Title: On Basic Principles of General Probabilistic Theories

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Abstract: We propose an operationally motivated definition of the physical equivalence of states in General Probabilistic Theories and consider the principle of the physical equivalence of pure states, which turns out to be equivalent to the symmetric structure of the state space. We further consider a principle of the decomposability with distinguishable pure states and give classification theorems of the state spaces for each principle, and derive the Bloch ball in 2 and 3 dimensional systems.

ON BASIC PRINCIPLES OF GENERAL PROBABILISTIC THEORIES

May 9-13,2011 Conceptual Foundations and Foils for Quantum Information Processing

Gen Kimura (Shibaura Institute of Technology, JAPAN)

collaboration with Koji Nuida (AIST, JAPAN)

OUTLINE

1: General Probabilistic Theories

2: Motivations and Goal

3: Symmetric GPT
  * Physical Equivalence of Pure States

4: Decomposability w.r.t. distinguishable Pure States

5: Obtain classical and quantum (Bloch Ball) in 3 dim.

6: What I don't know..

GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (1960), Araki (1961);Ludwig (1964-);Mielnik(1968), Devies and Lewis (1970), Gudder (1973), etc

* Operationally Most General Framework for Probability

State + Measurement ⇒ Probability

* Probabilistic Mixture of states (Convex Structure)
* Separation postulates for states and measurements
* Physical Topology measured by probabilities

GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (19 ... 3), etc

$\forall$ states ρ_1, ρ_2 and $p \in [0,1]$,
$\exists$ state $\rho = \langle p; \rho_1, \rho_2 \rangle$
as a preparation of ρ_1 with p and ρ_2 with $1-p$:

* Operationally M

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$\exists$ state $\rho = \langle p; \rho_1, \rho_2 \rangle$
as a preparation of ρ_1 with p and ρ_2 with $1-p$:

$$\Pr\{M = m \mid \langle p; \rho_1, \rho_2 \rangle\} = p\Pr\{M = m \mid \rho_1\} + (1-p)\Pr\{M = m \mid \rho_2\}$$

for any measurement M

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* Operationally Most General Framework for Probability

1 Separation Postulate for states:
$\forall$ measurement M and outcome m,
$\Pr\{m|M,\rho_1\} = \Pr\{m|M,\rho_2\}$, then $\rho_1 = \rho_2$

bability

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* Physical Topology

2 Separation Postulate for measurements:
$\forall$ states ρ,
$\Pr\{m|M_1,\rho\} = \Pr\{m|M_2,\rho\} \forall m$, then $M_1 = M_2$

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* Operationally Most General Framework for Probability

State + Measurement $\Rightarrow$ Probability

★ State space S is embedded into a convex subset in a real vector sp.
s.t. $\langle p; \rho_1, \rho_2 \rangle = p\rho_1 + (1-p)\rho_2$

★ Measurement is represented by effects:

$$E := (e_i \in \mathcal{E})_{i=1}^n, s.t. \sum_i e_i = u, \ e_i(\rho) : \text{prob. for } i\text{th output under } \rho$$

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$e \in \mathcal{E}$: effect on $\mathcal{S}$:
$\Leftrightarrow \ e : \mathcal{S} \to [0,1]$ affine:
$e(p\rho_1 + (1-p)\rho_2) = pe(\rho_1) + (1-p)e(\rho_2)$

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Weakest topology s.t.
all effects are continuous on $\mathcal{S}$.

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⇒ Theory of Convex Set and Affine Function

(PreCompact Convex Set in Locally Convex Hausdorff Topological Vector Space)

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If $\mathcal{S} = d < \infty$

$\mathcal{S}$ is a compact (i.e. closed bounded) convex of E^d

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If $\mathcal{S} = d < \infty$

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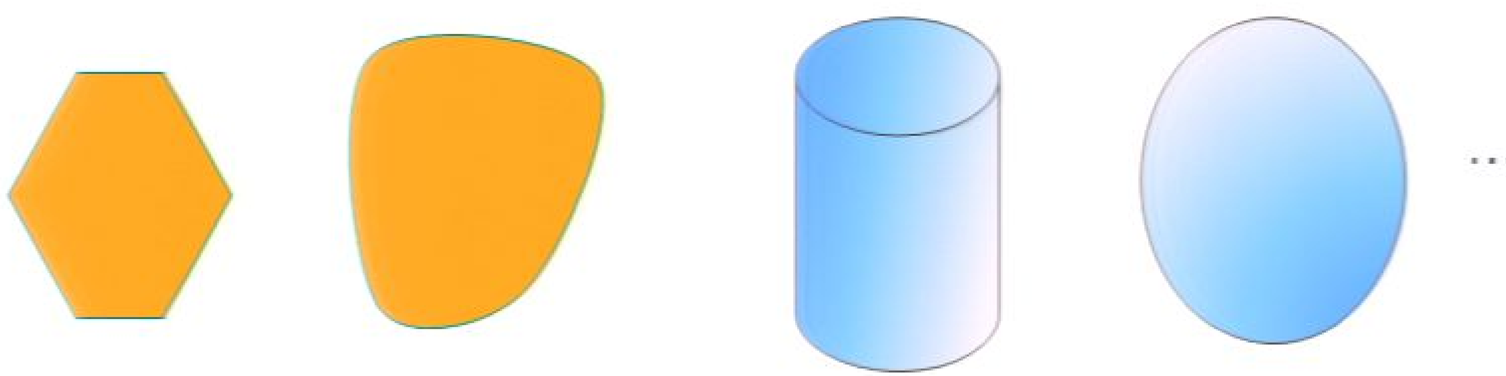
Any compact convex set you imagine,



you can consider its general probabilistic model.

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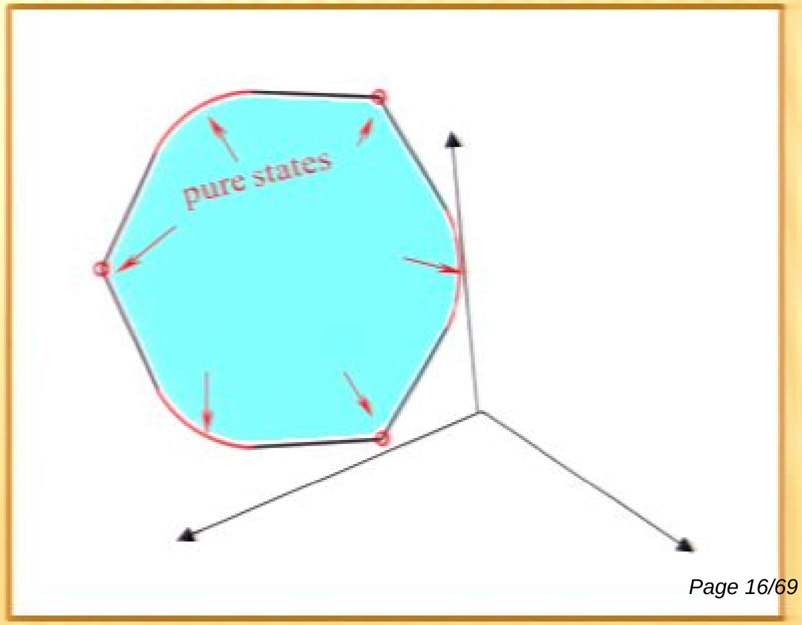
Pure State

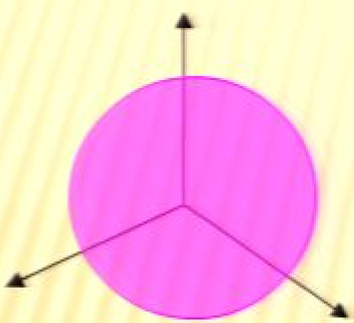
$\overset{\text{def}}{\Leftrightarrow}$ State which cannot be prepared by probabilistic mixtures of different states.

