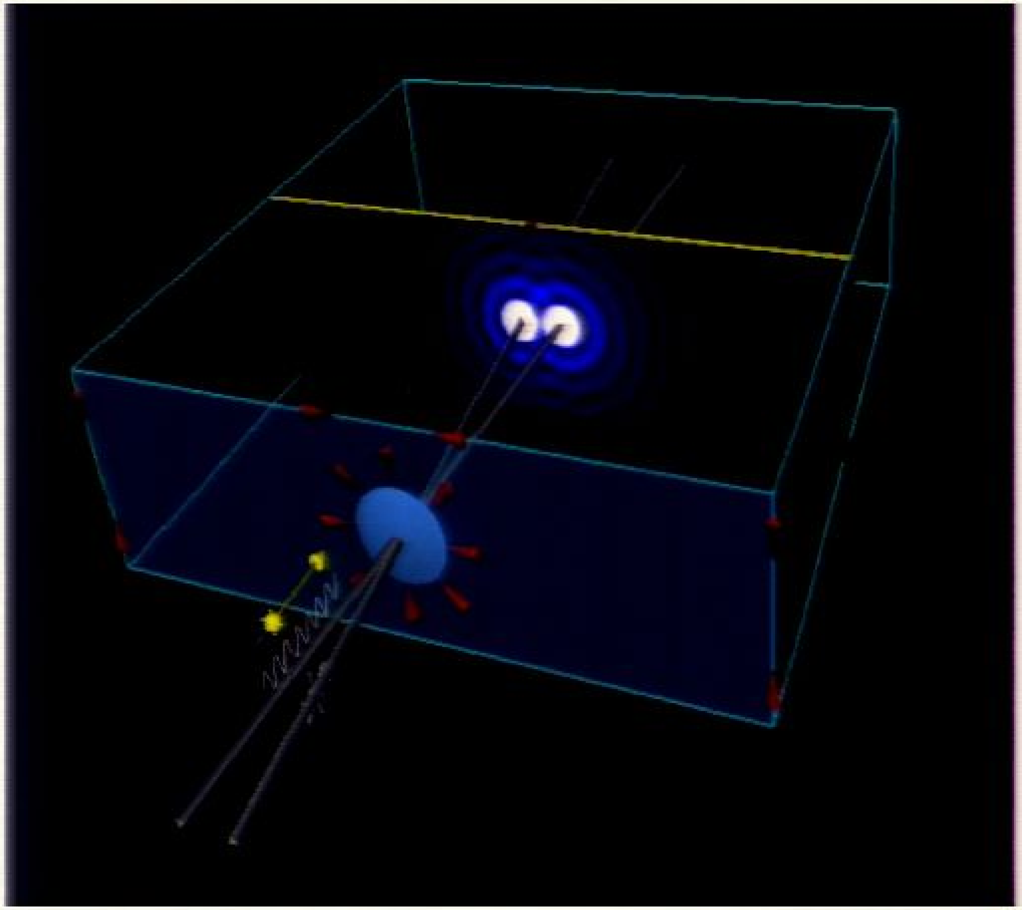
Title: Three-nucleon forces: From neutron matter to neutron stars

Date: Jun 20, 2011 11:20 AM

URL: http://pirsa.org/11060015

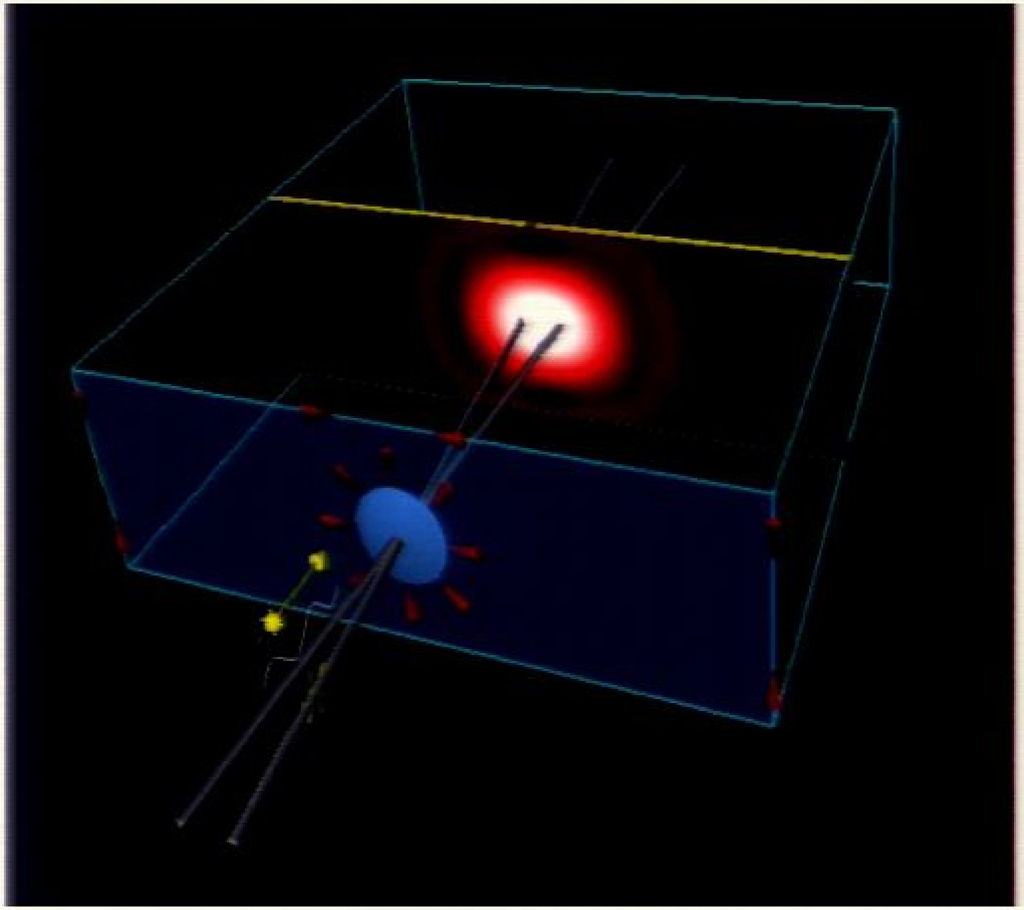
Abstract:

Wavelength and resolution



size of resolvable structures depends on the wavelength

Wavelength and resolution



size of resolvable structures depends on the wavelength

Question: Which resolution should we choose?

Resolution: The higher the better?

in the nuclear physics here we are interested in low-energy observables

(long-wavelength information!)

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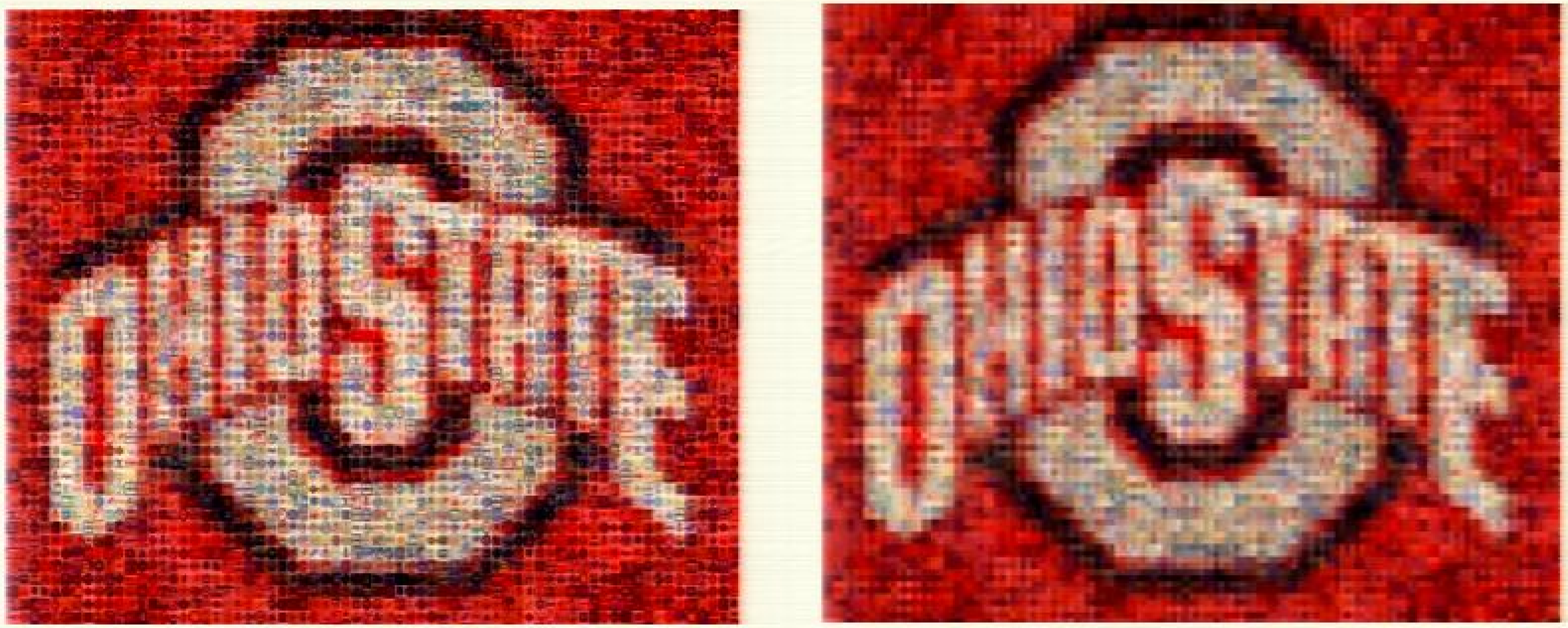
- resolution of very small (irrelevant) structures can obscure this information
- small details have nothing to do with long-wavelength information!

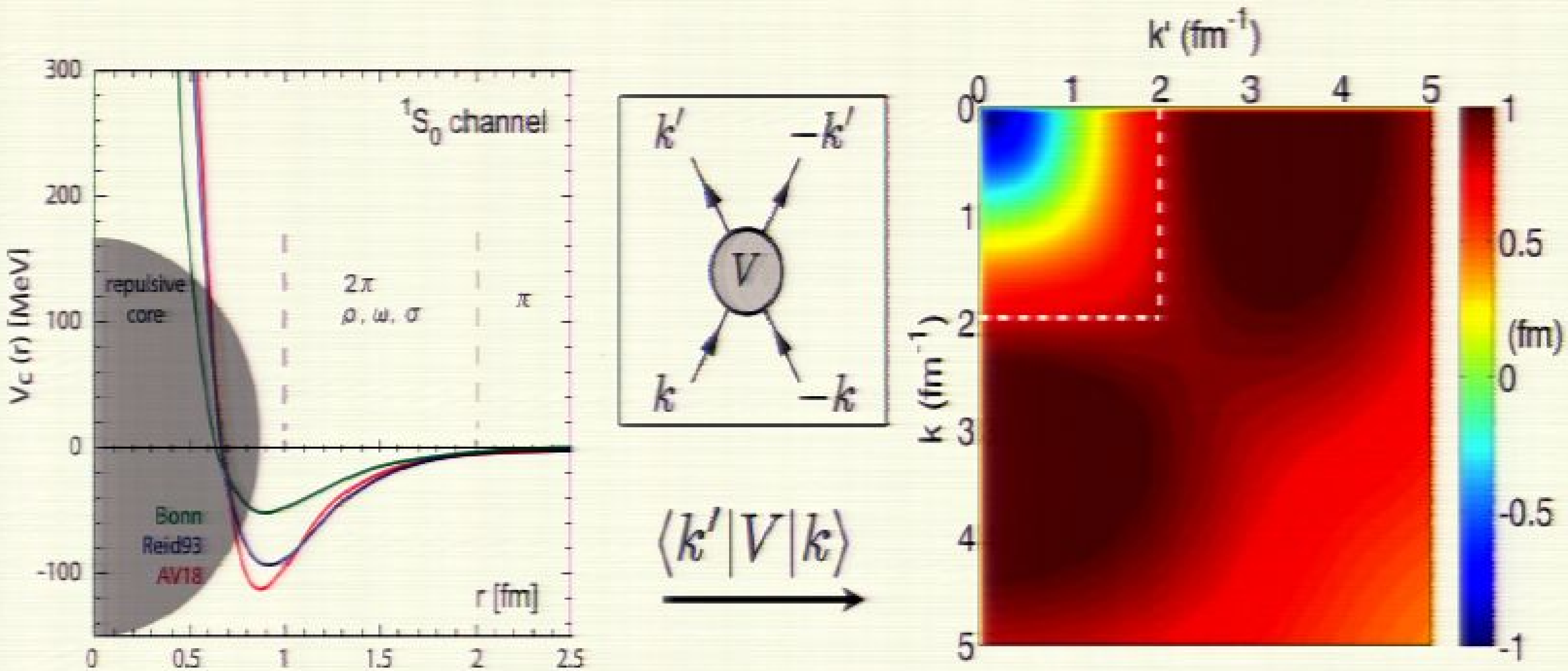
Strategy: Use a low-resolution version



- long-wavelength information is preserved
- distortion at small distance significantly reduced
- much less information necessary

In nuclear physics:
Use renormalization group (RG) to change resolution!

Problem: Traditional “hard” NN interactions



- constructed to fit scattering data (long-wavelength information!)
- “hard” NN interactions contain repulsive core at small relative distance
- strong coupling between low and high-momentum components, hard to solve!

