Title: Naturalness and Fine Tuning in Three Short Pieces

Date: May 26, 2011 04:00 PM

URL: http://pirsa.org/11050068

Abstract: Unexplained hierarchies and the quest for naturalness have driven model-building efforts in particle physics and cosmology for the past few decades. I will speak about various approaches to problems of 'unnatural'
fine tunings, in the context of supersymmetry, inflation and LHC phenomenology respectively.

Naturalness

For many decades now particle physics research in driven by questions of naturalness.

Fruitful! Gave us

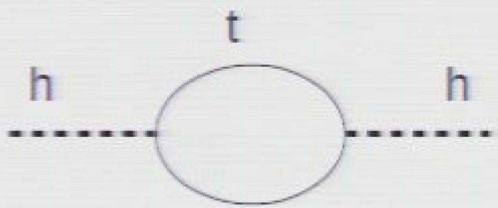
- Technicolor
- SUSY (and gauge coupling unification)
- Large/warped extra dimensions
- Little Higgs theories

Will not ask "Is this a valid motivation"?

NATURALNESS IN SUPERSYMMETRY

Chang, Kilic, RM hep-ph/0405267

Gauge Hierarchy Problem



$$\frac{3}{16\pi^2} h_t^2 \Lambda^2$$

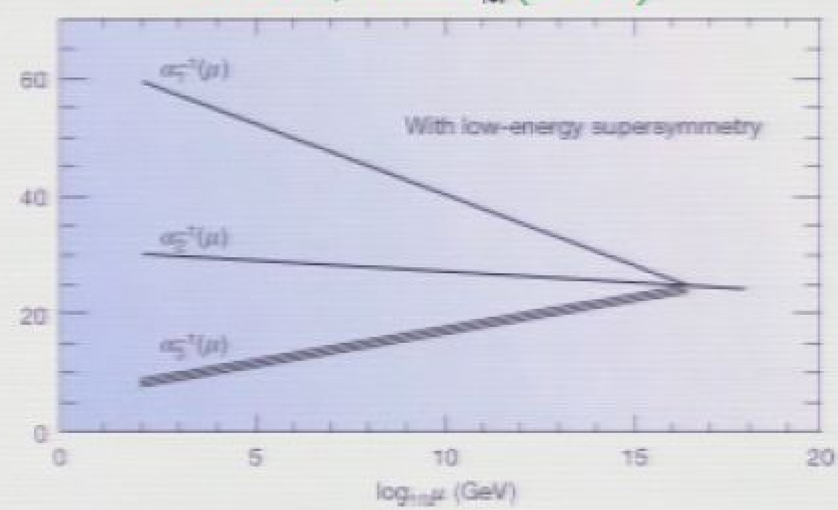
+

$$-\frac{3}{16\pi^2} h_t^2 \Lambda^2$$

SUSY

- Solves hierarchy problem
- Gauge coupling unification
- Stable LSP – dark matter
- Gives EWSB 'for free'

Wilczek, Nature (2005)



Little Hierarchy Problem

BUT no higgs at LEP!

$$m_{h^0}^2 \approx m_Z^2 + \frac{3}{8\pi^2} h_t^4 v^2 \log \frac{m_{\tilde{t}}^2}{m_t^2}$$

$$\delta m_{H_u}^2 \approx -\frac{3}{4\pi^2} m_{\tilde{t}}^2 \log \frac{\Lambda}{m_{\tilde{t}}}$$

Want heavy stop

Want light stop

TENSION

Live with 500 GeV stops and %-level fine-tuning?

Solution: increase the tree-level higgs mass

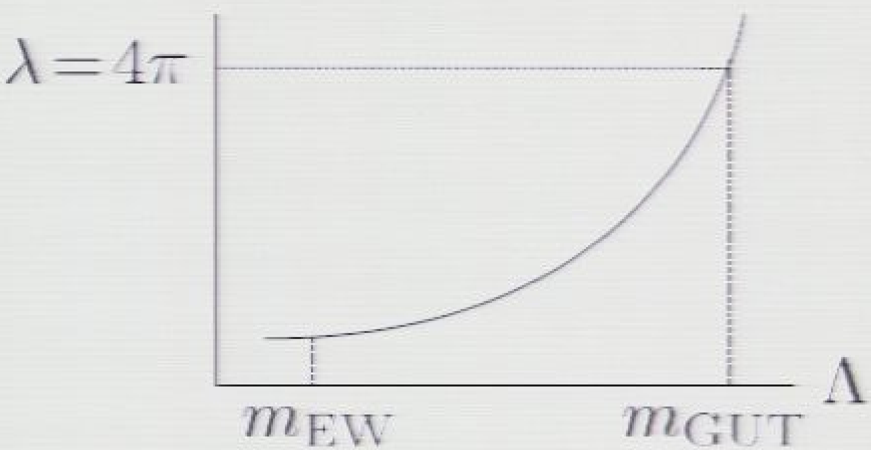
NMSSM

Add singlet with superpotential term $\lambda S H_u H_d$

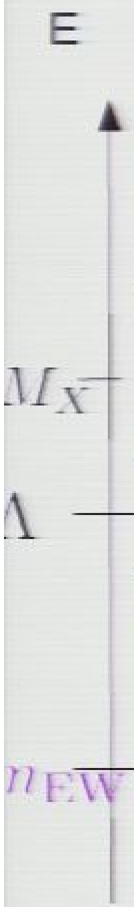
Gives additional contribution to quartic of $|\lambda|^2 |H_u H_d|^2$

BUT perturbativity:

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2}\left[4\lambda^2 + 3h_t^2 - 3g_2^2 - \frac{3}{5}g_1^2\right] + \cdots$$



$m_h \lesssim 150$ GeV



Composite Singlet

Harnik et. al.; hep-ph/0311349
Chang, Kilic, RM; hep-ph/0405267

Asymptotically free yukawas

$$W_\lambda = \lambda_1\, \phi X H_u + \lambda_2\, \phi^c X^c H_d + M_X X X^c + M_{\tilde{X}} \tilde{X} \tilde{X}^c + m\phi\phi^c + W_m$$

$$W_{\rm eff} = -\frac{\lambda_1 \lambda_2}{M_X} \phi\phi^c H_u H_d + m\phi\phi^c + \cdots$$

$$W_{\rm eff} = -\mathcal{O}(1)\,\sqrt{n}\frac{\lambda_1\lambda_2}{4\pi}\,\frac{\Lambda}{M_X} S H_u H_d + \frac{m\Lambda}{4\pi} S + W_m + W_{\rm dyn}$$

	$SU(3) \times SU(2)_L \times U(1)_Y$	$SU(4)_s$
ϕ	$(1,1,0)$	4
ϕ^c	$(1,1,0)$	$\bar{4}$
X	$(1,2,-\frac{1}{2})$	$\bar{4}$
X^c	$(1,2,\frac{1}{2})$	4
$\tilde{X}$	$(\bar{3},1,\frac{1}{3})$	$\bar{4}$
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The Lowdown

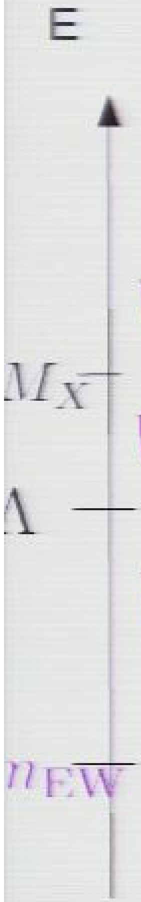
Total 10 flavors of SU(4) – in conformal window

Can use Seiberg duality to estimate size of yukawas at strong coupling scale

Run down to weak scale, $m_h \lesssim 260 - 350$ GeV

Gauge coupling unification manifest b/c only adding $5 + \bar{5}$ s of SU(5)

Not the most attractive way to solve a problem of aesthetics?



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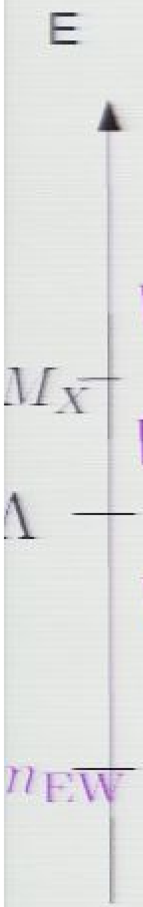

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NATURALNESS IN SLOW ROLL INFLATION

Gallicchio, RM 0911.5343

Inflation

Fine tuning

- Horizon problem
- Flatness problem
- Heavy relic problem

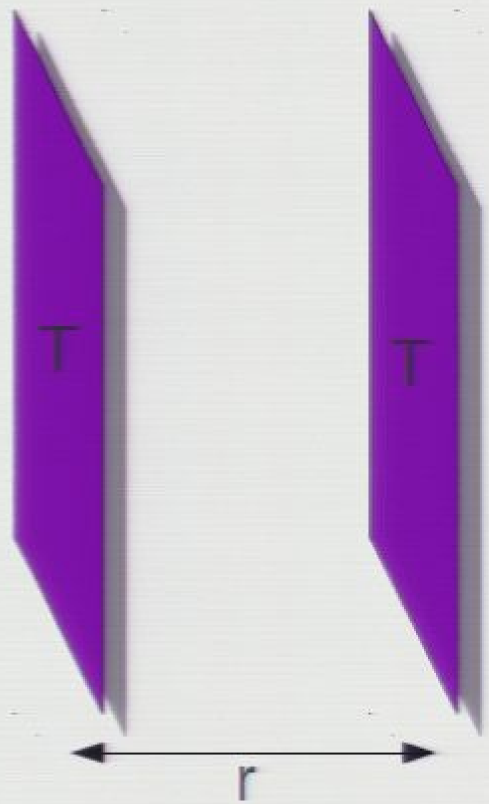
} 60 e-foldings of inflation

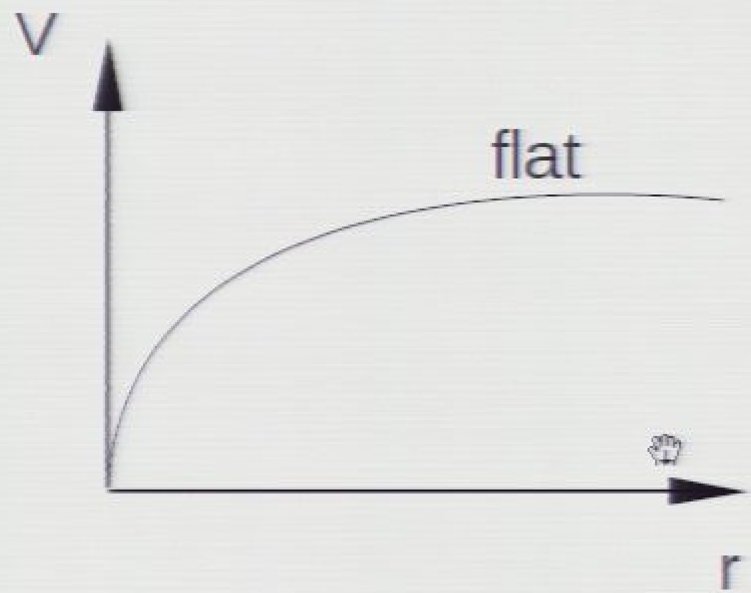
Field theory → slowly rolling scalar field

$$\epsilon = \left(M_{\rm PL} \frac{V'}{V} \right)^2 << 1 \qquad \eta = M_{\rm PL}^2 \frac{V''}{V} << 1$$

Fine tuning

Brane Inflation





In n extra dimensions, potential between branes

$$V = 2T + \frac{T^2}{M^{D-2}} \frac{1}{r^{n-2}}$$

Just Dimensional Analysis

$$V = 2T + \frac{T^2}{M^{D-2}} \frac{1}{L^{n-2}} \varphi(\theta)$$

Kinetic term for distance between branes:

$$\mathcal{L} = T\,(\partial_\mu \Delta Y)\,(\partial^\mu \Delta Y)$$

Canonically normalized inflaton field:

$$\Phi^2 = T\,(\Delta Y)^2 = T L^2 \theta^2$$

Gives η independent of all dimensionful params

$$\eta = \frac{M_{\rm PL}^2}{T} \frac{T^2}{M^{D-2}} \frac{1}{L^{n-2}} \frac{1}{T L^2} \varphi''(\theta) = \varphi''(\theta)$$

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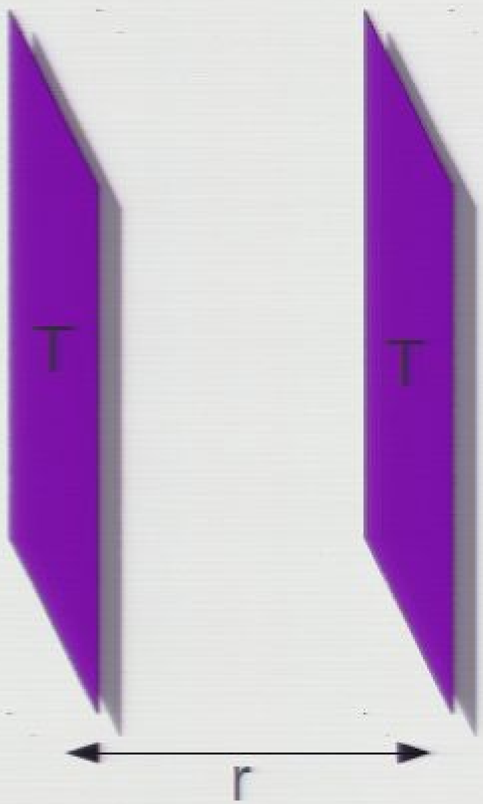

