Title: Quantum correlations with no causal order

Date: May 10, 2011 02:50 PM

URL: http://pirsa.org/11050039

Abstract: Much of the recent progress in understanding quantum theory has been achieved within an operational approach. Within this context quantum mechanics is viewed as a theory for making probabilistic predictions for measurement outcomes following specified preparations. However, thus far some of the essential elements of the theory Ã¢ÂÂ€ÂÂ“ space, time and causal structure Ã¢ÂÂ€ÂÂ“ elude such an operational formulation and are assumed to be fixed. Is it possible to extend the operational approach to quantum mechanics such that the notions of an underlying spacetime or causal structure are not assumed? What new phenomenology can follow from such an approach? We develop a framework for multipartite quantum correlations that does not presume these notions, but simply that experimenters in their local laboratories are free to perform arbitrary quantum operations. All known situations that respect definite causal order, including signalling and no-signalling correlations between space-like and time-like separated experiments, as well as probabilistic mixtures of these, can be expressed in this framework. Remarkably, we find quantum correlations which are neither causally ordered nor in a probabilistic mixture of definite causal orders. These correlations are shown to enable performing a communication task that is impossible if a fixed background time is assumed and the events are sufficiently localized in the time.

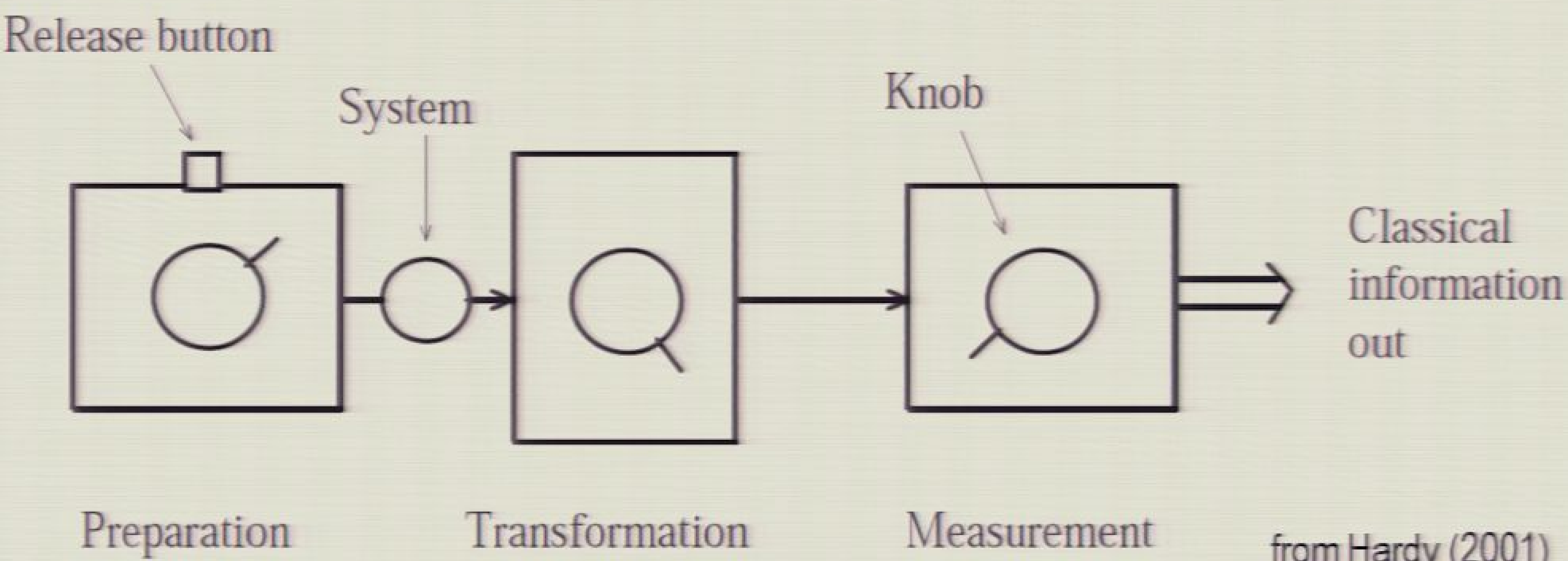
Faculty of Physics, University of Vienna &
Institute for Quantum Optics and Quantum Information, Vienna

Quantum correlations with no causal order

Ognyan Oreshkov, Fabio Costa, Časlav Brukner*

*anti-alphabetical order

Operational Approach

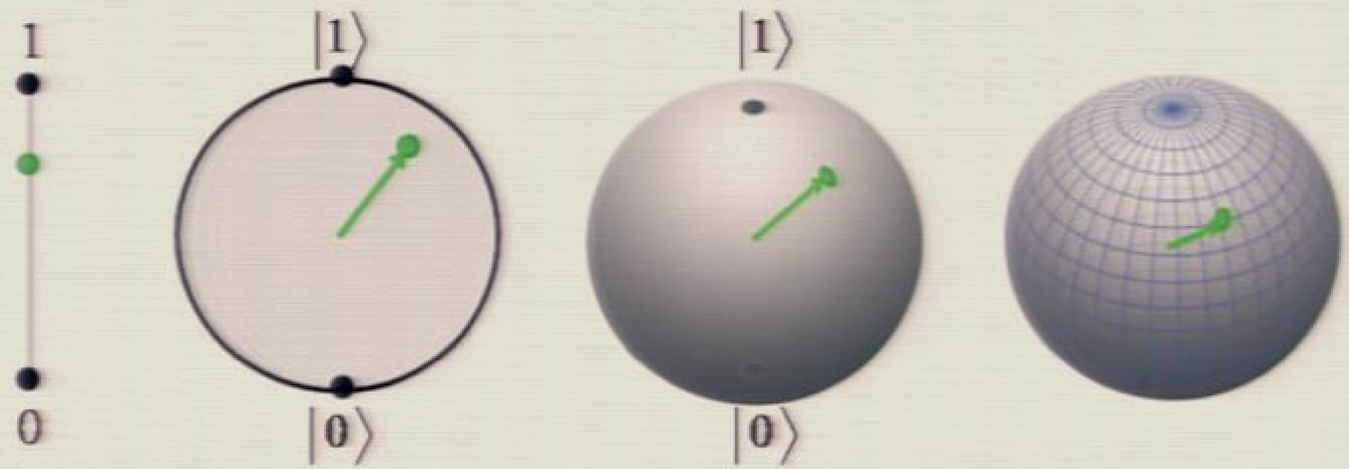


from Hardy (2001)

Last 10 years significant progress in understanding quantum theory within **operationalism** with primitive laboratory procedures as basic ingredients.

1 Bit Systems

$d=1$	$d=2$	$d=3$	d

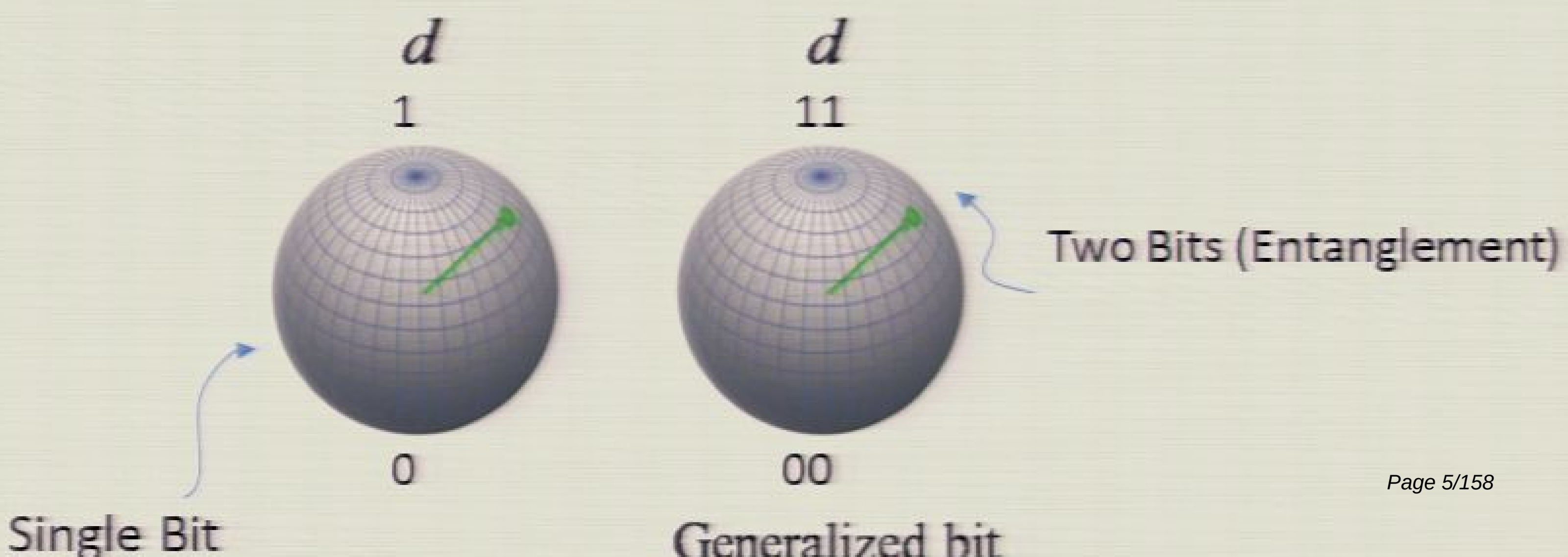


Classical bit	Real bit	Qubit	Generalized bit
$p\text{"0"}+(1-p)\text{"1"}$	$\rho \in \mathcal{D}(\mathbb{R}^2)$	$\rho \in \mathcal{D}(\mathbb{C}^2)$	

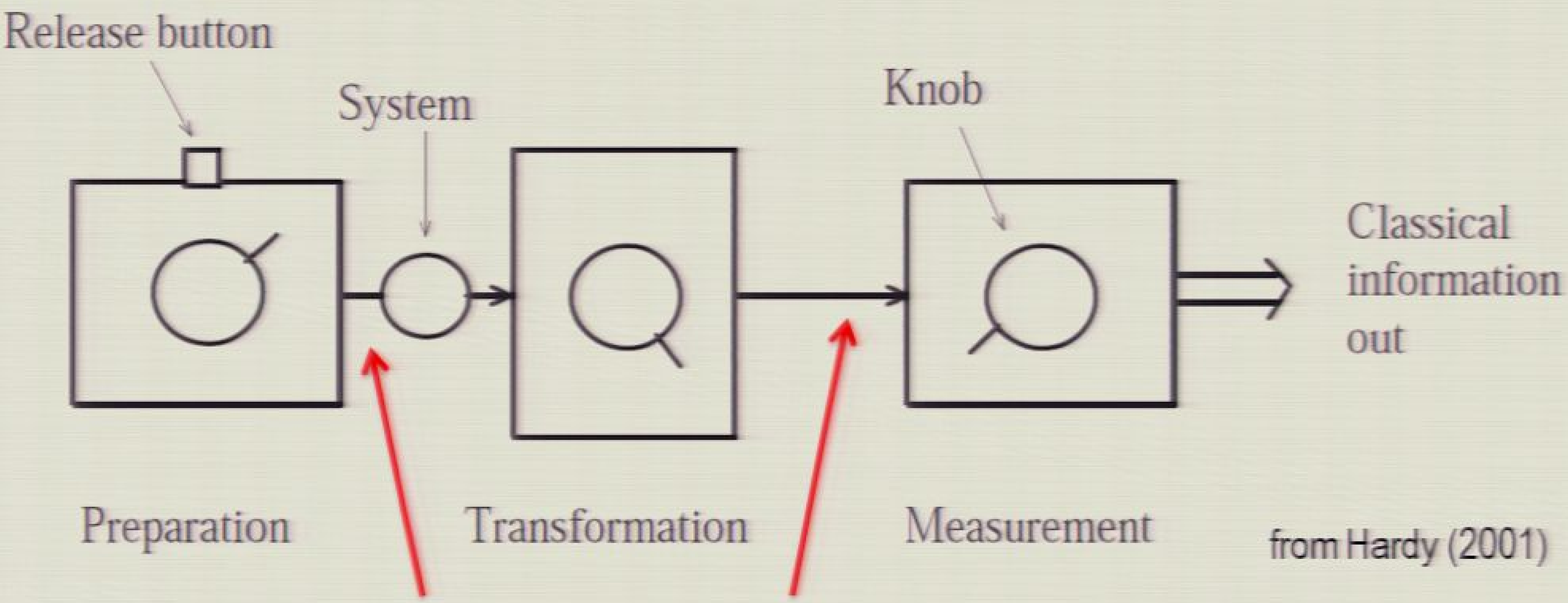
➡ d=1 or d=3

B. Dakic and Č. Brukner, quant-ph0911.0695, *Quantum Theory and Beyond: Is Entanglement Special?*

1. **Orthogonal Decomposition**: Any state of 1-bit system can be prepared by mixing two perfectly distinguishable states.
2. **Reversibility:** Between any two pure states there exists a reversible transformation
3. **Local Tomography**: The state of a composite system is completely determined by local measurements on its subsystems and their correlations.
4. **Subspace Axiom**: All 1-bit systems are equivalent **irrespectively of our (classical) notion of localizability**



Operational Approach



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Presupposes definite causal order

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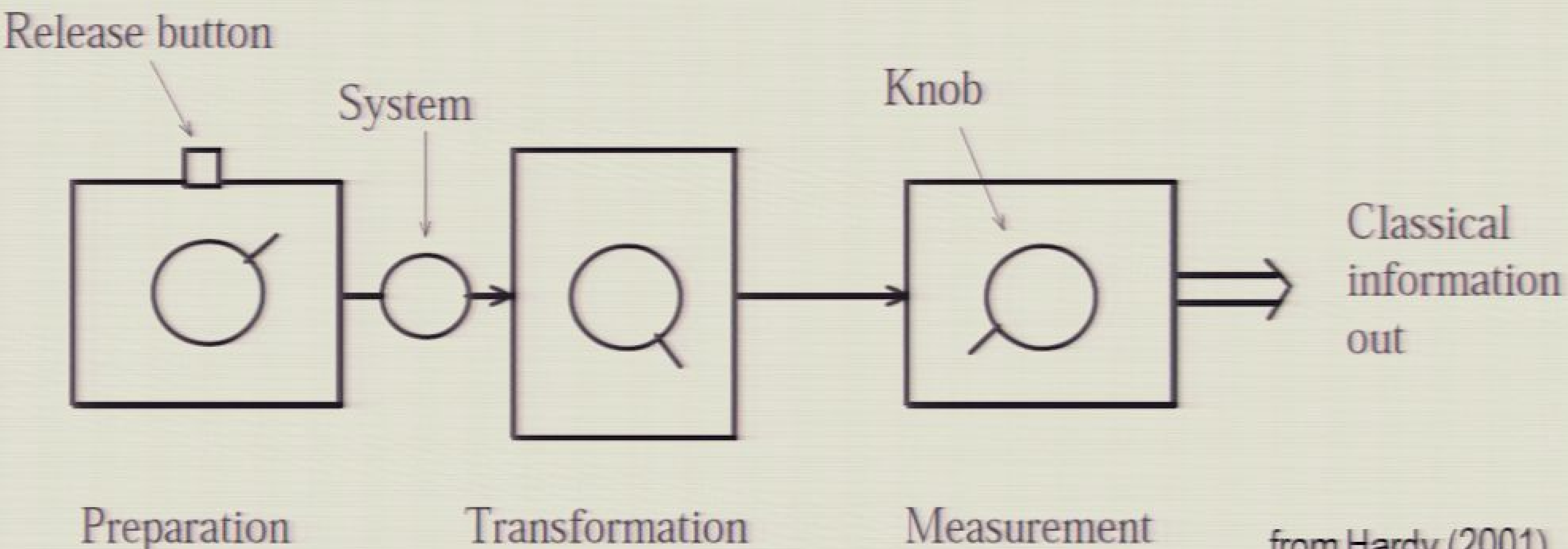

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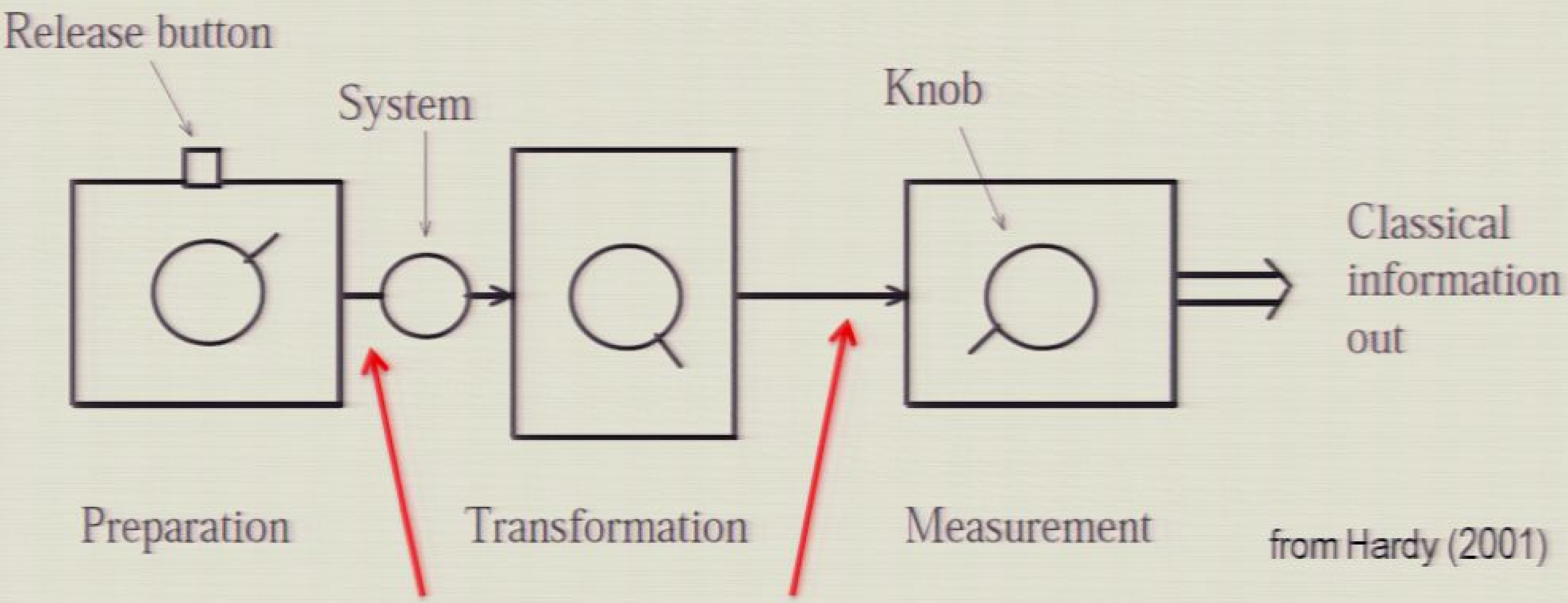

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Presupposes definite causal order

Questions

1. Is definite causal structure a necessary pre-assumption or does it follow from more primitive concepts?

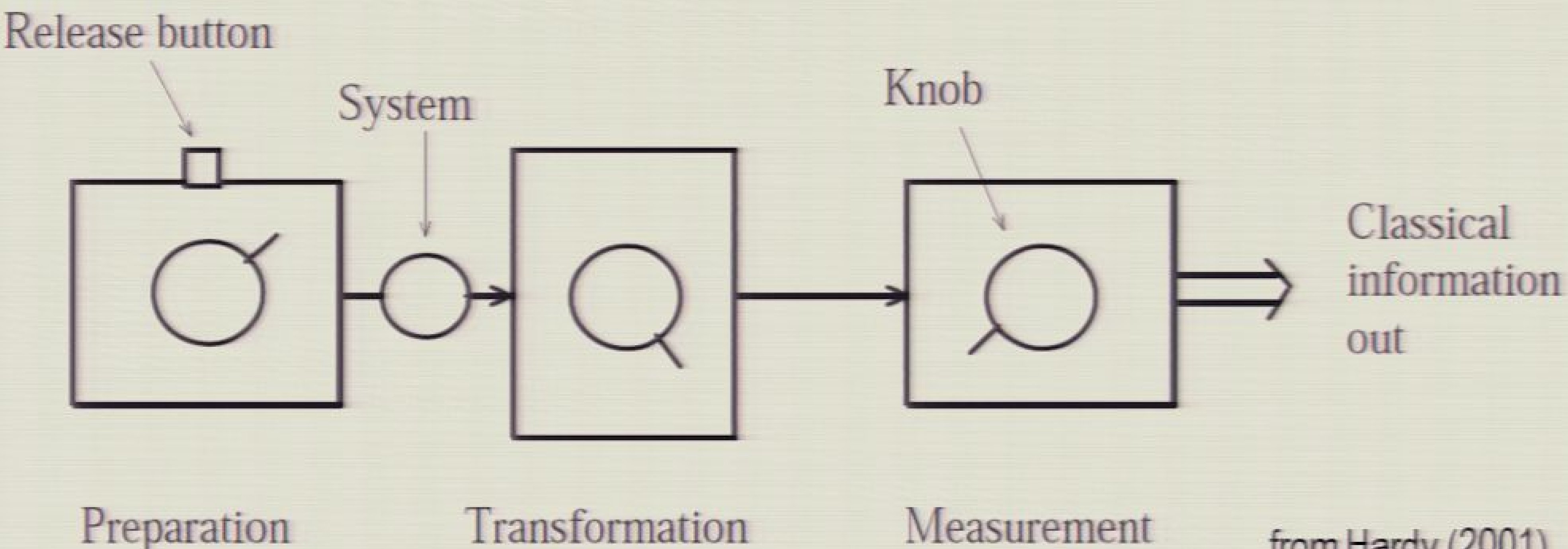
2. Is it possible to extend the operational approach to quantum mechanics such that the notions of an underlying space-time or causal structure are not assumed?

3. What new phenomenology can follow from such an approach?

Find a general framework for probabilistic theories with no pre-existing causal structure.

L. Hardy arXiv:gr-qc/0509120,
Probability Theories with Dynamic Causal Structure:
A New Framework for Quantum Gravity

Operational Approach



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Outline

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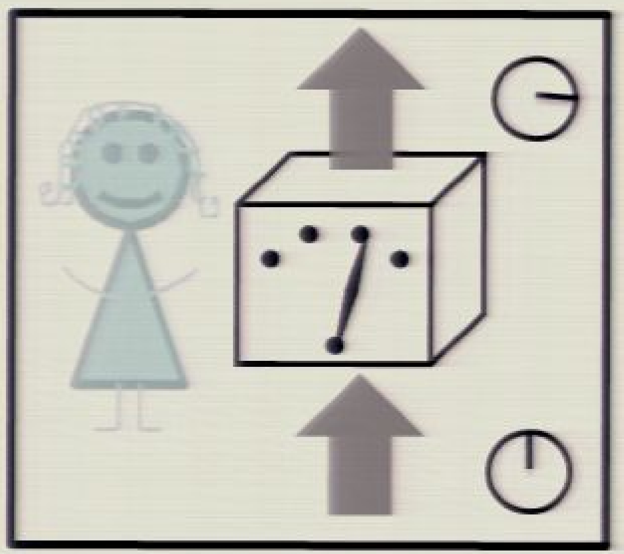
- Causal game → winning strategy with “non-causal” correlations
- Conclusions

Operational approach



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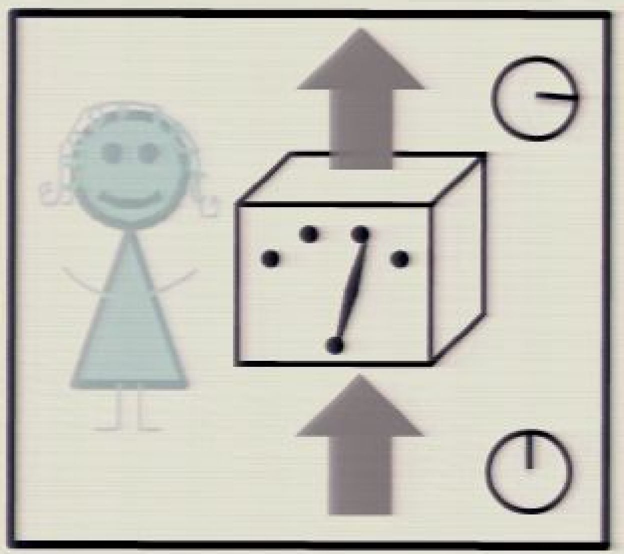
Operational approach



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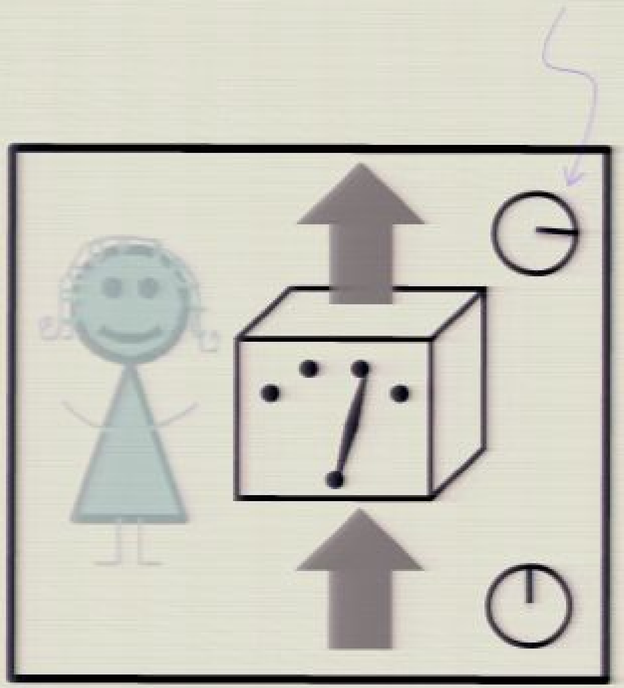

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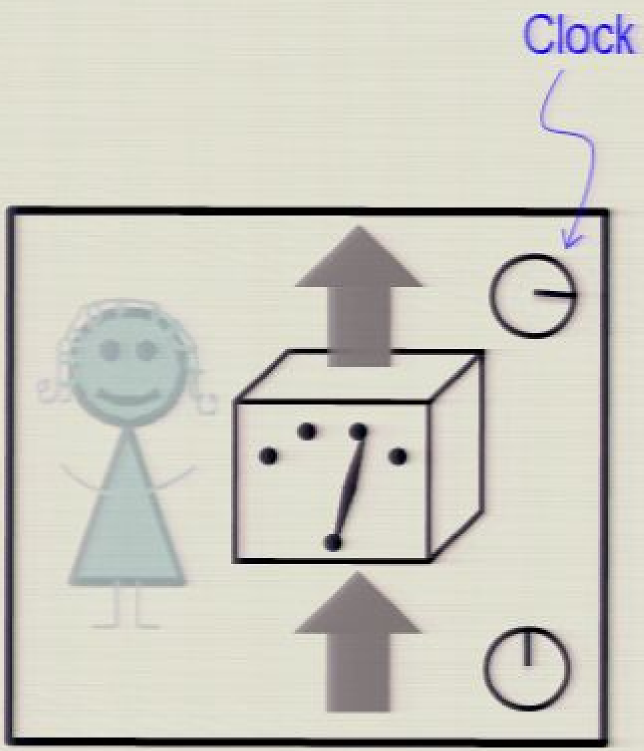
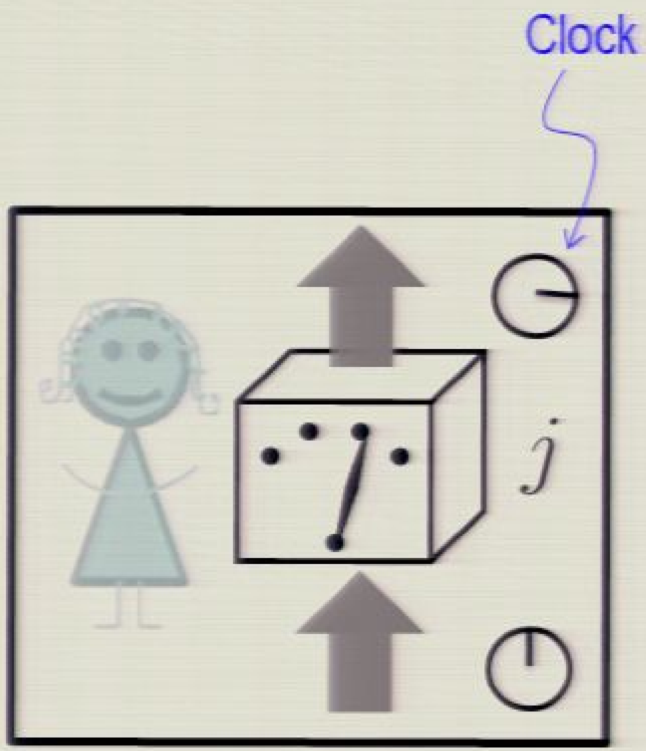


Input

A system enters the lab.

Operational approach

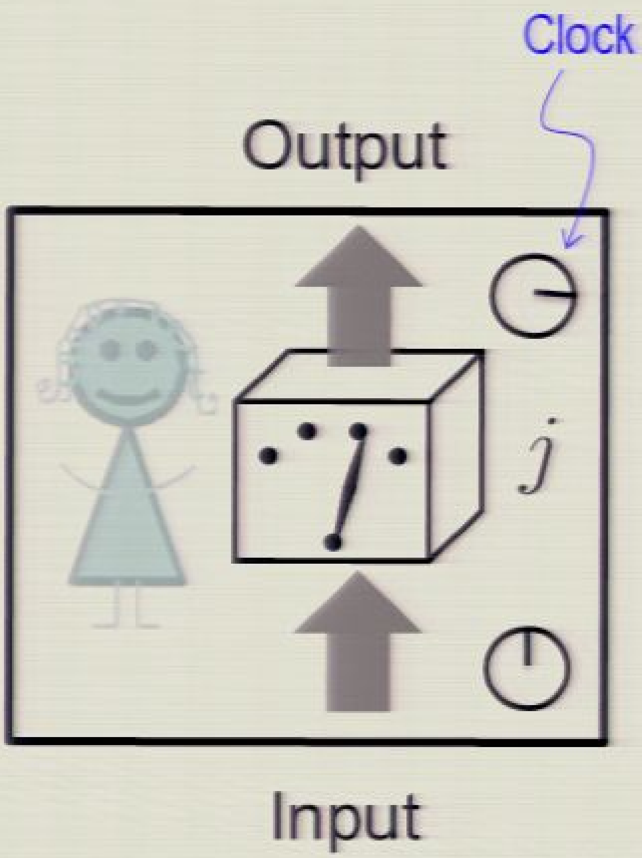


Input

One out of a set of possible transformations is performed.

A system enters the lab.

