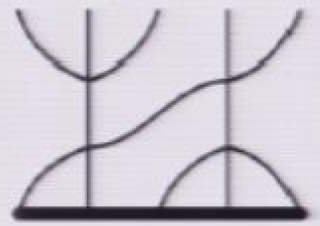
Title: Loop calculations in Superconformal Chern-Simons

Date: May 25, 2011 11:30 AM

URL: http://pirsa.org/11050070

Abstract: We discuss the calculation of higher loop contributions to the anomalous dimensions of operators in N=6 Superconformal Chern-Simons theories. These lead us to conjecture the form of an interpolating function of the 't Hooft coupling that is not determined by integrability.

Loop calculations in superconformal Chern-Simons 

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- Two "Very Interesting" superconformal field theories with integrable structures in planar limit
  - $\mathcal{N} = 4$ Super Yang-Mills
  - ABJM model (Aharony, Bergman, Jafferis and Maldacena): Chern-Simons w/ superconformal matter

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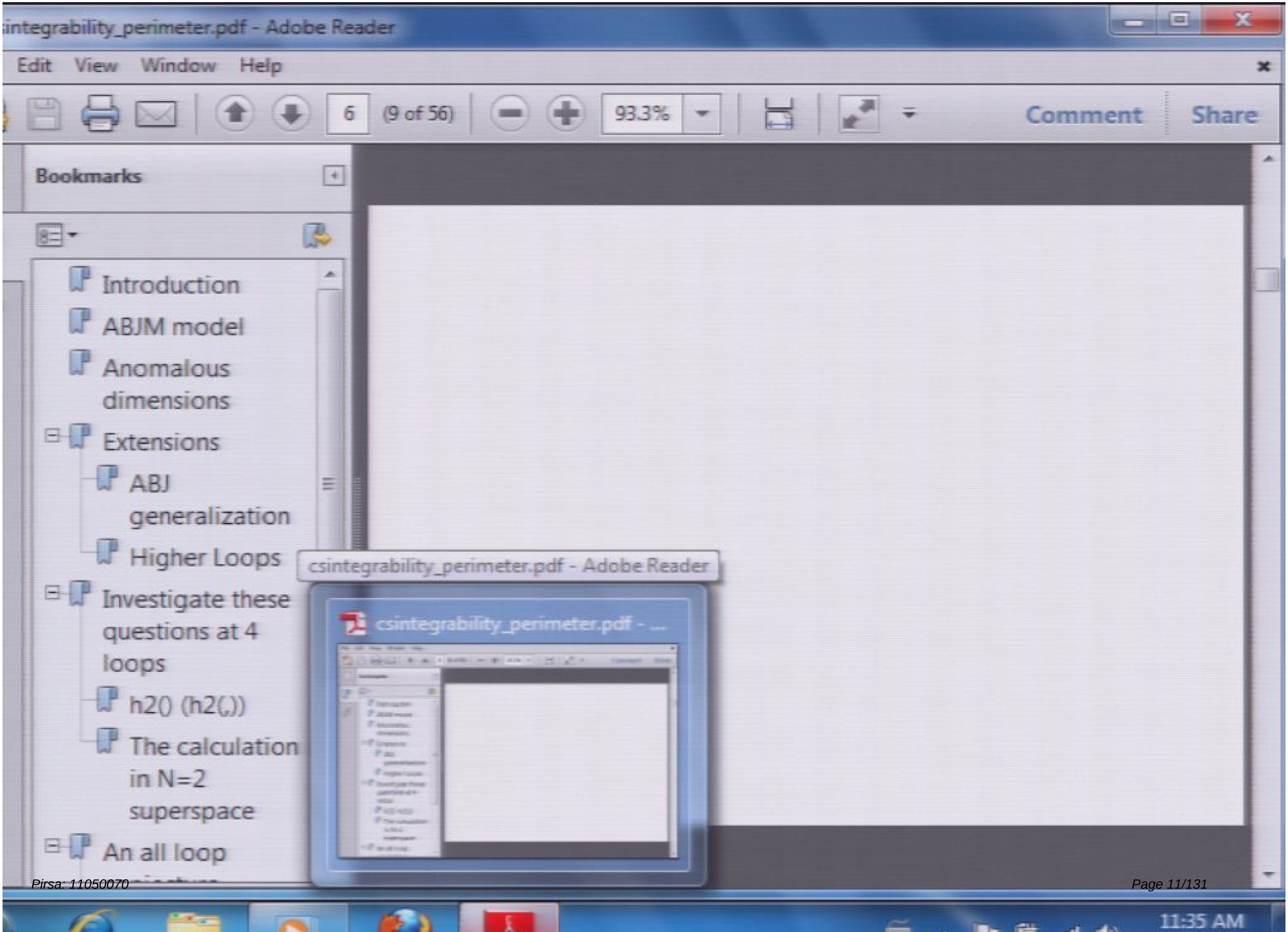

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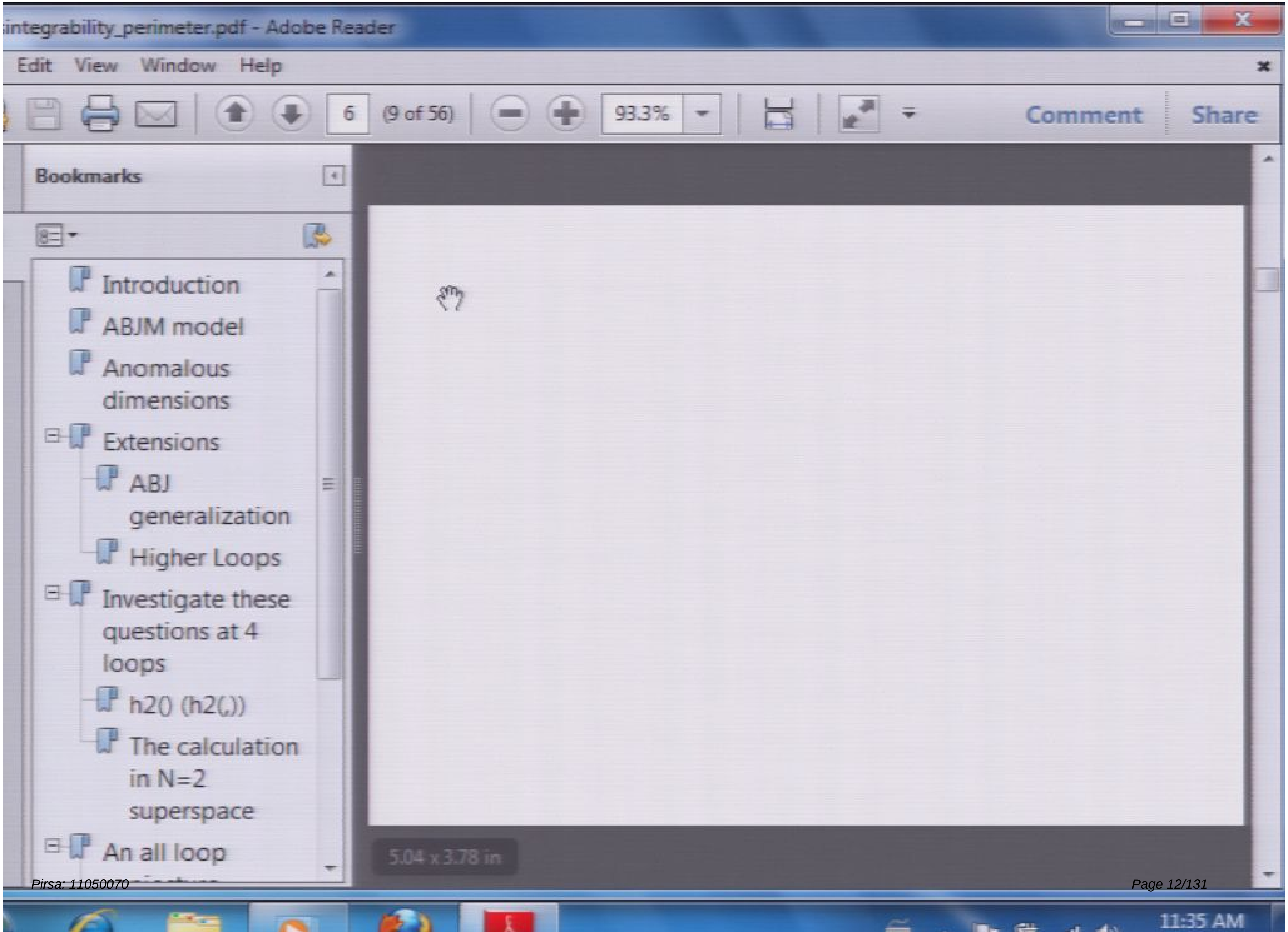
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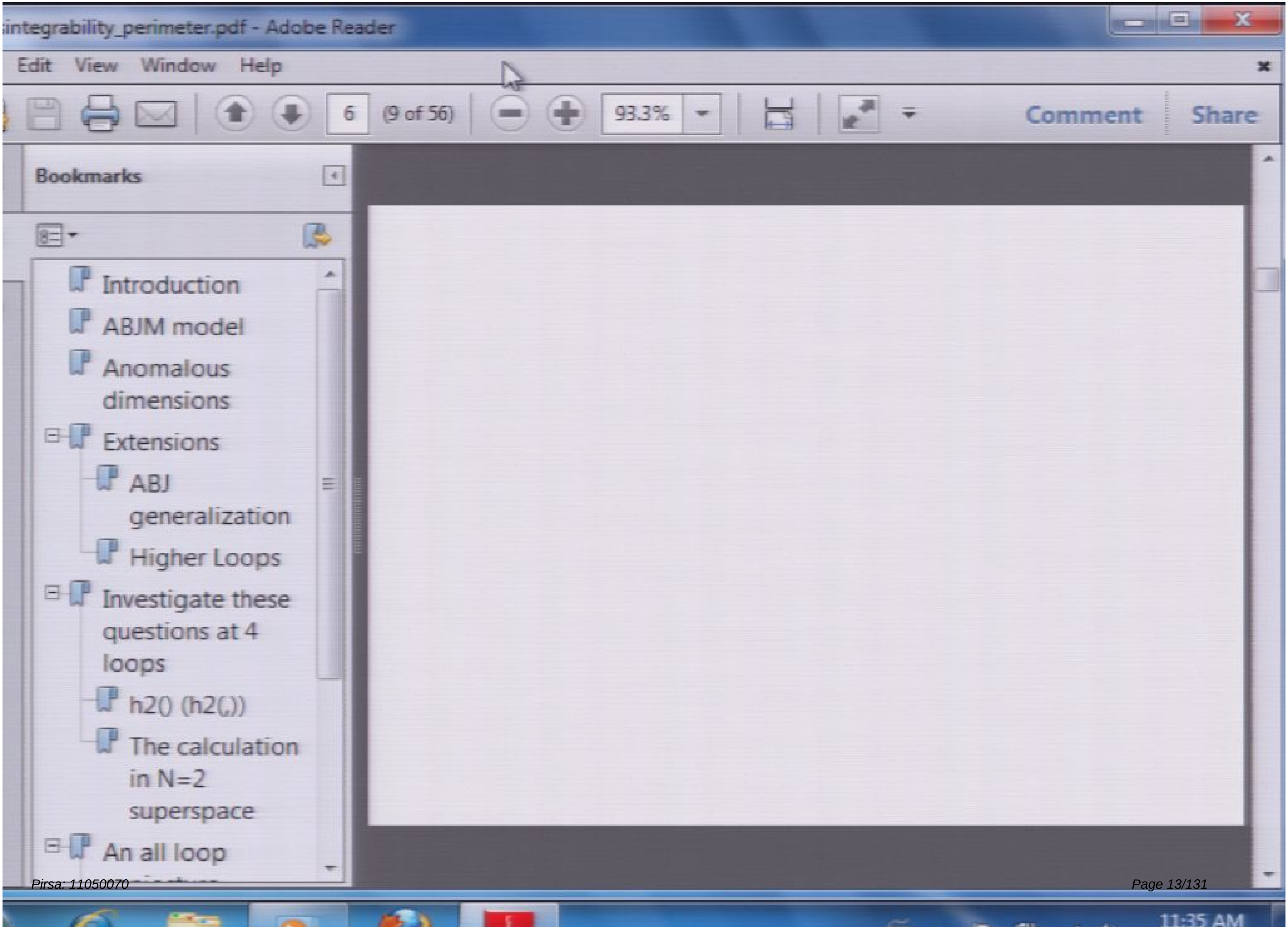
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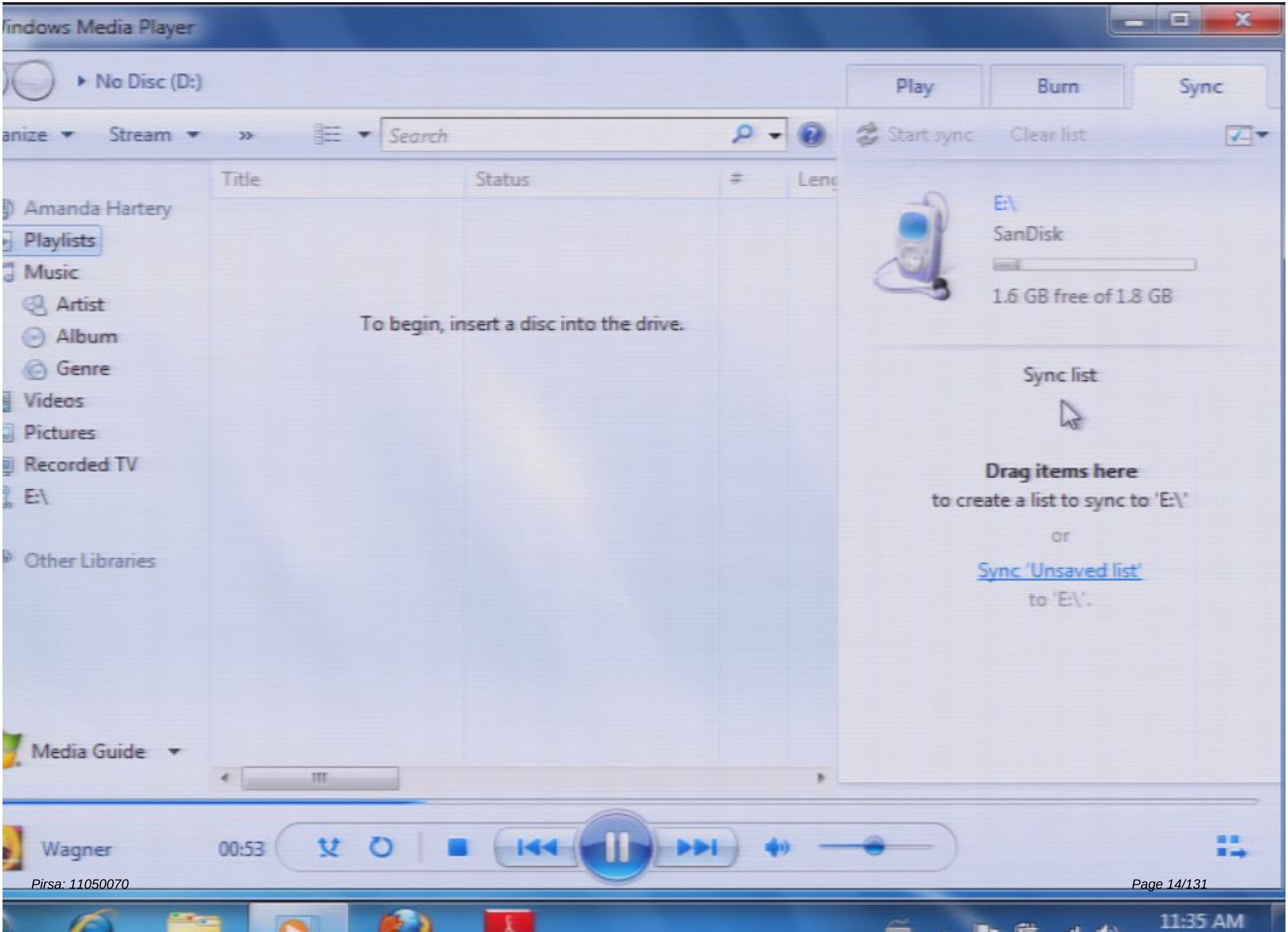

