Title: Local states and channels in causal theories

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Abstract: We will analyze different aspects of locality in causal operational probabilistic theories. We will first discuss the notion of local state and local objective information in operational probabilistic theories, and define an operational notion of discord that coincides with quantum discord in the case of quantum theory. Using such notion, we will show that the only theory in which all separable states have null discord is the classical one. We will then analyze locality of transformations, reviewing some general properties of no-signaling channels in causal theories. We will show that it is natural to define transformations on no-signaling channels that cannot be extended to all bipartite channels, and discuss the consequences of this fact on information processing.

Locality and causality in operational theories

Paolo Perinotti
Dipartimento di Fisica "A. Volta"
Università di Pavia

QUit quantum information theory group

Conceptual Foundations and Foils for Quantum Information Processing
Satellite Workshop, May 17 2010

In collaboration with

- G. M. D'Ariano

- G. Chiribella

- S. Facchini

- M. Zaopo

Outline

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- Causal theories
- Locality (different operational notions)

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- Localizable and non-signalling channels in causal theories with l. d.

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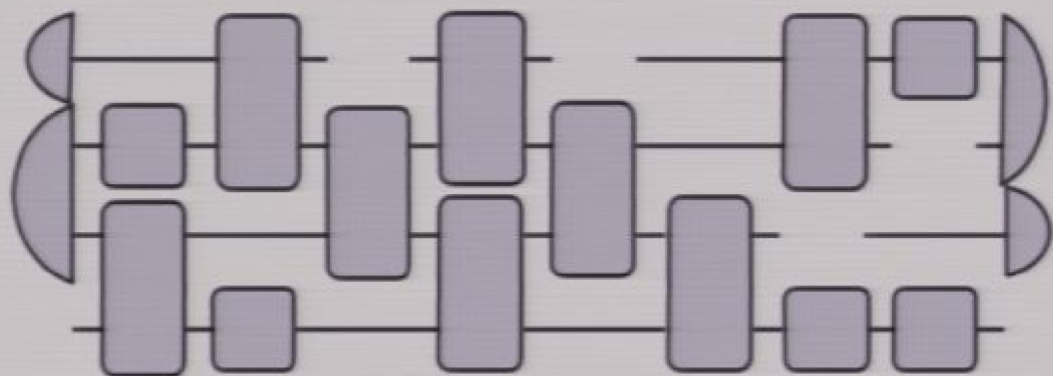
- Localizable and non-signalling channels in causal theories with l. d.
- Combs and higher order transformations in theories with purification

The operational language

- Operational theory: tests with composition rules



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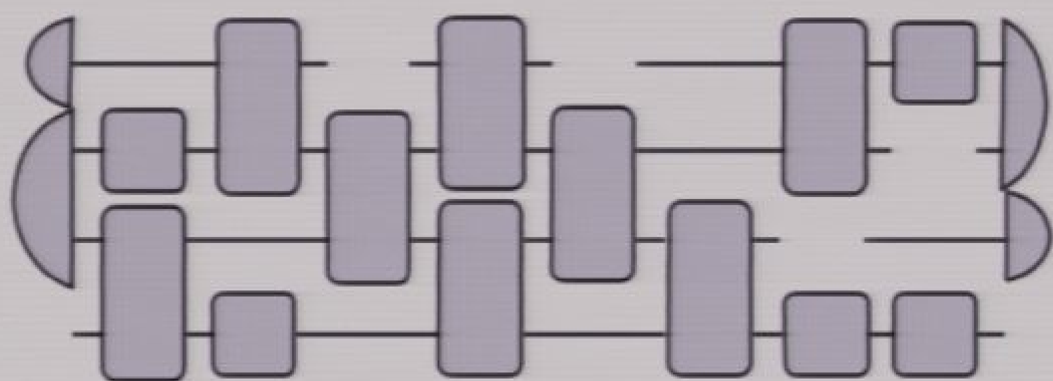

A : input label
B : output label

The operational language

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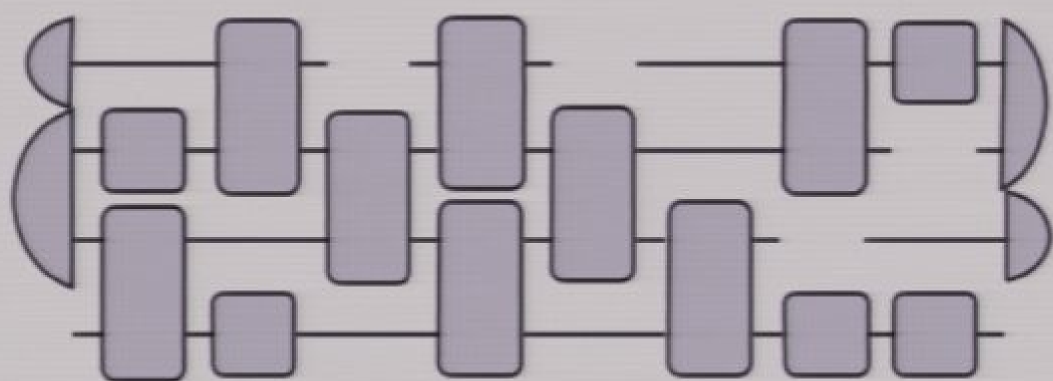

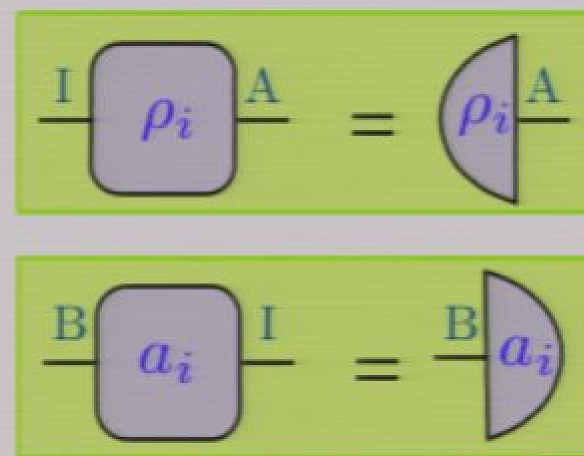

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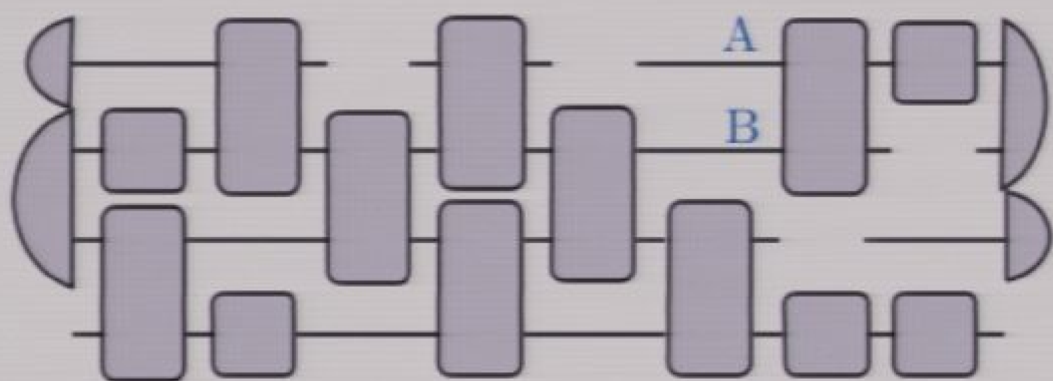

- C:=AB=BA
- (AB)C=A(BC)
- AI=IA=A

The operational language

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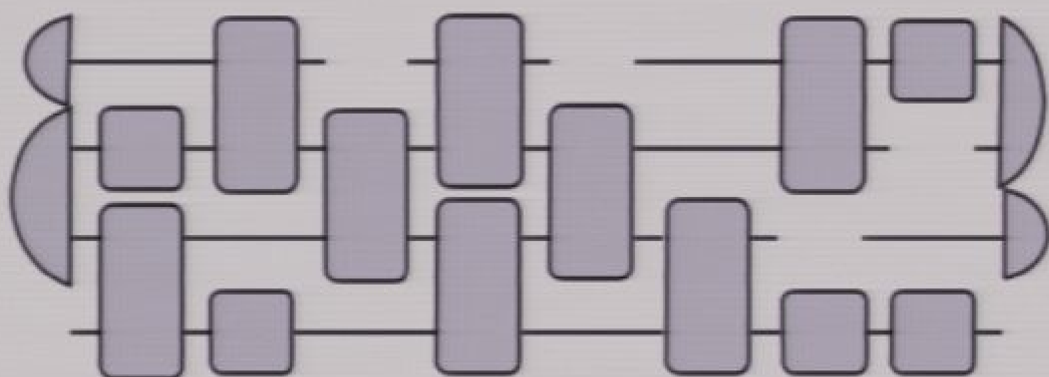
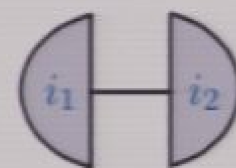


- For any system A there exists a unique test $\mathscr{I}_{\mathrm{A}}$ such that

The probabilistic structure

- Probabilistic theory

Every test of type I→I is a probability distribution



$= p(i_1, i_2)$

States are functionals on effects and viceversa

⇓

Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

$\mathfrak{T}_{\mathbb{R}}(\mathrm{A}, \mathrm{B})$ transformations are collections of linear maps

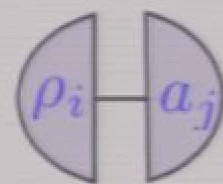
Causality

- Causality

Causality

- Causality

$$p_a(\rho_i) := \sum_j$$



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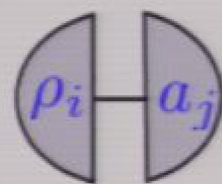

$$= p(\rho_i)$$

Causality

- Causality

$$p_a(\rho_i) := \sum_j \; (\rho_i \!-\! a_j) = p(\rho_i)$$

- Uniqueness of the deterministic effect $$\sum_j -a_j = \sum_k -b_k = -e$$

Causality

- Causality

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Causality

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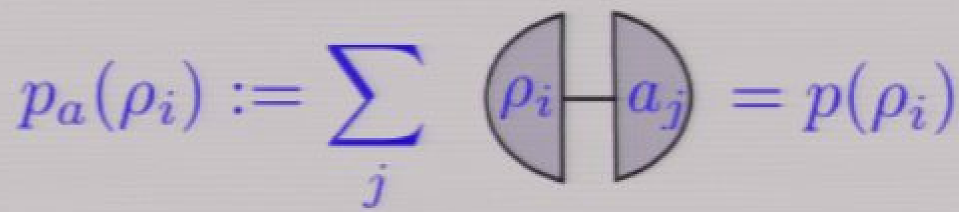

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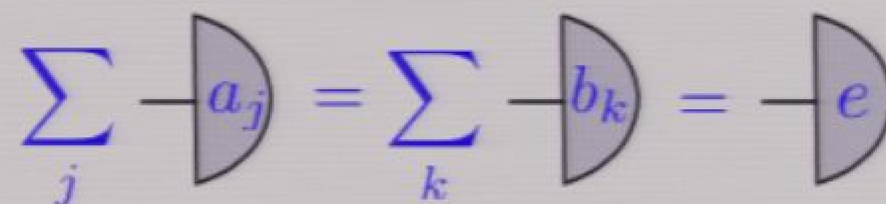

- All states are proportional to deterministic ones

Causality

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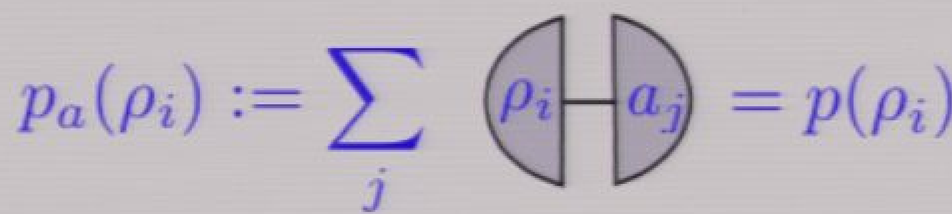

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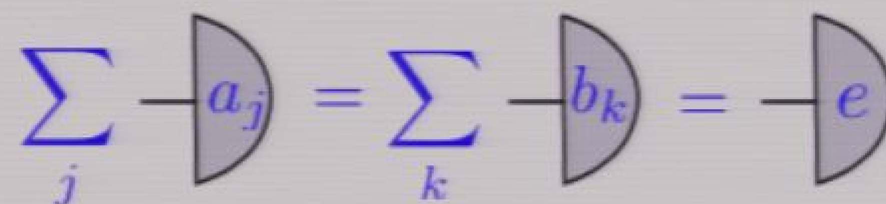
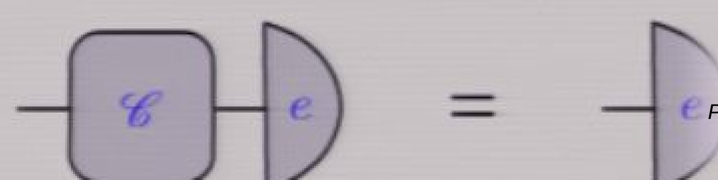


- All states are proportional to deterministic ones

- Unrestricted conditioning

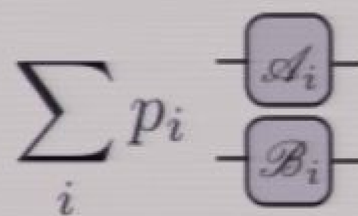
- Deterministic transformations



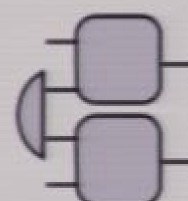
Locality properties of operational boxes

- Operationally locality of channels is classified by different notions:
- Factorized

- LOSR*



- Localizable**



- Non-signalling**

* G. Gutoski, Quantum Information and Computation, 9 739-764, (2009)
** D. Beckman, D. Gottesman, M. A. Nielsen, and J. Preskill, Phys. Rev A 64, 052309 (2001)

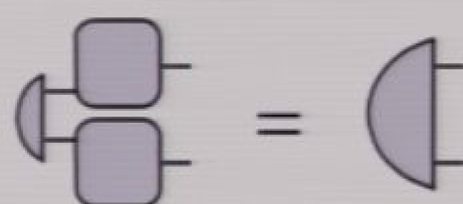
Locality properties of states in causal theories

- Factorized

- LOSR

$$\sum_i p_i$$

- Localizable, non-signalling and general bipartite states coincide



Locality properties of states in causal theories

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Separability

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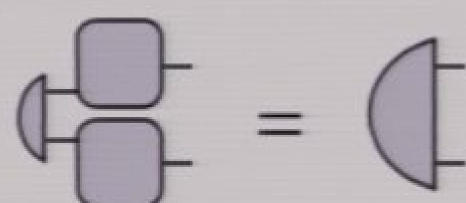


Non-locality without entanglement

- In "Quantum Nonlocality without entanglement"* the authors introduce a different notion of locality

- This definition is based on locality of the measurement of the eigenbasis

- The classical information encoded by a random source of distinguishable states can/cannot be accessed by LOCC

- How can we define a similar kind of non-locality in causal operational theories?