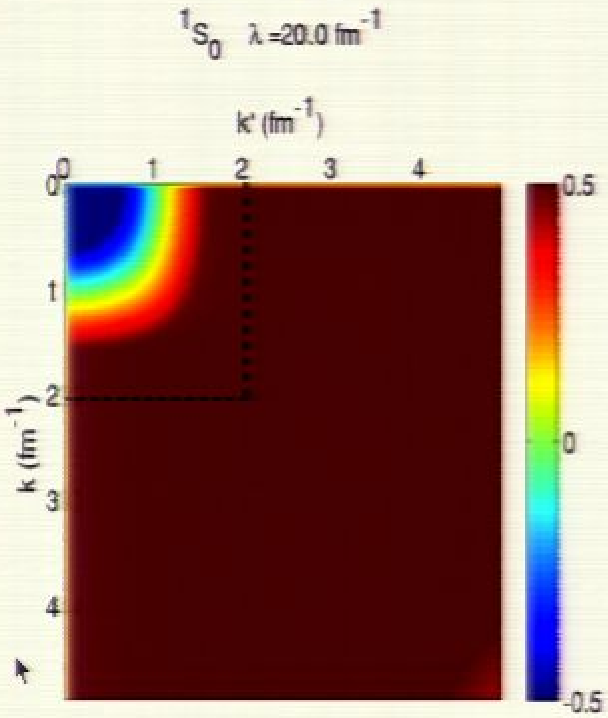
Claim:
Problems due to high resolution from interaction.
These interactions correspond to using beer coasters.

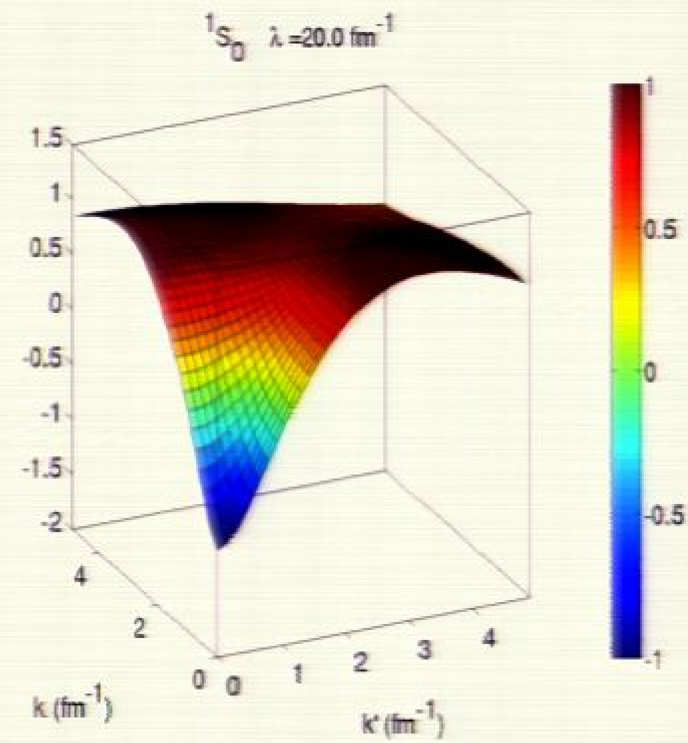
Changing the resolution: The (Similarity) Renormalization Group

- goal: generate unitary transformation of "hard" Hamiltonian

$$H_\lambda = U_\lambda H U_\lambda^\dagger$$ with the resolution parameter λ

- basic idea: change resolution in small steps: $\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$





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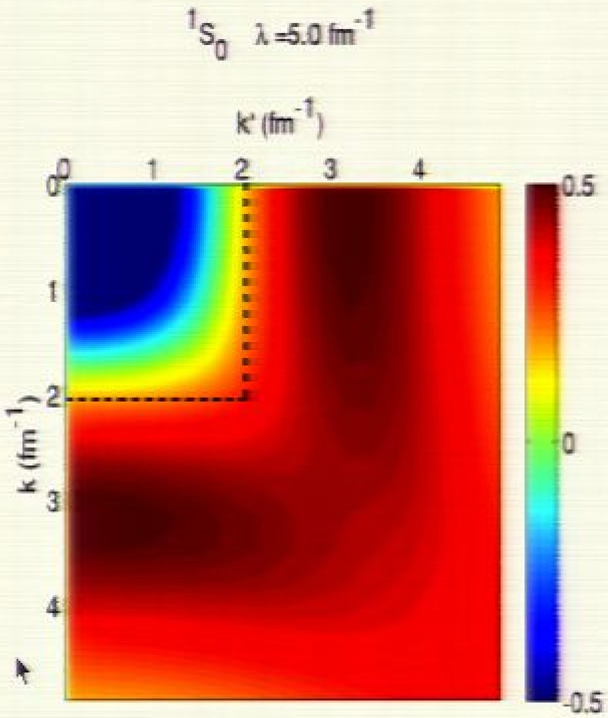

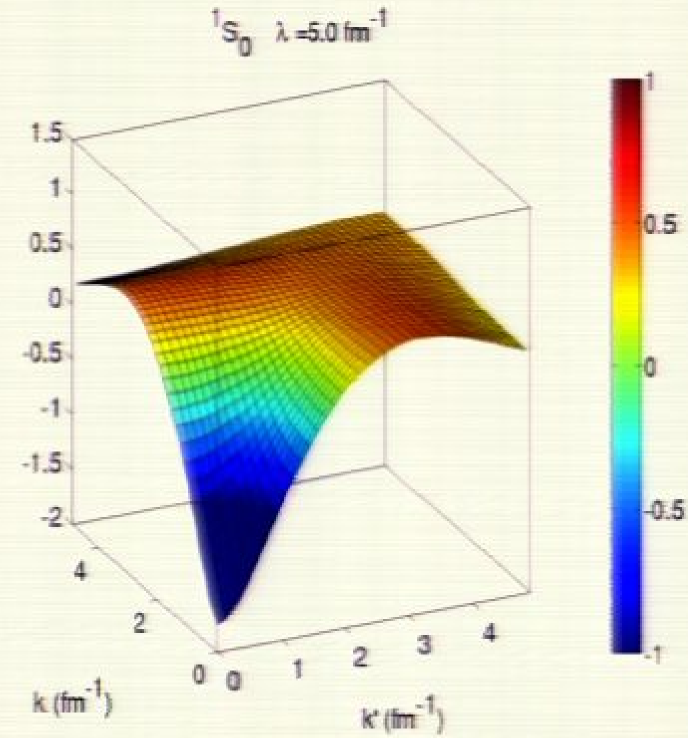

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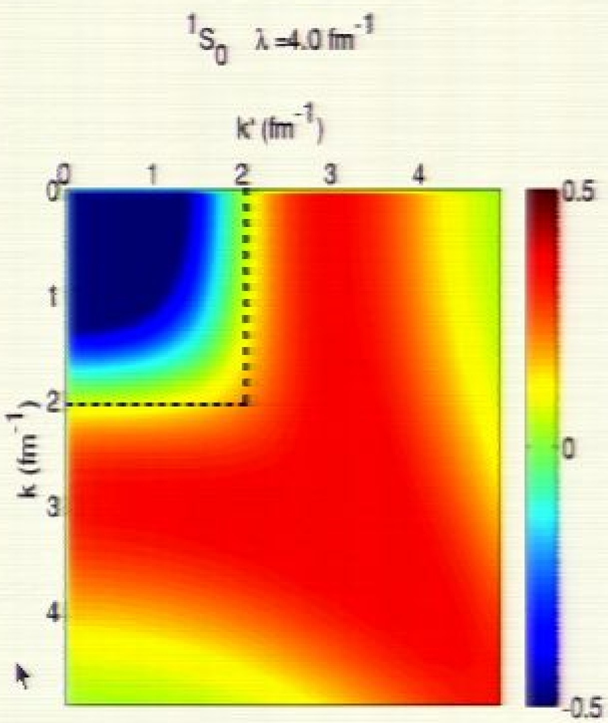

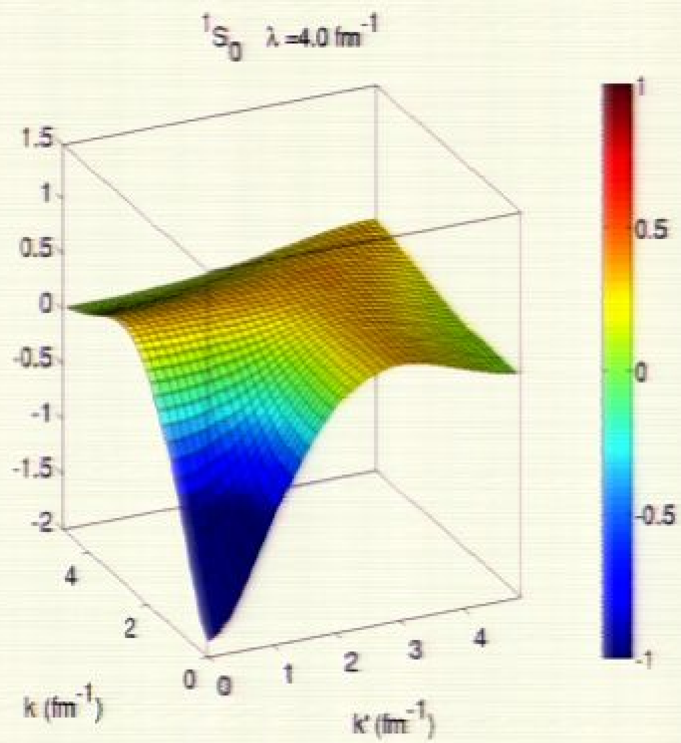

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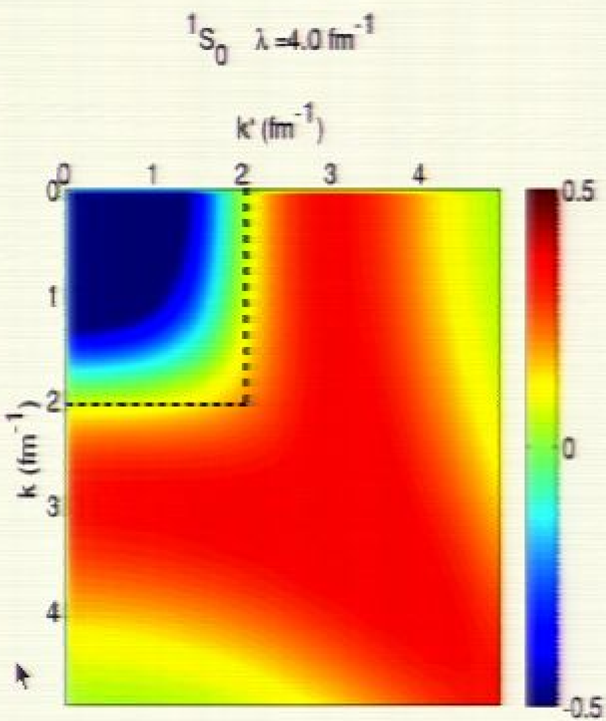

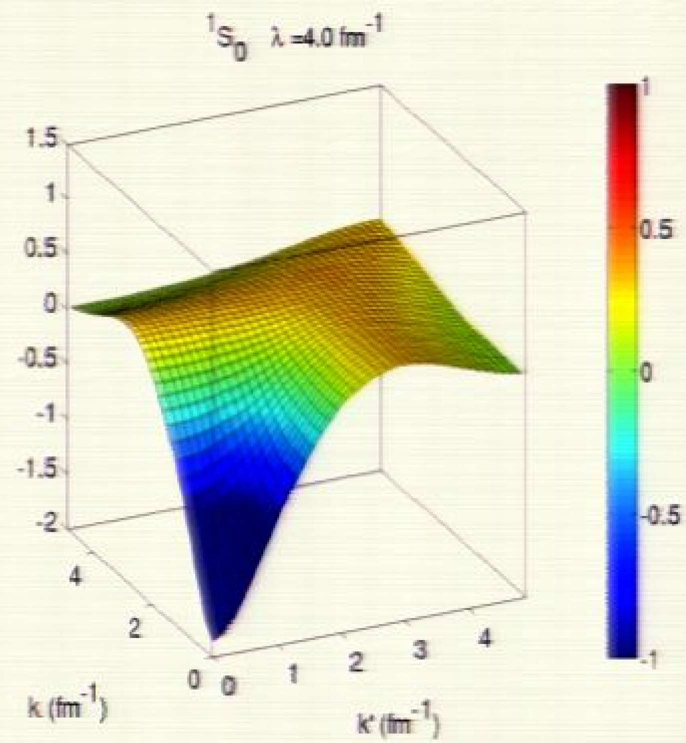

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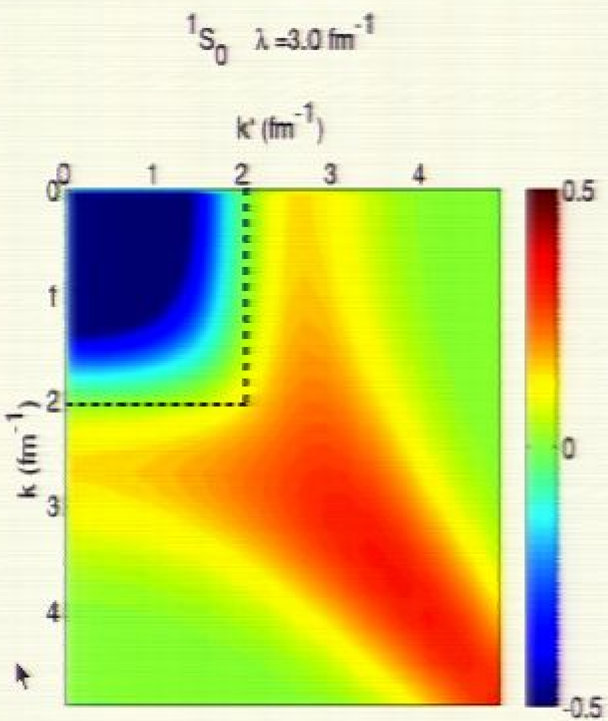