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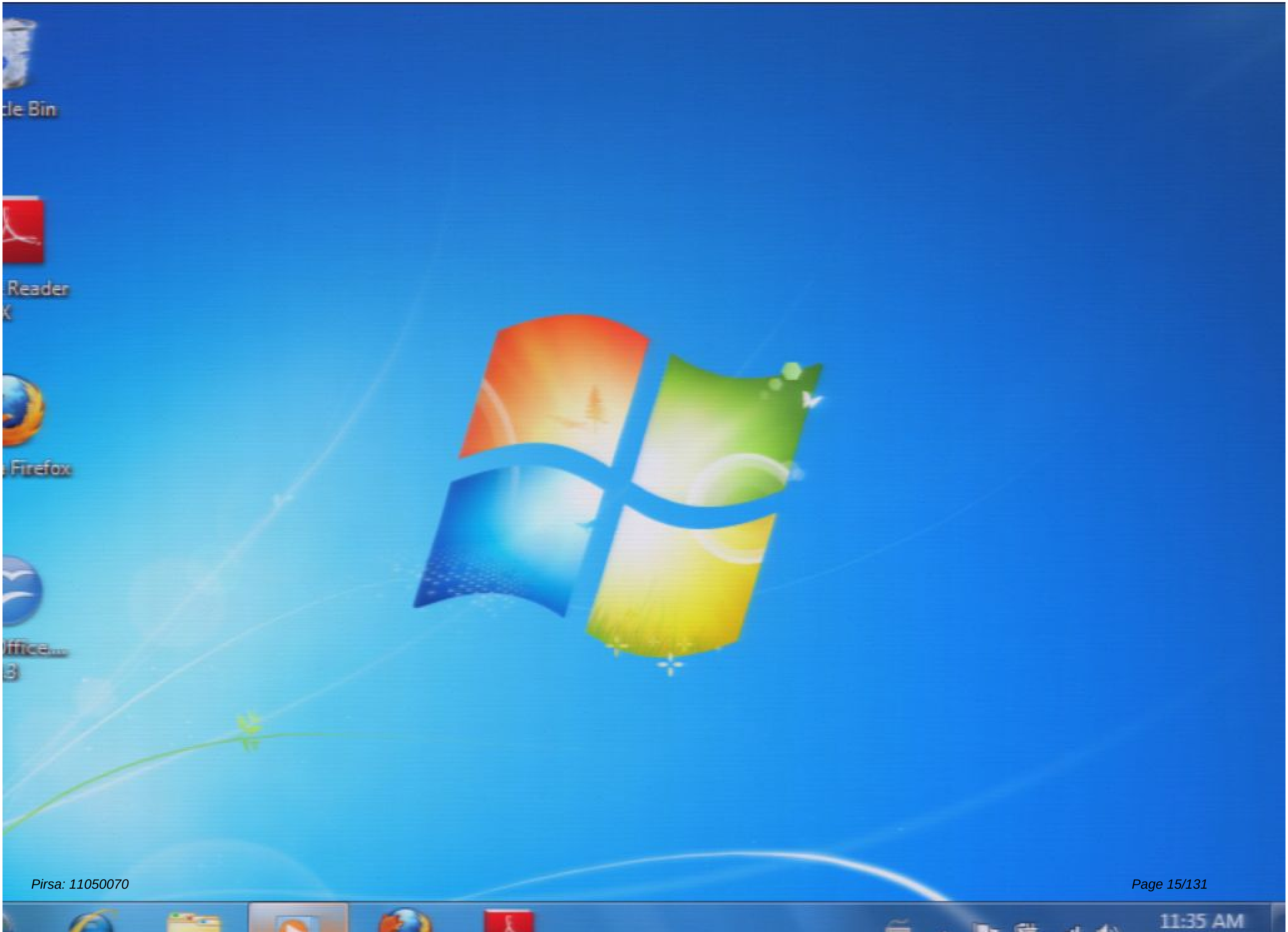
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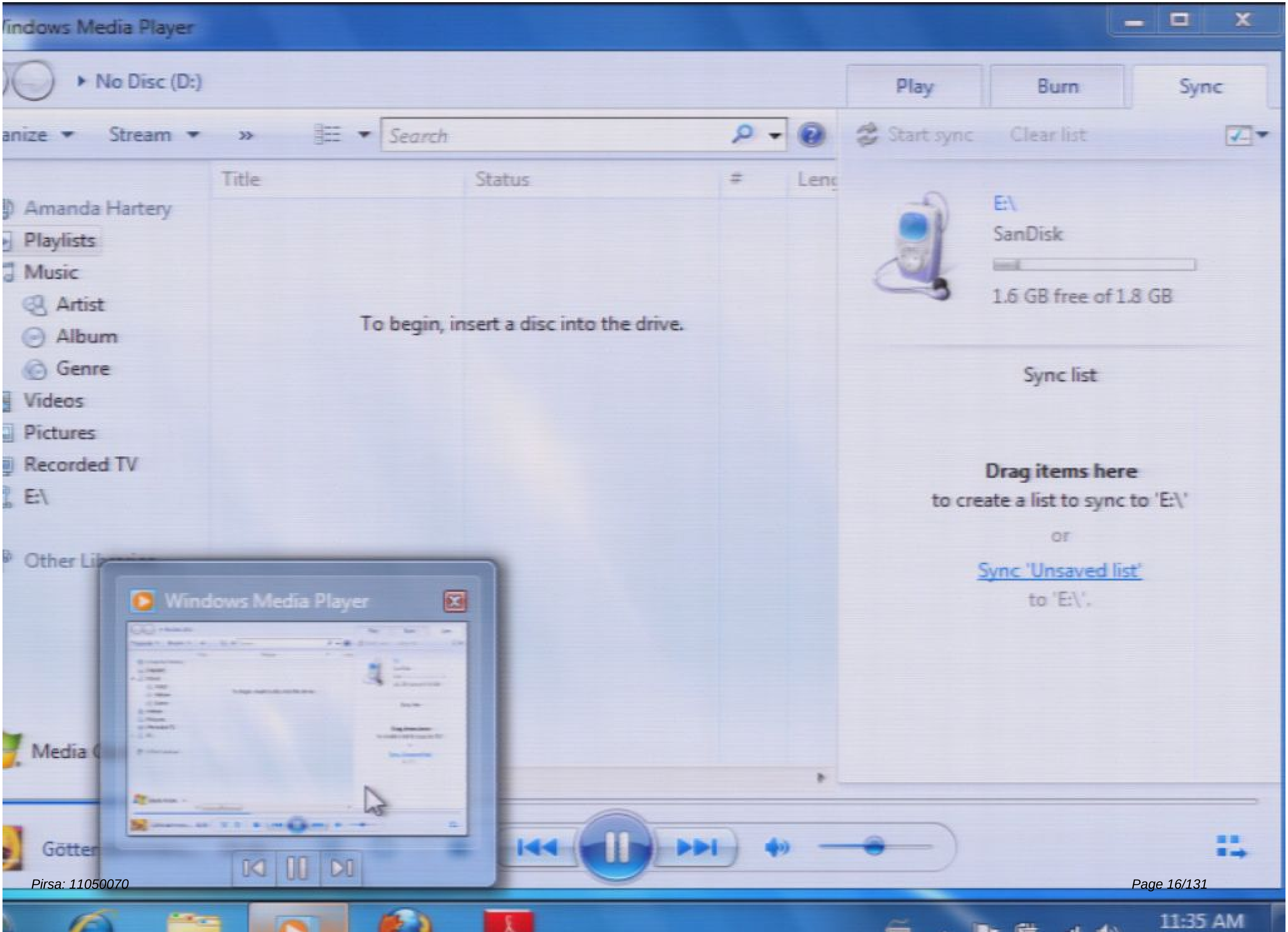
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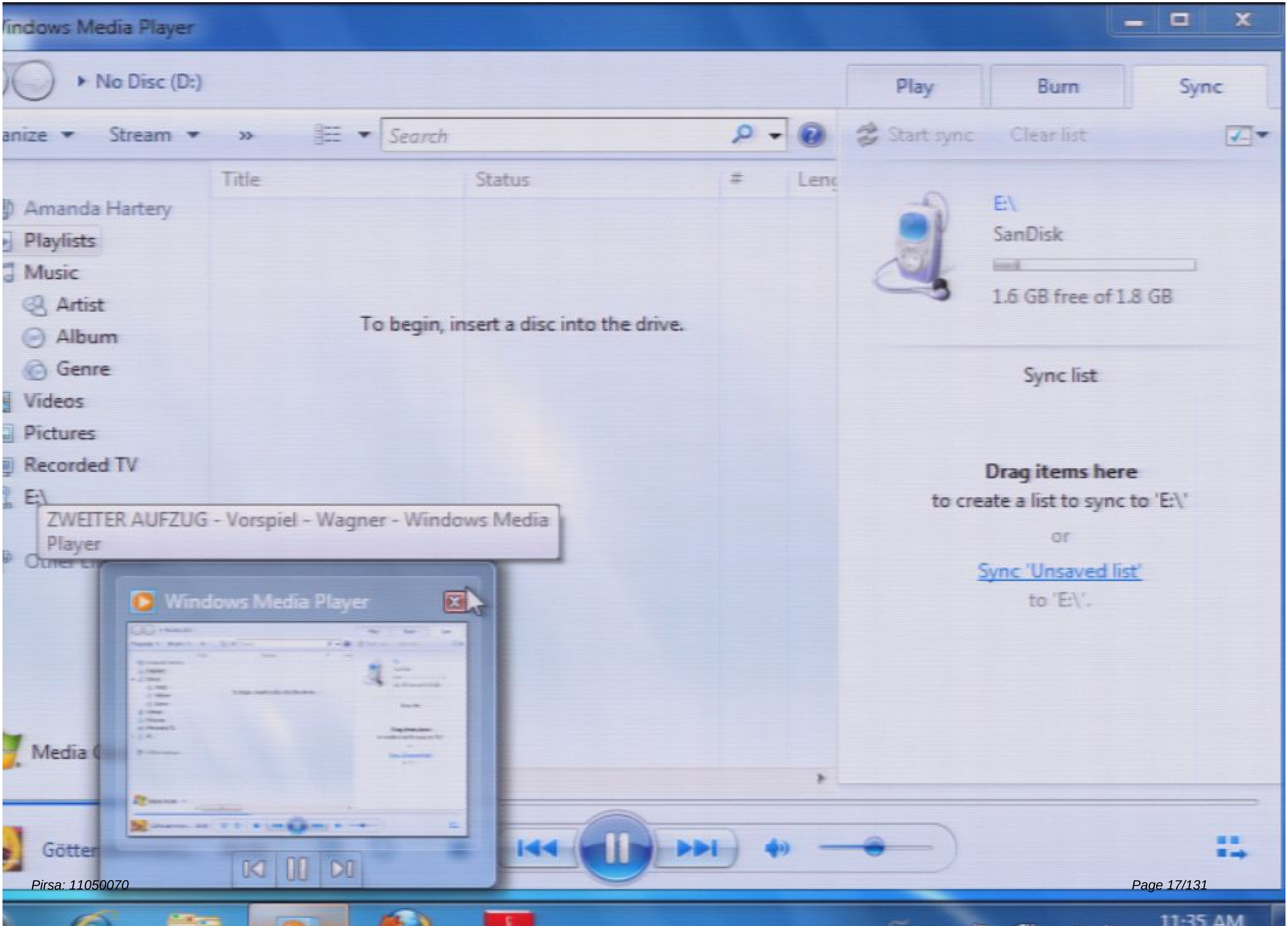
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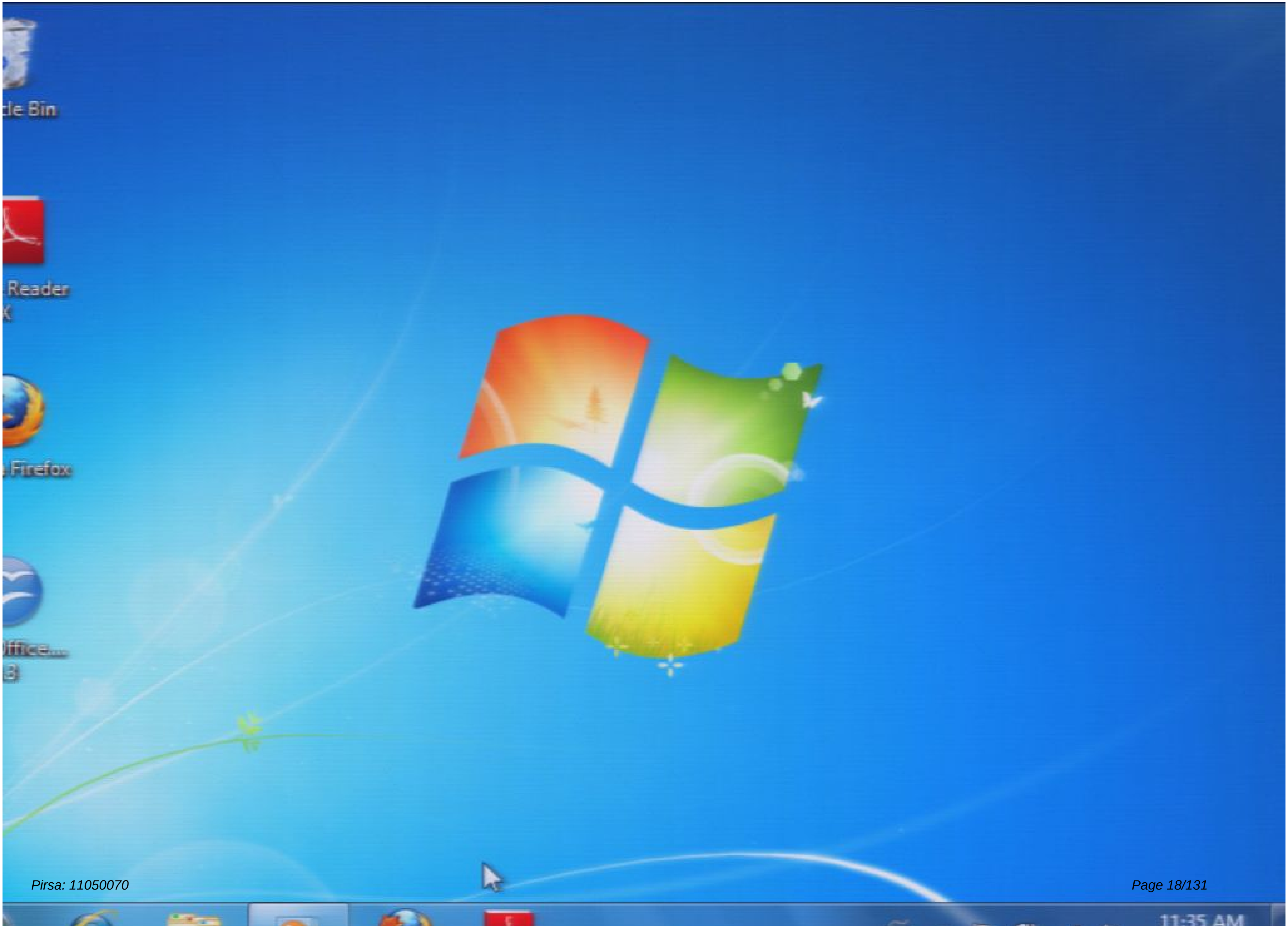
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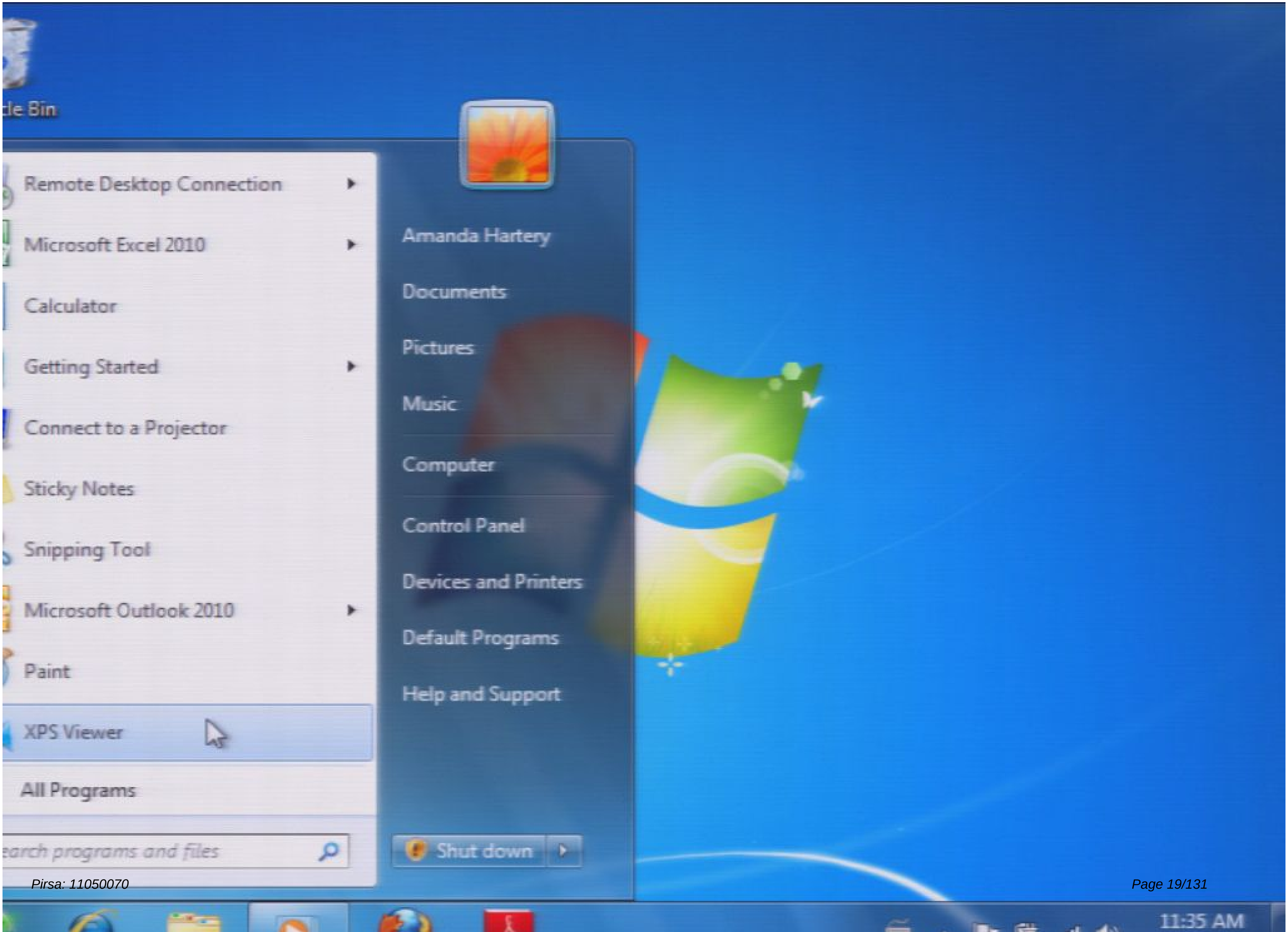
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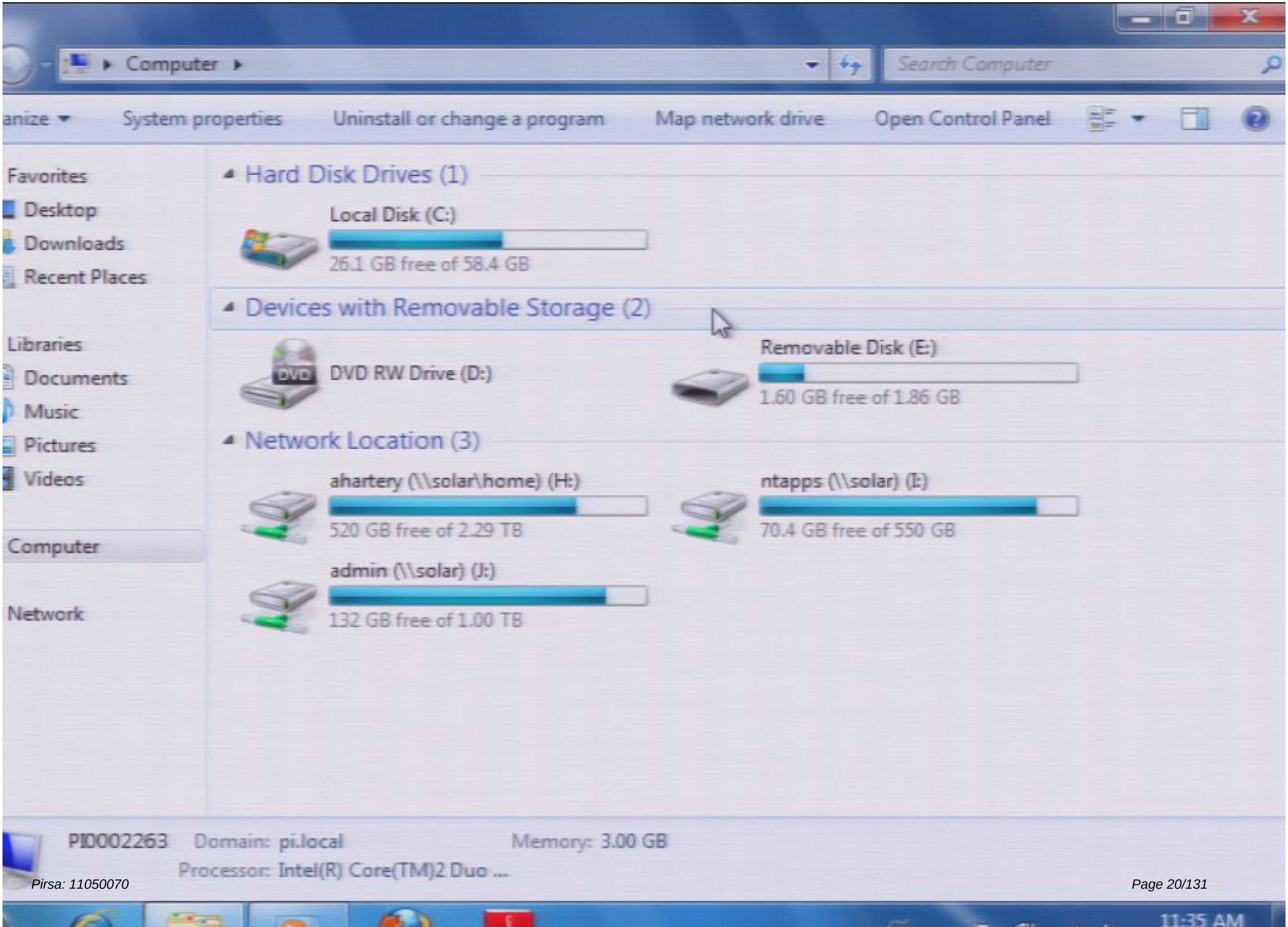
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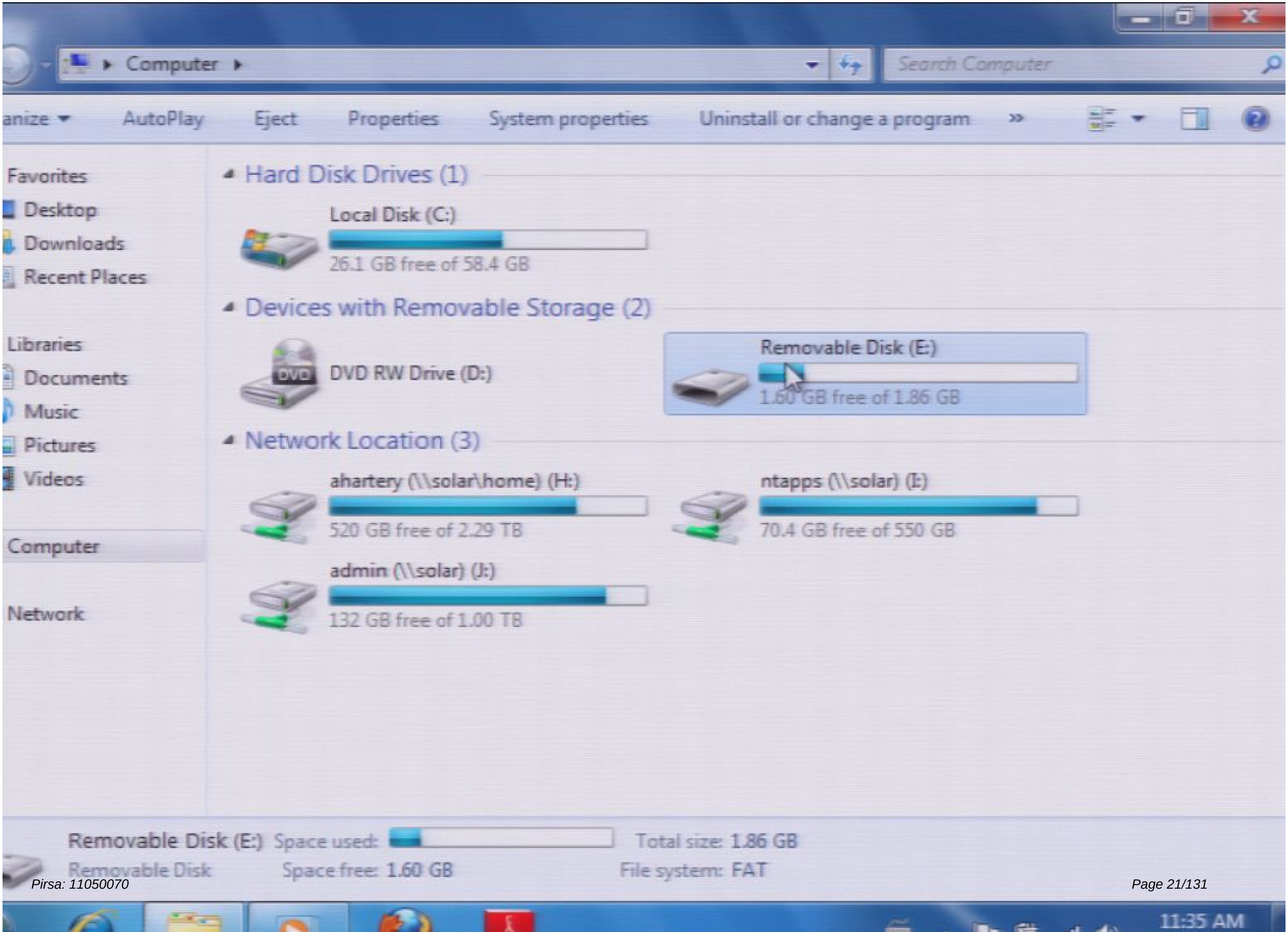
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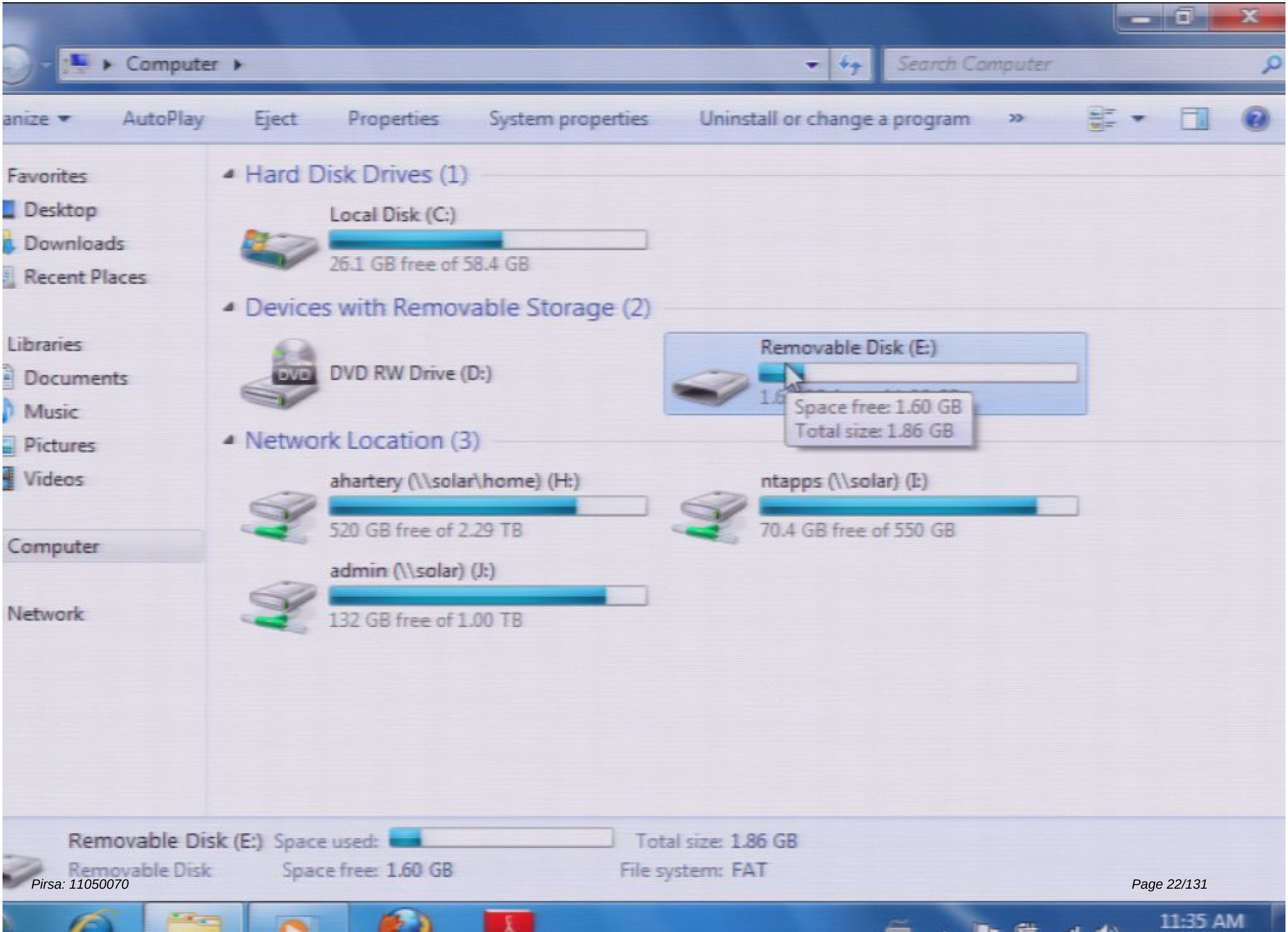
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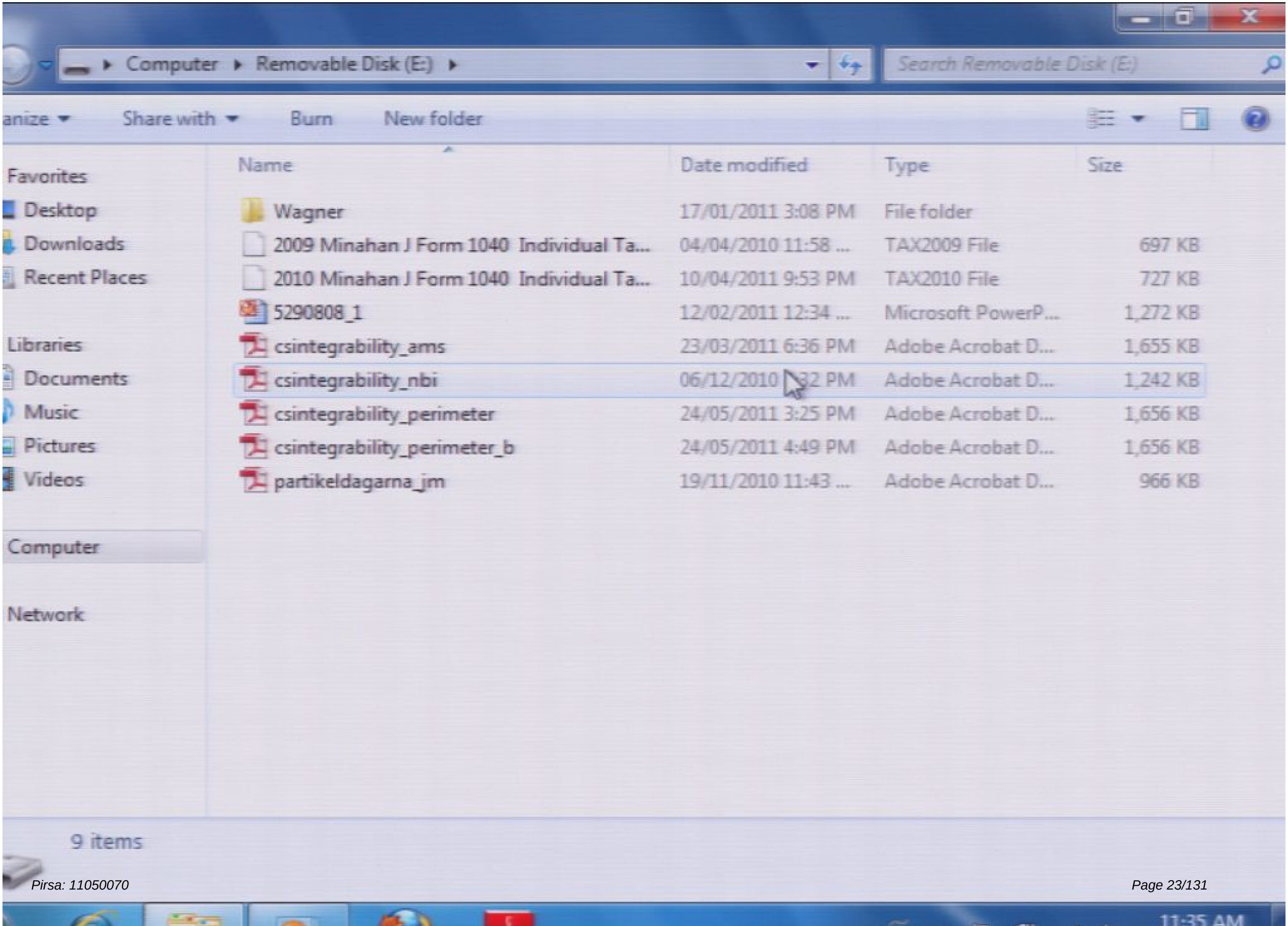
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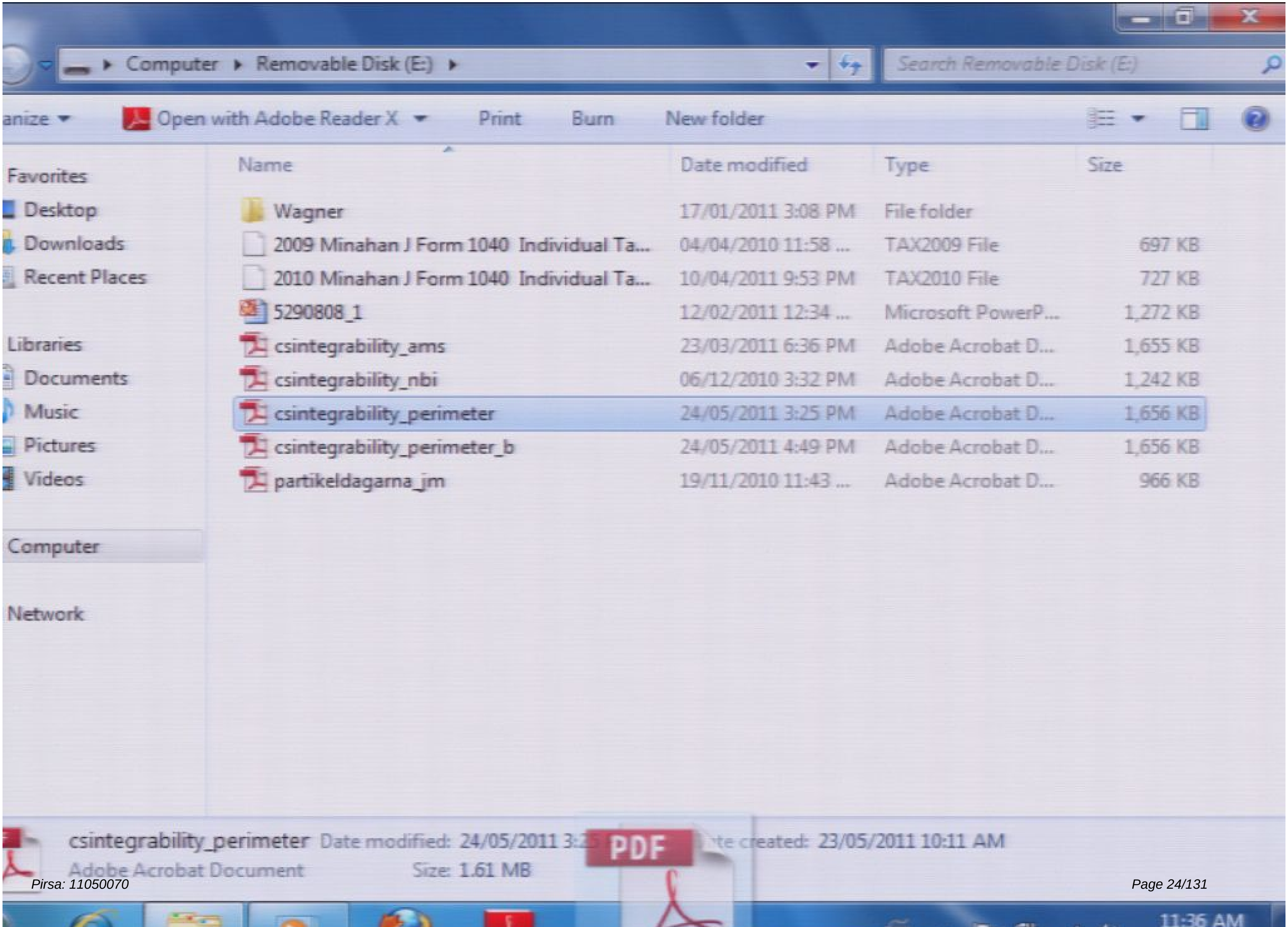
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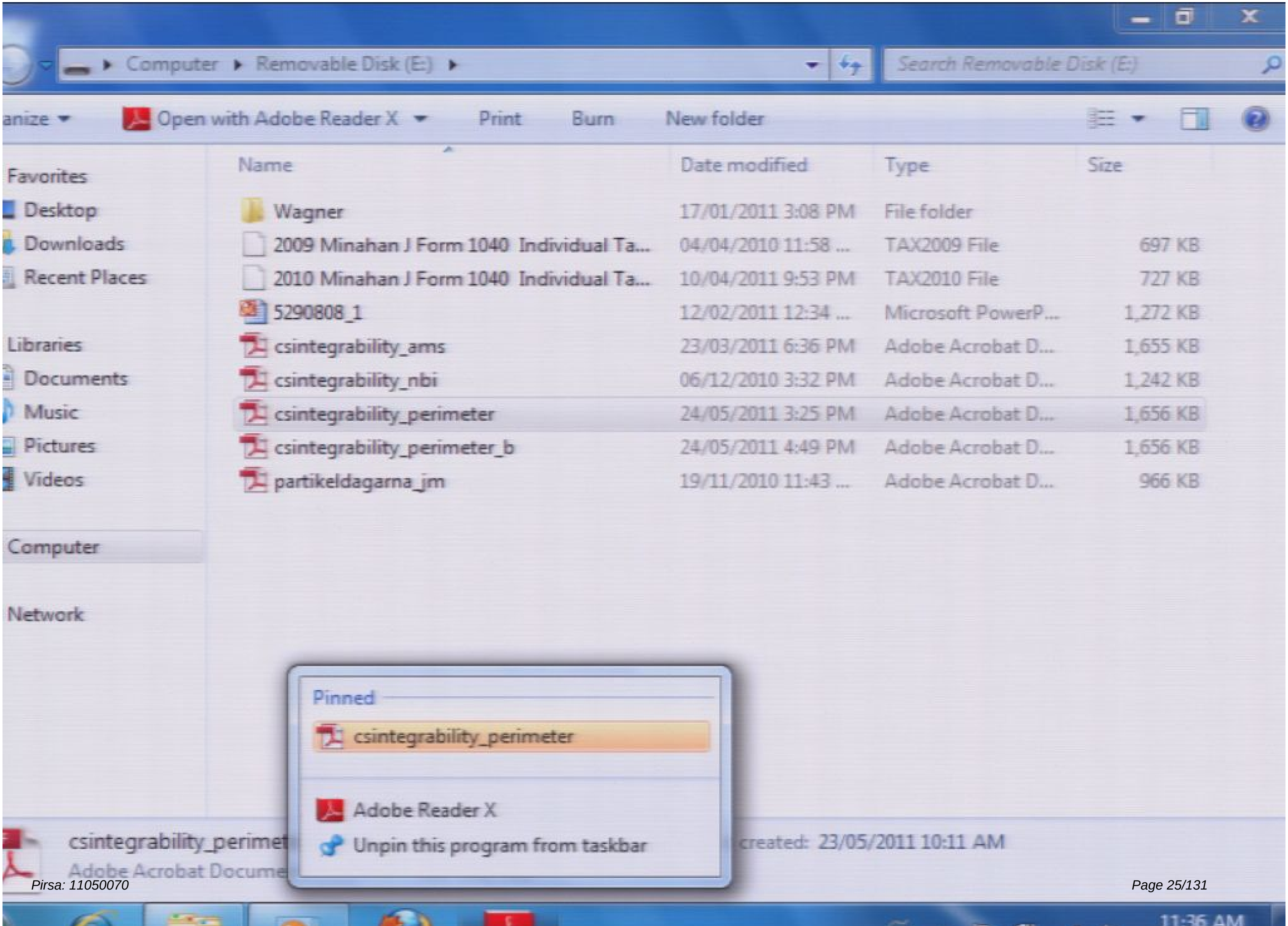
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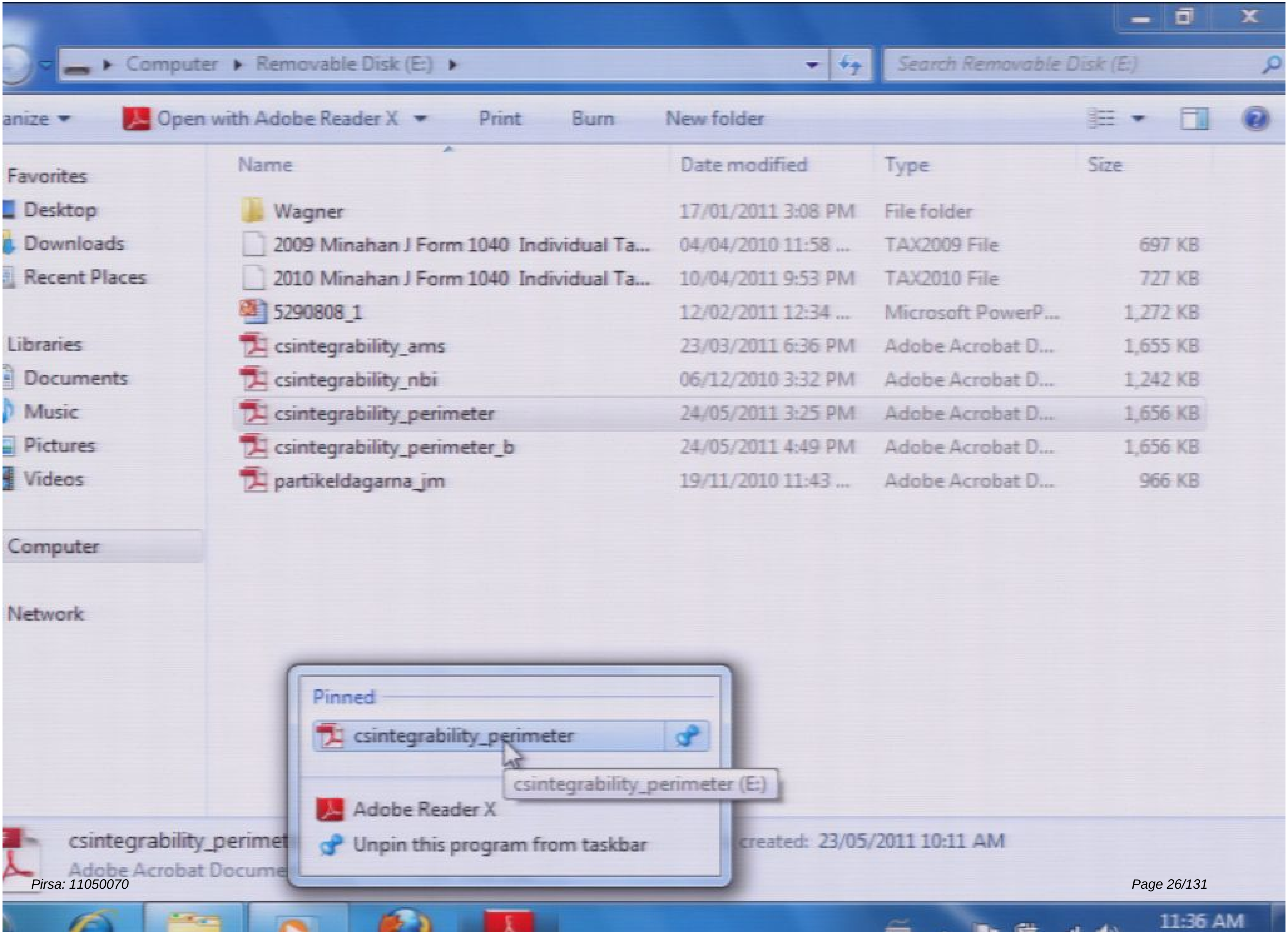
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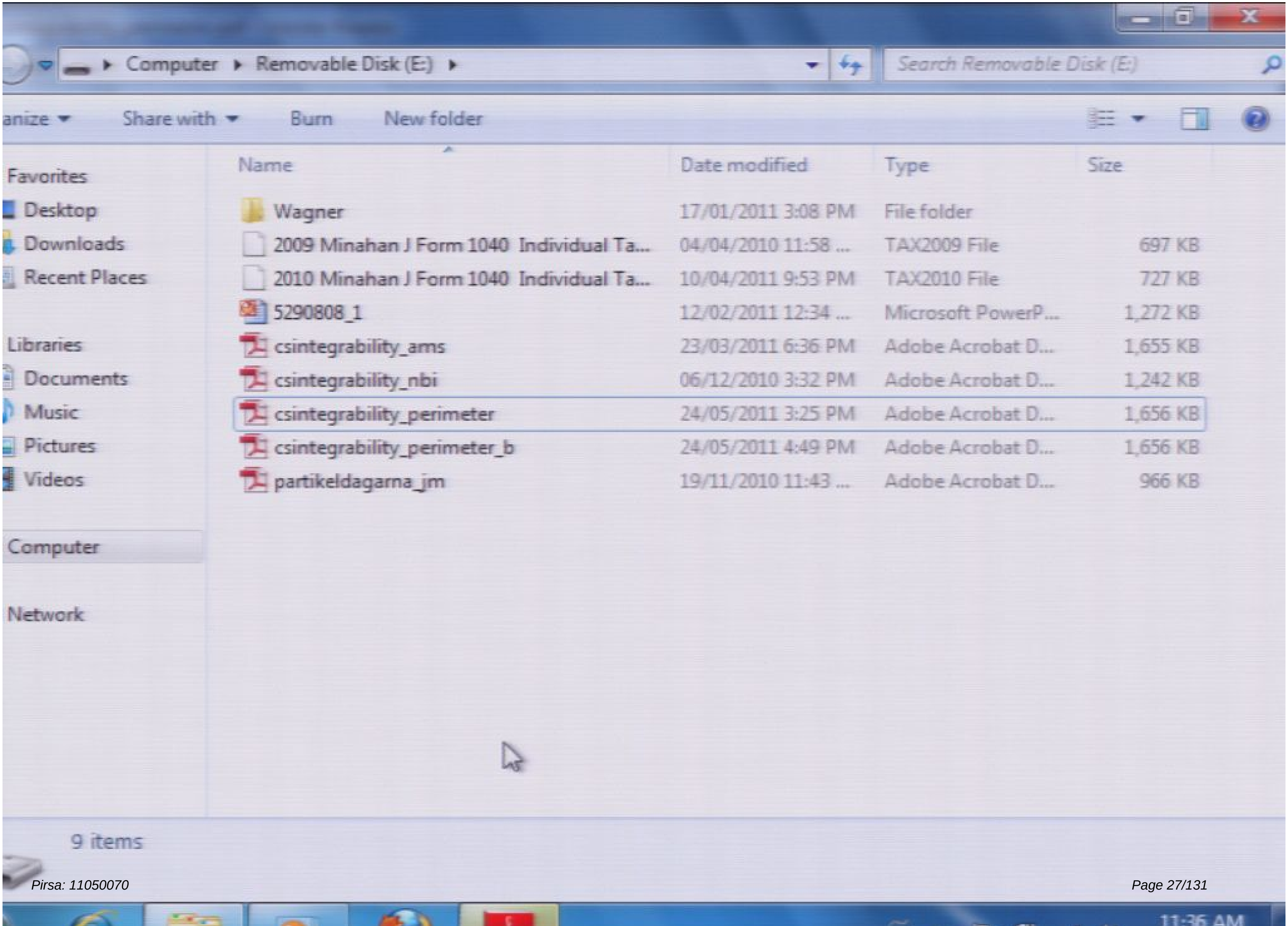
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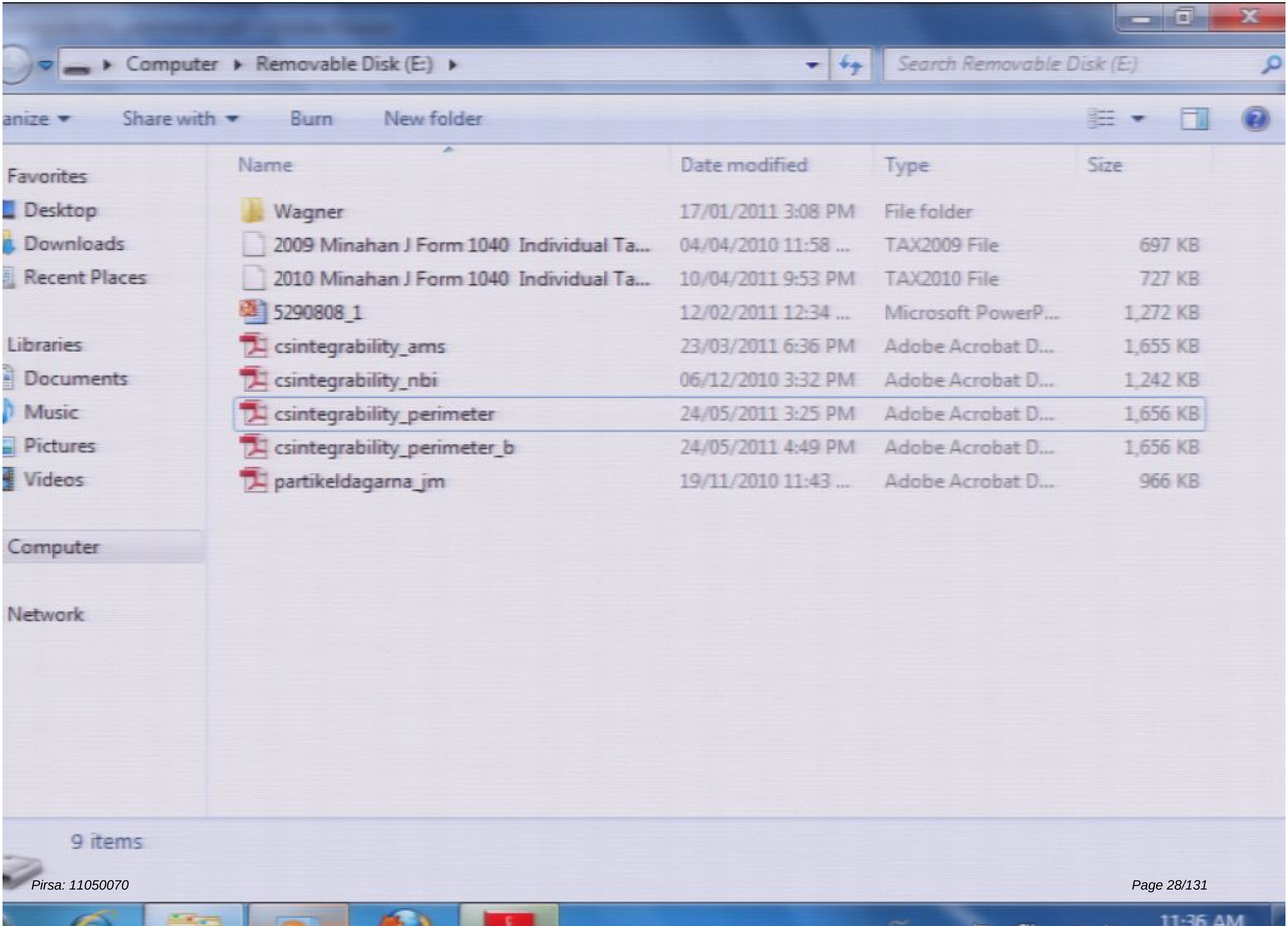
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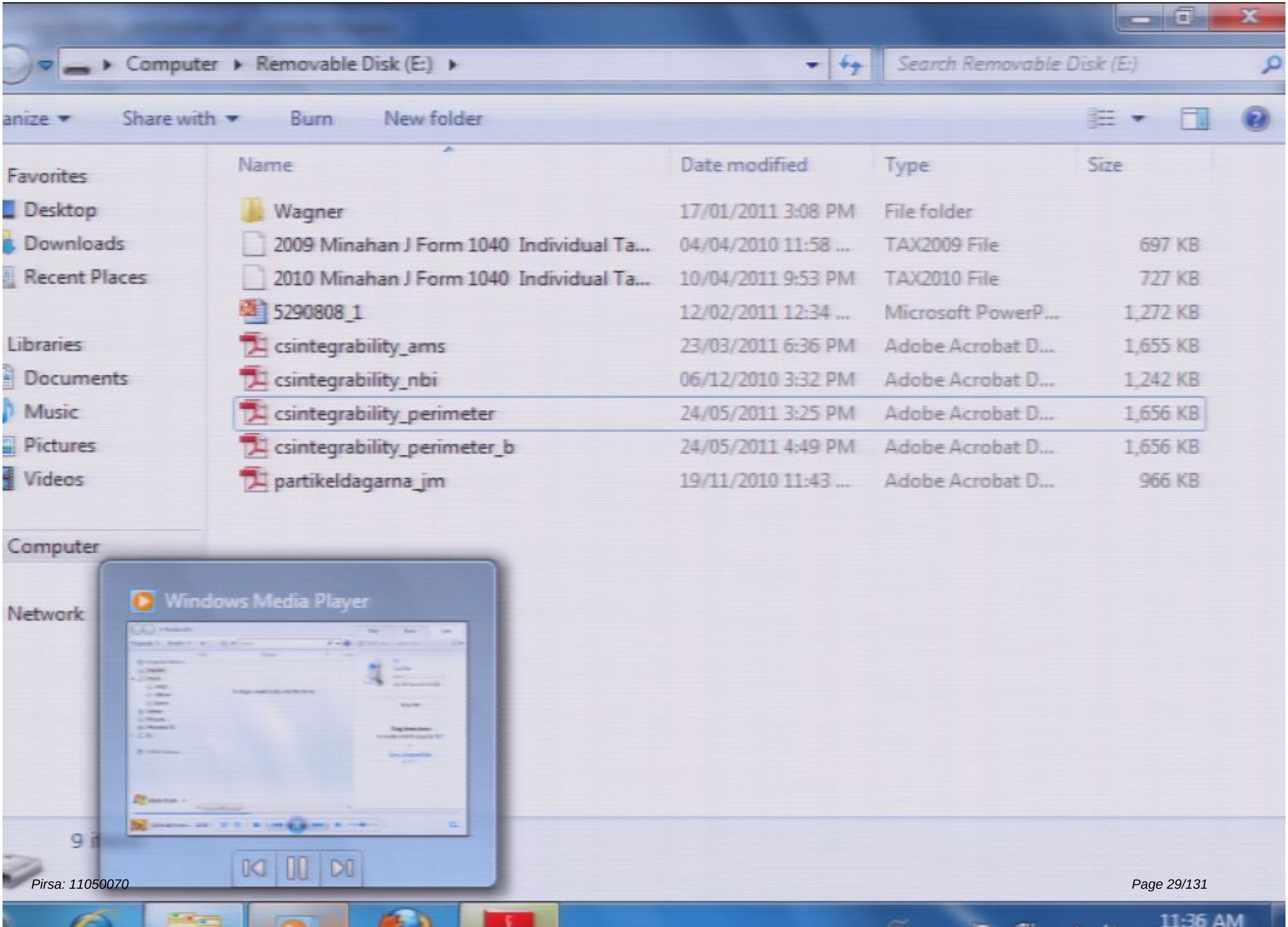

