Title: Part 3: Monte-Carlo approach to the gauge/gravity duality

Date: Jun 02, 2011 11:00 AM

URL: http://pirsa.org/11060009

Abstract: The gauge/gravity duality may give a nonperturbative formulation of superstring/M theory, and hence, if one can study the nonperturbative dynamics of the gauge theory, it would be useful to understand the nonperturbative aspects of superstring theory. Although researches in this direction were not successful for long time because of the notorious difficulties in lattice SUSY, however, recent progress made it possible; nonperturbative formulations free from the parameter fine-tuning were proposed, some of them are confirmed to work numerically, and nontrivial evidence for the validity of the gauge/gravity duality has been obtained.
In these talks I review the state of the art in this field. I start with reviewing basics of the Monte-Carlo. Then I explain how to put supersymmetric theories on computer and show actual numerical results.
1st talk : basics of Monte-Carlo simulation.
2nd talk : 1d SYM (matrix quantum mechanics).
3rd talk : how to put 2d, 3d and 4d SYM on computer.
In the talks I concentrate on basic ideas and omit technical details (e.g. algorithms to accelerate simulations). They will be explained after the talks if people are interested in.
References: 1st talk : standard textbooks e.g. Heinz J. Rothe, "Lattice Gauge Theories: An Introduction", Third Edition, World Scientific. 2nd talk : 0706.1647 [hep-lat], 0707.4454 [hep-th], 0811.2081 [hep-th], 0811.3102 [hep-th], 0911.1623 [hep-th], 1012.2913 [hep-th]. 3rd talk : hep-lat/0302017, hep-lat/0311021, 1010.2948 [hep-lat] (2d SYM); hep-th/0211139 (3d SYM); 1004.5513 [hep-lat], 1009.0901 [hep-lat] (4d SYM)

how to put
2d, 3d & 4d SYM
on computer

2D SYM ON LATTICE

Basic ideas

- Keep a few supercharges exact on lattice.
- Use it (and other discrete symmetries) to forbid SUSY breaking radiative corrections.
- Use Sugino's approach, which is slightly different from that of Kaplan-Unsal et al.
- Only "extended" SUSY can be realized for a technical reason. (4, 8 and 16 SUSY)
- Below we consider 16 SUSY theory.

$$S_0 = \frac{1}{g_{2d}^2}\int d^2x \,\mathrm{Tr}\Big\{F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2$$
$$+\Psi^T\left(D_1 + \gamma_2 D_2\right)\Psi + i\Psi^T\gamma_I\left[X^I,\Psi\right]\Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2}\int d^2x \,\mathrm{Tr}\Big\{-iB_A\Phi_A - \frac{1}{3}\epsilon_{ABC}B_A[B_B, B_C]$$
$$-\psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_-\Big\},$$

$$\Phi_1 = 2(-D_1X_3 - D_2X_4),\ \Phi_2 = 2(-D_1X_4 + D_2X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

$$Q^{(0)}_{\pm} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$
$$Q^{(0)}_{\mp} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \bar{H}_\mu,$$
$$Q^{(0)}_{\pm} \bar{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}[C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$
$$Q^{(0)}_{\pm} X_i = \rho_{\pm i}, \quad Q^{(0)}_{\pm} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$
$$Q^{(0)}_{\mp} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \bar{h}_i,$$
$$Q^{(0)}_{\pm} \bar{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}[C, \rho_{\pm i}] \pm \frac{1}{2}[X_i, \eta_{\pm}],$$
$$Q^{(0)}_{\pm} B_A = \chi_{\pm A}, \quad Q^{(0)}_{\pm} \chi_{\pm A} = \pm [\phi_{\pm}, B_A],$$
$$Q^{(0)}_{\mp} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$
$$Q^{(0)}_{\pm} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}[B_A, \eta_{\pm}] \mp \frac{1}{2}[C, \chi_{\pm A}],$$
$$Q^{(0)}_{\pm} C = \eta_{\pm}, \quad Q^{(0)}_{\pm} \eta_{\pm} = \pm [\phi_{\pm}, C],$$
$$Q^{(0)}_{\mp} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$
$$Q^{(0)}_{\pm} \phi_{\pm} = 0, \quad Q^{(0)}_{\mp} \phi_{\pm} = \mp \eta_{\pm}.$$

Nilpotency

$$\left(Q_+^{(0)}\right)^2 = \left(Q_-^{(0)}\right)^2 = \{Q_+^{(0)}, Q_-^{(0)}\} = 0$$

$\longrightarrow$ $Q_\pm^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$Q_{\pm}^{(0)} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$
$$Q_{\mp}^{(0)} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \tilde{H}_\mu,$$
$$Q_{\pm}^{(0)} \tilde{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}[C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$
$$Q_{\pm}^{(0)} X_i = \rho_{\pm i}, \quad Q_{\pm}^{(0)} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$
$$Q_{\mp}^{(0)} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \tilde{h}_i,$$
$$Q_{\pm}^{(0)} \tilde{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}[C, \rho_{\pm i}] \pm \frac{1}{2}[X_i, \eta_{\pm}],$$
$$Q_{\pm}^{(0)} B_A = \chi_{\pm A}, \quad Q_{\pm}^{(0)} \chi_{\pm A} = \pm [\phi_{\pm}, B_A],$$
$$Q_{\mp}^{(0)} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$
$$Q_{\pm}^{(0)} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}[B_A, \eta_{\pm}] \mp \frac{1}{2}[C, \chi_{\pm A}],$$
$$Q_{\pm}^{(0)} C = \eta_{\pm}, \quad Q_{\pm}^{(0)} \eta_{\pm} = \pm [\phi_{\pm}, C],$$
$$Q_{\mp}^{(0)} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$
$$Q_{\pm}^{(0)} \phi_{\pm} = 0, \quad Q_{\mp}^{(0)} \phi_{\pm} = \mp \eta_{\pm}.$$

$$S_0 = \frac{1}{g_{2d}^2} \int d^2x \, \mathrm{Tr}\Big\{ F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2$$
$$+\Psi^T \left(D_1 + \gamma_2 D_2\right)\Psi + i\Psi^T \gamma_I \left[X^I, \Psi\right] \Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2} \int d^2x \, \mathrm{Tr}\Big\{ -iB_A \Phi_A - \frac{1}{3}\epsilon_{ABC} B_A [B_B, B_C]$$
$$-\psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_- \Big\},$$

$$\Phi_1 = 2(-D_1 X_3 - D_2 X_4),\ \Phi_2 = 2(-D_1 X_4 + D_2 X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

Nilpotency

$$\left(Q_+^{(0)}\right)^2 = \left(Q_-^{(0)}\right)^2 = \{Q_+^{(0)}, Q_-^{(0)}\} = 0$$

$\longrightarrow$ $Q_\pm^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$Q^{(0)}_{\pm} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$
$$Q^{(0)}_{\mp} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \tilde{H}_\mu,$$
$$Q^{(0)}_{\pm} \tilde{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}[C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$
$$Q^{(0)}_{\pm} X_i = \rho_{\pm i}, \quad Q^{(0)}_{\pm} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$
$$Q^{(0)}_{\mp} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \tilde{h}_i,$$
$$Q^{(0)}_{\pm} \tilde{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}[C, \rho_{\pm i}] \pm \frac{1}{2}[X_i, \eta_{\pm}],$$
$$Q^{(0)}_{\pm} B_A = \chi_{\pm A}, \quad Q^{(0)}_{\pm} \chi_{\pm A} = \pm [\phi_{\pm}, B_A],$$
$$Q^{(0)}_{\mp} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$
$$Q^{(0)}_{\pm} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}[B_A, \eta_{\pm}] \mp \frac{1}{2}[C, \chi_{\pm A}],$$
$$Q^{(0)}_{\pm} C = \eta_{\pm}, \quad Q^{(0)}_{\pm} \eta_{\pm} = \pm [\phi_{\pm}, C],$$
$$Q^{(0)}_{\mp} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$
$$Q^{(0)}_{\pm} \phi_{\pm} = 0, \quad Q^{(0)}_{\mp} \phi_{\pm} = \mp \eta_{\pm}.$$

$$S_0 \;=\; \frac{1}{g_{2d}^2}\int d^2x\;\mathrm{Tr}\Big\{F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2 + \Psi^T\left(D_1 + \gamma_2 D_2\right)\Psi + i\Psi^T\gamma_I\left[X^I, \Psi\right]\Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} \;=\; \frac{1}{g_{2d}^2}\int d^2x\;\mathrm{Tr}\Big\{-iB_A\Phi_A - \frac{1}{3}\epsilon_{ABC}B_A[B_B, B_C] - \psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_-\Big\},$$

$$\Phi_1 = 2(-D_1X_3 - D_2X_4),\; \Phi_2 = 2(-D_1X_4 + D_2X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

Nilpotency

$$\left(Q_{+}^{(0)}\right)^2 = \left(Q_{-}^{(0)}\right)^2 = \{Q_{+}^{(0)}, Q_{-}^{(0)}\} = 0$$

$\longrightarrow$ $Q_{\pm}^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$
\begin{aligned}
Q_\pm U_\mu(x) &= i\psi_{\pm\mu}(x)U_\mu(x),\\
Q_\pm \psi_{\pm\mu}(x) &= i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),\\
Q_\mp \psi_{\pm\mu}(x) &= \frac{i}{2}\{\psi_{+\mu}(x),\psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),\\
Q_\pm \tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{\mp\mu}(x),\phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right]\\
&\quad \pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right]\\
&\quad \mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x),\psi_{\mp\mu}(x)\right]\\
&\quad +\frac{i}{2}\left[\psi_{\pm\mu}(x),\tilde{H}_\mu(x)\right]
\end{aligned}
$$

$$
D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)
$$

Sugino, 2003

$$\Phi_1(x) = 2\left(-D_1 X_3(x) - D_2 X_4(x)\right),$$
$$\Phi_2(x) = 2\left(-D_1^* X_4(x) + D_2^* X_3(x)\right),$$
$$\Phi_3(x) = \frac{i(U_{12}(x) - U_{21}(x))}{1 - \epsilon^{-2}||1 - U_{12}(x)||^2} + 2i[X_3(x), X_4(x)]$$

↑

admissibility condition

- Gauge part is not Tr(plaquette) but Tr(plaquette)^2
- To exclude wrong vacua with plaquette = -1,
  we employ the admissibility.
- The action is set to be zero unless

$||1 - U_{12}(x)|| < \epsilon$ for $\forall x$

Sugino, 2003

Absence of fine tuning (to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2}c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots\right)\int d^2x\, \mathcal{O}_p(x)$$

tree

up to log(a), where

$$\mathcal{O}_p(x) = \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \qquad p = \alpha + \beta + 3\gamma$$

- Only p=1,2 are dangerous.

$$\varphi, \varphi^2 \qquad (\partial\varphi \text{ is a total derivative})$$

$SU(2)_R$ allows only TrB_A and TrX_i.

Exact SUSY kills them.

φ^2 term is forbidden in a similar manner

$$\Phi_1(x) = 2\left(-D_1 X_3(x) - D_2 X_4(x)\right),$$
$$\Phi_2(x) = 2\left(-D_1^* X_4(x) + D_2^* X_3(x)\right),$$
$$\Phi_3(x) = \frac{i(U_{12}(x) - U_{21}(x))}{1 - \epsilon^{-2}||1 - U_{12}(x)||^2} + 2i[X_3(x), X_4(x)]$$

↑

admissibility condition

- Gauge part is not Tr(plaquette) but Tr(plaquette)^2
- To exclude wrong vacua with plaquette = -1, we employ the admissibility.
- The action is set to be zero unless

$||1 - U_{12}(x)|| < \epsilon$ for $\forall x$

Sugino, 2003

$$Q_\pm U_\mu(x) = i\psi_{\pm\mu}(x)U_\mu(x),$$
$$Q_\pm \psi_{\pm\mu}(x) = i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),$$
$$Q_\mp \psi_{\pm\mu}(x) = \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),$$
$$\begin{aligned} Q_\pm \tilde{H}_\mu(x) = &-\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right] \\ &\pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] \\ &\mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right] \\ &+\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right] \end{aligned}$$

$$D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)$$

Sugino, 2003

Nilpotency

$$\left(Q_+^{(0)}\right)^2 = \left(Q_-^{(0)}\right)^2 = \{Q_+^{(0)}, Q_-^{(0)}\} = 0$$

$\longrightarrow \; Q_\pm^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$
\begin{aligned}
Q_\pm U_\mu(x) &= i\psi_{\pm\mu}(x)U_\mu(x), \\
Q_\pm \psi_{\pm\mu}(x) &= i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x), \\
Q_\mp \psi_{\pm\mu}(x) &= \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x), \\
Q_\pm \tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\quad \pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\quad \mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right] \\
&\quad +\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]
\end{aligned}
$$

$$
D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)
$$

Sugino, 2003

Nilpotency

$$\left(Q_+^{(0)}\right)^2 = \left(Q_-^{(0)}\right)^2 = \{Q_+^{(0)}, Q_-^{(0)}\} = 0$$

$\longrightarrow$ $Q_\pm^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

But how?

... trial and error!

$$S_0 = \frac{1}{g_{2d}^2}\int d^2x \,\mathrm{Tr}\Big\{F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2 + \Psi^T\left(D_1 + \gamma_2 D_2\right)\Psi + i\Psi^T\gamma_I\left[X^I, \Psi\right]\Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2}\int d^2x \,\mathrm{Tr}\Big\{-iB_A\Phi_A - \frac{1}{3}\epsilon_{ABC}B_A[B_B, B_C] - \psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_-\Big\},$$

$$\Phi_1 = 2(-D_1X_3 - D_2X_4),\ \Phi_2 = 2(-D_1X_4 + D_2X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

$$Q_{\pm}^{(0)} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$

$$Q_{\mp}^{(0)} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \tilde{H}_\mu,$$

$$Q_{\pm}^{(0)} \tilde{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}[C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$

$$Q_{\pm}^{(0)} X_i = \rho_{\pm i}, \quad Q_{\pm}^{(0)} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$

$$Q_{\mp}^{(0)} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \tilde{h}_i,$$

$$Q_{\pm}^{(0)} \tilde{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}[C, \rho_{\pm i}] \pm \frac{1}{2}[X_i, \eta_{\pm}],$$

$$Q_{\pm}^{(0)} B_A = \chi_{\pm A}, \quad Q_{\pm}^{(0)} \chi_{\pm A} = \pm [\phi_{\pm}, B_A],$$

$$Q_{\mp}^{(0)} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$

$$Q_{\pm}^{(0)} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}[B_A, \eta_{\pm}] \mp \frac{1}{2}[C, \chi_{\pm A}],$$

$$Q_{\pm}^{(0)} C = \eta_{\pm}, \quad Q_{\pm}^{(0)} \eta_{\pm} = \pm [\phi_{\pm}, C],$$

$$Q_{\mp}^{(0)} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$

$$Q_{\pm}^{(0)} \phi_{\pm} = 0, \quad Q_{\mp}^{(0)} \phi_{\pm} = \mp \eta_{\pm}.$$

Nilpotency

$$\left(Q_{+}^{(0)}\right)^2 = \left(Q_{-}^{(0)}\right)^2 = \{Q_{+}^{(0)}, Q_{-}^{(0)}\} = 0$$

$\longrightarrow$ $Q_{\pm}^{(0)} S^{(0)} = 0$ can be seen manifestly.

Strategy

Realize this SUSY algebra on lattice.
Then the lattice action has two exact SUSY and $SU(2)_R$.

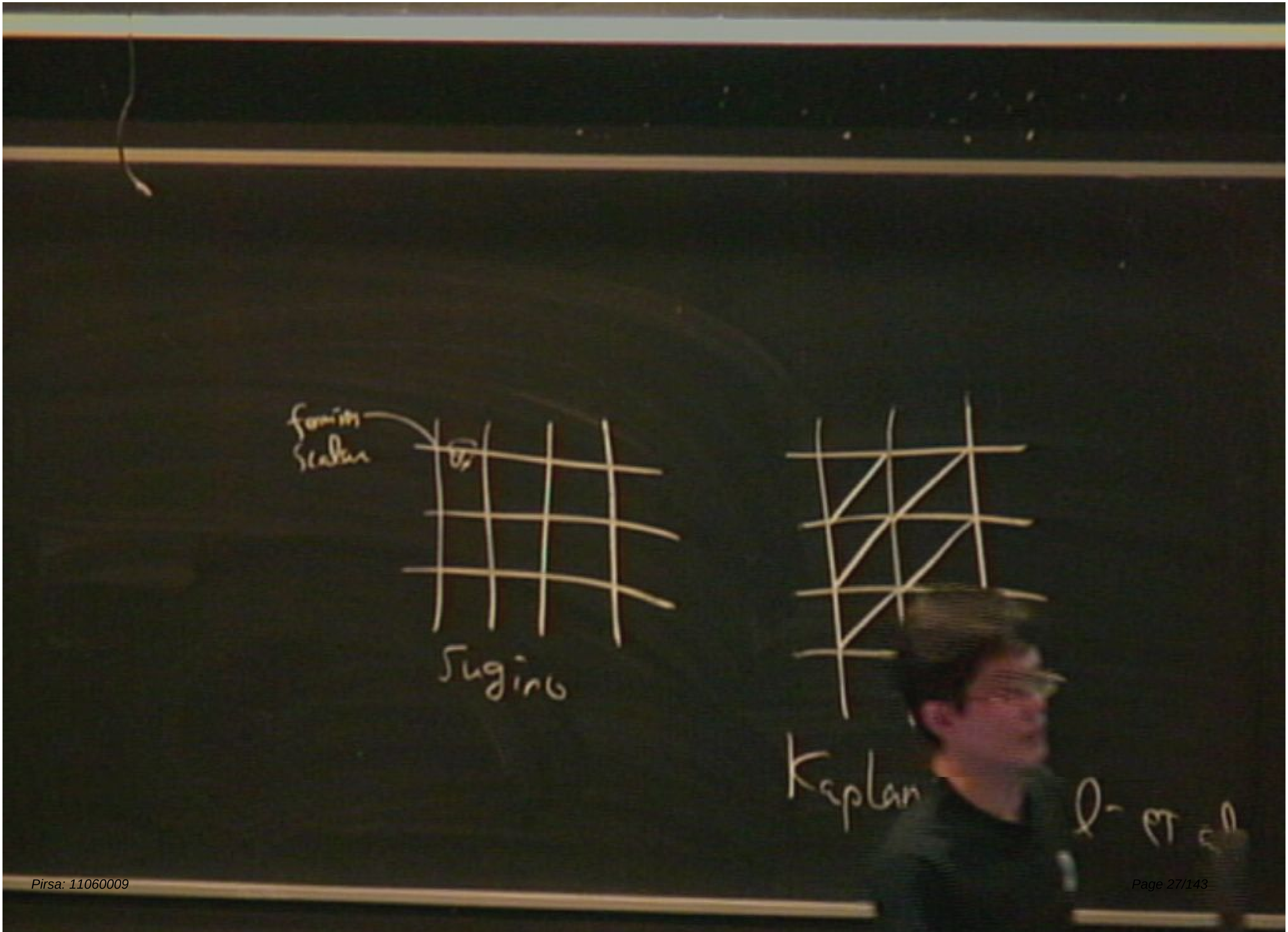
But how?

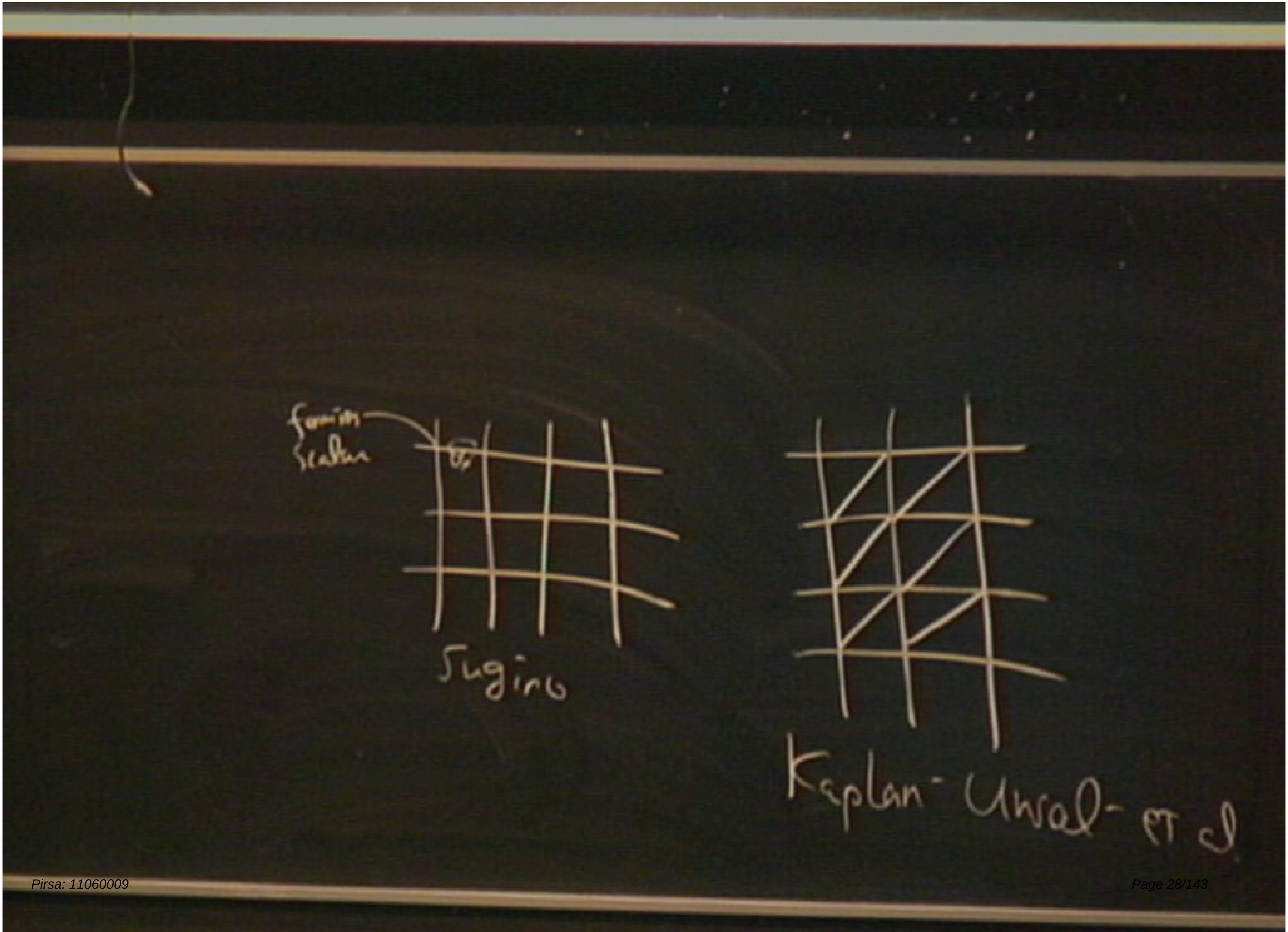
... trial and error!

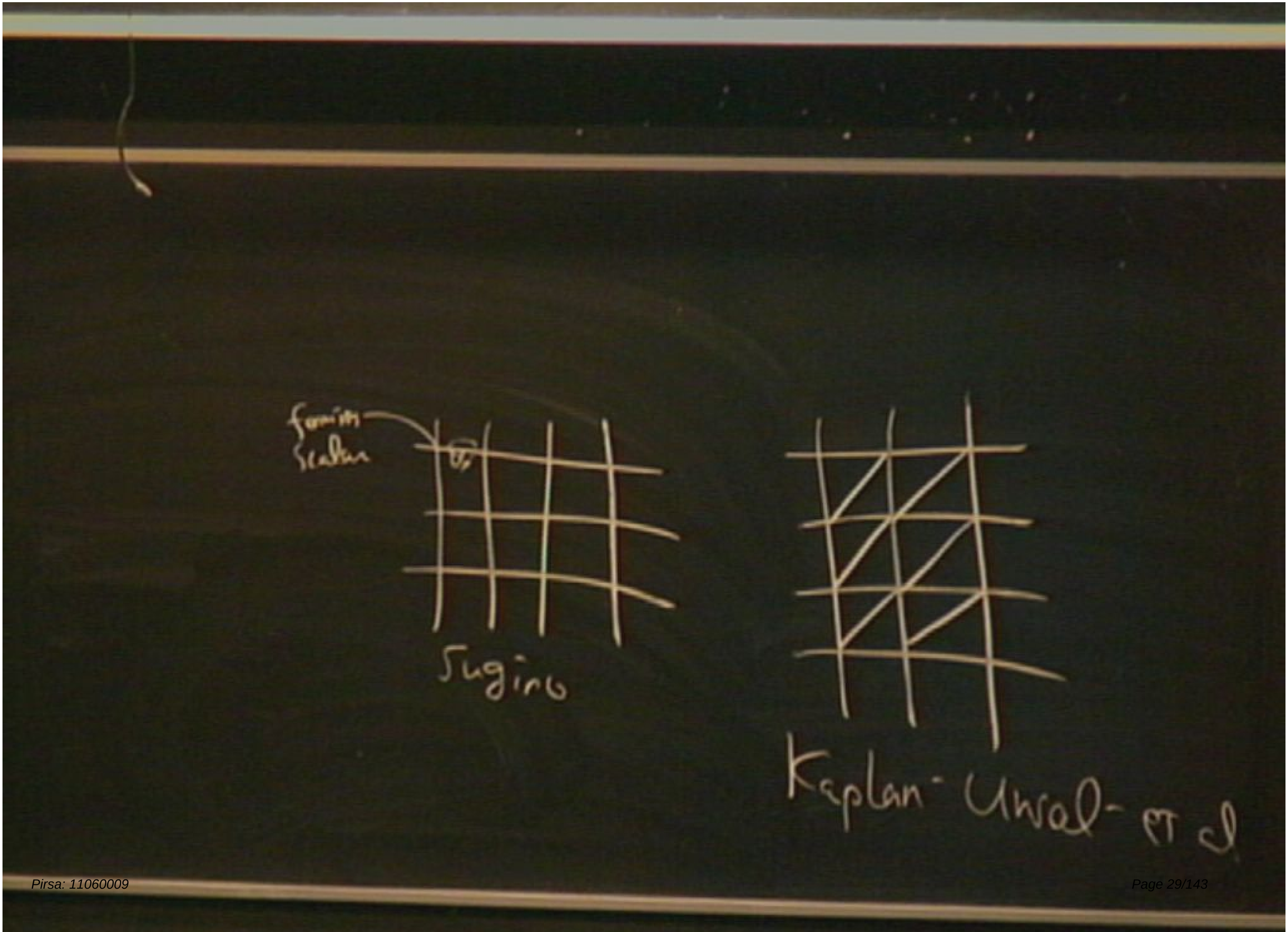
$$
\begin{aligned}
Q_\pm U_\mu(x) &= i\psi_{\pm\mu}(x)U_\mu(x),\\
Q_\pm \psi_{\pm\mu}(x) &= i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),\\
Q_\mp \psi_{\pm\mu}(x) &= \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),\\
Q_\pm \tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right]\\
&\quad \pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right]\\
&\quad \mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right]\\
&\quad +\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]
\end{aligned}
$$