Operational approach



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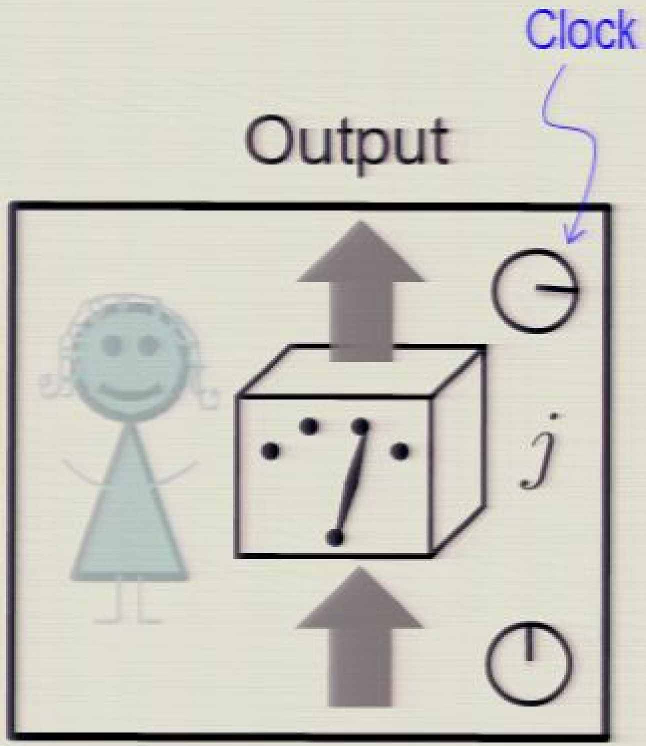

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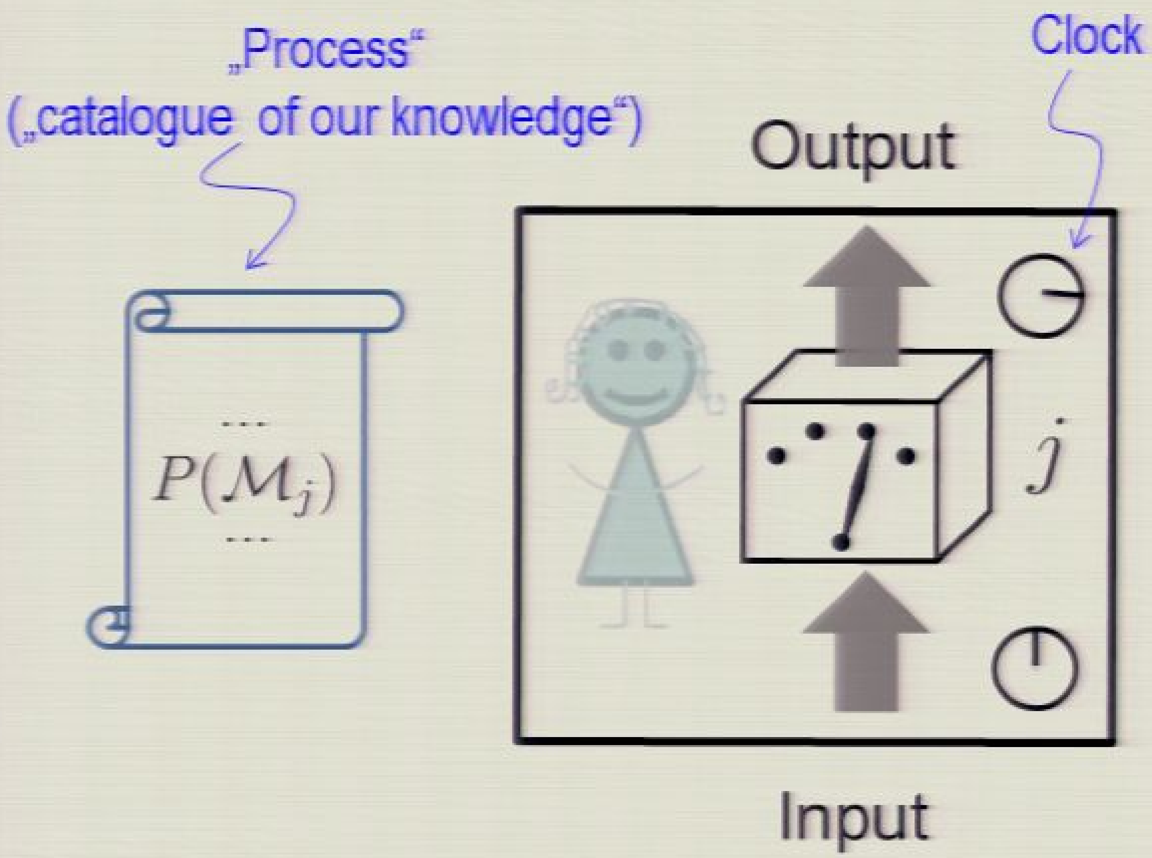
This is the **only** way how the labs interact with the "outside world".

Operational approach



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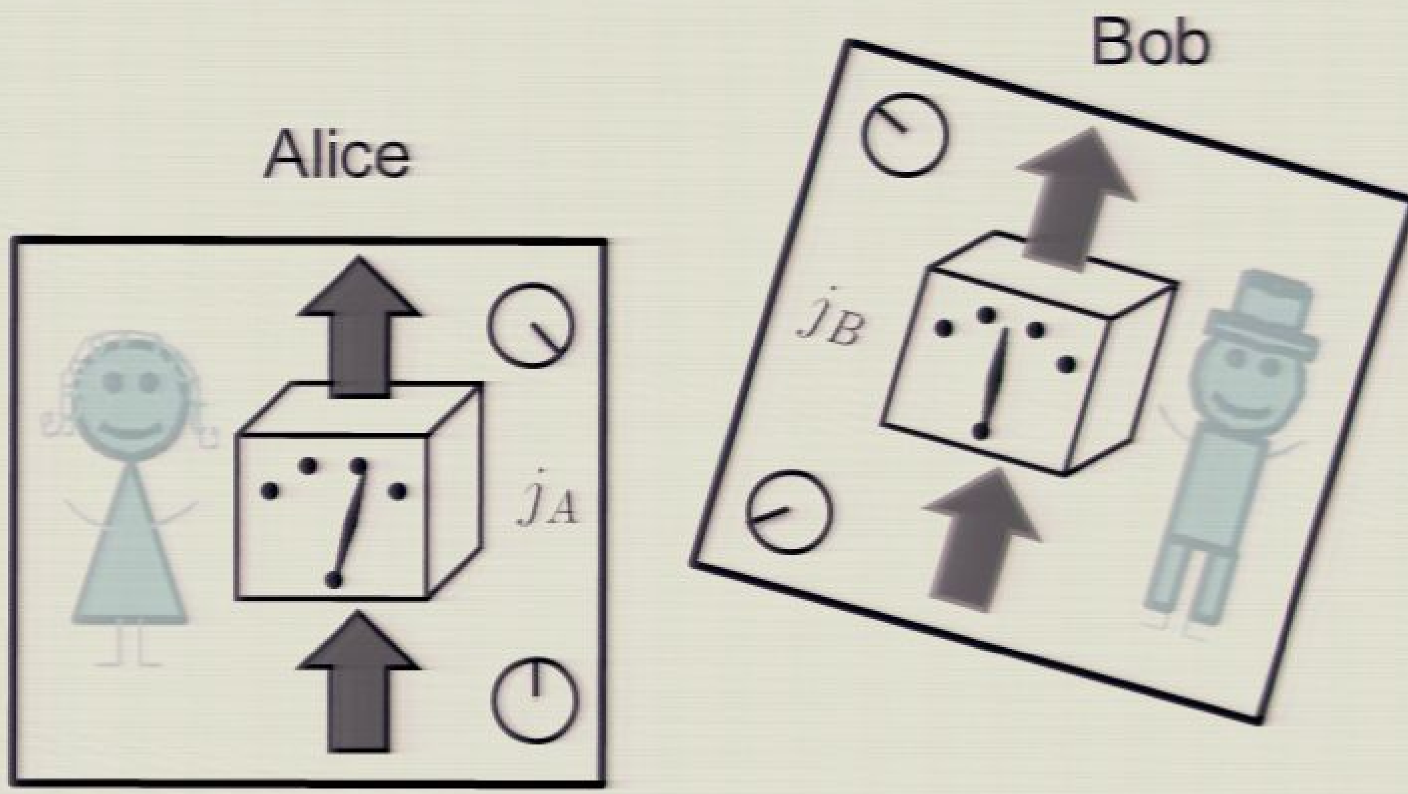

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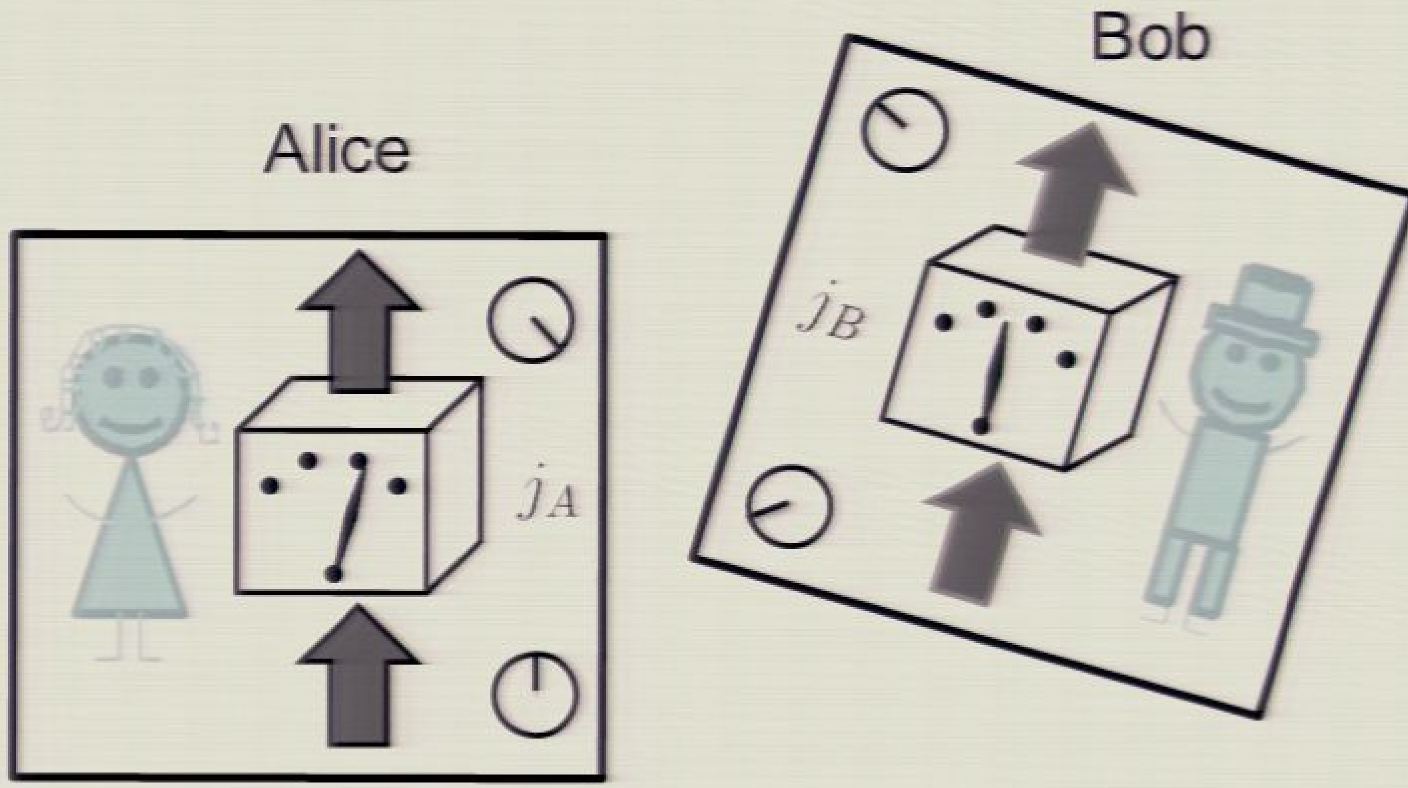
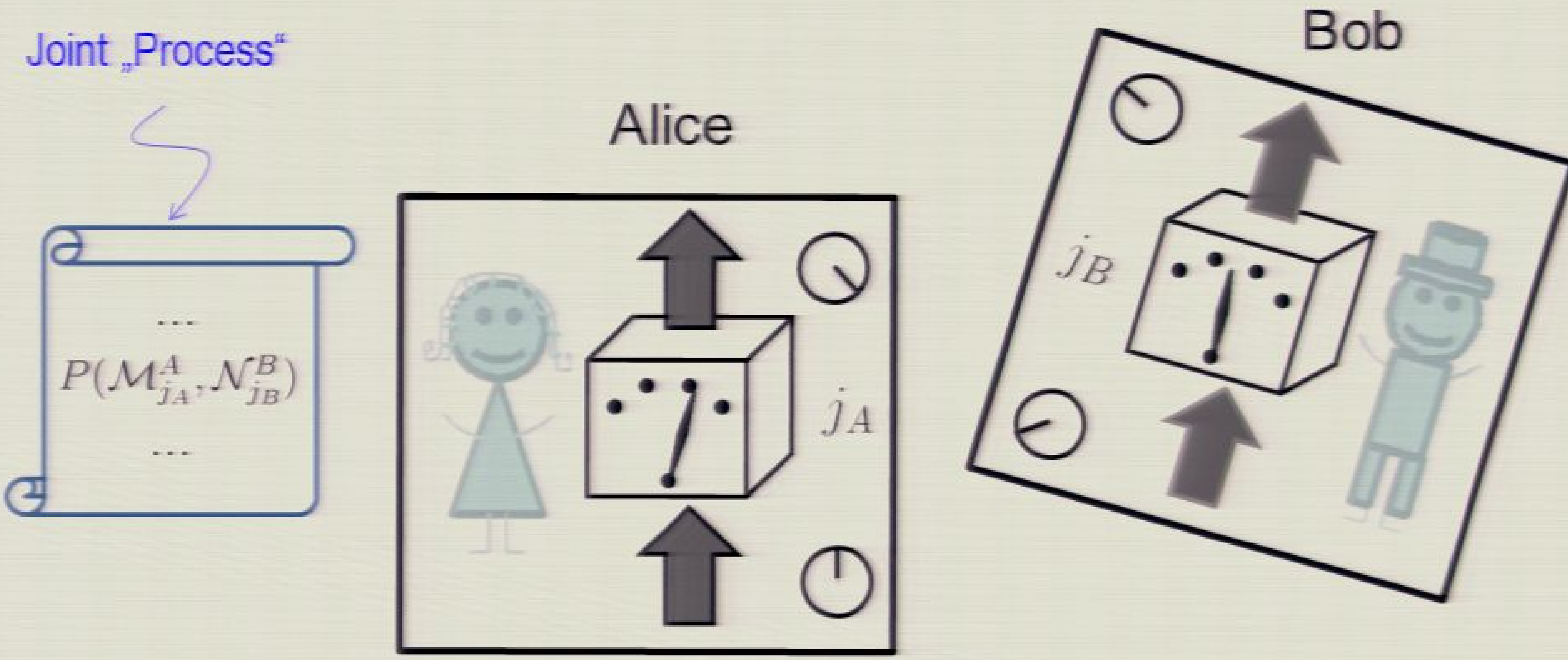


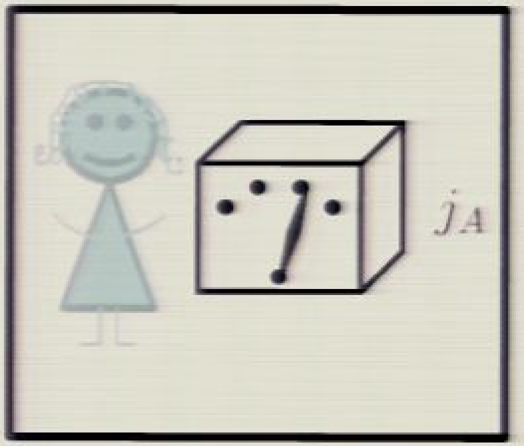
No prior assumption of pre-existing causal structure, in particular of the pre-existing background time.

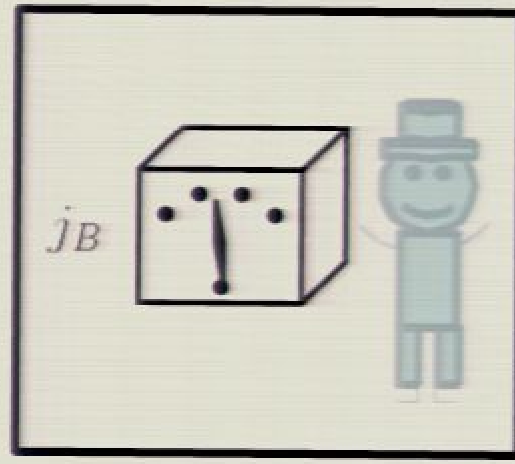
Operational approach



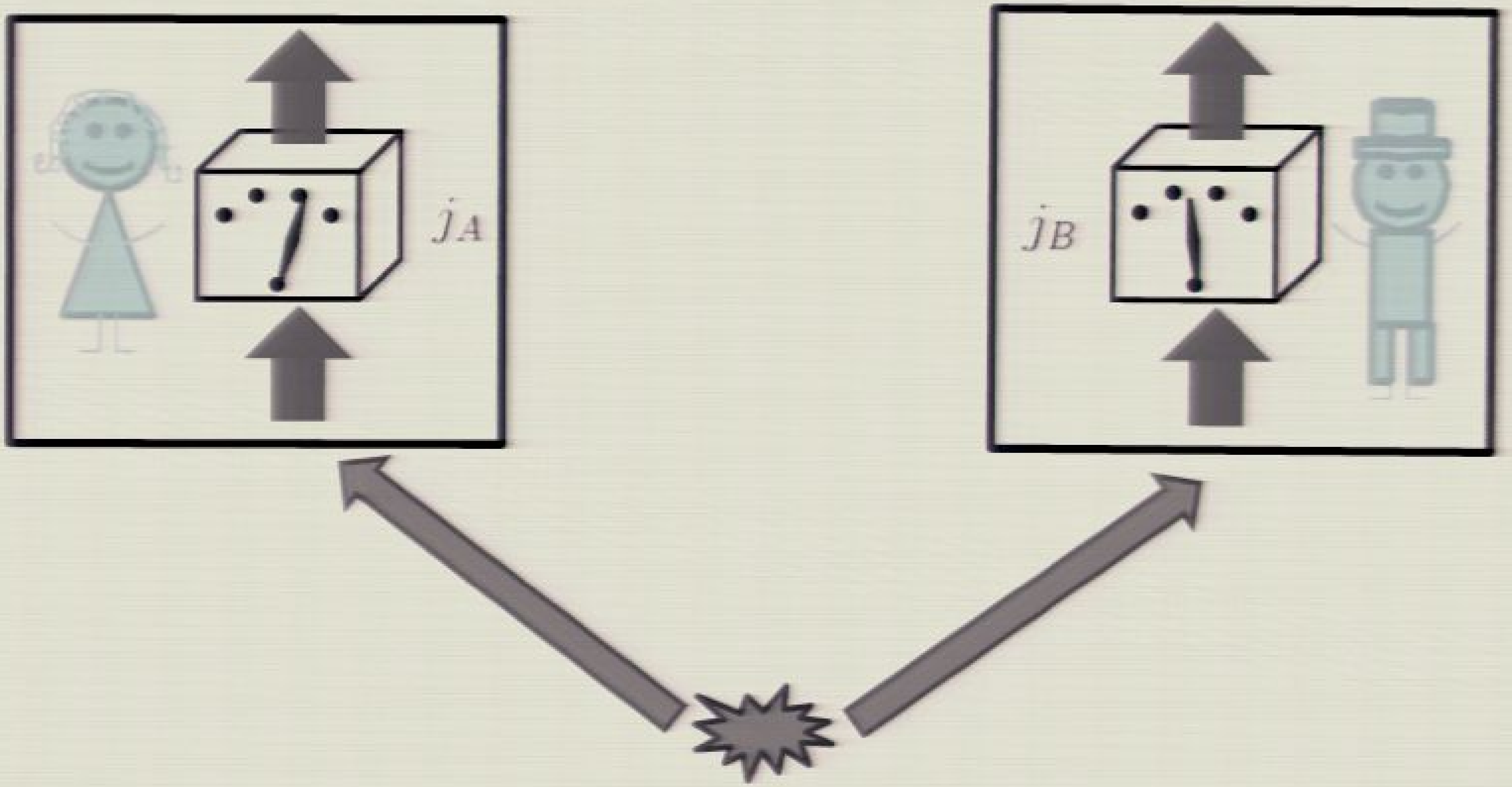
No prior assumption of pre-existing causal structure, in particular of the pre-existing background time.

Bipartite state

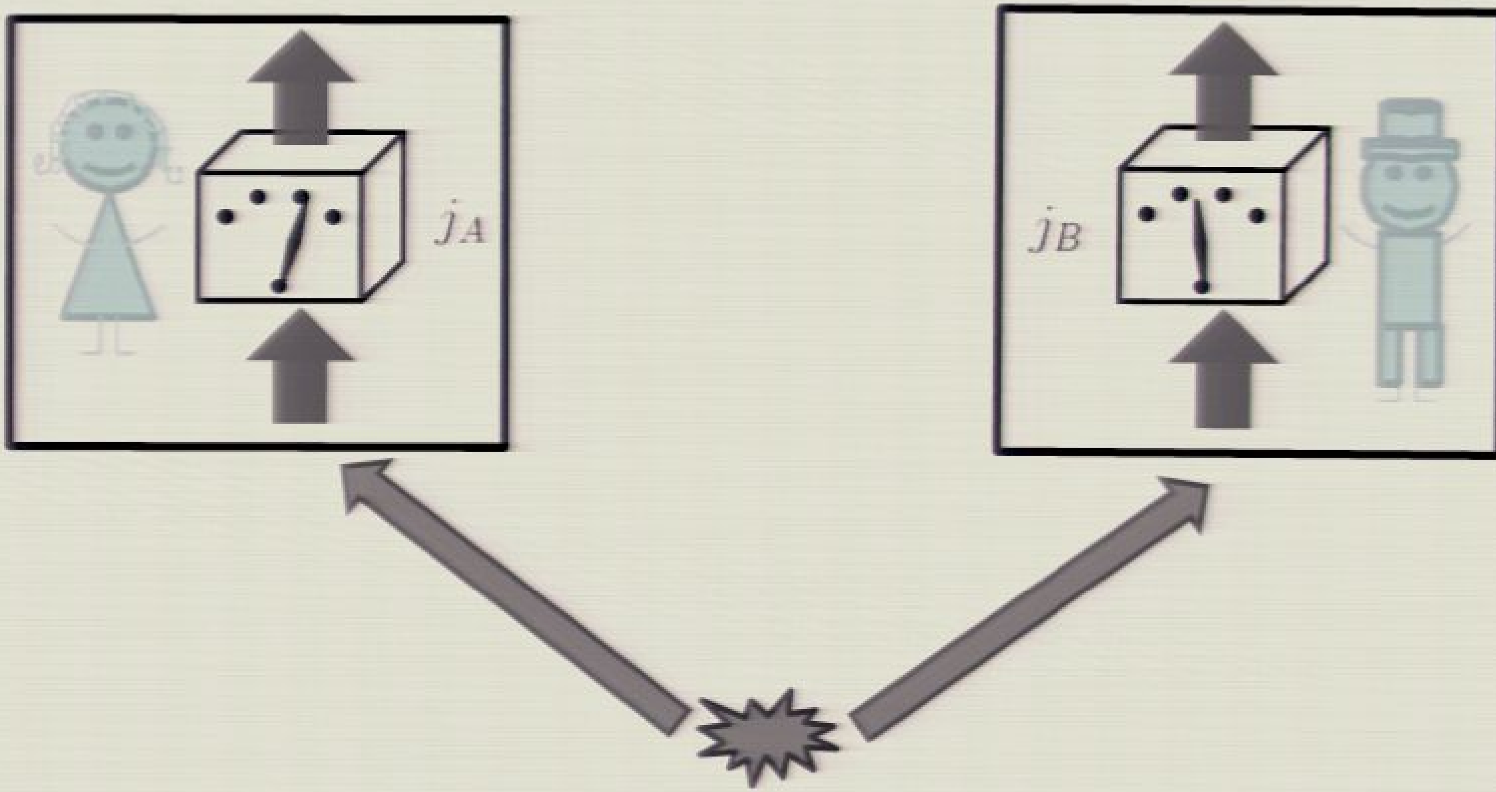




Bipartite state

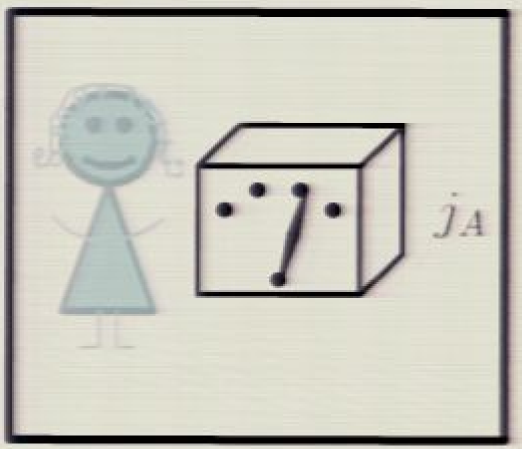


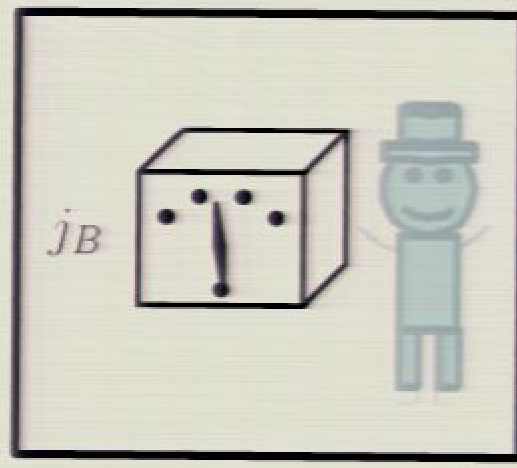
Bipartite state



Sharing a joint state; No signalling

Channel B→A





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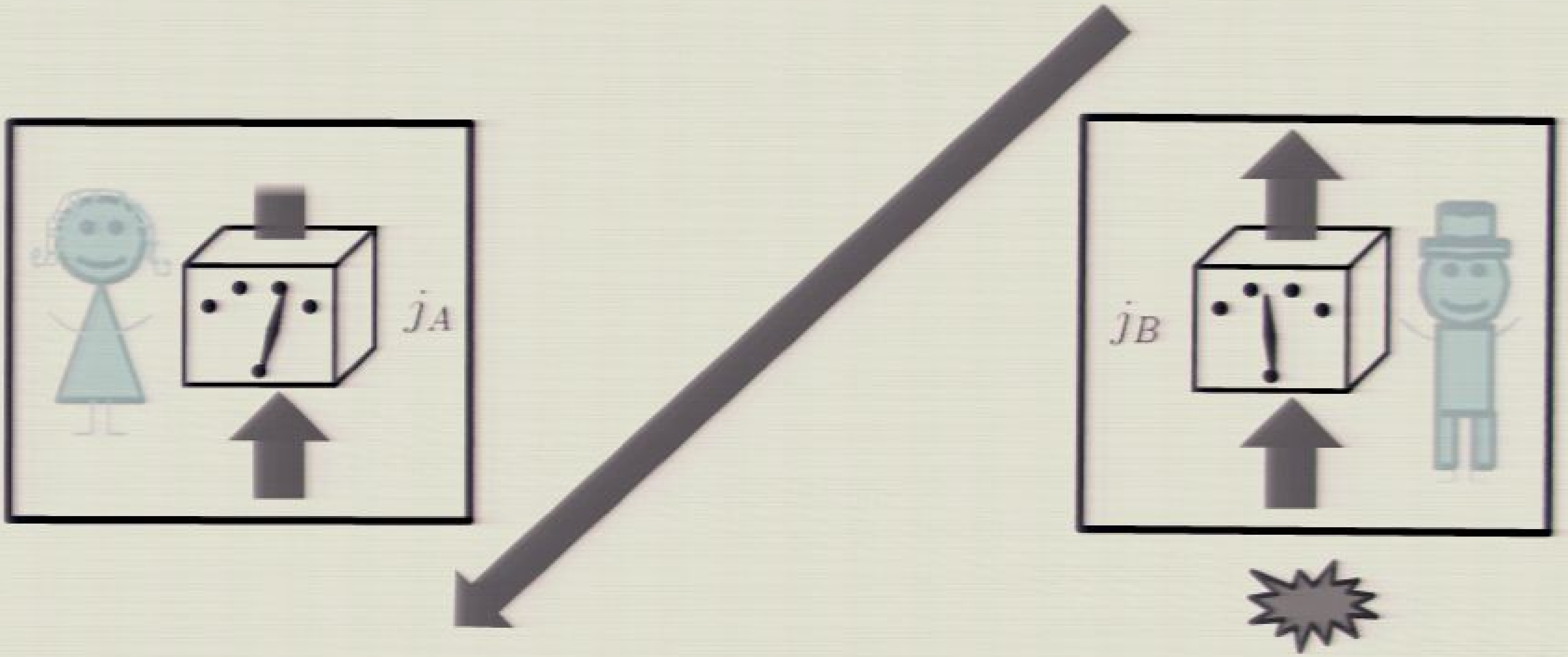

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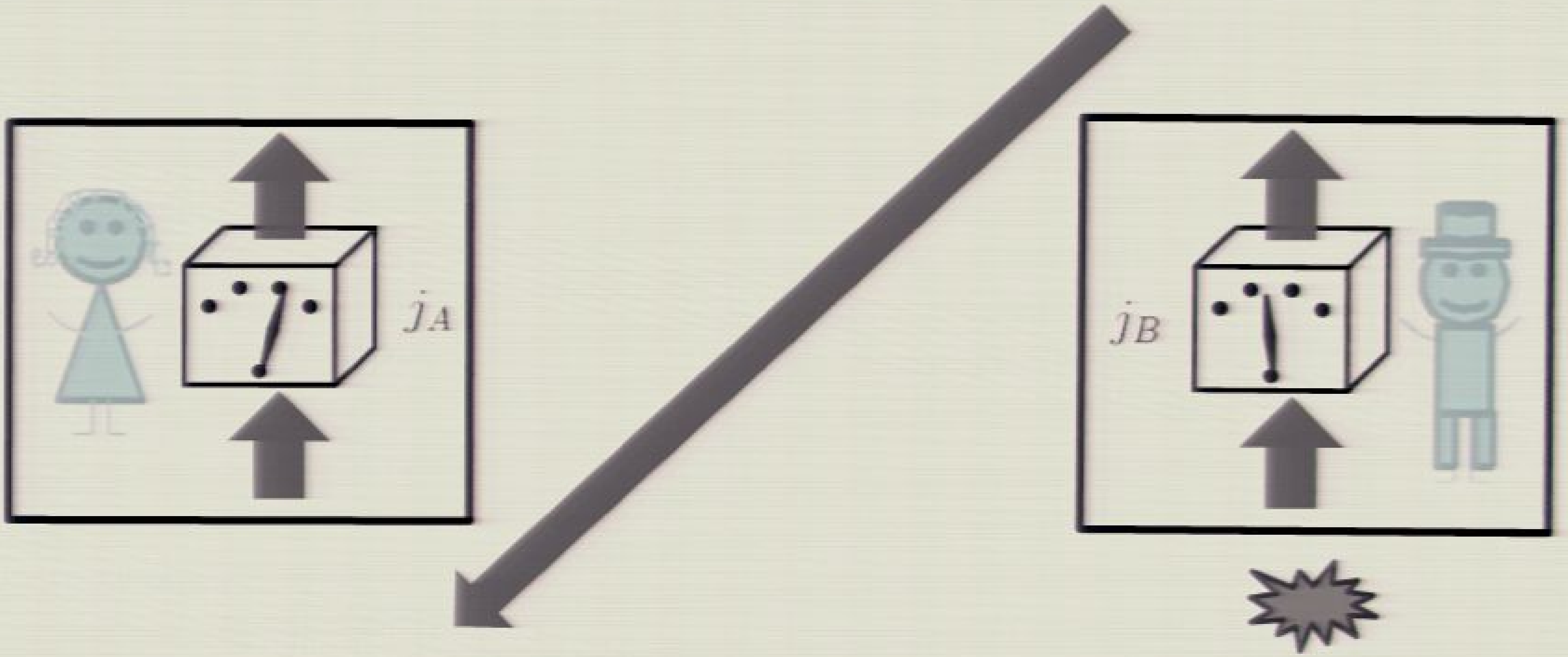
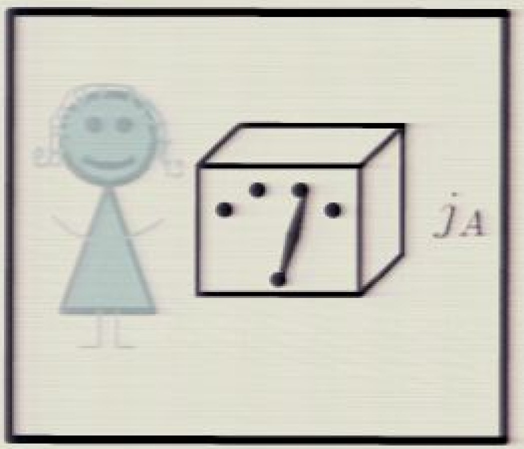
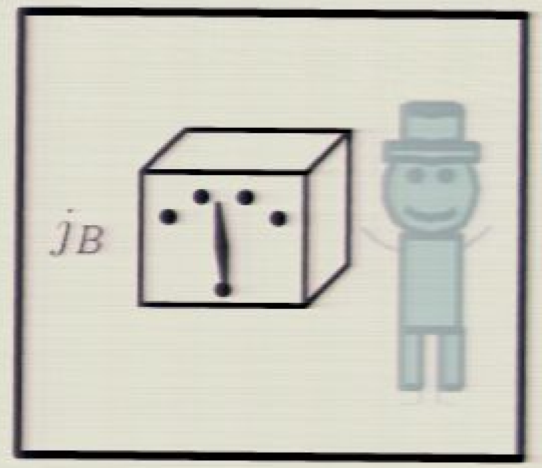