$\Leftrightarrow$ Extreme Point of State Space

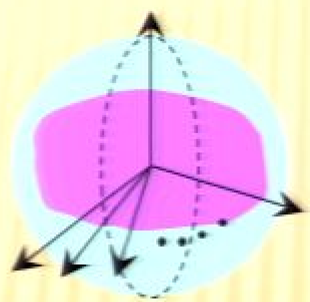


EXAMPLES OF GPT II

★ Quantum System: $\mathcal{S}_d^Q := \{\rho \in \mathcal{L}(\mathcal{H}_d) \ : \ \rho \geq 0, \ \mathrm{tr}\rho = 1\}$



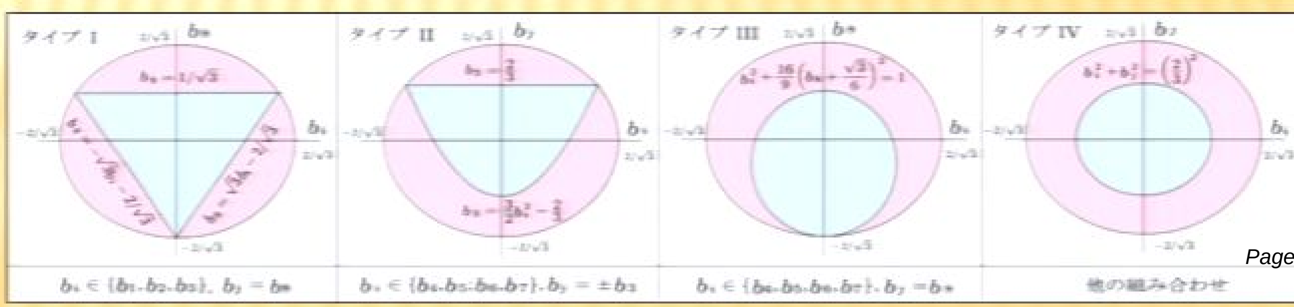
$d = 2$



$d \geq 3$

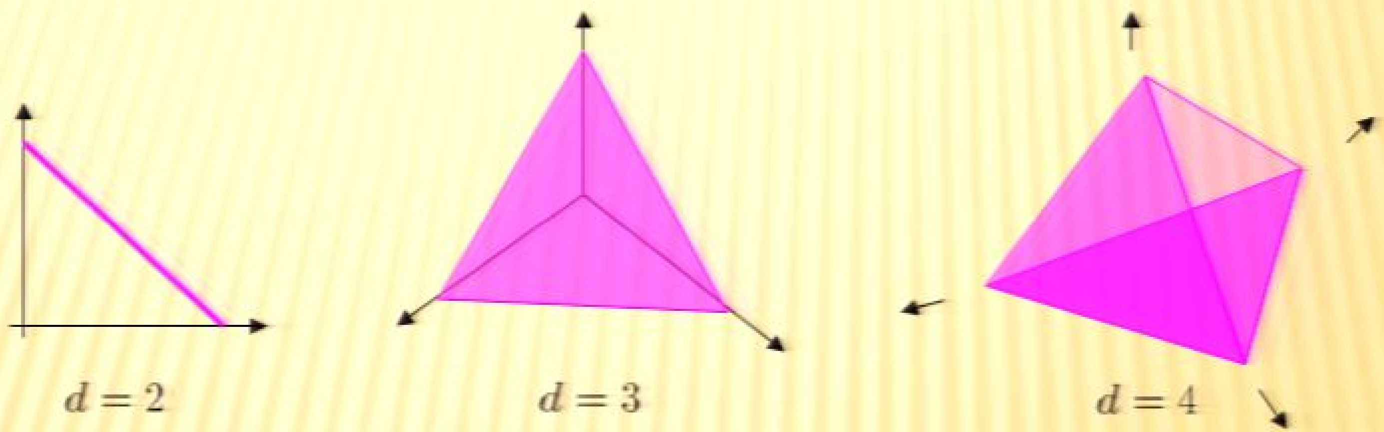
. . .

* ∞ Extreme Points



EXAMPLES OF GPT I

★ Classical System: $\mathcal{P}_d := \{\boldsymbol{p} = (p_1, \ldots, p_d) \in \mathbb{R}^d \mid p_i \geq 0,\ \sum_i p_i = 1\}$



$d = 2$ $d = 3$ $d = 4$ ...

Simplex of Affine Dimension d-1 with d Pure States

EXAMPLES OF GPT II

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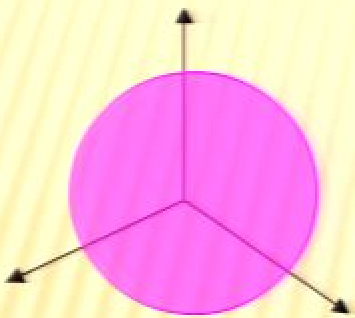

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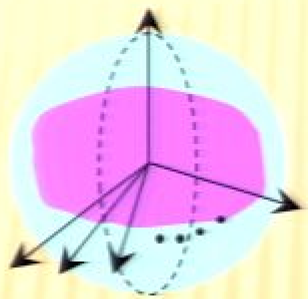

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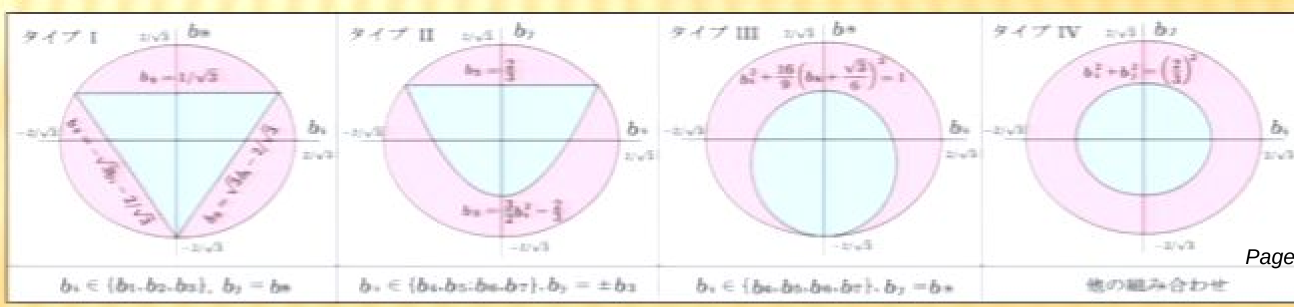
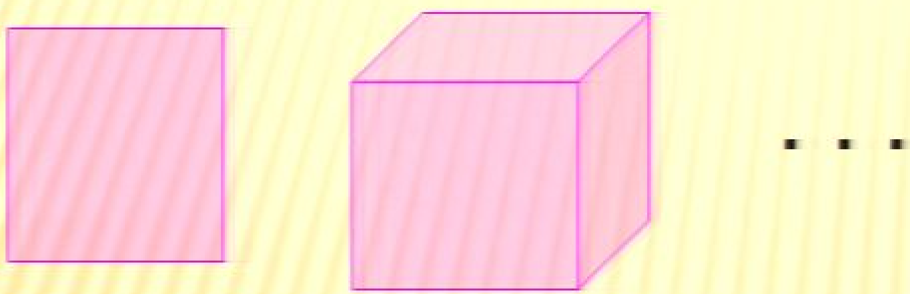