- A state is local if it encodes locally readable objective information

* C. H. Bennett, D. P. DiVincenzo, C. A. Fuchs, T. Mor, E. Rains, P. W. Shor, J. A. Smolin, and W. K. Wootters, Phys. Rev. A 59, 1070 (1999)

Objective information

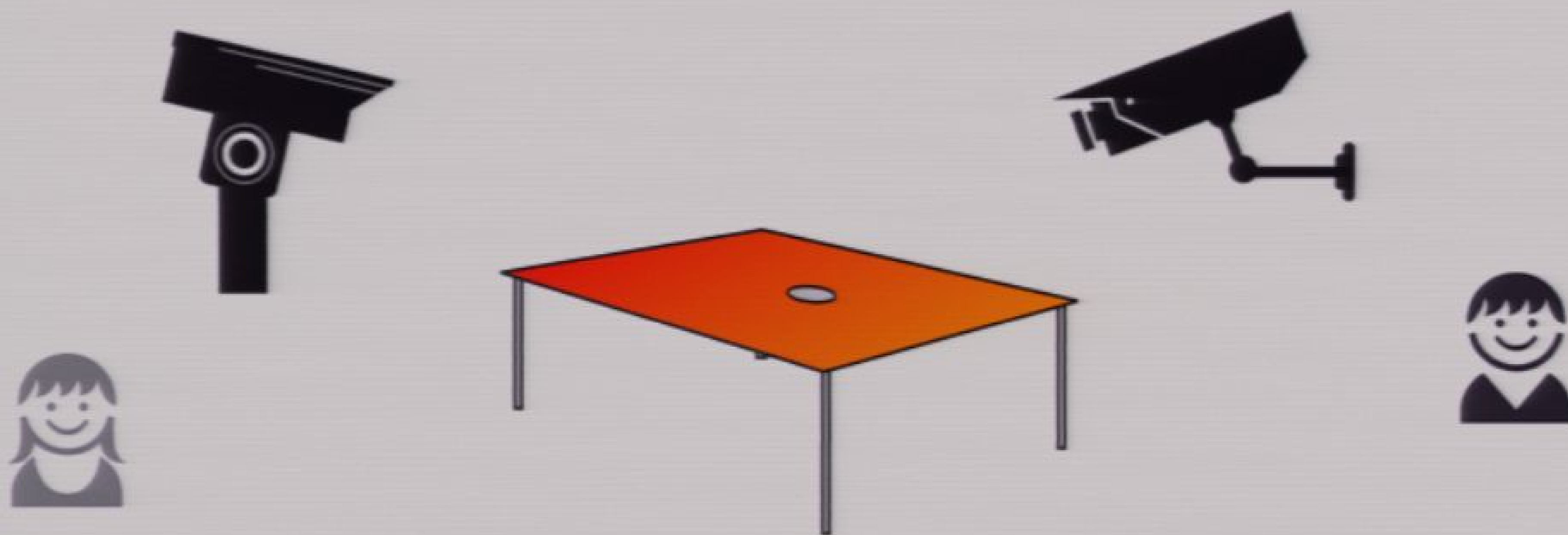
- Einstein, Podolski and Rosen: sufficient criterion for elements of reality

> *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

- A state ρ encodes objective information about the test $\{\mathscr{A}_i\}$ if
  - The test is repeatable $\mathscr{A}_i\mathscr{A}_j = \delta_{ij}\mathscr{A}_i$
  - The test does not disturb the state $\mathscr{A}\rho = \rho$ $\quad \mathscr{A} := \sum_i \mathscr{A}_i$
- The objective information is complete if $\mathscr{A}_i\rho$ is pure for every i

Example

- Consider a tossed coin before the {heads, tails} test has been performed



- Information about the upper side of the coin is objective

Consequences of the definition

- A state carries objective information if and only if

$$\rho = \sum_i p_i \rho_i \qquad \mathscr{A}_i \rho_j = \delta_{ij} \rho_j$$

- A state carries complete objective information if and only if

$$\rho = \sum_i p_i \psi_i \qquad \mathscr{A}_i \psi_j = \delta_{ij} \psi_j$$

Local objective information

- Local state in the sense of N. L. W. E.: a bipartite state ρ s. t.
  - The state ρ encodes complete objective information about $\{\mathscr{A}_i\}$
  - The test $\{\mathscr{A}_i\}$ can be measured by a LOCC procedure
- Conditions for locality/non locality without entanglement?
  - Work in progress
- The notion of objective information can be used to define discord

The standard notion of discord

- Definition

$$\delta(S:\mathcal{A})_{\{\Pi_j^{\mathcal{A}}\}} = I(S:\mathcal{A}) - J(S:\mathcal{A})_{\{\Pi_j^{\mathcal{A}}\}}$$

$$J(S:\mathcal{A})_{\{\Pi_j^{\mathcal{A}}\}} = H(S) - H(S|\{\Pi_j^{\mathcal{A}}\})$$

- Problems in general theories
  - Entropy is not uniquely defined
  - Entropy does not enjoy the same properties as in CPT and QT

Objective information and discord

- Null discord states: system + pointer after a measurement interaction
  - Complete objective information encoded in the pointer
- In causal theories

Definition 11 *In a causal operational probabilistic theory, a bipartite state ρ_{AB} has null discord if and only if it satisfies the following conditions*

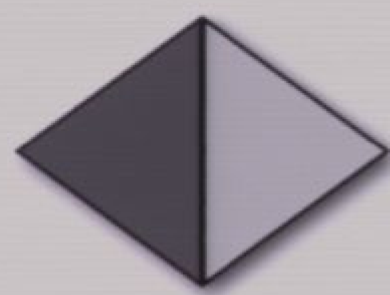
1. *ρ_{AB} is separable,*
2. *there exists a test $\{\mathscr{A}_k\}_{k\in X}$ on system A that provides complete objective information about the state ρ_A, and such that $\{\mathscr{A}_k \otimes \mathscr{I}\}_{k\in X}$ provides objective information on ρ_{AB}*

- Operational notion of discord

$$\mathscr{D}(\rho_{AB}) := \min_{\sigma\in\Omega_{AB}} \|\rho_{AB} - \sigma\|_{op}$$

Theorem

- Hypothesis: a state is separable if and only if it has null discord
- Thesis: the theory is simplicial



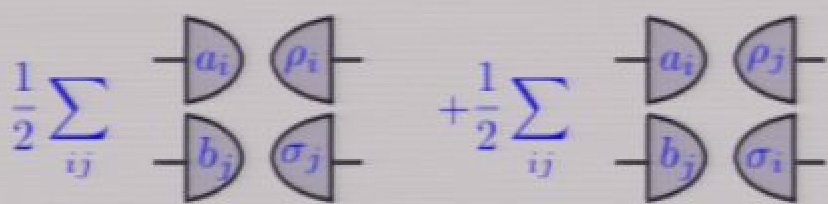
- Consequence: discord is the weakest signature of non-classicality
  - Shared by any theory

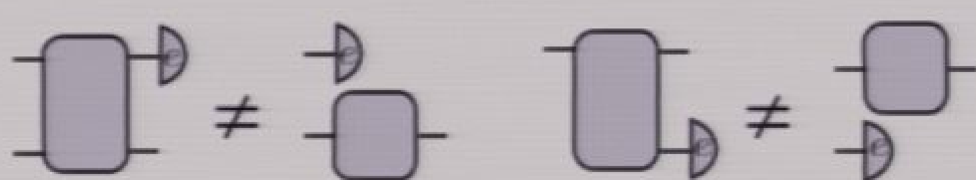
Locality and separability of channels

- For channels separability is not a relevant criterion
  - There are non-separable localizable channels

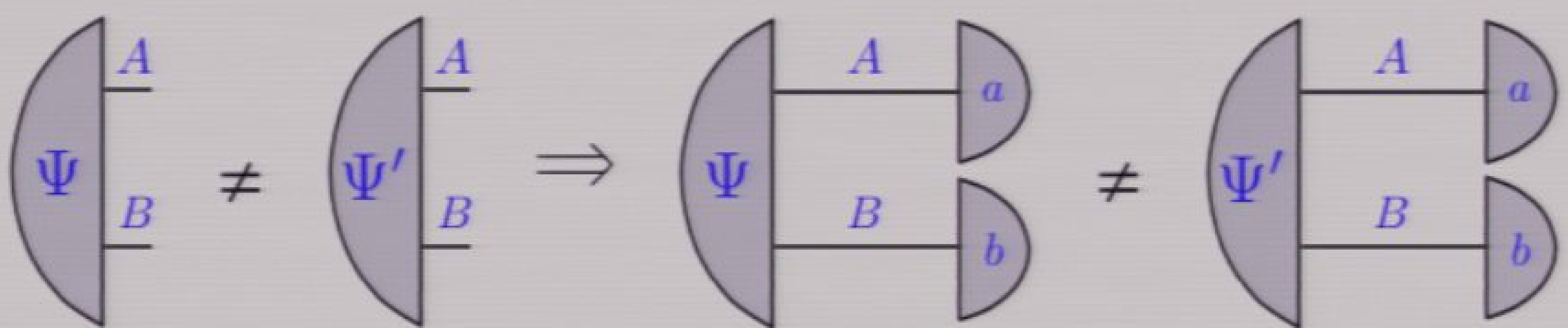
Example from quantum theory: PR box

  - There are separable channels that are signalling





Local discriminability



A property of channels in causal theories with l. d.

- In a causal theory a physical transformation is a channel if and only if

$$e_{\mathrm{B}} * \mathscr{C} = e_{\mathrm{A}}$$

- Every T satisfying the same condition can be decomposed as follows

$$\mathscr{T} \in \mathfrak{T}_{\mathbb{R}}(\mathrm{A}, \mathrm{B}), \quad e_{\mathrm{B}} * \mathscr{T} = e_{\mathrm{A}}$$

$$\Downarrow$$

$$\mathscr{T} = a\mathscr{T}_{+} - b\mathscr{T}_{-} \qquad \mathscr{T}_{\pm} \in \mathfrak{T}(\mathrm{A}, \mathrm{B})$$

$$e_{\mathrm{B}} * \mathscr{T}_{\pm} = e_{\mathrm{A}}$$

Linear span of local boxes

- Bipartite channels in causal theories with local discriminability

$$\mathscr{T} = \sum_i \mathscr{A}_i \otimes \mathscr{B}_i$$

- The transformations $\{\mathscr{A}_i\}, \{\mathscr{B}_i\}$ can be taken to be linearly independent

- Non-signalling implies $\quad e * \mathscr{A}_i = \lambda_i e \qquad e * \mathscr{B}_i = \mu_i e$

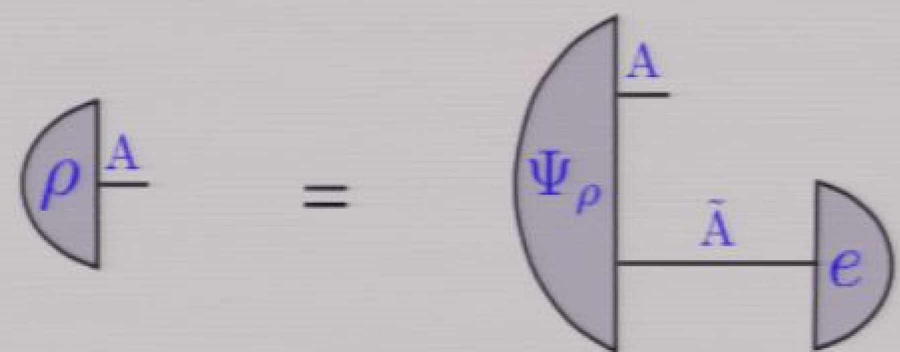
$$\mathscr{A}_i = a_i^+ \mathscr{A}_i^+ - a_i^- \mathscr{A}_i^- \qquad \mathscr{B}_i = b_i^+ \mathscr{B}_i^+ - b_i^- \mathscr{B}_i^-$$

- The span of local channels contains non-signalling channels

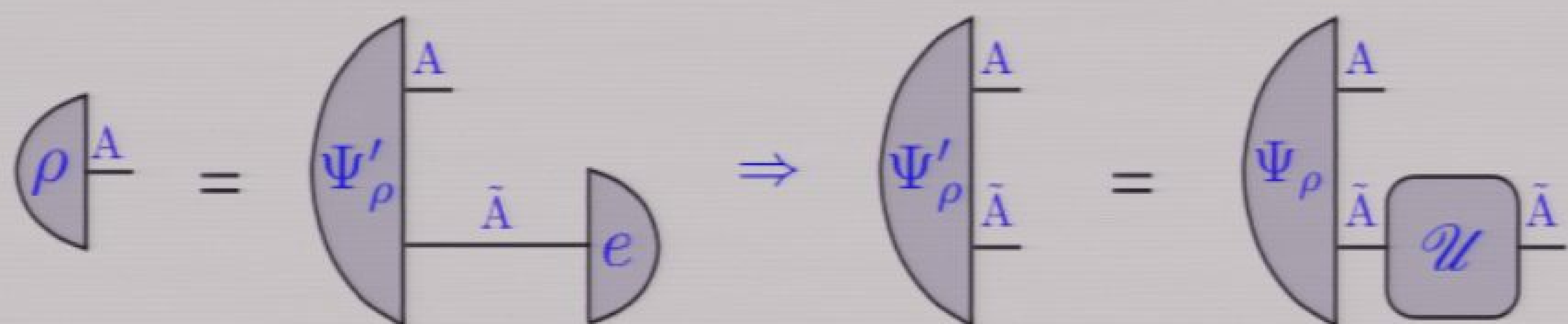
$$\mathscr{T} = \sum_j \mathscr{C}_j \otimes \mathscr{D}_j - \sum_k \mathscr{C}'_k \otimes \mathscr{D}'_k$$

