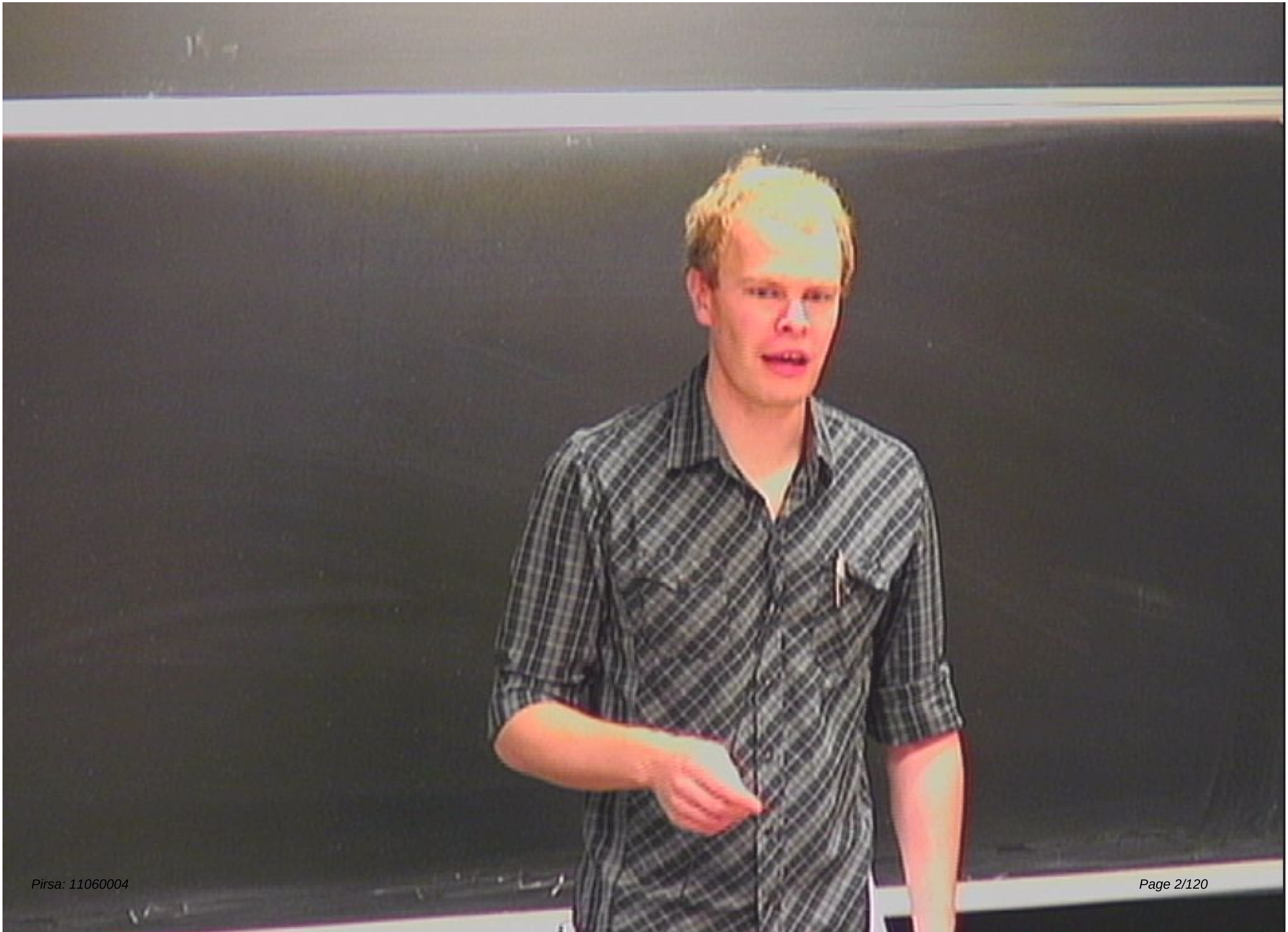
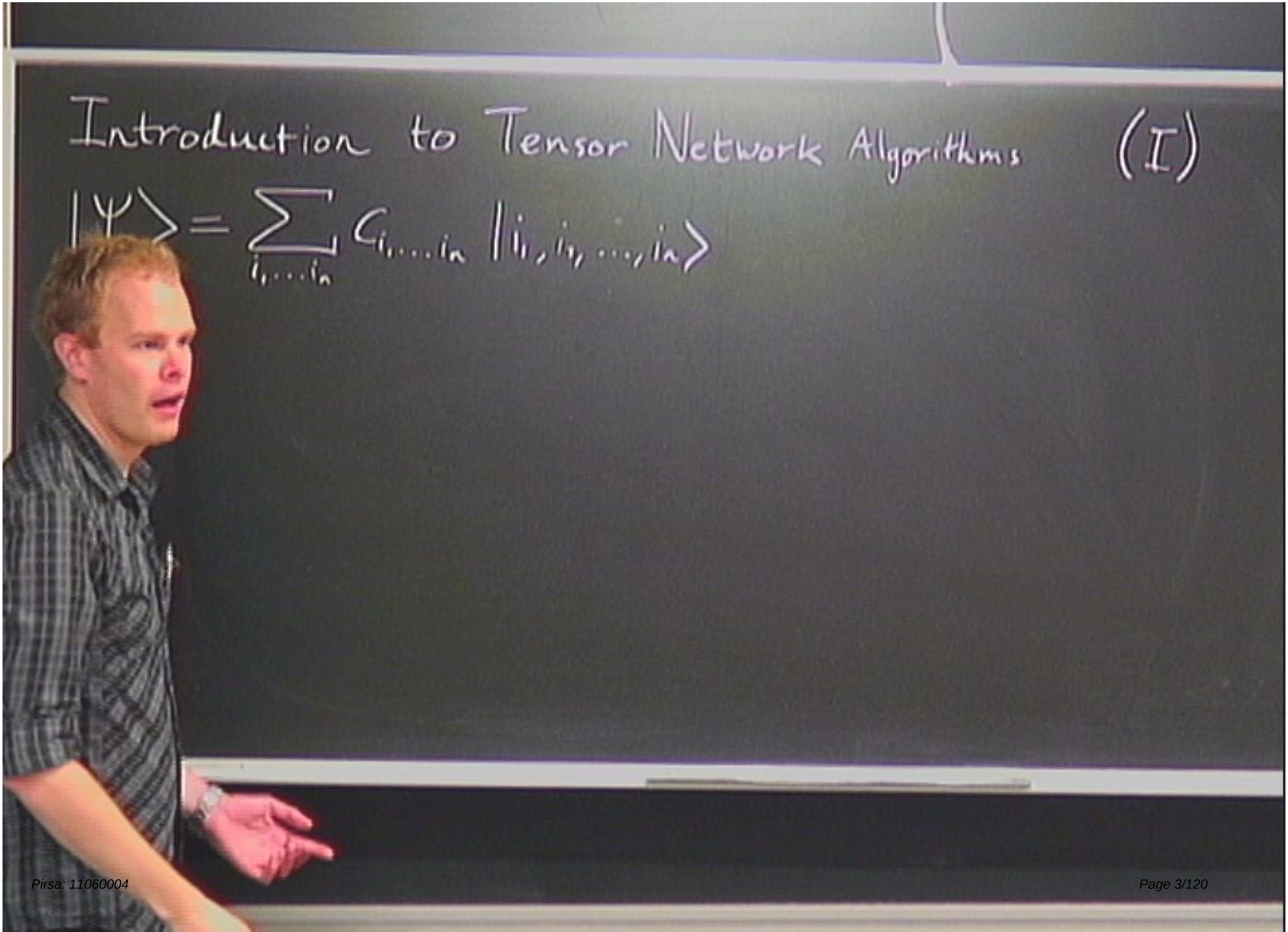
Title: Introduction to Tensor Network Algorithms - Lecture 1

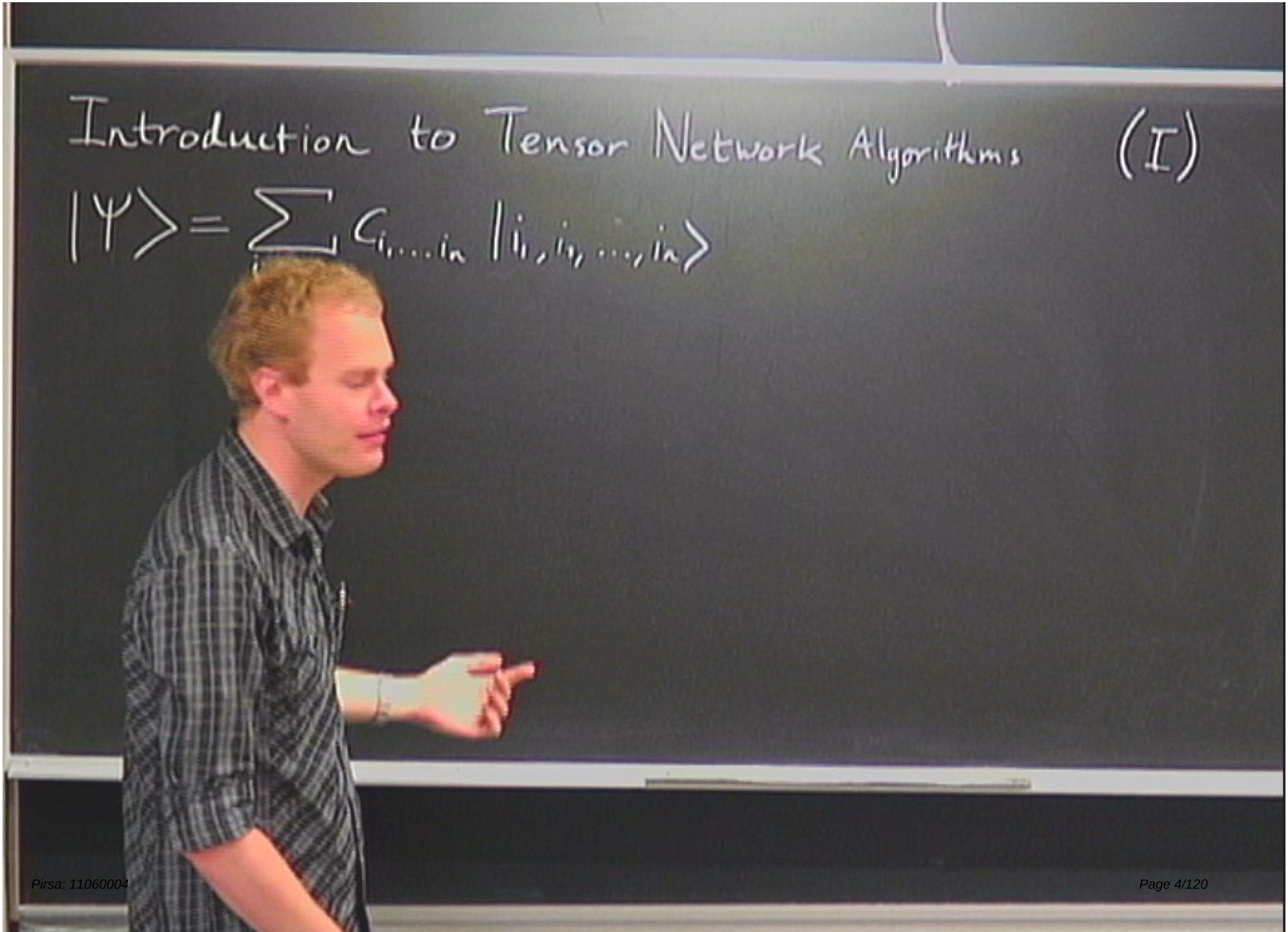
Date: Jun 01, 2011 11:00 AM

URL: http://pirsa.org/11060004

Abstract: Tensor network algorithms are a powerful technique for the study of quantum systems in condensed matter physics. In this short series of lectures, I will present an applied perspective on tensor network algorithms. Topics to be covered will include motivation and methodology, graphical notation, Matrix Product States (MPS) and the Time-Evolving Block Decimation (TEBD) algorithm, identifying the capabilities and limitations of tensor network algorithms, the Multi-scale Entanglement Renormalisation Ansatz (MERA) and the study of systems at criticality, and the exploitation of global internal symmetries. The intent of this lecture series is to provide attendees with the necessary theoretical background to be able to understand and implement the more common tensor network algorithms.




Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum_{i_1,\dots,i_n} C_{i_1\dots i_n} |i_1, i_2, \dots, i_n\rangle$


Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum C_{i_1 \ldots i_n} |i_1, i_2, \ldots, i_n\rangle$

# Introduction to Tensor Network Algorithms (I)

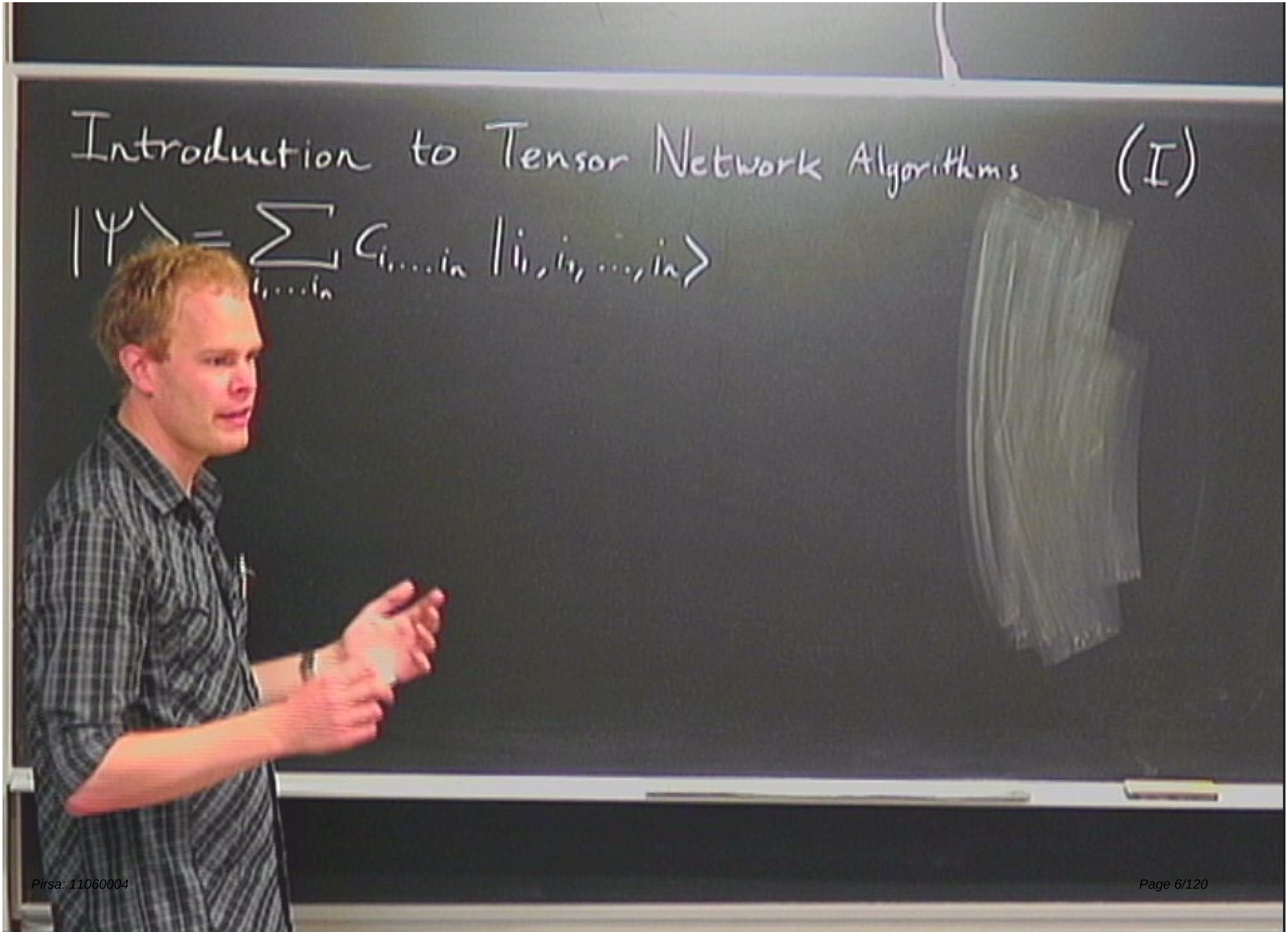
$$|\Psi\rangle = \sum_{i_1,\dots,i_n} C_{i_1,\dots,i_n} |i_1, i_2, \dots, i_n\rangle$$

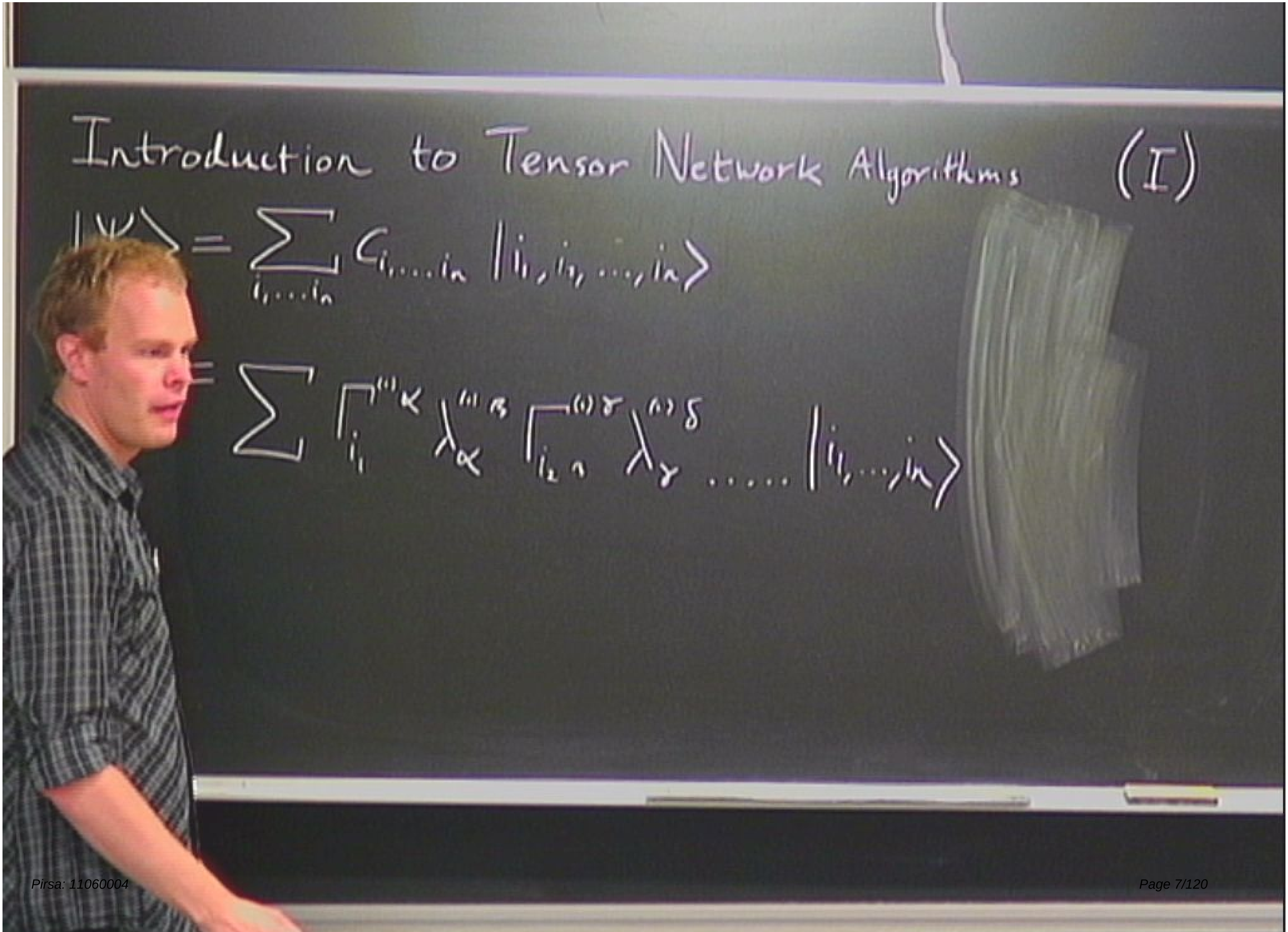
$$|\uparrow\uparrow\dots\rangle$$

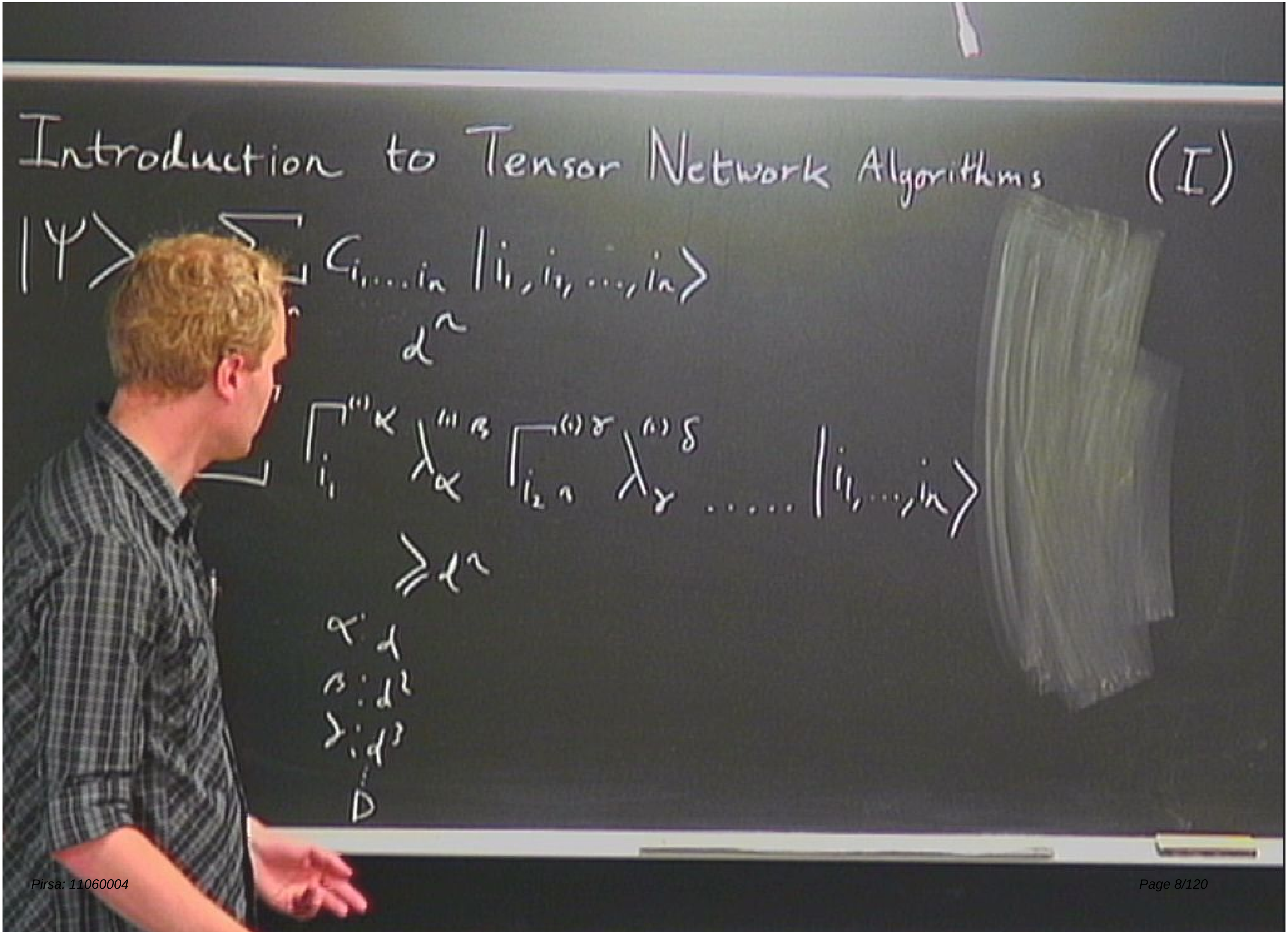
$$|\uparrow\downarrow\dots\rangle$$

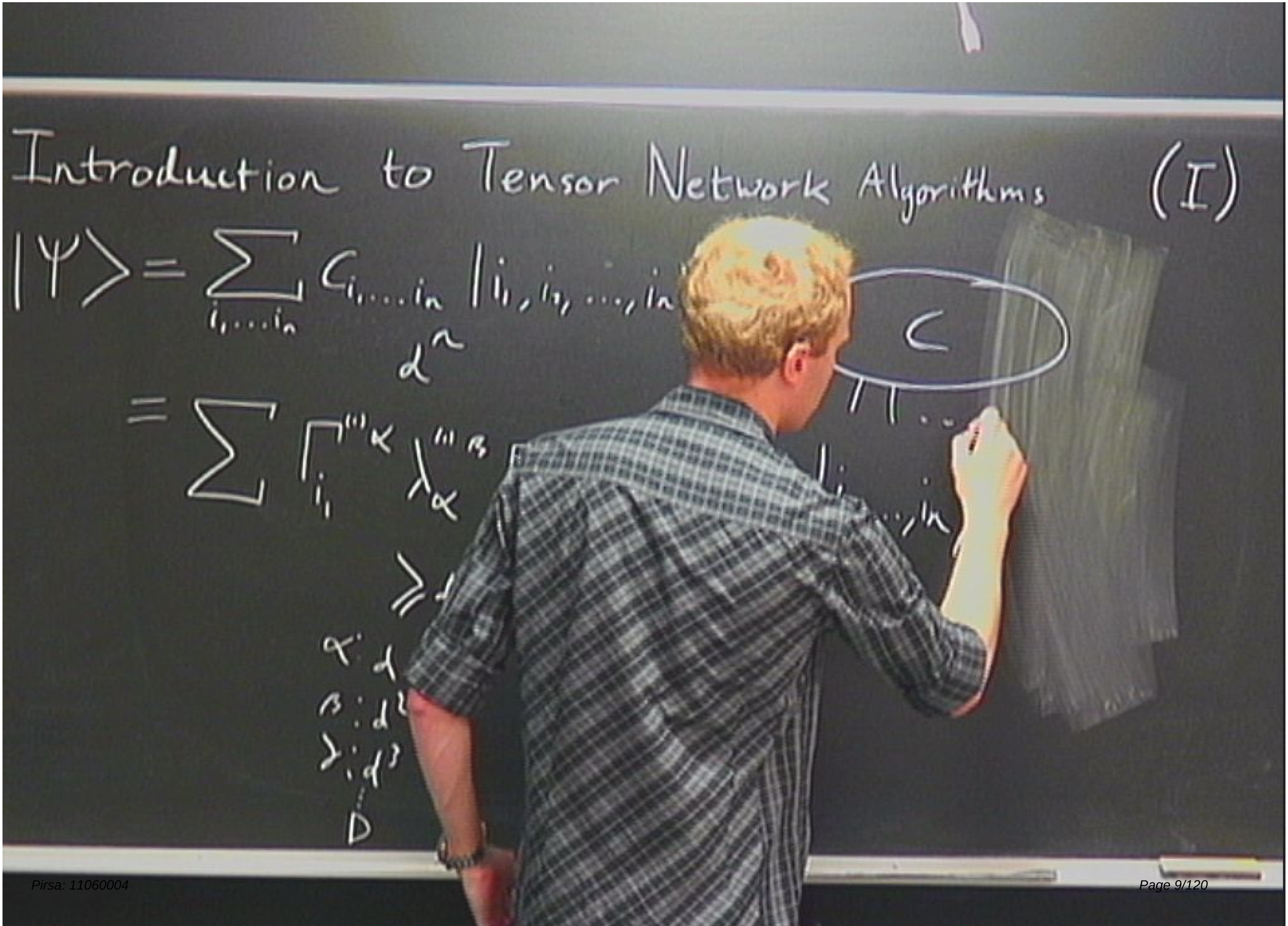
$$|\downarrow\uparrow\dots\rangle$$

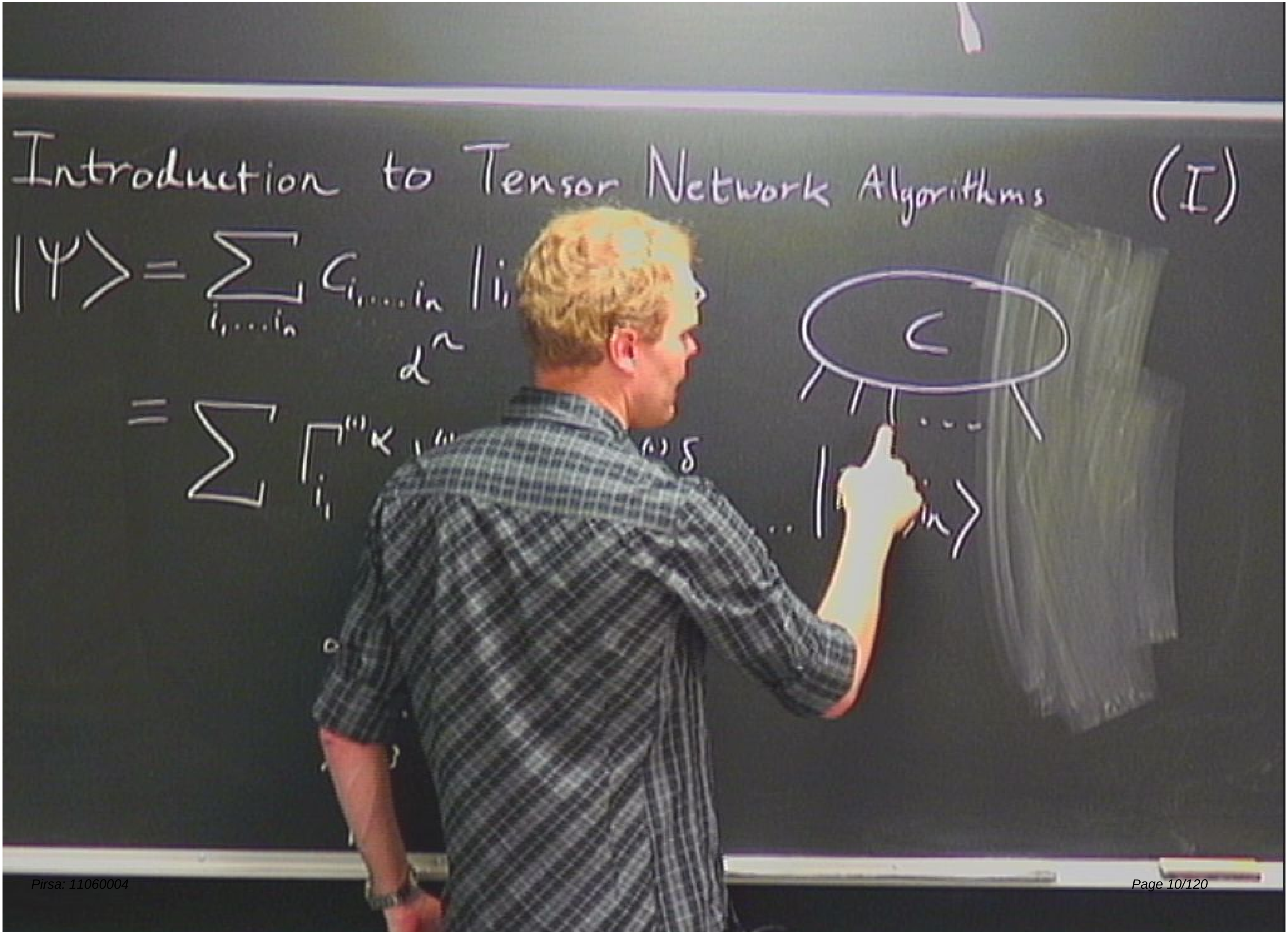
$$\vdots$$

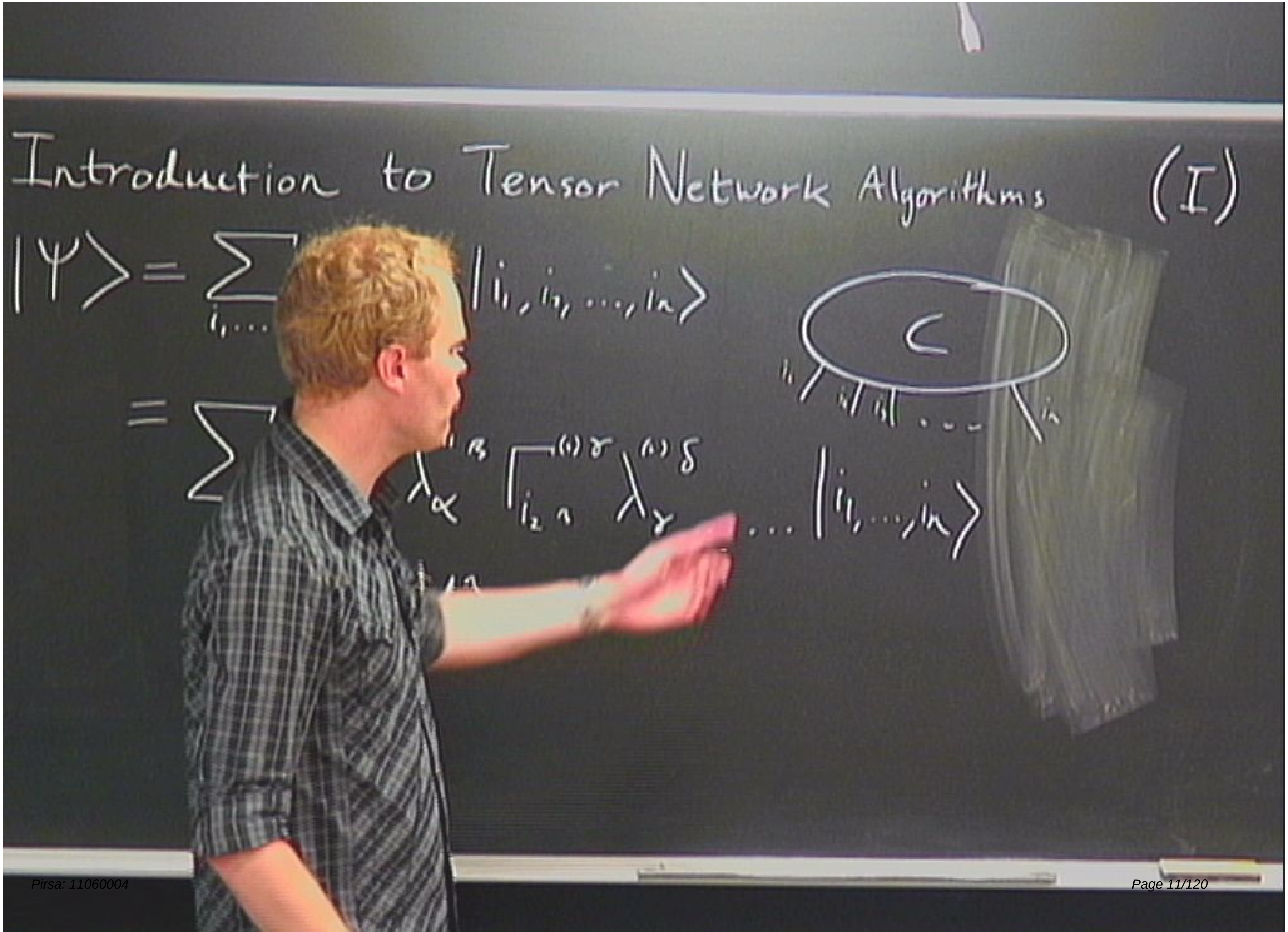

Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum_{i_1,\ldots,i_n} C_{i_1,\ldots,i_n} |i_1, i_2, \ldots, i_n\rangle$


Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum_{i_1,\ldots,i_n} C_{i_1,\ldots,i_n} |i_1, i_2, \ldots, i_n\rangle$
$= \sum \Gamma^{(1)\alpha}_{i_1} \lambda^{(1)\beta}_{\alpha} \Gamma^{(2)\gamma}_{i_2\,\beta} \lambda^{(2)\delta}_{\gamma} \ldots\ldots |i_1, \ldots, i_n\rangle$


Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum c_{i_1 \ldots i_n} |i_1, i_2, \ldots, i_n\rangle$
$d^n$
$\Gamma^{(1)\alpha}_{i_1} \lambda^{(1)\beta}_{\alpha} \Gamma^{(2)\gamma}_{i_2\,\beta} \lambda^{(2)\delta}_{\gamma} \ldots\ldots |i_1, \ldots, i_n\rangle$
$\geq d^n$
$\alpha: d$
$\beta: d^2$
$\gamma: d^3$
$D$


Introduction to Tensor Network Algorithms (I)
C


Introduction to Tensor Network Algorithms (I)
$|\Psi\rangle = \sum_{i_1,\dots,i_n} C_{i_1 \dots i_n} |i_1$
$d^n$
C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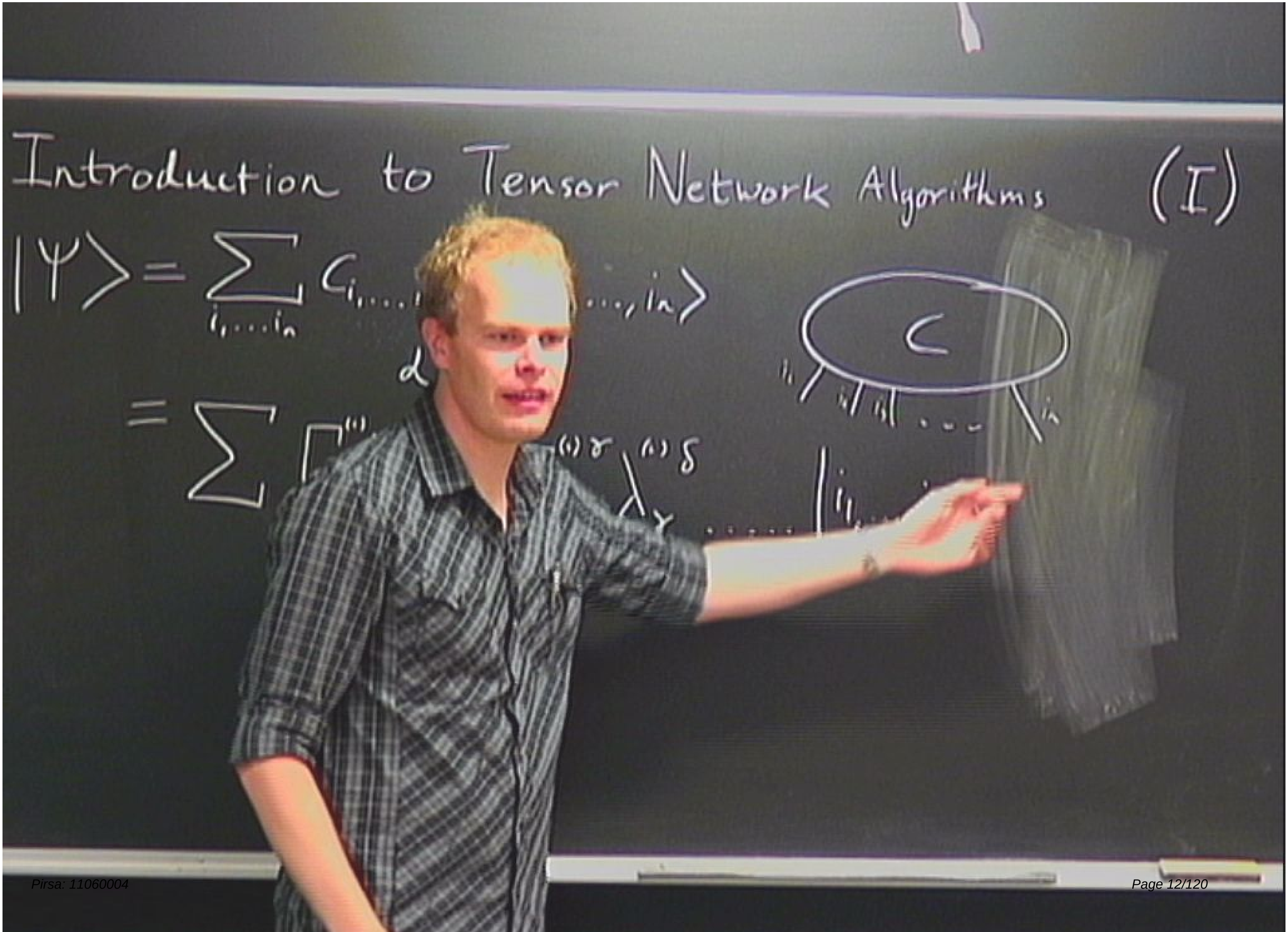
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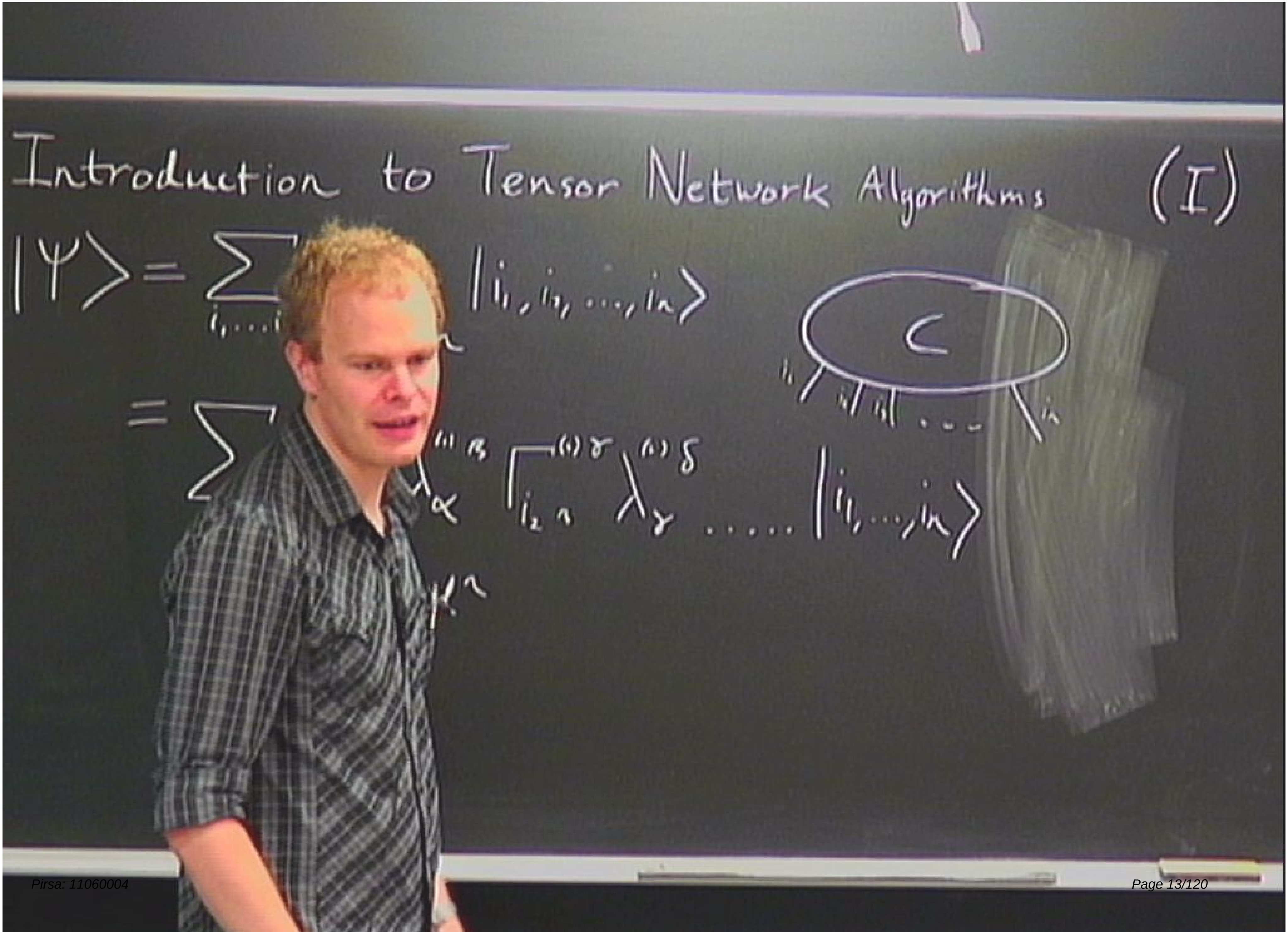
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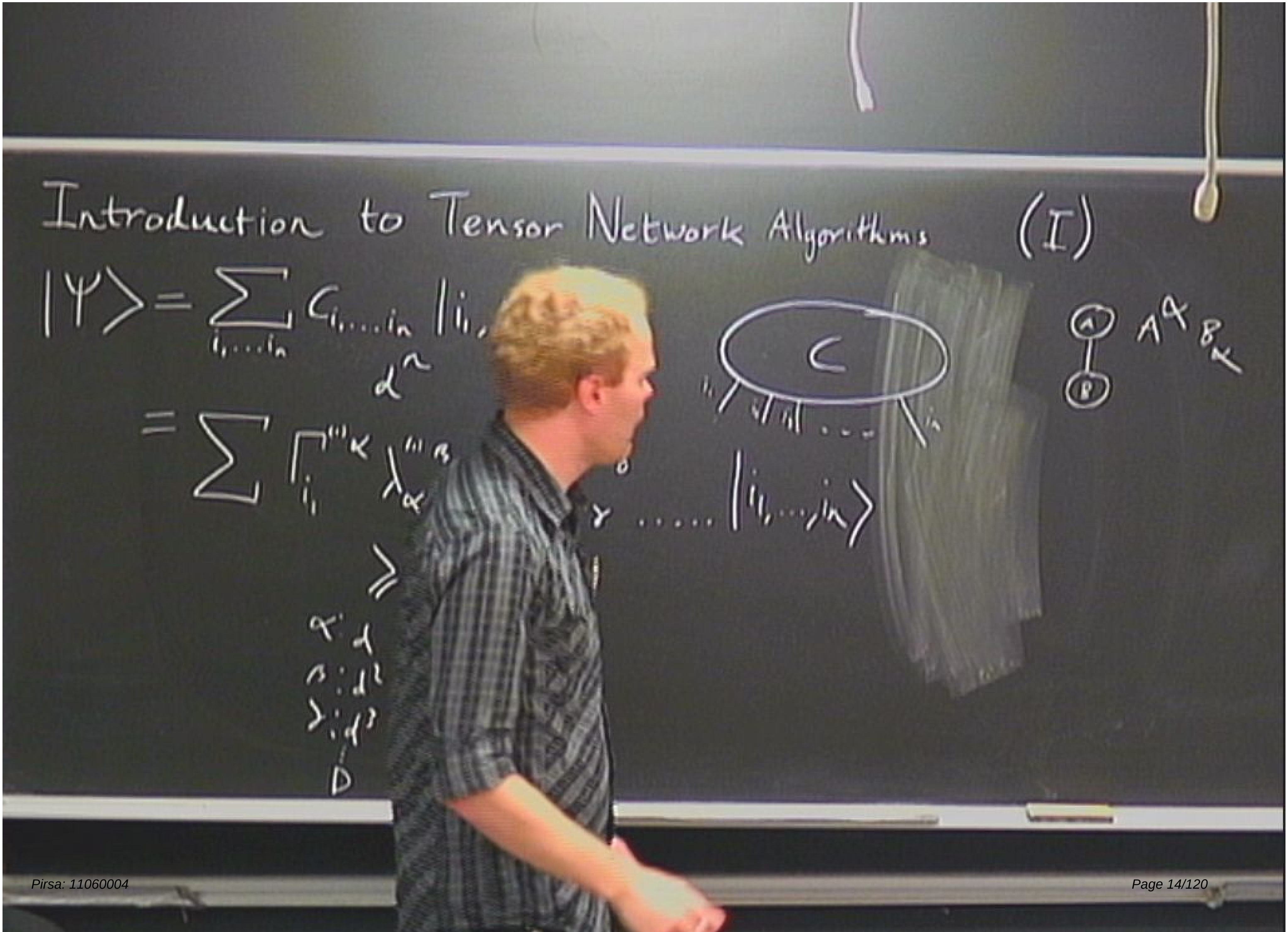
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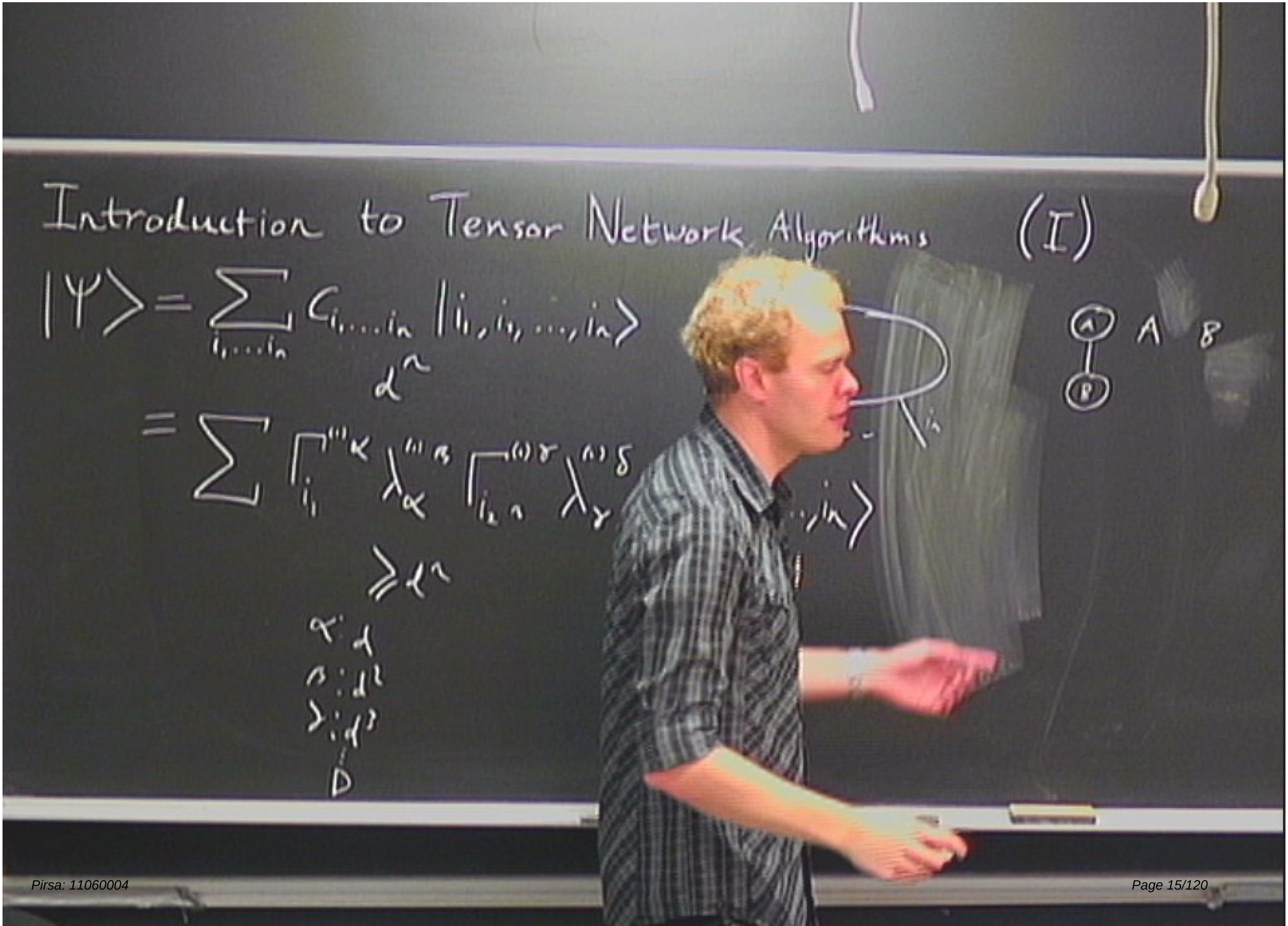
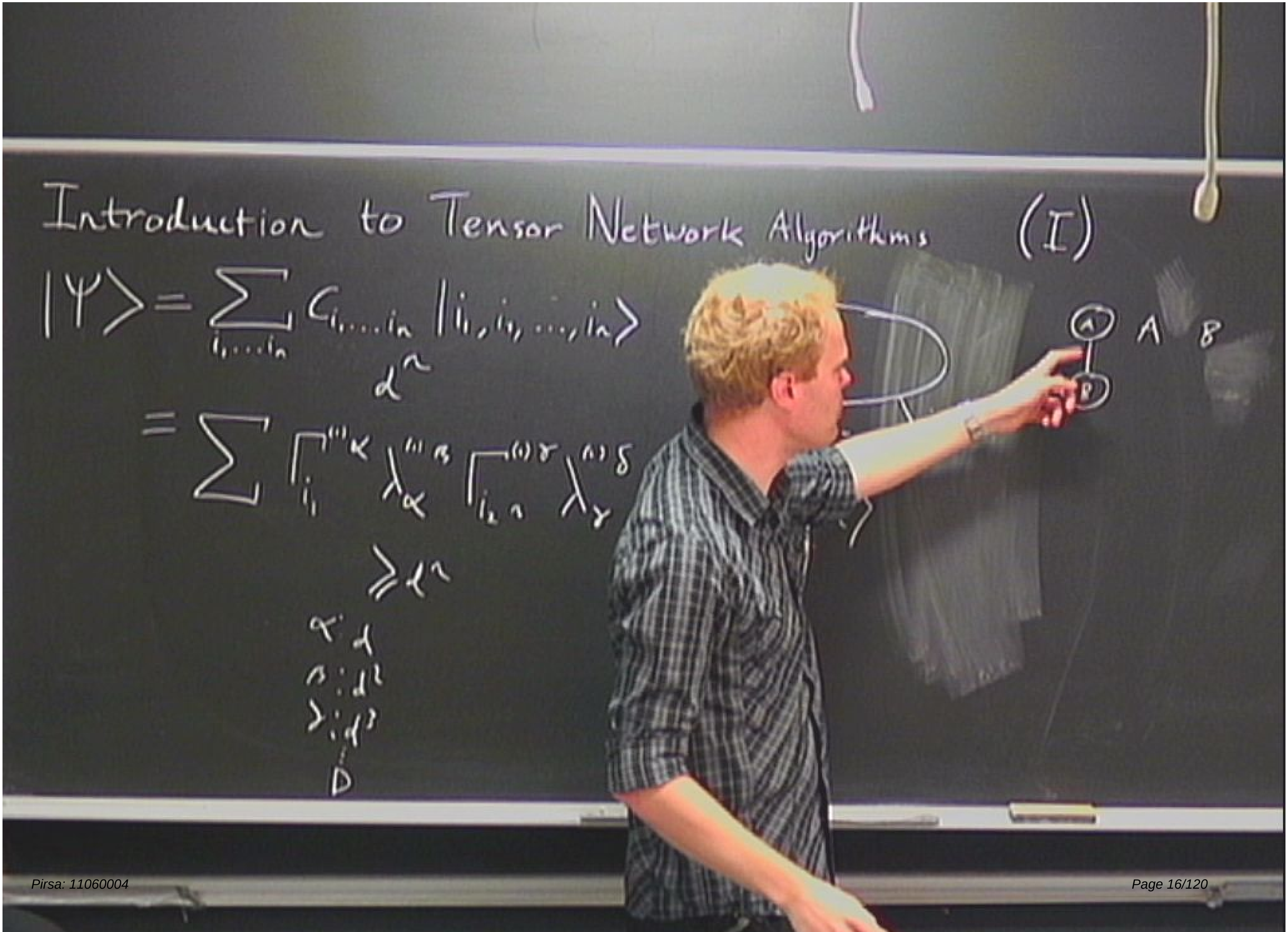
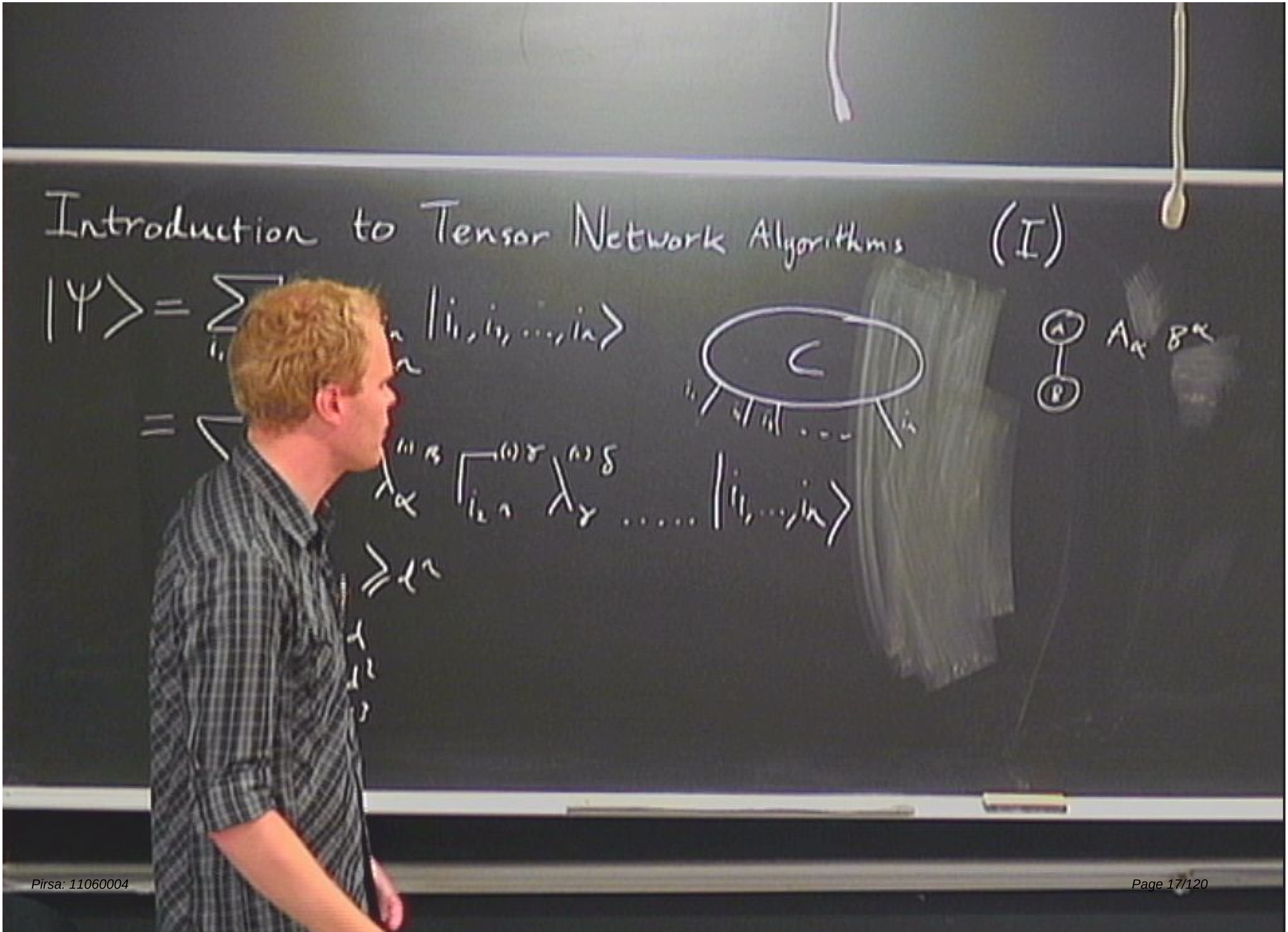
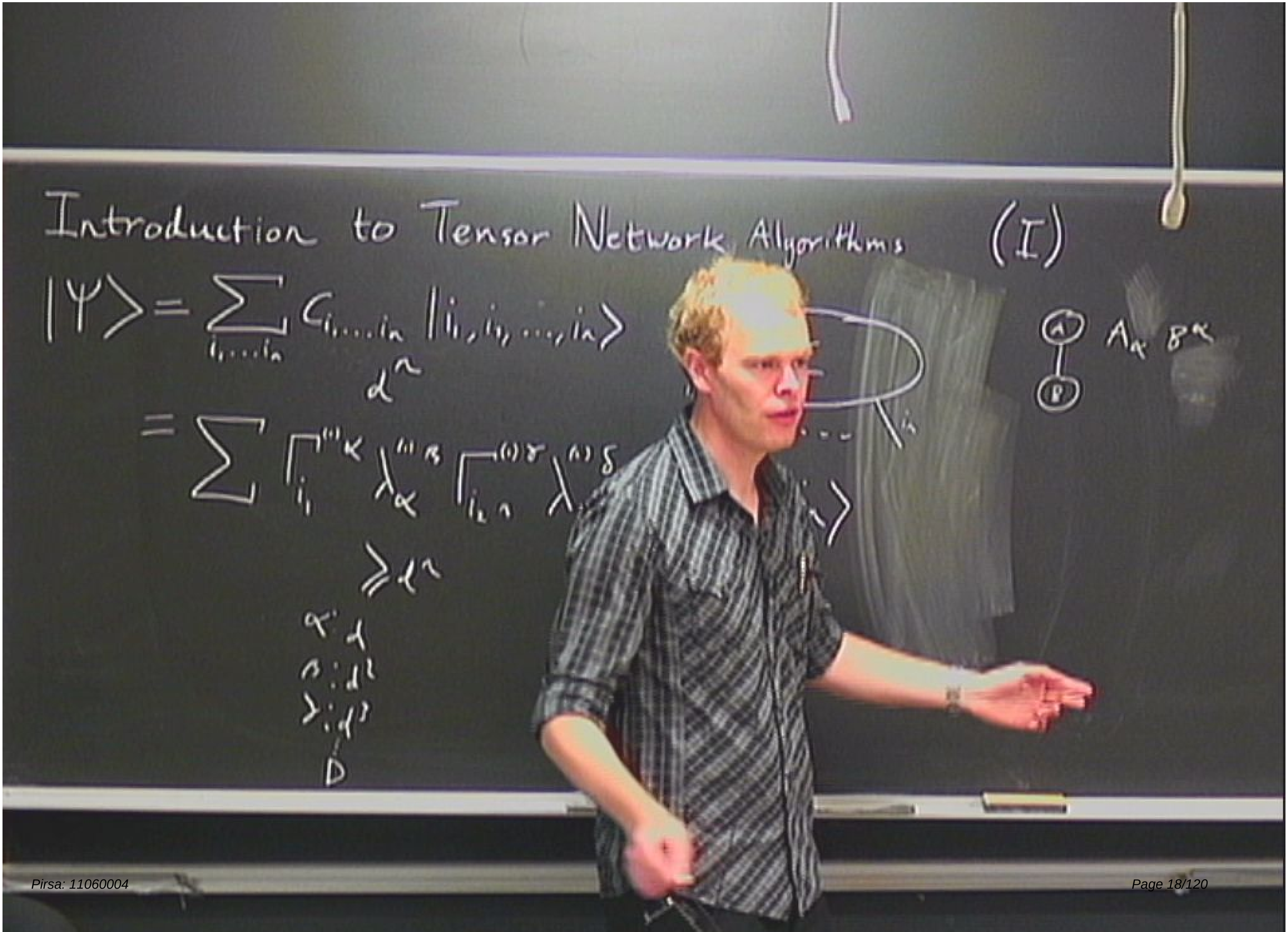