EXAMPLES OF GPT III

★ Hyper Cuboid System: $C_d = \{\boldsymbol{x} \in \mathbb{R}^d \mid 0 \leq x_i \leq 1\}$



$d = 2$ $d = 3$

* 2^d Extreme Points
* Affine Dimension d

EXAMPLES OF GPT III

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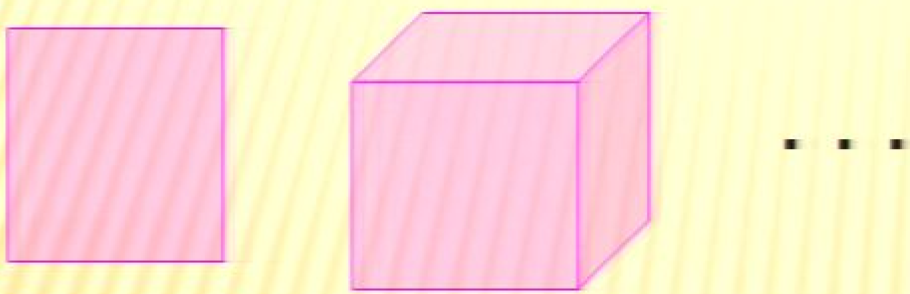

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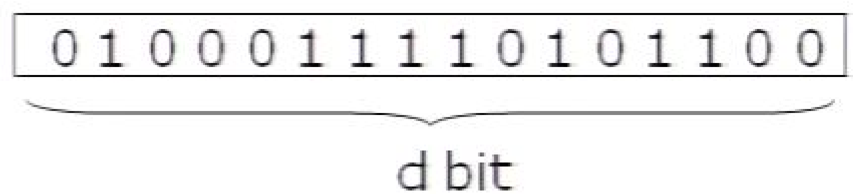

* 2^d Extreme Points
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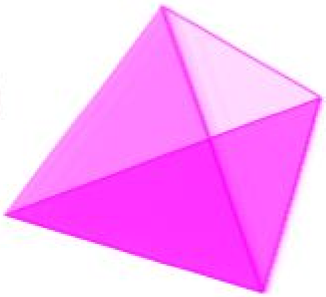
EXAMPLES OF GPT III

$C_d = \{\boldsymbol{x} \in \mathbb{R}^d \mid 0 \le x_i \le 1\}$

Classical



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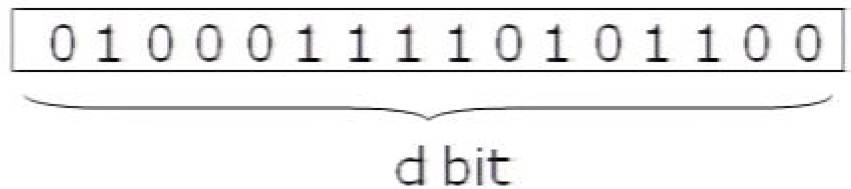

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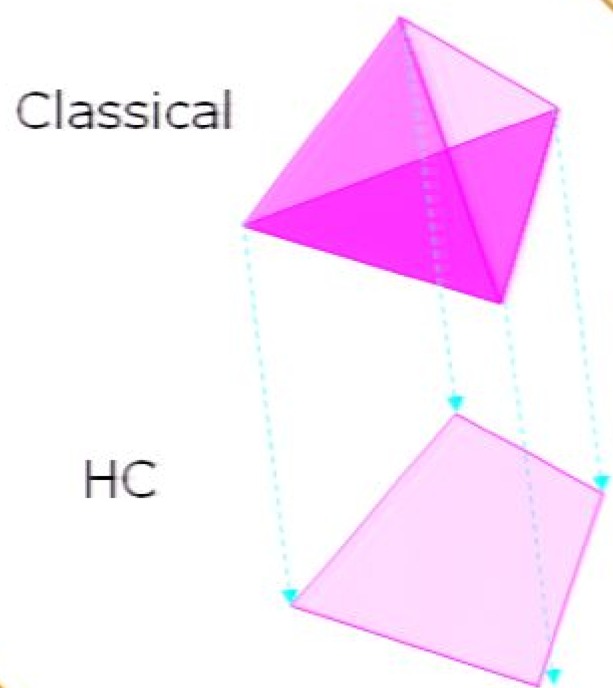

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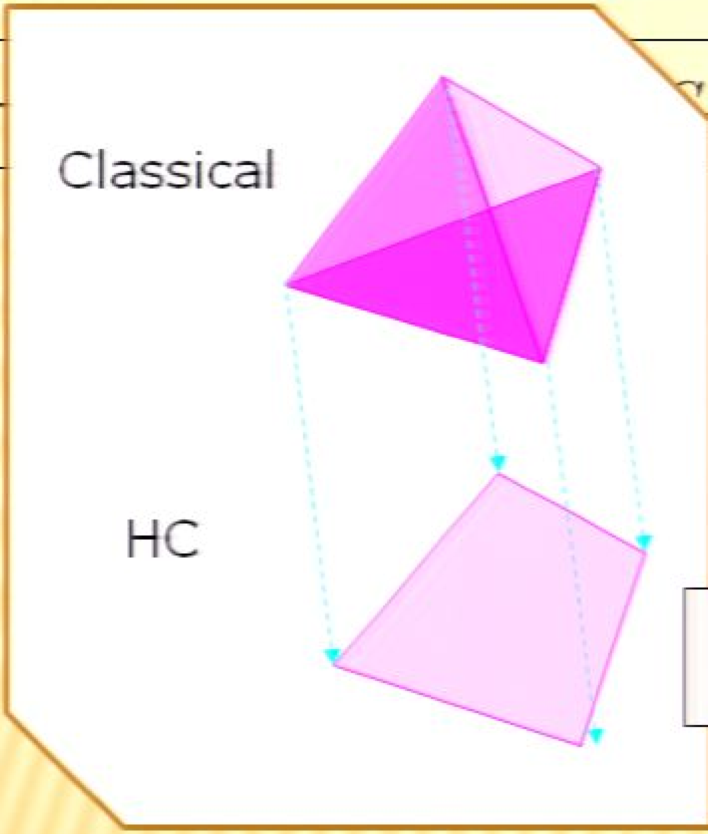

. . .

