Title: Nonlocal Games and Computational Complexity: A Survey

Date: May 13, 2011 02:00 PM

URL: http://pirsa.org/11050046

Abstract: A seminal work by Cleve, HÃÂƒÂ¸yer, Toner and Watrous (quant-ph/0404076) proposed a close connection between quantum nonlocality and computational complexity theory by considering nonlocal games and multi-prover interactive proof systems with entangled provers. It opened up the whole area of study of the computational nature of nonlocality. Since then, understanding nonlocality has been one of the major goals in computational complexity theory in the quantum setting. This talk gives a survey of this exciting area.

Nonlocal Games and Computational Complexity

Tsuyoshi Ito

IQC Institute for Quantum Computing

UNIVERSITY OF WATERLOO

Conceptual Foundations and Foils for Quantum Information Processing, May 2011

Quantum nonlocality

[Bell '64] [Clauser, Horne, Shimony, Holt '69]

Measurement in the quantum theory *cannot* be described by local hidden variable model

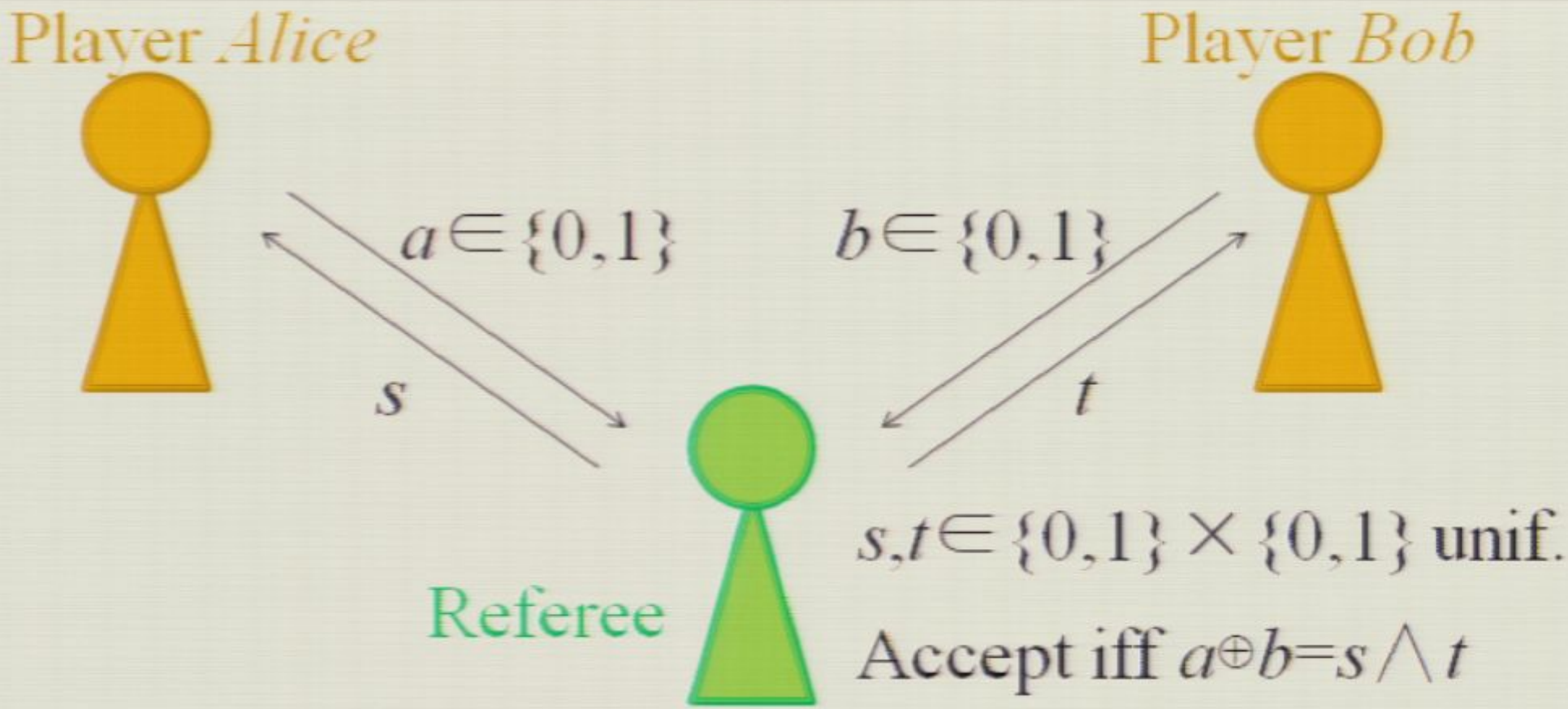
In LHV model, if $-1 \leq A_0, A_1, B_0, B_1 \leq 1$, then

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle \leq 2$$

In quantum theory, it can be

$$\langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle = 2\sqrt{2} > 2$$

CHSH game



Classical value $\omega(G) = 3/4$

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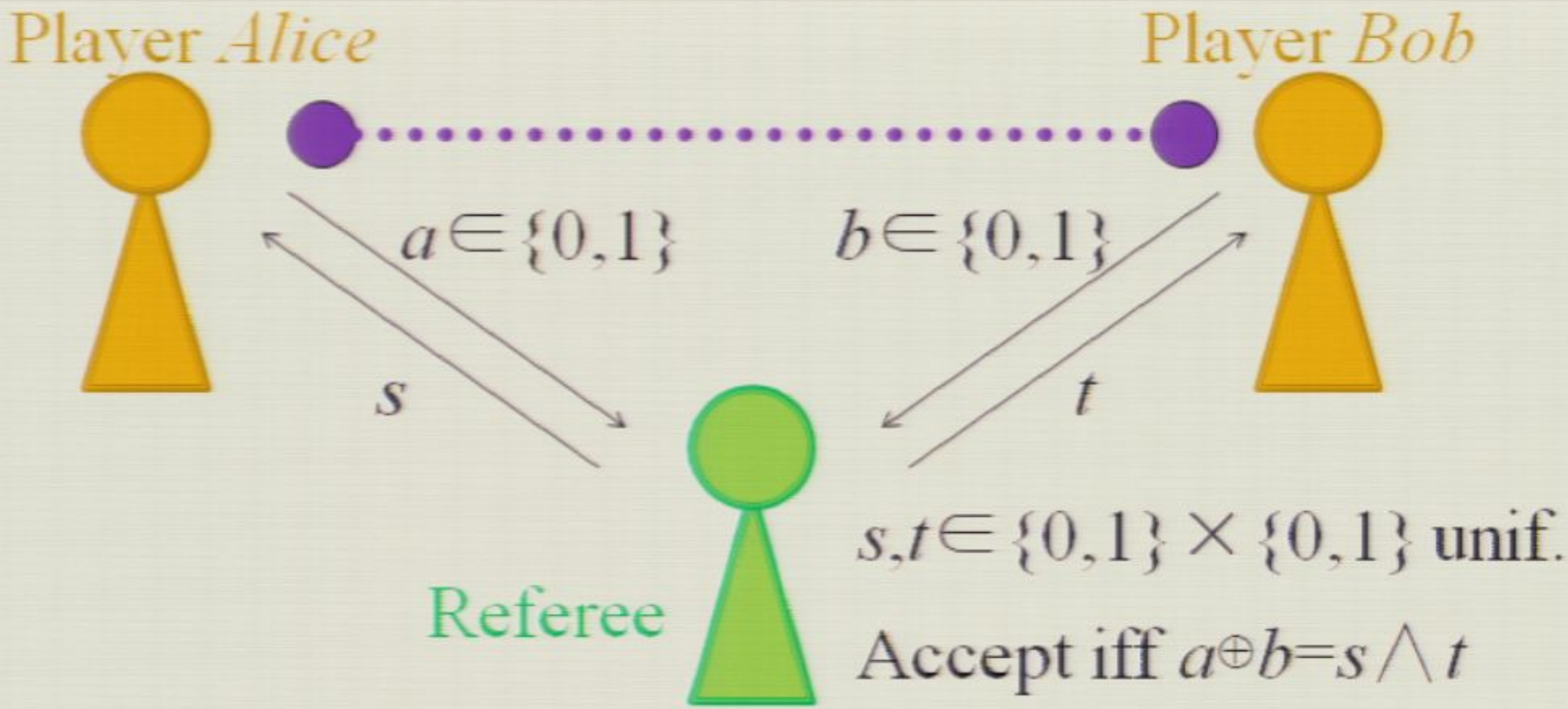
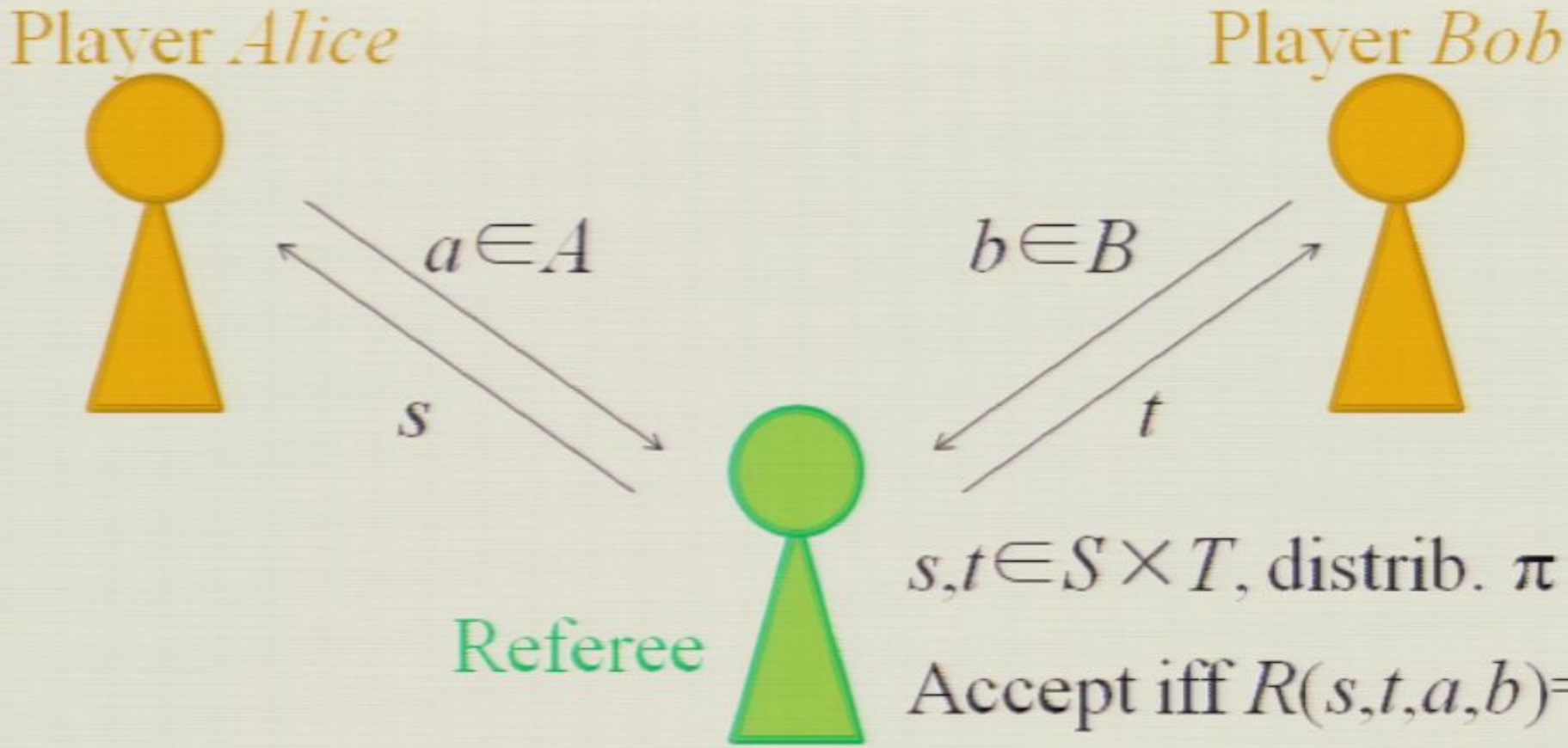


