

Title: Hints on integrability in the Holographic RG

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URL: <http://pirsa.org/11060010>

Abstract: The Polchinski equations for the Wilsonian renormalization group in the D -dimensional matrix scalar field theory can be written at large N in a Hamiltonian form. The Hamiltonian defines evolution along one extra holographic dimension (energy scale) and can be found exactly for the subsector of $\text{Tr}\phi^n$ (for all n) operators. We show that at low energies independently of the dimensionality D the Hamiltonian system in question reduces to the *integrable* effective theory. The obtained Hamiltonian system describes large wavelength KdV type (Burger--Hopf) equation with an external potential and is related to the effective theory obtained by Das and Jevicki for the matrix quantum mechanics.

CurXiv. 1006.1970

Musaev & Gahramanov & E.A

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arXiv:1006.1970

Musaev & Gaihramanov & E.A

$$\left\langle e^{i \sum_{in_i} \int g_{in_i}(x) G_{in_i} d^D x} \right\rangle \stackrel{N \rightarrow \infty}{\lambda \rightarrow \infty} \approx e^{i I_{\min}(g_n)_{\text{AdS}_{D+1}}}$$

arXiv:1006.1970

Musaev & Gahramanov & E.A

$$\langle e^{i \sum_{m=1}^n \int g_m(x) \mathcal{G}_{m_i} d^D x} \rangle_{T_{\mu\nu}} \approx e^{i I_{\min}(y_n)_{\text{AdS}_{D+1}}}$$

$$ds^2 = \frac{1}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$z \rightarrow 0 \quad z = z_0 \rightarrow 0$$

arXiv:1006.1970

Musaev & Gaihramanov & E.A

$$\left\langle e^{i \sum_{m1} \int g_{m1}(x) G_{m1} d^D x} \right\rangle \stackrel{N \rightarrow \infty}{\lambda \rightarrow \infty} \approx e^{i I_{\min}(g_{m1}(x, z_0))_{\text{AdS}_{D+1}}}$$

T_A

$$ds^2 = \frac{1}{z^2} (dz^2 + dx_j dx^j)$$

$$z \rightarrow 0 \quad z = z_0 \sim 0$$

arXiv:1006.1970

Musaev & Gahramanov & E.A

$$\langle e^{i \sum_{n=1}^N \int g_n(x) \mathcal{O}_n d^D x} \rangle$$

$T_{\mu\nu}$

$$\approx e^{i I_{\text{min}}(g_n(x, z_0))_{\text{AdS}_{D+1}}}$$

$$ds^2 = \frac{1}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$z \rightarrow 0 \quad z = z_0 \sim \infty$$

$$\int_{z_0}^{\infty} dz \mathcal{L}_1(g_n(z, \cdot))$$

arXiv:1006.1970

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$$\langle e^{i \sum_{in} \int g_{in}(x) G_{in} d^D x} \rangle_{T_{\mu\nu}}$$

$N \rightarrow \infty$
 $\lambda \rightarrow \infty$

$$\approx e^{i I_{\min}(g_{in}(x, z_0))_{AdS_{D+1}}}$$

$$\int_{z_0}^{\infty} dz \mathcal{L}(g_{in}(z))$$

$$ds^2 = \frac{1}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$z \rightarrow 0 \quad z = z_0 \sim \infty$$

arXiv:1006.1970

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$$\frac{\Lambda^2}{2\kappa} \langle e^{i \sum_{in} \int g_{in}(x) \mathcal{O}_{in} d^D x} \rangle_{T_{\mu\nu}} \Big|_{\lambda \rightarrow \infty}$$