Sending a state from B to A; Possibility of signalling

Channel A→B





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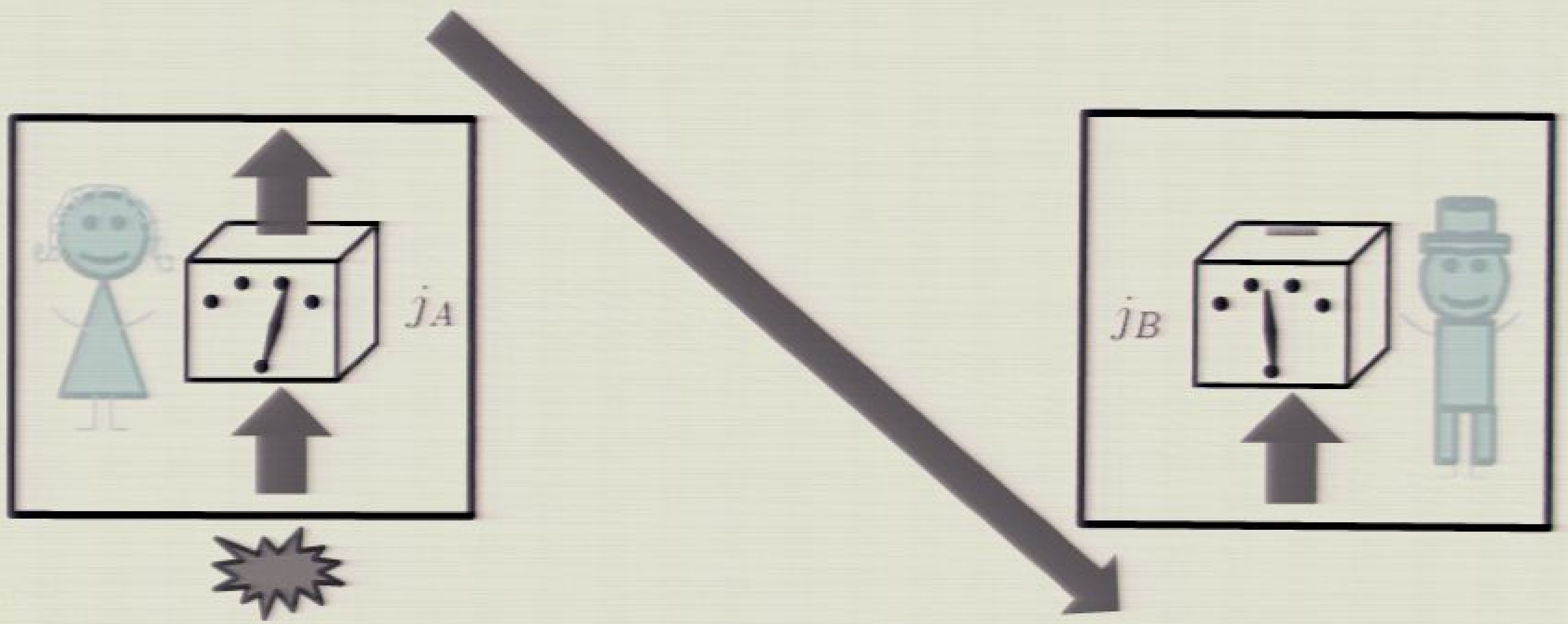

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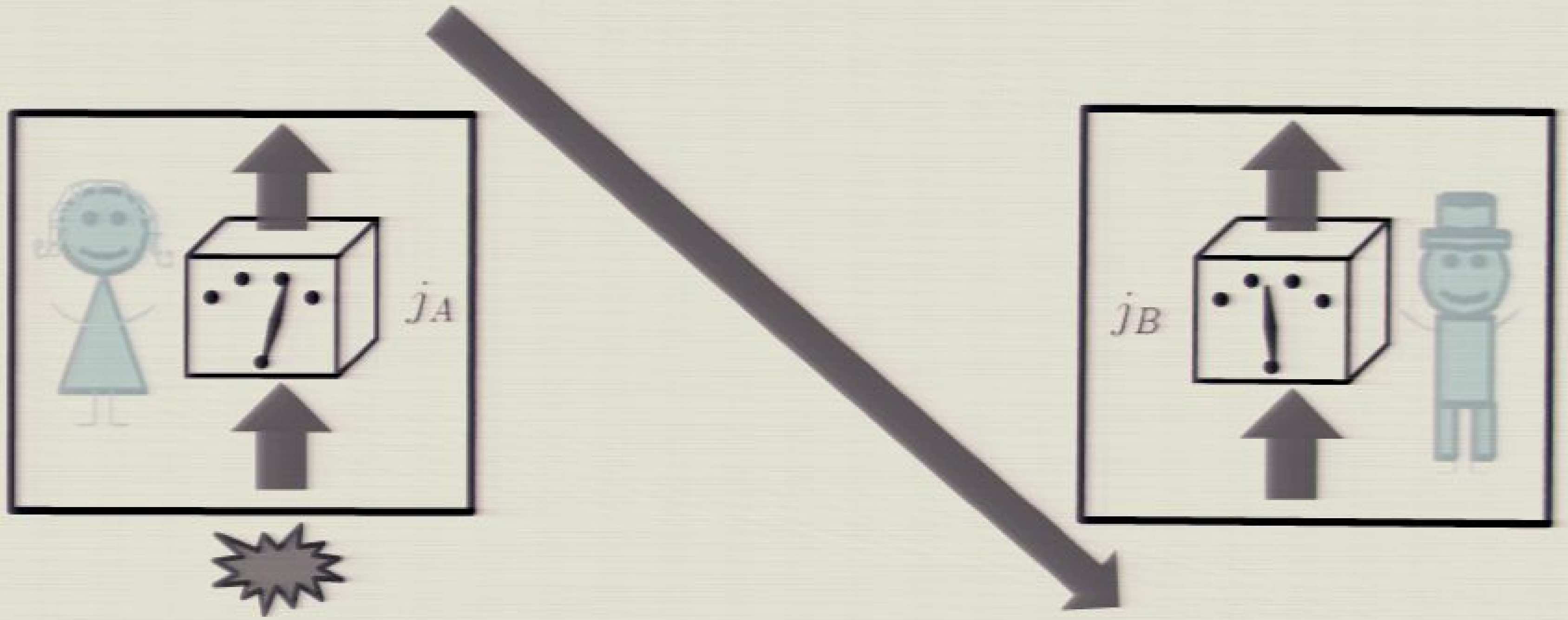
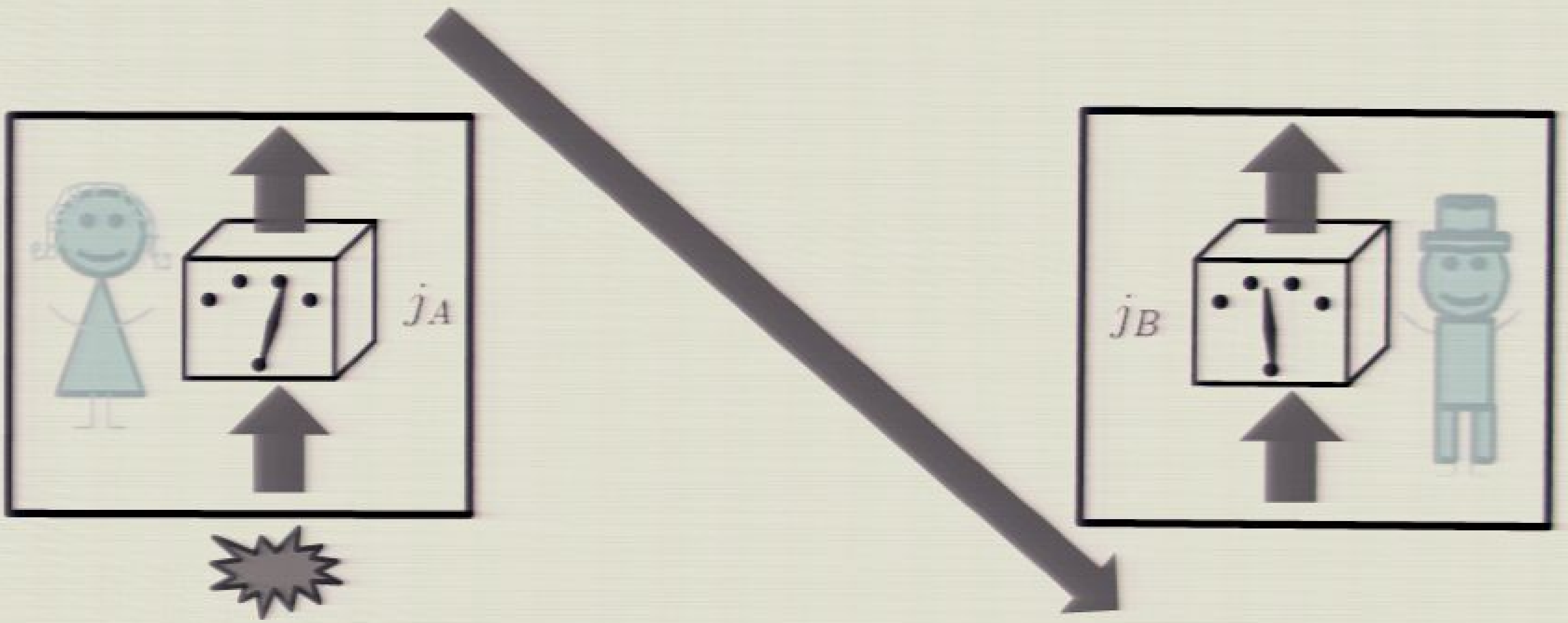
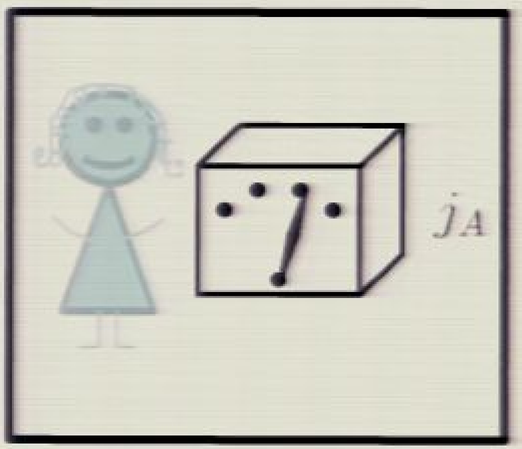


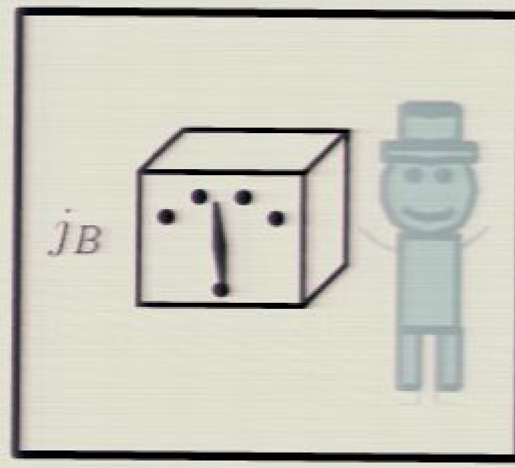
Channel A→B



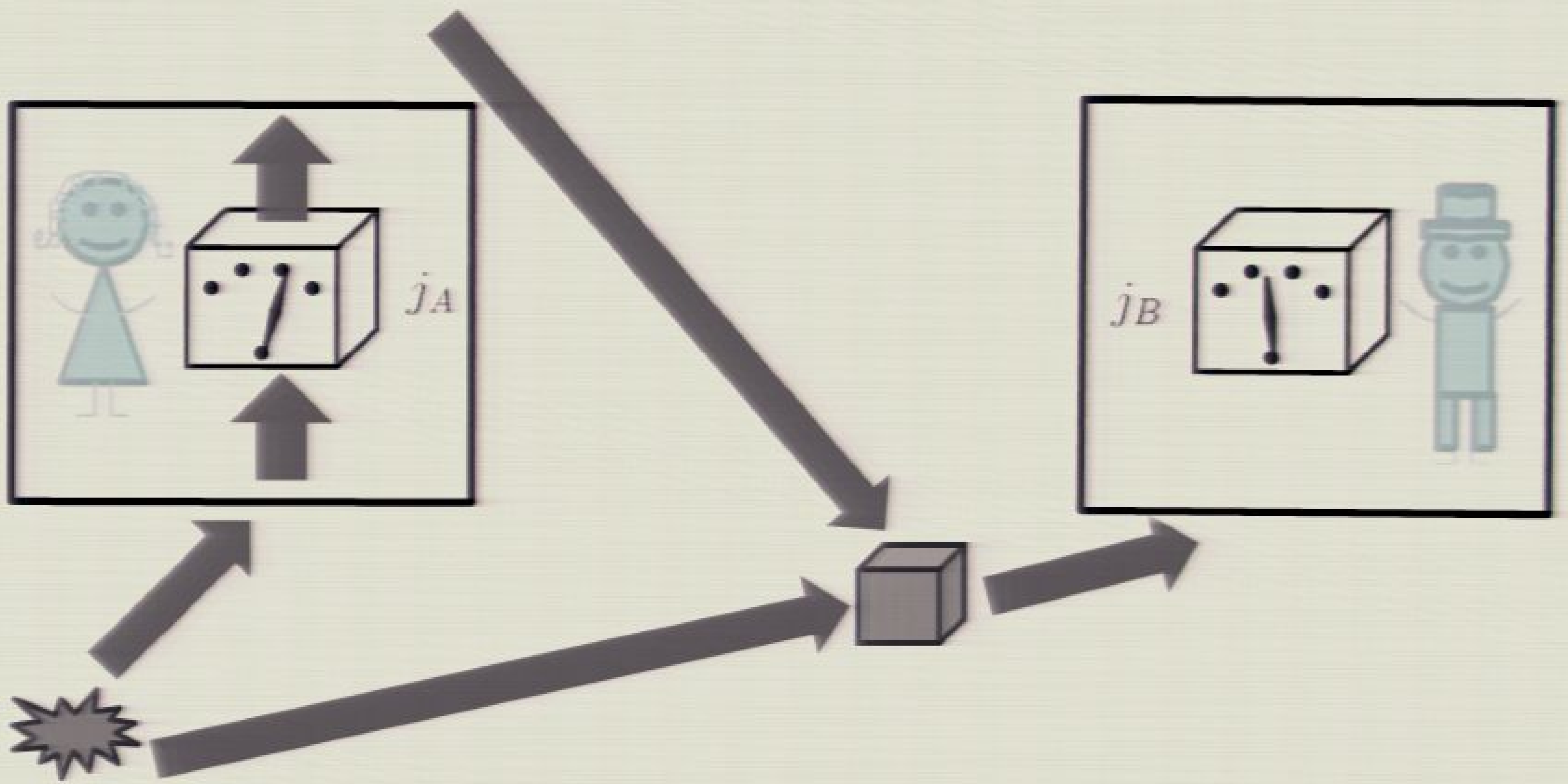
Sending a state from A to B; Possibility of signalling

Channel with memory





Channel with memory



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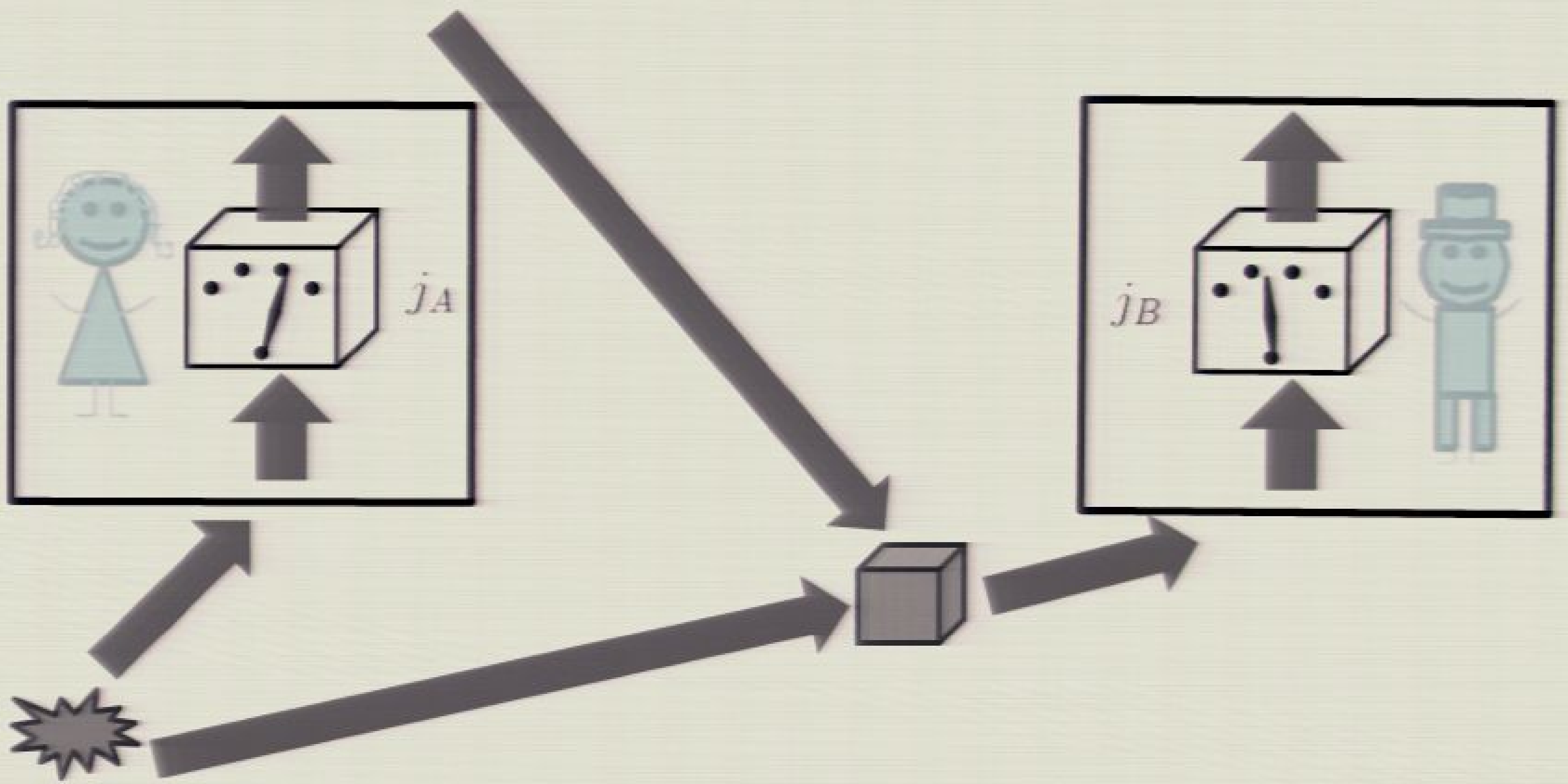
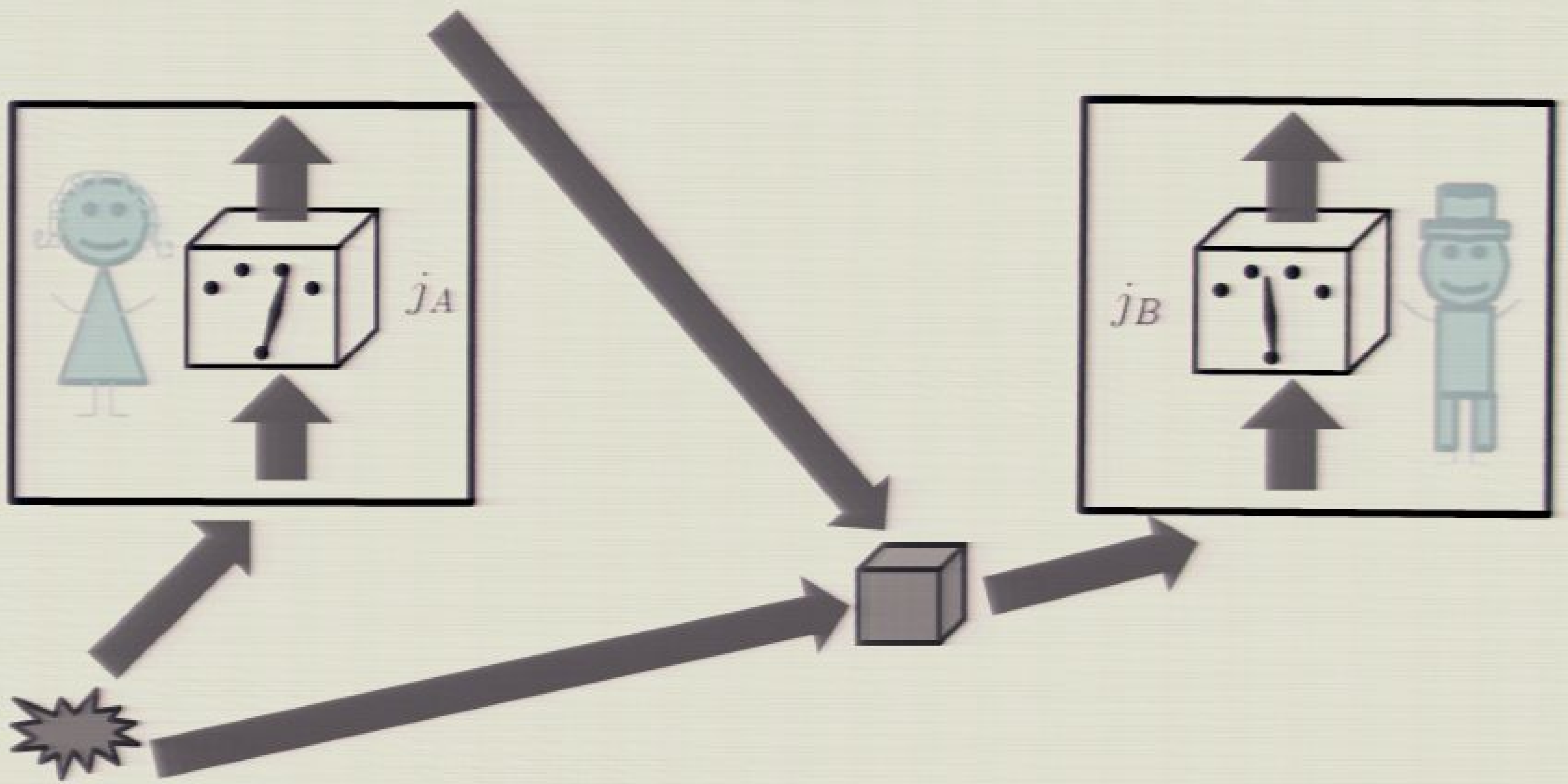


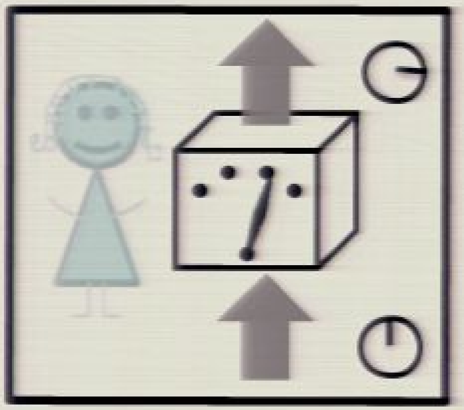
Channel with memory



Most general causally ordered situation; Signaling possible

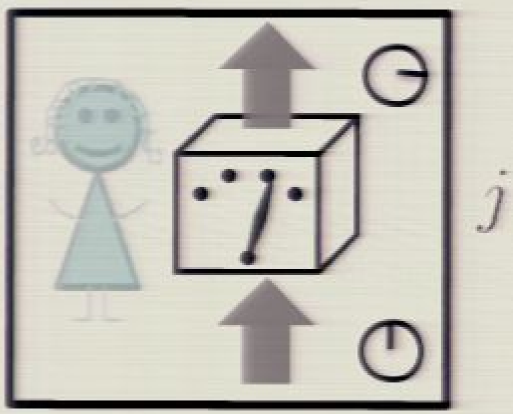
Mixtures of different orders also possible

Main premise



Main premise

Local descriptions agree with quantum mechanics



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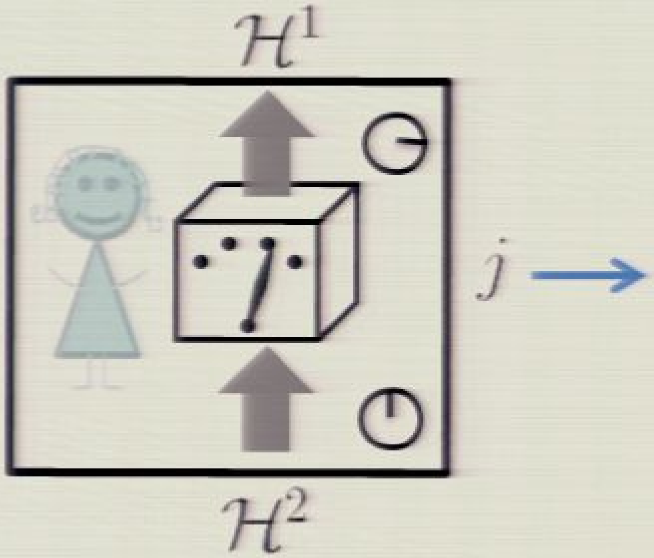

Transformations = completely positive (CP) trace non increasing maps

$$\mathcal{M}_j : \mathcal{L}(\mathcal{H}^2) \rightarrow \mathcal{L}(\mathcal{H}^1)$$

Main premise

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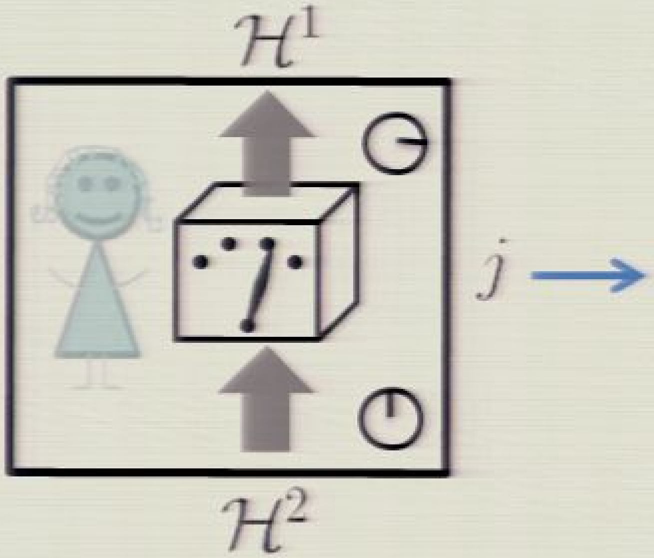

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Local algebra in both labs

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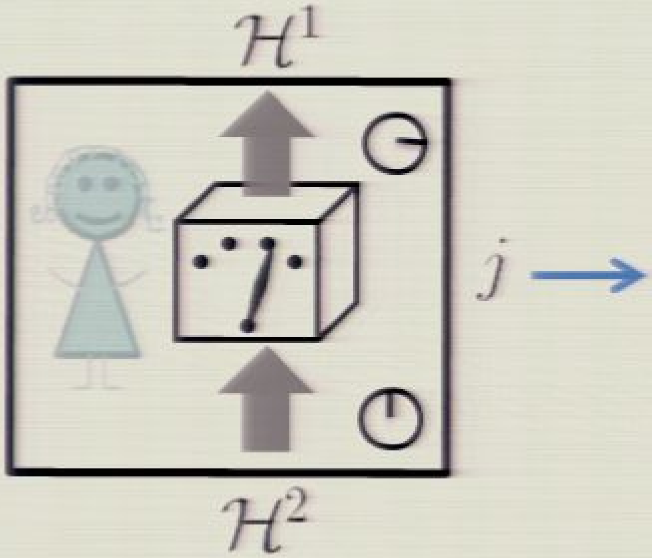
Local descriptions agree with quantum mechanics



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Convex Mixtures

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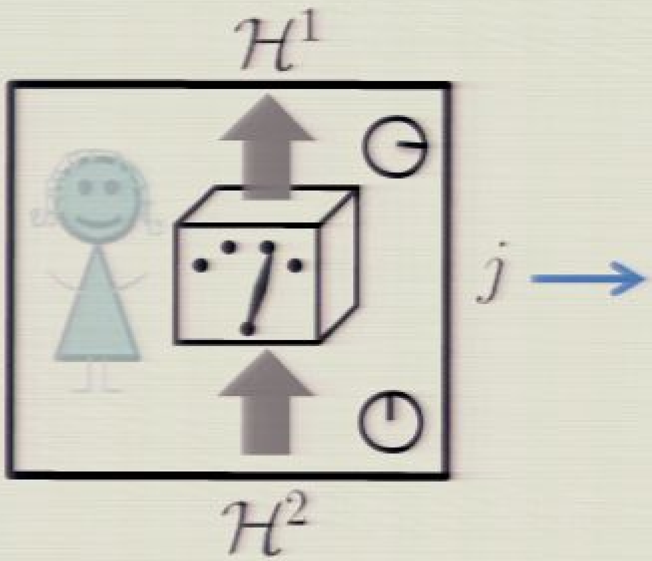