Introduction to Tensor Network Algorithms (I)
A B


Introduction to Tensor Network Algorithms (I)
|Ψ> = Σ C_{i1,...,in} |i1, i2, ..., in>
i1,...,in
d^n
= Σ Γ^{(1)α}_{i1} λ^{(1)β}_α Γ^{(2)γ}_{i2 β} λ^{(2)δ}_γ
≥ d^n
α : d
β : d^2
γ : d^3
D
A
B


Introduction to Tensor Network Algorithms (I)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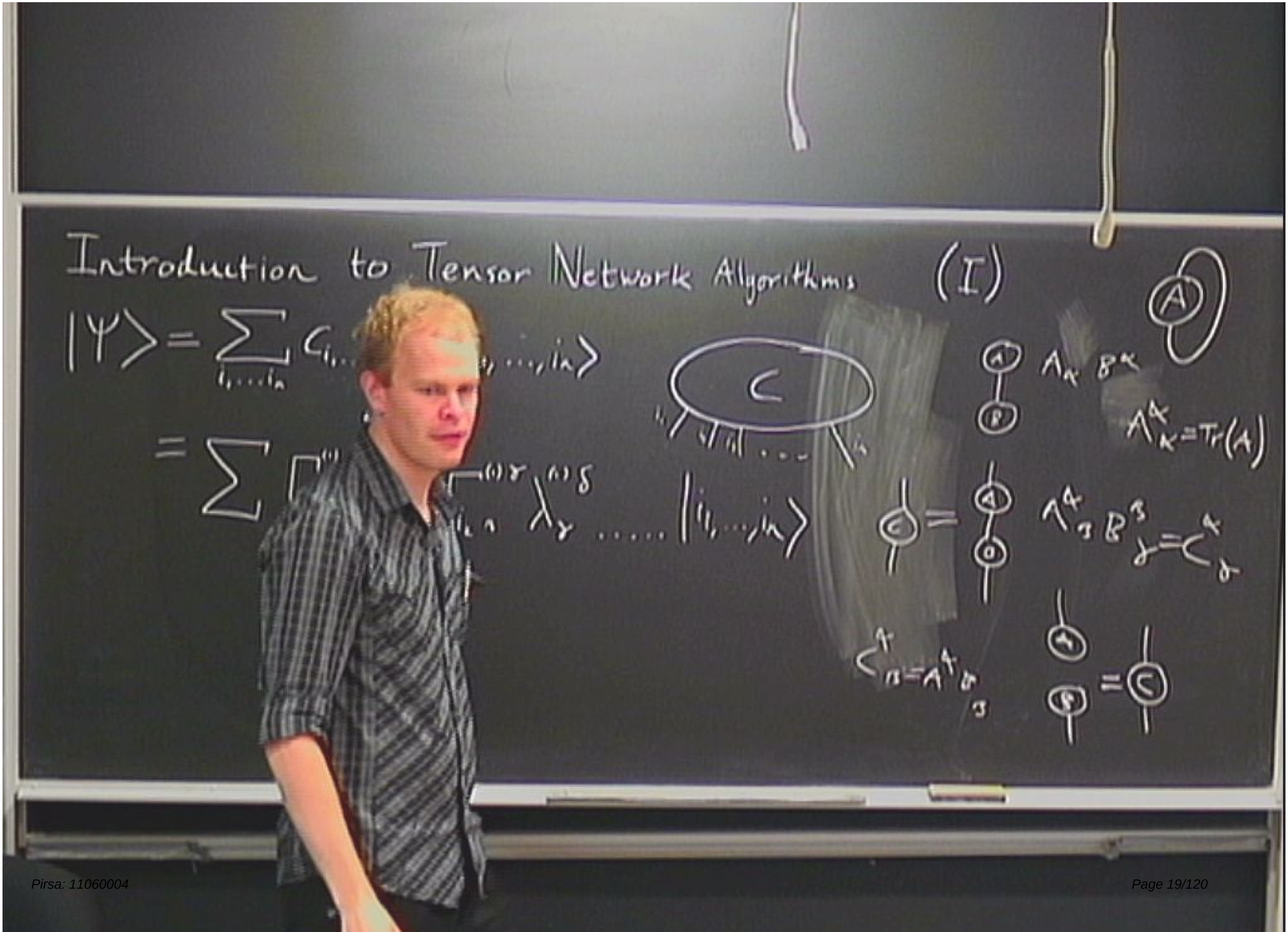
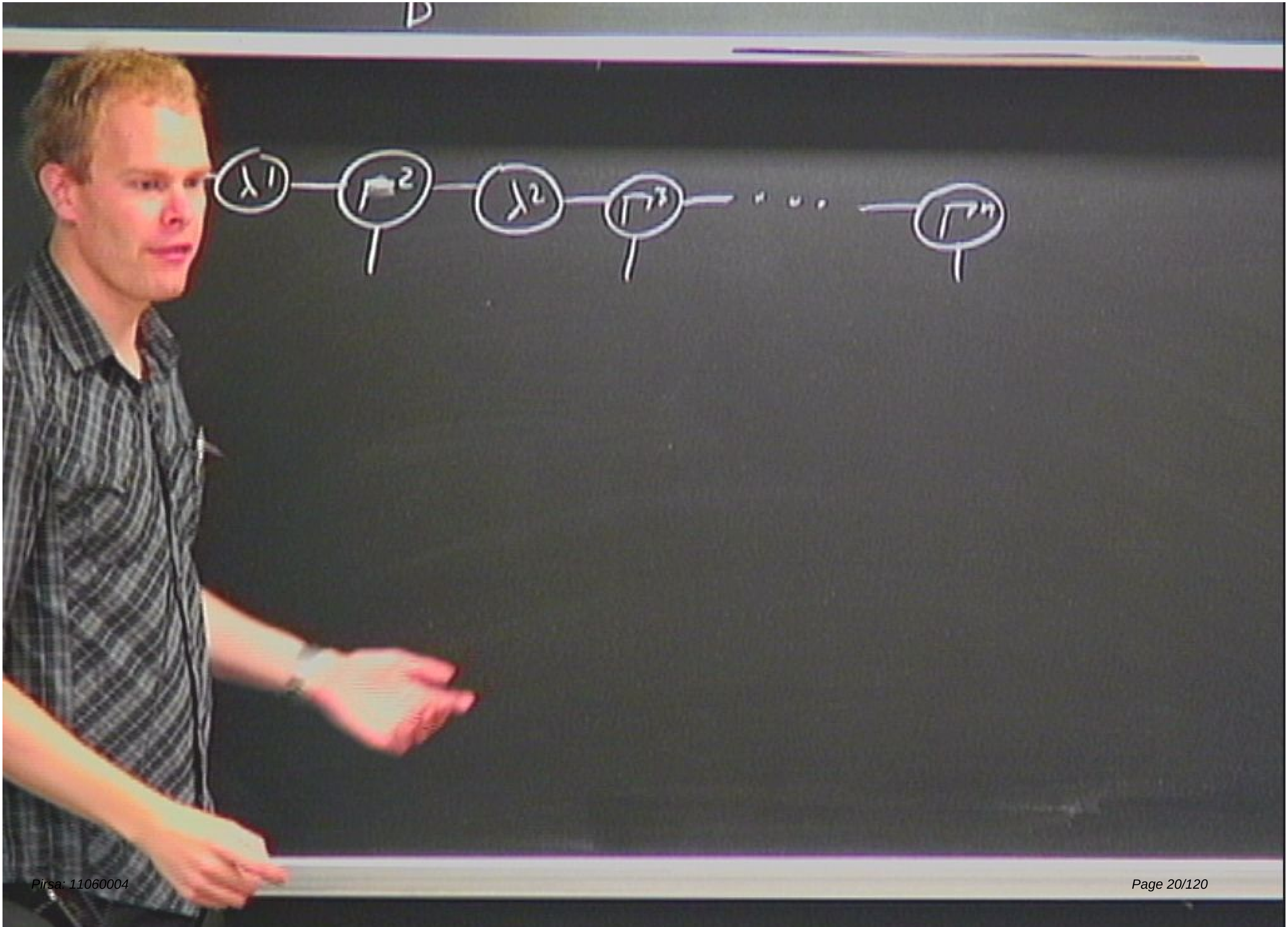
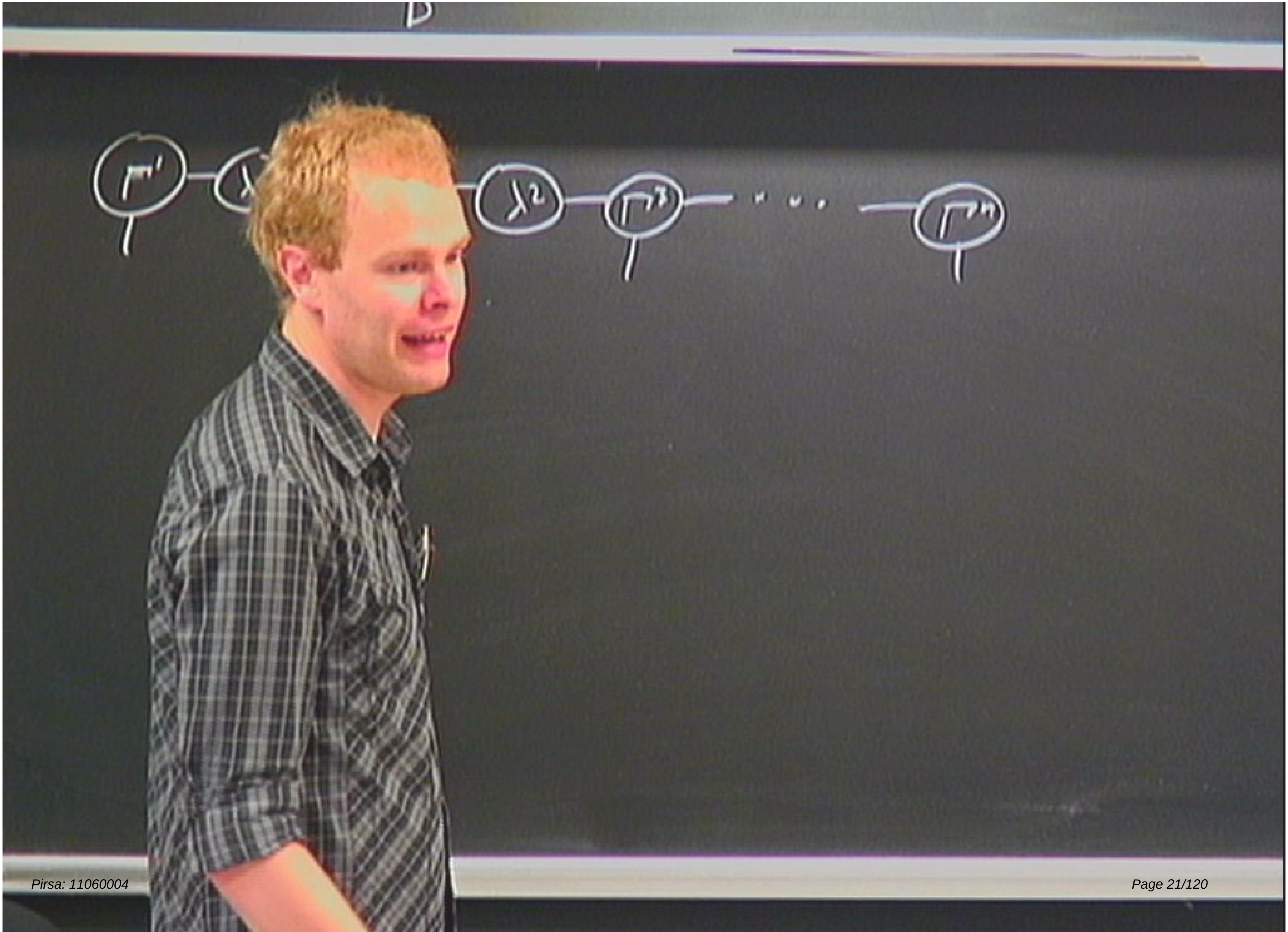
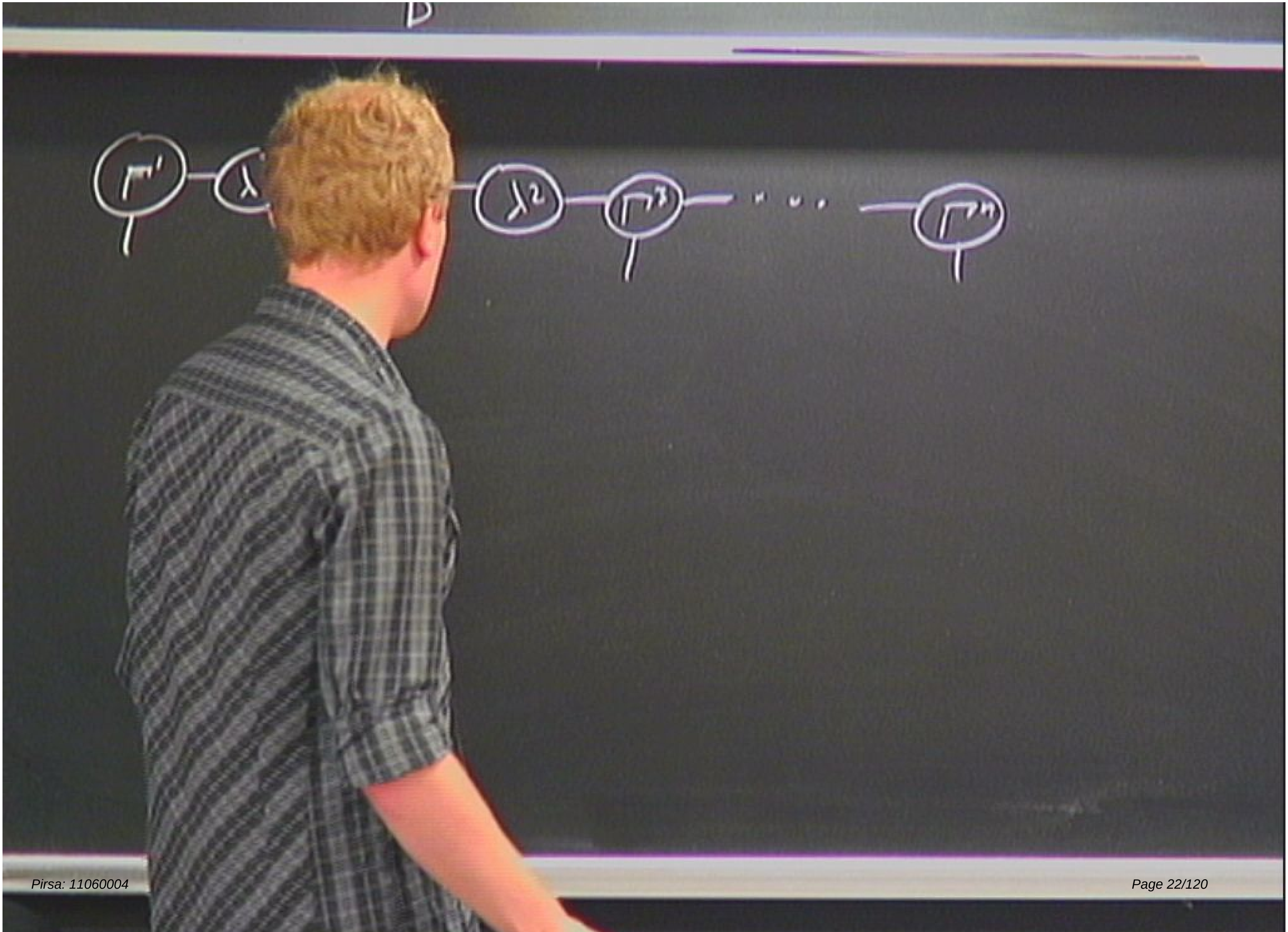
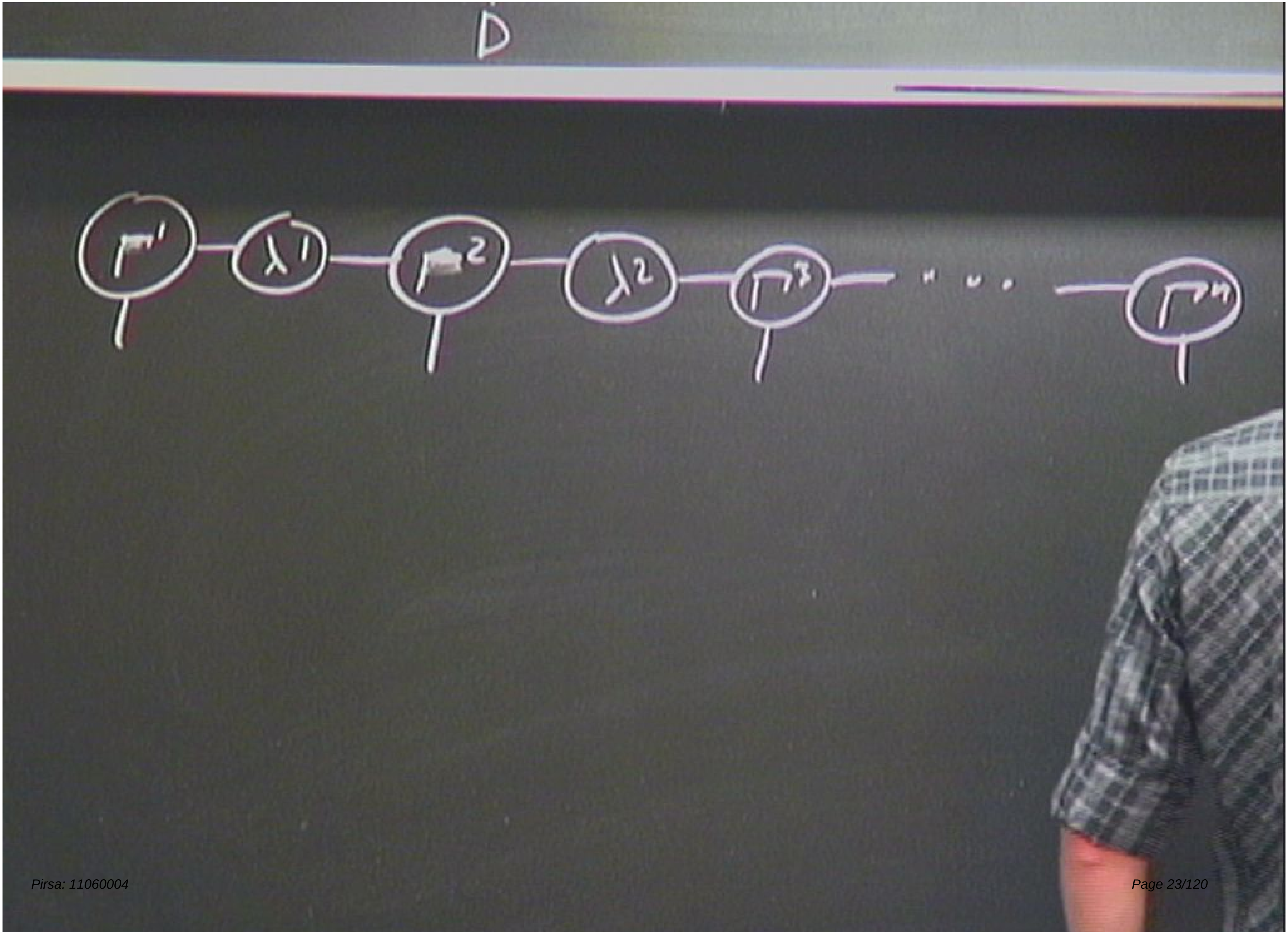

Introduction to Tensor Network Algorithms (I)
|Ψ> = Σ C
= Σ
C
A
B
$A_\alpha B^\alpha$
$A^\alpha_\alpha = Tr(A)$
$A^\alpha_\beta B^\beta_\gamma = C^\alpha_\gamma$
$C^\alpha_\beta = A^\alpha_\gamma B^\gamma_\beta$

