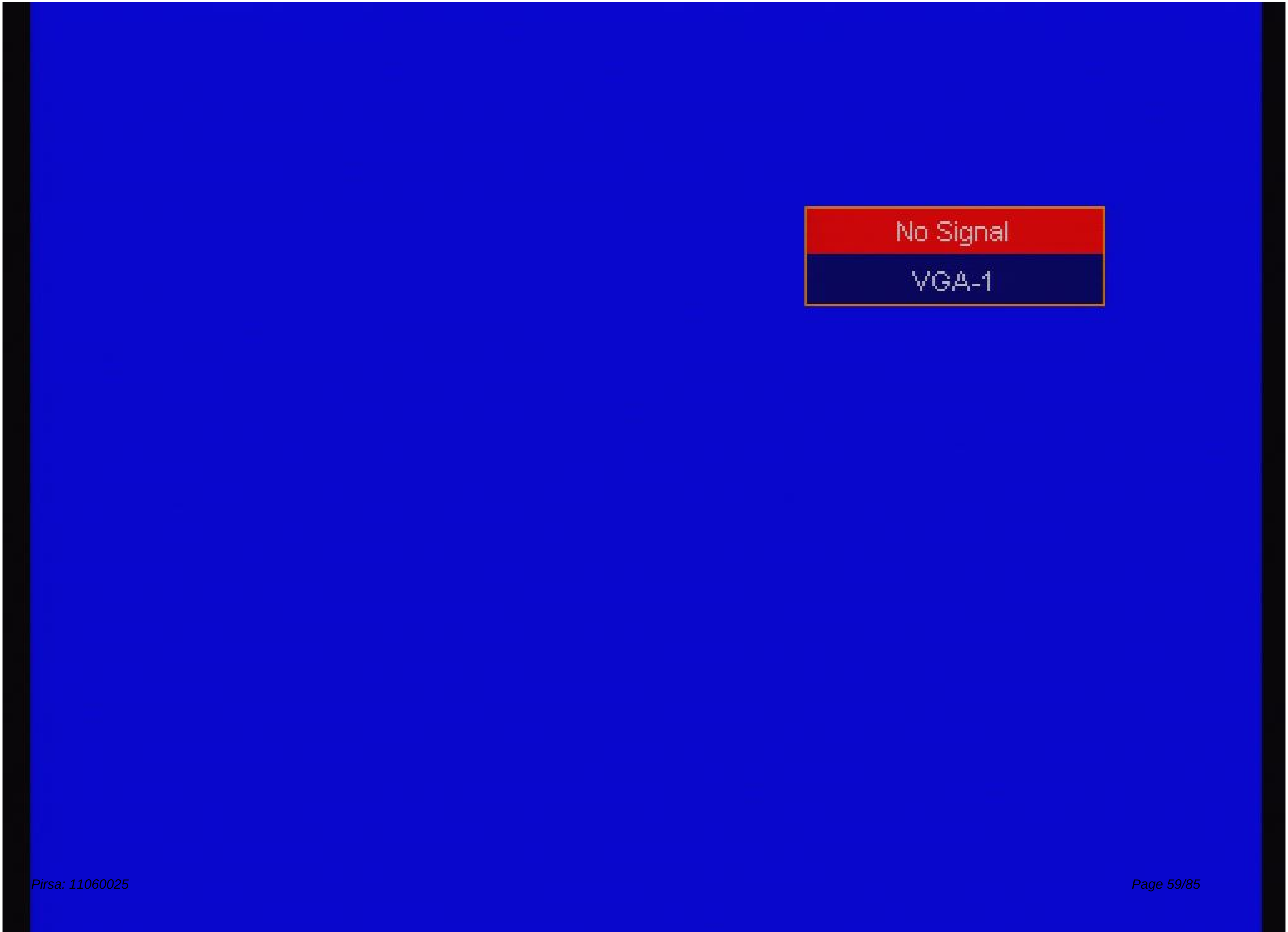
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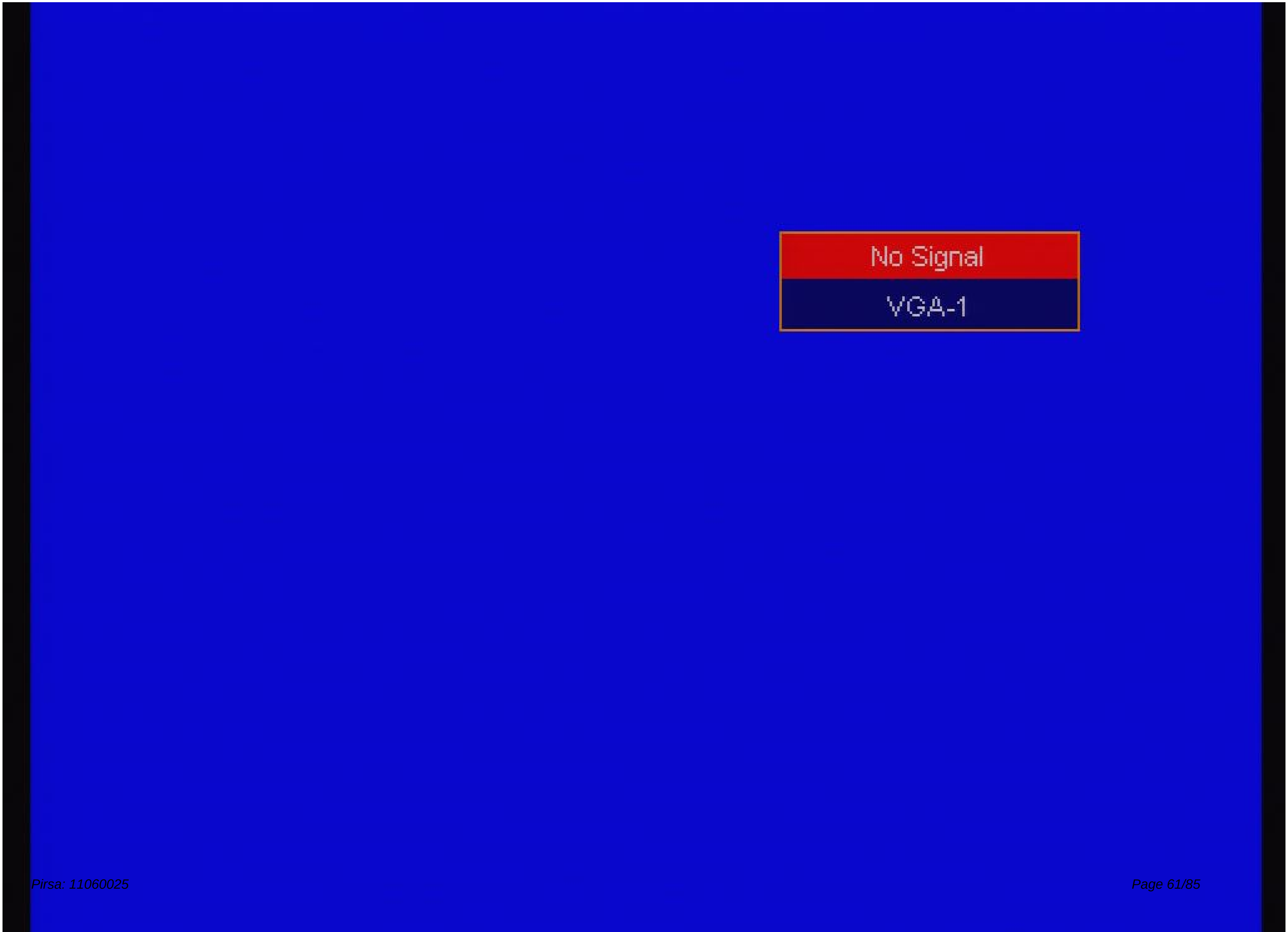
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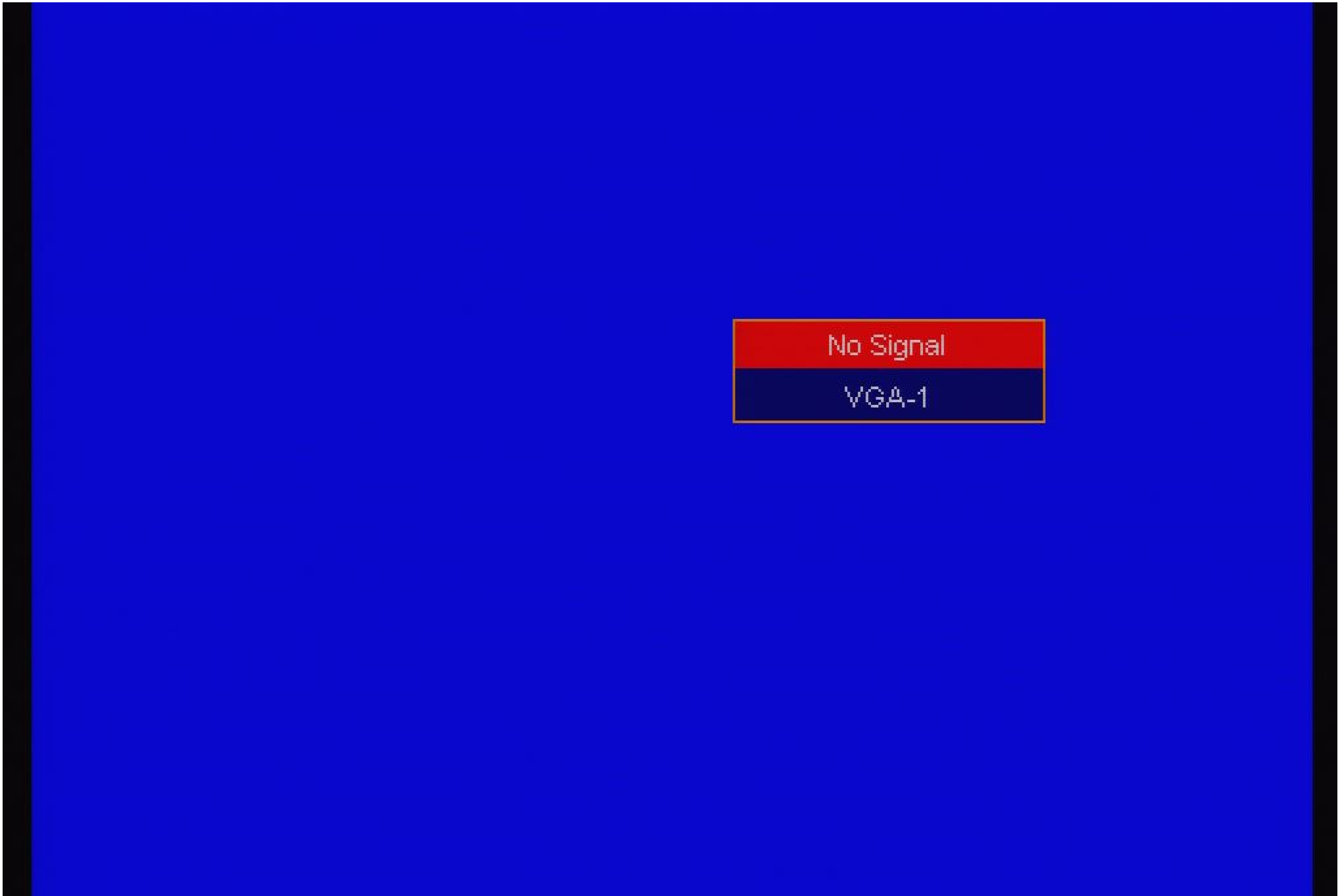