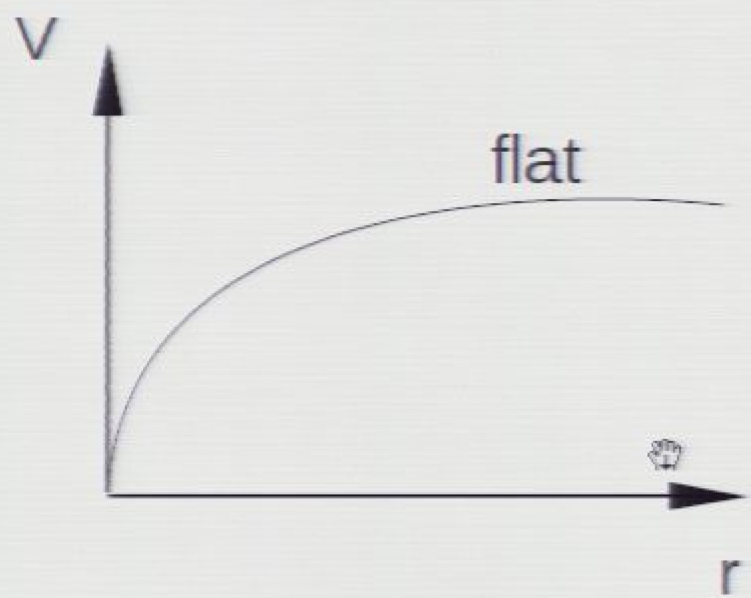

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Finite Volume Effects

For n flat, compact extra dimensions, Newtonian gravitational potential due to a brane at $\vec{y} = \vec{0}$

$$\nabla^2 \phi(\vec{y}) = \frac{T}{M^{D-2}} \delta^n(\vec{y})$$

Extra term needed to make this consistent for a closed space

Kachru et. al. hep-th/0308055: "...emerges naturally from curvature of spacetime"

$$\nabla^2 \phi(\vec{y}) = \frac{T}{M^{D-2}} \left(\delta^n(\vec{y}) - \frac{1}{L^n} \right)$$

Example: point mass on n-torus

Naïve Poisson equ in momentum space

$$\vec{k}^2 \tilde{\phi}_{\vec{n}} \sim m e^{i \vec{k}_n \vec{y}}$$

Don't integrate out zero mode

$$\vec{k}^2 \tilde{\phi}_{\vec{n}} \sim m \left(e^{i \vec{k}_n \vec{y}} - \delta_{\vec{n}.0} \right)$$

Corresponds exactly to modified Poisson equ!

$$\nabla^2 \phi(\vec{y}) = \frac{T}{M^{D-2}} \left(\delta^n(\vec{y}) - \frac{1}{L^n} \right)$$

(2) Finite Volume Effects

Factorizing out dimensionful quantities:

$$\nabla^2 \varphi(\vec{\theta}) = \left(\delta^n(\vec{\theta}) - 1\right)$$

Away from all sources

$$\left(\partial_1^2 + \cdots + \partial_n^2\right) \varphi(\vec{\theta}) = -1$$

Minimize by distributing evenly

$$|\varphi''(\theta)| > \frac{1}{n}$$

Gives hard lower limit for η !

Point Masses in (2+1)D Gravity

Staruszkiewicz; Deser et. al.; Gott and Alpert

Line element (recall 2 spatial dims conformal to flat space)

$$ds^2 = dt^2 - \omega(x,y)\left(dx^2 + dy^2\right)$$

Einstein equation for point mass at origin:

$$-\frac{1}{2}\nabla^2 \ln \omega = \frac{m}{M_{\rm PL}}\delta(\vec{r})$$

Solve this to obtain $\omega = |\vec{r}|^{-2m/M_{\rm PL}}$

Going to polar coordinates:

$$ds^2 = dt^2 - r^{-2m/M_{\rm PL}}\left(dr^2 + r^2 d\theta^2\right)$$

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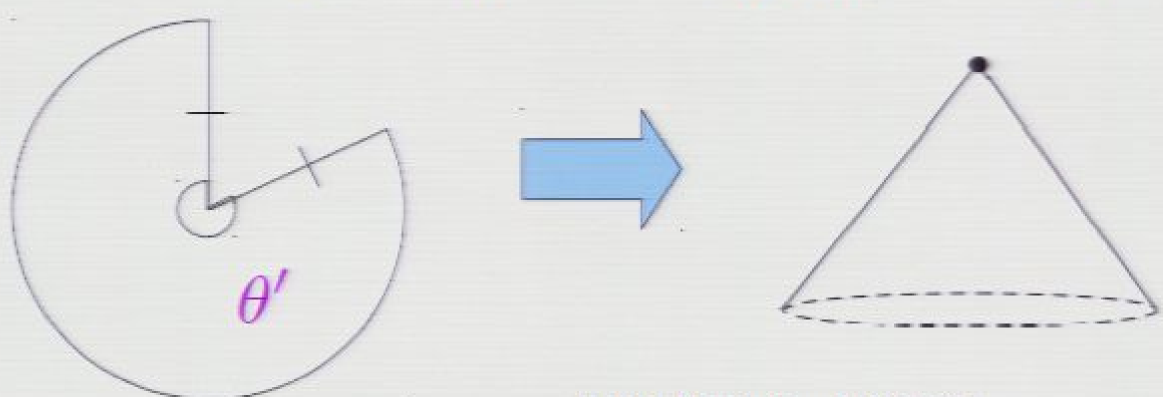
$$ds^2 = dt^2 - r^{-2m/M_{\rm PL}}\left(dr^2 + r^2 d\theta^2\right)$$

Making the following redefinitions:

$$\rho = \frac{1}{1-\frac{m}{M_{\rm PL}}} r^{1-\frac{m}{M_{\rm PL}}} \qquad \theta' = \left(1 - \frac{m}{M_{\rm PL}}\right)\theta$$

We obtain the metric for flat space

$$ds^2 = dt^2 - d\rho^2 - \rho^2 d\theta'^2 \quad \text{with} \quad 0 \le \theta' \le 2\pi\left(1 - \frac{m}{M_{\rm PL}}\right)$$



Infinite Flat 6D dimensions

Reproduce result perturbatively with 3-brane sources in 6D

$$\mathcal{L} = -\sqrt{|g|}\left(\frac{1}{2}R - \frac{1}{2}T^{MN}H_{MN}\right)$$

For brane with tension T located at $\vec{y} = \vec{0}$

$$T_{MN} = \begin{cases} T\,\delta(\vec{y})\,\eta_{\mu\nu} & M,N = 0,\cdots,3 \\ 0 & \text{otherwise} \end{cases}$$

Potential between 2 branes is $V = T_{MN}P^{MN\,SR}T_{SR}$

$$V \propto \eta_{\mu\nu}\left(\tfrac{1}{2}\,\eta^{MS}\eta^{NR} + \tfrac{1}{2}\,\eta^{MR}\eta^{NS} - \tfrac{1}{D-2}\,\eta^{MN}\eta^{SR}\right)\eta_{\sigma\rho}$$
$$= 0$$

4D Effective Theory

Can parametrize 6D metric perturbations as

$$H_{MN} = \begin{pmatrix} h_{\mu\nu} - 2\Phi\eta_{\mu\nu} & V_{\mu n} \\ V_{\nu m} & \phi_{(mn)} + 2\Phi\eta_{mn} \end{pmatrix}$$

Can choose 6 gauge fixing conditions to decouple fields

$$\partial^m V_{\mu m} = 0 \qquad \partial\phi_{(mn)} + 4\partial_n\Phi = 0$$

Branes couple to a tower of graviton and radion modes

$$V_h^{(i,j)} + V_\phi^{(i,j)} = 0$$

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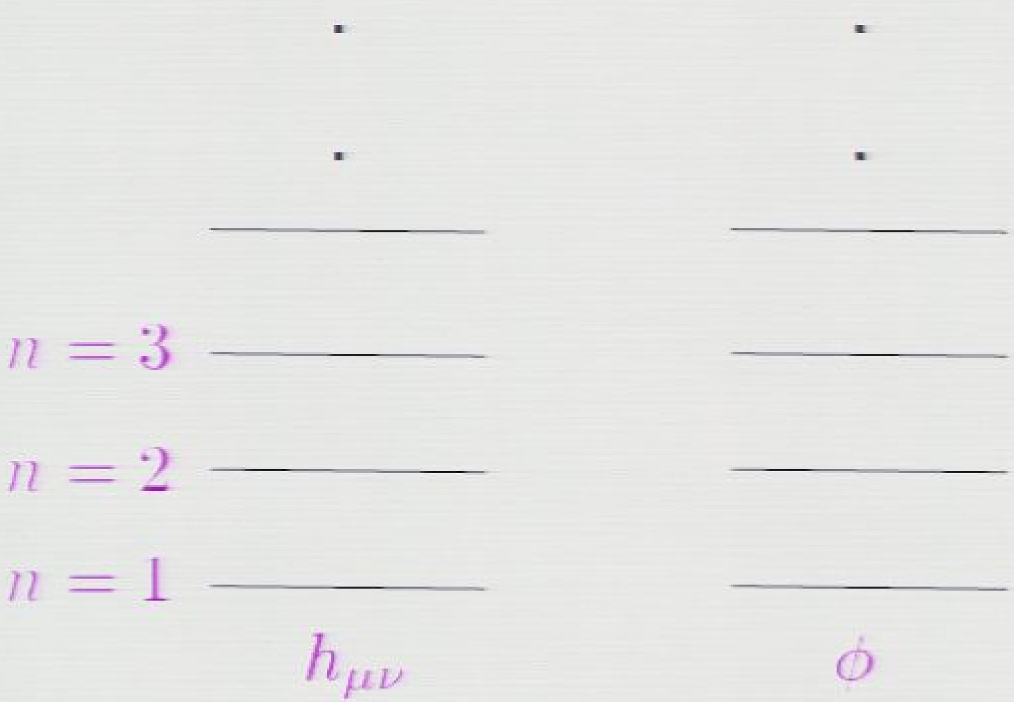
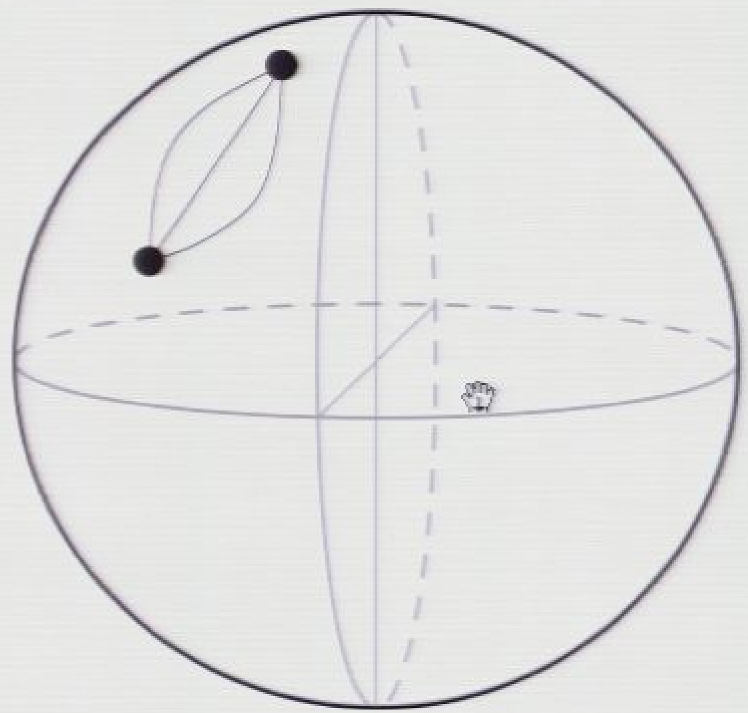
Results from level-by-level cancellation between exchange of 4D graviton and radion KK modes



Compact Spherical 6D

Branes source runaway potential for radion – need to stabilize

Do this on spherical background with CC and magnetic flux



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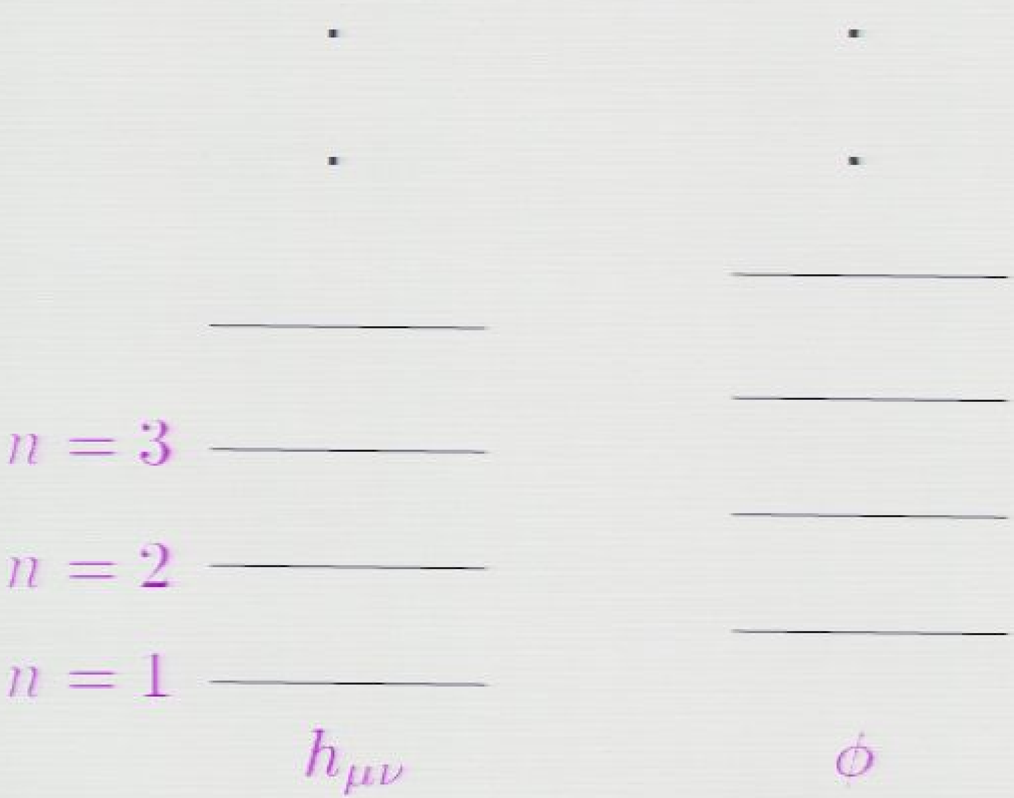
Freund, Rubin (1980)

4D Effective Theory?