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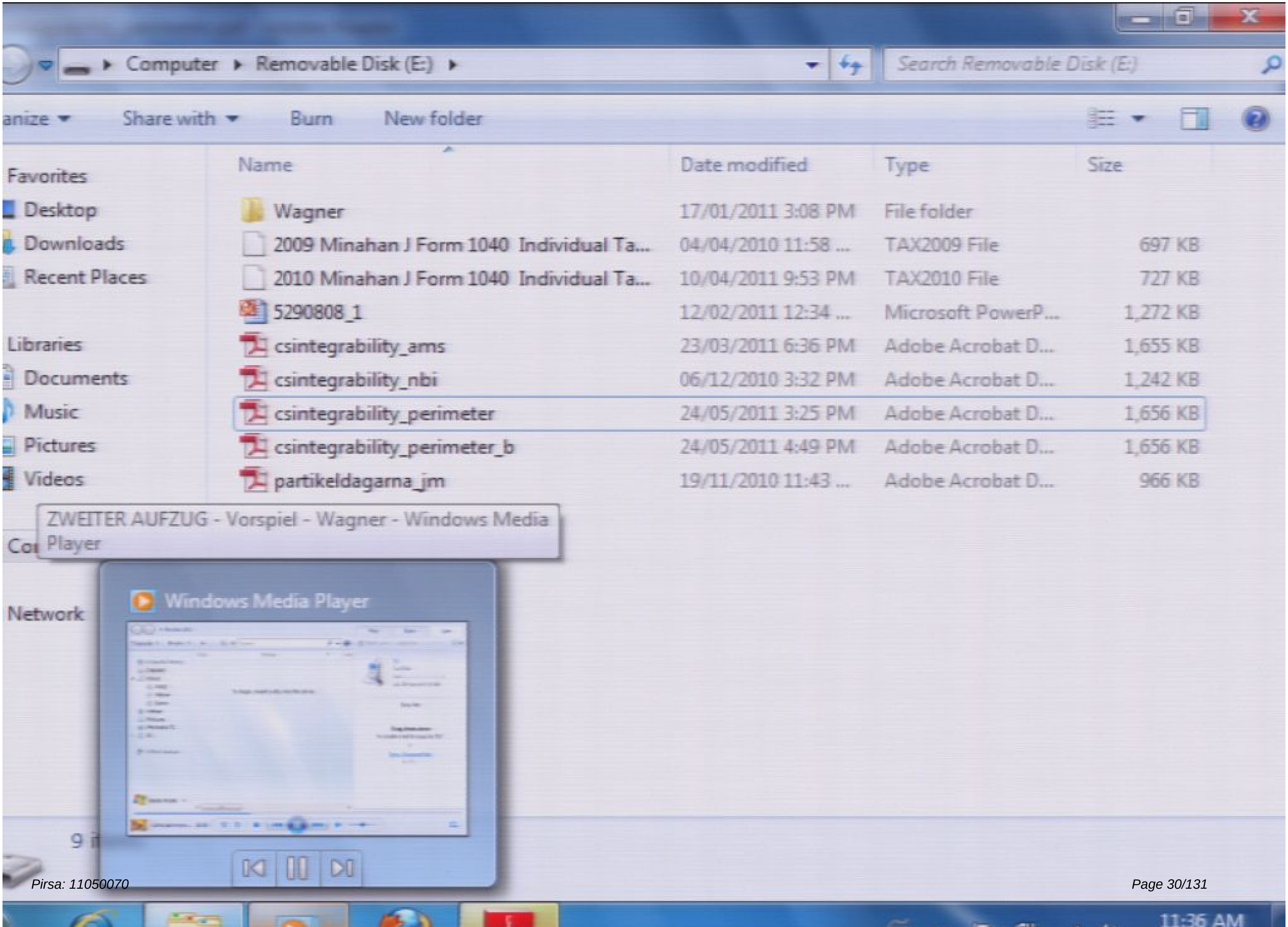
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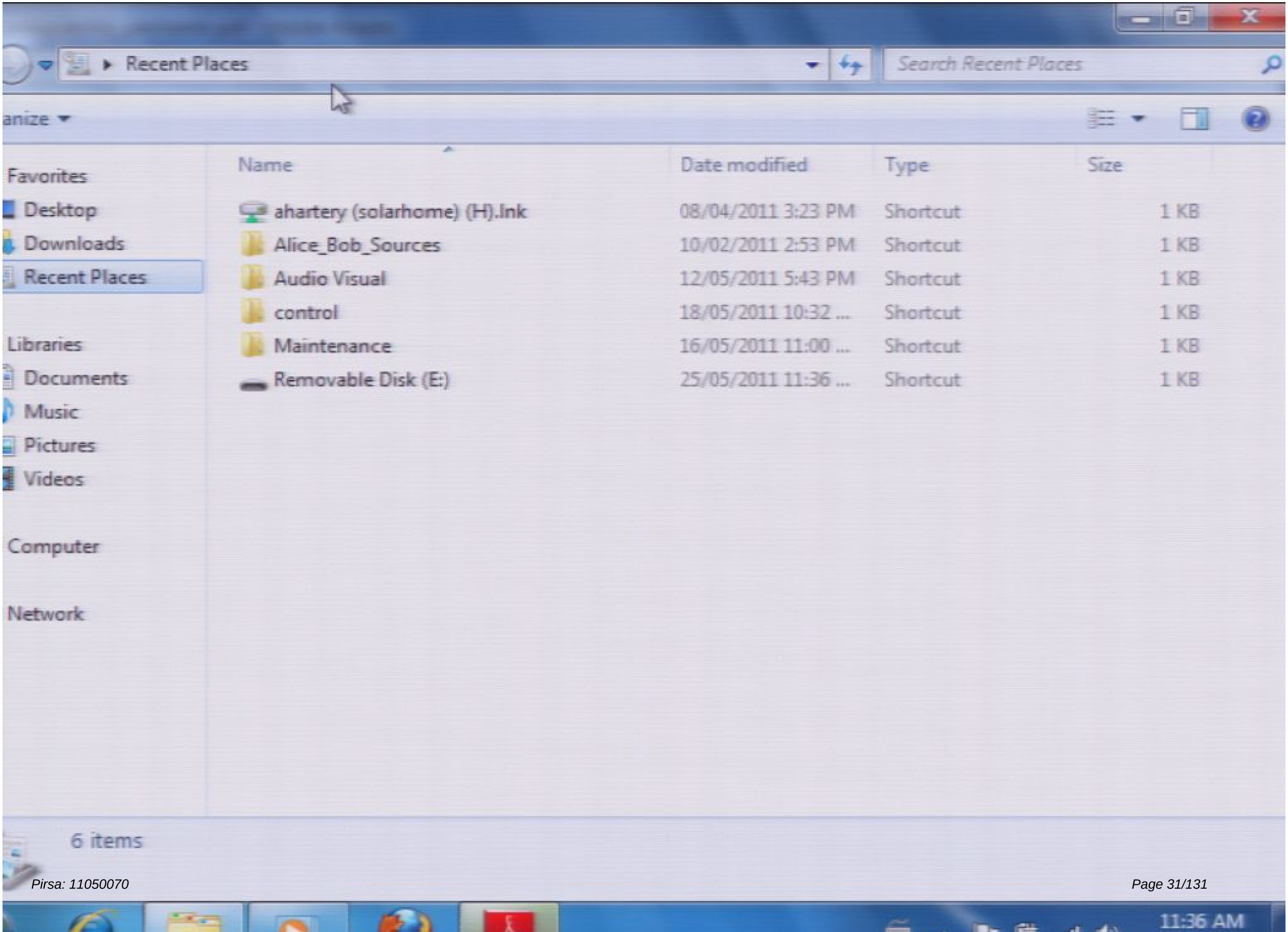
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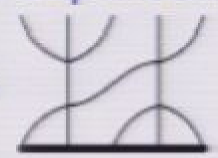

