

Title: Neutron Star Mass and Radius Constraints for the Dense Matter Equation of State

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Abstract: Recent discoveries, including a 2 solar mass pulsar, rapid cooling in the Cas A supernova remnant, and estimates of masses and radii from photospheric radius expansion bursts and thermal emissions from neutron stars, are able to constrain significantly the properties of dense matter. Implications for the pressure-density relation and properties of superfluids in neutron star interiors will be discussed.

Neutron Star Mass and Radius Constraints on the Dense Matter Equation of State

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20 June 2011

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MICRA 2011
Perimeter Institute

- ▶ **Neutron Star Structure**
 - ▶ Neutron Star Limits from General Relativity and Causality
 - ▶ Mass Measurements
 - ▶ $2 M_{\odot}$ Neutron Stars?
 - ▶ Limits to the Extent of Quark Matter
 - ▶ Neutron Star Radii
 - ▶ Relation to the Nuclear Symmetry Energy
 - ▶ Thermal Emission from Cooling Neutron Stars
 - ▶ Photospheric Radius Expansion X-Ray Bursters
 - ▶ The Universal Mass-Radius Relation and the Neutron Star EOS
 - ▶ Consistency with Neutron Matter Expectations
 - ▶ Implications for Other Laboratory Constraints
- ▶ **Time Permitting: The Evolution of Neutron Stars**
 - ▶ Neutron Star Cooling and the Direct Urca Process
 - ▶ Cas A: A Direct Detection of Core Superfluidity?

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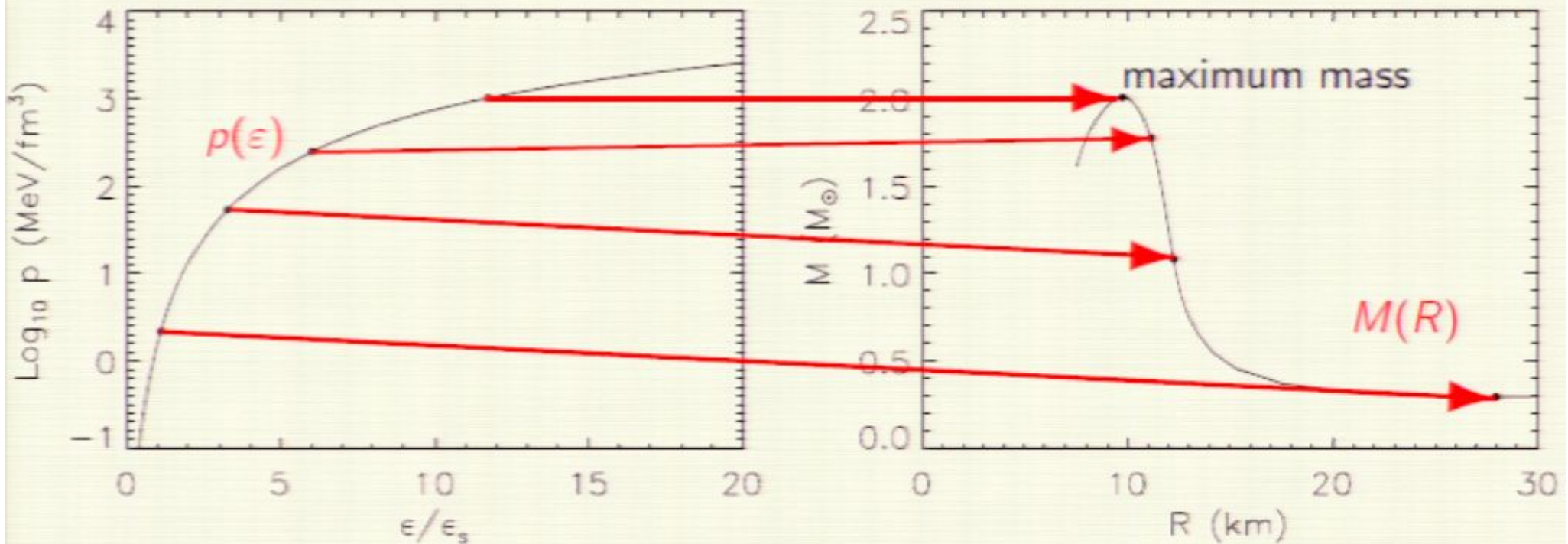
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Neutron Star Structure

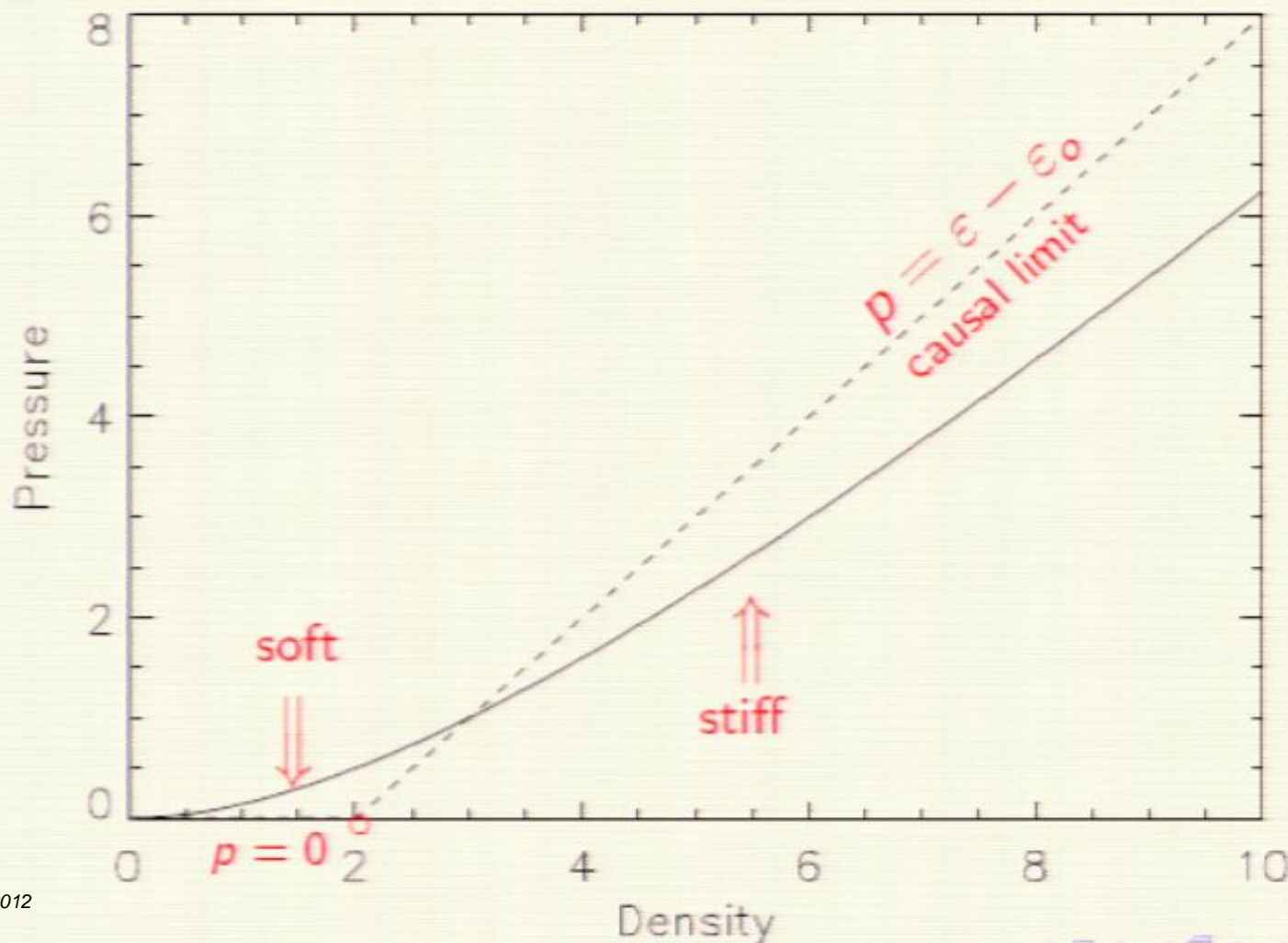
Tolman-Oppenheimer-Volkov equations

$$\frac{dp}{dr} = -\frac{G}{c^2} \frac{(m + 4\pi pr^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$
$$\frac{dm}{dr} = 4\pi \frac{\epsilon}{c^2} r^2$$



Extreme Properties of Neutron Stars

- ▶ The most compact and massive configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff" (Koranda, Stergioulas & Friedman 1997).



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

$$w = \epsilon / \epsilon_0$$

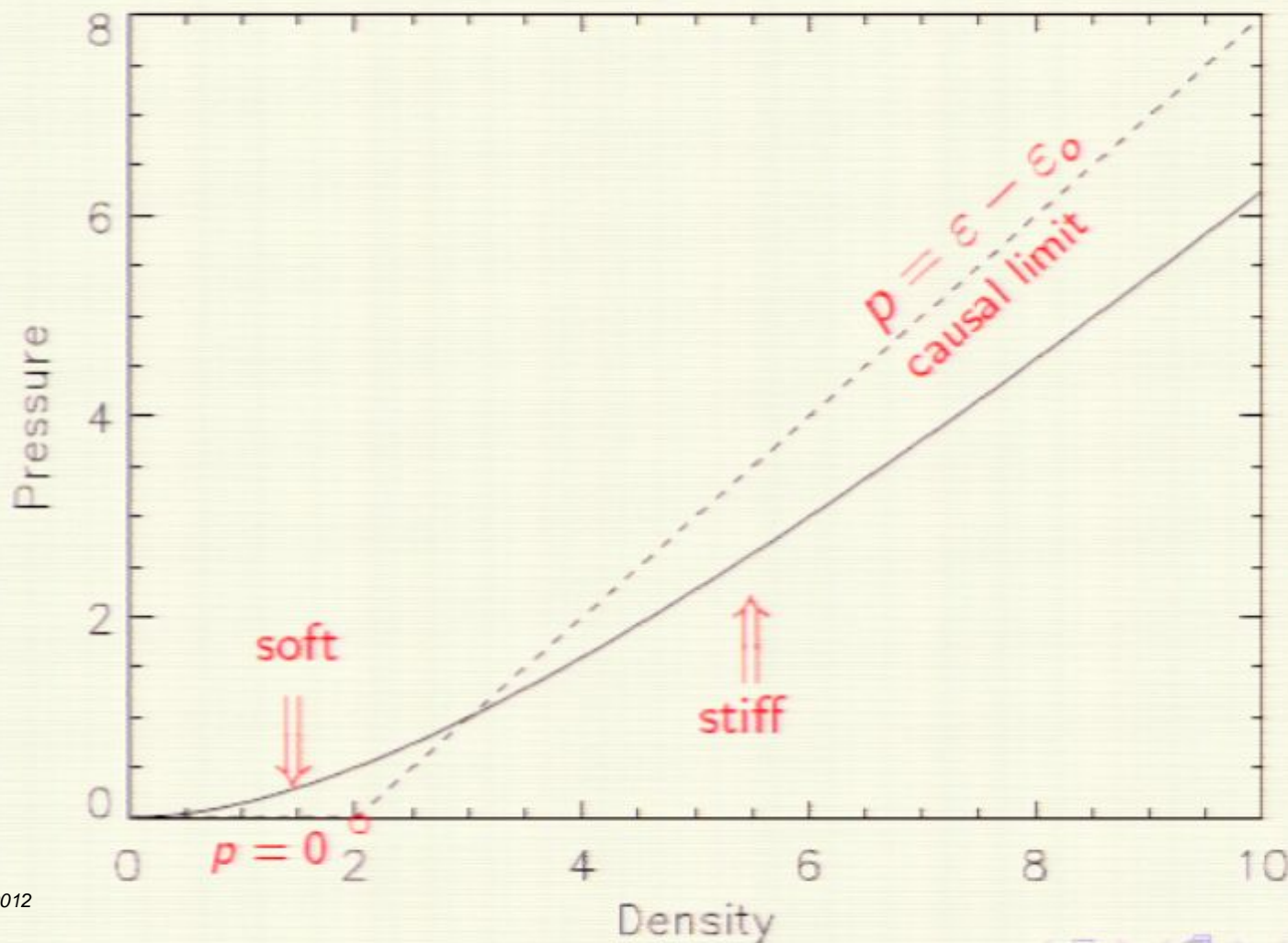
$$y = p / \epsilon_0$$

$$x = r \sqrt{G \epsilon_0} / c^2$$

$$z = m \sqrt{G^3 \epsilon_0} / c^2$$

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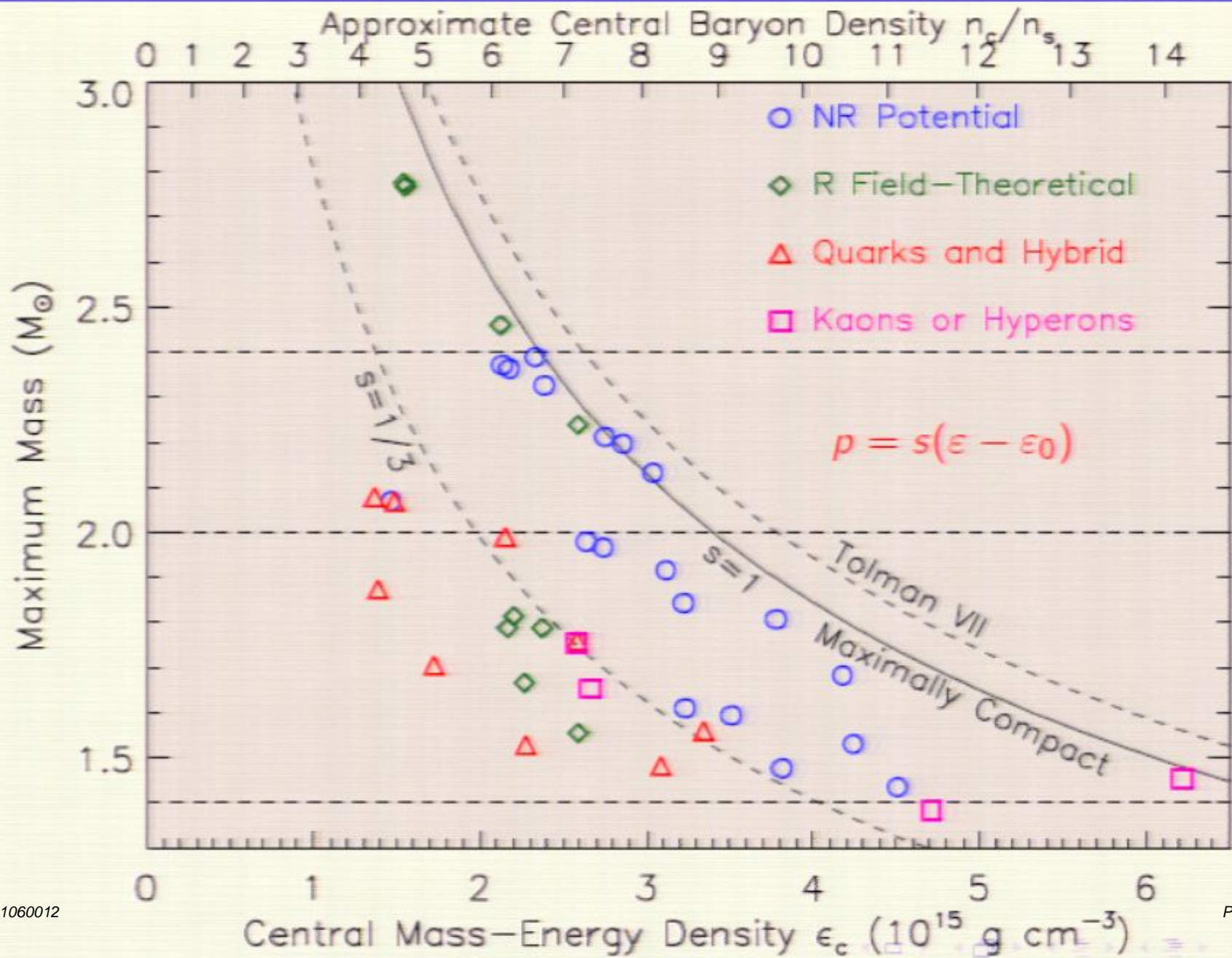
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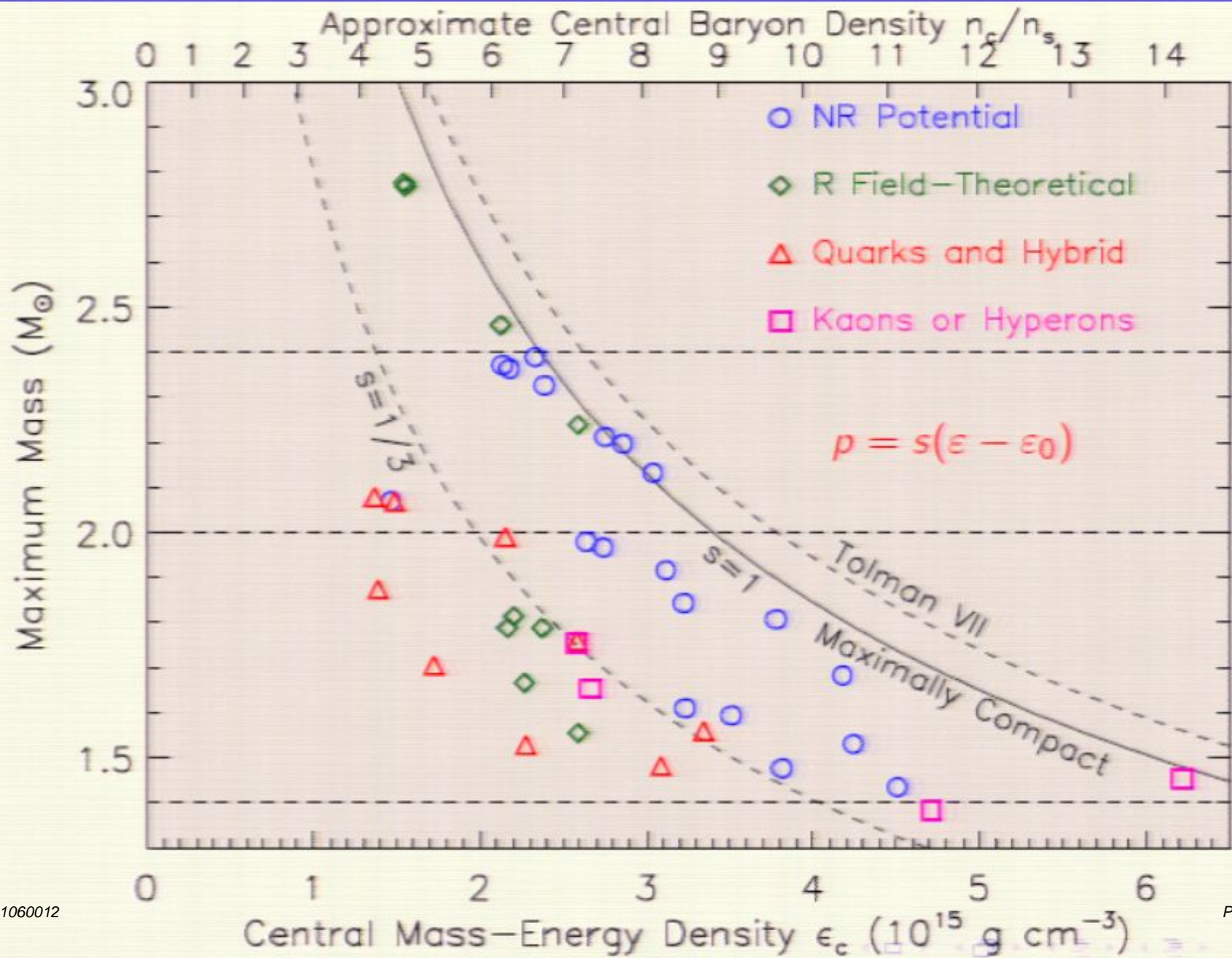
Extreme Properties of Neutron Stars

- ▶ $M_{max} = 4.1 (\epsilon_s/\epsilon_0)^{1/2} M_\odot$ (Rhoades & Ruffini 1974)
- ▶ $M_{B,max} = 5.41 (m_B c^2/\mu_o)(\epsilon_s/\epsilon_0)^{1/2} M_\odot$
- ▶ $R_{min} = 2.82 GM/c^2 = 4.3 (M/M_\odot) \text{ km}$
- ▶ $\mu_{B,max} = 2.09 \text{ GeV}$
- ▶ $\epsilon_{c,max} = 3.034 \epsilon_0 \simeq 51 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $\rho_{c,max} = 2.034 \epsilon_0 \simeq 34 (M_\odot/M_{largest})^2 \epsilon_s$
- ▶ $n_{B,max} \simeq 38 (M_\odot/M_{largest})^2 n_s$
- ▶ $BE_{max} = 0.34 M$
- ▶ $P_{min} = 0.74 (M_\odot/M_{sph})^{1/2} (R_{sph}/10 \text{ km})^{3/2} \text{ ms} =$
 $0.20 (M_{sph,max}/M_\odot) \text{ ms}$

Maximum Energy Density in Neutron Stars



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Mass-Radius Diagram and Theoretical Constraints

GR:

$$R > 2GM/c^2$$

$P < \infty$:

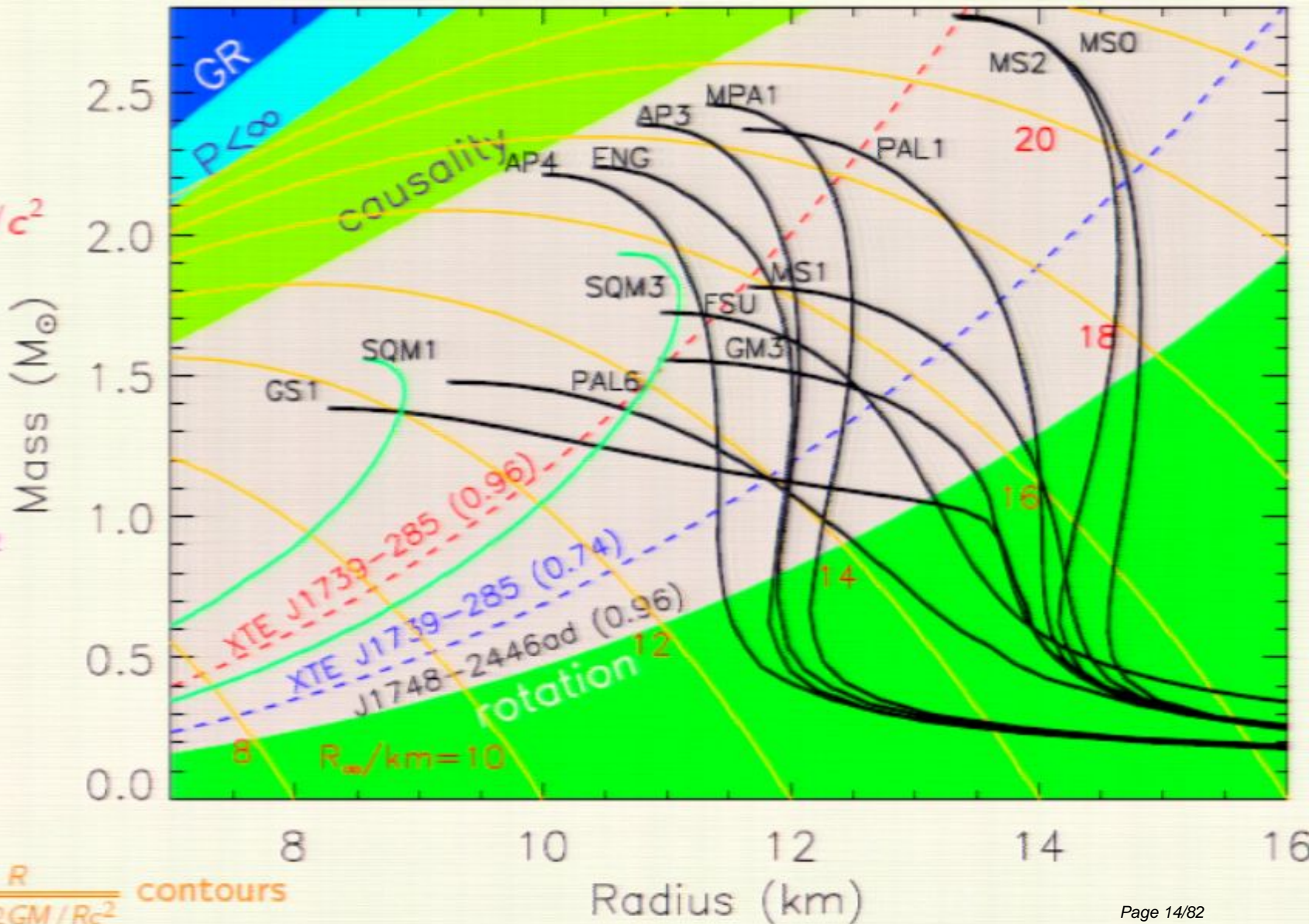
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causality:

$$R \gtrsim 2.9GM/c^2$$

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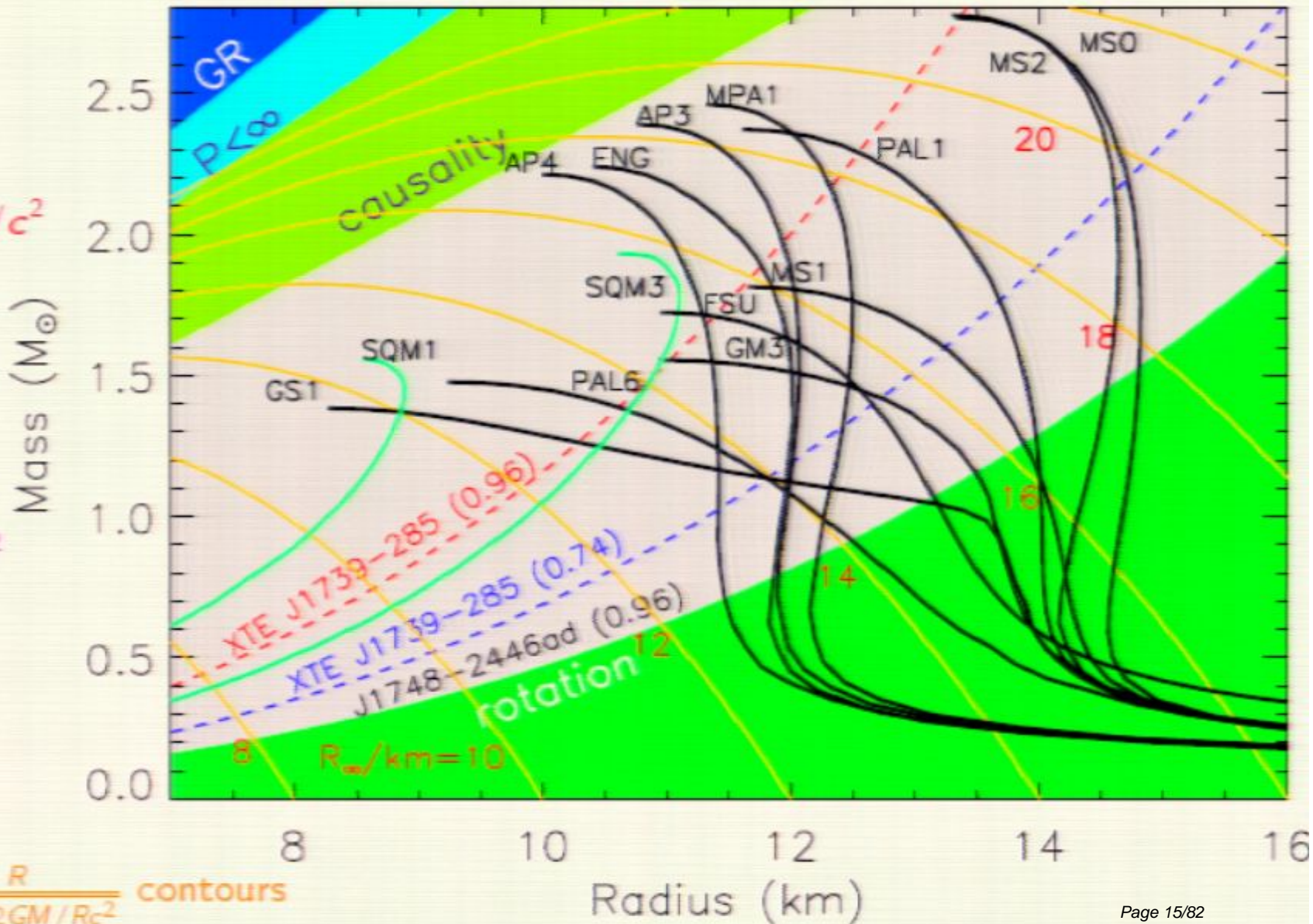
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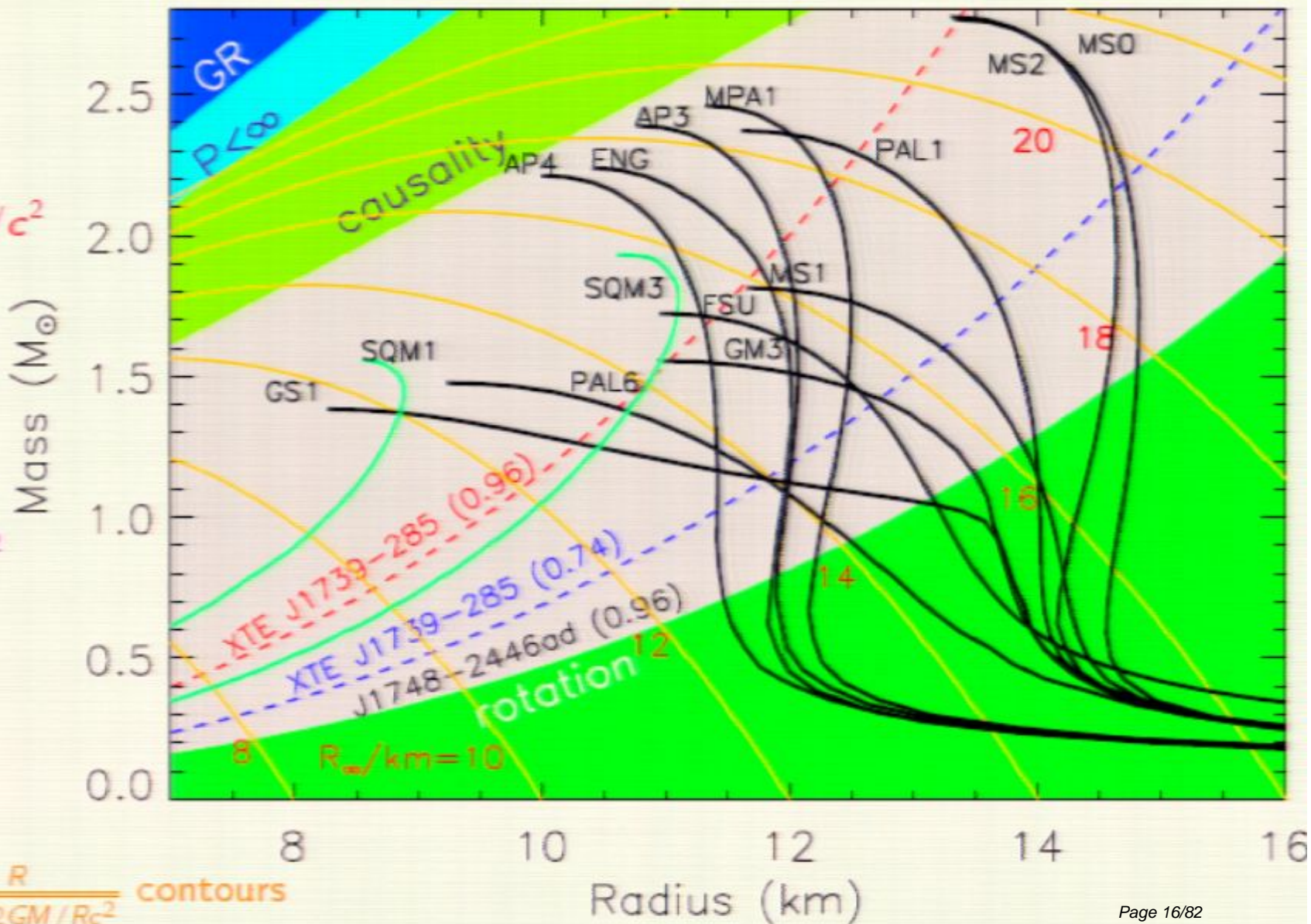
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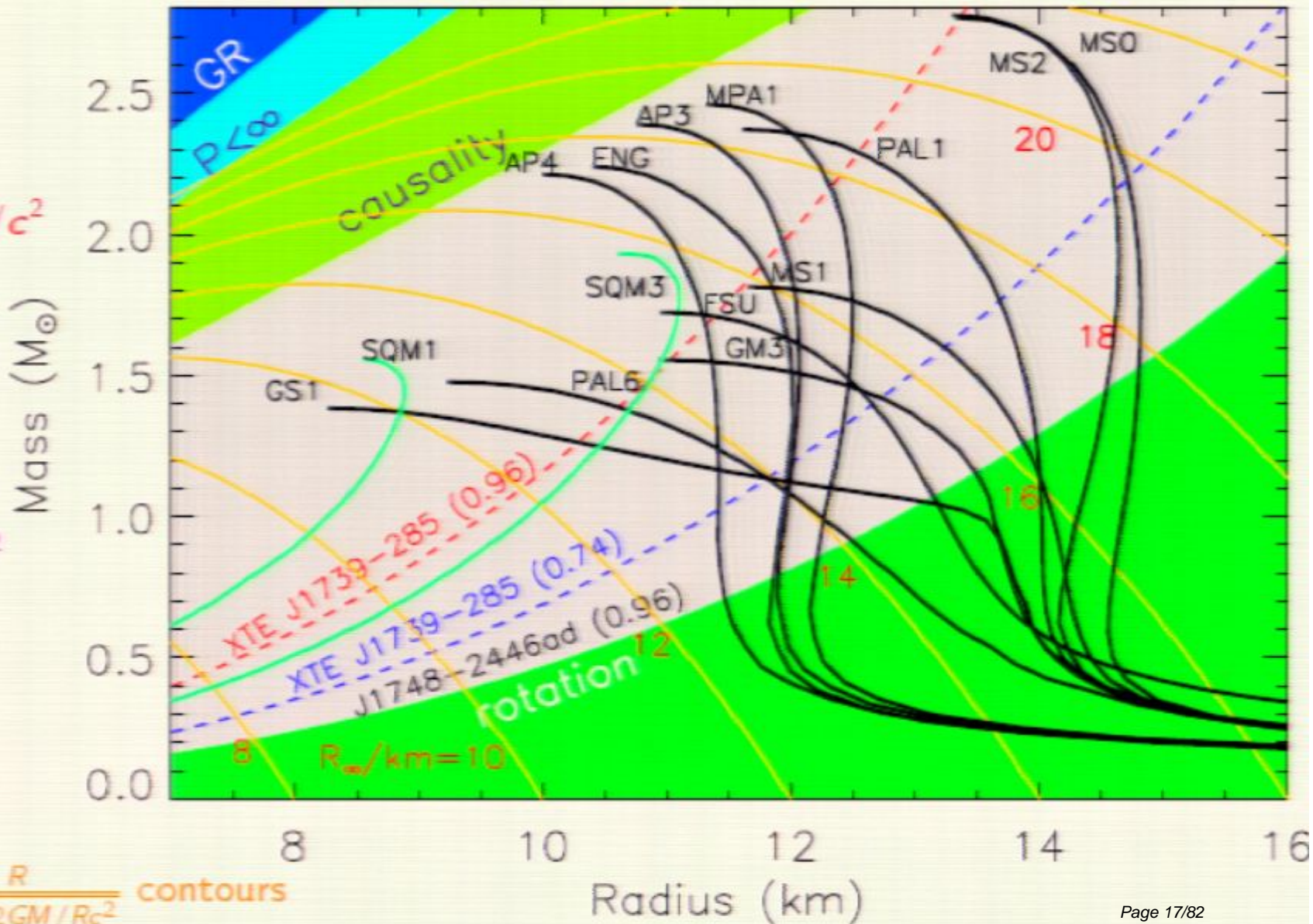
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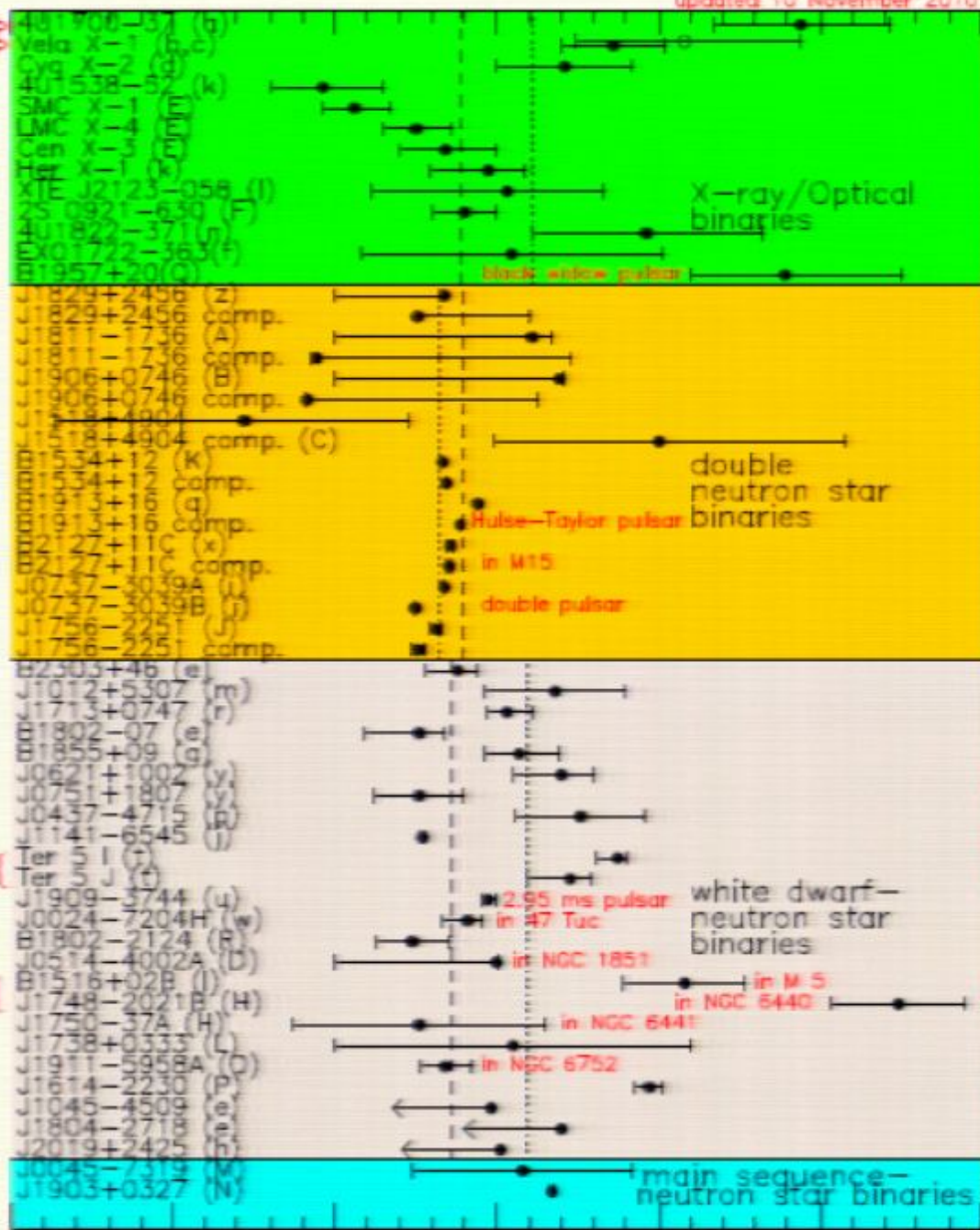
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$$R_{\infty} = \frac{R}{\sqrt{1-2GM/Rc^2}} \text{ contours}$$

