

Title: On Basic Principles of General Probabilistic Theories

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Abstract: We propose an operationally motivated definition of the physical equivalence of states in General Probabilistic Theories and consider the principle of the physical equivalence of pure states, which turns out to be equivalent to the symmetric structure of the state space. We further consider a principle of the decomposability with distinguishable pure states and give classification theorems of the state spaces for each principle, and derive the Bloch ball in 2 and 3 dimensional systems.

ON BASIC PRINCIPLES OF GENERAL PROBABILISTIC THEORIES

May 9-13, 2011 Conceptual Foundations and Foils for Quantum Information Processing

Gen Kimura (Shibaura Institute of Technology, JAPAN)

OUTLINE

1: General Probabilistic Theories

2: Motivations and Goal

3: Symmetric GPT

* Physical Equivalence of Pure States

4: Decomposability w.r.t. distinguishable Pure States

5: Obtain classical and quantum (Bloch Ball) in 3 dim.

6: What I don't know...

GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (1960), Araki (1961); Ludwig (1964-); Mielnik (1968), Davies and Lewis (1970), Gudder (1973), etc

* Operationally Most General Framework for Probability

State + Measurement \Rightarrow Probability

- * Probabilistic Mixture of states (Convex Structure)
- * Separation postulates for states and measurements
- * Physical Topology measured by probabilities

GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (19

3), etc

* **Operationally M**

\forall states ρ_1, ρ_2 and $p \in [0, 1]$,
 \exists state $\rho = \langle p; \rho_1, \rho_2 \rangle$
as a preparation of ρ_1 with p and ρ_2 with $1 - p$:

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GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (1963), etc

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 \exists state $\rho = \langle p; \rho_1, \rho_2 \rangle$
as a preparation of ρ_1 with p and ρ_2 with $1 - p$:
$$\Pr\{M = m \mid \langle p; \rho_1, \rho_2 \rangle\} =$$
$$p\Pr\{M = m \mid \rho_1\} + (1 - p)\Pr\{M = m \mid \rho_2\}$$

for any measurement M

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* Operationally Most General Framework for Probability

1 Separation Postulate for states:

\forall measurement M and outcome m ,

$\Pr\{m|M, \rho_1\} = \Pr\{m|M, \rho_2\}$, then $\rho_1 = \rho_2$

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* Separation postulates for states and measurements

* Physical Topology

2 Separation Postulate for measurements:

\forall states ρ ,

$\Pr\{m|M_1, \rho\} = \Pr\{m|M_2, \rho\} \forall m$, then $M_1 = M_2$

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★ State space S is embedded into a **convex** subset in a real vector sp.

$$\text{s.t. } \langle p; \rho_1, \rho_2 \rangle = p\rho_1 + (1-p)\rho_2$$

★ Measurement is represented by **effects**:

$$E := (e_i \in \mathcal{E})_{i=1}^n, \text{ s.t. } \sum_i e_i = u, e_i(\rho) : \text{prob. for } i\text{th output under } \rho$$

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$e \in \mathcal{E}$: effect on \mathcal{S} :

$\Leftrightarrow e : \mathcal{S} \rightarrow [0, 1]$ affine:

$$e(p\rho_1 + (1-p)\rho_2) = pe(\rho_1) + (1-p)e(\rho_2)$$

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Weakest topology s.t.
all effects are continuous on S .

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State + Measurement \Rightarrow Probability

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Theory of Convex Set and Affine Function

(PreCompact Convex Set in Locally Convex Hausdorff Topological Vector Space)

GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (1960), Araki (1961); Ludwig (1964-); Mielnik (1968), Davies and Lewis (1970), Gudder (1973), etc

If $\mathcal{S} = d < \infty$

\mathcal{S} is a compact (i.e. closed bounded) convex of E^d

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If $S = d < \infty$

S is a compact (i.e. closed bounded) convex of E^d

Any compact convex set you imagine,



...

you can consider its general probabilistic model.

