Title: Communication cost Vs Bell inequality violation

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Abstract: In 1964, John Bell proved that independent measurements on entangled quantum states lead to correlations that cannot be reproduced using local hidden variables. The core of his proof is that such distributions violate some logical constraints known as Bell inequalities. This remarkable result establishes the non-locality of quantum physics. Bell's approach is purely qualitative. This naturally leads to the question of quantifying quantum physics' non-locality. We will specifically consider two quantities introduced for this purpose. The first one is the maximum amount of Bell inequality violation, and the second one is the communication cost of simulating quantum distributions. In this talk, we prove that these two quantities are strongly related: the logarithm of the first is upper bounded by the second. We prove this theorem in the more general context of non-signalling distributions. This generalization gives us two clear benefits. First, the rich structure of the underlying affine space provides us with a very strong intuition. Secondly, non-signalling distributions capture traditional communication complexity of boolean functions. In that case, our theorem is equivalent to the factorization norm lower bound of Linial and Shraibman, for which we give an elementary proof.

Bell Inequality Violation VS Communication Cost

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Based on a work with Julien Degorre[1], Sophie Laplante[2], Jérémie Roland[3]

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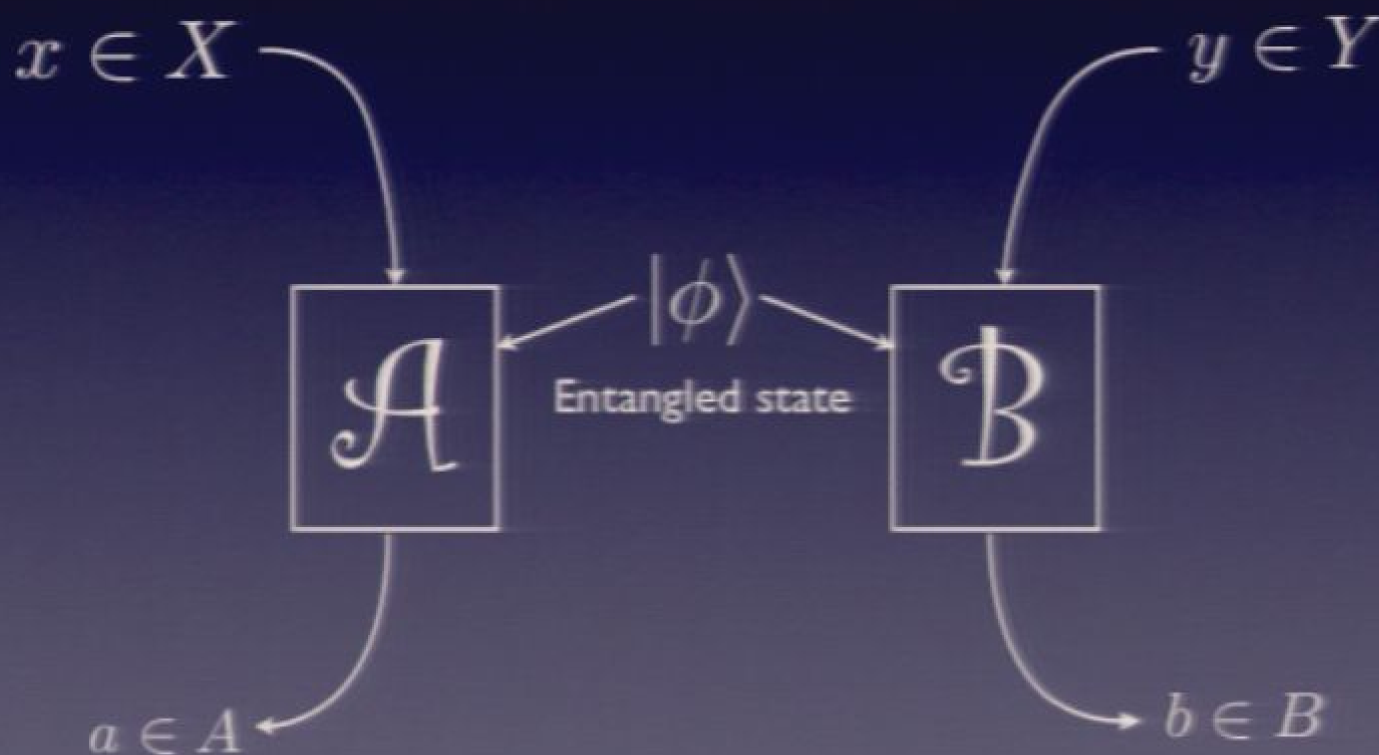
Physical motivation

EPR Experiment (Einstein, Podolsky, Rosen, 1935) :



Bell's theorem (mid 60'): the correlation of a and b is non-local

Can we quantify non-locality?

QUANTIFYING QUANTUM NONLOCALITY

Thesis by

Benjamin Francis Toner

In Partial Fulfillment of the Requirements

for the Degree of

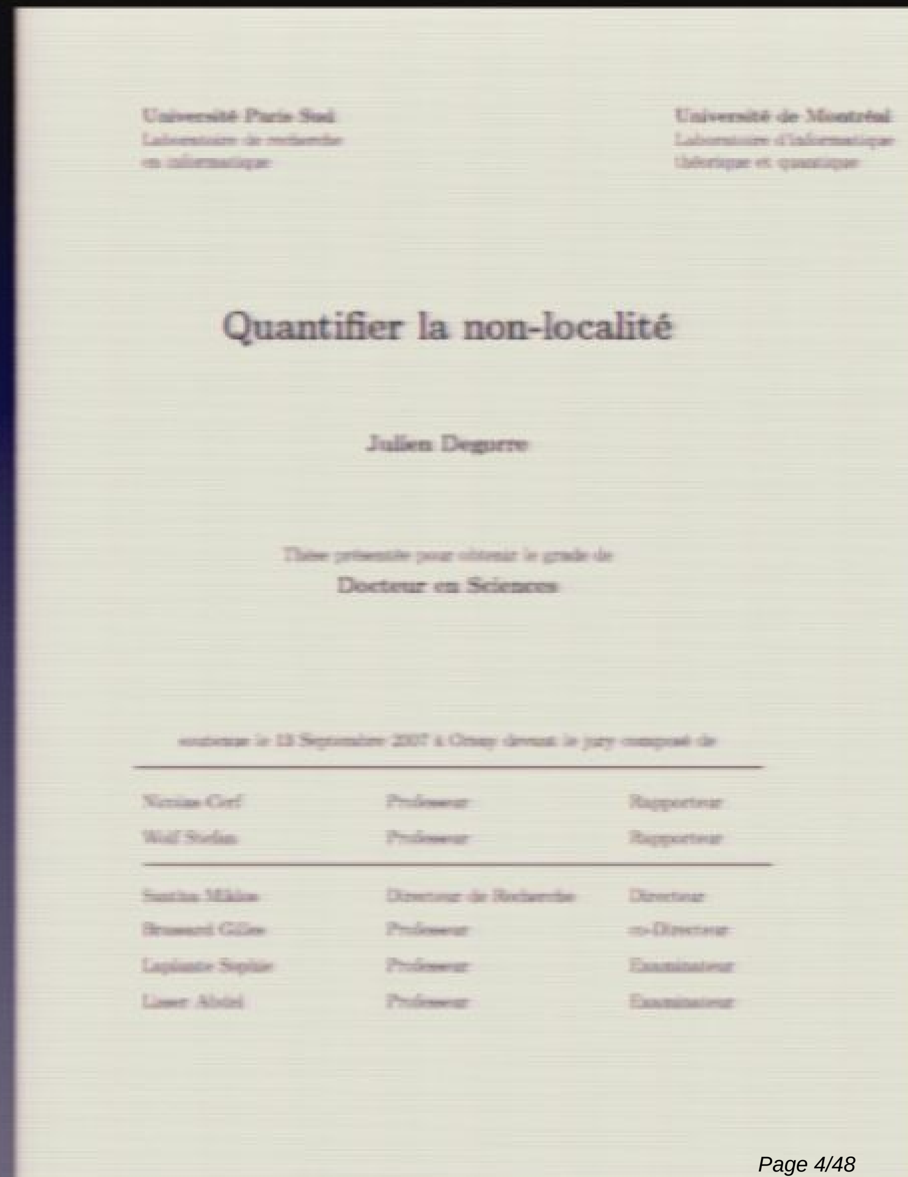
Doctor of Philosophy

California Institute of Technology

Pasadena, California

2007

(Defended June 19, 2006)



Université Paris-Sud
Laboratoire de recherche
en informatique

Université de Montréal
Laboratoire d'informatique
théorique et quantique

Quantifier la non-localité

Julien Degorre

Thèse présentée pour obtenir le grade de

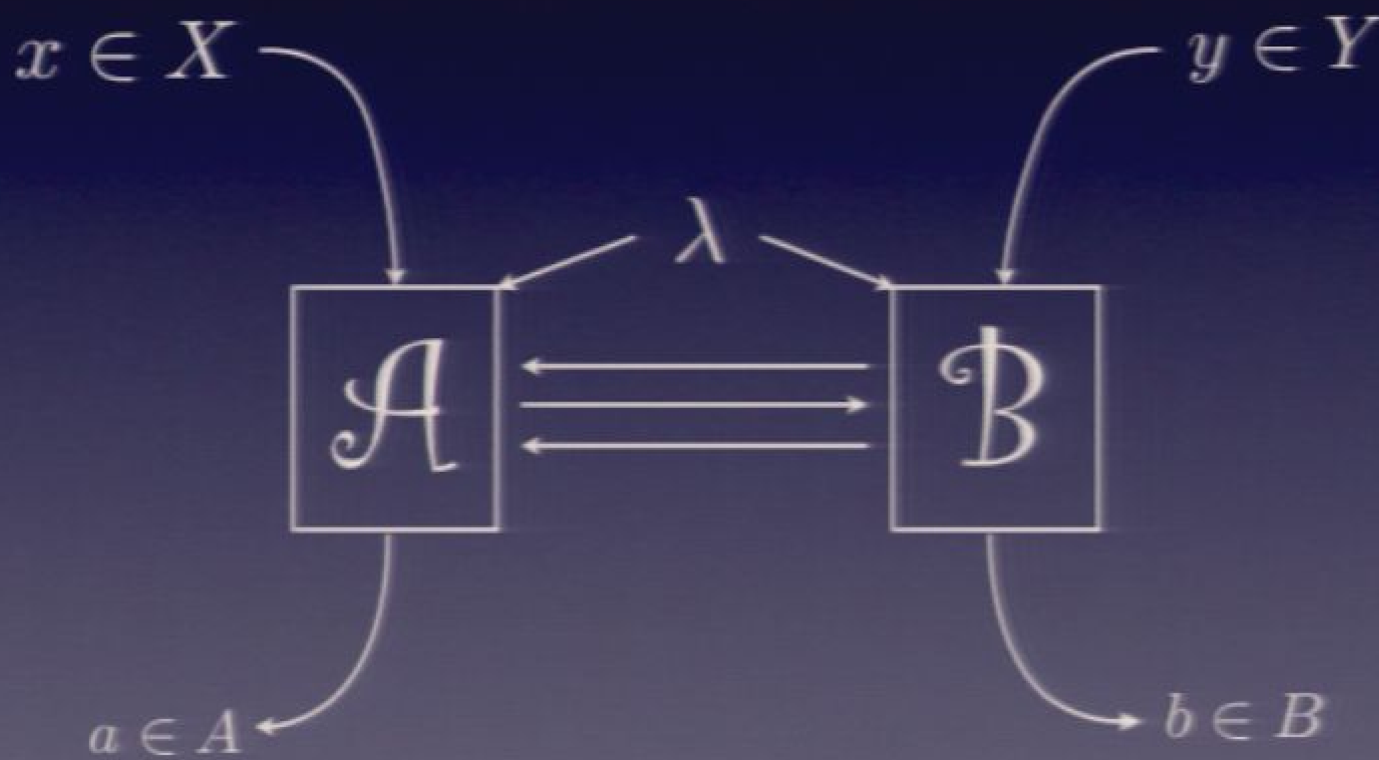
Docteur en Sciences

soutenue le 13 Septembre 2007 à Orsay devant le jury composé de

Nicolas Cerf	Professeur	Rapporteur
Wolf Stefan	Professeur	Rapporteur
Santha Miklos	Directeur de Recherche	Directeur
Brassard Gilles	Professeur	co-Directeur
Laplante Sophie	Professeur	Examinateur
[illegible]	Professeur	Examinateur

Communication Cost

Communication as a measure of non-locality



How much communication is required to simulate EPR like experiments?