Black hole? \Rightarrow
 Firm lower mass limit? \Rightarrow



$M > 1.68 M_{\odot}$ {
 95% confidence

Freire et al. 2007 {

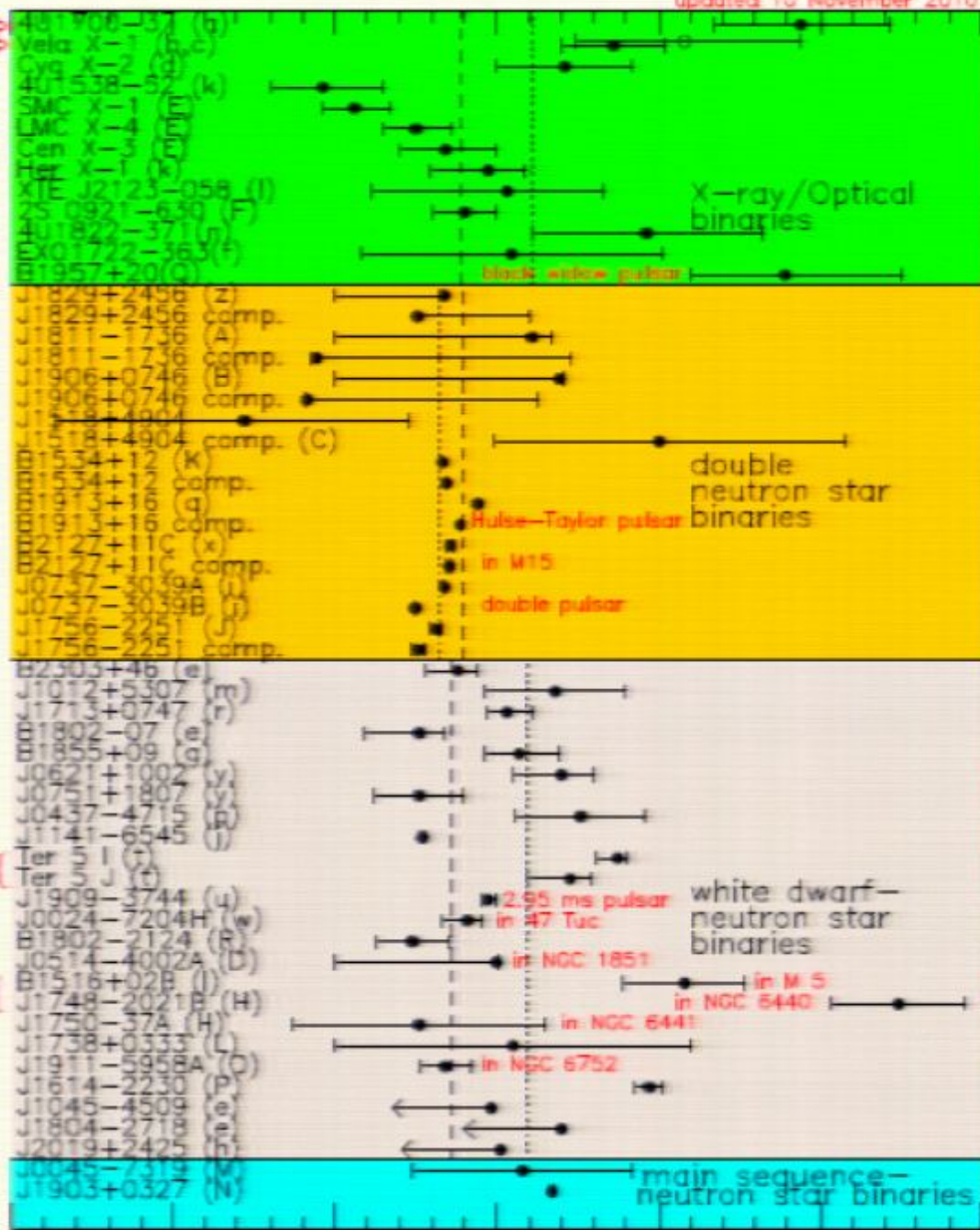
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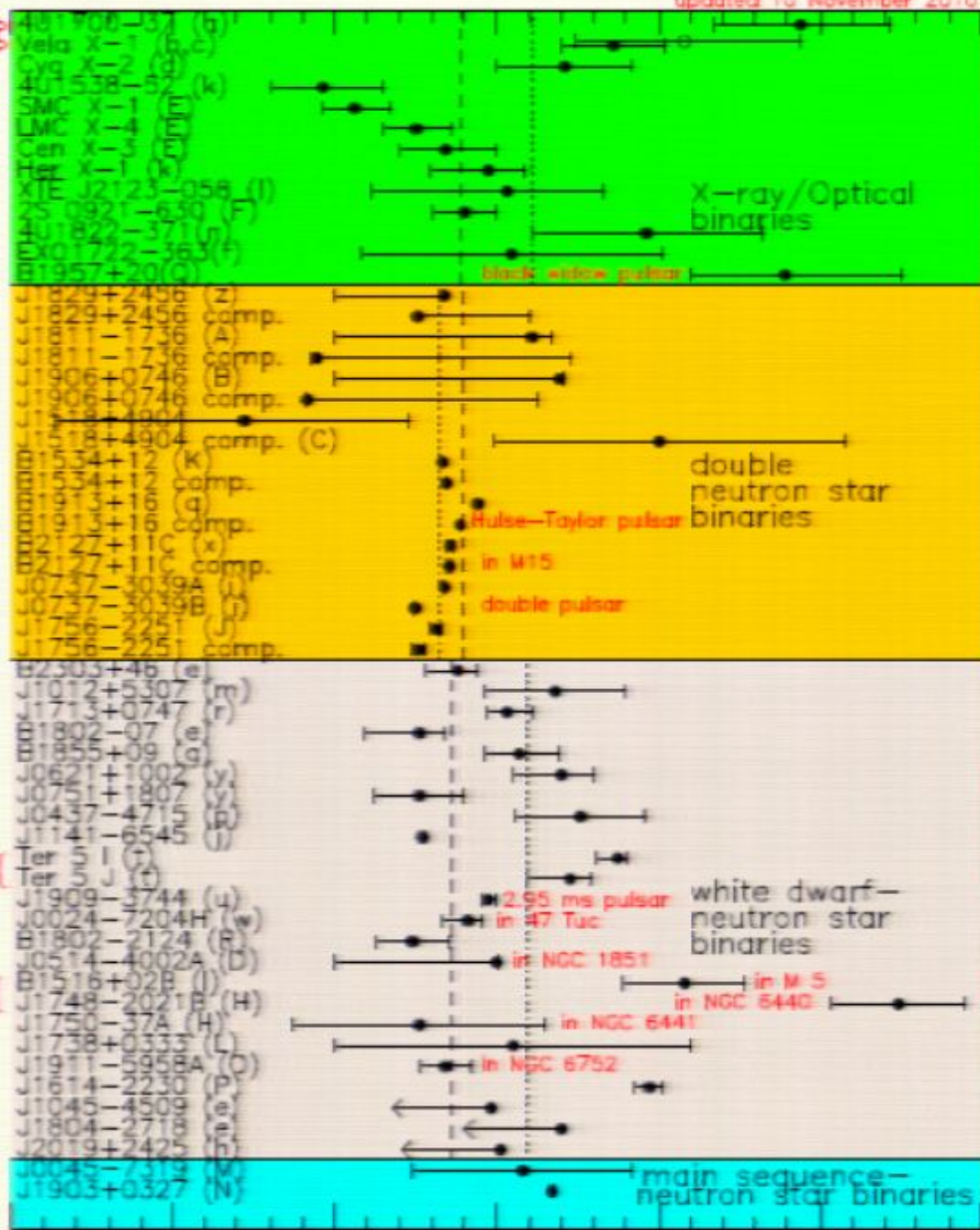
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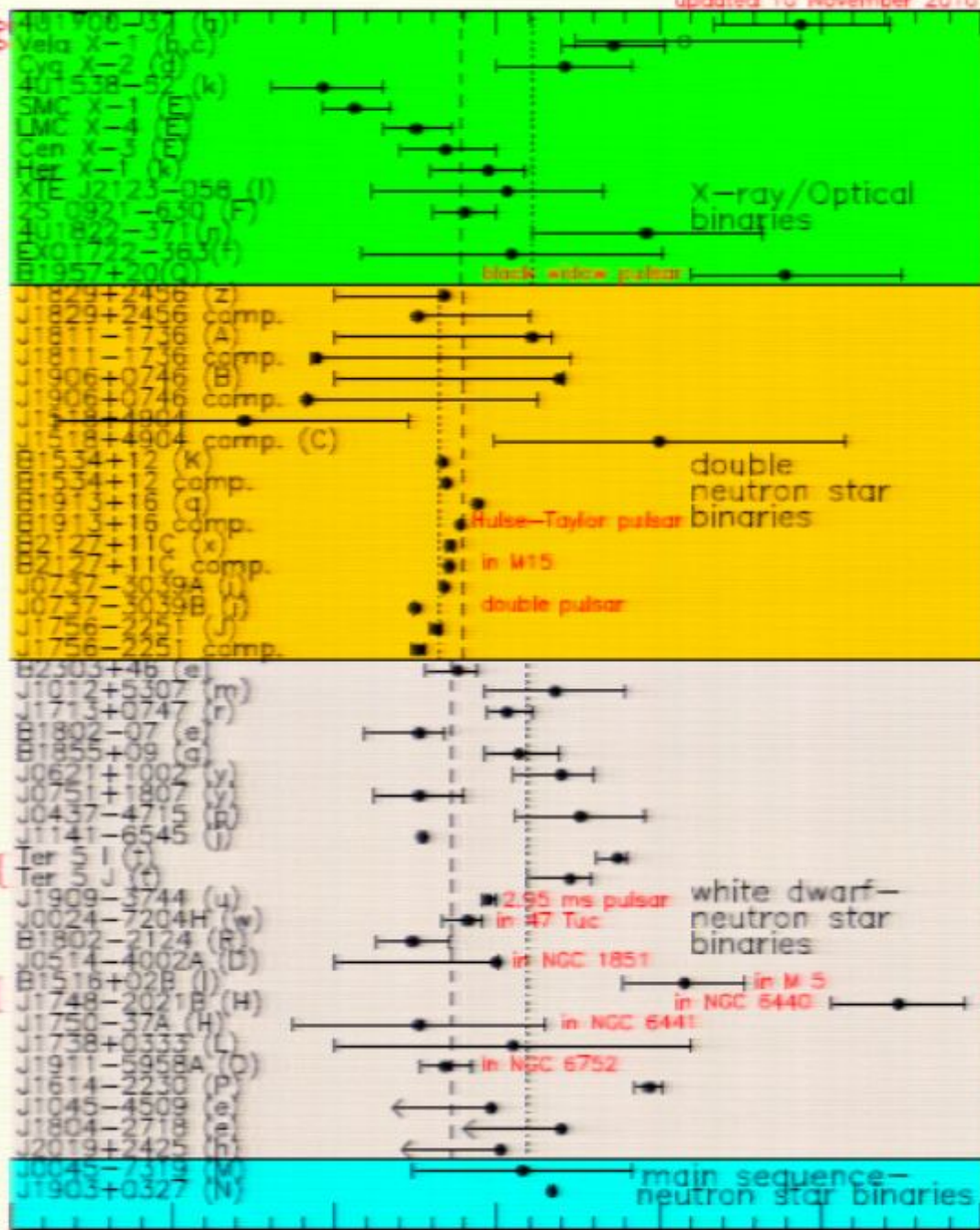
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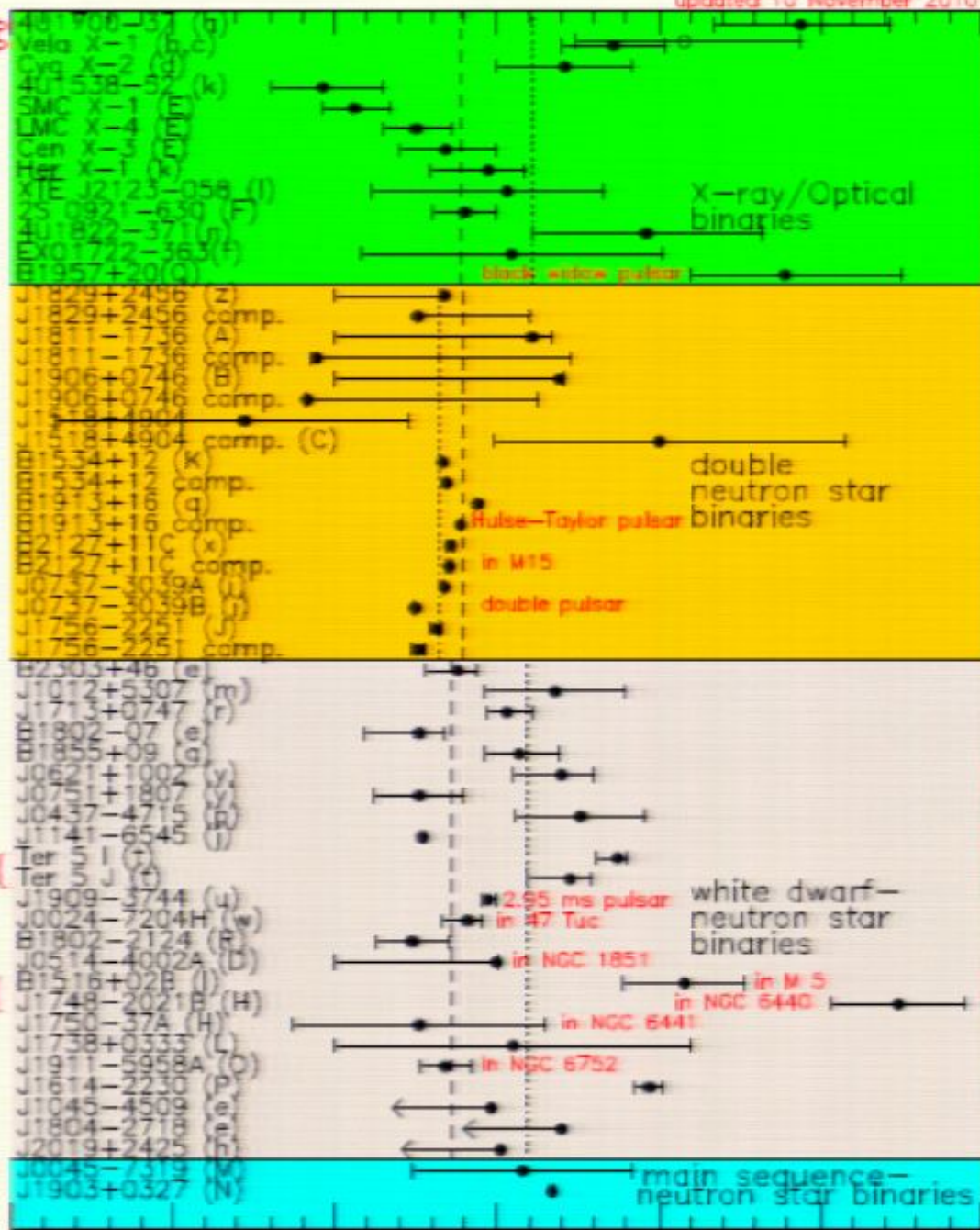
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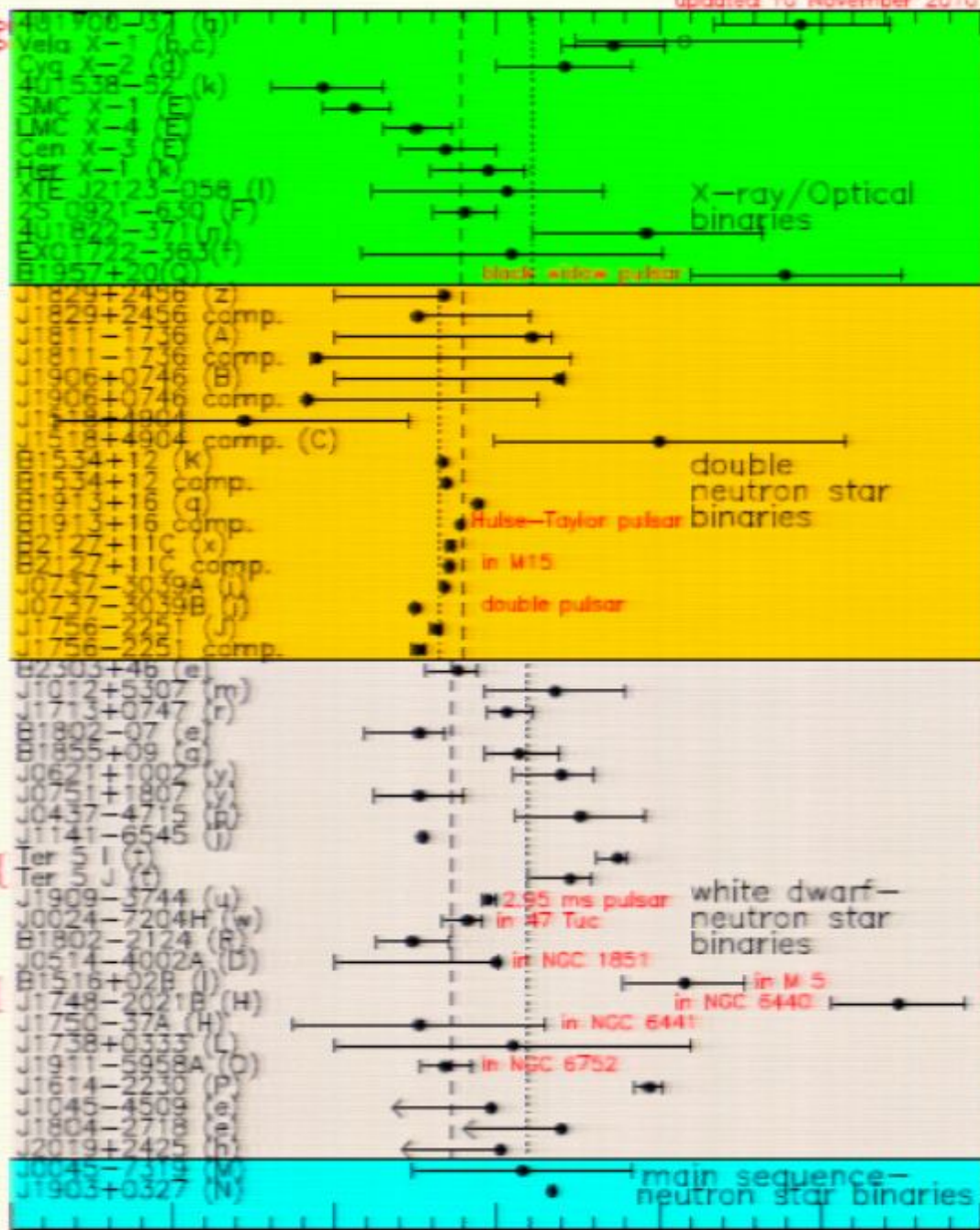
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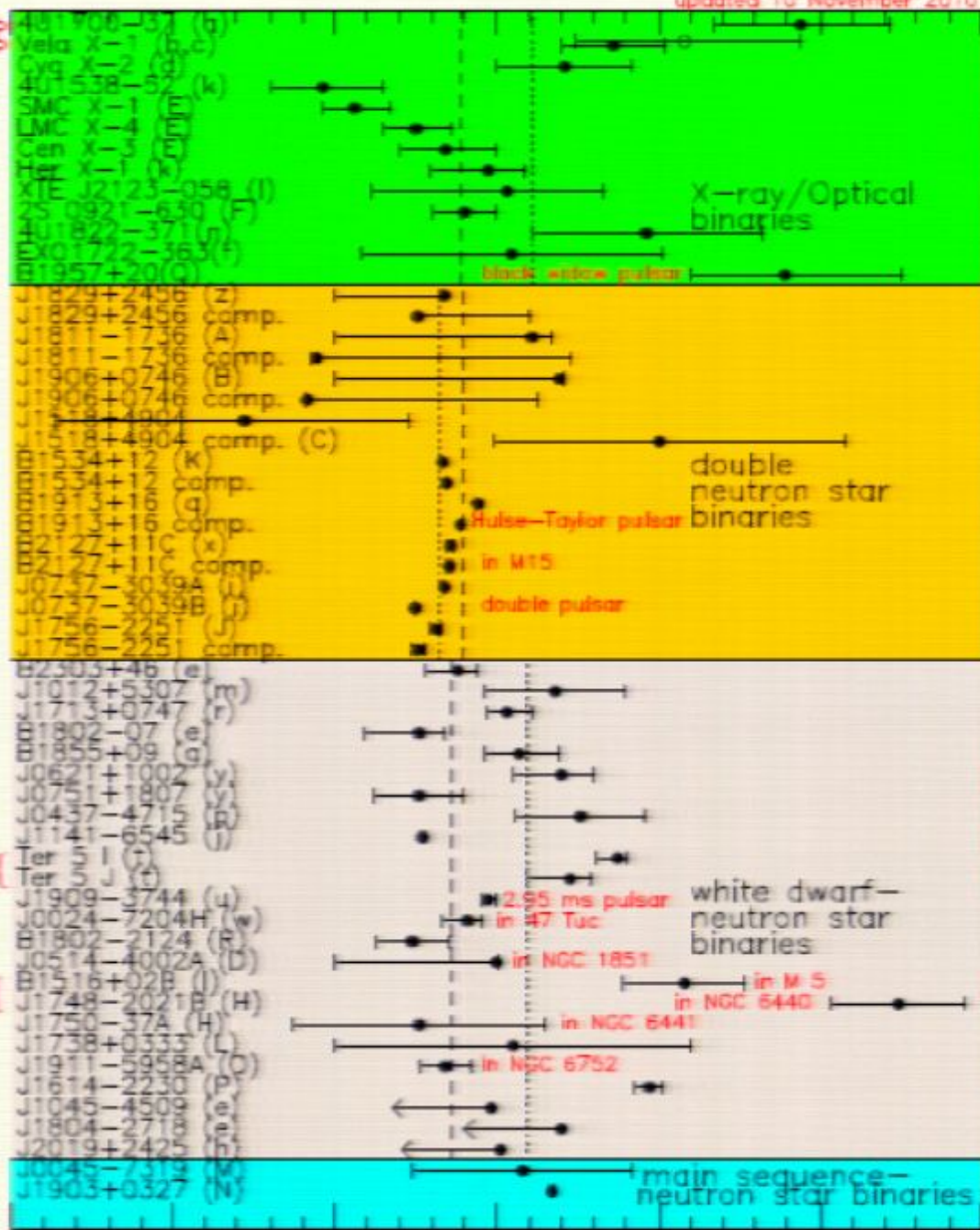
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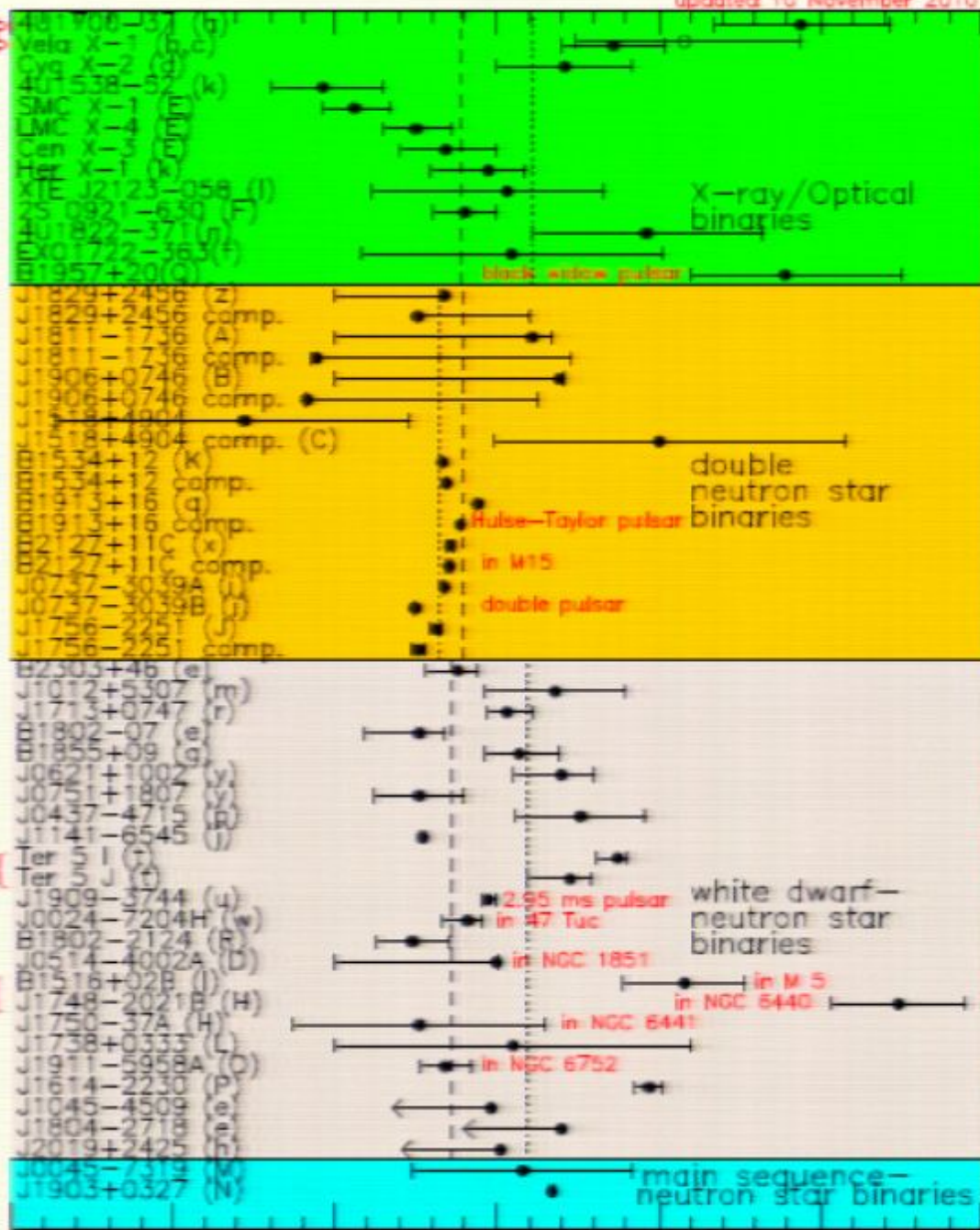
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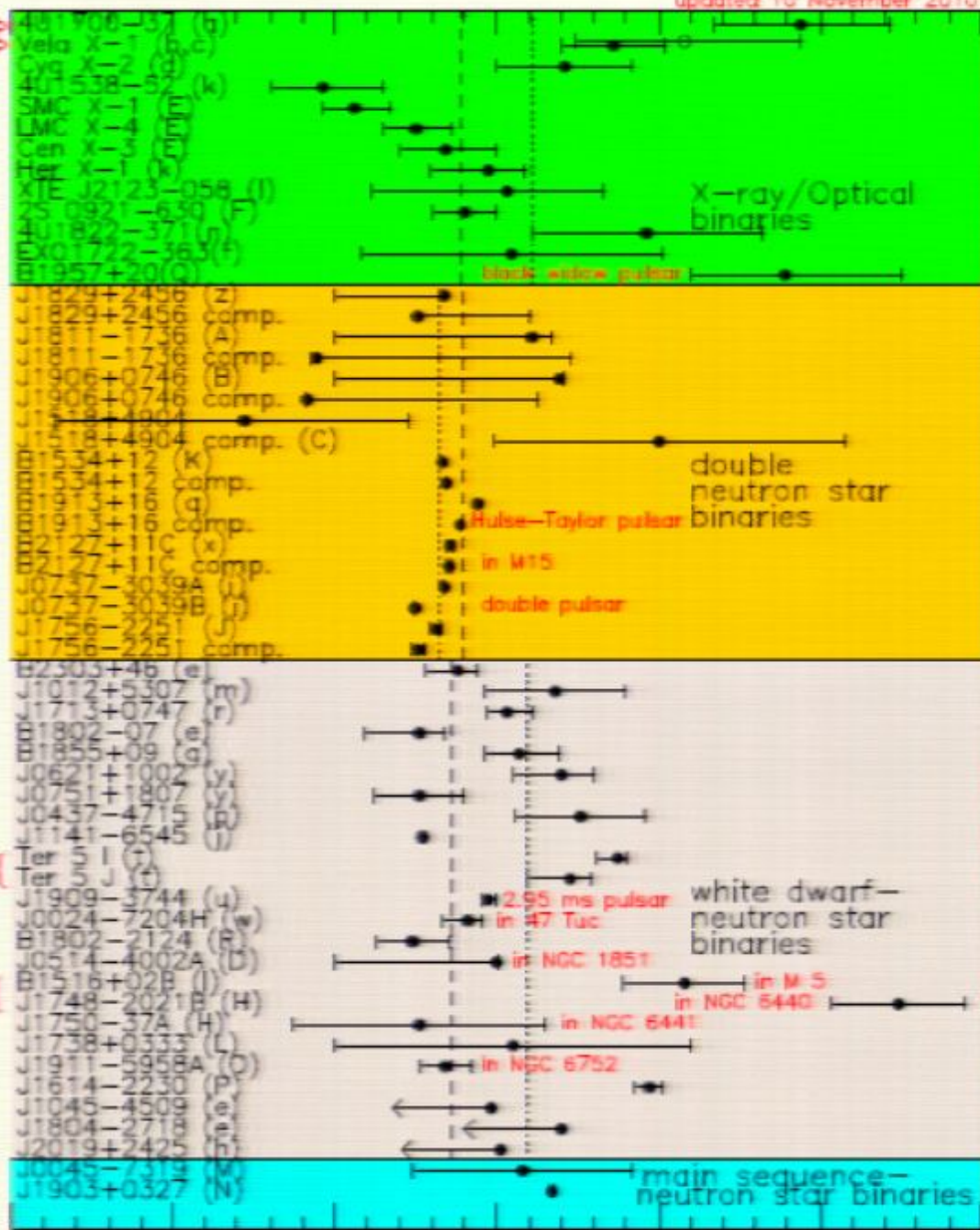
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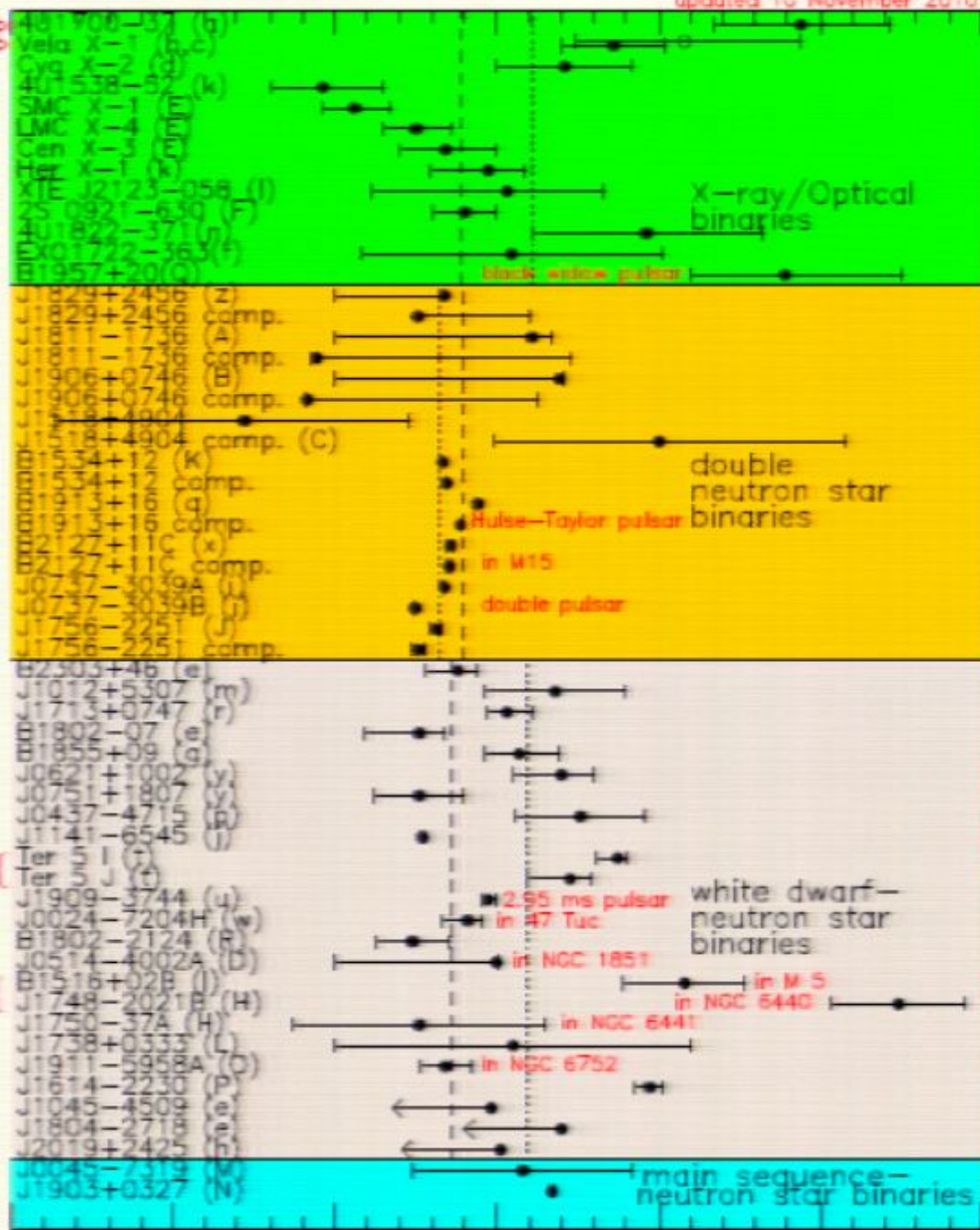
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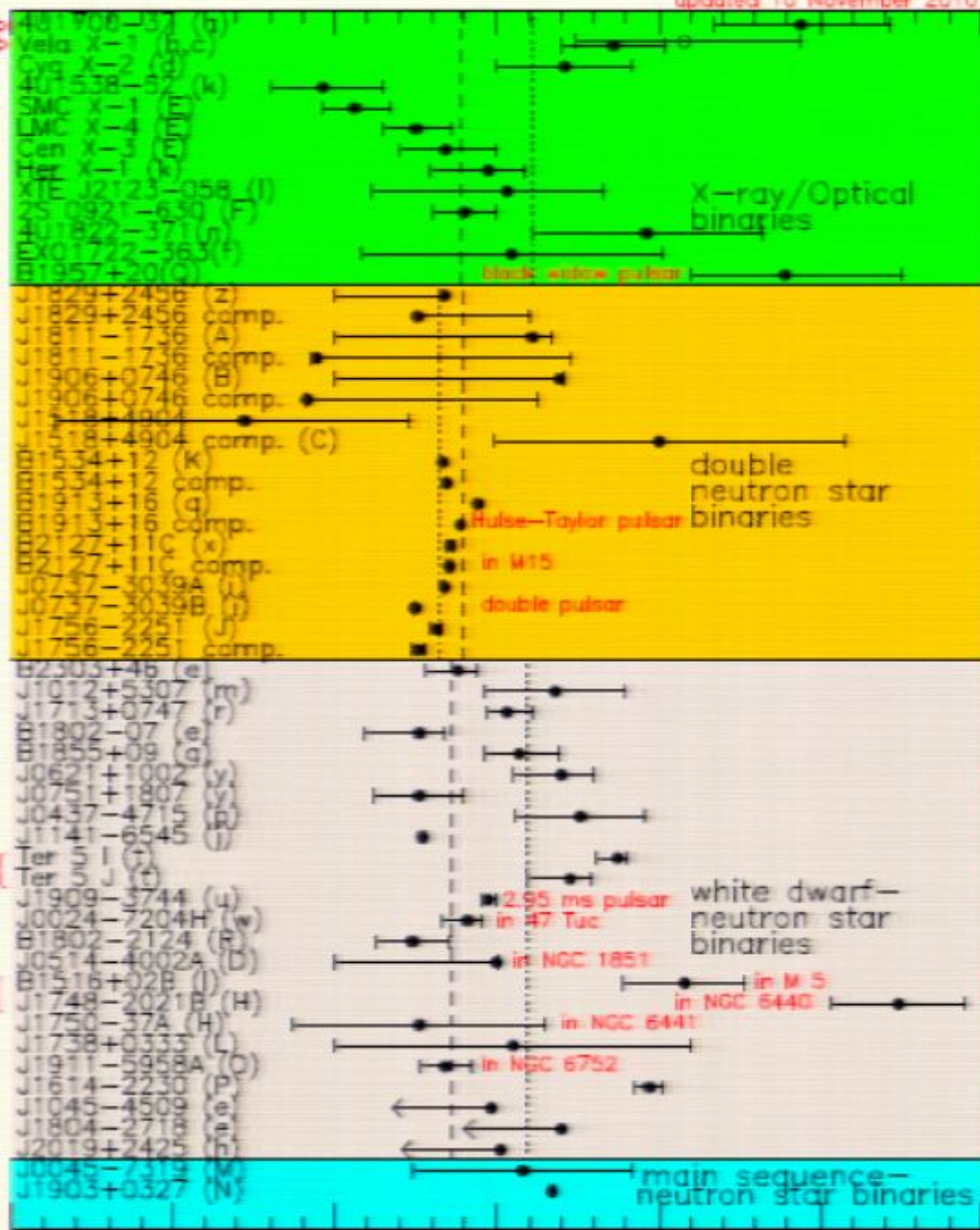
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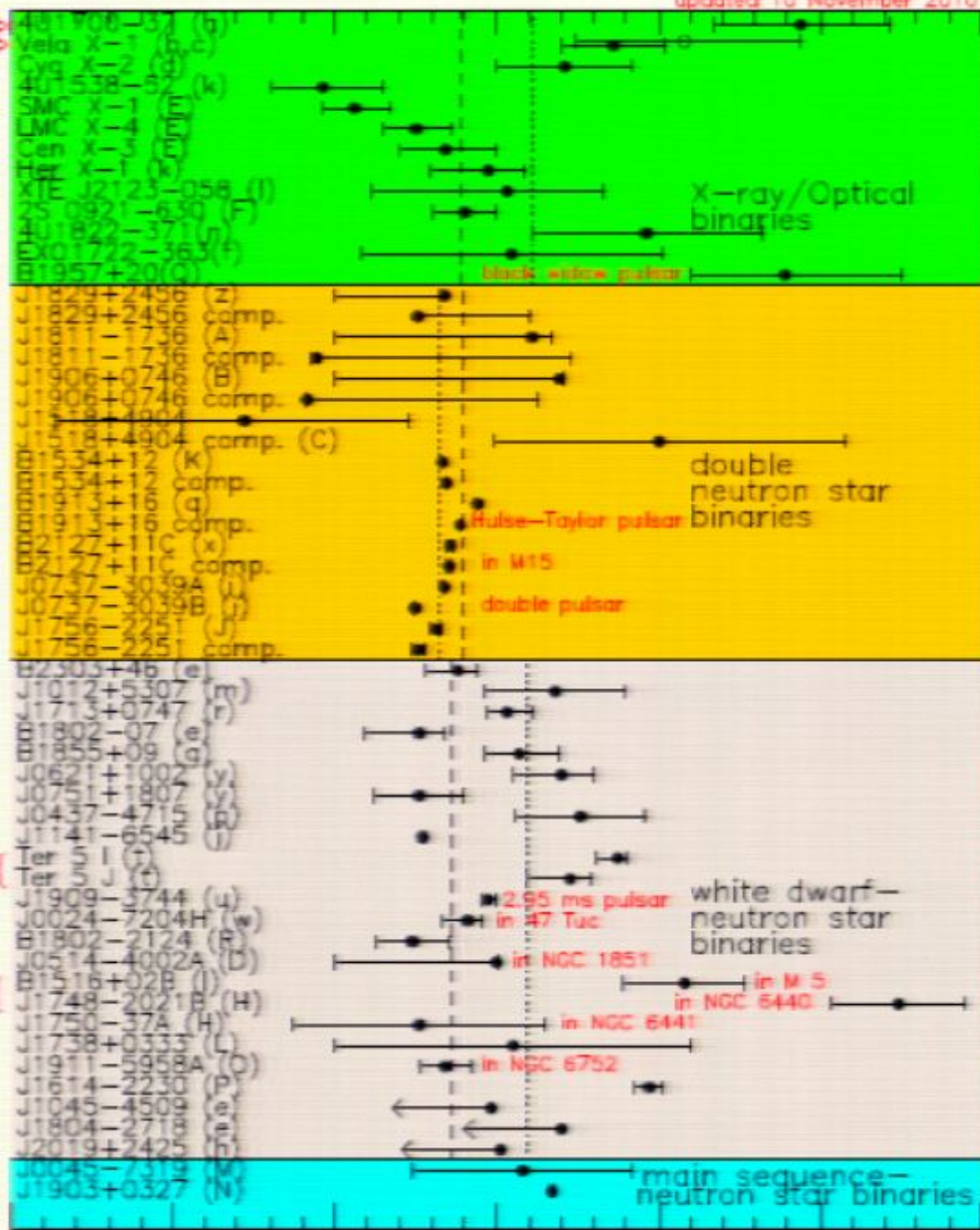
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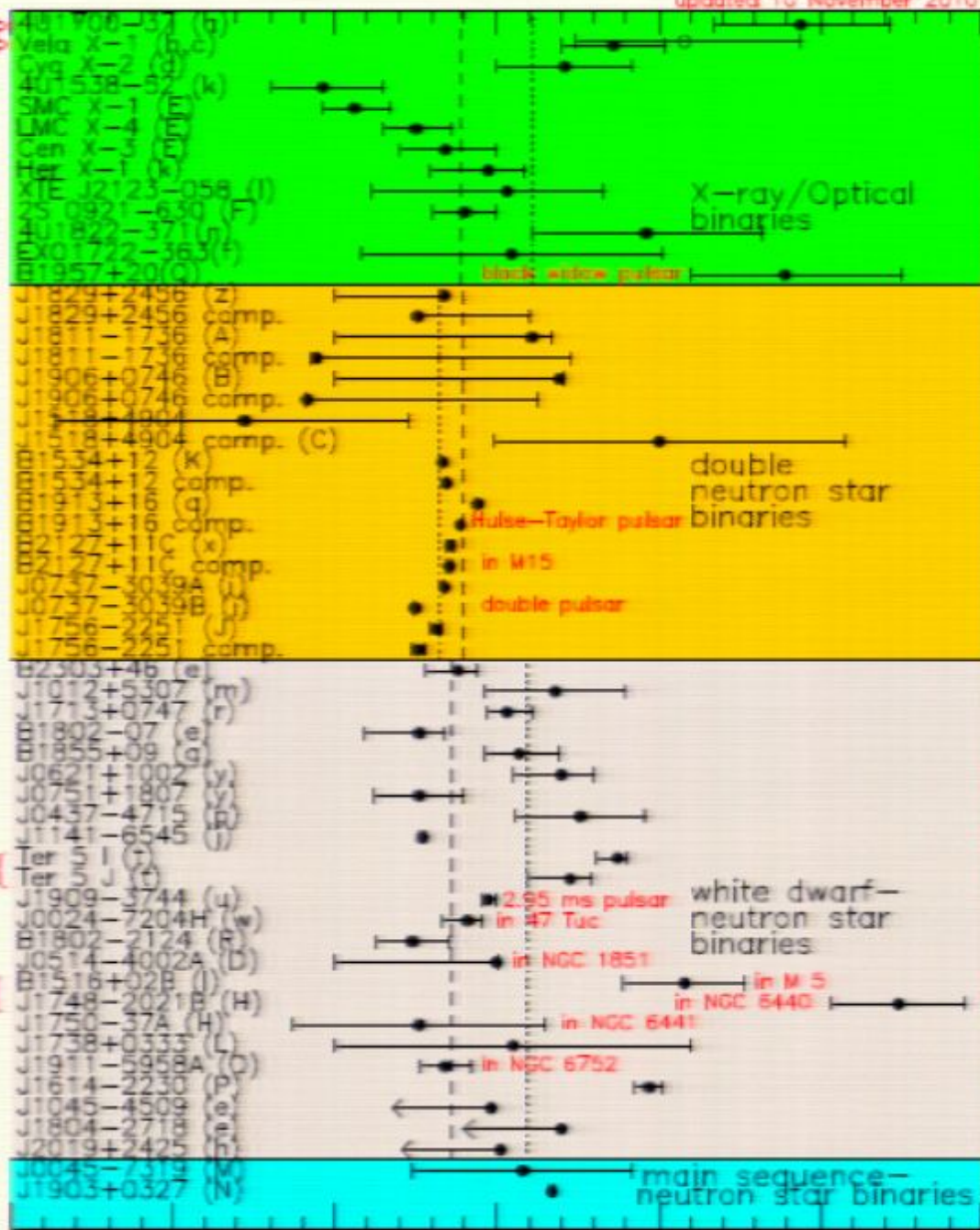
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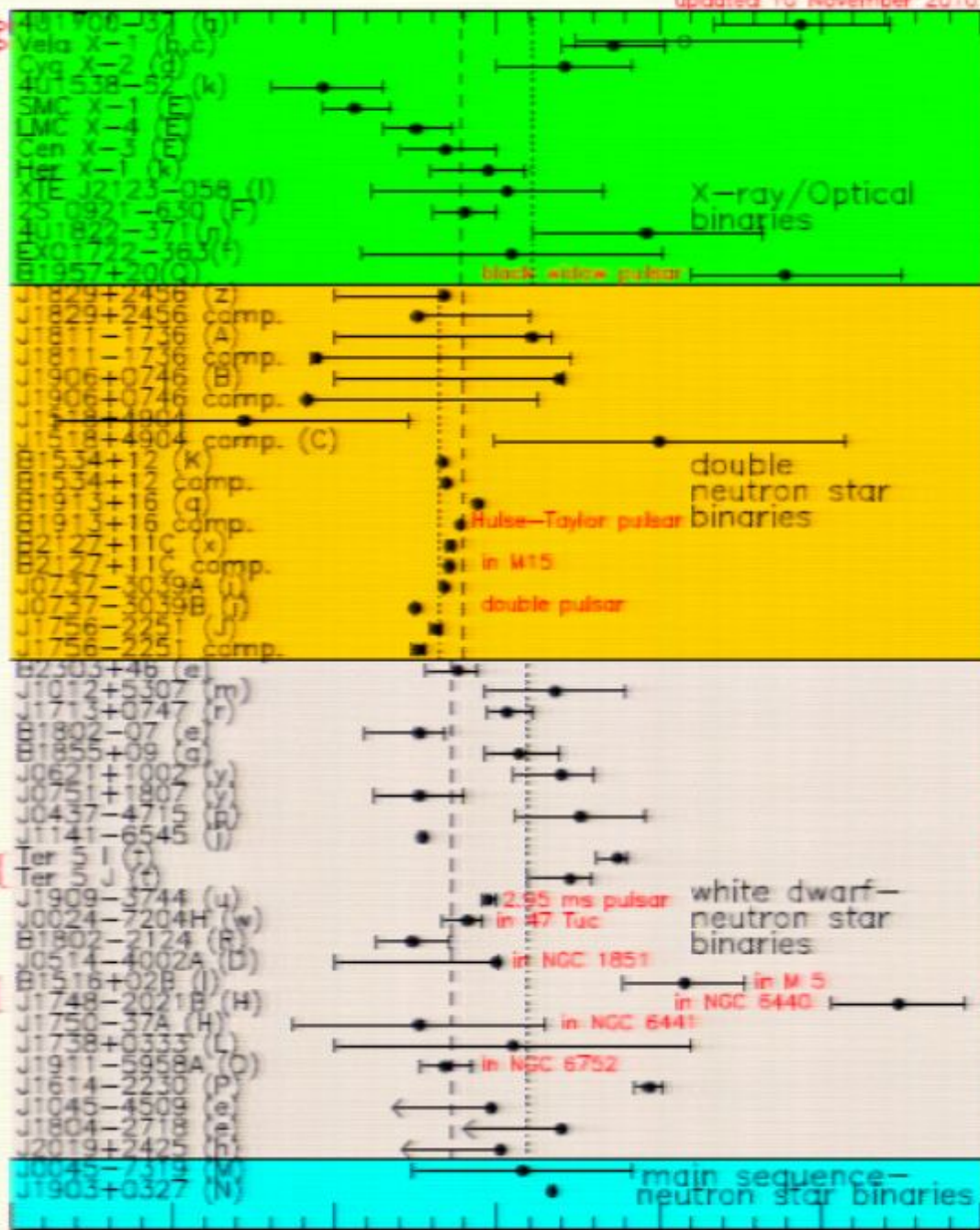
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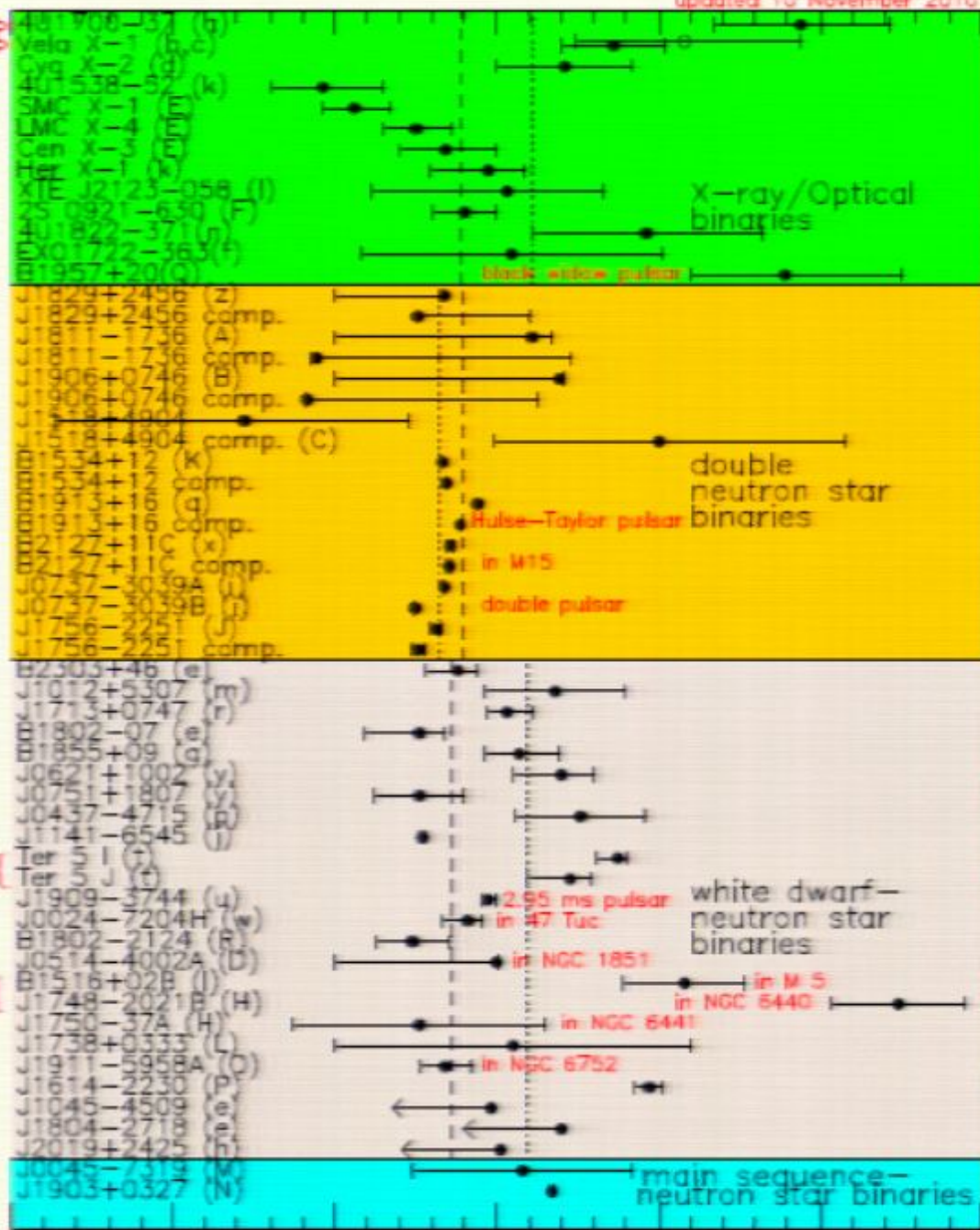
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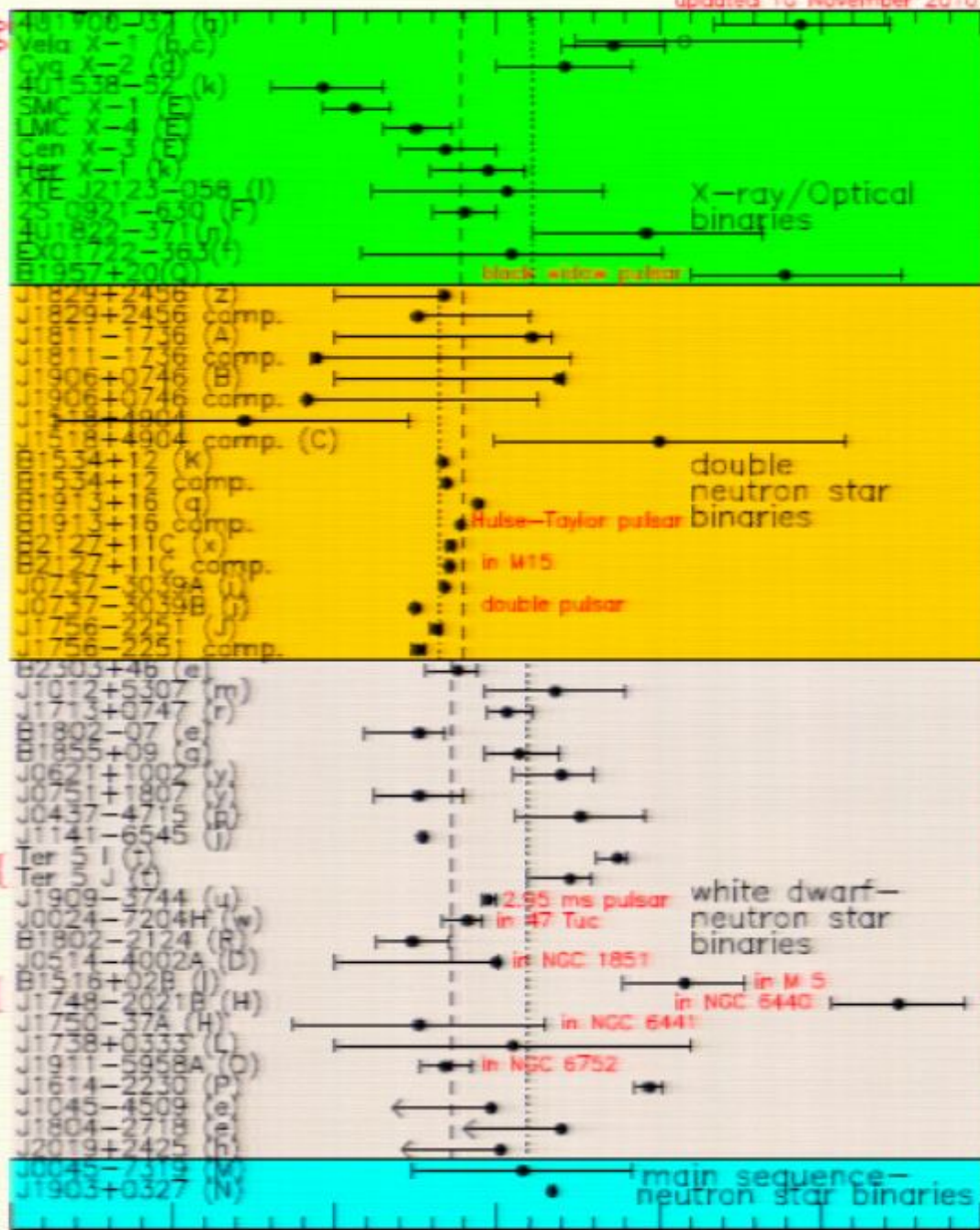
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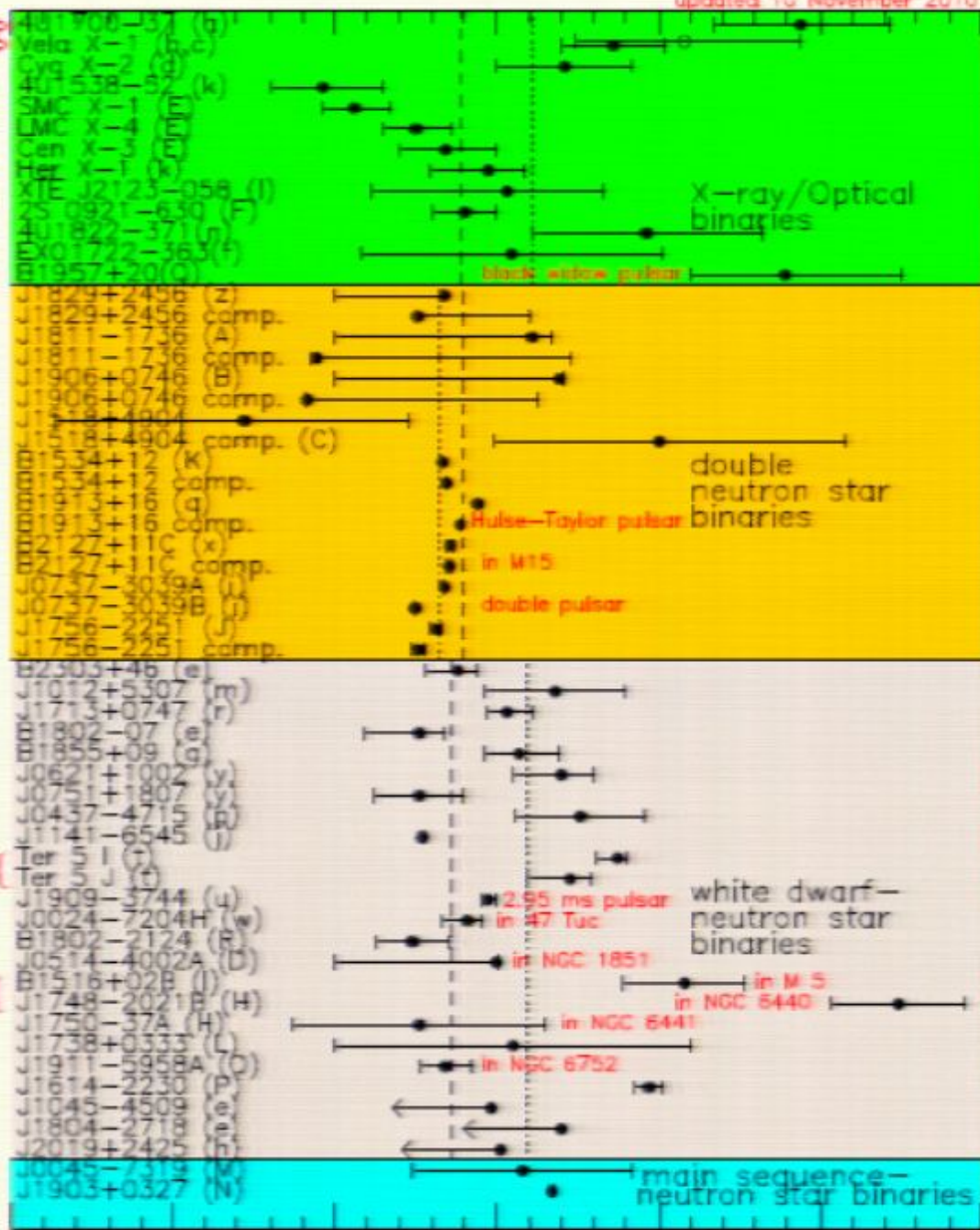
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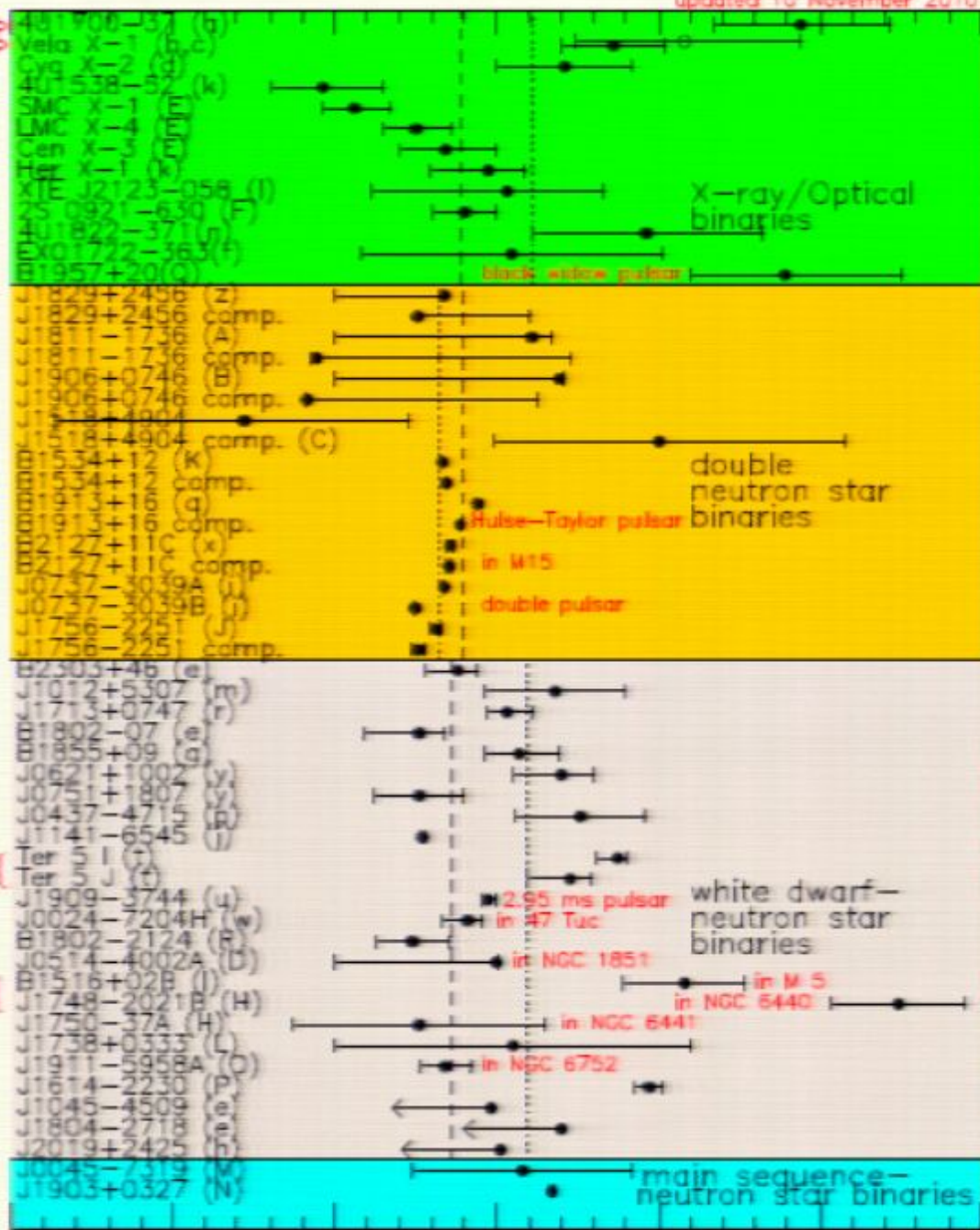
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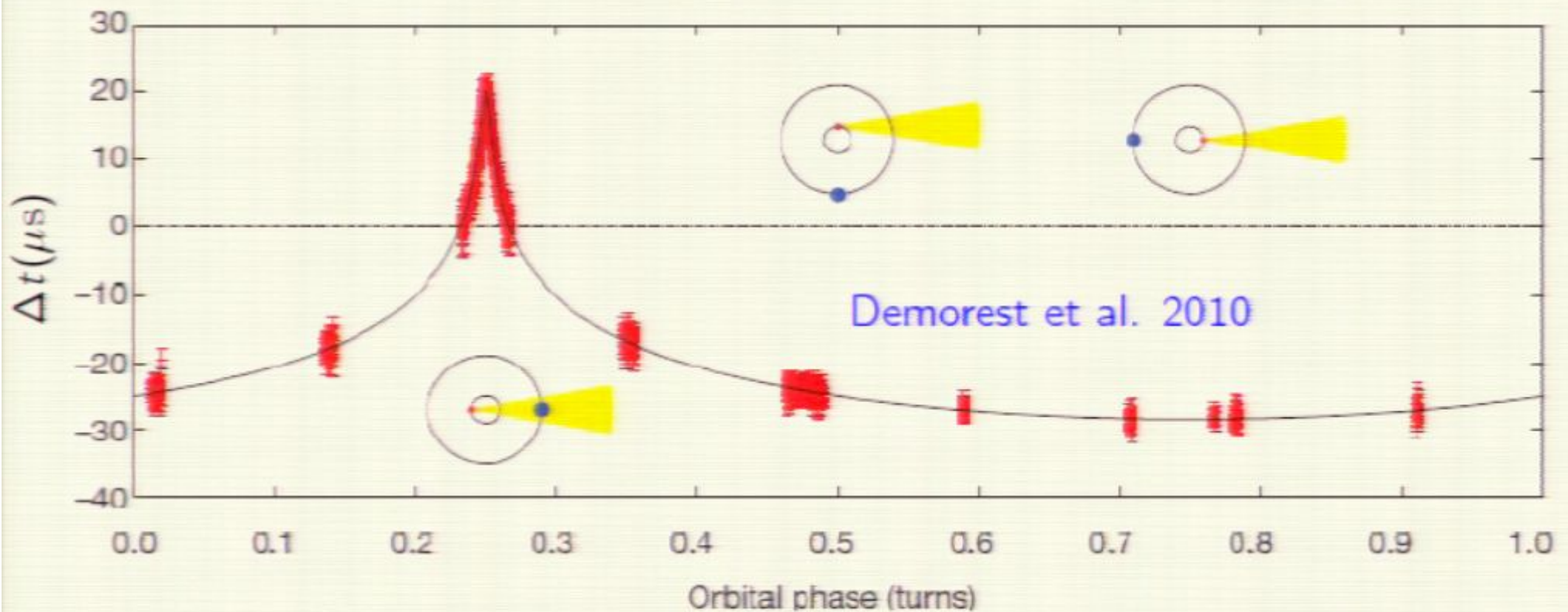
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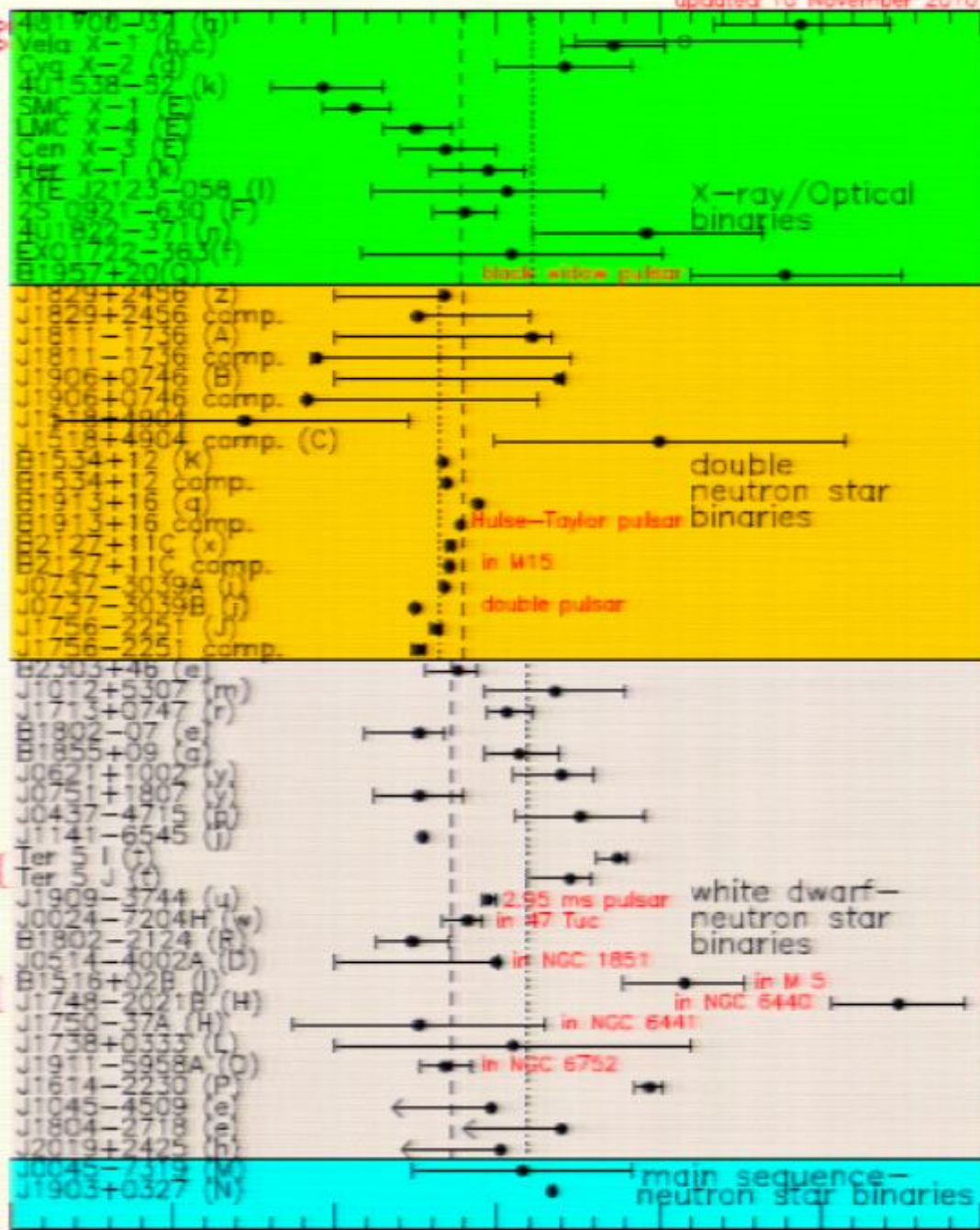
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PSR J1614-2230

