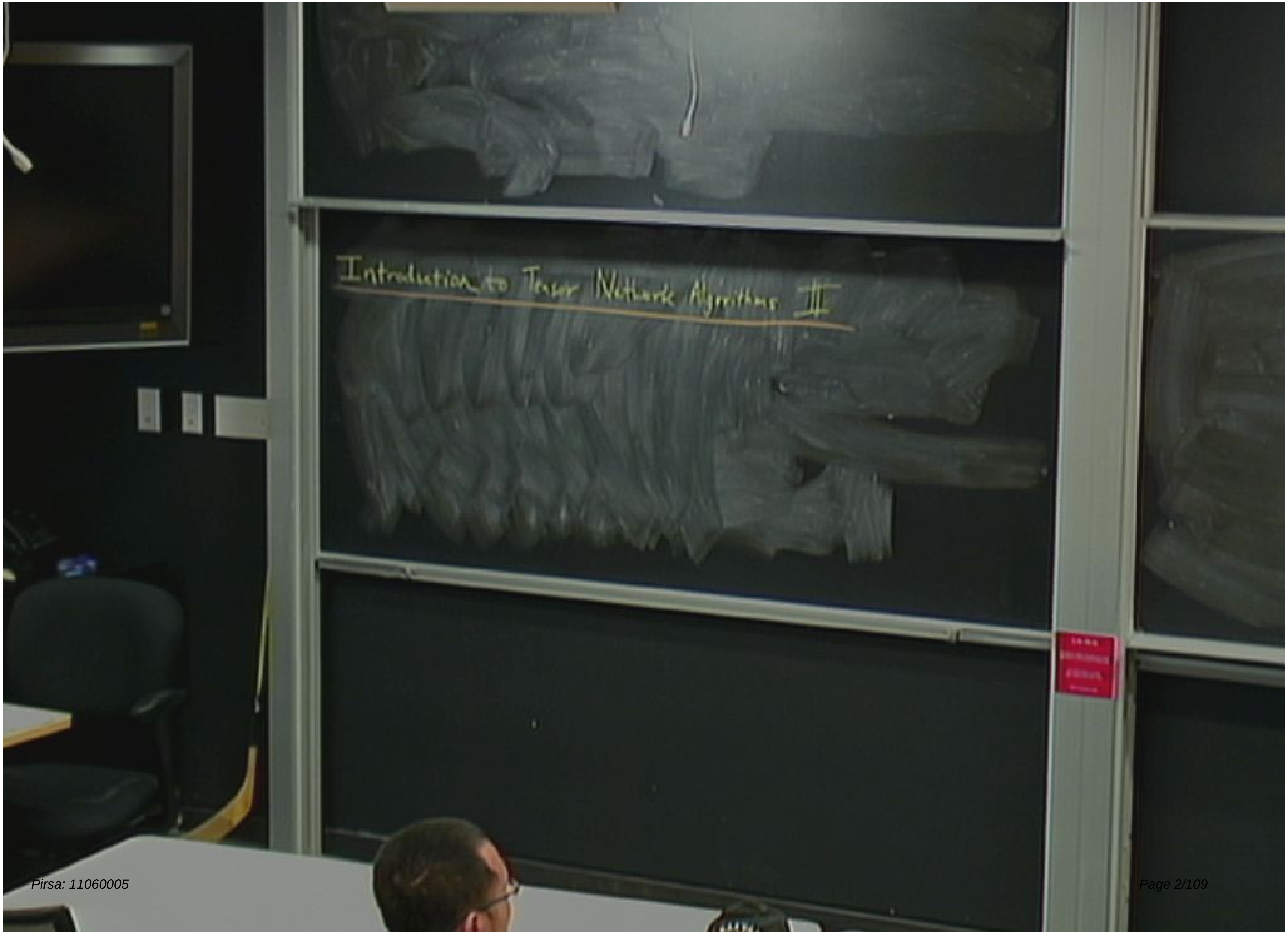
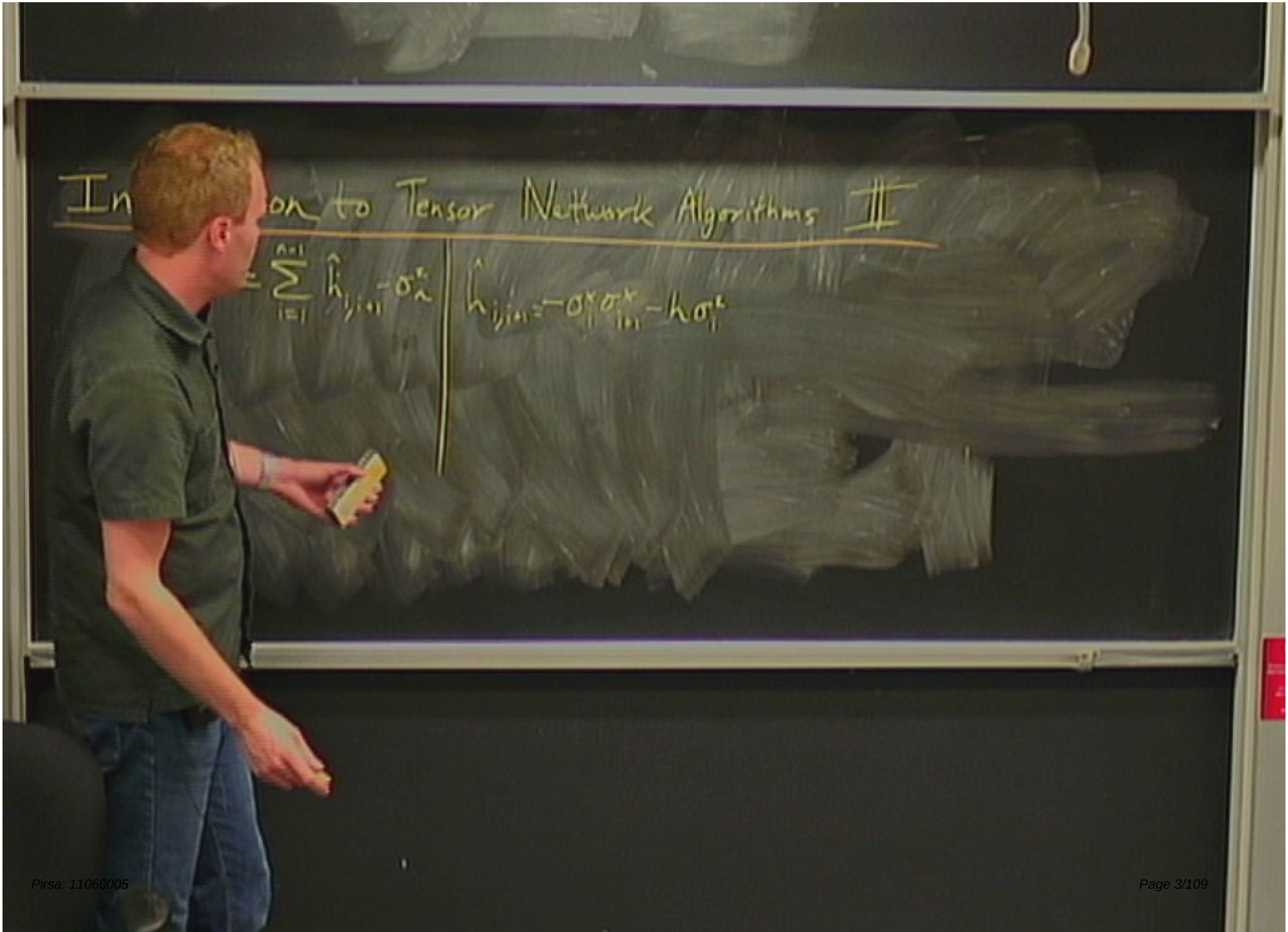
Title: Introduction to Tensor Network Algorithms - Lecture 2

Date: Jun 10, 2011 01:30 PM

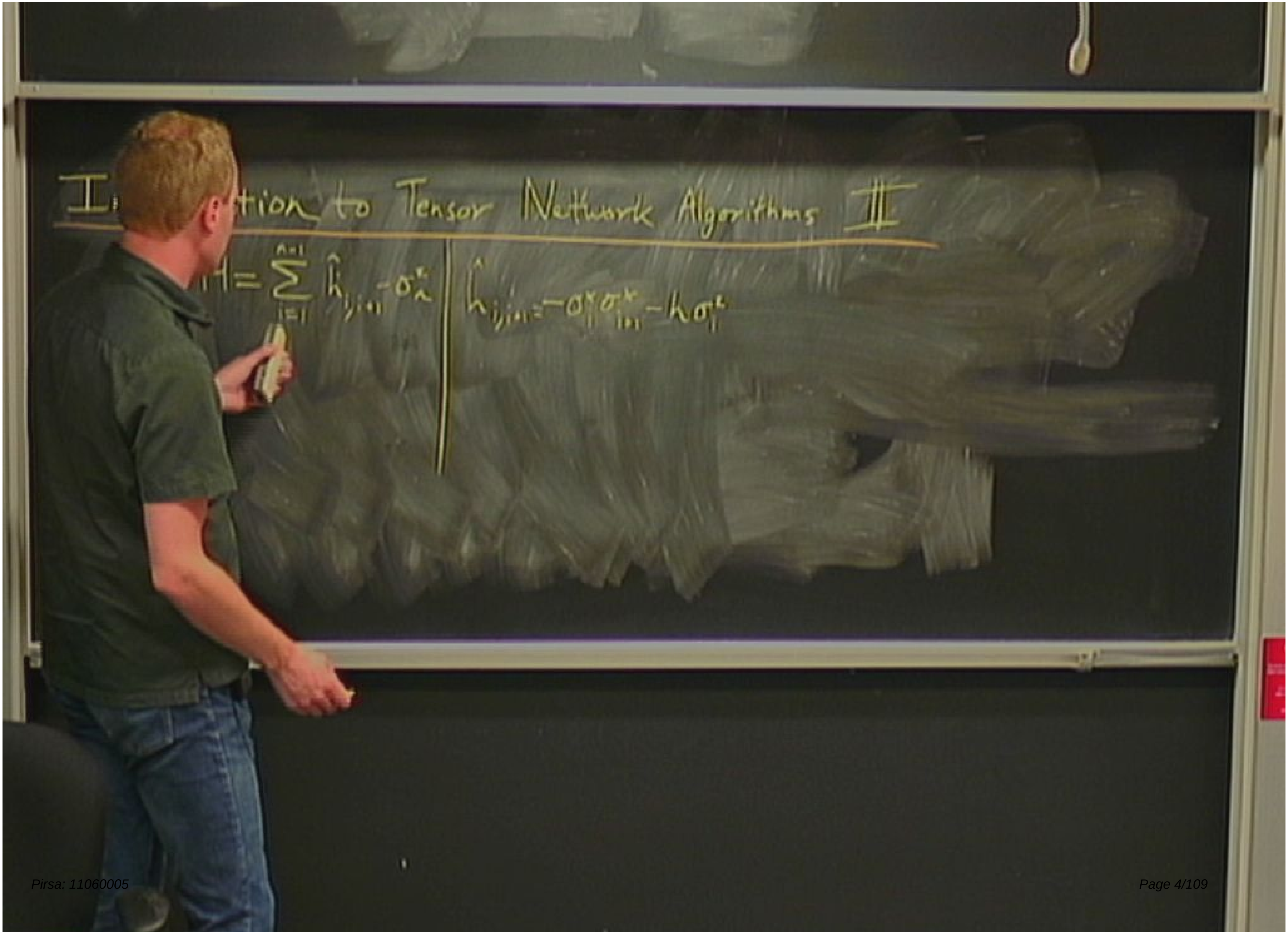
URL: http://pirsa.org/11060005

Abstract: Tensor network algorithms are a powerful technique for the study of quantum systems in condensed matter physics. In this short series of lectures, I will present an applied perspective on tensor network algorithms. Topics to be covered will include motivation and methodology, graphical notation, Matrix Product States (MPS) and the Time-Evolving Block Decimation (TEBD) algorithm, identifying the capabilities and limitations of tensor network algorithms, the Multi-scale Entanglement Renormalisation Ansatz (MERA) and the study of systems at criticality, and the exploitation of global internal symmetries. The intent of this lecture series is to provide attendees with the necessary theoretical background to be able to understand and implement the more common tensor network algorithms.


Introduction to Tensor Network Algorithms II



on to Tensor Network Algorithms II
= Σ_{i=1}^{n-1} ĥ_{i,i+1} − σ^x_n
ĥ_{i,i+1} = −σ^x_i σ^x_{i+1} − h σ^z_i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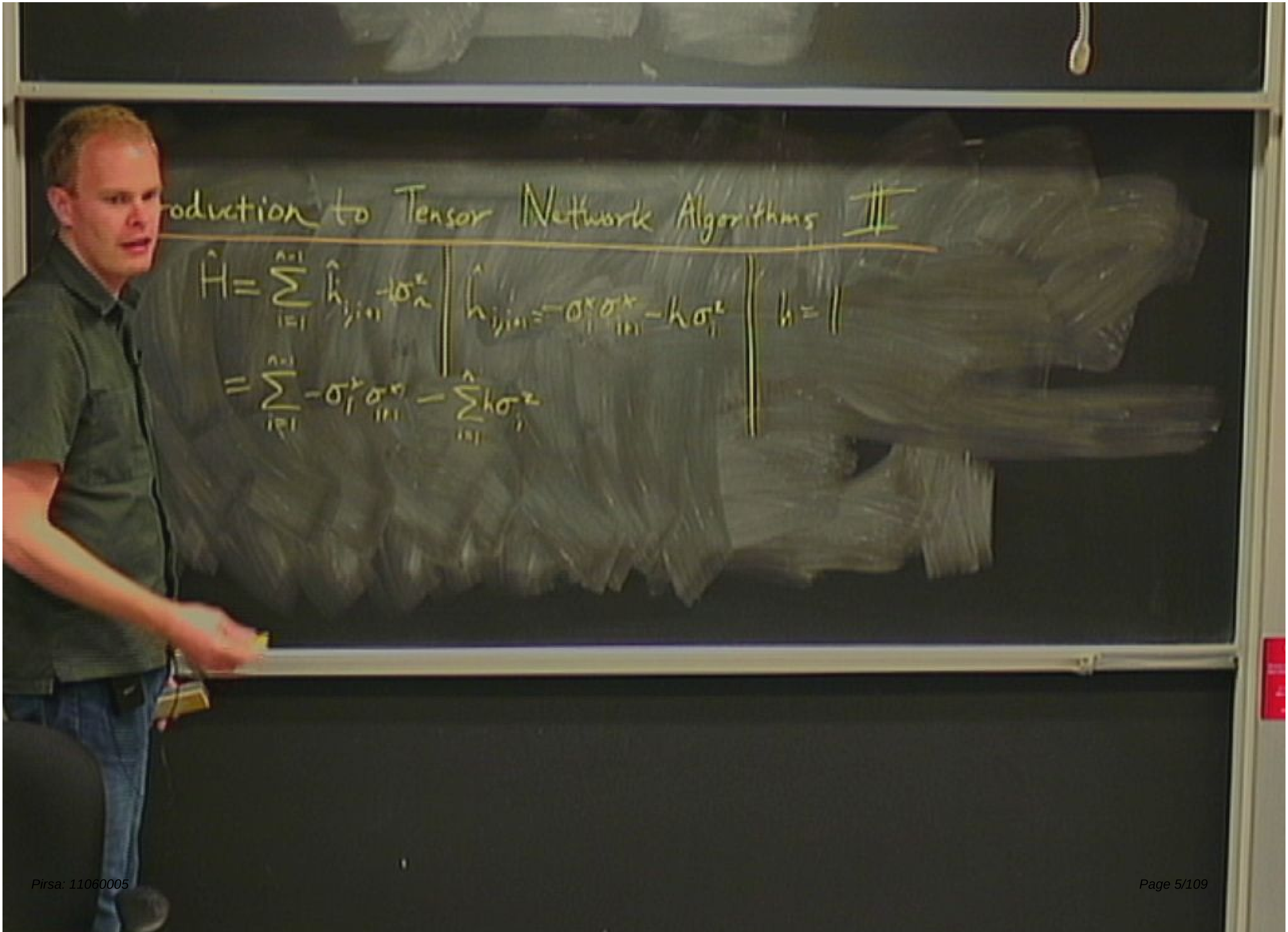
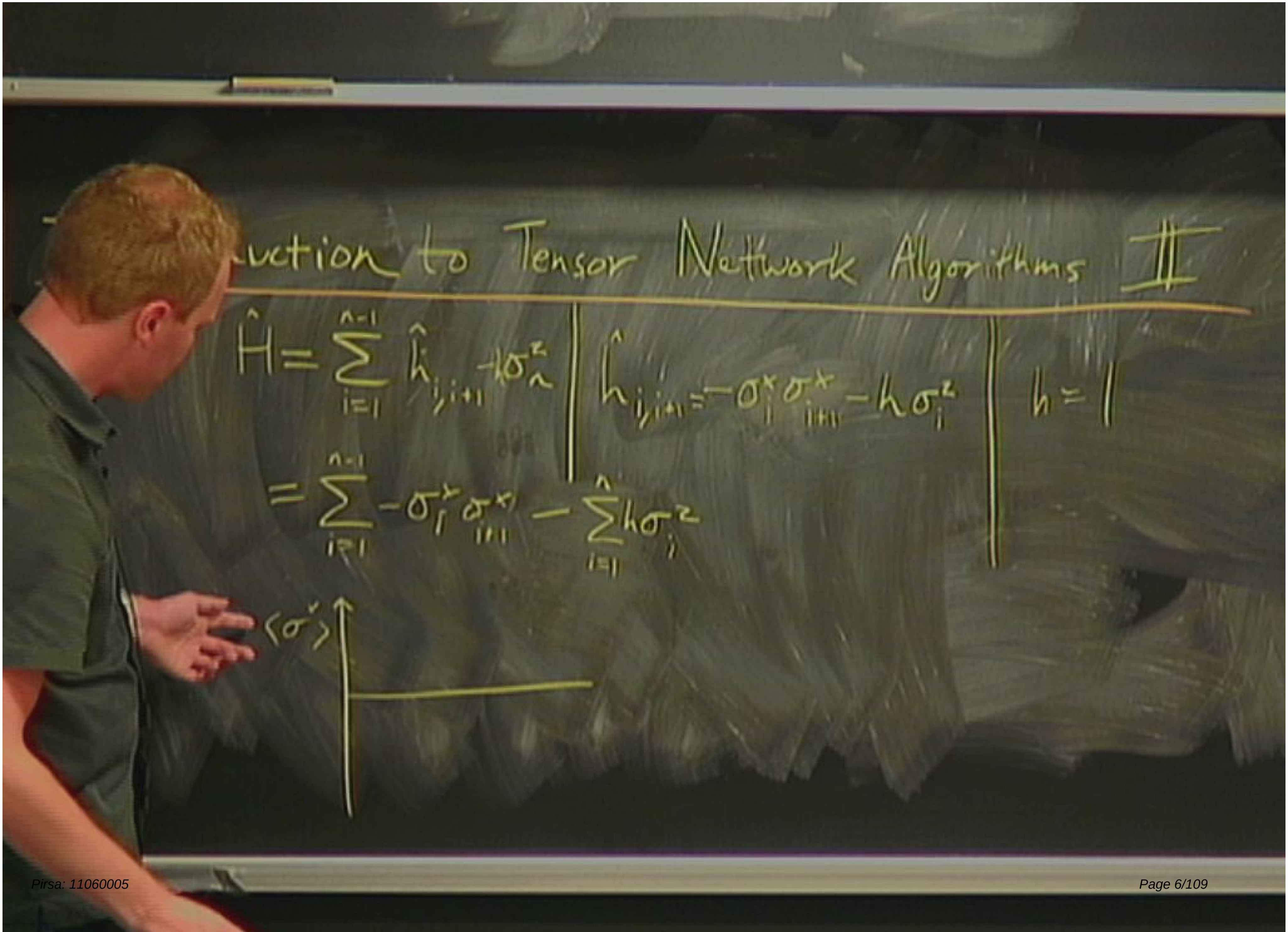

roduction to Tensor Network Algorithms II
$\hat{H} = \sum_{i=1}^{n-1} \hat{h}_{i,i+1} - h\sigma^z_n$
$\hat{h}_{i,i+1} = -\sigma^x_i \sigma^x_{i+1} - h\sigma^z_i$
$h = 1$
$= \sum_{i=1}^{n-1} -\sigma^x_i \sigma^x_{i+1} - \sum_{i=1}^{\hat{n}} h\sigma^z_i$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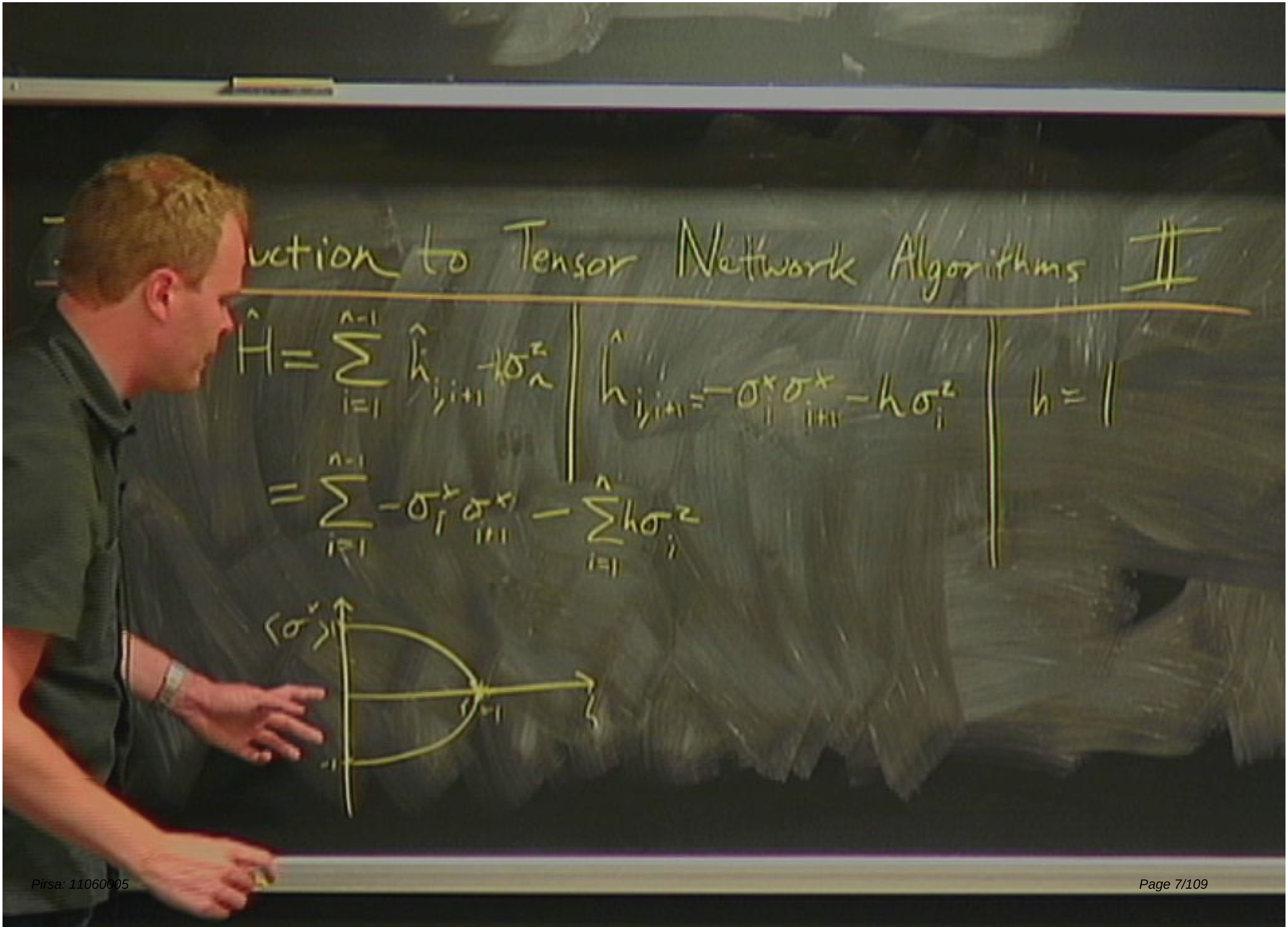
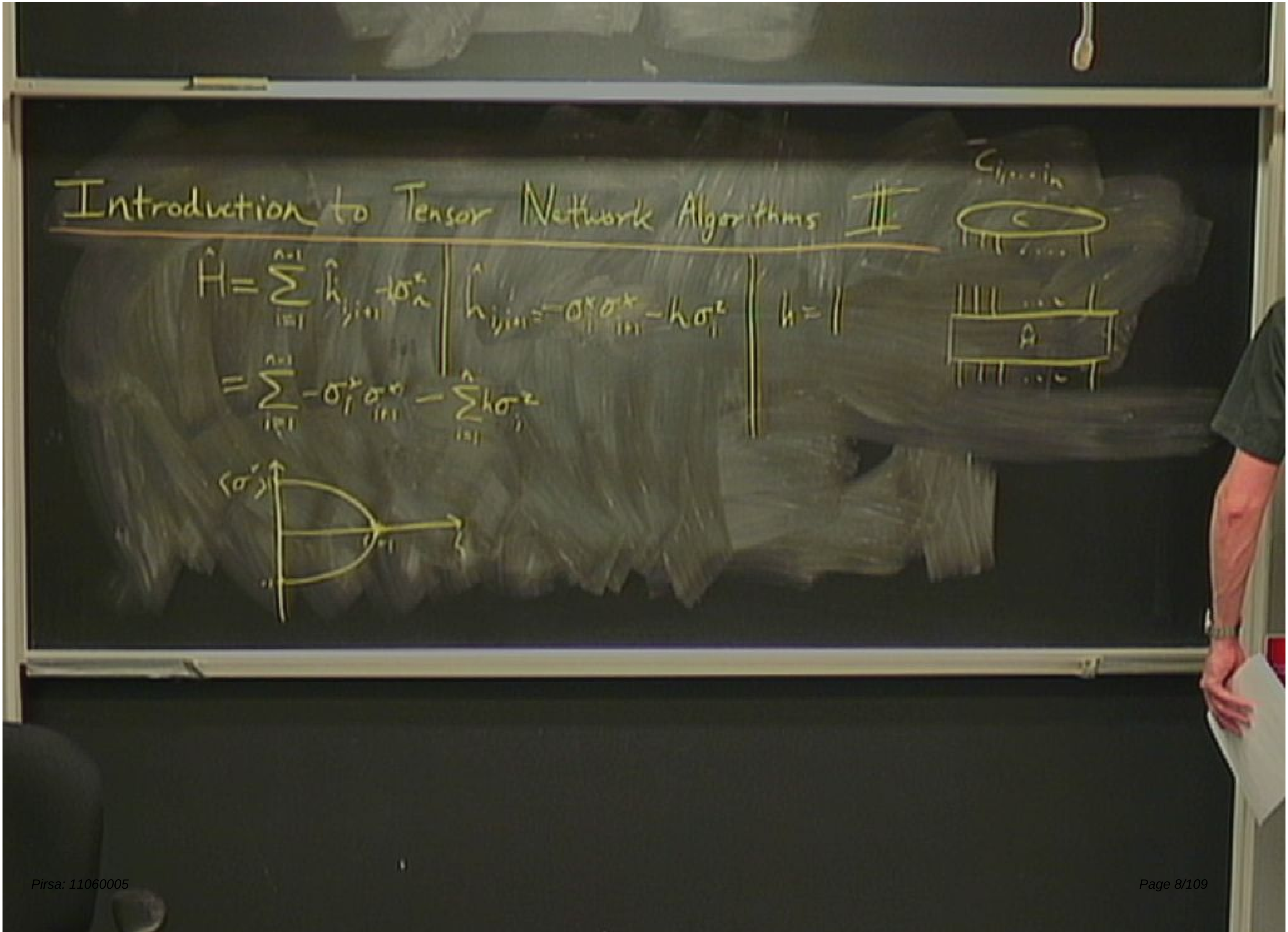
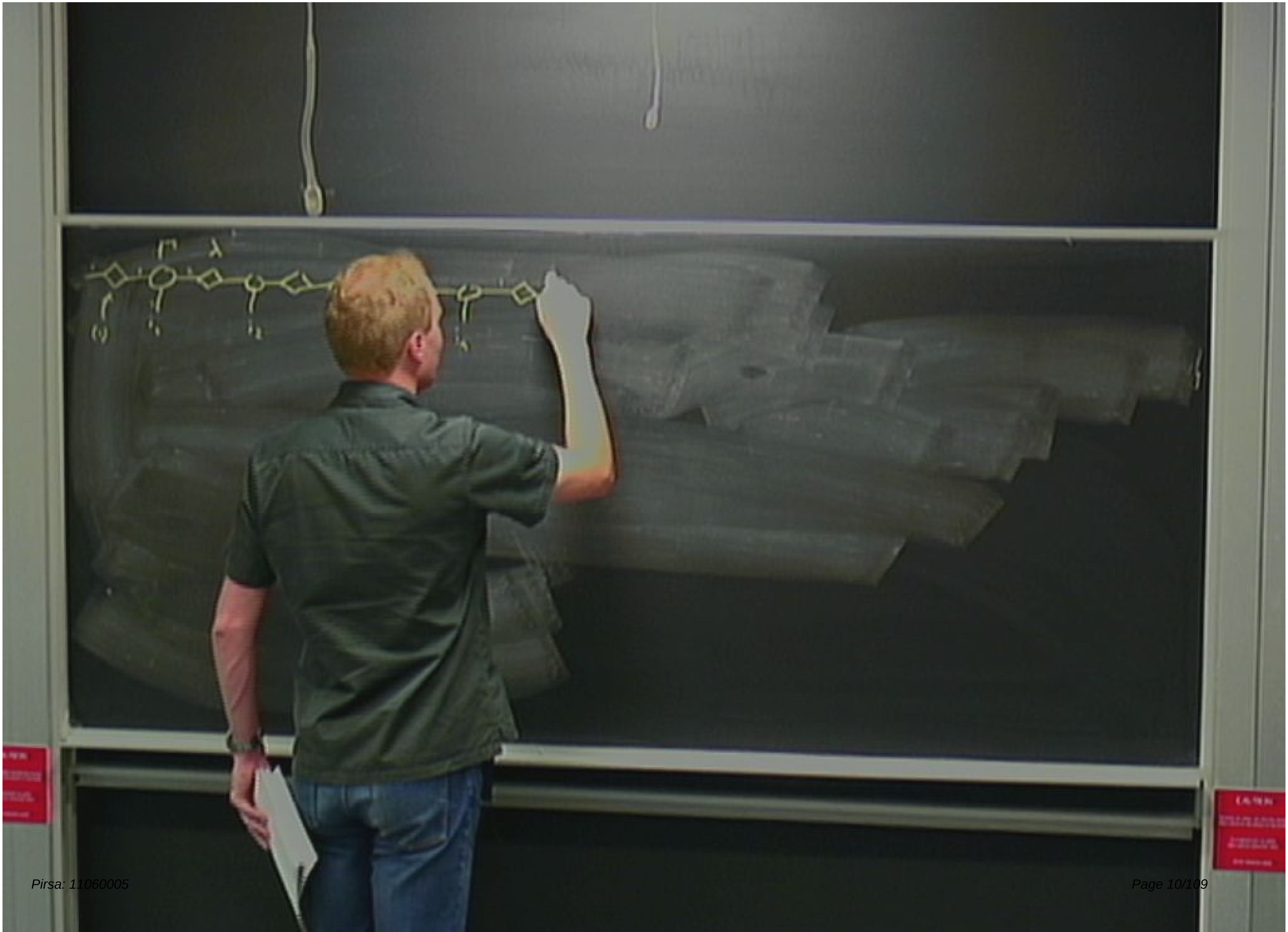
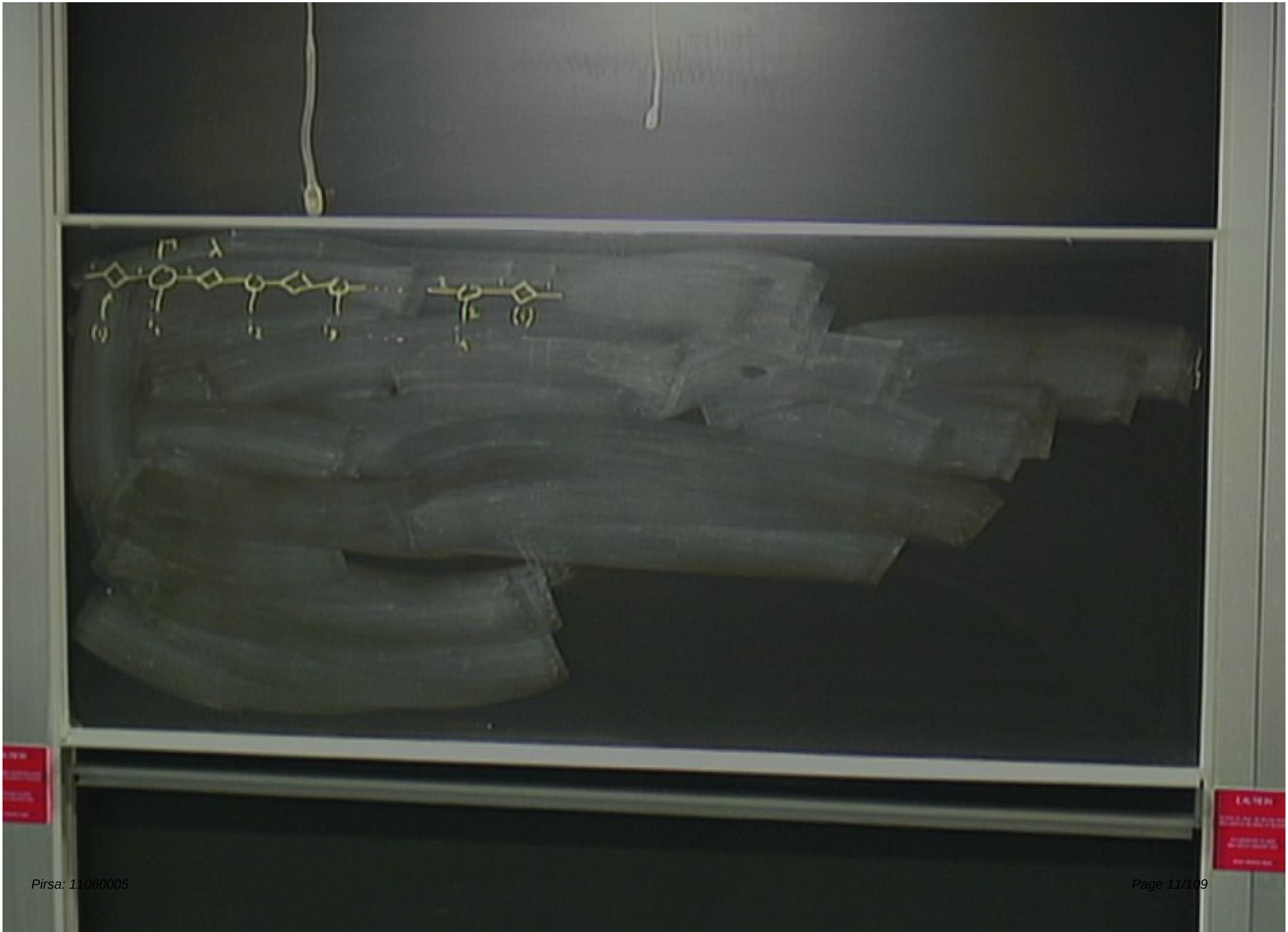
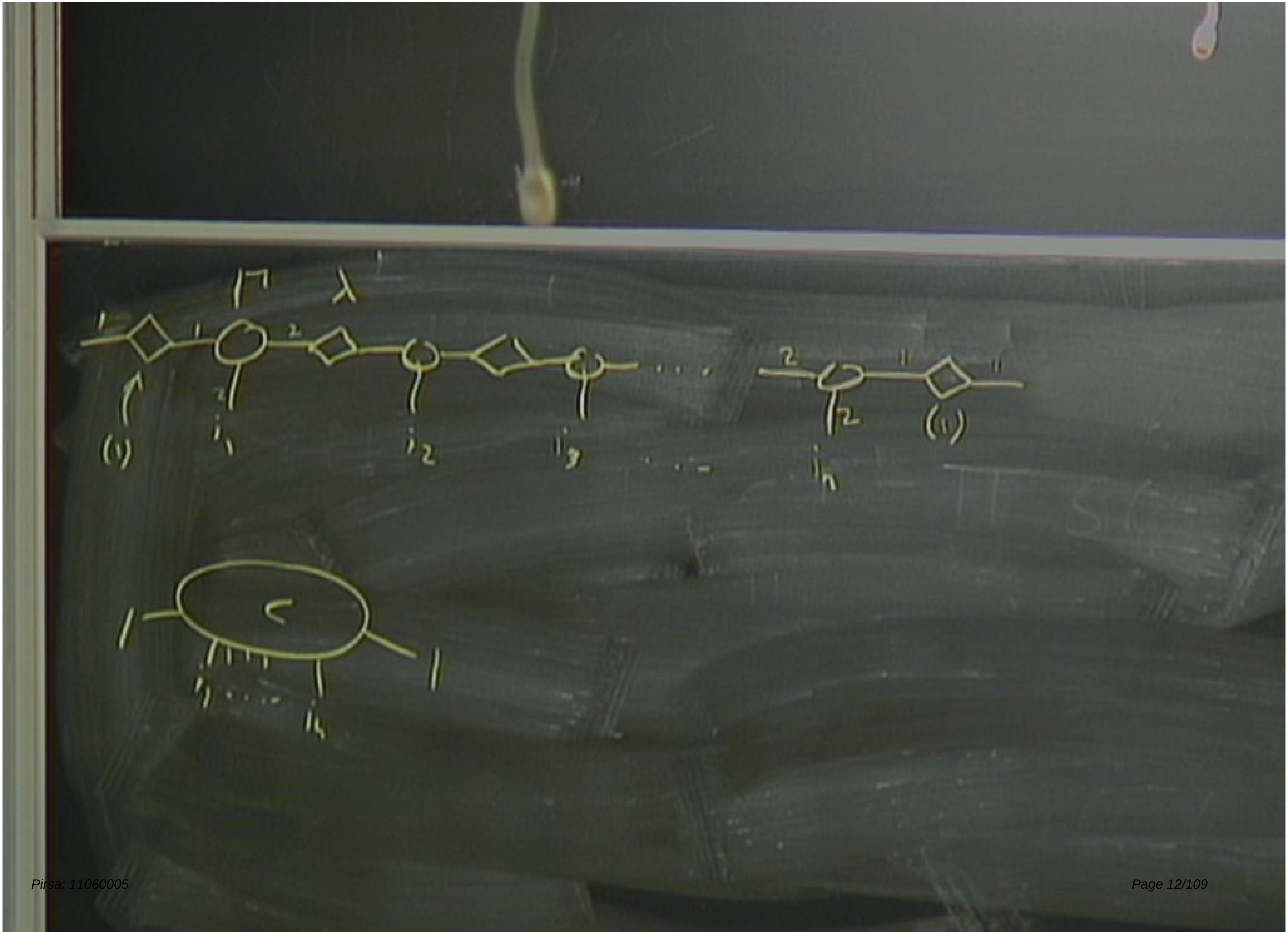
$\hat{H} = \sum_{i=1}^{n-1} \hat{h}_{i,i+1} - h\sigma^z_n$
$\hat{h}_{i,i+1} = -\sigma^x_i \sigma^x_{i+1} - h\sigma^z_i$
$h = 1$
$= \sum_{i=1}^{n-1} -\sigma^x_i \sigma^x_{i+1} - \sum_{i=1}^{n} h\sigma^z_i$
$\langle \sigma^x \rangle$

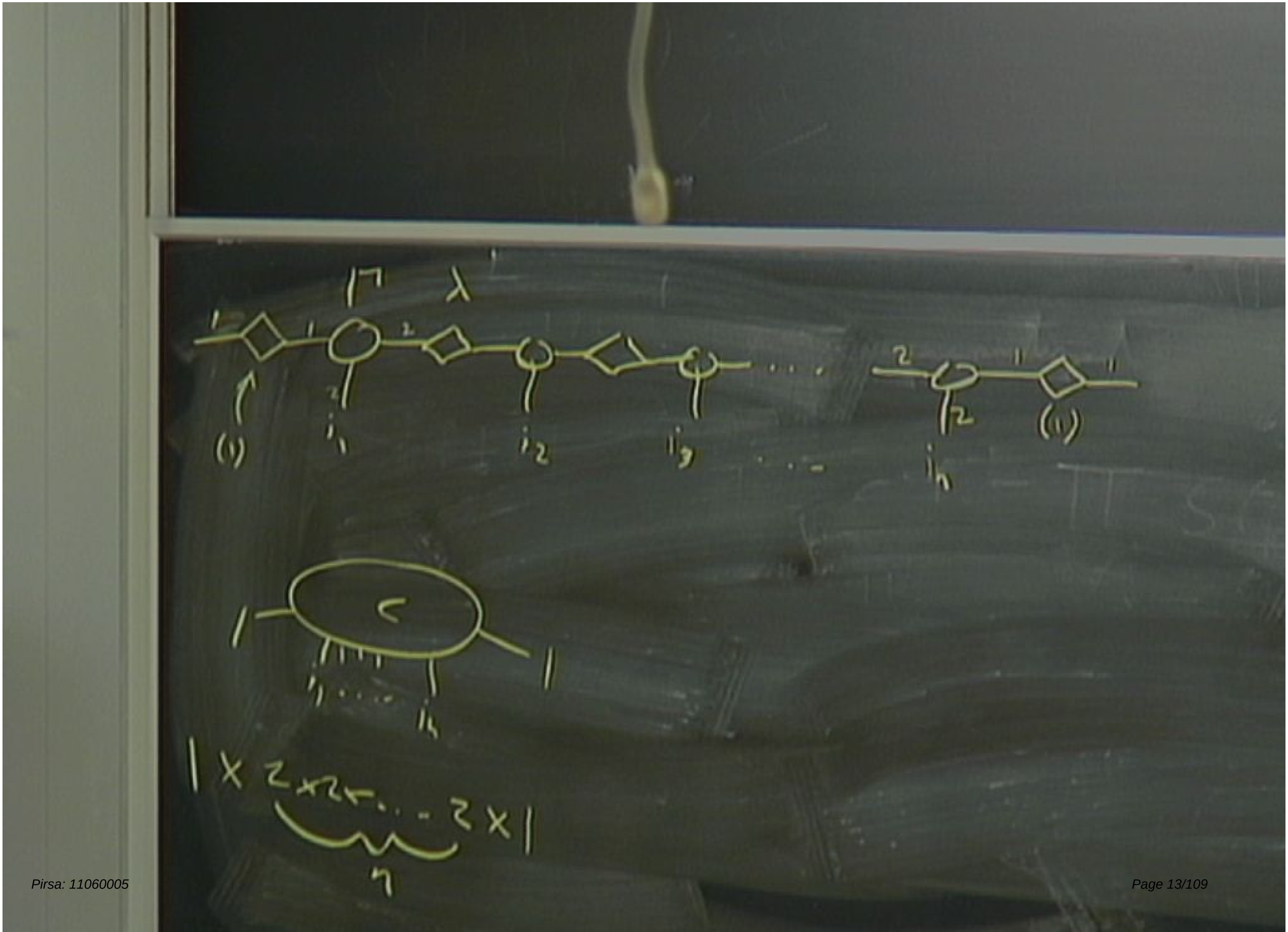

uction to Tensor Network Algorithms II
$\hat{H} = \sum_{i=1}^{n-1} \hat{h}_{i,i+1} + h\sigma^z_n$
$\hat{h}_{i,i+1} = -\sigma^x_i \sigma^x_{i+1} - h\sigma^z_i$
$h = 1$
$= \sum_{i=1}^{n-1} -\sigma^x_i \sigma^x_{i+1} - \sum_{i=1}^{n} h\sigma^z_i$
$\langle \sigma^x \rangle$
1
-1
i+1
i