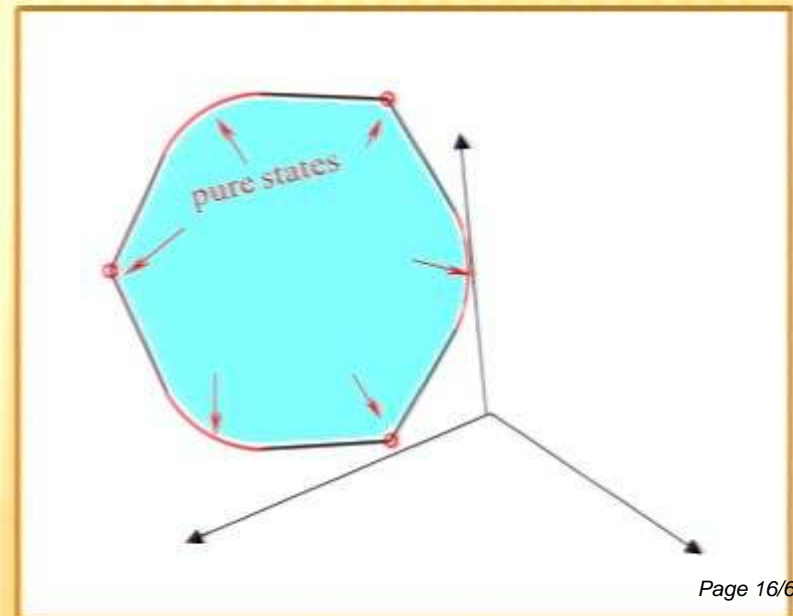
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Pure State

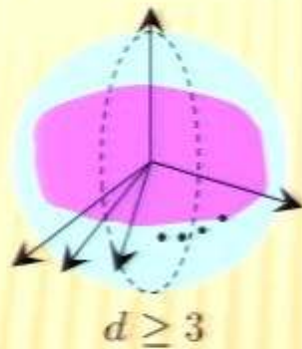
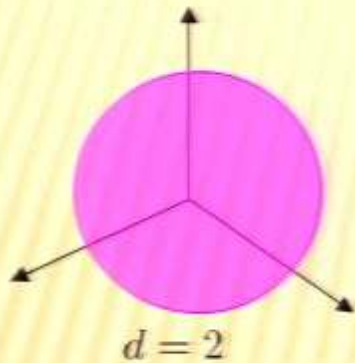
$\stackrel{\text{def}}{\Leftrightarrow}$ State which cannot be prepared by probabilistic mixtures of different states.

\Leftrightarrow Extreme Point of State Space



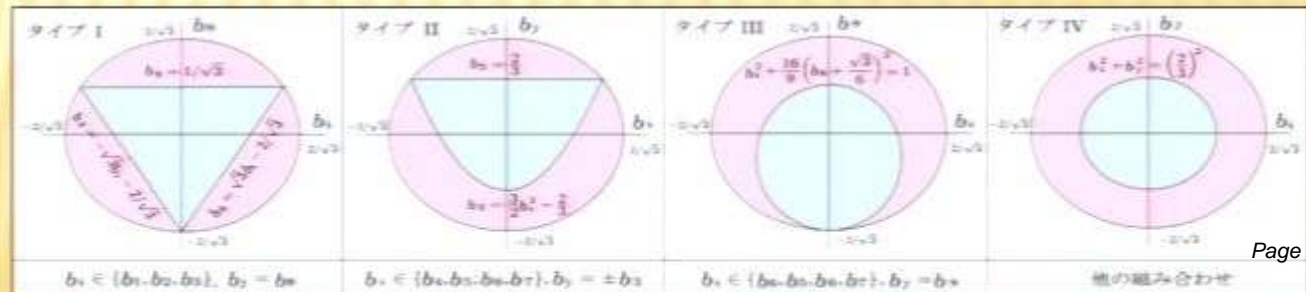
EXAMPLES OF GPT II

★ Quantum System: $S_d^Q := \{\rho \in \mathcal{L}(\mathcal{H}_d) : \rho \geq 0, \text{tr}\rho = 1\}$