3.15 ms pulsar in 8.69d orbit with $0.5 M_{\odot}$ white dwarf companion.
Shapiro delay tightly confines the edge-on inclination: $\sin i = 0.99984$
Pulsar mass is $1.97 \pm 0.04 M_{\odot}$
Distance > 1 kpc, $B \simeq 1.8 \times 10^8$ G



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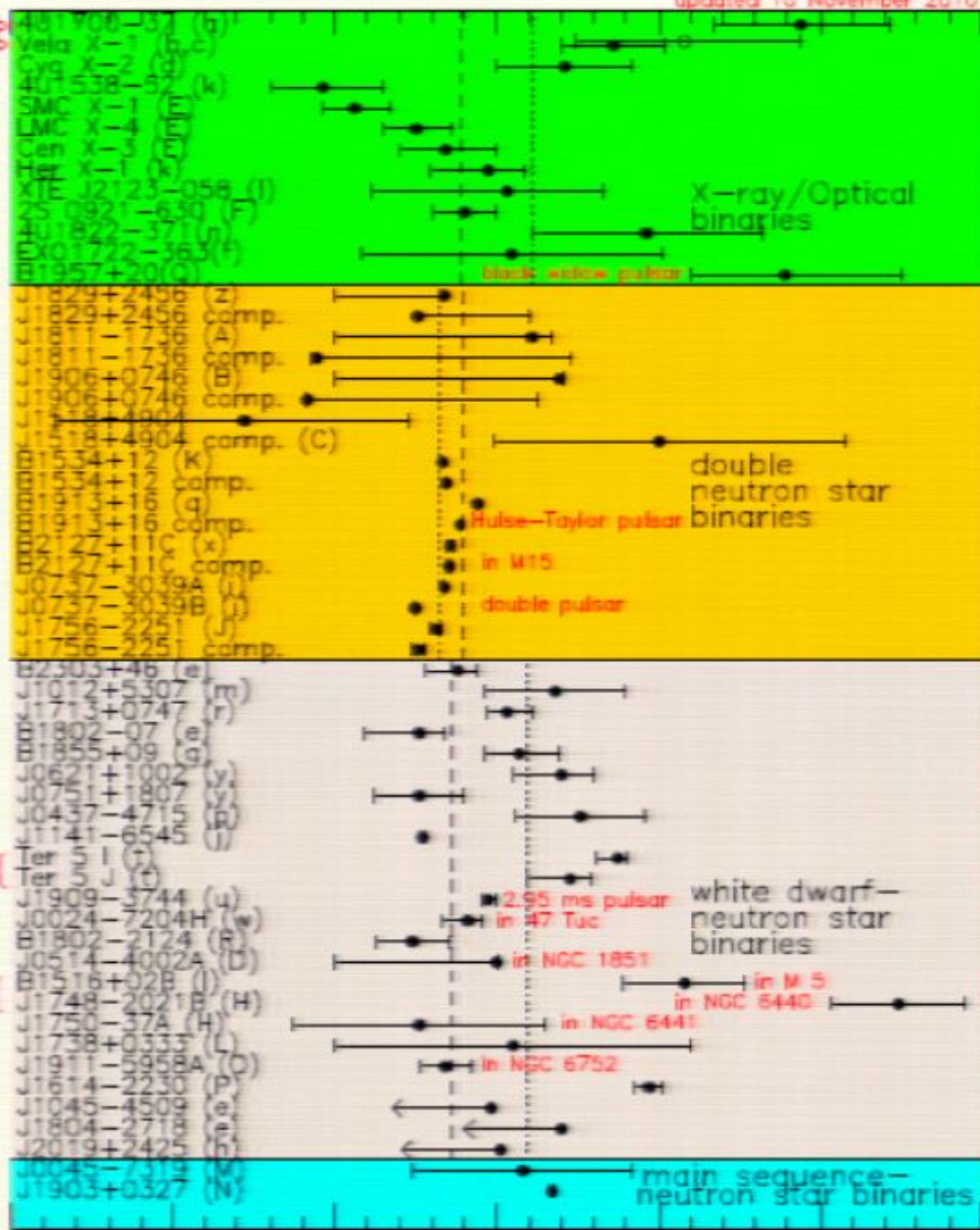
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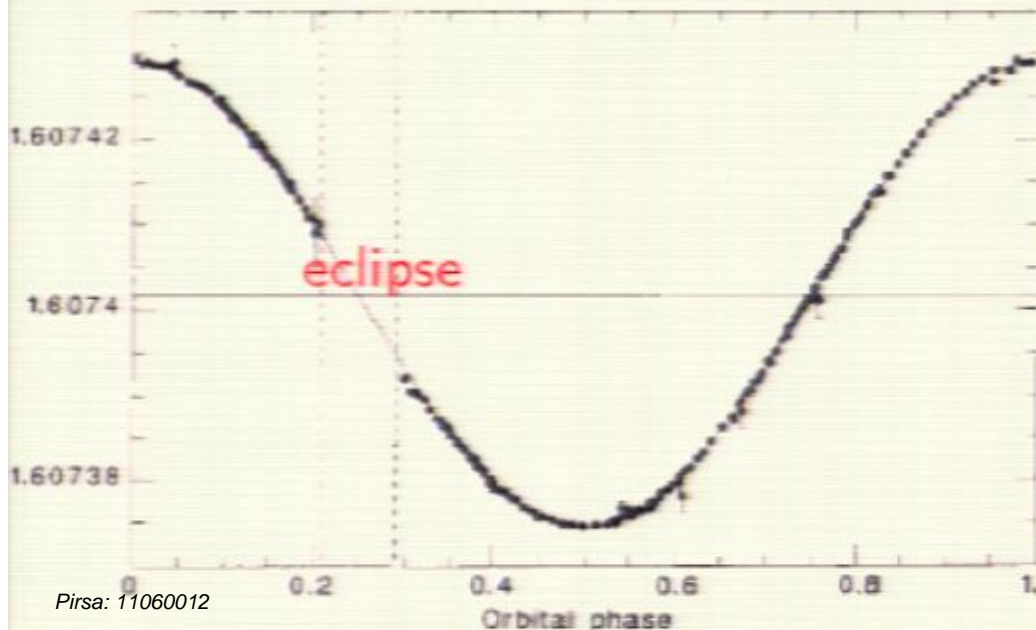
Black Widow Pulsar PSR B1957+20

1.6ms pulsar in circular 9.17h orbit with a $M_c \sim 0.03 M_\odot$ companion. Pulsar is eclipsed for 50-60 minutes each orbit; eclipsing object has a volume much larger than the companion or its Roche lobe.

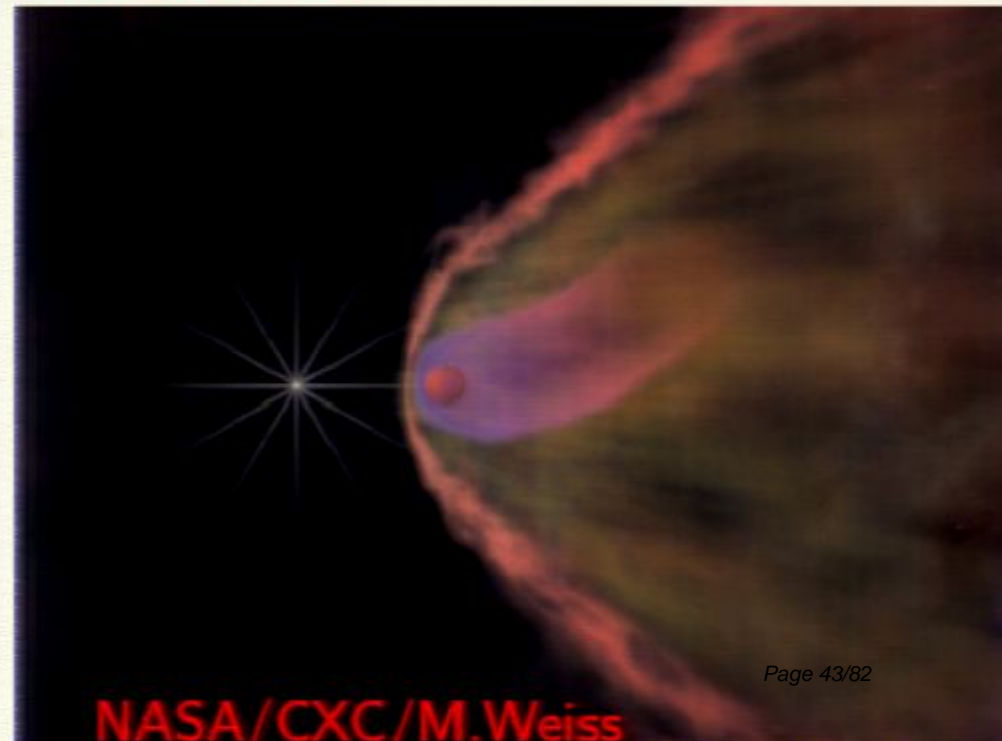
It is believed the companion is ablated by the pulsar leading to mass loss and an eclipsing plasma cloud. Companion nearly fills its Roche lobe. Ablation by pulsar leads to eventual disappearance of companion.

