Title: Part 2: Monte-Carlo approach to the gauge/gravity duality

Date: Jun 01, 2011 11:00 AM

URL: http://pirsa.org/11060008

Abstract: The gauge/gravity duality may give a nonperturbative formulation of superstring/M theory, and hence, if one can study the nonperturbative dynamics of the gauge theory, it would be useful to understand the nonperturbative aspects of superstring theory. Although researches in this direction were not successful for long time because of the notorious difficulties in lattice SUSY, however, recent progress made it possible; nonperturbative formulations free from the parameter fine-tuning were proposed, some of them are confirmed to work numerically, and nontrivial evidence for the validity of the gauge/gravity duality has been obtained. In these talks I review the state of the art in this field. I start with reviewing basics of the Monte-Carlo. Then I explain how to put supersymmetric theories on computer and show actual numerical results. 1st talk : basics of Monte-Carlo simulation. 2nd talk : 1d SYM (matrix quantum mechanics). 3rd talk : how to put 2d, 3d and 4d SYM on computer. In the talks I concentrate on basic ideas and omit technical details (e.g. algorithms to accelerate simulations). They will be explained after the talks if people are interested in. References: 1st talk : standard textbooks e.g. Heinz J. Rothe, "Lattice Gauge Theories: An Introduction", Third Edition, World Scientific. 2nd talk : 0706.1647 [hep-lat], 0707.4454 [hep-th], 0811.2081 [hep-th], 0811.3102 [hep-th], 0911.1623 [hep-th], 1012.2913 [hep-th]. 3rd talk : hep-lat/0302017, hep-lat/0311021, 1010.2948 [hep-lat] (2d SYM); hep-th/0211139 (3d SYM); 1004.5513 [hep-lat], 1009.0901 [hep-lat] (4d SYM)

Putting (supersymmetric) Yang-Mills on computer

Keywords

- Exact symmetry
- Radiative correction
- Parameter fine tuning

Plan

(1) Bosonic pure Yang-Mills   warm up
(Wilson's lattice gauge theory)

(2) QCD and Chiral symmetry
Wilson fermion → need for the fine tuning
Ginsparg-Wilson fermion → no finetuning

(3) super Yang-Mills & some numerical results

-- how can we avoid the fine tuning?

Today : 1d

Tomorrow:2d, 3d & 4d

warm-up example :

PURE YANG-MILLS (BOSONIC)

Wilson's lattice gauge theory

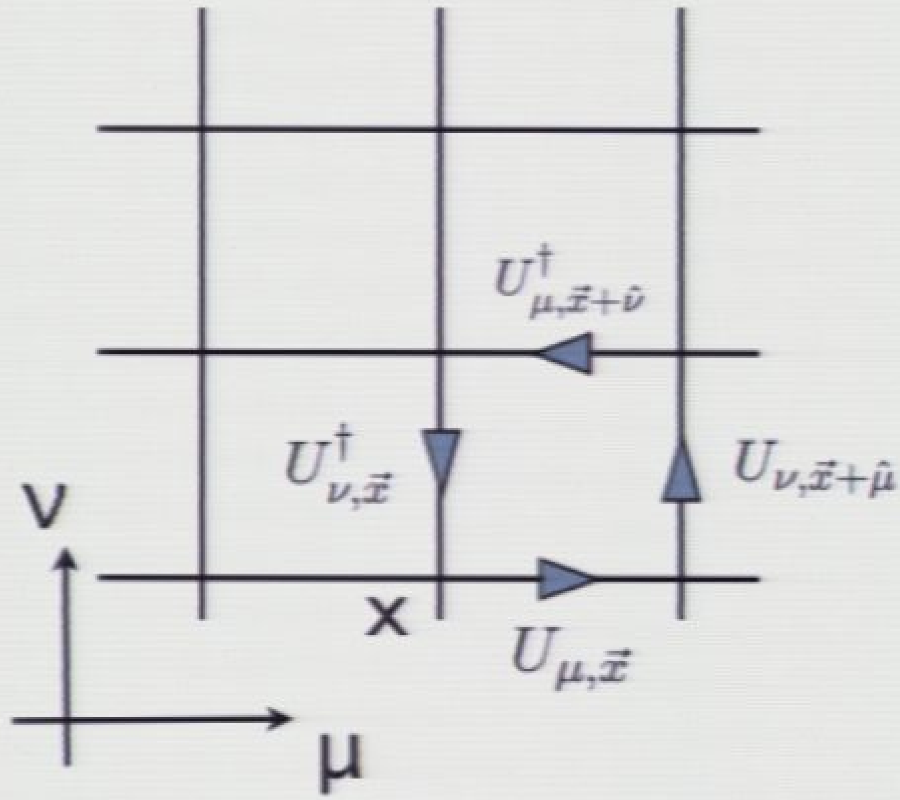
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}} \right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_{\mu}(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \rightarrow \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

Wilson's lattice gauge theory

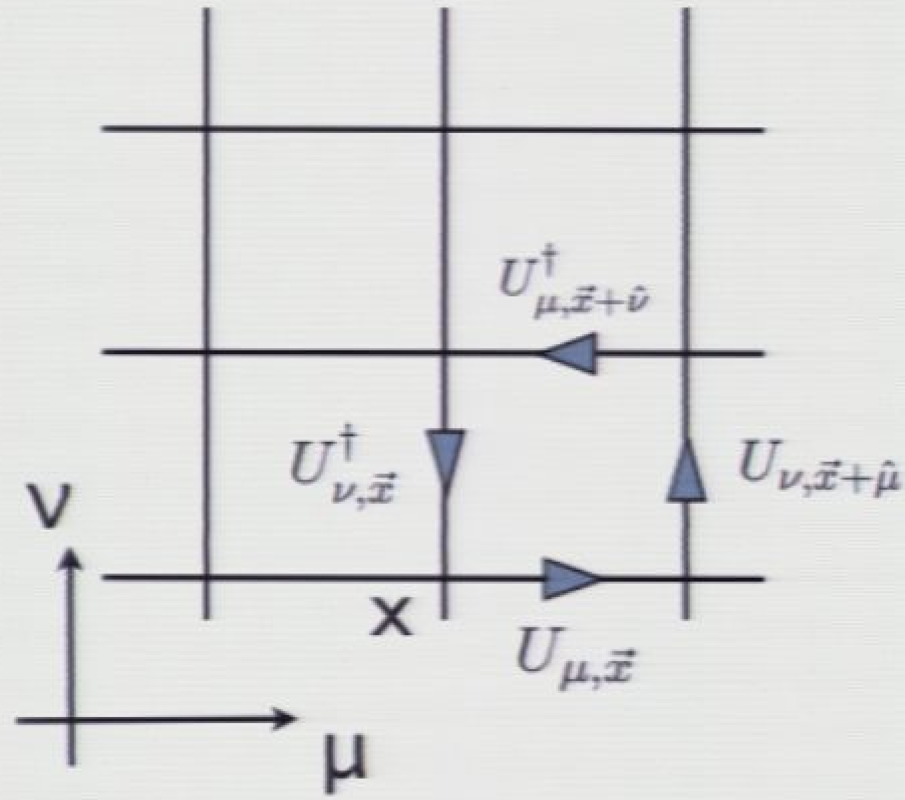
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr \left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}} \right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

'Exact' symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

Wilson's lattice gauge theory

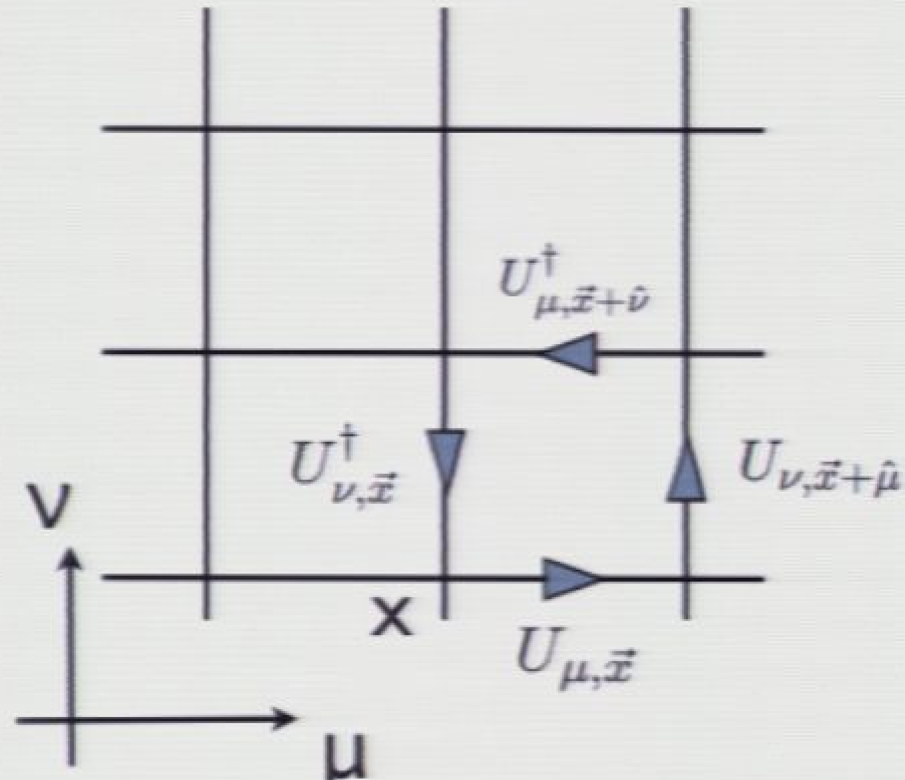
$$S = -\beta N \sum_{\vec{x}} \sum_{\mu \neq \nu} Tr\left(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U^{\dagger}_{\mu,\vec{x}+\hat{\nu}} U^{\dagger}_{\nu,\vec{x}}\right)$$

Unitary link variable

$$U_{\mu,\vec{x}} = e^{iaA_\mu(x)}$$

a : lattice spacing

$$\beta = 1/(g^2_{YM}(a) \cdot N)$$



$$S = \frac{1}{4g^2_{YM}} \int d^4x Tr F^2_{\mu\nu} + O(a^4)$$

‘Exact’ symmetries

- Gauge symmetry

$$U_{\mu,\vec{x}} \to \Omega(x) U_{\mu,\vec{x}} \Omega(x+\hat{\mu})^\dagger$$

- 90 degree rotation
- discrete translation
- Charge conjugation, parity

Continuum limit $a \to 0$ respects exact symmetries at discretized level.

Thanks to these exact symmetries, the continuum limit is gauge invariant, translational invariant, rotationally invariant, etc.

What happens if the gauge symmetry is explicitly (not spontaneously) broken, e.g. in the sharp momentum cutoff prescription?

- We are interested in low-energy, long-distance physics (compared to the lattice spacing a).
- So let us integrate out high frequency modes.

Then...

gauge symmetry breaking radiative corrections appear.

To kill them, one has to add counterterms to lattice action, whose coefficients must be fine-tuned!

'fine tuning problem'

This is the reason why we *must* preserve symmetries exactly.

Fermions and Chiral symmetry

Chiral symmetry

$$SU(N_f)_V \times U(1)_B \times SU(N_f)_A \times U(1)_A$$

anomalous

forbid the quark mass term
via a radiative correction.

but difficult to realize on lattice.

- Wilson fermion : chiral symetry is broken.

$$\gamma_5 D + D\gamma_5 = 0$$ otherwise the chiral anomaly is absent..

- Ginsparg-Wilson fermion : 'modified' chiral symmetry

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

Wilson fermion

$$S_F = \sum_{\vec{x}} \bar{\psi}_{\vec{x}} \psi_{\vec{x}} - \kappa \sum_{\vec{x},\mu} \bar{\psi}_{\vec{x}} \left\{ (1-\gamma_\mu) U_{\vec{x},\mu} \psi_{\vec{x}+\hat{\mu}} + (1+\gamma_\mu) U^\dagger_{\vec{x}-\hat{\mu},\mu} \psi_{\vec{x}-\hat{\mu}} \right\}$$

$$\kappa = \frac{1}{8 + 2am_0}$$

bare mass

Chiral symmetry is explicitly broken even when the bare mass is zero.

→ The radiative correction gives O(1/a) mass.

One has to fine tune κ so that the renormalized mass becomes small.

- In order to study the chiral symmetry breakdown, the Wilson fermion is not appropriate.
- Lattice simulation with Ginsparg-Wilson fermion was done and spontaneous chiral symmetry breakdown was confirmed.

(Fukaya et al, 2007)

Wilson fermion

$$S_F = \sum_{\vec{x}} \bar{\psi}_{\vec{x}} \psi_{\vec{x}} - \kappa \sum_{\vec{x},\mu} \bar{\psi}_{\vec{x}} \left\{ (1-\gamma_\mu) U_{\vec{x},\mu} \psi_{\vec{x}+\hat{\mu}} + (1+\gamma_\mu) U^\dagger_{\vec{x}-\hat{\mu},\mu} \psi_{\vec{x}-\hat{\mu}} \right\}$$

$$\kappa = \frac{1}{8 + 2am_0}$$

← bare mass

Chiral symmetry is explicitly broken
even when the bare mass is zero.

➡ The radiative correction gives O(1/a) mass.

One has to fine tune κ so that the renormalized mass becomes small.

- In order to study the chiral symmetry breakdown, the Wilson fermion is not appropriate.
- Lattice simulation with Ginsparg-Wilson fermion was done and spontaneous chiral symmetry breakdown was confirmed.

(Fukaya et al, 2007)

Super Yang-Mills

- In order to study the chiral symmetry breakdown, the Wilson fermion is not appropriate.
- Lattice simulation with Ginsparg-Wilson fermion was done and spontaneous chiral symmetry breakdown was confirmed.

(Fukaya et al, 2007)

Super Yang-Mills

'No-Go' for lattice SUSY

- SUSY algebra contains infinitesimal translation.

$$\{Q, \bar{Q}\} \sim \partial$$

- Infinitesimal translation is broken on lattice by construction.
- So it is impossible to keep all supercharges exactly on lattice.
- Still it is possible to preserve a part of supercharges. (subalgebra which does not contain ∂)

Strategy

Use other exact symmetries and/or a few exact SUSY to forbid SUSY breaking radiative correction.

- 1d : no problem thanks to UV finiteness. Lattice is not needed; momentum cutoff method is much more powerful.
  (M.H.-Nishimura-Takeuchi 2007)

- 2d : lattice with a few exact SUSY
  - no fine tuning at perturbative level (Cohen-Kaplan-Katz-Unsal 2003, Sugino 2003, Catterall 2003, D'Adda et al 2005, ...)
  - works even nonperturbatively (←simulation)

- 3d : perturbatively OK with 16 SUSY
  (Kaplan-Unsal 2005 + unpublished work by Unsal)
- 4d N=1 *pure* SYM : Chiral symmetry assures SUSY (Kaplan)
- 4d N=4 : "Hybrid" formulation:
  Lattice + fuzzy sphere

(M.H.-Matsuura-Sugino 2010, M.H. 2010)

•Large-N Eguchi-Kawai reduction : Ishii-Ishiki-Shimasaki-Tsuchiya, 2008
•Another Matrix model approach: Heckmann-Verlinde, 2011
•recent analysis of 4d lattice : Catterall-Dzienkowski-Giedt-Joseph-Wells, 2011
(Fine tuning is needed, but only for bare lattice couplings.)

SUSY matrix quantum mechanics

- D0-brane quantum mechanics
- Matrix model of M-theory

$$S = \frac{N}{\lambda}\int dt\ Tr\Big\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi]\Big\}$$

- Dimensional reduction of 4d N=4 (or 10d N=1)
- D0-brane effective action
- Matrix model of M-theory
- gauge/gravity duality → dual to black 0-brane

Simple but perhaps more interesting than AdS_5/CFT_4 from string theory point of view!

- Matrix quantum mechanics is UV finite.

No fine tuning!

(4d N=4 is also UV finite, but that relies on cancellations of the divergences...)

- Don't have to use lattice. Just fix the gauge & introduce momentum cutoff!
(M.H.-Nishimura-Takeuchi, 2007)

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

"(p+1)-d maximally supersymmetric U(N) YM and type II superstring on black p-brane background are equivalent"

p=3 : AdS_5/CFT_4

p<3 : nonAdS/nonCFT

large-N, strong coupling = SUGRA

finite coupling = α' correction

finite N = g_s correction

Gravity side

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

- Take the static diagonal gauge

$$A_0(t) = diag(\alpha_1, \cdots, \alpha_N)/\beta$$

$$\alpha_1, \cdots, \alpha_N \in (-\pi, \pi]$$

- Add Faddeev-Popov term

$$S_{FP} = -\sum_{a \neq b} \log \left| \sin \frac{\alpha_a - \alpha_b}{2} \right|$$

- Introduce momentum cutoff Λ

$$X_i(t) = \sum_{n=-\Lambda}^{\Lambda} \tilde{X}_i(n) e^{2\pi i n t/\beta}$$

Gravity side

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

"(p+1)-d maximally supersymmetric U(N) YM and type II superstring on black p-brane background are equivalent"

$p=3$: AdS_5/CFT_4

$p<3$: nonAdS/nonCFT

large-N, strong coupling = SUGRA

finite coupling = α' correction

finite N = g_s correction

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N}\, U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$\gg 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$\ll 1$

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature $$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature $$\lambda^{-1/3} T \ll 1.$$

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N} U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$>> 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$<< 1$

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature $$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature $$\lambda^{-1/3} T \ll 1.$$

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N}\, U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

$\gg 1$

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

$\ll 1$

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful*. It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature $$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature $$\lambda^{-1/3} T \ll 1.$$

The dictionary

Gravity	SYM
ADM mass	Energy density
minimal surface	Wilson/Polyakov loop
mass of field excitation	'generalized' conformal dimension

“moduli” problem

- There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

- In 1d (and 2d), it is not a “moduli space”; value of the configuration should be determined dynamically.

- SUGRA (N=∞) suggests the black zero-brae is stable. $$X_1 \simeq X_2 \simeq \cdots \simeq X_9 \simeq 0$$

- 1/N correction (g_s correction) should give an instability : Hawking radiation.

- Instability should disappear at large-N and/or at high temperature. And it does happen in simulations!

Gauge/gravity duality conjecture

(Maldacena 1997; Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

"(p+1)-d maximally supersymmetric U(N) YM and type II superstring on black p-brane background are equivalent"

p=3 : AdS_5/CFT_4

p<3 : nonAdS/nonCFT

large-N, strong coupling = SUGRA

finite coupling = α' correction

finite N = g_s correction

$$S = \frac{N}{\lambda} \int dt\ Tr \Big\{ \frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\bar{\psi} D_t \psi - \frac{1}{2}\bar{\psi}\gamma^i [X_i, \psi] \Big\}$$

- Dimensional reduction of 4d N=4 (or 10d N=1)
- D0-brane effective action
- Matrix model of M-theory
- gauge/gravity duality → dual to black 0-brane

Simple but perhaps more interesting than AdS_5/CFT_4 from string theory point of view!

black p-brane solution

(Horowitz-Strominger 1991)

- ‘t Hooft coupling can be set $\lambda=1$, by rescaling fields and coordinate.

Hawking temperature

$$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

‘strong coupling’ = low temperature

$$\lambda^{-1/3} T \ll 1.$$

“moduli” problem

- There is a flat direction even at quantum level.

$$[X_i, X_j] = 0$$

- In 1d (and 2d), it is not a “moduli space”; value of the configuration should be determined dynamically.

- SUGRA (N=∞) suggests the black zero-brae is stable. $$X_1 \simeq X_2 \simeq \cdots \simeq X_9 \simeq 0$$

- 1/N correction (g_s correction) should give an instability : Hawking radiation.

- Instability should disappear at large-N and/or at high temperature. *And it does happen in simulations!*

Numerical observation

- Because we are interested in the black hole, we take the initial configuration to be

$$X_1 = X_2 = \cdots = X_9 = 0$$

- The bound state of eigenvalues is (meta-) stable at large enough N and high enough T.
- The bound state appears again at low-temperature (T<0.25 for SU(2)).

← attraction due to fermion zero-mode.

(Aoki-Iso-Kawai-Kitazawa-Tada 1998)

More stable with pbc.

ADM mass vs energy density

$$E_{D0} \quad = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

$$\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)$$

at large-N & low temperature (strong coupling)

Numerical observation

- Because we are interested in the black hole, we take the initial configuration to be

$$X_1 = X_2 = \cdots = X_9 = 0$$

- The bound state of eigenvalues is (meta-) stable at large enough N and high enough T.
- The bound state appears again at low-temperature (T<0.25 for SU(2)).

← attraction due to fermion zero-mode.

(Aoki-Iso-Kawai-Kitazawa-Tada 1998)

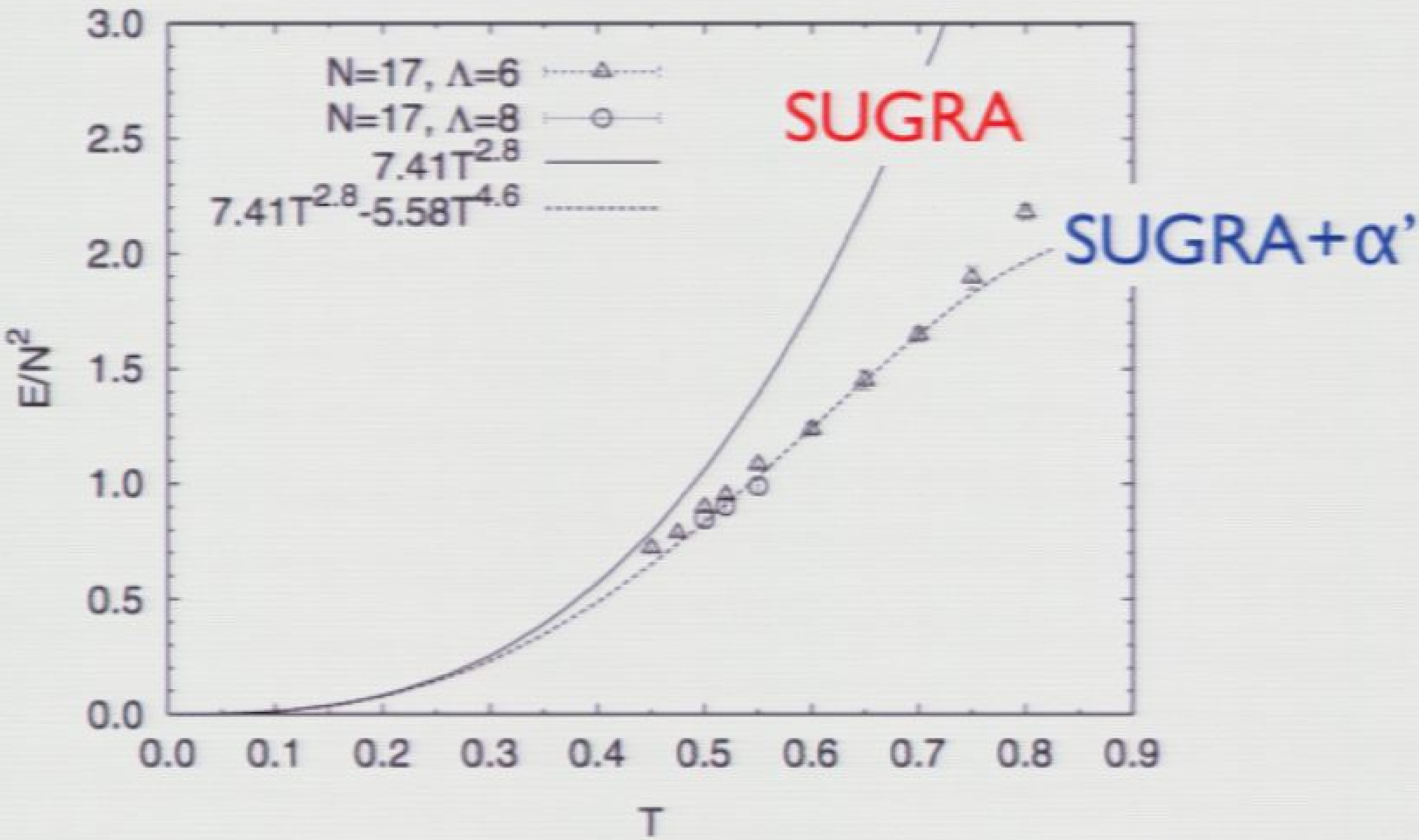
More stable with pbc.