Communication Cost

- Simulating quantum correlations requires *at most*
  - Maudlin 92: 1.17 bits on average
  - Brassard, Cleve, Tapp 99: 8 bits worst case
  - Steiner 99: 1.48 bits on average
  - Cerf, Gisin, Massar 00: 1.19 bits on average
  - Toner, Bacon 03: 1 bit worst case
  - Regev, Toner 07: 2 bits worst case, higher dimensions.

qubit pairs

maximally entangled

Motivations

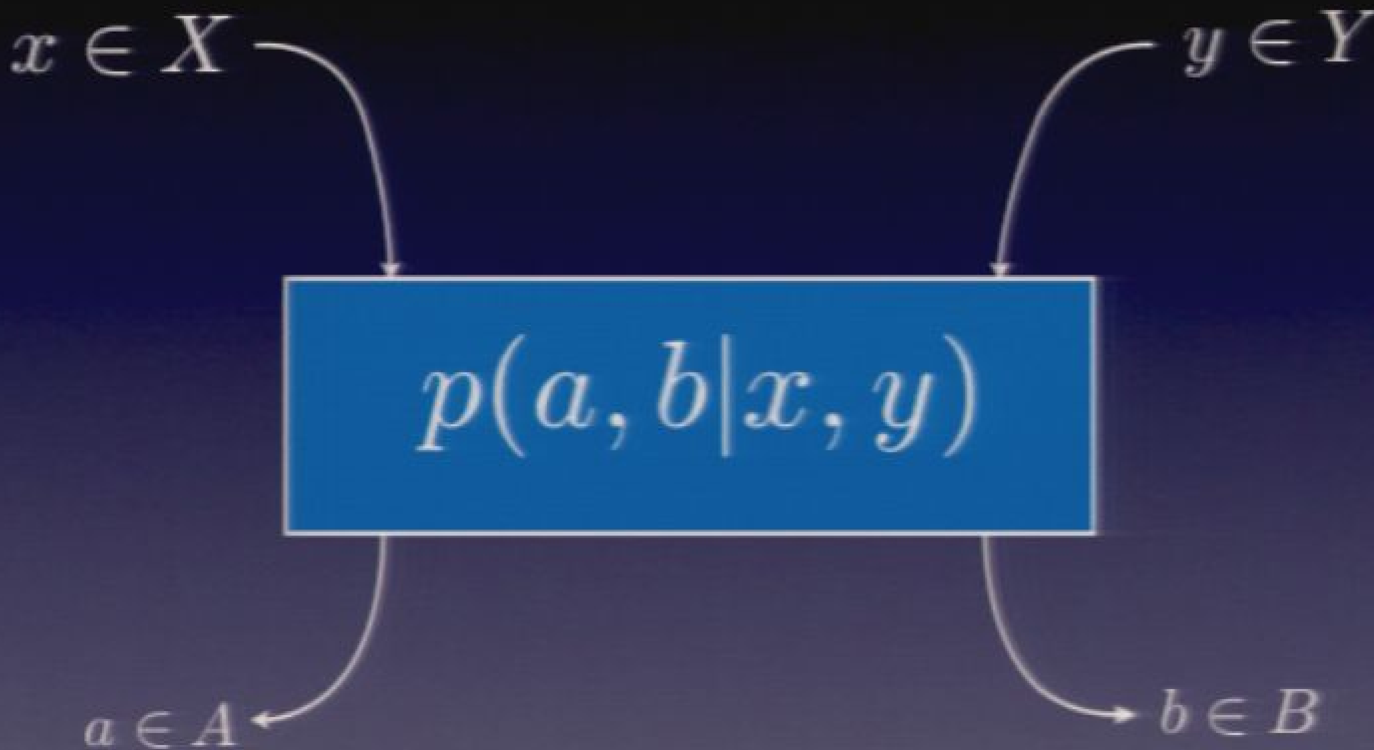
- From physics:
  - Quantify quantum non locality
  - Information theoretic perspective
- From computer science:
  - Give a unified framework for various problems in complexity
  - Better knowledge of the tool (communication complexity)

Understand the relation between non-locality and computation

Summary

1. The model of non-signaling distributions
2. Bell inequality violation
3. Lower bounds on communication cost
4. Gap between classical and quantum, upper bound on maximal violation of Bell inequalities
5. Upper bounds on communication cost

Non-signaling distributions



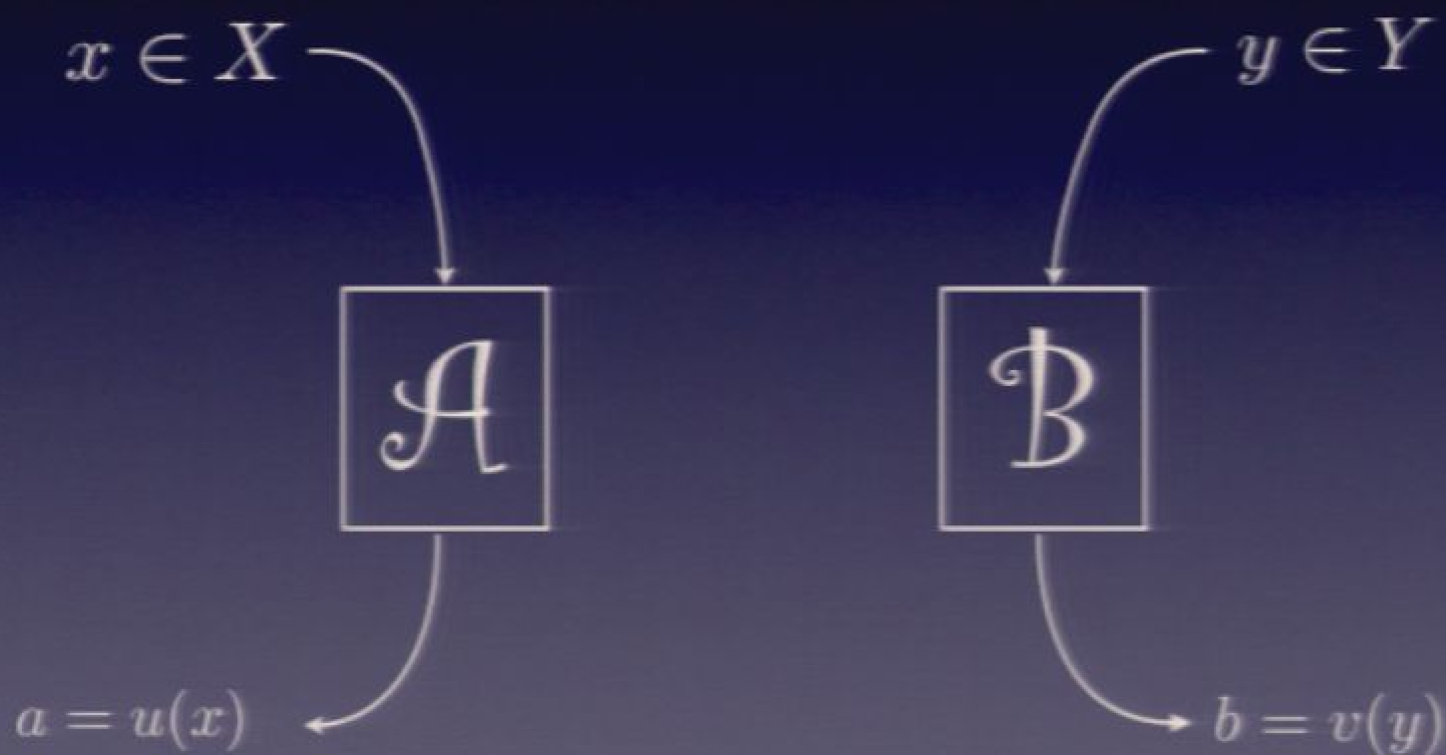
Non-signaling condition:

$$p(a|x,y) = p(a|x,y')$$

$$p(b|x,y) = p(b|x',y)$$

Local deterministic distributions

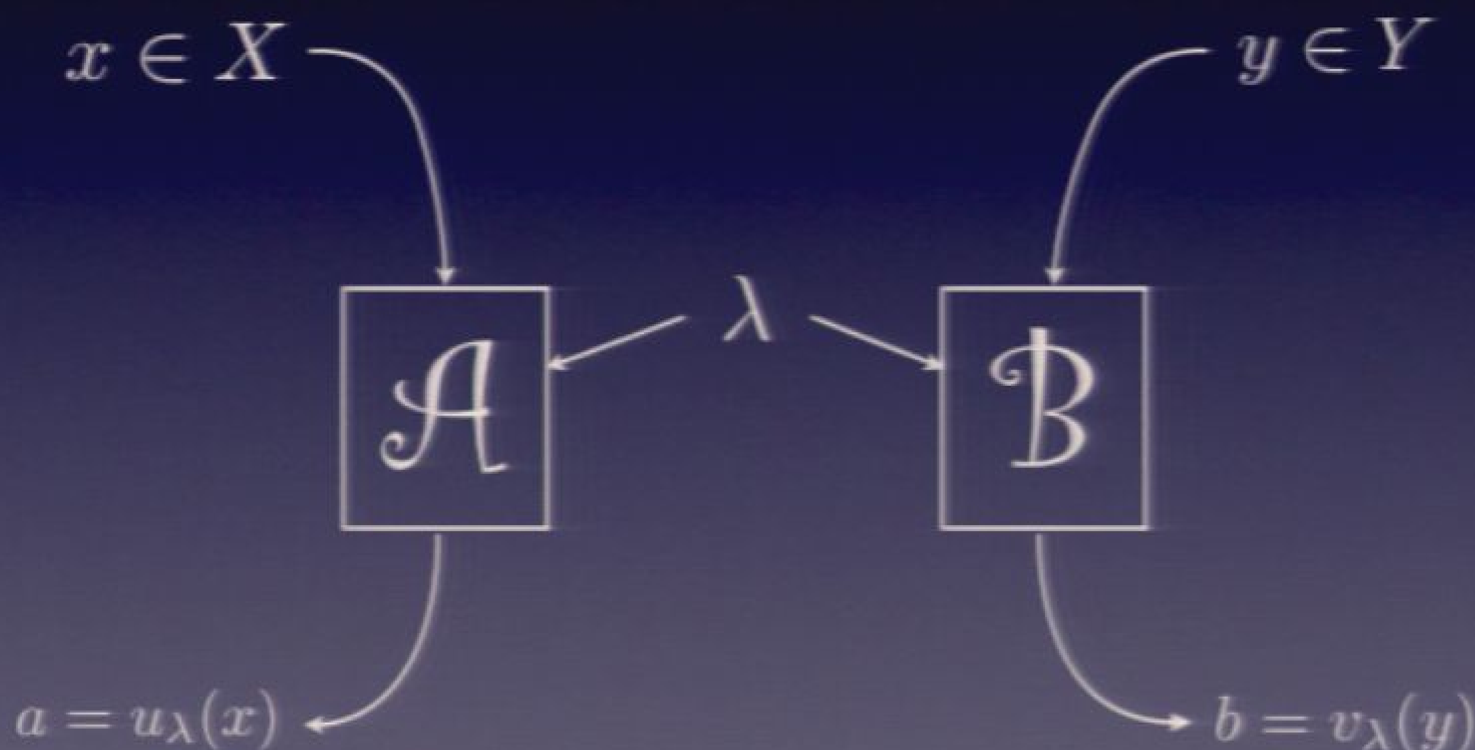
What can we do without any resource?