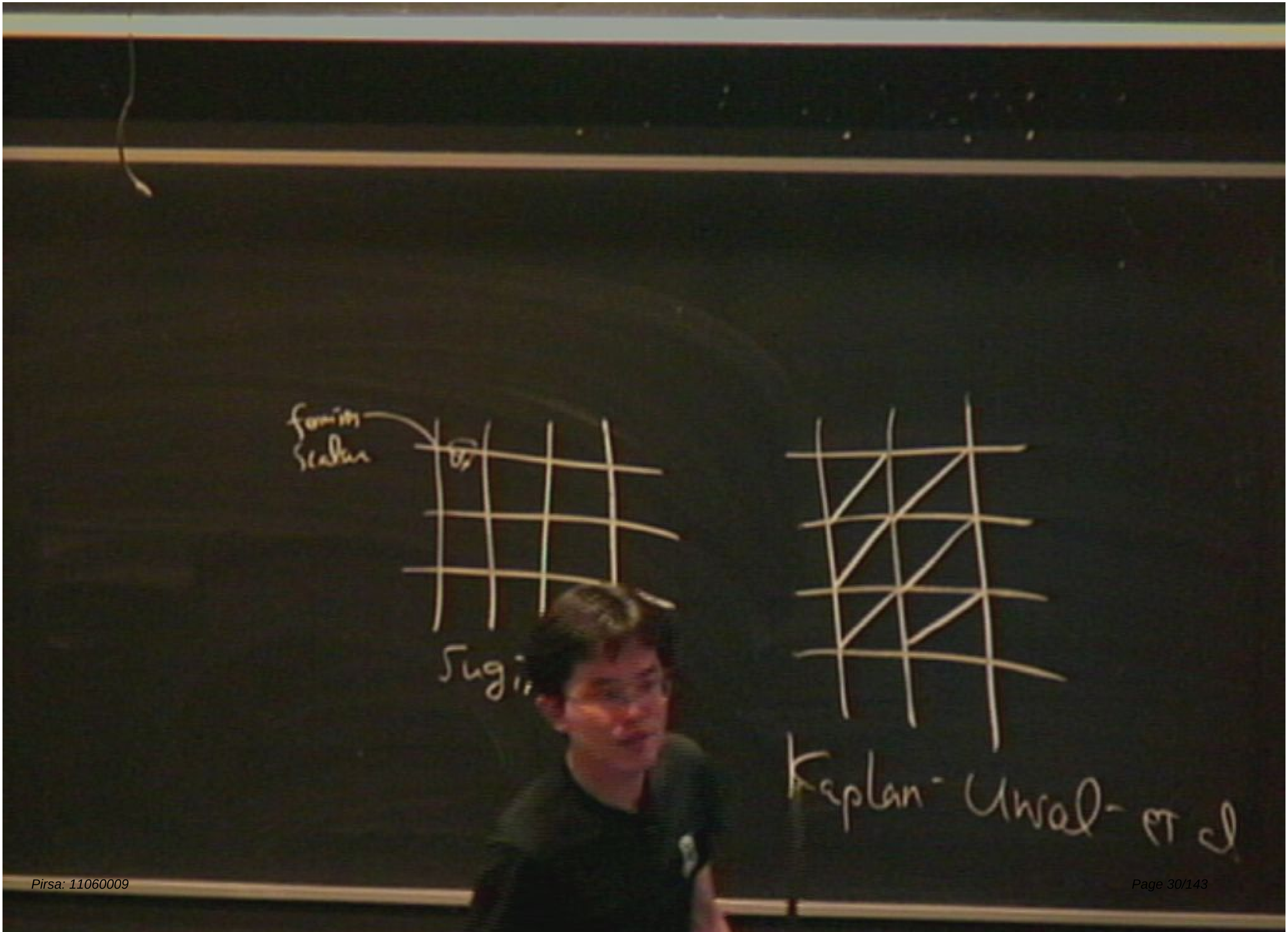
$$
D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)
$$