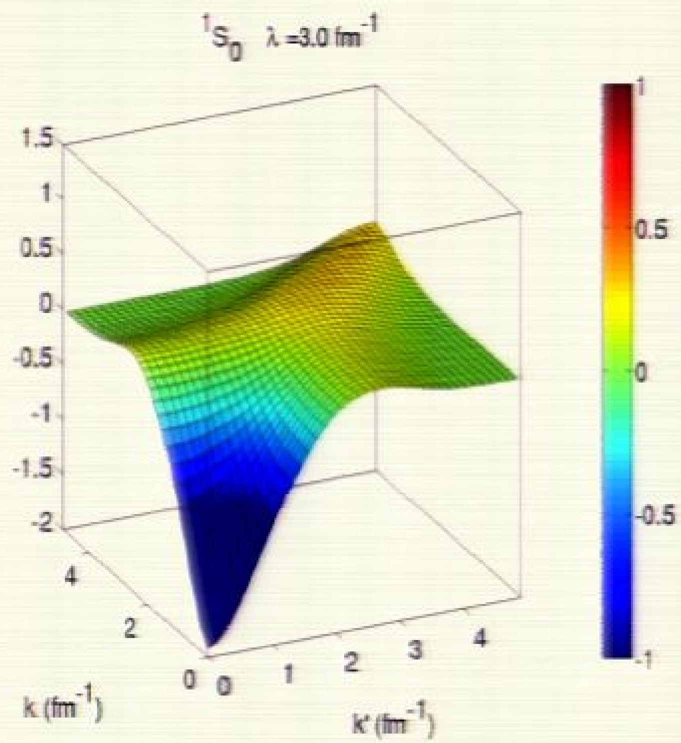

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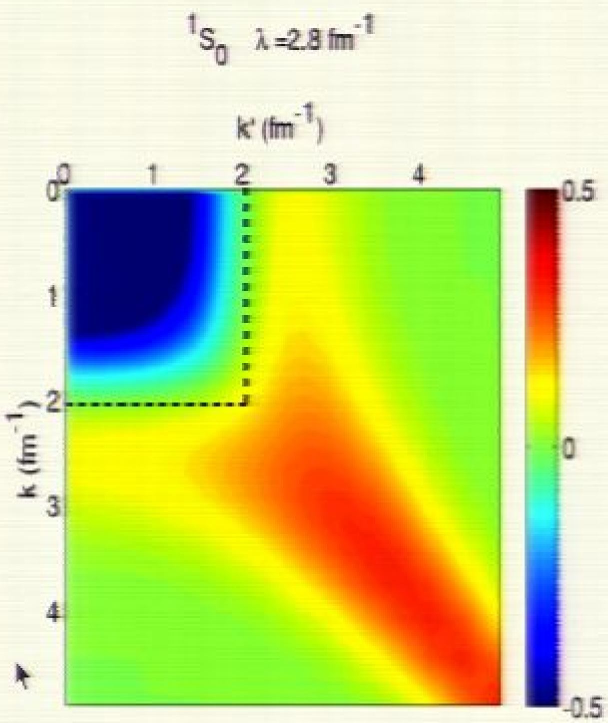

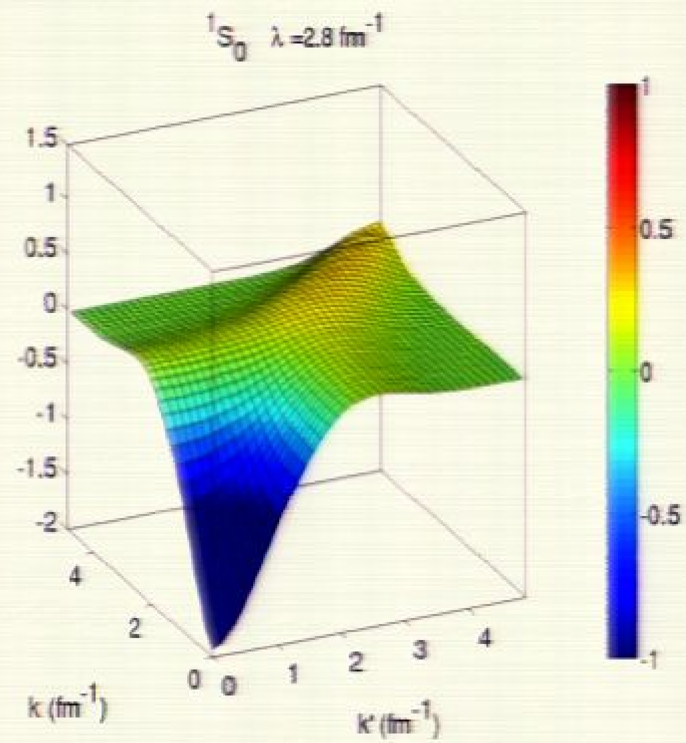

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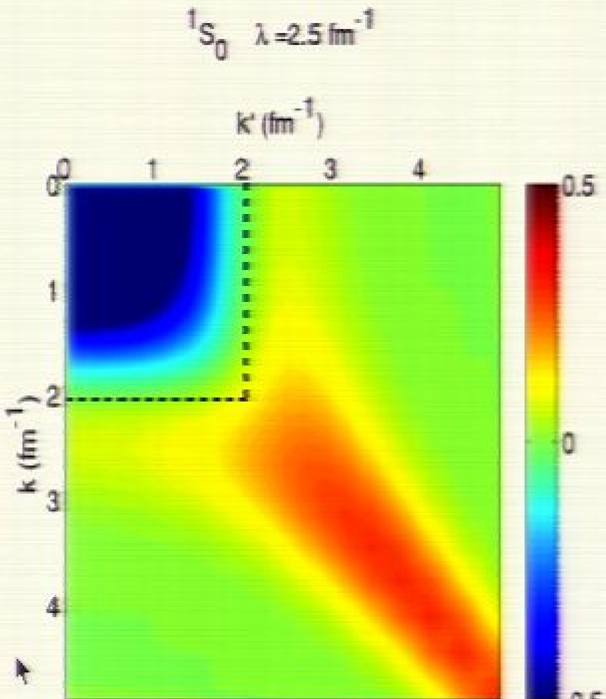

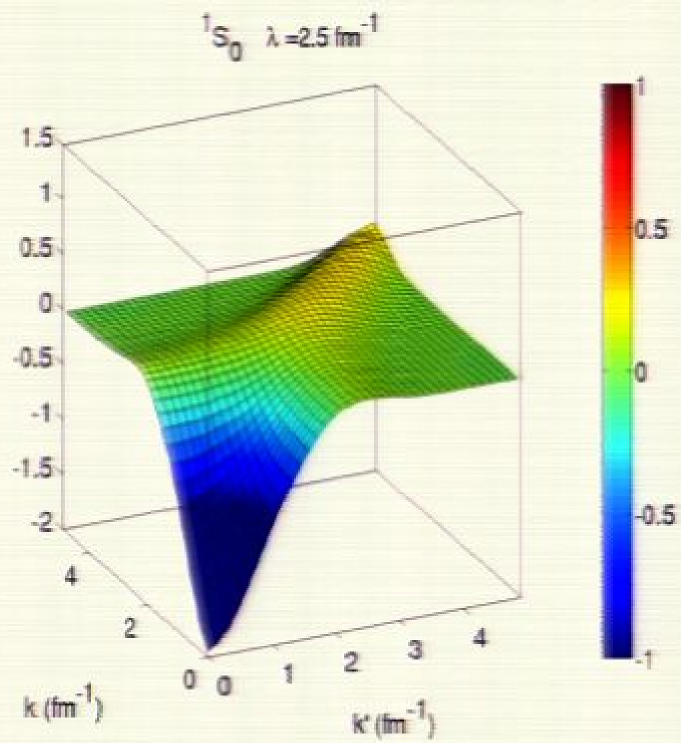

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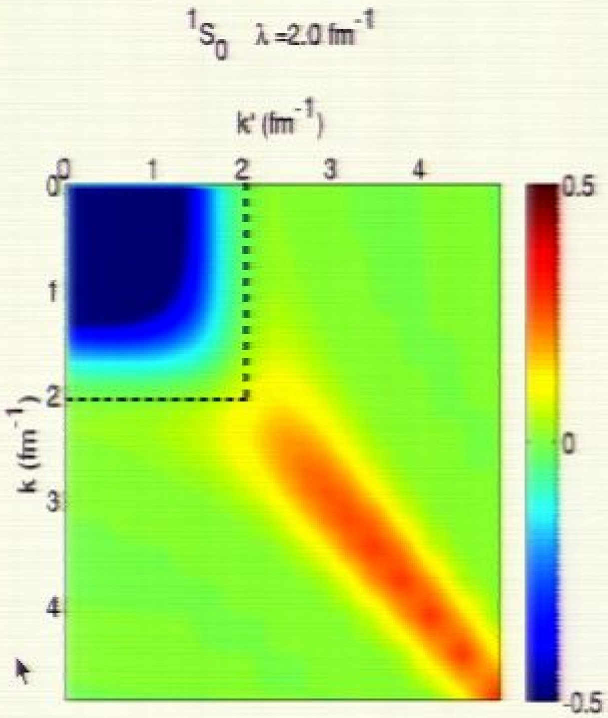
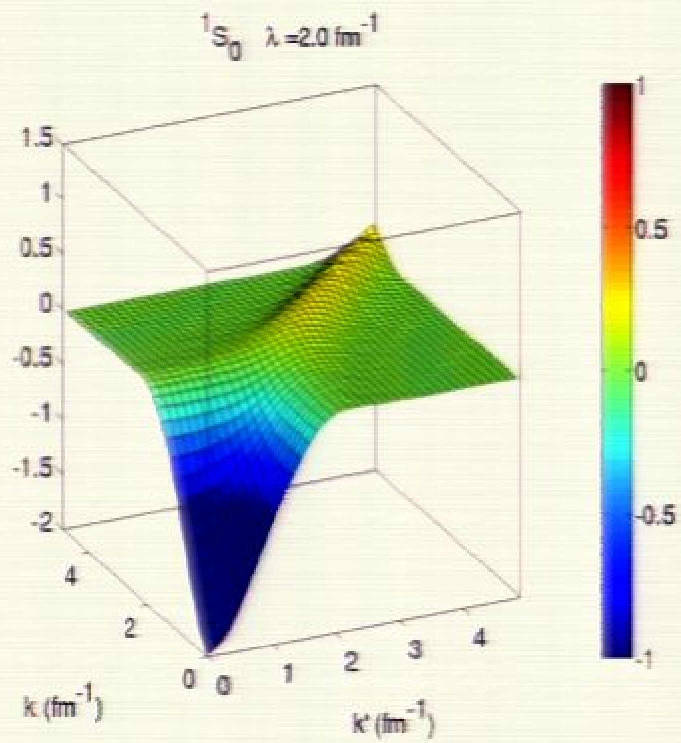
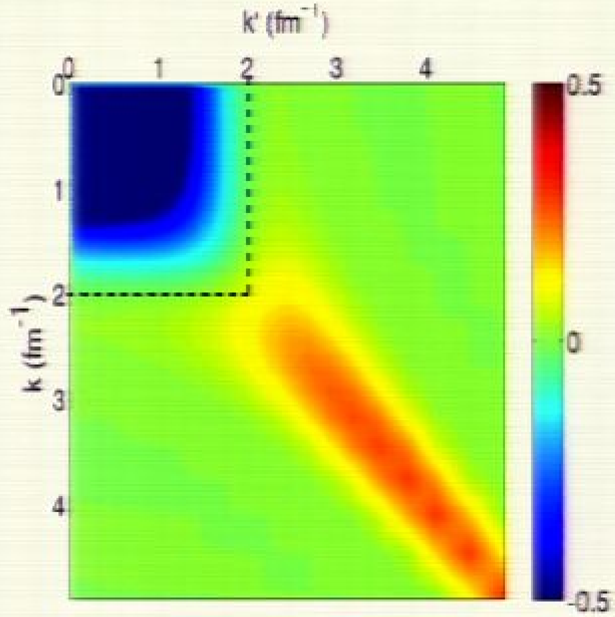




- SRG only one possibility, also: $V_{low\,k}$, UCOM, Lee-Suzuki...

Changing the resolution:
The (Similarity) Renormalization Group



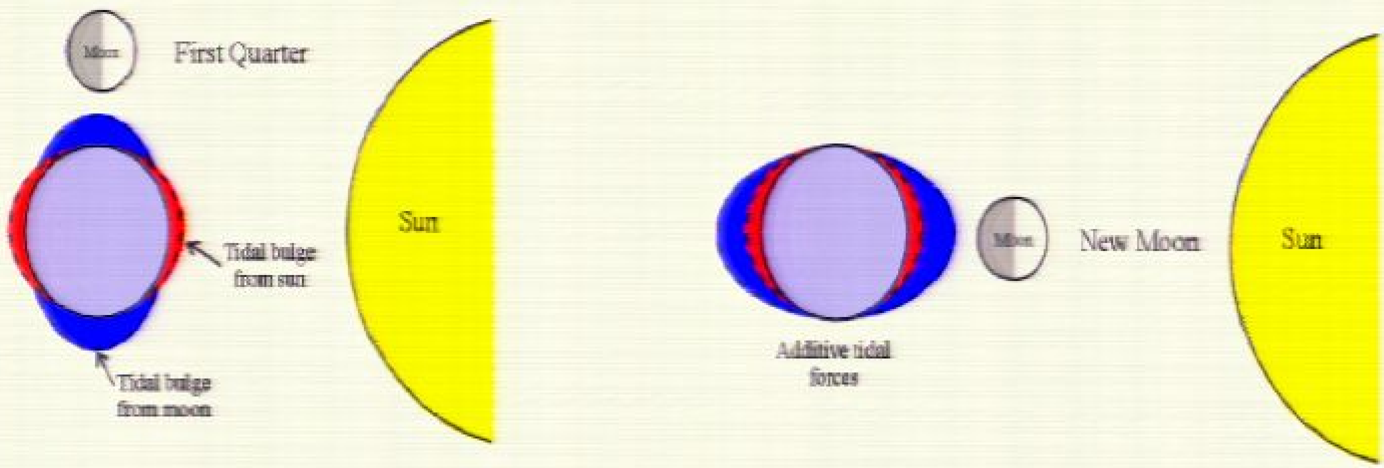
- elimination of coupling between low- and high momentum components, calculations much easier!
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions!