$$p(a, b|x, y) = \delta_{a=u(x)}\delta_{b=v(y)}$$

Local ~~deterministic~~ distributions

Add shared randomness

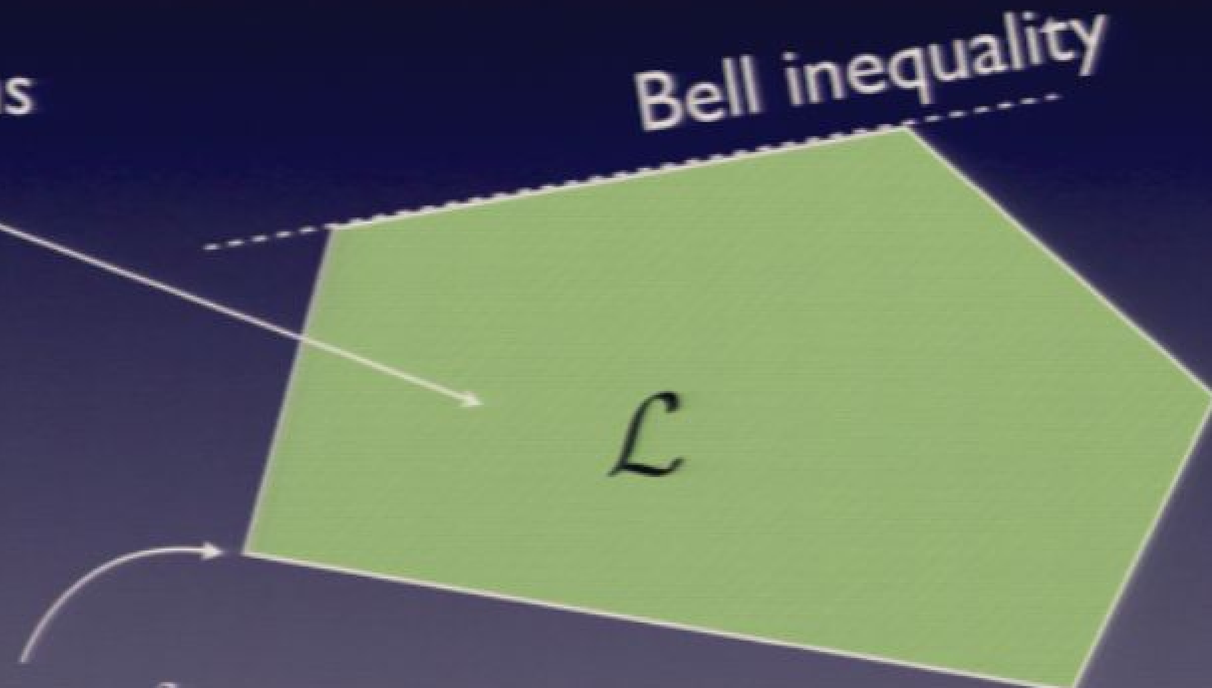


$$p(a,b|x,y) = \sum_\lambda p(\lambda)\delta_{a=u_\lambda(x)}\delta_{b=v_\lambda(y)}$$