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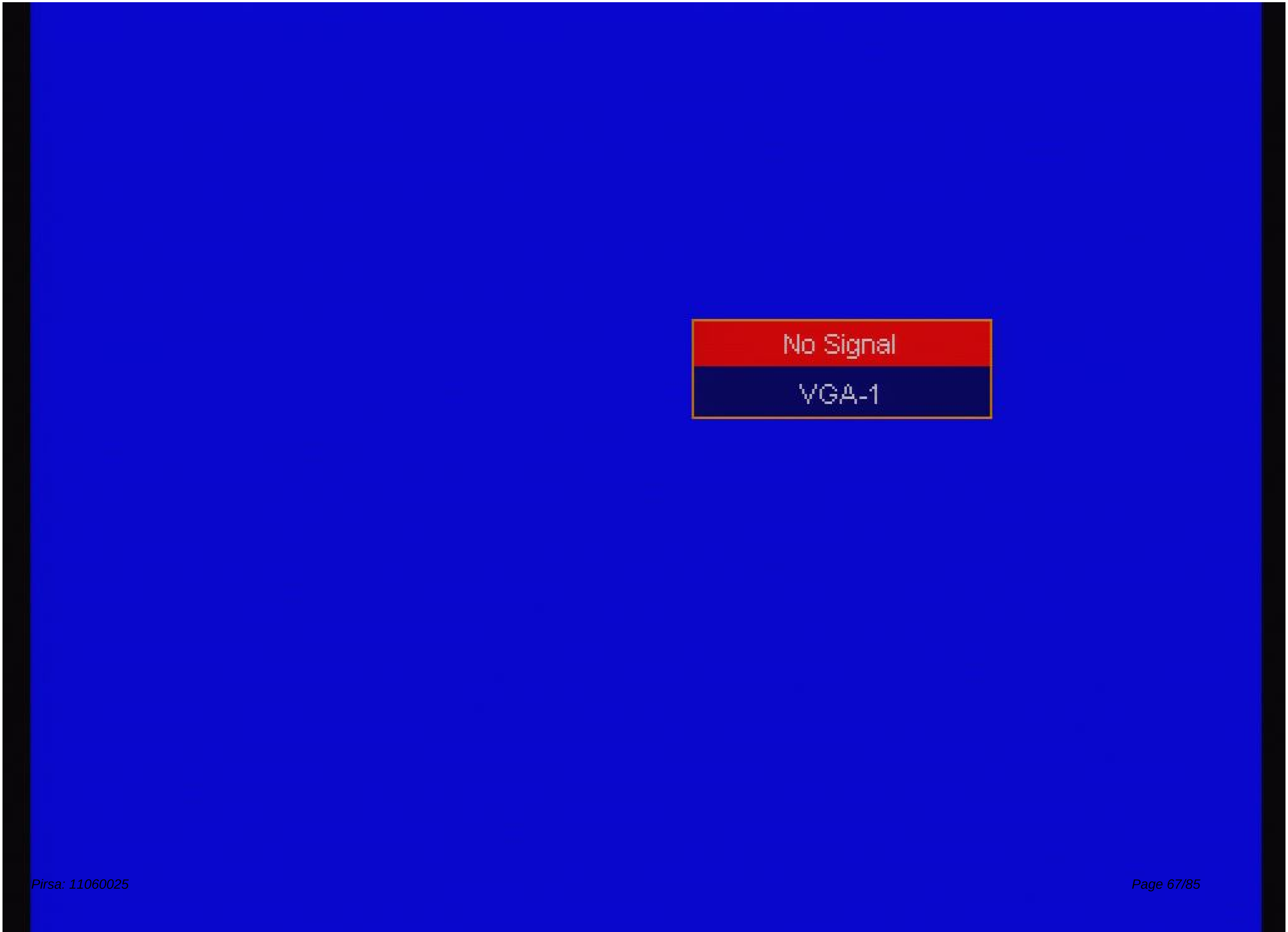
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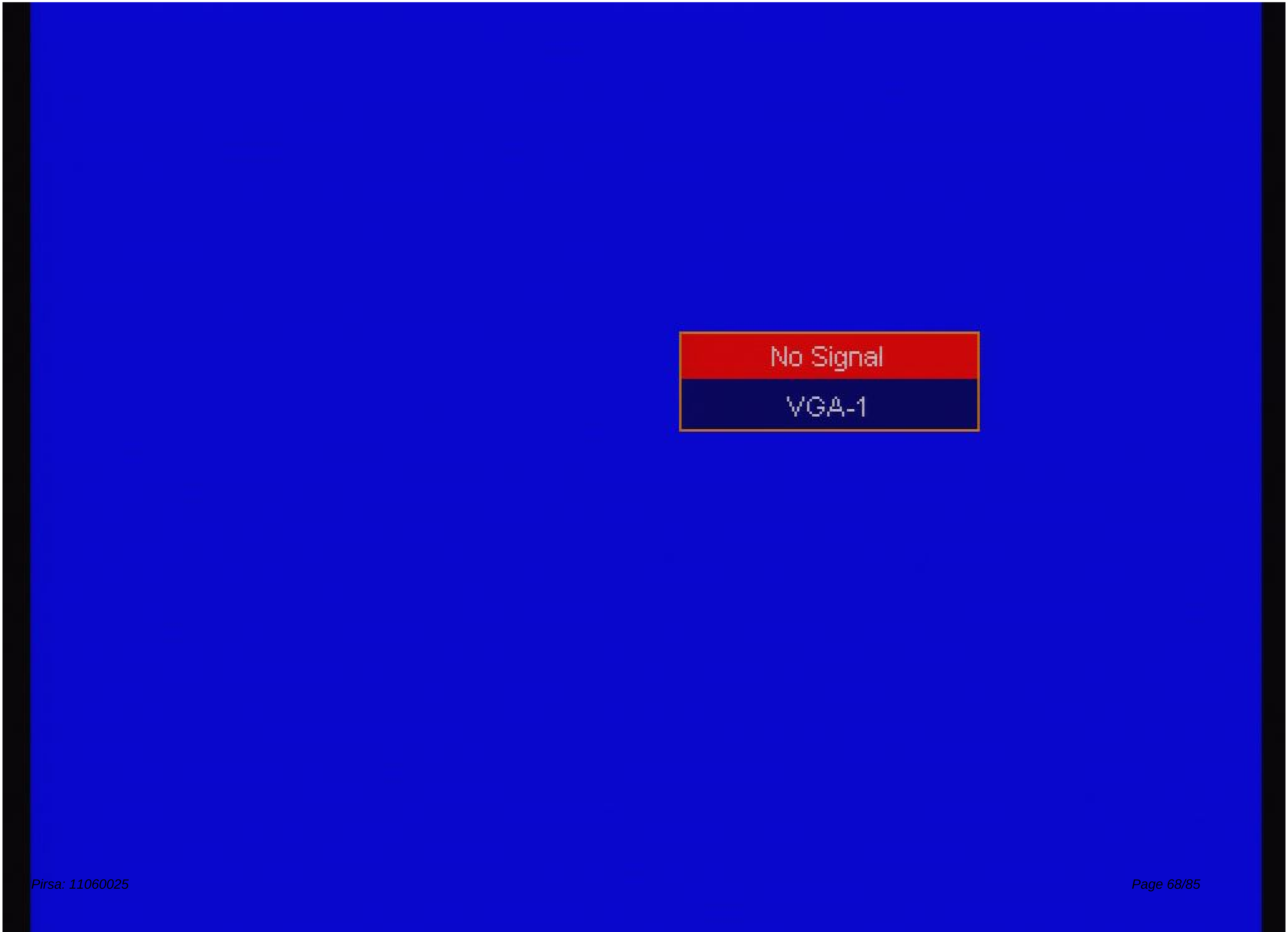