The optical light curve does not represent the center of mass of the companion, but the motion of its irradiated hot spot.

pulsar radial velocity



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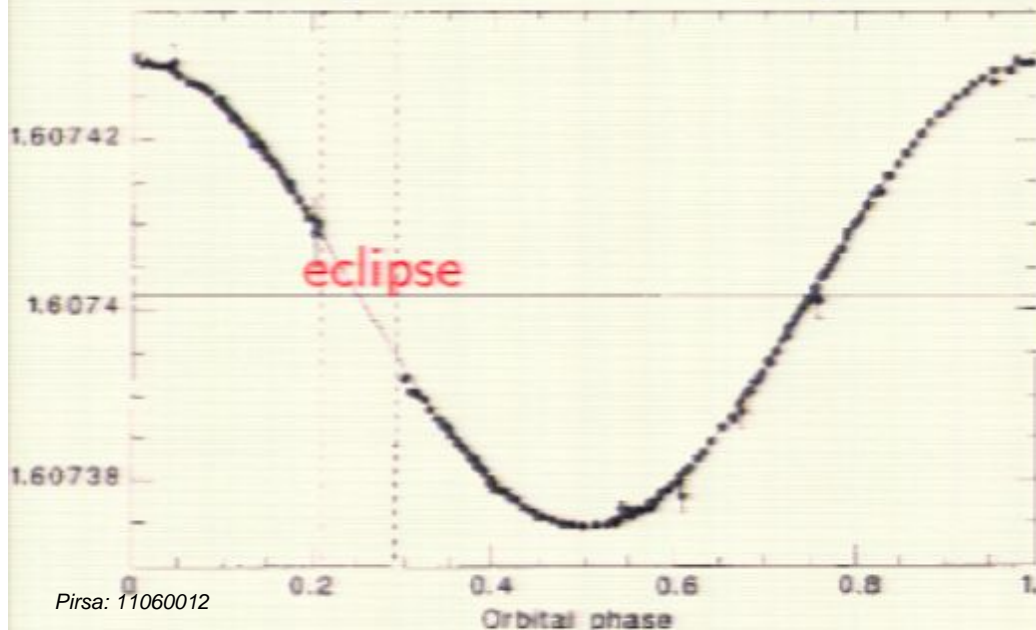
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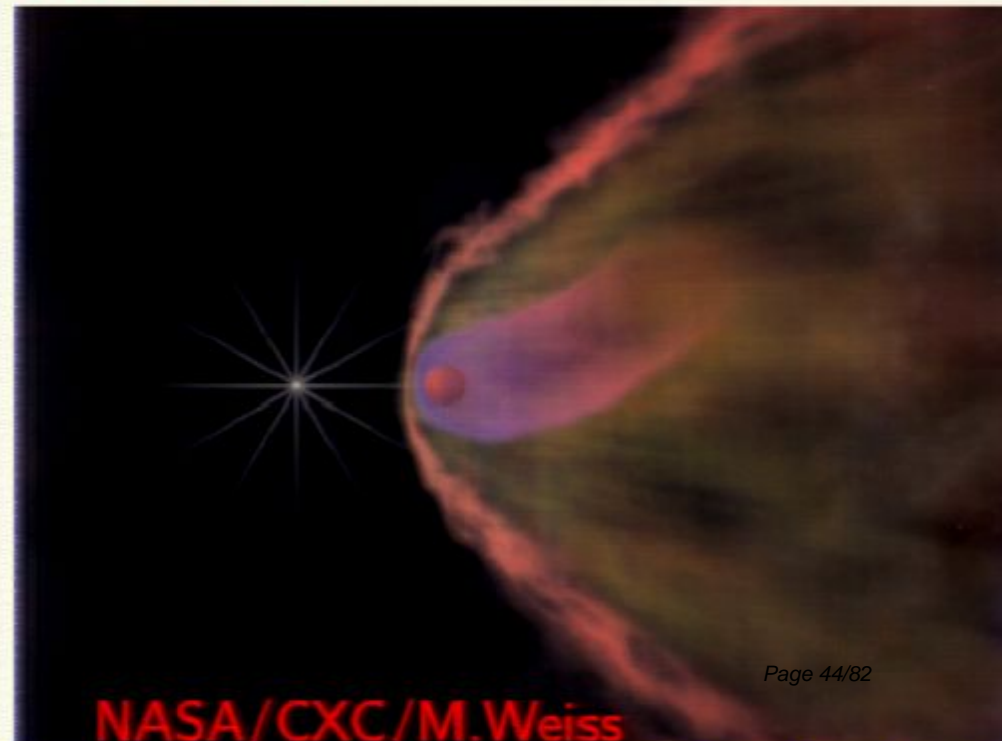
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Implications of Maximum Masses

$$M_{max} > 2 M_{\odot}$$

- ▶ Upper limits to energy density, pressure and baryon density:
 - ▶ $\epsilon < 13.1\epsilon_s$
 - ▶ $p < 8.8\epsilon_s$
 - ▶ $n_B < 9.8n_s$
- ▶ Lower limit to spin period:
 $P > 0.56$ ms
- ▶ Lower limit to neutron star radius:
 $R > 8.5$ km
- ▶ Upper limits to energy density, pressure and baryon density in the case of a quark matter core ($s = 1/3$):
 - ▶ $\epsilon < 7.7\epsilon_s$
 - ▶ $p < 2.0\epsilon_s$
 - ▶ $n_B < 6.9n_s$

$$M_{max} > 2.4 M_{\odot}$$

- ▶ Upper limits to energy density, pressure and baryon density:
 - ▶ $\epsilon < 8.9\epsilon_s$
 - ▶ $p < 5.9\epsilon_s$
 - ▶ $n_B < 6.6n_s$
- ▶ Lower limit to spin period:
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- ▶ Upper limits to energy density, pressure and baryon density in the case of a quark matter core ($s = 1/3$):
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Implications of Maximum Masses

$$M_{max} > 2 M_{\odot}$$

- ▶ Upper limits to energy density, pressure and baryon density:
 - ▶ $\epsilon < 13.1\epsilon_s$
 - ▶ $p < 8.8\epsilon_s$
 - ▶ $n_B < 9.8n_s$
- ▶ Lower limit to spin period:
 $P > 0.56$ ms
- ▶ Lower limit to neutron star radius:
 $R > 8.5$ km
- ▶ Upper limits to energy density, pressure and baryon density in the case of a quark matter core ($s = 1/3$):
 - ▶ $\epsilon < 7.7\epsilon_s$
 - ▶ $p < 2.0\epsilon_s$
 - ▶ $n_B < 6.9n_s$

$$M_{max} > 2.4 M_{\odot}$$

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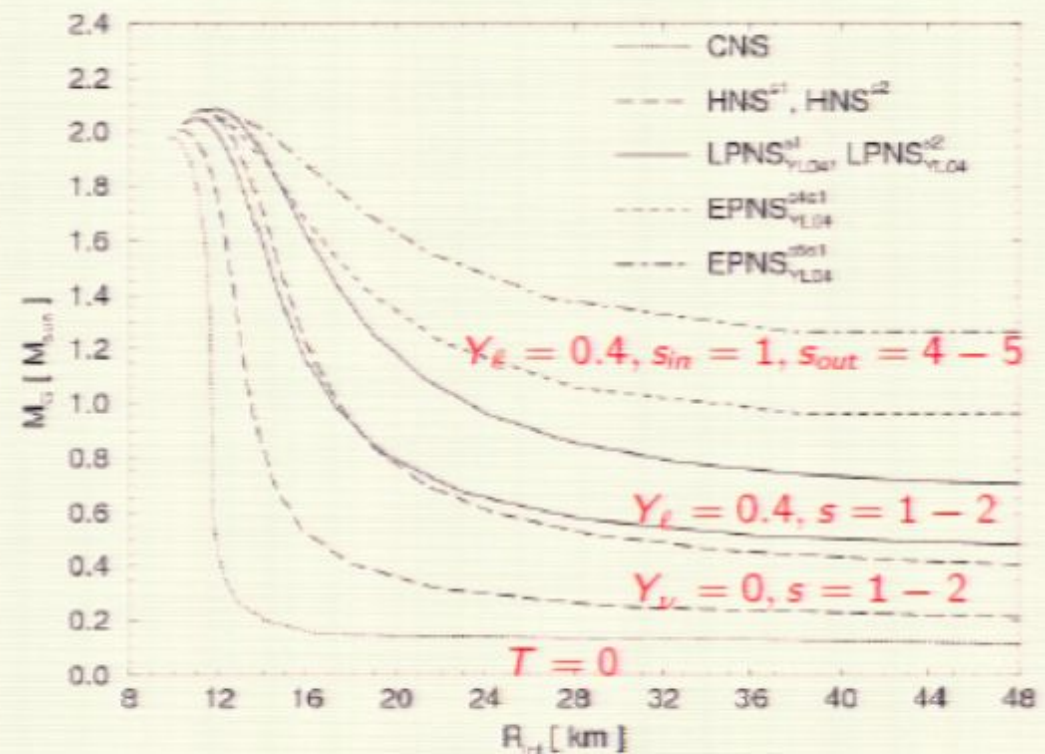
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Theoretical limit for minimum mass of neutron stars is about $0.09 M_{\odot}$.

Practical limit, on the basis that neutron stars form from lepton-rich proto-neutron stars, can't exceed of $1 M_{\odot}$ and could be close to $1.2 M_{\odot}$.

Recent refined mass determinations of X-ray pulsar binaries (Rawls, Orosz, McClintock and Torres (2011)):

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Strobel, Schaab & Weigel (1999)

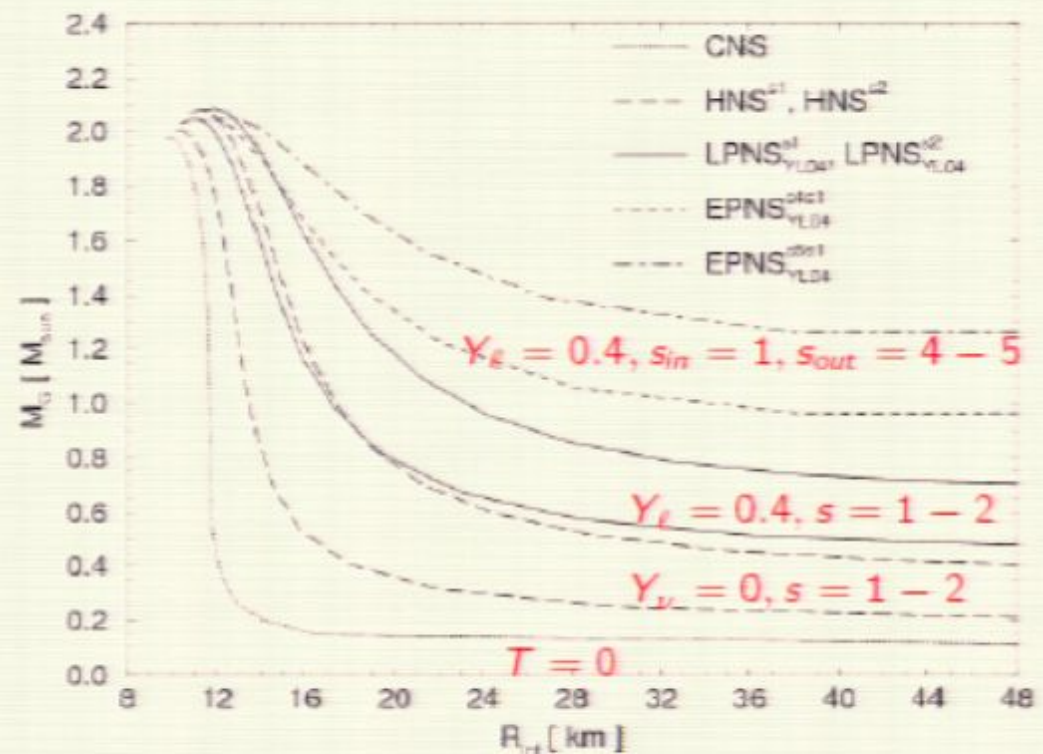
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Neutron Star Matter Pressure and the Radius

$$p \simeq Kn^\gamma$$

$$\gamma = d \ln p / d \ln n \sim 2$$

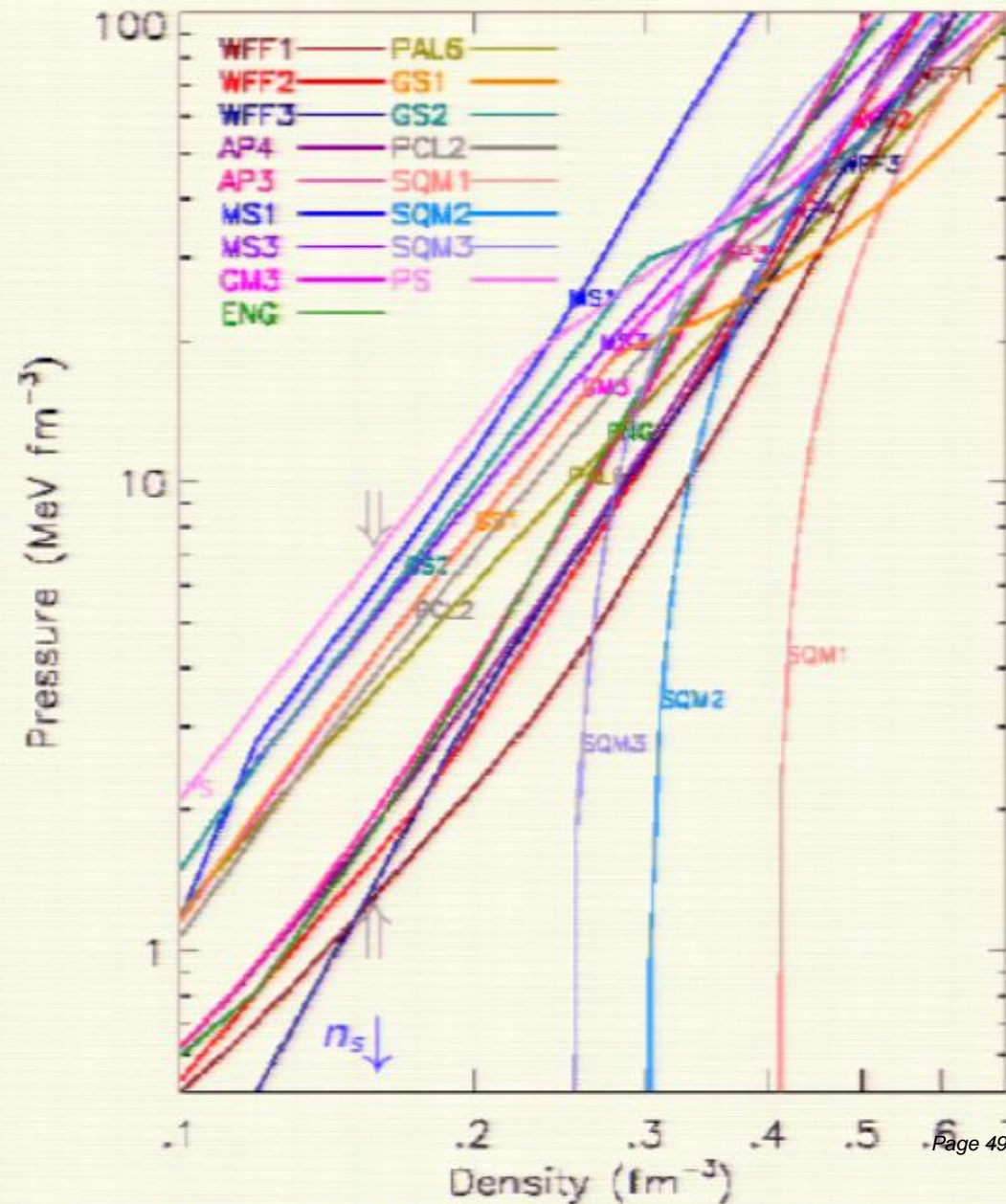
$$R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

$$R \propto p_f^{1/2} n_f^{-1} M^0$$

$$(1 < n_f/n_s < 2)$$

Wide variation:

$$1.2 < \frac{p(n_s)}{\text{MeV fm}^{-3}} < 7$$



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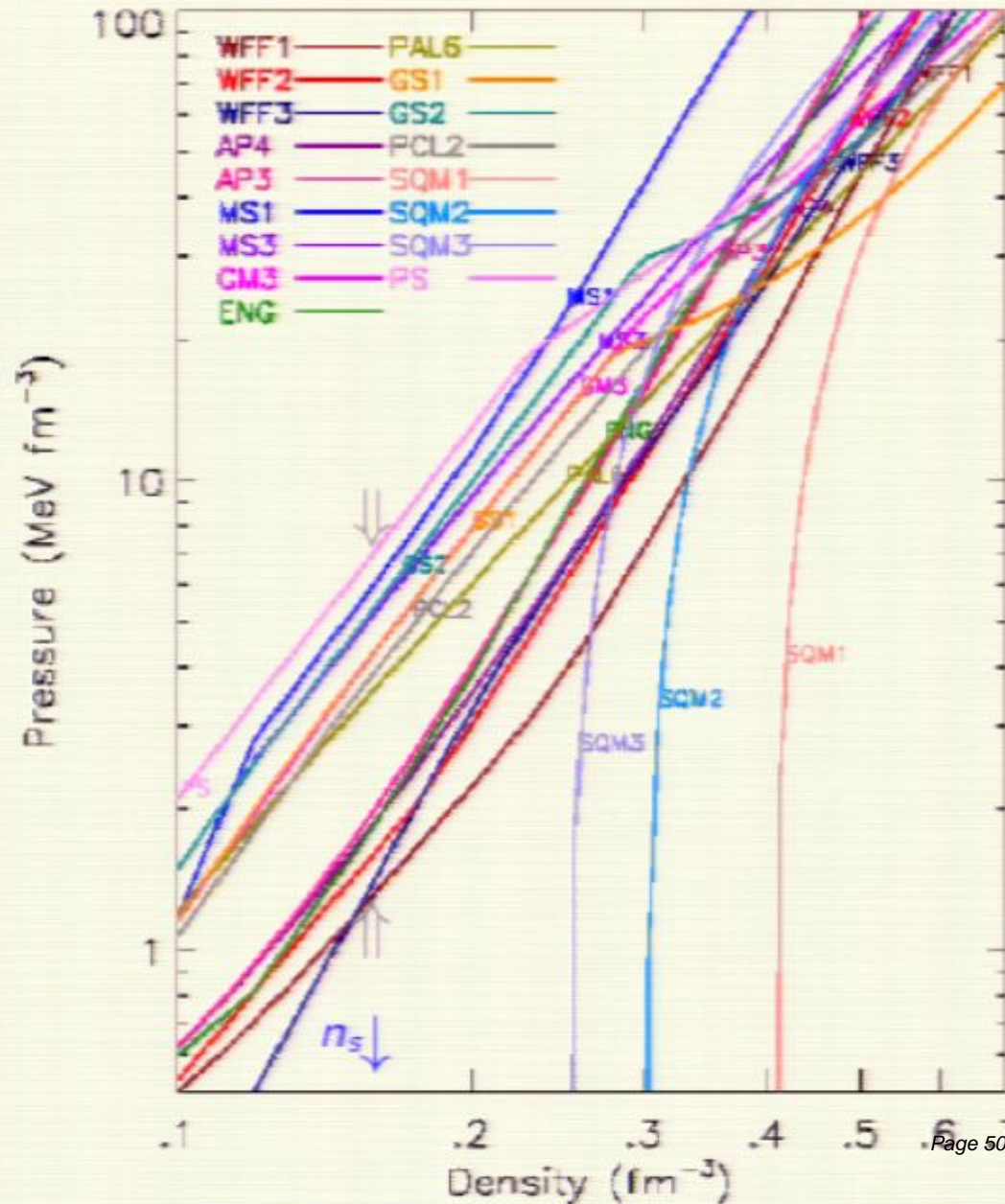
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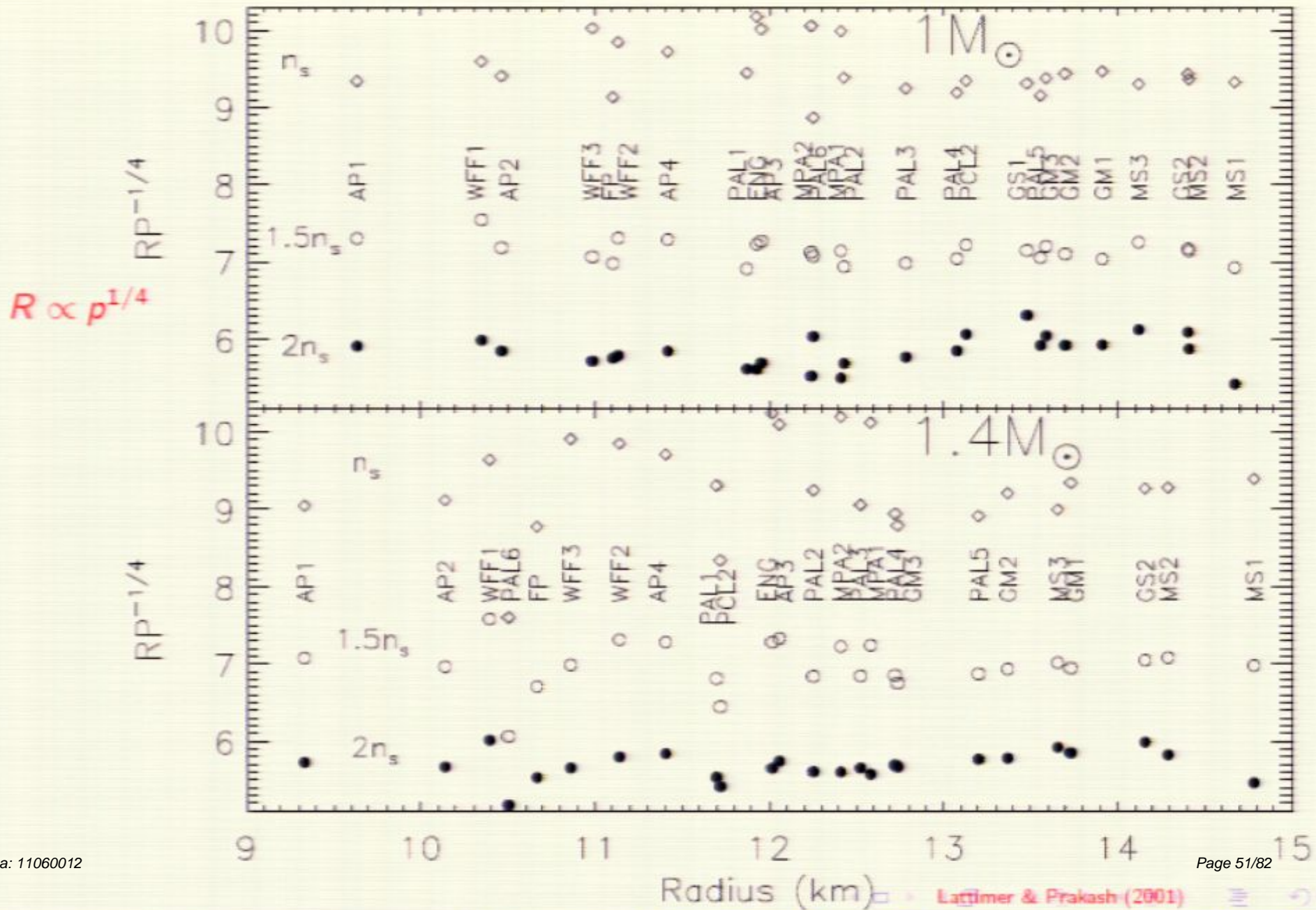
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The Radius – Pressure Correlation



The Pressure of Neutron Star Matter

Expansion of cold nucleonic matter energy near n_s and isospin symmetry $x = 1/2$:

$$E(n, x) \simeq E(n, 1/2) + E_{\text{sym}}(n)(1 - 2x)^2 + \frac{3\hbar c}{4} x (3\pi^2 n x)^{1/3},$$

$$P(n, x) \simeq n^2 \left[\frac{dE(n, 1/2)}{dn} + \frac{dE_{\text{sym}}}{dn} (1 - 2x)^2 \right] + \frac{\hbar c}{4} n x (3\pi^2 n x)^{1/3},$$

$$\mu_e = \hbar c (3\pi^2 n x)^{1/3}, \quad E(n, 1/2) \simeq -B + \frac{K}{18} \left(1 - \frac{n}{n_s} \right)^2.$$

Beta Equilibrium: $\left(\frac{\partial E}{\partial x} \right)_n = \mu_p - \mu_n + \mu_e = 0.$

$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{\text{sym}}}{\hbar c} \right)^3,$$

$$P_\beta \simeq \frac{Kn^2}{9n_0} \left(\frac{n}{n_s} - 1 \right) + n^2 (1 - 2x_\beta)^2 \frac{dE_{\text{sym}}}{dn} + E_{\text{sym}} n x_\beta (1 - 2x_\beta)$$

$$E_{\text{sym}}(n_s) \sim S_v \simeq 30 \text{ MeV}, \quad \hbar c \simeq 200 \text{ MeV/fm},$$

$$n \rightarrow n_s \implies x_\beta \rightarrow 0.04, \quad P_\beta \rightarrow n_s^2 \left(\frac{dE_{\text{sym}}}{dn} \right)_{n_s}.$$

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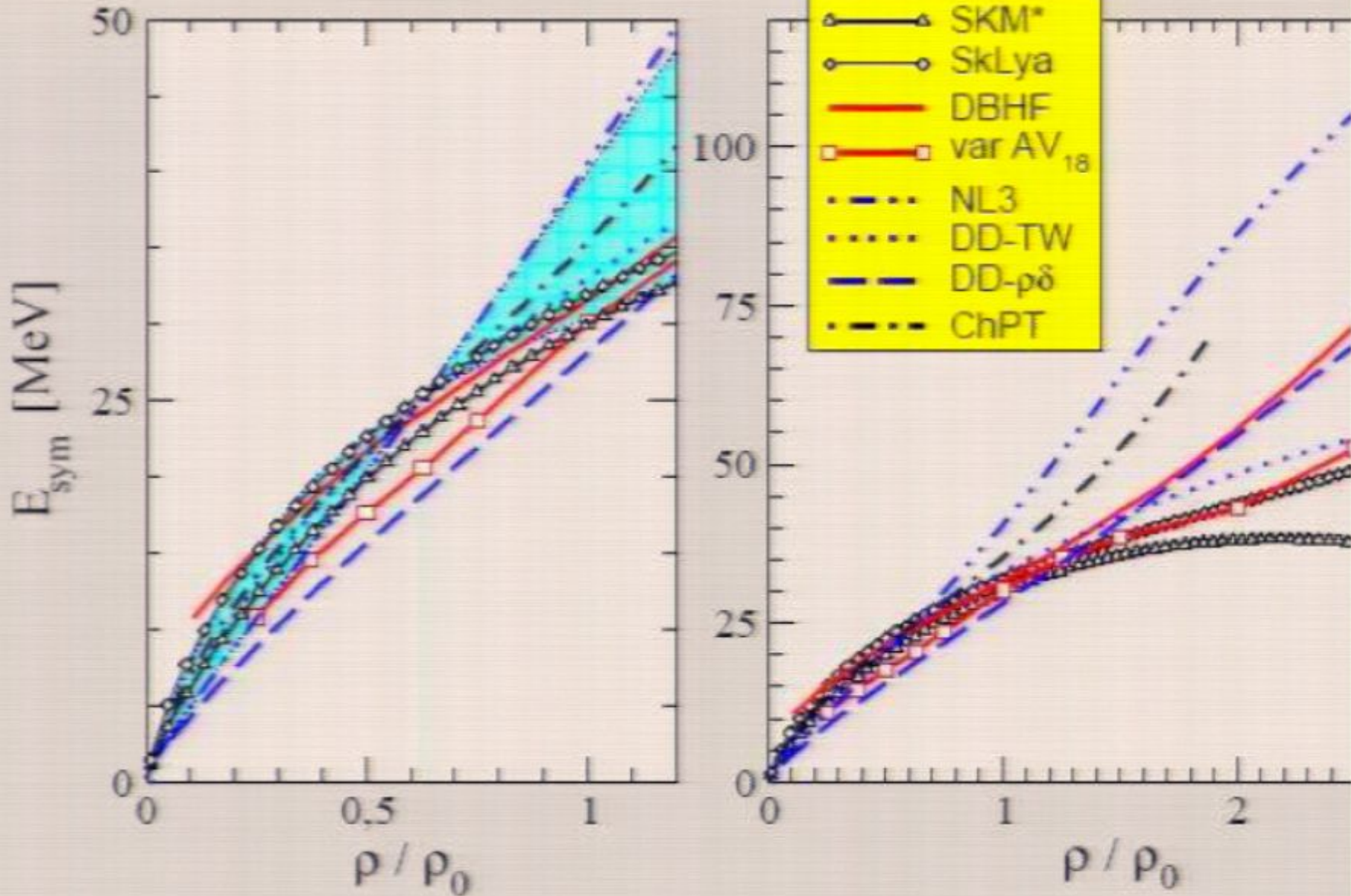
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The Uncertain $E_{sym}(n)$



Nuclear Structure Considerations

Information about $E_{sym}(n)$ can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

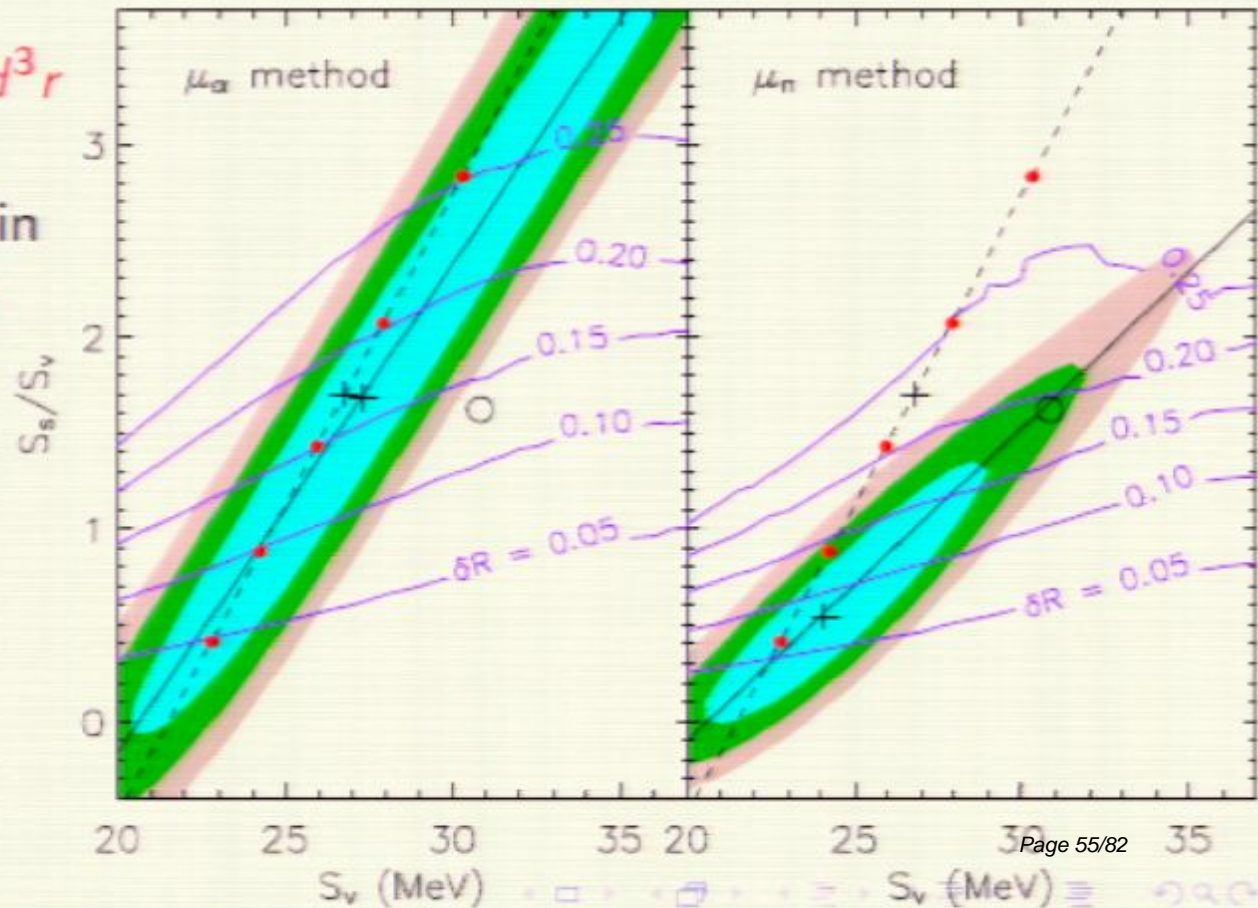
$$E(A, Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1 + (S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

$$S_s \propto S_v \int_0^R n \left(\frac{S_v}{E_{sym}(n)} - 1 \right) d^3r$$

Fitting binding energies results in a strong correlation between S_v and S_s , but not definite values.

Another observable: neutron skin thickness $\delta R \propto S_s/S_v$.