Classical value $\omega(G) = 3/4$

Entangled value $\omega^*(G) = \cos^2(\pi/8) \approx 0.85$

General form of 2-player 1-round game



$s,t \in S \times T$, distrib. π

Accept iff $R(s,t,a,b)=1$

Classical value $\omega(G)$, Entangled value $\omega^*(G)$

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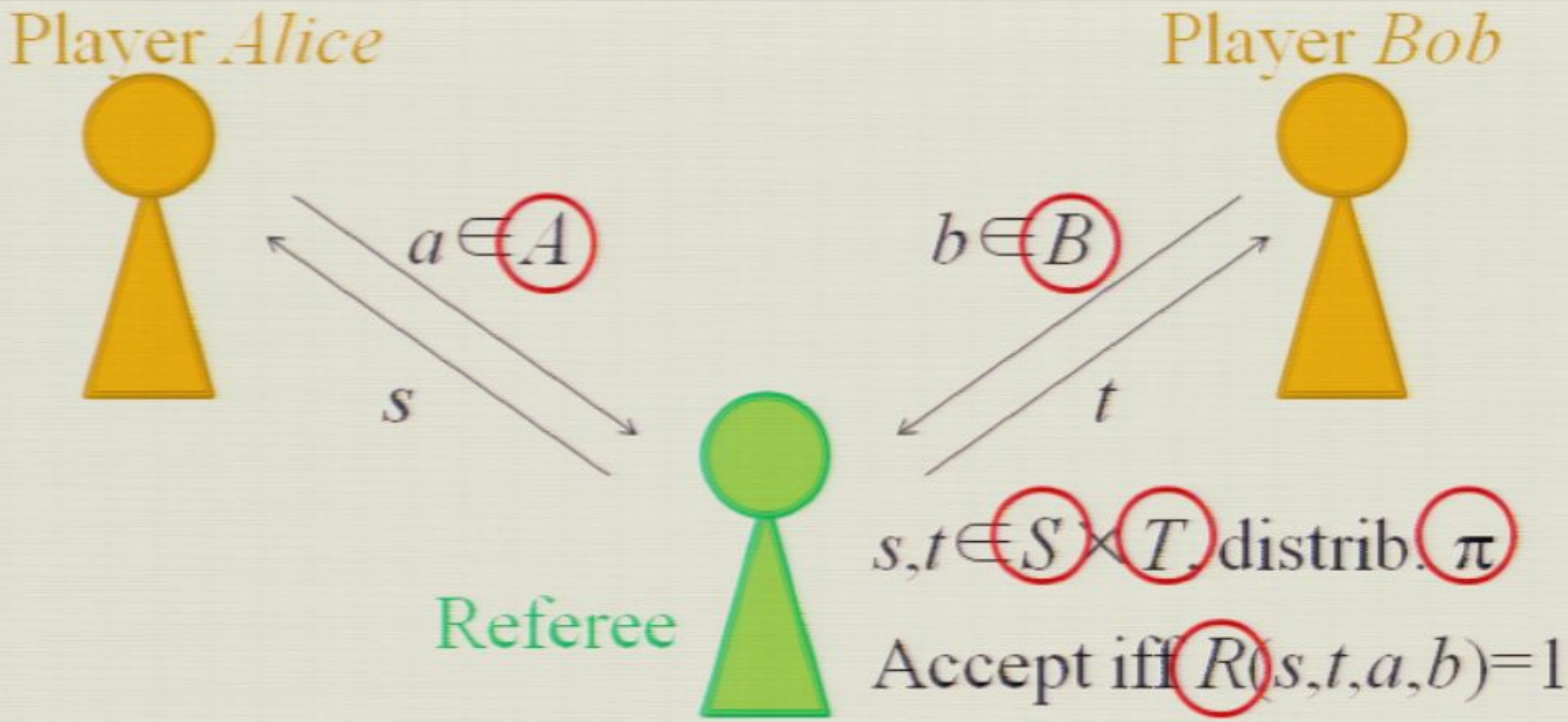
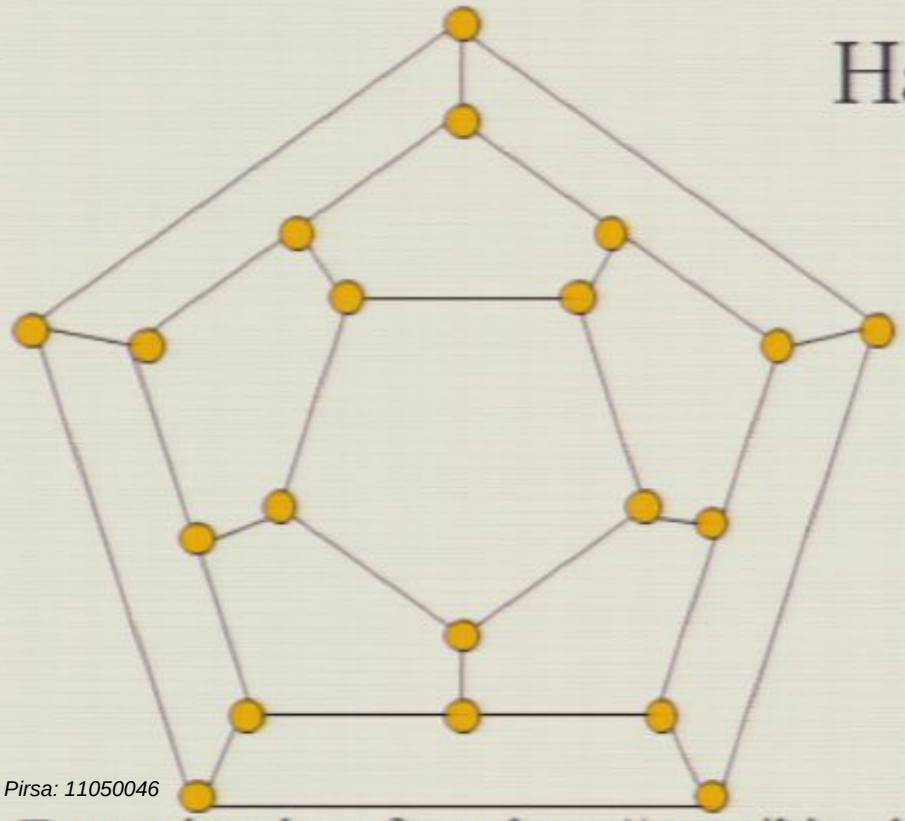


Classical value $\omega(G)$, Entangled value $\omega^*(G)$

Background in computational complexity theory

Complexity theory classifies problems by their inherent difficulty



Hamiltonian circuit problem:

Input: A graph G

Question: Does G have a circuit visiting every vertex exactly once?