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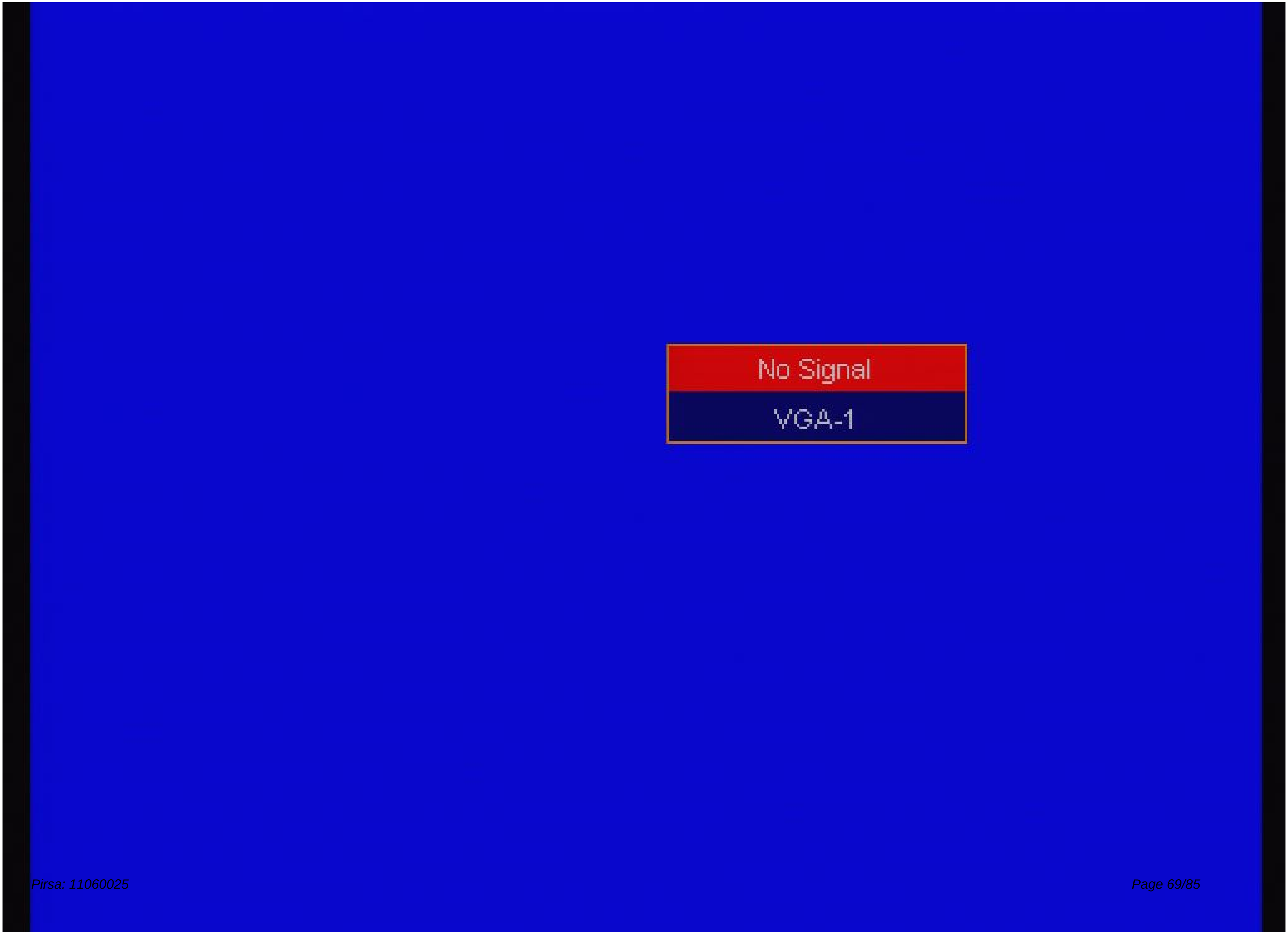
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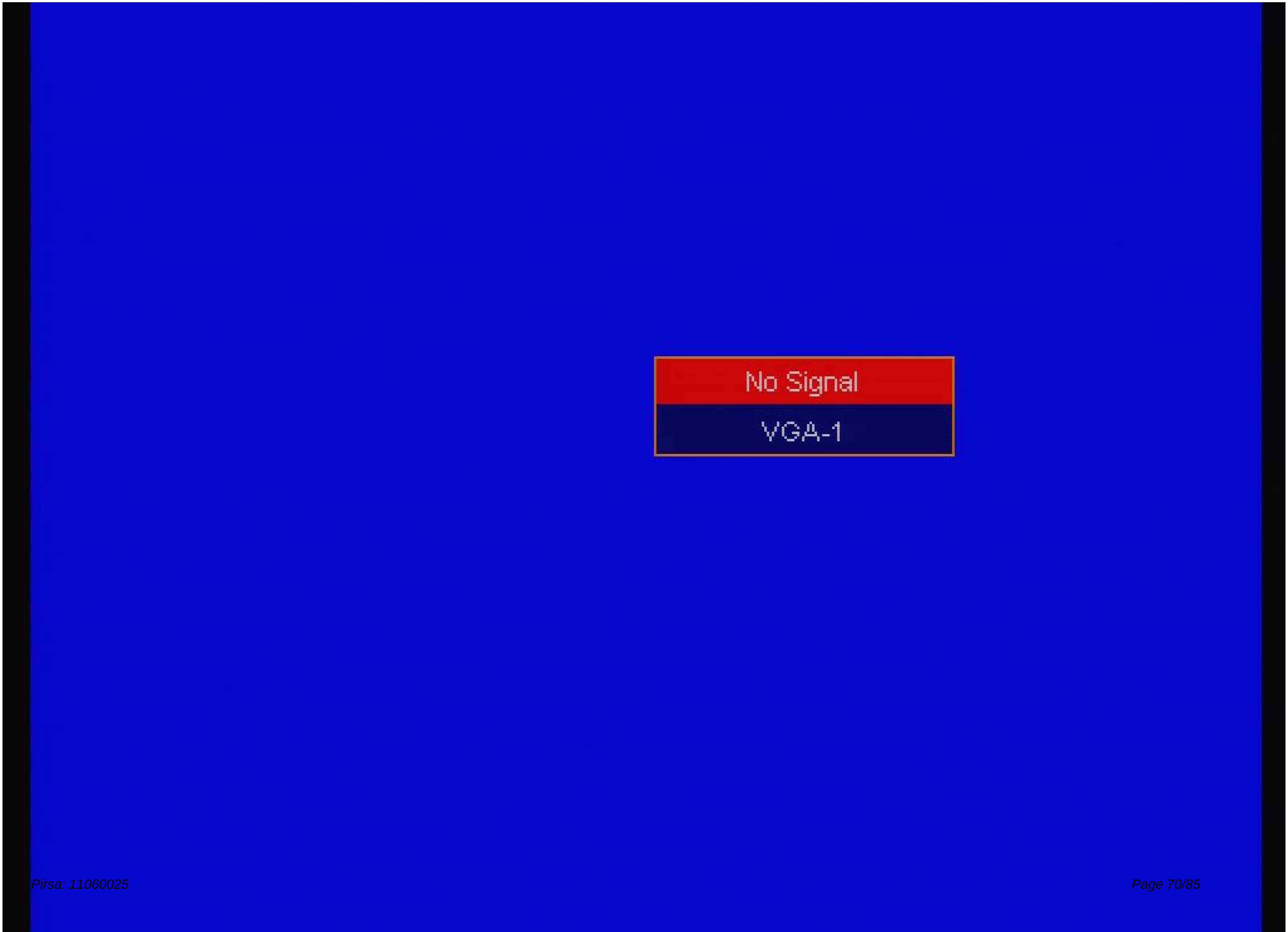