Stabilizing = giving radion a mass



= no mode-by-mode cancellation?

real story more subtle

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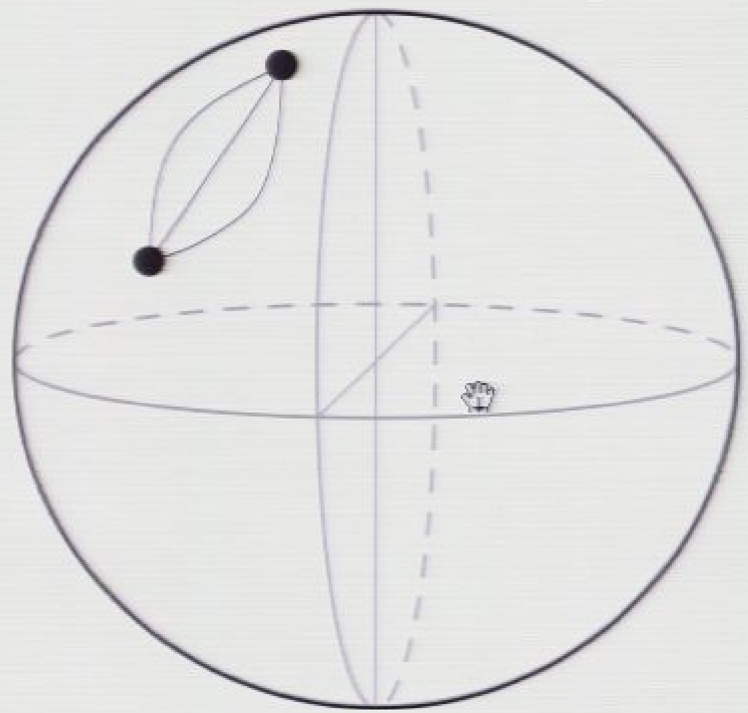

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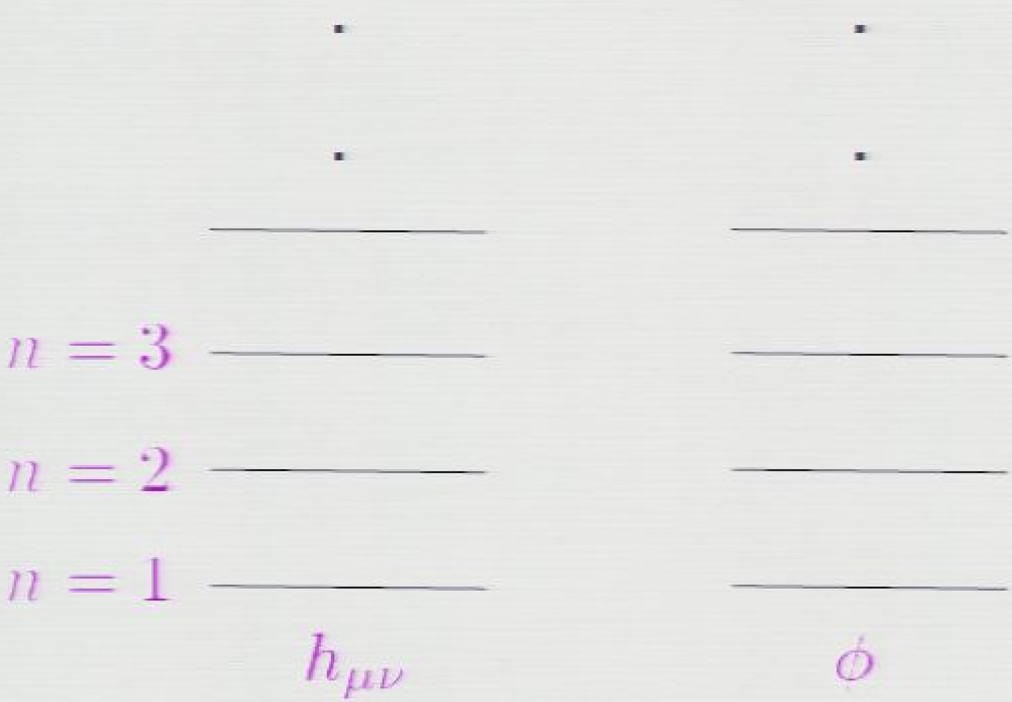

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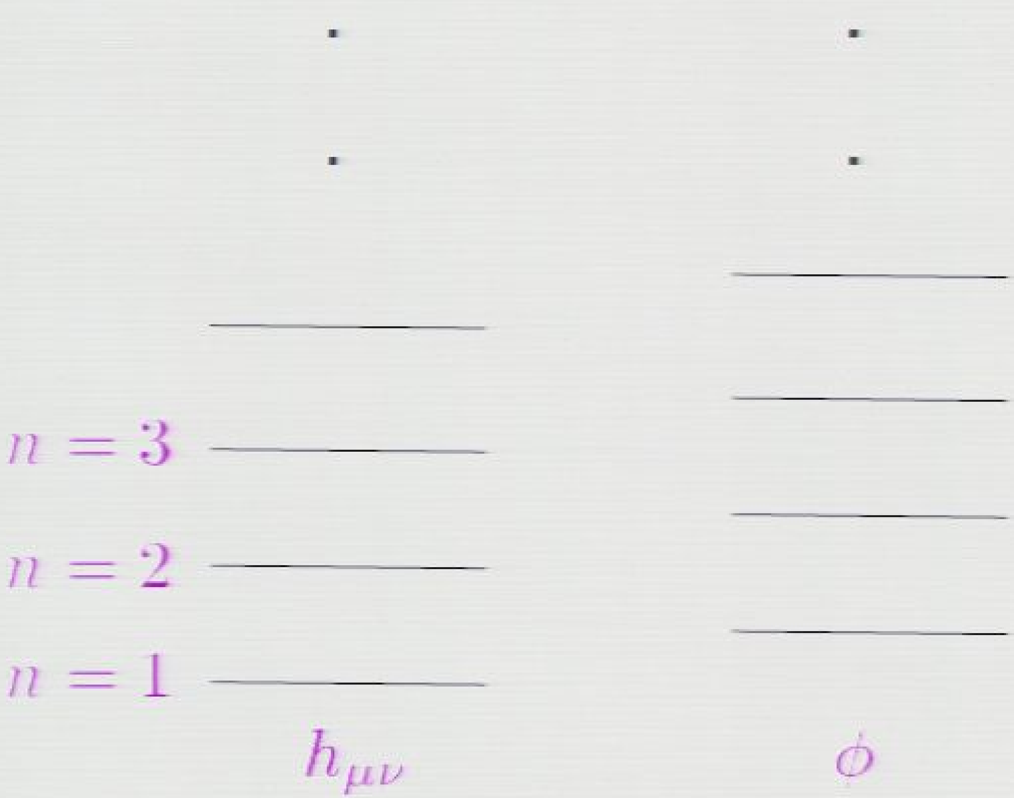

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Perturbations

Need to also perturb background field strength

$$F_{MN} = \bar{F}_{MN} + f_{MN}$$

where $$B\, f_{MN} = \nabla_M b_N - \nabla_N b_M$$

Solving Maxwell's equations

$$\bar{F}_{MN} = \begin{cases} 0 & \text{for M, N} = 0, \cdots, 3 \\ B\,\epsilon_{mn} & \text{otherwise} \end{cases}$$

Parametrize metric perturbations

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Gauge fix (fields not decoupled)

$$\nabla_n V^{n\mu} = 0 \qquad \nabla_n \phi^{(mn)} = 0 \qquad \nabla_n b^n = 0$$

Equations of Motion

Decompose fields in (scalar, vector, tensor) spherical harmonics on sphere

$$(-8\Psi)\, Y^{(l,m)}_{(mn)} = 0$$

Only field coupling to branes is zero for $l \geq 2$

For $l = 1$ get repulsive potential $V \sim \cos\theta$ from exchange of single mode! Symmetry reason?

Still no good b/c $\eta \sim \mathcal{O}(1)$ BUT...

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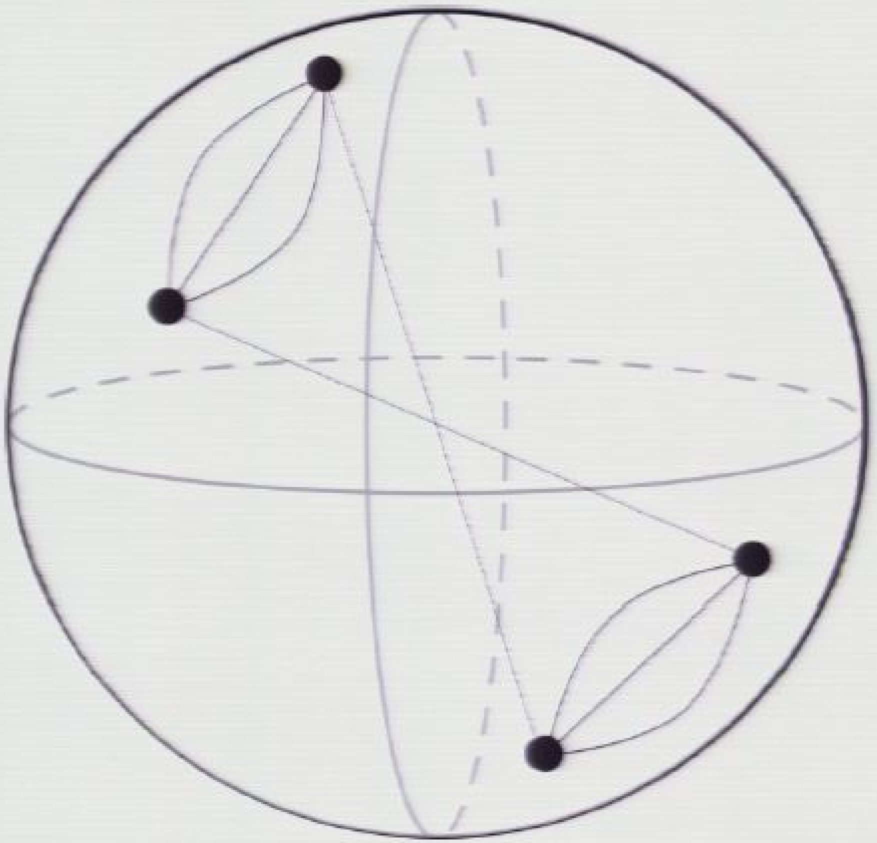

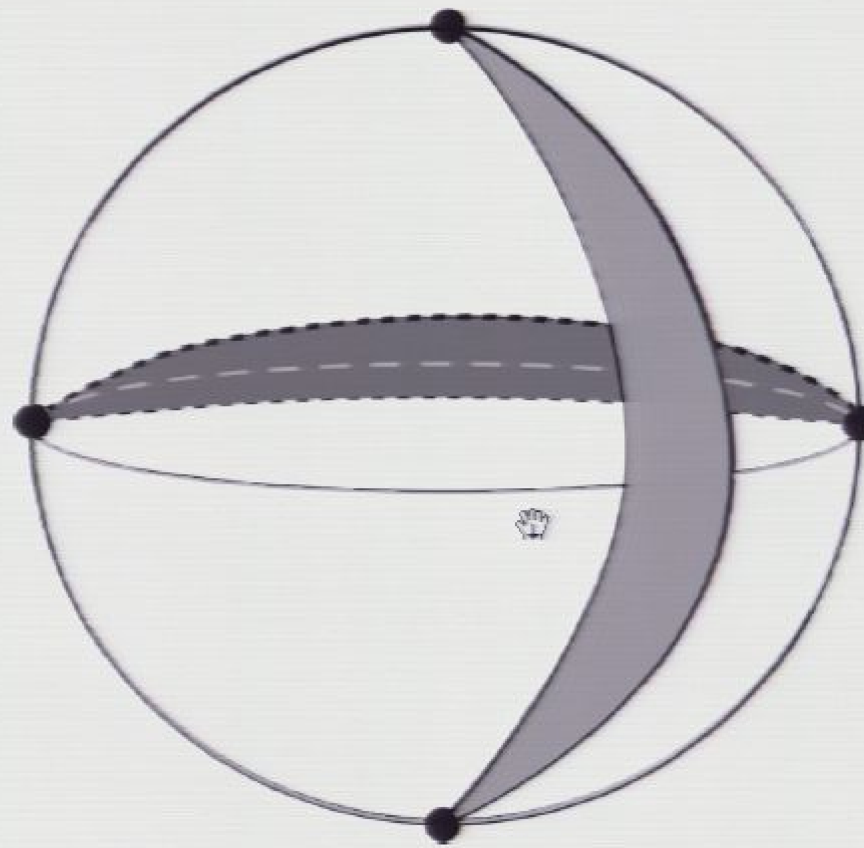

Gravitational potential =0

Slow roll from loop corrections?