Probability distribution =

Local distributions

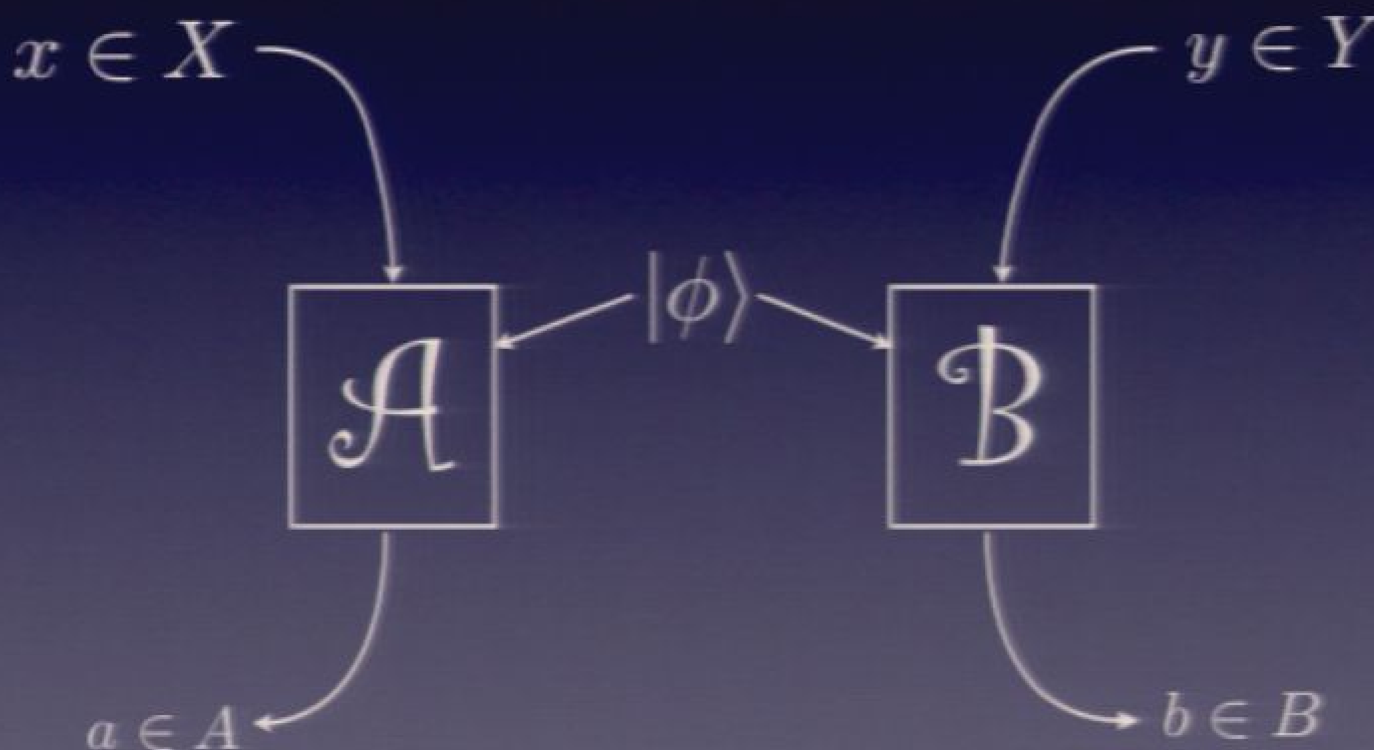
- Local distributions



$$p(a,b|x,y) = \delta_{a=u(x)}\delta_{b=v(y)}$$

Quantum distributions

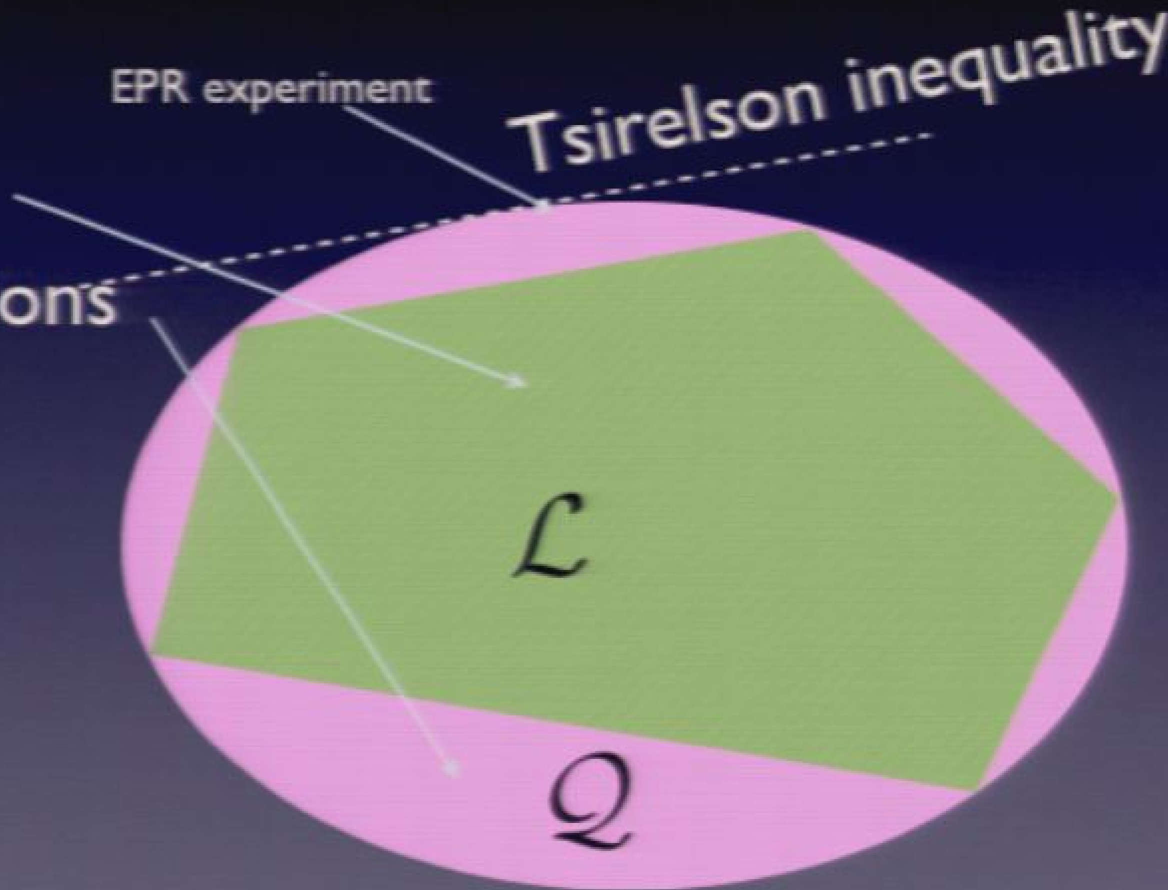
Give them entanglement



$$p(a, b|x, y) = \ldots$$

Structure of non signaling distributions

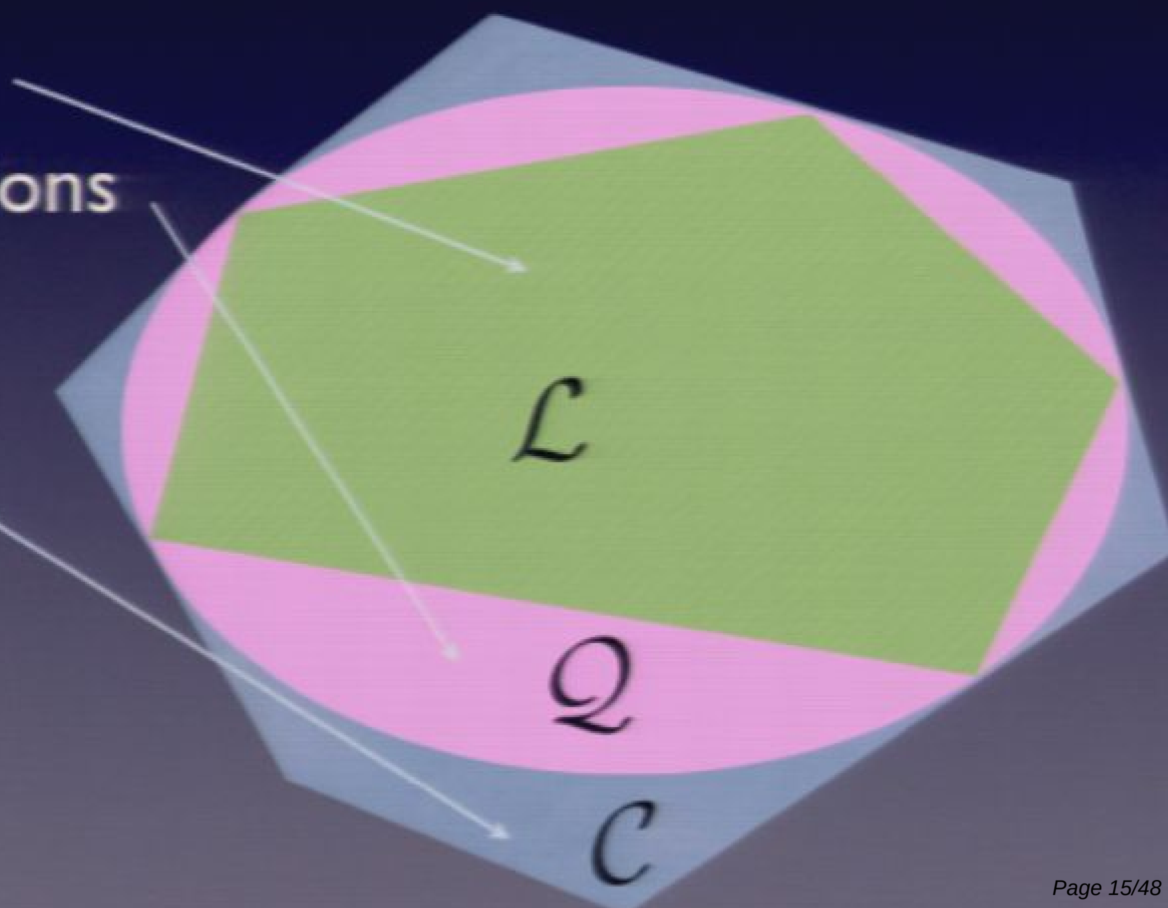
- Local distributions
- Quantum distributions



Structure of non signaling distributions

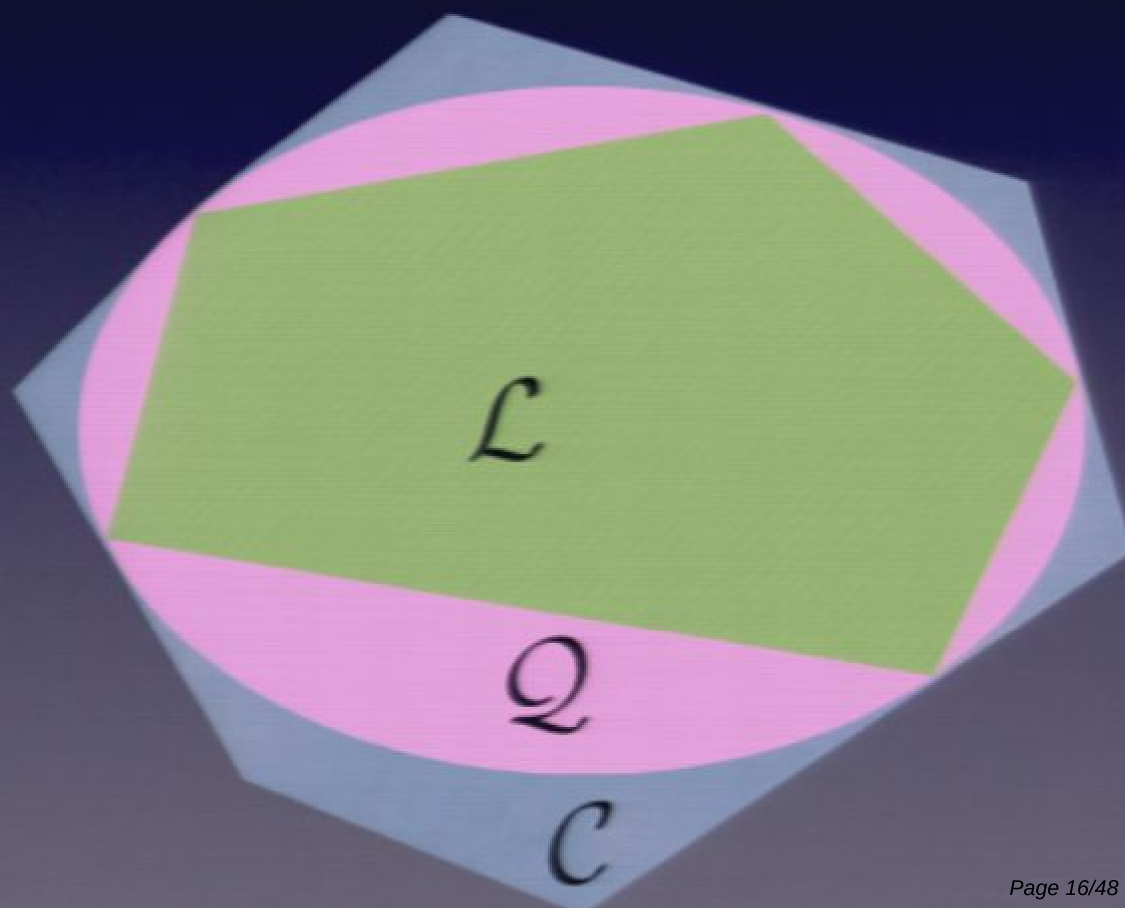
- Local distributions
- Quantum distributions
- Non signaling distributions

$$p(a|x,y) = p(a|x,y')$$
$$p(b|x,y) = p(b|x',y)$$



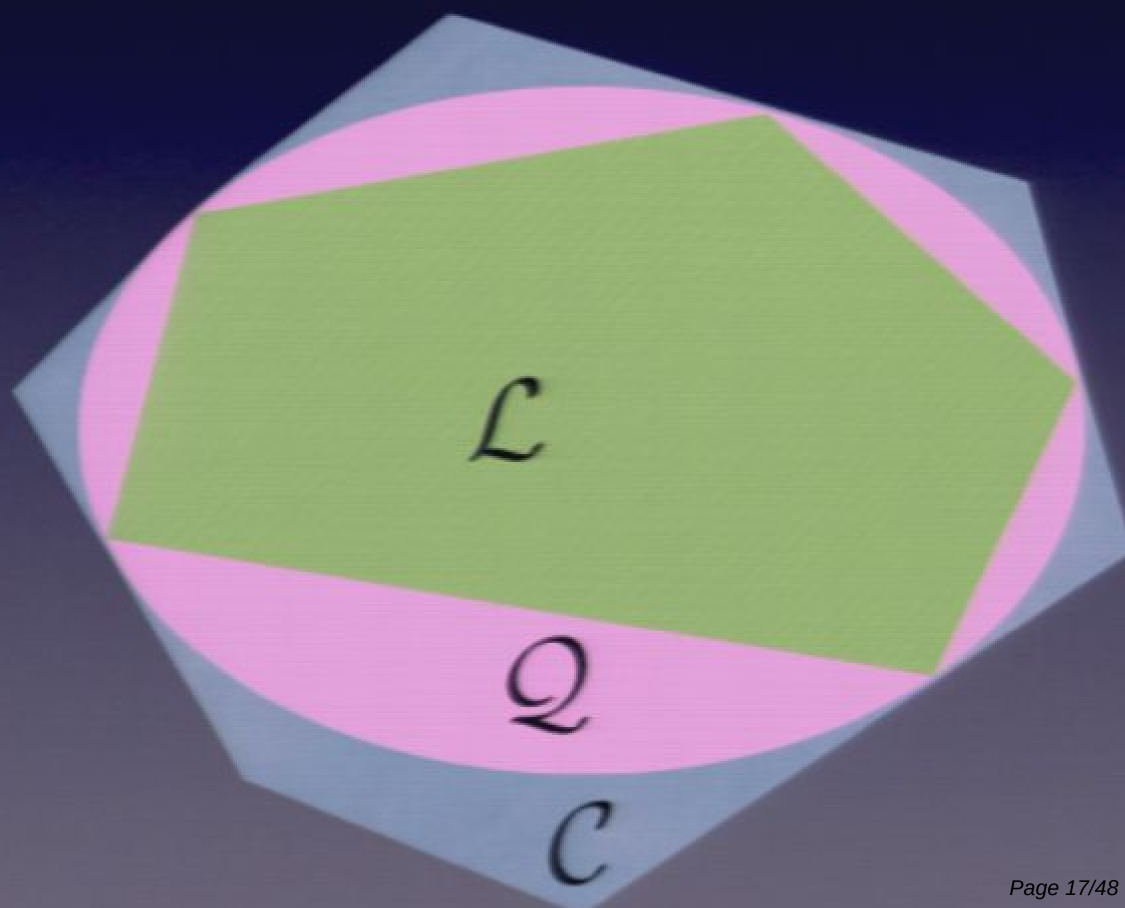
Structure of non signaling distributions

- Strict inclusions



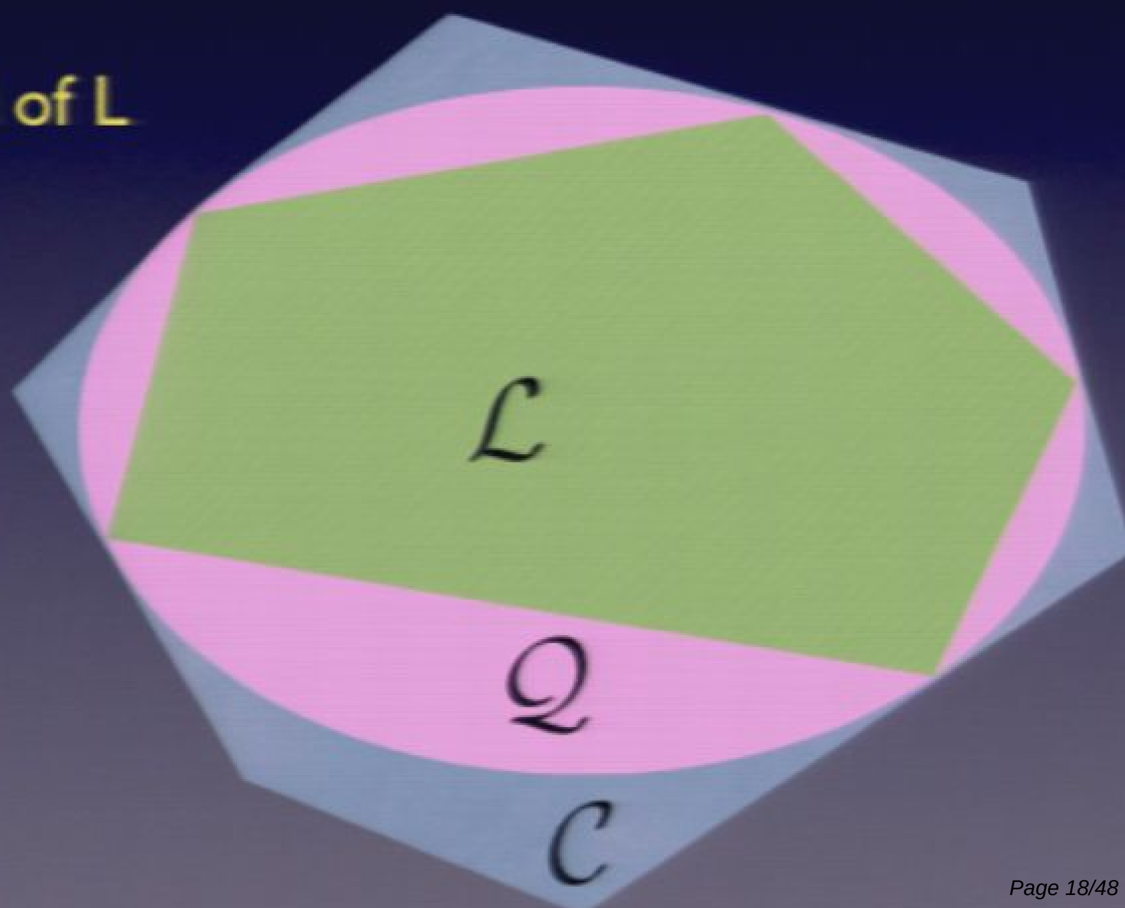
Structure of non signaling distributions