0 1 0 0 0 1 1 1 1 0 1 0 1 1 0 0

d bit

Prop: Hyper Cuboid system Cd can be realized by a
d bit classical system by restricting to 1 bit measurement

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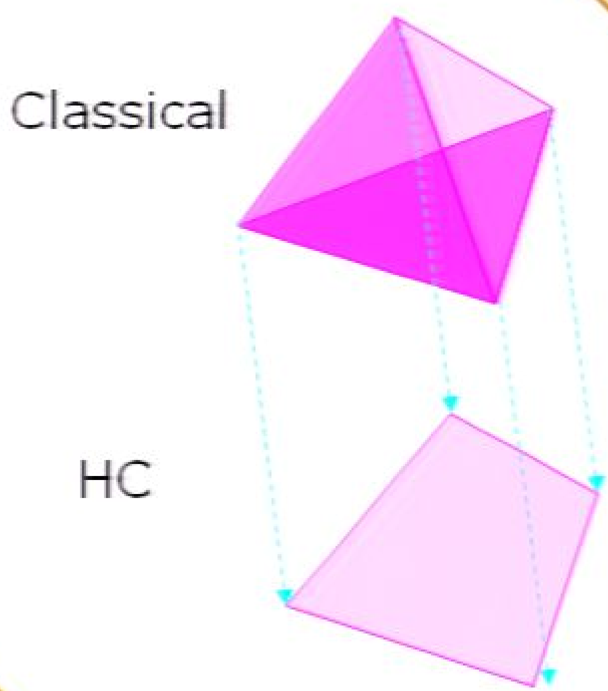

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Prop: Hyper Cuboid system Cd can be realized by a d bit classical system by restricting to 1 bit measurement

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Thm: [Holevo 1982] Any GPT can be realized by a Classical System with an appropriate restriction on measurements

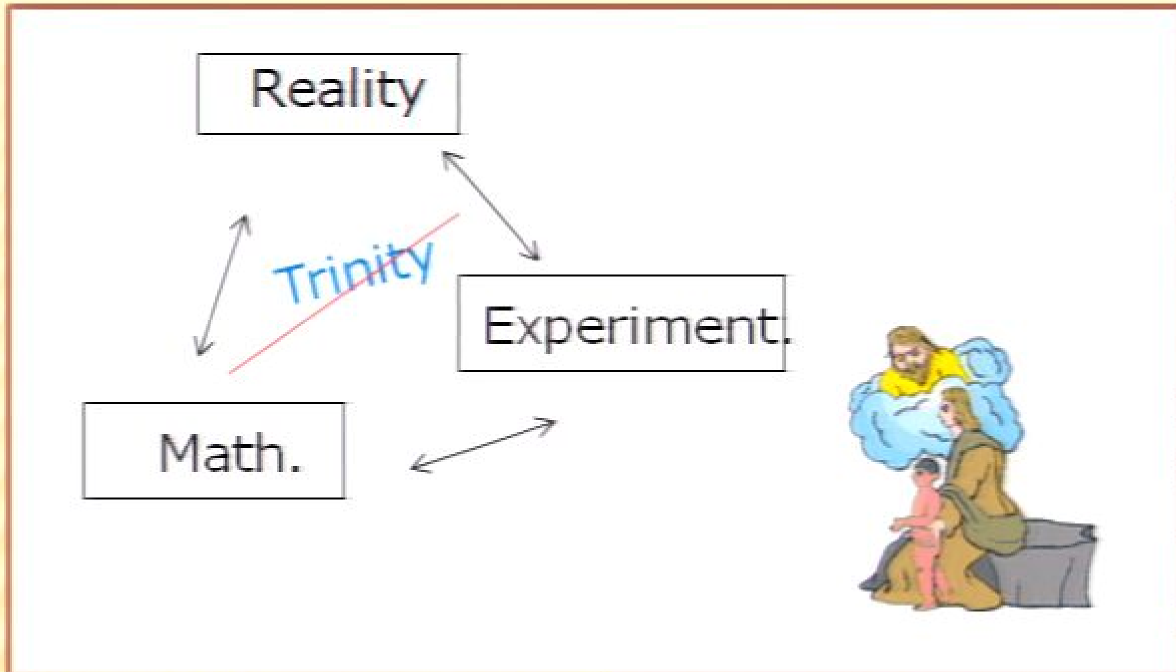
INFORMATION THEORY BASED ON GPT

✱ Classical Prob. Theory $\Rightarrow$ Classical Information Theory
✱ Quantum Theory $\Rightarrow$ Quantum Information Theory

GPT => GPT Information Theory

Motivations

- Seek for Physical Principles of Quantum Theory in terms of information languages
- Understand logical connections between physical principles and information processings.
- Preparation for "post" quantum
- (Classical/Quantum)Information theory under Measurement restrictions



SED ON GPT

ormation Theory
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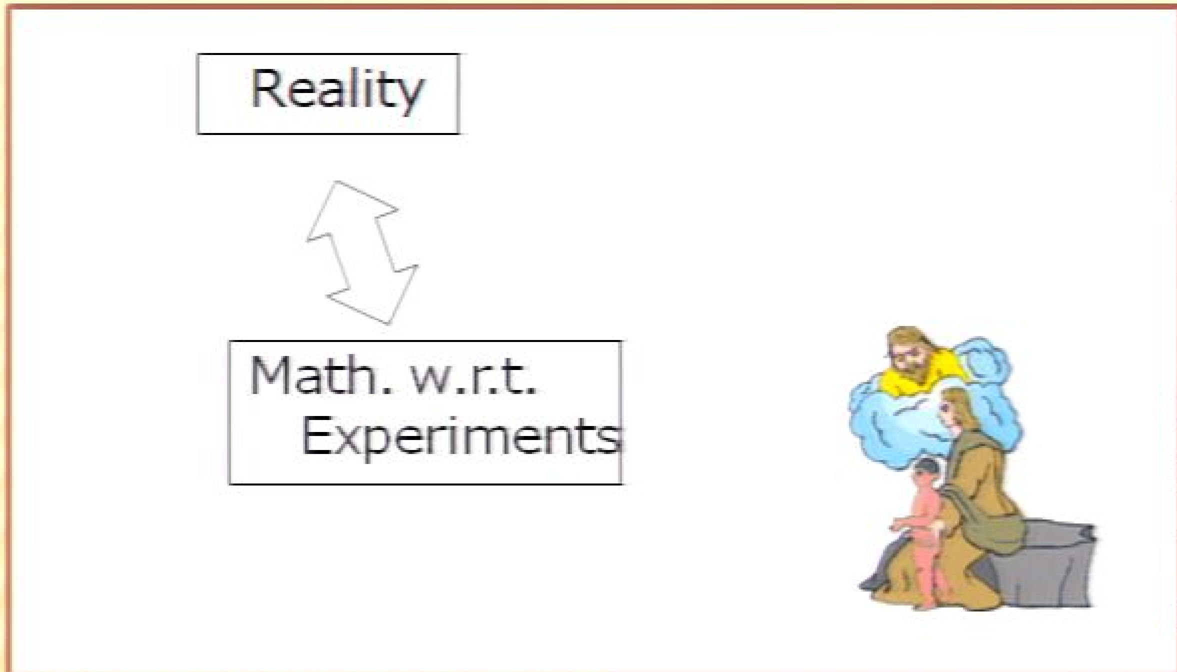

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INFORMATION THEORY BASED ON GPT

[1] Hardy (2001); arXiv:quant-ph/0101012v4
[2] Dakic and Brukner (2009); arXiv:0911.0695v1
[3] Masanes and Muller (2010); arXiv:1004.1483v2
[4] Chiribella, D'Ariano, and Perinotti (2010); arXiv:1011.6451v1

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THIS TALK…

"Natural" Physical Principles:

*1 Physical Equivalence of Pure States

$\Leftrightarrow$ Symmetricity of State Space (Davies)

$\Leftrightarrow$ Reversible Connections of Pure States (Hardy etc.)