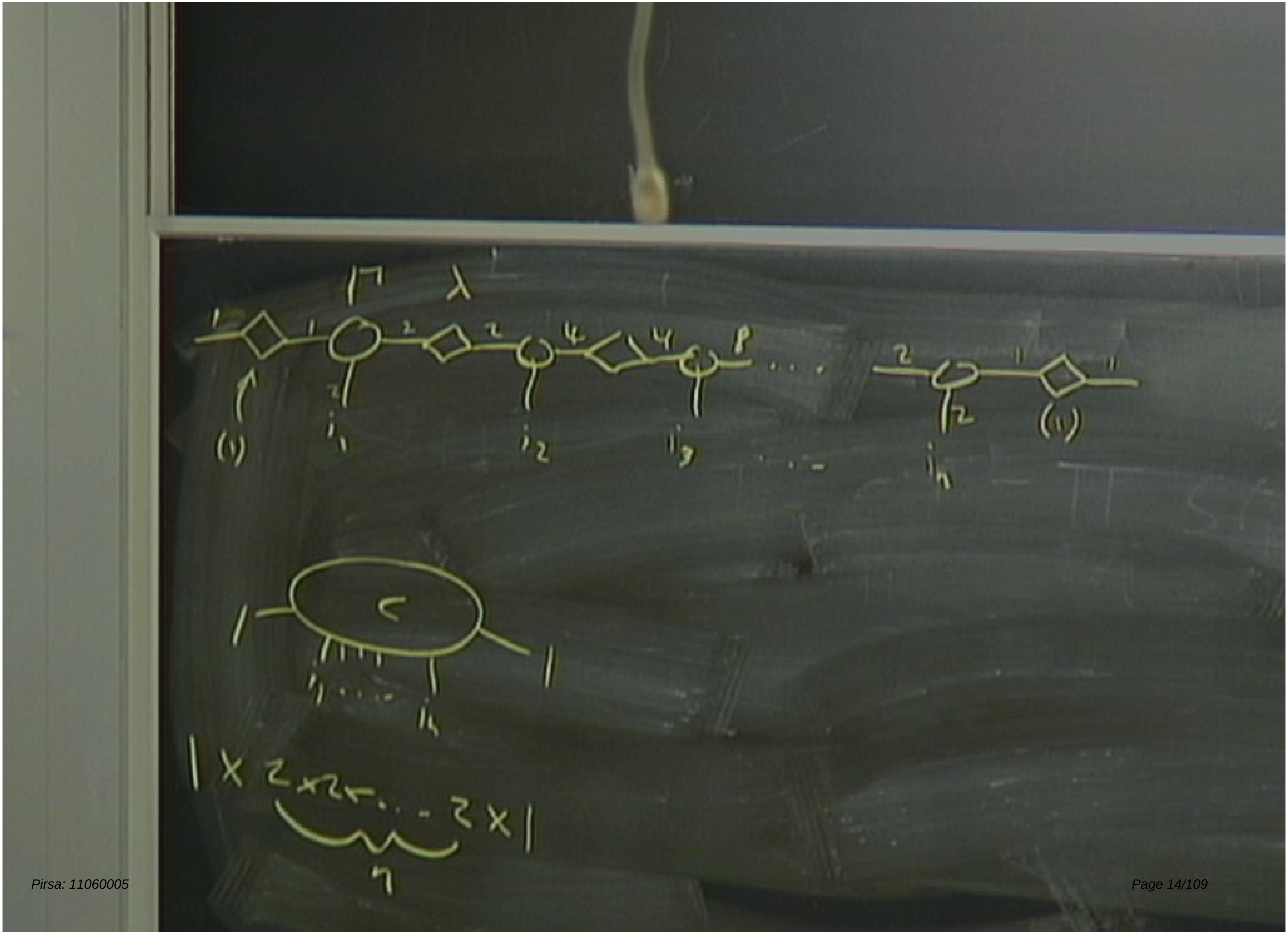

Introduction to Tensor Network Algorithms II
h = 1

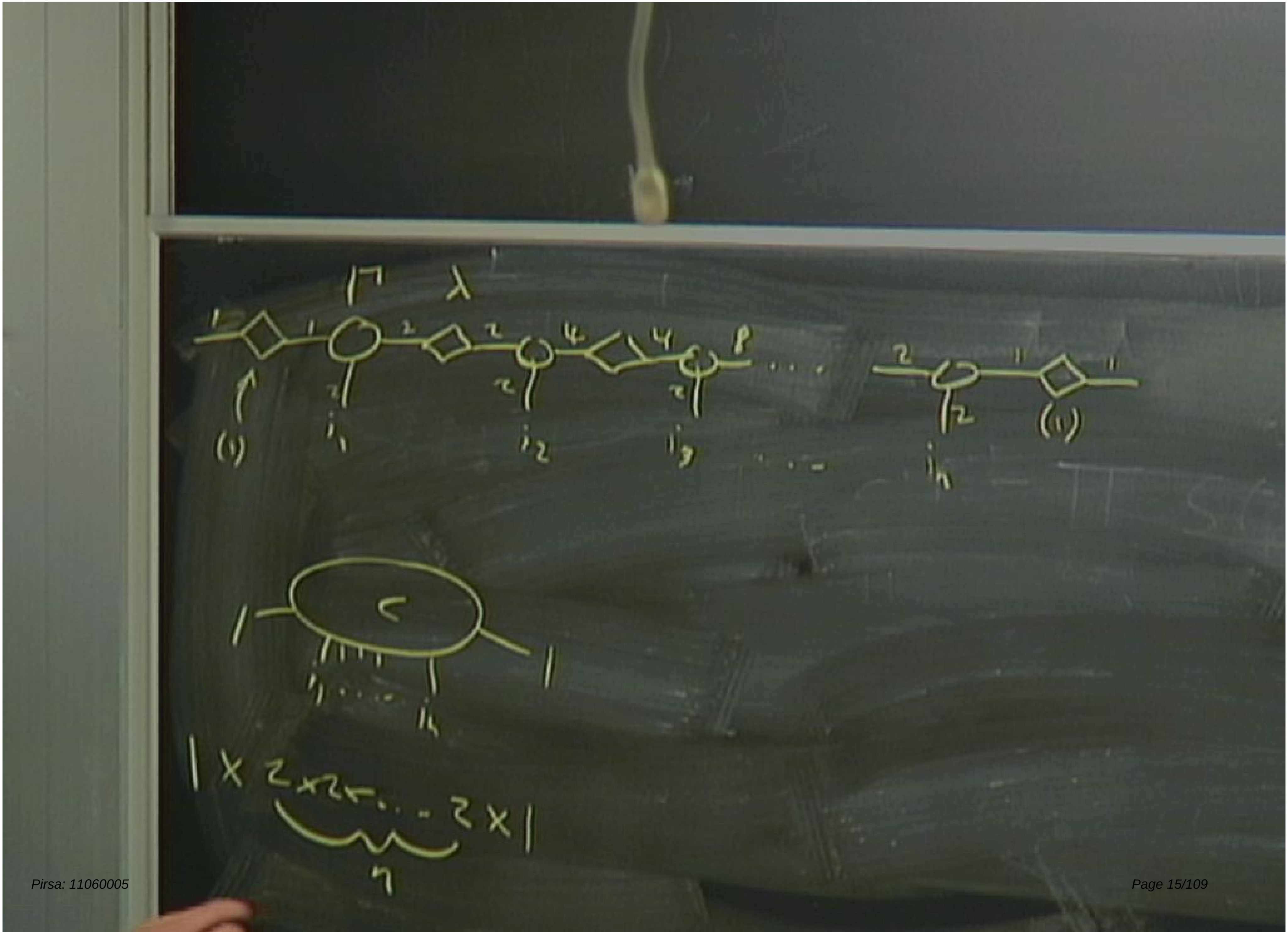




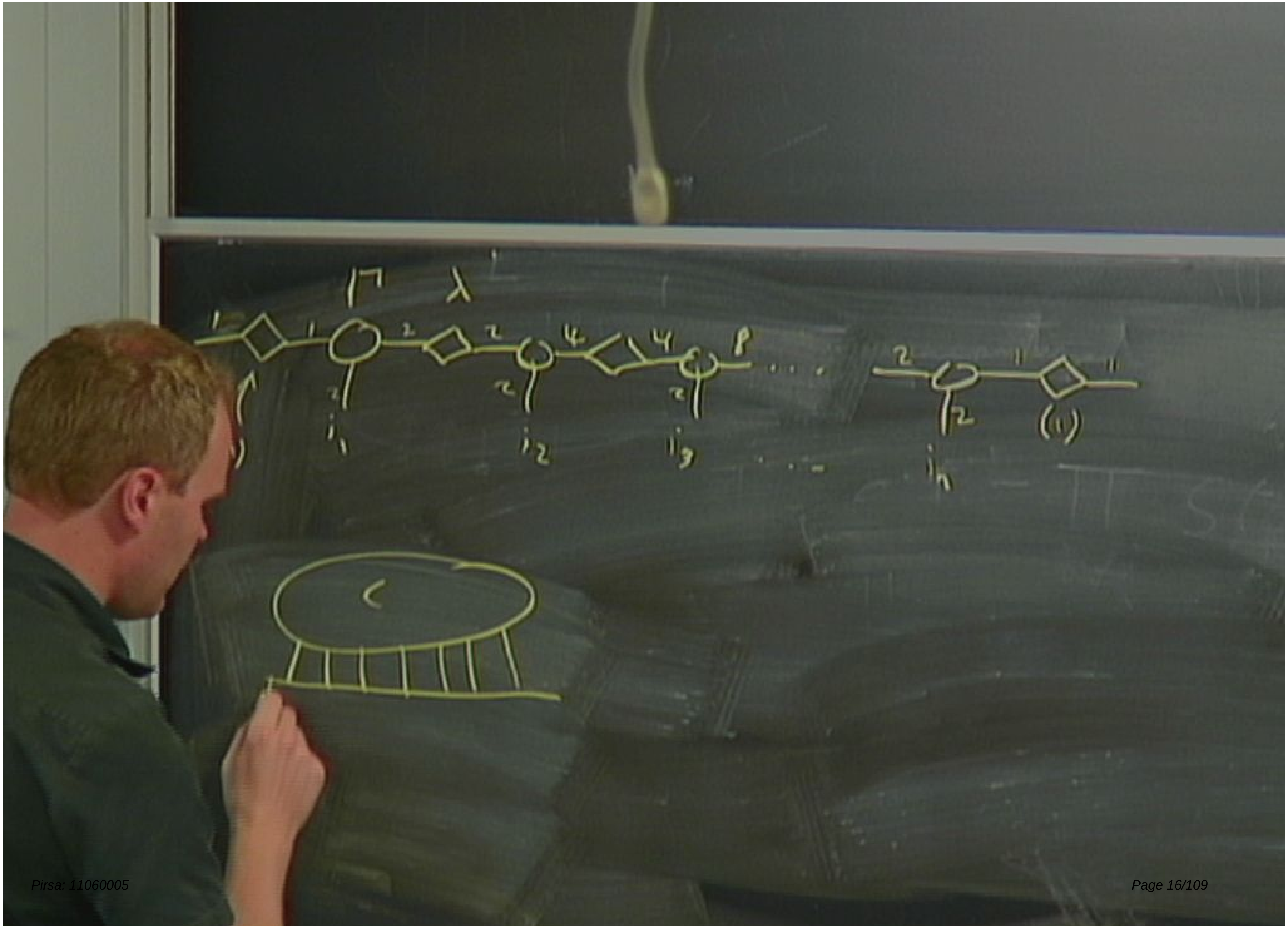

Γ
λ
(1)
i_1
i_2
i_3
i_n
(1)
i_1 ..... i_n

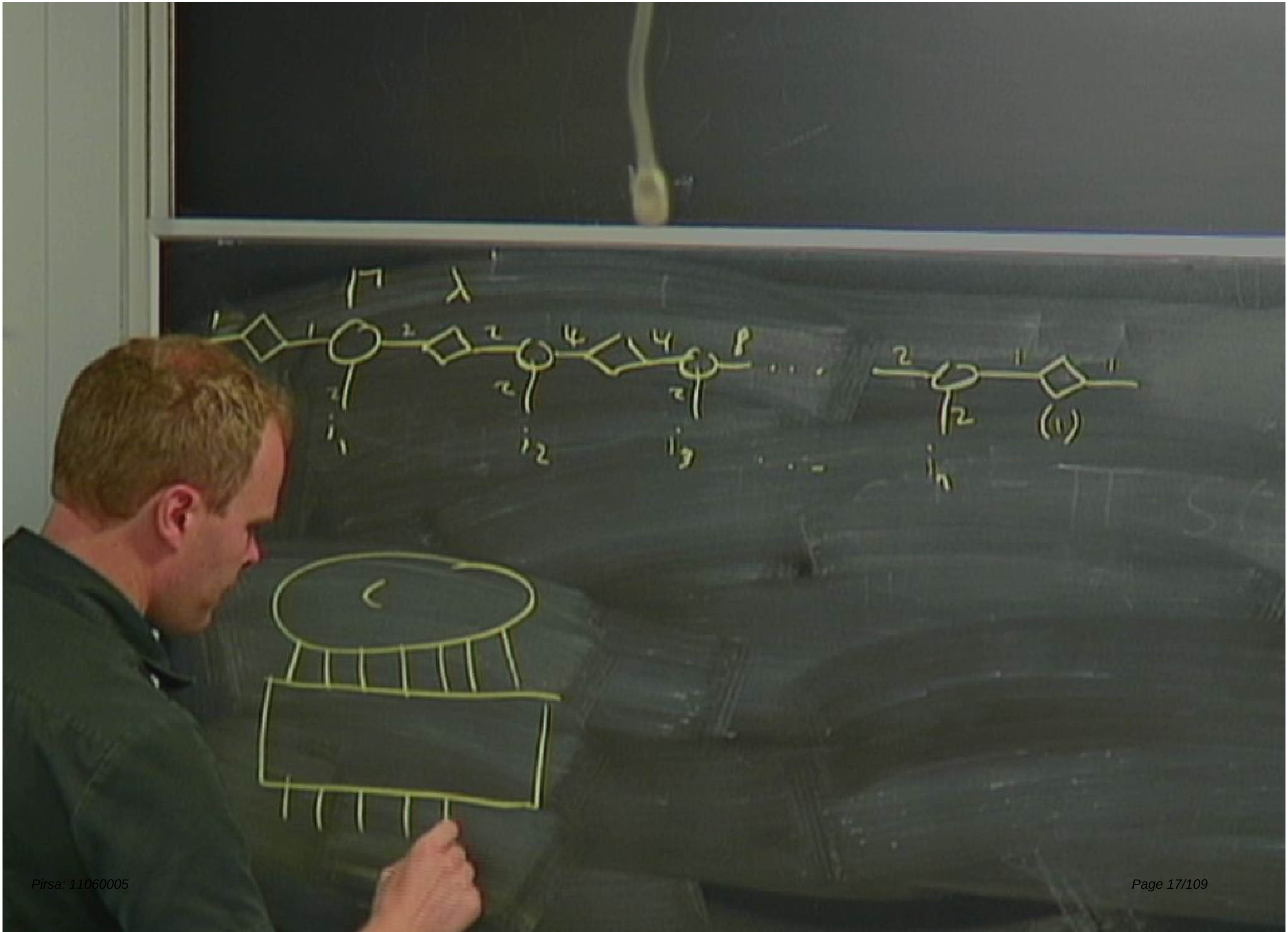

λ
(1)
(1)
1 x 2 x 2 x ... 2 x 1
n

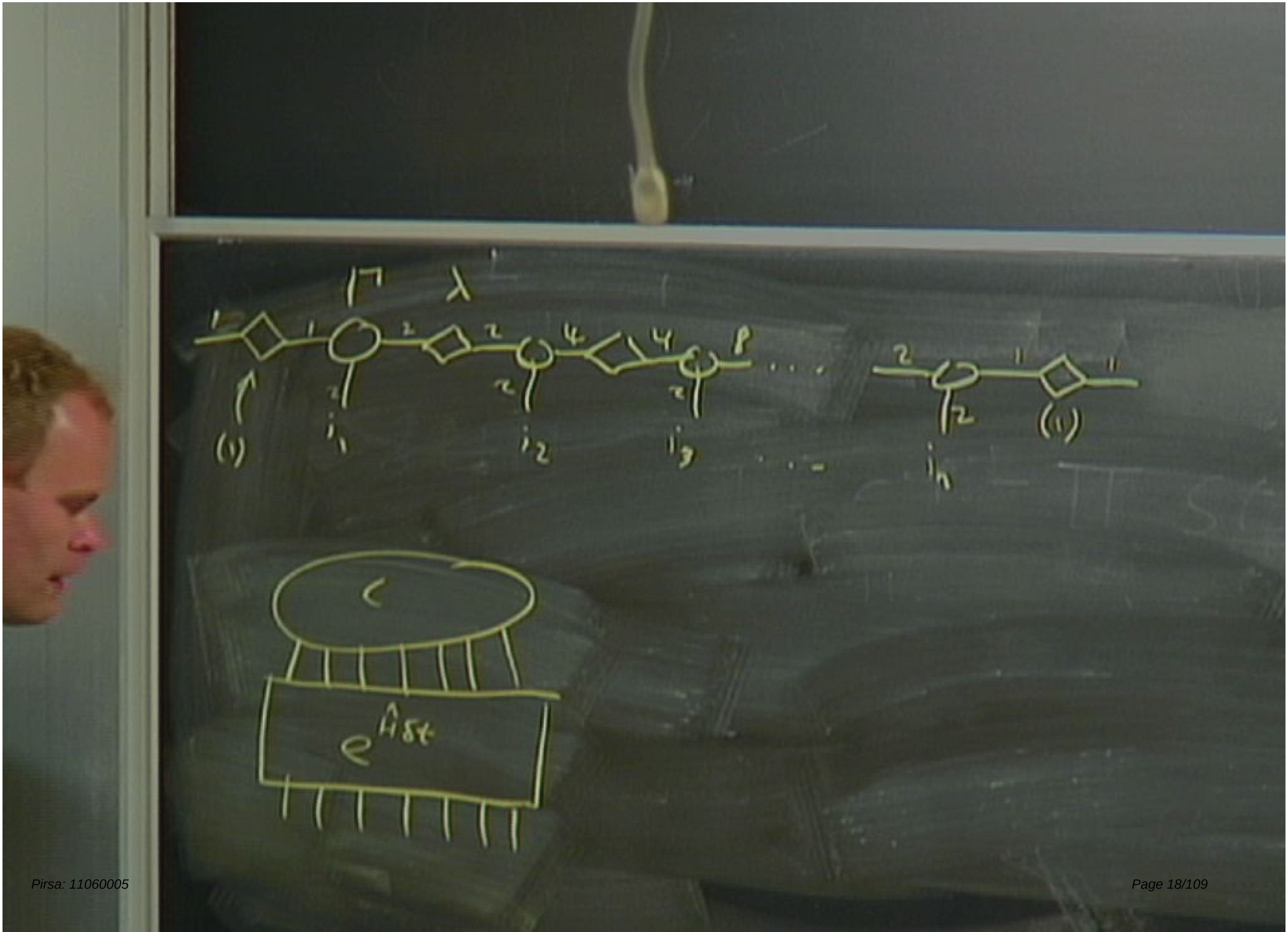

(1)
(1)
1 x 2 x 2 x ... 2 x 1
n

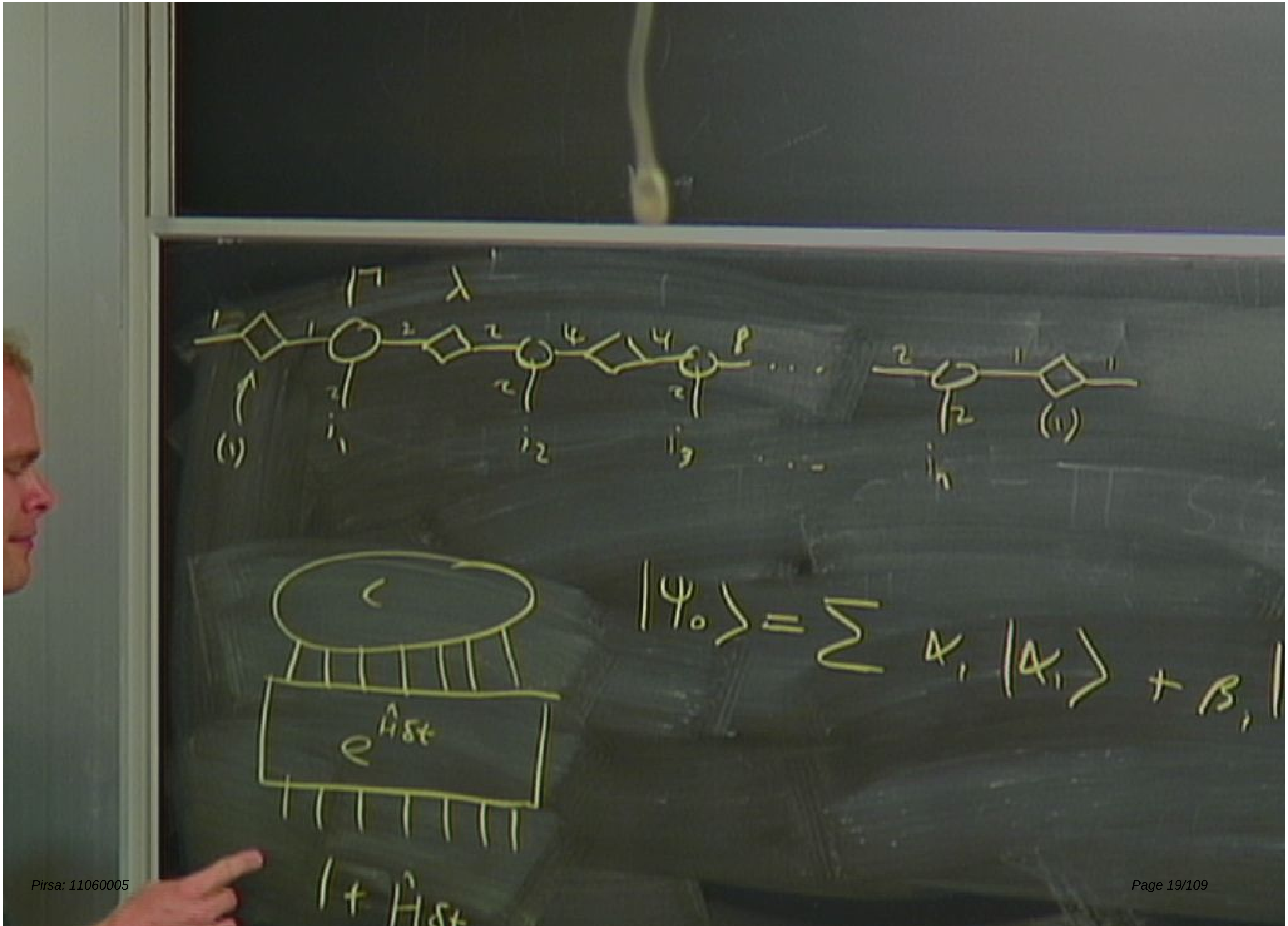


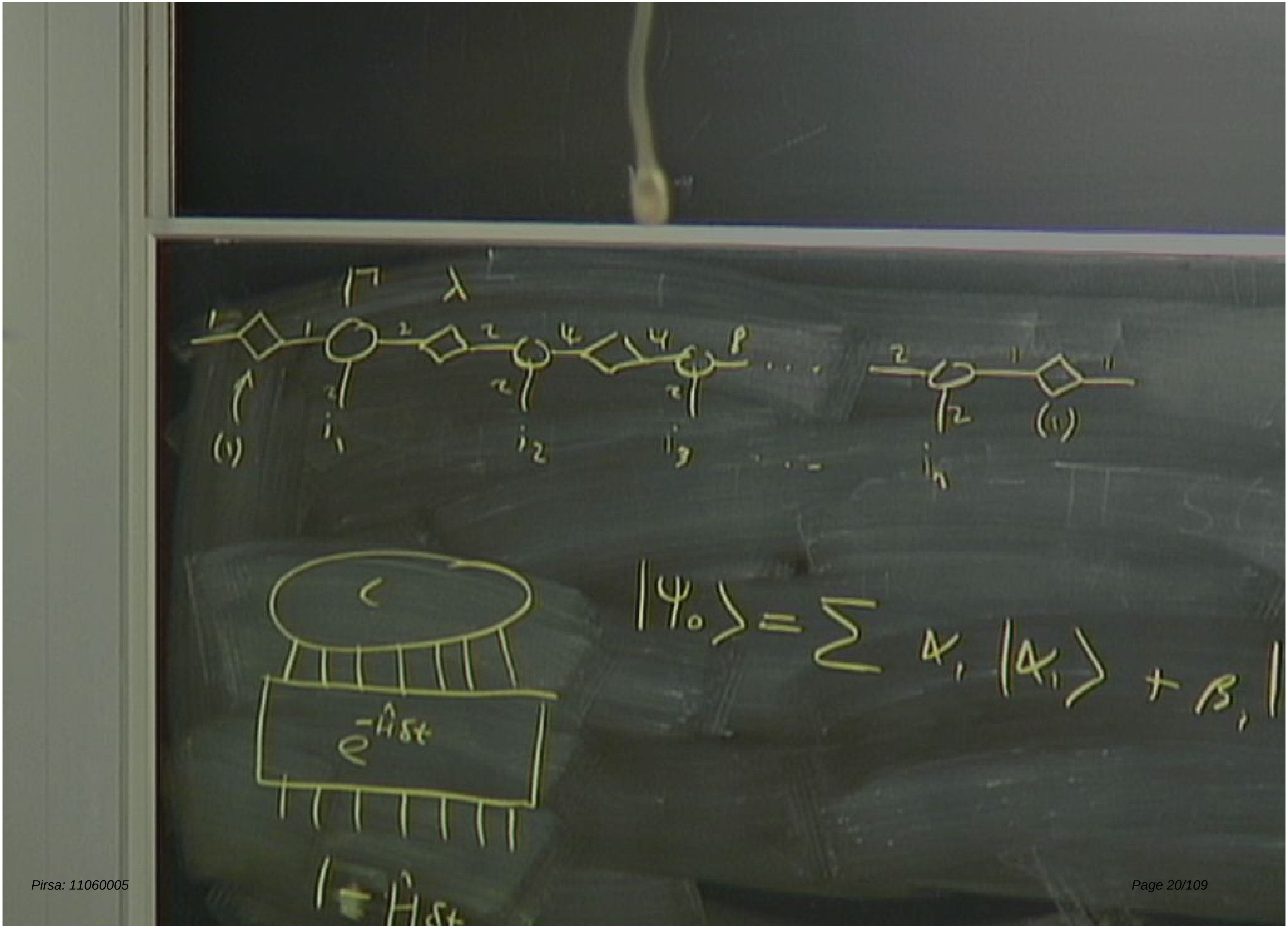
(1)
(1)
i1
i2
i3
in
C
1
1
1 x 2 x 2 x ... 2 x 1
n

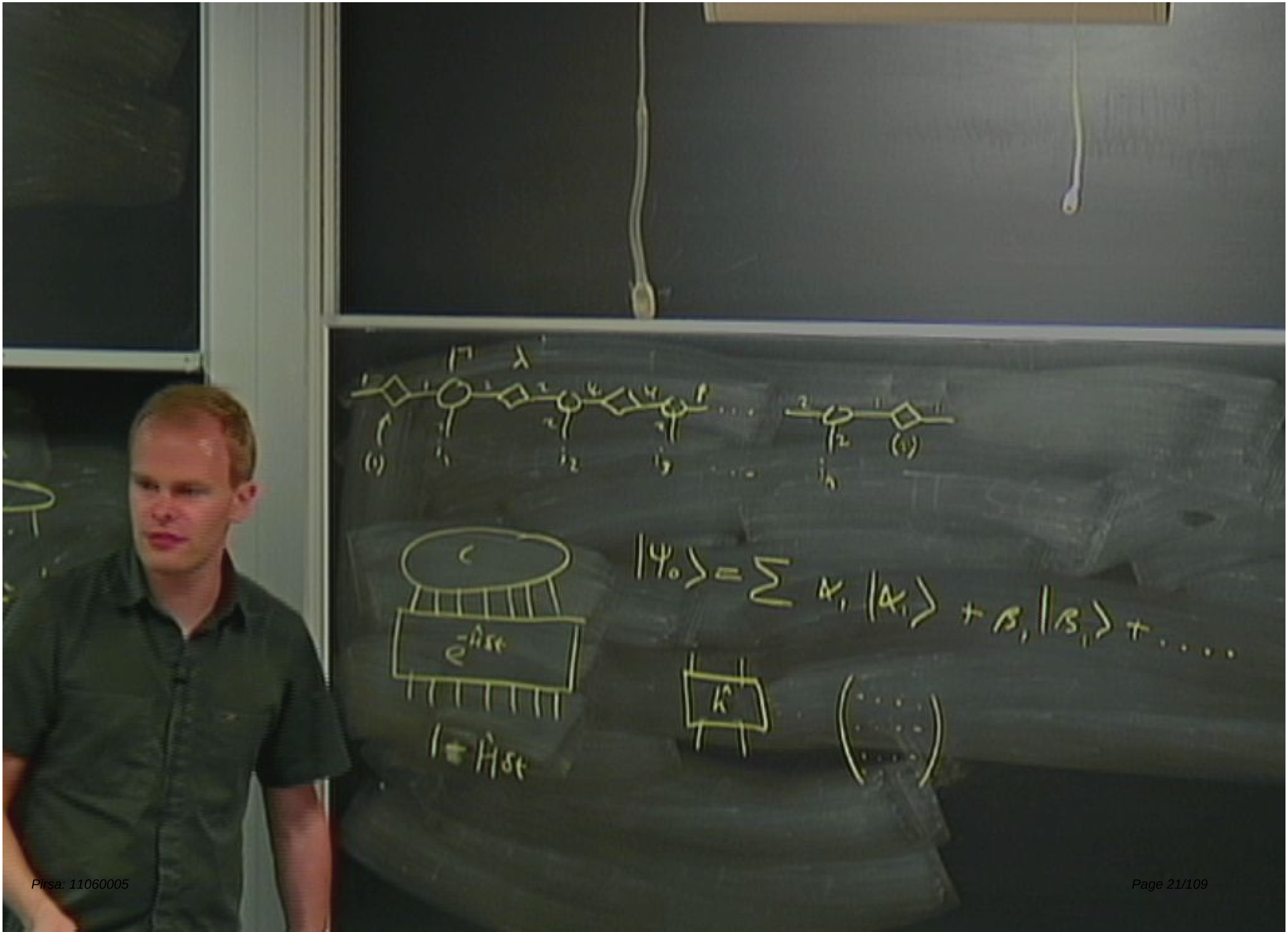


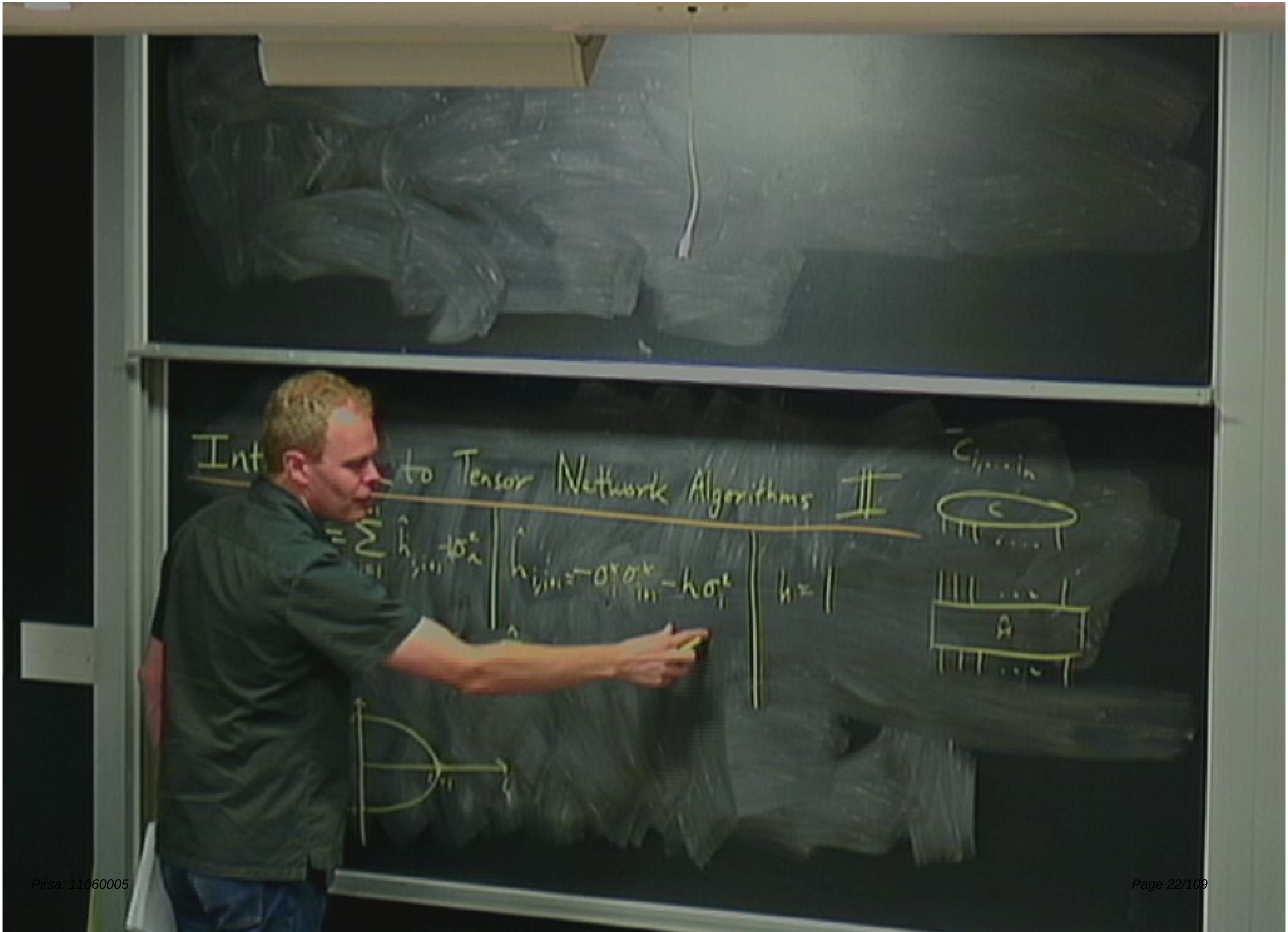

λ
i1
i2
i3
in
(11)
C




Γ
λ
(1)
i1
i2
i3
in
(11)
e^Ĥδt
|Ψ0⟩ = Σ α1 |α1⟩ + β1 |
1 + Ĥδt


Γ
λ
|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁ |
e^{-Ĥδt}
1 - Ĥδt

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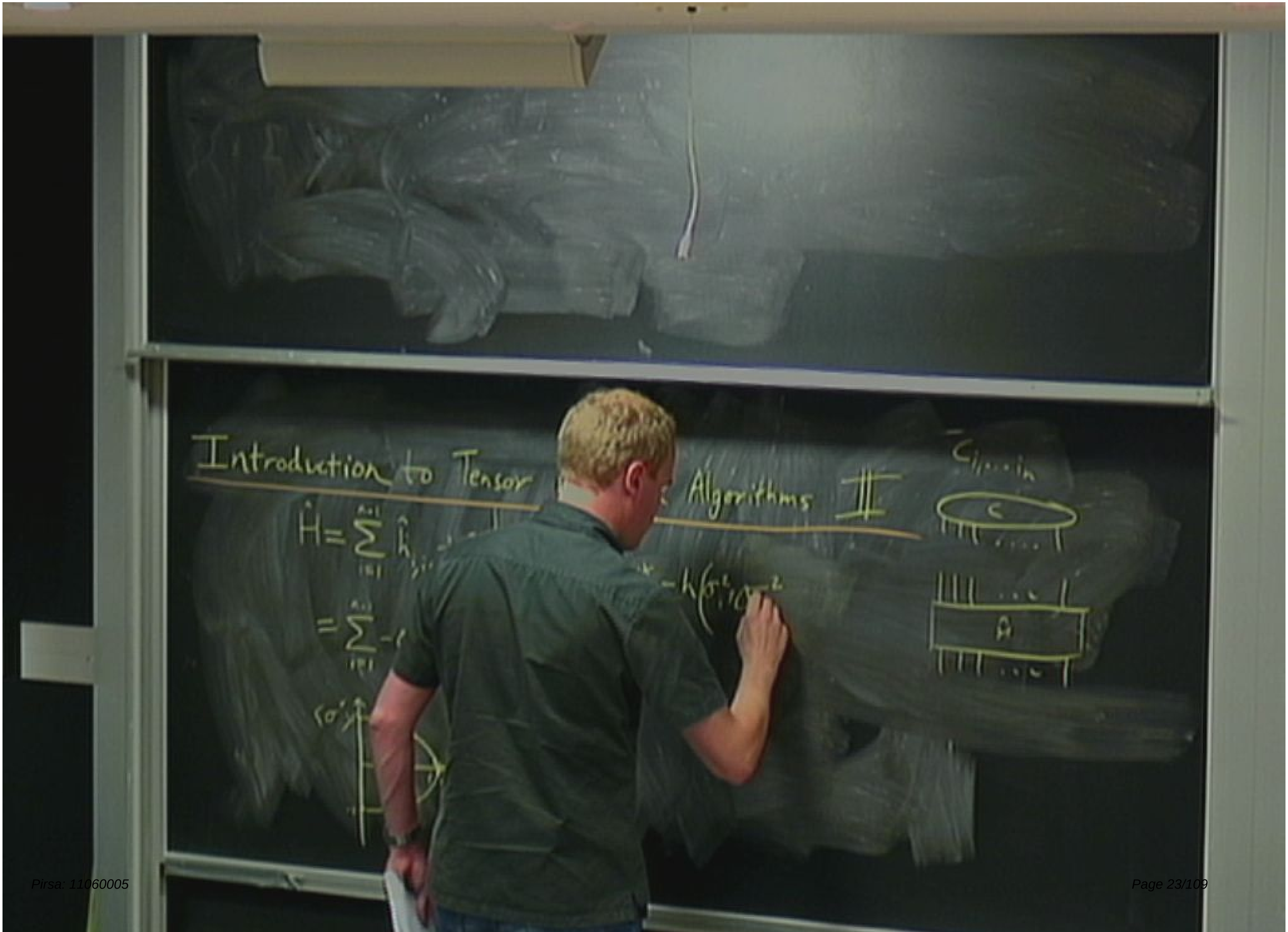
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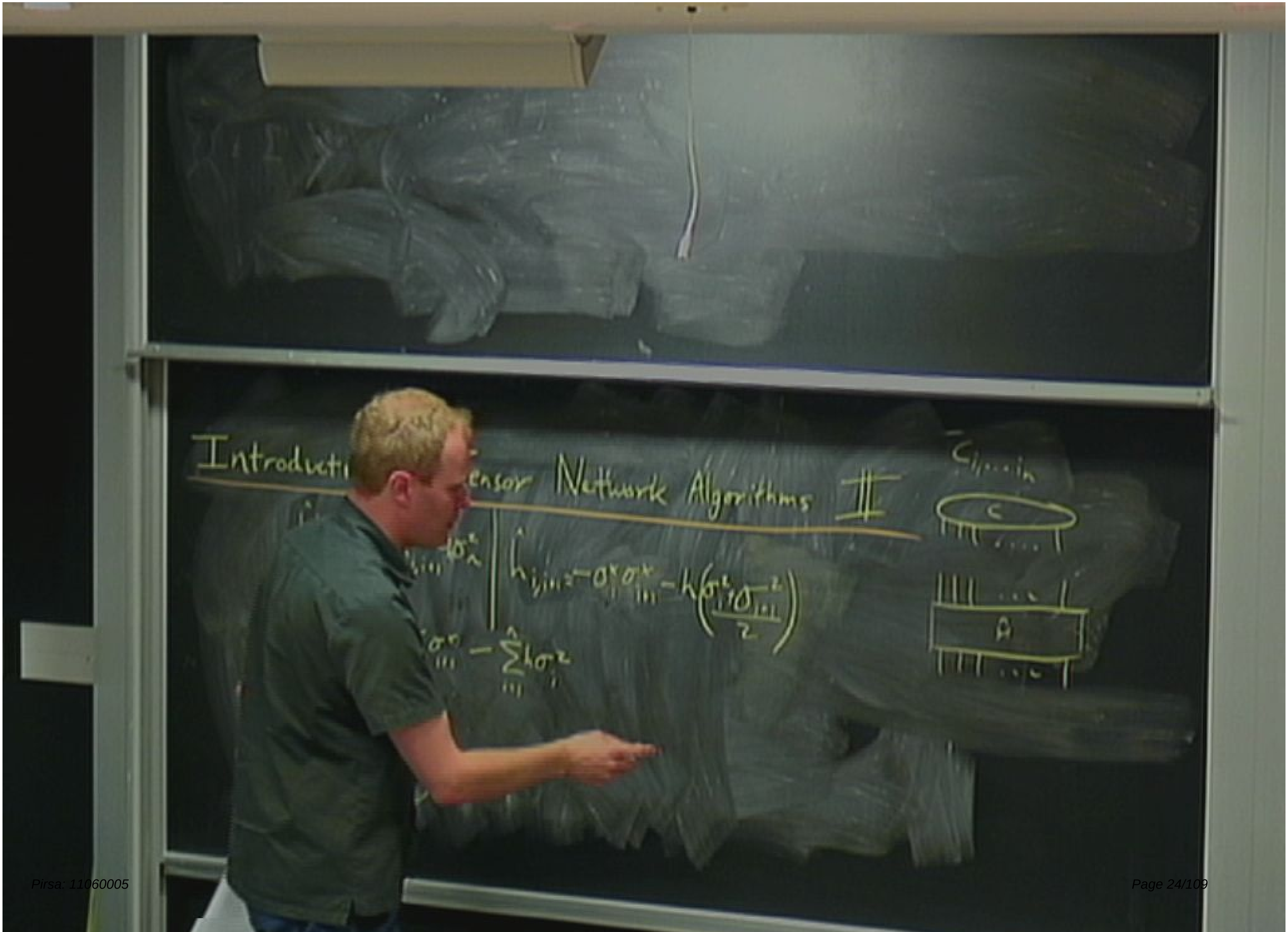
Introducti
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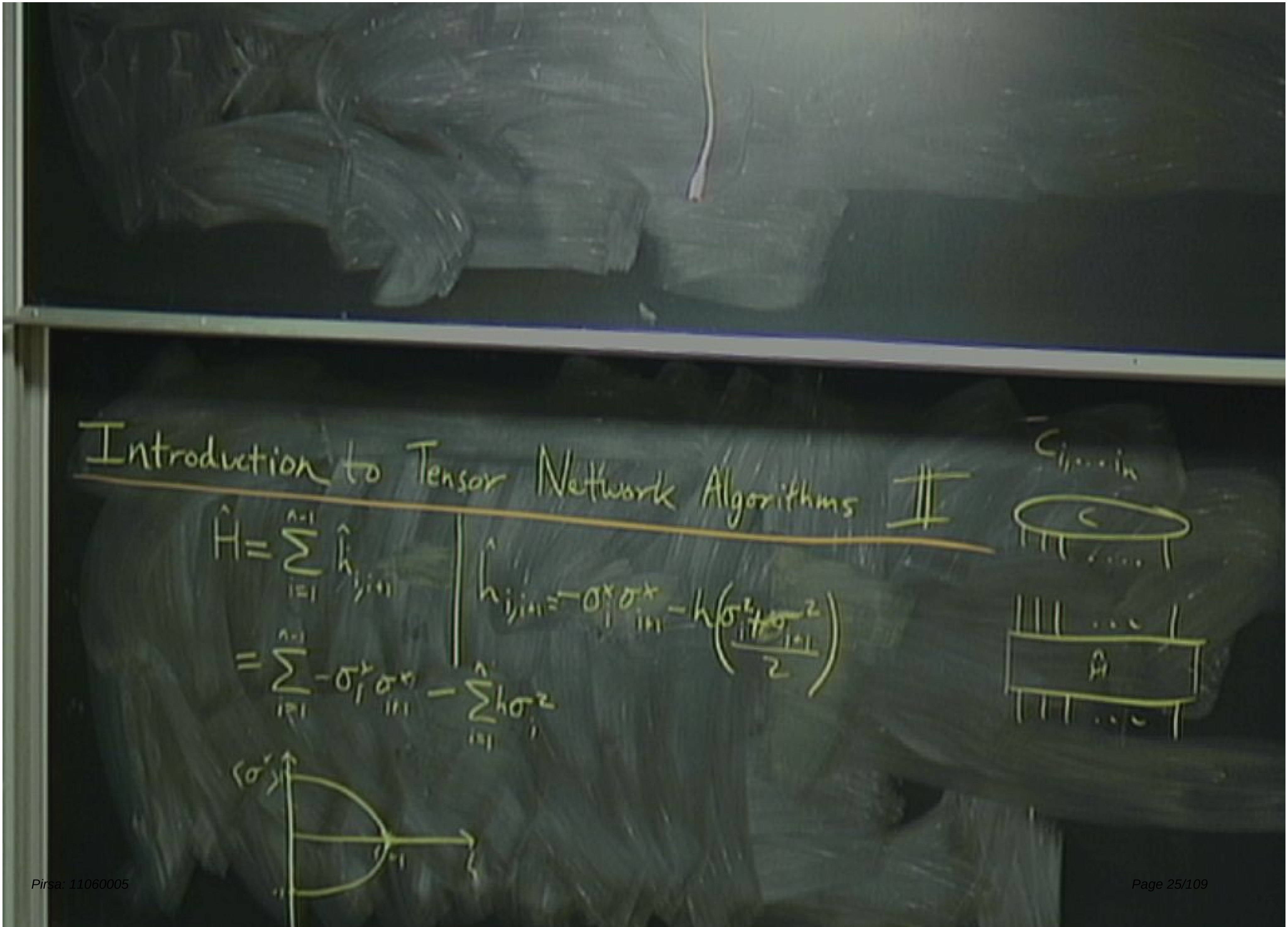
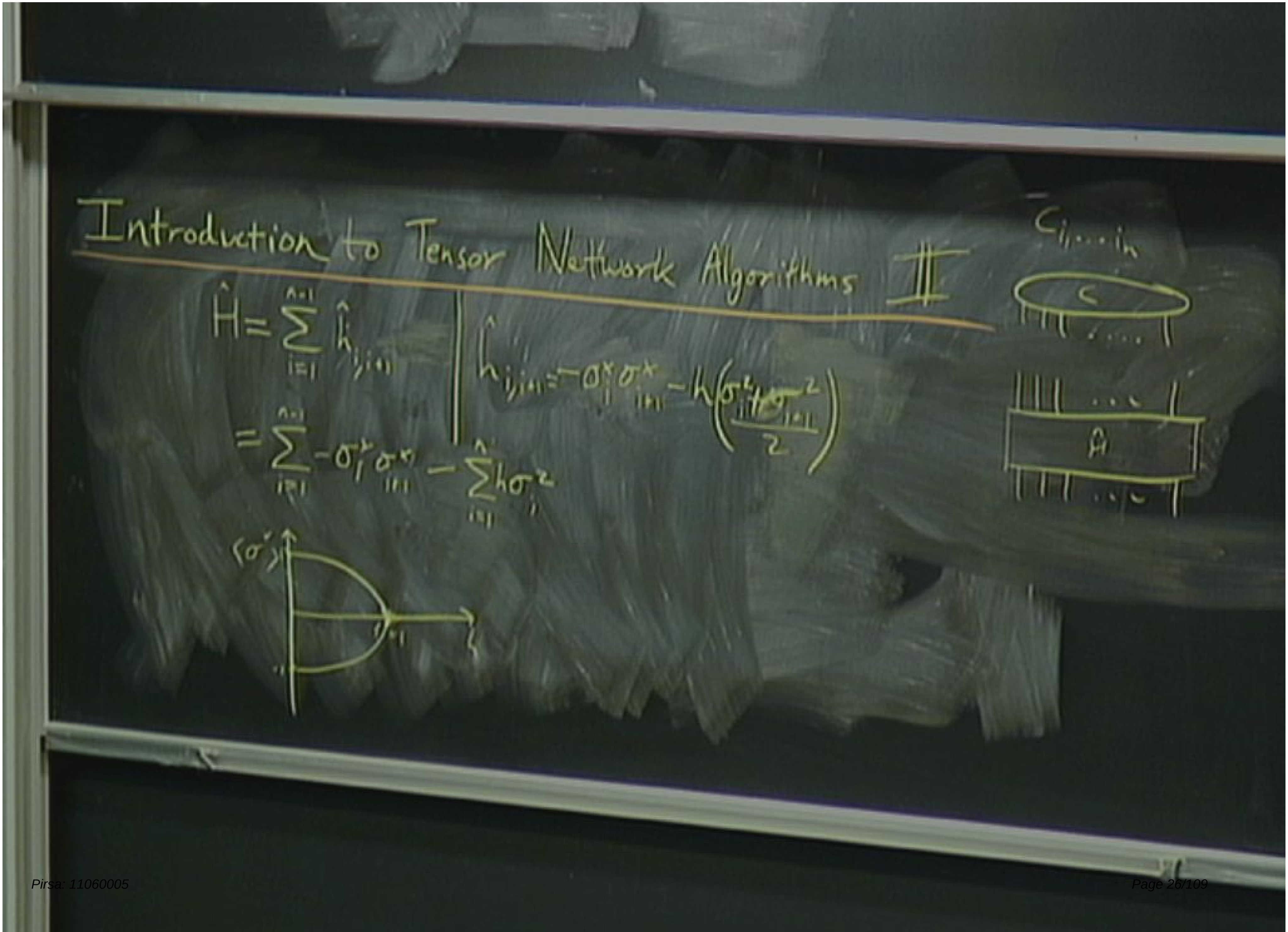
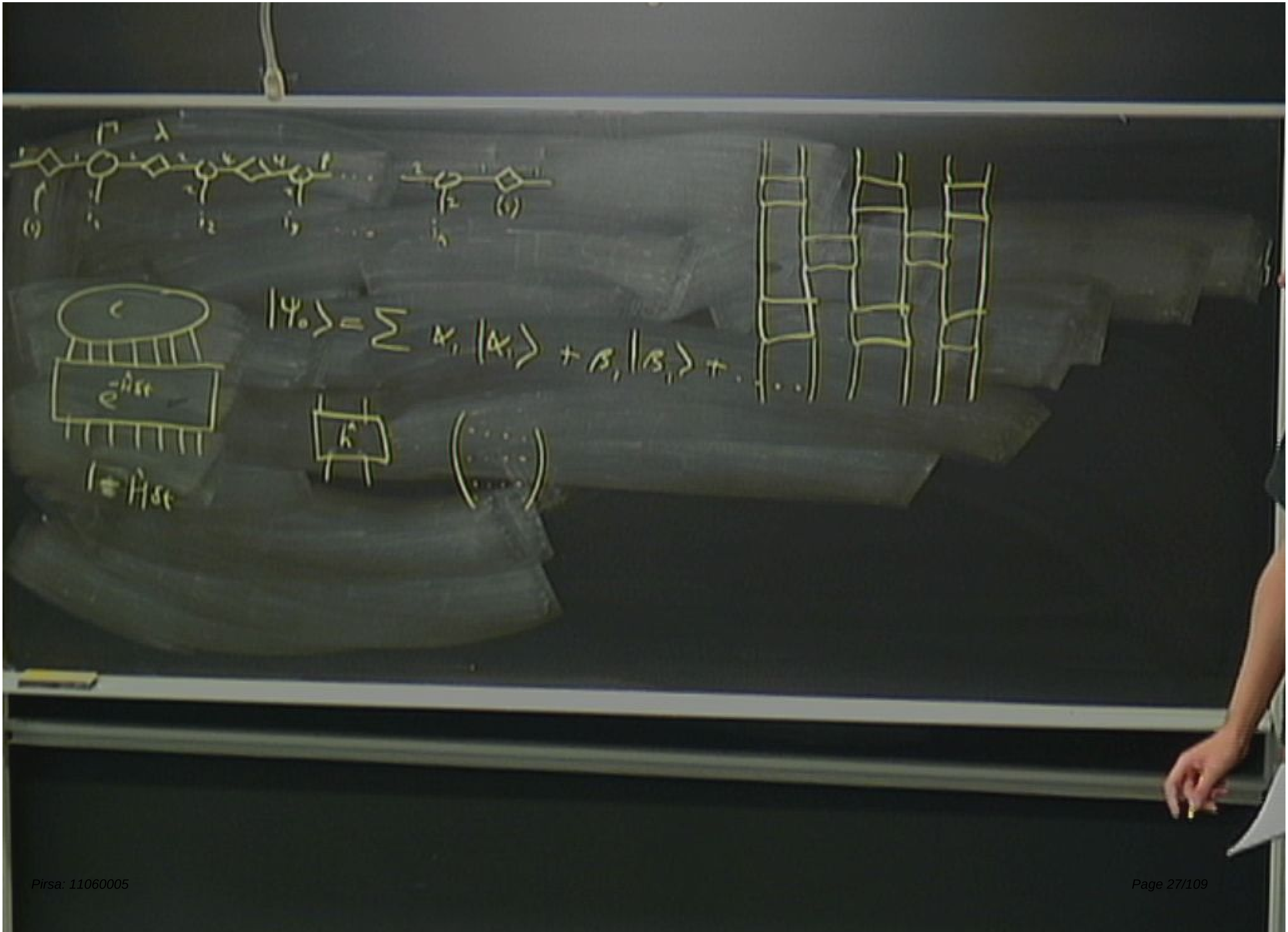
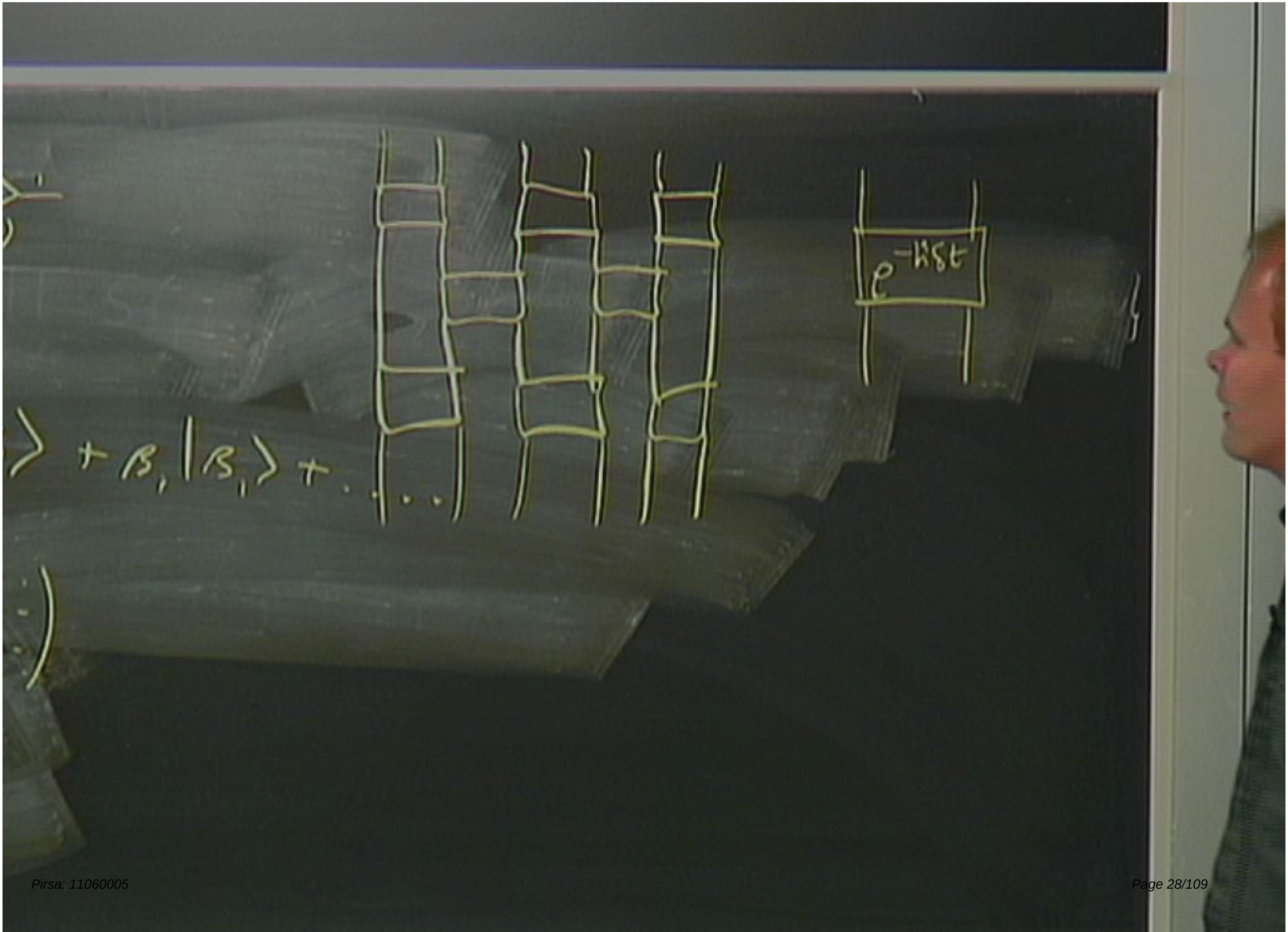
C
A