Example taken from http://en.wikipedia.org/wiki/Hamiltonian_path

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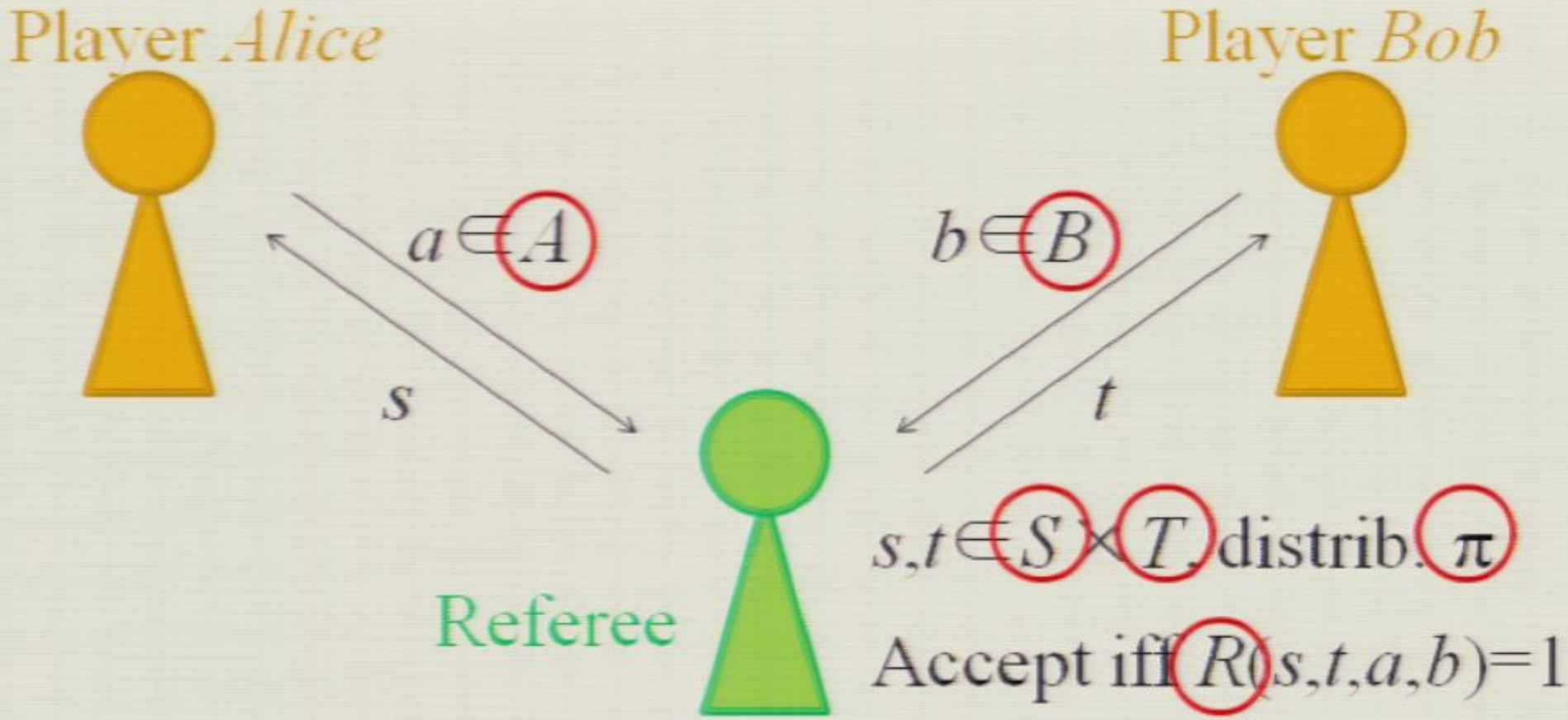

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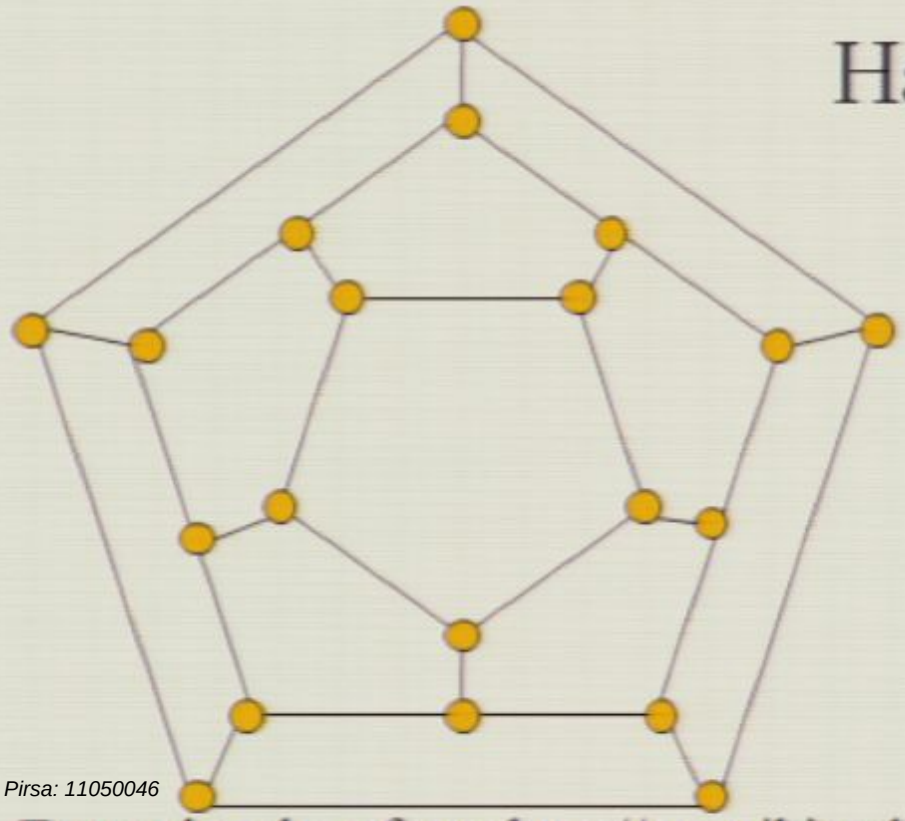

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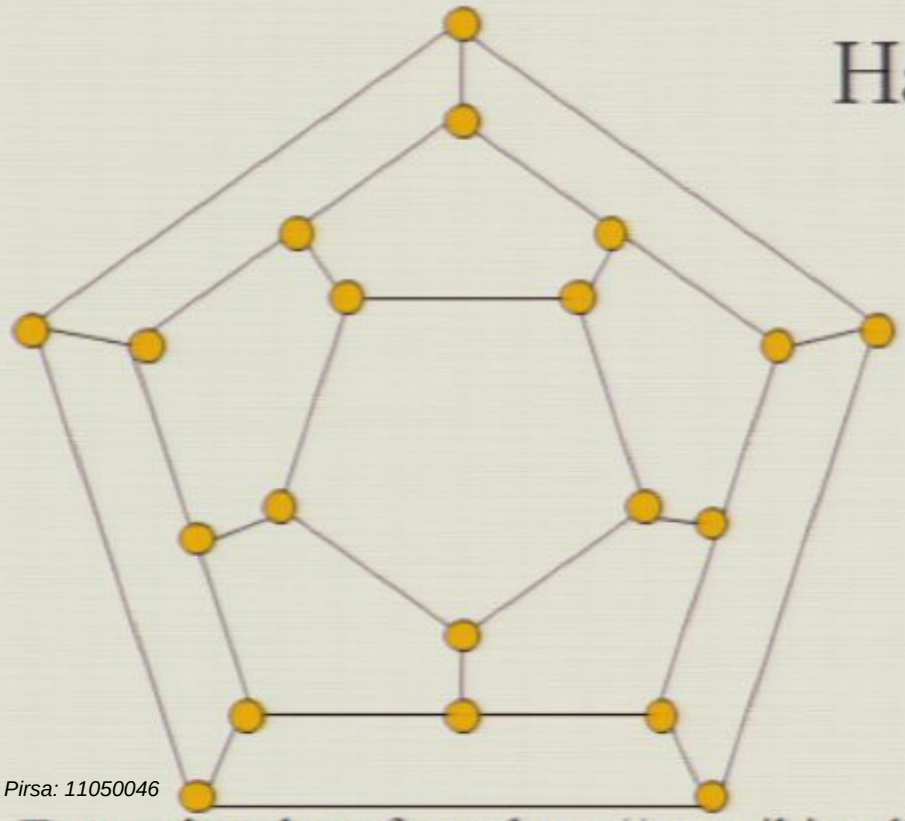

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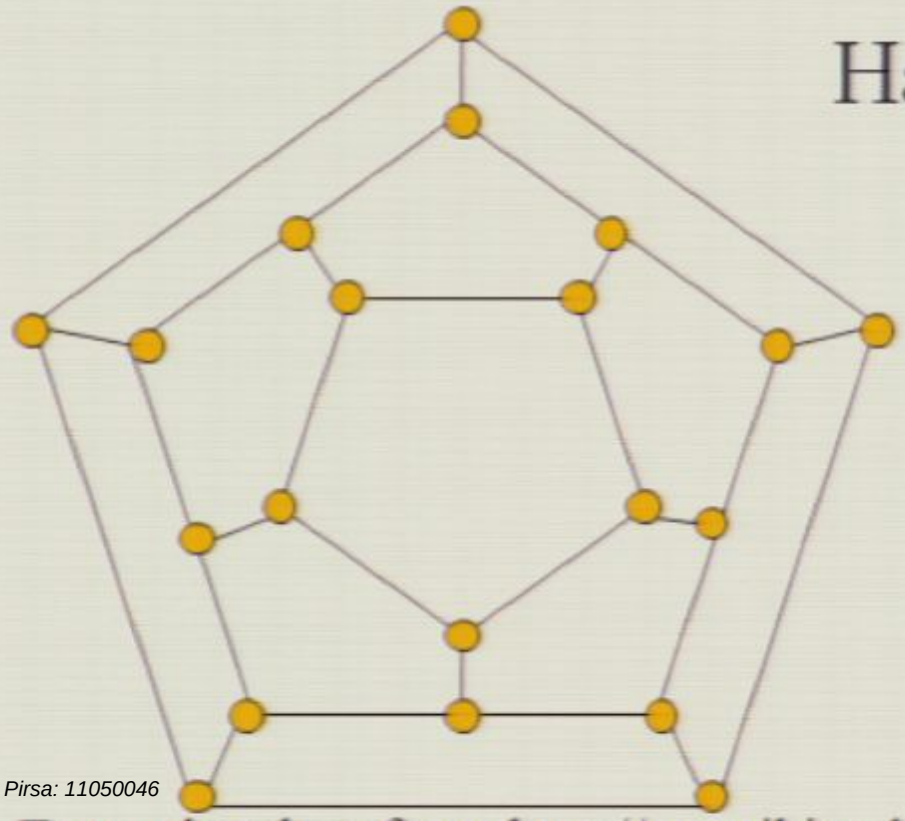

