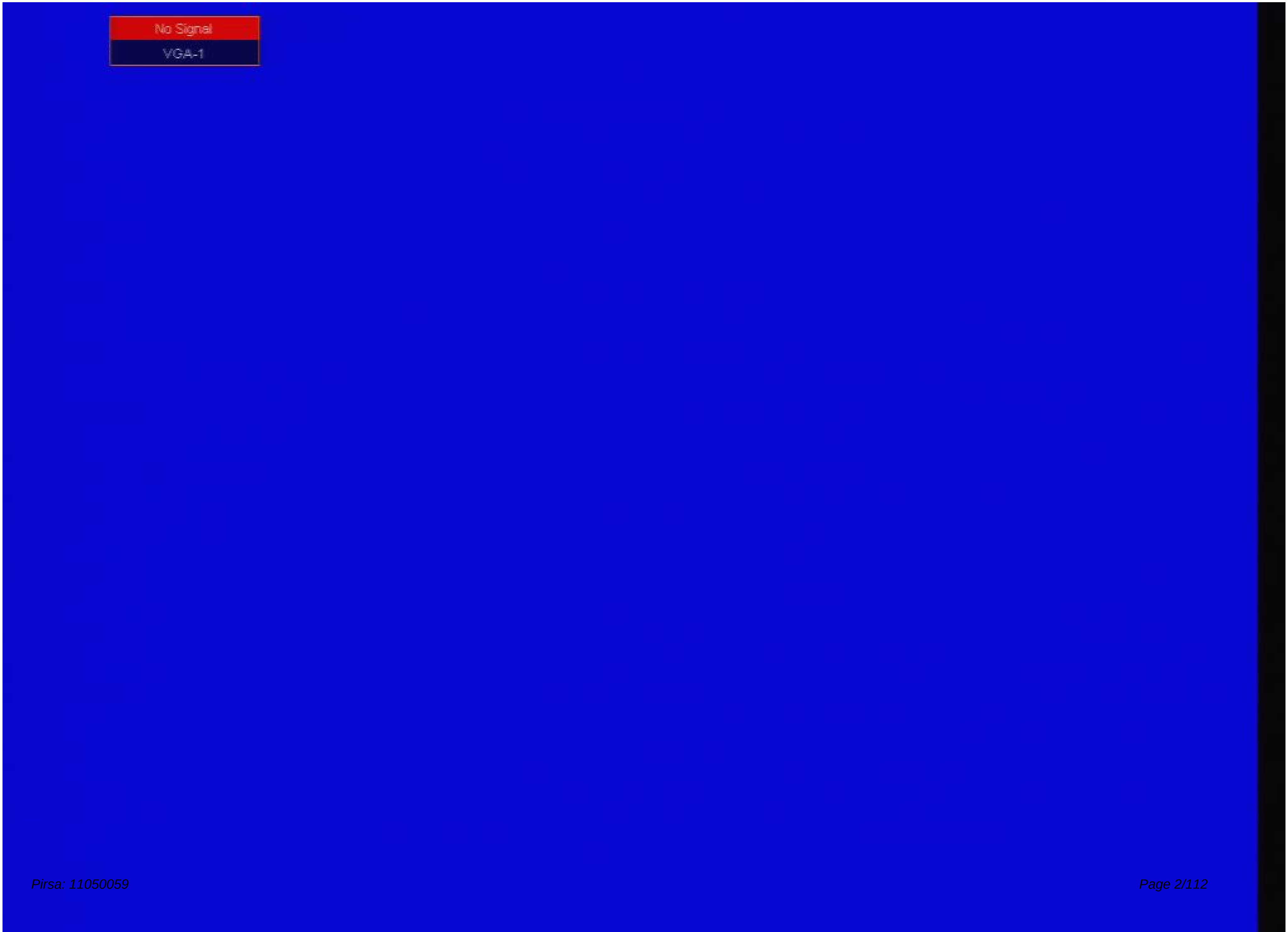
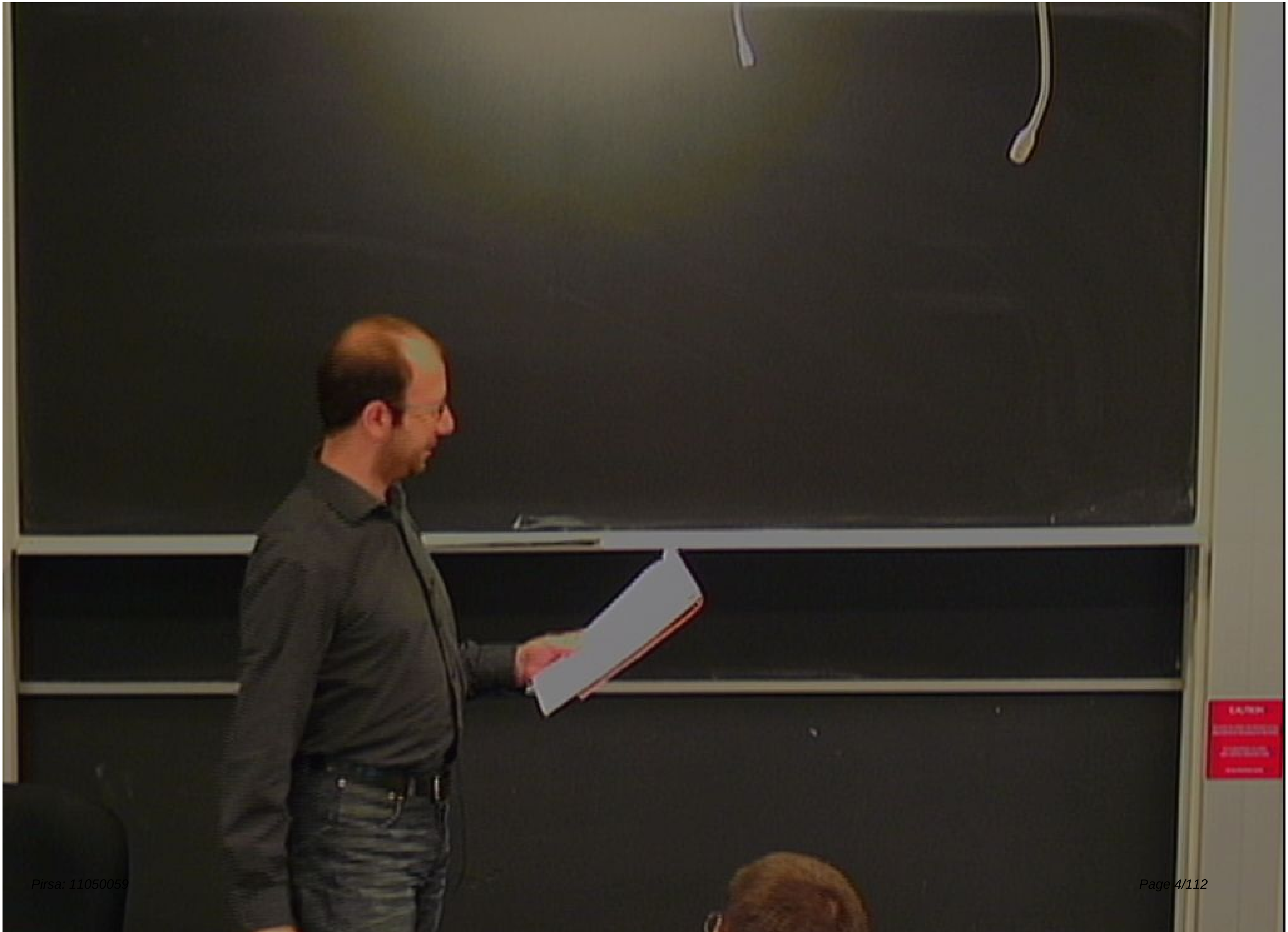
Title: An introduction to the pure-spinor formalism for the superstring - Part 1

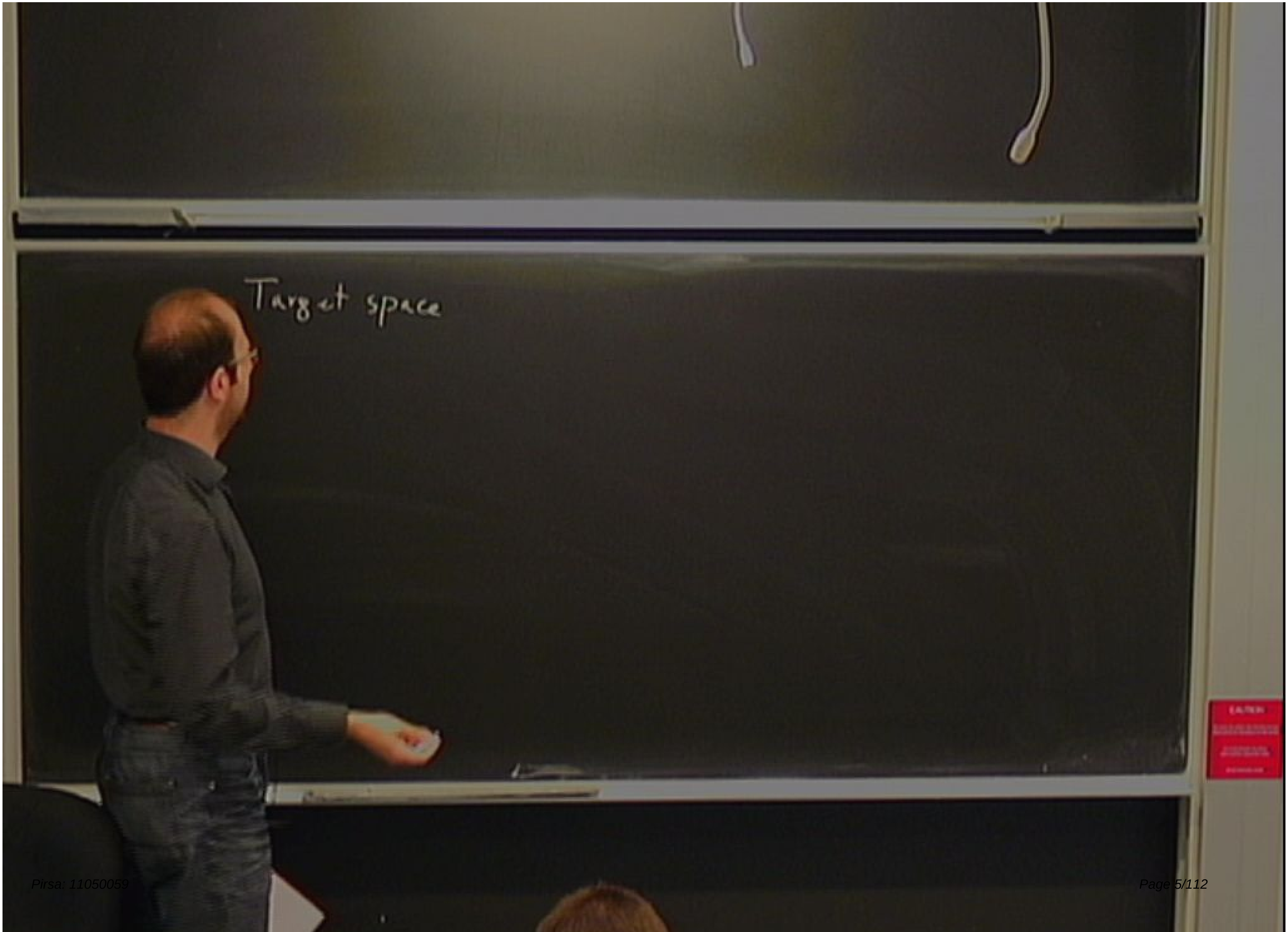
Date: May 11, 2011 11:00 AM

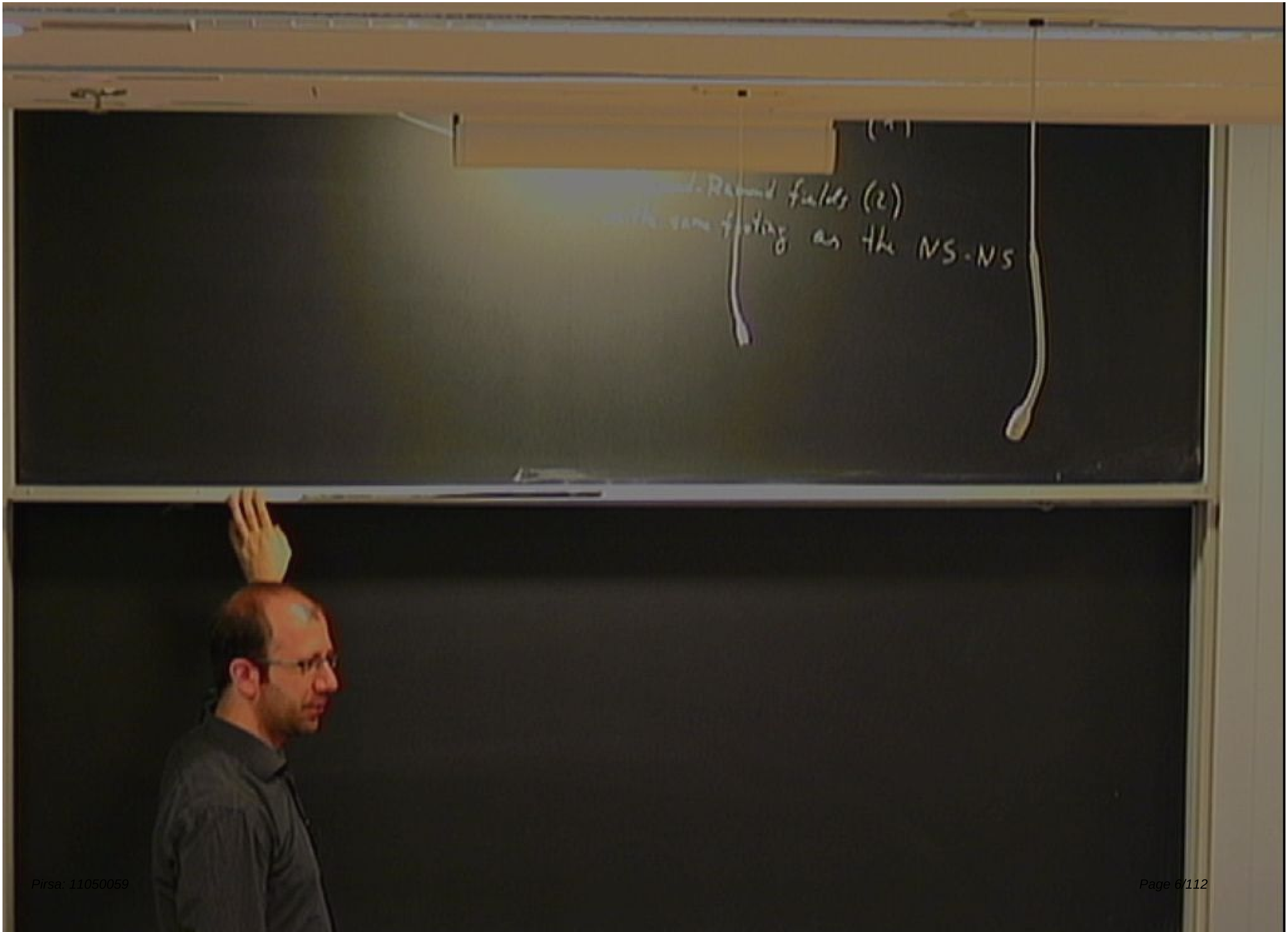
URL: http://pirsa.org/11050059

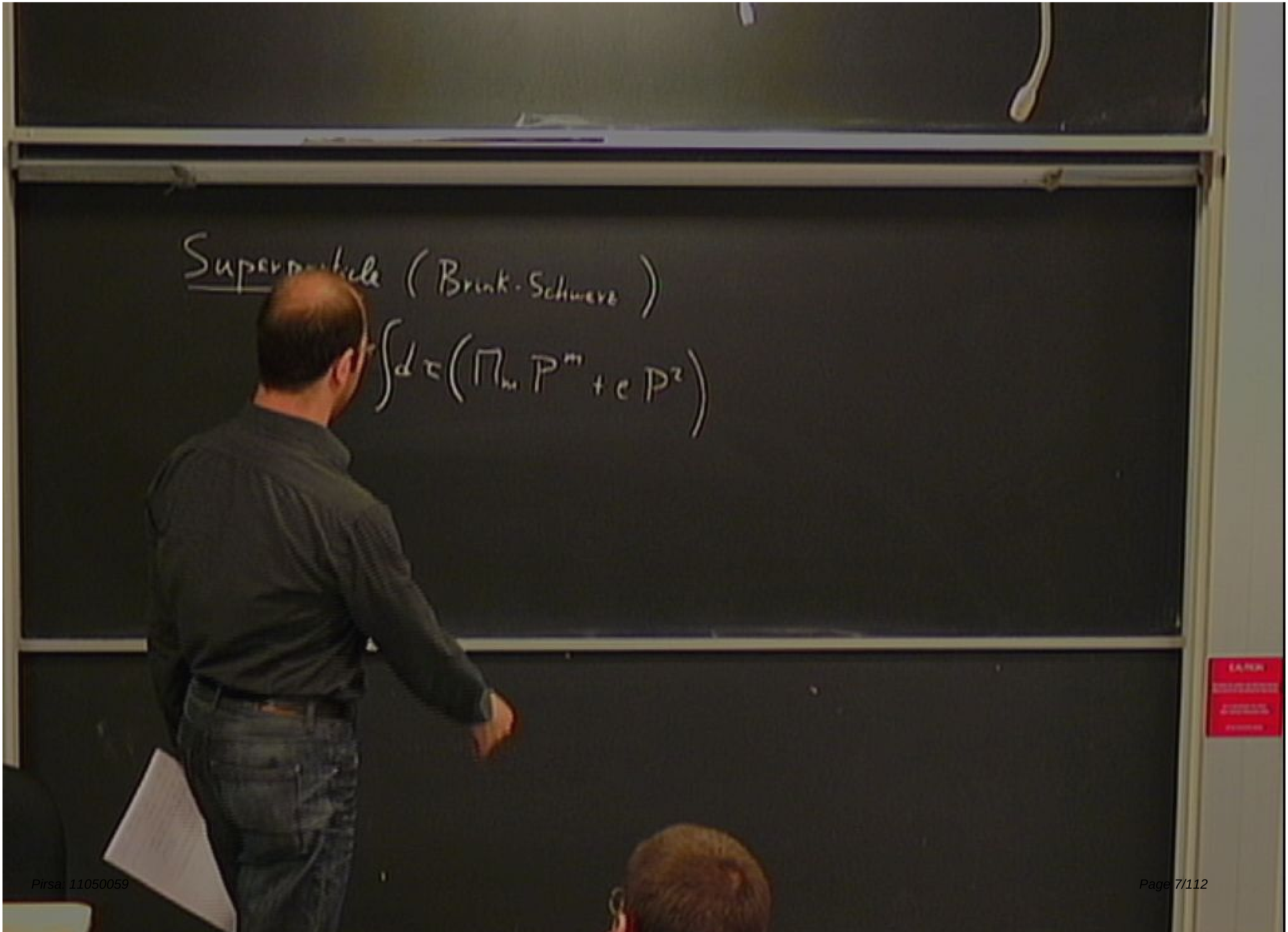
Abstract: Pure spinors, BRST cohomology and tree-level amplitudes

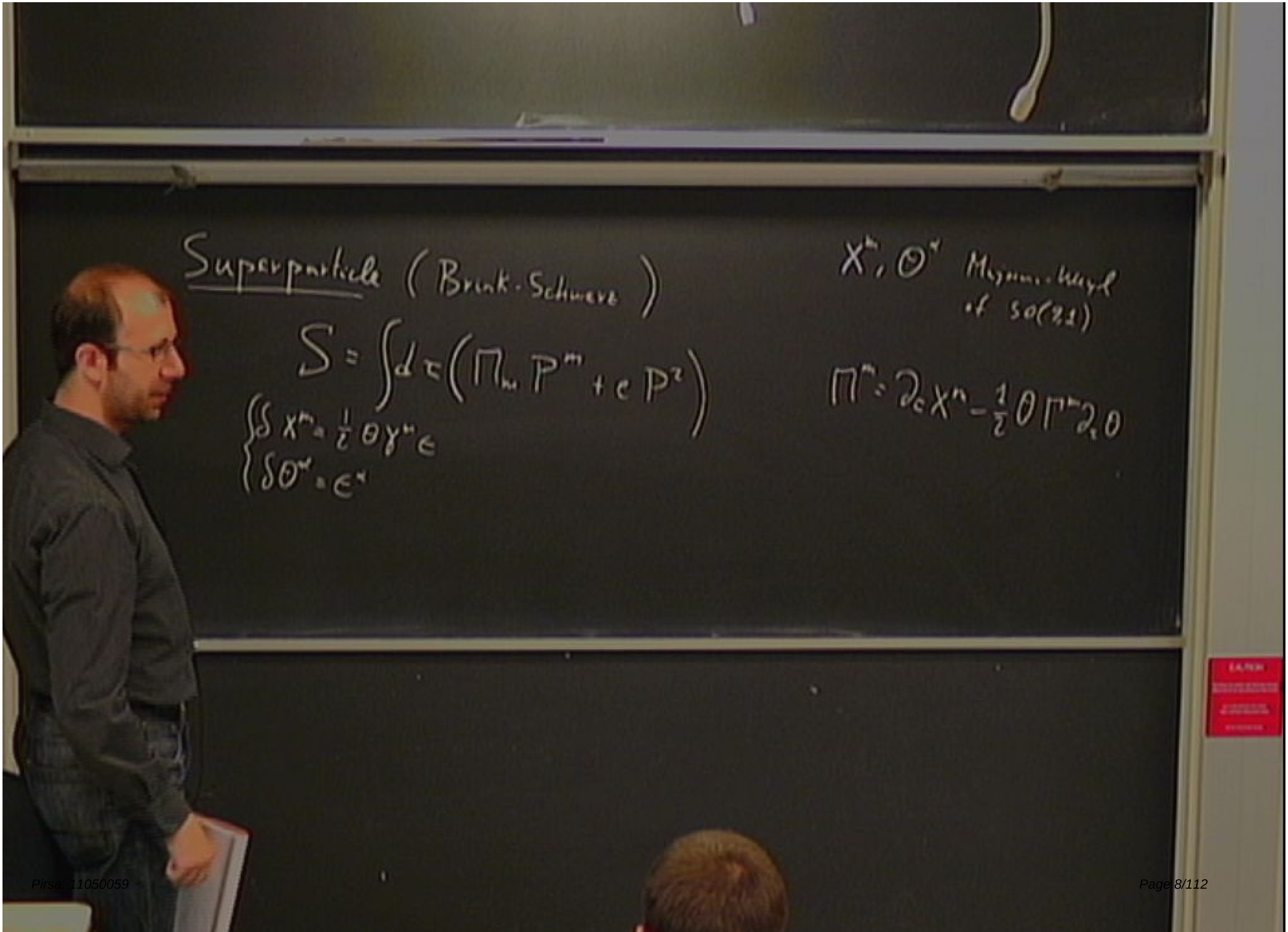

No Signal
VGA-1

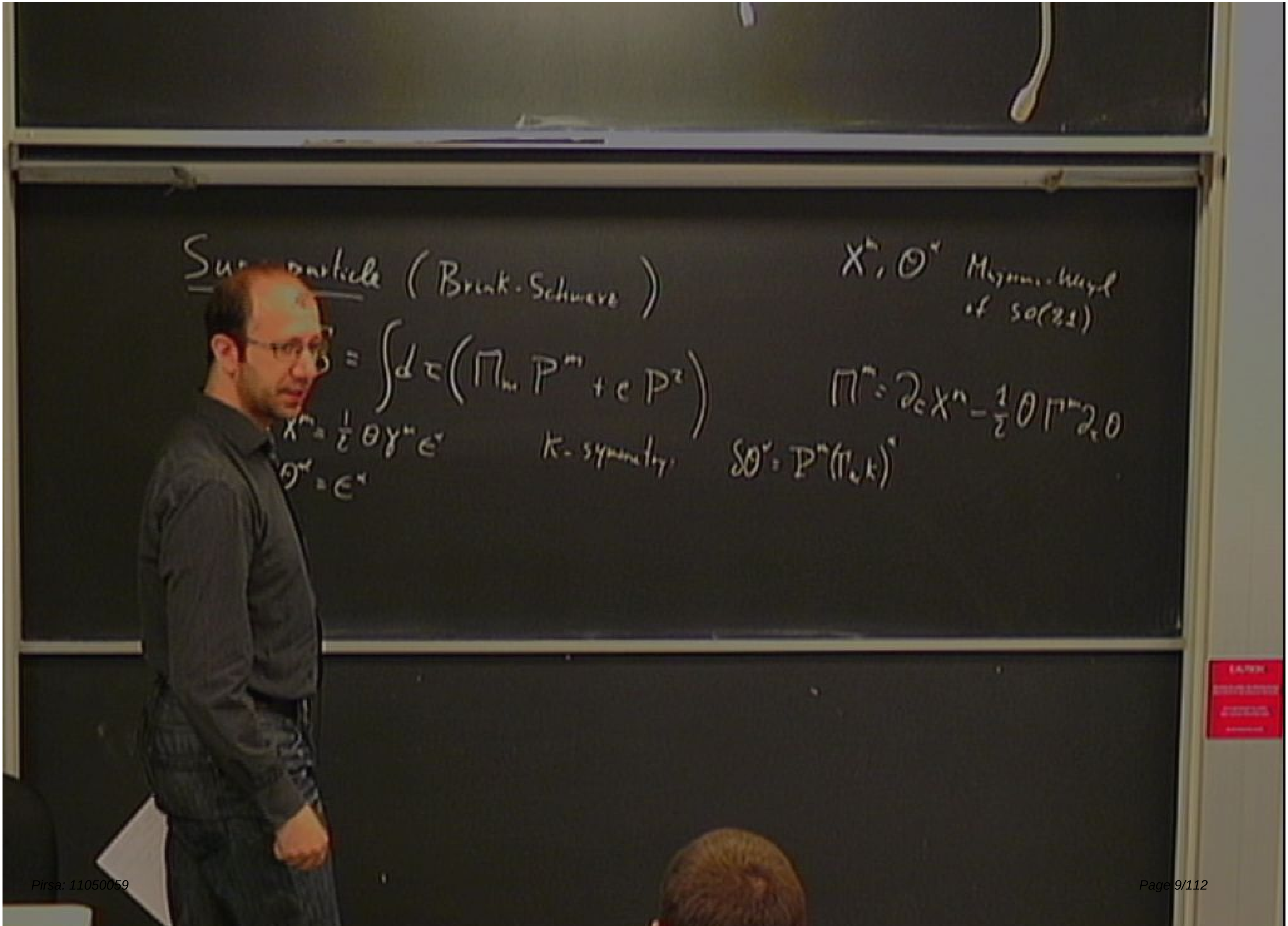


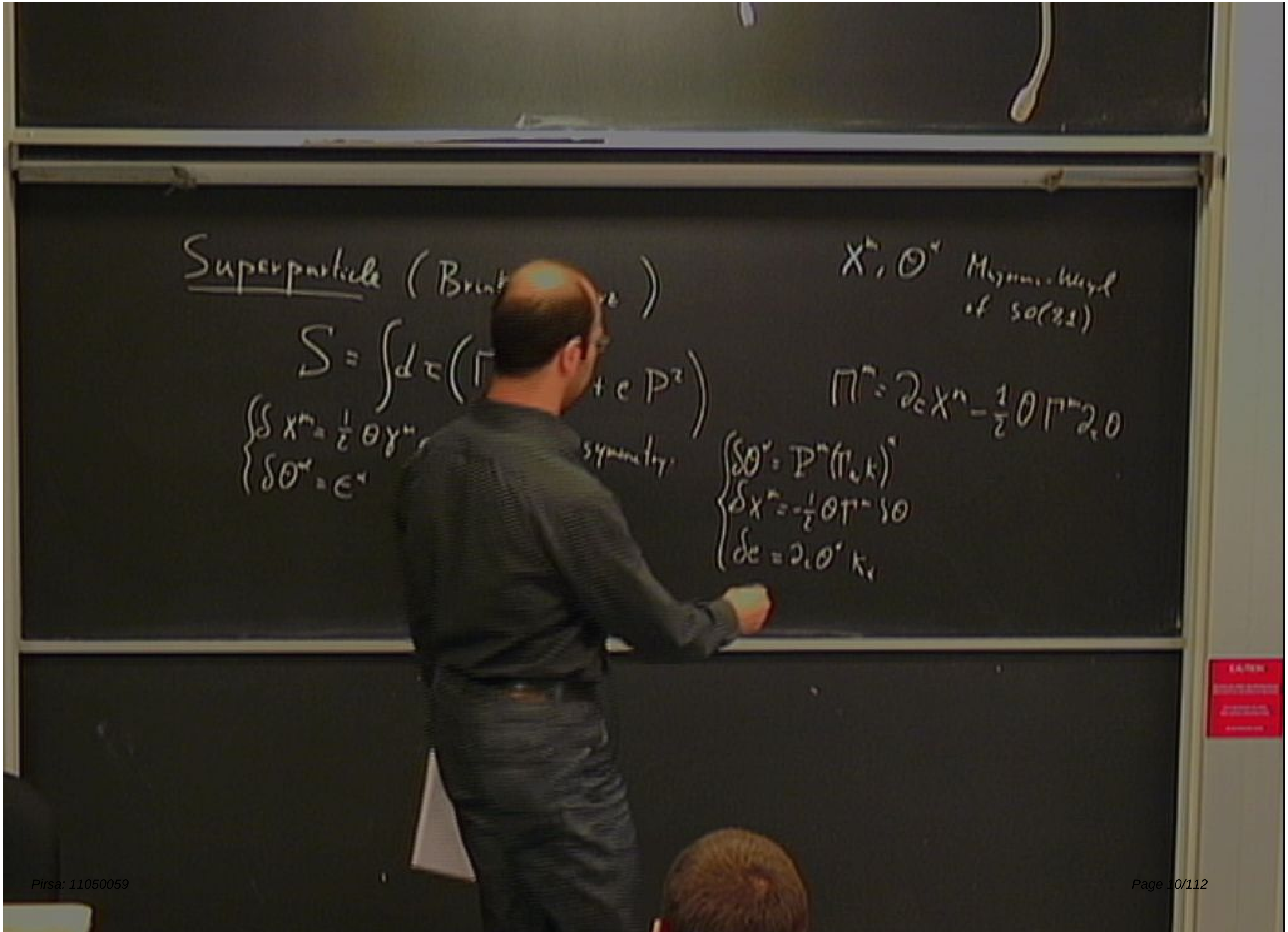

Target space

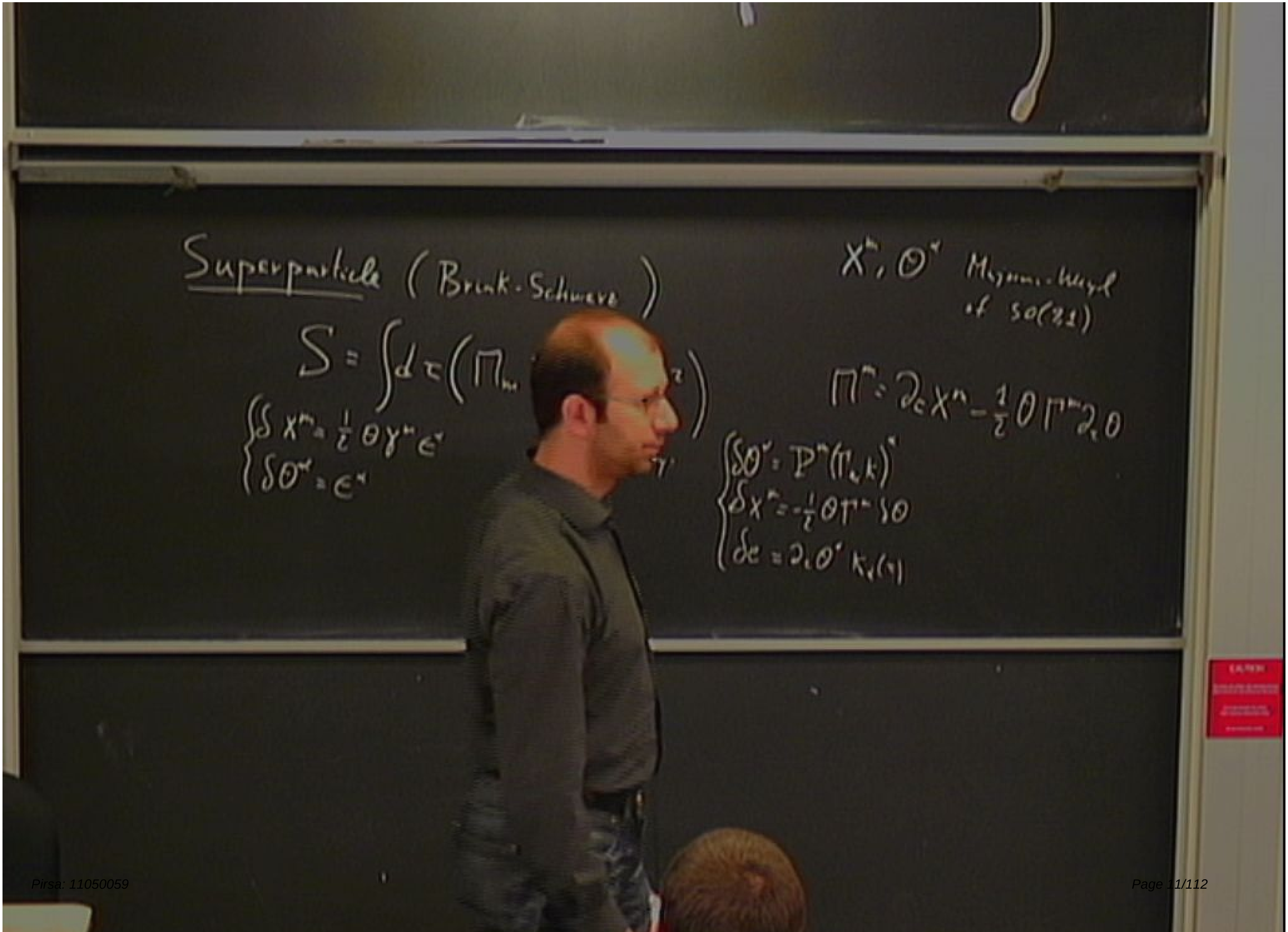

Ramond fields (2)
same footing as the NS-NS


(Brink-Schwarz)
$\int d\tau \left(\Pi_m P^m + e P^2 \right)$


Superparticle (Brink-Schwarz)
$S = \int d\tau \left(\Pi_m P^m + e P^2 \right)$
$\delta X^m = \frac{1}{2} \theta \gamma^m \epsilon$
$\delta \theta^\alpha = \epsilon^\alpha$
X^m, θ^α Majorana-Weyl of SO(9,1)
$\Pi^m = \partial_\tau X^m - \frac{1}{2} \theta \Gamma^m \partial_\tau \theta$

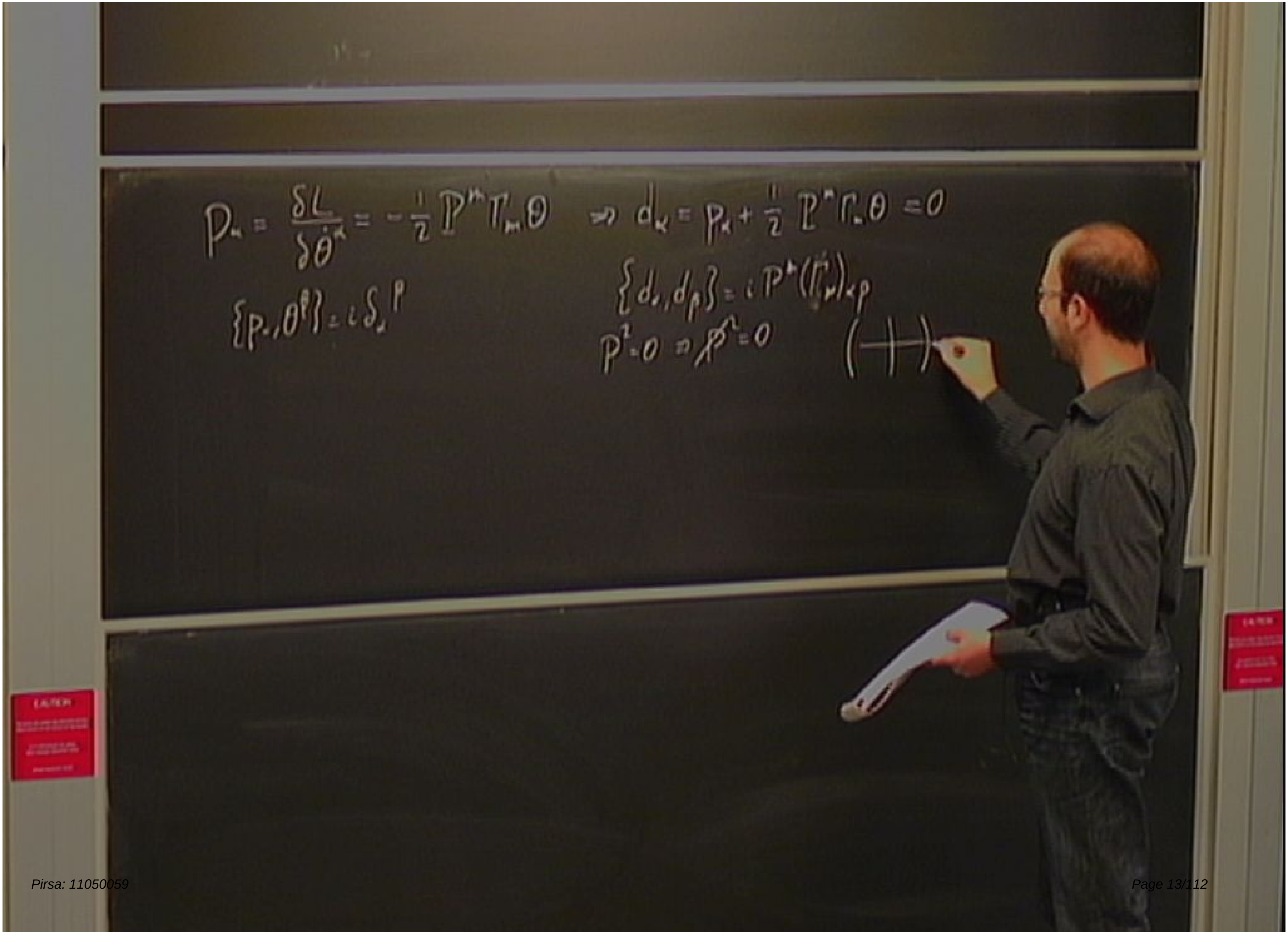

(Brink-Schwarz)
Majorana-Weyl
of SO(9,1)
K-symmetry

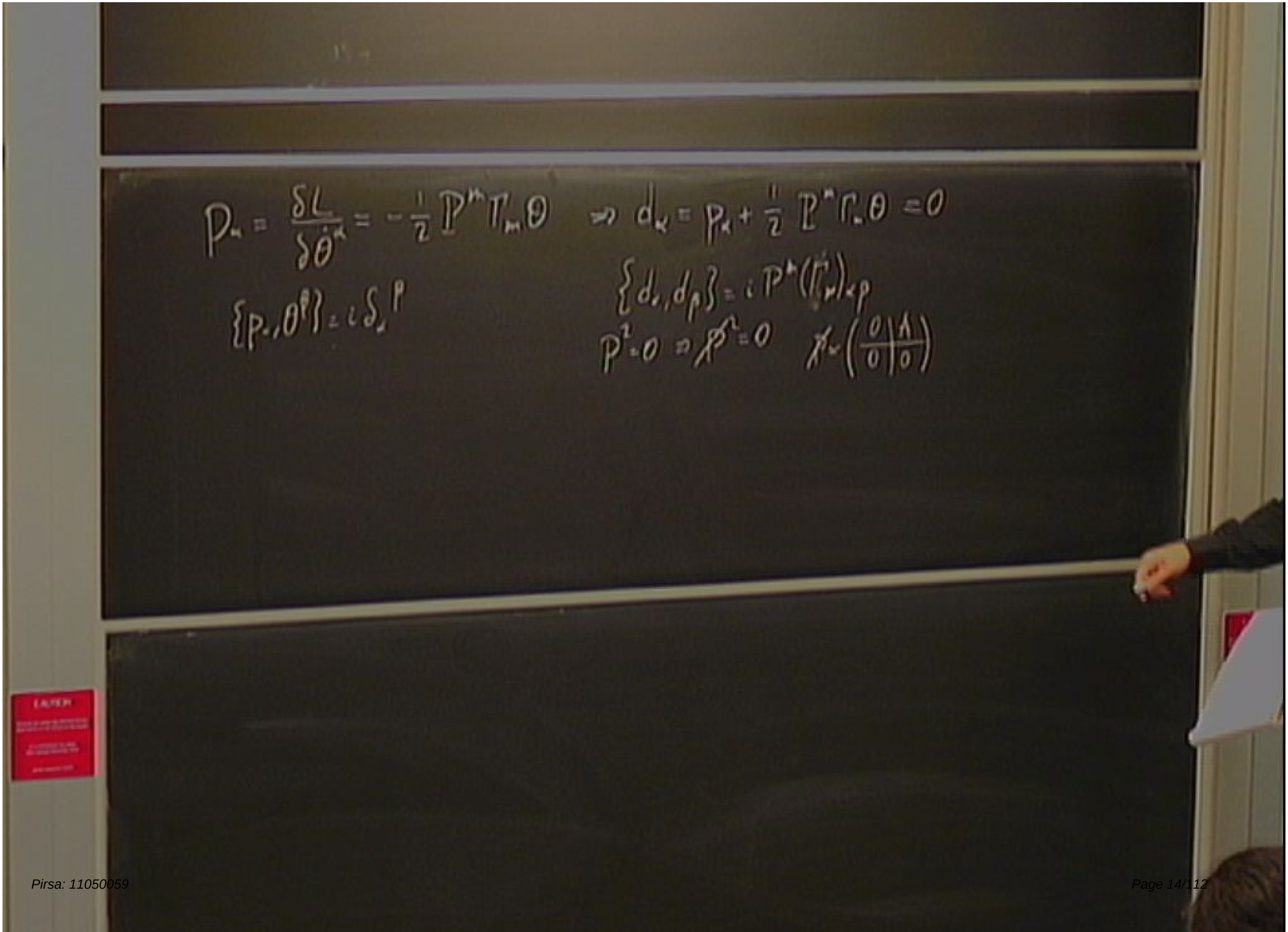

Superparticle
Majorana-Weyl
of SO(9,1)
symmetry

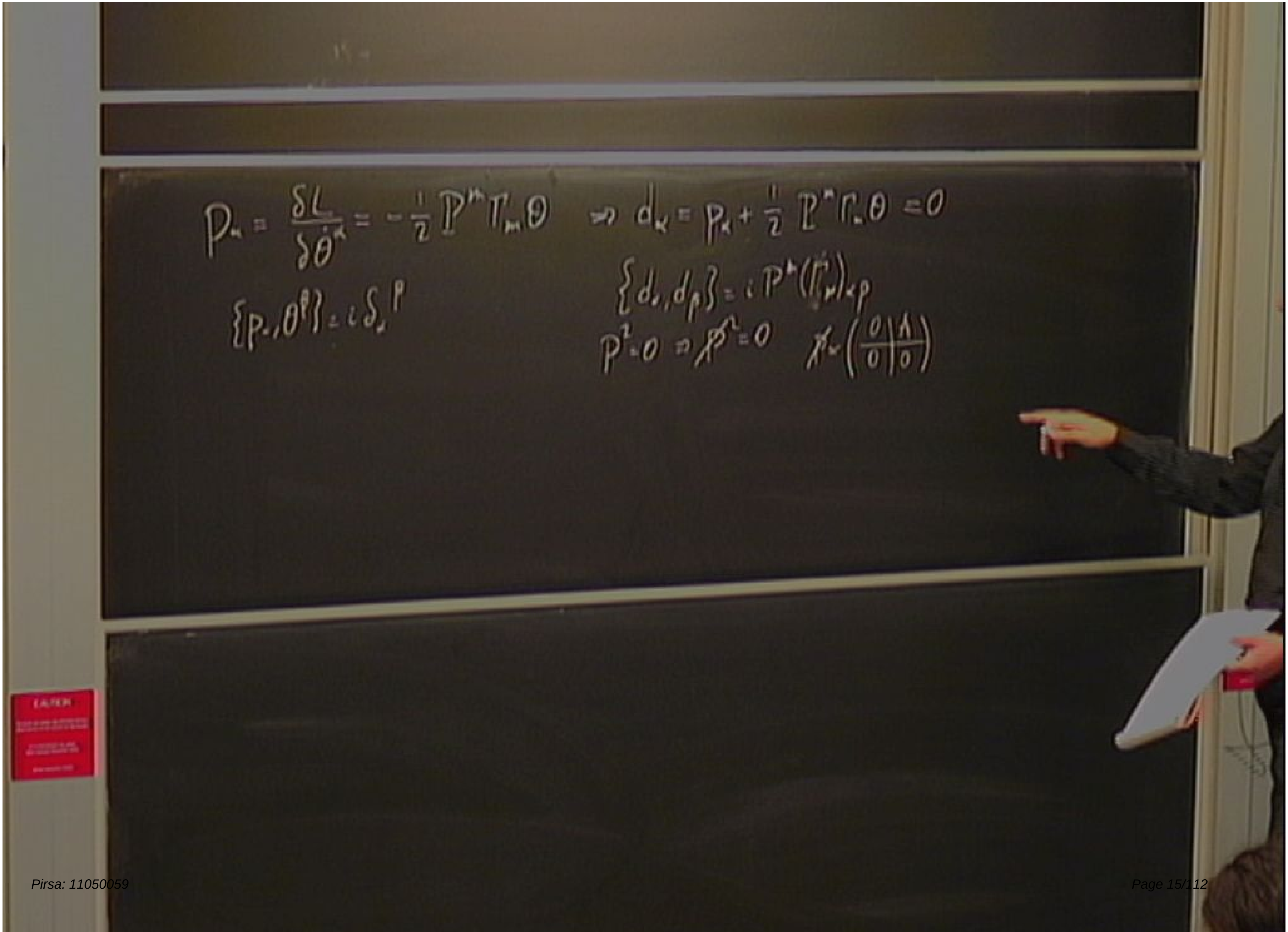

Superparticle (Brink-Schwarz)
δΘ^α = ε^α
Majorana-Weyl of SO(9,1)

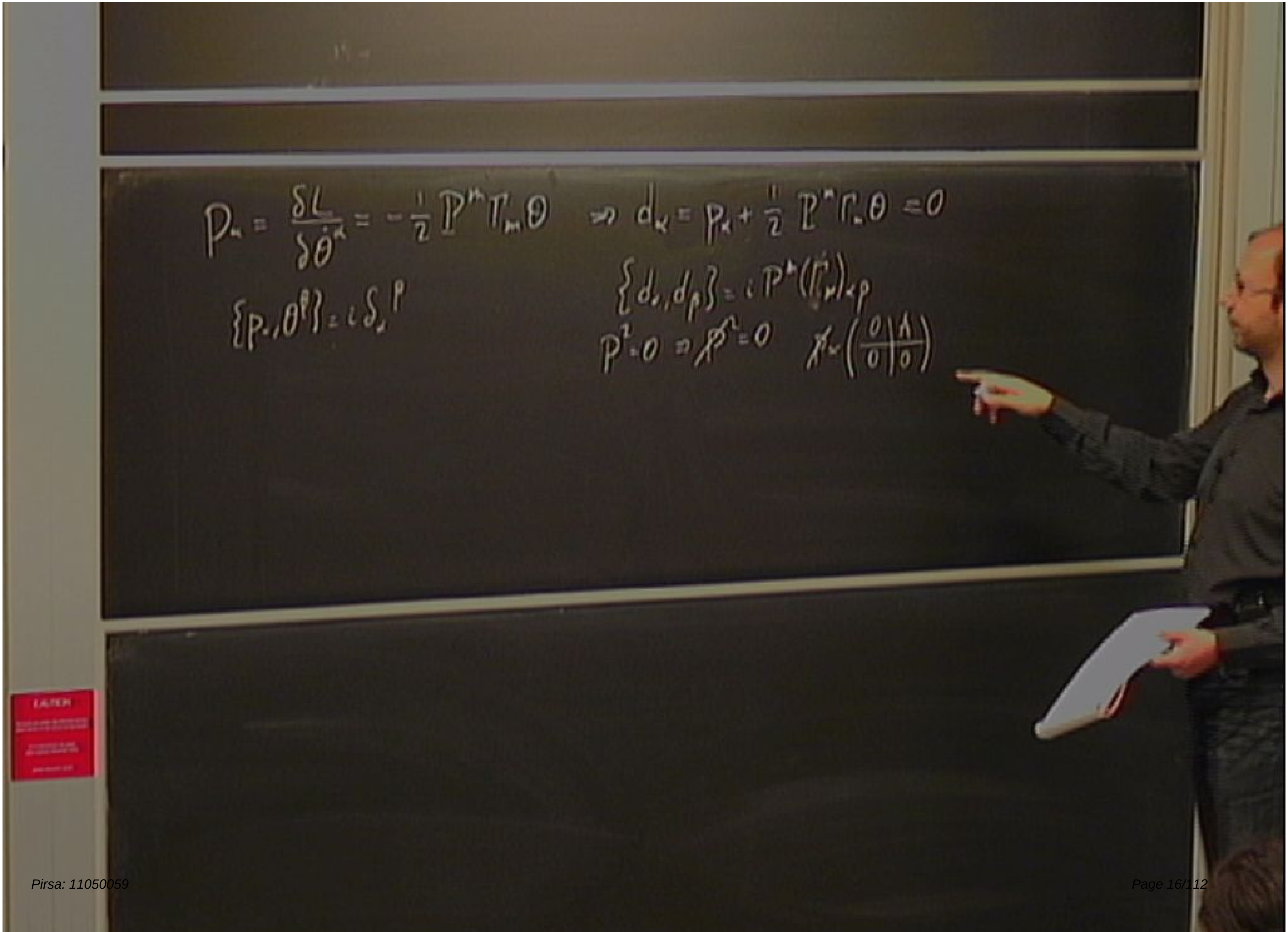
$$p_\alpha = \frac{\delta L}{\delta \dot{\theta}^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \quad \Rightarrow \quad d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$$

$$\{p_\alpha, \theta^\beta\} = i \delta_\alpha{}^\beta \qquad \{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$$


$p_\alpha = \frac{\delta L}{\delta \dot{\theta}^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \Rightarrow d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$
$\{p_\alpha, \theta^\beta\} = i \delta_\alpha{}^\beta$
$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$
$P^2 = 0 \Rightarrow \not{P}^2 = 0$


$p_\alpha = \frac{\delta L}{\delta \dot{\theta}^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \Rightarrow d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$
$\{p_\alpha, \theta^\beta\} = i \delta_\alpha{}^\beta$
$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$
$P^2 = 0 \Rightarrow \not{P}^2 = 0 \quad \not{P} \sim \begin{pmatrix} 0 & \lambda \\ 0 & 0 \end{pmatrix}$


$p_\alpha = \frac{\delta L}{\delta \dot\theta^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \Rightarrow d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$
$\{p_\alpha, \theta^\beta\} = i \delta_\alpha{}^\beta$
$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$
$P^2 = 0 \Rightarrow \not{P}^2 = 0 \quad \not{P} \sim \left(\begin{array}{c|c} 0 & \Lambda \\ \hline 0 & 0 \end{array}\right)$


$p_\alpha = \frac{\delta L}{\delta \dot\theta^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \Rightarrow d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$
$\{p_\alpha, \theta^\beta\} = i \delta_\alpha{}^\beta$
$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$
$P^2 = 0 \Rightarrow \not{P}^2 = 0$
$\not{P} \sim \left(\begin{array}{c|c} 0 & A \\ \hline 0 & 0 \end{array}\right)$