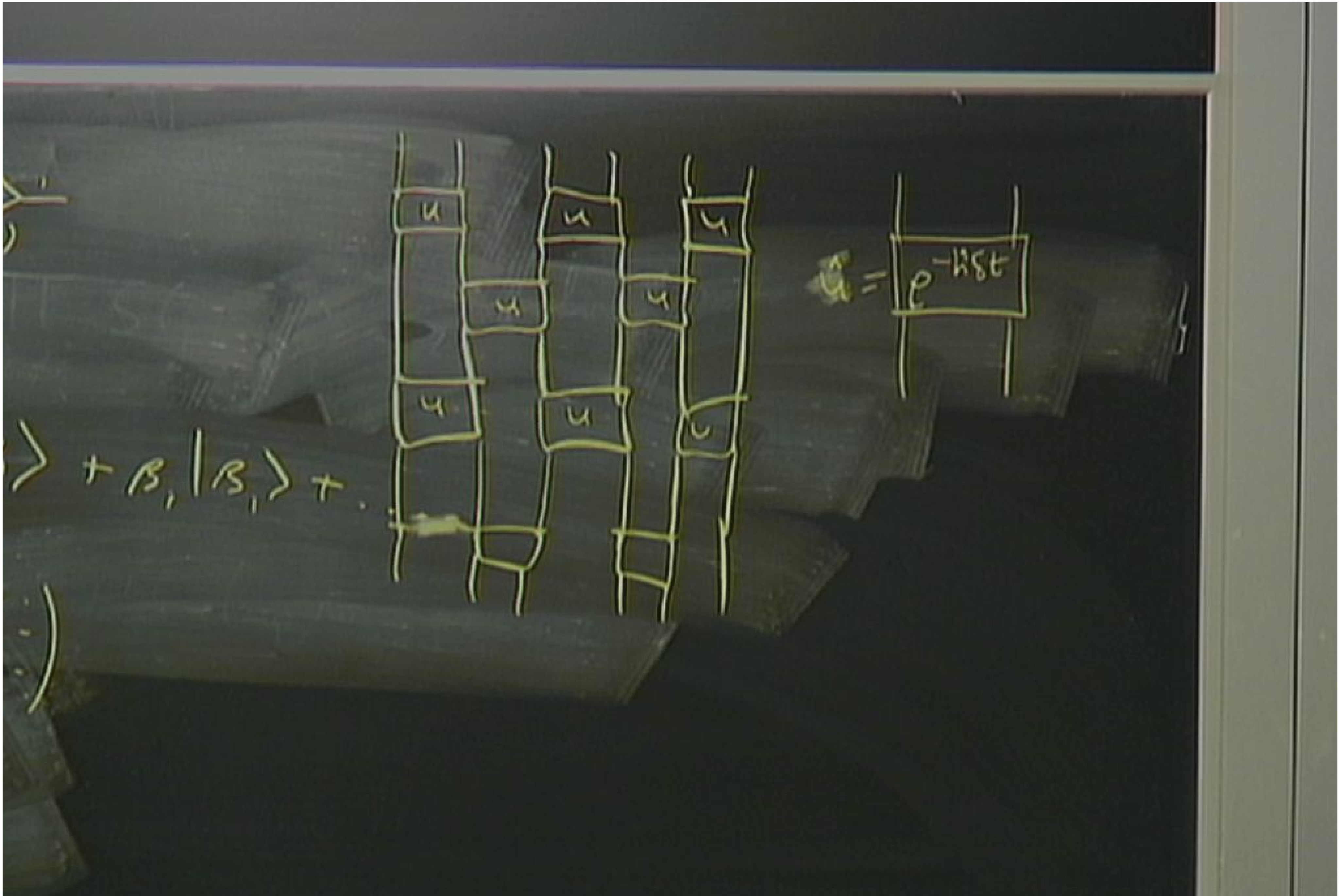

Introduction to Tensor Network Algorithms II
$\hat{H} = \sum_{i=1}^{n-1} \hat{h}_{i,i+1}$
$\hat{h}_{i,i+1} = -\sigma^x_i \sigma^x_{i+1} - h\left(\frac{\sigma^z_i + \sigma^z_{i+1}}{2}\right)$
$= \sum_{i=1}^{n-1} -\sigma^x_i \sigma^x_{i+1} - \sum_{i=1}^{n} h \sigma^z_i$
$c_{i_1 \ldots i_n}$
c
$\hat{H}$


Introduction to Tensor Network Algorithms II
c_{i1...in}
c
A

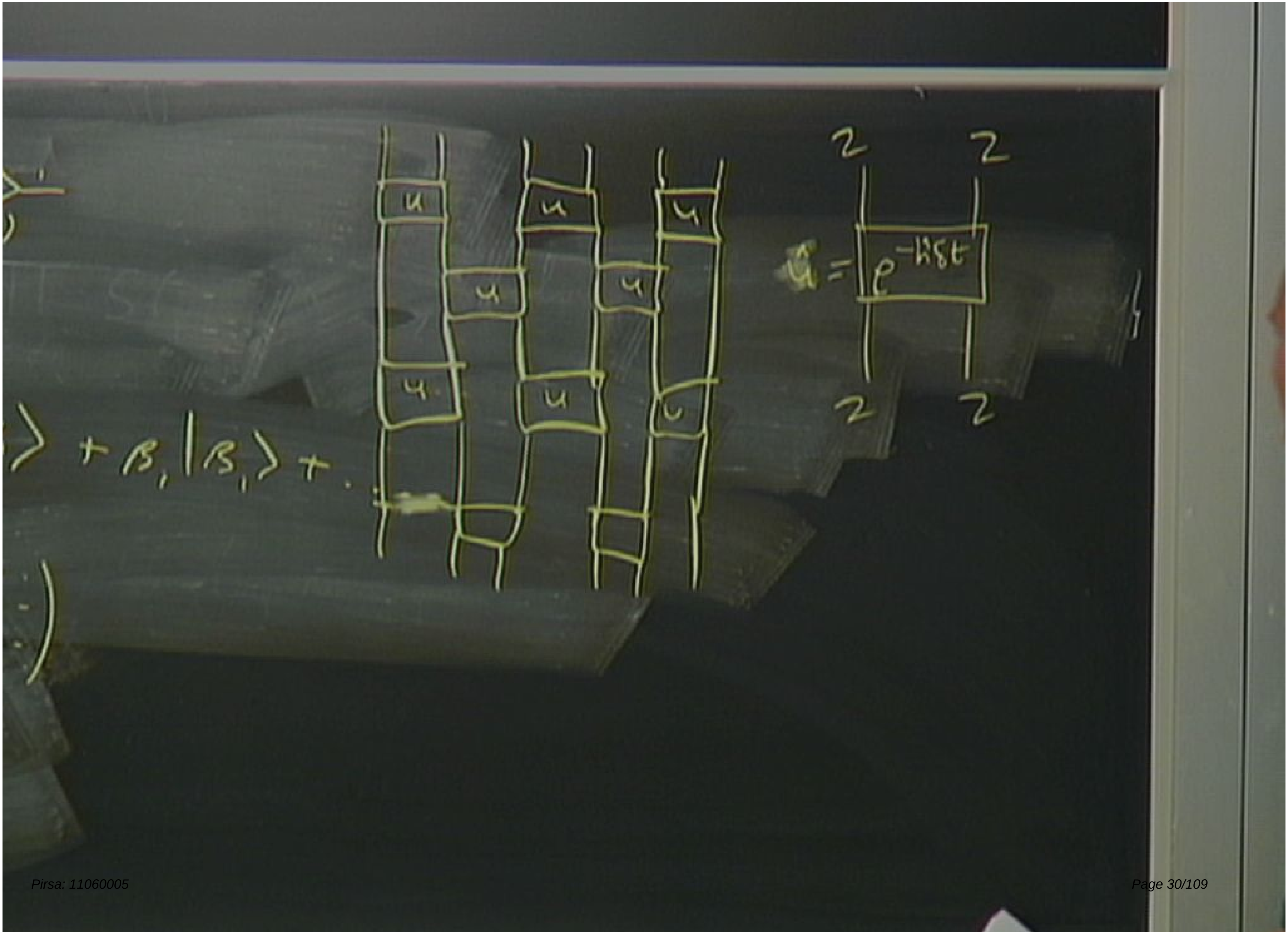

|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁ |β₁⟩ + ...

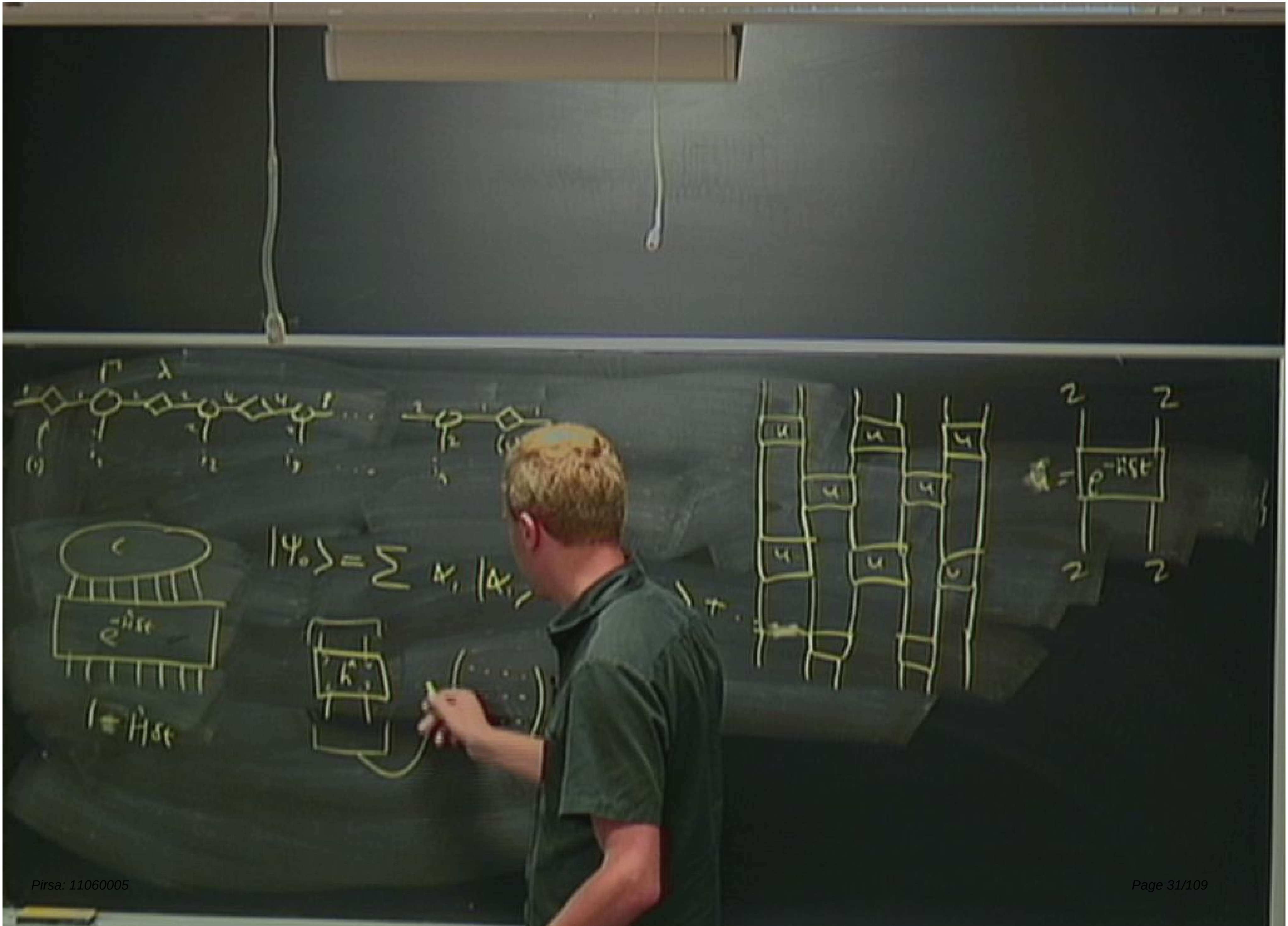


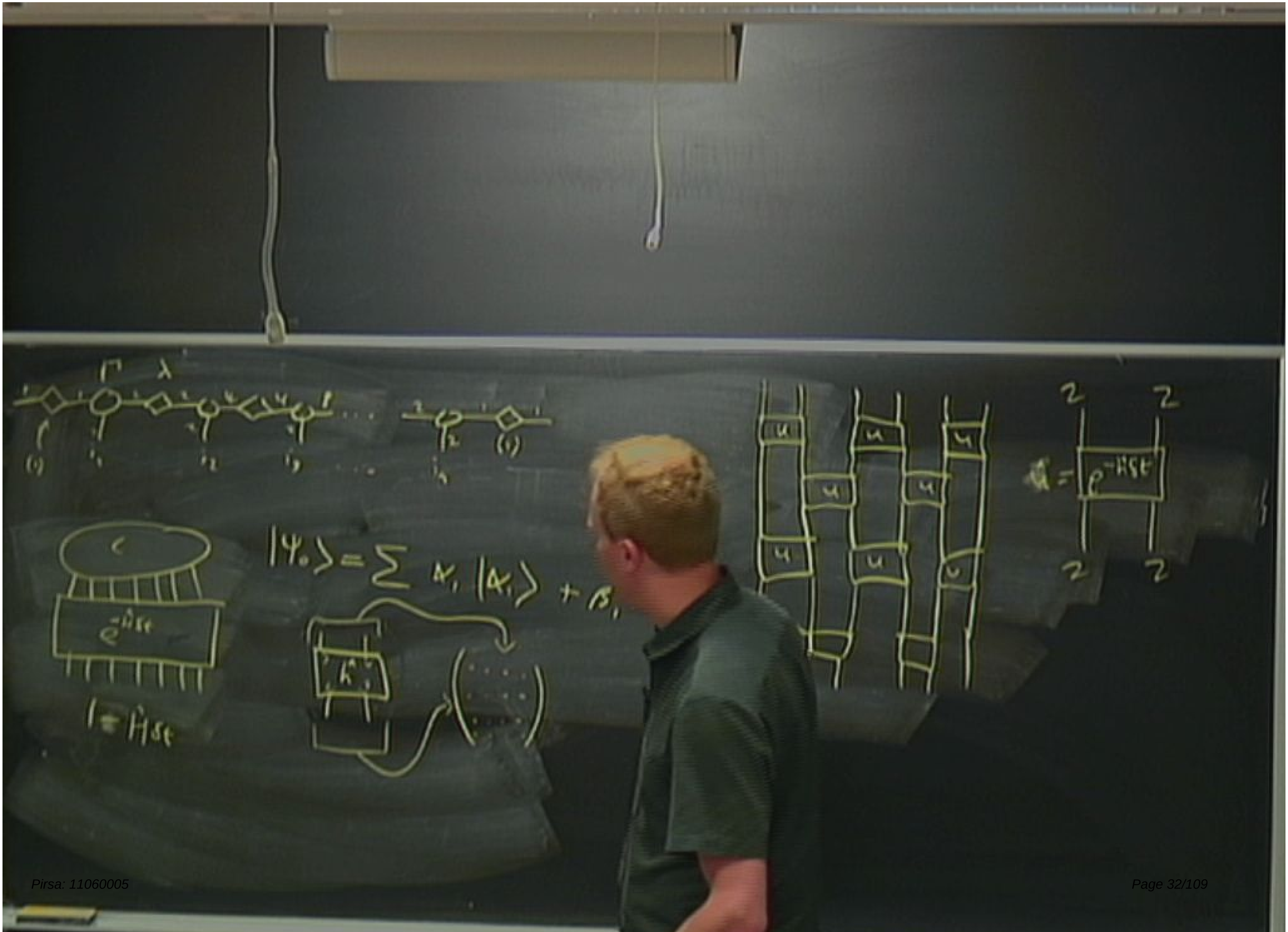
+ β₁|β₁⟩ + ...

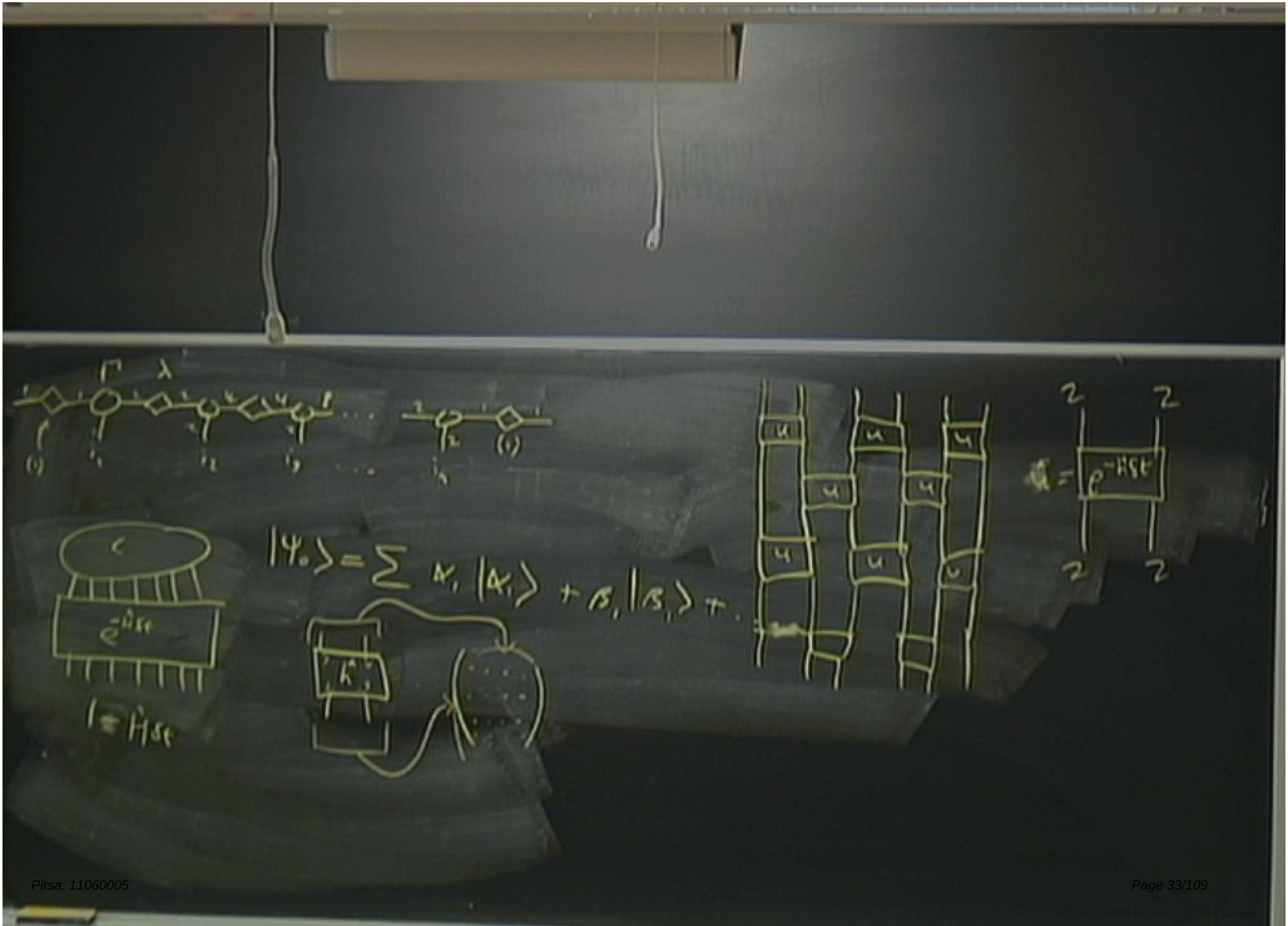


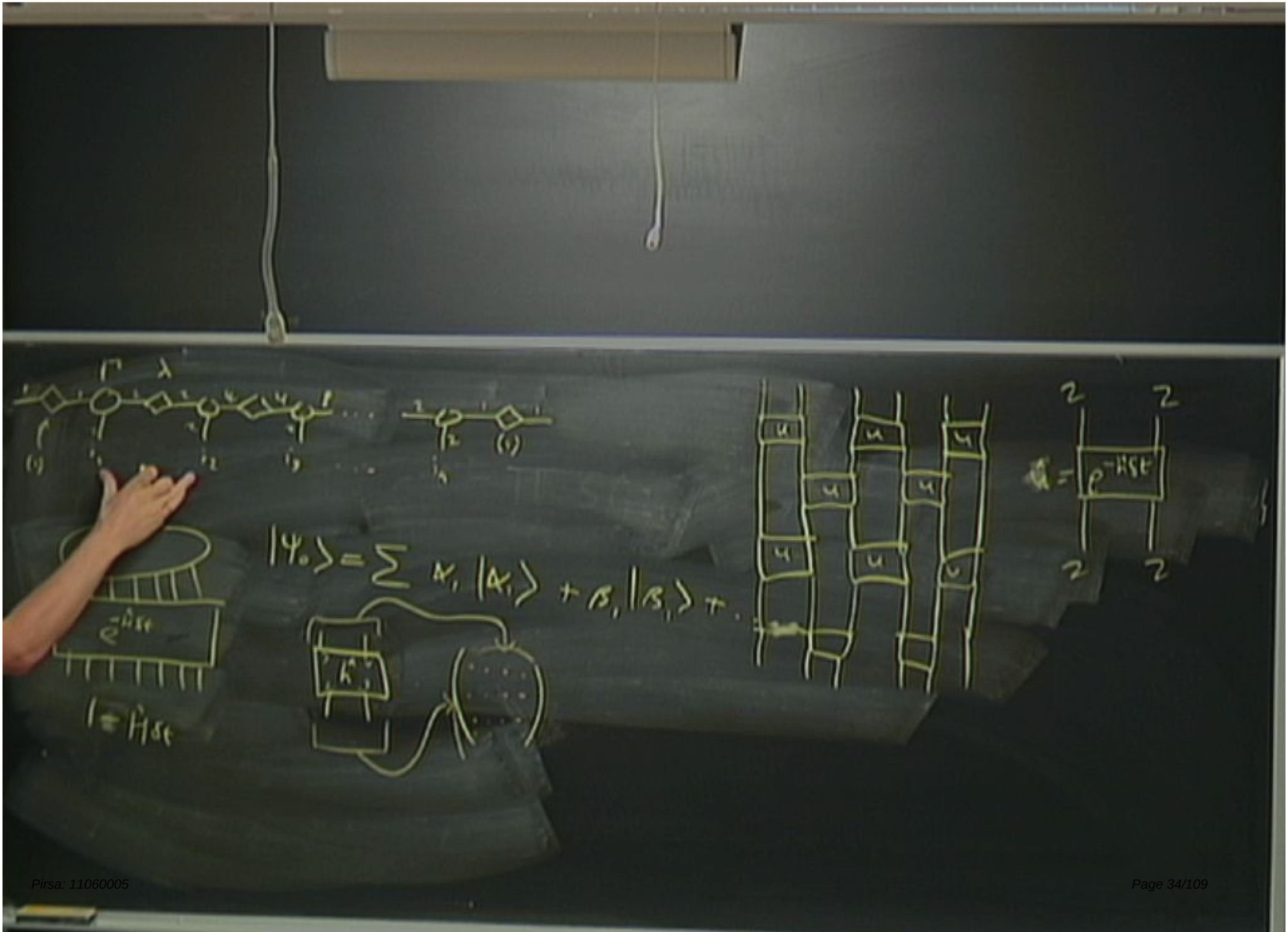
+ β₁ |β₁ > + . .
u
U = e^-iHδt

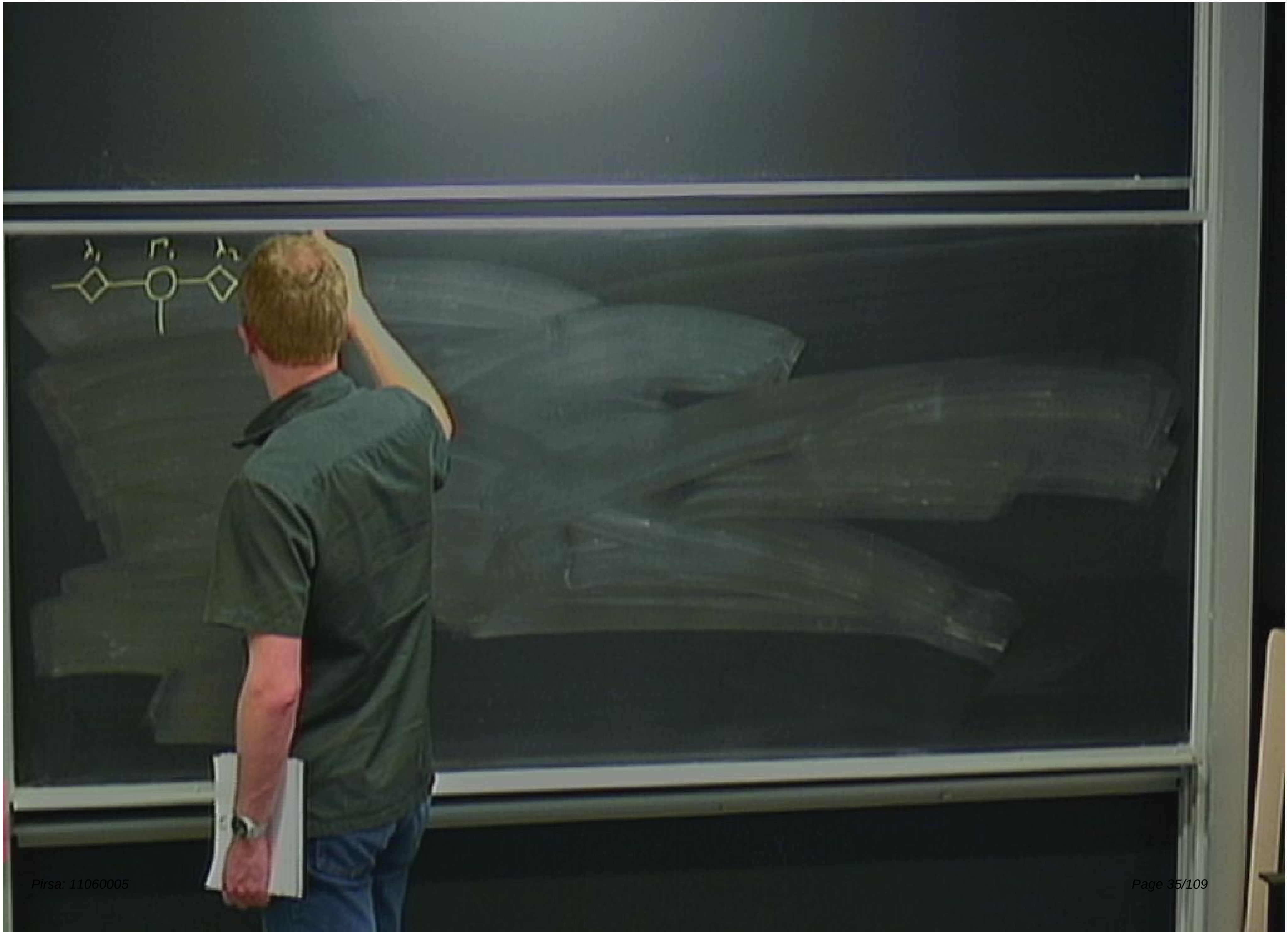

+ β₁|β₁⟩ + ...
u
u = e^{-ihδt}
2

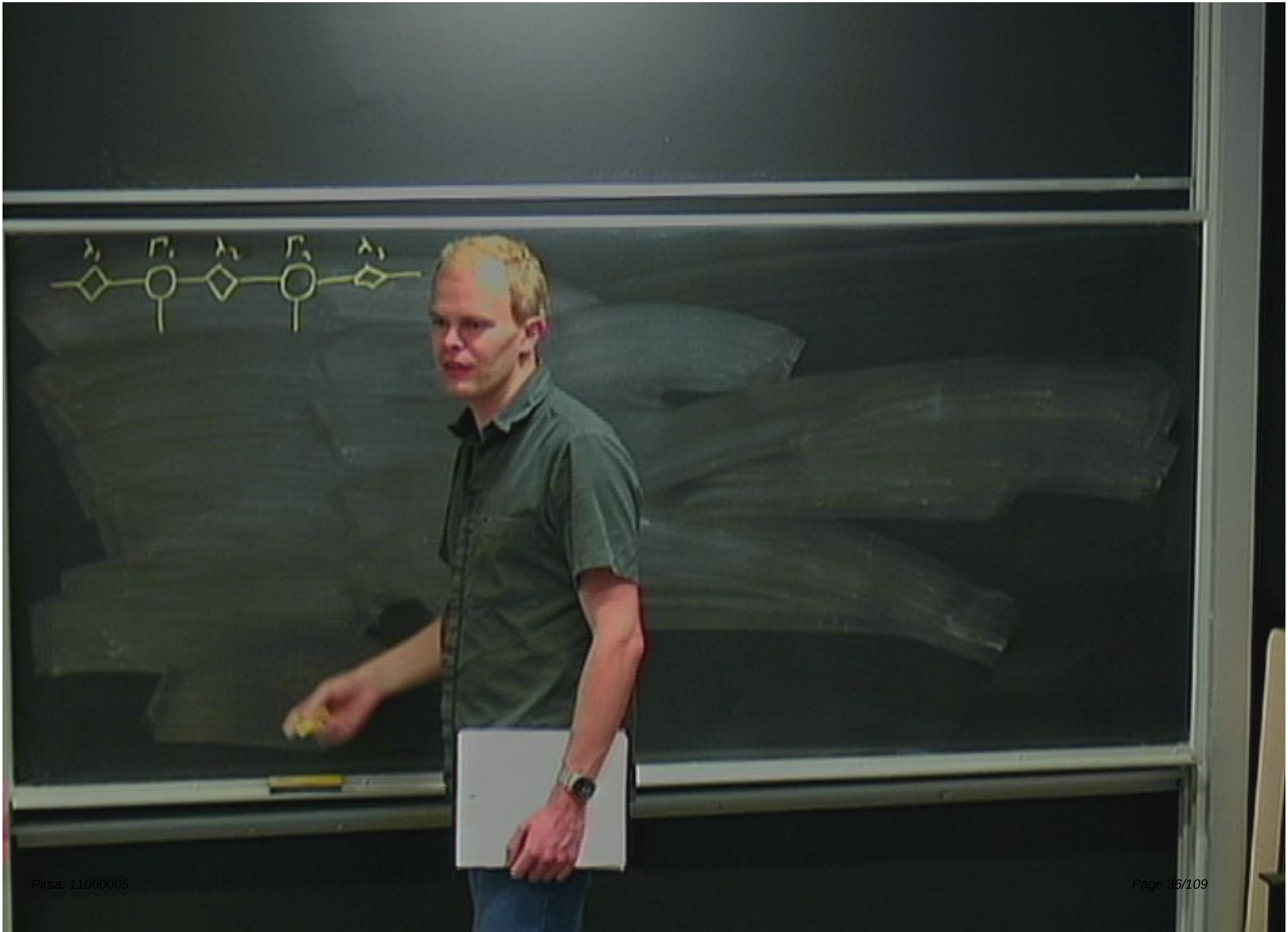


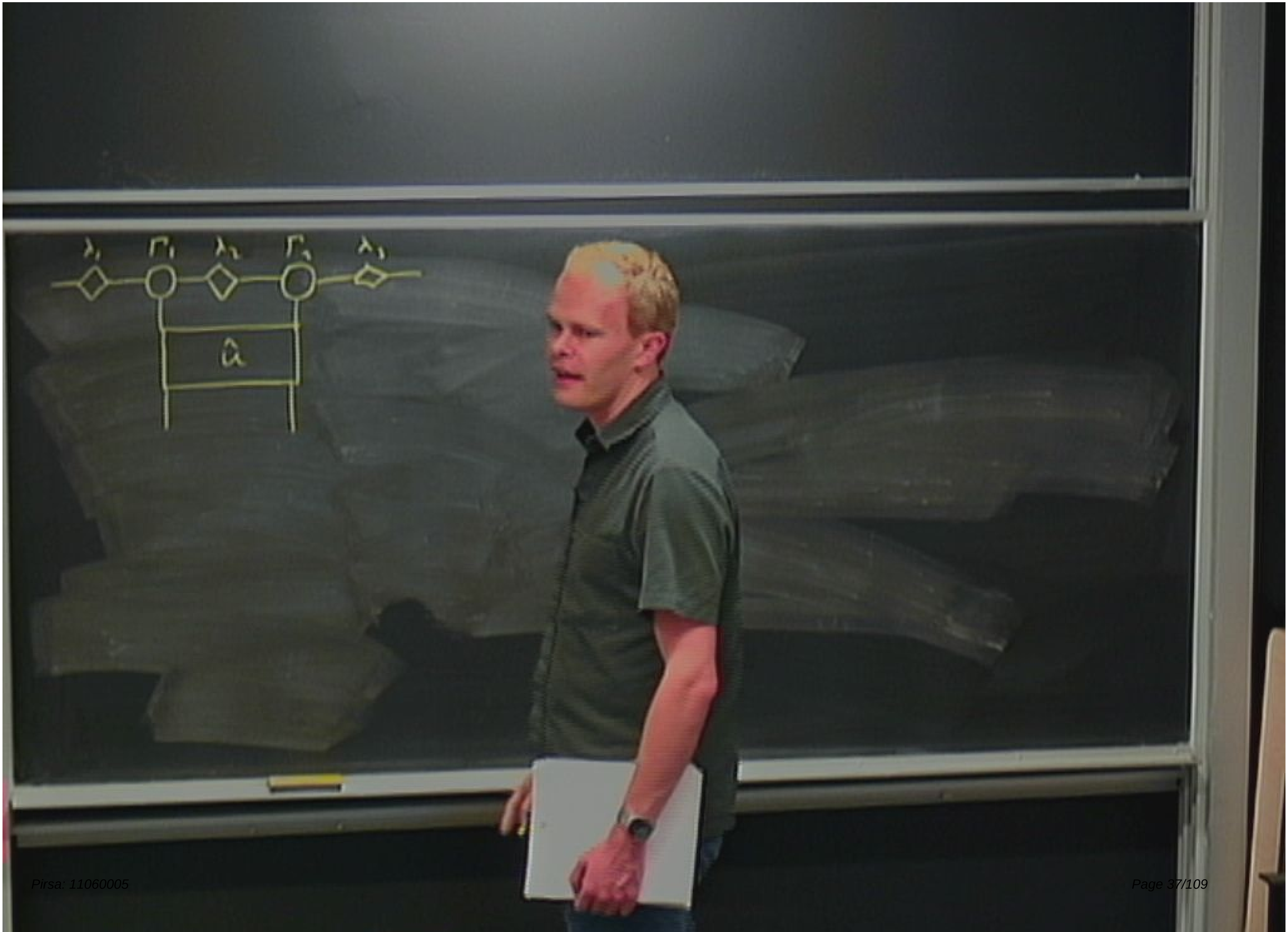

Γ
λ
e^{-iĤδt}
|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁

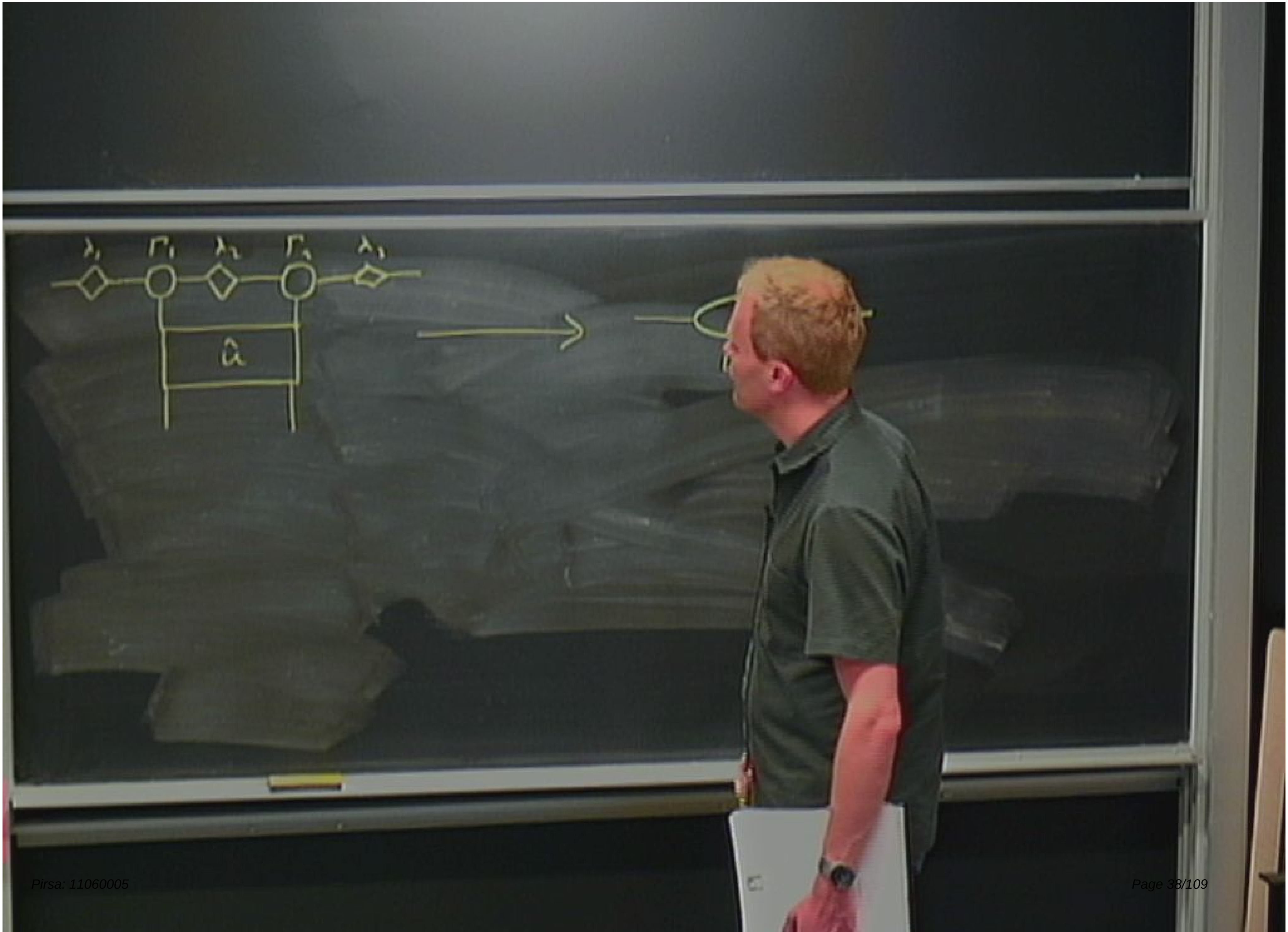


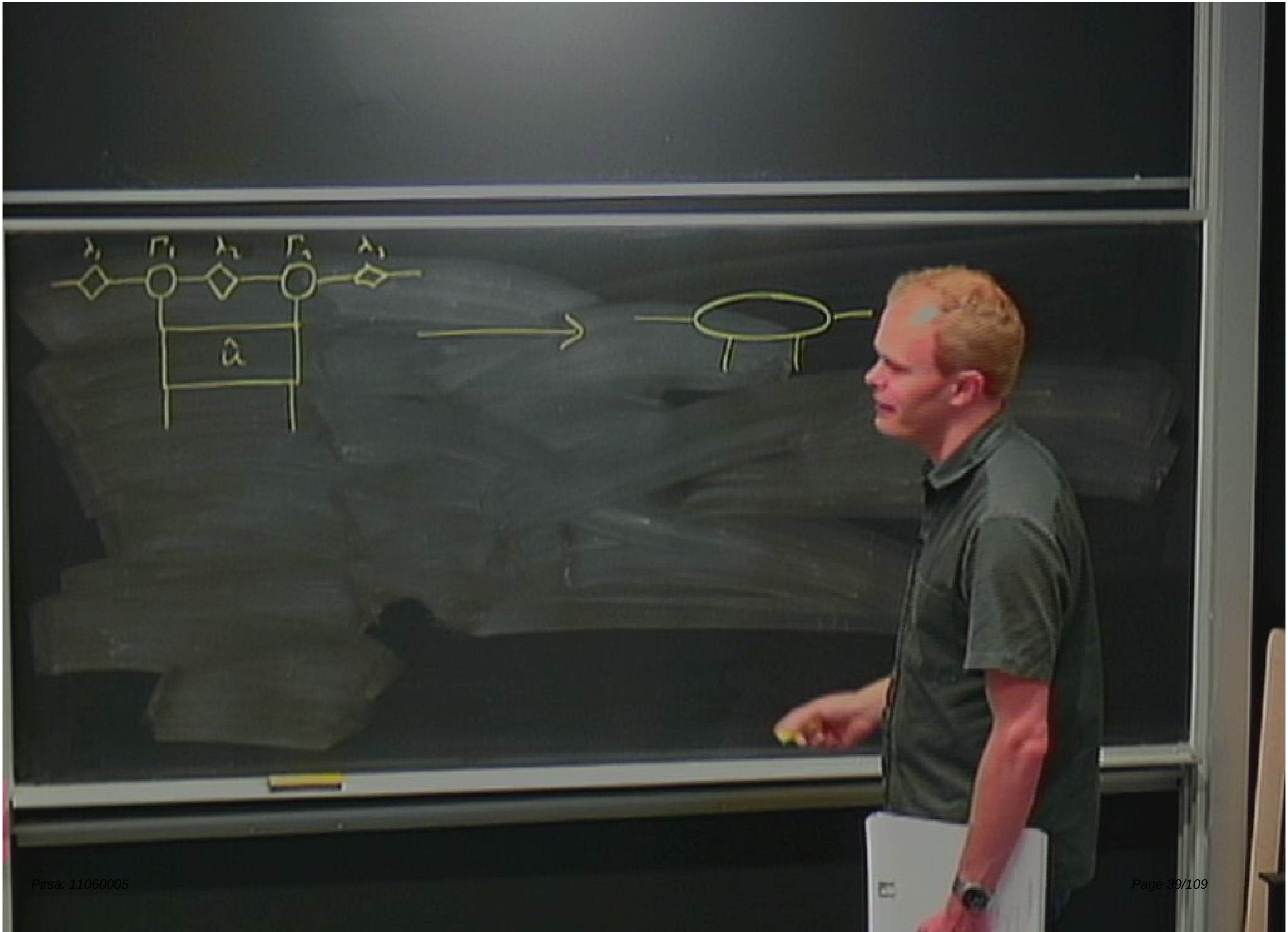

Γ λ
|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁ |β₁⟩ + ...

