Title: Quantum Theory as a Theory of Information Processing

Date: May 09, 2011 11:40 AM

URL: http://pirsa.org/11050040

Abstract: Quantum Theory can be derived from six operational axioms. We introduce the operational and probabilistic language that is used to formulate the principles. After the basic notions of system, state, effect and transformation are reviewed, the principles are stated, and their immediate consequences and interpretations are analyzed. Finally, some key results that represent milestones of the derivation are discussed, with particular focus on their implications on information processing and their relation with the standard quantum formalism. The global picture of the presentation highlights quantum theory as a particular operational language emerging from a background of information processing theories, thanks to the purification postulate that singles out the strictly quantum features of information.

In collaboration with

- G. M. D'Ariano

- G. Chiribella

G. Chiribella, G. M D'Ariano and PP, Phys. Rev. A **81**, 062348 (2010)

G. Chiribella, G. M. D'Ariano and PP, arXiv:1011.6451

Outline

- The operational and probabilistic framework

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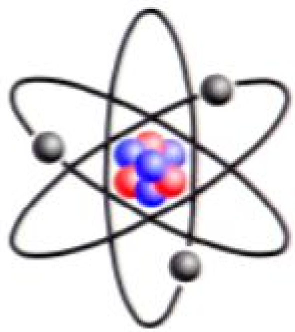
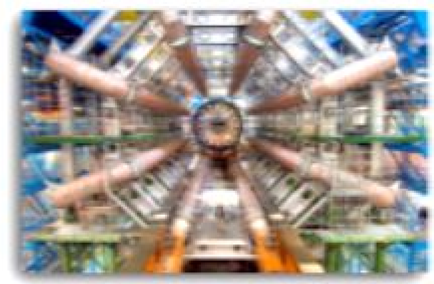
- The principles
- Milestones of the derivation

Why new axioms?

- Quantum theory is successful





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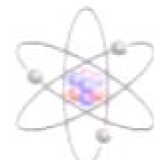

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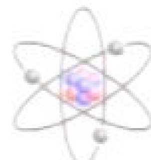


- QT is counterintuitive

- Interpretations of QT p

Why new axioms?

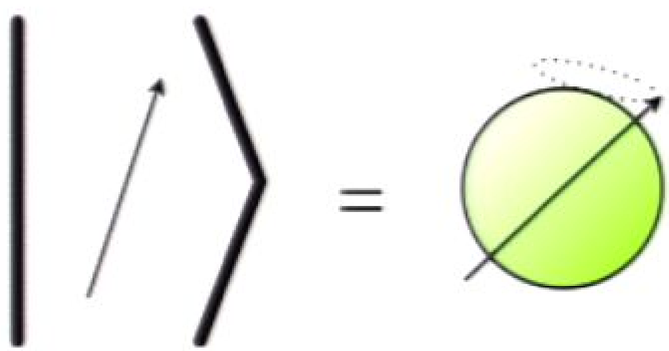
- Quantum theory is successful


- QT is counterintuitive

- Interpretations of QT produce debate

Why new axioms?

- Main reason for debate: axioms of QT are purely **mathematical**
- Foundational questions historically focused on **interpretation**
- E.g.: EPR paradox and Bell's theorem



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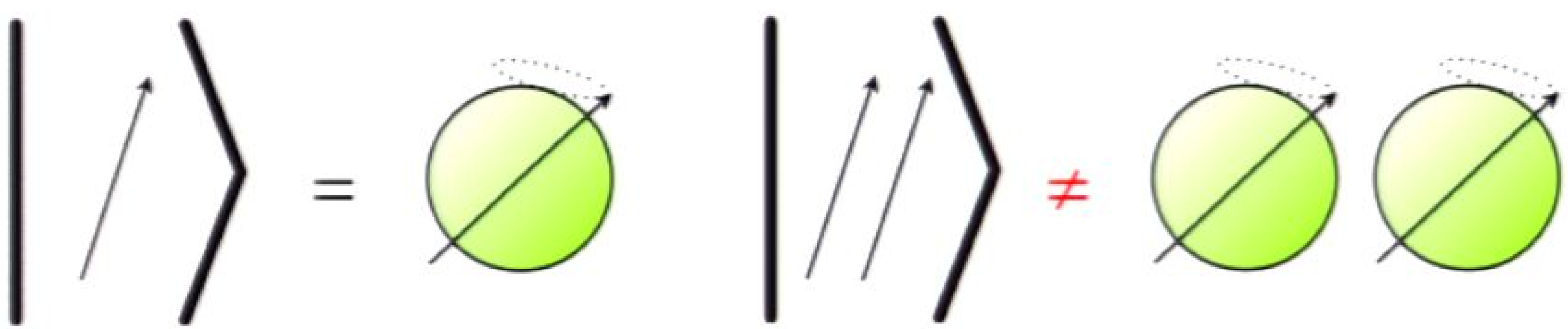

Quantum Information and Foundations

- Attitude completely reversed after the advent of QI theory
  - Exploit quantum theory:

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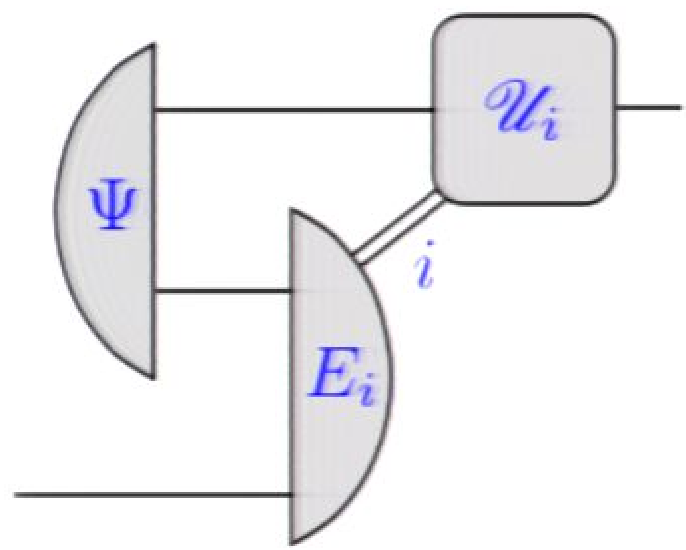
    - teleportation



Quantum Information and Foundations

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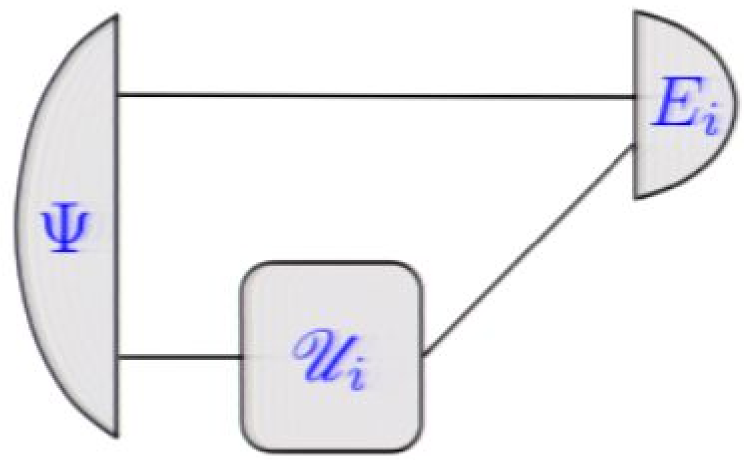
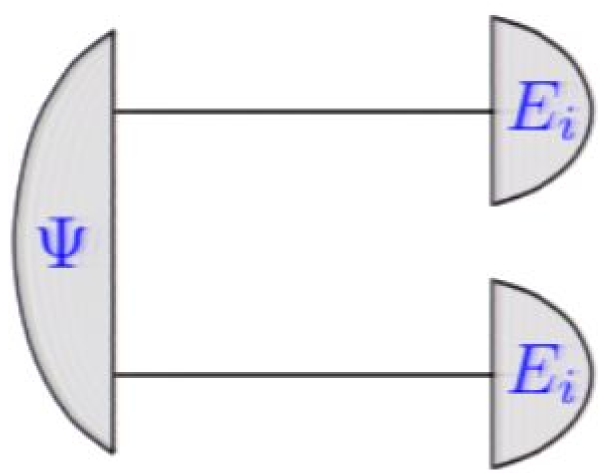


Quantum Information and Foundations

- Attitude completely reversed after the advent of QI theory
  - Exploit quantum theory:
    - teleportation
    - dense coding
    - key distribution
    - etc.