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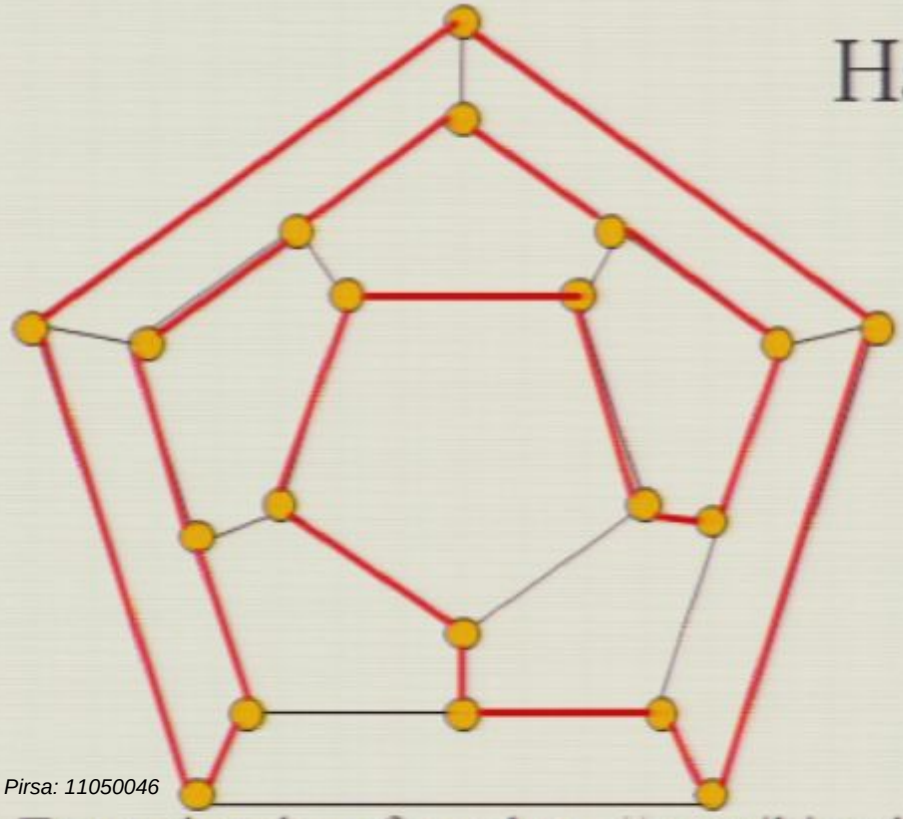

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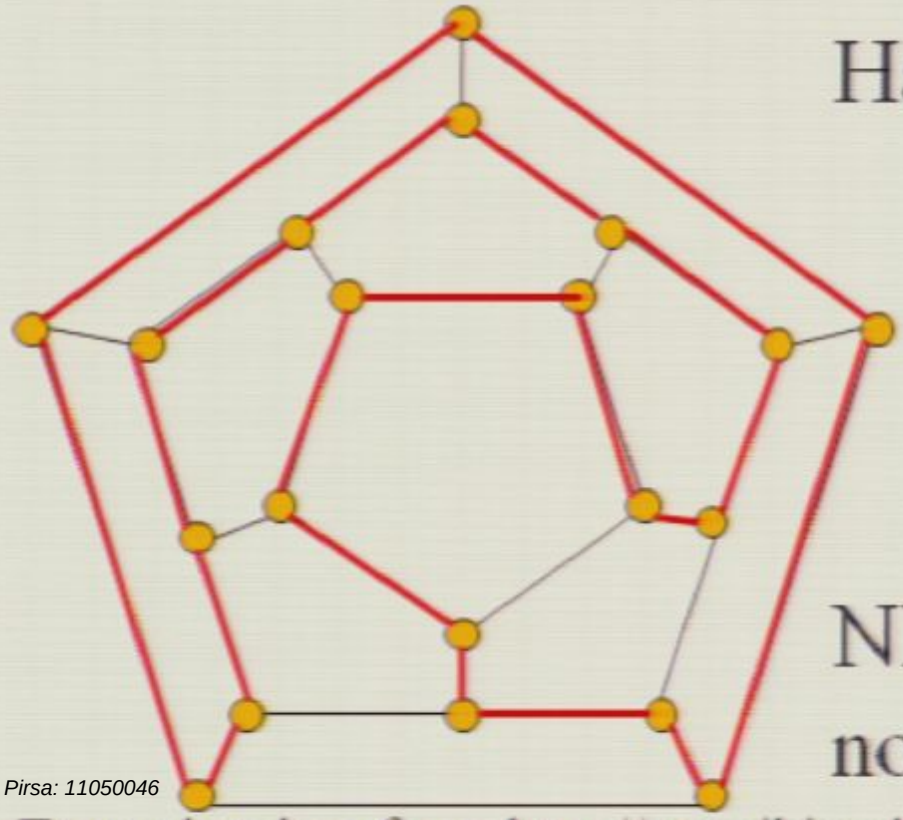

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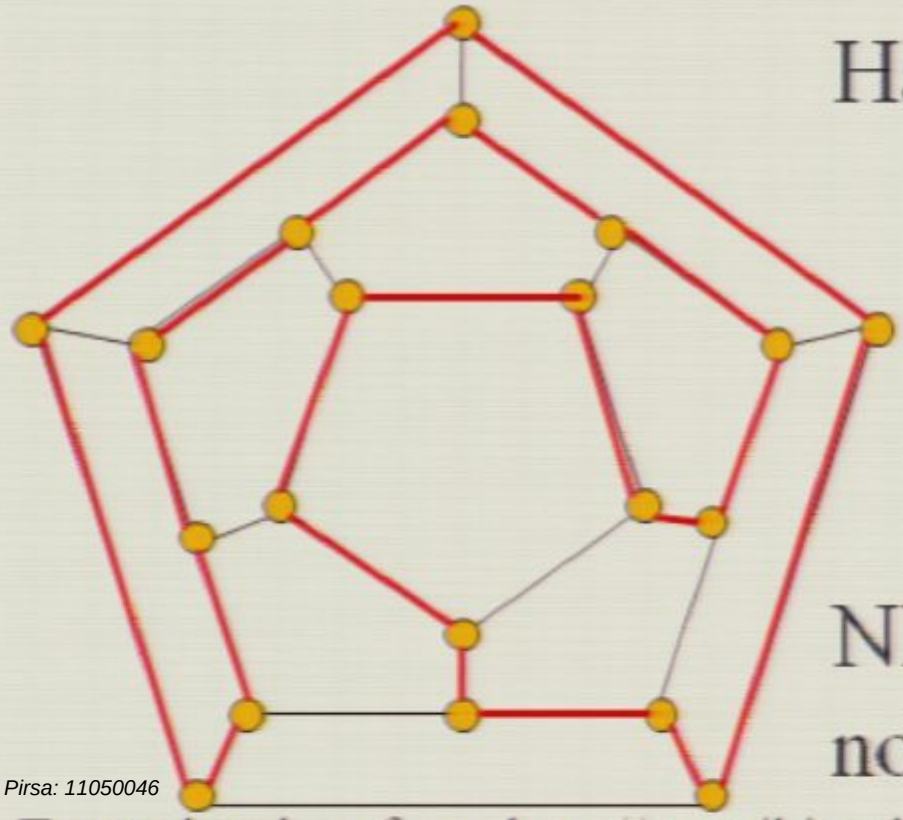

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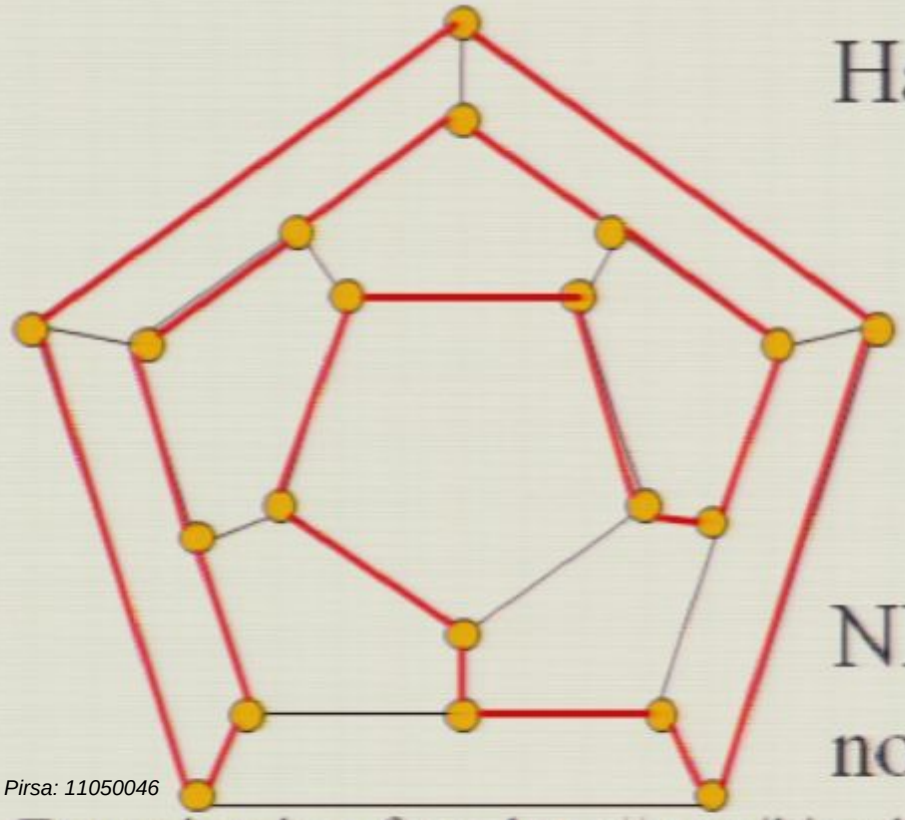