Purification

- For any state ρ there exists a purifying system Ã such that



- The purification is unique up to reversible transformations



Choi correspondence

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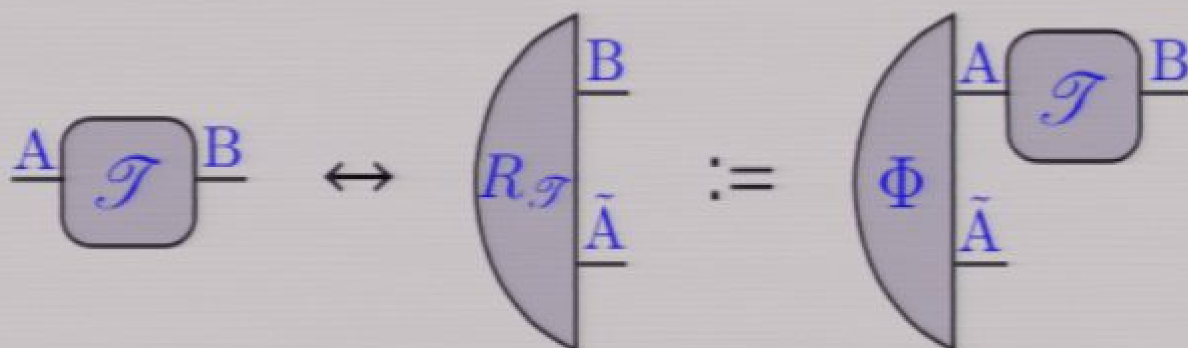
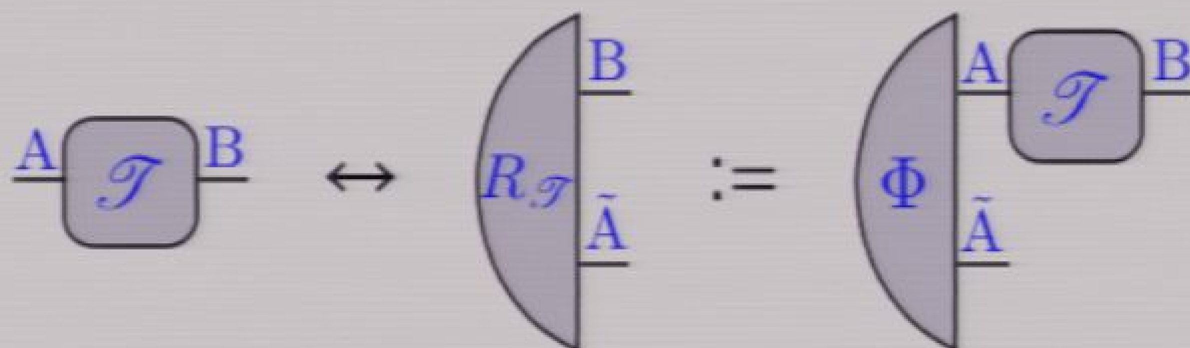


Choi correspondence

- Correspondence between bipartite states BÃ and transformations A→B
  - Deterministic transformations are in correspondence with some states



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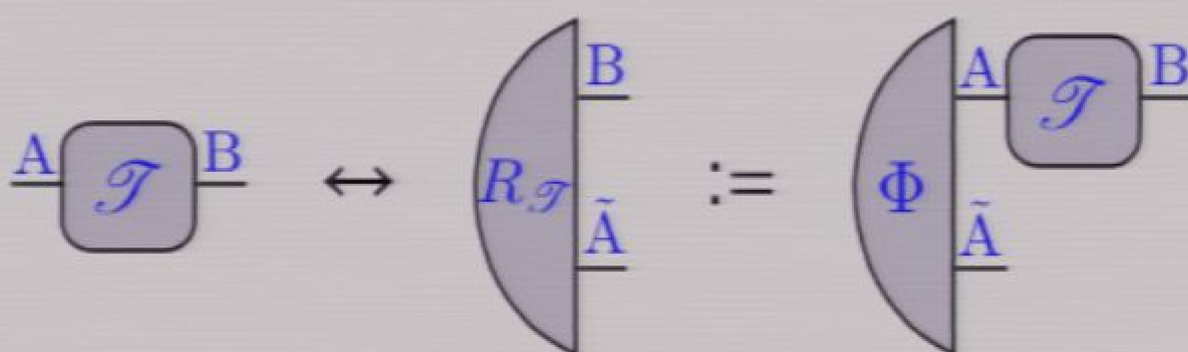
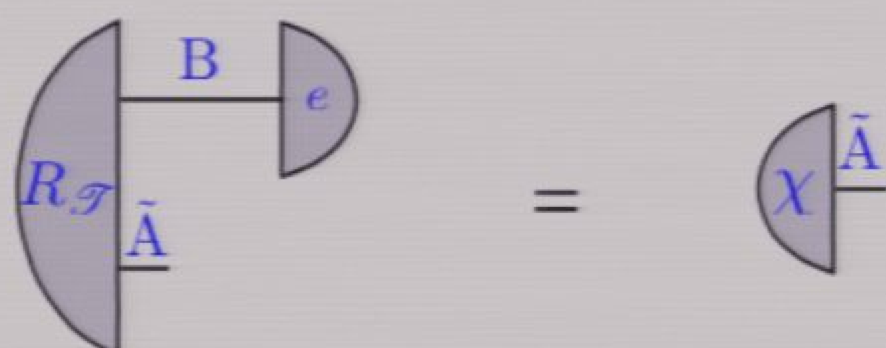
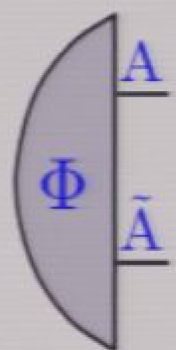




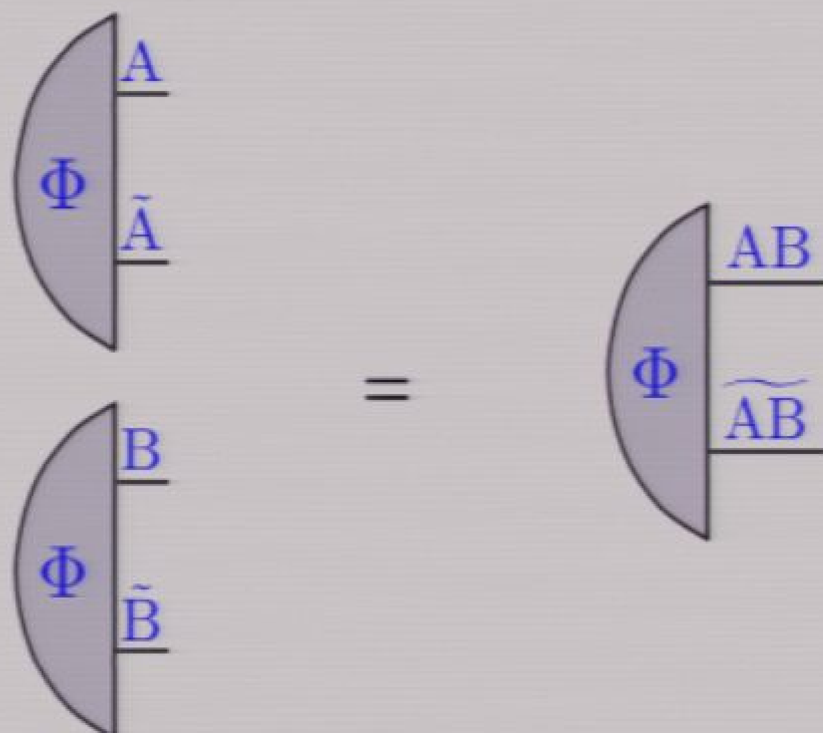
Faithful states

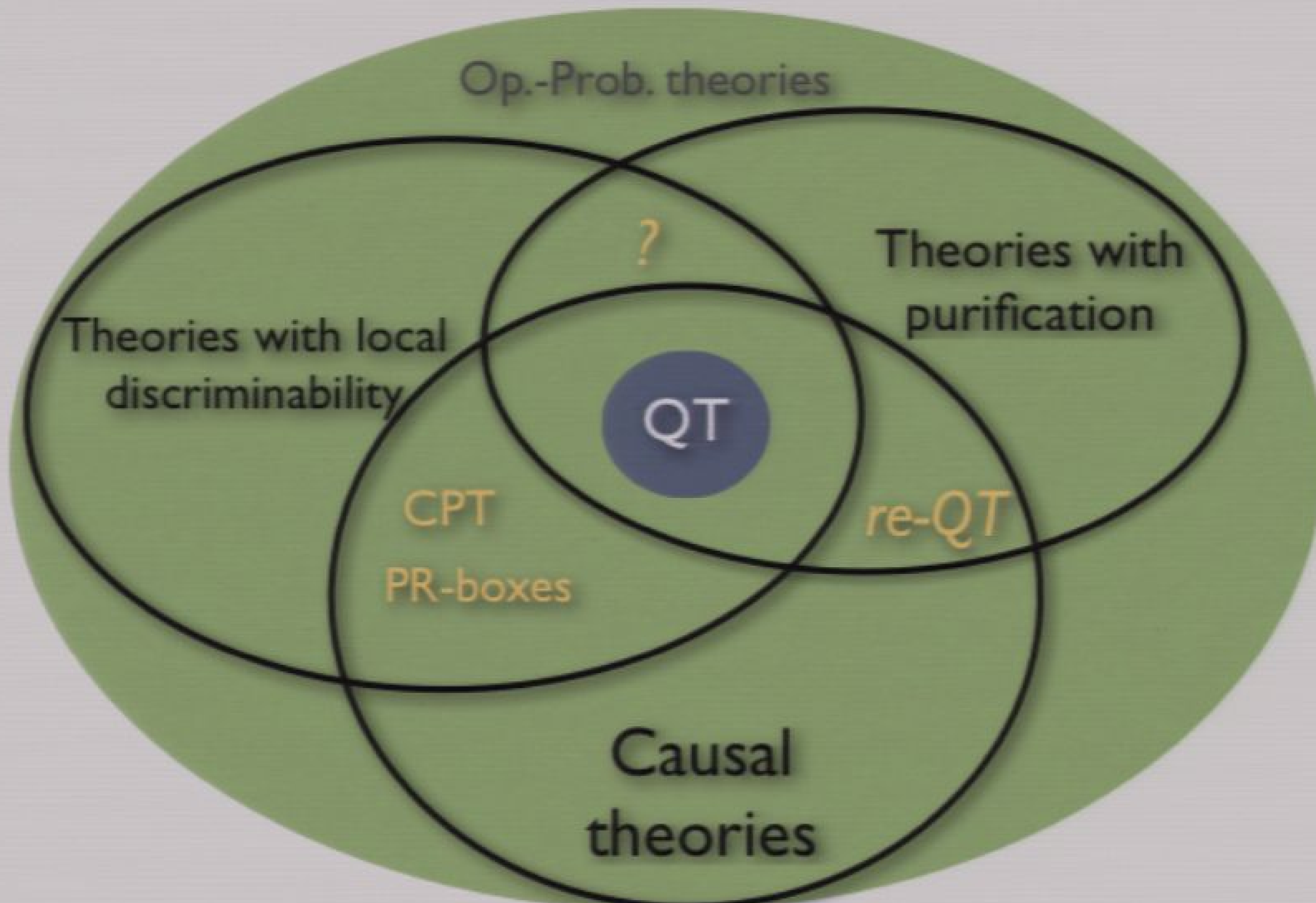
- The Choi correspondence holds through a **faithful** state



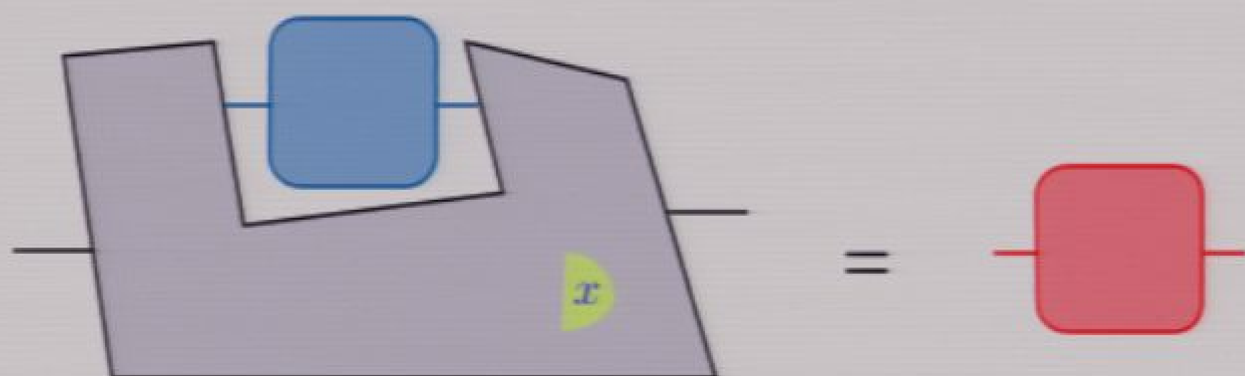
Faithful states

- The Choi correspondence holds through a faithful state
  - The composition of two faithful states is faithful




Op.-Prob. theories
?
Theories with purification
Theories with local discriminability
QT
CPT
re-QT
PR-boxes
Causal theories