- Strict inclusions
- C is a polytope



Structure of non signaling distributions

- Strict inclusions
- C is a polytope
- C is the **affine hull** of L



Structure of non signaling distributions

- Strict inclusions
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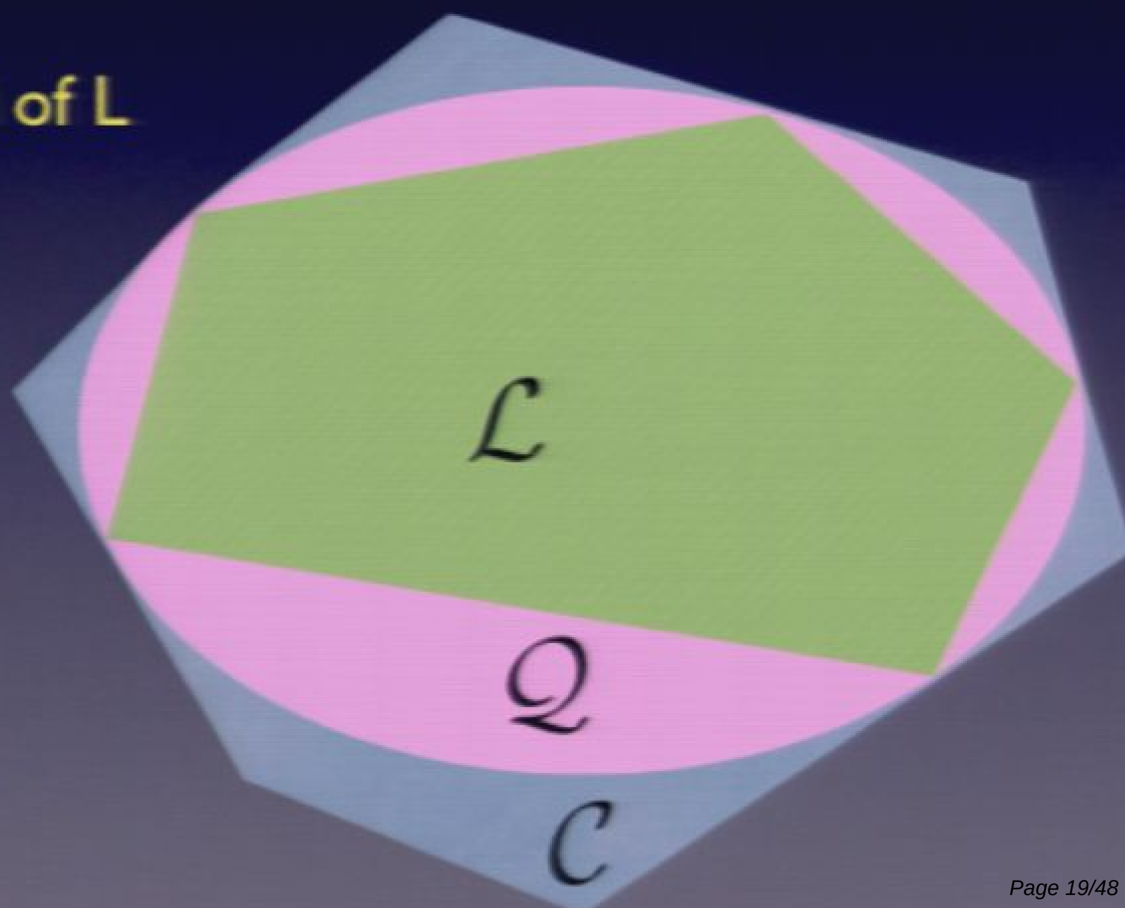
General proof in various contexts in [KlayRandallFoulis87] [Wilce 92], [Barrett07]



Writing boolean functions as a non-signaling distribution

For a boolean function

$$f : X \times Y \to \{+1, -1\}$$

Define the corresponding distribution,

$$p_f(a, b|x, y) = \begin{cases} 1/2 & \text{if } ab = f(x, y) \\ 0 & \text{otherwise} \end{cases}$$

(Like an XOR game)

Writing boolean functions as a non-signaling distribution

Encoding the equality function:

If $x = y$:

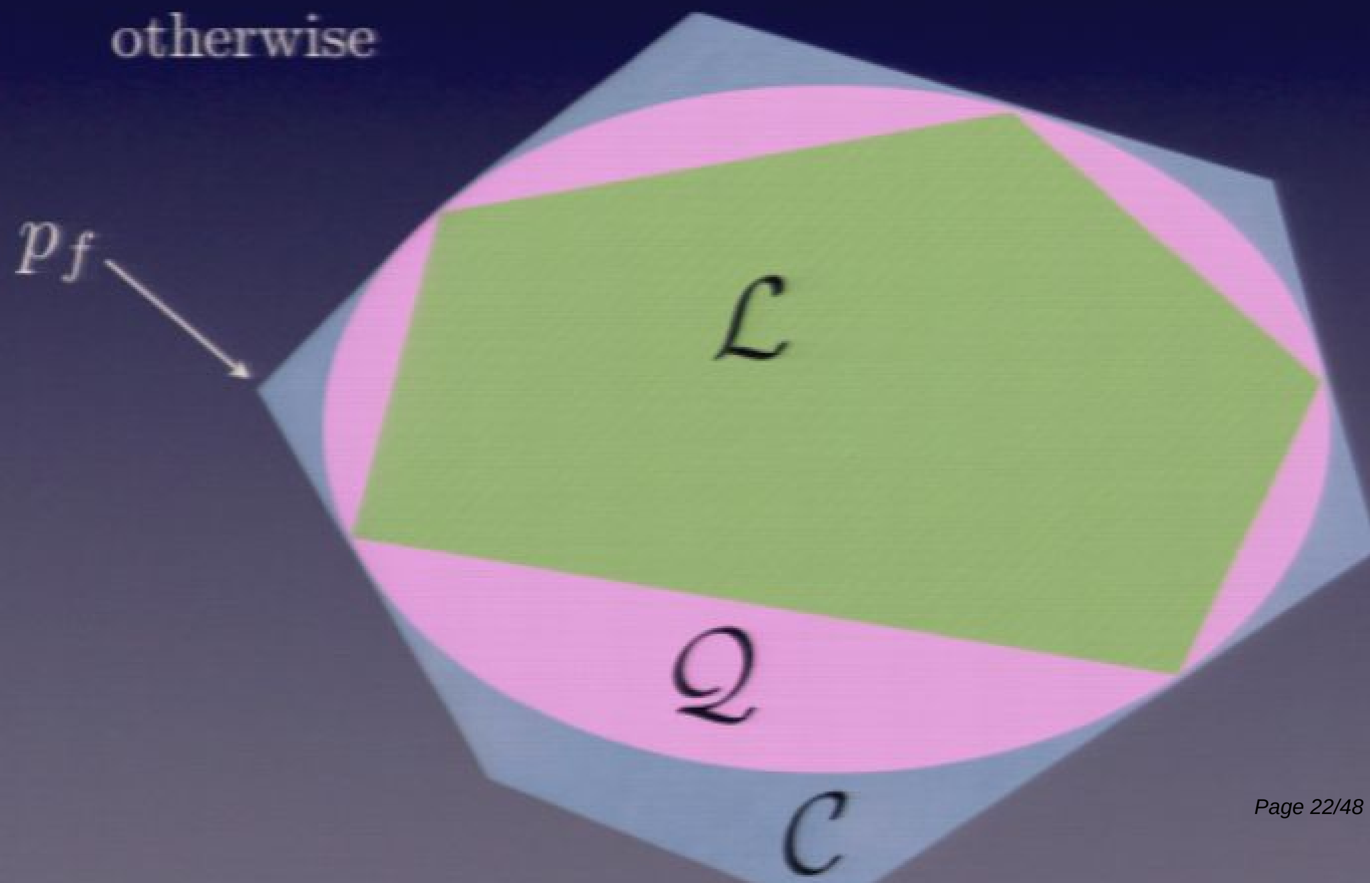
	$b = -1$	$b = 1$
$a = -1$	$1/2$	0
$a = 1$	0	$1/2$

If $x \neq y$:

	$b = -1$	$b = 1$
$a = -1$	0	$1/2$
$a = 1$	$1/2$	0

Structure of non signaling distributions

$$p_f(a,b|x,y) = \begin{cases} 1/2 & \text{if } ab = f(x,y) \\ 0 & \text{otherwise} \end{cases}$$