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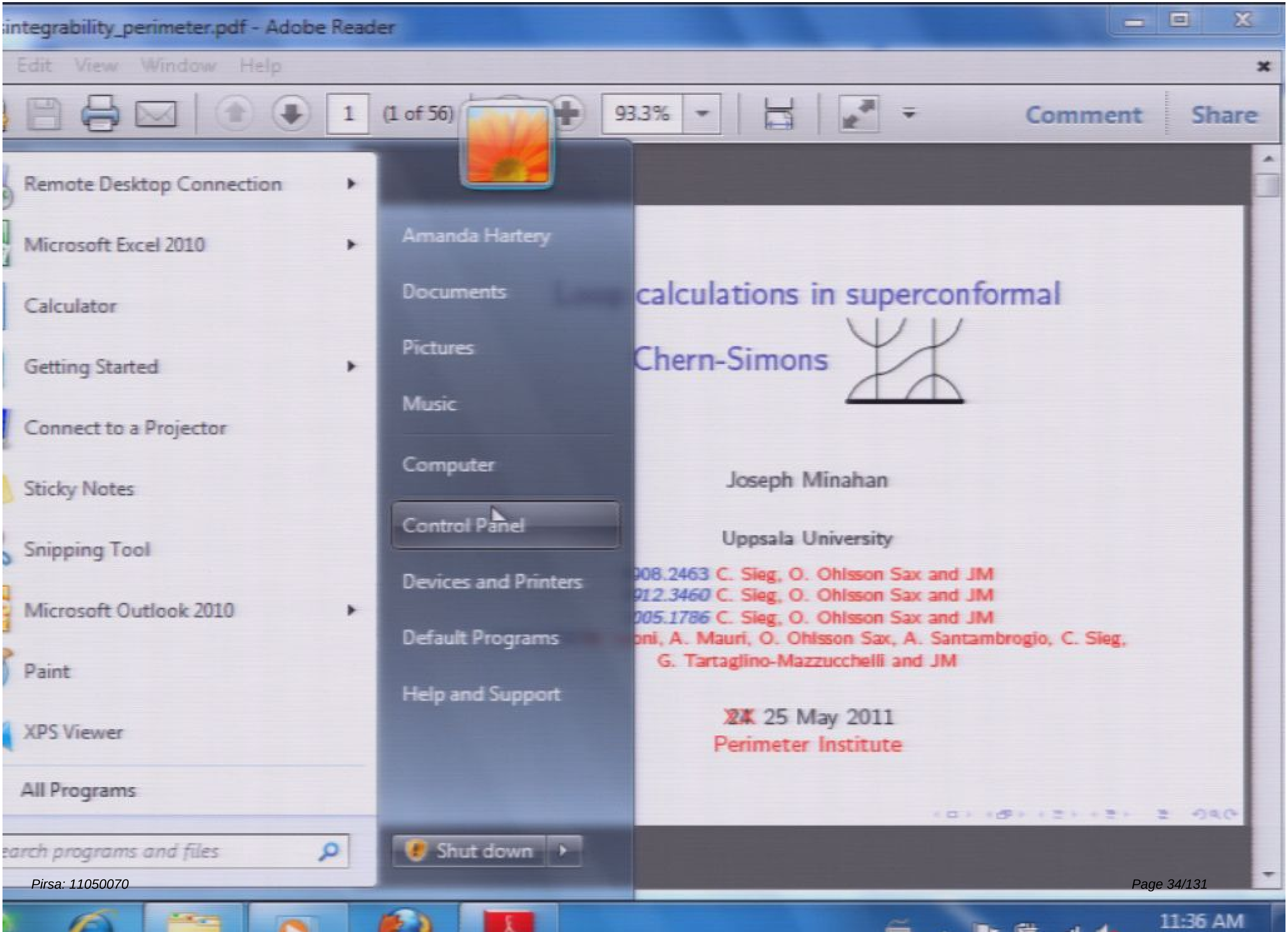
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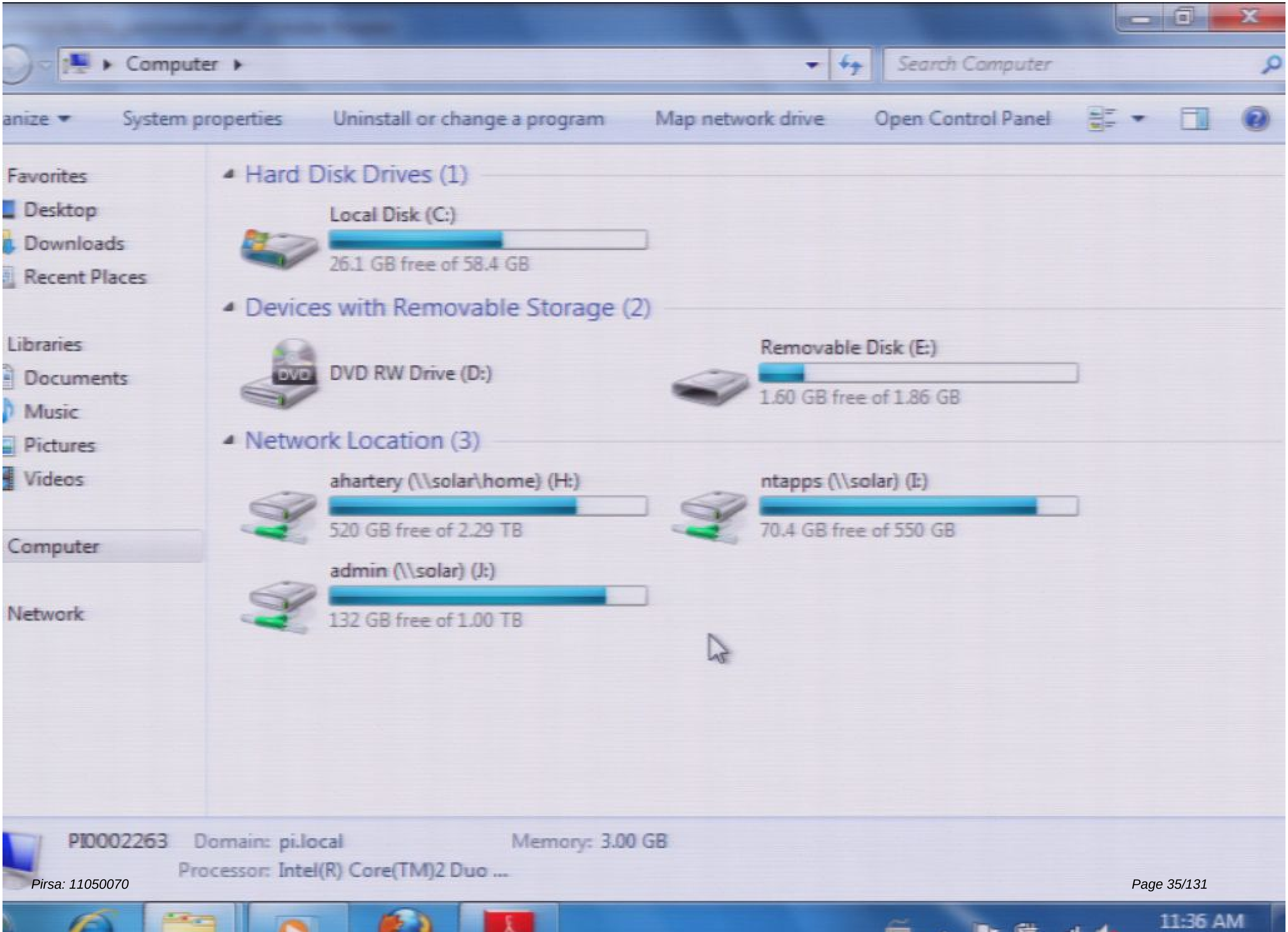
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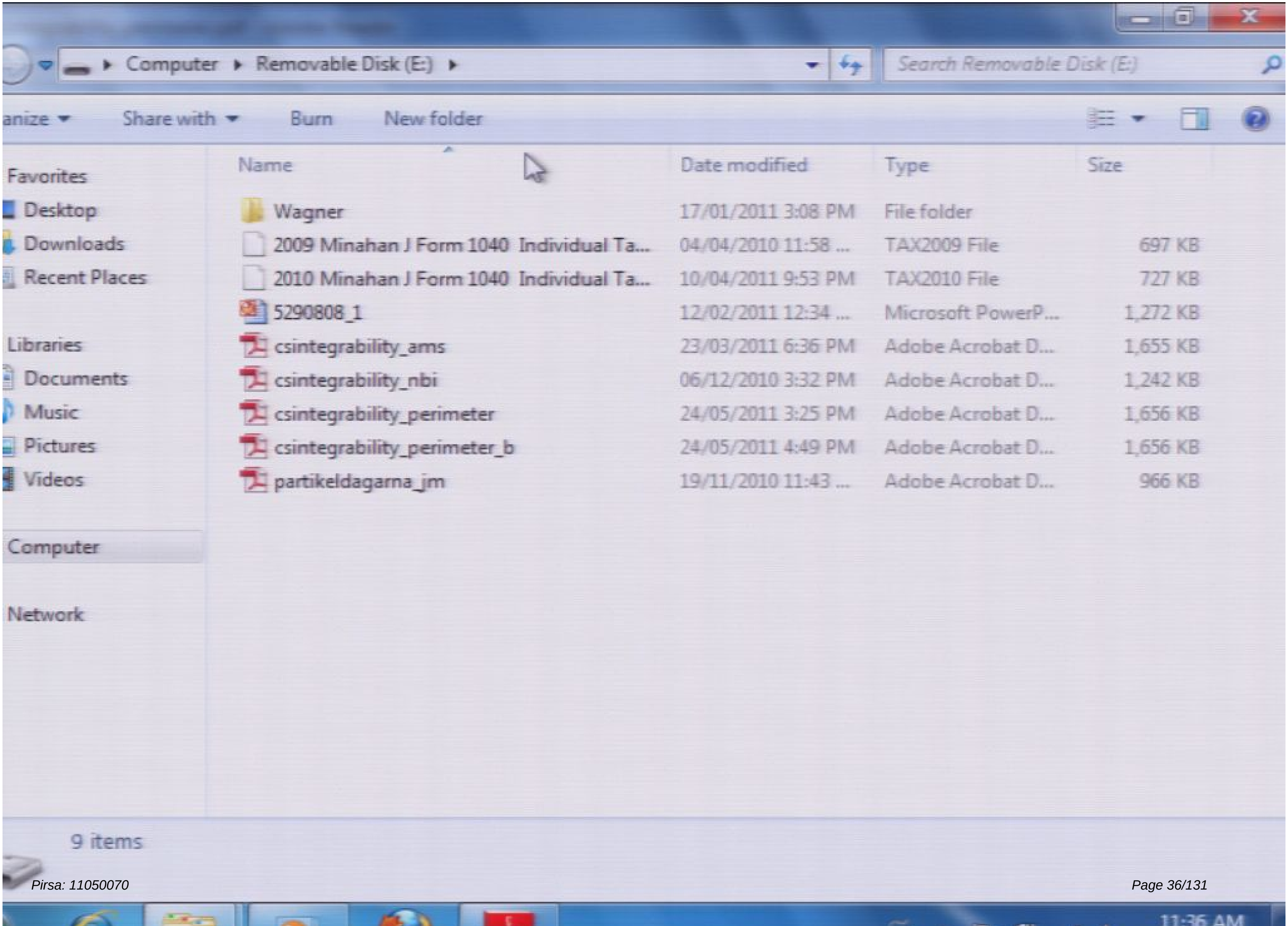
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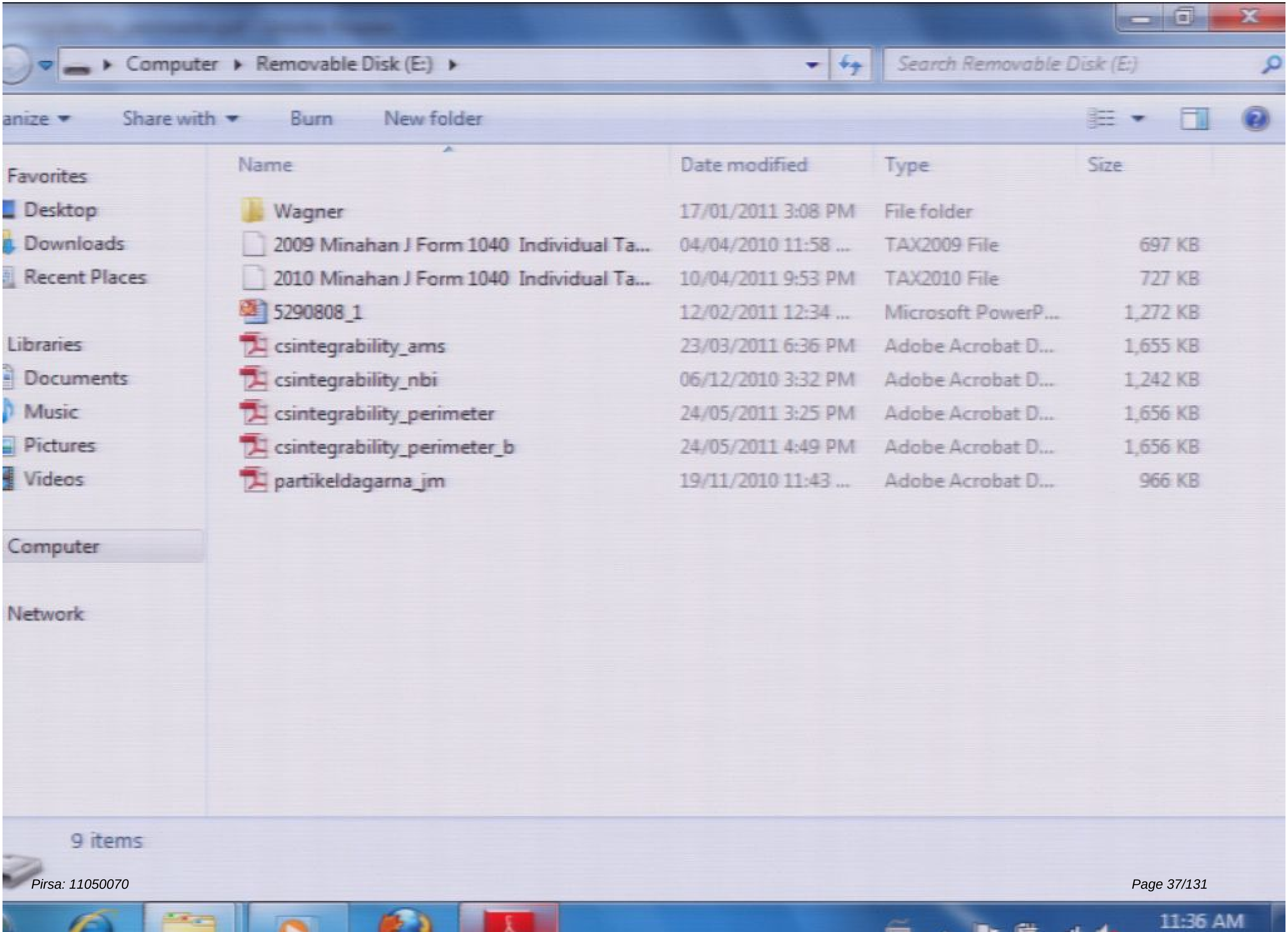
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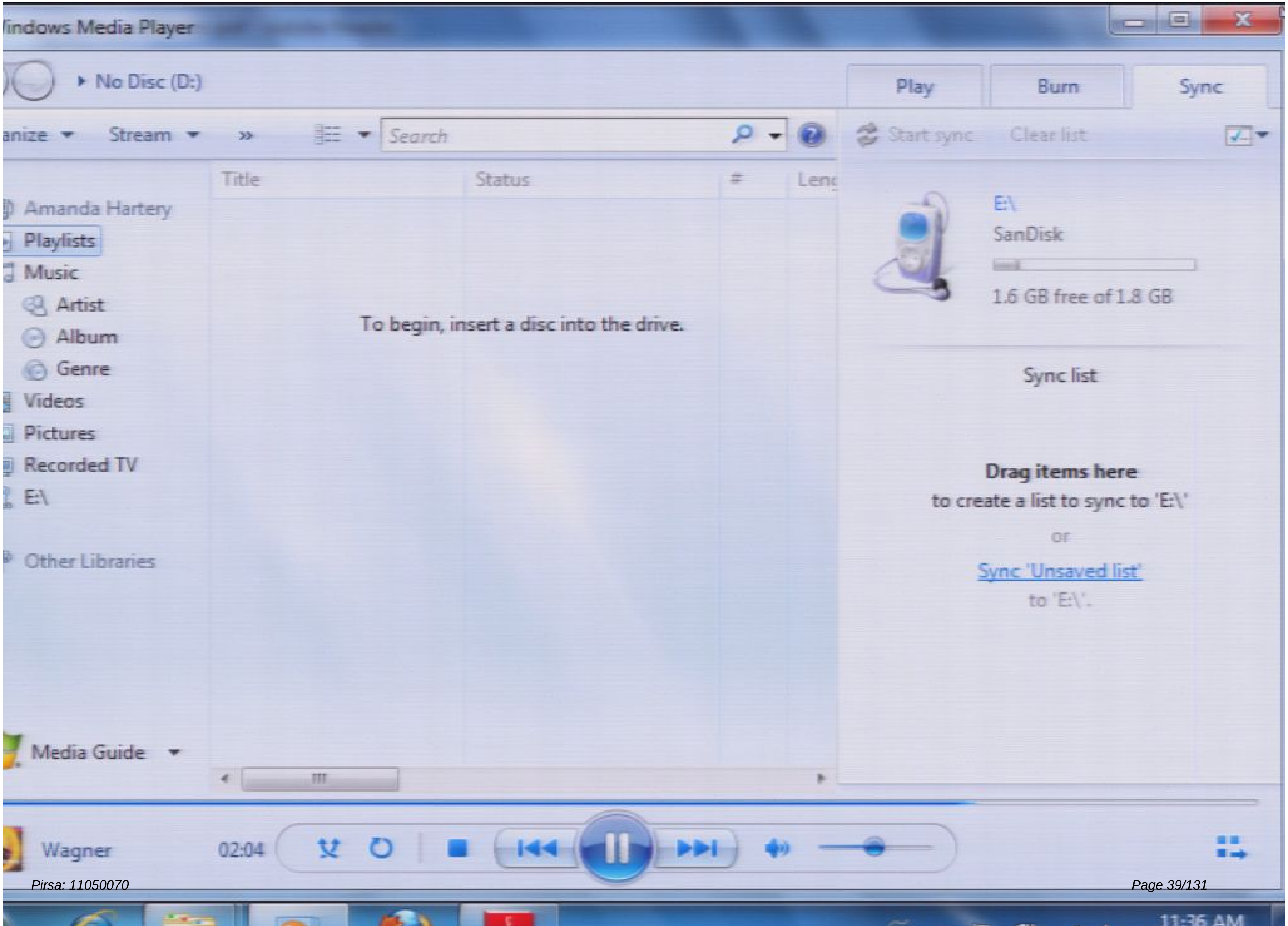

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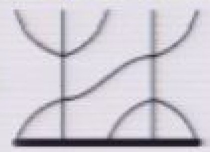

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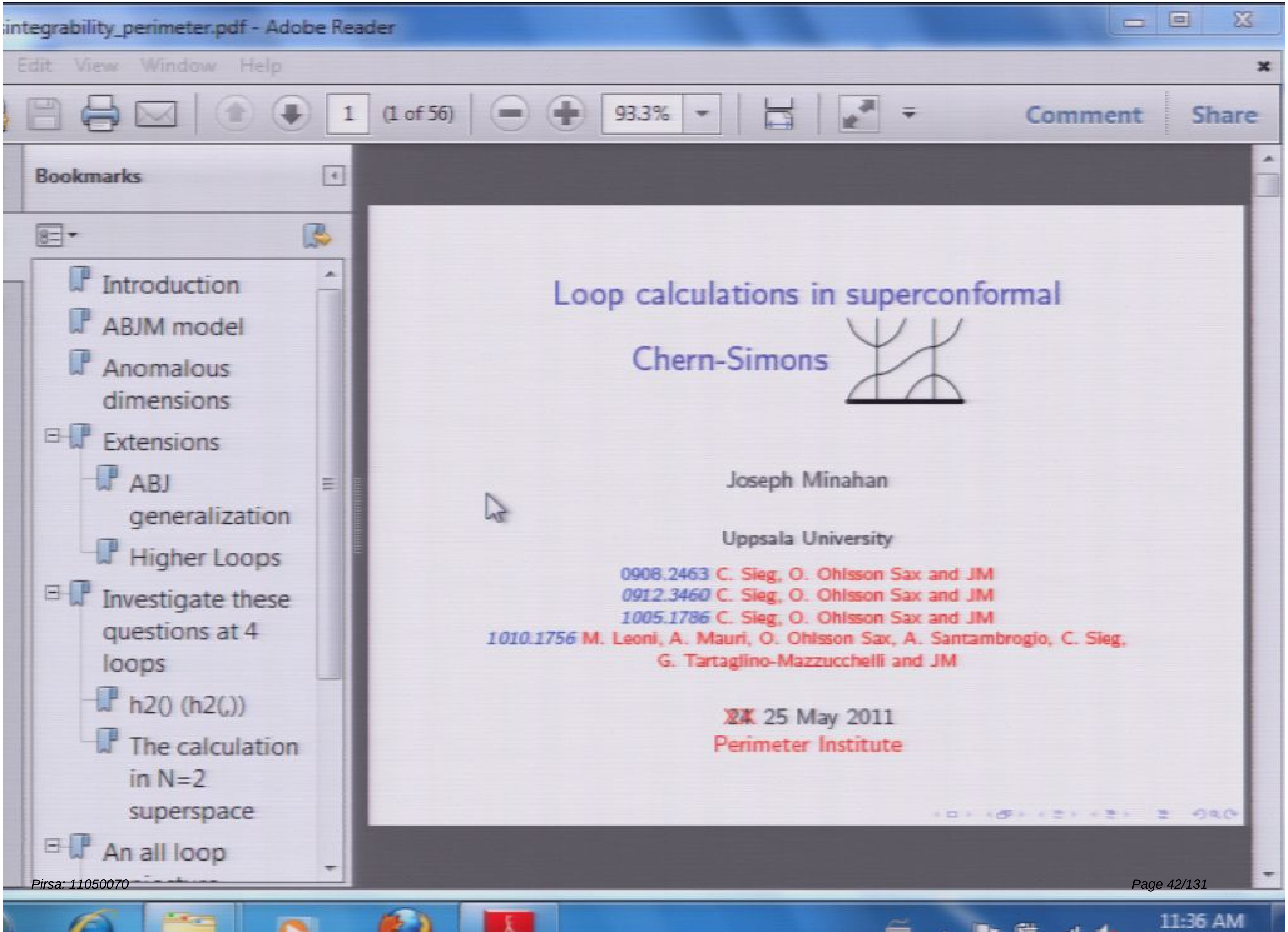
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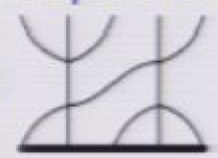

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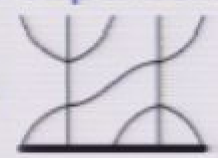

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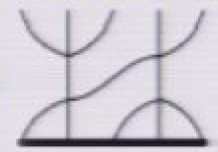

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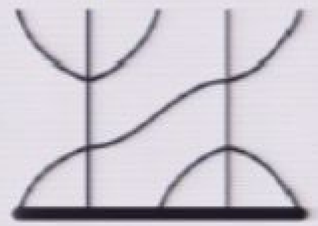

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ıtroduction

- Two "Very Interesting" superconformal field theories with integrable structures in planar limit
  - $\mathcal{N}=4$ Super Yang-Mills
  - ABJM model (Aharony, Bergman, Jafferis and Maldacena): Chern-Simons w/ superconformal matter
- Many similarities but some interesting differences
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ABJM model: Ingredients

- $U(N)\times U(N)$: A_μ, $\hat{A}_\mu$ CS action: levels k, $-k$
- Bifundamental matter fields: $(N,\bar{N})$, $(\bar{N},N)$
  - $SU(4)\simeq SO(6)$ R-symmetry
  - scalars: Y^A, $Y^\dagger_A$; fermions: ψ_A, ψ^A_c $A=1\dots4$

ABJM model: Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{k}{4\pi}\operatorname{tr}\Big[\varepsilon^{\mu\nu\lambda}\Big(A_\mu\partial_\nu A_\lambda+\frac{2}{3}A_\mu A_\nu A_\lambda-\hat{A}_\mu\partial_\nu\hat{A}_\lambda-\frac{2}{3}\hat{A}_\mu\hat{A}_\nu\hat{A}_\lambda\Big)\\
&+D_\mu Y^\dagger_A D^\mu Y^A+i\bar{\psi}^A\not{D}\psi_A\\
&+\frac{1}{12}Y^AY^\dagger_AY^BY^\dagger_BY^CY^\dagger_C+\frac{1}{12}Y^AY^\dagger_BY^BY^\dagger_CY^CY^\dagger_A\\
&-\frac{1}{2}Y^AY^\dagger_AY^BY^\dagger_CY^CY^\dagger_B+\frac{1}{3}Y^AY^\dagger_BY^CY^\dagger_AY^BY^\dagger_C\\
&-\frac{1}{2}Y^\dagger_AY^A\bar{\psi}^B\psi_B+Y^\dagger_AY^B\bar{\psi}^A\psi_B+\frac{1}{2}\bar{\psi}^AY^BY^\dagger_B\psi_A-\bar{\psi}^AY^BY^\dagger_A\psi_B\\
&+\frac{1}{2}\varepsilon^{ABCD}Y^\dagger_A\bar{\psi}_{cB}Y^\dagger_C\psi_D-\frac{1}{2}\varepsilon_{ABCD}Y^A\bar{\psi}^BY^CC\psi^D_c\Big]
\end{aligned}
$$

ABJM model: Features

- $OSp(6|4)$ superconformal symmetry (24 susys). Couplings are not renormalized.
- Gauge inv. ops are traces of alternating $(N, \bar{N})$ and $(\bar{N}, N)$ terms.

$$\mathcal{O}^{p_1 p_2 \cdots p_L}_{q_1 q_2 \cdots q_L} = \frac{1}{N^L} tr[\chi^{p_1} \bar{\chi}_{q_1} \chi^{p_2} \bar{\chi}_{q_2} \cdots \chi^{p_L} \bar{\chi}_{q_L}]$$

$\chi^p = D^k Y^A, D^k \psi_A \qquad \bar{\chi}_q = D^k Y^\dagger_A, D^k \psi^A_c$

- Chiral primary operators: $tr[(Y^1 Y^\dagger_4)^L] \qquad \Delta - J = 0$
  Y^1 and $Y^\dagger_4$ each have $J = \frac{1}{2}$, J is one of the three $SU(4)$ charges.
  In 3d, bare dimension is also $\frac{1}{2}$.
- Effective coupling $\sqrt{\frac{4\pi}{k}} \qquad$ 't Hooft: $\lambda = \frac{N}{k}$
  Planar limit requires large $k \Rightarrow k$ effectively continuous
- Parity invariance. Under parity CS and $\bar{\psi}\psi$ terms change sign. Including $A_\mu \leftrightarrow \hat{A}_\mu$ and charge conj. keeps terms invariant.
- Perturbative expansions are even in λ
- String dual on $AdS_4 \times CP_3$ (for large k) is classically integrable (Arutyunov and Frolov; Stefanski)

nomalous dimensions

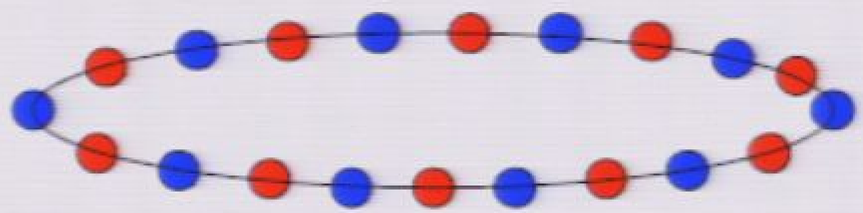
- Two-point functions:

$$\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = |x|^{-2\Delta},\ \ \Delta = \Delta_0 + \gamma$$
$$\mathcal{O}_{\rm ren} = Z(\Lambda)\mathcal{O}_{\rm bare} \Rightarrow \gamma = Z^{-1}\frac{\partial}{\partial \ln\Lambda}Z$$

- Mixing: $\mathcal{O}^I_{\rm ren} = Z^{IJ}(\Lambda)\mathcal{O}^J_{\rm bare} \Rightarrow \Gamma^{IJ} = (Z^{-1}\frac{\partial}{\partial \ln\Lambda}Z)^{IJ}$

$$\mathcal{O}^{p_1p_2\cdots p_L}_{q_1q_2\cdots q_L} = \frac{1}{N^L}tr[\chi^{p_1}\bar{\chi}_{q_1}\chi^{p_2}\bar{\chi}_{q_2}\cdots\chi^{p_L}\bar{\chi}_{q_L}] \Rightarrow \mathcal{V}_1\otimes\bar{\mathcal{V}}_1\otimes\mathcal{V}_2\otimes\bar{\mathcal{V}}_2\cdots\otimes\mathcal{V}_L\otimes\bar{\mathcal{V}}_L$$

$\mathcal{V}$ and $\bar{\mathcal{V}}$ are fundamental and anti-fundamental reps. of $OSP(6|4)$

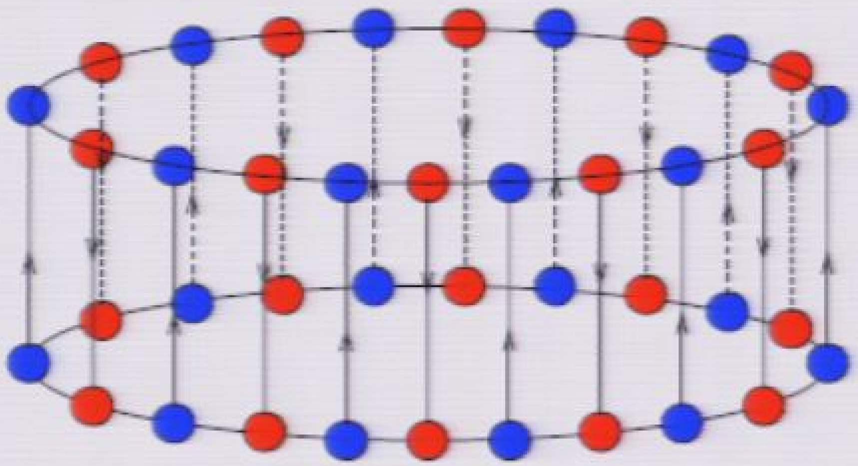


Spin chain with sites alternating between χ and $\bar{\chi}$

$$\Gamma = \mathcal{D} - \Delta_0 \equiv H$$

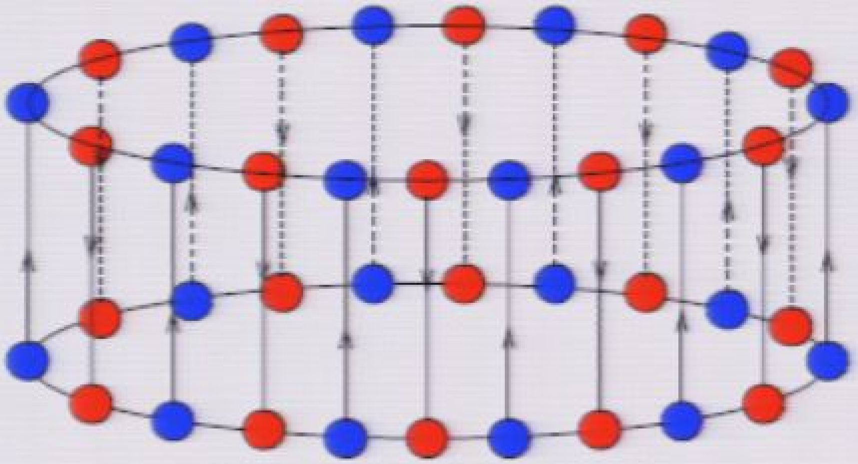
$$H : \mathcal{V}_1\otimes\bar{\mathcal{V}}_1\otimes\mathcal{V}_2\otimes\bar{\mathcal{V}}_2\cdots\otimes\mathcal{V}_L\otimes\bar{\mathcal{V}}_L \rightarrow \mathcal{V}_1\otimes\bar{\mathcal{V}}_1\otimes\mathcal{V}_2\otimes\bar{\mathcal{V}}_2\cdots\otimes\mathcal{V}_{L'}\otimes\bar{\mathcal{V}}_{L'}$$