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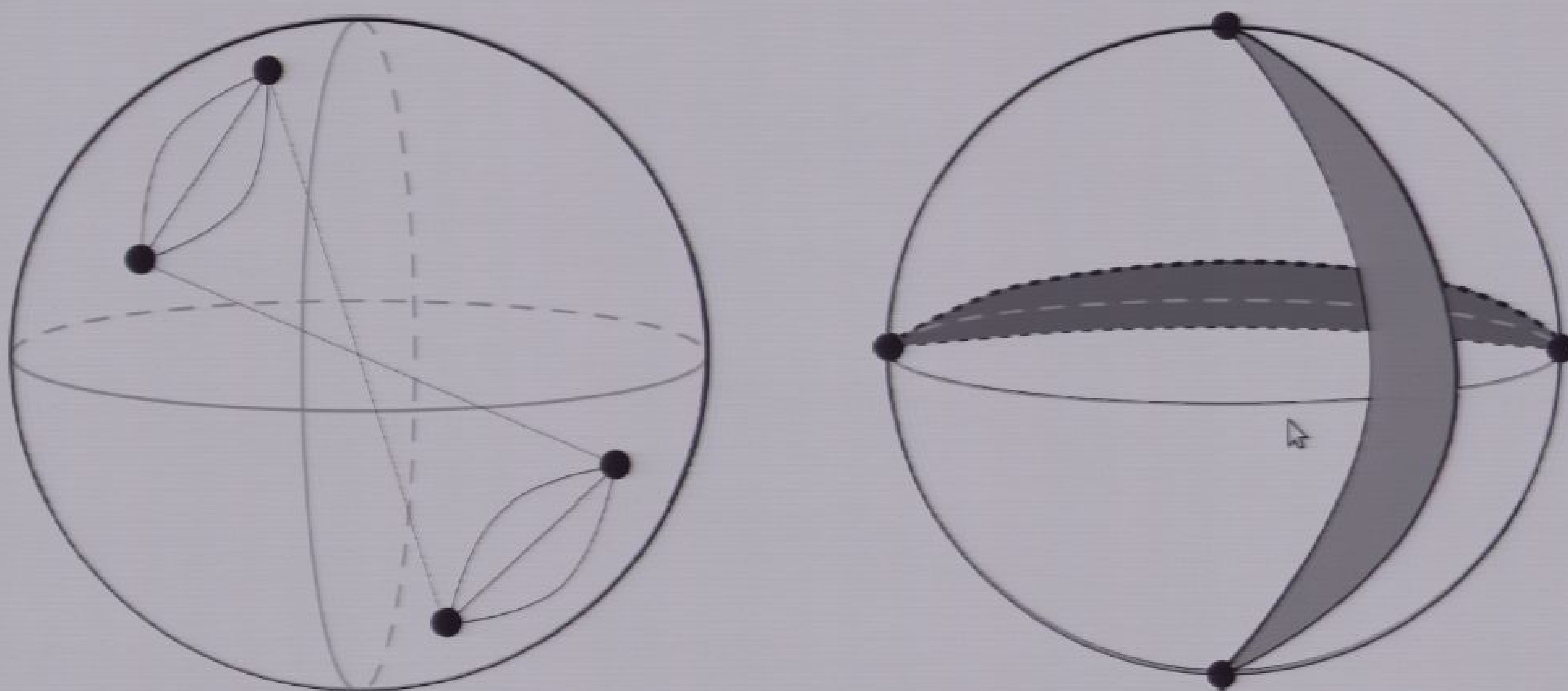

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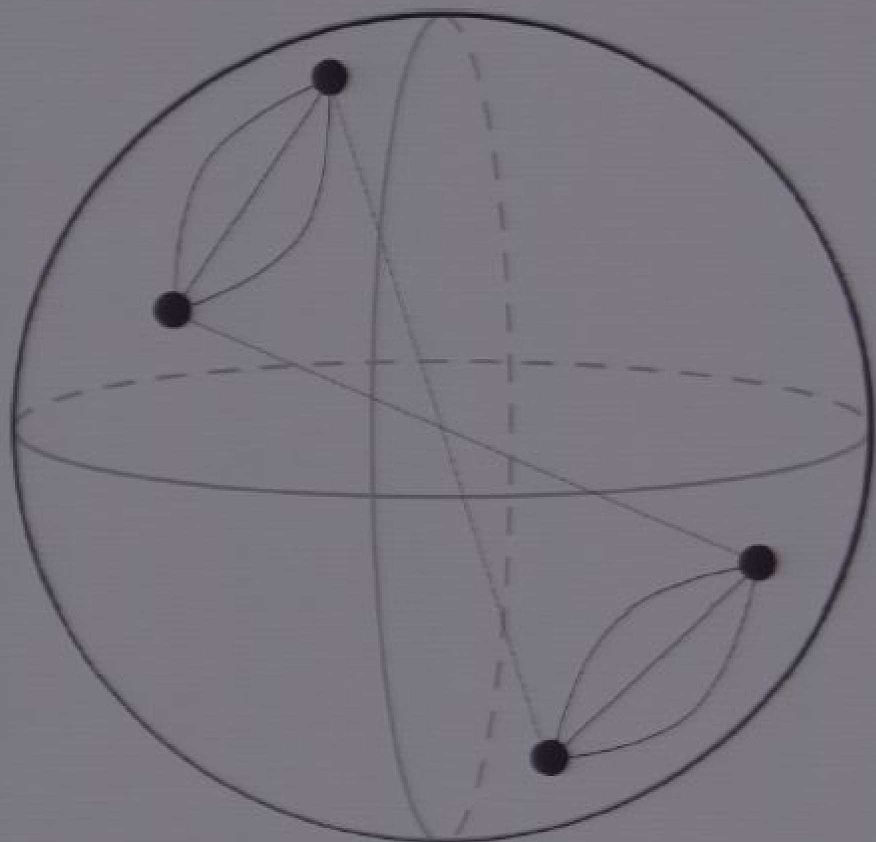

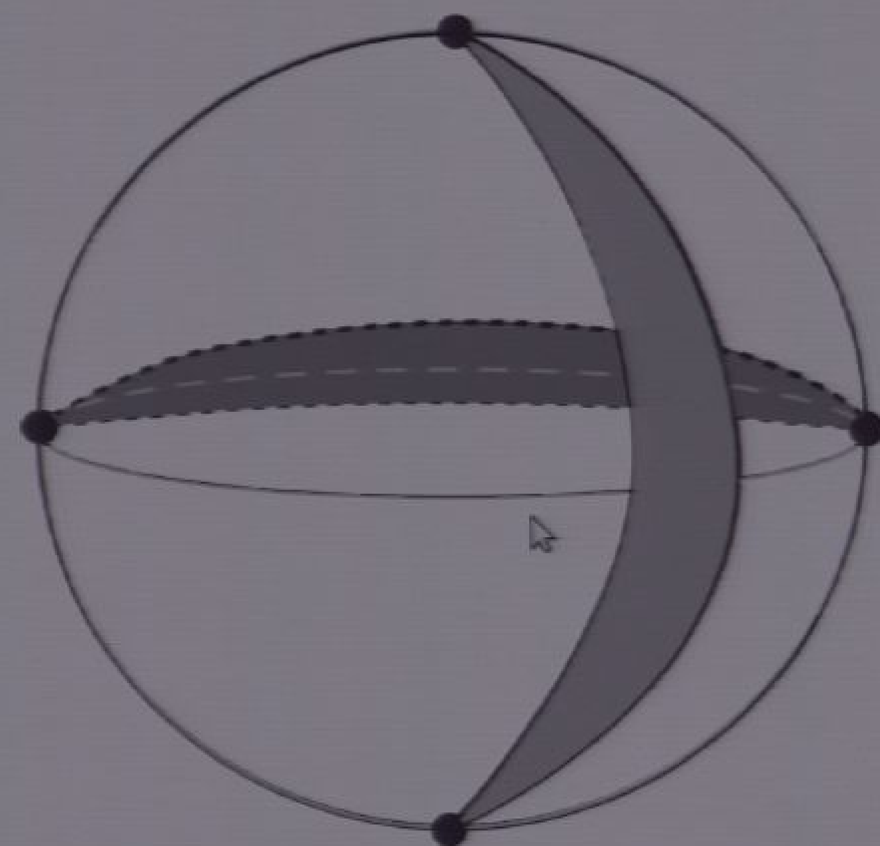

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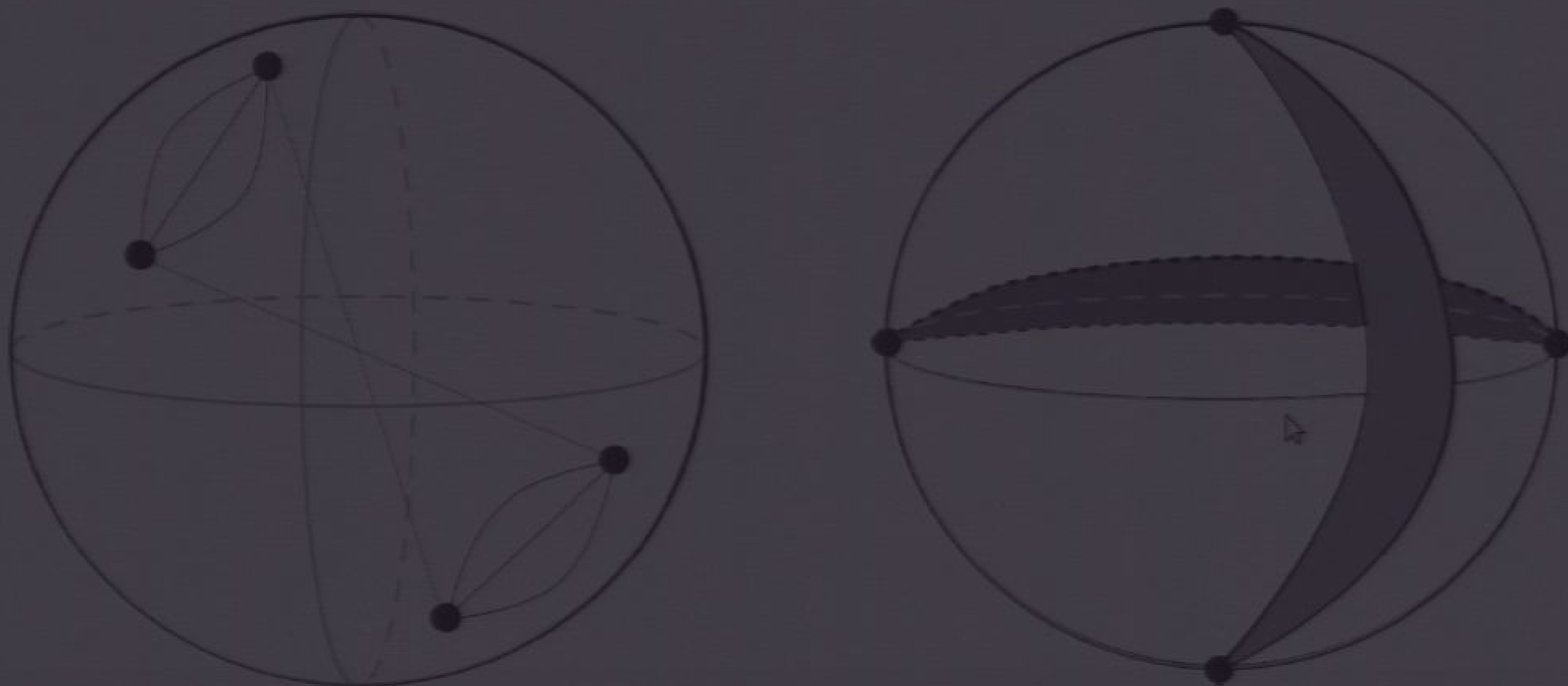

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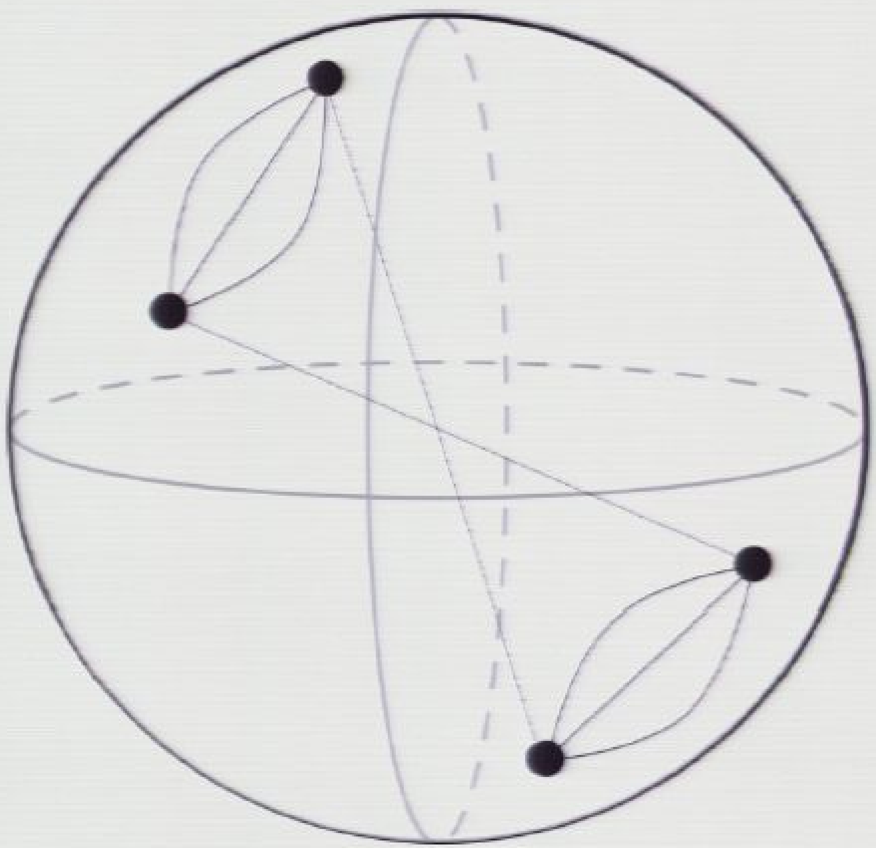