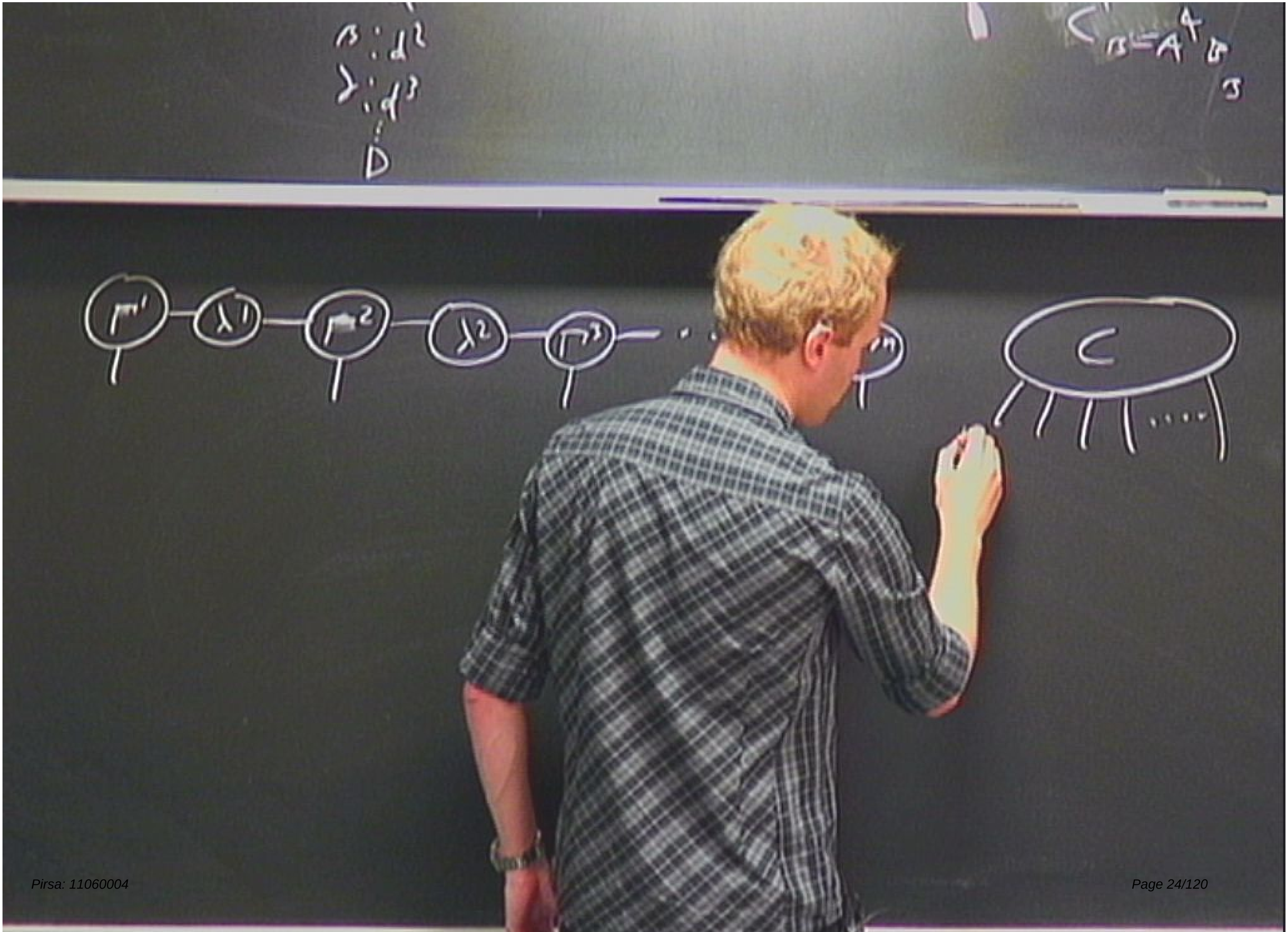

λ1
Γ2
λ2
Γ3
. . .
Γn

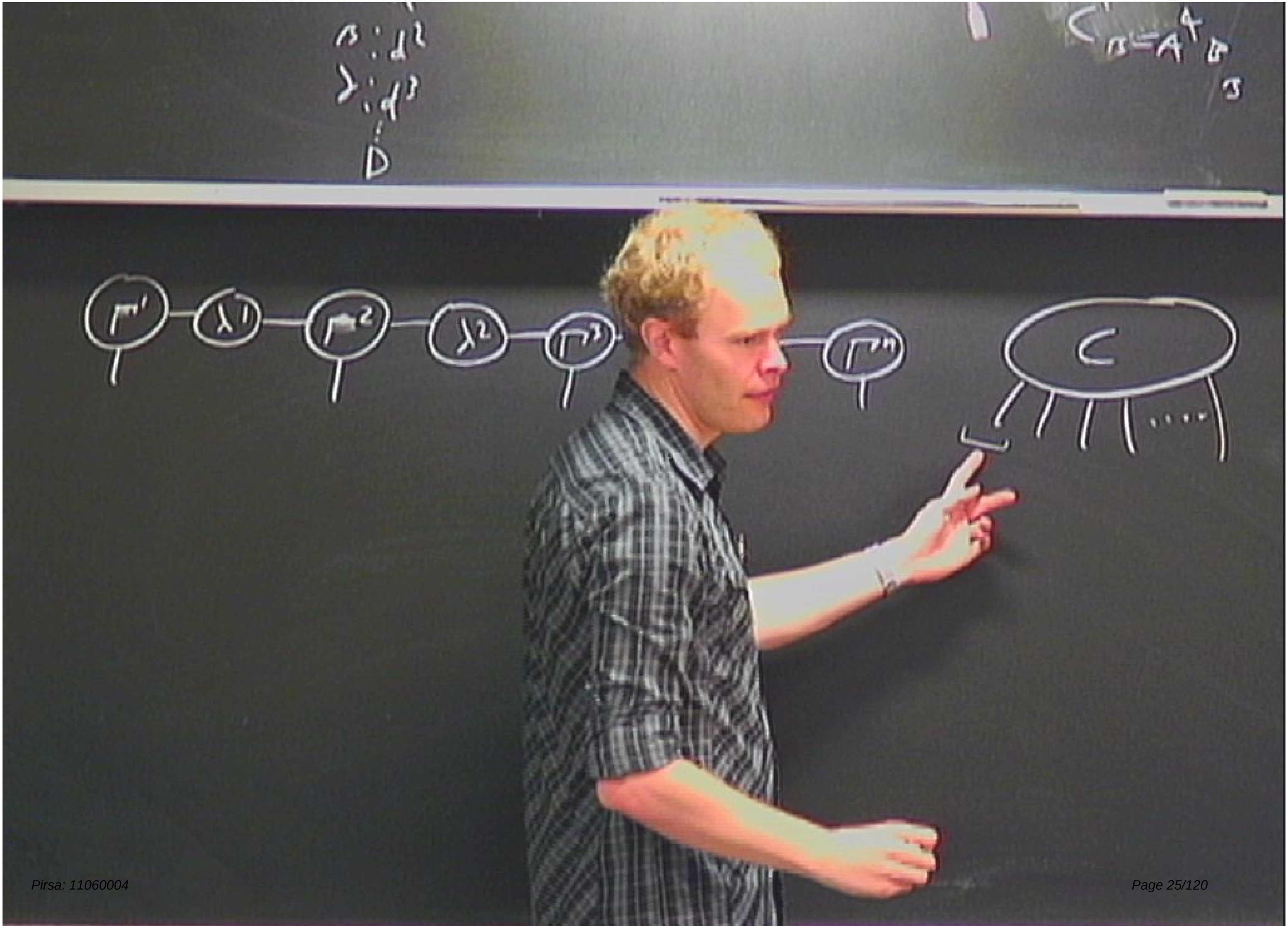


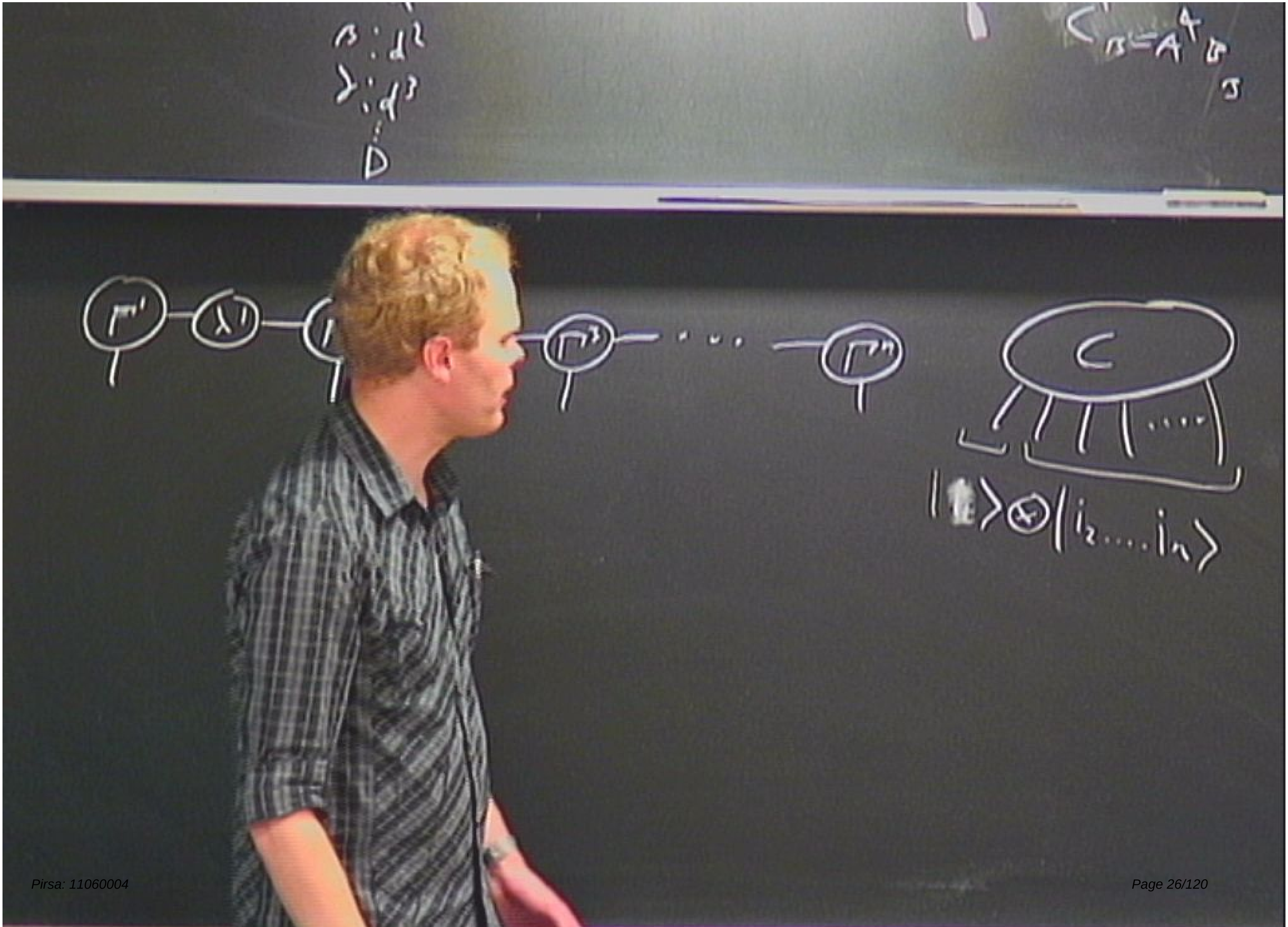


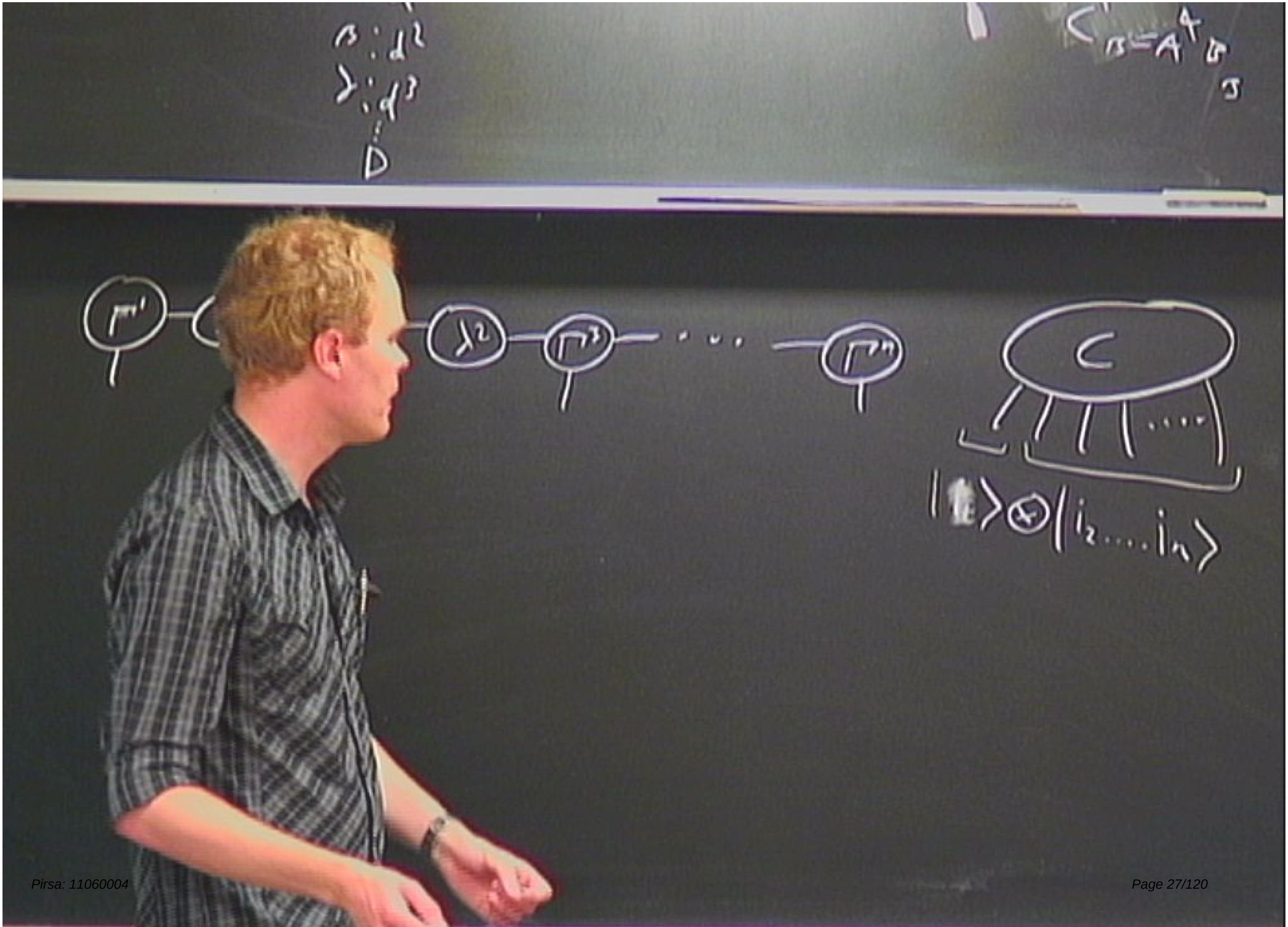


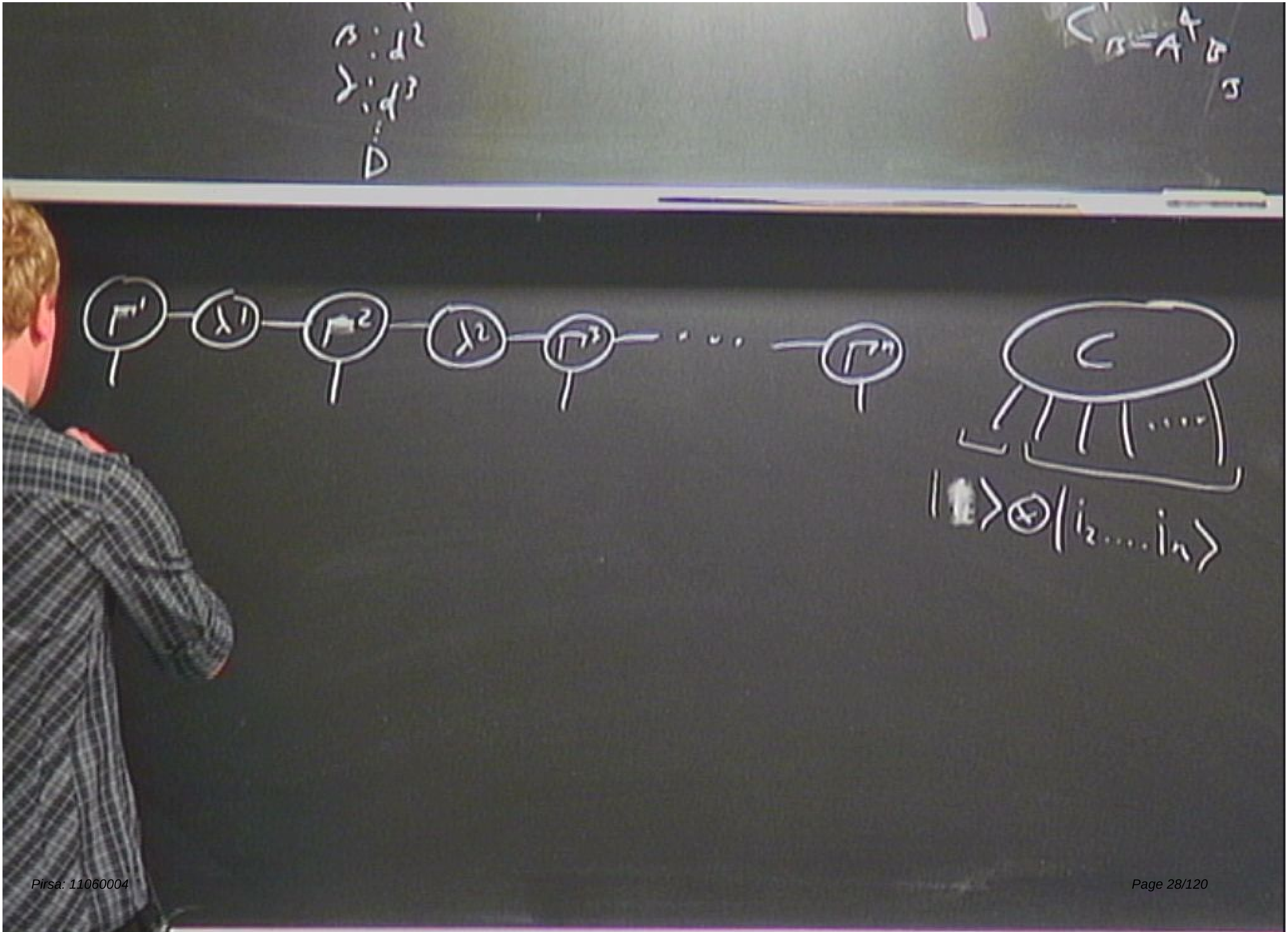
D
Γ¹
λ¹
Γ²
λ²
Γ³
Γⁿ




β : d²
γ : d³
D
Γ¹
λ1
Γ²
λ²
Γ³
Γⁿ
C



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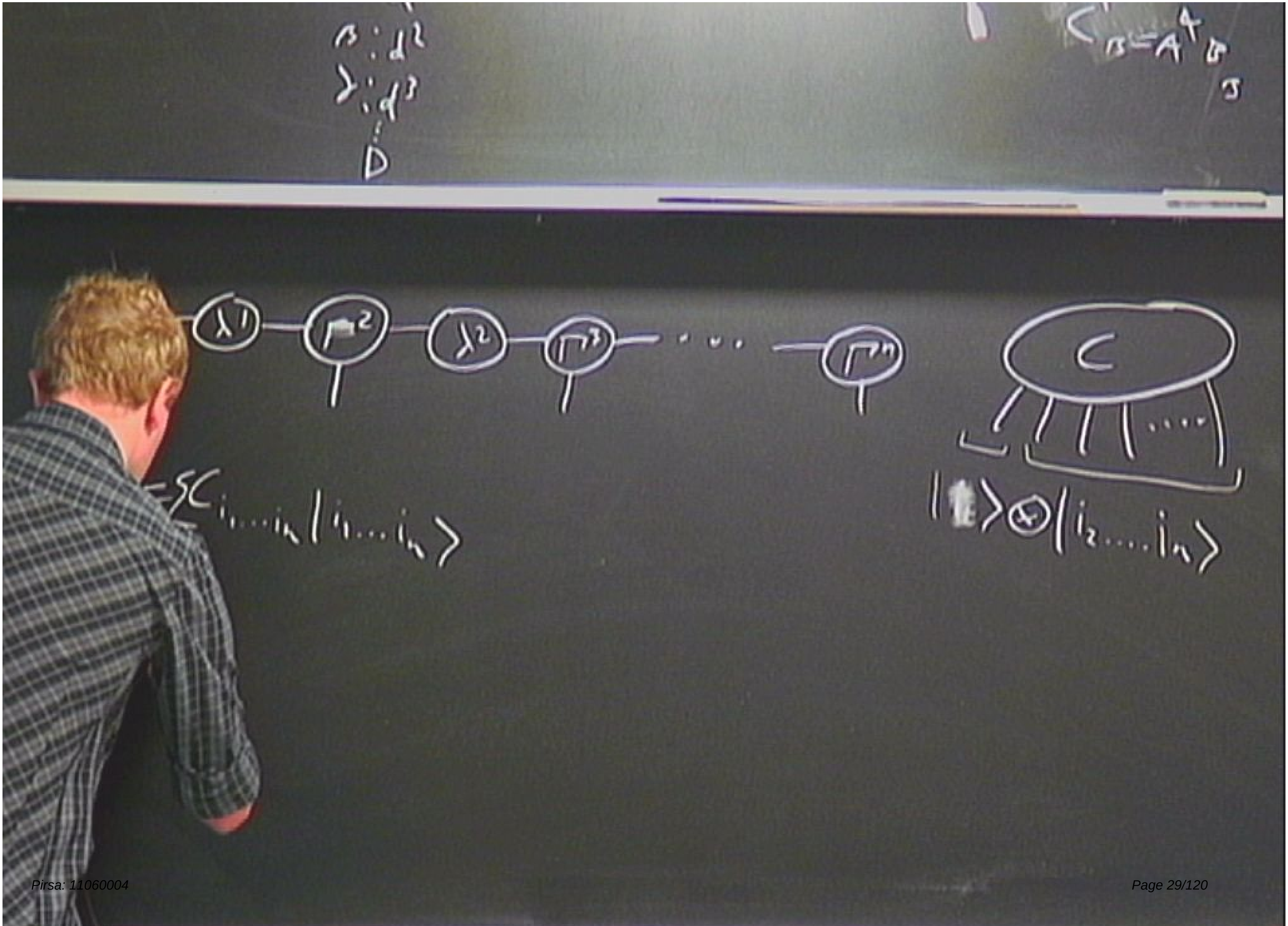
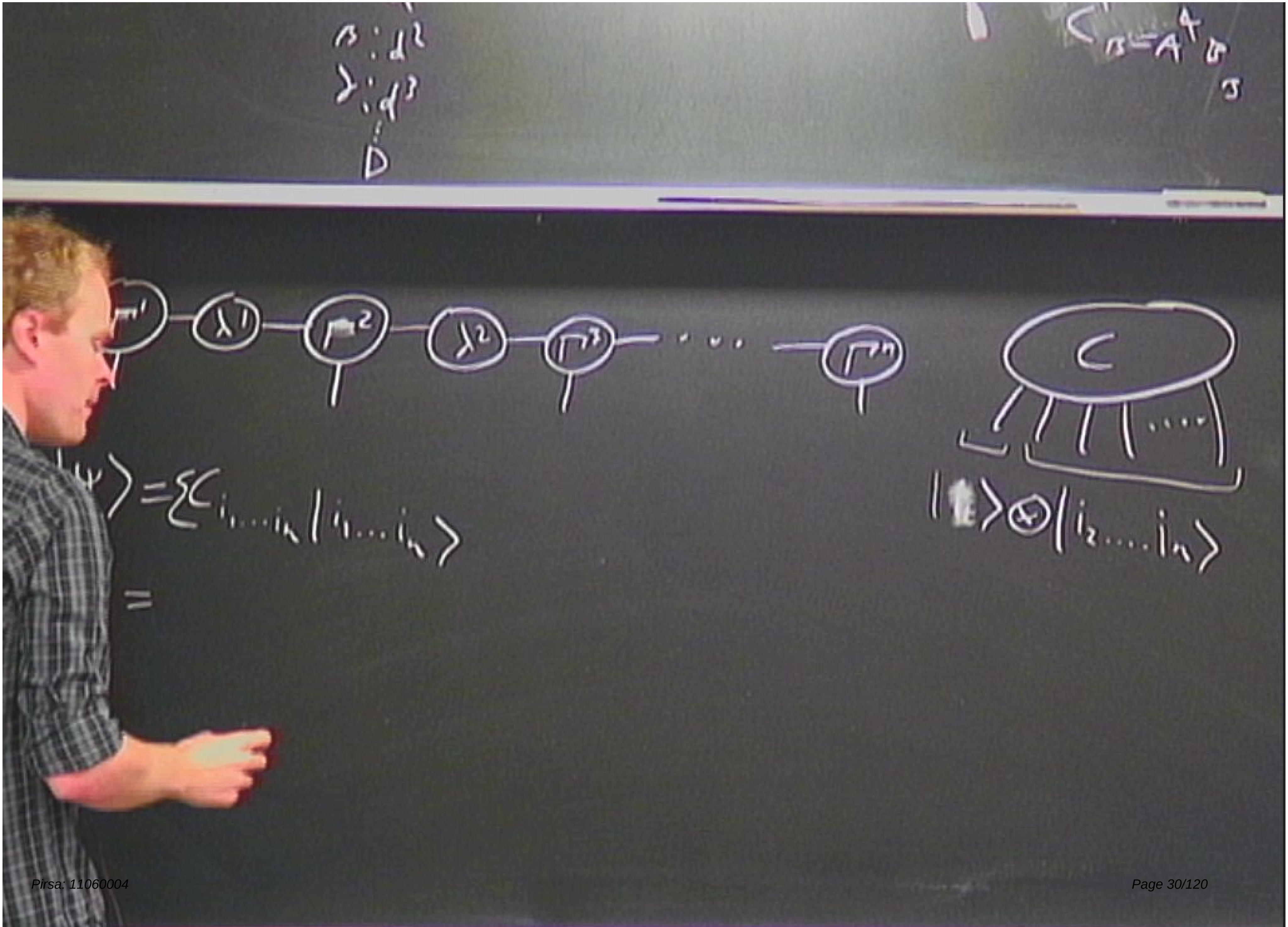
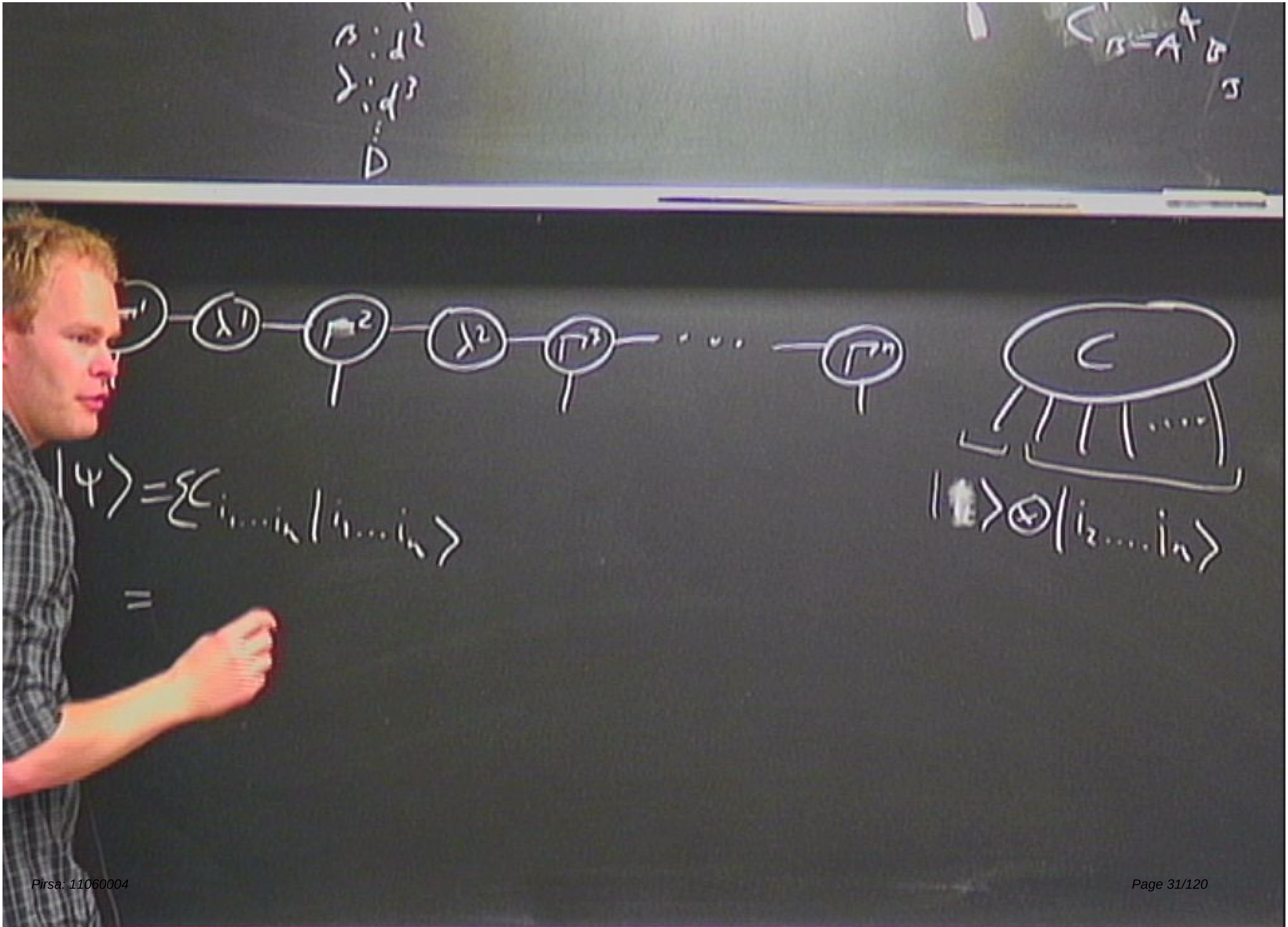
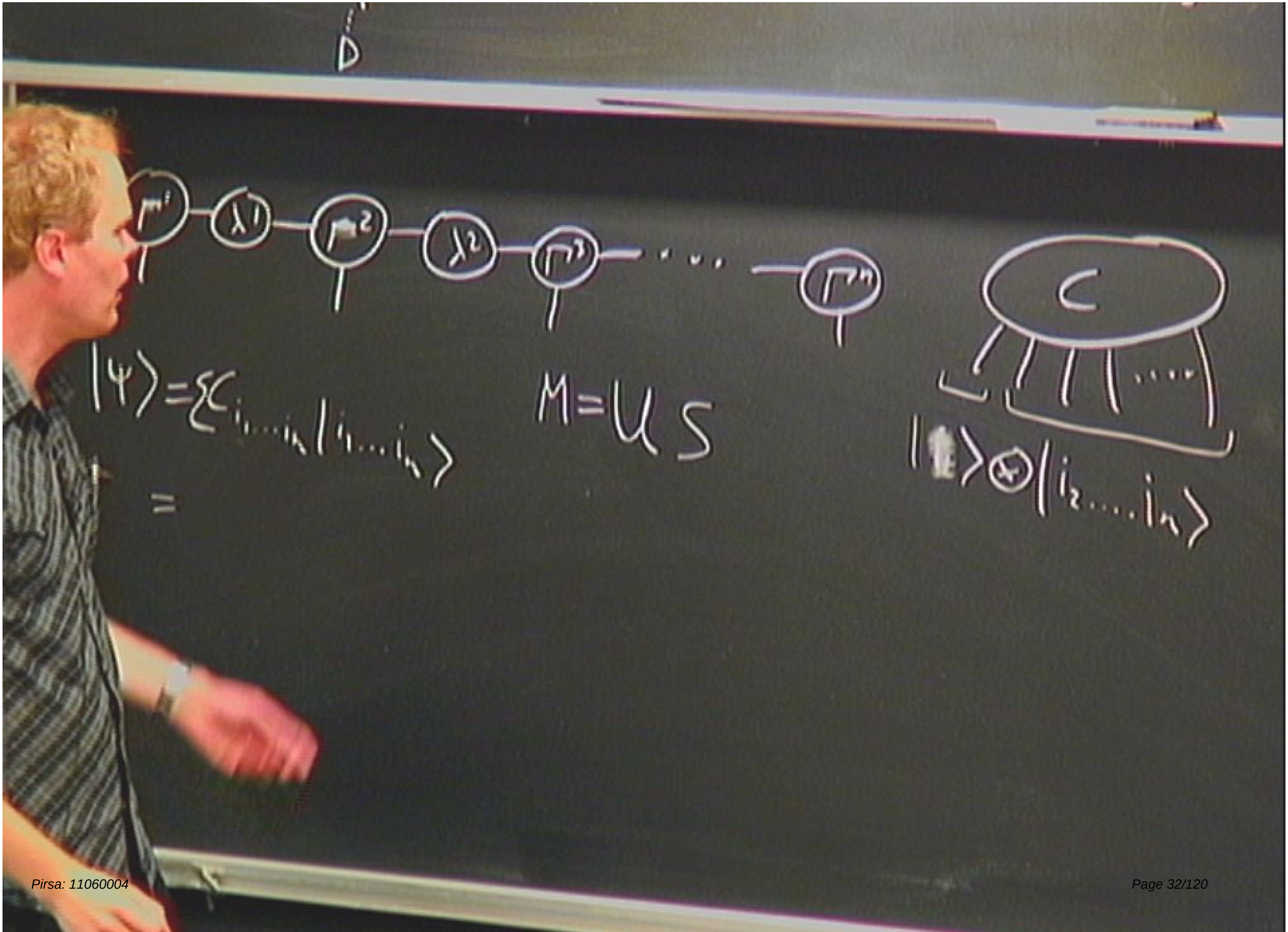
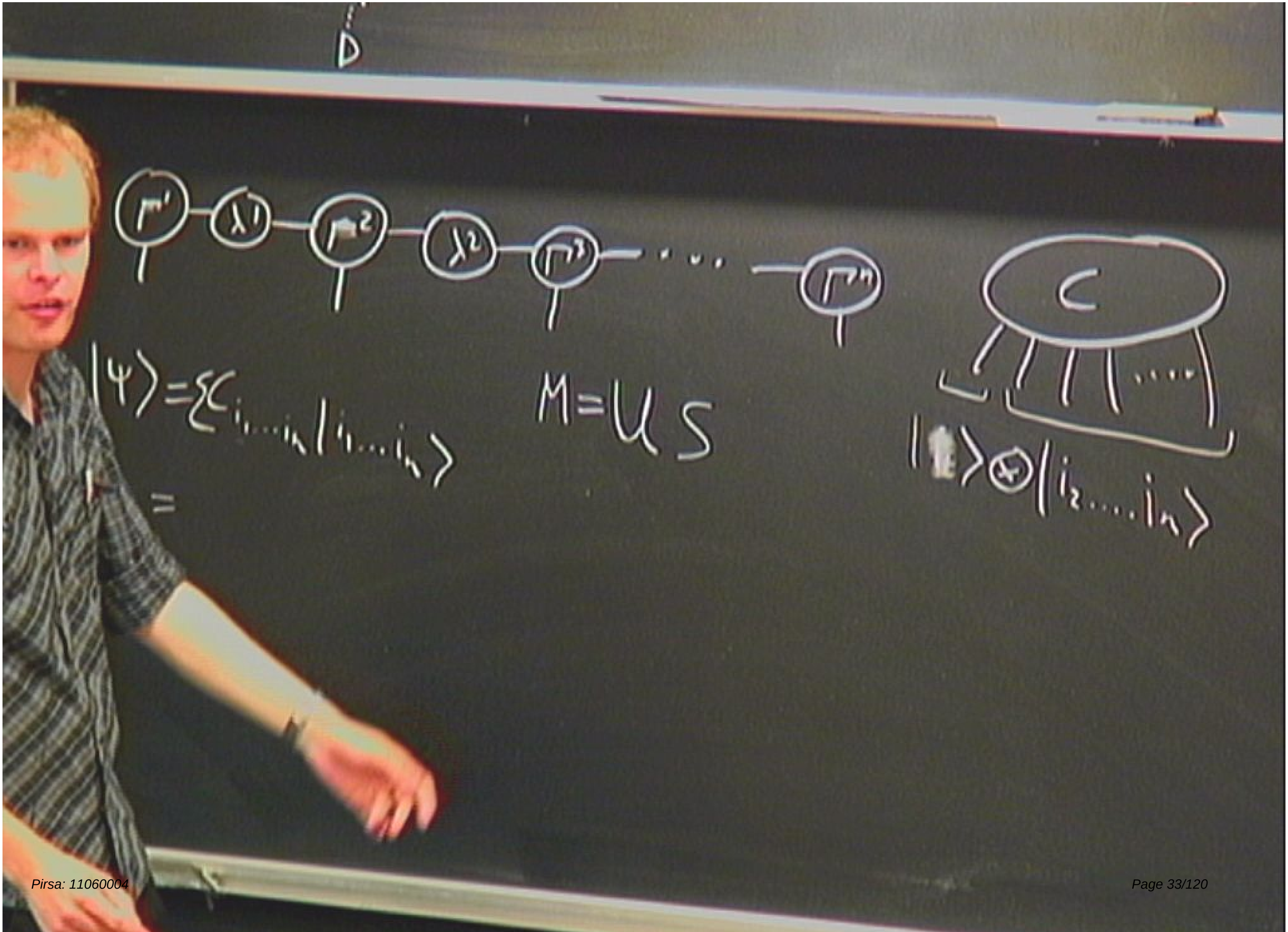
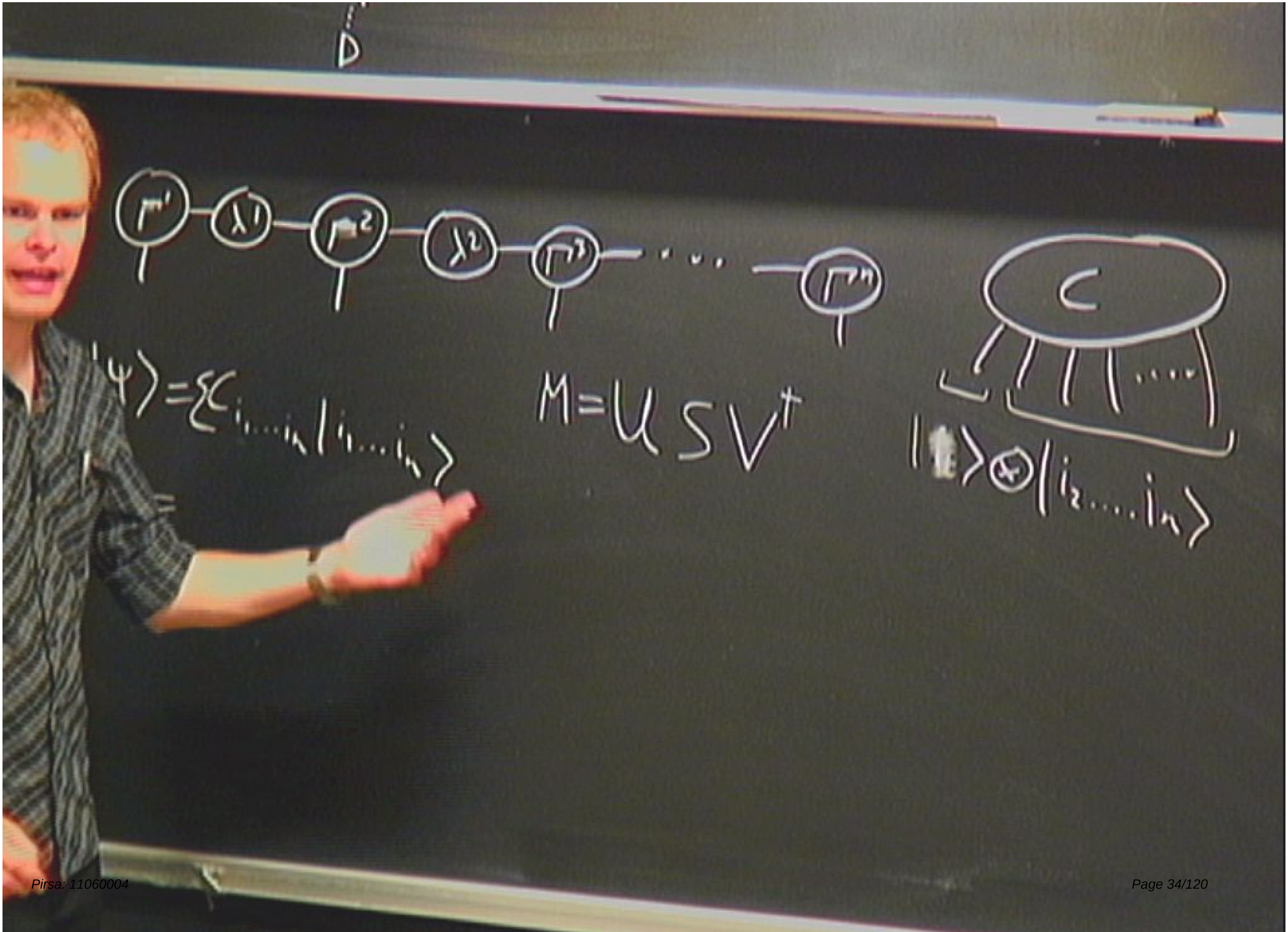


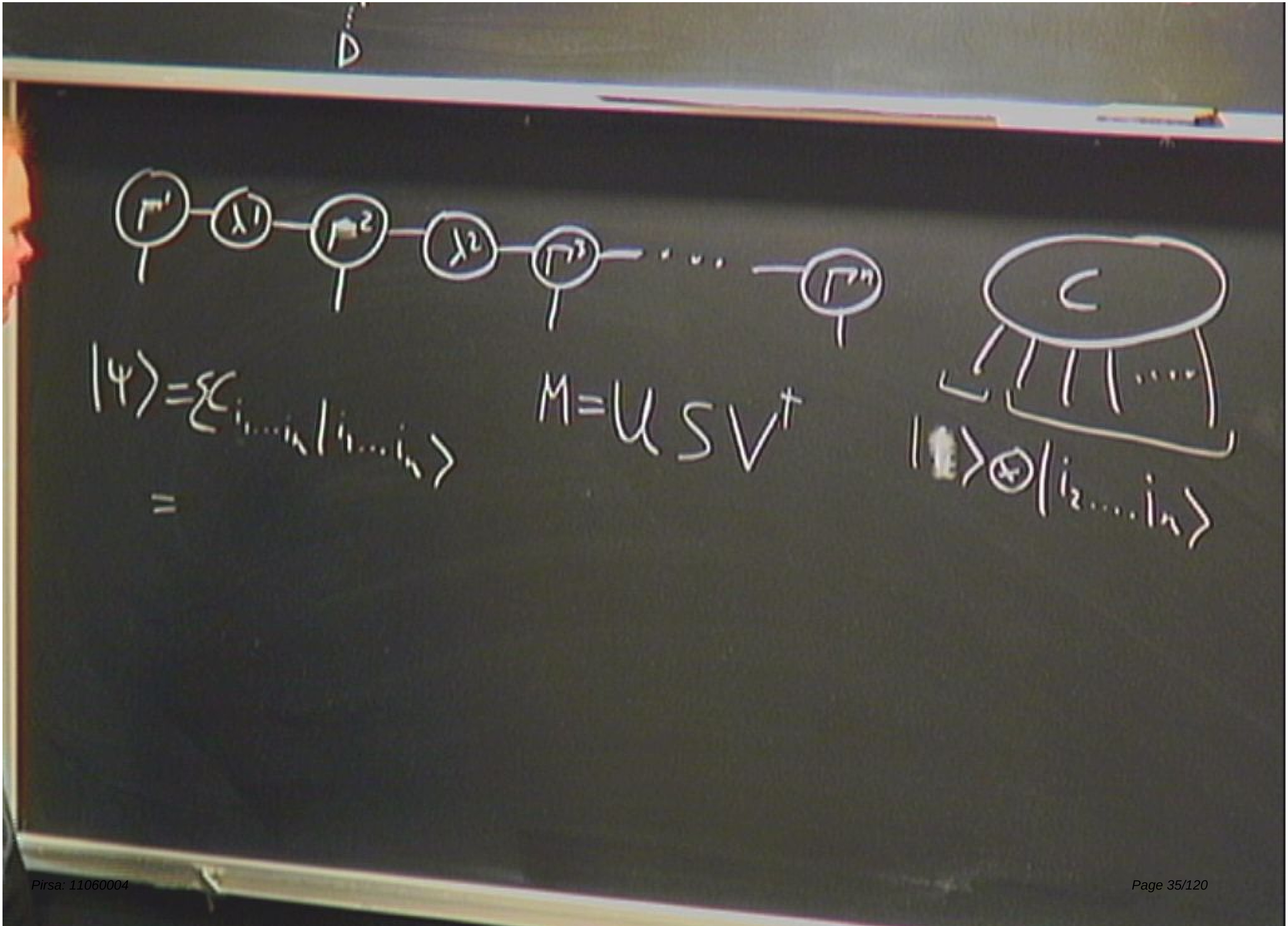


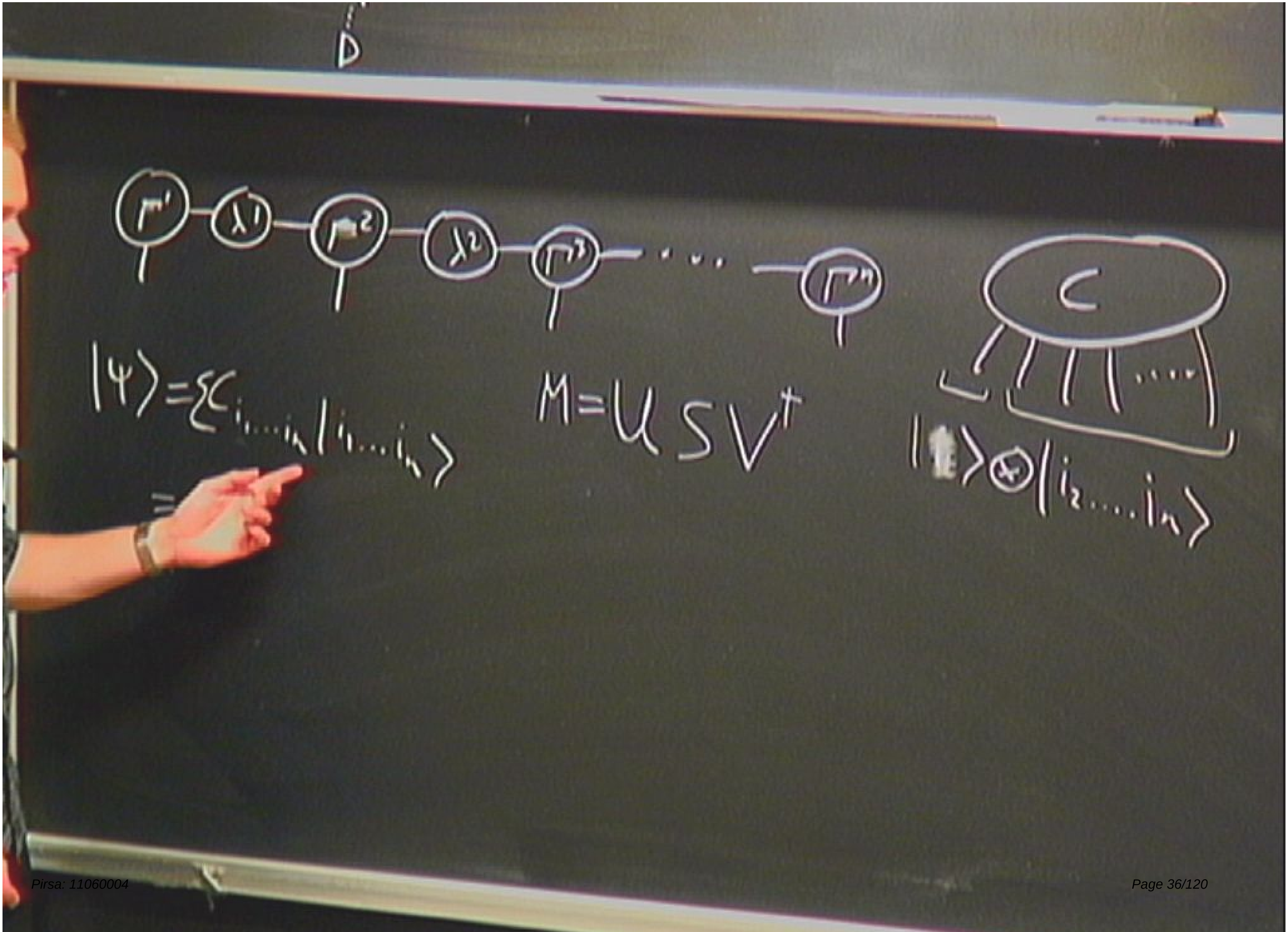

|ψ⟩ = Σ C_{i_1...i_n} |i_1...i_n⟩
=
C
|i_1⟩ ⊗ |i_2....i_n⟩

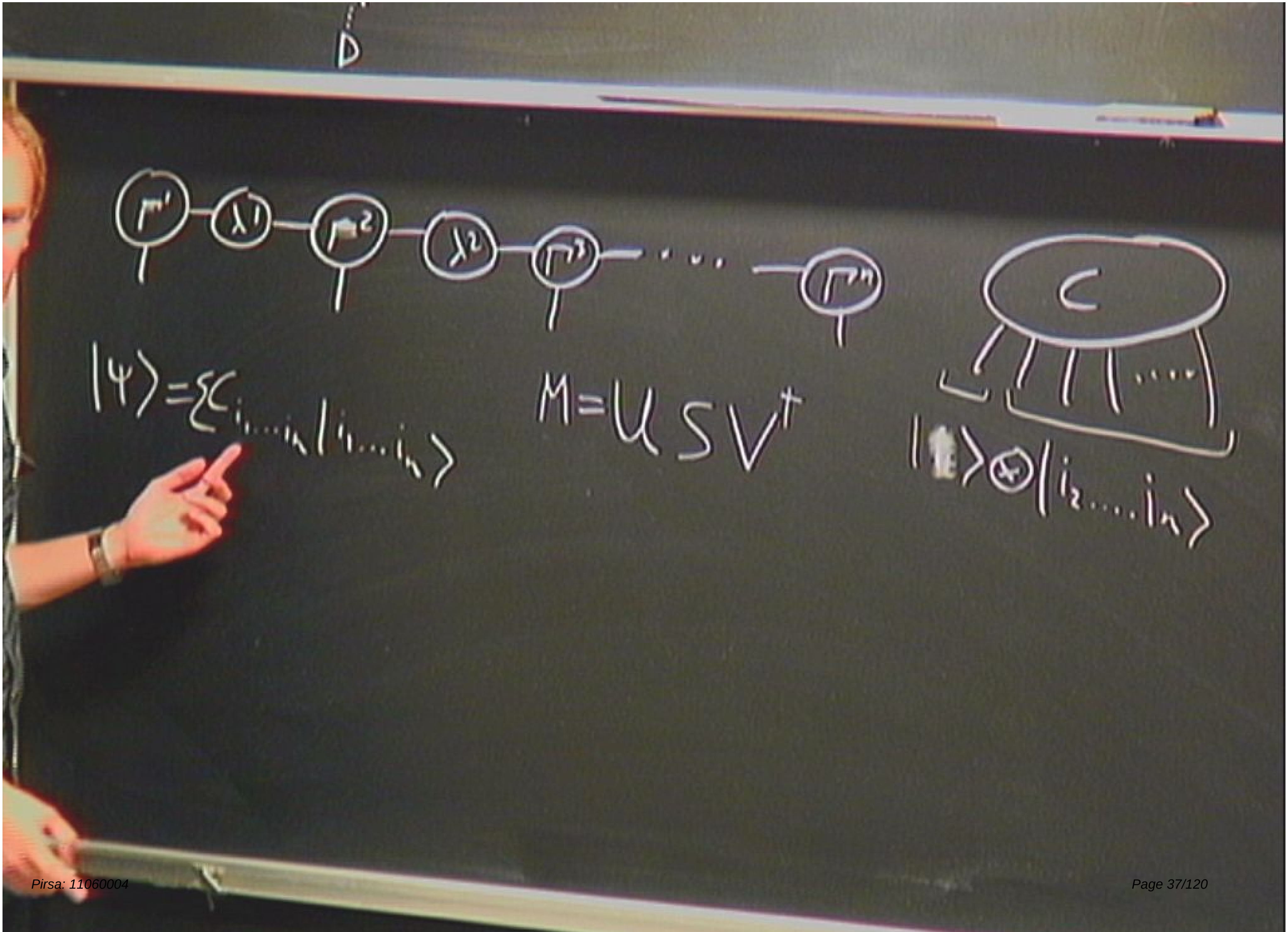

M=US

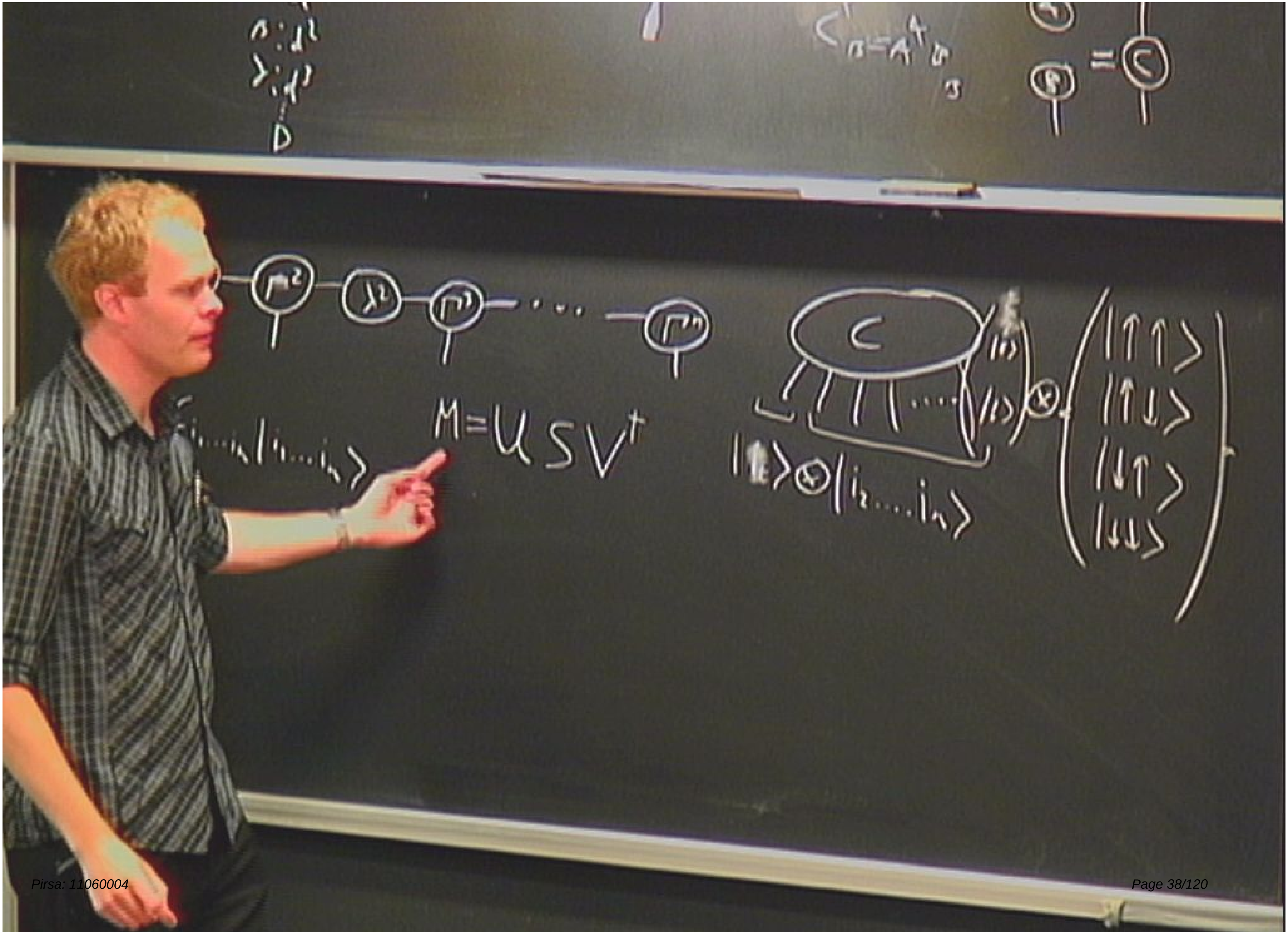

|ψ⟩ = Σ C_{i_1...i_n} |i_1...i_n⟩
M = U S
C
⊗ |i_2 ... i_n⟩

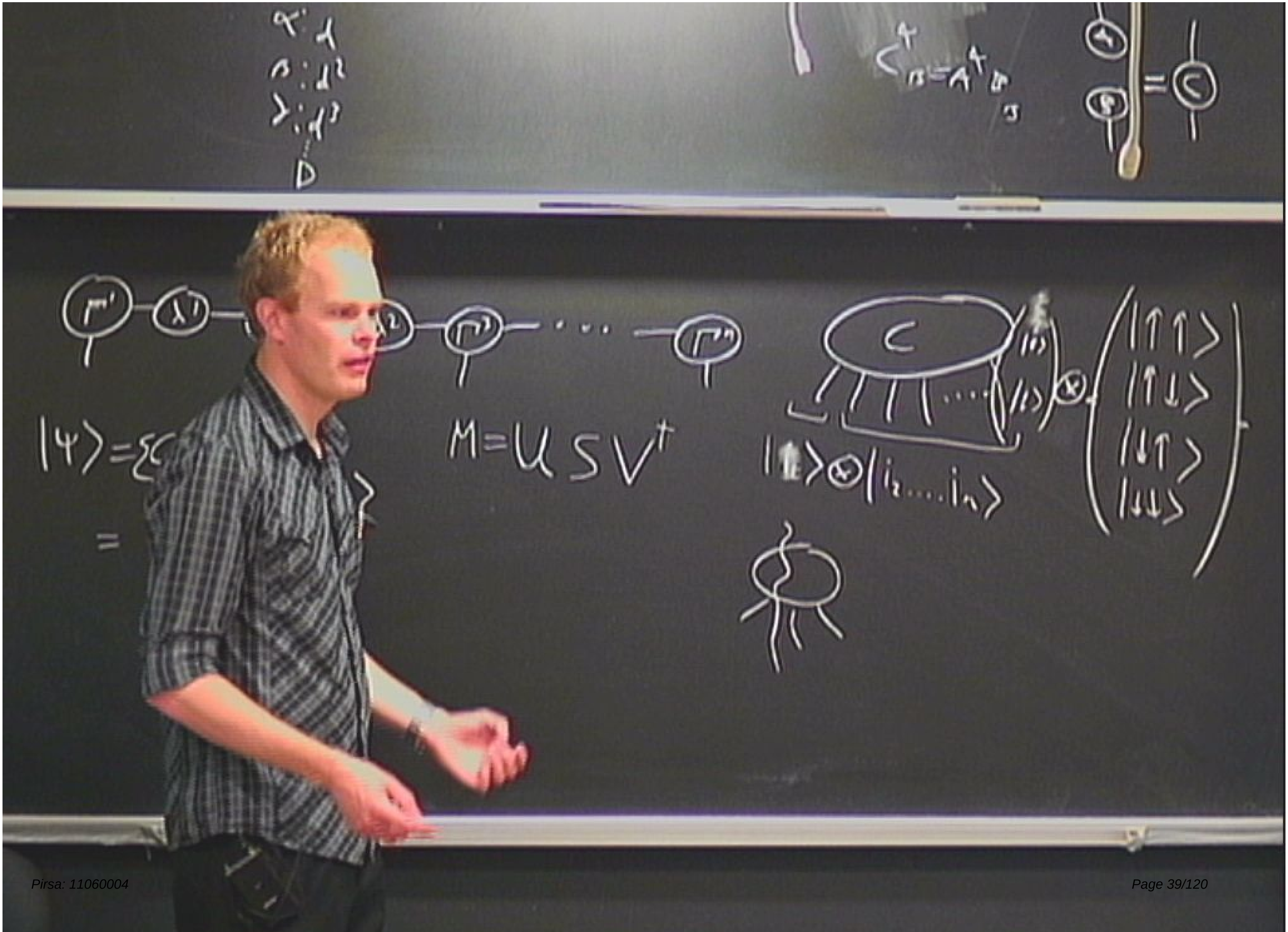

λ1
λ2
C
M=USV†

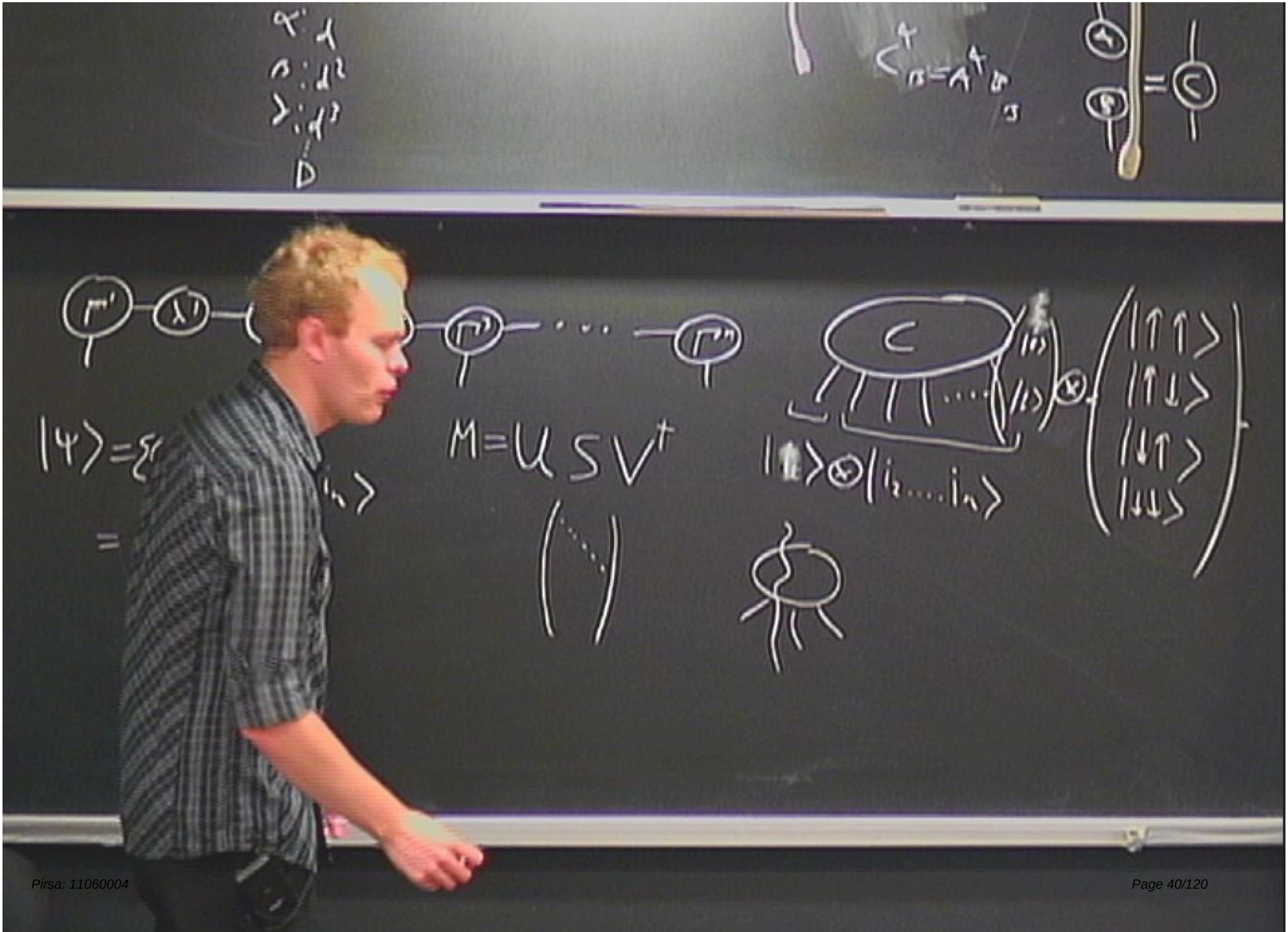

Γ¹ λ¹ Γ² λ² Γ³ ..... Γⁿ
C
|ψ⟩ = Σ C_{i₁...iₙ} |i₁...iₙ⟩
=
M = U S V†
|i₁⟩ ⊗ |i₂....iₙ⟩


λ1
λ2
C
|ψ⟩=ΣC i1...in |i1...in⟩
M=USV†
|i2....in⟩

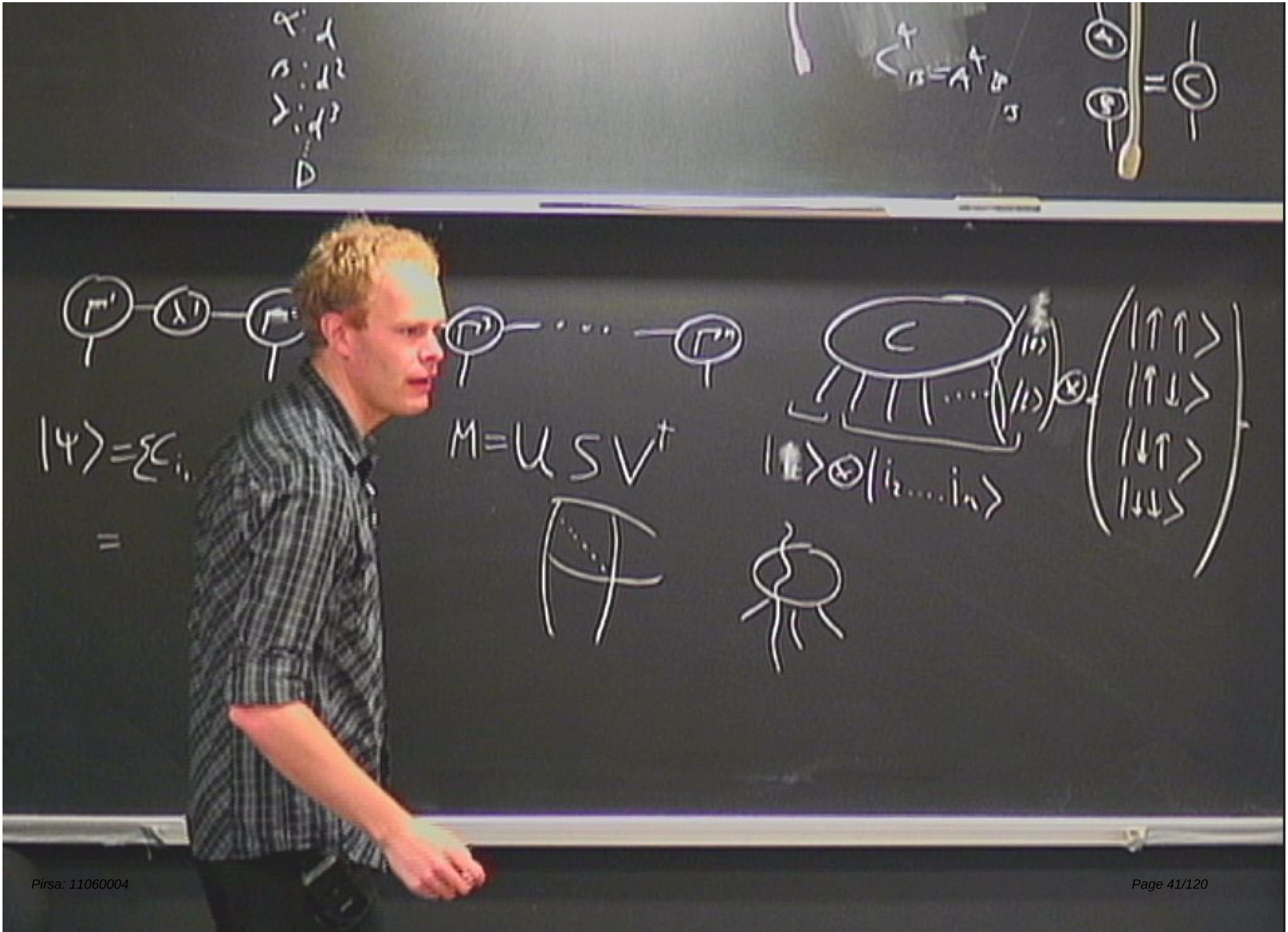

Γ¹ λ¹ Γ² λ² Γ³ ... Γⁿ
C
|ψ⟩ = ΣC_{i₁...iₙ}|i₁...iₙ⟩
M = USV†
⊗|i₂....iₙ⟩


M=USV†


M=USV†



M=USV†
|↑↑>
|↑↓>
|↓↑>
|↓↓>


M=USV†
|↑↑>
|↑↓>
|↓↑>
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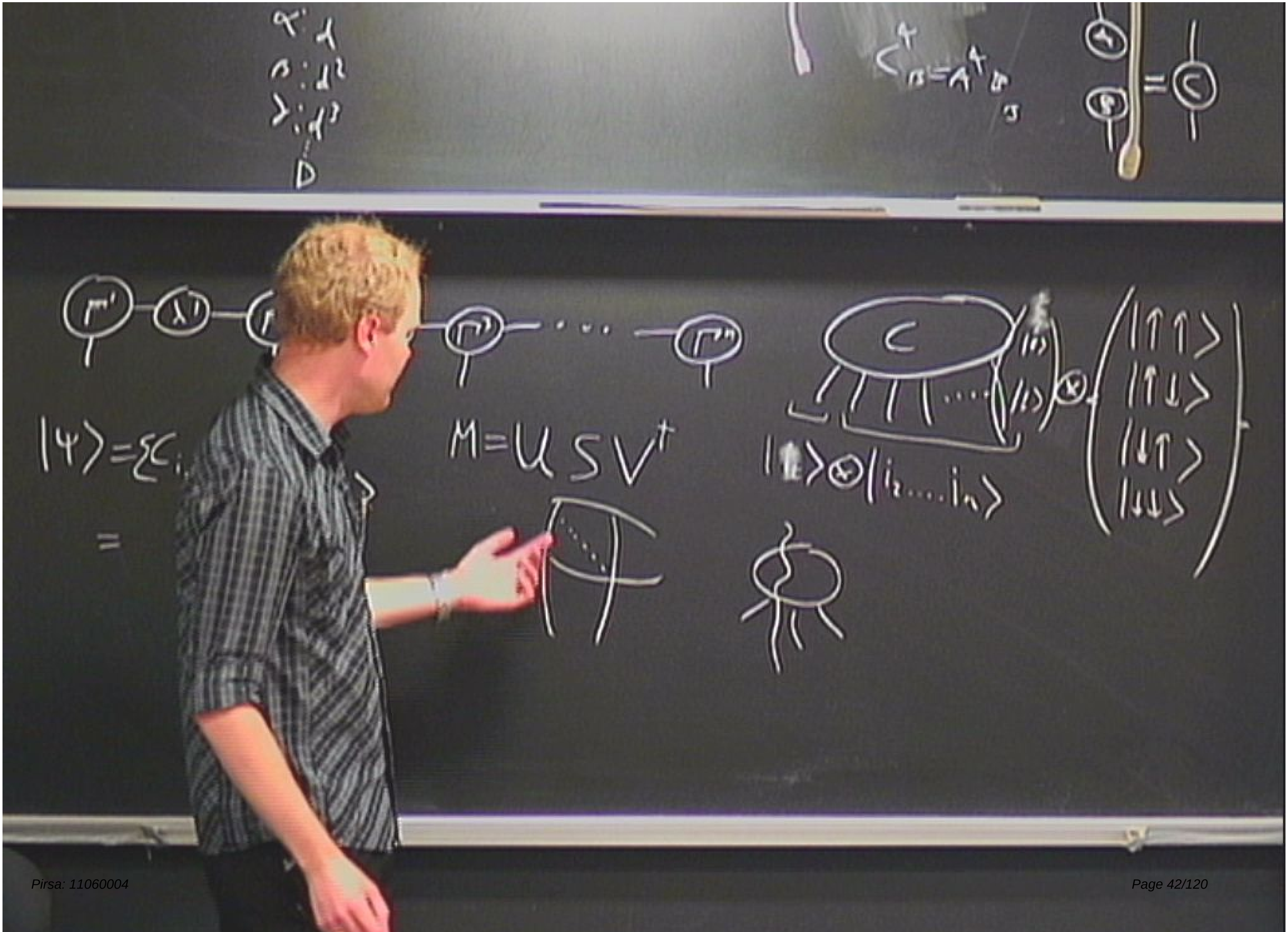
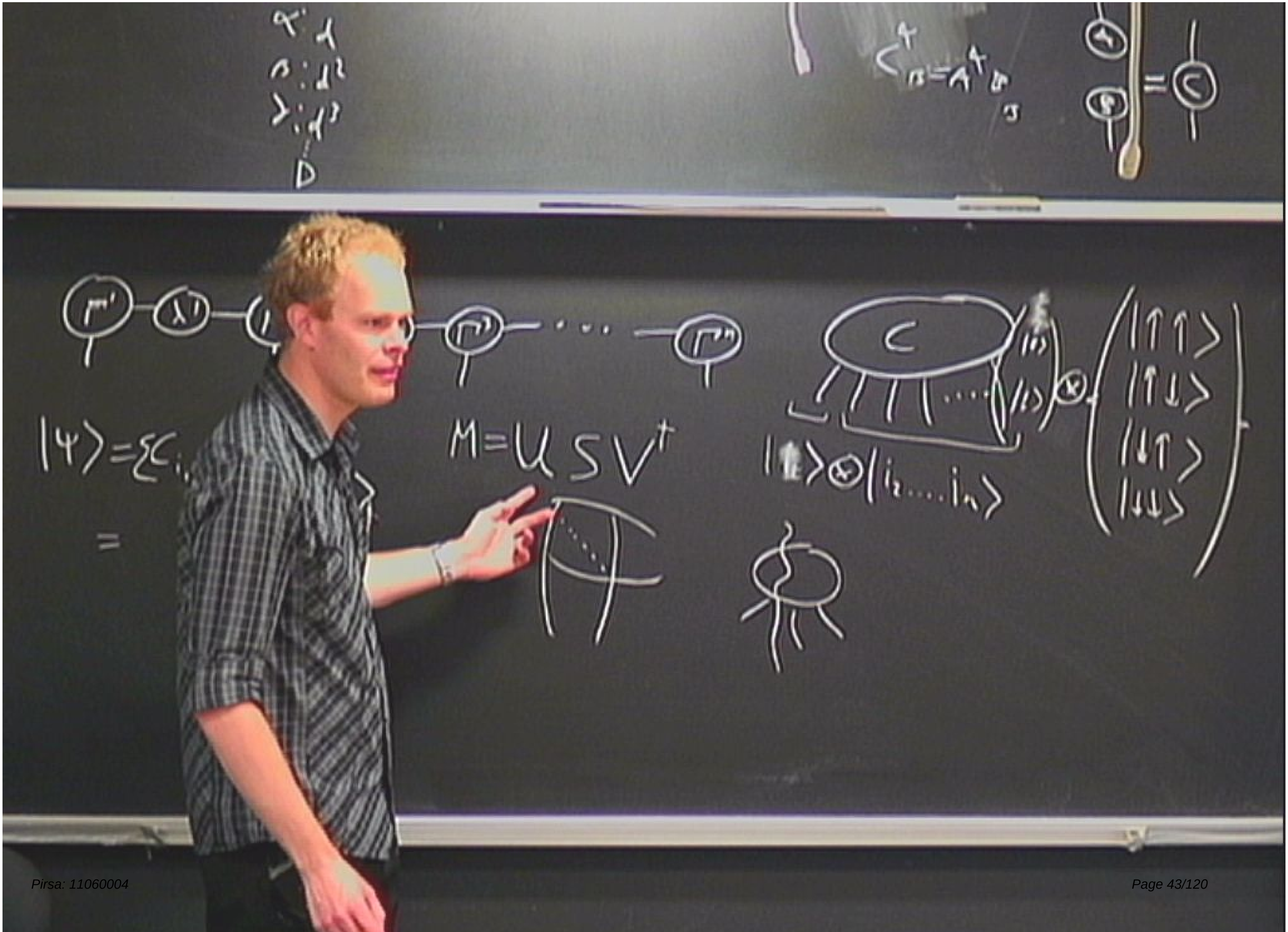
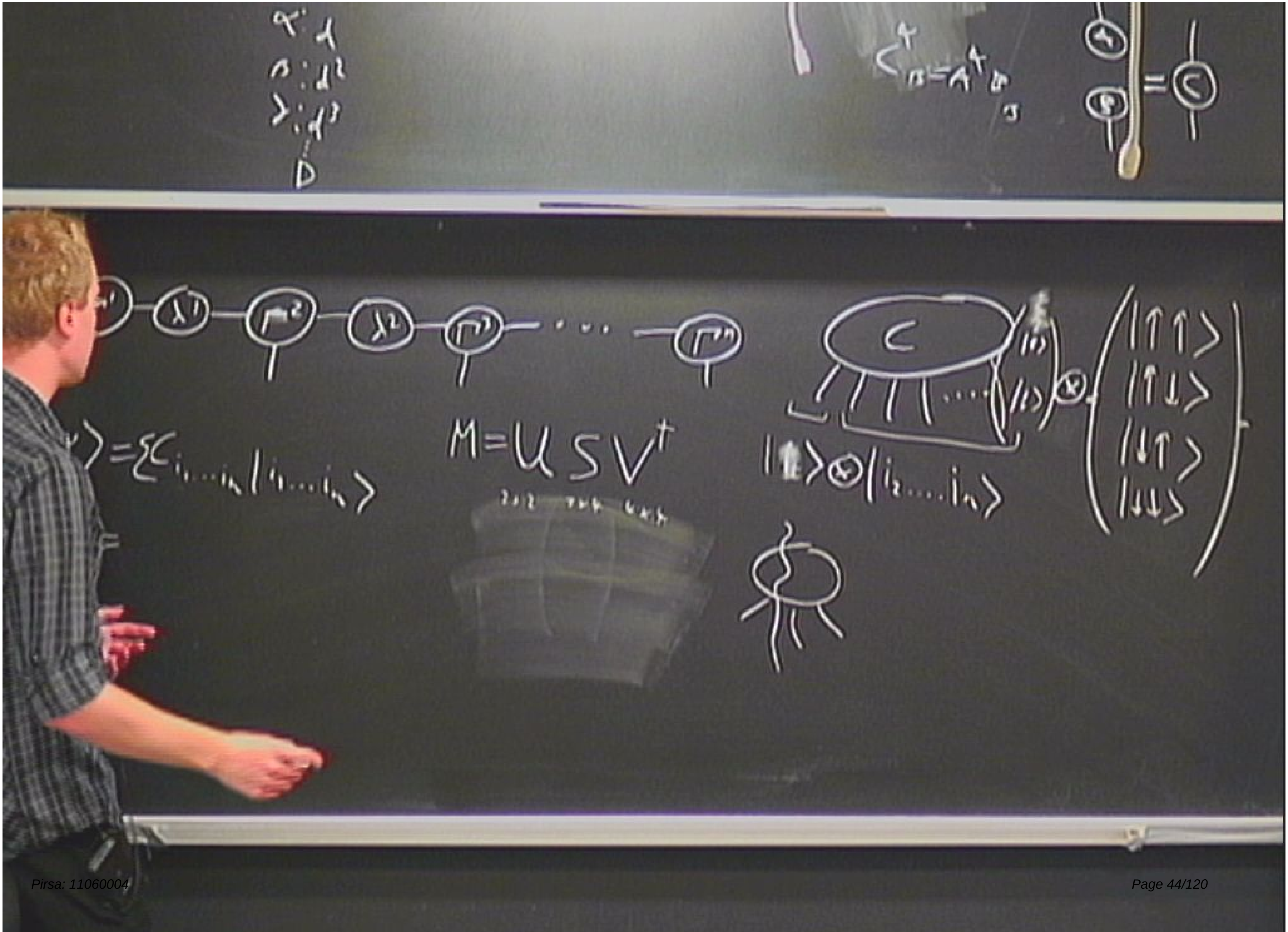
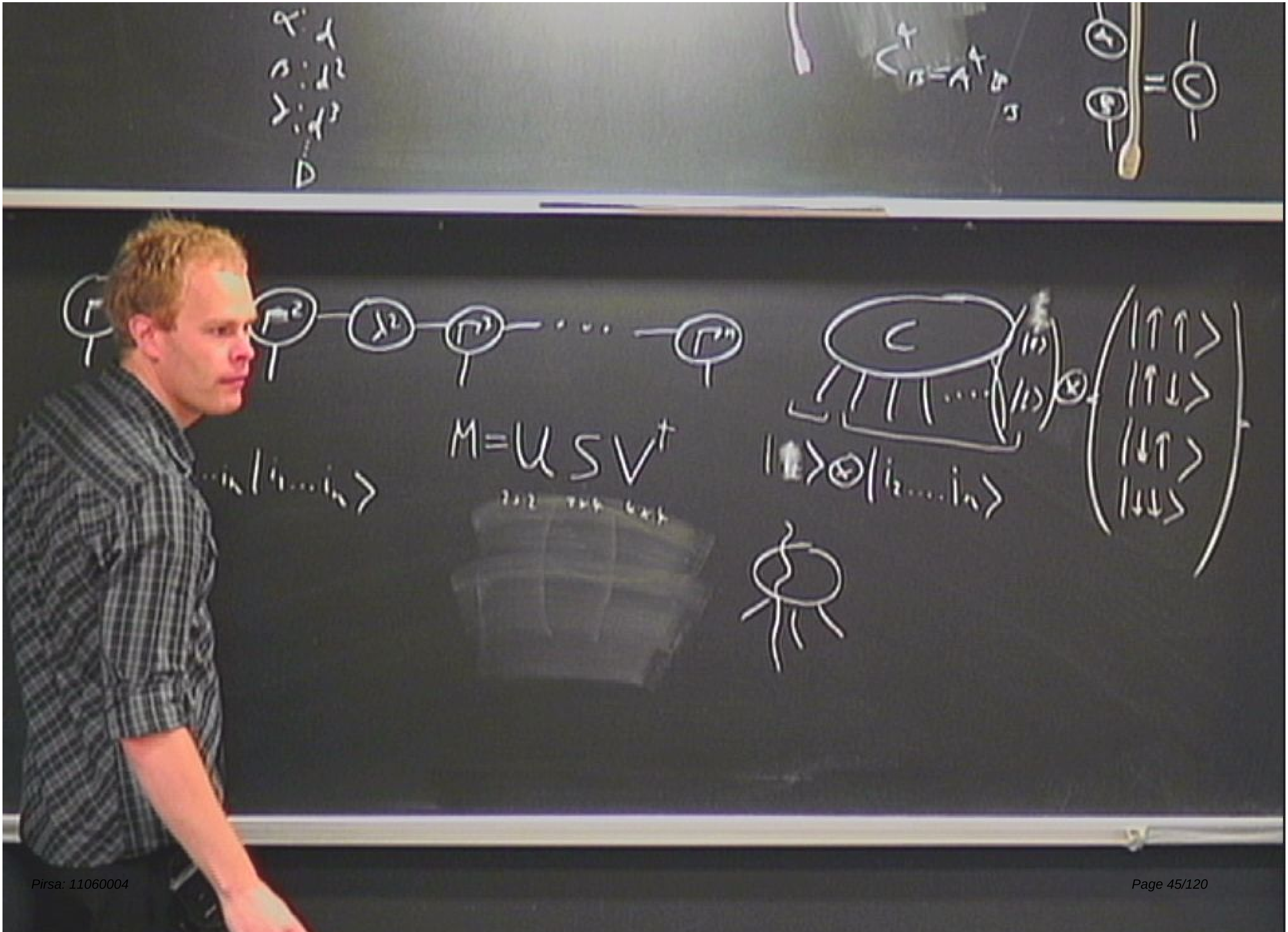
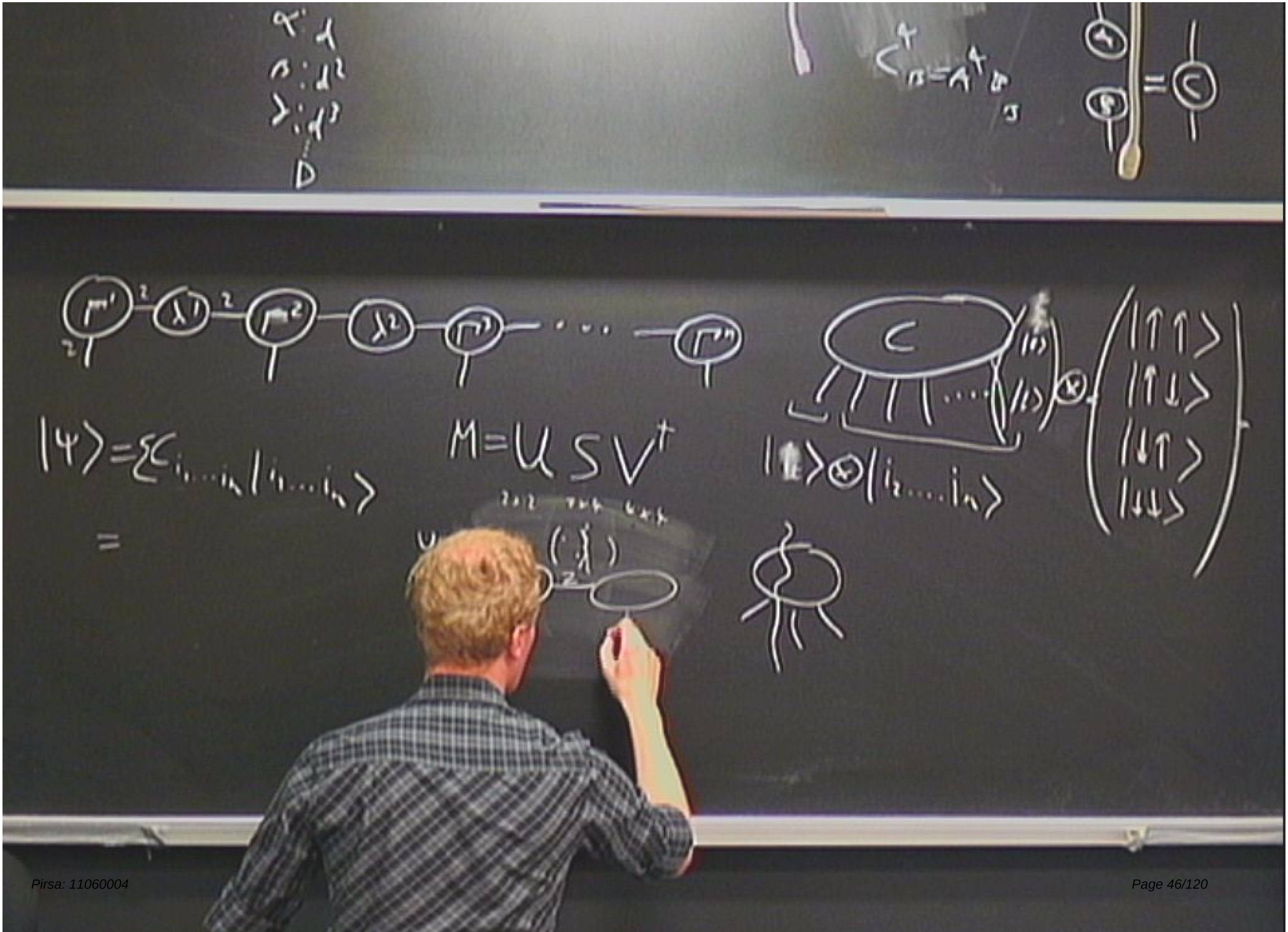