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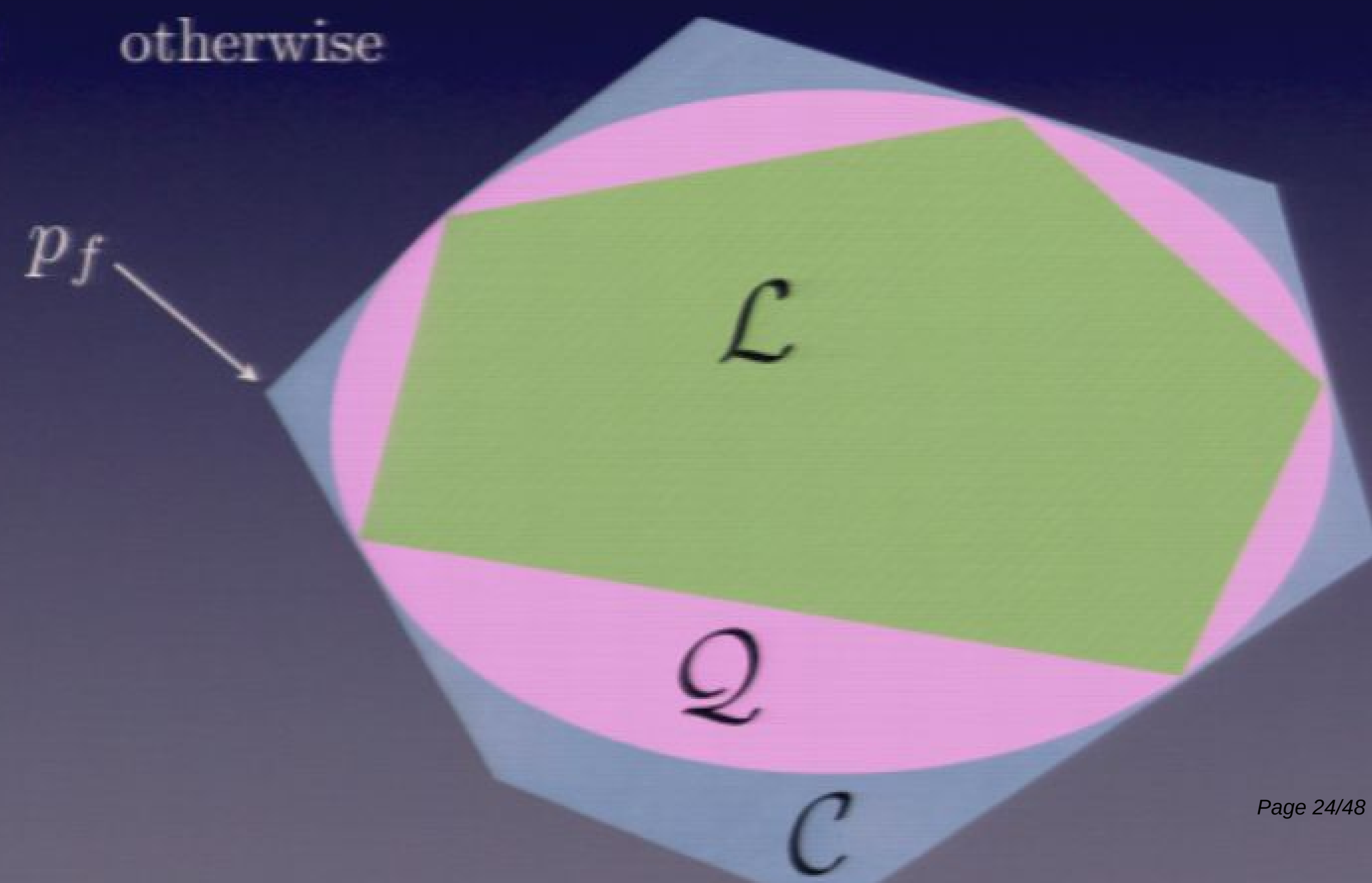
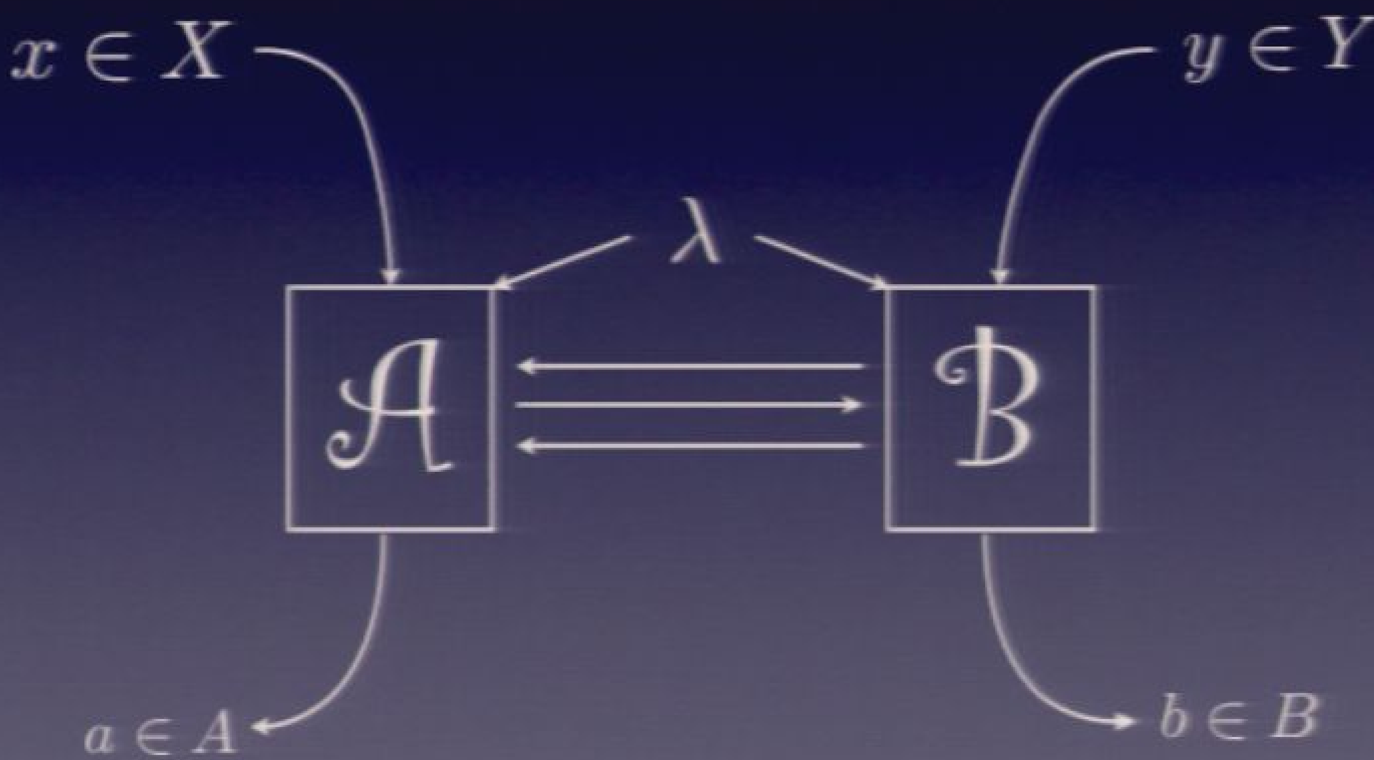


Communication Cost

Allow the players to communicate



How much communication is required to simulate a non signaling distribution?

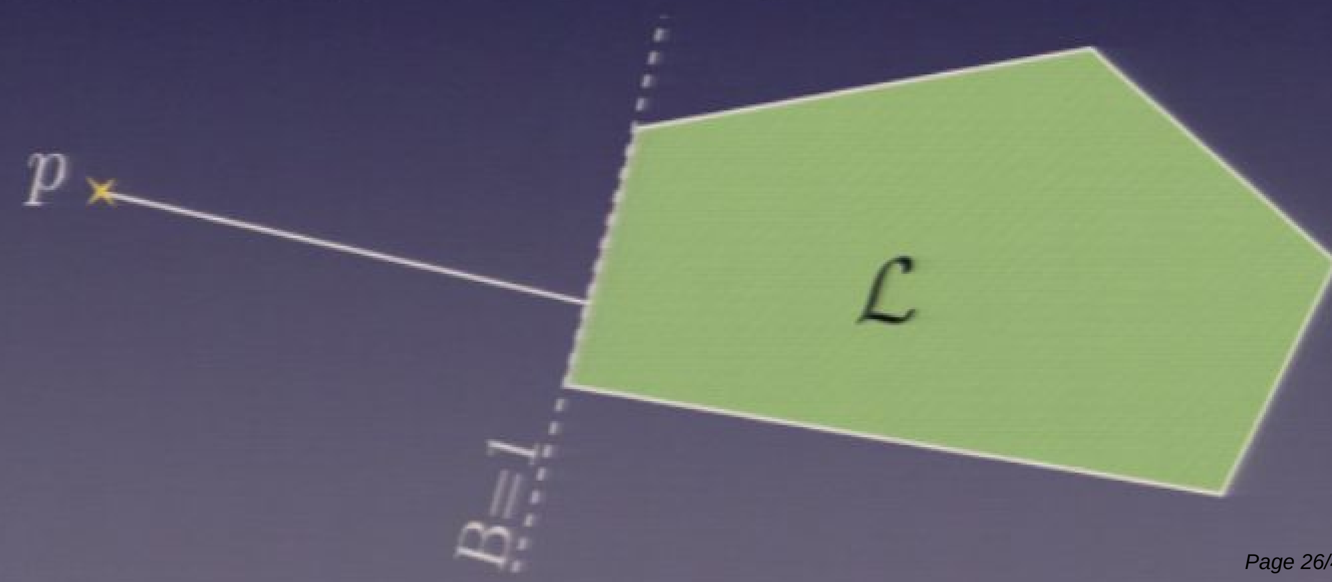
Bell inequality violation

Normalized Bell inequality:

$$\forall p_l \in \mathcal{L} : |B(p)| = |\sum_{a,b,x,y} B(a,b,x,y)p(x,y|a,b)| \leq 1$$

Bell inequality violation

$$\nu(p) = \max\{B(p) : |B(p_l)| \leq 1 \forall p_l \in \mathcal{L}\}$$



Bell inequality violation

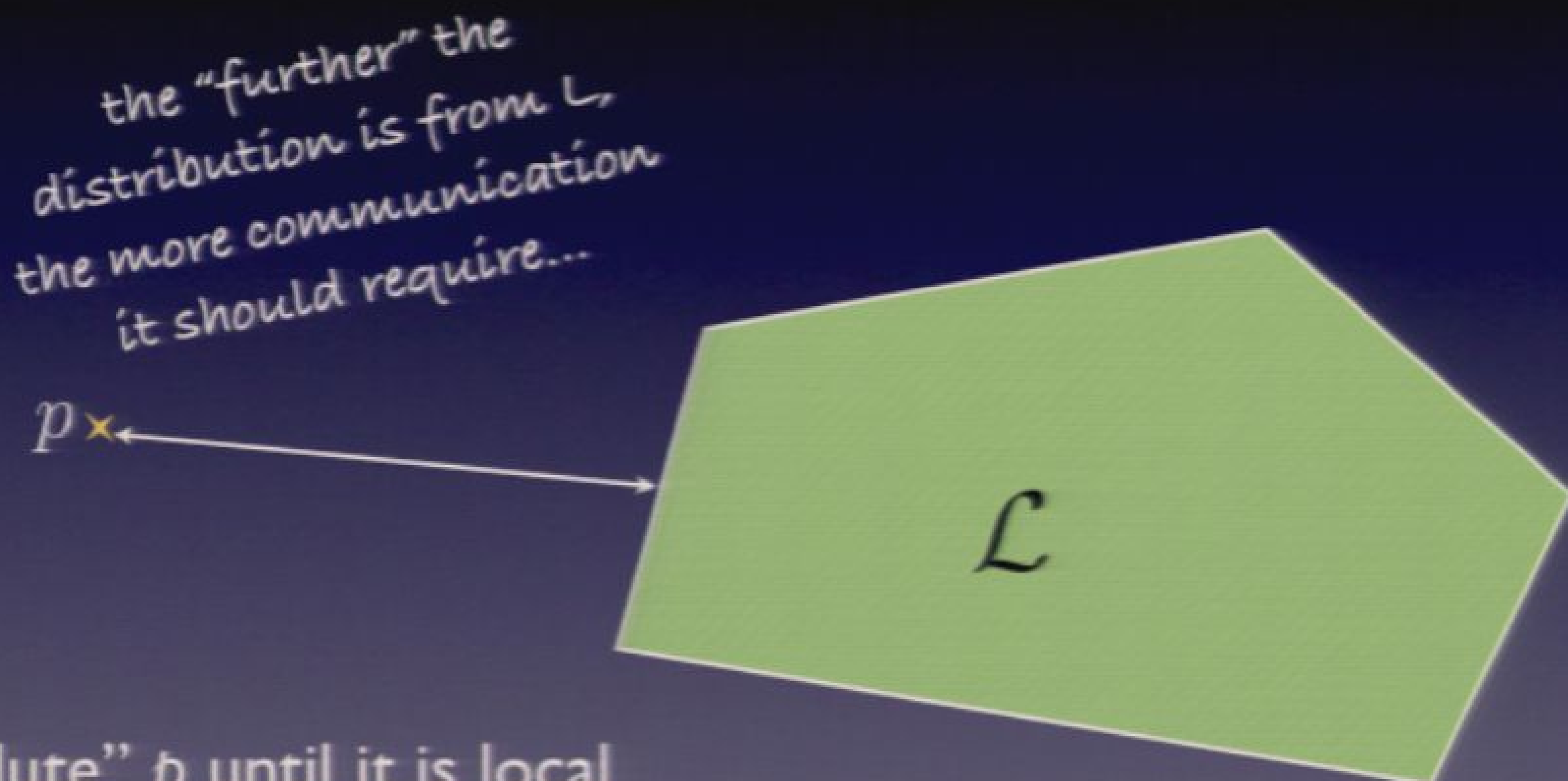
- Bell ('64): Some violation exists
- Clauser Horne Shimony Holt ('69): CHSH inequality
- Tsirelson ('80): Upper bound on maximal violation
- Pérez-García Wolfe Palazuelos Junge ('07): unbounded violation of tripartite states
- Junge Palazuelos Pérez-García Villanueva ('09): large violations of bipartite states
- Briët Buhrman Lee Vidick ('09): low violation of specific multipartite states
- Junge Palazuelos ('10): large violation with low entanglement
- Buhrman Regev Scarpa deWolf ('11): Near optimal violations

Lower bounds on communication

Theorem: $R_0(p) \geq \log \nu(p)$

Minimum amount of communication required to simulate p exactly

Lower bounds on communication



- "Dilute" p until it is local
- Use this local distribution in an affine model for the original distribution

Diluting the distribution

Assume there is a t bit protocol (A, B) for p:

- *The players pick a random transcript T, using shared randomness.*
- *If T is consistent with x, Alice outputs A(T,x)*
- *If T is consistent with y, Bob outputs B(T,y)*
- *Otherwise, they output according to their marginals.*

$$p'(a,b|x,y) = \frac{1}{2^t} p(a,b|x,y) + (1 - \frac{1}{2^t}) p(a|x) p(b|y)$$

This distribution is local (doesn't require communication)

Diluting the distribution

$$p'(a,b|x,y) = \frac{1}{2^t}p(a,b|x,y) + (1-\frac{1}{2^t})p(a|x)p(b|y)$$

Diluting the distribution

$$p'(a,b|x,y) = \frac{1}{2^t}p(a,b|x,y) + (1 - \frac{1}{2^t})p(a|x)p(b|y)$$

$$\downarrow$$

$$p(a,b|x,y) = 2^t p'(a,b|x,y) - (2^t - 1)p(a|x)p(b|y)$$

We get p as an affine combination of local distributions.