$$\int_{z_0}^{\infty} dz \mathcal{L}(g_n(z))$$

$$e^{i \text{I}_{\text{min}}(g_n(x, z_0))}_{\text{AdS}_{D+1}}$$

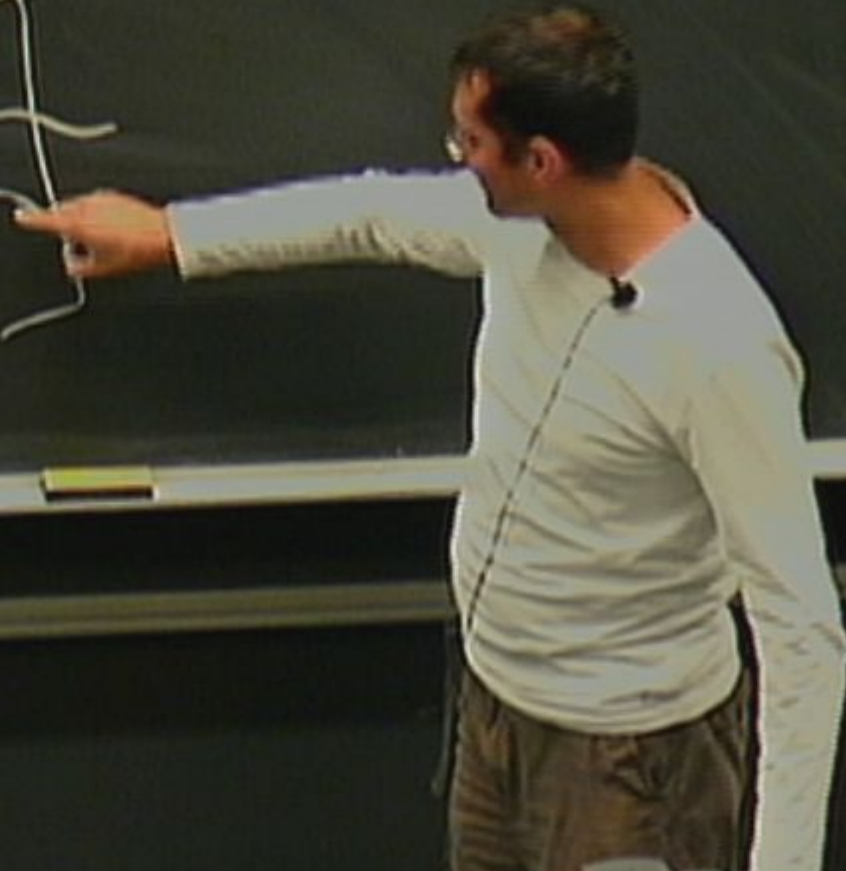
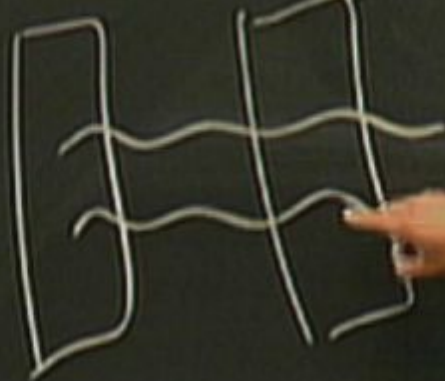
$$ds^2 = \frac{1}{z^2} (dz^2 + dx_\mu dx^\mu)$$