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- Physical axioms
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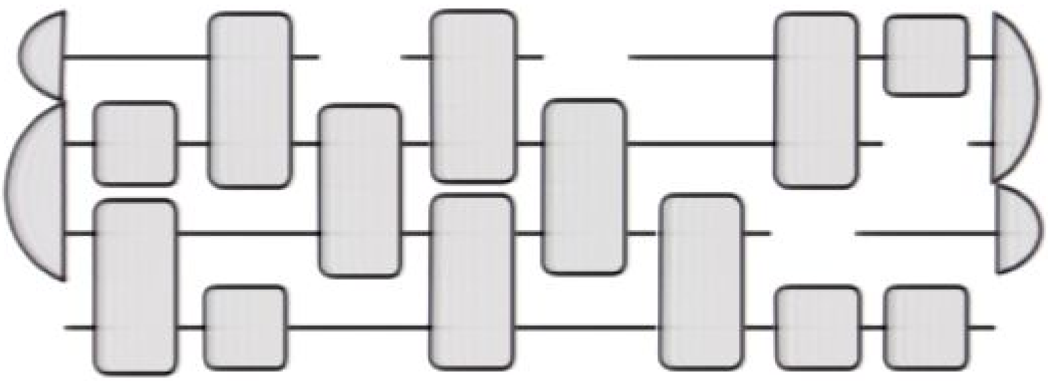
  - The axioms must be stated in an operational language
  - The axioms can be translated into a suitable mathematical language
- Axioms about information processing capabilities

The operational language

- Operational theory: tests with composition rules



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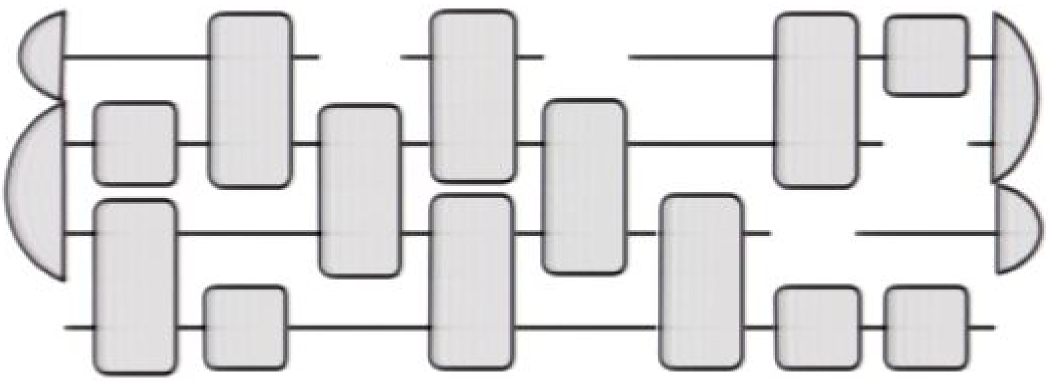

i∈X: outcome

$\mathscr{C}_i$: event of the test

The operational language

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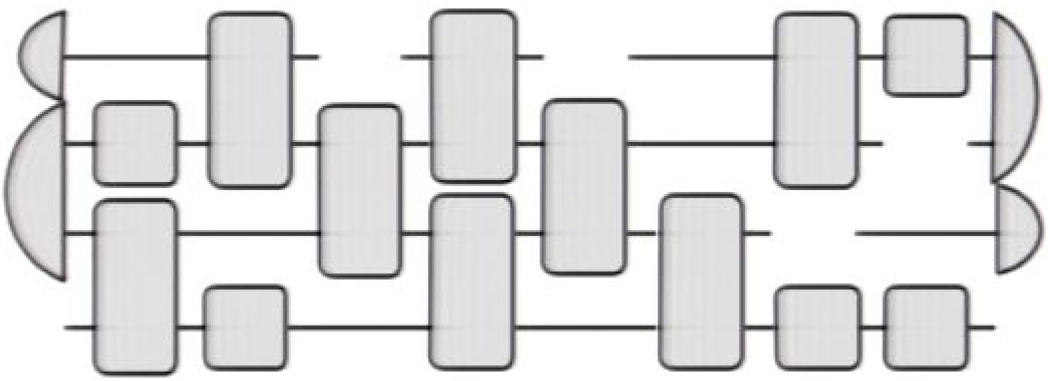

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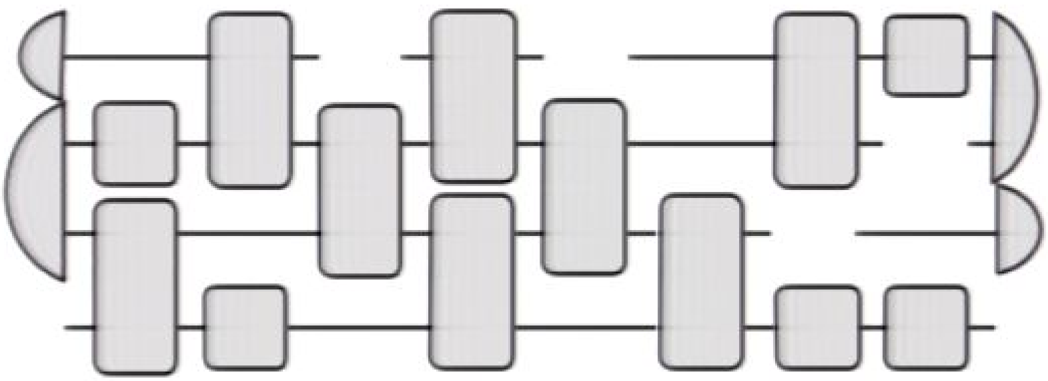

A : input label
B : output label

The operational language

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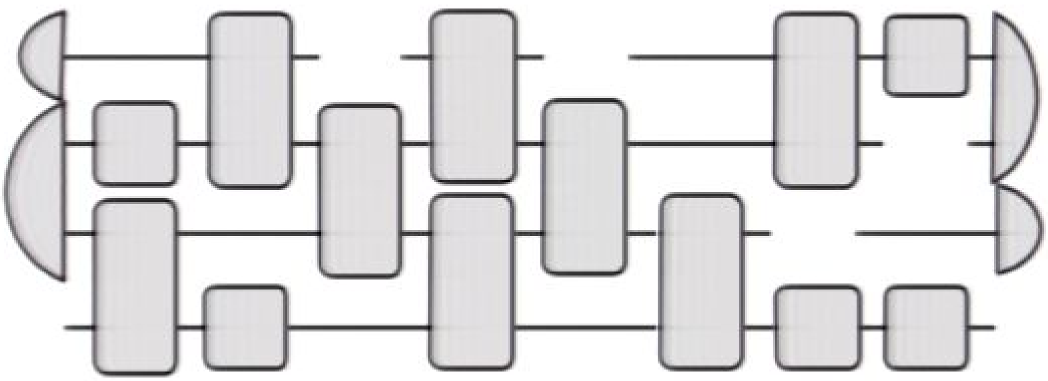

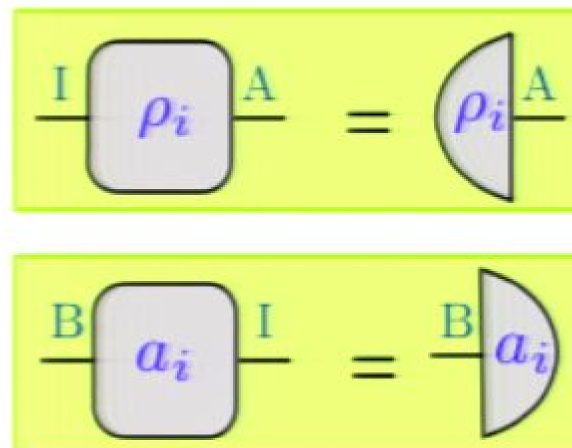

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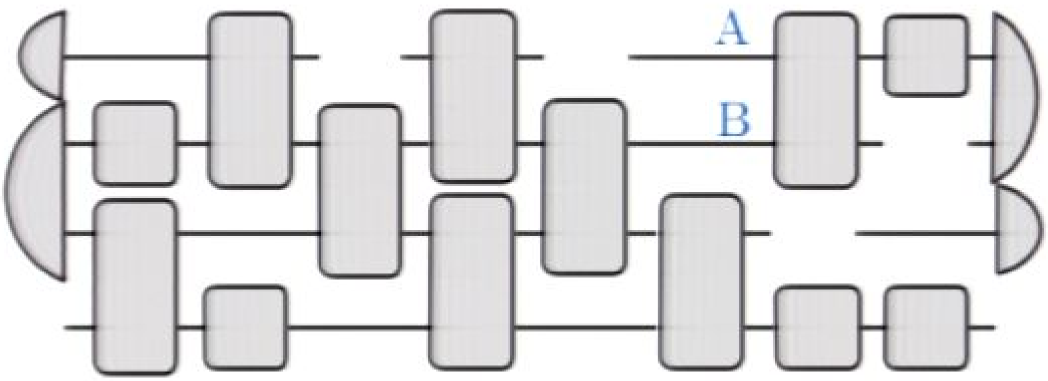
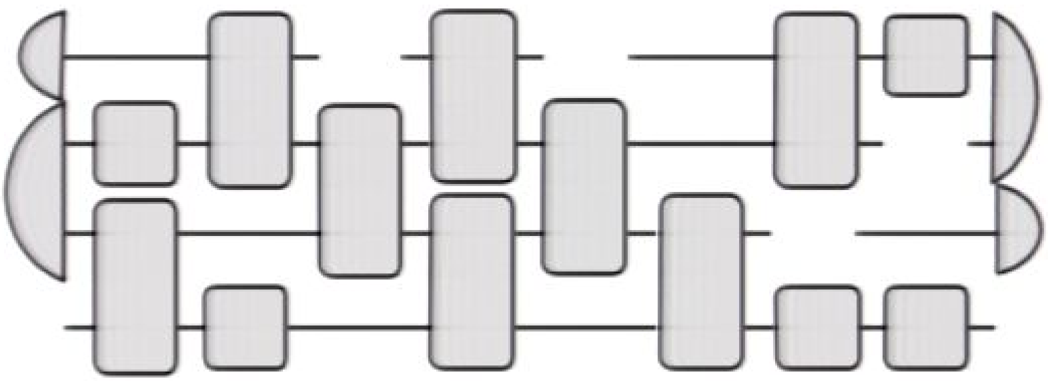