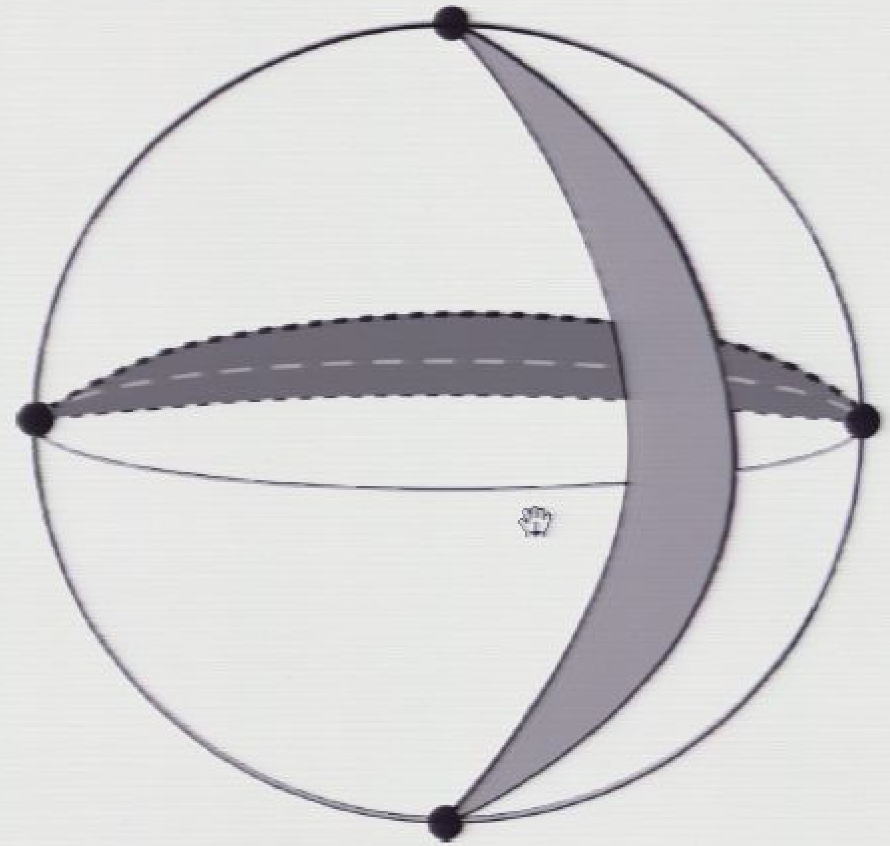

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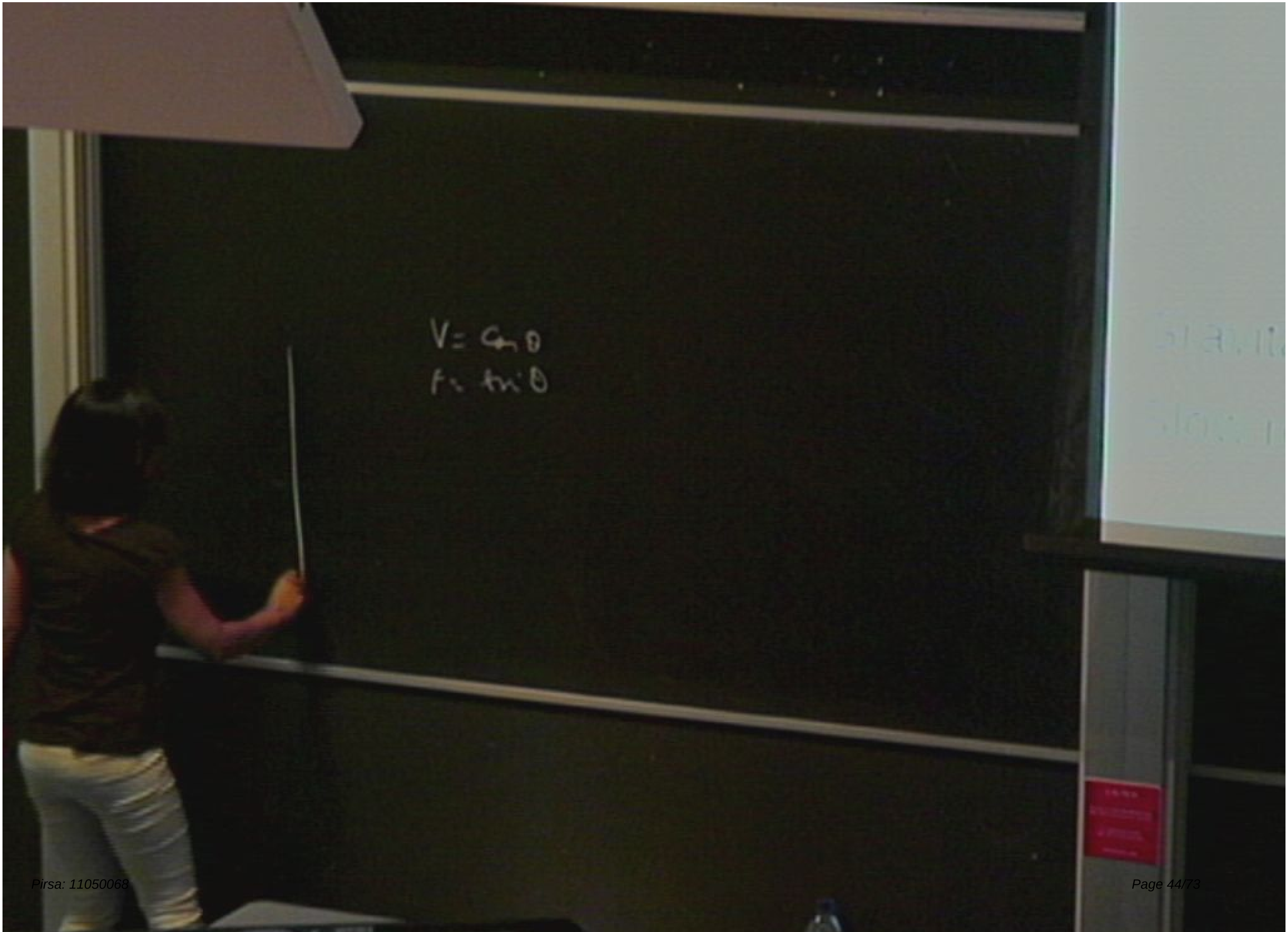