$$z \rightarrow 0 \quad z = z_0 \sim 0$$

$$\dot{g}_{m3} = \beta_{m3}(\{g\})$$

$\Lambda \rightarrow \infty$

$\log \Lambda$



Hermitian matrix



Hermitian. matrix field theory

$$S_0 = \int d^D x \text{Tr} \left[(\partial_\mu \Phi)^2 + m^2 \Phi^2 \right]$$

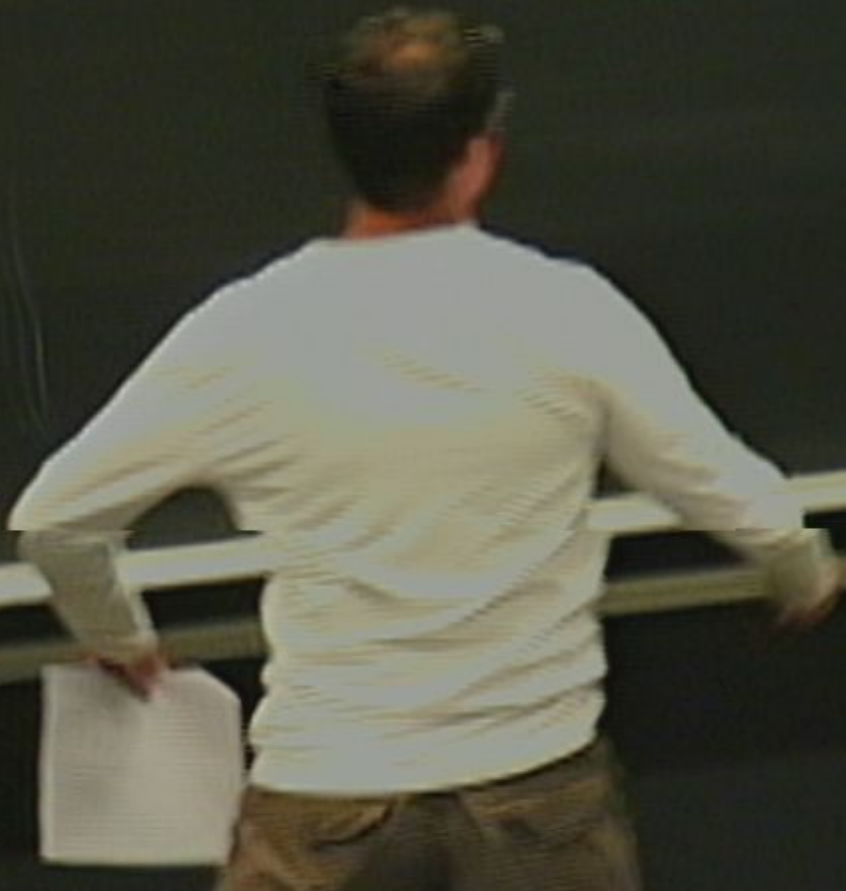
Hermitian matrix field theory

$$S = \int d^D x \text{Tr} \left[(\partial_\mu \Phi)^2 + m^2 \Phi^2 + \lambda \Phi^4 \right] + i \int d^D x g_{\mu\nu} \Theta_{\mu\nu}$$

$$\Theta_{\mu\nu} = \text{Tr} \left[\partial_\mu \Phi \partial_\nu \Phi - \eta_{\mu\nu} (\dots) \right]$$

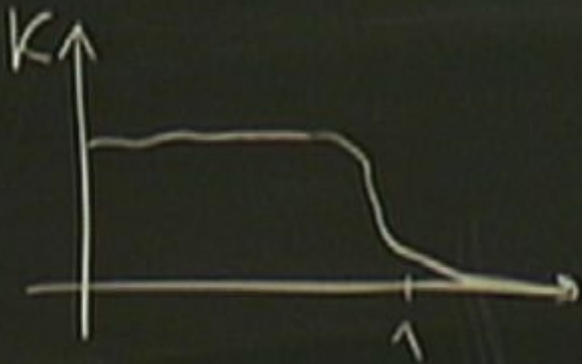
$$e^{\text{I}_{\text{ess}}(g)} = \int \mathcal{D}\Phi e^{-S}$$

$$S = \int d^D p \text{Tr} \left\{ \Phi(p) [i \not{p} + m] \Phi(-p) + \prod_{i=1}^4 \Phi_{p_i} \delta(p_1 \dots p_4) \right\}$$



$$S = \int d^D p \text{Tr} \left\{ \Phi(p) \left[p^2 + m^2 \right] \Phi(-p) + \left(\frac{1}{\Lambda} \right)^2 \Phi(p_1) \Phi(p_2) \Phi(p_3) \Phi(p_4) \delta(p_1 \dots p_4) + \dots \right.$$

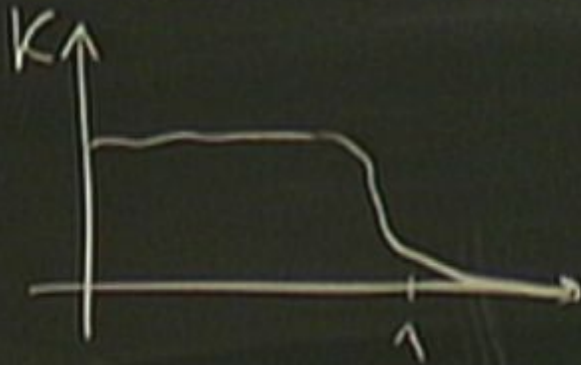
\uparrow
 $K^{-1}(p/\Lambda^2)$



$$S_{\Lambda} = \int d^D p \text{Tr} \left\{ \Phi(p) \left[p^2 + m^2 \right] \Phi(-p) + \lambda \left(\prod_{i=1}^n \Phi_{p_i} \right) \delta(p_1 \dots p_n) + \dots \right\}$$

\uparrow
 $K^{-1}(p/\Lambda)$

\rightarrow



$$12_1 I = 0$$

$$\delta \mathcal{L} = 0$$

$$\delta S_I = -\frac{1}{2} \int \frac{d^D p}{p^2 + m^2} \delta \mathcal{L}(p) \times$$

$$\left[\frac{1}{2} \frac{\delta^2 S_I}{\delta \phi^i(-p) \delta \phi^j(p)} + \frac{\delta S_I}{\delta \phi^i(-p)} \frac{\delta S_I}{\delta \phi^j(p)} + \dots \right]$$

$$\delta \mathcal{L} = 0$$

$$\delta S_I = -\frac{1}{2} \int \frac{d^D p}{p^2 + m^2} \delta \mathcal{L}(p)$$

$$\left[\frac{1}{N} \frac{\delta^2 S_I}{\delta \phi^i(-p) \delta \phi^j(p)} + \frac{\delta S_I}{\delta \phi^i(-p)} \frac{\delta S_I}{\delta \phi^j(p)} + \dots \right]$$