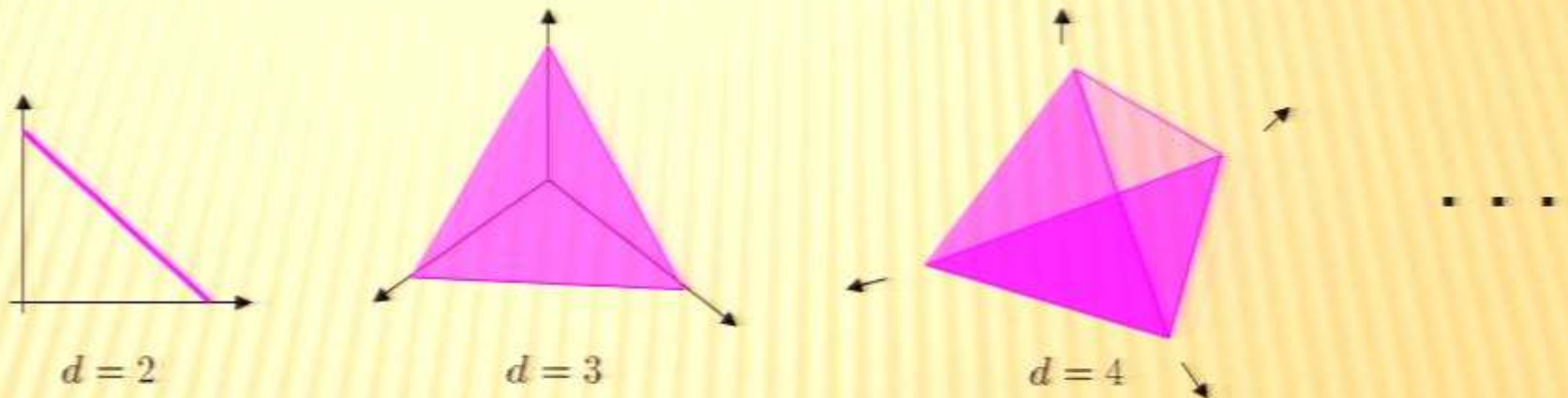
...

* ∞ Extreme Points



EXAMPLES OF GPT I

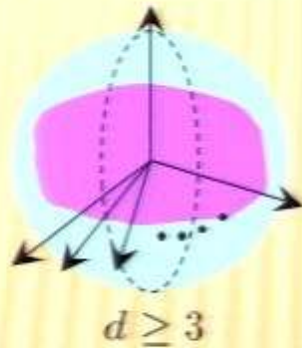
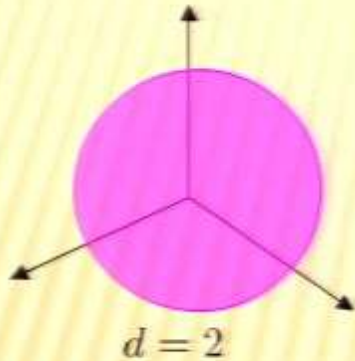
★ Classical System: $\mathcal{P}_d := \{\mathbf{p} = (p_1, \dots, p_d) \in \mathbb{R}^d \mid p_i \geq 0, \sum_i p_i = 1\}$