*2 Decomposability w.r.t. Distinguishable Pure States

$\Rightarrow$ Classification of GPTs for each Principle

* Kimura, Nuida, Imai (2010); arXiv:1012.5361v2

* Kimura, Nuida, (2010); arXiv:1012.5350v2

THIS TALK...

What We also Assume:

* Existence of Objective (Classical) World
* Causality
* All Mathematically Well-defined Measurements:

$$(e_i \in \mathcal{E})_i \text{ s.t. } \sum_i e_i = u$$

are physically realizable.

SYMMETRICITY OF STATE SPACE

[P1] Group of Affine Bijection on S acts transitively on ∂S

(Davies 1974)

⇔ [P1′] Any pure states are connected by Reversible Transformation

(Hardy, …)

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⇔ [P1″] Any pure states are Physically Equivalent

meaning that there are no "special" Pure States

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PHYSICAL EQUIVALENCE OF STATES

State s1 and s2 have physically the same properties if

* For any measurement E1,
there uniquely exists measurement E2 s.t.
prob. dist of E1 under s1 equals prob. dist. of E2 under s2
* The correspondence should preserves
probabilistic mixture of measurements

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[Def] State s_1 and s_2 are physically equivalent iff there exists unit preserving affine bijection Λ on $\mathcal{E}$ such that

$$e(s_1) = \Lambda(e)(s_2) \ \forall e \in \mathcal{E}$$

* Physical Equivalence is an equivalence relation
* Physical Equivalence in QM is unitary equivalence:

$$\rho_1 \simeq \rho_2 \Leftrightarrow \exists unitary\ U s.t. \rho_1 = U \rho_2 U^\dagger$$

PHYSICAL EQUIVALENCE OF STATES

[Lem] For any affine functionaol $\Phi : \mathcal{E} \to [0, 1]$ satisfying $\Phi(u) = 1$ and $\Phi(0) = 0$, there uniquely exists a state $s \in \mathcal{S}$ such that $\Phi(e) = e(s)$ for any $e \in \mathcal{E}$.

> [Thm] State s_1 and s_2 are physically equivalent iff there exists an affine bijection Ψ on $\mathcal{S}$ such that $s_1 = \Psi(s_2)$.

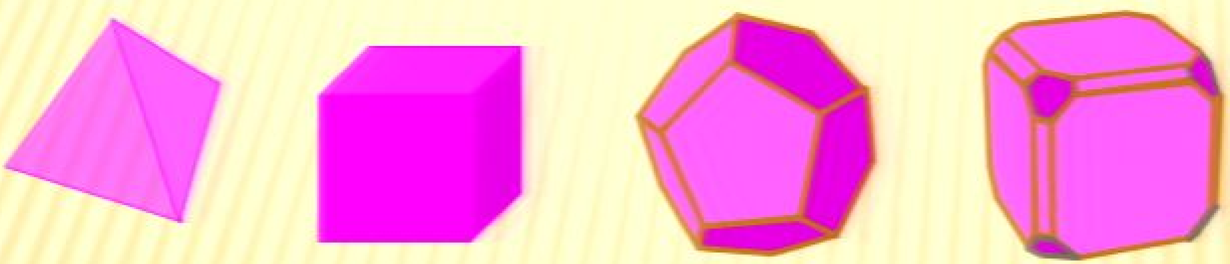
[P1′] Any pure states are connected by Reversible Transformation

$\Leftrightarrow$ [P1″] Any pure states are Physically Equivalent

We call Symmetric GPT if it satisfy [P1] ([P1′] or [P1″])

CLASSIFICATION OF SYMMETRIC GPTS

[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorhic to Isogonal Figure



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CLASSIFICATION OF SYMMETRIC GPTS

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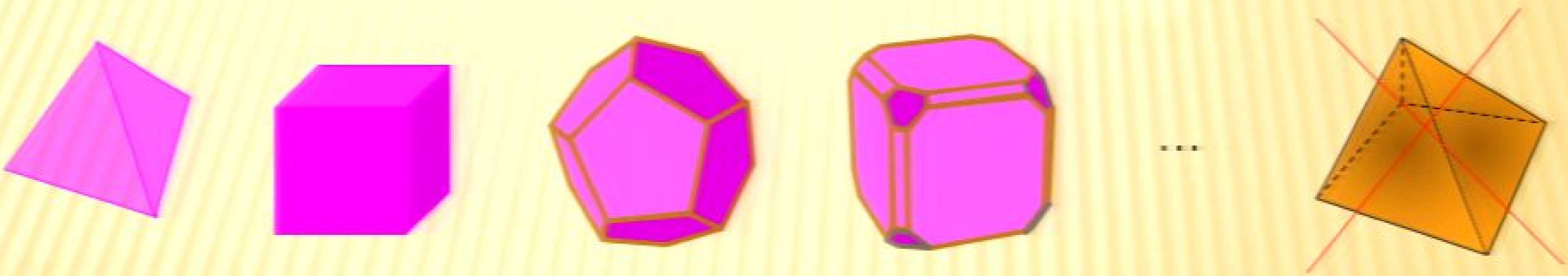

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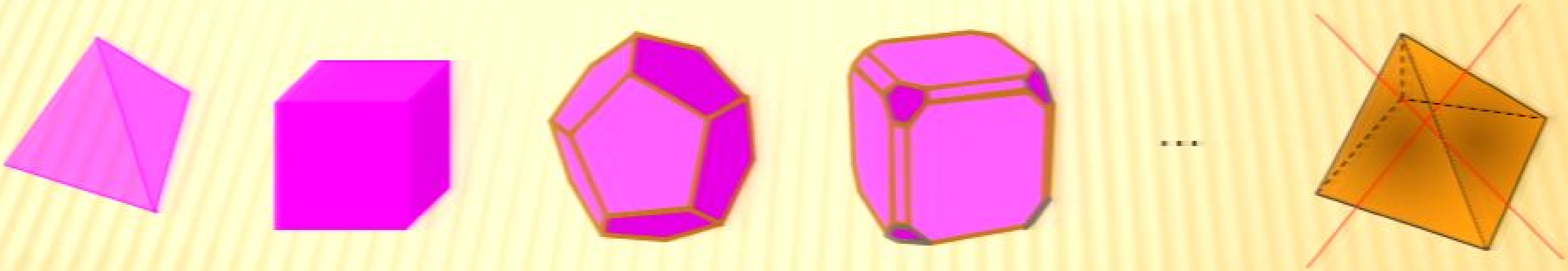
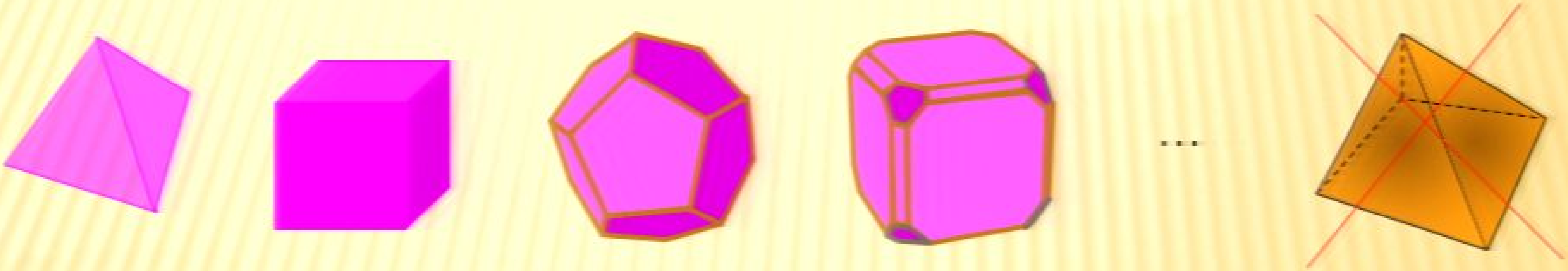


[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorhic to a disk.