- Blue: $\Delta E < 0.01$ MeV/b
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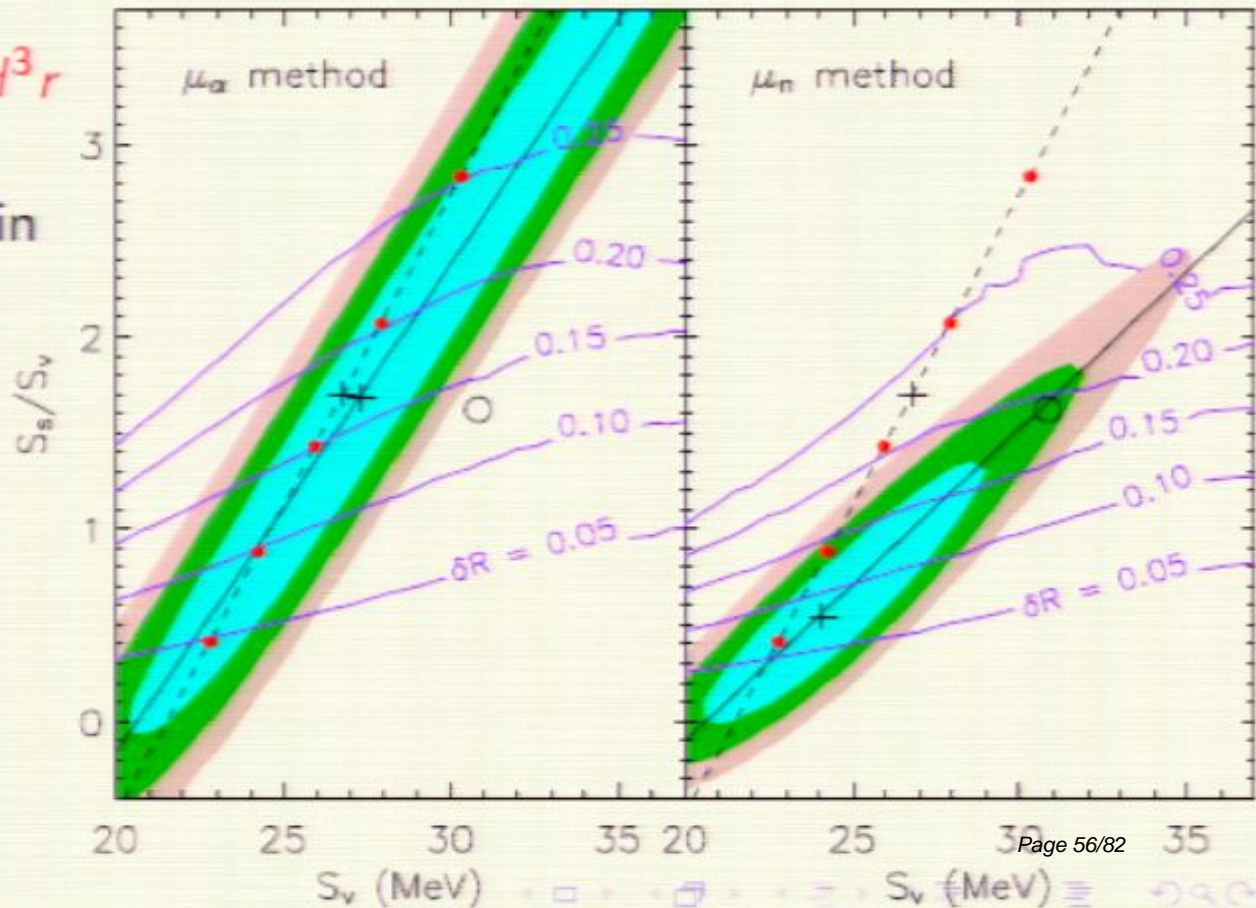
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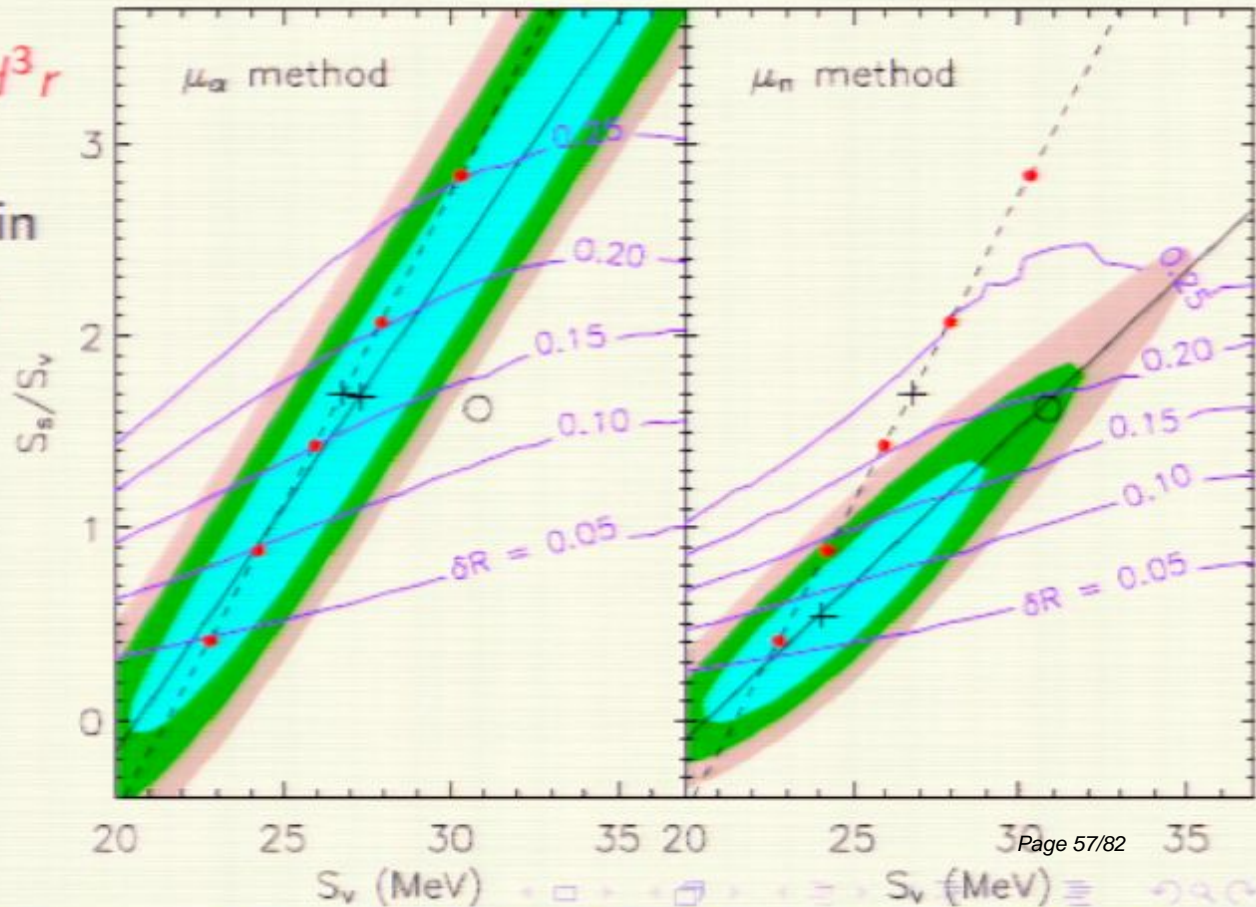
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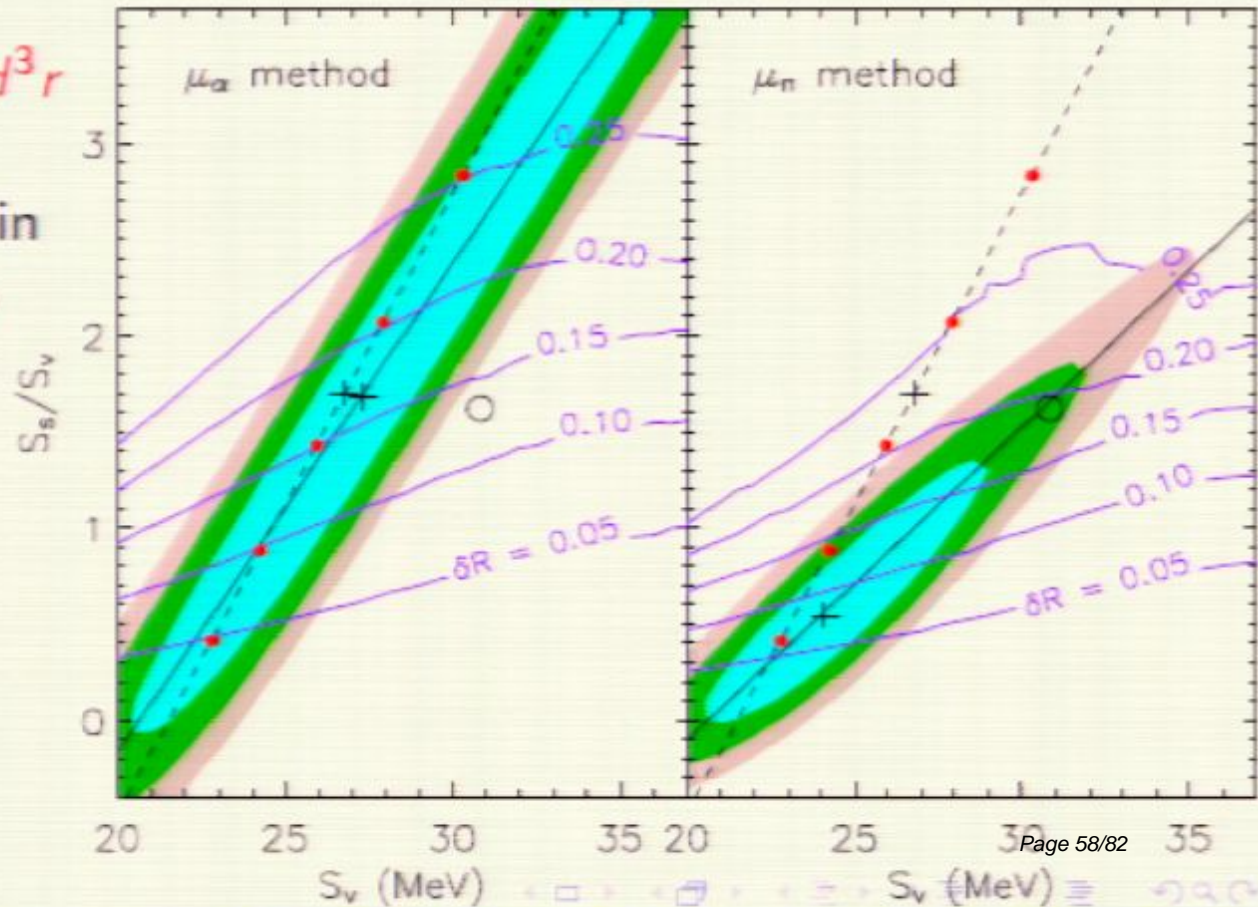
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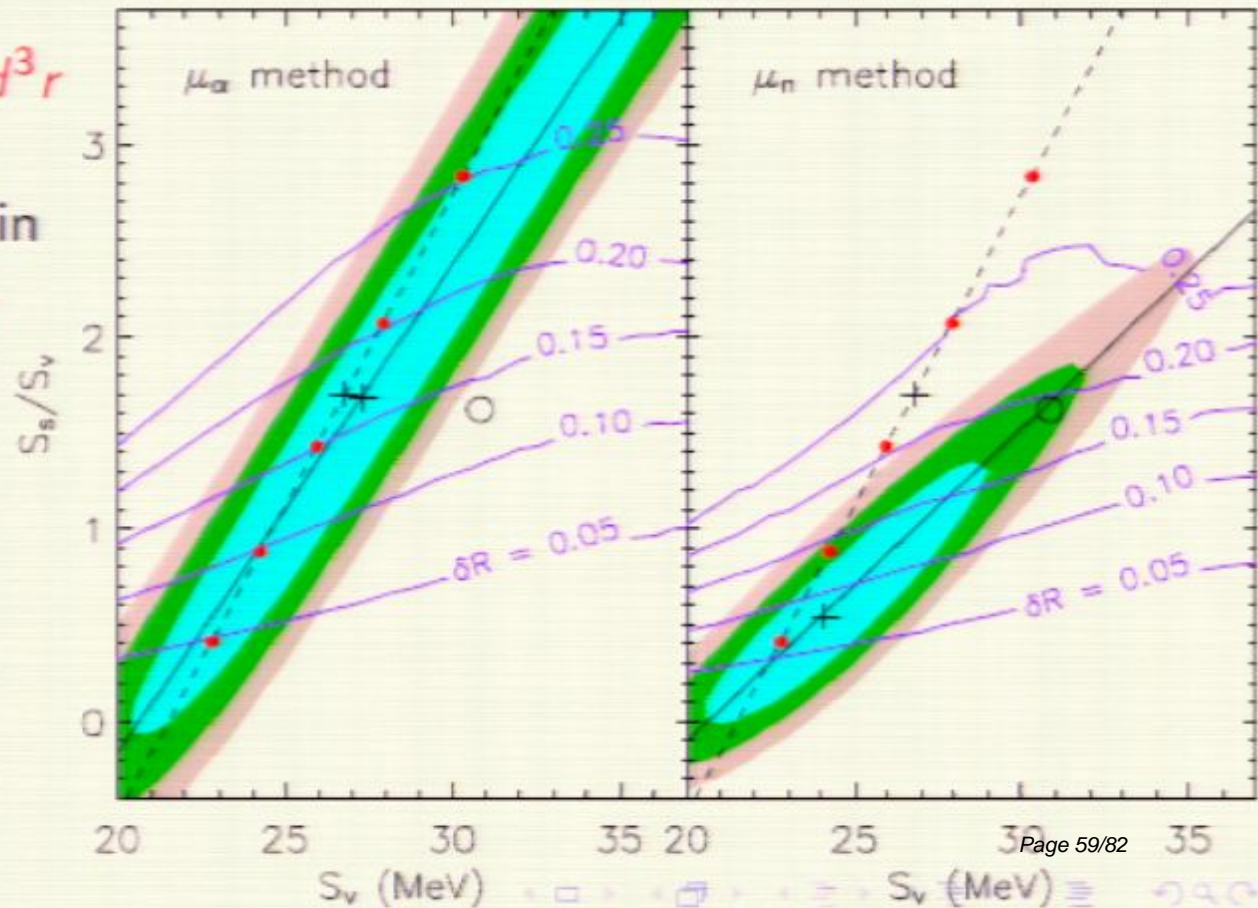
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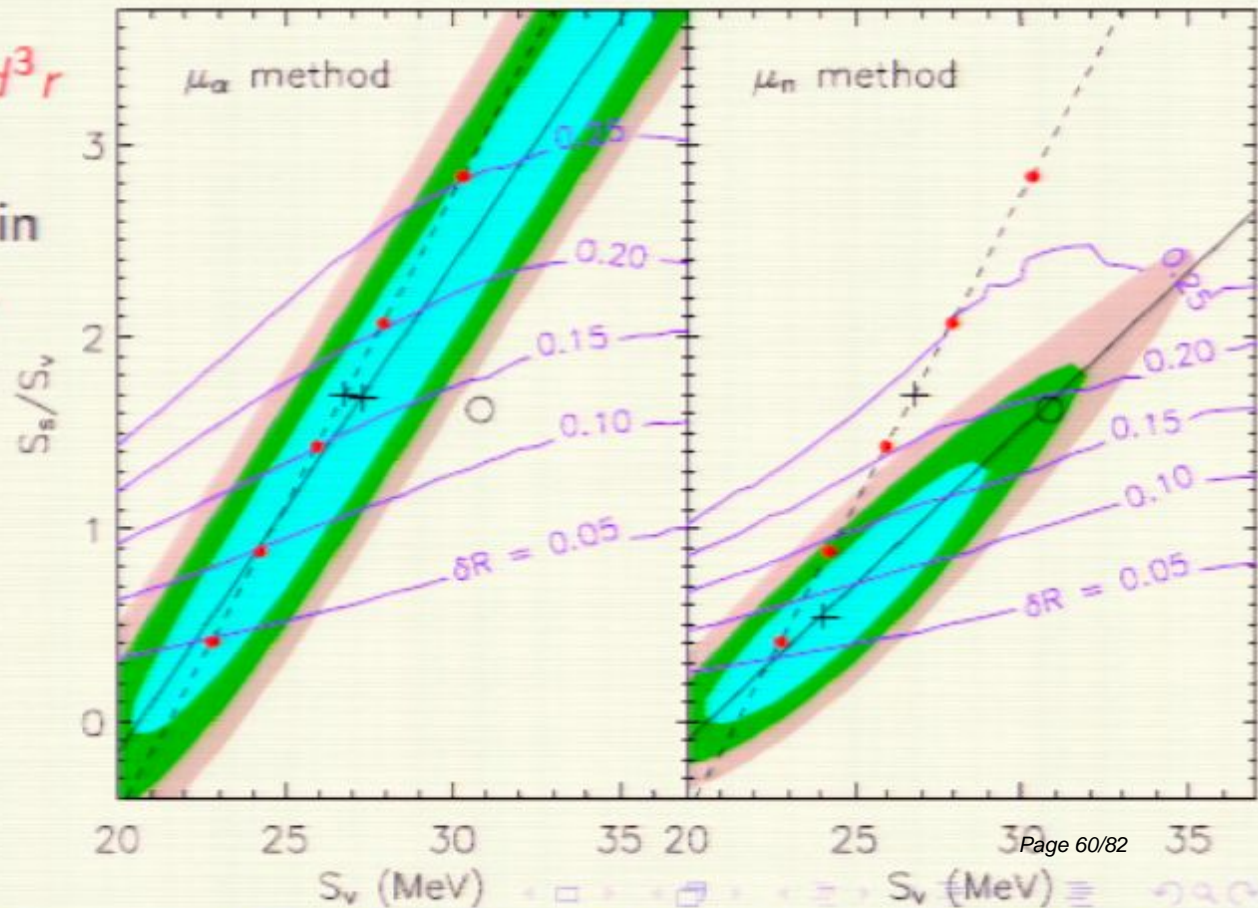
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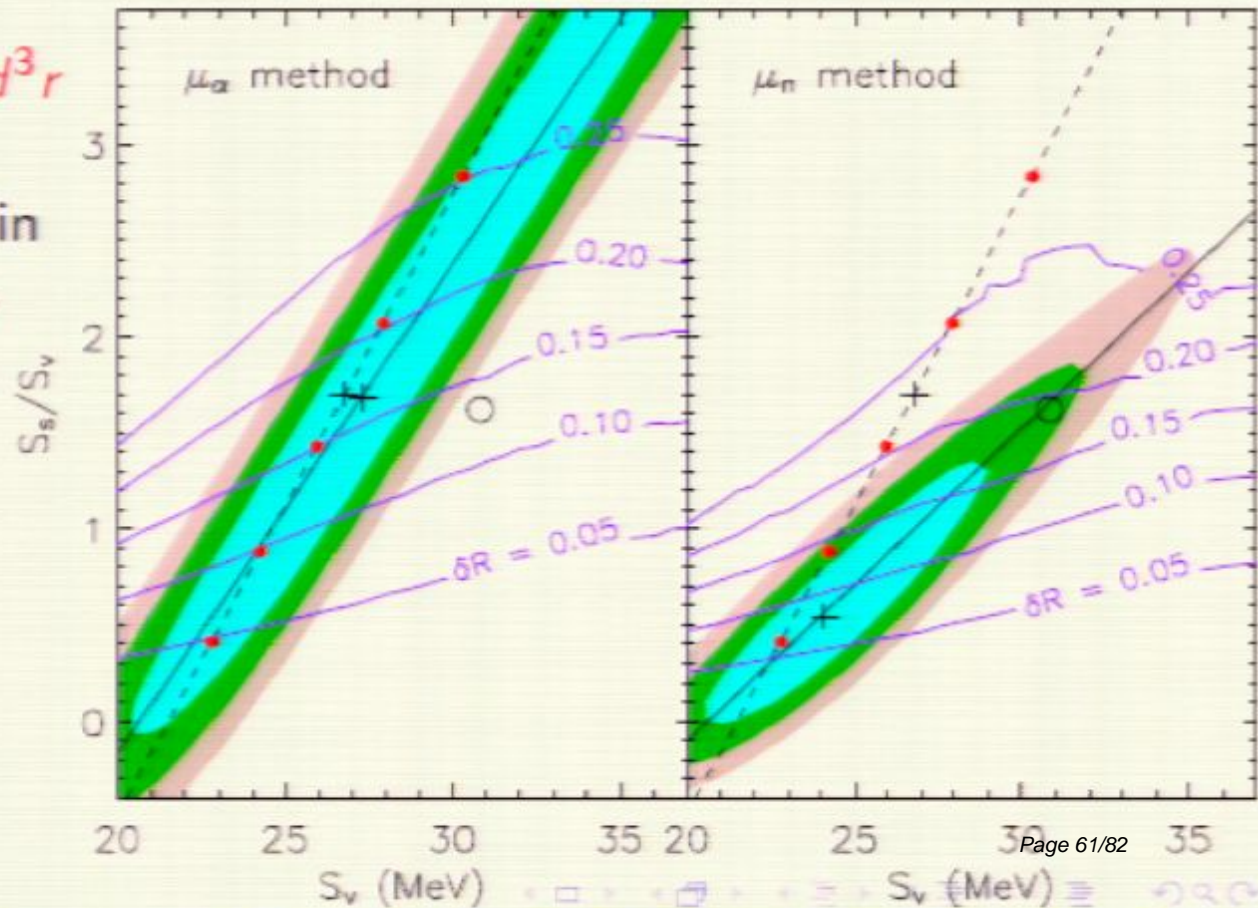
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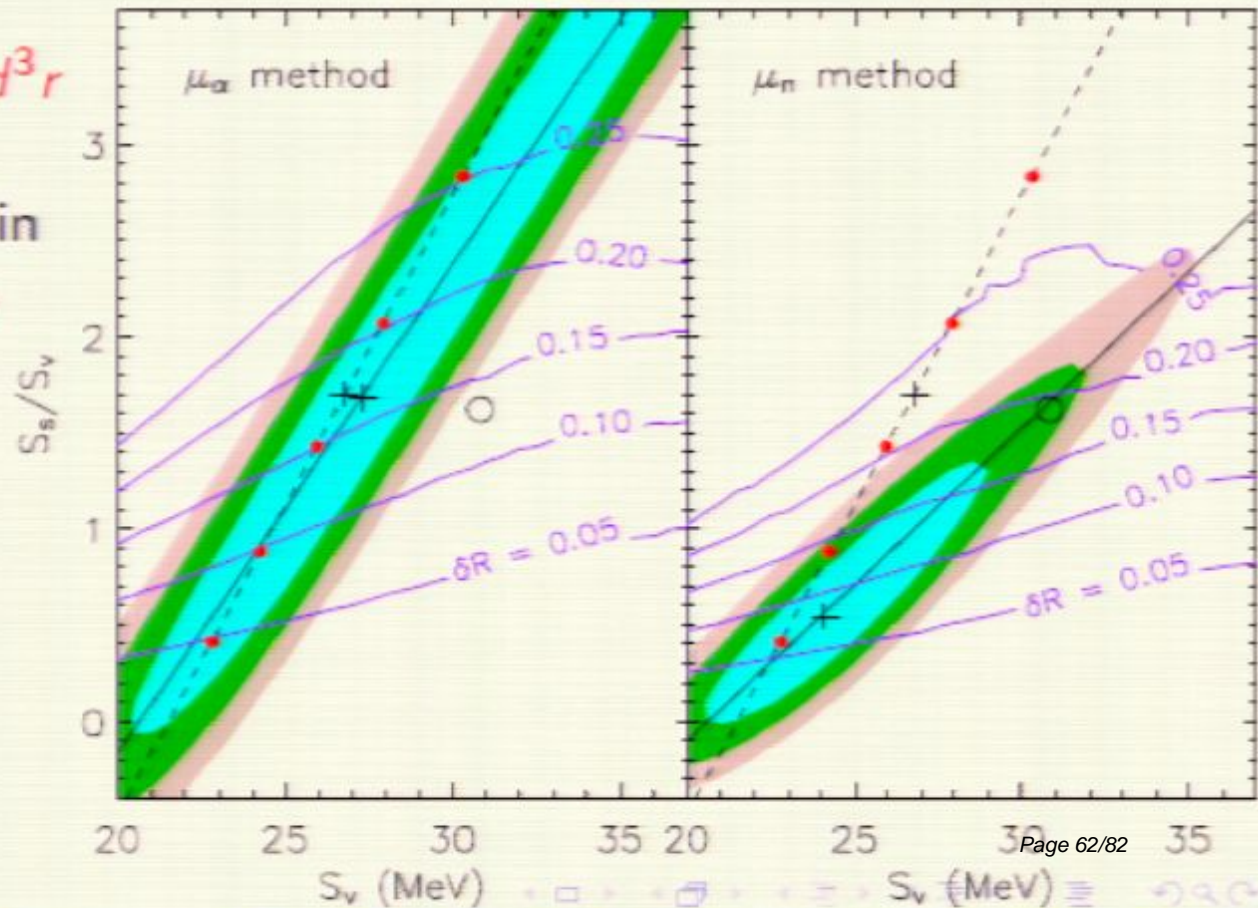
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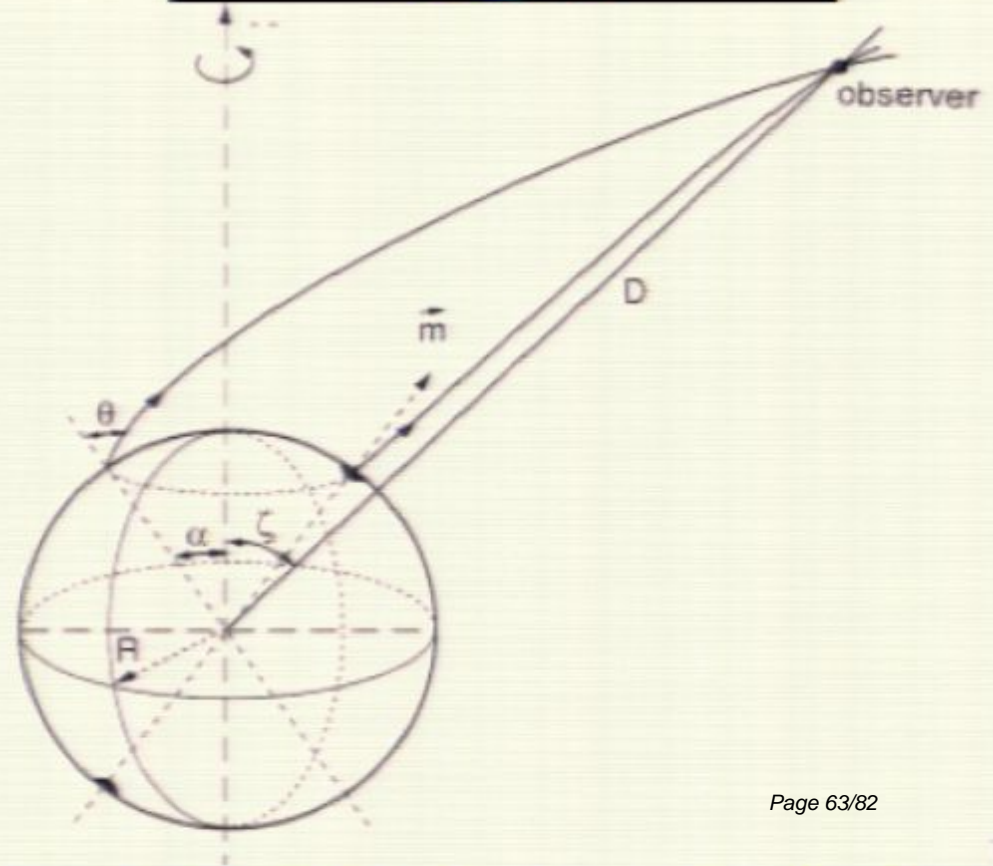


Radiation Radius

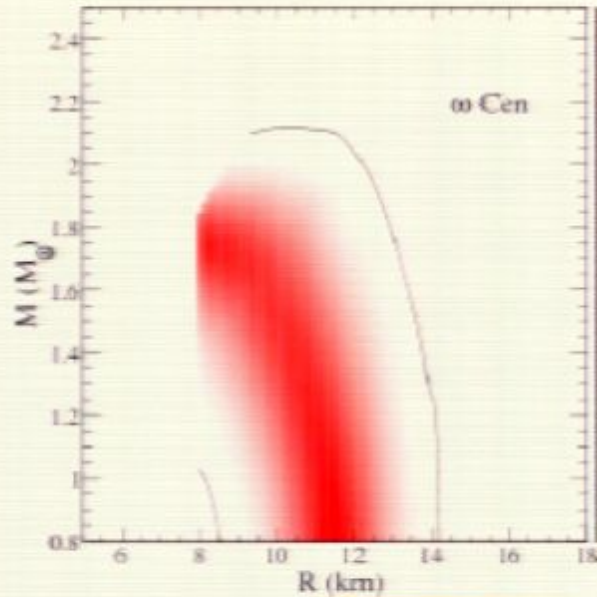
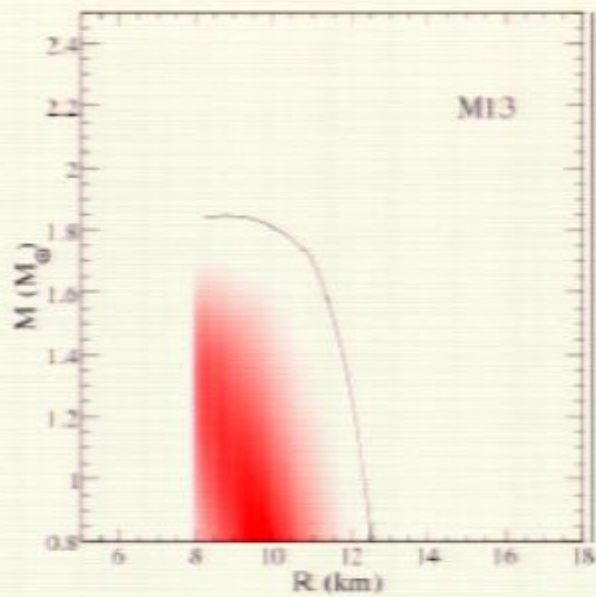
- ▶ The measurement of flux and temperature yields an apparent angular size (pseudo-BB):

$$\frac{R_{\infty}}{d} = \frac{R}{d} \frac{1}{\sqrt{1 - 2GM/Rc^2}}$$

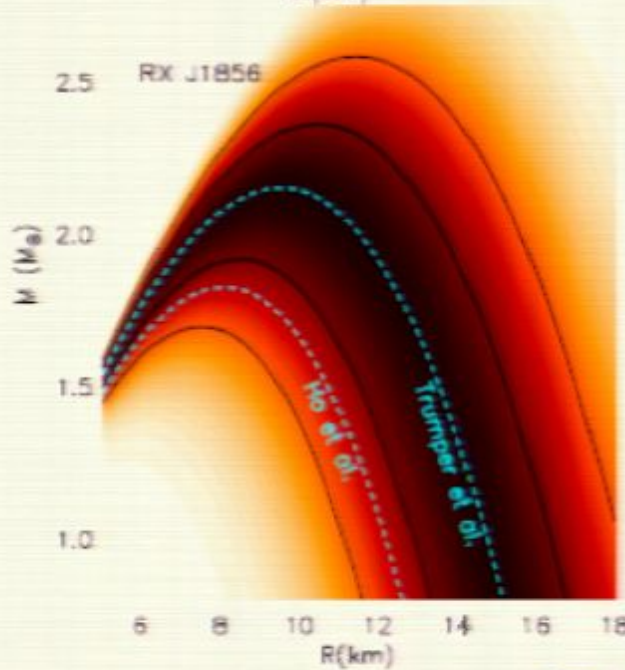
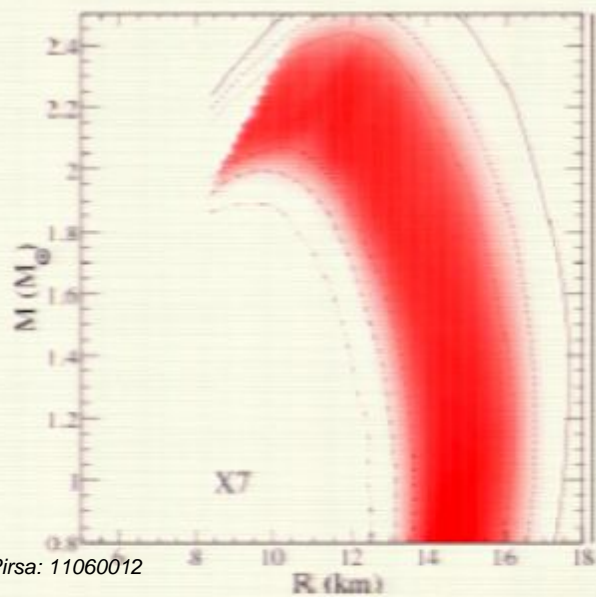
- ▶ Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- ▶ Best chances for accurate radii:
 - ▶ Nearby isolated neutron stars (parallax measurable)
 - ▶ Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)