Maps on transformations: supermaps



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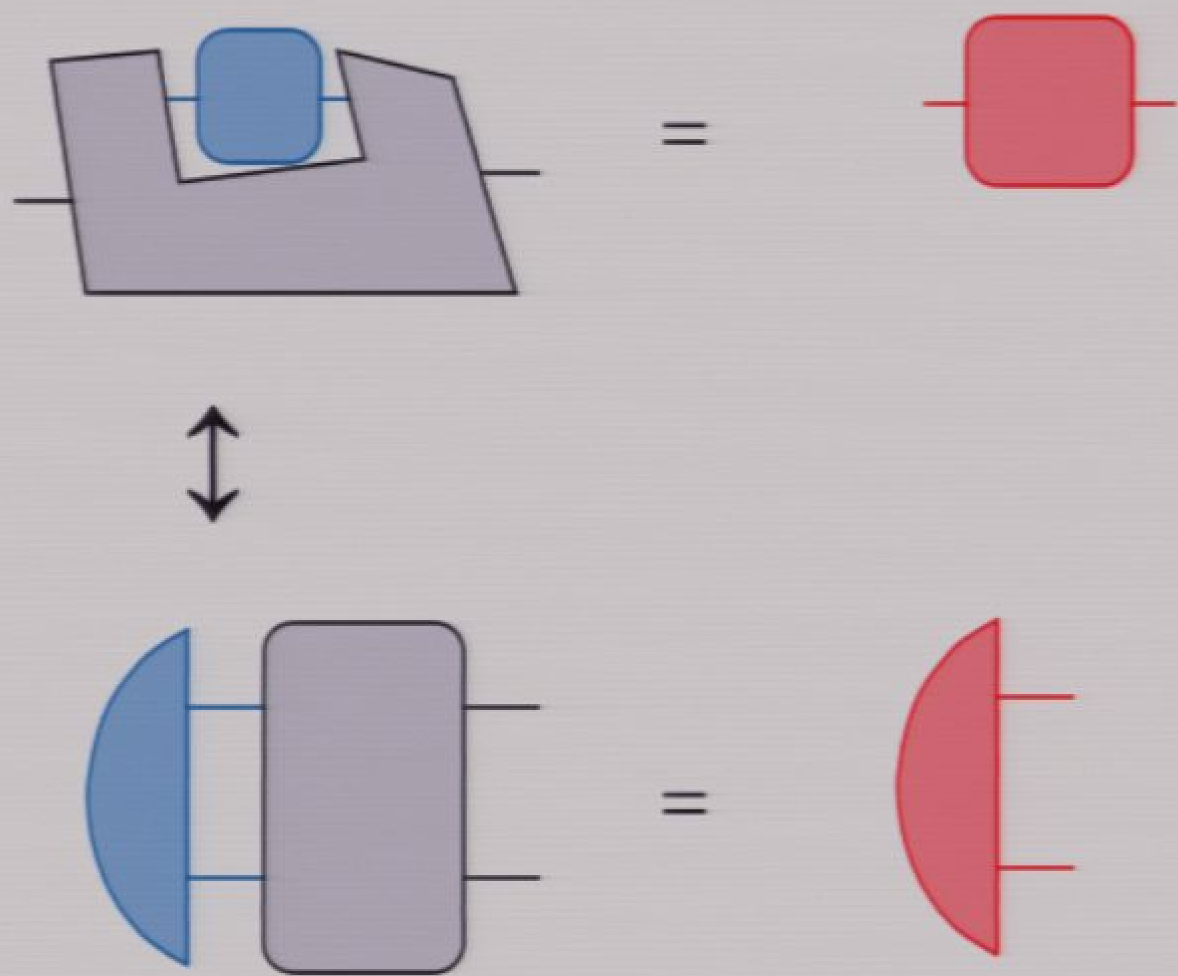

Maps on transformations: supermaps

Admissibility conditions

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Admissibility conditions

- Linear → preservation of convex combinations (probabilities)

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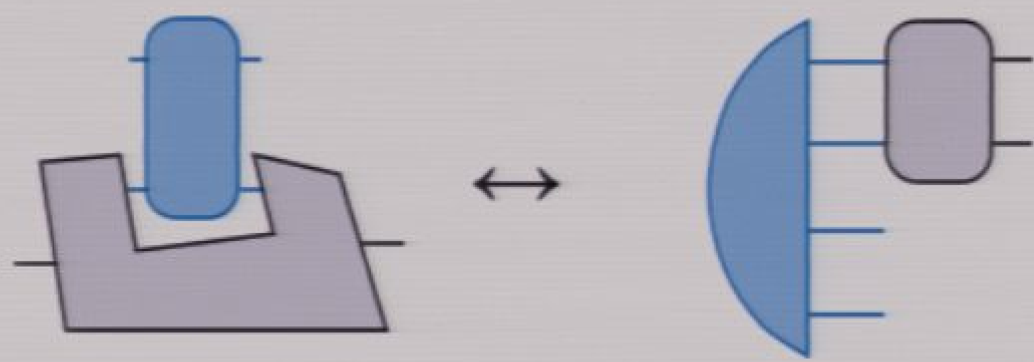

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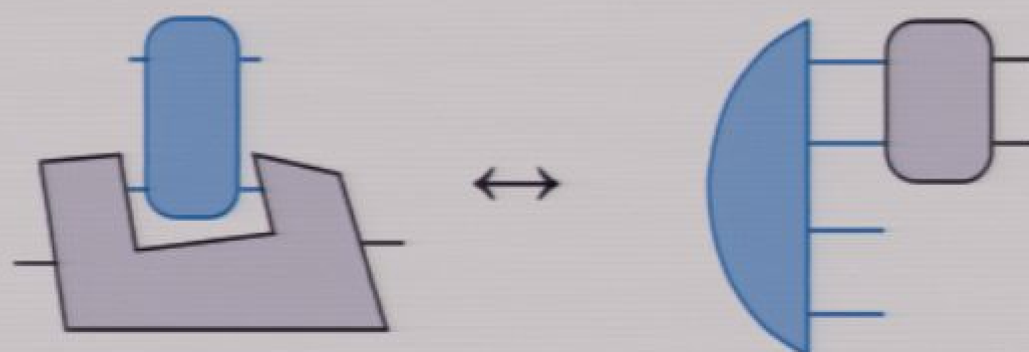
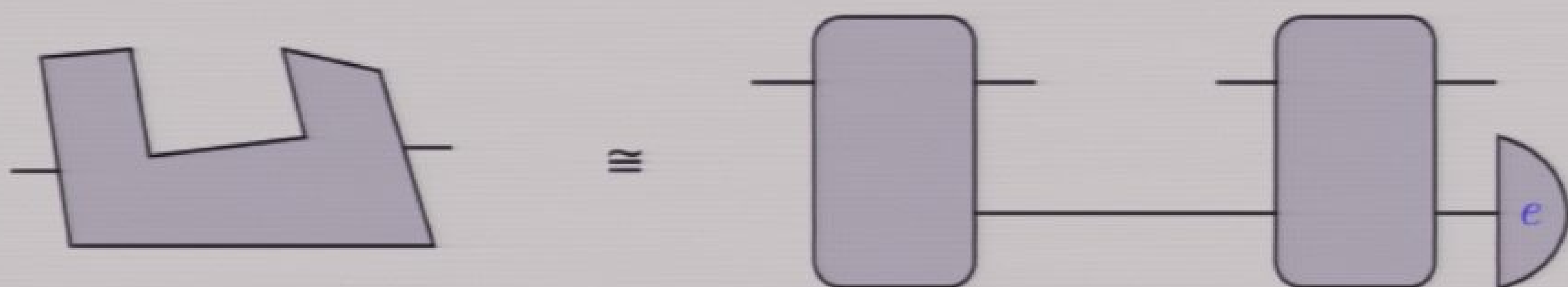


- Deterministic → preservation of normalisation

Realisation theorem

Admissibility conditions

⇕



Realisation theorem

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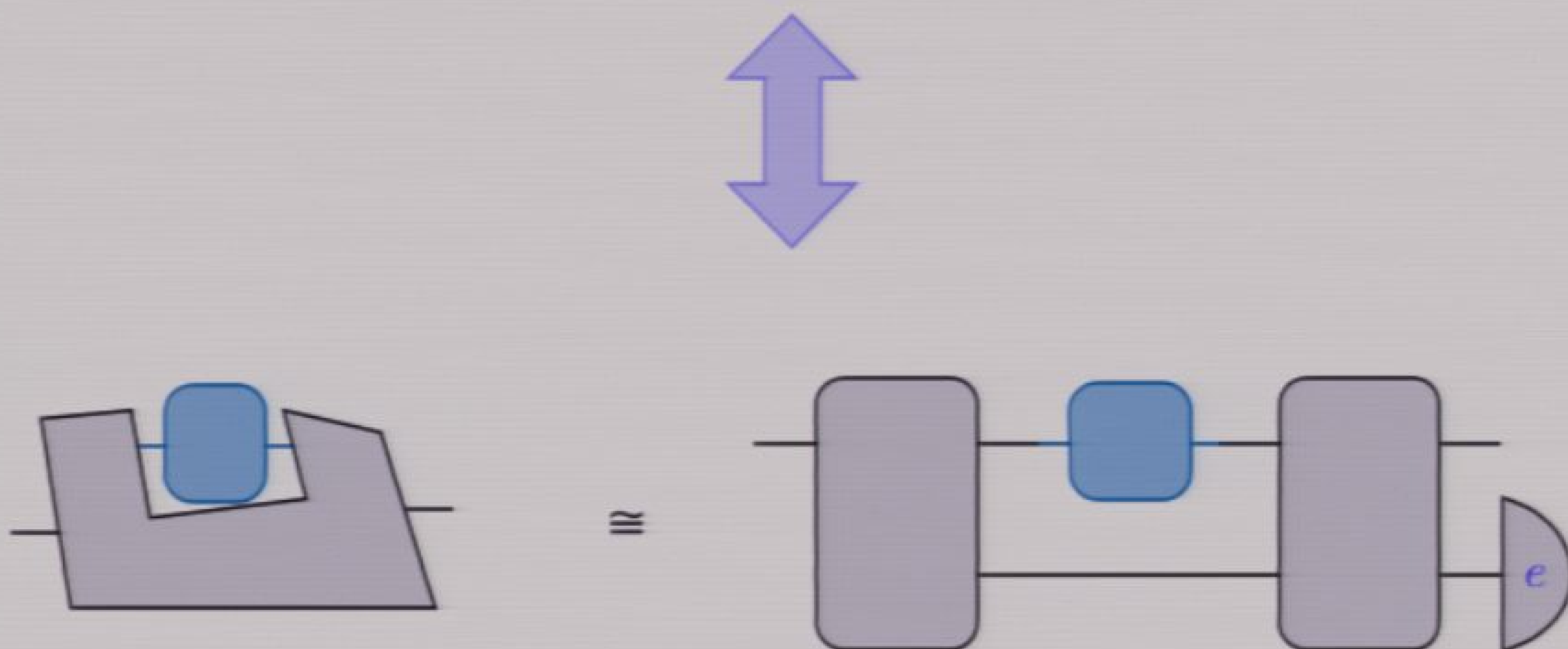

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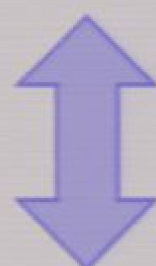

Testers

- A measurement on a transformation provides probabilities at the output
- A probability is a transformation of the trivial system I
- Realisation theorem: collection of supermaps with the following scheme

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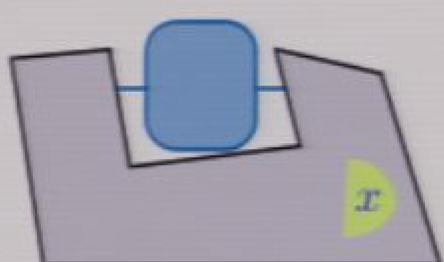

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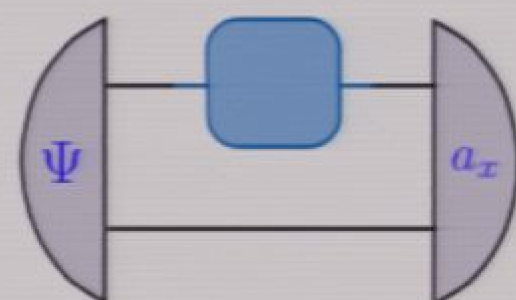
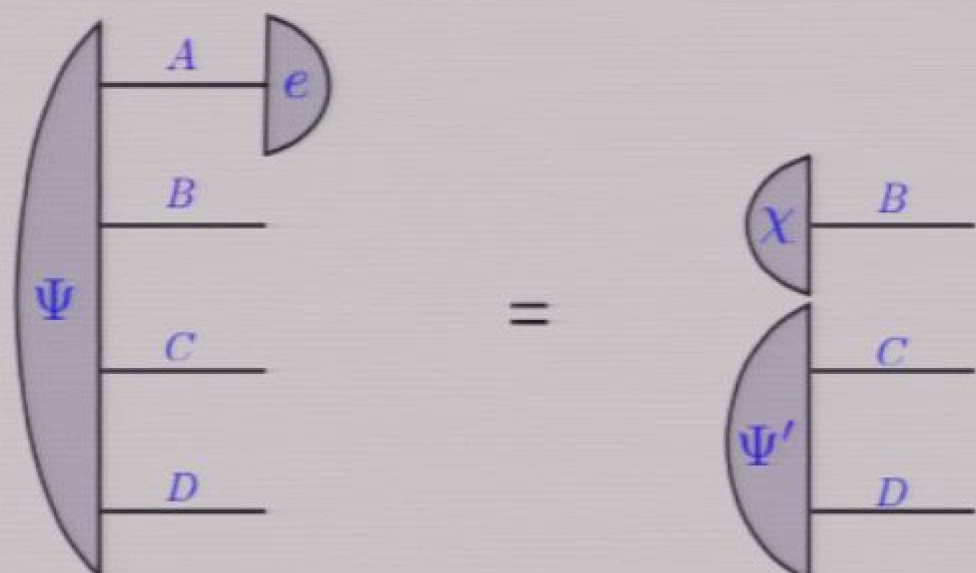


Supermaps and states

- Supermaps are in correspondence with states
  - Deterministic supermaps are in correspondence with **some** states



  - The cones coincide

Second-order theory

Second-order theory

- System ty

Second-order theory

- System types are A→B

A→B → A B

- States of the system A→B are transformations from A to B

Second-order theory

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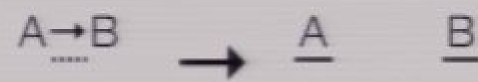

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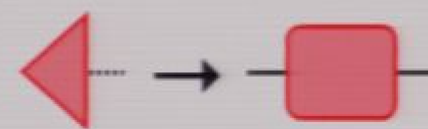

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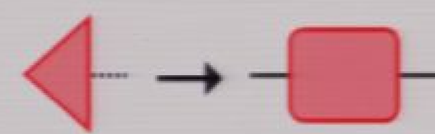
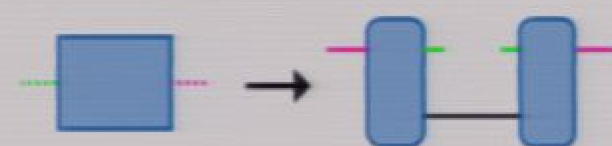