ADM mass vs energy density

$$E_{D0} \quad = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

$$\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)$$

at large-N & low temperature (strong coupling)



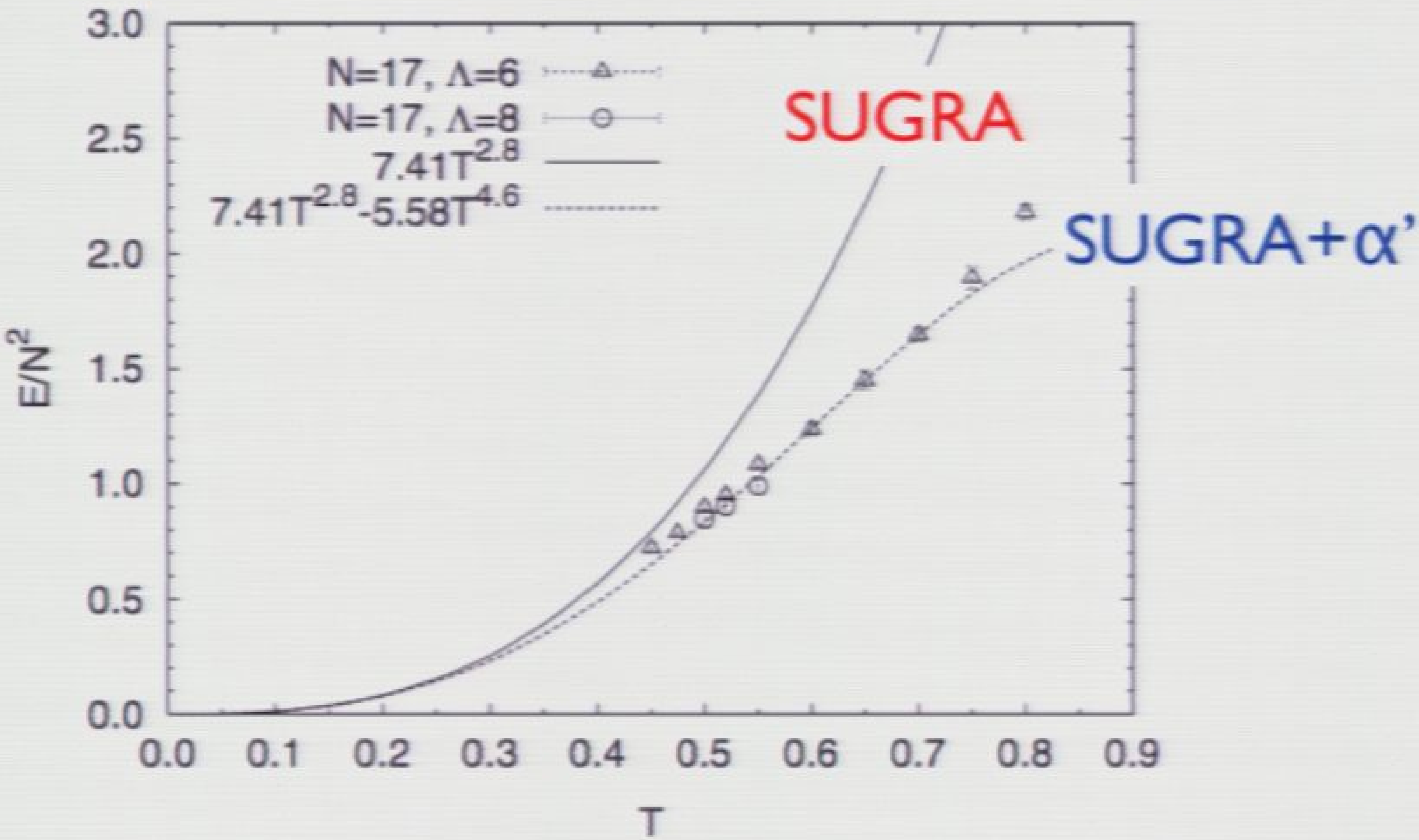
Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,

M.H.-Hyakutake-Nishimura-Takeuchi 2008

α’ correction

- deviation from the strong coupling (low temperature) corresponds to the α’ correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

‘prediction’ by SYM simulation



Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,
M.H.-Hyakutake-Nishimura-Takeuchi 2008

Numerical observation

- Because we are interested in the black hole, we take the initial configuration to be

$$X_1 = X_2 = \cdots = X_9 = 0$$

- The bound state of eigenvalues is (meta-) stable at large enough N and high enough T.

- The bound state appears again at low-temperature (T<0.25 for SU(2)).

← attraction due to fermion zero-mode.

(Aoki-Iso-Kawai-Kitazawa-Tada 1998)

More stable with pbc.

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature
$$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature
$$\lambda^{-1/3} T \ll 1.$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature
$$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature
$$\lambda^{-1/3} T \ll 1.$$

The dictionary

Gravity	SYM
ADM mass	Energy density
minimal surface	Wilson/Polyakov loop
mass of field excitation	'generalized' conformal dimension

black p-brane solution

(Horowitz-Strominger 1991)

$$ds^2 = \alpha' \left\{ \frac{U^{\frac{7-p}{2}}}{g_{YM}\sqrt{d_p N}} \left[-\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right) dt^2 + \sum_{i=1}^{p} dy_i^2 \right] + \frac{g_{YM}\sqrt{d_p N}}{U^{\frac{7-p}{2}}\left(1 - \frac{U_0^{7-p}}{U^{7-p}}\right)} dU^2 + g_{YM}\sqrt{d_p N}\, U^{\frac{p-3}{2}} d\Omega_{8-p}^2 \right\},$$

>> 1

$$e^{\phi} = (2\pi)^{2-p} g_{YM}^2 \left(\frac{g_{YM}^2 d_p N}{U^{7-p}} \right)^{\frac{3-p}{4}}, \qquad d_p = 2^{7-2p} \pi^{\frac{9-3p}{2}} \Gamma\left(\frac{7-p}{2}\right),$$

<< 1

SUGRA is valid at

$$\lambda^{1/3} N^{-4/21} \ll U \ll \lambda^{1/3} \quad (p = 0)$$

Difference from AdS/CFT

- When p<3, 't Hooft coupling λ is *dimensionful.* It sets the length scale of the theory.
- 't Hooft coupling can be set λ=1, by rescaling fields and coordinate.

Hawking temperature $$T_{D0} = \frac{7}{4\pi\sqrt{d_0\lambda}} U_0^{\frac{5}{2}}$$

'strong coupling' = low temperature $$\lambda^{-1/3} T \ll 1.$$

ADM mass vs energy density

$$E_{D0} \quad = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

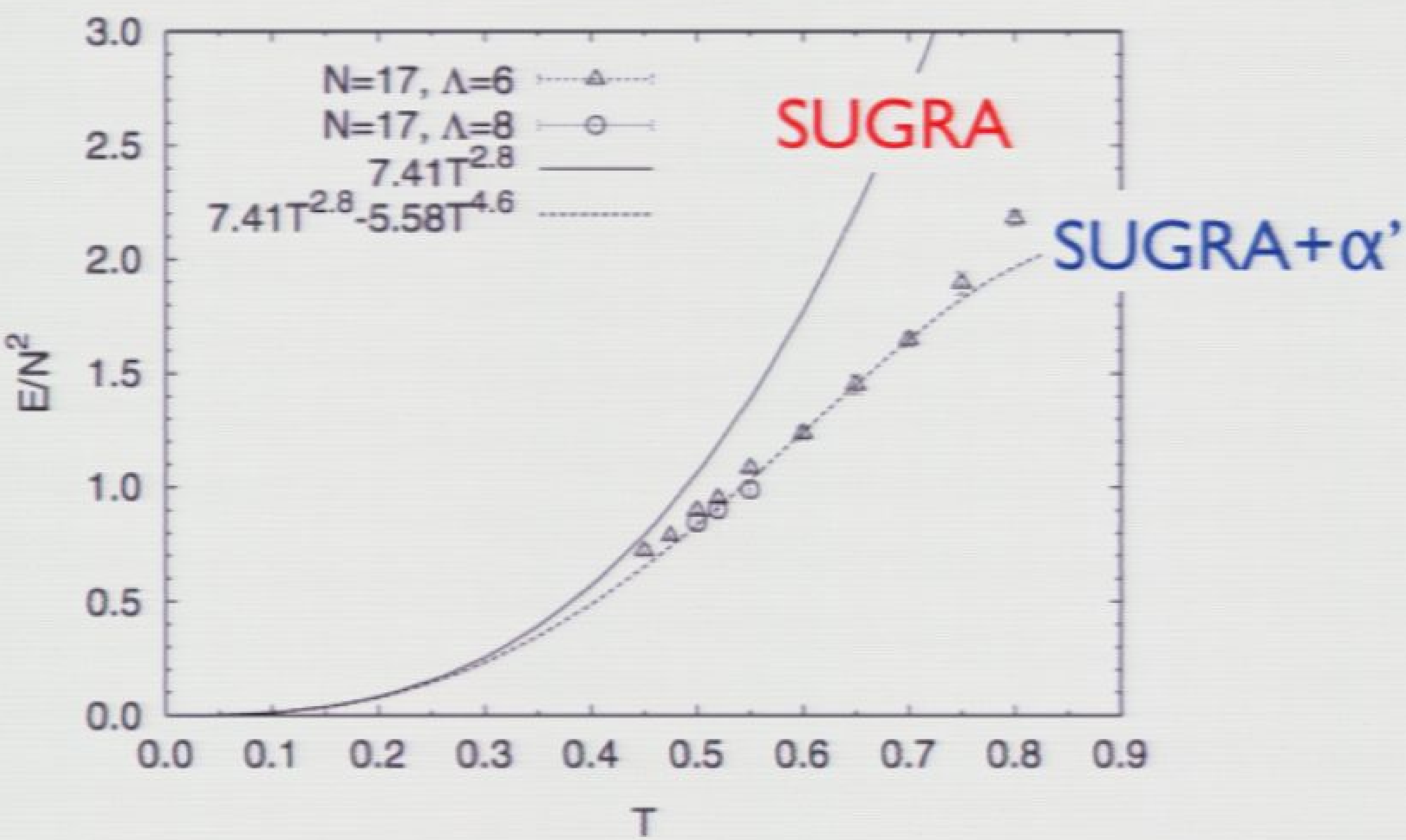
$$\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)$$

at large-N & low temperature (strong coupling)

α’ correction

- deviation from the strong coupling (low temperature) corresponds to the α’ correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2=7.41T^{2.8} - 5.58T^{4.6}$

‘prediction’ by SYM simulation



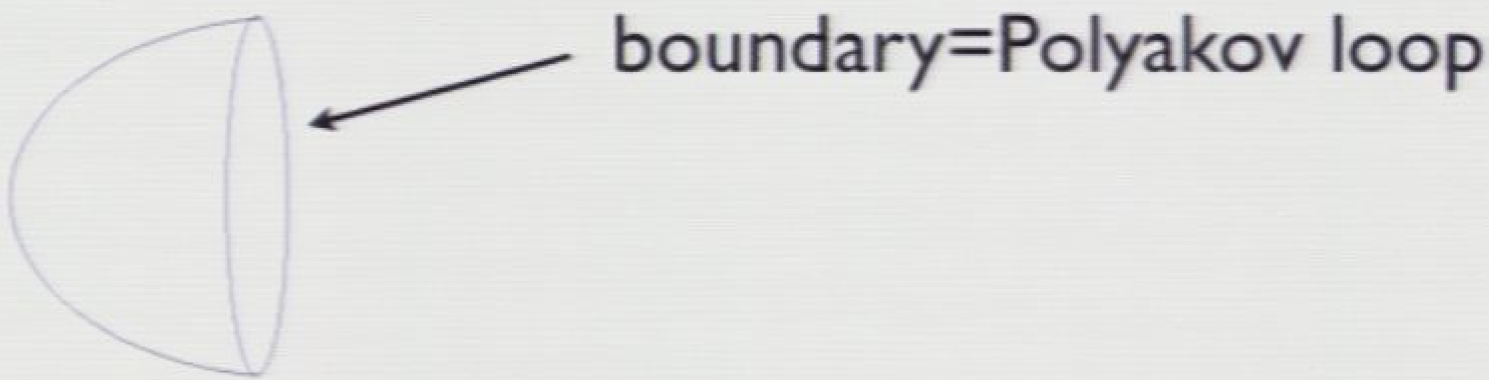
M.H.-Hyakutake-Nishimura-Takeuchi 2008

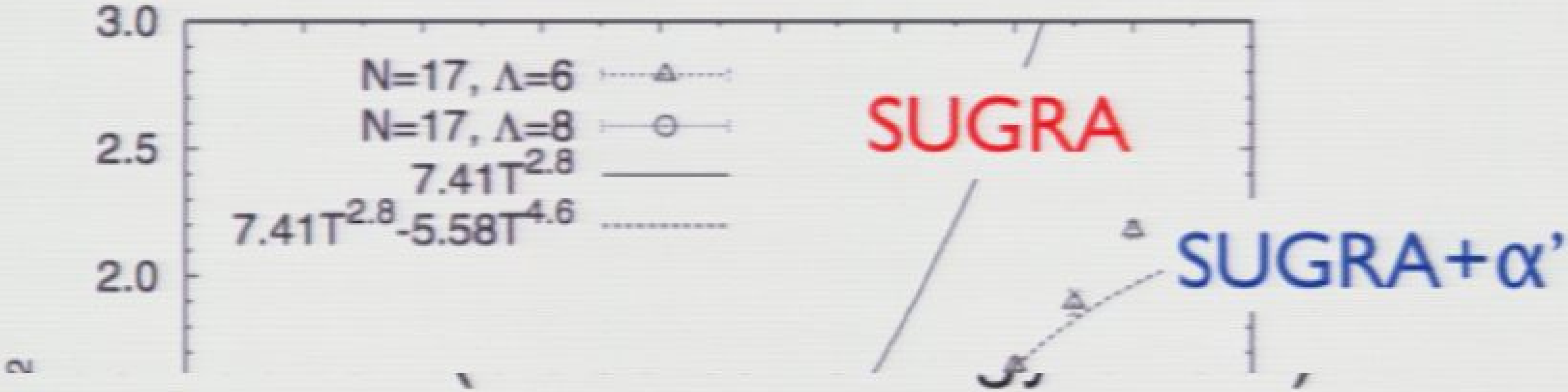
Polyakov loop with scalar

(Rey-Yee 1998; Maldacena 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$





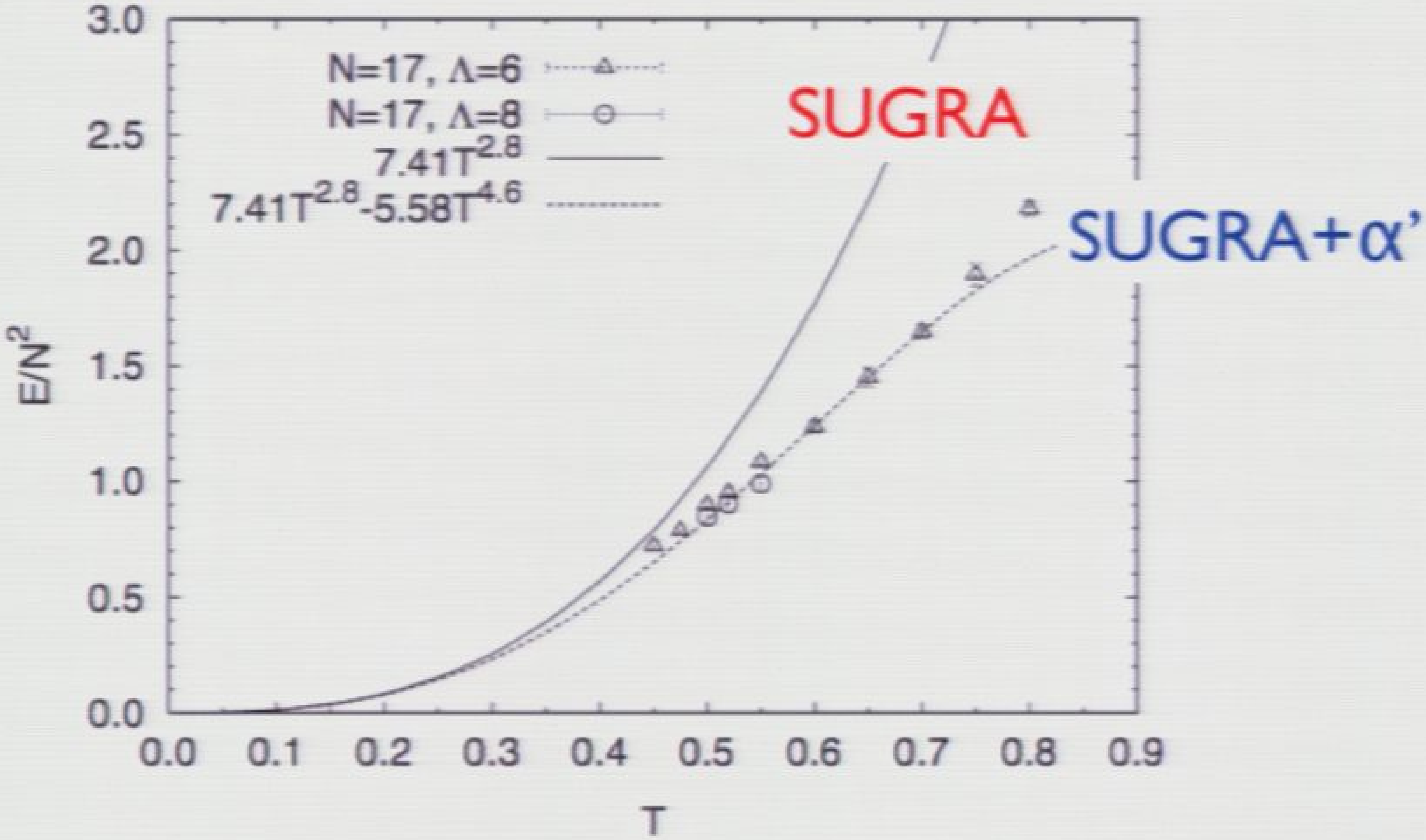
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2=7.41T^{2.8} - 5.58T^{4.6}$

'prediction' by SYM simulation

α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

'prediction' by SYM simulation



M.H.-Hyakutake-Nishimura-Takeuchi 2008

Correlation functions (GKPW relation)

- AdS/CFT (D3-brane) $\rightarrow$ GKPW relation

  (Gubser-Klebanov-Polyakov 1998, Witten 1998)

- Similar relation in D0-brane theory :

  "generalized" conformal dimension
  $\Leftrightarrow$ mass of field excitations

  (Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

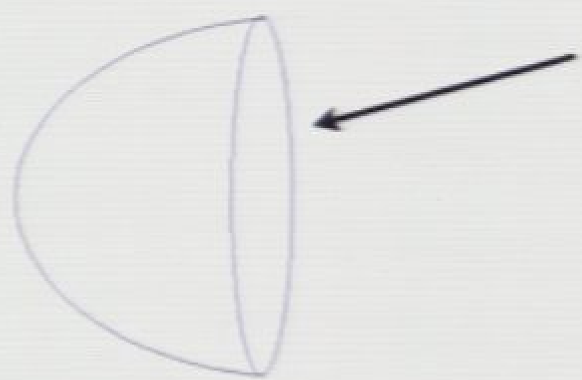
calculable via SUGRA

Polyakov loop with scalar

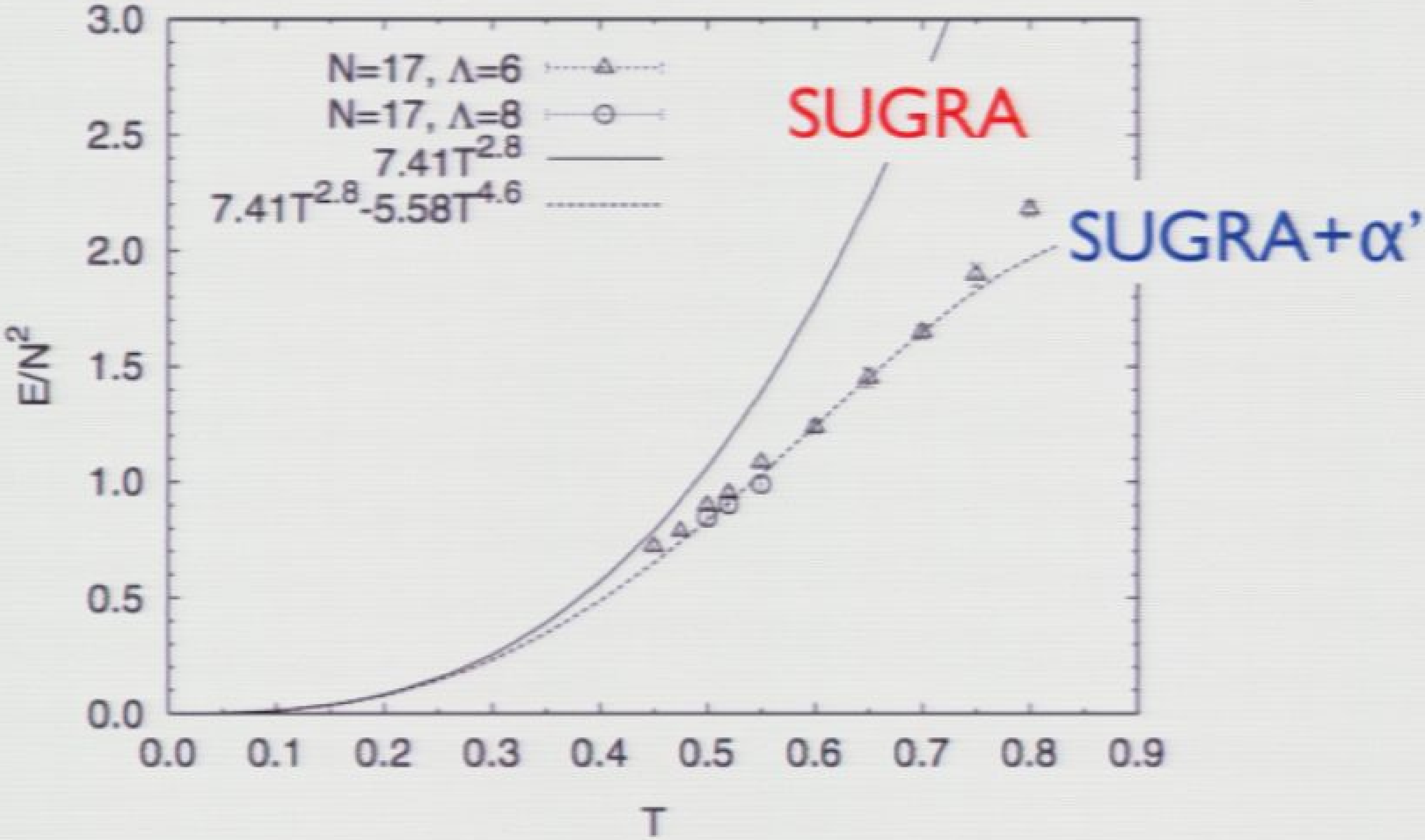
(Rey-Yee 1998; Maldacena 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle\log W\rangle \sim \text{area of minimal surface}$$



boundary=Polyakov loop

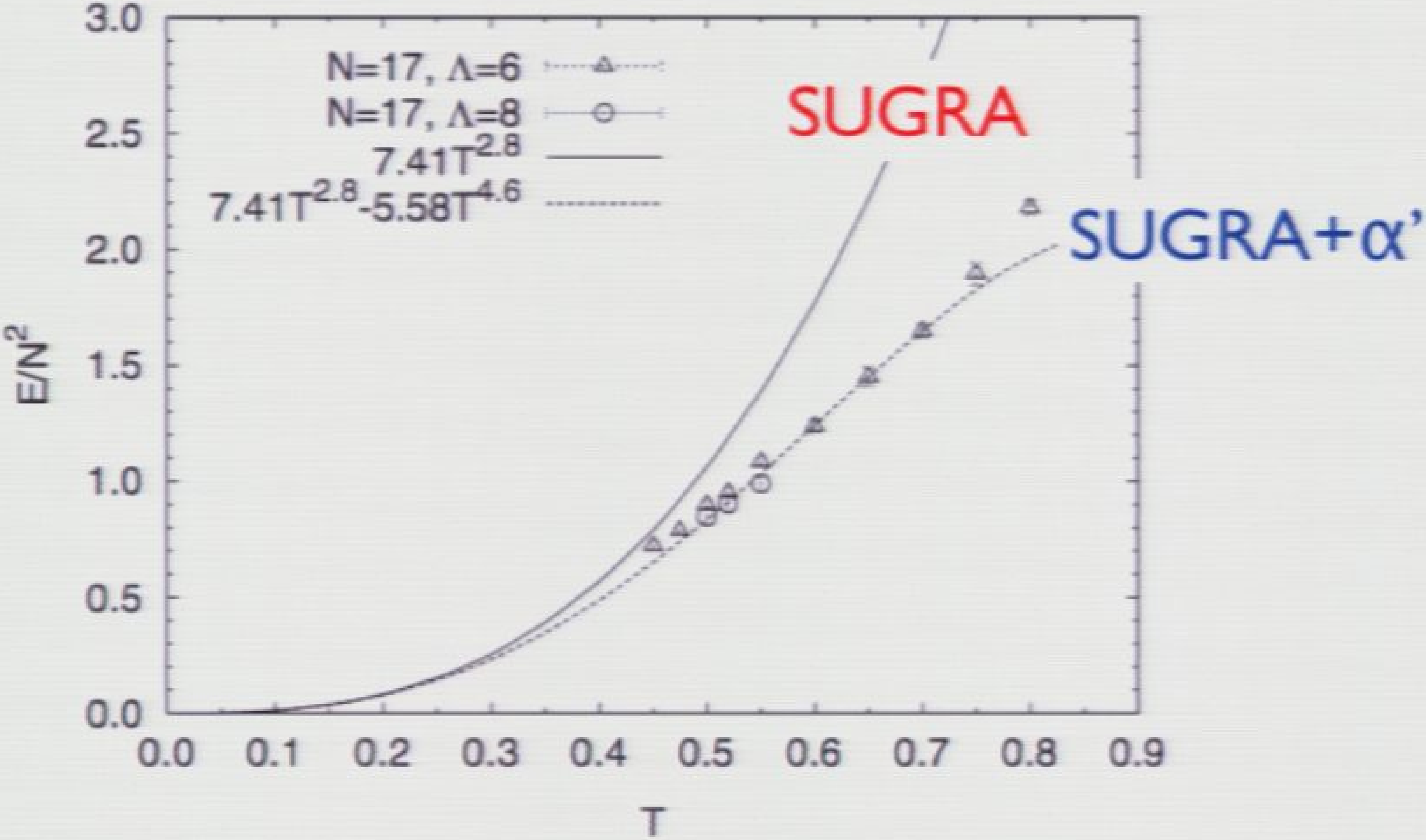


M.H.-Hyakutake-Nishimura-Takeuchi 2008

α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

'prediction' by SYM simulation



Anagnostopoulos-M.H.-Nishimura-Takeuchi 2007,

M.H.-Hyakutake-Nishimura-Takeuchi 2008

ADM mass vs energy density

$$E_{D0} = \frac{9}{2^{11}\pi^{\frac{13}{2}}\Gamma(\frac{9}{2})\lambda^2} N^2 U_0^7$$