Simplex of Affine Dimension $d-1$ with d Pure States

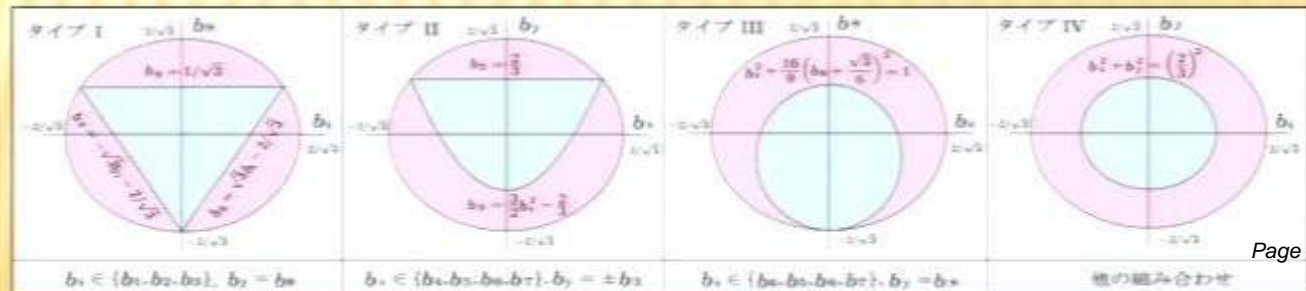
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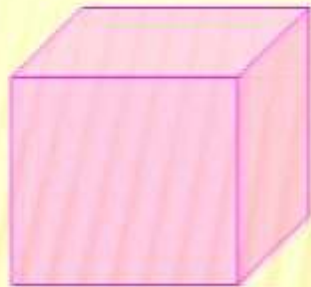


EXAMPLES OF GPT III

★ Hyper Cuboid System: $C_d = \{x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1\}$



$d = 2$



$d = 3$

...

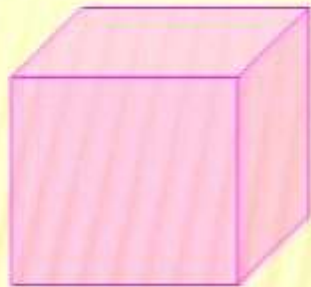
- * 2^d Extreme Points
- * Affine Dimension d

EXAMPLES OF GPT III

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$d = 3$

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0 1 0 0 0 1 1 1 1 0 1 0 1 1 0 0

d bit

- * 2^d Extreme Points
- * Affine Dimension d

EXAMPLES OF GPT III

Classical



$$\mathcal{C}_d = \{x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1\}$$

...

0 1 0 0 0 1 1 1 1 0 1 0 1 1 0 0

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EXAMPLES OF GPT III

Classical



HC



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...

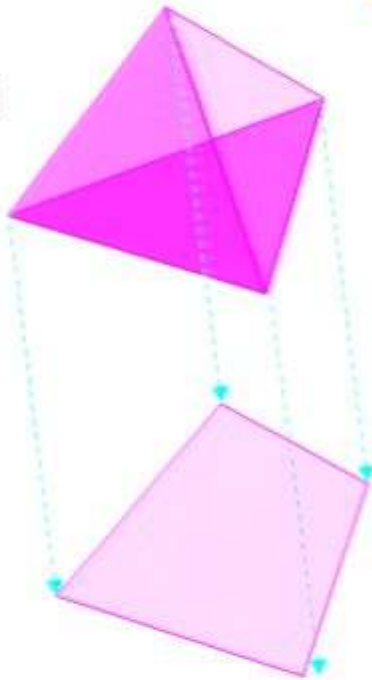
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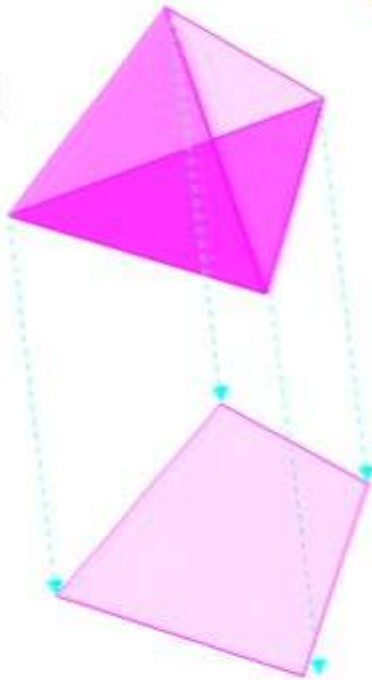
d bit

