Title: From operational axioms to quantum theory - and beyond?

Date: May 09, 2011 02:50 PM

URL: http://pirsa.org/11050033

Abstract: Usually, quantum theory (QT) is introduced by giving a list of abstract mathematical postulates, including the Hilbert space formalism and the Born rule. Even though the result is mathematically sound and in perfect agreement with experiment, there remains the question why this formalism is a natural choice, and how QT could possibly be modified in a consistent way. My talk is on recent work with Lluis Masanes, where we show that five simple operational axioms actually determine the formalism of QT uniquely. This is based to a large extent on Lucien Hardy's seminal work. We start with the framework of "general probabilistic theories", a simple, minimal mathematical description for outcome probabilities of measurements. Then, we use group theory and convex geometry to show that the state space of a bit must be a 3D (Bloch) ball, finally recovering the Hilbert space formalism. There will also be some speculation on how to find natural post-quantum theories by dropping one of the axioms.

Outline

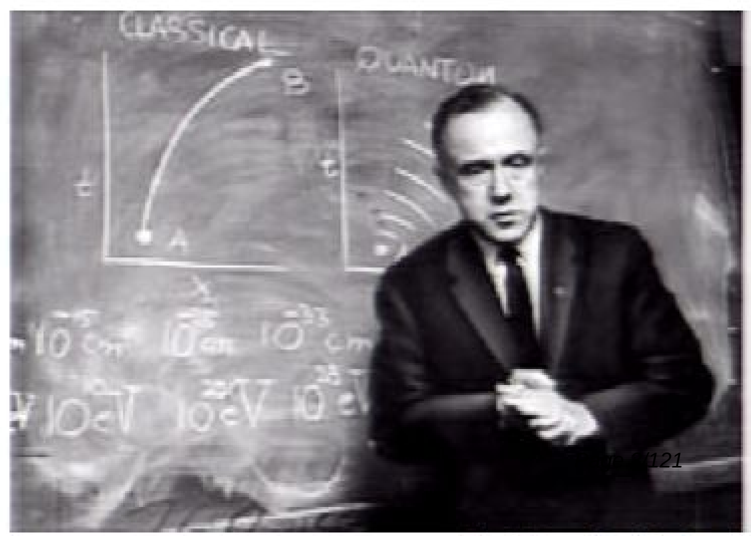
1. Motivation Why??
2. General Probabilistic Theories Physics?
3. The Axioms What do they mean?
4. Derivation of the Hilbert space formalism Why are qubits 3D-balls??
5. What's beyond QT?

1. Motivation

John A. Wheeler, New York Times, Dec. 12 2000:

„Quantum physics [...] has explained the structure of atoms and molecules, [...] the behavior of semiconductors [...] and the comings and goings of particles from neutrinos to quarks.

Successful, yes, but mysterious, too. Why does the quantum exist?“



I. Motivation

ANNALS OF PHYSICS 194, 336–386 (1989)

Testing Quantum Mechanics

STEVEN WEINBERG*

Theory Group, Department of Physics, University of Texas, Austin, Texas 78712

Received March 6, 1989

This paper presents a general framework for introducing nonlinear corrections into ordinary quantum mechanics, that can serve as a guide to experiments that would be sensitive to such corrections. In the class of generalized theories described here, the equations that determine the time-dependence of the wave function are no longer linear, but are of Hamiltonian type. Also, wave functions that differ by a constant factor represent the same physical state and satisfy the same time-dependence equations. As a result, there is no difficulty in combining separated subsystems. Prescriptions are given for determining the states in which observables have definite values and for calculating the expectation values of observables for general states, but the calculation of probabilities requires detailed analysis

I. Motivation

ANNALS OF PHYSICS 194, 336–386 (1989)

Volume 143, number 1,2 PHYSICS LETTERS A 1 January 1990

WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

N. GISIN
Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

Received 16 October 1989; accepted for publication 3 November 1989
Communicated by J.P. Vigier

We show with an example that Weinberg's general framework for introducing non-linear corrections into quantum mechanics allows for arbitrarily fast communications.