$$\int \sum_{\gamma_n} \left[\sum_{\alpha_n} \alpha_n \mathcal{O}_n + \sum_{n,m} \gamma_{nm} \mathcal{O}_n \mathcal{O}_m \right]$$

$$\langle \sum_{n=1}^{\Lambda} \dot{g}_n \mathcal{O}_n \rangle = \left\langle \sum_{n=1}^{\Lambda} \alpha_n \mathcal{O}_n + \sum_{n,m} \gamma_{nm} \underbrace{\mathcal{O}_n \mathcal{O}_m}_{C_{nm}^k \mathcal{O}_k} \right\rangle$$

$\Phi(p)$

$\Lambda \leq p \leq \Delta$

$$\dot{g}_n = \beta_n(g)$$

$$\langle \sum_{n \in \mathbb{Z}} \dot{g}_n \mathcal{O}_n \rangle = \left\langle \sum_{n \in \mathbb{Z}} \alpha_n \mathcal{O}_n + \sum_{n, m} \gamma_{nm} \underbrace{\mathcal{O}_n \mathcal{O}_m}_{C_{nm}^k \mathcal{O}_k} \right\rangle$$

$\Phi(p)$

$\mathcal{J} \leq p \leq \Delta$

$$\dot{g}_n = \beta_n(g)$$

Hermitian matrix field theory

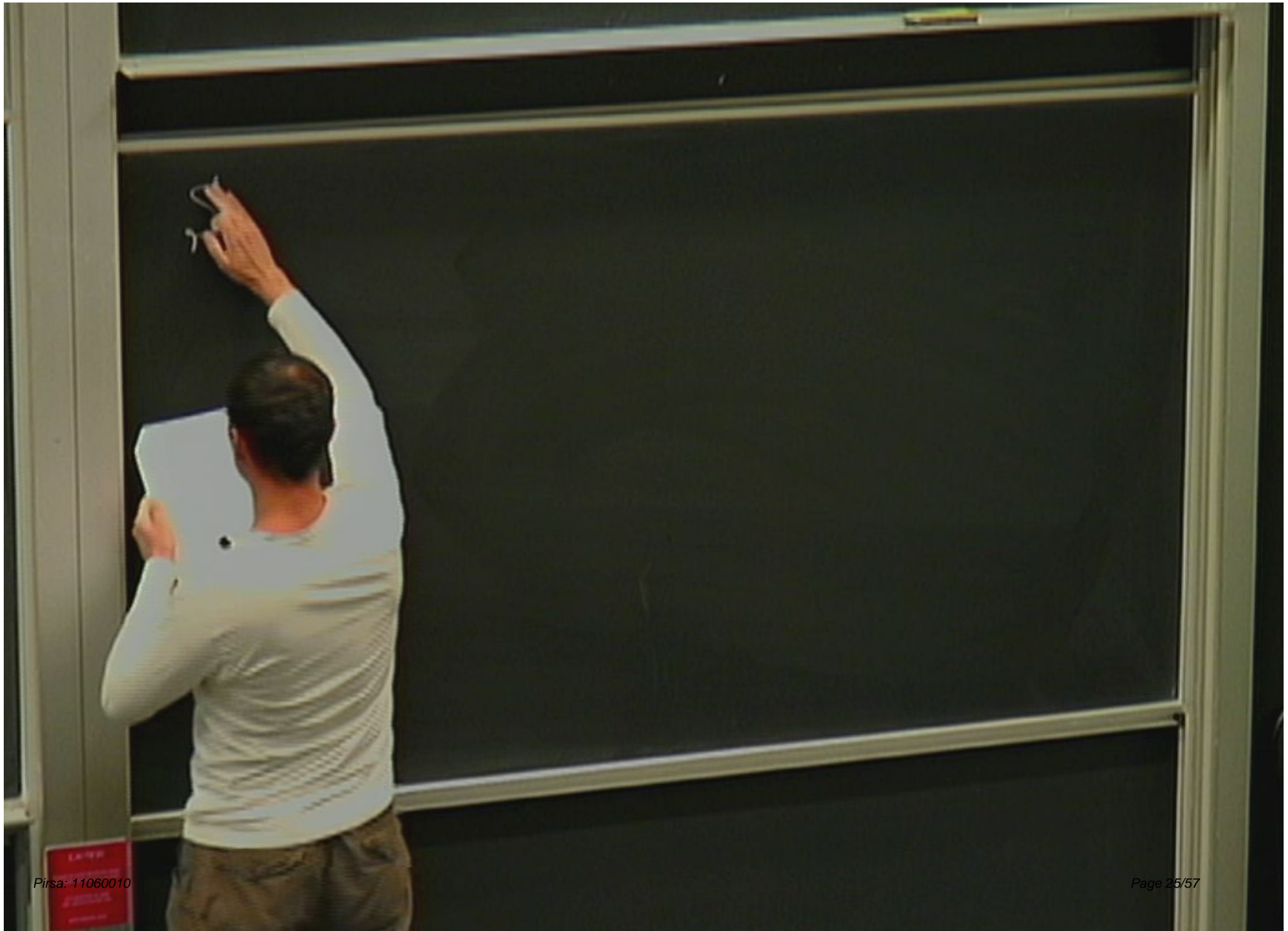
$$S = \int d^D x \operatorname{Tr} \left[(\partial_\mu \Phi)^2 + m^2 \Phi^2 \right] + \sum_n \int d^D x g_n(x) \operatorname{Tr} \Phi^n(x)$$