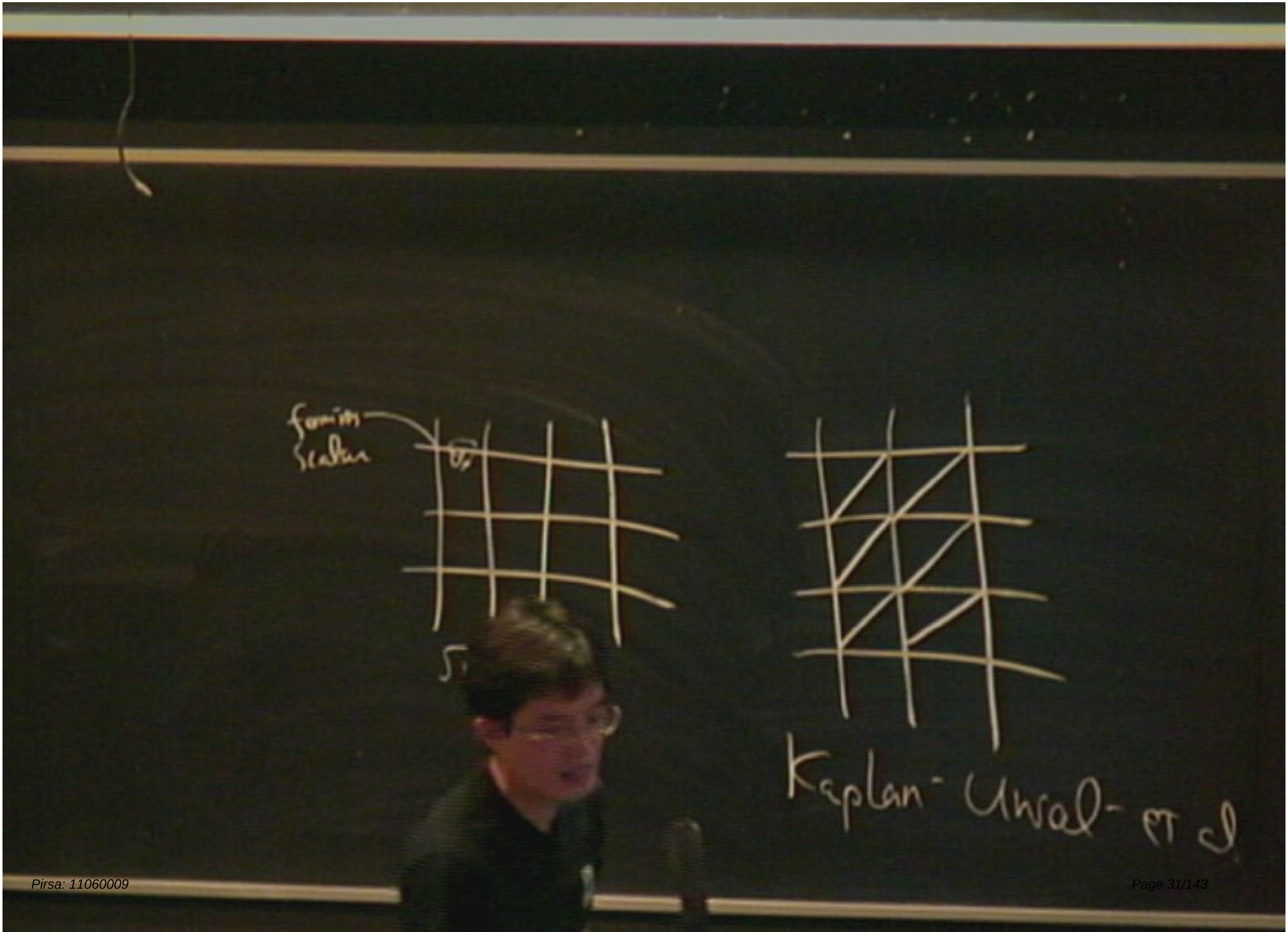
Sugino, 2003

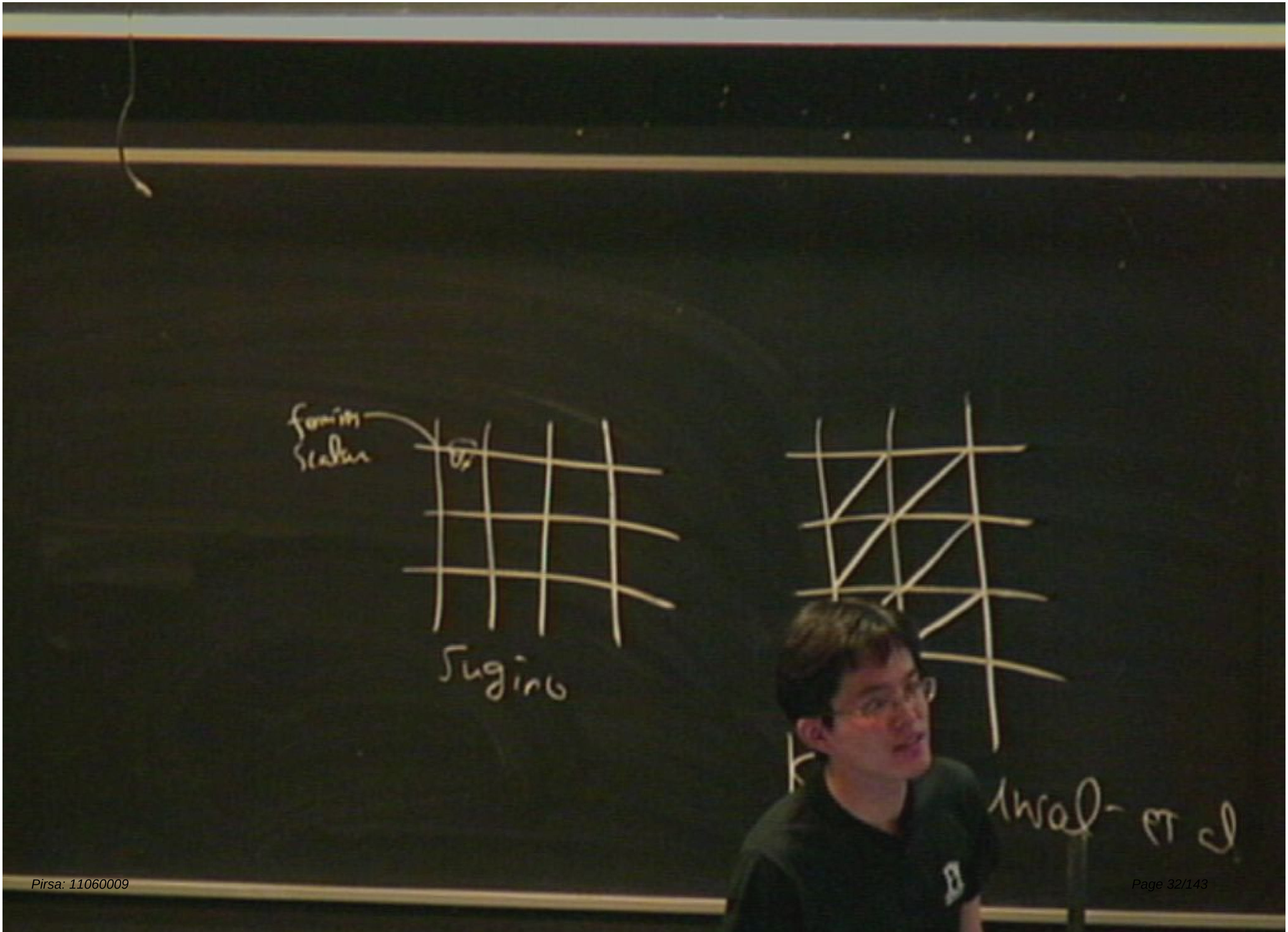

fermion
scalar
Sugino
Kaplan

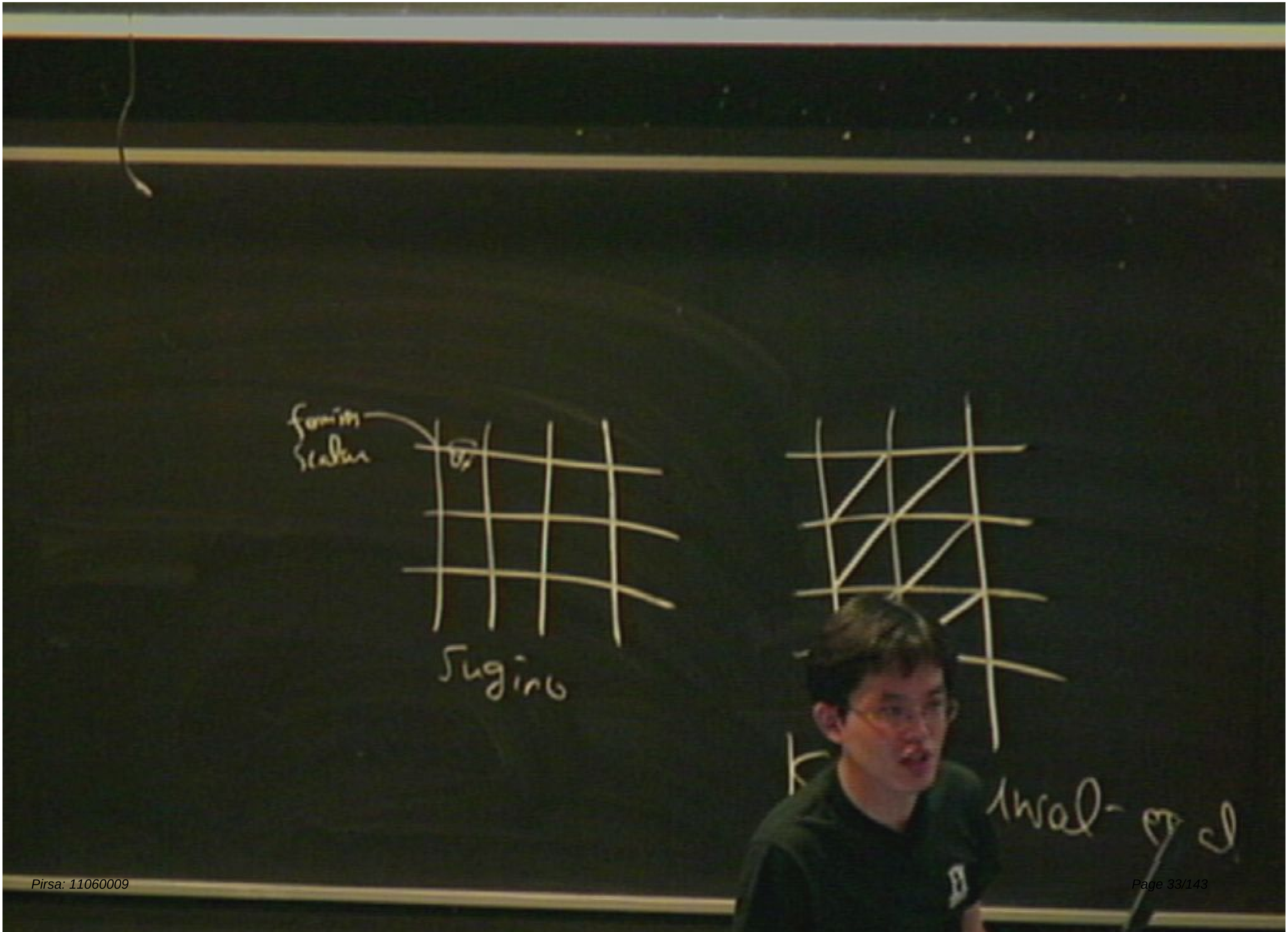

fermion
scalar
Sugino
Kaplan- Unsal- et al

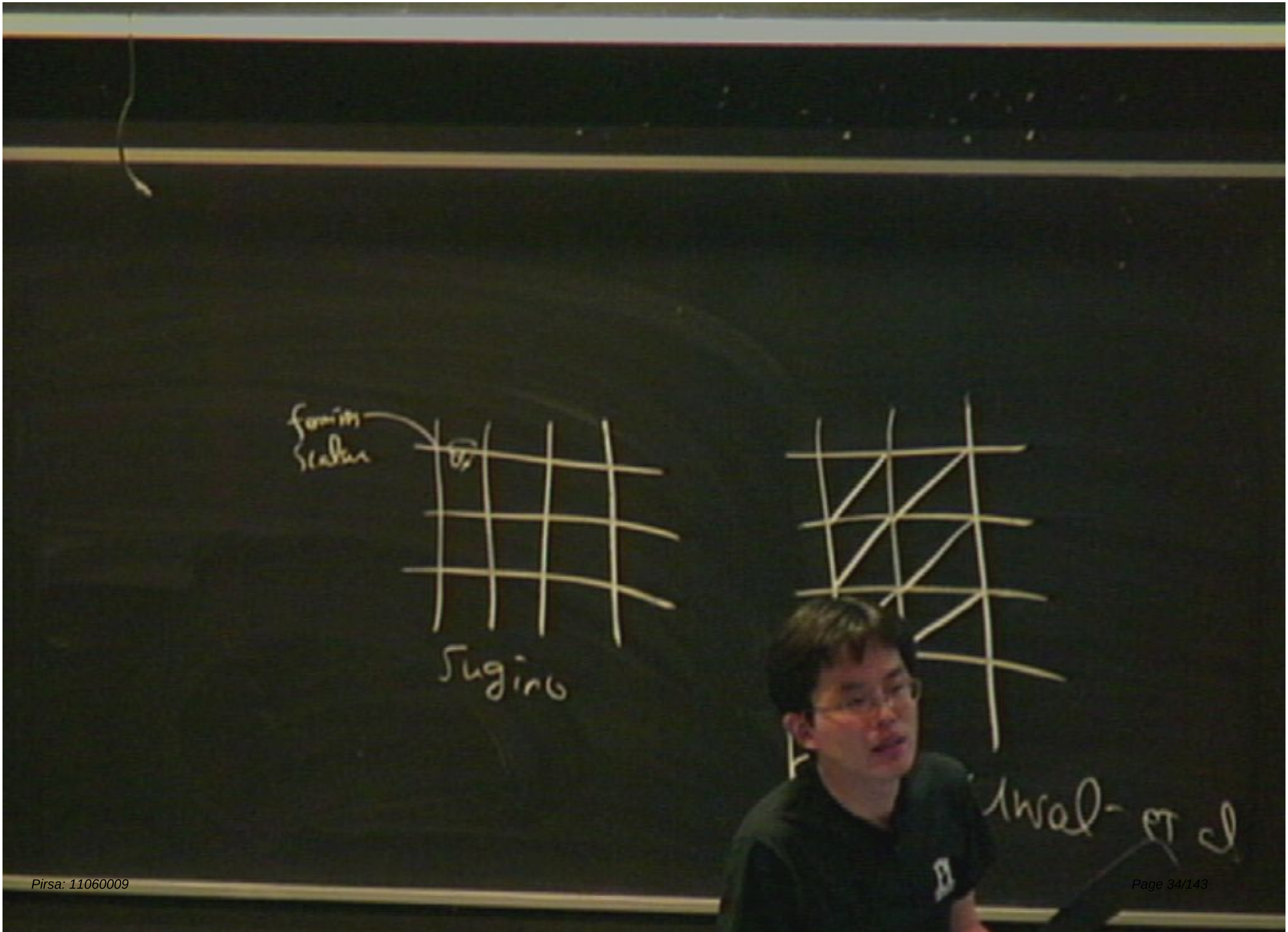

fermion
scalar
Sugino
Kaplan-Unsal- et al

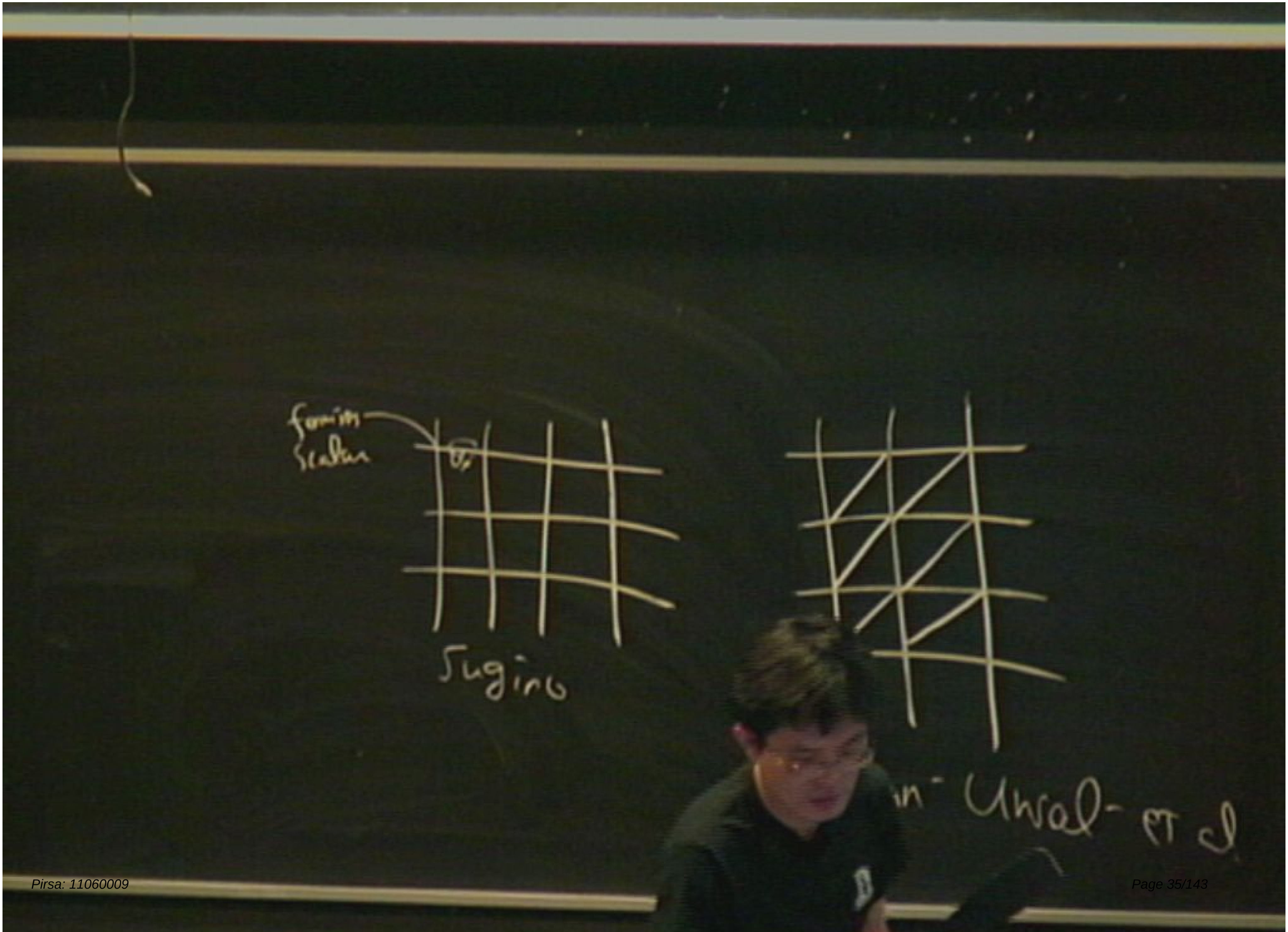

fermion
scalar
Kaplan- Unsal- et al


fermion
scalar
Kaplan- Unsal- et al


fermion
scalar
Sugino

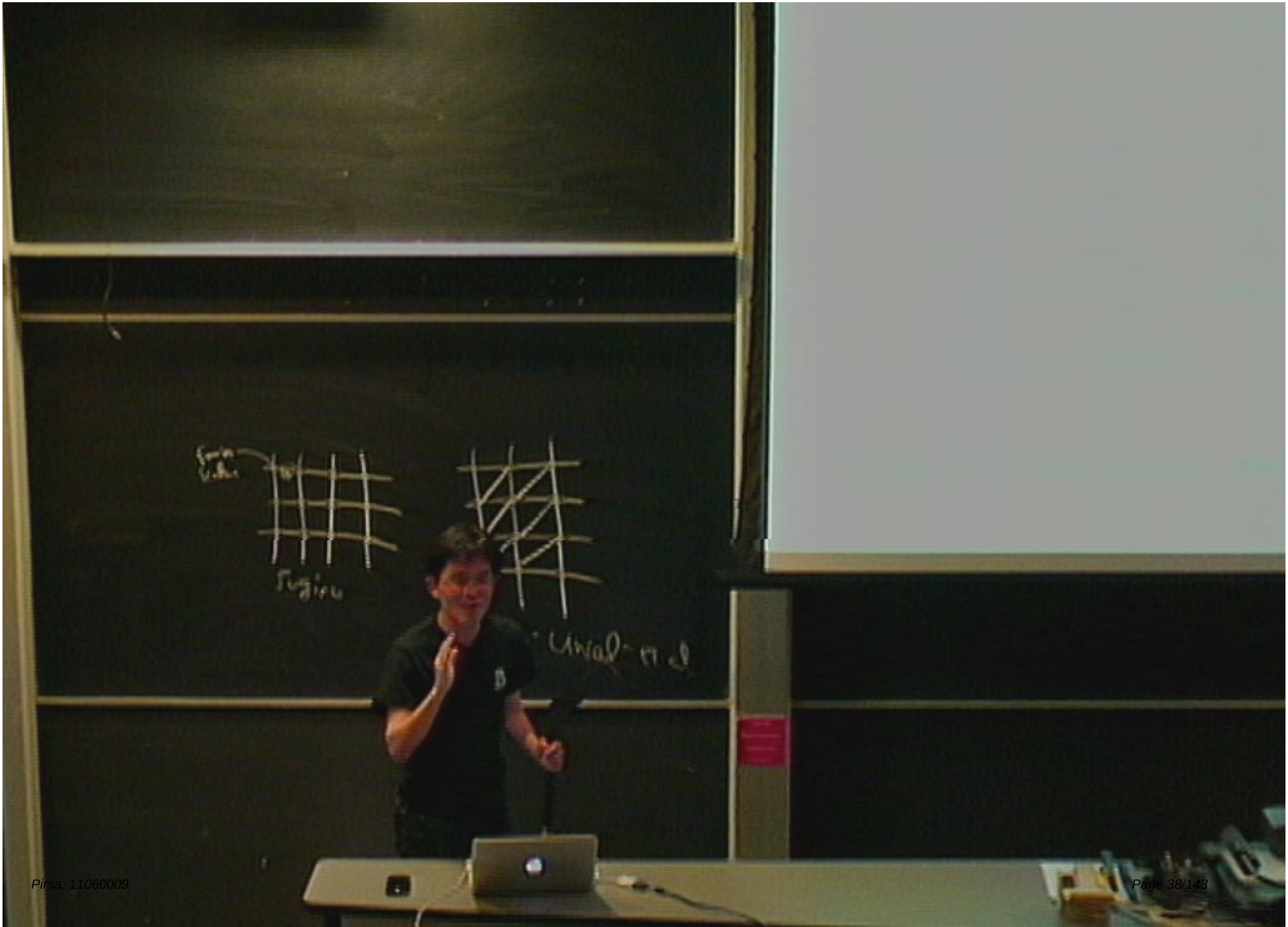

fermion
scalar
Sugino

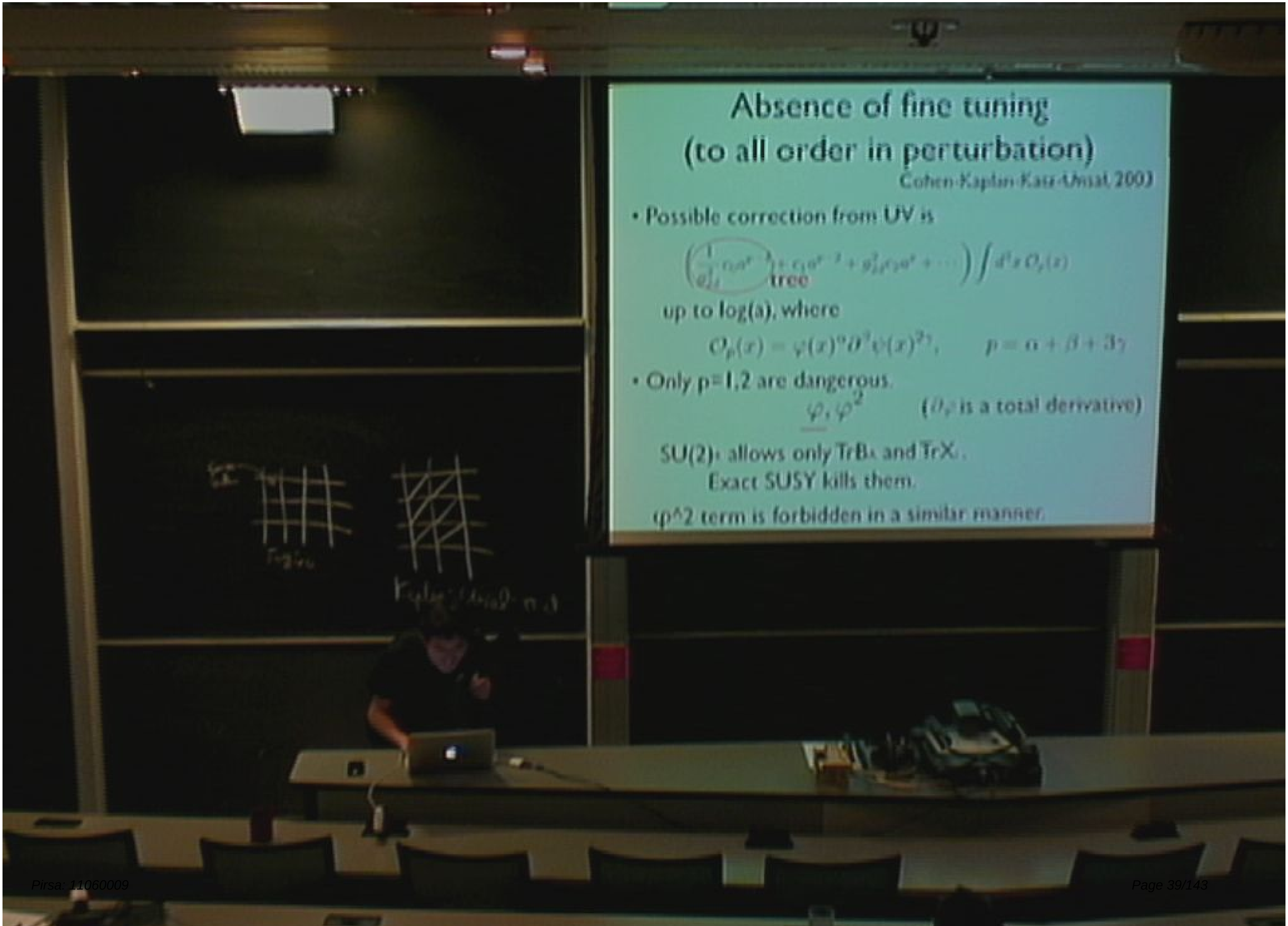

fermion
scalar
Sugino

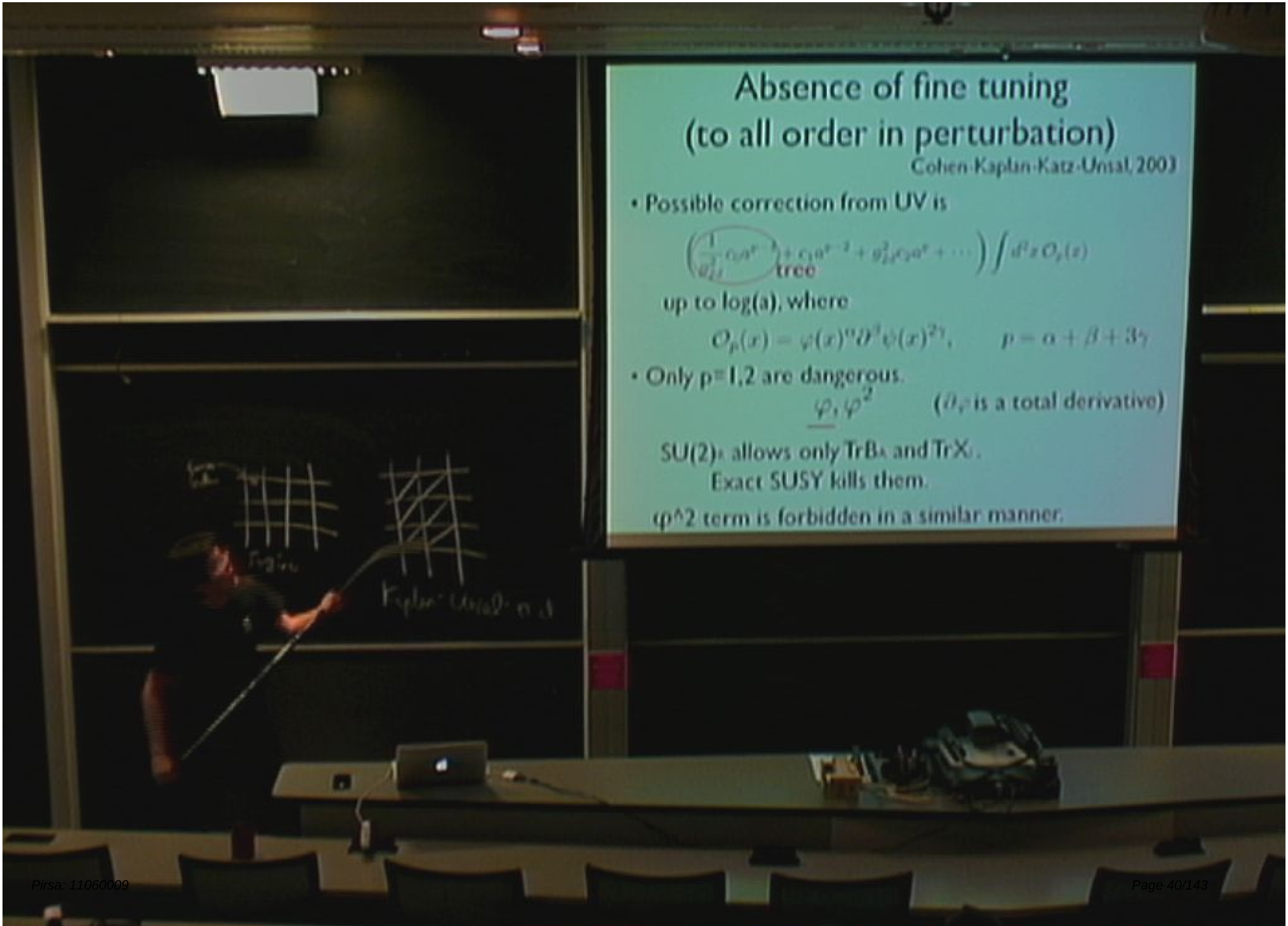

Sugino

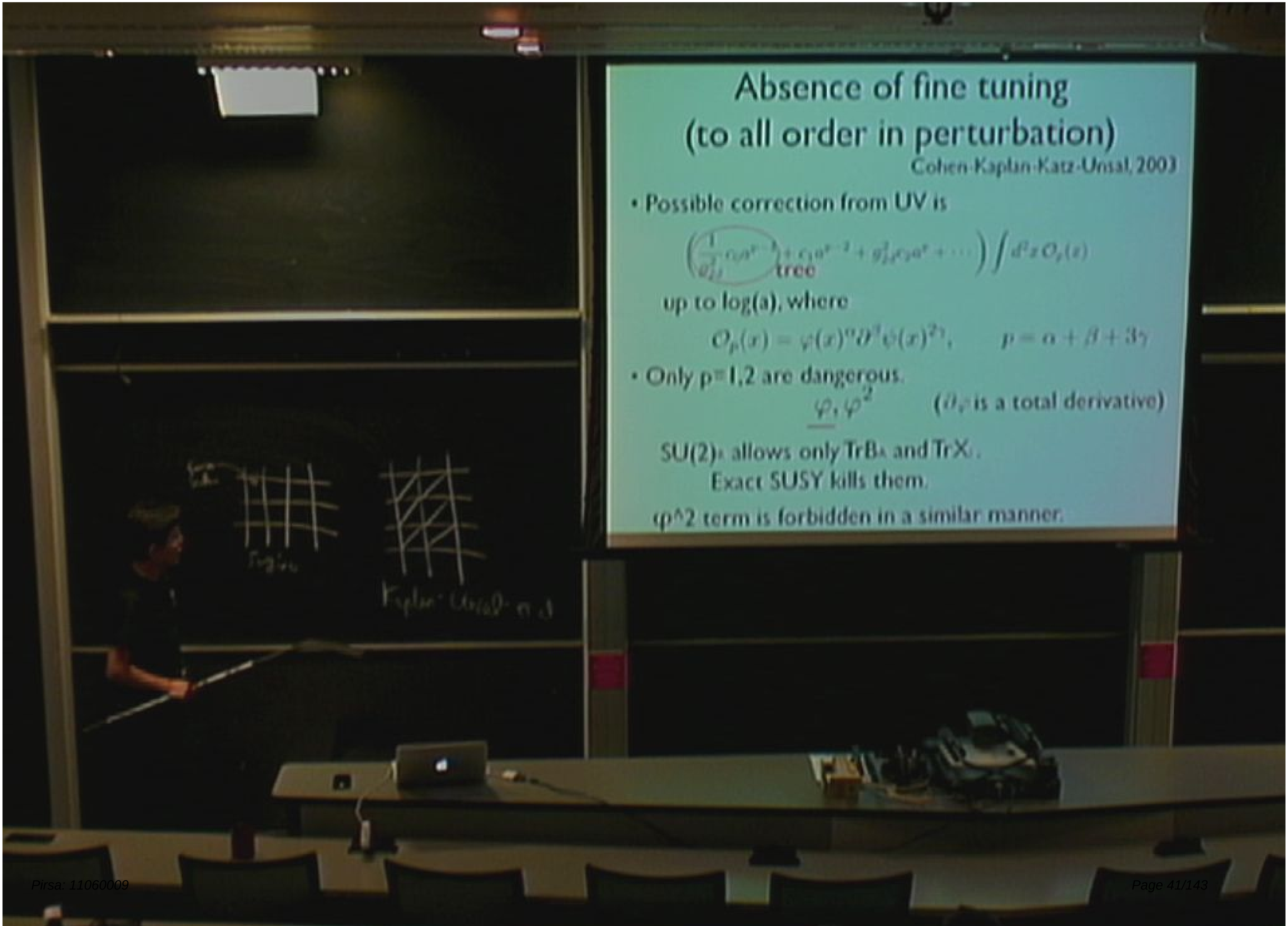

fermion
scalar
Kaplan- Unsal- et al.

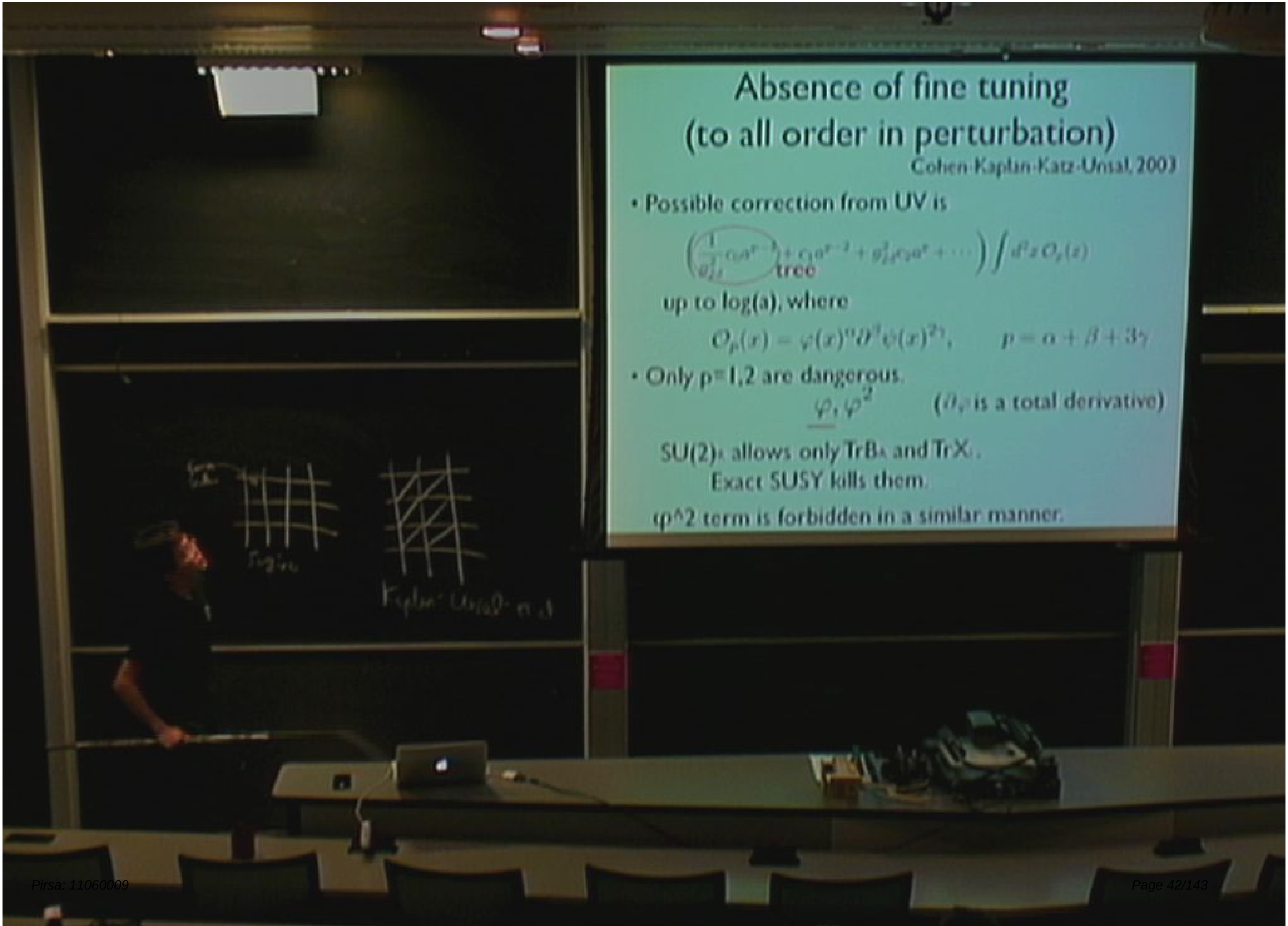

fermion
scalar
Sugino
Kaplan- Unsal- et al

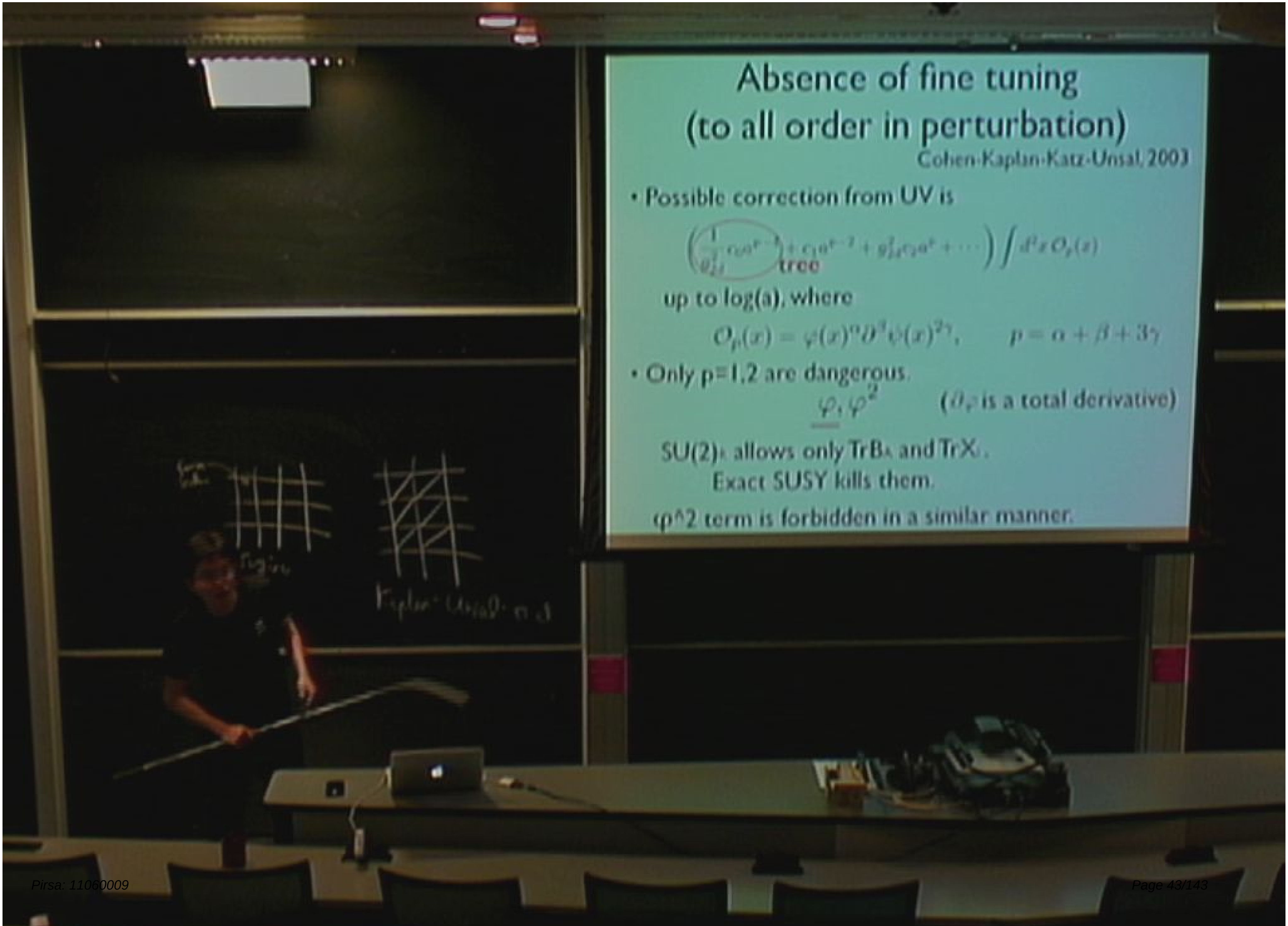



Absence of fine tuning
(to all order in perturbation)
Cohen-Kaplan-Katz-Unsal, 2003
• Possible correction from UV is
tree
up to log(a), where
$\mathcal{O}_p(x) = \varphi(x)^\alpha \partial^\beta \varphi(x)^{2\gamma}, \quad p = \alpha + \beta + 3\gamma$
• Only p=1,2 are dangerous.
φ, φ^2
is a total derivative)
SU(2)$_R$ allows only TrB$_R$ and TrX$_R$.
Exact SUSY kills them.
φ^2 term is forbidden in a similar manner.


Absence of fine tuning
(to all order in perturbation)
Cohen-Kaplan-Katz-Unsal, 2003
• Possible correction from UV is
tree
up to log(a), where
• Only p=1,2 are dangerous.
(is a total derivative)
Exact SUSY kills them.
φ^2 term is forbidden in a similar manner.


Absence of fine tuning
(to all order in perturbation)
Cohen-Kaplan-Katz-Unsal, 2003
• Possible correction from UV is
tree
up to log(a), where
$\mathcal{O}_p(x) = \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \quad p = \alpha + \beta + 3\gamma$
• Only p=1,2 are dangerous.
φ, φ^2
(∂_φ is a total derivative)
SU(2)R allows only TrBA and TrX.
Exact SUSY kills them.
φ^2 term is forbidden in a similar manner.


Absence of fine tuning
(to all order in perturbation)
Cohen-Kaplan-Katz-Unsal, 2003
• Possible correction from UV is
tree
up to log(a), where
$p = \alpha + \beta + 3\gamma$
• Only p=1,2 are dangerous.
φ, φ^2
is a total derivative)
SU(2)R allows only TrBR and TrXR.
Exact SUSY kills them.
φ^2 term is forbidden in a similar manner.


Absence of fine tuning
(to all order in perturbation)
Cohen-Kaplan-Katz-Unsal 2003
• Possible correction from UV is
tree
up to log(a), where
• Only p=1,2 are dangerous.
(is a total derivative)
SU(2)R allows only TrBR and TrXR.
Exact SUSY kills them.
φ^2 term is forbidden in a similar manner.

Absence of fine tuning
(to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2}c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots\right)\int d^2x\, \mathcal{O}_p(x)$$

tree

up to log(a), where

$$\mathcal{O}_p(x) = \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \qquad p = \alpha + \beta + 3\gamma$$

- Only p=1,2 are dangerous.

φ, φ^2 ($\partial\varphi$ is a total derivative)

SU(2)$_R$ allows only TrB_A and TrX_i.

Exact SUSY kills them.

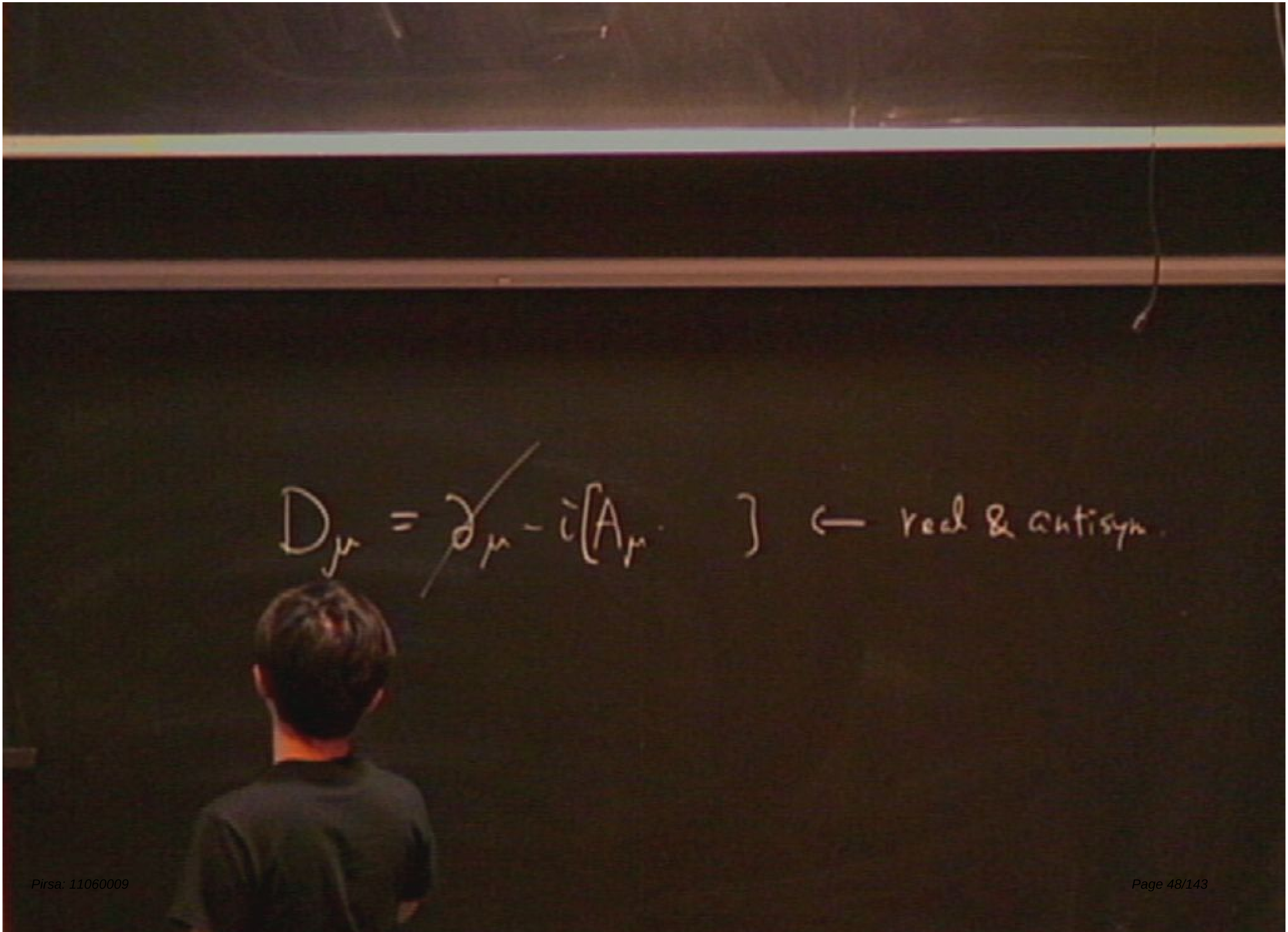
φ^2 term is forbidden in a similar manner

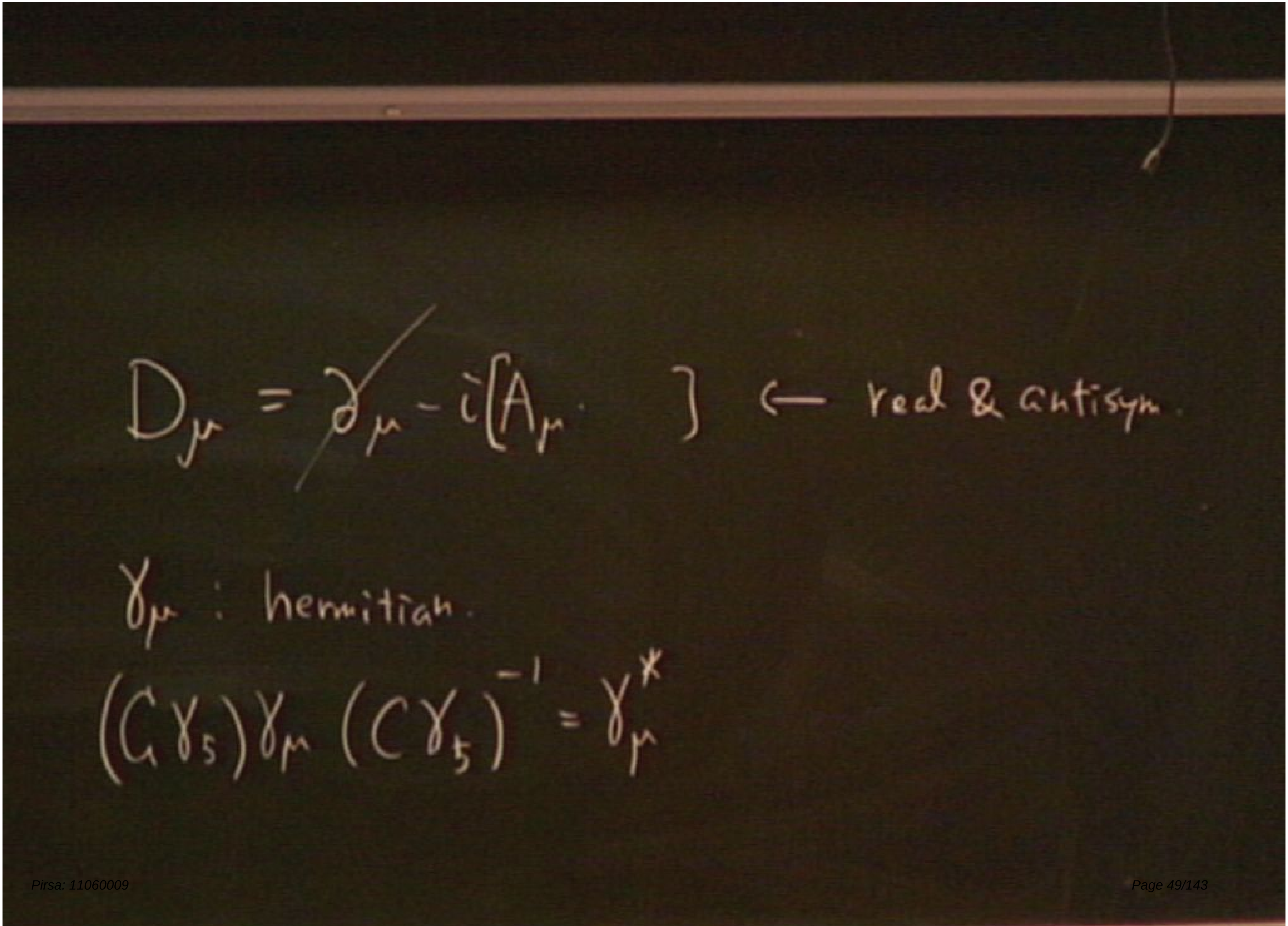
Does it work at nonperturbative level?