Prop: Hyper Cuboid system C_d can be realized by a d bit classical system by restricting to 1 bit measurement

- * 2^d Extreme Points
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EXAMPLES OF GPT III

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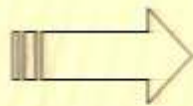
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Prop: Hyper Cuboid system \mathcal{C}_d can be realized by a d bit classical system by restricting to **1 bit measurement**

Thm: [Holevo 1982] Any GPT can be realized by a **Classical System** with an appropriate **restriction on measurements**

INFORMATION THEORY BASED ON GPT

- * Classical Prob. Theory \Rightarrow Classical Information Theory
- * Quantum Theory \Rightarrow Quantum Information Theory

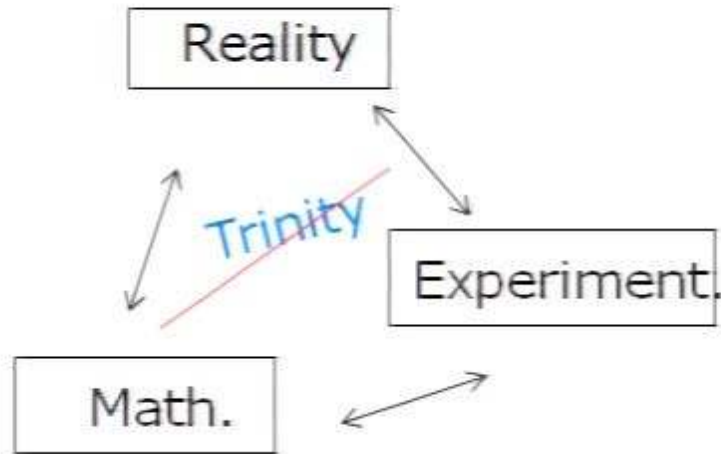


GPT \Rightarrow GPT Information Theory

Motivations

- ⇨ Seek for **Physical Principles** of Quantum Theory
in terms of **information languages**
- ⇨ Understand logical connections between
physical principles and information processings.
- ⇨ Preparation for “post” quantum
- ⇨ (Classical/Quantum)Information theory under Measurement restrictions

BASED ON GPT



Information Theory
Information Theory

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Reality



Math. w.r.t.
Experiments



BASED ON GPT

Information Theory
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INFORMATION THEORY BASED ON GPT

- [1] Hardy (2001); arXiv:quant-ph/0101012v4
- [2] Dakic and Brukner (2009); arXiv:0911.0695v1
- [3] Masanes and Muller (2010); arXiv:1004.1483v2
- [4] Chiribella, D'Ariano, and Perinotti (2010); arXiv:1011.6451v1

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THIS TALK...

“Natural” Physical Principles:

***1 Physical Equivalence of Pure States**

⇔ Symmetricity of State Space (Davies)

⇔ Reversible Connections of Pure States (Hardy etc.)

***2 Decomposability w.r.t. Distinguishable Pure States**

⇒ Classification of GPTs for each Principle

THIS TALK...

What We **also** Assume:

- * Existence of Objective (Classical) World
- * Causality
- * All Mathematically Well-defined Measurements:

$$(e_i \in \mathcal{E})_i \text{ s.t. } \sum_i e_i = u$$

are physically realizable.

SYMMETRICITY OF STATE SPACE

[P1] Group of Affine Bijection on S acts transitively on ∂S

(Davies 1974)

\Leftrightarrow [P1'] Any pure states are connected by Reversible Transformation

(Hardy, ...)