- Transformations (A→B)→(C→D) are supermaps



Second-order theory

- System types are A→B

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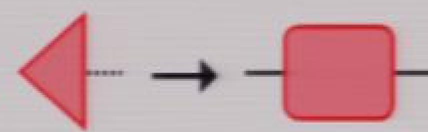

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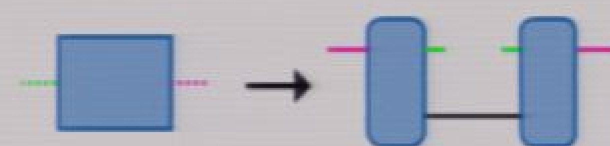
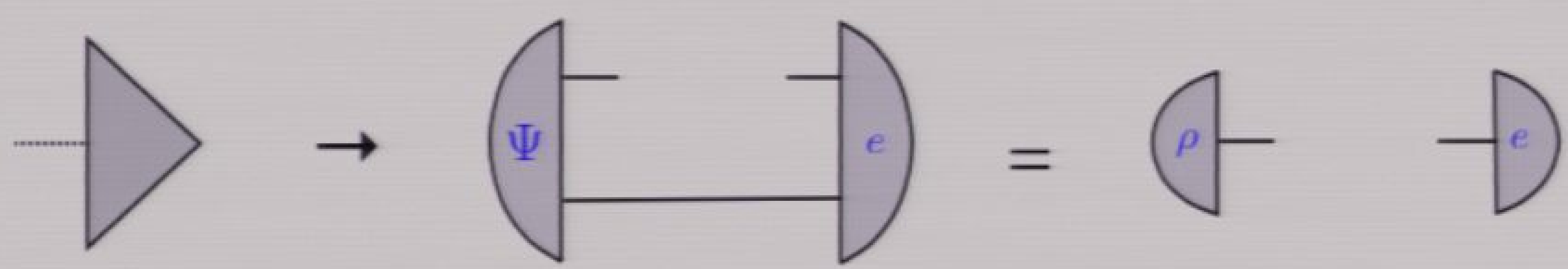


- Effects are testers

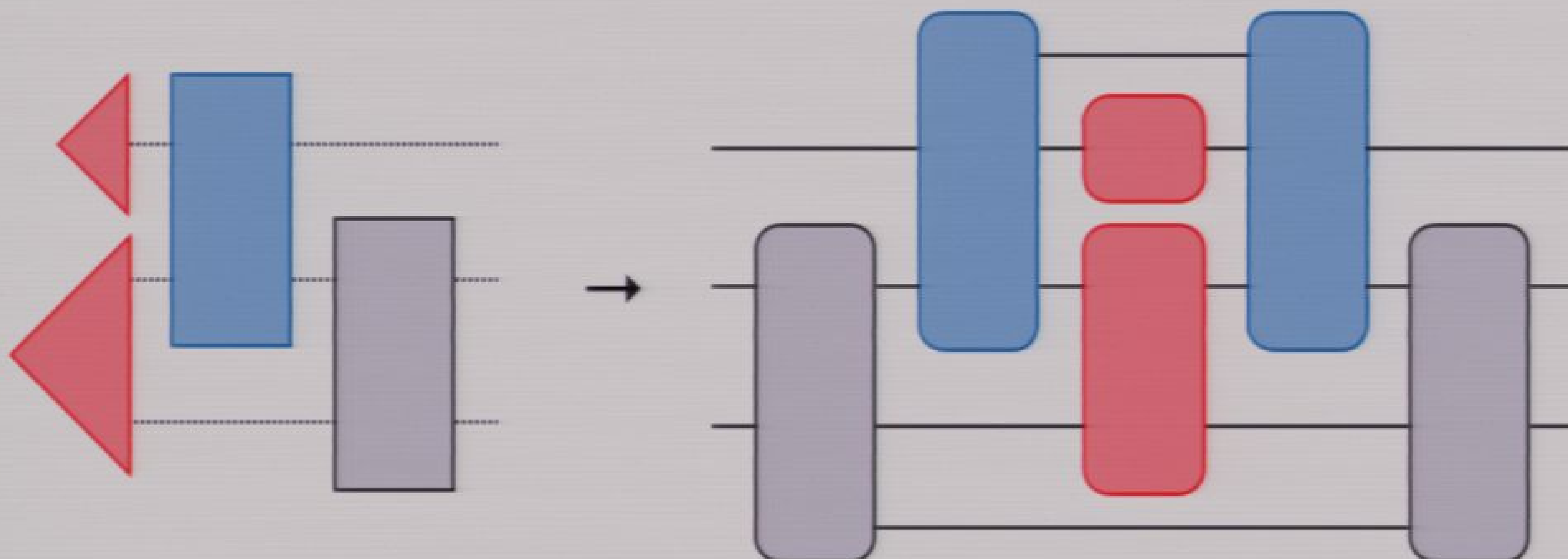
Second-order theory

- Deterministic effect ↔ deterministic tester
- The second order theory is non causal

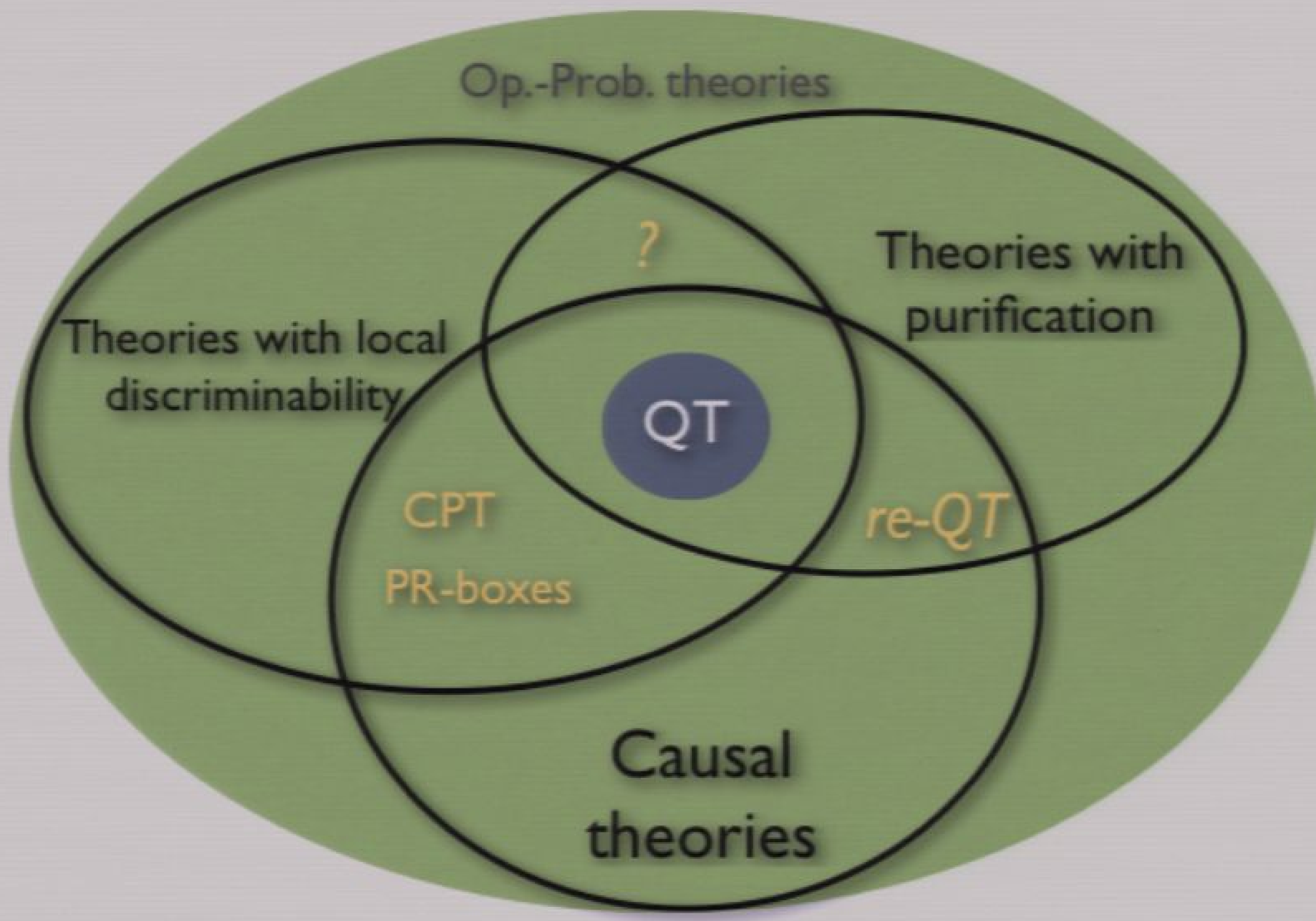


Second order theory

- Circuits of a second order theory are perfectly simulated in a causal theory



Are all non-causal theories of this kind?

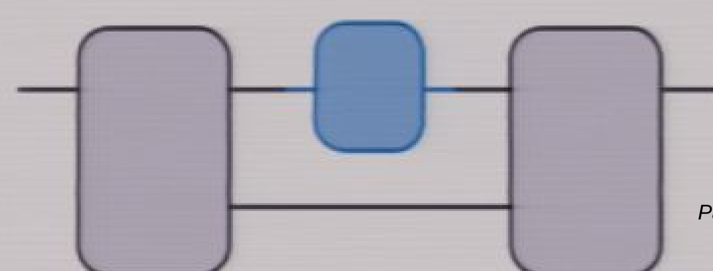


Op.-Prob. theories
?
Theories with purification
Theories with local discriminability
QT
CPT
re-QT
PR-boxes
Causal theories

The hierarchy of combs

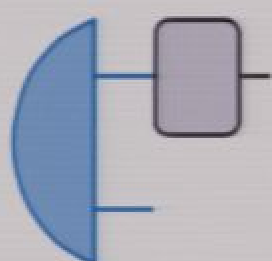
- Consider the following recursively defined hierarchy of transformations
  - **1-Combs:** transformations in a causal theory with purification
  - **N-Combs:** transformations from N-1-combs to 1-Combs

- Example: 2-Combs are supermaps



The hierarchy of combs

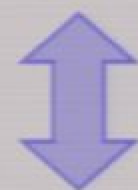
- 1-Combs are in correspondence with states
- If N-1-combs are in correspondence with states, N-combs are in correspondence with transformations, hence with states
- Admissibility conditions:
  - Linear
  - CP

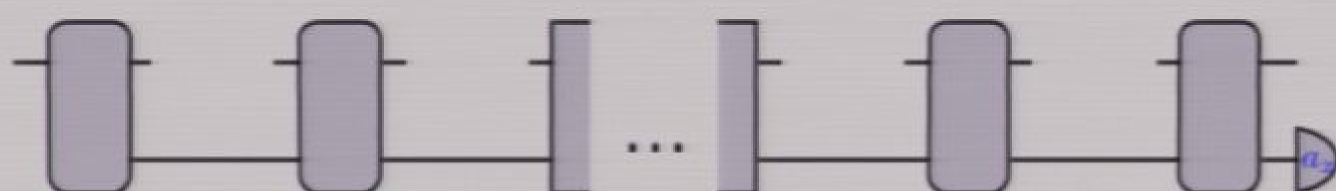


  - Deterministic → Deterministic comb mapped to Deterministic transformations

Realisation Theorem

Admissibility conditions





Realisation Theorem

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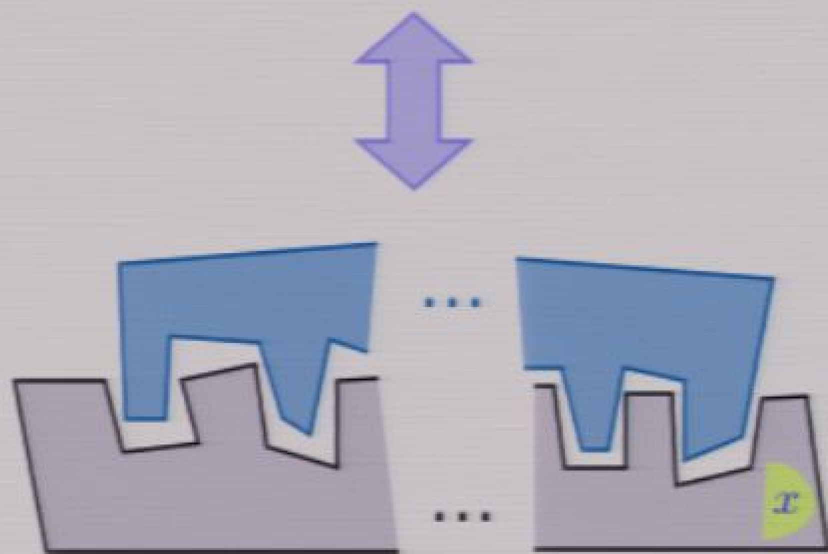
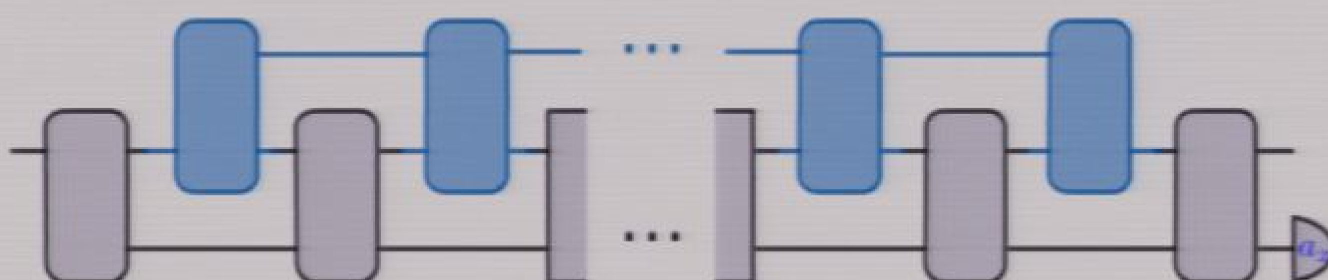