4 SUSY model (dimensional redcution of 4d N=1) has been studied extensively.

- Conservation of supercurrents. (Kanamori-Suzuki 2008)
- Comparison to analytic results at small volume. (M.H.-Kanamori 2009)
- Comparison to Cohen-Kaplan-Katz-Unsal model. (M.H.-Kanamori 2010)

All results supports that the correct continuum limit is obtained without performing any fine tuning.


Dμ = ∂μ - i[Aμ,] ← real & antisym.


$D_\mu = \partial_\mu - i[A_\mu \quad] \leftarrow$ real & antisym.
γ_μ : hermitian.
$(C\gamma_5)\gamma_\mu(C\gamma_5)^{-1} = \gamma_\mu^*$

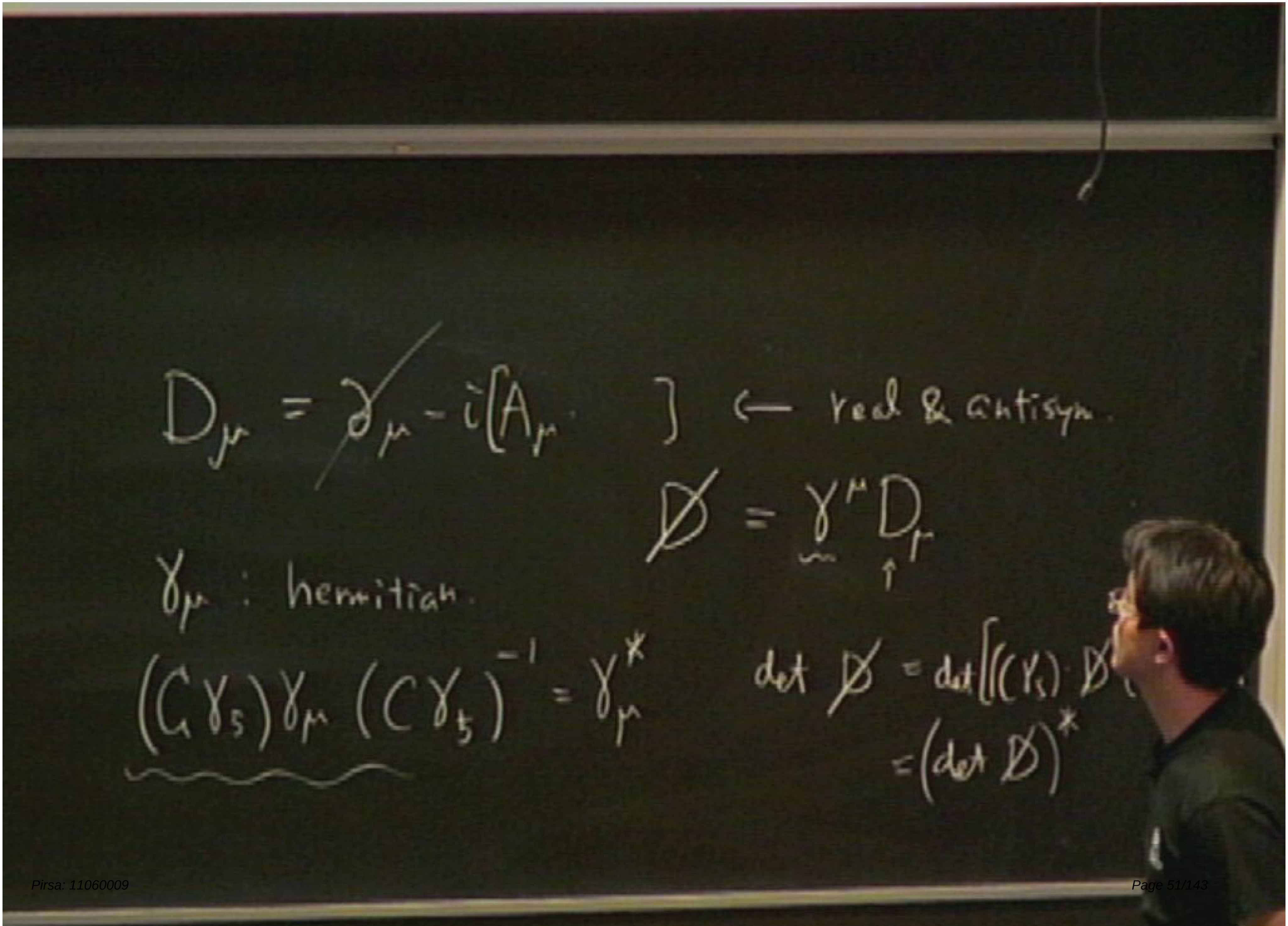
$D_\mu = \partial_\mu - i[A_\mu \quad]$ ← real & antisym.

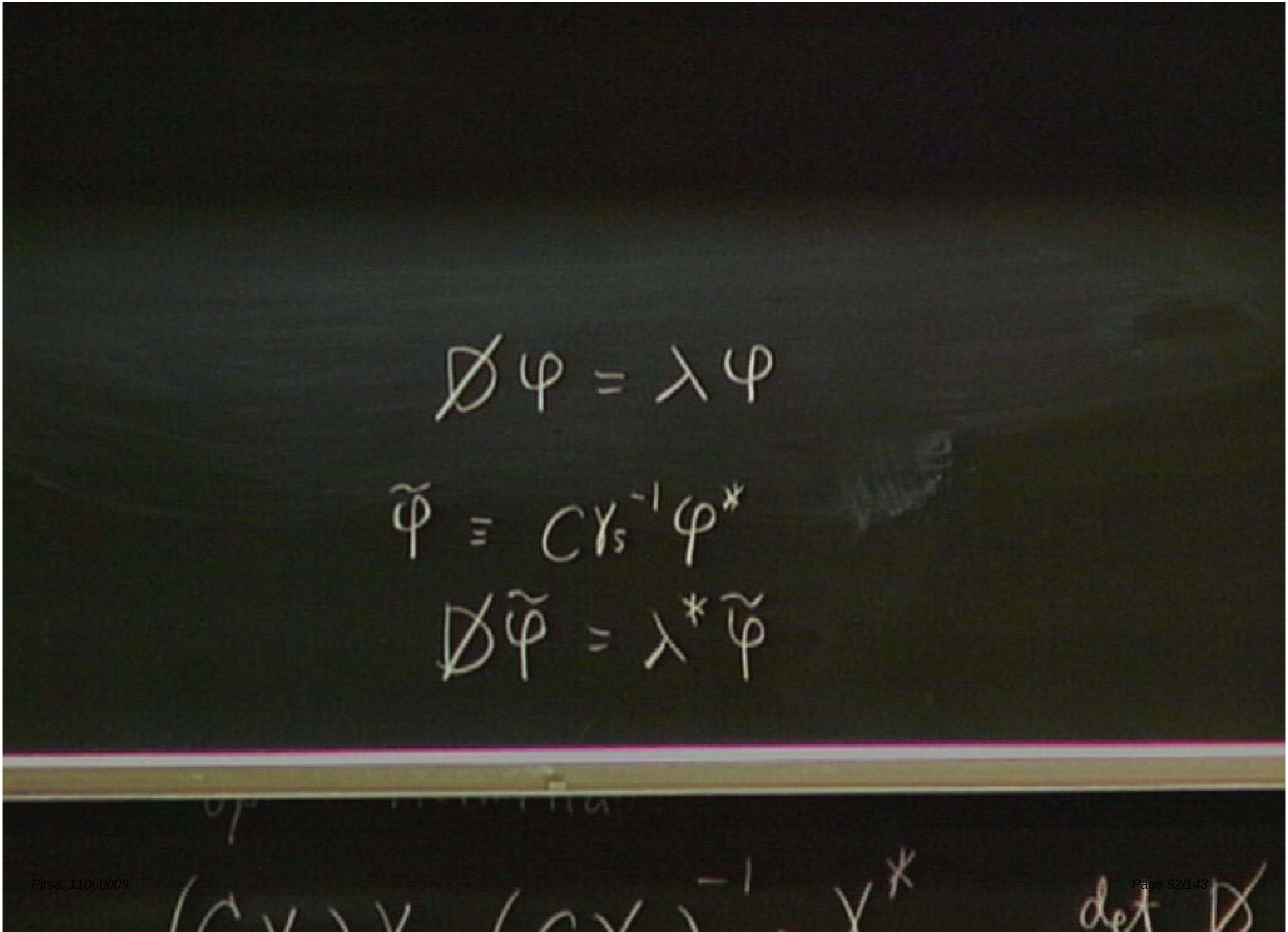
$$\not{D} = \gamma^\mu D_\mu$$

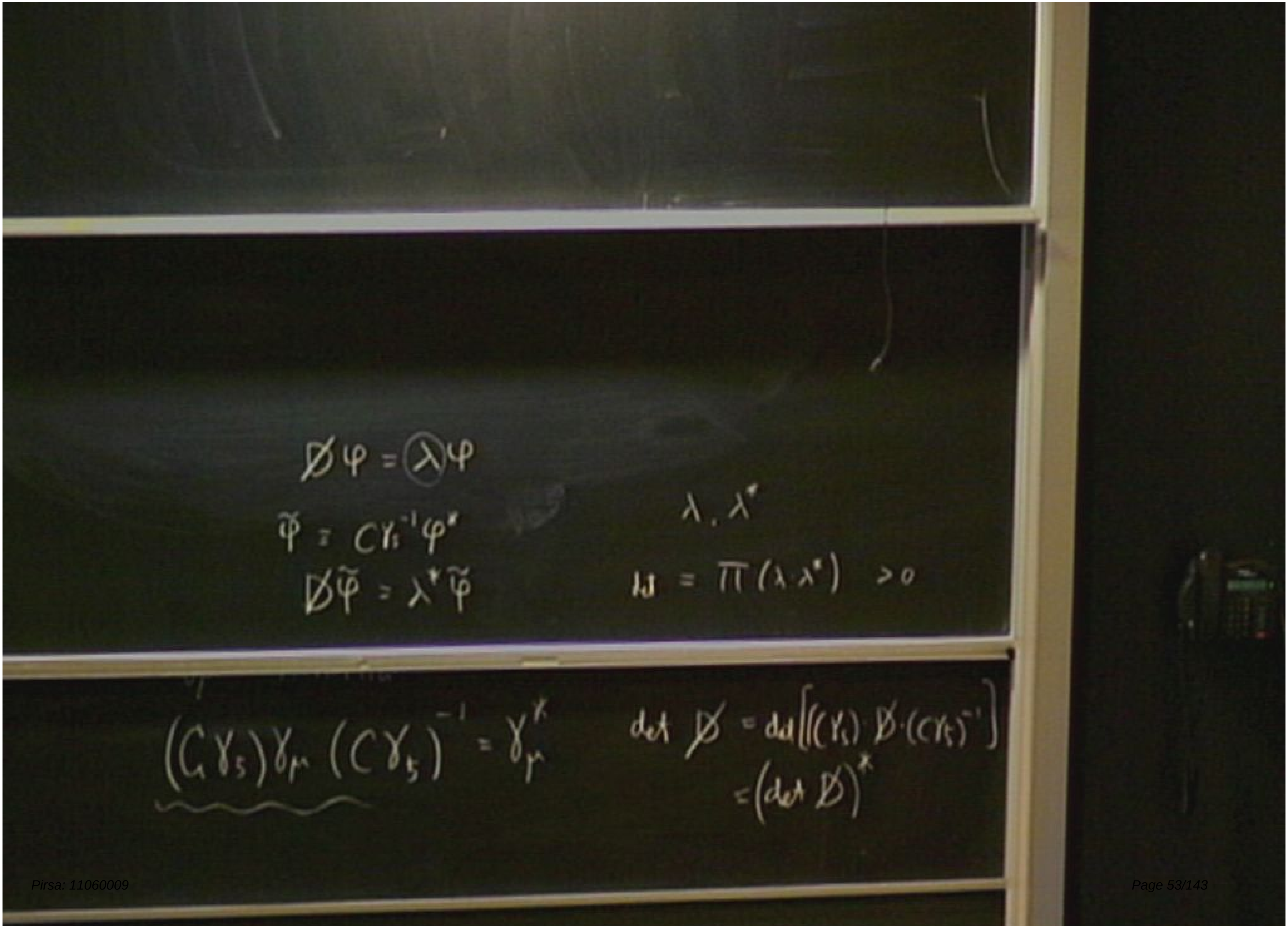
γ_μ : hermitian.

$$(C\gamma_5)\gamma_\mu(C\gamma_5)^{-1} = \gamma_\mu^*$$

$$\det \not{D} = \det\left[(C\gamma_5)\cdot\not{D}\cdot(C\gamma_5)^{-1}\right] = (\det \not{D})^*$$


$D_\mu = \partial_\mu - i[A_\mu, \ \ ]$ ← real & antisym.
$\not{D} = \gamma^\mu D_\mu$
γ_μ : hermitian.
$(C\gamma_5)\gamma_\mu(C\gamma_5)^{-1} = \gamma_\mu^*$
$\det \not{D} = \det[(C\gamma_5) \not{D}$
$= (\det \not{D})^*$

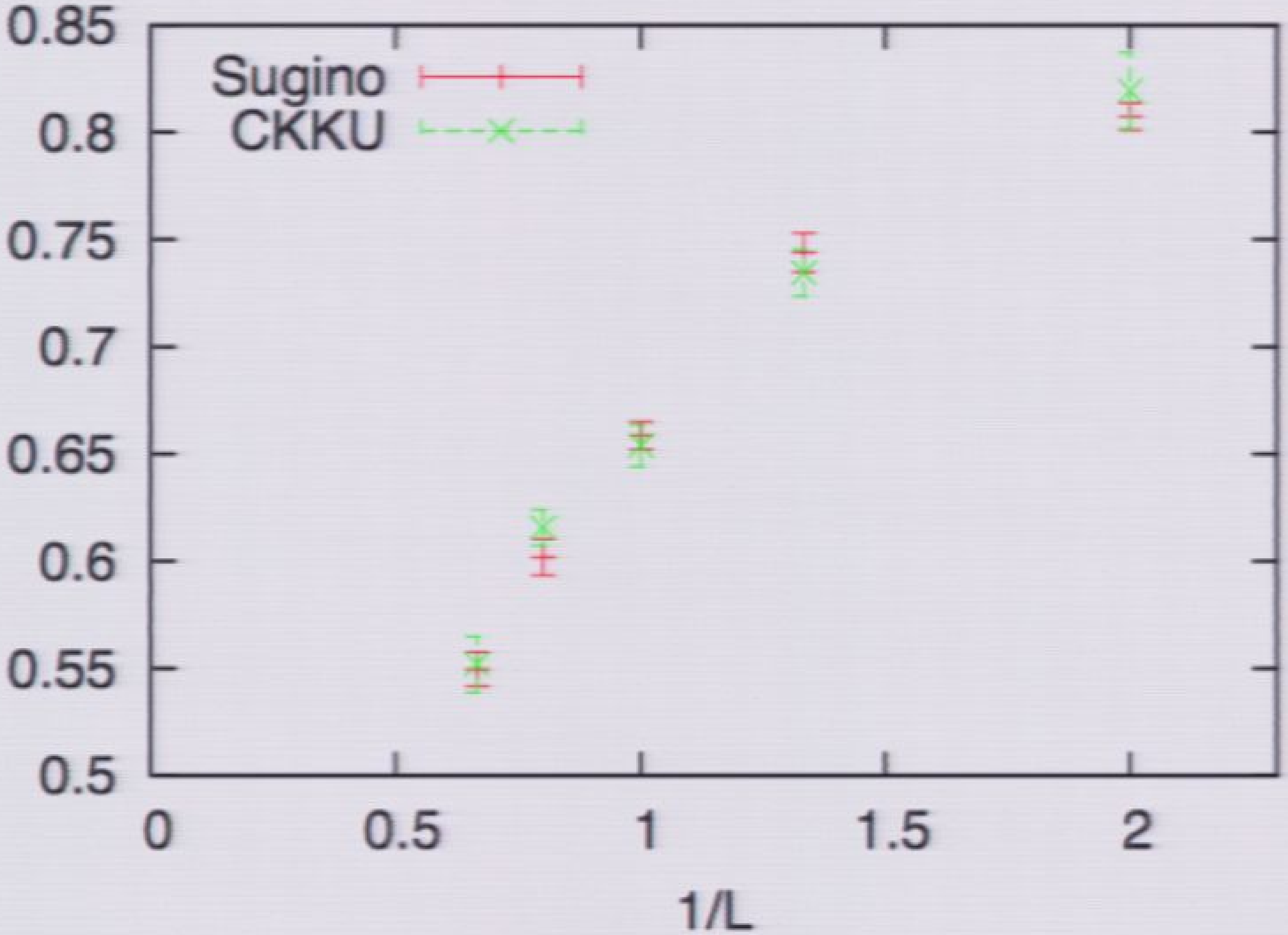

$\not{D}\varphi = \lambda\varphi$
$\tilde{\varphi} \equiv C\gamma_5^{-1}\varphi^*$
$\not{D}\tilde{\varphi} = \lambda^*\tilde{\varphi}$


$\not{\partial}\varphi = \lambda\varphi$
$\tilde{\varphi} = C\gamma_5^{-1}\varphi^*$
$\not{\partial}\tilde{\varphi} = \lambda^*\tilde{\varphi}$
λ, λ^*
$= \Pi(\lambda \cdot \lambda^*) > 0$
$(C\gamma_5)\gamma_\mu(C\gamma_5)^{-1} = \gamma_\mu^*$
$\det \not{\partial} = \det[(C\gamma_5)\not{\partial}(C\gamma_5)^{-1}]$
$= (\det \not{\partial})^*$

4 SUSY model (dimensional redcution of 4d N=1) has been studied extensively.

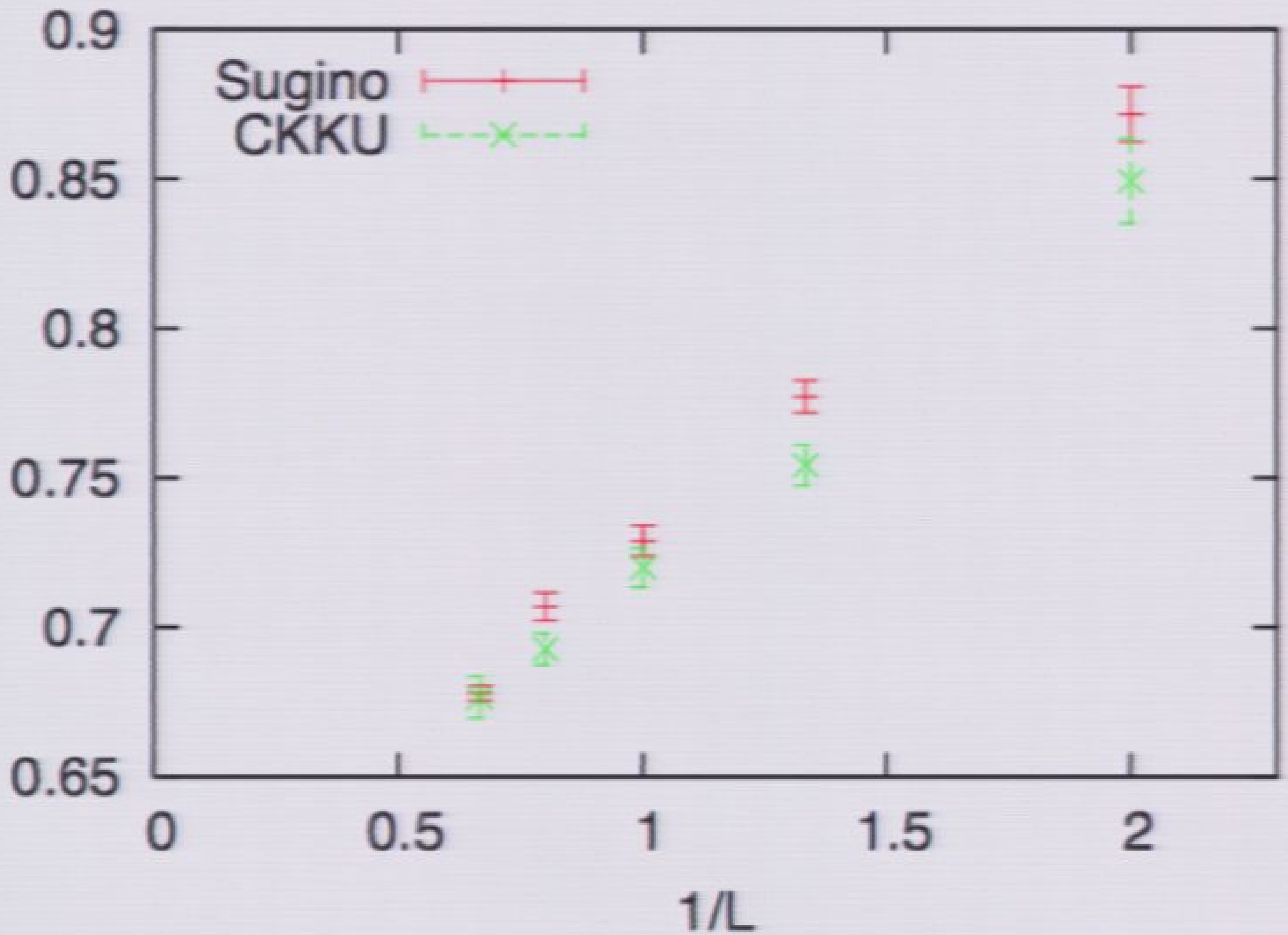
- Conservation of supercurrents. (Kanamori-Suzuki 2008)
- Comparison to analytic results at small volume. (M.H.-Kanamori 2009)
- Comparison to Cohen-Kaplan-Katz-Unsal model. (M.H.-Kanamori 2010)

All results supports that the correct continuum limit is obtained without performing any fine tuning.



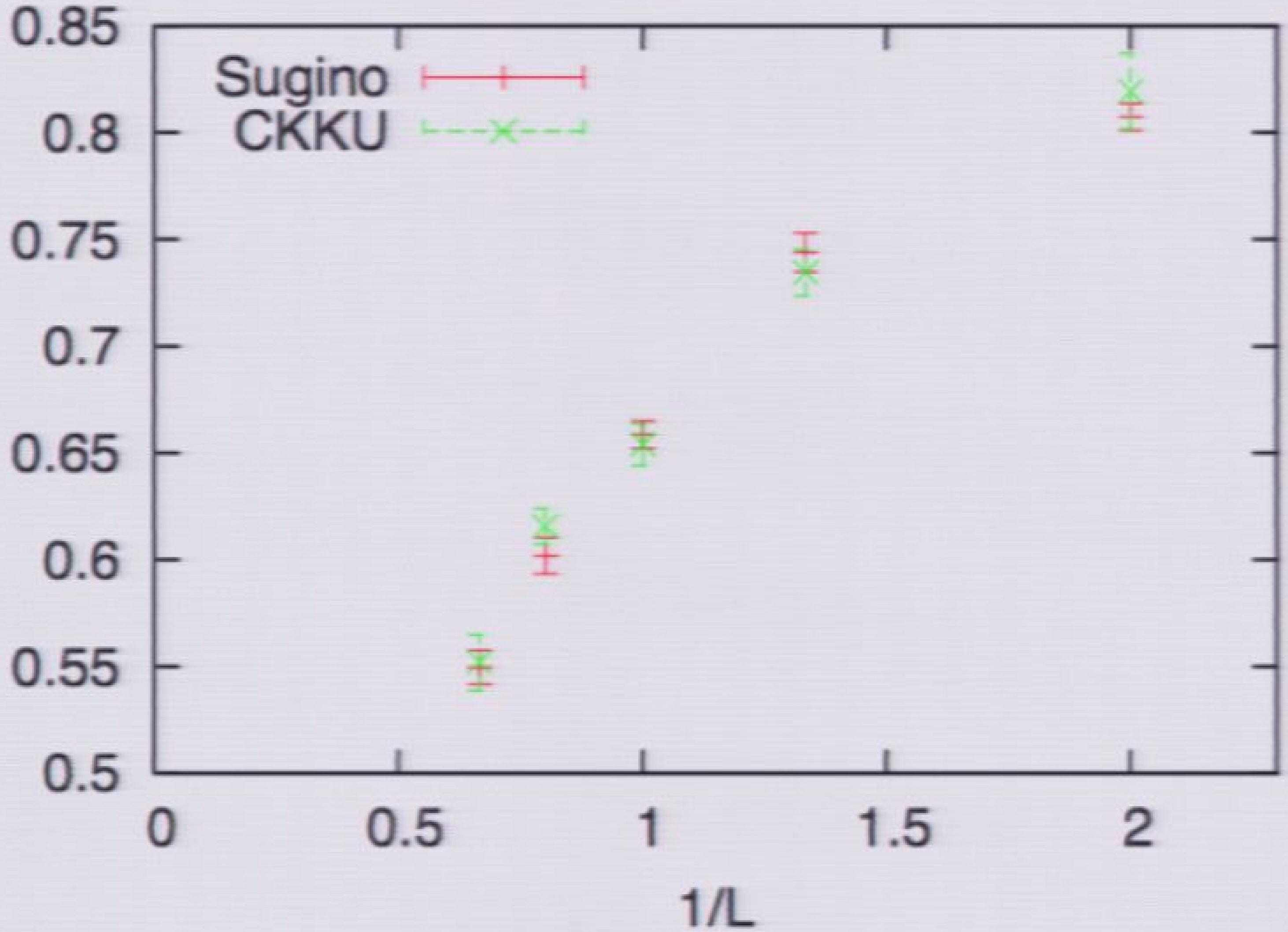
Polyakov loop vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)