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\Leftrightarrow [P1''] Any pure states are Physically Equivalent

meaning that there are no "special" Pure States

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PHYSICAL EQUIVALENCE OF STATES

State s_1 and s_2 have physically the same properties if

- * For any measurement E_1 ,
there uniquely exists measurement E_2 s.t.
prob. dist of E_1 under s_1 equals prob. dist. of E_2 under s_2
- * The correspondence should preserve
probabilistic mixture of measurements

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[Def] State s_1 and s_2 are physically equivalent iff there exists unit preserving affine bijection Λ on \mathcal{E} such that

$$e(s_1) = \Lambda(e)(s_2) \quad \forall e \in \mathcal{E}$$

- * Physical Equivalence is an equivalence relation
- * Physical Equivalence in QM is unitary equivalence:

$$\rho_1 \simeq \rho_2 \Leftrightarrow \exists \text{unitary } U \text{ s.t. } \rho_1 = U \rho_2 U^\dagger$$

PHYSICAL EQUIVALENCE OF STATES

[Lem] For any affine functional $\Phi : \mathcal{E} \rightarrow [0, 1]$ satisfying $\Phi(u) = 1$ and $\Phi(0) = 0$, there uniquely exists a state $s \in \mathcal{S}$ such that $\Phi(e) = e(s)$ for any $e \in \mathcal{E}$.

[Thm] State s_1 and s_2 are physically equivalent iff there exists an affine bijection Ψ on \mathcal{S} such that $s_1 = \Psi(s_2)$.

[P1'] Any pure states are connected by **Reversible Transformation**

\Leftrightarrow [P1''] Any pure states are **Physically Equivalent**

We call **Symmetric GPT** if it satisfy [P1] ([P1'] or [P1''])

CLASSIFICATION OF SYMMETRIC GPTS

[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorphic to Isogonal Figure



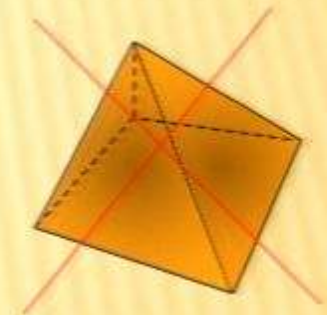
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CLASSIFICATION OF SYMMETRIC GPTS

[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorphic to Isogonal Figure



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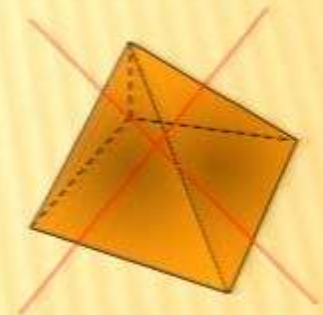


CLASSIFICATION OF SYMMETRIC GPTS

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[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorphic to a disk.

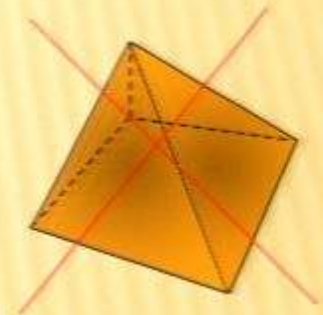


CLASSIFICATION OF SYMMETRIC GPTS

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...



[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorphic to a disk.

[Thm] State Space of 3-dimensional Symmetric GPT with infinite numbers of pure states is either a ball or a circular cylinder.

Dim $S = 2$



Dim $S = 3$



or



DIM $S = 3$



...

Classical Sys.

or



or



Quantum Sys.
(Bloch Ball)

DIM S = 3



Classical Sys.

or



or



Quantum Sys.
(Bloch Ball)

DIM S = 3



Classical Sys.



or



or



Quantum Sys.
(Bloch Ball)



ANOTHER PRINCIPLES

[P2] Decomposability w.r.t. Distinguishable Pure States:

For any state s , there exists a set of distinguishable pure states $\{s_i\}_i$ such that $s = \sum_i p_i s_i$ with some probability dist. $(p_i)_i$

Any state can be prepared as an ensemble of distinguishable pure states

In QM, this corresponds to Eigenvalue Decomposition of Density Op.

ANOTHER PRINCIPLES

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[P3] Dynamical Generation of State Space by Classical System:

$$\mathcal{S} = \cup_{\Lambda \in G} \Lambda(\text{Simplex})$$

ANOTHER PRINCIPLE

[P2] Decomposability w.r.t.