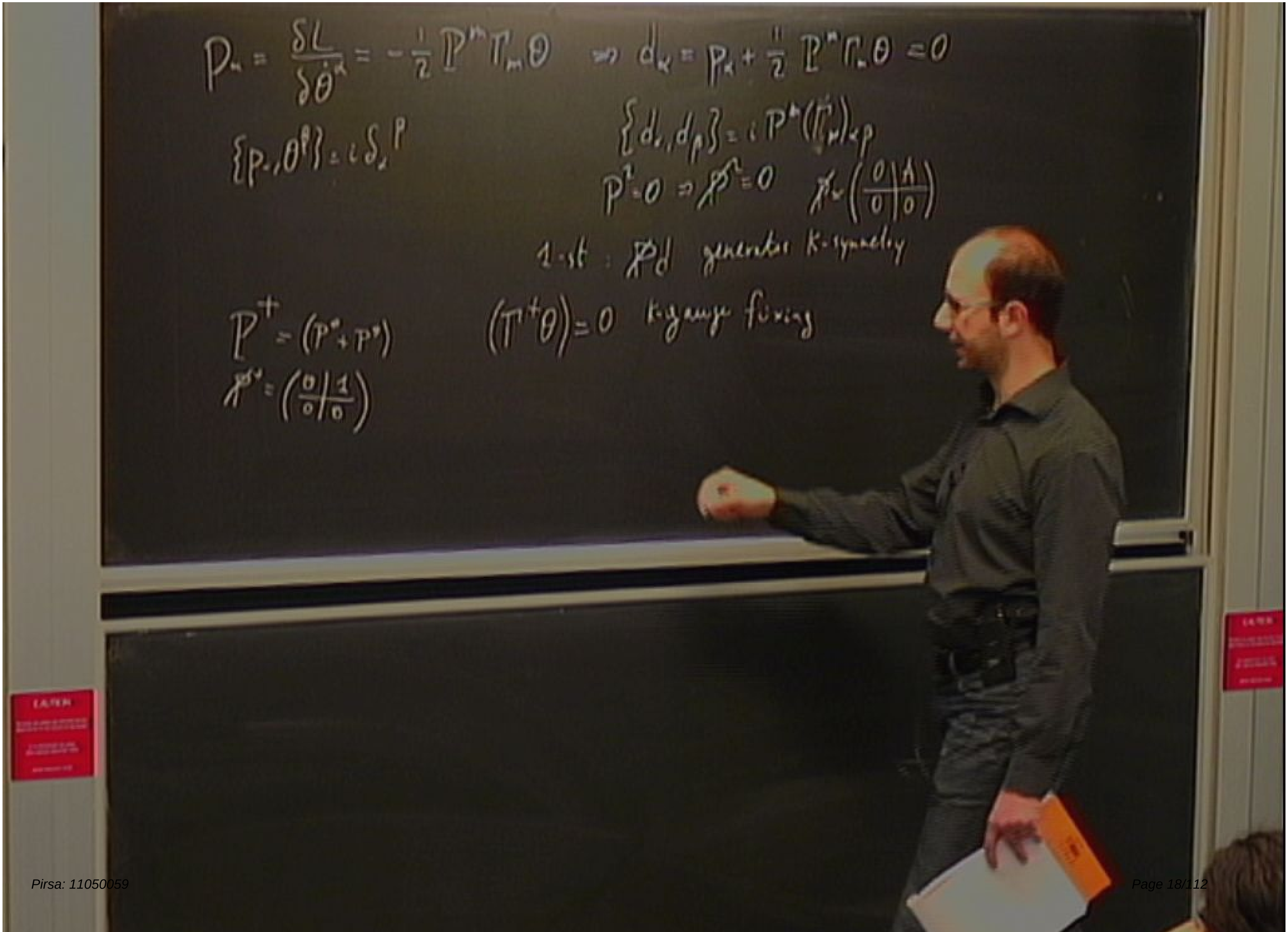
$$p_\alpha = \frac{\delta L}{\delta \dot{\theta}^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \;\Rightarrow\; d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$$

$$\{p_\alpha, \theta^\beta\} = i\,\delta_\alpha{}^\beta$$

$$\{d_\alpha, d_\beta\} = i\, P^m (\Gamma_m)_{\alpha\beta}$$

$$P^2 = 0 \Rightarrow \not{P}^2 = 0 \qquad \not{P} \sim \left(\begin{array}{c|c} 0 & A \\ \hline 0 & 0 \end{array}\right)$$

1-st : $\not{P} d$ generates K-symmetry


$P_\alpha = \frac{\delta L}{\delta \dot\theta^\alpha} = -\frac{1}{2} P^m \Gamma_m \theta \Rightarrow d_\alpha = p_\alpha + \frac{1}{2} P^m \Gamma_m \theta = 0$
$\{p_\alpha, \theta^\beta\} = i\delta_\alpha{}^\beta$
$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$
$P^2 = 0 \Rightarrow \not{P}^2 = 0$
1-st : $\not{P}d$ generates κ-symmetry
$P^+ = (P^0 + P^9)$
$(\Gamma^+\theta) = 0$ κ-gauge fixing

$$\{p_\alpha, \theta^\beta\} = i\,\delta_\alpha{}^\beta$$

$$\{d_\alpha, d_\beta\} = i P^m (\Gamma_m)_{\alpha\beta}$$

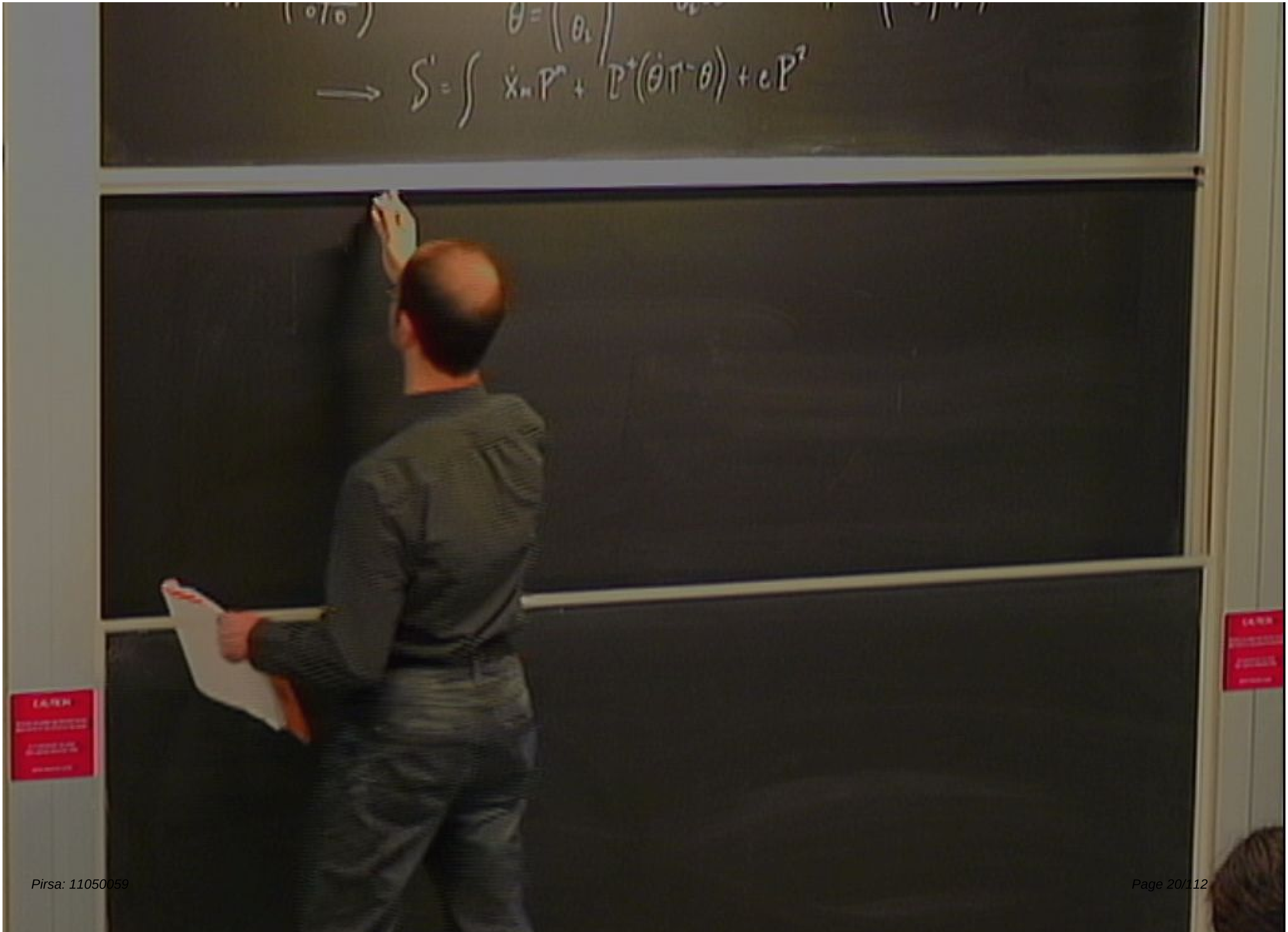
$$P^2 = 0 \Rightarrow \not{P}^2 = 0 \qquad \not{P} = \left(\begin{array}{c|c} 0 & A \\ \hline 0 & 0 \end{array}\right)$$

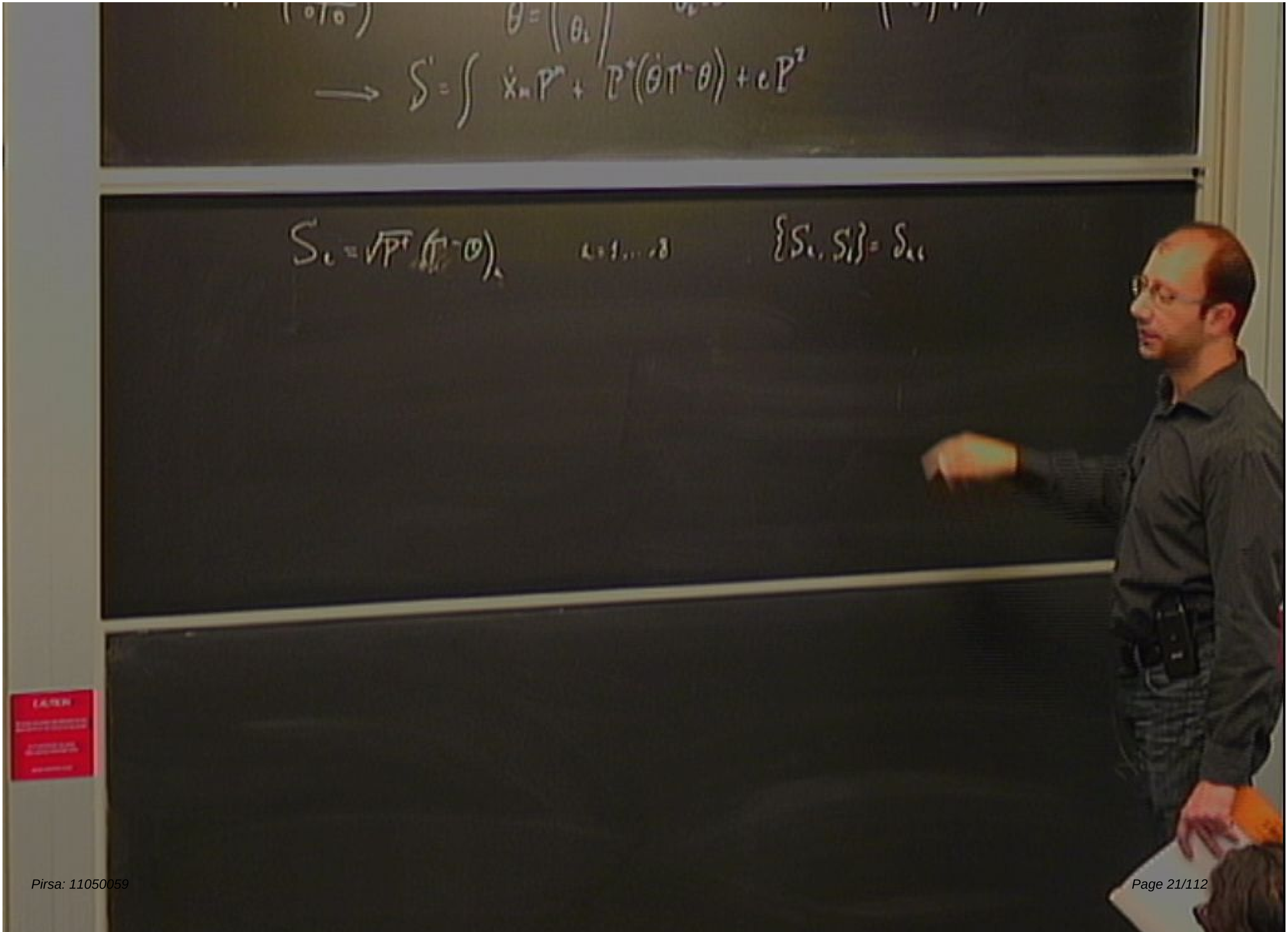
1-st : $\not{P} d$ generates K-symmetry

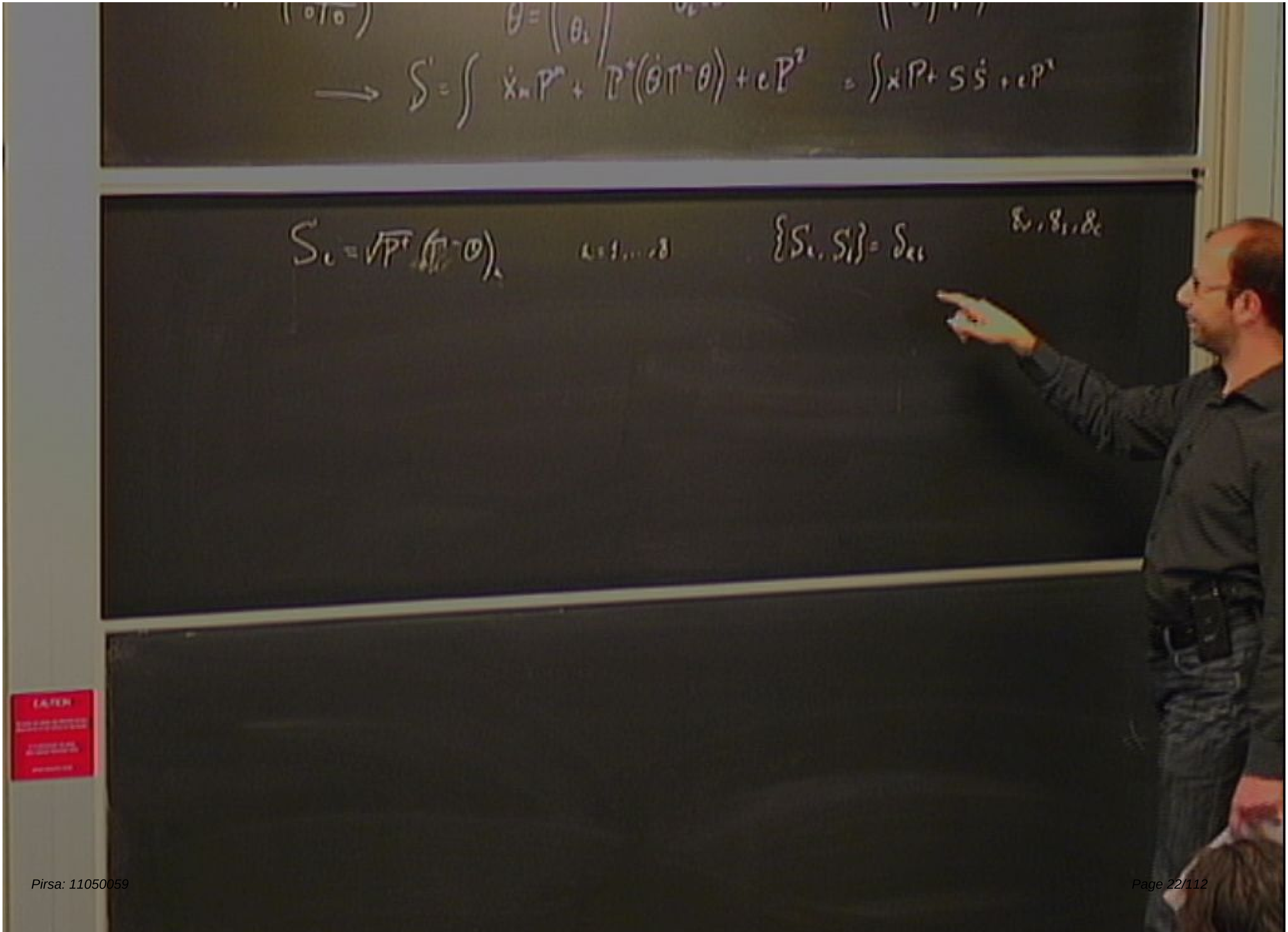
$$P^+ = (P^0 + P^9) \qquad (\Gamma^+ \theta) = 0 \quad \text{K-gauge fixing}$$

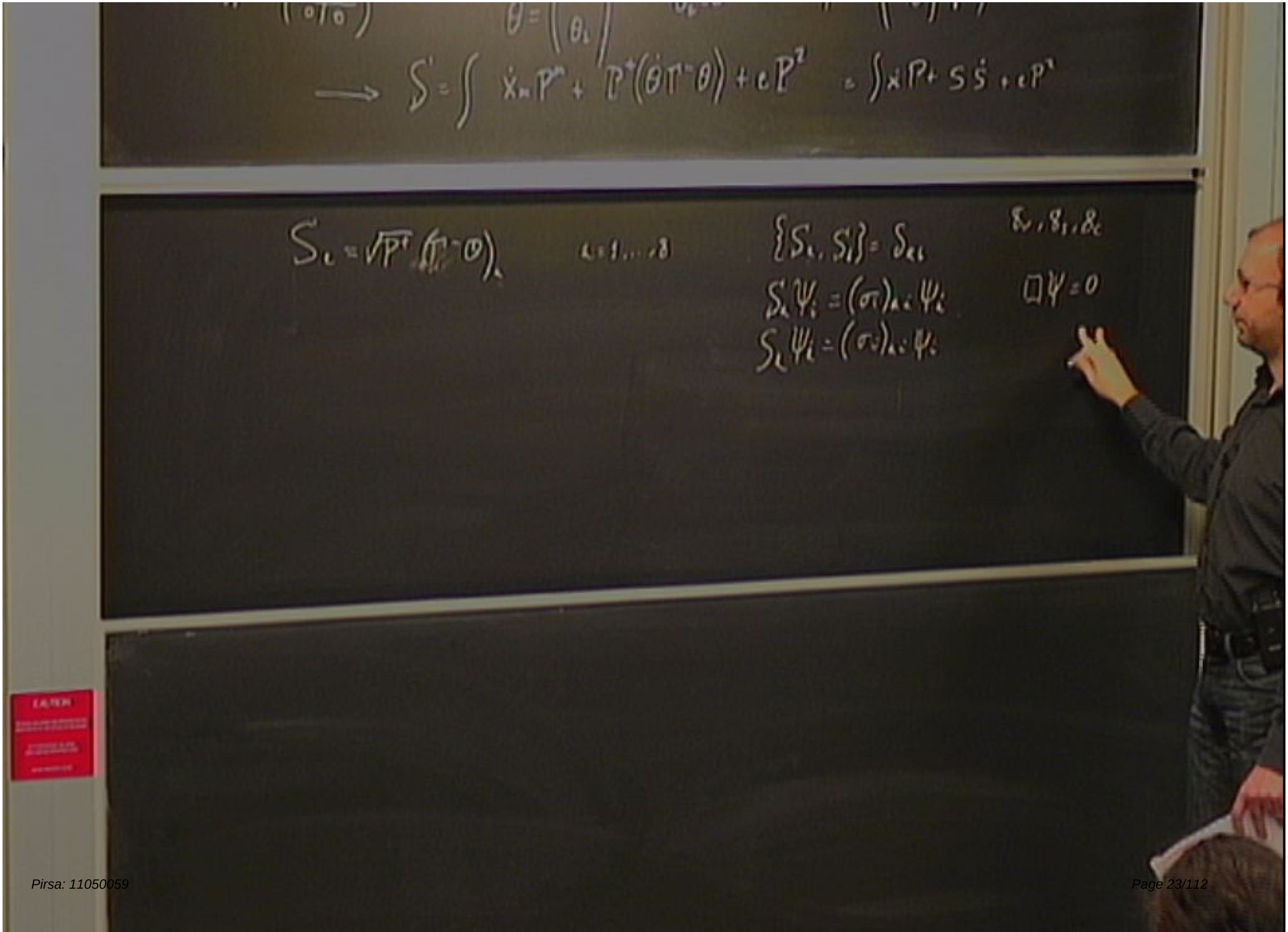
$$\not{P}^+ = \left(\begin{array}{c|c} 0 & 1 \\ \hline 0 & 0 \end{array}\right) \qquad \theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \qquad \theta_2 = 0 \qquad \Gamma^i = \left(\begin{array}{c|c} \gamma^i & 0 \\ \hline 0 & \gamma^i \end{array}\right)$$

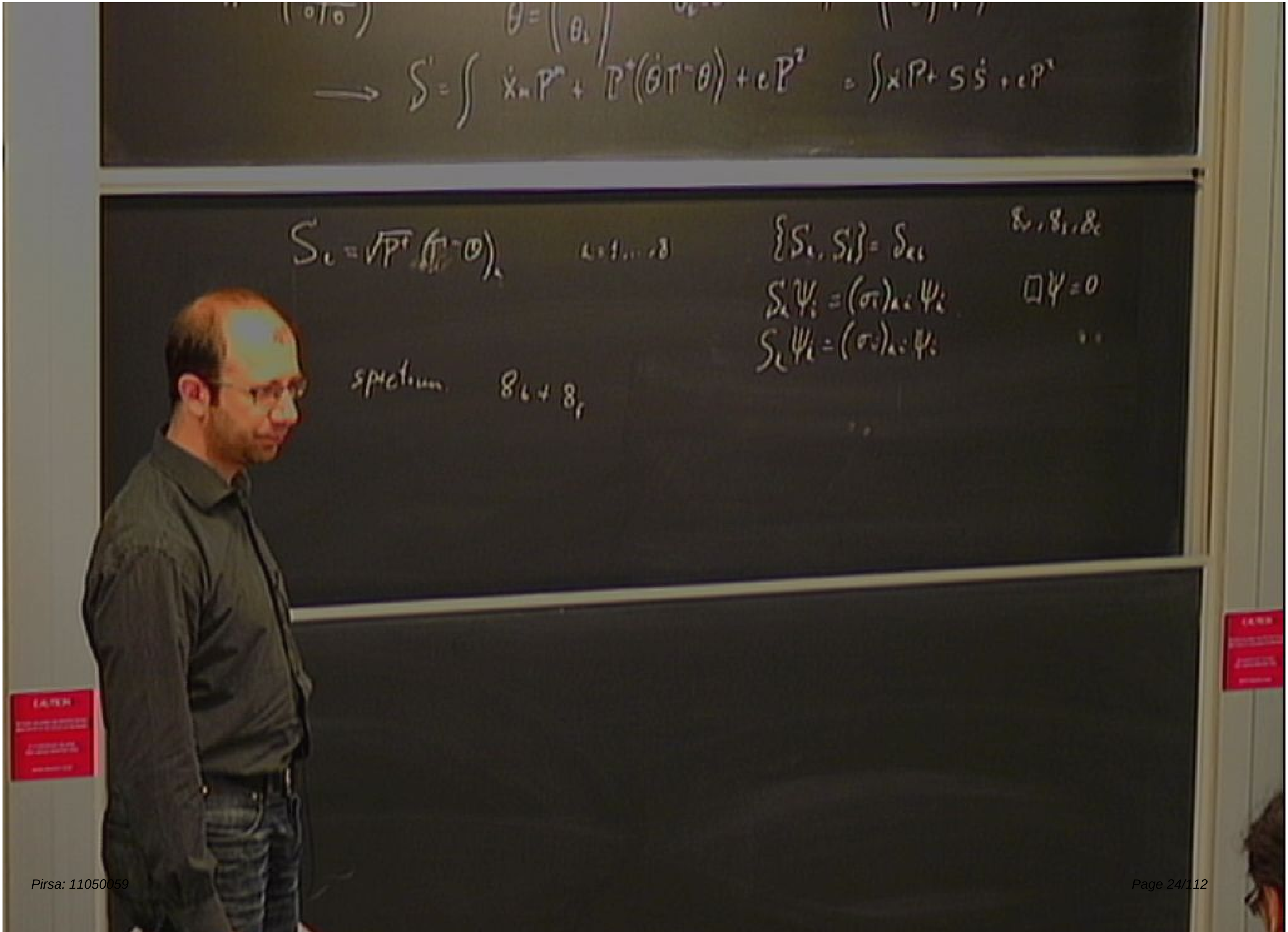
$$\longrightarrow S' = \int \dot{X}_m P^m + P^+(\dot{\theta}\,\Gamma^- \theta) + e P^2$$

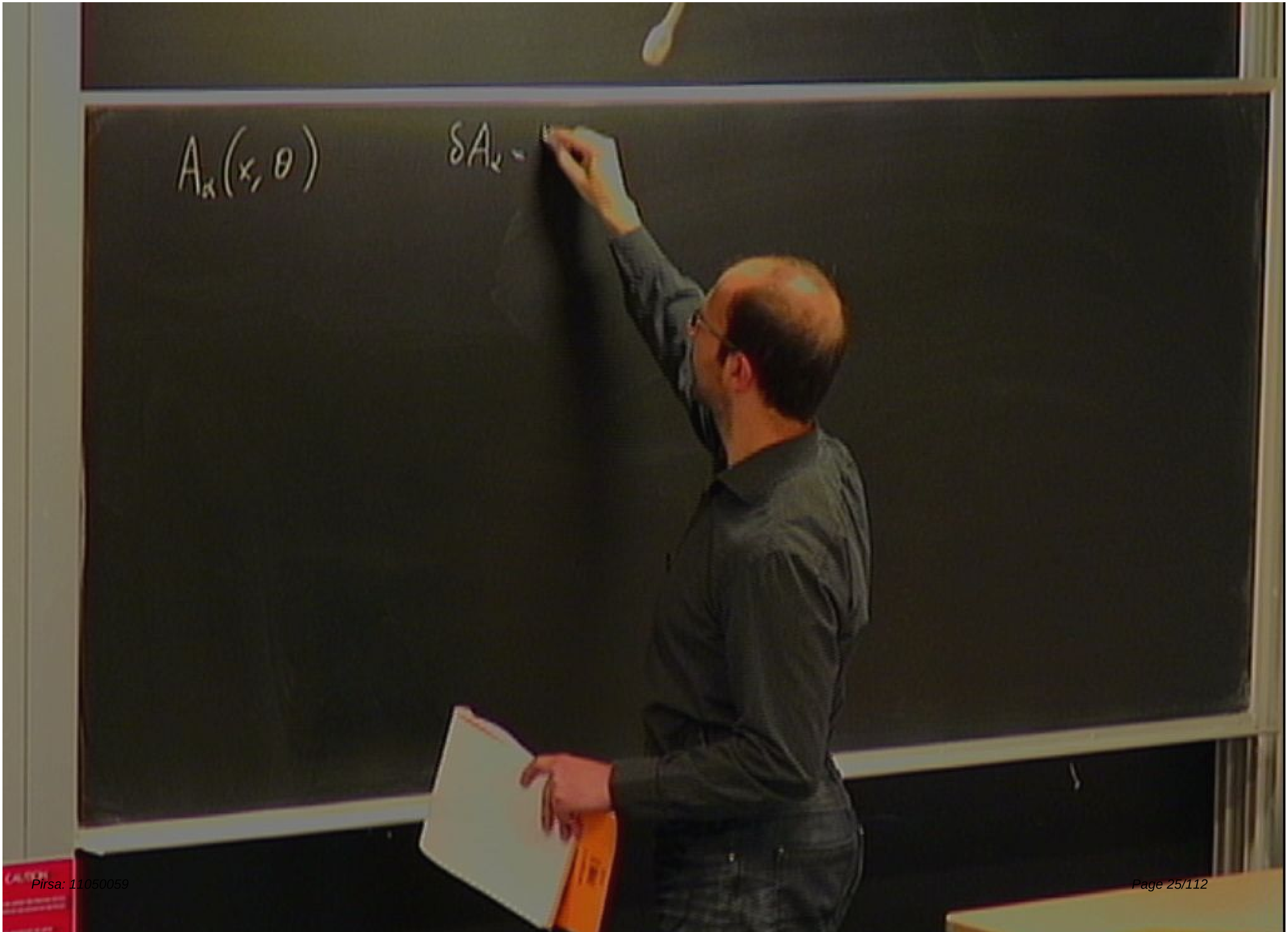

→ S' = ∫ Ẋ_m P^m + P^+(θ̇Γ^-θ) + eP^2

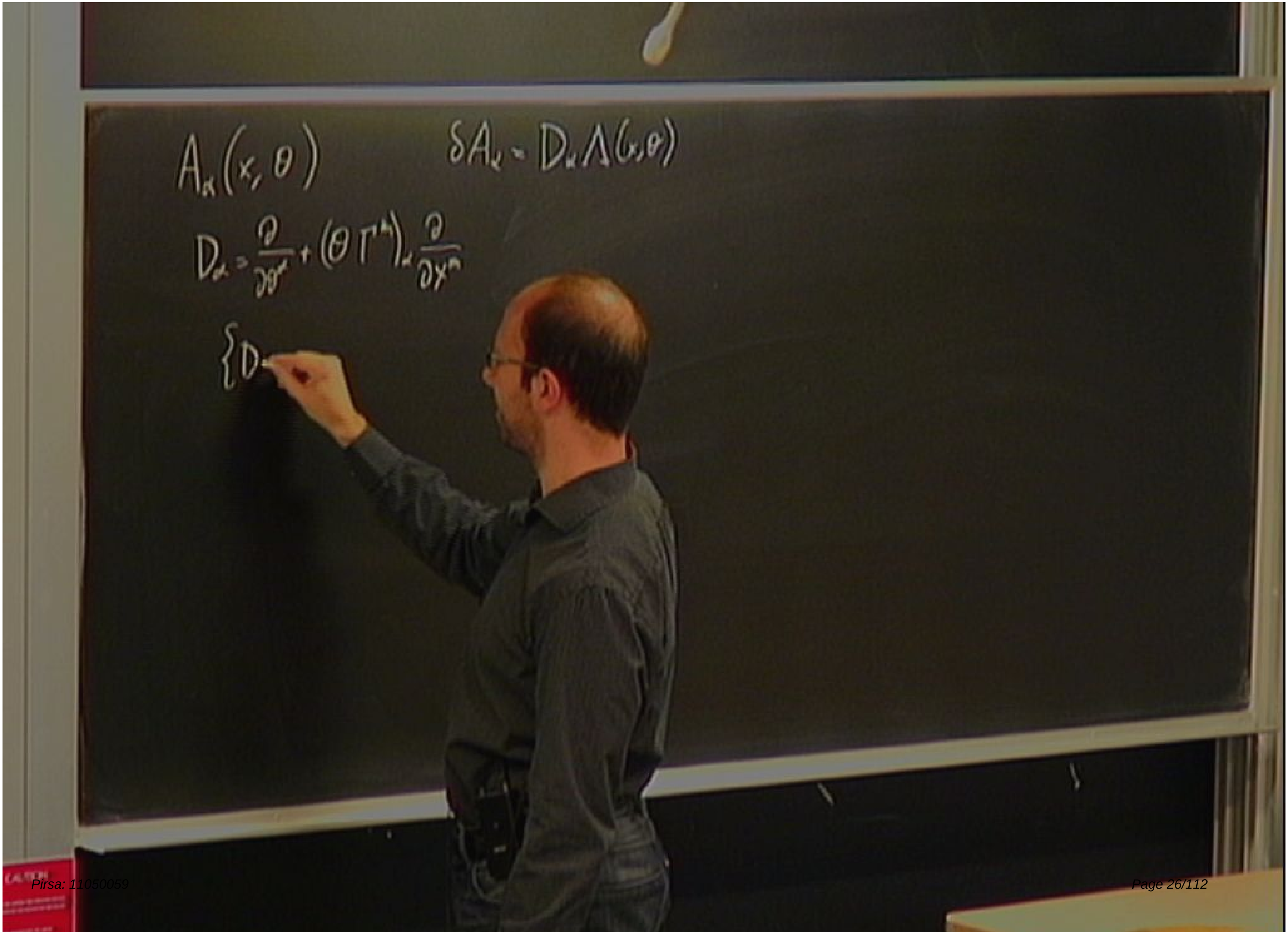

S' = ∫ Ẋ_μ P^μ + P^+(θ̇Γ^-θ) + e P^2
S_a = √P^+ (Γ^- θ)_a
a = 1, ..., 8
{S_a, S_b} = δ_ab

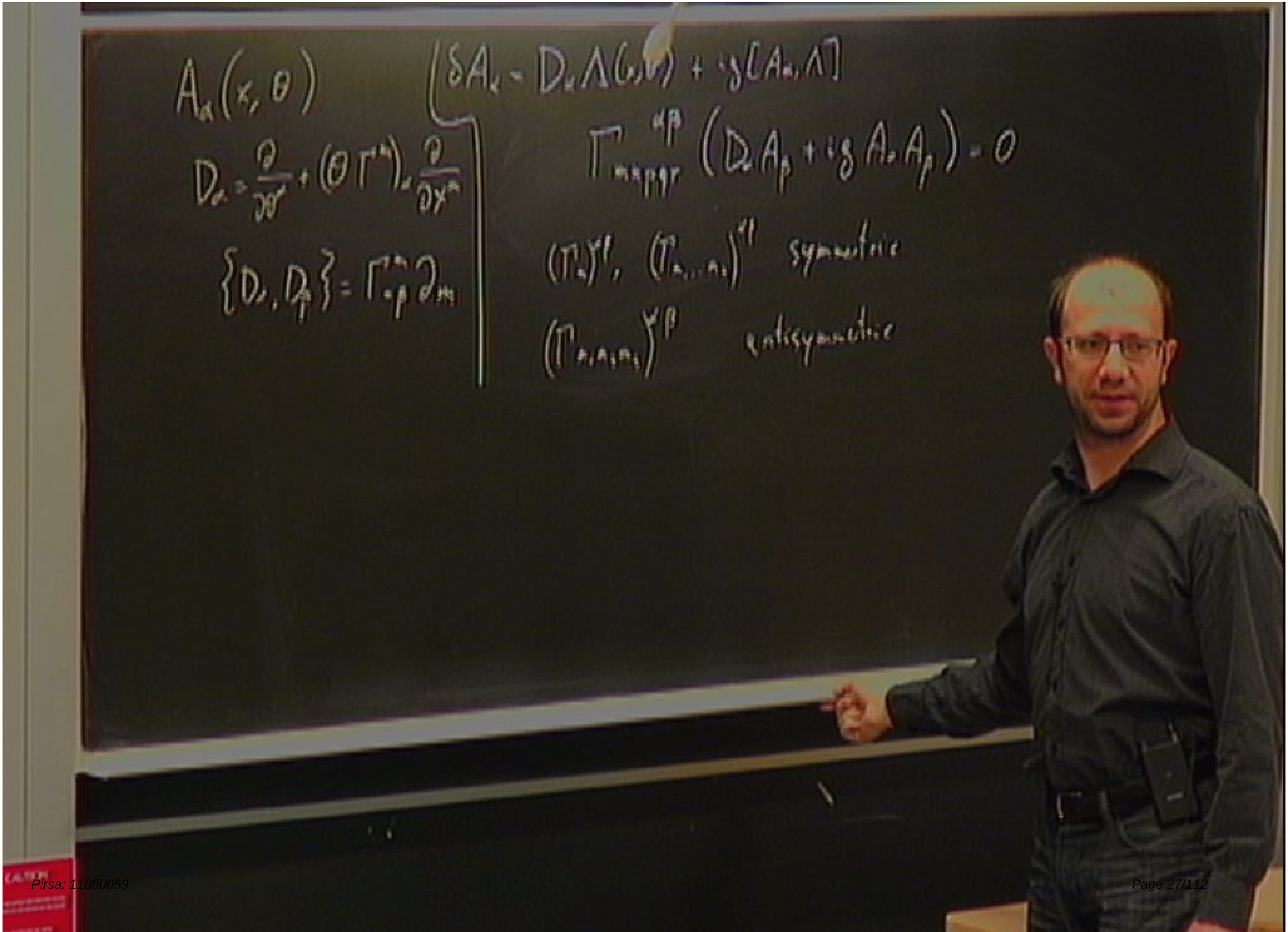

$\longrightarrow S' = \int \dot{x}_\mu P^\mu + P^+(\dot{\theta}\Gamma^-\theta) + e P^2 = \int \dot{x} P + S\dot{S} + e P^2$
$S_a = \sqrt{P^+}(\Gamma^-\theta)_a$
$a = 1, \ldots, 8$
$\{S_a, S_b\} = \delta_{ab}$
$8_v, 8_s, 8_c$

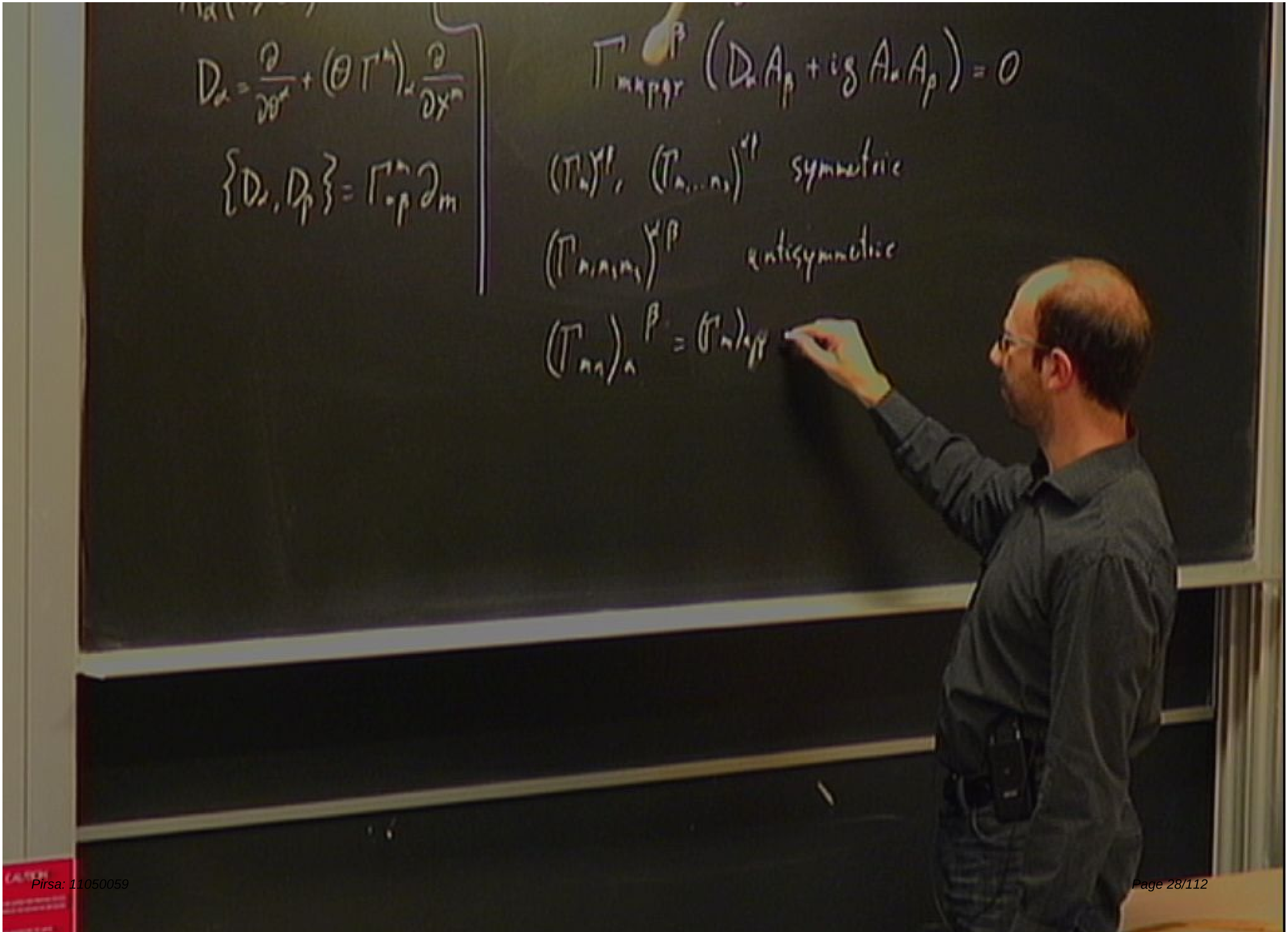

CAUTION

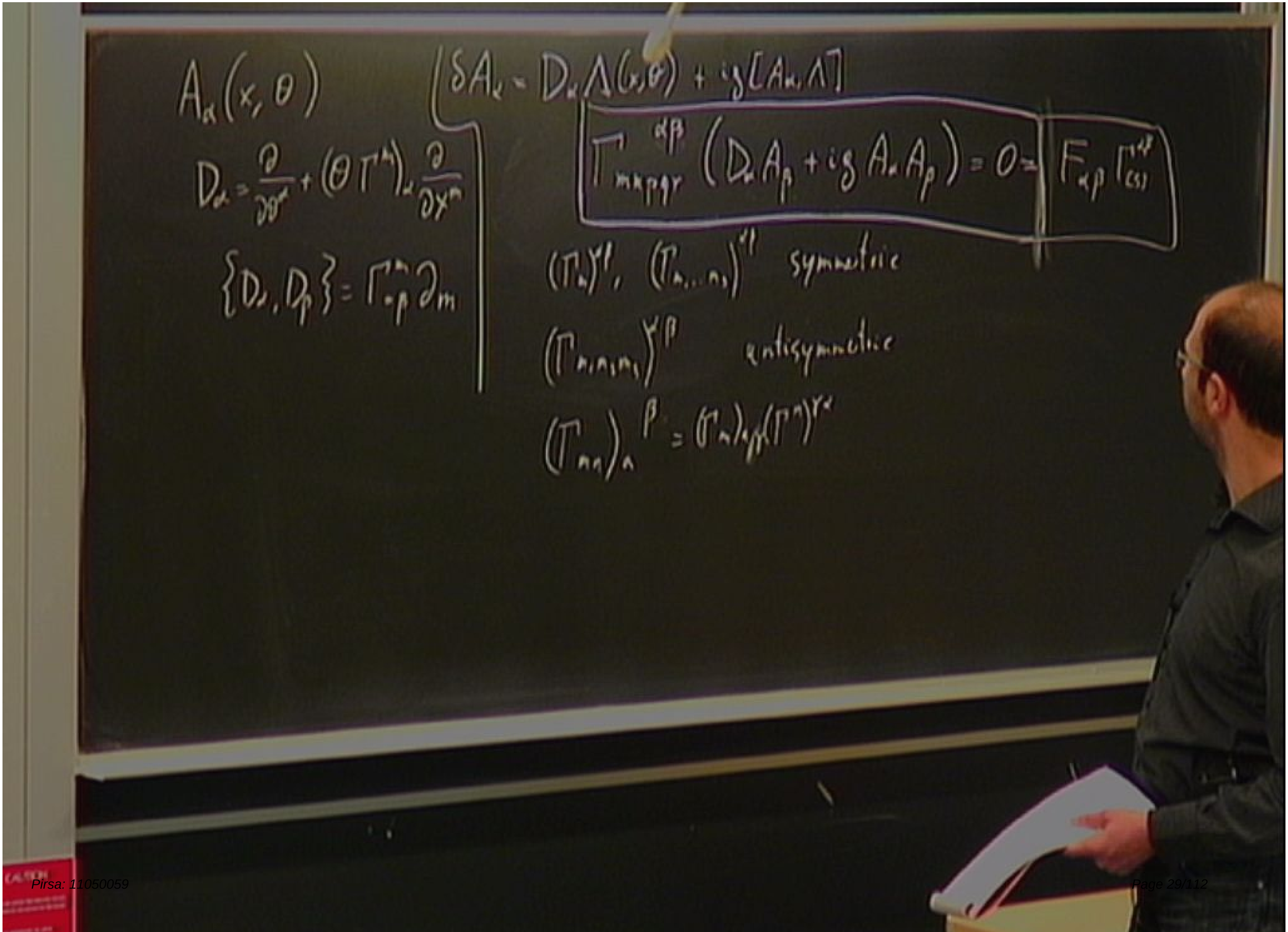

spectrum
$8_b + 8_f$
$\Box\psi = 0$

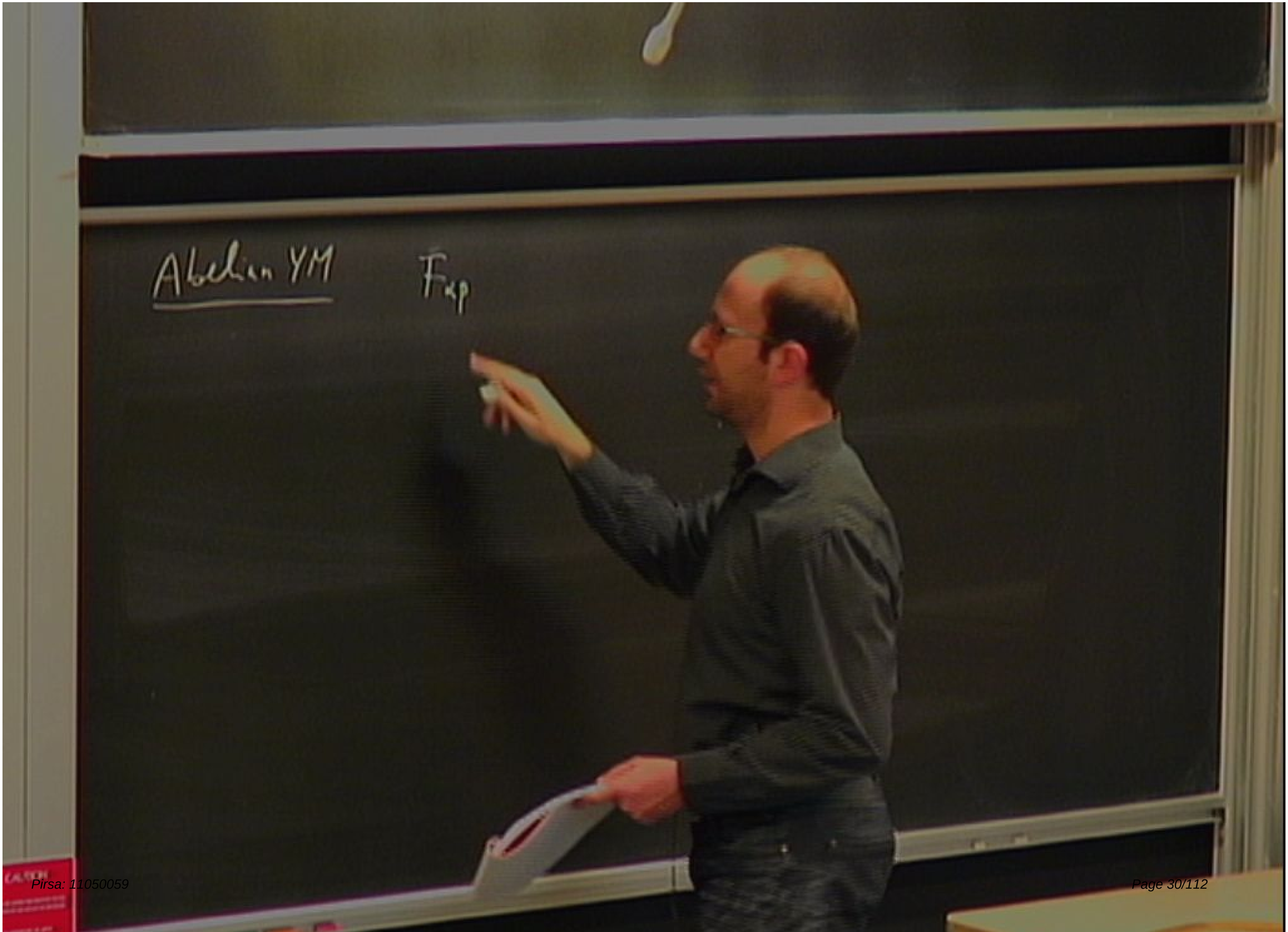

$A_\alpha(x, \theta)$
$\delta A_\alpha =$

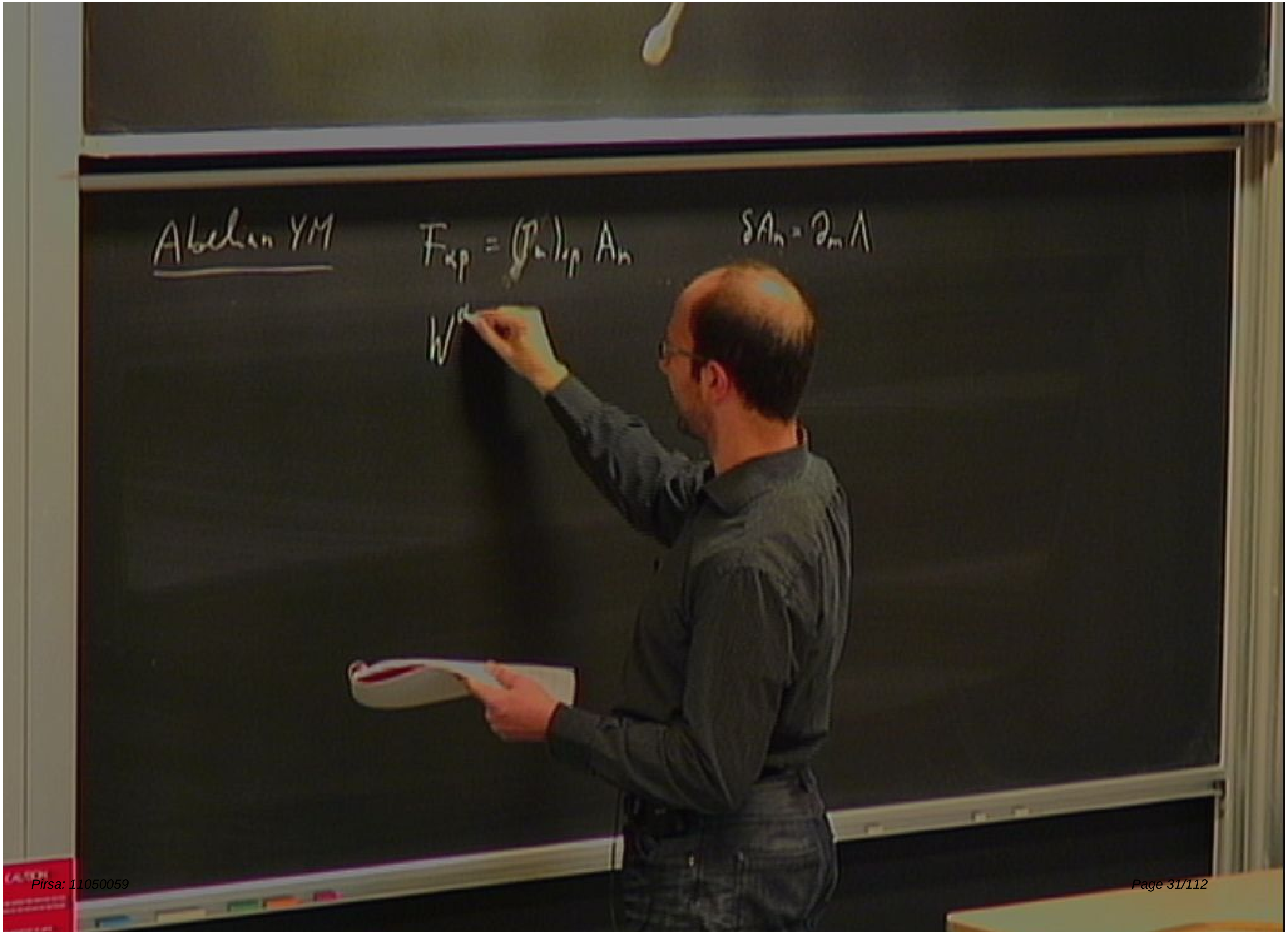

$A_\alpha(x, \theta)$
$\delta A_\alpha = D_\alpha \Lambda(x,\theta)$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\theta\Gamma^m)_\alpha \frac{\partial}{\partial x^m}$
$\{D$