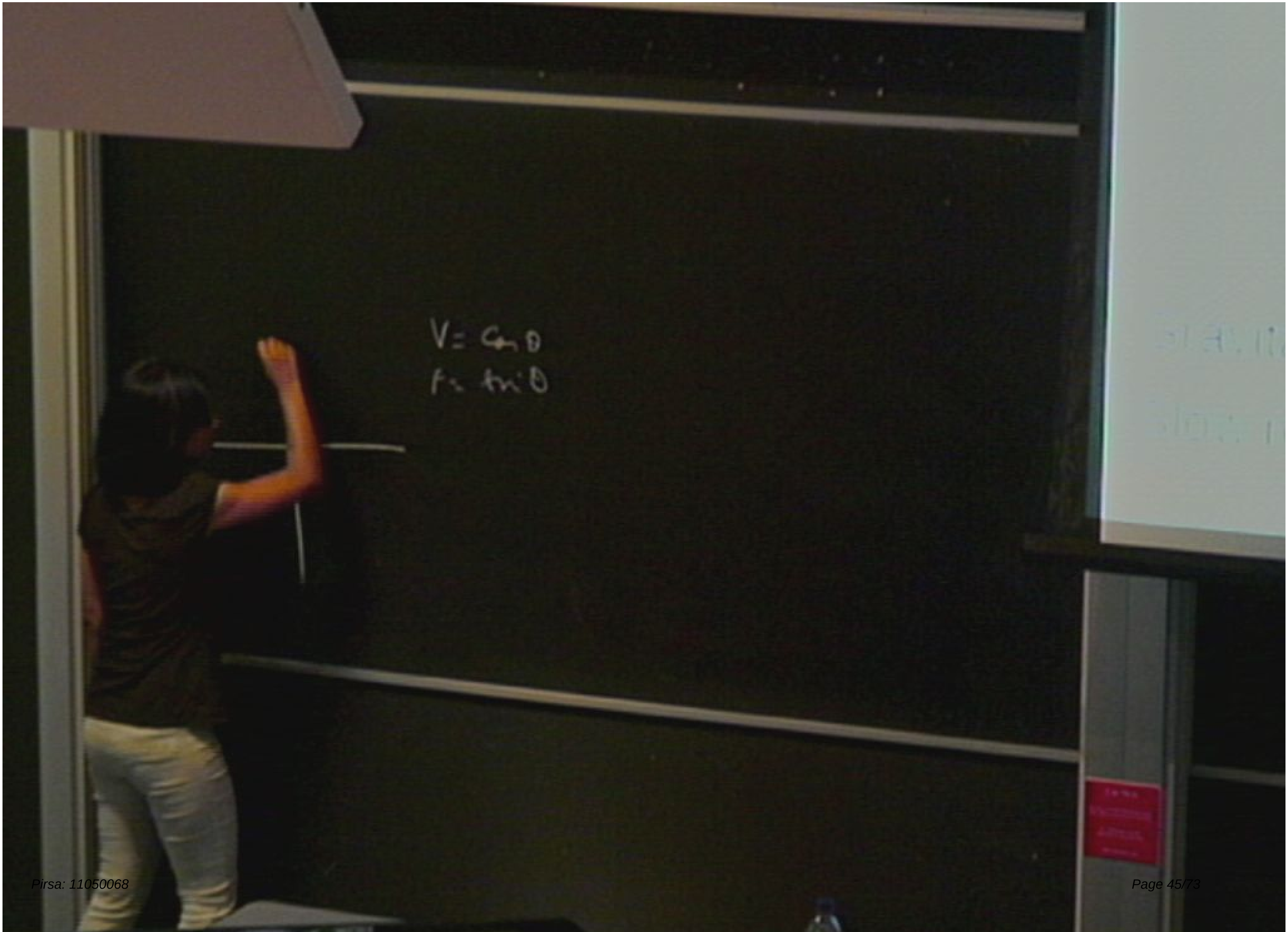
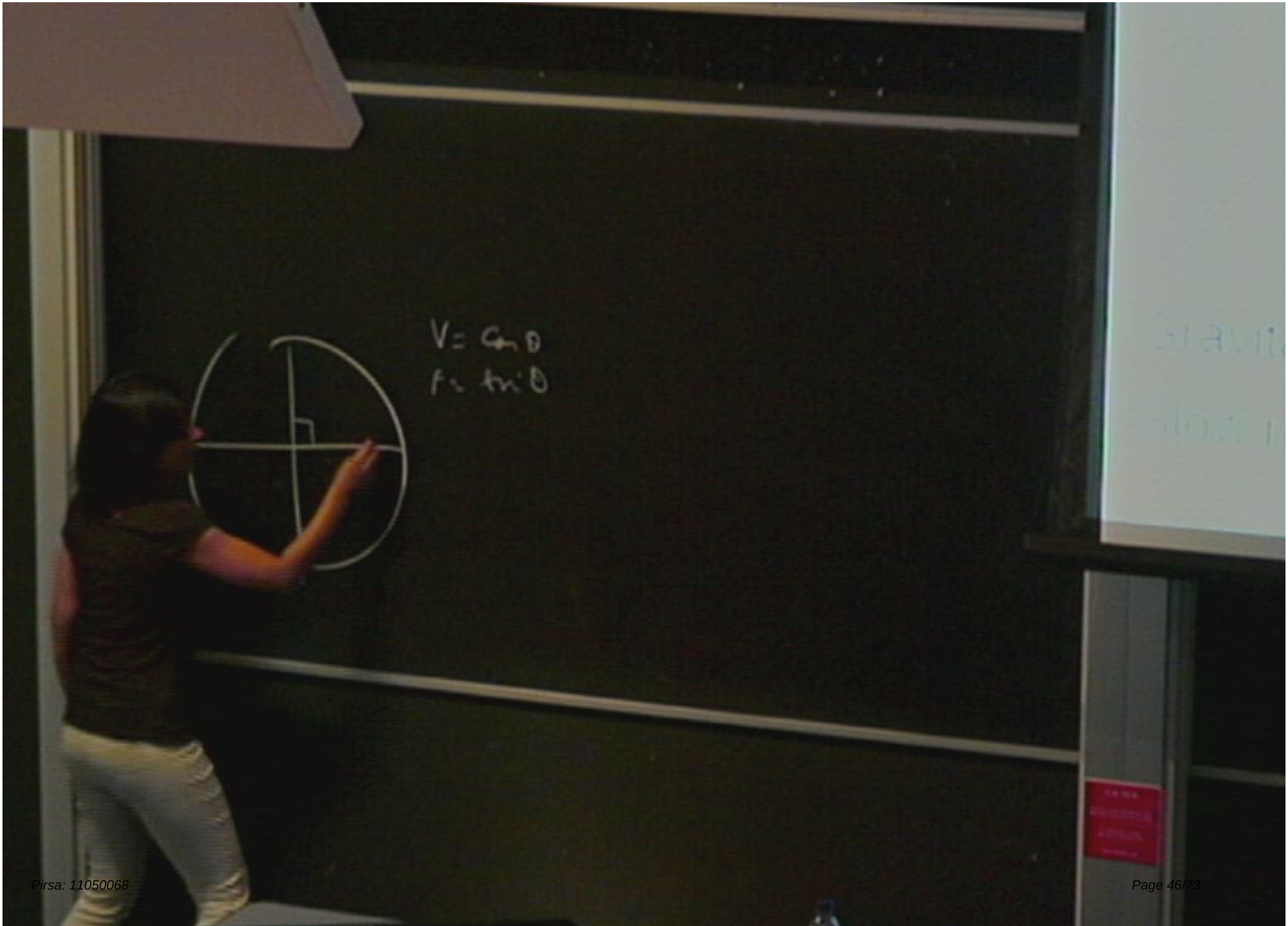
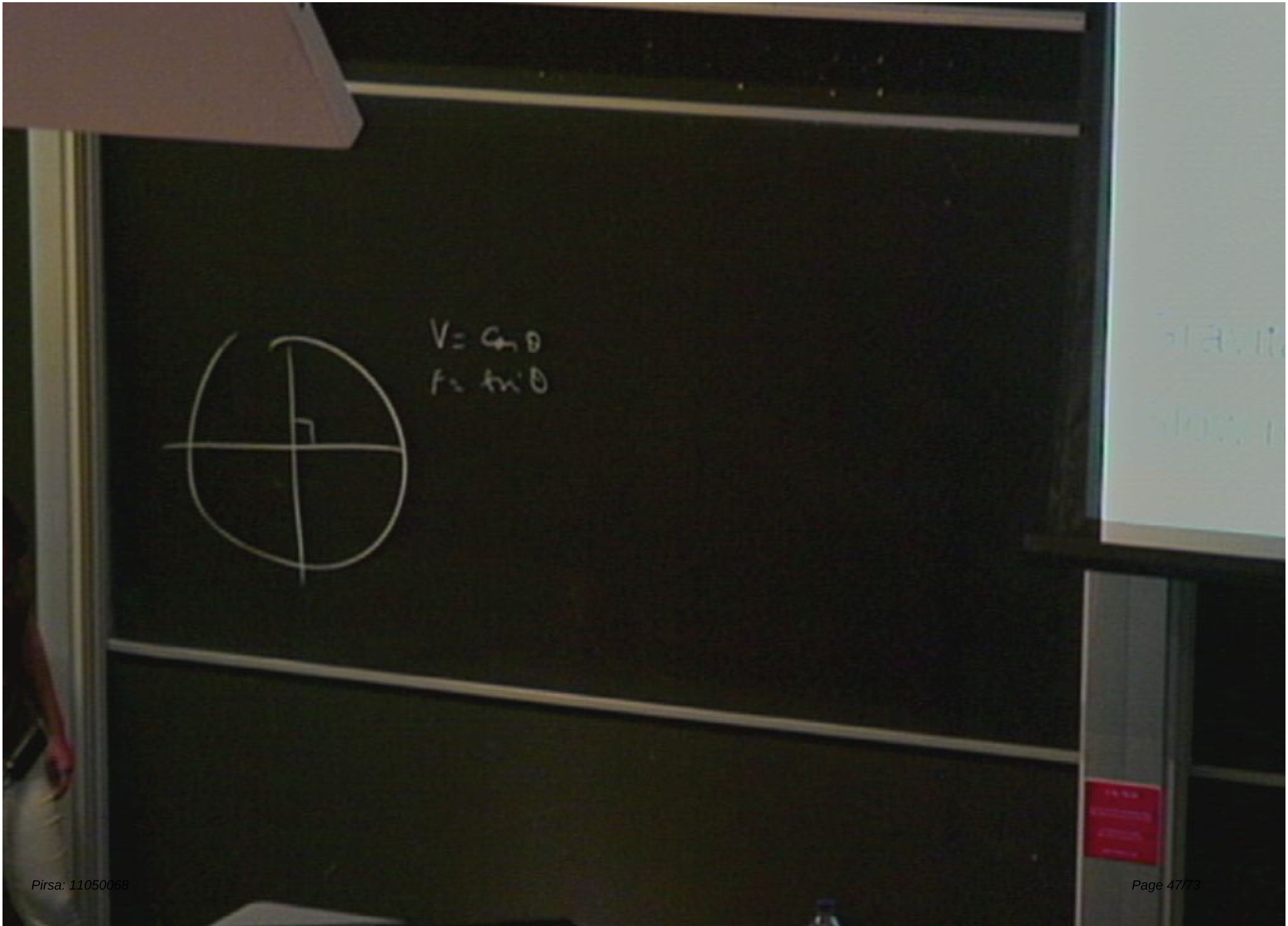
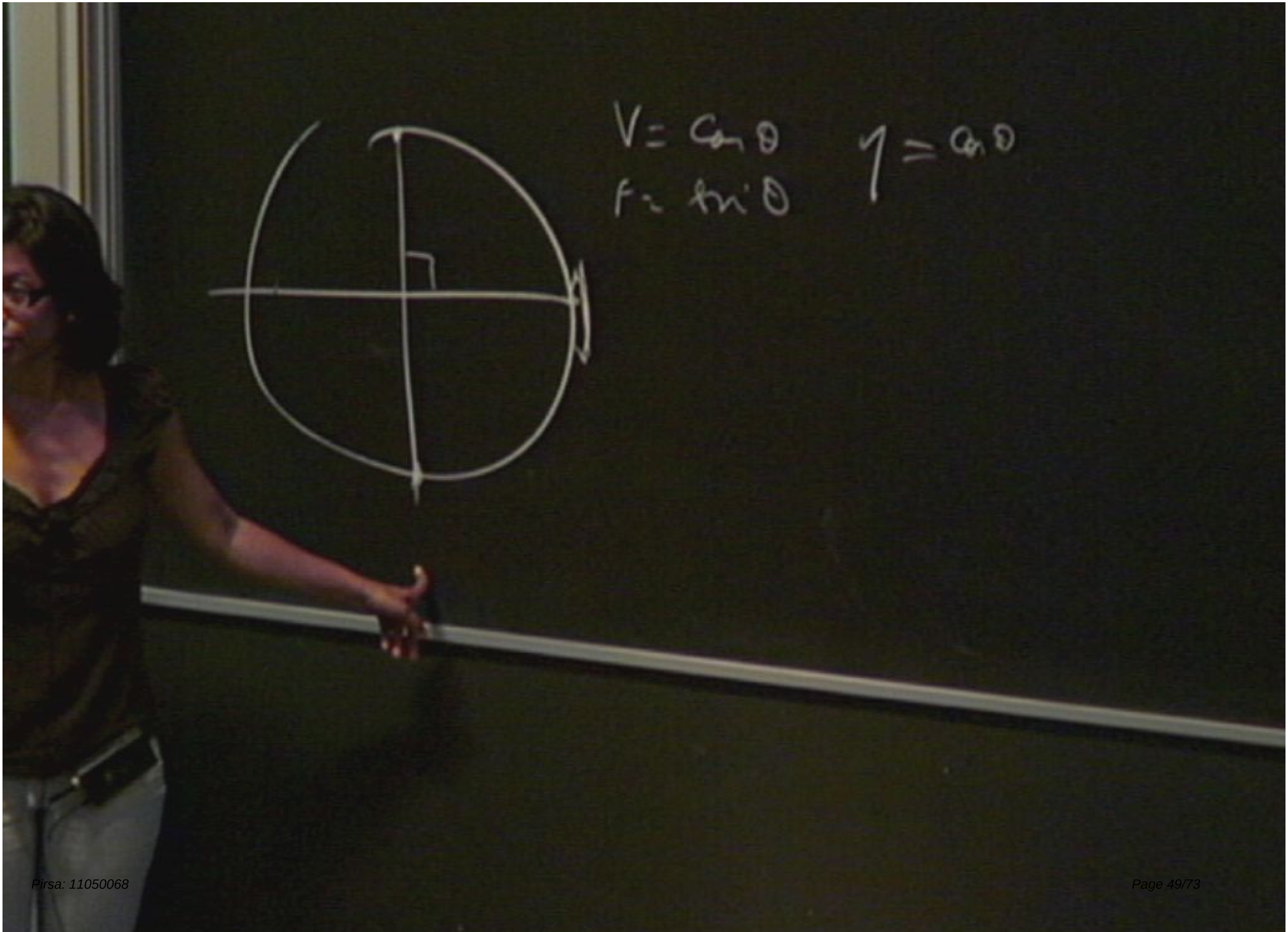