$$S_{\Lambda} = i \int d^D p \operatorname{Tr} \left\{ \Phi(p) \left[p^2 + m^2 \right] K^{-1} \left(\frac{p}{\Lambda} \right) \Phi(-p) \right\} +$$

$$+ i \sum_{n=0}^{\infty} \int d^D k_1 \dots d^D k_n \operatorname{Tr} \left[\Phi(k_1) \dots \Phi(k_n) \right] g_n(-k_1, \dots, -k_n)$$

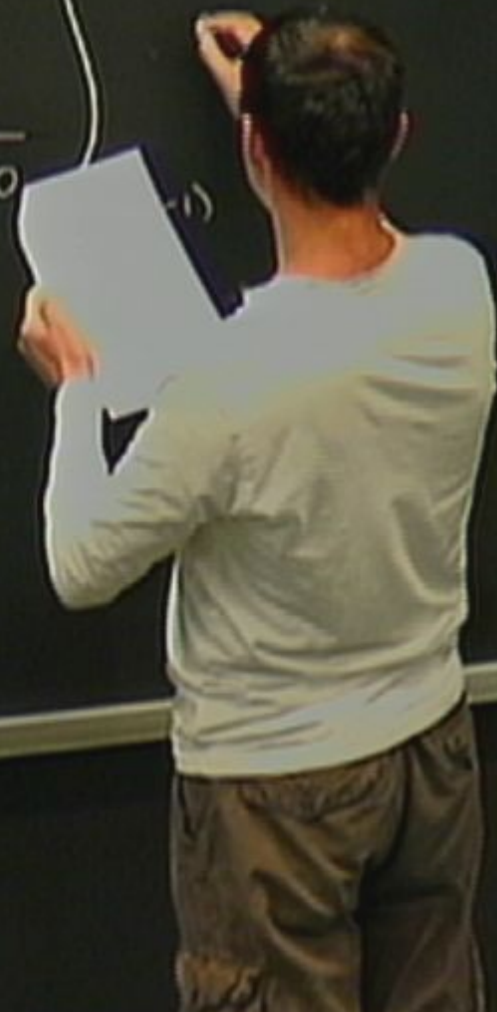
$$S_n = N \int d^D p \operatorname{Tr} \left\{ \Phi(p) [p^2 + m^2] K^{-1}(p/\Lambda) \Phi(-p) \right\} +$$

$$+ N \sum_{n=0}^{\infty} \int d^D \kappa_1 \dots d^D \kappa_n \operatorname{Tr} \left[\Phi(\kappa_1) \dots \Phi(\kappa_n) \right] \frac{g_n(-\kappa_1, \dots, -\kappa_n)}{g_n(\kappa_1, \dots, \kappa_n)}$$



$$\sum_{e=1}^{\infty} \int_{\kappa(e)} T_e(\{\kappa_e\}) g_e(-\kappa_e) =$$

$$= -\frac{1}{2} \int_p \frac{\dot{\kappa}(p/\Lambda^2)}{p^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]$$



$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} T_e(\{\kappa_e\}) g_e(-\kappa_e) = \\
& = -\frac{1}{P} \int_P \frac{k(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \quad \left(\sum_{s=1}^{m-1} \kappa_{m-s-1} \right) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)})^+
\end{aligned}$$