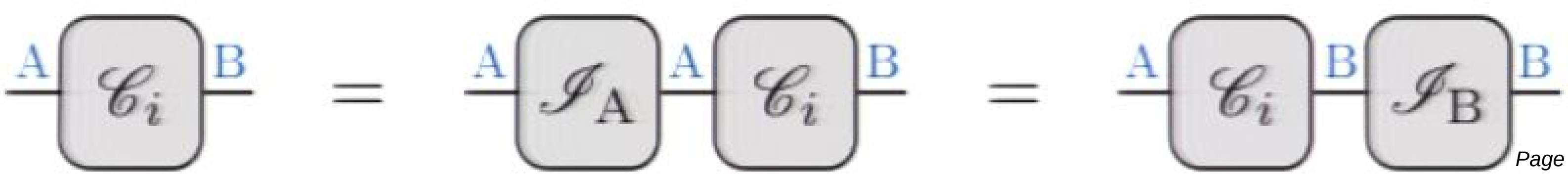
- C:=AB=BA
- (AB)C=A(BC)
- AI=IA=A

The operational language

- Operational theory: tests with composition rules

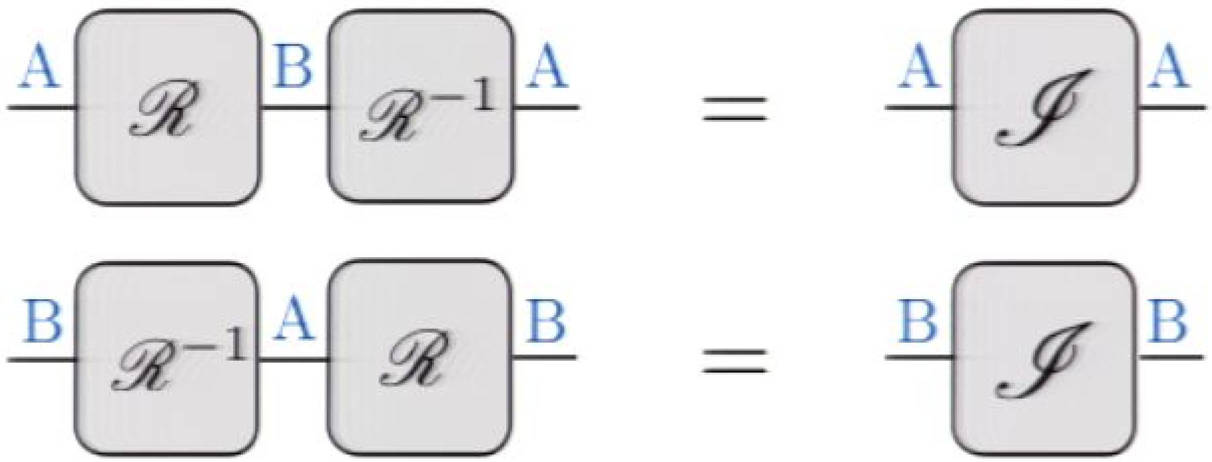


- For any system A there exists a unique test $\mathscr{I}_{\mathrm{A}}$ such that



Reversible tests

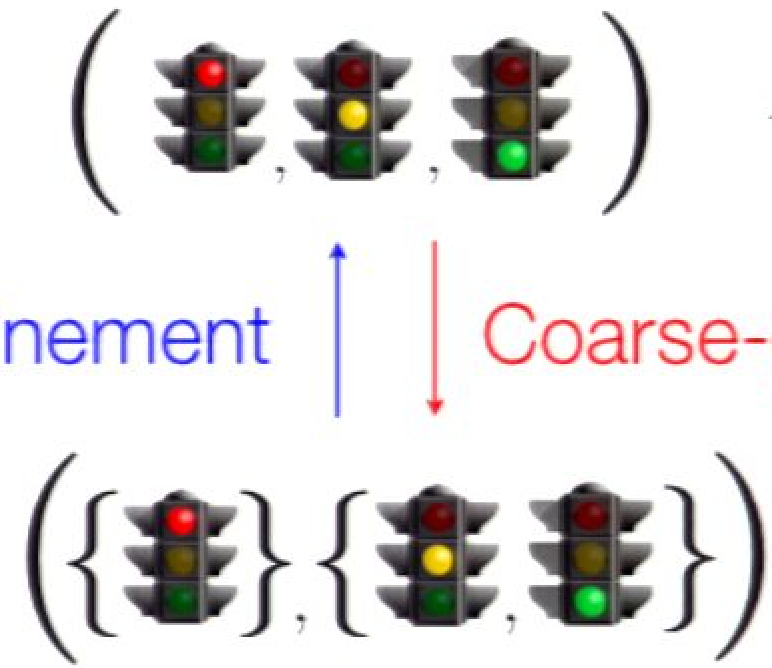
- A deterministic test R is reversible if there exists R^{-1} such that



Coarse-Graining and Refinement

Example

$(\mathscr{C}_i)_{i \in X} =$ $\quad X = (r, y, g)$

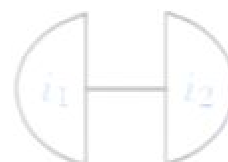


Refinement ↑ ↓ Coarse-graining

$(\mathscr{D}_j)_{j \in Y} =$ $\quad Y = (r, \bar{r}) = (\{r\}, \{y, g\})$

The probabilistic structure

- Probabilistic theory

Every test of type I→I is a probability distribution 

$= p(i_1, i_2)$

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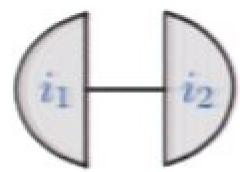
Every test of type I→I is a probability distribution 

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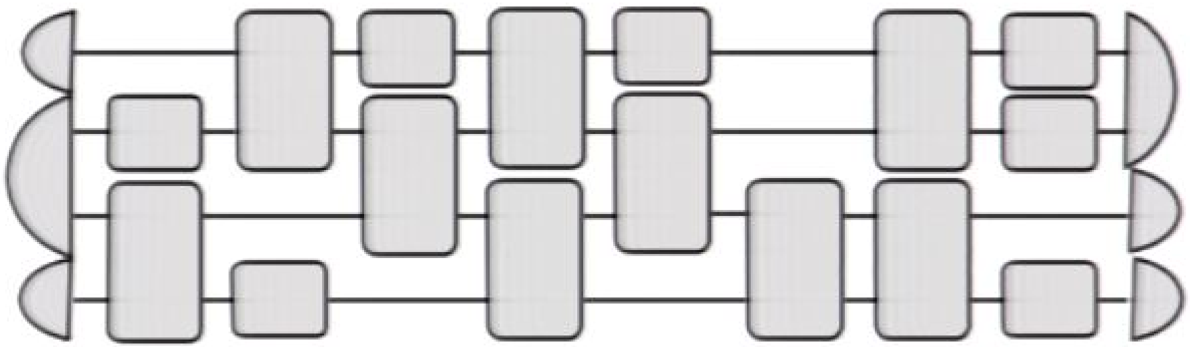

$$= \quad p(\boldsymbol{i})$$

The probabilistic structure

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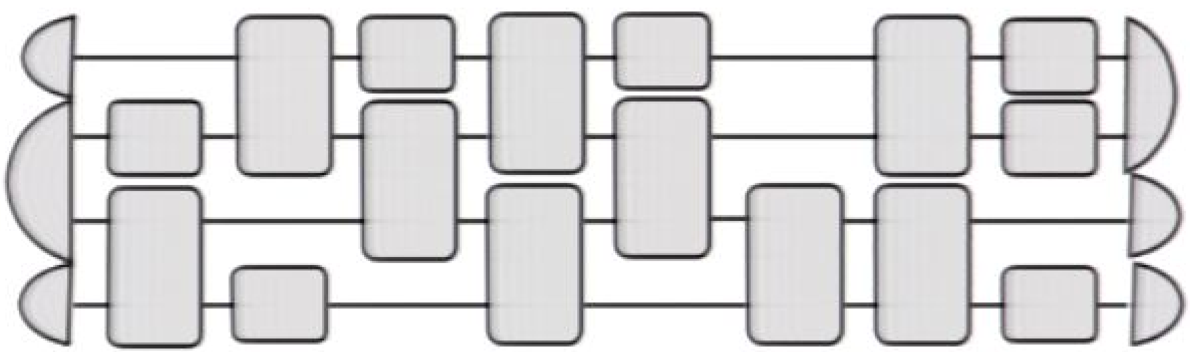

$$= \quad p(\boldsymbol{i})$$

- Deterministic theories are a special case in which $p \in \{0,1\}$

Probabilistic Theory

States are functionals on effects and viceversa

Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

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System A ⟷ dimension D_{A}

Probabilistic Theory

States are functionals on effects and viceversa

Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

System A $\longleftrightarrow$ dimension D_{A}

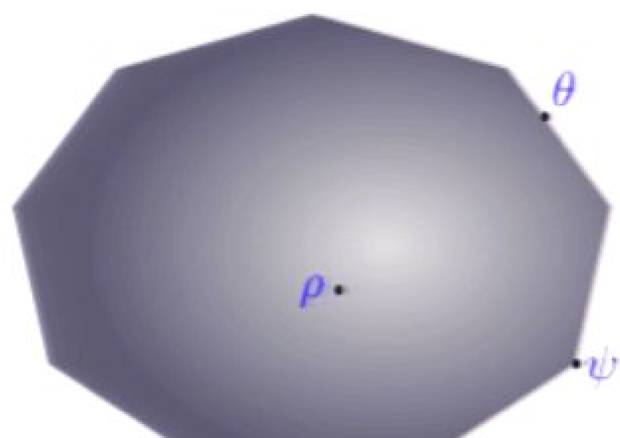
Events are linear maps: transformations

Coarse-graining is represented by the sum

Pure and internal states

Atomic is a transformation that cannot be refined

- An atomic preparation event ψ is a **pure state**
- A state θ is **mixed** if it is not pure
- A state ρ is **fully mixed** iff F_ρ spans the whole state space