M=USV†
|ψ>=ΣC
|i1>⊗|i2....in>
|↑↑>
|↑↓>
|↓↑>
|↓↓>


M=USV†
|i1>⊗|i2....in>




M=USV†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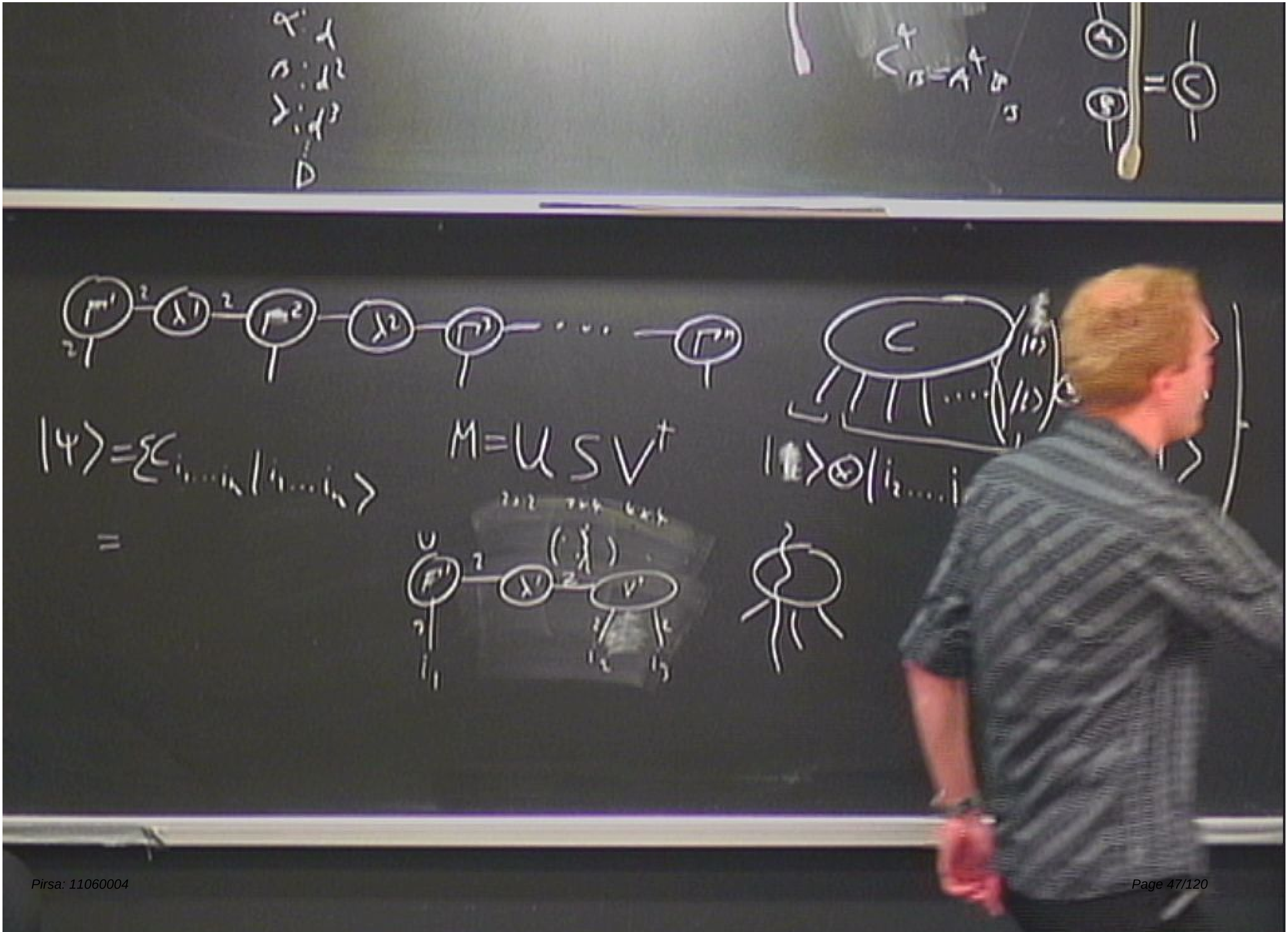
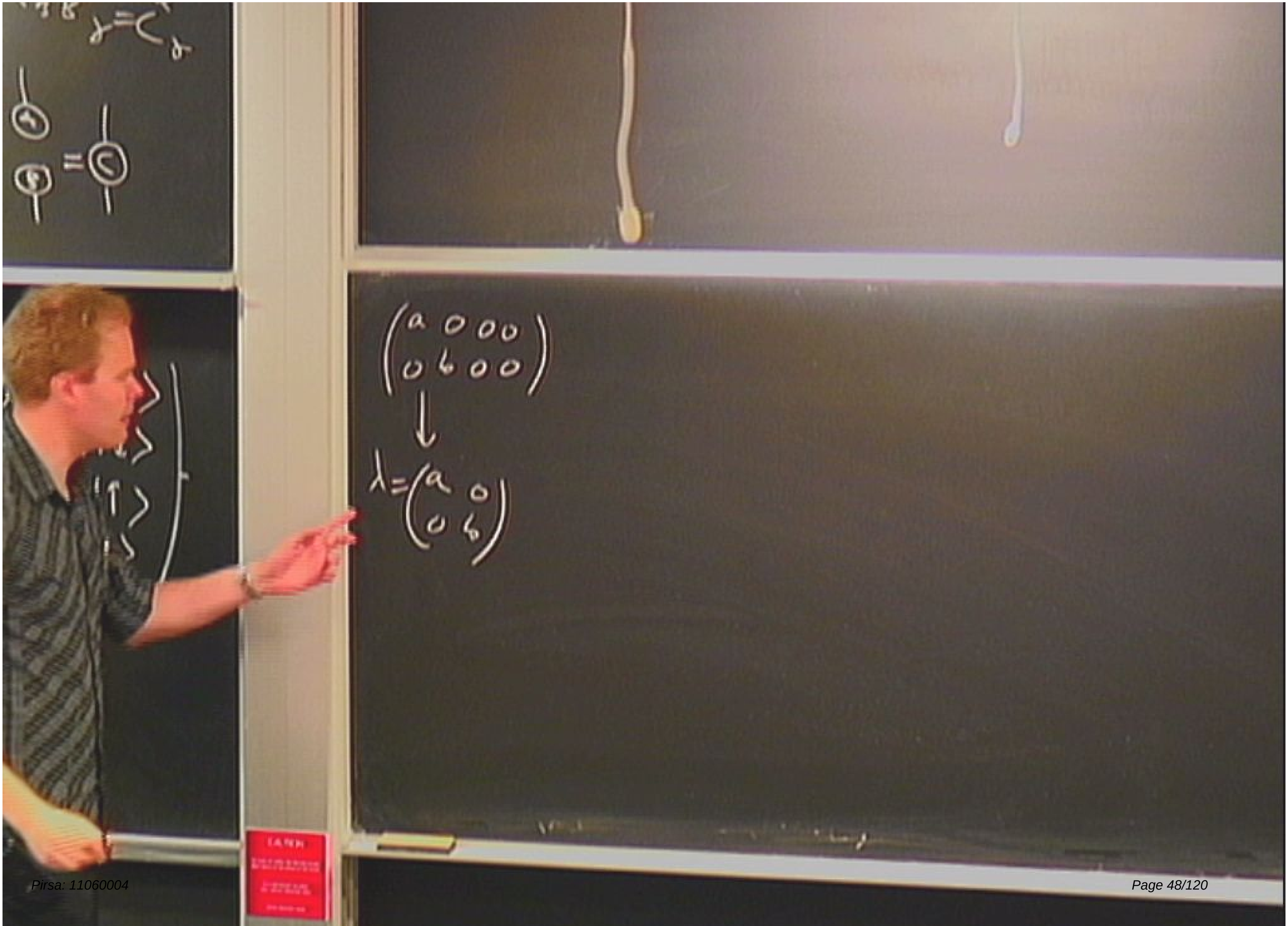
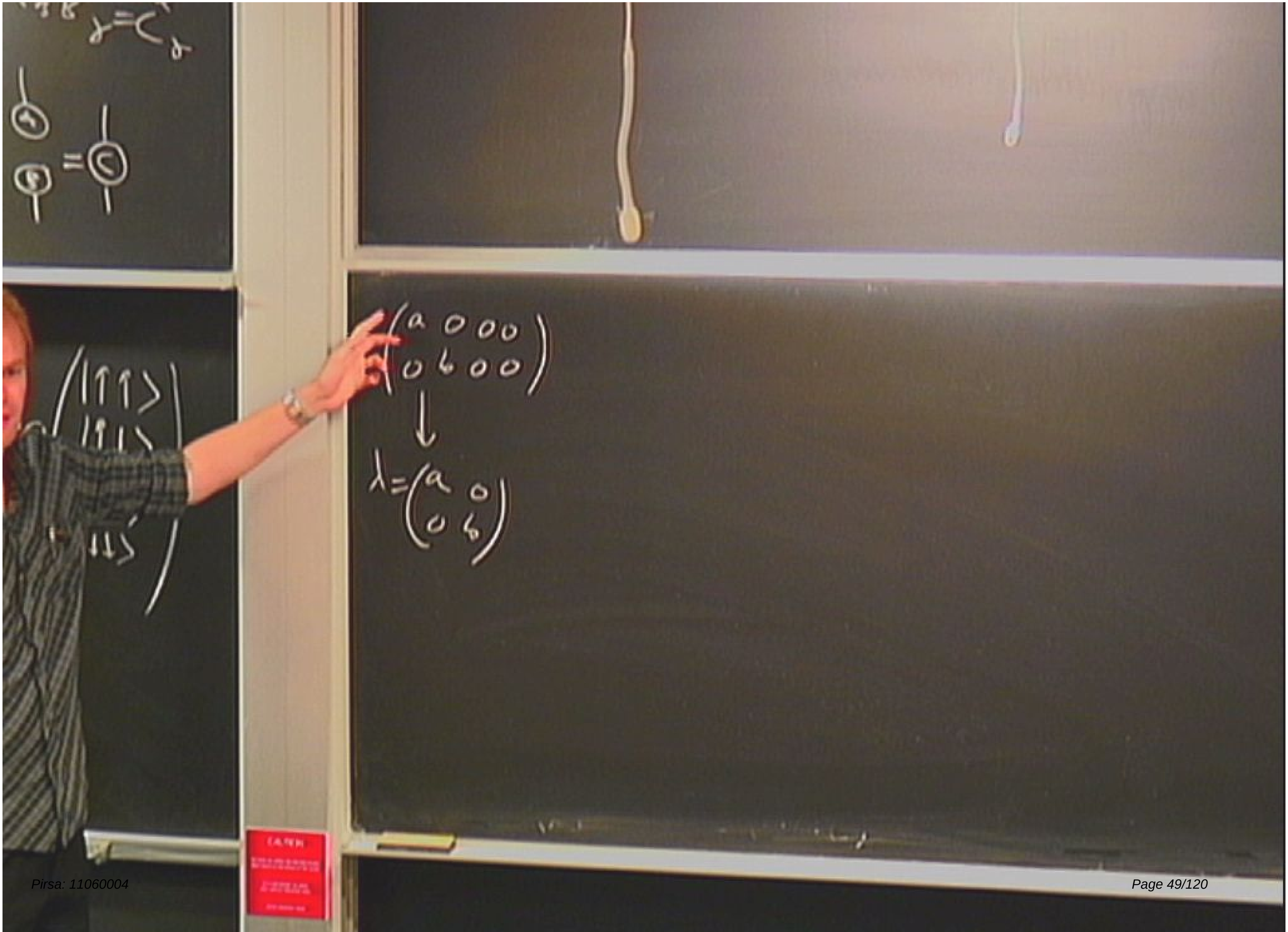
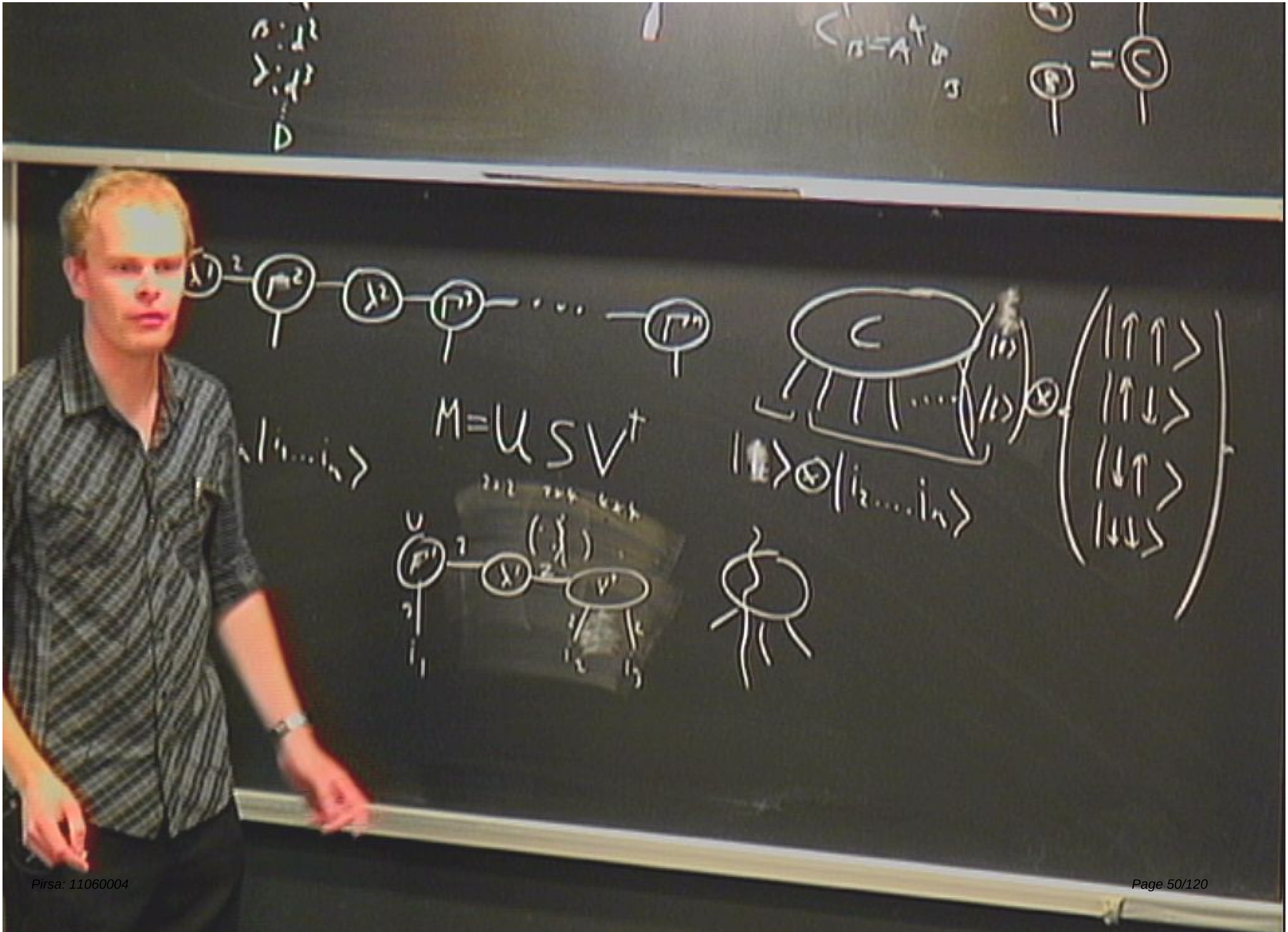
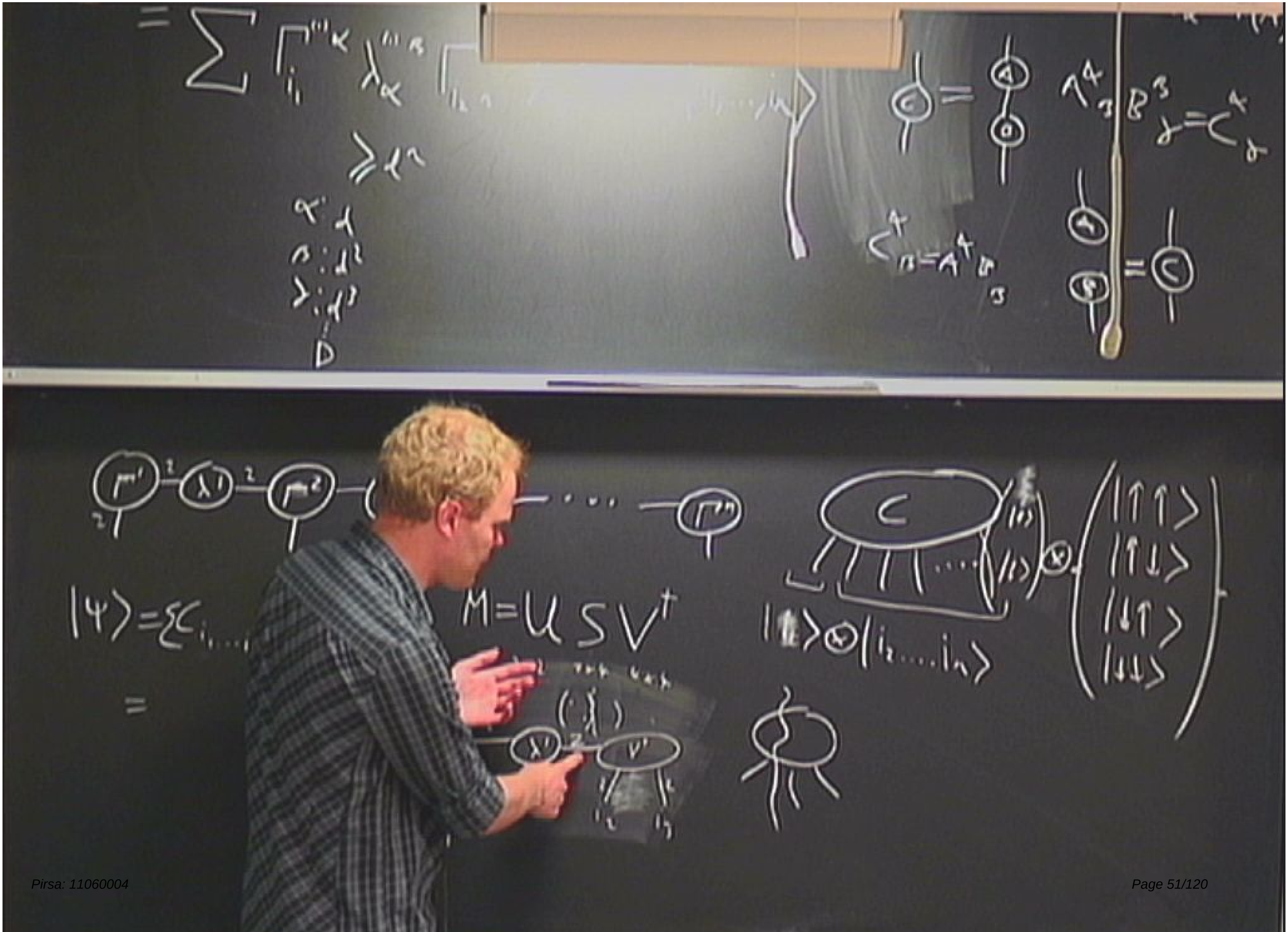
M=USV†
|i_1>⊗|i_2....i_n>

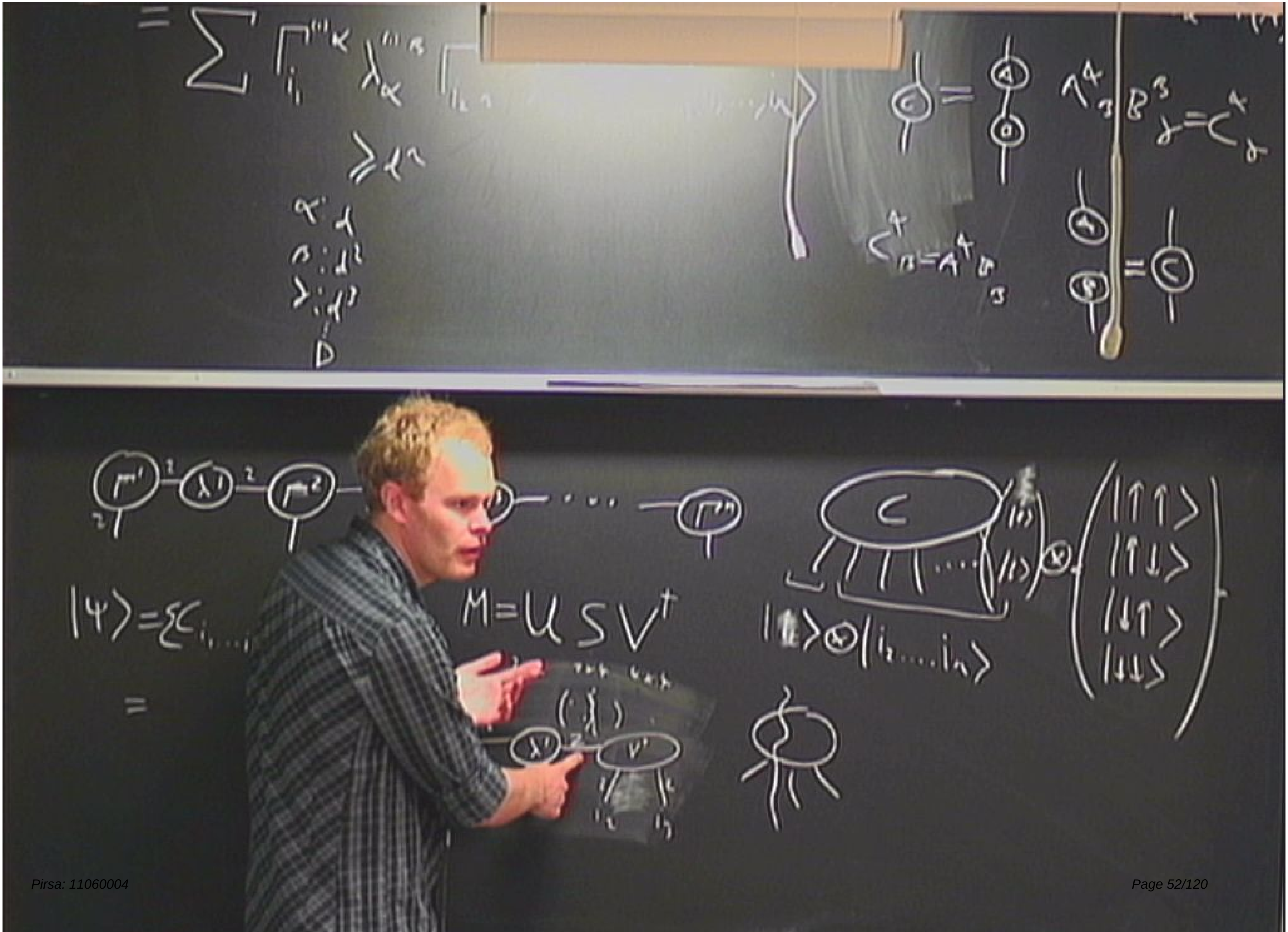

|ψ⟩ = Σ C_{i_1...i_n} |i_1...i_n⟩
M = U S V†


a 0 0 0
0 b 0 0
λ= a 0
0 b




M=USV†

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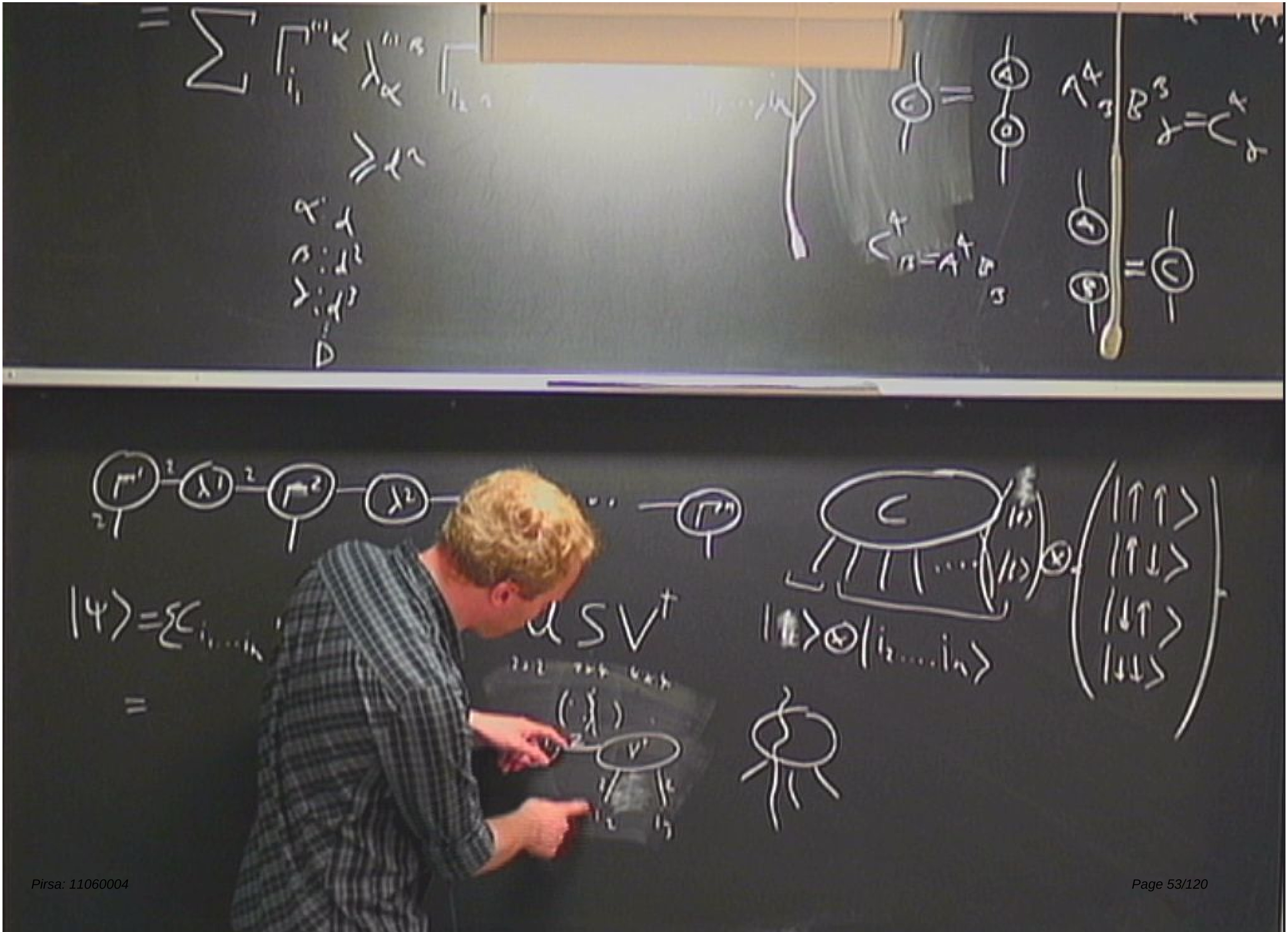
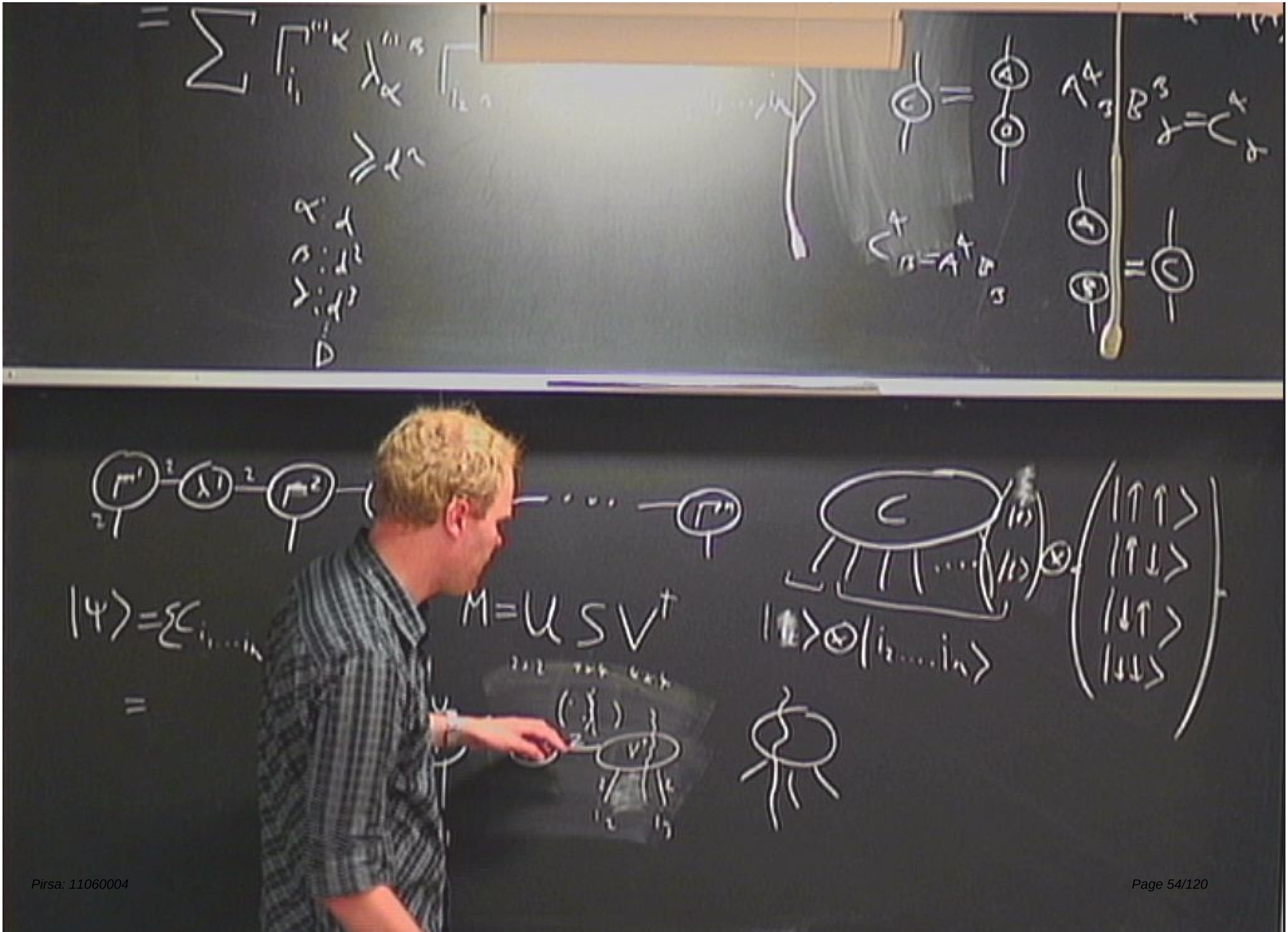
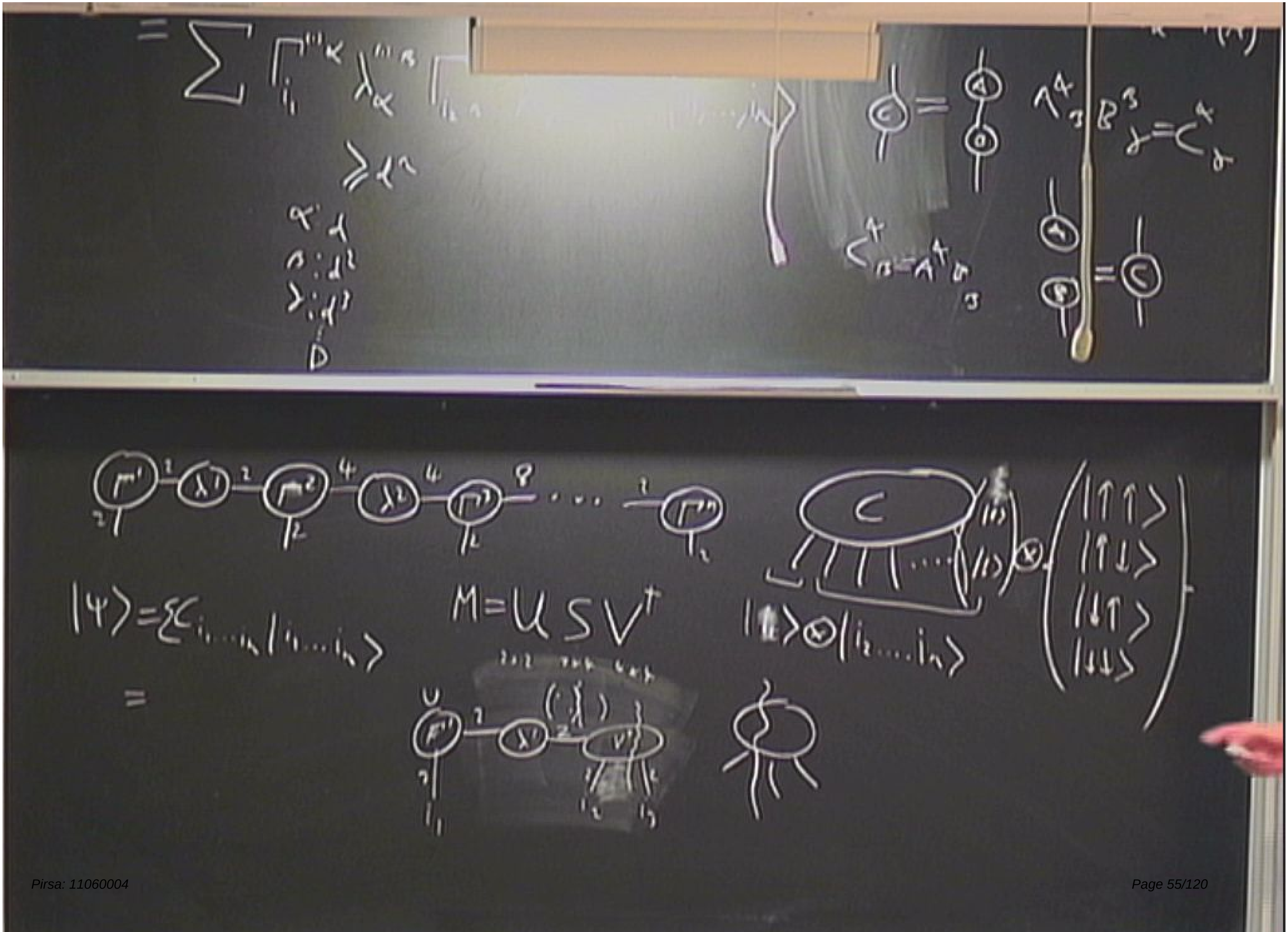
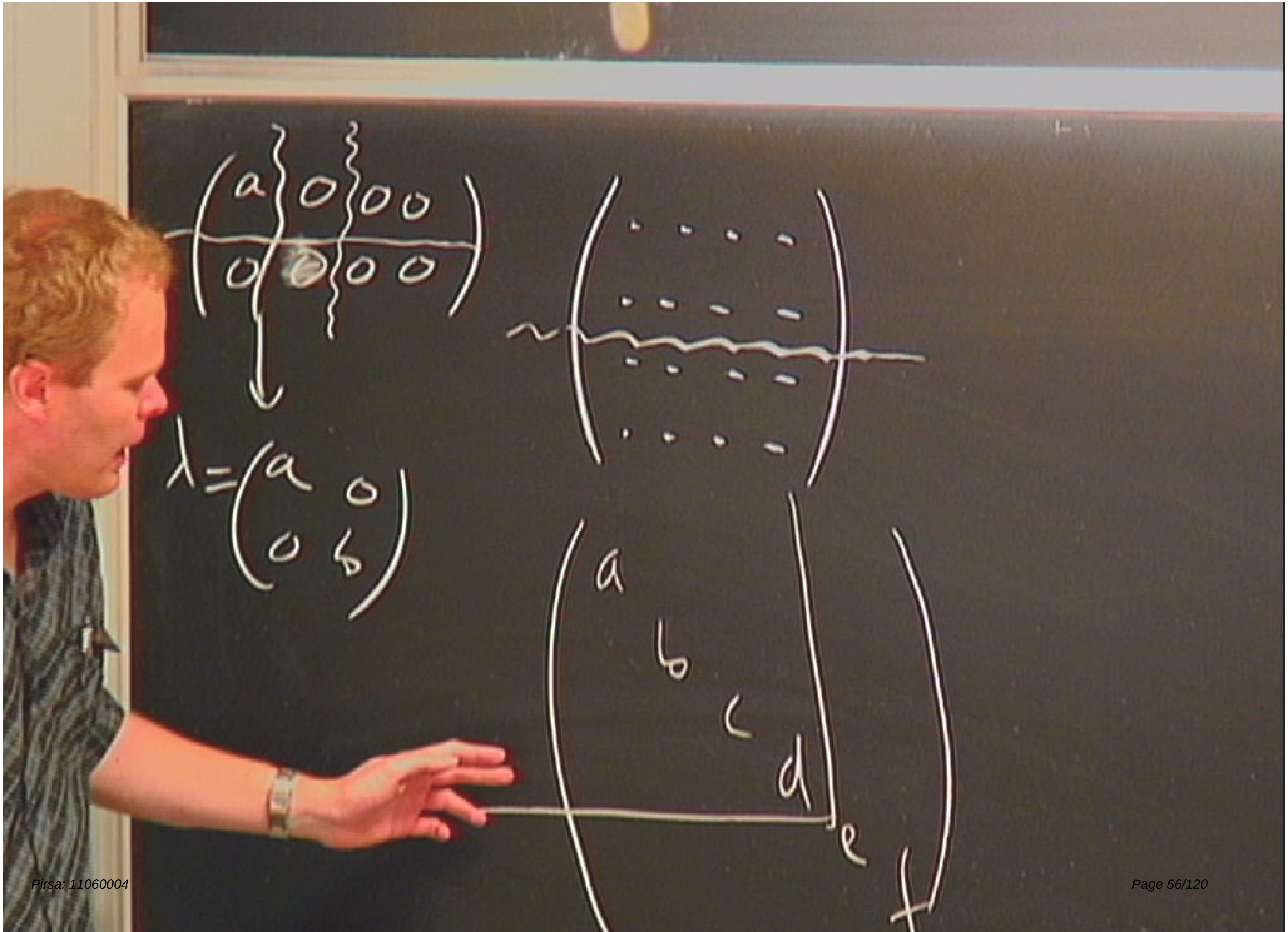
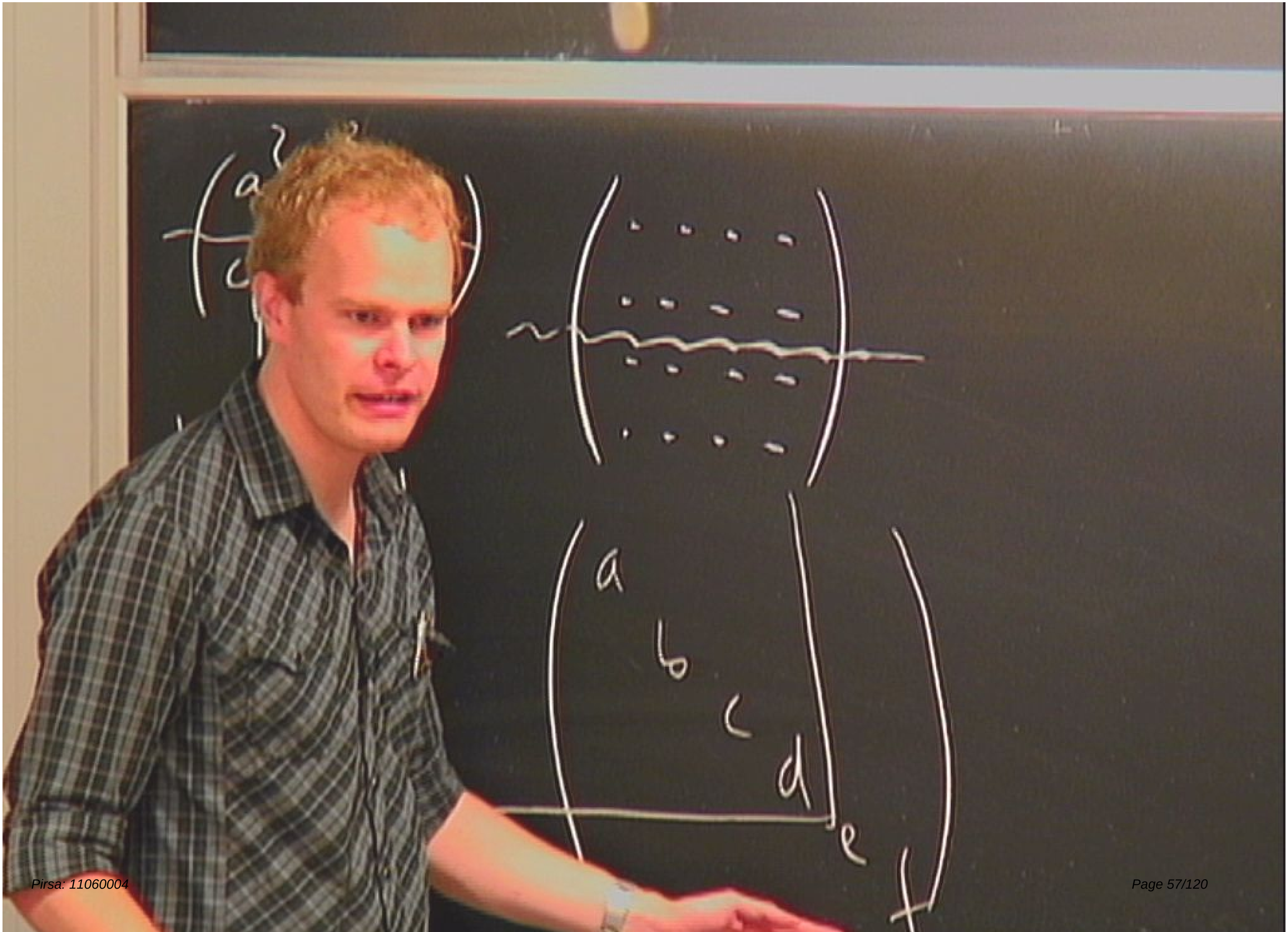
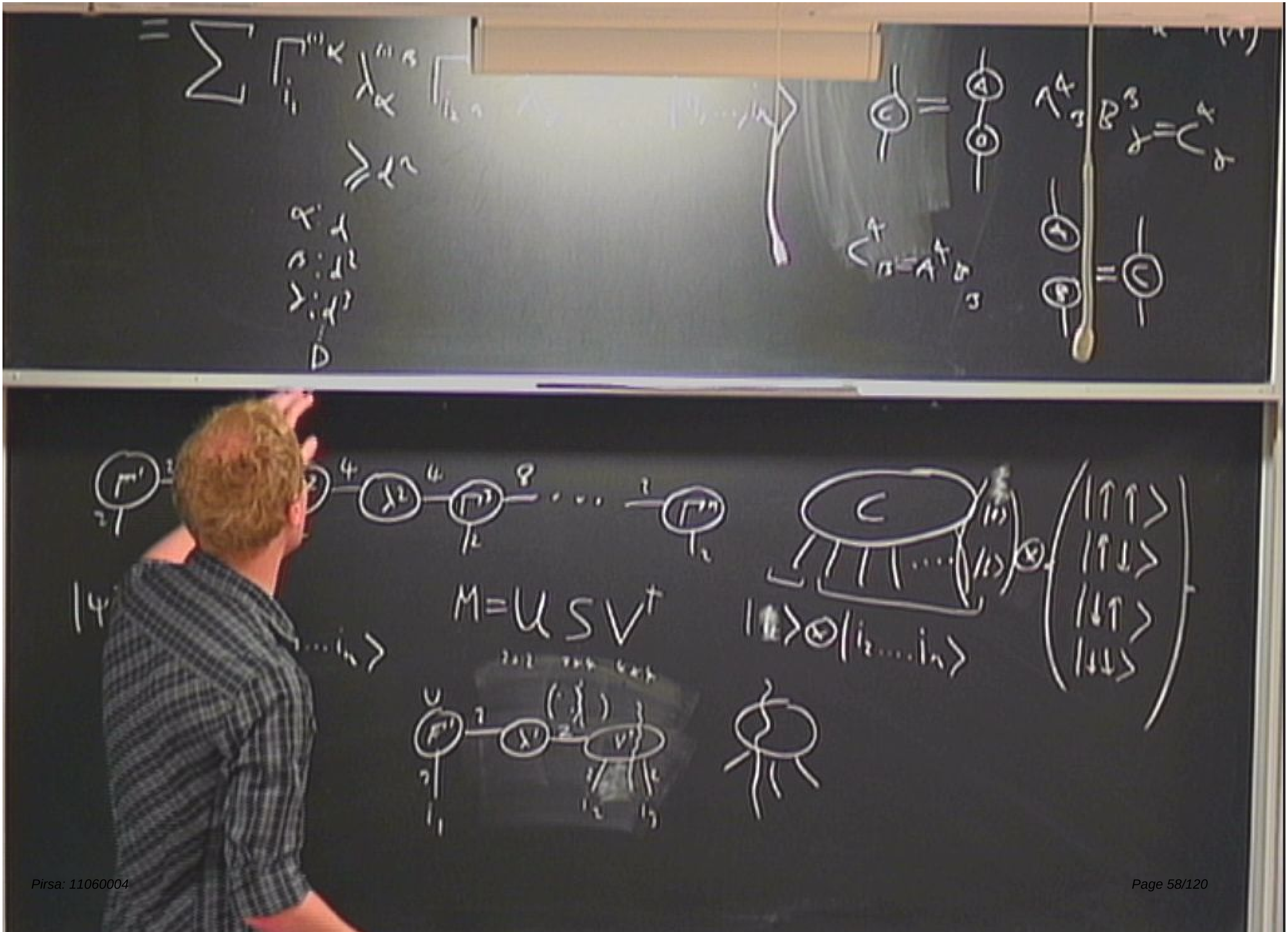


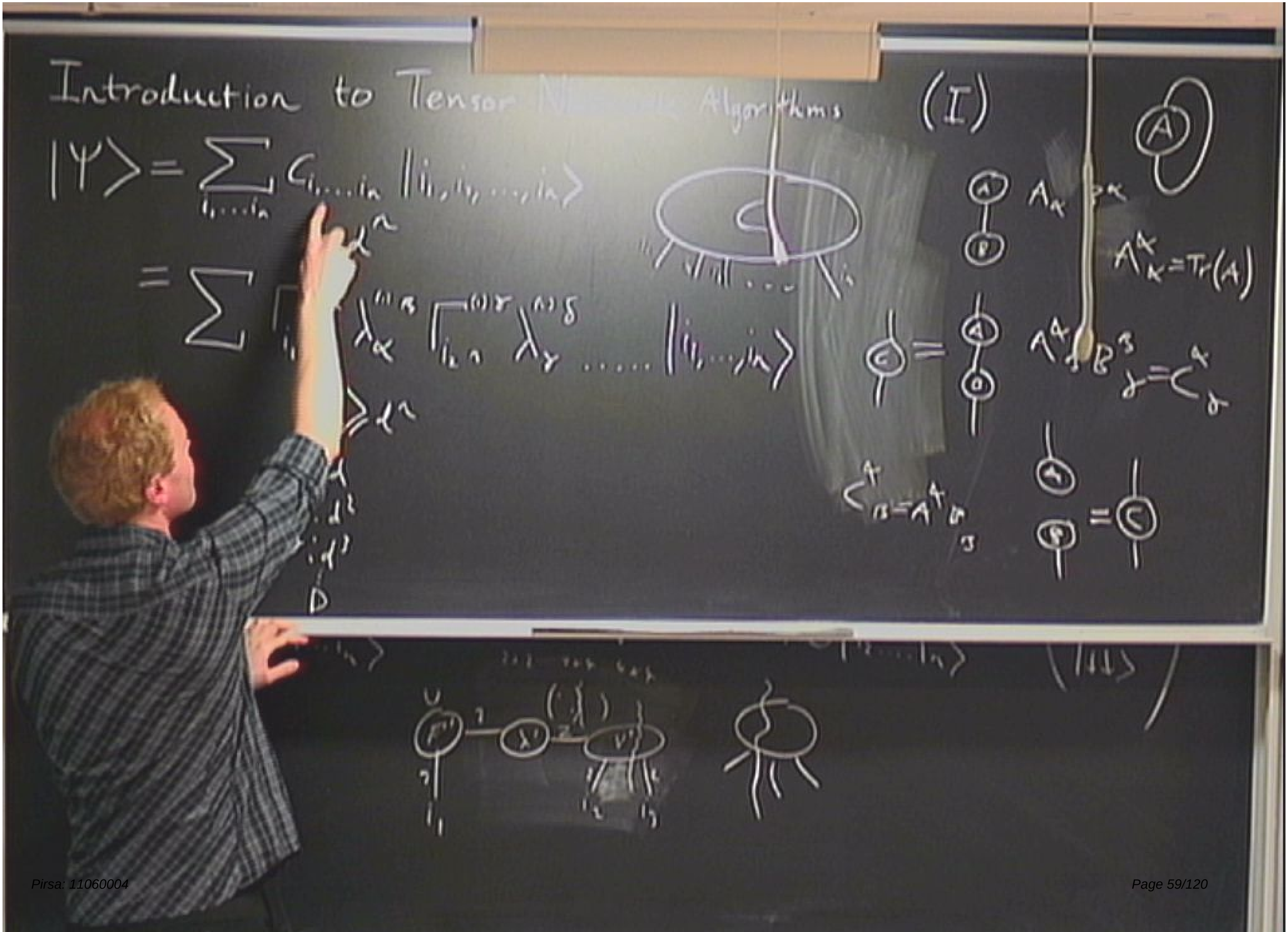

M=USV†

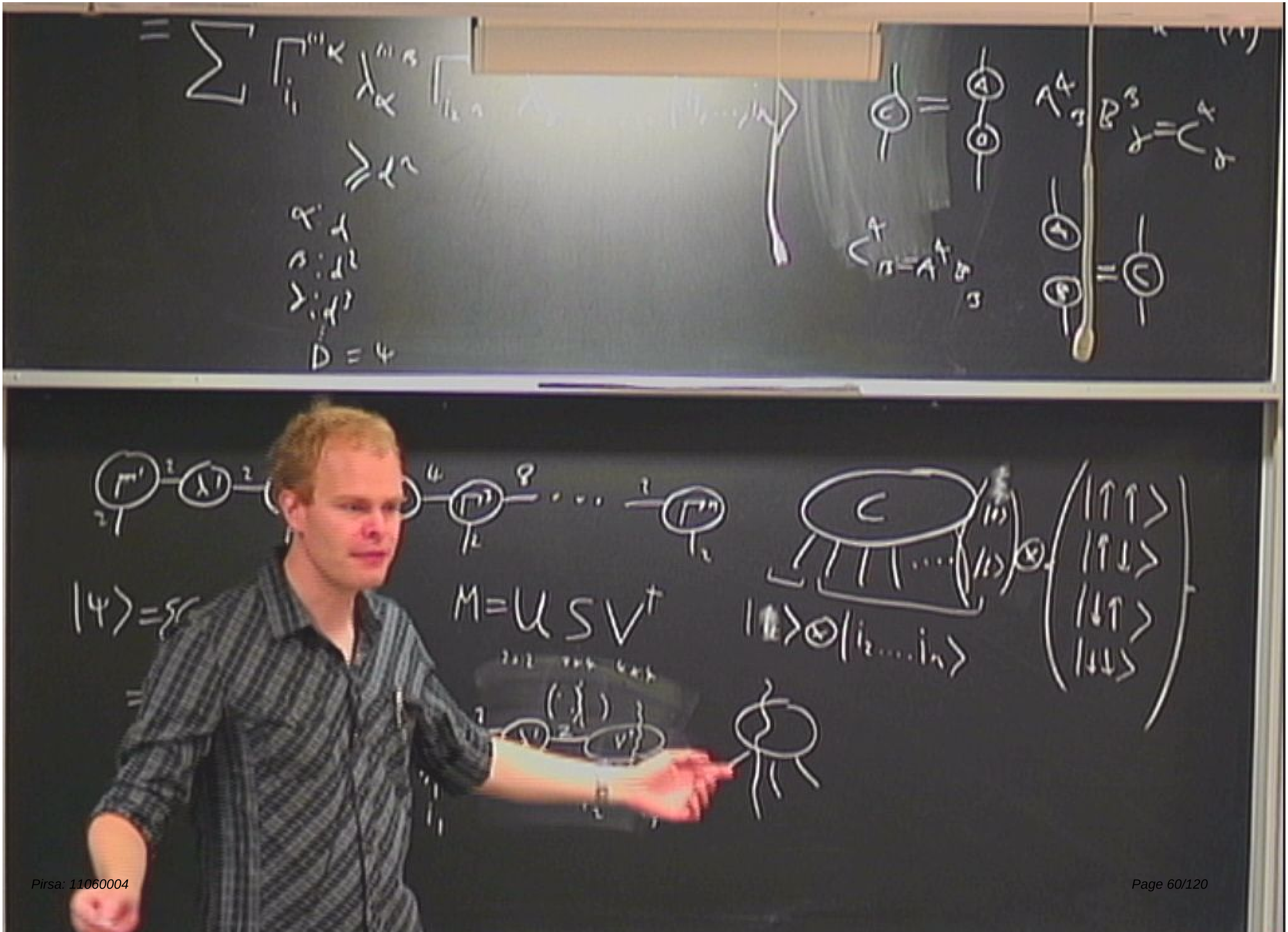

M=USV†
|ψ>=ΣC i1...in |i1...in>

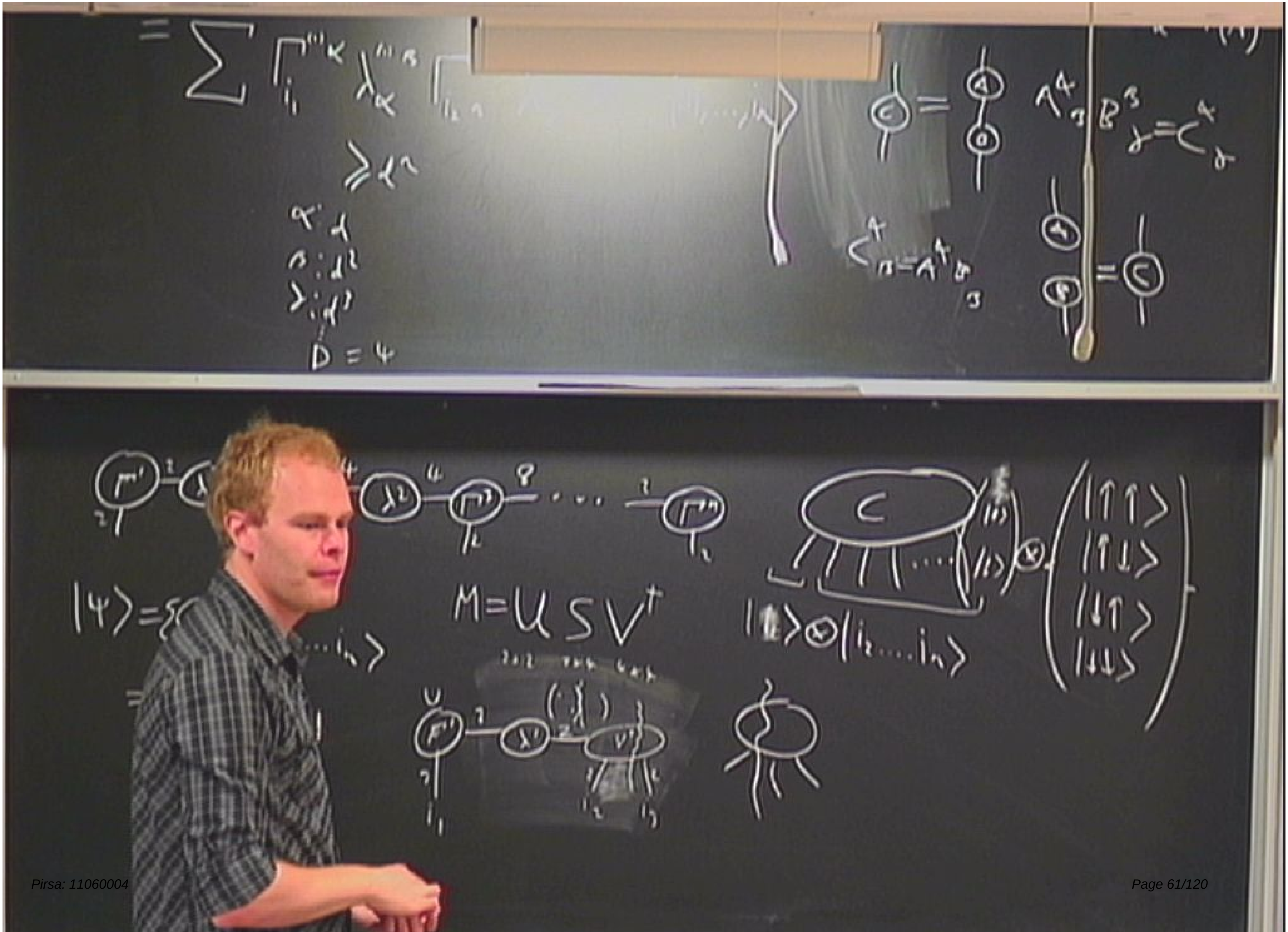

λ = (a 0 / 0 b)