$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} T_e(\{\kappa_e\}) g_e(-\kappa_e) = \\
& = -\frac{1}{2} \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \quad \times T_{m-s-1}(\{\kappa_{m-s-1}\}) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)}) + \\
& \quad + \sum_{e,j=1}^{\infty} \int_{\kappa(e-1)\kappa(j-1)} (e,j) T_{e,j+2}(\{\kappa_{e-1}\}, \{\kappa_{j-1}\}) \times g_e(-\kappa_{(e+1)}) g_j(-\kappa_{(j+1)})
\end{aligned}$$

Hermitian matrix field theory

$$S = \int d^D x \text{Tr} \left[(\partial_\mu \Phi)^2 + m^2 \Phi^2 \right] + \sum_n \int d^D x g_n(x) \text{Tr} \Phi^n(x)$$

$$e^{\mathcal{I}(g)} \rightarrow e^{\mathcal{J}(\tau)}$$

$$\mathcal{J}(\tau) = \left(\mathcal{I}(g) - \int g \tau \right) \quad \boxed{\tau = \frac{\partial \mathcal{I}}{\partial g}}$$

Hermitian matrix field theory

$$S_g = \int d^D x \text{Tr} \left[(\partial_\mu \Phi)^2 + m^2 \Phi^2 \right] + \sum_n \int d^D x g_n(x) \text{Tr} \Phi^n(x)$$

$$e^{\mathcal{I}(g)} \rightarrow e^{\mathcal{J}(T)}$$

$$p = \frac{\partial S}{\partial q}$$

$$\mathcal{J}(T) = \left(\mathcal{I}(g) - \int g T \right)$$

$$T = \langle \text{Tr} \Phi \cdot \Phi \rangle = \text{Tr} \Phi^2$$

$$T = \frac{\partial \mathcal{I}}{\partial g}$$

$$+iN \sum_{\nu=0}^{\infty} \int d^D k_1 \dots d^D k_n \text{Tr} [\Phi(k_1) \dots \Phi(k_n)] g_n(-k_1, \dots, -k_n)$$

$$\int \mathcal{D}\Phi e^{-S_0 + \sum \left(g_n(\Sigma k) \text{Tr} [\Phi \dots \Phi] \right)}$$

$$\sum_{l,j=1}^n \int_{\substack{g_{l-1} \leq k_{l-1} \\ g_l \leq k_l}} (l_j) T_{l,j+2}(\{k_{l-1}\}, \{g_{l-1}\}) * g_l(-k_{l-1}, \dots) \delta(k_{l-1})$$

$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} \left(T_e(\{\kappa_e\}) \right) g_e(-\kappa_e) = \\
& = - \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \quad \left(\{\kappa_{m-s-1}\} \right) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)}) + \\
& \quad (e_j) T_{e_j+2}(\{\kappa_{e-1}\}, \{\eta_{e-1}\}) * g_e(-\kappa_{e+P}) \delta(\eta_{e+1})
\end{aligned}$$

$$\left\{ \begin{aligned} T &= - \frac{\partial H(q, T)}{\partial q} \\ q &= \frac{\partial H(q, T)}{\partial T} \end{aligned} \right.$$

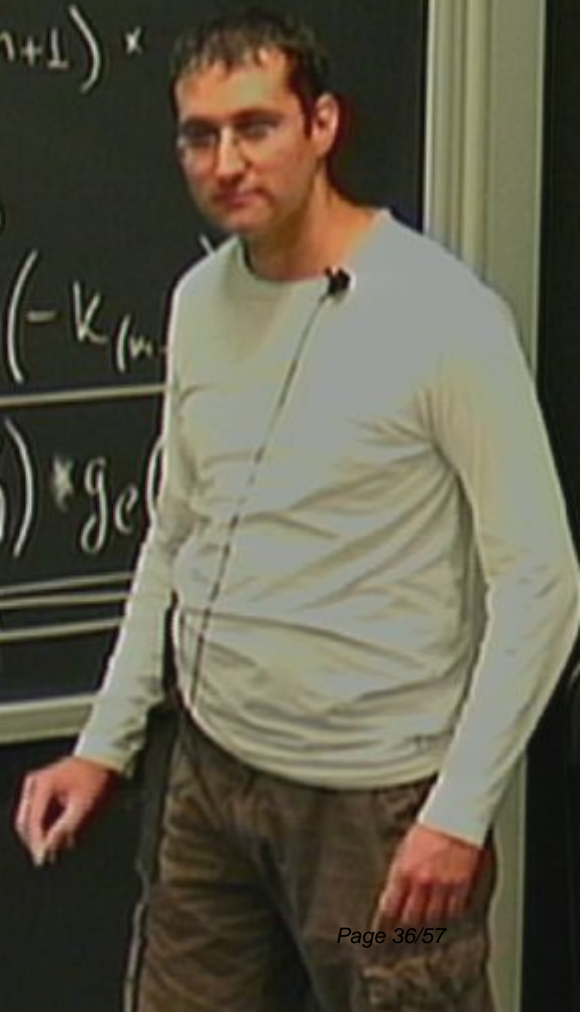
$$\begin{cases} T = - \frac{\partial H(q, T)}{\partial q} \\ q = \frac{\partial H(q, T)}{\partial T} \end{cases}$$