The principles

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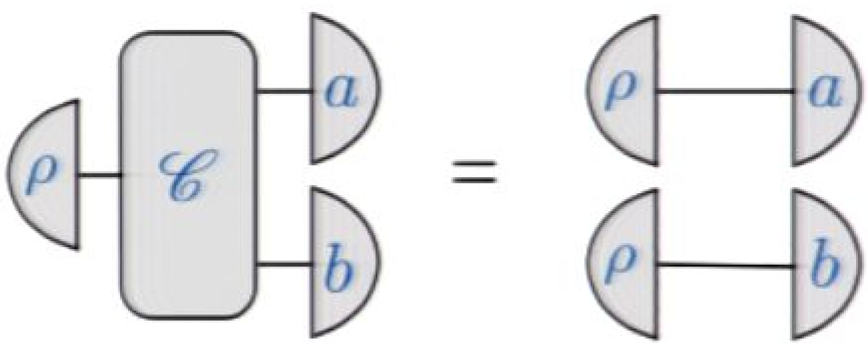
- Operations are specified by special probabilities they allow for
    - Example: “The state of a single system cannot be cloned”



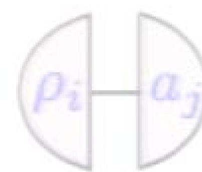
- The cloning operation is defined through probabilities p(a,b,ρ)

Causality

The probability of preparations is independent of the choice of observations

G. Chiribella, G. M D'Ariano and PP, Phys. Rev. A **81**, 062348 (2010)

Causality

- No-backwards signalling $p_a(\rho_i) := \sum_j$ 

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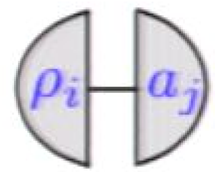

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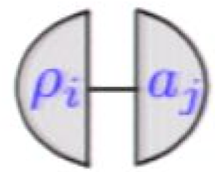
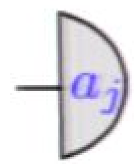
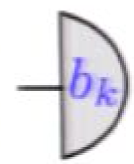
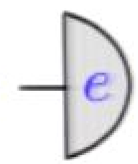
- No-backwards signalling $p_a(\rho_i) := \sum_j$ 

$= p(\rho_i)$

- Uniqueness of the deterministic effect $\sum_j$ 

$= \sum_k$ 

$=$ 

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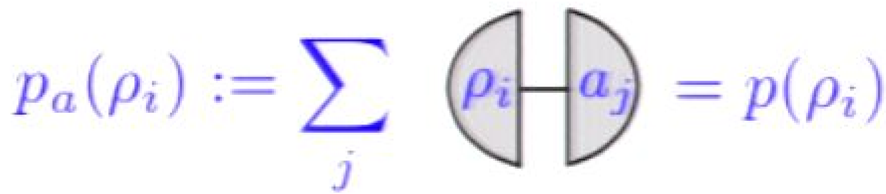
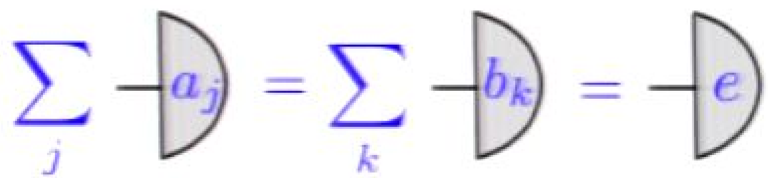
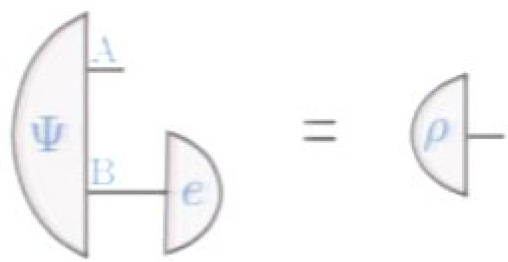


- **Uniqueness of the deterministic effect**



- The marginal state is uniquely defined



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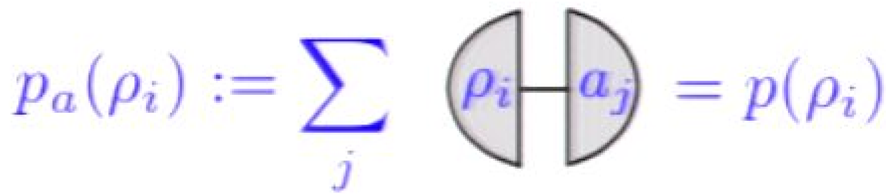

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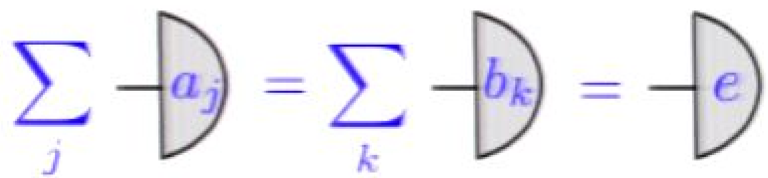

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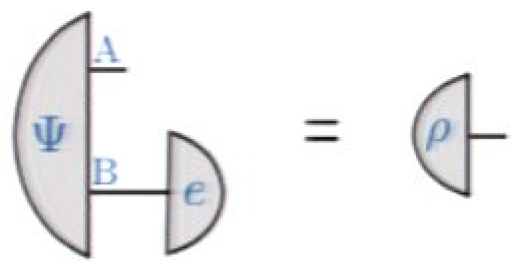
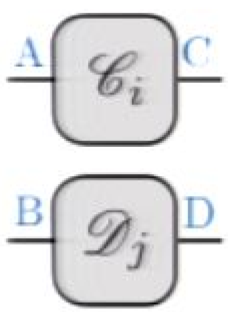


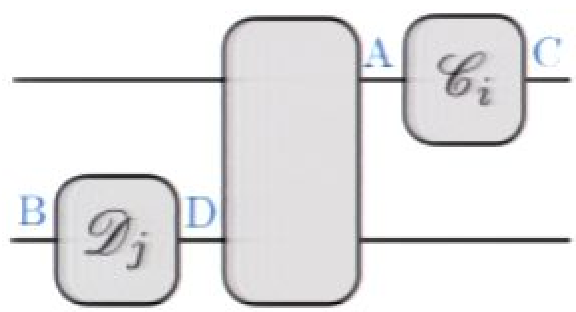
No-signalling

In a causal theory it is impossible to signal from a system A to another B without an interaction

$p(i|\mathscr{D}) = p(i)$



$p(i|\mathscr{D}) \neq p(i)$

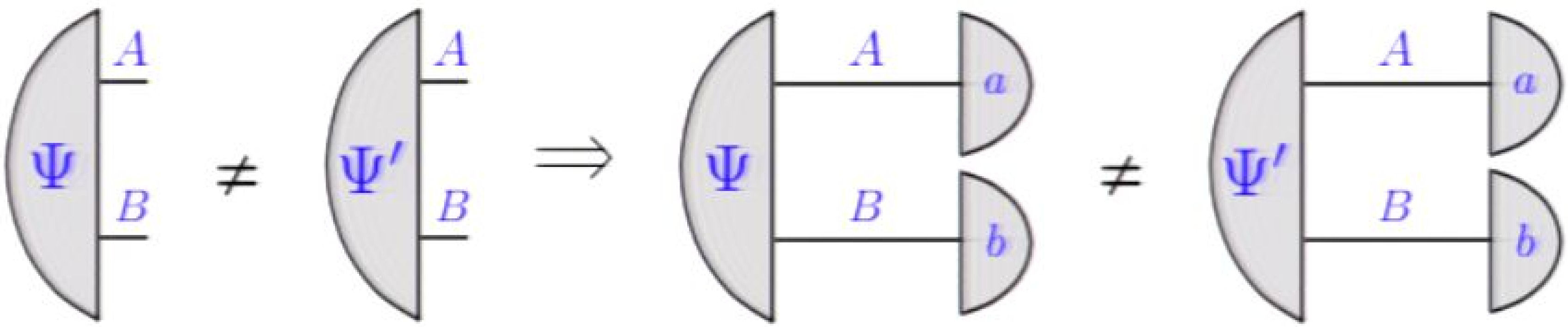


Local discriminability

If two bipartite states are different, they give different probabilities for at least one product experiment

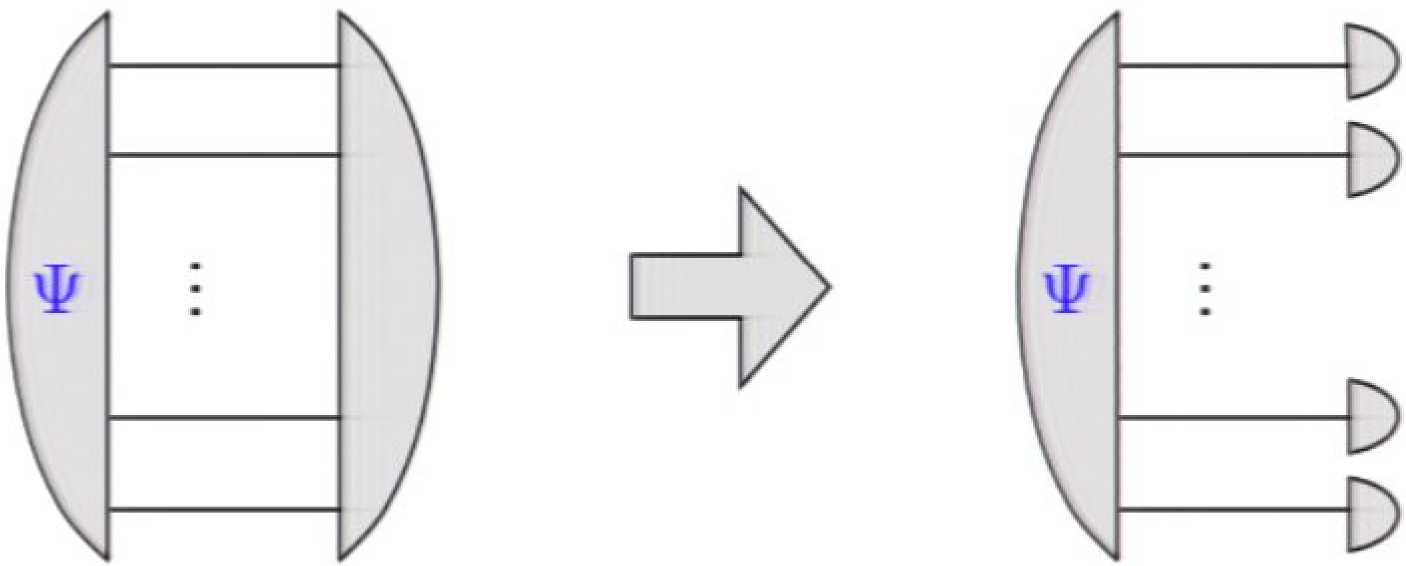
G. M. D'Ariano, arXiv:quant-ph/0603011

Local discriminability



This property gives rise to the tensor product structure for state spaces of composite systems