Tree level correlator



$$[H^{(0)}, \mathcal{O}] = 0.$$

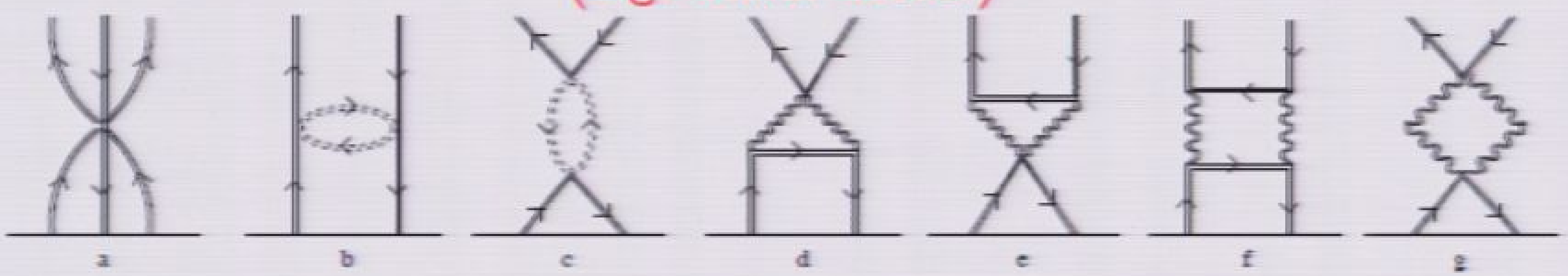
Tree level correlator



$$[H^{(0)}, \mathcal{O}] = 0.$$

Two loop corrections

(e.g. scalar sector)



Operator mixing via two loop planar graphs

2 Loops : $SU(2) \times SU(2)$ closed sector

- Start with groundstate (chiral primary): $[H, \mathrm{tr}[(Y^1 Y_4^\dagger)^L]] = 0$
- Add impurities (magnons): $Y^1 \to Y^2$ (odd), $Y_4^\dagger \to Y_3^\dagger$ (even)
- $H^{(2)} =$
  $\lambda^2 \sum_{\ell=1}^{2L} (1 - P_{\ell,\ell+2})$ K. Zarembo and JM; Gaiotto, Giombi and Yin; Bak and Rey
- 2 independent spin chains: Heisenberg ferromagnets

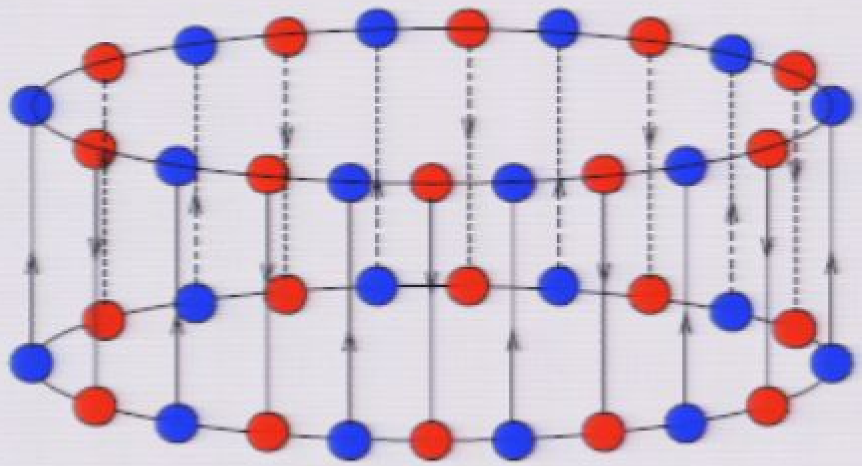
$$\text{Bethe eqs:} \quad \left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1,k\neq j}^{M_u} \frac{u_j - u_k + i}{u_j - u_k - i}$$

$$\left(\frac{v_j + i/2}{v_j - i/2}\right)^L = \prod_{k=1,k\neq j}^{M_v} \frac{v_j - v_k + i}{v_j - v_k - i}$$

$$e^{ip_j} = \frac{u_j + i/2}{u_j - i/2}; \quad e^{i\tilde{p}_j} = \frac{v_j + i/2}{v_j - i/2}; \qquad \text{Trace cond}: \prod_{j=1}^{M_u} e^{ip_j} \prod_{j=1}^{M_v} e^{i\tilde{p}_j} = 1,$$

$$\varepsilon(p) = 4\lambda^2 \sin^2 \frac{p}{2} \qquad \gamma = \sum_{j=1}^{M_u} \varepsilon(p_j) + \sum_{j=1}^{M_v} \varepsilon(\tilde{p}_j)$$

Tree level correlator



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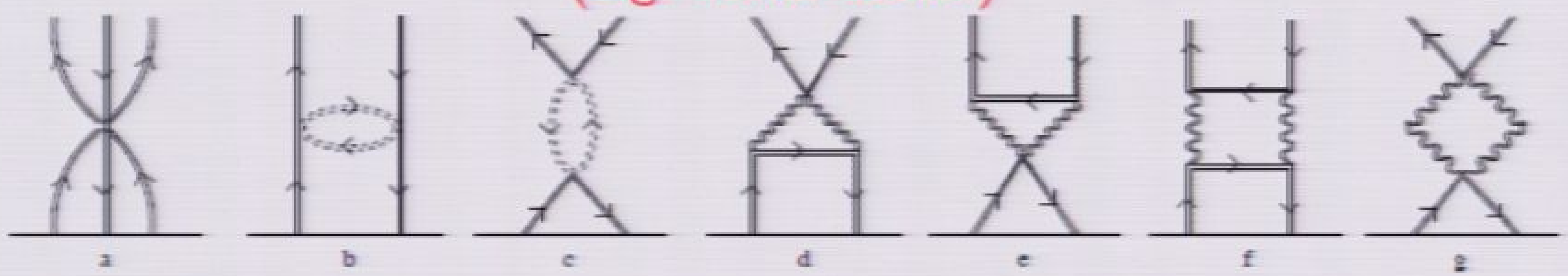
Two loop corrections

(e.g. scalar sector)



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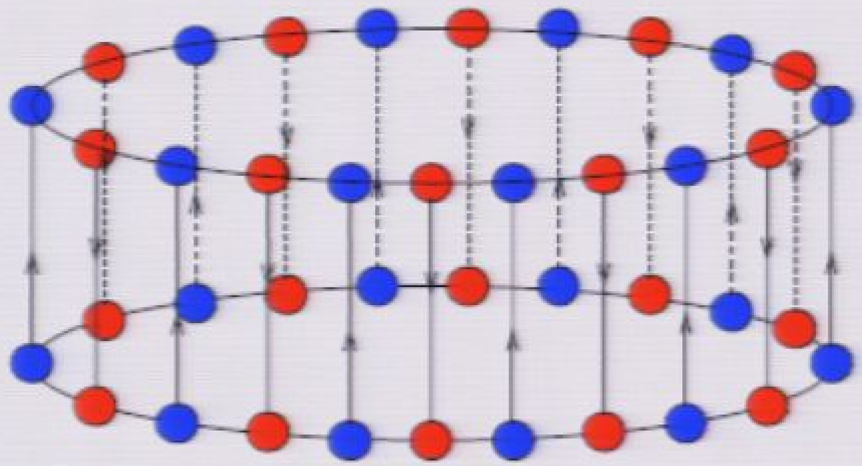

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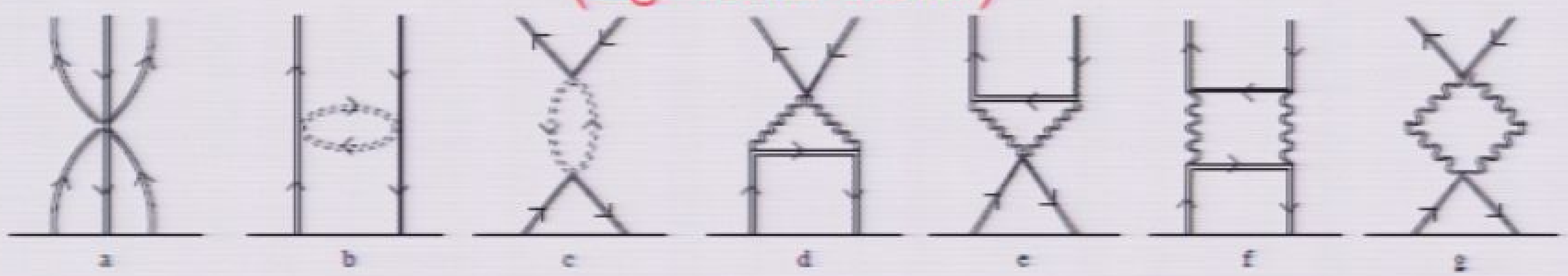

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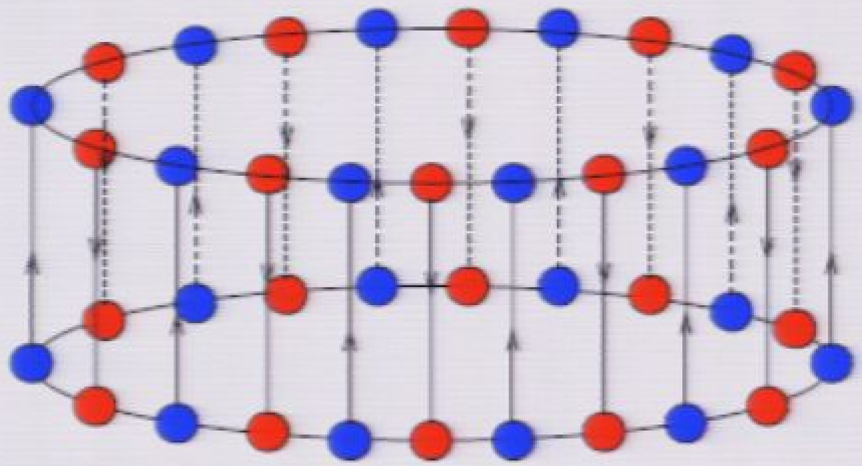

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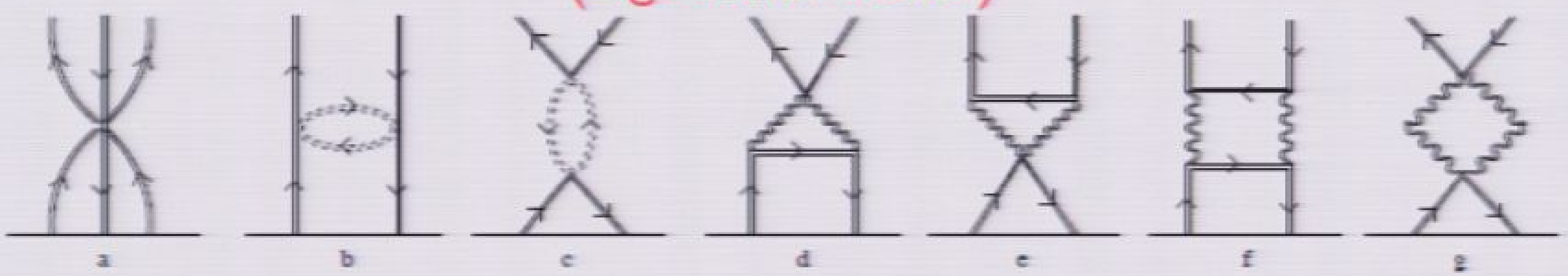

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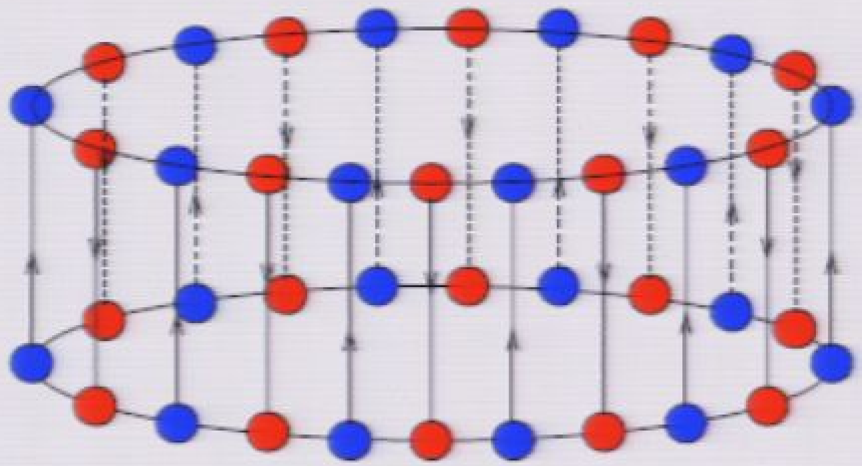

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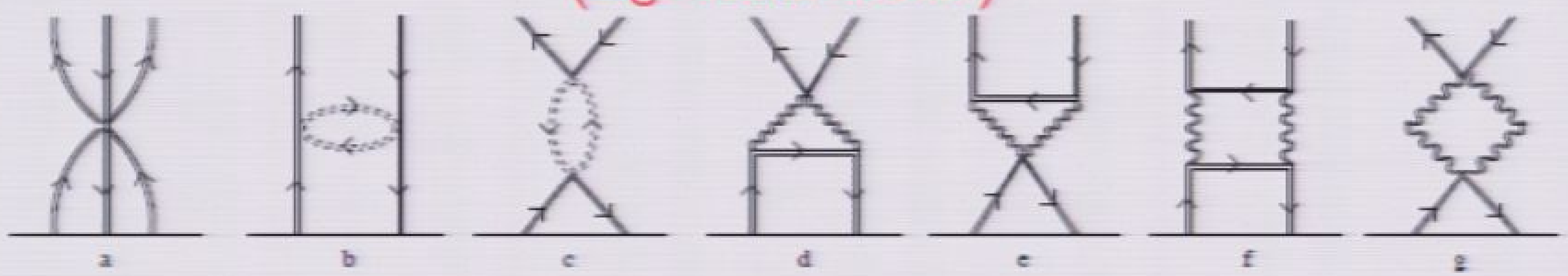

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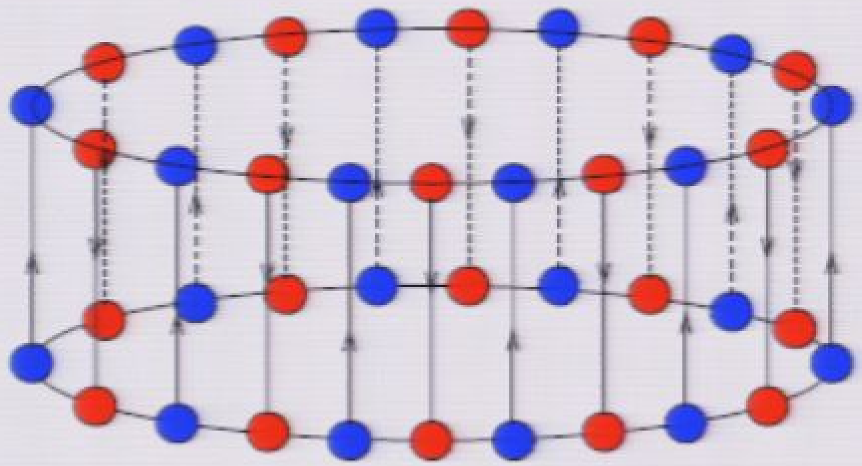

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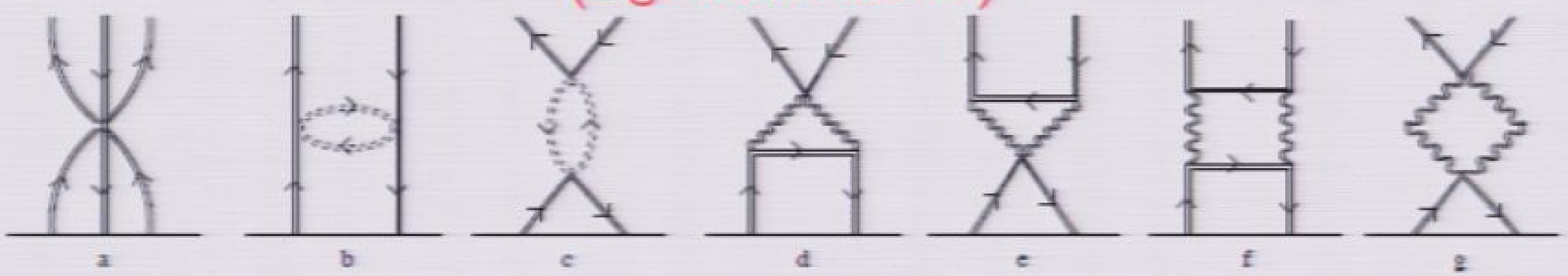

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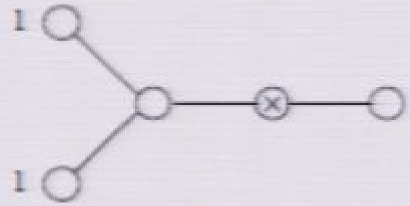
Full $OSp(6|4)$ sector



$$\left(\frac{u_j + i/2}{u_j - i/2}\right)^L = \prod_{k=1,k\neq j}^{M_u} \frac{u_j - u_k + i}{u_j - u_k - i} \prod_{k=1}^{M_r} \frac{u_j - r_k - i/2}{u_j - r_k + i/2}$$

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ABJ (Aharony, Bergman, Jafferis): Extend to $U(N) \times U(M)$

- Still has $\mathcal{N} = 6$ supersymmetry
- Two 't Hooft parameters: $\lambda = \frac{N}{k}$, $\hat{\lambda} = \frac{M}{k}$
- Background $B \sim \frac{N-M}{k} J$ for string dual: no longer clear if string theory is integrable, B effectively adds a θ term to sigma model.
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Extension to higher loops

- n loops: interactions reach over n sites
- Odd and even magnons each transform in $SU(2|2)$ short rep.
  $\varepsilon(p) = \sqrt{\left(\frac{1}{2}\right)^2 + 4h^2(\lambda)\sin^2\frac{p}{2}} - \frac{1}{2}$ Beisert
- A proposed set of Bethe equations depending on $h(\lambda)$ Gromov and Vieira
  - $h^2(\lambda)$ is not determined by integrability
  - $h^2(\lambda) \approx \lambda^2$, $\lambda << 1$ 2 loop perturbation theory;
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$h^2(\lambda, \hat{\lambda})$ to 4 loop order: $\bar{\lambda} = \sqrt{\lambda\hat{\lambda}}$, $\sigma = \frac{\lambda - \hat{\lambda}}{\bar{\lambda}}$

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- H determined by dispersion relation (in $SU(2) \times SU(2)$ sector)
- Convenient to use antisymmetric projectors (Flavor Structures):
  - $\{a_1, a_2, \ldots, a_m\} = \sum_{i=0}^{L-1} P_{2i+a_1,\, 2i+a_1+2} P_{2i+a_2,\, 2i+a_2+2} \ldots P_{2i+a_m,\, 2i+a_m+2}$
  - $\chi(a) = \{a\} - \{\}$, $\chi(a, b) = \{a, b\} - \{a\} - \{b\} + \{\}$
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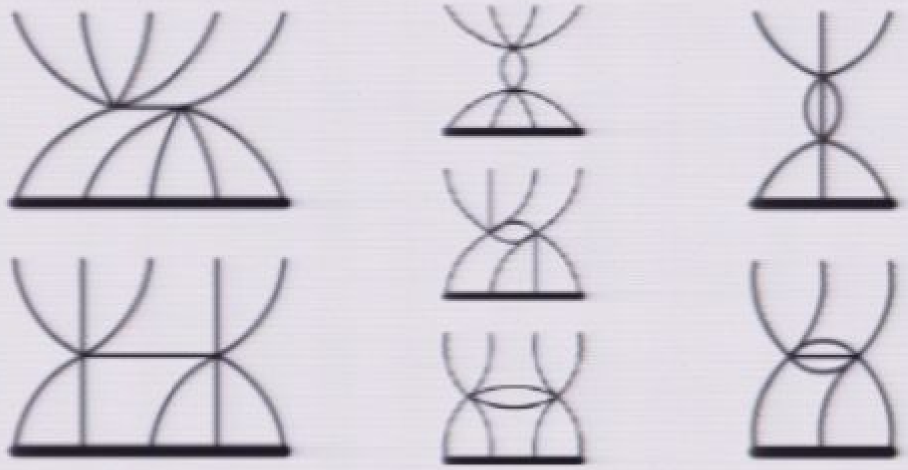
Use flavor structures:

Consider all 4-loop graphs that lead to $\chi(1)$ and $\chi(2)$ flavor structures to find $h_4(\sigma)$.

4-loop Diagrams (two scalar vertices)



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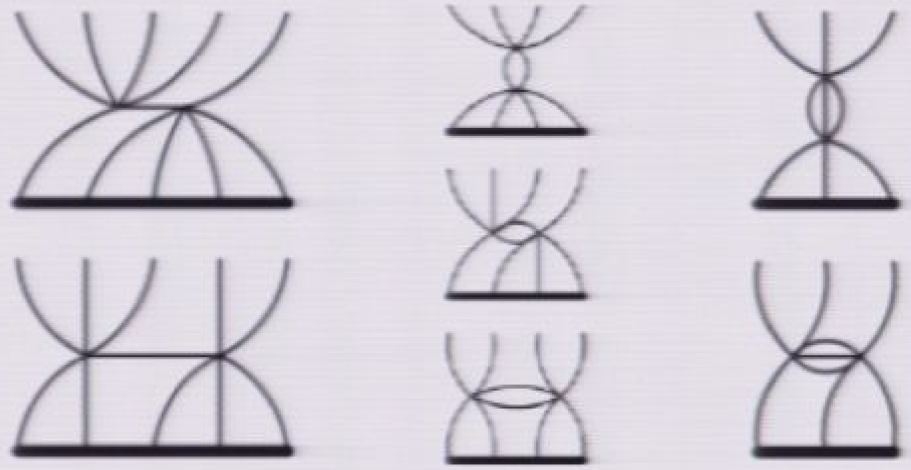
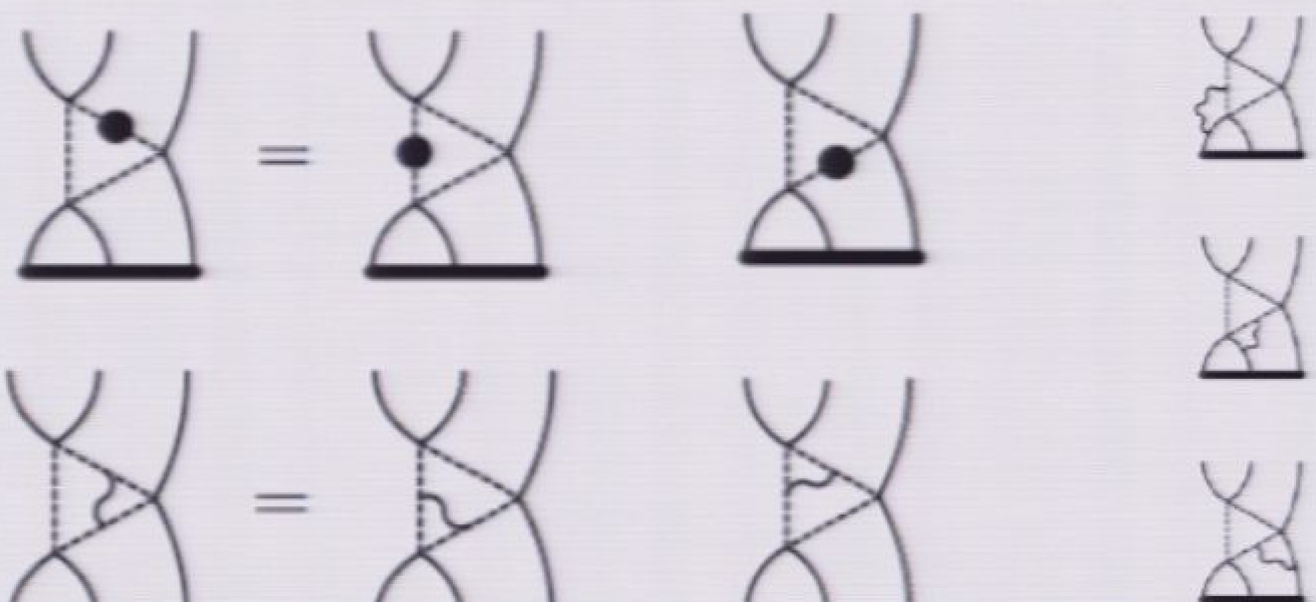
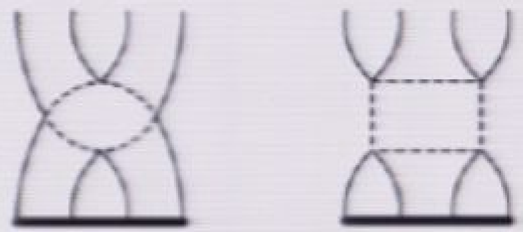
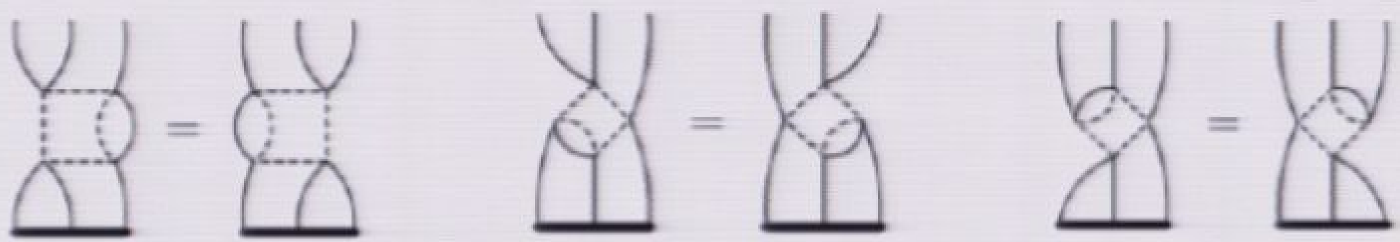


4-loop Diagrams (Triangles: 3 fermion vertices)



4-loop Diagrams (4 fermion vertices (no crossed))