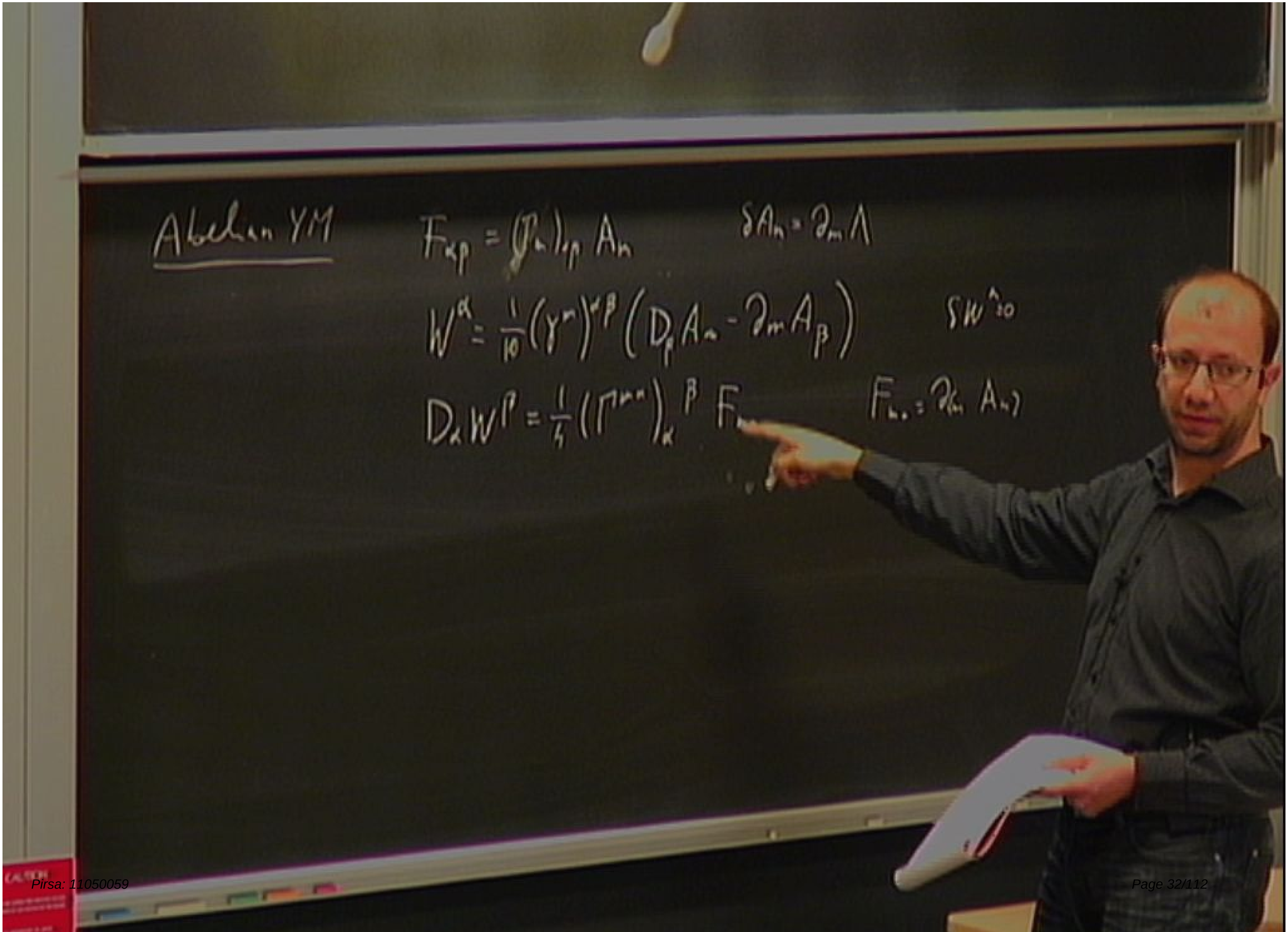

$A_\alpha(x,\theta)$
$\delta A_\alpha = D_\alpha \Lambda(x,\theta) + ig[A_\alpha, \Lambda]$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\theta\Gamma^m)_\alpha \frac{\partial}{\partial x^m}$
$\Gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + ig A_\alpha A_\beta) = 0$
$\{D_\alpha, D_\beta\} = \Gamma^m_{\alpha\beta}\partial_m$
$(\Gamma_m)^{\alpha\beta}$, $(\Gamma_{m_1 \ldots m_5})^{\alpha\beta}$ symmetric
$(\Gamma_{m_1 m_2 m_3})^{\alpha\beta}$ antisymmetric

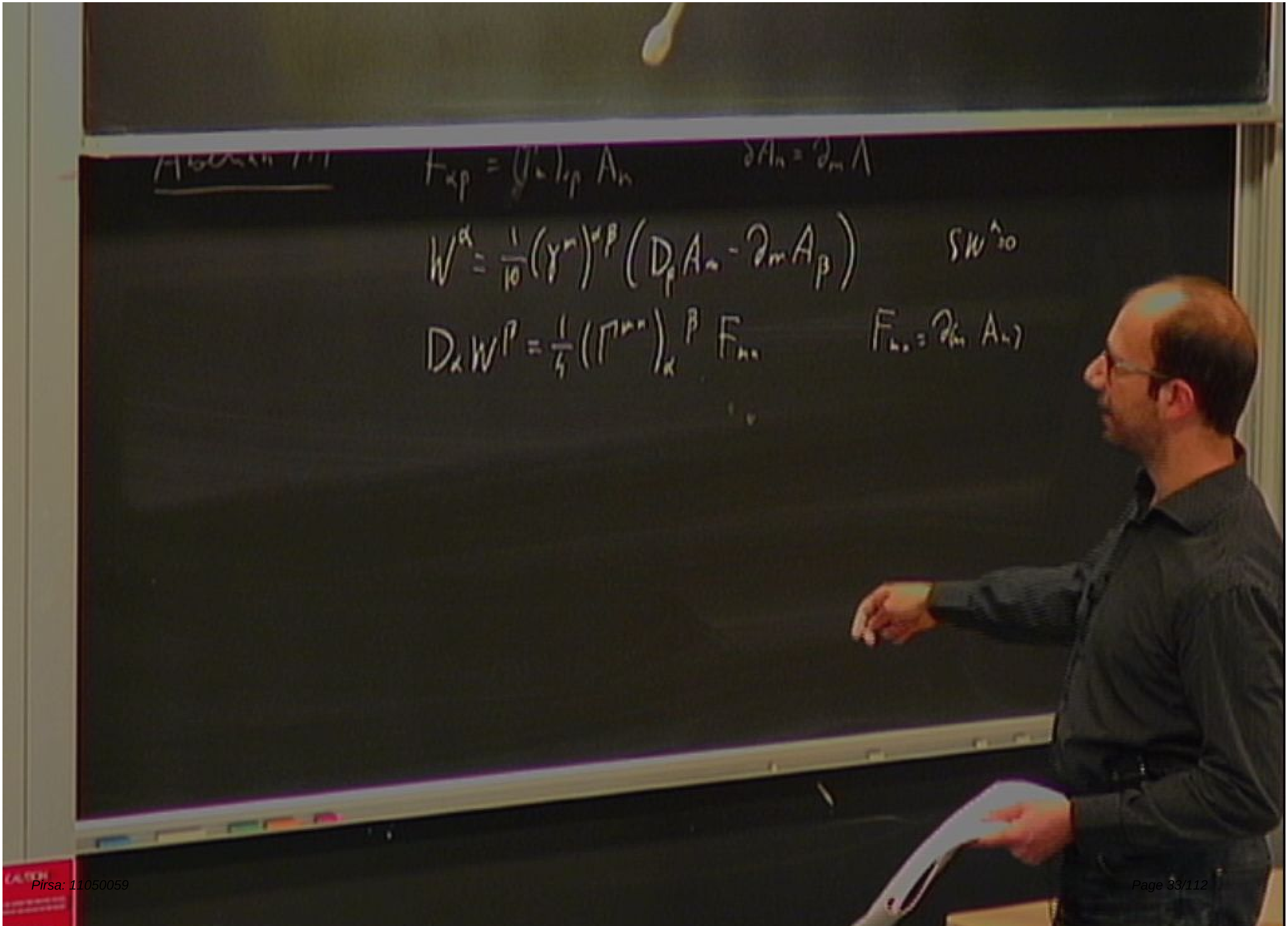

$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\theta\Gamma^m)_\alpha \frac{\partial}{\partial x^m}$
$\{D_\alpha, D_\beta\} = \Gamma^m_{\alpha\beta}\partial_m$
$\Gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + ig A_\alpha A_\beta) = 0$
$(\Gamma_m)^{\alpha\beta}$, $(\Gamma_{m_1 \dots m_5})^{\alpha\beta}$ symmetric
$(\Gamma_{m_1 m_2 m_3})^{\alpha\beta}$ antisymmetric
$(\Gamma_{mn})_\alpha{}^\beta = (\Gamma_m)_{\alpha\gamma}$

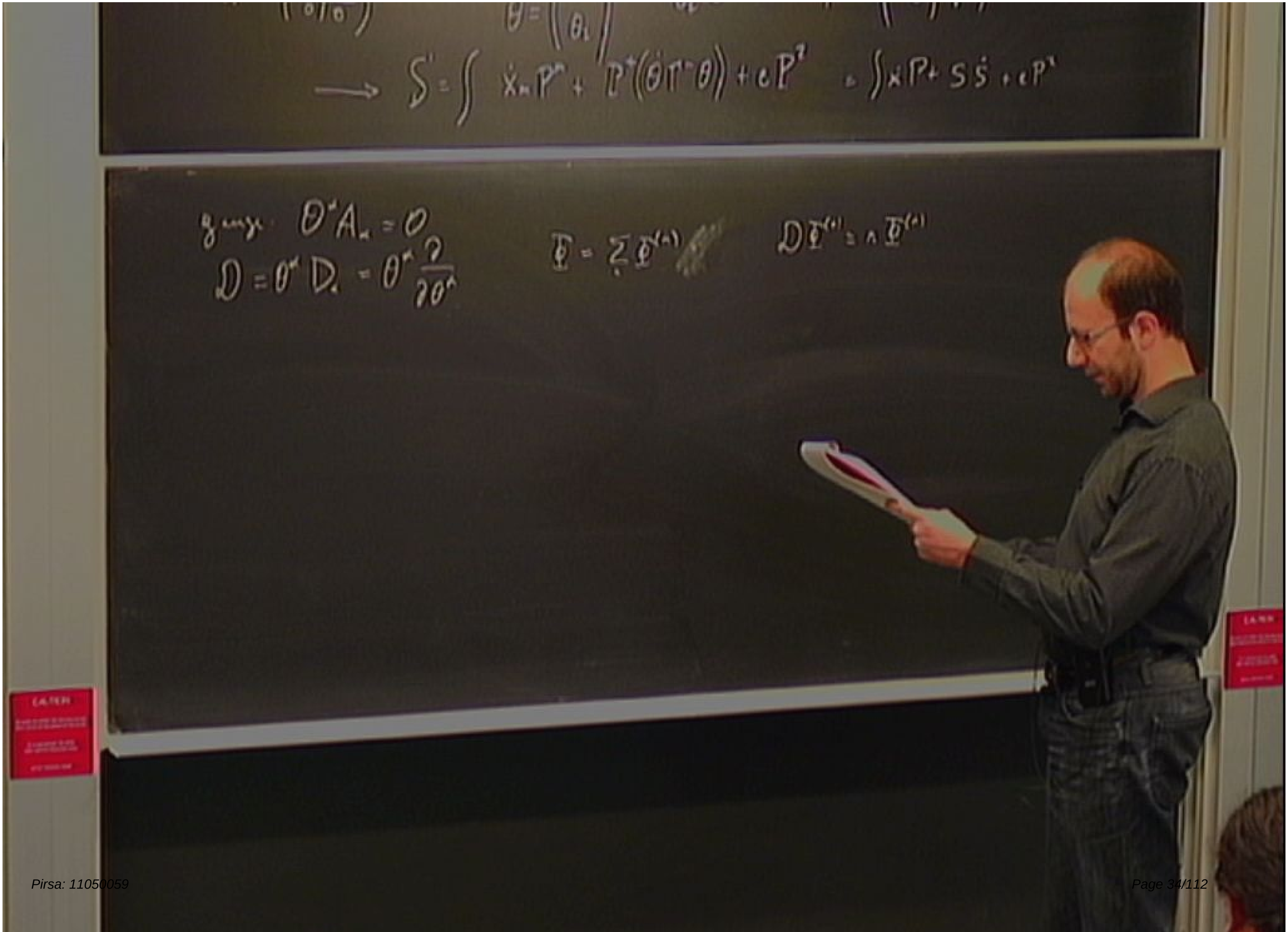

$A_\alpha(x,\theta)$
$\delta A_\alpha = D_\alpha \Lambda(x,\theta) + ig[A_\alpha, \Lambda]$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\theta\Gamma^m)_\alpha \frac{\partial}{\partial x^m}$
$\Gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + ig A_\alpha A_\beta) = 0 = F_{\alpha\beta}\Gamma_{(5)}^{\alpha\beta}$
$\{D_\alpha, D_\beta\} = \Gamma^m_{\alpha\beta}\partial_m$
$(\Gamma_m)^{\gamma\beta}$, $(\Gamma_{m_1 \dots m_5})^{\alpha\beta}$ symmetric
$(\Gamma_{m_1 m_2 m_3})^{\gamma\beta}$ antisymmetric
$(\Gamma_{mn})_\alpha{}^\beta = (\Gamma_m)_{\alpha\gamma}(\Gamma^n)^{\gamma\beta}$

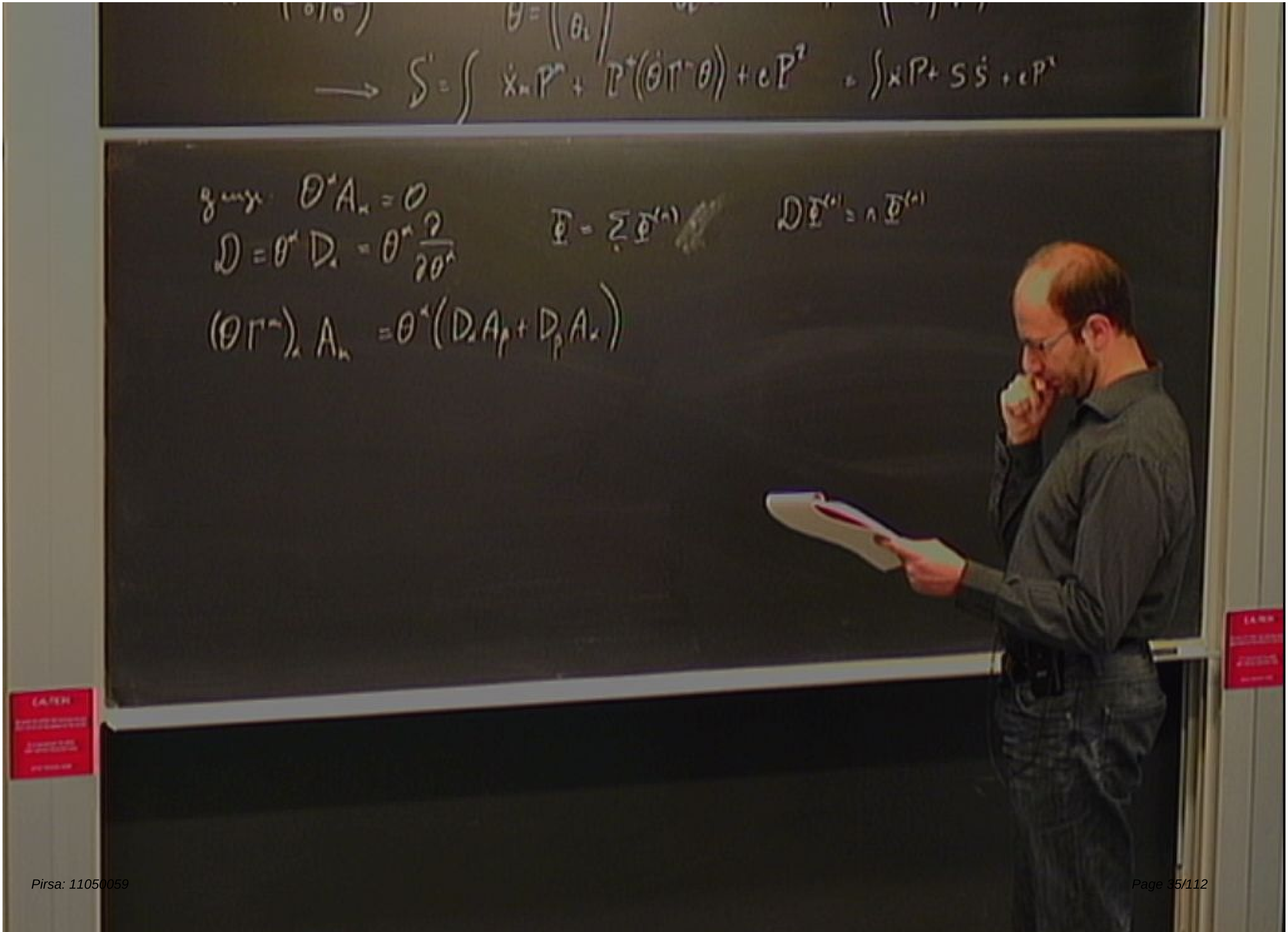

Abelian YM

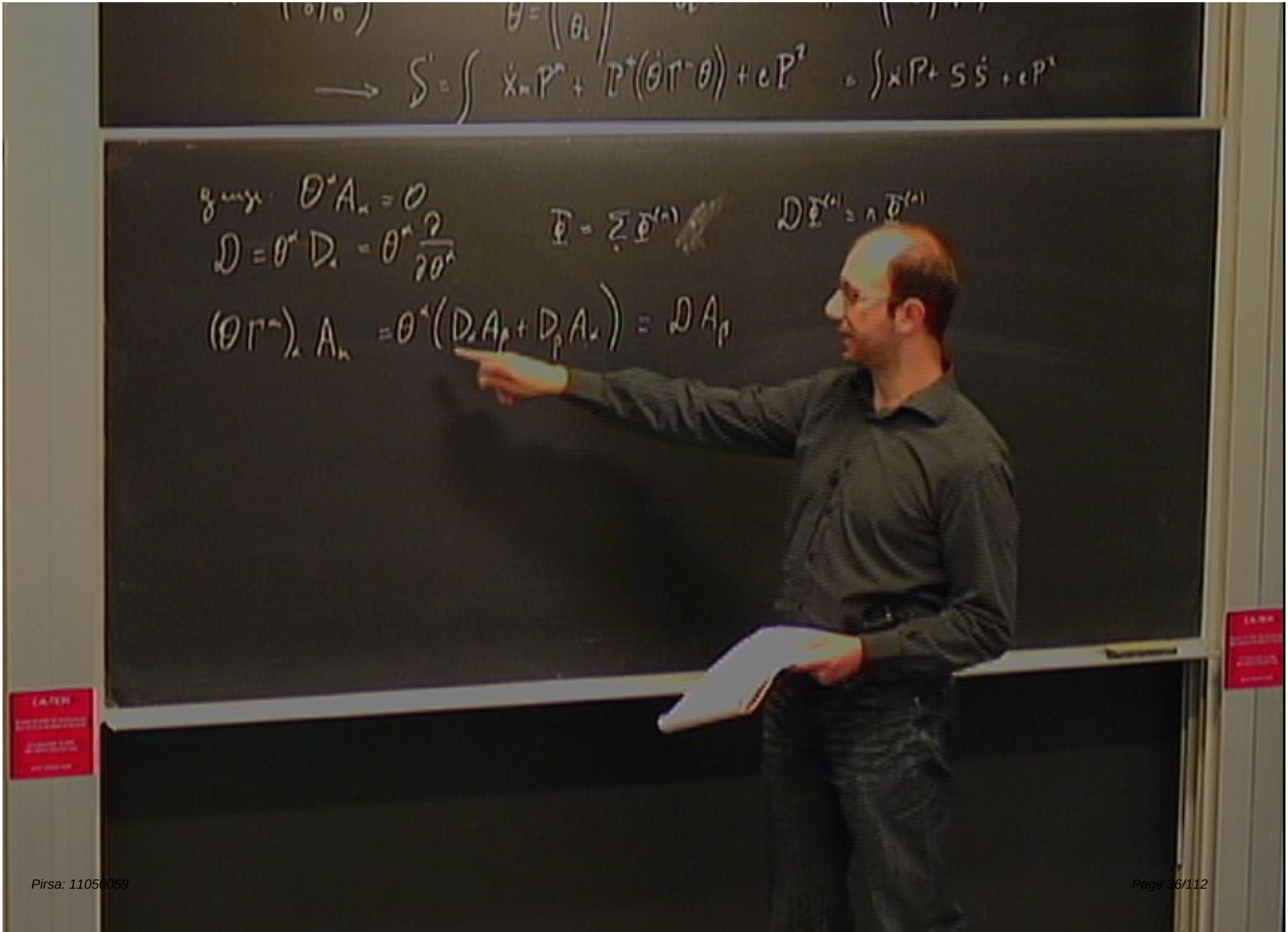

Abelian YM
δA_m = ∂_m Λ

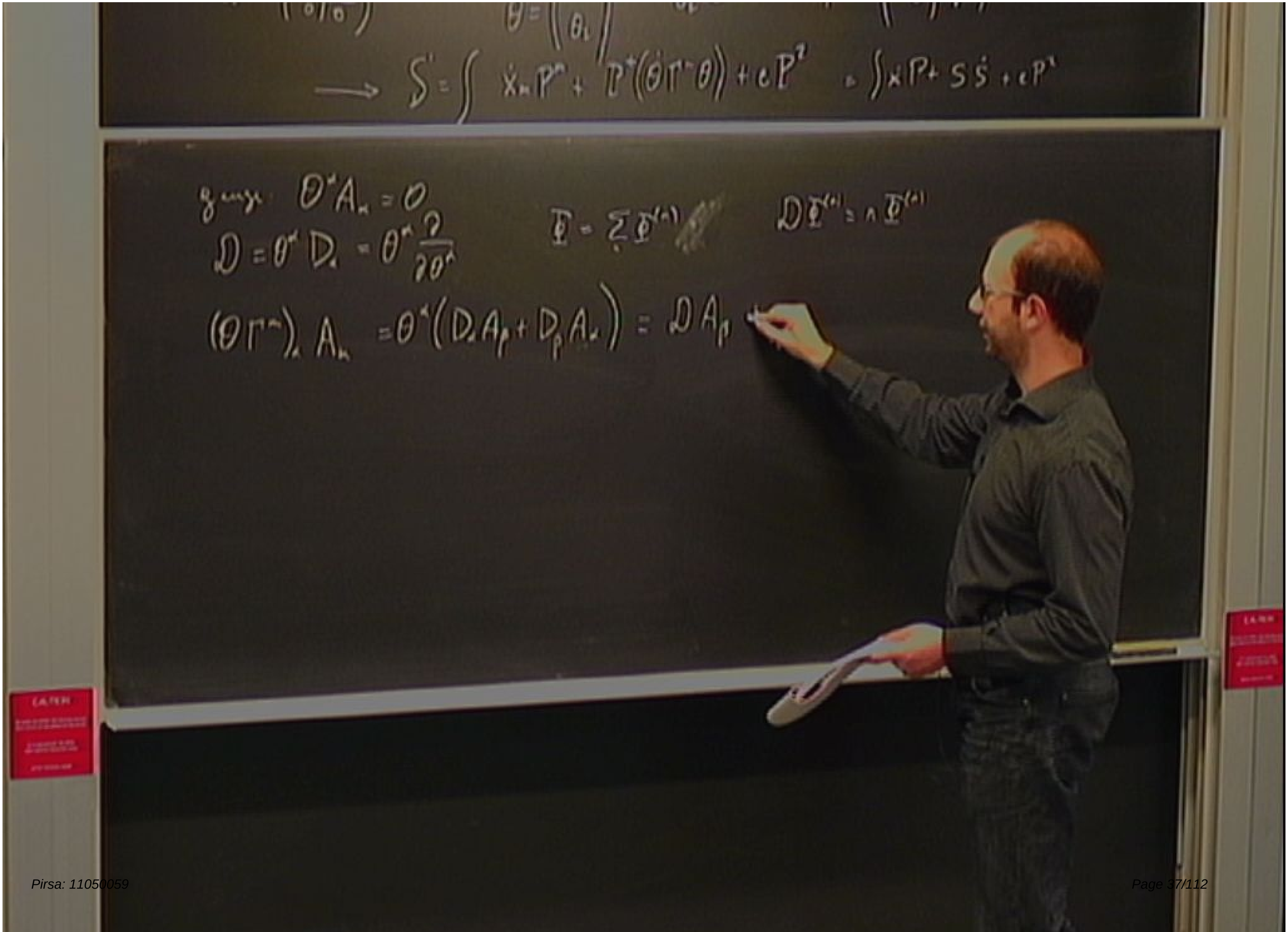

Abelian YM
F_αβ = (Γ^m)_αβ A_m
δA_m = ∂_m Λ
W^α = 1/10 (γ^m)^αβ (D_β A_m − ∂_m A_β)
δW^α = 0
D_α W^β = 1/4 ((Γ^mn)_α^β F_mn
F_mn = ∂_[m A_n]

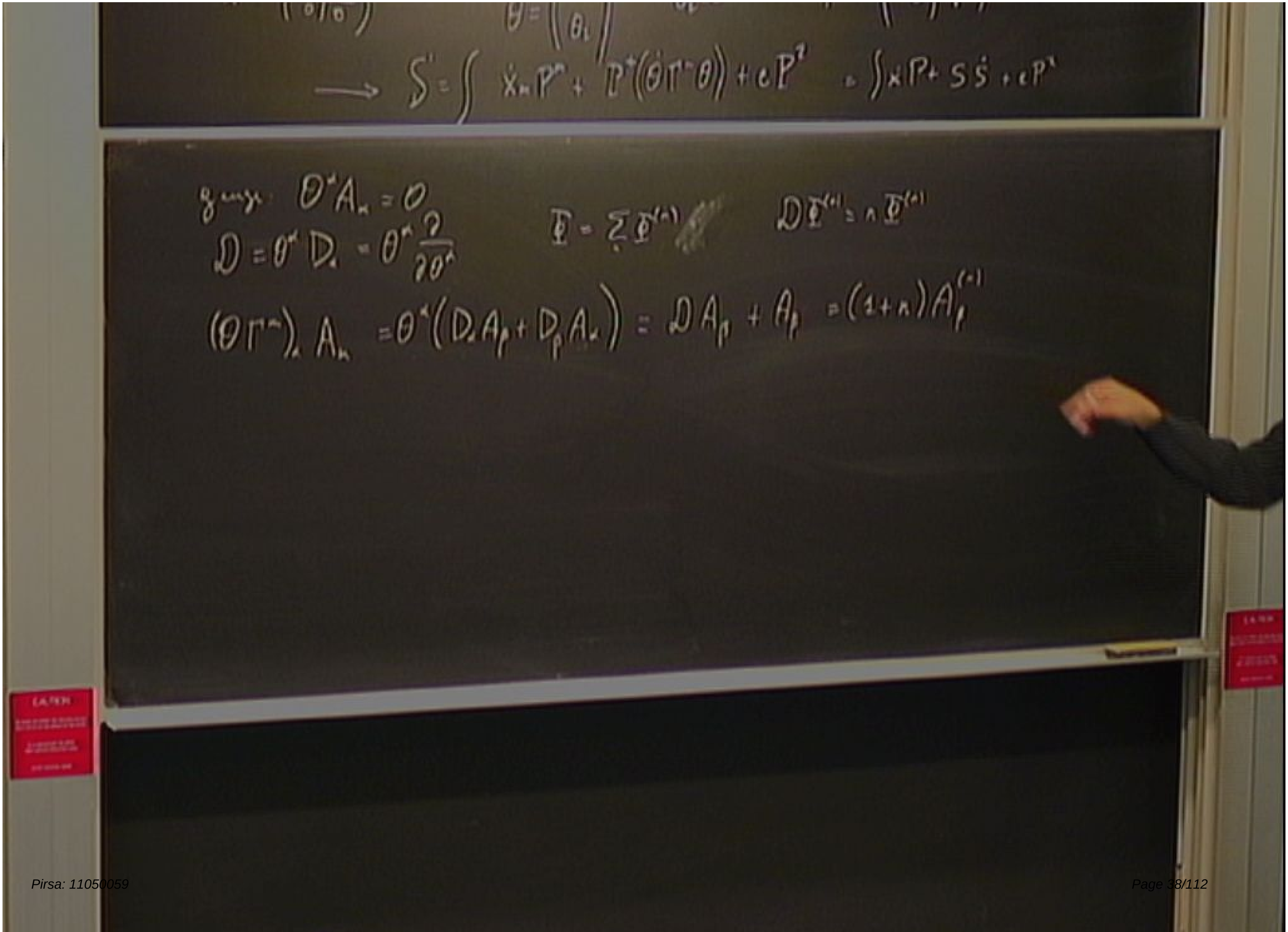

Fαβ = (γm)αβ Am
δAm = ∂m Λ
Wα = 1/10 (γm)αβ (Dβ Am - ∂m Aβ)
Dα Wβ = 1/4 ((Γmn))α β Fmn
Fmn = ∂[m An]

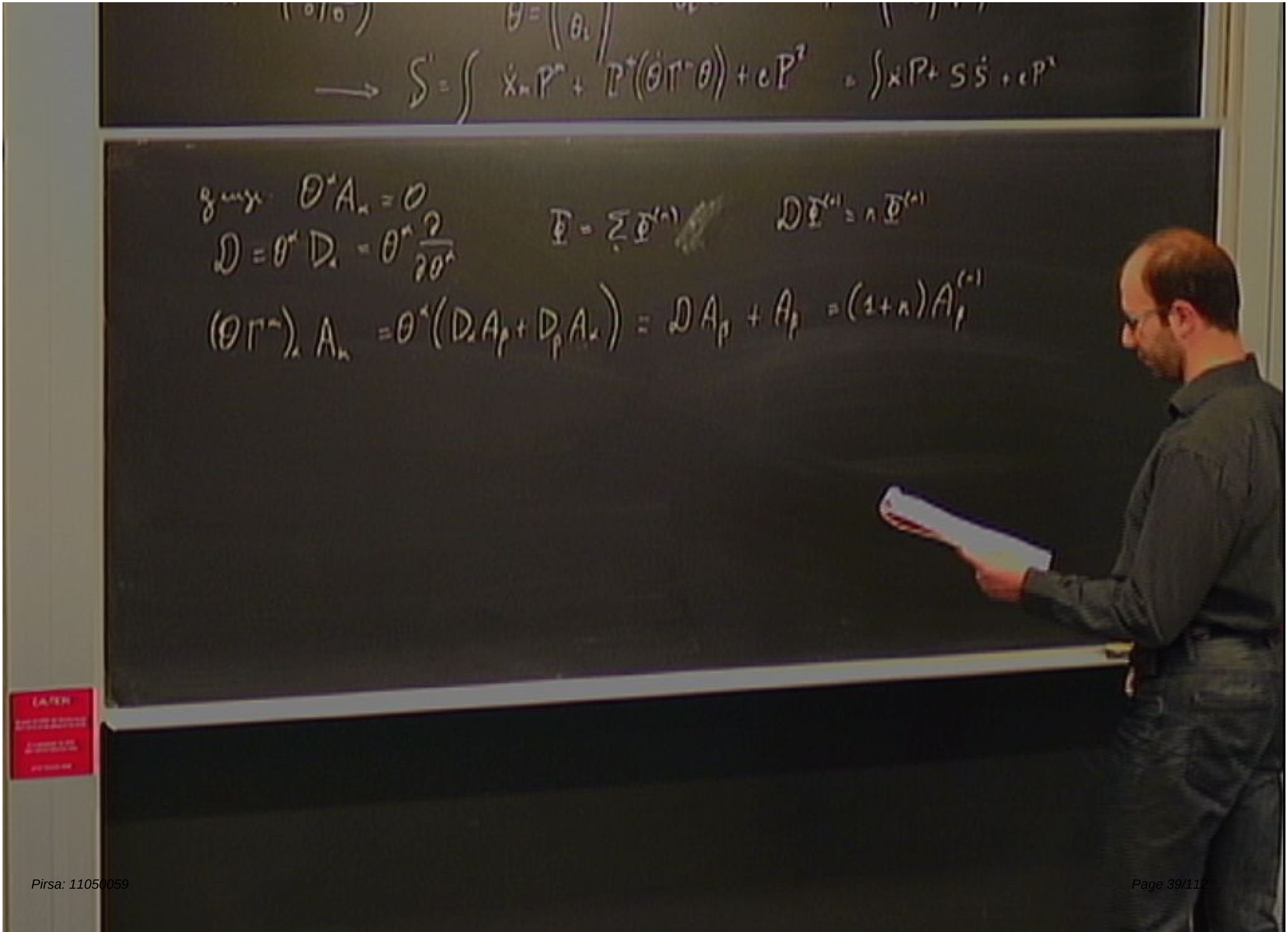

gauge $\theta^\alpha A_\alpha = 0$
$\mathcal{D} = \theta^\alpha D_\alpha = \theta^\alpha \frac{\partial}{\partial \theta^\alpha}$
$\Phi = \sum_n \Phi^{(n)}$
$\mathcal{D}\Phi^{(n)} = n\,\Phi^{(n)}$

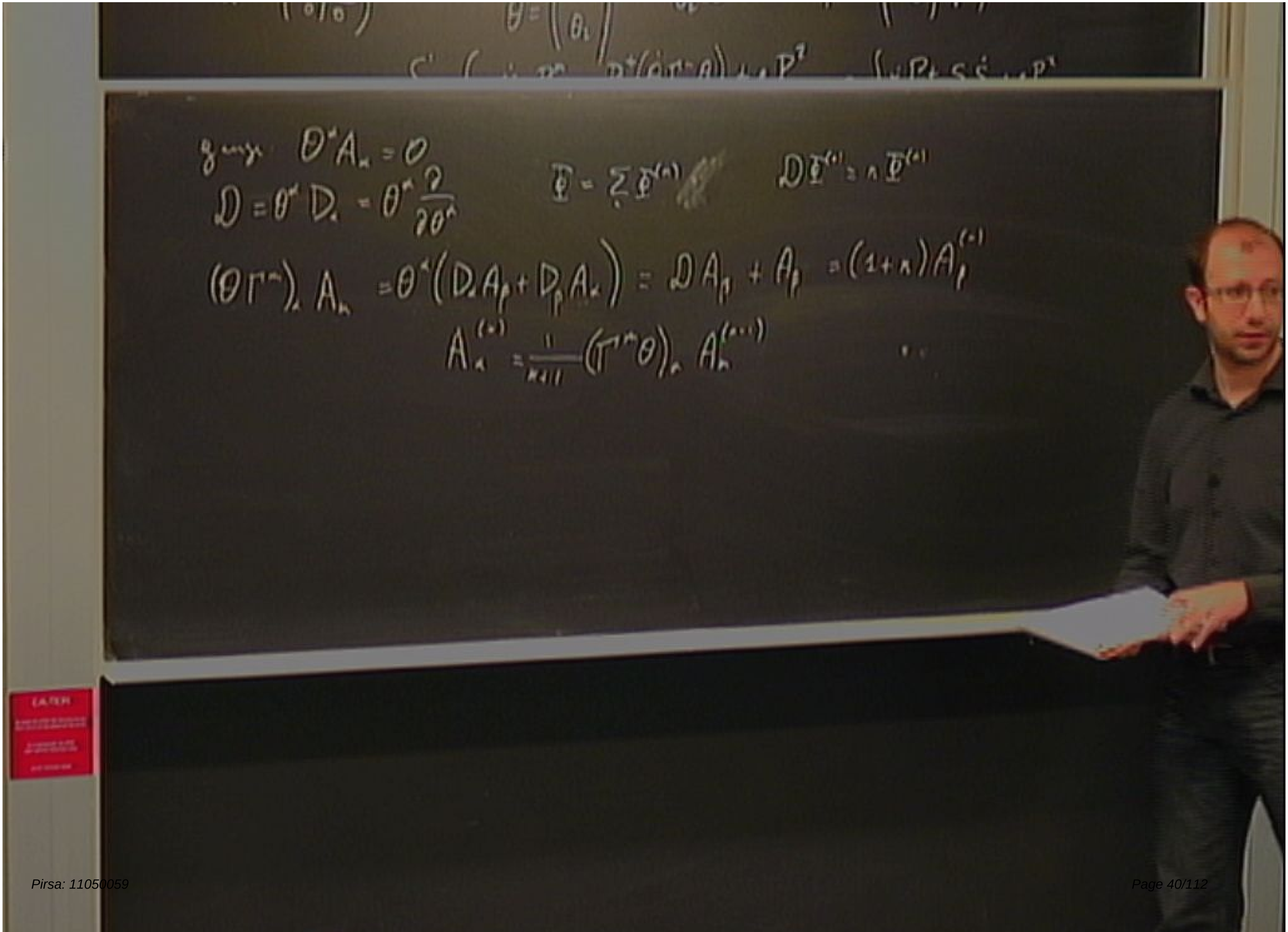

gauge: θ^α A_α = 0
D = θ^α D_α = θ^α ∂/∂θ^α
(θΓ^m)_α A_m = θ^α(D_α A_β + D_β A_α)

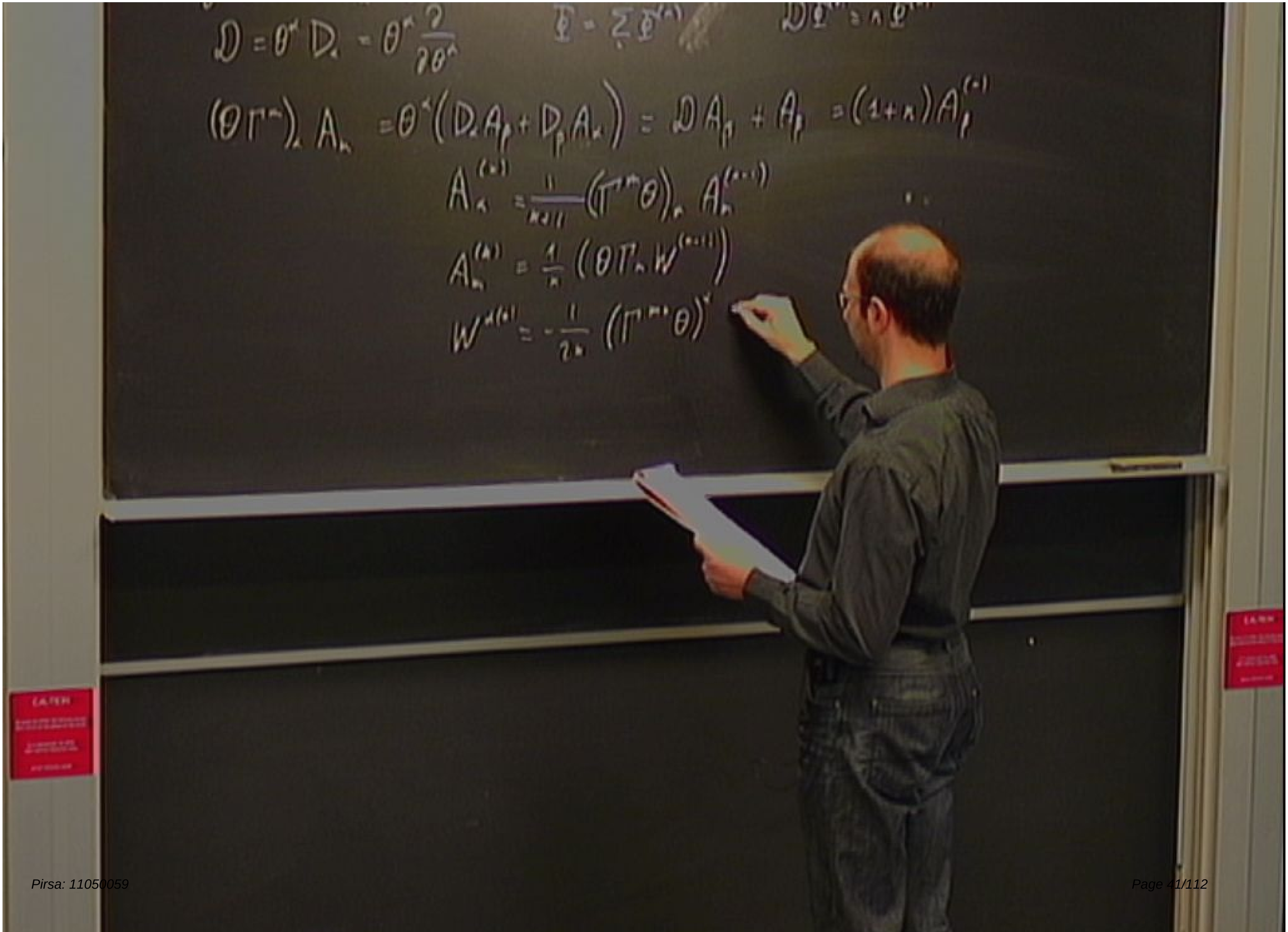





$\longrightarrow S' = \int \dot{X}_m P^m + P^+(\dot{\theta}\Gamma^-\theta) + cP^2 = \int \dot{x} P + S\dot{S} + cP^2$
gauge: $\theta^\alpha A_\alpha = 0$
$\mathcal{D} = \theta^\alpha D_\alpha = \theta^\alpha \frac{\partial}{\partial \theta^\alpha}$
$\Phi = \sum_n \Phi^{(n)}$
$\mathcal{D}\Phi^{(n)} = n\Phi^{(n)}$
$(\theta\Gamma^m)_\alpha A_m = \theta^\alpha(D_\alpha A_\beta + D_\beta A_\alpha) = \mathcal{D}A_\beta + A_\beta = (1+n)A_\beta^{(n)}$


S = ∫ ẋ_m P^m + P^+ (θ Γ^- θ) + e P^2 = ∫ ẋ P + S Ṡ + e P^2
gauge θ^α A_α = 0
D = θ^α D_α = θ^α ∂/∂θ^α
Φ = Σ Φ^(n)
D Φ^(n) = n Φ^(n)
(θ Γ^m)_α A_m = θ^α (D_α A_β + D_β A_α) = D A_β + A_β = (1+n) A_β^(n)

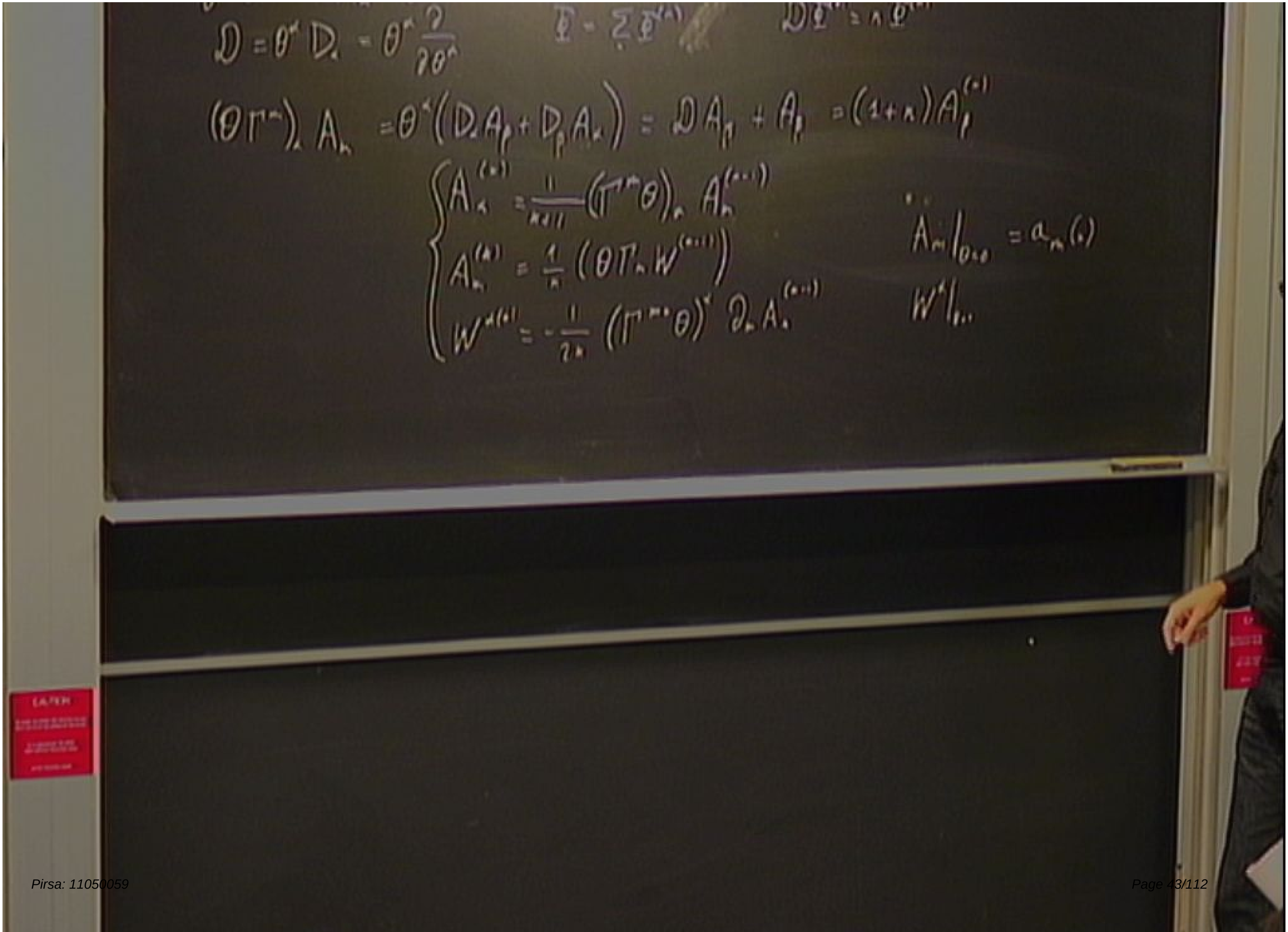

gauge: θ^α A_α = 0
𝒟 = θ^α D_α = θ^α ∂/∂θ^α
Φ = Σ Φ^(n)
𝒟Φ^(n) = n Φ^(n)
(θΓ^m)_α A_m = θ^α(D_α A_β + D_β A_α) = 𝒟A_β + A_β = (1+n) A_β^(n)
A_α^(n) = 1/(n+1) (Γ^m θ)_α A_m^(n-1)

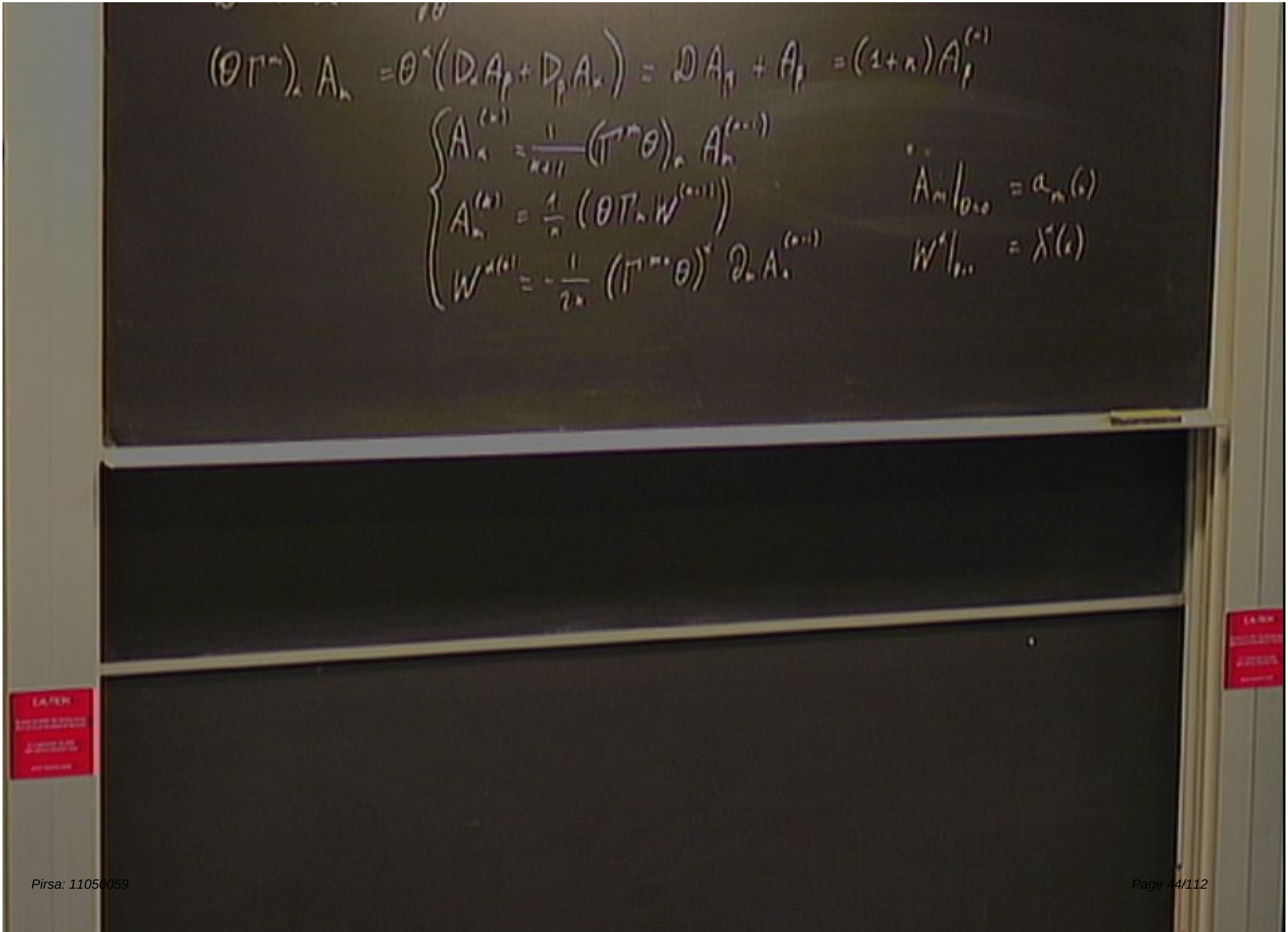

𝒟 = θ^α D_α = θ^α ∂/∂θ^α
Φ = Σ Φ^(n)
(θΓ^m)_α A_m = θ^α(D_α A_β + D_β A_α) = 𝒟 A_β + A_β = (1+n) A_β^(n)
A_α^(n) = 1/(n+1) (Γ^m θ)_α A_m^(n-1)
A_m^(n) = 1/n (θ Γ_m W^(n-1))
W^α(n) = -1/(2n) (Γ^mn θ)^α

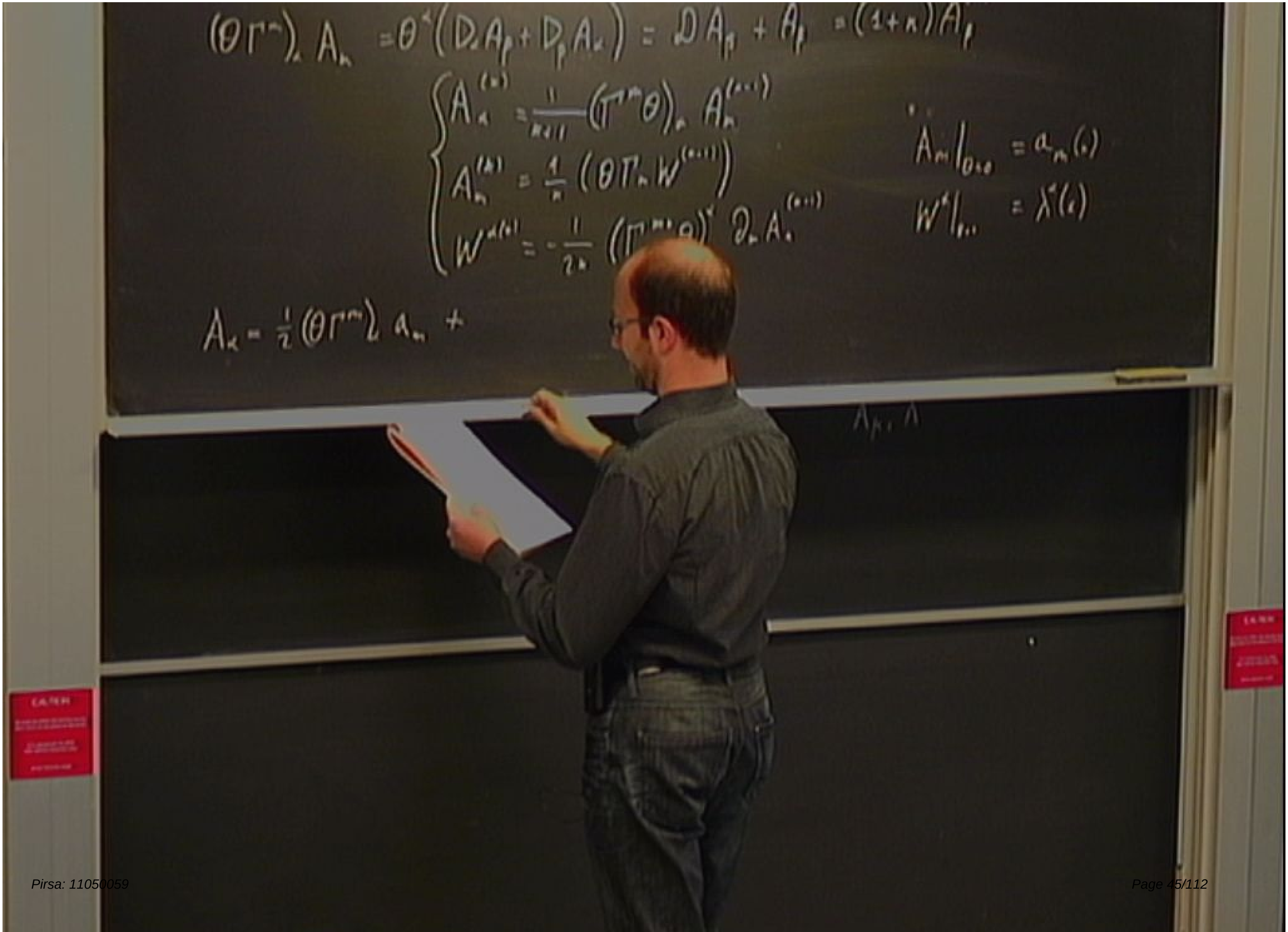
$$\mathcal{D} = \theta^\alpha D_\alpha = \theta^\alpha \frac{\partial}{\partial \theta^\alpha} \qquad \Phi = \sum \Phi^{(n)} \qquad \mathcal{D}\Phi^{(n)} = n\,\Phi^{(n)}$$