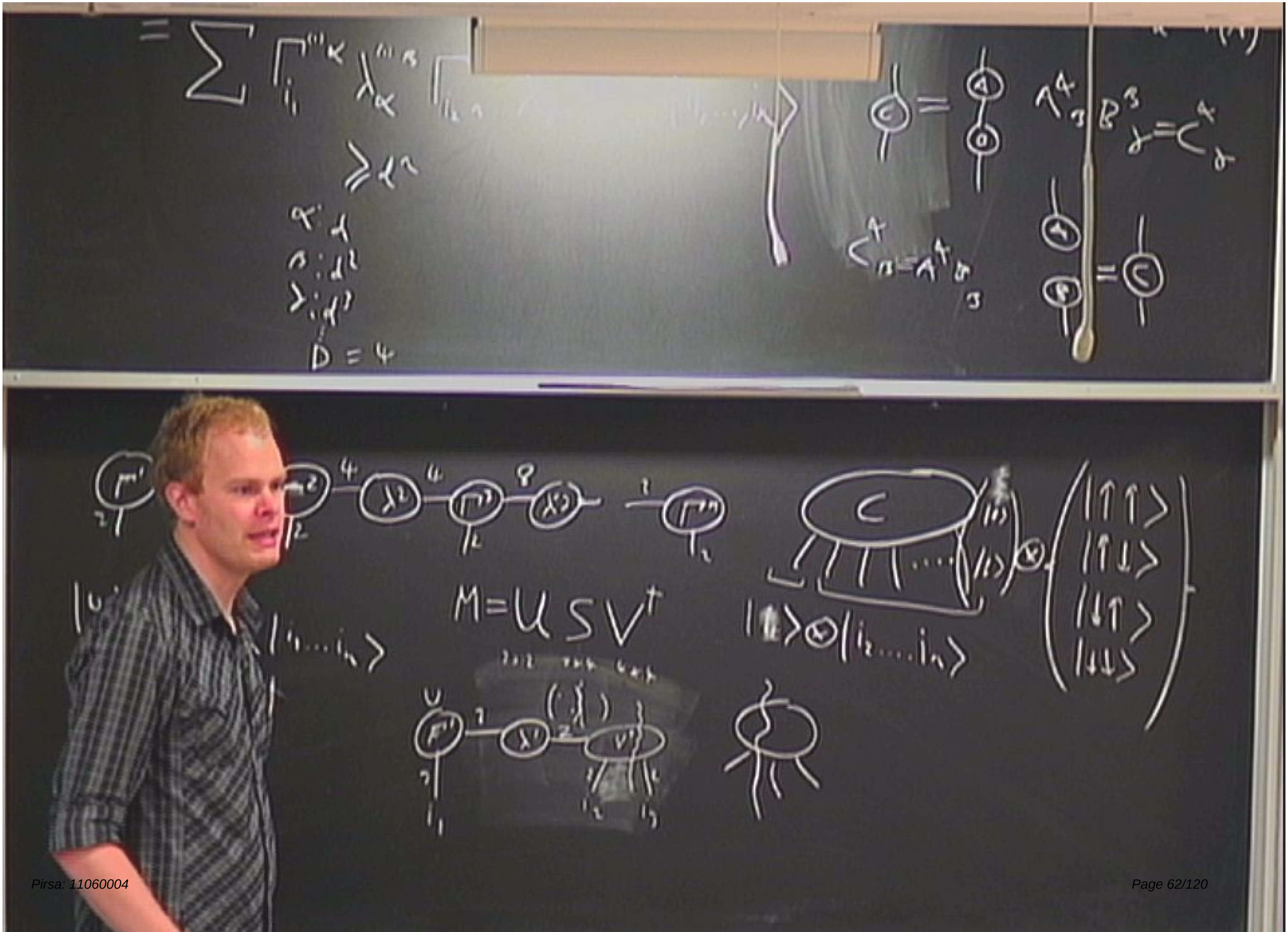


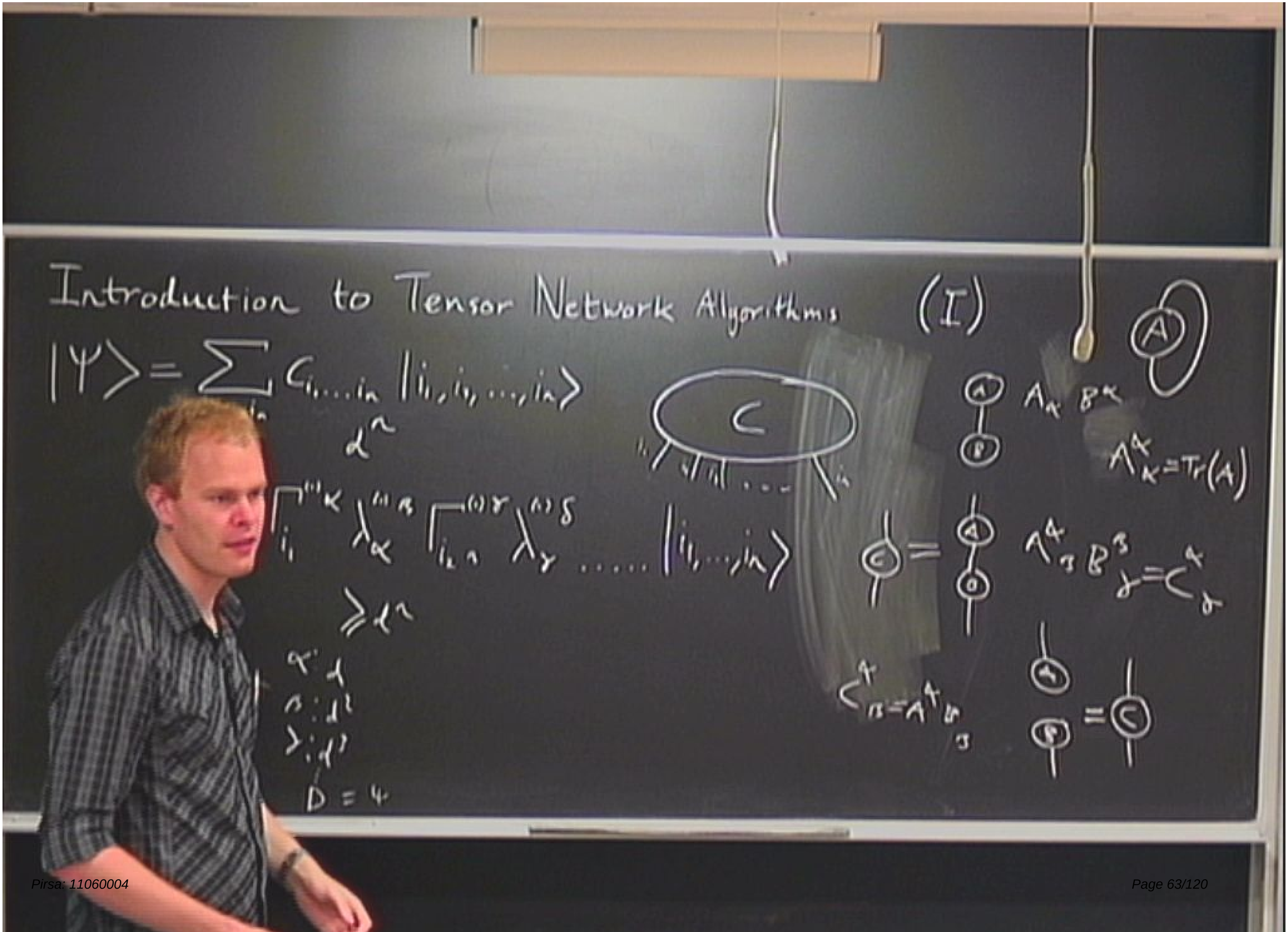

M=USV†

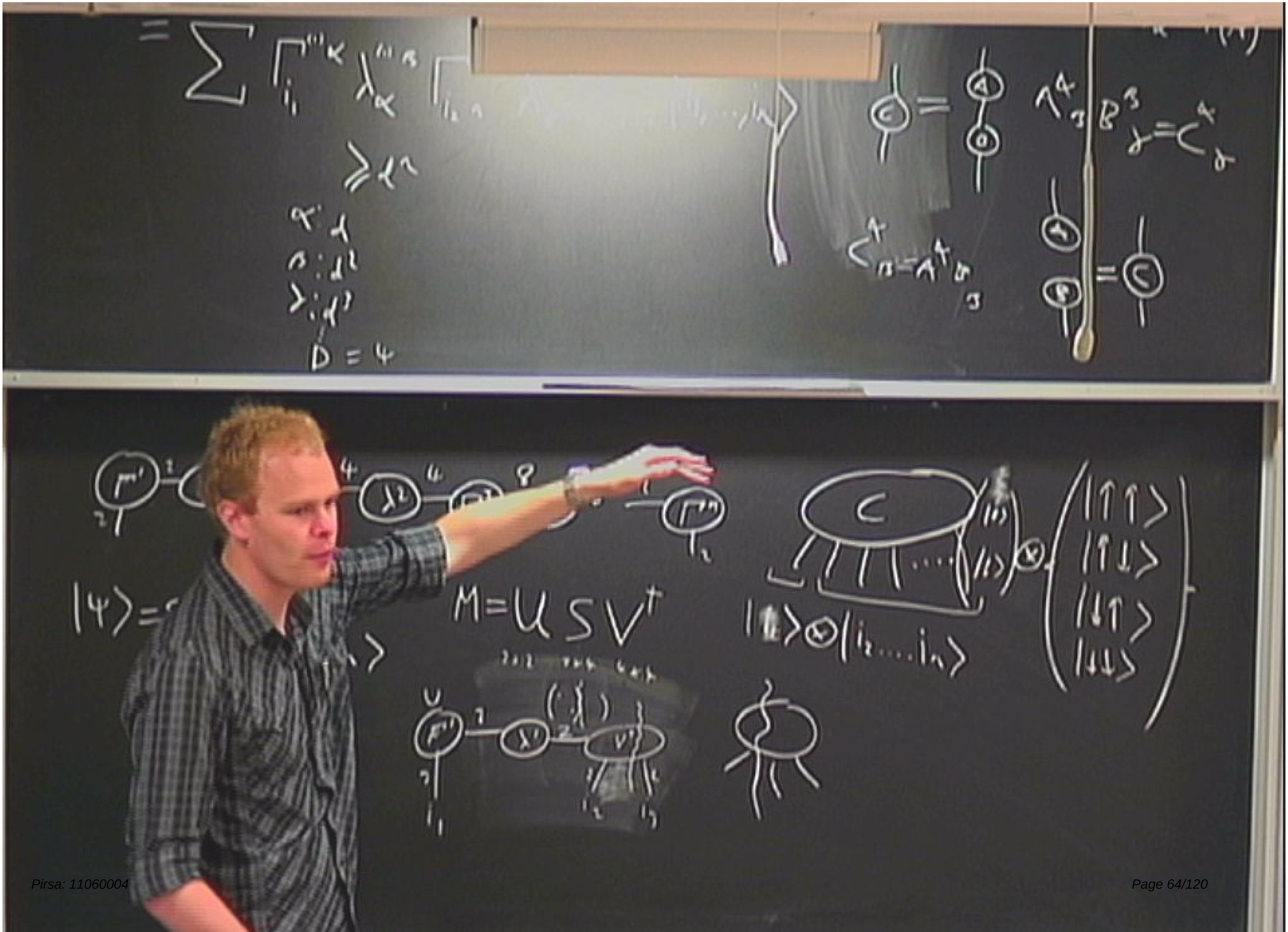

Introduction to Tensor
Algorithms
(I)

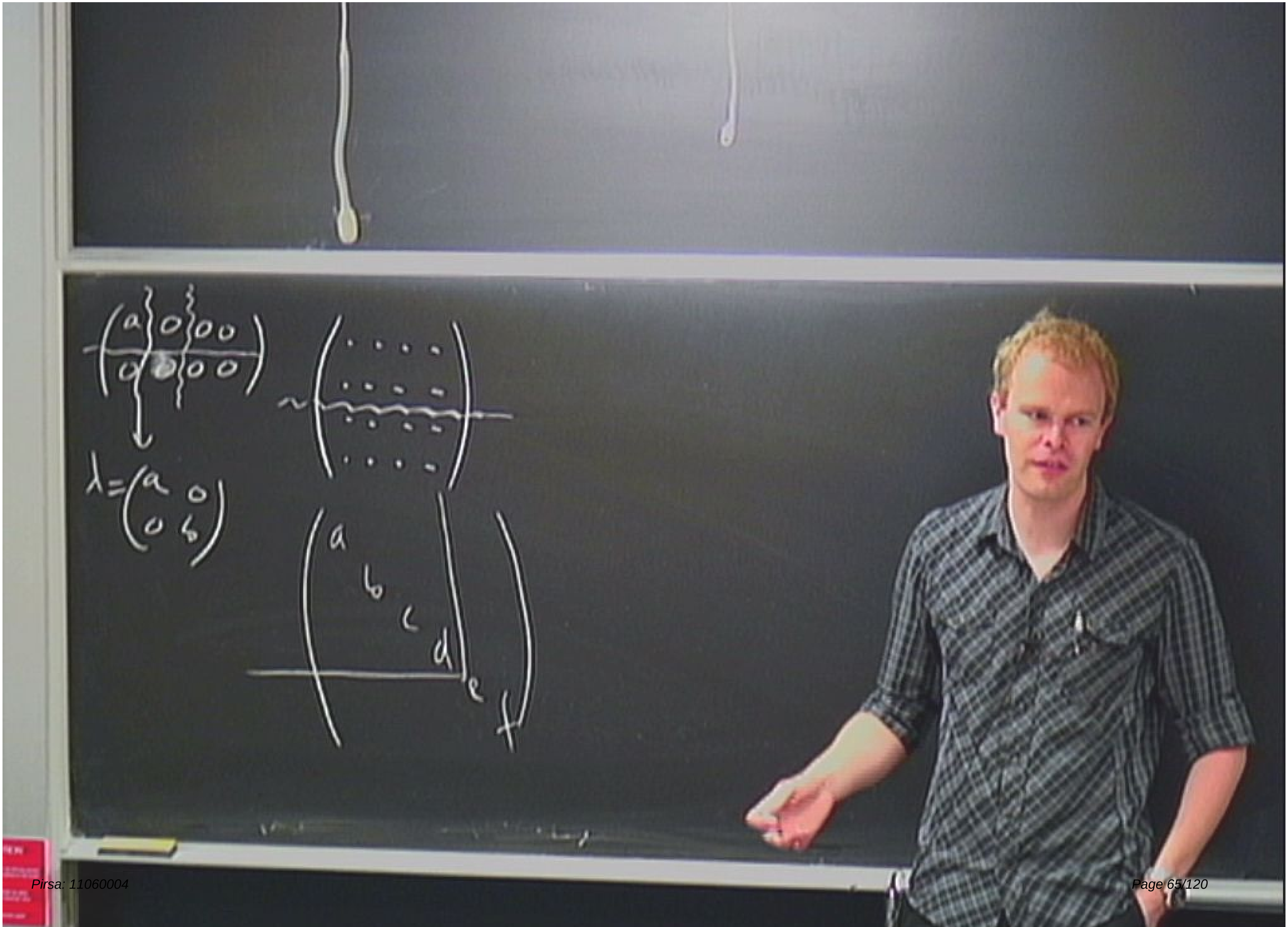

D = 4
M=USV†

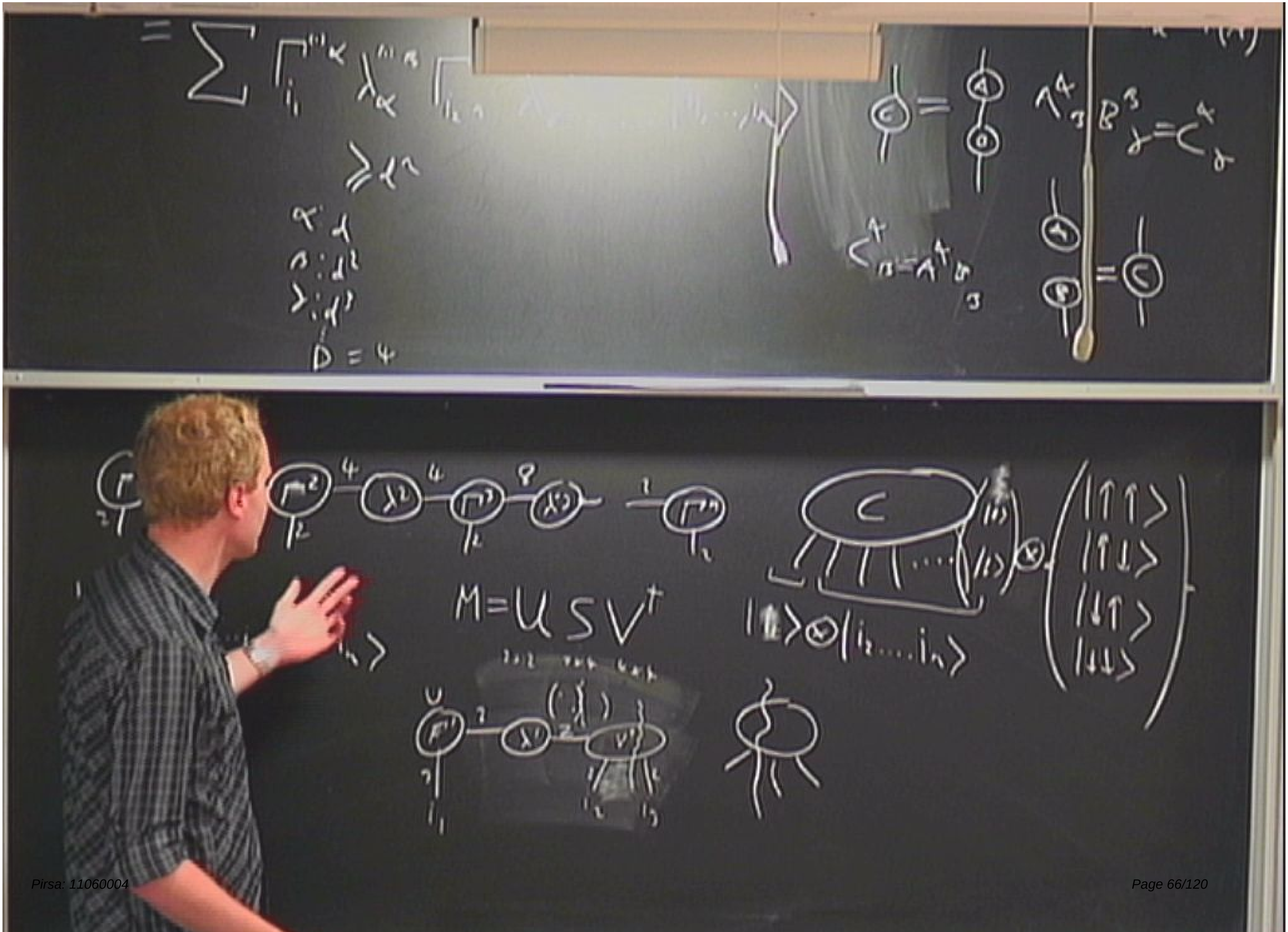

M=USV†


M=USV†
D = 4


Introduction to Tensor Network Algorithms (I)
C
D = 4


M=USV†
D = 4

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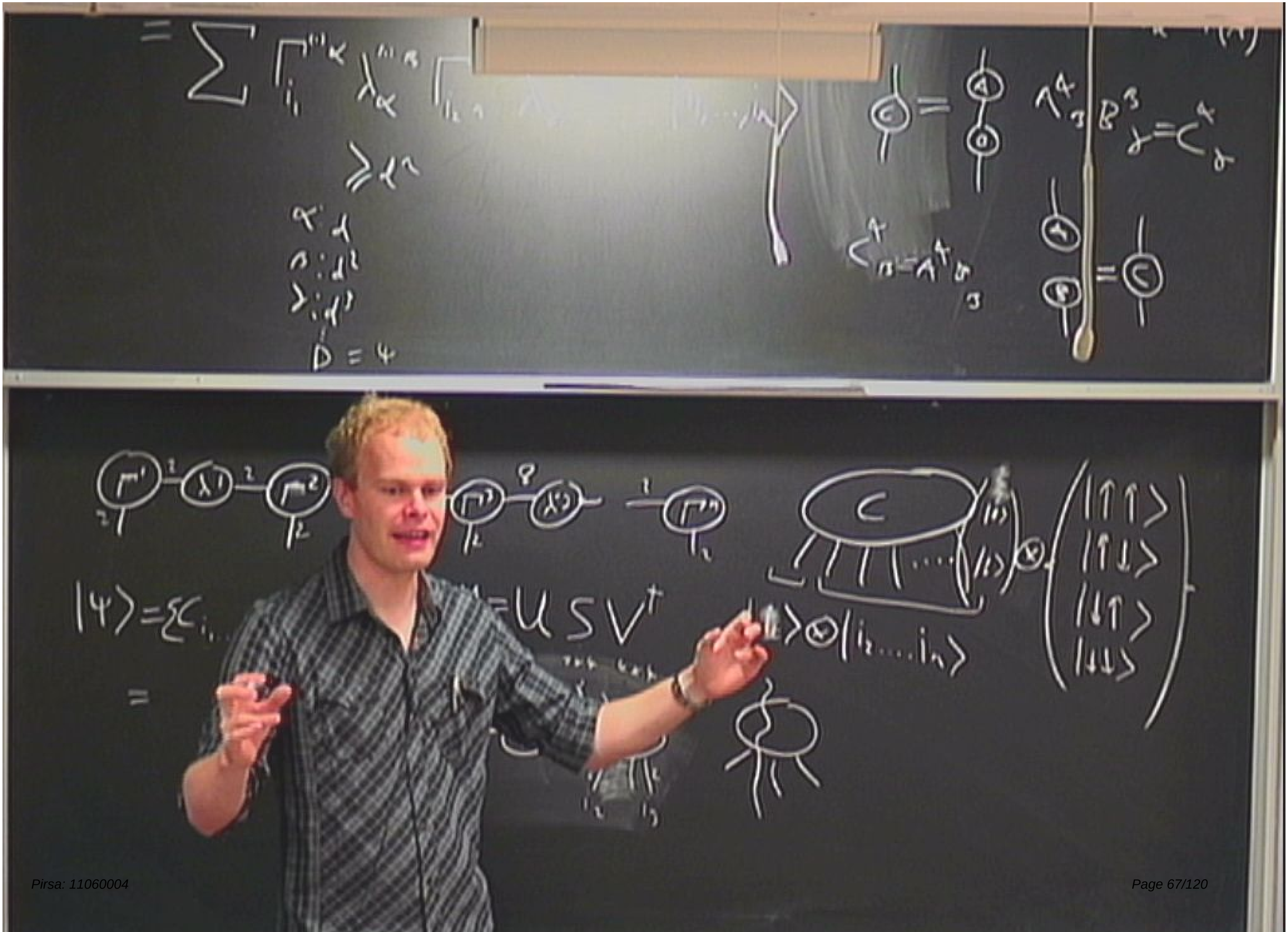
D = 4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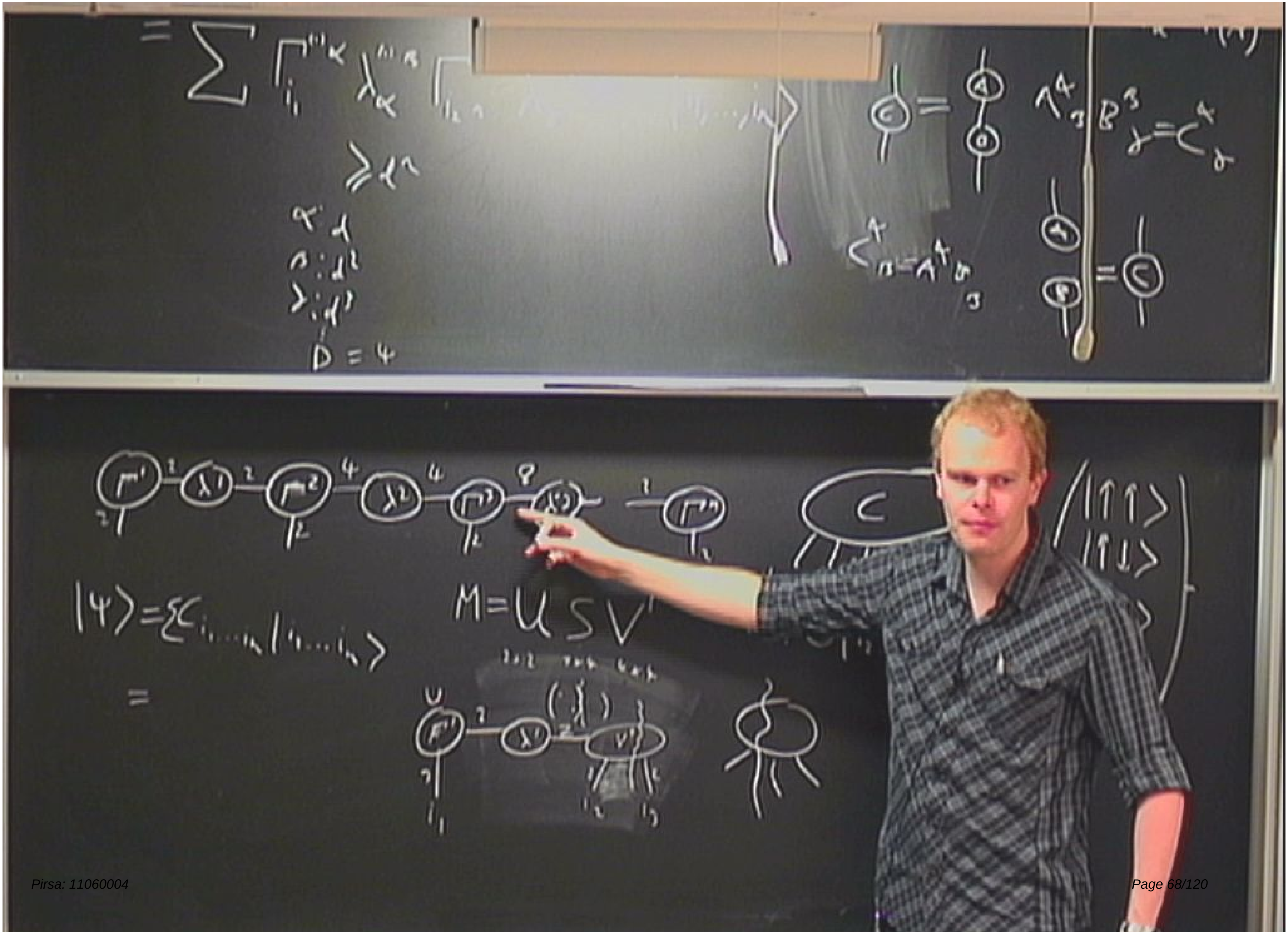

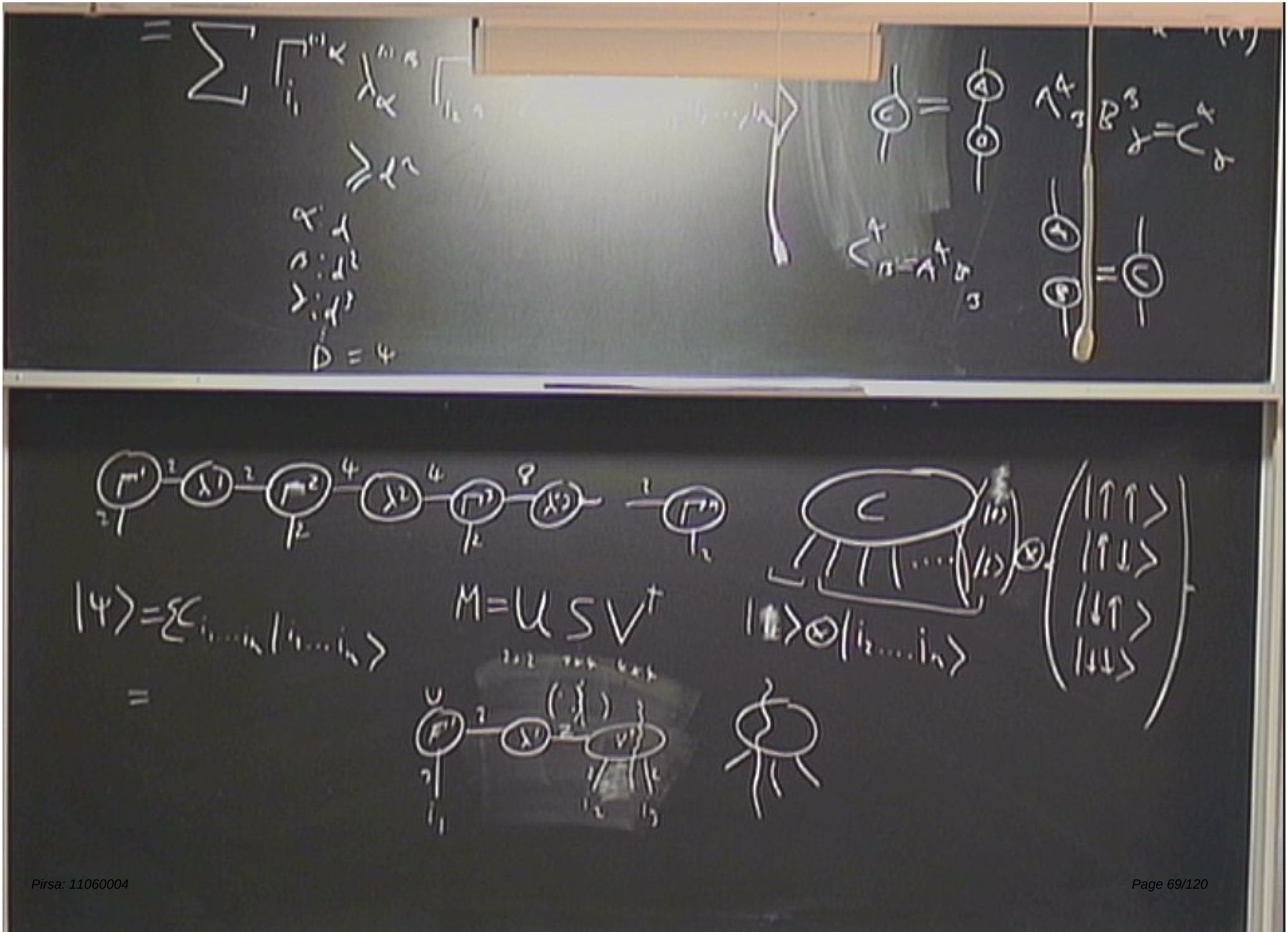
D = 4
M=USV†
|↑↑>
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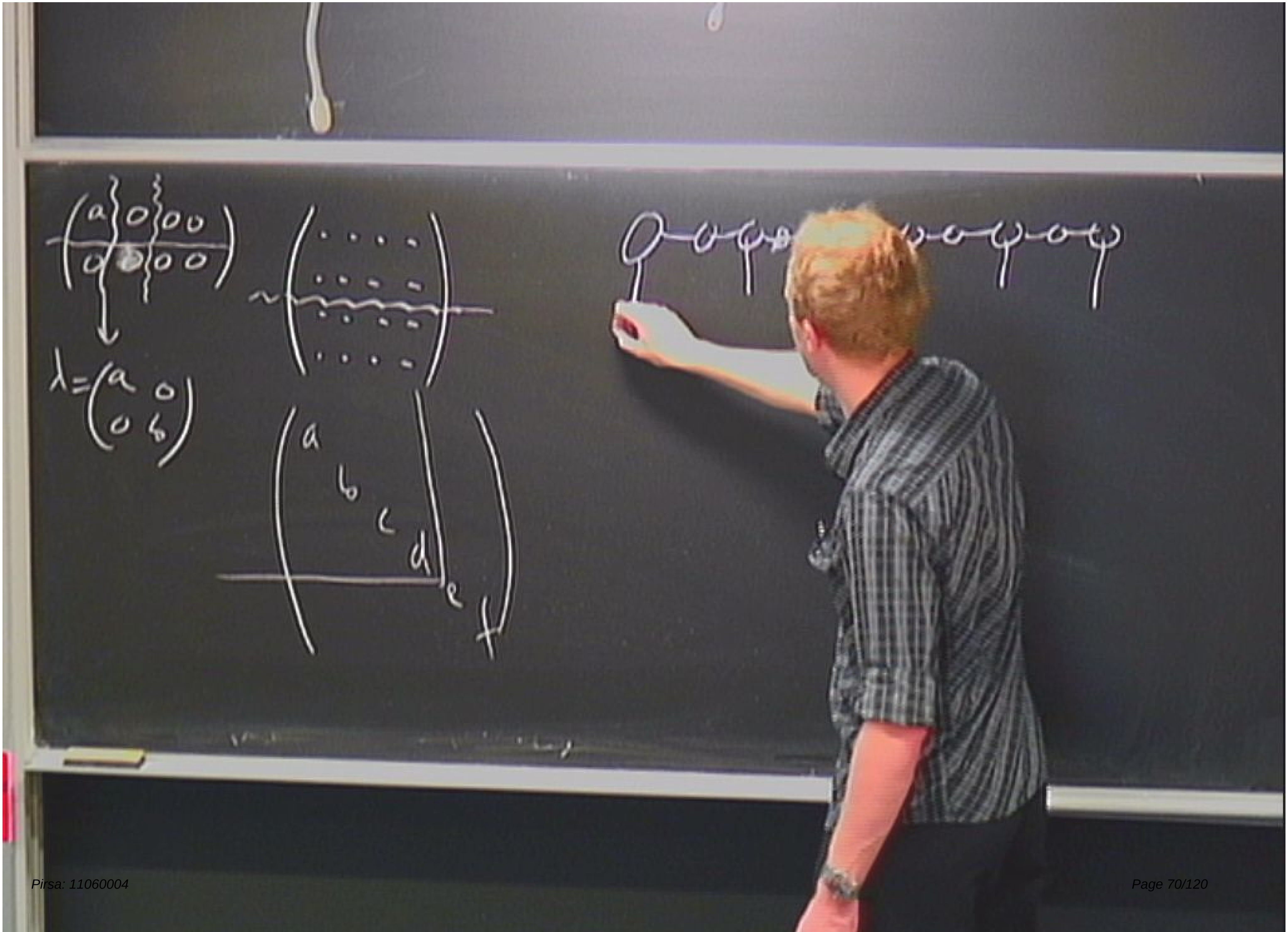
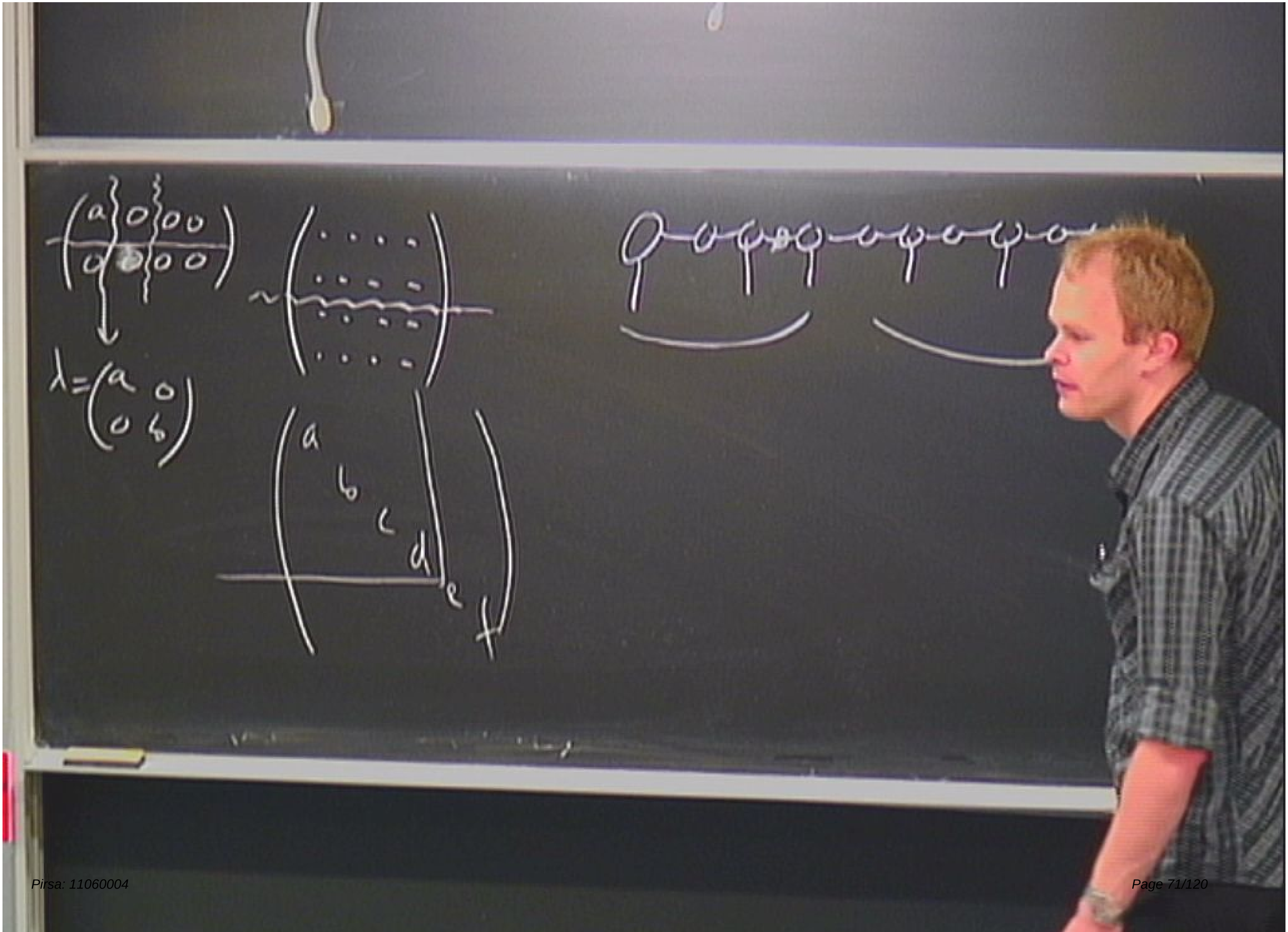
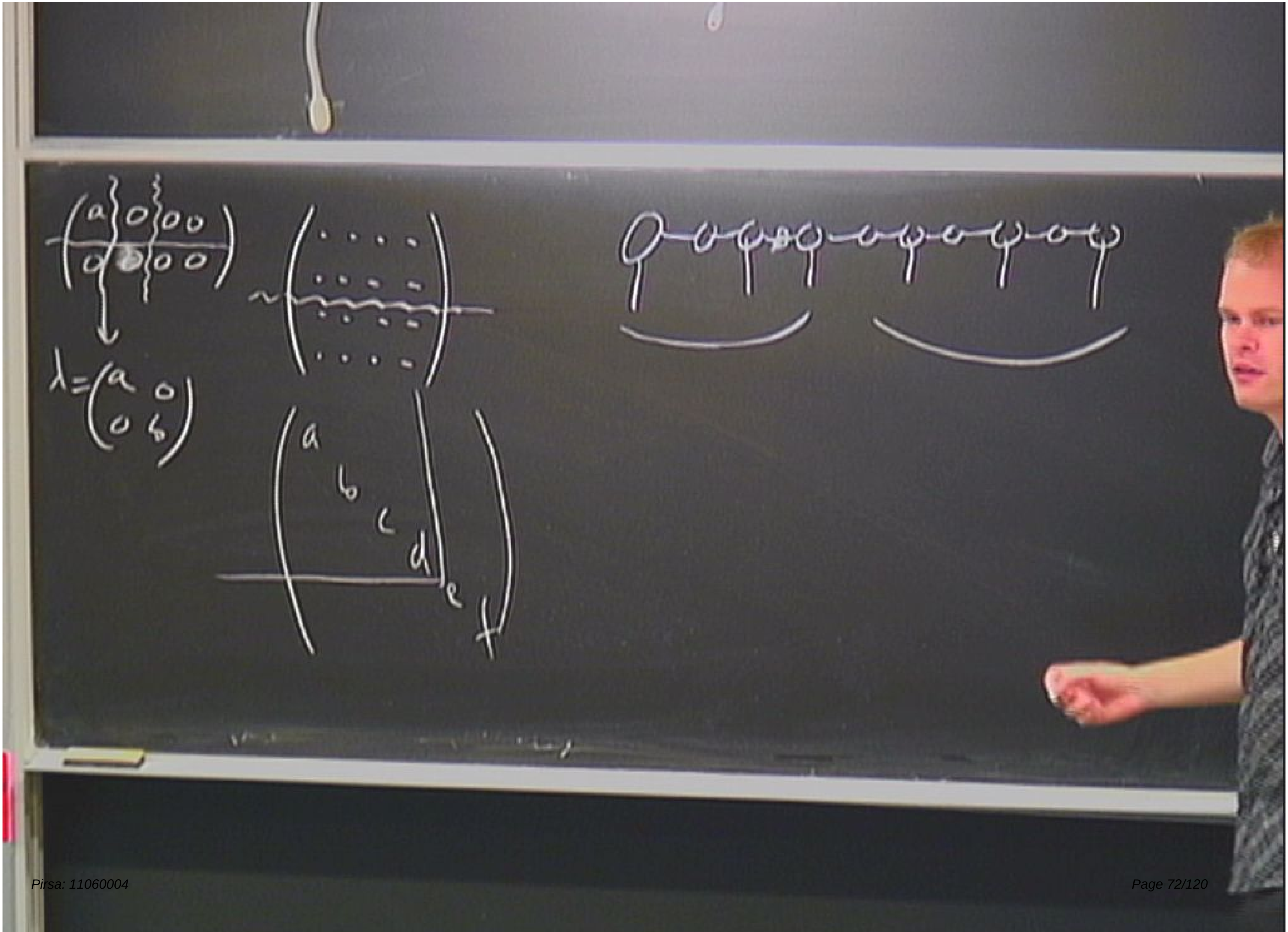
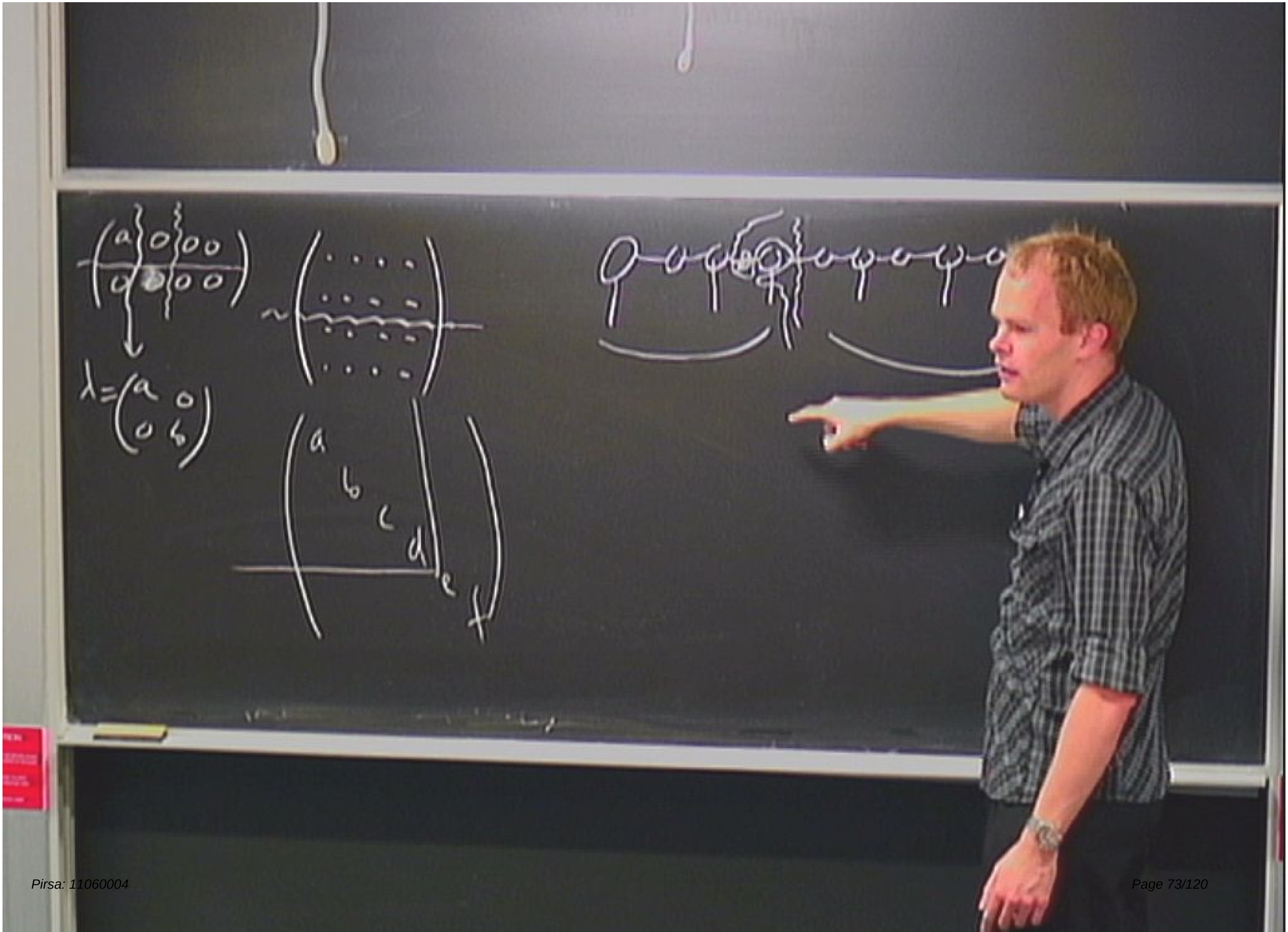
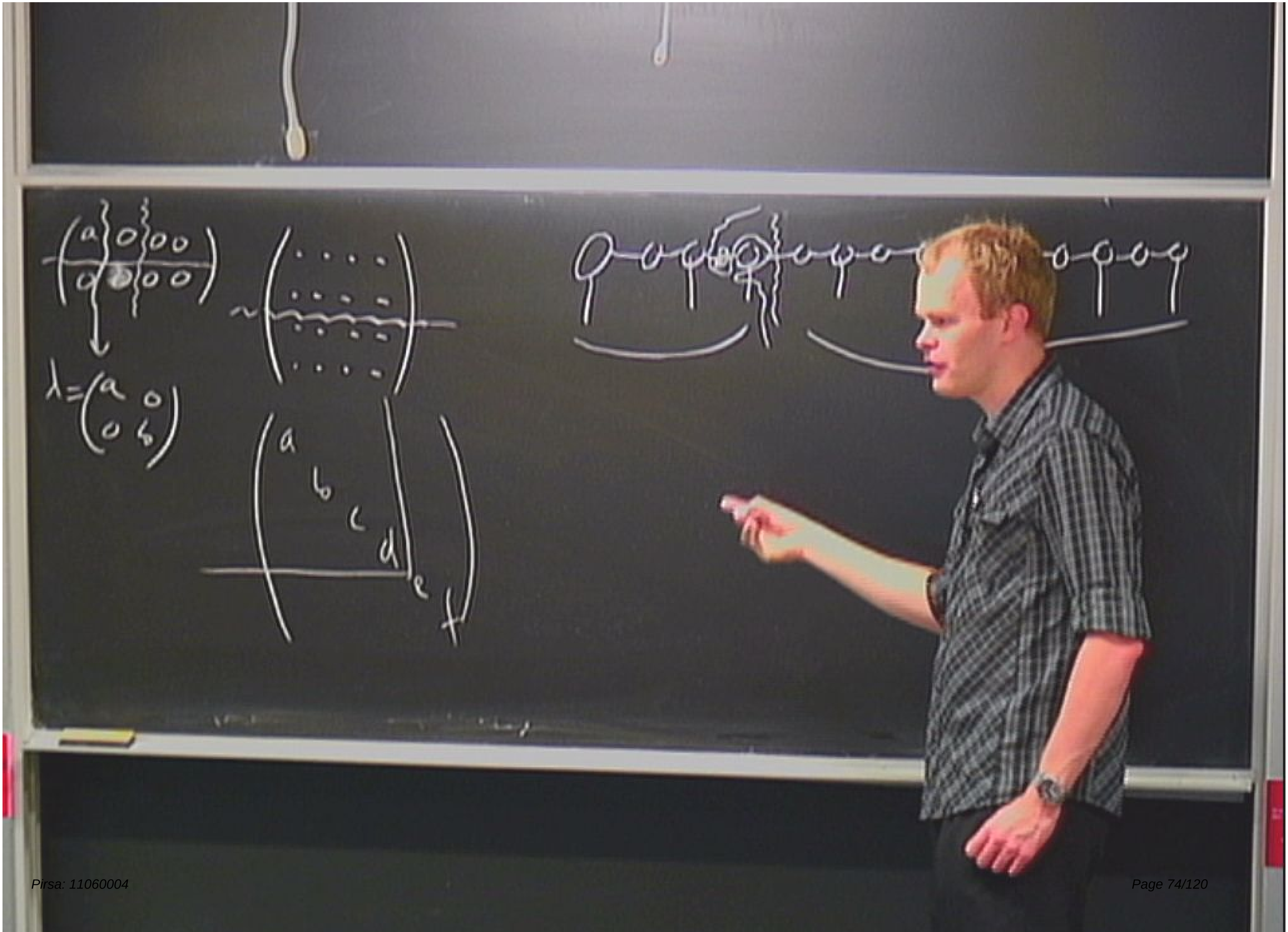
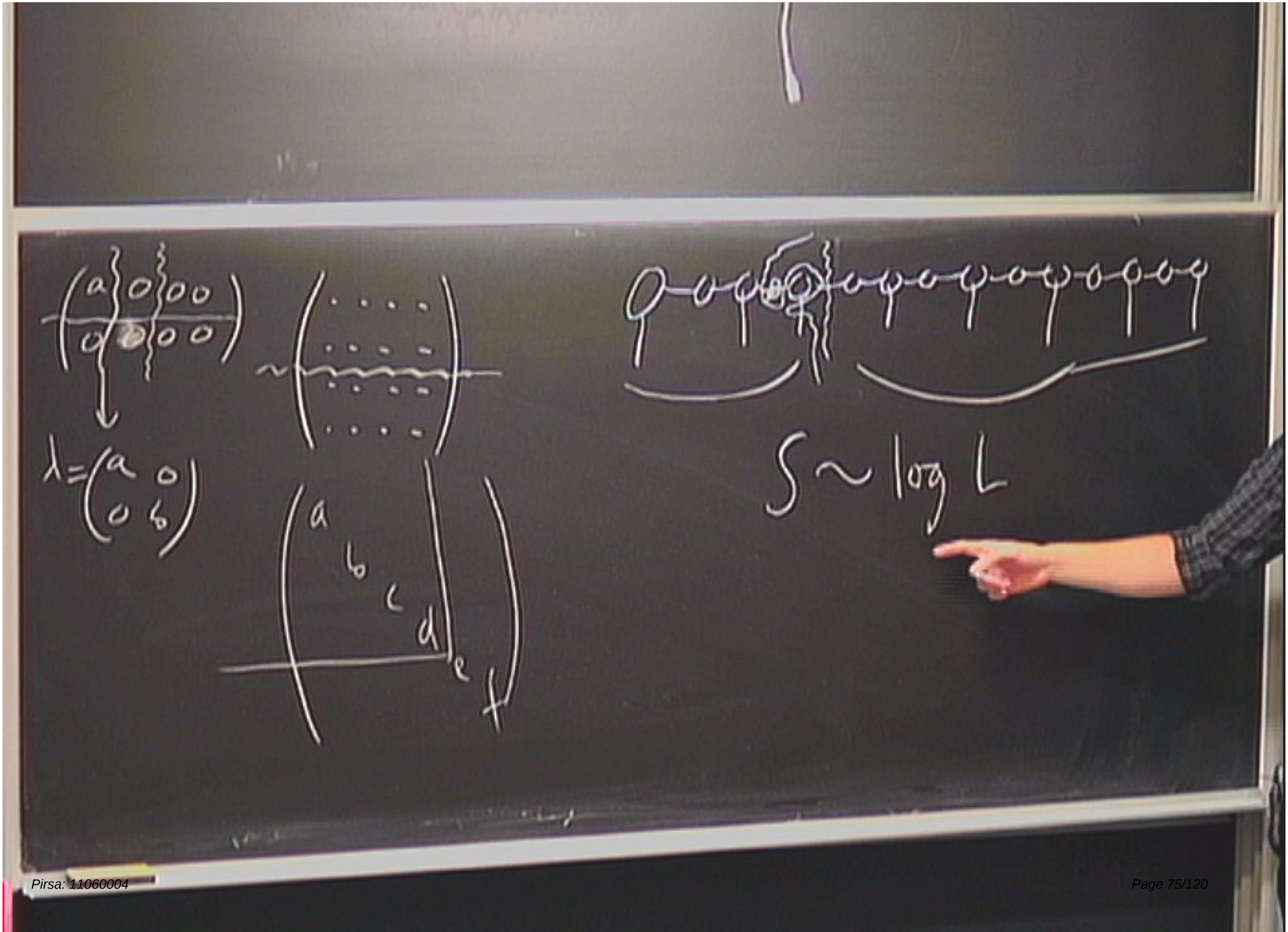