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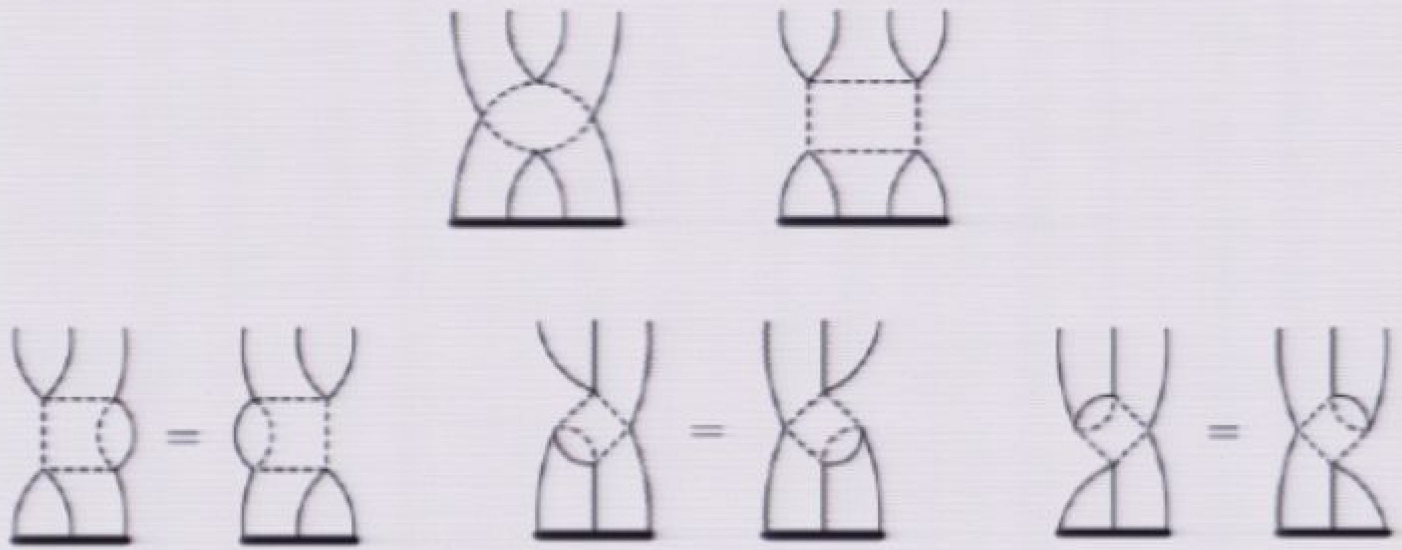
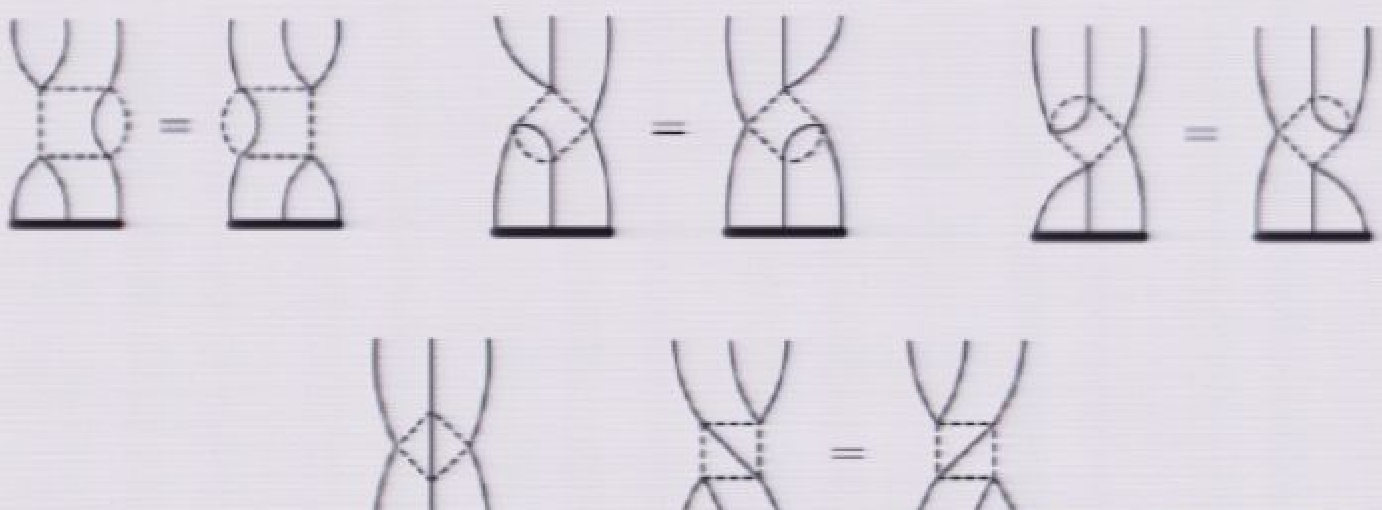
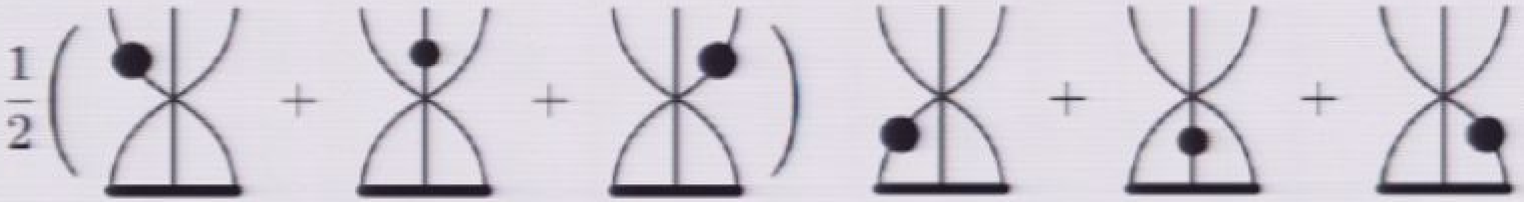
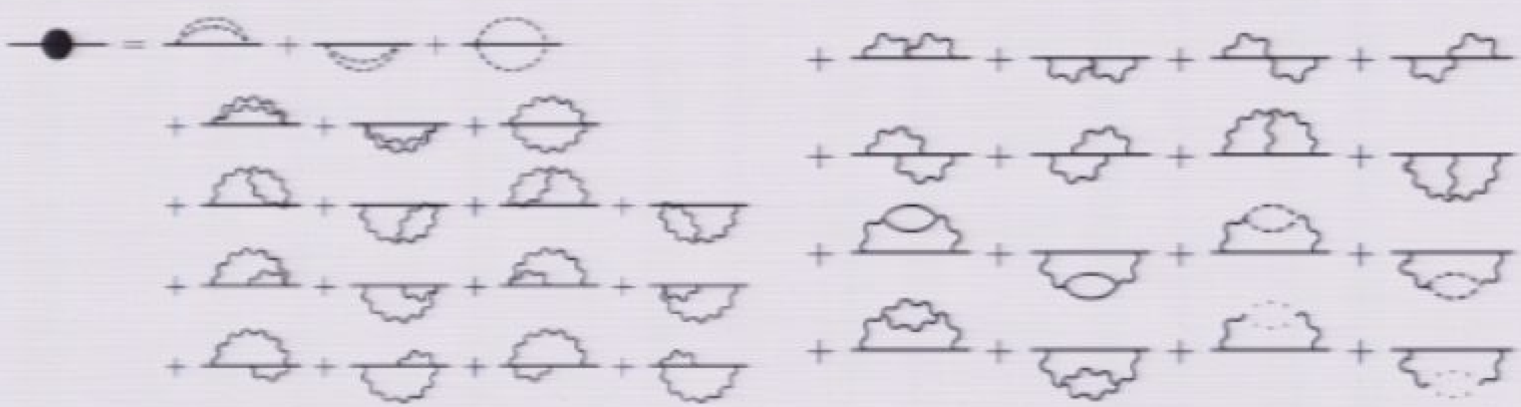


4-loop Diagrams (4 fermion vertices (2 crossed))



4-loop Diagrams (scalar vert. w/ self energies)





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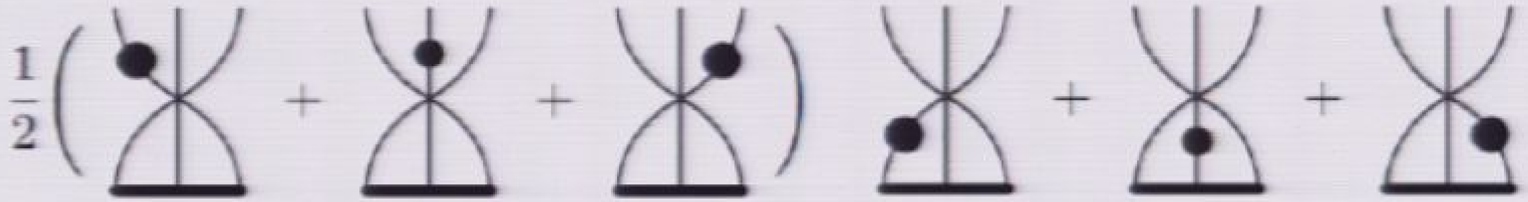
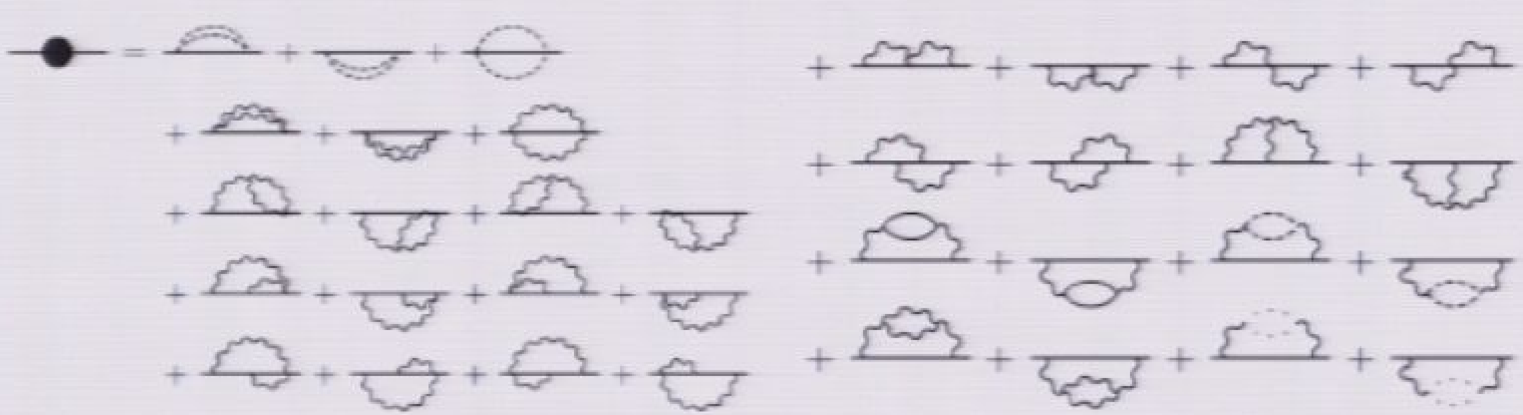
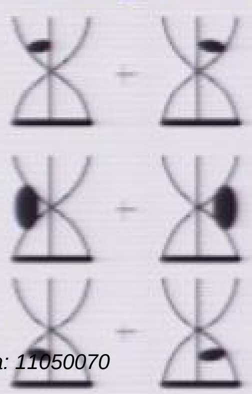
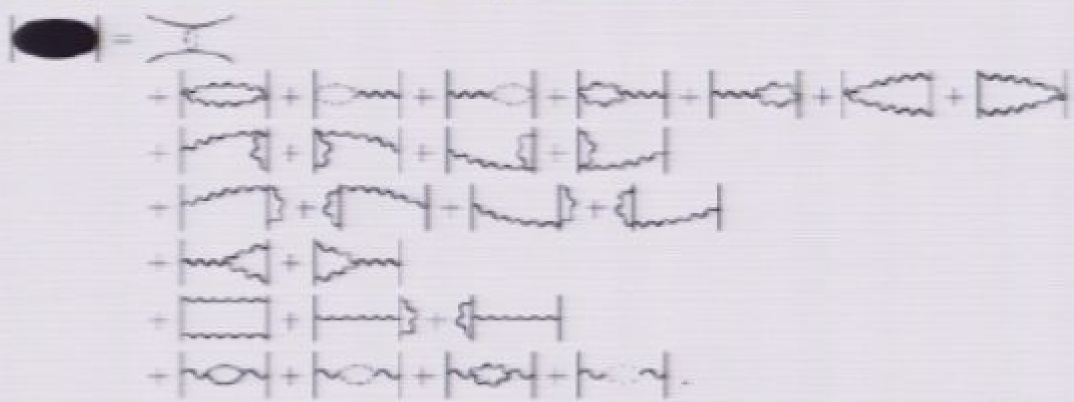


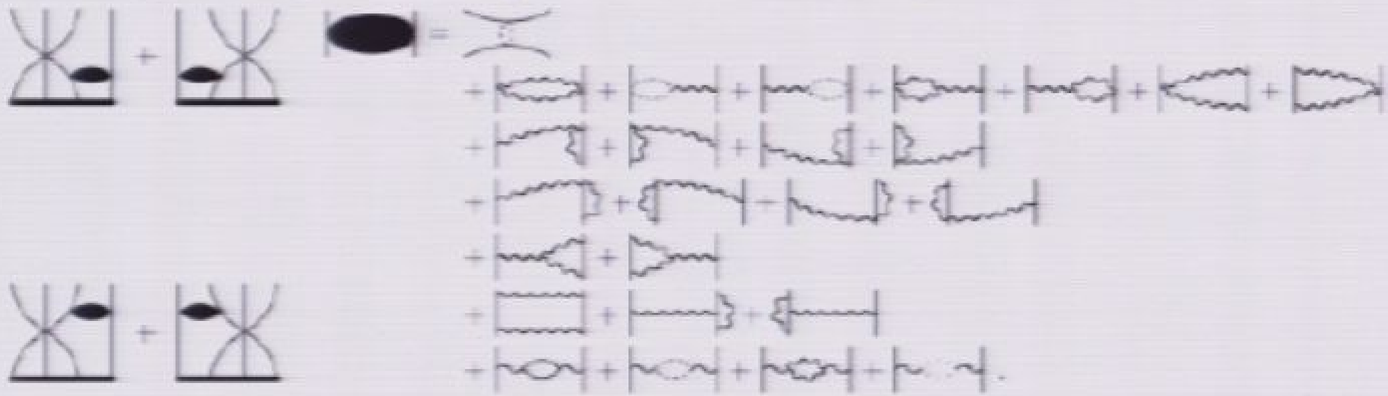


4-loop Diagrams (scalar vert. w/ blob)





4-loop Diagrams (scalar vert. w/ blob attached to neighbor)



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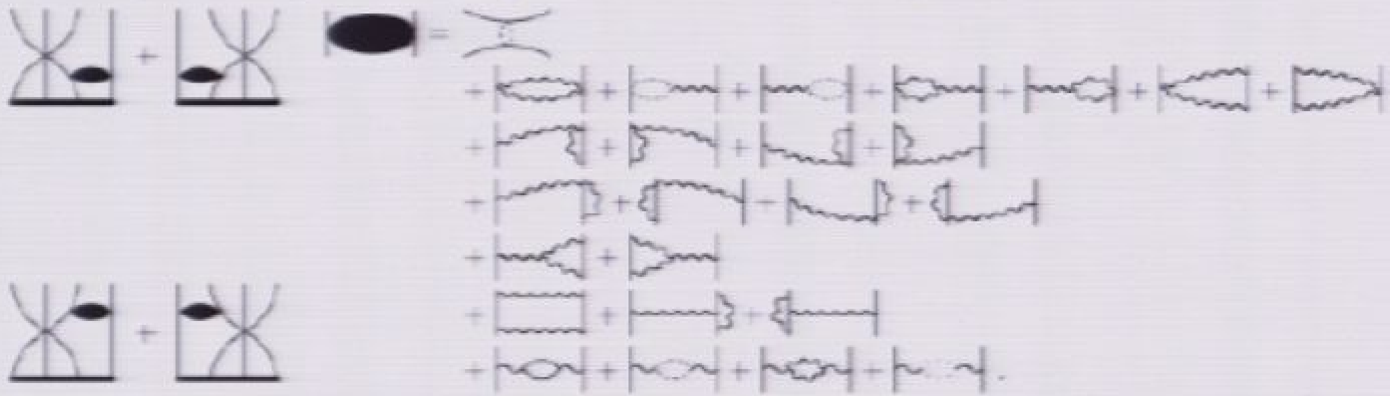
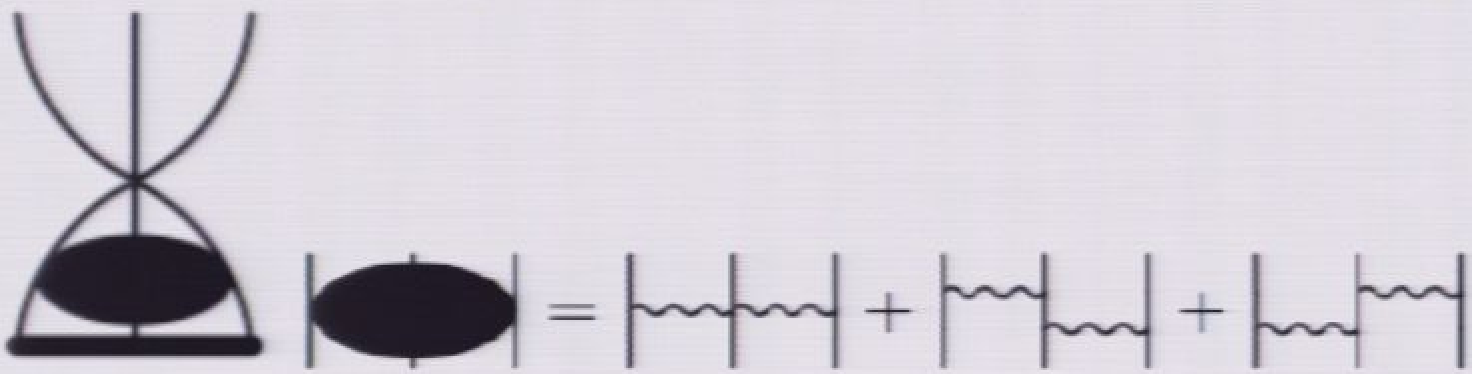


4-loop Diagrams (scalar vert. w/ double wide blob)



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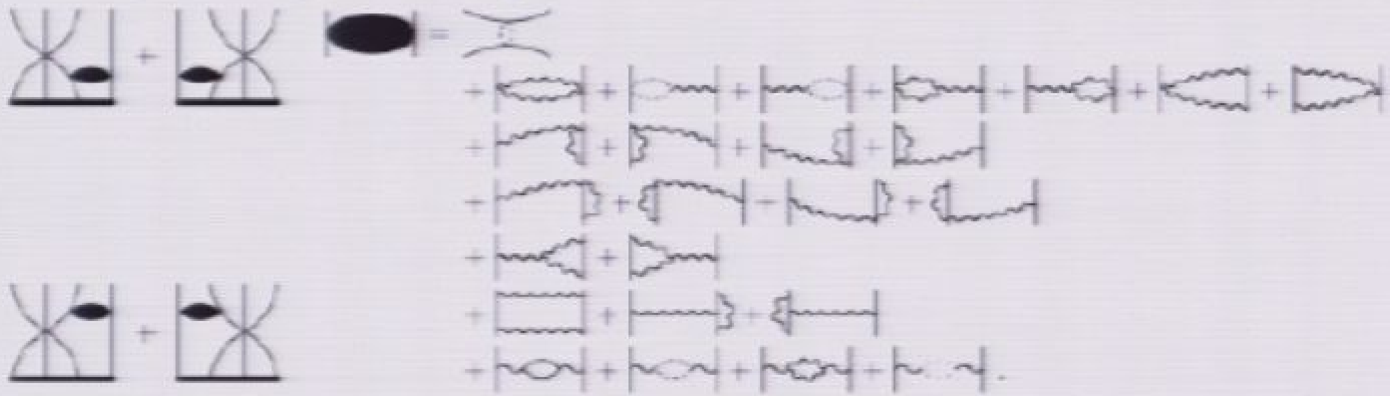

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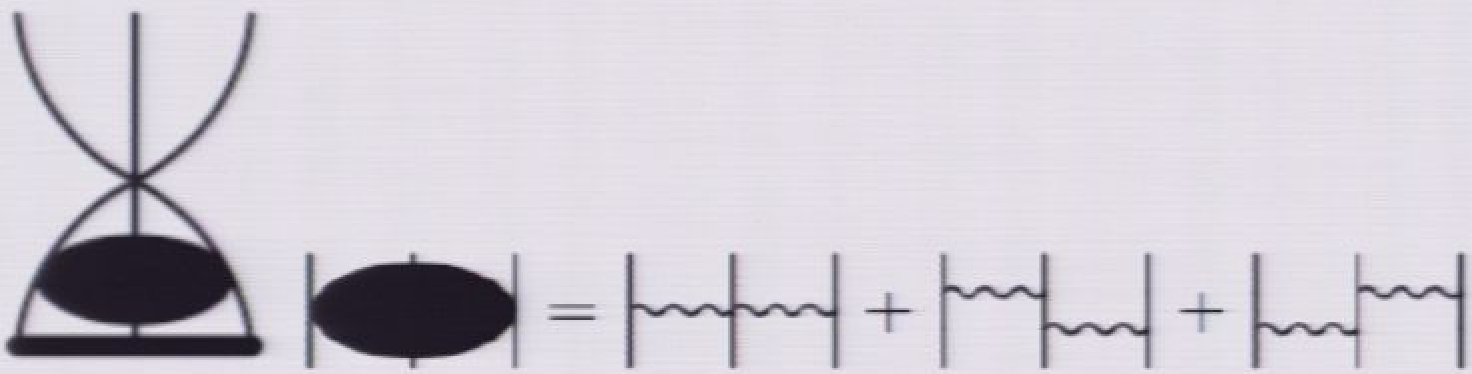

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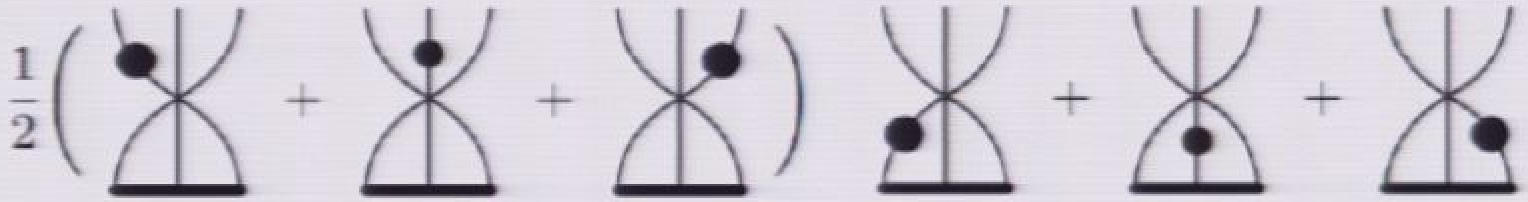

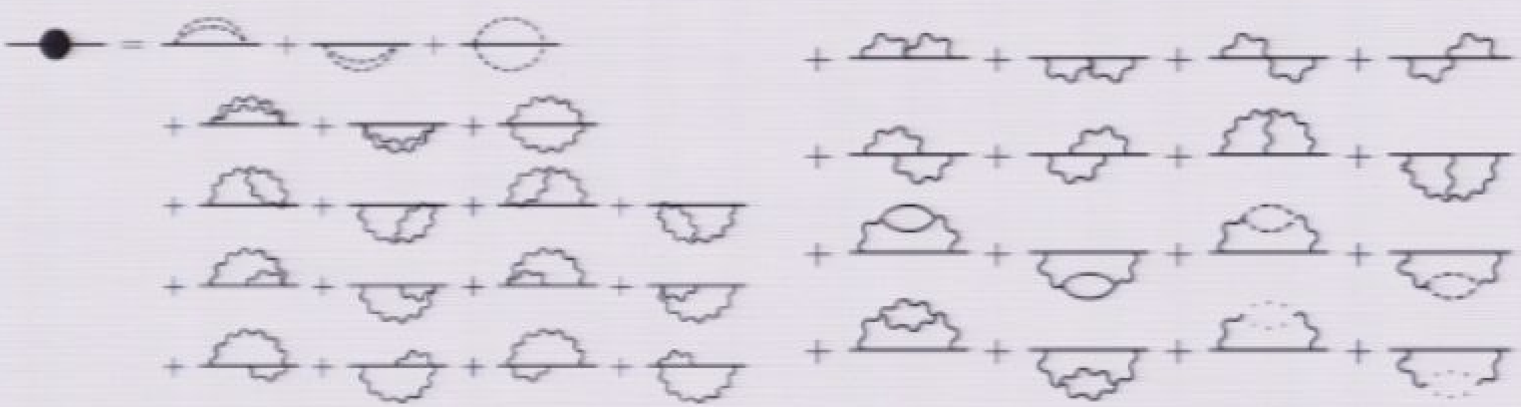

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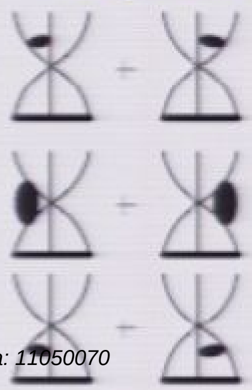

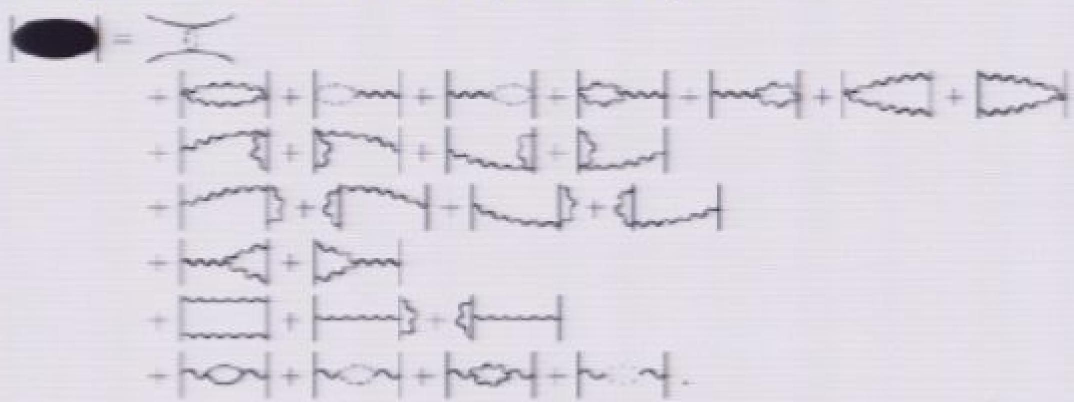

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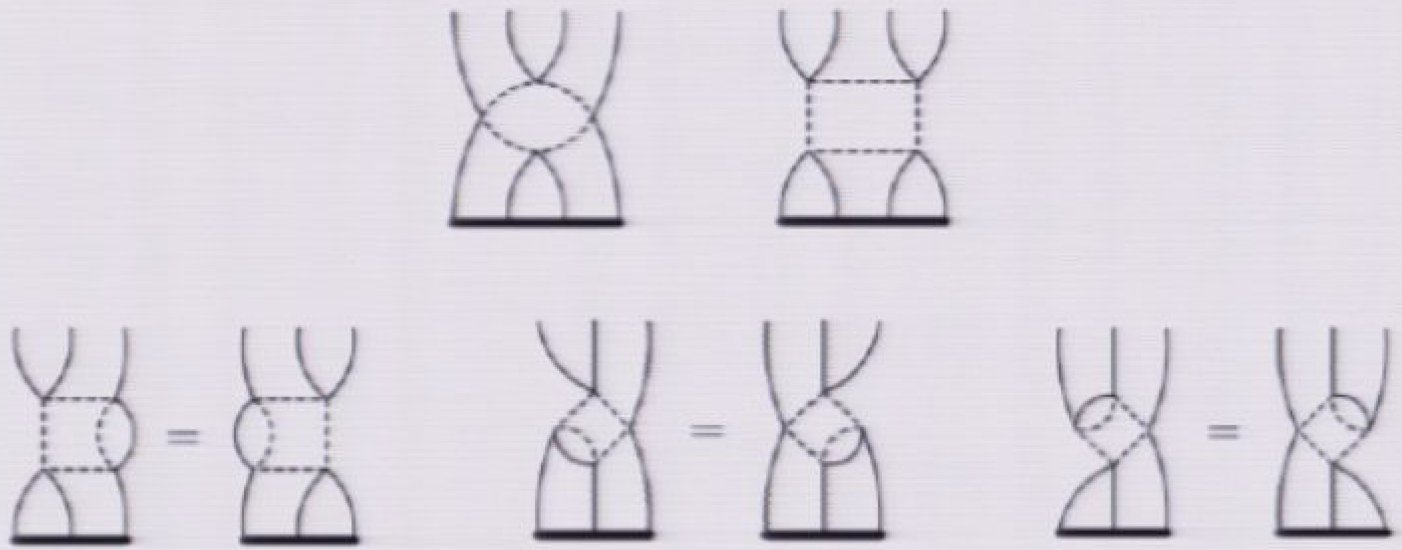
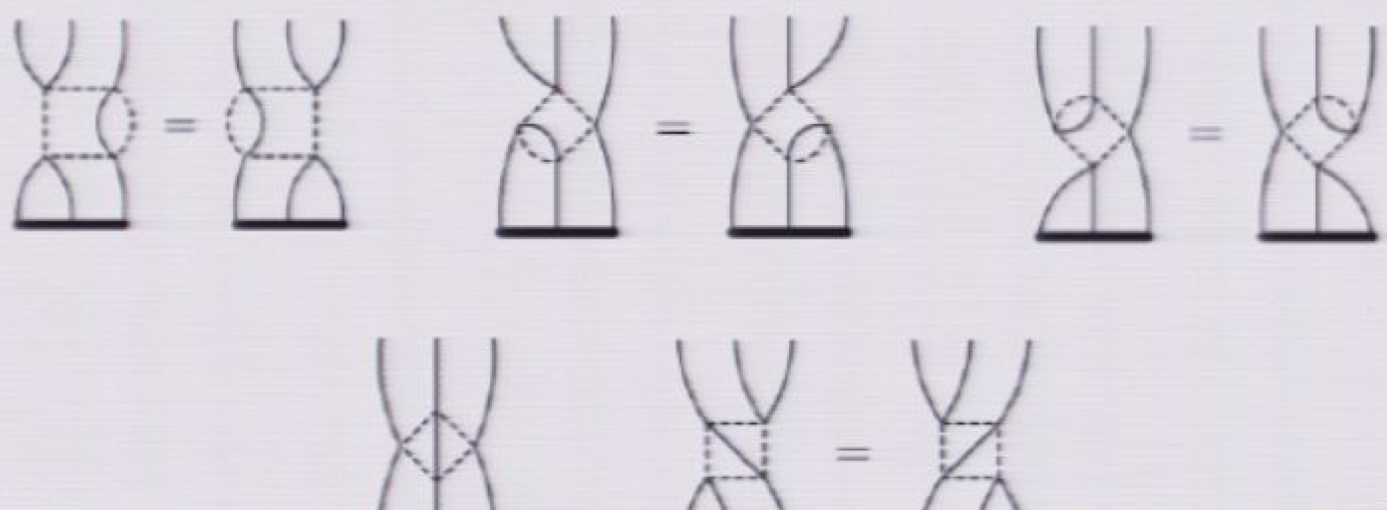