Dim S = 2

CLASSIFICATION OF SYMMETRIC GPTS

[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorhic to Isogonal Figure

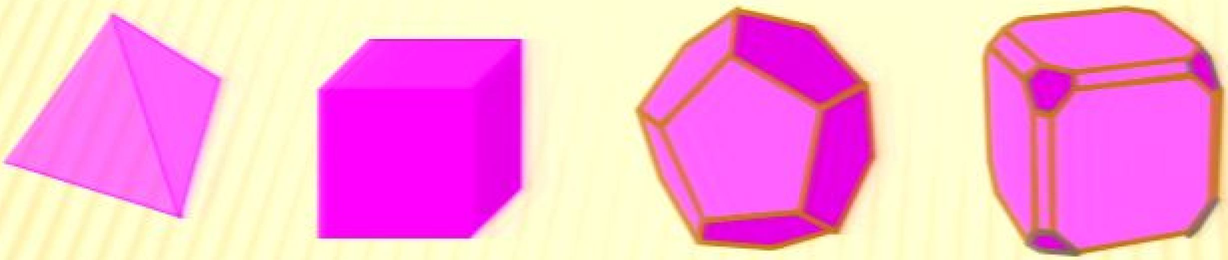


[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorhic to a disk.

[Thm] State Space of 3-dimensional Symmetric GPT with infinite numbers of pure states is either a ball or a circular cylinder.

Dim S = 2  Dim S = 3  or 

DIM S = 3



...

Classical Sys.

or

or

Quantum Sys.
(Bloch Ball)

DIM S = 3

Classical Sys.

or

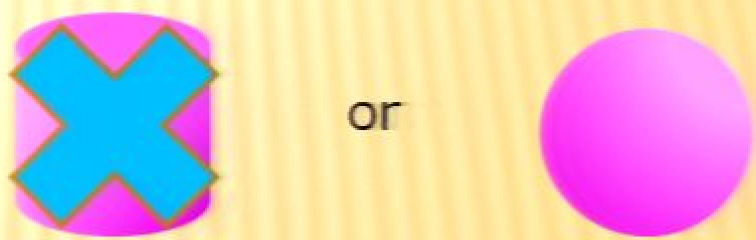
or

Quantum Sys.
(Bloch Ball)

DIM S = 3

Classical Sys.

or



or

Quantum Sys.
(Bloch Ball)

[P2] Decomposability w.r.t. Distinguishable Pure States

ANOTHER PRINCIPLES

[P2] Decomposability w.r.t. Distinguishable Pure States:

For any state s, there exists a set of distinguishable pure states $\{s_i\}_i$ such that $s = \sum_i p_i s_i$ with some probability dist. $(p_i)_i$

Any state can be prepared as an ensemble of distinguishable pure states

In QM, this corresponds to Eigenvalue Decomposition of Density Op.

ANOTHER PRINCIPLES

[P2] Decomposability w.r.t. Distinguishable Pure States:

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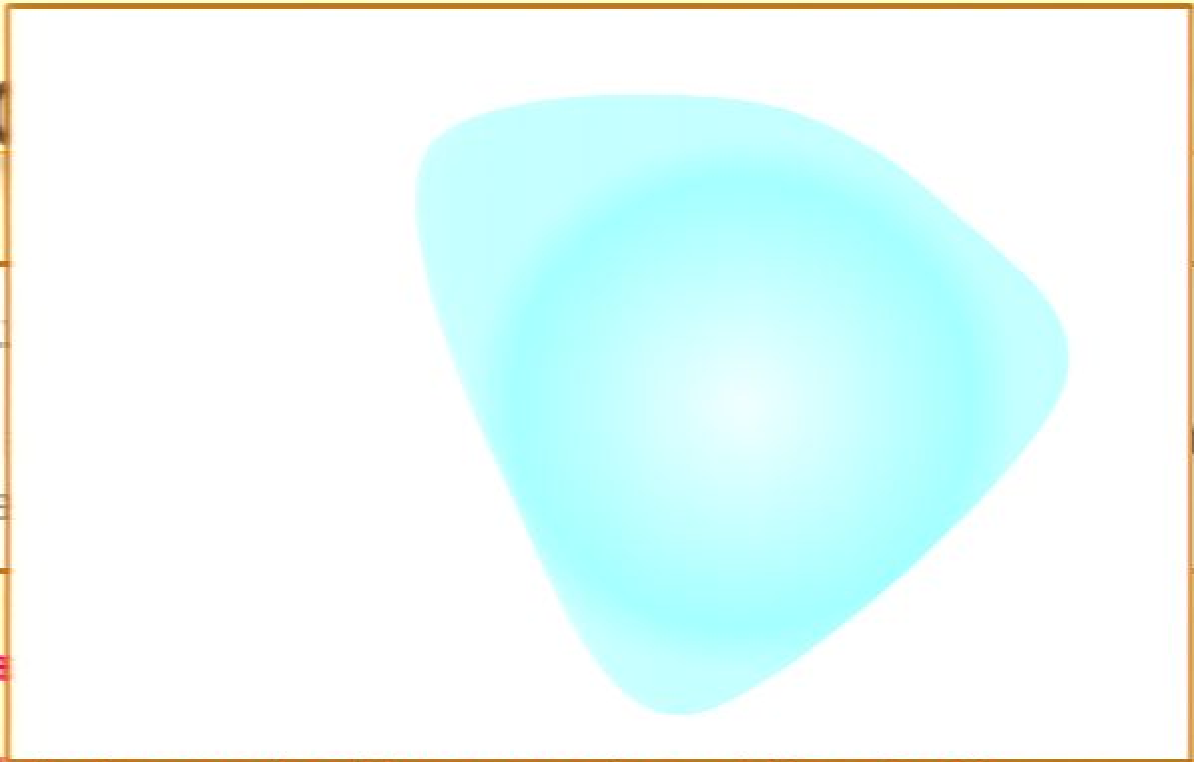