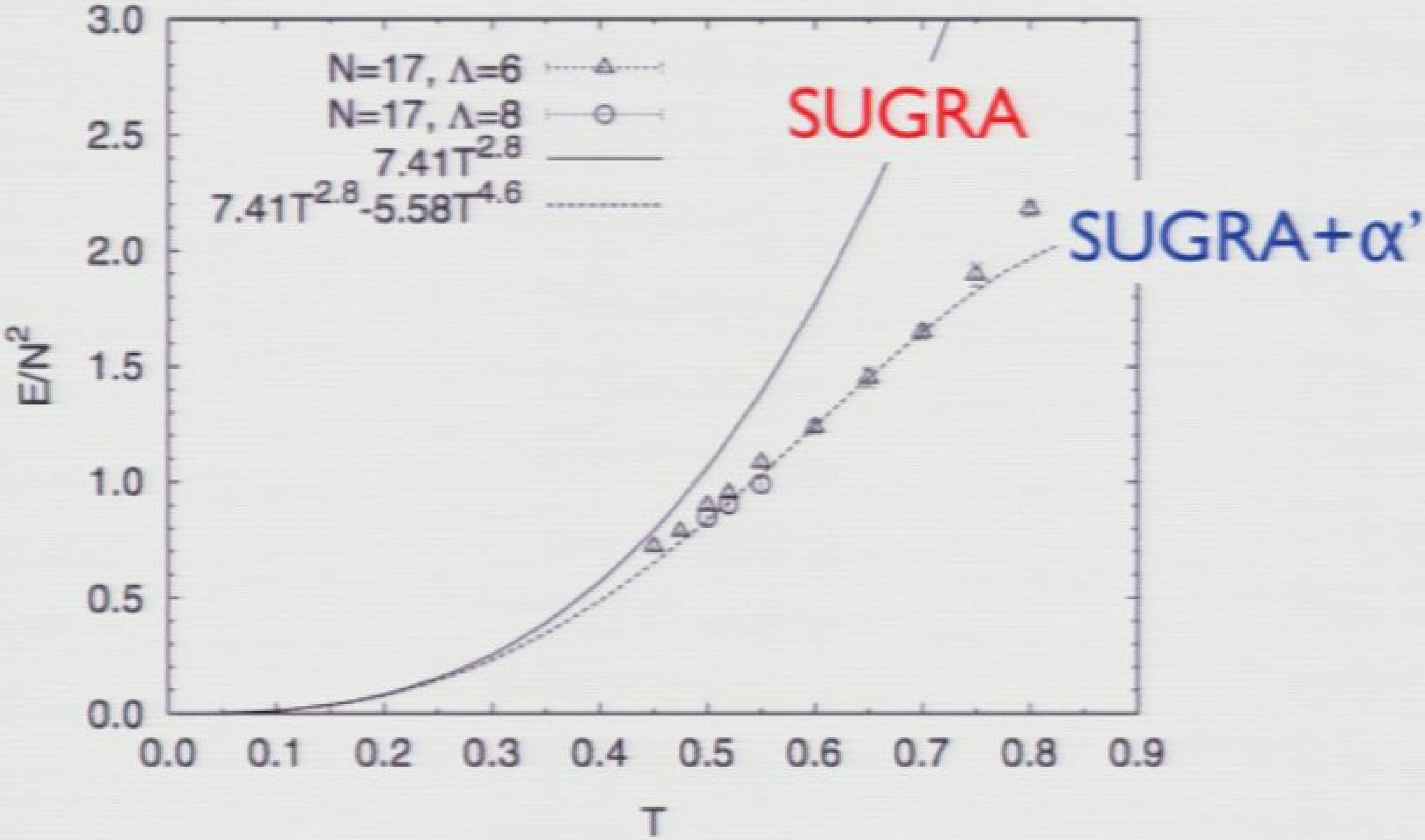
$$\frac{1}{N^2} E_{D0} \sim 7.4\, T^{2.8} \quad (\lambda = 1)$$

at large-N & low temperature (strong coupling)

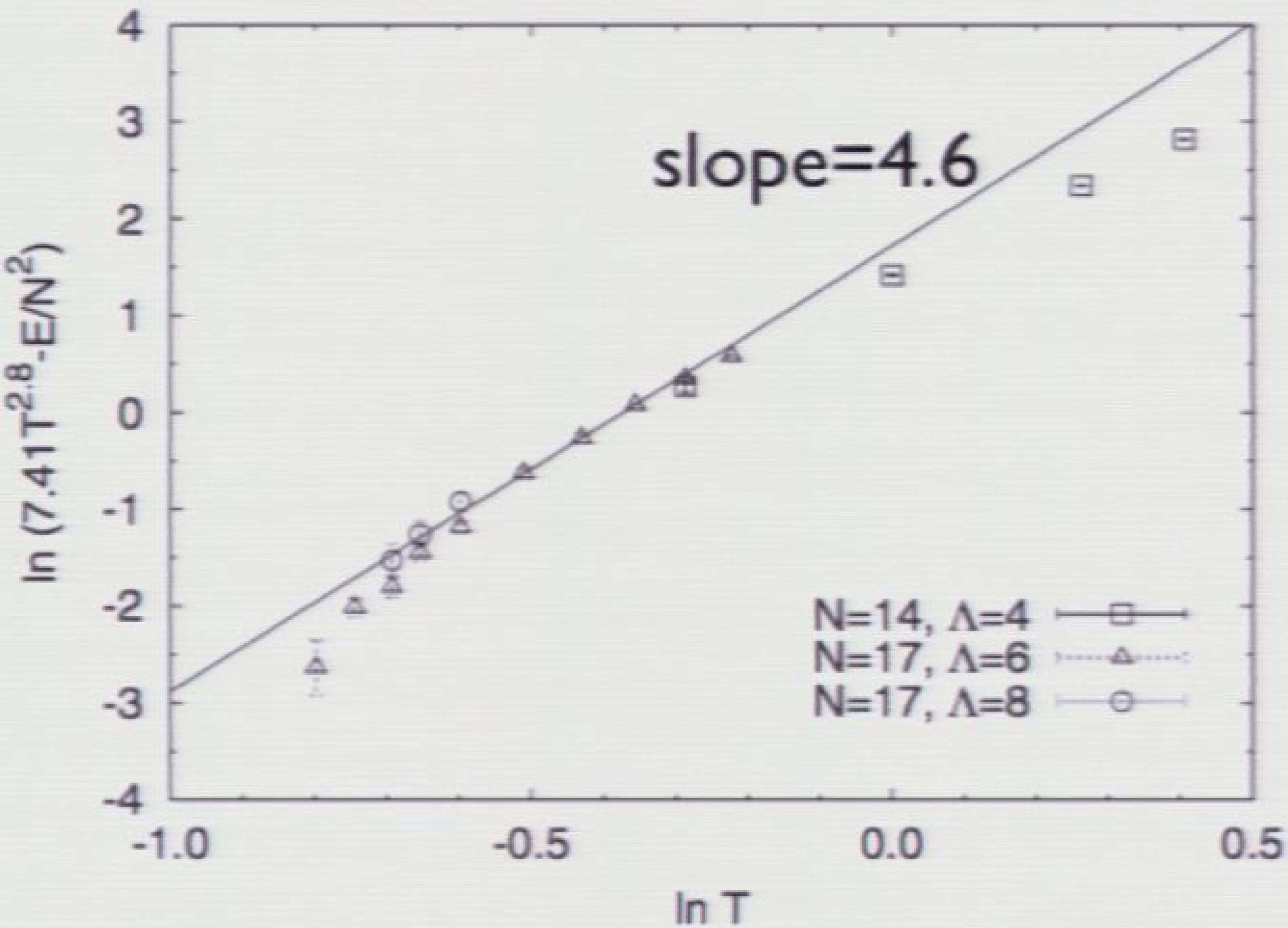
α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

'prediction' by SYM simulation



M.H.-Hyakutake-Nishimura-Takeuchi 2008



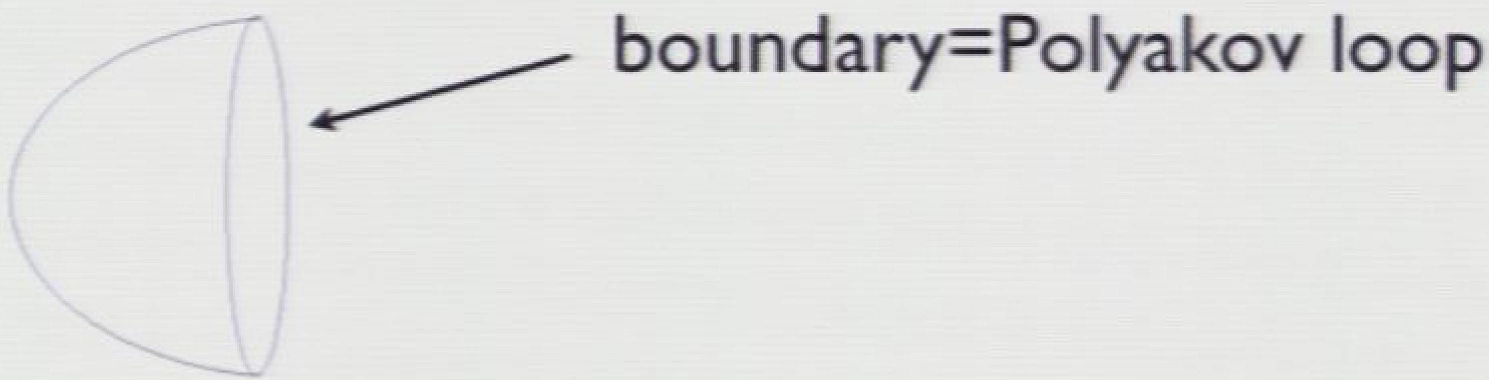
M.H.-Hyakutake-Nishimura-Takeuchi 2008

Polyakov loop with scalar

(Rey-Yee 1998; Maldacena 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$



α' correction

- deviation from the strong coupling (low temperature) corresponds to the α' correction (classical stringy effect).
- The α' correction to SUGRA starts from $(\alpha')^3$ order
- Correction to the BH mass : $(\alpha'/R^2)^3 \sim T^{1.8}$
- $E/N^2 = 7.41T^{2.8} - 5.58T^{4.6}$

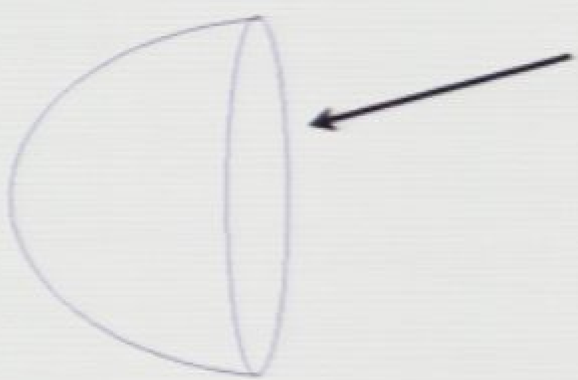
'prediction' by SYM simulation

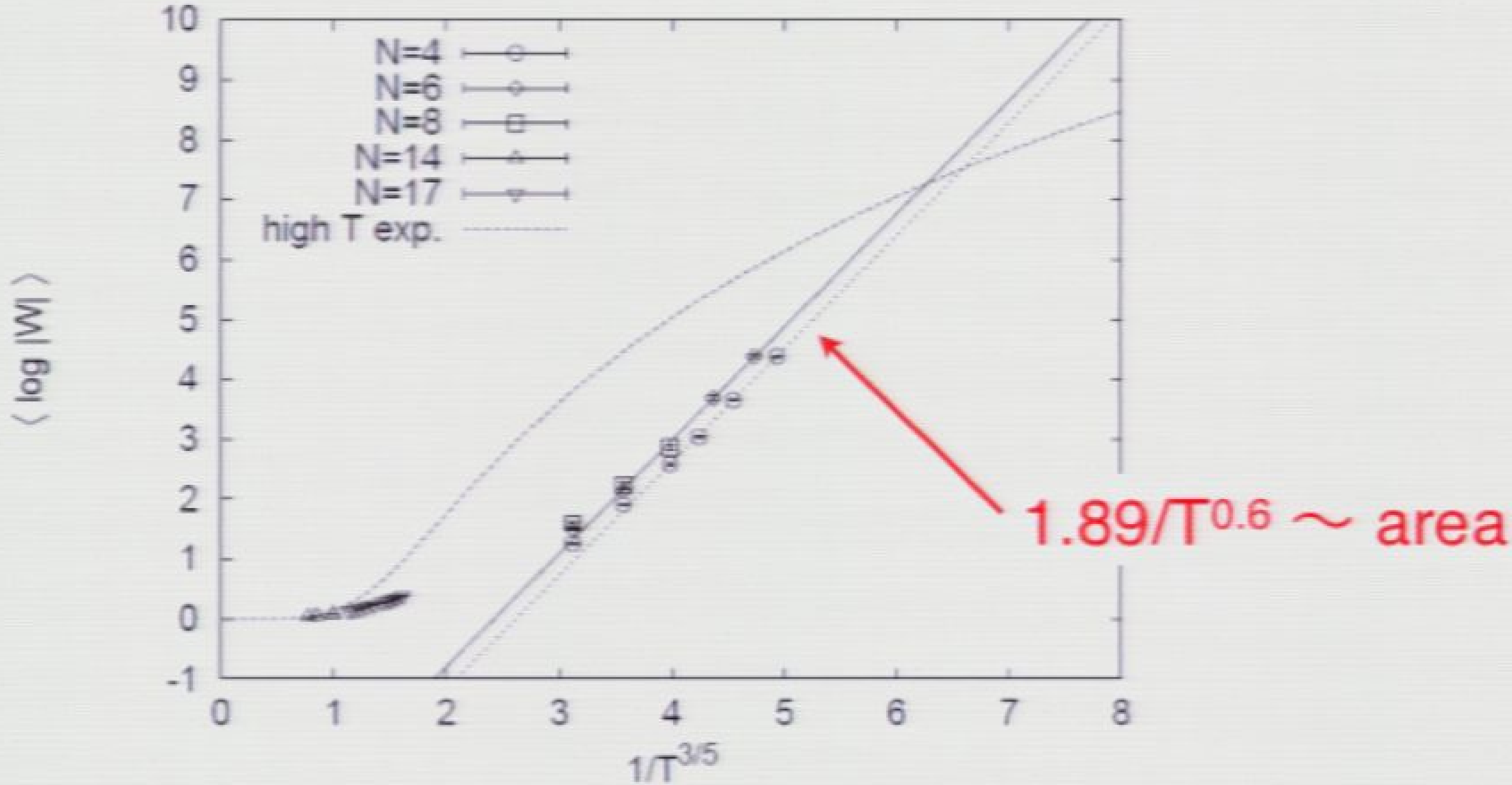
Polyakov loop with scalar

(Rey-Yee 1998; Maldacena 1998)

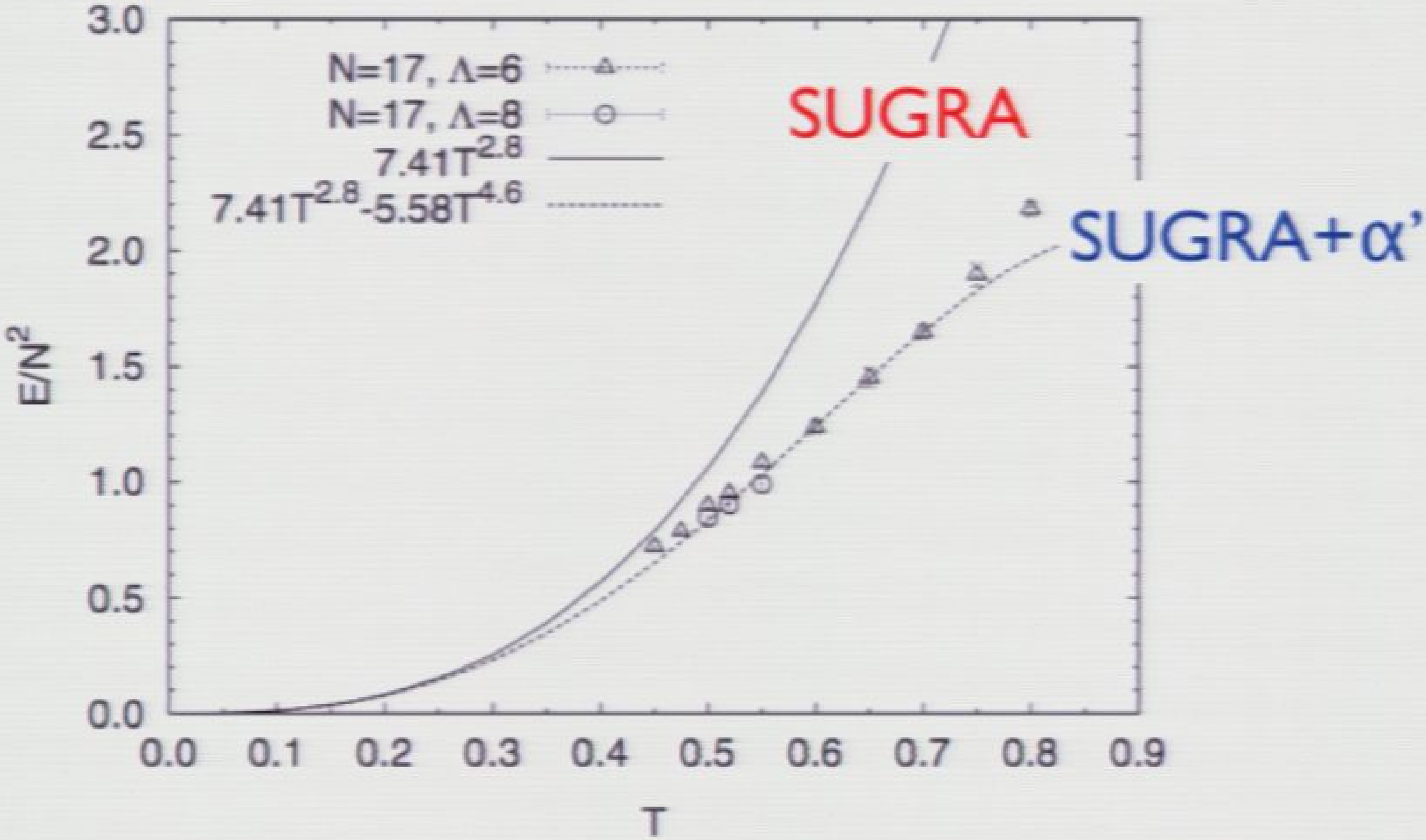
$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$