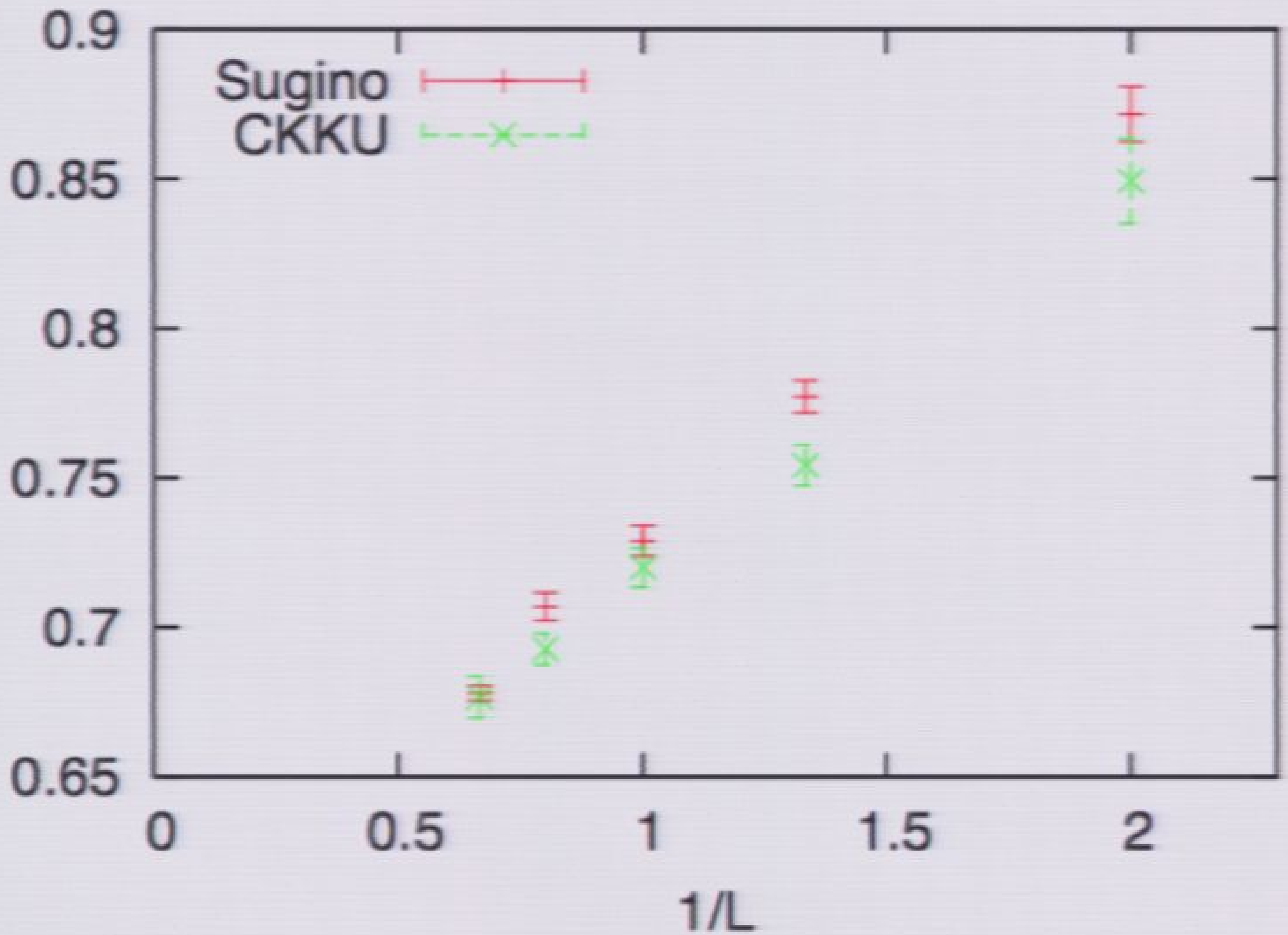
$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)



Polyakov loop vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)



$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

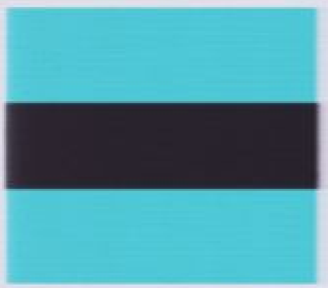
SU(2), periodic b.c. (M.H.-Kanamori 2010)

Application :
black hole/black string transition

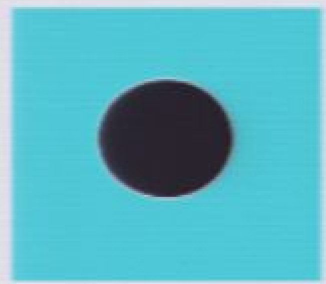
Susskind, Barbon-Kogan-Rabinovici,
Li-Martinec-Sahakian,
Aharony-Marsano-Minwalla-Wiseman,…

SYM simulation : Catterall-Wiseman, 2010

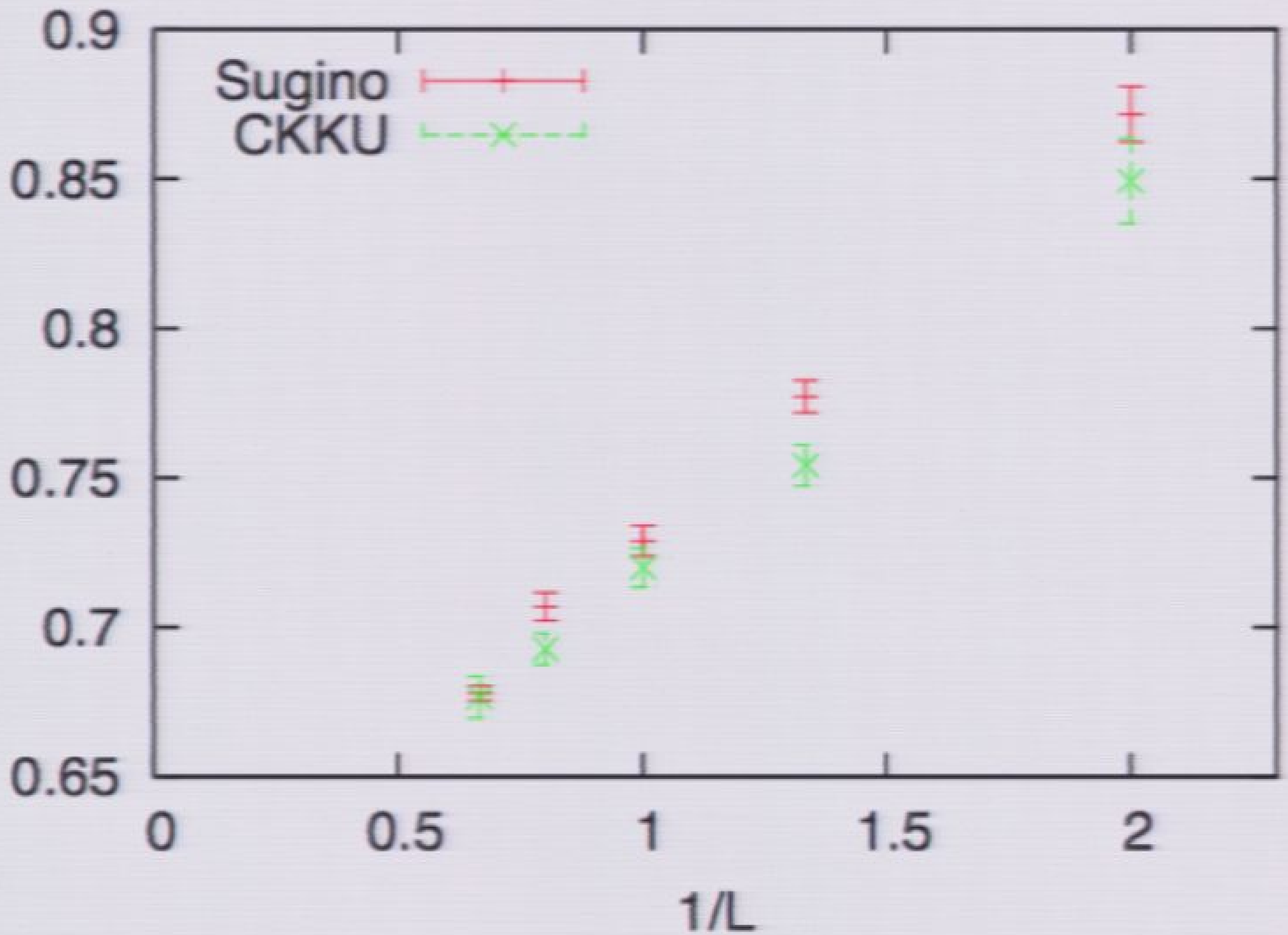
- Consider 2d U(N) SYM on a spatial circle. It describes N D1-branes in $R^{1,8} \times S^1$, winding on S^1.
- T-dual picture : N D0-branes in $R^{1,8} \times S^1$.
- Wilson line phase = position of D0



uniform distribution
= 'black string'

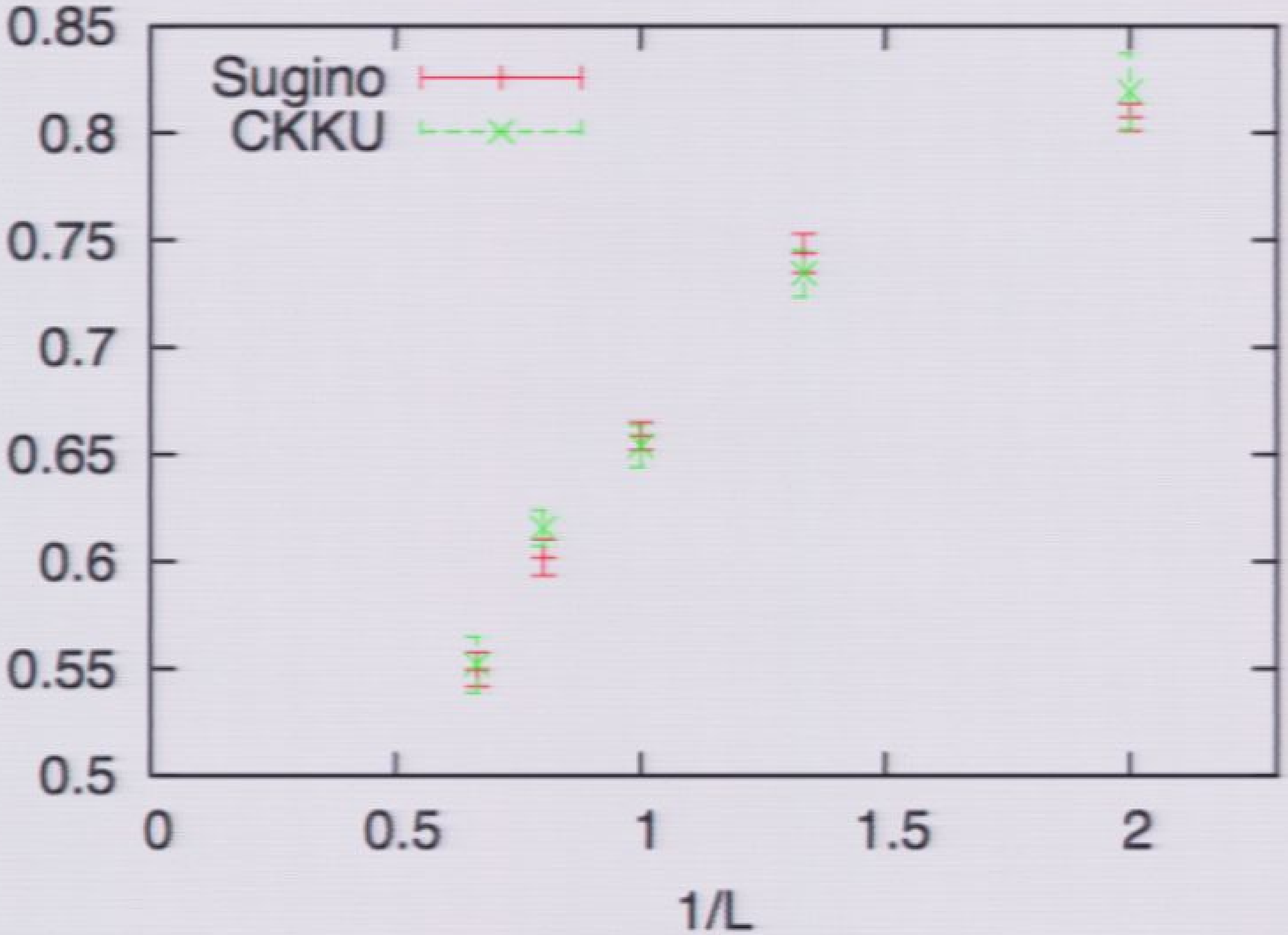


localized distribution
= 'black hole'



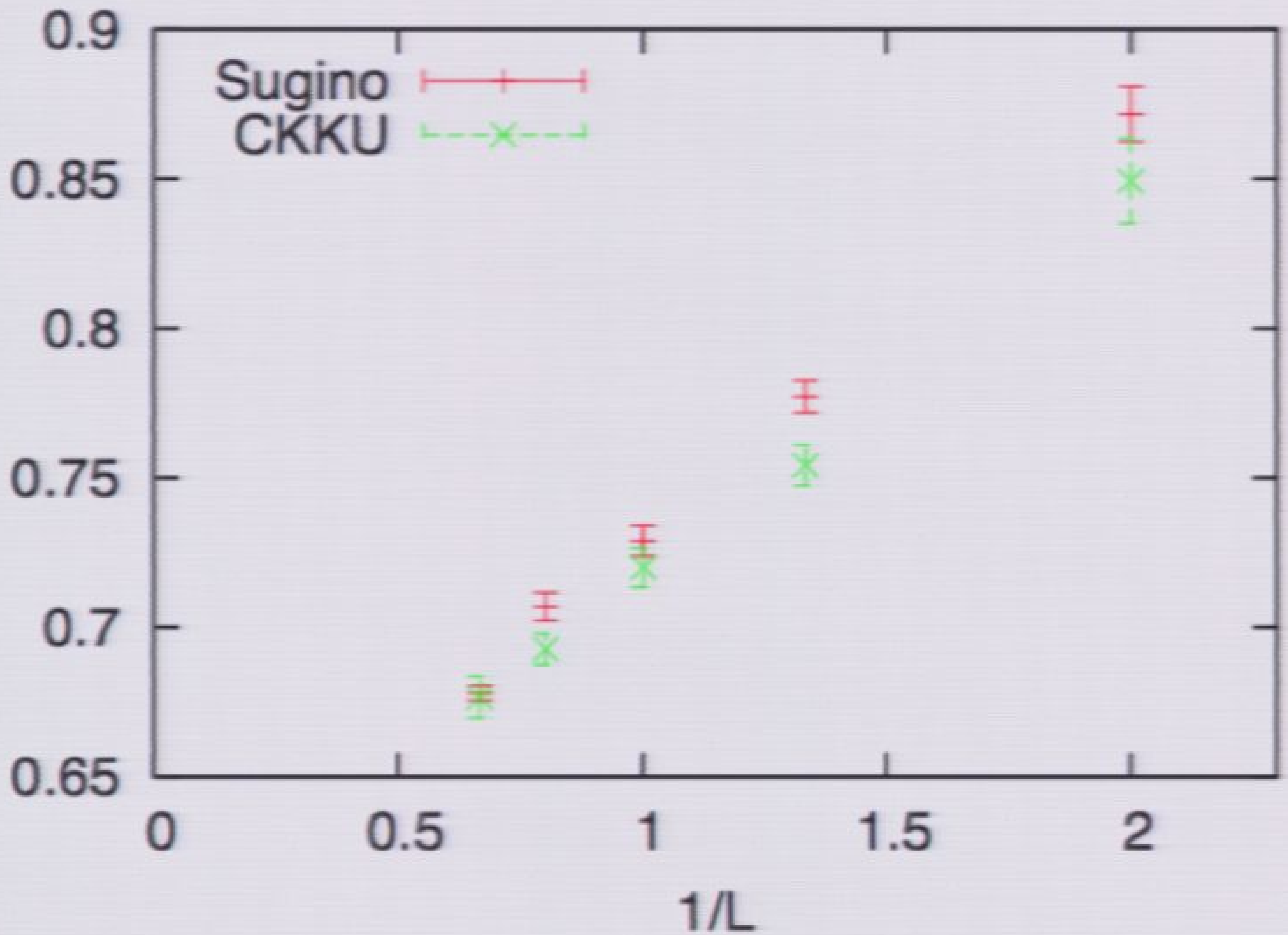
$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)



Polyakov loop vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)



$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)

Application :
black hole/black string transition

Susskind, Barbon-Kogan-Rabinovici,
Li-Martinec-Sahakian,
Aharony-Marsano-Minwalla-Wiseman,…

SYM simulation : Catterall-Wiseman, 2010

- Consider 2d U(N) SYM on a spatial circle. It describes N D1-branes in $R^{1,8} \times S^1$, winding on S^1.
- T-dual picture : N D0-branes in $R^{1,8} \times S^1$.
- Wilson line phase = position of D0

uniform distribution = 'black string'

localized distribution = 'black hole'

Absence of fine tuning
(to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2}c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots\right)\int d^2x\,\mathcal{O}_p(x)$$

tree

up to log(a), where

$$\mathcal{O}_p(x) = \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \qquad p = \alpha + \beta + 3\gamma$$

- Only p=1,2 are dangerous.

$$\varphi, \varphi^2 \qquad (\partial\varphi \text{ is a total derivative})$$

$SU(2)_R$ allows only TrB_A and TrX_i.

Exact SUSY kills them.

φ^2 term is forbidden in a similar manner.

$$Q_\pm U_\mu(x) = i\psi_{\pm\mu}(x)U_\mu(x),$$
$$Q_\pm \psi_{\pm\mu}(x) = i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),$$
$$Q_\mp \psi_{\pm\mu}(x) = \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),$$
$$Q_\pm \tilde{H}_\mu(x) = -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right]$$
$$\pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right]$$
$$\mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right]$$
$$+\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]$$

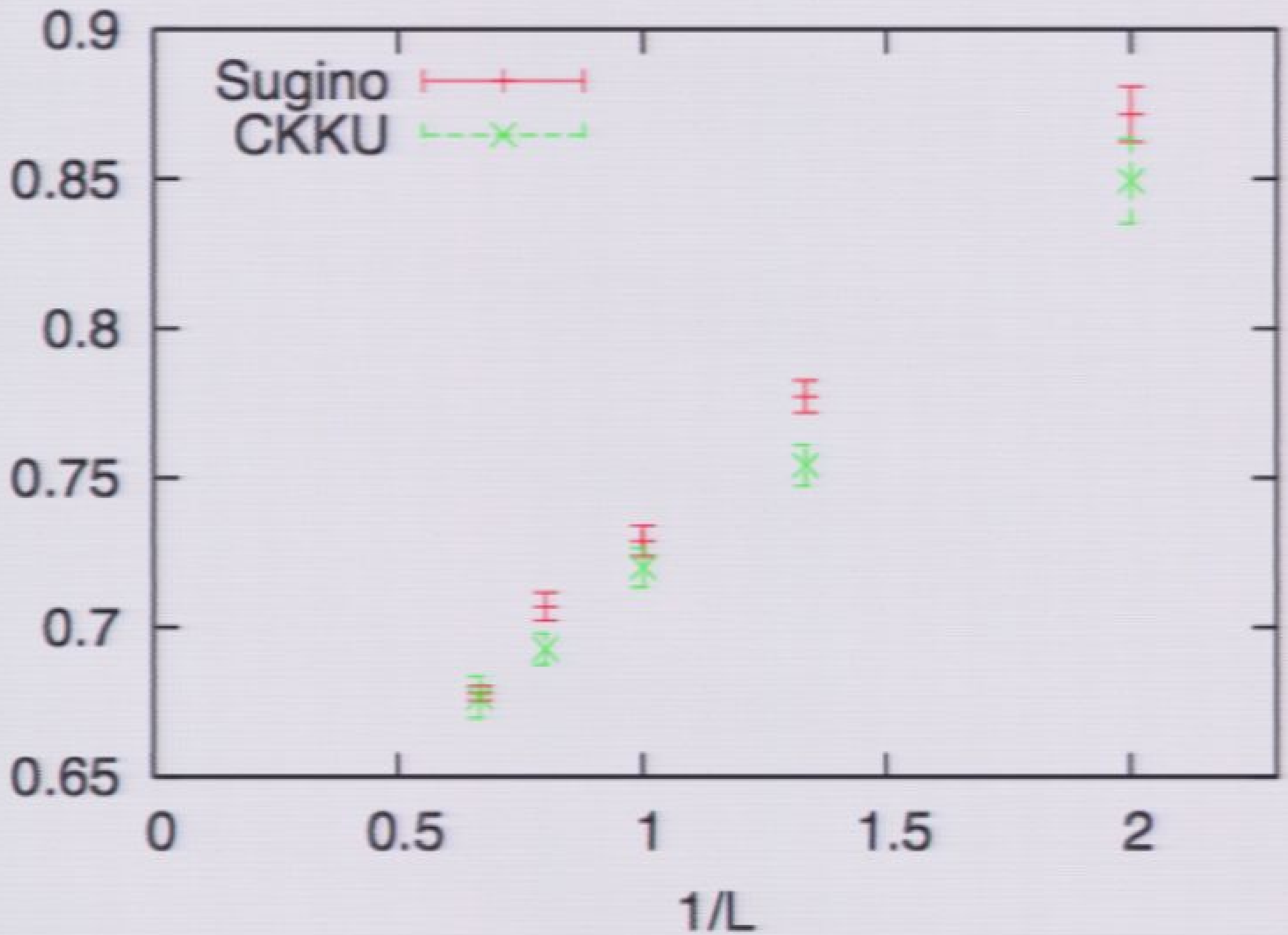
$$D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)$$

Sugino, 2003

$$Q_\pm U_\mu(x) = i\psi_{\pm\mu}(x)U_\mu(x),$$
$$Q_\pm \psi_{\pm\mu}(x) = i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),$$
$$Q_\mp \psi_{\pm\mu}(x) = \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),$$
$$\begin{aligned} Q_\pm \tilde{H}_\mu(x) = &-\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right] \\ &\pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] \\ &\mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right] \\ &+\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right] \end{aligned}$$

$$D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)$$

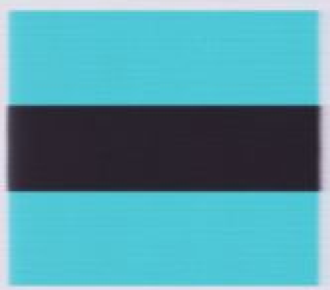
Sugino, 2003



$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)

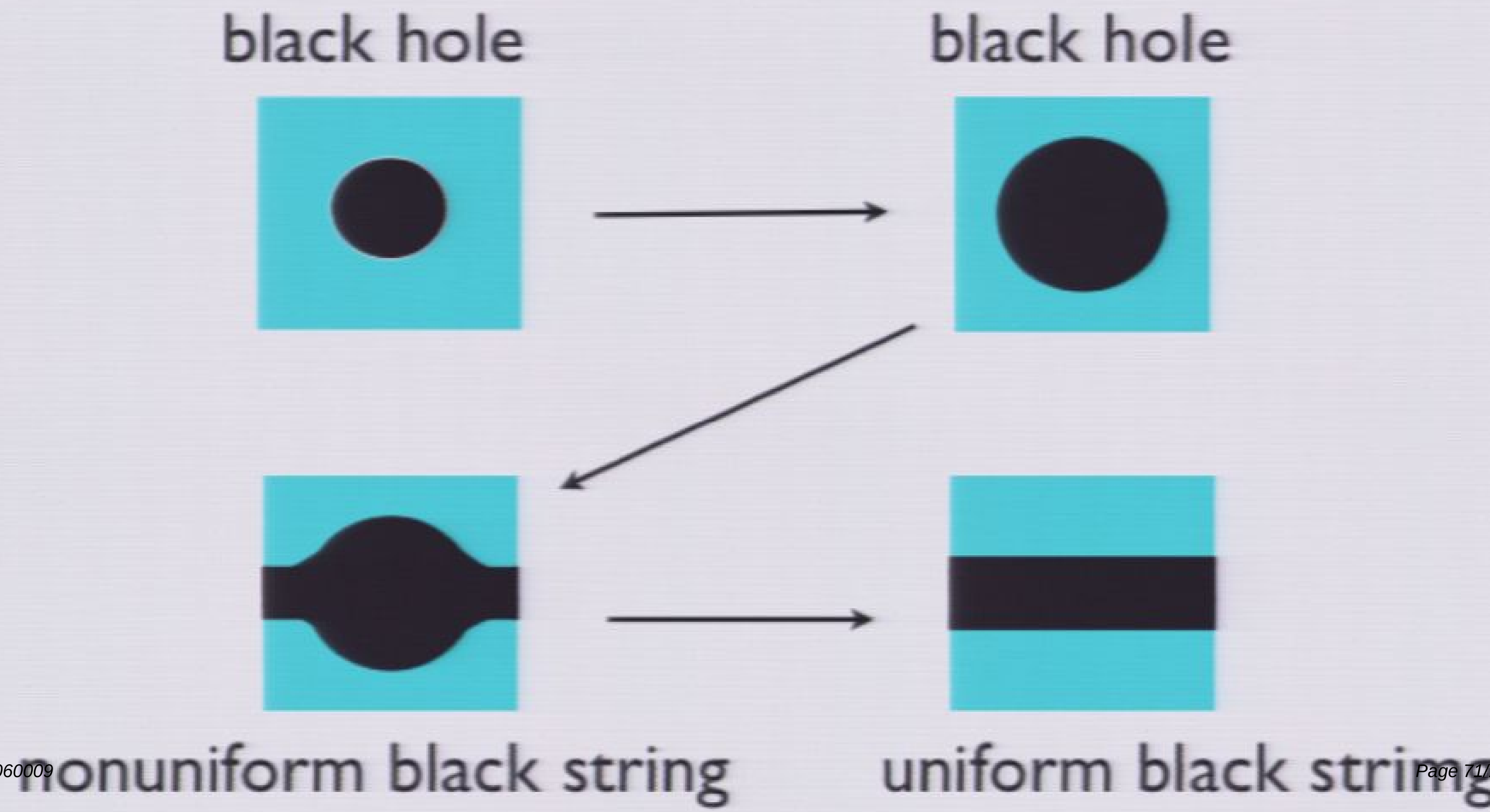
- Consider 2d U(N) SYM on a spatial circle. It describes N D1-branes in $R^{1,8} \times S^1$, winding on S^1.
- T-dual picture : N D0-branes in $R^{1,8} \times S^1$.
- Wilson line phase = position of D0



uniform distribution
= 'black string'

localized distribution
= 'black hole'

Fix the mass (or temparature)
and shrink the compactification radius.
Then...