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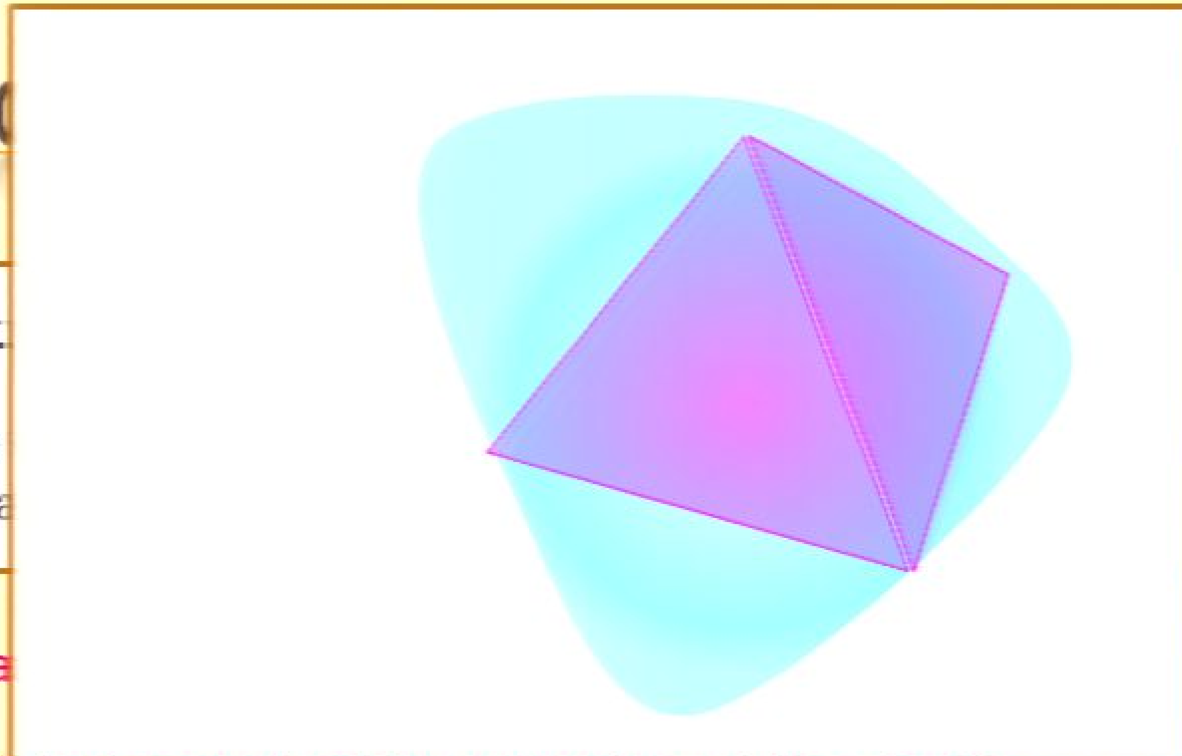

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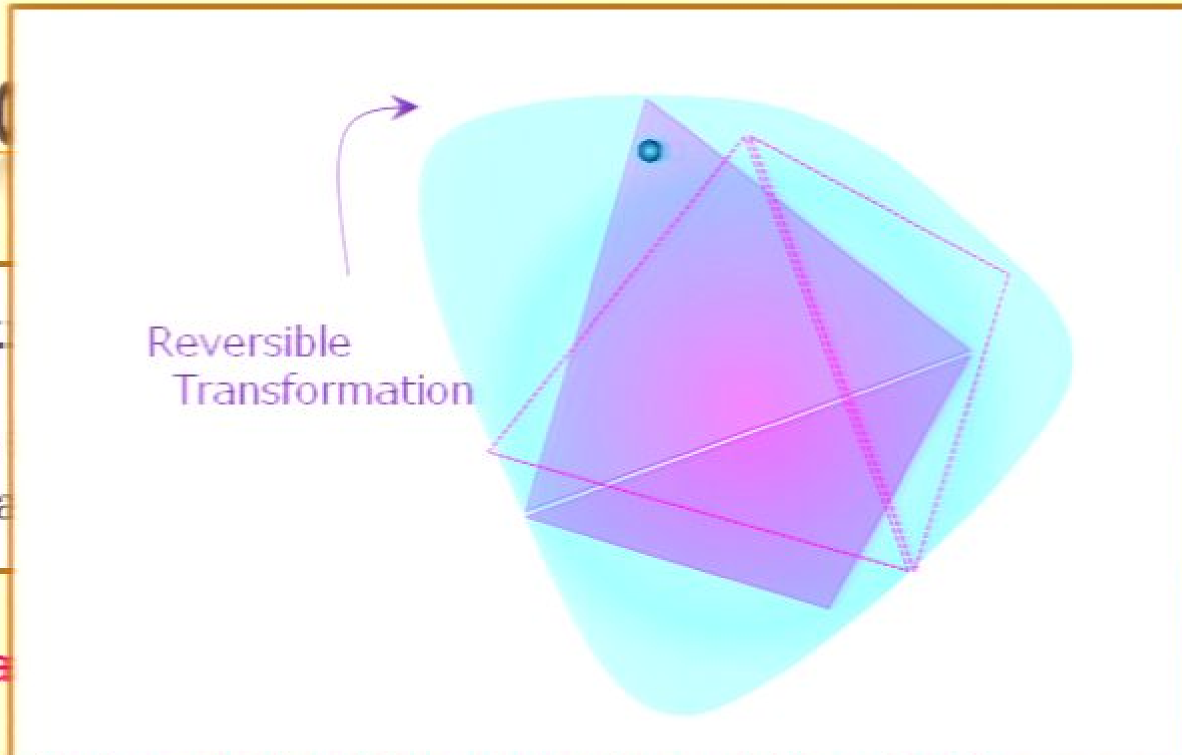

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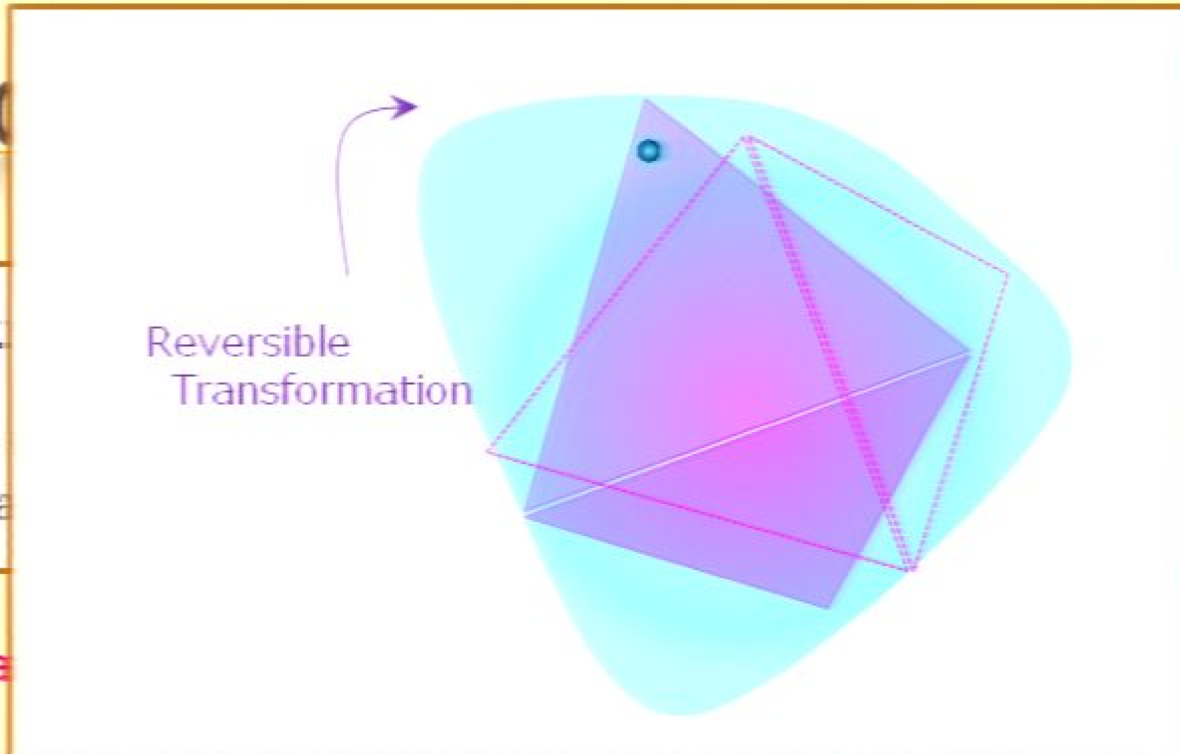