M.H.-Miwa-Nishimura-Takeuchi, 2008



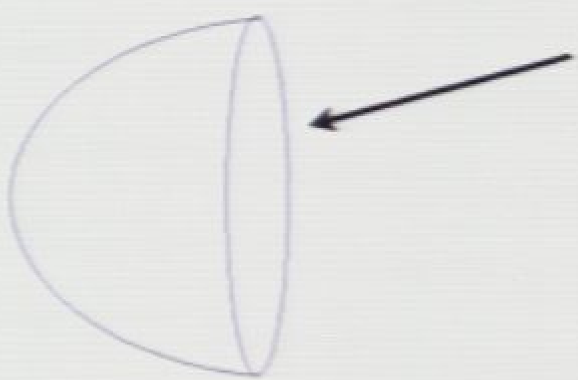
M.H.-Hyakutake-Nishimura-Takeuchi 2008

Polyakov loop with scalar

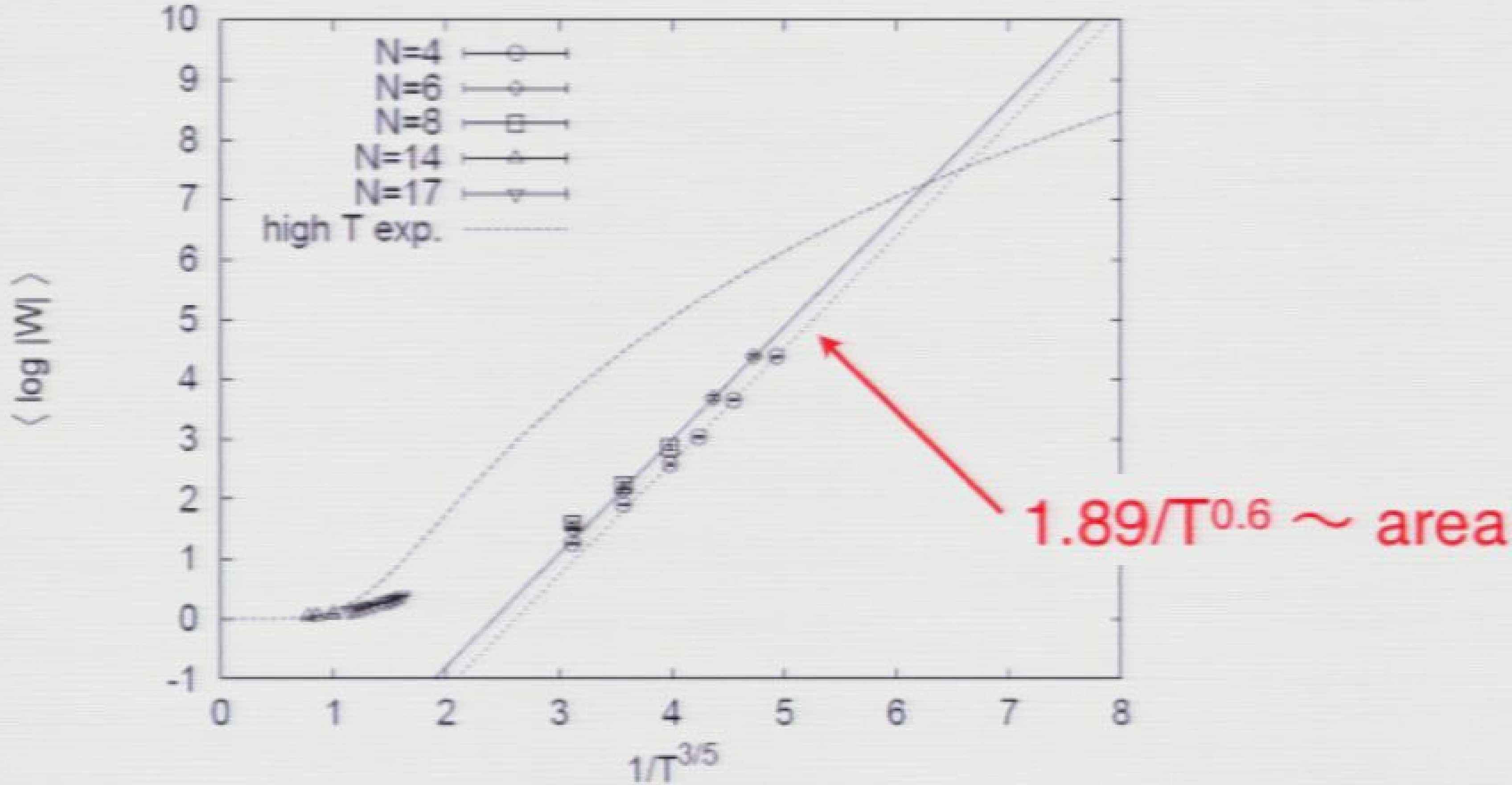
(Rey-Yee 1998; Maldacena 1998)

$$W = Tr\mathrm{P}\exp\left(\int (iA + X)dt\right)$$

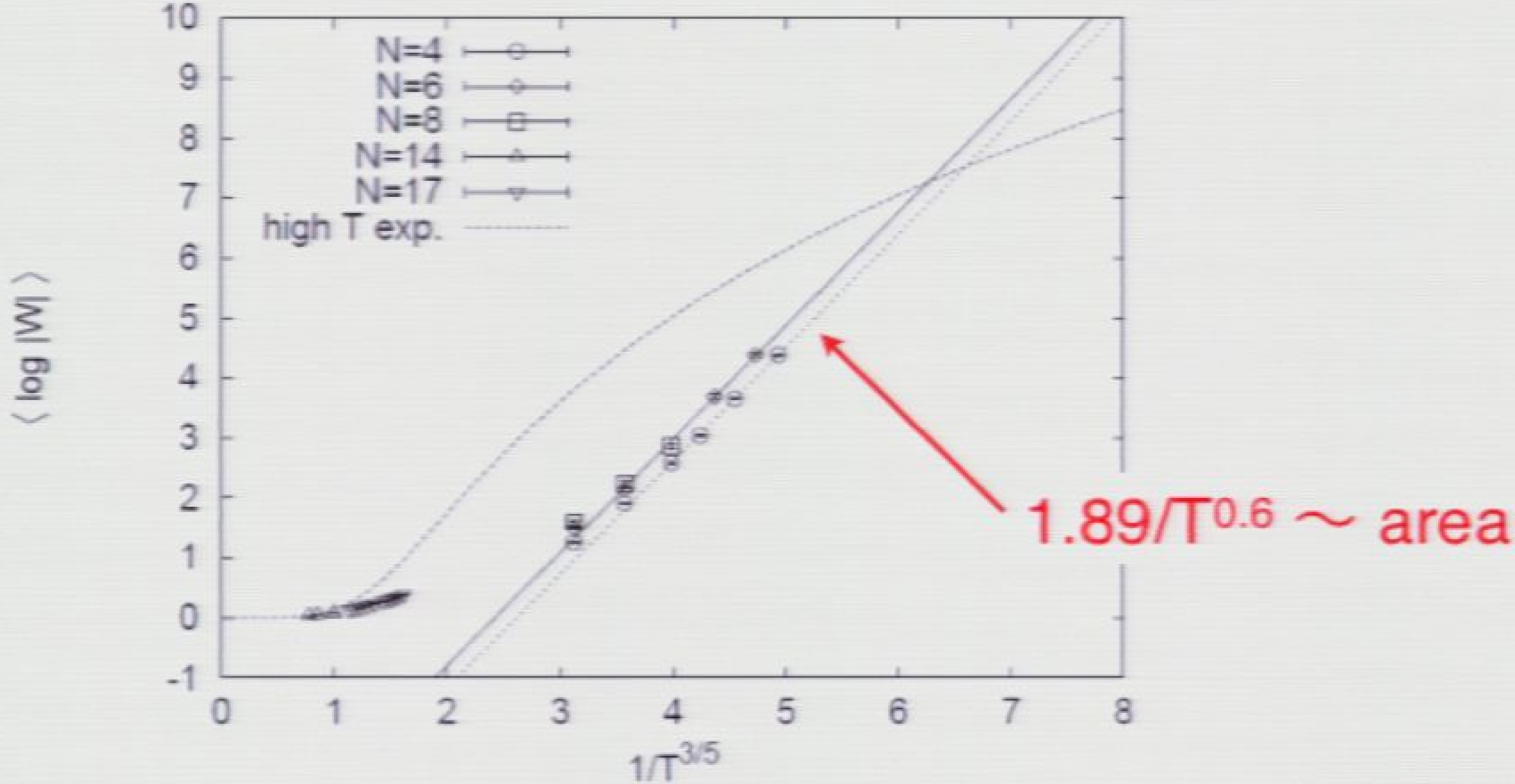
$$\log\langle W\rangle \sim \langle \log W\rangle \sim \text{area of minimal surface}$$



boundary=Polyakov loop



M.H.-Miwa-Nishimura-Takeuchi, 2008



M.H.-Miwa-Nishimura-Takeuchi, 2008

Correlation functions (GKPW relation)

- AdS/CFT (D3-brane) → GKPW relation

  (Gubser-Klebanov-Polyakov 1998, Witten 1998)

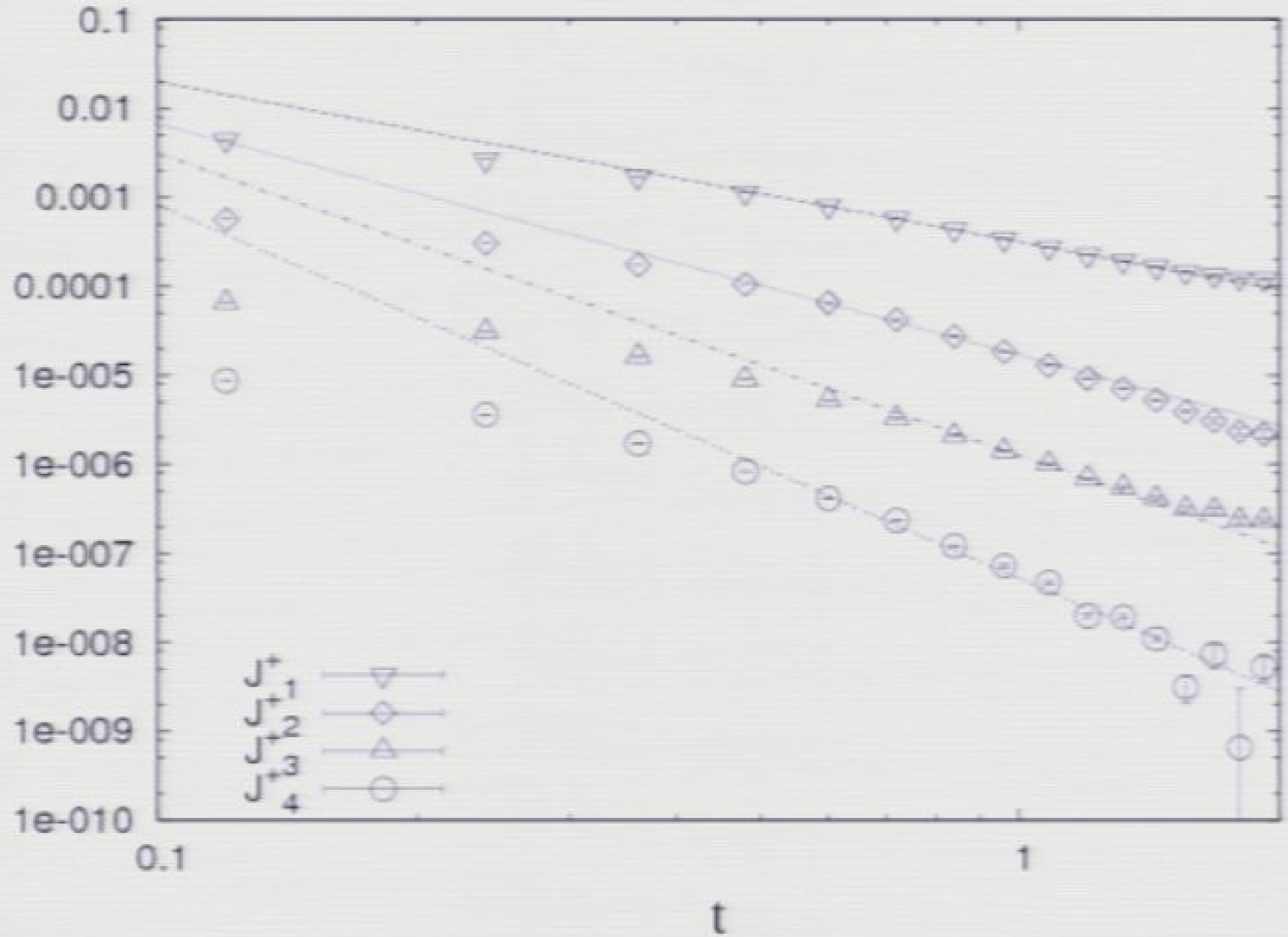
- Similar relation in D0-brane theory :

  "generalized" conformal dimension
  ⇔ mass of field excitations

  (Sekino-Yoneya 1999)

$$\langle \mathcal{O}(t)\mathcal{O}(0)\rangle \sim t^{-p}$$

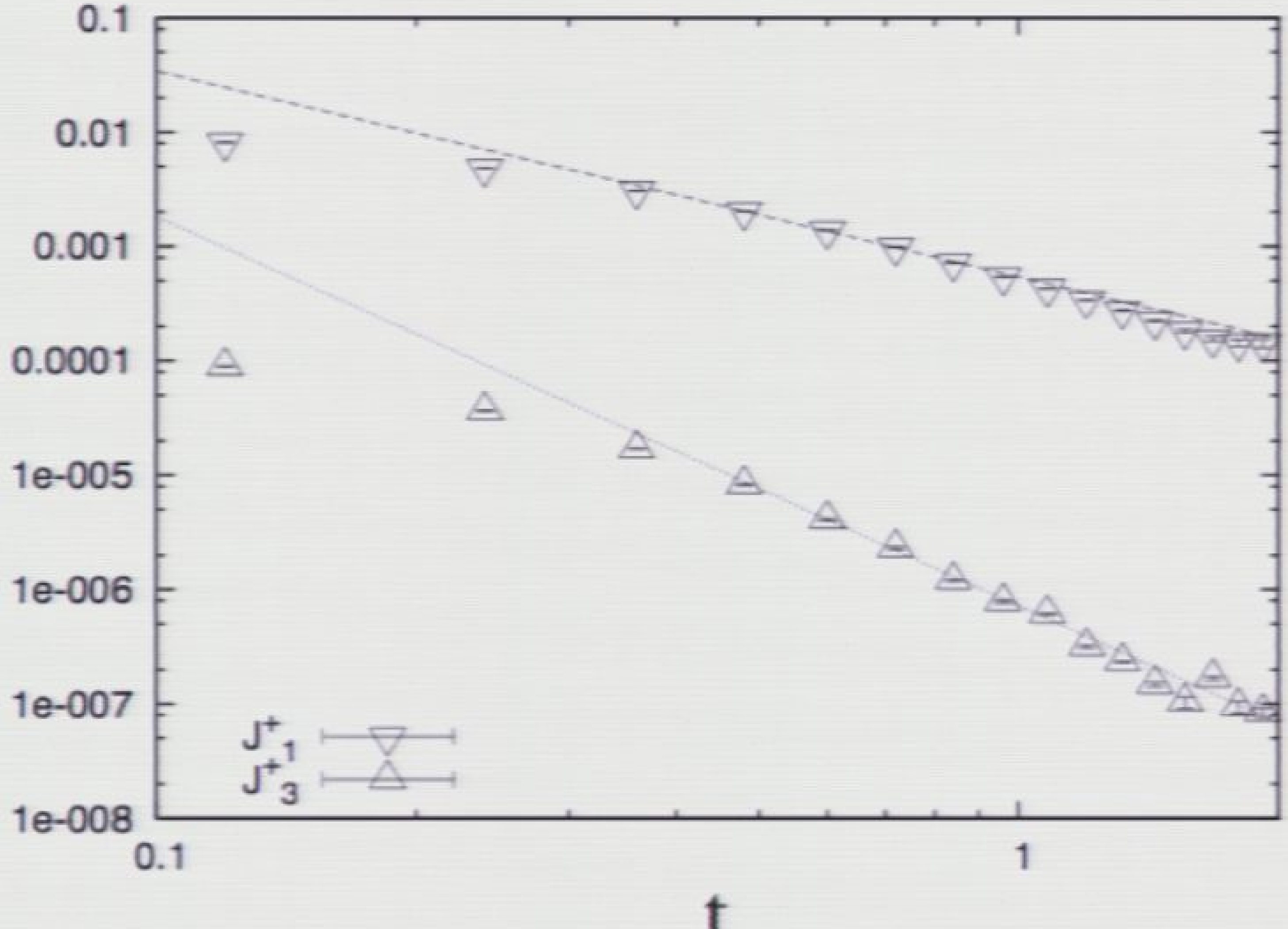
calculable via SUGRA



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N}\cdot Str\left(F_{ij}X_{i_1}\cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya 2009)



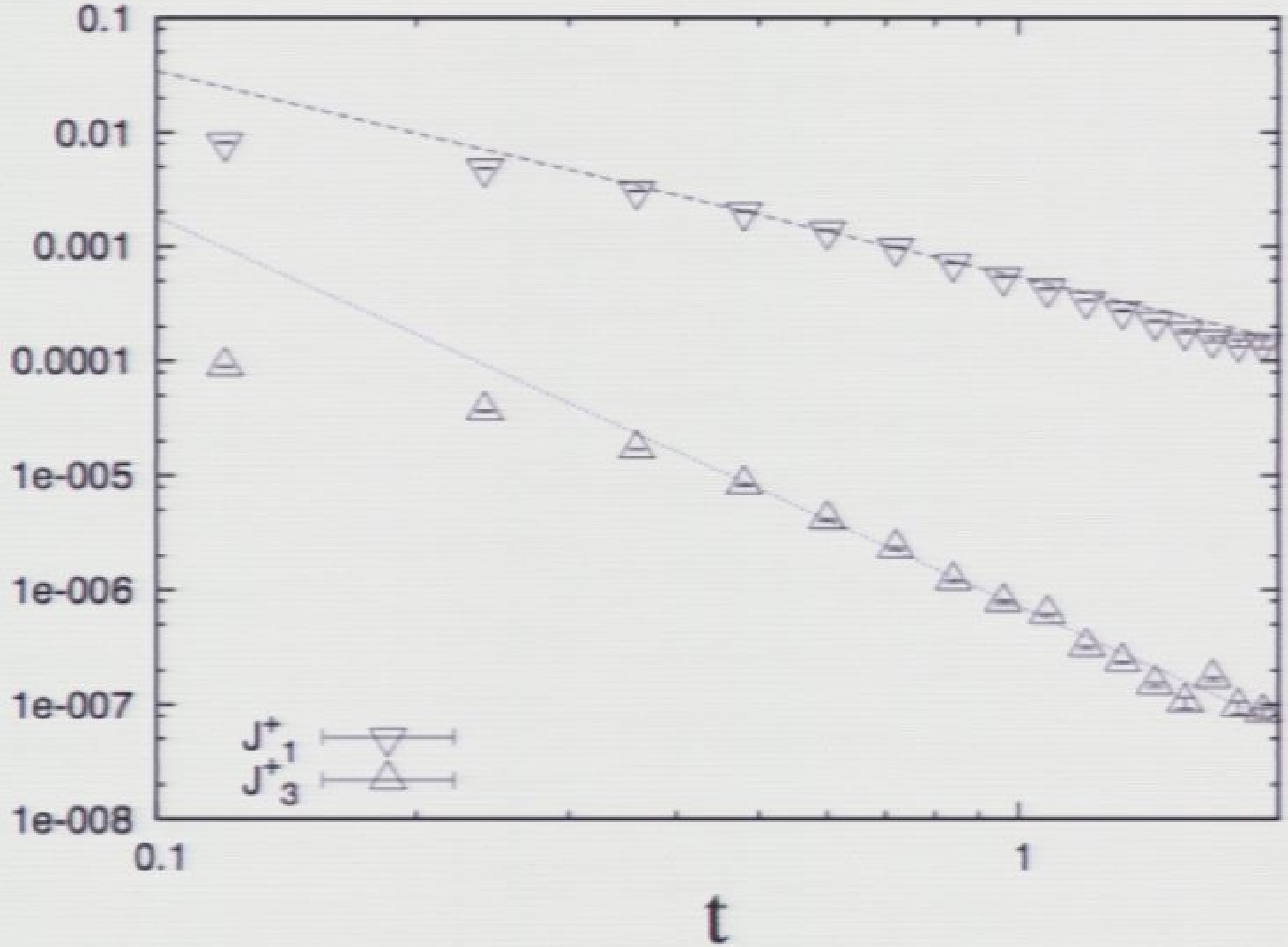
two-point functions, SU(2), pbc

(M.H.-Nishimuea-Sekino-Yoneya, to appear soon)

No sign problem?

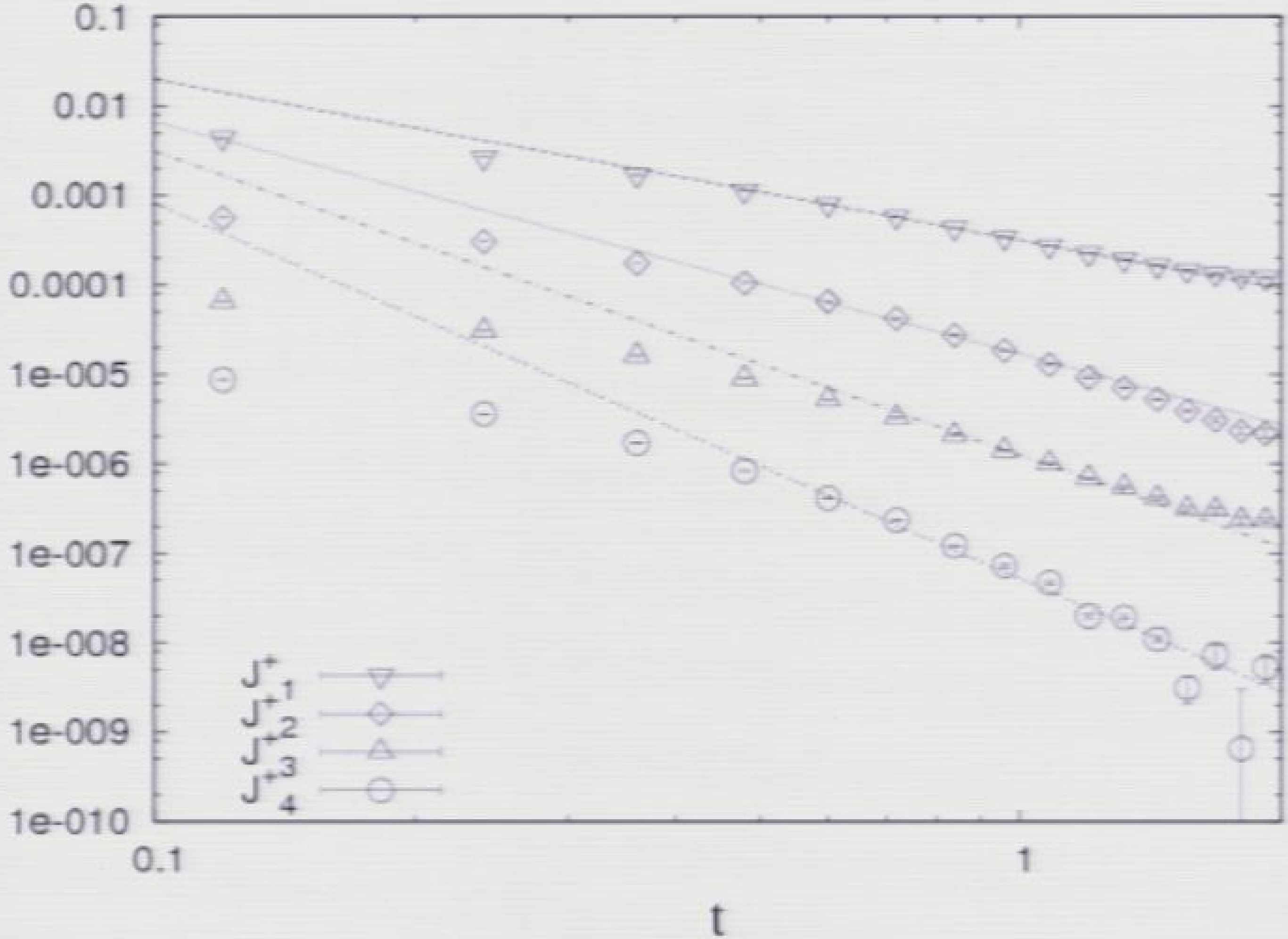
- Down to rather low temperature ($T \sim 0.4$), determinant is almost real positive.
- Even at lower temperature, phase quench works rather well; the results agree with the gravity prediction.
- With periodic b.c. for fermion, sign always fluctuates, except for SU(2). But phase quench reproduces gravity prediction.

There must be a reason for this 'miracle'..



two-point functions, SU(2), pbc

(M.H.-Nishimuea-Sekino-Yoneya, to appear soon)



two-point functions, SU(3), pbc

$$J^{+ij}_{l;i_1\cdots i_l} \equiv \frac{1}{N} \cdot Str\left(F_{ij} X_{i_1} \cdots X_{i_l}\right) \qquad (F_{ij} \equiv -i[X_i, X_j])$$

(M.H.-Nishimuea-Sekino-Yoneya 2009)

No sign problem?

- Down to rather low temperature ($T \sim 0.4$),

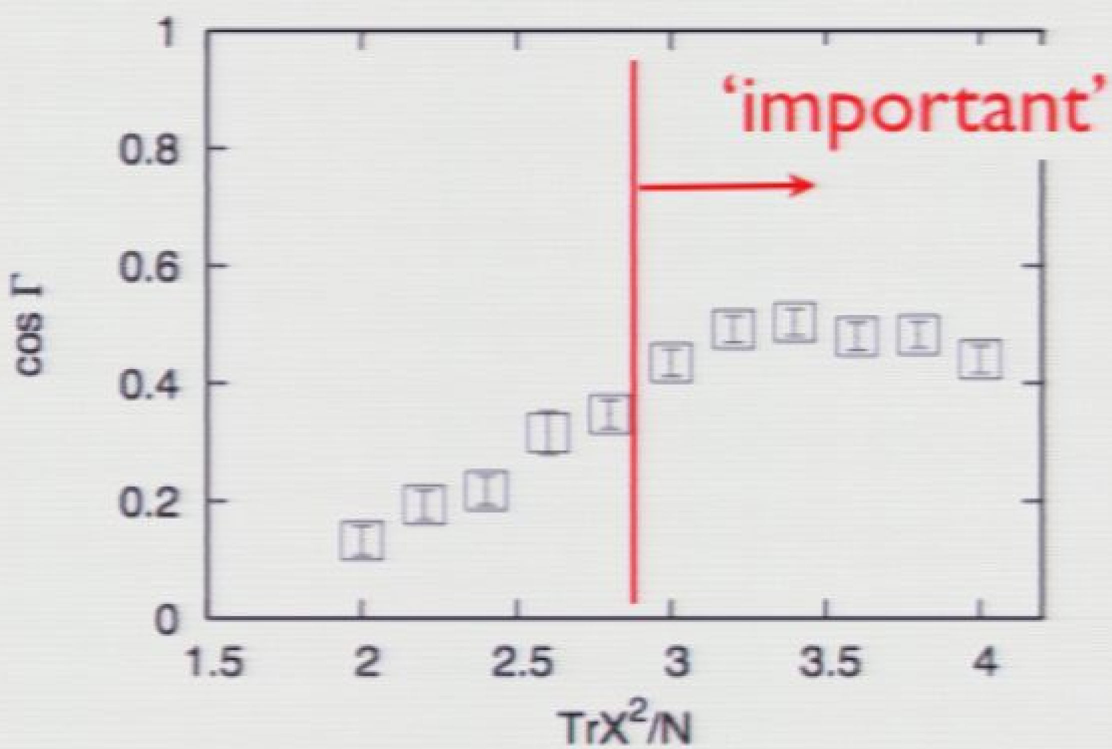
  determinant is almost real positive.