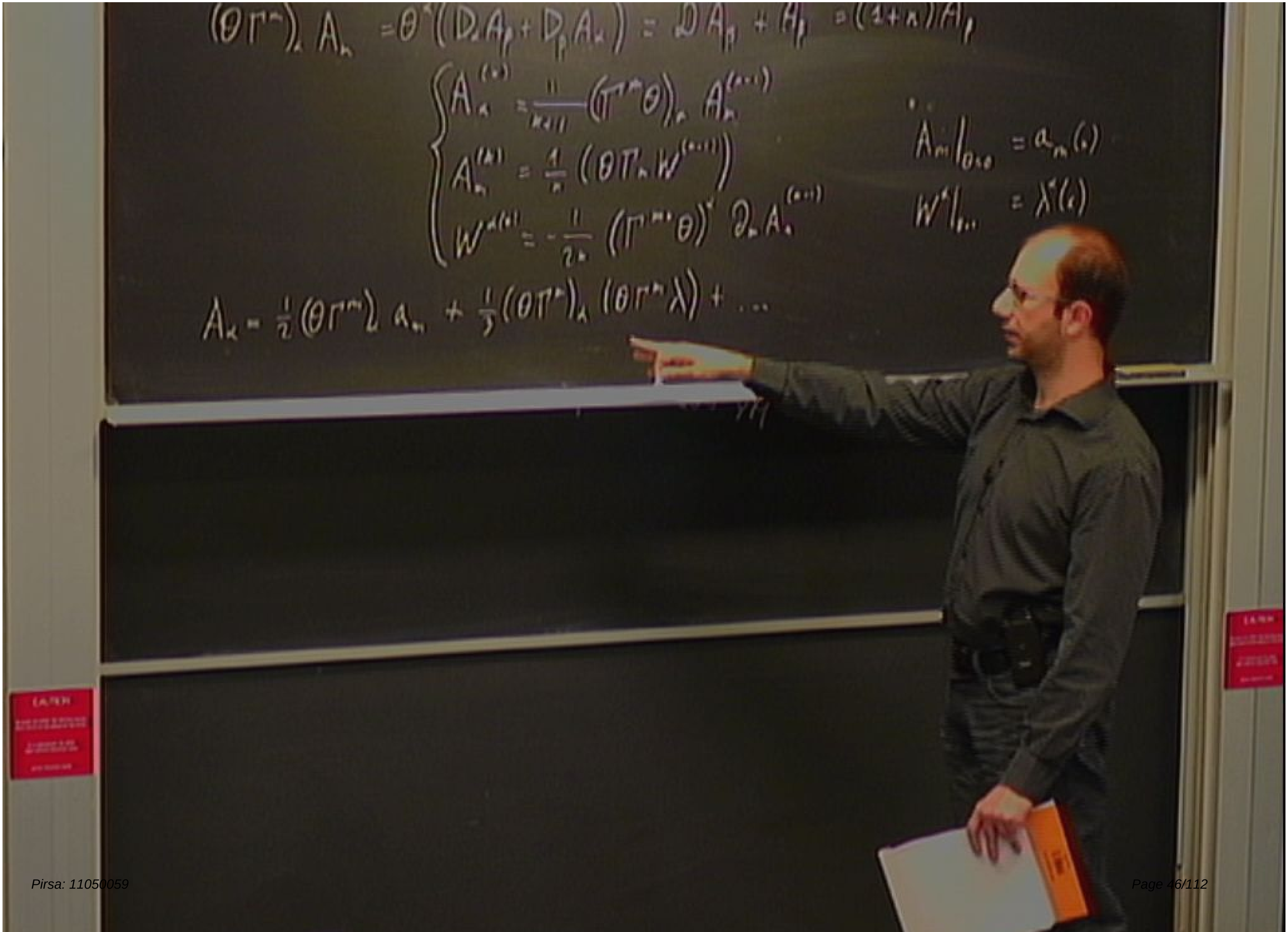
$$(\theta\Gamma^m)_\alpha A_m = \theta^\alpha \left(D_\alpha A_\beta + D_\beta A_\alpha \right) = \mathcal{D} A_\beta + A_\beta = (1+n) A_\beta^{(n)}$$

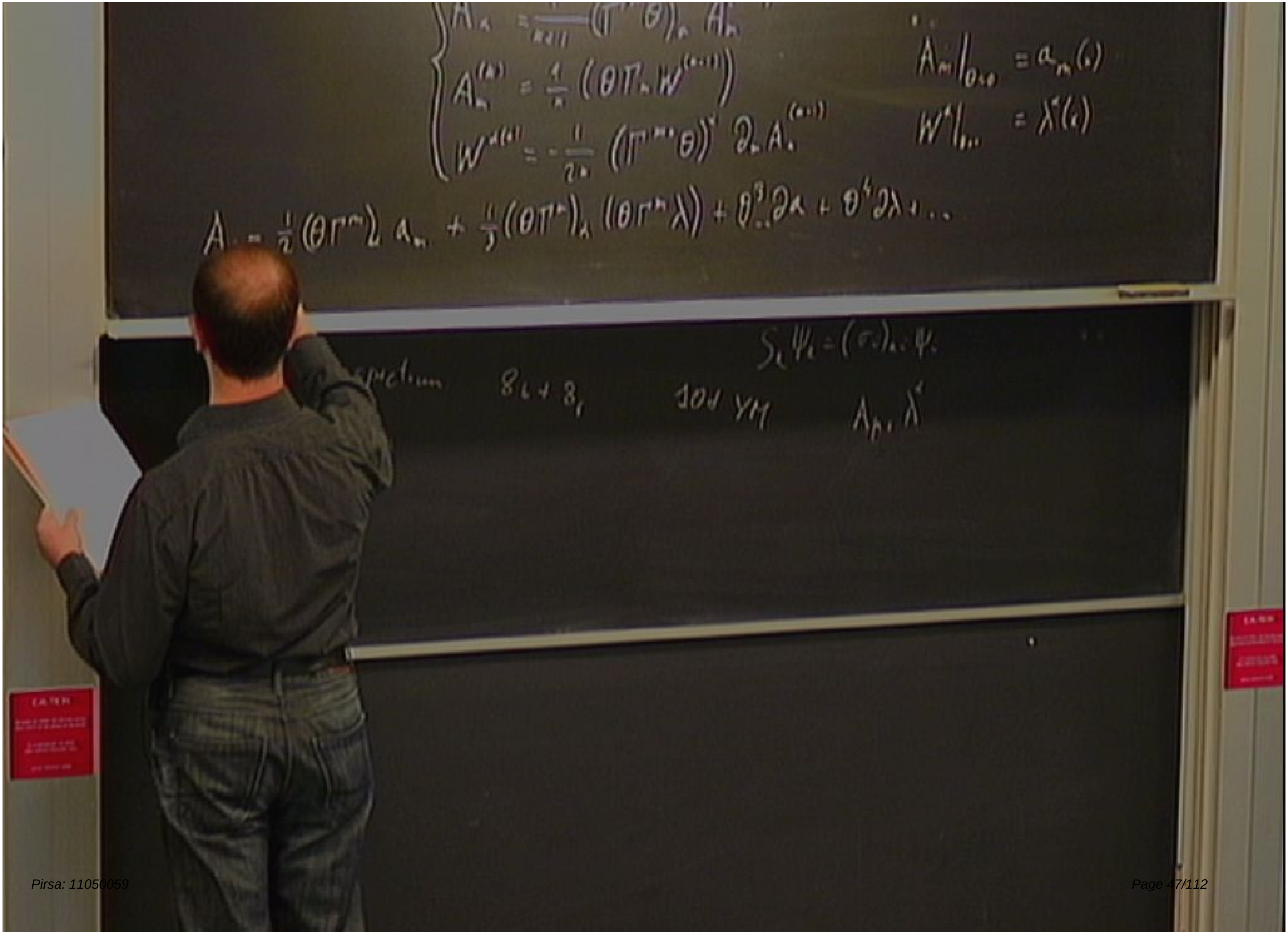
$$\begin{cases} A_\alpha^{(n)} = \frac{1}{n+1} (\Gamma^m \theta)_\alpha A_m^{(n-1)} \\ A_m^{(n)} = \frac{1}{n} \left(\theta \Gamma_m W^{(n-1)} \right) \\ W^{\alpha(n)} = -\frac{1}{2n} (\Gamma^{mn} \theta)^\alpha \partial_m A_n^{(n-1)} \end{cases}$$

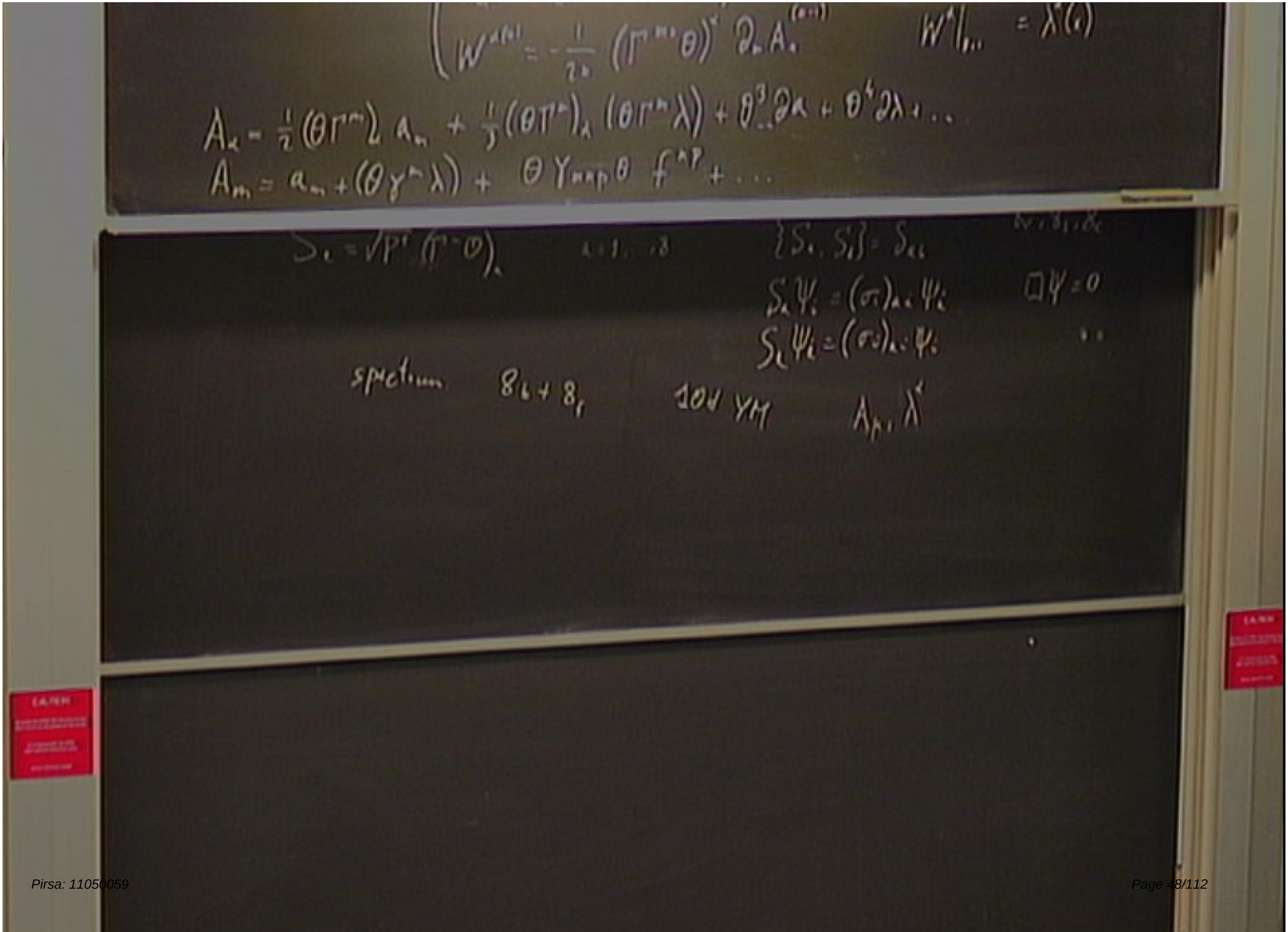

$\mathcal{D} = \theta^\alpha D_\alpha = \theta^\alpha \frac{\partial}{\partial\theta^\alpha}$
$(\theta\Gamma^m)_\alpha A_m = \theta^\alpha(D_\alpha A_\beta + D_\beta A_\alpha) = \mathcal{D} A_\beta + A_\beta = (1+n) A_\beta^{(n)}$
$A_\alpha^{(n)} = \frac{1}{n+1}(\Gamma^m\theta)_\alpha A_m^{(n-1)}$
$A_m^{(n)} = \frac{1}{n}(\theta\Gamma_m W^{(n-1)})$
$W^{\alpha(n)} = -\frac{1}{2n}(\Gamma^{mn}\theta)^\alpha \partial_m A_n^{(n-1)}$
$A_m|_{\theta=0} = a_m(x)$
$W^\alpha|_{\theta=0}$

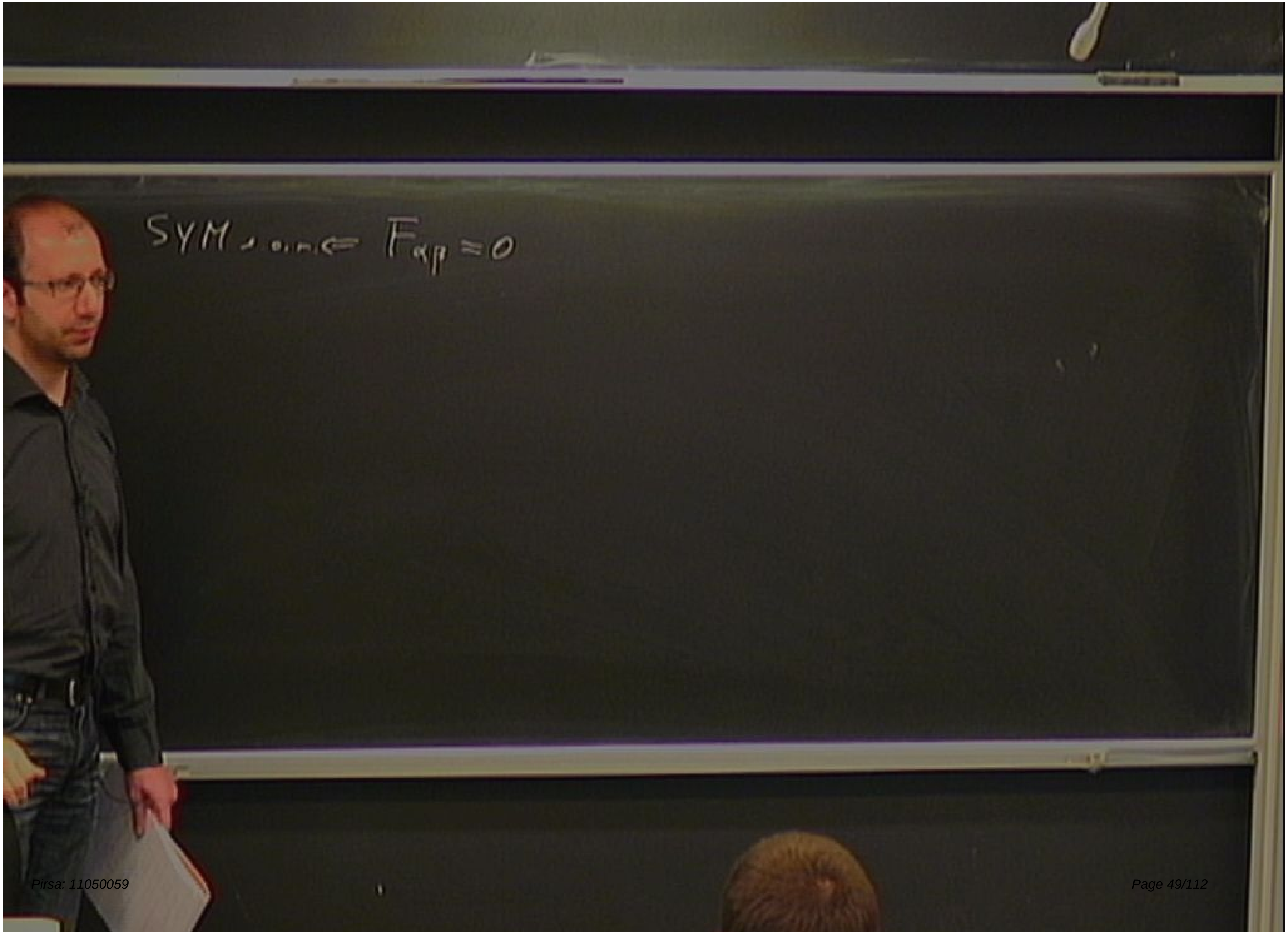

$(\theta\Gamma^{m})_\alpha A_m = \theta^\alpha(D_\alpha A_\beta + D_\beta A_\alpha) = \mathcal{D}A_\beta + A_\beta = (1+n)A_\beta^{(n)}$
$A_\alpha^{(n)} = \frac{1}{n+1}(\Gamma^m\theta)_\alpha A_m^{(n-1)}$
$A_m^{(n)} = \frac{1}{n}(\theta\Gamma_m W^{(n-1)})$
$W^{\alpha(n)} = -\frac{1}{2n}(\Gamma^{mn}\theta)^\alpha \partial_m A_n^{(n-1)}$
$A_m|_{\theta=0} = a_m(x)$
$W^\alpha|_{\theta=0} = \lambda^\alpha(x)$


(θΓ^m)_α A_m = θ^α(D_α A_β + D_β A_α) = D A_β + A_β = (1+κ) A_β
A_α^(k) = 1/(k+1) (Γ^m θ)_α A_m^(k-1)
A_m^(k) = 1/k (θ Γ_m W^(k-1))
W^α(k) = -1/(2k) (Γ^mn θ)^α ∂_m A_n^(k-1)
A_m|_θ=0 = a_m(x)
W^α|_θ=0 = λ^α(x)
A_α = 1/2 (θΓ^m)_α a_m +
A_α, A


$A_m|_{\theta=0} = a_m(x)$
$W^\alpha|_{\theta=0} = \lambda^\alpha(x)$
$A_\alpha = \frac{1}{2}(\theta\Gamma^m)_\alpha a_m + \frac{1}{3}((\theta\Gamma^m)_\alpha(\theta\Gamma_m\lambda)) + \dots$



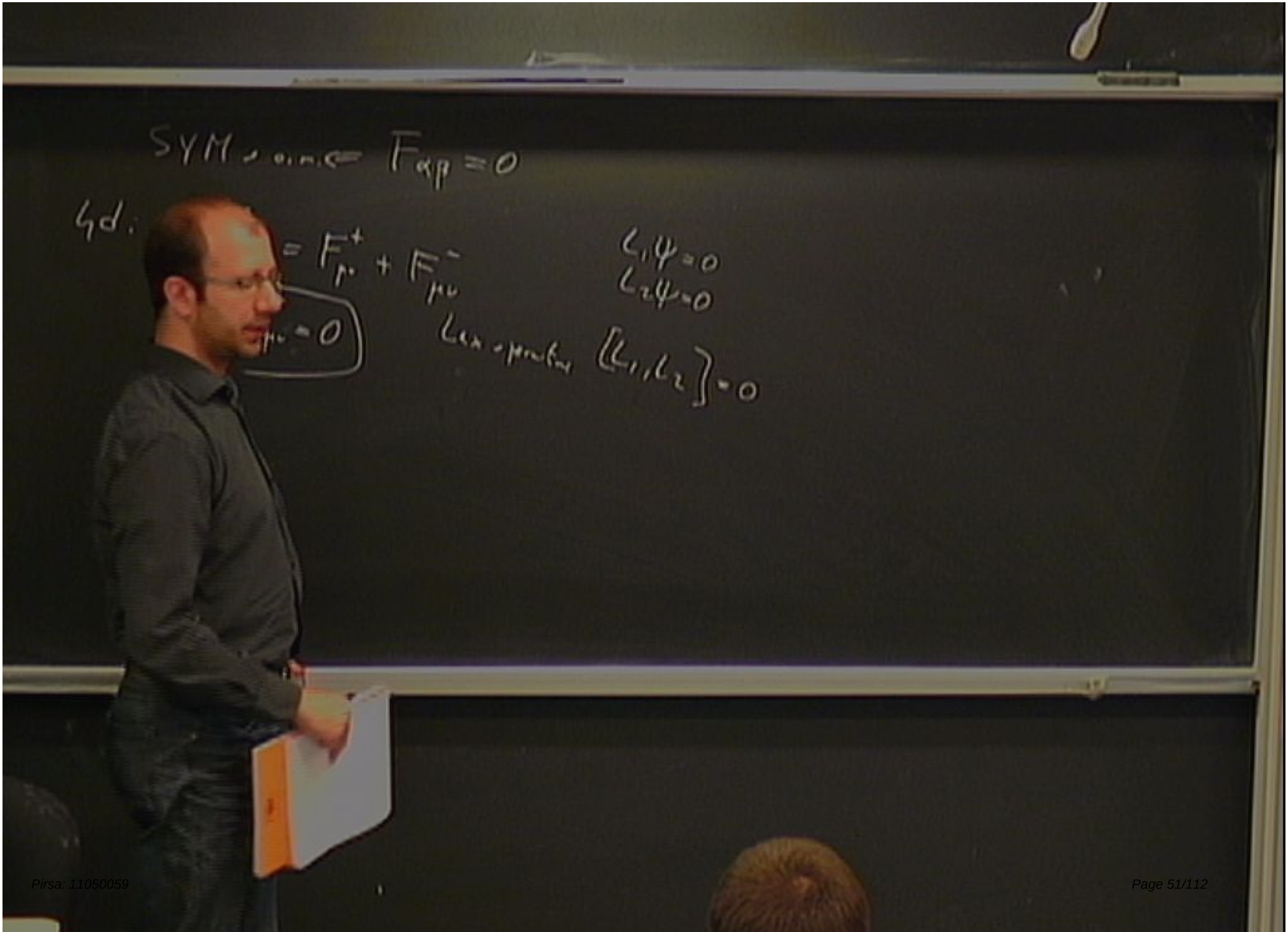

A_α = 1/2 (θΓ^m)_α a_m + 1/3 ((θΓ^m)_α (θΓ^m λ)) + θ^3 ∂a + θ^4 ∂λ + ...
A_m = a_m + (θγ^m λ) + θ γ_mnp θ f^np + ...
{S_a, S_b} = δ_ab
□ψ = 0
spectrum 8_b + 8_f
10d YM
A_μ, λ^α

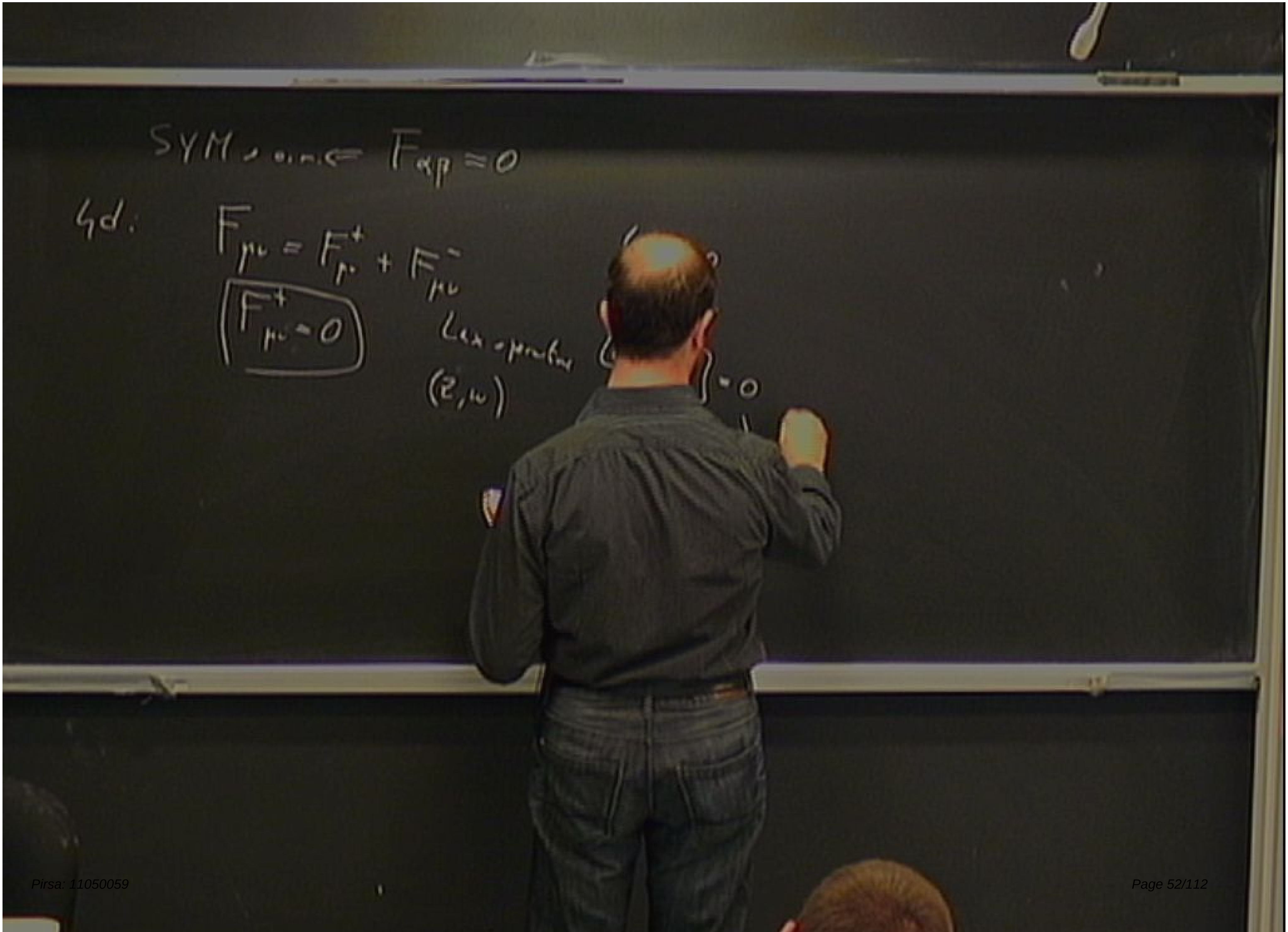

SYM ⇐ $F_{\alpha\beta} = 0$

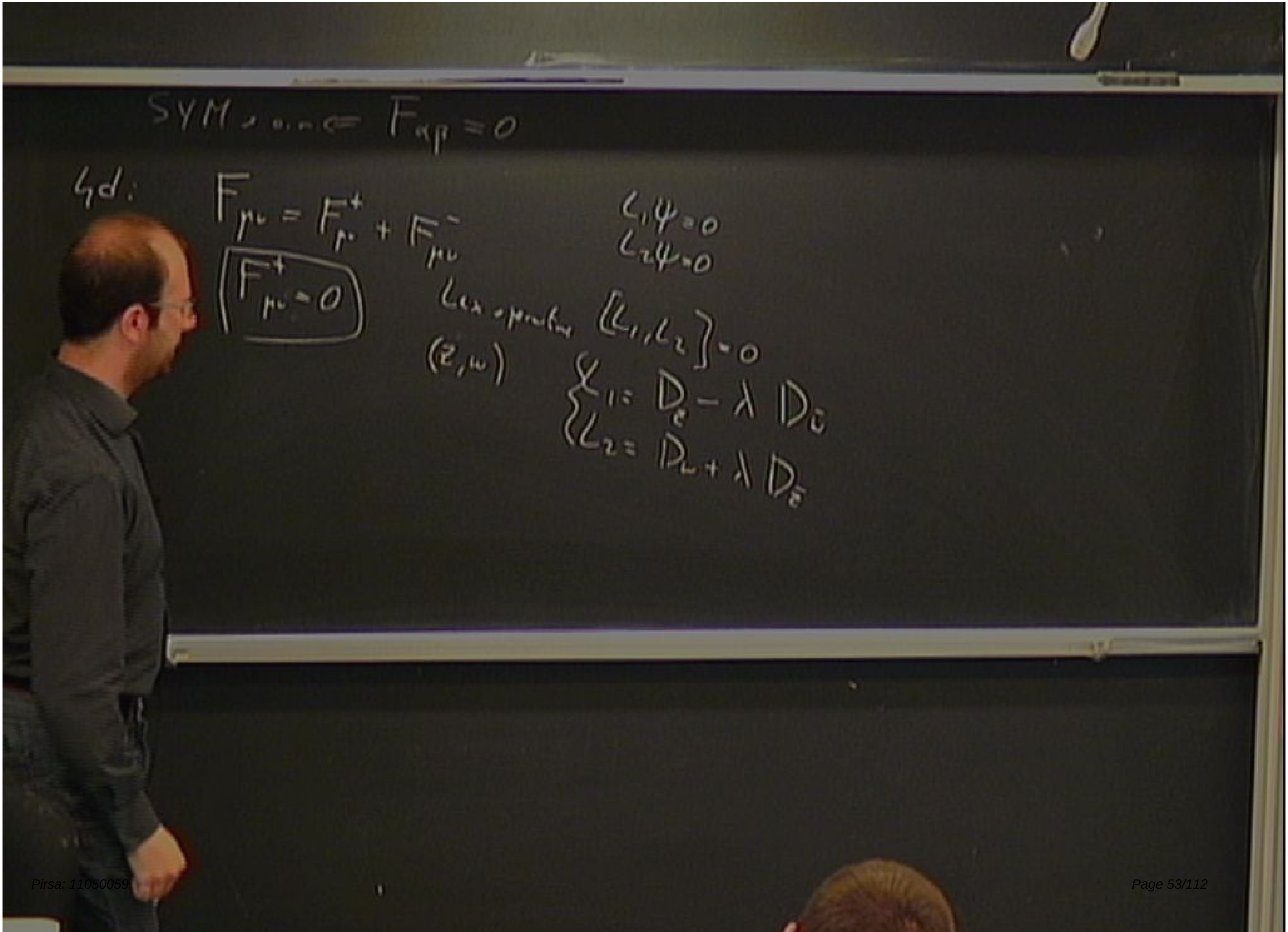
SYM, a.m.c $\Leftarrow F_{\alpha\beta} = 0$

4d: $F_{\mu\nu} = F^+_{\mu\nu} + F^-_{\mu\nu}$

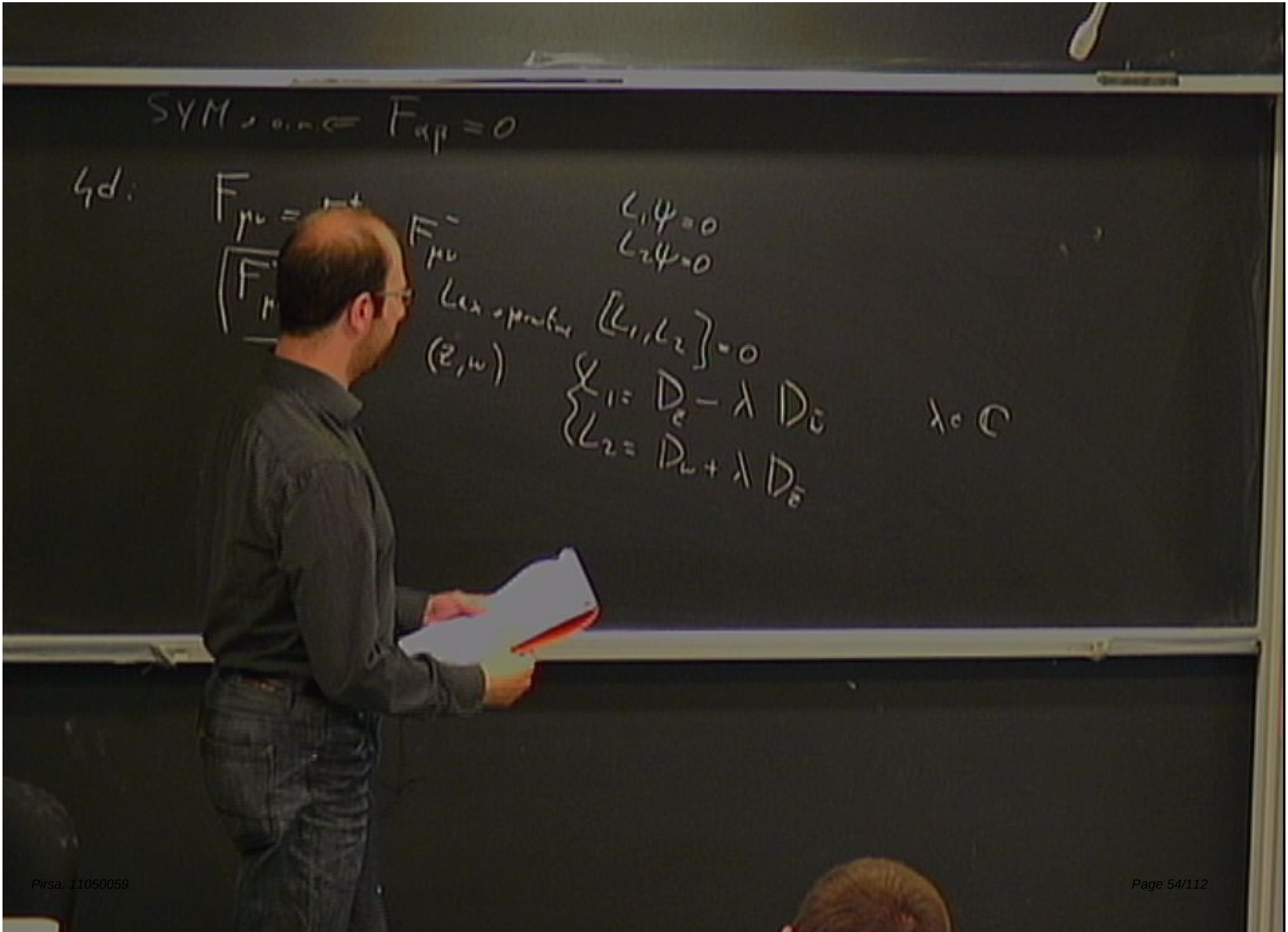
$$\boxed{F^+_{\mu\nu} = 0}$$

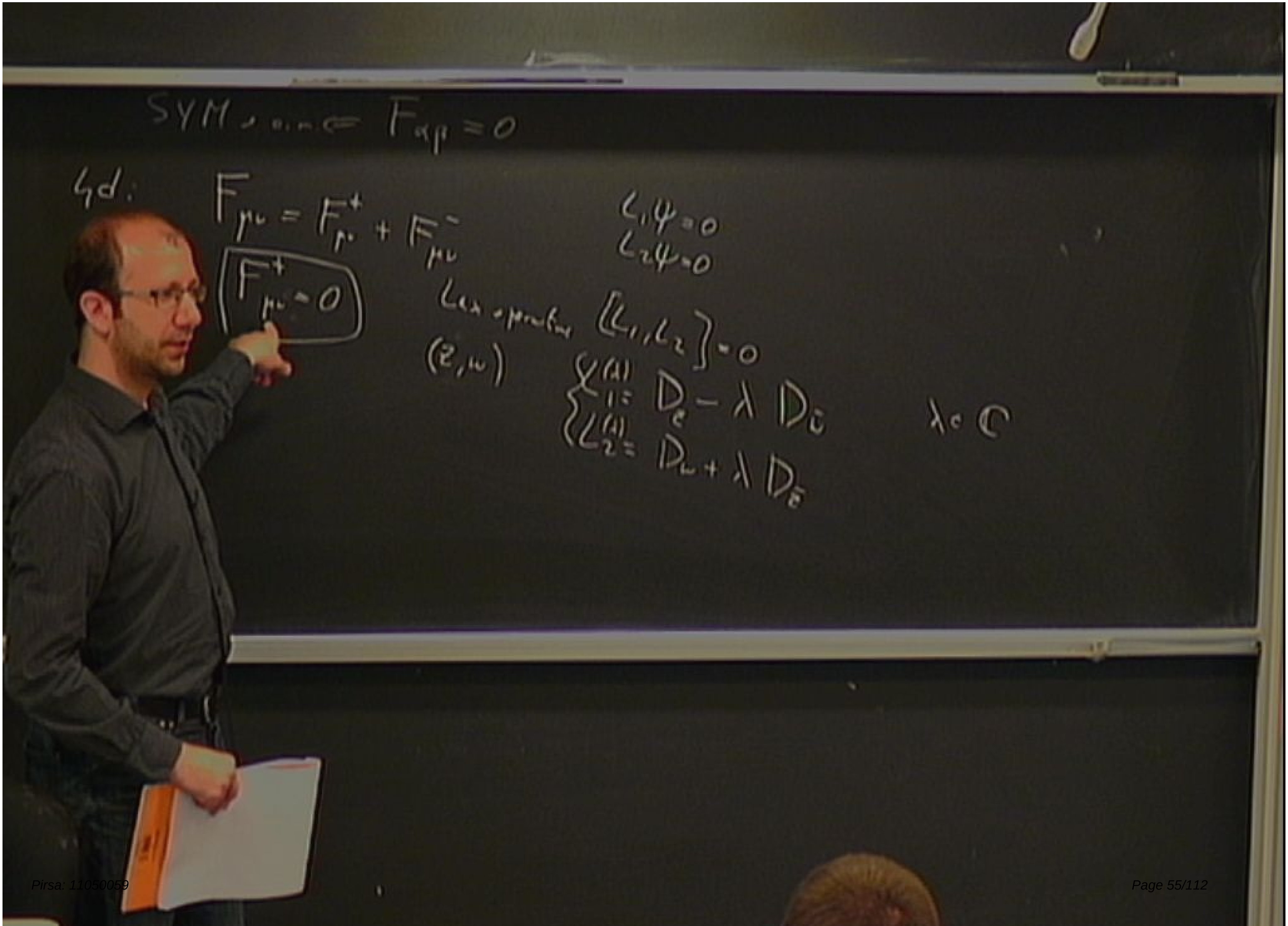

SYM e.o.m ⇐ $F_{\alpha\beta}=0$
4d:
$= F^+_{\mu\nu} + F^-_{\mu\nu}$
$L_1\psi=0$
$L_2\psi=0$
$=0$
Lax operator
$[L_1,L_2]=0$


SYM, e.o.m. ⇔ Fαβ = 0
4d:
Fμν = F+μν + F-μν
F+μν = 0
Lax operator
(2,w)
= 0

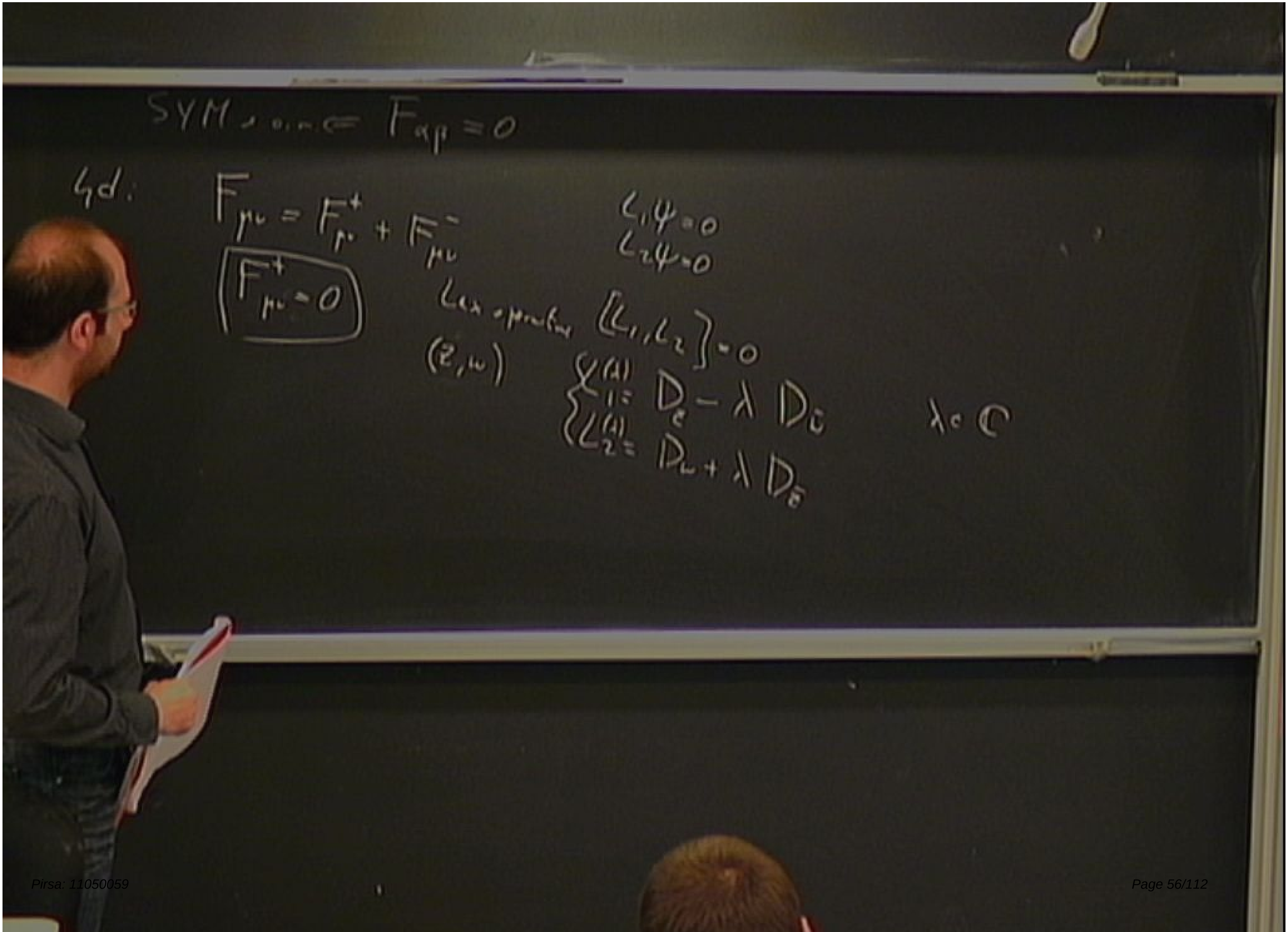


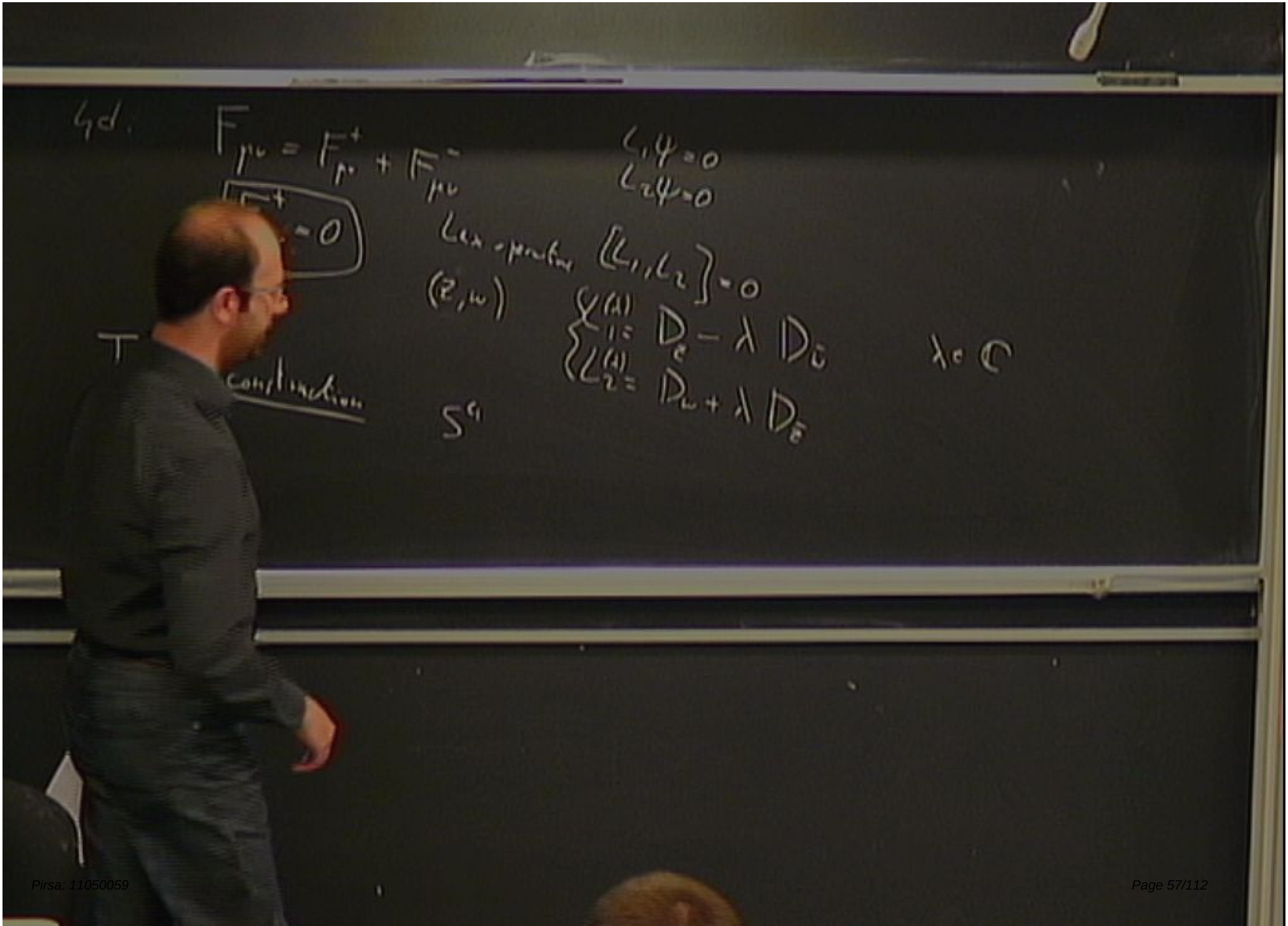
SYM e.o.m ⟸ $F_{\alpha\beta} = 0$
4d:
$F_{\mu\nu} = F^{+}_{\mu\nu} + F^{-}_{\mu\nu}$
$F^{+}_{\mu\nu} = 0$
$L_1 \psi = 0$
$L_2 \psi = 0$
Lax operator $[L_1, L_2] = 0$
(z, w)
$L_1 = D_z - \lambda D_{\bar{w}}$
$L_2 = D_w + \lambda D_{\bar{z}}$

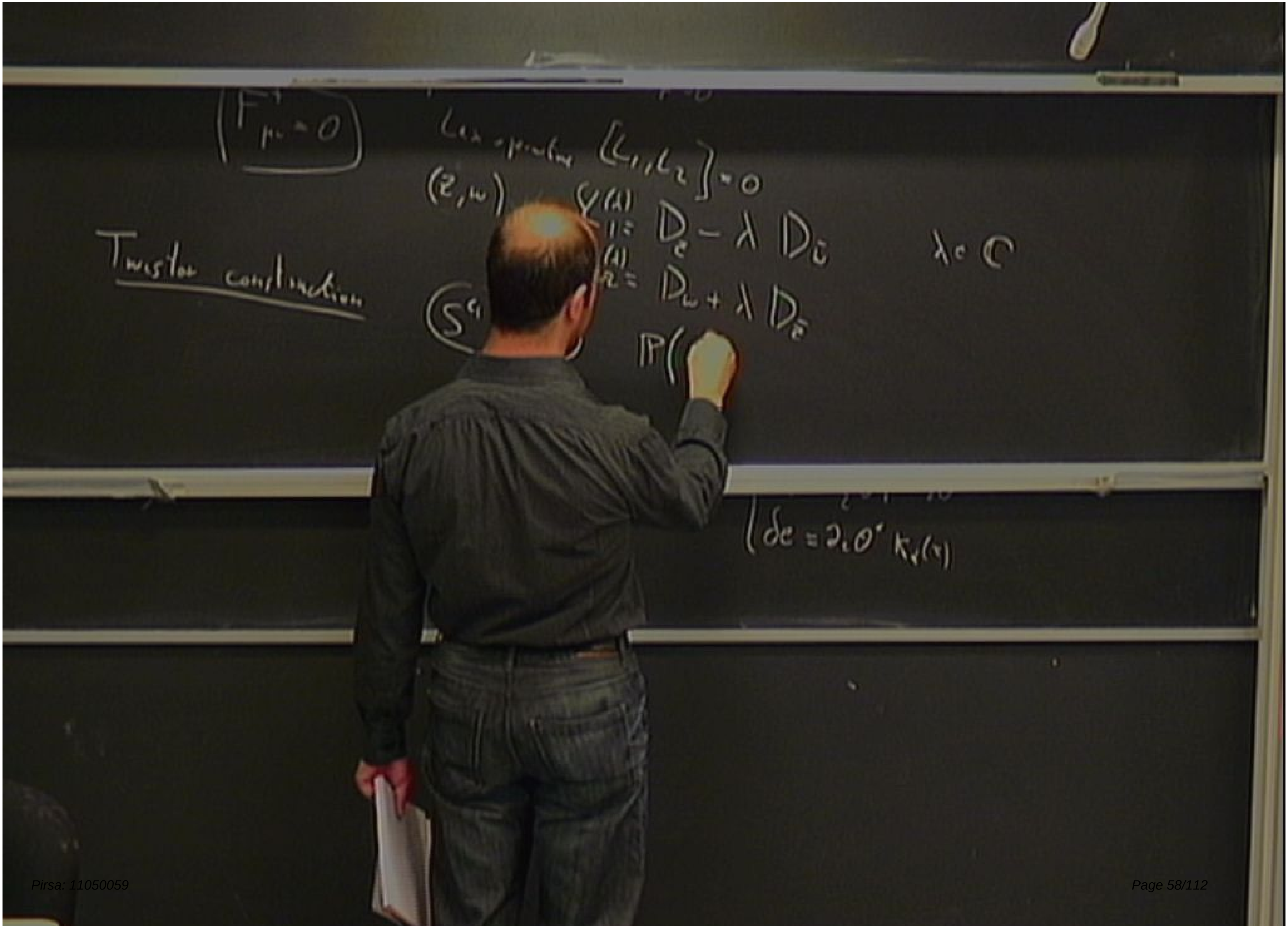

SYM
$F_{\alpha\beta}=0$
4d:
$L_1\psi=0$
$L_2\psi=0$
$[L_1,L_2]=0$
(z,w)
$L_1 = D_z - \lambda D_{\bar{w}}$
$L_2 = D_w + \lambda D_{\bar{z}}$
$\lambda \in \mathbb{C}$

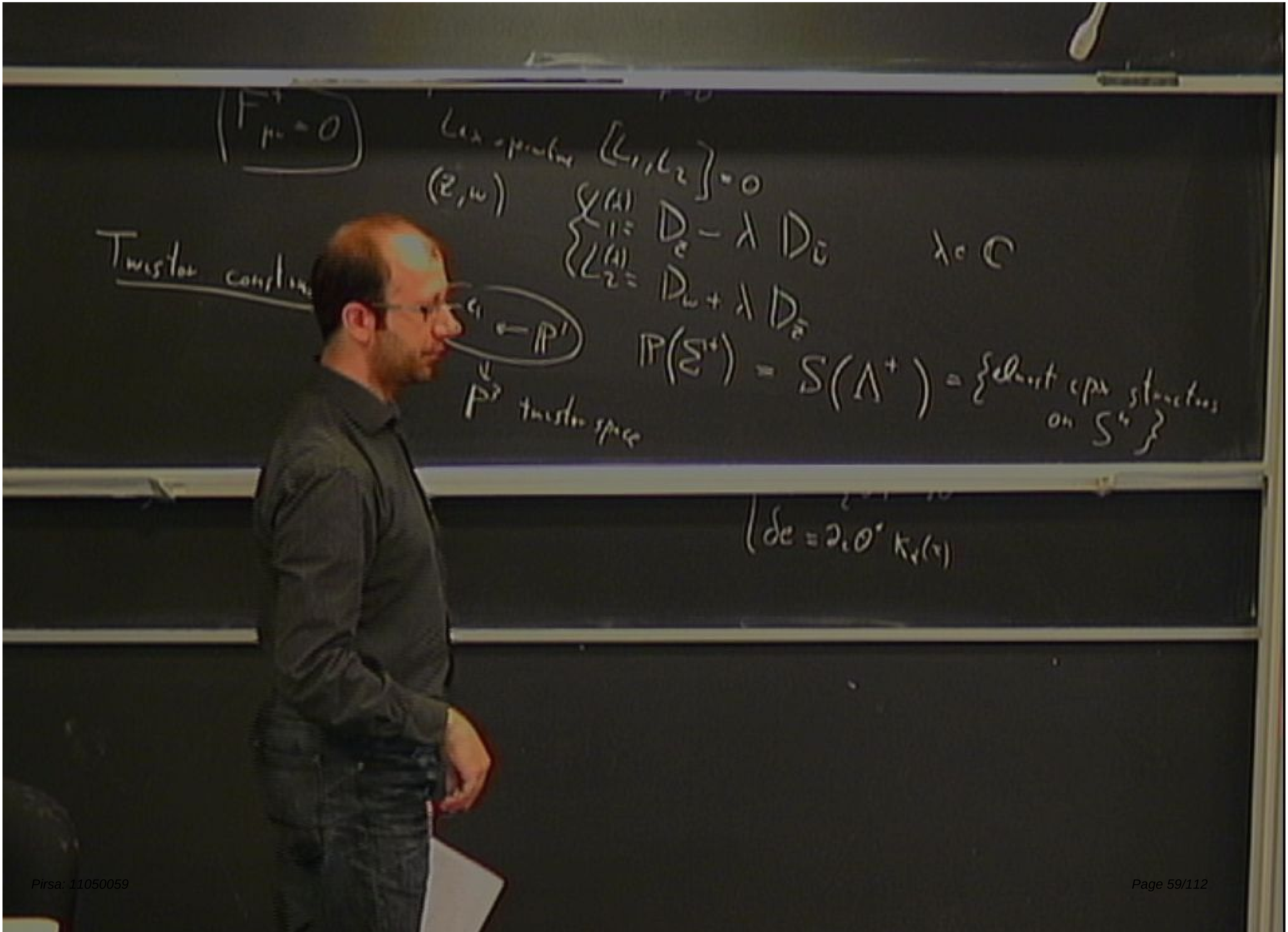


4d:
F_μν = F⁺_μν + F⁻_μν
F⁺_μν = 0
L₁ψ = 0
L₂ψ = 0
Lax operator [L₁, L₂] = 0
(z, w)
L₁^(λ) = D_z − λ D_w̄
L₂^(λ) = D_w + λ D_z̄
λ ∈ ℂ