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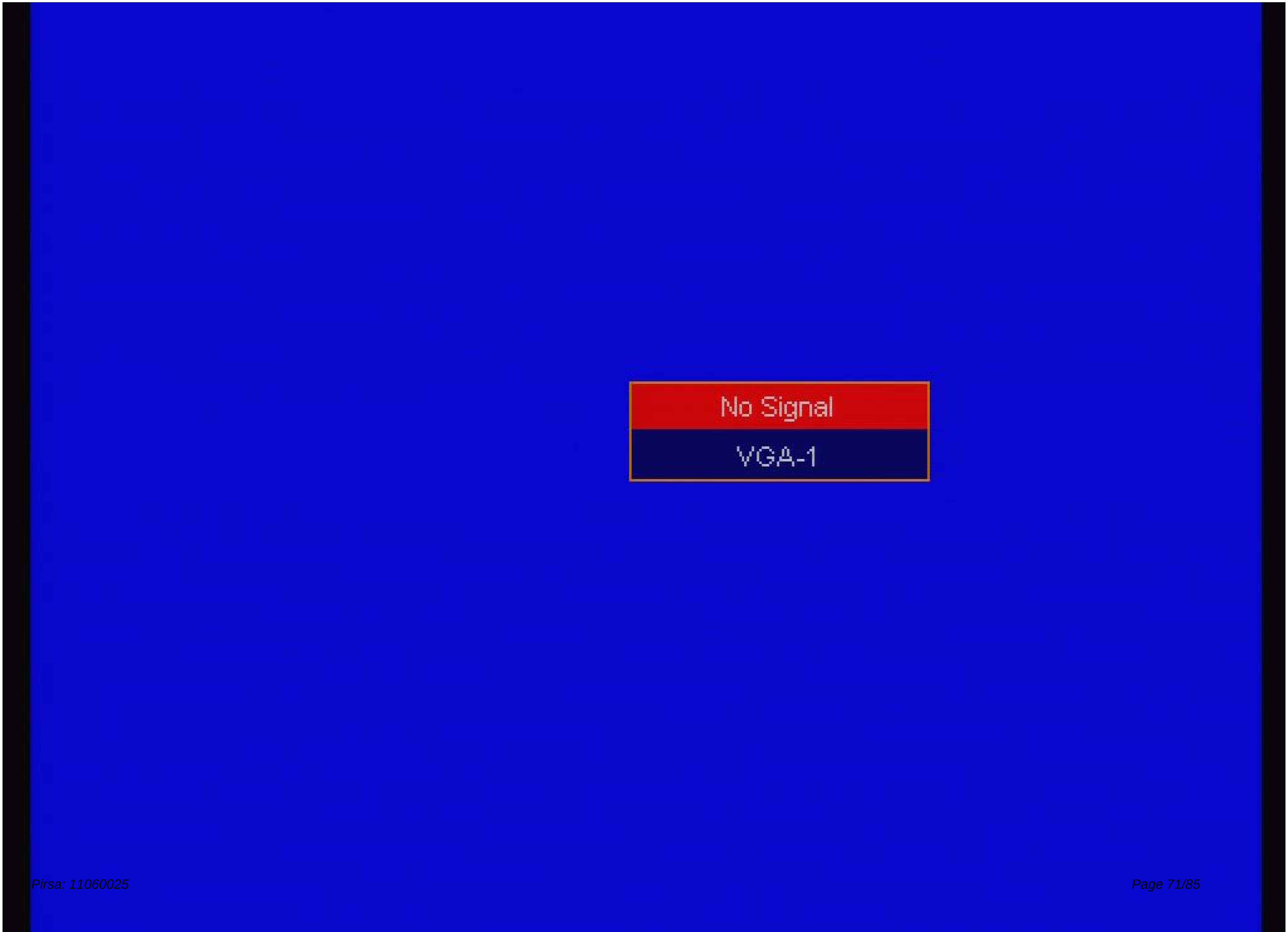
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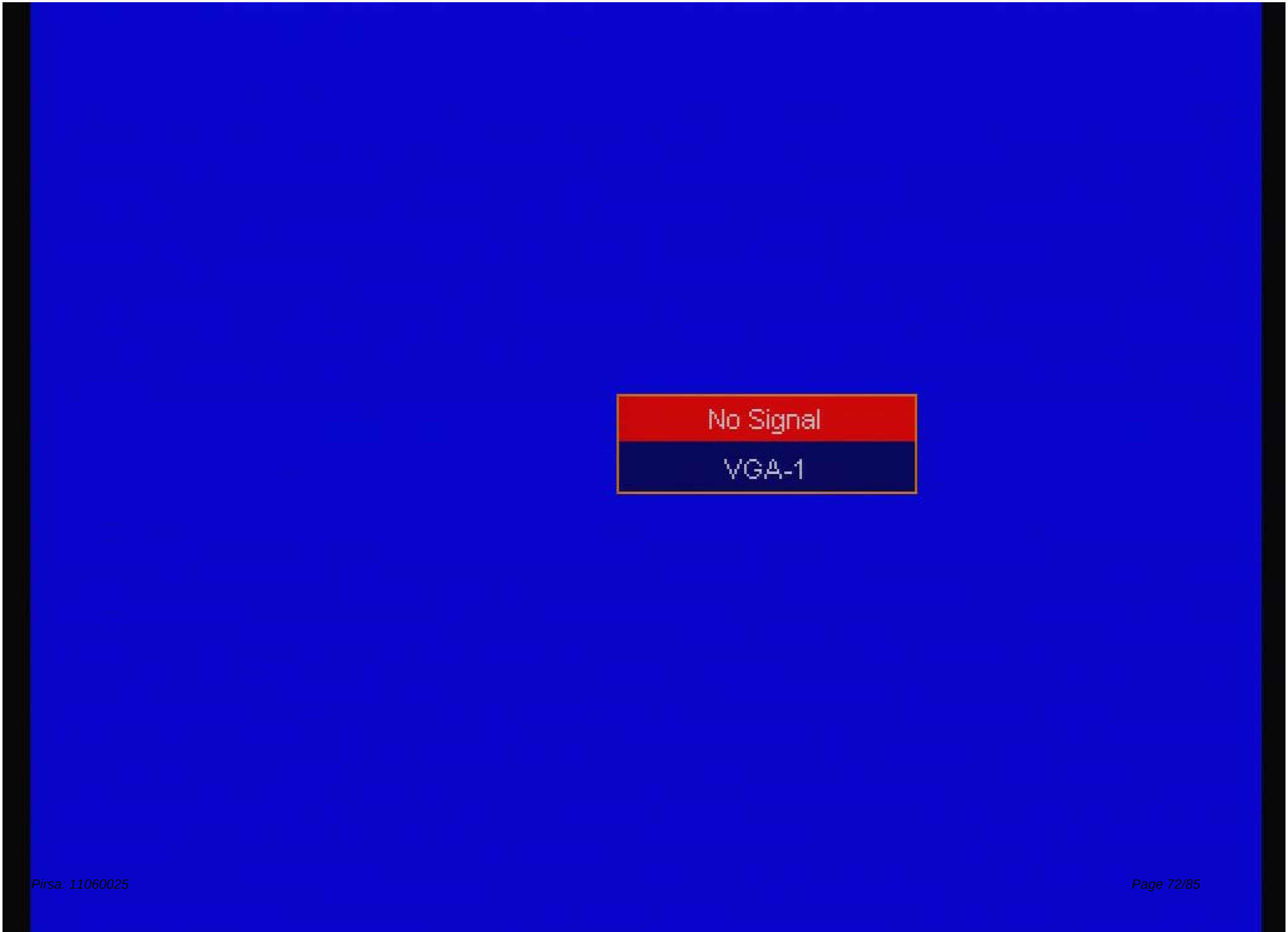
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