Counterpart in SYM
= center symmetry breakdown

- Wilson line phase = position of D0

$$W = diag(e^{i\theta_1,\cdots,e^{i\theta_N}})$$

- Center symmetry

$$\theta_i \to \theta_i + const.$$

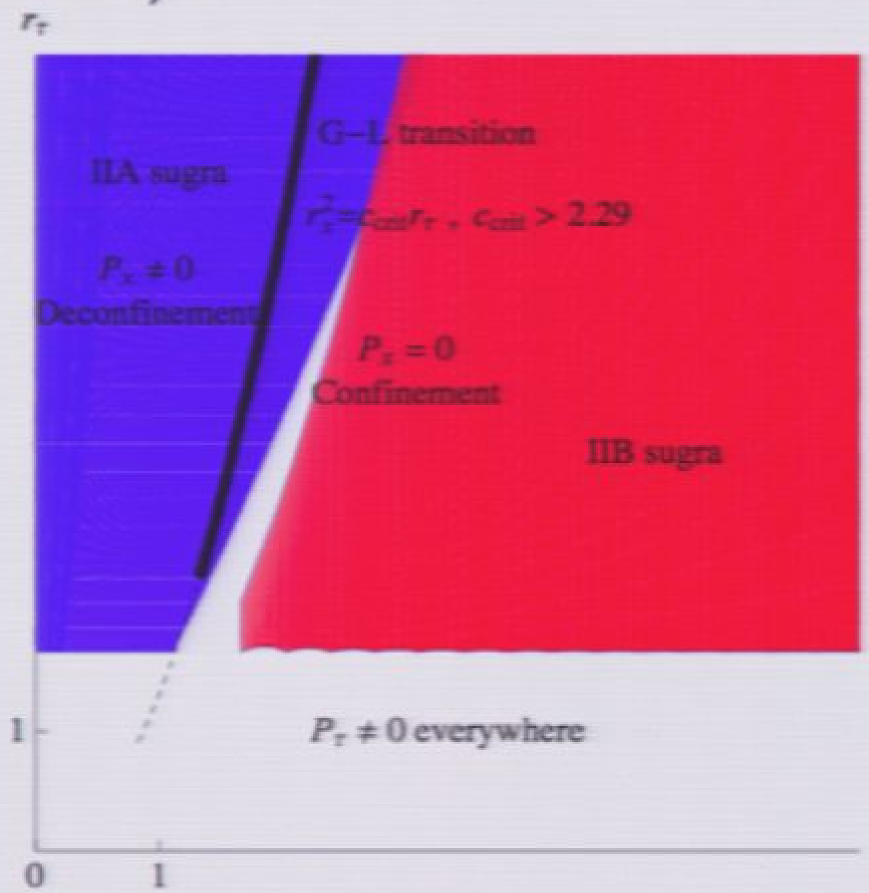
Uniform = center unbroken $\left\langle \frac{1}{N} TrW \right\rangle = 0$

Non-uniform = center broken $\left\langle \frac{1}{N} TrW \right\rangle \neq 0$

Phase diagram

(Theoretical prediction)

(Temperature)$^{-1}$



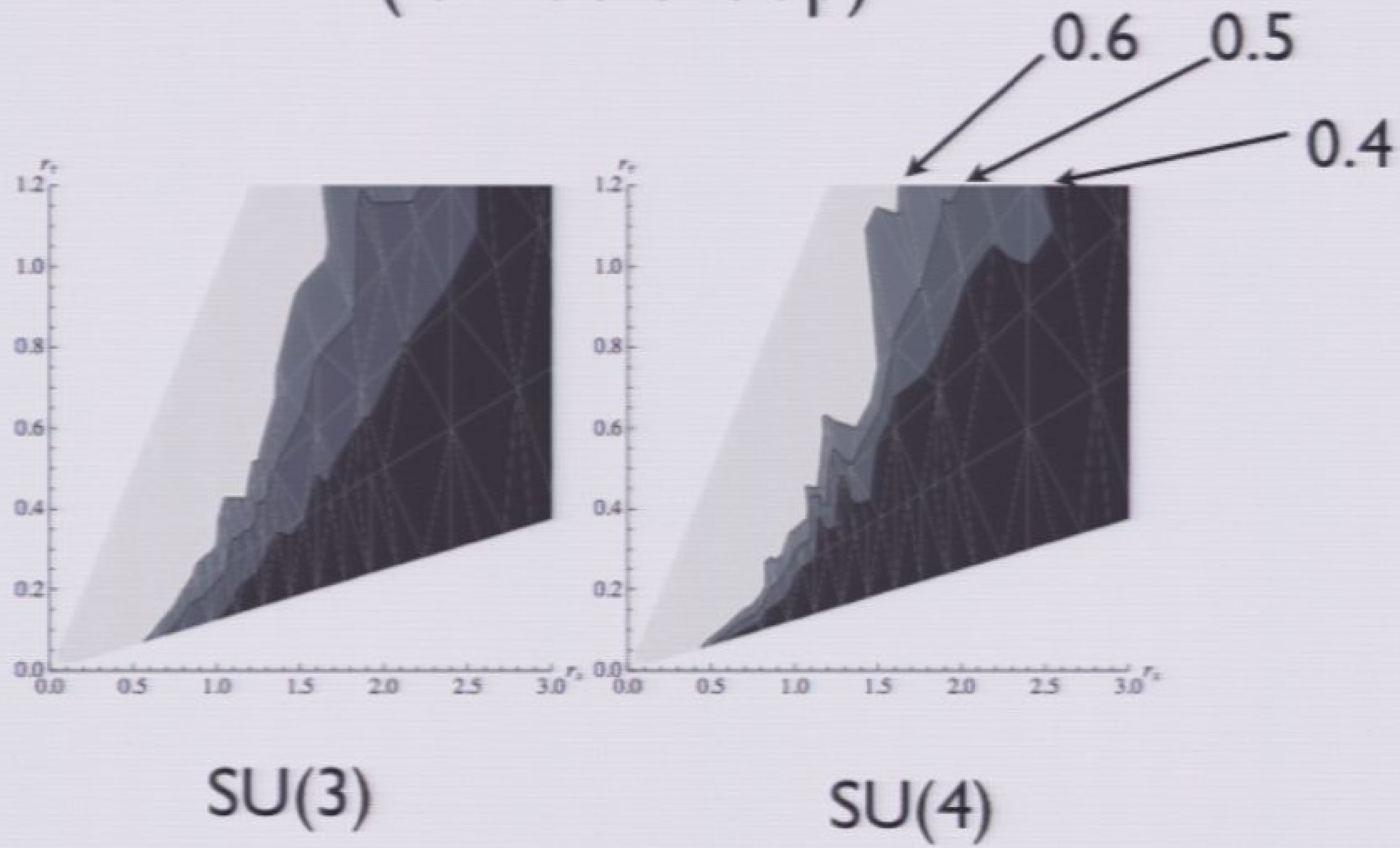
r_x radius of spatial circle

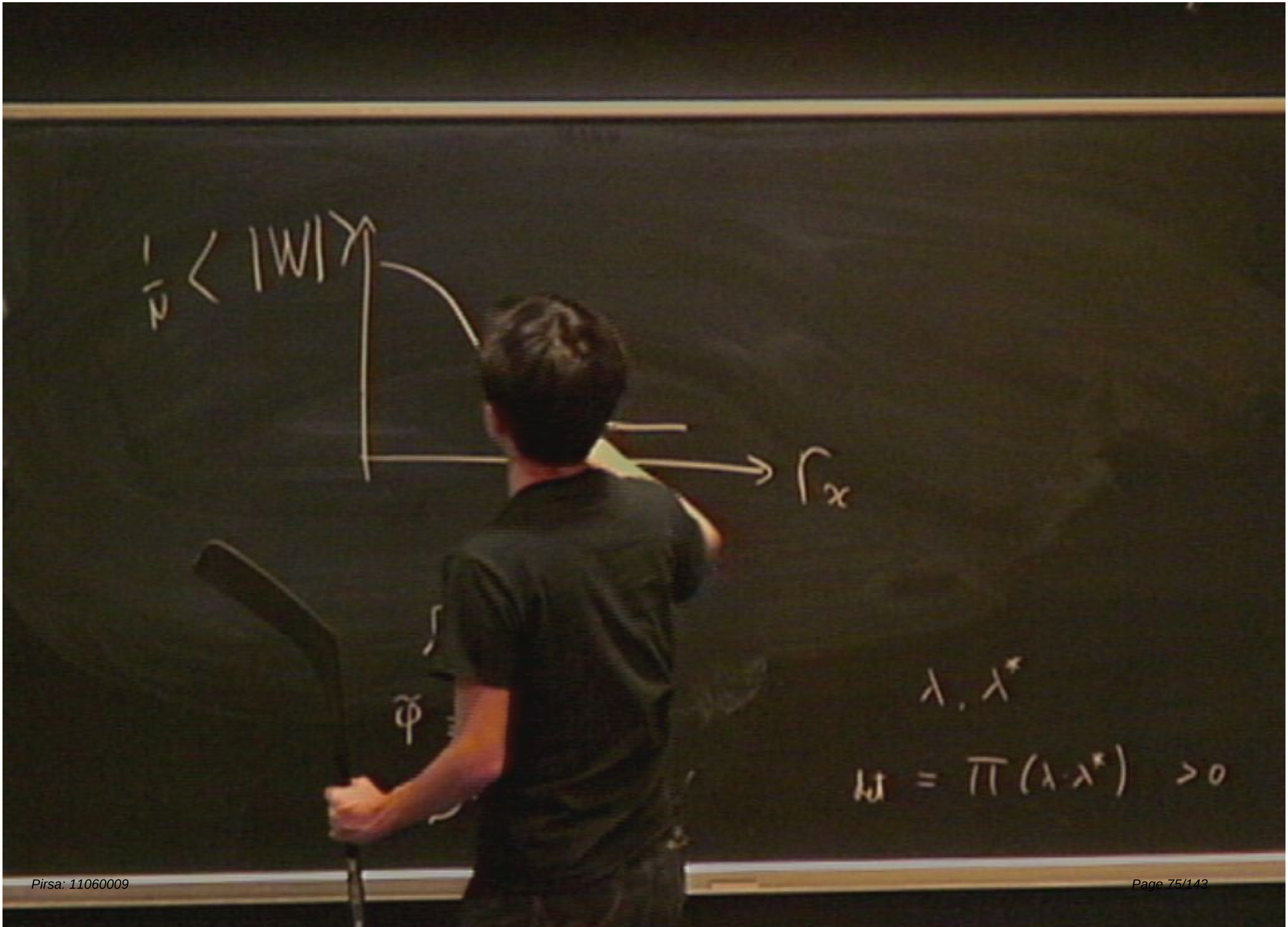
Low temperature:
1st order
BH→uniform BS
(Aharony et al, 2004)

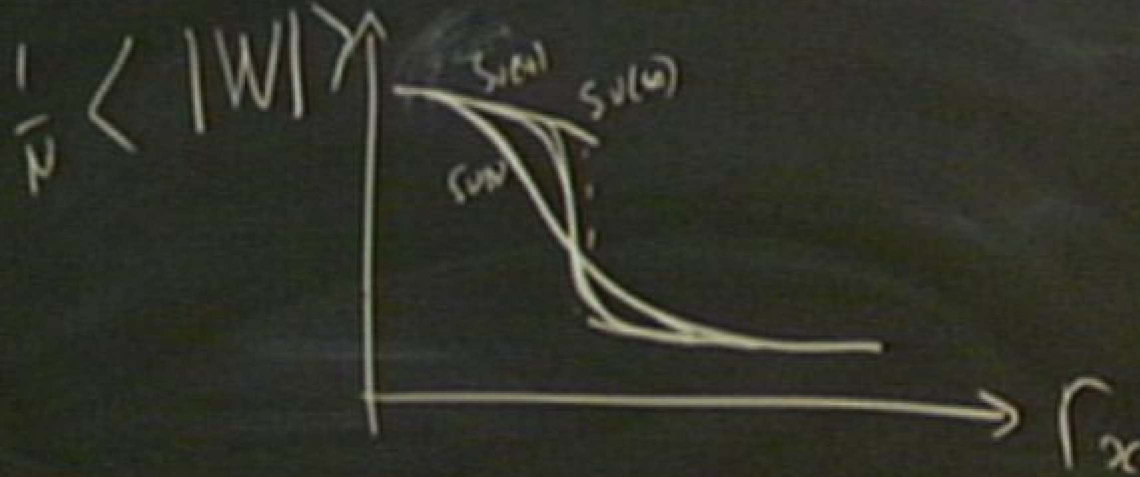
High temperature:
2nd + 3rd
BH→nonuniform BS
→uniform BS
(Kawahara et al, 2007)

Figure from Catterall Wiseman 2010

Value of spatial Wilson loop ('t Hooft loop)




λ , λ*



$$\not{D}\varphi = \lambda\varphi$$

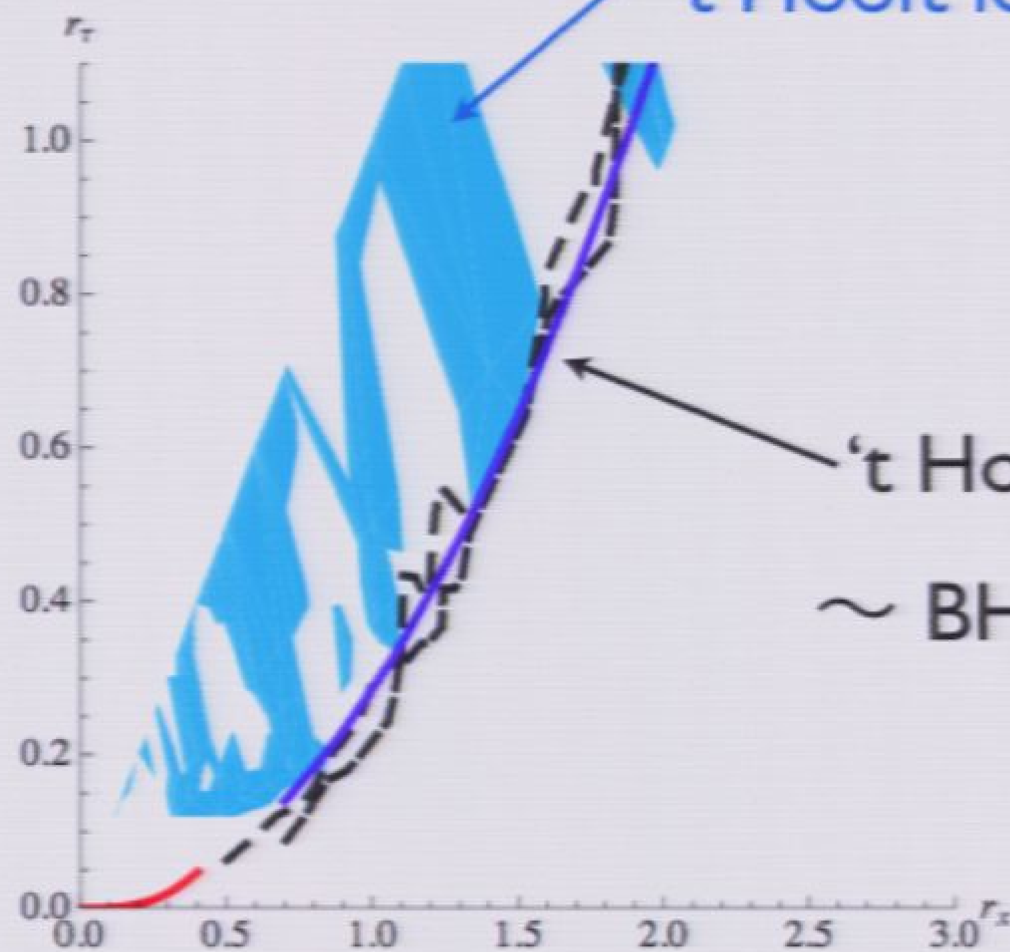
$$\tilde{\varphi} \equiv C\gamma_5^{-1}\varphi^*$$

$$\not{D}\tilde{\varphi} = \lambda^*\tilde{\varphi}$$

$$\lambda, \lambda^*$$

$$\det = \prod(\lambda\cdot\lambda^*) > 0$$

SU(4) gives bigger value of
‘t Hooft loop than SU(3)



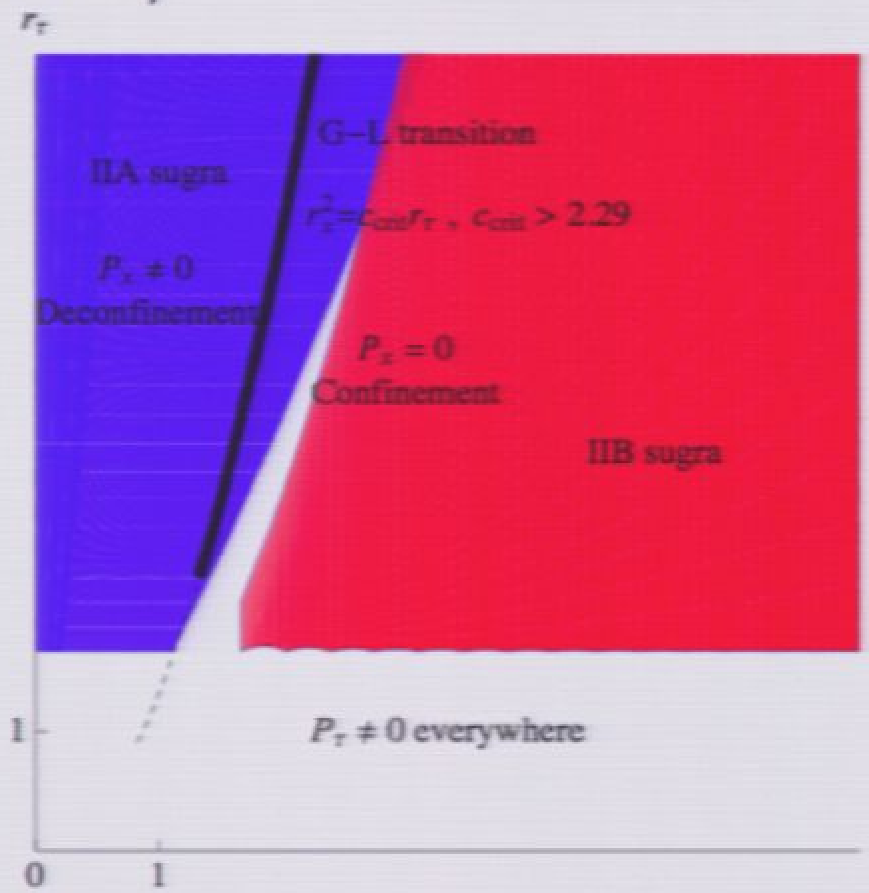
‘t Hooft loop = 0.5

∼ BH/BS transition

Phase diagram

(Theoretical prediction)

(Temperature)$^{-1}$



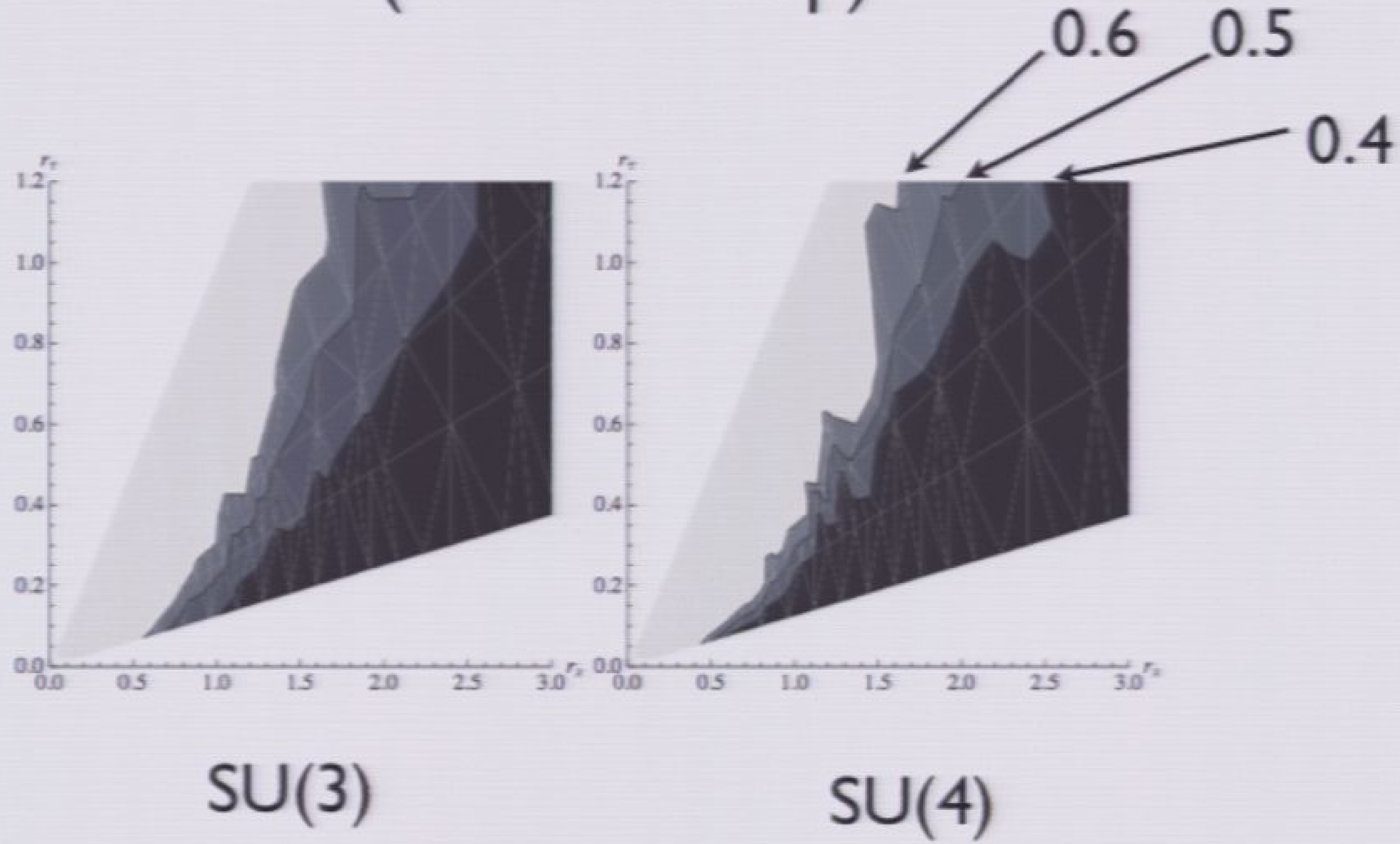
radius of spatial circle

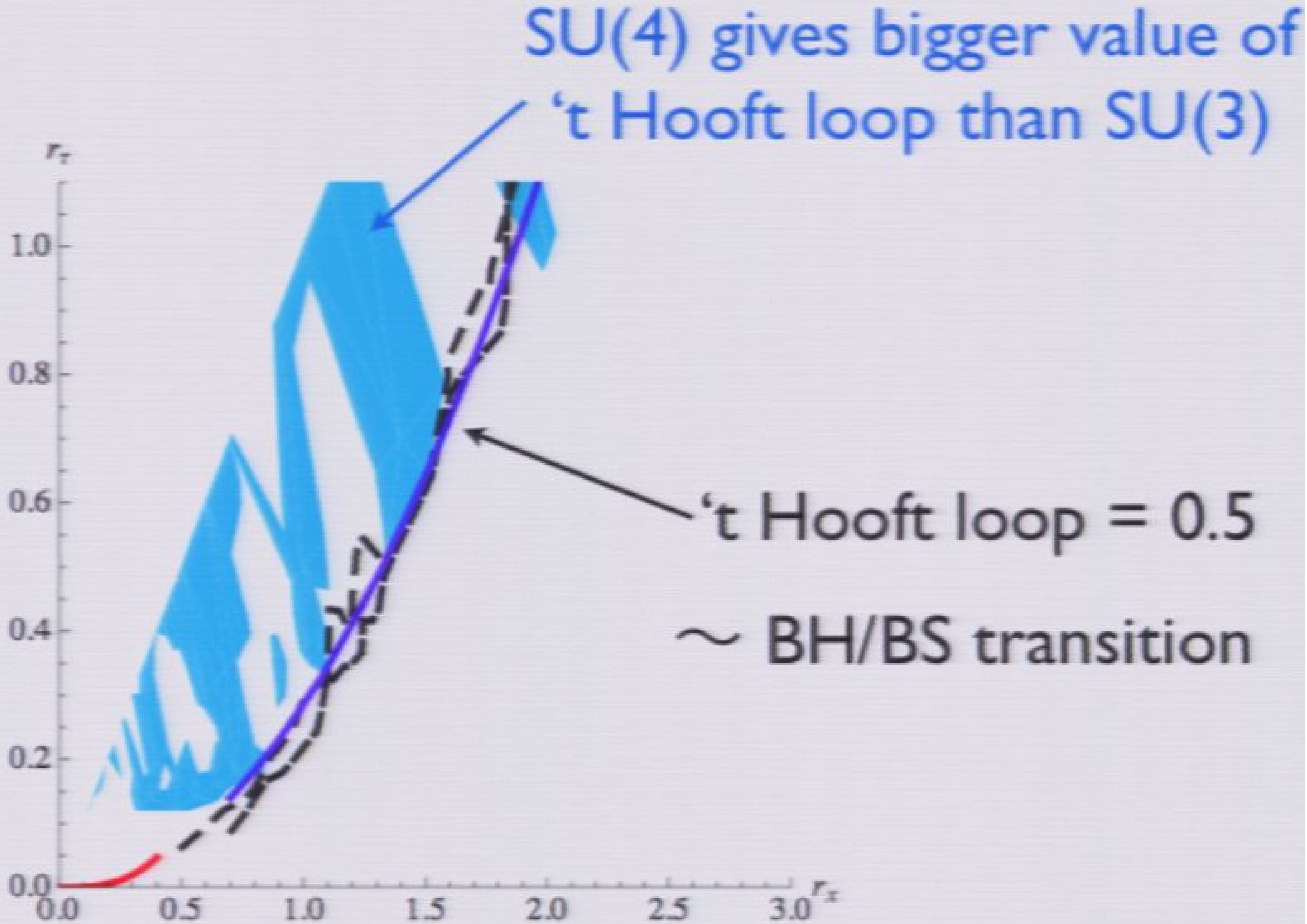
Low temperature:
1st order
BH→uniform BS
(Aharony et al, 2004)

High temperature:
2nd + 3rd
BH→nonuniform BS
→uniform BS
(Kawahara et al, 2007)

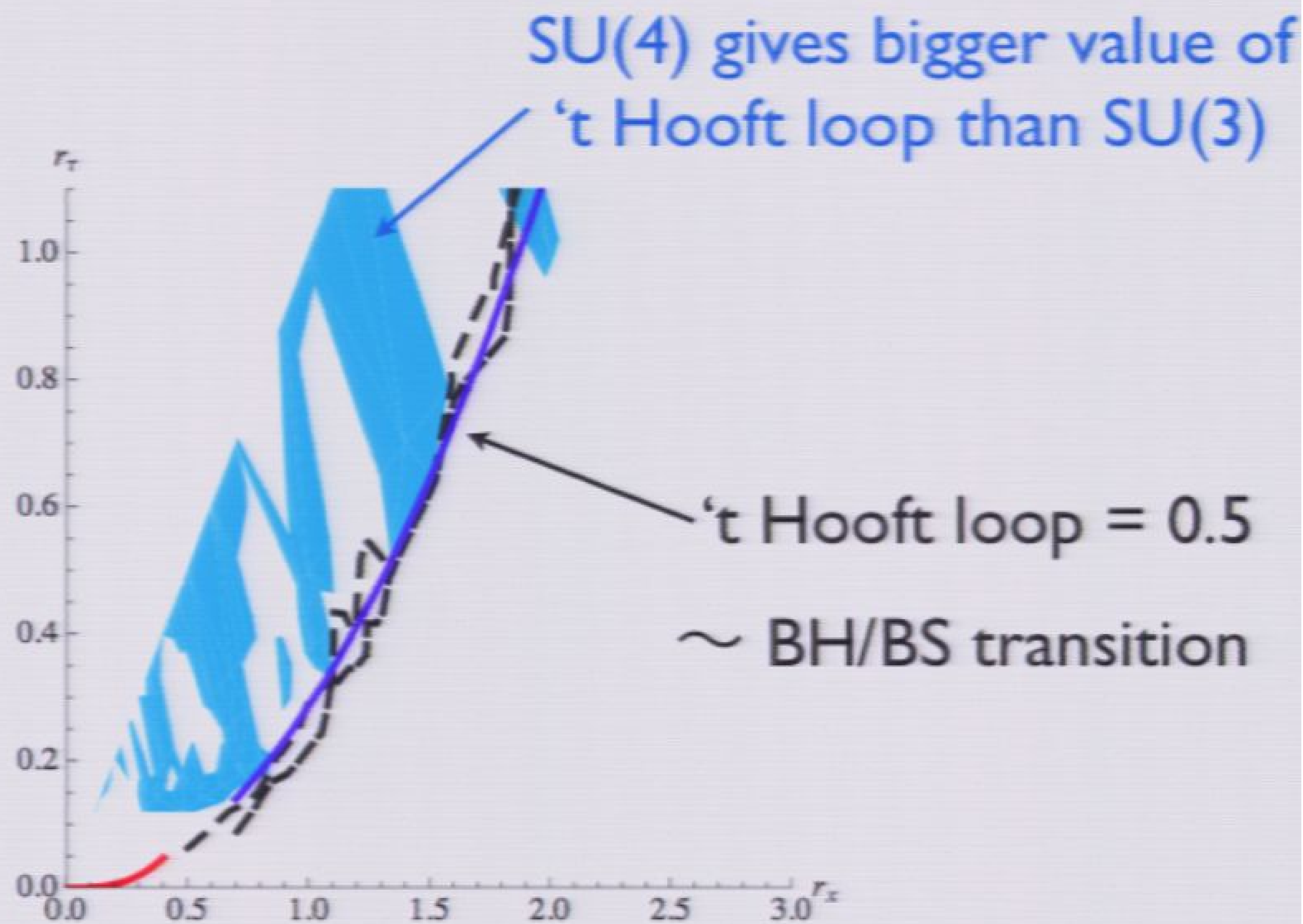
Figure from Catterall-Wiseman 2010

Value of spatial Wilson loop
('t Hooft loop)