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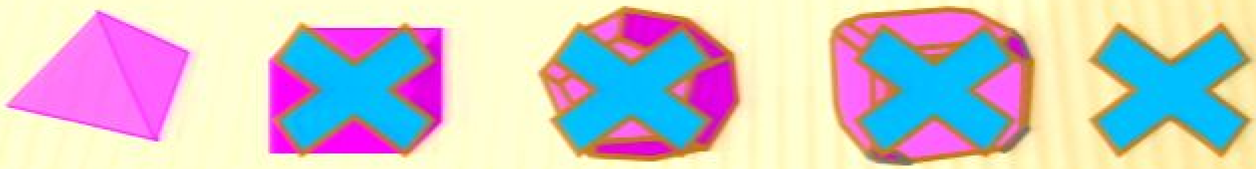

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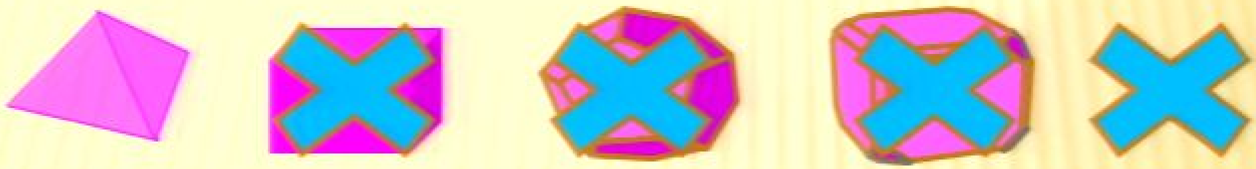

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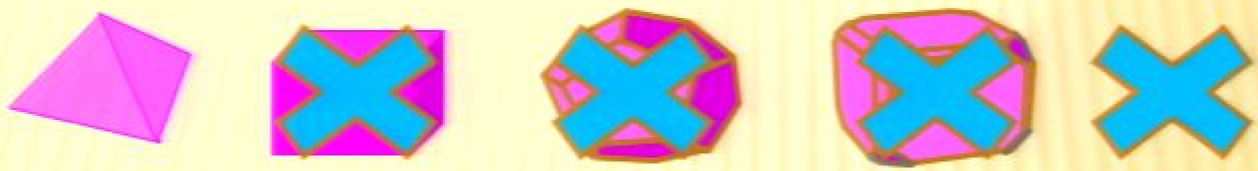

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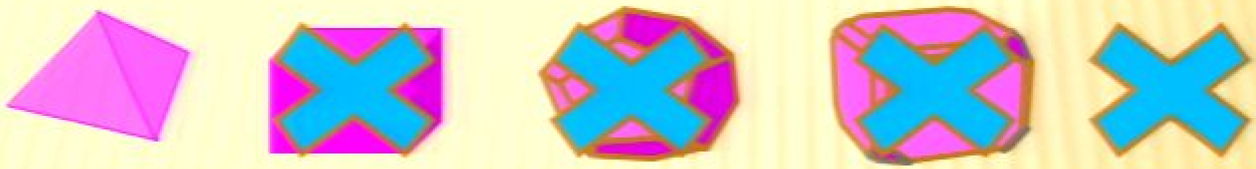

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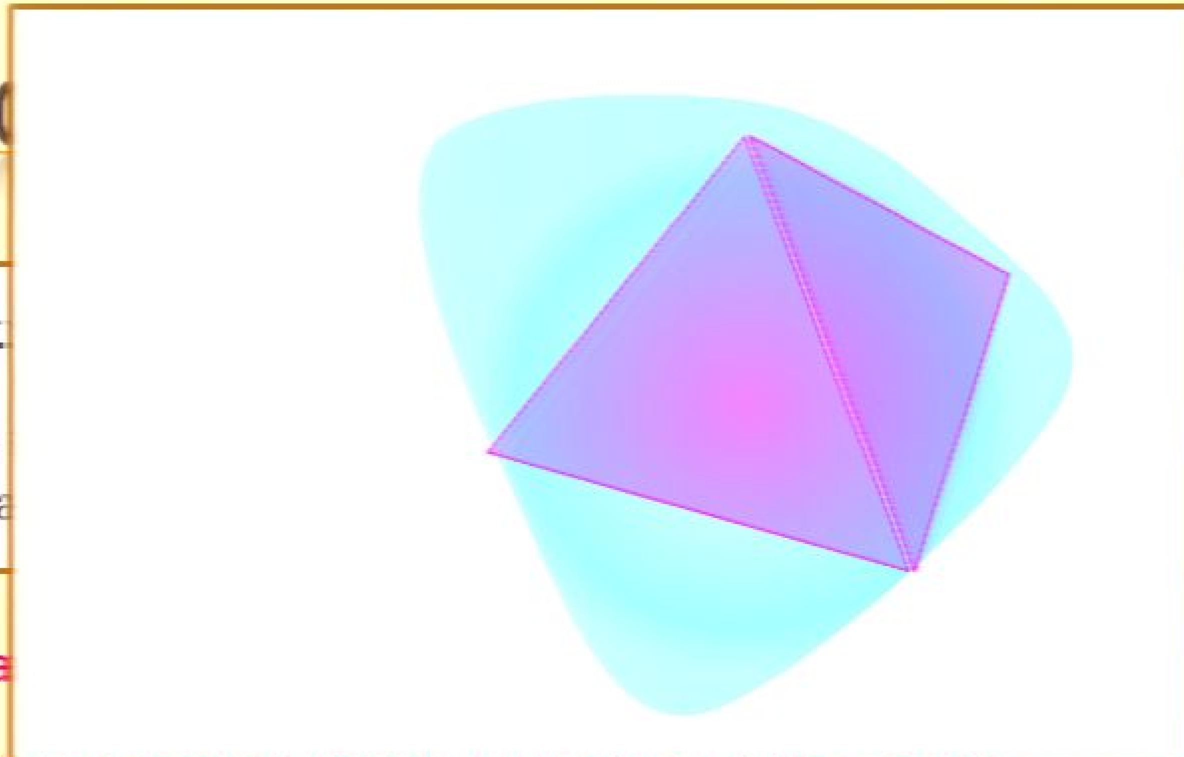

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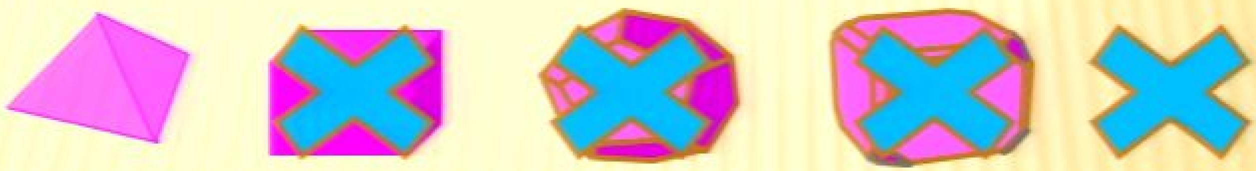

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