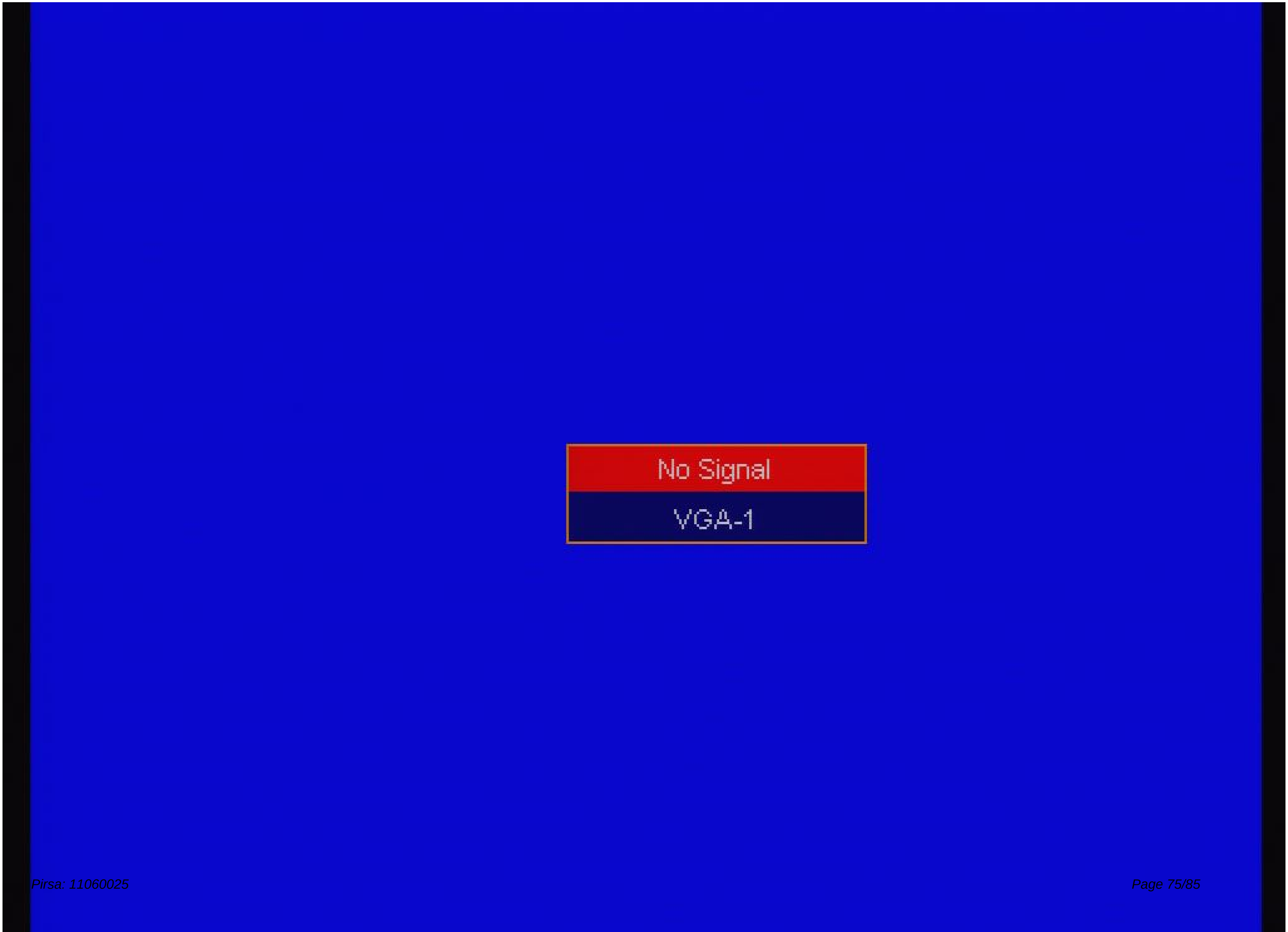
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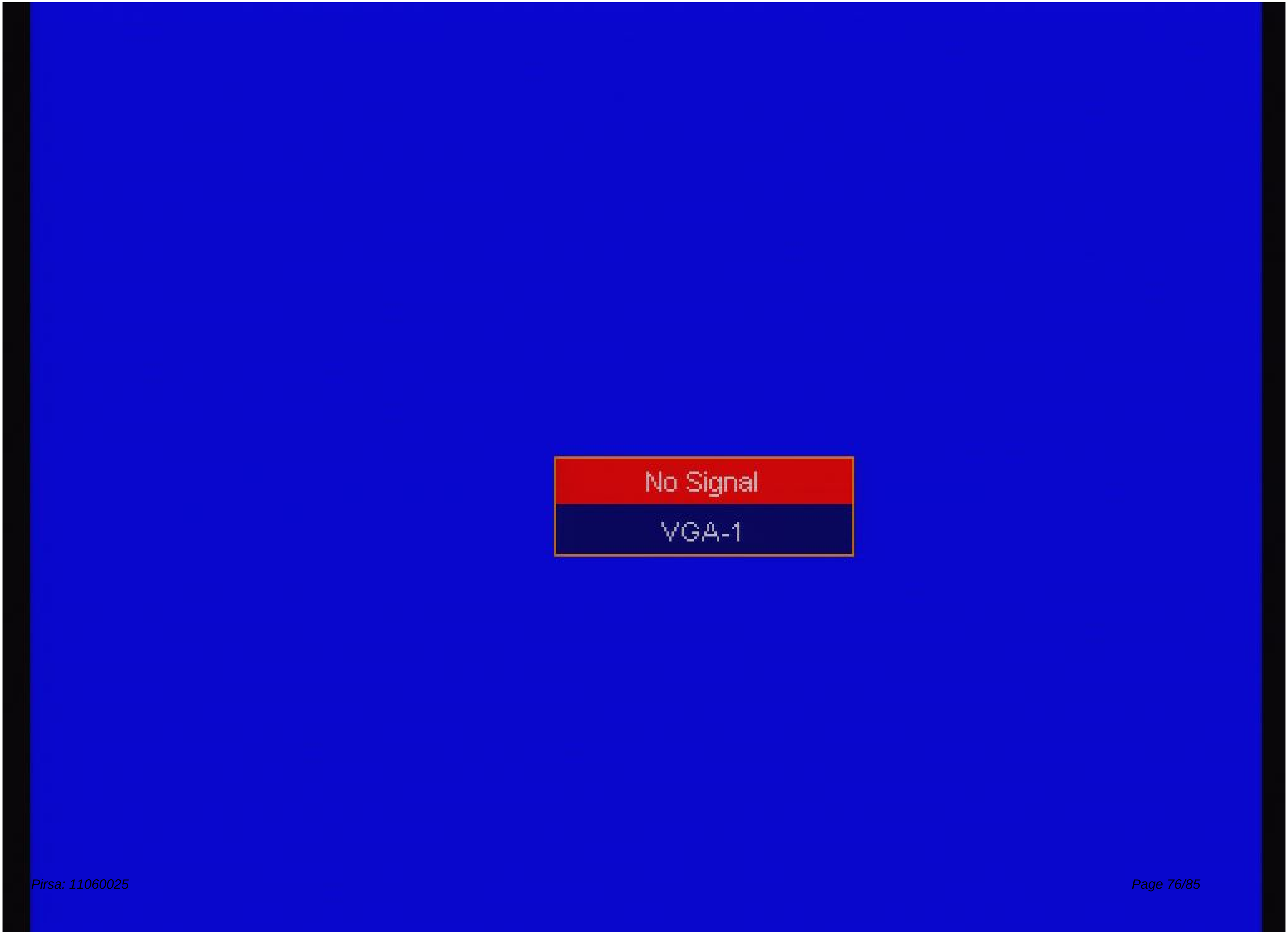
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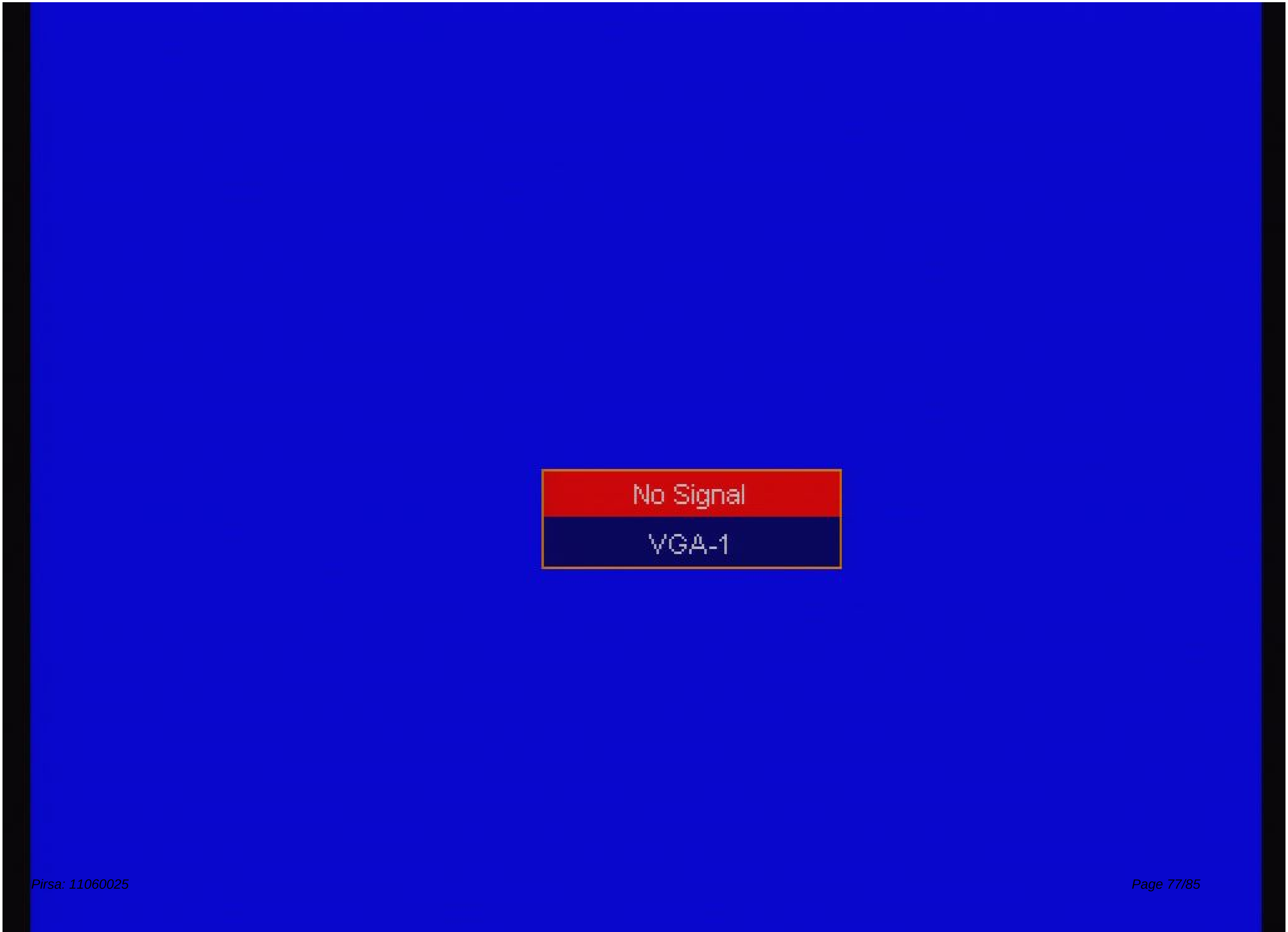