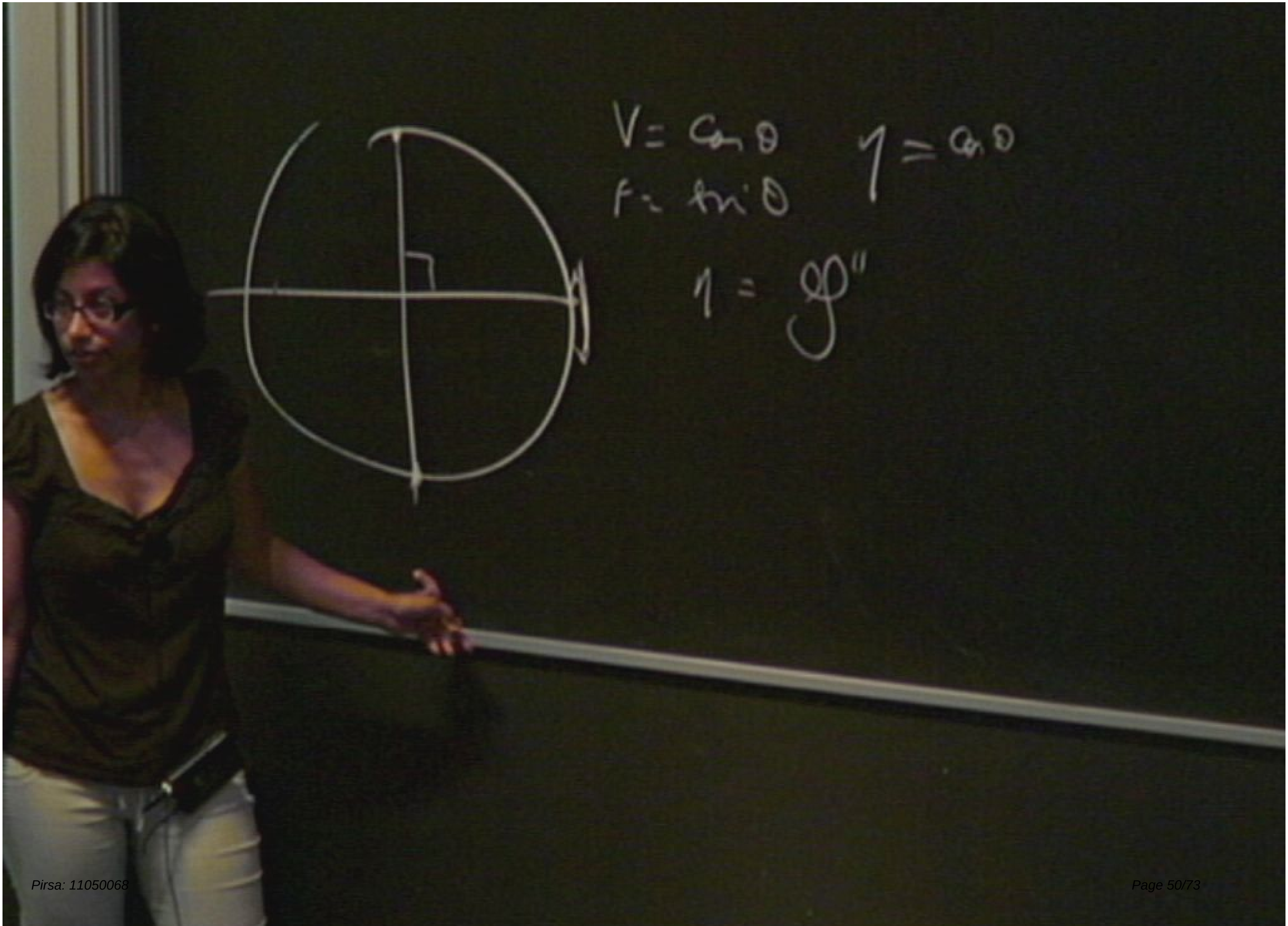




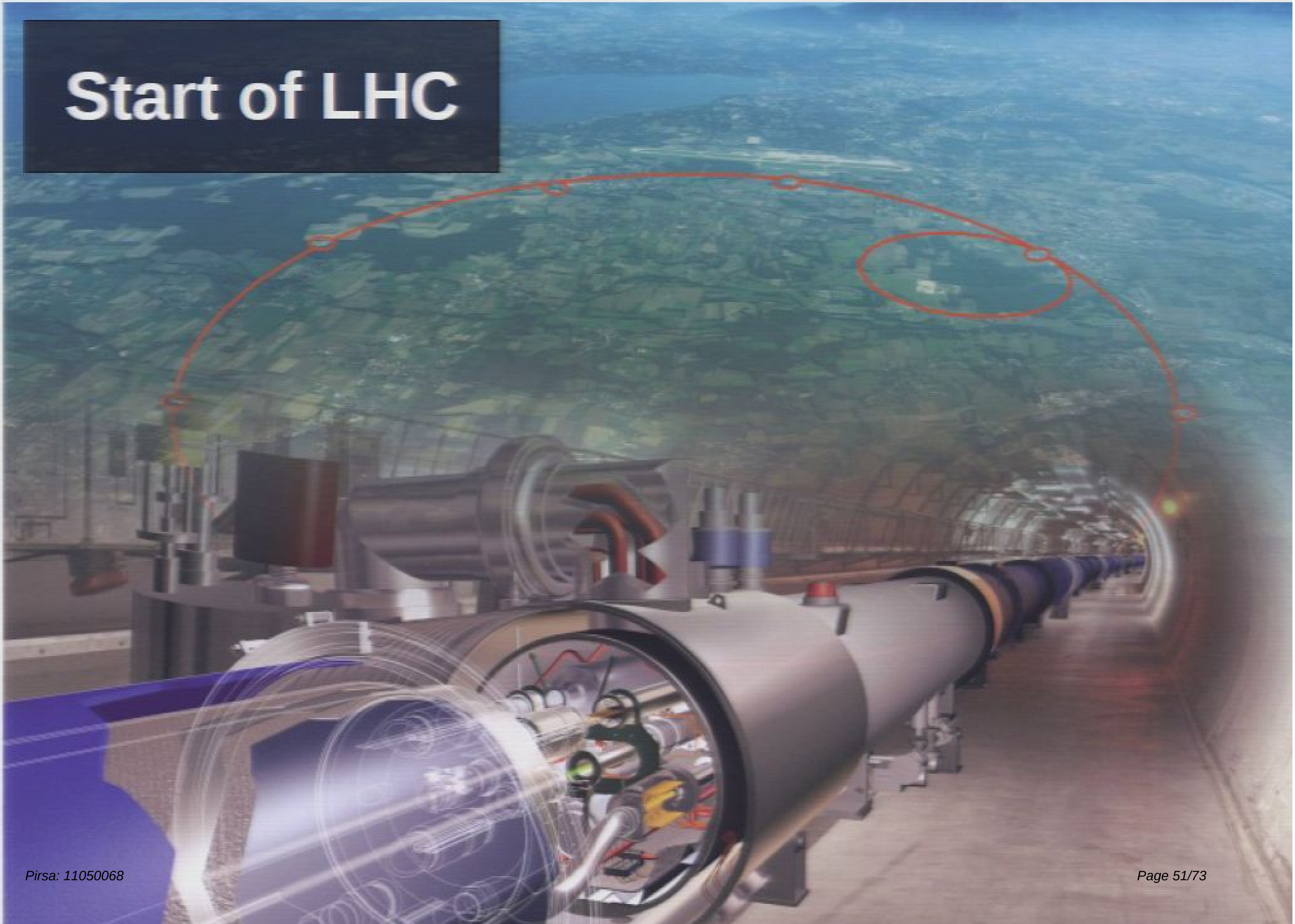


V = Cos θ
F = Sin θ


V = cos θ
F = sin θ
γ = cos θ


V = cos θ
F = sin θ

Start of LHC



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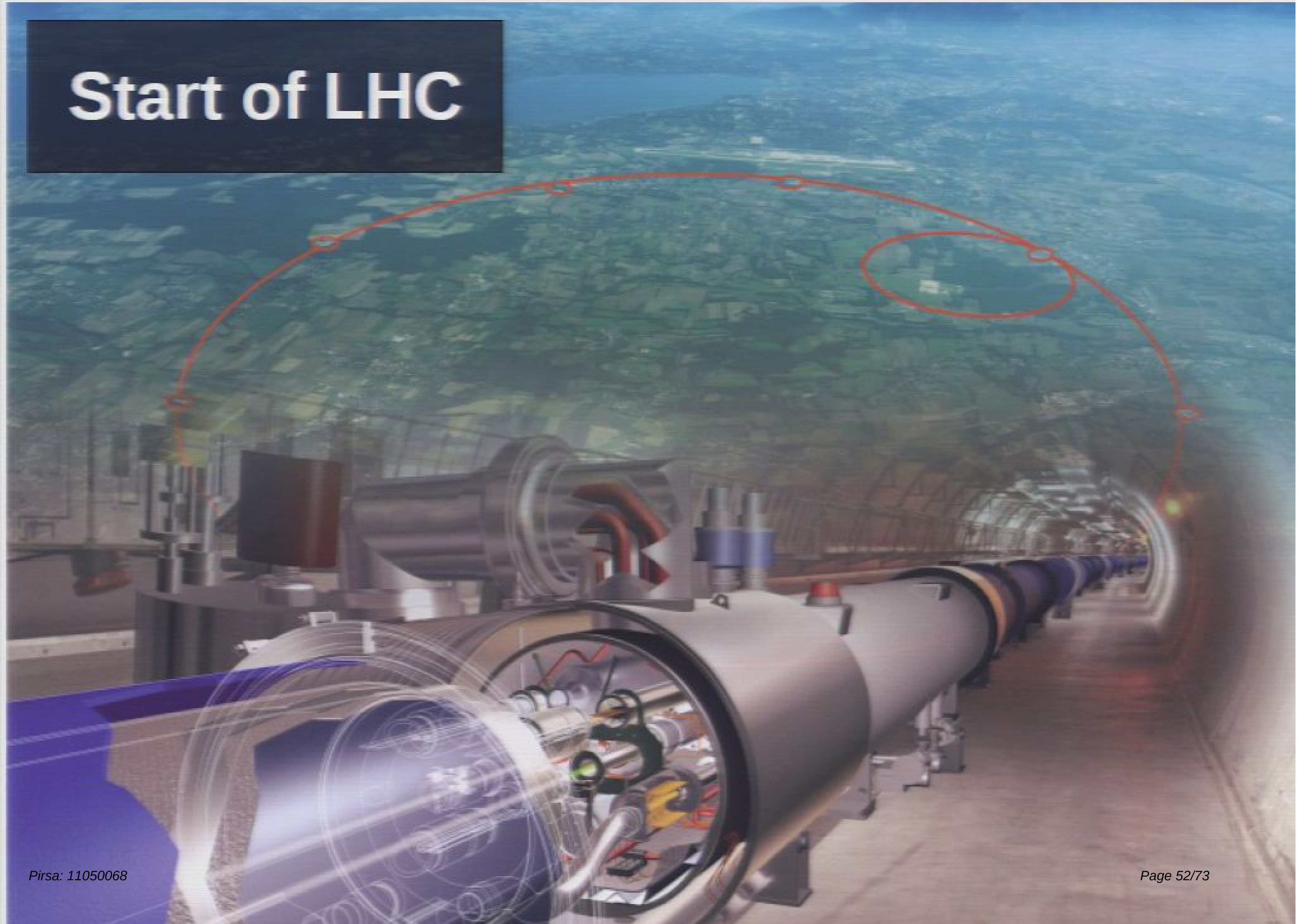


NATURALNESS, WHITHER?

COUNTING DARK MATTER AT THE LHC

Giudice, Gripaios, RM; to appear

Counting DM at the LHC

How to establish connection between DM and invisible particles at the LHC?

- Measure all properties
- Compute relic densities
- Compare with cosmology

UNFEASIBLE?!

Counting DM at the LHC

How to establish connection between DM and invisible particles at the LHC?

- Measure all properties
- Compute relic densities — UNFEASIBLE?!
- Compare with cosmology

Count number of invisibles

If singly produced, not stabilized by symmetry, must decay by reverse process!

Contains information on

# of particles in final state

> Phase space integral:
> KINEMATICS

$$\sigma\left(pp \to CD\right) = \sum_{a,b=q,g} \int f_a f_b \int_{\mathrm{PS}} \left| M\left(ab \to CD\right)\right|^2$$

How to extract this independently of 'nuisance parameters'?

Transverse Mass

Define transverse mass for M ⇨ m V +n I

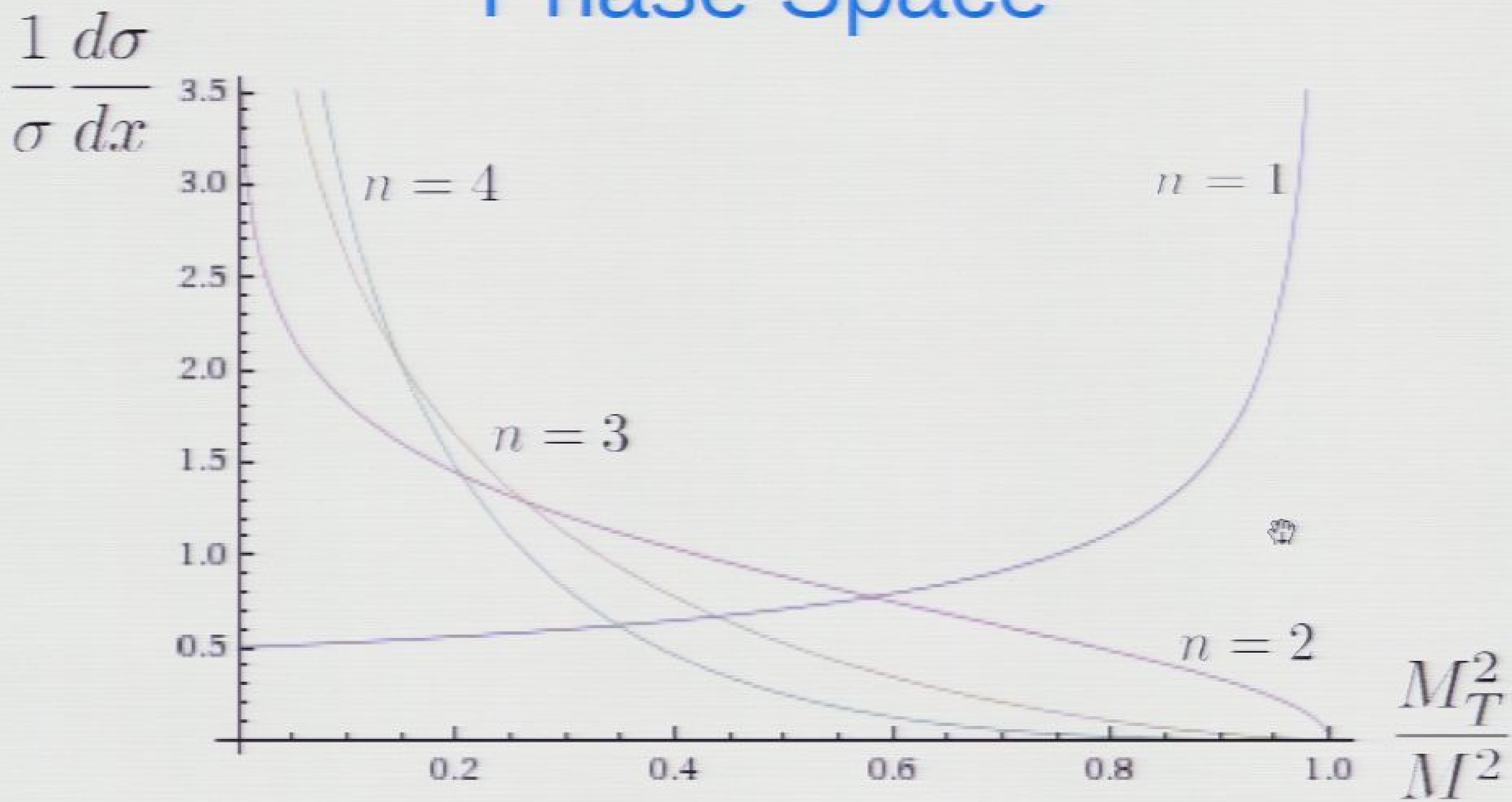
$$m_T^2 = m_V^2 + 2\left(\sqrt{\not{p}_T^2\,(p_T^2 + m_V^2)} - \not{p}_T . p_T\right)$$

- Has endpoint at $m_T = M$
- Can show that to be at endpoint, all invisibles need to be aligned in lab frame

Can compute end-point behavior from phase space

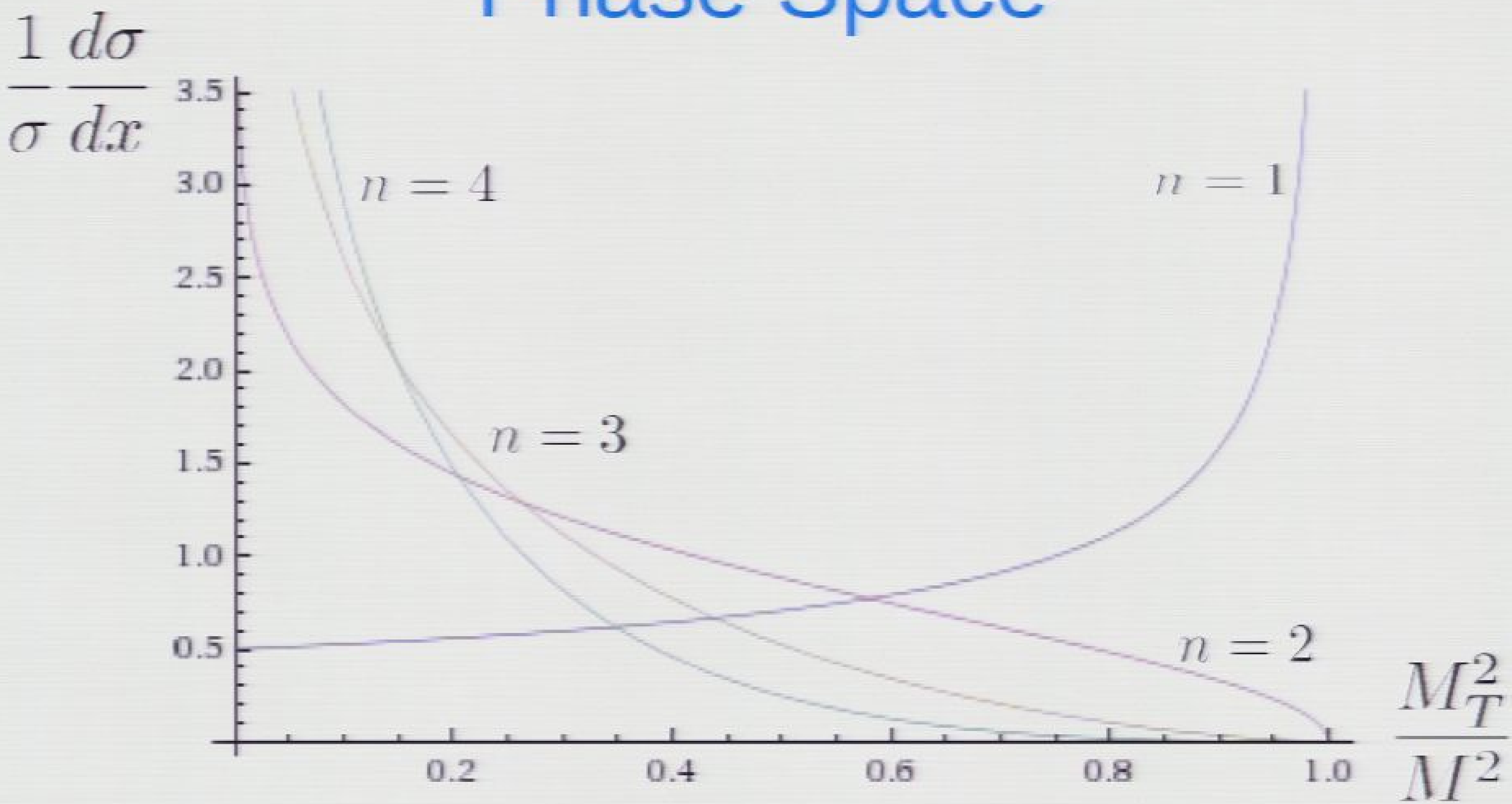
For massless invisibles: $\frac{M^2}{\Phi_{n-m}} \frac{d\Phi_{n+m}}{dm_T^2} \sim \left(1 - \frac{m_T^2}{M^2}\right)^{n-\frac{3}{2}}$

Phase Space



HUGE difference between endpoint behaviour for n=1 and n=2!