Reduction of measurement complexity



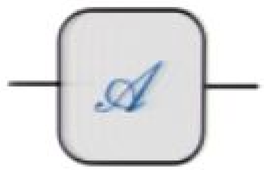
State reconstruction can be carried locally

Atomicity of the composition

The sequential composition of two atomic operations is atomic

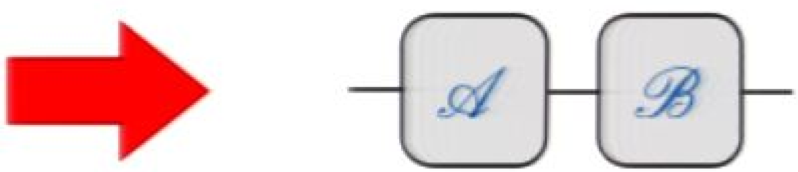
G. M. D'Ariano, arXiv:0807.4383

Atomicity of the composition

If 

, 

are atomic



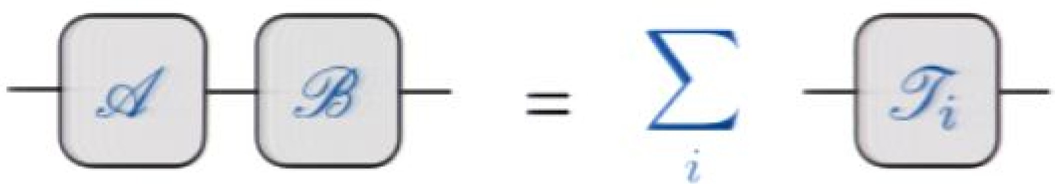
is atomic

Atomicity of the composition

What if atomicity is removed?

Full information about

does not provide full information about



Perfect distinguishability

Every state that is not completely mixed can be perfectly distinguished from some other state

G. Chiribella, G. M. D'Ariano and PP, arXiv:1011.6451

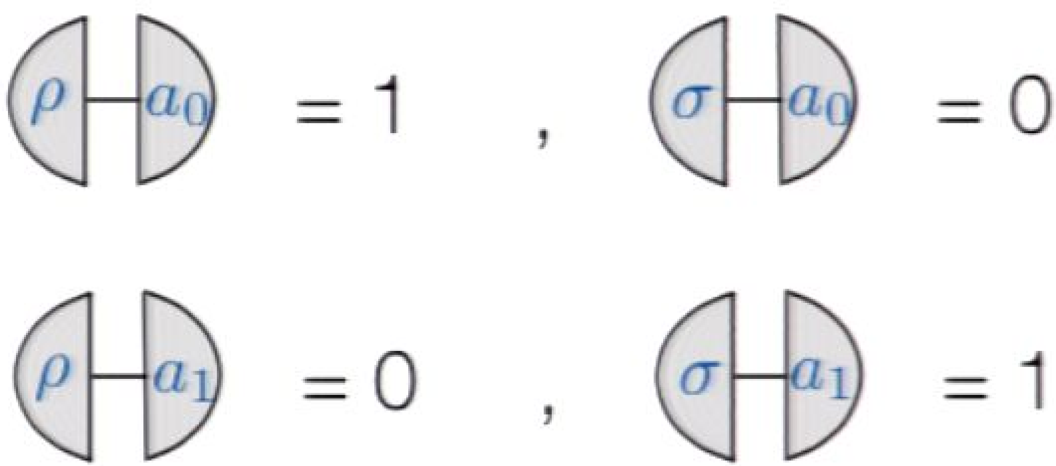
Perfect distinguishability

$= 1$,

$= 0$

Perfect distinguishability

If ρ is not completely mixed, there exist σ and an observation-test (a_0, a_1) such that



Falsifiable propositions and bits

- Perfect distinguishability introduces the existence of falsifiable propositions

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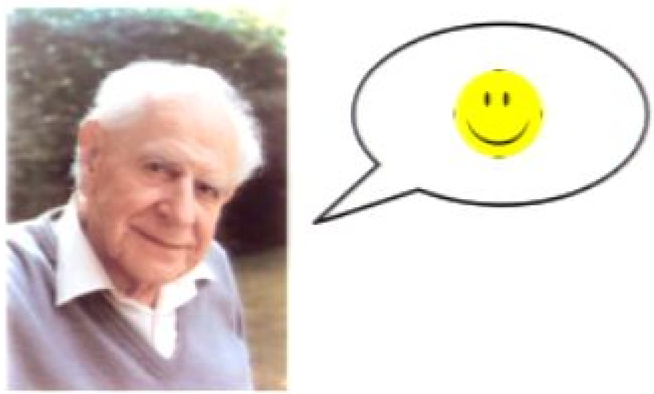

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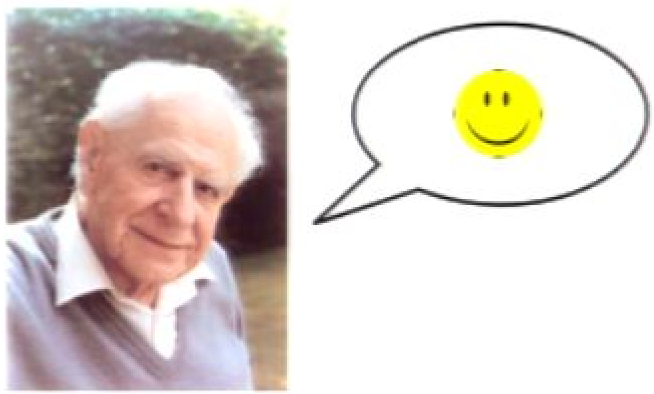
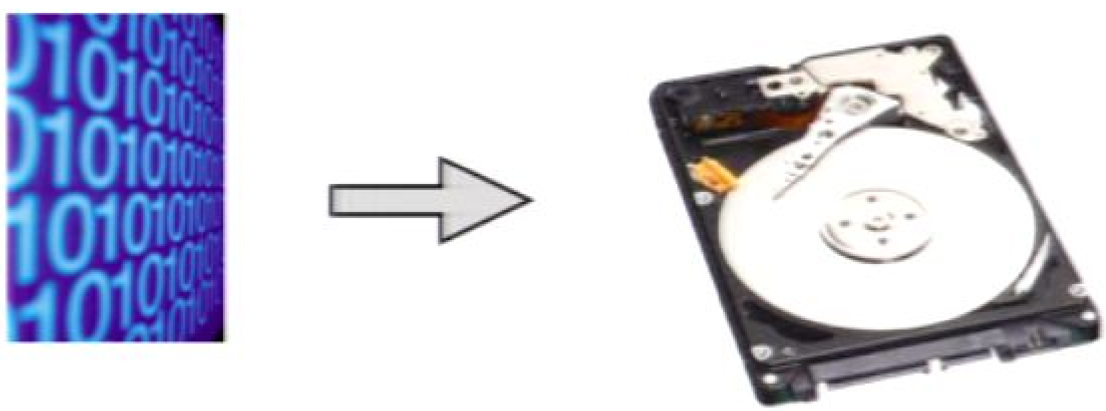


- Distinguishability grants encoding of classical bits



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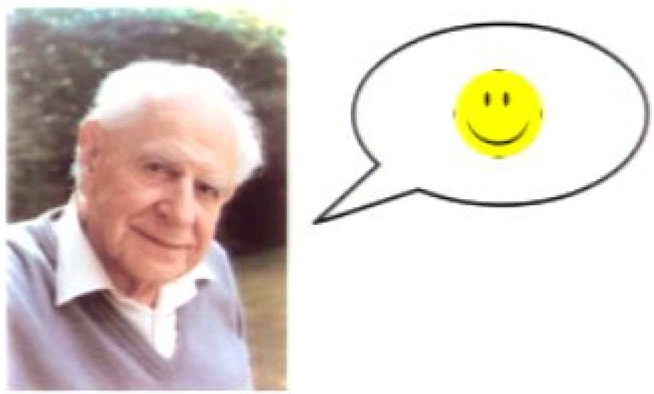

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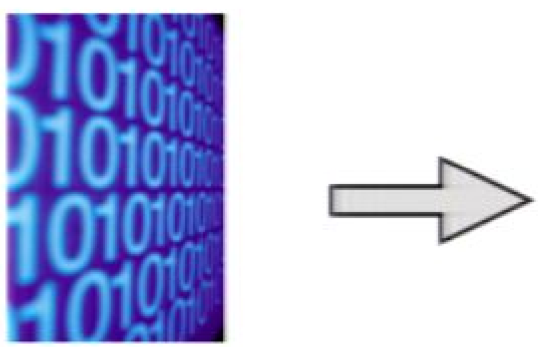