Lower bounds on communication

$$p(a,b|x,y) = 2^t p'(a,b|x,y) - (2^t - 1) p(a|x)p(b|y)$$

$\in \mathcal{L}$ $\qquad\qquad$ $\in \mathcal{L}$

Distance to L: sum of absolute values of coefficients

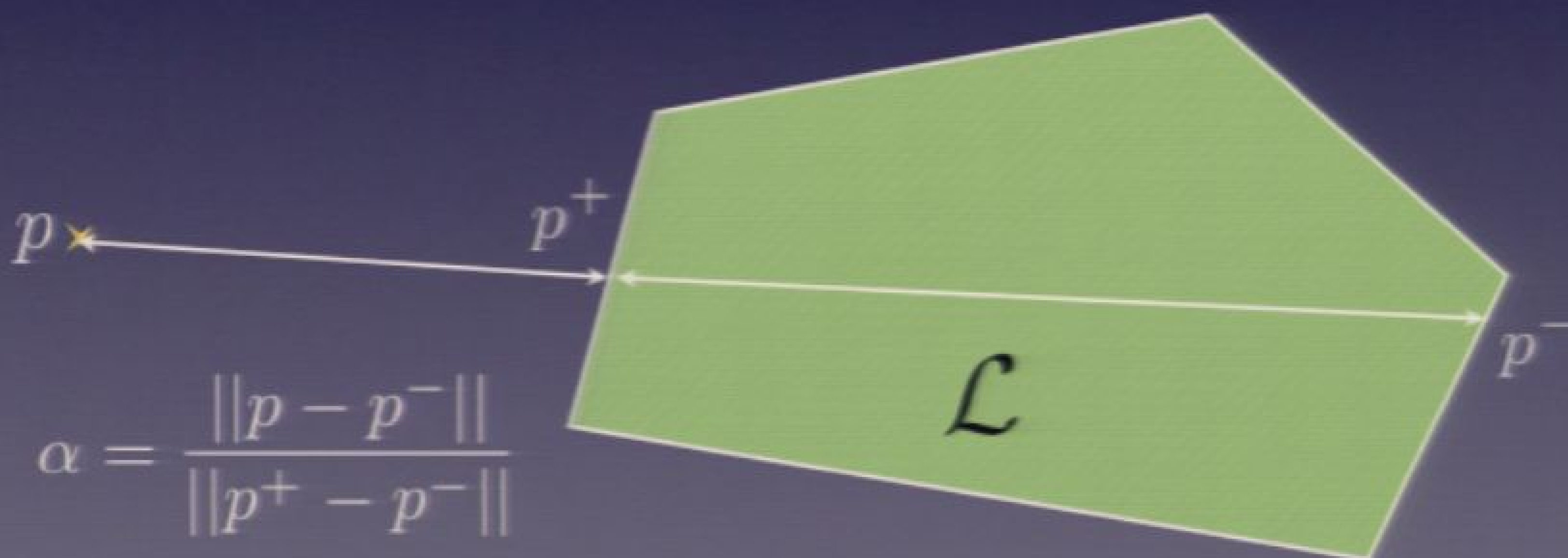
$$\tilde{\nu}(p) = \min\{\sum_{p_l \in \mathcal{L}} |q_l| : p = \sum_{p_l \in \mathcal{L}} q_l p_l, \sum_{p_l \in \mathcal{L}} q_l = 1\} \leq 2^{t+1} - 1$$

$$\Rightarrow \quad R_0(p) \geq \log \tilde{\nu}(p)$$

Lower bounds on communication

$$\tilde{\nu}(p) = \min\{\sum_{p_l\in\mathcal{L}} |q_l| : p = \sum_{p_l\in\mathcal{L}} q_l p_l, \sum_{p_l\in\mathcal{L}} q_l = 1\}$$

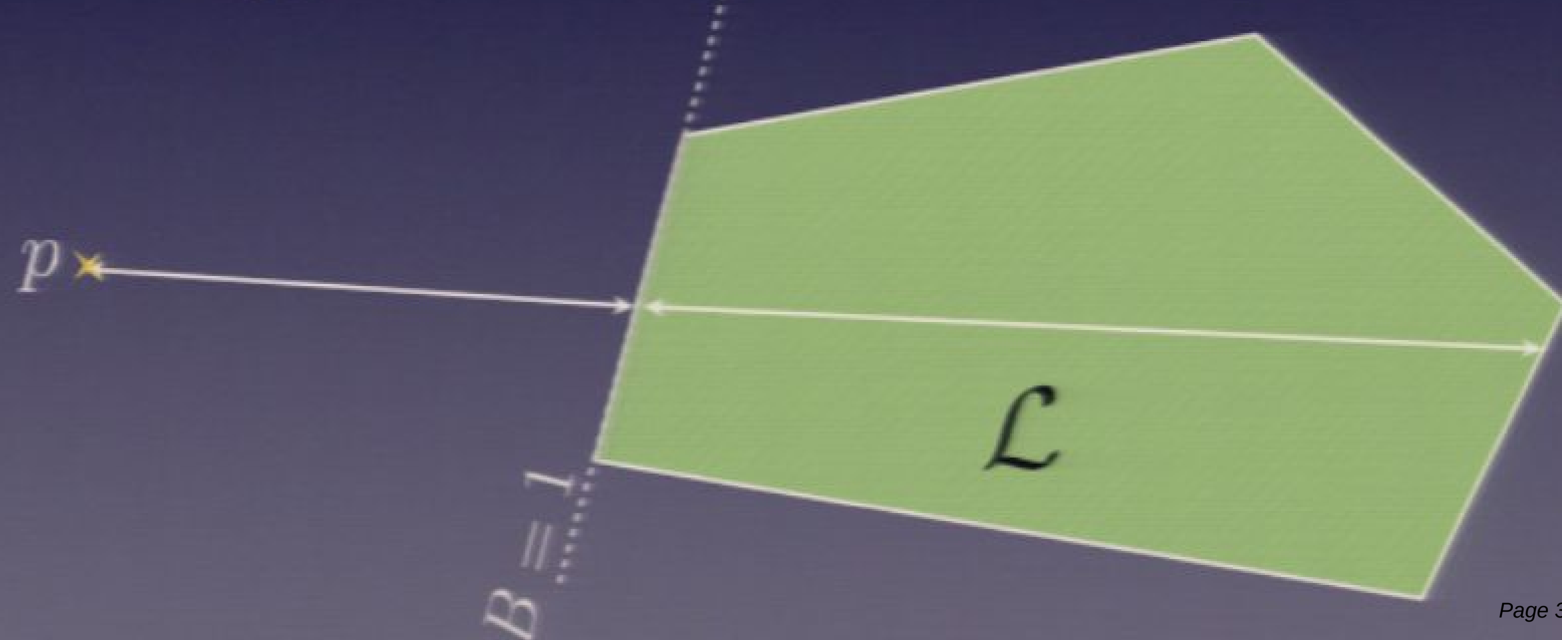
$$= \min\{2\alpha - 1 : p = \alpha p^+ + (1-\alpha)p^-\}$$



$$\alpha = \frac{||p - p^-||}{||p^+ - p^-||}$$

Lower bounds on communication

$$
\begin{aligned}
\tilde{\nu}(p) &= \min\{\sum_{p_l \in \mathcal{L}} |q_l| : p = \sum_{p_l \in \mathcal{L}} q_l p_l, \sum_{p_l \in \mathcal{L}} q_l = 1\} \\
&= \max\{B(p) : |B(p_l)| \leq 1 \forall p_l \in \mathcal{L}\} \\
&= \nu(p)
\end{aligned}
$$



Lower bounds on communication

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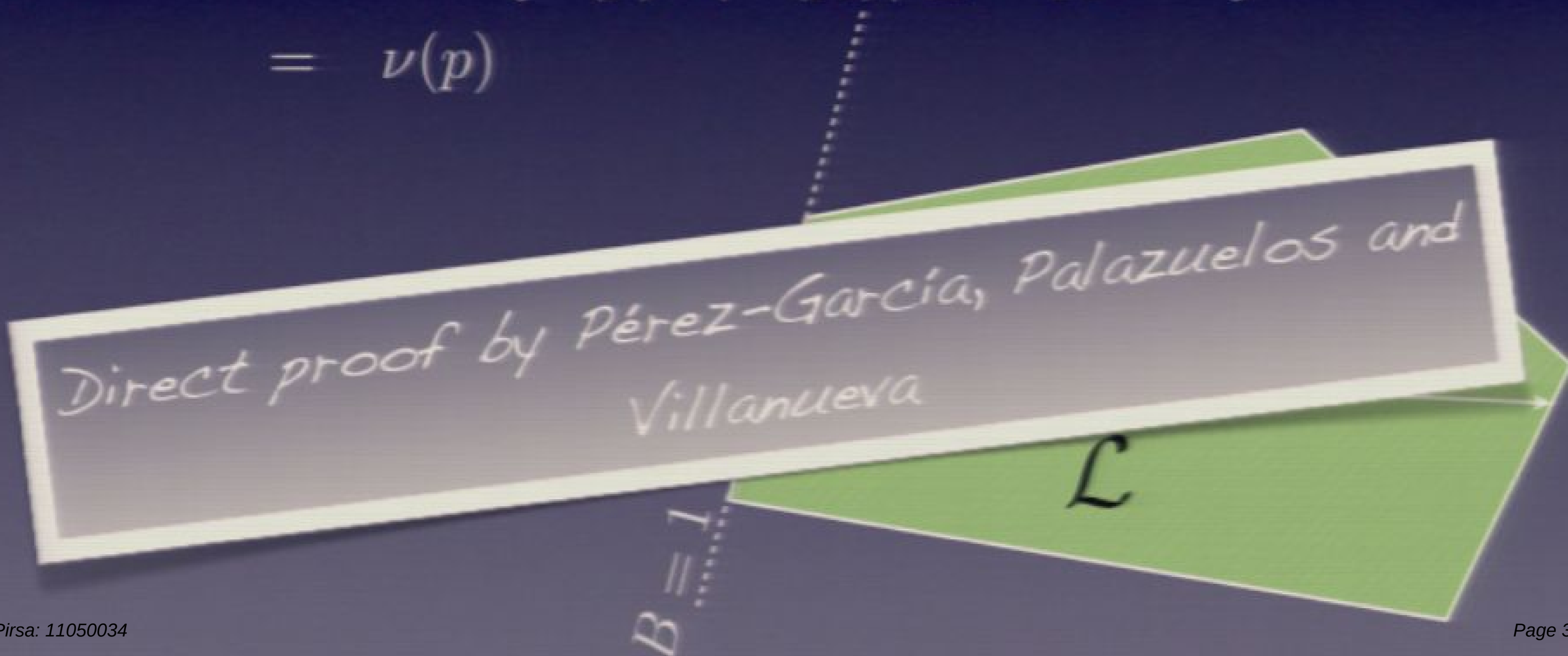