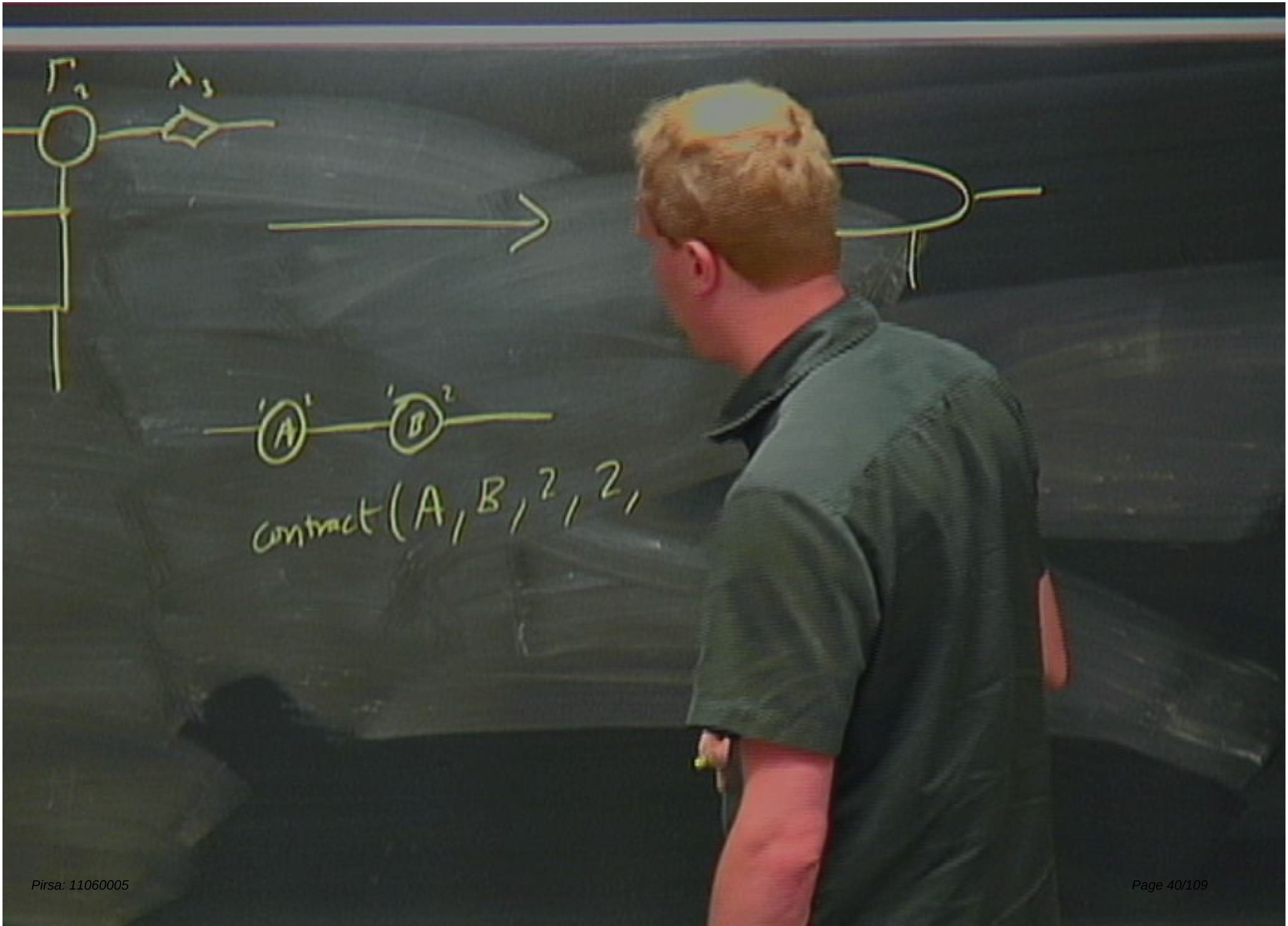


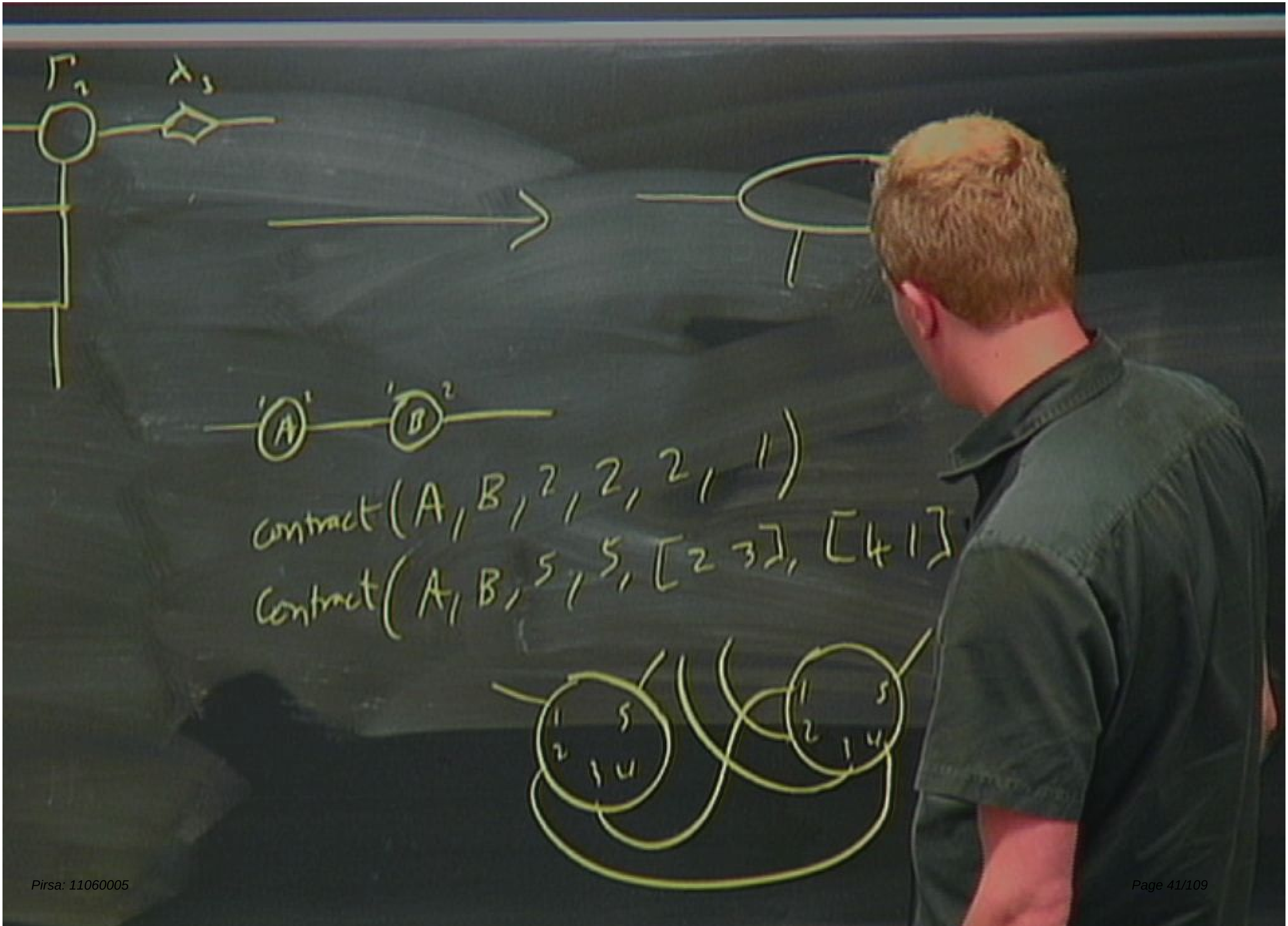

λ_1 Γ_1 λ_2 Γ_2 λ_3

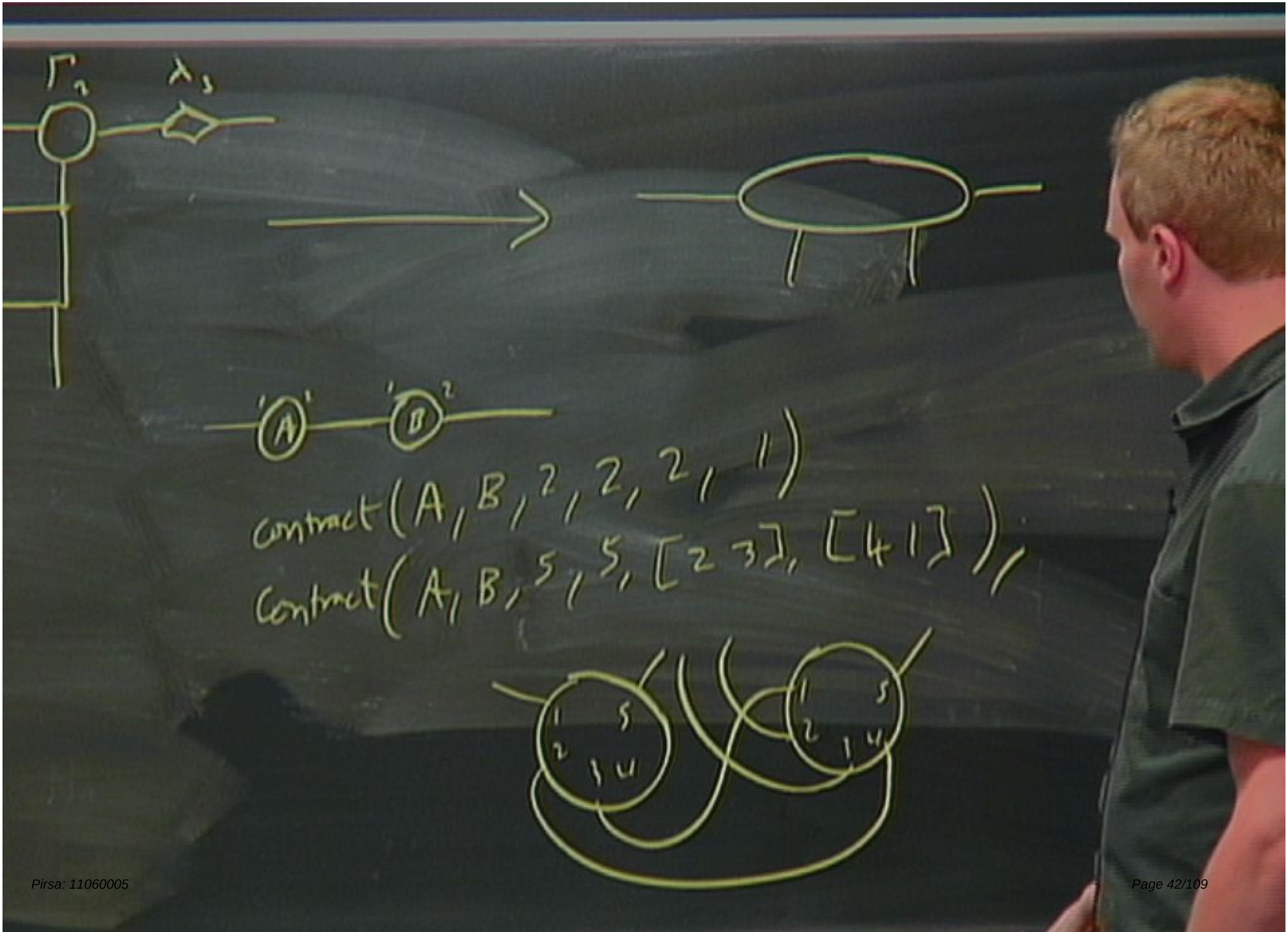

λ_1
Γ_1
λ_2
Γ_2
λ_3
$\hat{u}$

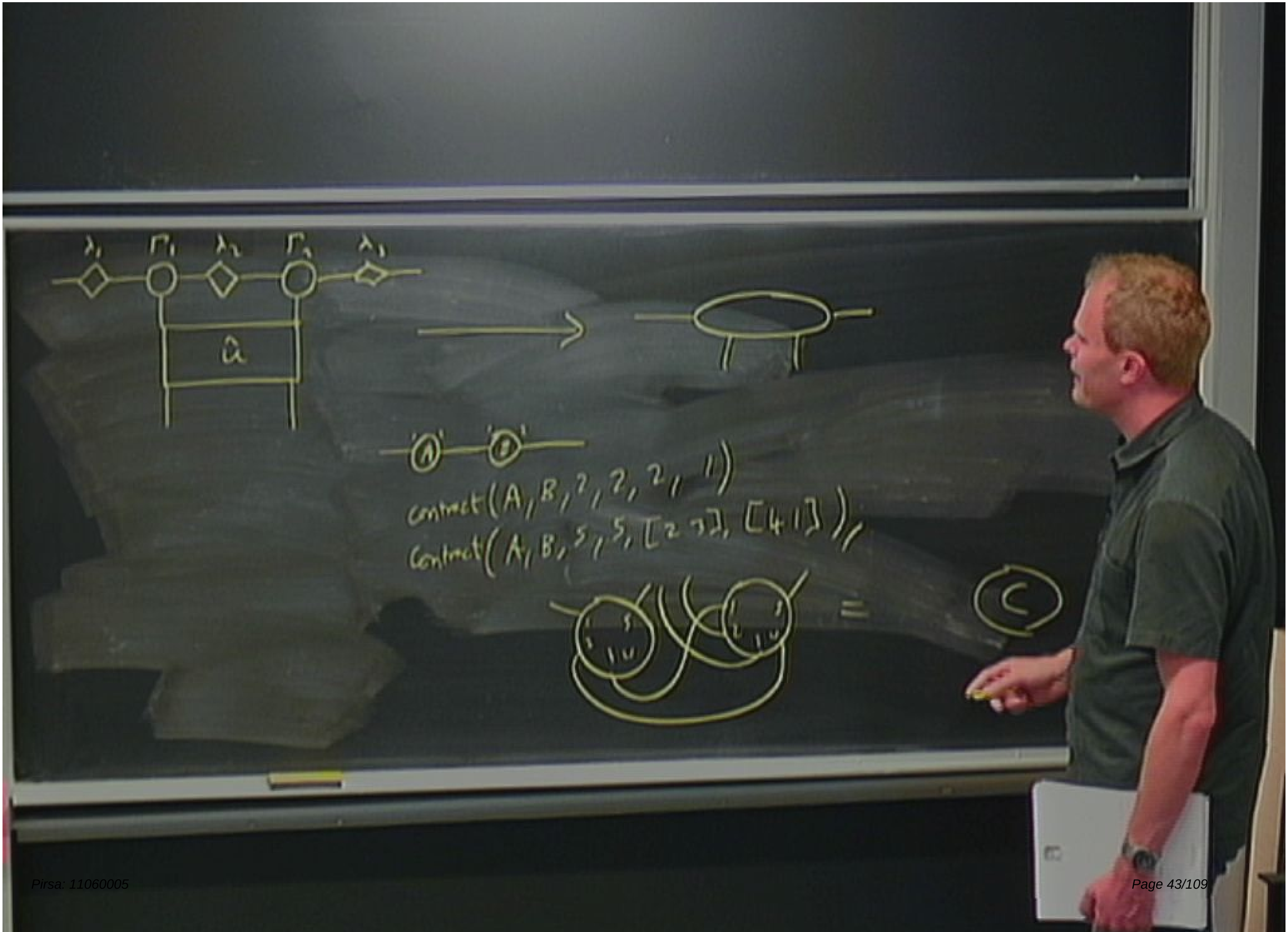


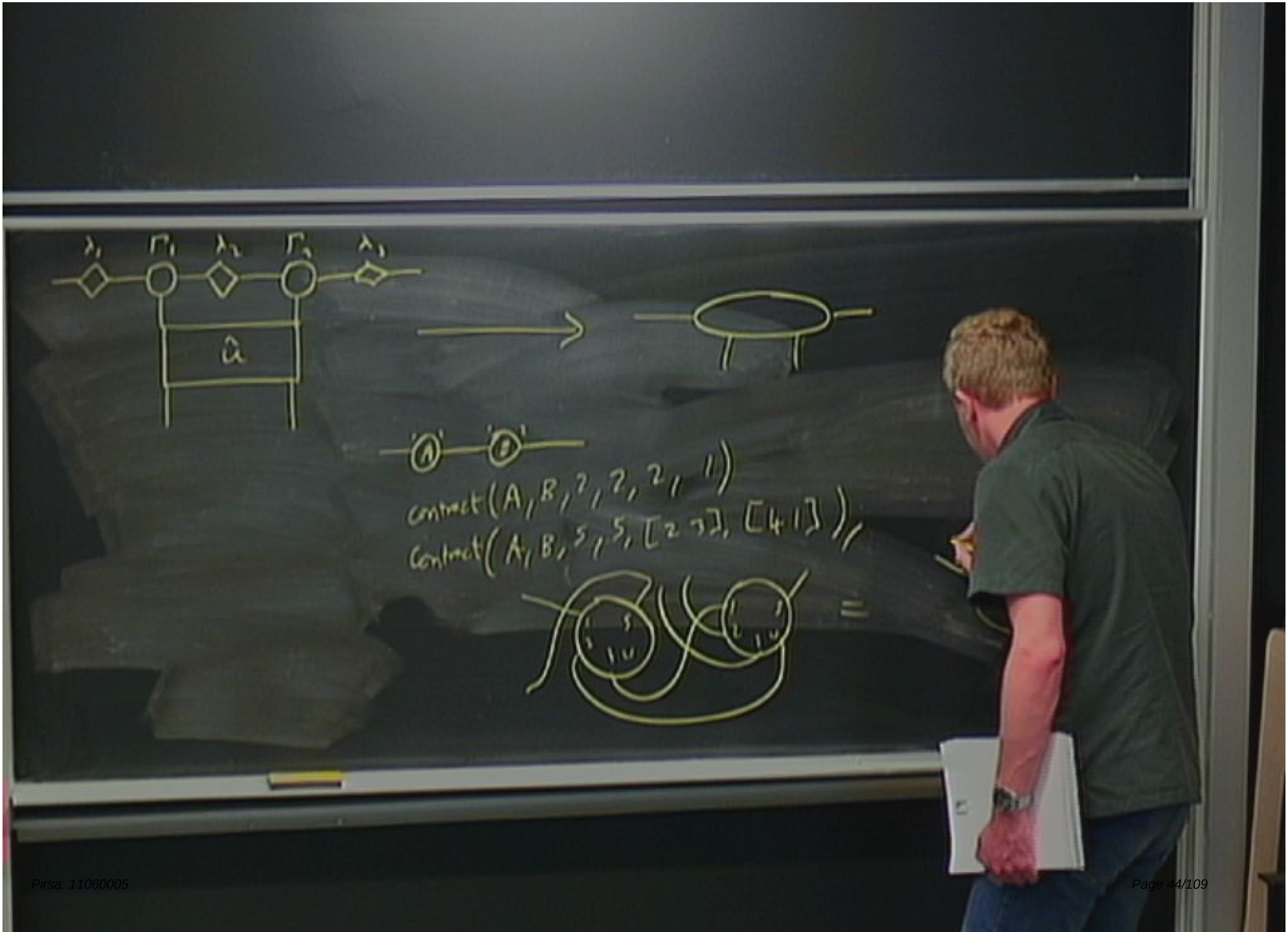

λ₁
Γ₁
λ₂
Γ₂
λ₃
û


A
B
contract(A, B, 2, 2,

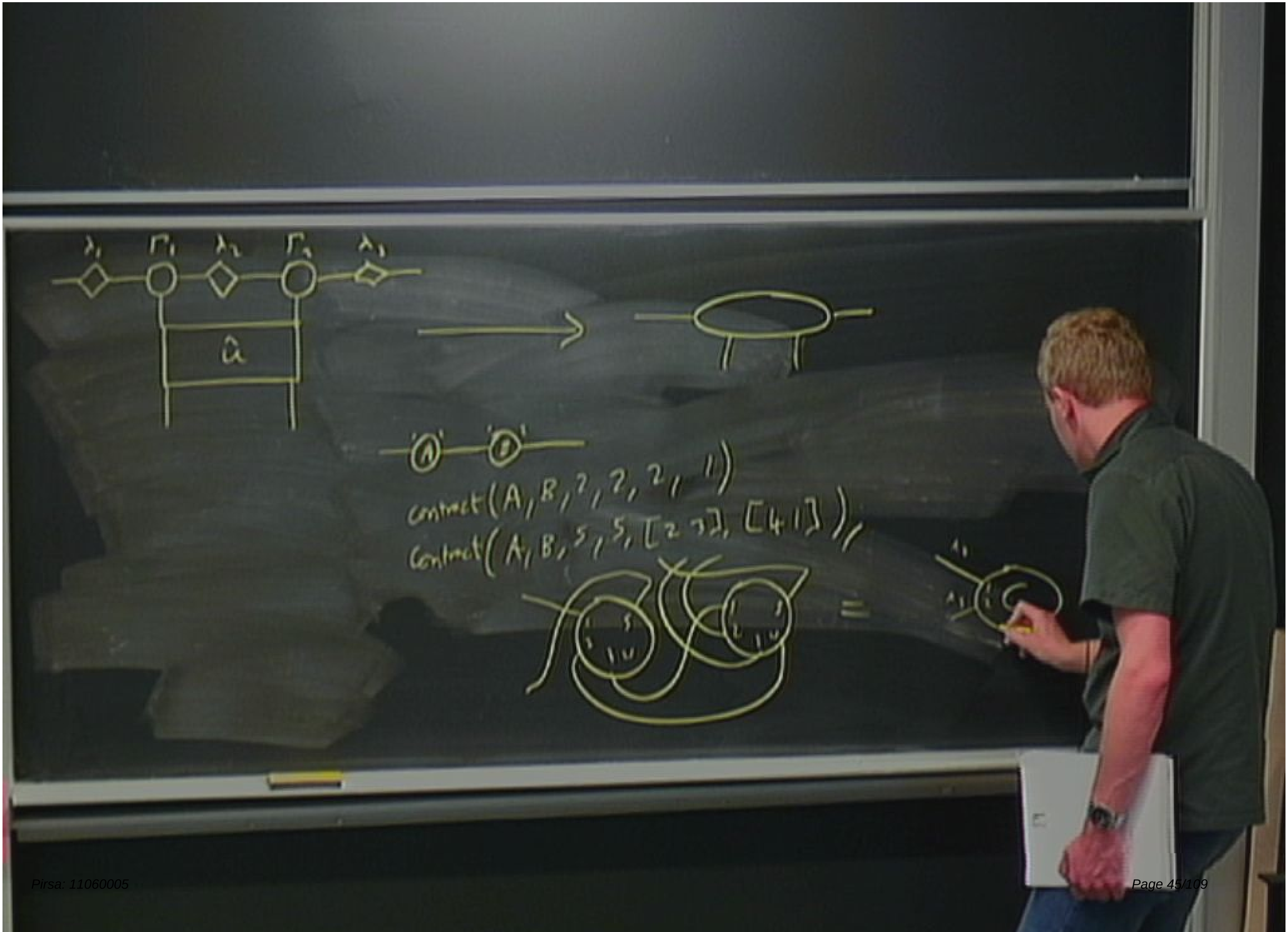

A
B
contract(A, B, 2, 2, 2, 1)
Contract(A, B, 5, 5, [2 3], [4 1])

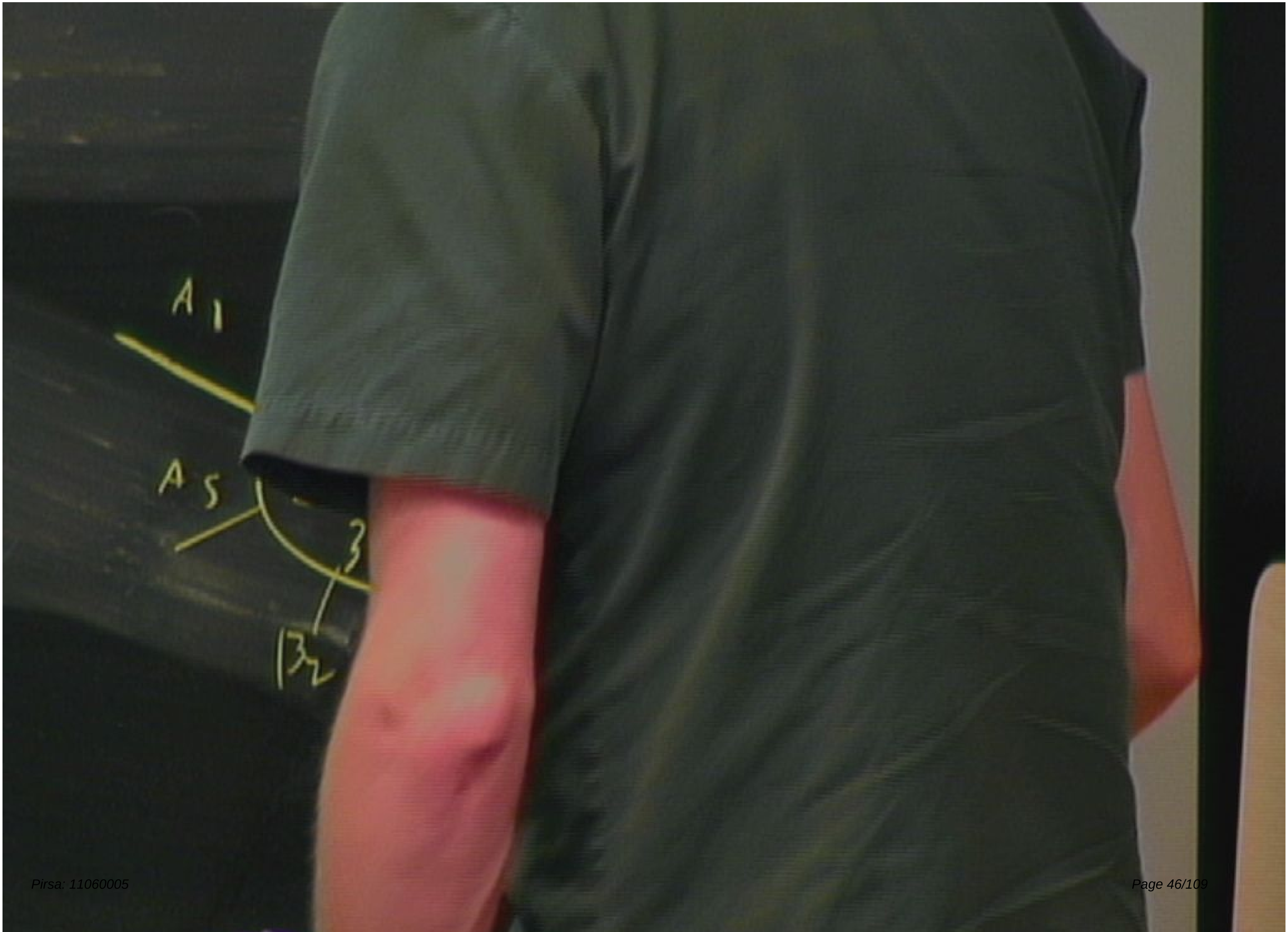

contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 3], [4 1]),

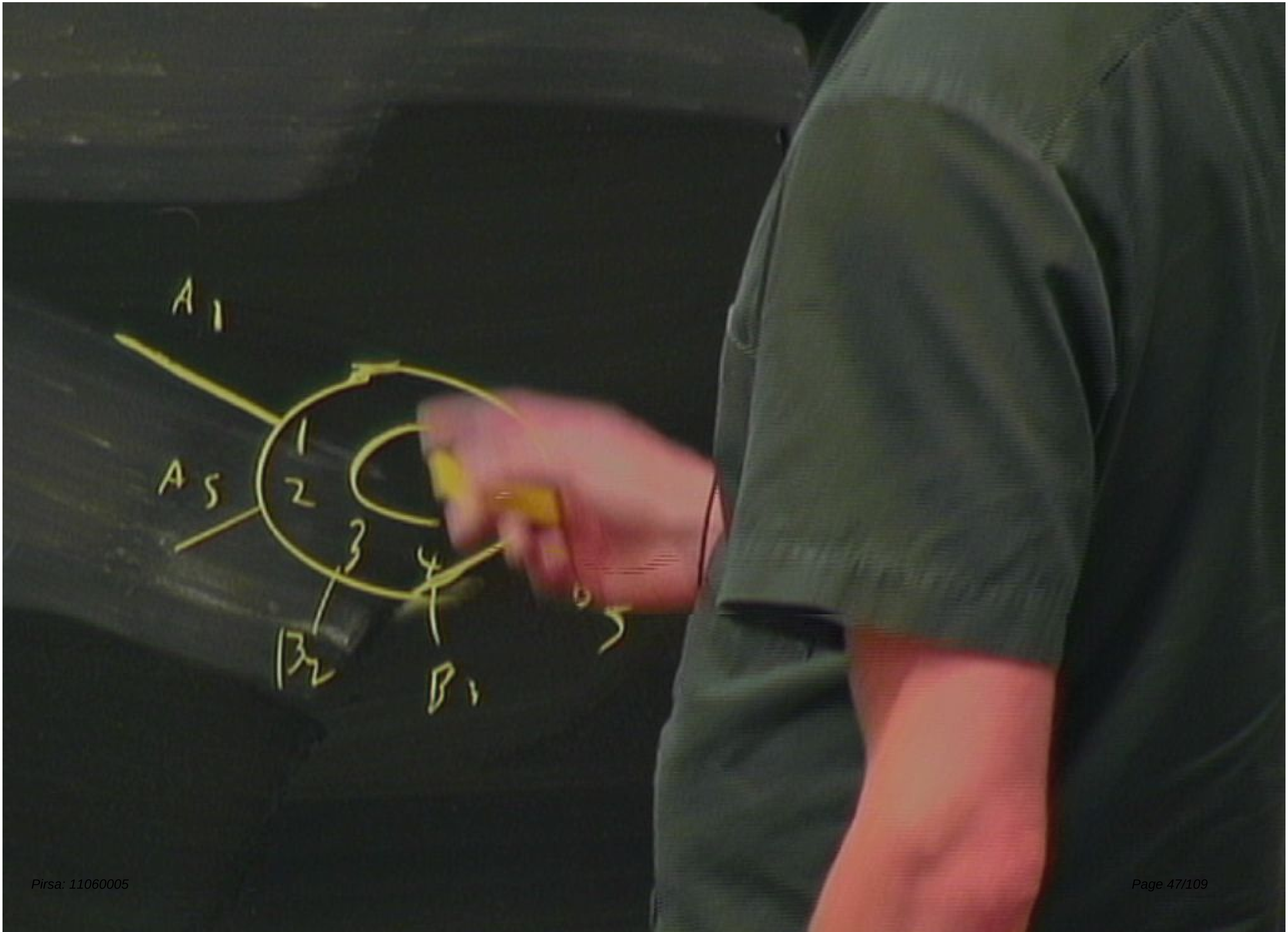

λ1 Γ1 λ2 Γ2 λ3
û
A B
contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 7], [4 1])
C

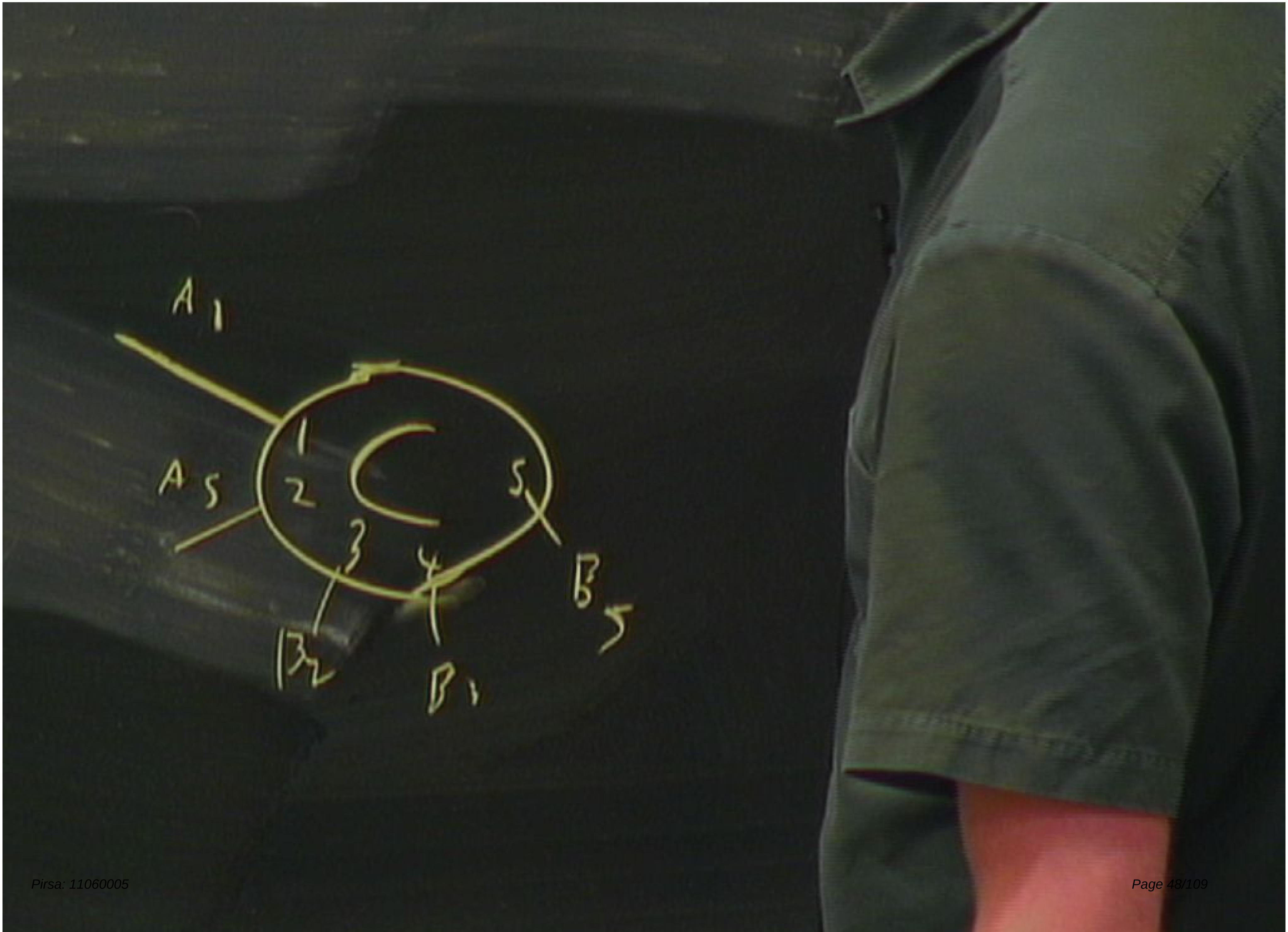


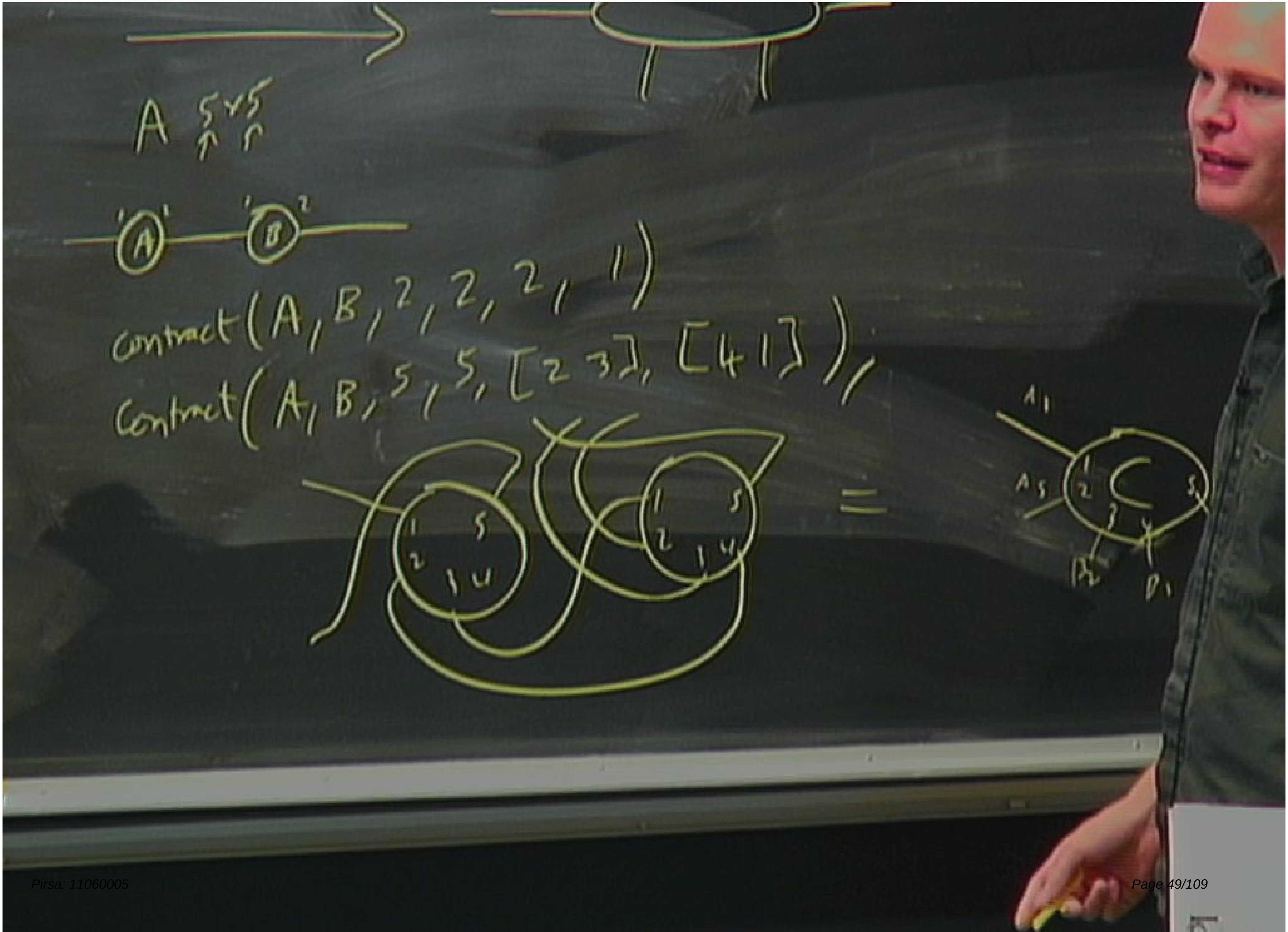
λ1 Γ1 λ2 Γ2 λ3
contract(A, B, 2, 2, 2, 1))
contract(A, B, 5, 5, [2 3], [4 1]),