- Recovery of logic and truth values in a probabilistic framework

Efficient lossless compression

For every state there exists an ideal compression scheme

G. Chiribella, G. M. D'Ariano and PP, arXiv:1011.6451

Compression

Lossless $\mathscr{D}: \quad \mathrm{C} \to \mathrm{A} \qquad \mathscr{D}\mathscr{E}\rho_i = \rho_i \qquad \sum$

Efficient

Ideal = lossless + efficient

Compression

A compression for state ρ of system A is

$$\mathscr{E}: \quad \mathrm{A} \to \mathrm{C}$$

Lossless $\mathscr{D}: \quad \mathrm{C} \to \mathrm{A} \qquad \mathscr{D}\mathscr{E}\rho_i = \rho_i \qquad \sum_i \rho_i = \rho$

Efficient $\tau \in \mathsf{St}(\mathrm{C}) \qquad \Rightarrow \qquad \tau = \mathscr{E}(\sigma), \quad \sigma \in F_\rho$

Ideal = lossless + efficient

Subsystems

Compression implies that every face is a subsystem

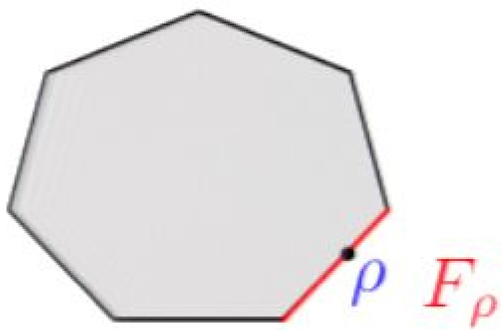
E.g.: in quantum theory $\mathrm{C}_\rho = \mathcal{S}upp(\rho)$

$$\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow \mathrm{C}_\rho \text{ is a Bloch sphere}$$

$$\mathscr{C}\rho = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

Faces

- The refinement set of a state ρ is a **face** F_ρ of the convex set



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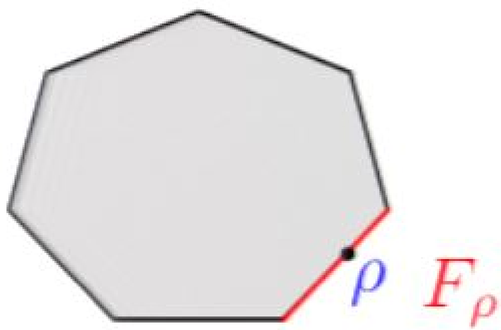


- Every face is equivalent to the state space of a smaller system

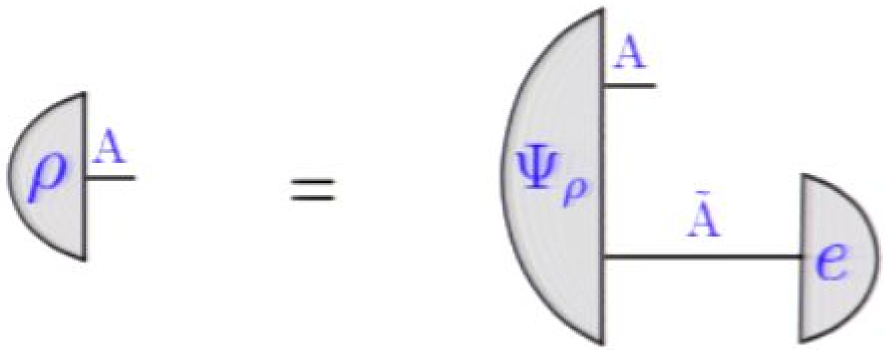
Purification

Every state has a purification. For fixed purifying system, every two purifications of the same state are connected by a reversible transformation on the purifying system

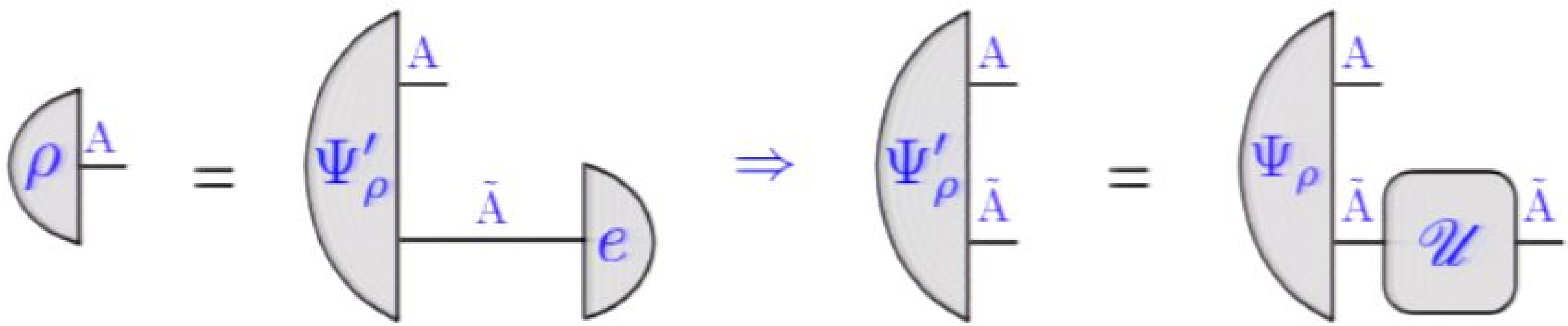
G. Chiribella, G. M D'Ariano and PP, Phys. Rev. A 81, 062348 (2010)

Purification

- For any state ρ there exists a **purifying** system Ã such that

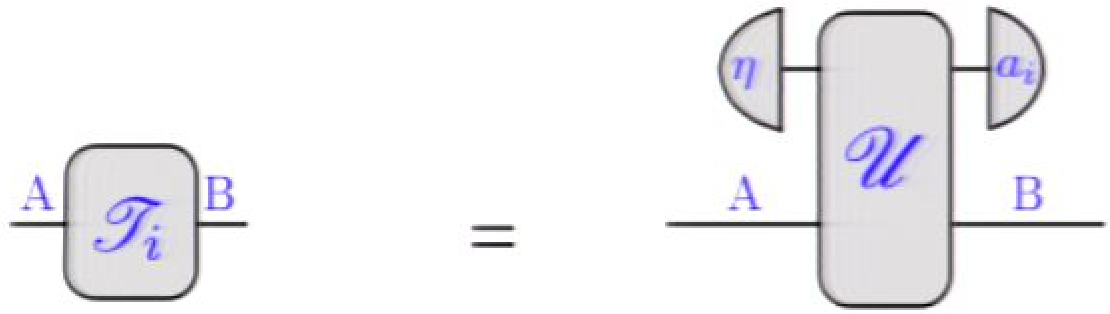


- The purification is unique up to reversible transformations



Purification and reversibility

- Every test can be dilated to a reversible transformation + measurement

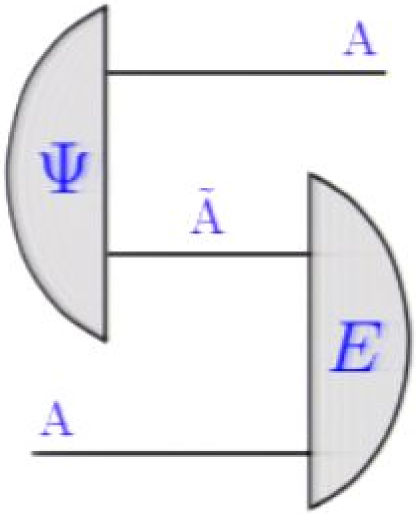


- This does not imply that irreversibility necessarily stems from marginalization

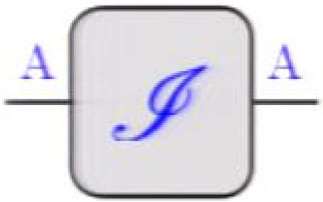
- In principle any purifying system could be included in the physical description

Consequences of purification

Probabilistic teleportation



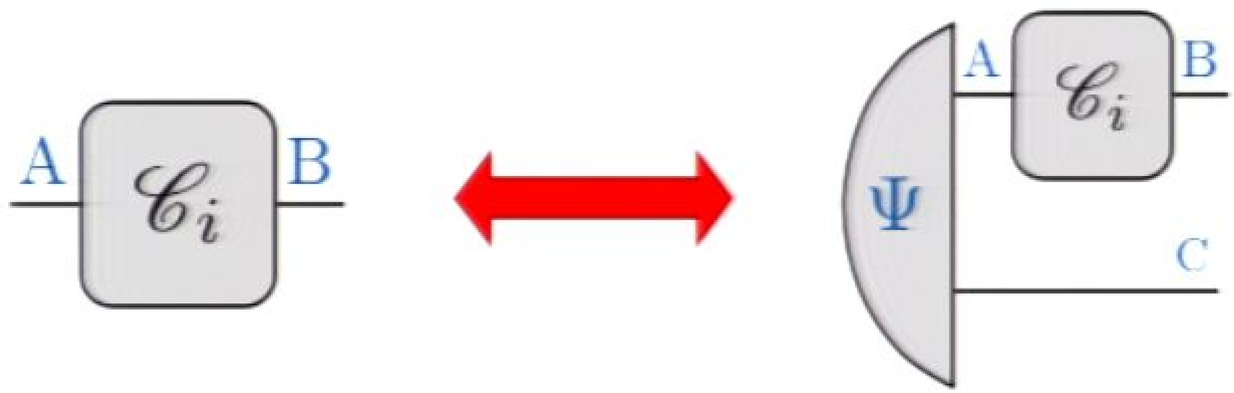
$$= \quad p_{\mathrm{A}}$$



$$p_{\mathrm{A}} \leq \frac{1}{D_{\mathrm{A}}}$$

Consequences of purification

- Choi isomorphism



Sufficiency of states

Let Θ, Θ' be two theories satisfying the purification postulate. If Θ and Θ' have the same sets of normalized states, then $\Theta' = \Theta$.