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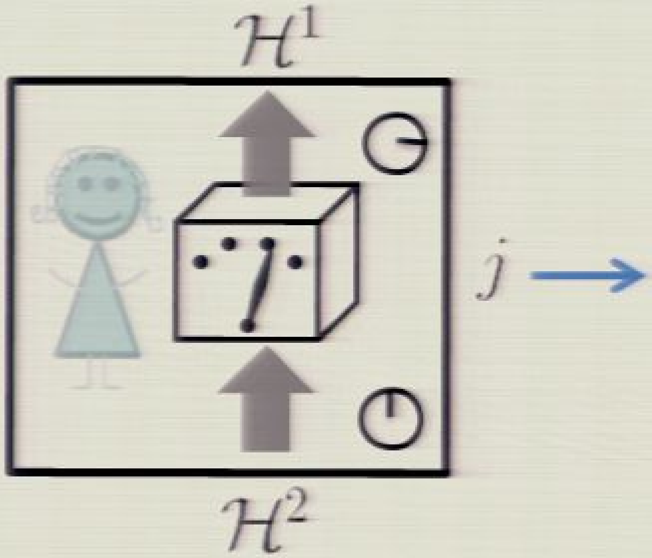
Local algebra in both labs

Convex Mixtures $$P(q\mathcal{M} + (1-q)\mathcal{N}) = qP(\mathcal{M}) + (1-q)P(\mathcal{N})$$

Main premise

Local descriptions agree with quantum mechanics



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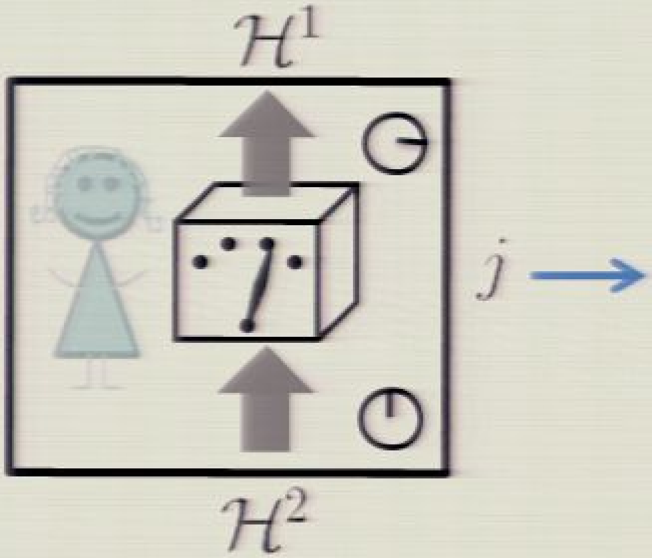
Local algebra in both labs

Convex Mixtures $$P(q\mathcal{M} + (1-q)\mathcal{N}) = qP(\mathcal{M}) + (1-q)P(\mathcal{N})$$

Distributivity $$P(\mathcal{M} + \mathcal{N}) = P(\mathcal{M}) + P(\mathcal{N})$$

Main premise

Local descriptions agree with quantum mechanics



Transformations = completely positive (CP) trace non increasing maps

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Local algebra in both labs

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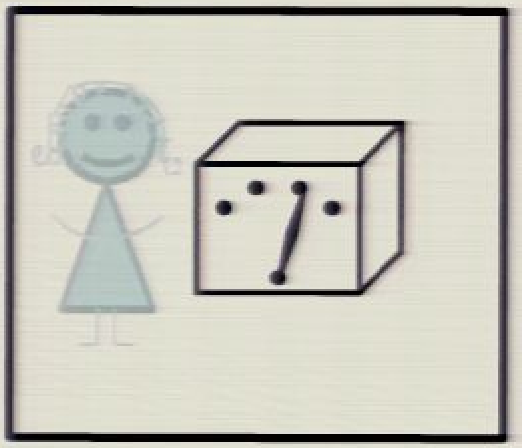
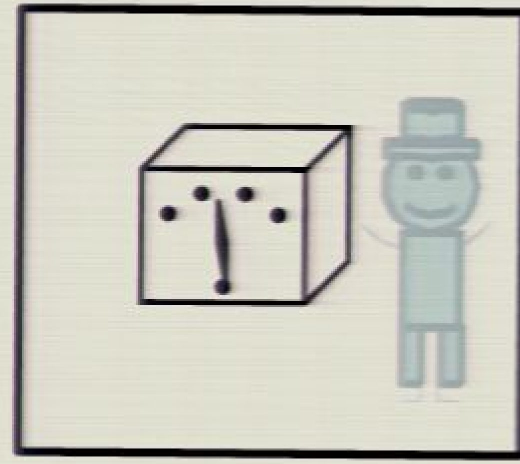
Distributivity $$P(\mathcal{M} + \mathcal{N}) = P(\mathcal{M}) + P(\mathcal{N})$$

⇒ Probabilities are linear functions of CP maps

Two parties





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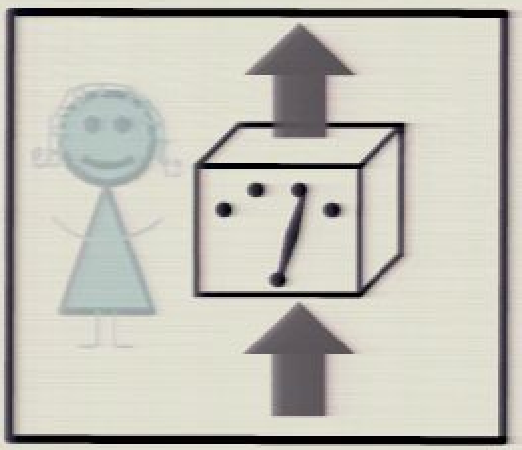

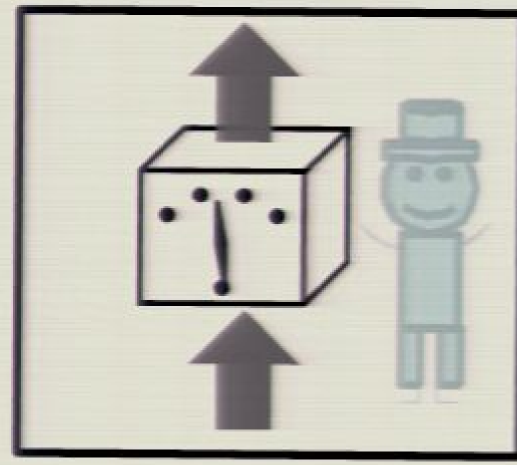

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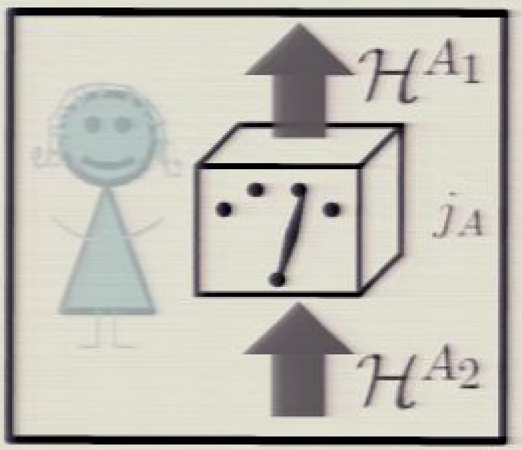
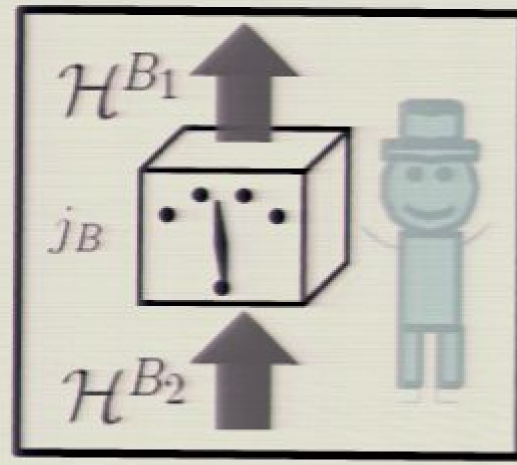
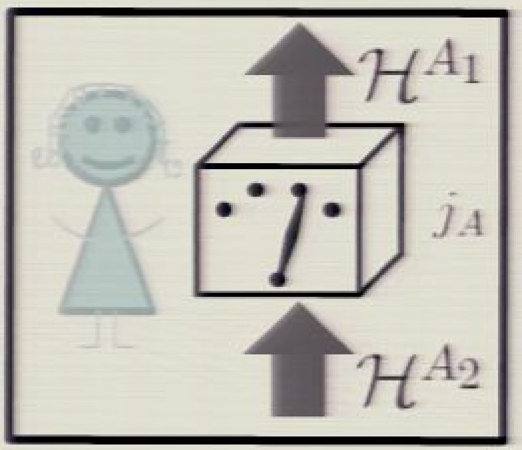




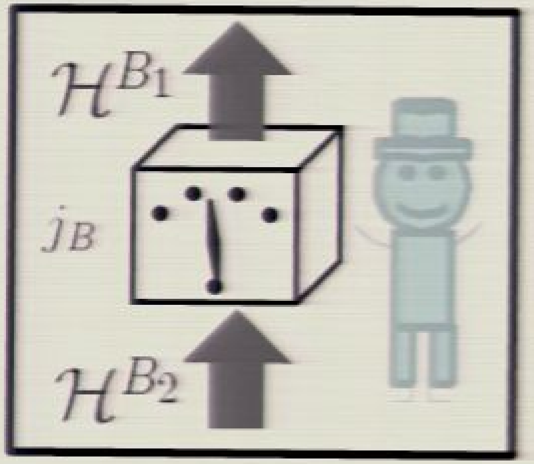
$$\mathcal{M}^A_{j_A} : \mathcal{L}(\mathcal{H}^{A_2}) \to \mathcal{L}(\mathcal{H}^{A_1})$$

$$\mathcal{M}^B_{j_B} : \mathcal{L}(\mathcal{H}^{B_2}) \to \mathcal{L}(\mathcal{H}^{B_1})$$

Two parties



$$\mathcal{M}^A_{j_A} : \mathcal{L}(\mathcal{H}^{A_2}) \to \mathcal{L}(\mathcal{H}^{A_1})$$

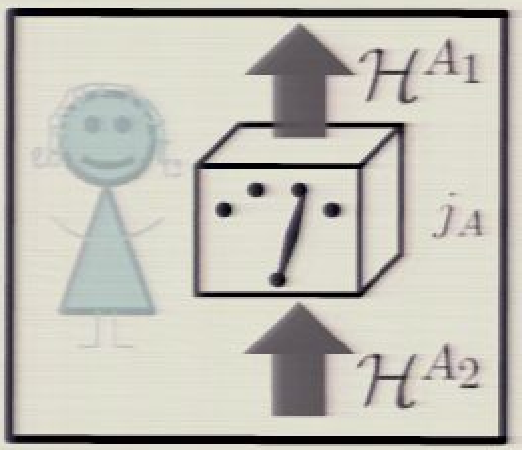


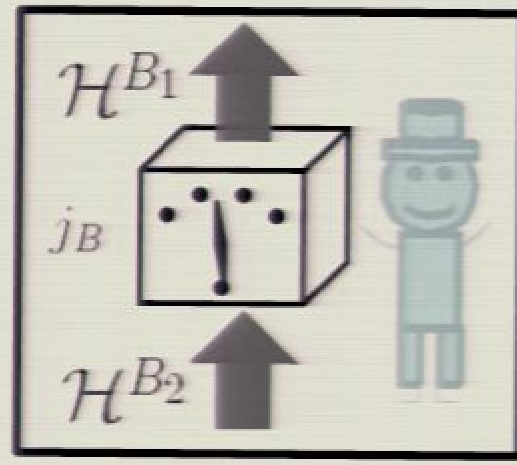
$$\mathcal{M}^B_{j_B} : \mathcal{L}(\mathcal{H}^{B_2}) \to \mathcal{L}(\mathcal{H}^{B_1})$$

Probabilities are bilinear functions of the CP maps

$$P(\mathcal{M}^A, \mathcal{M}^B) = \omega(\mathcal{M}^A, \mathcal{M}^B)$$

Two parties





$$\mathcal{M}^A_{j_A} : \mathcal{L}(\mathcal{H}^{A_2}) \to \mathcal{L}(\mathcal{H}^{A_1}) \qquad \mathcal{M}^B_{j_B} : \mathcal{L}(\mathcal{H}^{B_2}) \to \mathcal{L}(\mathcal{H}^{B_1})$$

Probabilities are bilinear functions of the CP maps

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Question: how to characterize the most general probability distributions?

Choi-Jamiołkowski isomorphism

Choi-Jamiołkowski isomorphism

CP maps ⟷ Bipartite positive operators

$\mathcal{M} : \mathcal{L}(\mathcal{H}^2) \rightarrow \mathcal{L}(\mathcal{H}^1)$ ⟷ $\rho^{12}_{\mathcal{M}} \in \mathcal{L}(\mathcal{H}^1) \otimes \mathcal{L}(\mathcal{H}^2)$

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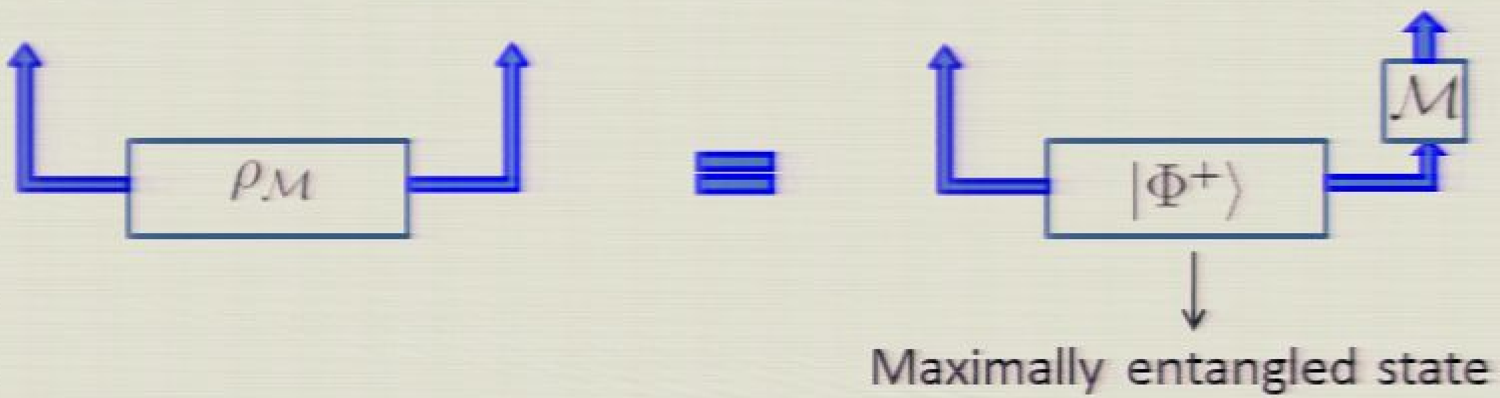

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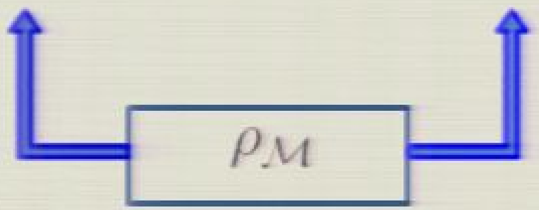

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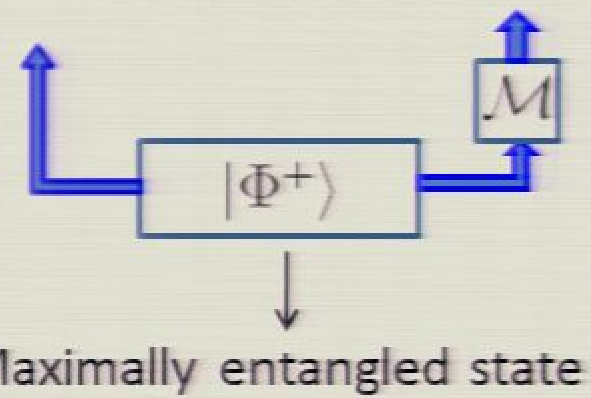

$$\rho_{\mathcal{M}}^{12} = \mathcal{M} \otimes \mathcal{I}(|\Phi^+\rangle\langle\Phi^+|)$$

$$|\Phi^+\rangle = \sum_i |i\rangle|i\rangle$$

$$|i\rangle \in \mathcal{H}^1$$

Choi-Jamiołkowski isomorphism

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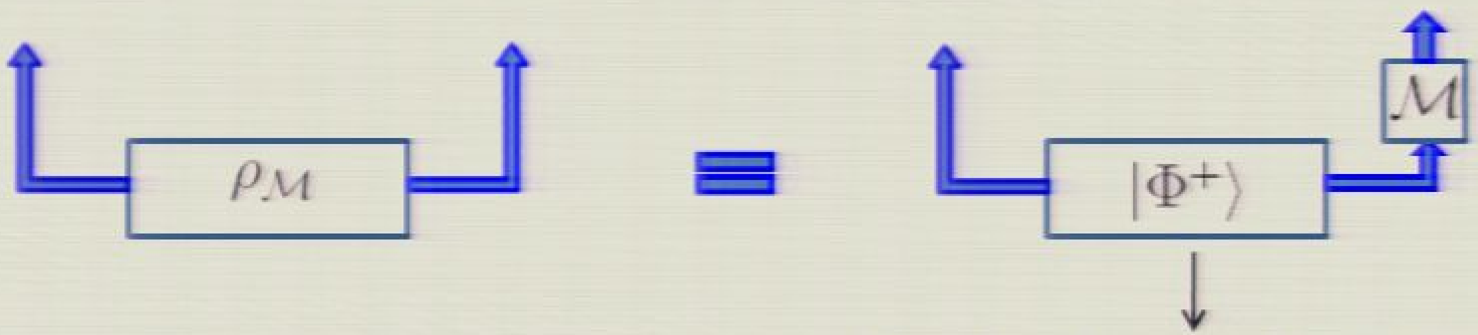

Maximally entangled state

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Examples

Choi-Jamiołkowski isomorphism

CP maps ⟷ Bipartite positive operators

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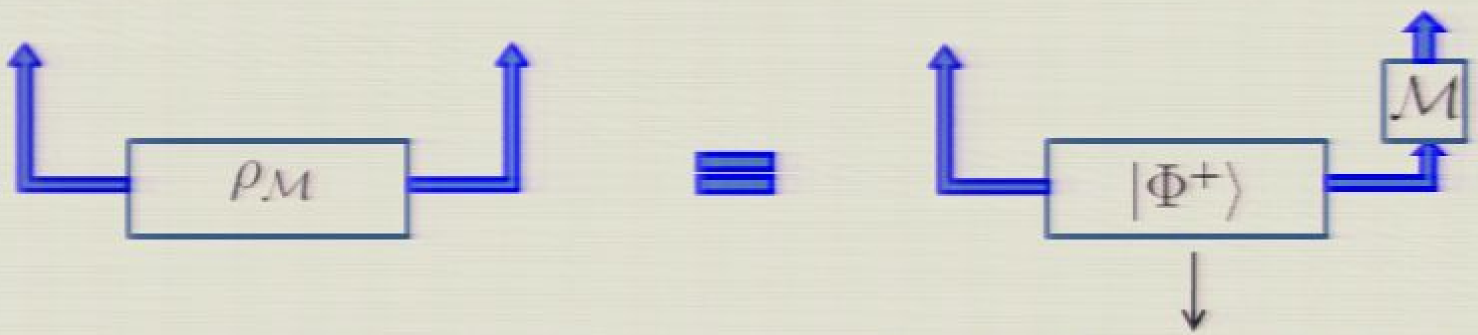

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Examples

Projection on a pure state $|\psi\rangle$
and its repreparation

$$\rho^{12} = |\psi\rangle\langle\psi|^1 \otimes |\psi\rangle\langle\psi|^2$$

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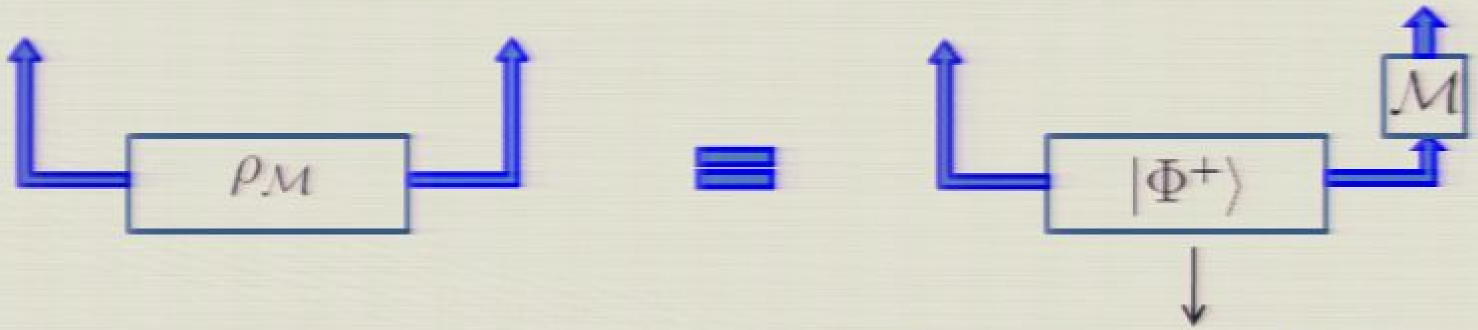

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Examples

Projection on a pure state $|\psi\rangle$ and its repreparation

$$\rho^{12}=|\psi\rangle\langle\psi|^1\otimes|\psi\rangle\langle\psi|^2$$

Preparation of a new state σ

$$\rho^{12}=\sigma^1\otimes\mathbb{1}^2$$

Bipartite probabilities

Bipartite probabilities

Bilinear functions of CP maps → Bilinear functions of positive operators

$$P(\mathcal{M}^A, \mathcal{M}^B) = \omega(\mathcal{M}^A, \mathcal{M}^B)$$

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Representation

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[W^{A_1 A_2 B_1 B_2}\left(\rho_{\mathcal{M}^A}^{A_1 A_2} \otimes \rho_{\mathcal{M}^B}^{B_1 B_2}\right)\right]$$

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„Process“ Matrix

Bipartite probabilities

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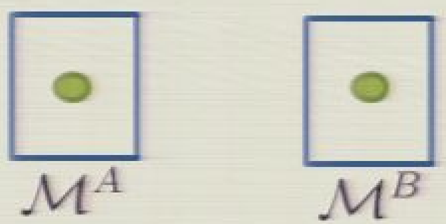

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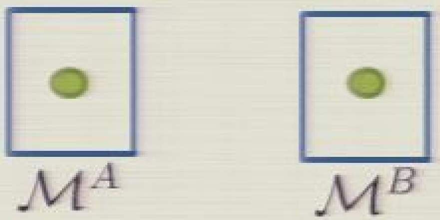

$P(\mathcal{M}^A, \mathcal{N}^B) \geq 0$ ➡ $W^{A_1 A_2 B_1 B_2}$ POPT

Bipartite probabilities

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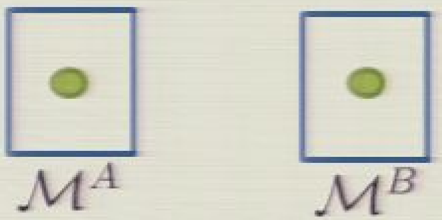

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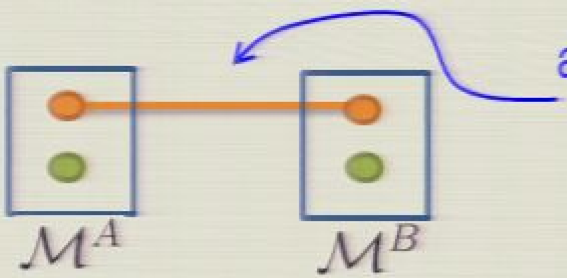

ancillary entangled state do not fix the causal order

Bipartite probabilities

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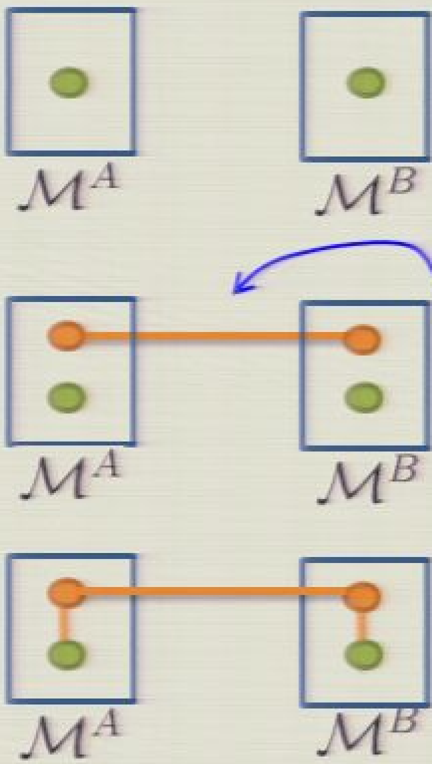

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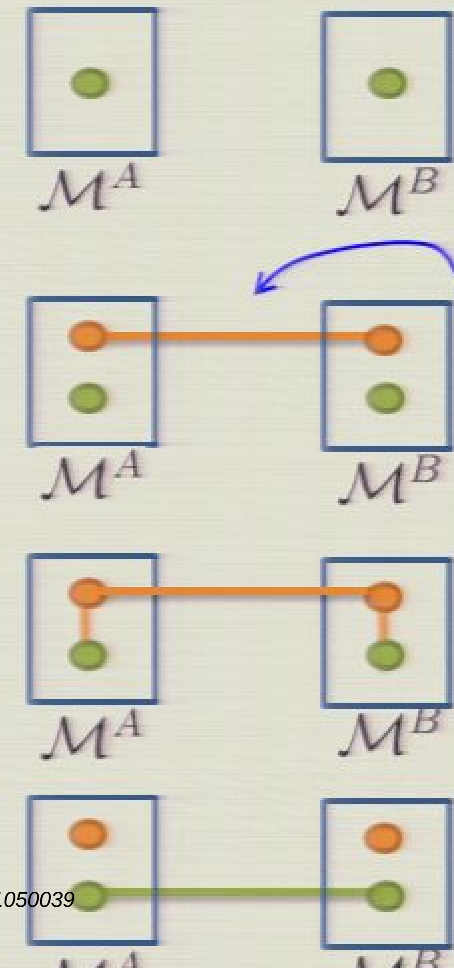

$P(\mathcal{M}^A, \mathcal{N}^B) \geq 0 \Rightarrow W^{A_1 A_2 B_1 B_2}$ POPT

ancillary entangled state do not fix the causal order

Bipartite probabilities

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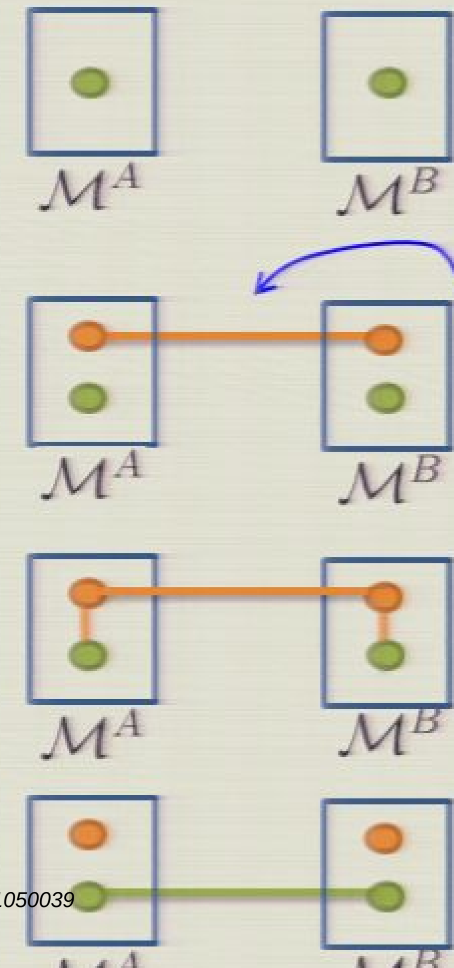

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ancillary entangled state do not fix the causal order

1. Probability positive: $W^{A_1 A_2 B_1 B_2} \geq 0$

Bipartite probabilities

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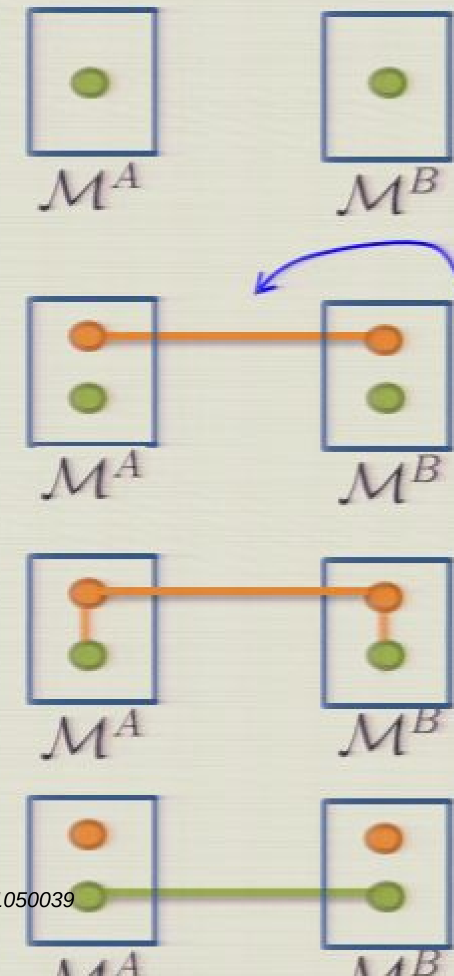

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ancillary entangled state do not fix the causal order

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H. Barnum, C. A. Fuchs, J. M. Renes, and A. Wilce,

Bipartite probabilities

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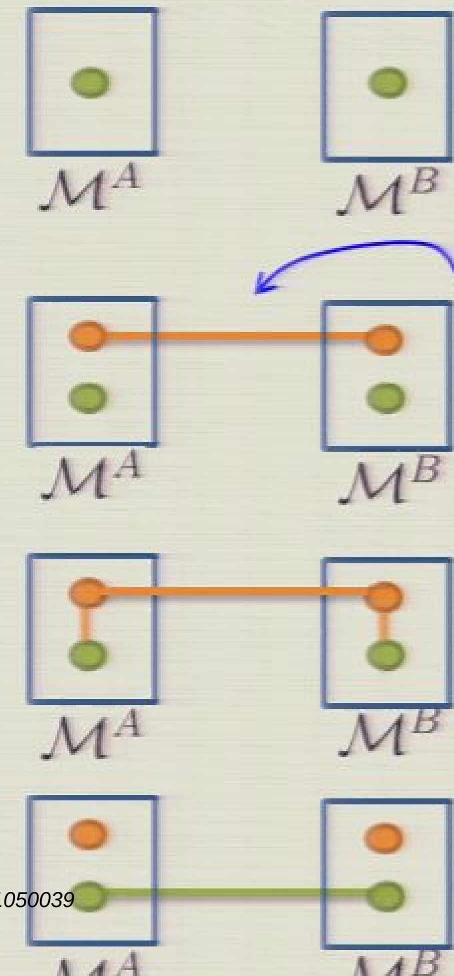

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ancillary entangled state do not fix the causal order