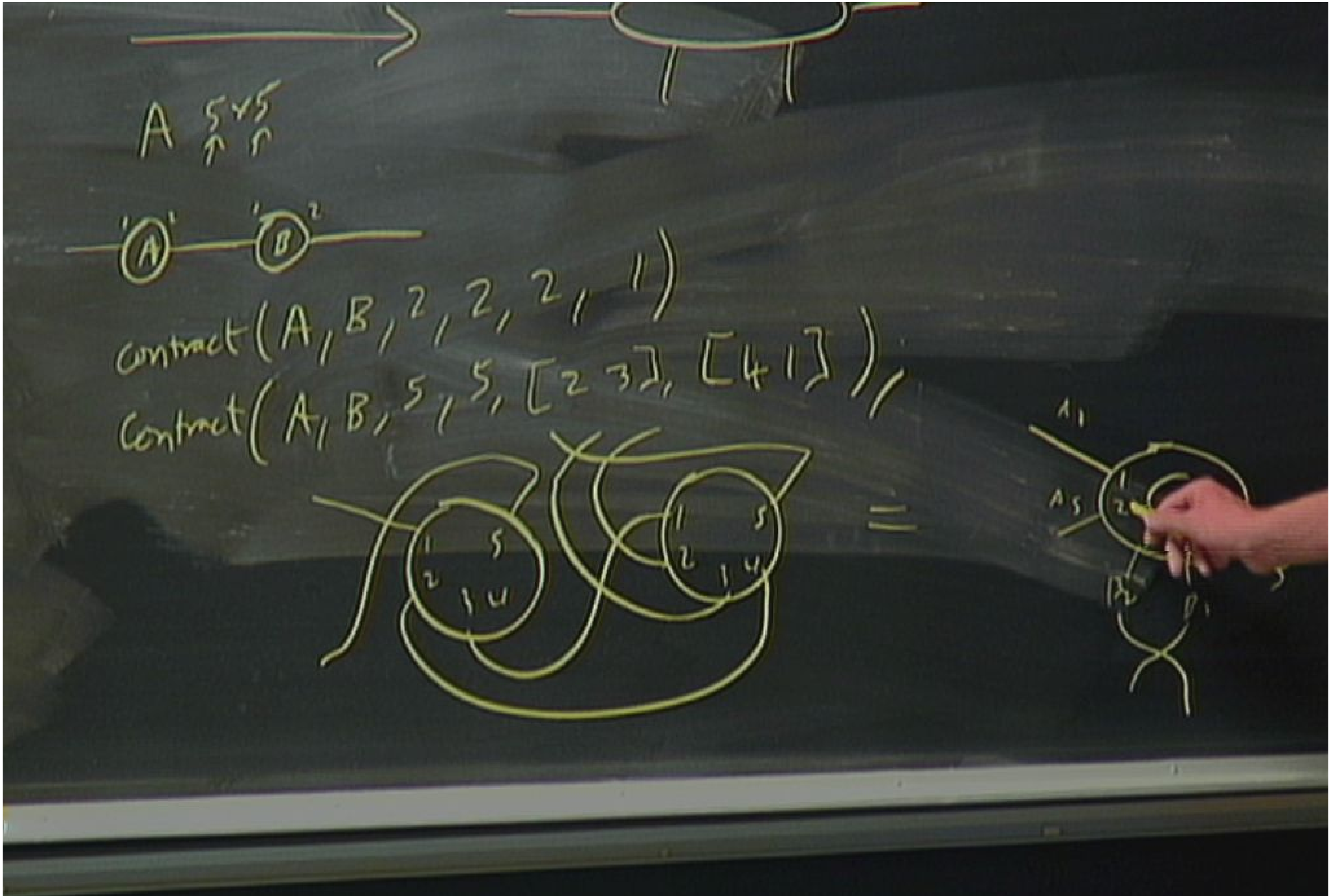

contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 3], [4 1]),

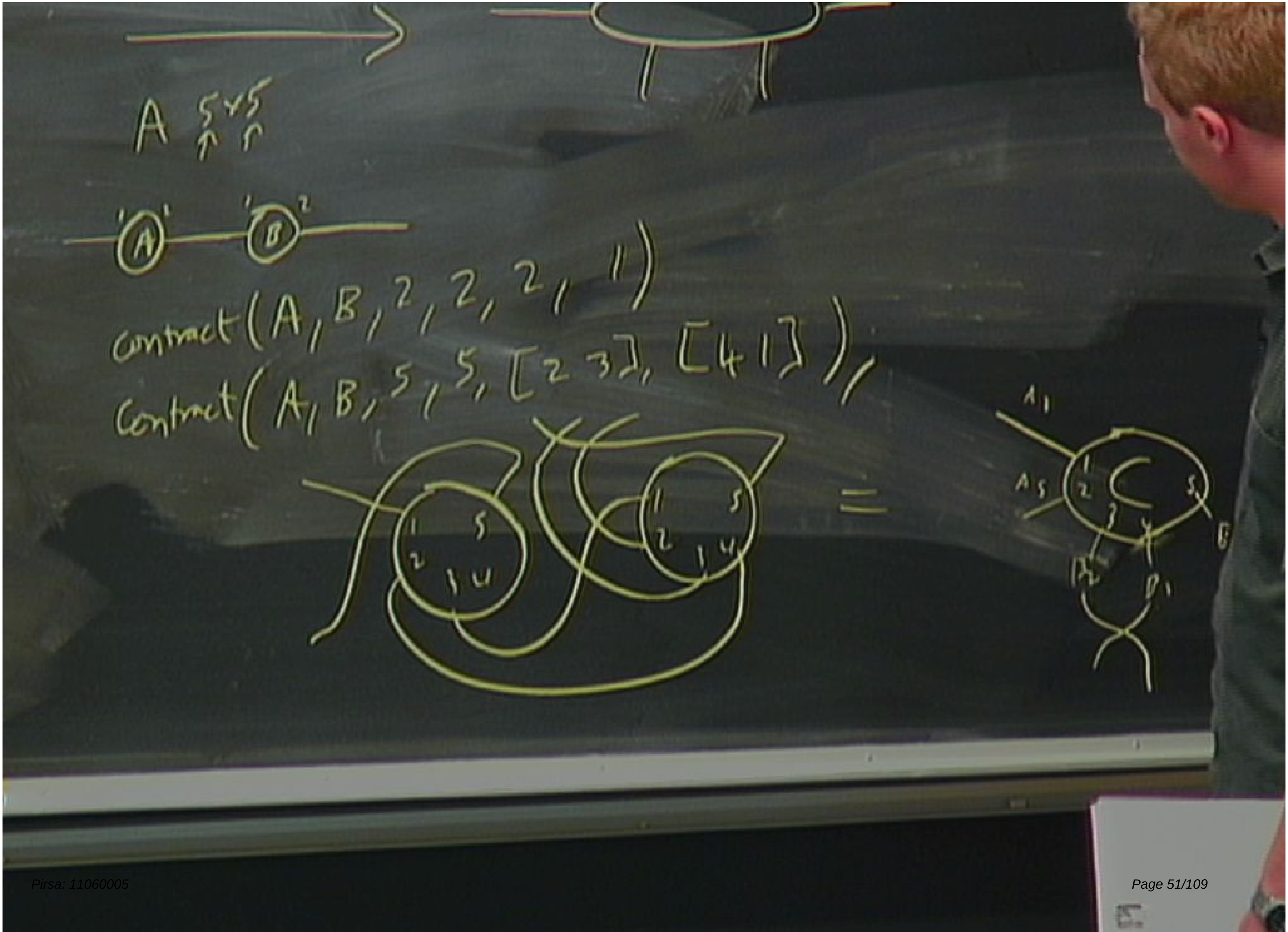

A 1
A 5
3
13/2

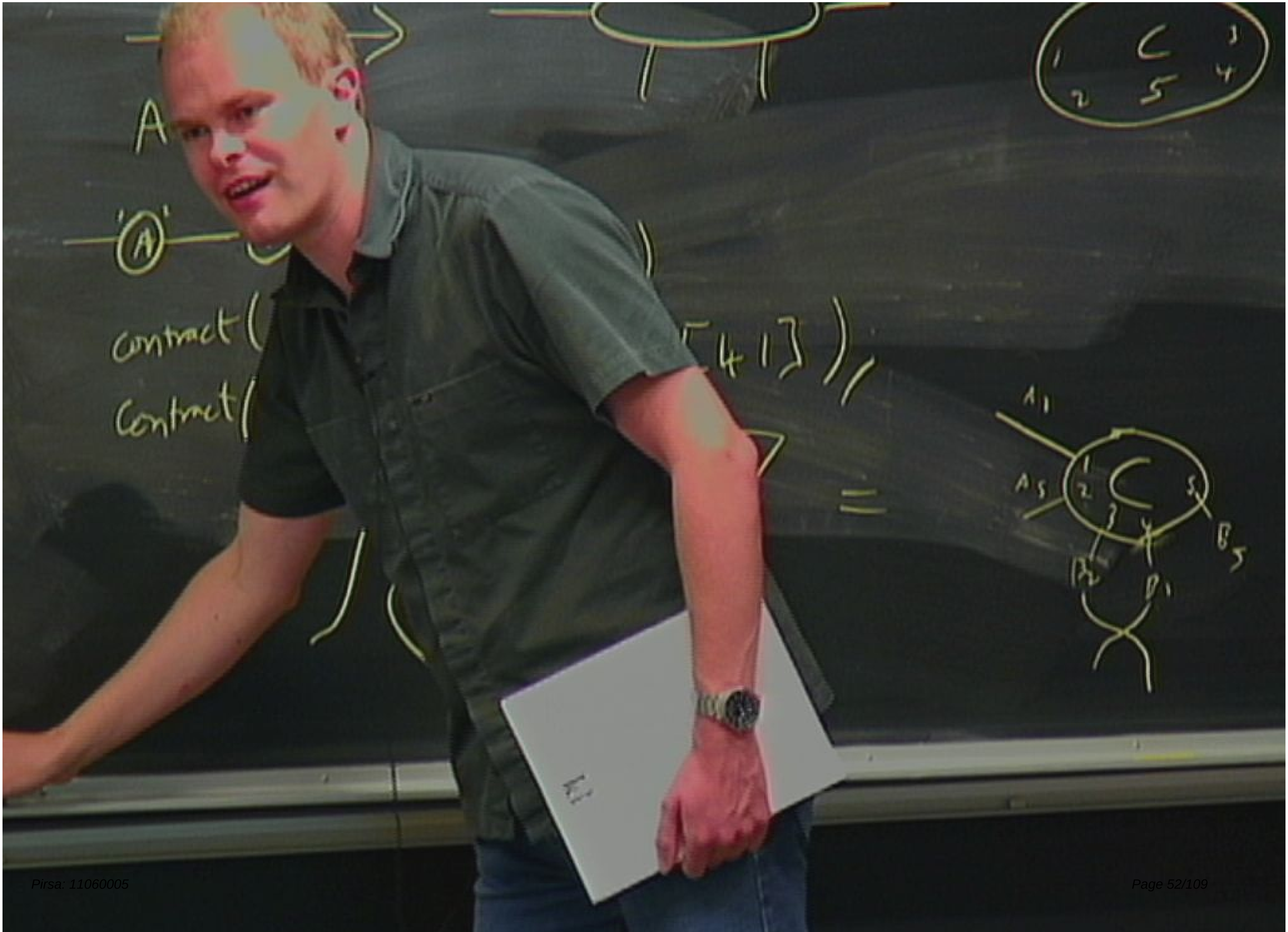

A 1
A 5
1
2
3
4
B2
B1

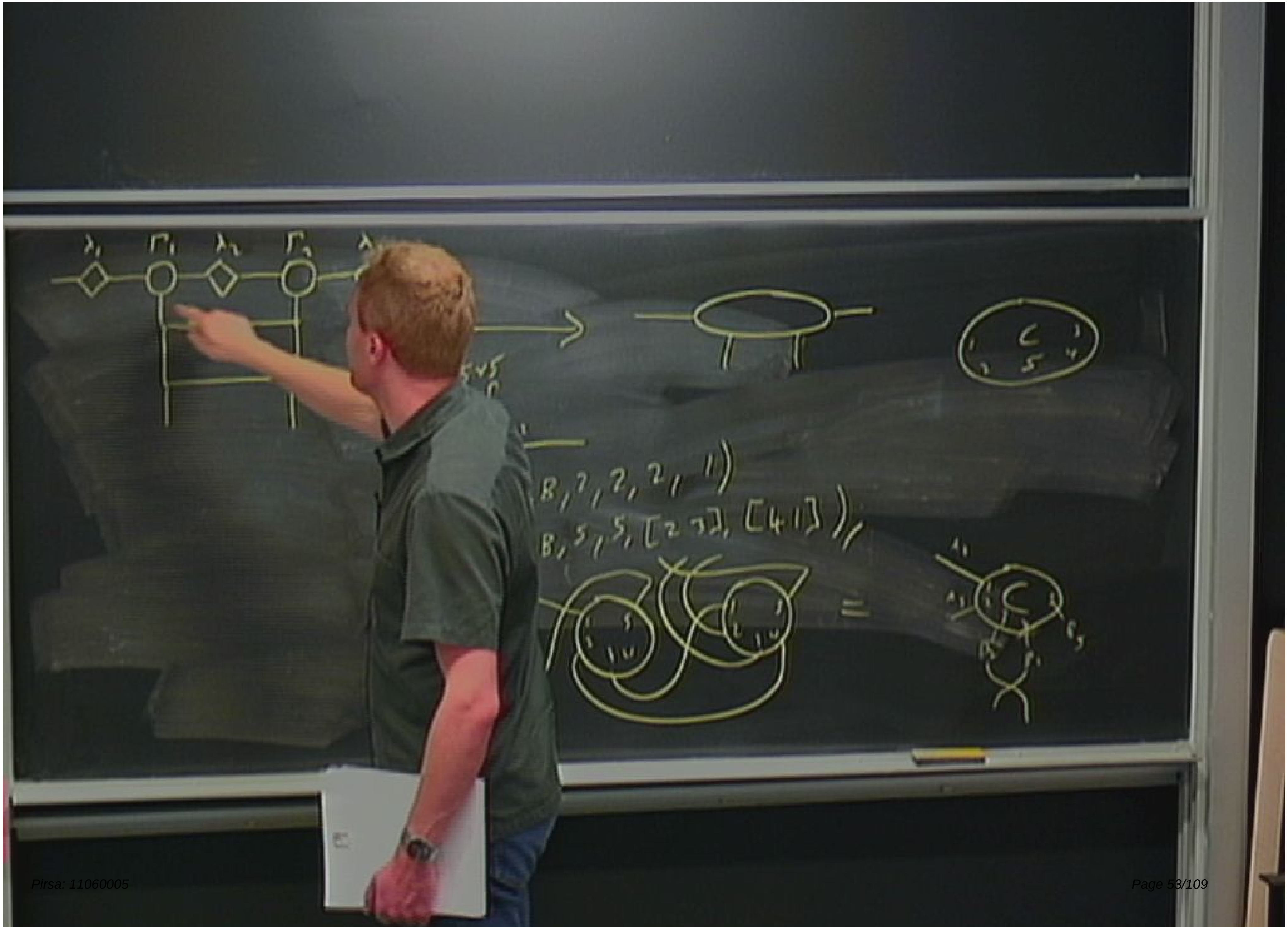

A 1
A 5
1
2
C
3
4
5
B 5
B 2
B 1

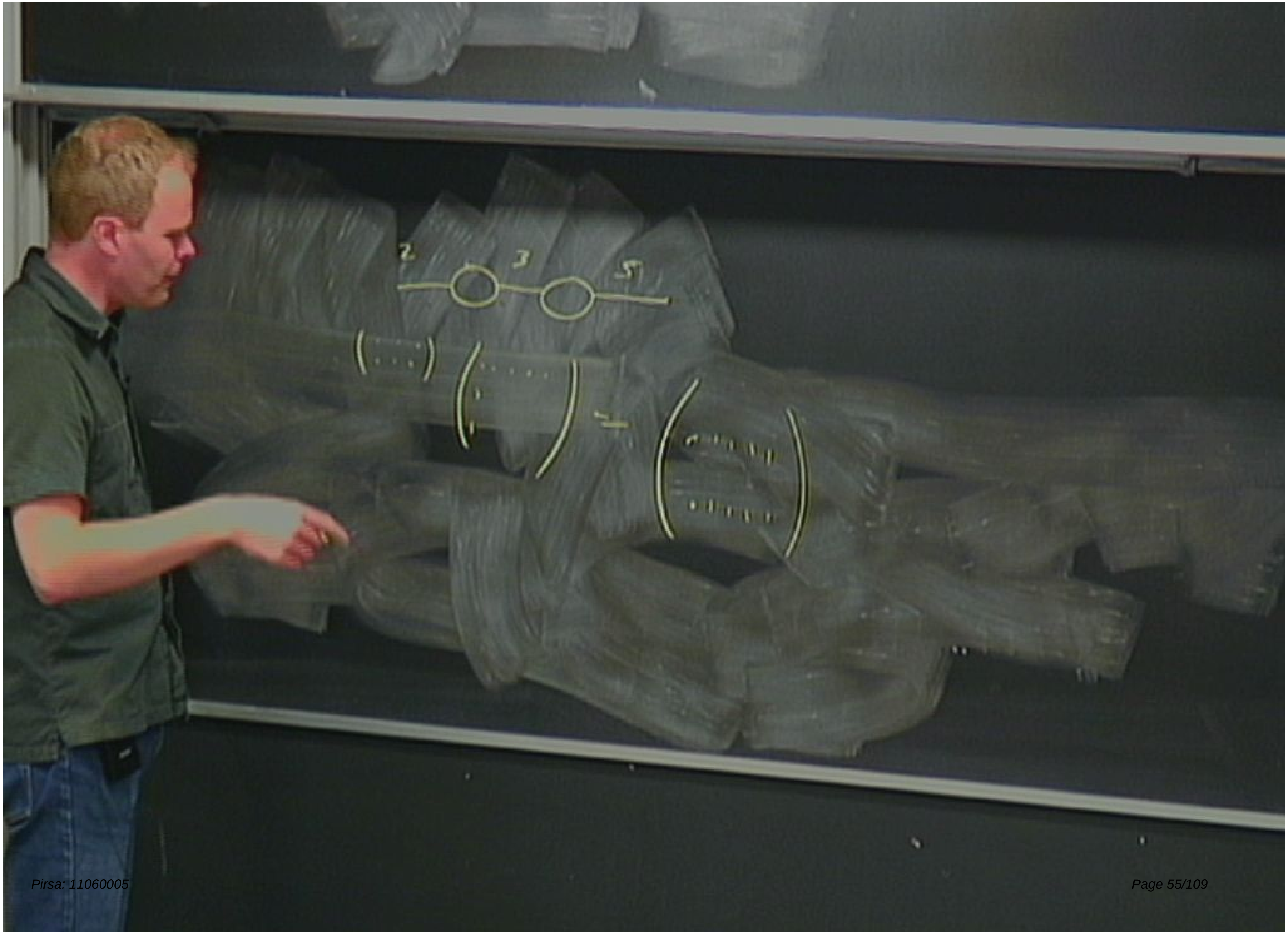

A 5×5
contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 3], [4 1]),

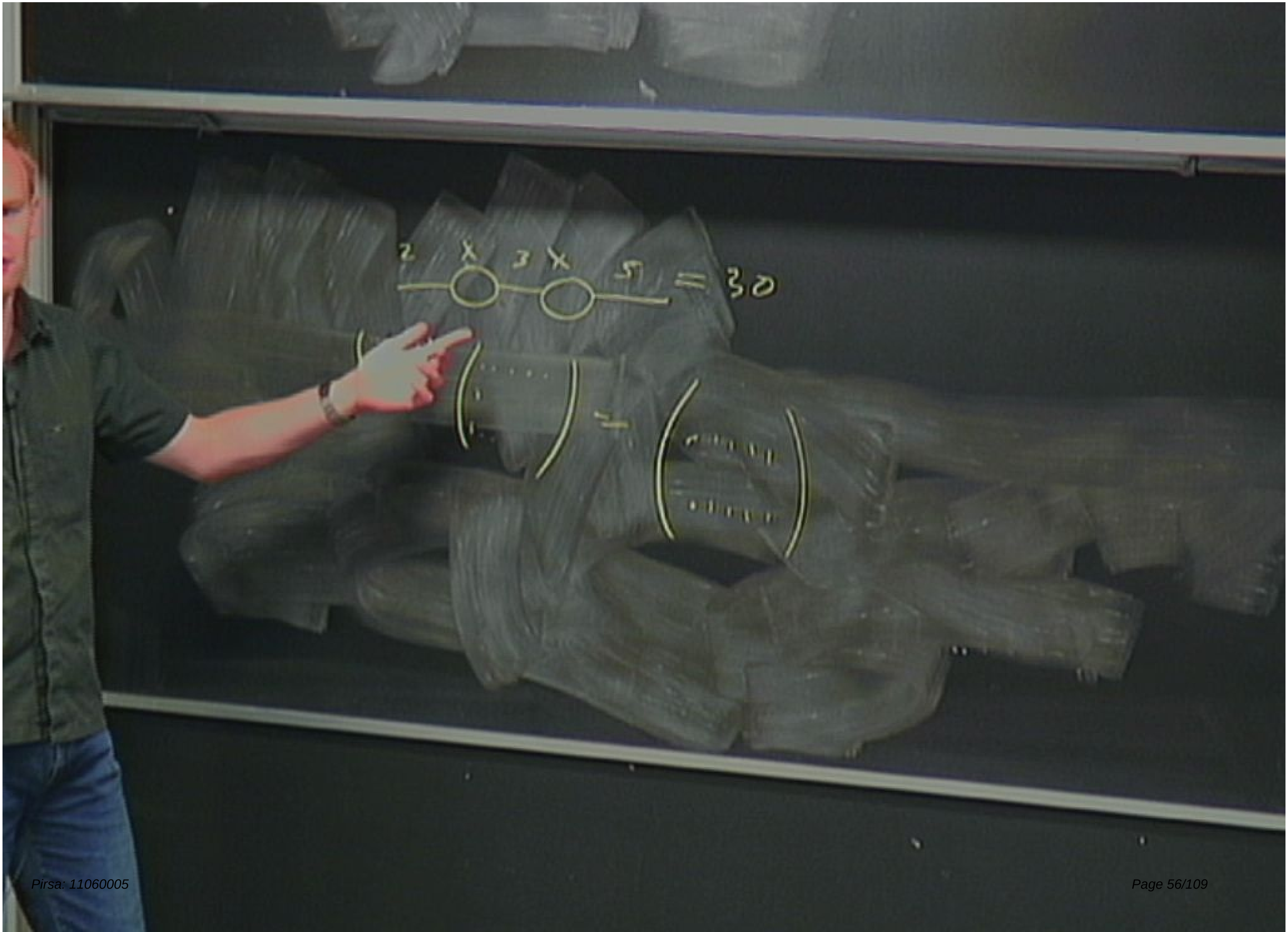

A 5×5
contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 3], [4 1]),

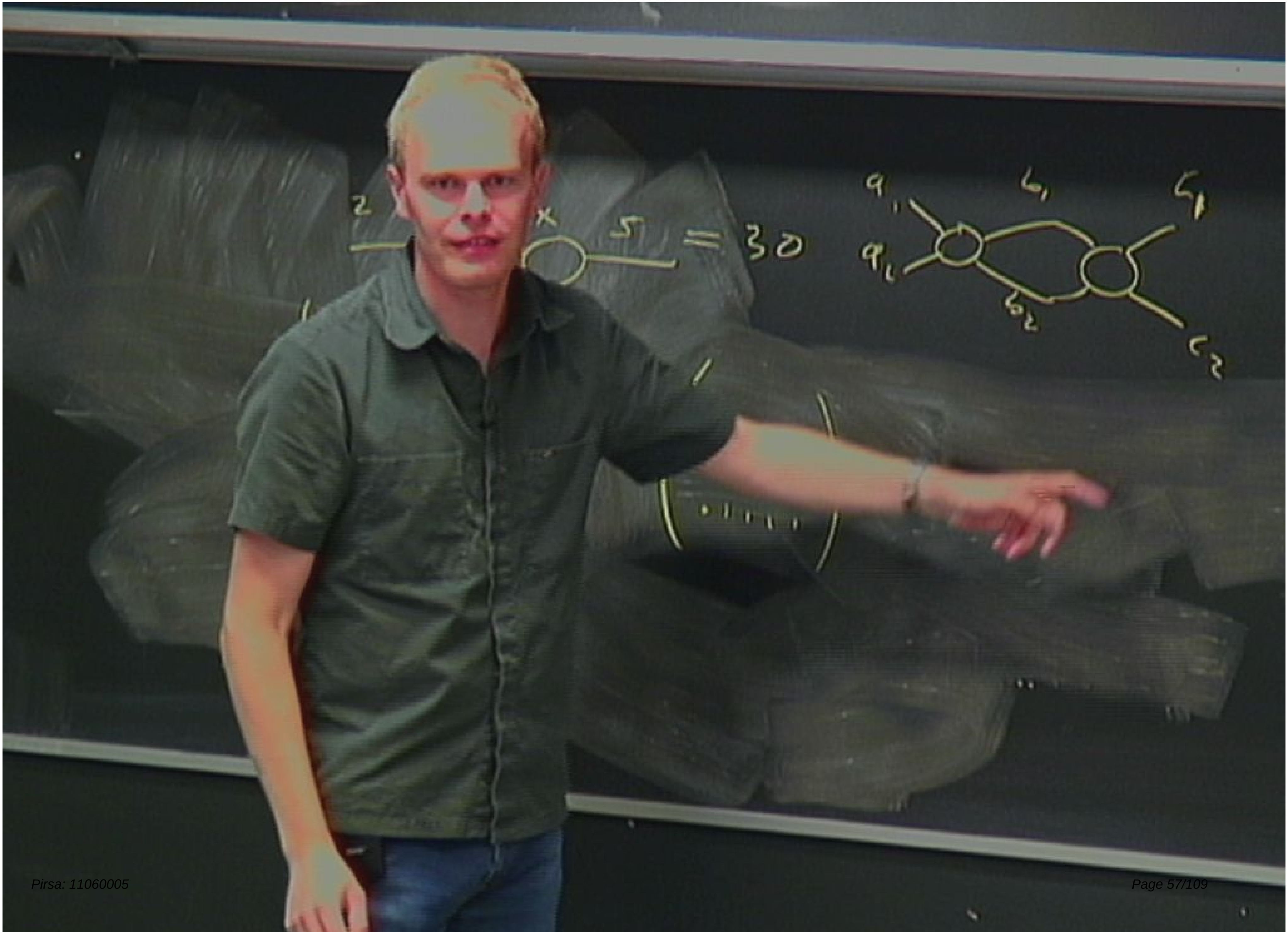

A 5×5
contract(A, B, 2, 2, 2, 1)
Contract(A, B, 5, 5, [2 3], [4 1]),
=

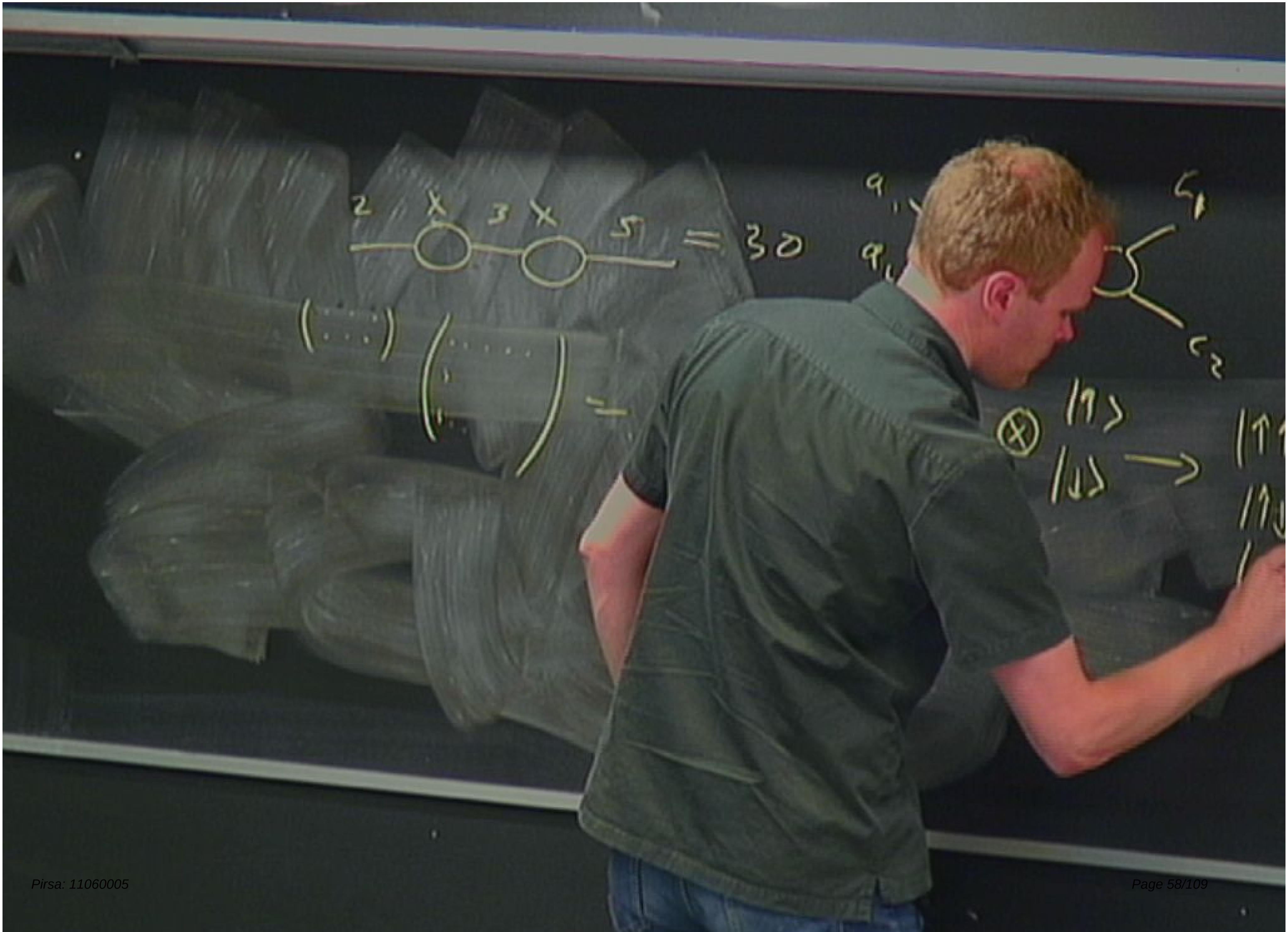

A
contract(
Contract(
[4 1]),
=
A1
A5
B5
B1
B2

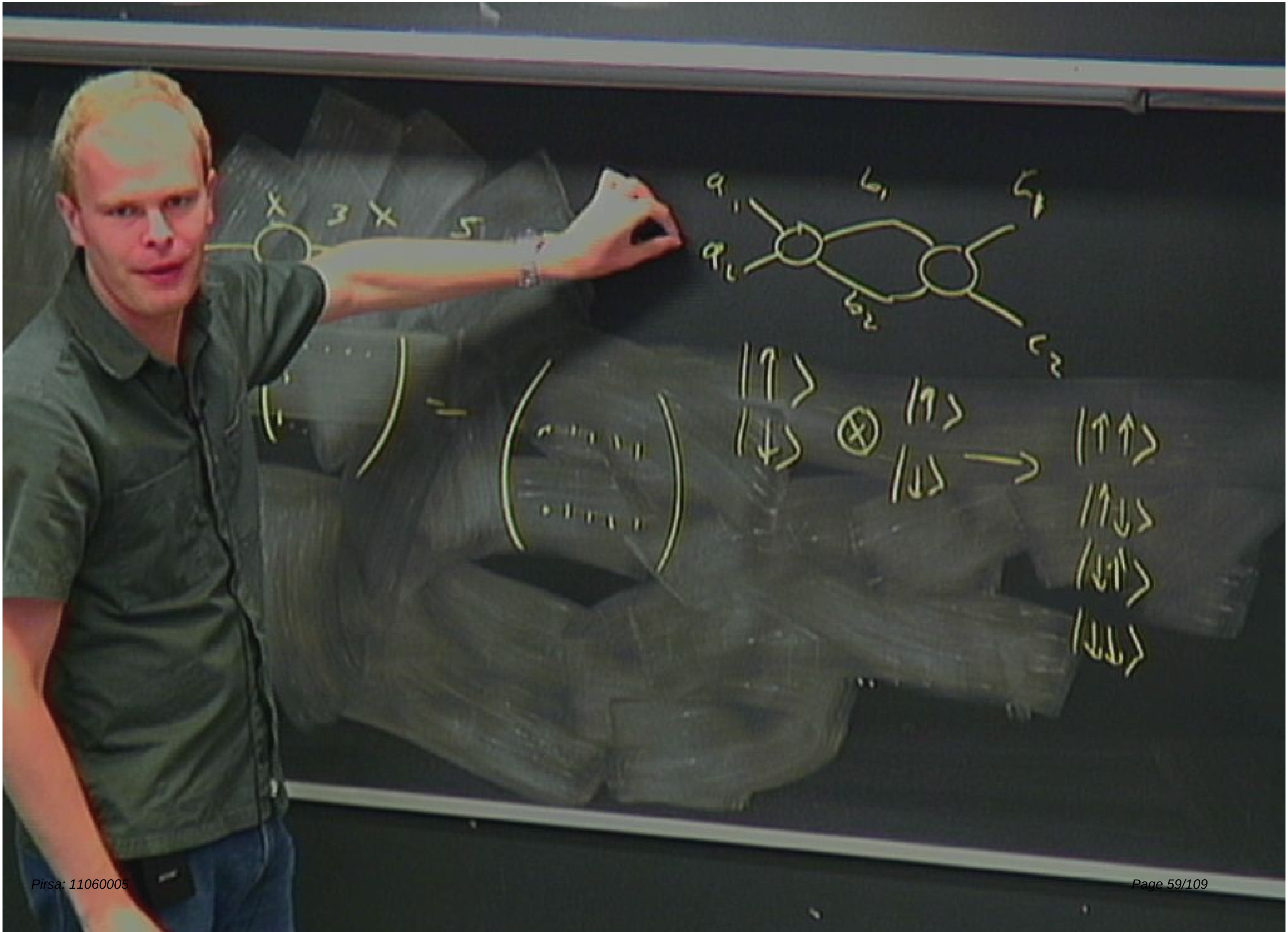


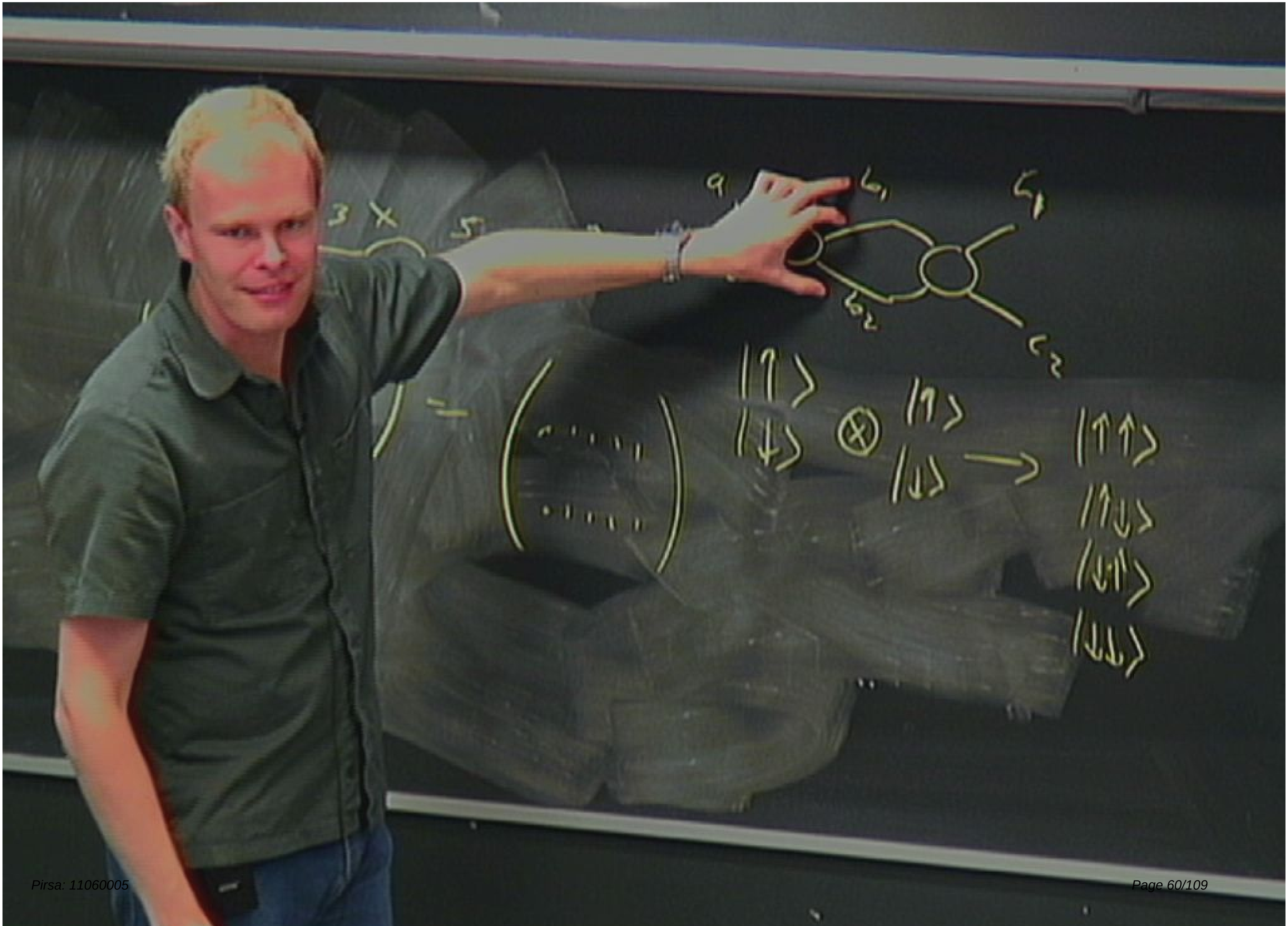


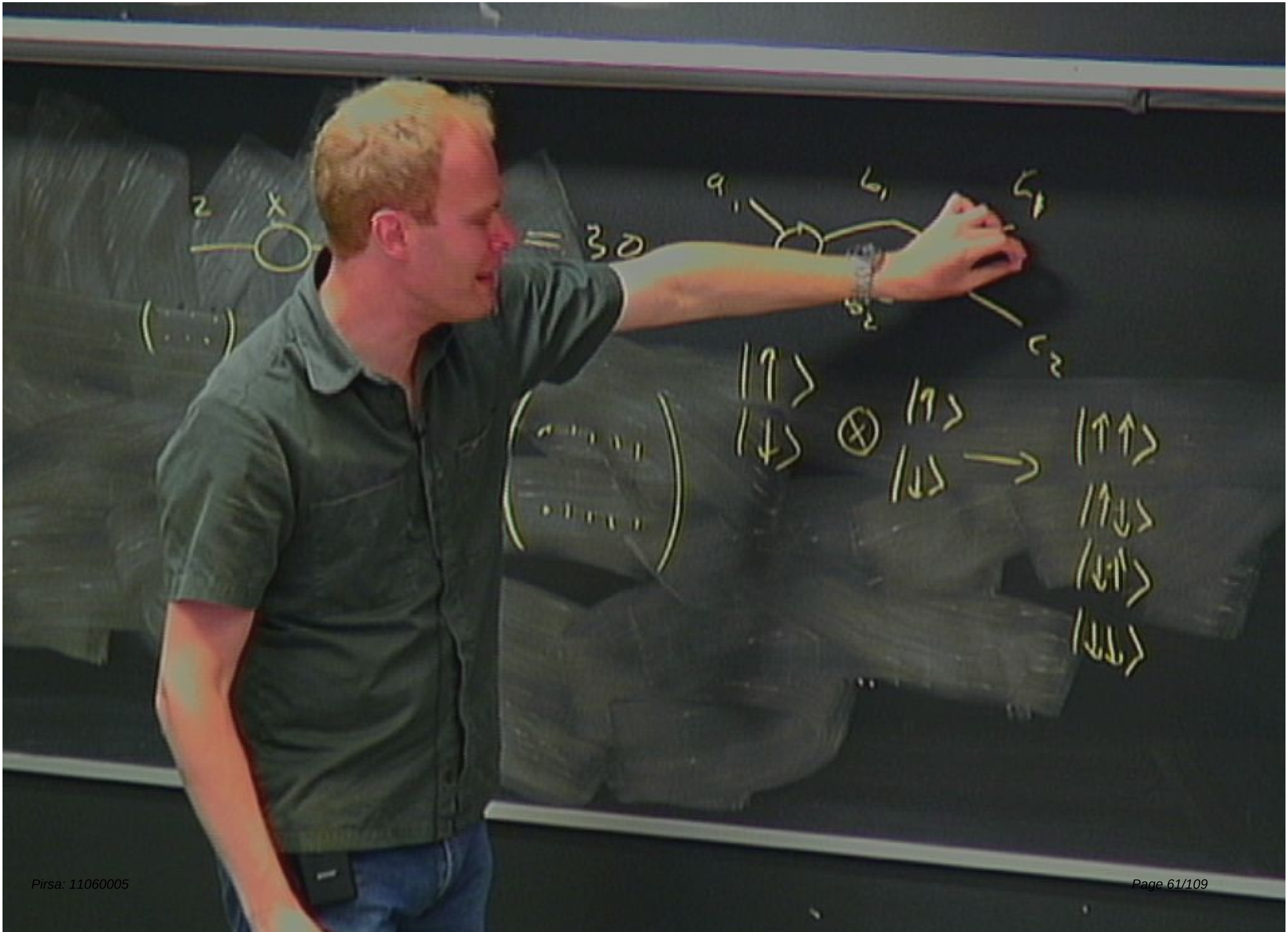

2 X 3 X 5 = 30

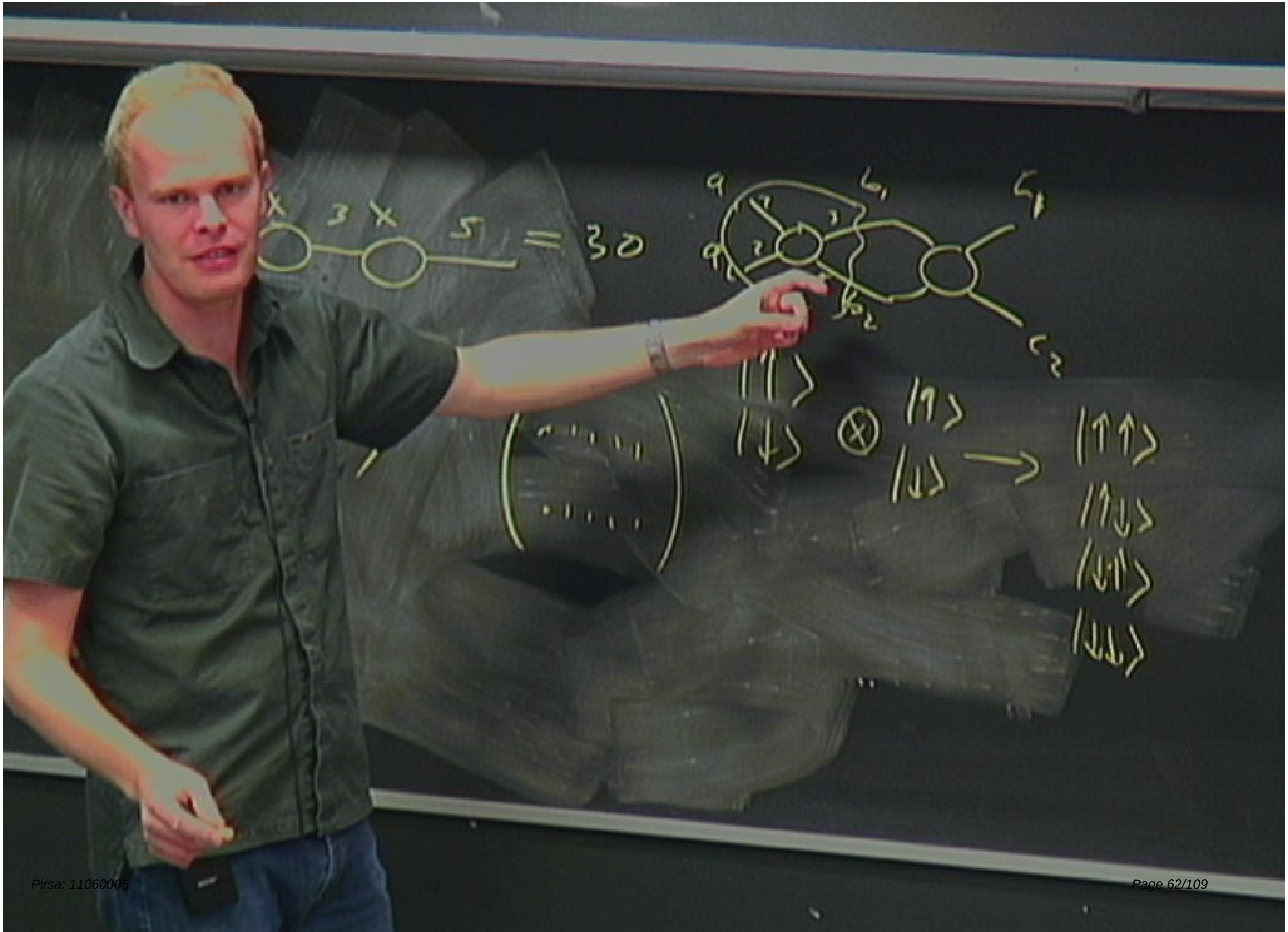

= 30


2 X 3 X 5 = 30
|↑⟩
|↓⟩

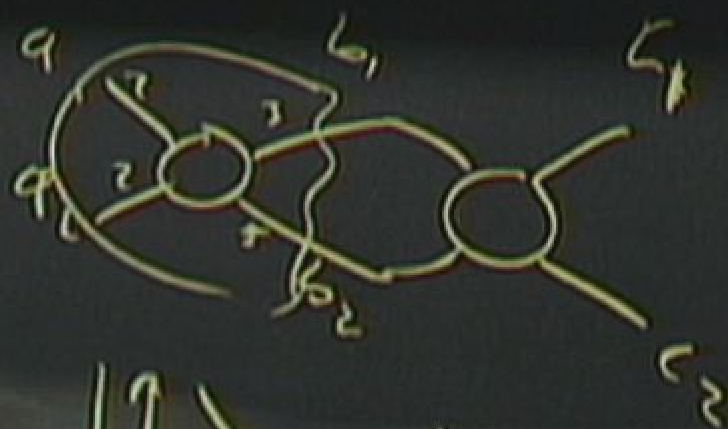




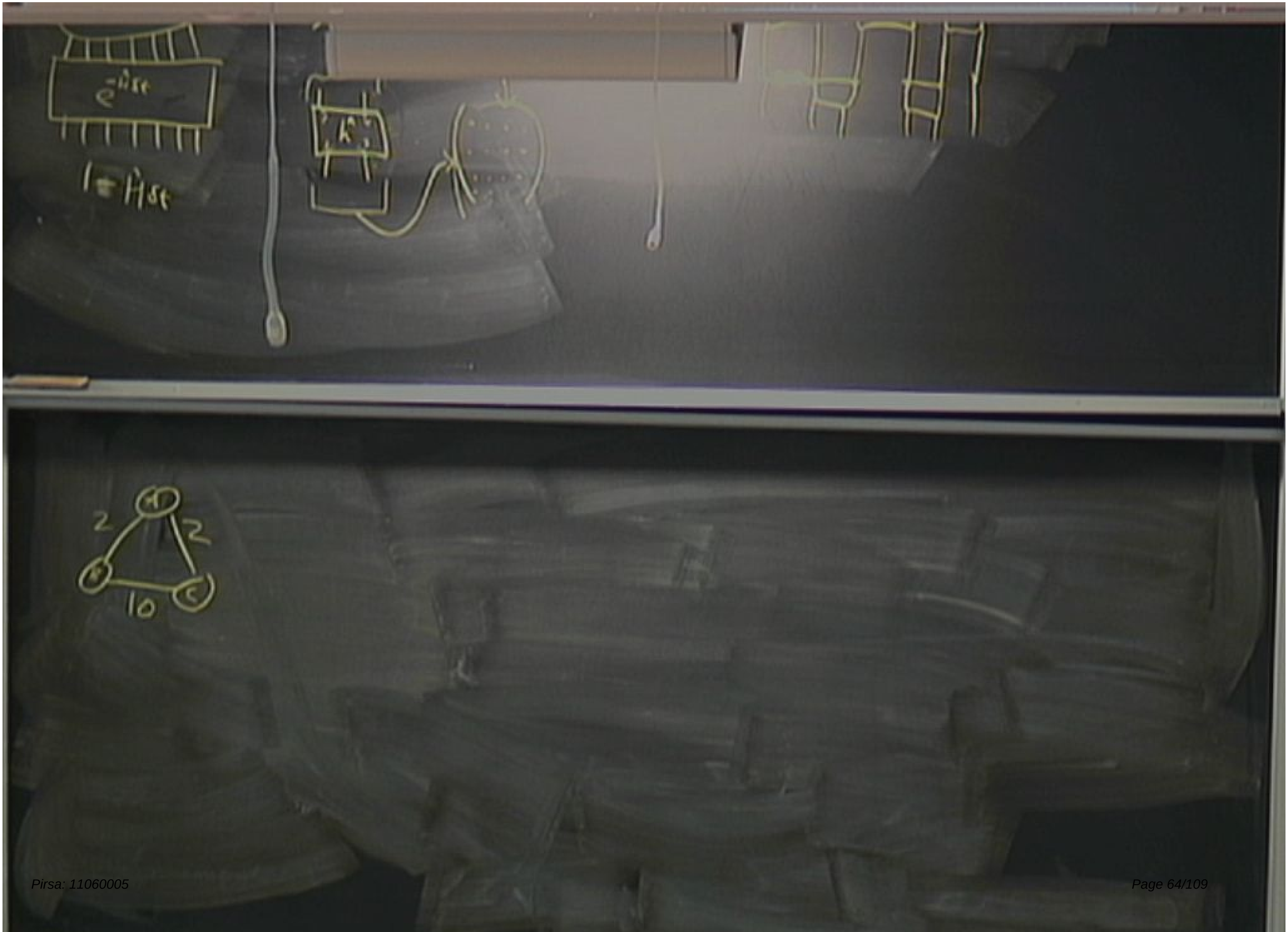


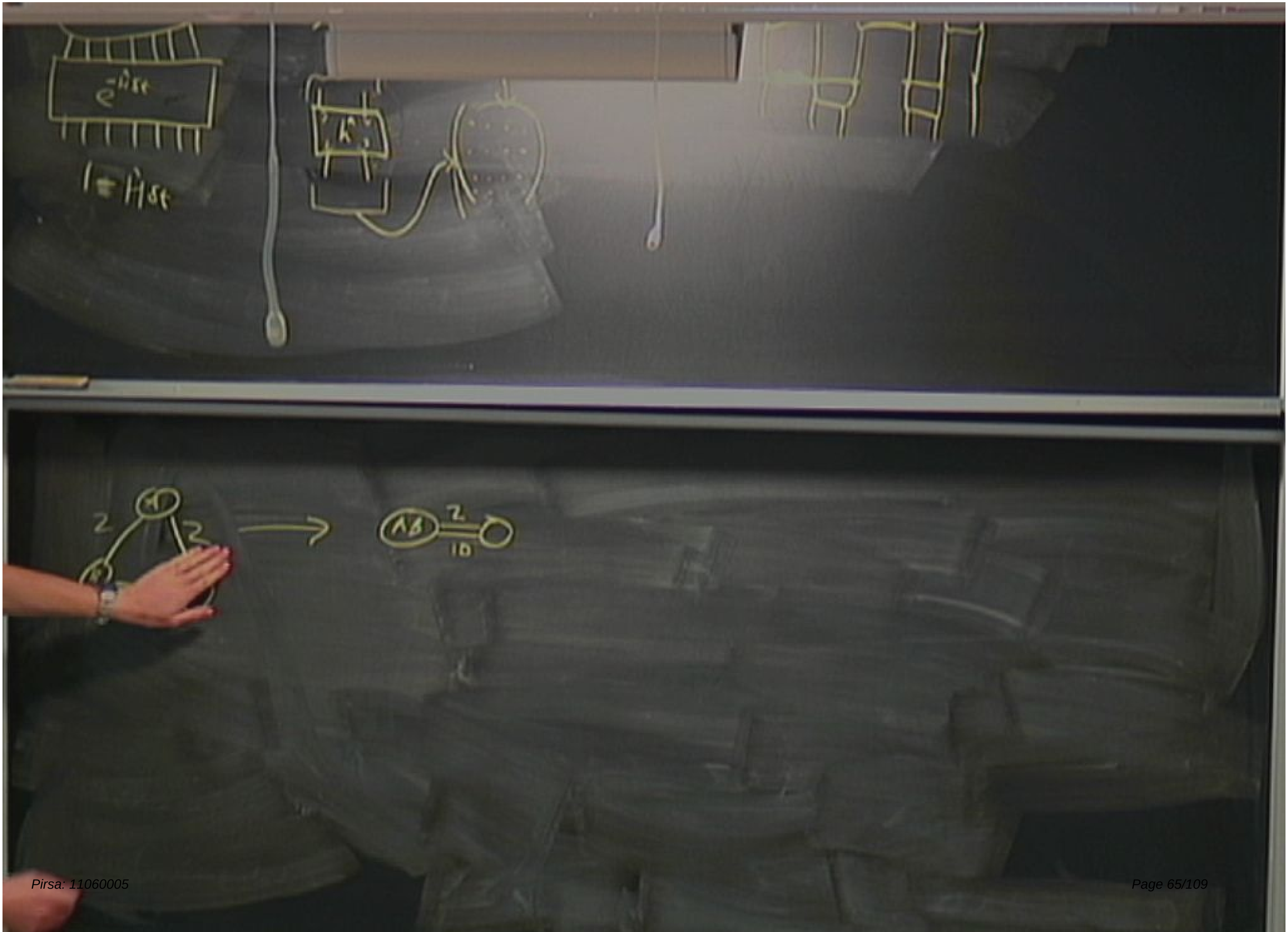

3 x 5 = 30
|↑↑⟩
|↑↓⟩
|↓↑⟩
|↓↓⟩

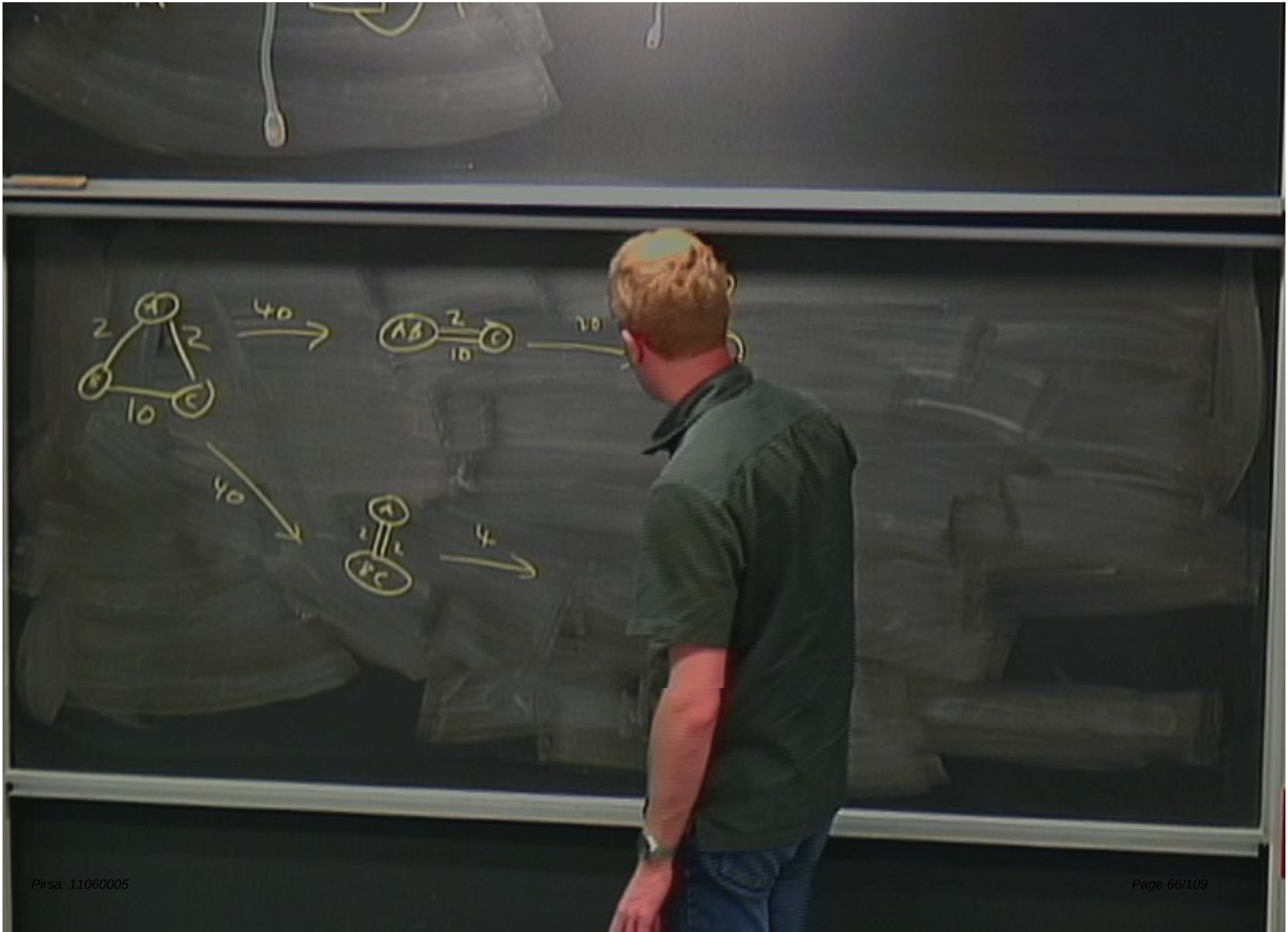
$S = 30$

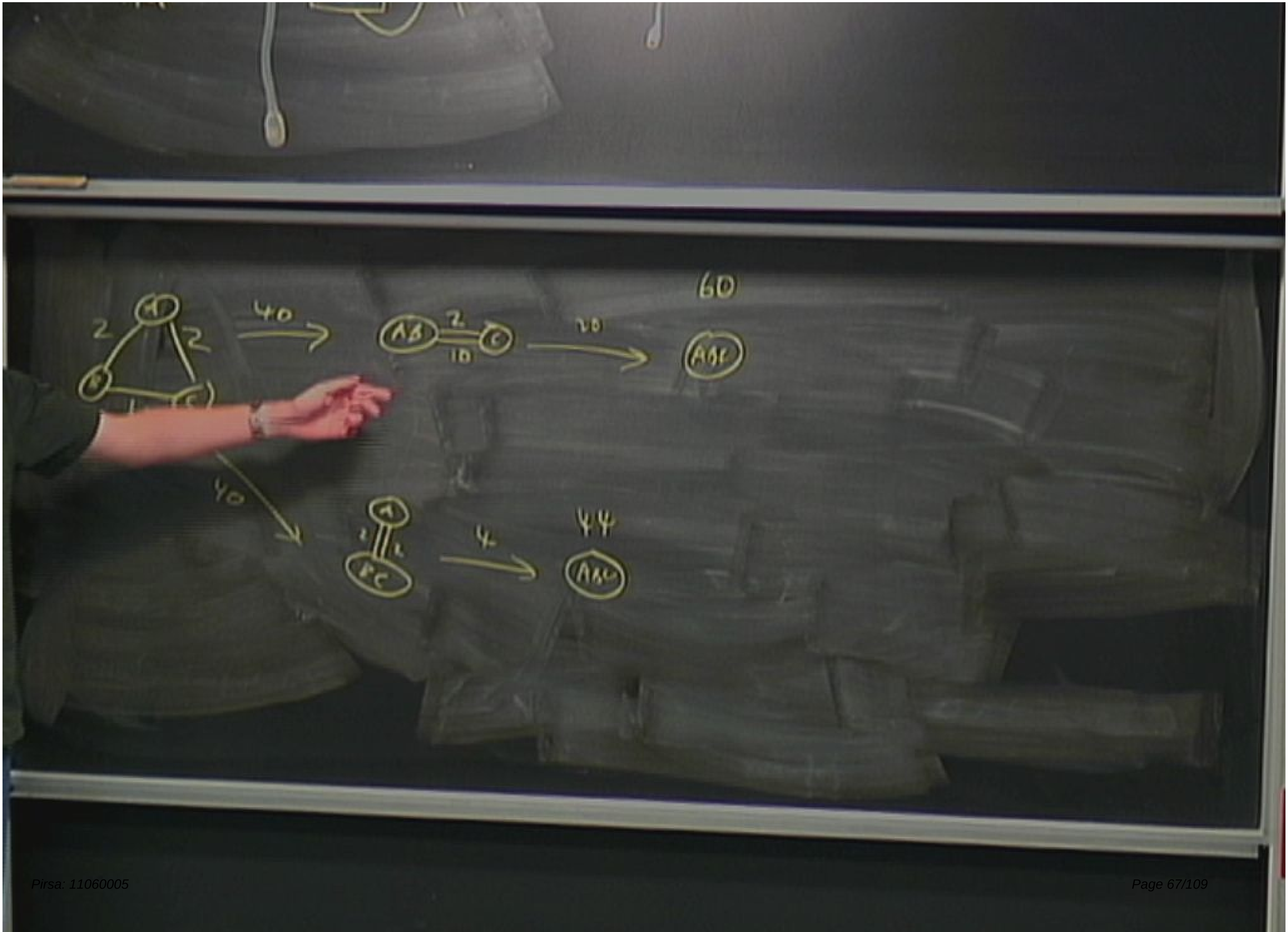


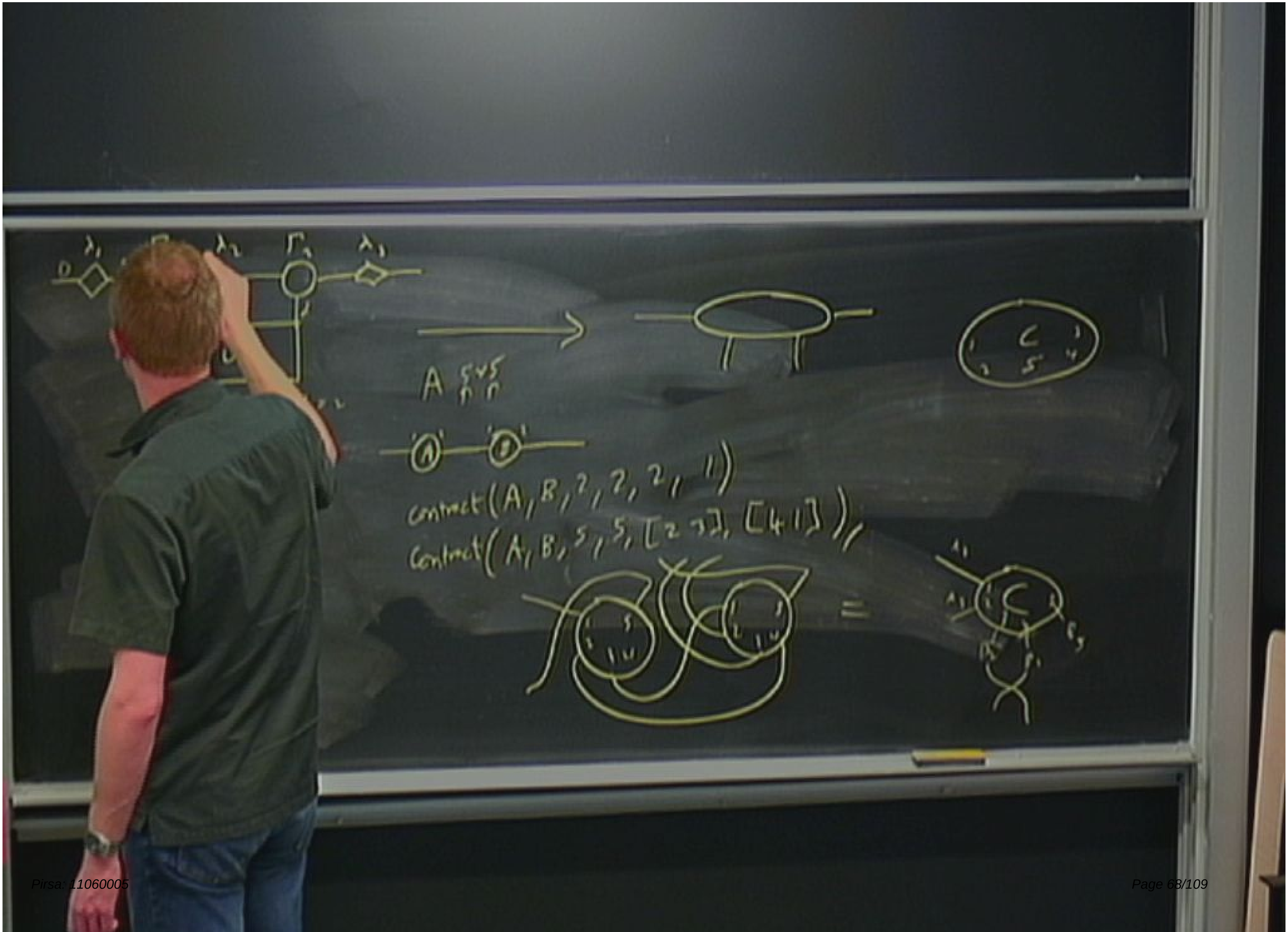
$$\begin{pmatrix} \cdots \\ \cdots \end{pmatrix} \quad \begin{matrix} |\uparrow\rangle \\ |\downarrow\rangle \end{matrix} \otimes \begin{matrix} |\uparrow\rangle \\ |\downarrow\rangle \end{matrix} \longrightarrow \begin{matrix} |\uparrow\uparrow\rangle \\ |\uparrow\downarrow\rangle \\ |\downarrow\uparrow\rangle \\ |\downarrow\downarrow\rangle \end{matrix}$$