Higher-order maps

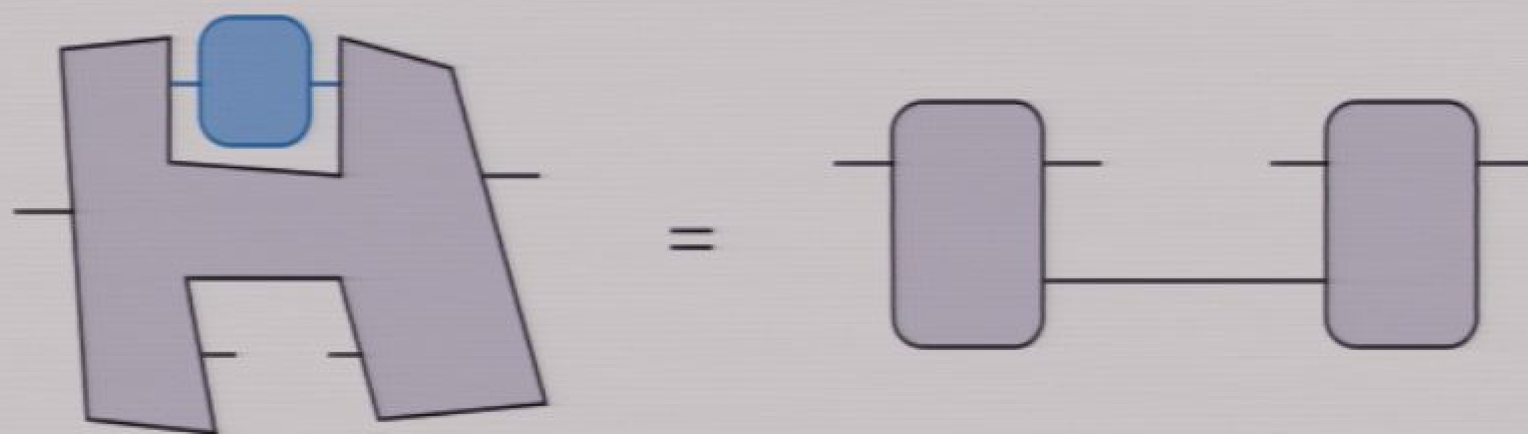
- We want to define maps g from N-combs {x} to M-combs $\{g_x\}$
    - g_x is a map from (M-1)-combs {y} to transformations
- We use an "Uncurrying" procedure

$$h(x, y) := g_x(y)$$

- A map from N-combs x to M-combs g_x is equivalent to a map from couples (x,y) to transformations (1-combs)

Example

- Maps from 1-combs to 2-combs are equivalently defined as maps from couples of transformations to transformations



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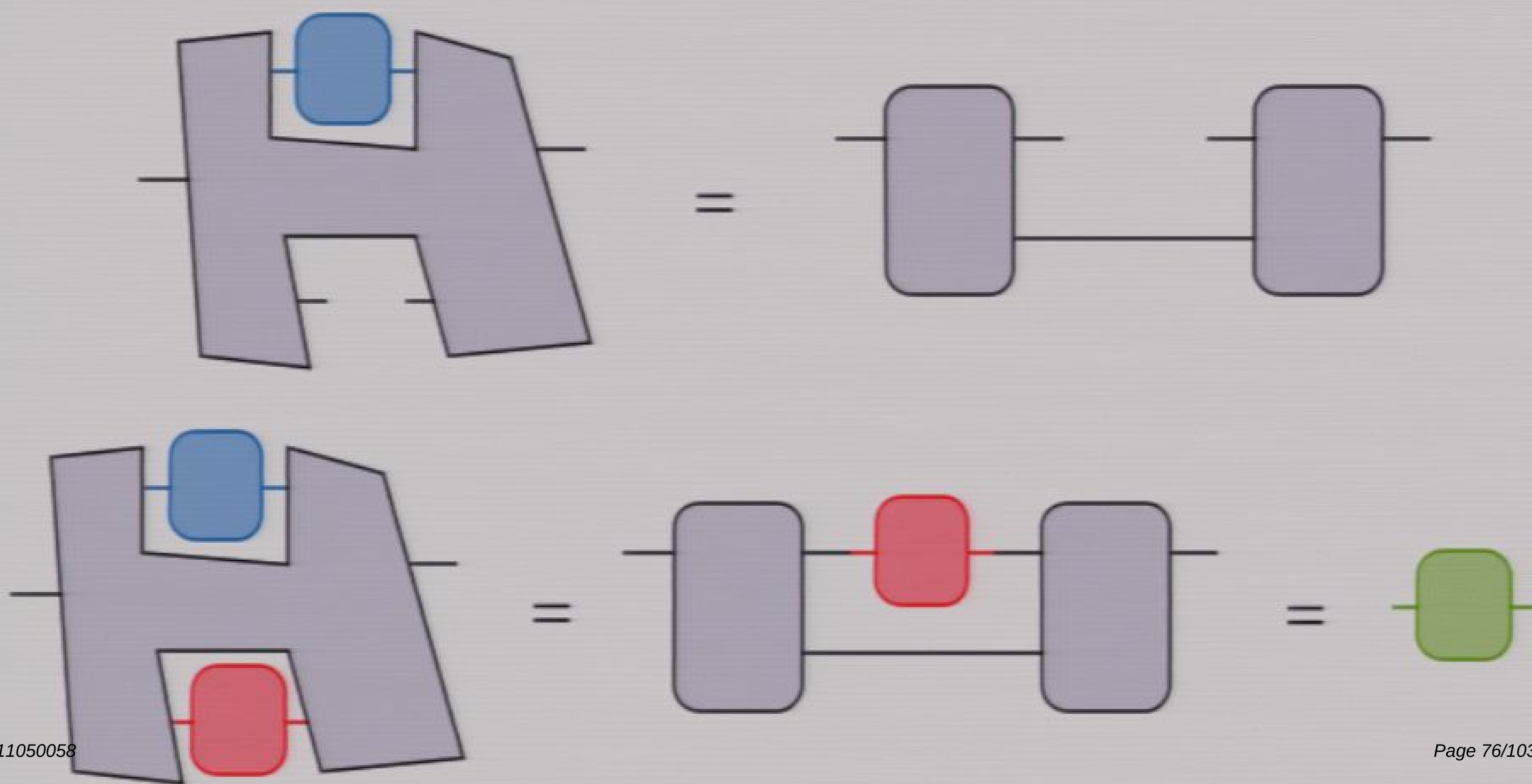
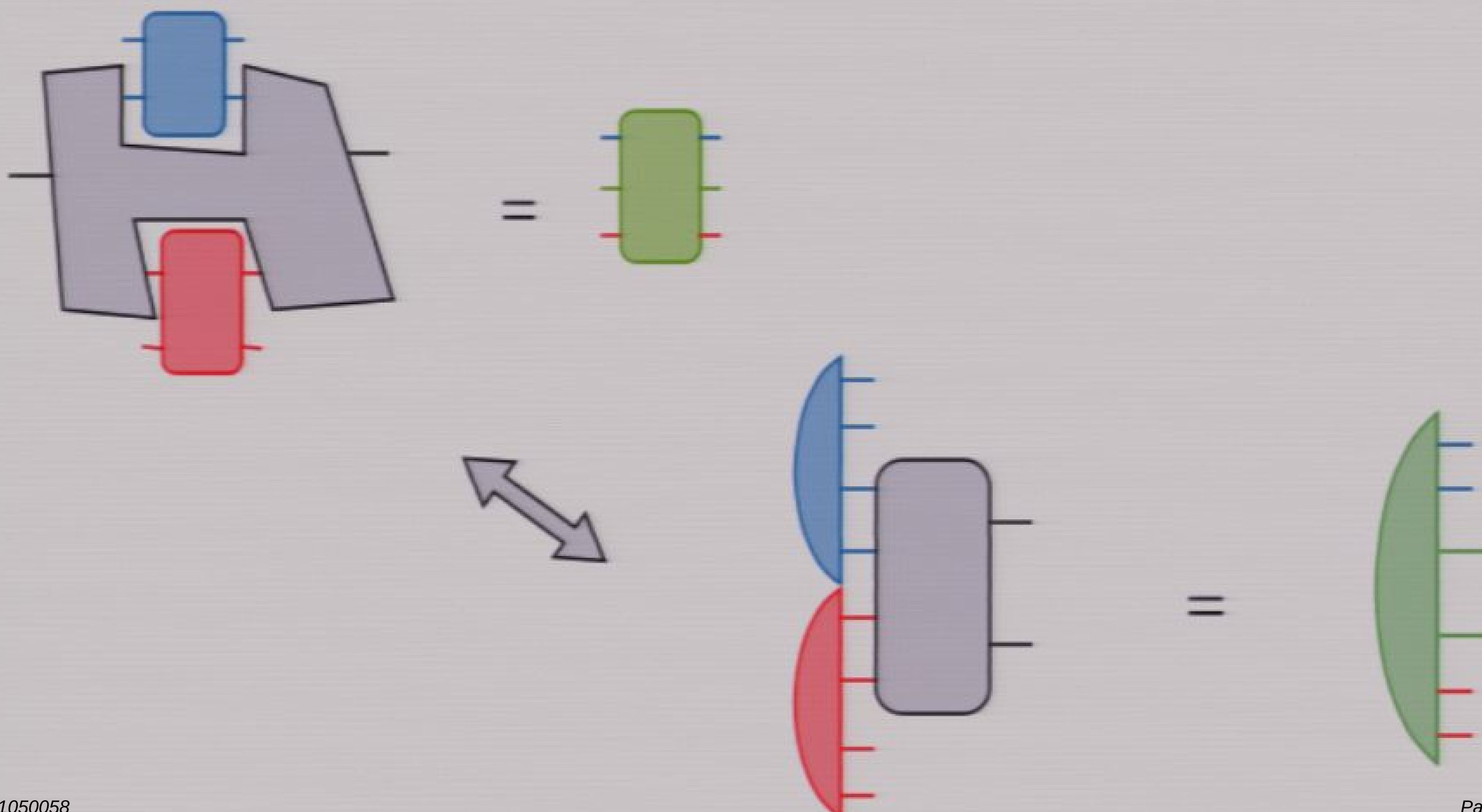


The type of 1→2 maps

- Admissibility conditions on the uncurried map
    - Linearity
    - Complete Positivity
    - Normalization
- Imposed on factorized transformations

Admissibility on non-signalling channels

- Complete positivity using Choi and parallel composition of faithful states

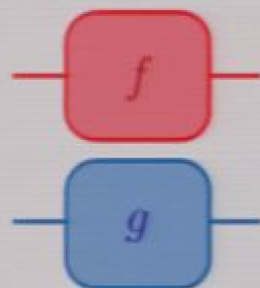


Admissibility on non-signalling channels

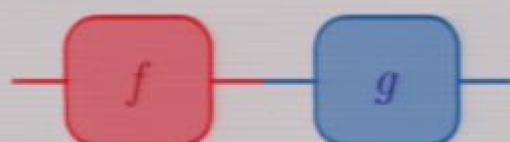
- Normalization is a linear constraint
- The non-signalling channels belong to the linear span of factorized channels
  - Admissible maps are normalized on non-signalling channels

The switch algorithm

- Input: x∈{0,1},



- If x=1, then do



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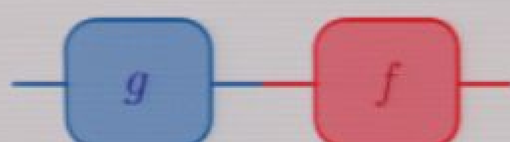
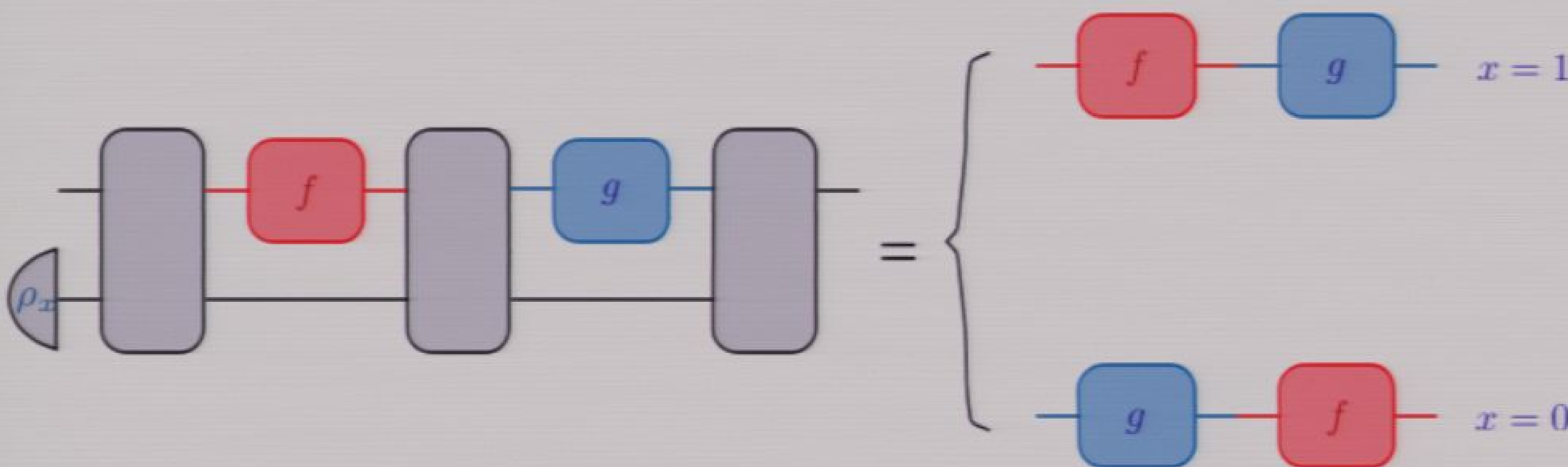
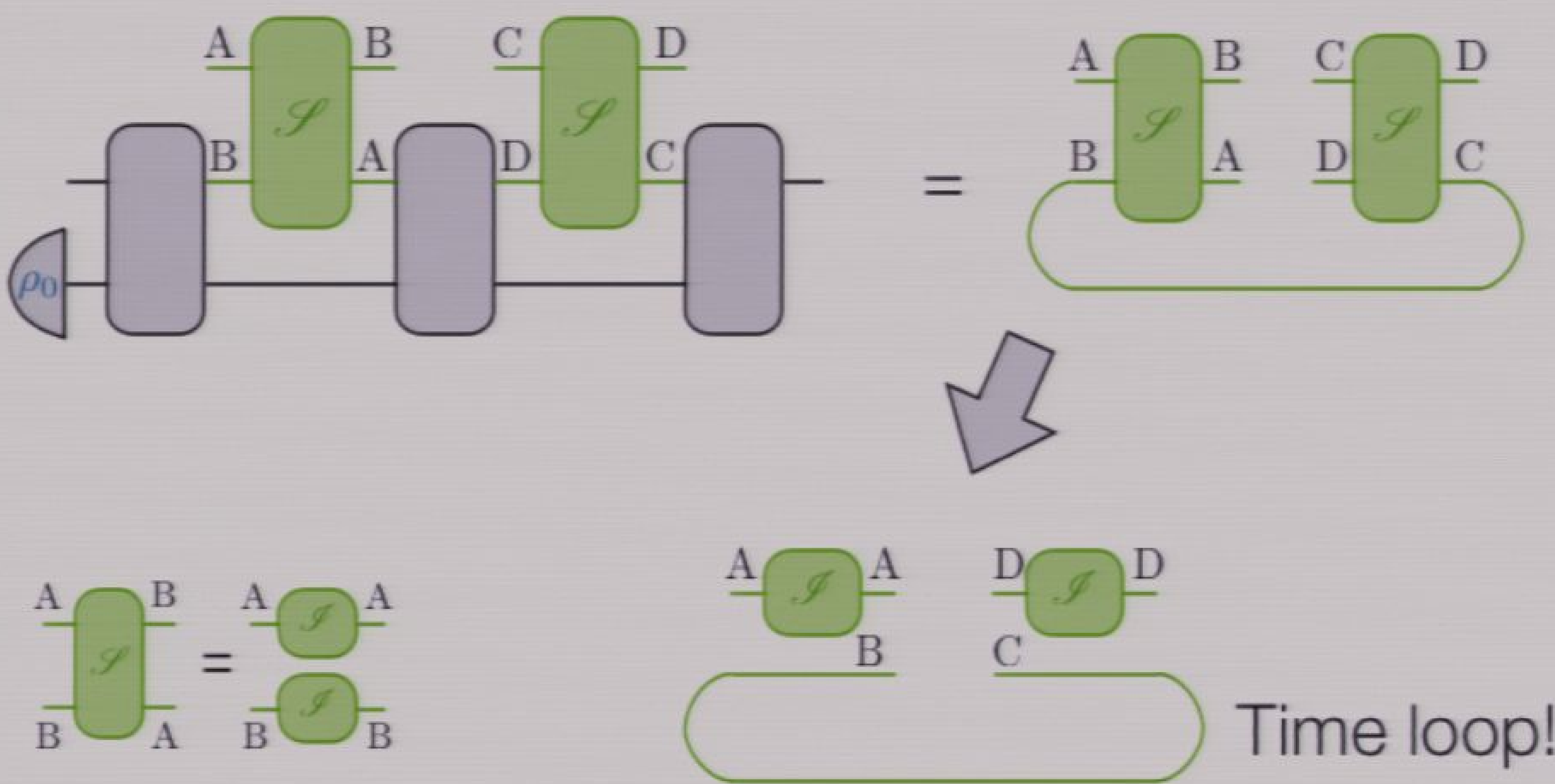