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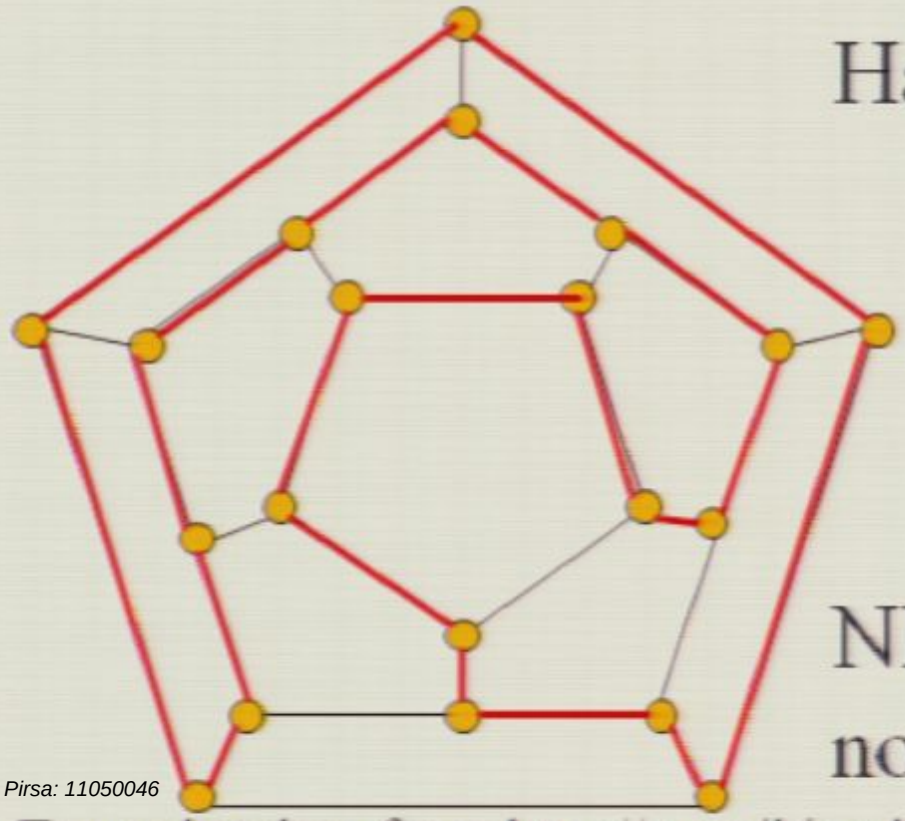

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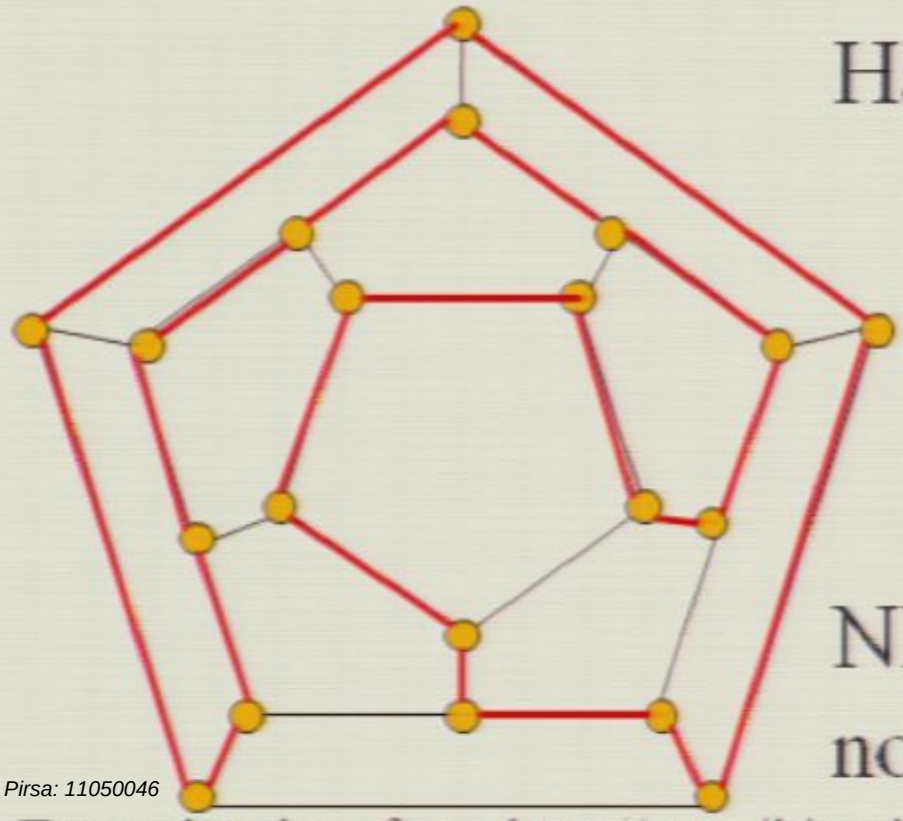

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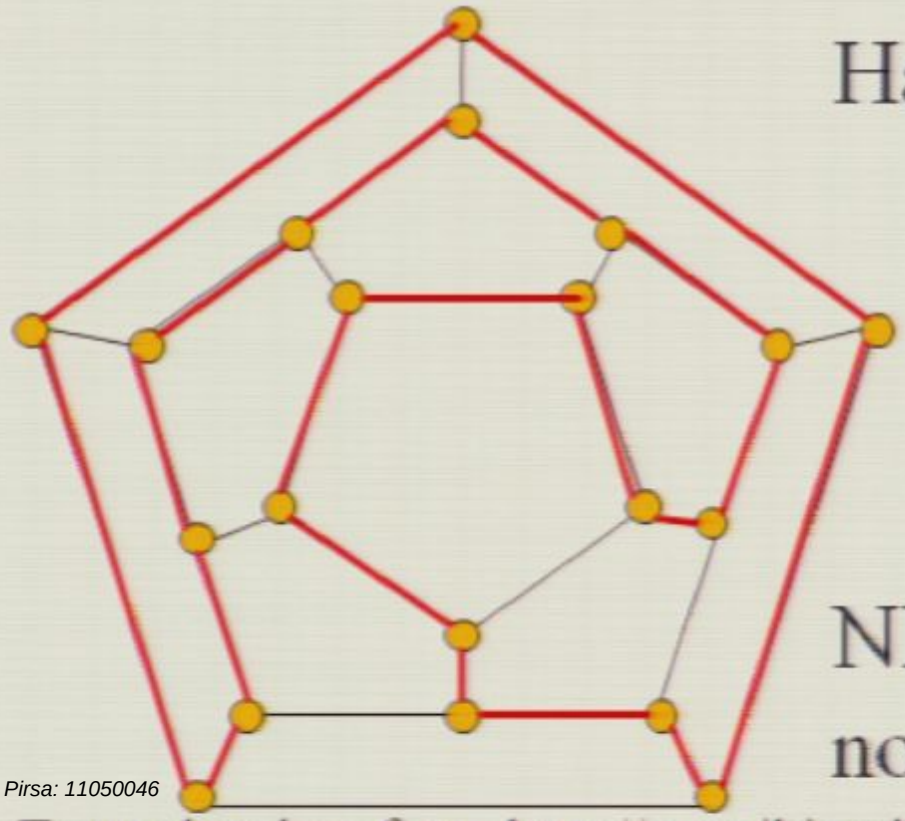