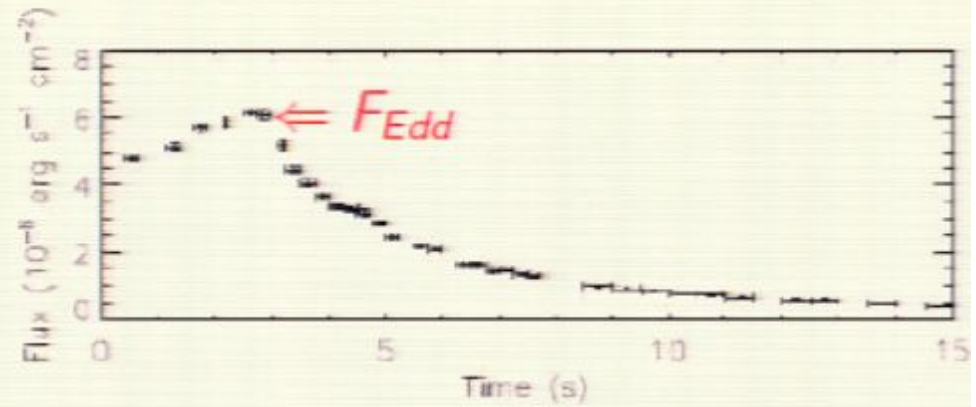
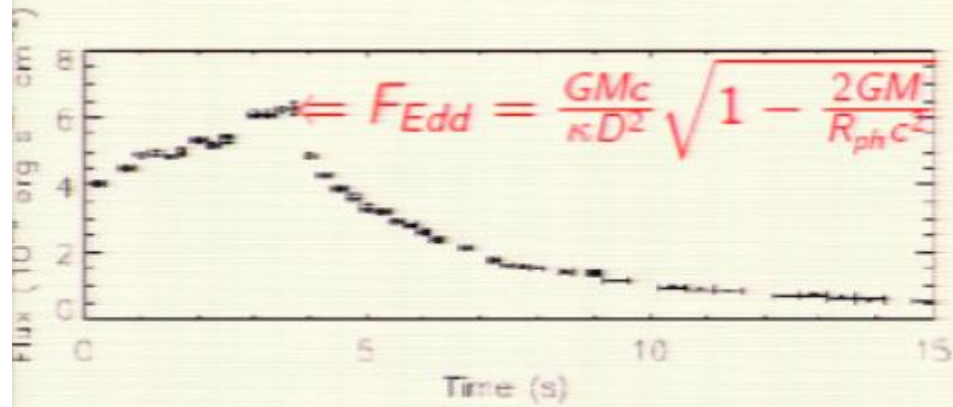
Inferred M-R Probability Estimates from Thermal Sources



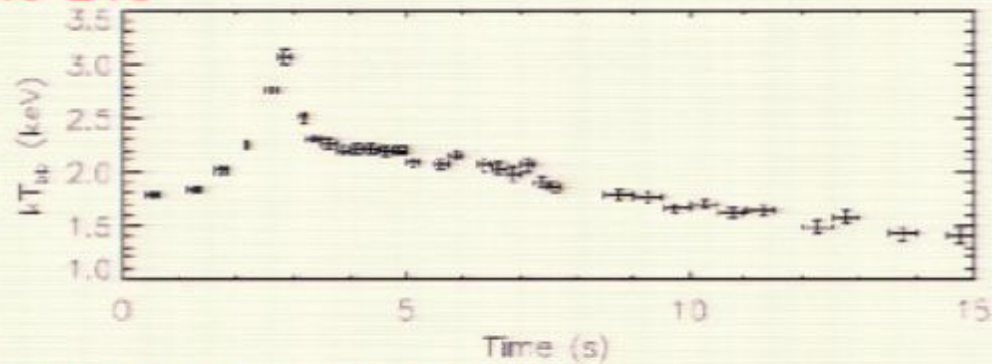
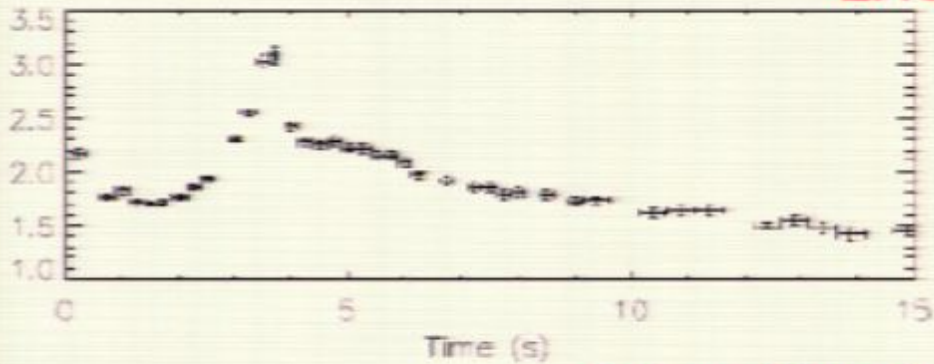
Steiner, Lattimer & Brown 2010



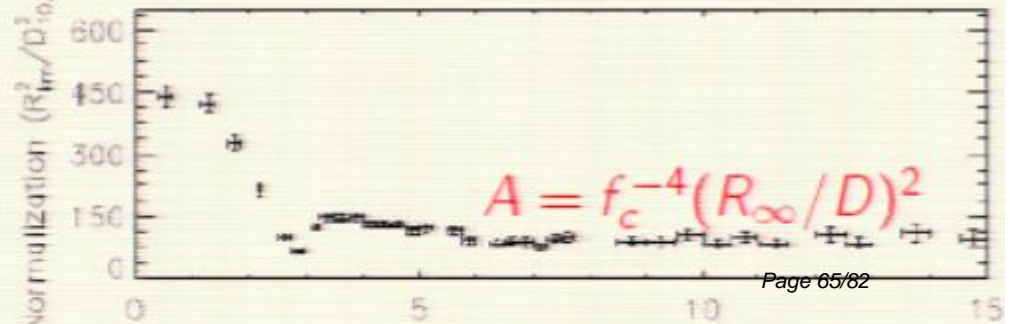
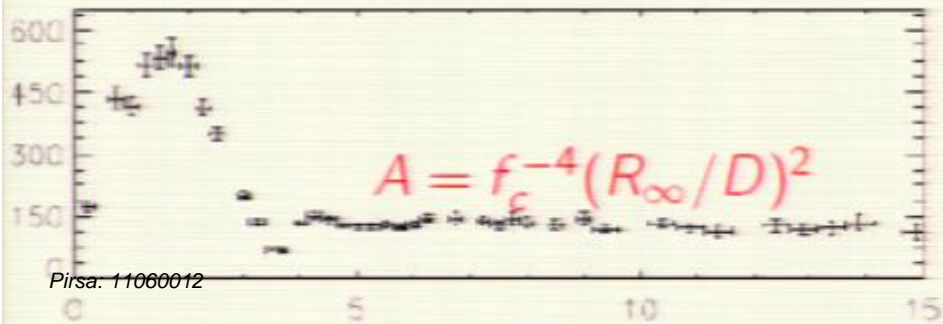
Photospheric Radius Expansion X-Ray Bursts



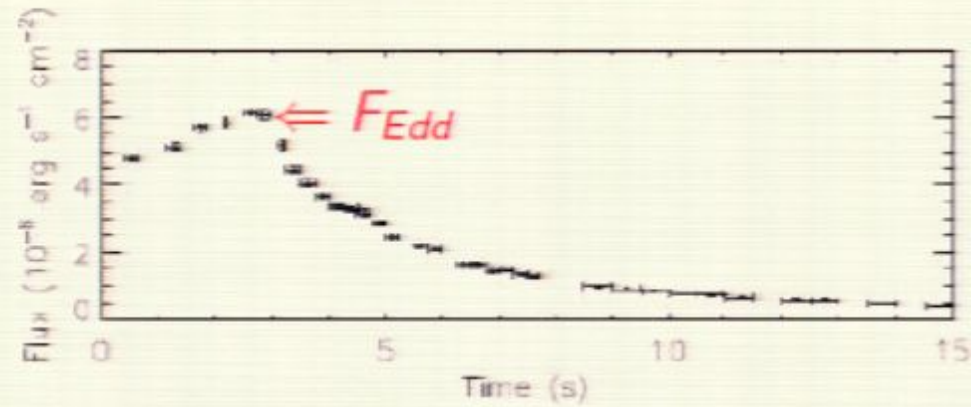
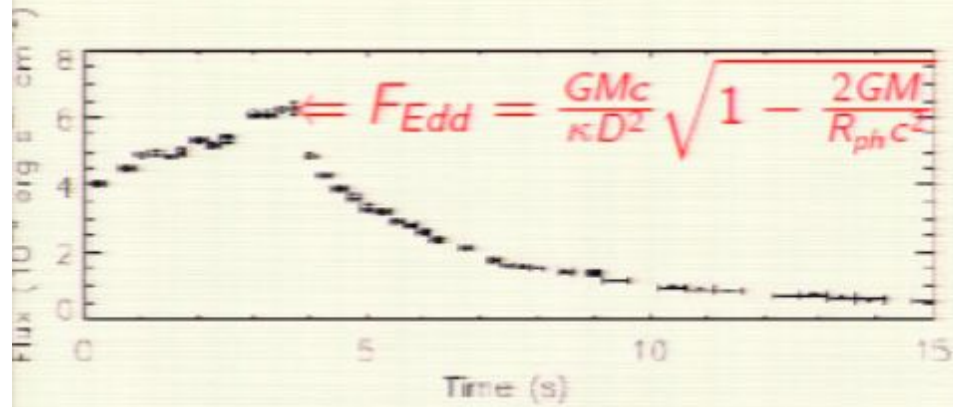
EXO 1745-248



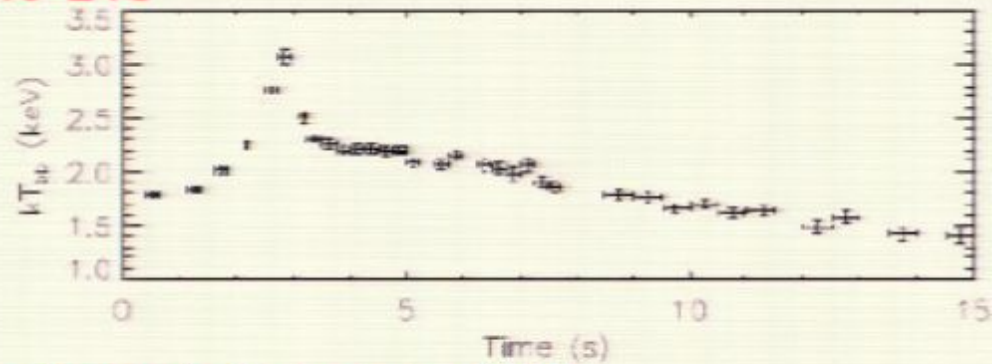
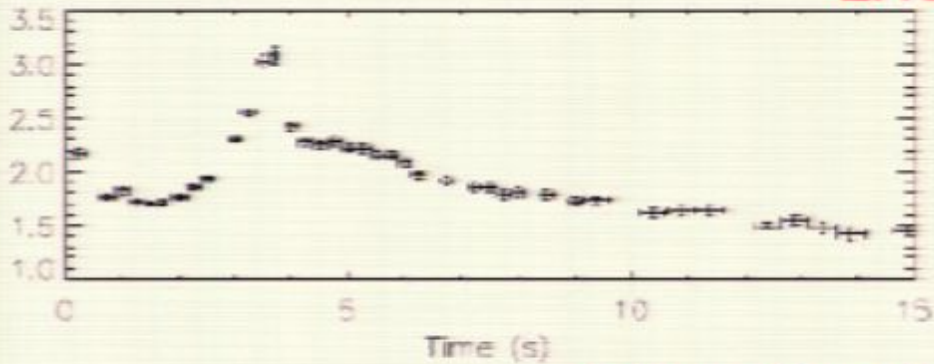
Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)



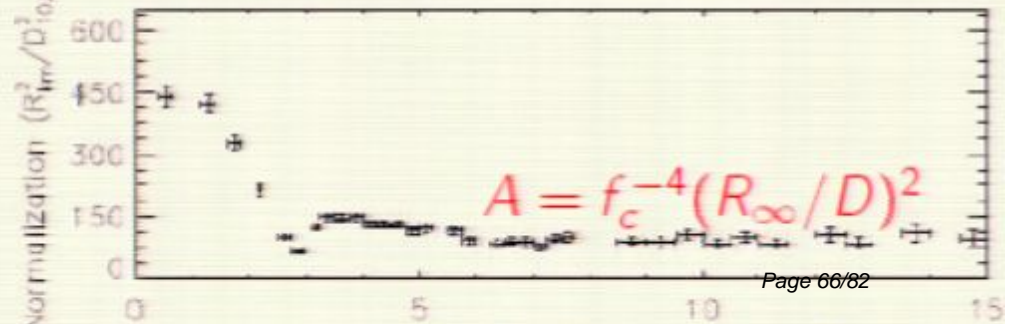
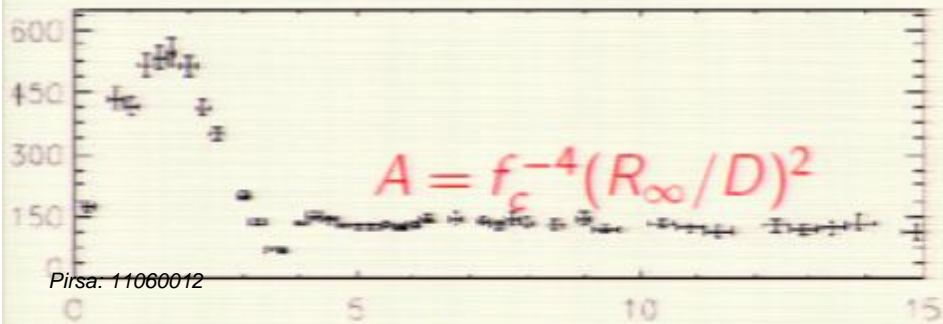
Photospheric Radius Expansion X-Ray Bursts



EXO 1745-248



Galloway, Muno, Hartman, Psaltis & Chakrabarty (2006)



Systematics with $R_{ph} = R$

$$F_{Edd} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2 \frac{GM}{R_{ph} c^2}} = \frac{GMc}{\kappa D^2} \sqrt{1 - 2\beta}$$

$$\kappa \simeq 0.2(1 + X) \text{ cm}^2 \text{ g}^{-1}$$

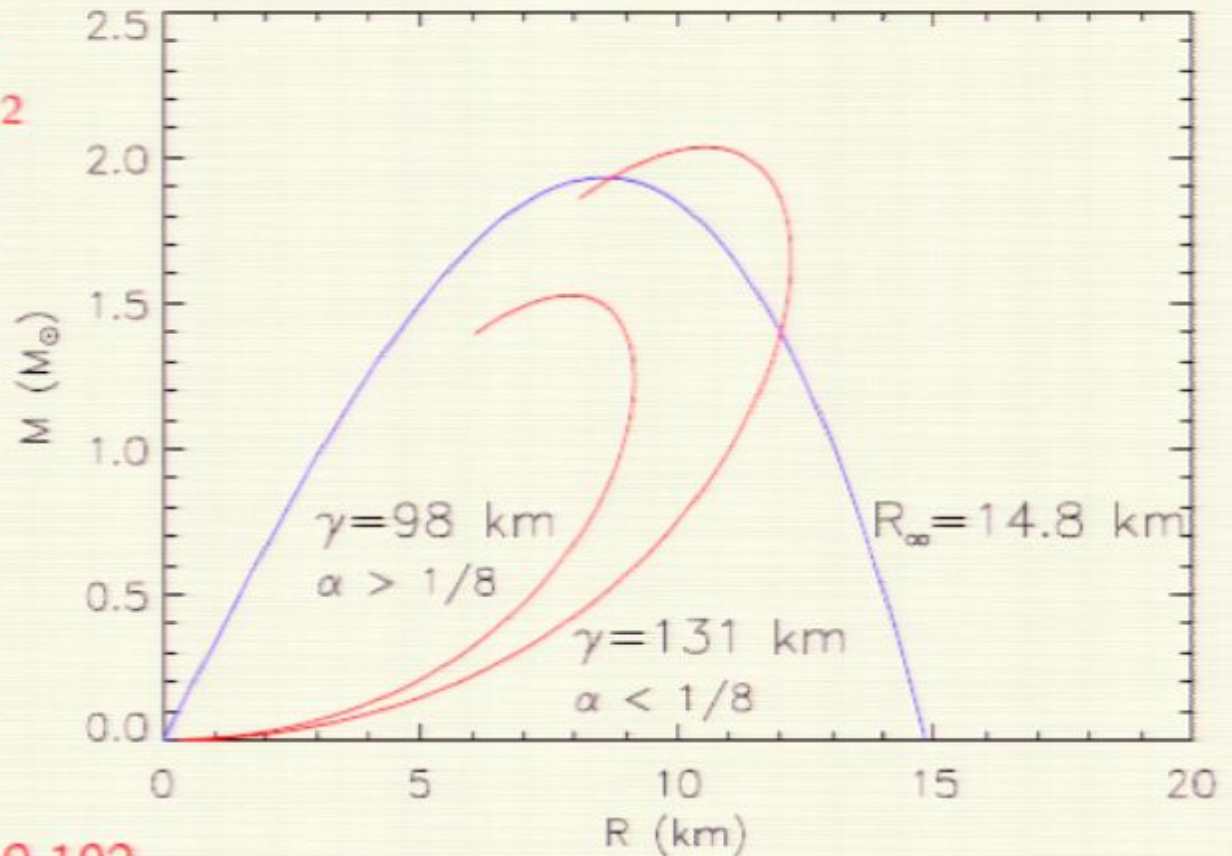
$$A = \frac{F_{\infty}}{\sigma T_{\infty}^4} = f_c^{-4} \left(\frac{R_{\infty}}{D} \right)^2$$

$$\alpha = \frac{F_{Edd} \kappa D}{\sqrt{A} c^3 f_c^2} = \beta(1 - 2\beta)$$

$$\gamma = \frac{Ac^3 f_c^4}{F_{Edd} \kappa} = \frac{R}{\beta(1 - 2\beta)^{3/2}}$$

$$\beta = \frac{1}{4} \pm \frac{1}{4} \sqrt{1 - 8\alpha}$$

$$R_{\infty} = \alpha \gamma$$



If $R_{ph} \gg R$, $\alpha < 1/\sqrt{27} \simeq 0.192$

M – R Probability Estimates from PRE Bursts

EXO 1745-248
 $\alpha = 0.14 \pm 0.01$

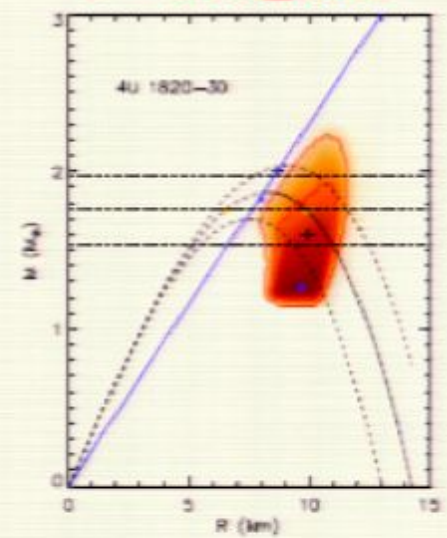
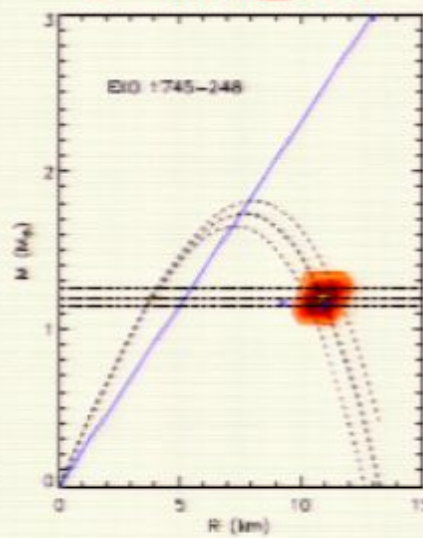
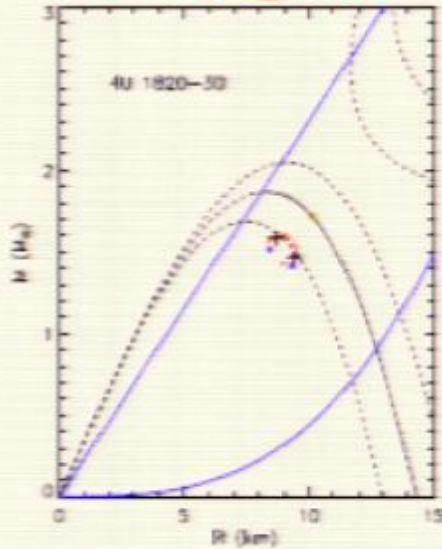
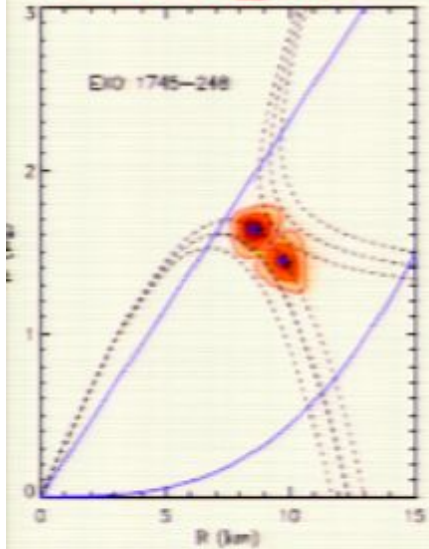
$R_{ph} = R$

4U 1820-30
 $\alpha = 0.18 \pm 0.02$

EXO 1745-248
 $\alpha = 0.14 \pm 0.01$

$R_{ph} > R$

4U 1820-30
 $\alpha = 0.18 \pm 0.02$

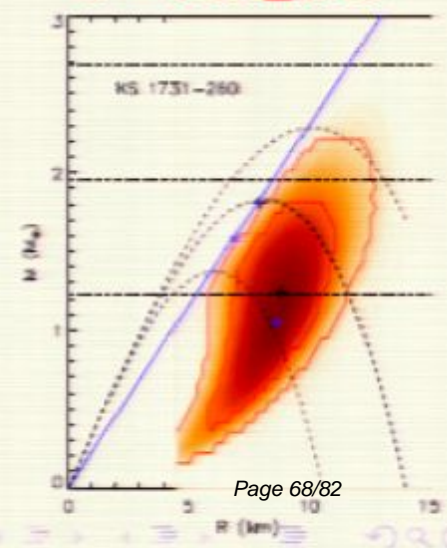
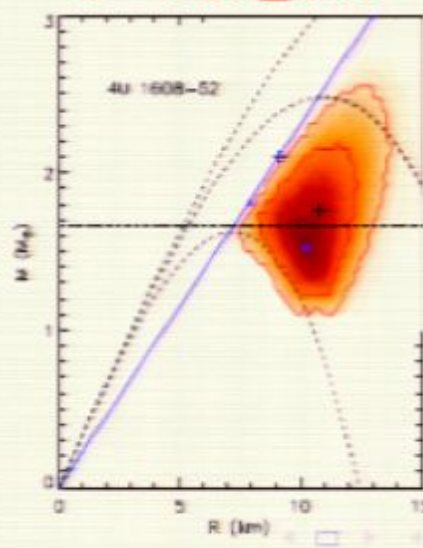
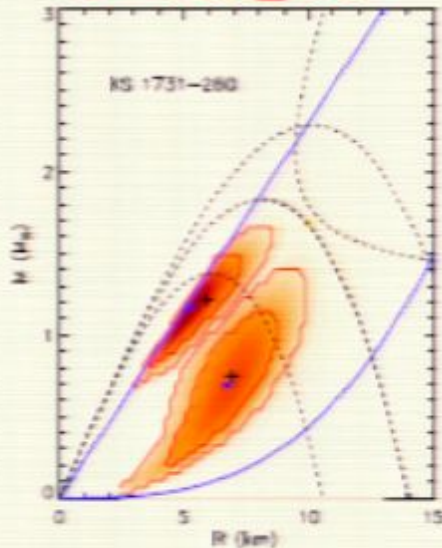
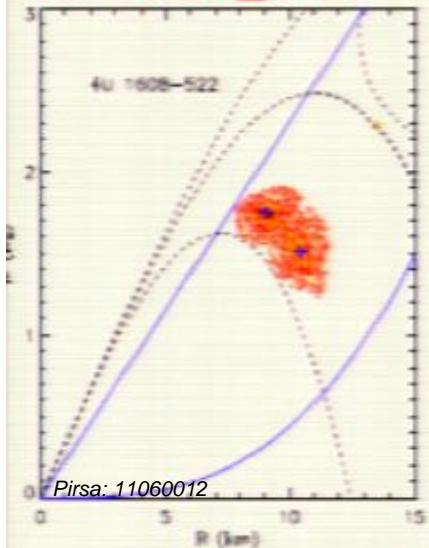


4U 1608-52
 $\alpha = 0.26 \pm 0.10$

Özel et al. 2009, 2010, 2011
 $\alpha = 0.21 \pm 0.06$

4U 1608-52 Steiner, Lattimer & Brown 2010, 2011
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M – R Probability Estimates from PRE Bursts

EXO 1745-248
 $\alpha = 0.14 \pm 0.01$

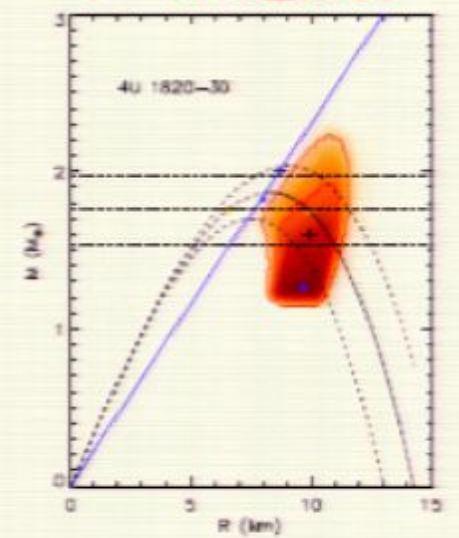
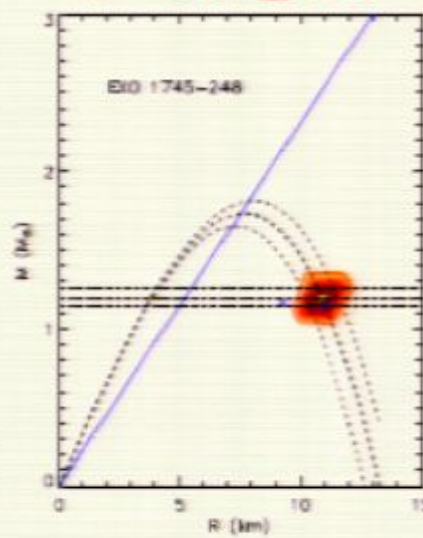
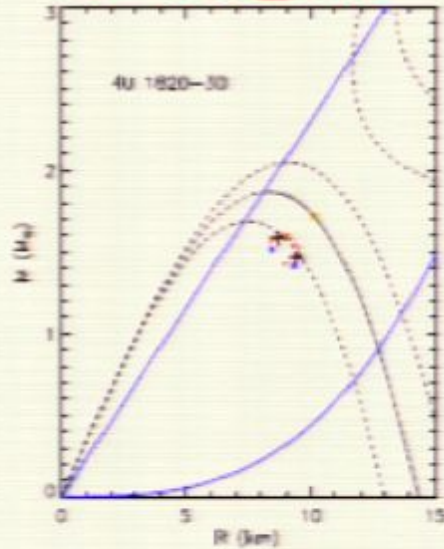
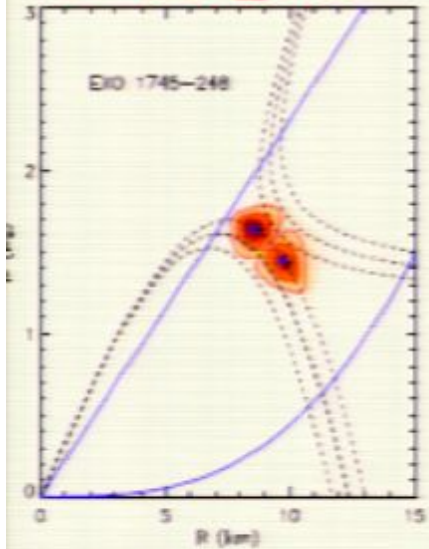
$R_{ph} = R$

4U 1820-30
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 $\alpha = 0.14 \pm 0.01$