Lower bounds on *quantum* communication

$$\gamma_2(p) = \min\{\sum_{p_l \in X} |q_l| : p = \sum_{p_l \in \mathcal{Q}} q_l p_l, \sum_{p_l \in \mathcal{Q}} q_l = 1\}$$

$$= \max\{B(p) : |B(p_l)| \leq 1 \, \forall p_l \in \mathcal{Q}\}$$

Maximal violation of a (normalized) Tsirelson inequality

Gap for boolean distributions with uniform marginals

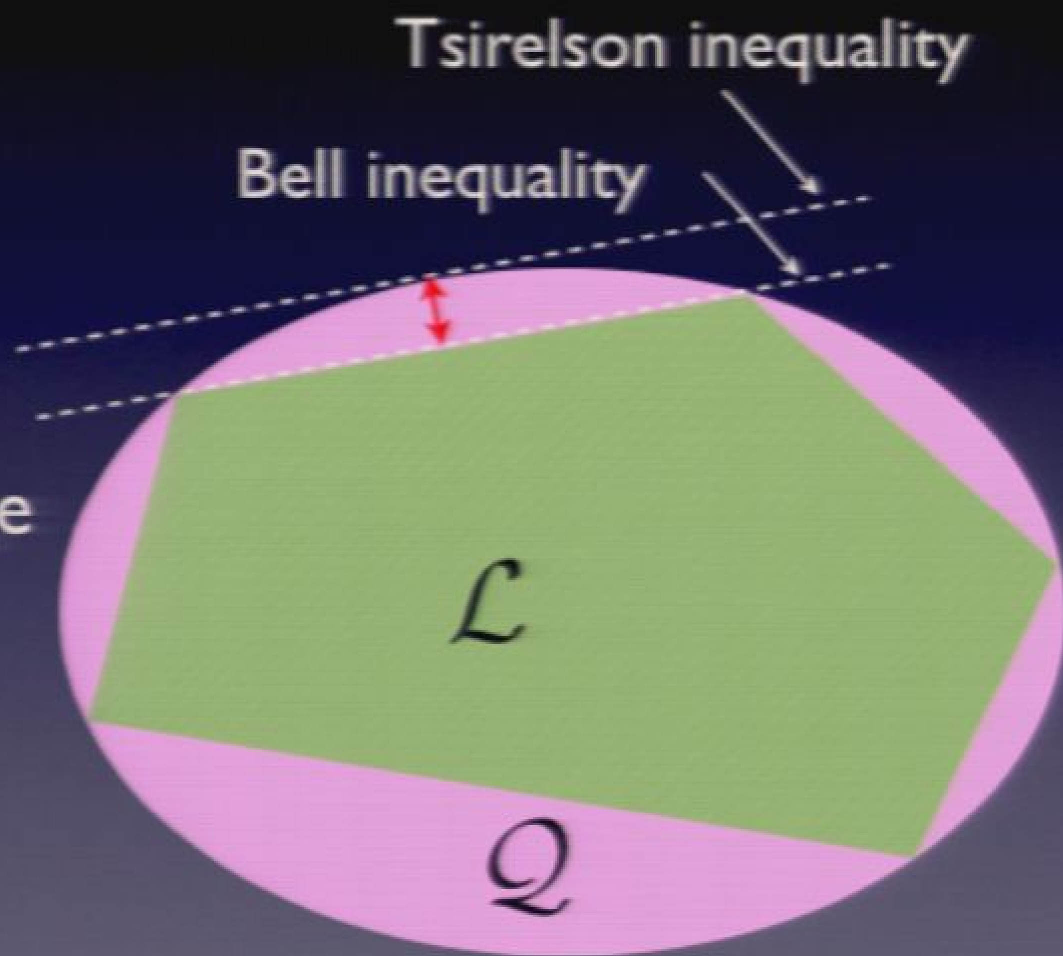
- Includes maximally entangled states, boolean functions.
- Gap between classical and quantum at most K_G
  - Tisirelson's theorem: quantum strategy <=> inner product over real vectors,
  - local deterministic strategy (classical) <=> inner product over +/-1 vectors),
  - Grothendieck's inequality

Gap between ν and γ_2

- If p(a,b|x,y) over outcomes $A \times B$

$$\nu(p) \leq O(AB\gamma_2(p))$$

- Cannot prove large gaps between classical and quantum.



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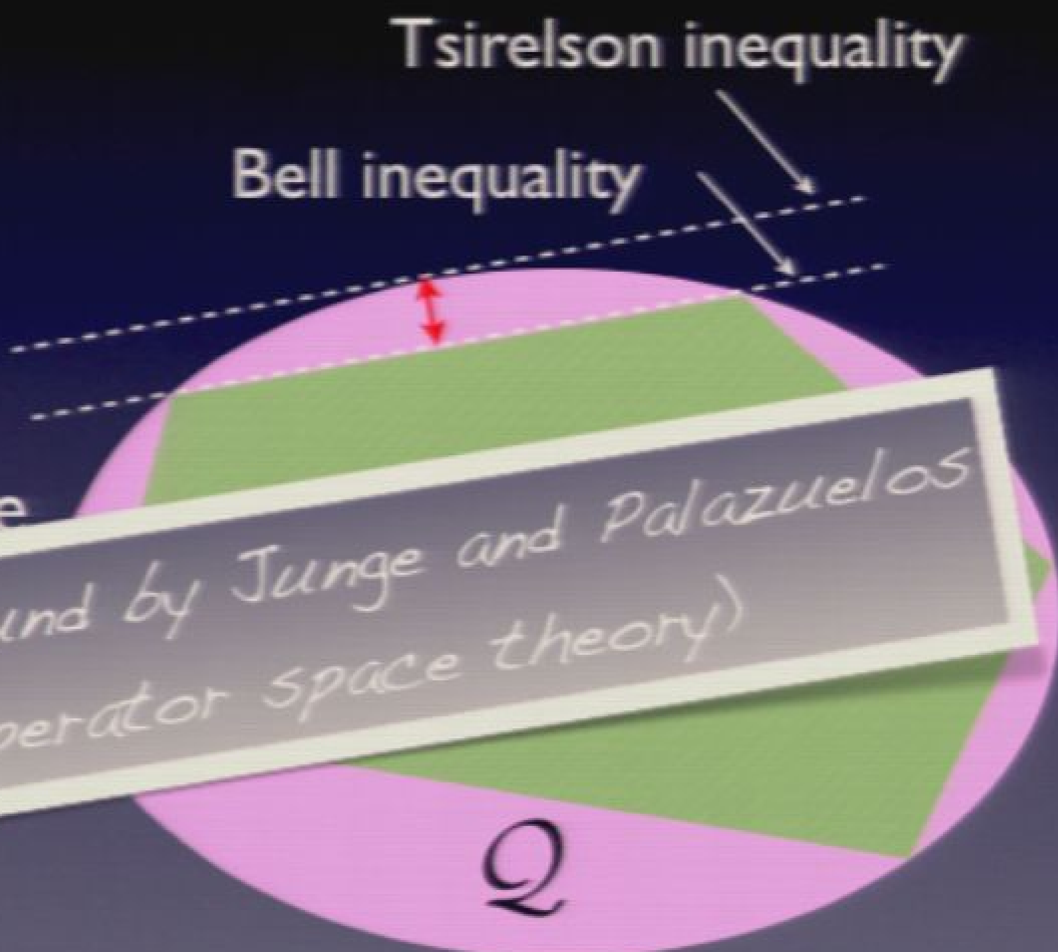
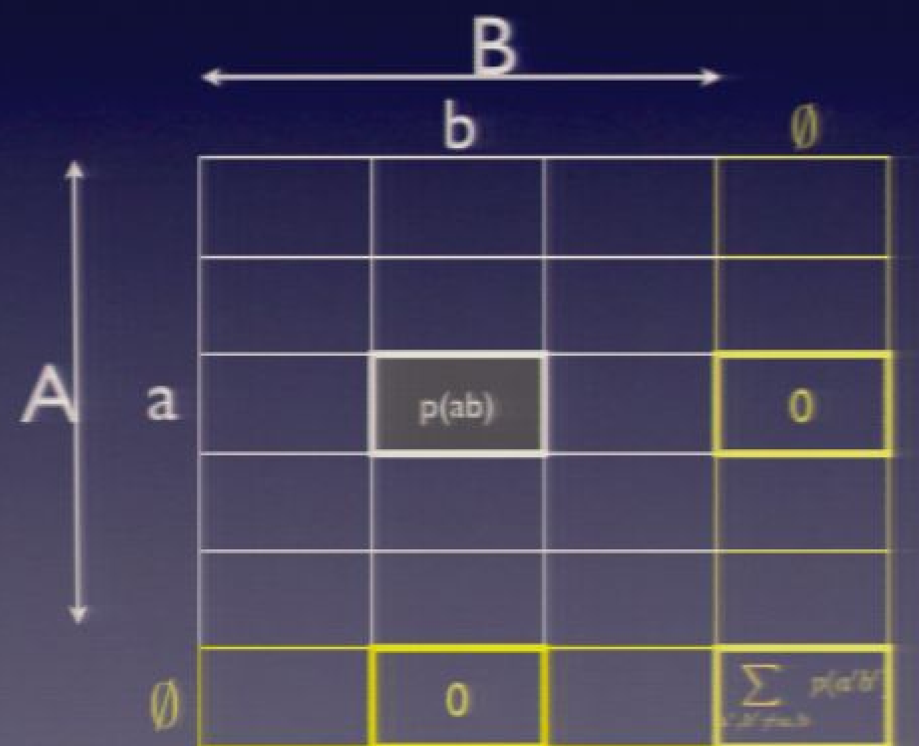


Better upper bound by Junge and Palazuelos (using Operator space theory)

Gap between ν and γ_2

- Proof idea : reduce to affine combination of boolean distributions: $p = \sum p_{ab} - (AB-1)p_{\emptyset}$



By a composition principle,

$\nu(p) \leq O(AB\gamma_2(p))$

Gap between ν and γ_2

- Deutch-Josza equality problem [BCT98]:
  - A, B given n-bit strings
  - Output a, b in [n] such that a=b if x=y, and a$\neq$b if d(x,y)=n/2
- Classical $\Omega(n)$, quantum O(1), $\gamma_2 = O(1)$
- Best classical lower bound with our method is at most O(log(AB)) = O(log(n))
- Example of distribution where rectangle bound is better than [LS07]

Upper bounds

simultaneous messages

$$R_\delta^{||,pub}(p) = O((AB)^{O(1)}\nu(p)^2)$$
$$R_\delta^{||,ent}(p) = O((AB)^{O(1)}\gamma_2(p)^2)$$

Some corollaries :

- Any quantum distribution can be approximated with constant communicaton [SZ08]
- $R_\delta^{||,pub}(f) = O(2^{2Q_\epsilon^*(f)})$ [SZ08]
- Using Newman+fingerprinting, $Q_\delta^{||}(f) = O(\log(n)2^{4Q_\epsilon^*(f)})$ [GKdW06]

Proof idea

$$\nu(p) = \Lambda$$

$$p = q^+ p^+ - q^- p^-$$

$$q^+ + q^- = \Lambda, p^+, p^- \in \mathcal{L}$$

Simultaneous protocol to approximate p:

- A and B send the referee "enough" samples of p^+, p^- (Chernov, McDiarmid inequality)
- Referee estimates p^+ and p^- and uses this to estimate p,
- Referee outputs according to this estimate

Conclusion

- Lower bounds on classical, quantum communication for any non-signaling ditribution (arbitrary I/O, including marginals)
- Interpretation by Bell, Tsirelson inequality violations
- New proof of Linial and Shraibman's factorization norm lower bound (implies rank, Fourier method, discrepancy, etc...)

Open Problems

- Quantitive approach for detector efficiency or other loopholes
- Improve the gap between the upper and lower bound (for specific classes of functions)

- What we have done:

  Bell Inequalities VS Communication
- What we really want:

  Bell Inequalities VS Information

Upper bounds

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