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P, NP, interactive proofs

P

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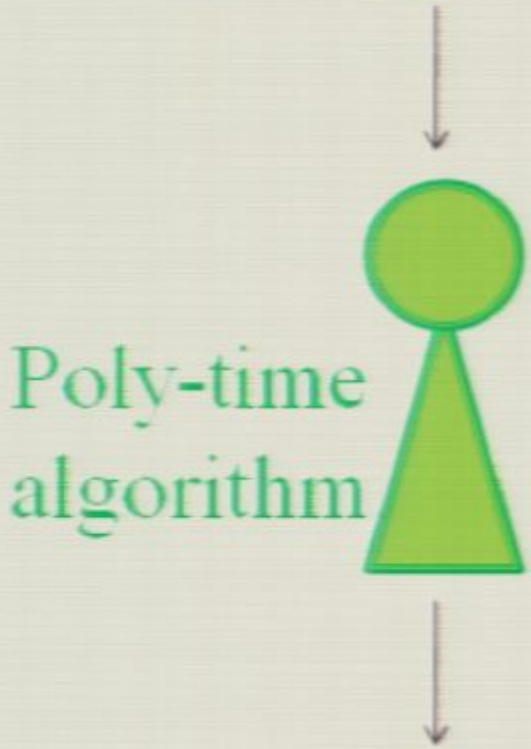
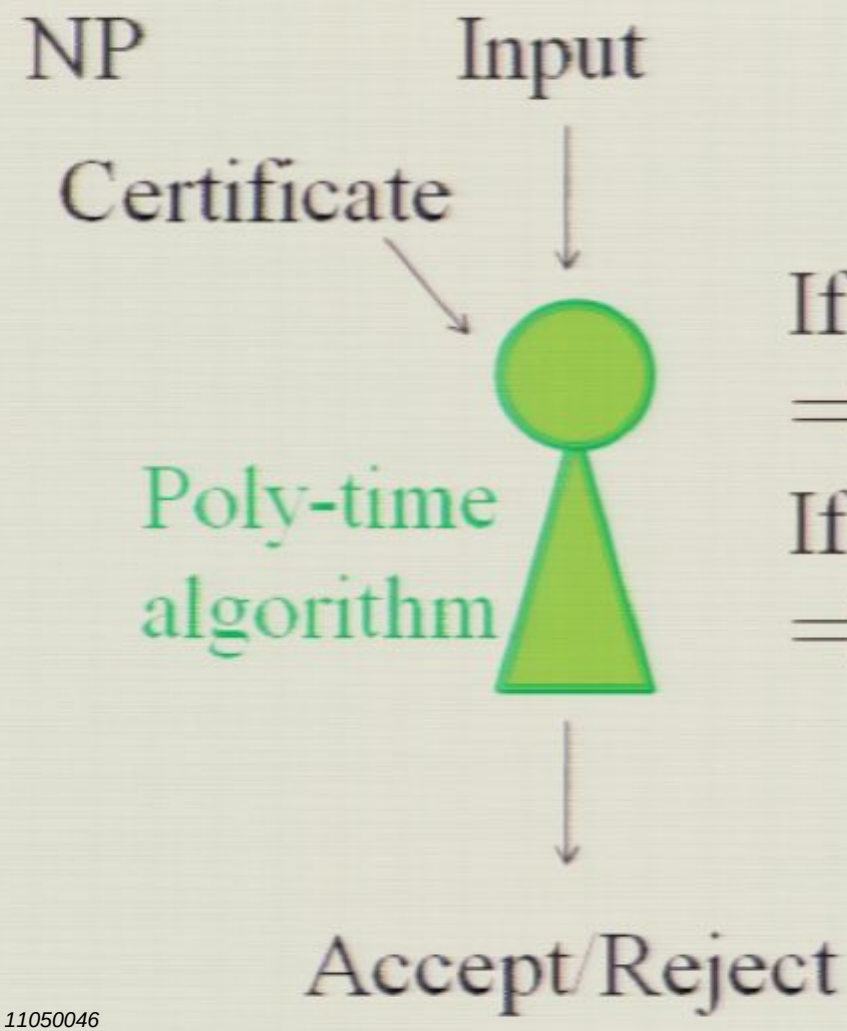


If the correct answer is yes
$\Rightarrow$ Accept
If the correct answer is no
$\Rightarrow$ Reject

Accept/Reject

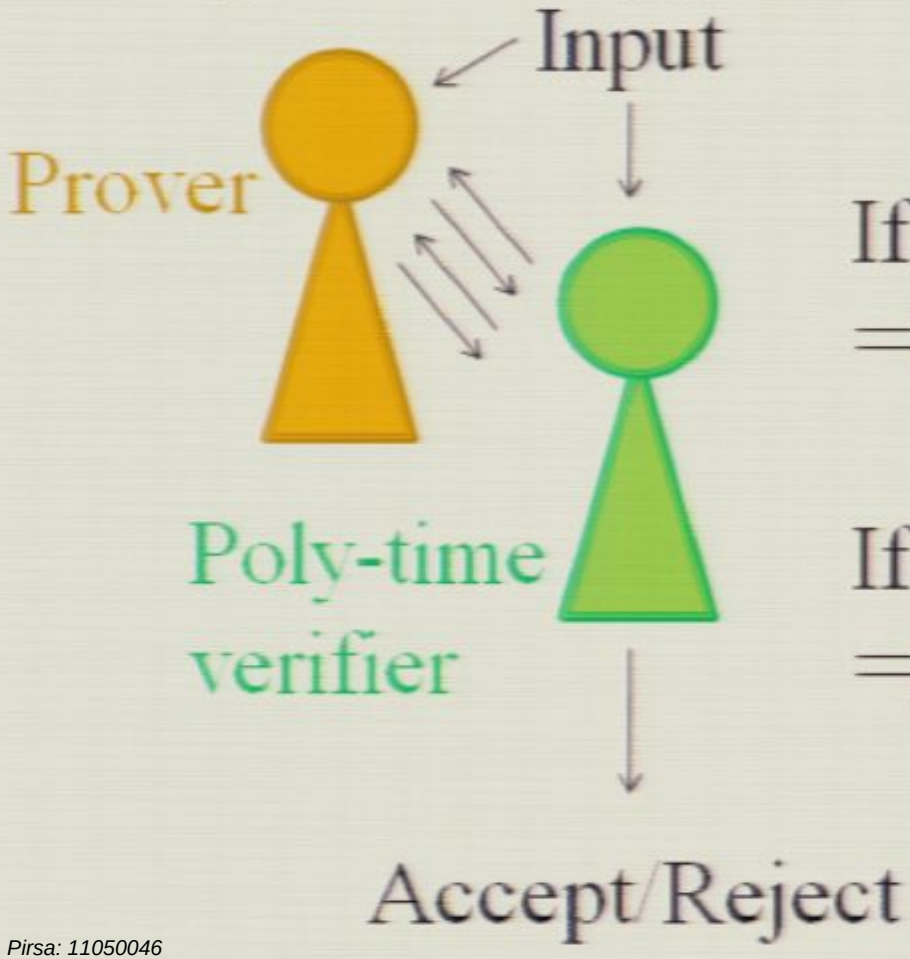
P, NP, interactive proofs



If the correct answer is yes
$\Rightarrow \exists$ accepted certificate
If the correct answer is no
$\Rightarrow \forall$ certificate is rejected

P, NP, interactive proofs

IP [Babai '85] [Goldwasser, Micali, Rackoff '85]

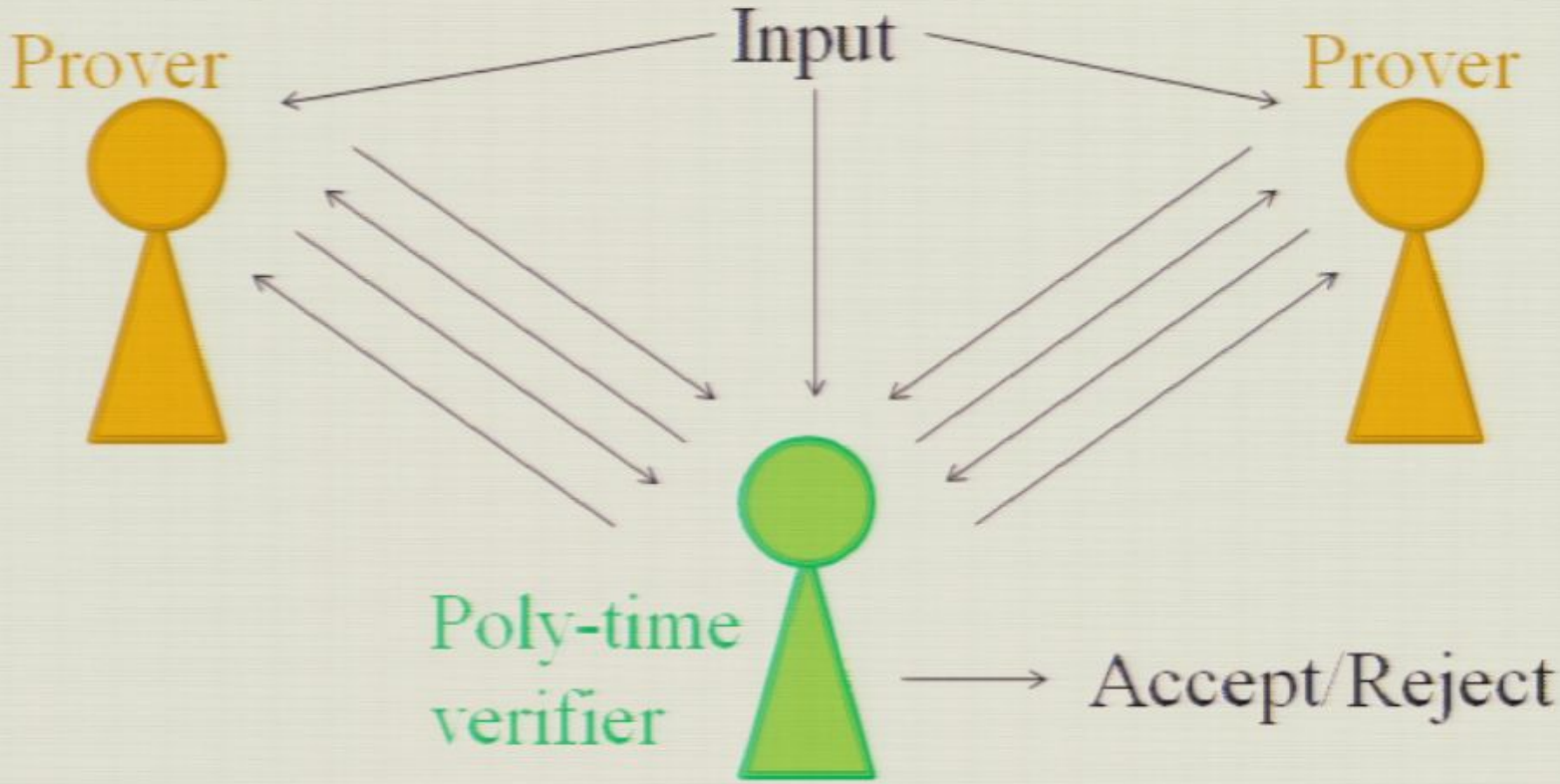


If the correct answer is yes
$\Rightarrow \exists$ prover is accepted
with high prob.

If the correct answer is no
$\Rightarrow \forall$ prover is rejected
with high prob.