SYM ⇐ F_αβ = 0
4d:
F_μν = F⁺_μν + F⁻_μν
F⁺_μν = 0
L₁ψ = 0
L₂ψ = 0
Lax operator
[L₁, L₂] = 0
(z, w)
L₁^(λ) = D_z − λ D_w̄
L₂^(λ) = D_w + λ D_z̄
λ ∈ ℂ


4d.
$F_{\mu\nu} = F^+_{\mu\nu} + F^-_{\mu\nu}$
$L_1\psi = 0$
$L_2\psi = 0$
Lax operator $[L_1, L_2] = 0$
(z, w)
$L_1^{(\lambda)} = D_z - \lambda D_{\bar{w}}$
$\lambda \in \mathbb{C}$
$L_2^{(\lambda)} = D_w + \lambda D_{\bar{z}}$
construction
S^4


Twistor construction
Lax operator
(z,w)
λ ∈ ℂ


Lax operator
Twistor
twistor space

SYM ... $F_{\alpha\beta} = 0$

4d: $F_{\mu\nu} = F^+_{\mu\nu} + F^-_{\mu\nu}$

$\boxed{F^+_{\mu\nu} = 0}$

$L_1 \psi = 0$

$L_2 \psi = 0$

Lax operator $[L_1, L_2] = 0$

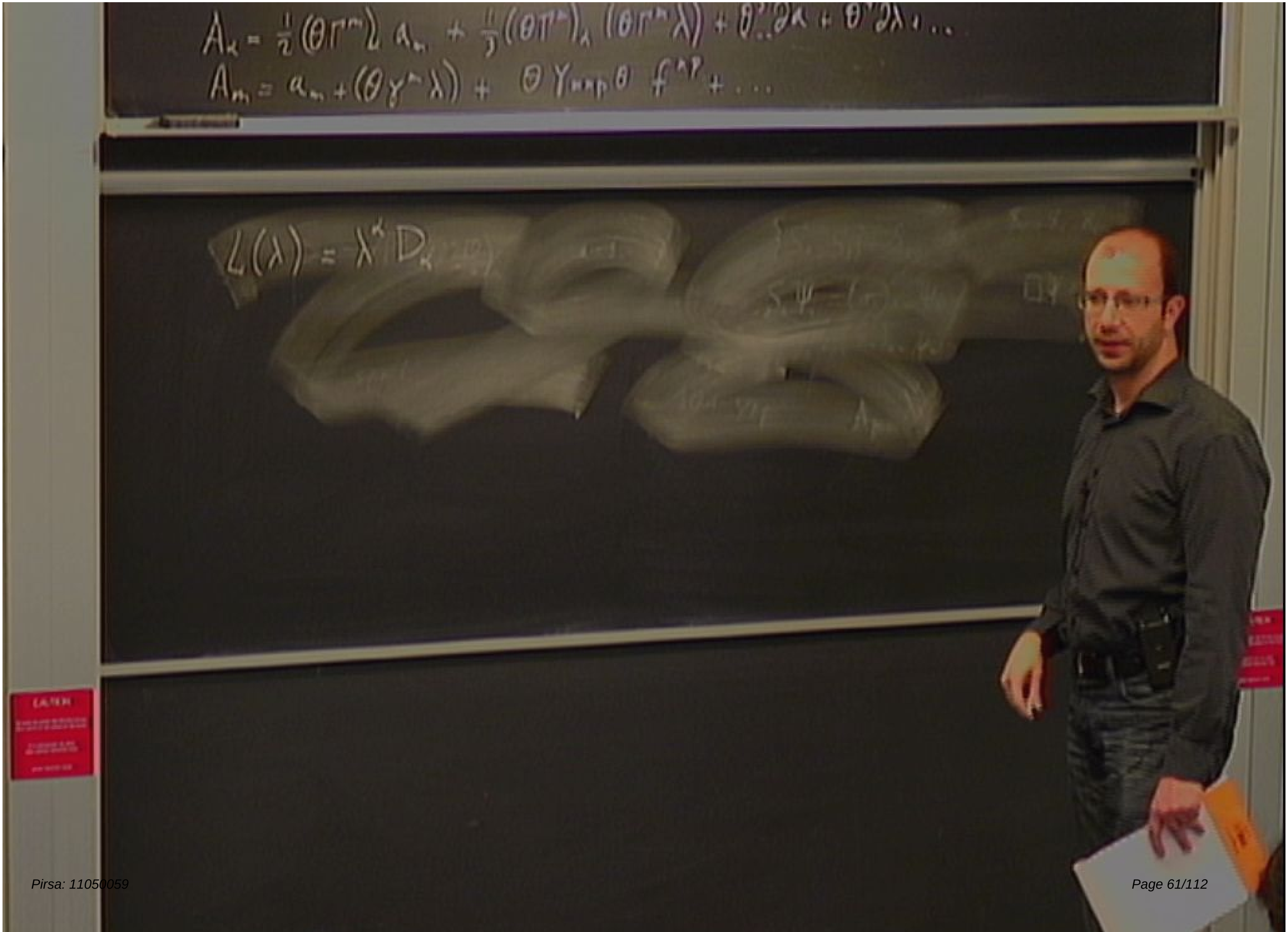
(z, w) $\begin{cases} L_1^{(\lambda)} = D_z - \lambda D_{\bar{w}} \\ L_2^{(\lambda)} = D_w + \lambda D_{\bar{z}} \end{cases}$ $\lambda \in \mathbb{C}$

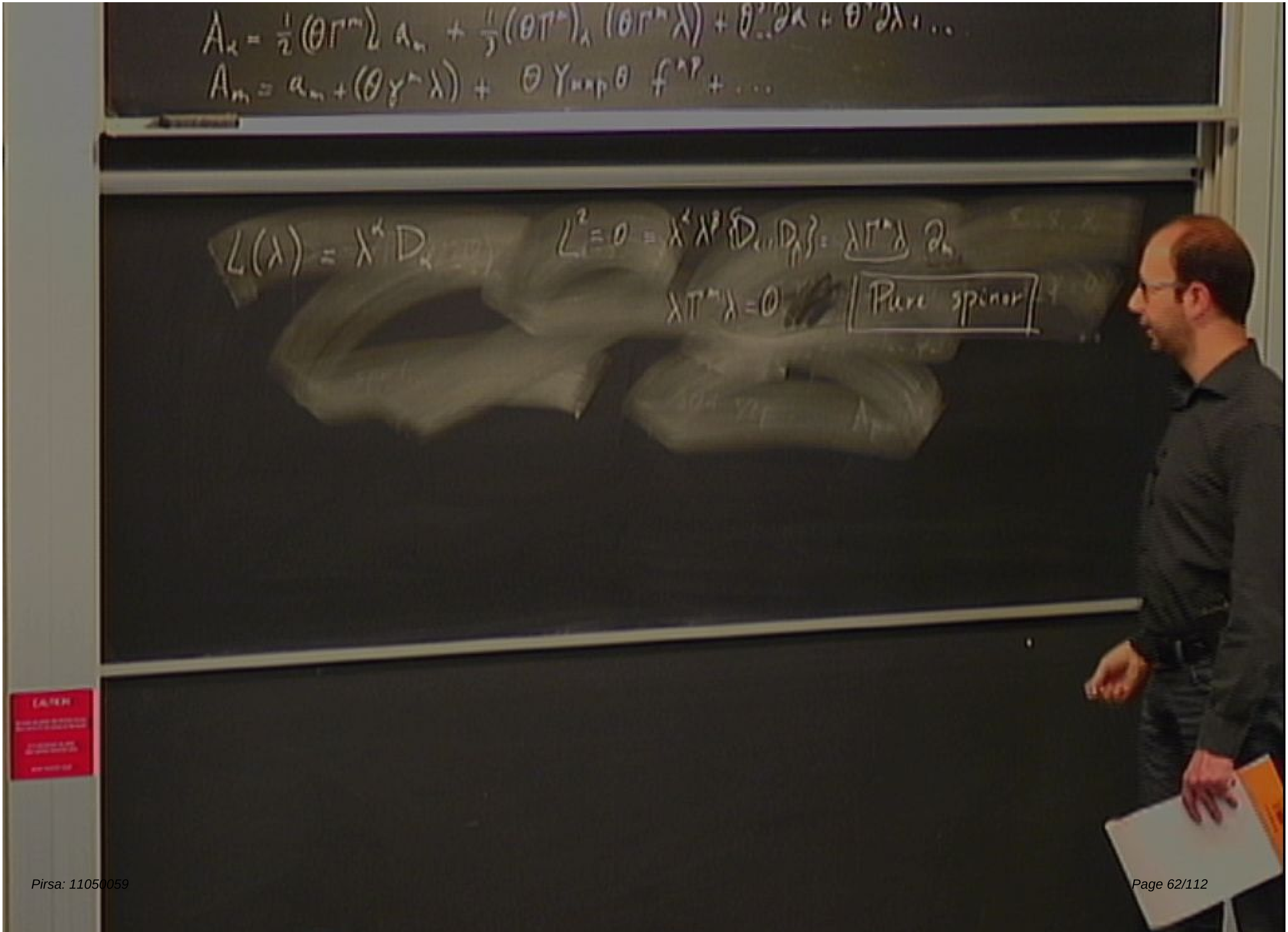
Twistor construction

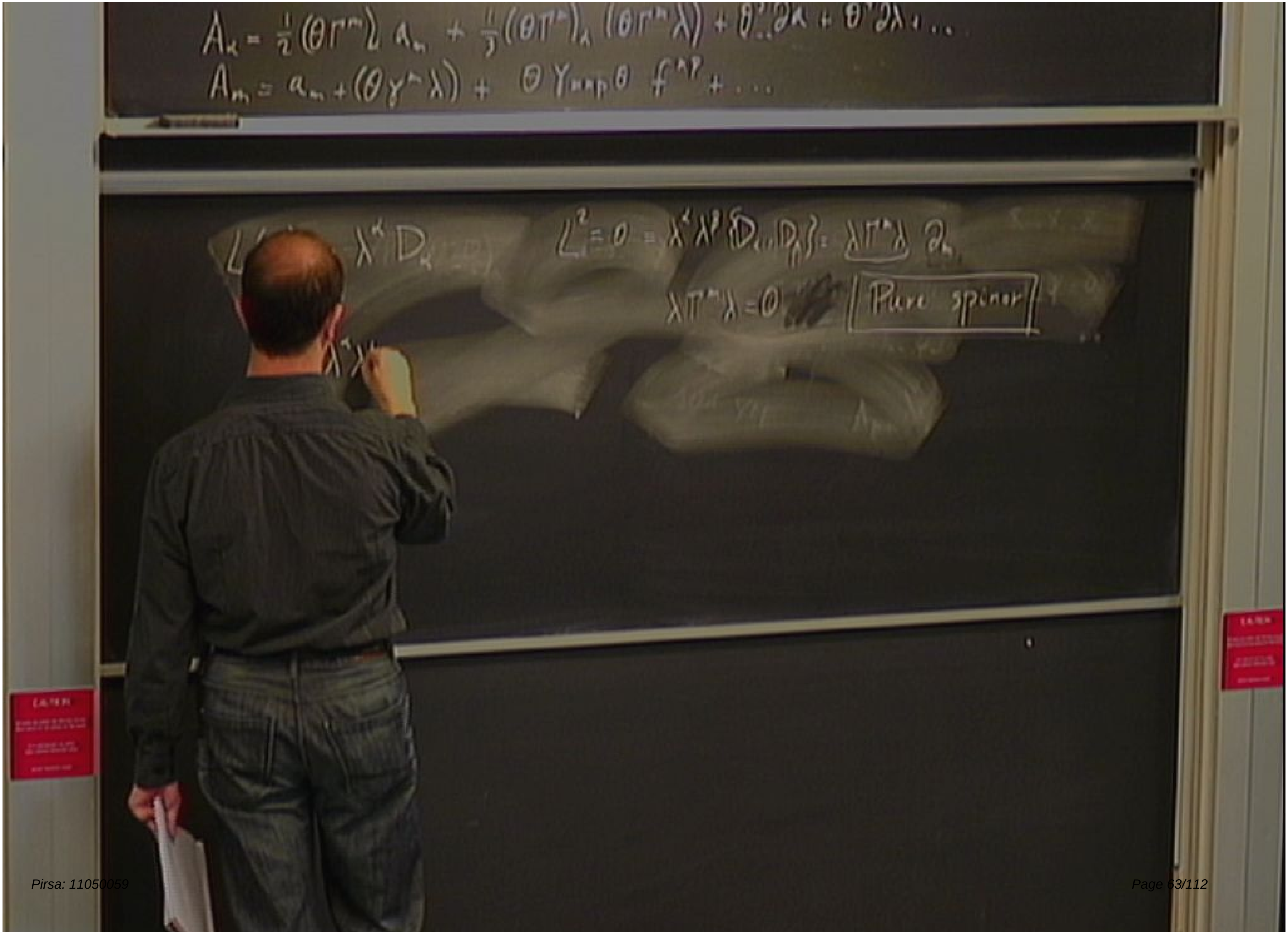
$S^4 \leftarrow \mathbb{P}^1$

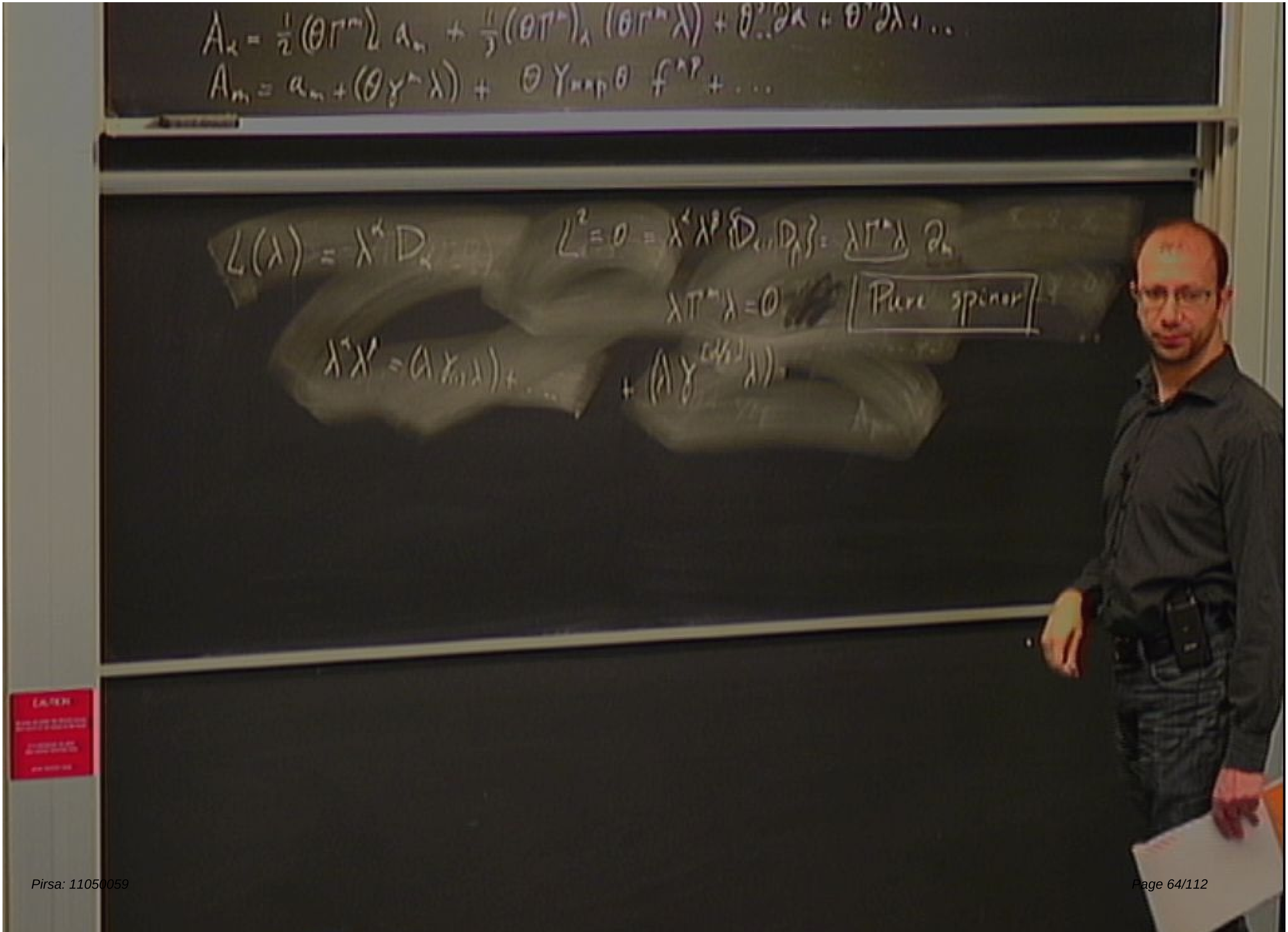
$\mathbb{P}^3$ twistor space

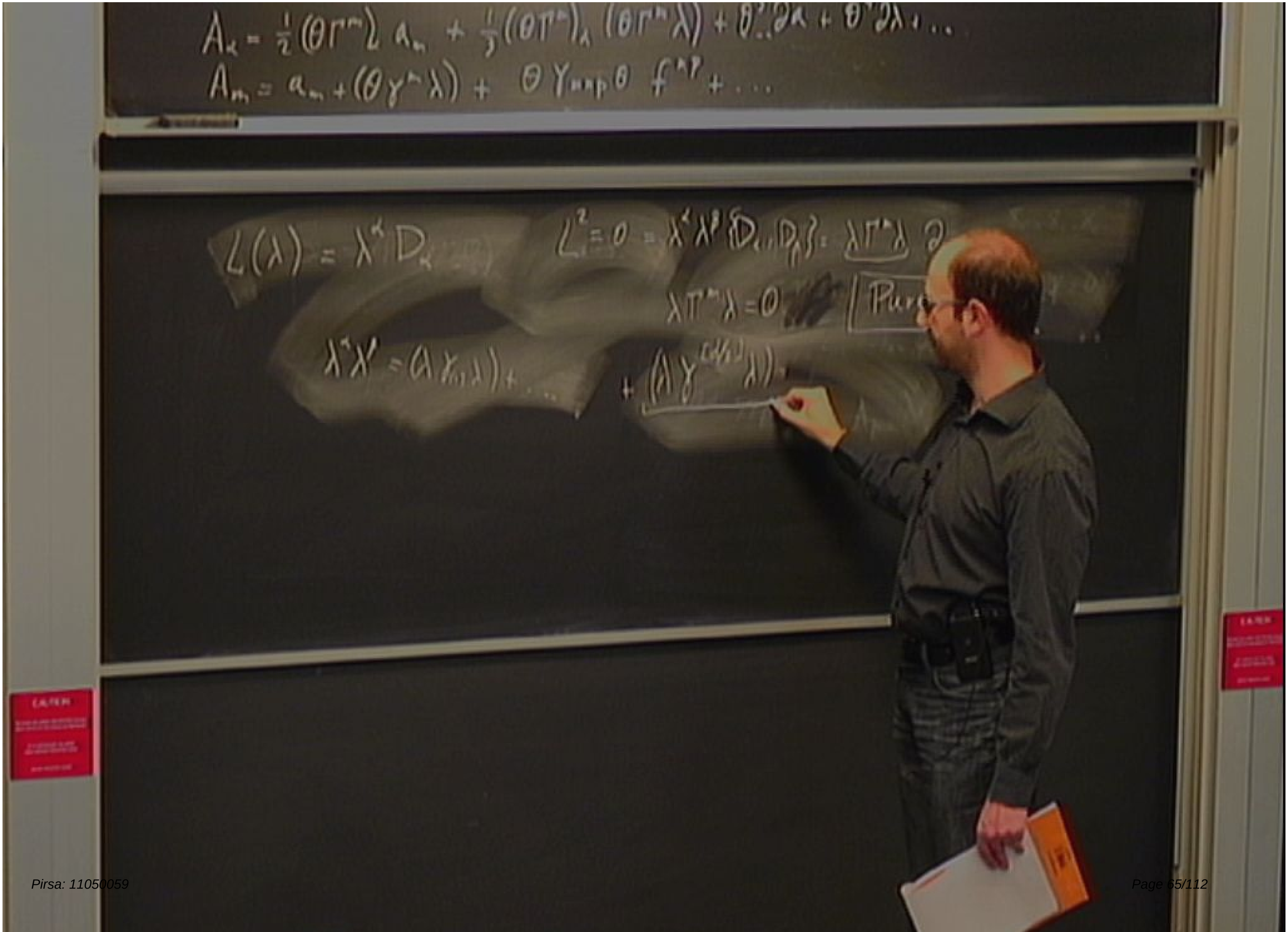
$\mathbb{P}(\Sigma^+) = S(\Lambda^+) = \{$almost cpx structures on $S^4\}$

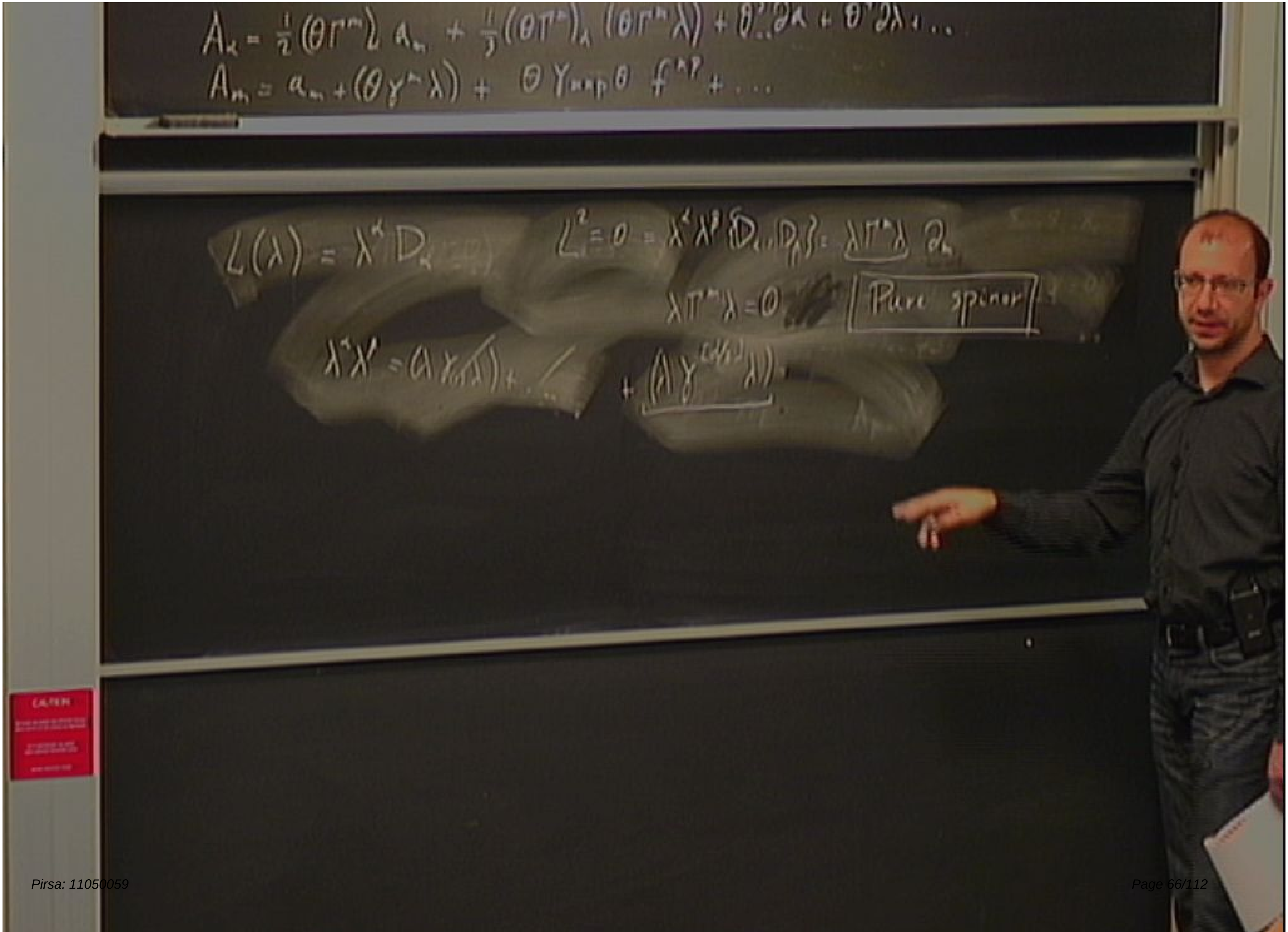

L(λ) = λ^α D_α

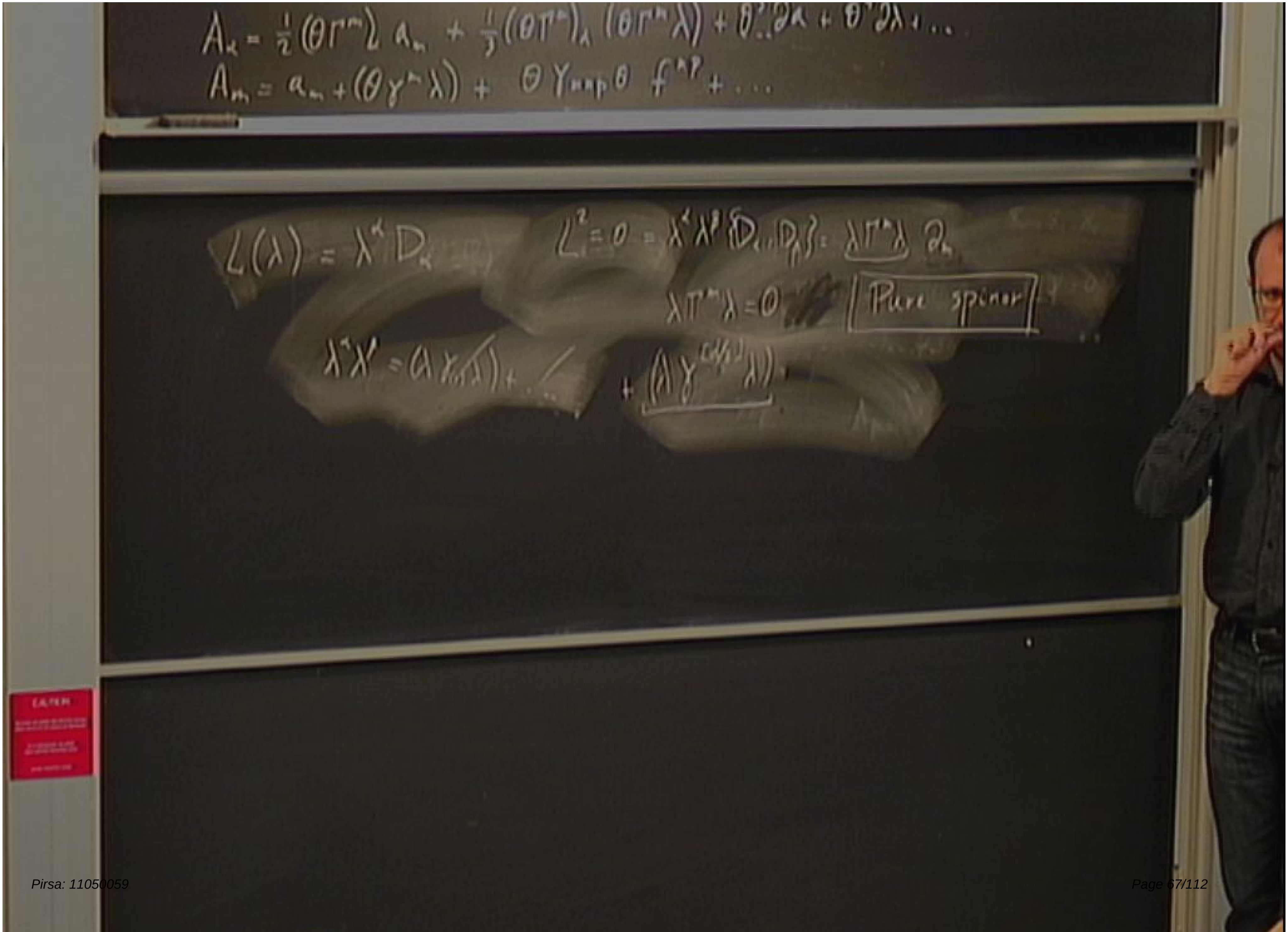

A_m = a_m + (θγ^m λ) + θγ_mnp θ f^np + ...
L(λ) = λ^α D_α
L² = 0 = λ^α λ^β {D_α, D_β} = λΓ^m λ ∂_m
λΓ^m λ = 0
Pure spinor


Pure spinor

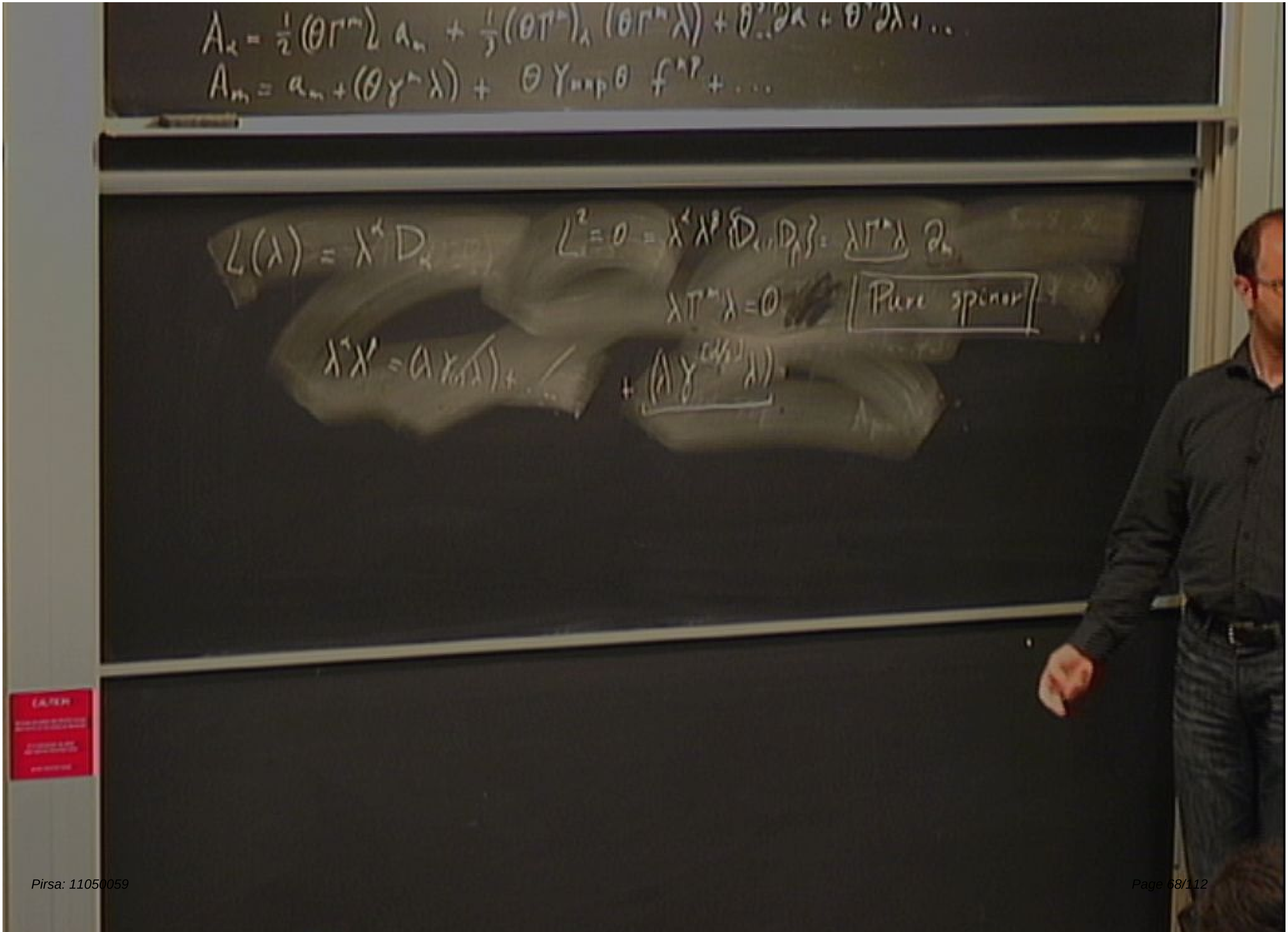

Pure spinor

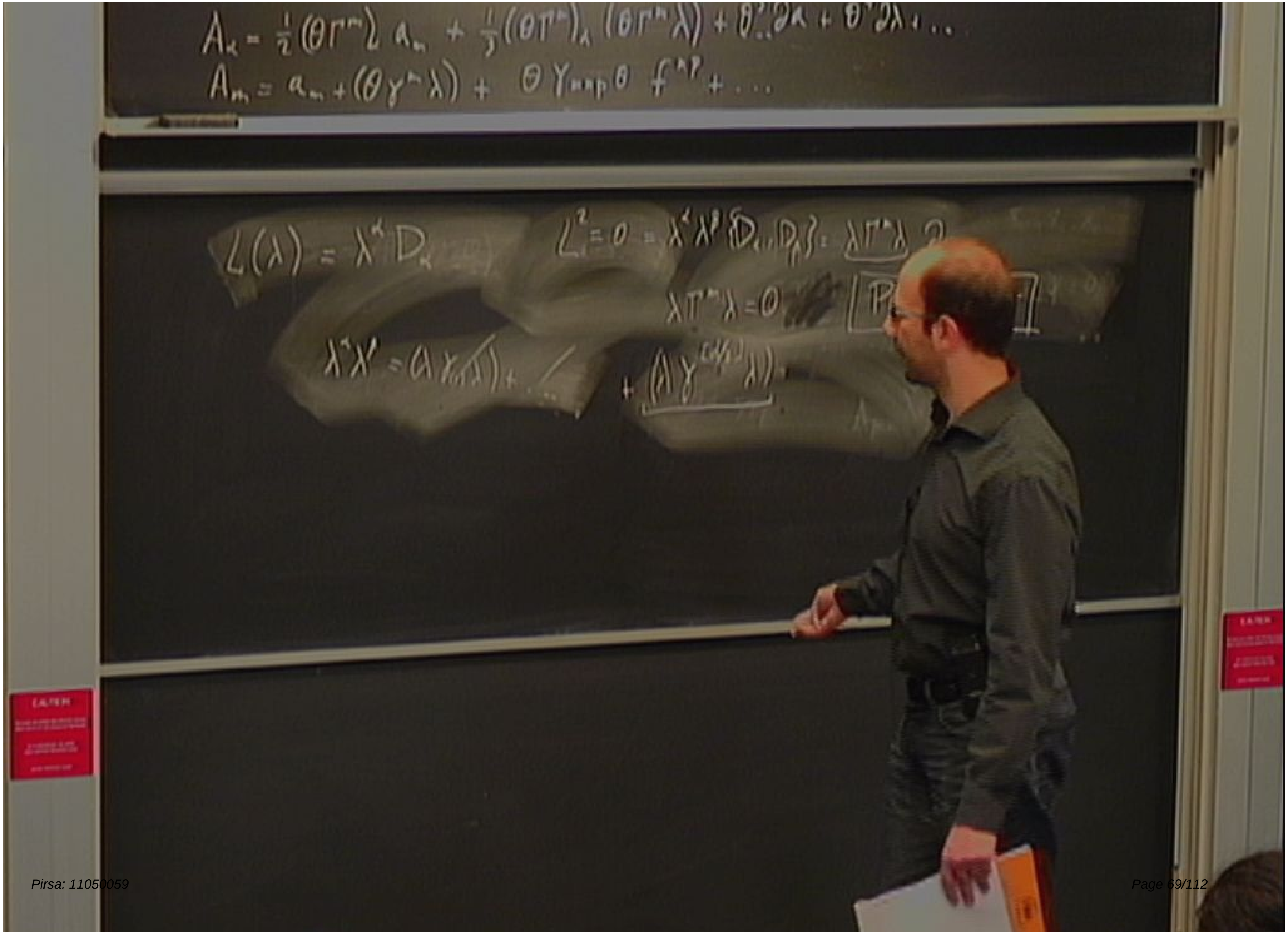

L(λ) = λ^α D_α
λΓ^m λ = 0
Pure

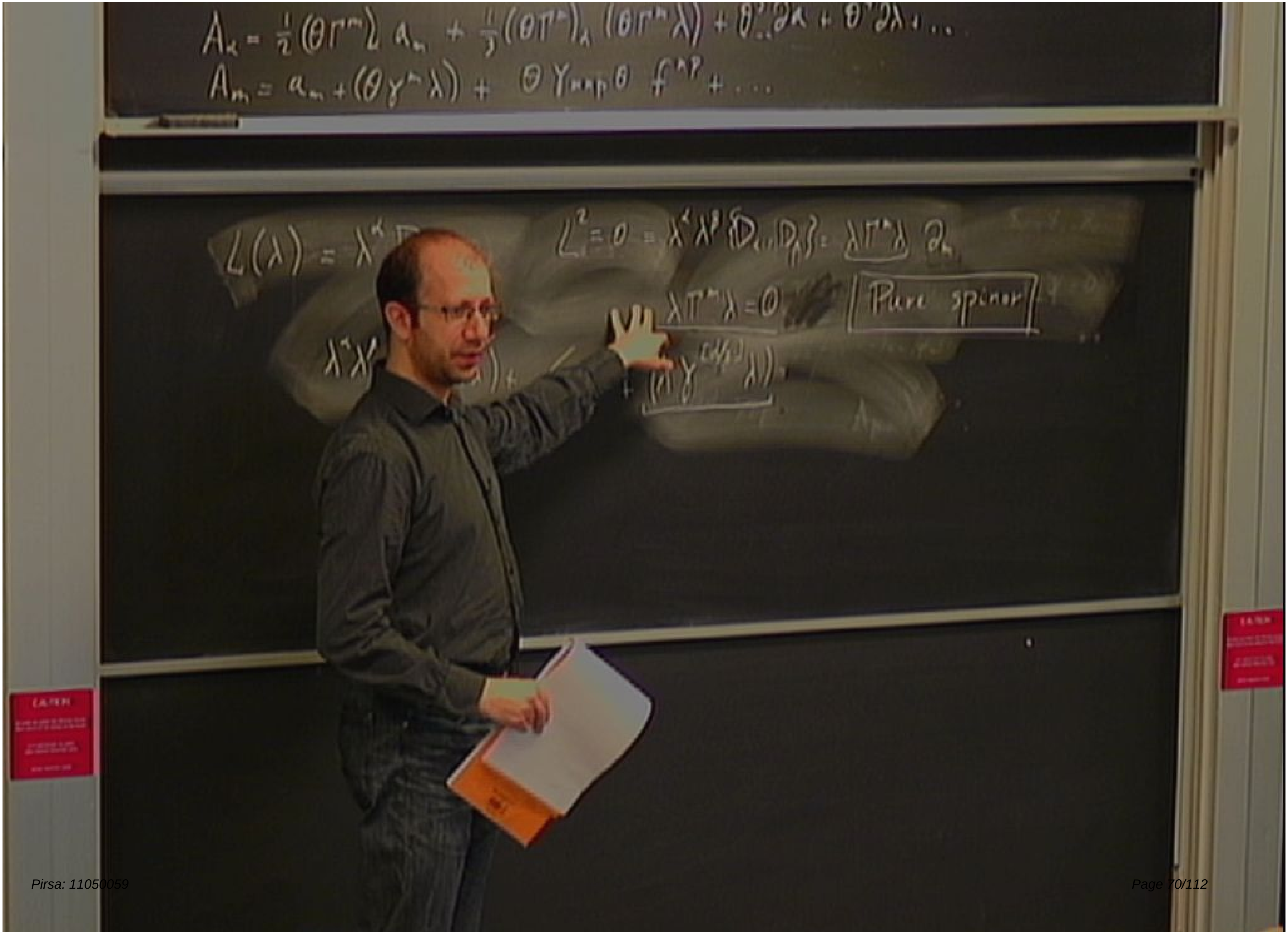

Pure spinor

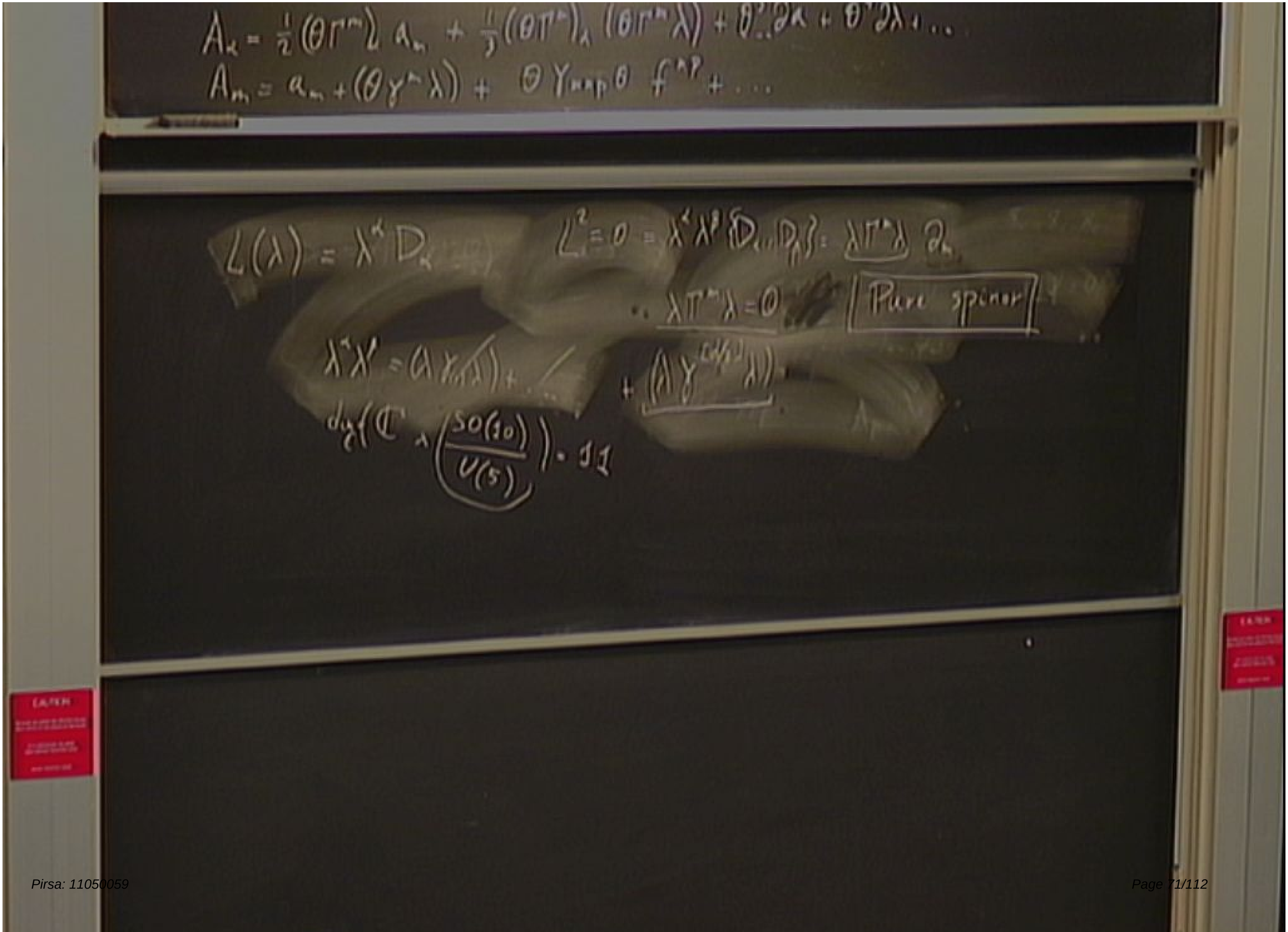


$A_\alpha = \frac{1}{2}(\theta\Gamma^m)_\alpha a_m + \frac{1}{3}((\theta\Gamma^m)_\alpha(\theta\Gamma^m\lambda)) + \ldots$
$A_m = a_m + (\theta\gamma^m\lambda) + \theta\gamma_{mnp}\theta f^{np} + \ldots$
$L(\lambda) = \lambda^\alpha D_\alpha$
$\lambda\Gamma^m\lambda = 0$
Pure spinor