4-loop Diagrams (4 fermion vertices (2 crossed))



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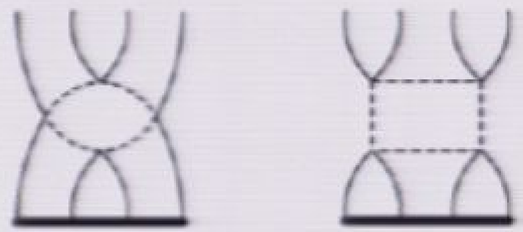
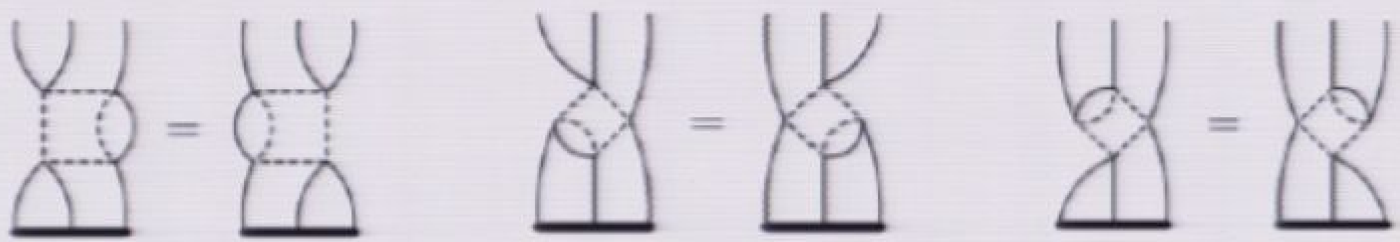
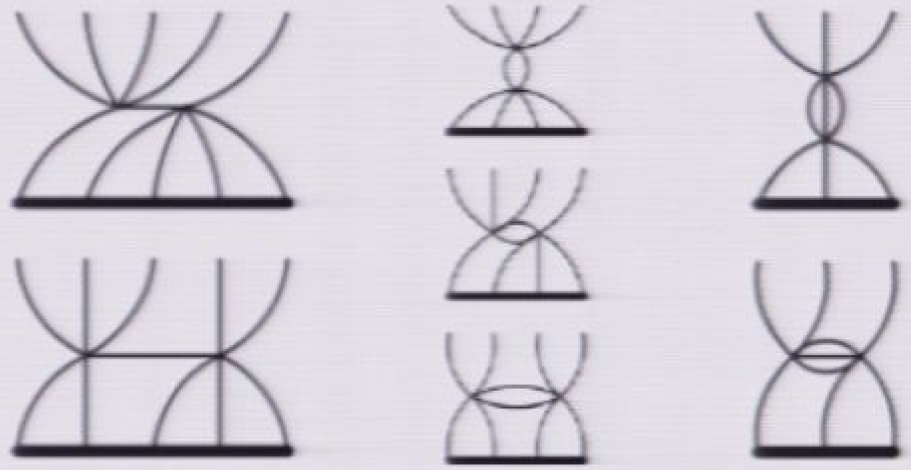
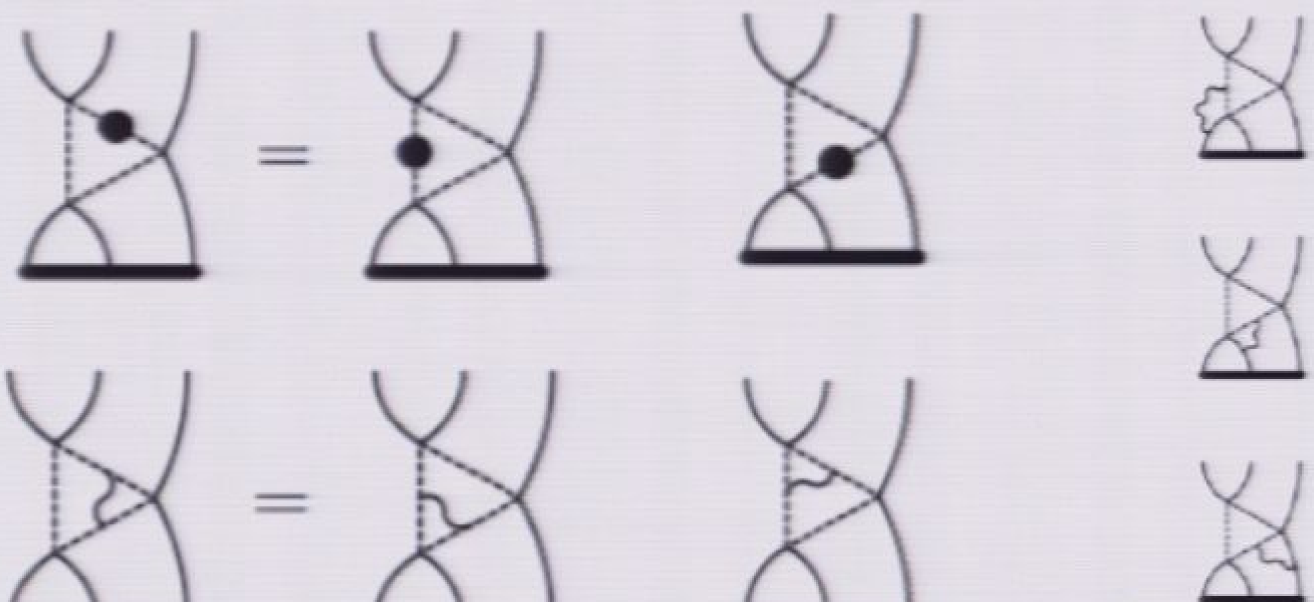




4-loop Diagrams (two scalar vertices)



4-loop Diagrams (Triangles: 3 fermion vertices)



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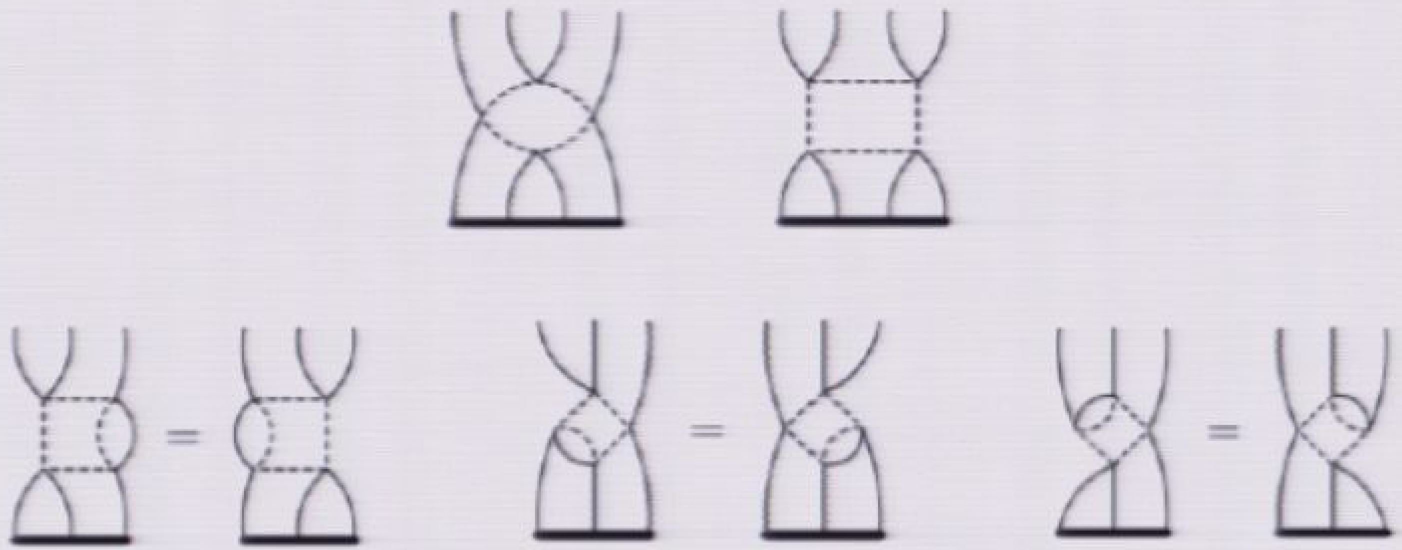

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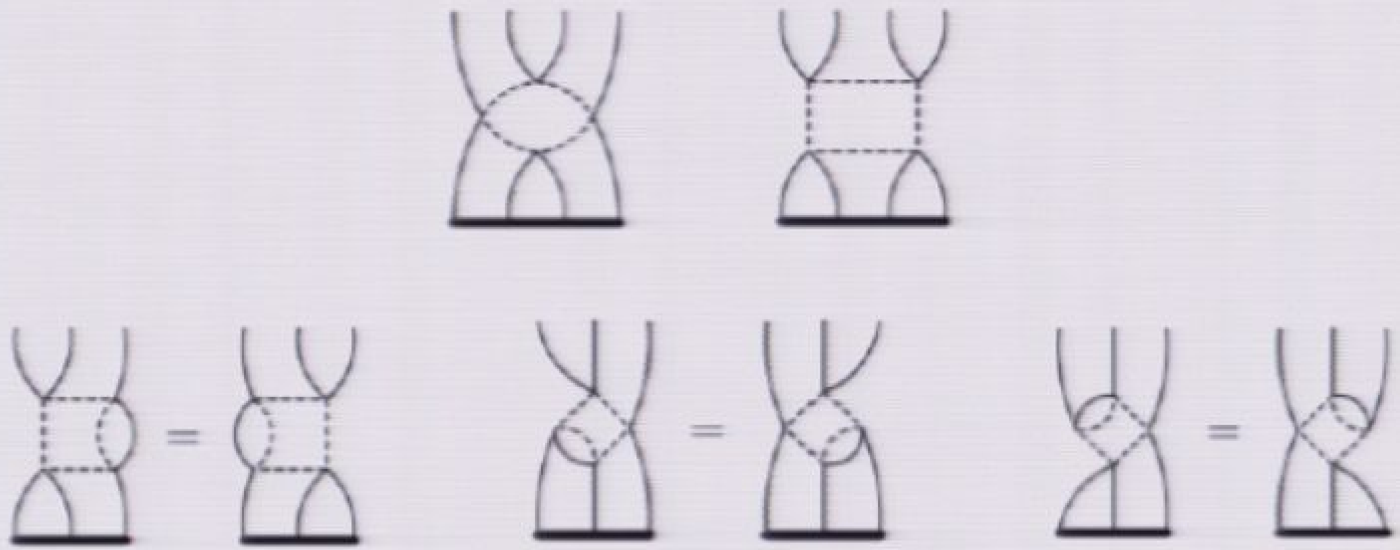

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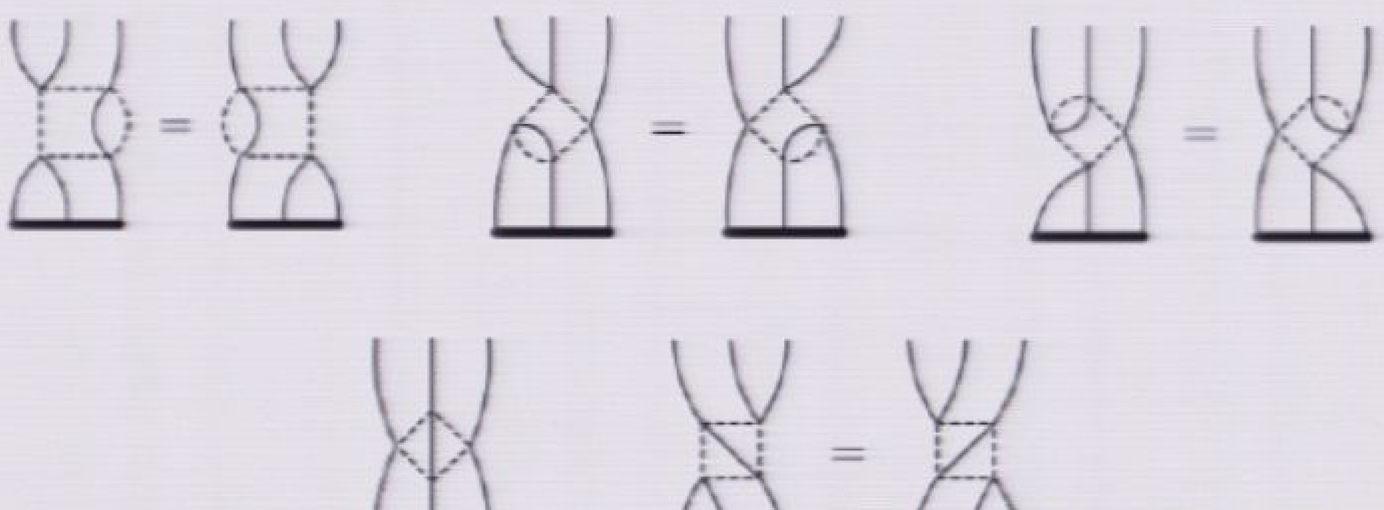
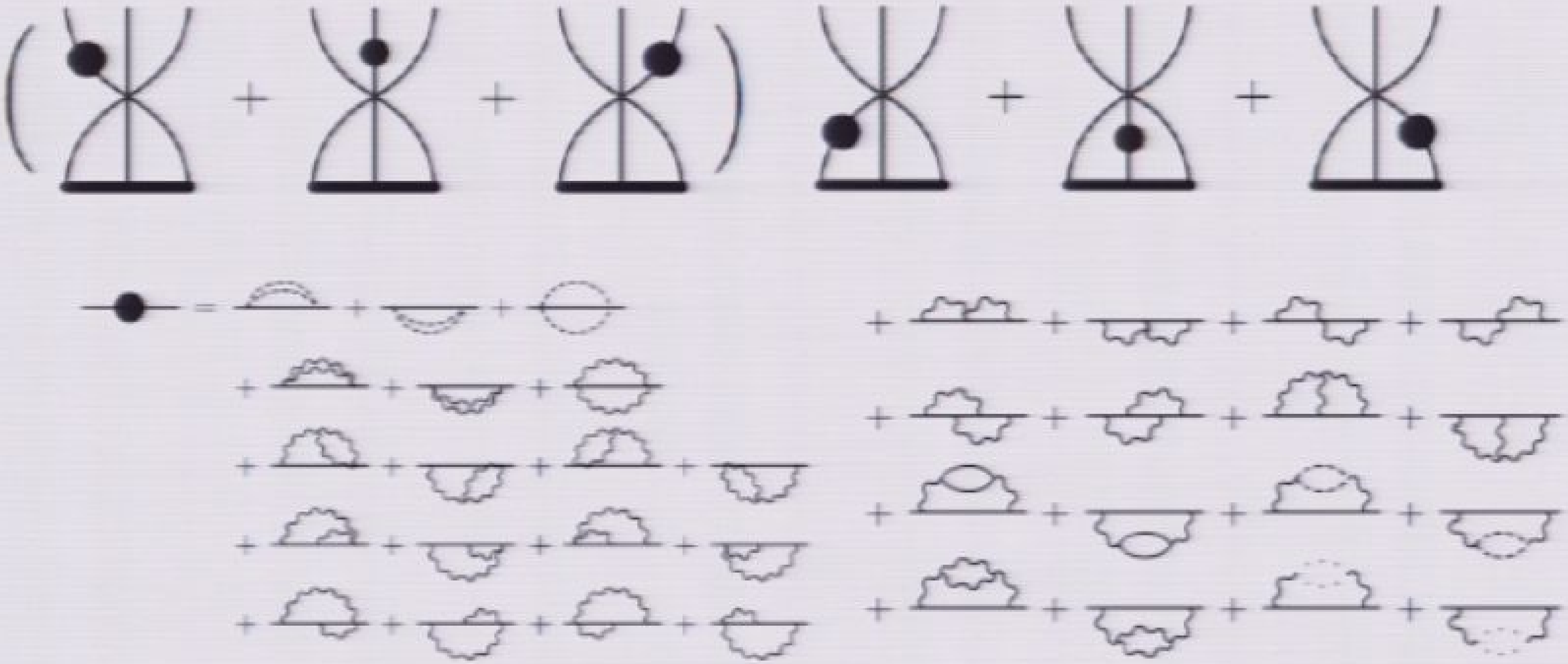
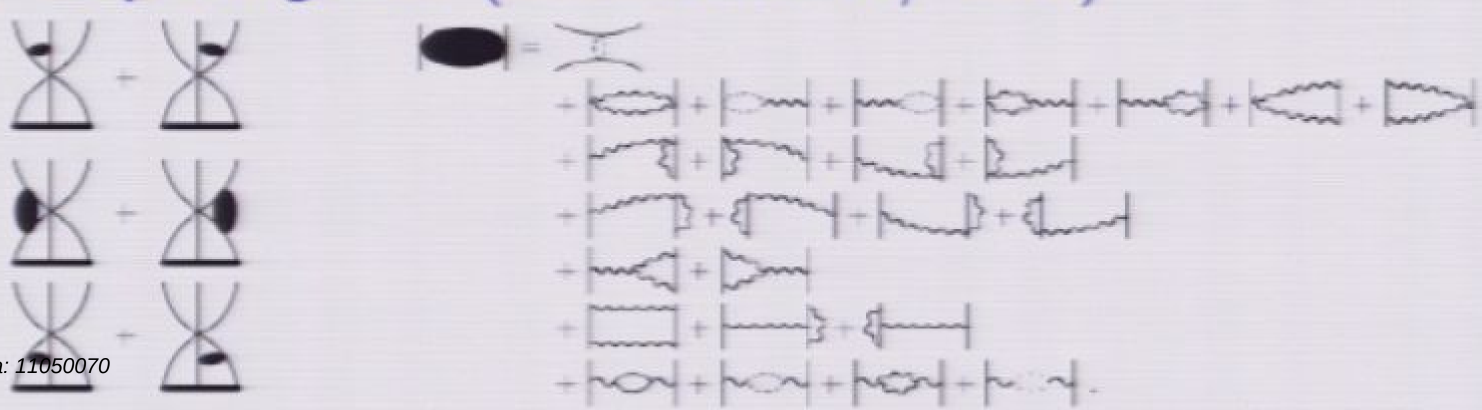


4-loop Diagrams (scalar vert. w/ self energies)

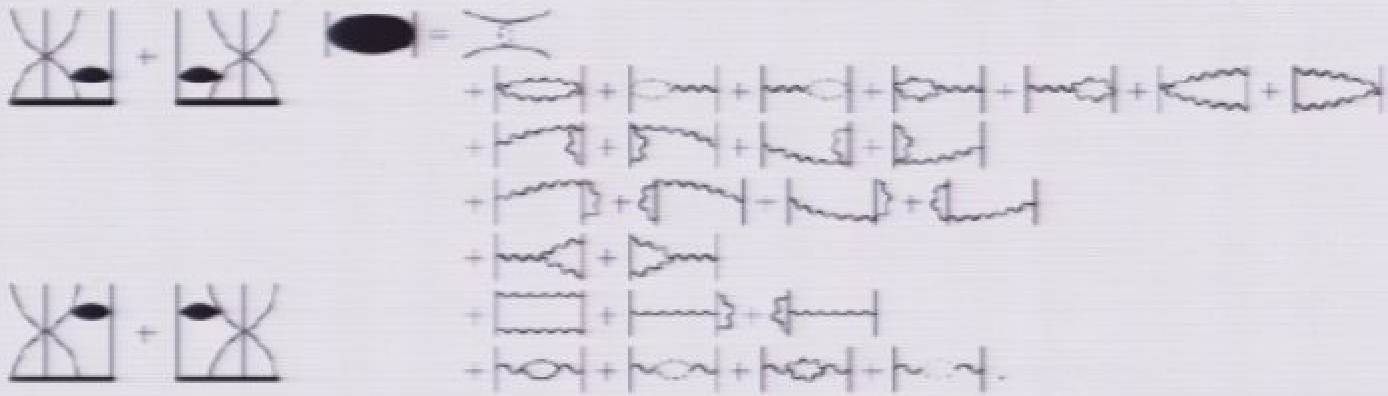
$\frac{1}{2}$



4-loop Diagrams (scalar vert. w/ blob)



4-loop Diagrams (scalar vert. w/ blob attached to neighbor)



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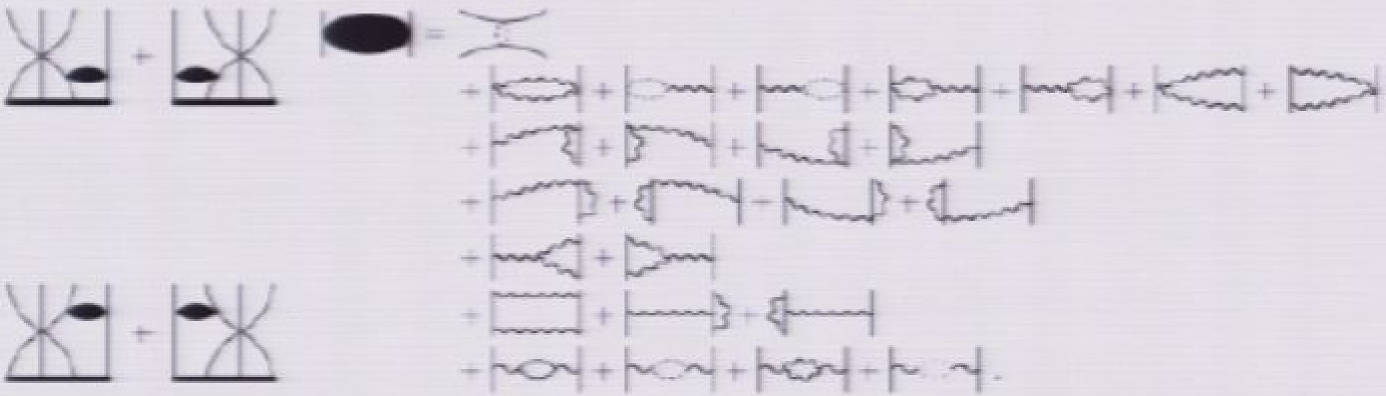

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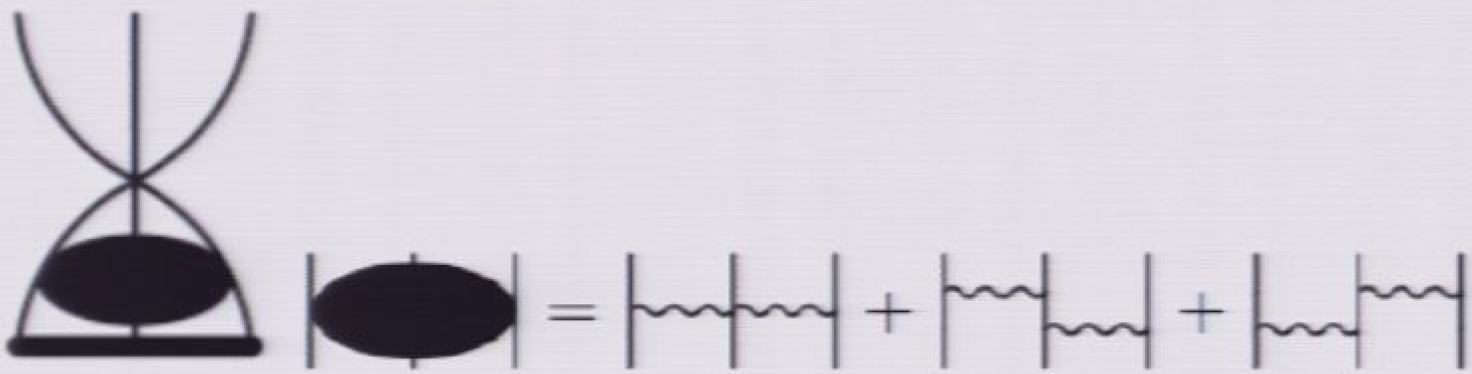

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Comments

- Sign of h_4 is reasonable to match large λ behavior.
- Parity is preserved in this sector ($h_4(\sigma) = h_4(-\sigma)$).
- h_4 and $h_{4,\Delta}$ have highest transcendality (2). Conjectured to continue at higher loops.
- $h^2(\lambda,\hat{\lambda}) = \lambda\hat{\lambda} - \zeta(2)\lambda\hat{\lambda}(\lambda+\hat{\lambda})^2 + \dots$

'he calculation in $N = 2$ superspace

Vector superfields: V, $\hat{V}$

Chiral superfields: Z^A, W_A Anti-chiral fields: $\bar{Z}_A$, $\bar{W}^A$ $A = 1, 2$

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$$S_{CS} = \frac{k}{4\pi} \int d^3x\, d^4\theta \int_0^1 dt \Big[\mathrm{tr}\, V \bar{\mathrm{D}}^\alpha \mathrm{e}^{-tV} \mathrm{D}_\alpha \mathrm{e}^{tV} - \mathrm{tr}\, \hat{V} \bar{\mathrm{D}}^\alpha \mathrm{e}^{-t\hat{V}} \mathrm{D}_\alpha \mathrm{e}^{t\hat{V}} \Big] ,$$

$$S_{\mathrm{mat}} = \frac{k}{4\pi} \int d^3x\, d^4\theta\, \mathrm{tr} \left(\bar{Z}_A \mathrm{e}^V Z^A \mathrm{e}^{-\hat{V}} + \bar{W}^B \mathrm{e}^{\hat{V}} W_B \mathrm{e}^{-V} \right) ,$$

$$S_{\mathrm{pot}} = \frac{k}{4\pi} \frac{i}{2} \Big[\int d^3x\, d^2\theta \epsilon_{AC} \epsilon^{BD}\, \mathrm{tr}\, Z^A W_B Z^C W_D + \int d^3x\, d^2\bar{\theta} \epsilon^{AC} \epsilon_{BD}\, \mathrm{tr}\, \bar{Z}_A \bar{W}^B \bar{Z}_C \bar{W}^D \Big] ,$$

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$$+ \int d^3x\, d^2\bar{\theta} \epsilon^{AC} \epsilon_{BD}\, \mathrm{tr}\, \bar{Z}_A \bar{W}^B \bar{Z}_C \bar{W}^D \Big] ,$$

Operators in the $SU(2) \times SU(2)$ sector are composed of chiral fields only.

$$\frac{1}{N^L} \mathrm{tr}(Z^{A_1} W_{B_1} Z^{A_2} W_{B_2} \ldots Z^{A_L} W_{B_L})$$

Antisymmetric projectors appear naturally

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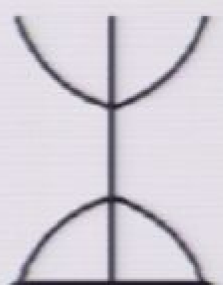
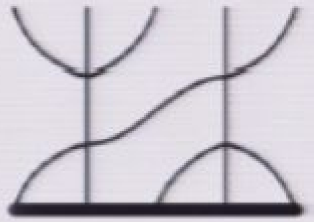
Antisymmetric projectors appear naturally

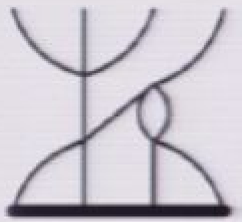
Two loops:  $\sim$ $\chi(1)$ acting on (odd,even,odd)
$\chi(2)$ acting on (even,odd,even)

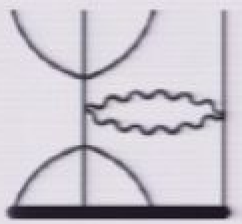
our Loops

5 sites:
(odd,...,odd)



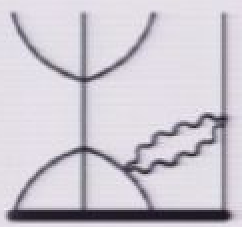
$\sim \lambda^2 \hat{\lambda}^2 \chi(1,3)$: No contribution to $h_4(\sigma)$

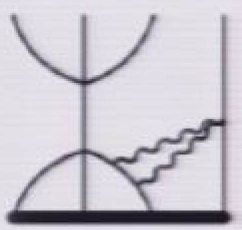




4 sites:
(odd,...,even)

$\sim \lambda^3 \hat{\lambda}\, \chi(1)$: Contributes to $h_4(\sigma)$





our Loops: 3 sites: (odd,even,odd)

$\sim \lambda^3 \hat{\lambda}\, \chi(1)$

$\sim \lambda^2 \hat{\lambda}^2\, \chi(1)$

$\sim$ mixed $\chi(1)$

n all loop conjecture

[$\sigma = 0$ (ABJM)]

Define: $t = 2\pi i\lambda \qquad g(t) = (2\pi)^2 h^2(\lambda)$

Dispersion:

$$\varepsilon(p) = \sqrt{\left(\frac{1}{2}\right)^2 + \frac{g(t)}{\pi^2}\sin^2\frac{p}{2}} - \frac{1}{2}$$

Similar to $\mathcal{N} = 4$ SYM, $g(t) \to \lambda_{\mathsf{SYM}}$

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Conjecture:

$$g(t) = -(1-t)\log(1-t) - (1+t)\log(1+t)$$

- Branch cuts at $t = \pm 1$
- Weak coupling:
  $g(t) = -t^2 - \frac{1}{3}t^4 + \dots = (2\pi)^2(\lambda^2 - 4\zeta(2)\lambda^4 + \dots)$
- Strong coupling:
  $g(t) = -i\pi t - 2\log t - 2 + \dots = (2\pi)^2(\frac{1}{2}\lambda - \frac{1}{(2\pi)^2}(\log 2\pi\lambda + 2))$
- Matches the classical string prediction, but no one-loop contribution (consistent with one regularization scheme in the literature)

dual 't Hooft parameter?