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Any state can be prepared

In QM, this corresponds to Eigenvalue Decomposition of Density Op.

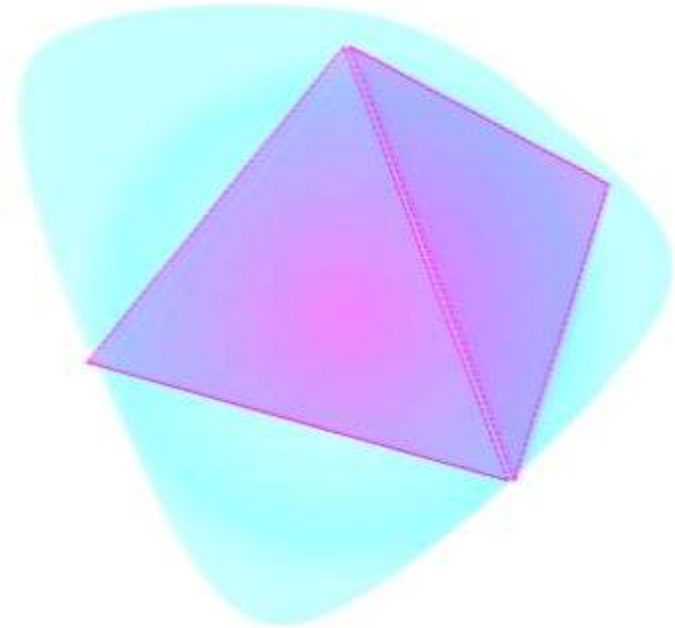
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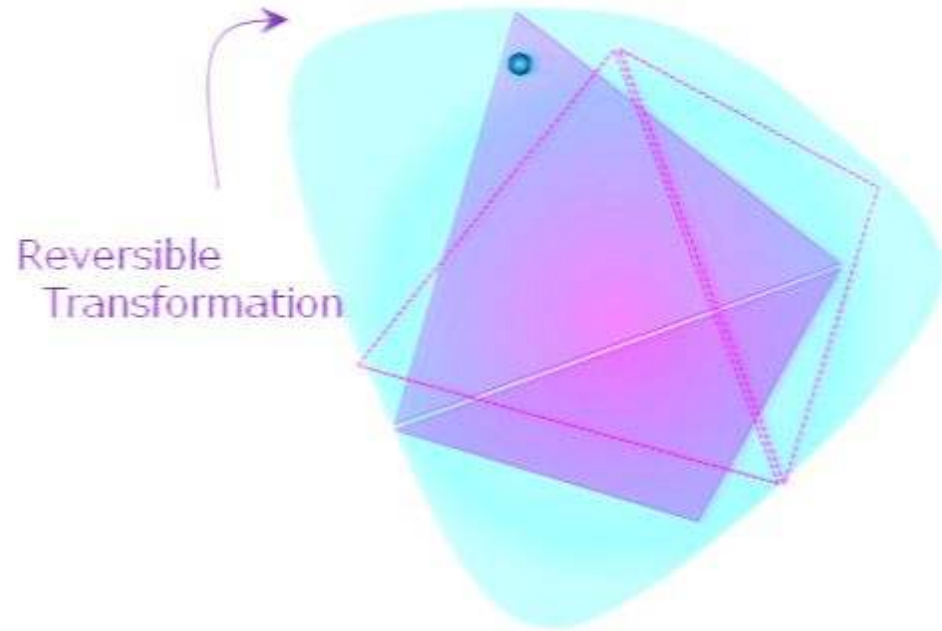
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SYMMETRIC GPT WITH [P2] (OR [P3])