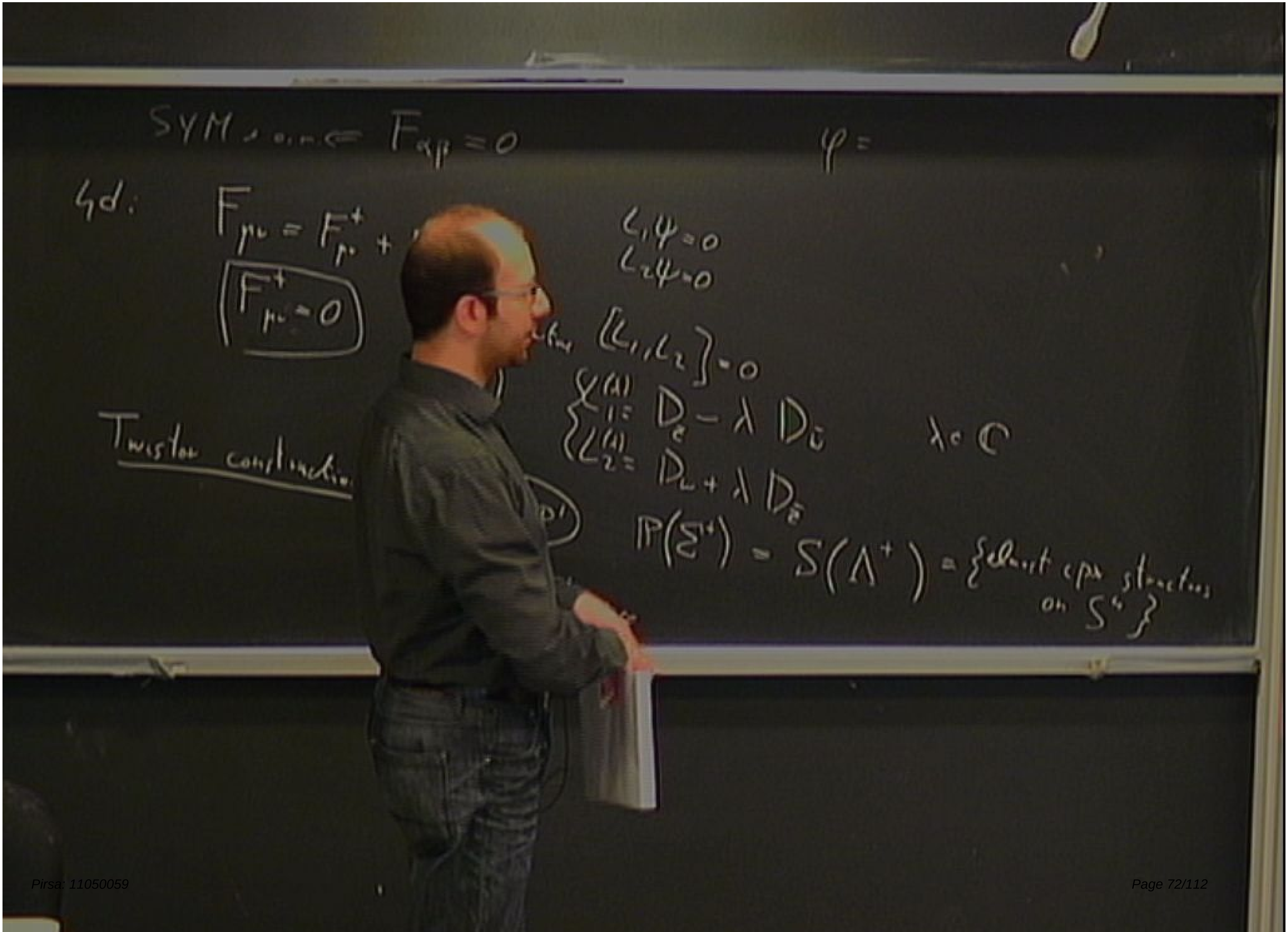

Pure spinor


L(λ) = λ^α D_α
λΓ^m λ = 0

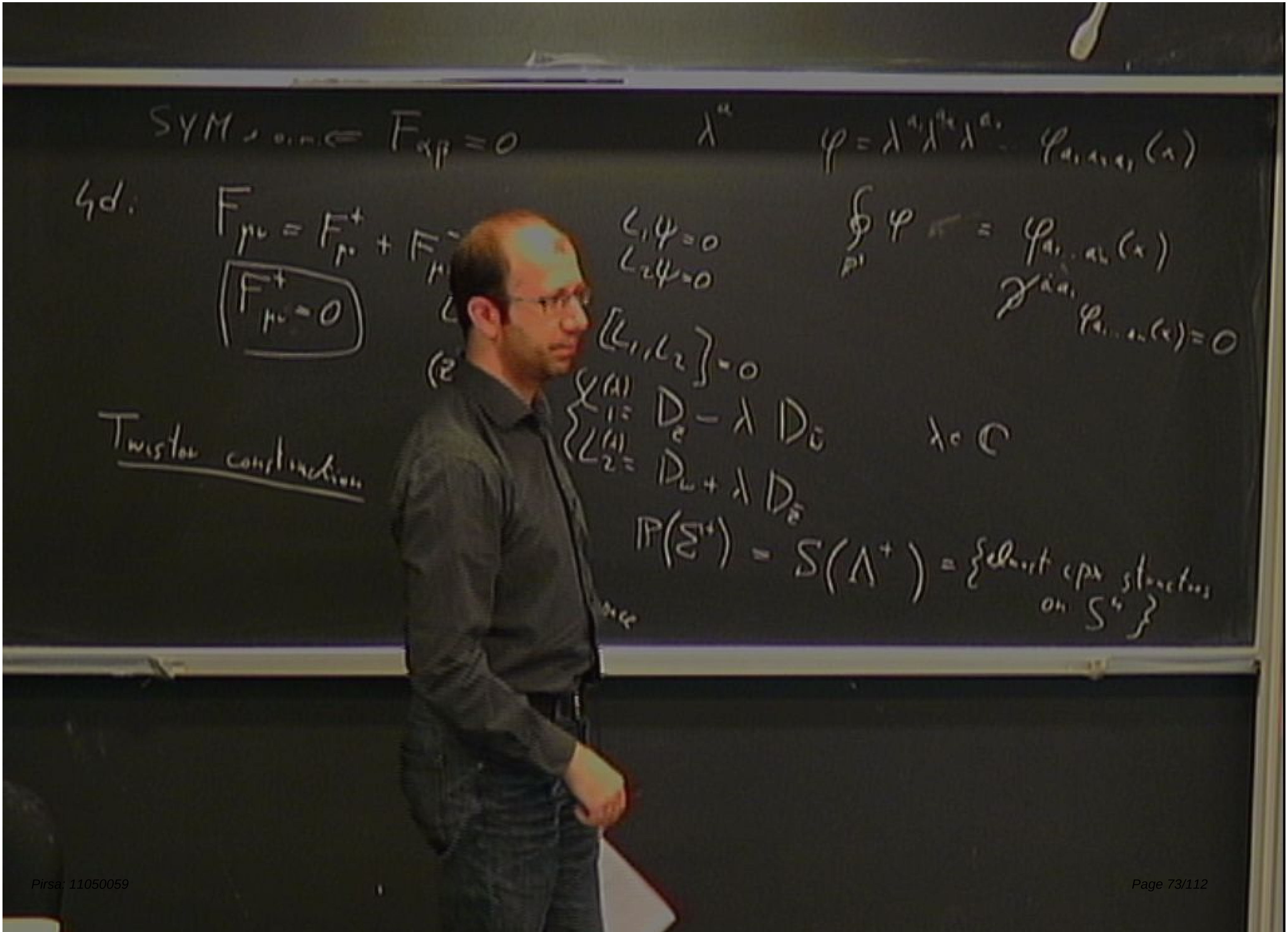

Pure spinor

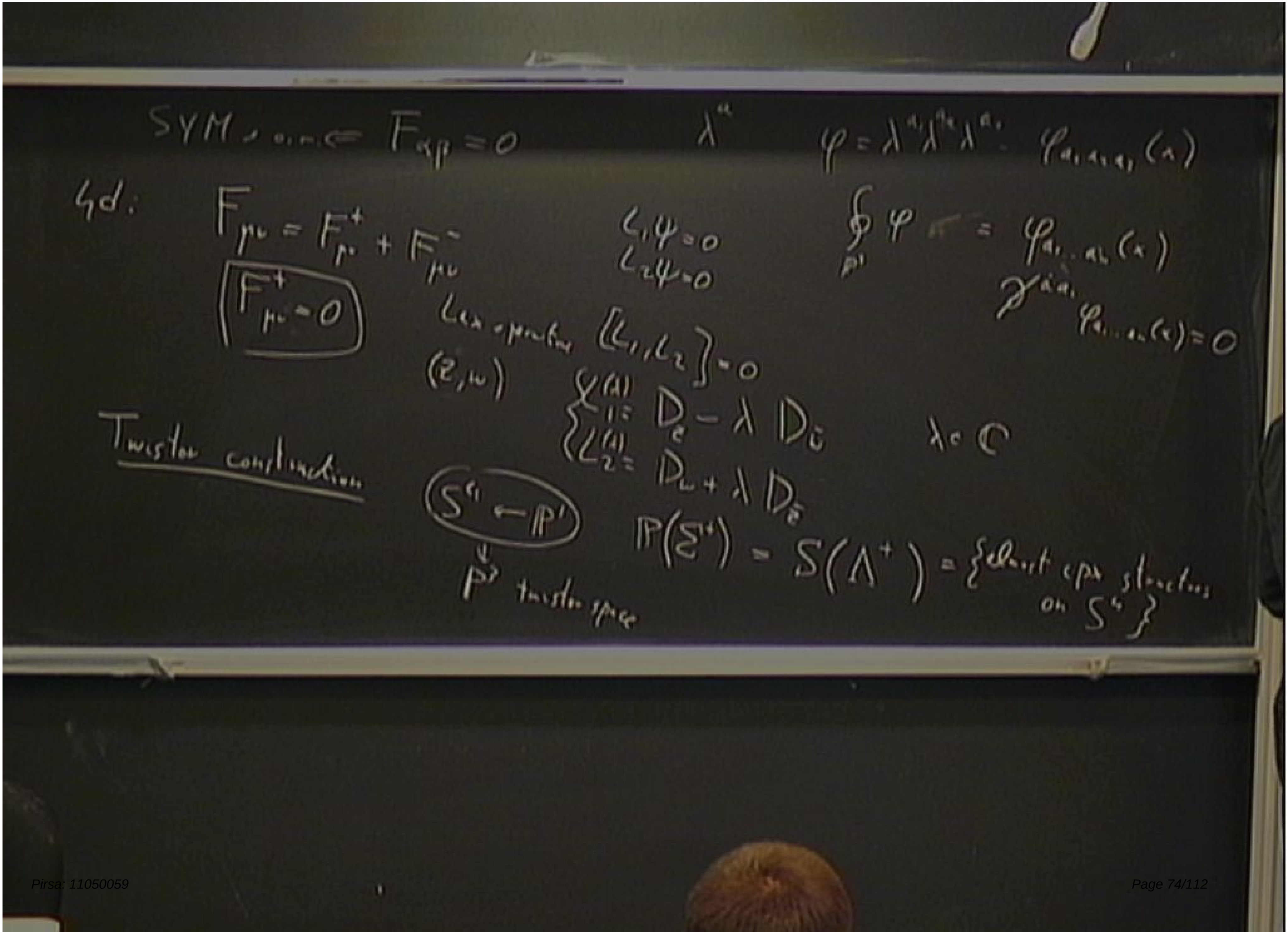


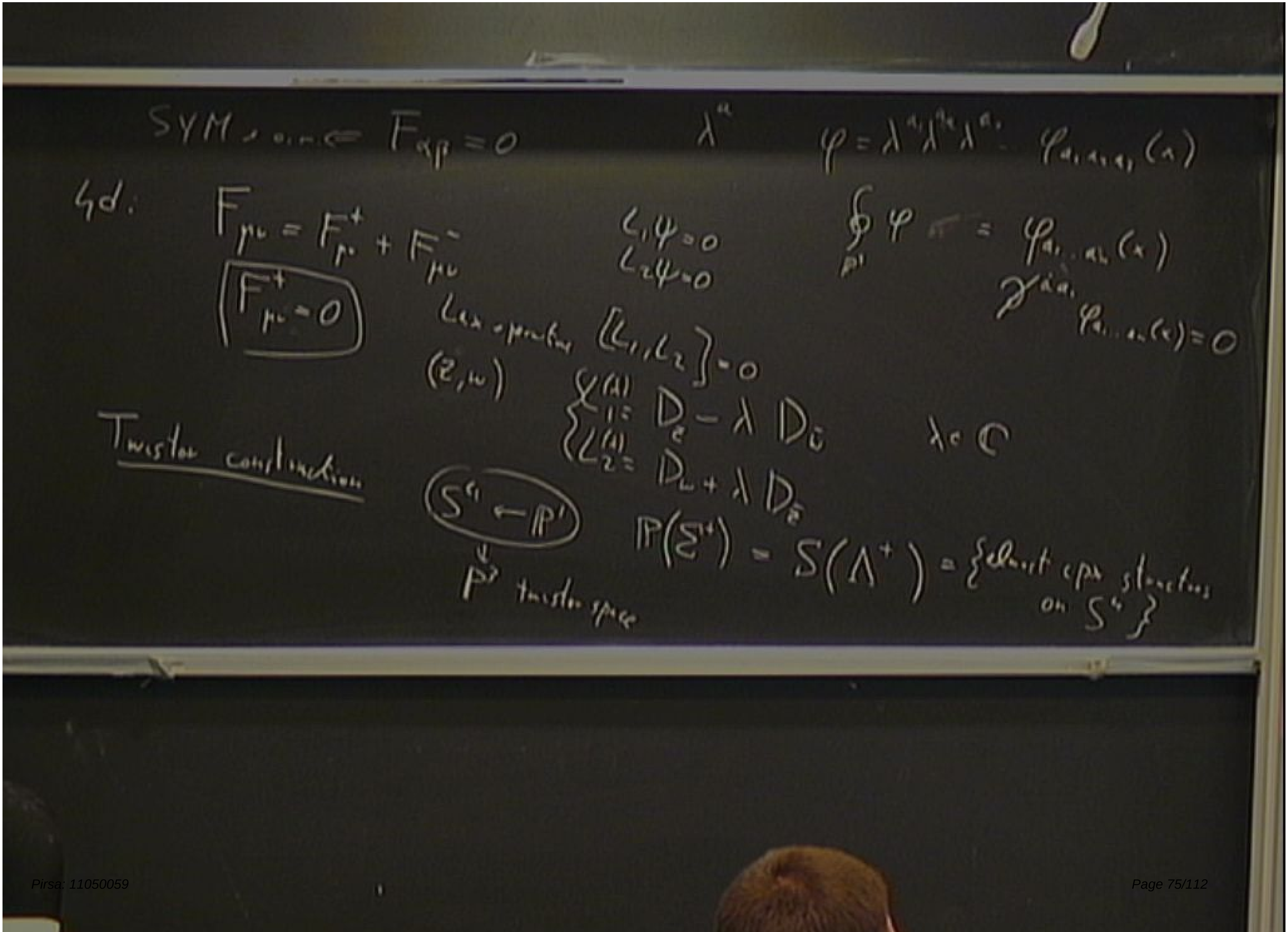
$$d_{\mathbb{C}}\left(\mathbb{C}^* \times \frac{SO(10)}{U(5)}\right) = 11$$

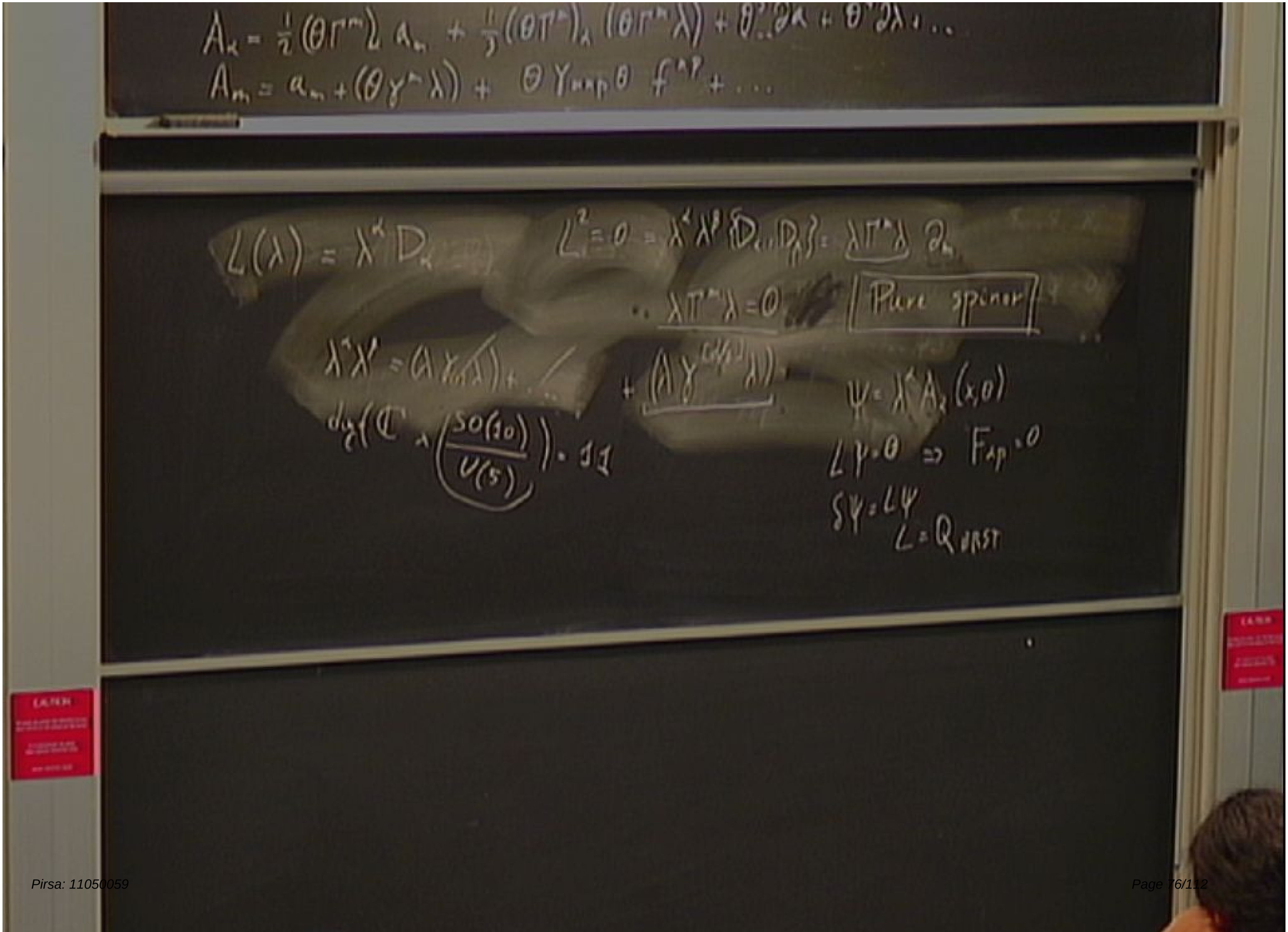


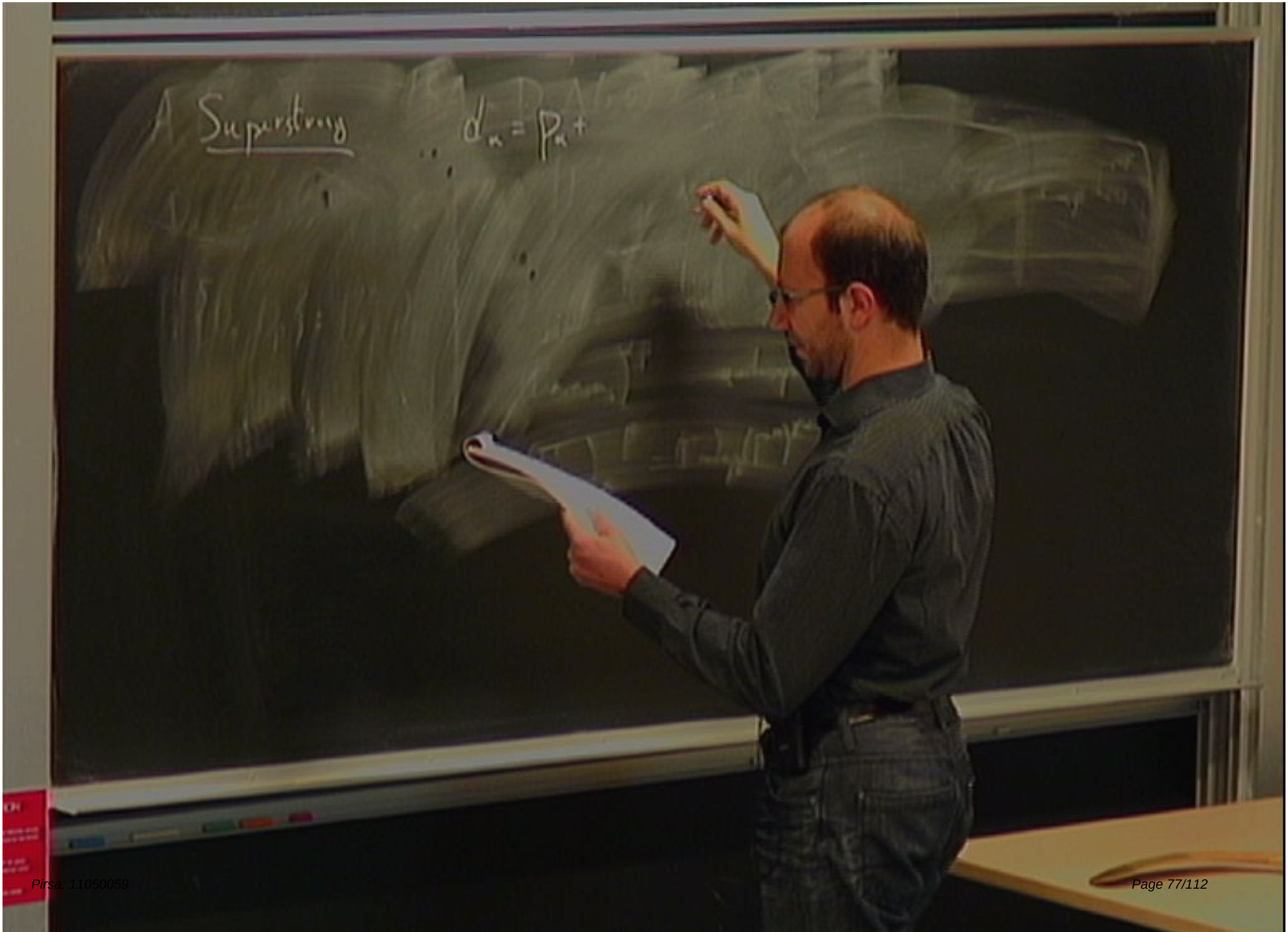
SYM
4d:
L₁ψ = 0
L₂ψ = 0
Twistor construction
λ ∈ ℂ

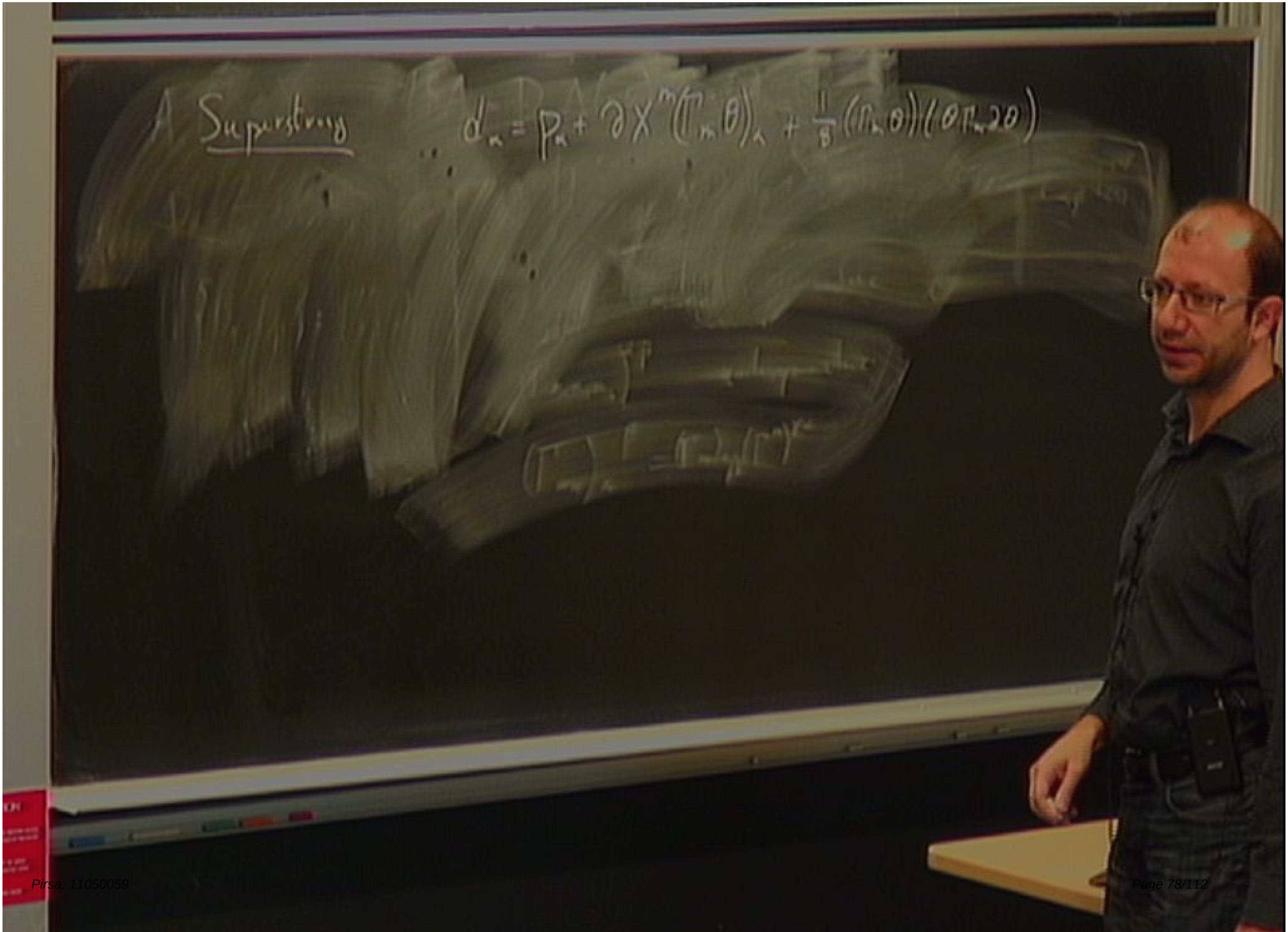

SYM
Fαβ = 0
λα
4d:
Fμν = F+μν + F-μν
F+μν = 0
L1ψ = 0
L2ψ = 0
[L1, L2] = 0
λ ∈ ℂ
Twistor construction
P(Σ+) = S(Λ+) = {almost cpx structures on S4}

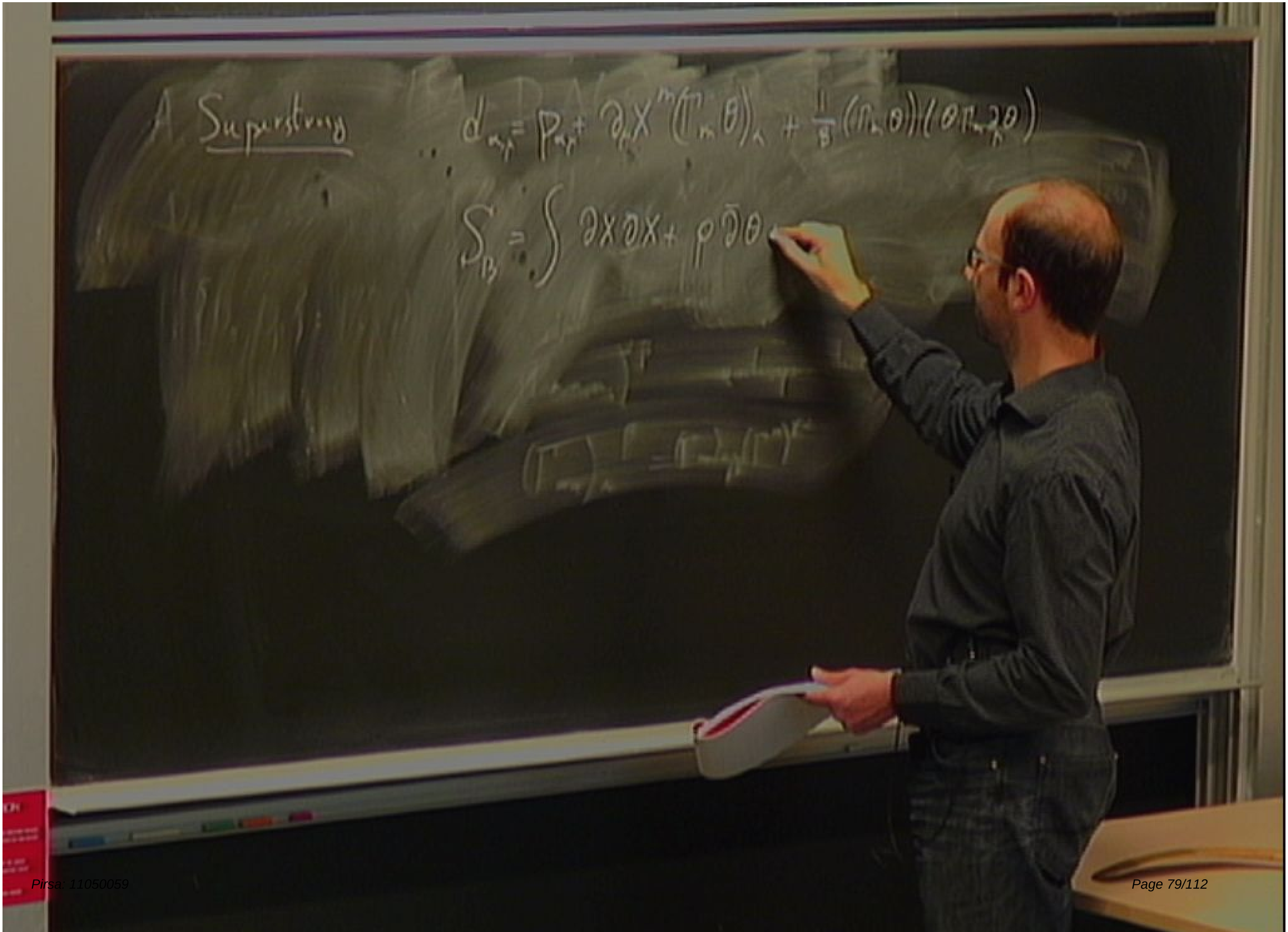

SYM
4d:
Twistor construction
twistor space
Lax operator
on S⁴

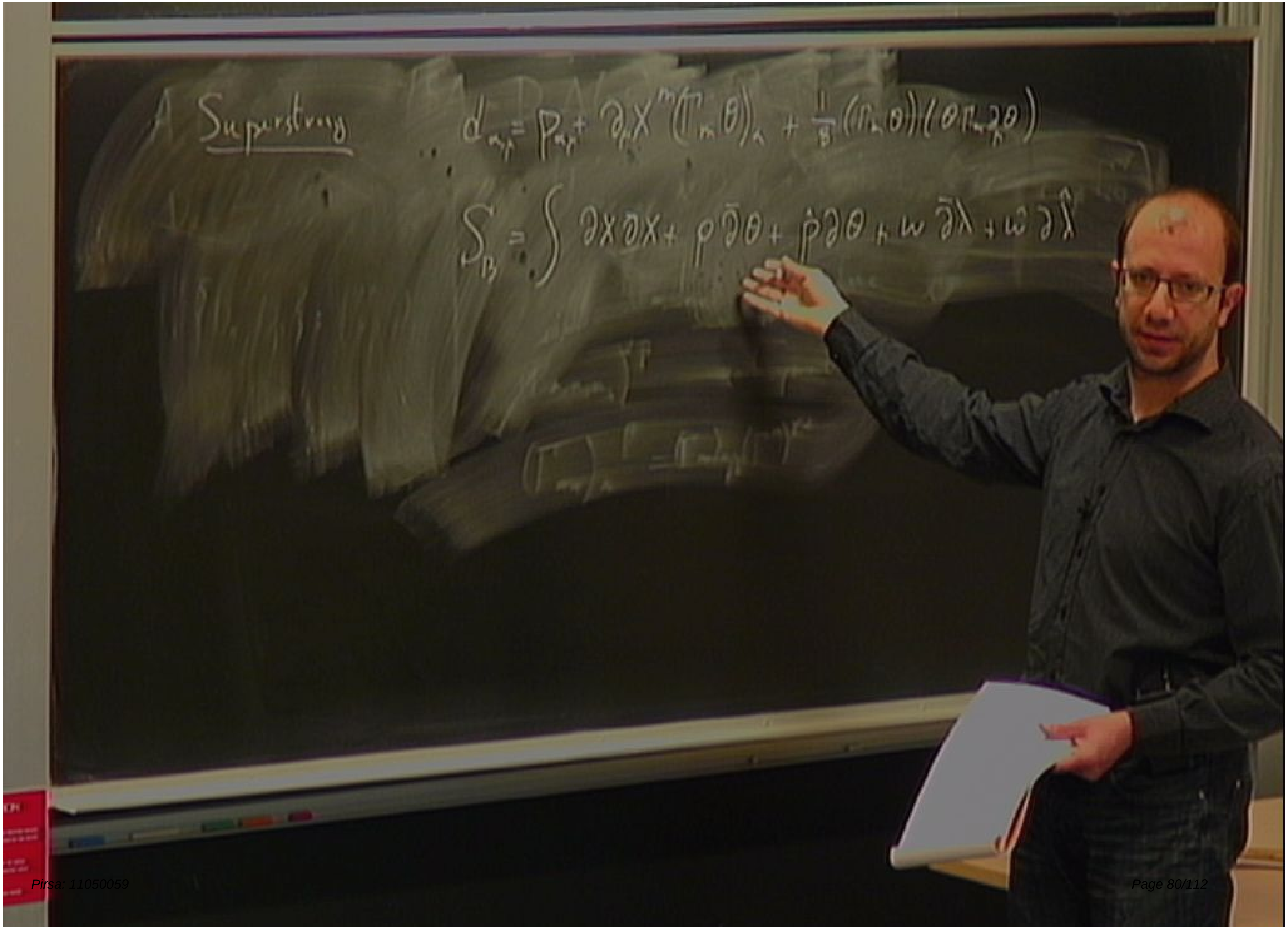

SYM ⇐ Fαβ = 0
λα
φ = λα1 λα2 λα3 φα1α2α3(x)
4d:
Fμν = F+μν + F−μν
F+μν = 0
L1ψ = 0
L2ψ = 0
∮P1 φ = φα1...αn(x)
Lax operator [L1, L2] = 0
(z, w)
L1(λ) = Dz − λ Dw̄
L2(λ) = Dw + λ Dz̄
λ ∈ ℂ
Twistor construction
S4 ← P1
P3 twistor space
P(Σ+) = S(Λ+) = {almost cpx structures on S4}

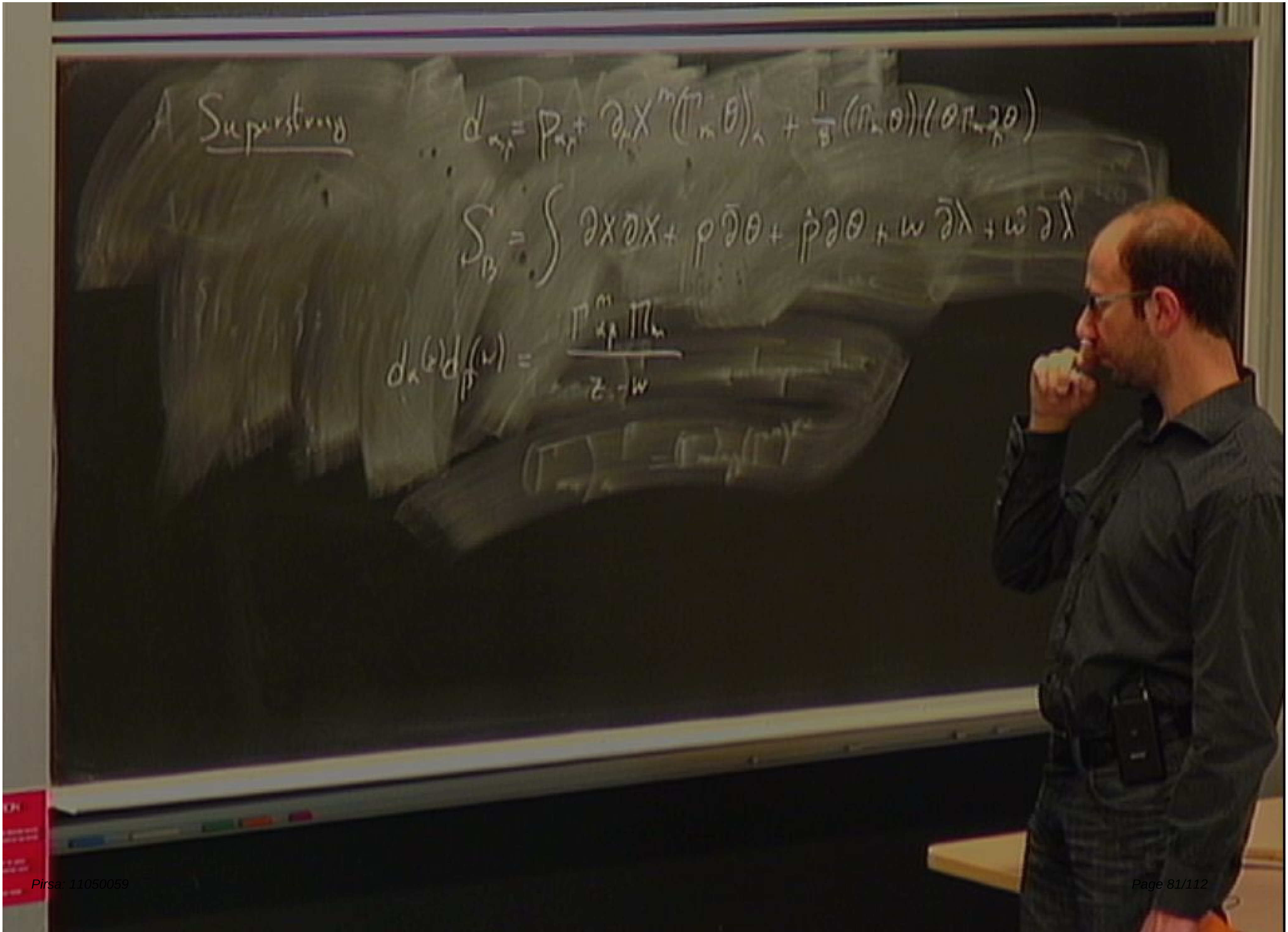

$A_\alpha = \frac{1}{2}(\theta\Gamma^m)_\alpha a_m + \frac{1}{3}((\theta\Gamma^m)_\alpha(\theta\Gamma^m\lambda)) + \theta^2 \partial a + \theta^3 \partial\lambda + \dots$
$A_m = a_m + (\theta\gamma^m\lambda) + \theta\gamma_{mnp}\theta f^{np} + \dots$
$L(\lambda) = \lambda^\alpha D_\alpha$
$L^2 = 0 = \lambda^\alpha\lambda^\beta\{D_\alpha, D_\beta\} = \lambda\Gamma^m\lambda\, \partial_m$
$\therefore \lambda\Gamma^m\lambda = 0$
Pure spinor
$\lambda^\alpha\lambda^\beta = (\lambda\gamma_m\lambda) + (\lambda\gamma^{[5]}\lambda)$
$\dim_{\mathbb{C}}\left(\mathbb{C}^* \times \frac{SO(10)}{U(5)}\right) = 11$
$\psi = \lambda^\alpha A_\alpha(x,\theta)$
$L\psi = 0 \Rightarrow F_{\alpha\beta} = 0$
$\delta\psi = L\psi$
$L = Q_{BRST}$
CAUTION

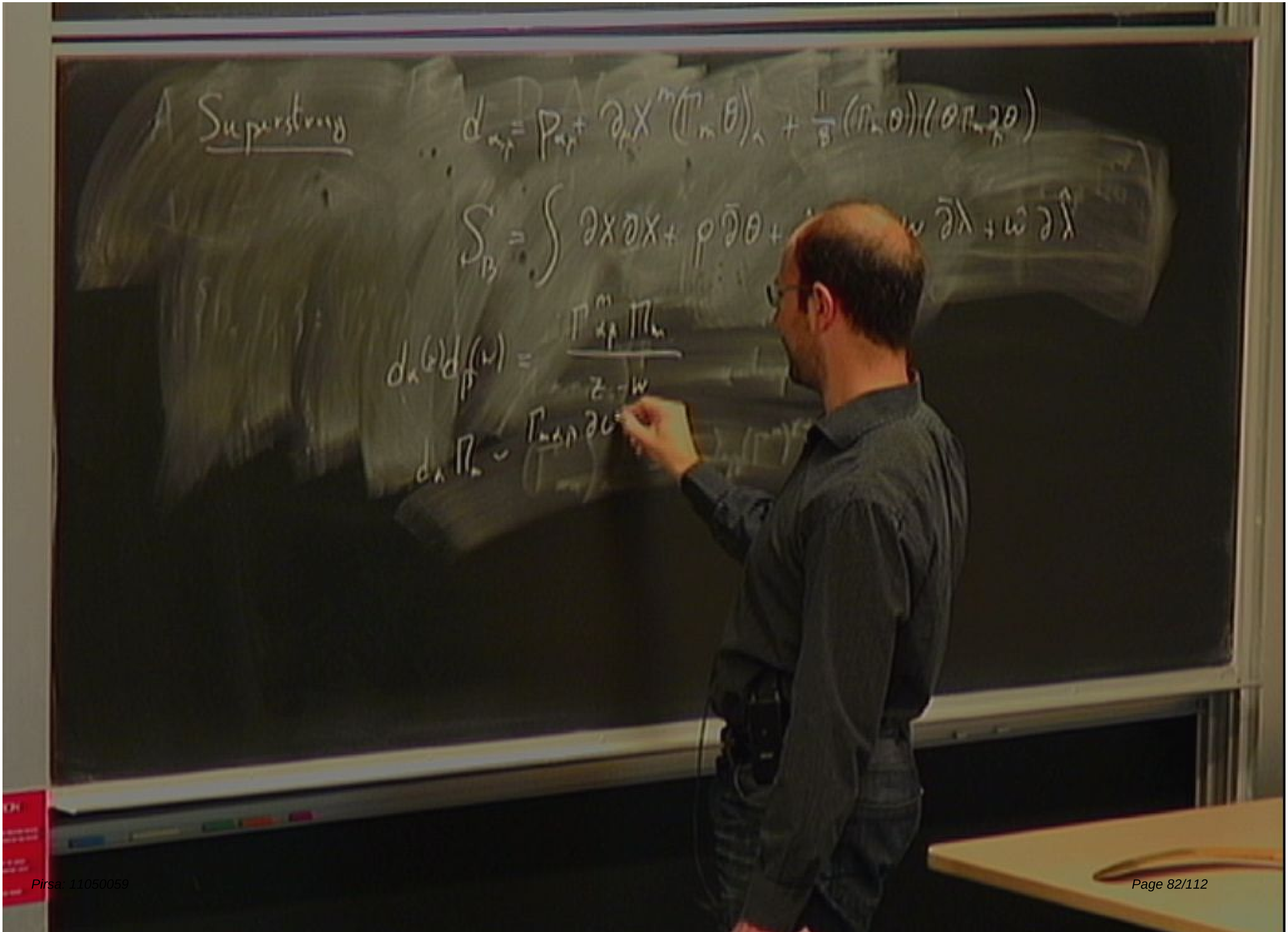

A Superstring
$d_\alpha = p_\alpha +$

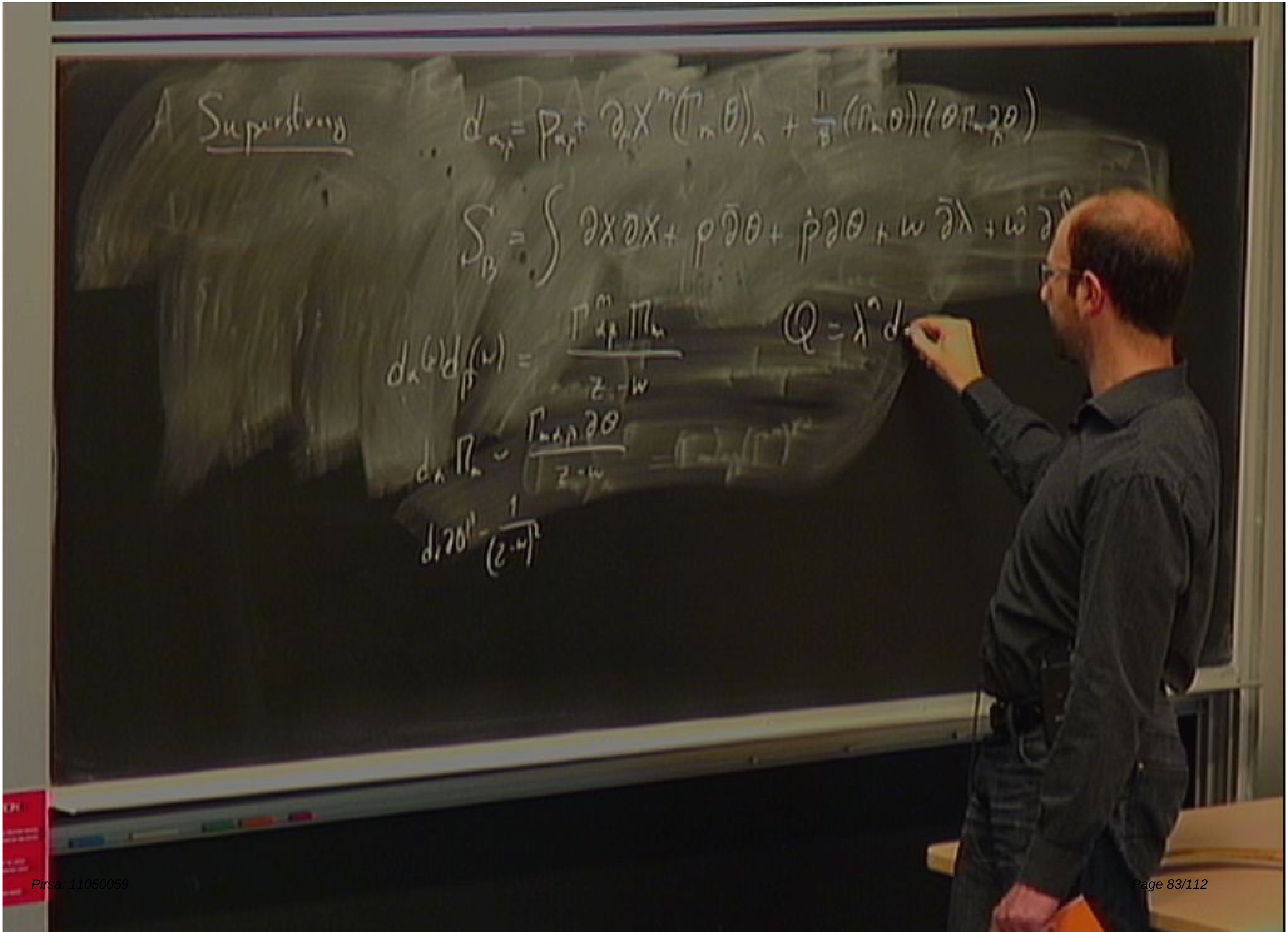

A Superstring
$d_\alpha = p_\alpha + \partial X^m(\Gamma_m\theta)_\alpha + \frac{1}{8}((\Gamma_m\theta)(\theta\Gamma_m\partial\theta))$

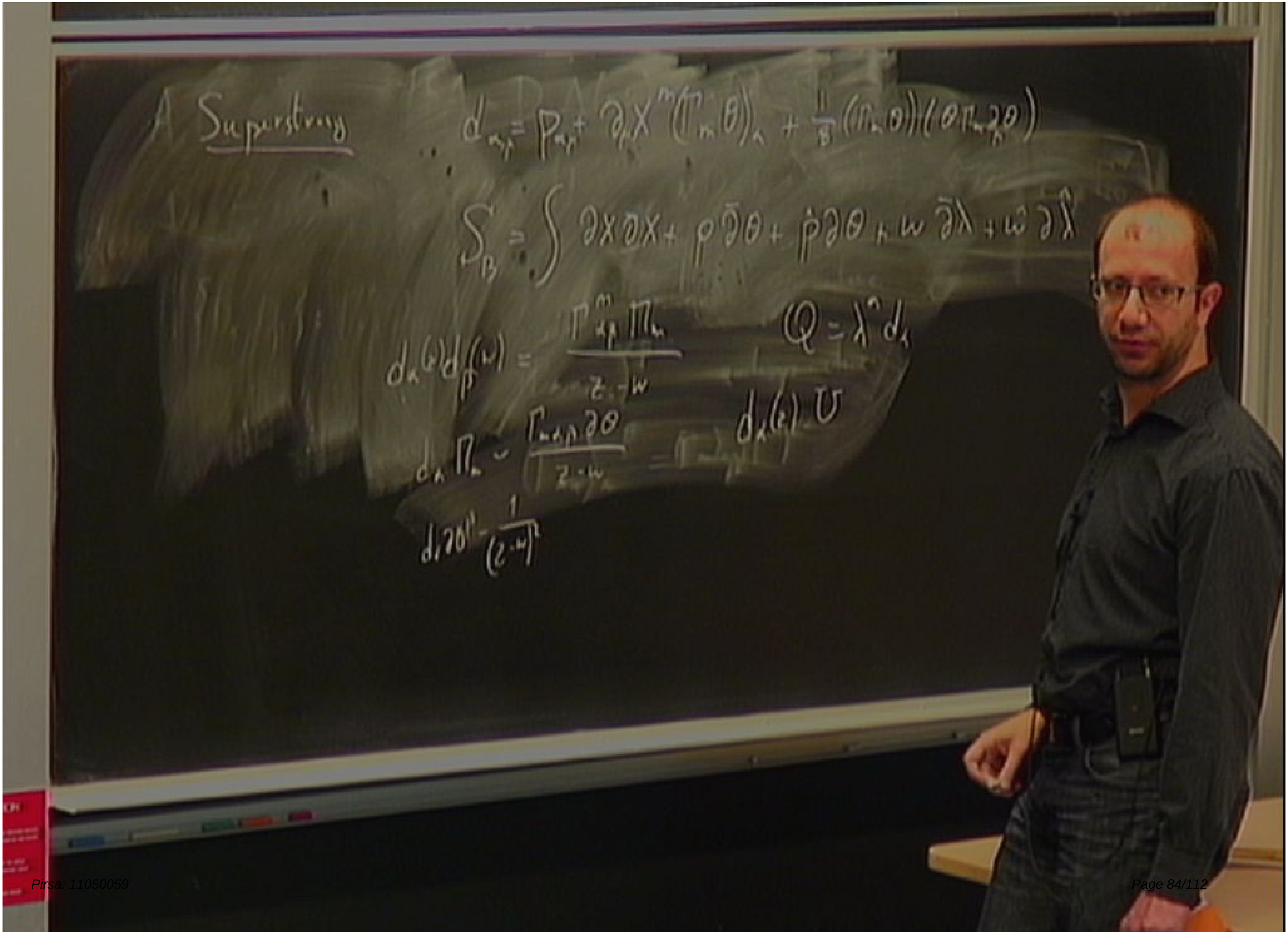

A Superstring
d = P + ∂X^m(Γ_m θ) + 1/8 (Γ^m θ)(θ Γ_m ∂θ)
S_B = ∫ ∂X∂X + p∂θ

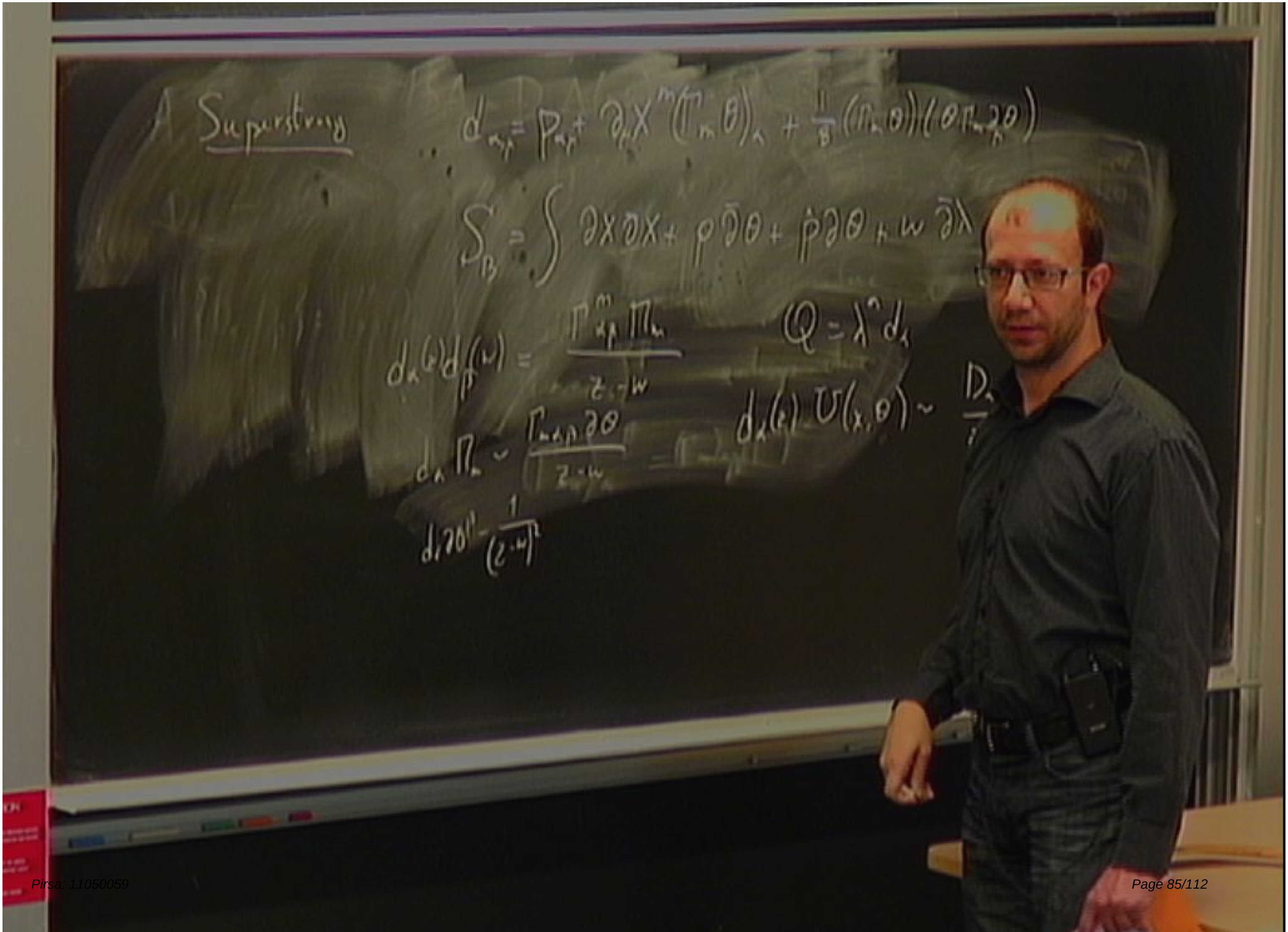

Superstring
$d_{\alpha\mu} = p_{\alpha\mu} + \partial_\mu X^m (\Gamma_m \theta)_\alpha + \frac{1}{8}(\Gamma_m \theta)(\theta \Gamma_m \partial_\mu \theta)$
$S_B = \int \partial X \bar\partial X + p \bar\partial \theta + \hat p \partial \hat\theta + w \bar\partial \lambda + \hat w \partial \hat\lambda$