Multi-prover interactive proofs

[Ben-Or, Goldwasser, Kilian, Wigderson '88]



Multi-prover interactive proofs

[Ben-Or, Goldwasser, Kilian, Wigderson '88]

MIP system defines a multi-player multi-round game of exponential size for each input

Classical value = Maximum acceptance probability

Results in the classical case

[Feige, Lovász '92]

MIP = NEXP, even with 2 provers, 1 round, exp-small 1-sided error

In terms of games: Given a 2-player 1-round game G with $\leq n$ questions and $\leq n$ answers, deciding whether

$$\omega(G) = 1 \quad \text{or} \quad \omega(G) \leq 1/n$$

is NP-complete

This is used to prove hardness results for many approximation problems

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Computational complexity of entangled value of games

[Cleve, Hoyer, Toner, Watrous '04]

$\omega(G)$ is hard to compute, then what about $\omega^*(G)$?

Naïve thought: $\omega^*(G)$ looks at least as hard as $\omega(G)$ to compute ($\Rightarrow$ NP-hard), because $\omega^*(G)$ searches in the larger set of strategies for players…?

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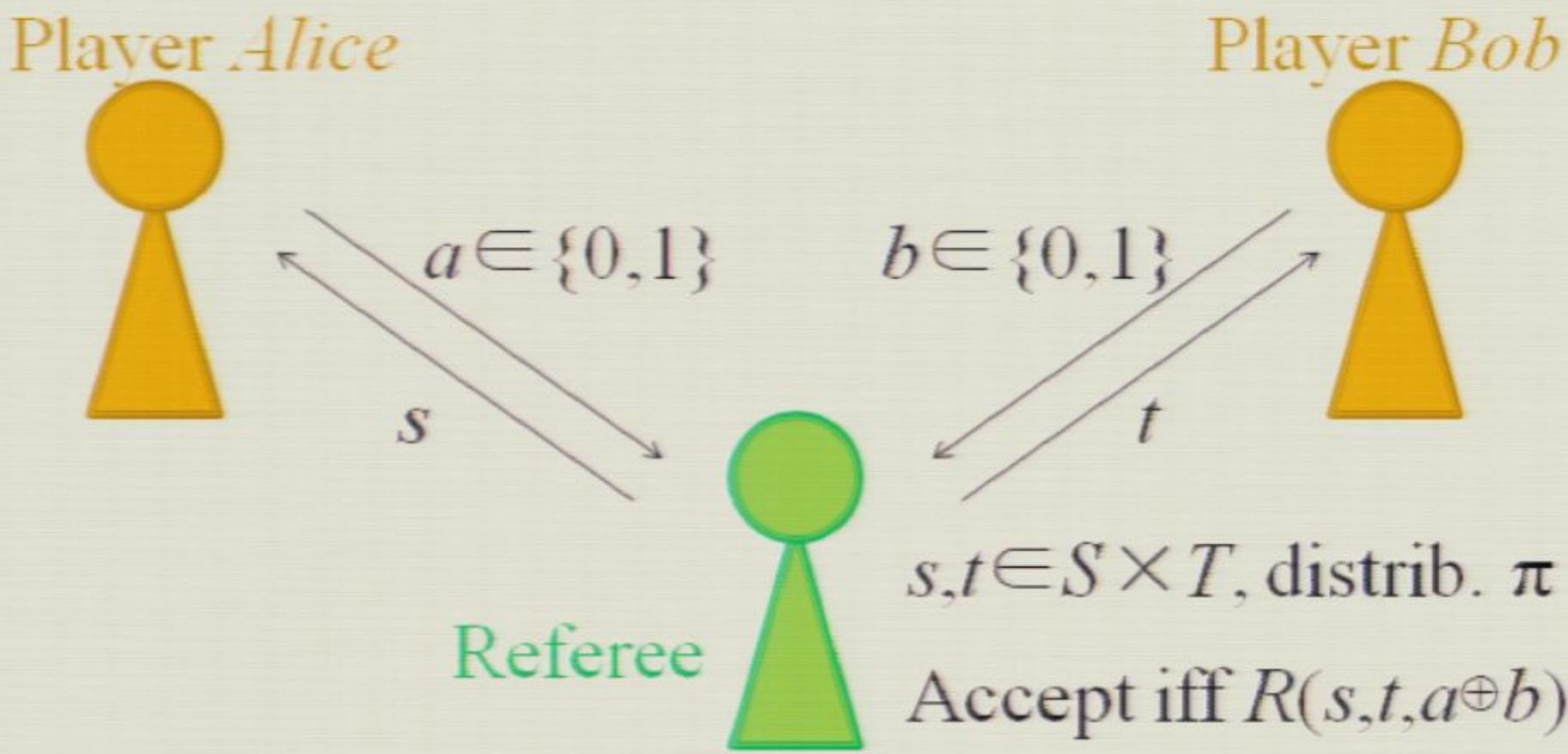
2-player XOR games



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Accept iff $R(s,t,a \oplus b)=1$

2-player XOR games

[Cleve, Hoyer, Toner, Watrous '04]

For 2-player XOR game G,

- $\omega^*(G)$ can be computed efficiently (to a polynomial number of digits) (based on [Tsirelson '80])
- Deciding whether $\omega(G) \geq 0.75$ or $\omega(G) \leq 0.70$ is NP-complete (based on [Håstad '97])

This means: Allowing more power to players sometimes makes the problem easier

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Hardness of computing entangled values

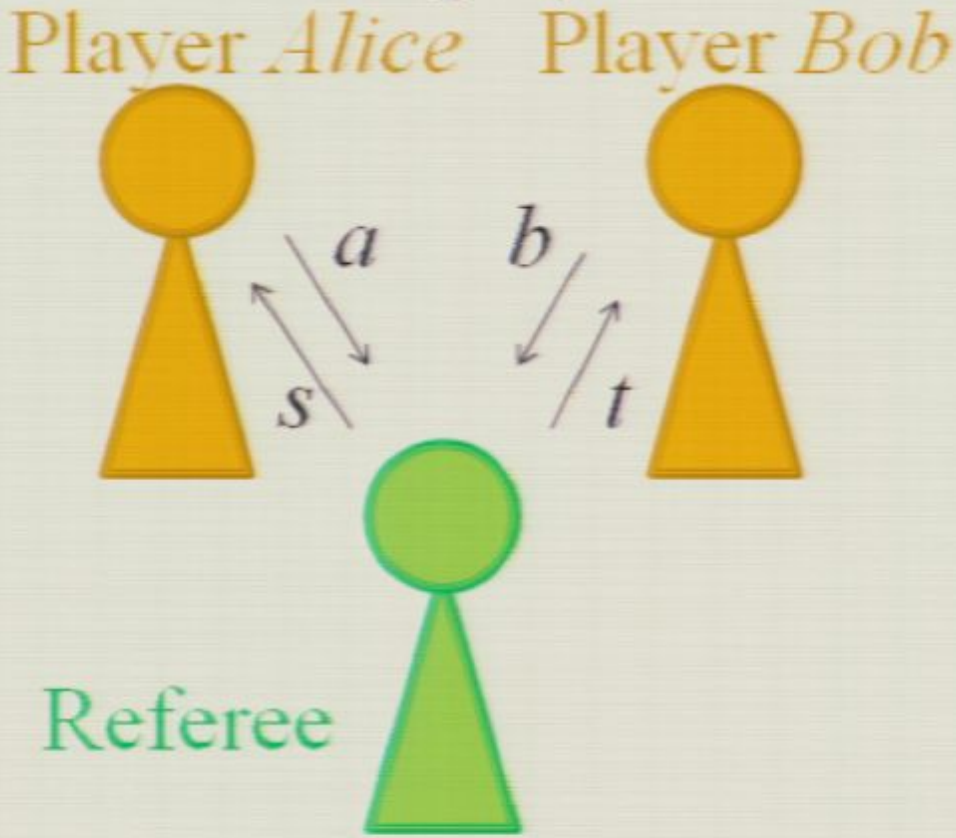
Theorem
[Kempe, Kobayashi, Matsumoto, Toner, Vidick '07]:
Given a 3-player 1-round game G,
deciding whether $\omega^*(G)=1$ or not is NP-hard.

n computational complexity theory,
hardness such as this theorem is proved by comparing
he difficulty of two problems via a *reduction*.

Reduction

[Kempe, Kobayashi, Matsumoto, Toner, Vidick '07]