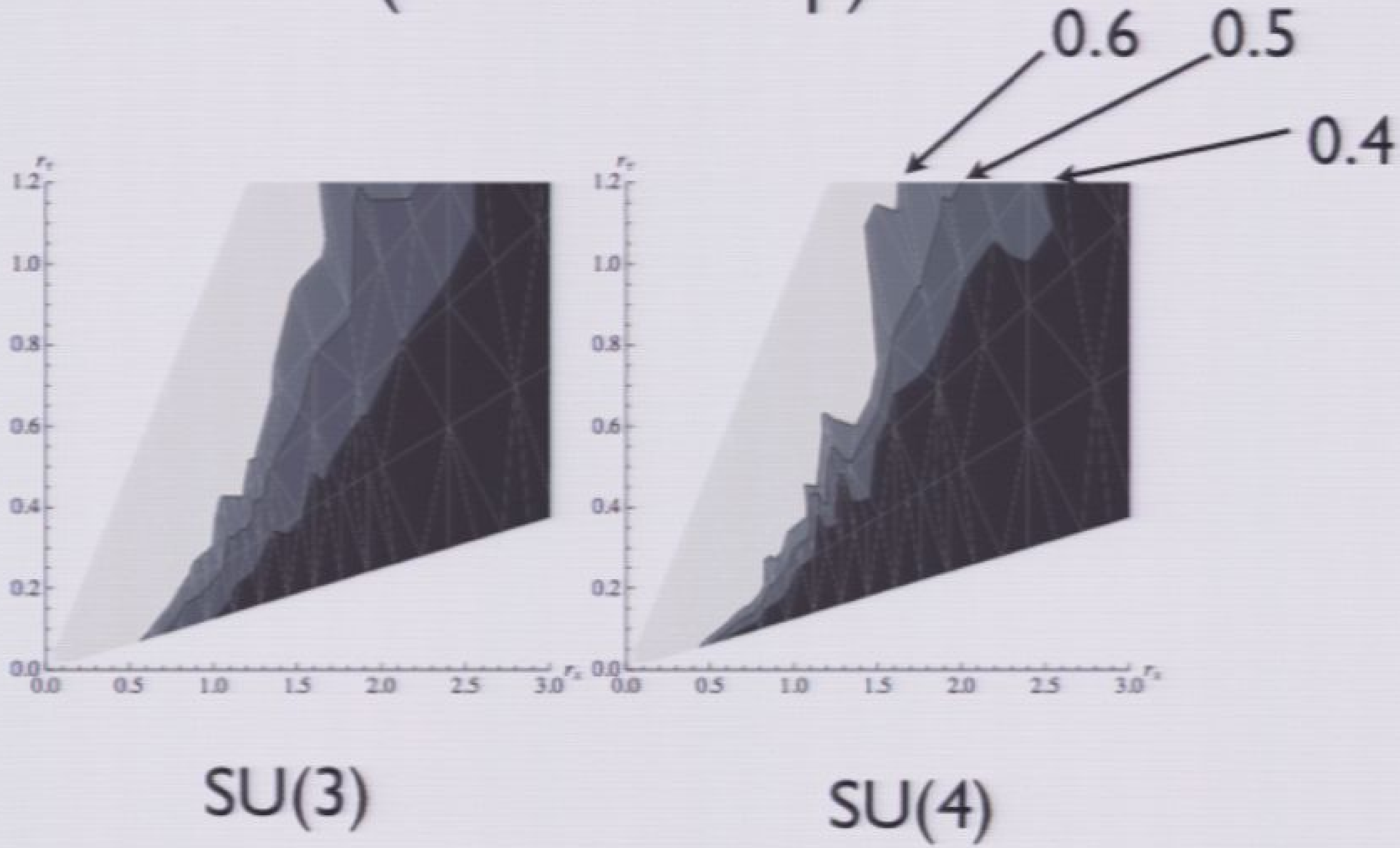



SU(4) gives bigger value of
't Hooft loop than SU(3)
r_τ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
r_x
't Hooft loop = 0.5
~ BH/BS transition

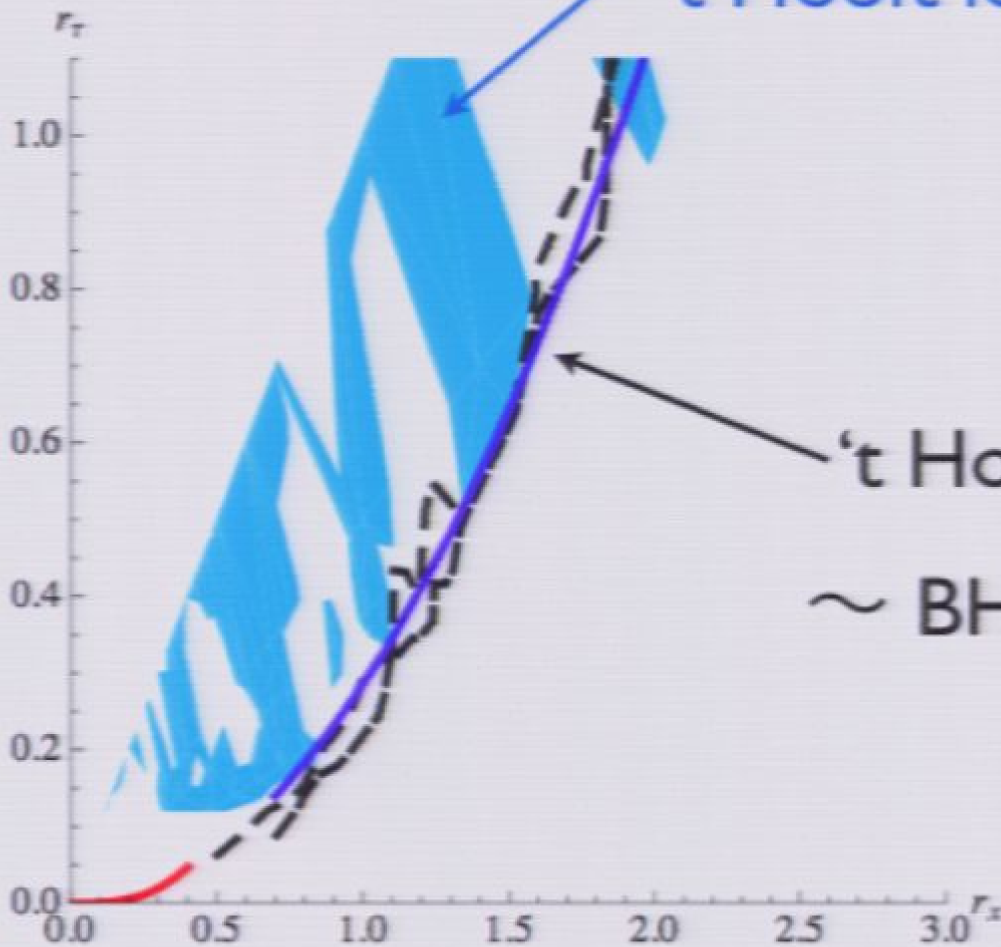
4D N=4 SYM


SU(4) gives bigger value of
't Hooft loop than SU(3)
't Hooft loop = 0.5
~ BH/BS transition
r_T
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
r_x

Value of spatial Wilson loop ('t Hooft loop)



SU(4) gives bigger value of
't Hooft loop than SU(3)



't Hooft loop = 0.5

~ BH/BS transition

4D N=4 SYM

Outline

- Main idea
- lattice formulation of ‘2d BMN’
- uplift to 4d

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\, Tr \left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc} X_a X_b X_c + \frac{\mu^2}{18} X_a^2 + \frac{\mu^2}{72} X_i^2 \right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Fuzzy sphere solution

$$-[X_b,[X_a,X_b]] + i\mu\epsilon^{abc}X_bX_c + \frac{\mu^2}{9}X_a = 0$$

$$\rightarrow \quad X_a = \frac{\mu}{3}J_a, \quad [J_a,J_b] = i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains (1+2)-d SYM on *noncommutative space*.

  D2 out of D0 (Myers effect)
- With maximal SUSY, commutative limit of the noncommutative space should be smooth.

(suggested by Matusis-Susskind-Toumbas '00; a proof in progress by M.H. and H. Shimada)

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\, Tr\left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc}X_a X_b X_c + \frac{\mu^2}{18}X_a^2 + \frac{\mu^2}{72}X_i^2\right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Fuzzy sphere solution

$$-[X_b,[X_a,X_b]]+i\mu\epsilon^{abc}X_bX_c+\frac{\mu^2}{9}X_a=0$$

$$\rightarrow\quad X_a=\frac{\mu}{3}J_a,\quad [J_a,J_b]=i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains (1+2)-d SYM on *noncommutative space*.

  D2 out of D0 (Myers effect)

- With maximal SUSY, commutative limit of the noncommutative space should be smooth.

(suggested by Matusis-Susskind-Toumbas '00; a proof in progress by M.H. and H. Shimada)

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\, Tr \left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc} X_a X_b X_c + \frac{\mu^2}{18} X_a^2 + \frac{\mu^2}{72} X_i^2 \right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Fuzzy sphere solution

$$-[X_b,[X_a,X_b]]+i\mu\epsilon^{abc}X_bX_c+\frac{\mu^2}{9}X_a=0$$

$$\rightarrow \quad X_a=\frac{\mu}{3}J_a, \quad [J_a,J_b]=i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains (1+2)-d SYM on *noncommutative space*.
  D2 out of D0 (Myers effect)
- With maximal SUSY, commutative limit of the noncommutative space should be smooth.

(suggested by Matusis-Susskind-Toumbas '00; a proof in progress by M.H. and H. Shimada)

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\, Tr\left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc}X_a X_b X_c + \frac{\mu^2}{18}X_a^2 + \frac{\mu^2}{72}X_i^2\right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Fuzzy sphere solution

$$-[X_b,[X_a,X_b]] + i\mu\epsilon^{abc}X_bX_c + \frac{\mu^2}{9}X_a = 0$$

$$\rightarrow \quad X_a = \frac{\mu}{3}J_a, \quad [J_a,J_b] = i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains (1+2)-d SYM on *noncommutative space*.

  D2 out of D0 (Myers effect)

- With maximal SUSY, commutative limit of the noncommutative space should be smooth.

(suggested by Matusis-Susskind-Toumbas '00; a proof in progress by M.H. and H. Shimada)

Q. How can we construct 4d theory (D3-brane) ?

A. From D1-branes through the Myers effect.

Crucial point

D1-brane theory (2d SYM) can be formulated on lattice without fine tuning!

- Take 2d continuum limit first, then large-N
- Similar anisotropic continuum limit was taken on 4 lattice, in order to reduce the number of fine tuning parameters. (Kaplan-Katz-Unsal, 2003)
- Analogous to "deconstruction" of 5d out of 4d (Arkani Hamed-Cohen-Georgi, 2001)

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\, Tr\left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc}X_a X_b X_c + \frac{\mu^2}{18}X_a^2 + \frac{\mu^2}{72}X_i^2\right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Fuzzy sphere solution

$$-[X_b,[X_a,X_b]] + i\mu\epsilon^{abc}X_bX_c + \frac{\mu^2}{9}X_a = 0$$

$$\rightarrow \quad X_a = \frac{\mu}{3}J_a, \quad [J_a,J_b] = i\epsilon_{abc}J_c$$

- preserves 16 SUSY. Around it one obtains

  (1+2)-d SYM on *noncommutative space*.

  D2 out of D0 (Myers effect)

- With maximal SUSY, commutative limit of

  the noncommutative space should be smooth.

(suggested by Matusis-Susskind-Toumbas '00; a proof in progress by M.H. and H. Shimada)

Q. How can we construct 4d theory (D3-brane) ?

A. From D1-branes through the Myers effect.

Crucial point

D1-brane theory (2d SYM) can be formulated on lattice without fine tuning!

- Take 2d continuum limit first, then large-N
- Similar anisotropic continuum limit was taken on 4 lattice, in order to reduce the number of fine tuning parameters. (Kaplan-Katz-Unsal, 2003)
- Analogous to "deconstruction" of 5d out of 4d (Arkani Hamed-Cohen-Georgi, 2001)

Outline

- Main idea
- lattice formulation of '2d BMN'
- uplift to 4d

"BMN deformation"

$$\Delta S = \frac{1}{g_{2d}^2}\int d^2x\, \mathrm{Tr}\Big\{ \frac{2M^2}{81}\left(B_A^2 + X_i^2\right) - \frac{M}{2} C[\phi_+, \phi_-] + \frac{M^2}{9}\left(\frac{C^2}{4} + \phi_+\phi_-\right) + \frac{2M}{3}\psi_{+\mu}\psi_{-\mu} + \frac{2M}{9}\rho_{+i}\rho_{-i} + \frac{4M}{9}\chi_{+A}\chi_{-A} - \frac{M}{6}\eta_+\eta_- - \frac{4iM}{9}B_3\left(F_{12} + i[X_3, X_4]\right)\Big\}$$

$$\Delta Q_\pm \tilde{H}_\mu = \frac{M}{3}\psi_{\pm\mu}, \qquad \Delta Q_\pm \tilde{h}_i = \frac{M}{3}\rho_{\pm i},$$

$$\Delta Q_\pm H_A = \frac{M}{3}\chi_{\pm A}, \qquad \Delta Q_\pm \eta_\pm = \frac{2M}{3}\phi_\pm,$$

$$\Delta Q_\mp \eta_\pm = \pm\frac{M}{3}C.$$

M.H.-Matsuura-Sugino, 2010

$$Q_+^2 = \frac{M}{3} J_{++}, \quad Q_-^2 = -\frac{M}{3} J_{--},$$
$$\{Q_+, Q_-\} = -\frac{M}{3} J_0,$$

J : SU(2)R generator;
fermions with +/- form doublets

$$S = \left(Q_+ Q_- - \frac{M}{3} \right) \mathcal{F}$$

Q-closed!

M.H.-Matsuura-Sugino, 2010

Absence of fine tuning (to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2} c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots \right) \int d^2x \, \mathcal{O}_p(x)$$

tree

up to log(aM), where

$$\mathcal{O}_p(x) = M^m \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \quad p \equiv m+\alpha+\beta+3\gamma$$

- Only p=1,2 are dangerous.

$$\varphi,\ M\varphi,\ \varphi^2 \qquad (\partial\varphi \text{ is a total derivative})$$

$SU(2)_R$ allows only TrB_A and TrX_i .

Exact SUSY kills them.

φ^2 term is forbidden in a similar manner

$$Q_+^2 = \frac{M}{3} J_{++}, \quad Q_-^2 = -\frac{M}{3} J_{--},$$
$$\{Q_+, Q_-\} = -\frac{M}{3} J_0,$$

J : SU(2)R generator;
fermions with +/- form doublets

$$S = \left(Q_+ Q_- - \frac{M}{3} \right) \mathcal{F}$$

Q-closed!

M.H.-Matsuura-Sugino, 2010

Absence of fine tuning (to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2}c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots\right)\int d^2x\, \mathcal{O}_p(x)$$

tree

up to log(aM), where

$$\mathcal{O}_p(x) = M^m \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \quad p \equiv m+\alpha+\beta+3\gamma$$

- Only p=1,2 are dangerous.

$$\varphi,\ M\varphi,\ \varphi^2$$

($\partial\varphi$ is a total derivative)

$SU(2)_R$ allows only TrB_A and TrX_i .

Exact SUSY kills them.

φ^2 term is forbidden in a similar manner

Outline

- Main idea
- lattice formulation of '2d BMN'
- uplift to 4d

- Firstly, take a continuum limit as 2d theory.
- Secondly, take a k-coincident fuzzy sphere solution. Then U(k) SYM on fuzzy sphere is obtained.

$$L_a = L_a^{(n)} \otimes \mathbf{1}_k \qquad N=k(2n+1)$$

noncommutativity : $\theta \sim 1/(M^2 n)$

UV/IR momentum cutoff : Mn, M

Coupling constant : $g_{4d}^2 = 4\pi\theta g_{2d}^2$

- Take a flat noncommutative space limit, i.e. N→∞ fixing Θ

- Because 14 of 16 SUSYs are softly broken, additional "UV divergence" seems to appear.

$$M^p (\log N)^q$$

- But now M goes to zero as $M \sim 1/\sqrt{N}$ and hence there is no "UV divergence". So SYM on flat noncommutative space is obtained.

- In the end we take the commutative limit.

- 4d N=4 can be formulated without requiring parameter fine tuning, at least to all order in perturbation.
- UV finiteness is the key to justify the use of fuzzy sphere. Other UV finite theories may be formulated in a similar manner.
- Simulation? -- hopefully in near future! Difficult in this summer and winter because resources in Tokyo metropolitan area are not fully available because of the earthquake :(

- Take a flat noncommutative space limit, i.e. $N \to \infty$ fixing Θ
- Because 14 of 16 SUSYs are softly broken, additional "UV divergence" seems to appear.

$$M^p (\log N)^q$$

- But now M goes to zero as $M \sim 1/\sqrt{N}$ and hence there is no "UV divergence". So SYM on flat noncommutative space is obtained.
- In the end we take the commutative limit.

- 4d N=4 can be formulated without requiring parameter fine tuning, at least to all order in perturbation.
- UV finiteness is the key to justify the use of fuzzy sphere. Other UV finite theories may be formulated in a similar manner.
- Simulation? -- hopefully in near future! Difficult in this summer and winter because resources in Tokyo metropolitan area are not fully available because of the earthquake :(

- Take a flat noncommutative space limit, i.e. N $\to \infty$ fixing Θ

- Because 14 of 16 SUSYs are softly broken, additional "UV divergence" seems to appear.

$$M^p (\log N)^q$$

- But now M goes to zero as $M \sim 1/\sqrt{N}$ and hence there is no "UV divergence". So SYM on flat noncommutative space is obtained.

- In the end we take the commutative limit.

- 4d N=4 can be formulated without requiring parameter fine tuning, at least to all order in perturbation.
- UV finiteness is the key to justify the use of fuzzy sphere. Other UV finite theories may be formulated in a similar manner.
- Simulation? -- hopefully in near future! Difficult in this summer and winter because resources in Tokyo metropolitan area are not fully available because of the earthquake :(

- Take a flat noncommutative space limit, i.e. N→∞ fixing Θ
- Because 14 of 16 SUSYs are softly broken, additional "UV divergence" seems to appear.

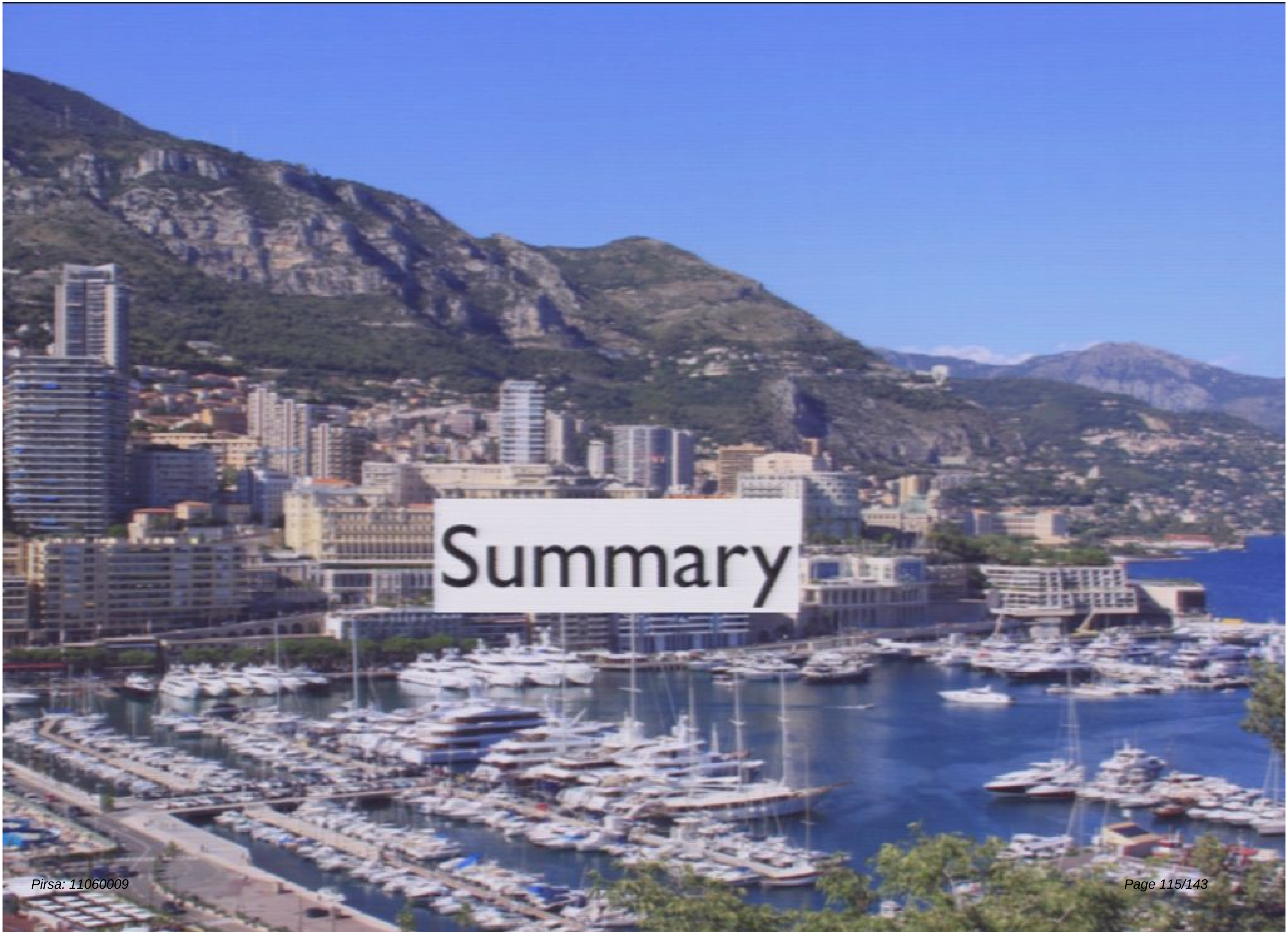
$$M^p(\log N)^q$$

- But now M goes to zero as $M \sim 1/\sqrt{N}$ and hence there is no "UV divergence". So SYM on flat noncommutative space is obtained.
- In the end we take the commutative limit.

- 4d N=4 can be formulated without requiring parameter fine tuning, at least to all order in perturbation.

- UV finiteness is the key to justify the use of fuzzy sphere. Other UV finite theories may be formulated in a similar manner.

- Simulation? -- hopefully in near future! Difficult in this summer and winter because resources in Tokyo metropolitan area are not fully available because of the earthquake :(



Summary

- Take a flat noncommutative space limit, i.e. $N \to \infty$ fixing Θ
- Because 14 of 16 SUSYs are softly broken, additional "UV divergence" seems to appear.

$$M^p(\log N)^q$$

- But now M goes to zero as $M \sim 1/\sqrt{N}$ and hence there is no "UV divergence". So SYM on flat noncommutative space is obtained.
- In the end we take the commutative limit.

Absence of fine tuning (to all order in perturbation)

Cohen-Kaplan-Katz-Unsal, 2003

- Possible correction from UV is

$$\left(\frac{1}{g_{2d}^2}c_0 a^{p-4} + c_1 a^{p-2} + g_{2d}^2 c_2 a^p + \cdots\right)\int d^2x\, \mathcal{O}_p(x)$$

tree

up to log(aM), where

$$\mathcal{O}_p(x) = M^m \varphi(x)^\alpha \partial^\beta \psi(x)^{2\gamma}, \quad p \equiv m+\alpha+\beta+3\gamma$$

- Only p=1,2 are dangerous.

$\varphi,\ M\varphi,\ \varphi^2$ ($\partial\varphi$ is a total derivative)

$SU(2)_R$ allows only TrB_A and TrX_i .

Exact SUSY kills them.

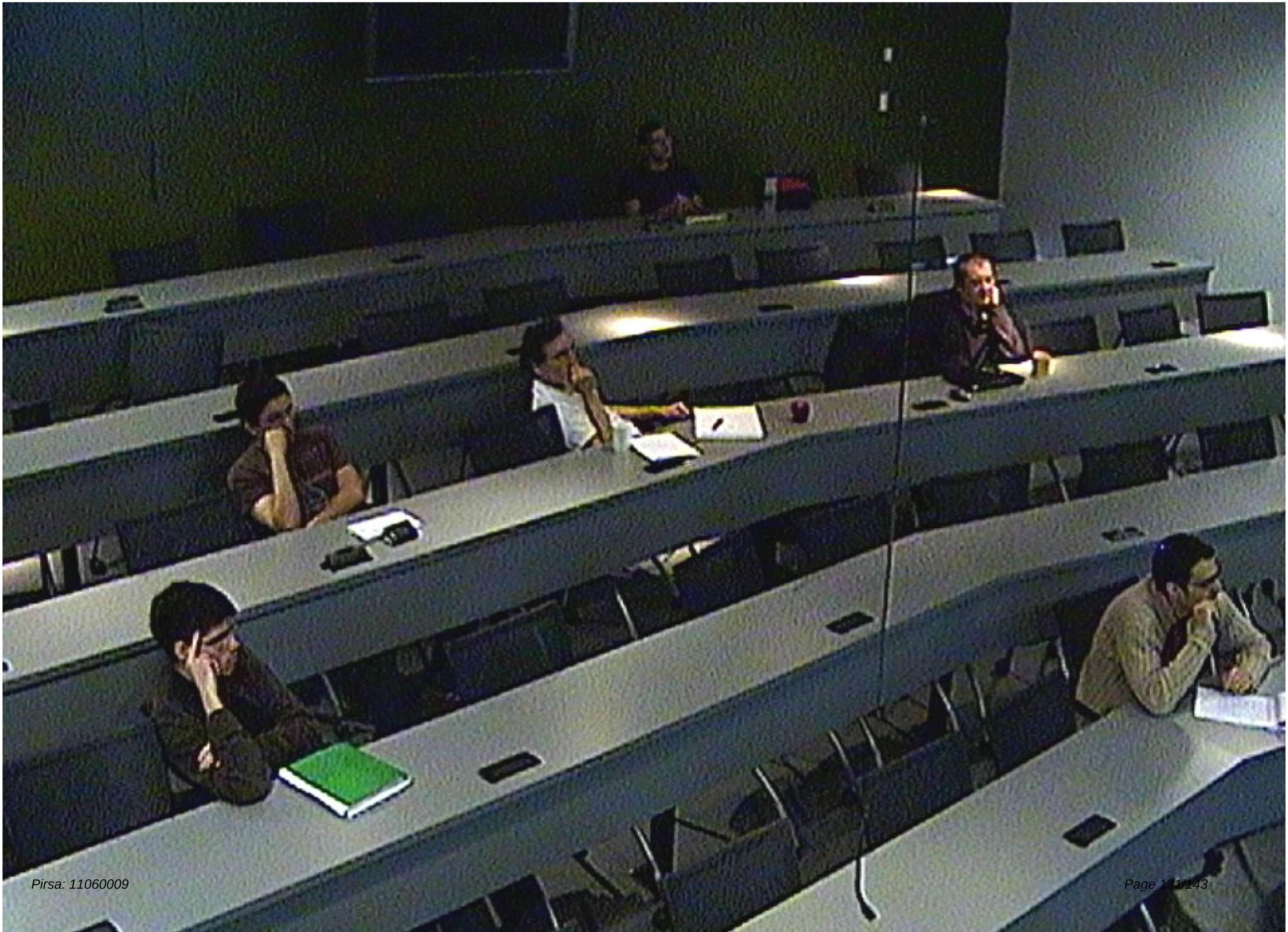
φ^2 term is forbidden in a similar manner.