$$H = \int d^D k \left(\dots \right)$$

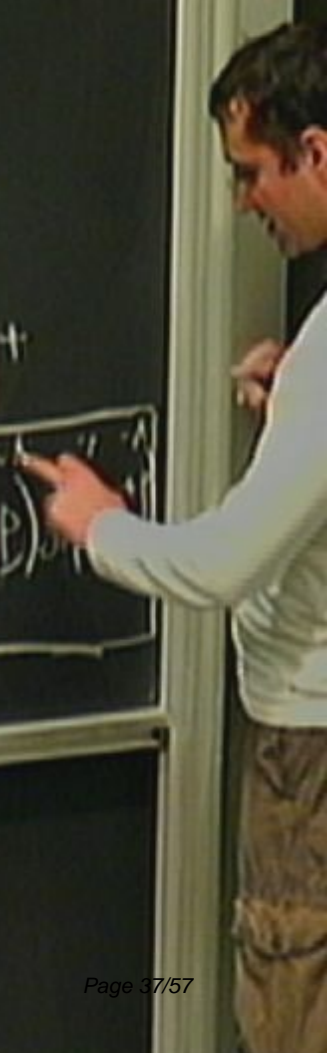
$$\delta(p^2 - \Lambda^2) \sim \dot{K} \left(\frac{p^2}{\Lambda^2} \right)$$



$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} \left(T_e(\{\kappa_e\}) g_e(-\kappa_e) \right) = \\
& = -\frac{1}{2} \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \times T_{m-s-1}(\{\kappa_{m-s-1}\}) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{m+1}) \\
& + \sum_{e,j=1}^{\infty} \int_{\substack{\kappa(e-1) \\ \kappa(e+1)}} (e_j) T_{e,j+2}(\{\kappa_{e-1}\}, \{\kappa_{e+1}\}) g_e
\end{aligned}$$



$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa_e} \left(T_e(\{\kappa_e\}) g_e(-\kappa_e) \right) = \\
& = -\frac{1}{2} \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa_{(m-1)}} (m+1) \times \\
& \times T_{m-s-1}(\{\kappa_{m-s-1}\}) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)}) + \\
& + \sum_{e,j=1}^{\infty} \int_{\kappa_{(e-1)} \kappa_{(j-1)}} (e,j) T_{e+j+2}(\{\kappa_{e-1}\}, \{\kappa_{j-1}\}) * g_e(-\kappa_{(e,j)})
\end{aligned}$$



$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} \left(T_e(\{\kappa_e\}) g_e(-\kappa_e) \right) = \\
& = -\frac{1}{2} \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \quad \times T_{m-s-1}(\{\kappa_{m-s-1}\}) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)}) + \\
& \quad + \sum_{e,j=1}^{\infty} \int_{\substack{\kappa(e-1) \\ \kappa(e-1)}}^{\kappa(e)} (e_j) T_{e,j+2}(\{\kappa_{e-1}\}, \{\kappa_{j+1}\}) * g_e(-\kappa_e)
\end{aligned}$$

$$\begin{aligned}
& \sum_{e=1}^{\infty} \int_{\kappa(e)} \left(T_e(\{\kappa_e\}) g_e(-\kappa_e) \right) = \\
& = -\frac{1}{2} \int_P \frac{\dot{\kappa}(P/\Lambda)}{P^2 + m^2} \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{\kappa(m-1)} (m+1) \times \\
& \quad \times T_{m-s-1}(\{\kappa_{m-s-1}\}) T_s(\{\kappa_s\}) g_{m+1}(-\kappa_{(m-1)}) + \\
& \quad + \sum_{e,j=1}^{\infty} \int_{\substack{\kappa(e-1) \\ \kappa(e-1)}}^{(e,j)} T_{e,j/2}(\{\kappa_{e-1}\}, \{q_{j+1}\}) g_e(-\kappa_{(e-1)}) \dots
\end{aligned}$$

$$\pi(\xi) = \frac{1}{N} \int_{p_1 \dots p_n} \delta(p)$$



$$\pi_n(\xi) = \frac{1}{N} \int_{p_1 \dots p_n} \delta(p - p_1 - \dots - p_n) T_n(\{p_n\})$$