No-switch theorem

Suppose a circuit exists that performs the SWITCH

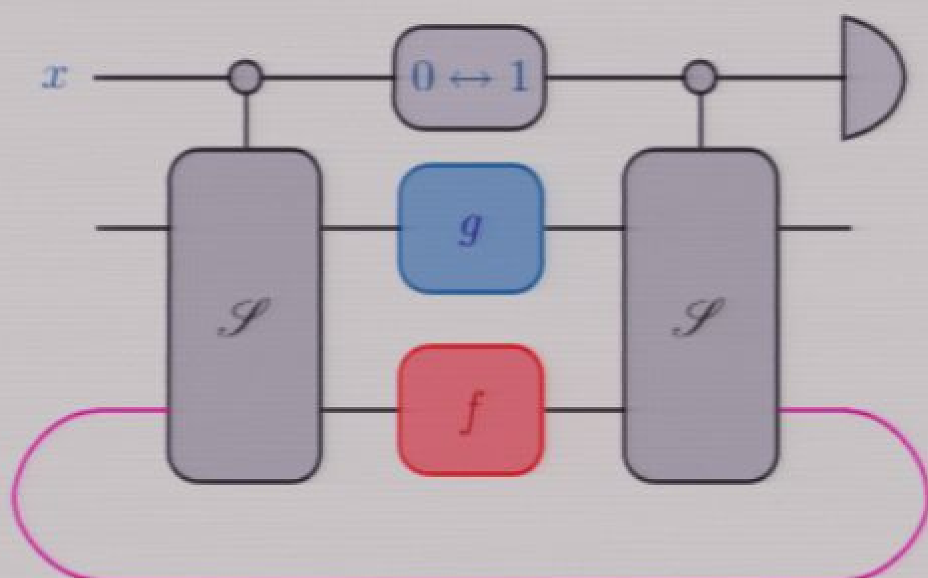


No-switch theorem



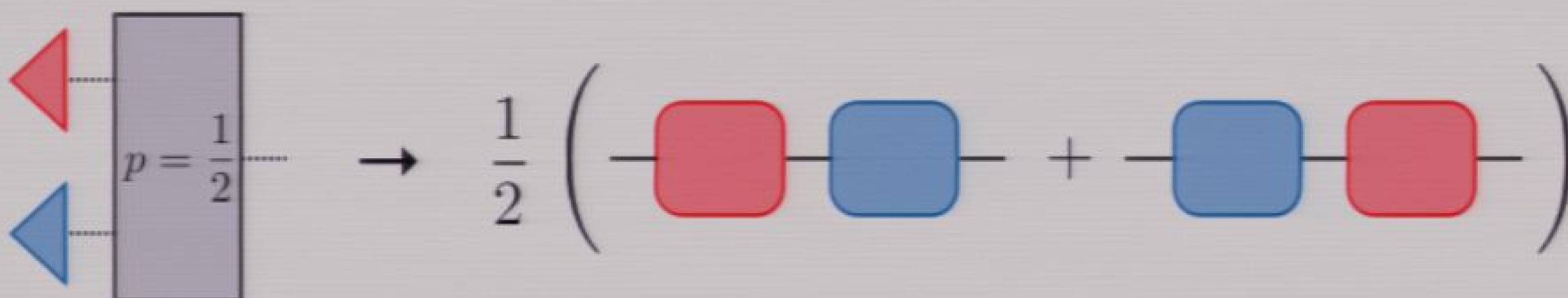
Equivalence of switch and time loops

- If we had access to a time loop we could make a circuit for the SWITCH



Higher order theory

- Higher order maps **are not** perfectly simulated in the underlying causal theory
- There exist non-circuital maps that are operationally well defined
- We lack an operational representation for convex combinations of circuits



- Analogously for superpositions
  - Is there a categorical solution to the problem?

Conjecture

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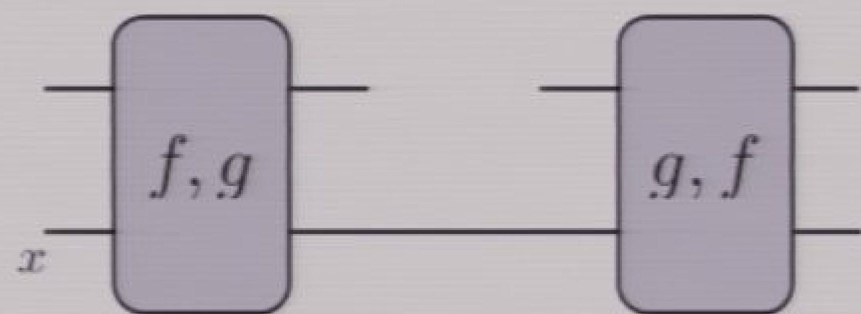

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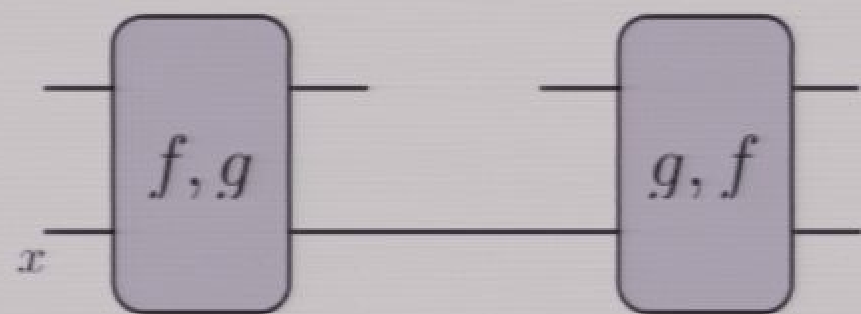

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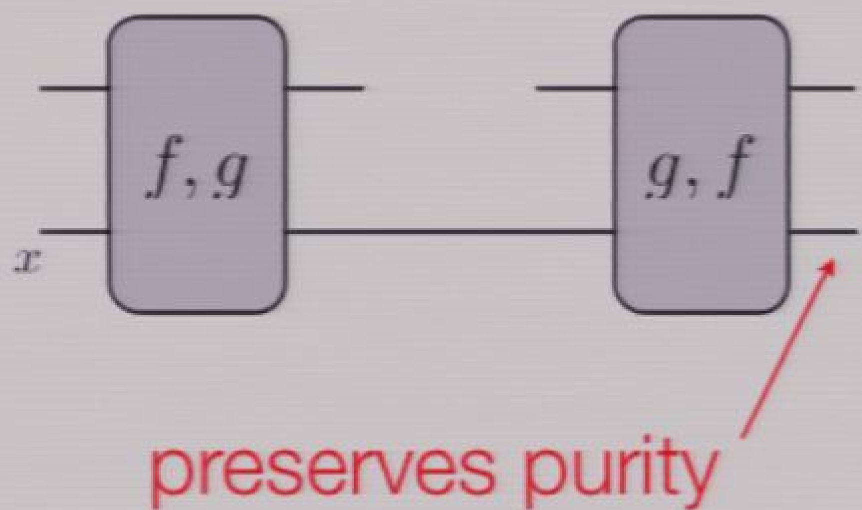

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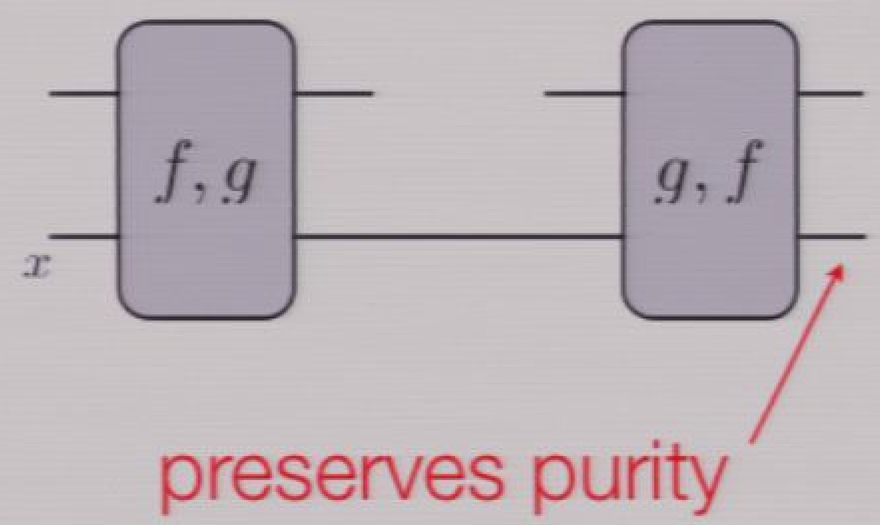

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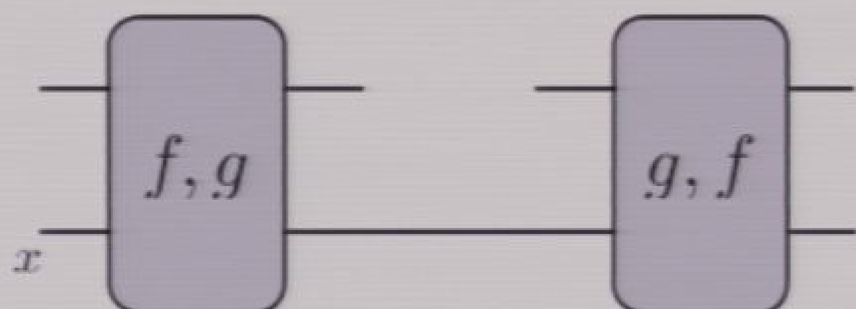

Can all non-causal maps be obtained by combs provided we allow for this special “oracle”?

Higher-order maps

- Higher-order maps are in correspondence with multipartite states
- The purifications of such states are still admissible higher-order maps
    - Higher-order maps are not only combs
    - Higher-order maps are not only convex combinations of combs having different causal structures
    - Are all admissible maps “operational”?