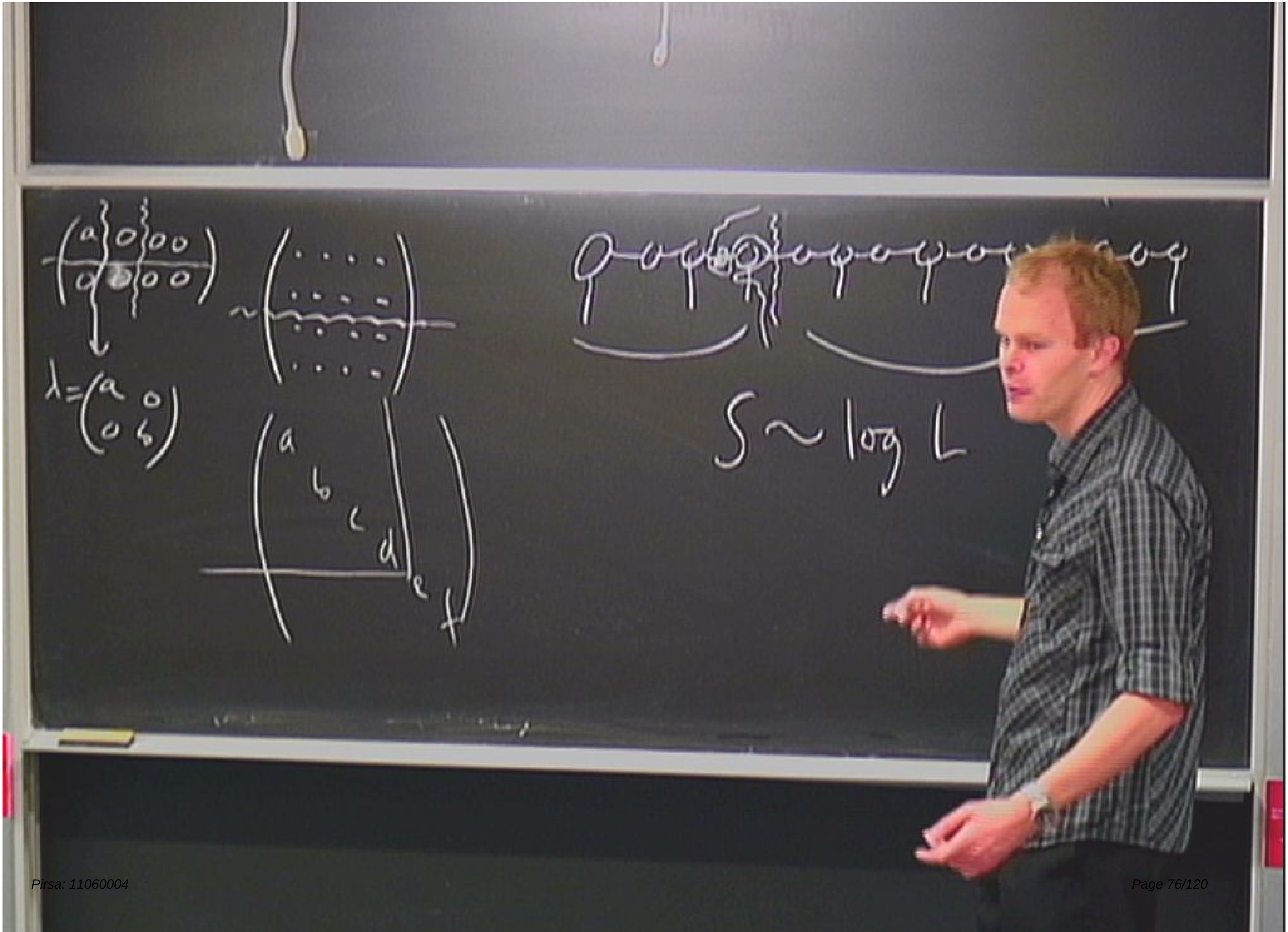









S ~ log L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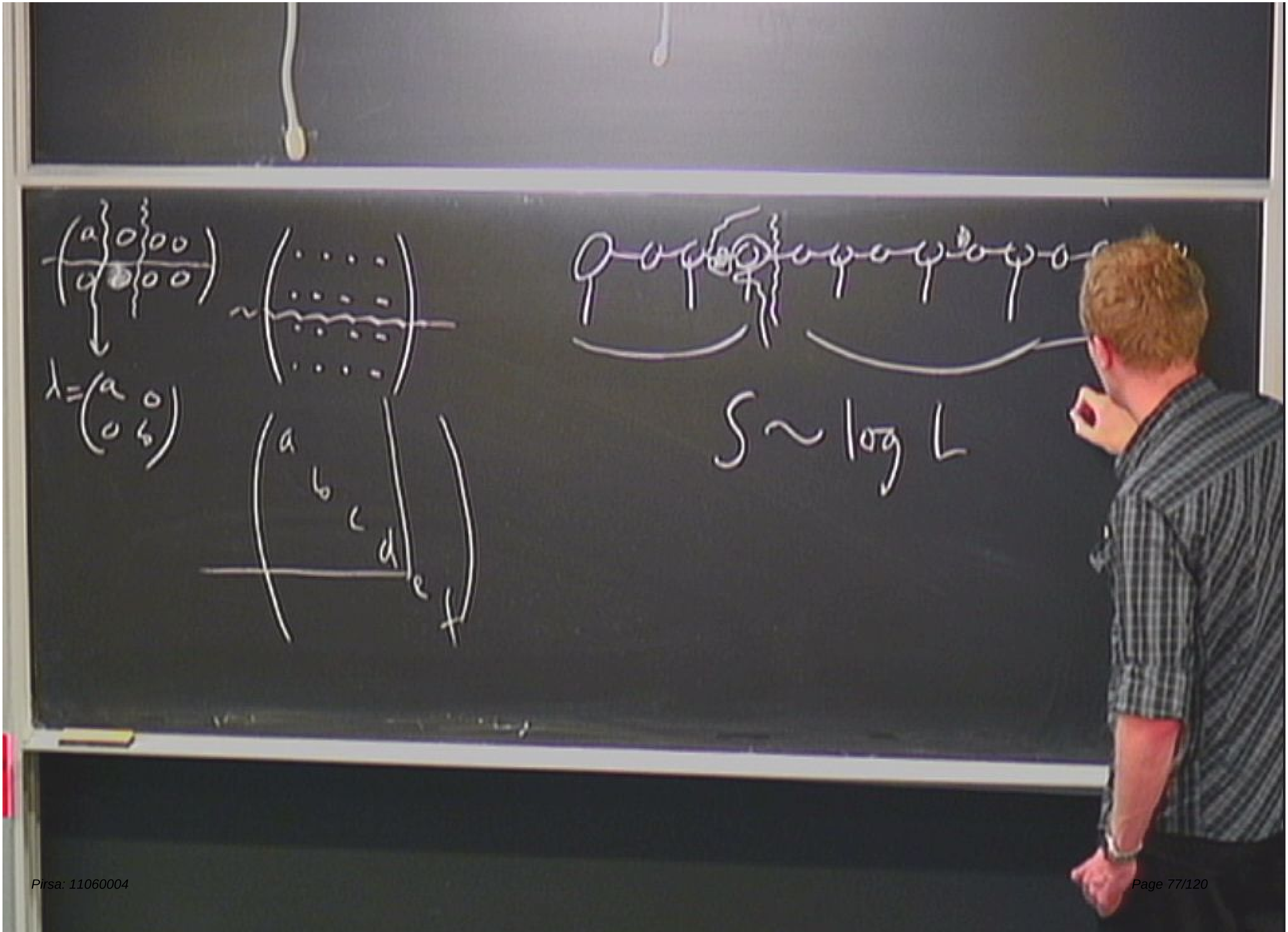
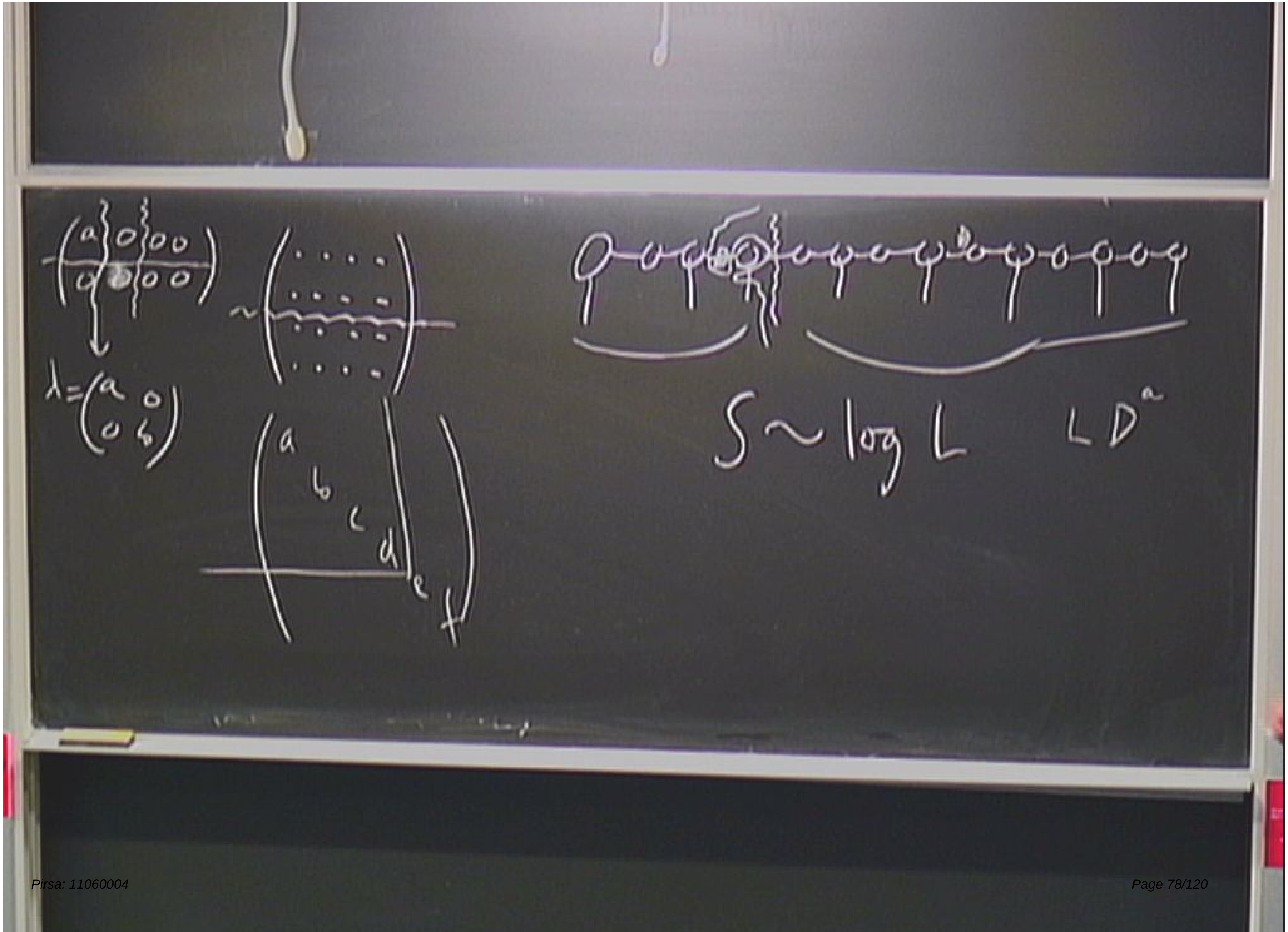
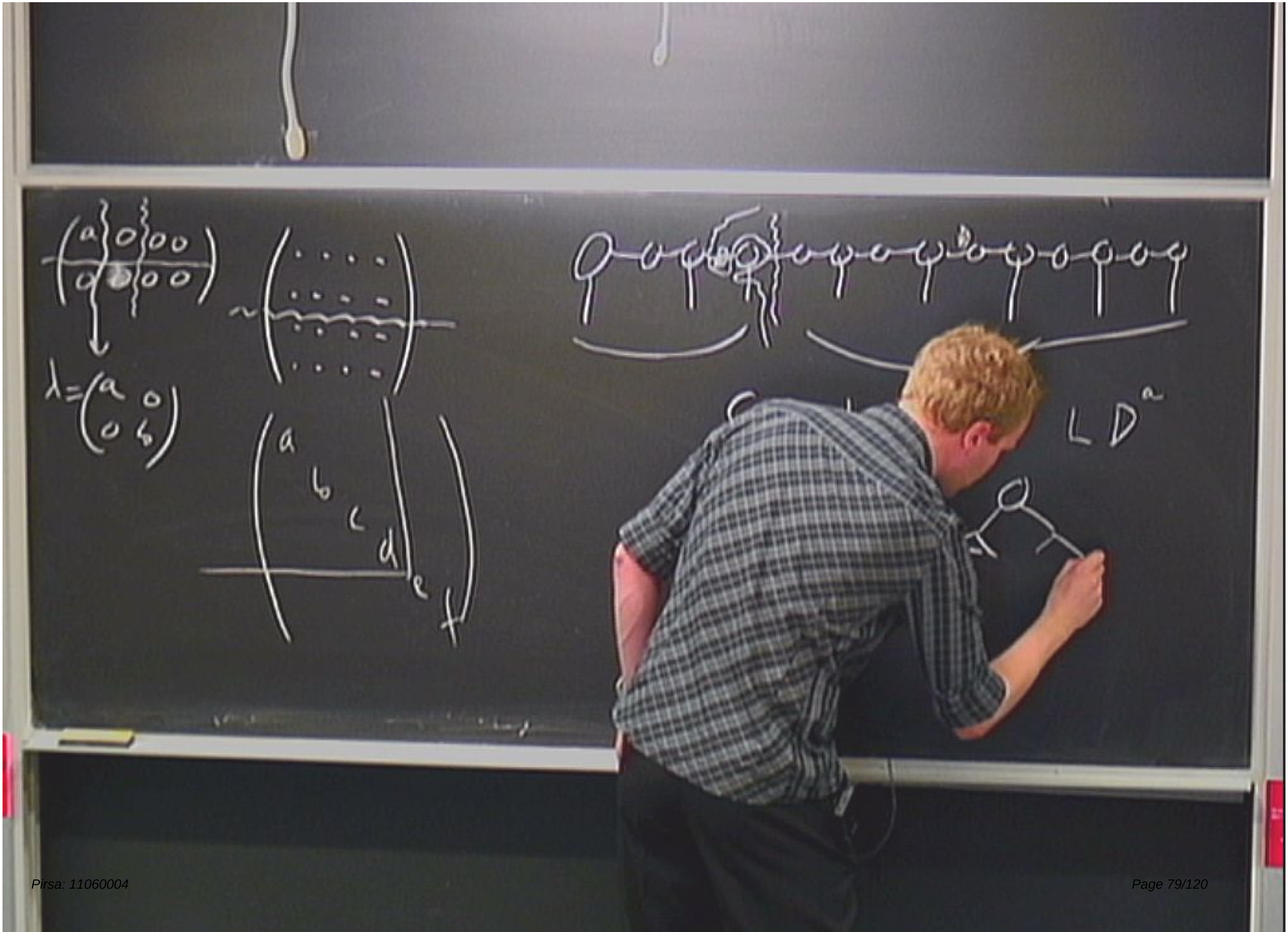
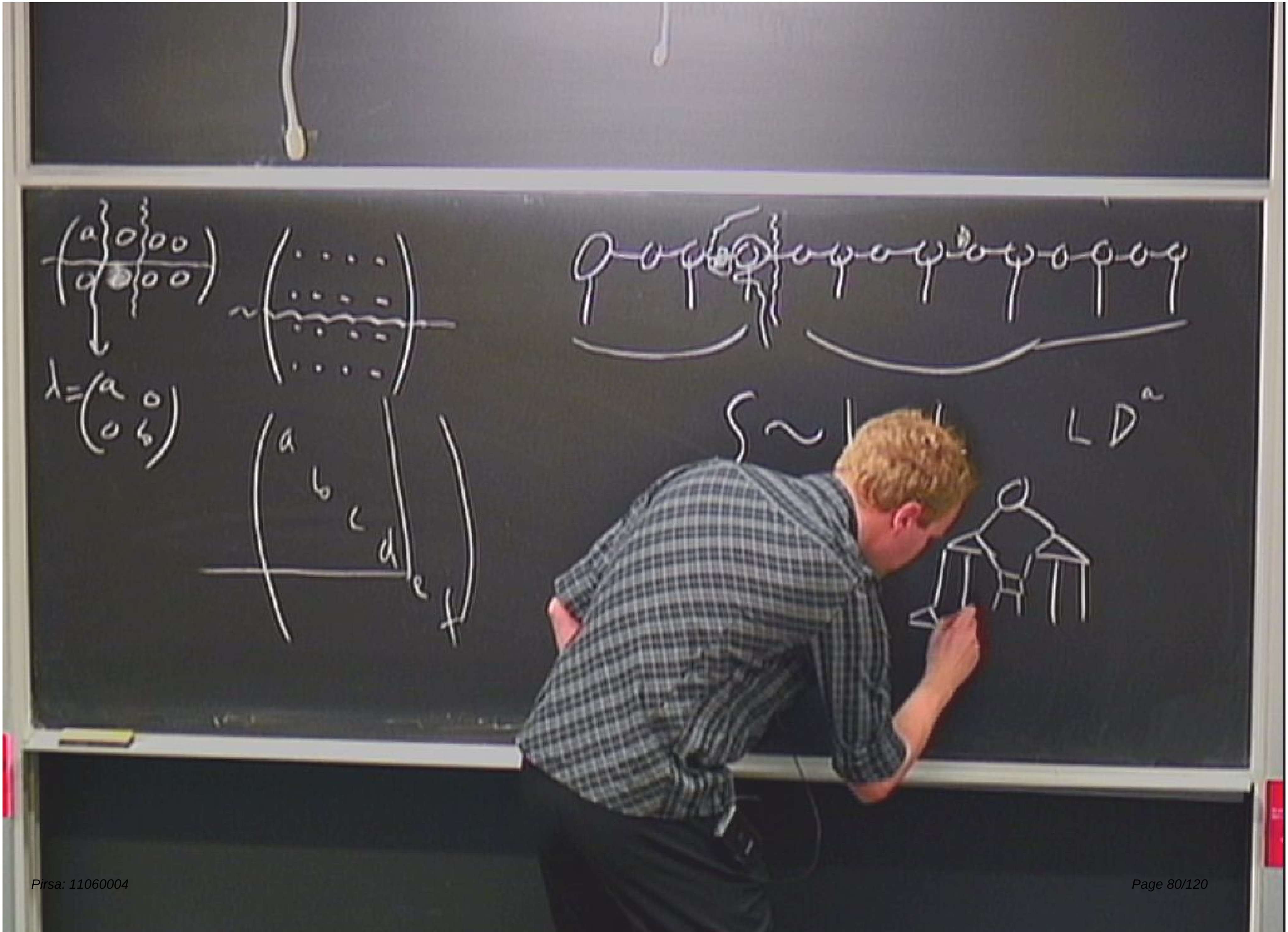
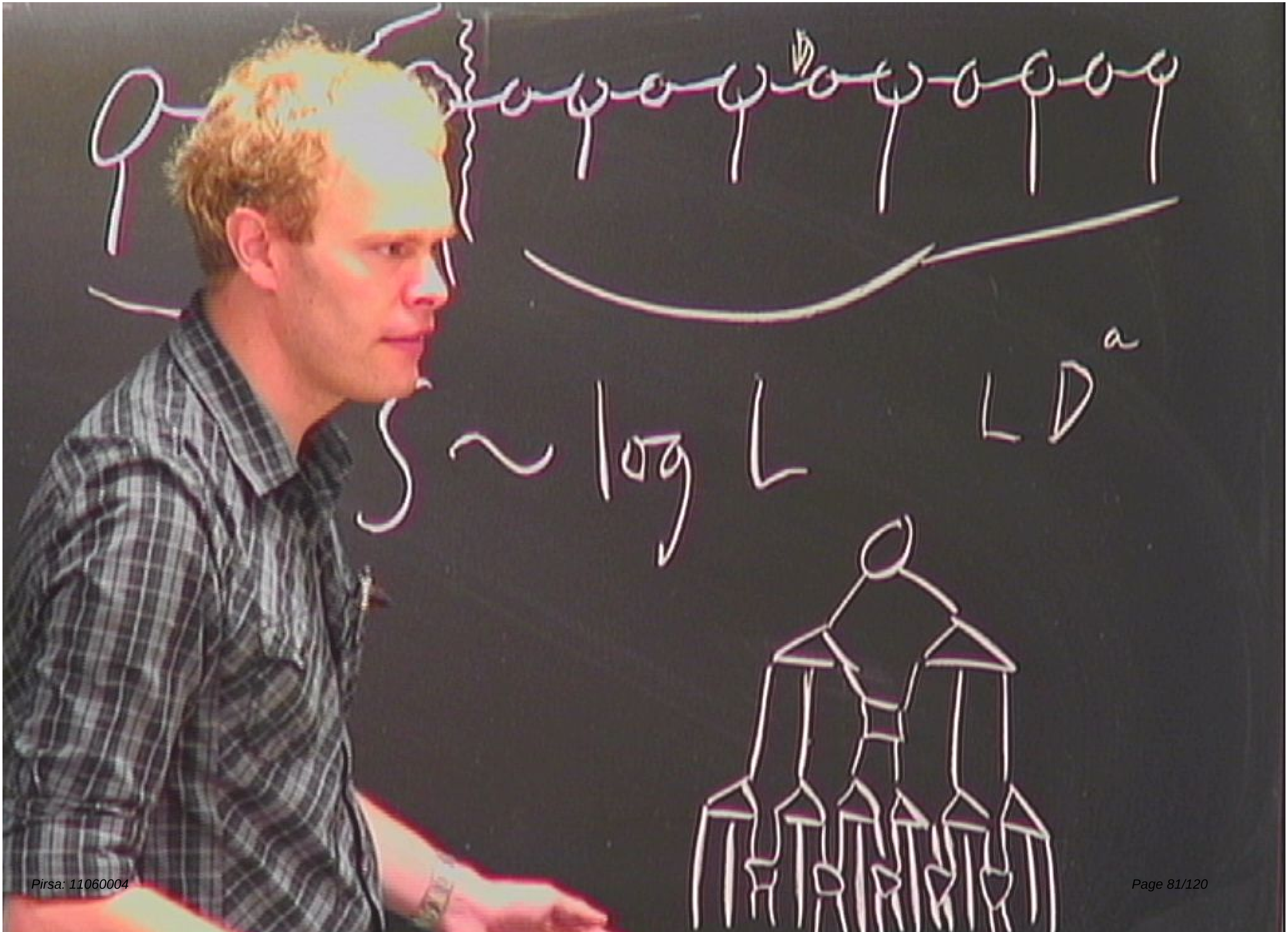
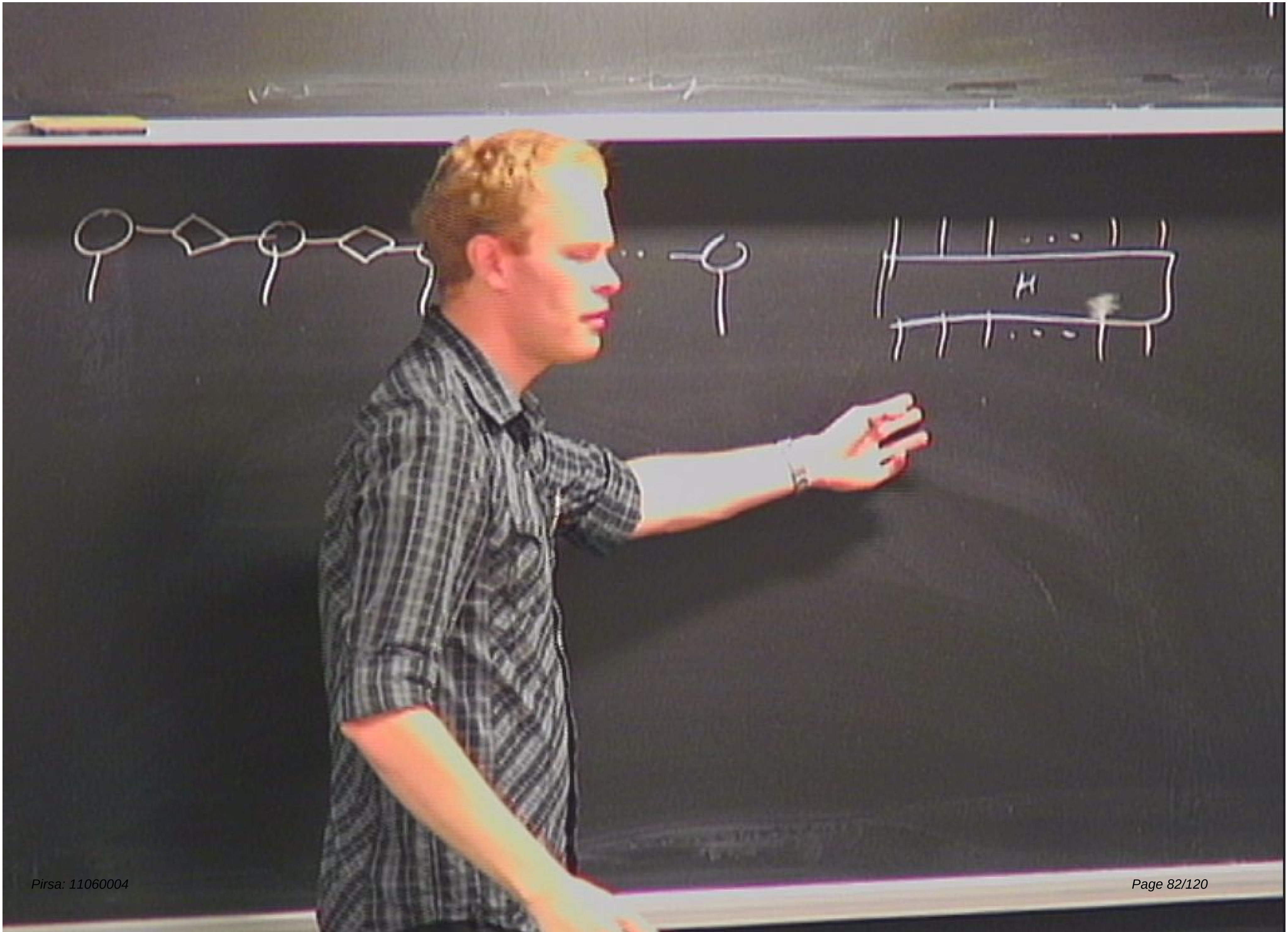