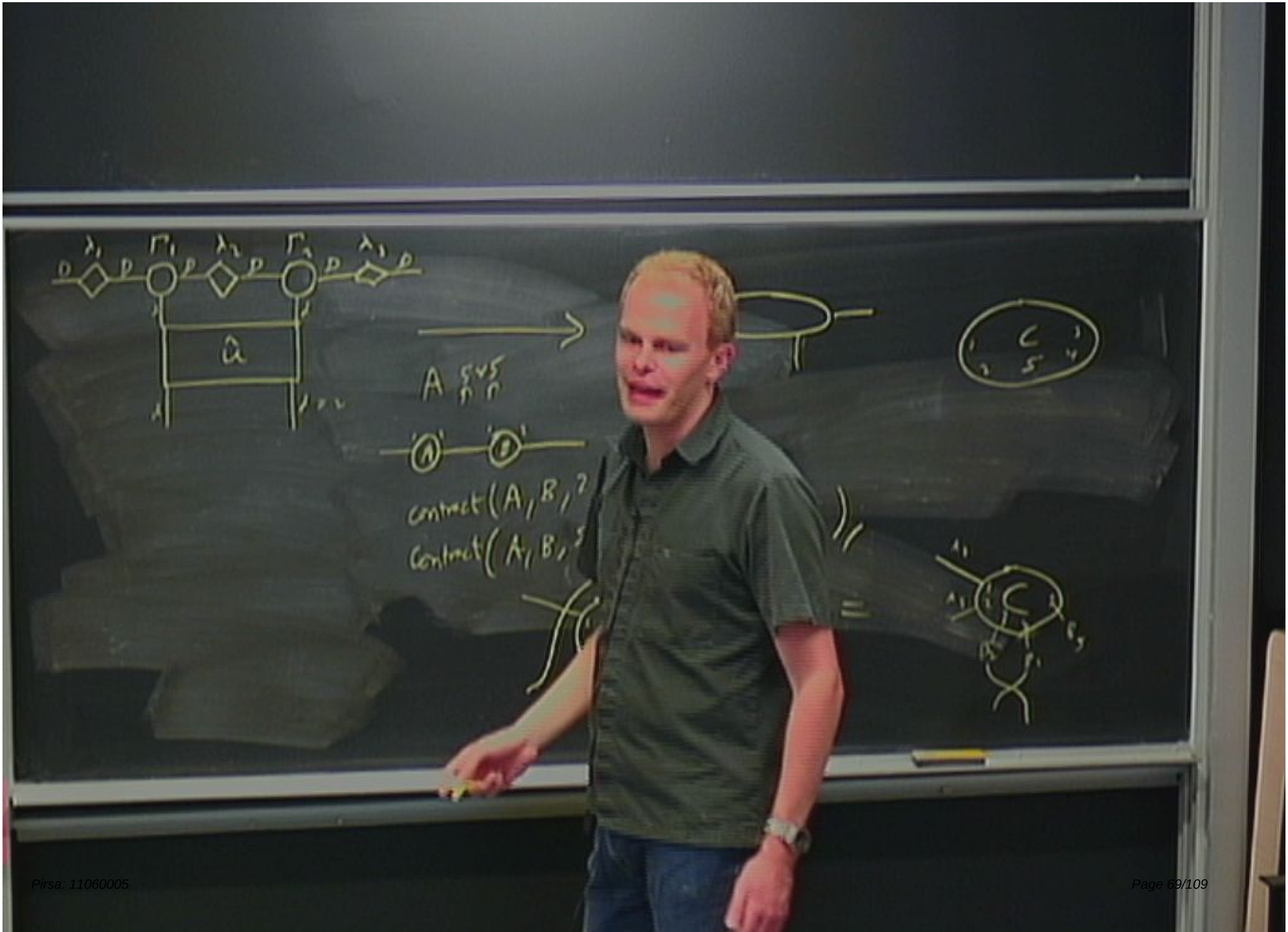

2
2
10

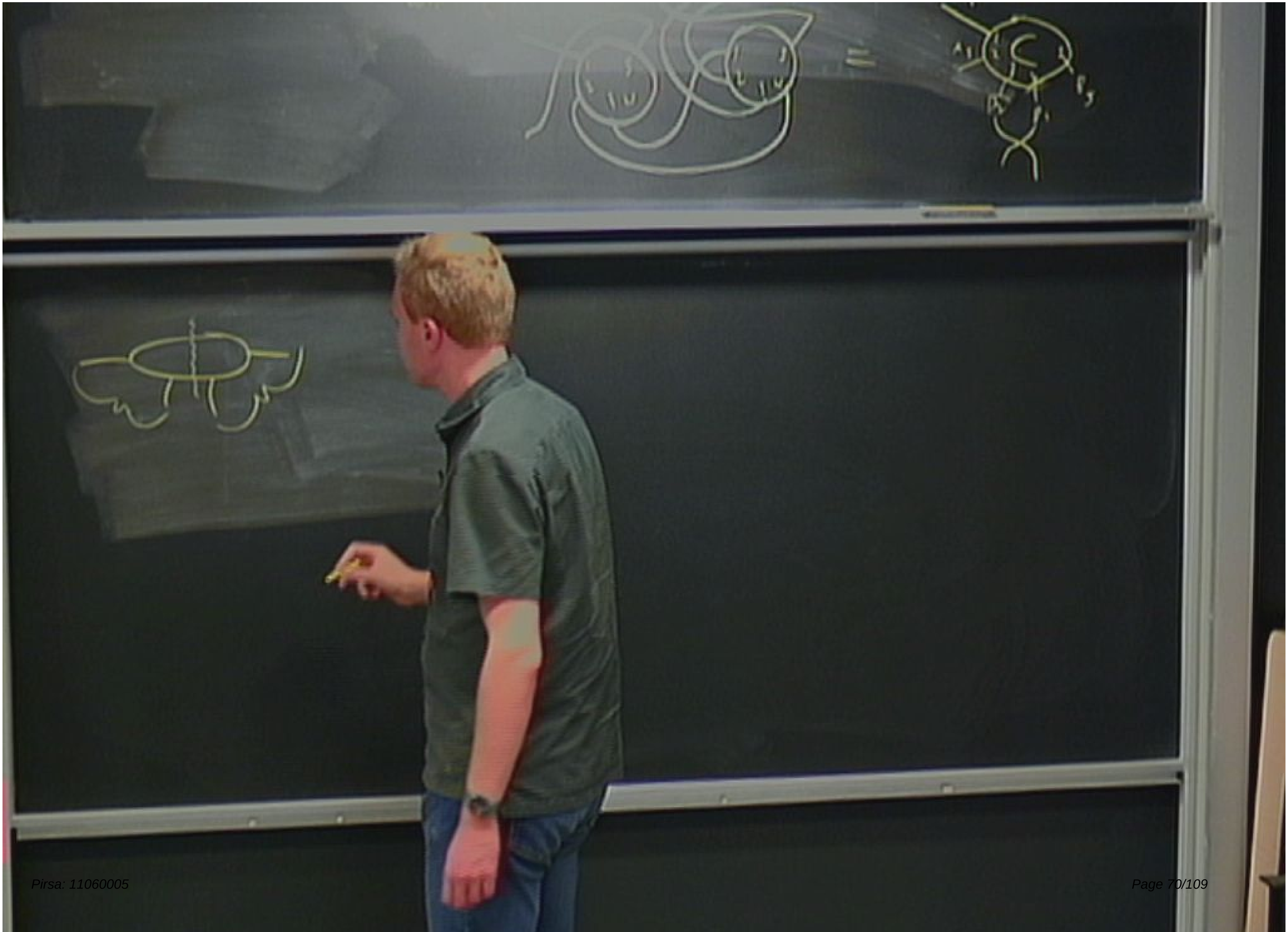

AB
2
10

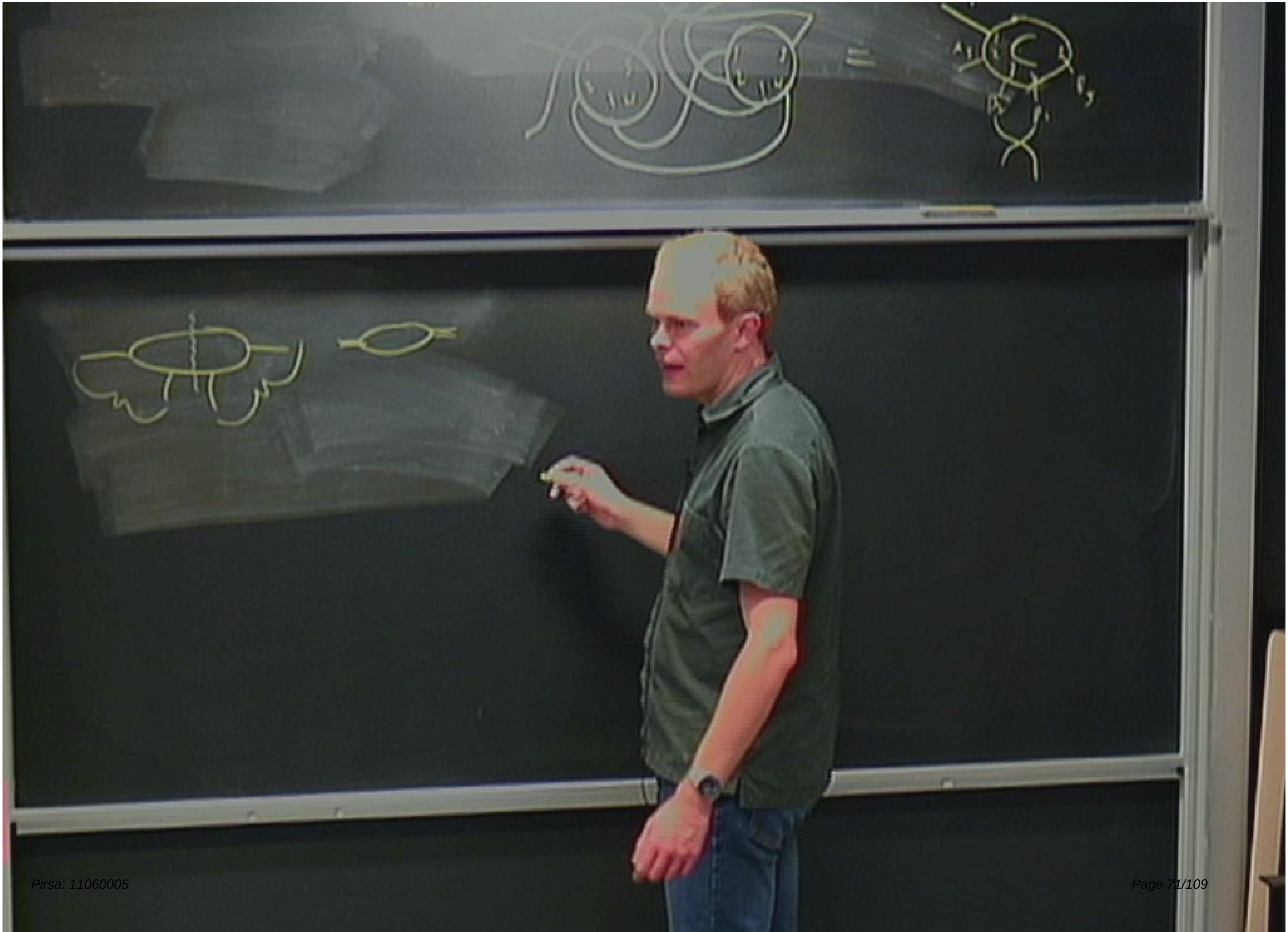

2
2
10
40
AB
2
10
C
20
40
4

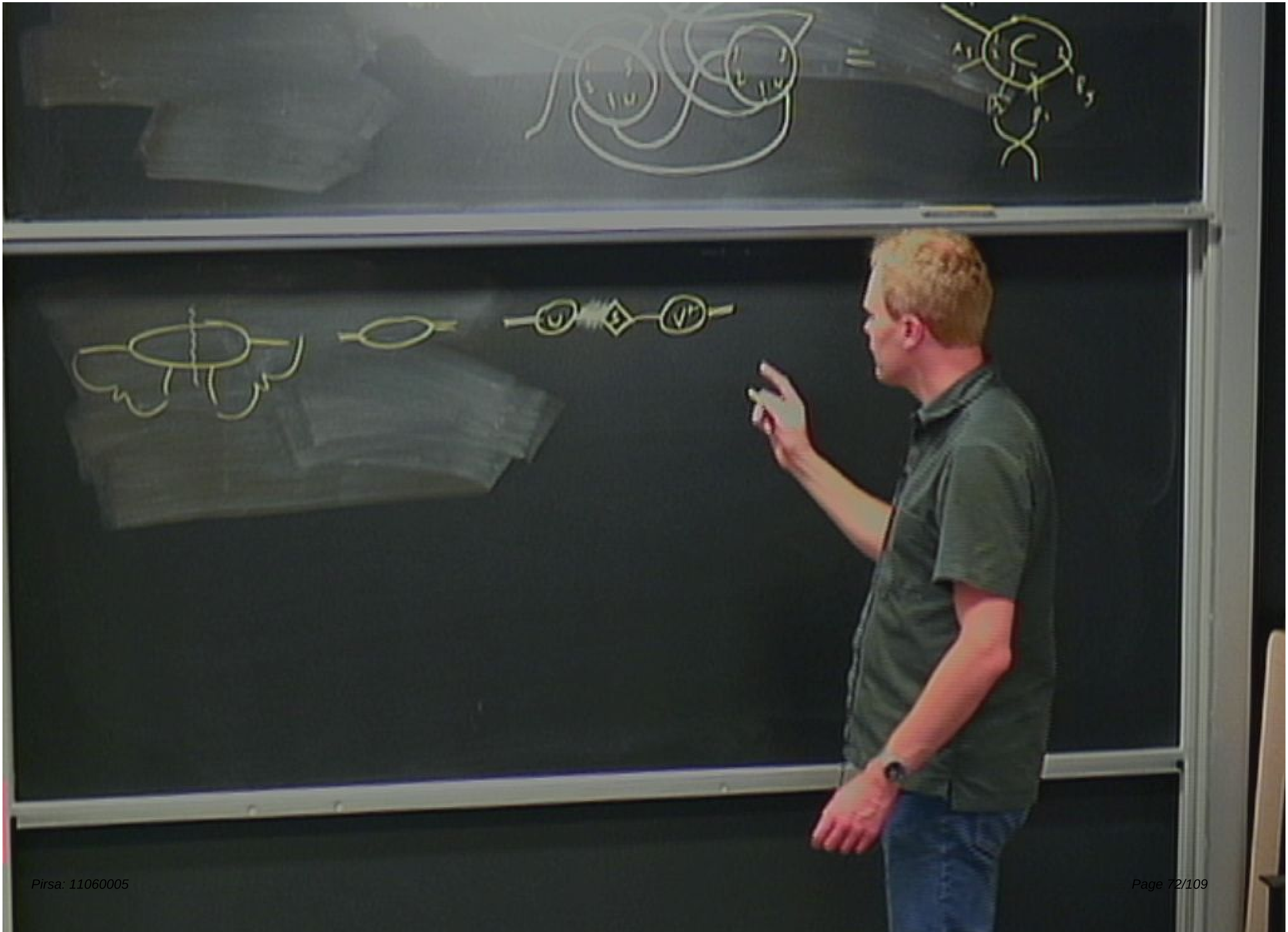

2
2
40
AB
2
C
10
20
60
ABC
40
A
BC
ABC

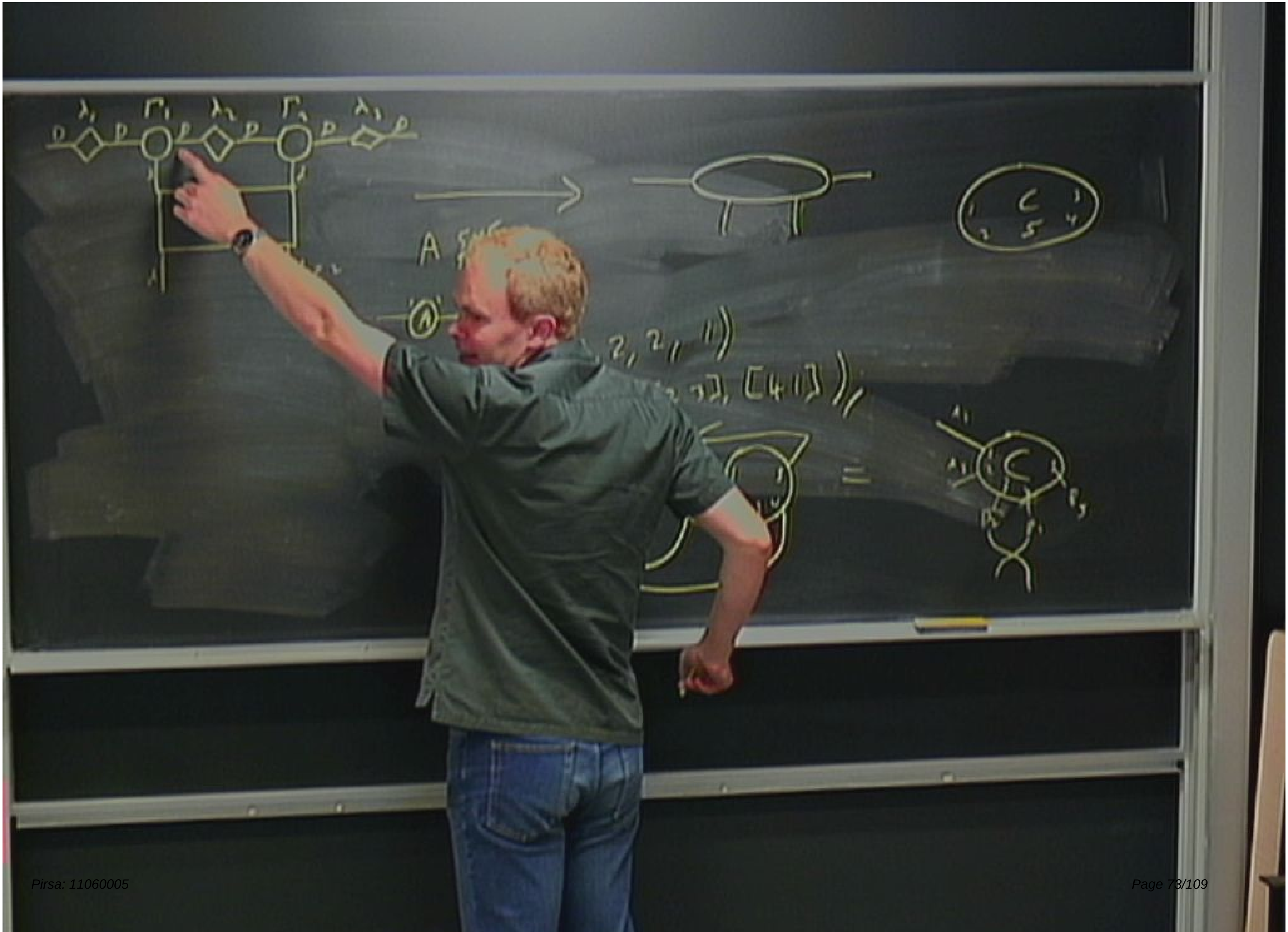

contract(A, B, 2, 2, 2, 1)
contract(A, B, 5, 5, [2 3], [4 1]),

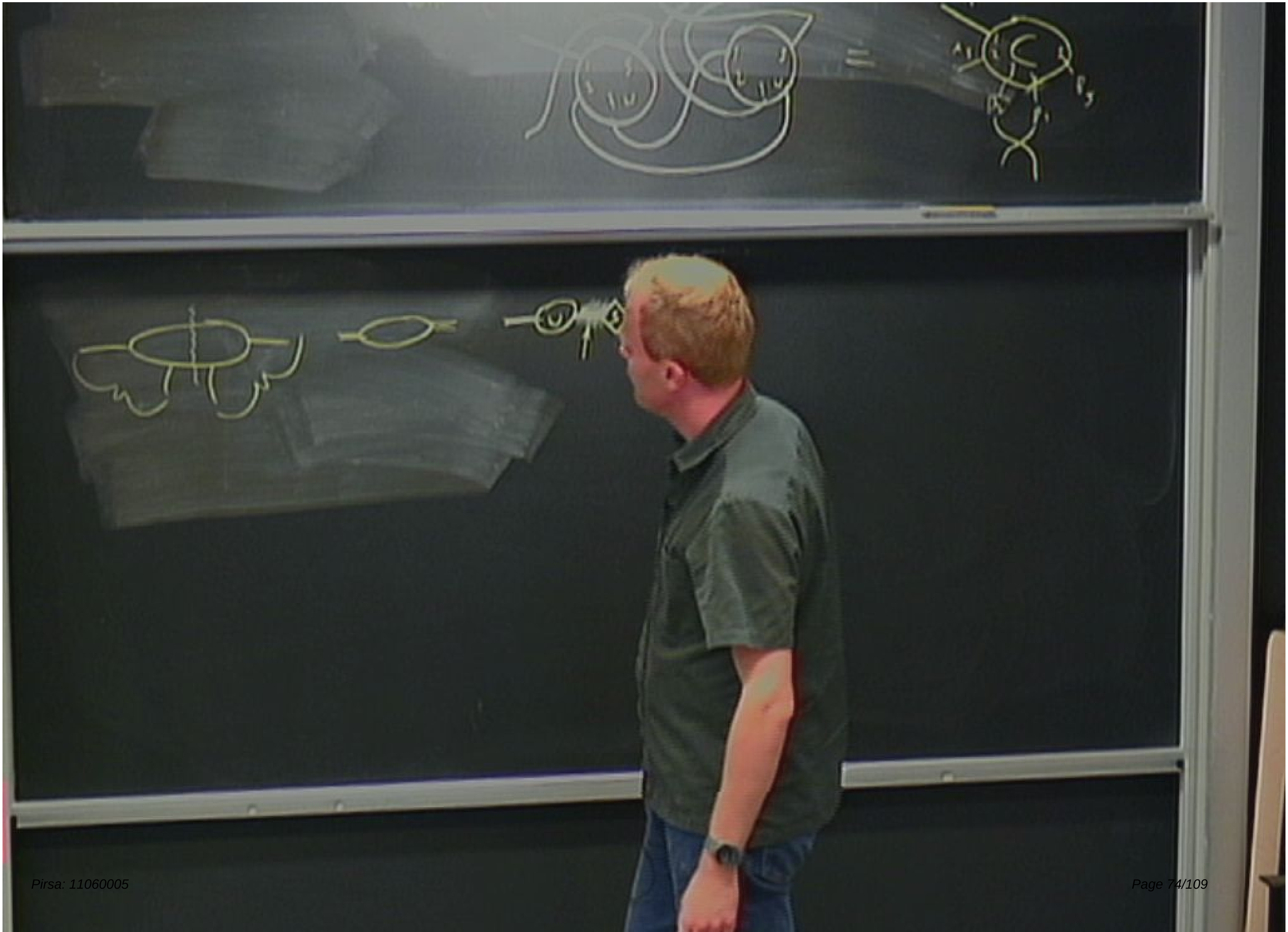

contract(A, B,
Contract(A, B,

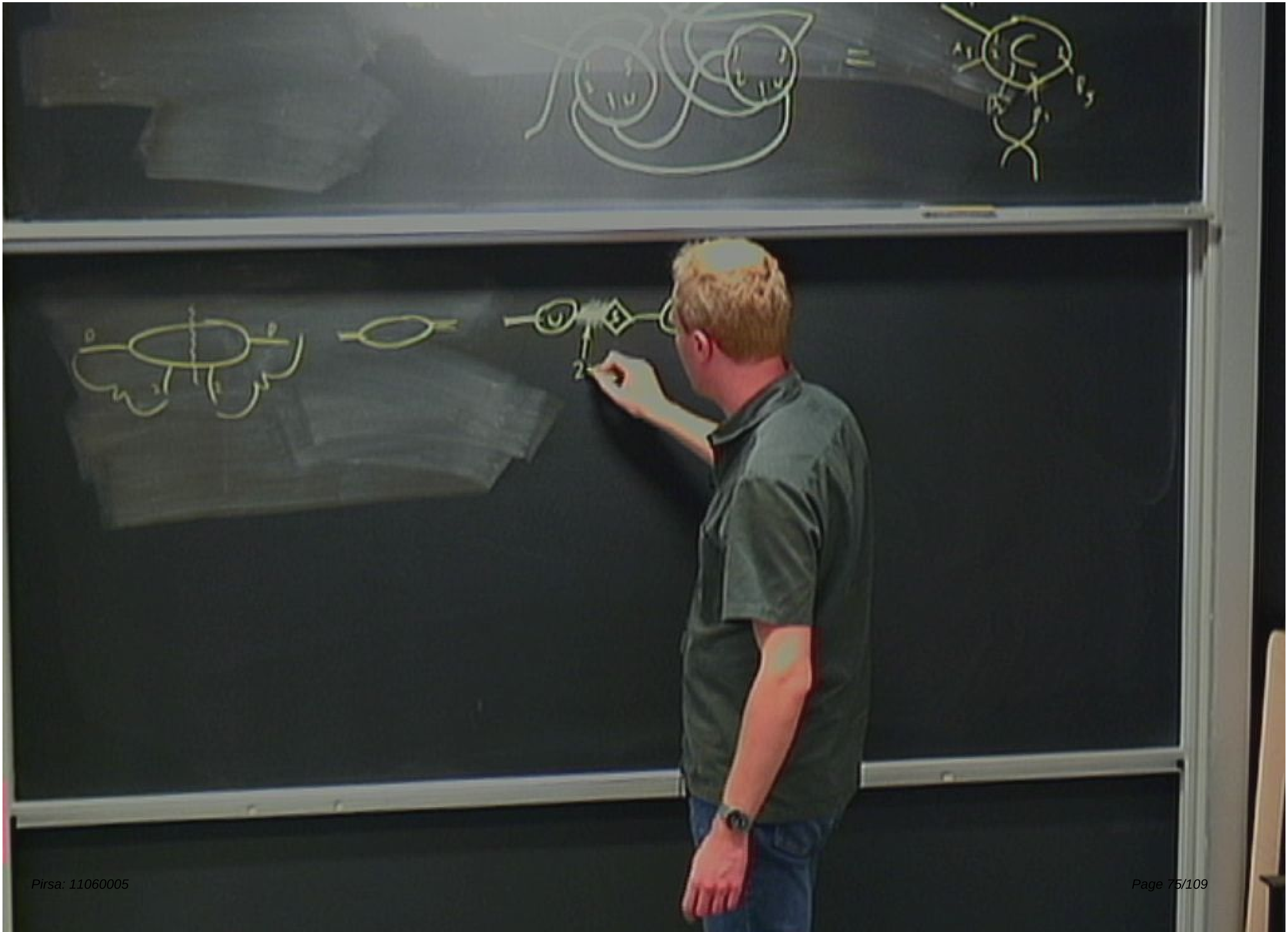


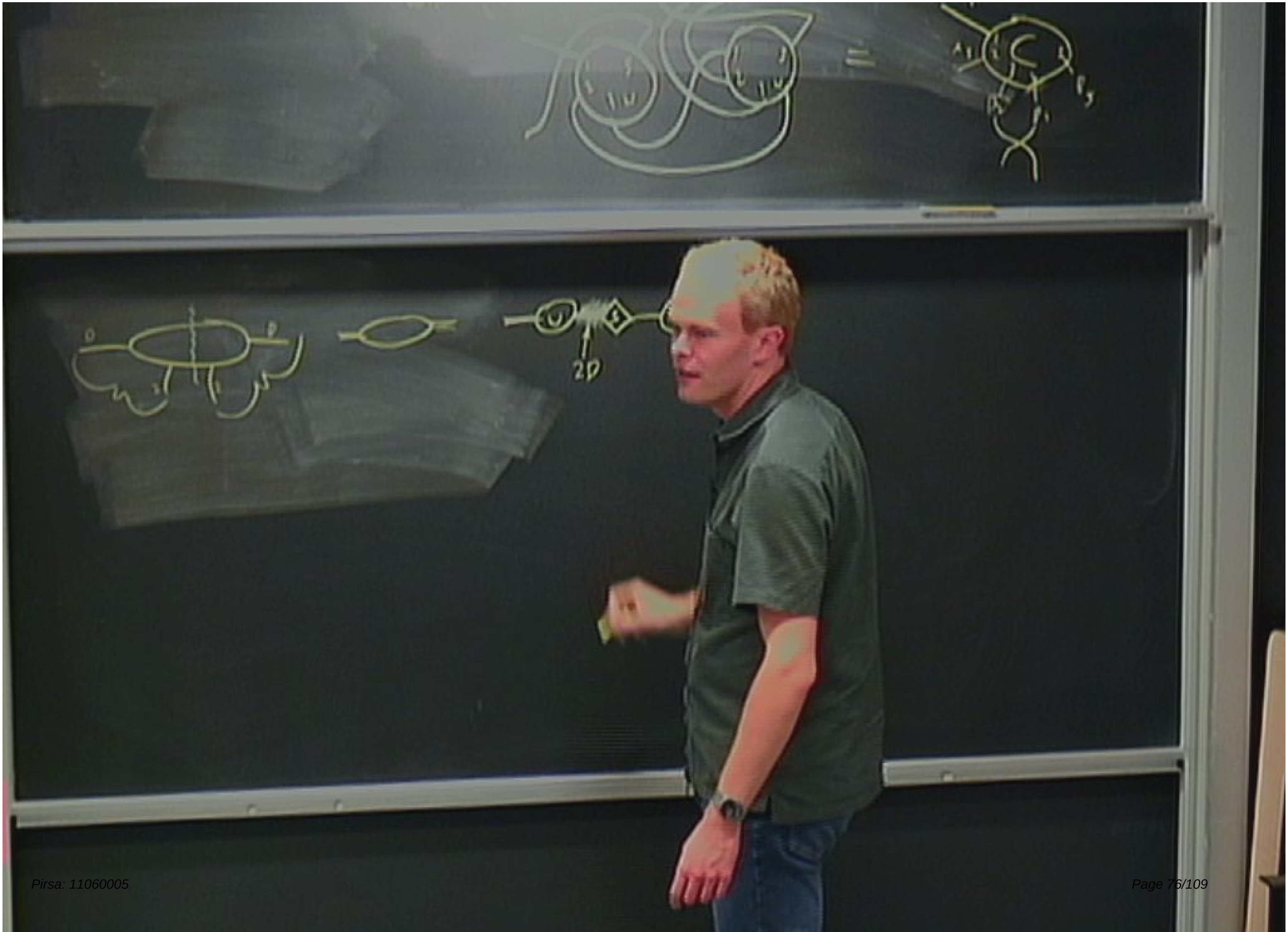


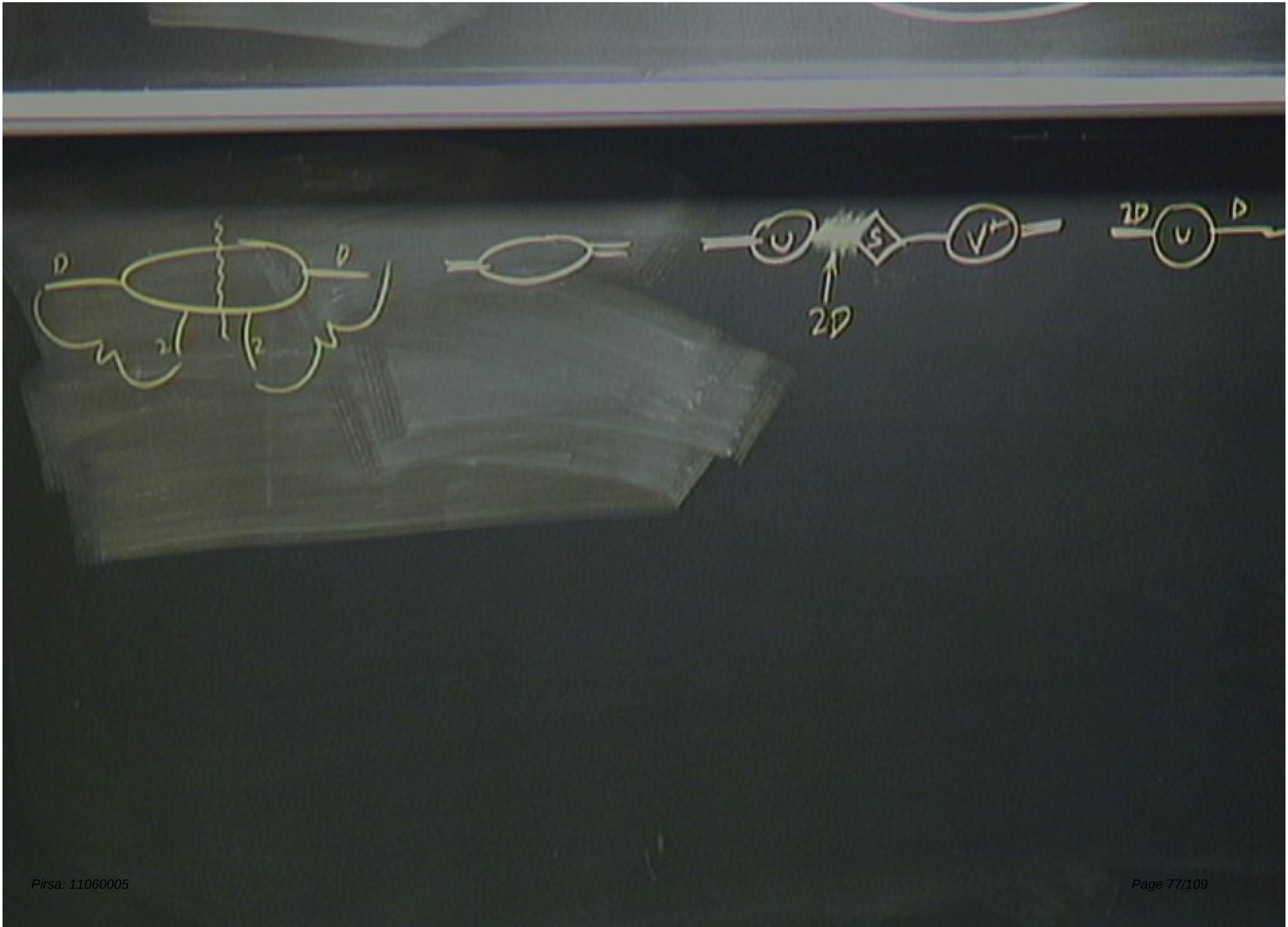


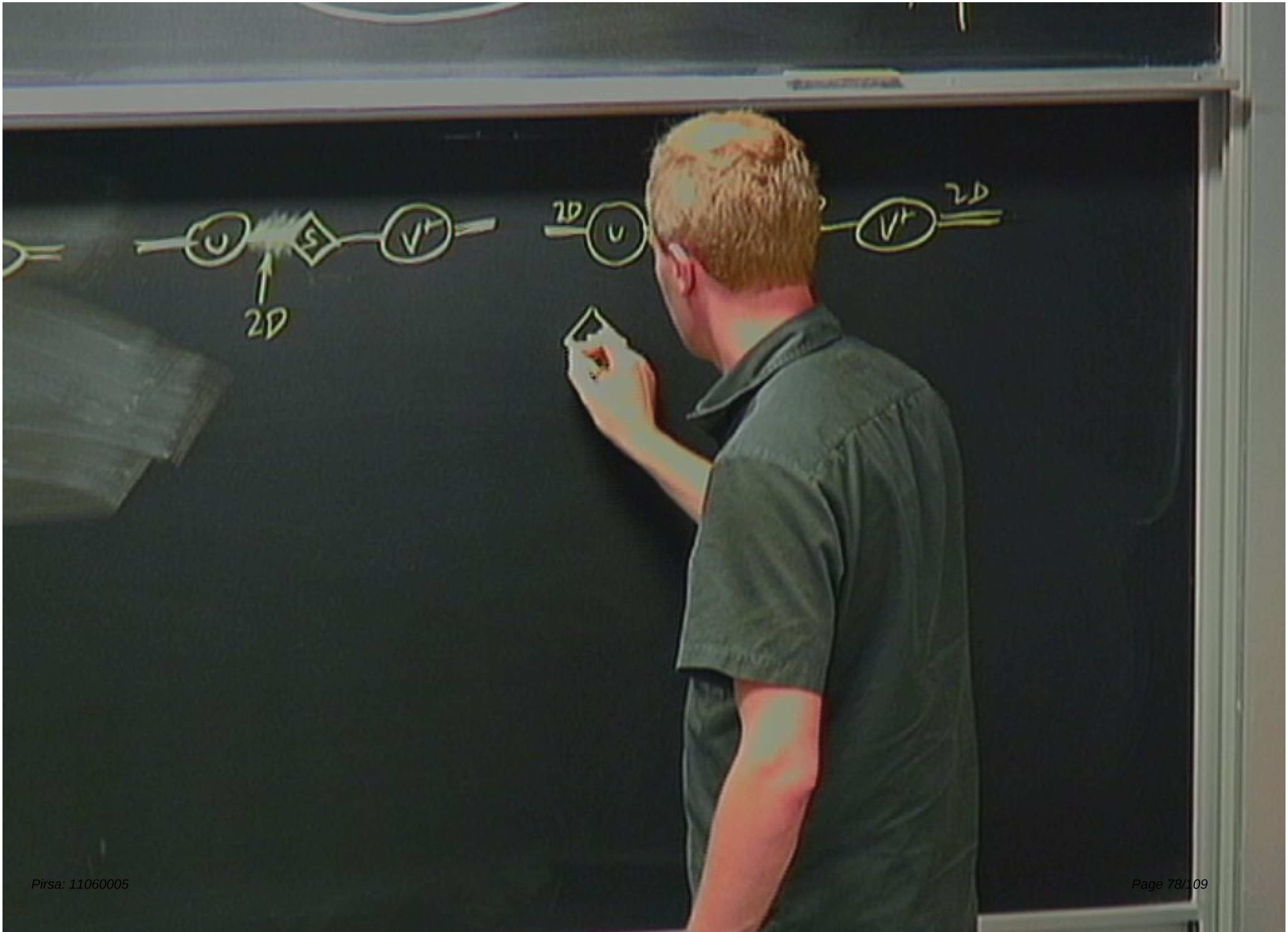


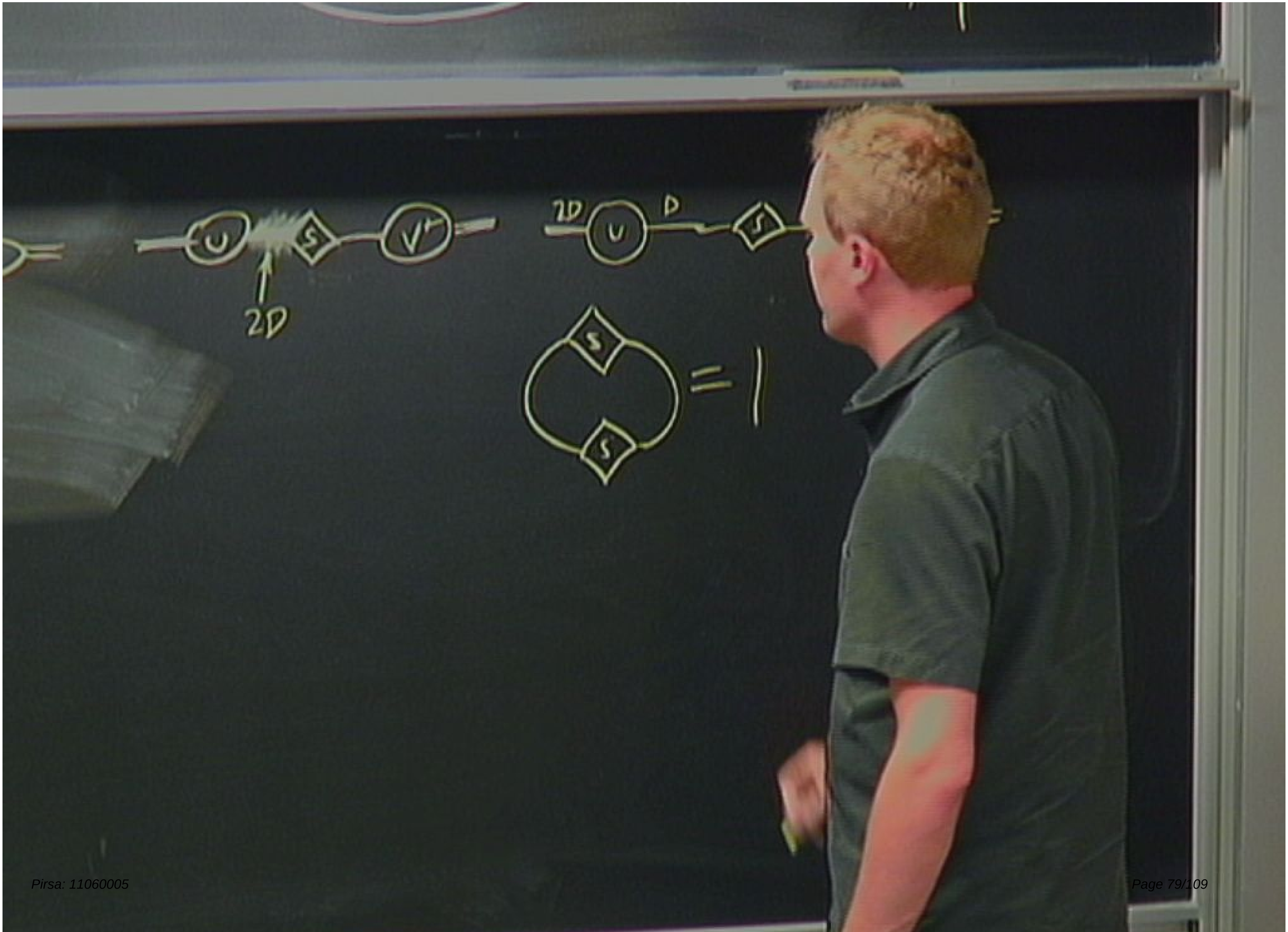


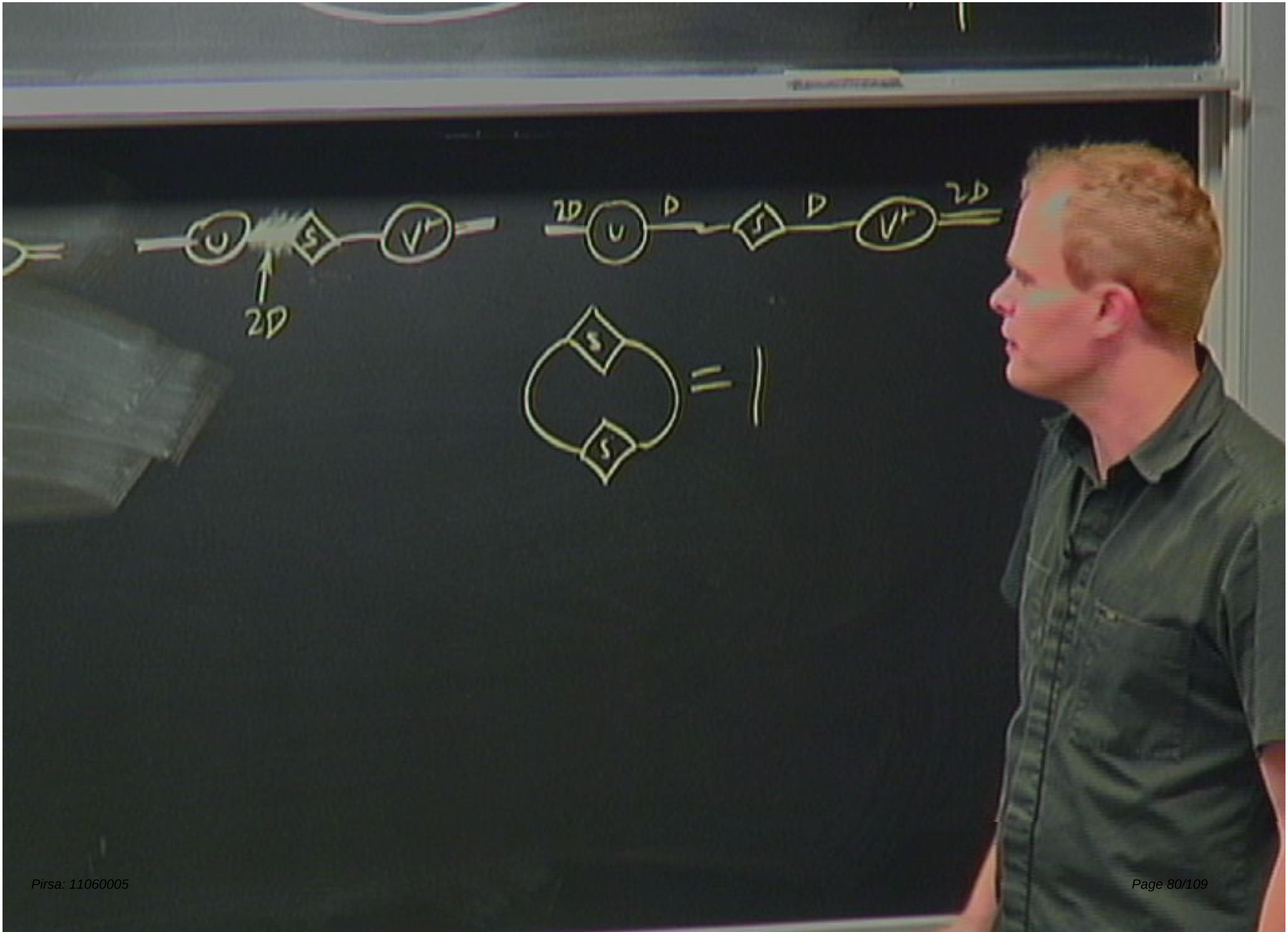



2D

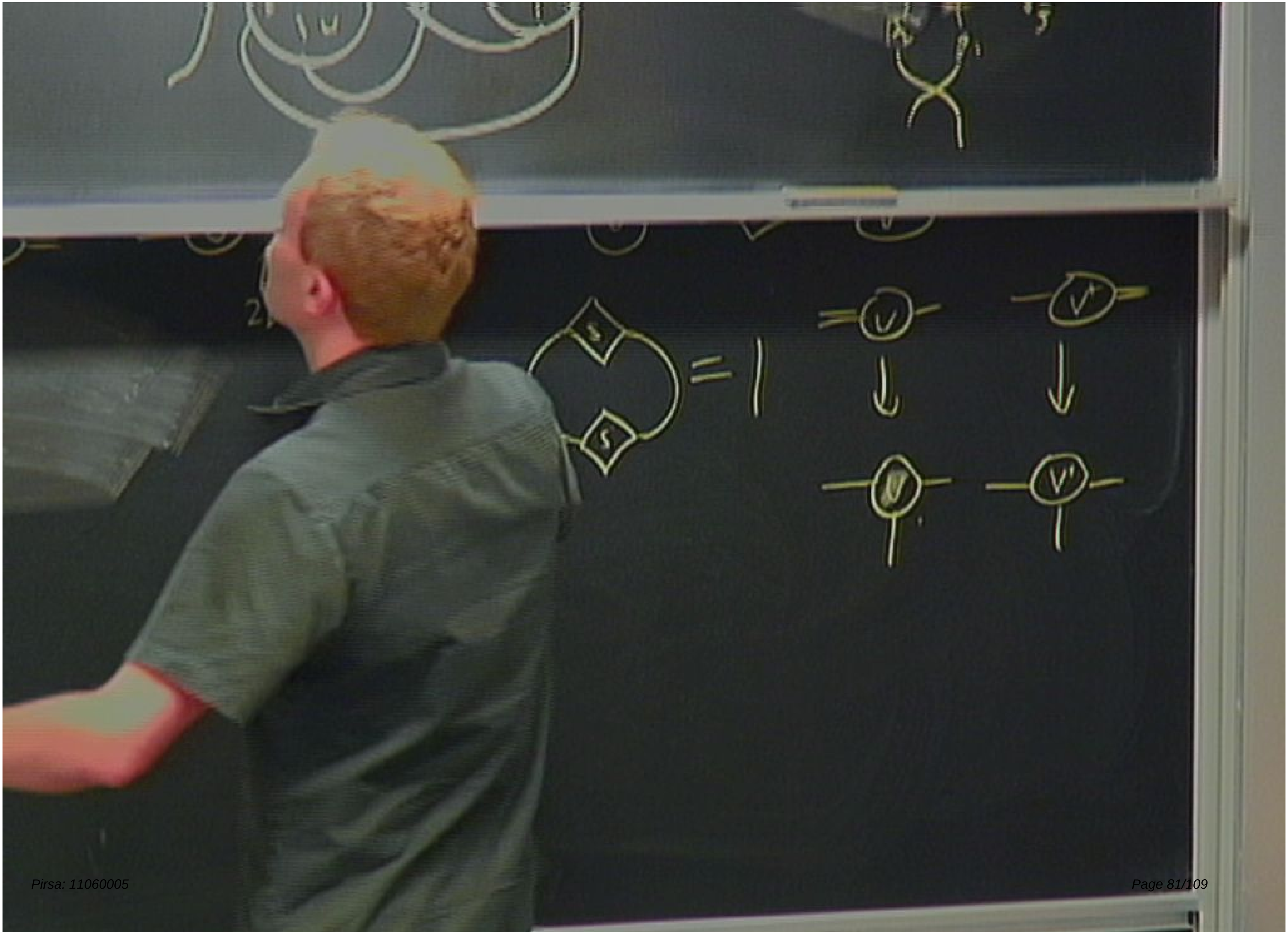

D
D
2
2
U
S
V
2D
2D
U
D