Why are there 3N forces?

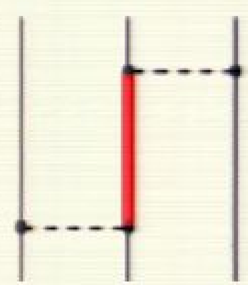
Classical analog

Tidal effects lead to 3N forces in earth-sun-moon system:



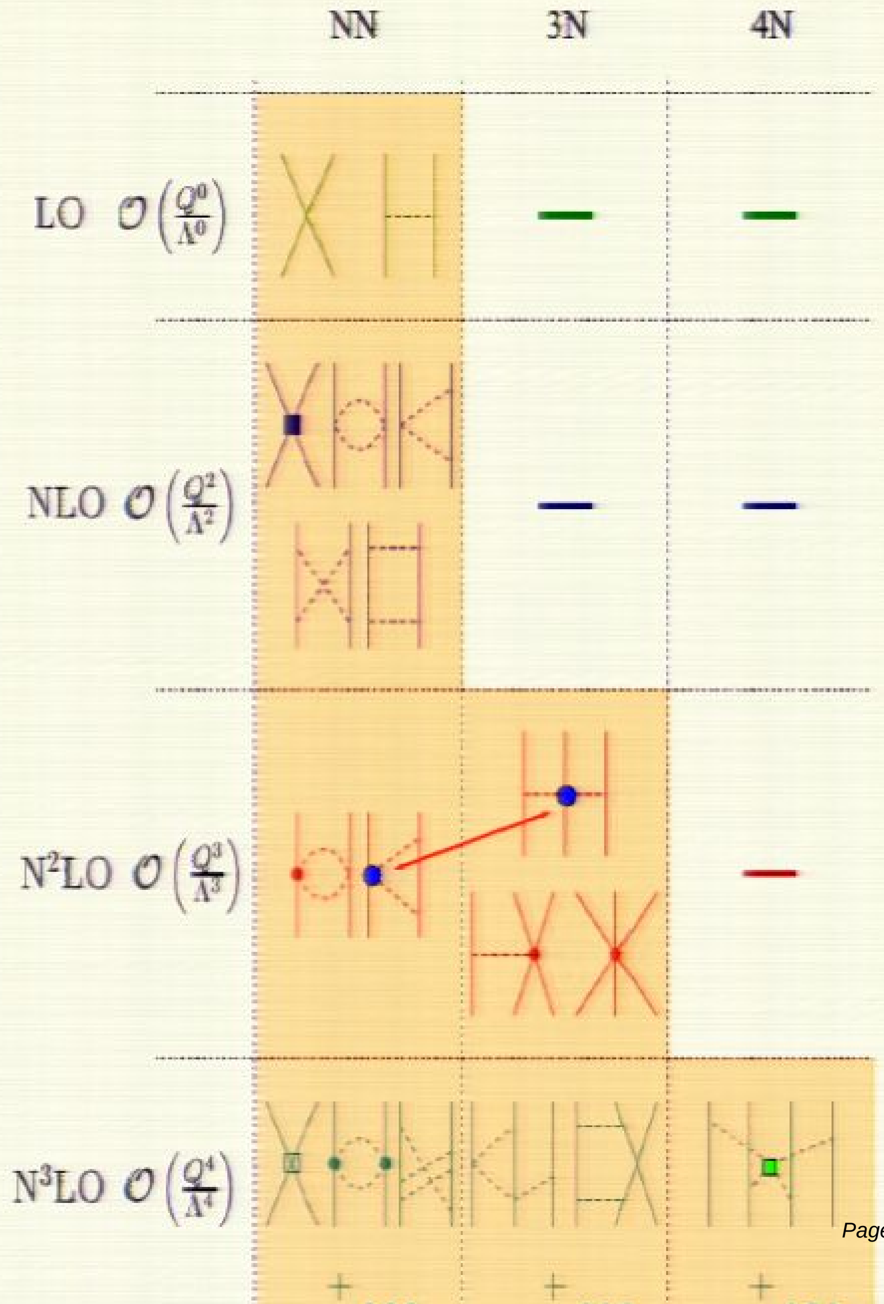
- force between earth and moon depends on the position of sun
- tidal deformations are internal excitations

- nucleons are composite particles, can also be excited
- change of resolution changes the excitations that can be described explicitly ⟶ change of 3N force
- three-nucleon forces are crucial at low resolutions!



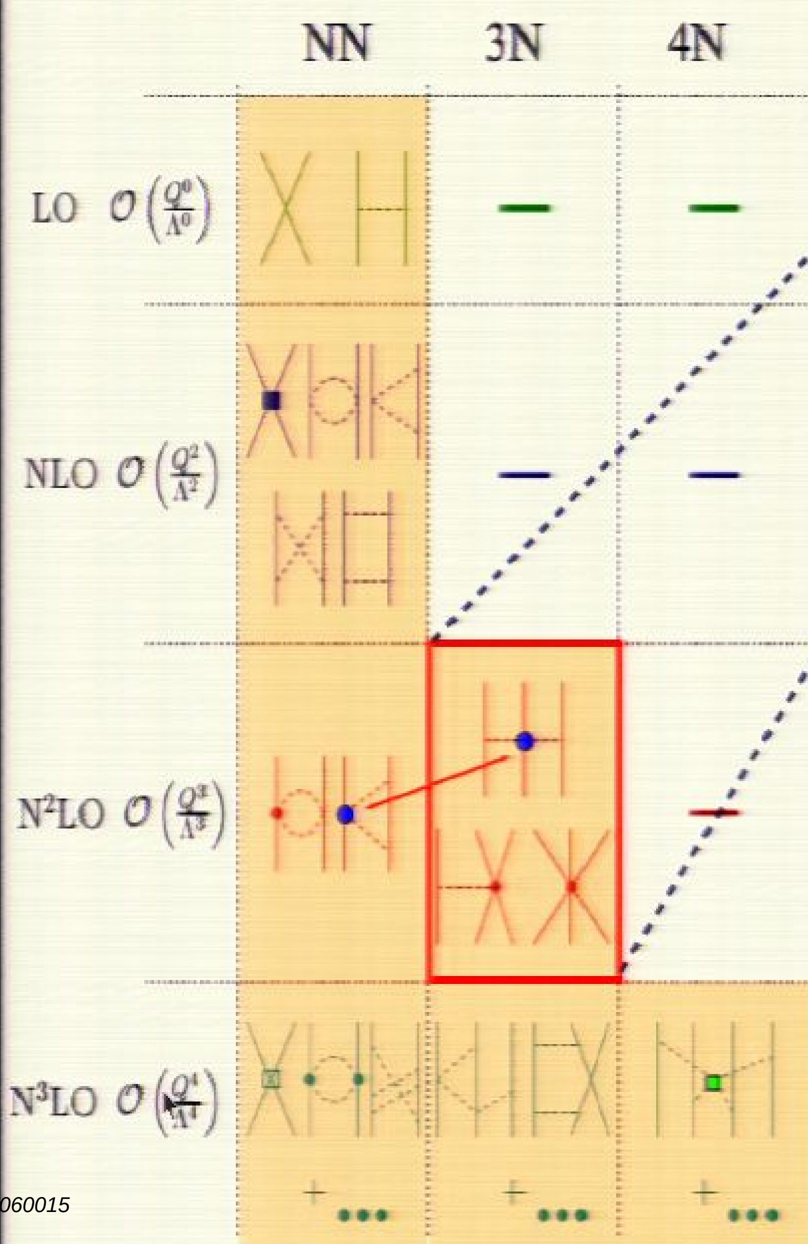
Nuclear interactions from chiral effective field theory

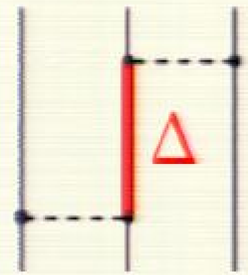
- choose effective degrees of freedom: here nucleons and pions
- short-range physics captured in few short-range couplings
- separation of scales: Q << Λ_b, breakdown scale Λ_b~500 MeV
- power-counting: expand in powers Q/Λ_b
- systematic: work to desired accuracy, obtain error estimates

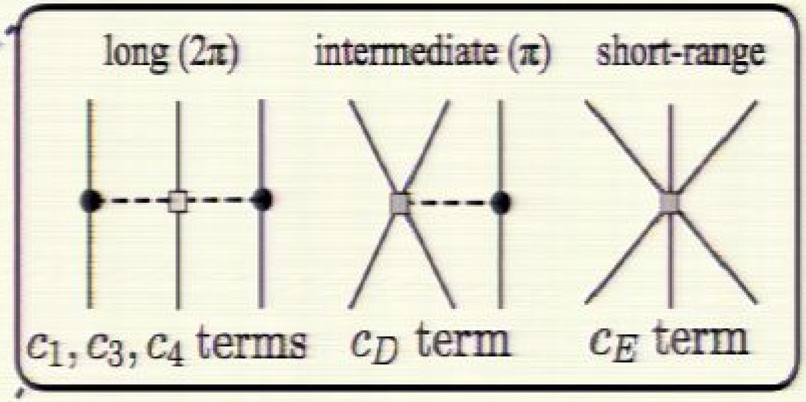


Plan: Use EFT interactions as input to RG evolution.

Leading order chiral 3N forces







- large uncertainties in 2π coupling constants at present:

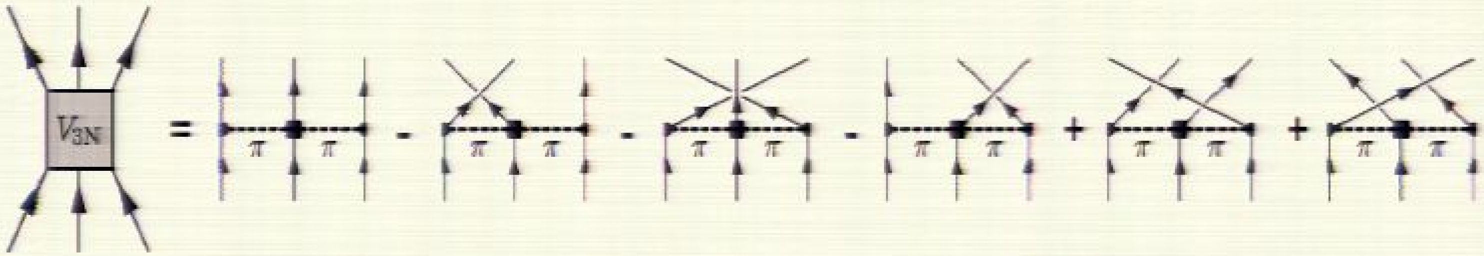
$$c_1 = -0.9^{+0.2}_{-0.5}\,,\; c_3 = -4.7^{+1.5}_{-1.0}\,,\;\; c_4 = 3.5^{+0.5}_{-0.2}$$

leads to theoretical uncertainties in many-body observables

- c_D and c_E have to be determined in $A \geq 3$ systems (do not contribute in pure neutron matter!)

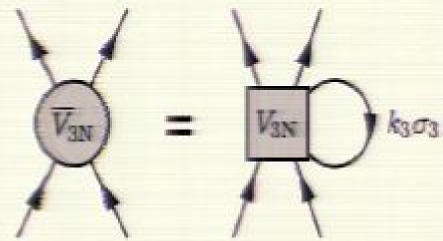
Chiral 3N interaction as density-dependent two-body interaction

(1) calculate antisymmetrized 3N interaction



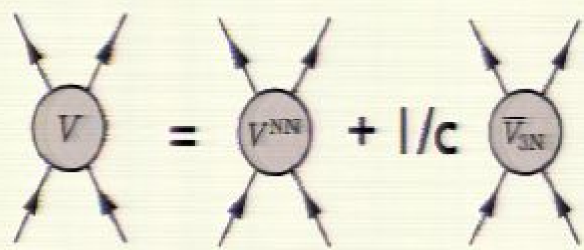
(2) construct effective density-dependent NN interaction

Basic idea:
Sum one particle over occupied states in the Fermi sea



(3) combine with free-space NN interaction

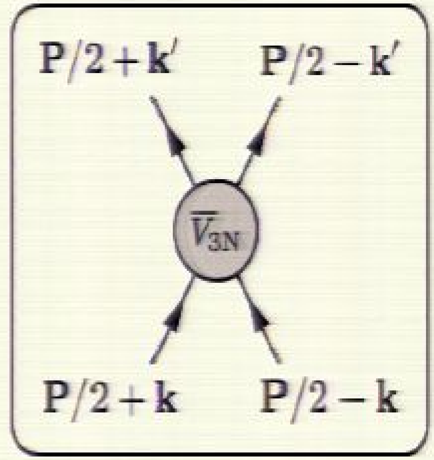
combinatorial factor c depends on type of diagram!