S ~ log L

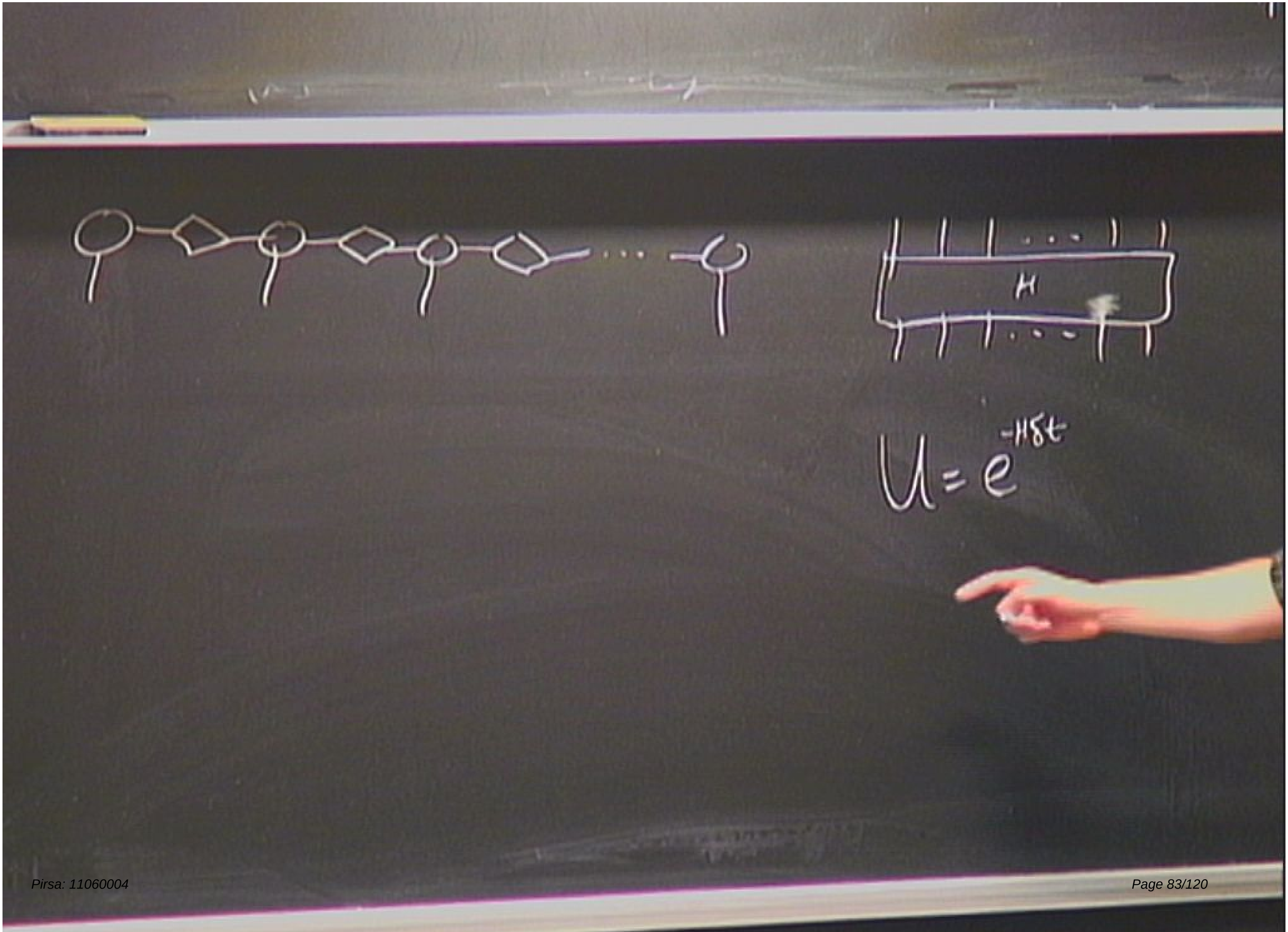

λ = (a 0 / 0 b)
S ~ log L
L D^a

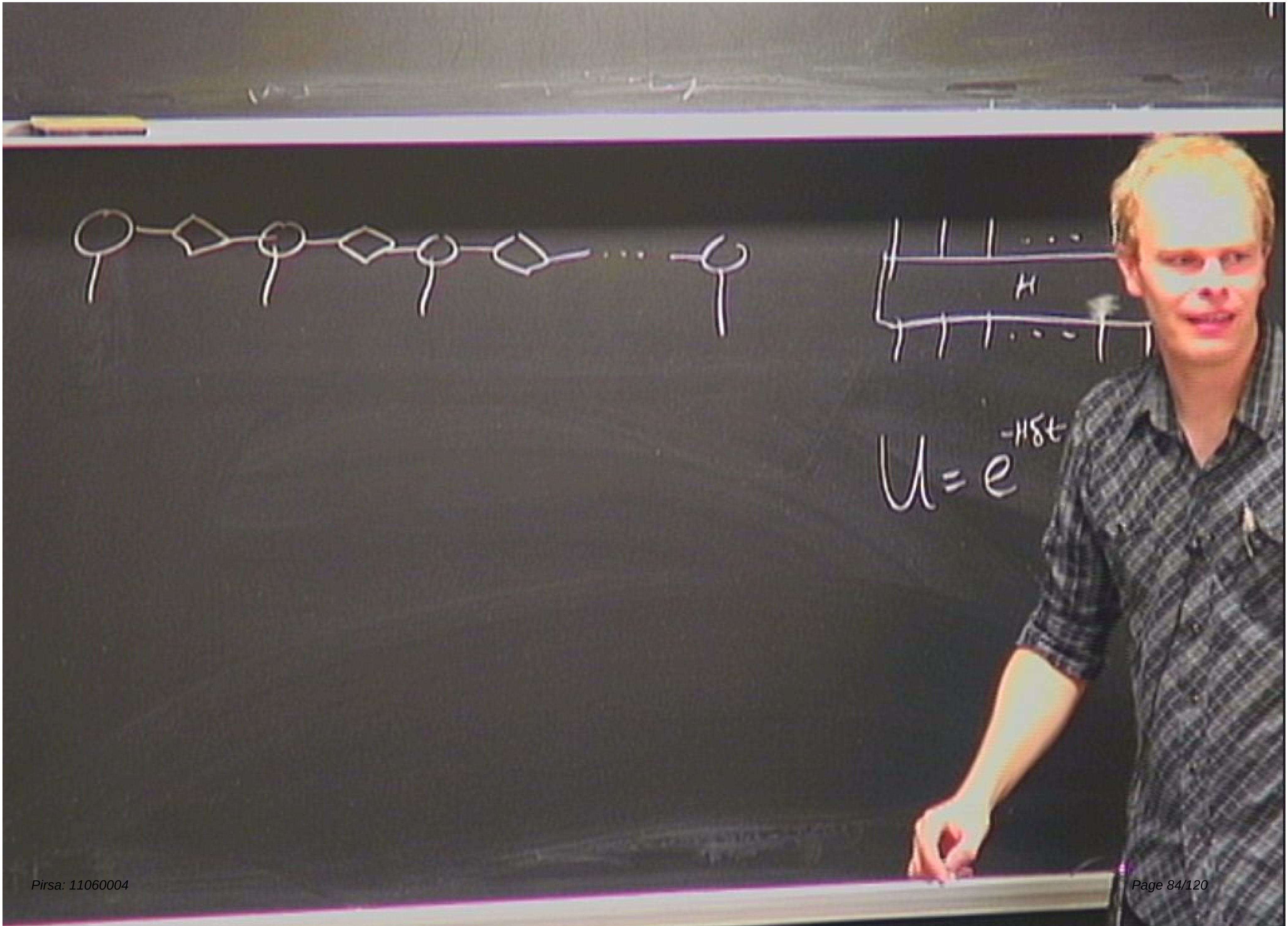


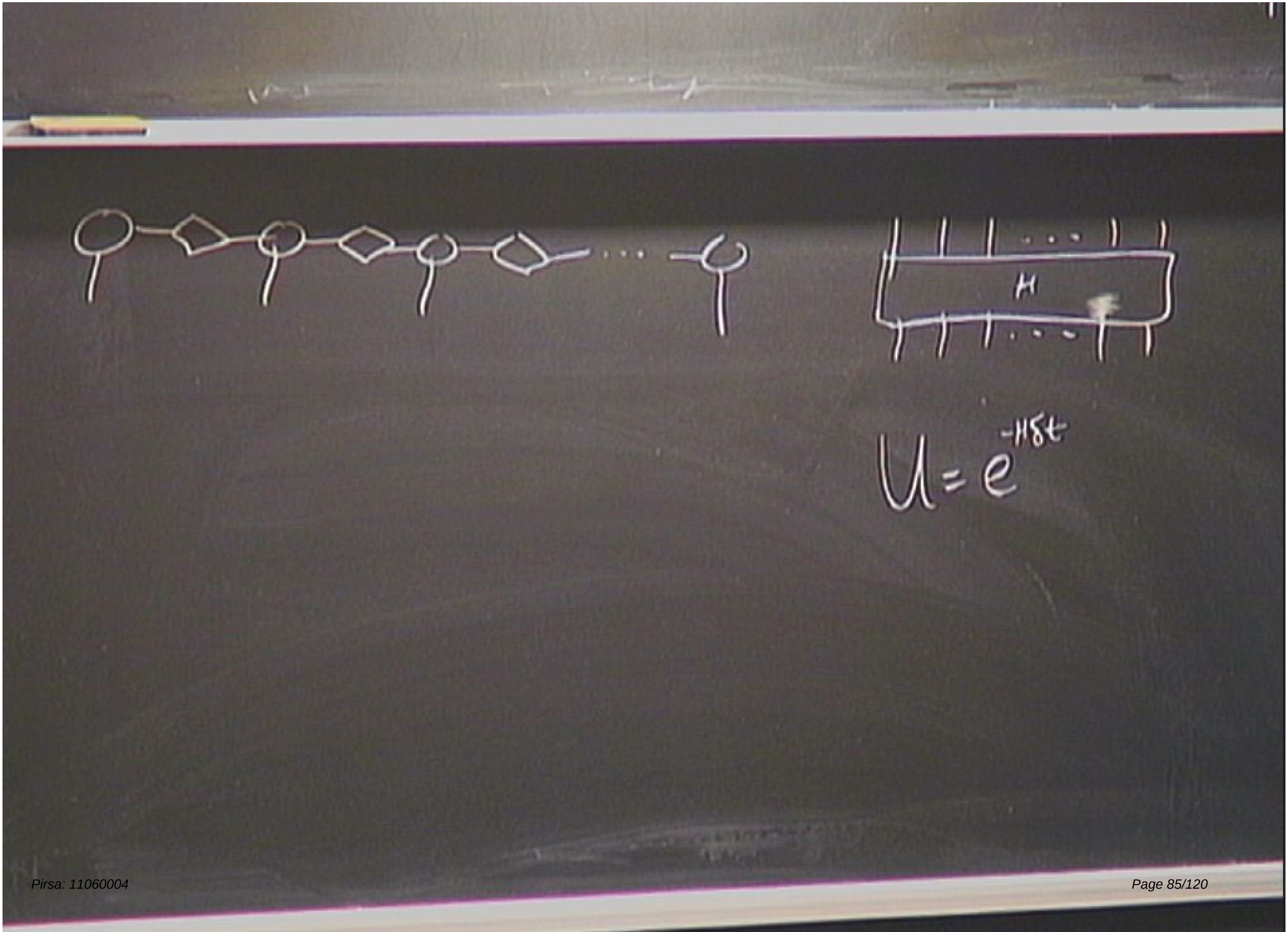


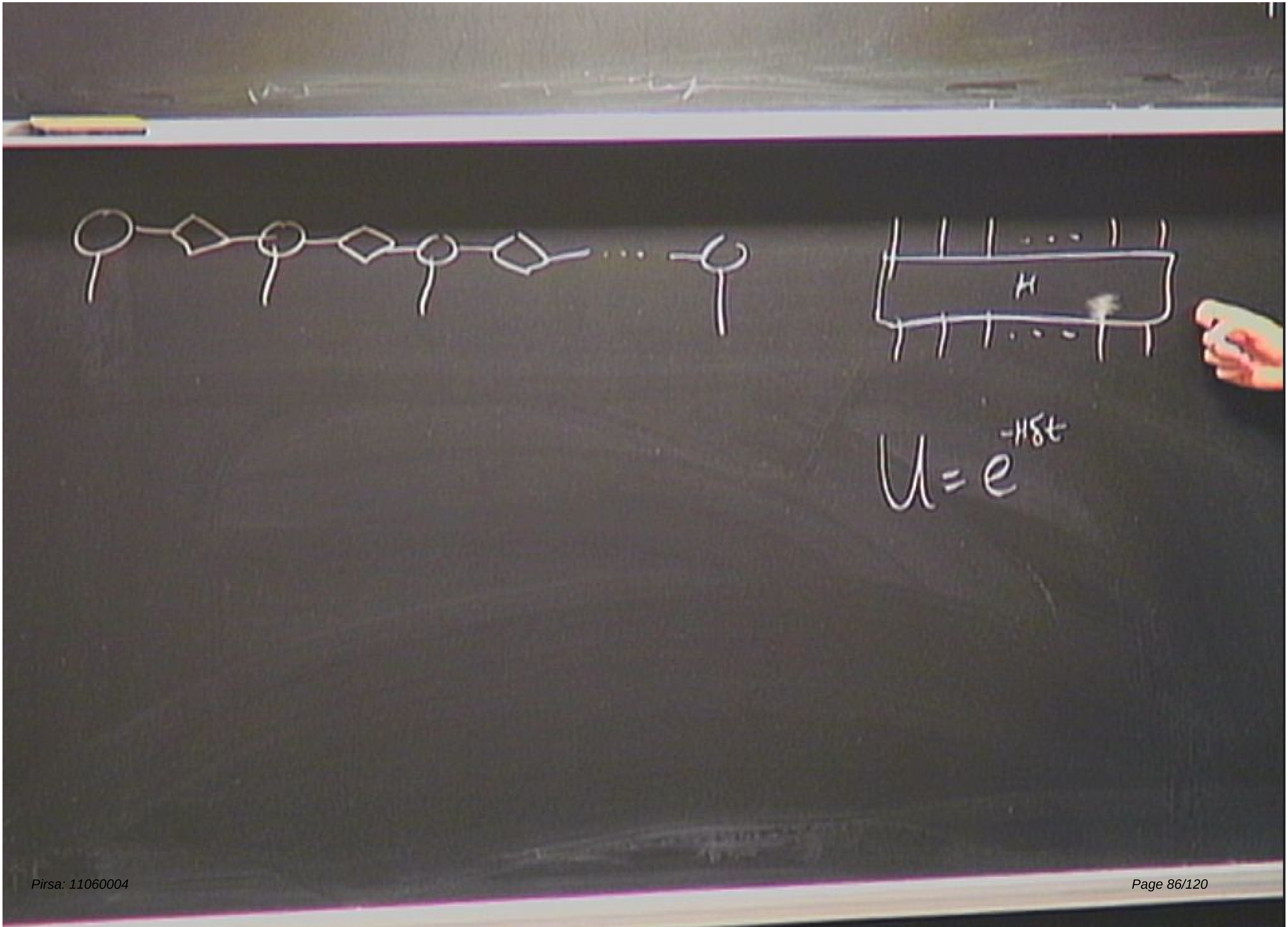

S ~ log L
LD^a

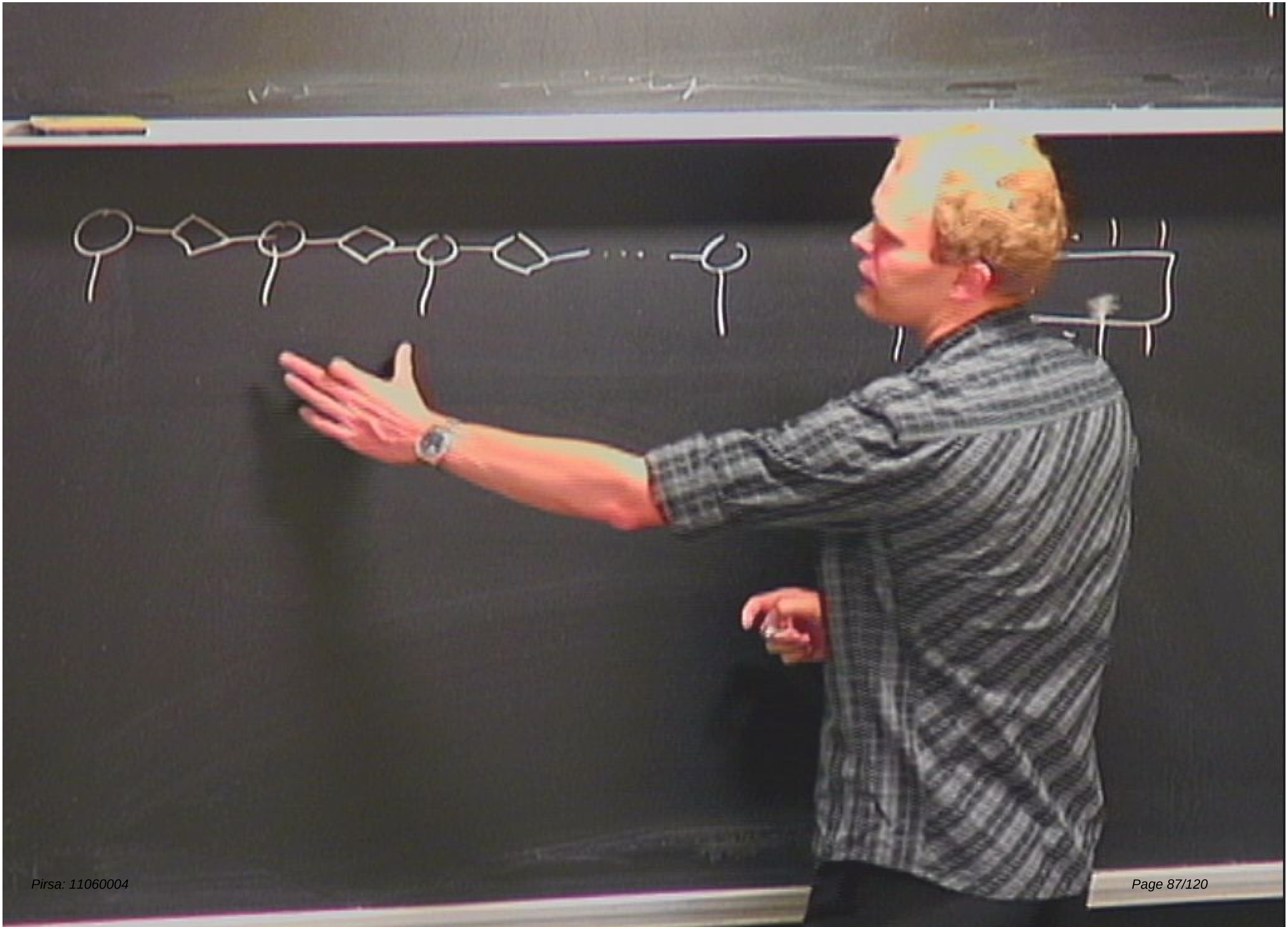

H

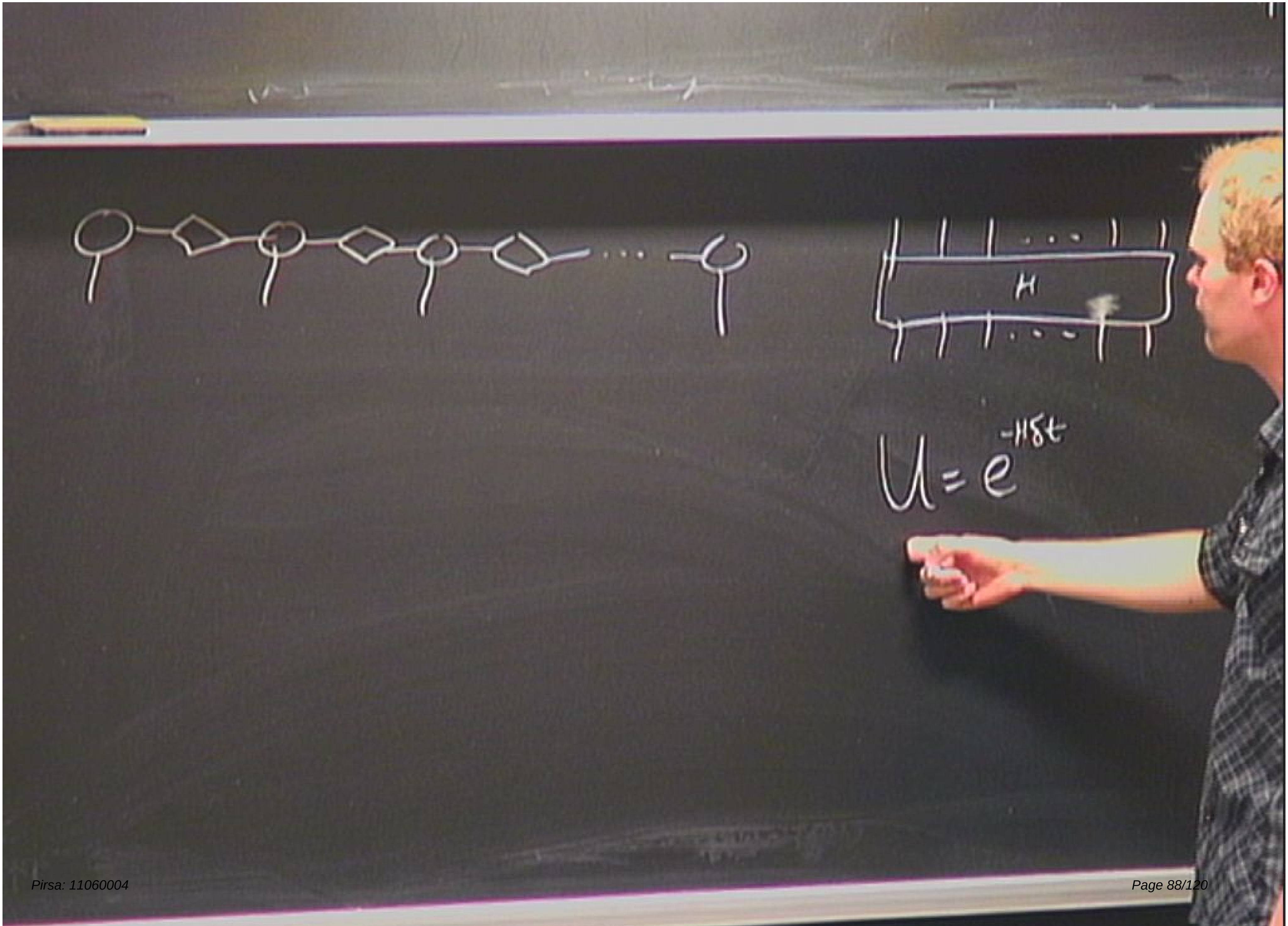

H
$U = e^{-iH\delta t}$

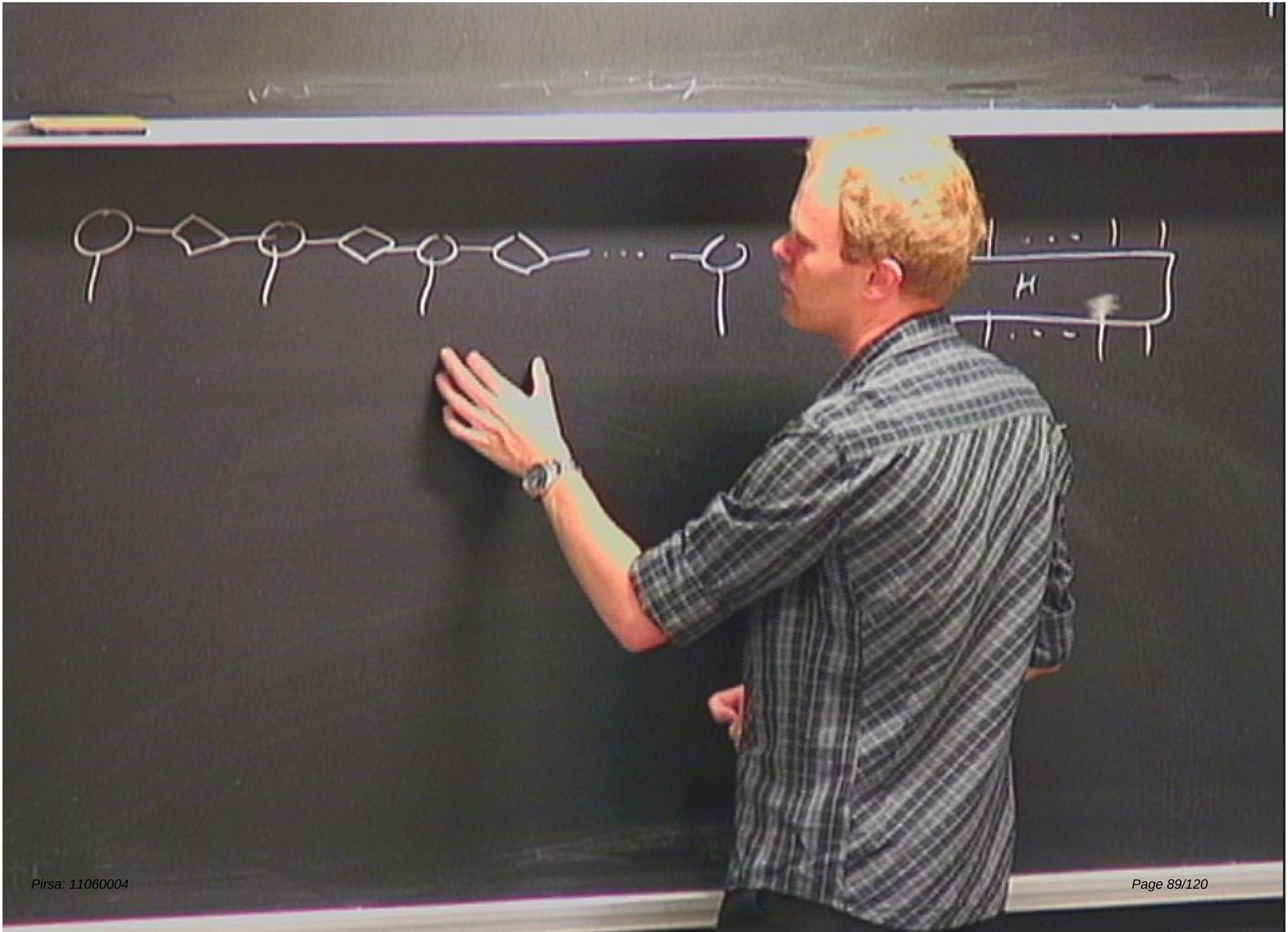

H
U=e -iHδt

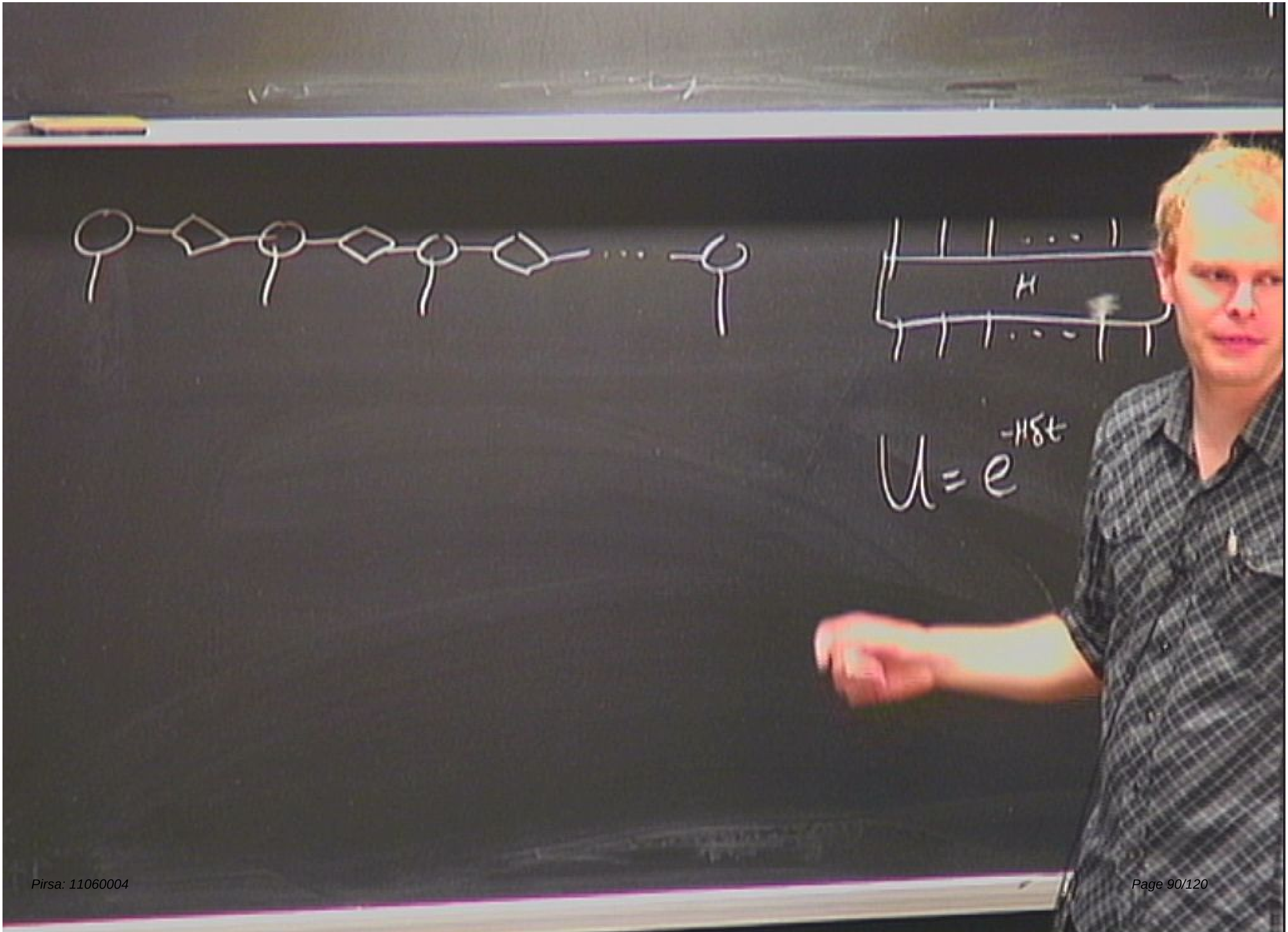

H
U = e^{-iHδt}

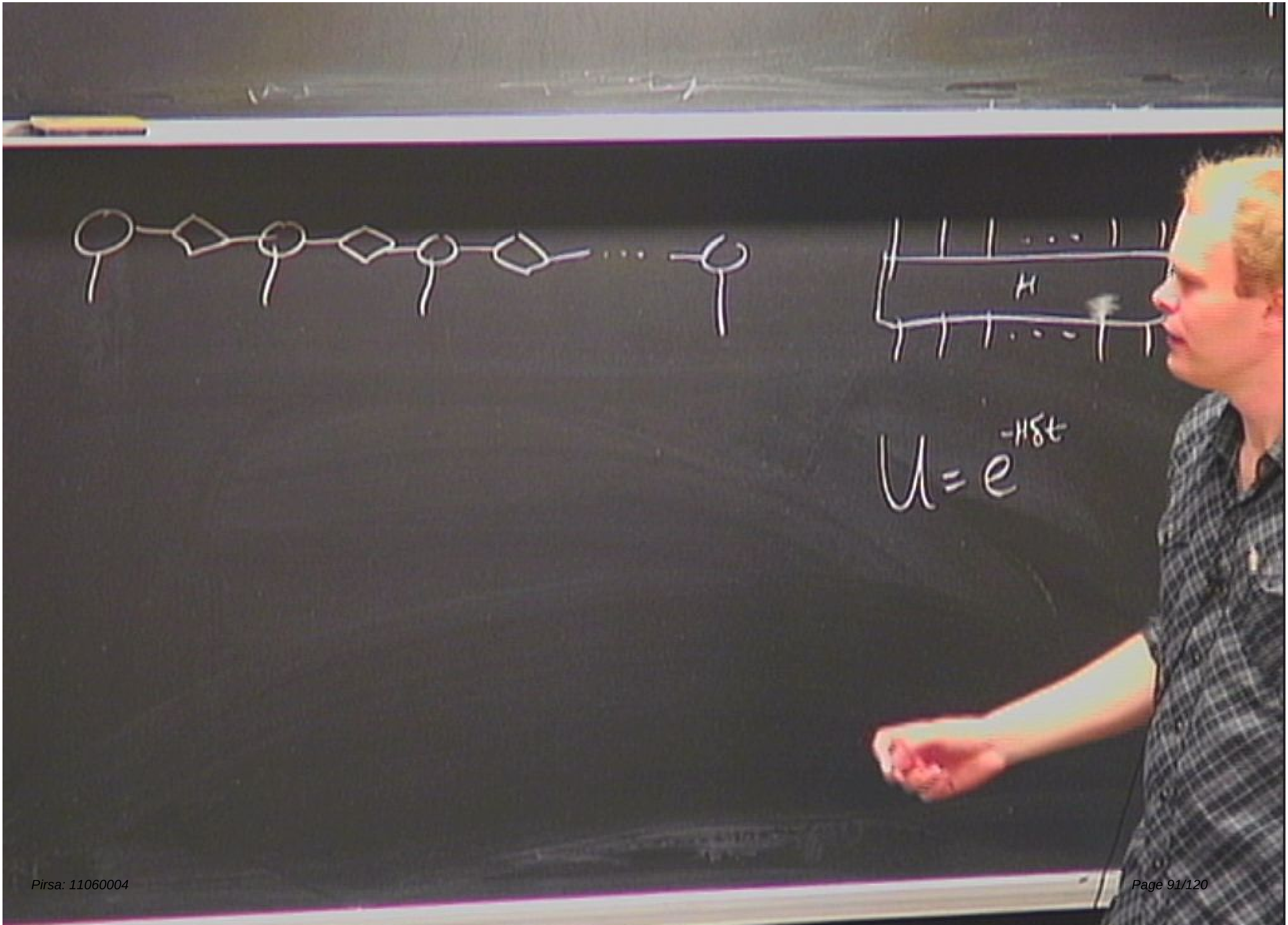

H
U = e^{-iHδt}

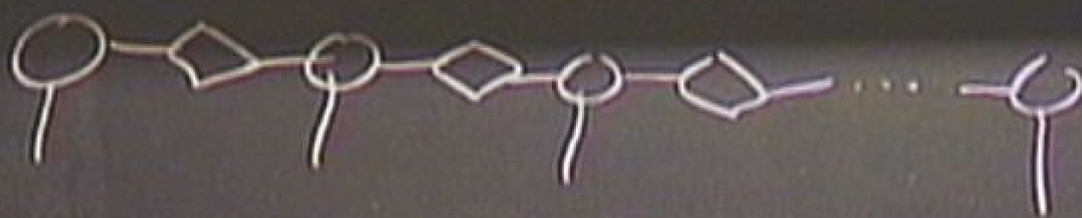



H
$U = e^{-iH\delta t}$

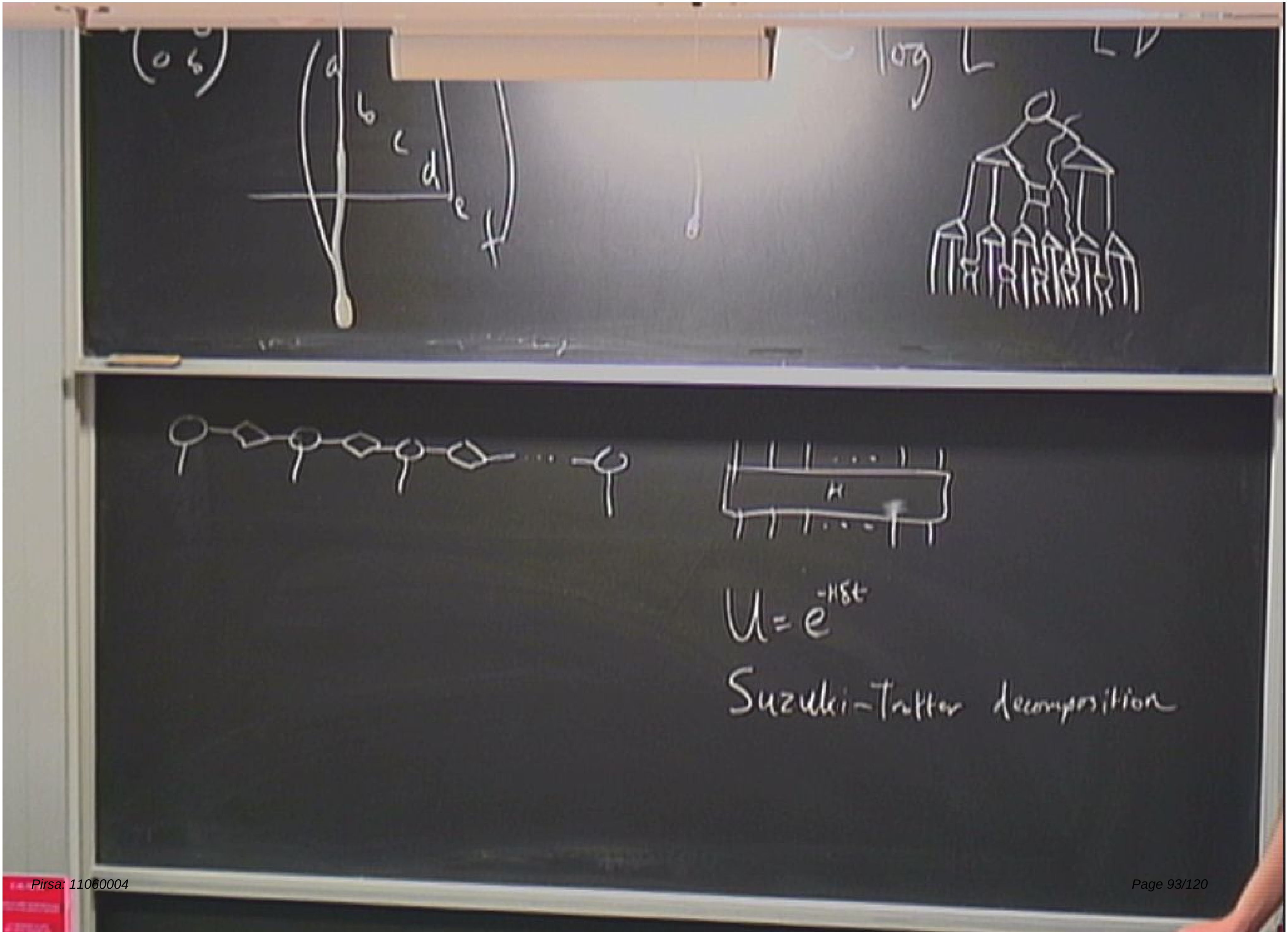

H

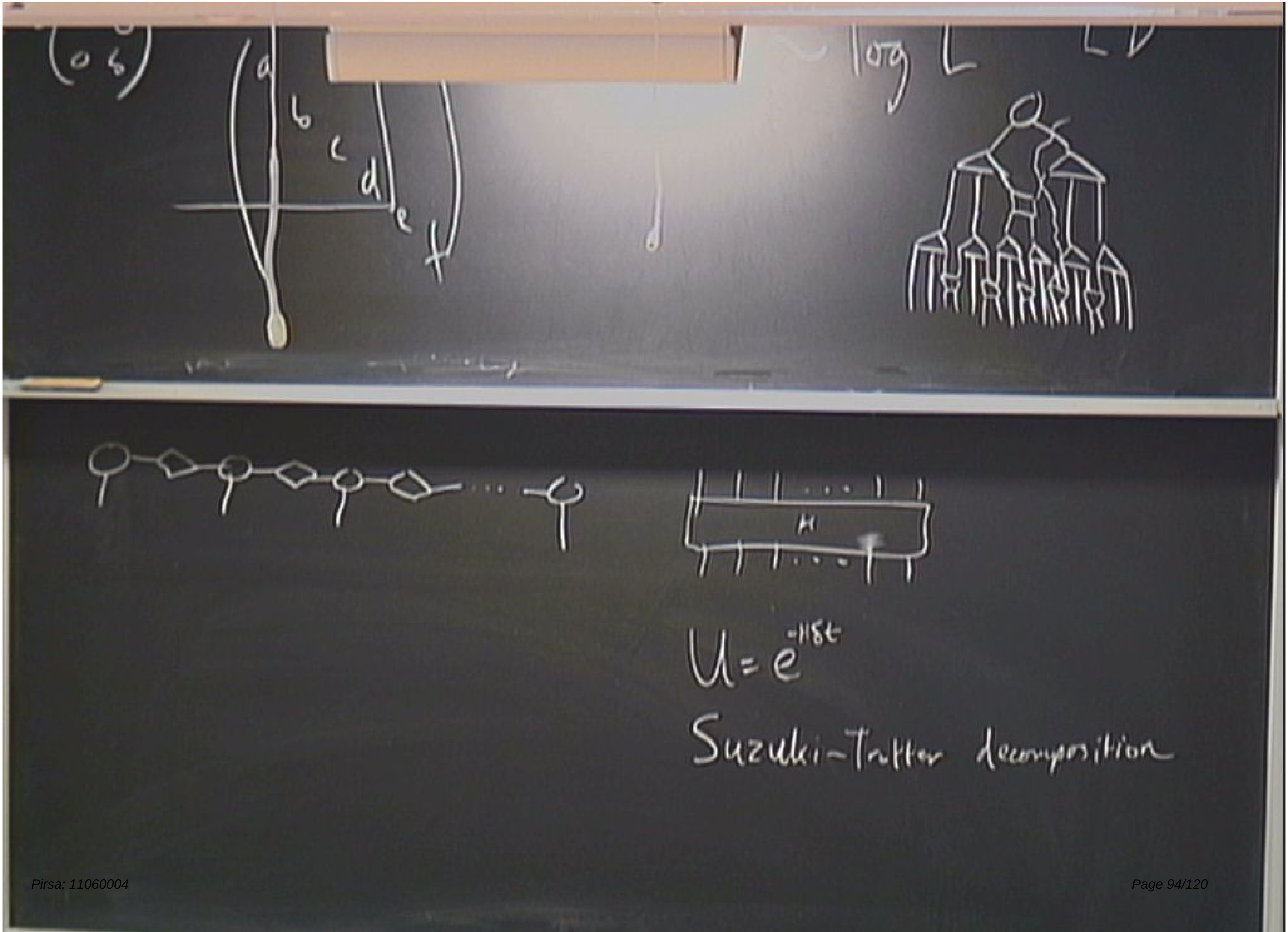

H
U = e^-iHδt

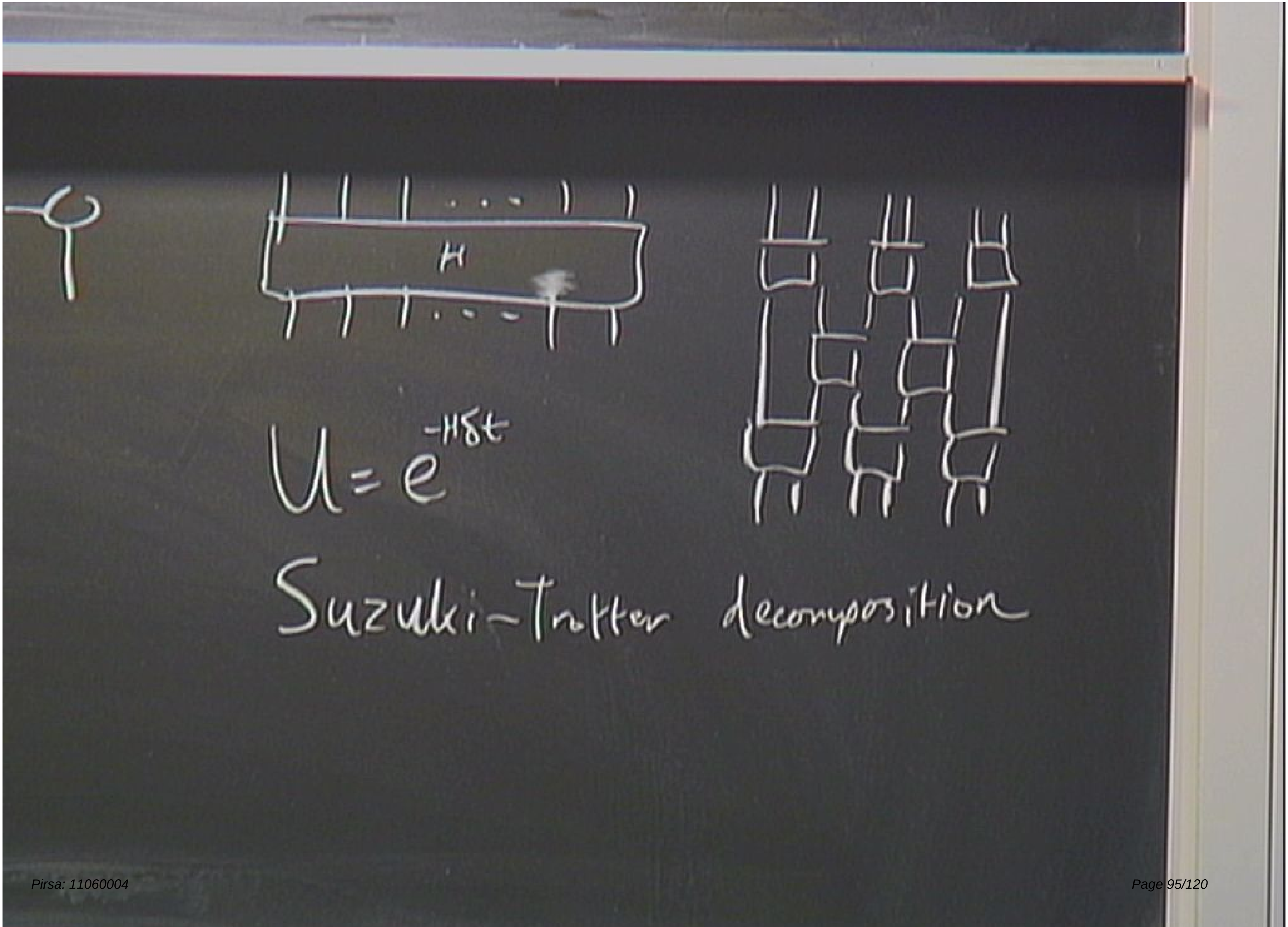

H
U = e^{-iHδt}

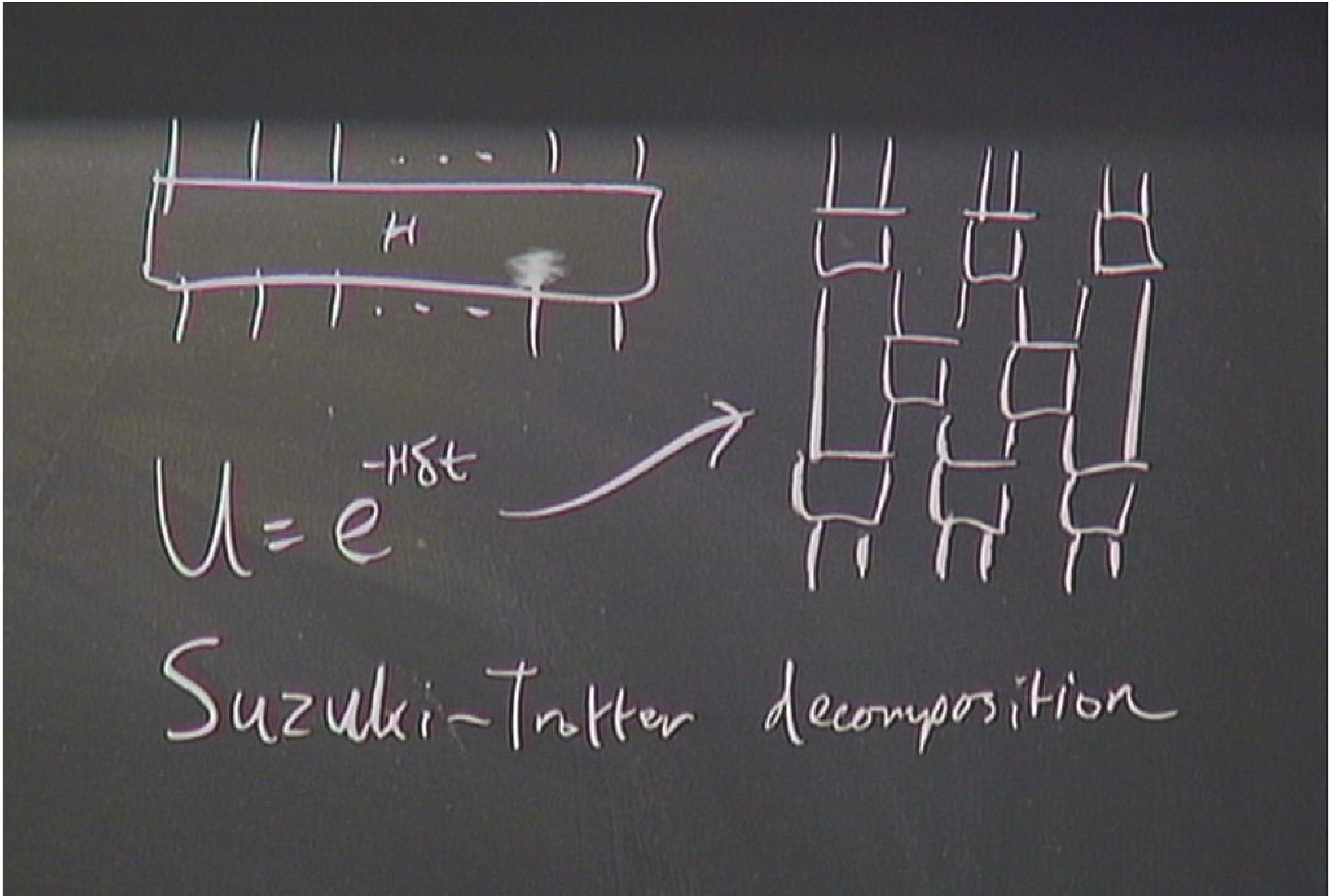


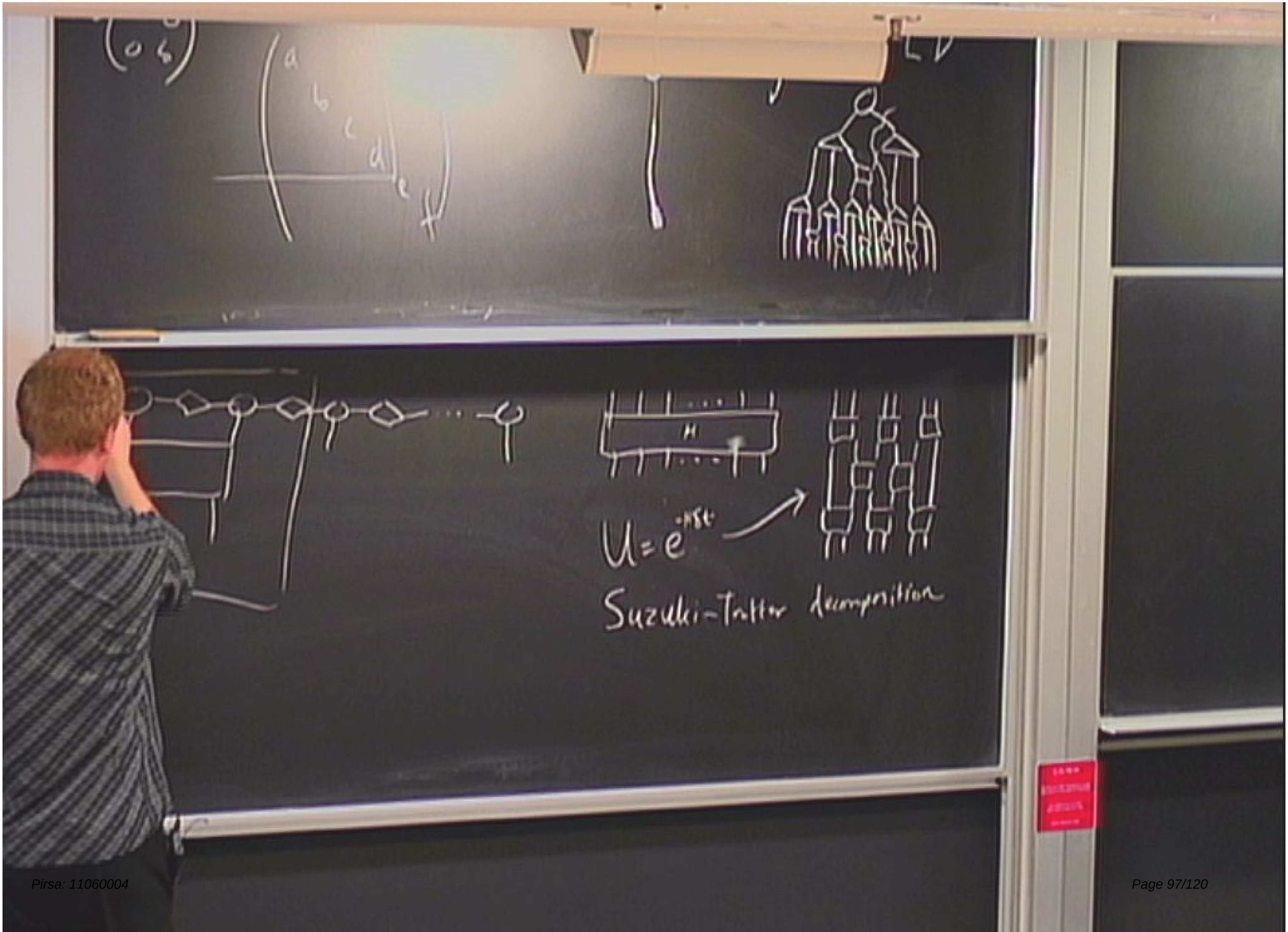

$$U = e^{-iH\delta t}$$

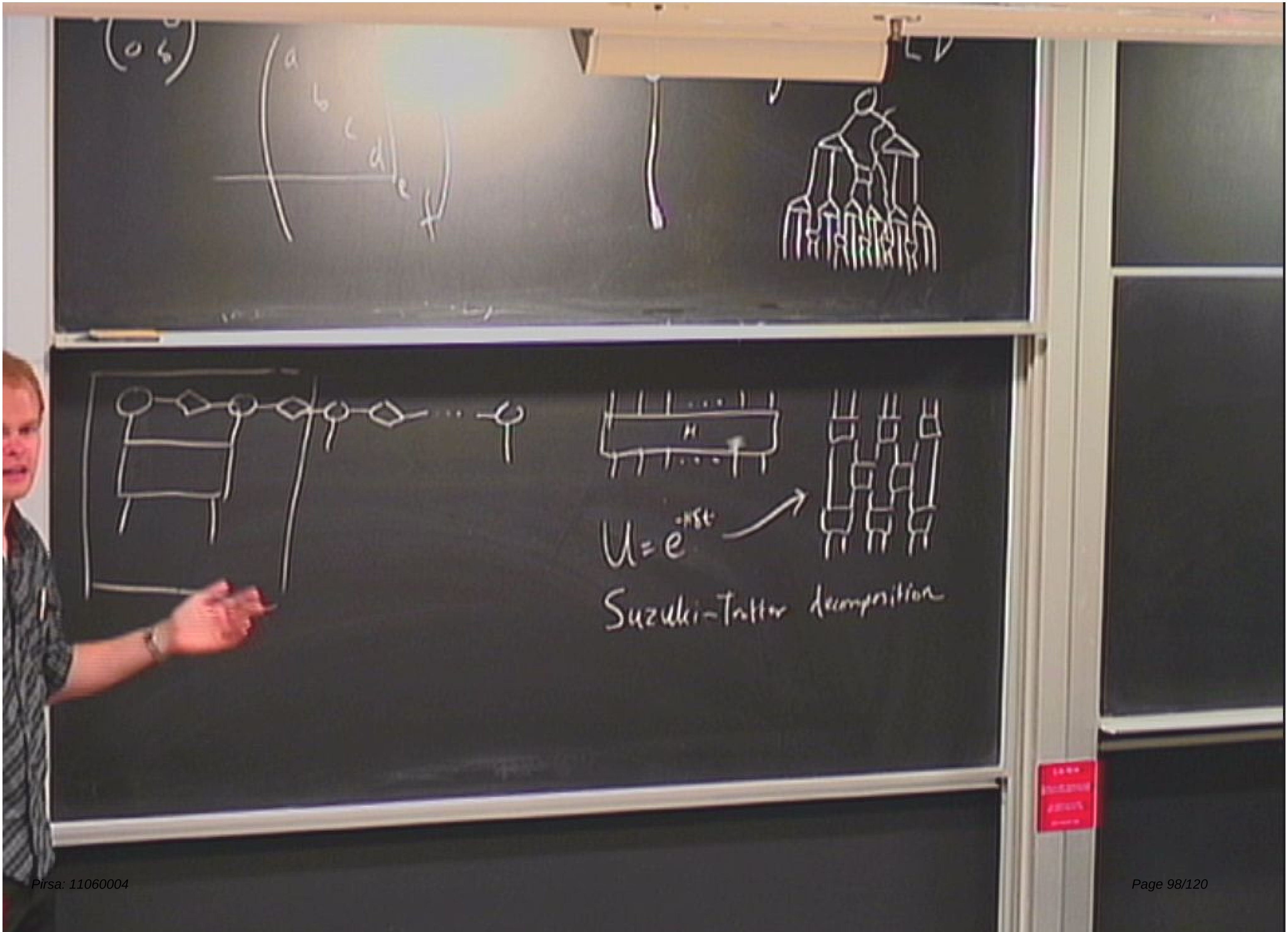

~ log L
H
U = e^{-iHδt}
Suzuki-Trotter decomposition

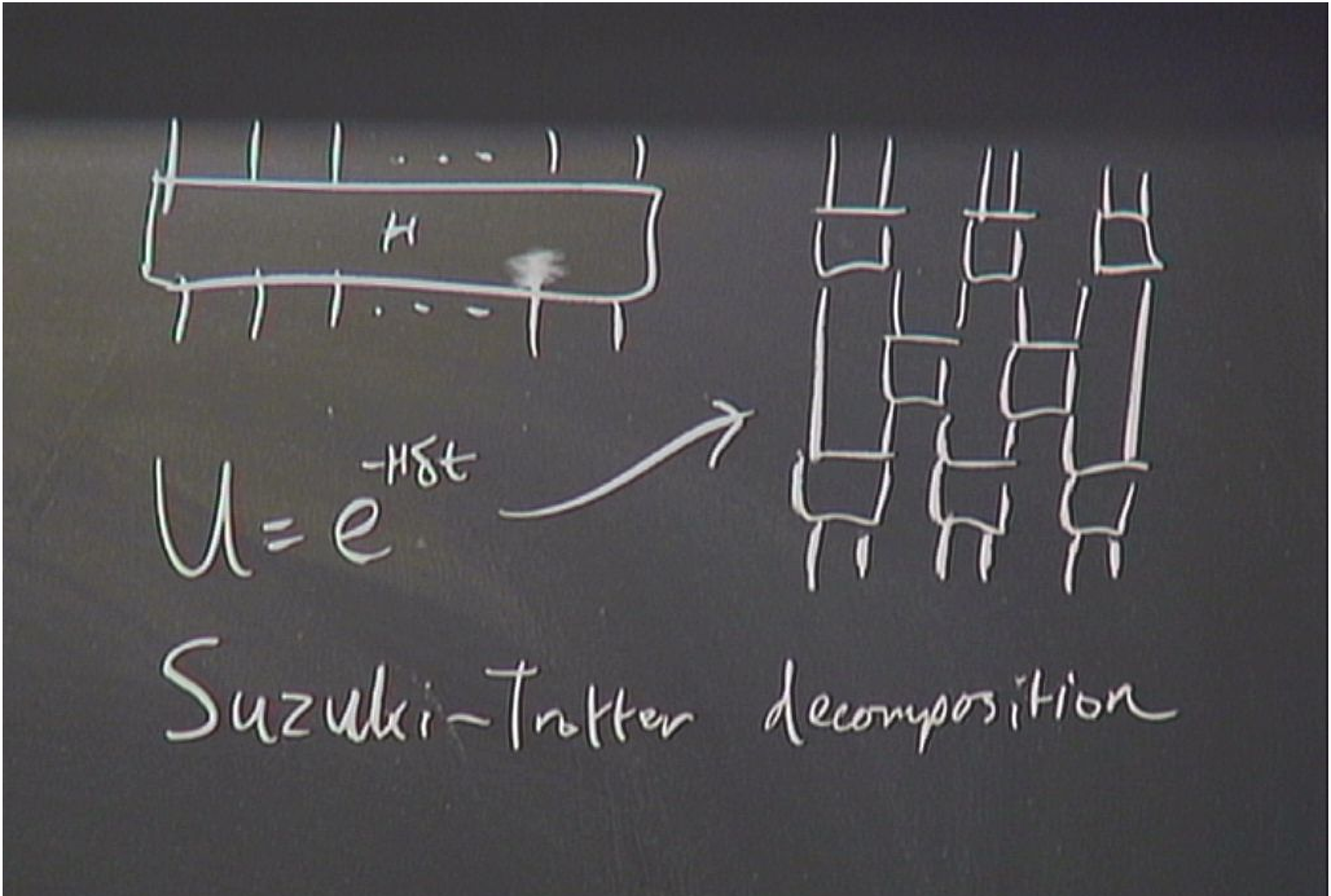

~ log L
H
U = e^{-iHδt}
Suzuki-Trotter decomposition

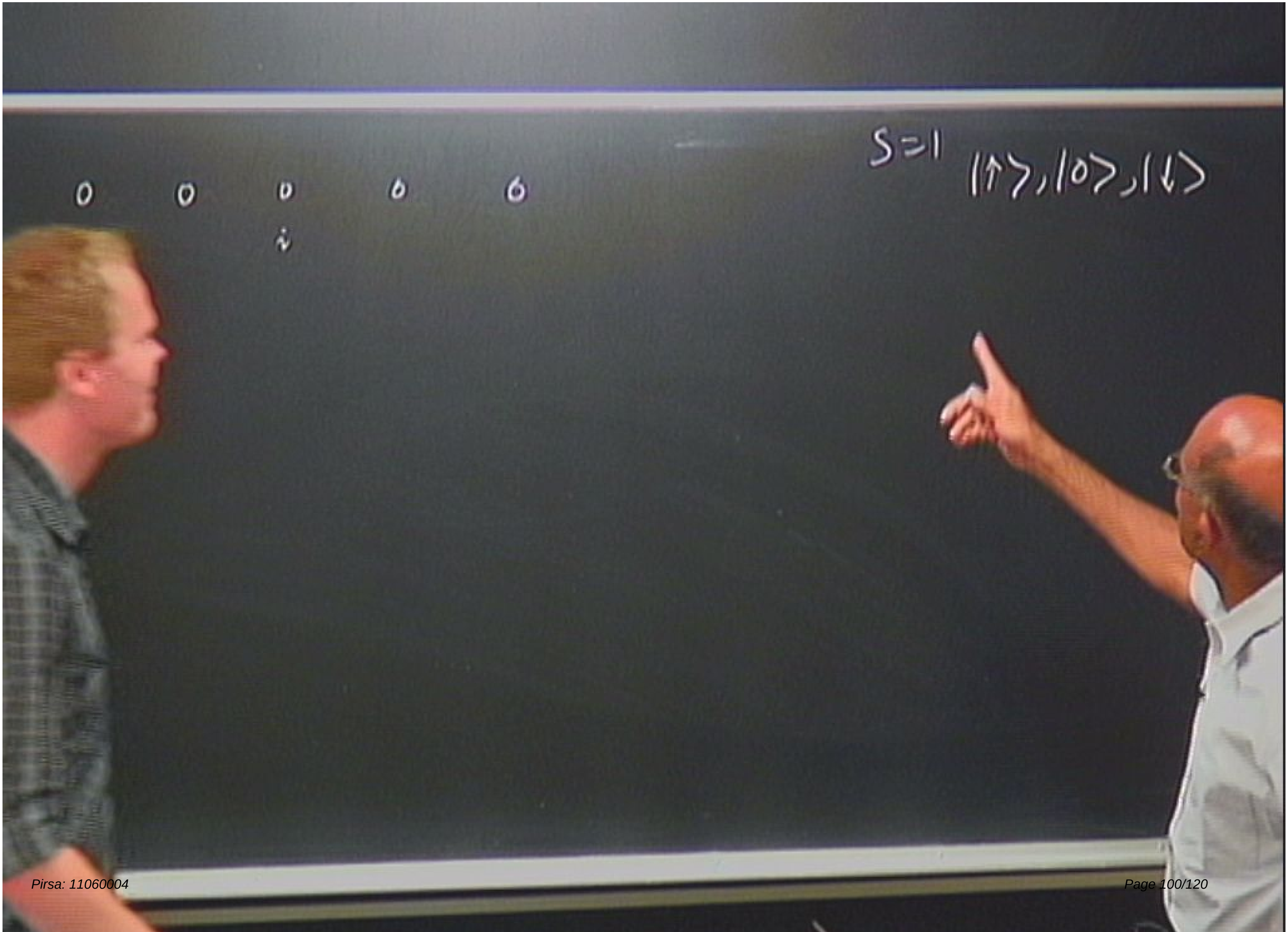

H
$U = e^{-H\delta t}$
Suzuki-Trotter decomposition


H
U = e^{-Hδt}
Suzuki-Trotter decomposition


H
U=e^-iHδt
Suzuki-Trotter decomposition


H
Suzuki-Trotter decomposition


H
$U = e^{-H\delta t}$
Suzuki-Trotter decomposition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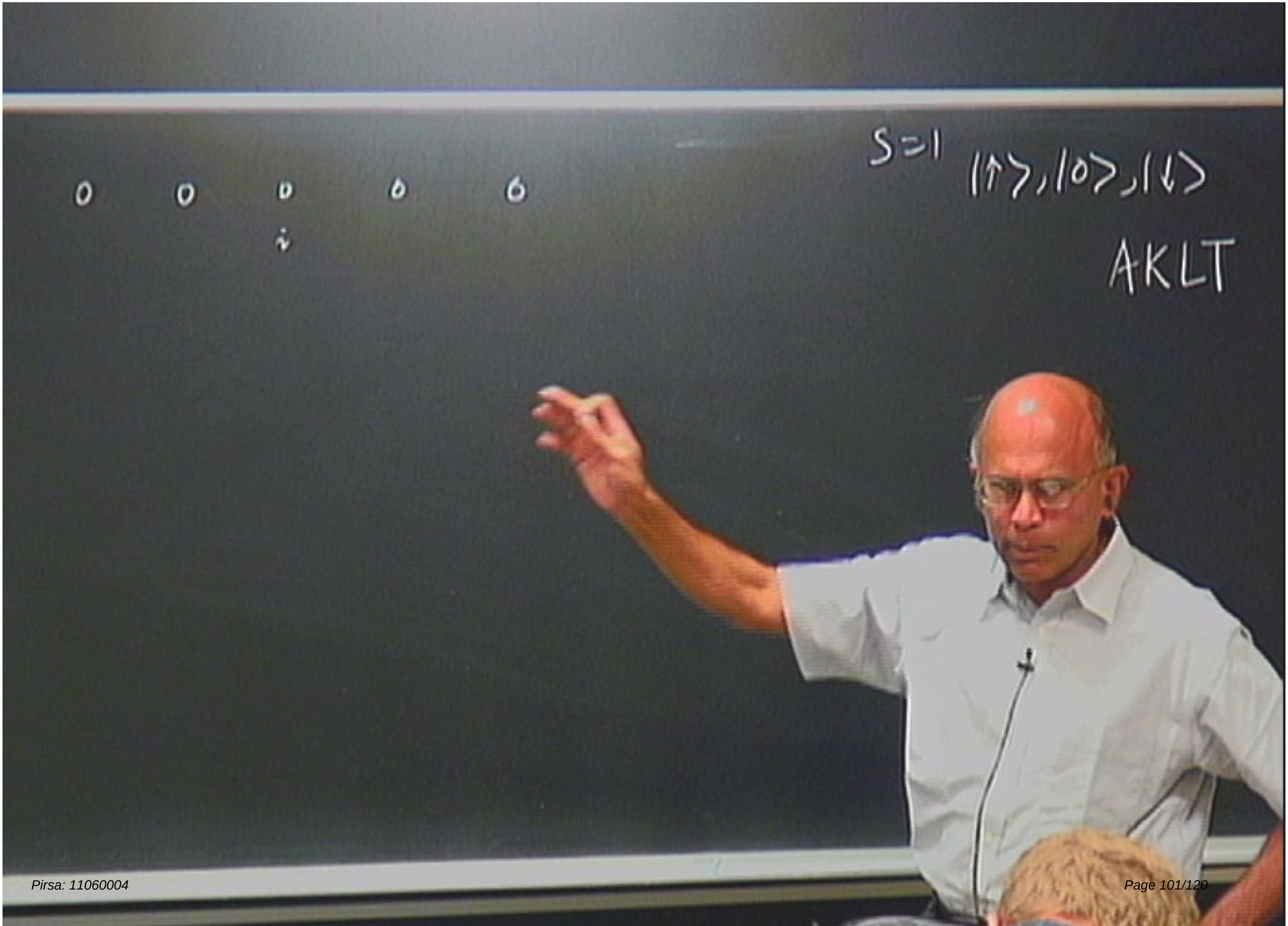
i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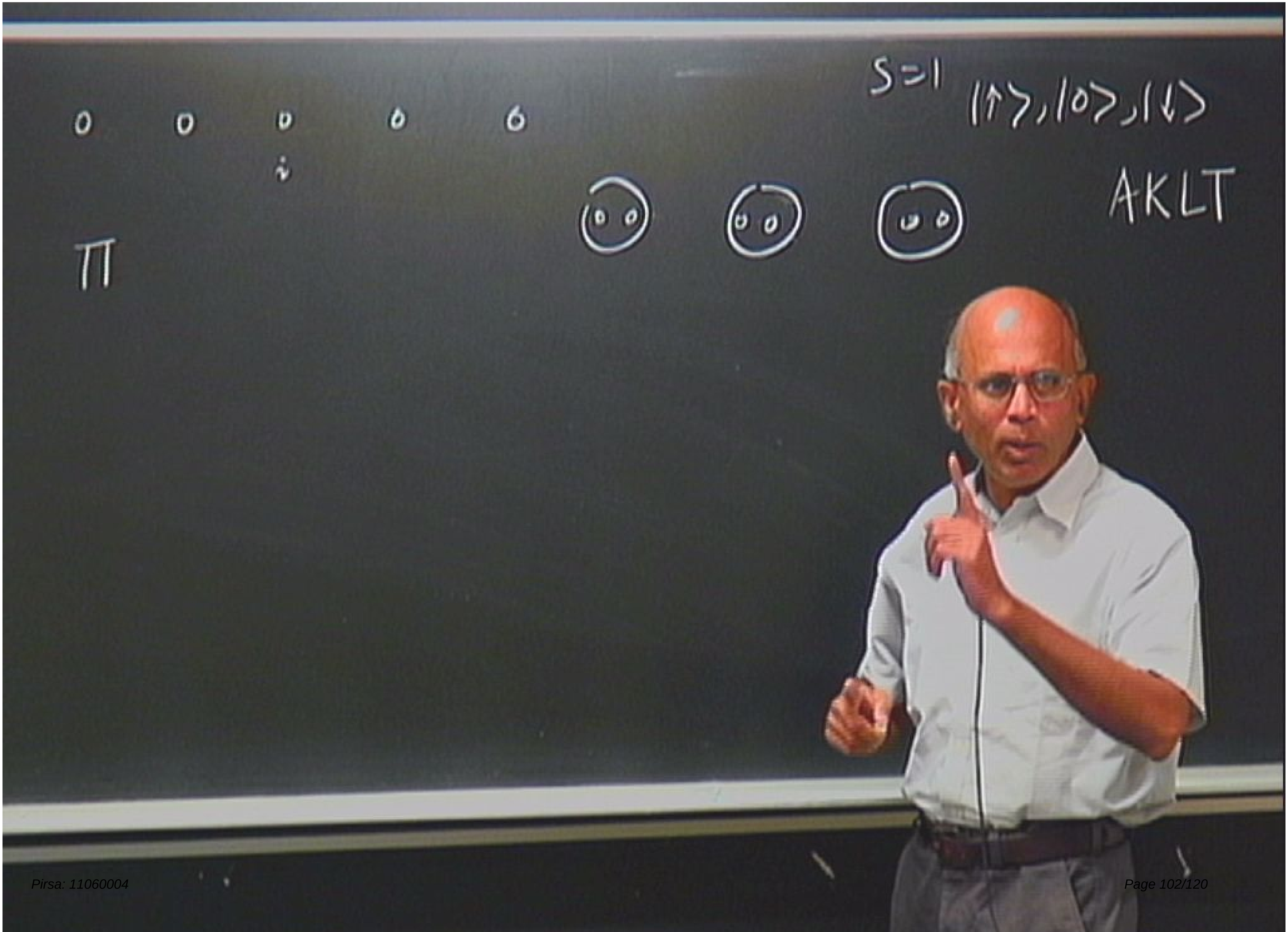
AKLT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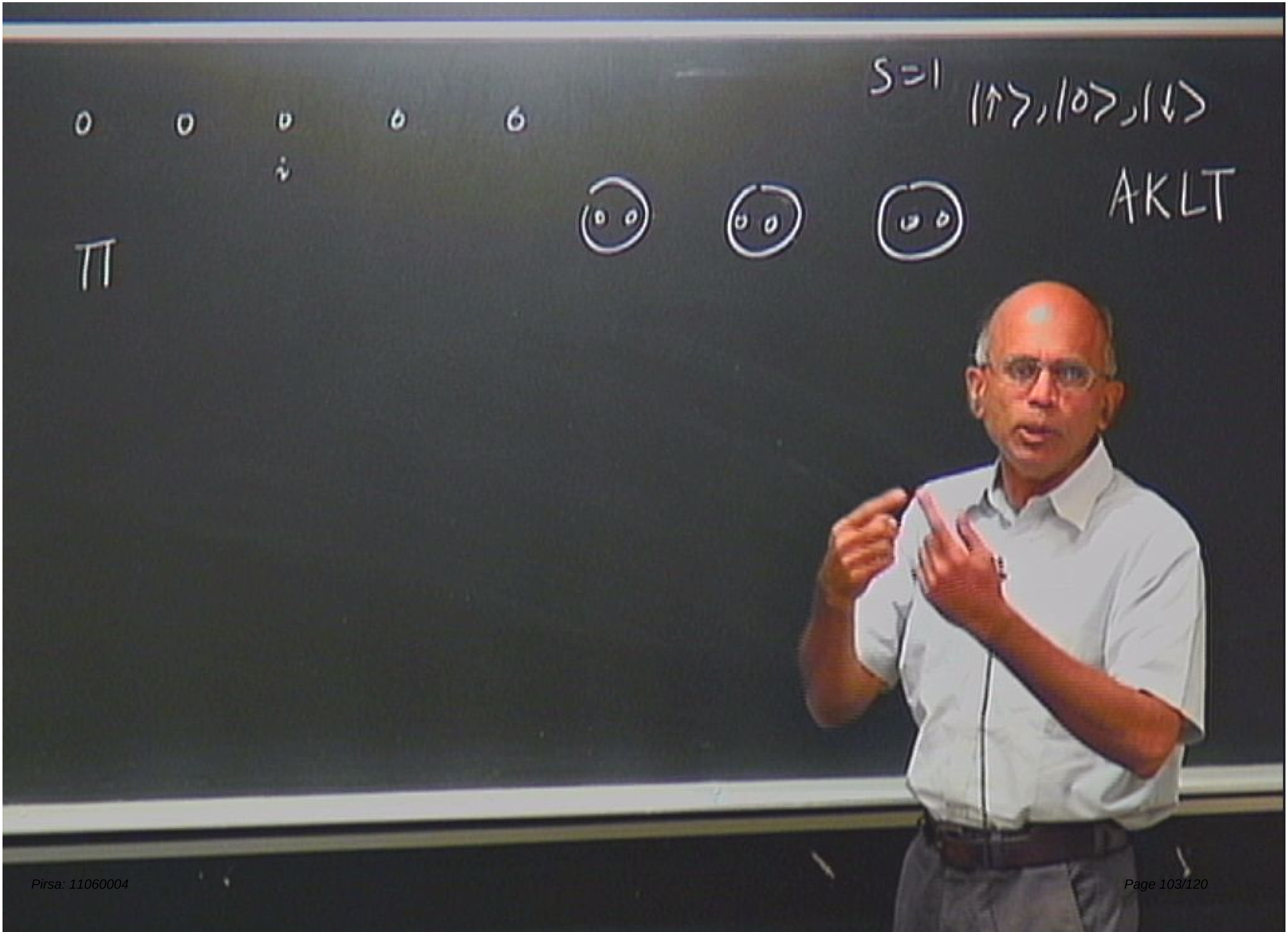
AKLT
π
i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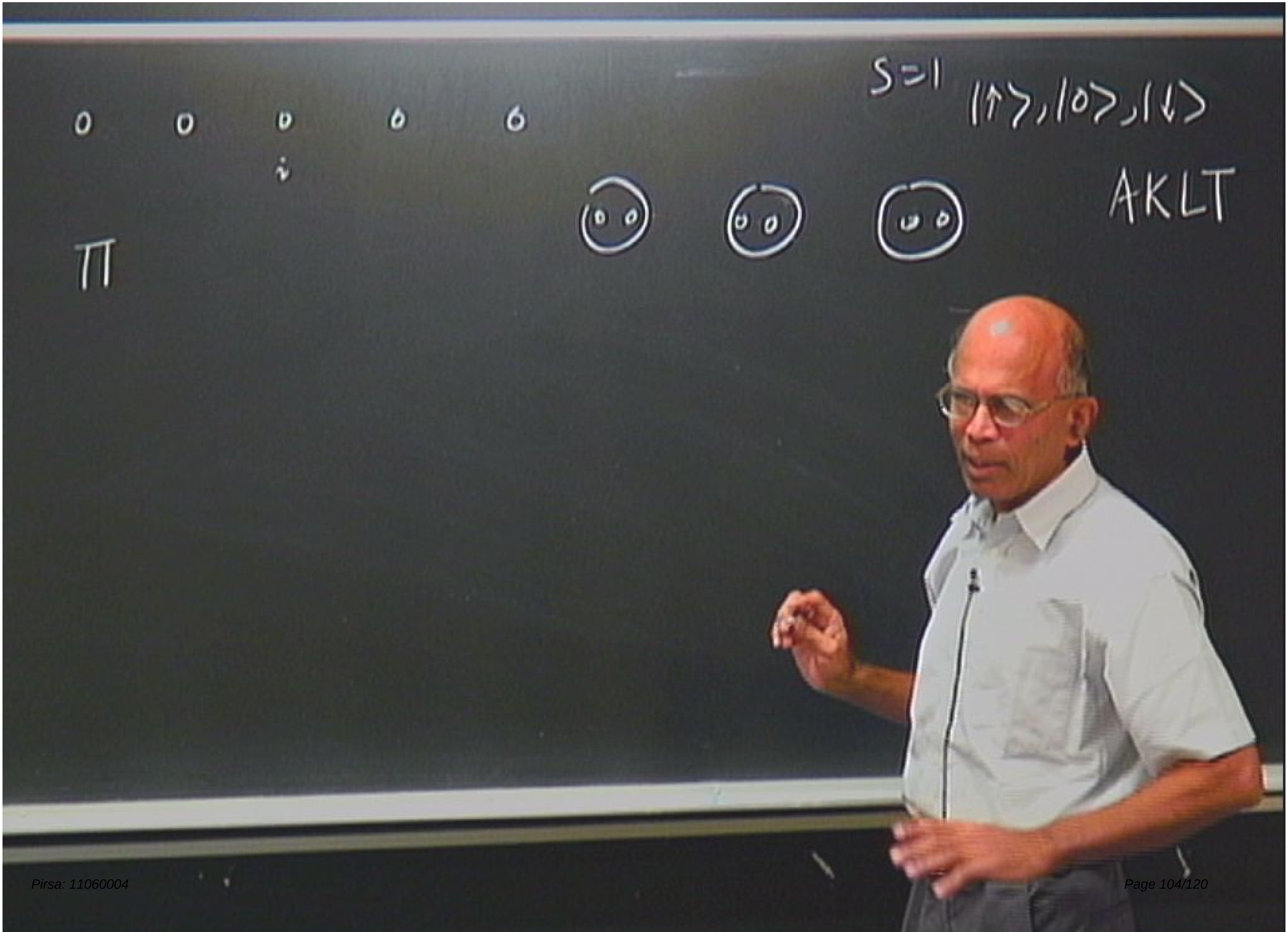
AKLT
π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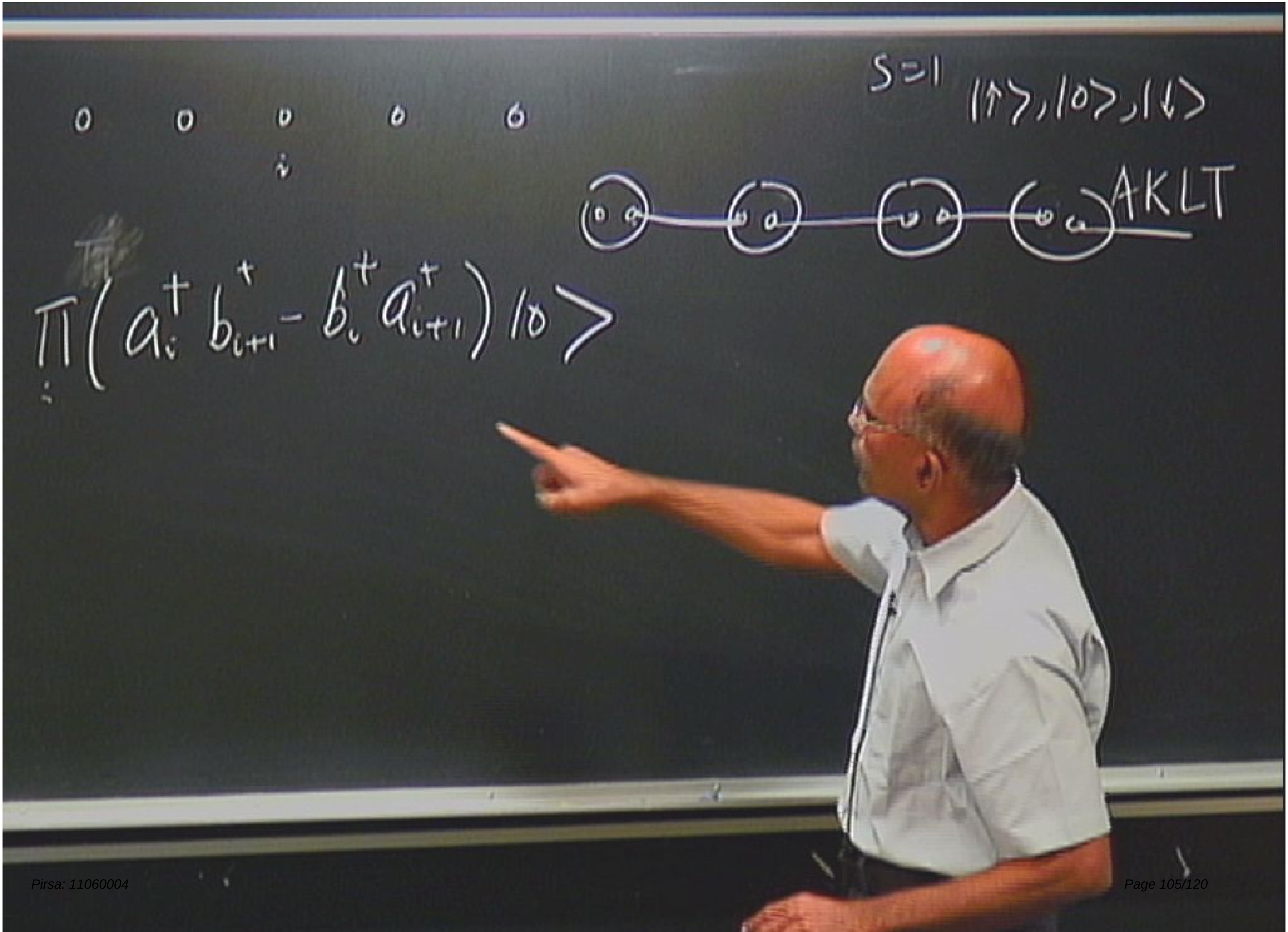
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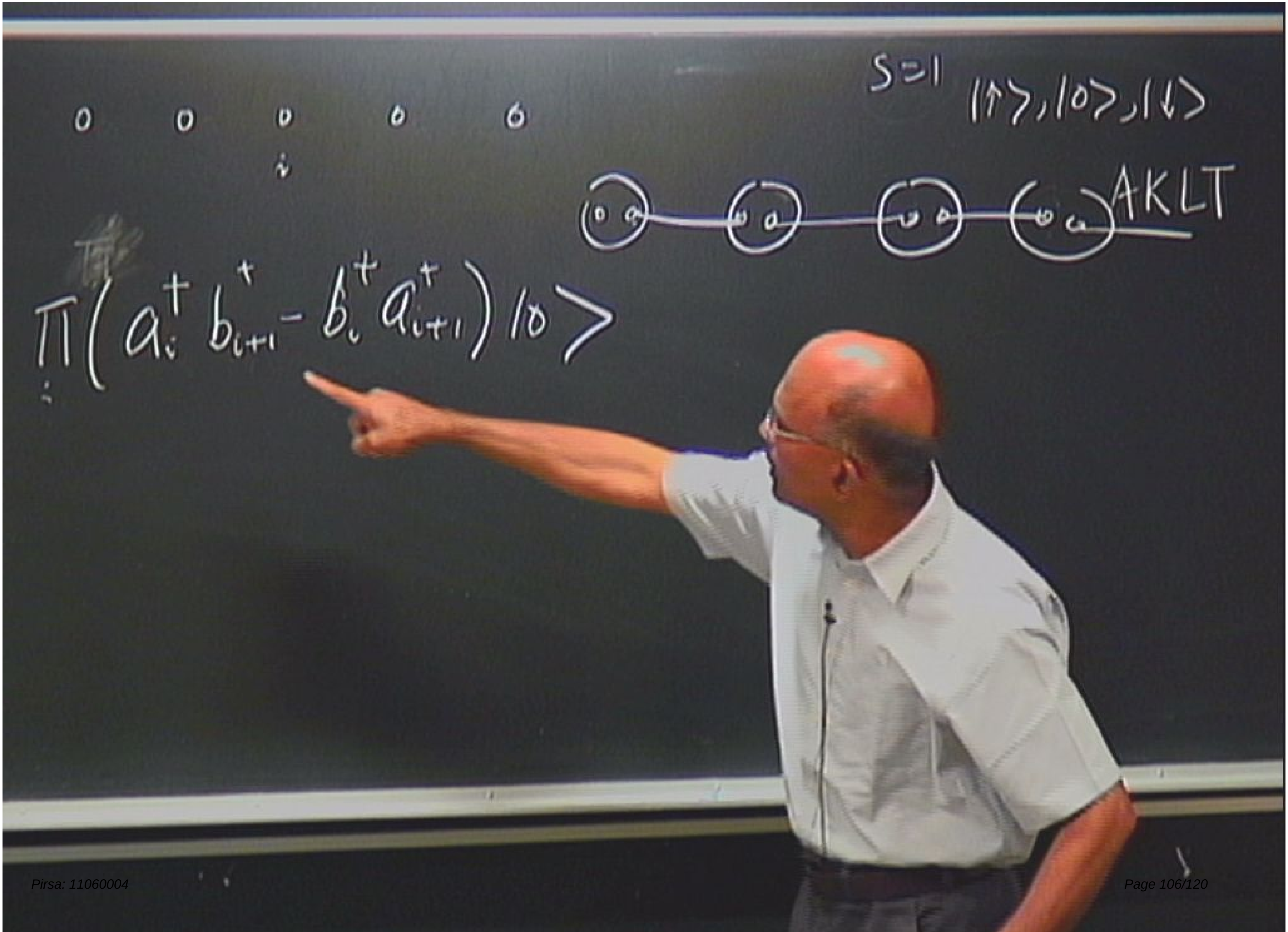
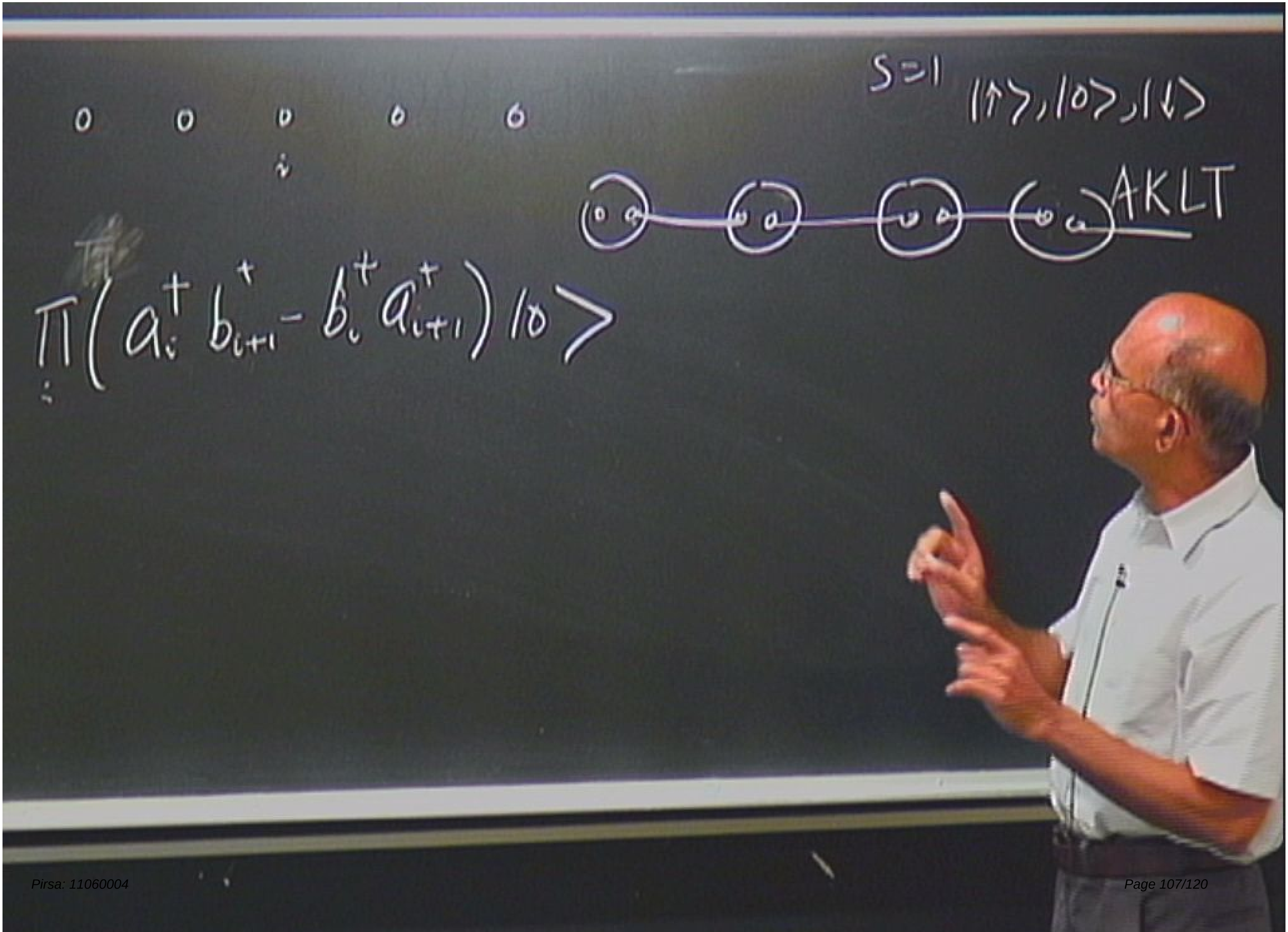
AKLT
i
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$