- Even at lower temperature, phase quench works rather well; the results agree with the gravity prediction.

- With periodic b.c. for fermion, sign always fluctuates, except for SU(2). But phase quench reproduces gravity prediction.

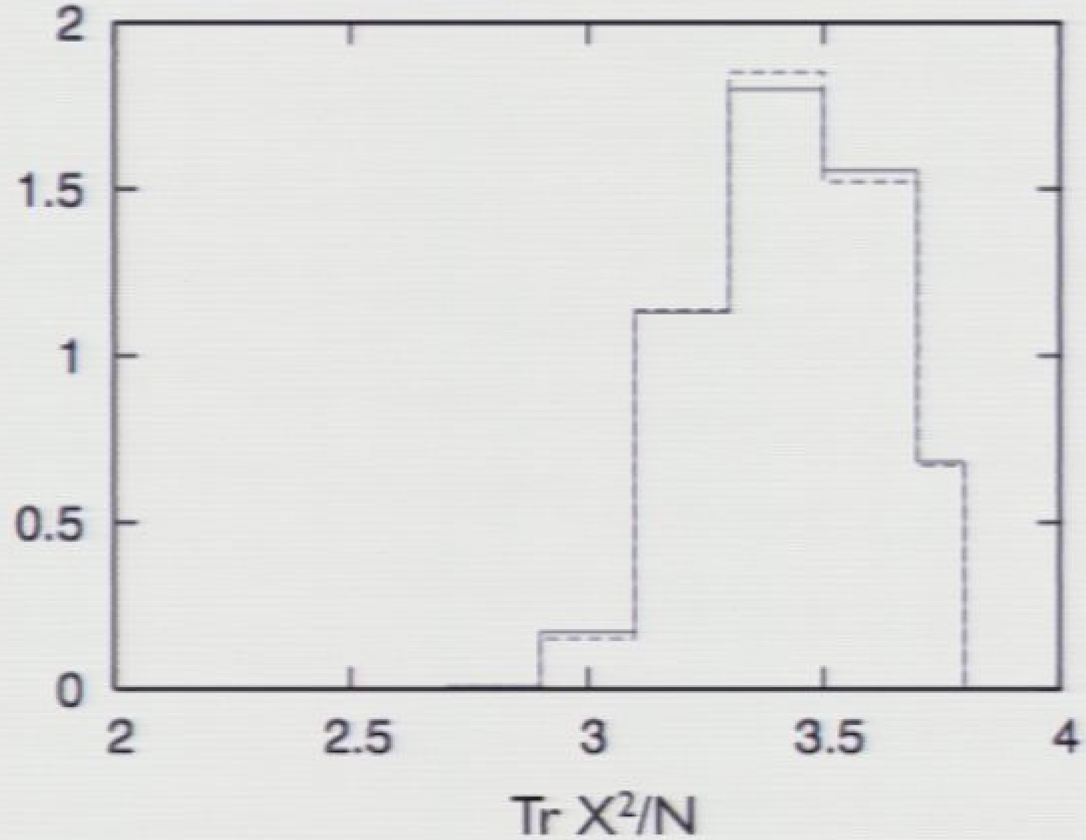
There must be a reason for this 'miracle'..

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

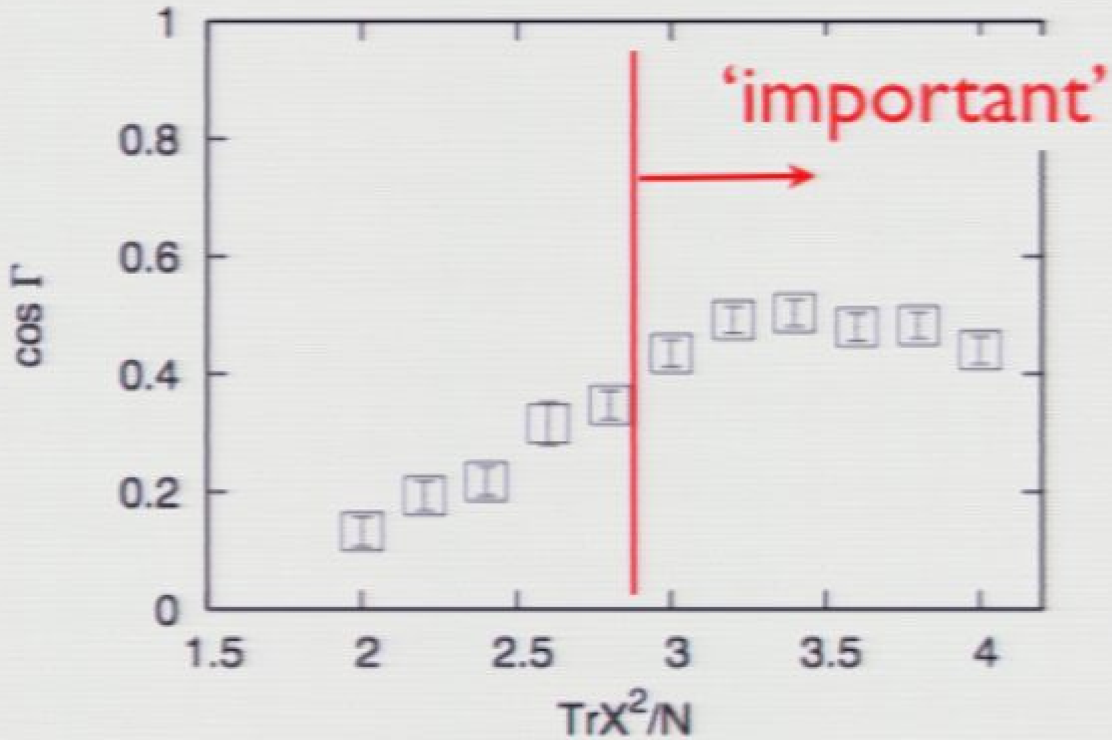
(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

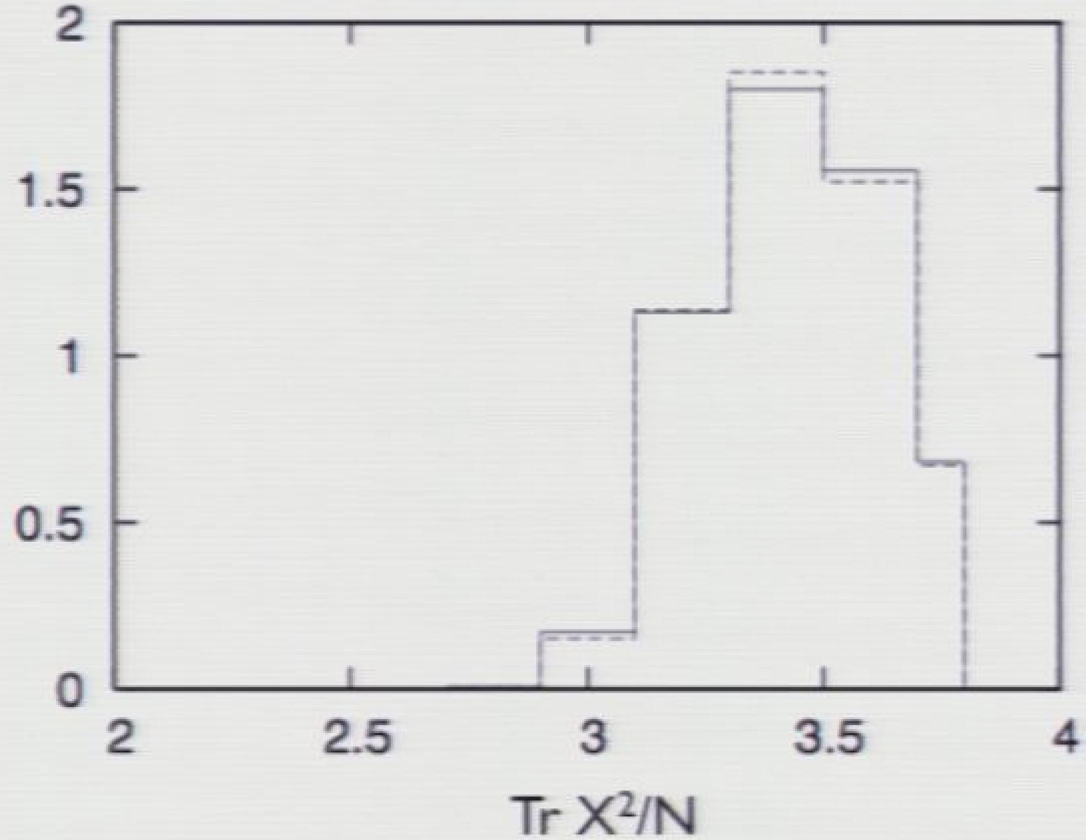
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

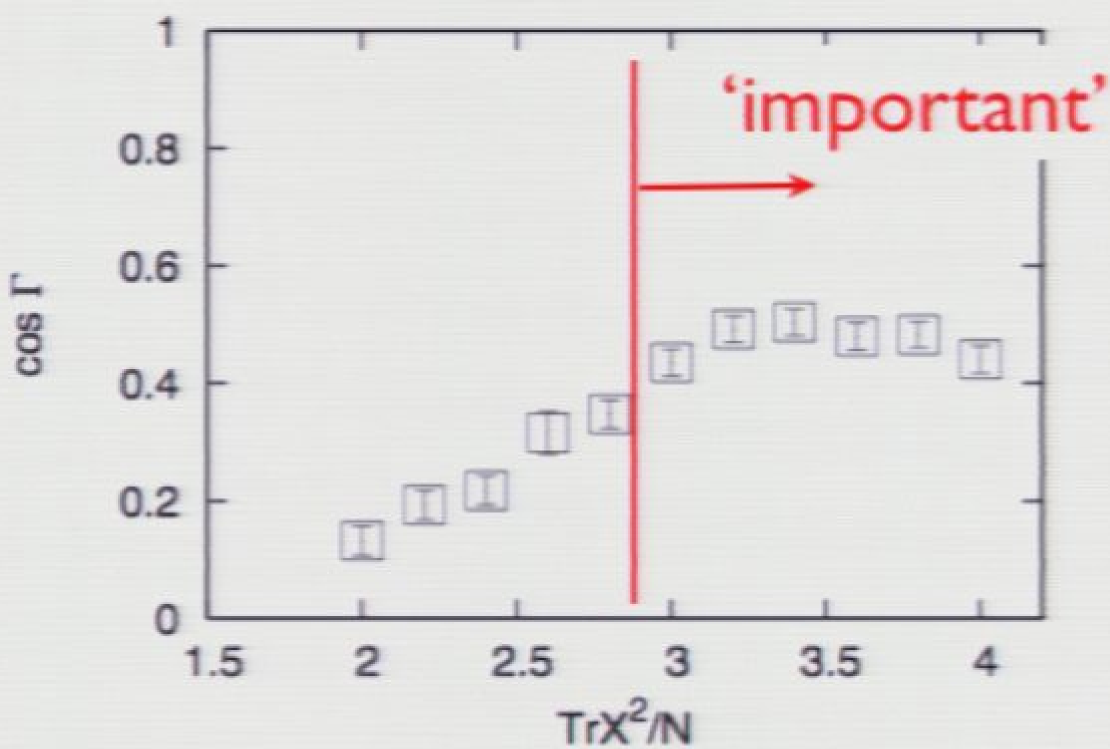
(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

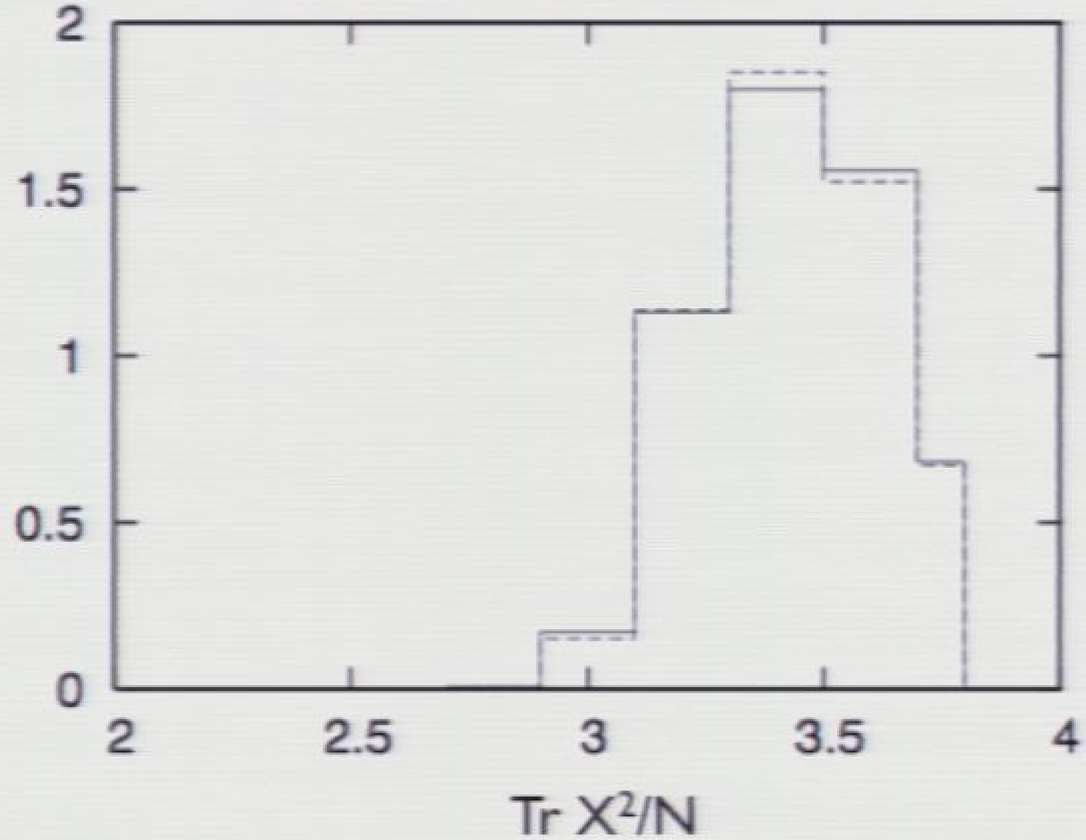
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

(M.H.-Nishimura-Sekino-Yoneya, to appear)



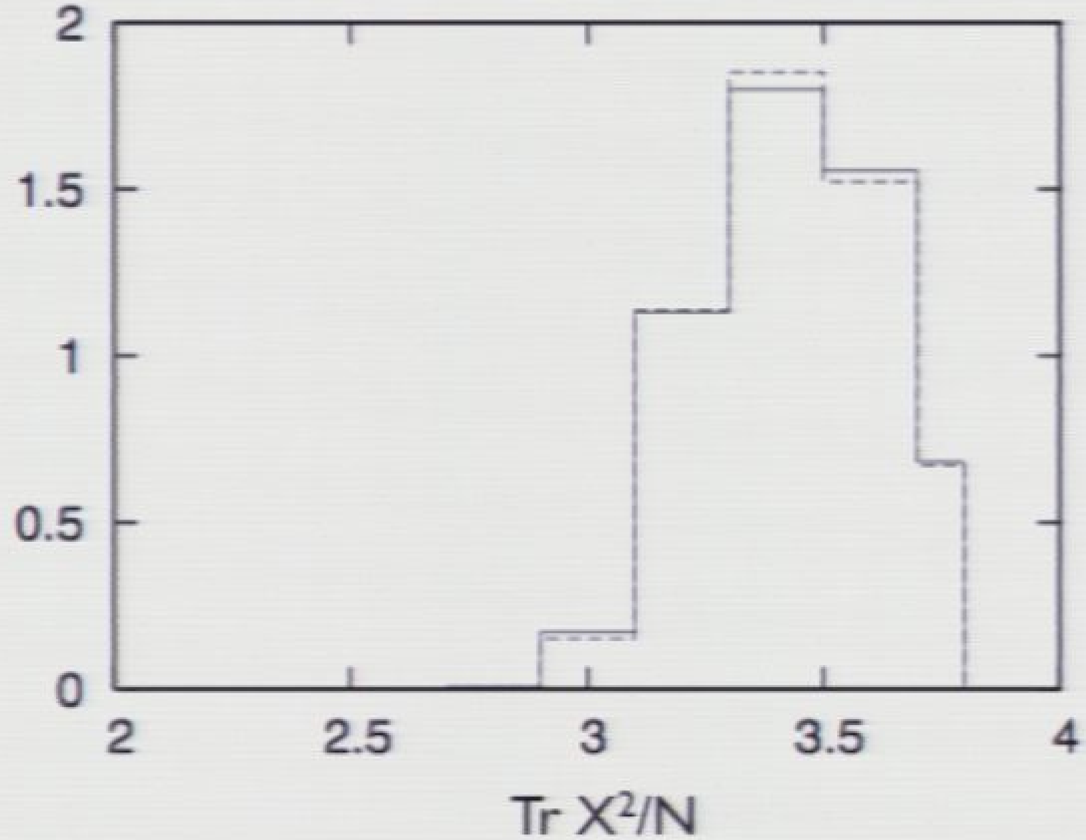
before & after the reweighting

(M.H.-Nishimura-Sekino-Yoneya, to appear)

- If <phase> is constant, phase quench approximation is exact.

$$\begin{aligned}\langle TrX^2\rangle &= \frac{\langle TrX^2\cdot phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}}\\ &= \frac{\langle TrX^2\rangle_{P.Q.}\langle phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}}\\ &= \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

$$\begin{aligned}\langle TrX^2\cdot phase\rangle_{P.Q.} &= \int dx\rho(x)x\cdot\langle phase\rangle_{P.Q.,x}\\ &= c\int dx\rho(x)x\\ &= c\cdot\langle TrX^2\rangle_{P.Q.}\end{aligned}$$



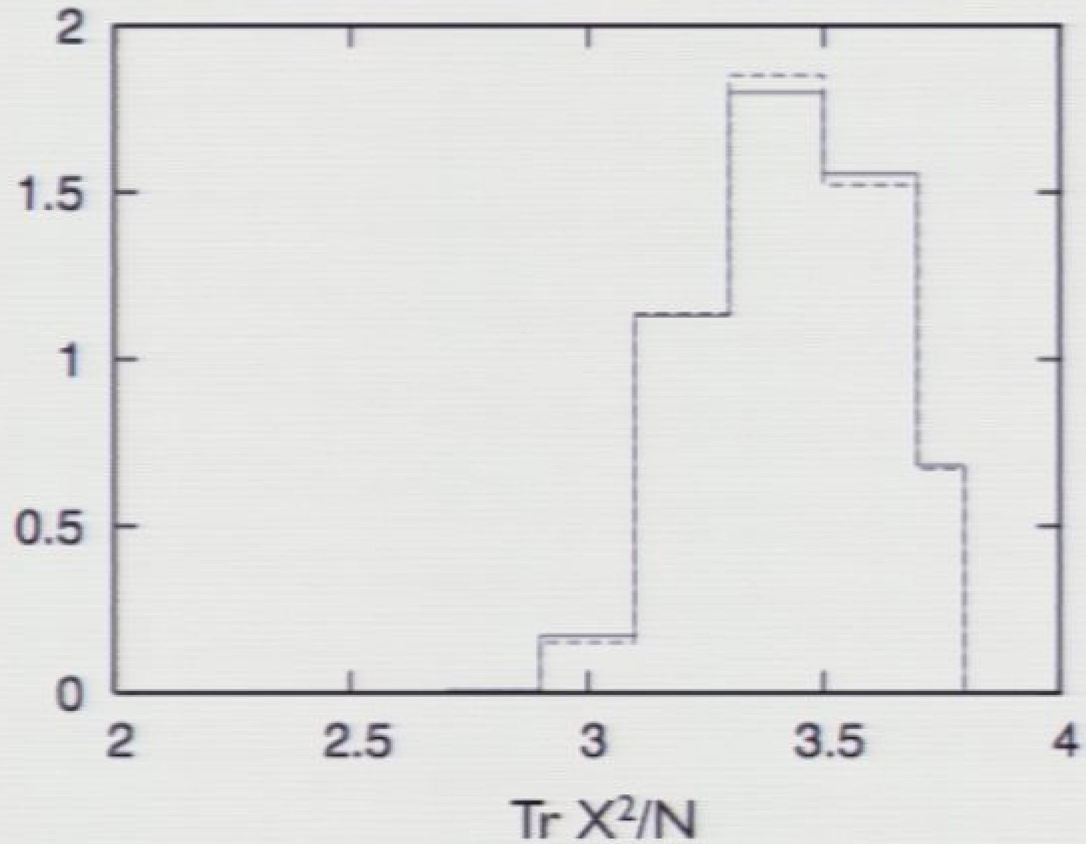
before & after the reweighting

(M.H.-Nishimura-Sekino-Yoneya, to appear)

- If <phase> is constant, phase quench approximation is exact.

$$\begin{aligned}\langle TrX^2\rangle &= \frac{\langle TrX^2 \cdot phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \frac{\langle TrX^2\rangle_{P.Q.}\langle phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

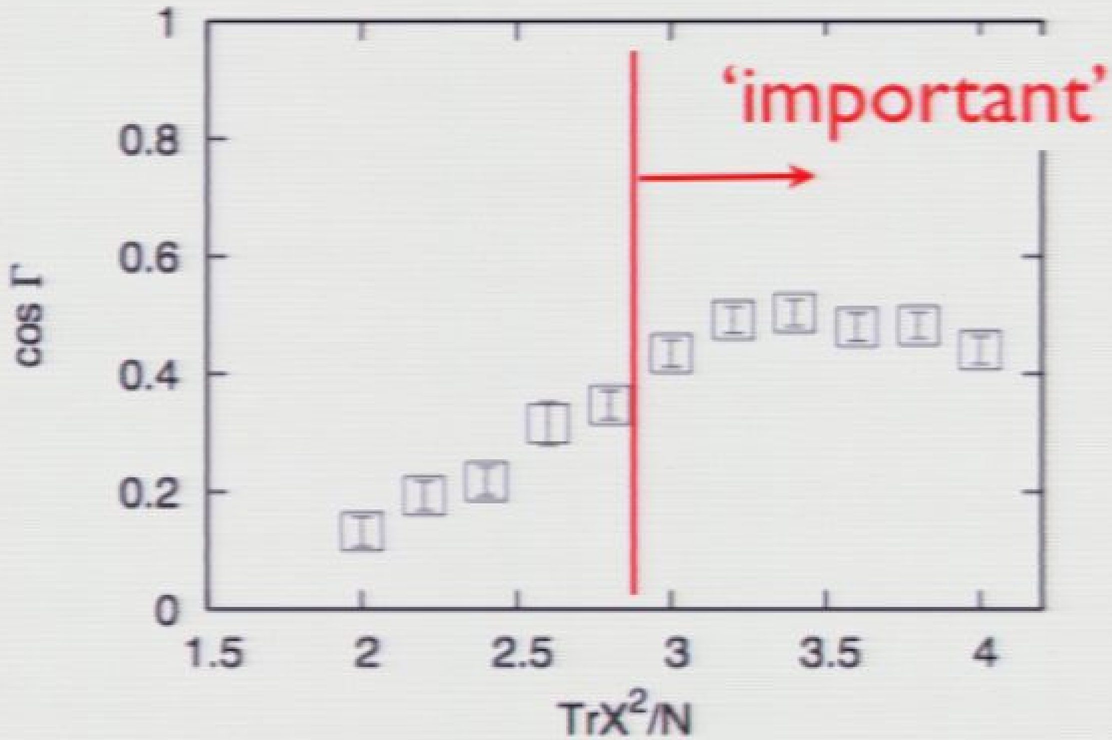
$$\begin{aligned}\langle TrX^2 \cdot phase\rangle_{P.Q.} &= \int dx\rho(x)x \cdot \langle phase\rangle_{P.Q.,x} \\ &= c\int dx\rho(x)x \\ &= c \cdot \langle TrX^2\rangle_{P.Q.}\end{aligned}$$



before & after the reweighting

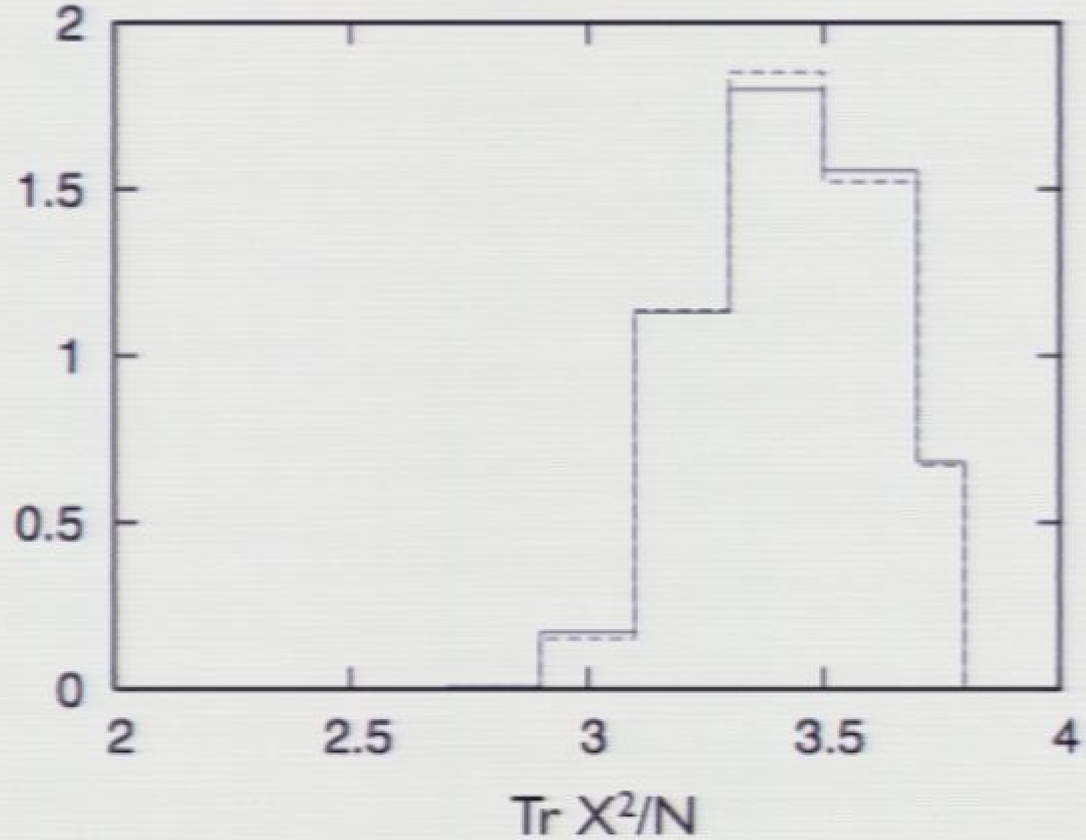
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

(M.H.-Nishimura-Sekino-Yoneya, to appear)

- If <phase> is constant, phase quench approximation is exact.