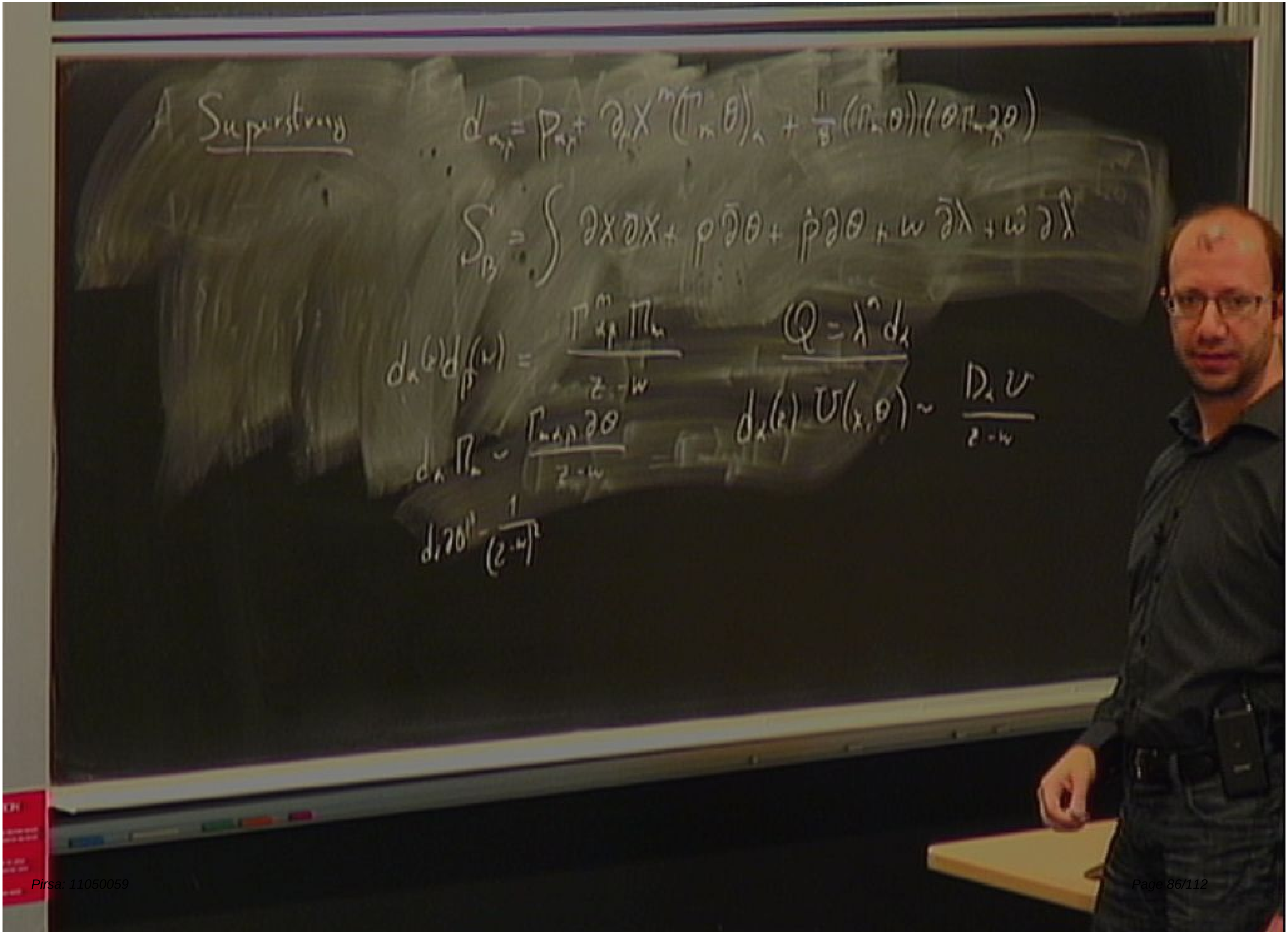

A Superstring

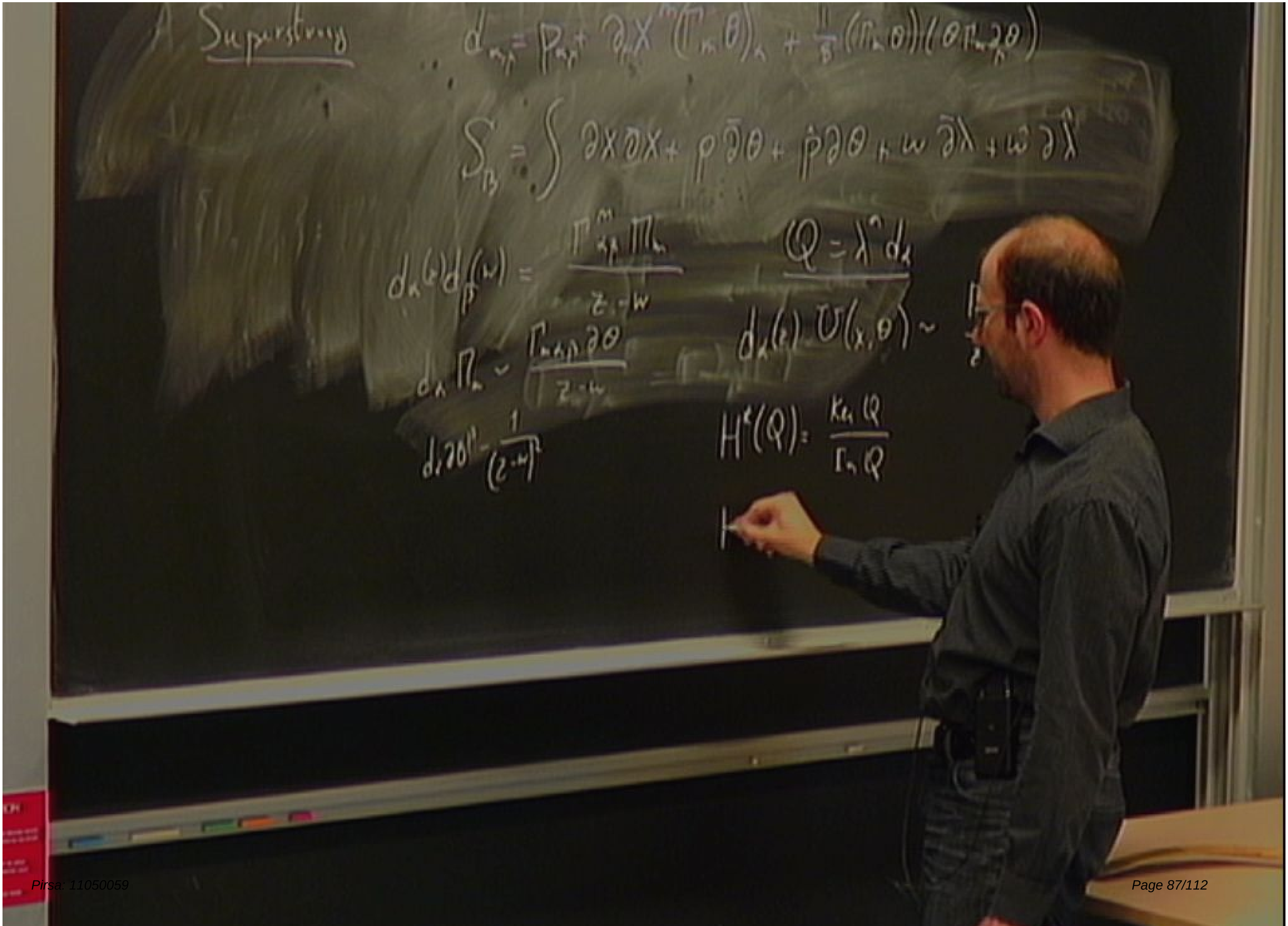

Superstring

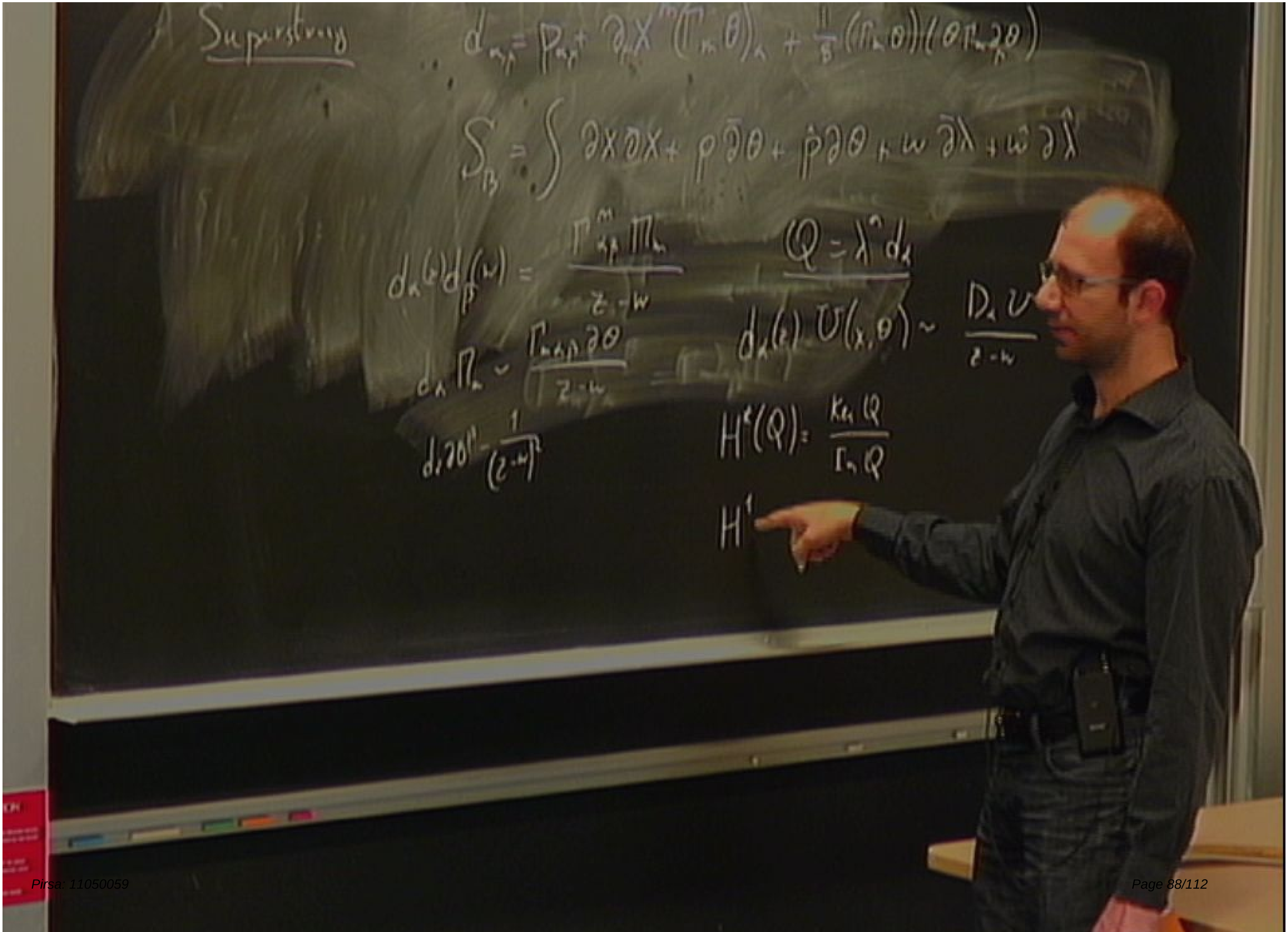

A Superstring
Q = λ^α d_α

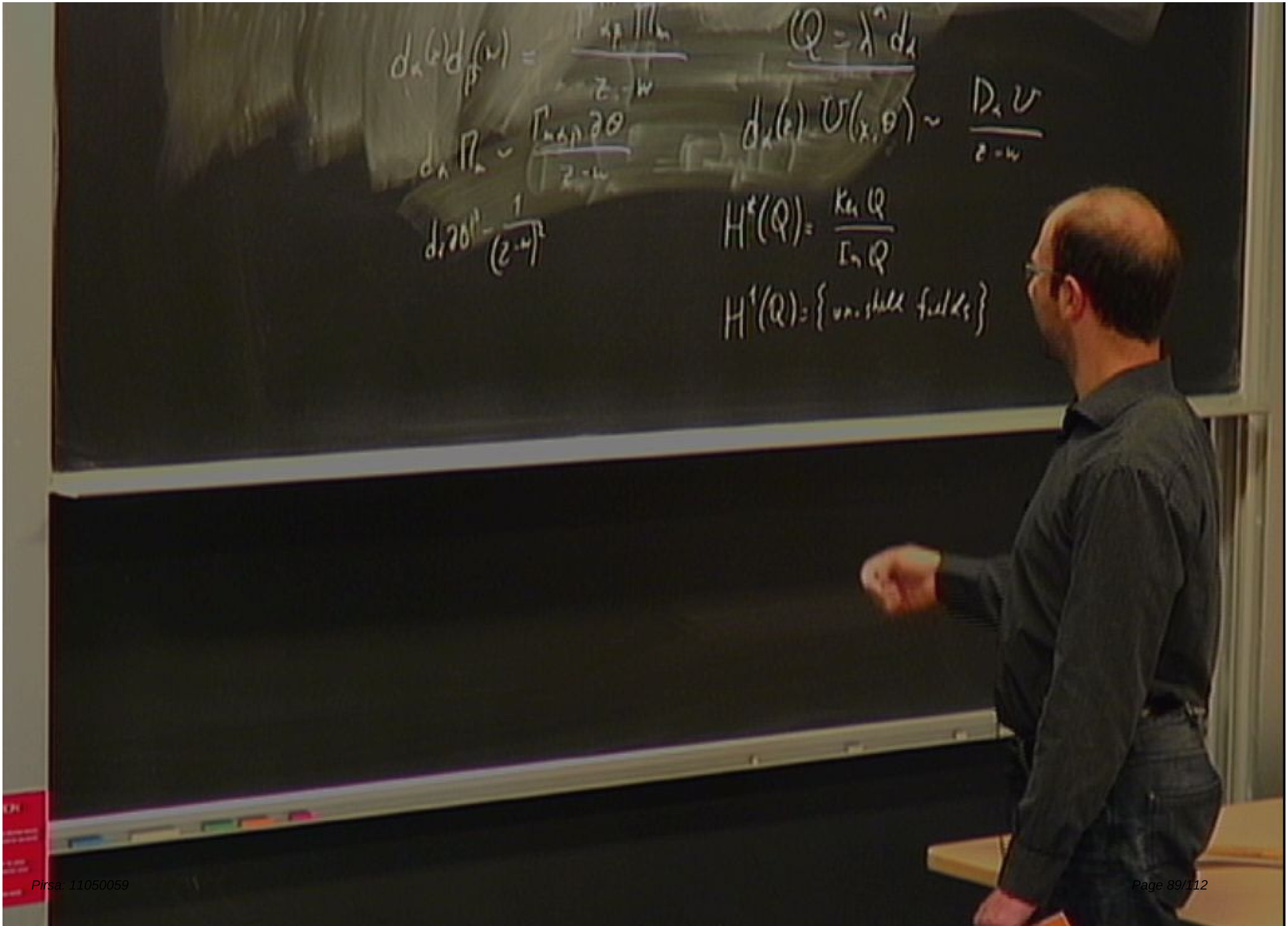

A Superstring
$d_{\alpha\mu} = p_{\alpha\mu} + \partial_\mu X^m (\Gamma_m \theta)_\alpha + \frac{1}{8}(\Gamma_m \theta)(\theta \Gamma_m \partial_\mu \theta)$
$S_B = \int \partial X \bar\partial X + p \bar\partial \theta + \hat p \partial \theta + w \bar\partial \lambda + \hat w \partial \hat\lambda$
$d_\alpha(z) d_\beta(w) = \frac{\Gamma^m_{\alpha\beta} \Pi_m}{z-w}$
$Q = \lambda^\alpha d_\alpha$
$d_\alpha \Pi_m \sim \frac{\Gamma_{m\alpha\beta} \partial\theta}{z-w}$
$d_\alpha(z) V$
$d_\alpha \partial\theta^\beta \sim \frac{1}{(z-w)^2}$

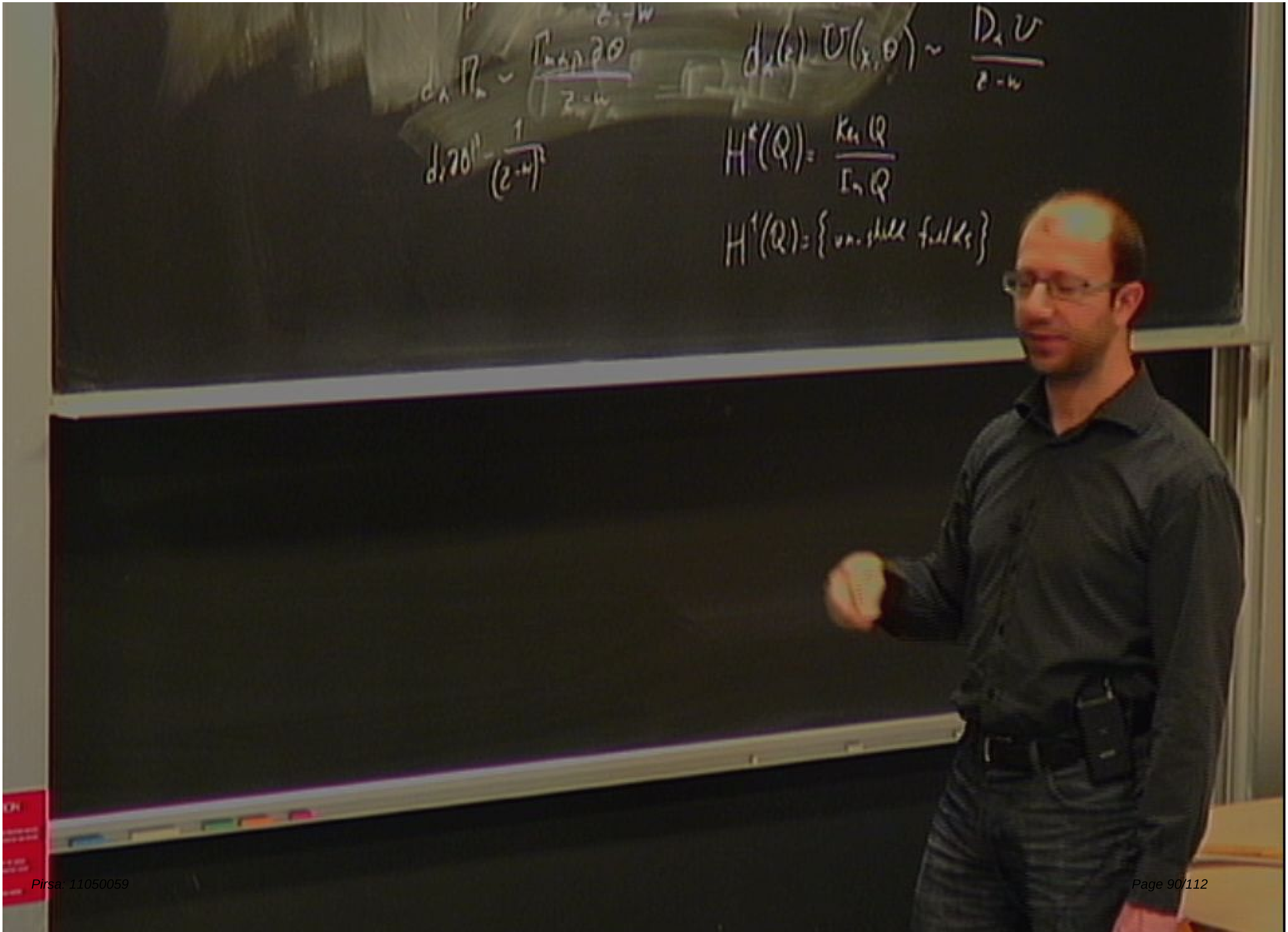

Superstring

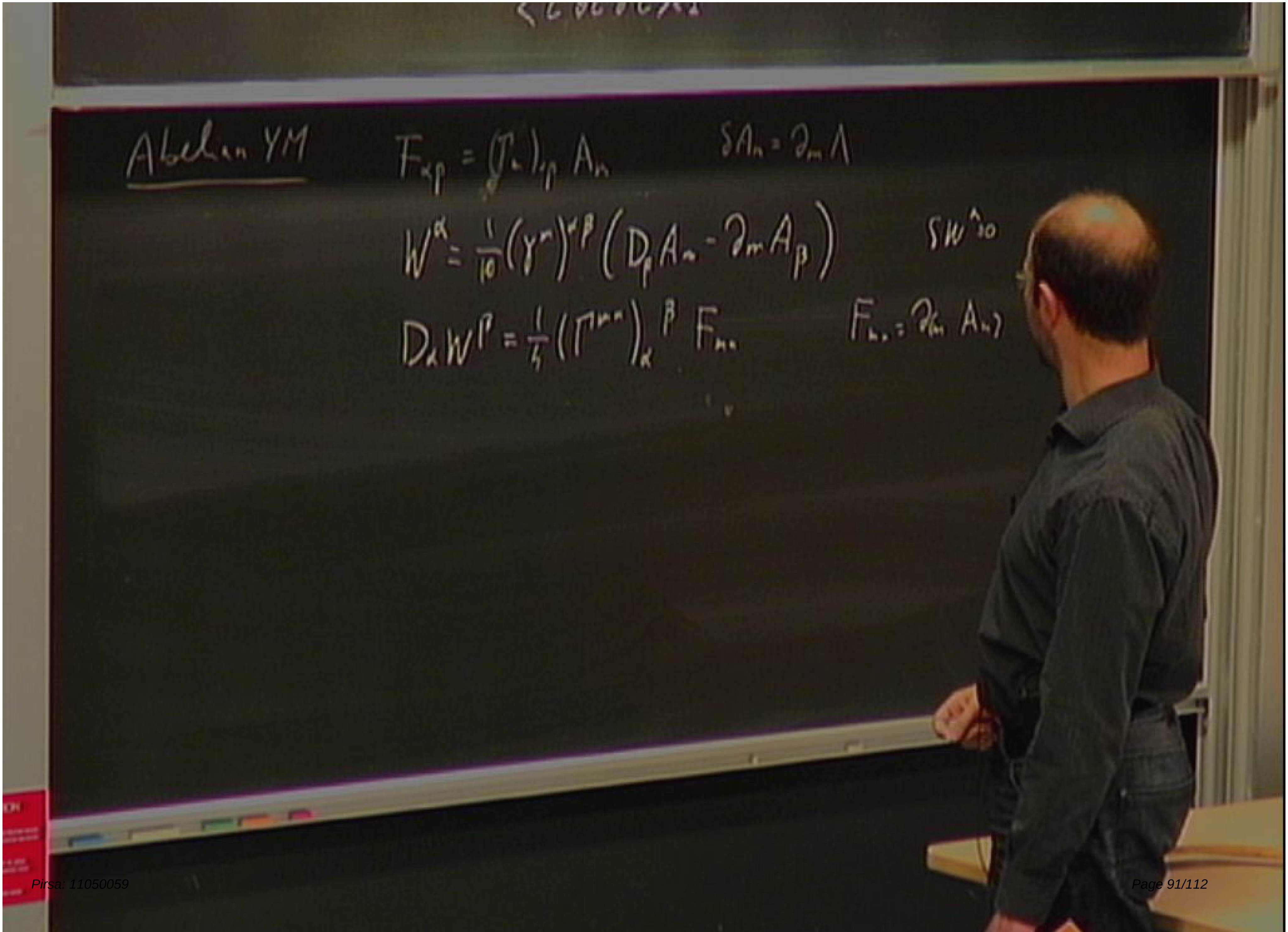

A Superstring
$d_{\alpha} = p_{\alpha} + \partial X^m (\Gamma_m \theta)_\alpha + \frac{1}{8}(\Gamma_m\theta)_\alpha(\theta\Gamma_m\partial\theta)$
$S_B = \int \partial X \bar\partial X + p\bar\partial\theta + \hat p \partial \hat\theta + w\bar\partial\lambda + \hat w \partial\hat\lambda$
$d_\alpha(z) d_\beta(w) = \frac{\Gamma^m_{\alpha\beta}\Pi_m}{z-w}$
$Q = \lambda^\alpha d_\alpha$
$d_\alpha(z)\, U(x,\theta) \sim \frac{D_\alpha U}{z-w}$
$d_\alpha \Pi_m \sim \frac{\Gamma_{m\alpha\beta}\partial\theta^\beta}{z-w}$
$d_\alpha \partial\theta^\beta \sim \frac{1}{(z-w)^2}$

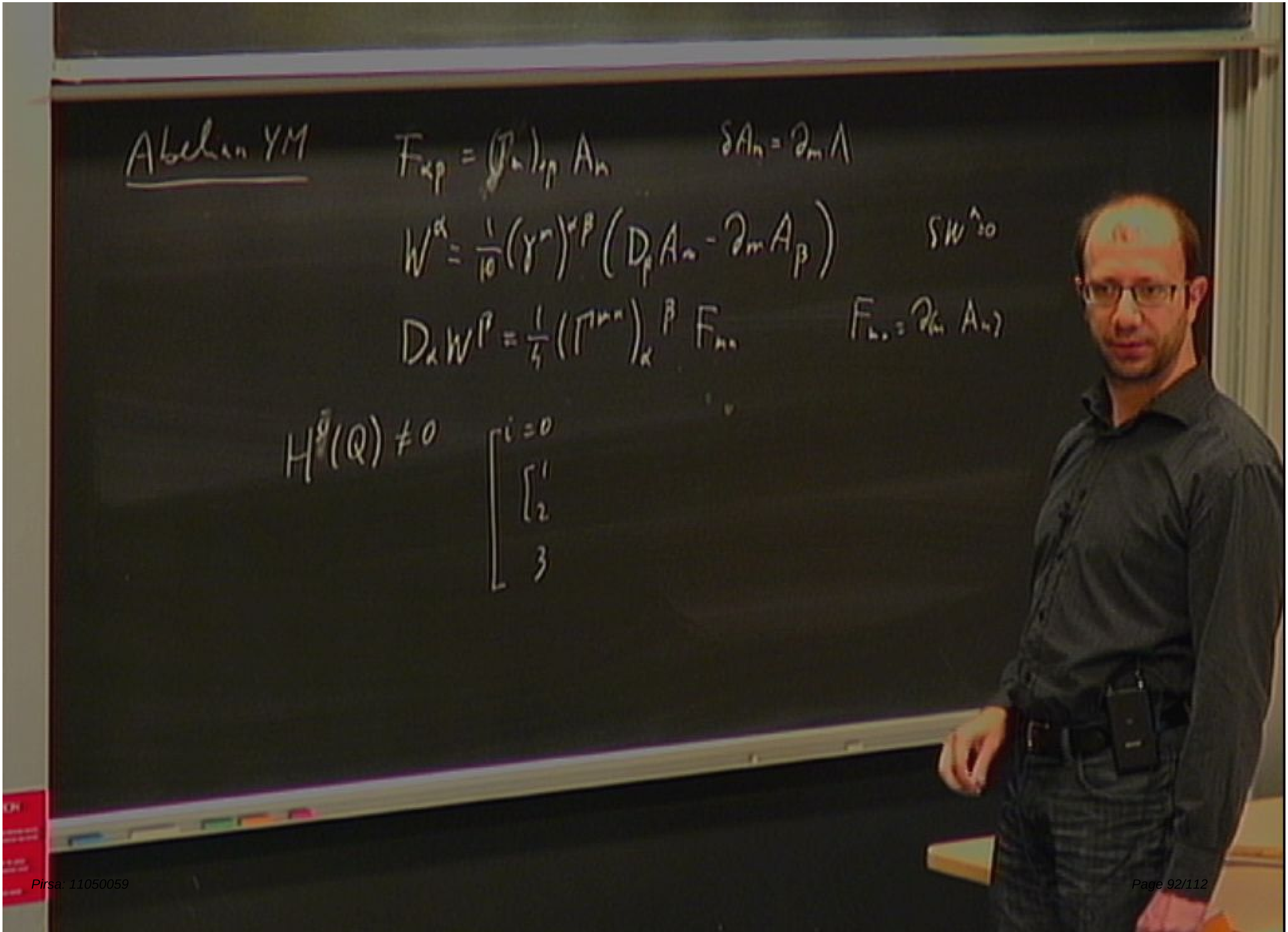

Superstring
Q = λ^α d_α
H*(Q) = Ker Q / Im Q

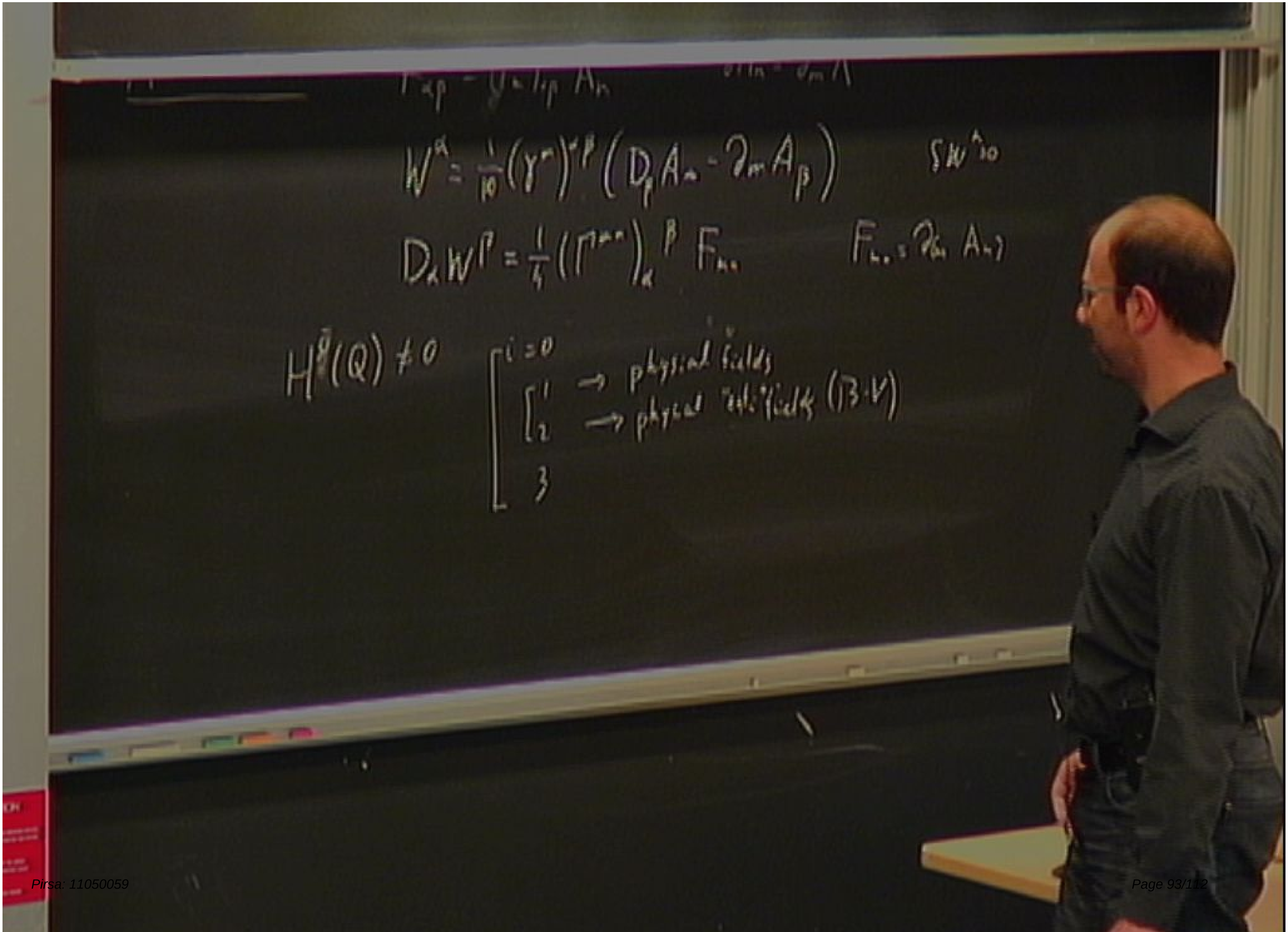

Superstring
Q = λ^α d_α

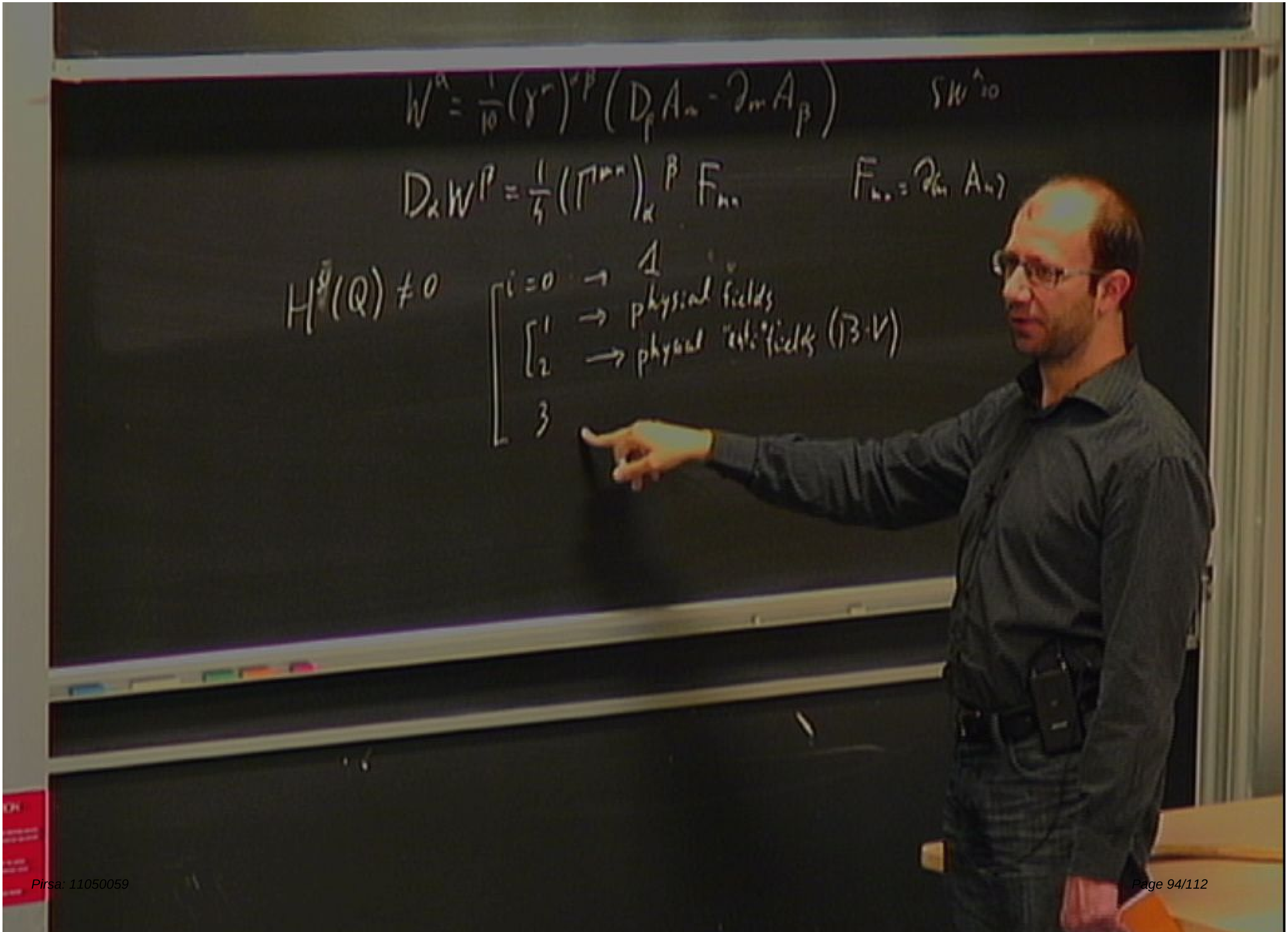

Q = λ^α d_α
d_α(z) U(x,θ) ~ D_α U / (z - w)
H^2(Q) = Ker Q / Im Q
H^1(Q) = {on-shell fields}

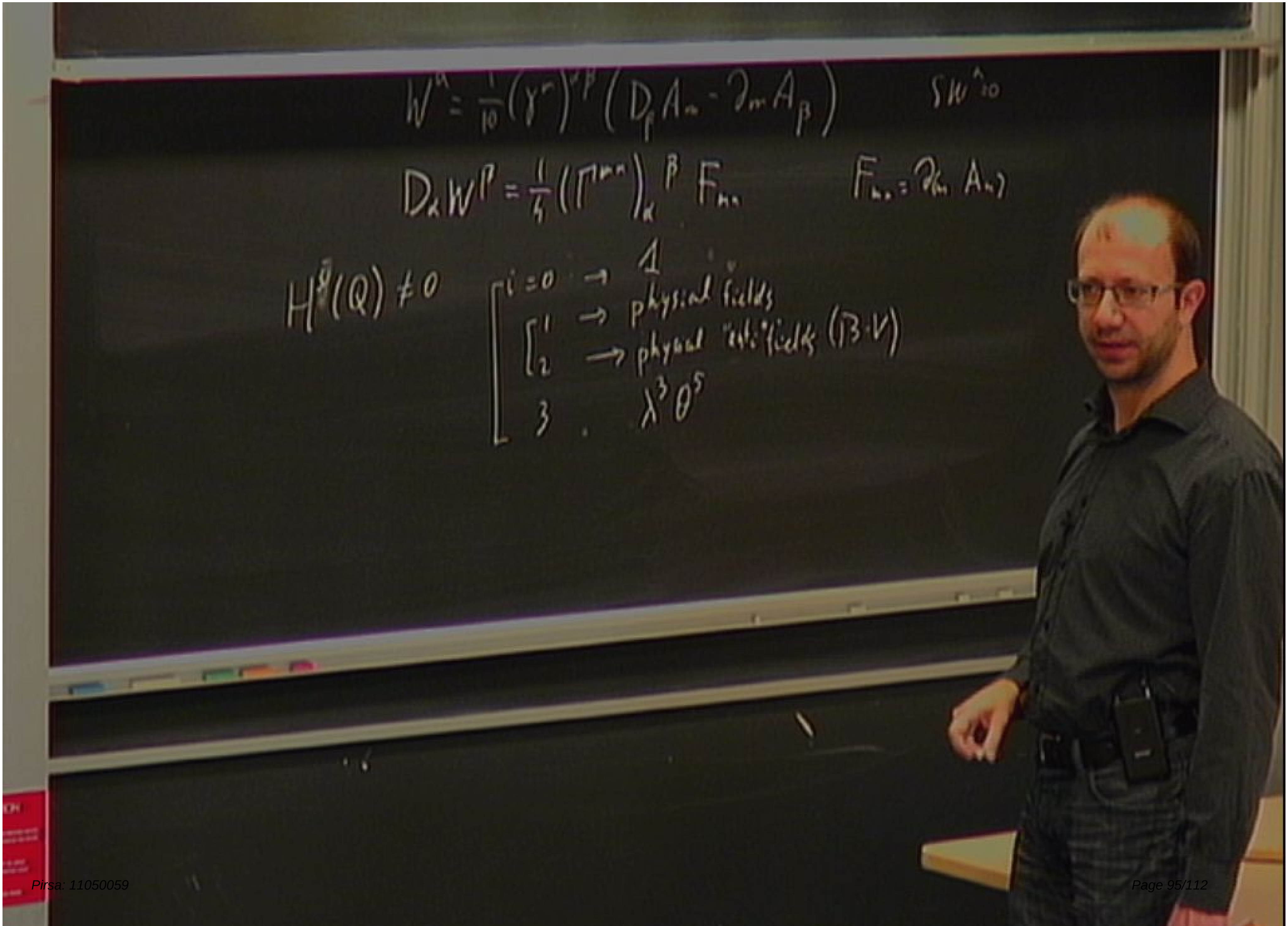

{on-shell fields}

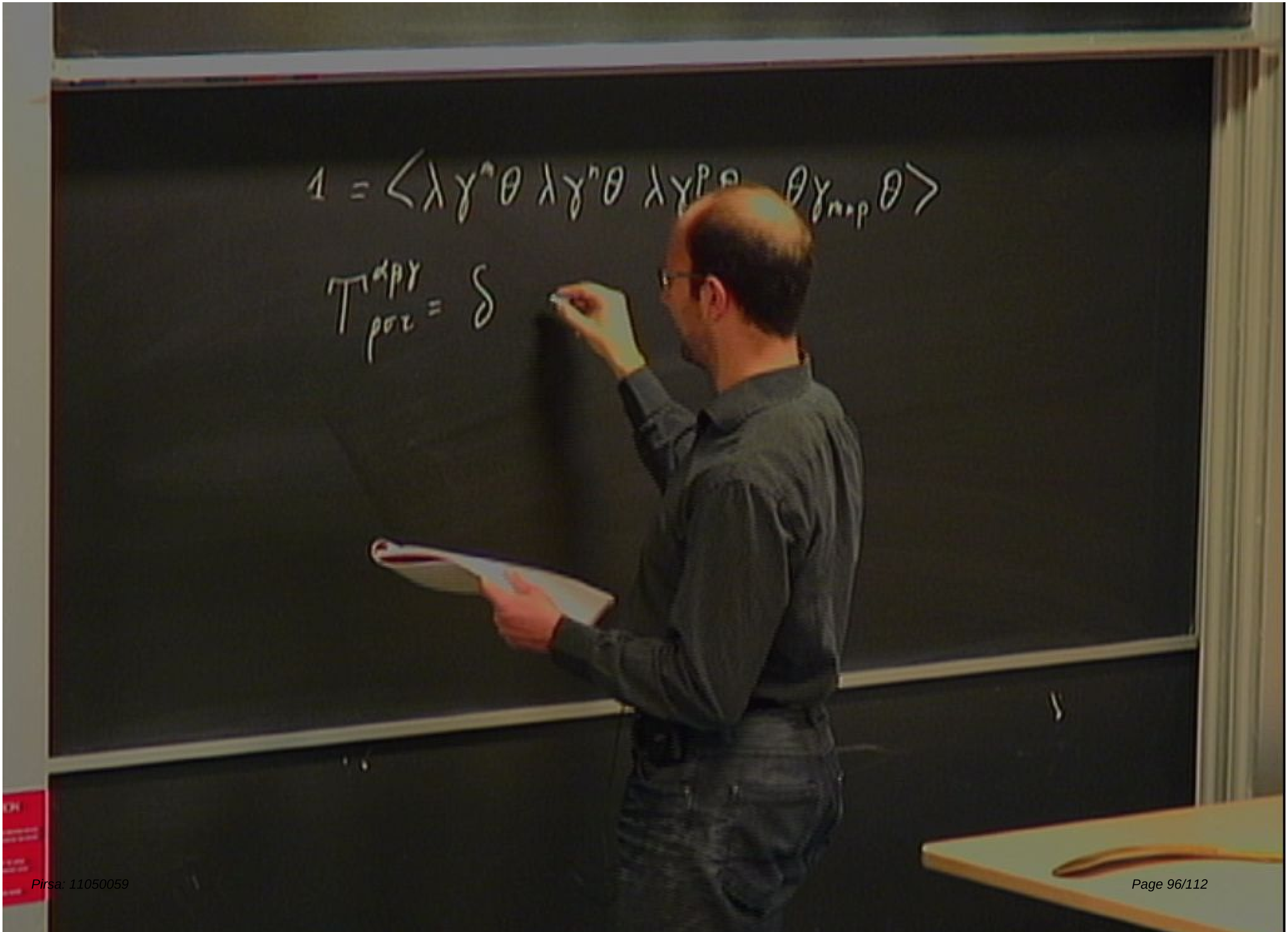

Abelian YM
$F_{\alpha\beta} = (\Gamma_m)_{\alpha\beta} A_m$
$\delta A_m = \partial_m \Lambda$
$W^\alpha = \frac{1}{10}(\gamma^m)^{\alpha\beta}(D_\beta A_m - \partial_m A_\beta)$
$\delta W^\alpha = 0$
$D_\alpha W^\beta = \frac{1}{4}(\Gamma^{mn})_\alpha{}^\beta F_{mn}$
$F_{mn} = \partial_{[m} A_{n]}$

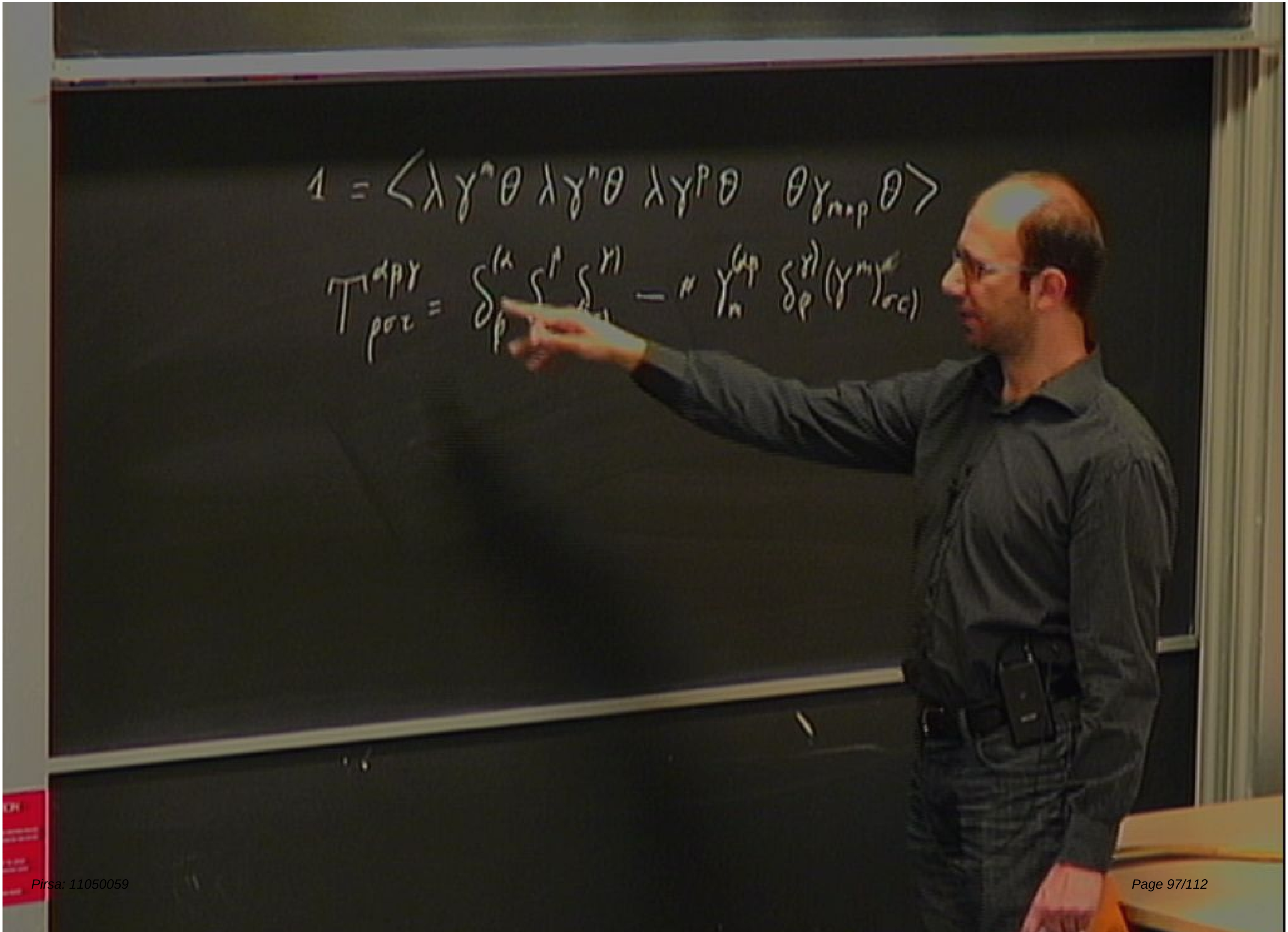

Abelian YM
F_{αβ} = (γ^m)_{αβ} A_m
δA_m = ∂_m Λ
W^α = 1/10 (γ^m)^{αβ} (D_β A_m − ∂_m A_β)
δW^α = 0
D_α W^β = 1/4 ((Γ^{mn})_α^β F_{mn}
F_{mn} = ∂_{[m} A_{n]}
H^g(Q) ≠ 0
i = 0
1
2
3

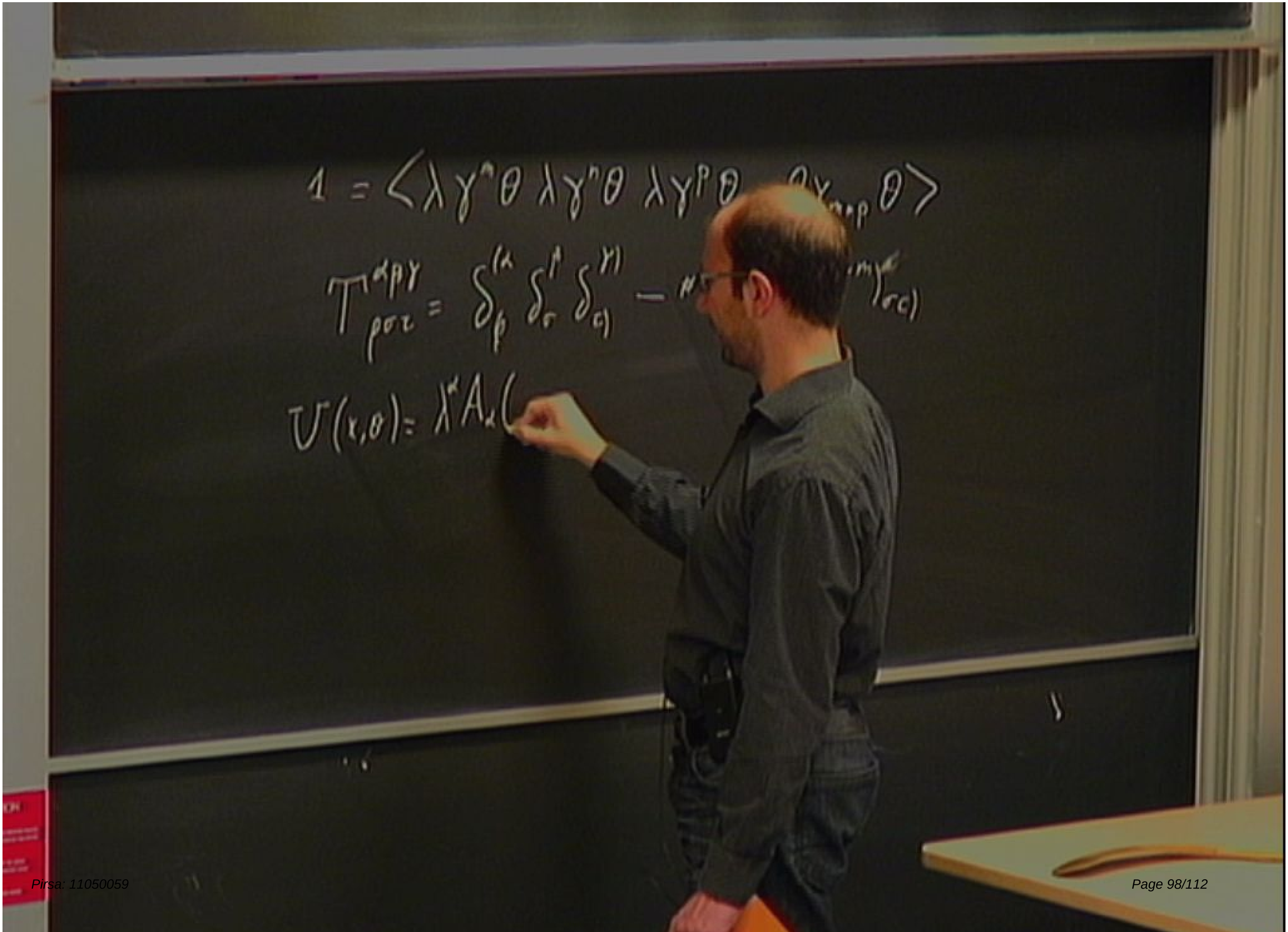

W^α = 1/10 (γ^m)^{αβ} (D_β A_m - ∂_m A_β)
δW^α = 0
D_α W^β = 1/4 ((Γ^{mn}))_α^β F_{mn}
F_{mn} = ∂_{[m} A_{n]}
H^i(Q) ≠ 0
i = 0
1 → physical fields
2 → physical anti-fields (B-V)
3