Properties of the effective interaction $\overline{V}_{3N}$

General momentum dependence:

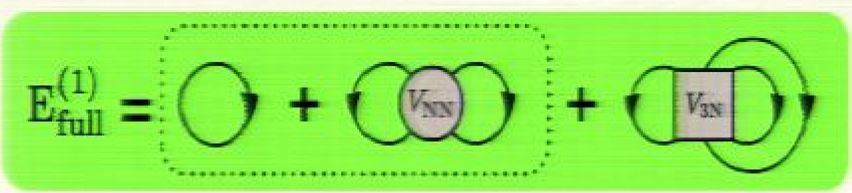
$$\overline{V}_{3N} = \overline{V}_{3N}(\mathbf{k}, \mathbf{k}', \mathbf{P})$$

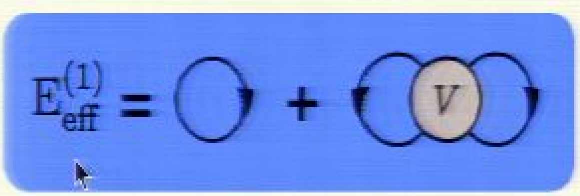


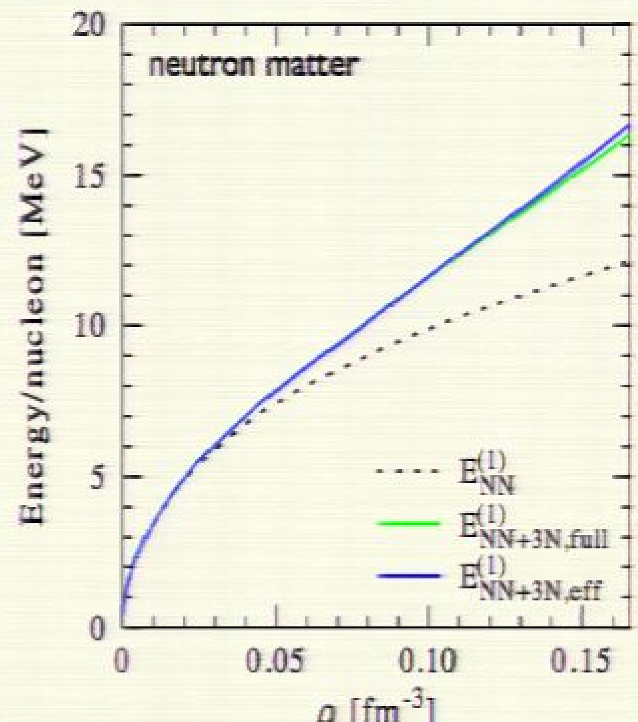
- $\mathbf{P}$-dependence much weaker than $\mathbf{k}, \mathbf{k}'$-dependence!
- neglect $\mathbf{P}$-dependence, set $\mathbf{P} = 0$
- matrix elements have the same form like free-space

NN interaction matrix elements

- straightforward to include in existing many-body schemes



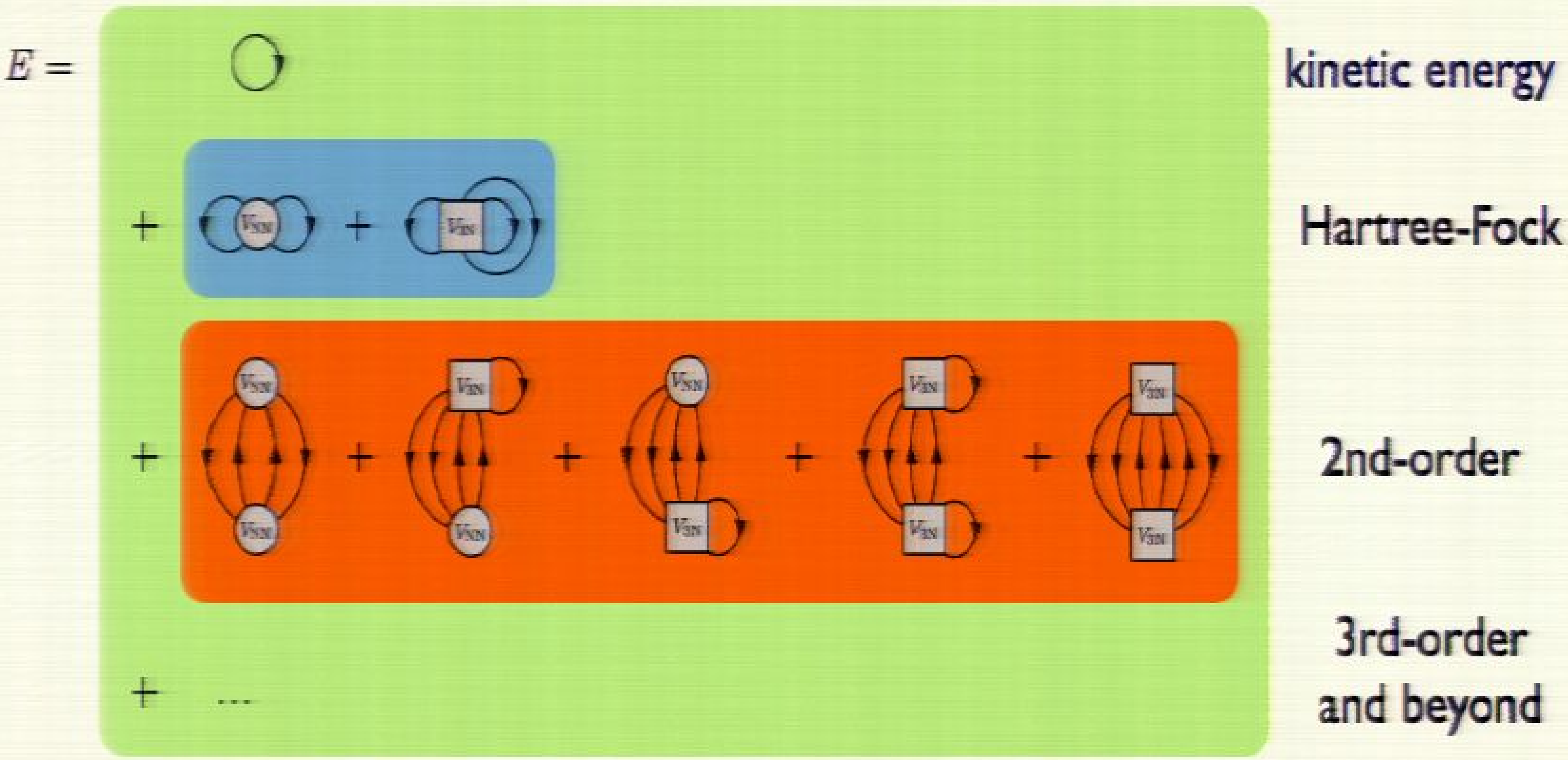




Equation of state: Many-body perturbation theory

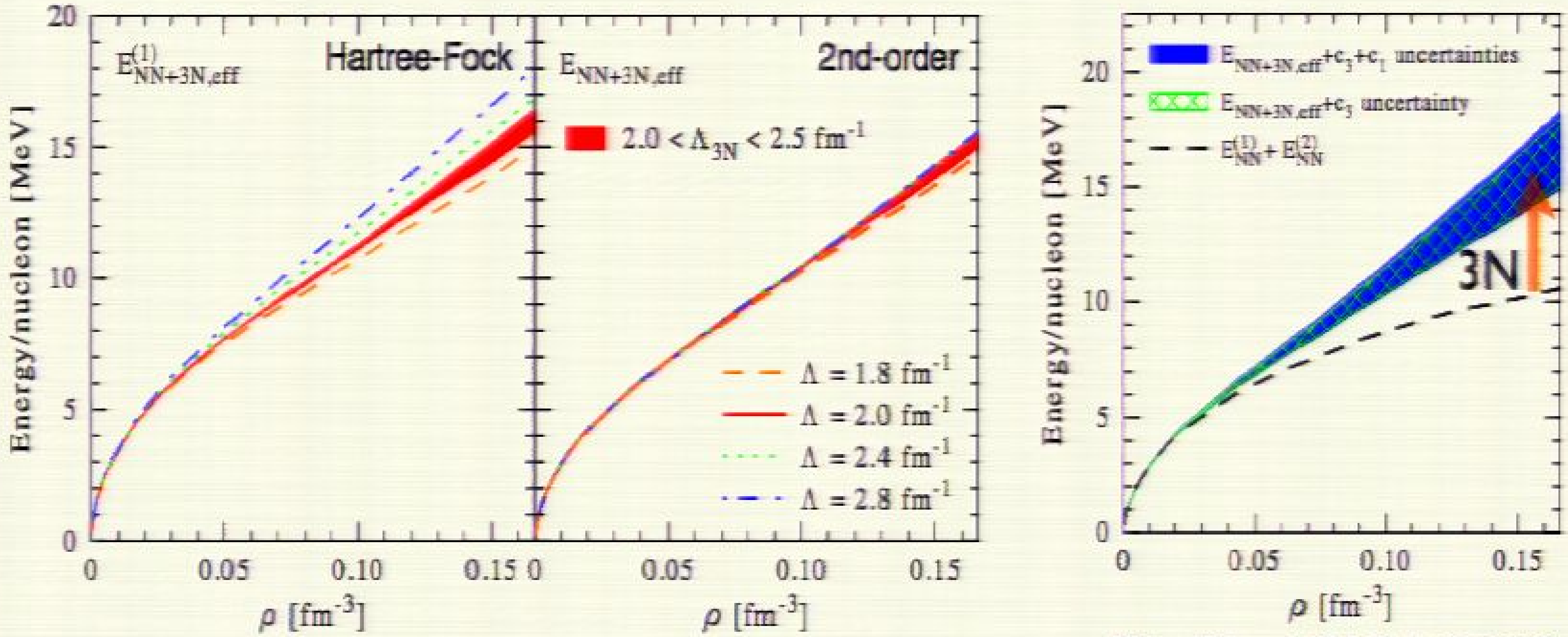
central quantity of interest: energy per particle E/N

$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \dots$$



- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of 3N interaction contributions

Equation of state of pure neutron matter



KH and Schwenk PRC 82, 014314 (2010)

- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence

Neutron star radius constraints

Problem: Solution of TOV equation requires EOS up to very high densities. Radius of a typical NS ($M \sim 1.4\, M_\odot$) theoretically not well constrained.

But: Radius of NS is relatively insensitive to high density region.

---

incorporation of beta-equilibrium: neutron matter ⟶ neutron star matter

parametrize piecewise high-density extensions of EOS:

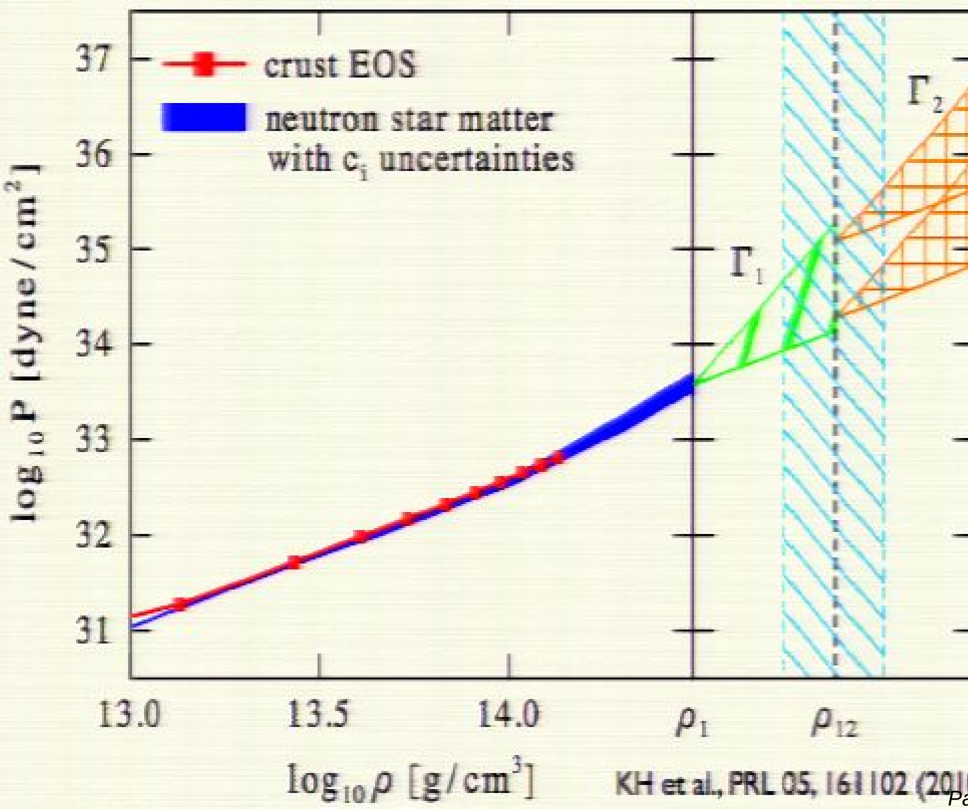
- use polytropic ansatz

$$p \sim \rho^\Gamma$$

- range of parameters

$$\Gamma_1, \rho_{12}, \Gamma_2$$

limited by physics!