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- Sufficient condition for QT: states are density matrices

Pure state - atomic effect duality

For any pure state ψ there exists a unique atomic effect a_ψ

s.t.

$= 1$

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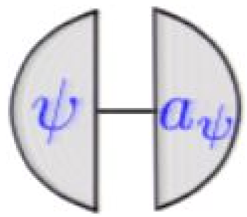

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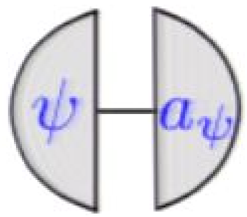

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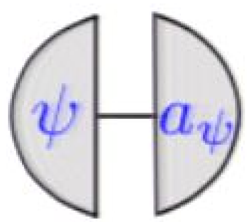

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Discriminability dimension

A set of perfectly distinguishable states $\{\psi_i\}_{i=1}^k$ is maximal if for any state σ the set $\{\sigma\} \cup \{\psi_i\}_{i=1}^k$ is not perfectly discriminable

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Spectral Decomposition

For any state ρ there is a maximal set $\{\psi_i\}_{i=1}^{d_{\mathrm{A}}}$

such that $$\rho = \sum_{i=1}^{d_{\mathrm{A}}} p_i \psi_i$$

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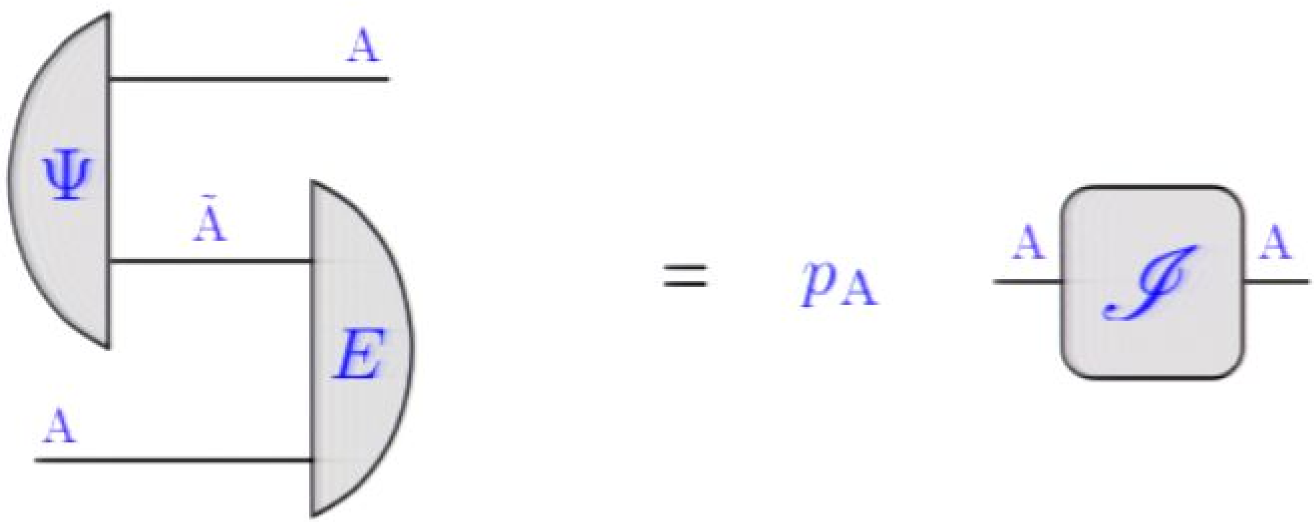
For any $a \in \mathfrak{E}_\mathbb{R}(A)$ there is a p.d. observation-test $\{a_i\}_{i=1}^{d_\mathrm{A}}$

such that $$a = \sum_{i=1}^{d_\mathrm{A}} \lambda_i a_i$$

Relation between dimensions



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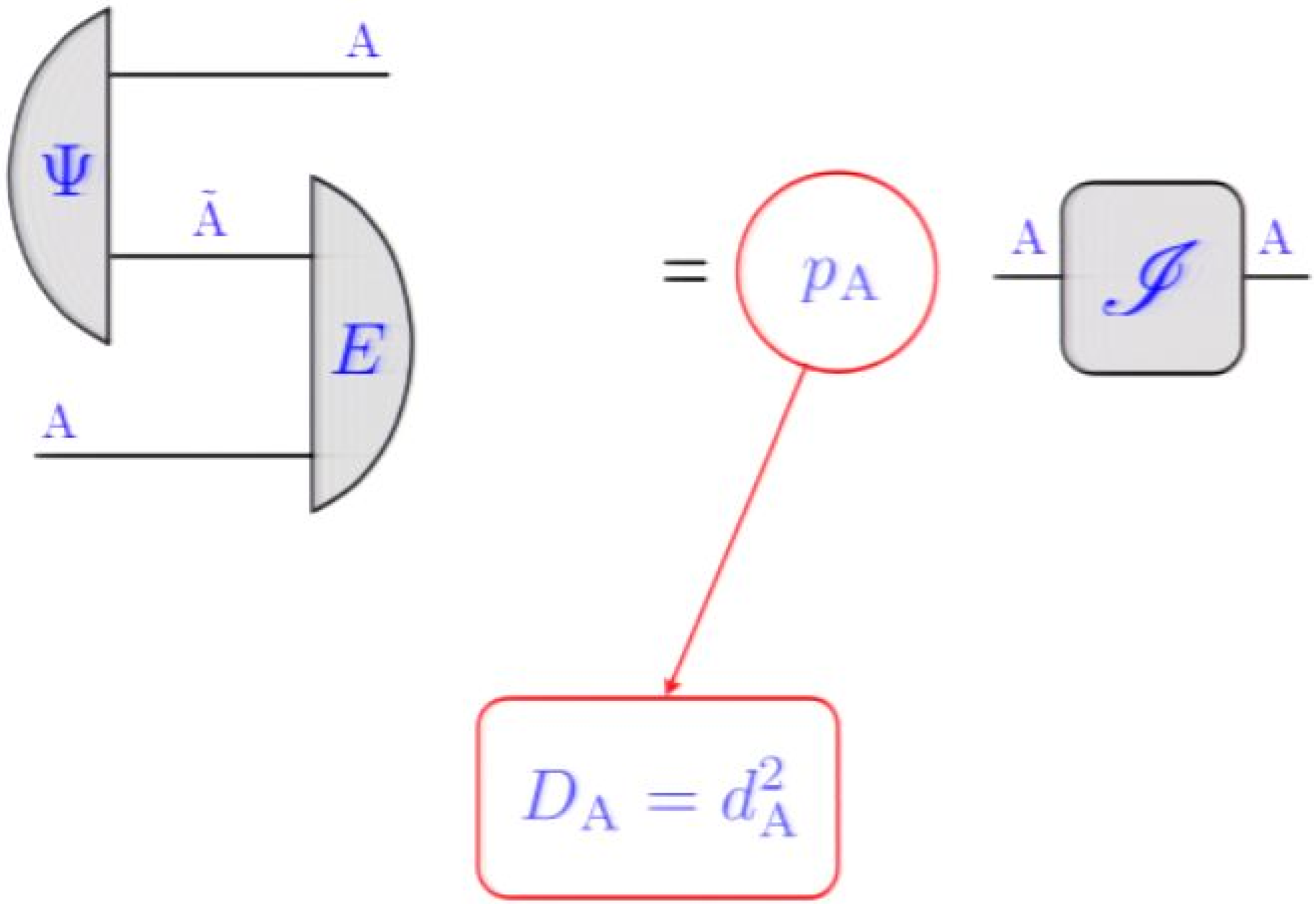


Matrix representation of states

$$D_{\mathrm{A}} = d_{\mathrm{A}}^2$$

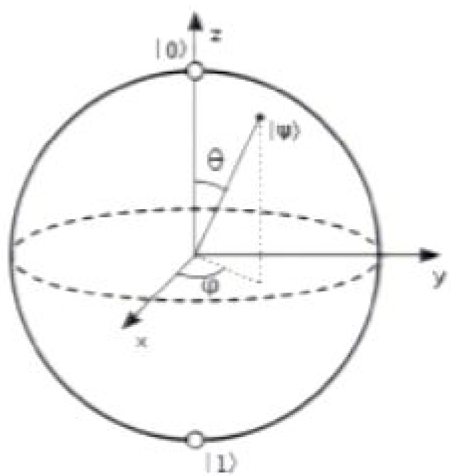
The states can be represented as square real matrices $\tilde{M}$

The states can be represented as Hermitian matrices

$$\tilde{M} \mapsto M := \frac{1}{2}\{(\tilde{M} + \tilde{M}^T) + i(\tilde{M} - \tilde{M}^T)\}$$

The qubit

For $d_{\mathrm{A}} = 2$ the states of system A are qubit states



$$\mathscr{U}\rho = U S_{\rho} U^{\dagger}$$

Projections and superposition principle

$$F_\rho \quad \leftrightarrow \quad F_\rho^\perp$$

Definition: projection $\begin{cases} \Pi_F \rho = \rho & \rho \in F \\ \Pi_F \rho = 0 & \rho \in F^\perp \end{cases}$