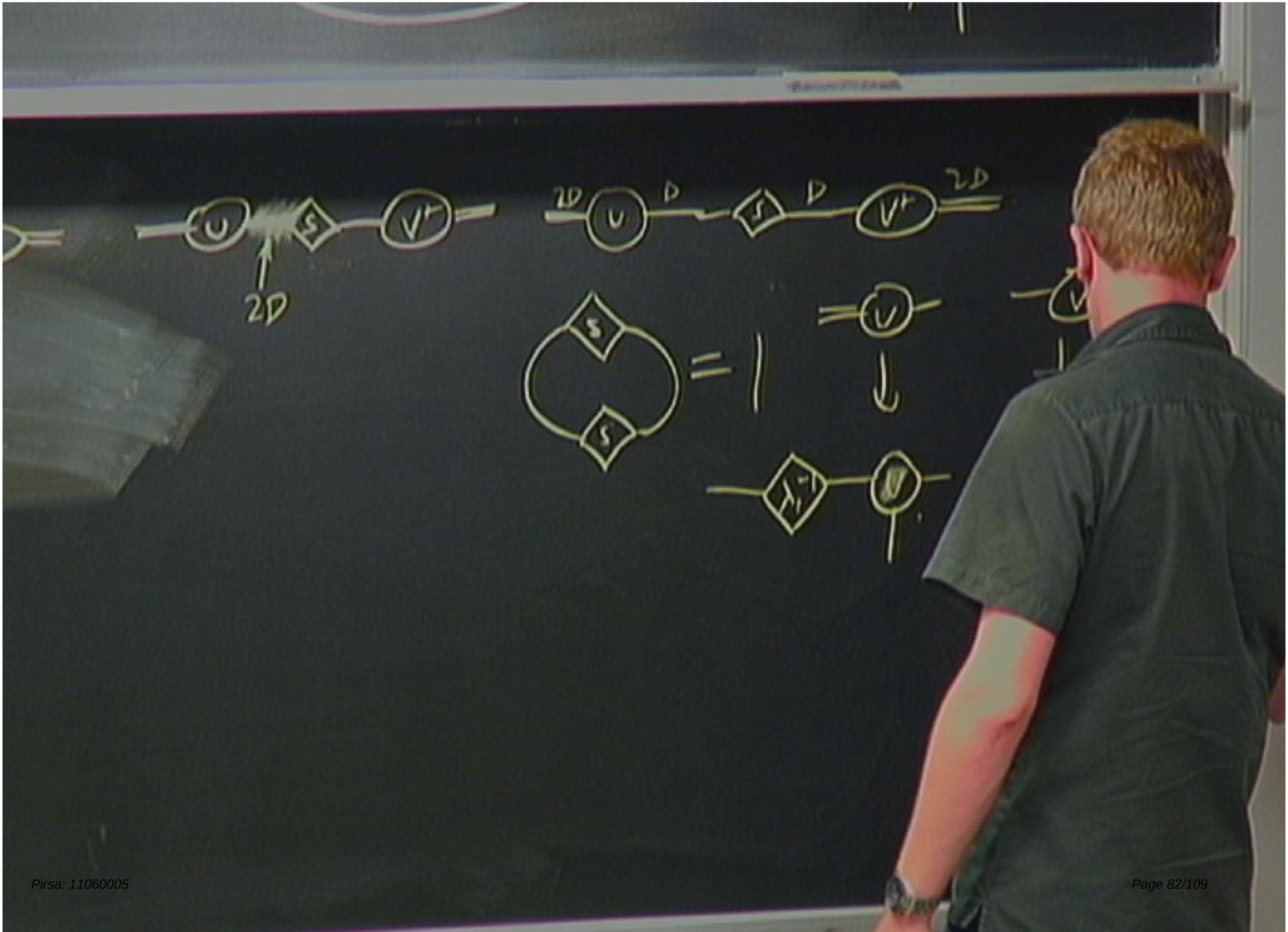

U
S
V†
2D
2D
U
V†
2D
2D


U
S
V†
2D
2D
U
D
S
S
S
= 1

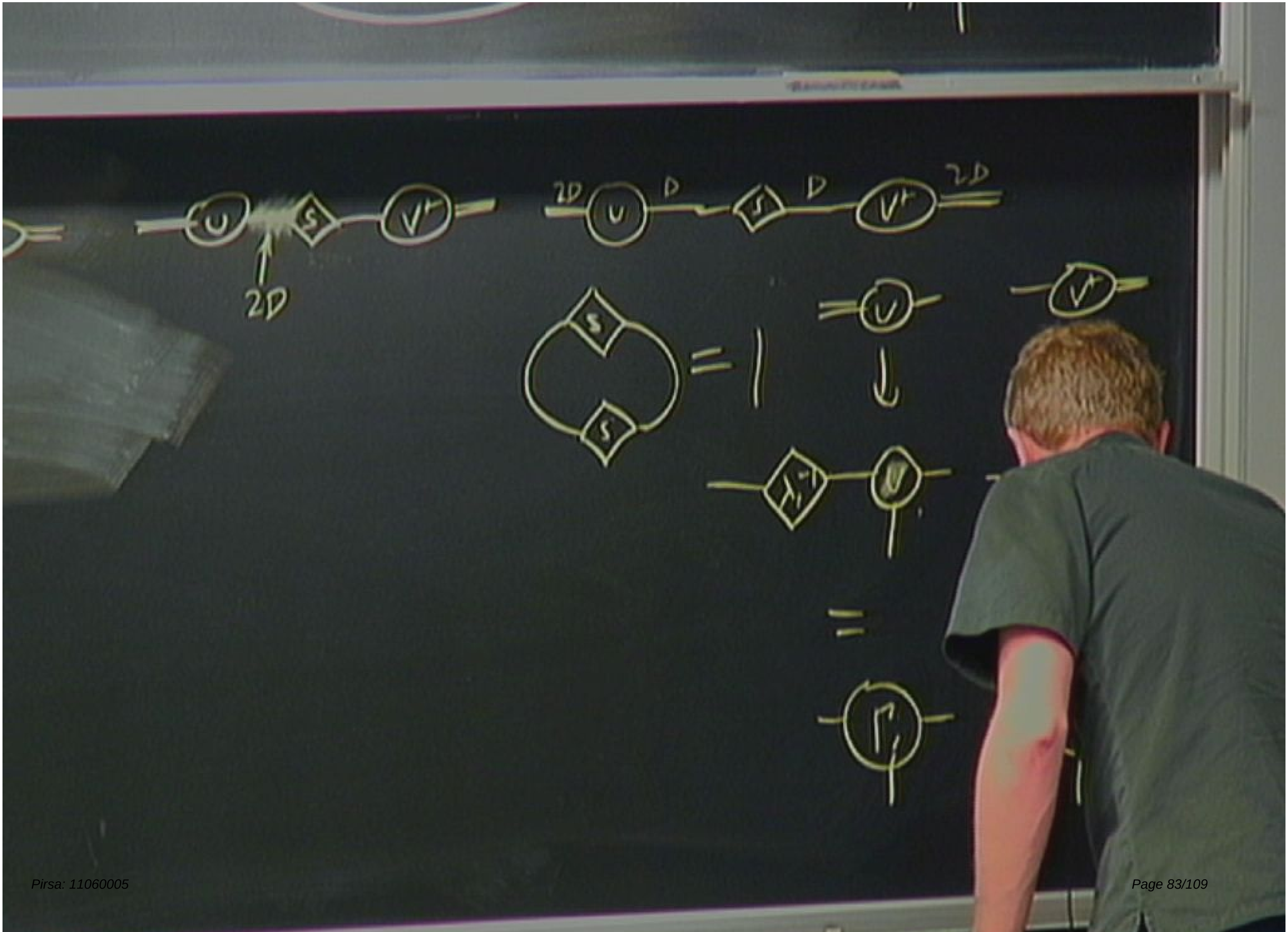


U
S
V†
2D
2D
U
D
S
D
V†
2D
S
S
= 1

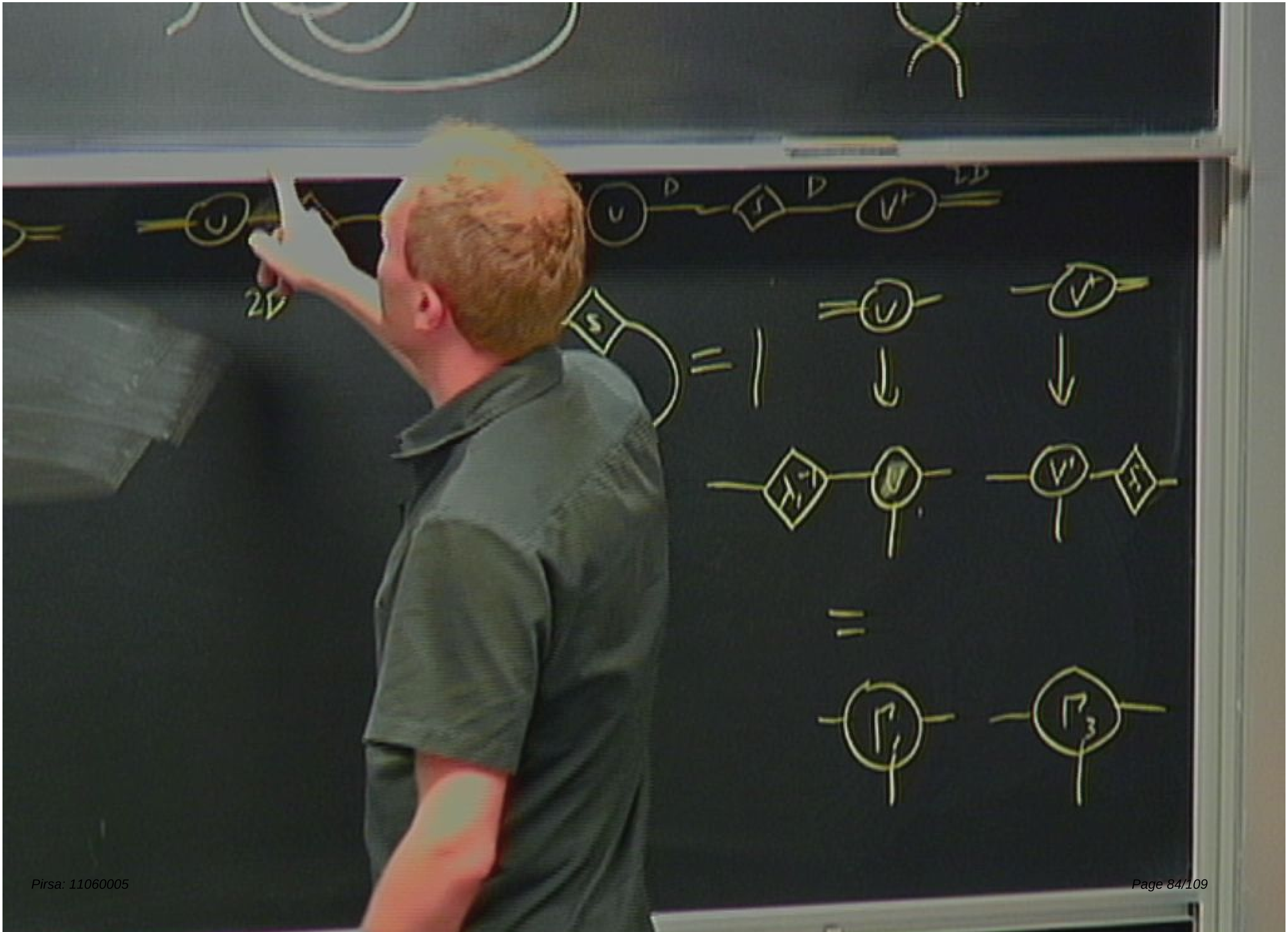


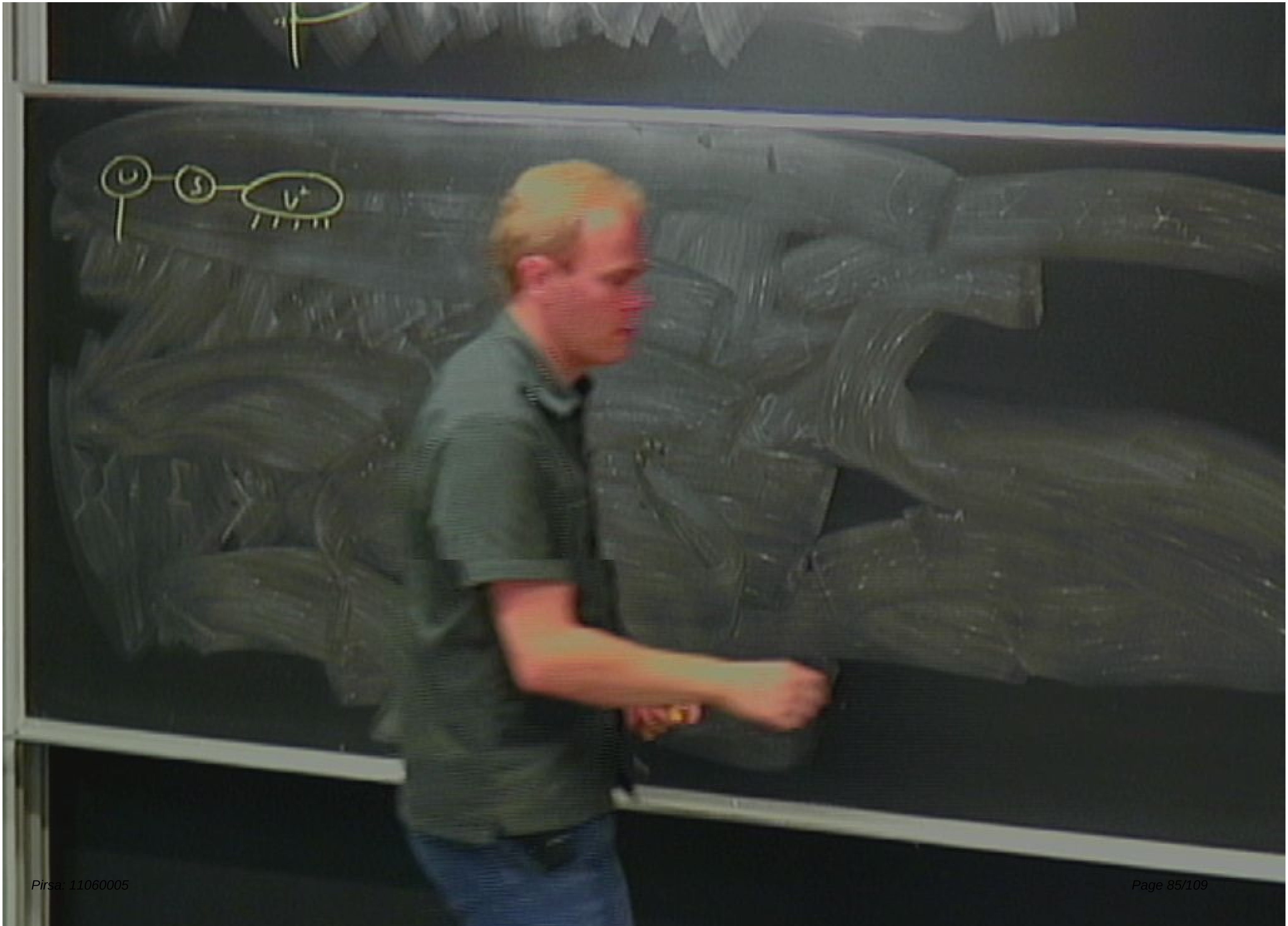


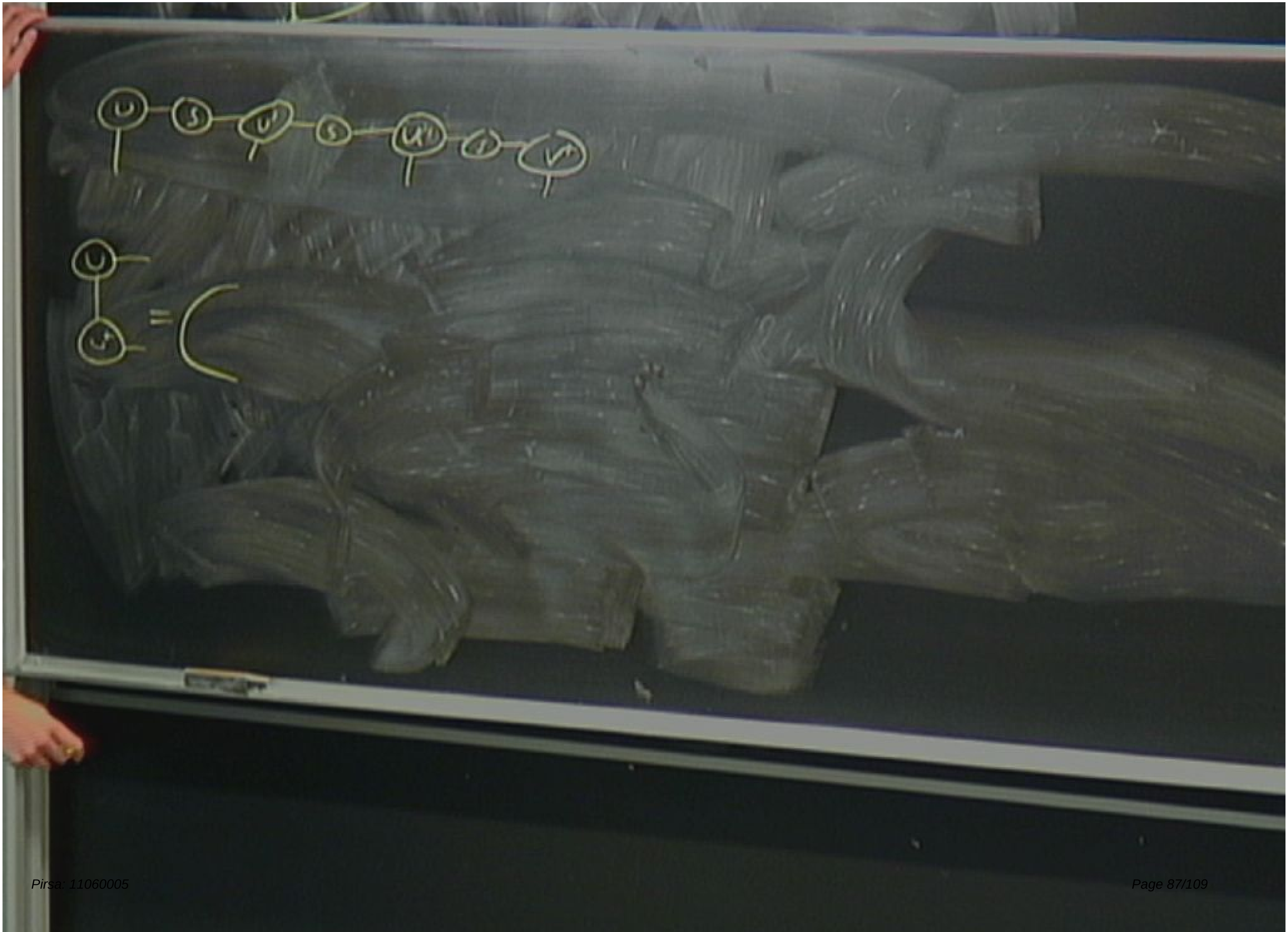
2D
D
D
2D
2D

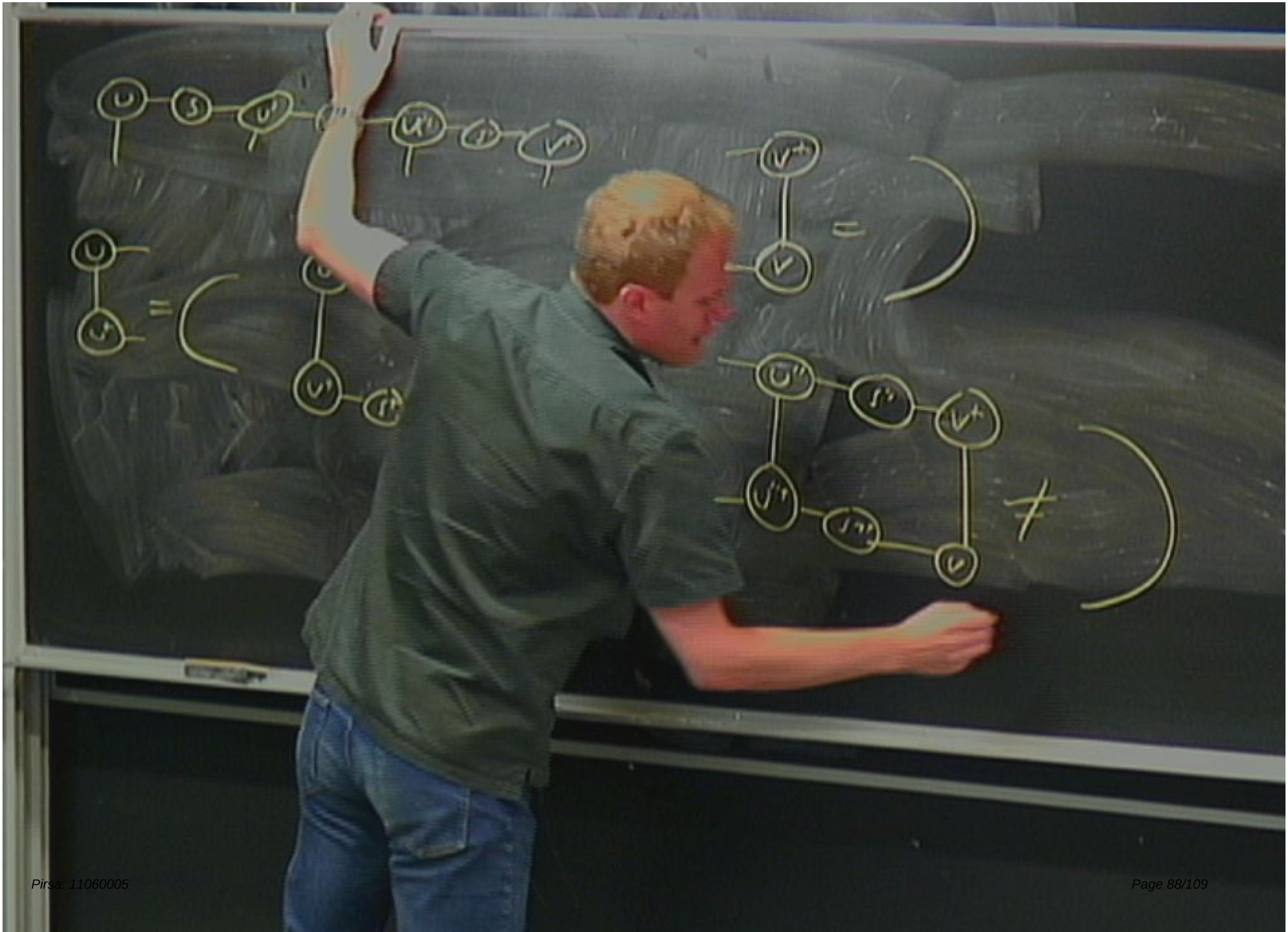


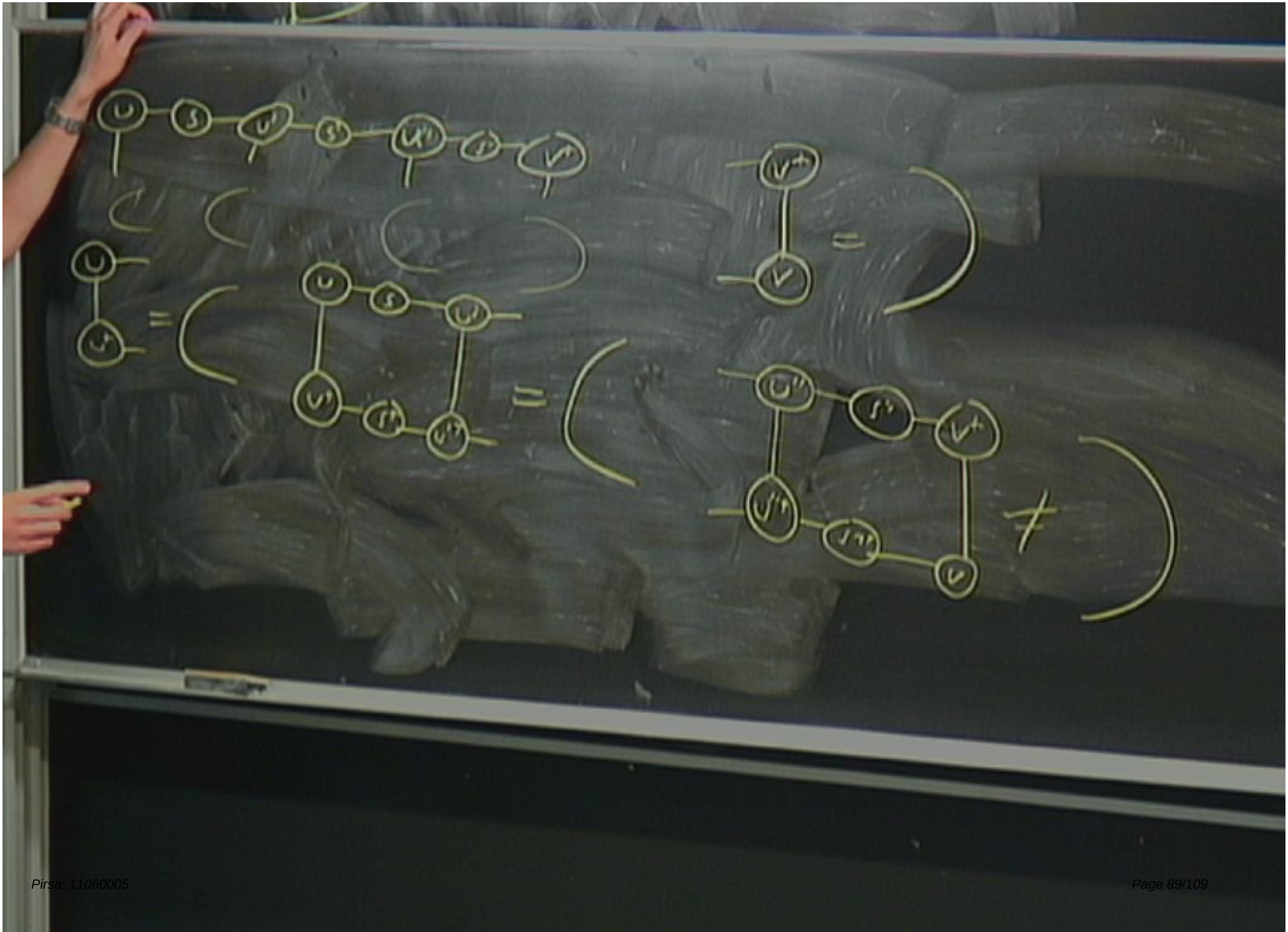
2D
U
S
V†
2D
D
D
2D

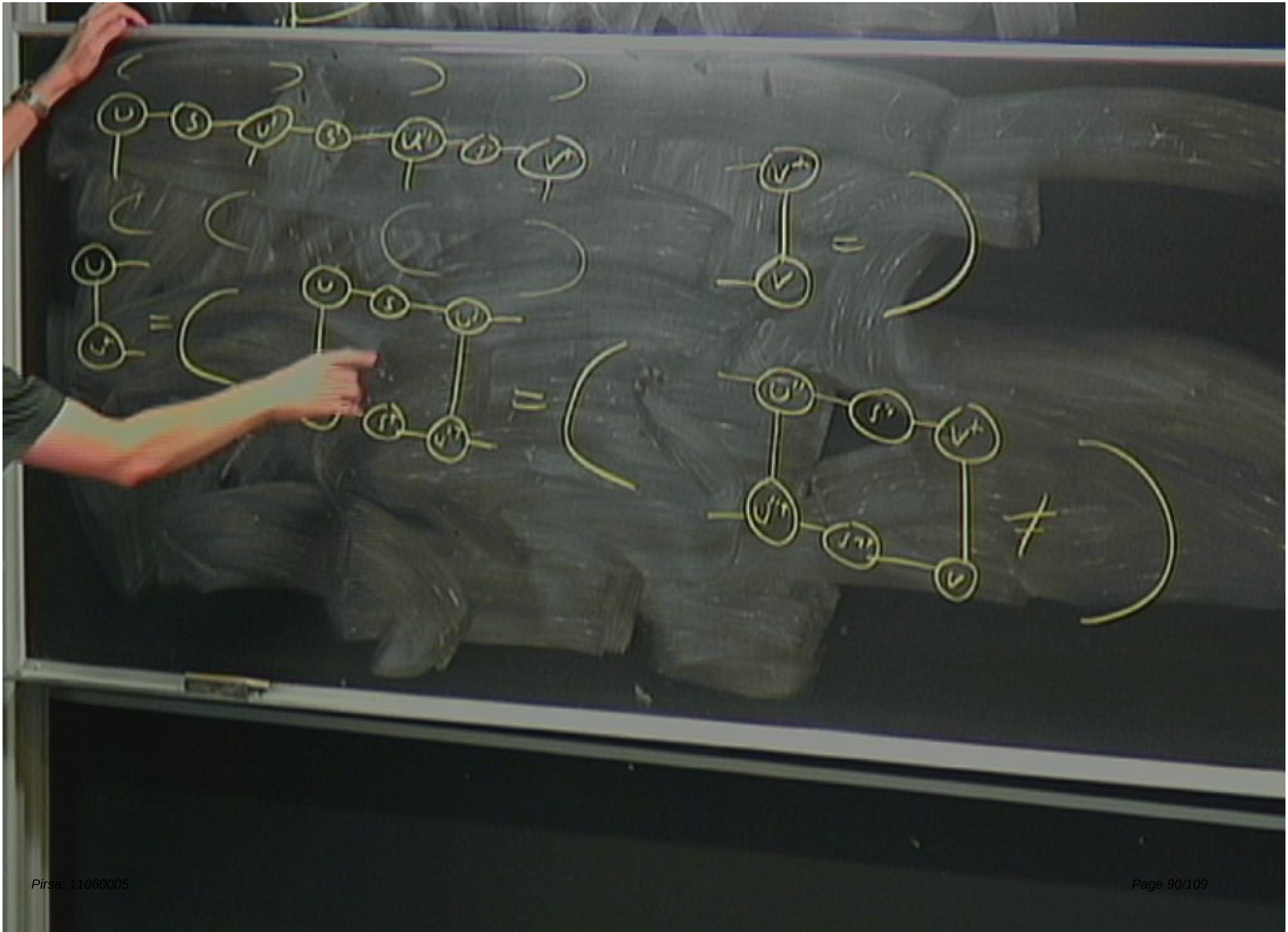


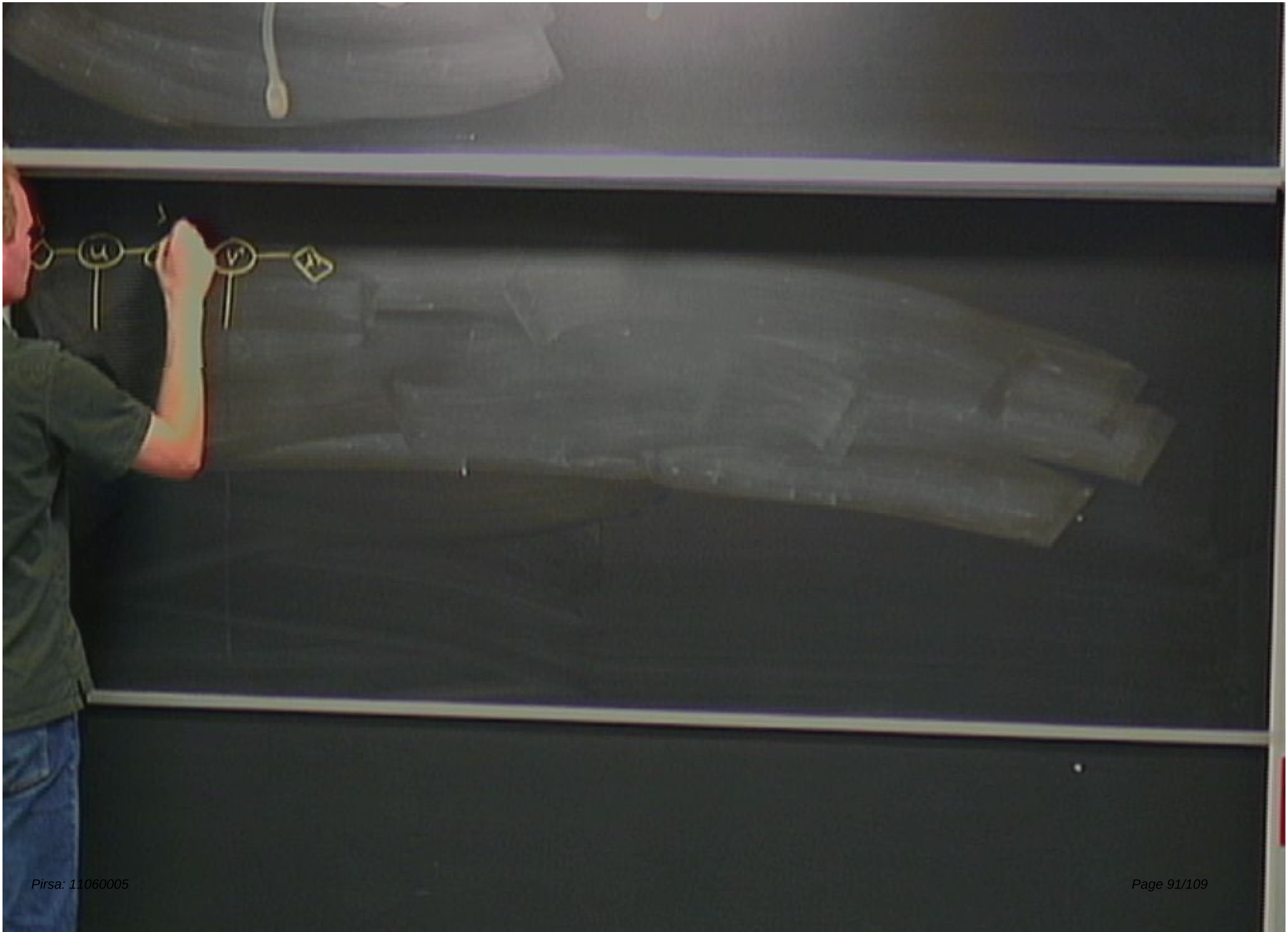


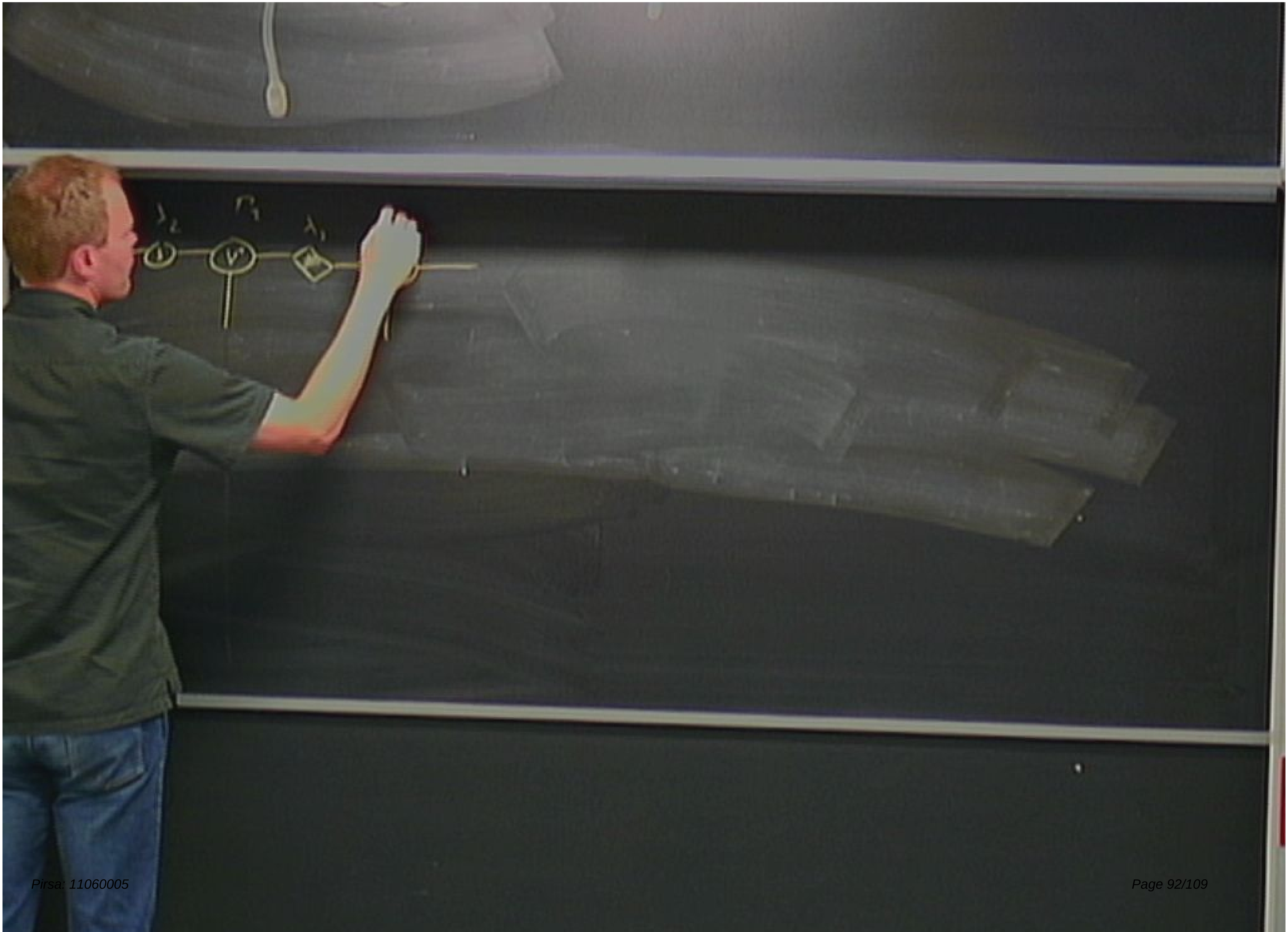


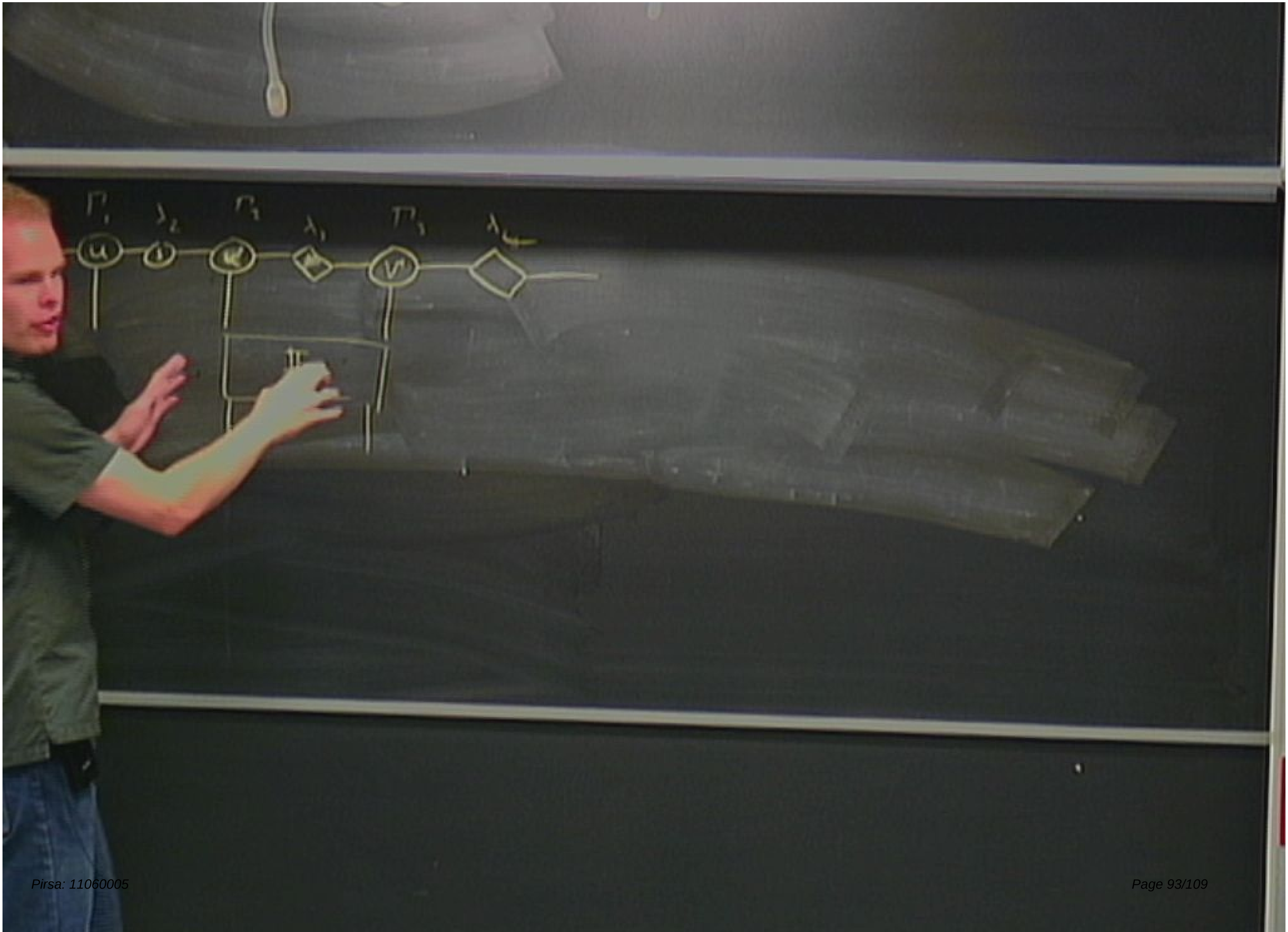


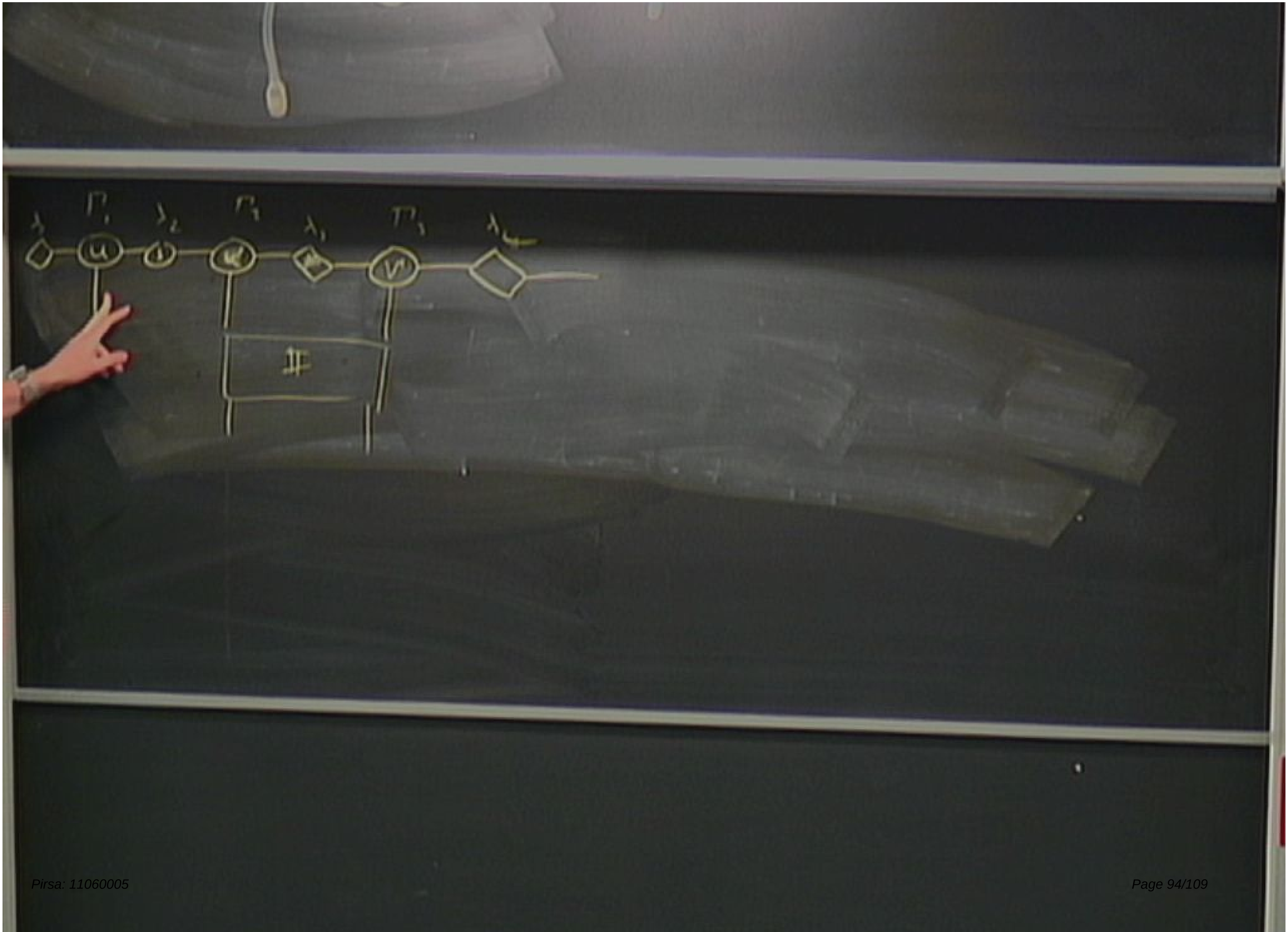


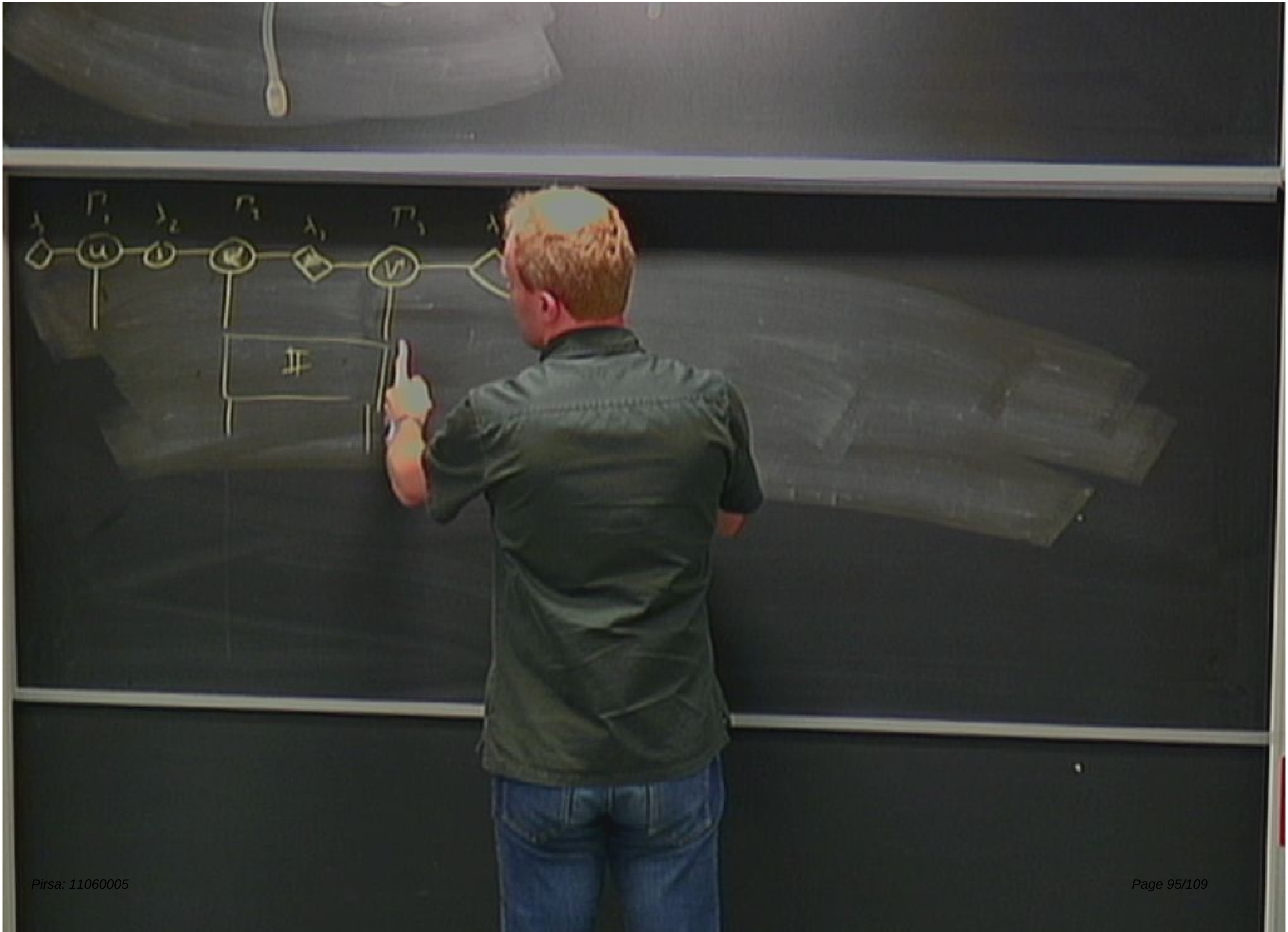


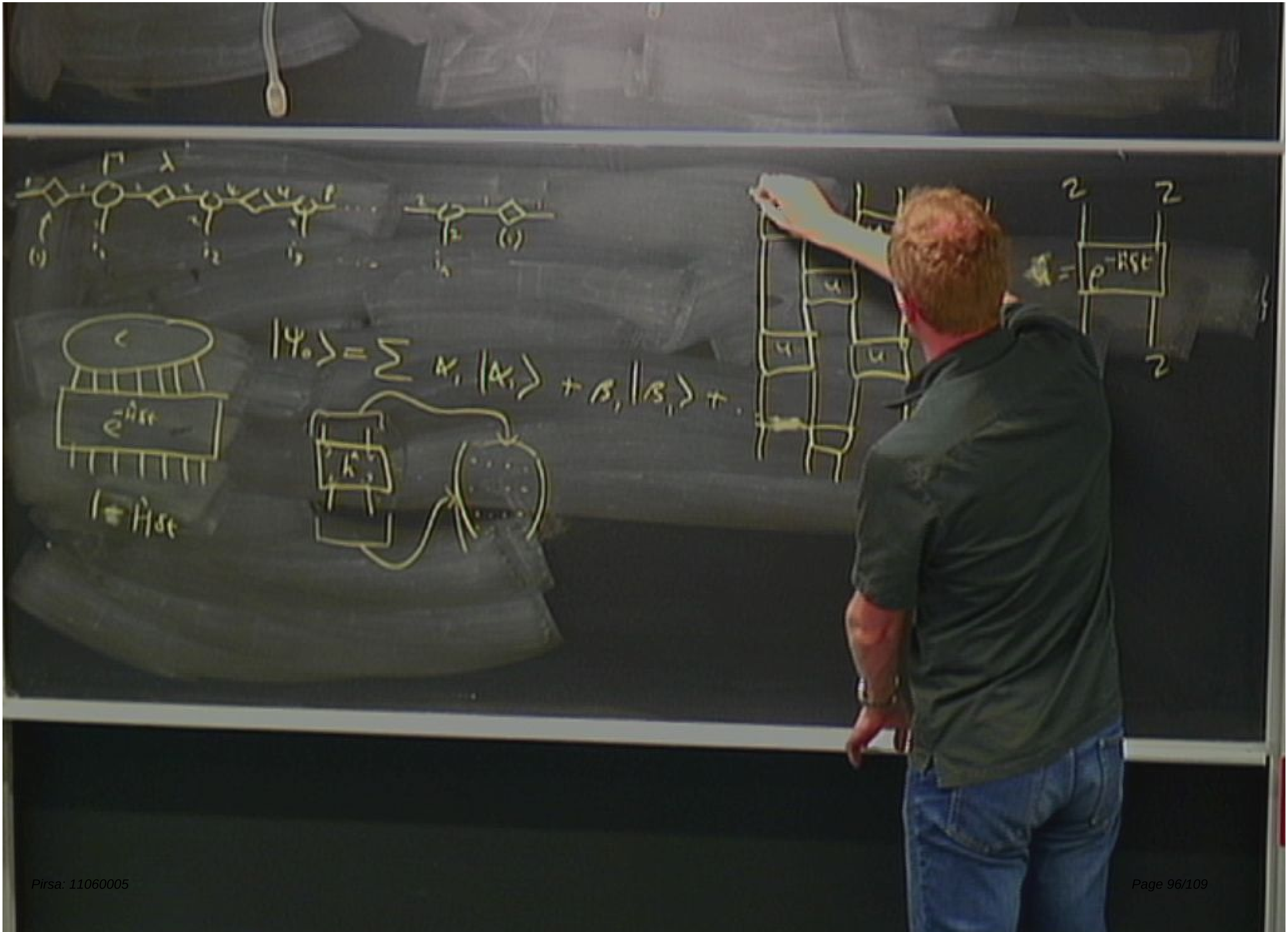


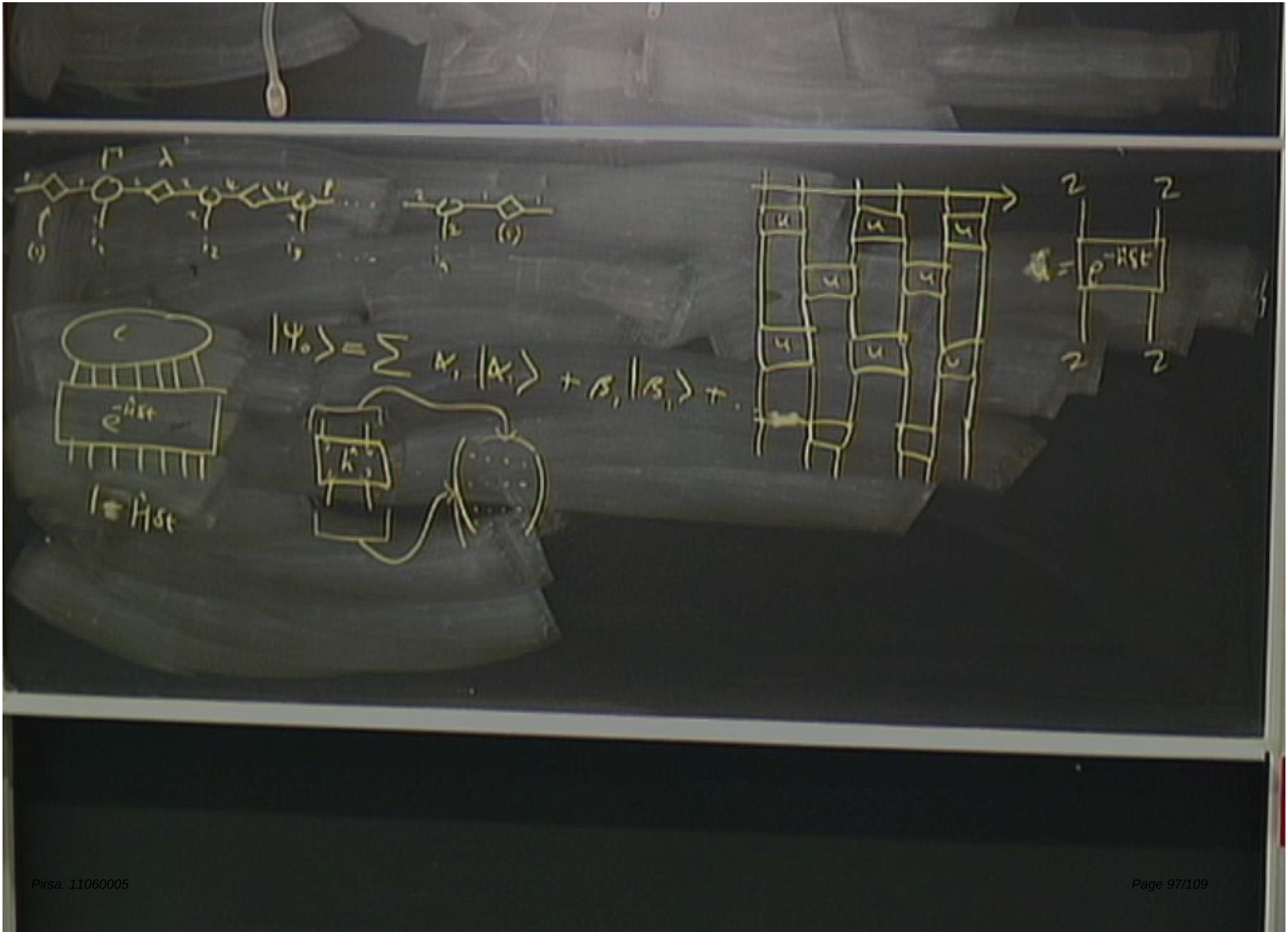