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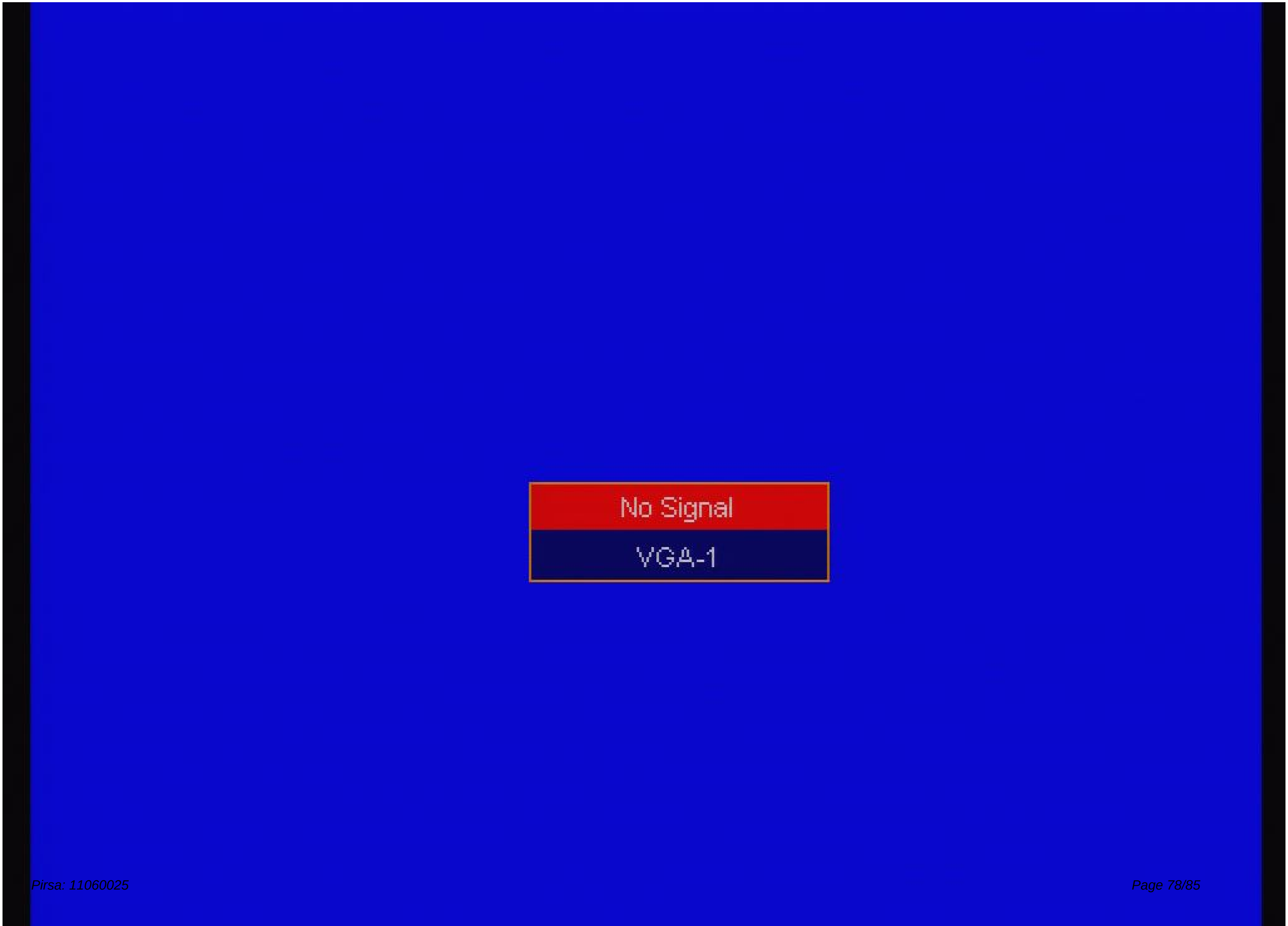
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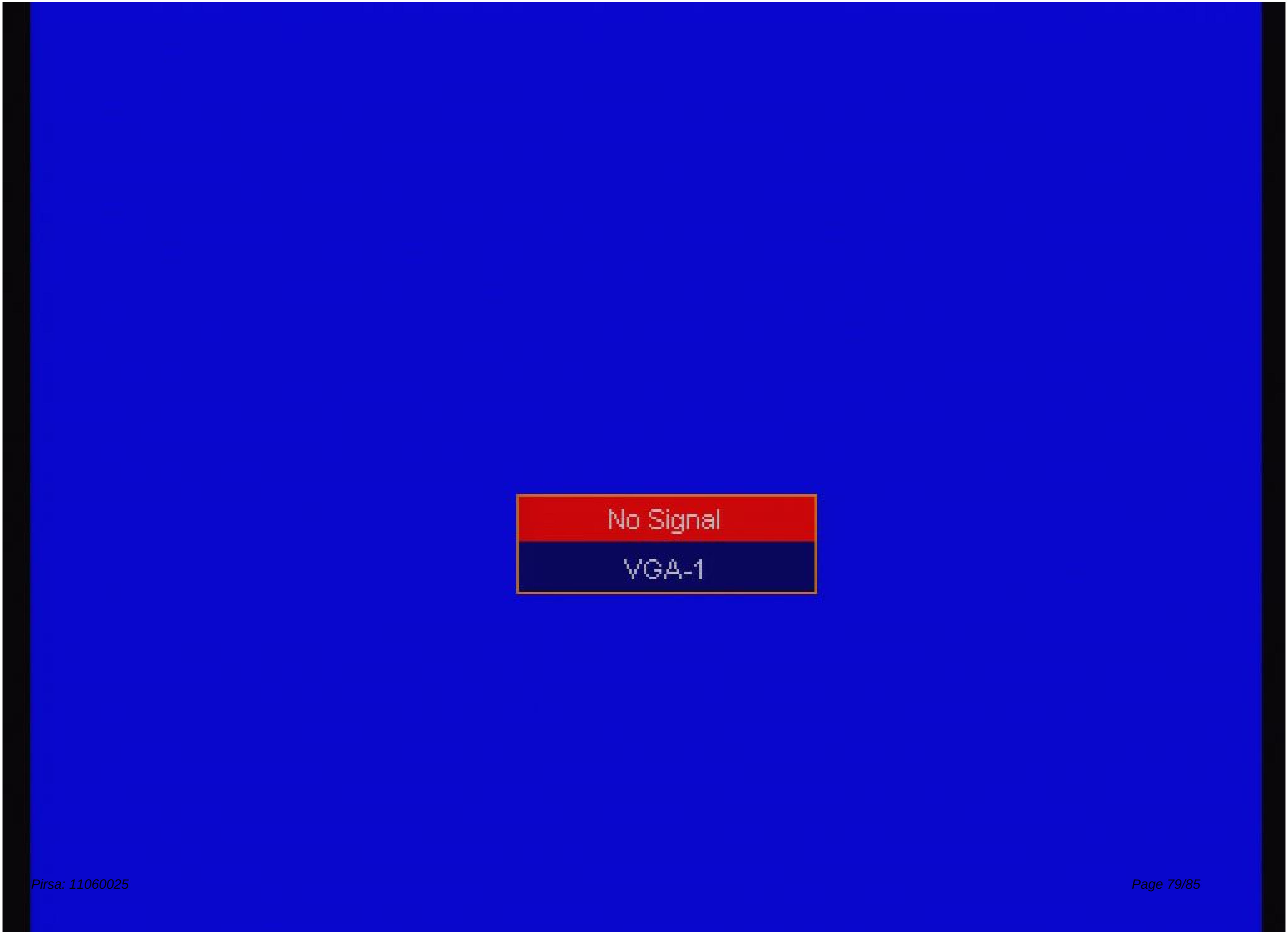