1. Probability positive: $W^{A_1A_2B_1B_2} \geq 0$

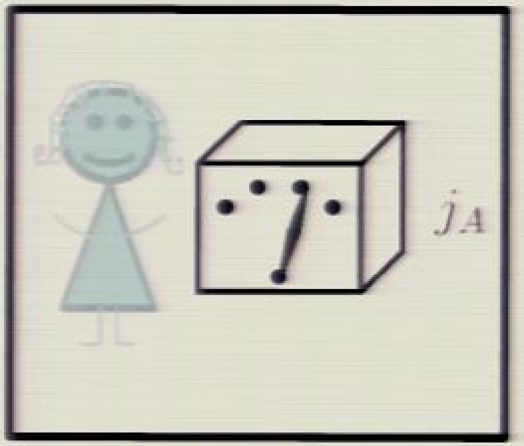
2. Probability 1 on all CPTP maps:

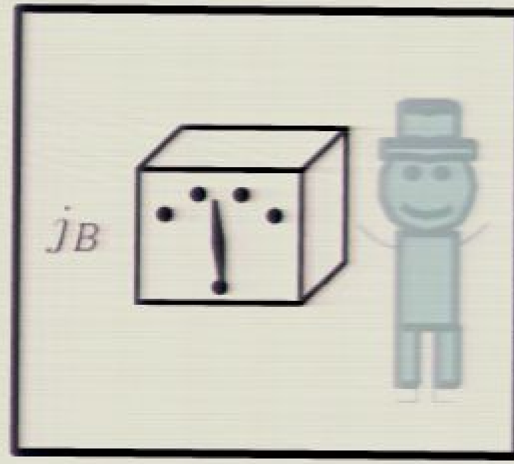
$$\mathrm{Tr}\left[W^{A_1A_2B_1B_2}\left(\rho_{\mathcal{E}^A}^{A_1A_2} \rho_{\mathcal{E}^B}^{B_1B_2}\right)\right] = 1$$

$$\forall \rho^{A_1A_2}, \rho^{B_1B_2} > 0, \ \mathrm{Tr}_1\rho^{A_1A_2} = \mathbb{1}^{A_2}, \mathrm{Tr}_1\rho^{B_1B_2} = \mathbb{1}^{B_2}$$

H. Barnum, C. A. Fuchs, J. M. Renes, and A. Wilce,

Example: Bipartite state





Sharing a joint state,
No signalling

$$W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1 B_1} \otimes \left(\rho^{A_2 B_2}\right)^T$$

$$P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[\mathcal{M}^A \otimes \mathcal{M}^B(\rho^{A_2 B_2})\right]$$

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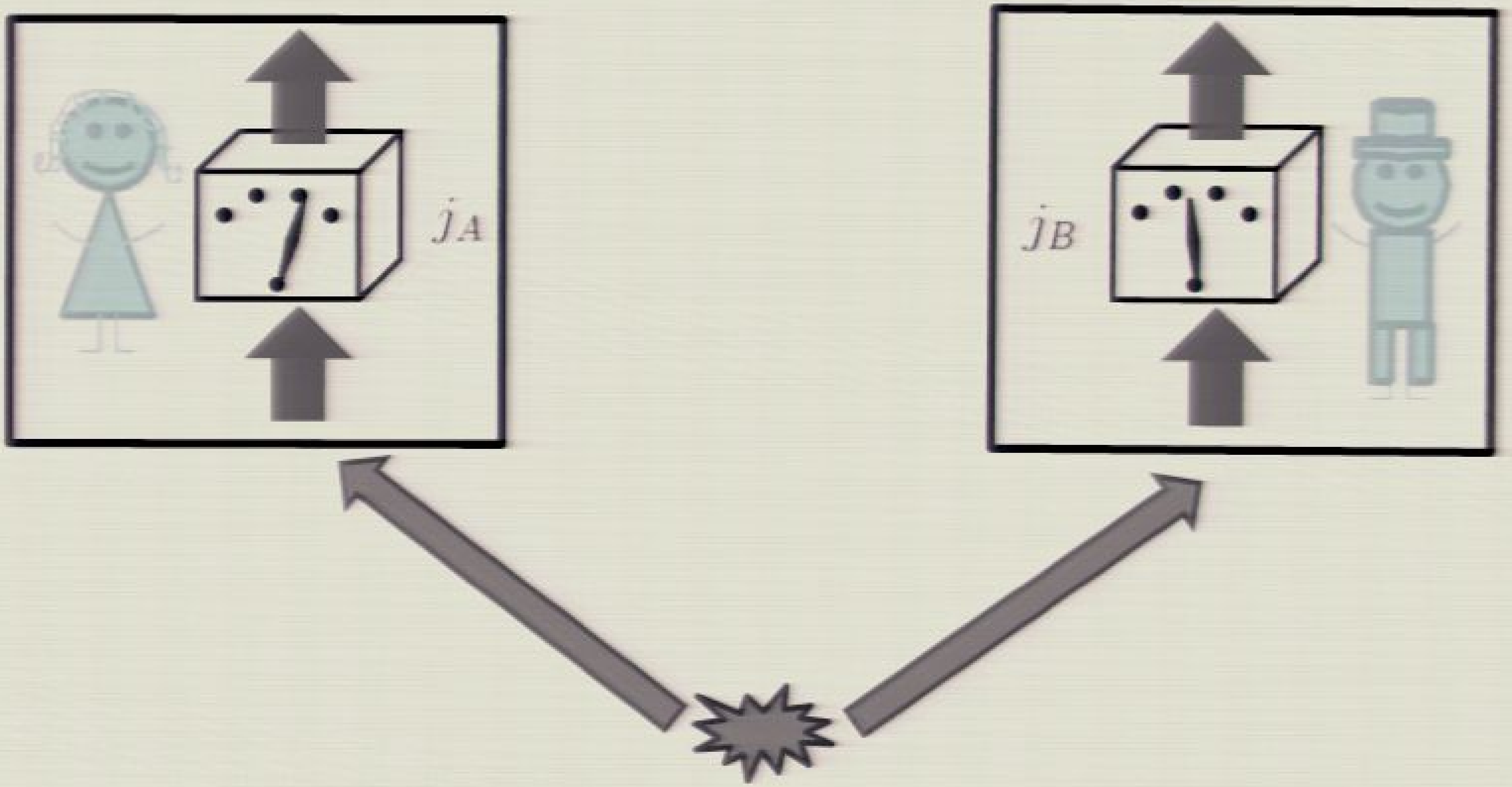

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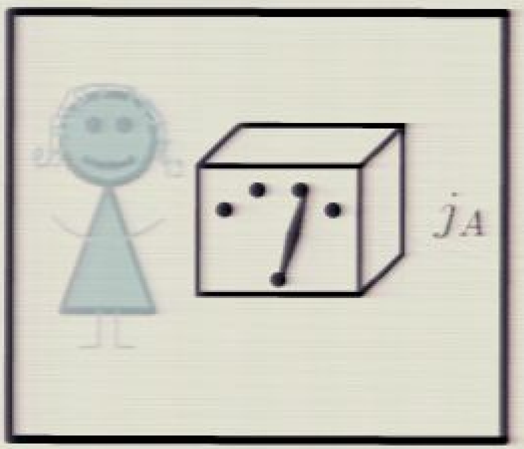
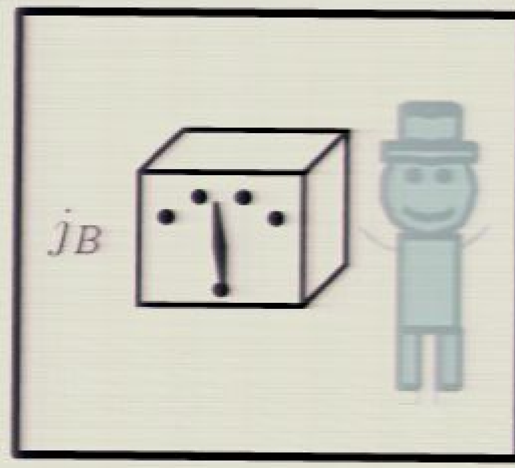
Example: Channel B→A





Sending a state from B to A, $W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1} \otimes \left(\rho_{\mathcal{E}}^{A_2 B_1}\right)^T \rho_0^{B_2^T}$

Possibility of signalling $P(\mathcal{M}^A, \mathcal{M}^B) = \mathrm{Tr}\left[\mathcal{M}^A \circ \mathcal{E} \circ \mathcal{M}^B \left(\rho_0^{B_2}\right)\right]$

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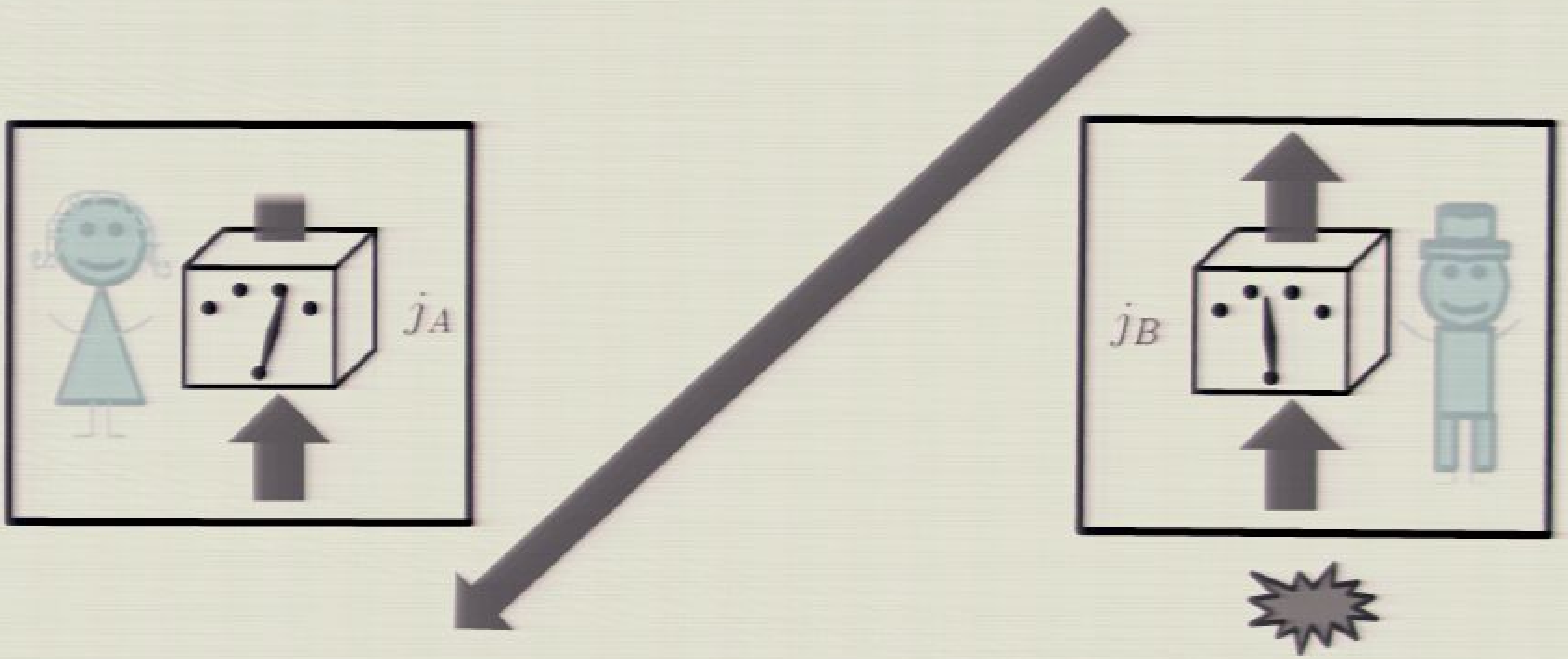

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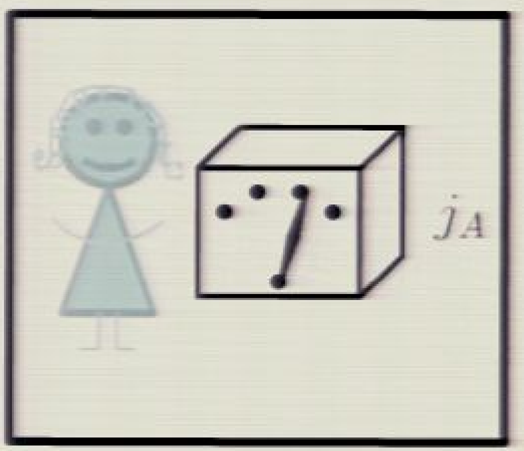
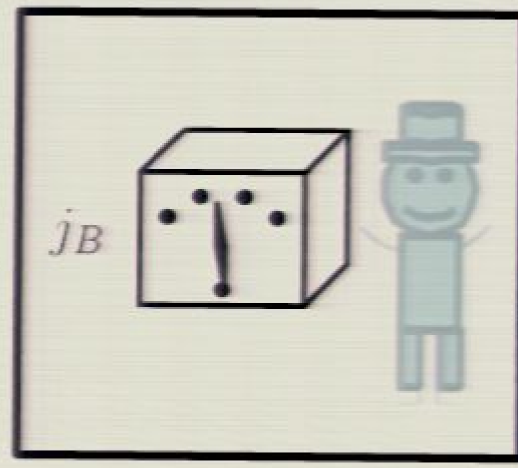
Channel with memory





Possiblity of signaling $W^{A_1A_2B_1B_2} = \mathbb{1}^{A_1} \otimes W^{A_2B_1B_2}$

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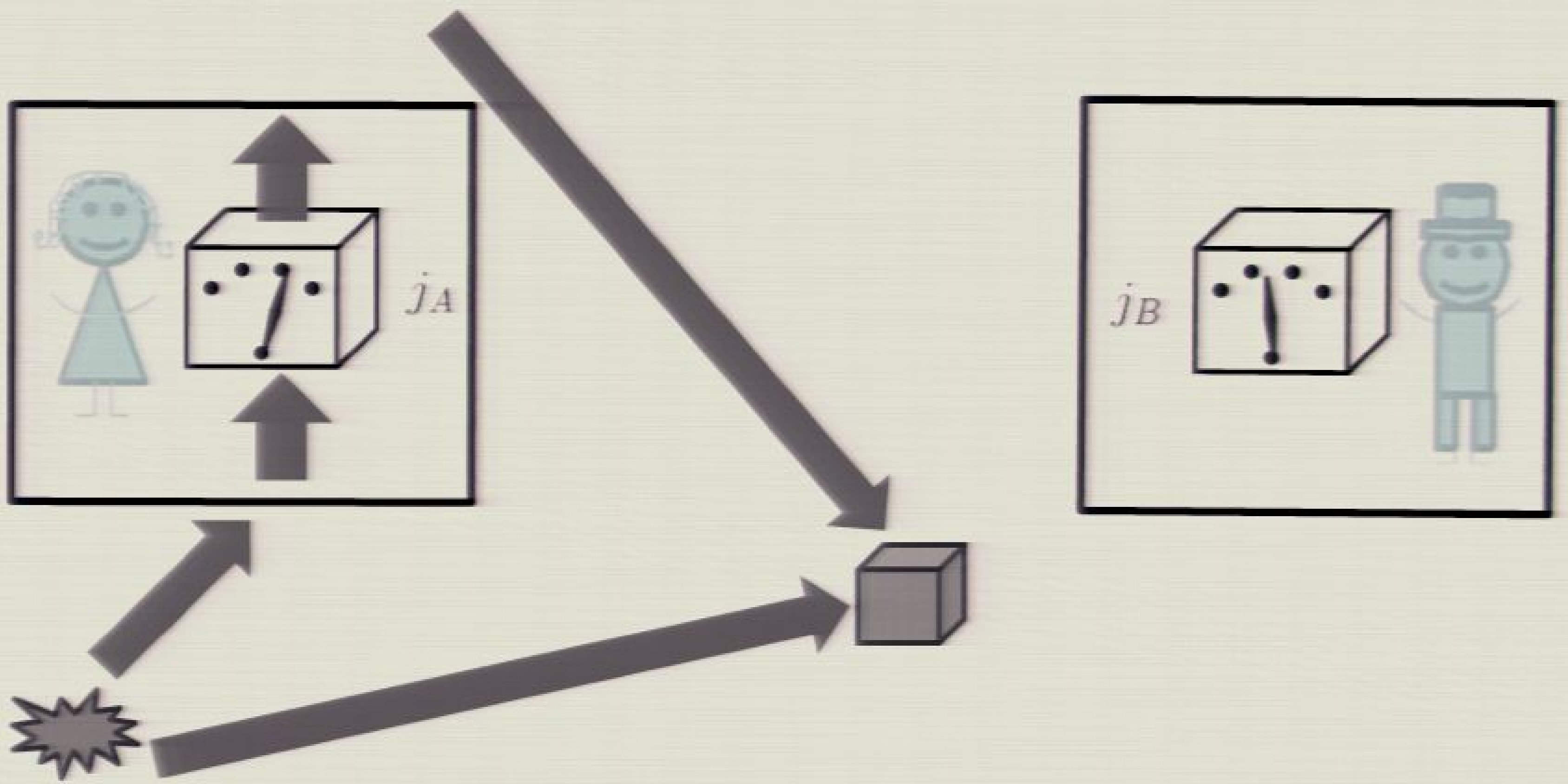

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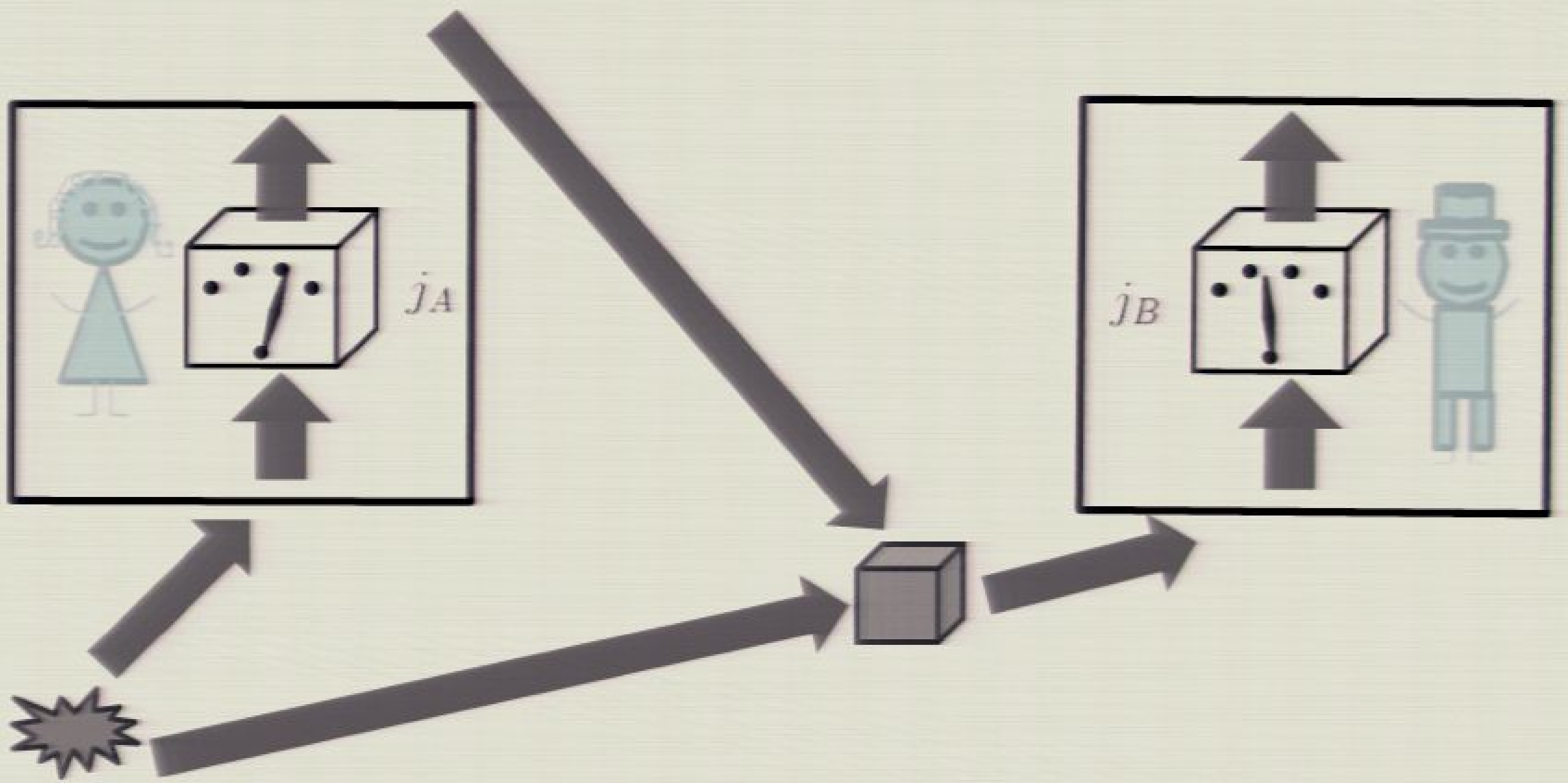

Possiblity of signaling $W^{A_1 A_2 B_1 B_2} = \mathbb{1}^{A_1} \otimes W^{A_2 B_1 B_2}$

Causal order - notation

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- $A \preccurlyeq B$: "A is in the causal past of B" *A* can signal to *B*

Causal order - notation

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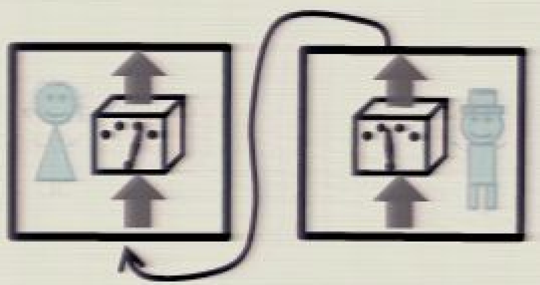

Causal order - notation

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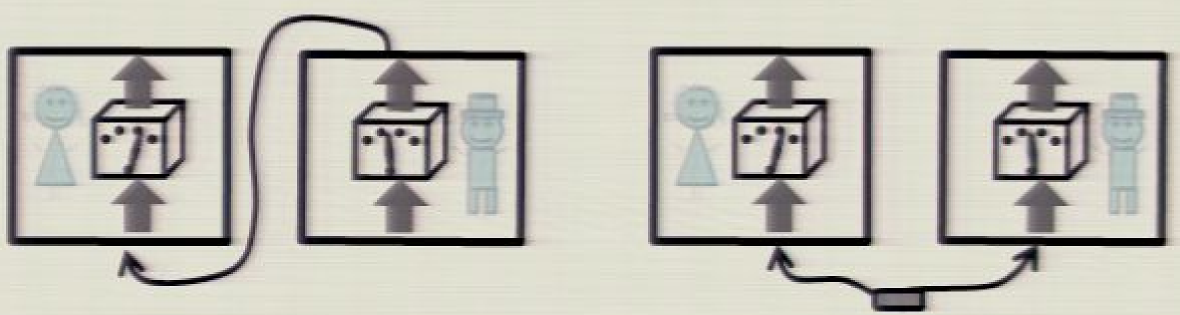

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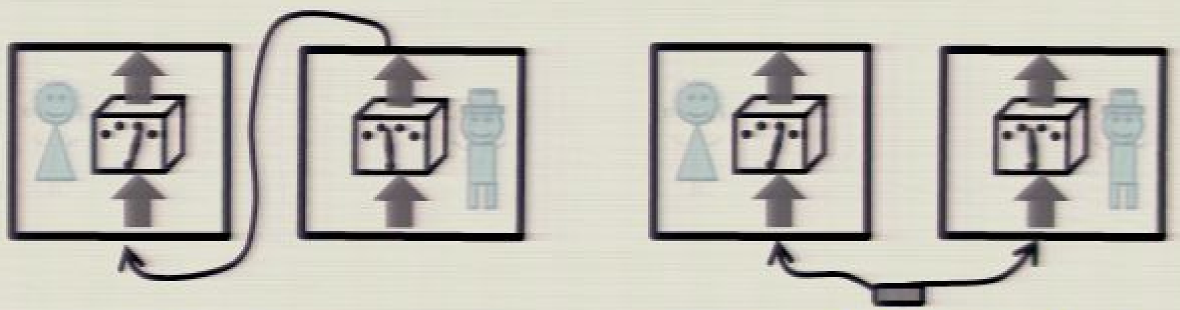

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Causal order - notation

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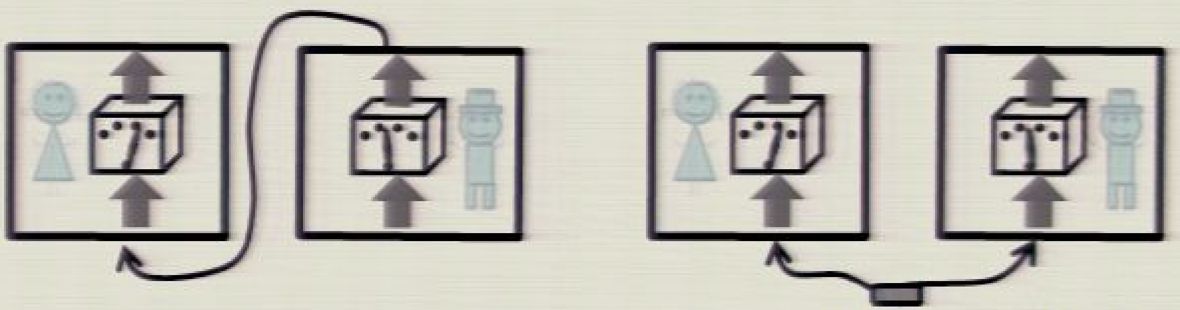

- $B \npreccurlyeq A$: "B is not in the causal past of A" — *B cannot signal to A*
- $A \npreccurlyeq B$ & $B \npreccurlyeq A$: "A and B causally independent"

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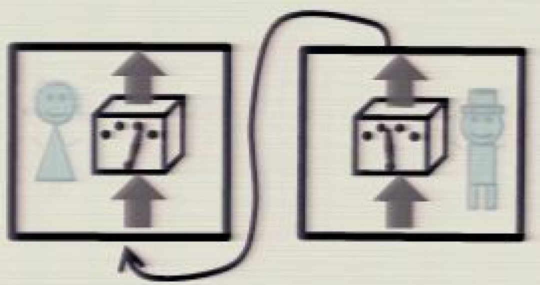

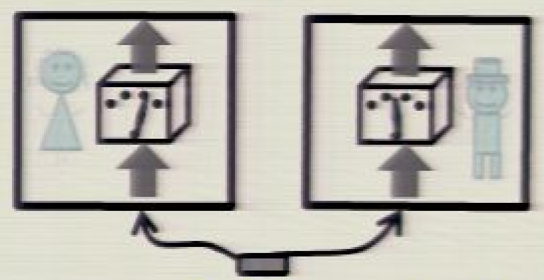

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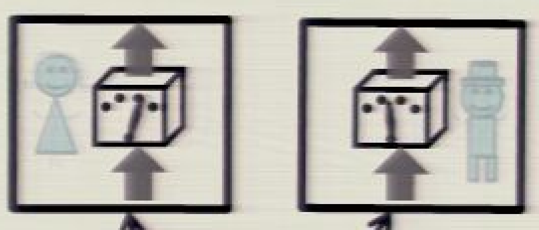

Terms appearing in process matrix

$$W^{A_1 A_2 B_1 B_2} = \sum_{\mu_1,\dots,\mu_4} a_{\mu_1\dots\mu_4} \sigma_{\mu_1}^{A_1} \otimes \dots \otimes \sigma_{\mu_4}^{B_2}$$

$\sigma_i^{A_1} \otimes \mathbb{1}^{rest}$ type A_1

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1. Probability positive & 2. Probability 1on all CPTP maps

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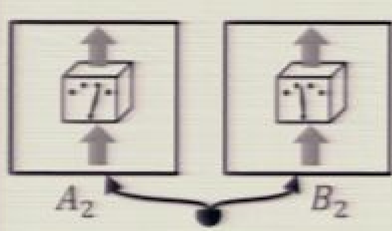
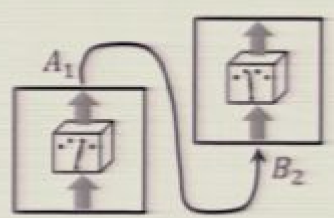
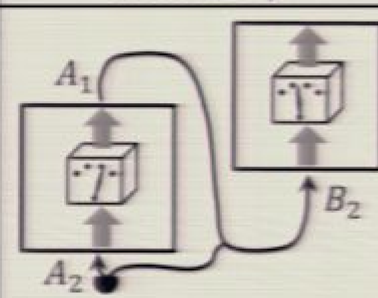
1. Probability positive & 2. Probability 1on all CPTP maps

$B \npreceq A$	$A_2,\ B_2,\ A_2B_2$	A_1B_2	$A_1A_2B_2$
$A \npreceq B$		A_2B_1	$A_2B_1B_2$
Causal order	States	Channels	Channels with memory
			

Most general causally separable situation: probabilistic mixture of ordered ones:

$$W^{A_1 A_2 B_1 B_2} = qW^{B \not\preceq A} + (1-q)W^{A \not\preceq B}$$

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Signalling only from
A to B or causally
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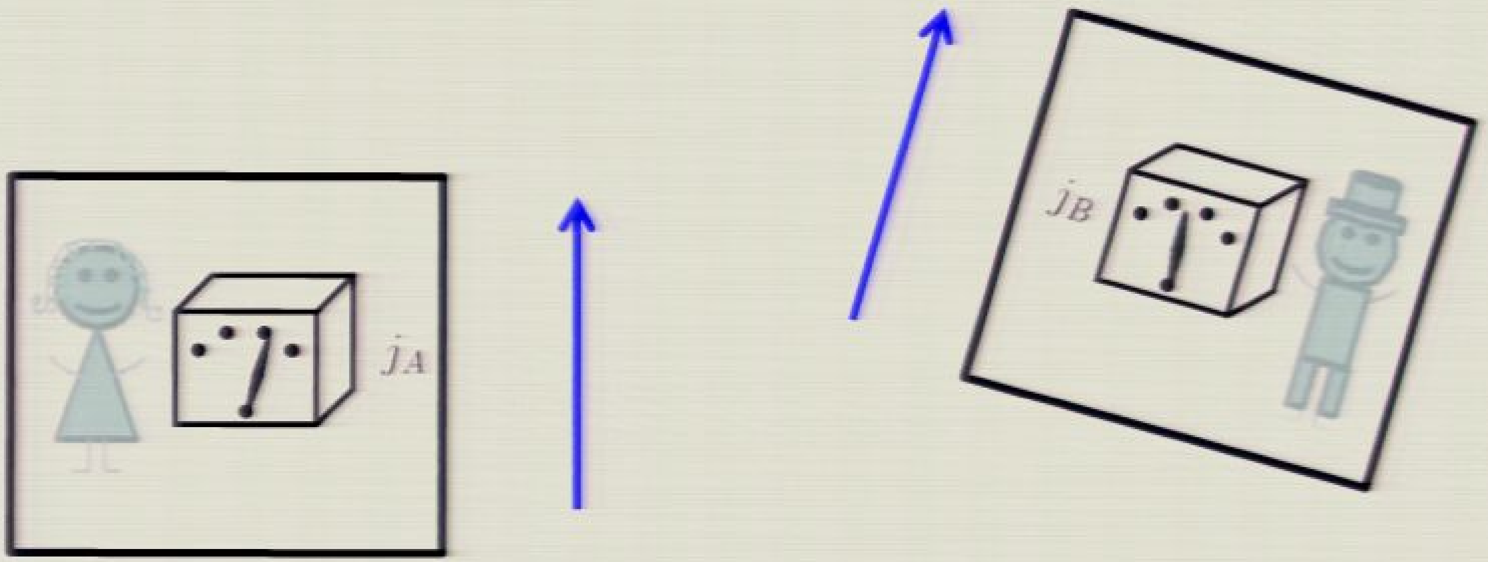
Signalling only from A to B or causally independent

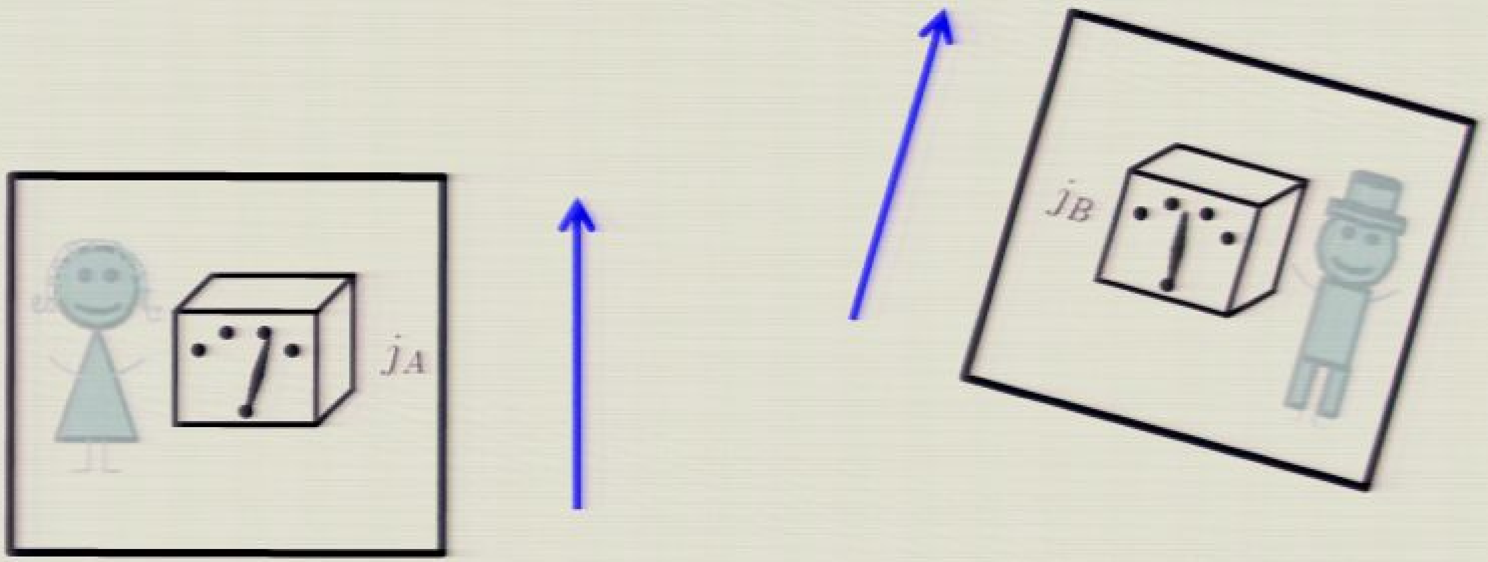
Signalling only from B to A or causally independent

Do all possible processes W respect definite causal order?