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Classification: open problem

Conclusion

- Locality, local objective information, discord
- Factorized and non-signalling channels
- Supermaps and combs: non-causal theories
- The switch algorithm and the universality conjecture
- Non-causal theories without an immediate causal representation

Conjecture

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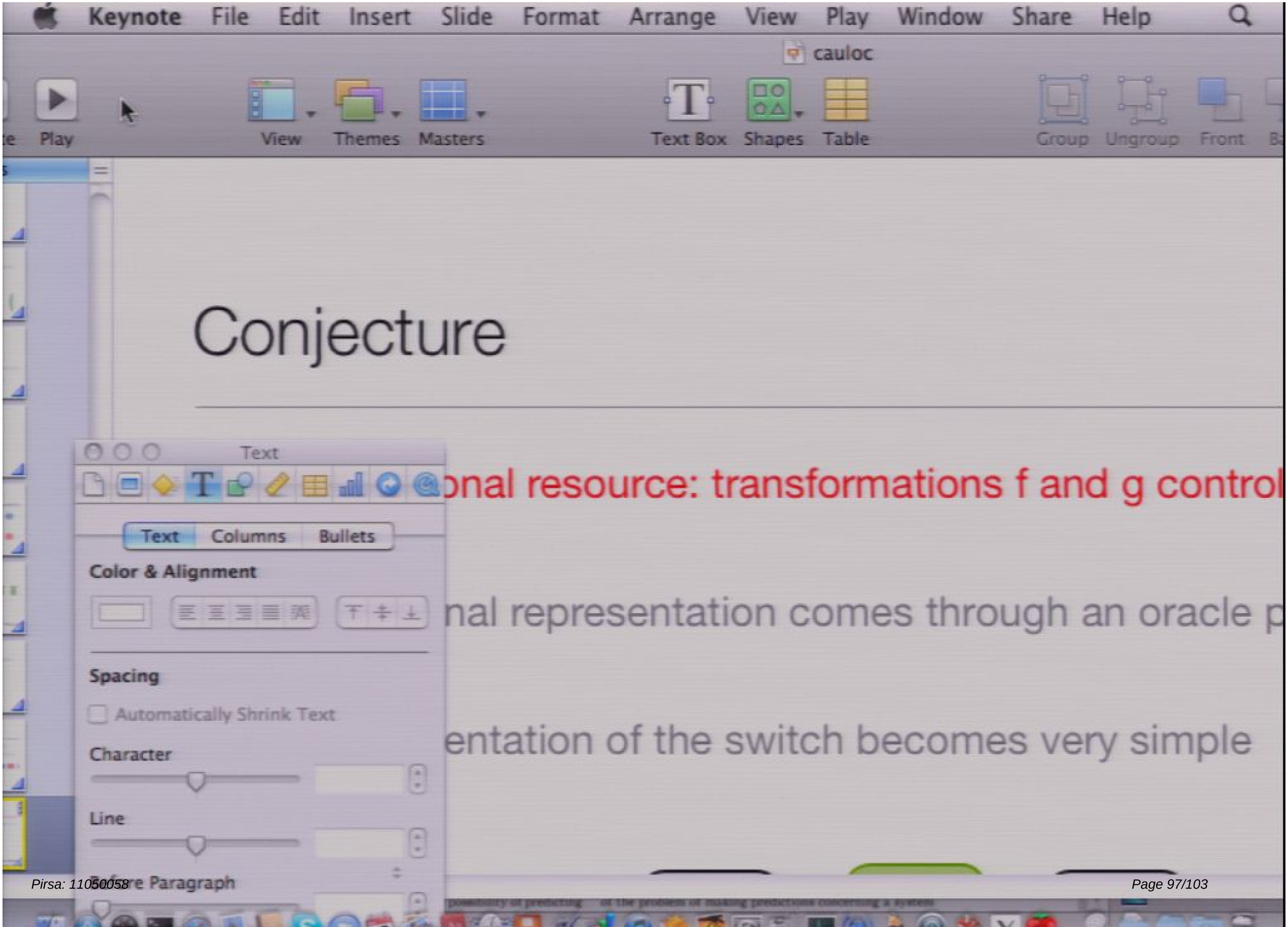
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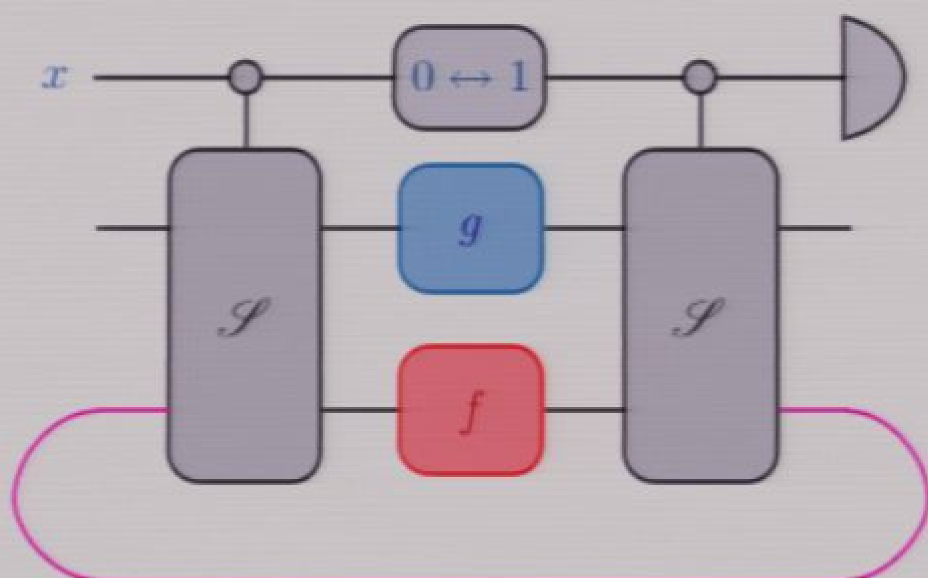

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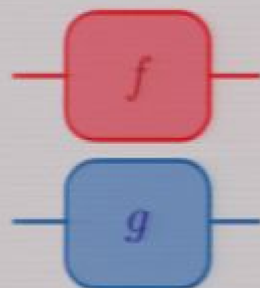

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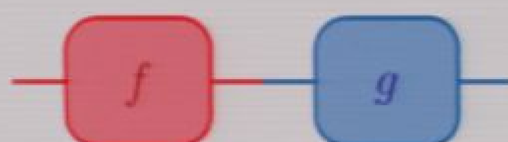

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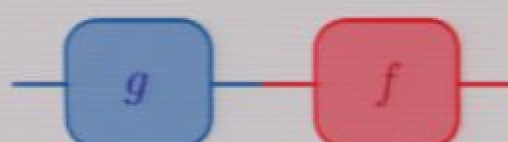

Second-order theory

- Deterministic effect ↔ deterministic tester
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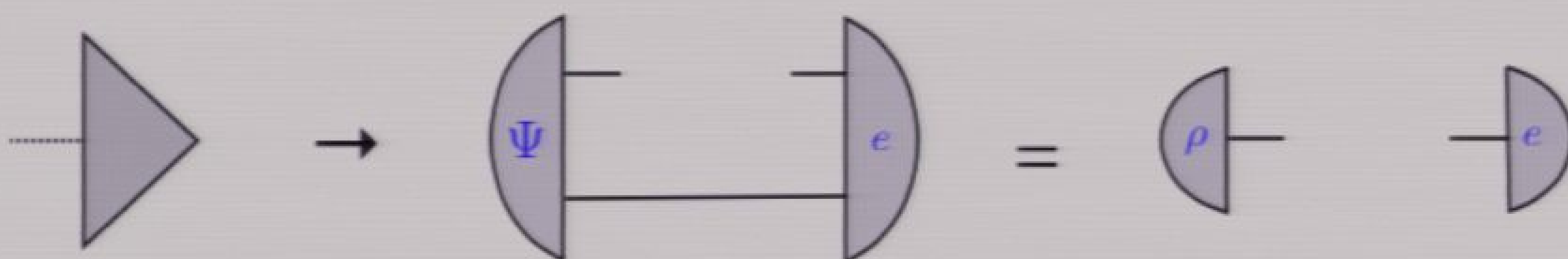

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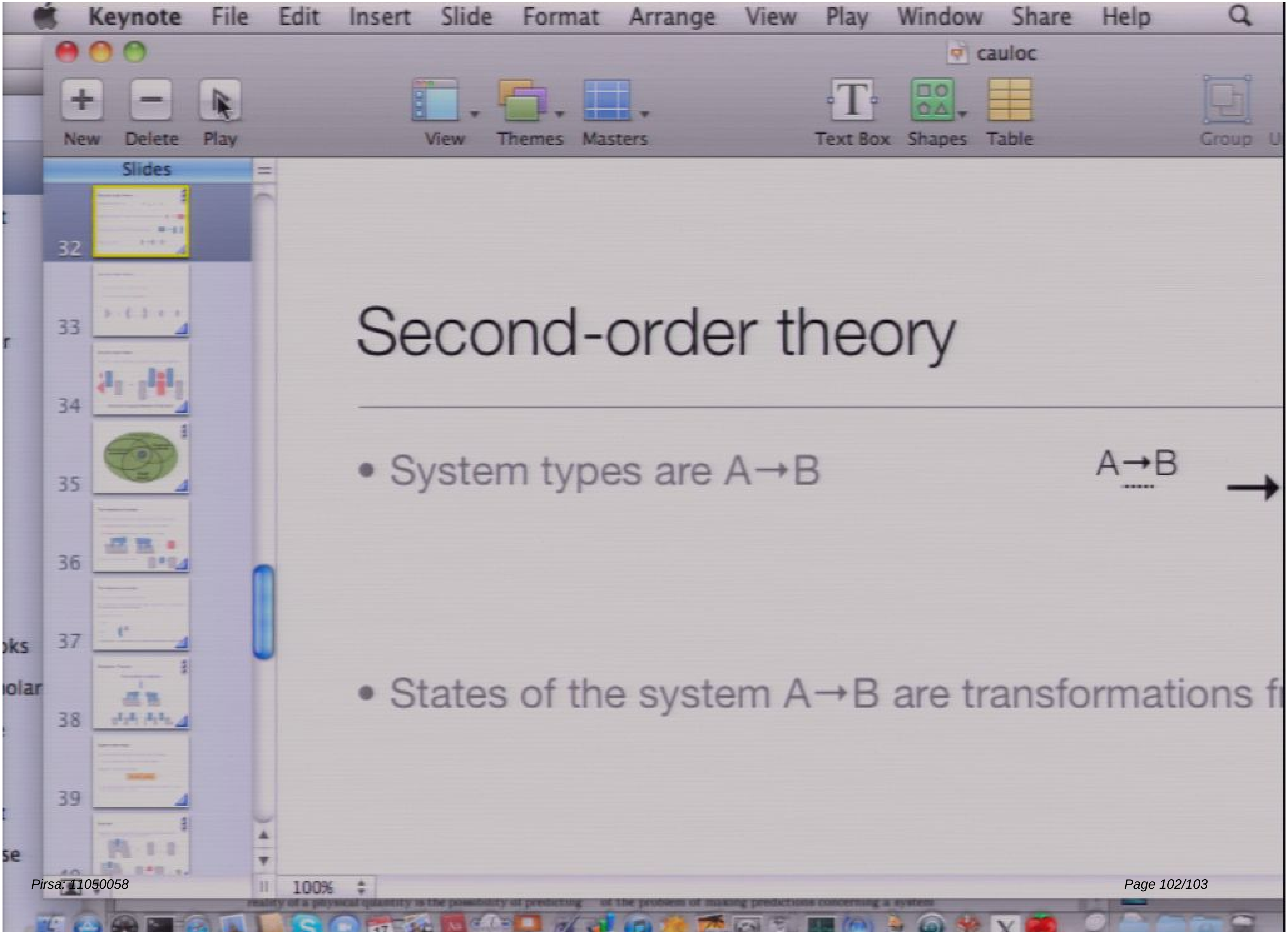
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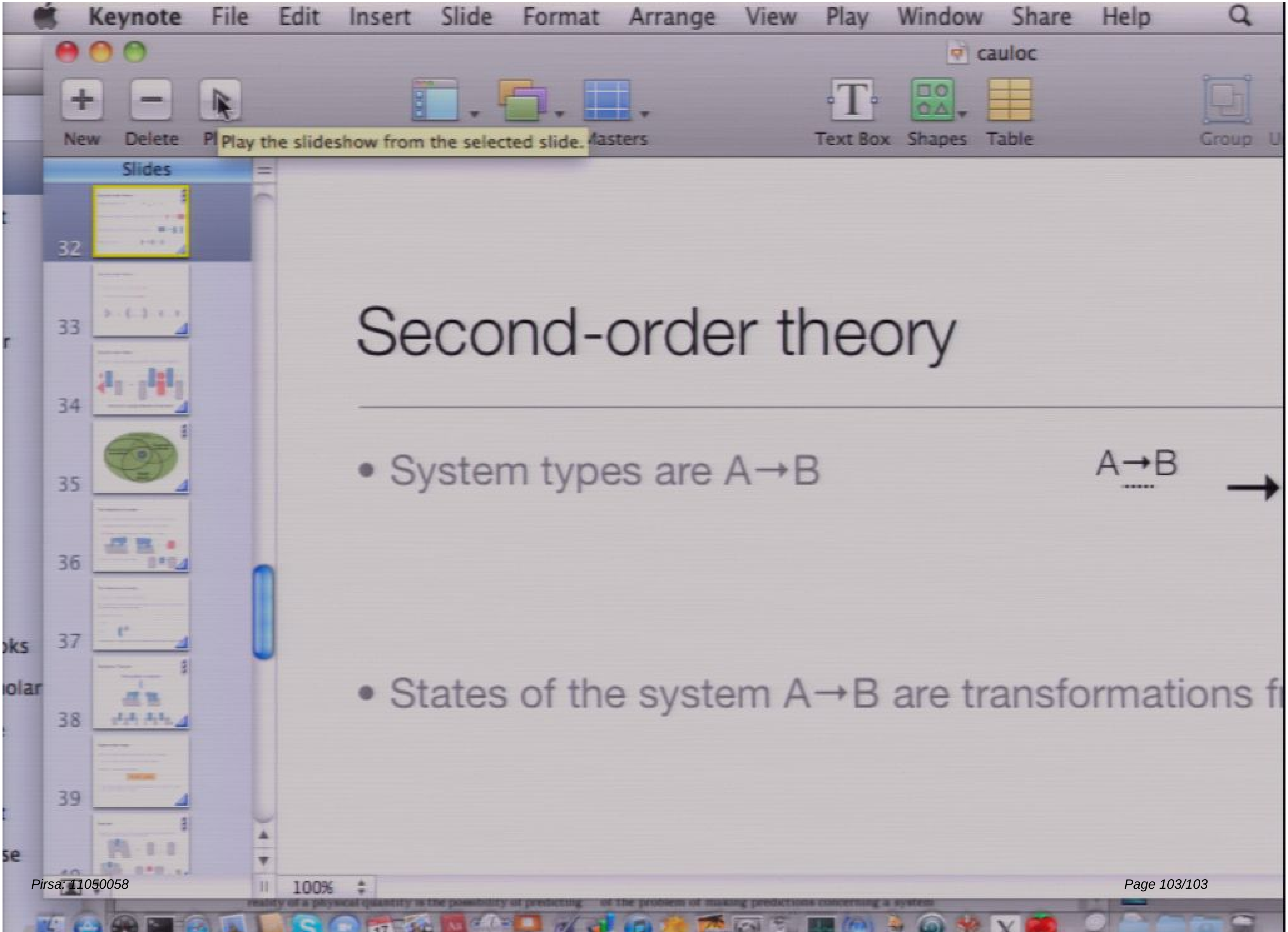
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