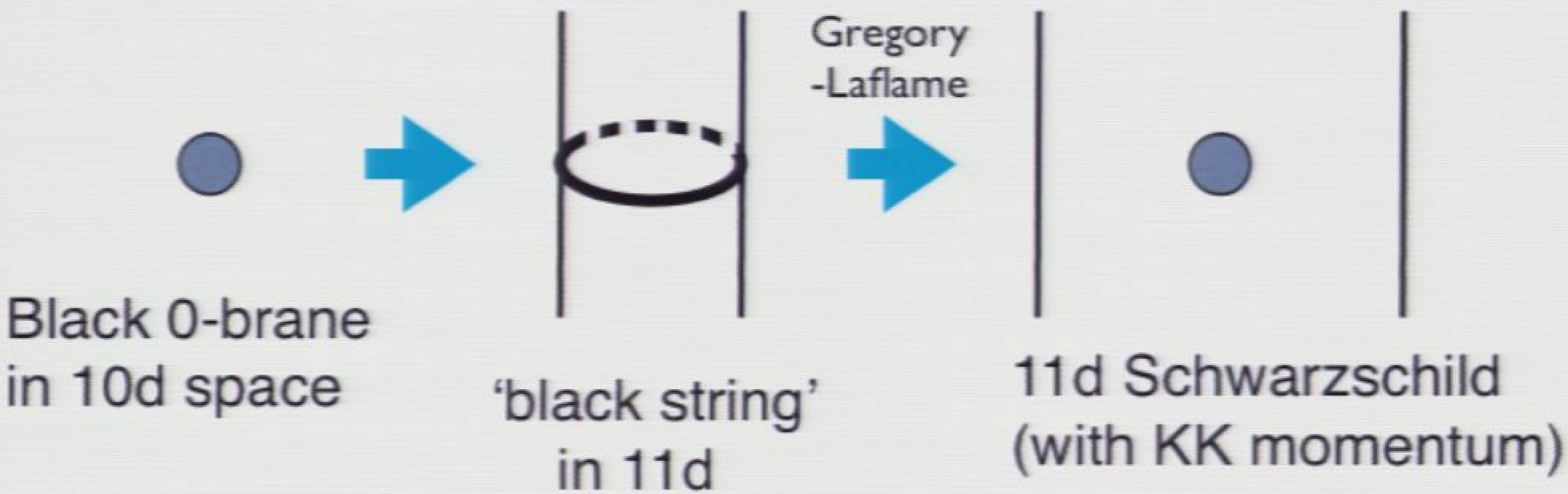
$$\begin{aligned}\langle TrX^2\rangle &= \frac{\langle TrX^2 \cdot phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \frac{\langle TrX^2\rangle_{P.Q.}\langle phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

$$\begin{aligned}\langle TrX^2 \cdot phase\rangle_{P.Q.} &= \int dx\rho(x)x \cdot \langle phase\rangle_{P.Q.,x} \\ &= c\int dx\rho(x)x \\ &= c \cdot \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

1/N correction

(M.H.-Ishiki-Nishimura-Hyakutake, in progress)

- If 1/N correction fully describe string & M-theory corrections...

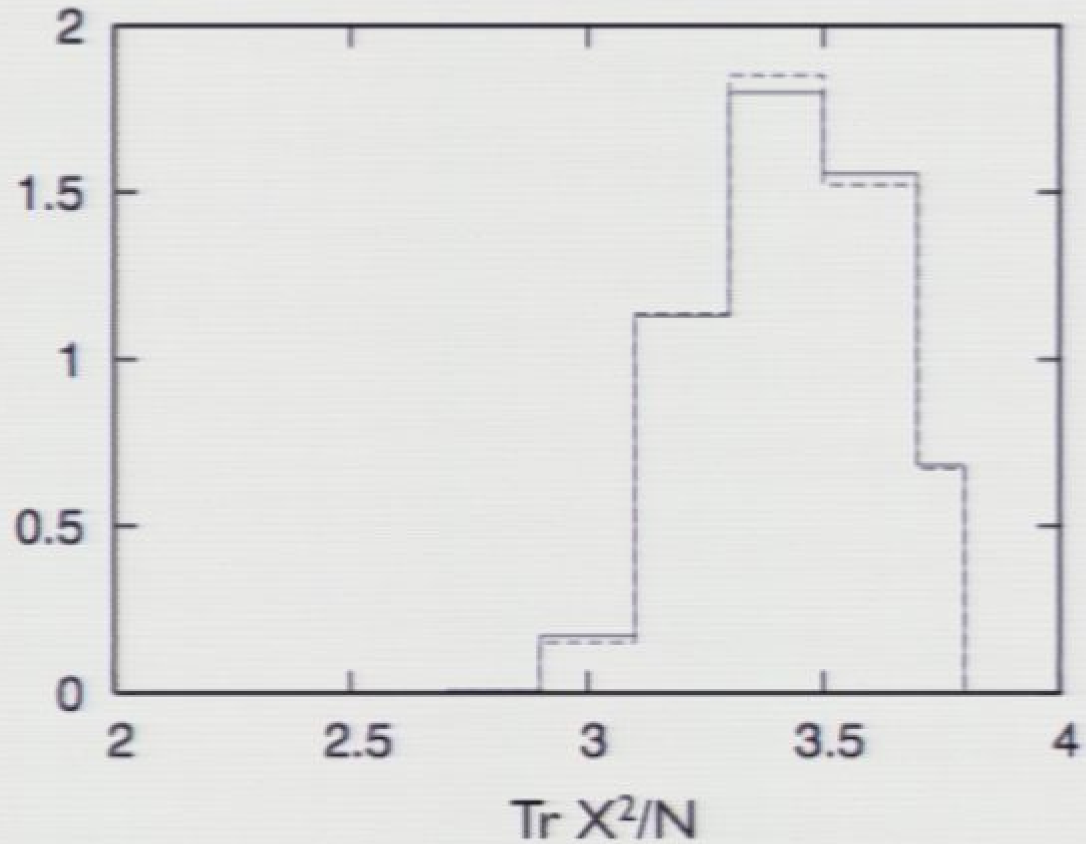


(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

- If <phase> is constant, phase quench approximation is exact.

$$\begin{aligned}\langle TrX^2\rangle &= \frac{\langle TrX^2\cdot phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}}\\ &= \frac{\langle TrX^2\rangle_{P.Q.}\langle phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}}\\ &= \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

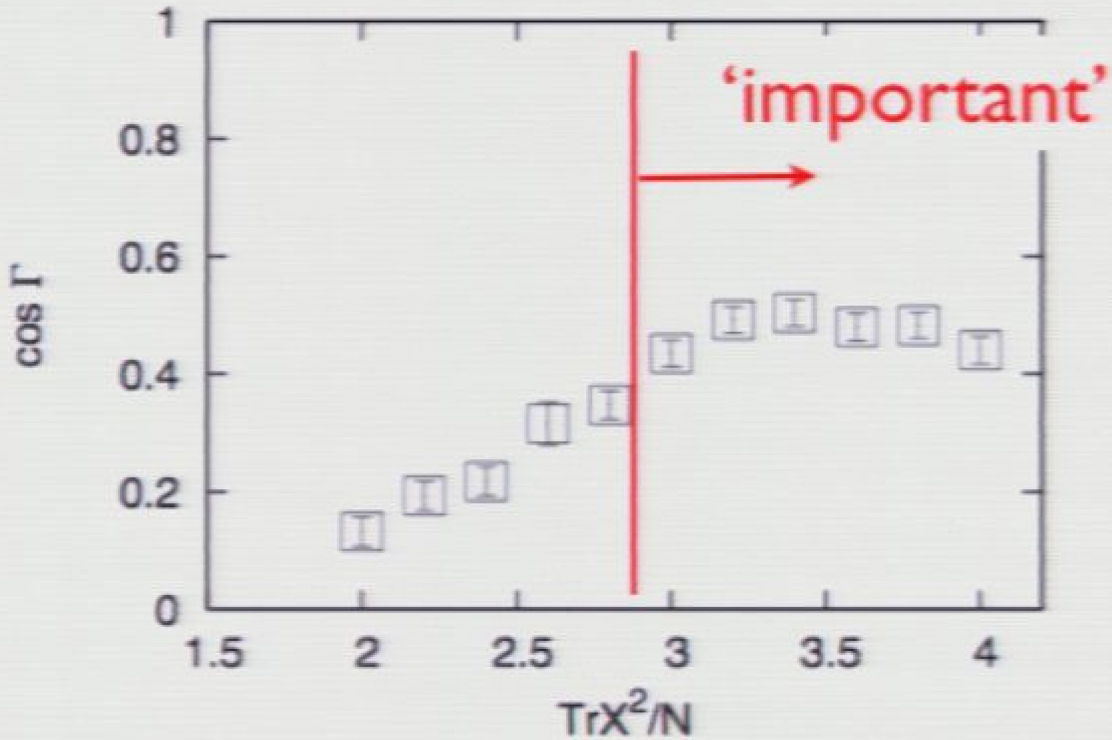
$$\begin{aligned}\langle TrX^2\cdot phase\rangle_{P.Q.} &= \int dx\rho(x)x\cdot\langle phase\rangle_{P.Q.,x}\\ &= c\int dx\rho(x)x\\ &= c\cdot\langle TrX^2\rangle_{P.Q.}\end{aligned}$$



before & after the reweighting

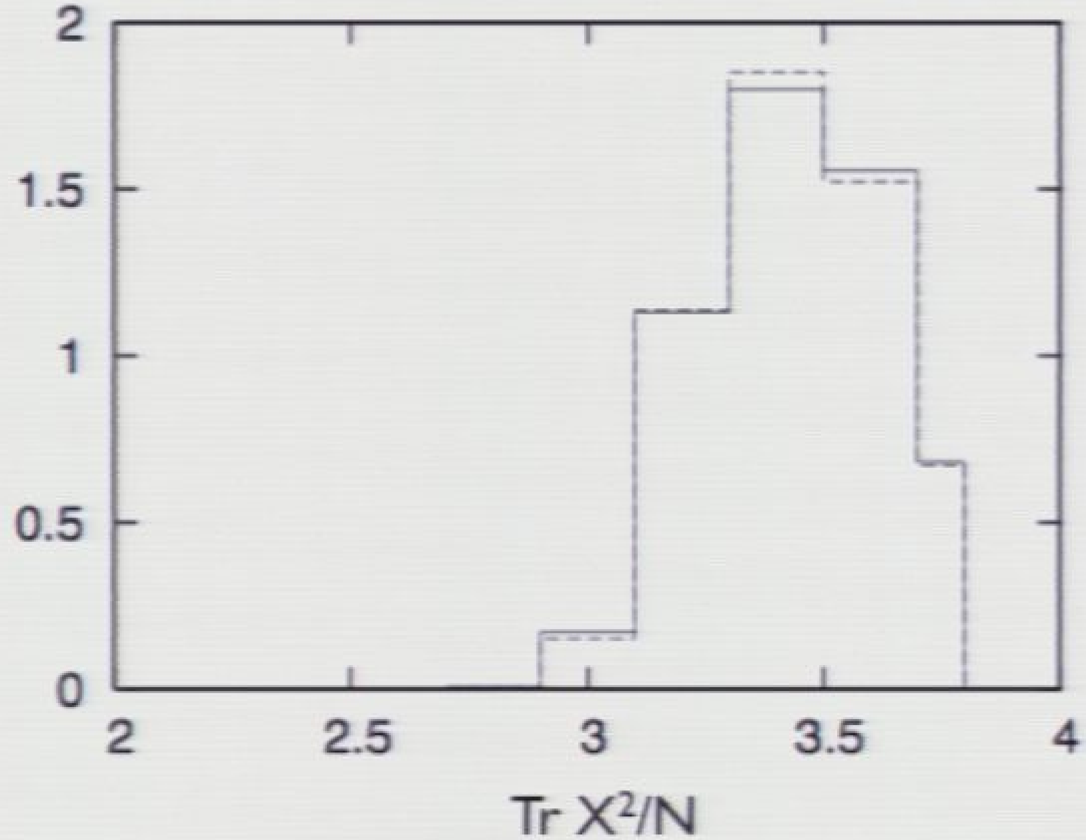
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

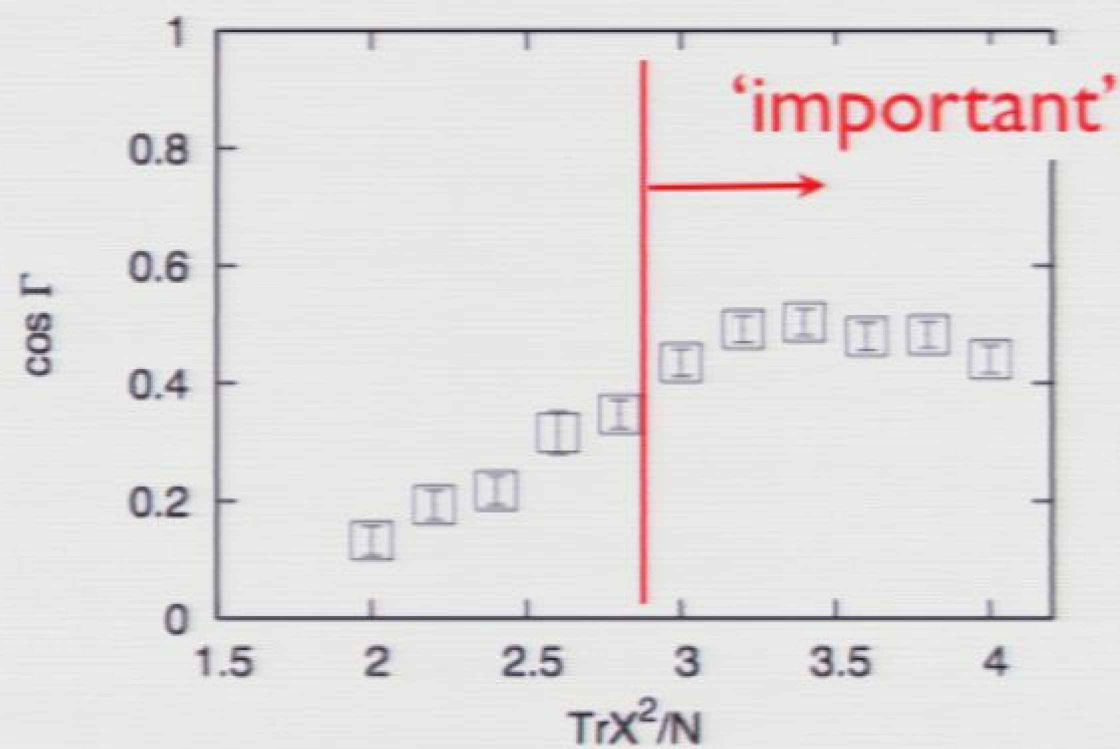
(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

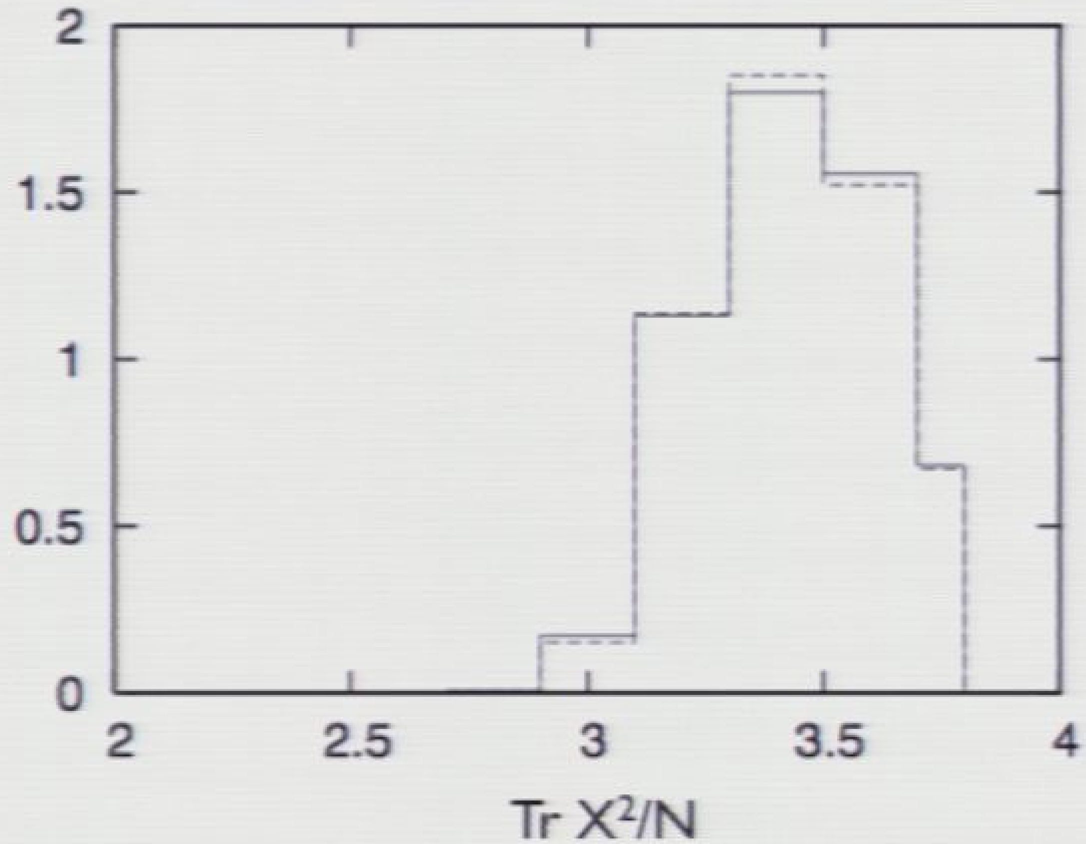
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

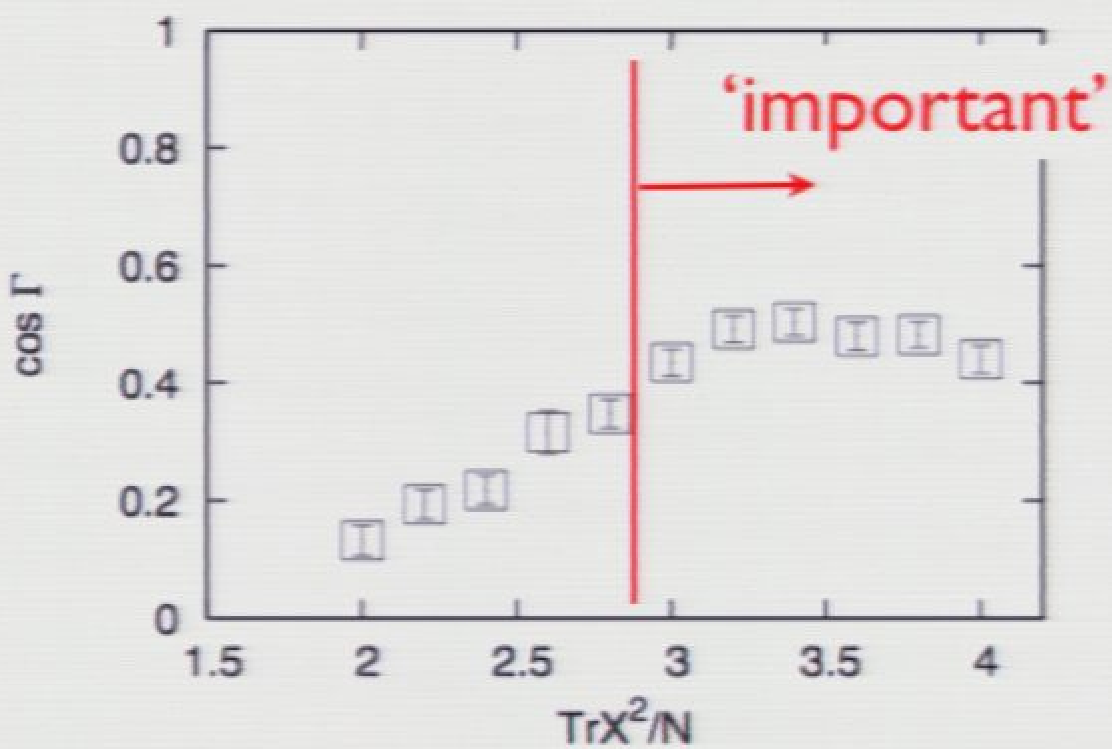
(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

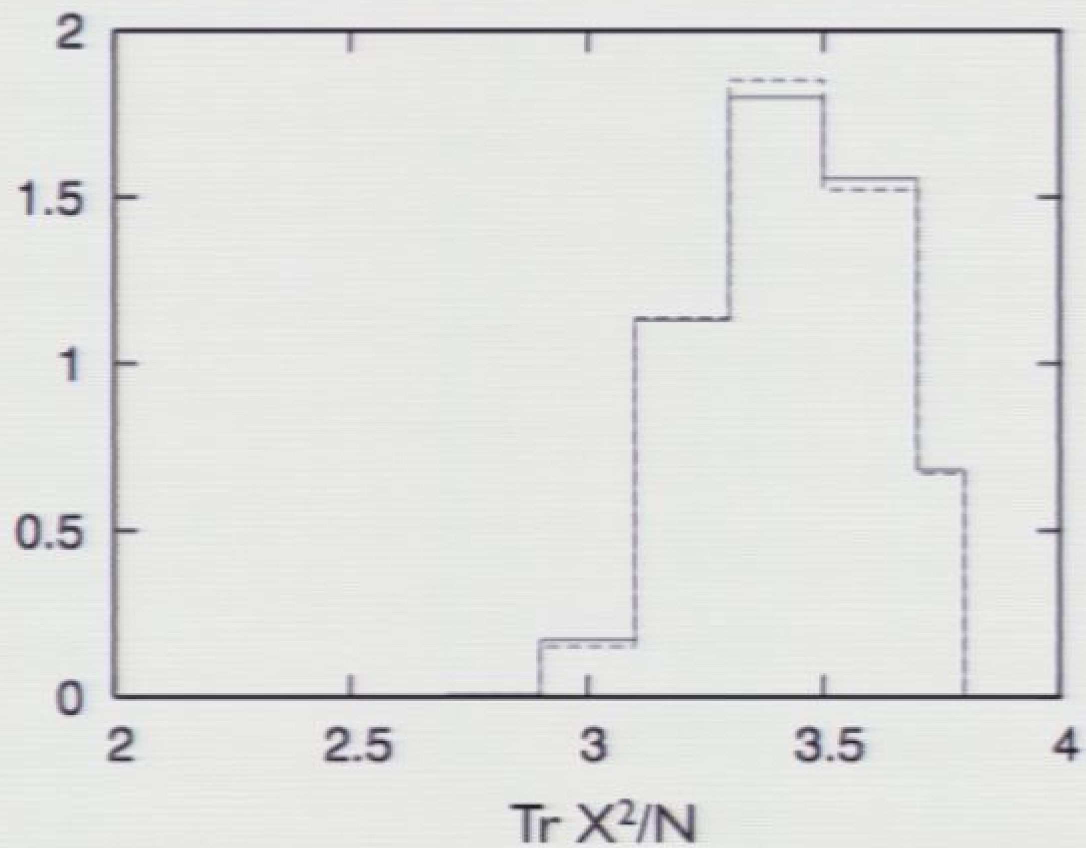
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