Causal Game

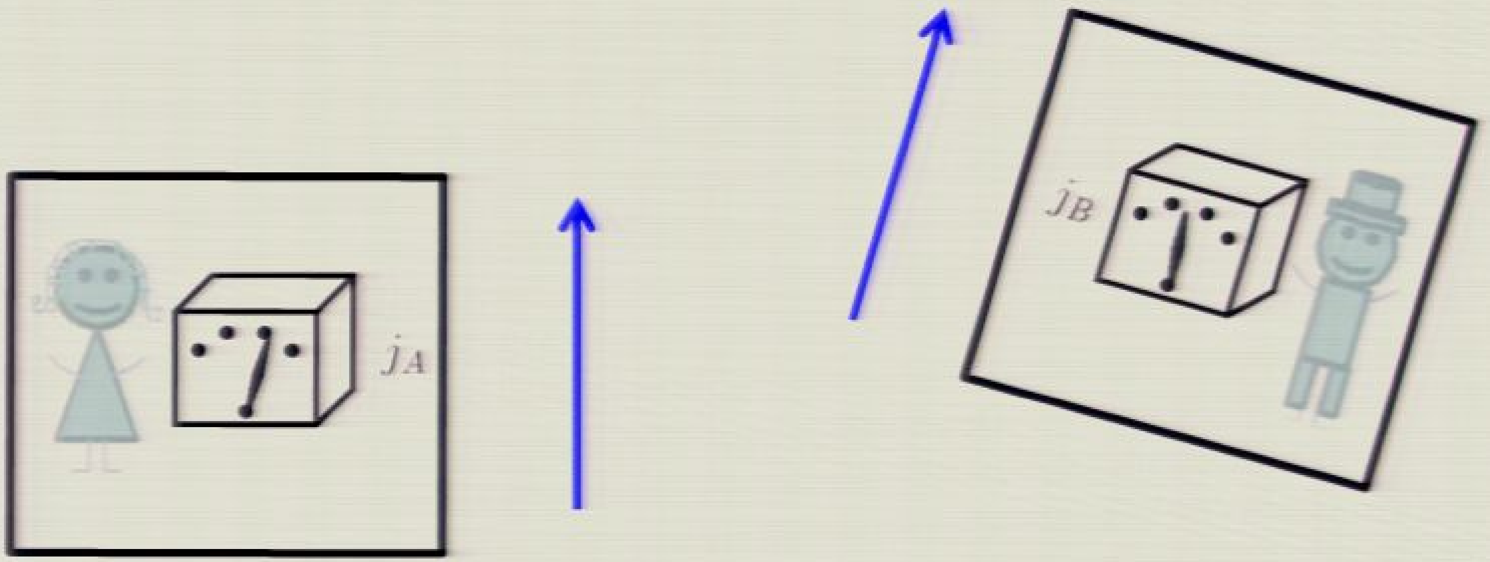


Causal Game



- Alice is given bit **a** and Bob bit **b**.

Causal Game



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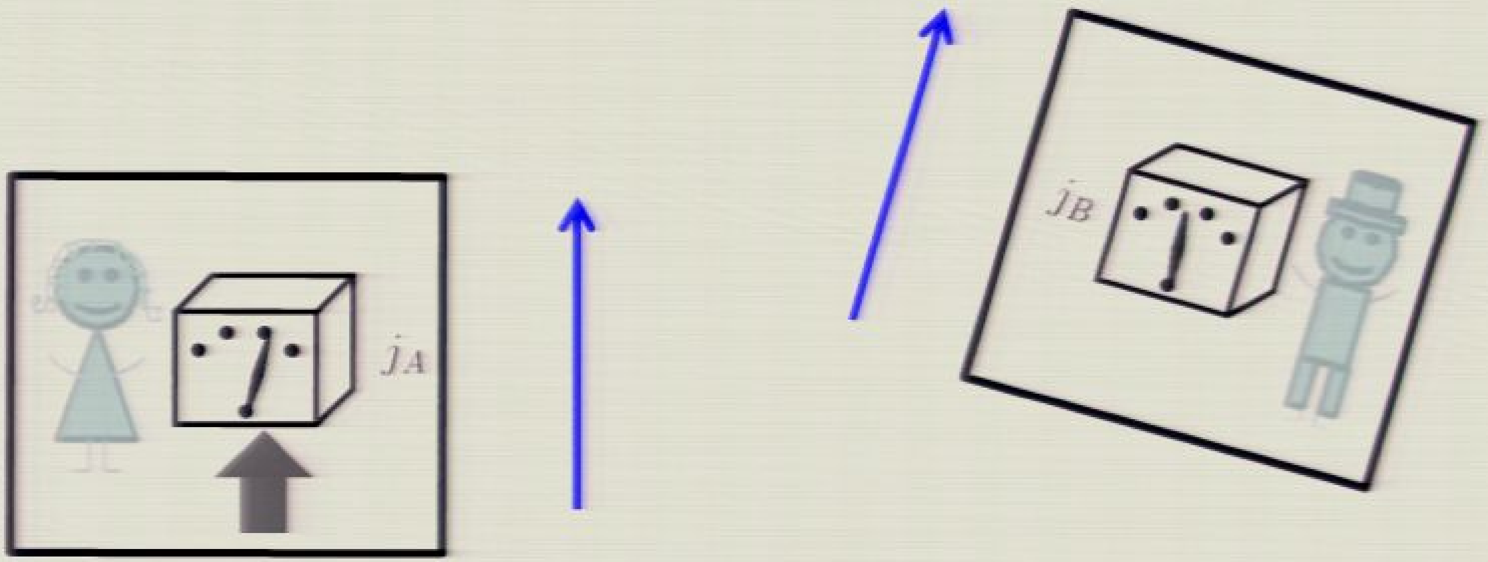

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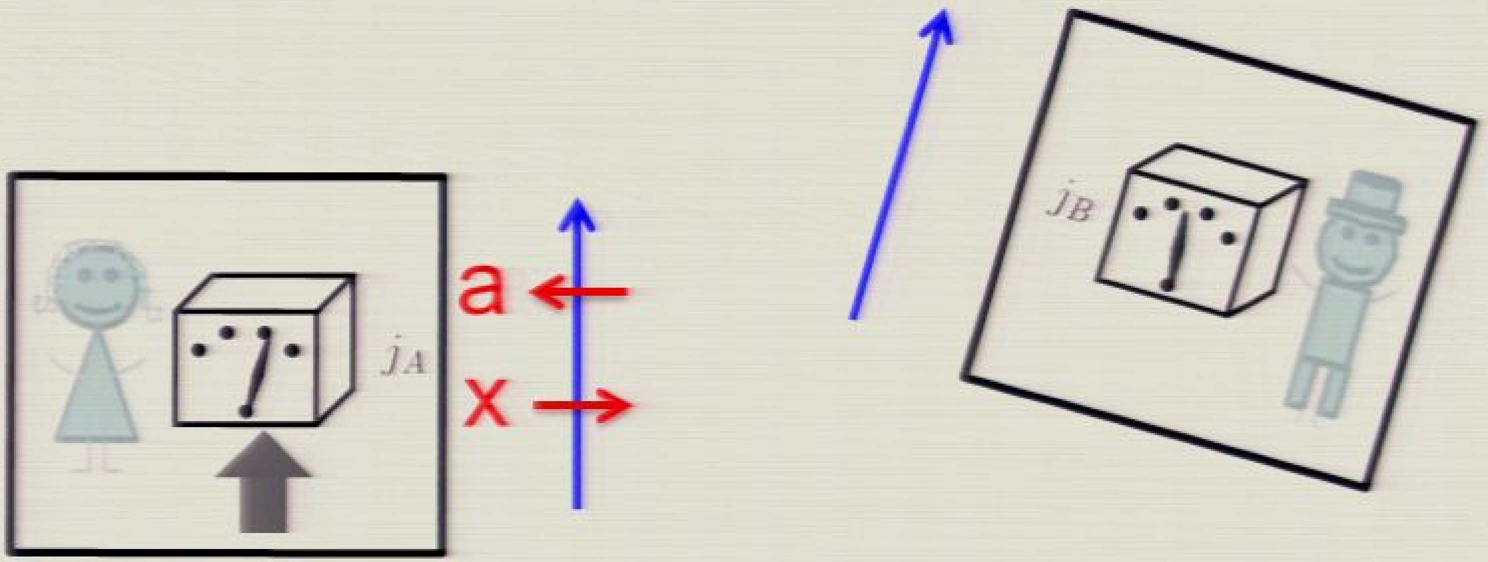

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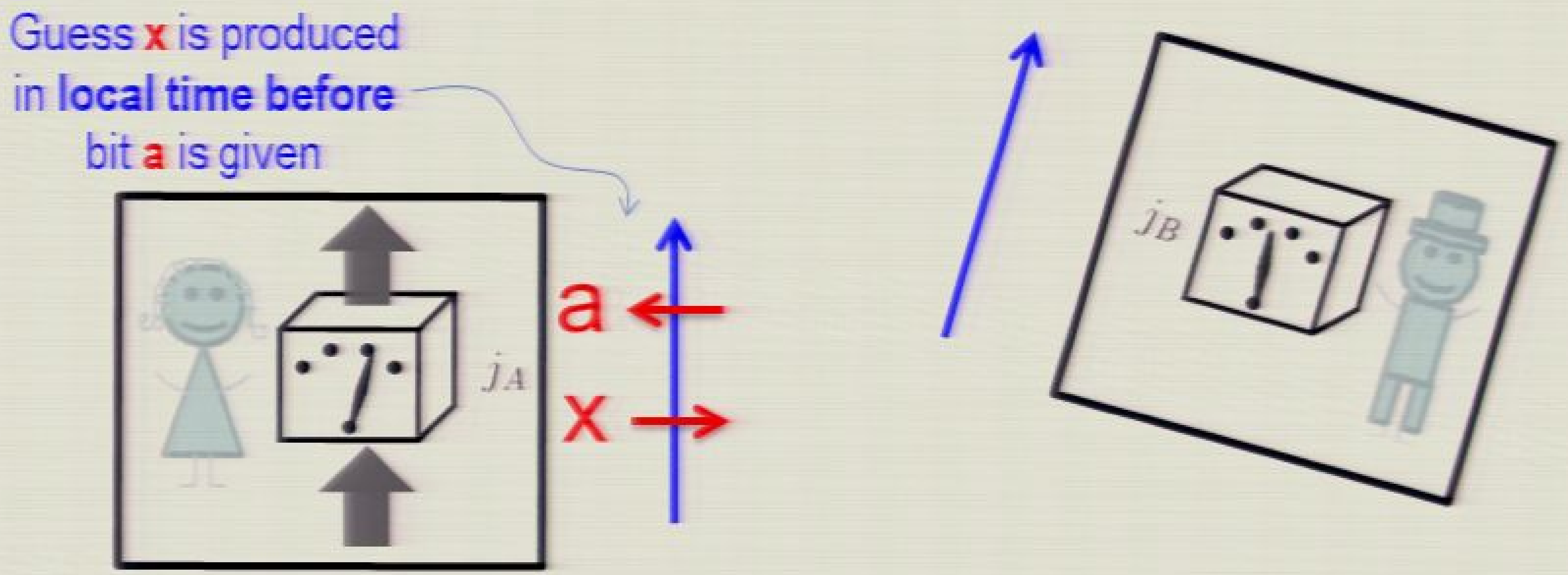

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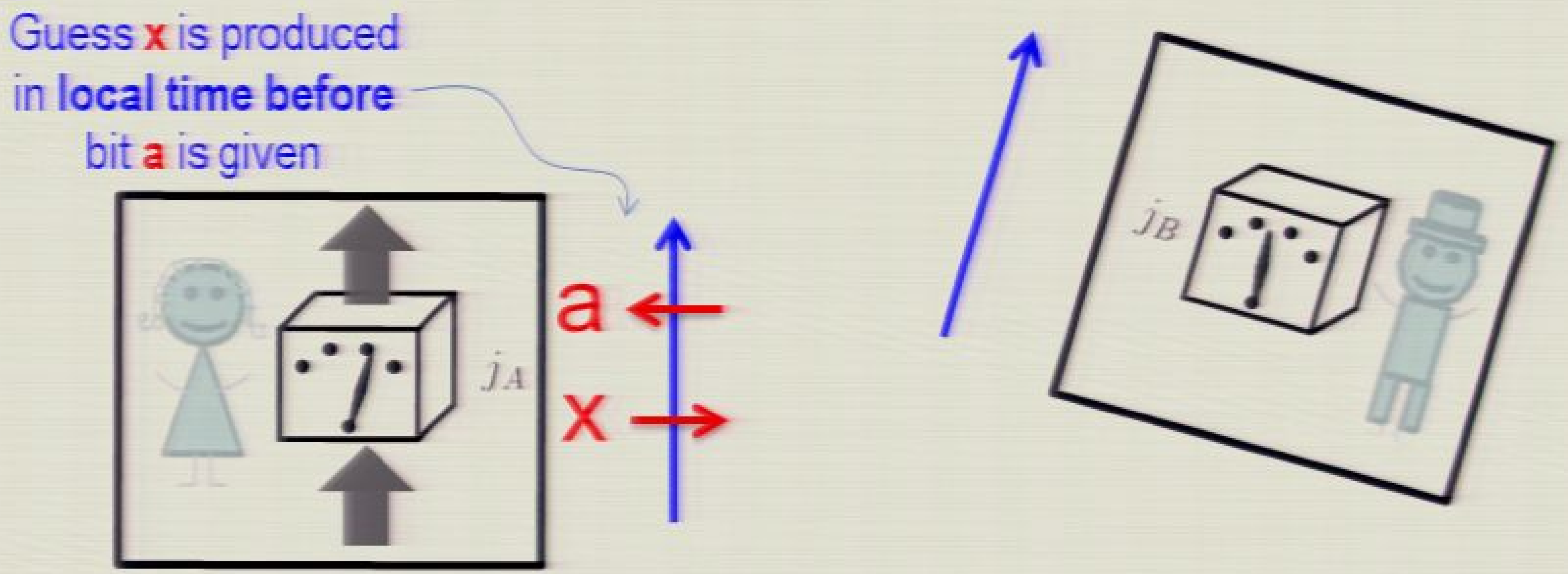

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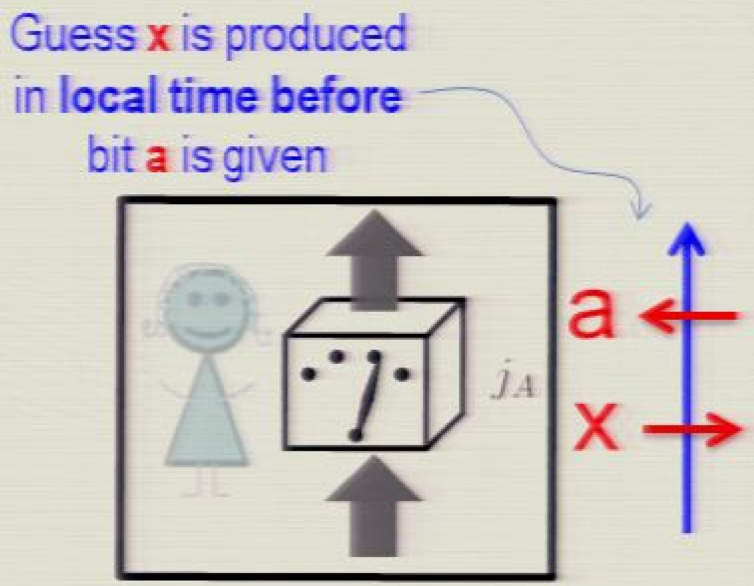
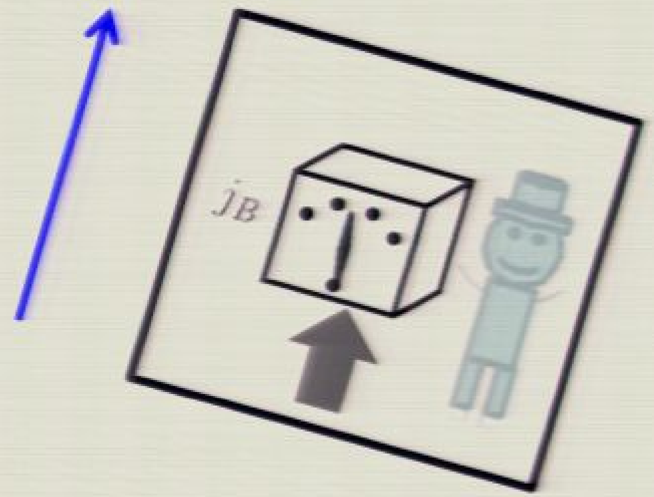
Causal Game





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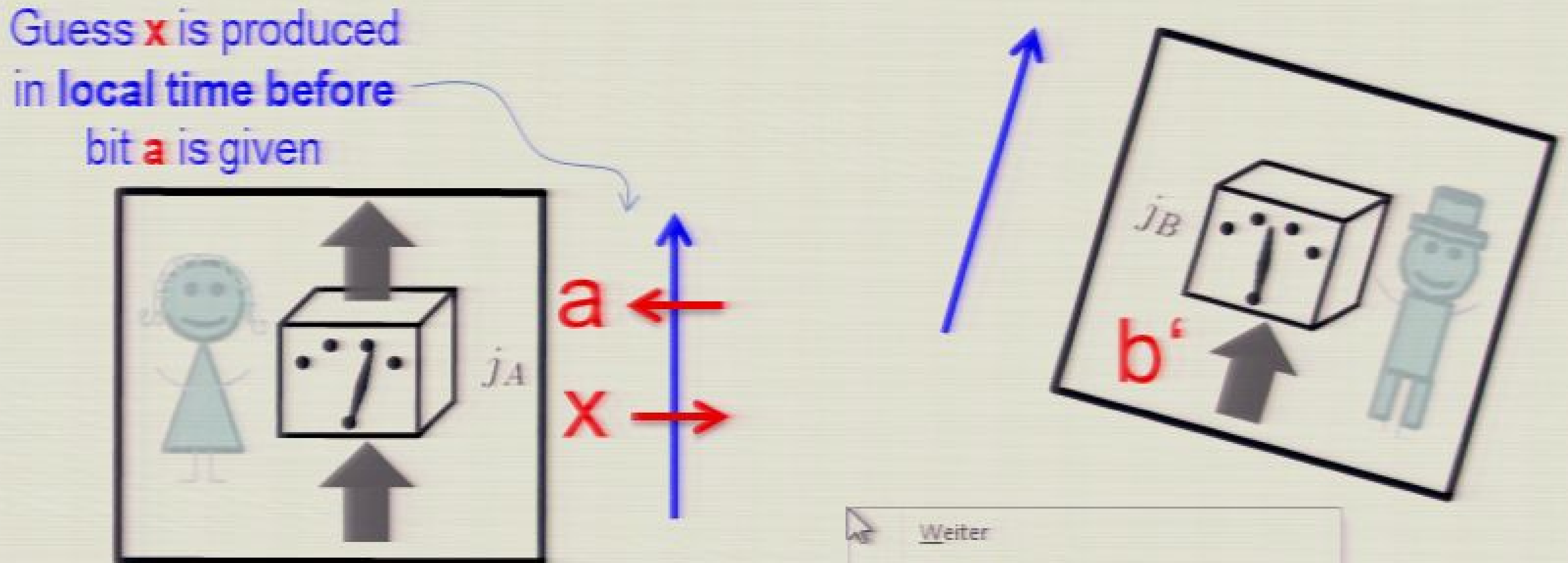

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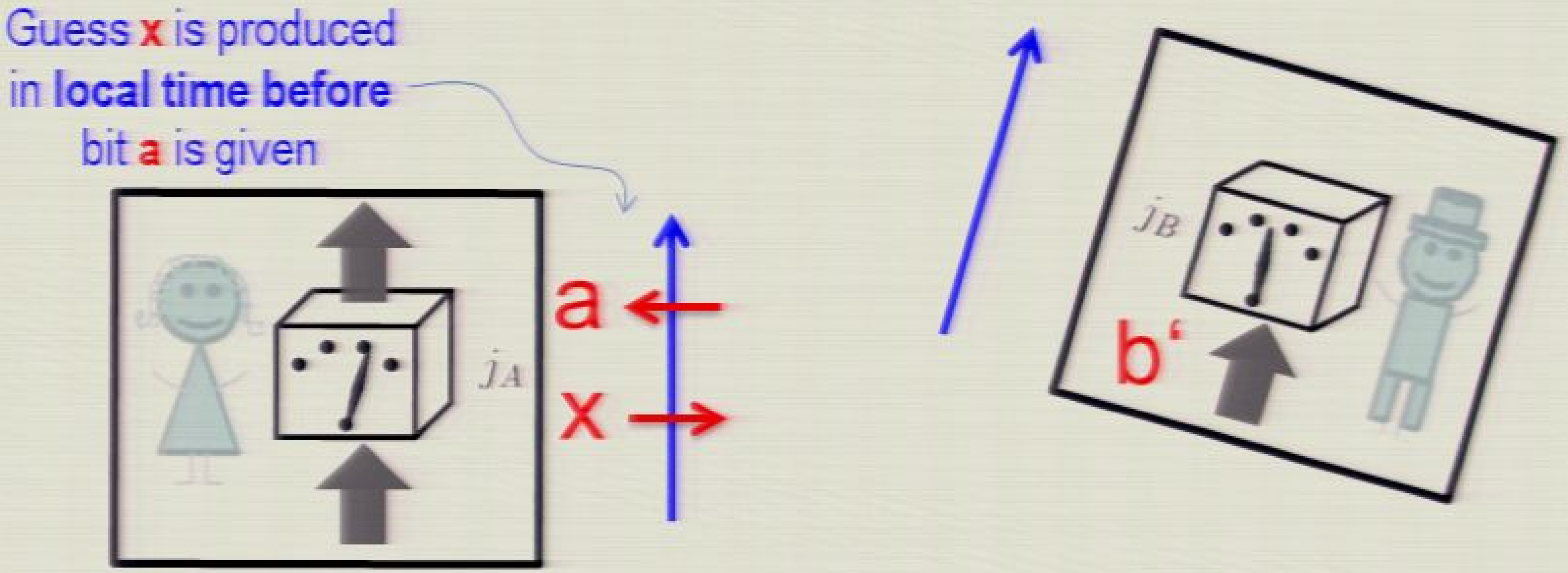
Weiter
Zurück
Zuletzt angesehen
Gehe zu Folie
Gehe zu Abschnitt
Zielgruppenorientierte Präsentation
Bildschirm
Zeigeroptionen
Hilfe
Anhalten
Präsentation beenden

Causal Game



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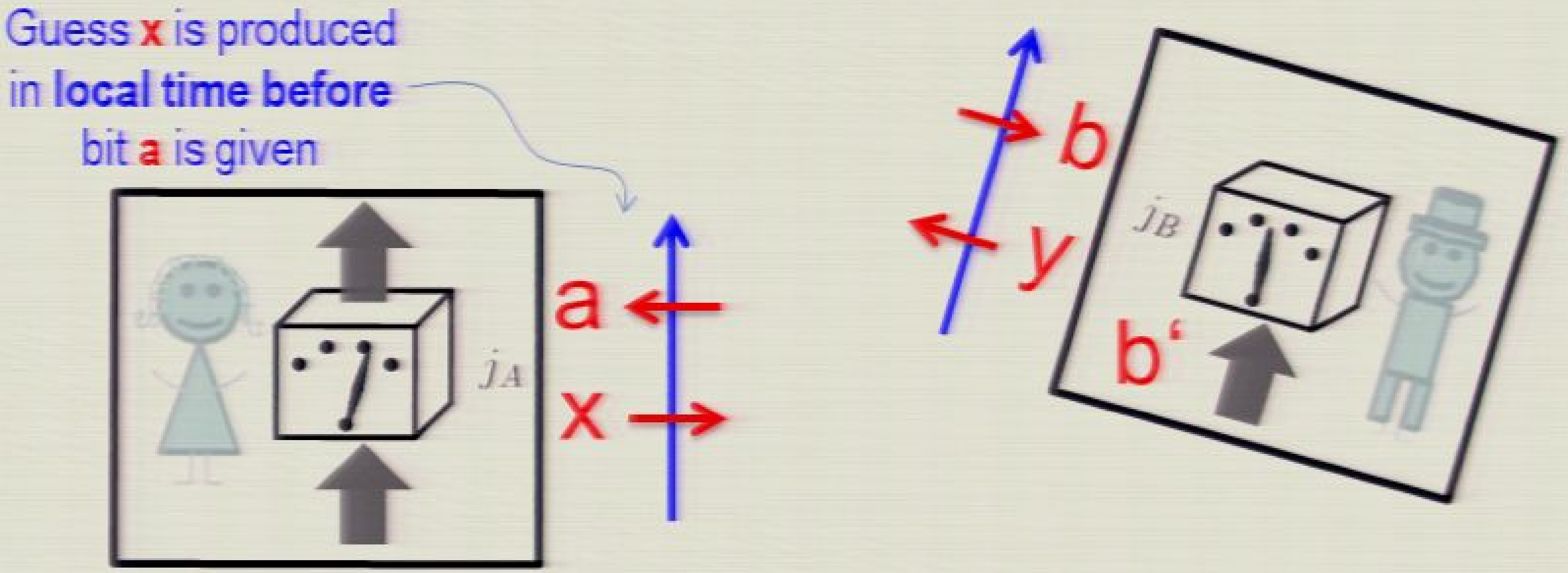

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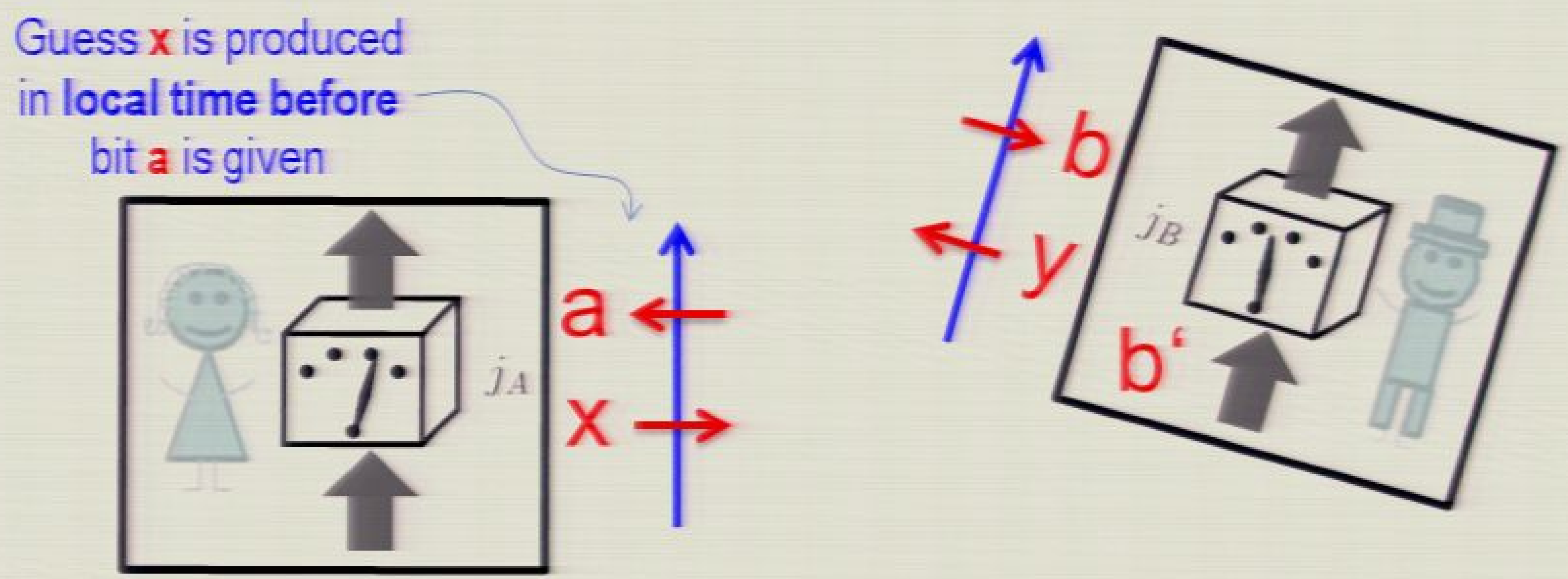