Phase Space



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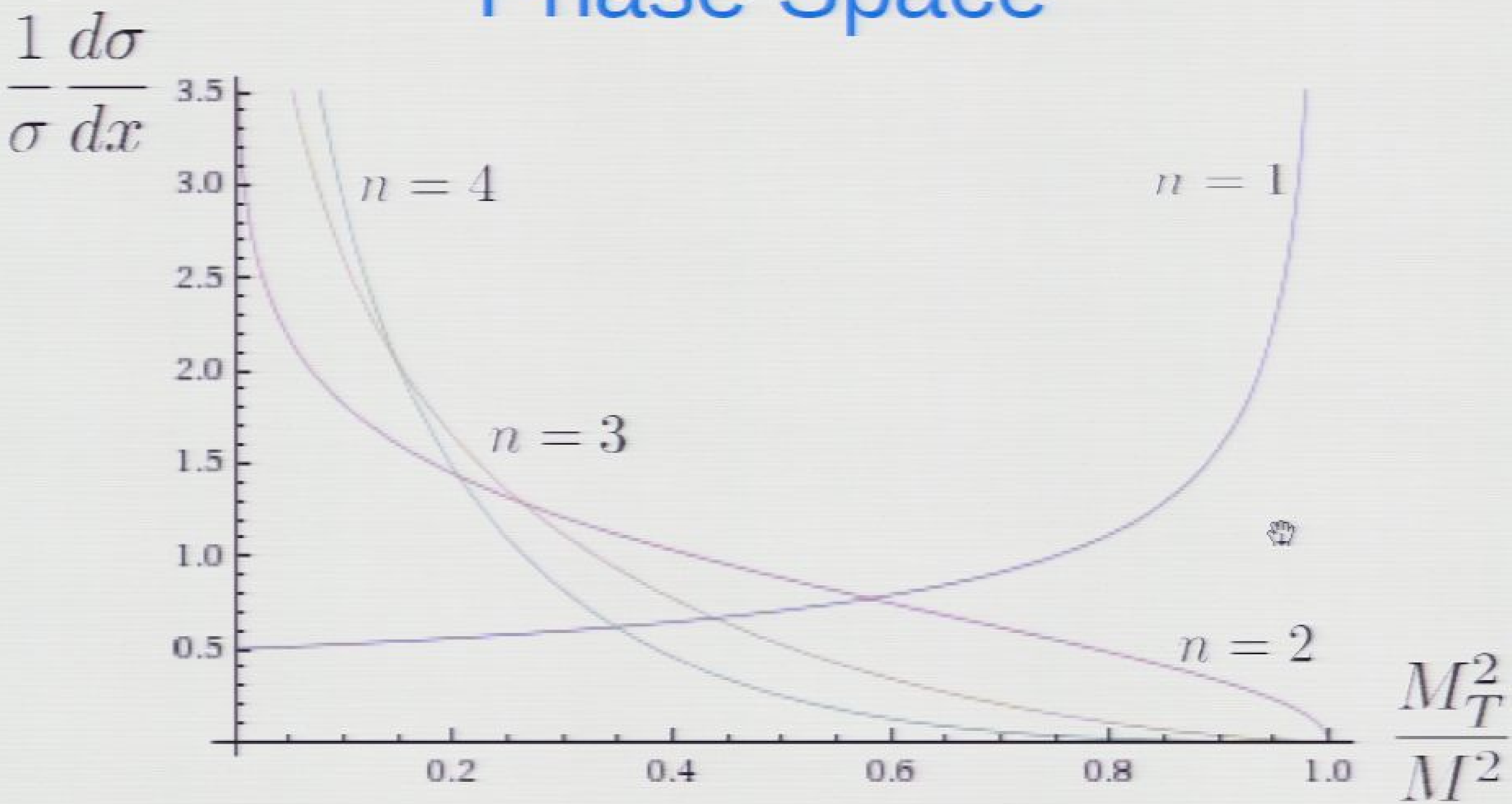
Phase Space



Power law fall-off independent of # and mass of visibles, cascades with onshell intermediates, longitudinal boosts of CM frame

Transverse Mass (reprise)

If DM stabilized by some symmetry, expect new physics to be pair produced at LHC

More sensible to use m_{T_2} since manifestly invariant under back-to-back boosts of parents

$$m_{T_2}^2 = 2\left(|p_{1T}|\,|p_{2T}| + p_{1T}\cdot p_{2T}\right)$$

Has same power-law fall-off near endpoint

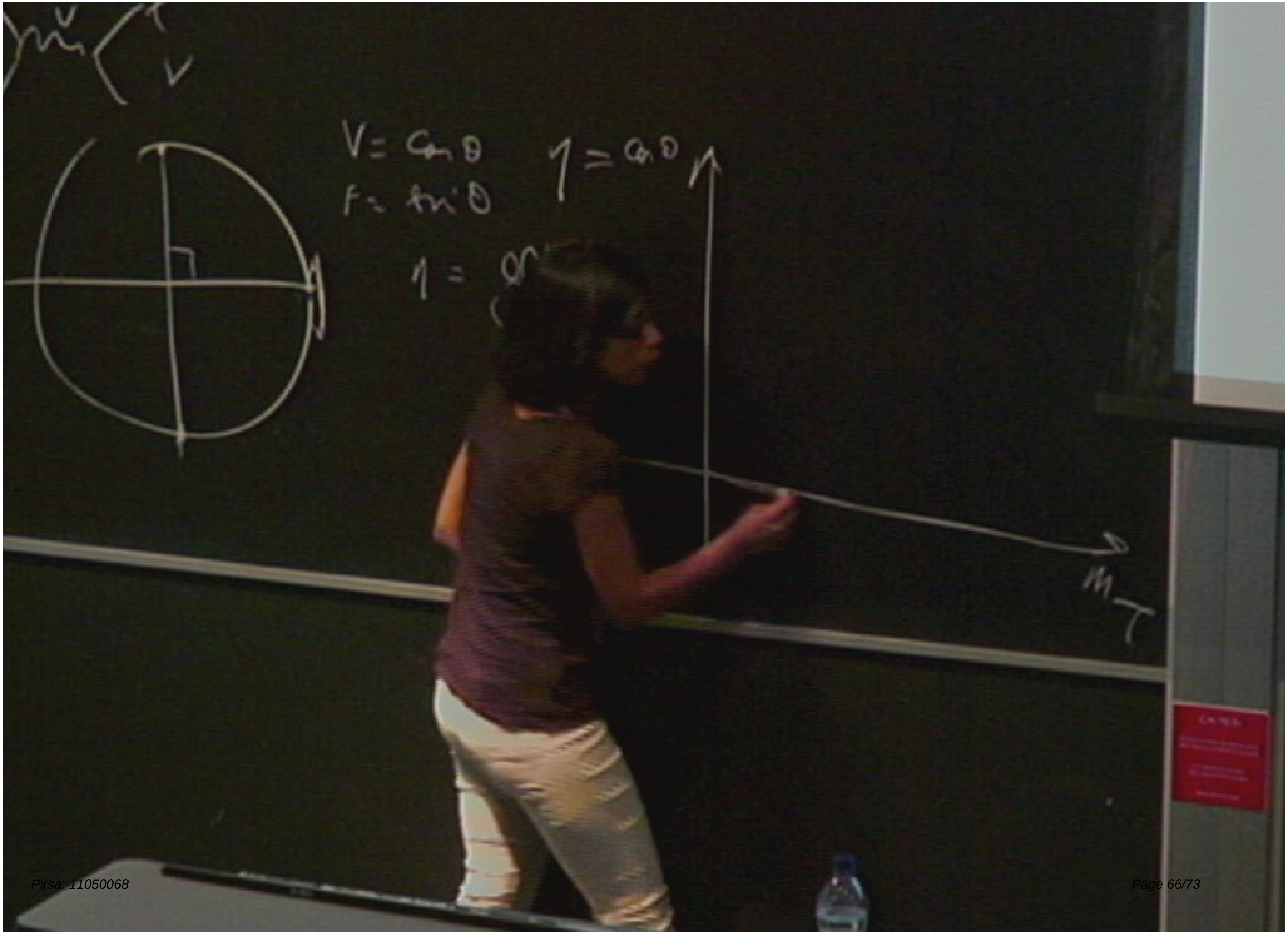
Proposal

Measure # of invisibles by fitting transverse mass distributions near endpoint

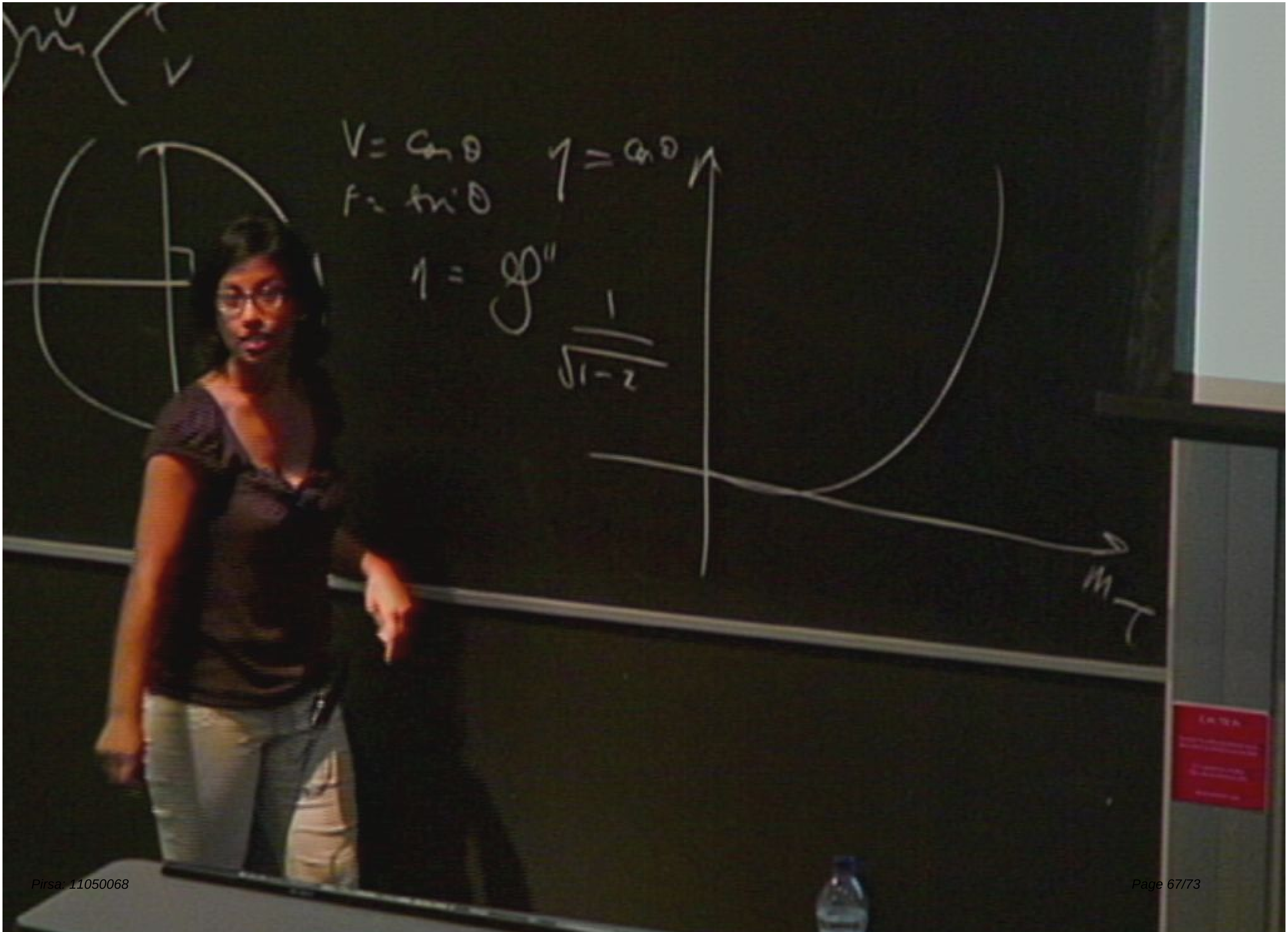
Can get information on DM stabilization symmetry, and decay topologies

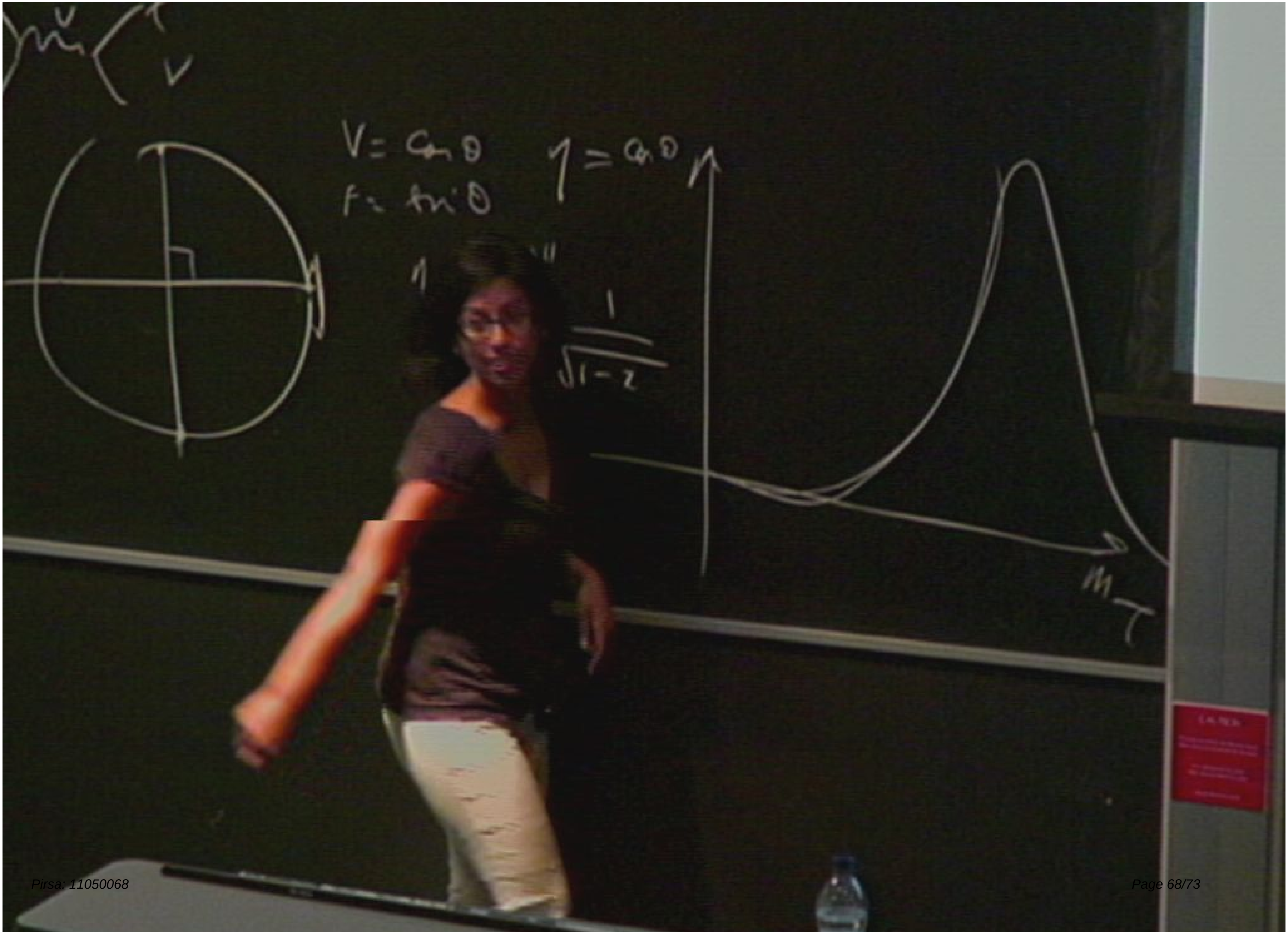
Caveats:

- Massive invisibles
- Finite widths
- Experimental resolutions



V = cos θ
F = sin θ
1 = cos θ


V = cos θ



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