(M.H.-Nishimura-Sekino-Yoneya, to appear)



before & after the reweighting

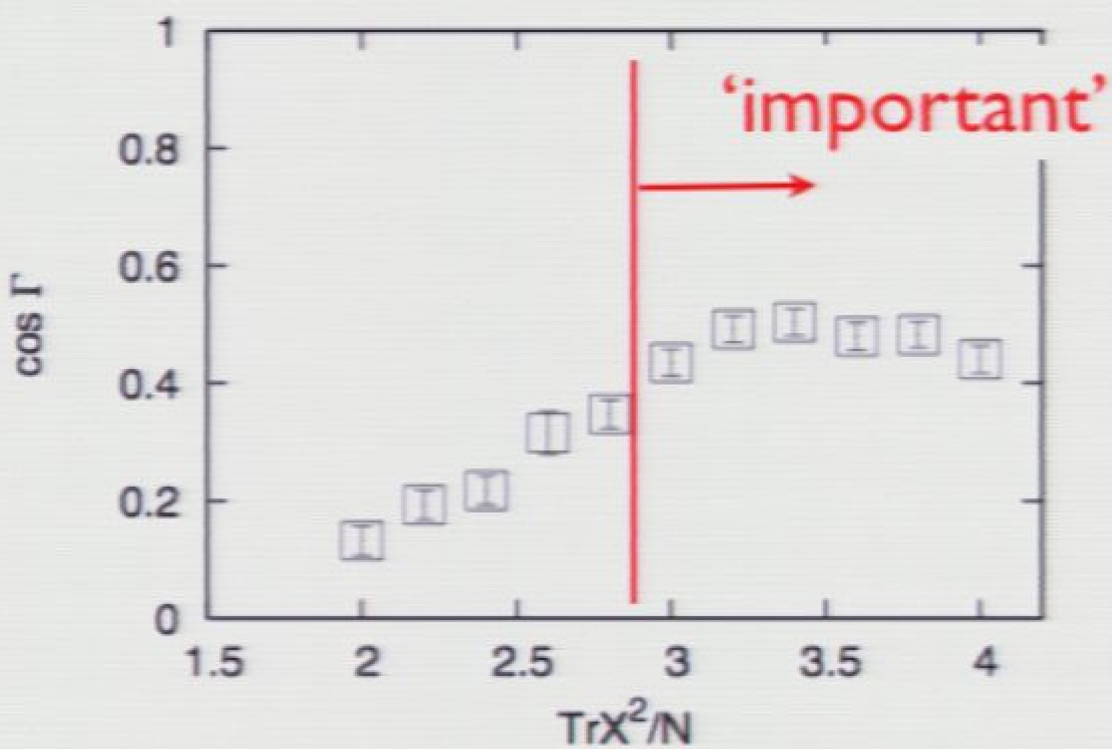
(M.H.-Nishimura-Sekino-Yoneya, to appear)

- If <phase> is constant, phase quench approximation is exact.

$$\begin{aligned}\langle TrX^2\rangle &= \frac{\langle TrX^2 \cdot phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \frac{\langle TrX^2\rangle_{P.Q.}\langle phase\rangle_{P.Q.}}{\langle phase\rangle_{P.Q.}} \\ &= \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

$$\begin{aligned}\langle TrX^2 \cdot phase\rangle_{P.Q.} &= \int dx\rho(x)x \cdot \langle phase\rangle_{P.Q.,x} \\ &= c\int dx\rho(x)x \\ &= c \cdot \langle TrX^2\rangle_{P.Q.}\end{aligned}$$

- Suppose phase and the value of the observable x are not correlated, that is,



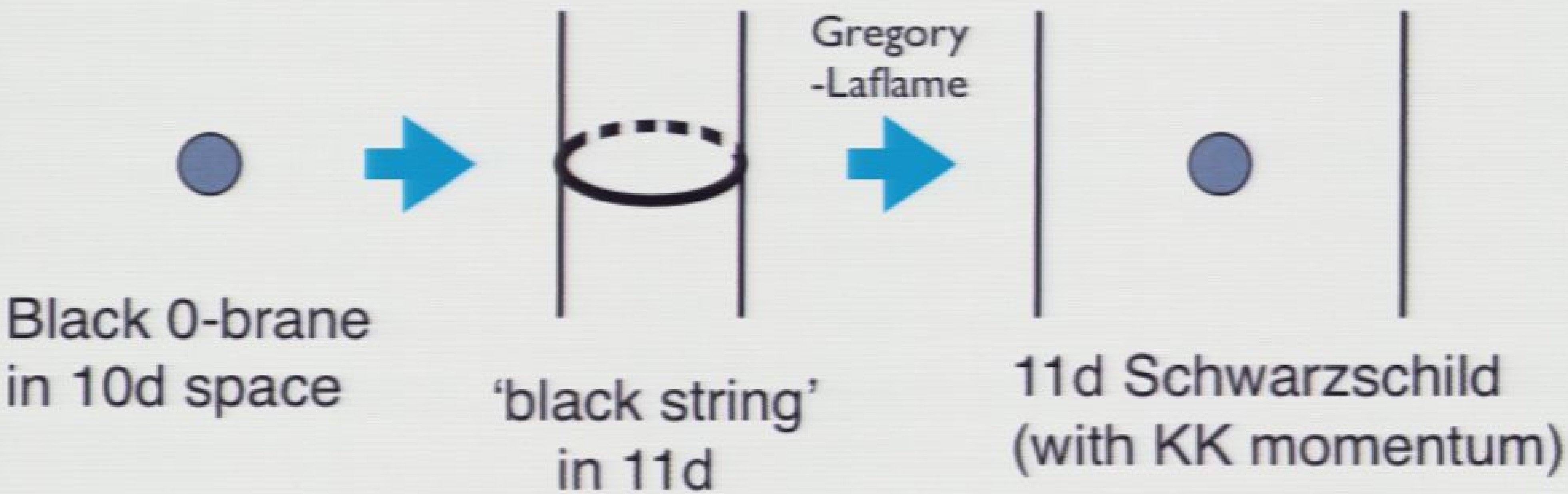
<phase> for fixed TrX^2/N

(M.H.-Nishimura-Sekino-Yoneya, to appear)

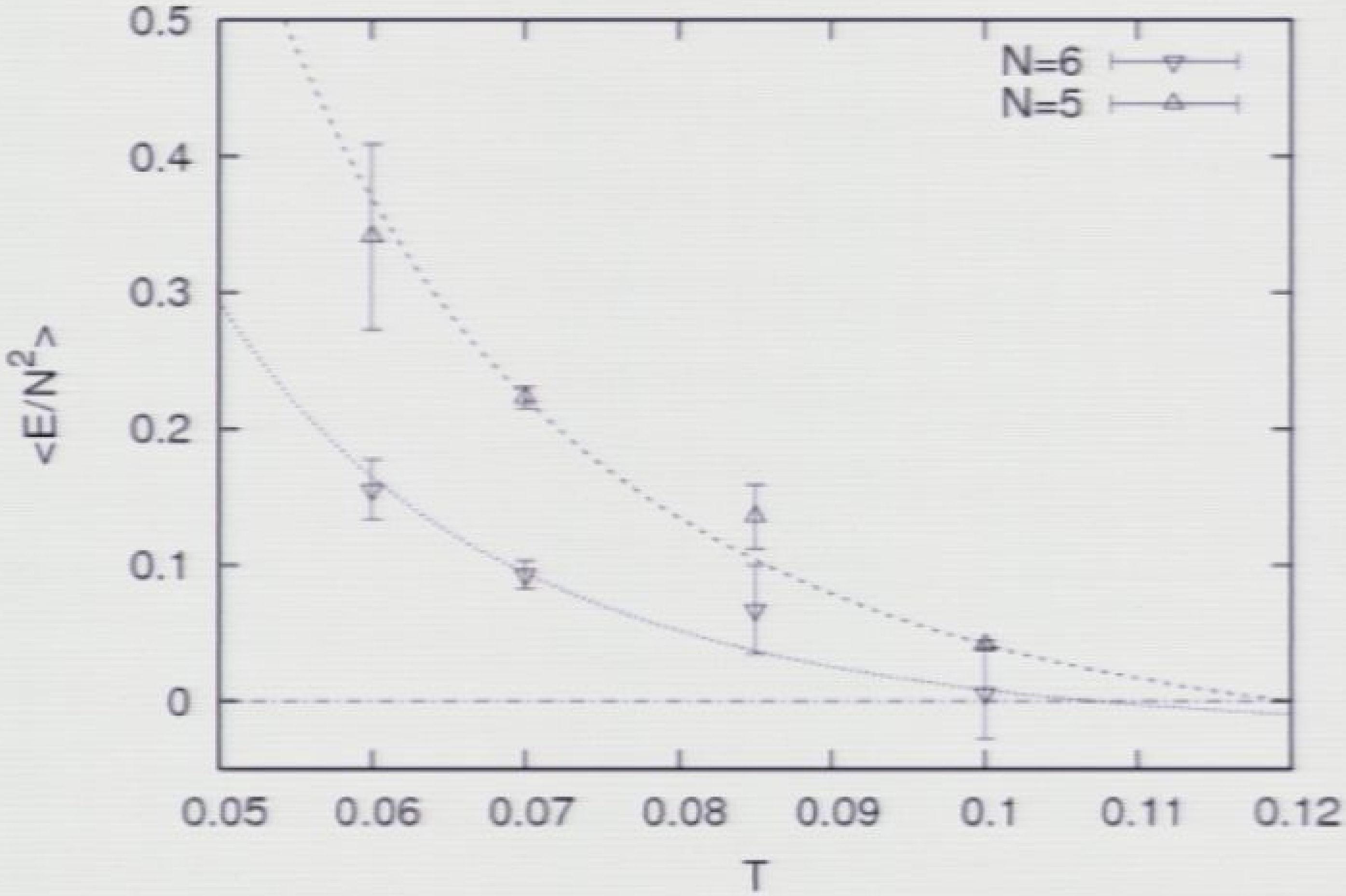
1/N correction

(M.H.-Ishiki-Nishimura-Hyakutake, in progress)

- If 1/N correction fully describe string & M-theory corrections...



(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)



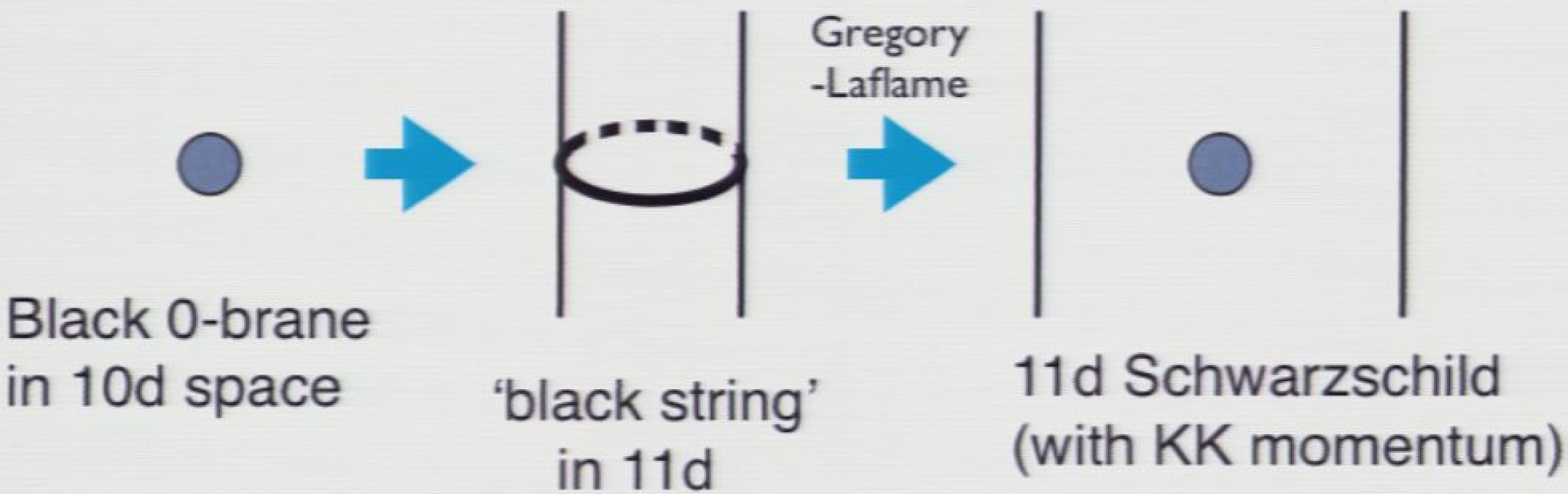
$$E/N^2 = 7.41T^{2.8} + a\,T^{0.4}/N^2 + bT^{-2.6}/N^4 + \ldots$$

$$a = -5.4 \pm 0.3,\ b = 0.182 \pm 0.005$$

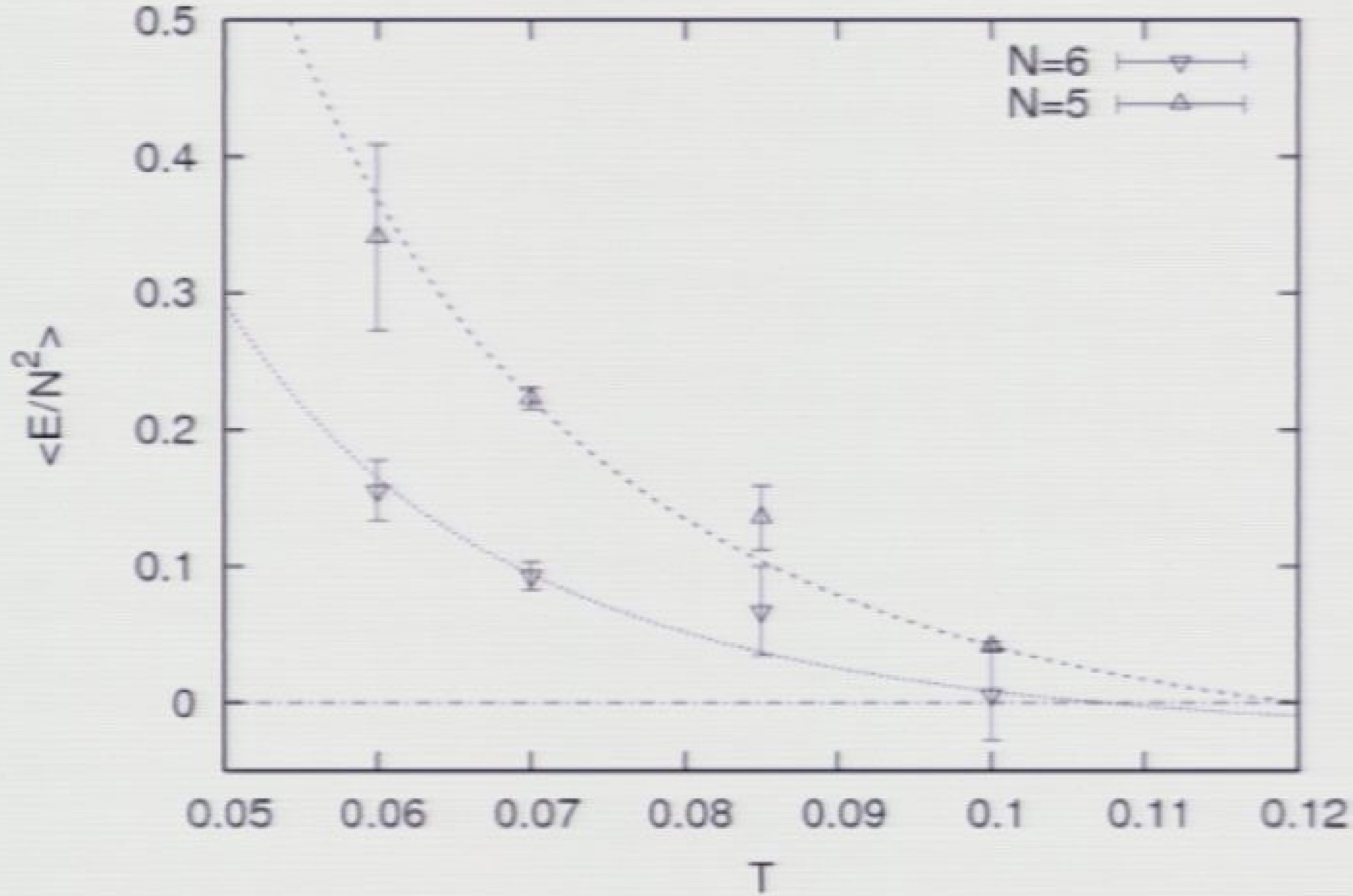
1/N correction

(M.H.-Ishiki-Nishimura-Hyakutake, in progress)

- If 1/N correction fully describe string & M-theory corrections...



(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)



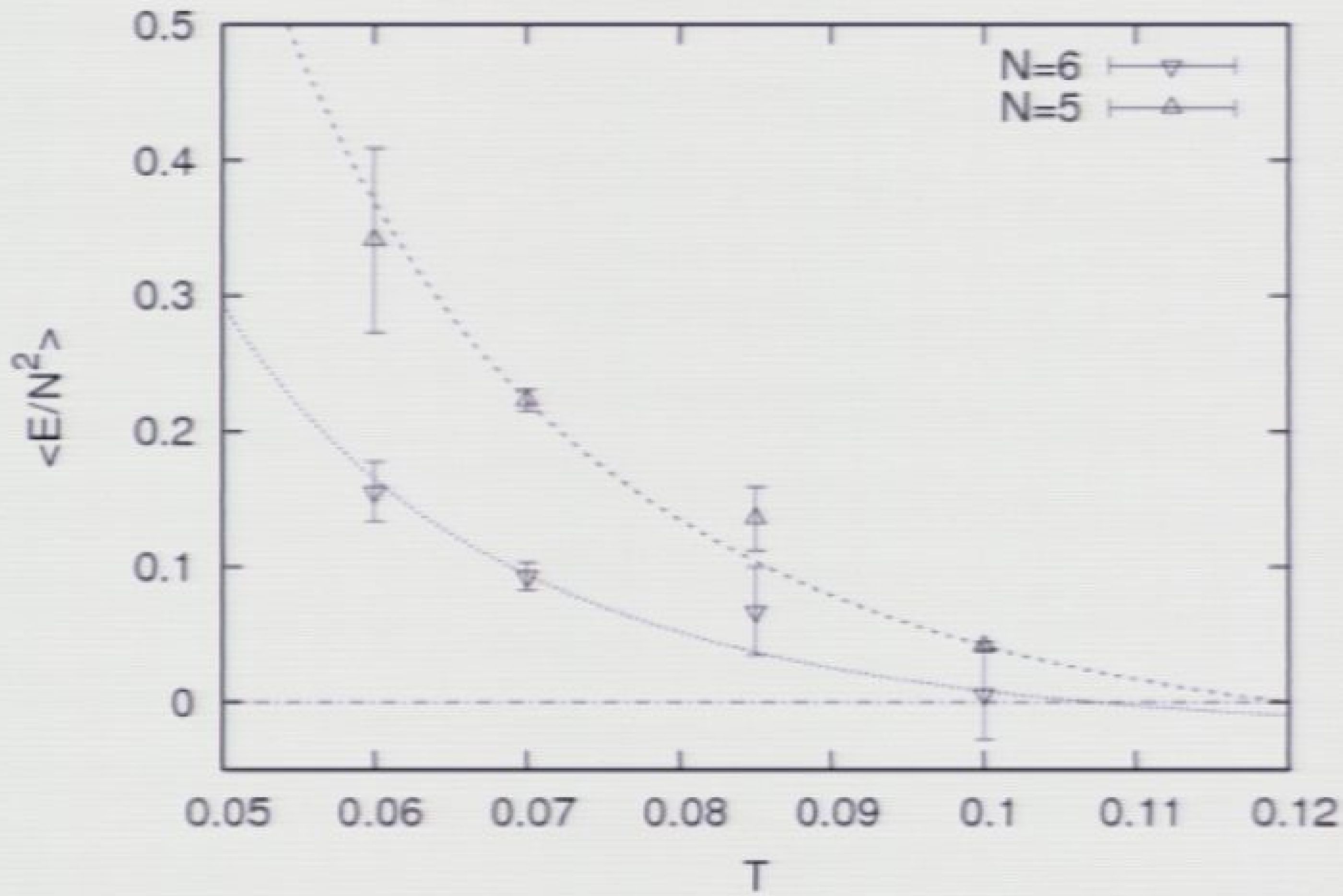
$$E/N^2 = 7.41 T^{2.8} + a\, T^{0.4}/N^2 + b T^{-2.6}/N^4 + \ldots$$

$$a = -5.4 \pm 0.3,\ b = 0.182 \pm 0.005$$

Summary

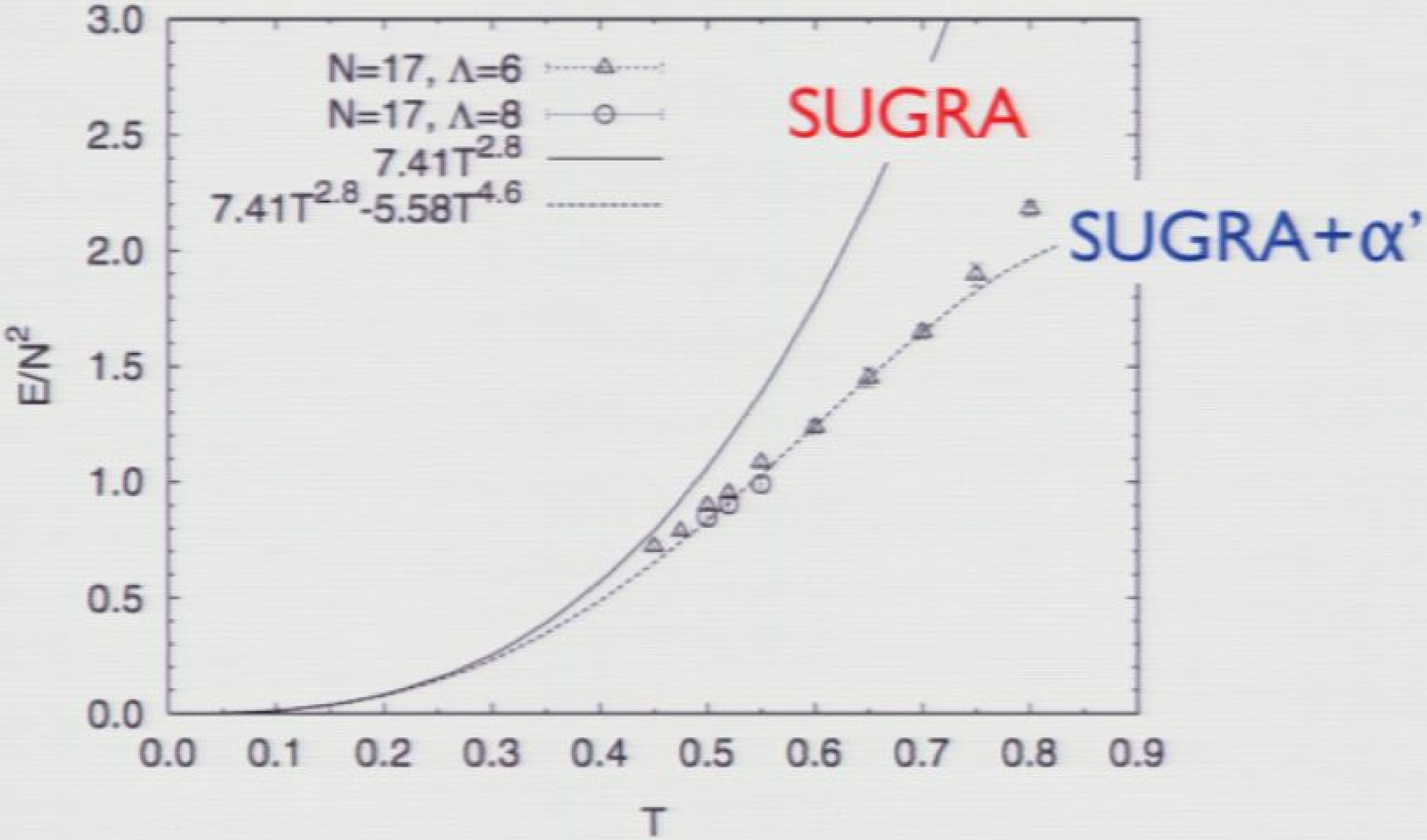
- Monte Carlo is powerful.
- strong coupling vs SUGRA is OK
- finite coupling vs α'-correction is OK
- 1/N correction vs g_s-correction : *promising!*