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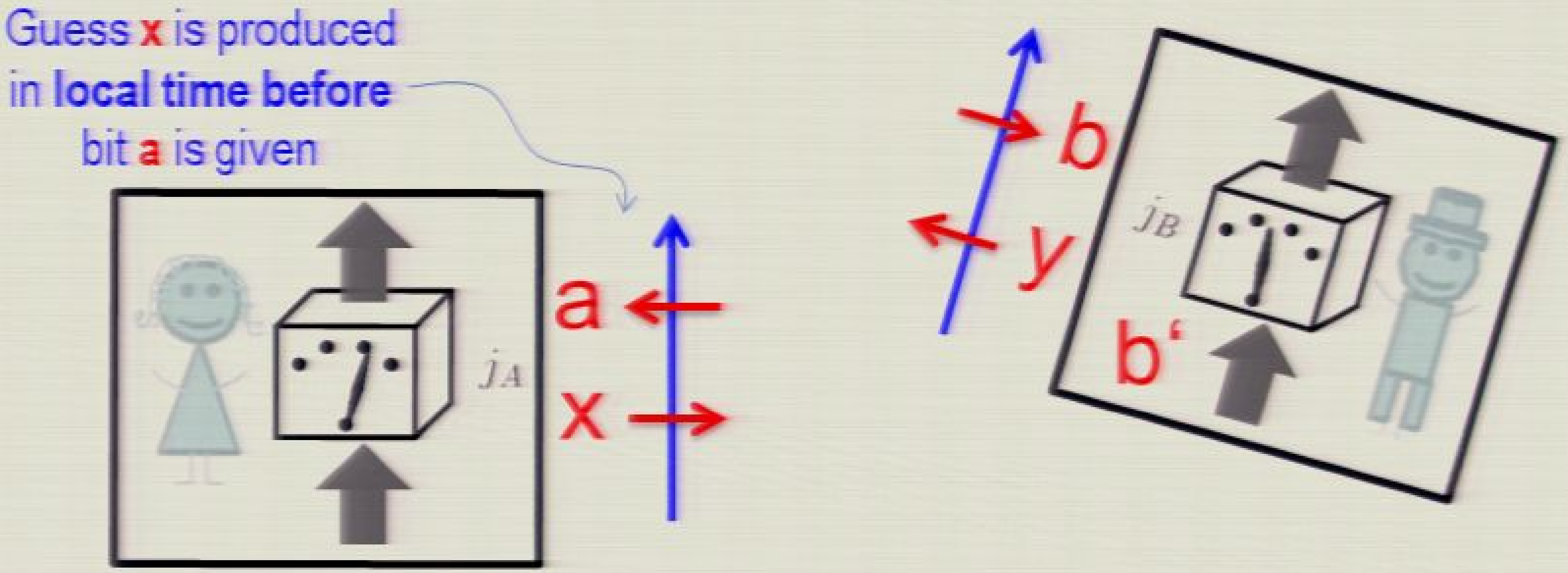

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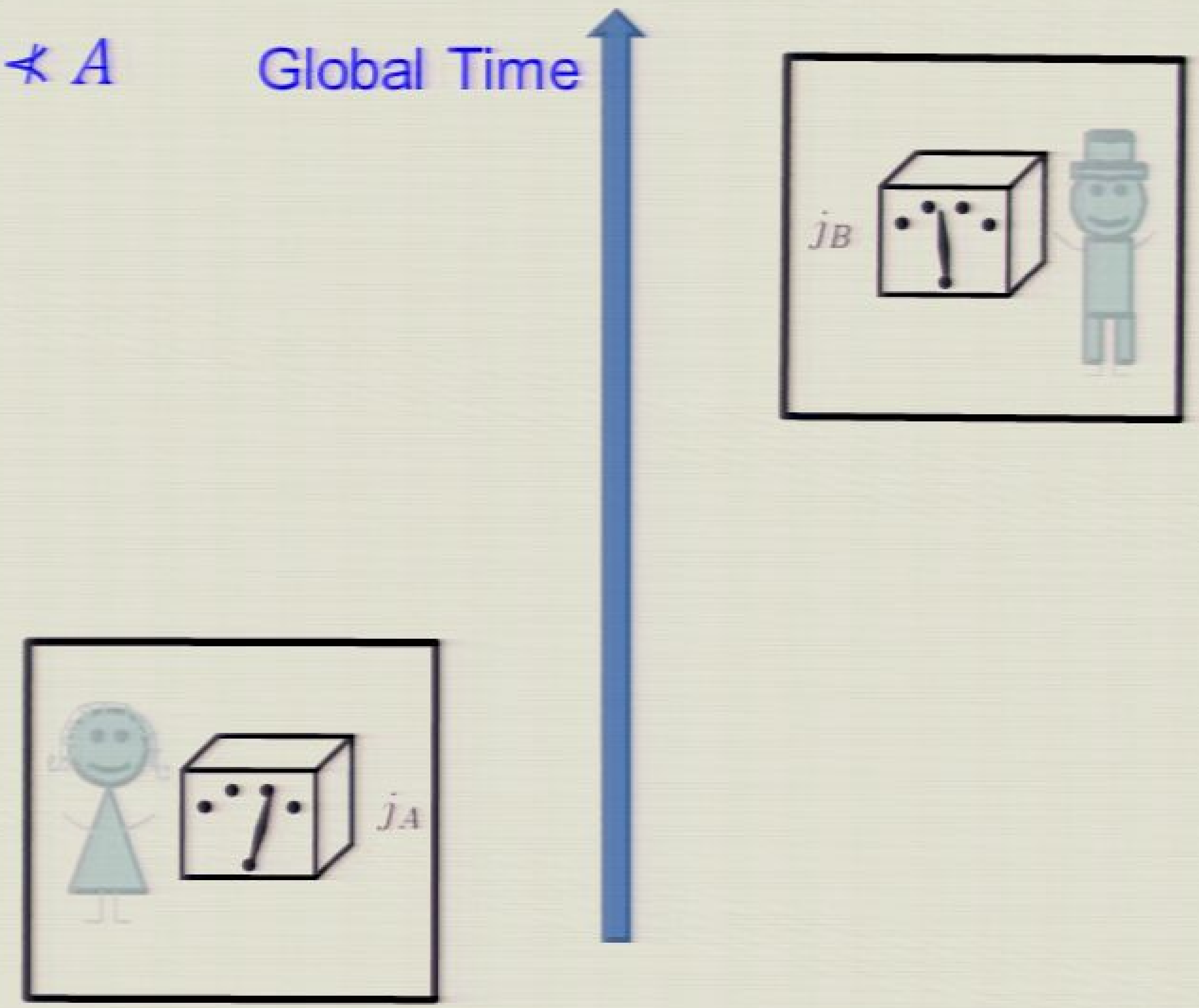
- Bob is given an additional bit **b'** that tells him whether he should guess her bit (**b'=1**) or she should guess his bit (**b'=0**).
- The goal is to maximize the probability for correct guess:

$$p_{succ} := \frac{1}{2}\left[P(x = b|b' = 0) + P(y = a|b' = 1)\right]$$

Causally ordered situation

Case: $B \nprec A$



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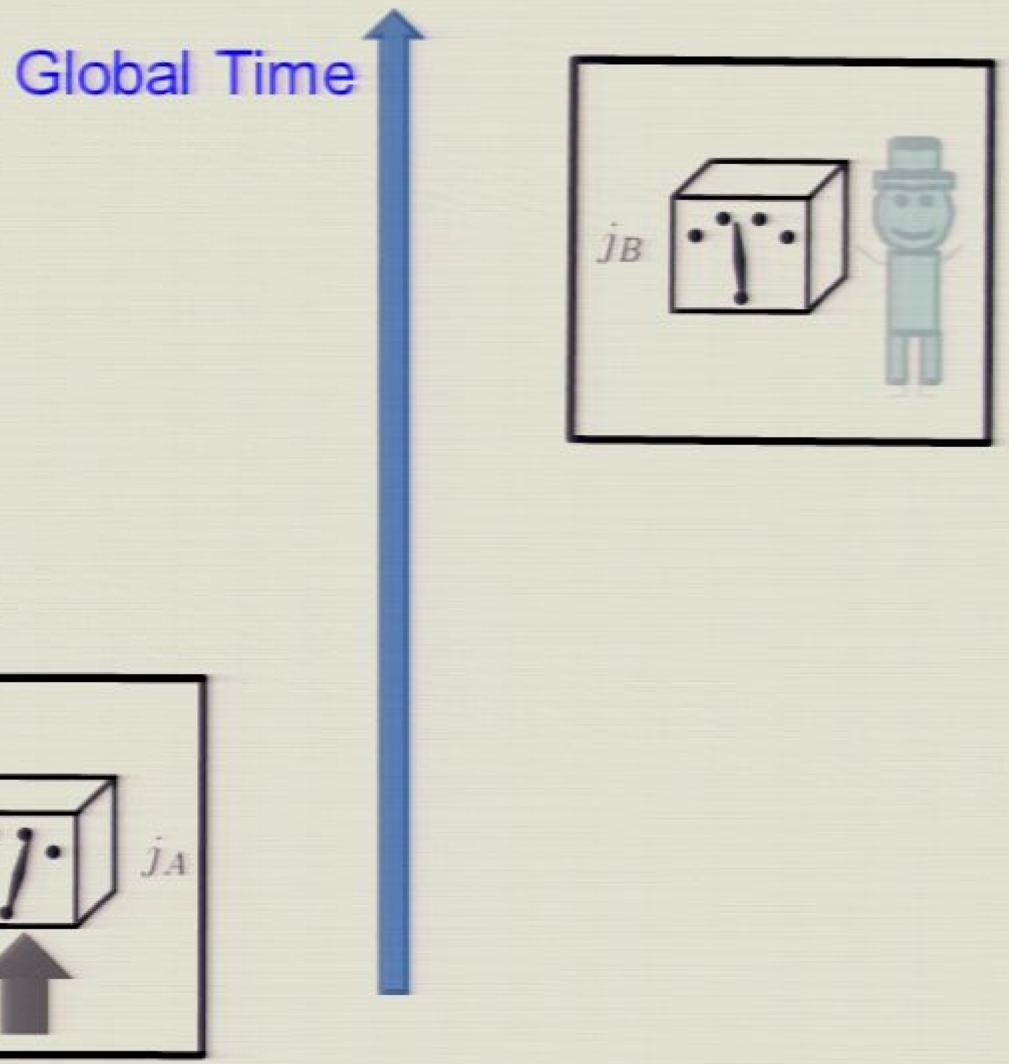

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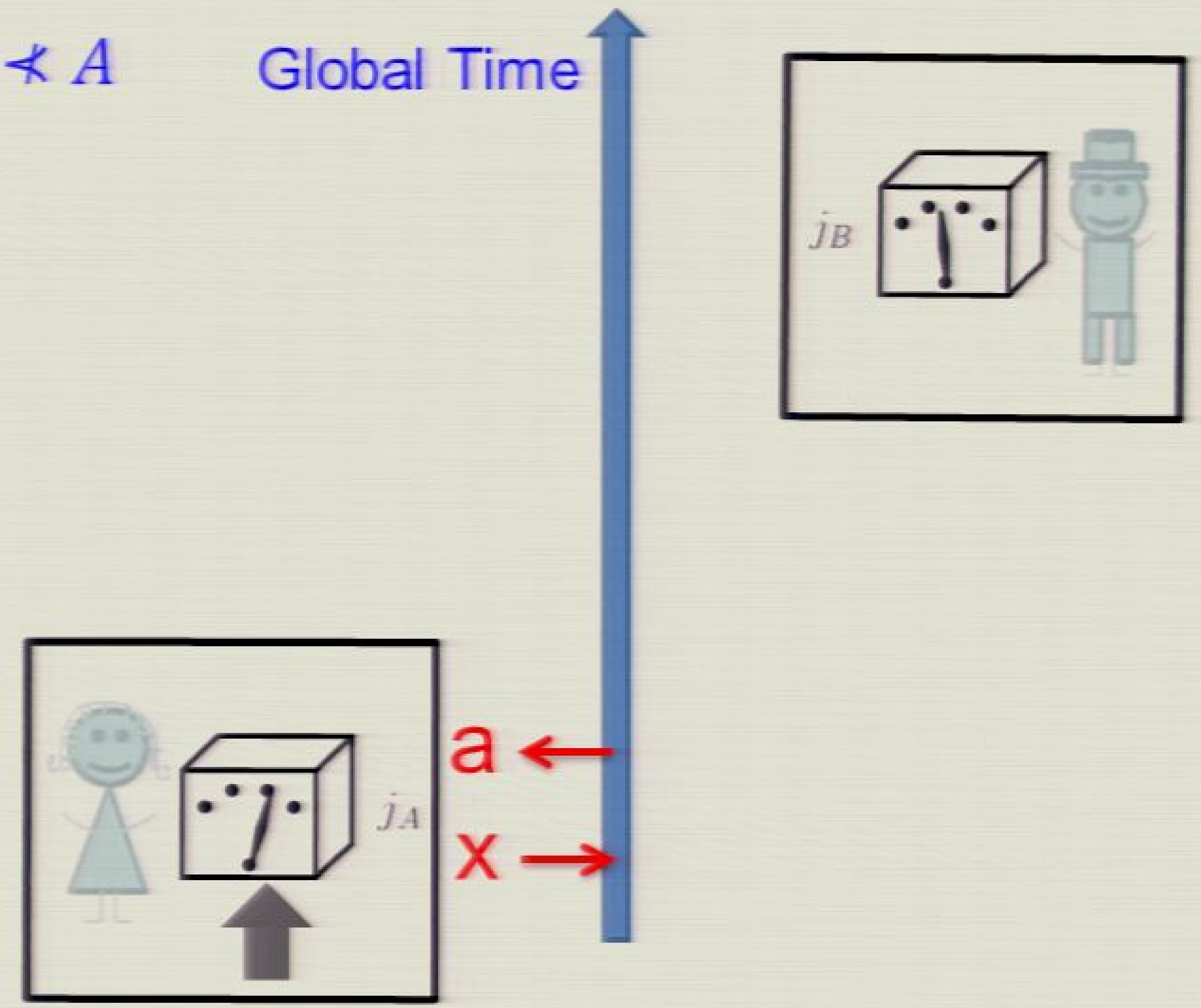

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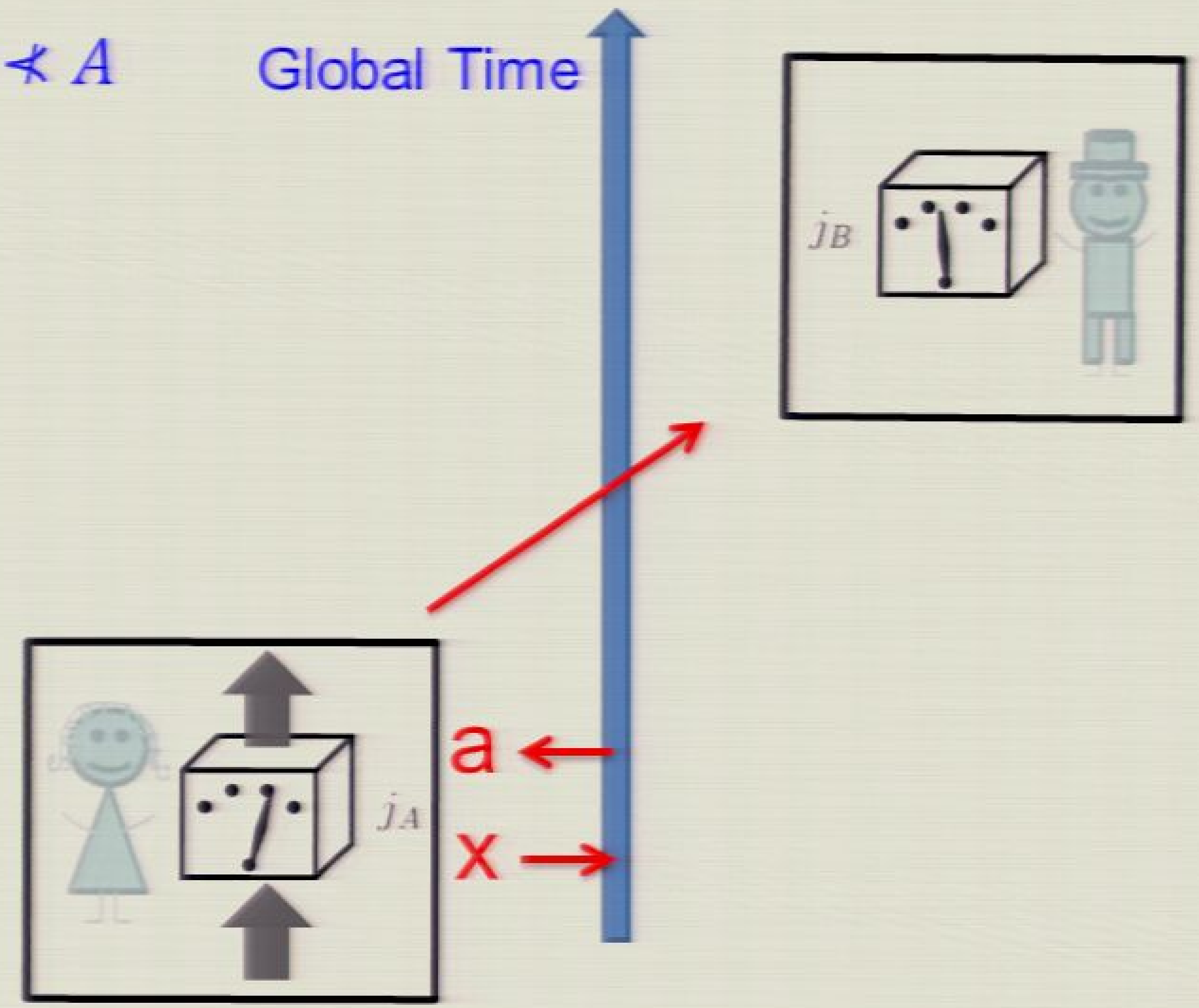

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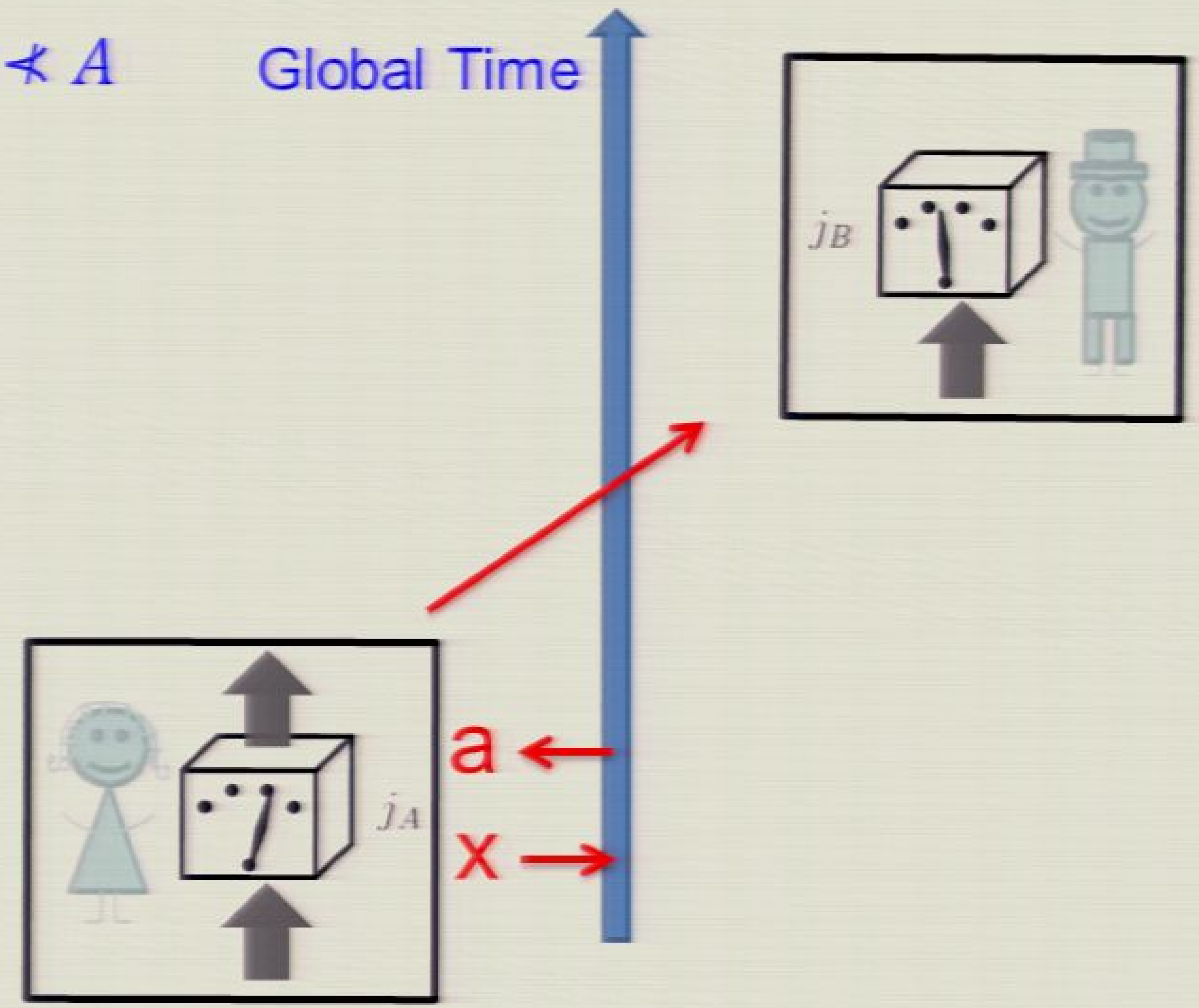

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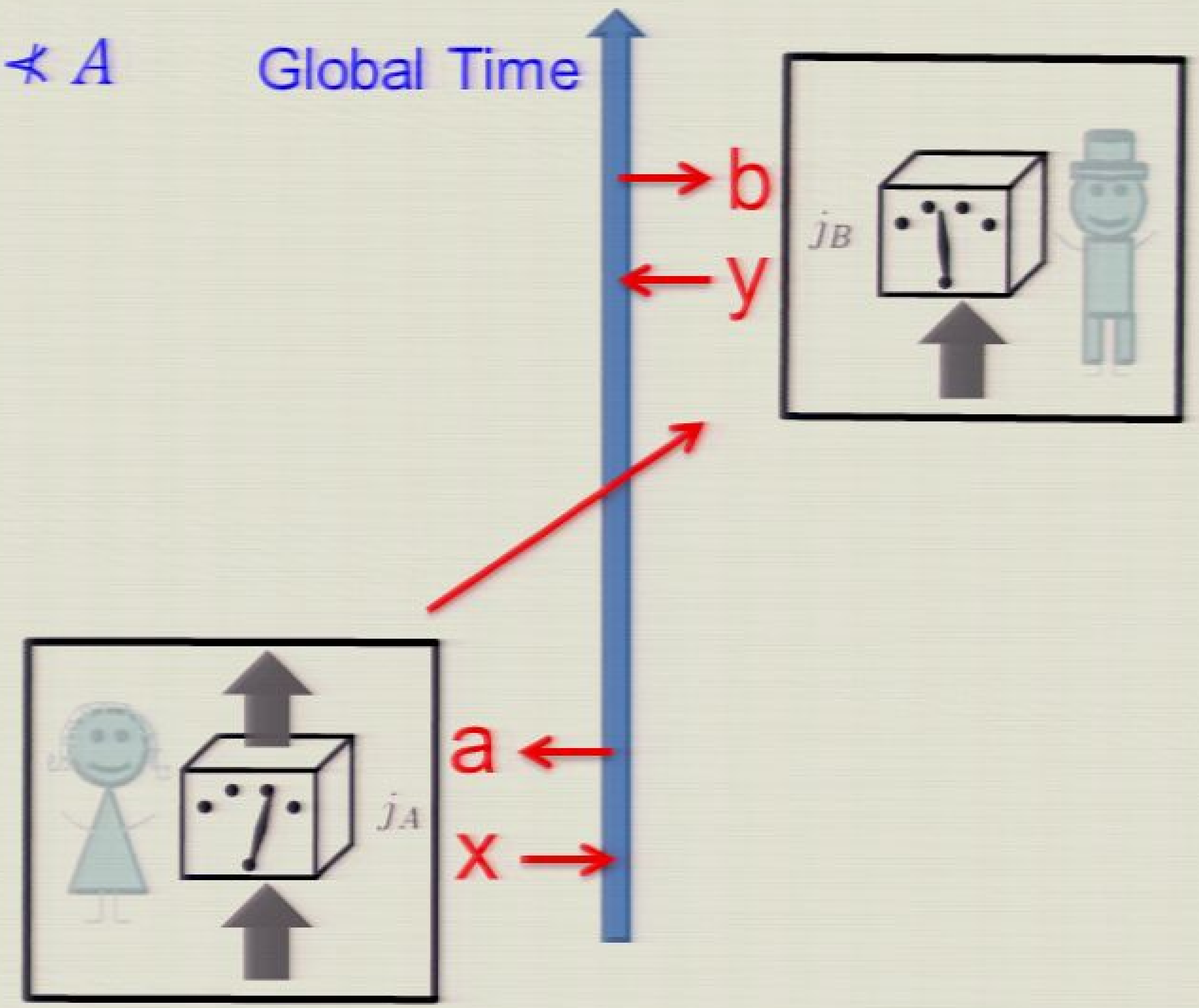

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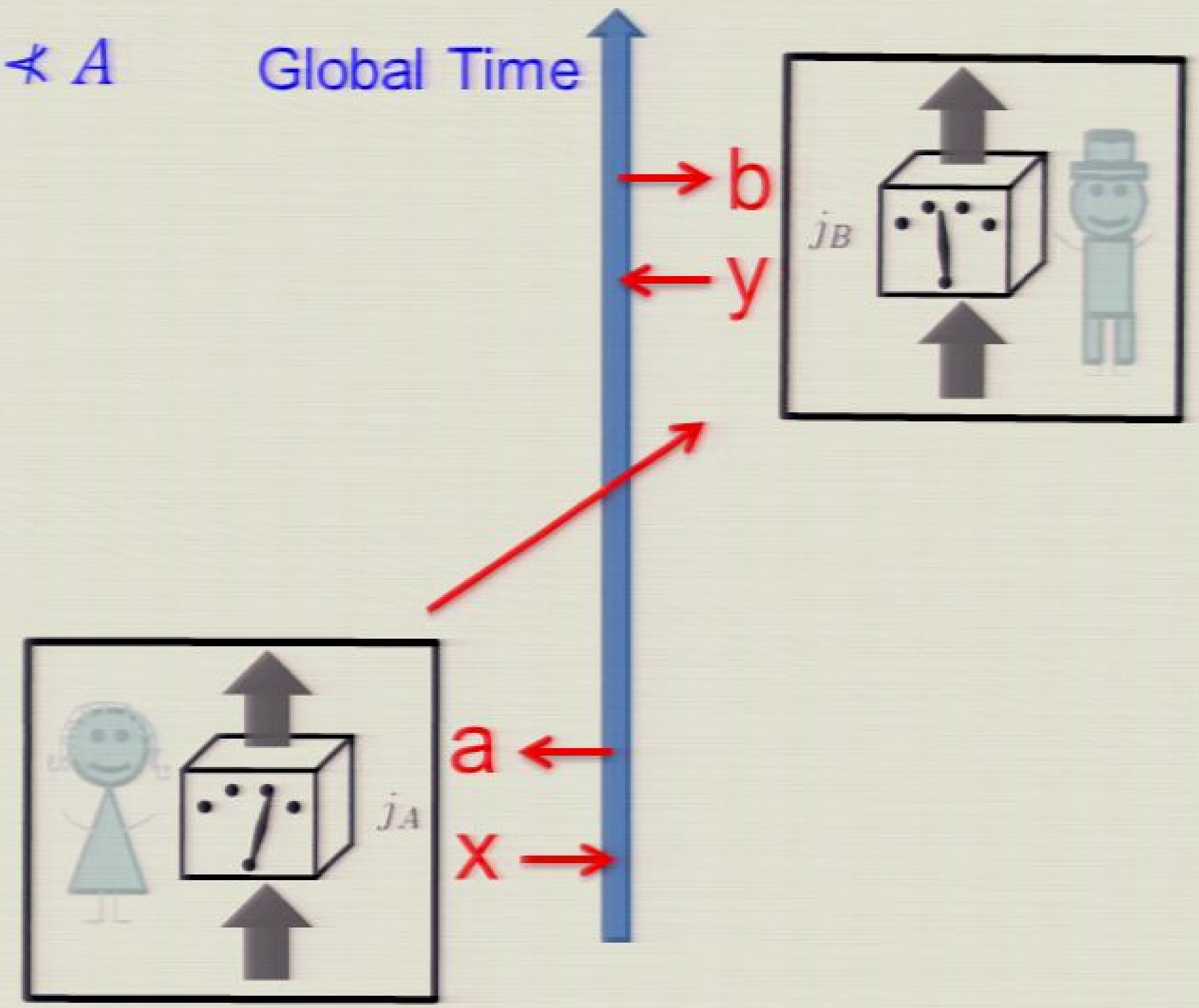
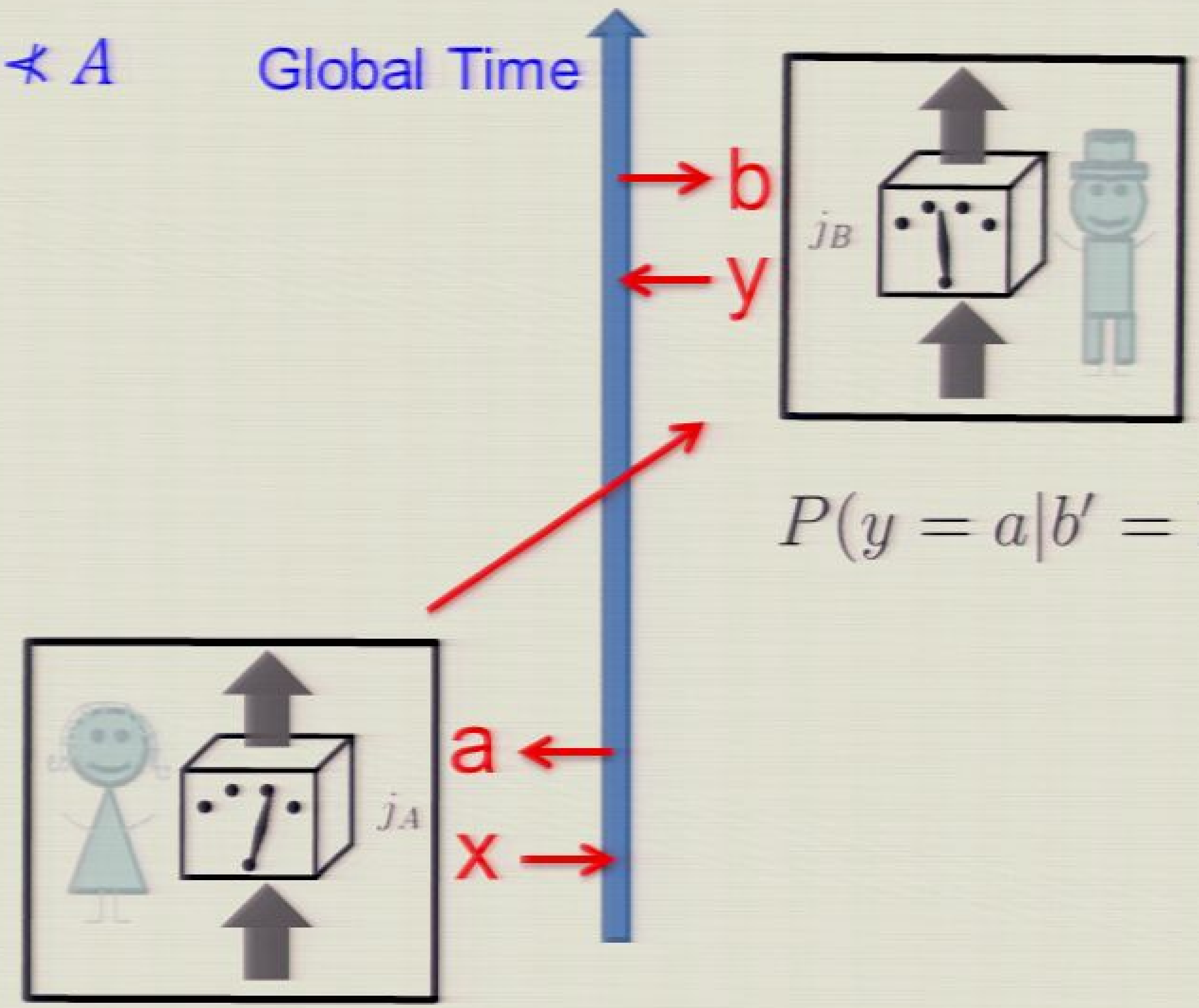