$R_{ph} > R$

4U 1820-30
 $\alpha = 0.18 \pm 0.02$

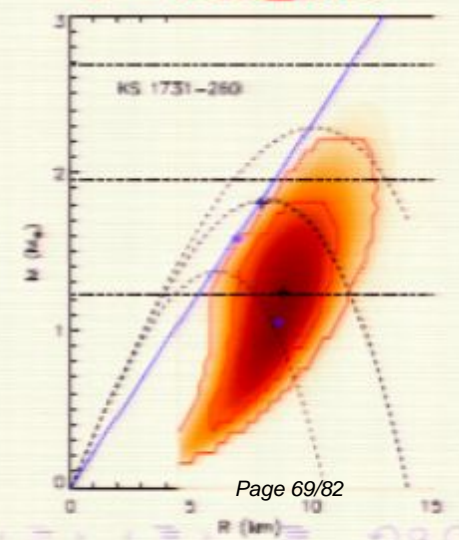
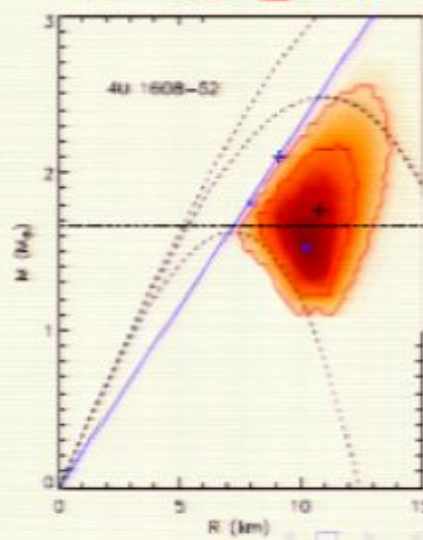
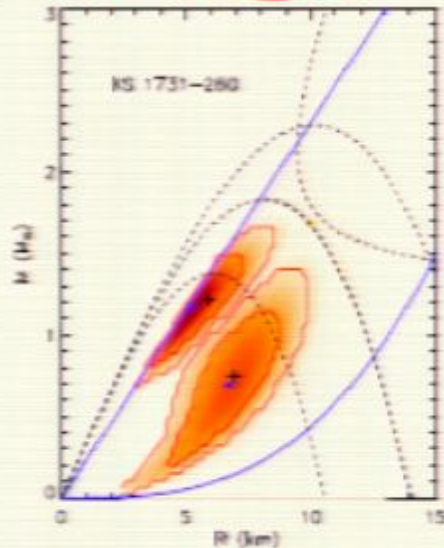
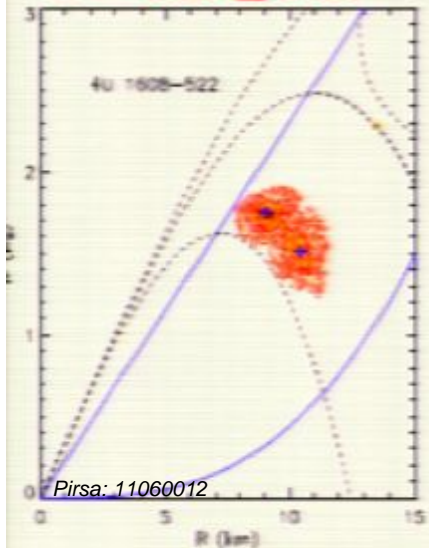


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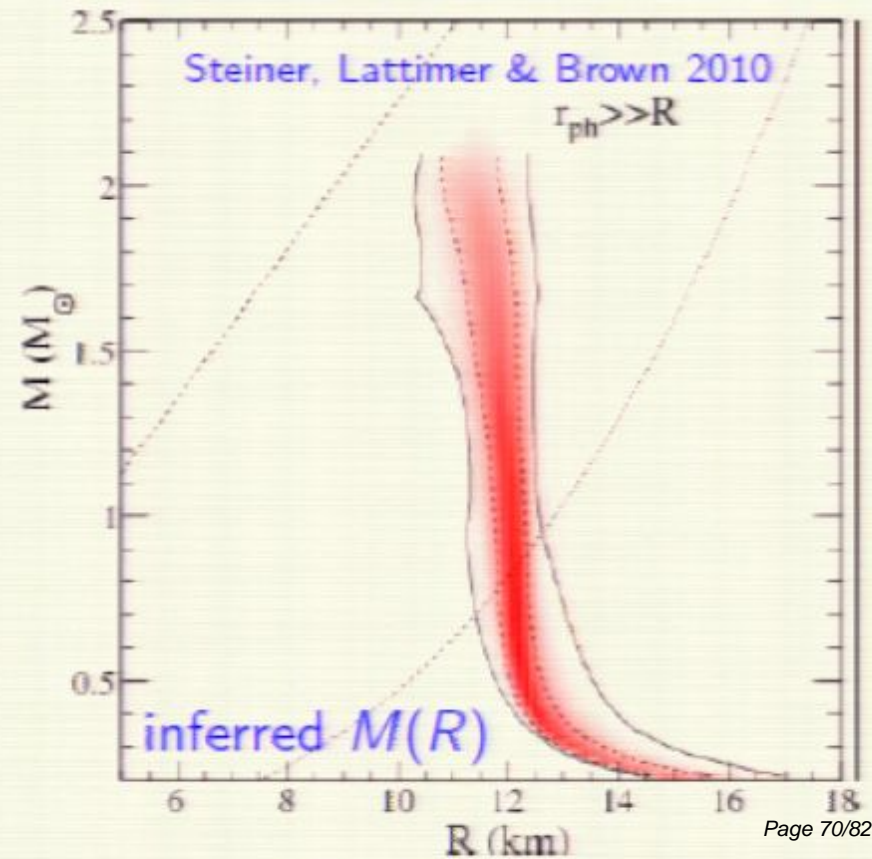
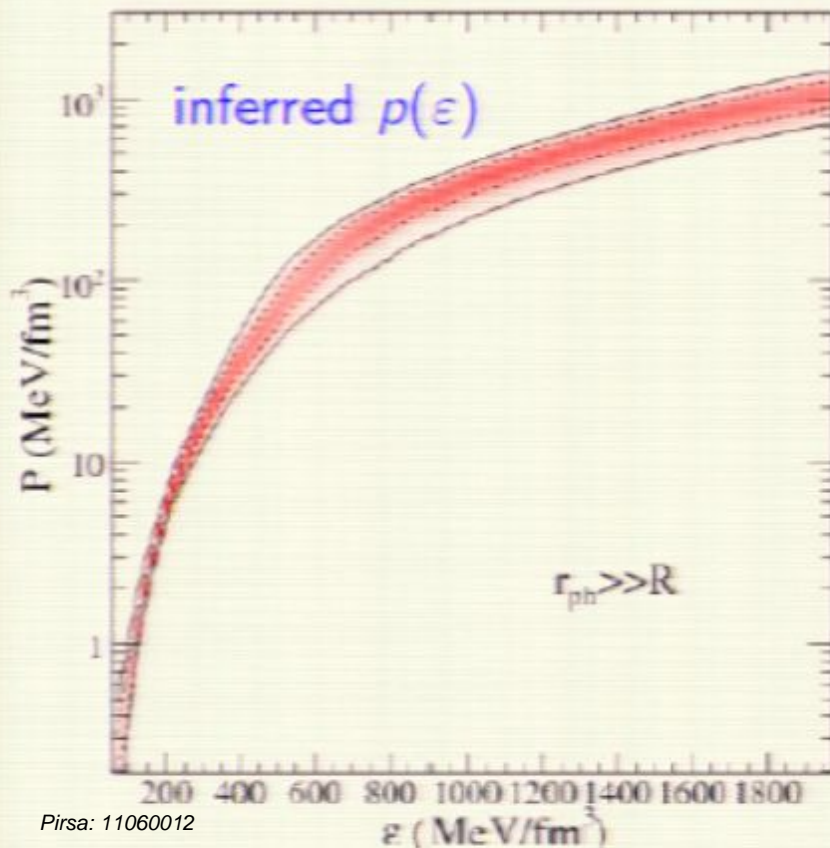
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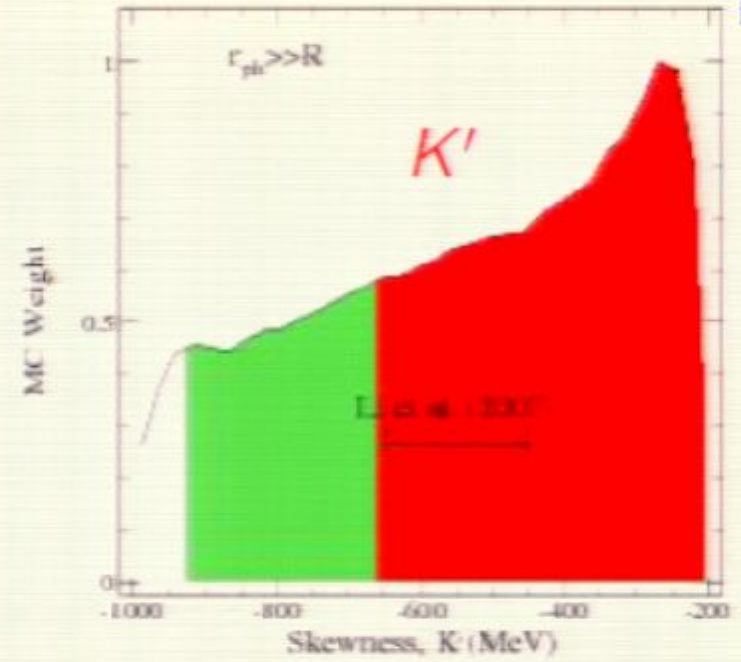
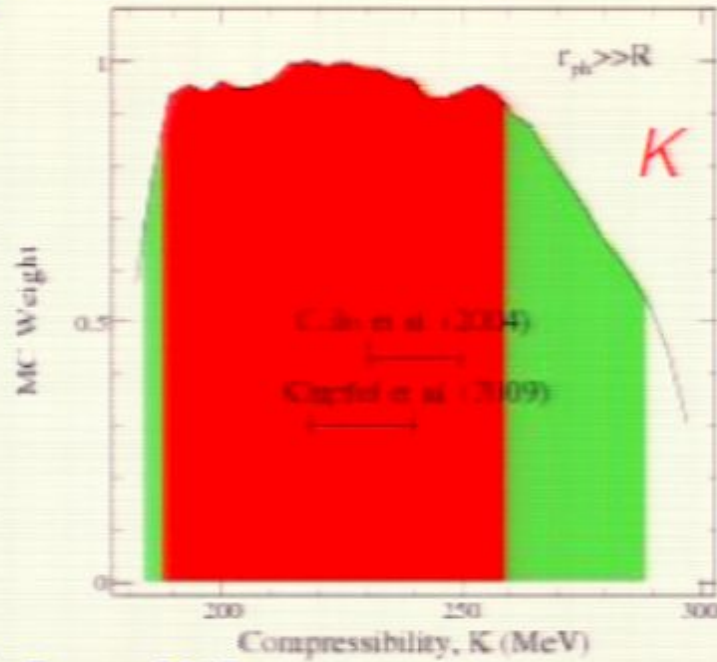


Bayesian TOV Inversion

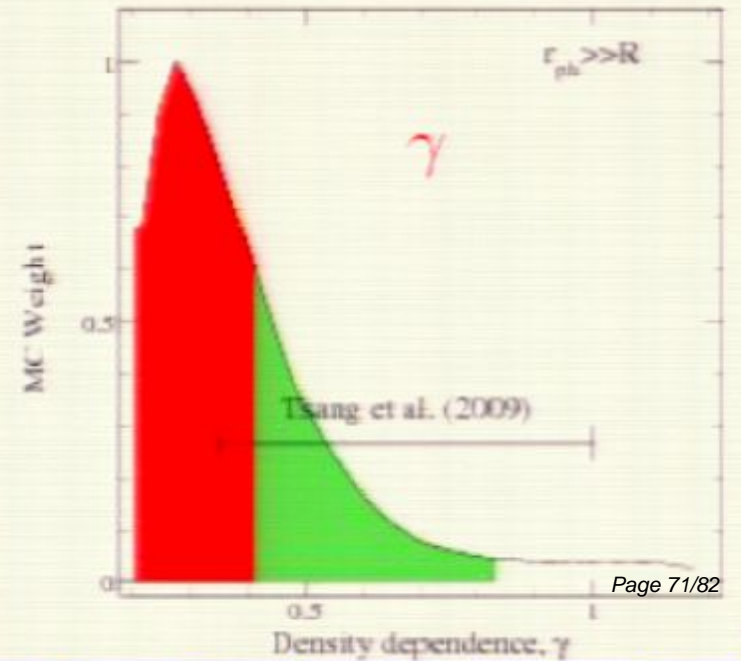
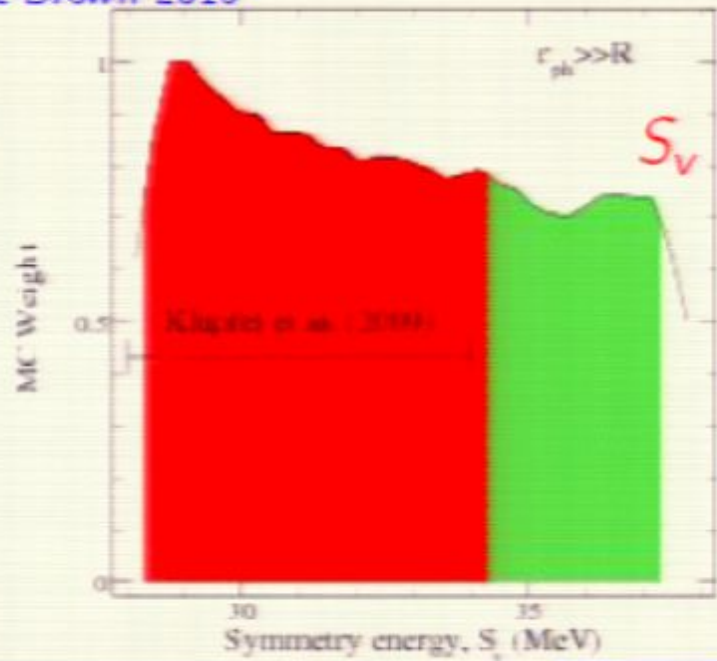
- ▶ $\varepsilon < 0.5\varepsilon_0$: Known crustal EOS
- ▶ $0.5\varepsilon_0 < \varepsilon < \varepsilon_1$: EOS parametrized by K, K', S_v, γ
- ▶ $\varepsilon_1 < \varepsilon < \varepsilon_2$: n_1 ; $\varepsilon > \varepsilon_2$: Polytropic EOS with n_2
- ▶ EOS parameters ($K, K', S_v, \gamma, \varepsilon_1, n_1, \varepsilon_2, n_2$) uniformly distributed
- ▶ M and R probability distributions for 7 neutron stars treated equally.



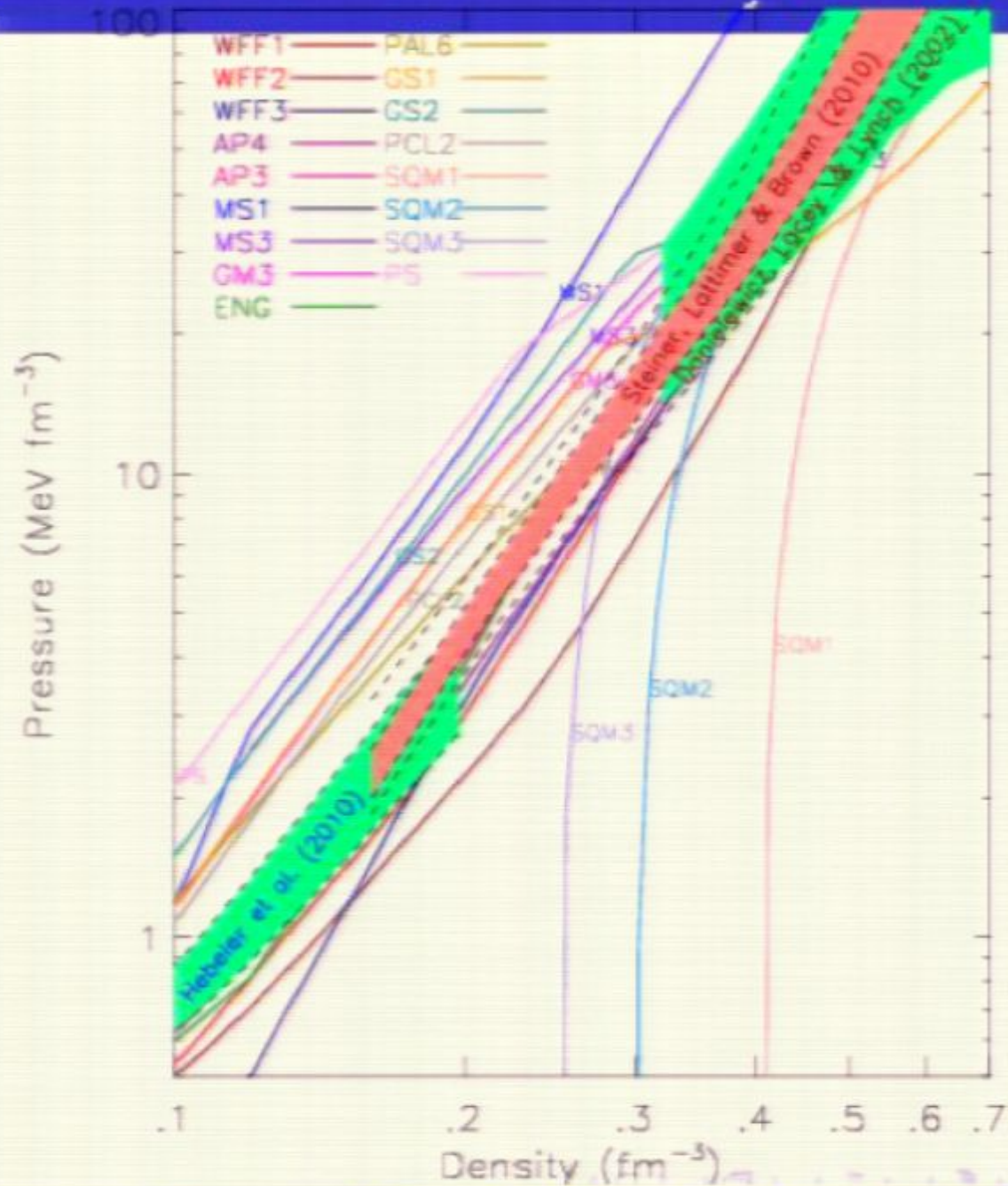
Inferred Model EOS Parameters



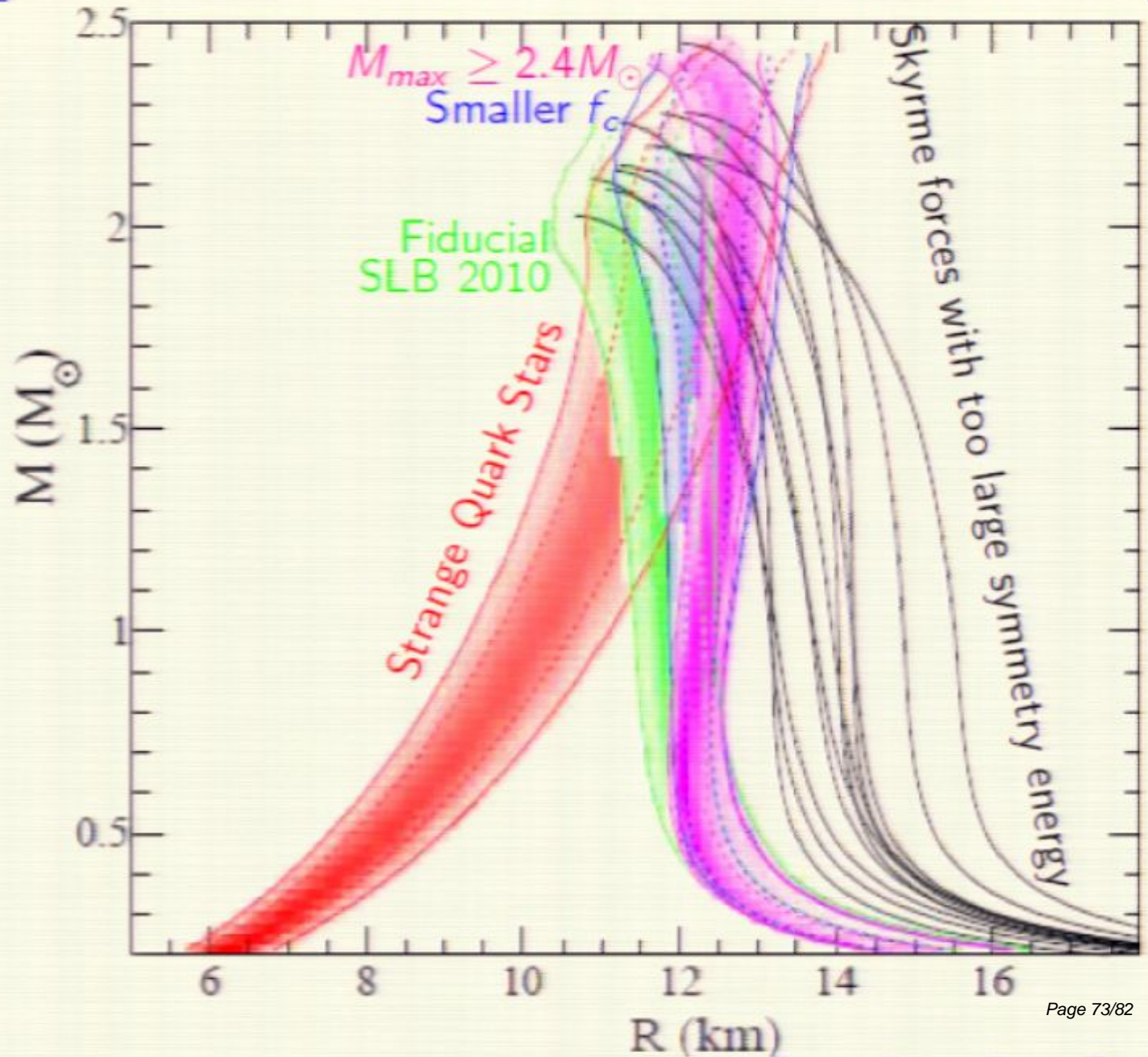
Steiner, Lattimer & Brown 2010



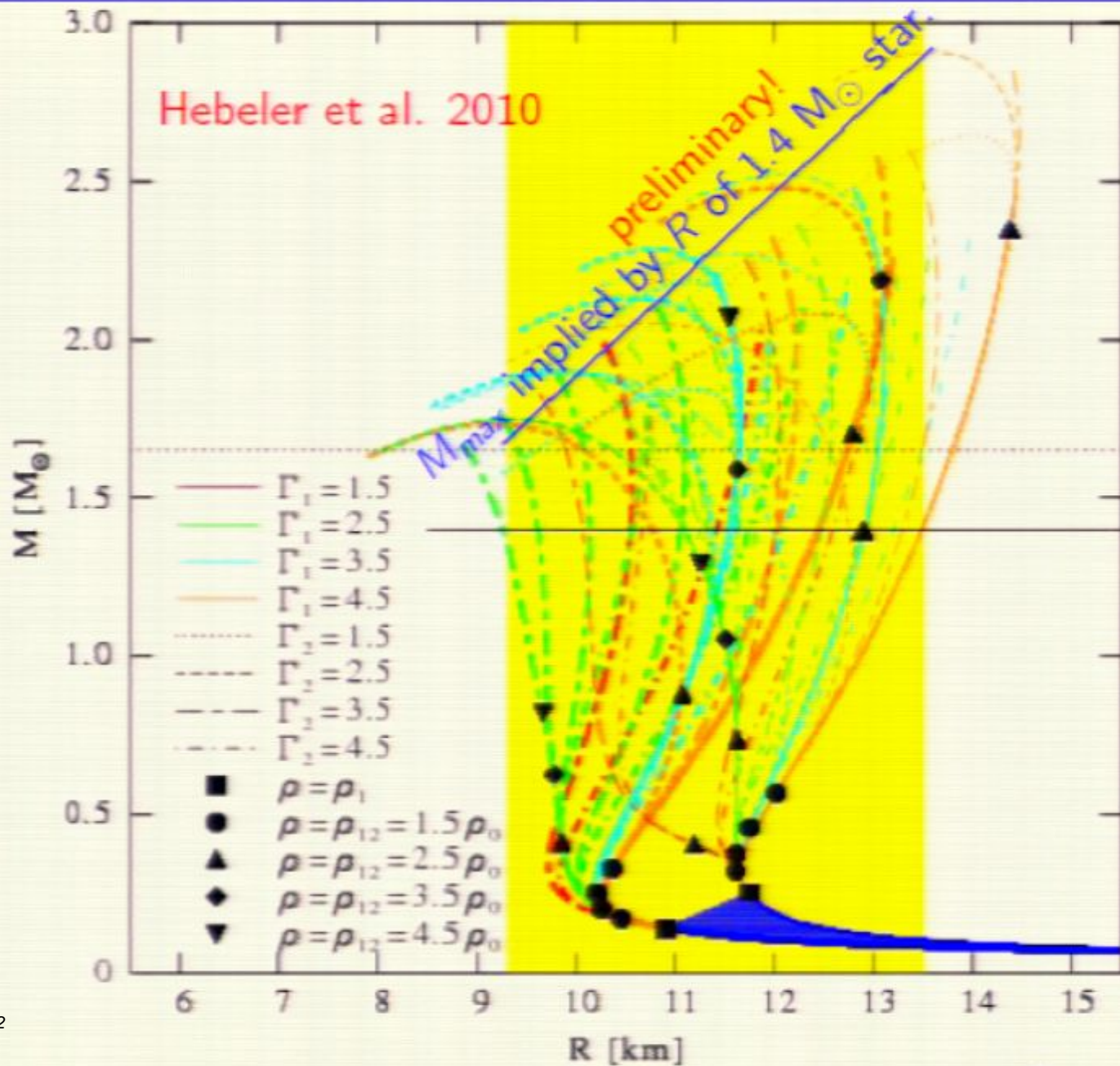
Consistency with Neutron Matter and Heavy-Ion Collisions



With More Extreme Assumptions



Radius and Maximum Mass Limits



Neutron Matter and the Symmetry Energy

- ▶ Fits to nuclear binding energies result in a strong, nearly linear, correlation between volume and surface symmetry energy coefficients of the liquid droplet model.
- ▶ This correlation is dependent on the nature of the liquid droplet model and how it treats the interaction between the Coulomb effects on the nuclear surface, and does not translate directly into a correlation between S_V and $L = 3(dS_V/d \ln n)_{n_\varepsilon}$.
- ▶ Finite nucleus models, such as Thomas-Fermi and Hartree or Hartree-Fock, for a particular nuclear interaction, can be fit to binding energies to obtain the correlation between S_V and L .
- ▶ Neutron matter studies (Hebeler & Schwenk; Carlson et al.) indicate that E_{sym} and $dE_{sym}/d \ln n)_{n_\varepsilon}$ are also correlated.
- ▶ Comparing these correlations could constrain the properties of the symmetry energy. It could be dependent on the nature of the nuclear interaction model, but this has not been thoroughly explored.

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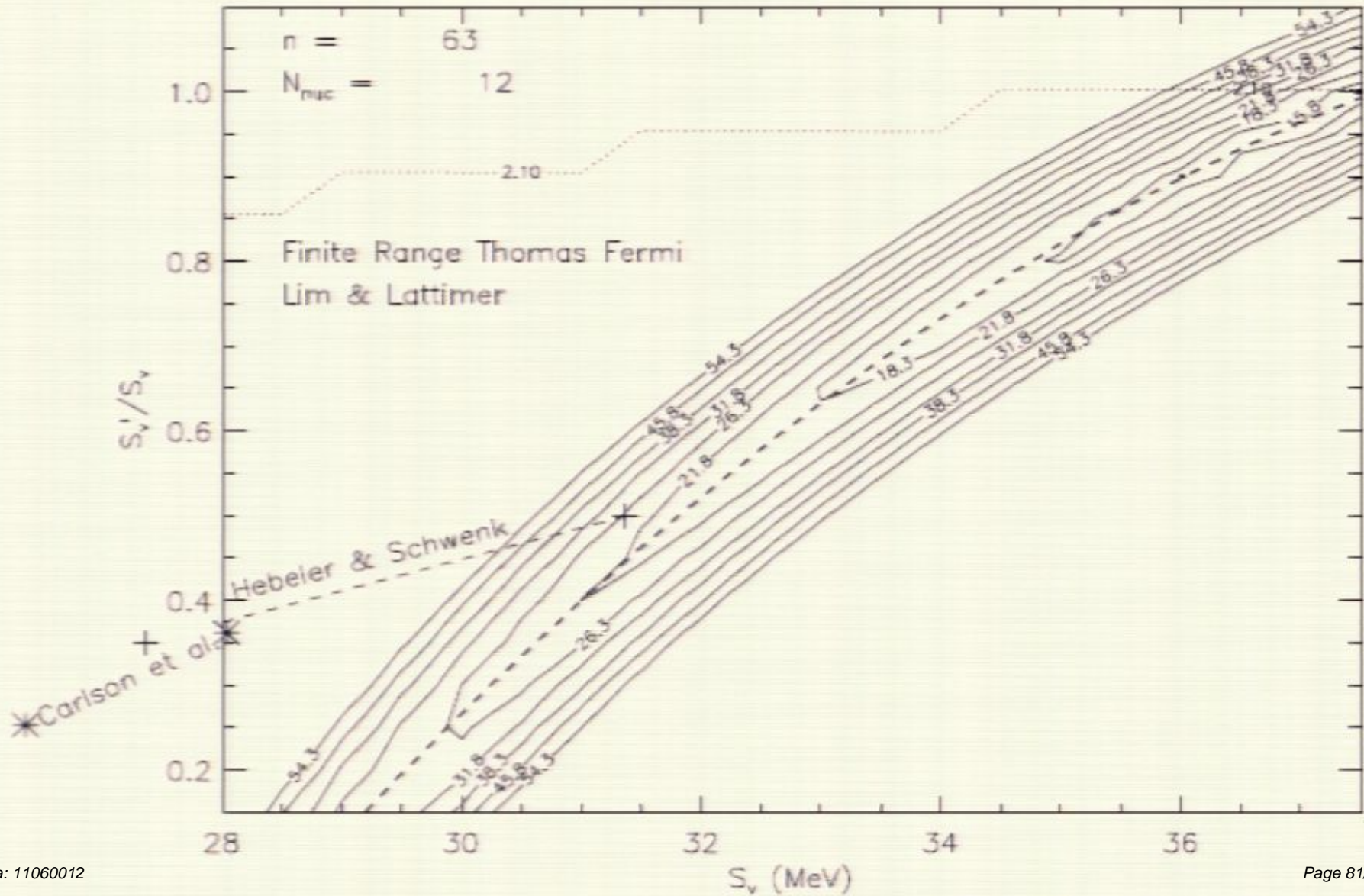
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Neutron Matter and Mass Fit Symmetry Correlations



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