Recently Weinberg has proposed a general framework for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

to know what such an apparatus is... do you know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions z and u are in the xz-plane orthogonal to the incoming flow of particles, and are 45° from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of

I. Motivation

ANNALS OF PHYSICS 194, 336–386 (1989)

Volume 143, number 1,2 PHYSICS LETTERS A 1 January 1990

WEINBERG'S NON-LINEAR QUANTUM MECHANICS AND SUPRALUMINAL COMMUNICATIONS

N. GISIN
Group on Applied Physics, University of Geneva, 1211 Geneva 4, Switzerland

Received 16 October 1989; accepted for publication 3 November 1989
Communicated by J.P. Vigier

We show with an example that Weinberg's general framework for introducing non-linear corrections into quantum mechanics allows for arbitrarily fast communications.

Recently Weinberg has proposed a general framework for introducing non-linear corrections into ordinary quantum mechanics [1,2]. Although we fully support his emphasis on the importance of testing quantum mechanics, we would like in this Letter to draw attention to the difficulty of modifying quantum mechanics without introducing arbitrarily fast actions at a distance. Below we show how to construct, within Weinberg's framework, an arbitrarily fast telephone line. In ordinary quantum mechanics

to know what such an apparatus is... do you know what is inside your phone?) In order to simplify we consider only a single-bit message. The two directions z and u are in the xz-plane orthogonal to the incoming flow of particles, and are 45° from each other. The way the inhomogeneous magnetic field acts on the particles is well-known from experimental evidence. After the apparatus there are two counters. For each particle one of the counters will click. This click will be amplified until all readers of

It is difficult to modify quantum theory

Our results:

- A derivation of the full quantum formalism from operational / physical axioms.
- Methods to construct natural consistent modifications of quantum theory.

Our results:

• A derivation of the full quantum formalism from operational / physical axioms.

• Methods to construct natural consistent modifications of quantum theory.

Builds on:

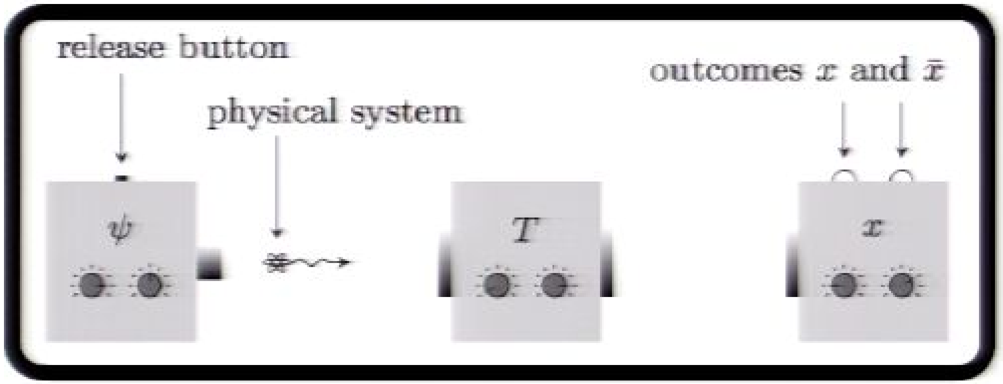
• L. Hardy, *Quantum Theory From Five Reasonable Axioms*, 2001

• B. Dakić and Č. Brukner, *Quantum Theory and Beyond: Is Entanglement Special?*, 2009

See also:

• G. Chiribella et al., *Informational derivation of Q.T.*, 2010

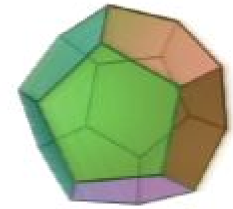
Basic physical / operational assumptions