KH et al., PRL 05, 161102 (2010)

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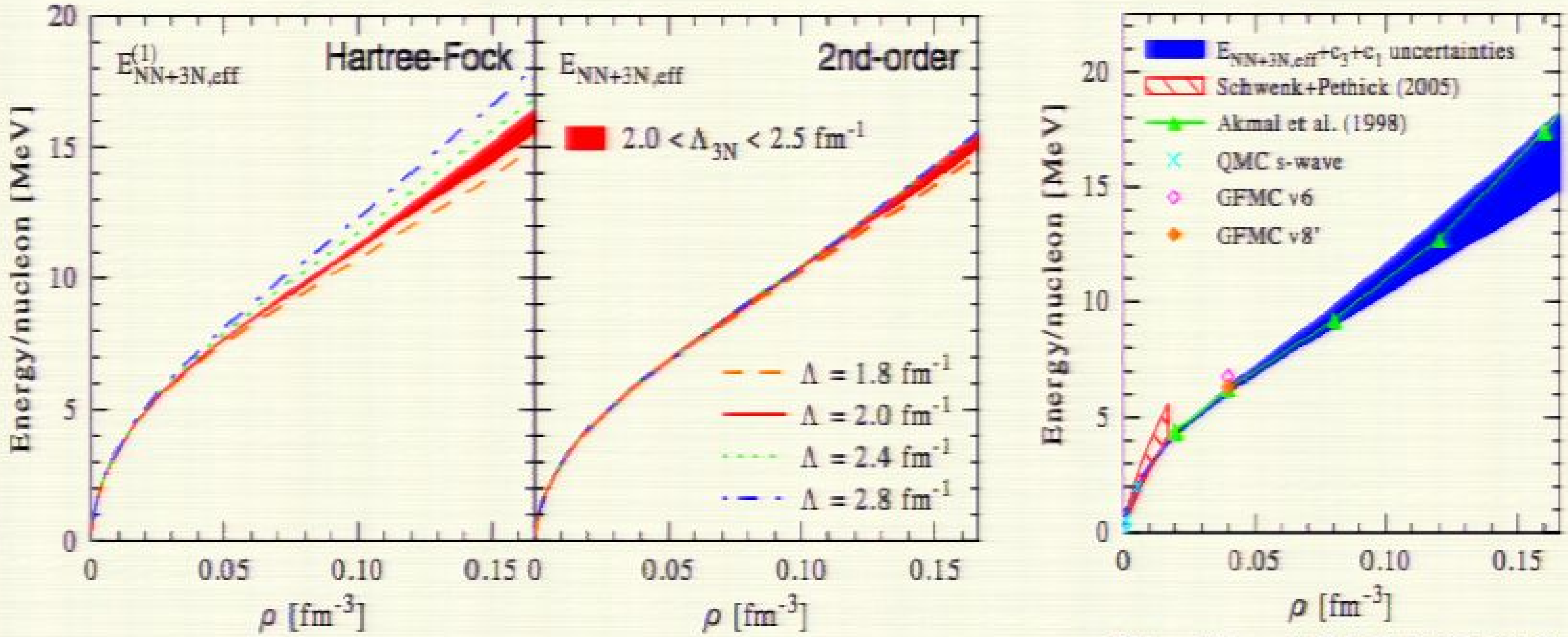

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- significantly reduced cutoff dependence at 2nd order perturbation theory
- small resolution dependence indicates converged calculation
- energy sensitive to uncertainties in 3N interaction
- variation due to 3N input uncertainty much larger than resolution dependence
- good agreement with other approaches (different NN interactions)

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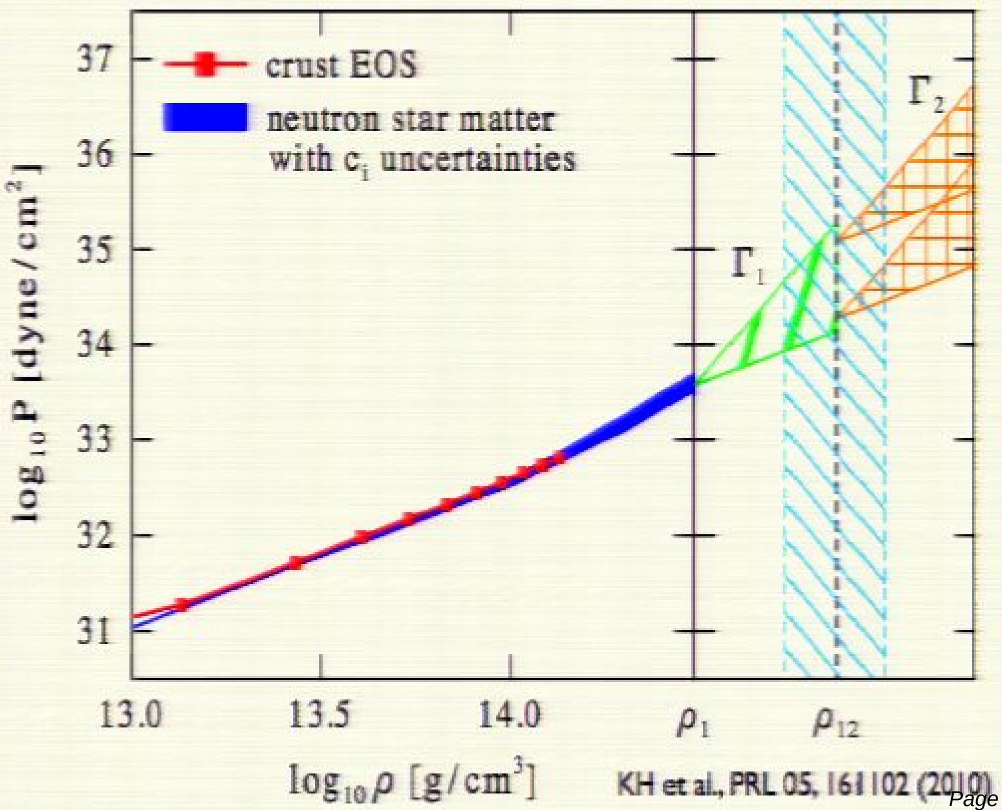

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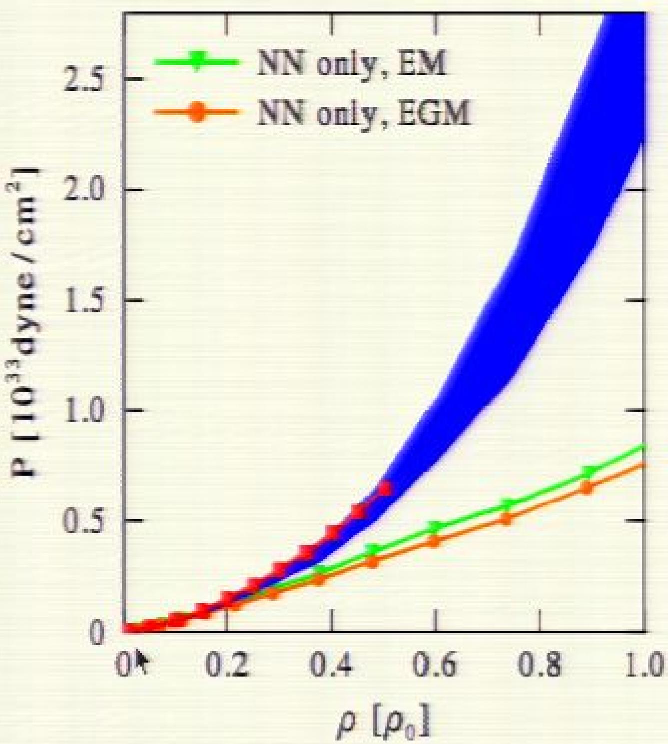
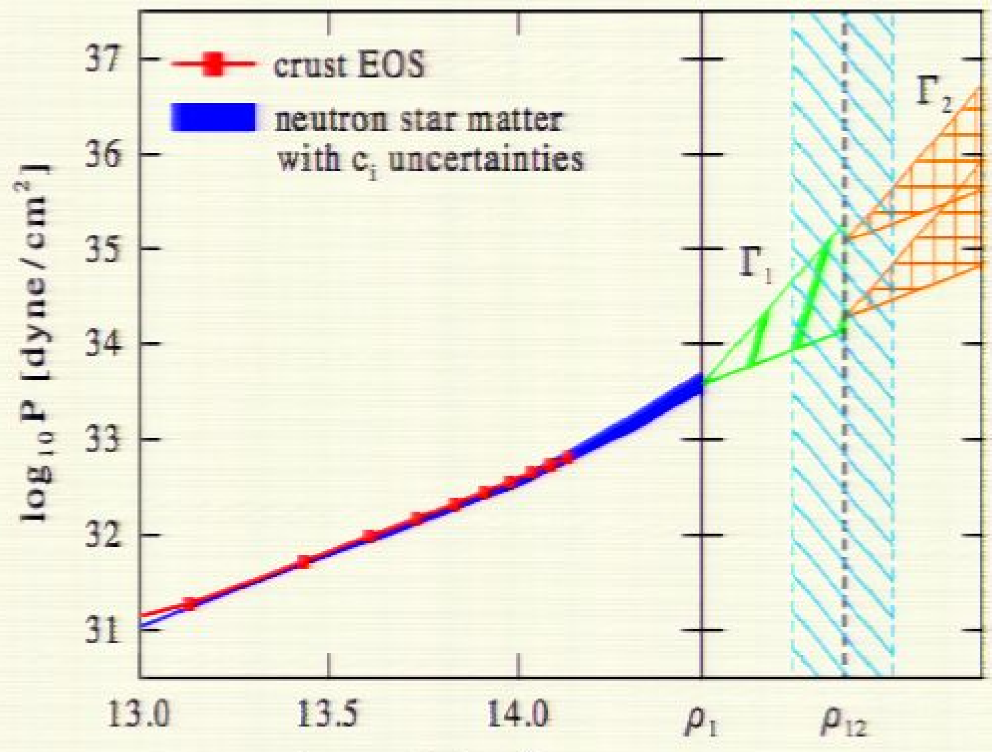




KH et al., PRL 05, 161102 (2010)

without 3N forces EOS differs significantly from crust EOS around $\rho_0/2$

Neutron star radius constraints

use the constraints:

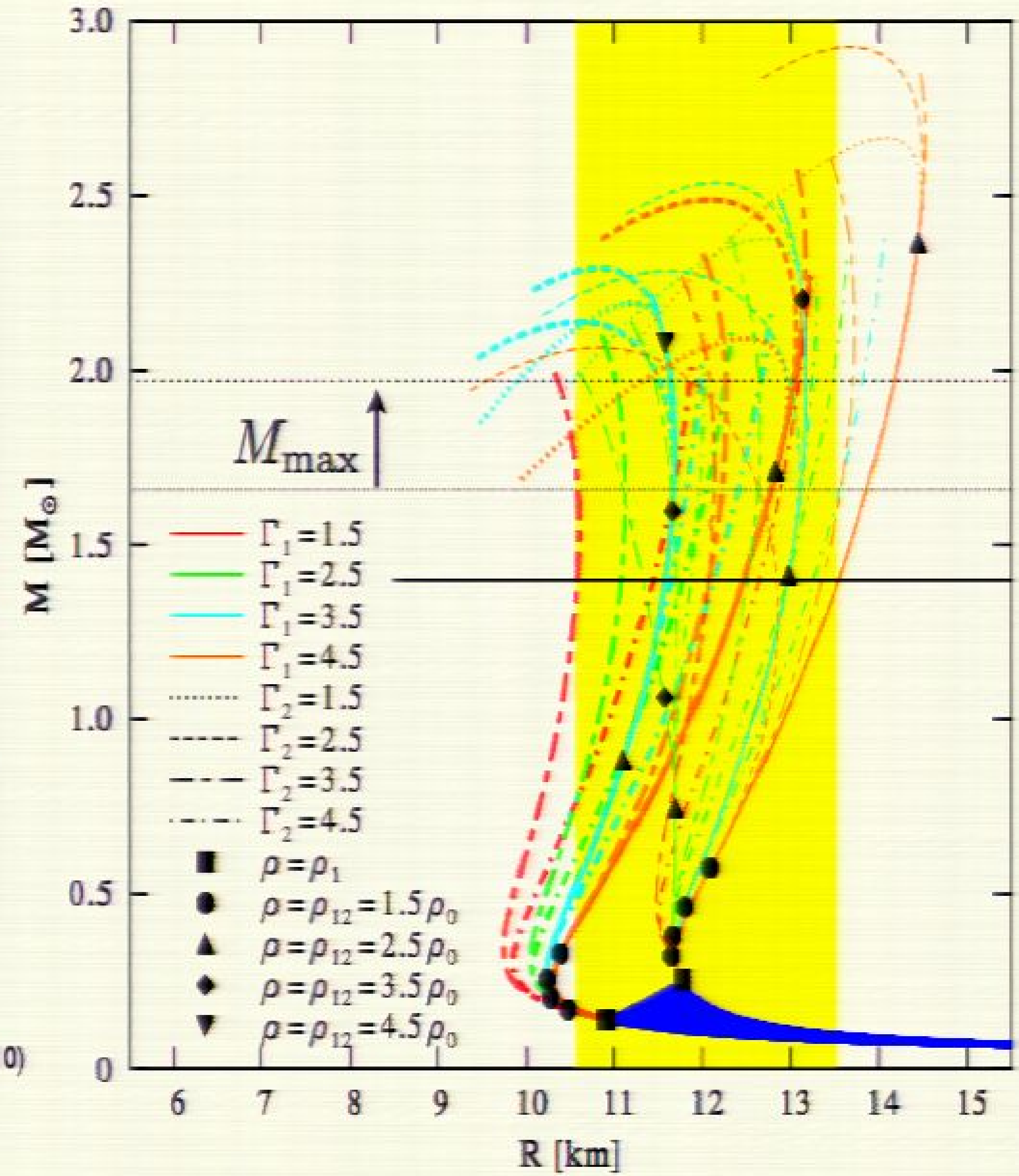
recent NS observation

$M_{\mathrm{max}} > 1.97\, M_{\odot}$

causality

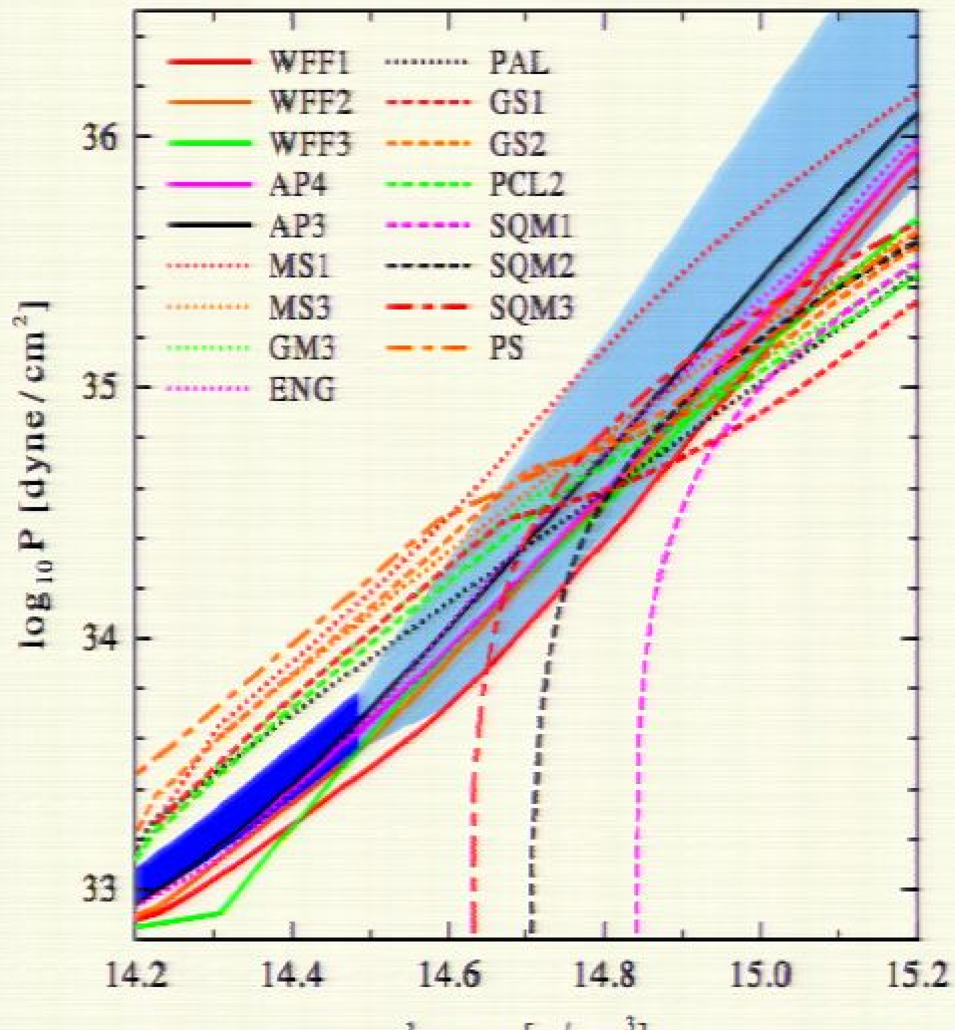
$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

KH et al., PRL 05, 161102 (2010)



- low-density part of EOS sets scale for allowed high-density extensions
- radius constraint after incorporating crust corrections: $10.5 - 13.5\,\mathrm{km}$

Constraints on neutron star equations of state

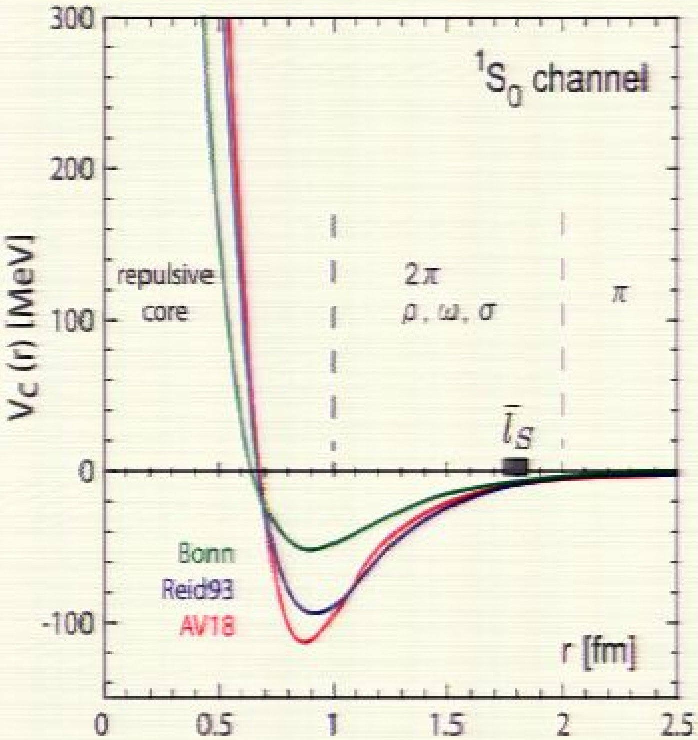


KH et al., PRL 05, 161102 (2010)

- $1.97 M_\odot$ neutron star and causality constrain nuclear equation of state at high densities (esp. lower bound)

- very stiff EOS lead to low central densities in typical ns ($\rho \sim (2-2.5)\rho_0$)

Equation of state of symmetric nuclear matter, evolution of 3NFs



"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."
Hans Bethe (1971)

- empirical saturation at $n_S \sim 0.16\,\mathrm{fm}^{-3}$ and $E_{\mathrm{binding}}/N \sim -16\,\mathrm{MeV}$

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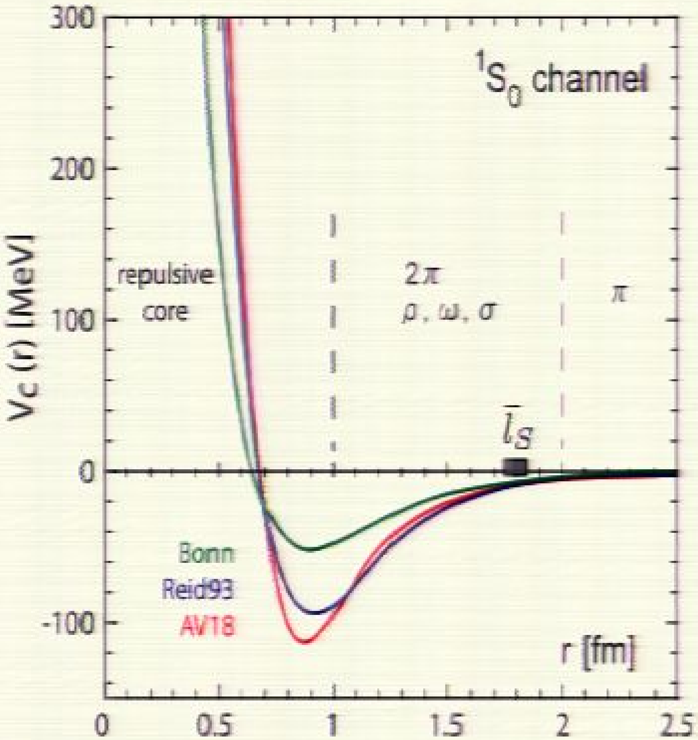
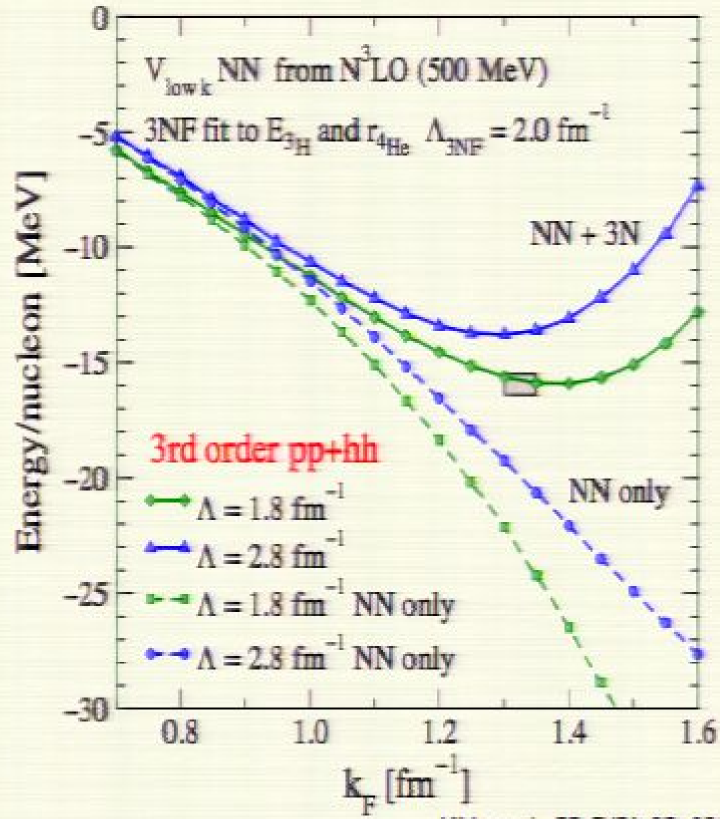
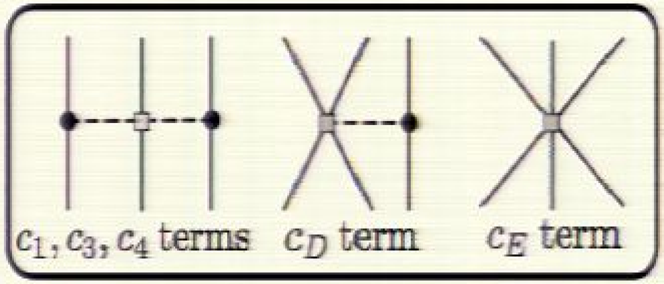




KH et al., PRC(R) 83, 031301 (2011)

- empirical saturation at $n_S \sim 0.16\,\mathrm{fm}^{-3}$ and $E_{\mathrm{binding}}/N \sim -16\,\mathrm{MeV}$
- nuclear saturation delicate due to cancellations of large kinetic and potential energy contributions
- 3N forces are essential!

SRG evolution of 3N interactions



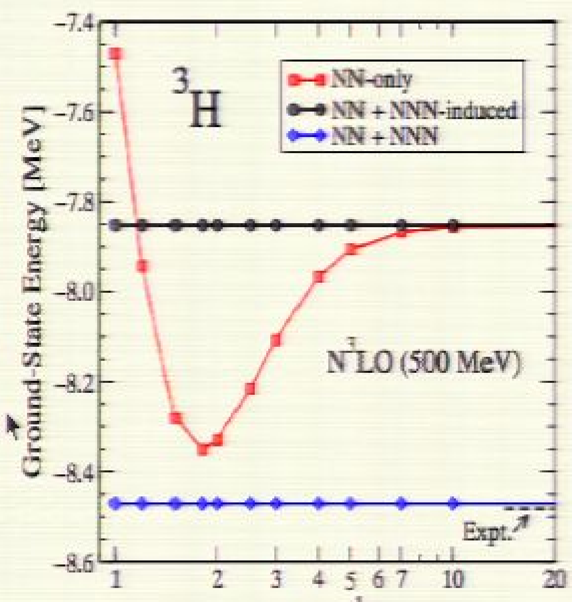
- **So far:**

fit intermediate (c_D) and short-range (c_E) 3NF couplings to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

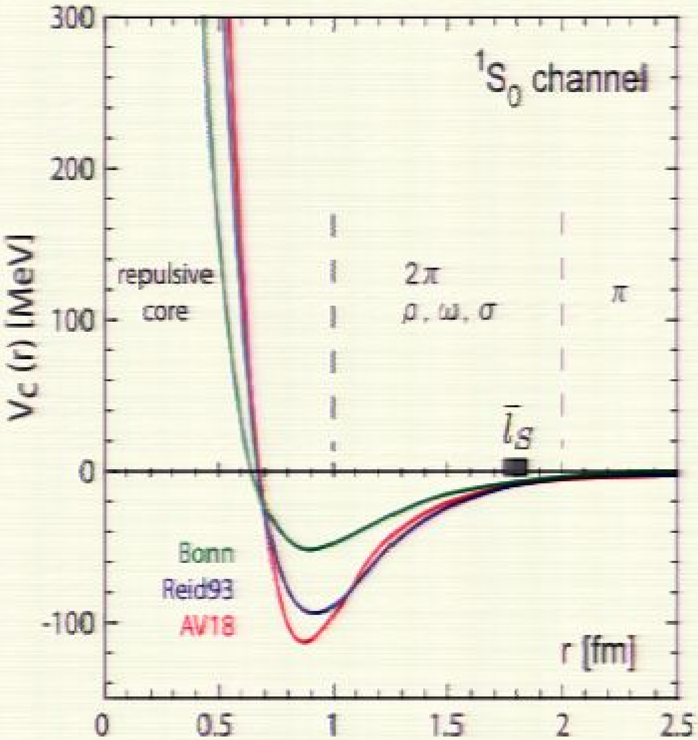
- **Ideal case:** evolve 3NF consistently to lower resolution

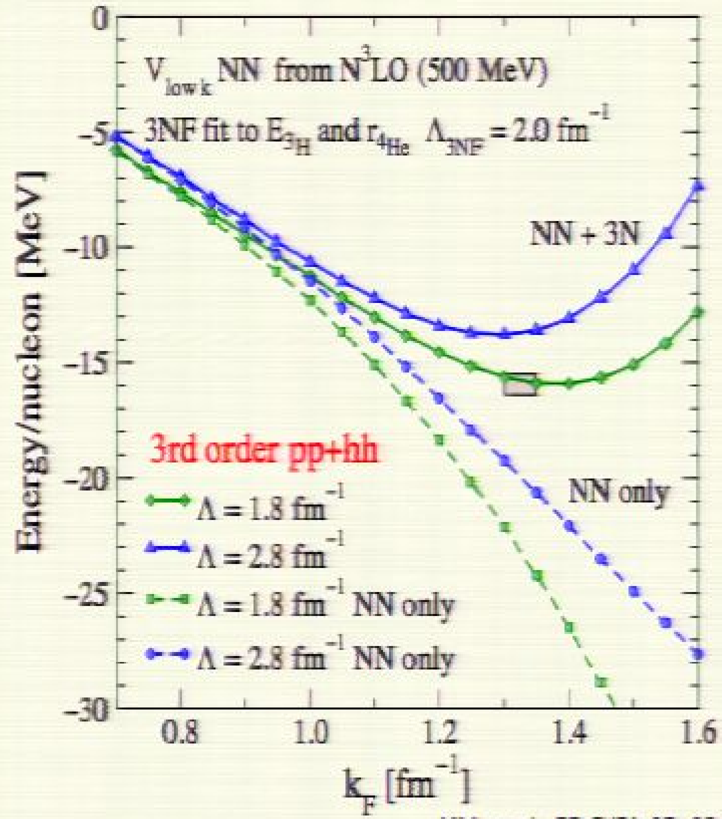
⋆ has been achieved using oscillator basis states, promising results in light nuclei; however, not suitable for use in infinite systems