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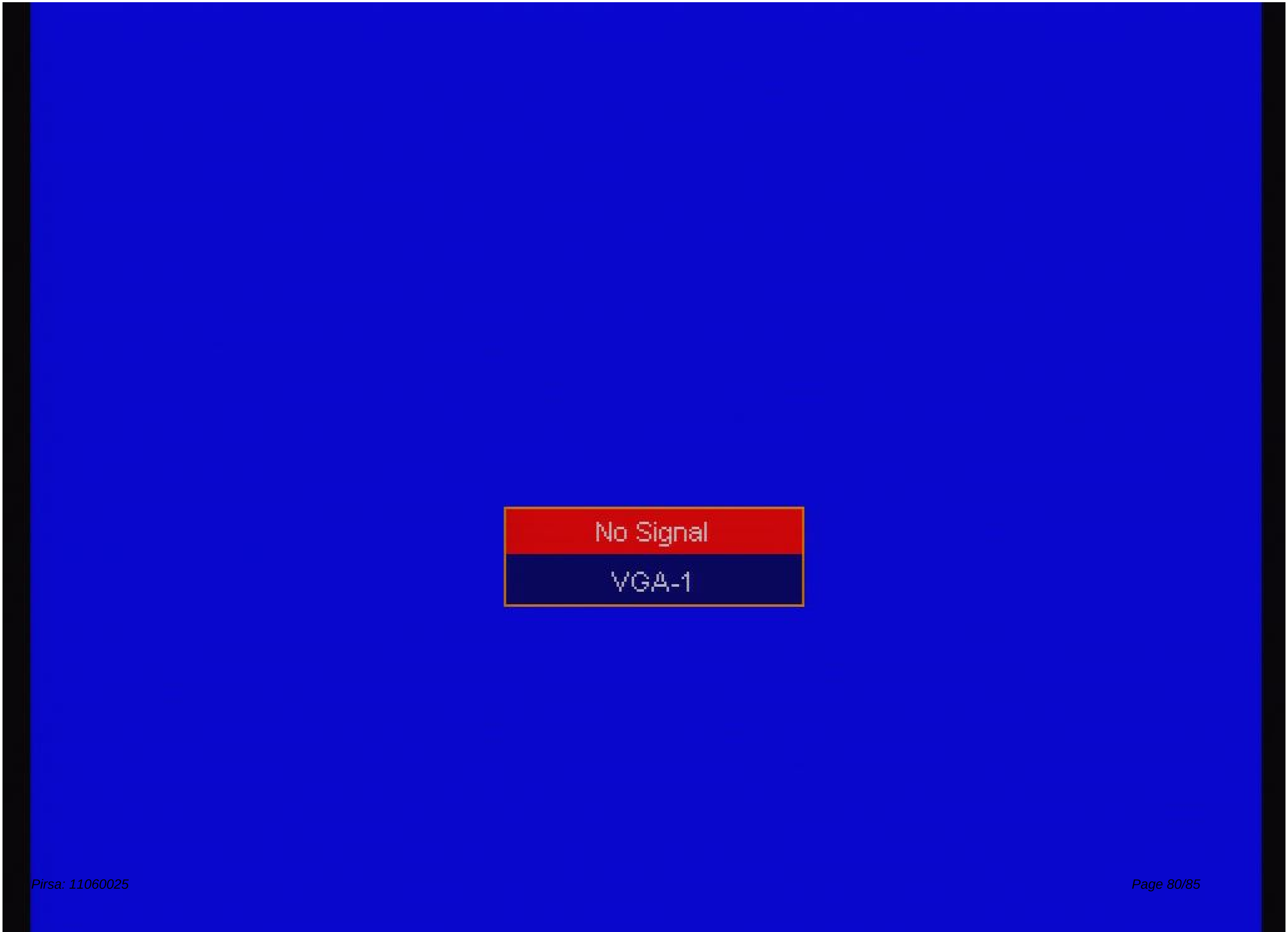
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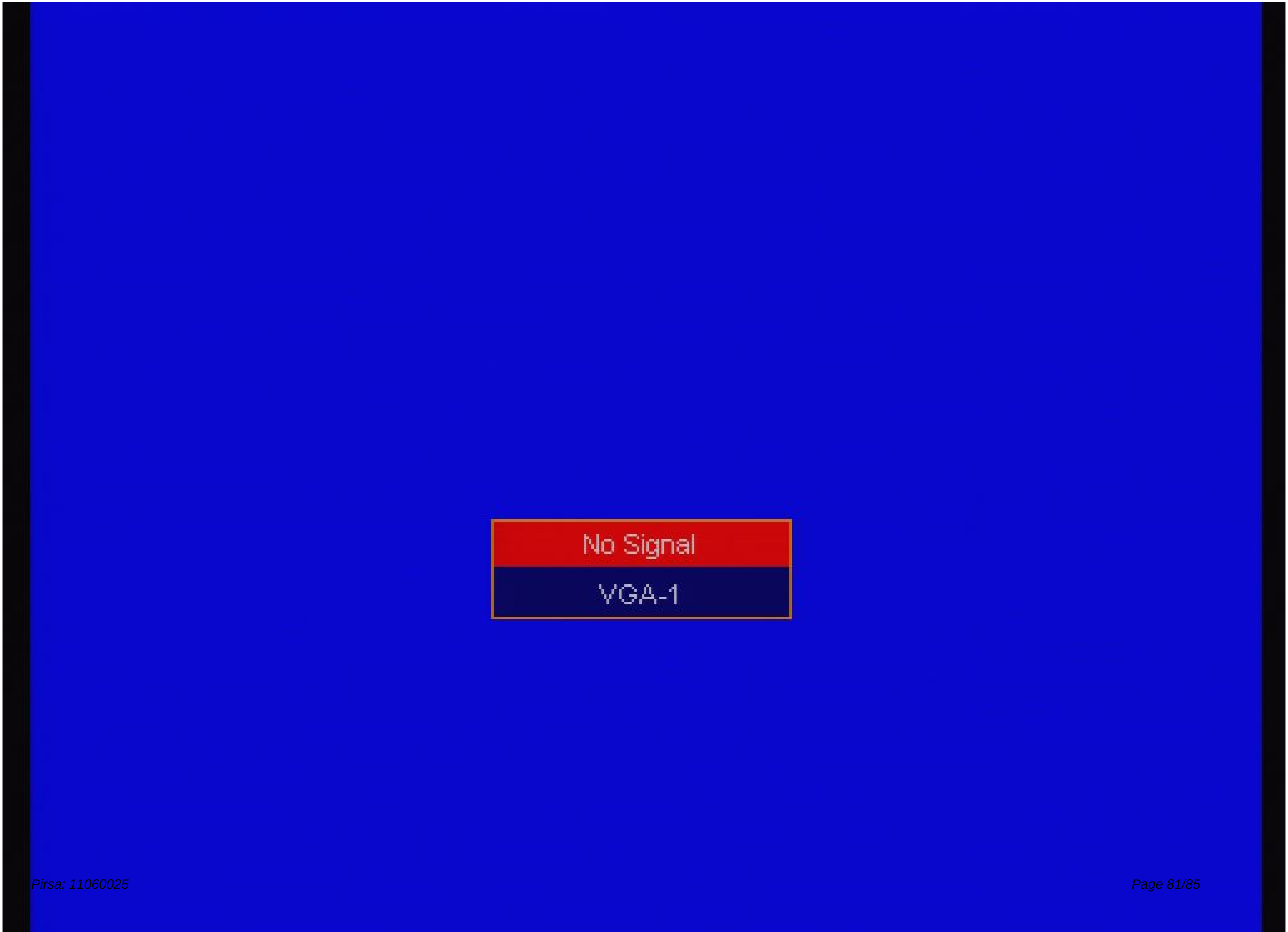