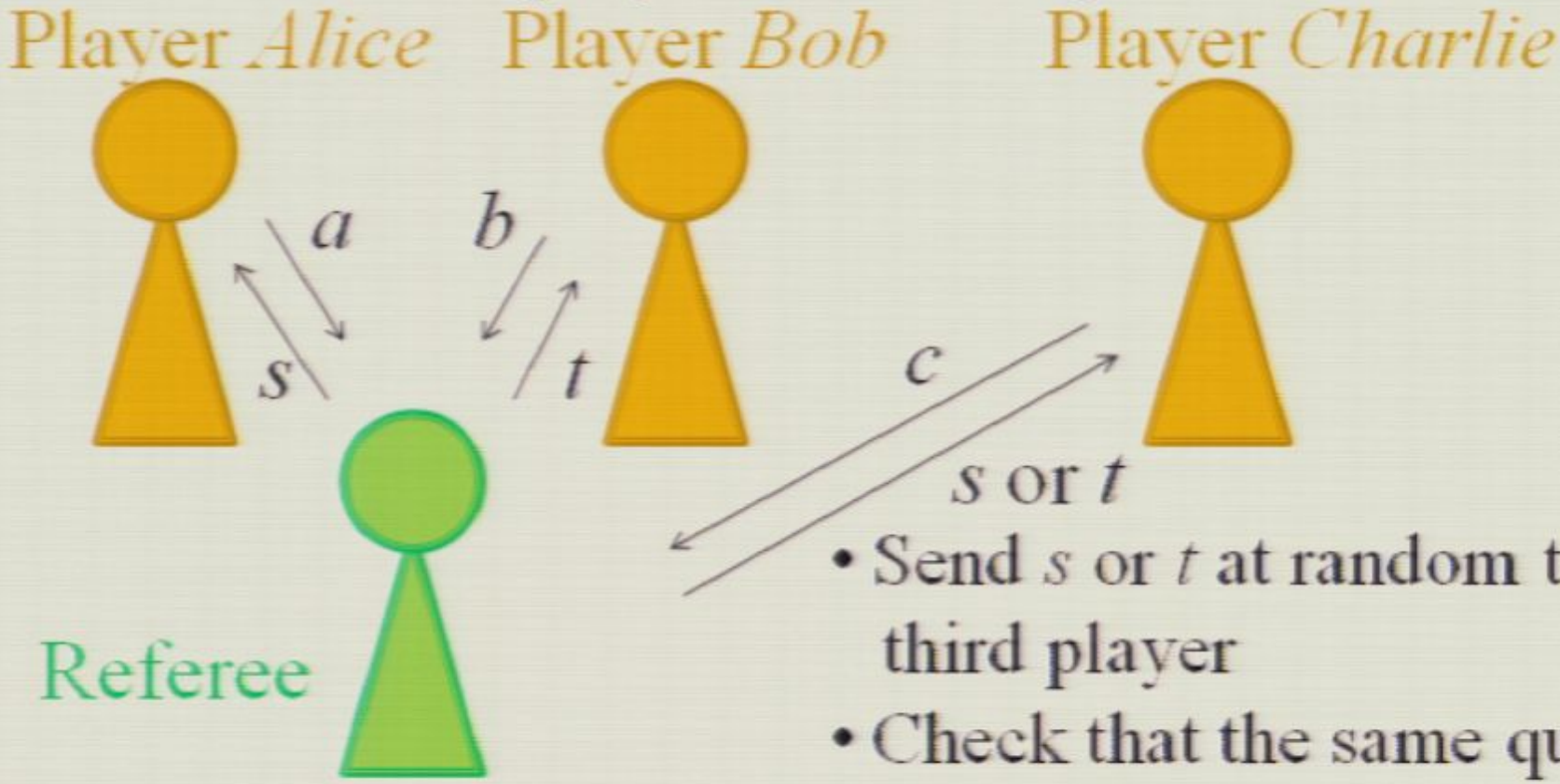
Start with 2-player 1-round game G:



Reduction

[Kempe, Kobayashi, Matsumoto, Toner, Vidick '07]

Convert G to 3-player 1-round game G':



- Send s or t at random to a third player
- Check that the same questions lead to the same answers

Reduction

[Kempe, Kobayashi, Matsumoto, Toner, Vidick '07]

Prove $\omega(G)=1 \Leftrightarrow \omega^*(G')=1$

by considering what a strategy in G'

with acceptance prob. 1 looks like

$\Downarrow$

Deciding whether $\omega^*(G')=1$ or not is as hard as deciding whether $\omega(G)=1$ or not

$\Downarrow$

Since deciding whether $\omega(G)=1$ or not is NP-complete, deciding whether $\omega^*(G')=1$ or not is NP-hard

NP-hardness via reductions

In general, proving the hardness of computing $\omega^*(G)$ requires suitable *transformations among games*.

Hardness of computing entangled values

Later improvements:

- NP-hard even with binary answers [Ito, Kobayashi, Preda, Sun, Yao '08]
- NP-hard even with 2 players [Ito, Kobayashi, Matsumoto '09]

Hardness of approximation

[Kempe, Kobayashi, Matsumoto, Toner, Vidick '07]

It can be proved that

$\omega(G) \leq 1/2 \Rightarrow \omega^*(G') \leq 1-c/n^2$ for some $c>0$

$\Downarrow$

Since deciding whether $\omega(G)=1$ or $\omega(G) \leq 1/2$ is NP-complete, deciding whether $\omega^*(G')=1$ or $\omega^*(G) \leq 1-c/n^2$ is NP-hard

The analogous results hold for 3-player 1-bit-answer games and 2-player games [IKPSY'08] [IKM'09]

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It can be proved that

$\omega(G) \leq 1/2 \Rightarrow \omega^*(G') \leq 1-c/n^2$ for some $c>0$

$\Downarrow$

Since deciding whether $\omega(G)=1$ or $\omega(G) \leq 1/2$ is NP-complete, deciding whether $\omega^*(G')=1$ or $\omega^*(G) \leq 1-c/n^2$ is NP-hard

The analogous results hold for 3-player 1-bit-answer games and 2-player games [IKPSY'08] [IKM'09]

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2 provers vs 3 players

"Almost commuting vs. nearly commuting" conjecture implies a better hardness result for 3-player 1-round games [KKMTV '07]

$\Updownarrow$

However, no known reductions are sufficient for a better hardness result for 2-player 1-round games [IKM '09]

2 and 3 players may be significantly different when the complexity of nonlocal games is concerned

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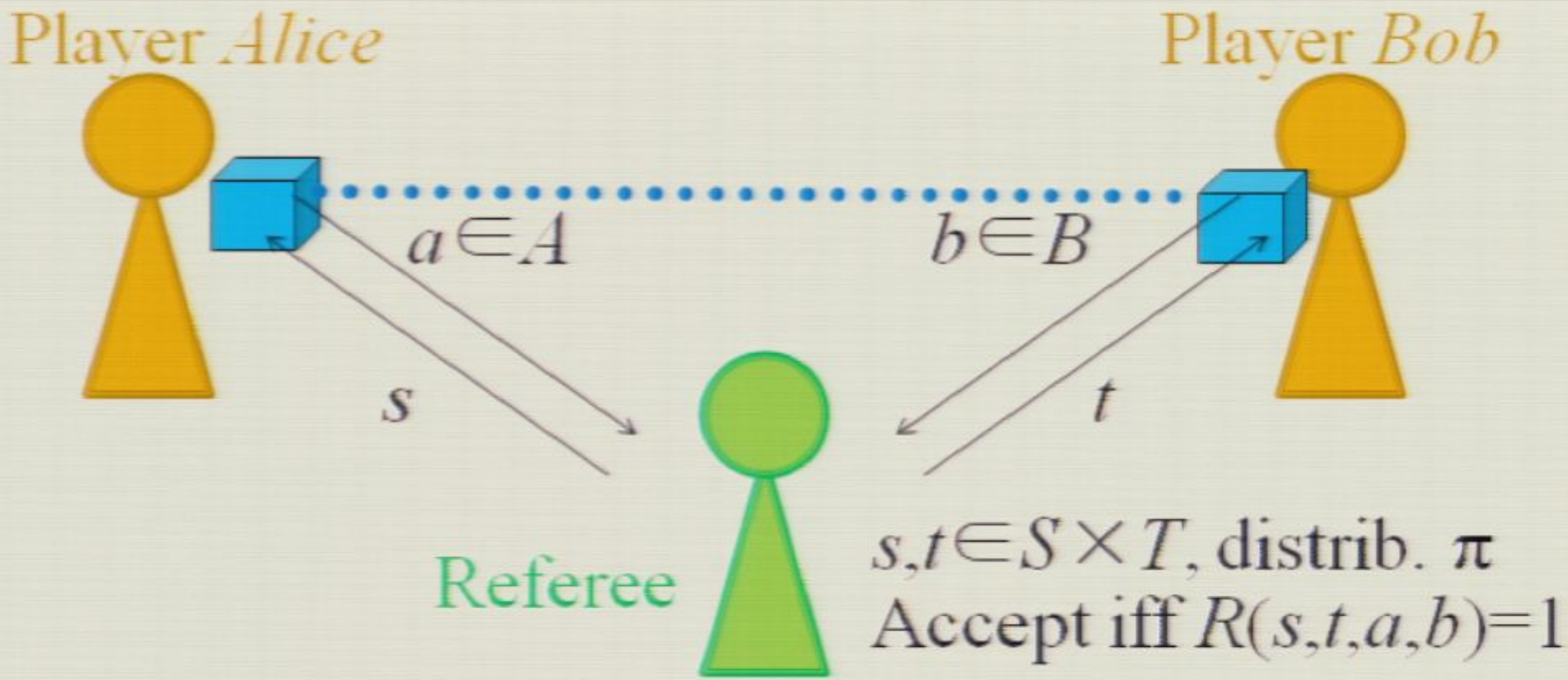
No-signaling value



No-signaling value $\omega_{ns}(G)$: Arbitrary prob. dist. as long as it cannot be used for signaling between players

No-signaling value

$\omega_{ns}(G)$ can be computed in polynomial time via linear programming

With 2 players and 1 round, it is even better than merely poly-time;
$\omega_{ns}(G)$ for exponential size game
can be computed in PSPACE [Ito '09]

Embarrassing(?) open problem

Find *some* (even exponential-time or less efficient) algorithm which decides whether $\omega^*(G)=1$ or not when given a nonlocal game G (or prove that it is undecidable).

Any computable upper bound on the required dimension of shared quantum state will yield such an algorithm.

Parallel repetition / Gap amplification

If G is 2-player XOR game, repeating G for t times in parallel reduces $2\omega^*(G)-1$ exponentially in t [Cleve, Slofstra, Unger, Upadhyay '07]

Every 2-player 1-round game G can be efficiently converted to another 2-player 1-round G' so that
$\omega^*(G)=1 \Rightarrow \omega^*(G')=1$ and
$\omega^*(G)\leq 0.99 \Rightarrow \omega^*(G')\leq 0.01$ [Kempe, Vidick '11]

Summary

The *classical* value of a game was very well studied in the complexity theory and is important in various inapproximability results.

The complexity of computing/approximating the *entangled* value is still largely unknown, although it is known in certain special cases.

The ability to control the game value by *transformation* is the key to prove hardness results.

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