Causally ordered situation

Case: $B \nprec A$



$$P(y = a | b' = 1) = 1$$

Causally ordered situation

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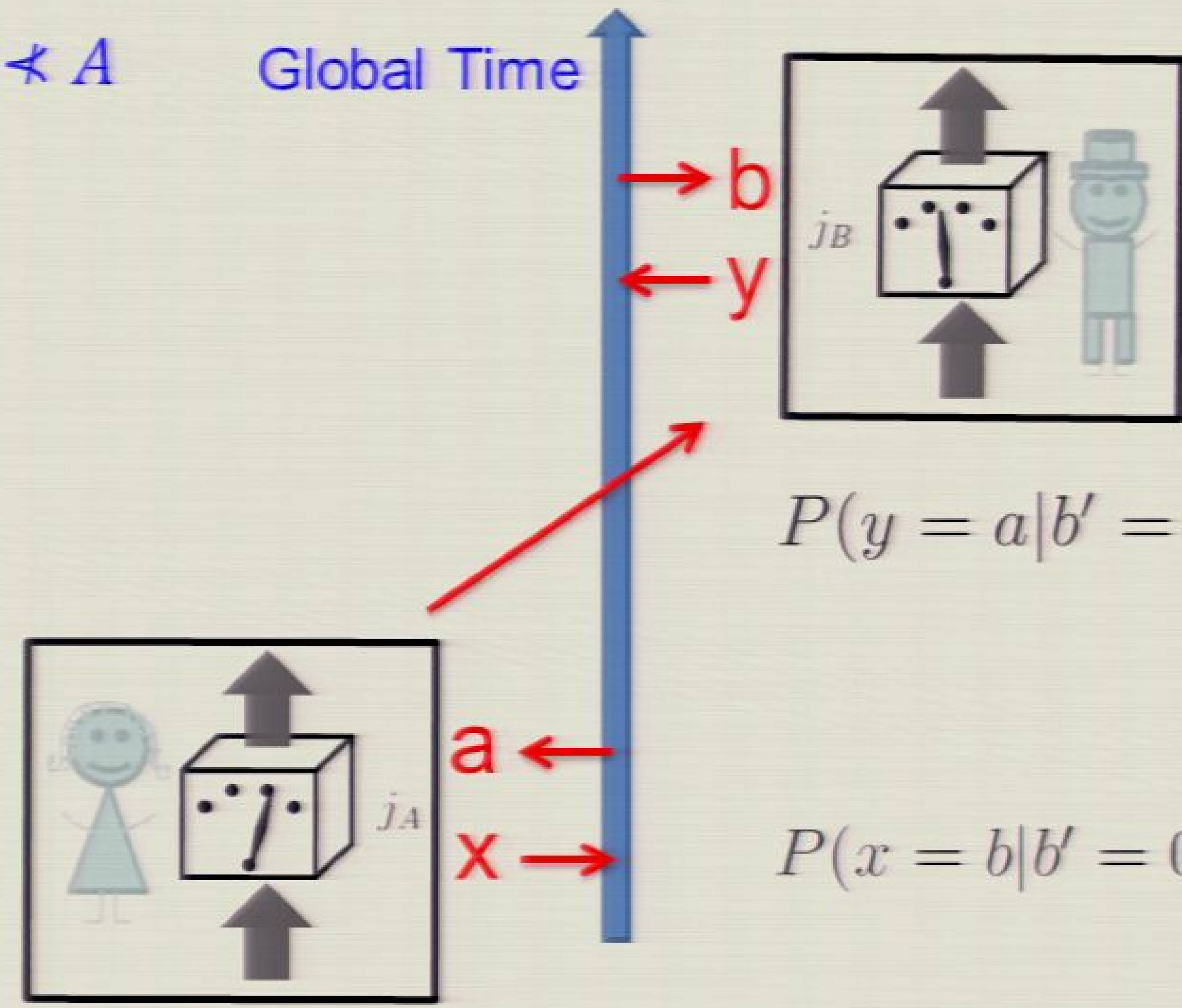

$$P(y = a|b' = 1) = 1$$

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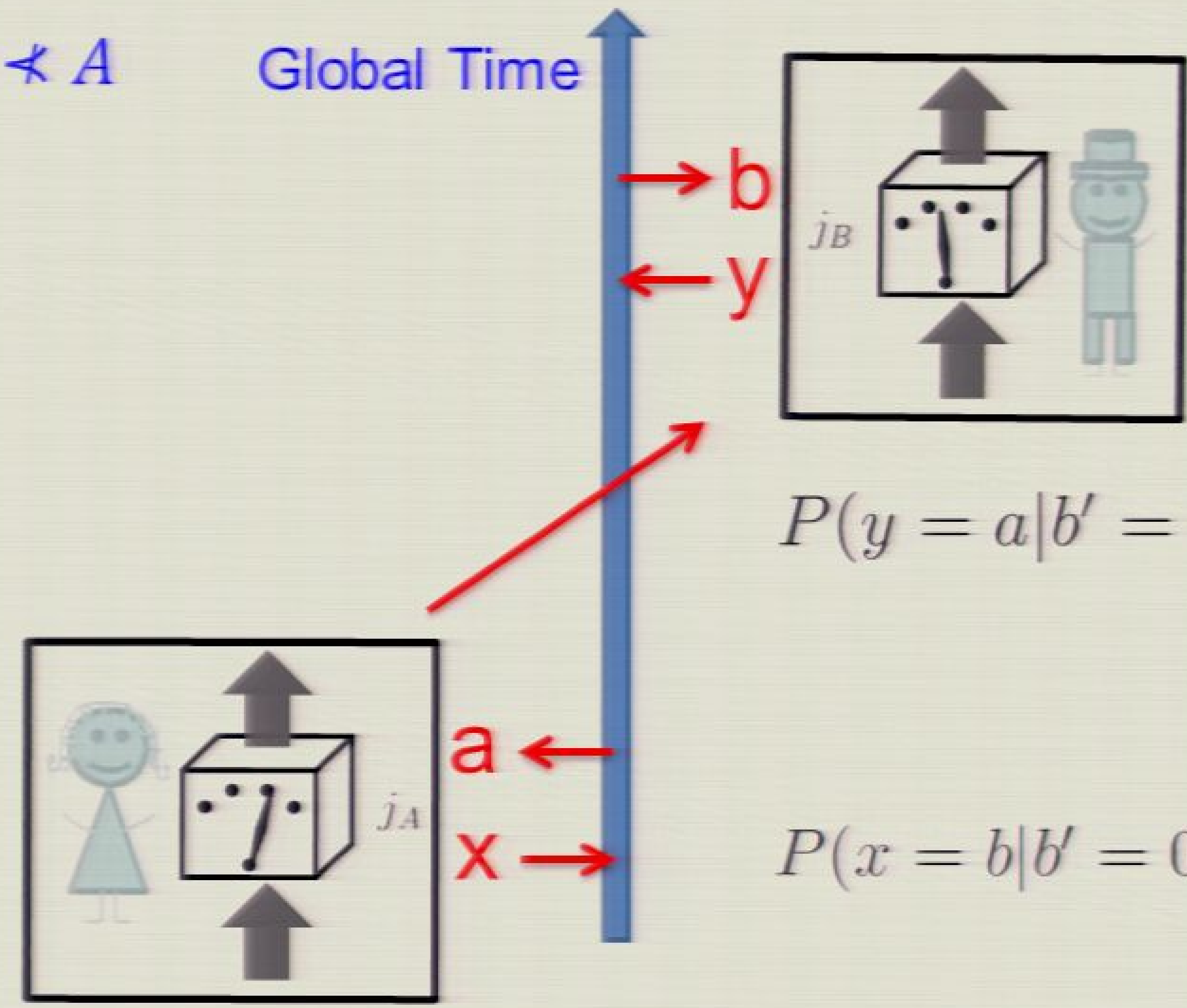

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$$p_{succ} = P(x = b|b' = 0) + P(y = a|b' = 1) \leq \frac{3}{4}$$

Causally non-separable situation

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$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

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The probability of success is

$$p_{succ} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$$

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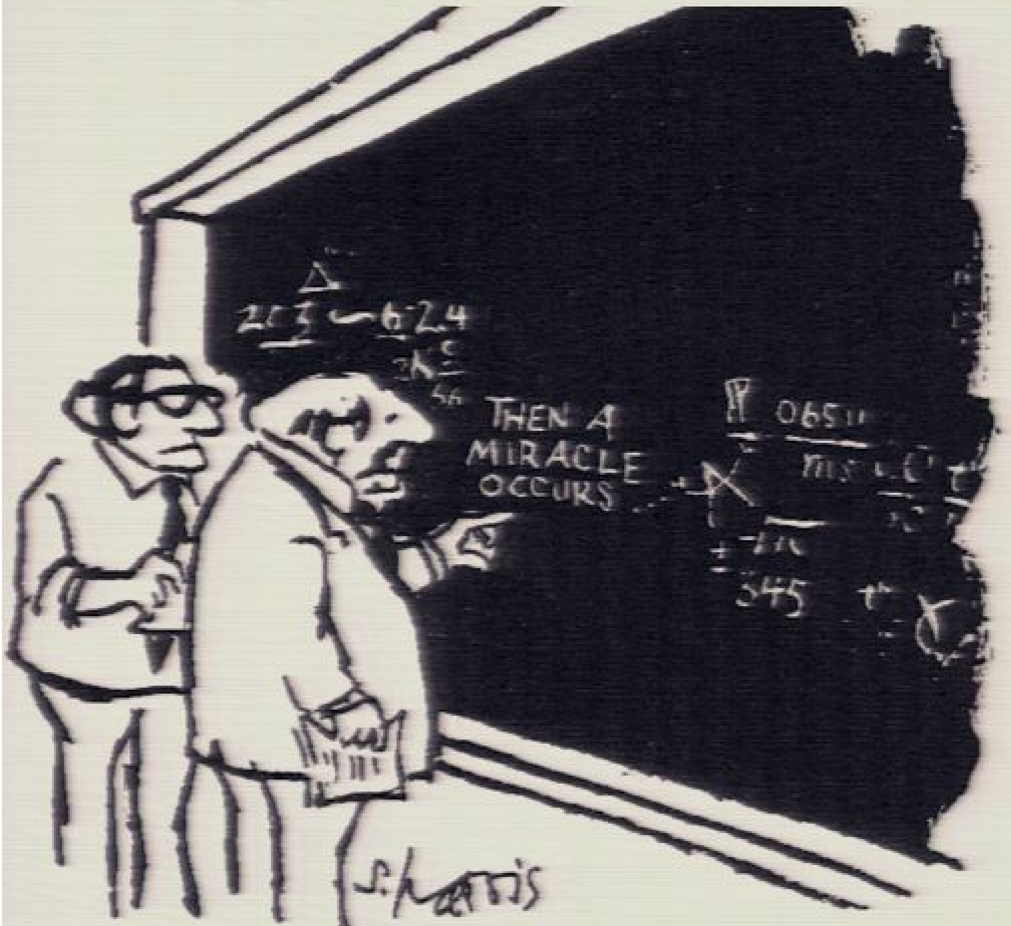

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A causally non-separable example

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A causally non-separable example

Repreparation ↙ Measurement ↙

Alice's CP map: $|z_a\rangle\langle z_a|^{A_1} \otimes |z_x\rangle\langle z_x|^{A_2} \quad x, a = \pm 1$

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Not seen by Bob

A causally non-separable example

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Choosen by Alice   Not seen by Bob

A causally non-separable example

Repreparation — Measurement

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A causally non-separable example

Repreparation Measurement

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Bob effectively „sees":

$$\widetilde{W}^{B_1B_2} = \sum_x \mathrm{Tr}_{A_1A_2}\left[W^{A_1A_2B_1B_2}\left(|z_a\rangle\langle z_a|^{A_1} \otimes |z_x\rangle\langle z_x|^{A_2}\right)\right]$$

$$W^{A_1A_2B_1B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_x^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

$\langle z_a|\sigma_z|z_a\rangle = a$ $\langle z_\pm|\sigma_z|z_\pm\rangle = \pm 1$

Choosen by Alice Not seen by Bob

Bob receives the state: $\widetilde{W}^{B_1B_2} = \frac{1}{2}(\mathbb{1} + a\frac{1}{\sqrt{2}}\sigma_z^{B_2})$

A causally non-separable example

Repreparation   Measurement

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$\langle z_a|\sigma_z|z_a\rangle = a$ — Choosen by Alice

$\langle z_\pm|\sigma_z|z_\pm\rangle = \pm 1$ — Not seen by Bob

⇒ Bob receives the state: $\widetilde{W}^{B_1 B_2} = \frac{1}{2}(\mathbb{1} + a\frac{1}{\sqrt{2}}\sigma_z^{B_2})$

If Bob wants to read ($b' = 1$), he measures in the z basis and achieves

$$P(y = a | b' = 1) = \frac{2+\sqrt{2}}{4}$$

A causally non-separable example

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

A causally non-separable example

Repreparation Measurement

If Bob wants to send ($b' = 0$)

Bob's CP map: $|z_{by}\rangle\langle z_{by}|^{B_1} \otimes |x_y\rangle\langle x_y|^{B_2} \quad y, b = \pm 1$

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

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$$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$$

Not seen by Alice

A causally non-separable example

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Bob's CP map: $|z_{by}\rangle\langle z_{by}|^{B_1} \otimes |x_y\rangle\langle x_y|^{B_2} \quad y, b = \pm 1$

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$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$

Not seen by Alice

The encoding correlated with the detection result

⟹ Alice receives the state $\widetilde{W}^{A_1A_2} = \frac{1}{2}(\mathbb{1} + b\frac{1}{\sqrt{2}}\sigma_z^{A_2})$

A causally non-separable example

If Bob wants to send ($b' = 0$)

Repreparation Measurement

Bob's CP map: $|z_{by}\rangle\langle z_{by}|^{B_1} \otimes |x_y\rangle\langle x_y|^{B_2} \quad y, b = \pm 1$

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_x^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$

Not seen by Alice

The encoding correlated with the detection result

Alice receives the state $\widetilde{W}^{A_1 A_2} = \frac{1}{2}(\mathbb{1} + b\frac{1}{\sqrt{2}}\sigma_z^{A_2})$

She can read Bob's sent bit with probability

$$P(x = b|b' = 0) = \frac{2+\sqrt{2}}{4}$$

Where to look for non-causal processes?

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1. Closed time-like curves?: The entire process W is such that the information is sent back in time through a noisy channel.

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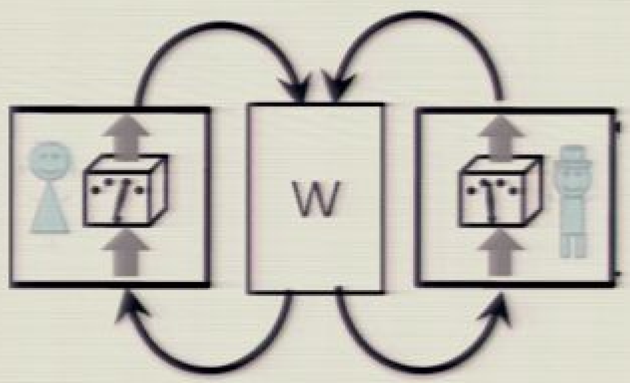

Where to look for non-causal processes?

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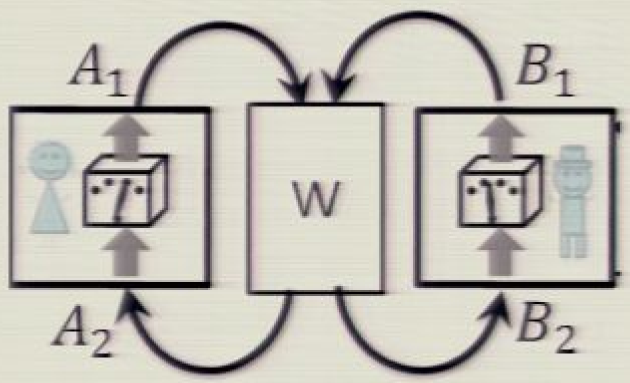

Where to look for non-causal processes?

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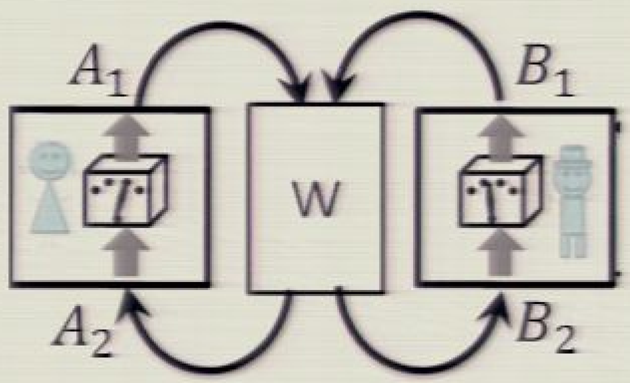


1. "Superpositions of space-time"?

Conclusions

- [Not shown]: In the classical limit all correlations are causally ordered
- Unified framework for both signalling ("time-like") and non-signalling ("space-like") quantum correlations
- Situations where a causal ordering between laboratory operations is not definite → Suggests that causal ordering might not be a necessary element of quantum theory
- What one needs to do in the lab to realize the "processes"? New resource for quantum information processing?

Magdalena Zych

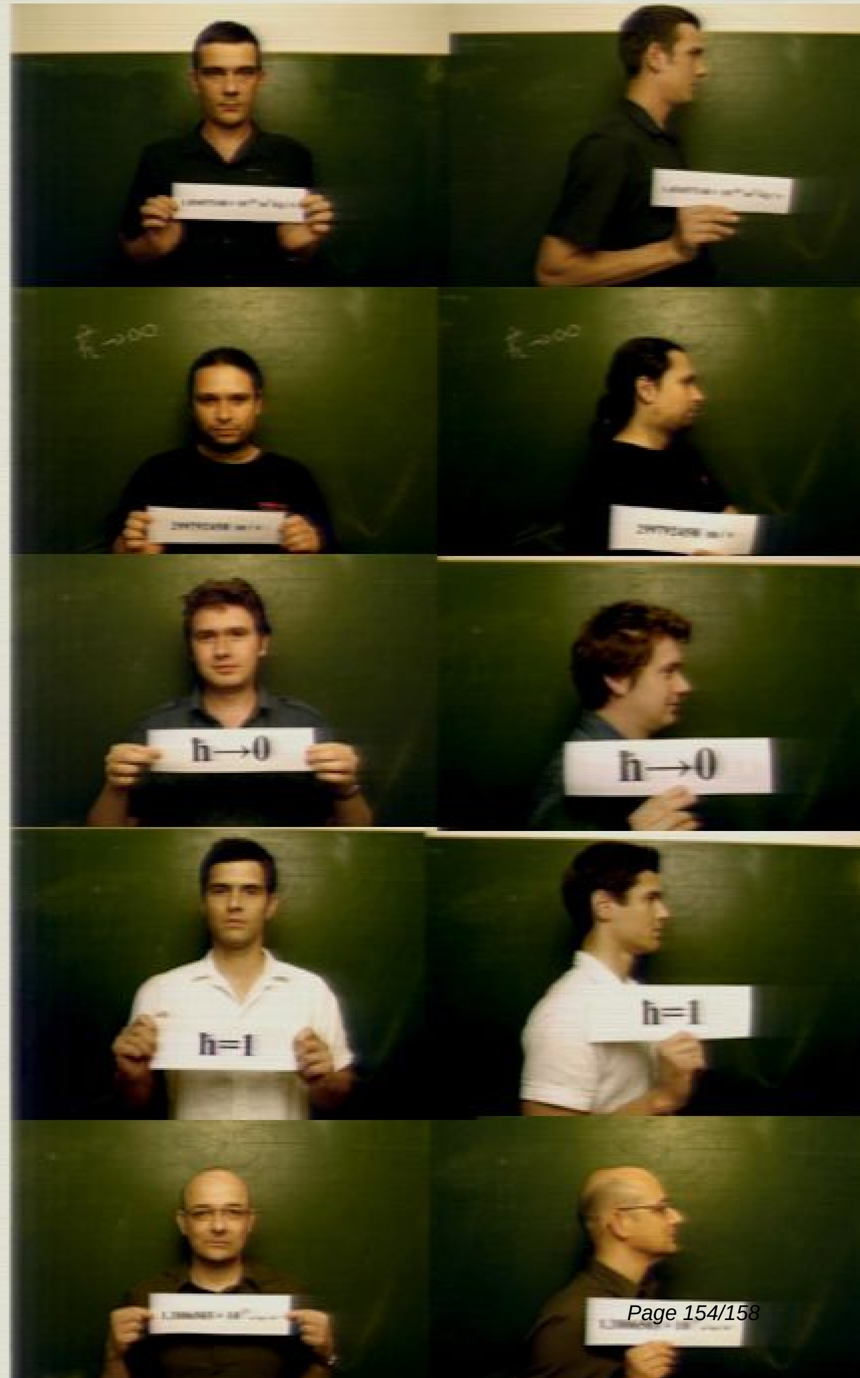
Borivoje Dakic

Fabio Costa

Igor Pikovski

Ognyan Oreshkov (U. Brüssels)

C.B.



CoQuS Complex Quantum Systems FWF FQXi

Borivoje
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Poster! → Fabio
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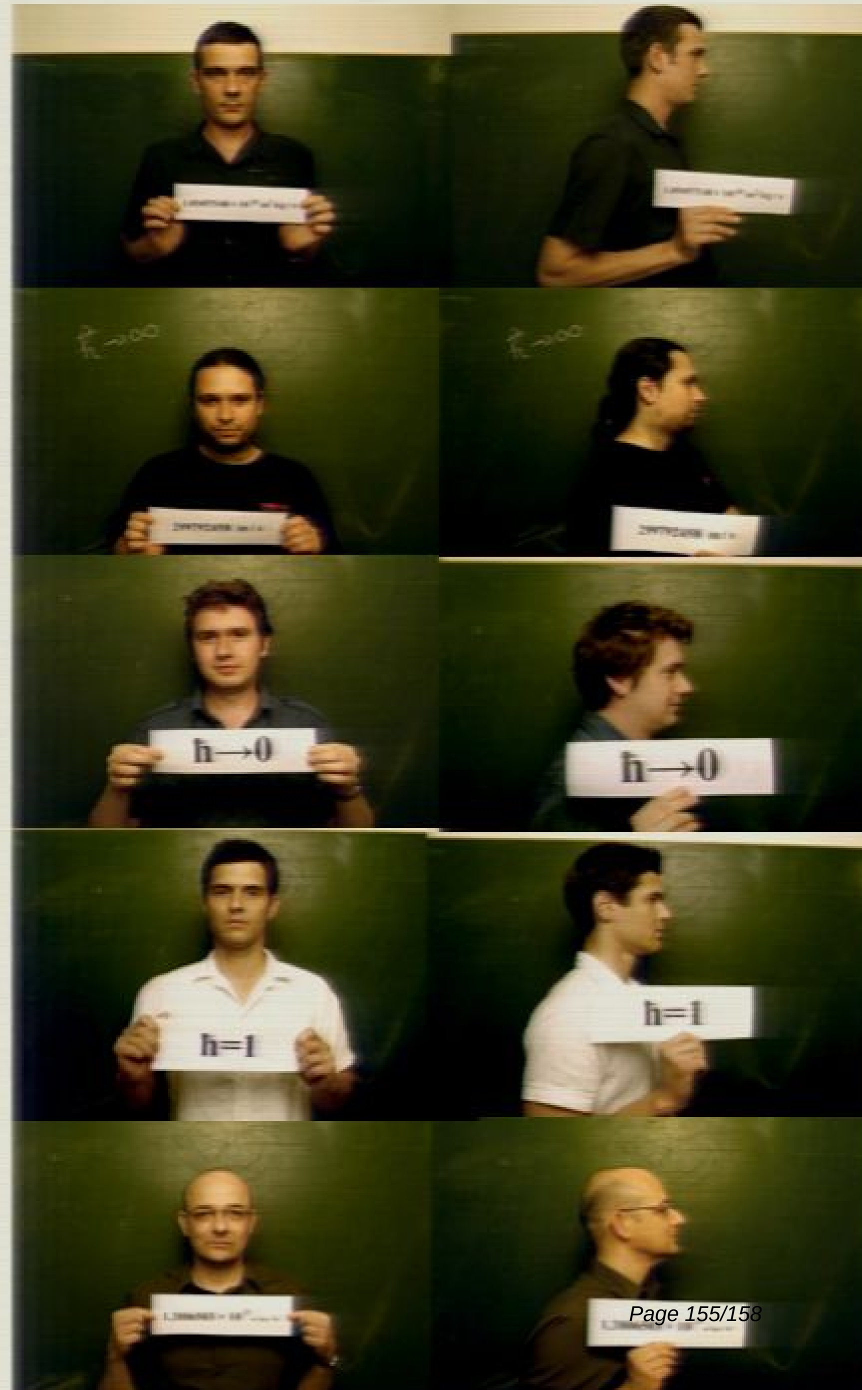

Thank you for your attention

Borivoje Dakic

Poster! → Fabio Costa

Igor Pikovski

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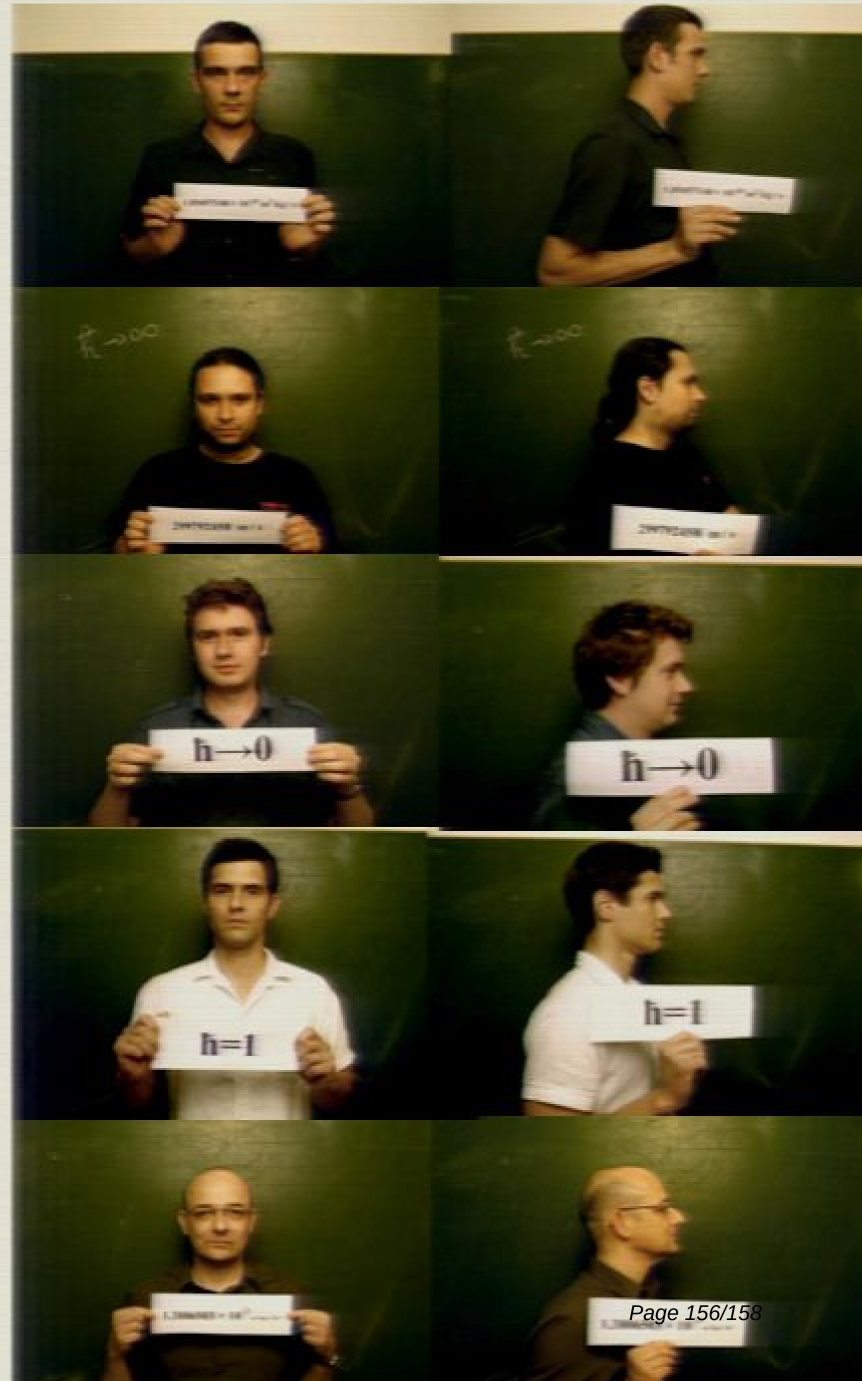

Magdalena Zych

CoQuS Complex Quantum Systems FWF FQXi

A causally non-separable example

If Bob wants to send ($b' = 0$)

Bob's CP map: $|z_{by}\rangle\langle z_{by}|^{B_1} \otimes |x_y\rangle\langle x_y|^{B_2} \quad y, b = \pm 1$

Repreparation — Measurement

$$W^{A_1 A_2 B_1 B_2} = \frac{1}{4}\left[\mathbb{1} + \frac{1}{\sqrt{2}}\left(\sigma_z^{A_1}\sigma_z^{B_2} + \sigma_z^{A_2}\sigma_z^{B_1}\sigma_x^{B_2}\right)\right]$$

$\langle x_\pm|\sigma_z|x_\pm\rangle = 0$

Not seen by Alice

The encoding correlated with the detection result

A causally non-separable example

If Bob wants to send (*b*‘ = 0)

Repreparation → Measurement

Bob‘s CP map: $|z_{by}\rangle\langle z_{by}|^{B_1} \otimes |x_y\rangle\langle x_y|^{B_2} \quad y, b = \pm 1$

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