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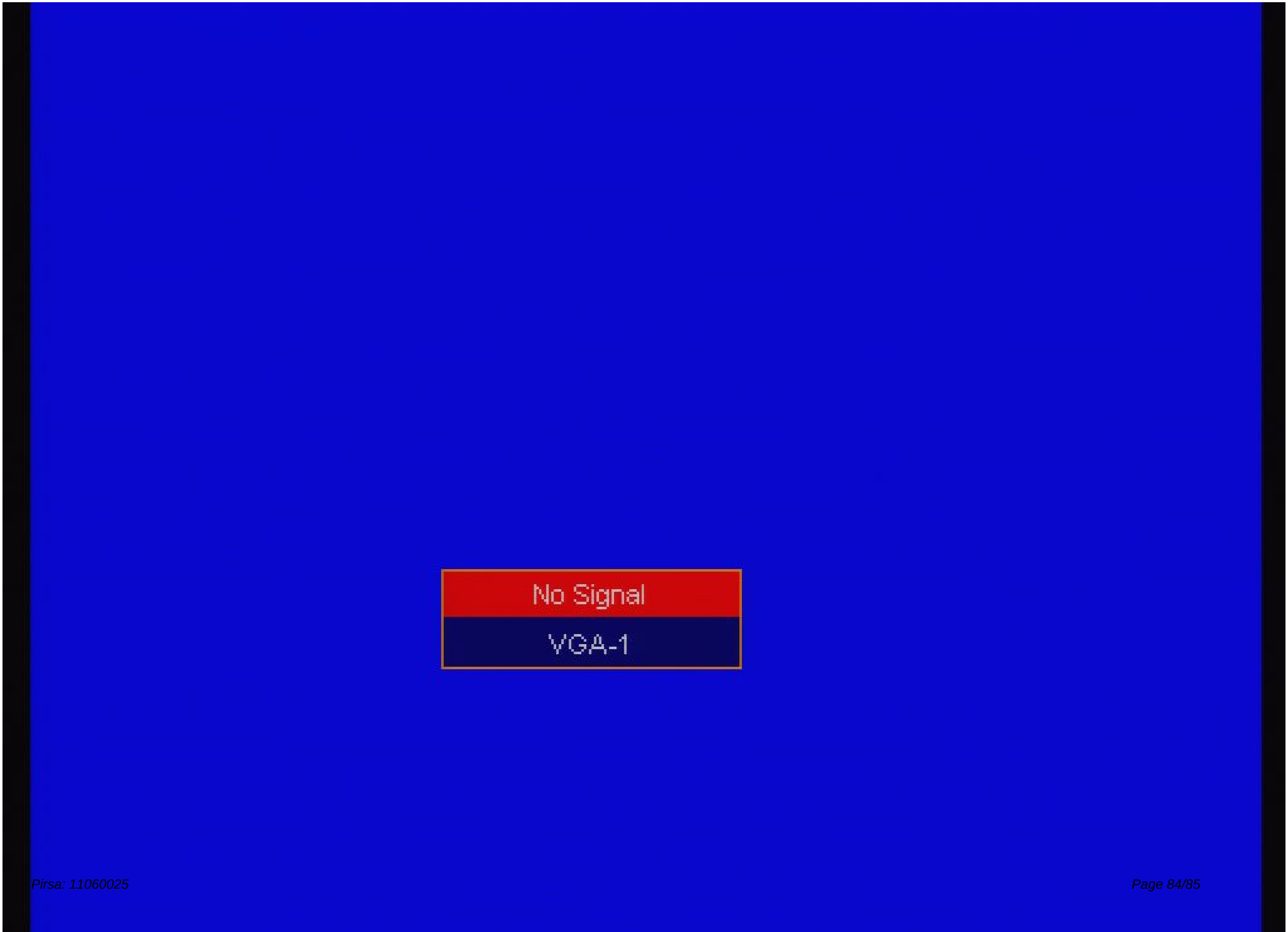
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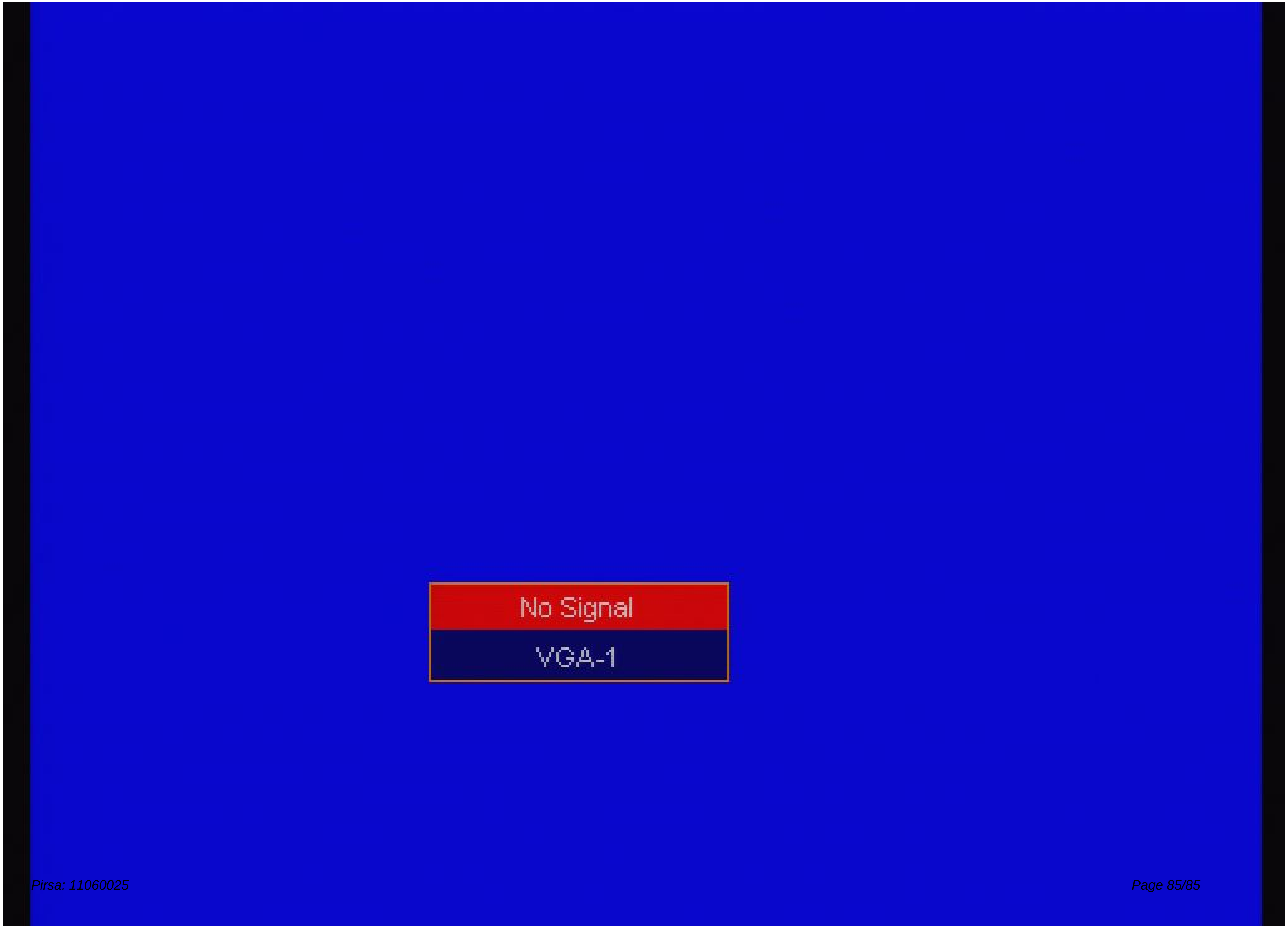
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