S=1
$|\uparrow\rangle, |0\rangle, |\downarrow\rangle$
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger)|0\rangle$


S=1
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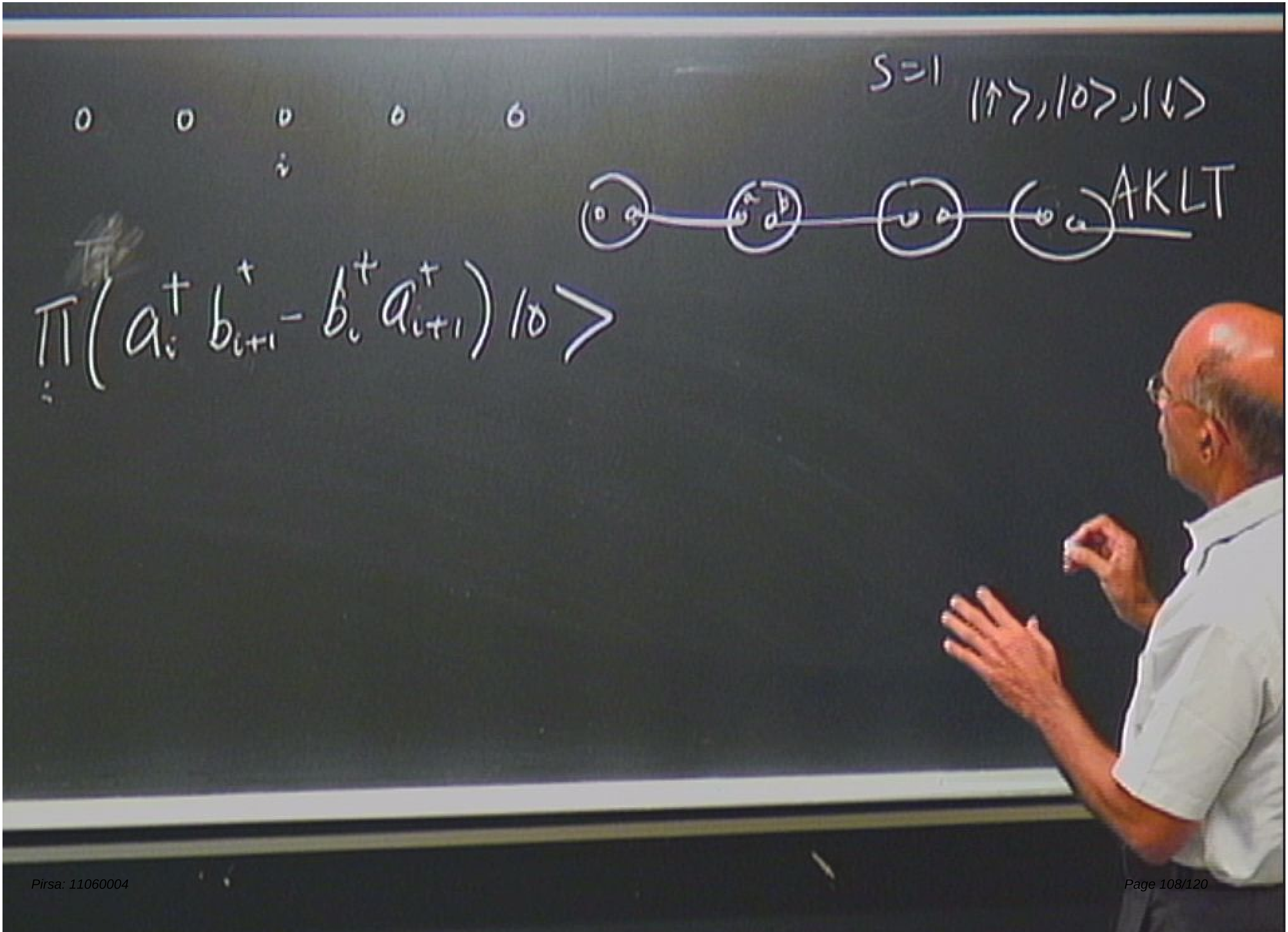
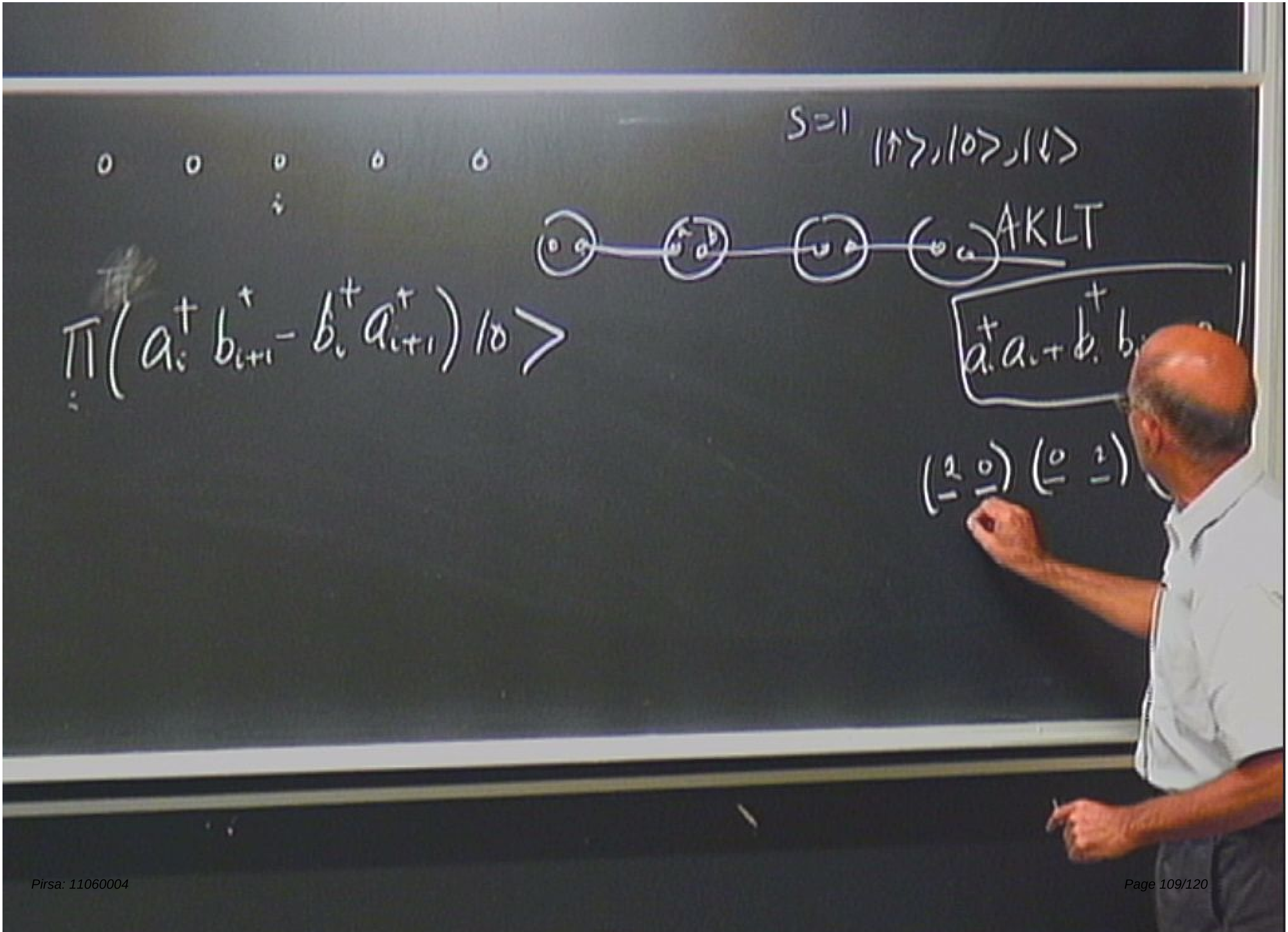
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$


S=1
$|\uparrow\rangle, |0\rangle, |\downarrow\rangle$
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger)|0\rangle$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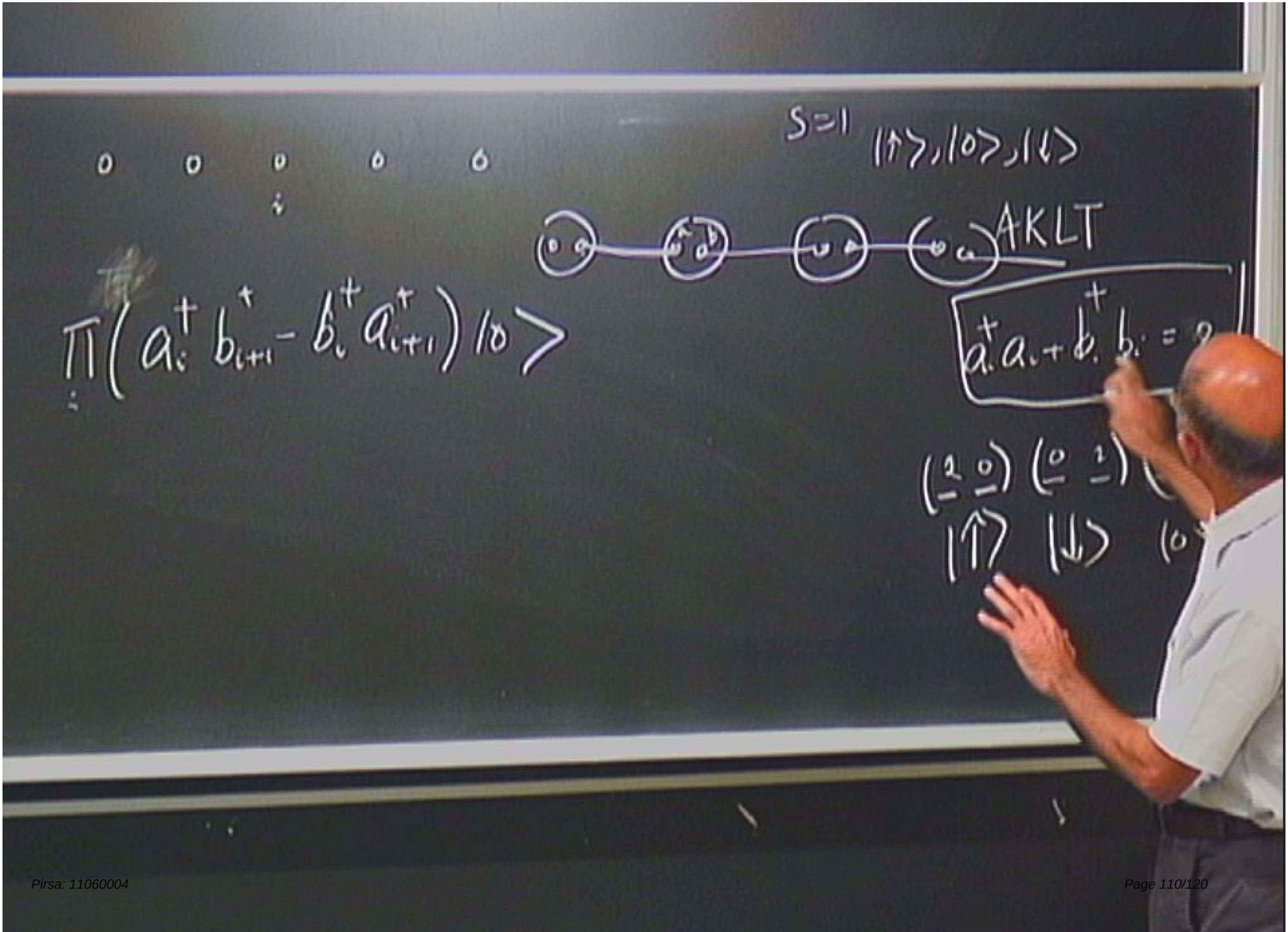
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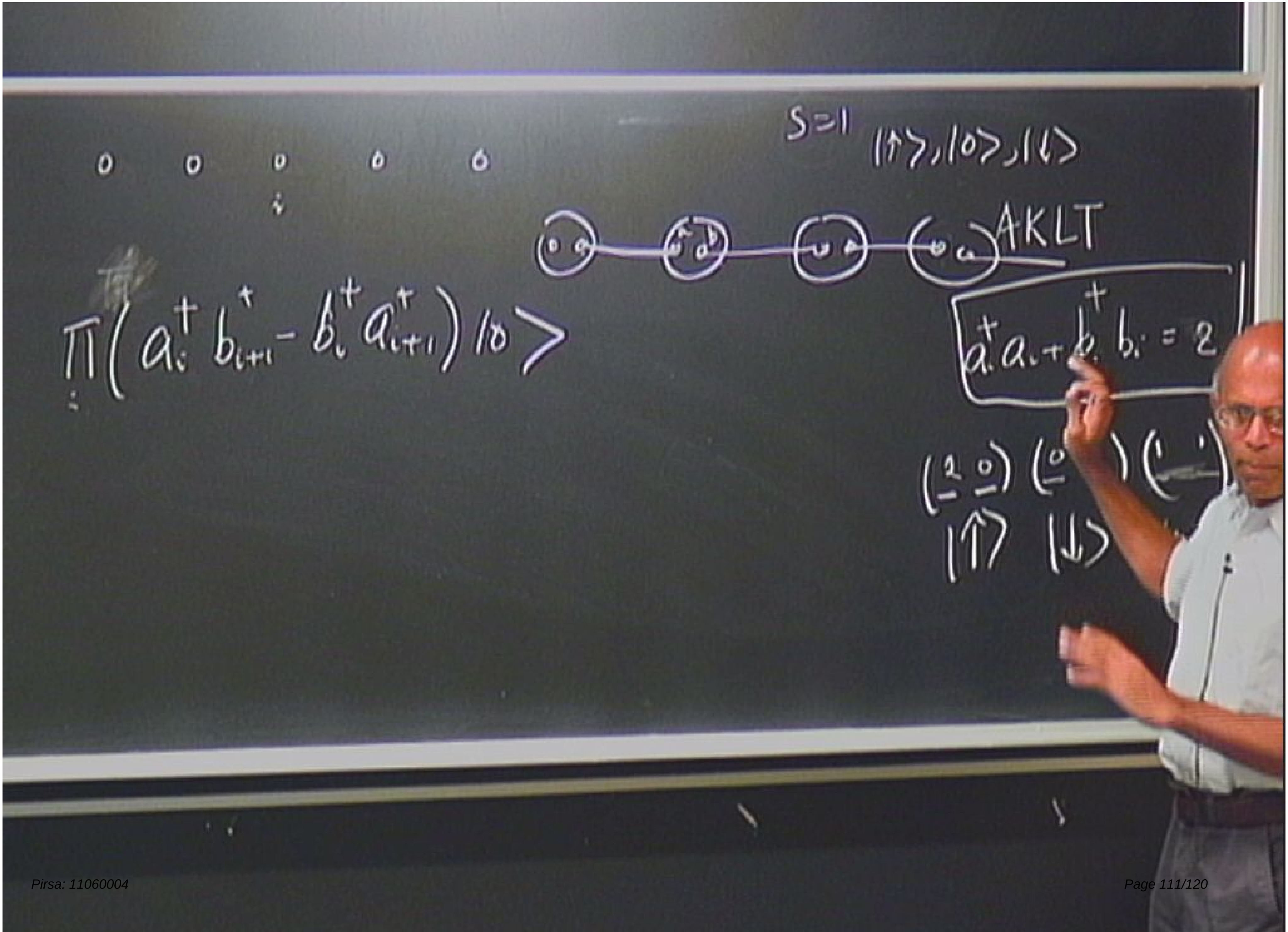
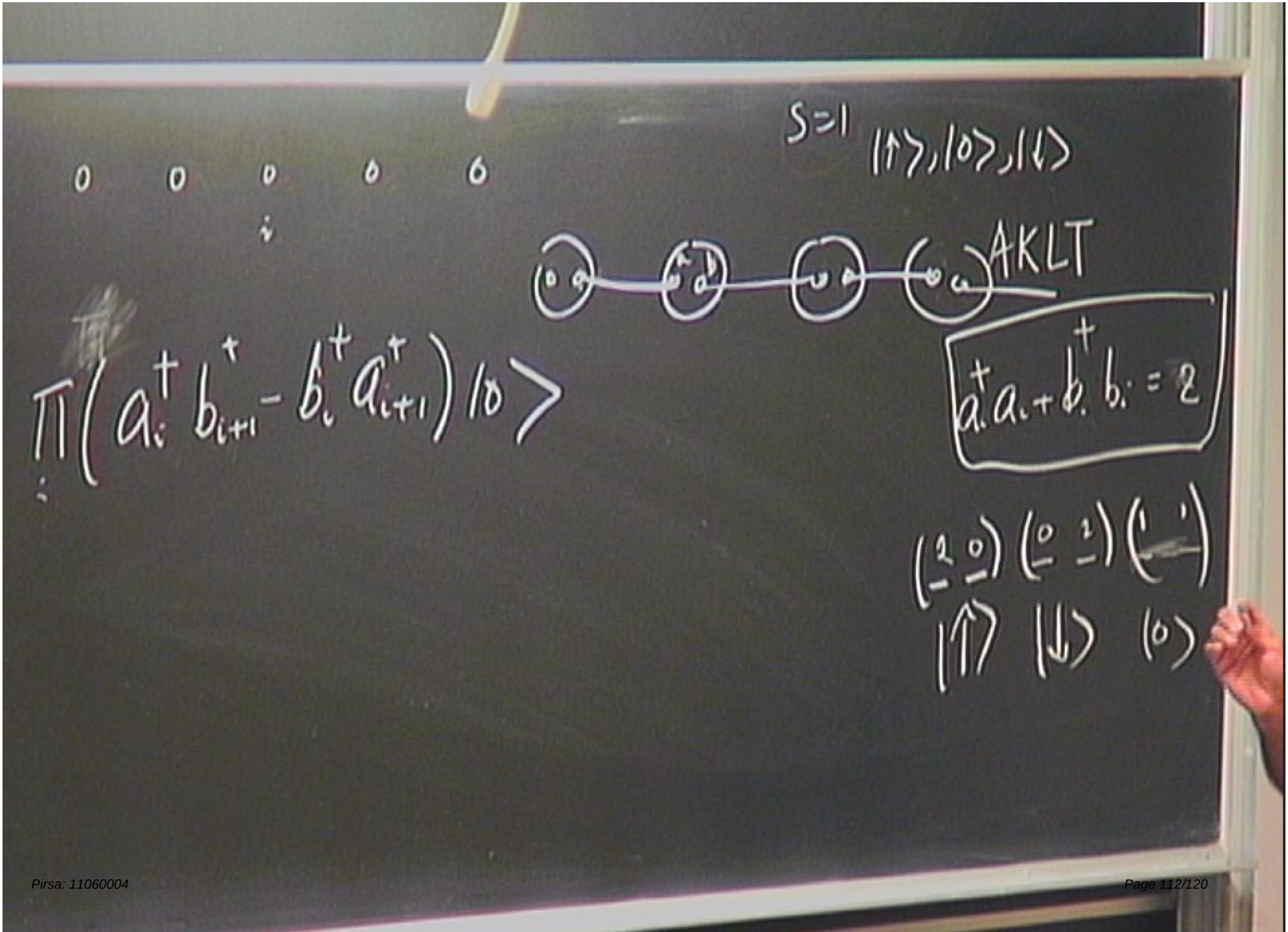
AKLT
Π(a_i† b_i+1† − b_i† a_i+1†) |0>
a_i† a_i + b_i† b_i = 2
|↑>
|↓>


S=1
|↑>, |0>, |↓>
AKLT
Π (a_i^† b_{i+1}^† − b_i^† a_{i+1}^†) |0>
a_i^† a_i + b_i^† b_i = 2
|↑>
|↓>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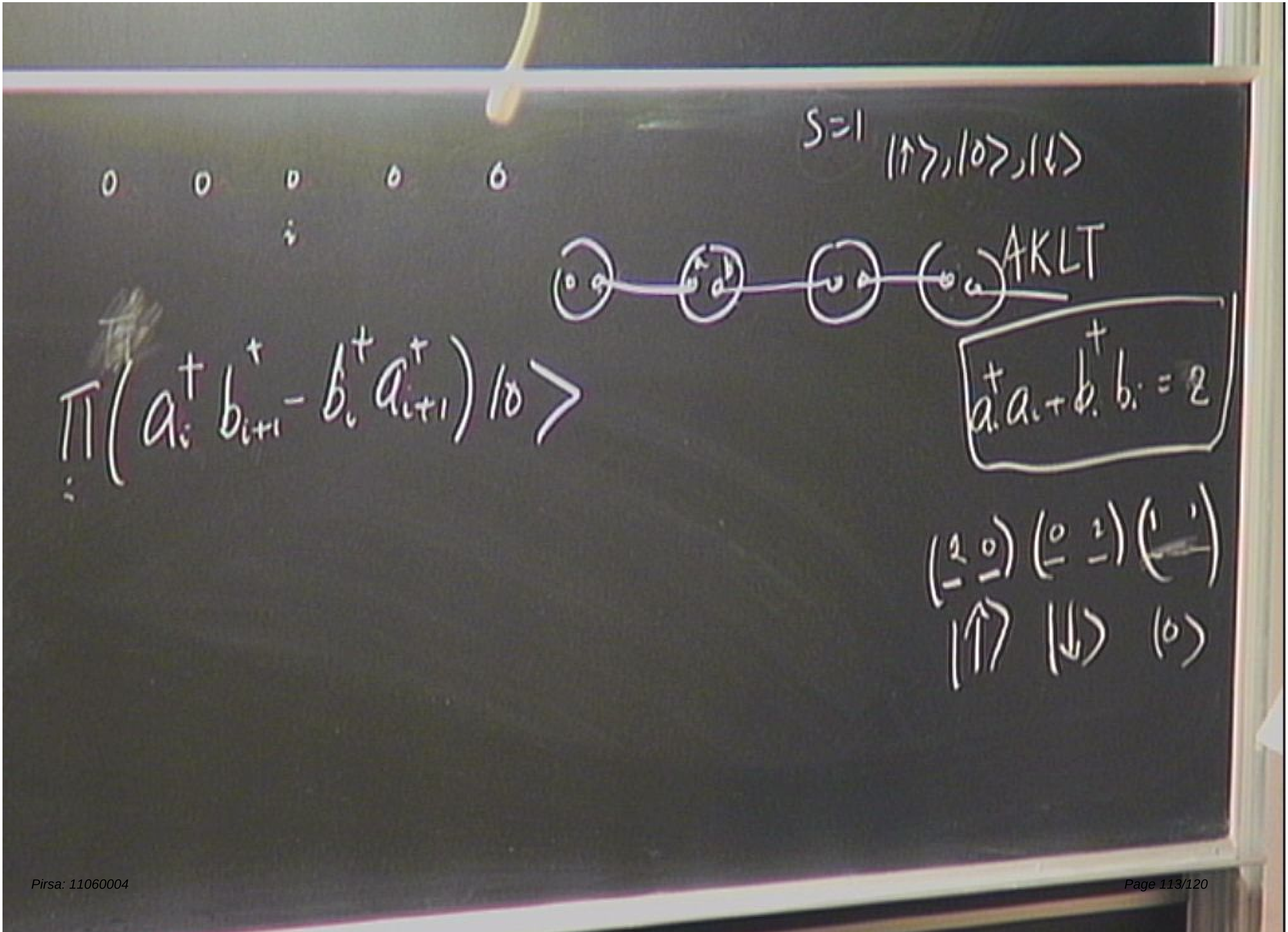
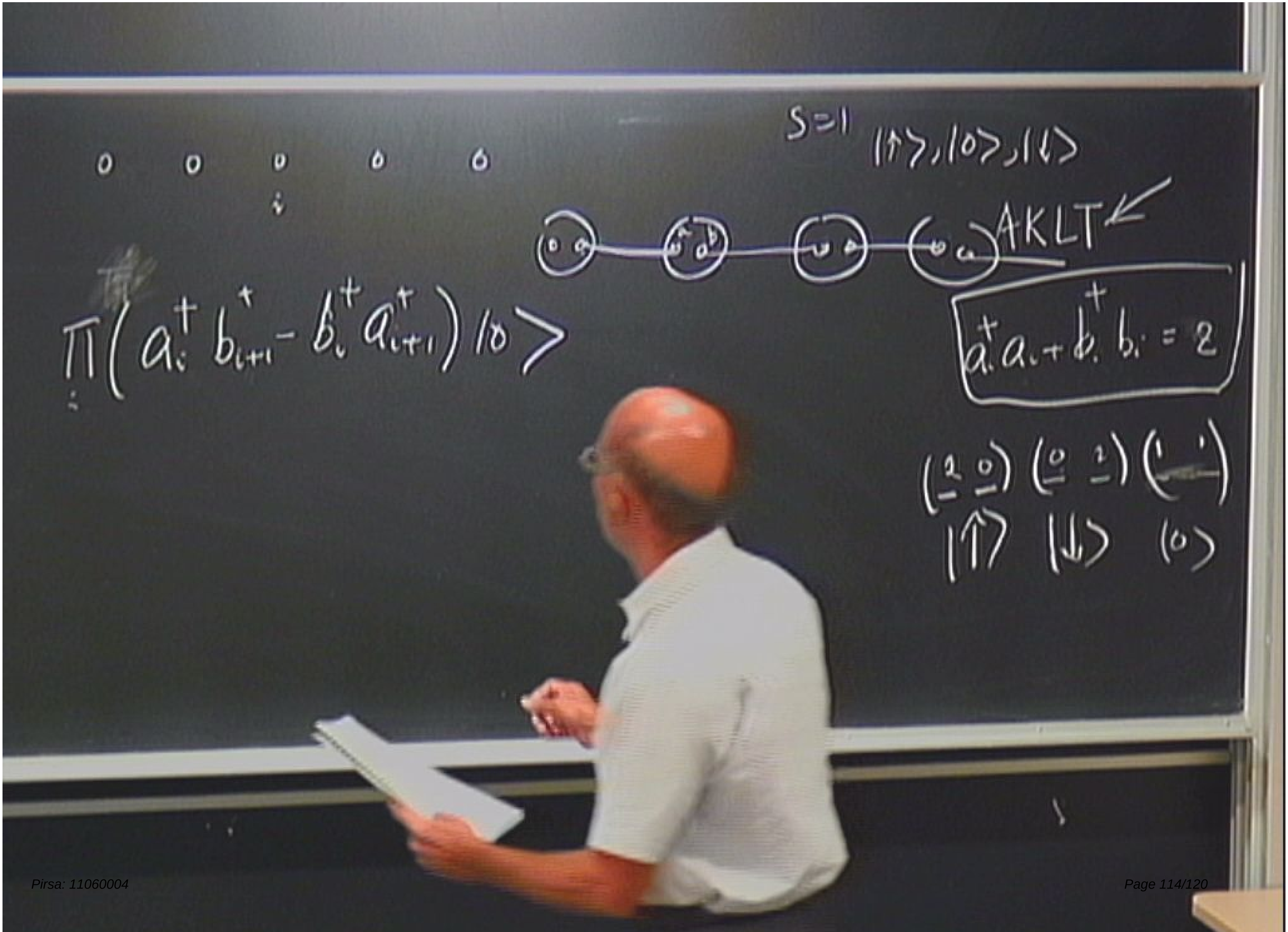
AKLT
Π (a_i^† b_{i+1}^† − b_i^† a_{i+1}^†) |0>
a_i^† a_i + b_i^† b_i = 2
|↑>  |↓>  |0>


S=1 |↑>, |0>, |↓>
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
|↑> |↓> |0>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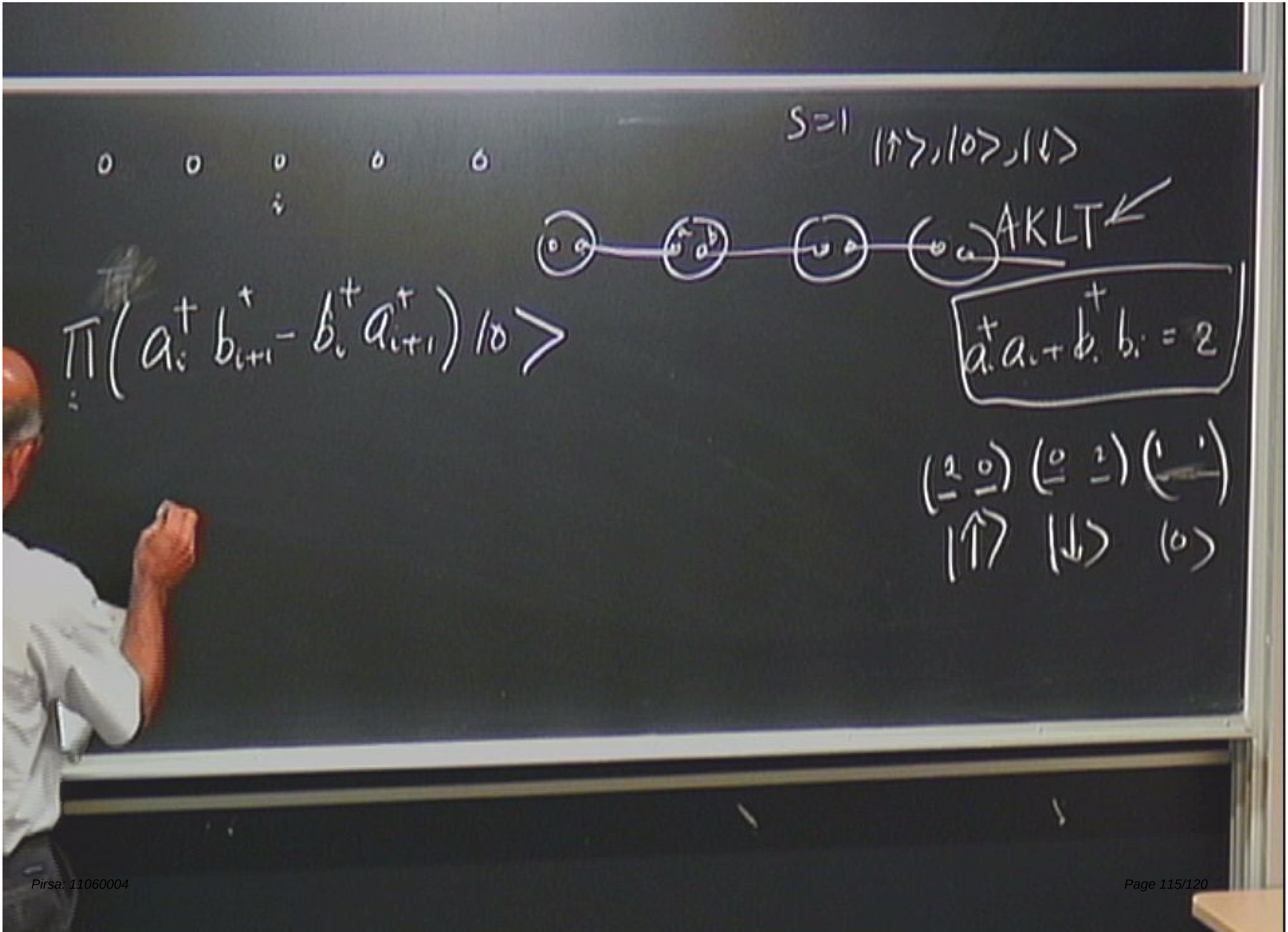
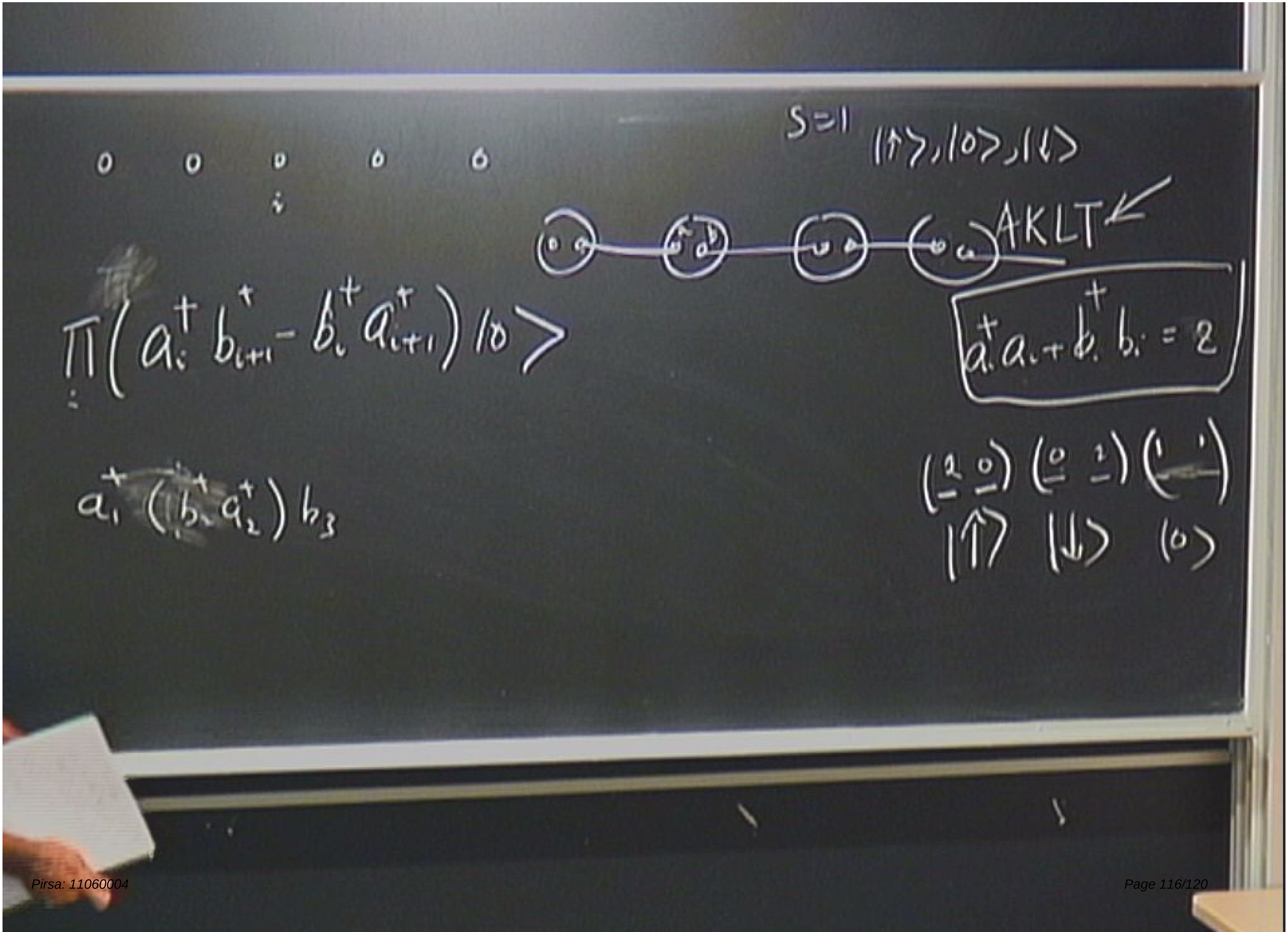
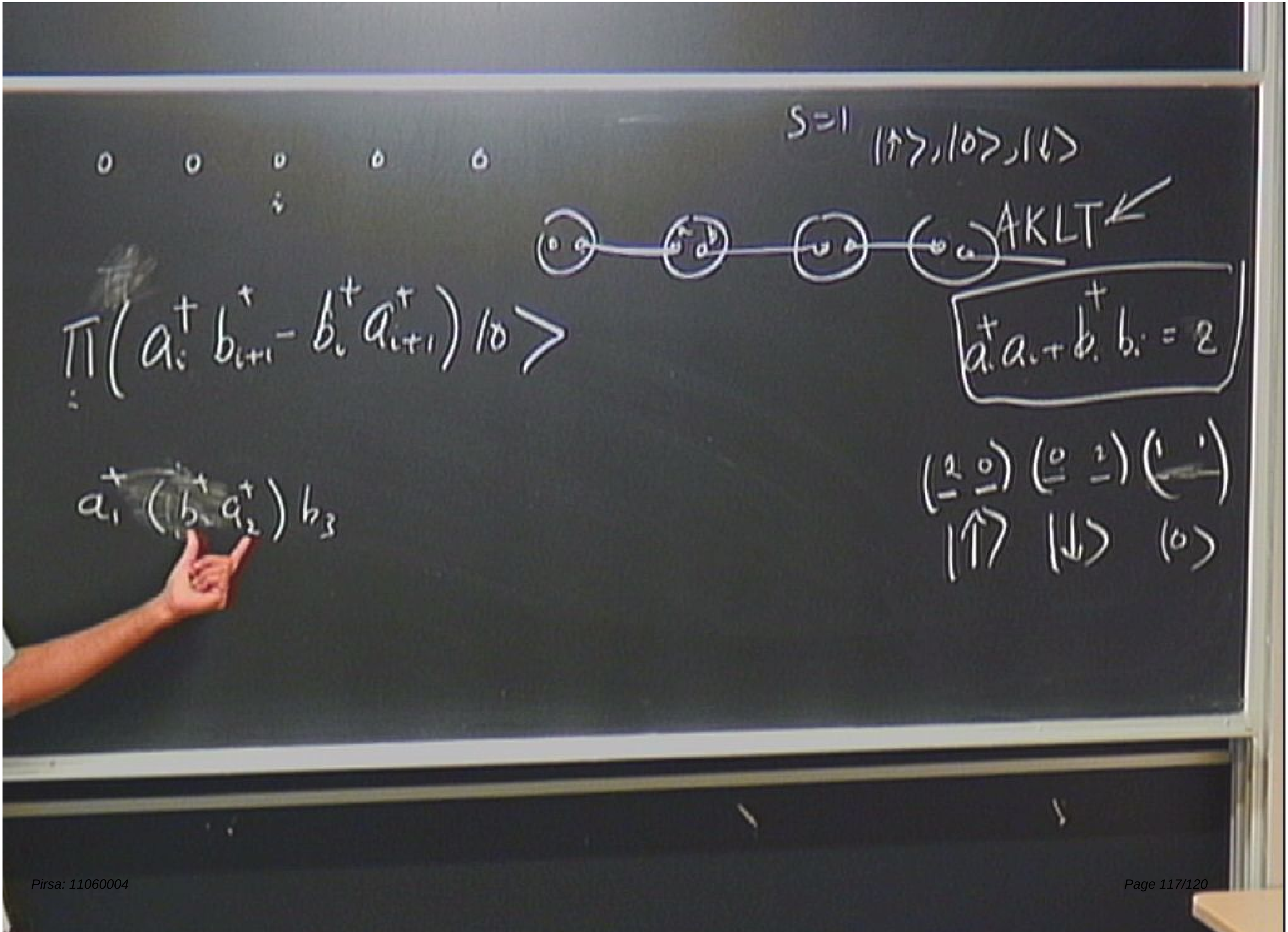
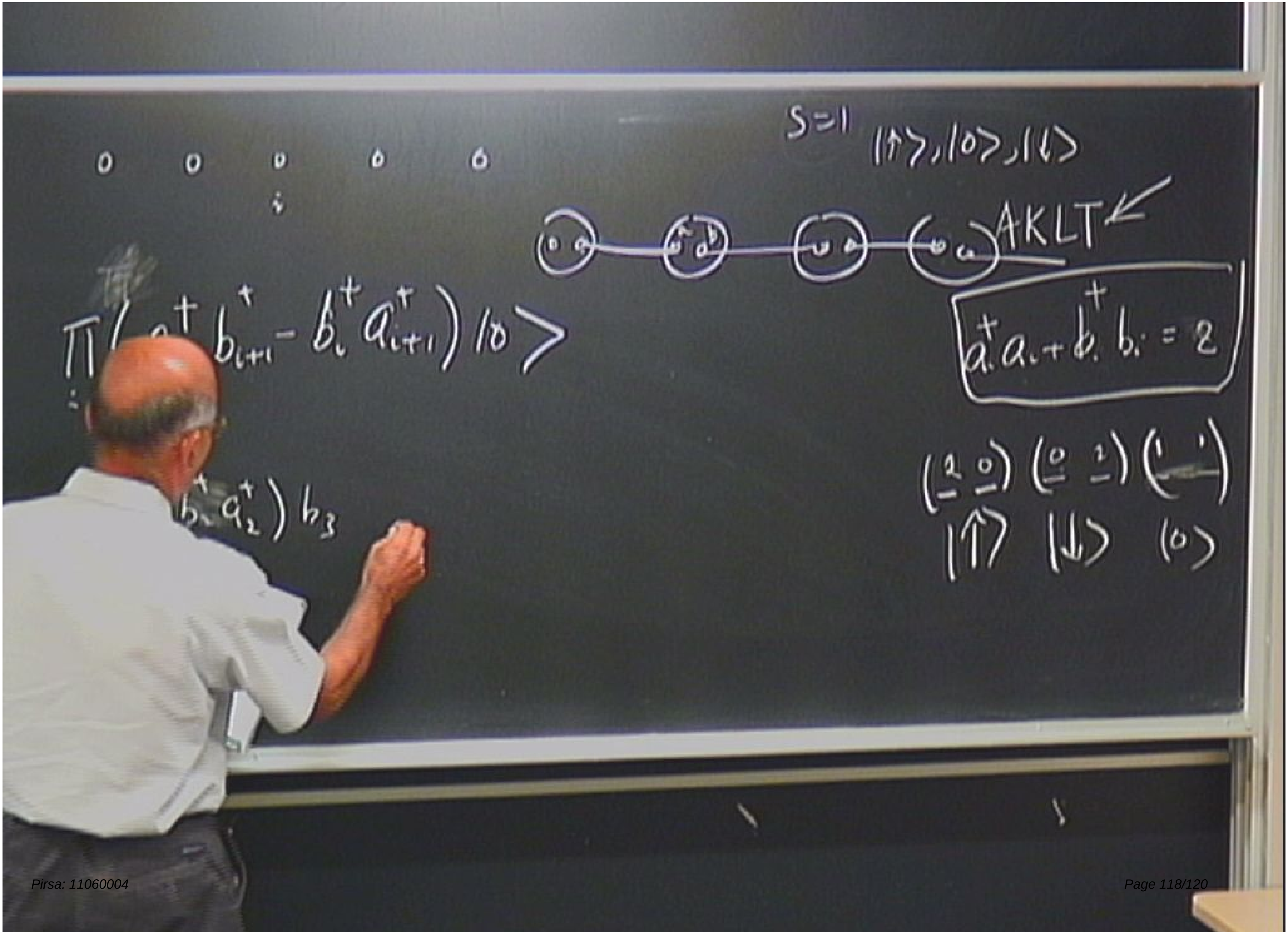

S=1
|↑>, |0>, |↓>
AKLT
$\prod_i (a_i^\dagger b_{i+1} - b_i^\dagger a_{i+1}) |0\rangle$
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
|↑> |↓> |0>



S=1
|↑>, |0>, |↓>
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
$a_1^\dagger (b_2^\dagger a_2^\dagger) b_3$
|↑> |↓> |0>


S=1
|↑>, |0>, |↓>
AKLT
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$
$a_1^\dagger (b_2^\dagger a_2^\dagger) b_3$
|↑> |↓> |0>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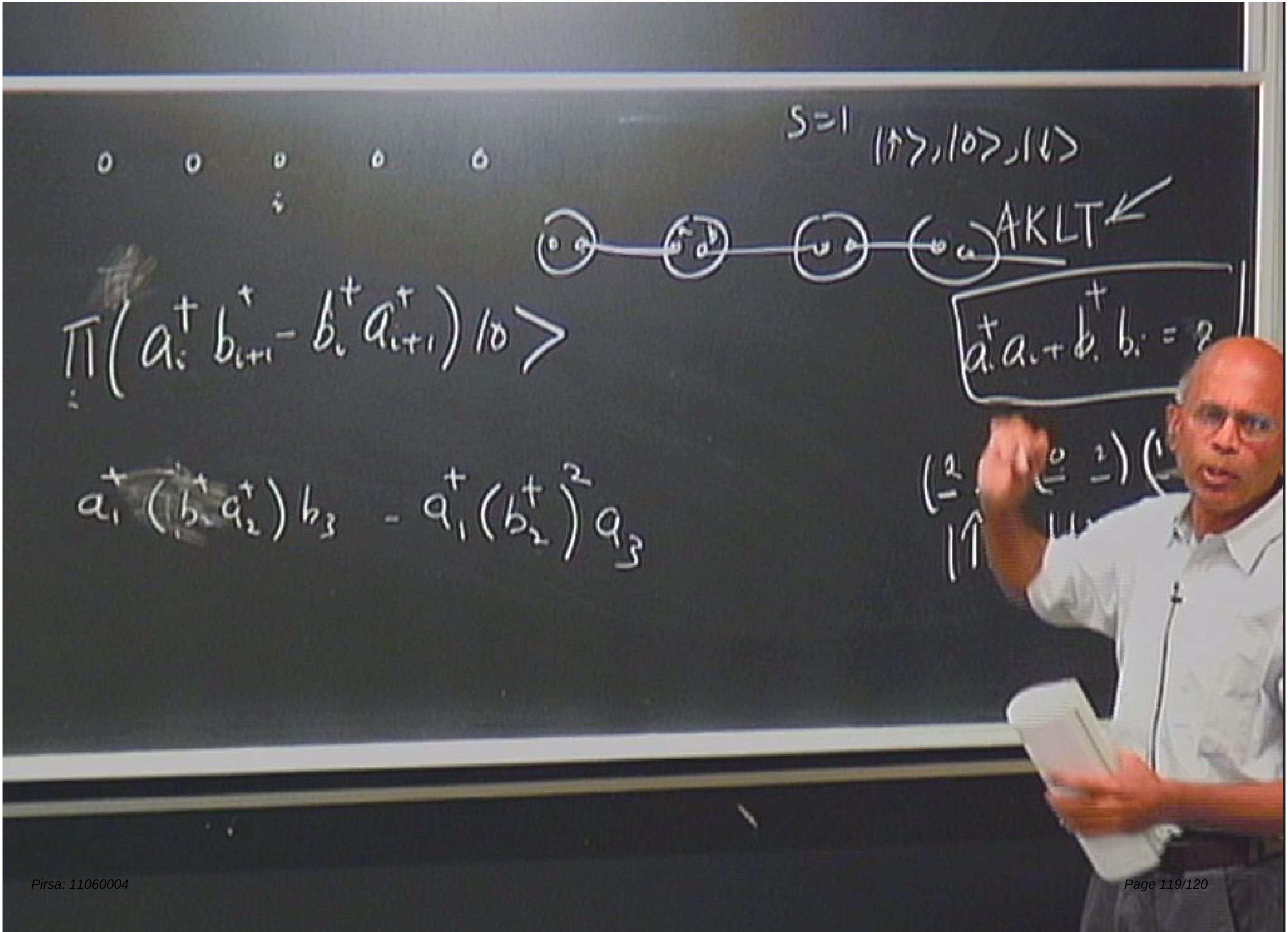
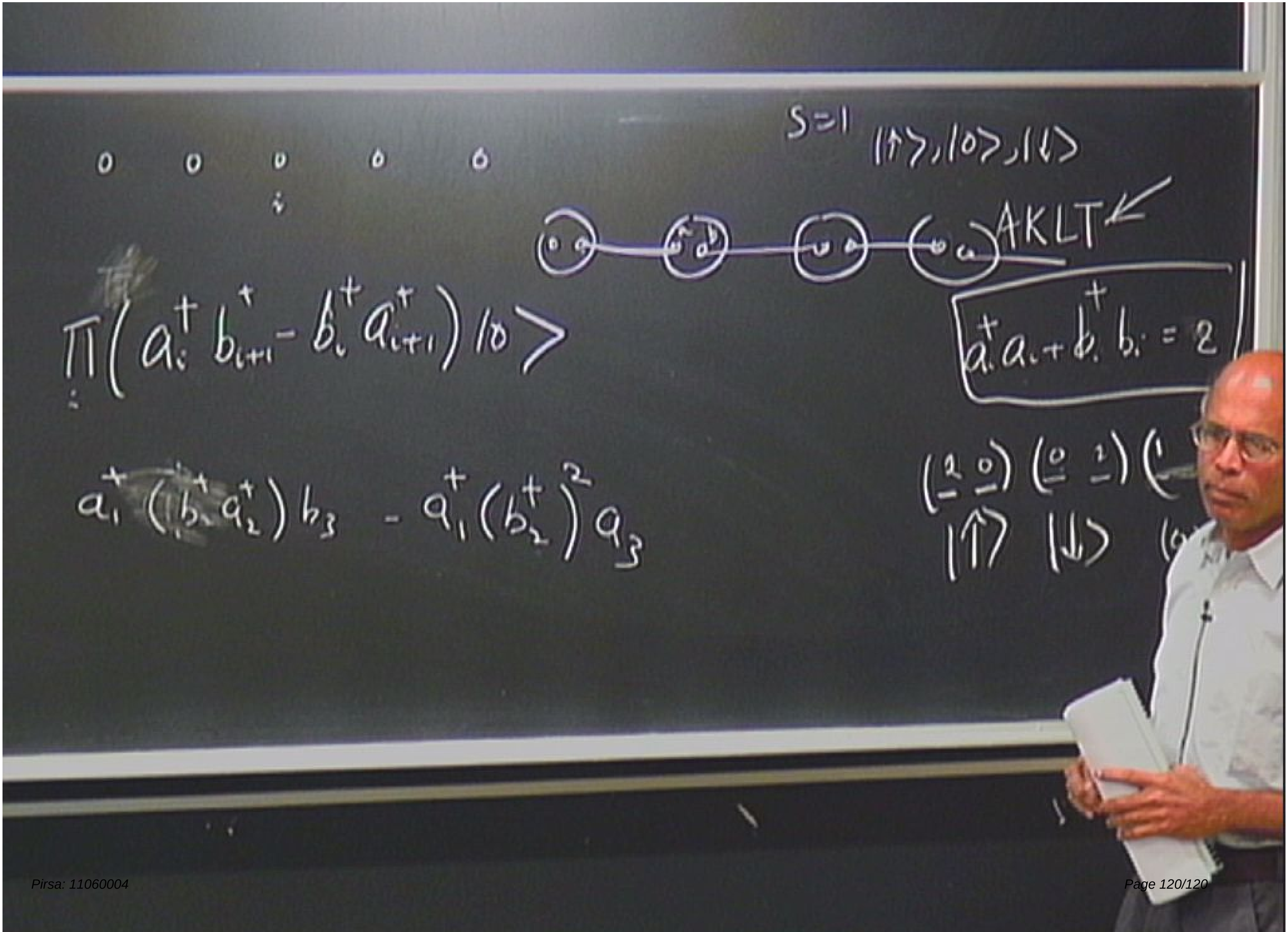
AKLT


S=1
|↑>, |0>, |↓>
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
$a_1^\dagger (b_2^\dagger a_2^\dagger) b_3 - a_1^\dagger (b_2^\dagger)^2 a_3$


S=1
|↑>, |0>, |↓>
AKLT
$\prod_i (a_i^\dagger b_{i+1}^\dagger - b_i^\dagger a_{i+1}^\dagger) |0>$
$a_i^\dagger a_i + b_i^\dagger b_i = 2$
$a_1^\dagger (b_2^\dagger a_2^\dagger) b_3 - a_1^\dagger (b_2^\dagger)^2 a_3$
|↑>
|↓>