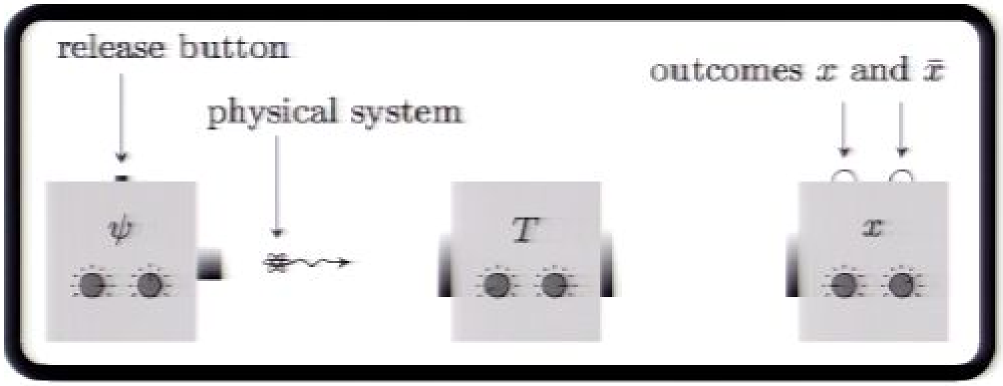


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

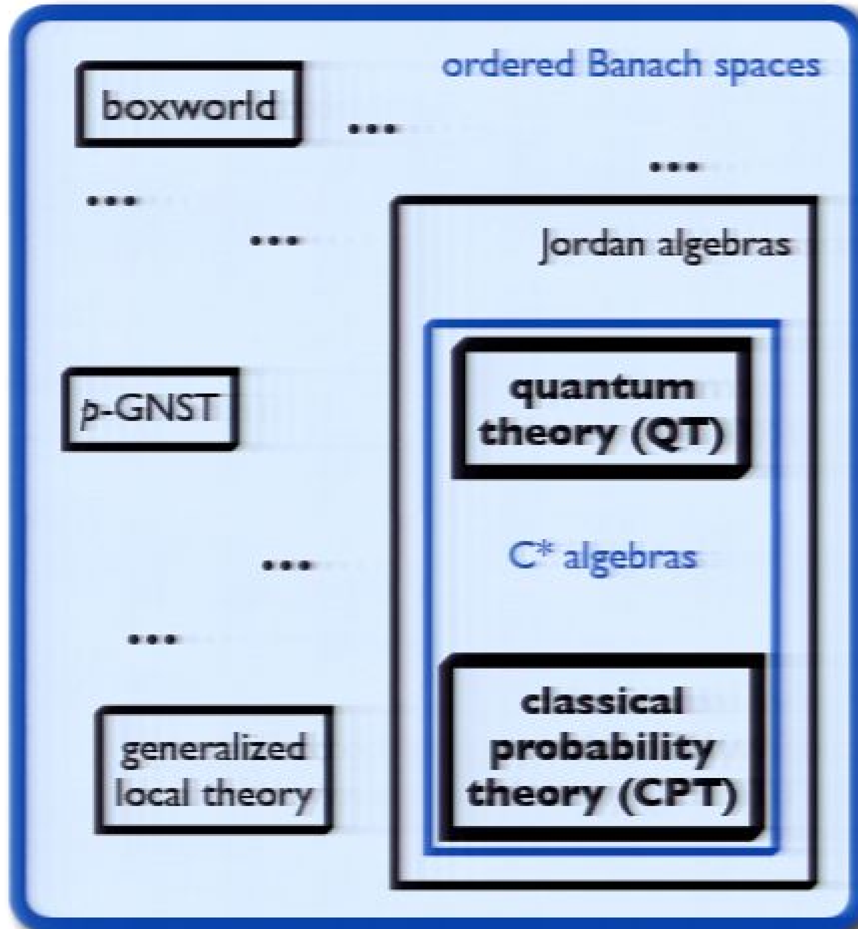
Basic physical / operational assumptions