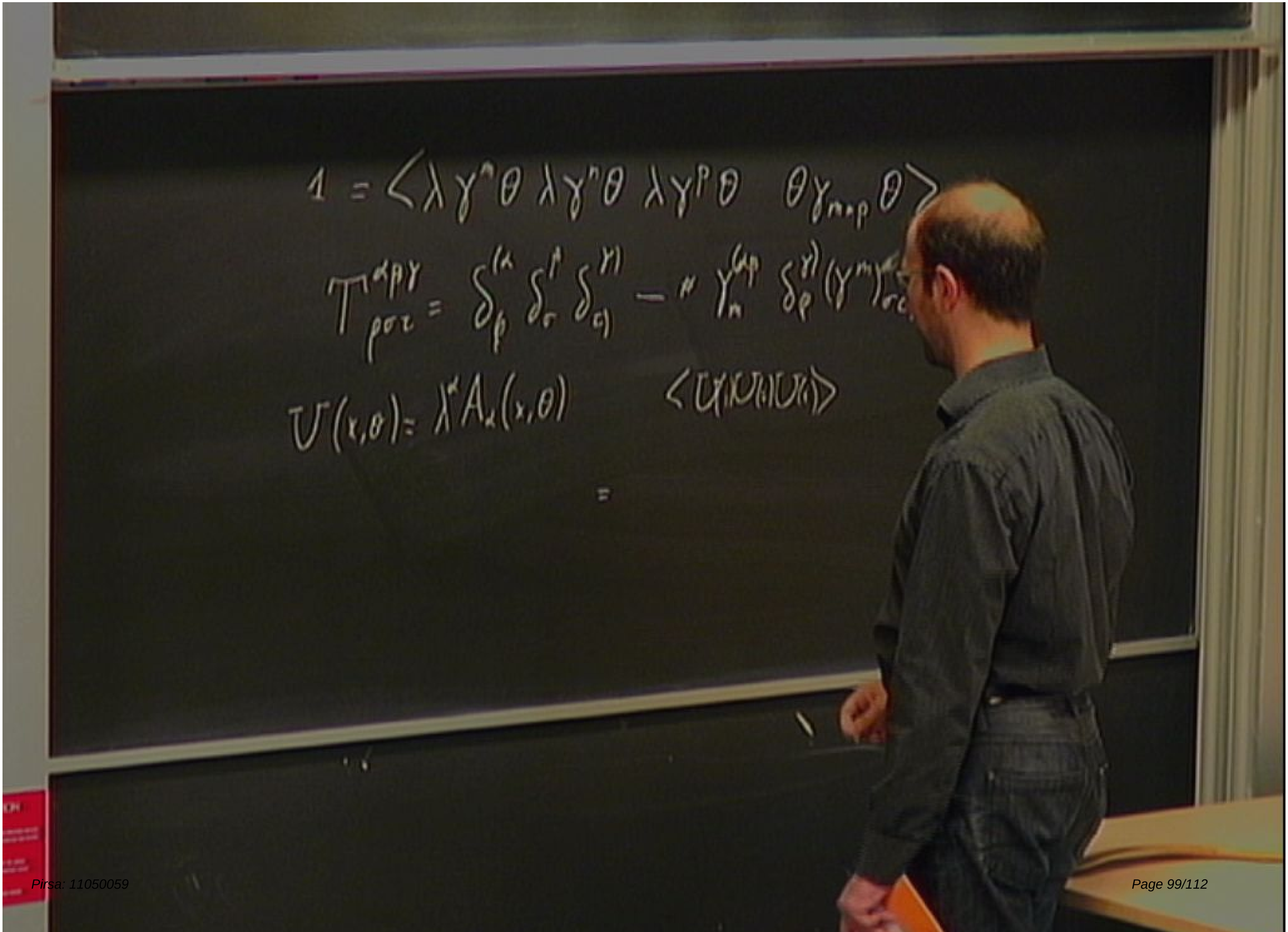


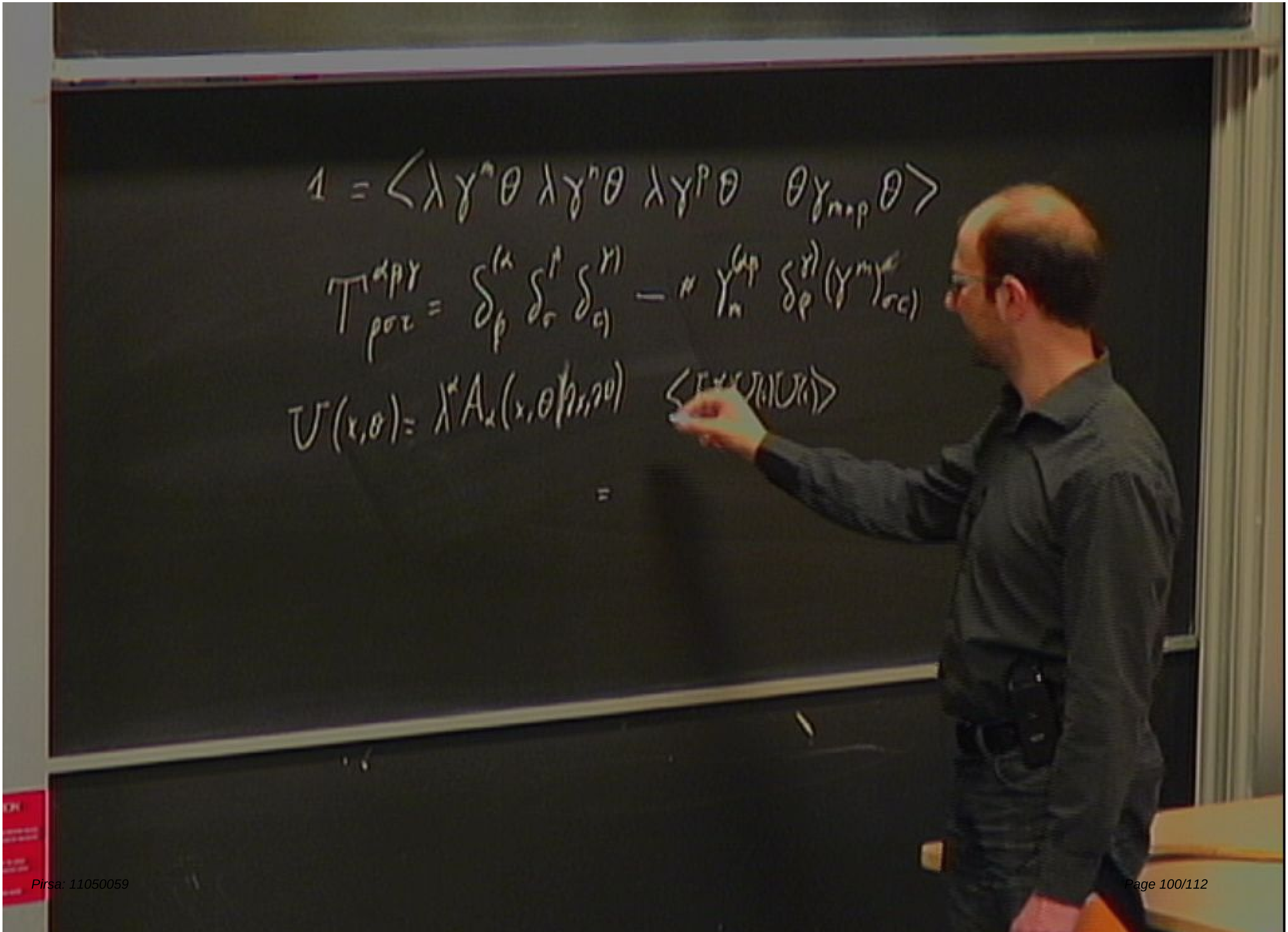

W^α = 1/10 (γ^m)^{αβ} (D_β A_m - ∂_m A_β)
D_α W^β = 1/4 ((Γ^{mn})_α^β F_{mn}
F_{mn} = ∂_{[m} A_{n]}
H^i(Q) ≠ 0
i = 0 → 1
1 → physical fields
2 → physical anti-fields (B-V)
3 λ^3 θ^5

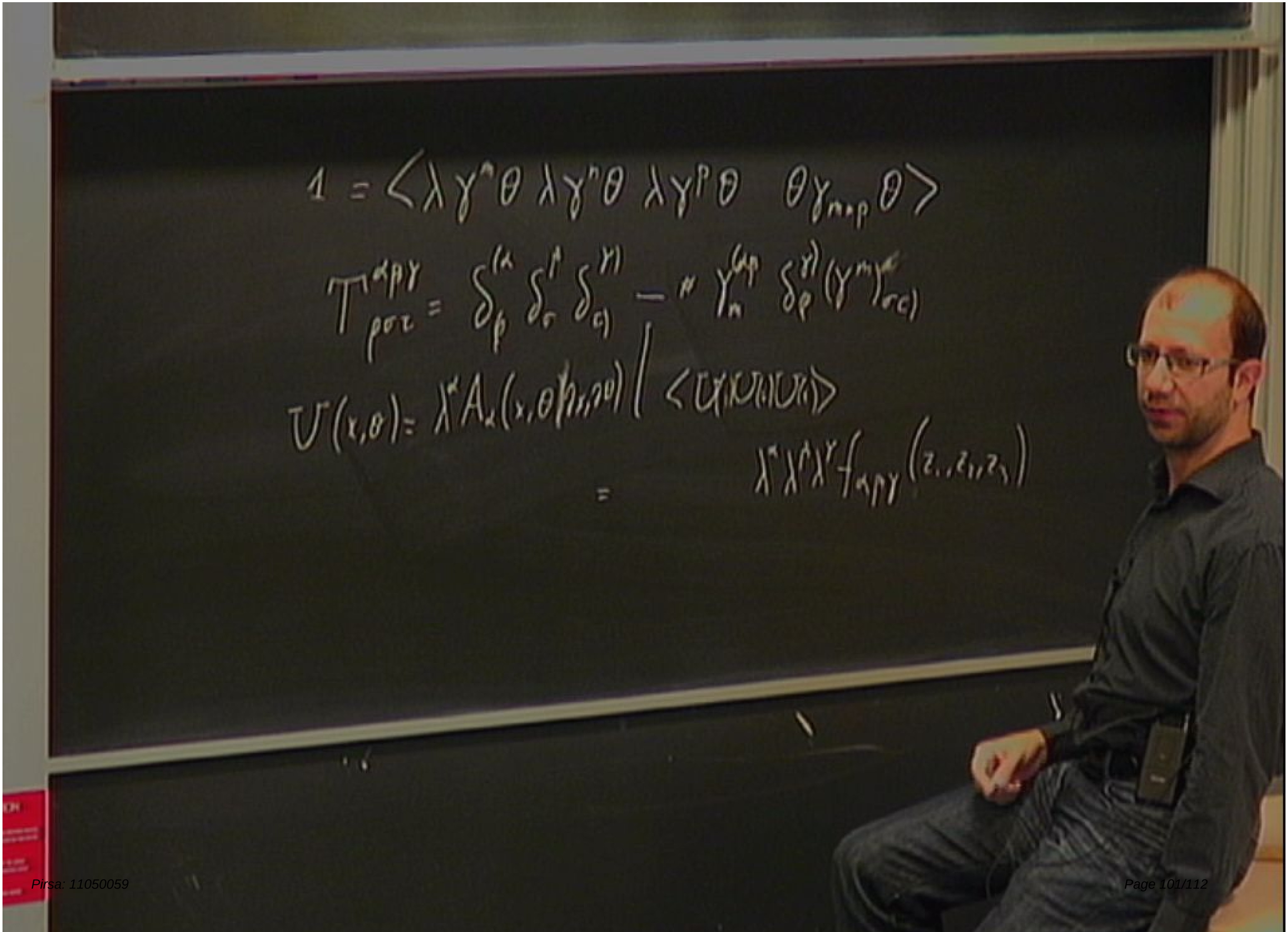

1 = ⟨λγ^m θ λγ^n θ λγ^p θ θγ_mnp θ⟩
T^αβγ_ρστ = δ

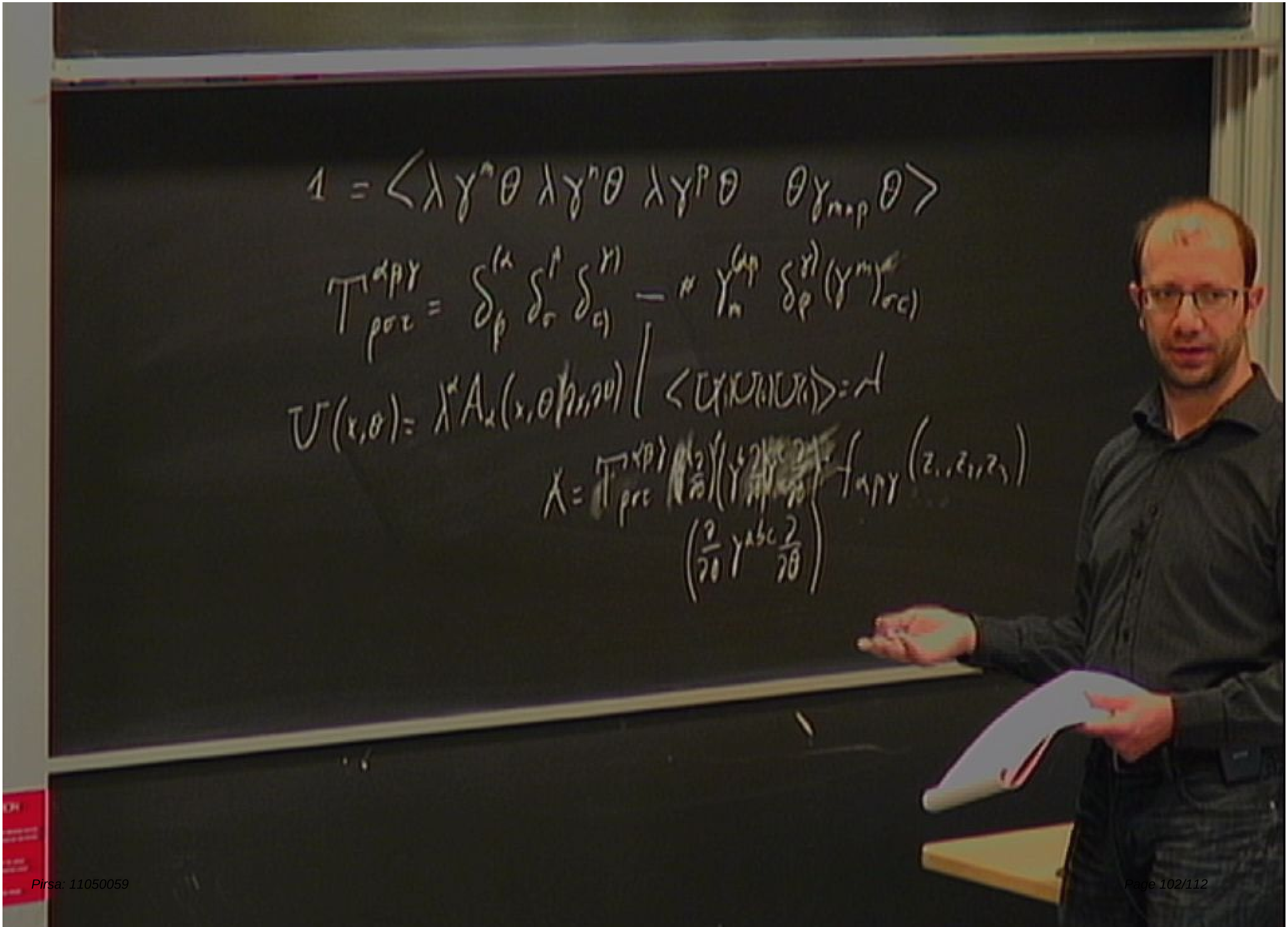

1 = ⟨λγ^m θ λγ^n θ λγ^p θ θγ_mnp θ⟩

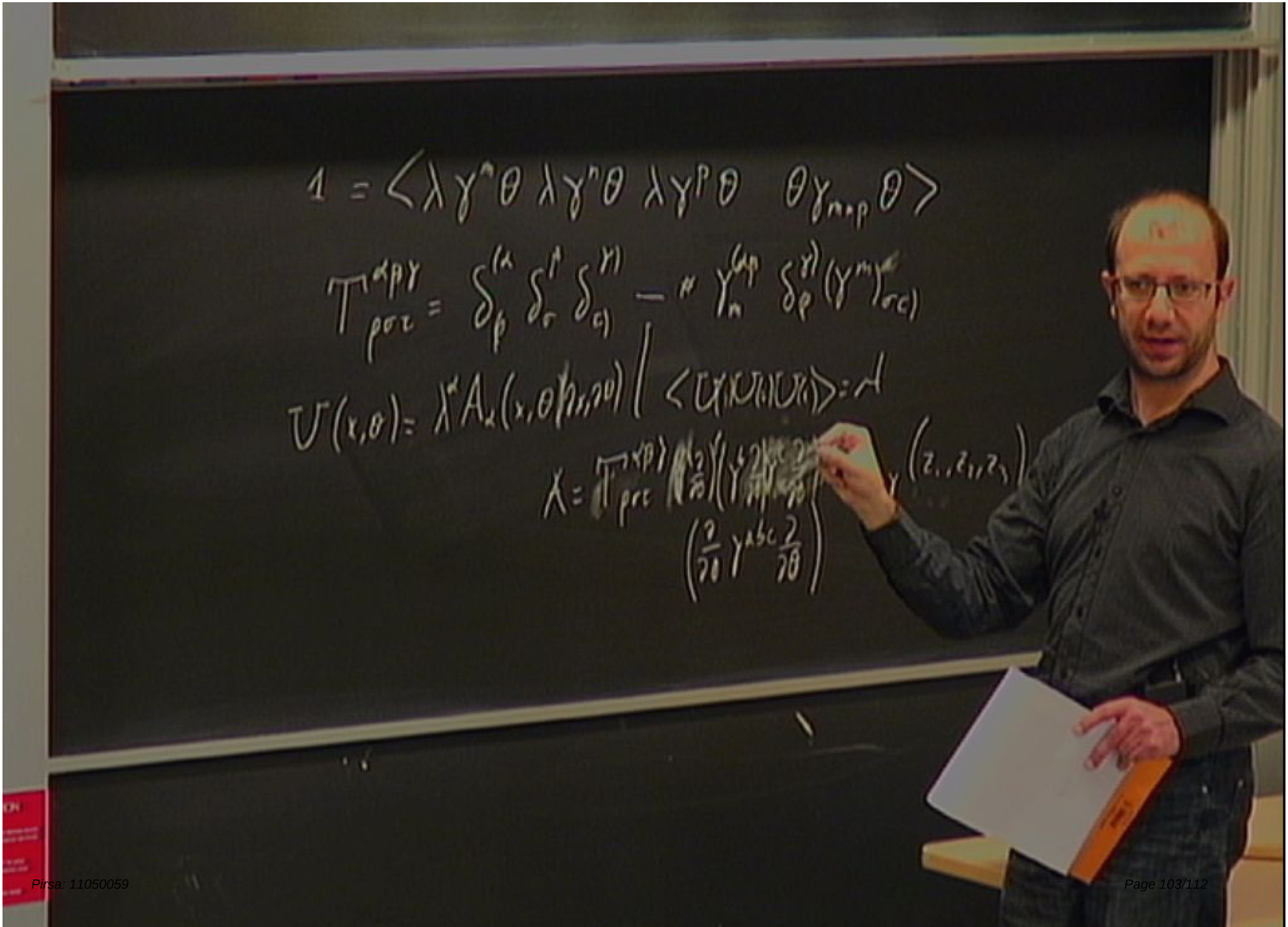

1 = <λγ^m θ λγ^n θ λγ^p θ θγ_mnp θ>
T^αβγ_ρστ = δ_ρ^(α δ_σ^β δ_τ)^γ) −
U(x,θ) = λ^α A_α(

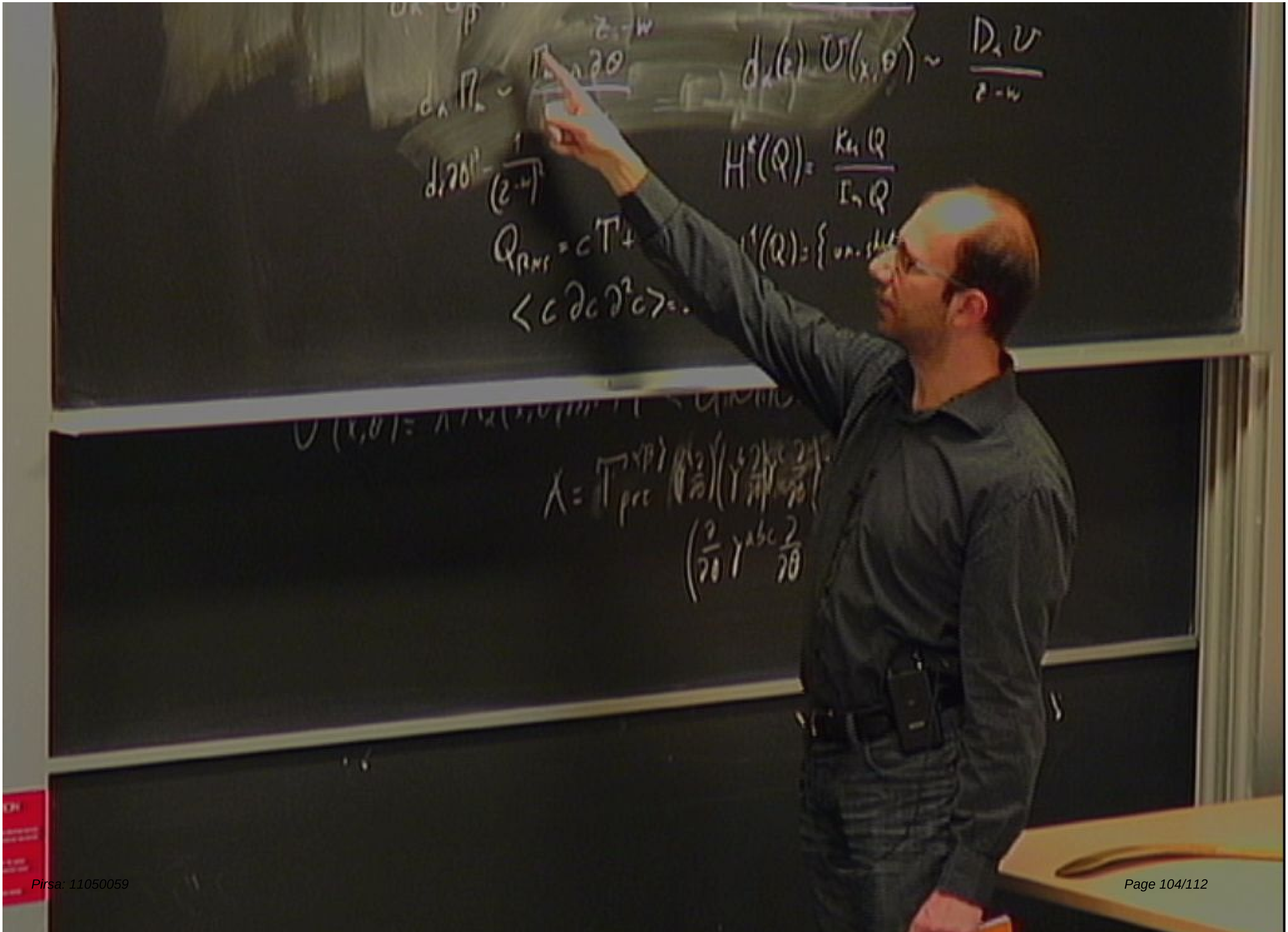

1 = ⟨λγ^m θ λγ^n θ λγ^p θ θγ_mnp θ⟩
U(x,θ) = λ^α A_α(x,θ)
⟨U(x_1)U(x_2)U(x_3)⟩

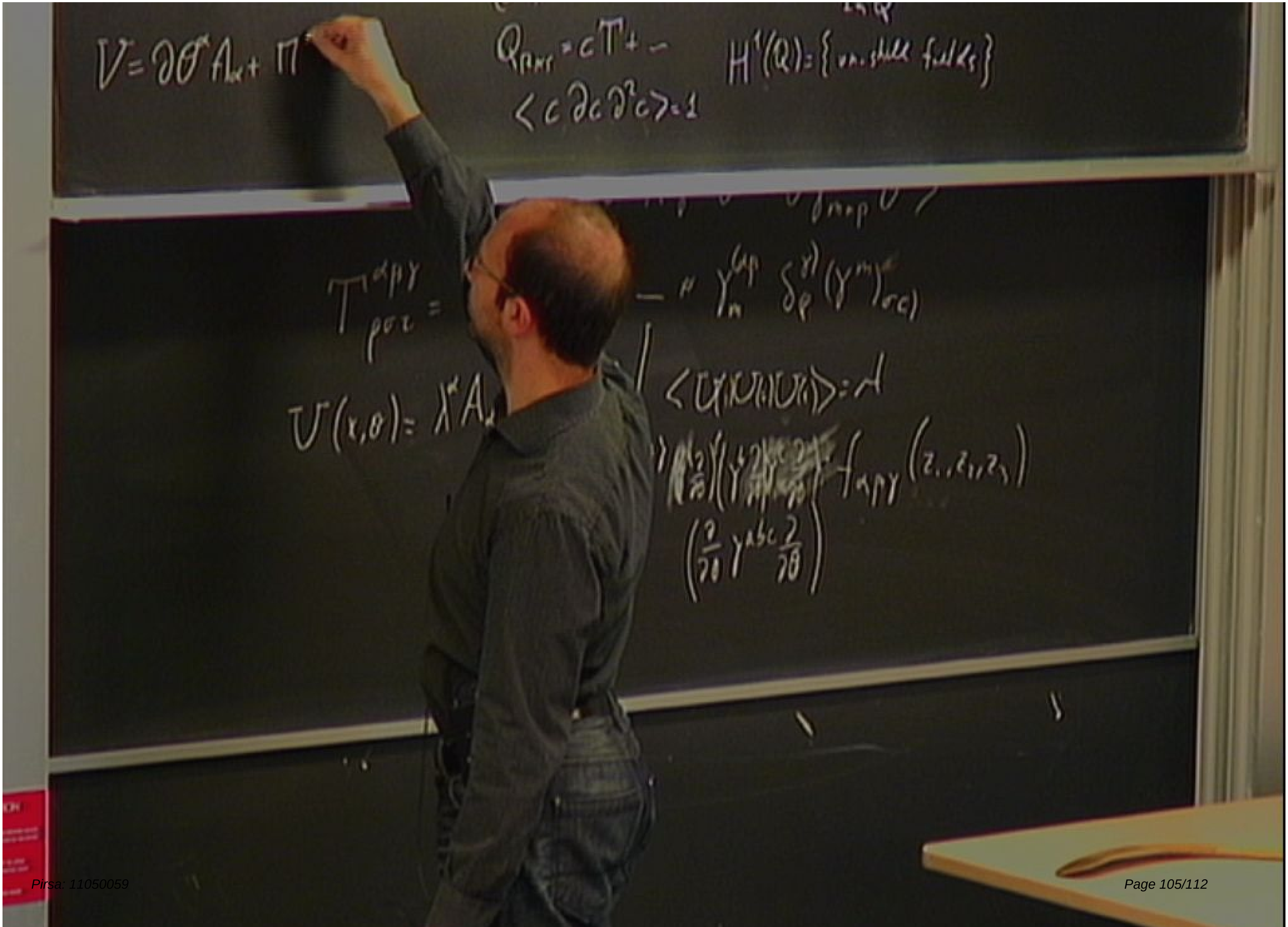

$1 = \langle \lambda\gamma^m\theta\, \lambda\gamma^n\theta\, \lambda\gamma^p\theta \;\; \theta\gamma_{mnp}\theta \rangle$
$T^{\alpha\beta\gamma}_{\rho\sigma\tau} = \delta^{(\alpha}_{\rho}\delta^{\beta}_{\sigma}\delta^{\gamma)}_{\tau)} - \gamma^{(\alpha\beta}_{m}\delta^{\gamma)}_{\rho}(\gamma^m)_{\sigma\tau)}$

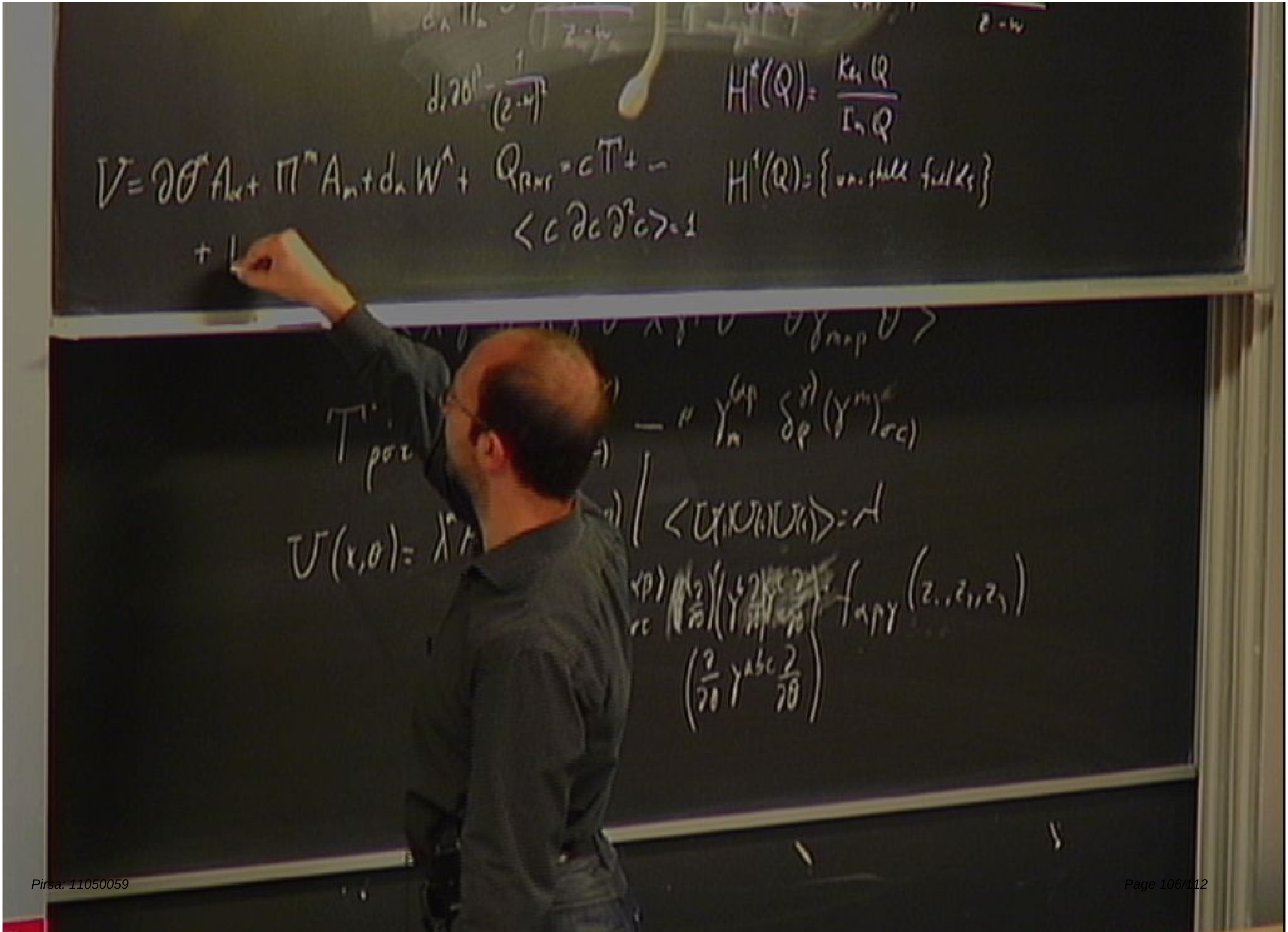

1 = <λγ^m θ λγ^n θ λγ^p θ θγ_{mnp} θ>
T^{αβγ}_{ρστ} = δ^{(α}_ρ δ^β_σ δ^{γ)}_{τ)} − γ^{(αβ}_m δ^{γ)}_{(ρ} (γ^m)_{στ)}
U(x,θ) = λ^α A_α(x,θ)
λ^α λ^β λ^γ f_{αβγ}(z_1, z_2, z_3)
=

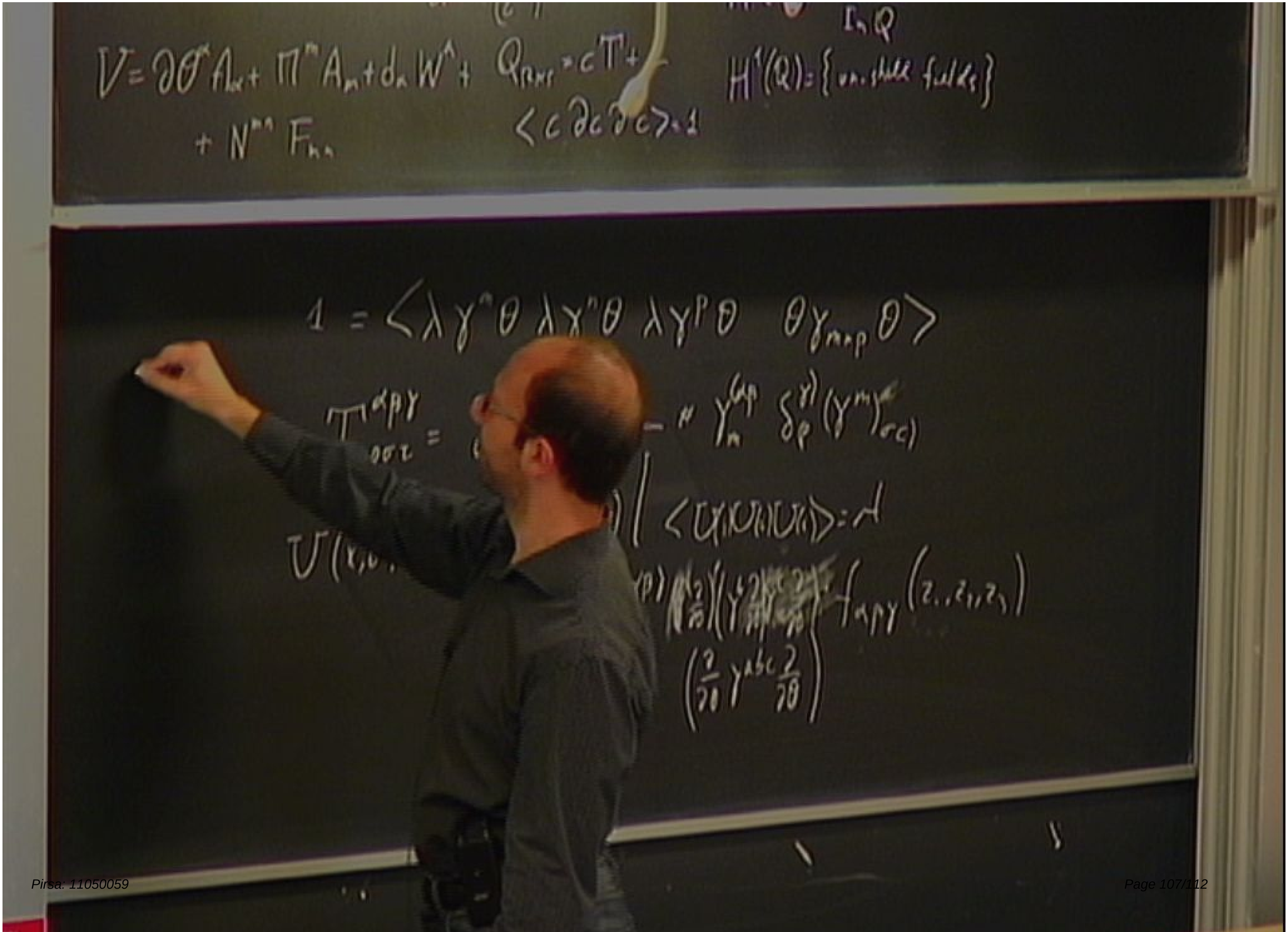

1 = <λγ^m θ λγ^n θ λγ^p θ θγ_{mnp} θ>
U(x,θ) = λ^α A_α(x,θ)

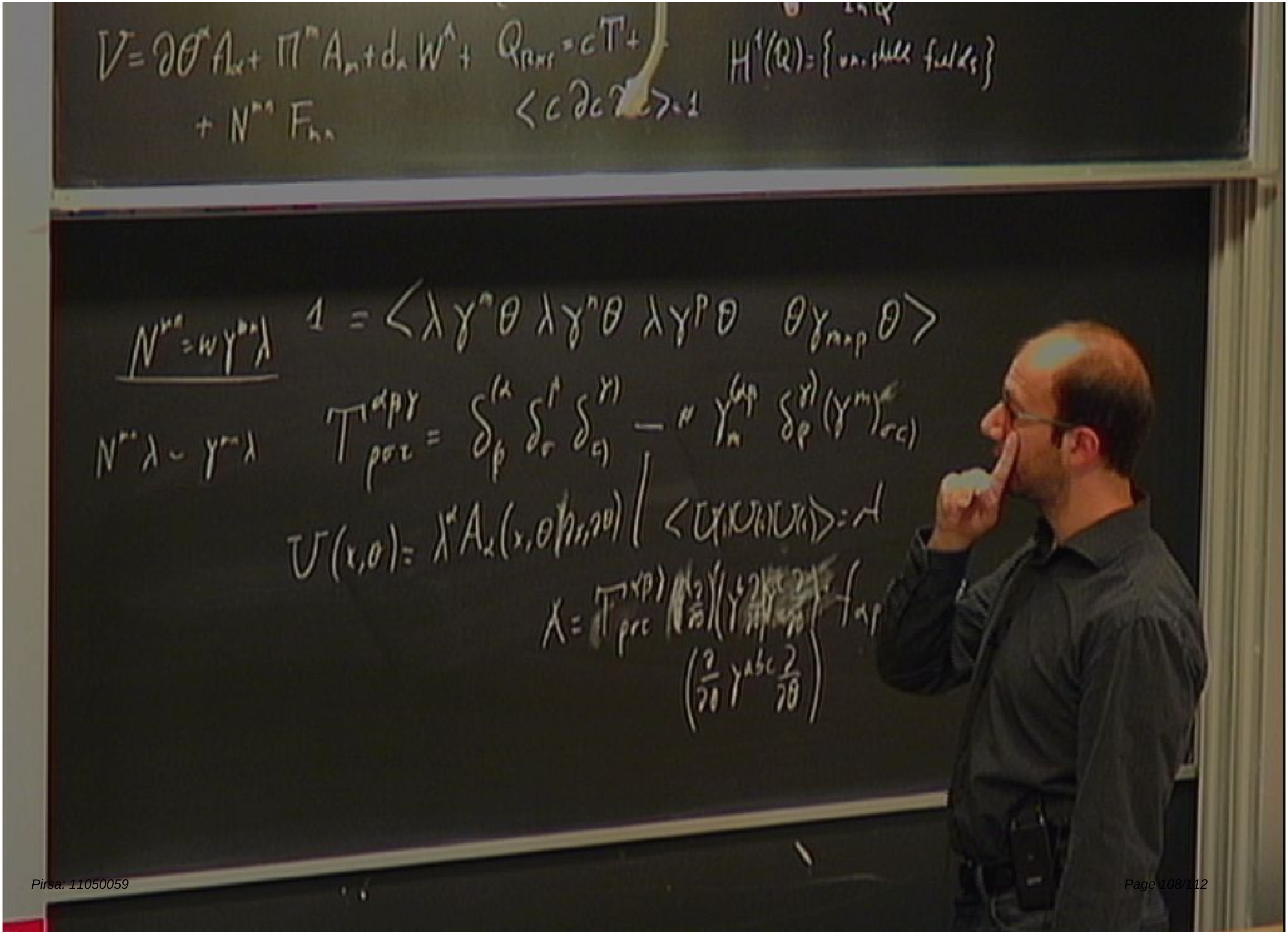

$1 = \langle \lambda\gamma^m\theta\, \lambda\gamma^n\theta\, \lambda\gamma^p\theta\ \ \theta\gamma_{mnp}\theta \rangle$
$T^{\alpha\beta\gamma}_{\rho\sigma\tau} = \delta^{(\alpha}_{\rho}\delta^{\beta}_{\sigma}\delta^{\gamma)}_{\tau)} - \gamma^{(\alpha\beta}_{m}\delta^{\gamma)}_{\rho}(\gamma^m)_{\sigma\tau)}$
$U(x,\theta) = \lambda^\alpha A_\alpha(x,\theta)$
$\left(\frac{\partial}{\partial\theta}\gamma^{abc}\frac{\partial}{\partial\theta}\right)$

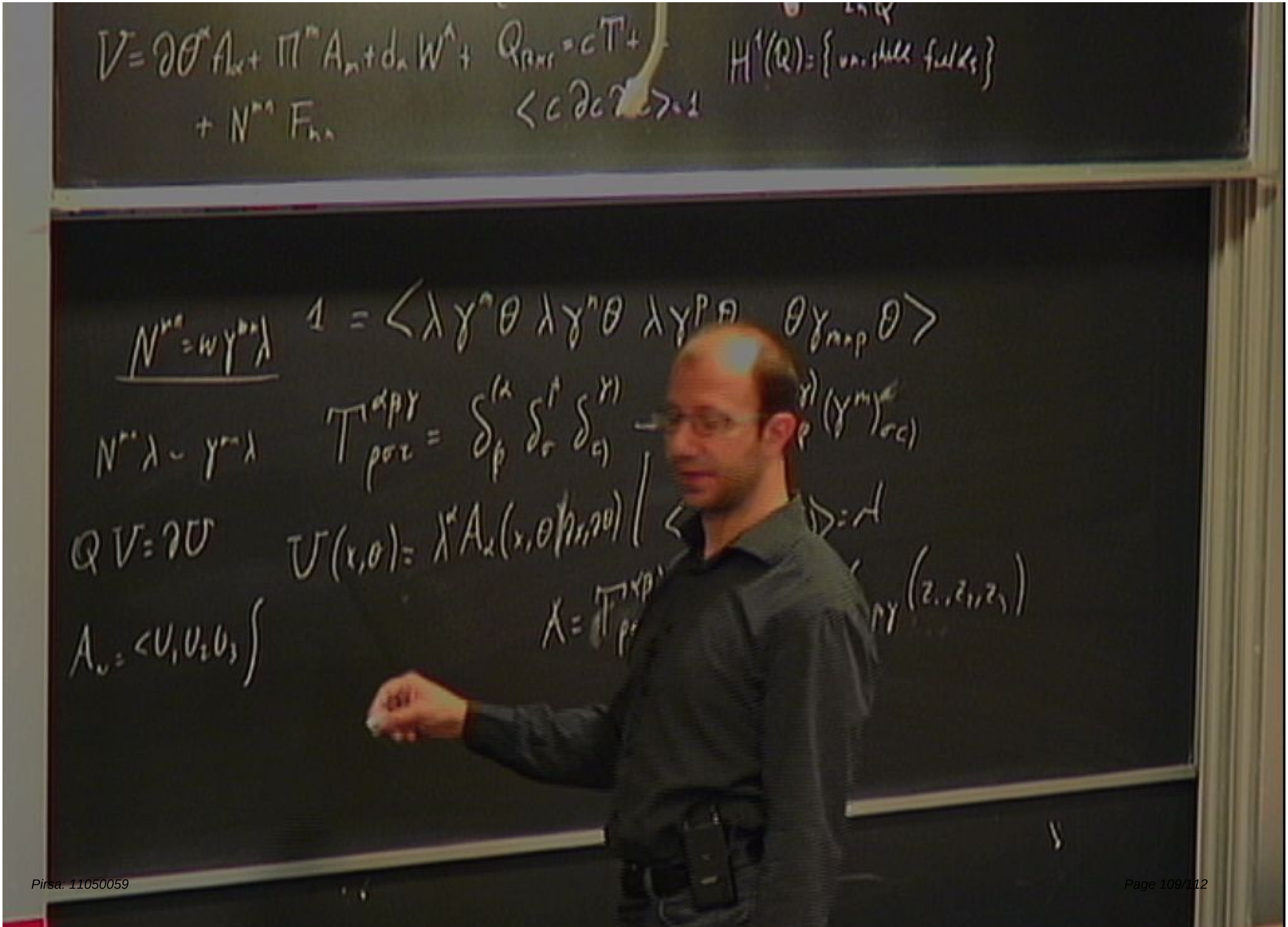


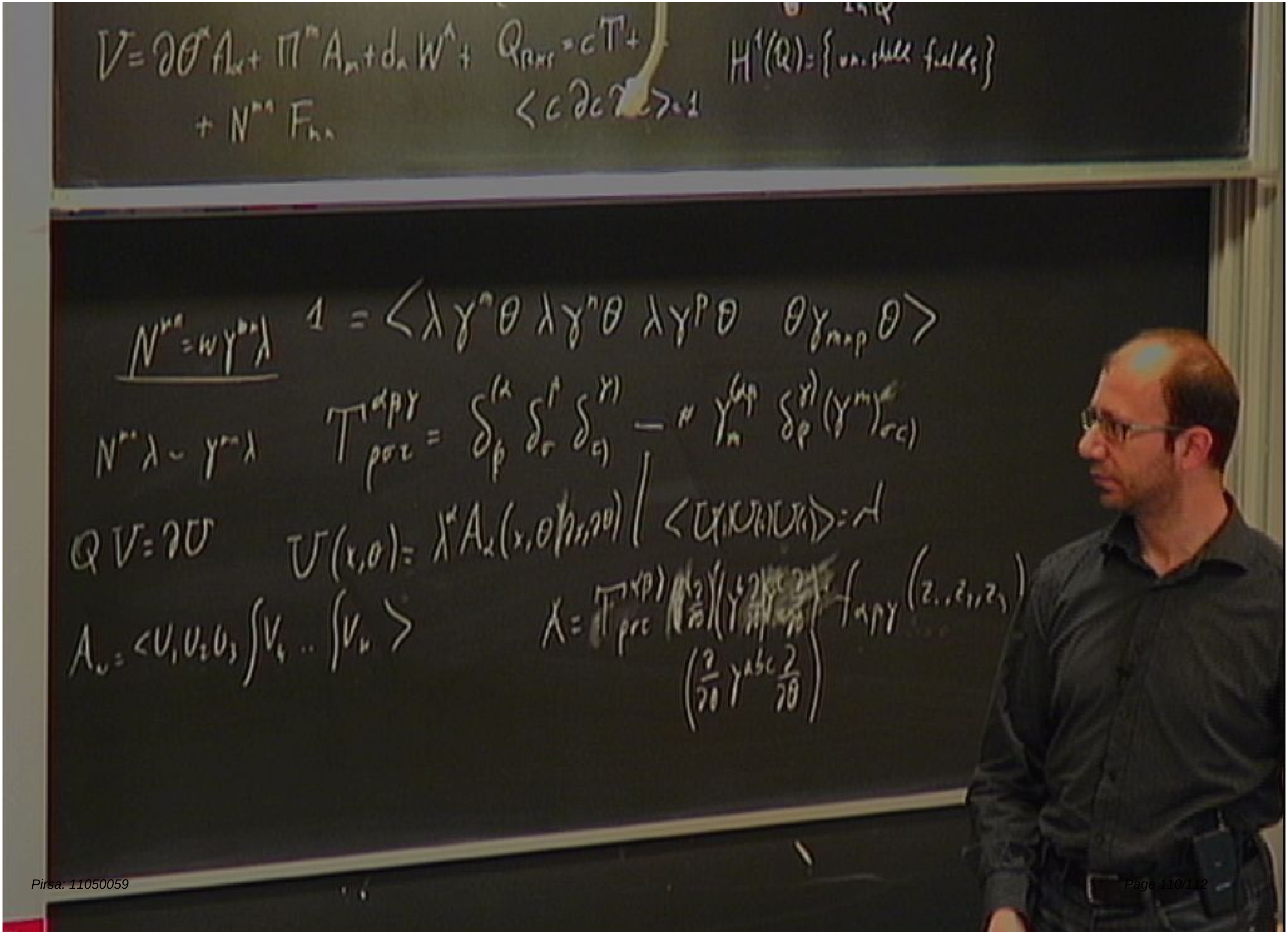


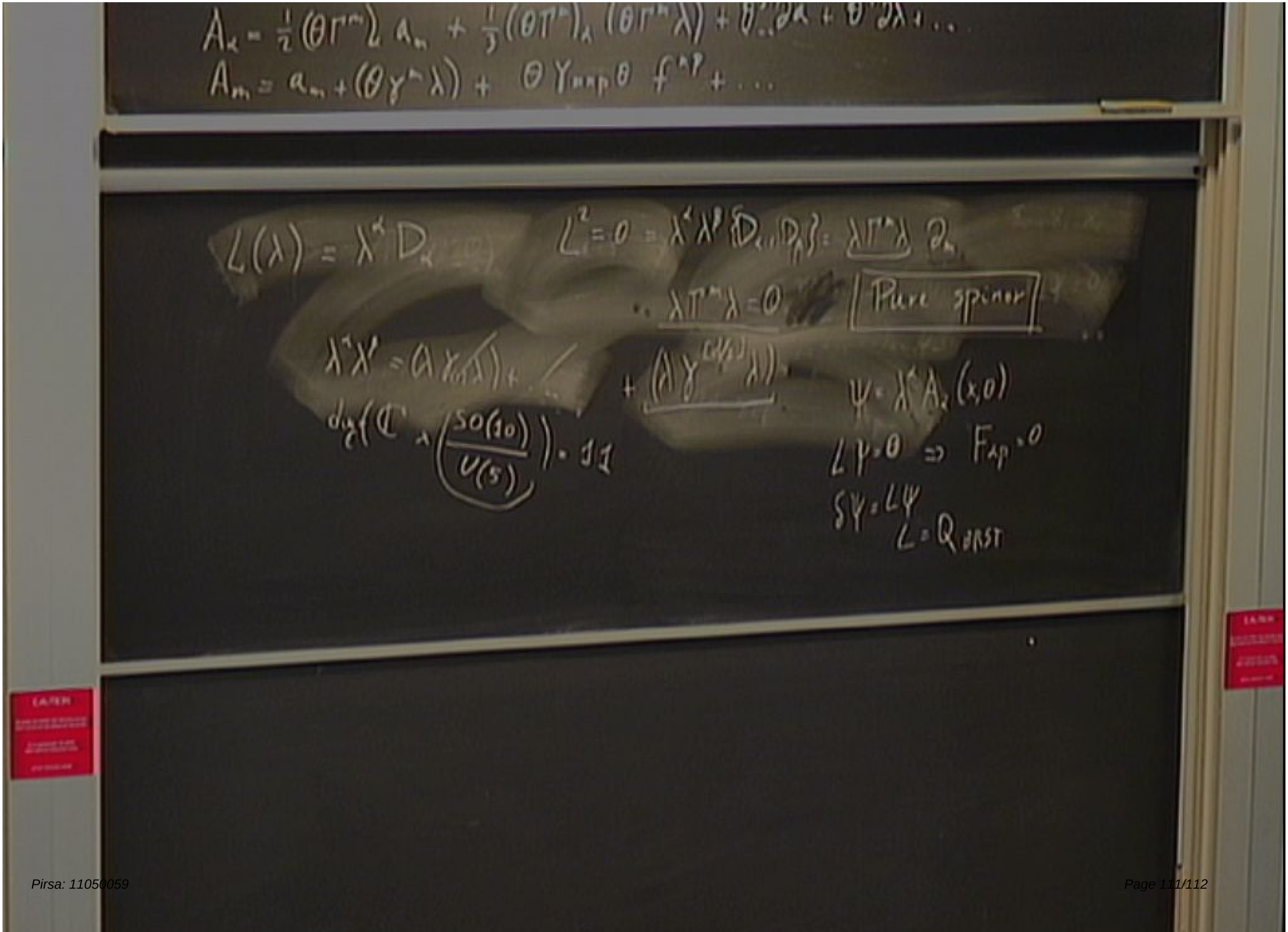


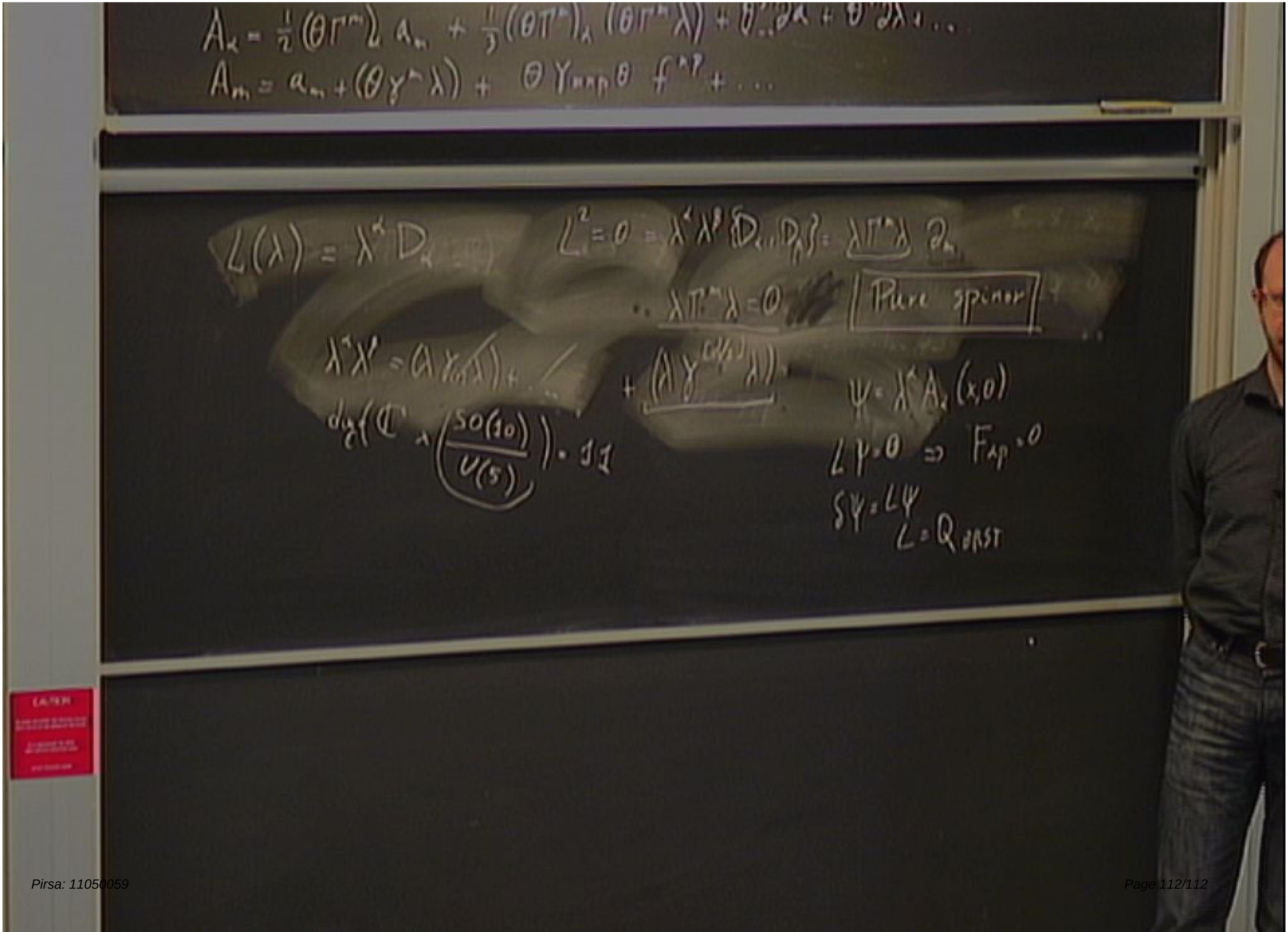





$V = \partial\theta^\alpha A_\alpha + \Pi^m A_m + d_\alpha W^\alpha + N^{mn} F_{mn}$
$Q_{BRST} = cT + \ldots$
$\langle c \partial c \partial^2 c \rangle = 1$
$H^1(Q) = \{$unphysical fields$\}$
$N^{mn} = w\gamma^{mn}\lambda$
$1 = \langle \lambda\gamma^m\theta\, \lambda\gamma^n\theta\, \lambda\gamma^p\theta\, \theta\gamma_{mnp}\theta \rangle$
$N^{mn}\lambda \sim \gamma^{mn}\lambda$
$QV = \partial U$
$U(x,\theta) = \lambda^\alpha A_\alpha(x,\theta)$
$A_{\nu} = \langle U_1 U_2 U_3 \int$


N^{mn} = w γ^{mn} λ
QV = ∂U
A_n = <U_1 U_2 U_3 ∫V_4 ... ∫V_n>


Pure spinor


Pure spinor