Tomorrow : 2d,3d and 4d SYMs

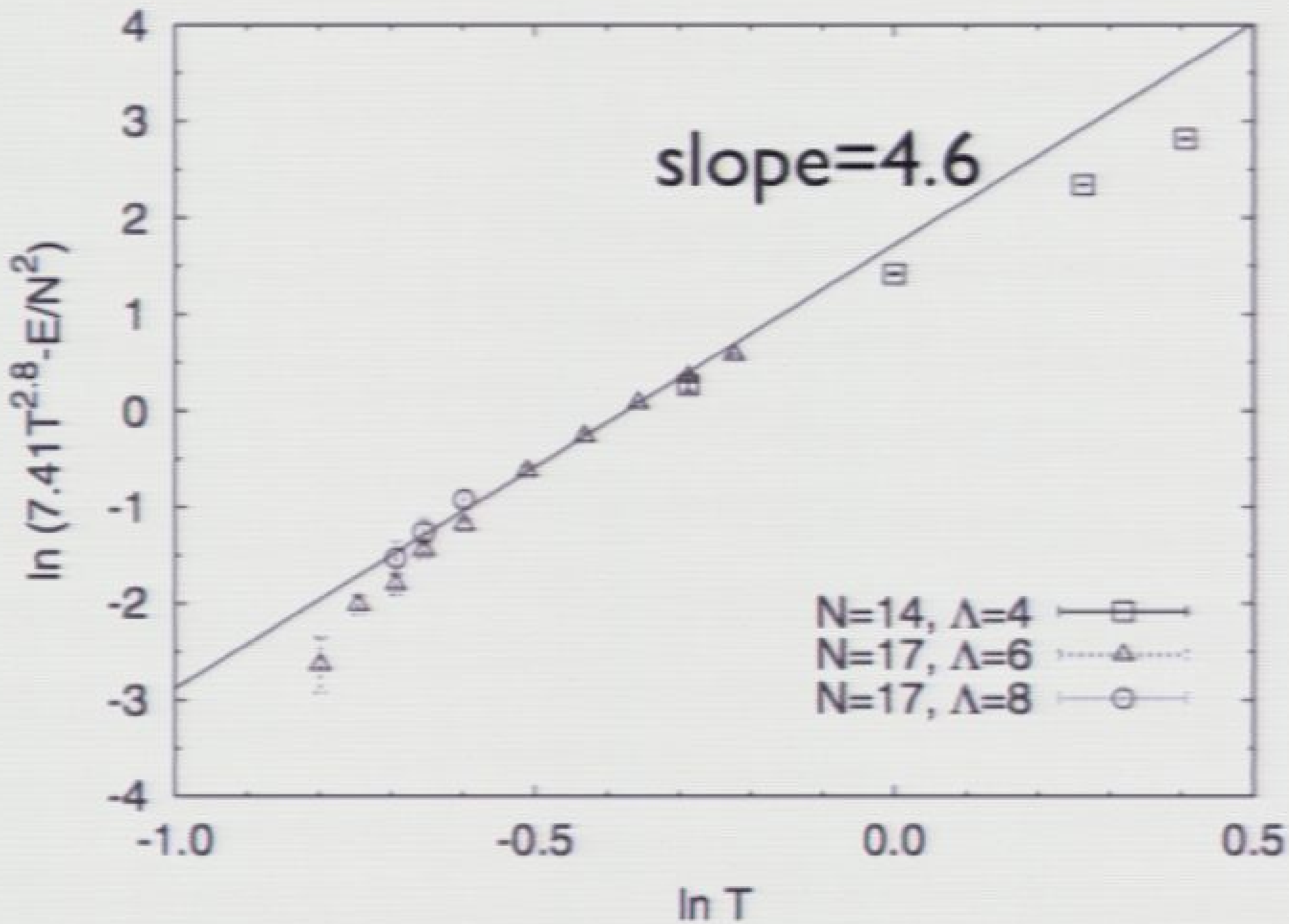


$$E/N^2=7.41T^{2.8} + a\,T^{0.4}/N^2+bT^{-2.6}/N^4+\ldots$$

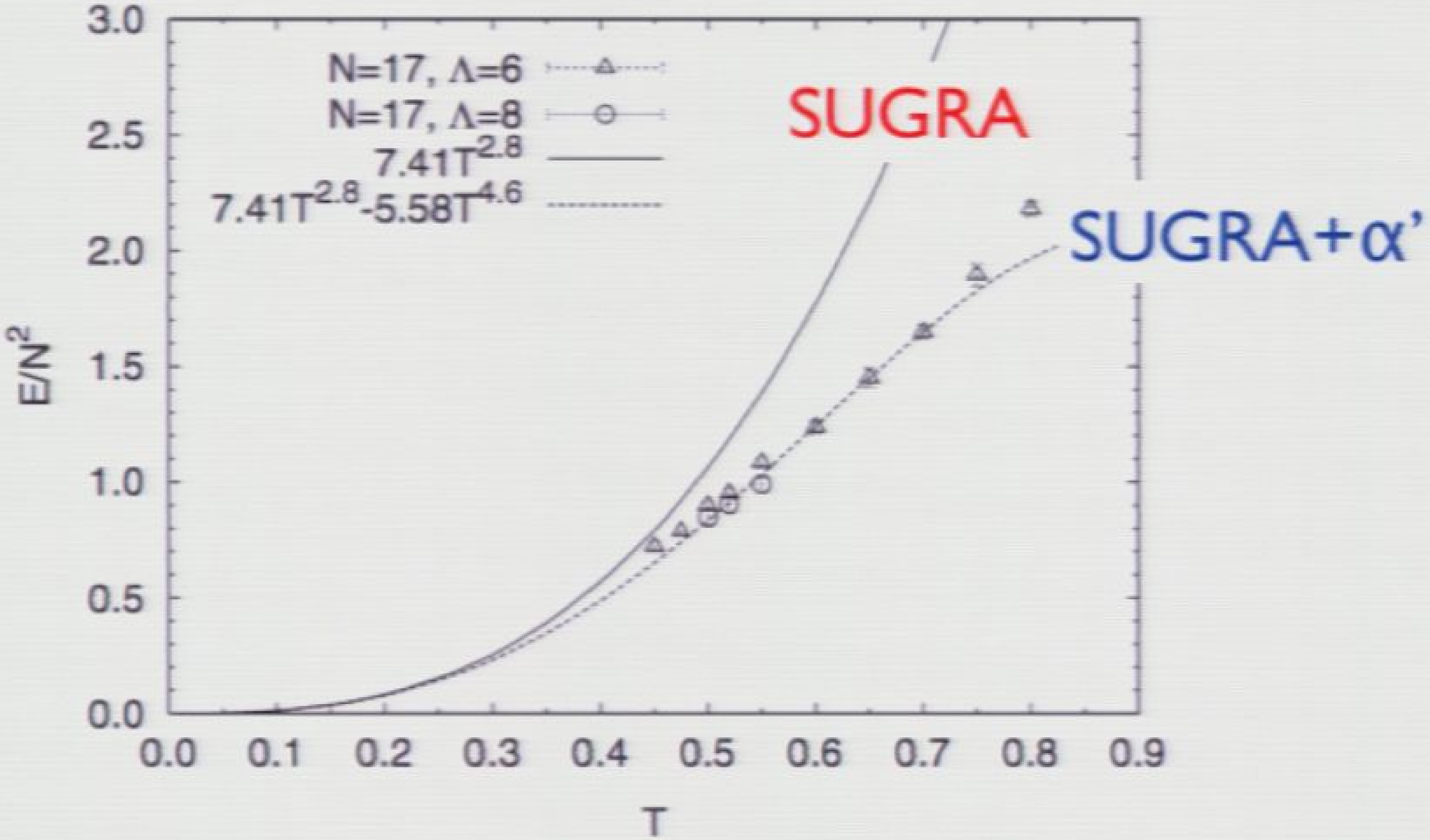
$$a=-5.4\pm0.3,\ b=0.182\pm0.005$$



M.H.-Hyakutake-Nishimura-Takeuchi 2008



M.H.-Hyakutake-Nishimura-Takeuchi 2008



M.H.-Hyakutake-Nishimura-Takeuchi 2008

No sign problem?

- Down to rather low temperature ($T \sim 0.4$),

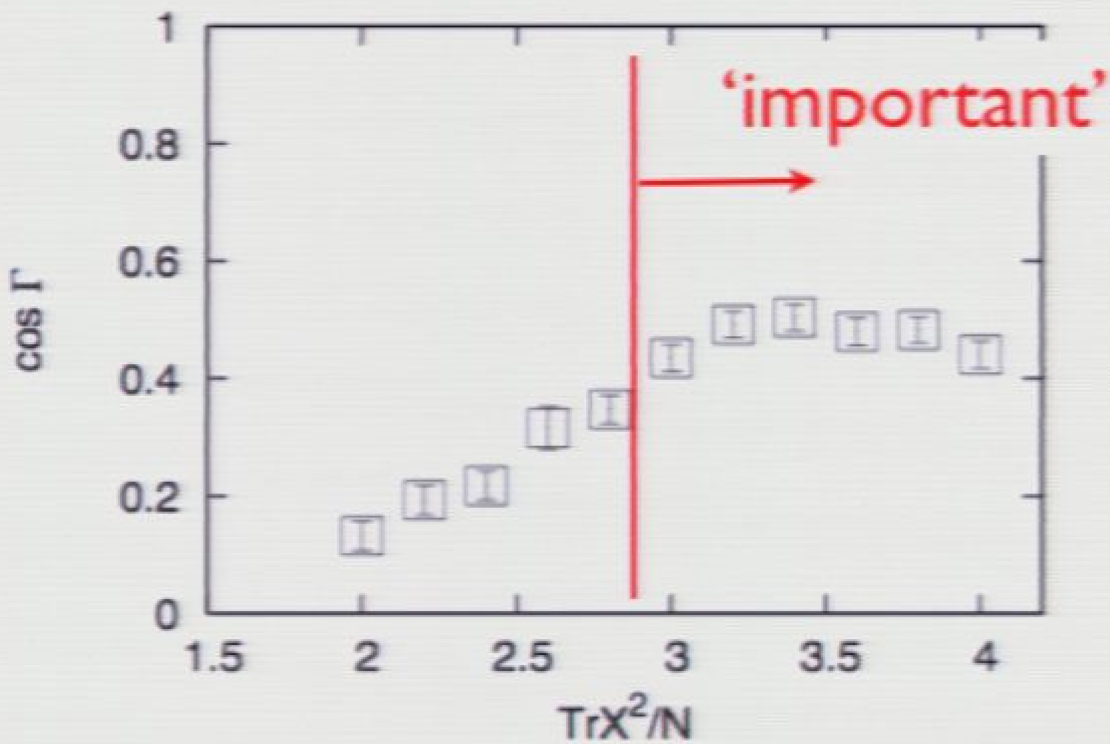
  determinant is almost real positive.

- Even at lower temperature, phase quench works rather well; the results agree with the gravity prediction.

- With periodic b.c. for fermion, sign always fluctuates, except for SU(2). But phase quench reproduces gravity prediction.

There must be a reason for this 'miracle'..

- Suppose phase and the value of the observable x are not correlated, that is,



<phase> for fixed TrX^2/N

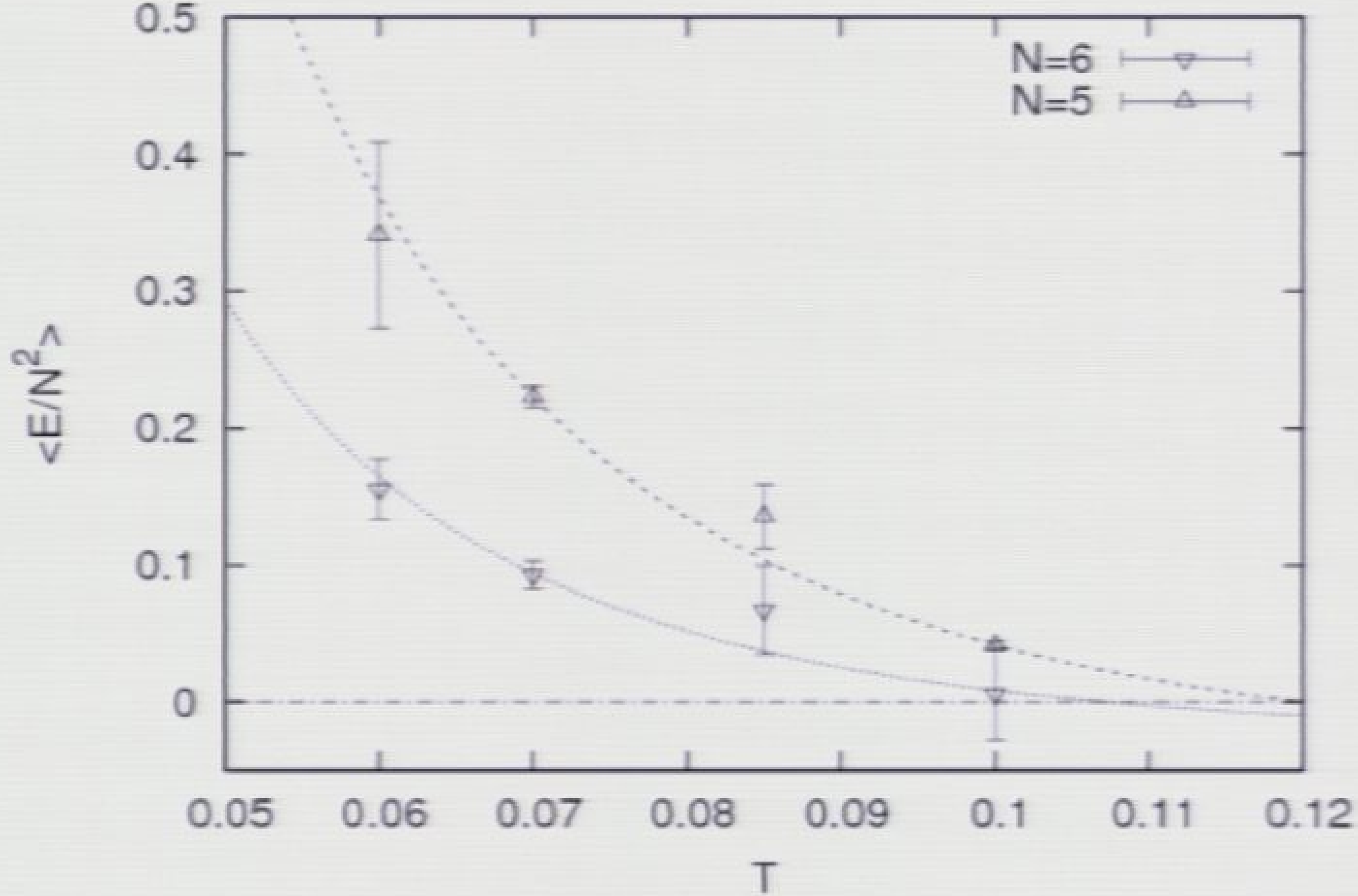
(M.H.-Nishimura-Sekino-Yoneya, to appear)

1/N correction

(M.H.-Ishiki-Nishimura-Hyakutake, in progress)

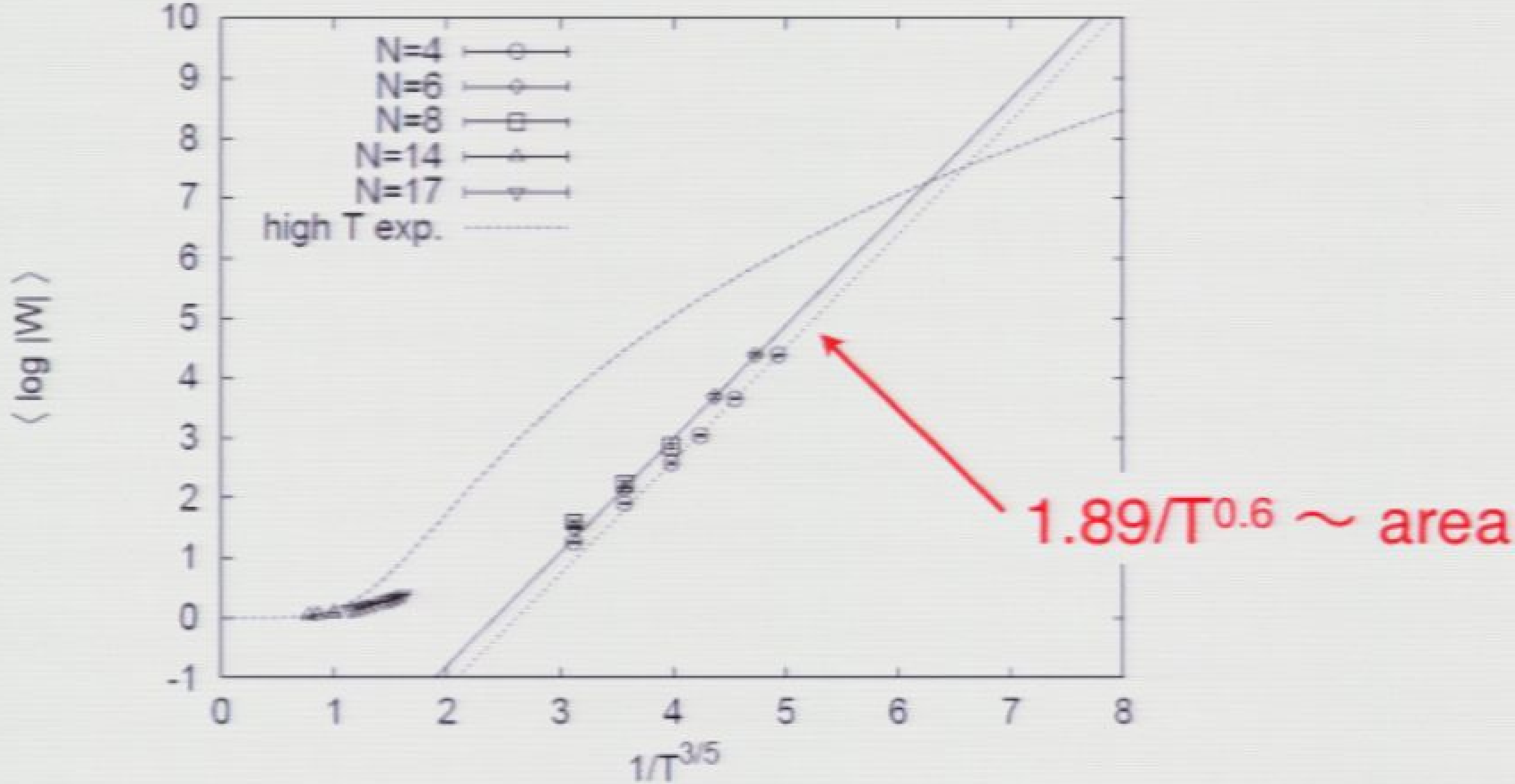
- If 1/N correction fully describe string & M-theory corrections...

(Itzhaki-Maldacena-Sonnenschein-Yankielowicz 1998)

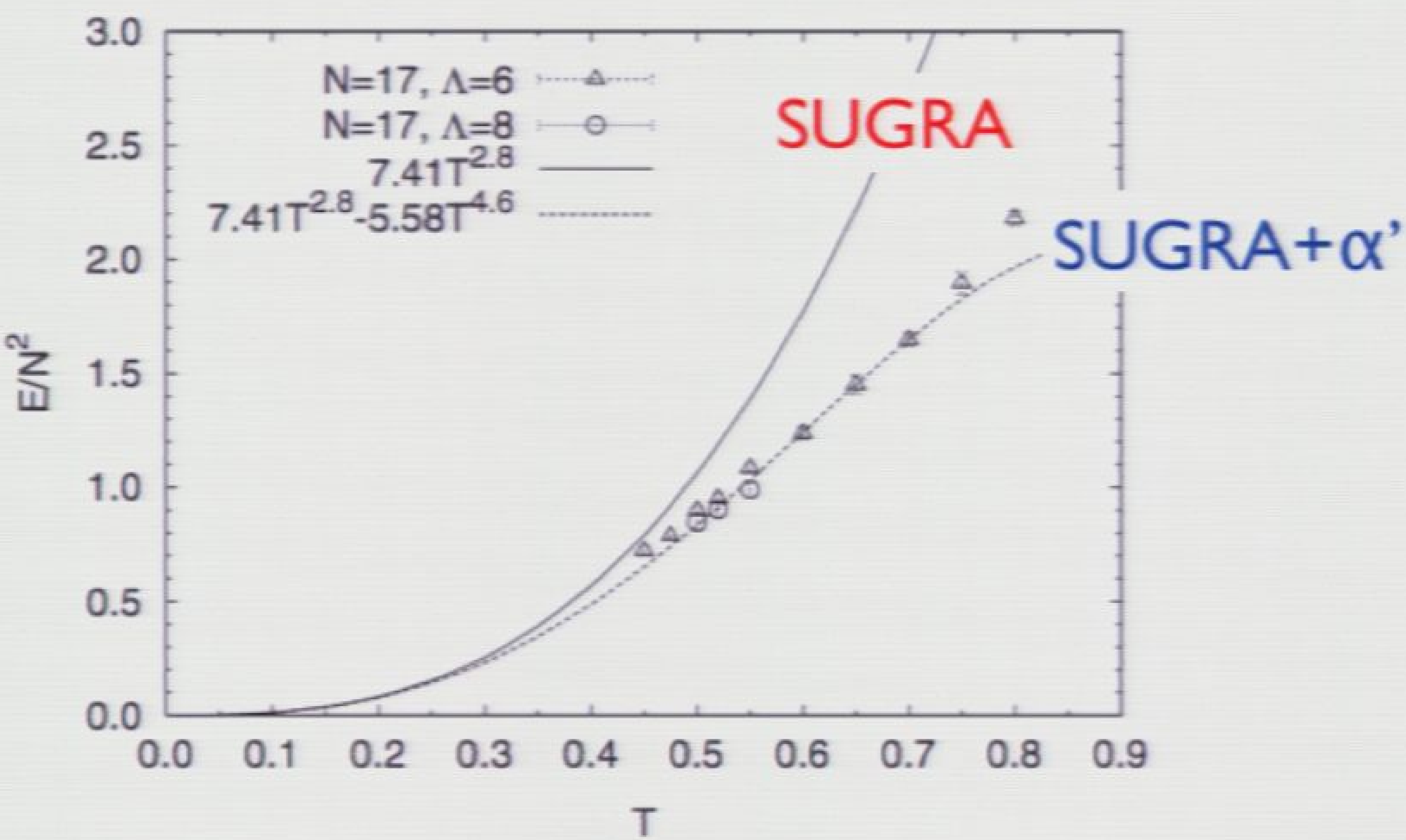


$$E/N^2 = 7.41T^{2.8} + a\,T^{0.4}/N^2 + bT^{-2.6}/N^4 + \ldots$$

$$a = -5.4 \pm 0.3,\ b = 0.182 \pm 0.005$$



M.H.-Miwa-Nishimura-Takeuchi, 2008



M.H.-Hyakutake-Nishimura-Takeuchi 2008