Jurgenson, Navratil, Furnstahl, PRL 103, 082501(2009)

Equation of state of symmetric nuclear matter, evolution of 3NFs





KH et al., PRC(R) 83, 031301 (2011)

- empirical saturation at $n_S \sim 0.16\,\mathrm{fm}^{-3}$ and $E_{\mathrm{binding}}/N \sim -16\,\mathrm{MeV}$
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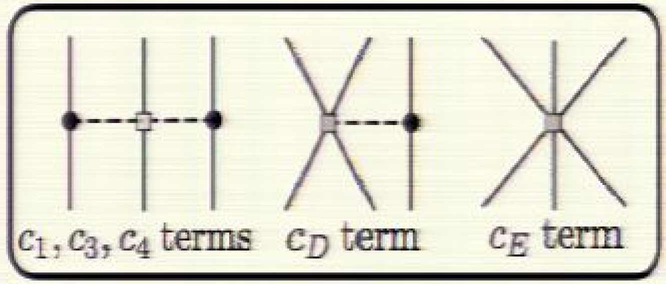
SRG evolution of 3N interactions



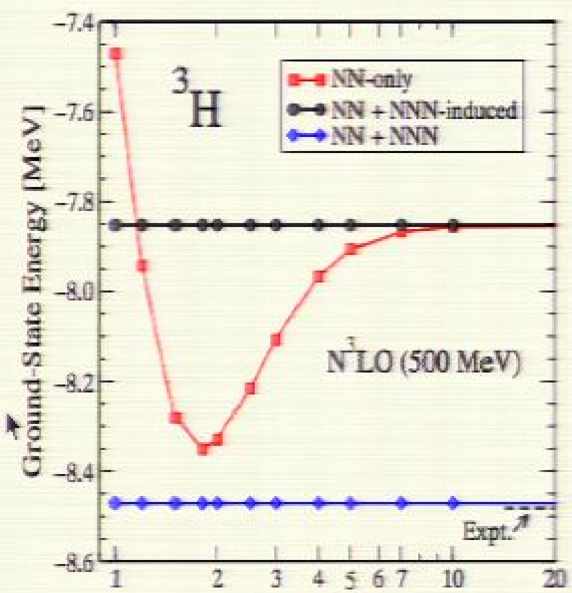
- **So far:**

fit intermediate (c_D) and short-range (c_E) 3NF couplings to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \quad \text{and} \quad r_{^4\mathrm{He}} = 1.95 - 1.96\,\mathrm{fm}$$

- **Ideal case:** evolve 3NF consistently to lower resolution

  ⋆ has been achieved using oscillator basis states, promising results in light nuclei; however, not suitable for use in infinite systems



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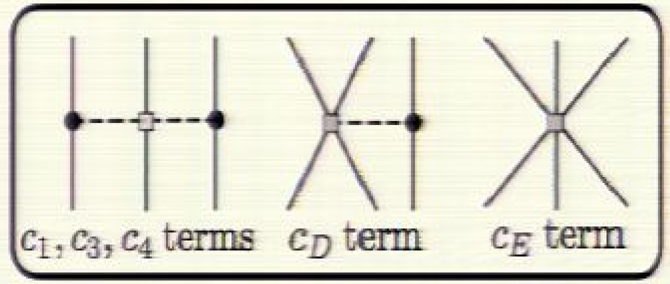

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  * current project: evolve 3NF in momentum basis

    - similar technology to solving the A=3 Faddeev equations

    - allows systematic investigation of flow of 3NF couplings and provides matrix elements for infinite matter many-body perturbation theory

    - makes it possible to study the evolution of operators like densities

Conclusions

- low-resolution interactions allow simpler, perturbative calculations for nuclear systems

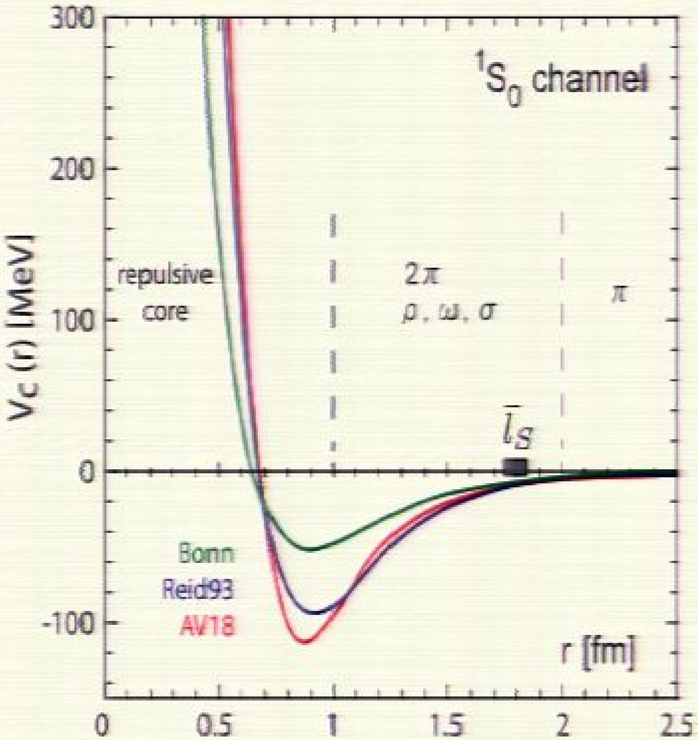
- 3N interactions are essential at low resolution

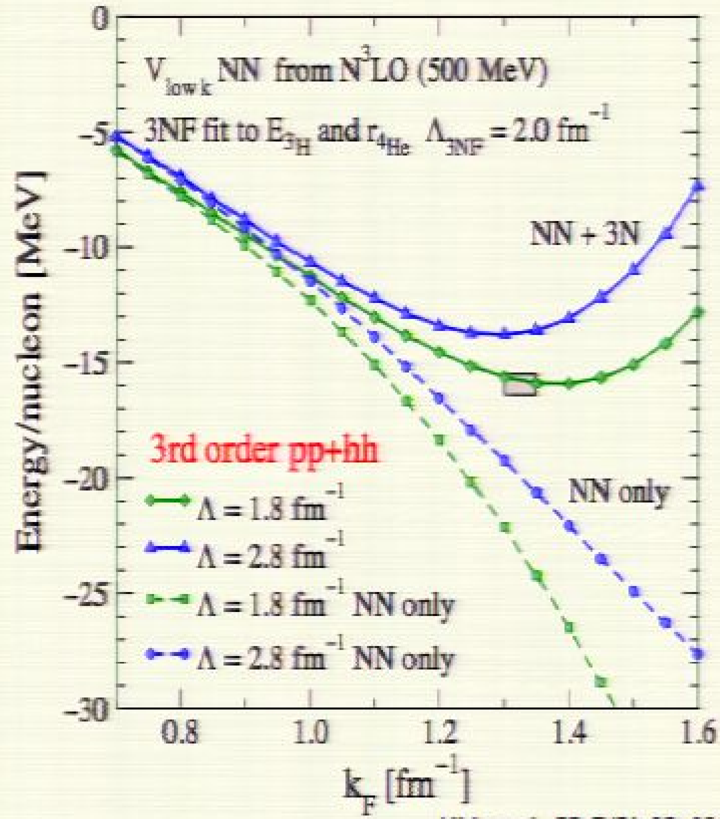
- derivation of density-dependent effective NN interactions from chiral 3N interactions

- effective NN interaction efficient to use and accounts for 3N effects in neutron and nuclear matter to good approximation

- constraints for the neutron star equation of state and radii of neutron stars

Equation of state of symmetric nuclear matter, evolution of 3NFs





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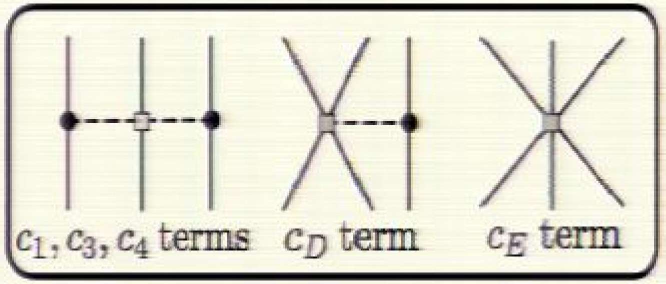

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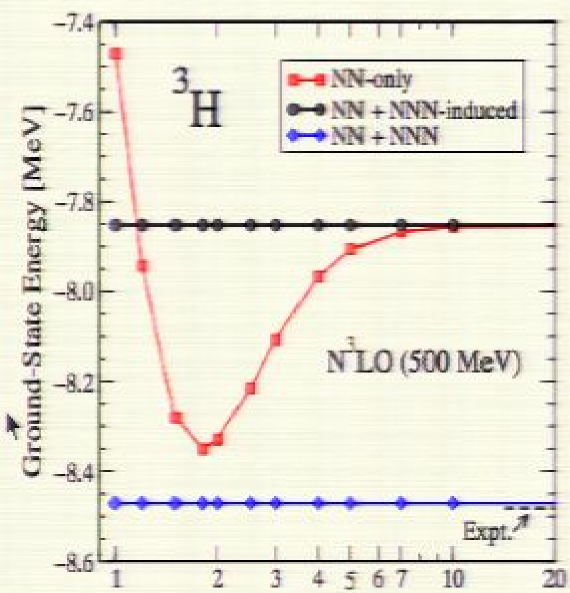

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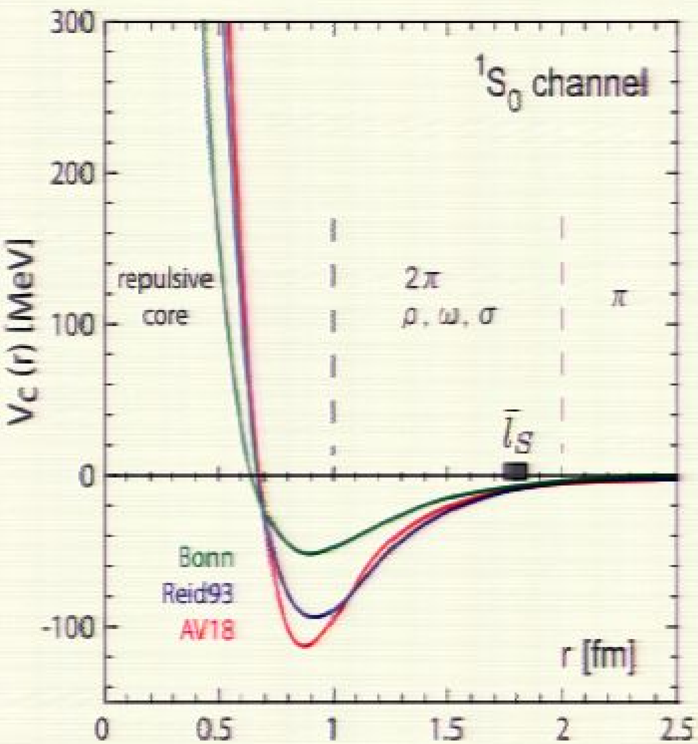

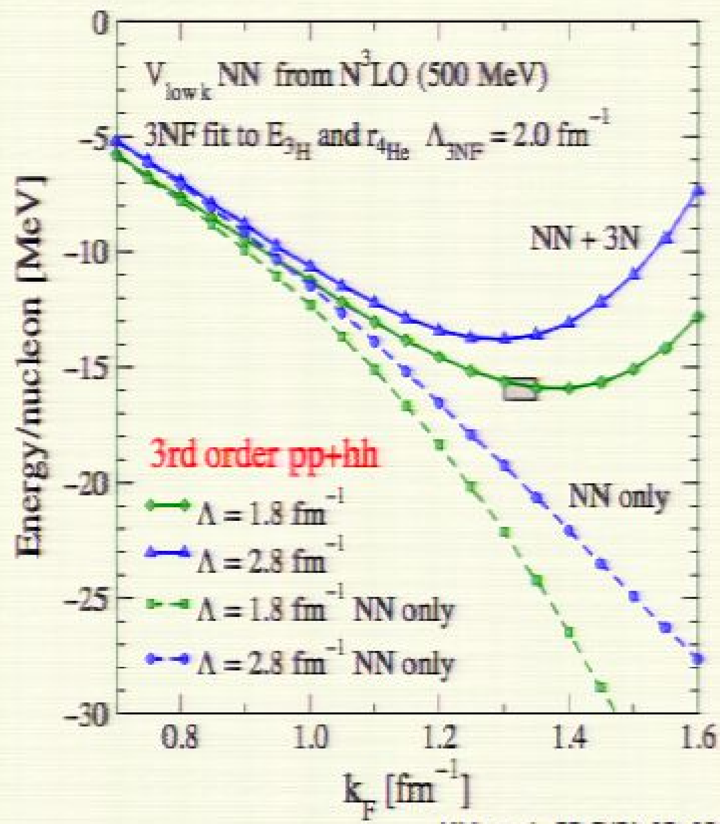

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