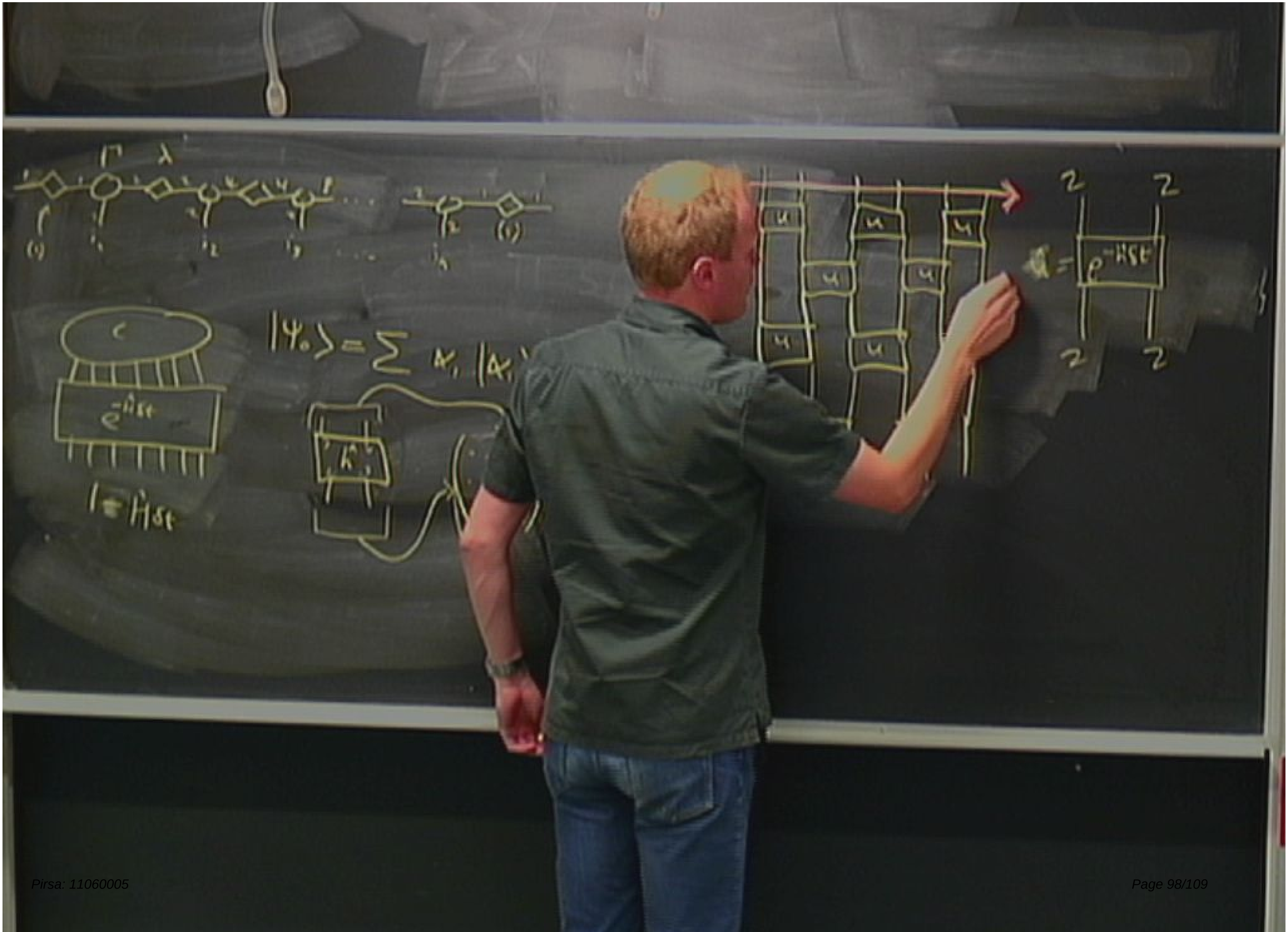


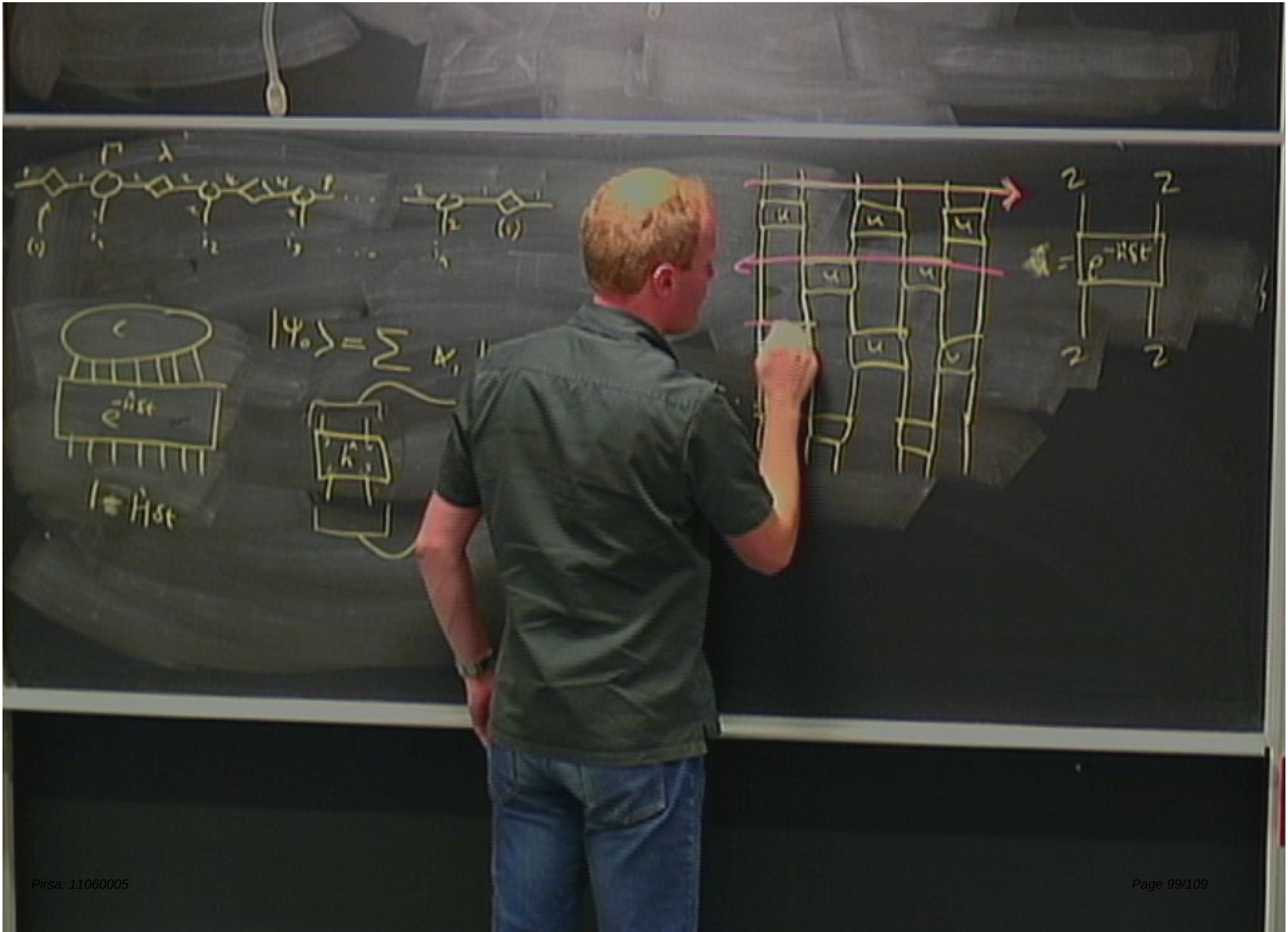


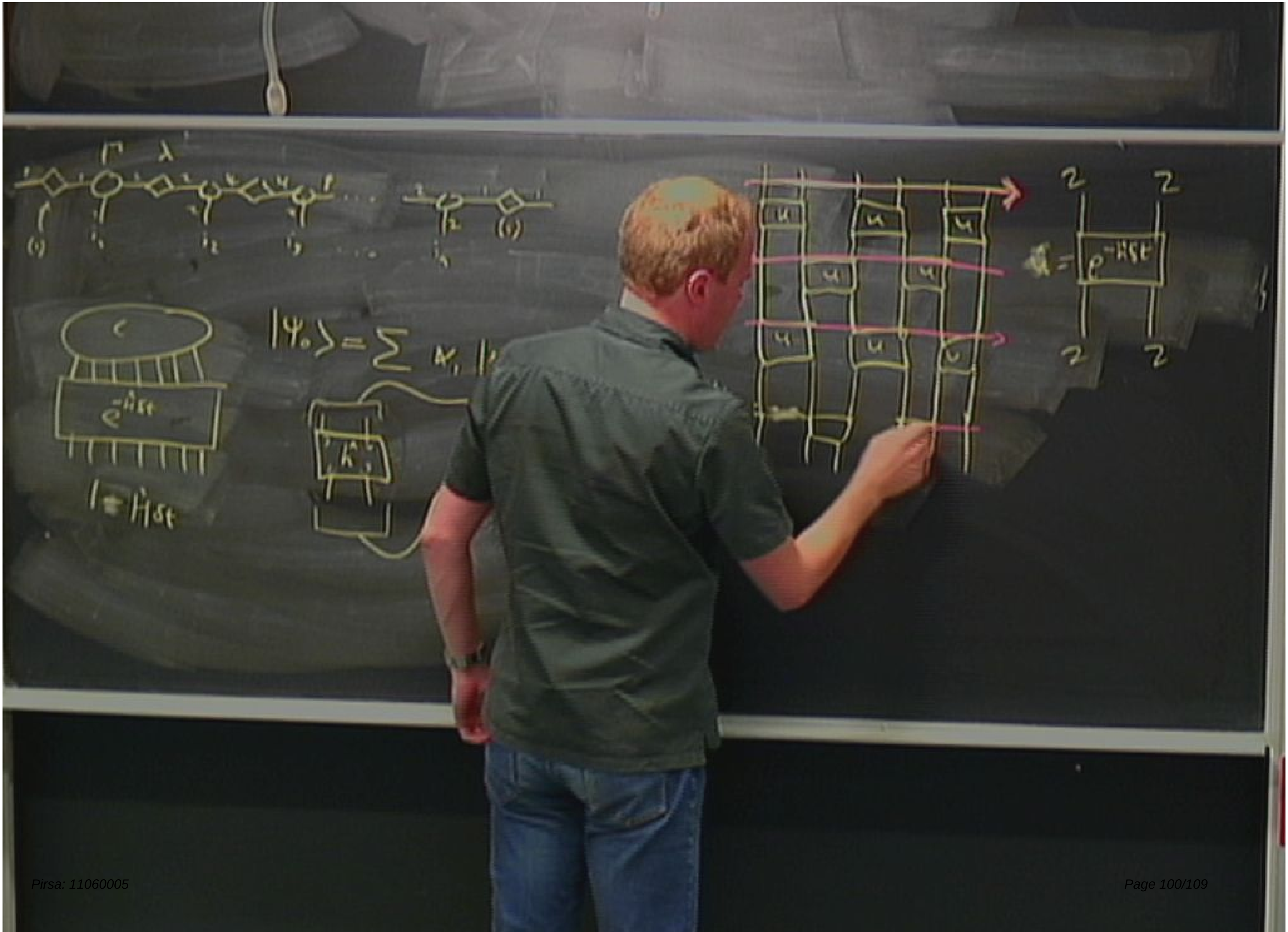


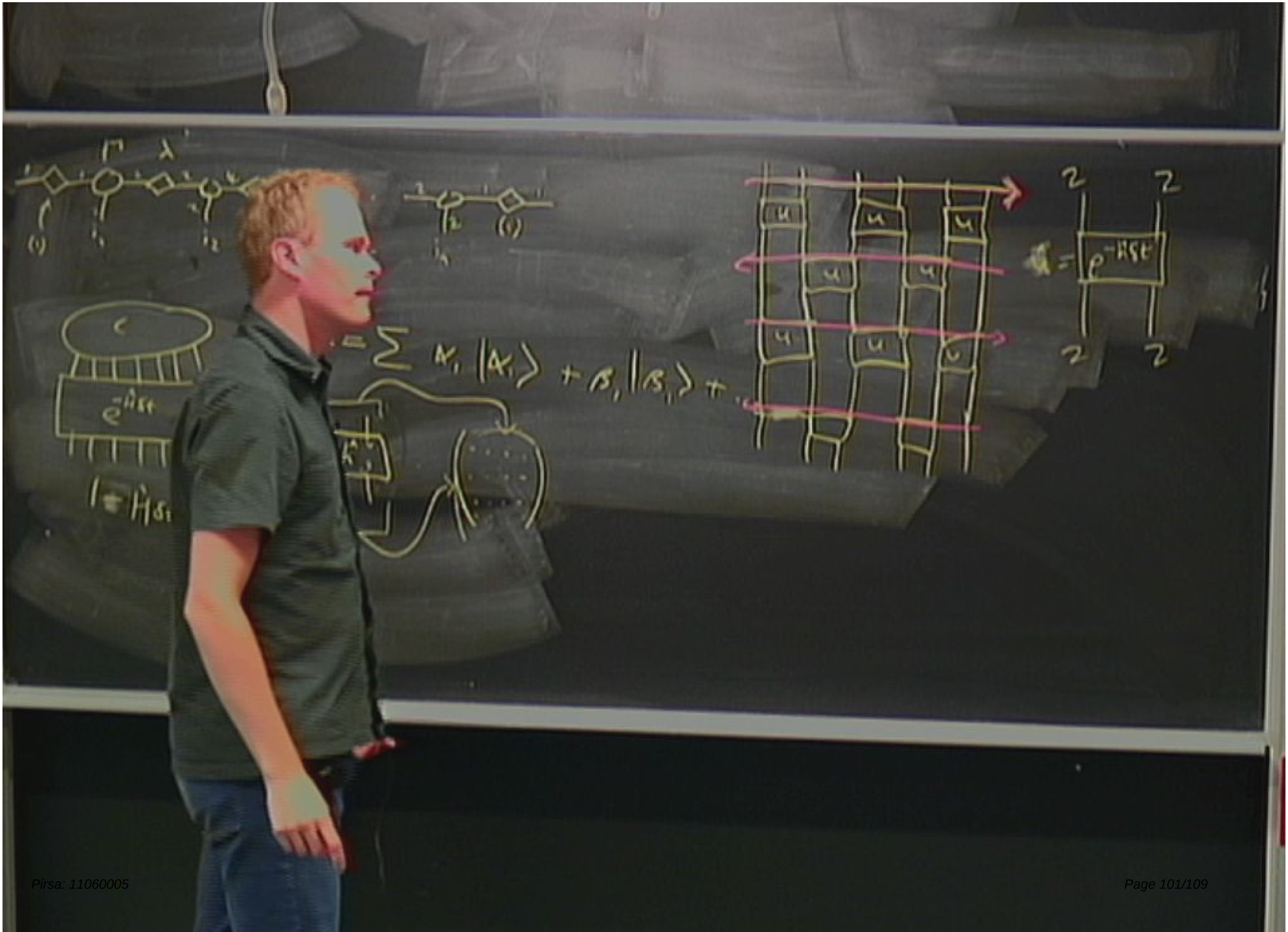

|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁ |β₁⟩ + .

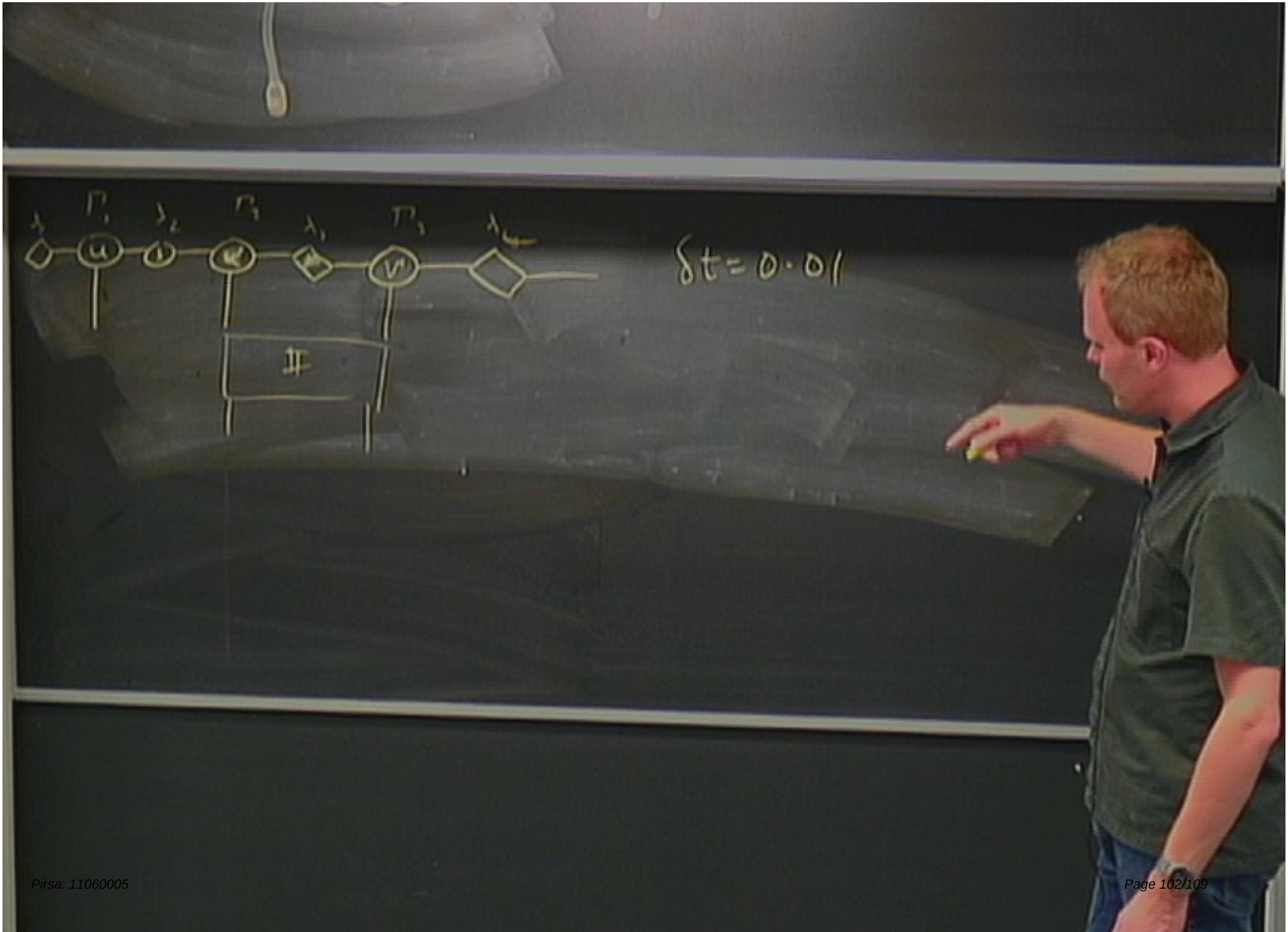

|Ψ₀⟩ = Σ α₁ |α₁⟩ + β₁ |β₁⟩ + ...
e^{-iHδt}

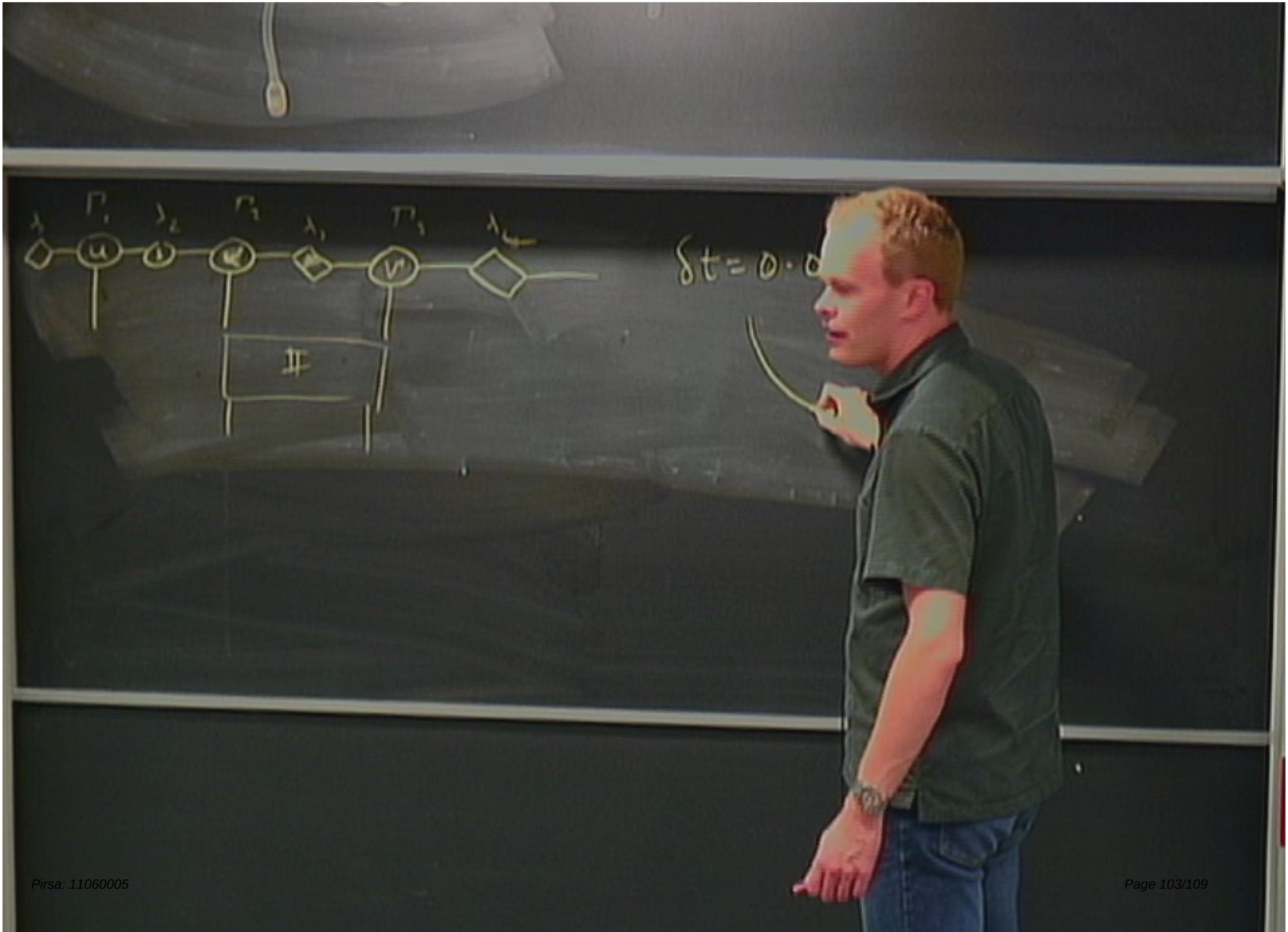


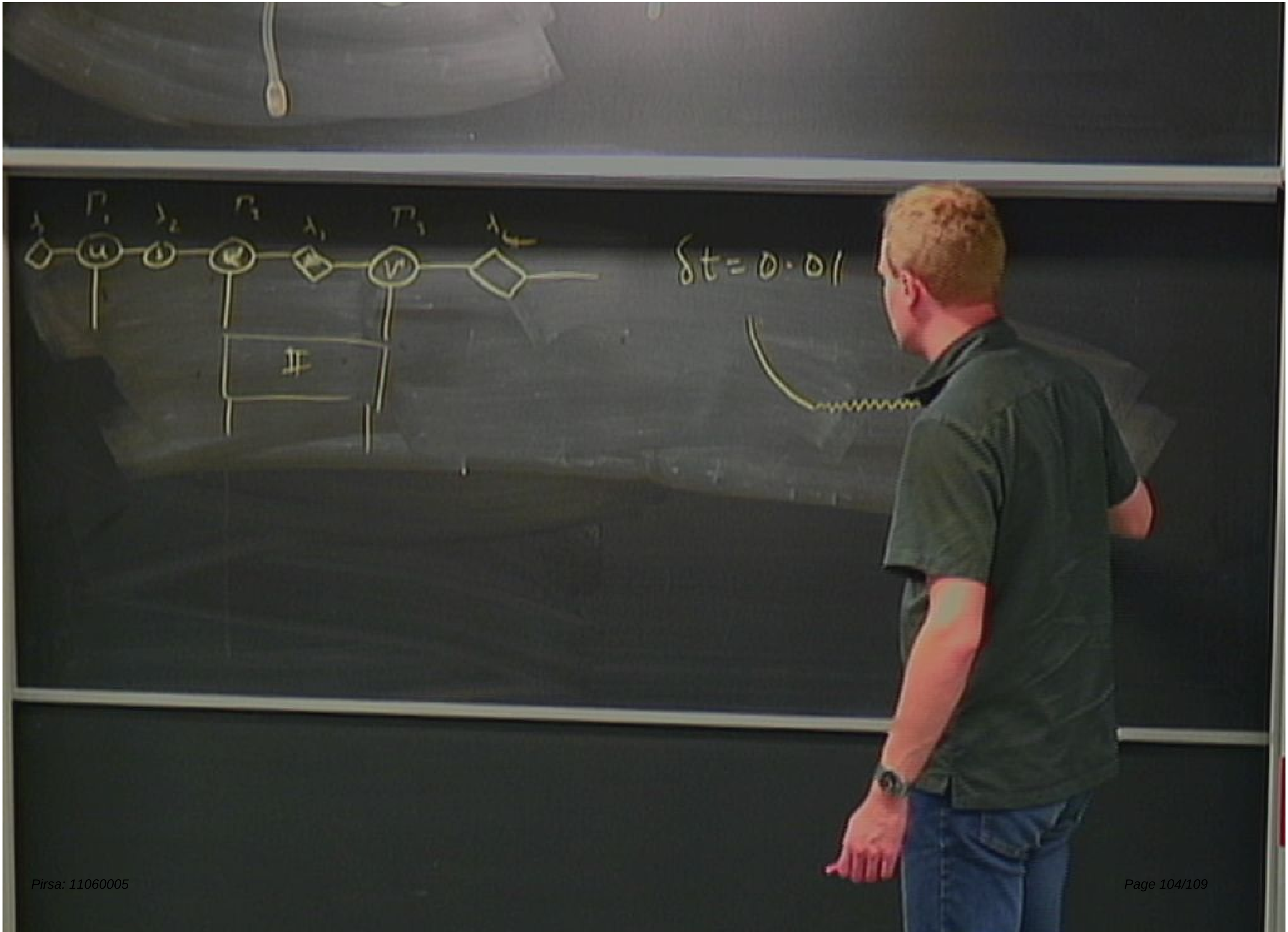


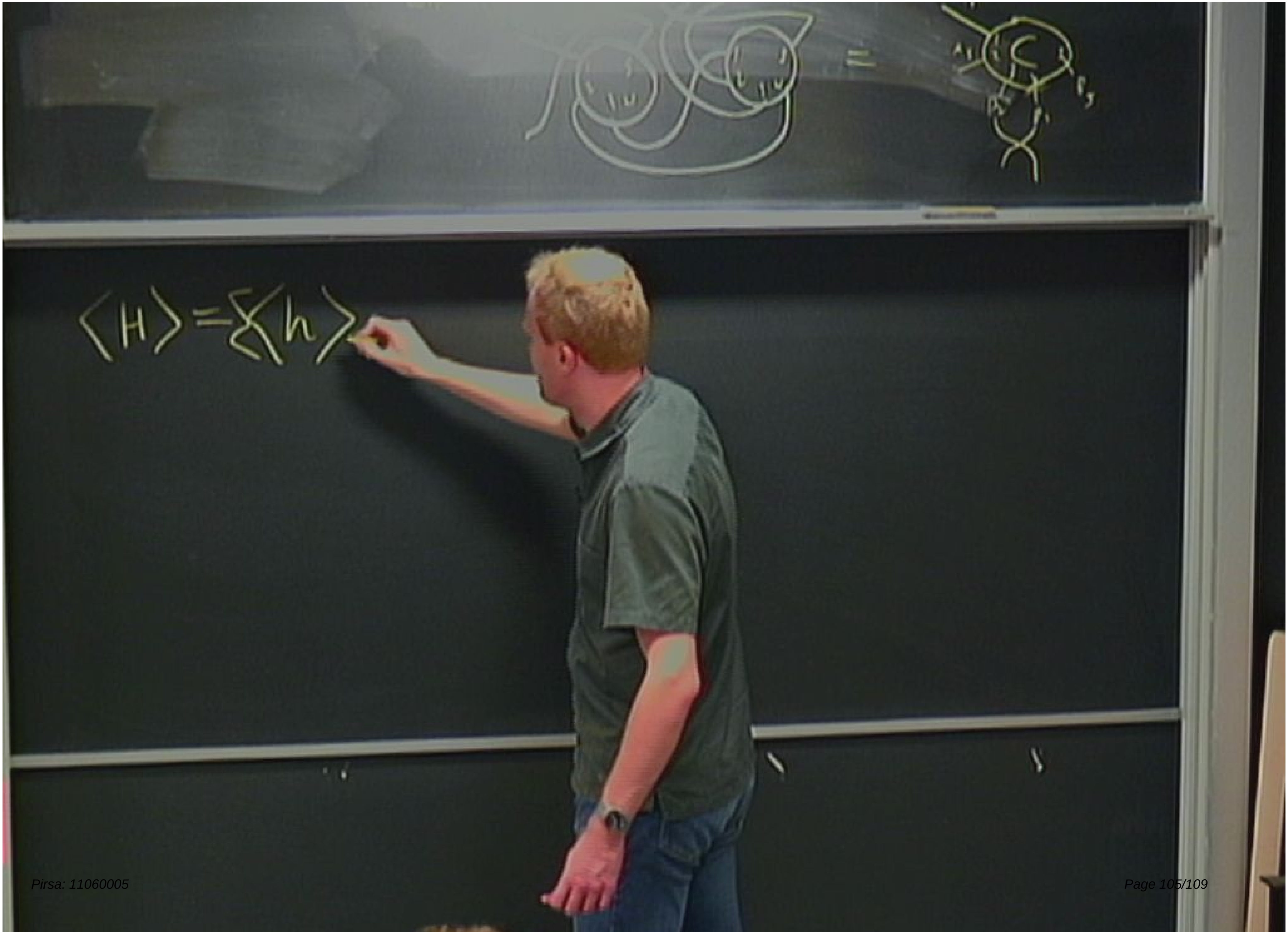


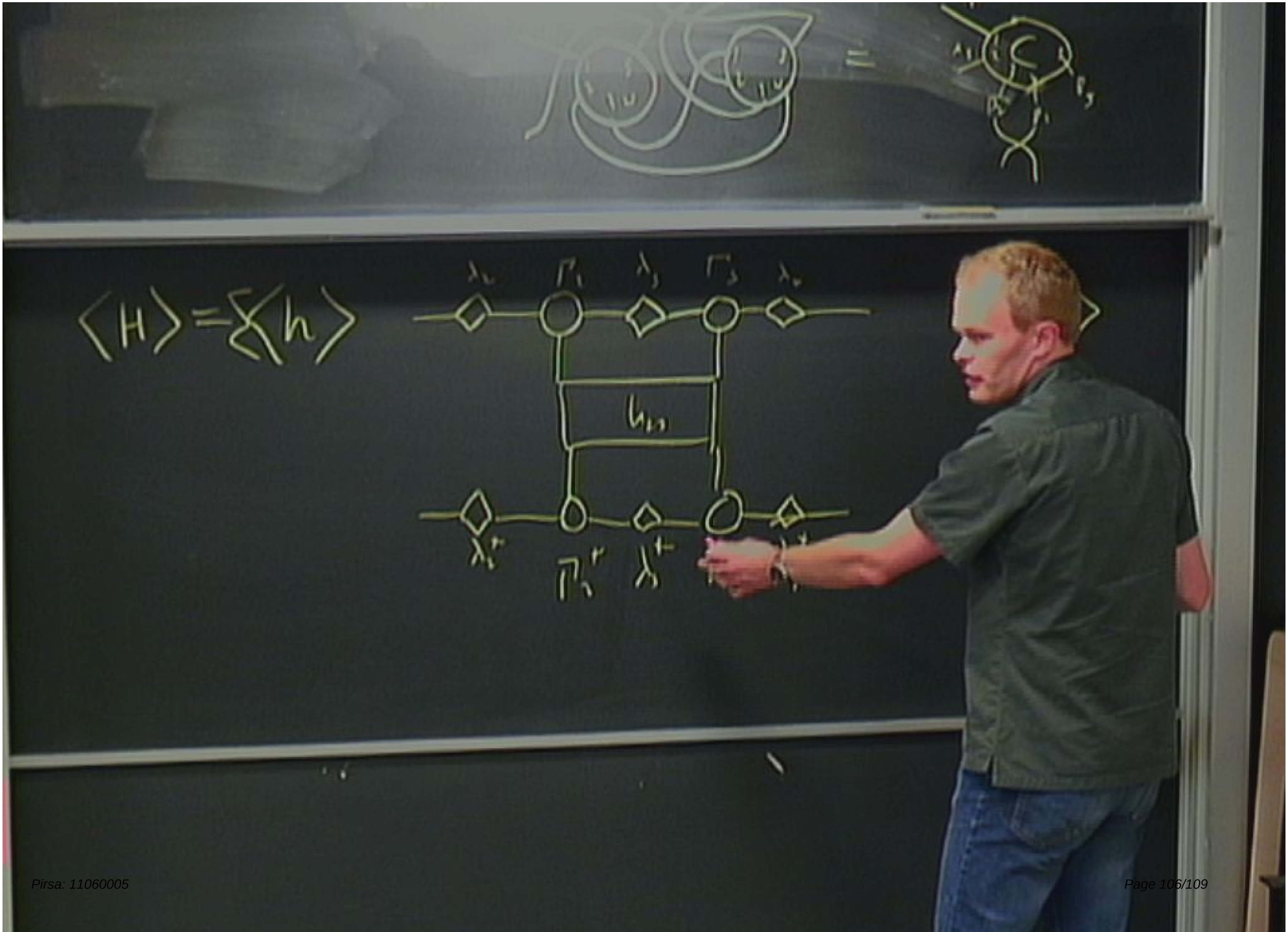


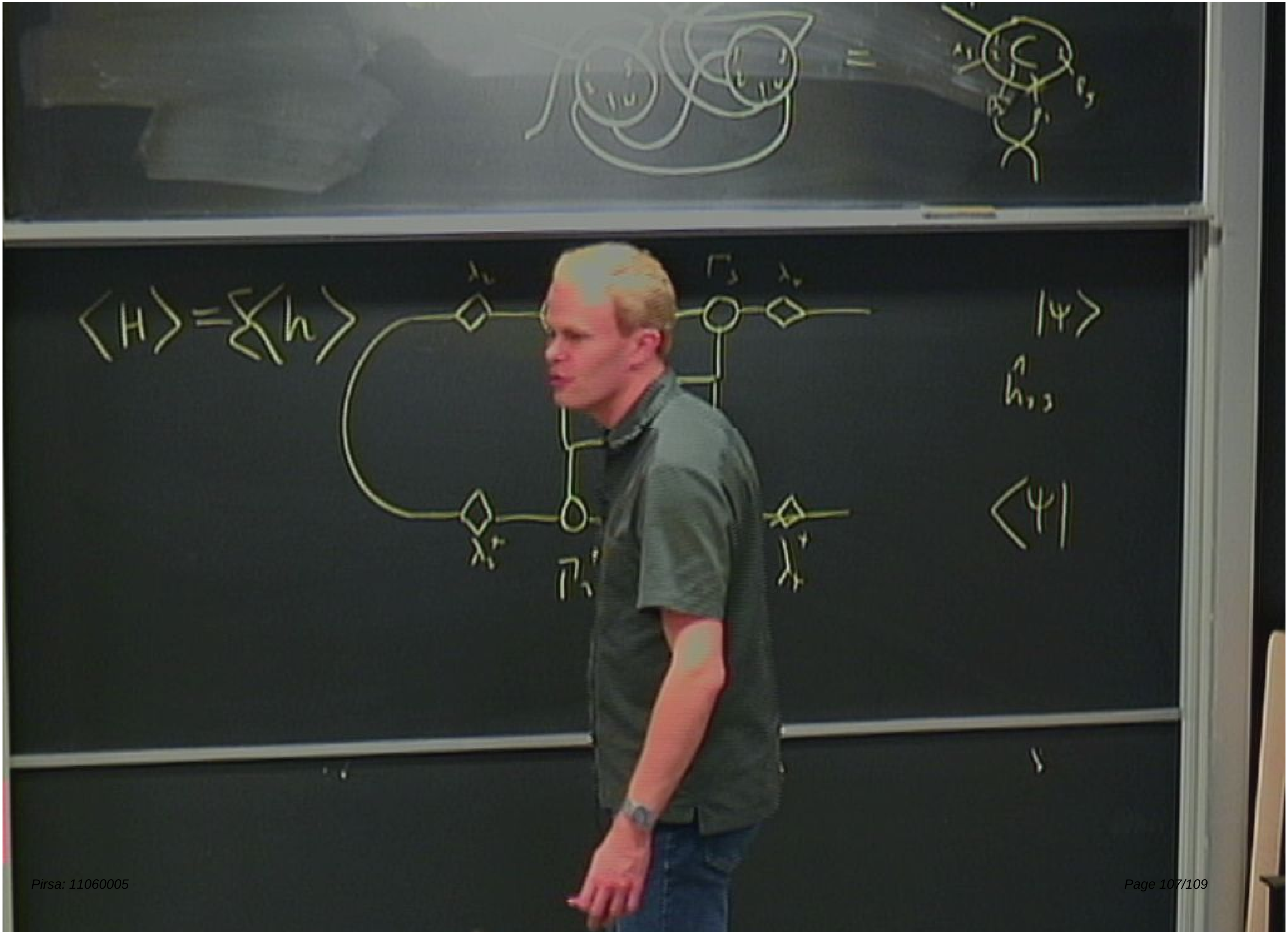

δt = 0·01

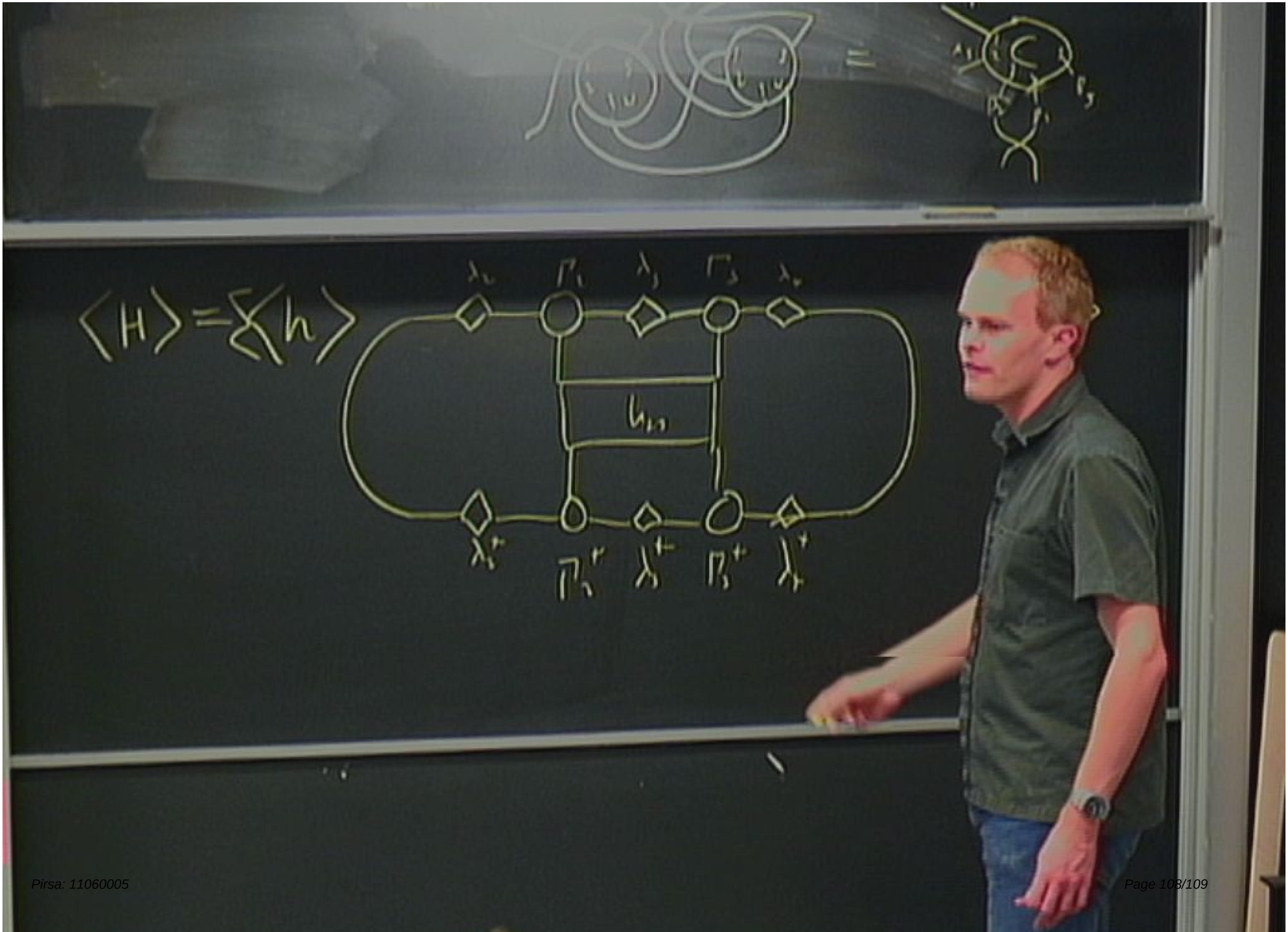

δt = 0·0

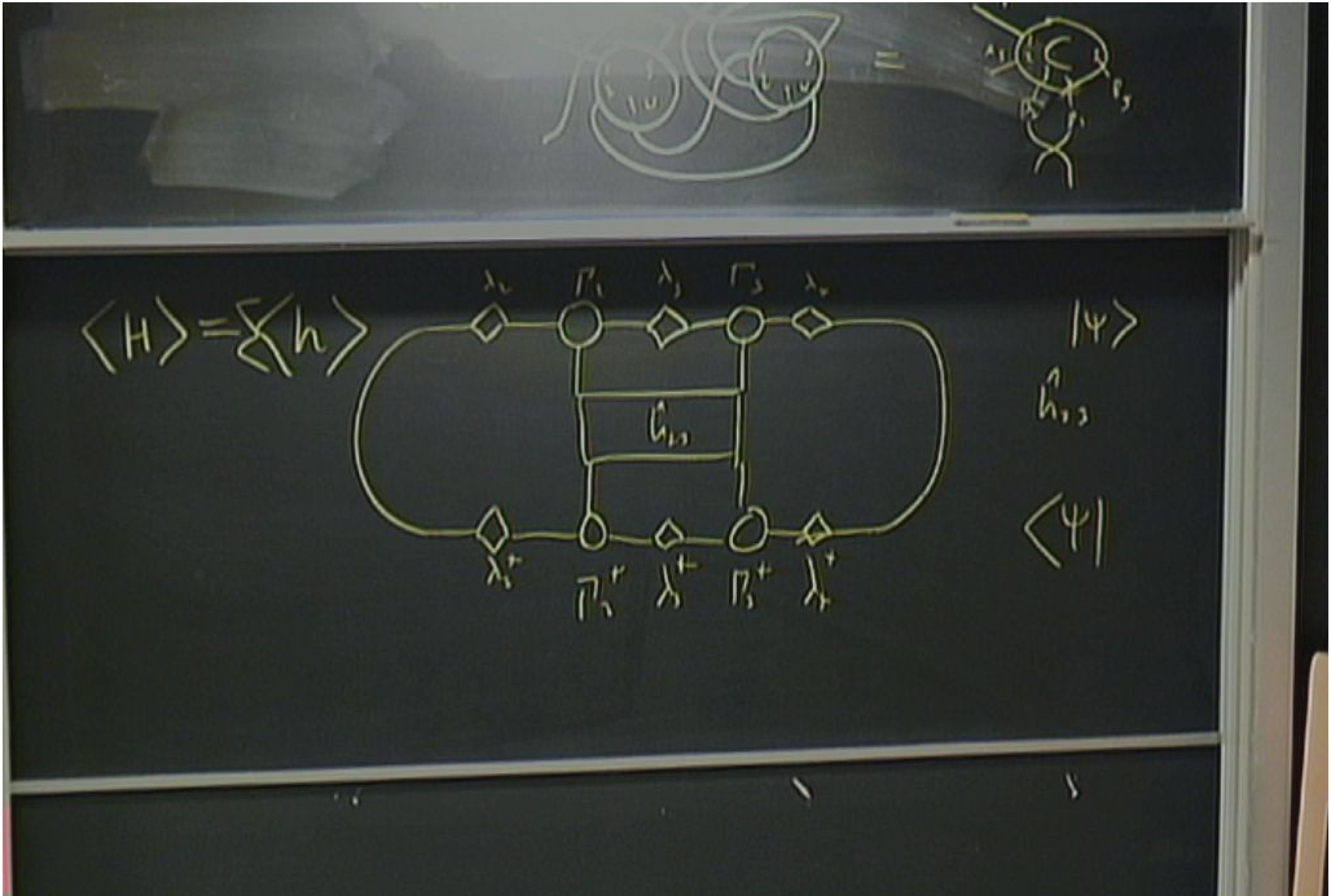

δt = 0·01


⟨H⟩ = Σ⟨h⟩


⟨H⟩ = Σ⟨h⟩


⟨H⟩ = Σ⟨h⟩
|Ψ⟩
⟨Ψ|




$\langle H \rangle = \sum \langle h \rangle$
$|\Psi\rangle$
$\langle \Psi|$