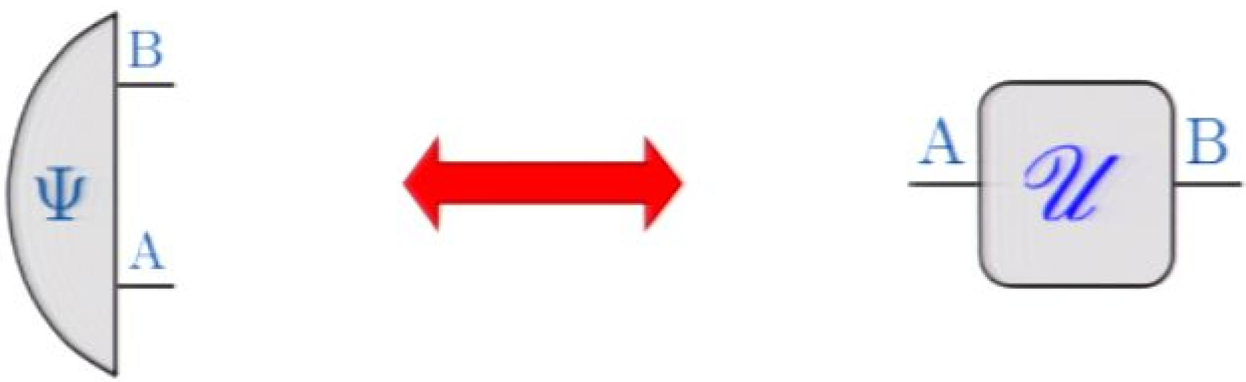
Π_F exists and is unique

For all $\{p_i\}$ and $\{a_i\}$ there is a pure state ψ

such that $(a_i|\psi) = p_i$

Equivalence of systems

Two systems A, B with $d_A = d_B$ are equivalent



The qudit

$$S_\rho = \begin{pmatrix} * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{pmatrix}$$

Using projections we can embed qutrits in larger systems and prove that the matrices representing states are all the density matrices

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Concluding remarks

- Six axioms
  - They only refer to concepts from the operational-probabilistic framework
  - They do not have a mathematical statement
  - We did not check for possible simplifications or dependencies
  - We did not care about having **fewer** axioms

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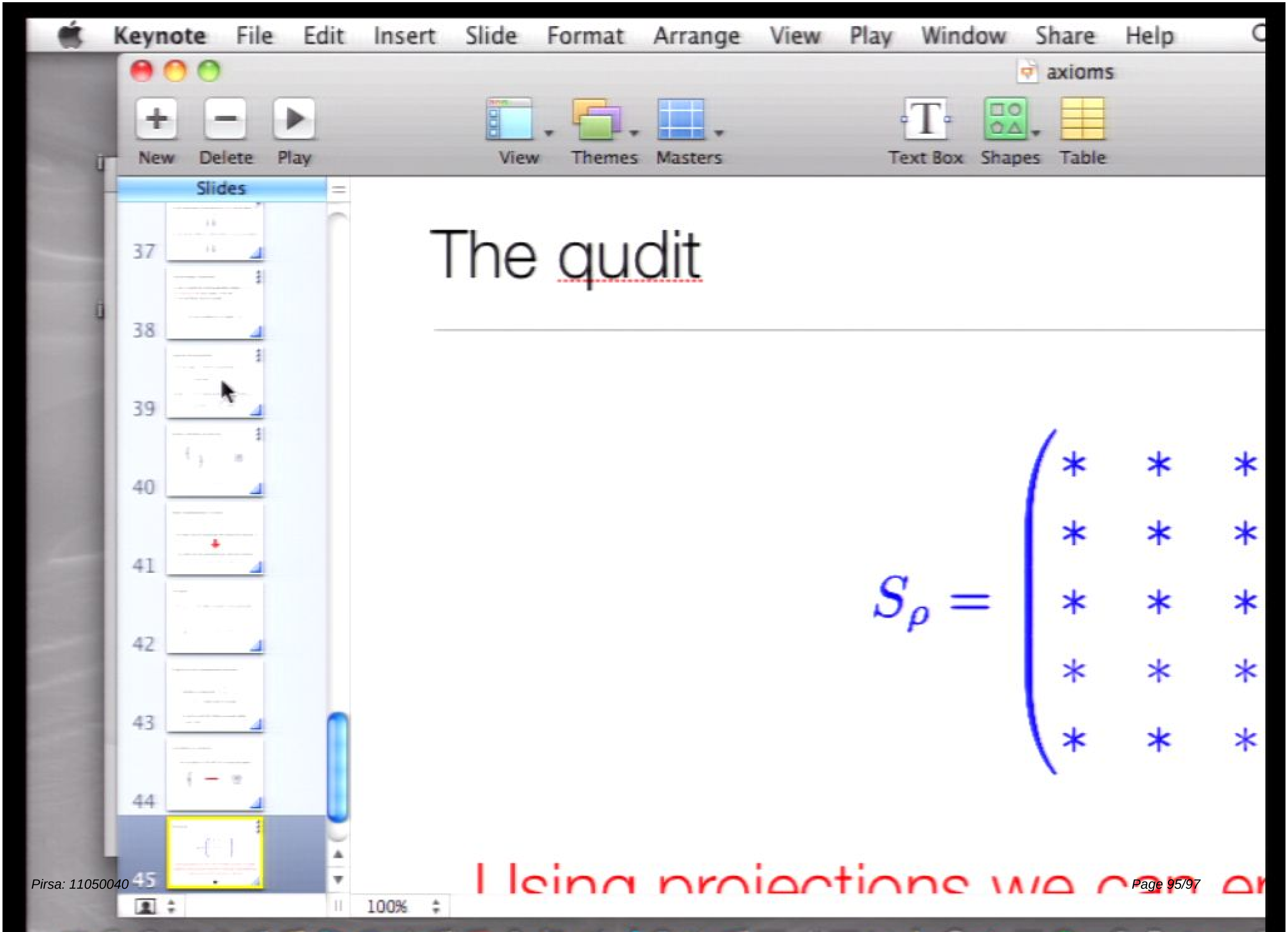
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100%

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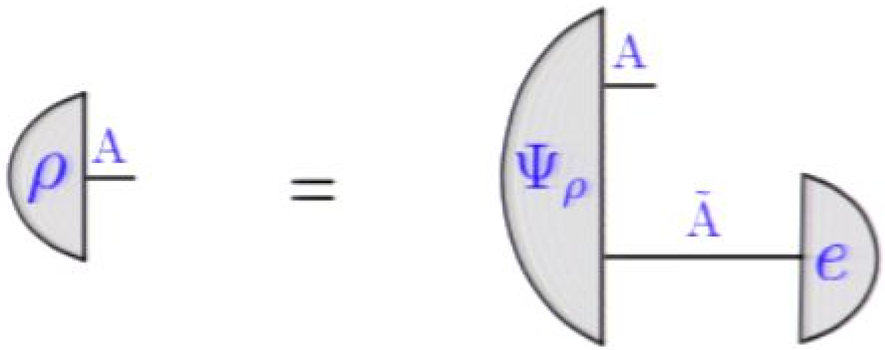

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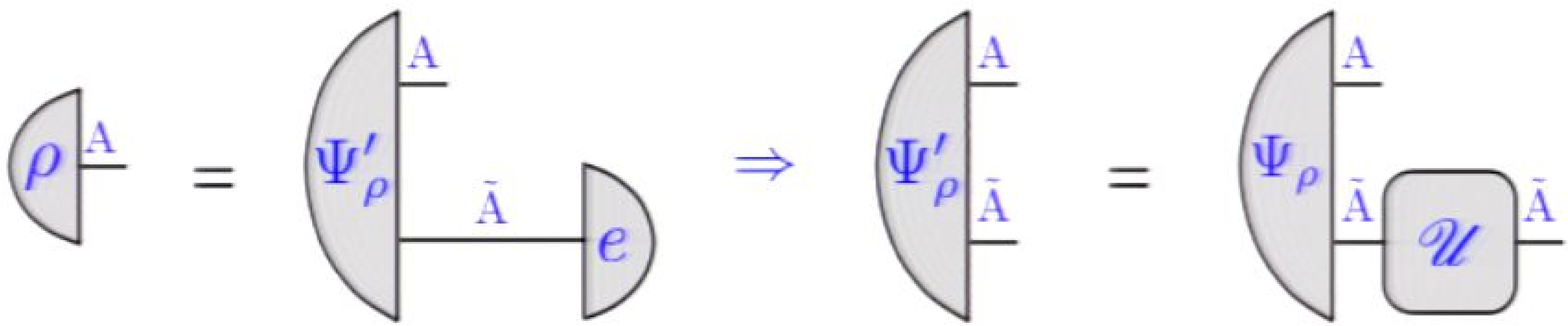

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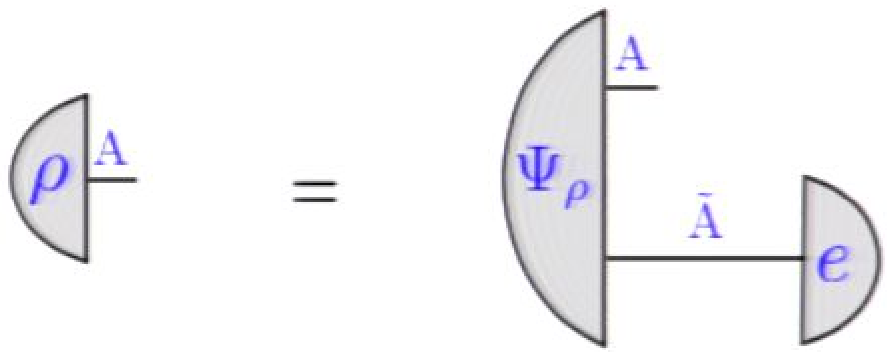

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