- $2\pi\lambda = -\frac{i}{2}(t - \bar{t})$ (for imag. t)
- Define a prepotential: $\mathcal{F}(t) = \frac{i}{2\pi}(1-t)^2 \log \frac{(1-t)^2}{e}$
- Dual coordinate: $t_D = \frac{\partial \mathcal{F}(t)}{\partial t} = -\frac{i}{\pi}(1-t)\log(1-t)^2$
- Dual 't Hooft: $2\pi\lambda_D = -\frac{i}{2}(t_D - \bar{t}_D)$

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- $g(t) = 2\pi^2\lambda_D$
- Dispersion: $\varepsilon(p) = \sqrt{\left(\frac{1}{2}\right)^2 + 2\lambda_D \sin^2\frac{p}{2}} - \frac{1}{2}$
- Strong coupling: $\lambda_D \approx \lambda$

ABJ?) An interesting limit: "Maximal" ABJ

- Let $1 >> \lambda >> \hat{\lambda}$, rescale: $H = \frac{\hat{\lambda}}{\pi}\mathcal{H}$
- In this scaling limit for the $SU(2)\times SU(2)$ sector (up to 4 loops)

$$\mathcal{H} = \pi\lambda\left(1 - \zeta(2)\lambda^2\right)\sum_{\ell=1}^{2L}(1 - P_{\ell,\ell+2})$$

  Two local Heisenberg spin chains

- Local $\mathcal{H}$ persists to all loops *in this sector*

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Thanks!

Don't forget to register!

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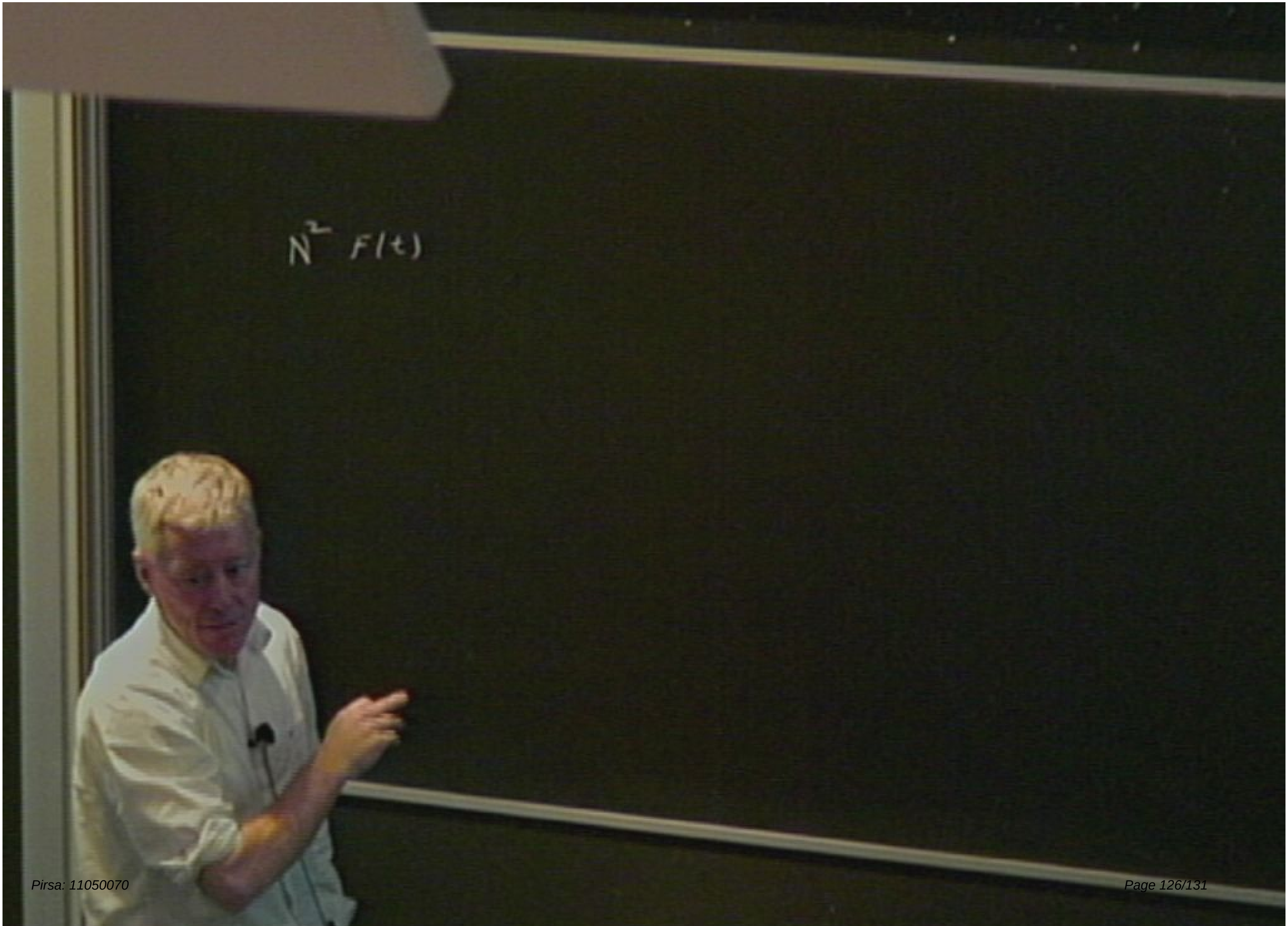
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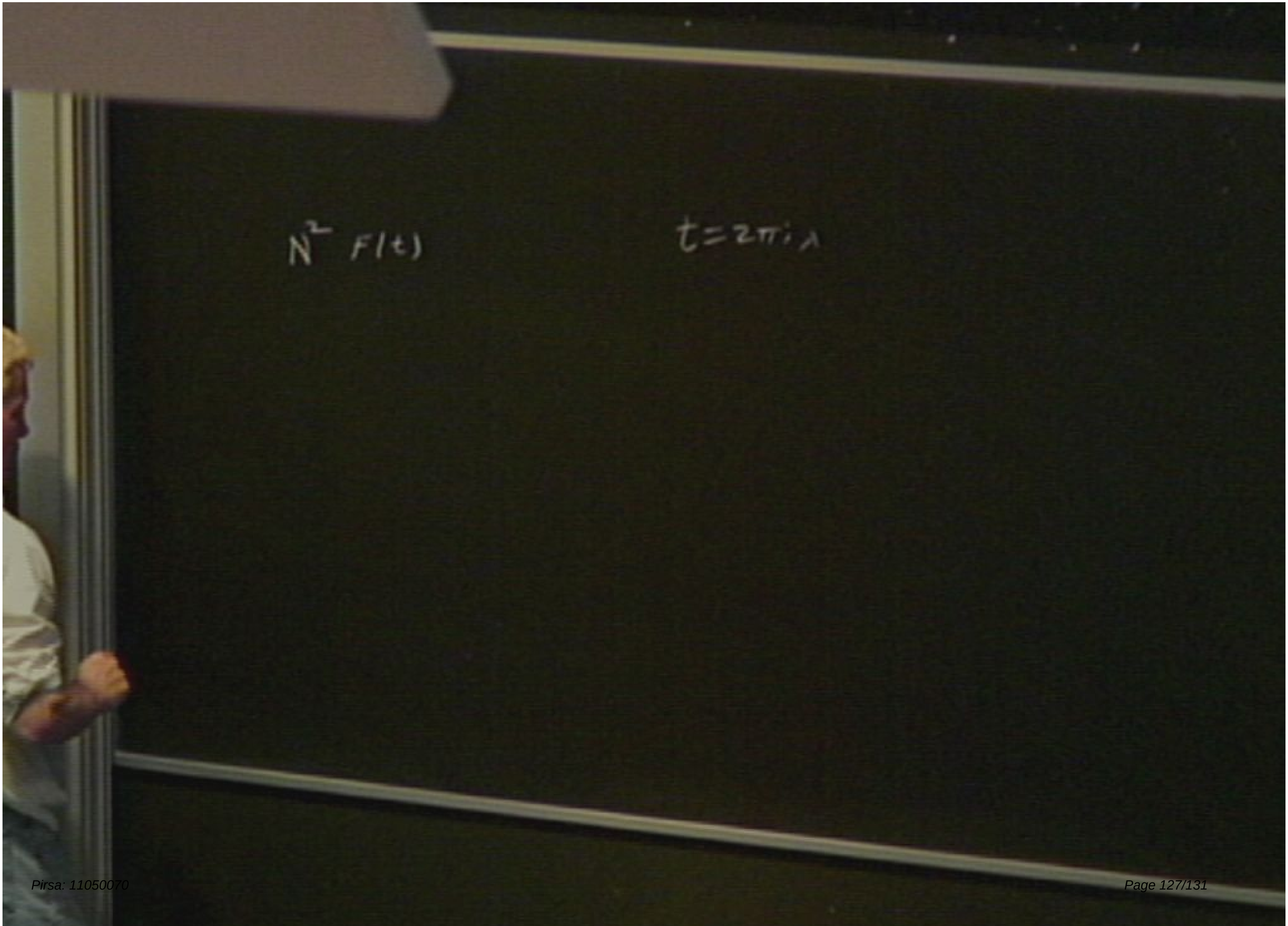
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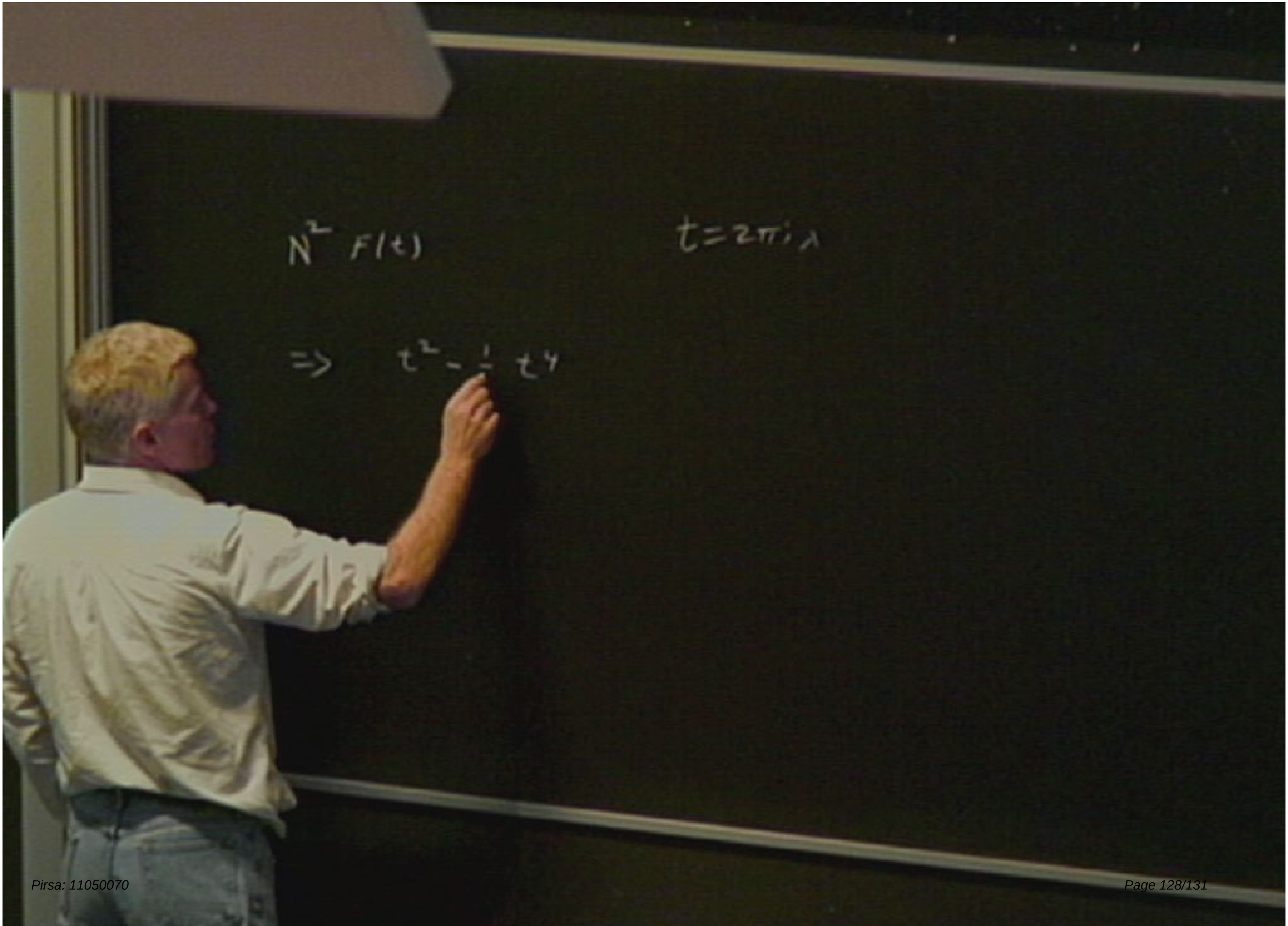
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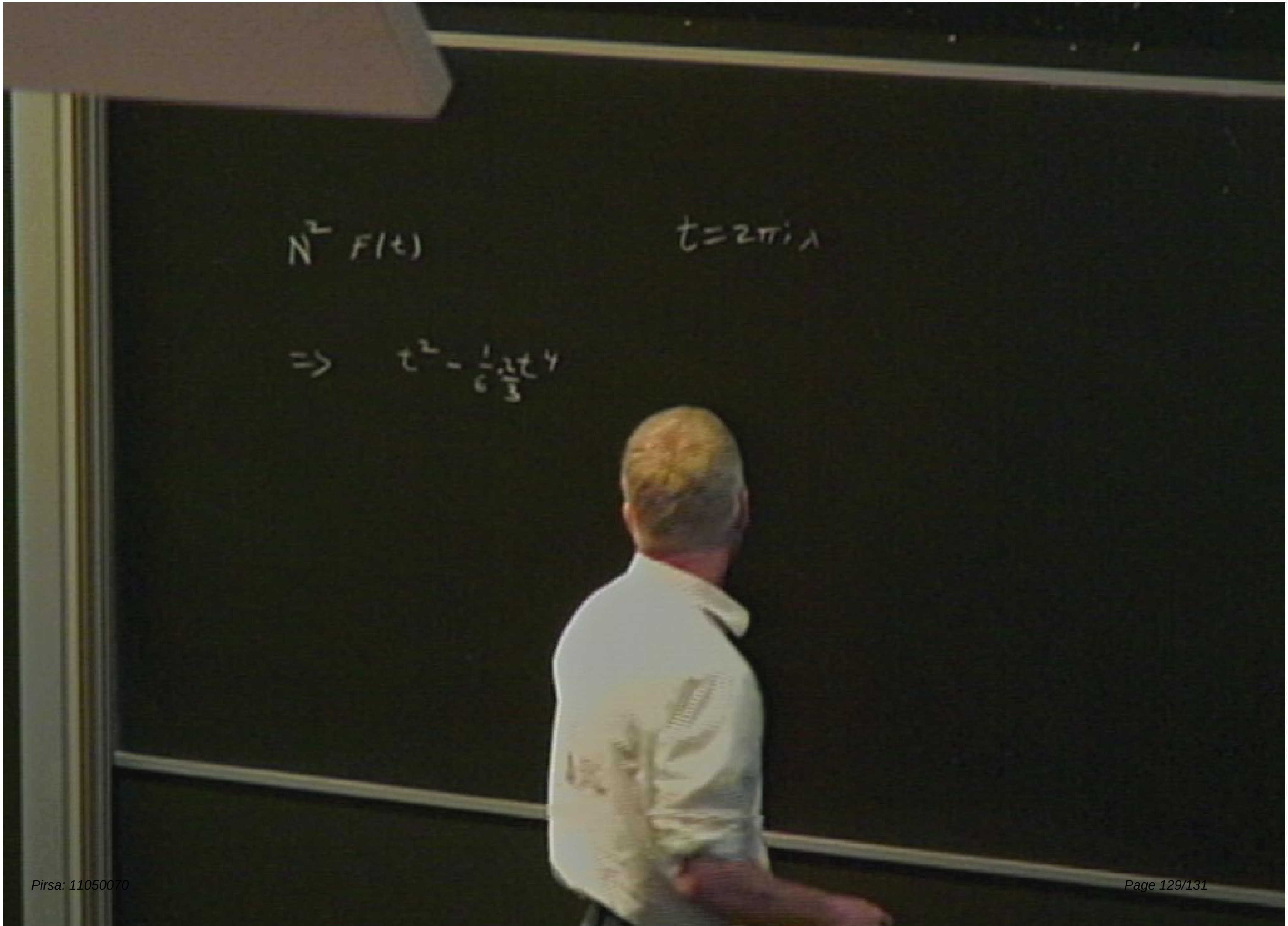
=> t^2 - 1/ t^4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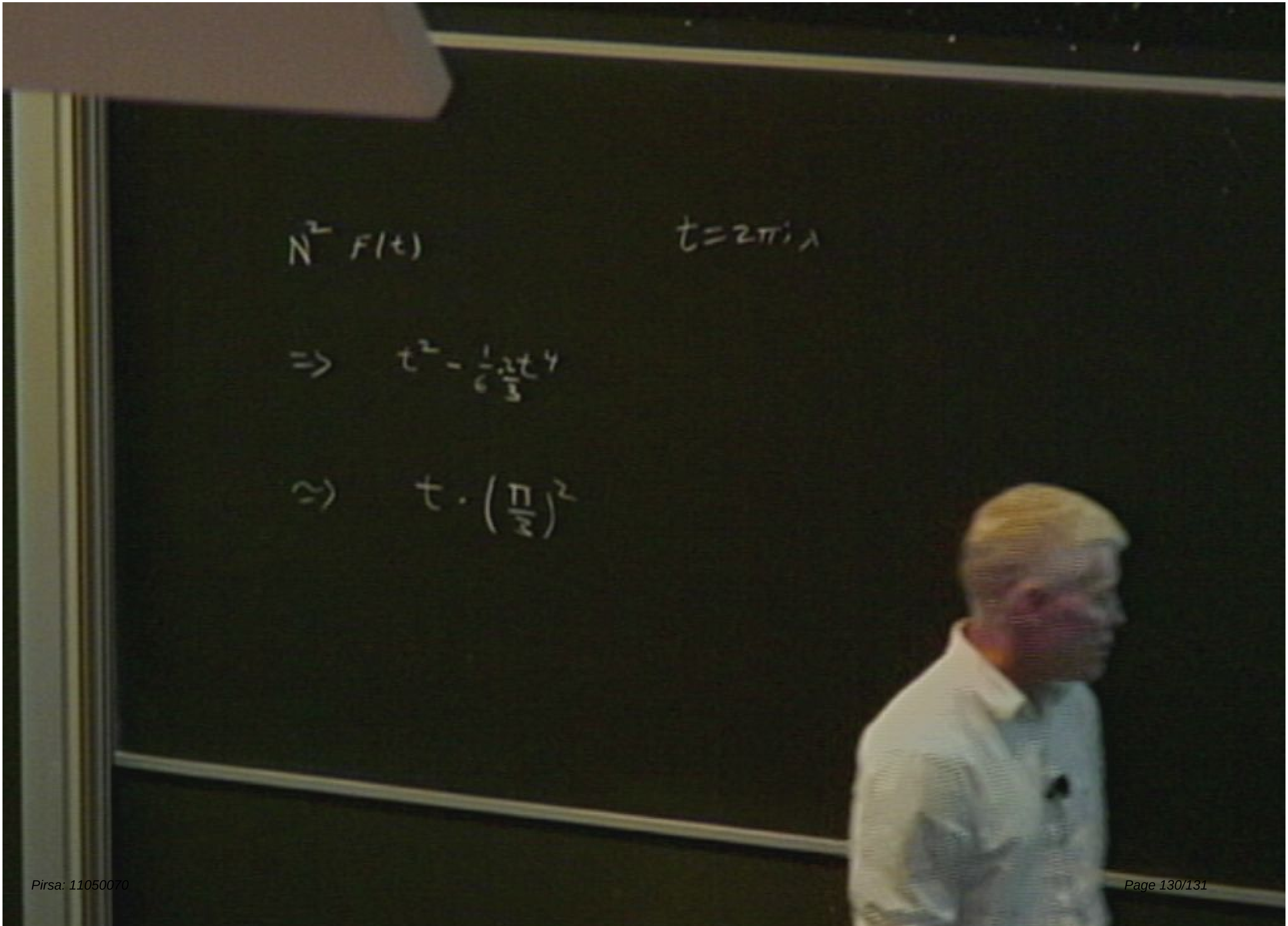

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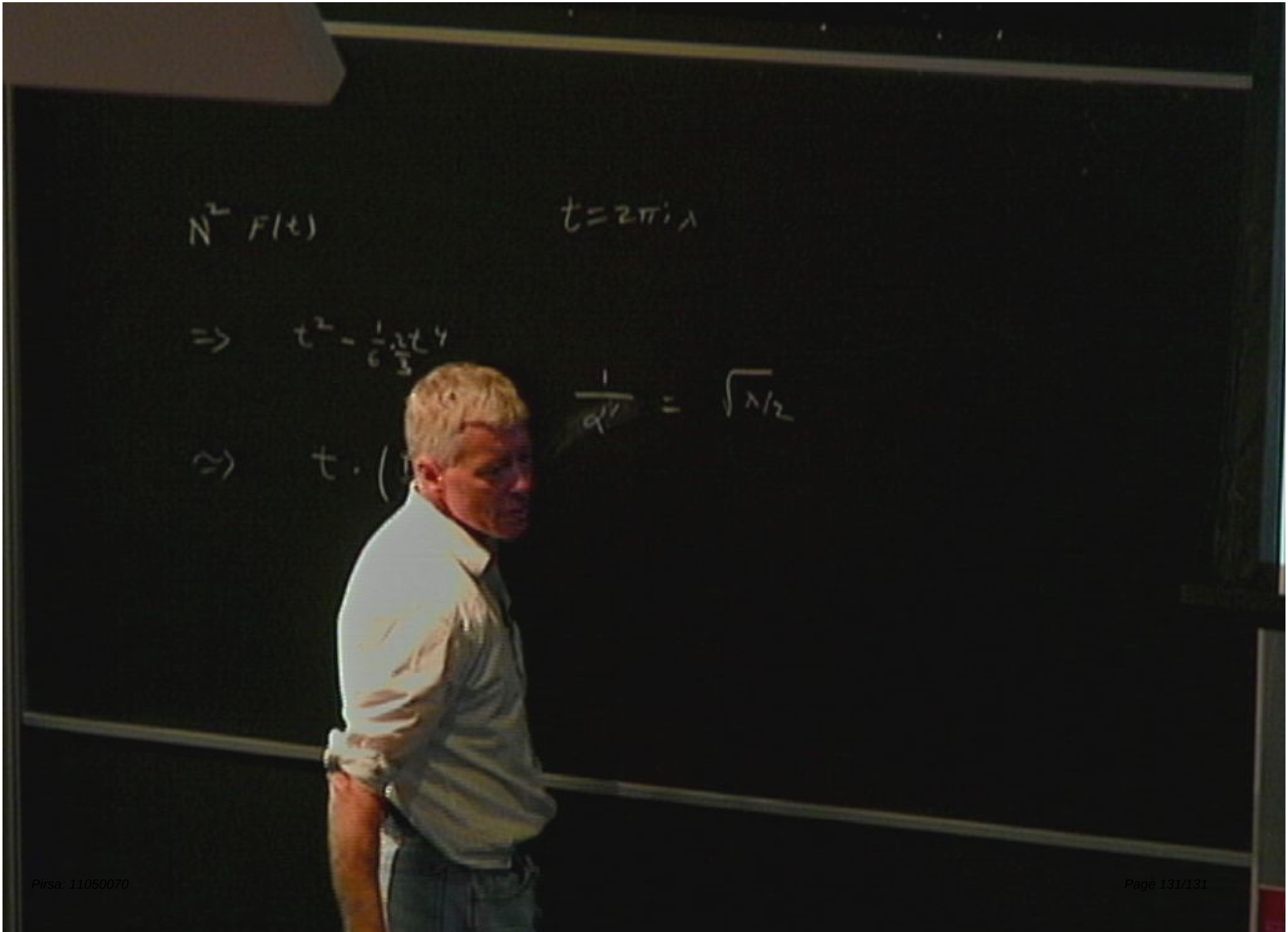
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