$$\Pi_n^{\delta} = \frac{1}{N} \int_{p_1 \dots p_n} \delta(p - p_1 - \dots - p_n) T_n(\{p_n\})$$

$$\pi(\xi) = \frac{1}{N} \int_{p_1 \dots p_n} \delta(p - p_1 - \dots - p_n) T_n(\{p_n\})$$

$$\pi(\Lambda, S)$$

$$\pi(\sigma) = \frac{1}{N} \int_{p_1 \dots p_n} \delta(p - p_1 - \dots - p_n) T_n(\{p_n\})$$

$$\pi(\wedge, \sigma, x) = \sum_k \sigma^{-(k+1)} \pi_k(\wedge, x)$$

$$g(\wedge, \sigma, x) = \sum_k \sigma^k g_k(\wedge, x)$$

$$H = \int_{-\pi}^{\pi} dG \int d^D x \Pi^2 \cdot g'$$

$$g' = \partial_G g$$

$$H = \int_{-\pi}^{\pi} dG \int d^D x \Pi^2 \cdot g'$$

$$g' = \partial_G g$$

$$H = \int_{-\pi}^{\pi} dG \left(\int d^D x \right) \pi^2 \cdot g'$$

$$g' = \partial_G g$$

$$H = \int_{-\pi}^{\pi} d\phi \left(\int d^D x \right) \pi^2 \cdot g'$$

$$g' = \partial_\phi g$$

$$|g| = 2\pi g', \quad \dot{\pi} = 2\pi \pi'$$

$$-\partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) + \frac{1}{2} \partial_\phi \left(\frac{\partial \mathcal{L}}{\partial g'} \right) = 0$$

$$H = \int_{-\infty}^{\infty} dG \left(\int d^D x \right) \pi^2 \cdot g'$$

$$g' = \partial_G g$$

$$j = 2\pi g', \quad \dot{\pi} = 2\pi \pi'$$

$$-\partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{g}'} \right) + \frac{1}{2} \partial_G \left(\frac{\partial \mathcal{L}}{\partial g'} \right)^2 = 0$$

$$H = \int_{-\hbar}^{\hbar} dG \left(\int d^D x \right) \pi^2 \cdot g'$$

$$g' = \partial_\mu g$$

$$j = 2\pi g', \quad \dot{\pi} = 2\pi \pi'$$

$$-\partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{g}'} \right) + \frac{1}{2} \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial g'} \right)^2 = 0$$

$$\sum_{e=1}^{\infty} \int_{k(e)} T_e(\{k_e\}) g_e(-k_e) =$$

$$= -\frac{1}{2} \int_P \left(\frac{k(P/\Lambda)}{P^2 + m^2} \right) \left[\frac{1}{N} \sum_{m=1}^{\infty} \sum_{s=0}^{m-1} \right]_{k(m-1)} (m+1) \times$$

$$\times T_{m-s-1}(\{k_{m-s-1}\}) T_s(\{k_s\}) g_{m+1}(-k_{(m-1)}) +$$

$$+ \sum_{e,j=1}^{\infty} \int_{k(e-1)k(j-1)} (e,j) T_{e,j/2}(\{k_{e-1}\}, \{k_{j-1}\}) g_e(-k_{(e,j)})$$

$$H = \int_{-\pi}^{\pi} d\phi \left(\int d^D x \right) \pi^2 \cdot g'$$

$$g' = \partial_\phi g$$

$$j = 2\pi g', \quad \dot{\pi} = 2\pi \pi'$$

$$-\partial_t \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) + \frac{1}{2} \partial_\phi \left(\frac{\partial \mathcal{L}}{\partial g'} \right)^2 = 0$$



$$g(\{k\})$$

$$T(\{k\})$$

$$-H$$

$$g(\{k\})$$

$$T(\{k\})$$

$$H = \iiint (T^2 g + g^2 T)$$

$$g(\{k\})$$

$$T(\{k\})$$

$$\underline{N \rightarrow \infty}$$

$$H = \iiint (T^2 g + g^2 T)$$

$$g(\{x\})$$

$$T(\{x\})$$

$$\underline{N \rightarrow \infty}$$

$$p^2 x + x' p$$

$$H = \int \int (T^2 g + g^2 T)$$

$$\{g, T\} = \delta(,)$$

$$g(\{x\})$$

$$T(\{x\})$$

$$\underline{N \rightarrow \infty}$$

$$p^2 x + x' p$$

$$H = \iiint (T^2 g + g^2 T)$$

$$\{g, T\} = \delta(,)$$