"BMN deformation"

$$\Delta S = \frac{1}{g_{2d}^2} \int d^2x \, \mathrm{Tr}\Big\{ \frac{2M^2}{81} (B_A^2 + X_i^2)$$
$$- \frac{M}{2} C[\phi_+, \phi_-] + \frac{M^2}{9} \left(\frac{C^2}{4} + \phi_+ \phi_- \right)$$
$$+ \frac{2M}{3} \psi_{+\mu} \psi_{-\mu} + \frac{2M}{9} \rho_{+i} \rho_{-i} + \frac{4M}{9} \chi_{+A} \chi_{-A}$$
$$- \frac{M}{6} \eta_+ \eta_- - \frac{4iM}{9} B_3 (F_{12} + i[X_3, X_4]) \Big\}$$

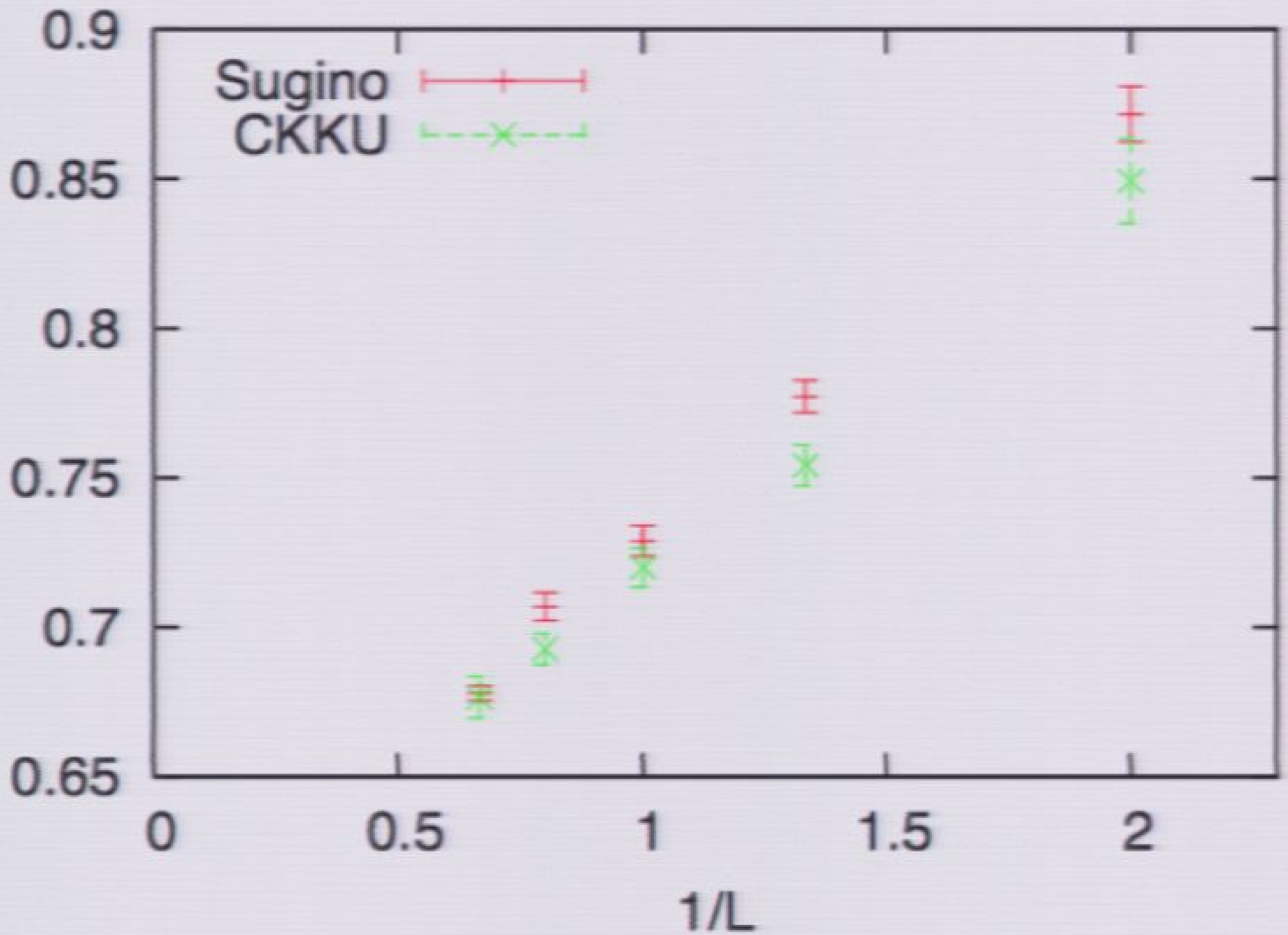
$$\Delta Q_\pm \tilde{H}_\mu = \frac{M}{3} \psi_{\pm\mu}, \qquad \Delta Q_\pm \tilde{h}_i = \frac{M}{3} \rho_{\pm i},$$
$$\Delta Q_\pm H_A = \frac{M}{3} \chi_{\pm A}, \qquad \Delta Q_\pm \eta_\pm = \frac{2M}{3} \phi_\pm,$$
$$\Delta Q_\mp \eta_\pm = \pm \frac{M}{3} C.$$

M.H.-Matsuura-Sugino, 2010



Outline

- Main idea
- lattice formulation of '2d BMN'
- uplift to 4d



$\sqrt{Tr(X_i(x))^2/N}$ vs compactification radius

SU(2), periodic b.c. (M.H.-Kanamori 2010)

$$\Phi_1(x) = 2\left(-D_1 X_3(x) - D_2 X_4(x)\right),$$
$$\Phi_2(x) = 2\left(-D_1^* X_4(x) + D_2^* X_3(x)\right),$$
$$\Phi_3(x) = \frac{i(U_{12}(x) - U_{21}(x))}{1 - \epsilon^{-2}||1 - U_{12}(x)||^2} + 2i[X_3(x), X_4(x)]$$

↑

admissibility condition

- Gauge part is not Tr(plaquette) but Tr(plaquette)^2
- To exclude wrong vacua with plaquette = -1, we employ the admissibility.
- The action is set to be zero unless

$||1 - U_{12}(x)|| < \epsilon$ for $\forall x$

Sugino, 2003

$$Q^{(0)}_{\pm} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$

$$Q^{(0)}_{\mp} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \bar{H}_\mu,$$

$$Q^{(0)}_{\pm} \bar{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2}[C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$

$$Q^{(0)}_{\pm} X_i = \rho_{\pm i}, \quad Q^{(0)}_{\pm} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$

$$Q^{(0)}_{\mp} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \bar{h}_i,$$

$$Q^{(0)}_{\pm} \bar{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2}[C, \rho_{\pm i}] \pm \frac{1}{2}[X_i, \eta_{\pm}],$$

$$Q^{(0)}_{\pm} B_A = \chi_{\pm A}, \quad Q^{(0)}_{\pm} \chi_{\pm A} = \pm [\phi_{\pm}, B_A],$$

$$Q^{(0)}_{\mp} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$

$$Q^{(0)}_{\pm} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2}[B_A, \eta_{\pm}] \mp \frac{1}{2}[C, \chi_{\pm A}],$$

$$Q^{(0)}_{\pm} C = \eta_{\pm}, \quad Q^{(0)}_{\pm} \eta_{\pm} = \pm [\phi_{\pm}, C],$$

$$Q^{(0)}_{\mp} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$

$$Q^{(0)}_{\pm} \phi_{\pm} = 0, \quad Q^{(0)}_{\mp} \phi_{\pm} = \mp \eta_{\pm}.$$

$$Q_\pm U_\mu(x) = i\psi_{\pm\mu}(x)U_\mu(x),$$
$$Q_\pm \psi_{\pm\mu}(x) = i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x),$$
$$Q_\mp \psi_{\pm\mu}(x) = \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x),$$
$$Q_\pm \tilde{H}_\mu(x) = -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right]$$
$$\pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right]$$
$$\mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right]$$
$$+\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]$$

$$D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)$$

Sugino, 2003

$$Q_{\pm}^{(0)} A_\mu = \psi_{\pm\mu}, \quad Q_{\pm}\psi_{\pm\mu} = \pm i D_\mu \phi_{\pm},$$

$$Q_{\mp}^{(0)} \psi_{\pm\mu} = \frac{i}{2} D_\mu C \mp \tilde{H}_\mu,$$

$$Q_{\pm}^{(0)} \tilde{H}_\mu = [\phi_{\pm}, \psi_{\mp\mu}] \mp \frac{1}{2} [C, \psi_{\pm\mu}] \mp \frac{i}{2} D_\mu \eta_{\pm},$$

$$Q_{\pm}^{(0)} X_i = \rho_{\pm i}, \quad Q_{\pm}^{(0)} \rho_{\pm i} = \mp [X_i, \phi_{\pm}],$$

$$Q_{\mp}^{(0)} \rho_{\pm i} = -\frac{1}{2}[X_i, C] \mp \tilde{h}_i,$$

$$Q_{\pm}^{(0)} \tilde{h}_i = [\phi_{\pm}, \rho_{\mp i}] \mp \frac{1}{2} [C, \rho_{\pm i}] \pm \frac{1}{2} [X_i, \eta_{\pm}],$$

$$Q_{\pm}^{(0)} B_A = \chi_{\pm A}, \quad Q_{\pm}^{(0)} \chi_{\pm A} = \pm[\phi_{\pm}, B_A],$$

$$Q_{\mp}^{(0)} \chi_{\pm A} = -\frac{1}{2}[B_A, C] \mp H_A,$$

$$Q_{\pm}^{(0)} H_A = [\phi_{\pm}, \chi_{\mp A}] \pm \frac{1}{2} [B_A, \eta_{\pm}] \mp \frac{1}{2} [C, \chi_{\pm A}],$$

$$Q_{\pm}^{(0)} C = \eta_{\pm}, \quad Q_{\pm}^{(0)} \eta_{\pm} = \pm [\phi_{\pm}, C],$$

$$Q_{\mp}^{(0)} \eta_{\pm} = \mp [\phi_{+}, \phi_{-}],$$

$$Q_{\pm}^{(0)} \phi_{\pm} = 0, \quad Q_{\mp}^{(0)} \phi_{\pm} = \mp \eta_{\pm}.$$

$$S_0 = \frac{1}{g_{2d}^2}\int d^2x\, \mathrm{Tr}\Big\{F_{12}^2 + \left(D_\mu X^I\right)^2 - \frac{1}{2}\left[X^I, X^J\right]^2$$
$$+\Psi^T\left(D_1+\gamma_2 D_2\right)\Psi + i\Psi^T\gamma_I\left[X^I,\Psi\right]\Big\}$$

Q-exact form

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2}\int d^2x\, \mathrm{Tr}\Big\{-iB_A\Phi_A - \frac{1}{3}\epsilon_{ABC}B_A[B_B, B_C]$$
$$-\psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_-\Big\},$$

$$\Phi_1 = 2(-D_1X_3 - D_2X_4),\ \Phi_2 = 2(-D_1X_4 + D_2X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

$$
\begin{aligned}
Q_\pm U_\mu(x) &= i\psi_{\pm\mu}(x)U_\mu(x), \\
Q_\pm \psi_{\pm\mu}(x) &= i\psi_{\pm\mu}(x)\psi_{\pm\mu}(x) \pm iD_\mu\phi_\pm(x), \\
Q_\mp \psi_{\pm\mu}(x) &= \frac{i}{2}\{\psi_{+\mu}(x), \psi_{-\mu}(x)\} + \frac{i}{2}D_\mu C(x) \mp \tilde{H}_\mu(x), \\
Q_\pm \tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{\mp\mu}(x), \phi_\pm(x) + U_\mu(x)\phi_\pm(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\qquad \pm\frac{1}{4}\left[\psi_{\pm\mu}(x), C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] \\
&\qquad \mp\frac{i}{2}D_\mu\eta_\pm(x) \pm \frac{1}{4}\left[\psi_{\pm\mu}(x)\psi_{\pm\mu}(x), \psi_{\mp\mu}(x)\right] \\
&\qquad +\frac{i}{2}\left[\psi_{\pm\mu}(x), \tilde{H}_\mu(x)\right]
\end{aligned}
$$

$$
D_\mu A(x) \equiv U_\mu(x)A(x+\hat{\mu})U_\mu(x)^\dagger - A(x)
$$

Sugino, 2003

$$\Phi_1(x) = 2\left(-D_1 X_3(x) - D_2 X_4(x)\right),$$
$$\Phi_2(x) = 2\left(-D_1^* X_4(x) + D_2^* X_3(x)\right),$$
$$\Phi_3(x) = \frac{i(U_{12}(x) - U_{21}(x))}{1 - \epsilon^{-2}||1 - U_{12}(x)||^2} + 2i[X_3(x), X_4(x)]$$

↑

admissibility condition

- Gauge part is not Tr(plaquette) but Tr(plaquette)^2
- To exclude wrong vacua with plaquette = -1,
  we employ the admissibility.
- The action is set to be zero unless

$||1 - U_{12}(x)|| < \epsilon$ for $\forall x$

Sugino, 2003

$$S_0 = \frac{1}{g_{2d}^2} \int d^2x \, \mathrm{Tr}\Big\{ F_{12}^2 + (D_\mu X^I)^2 - \frac{1}{2}[X^I, X^J]^2$$
$$+\Psi^T (D_1 + \gamma_2 D_2) \Psi + i\Psi^T \gamma_I [X^I, \Psi] \Big\}$$

<u>Q-exact form</u>

$$S_0 = Q_+^{(0)} Q_-^{(0)} \mathcal{F}^{(0)},$$

$$\mathcal{F}^{(0)} = \frac{1}{g_{2d}^2} \int d^2x \, \mathrm{Tr}\Big\{ -iB_A \Phi_A - \frac{1}{3}\epsilon_{ABC} B_A [B_B, B_C]$$
$$-\psi_{+\mu}\psi_{-\mu} - \rho_{+i}\rho_{-i} - \chi_{+A}\chi_{-A} - \frac{1}{4}\eta_+\eta_- \Big\},$$

$$\Phi_1 = 2(-D_1 X_3 - D_2 X_4), \; \Phi_2 = 2(-D_1 X_4 + D_2 X_3),$$

$$\Phi_3 = 2(-F_{12} + i[X_3, X_4]).$$

Counterpart in SYM = center symmetry breakdown

- Wilson line phase = position of D0

$$W = diag(e^{i\theta_1, \cdots, e^{i\theta_N}})$$

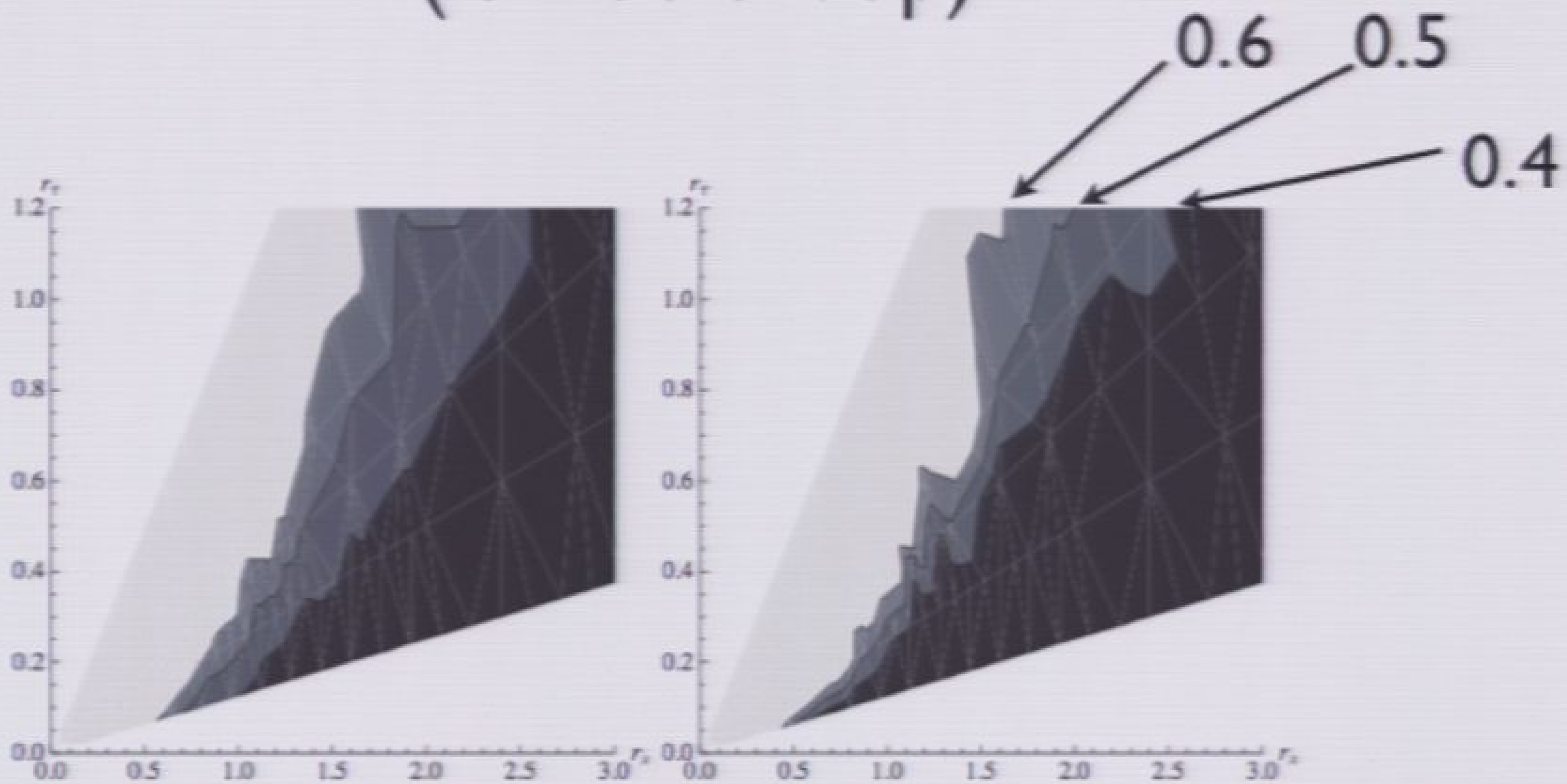
- Center symmetry

$$\theta_i \to \theta_i + const.$$

Uniform = center unbroken $\left\langle \frac{1}{N} TrW \right\rangle = 0$

Non-uniform = center broken $\left\langle \frac{1}{N} TrW \right\rangle \neq 0$

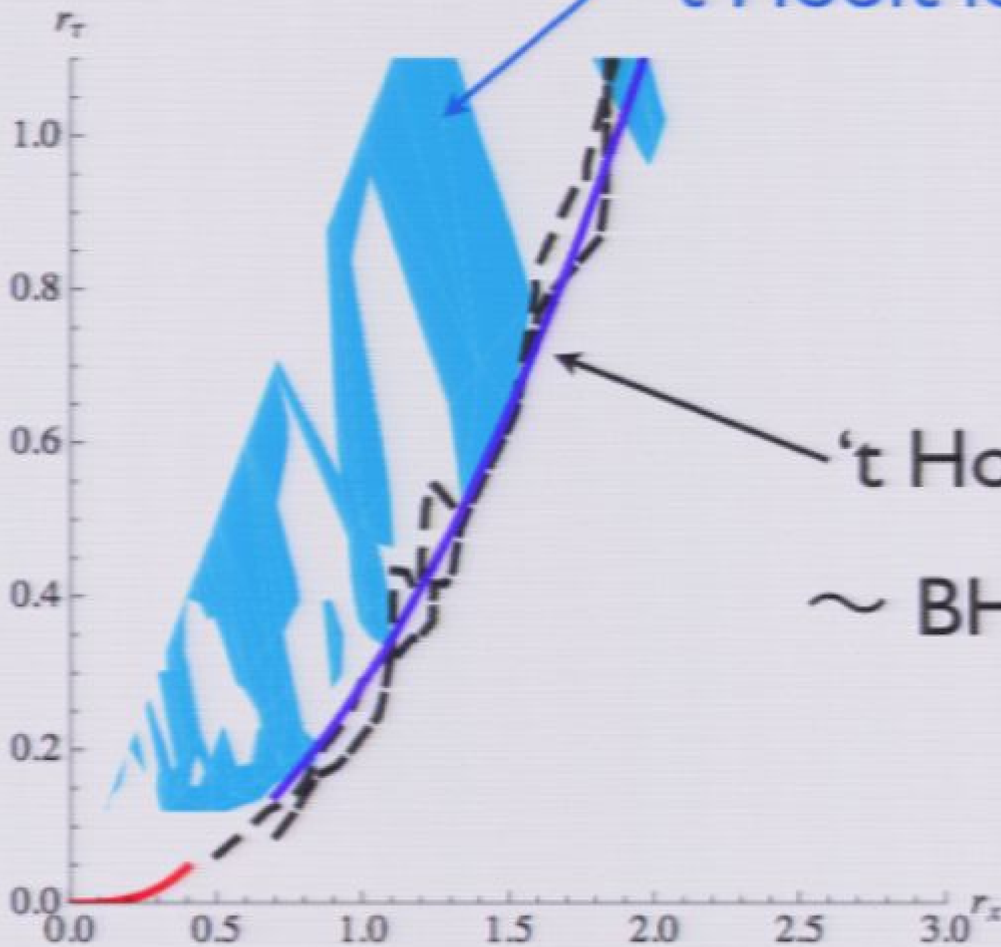
Value of spatial Wilson loop
('t Hooft loop)



SU(3)

SU(4)

SU(4) gives bigger value of 't Hooft loop than SU(3)



't Hooft loop = 0.5

$\sim$ BH/BS transition

Outline

- Main idea
- lattice formulation of '2d BMN'
- uplift to 4d

Fuzzy sphere formulation of 3d maximal SYM

(Maldacena-Sheikh Jabbari-van Raamsdonk, 2003)

- Start with the Berenstein-Maldacena-Nastase Matrix model, which can be formulated without fine tuning.

$$S = \int dt\ Tr\left(\frac{1}{2}(D_t X_I)^2 - \frac{1}{4}[X_I, X_J]^2 + \frac{i\mu}{3}\epsilon^{abc}X_a X_b X_c + \frac{\mu^2}{18}X_a^2 + \frac{\mu^2}{72}X_i^2\right)$$

I,J=1,...,9; a,b,c=1,2,3; i=4,...,9

- BMN model has (modified) 16 SUSY

Q. How can we construct 4d theory (D3-brane) ?

A. From D1-branes through the Myers effect.

Crucial point

D1-brane theory (2d SYM) can be formulated on lattice without fine tuning!

- Take 2d continuum limit first, then large-N
- Similar anisotropic continuum limit was taken on 4 lattice, in order to reduce the number of fine tuning parameters. (Kaplan-Katz-Unsal, 2003)
- Analogous to "deconstruction" of 5d out of 4d

(Arkani Hamed-Cohen-Georgi, 2001)

$$Q_+^2 = \frac{M}{3} J_{++}, \quad Q_-^2 = -\frac{M}{3} J_{--},$$
$$\{Q_+, Q_-\} = -\frac{M}{3} J_0,$$

J : SU(2)R generator;
fermions with +/- form doublets

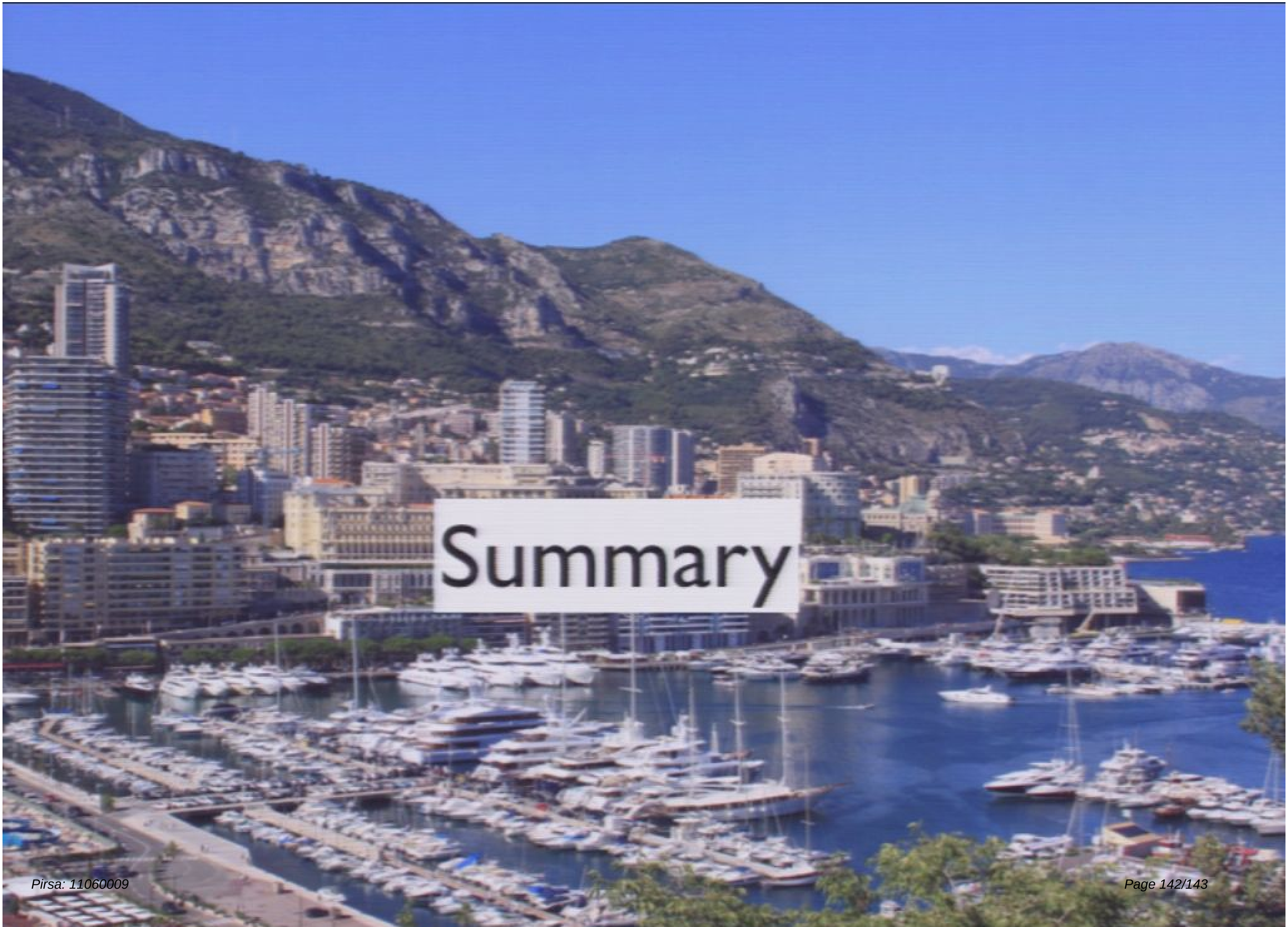
$$S = \left(Q_+ Q_- - \frac{M}{3} \right) \mathcal{F}$$

Q-closed!

M.H.-Matsuura-Sugino, 2010

- 4d N=4 can be formulated without requiring parameter fine tuning, at least to all order in perturbation.
- UV finiteness is the key to justify the use of fuzzy sphere. Other UV finite theories may be formulated in a similar manner.
- Simulation? -- hopefully in near future! Difficult in this summer and winter because resources in Tokyo metropolitan area are not fully available because of the earthquake :(

Summary



- maximal SYM can be put on computer, if one does not stick on lattice.
- Sign problem? No problem (most likely).
- 1d (non-lattice) : nice & precise results.
- 2d (lattice) : ongoing.
- 3d, 4d (fuzzy sphere) : coming soon.
- For other theories (e.g. SUSY QCD) new ideas are needed.

Does gauge/gravity duality hold at finite N?

'simulation of quantum superstring' is within reach.