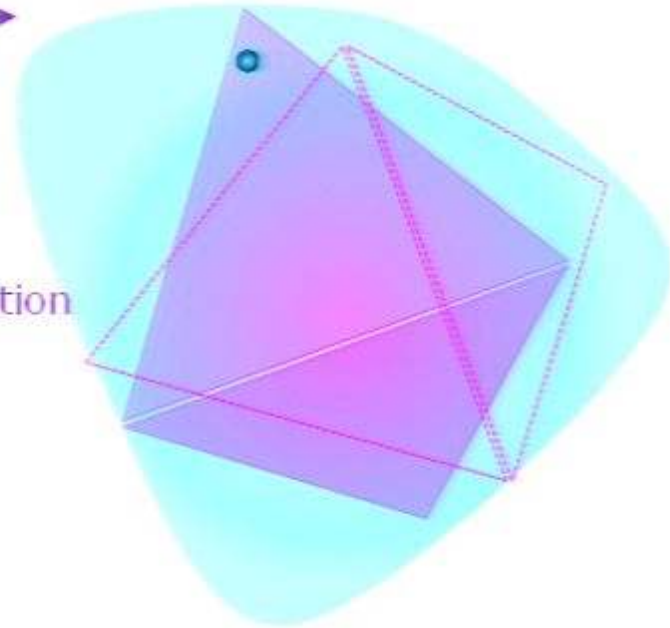
[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.

ANOTHER PRINCIPLE

[P2] Decomposability w.r.t.

For any state s , there exists
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Reversible
Transformation



Any state can be prepared

In QM, this corresponds to Eigenvalue Decomposition of Density Op.

[P3] Dynamical Generation of State Space by Classical System:

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SYMMETRIC GPT WITH [P2] (OR [P3])

[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.

SYMMETRIC GPT WITH [P2] (OR [P3])

[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.



Classical Sys.

SYMMETRIC GPT WITH [P2] (OR [P3])

[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.



Classical Sys.

[Prop] (Dim $S = 3$) Circular Cylinder does not satisfy [P2].

SYMMETRIC GPT WITH [P2] (OR [P3])

[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.



Classical Sys.

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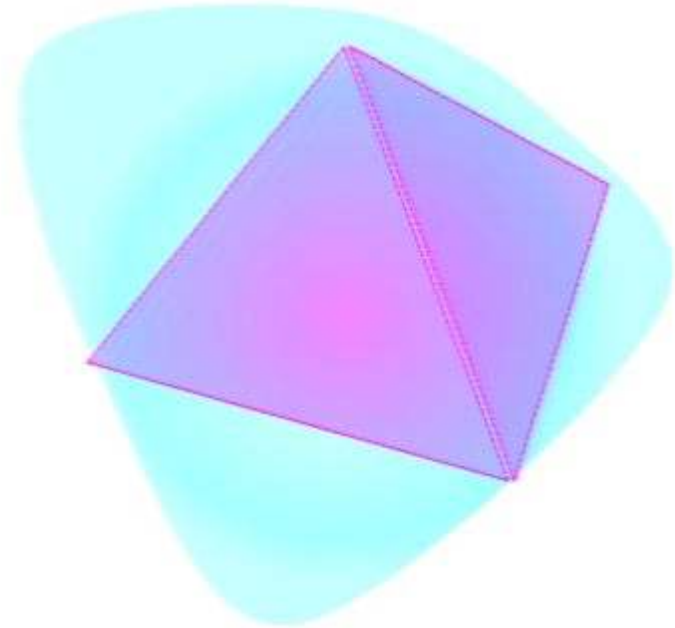
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ANOTHER PRINCIPLE

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In QM, this corresponds to Eigenvalue Decomposition of Density Op.

[P3] Dynamical Generation of State Space by Classical System:

$$\mathcal{S} = \cup_{\Lambda \in G} \Lambda(\text{Simplex})$$

ANOTHER PRINCIPLES

[P2] Decomposability w.r.t. Distinguishable Pure States:

For any state s , there exists a set of distinguishable pure states $\{s_i\}_i$ such that $s = \sum_i p_i s_i$ with some probability dist. $(p_i)_i$



Any state can be prepared as an ensemble of distinguishable pure states

In QM, this corresponds to Eigenvalue Decomposition of Density Op.

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