General probabilistic theories



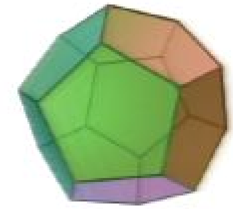


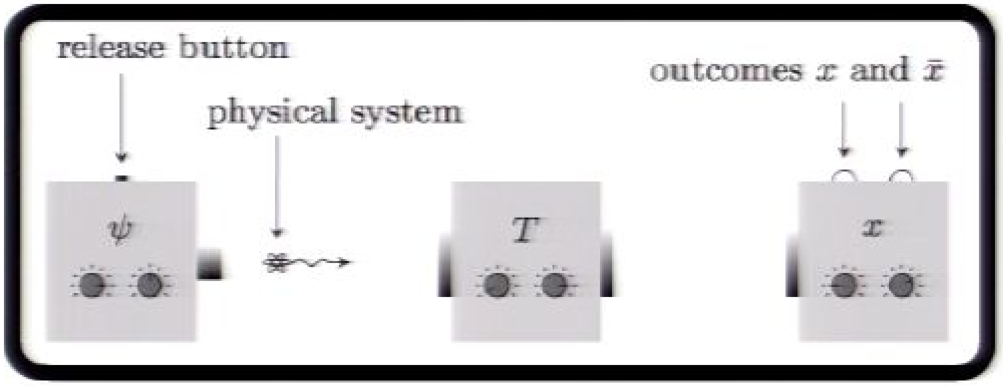
- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories

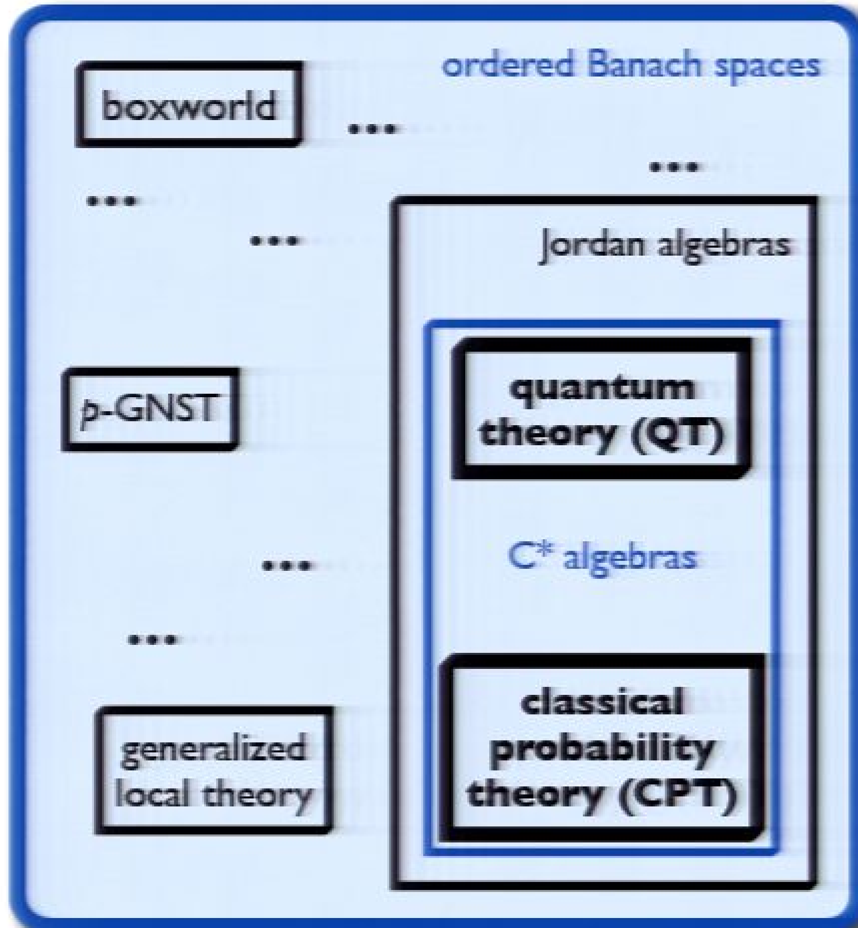




- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

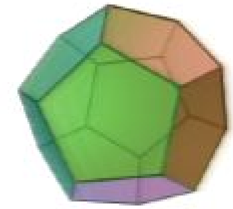
The Axioms:

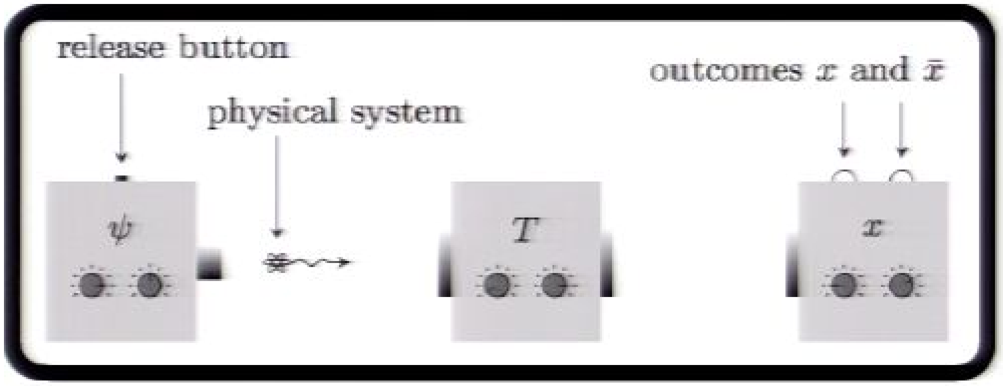
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed



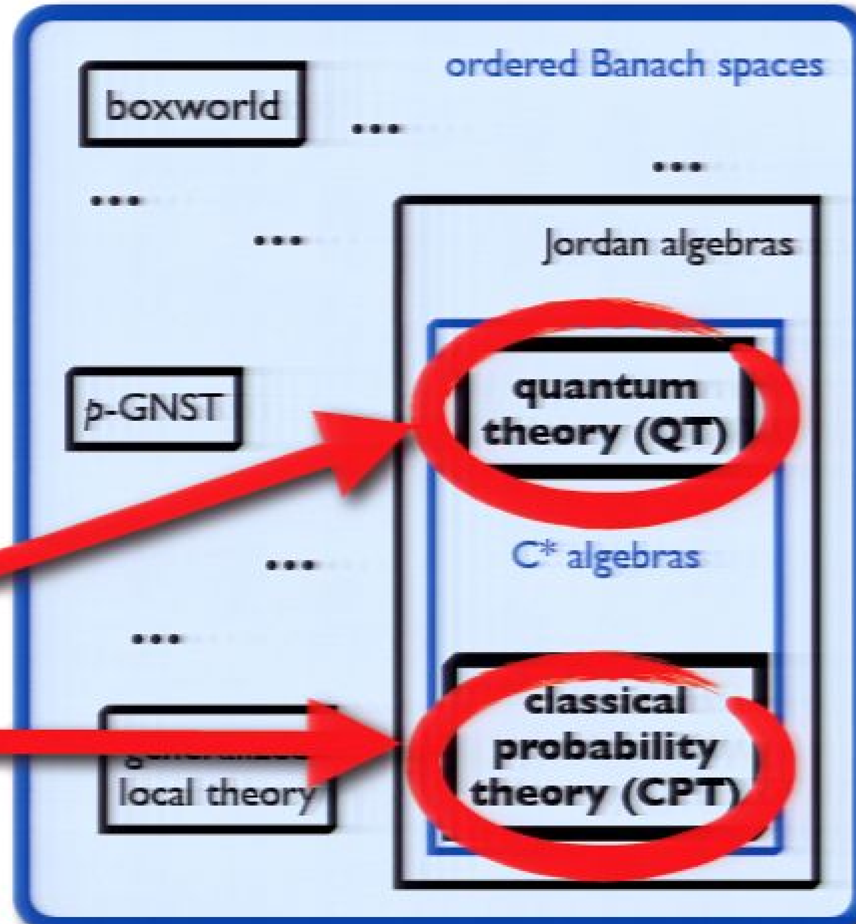
- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories





- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.



The Axioms:

I. Local tomography

II. Reversibility

III. Subspace axiom

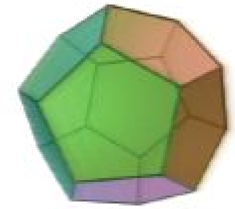
IV. Finite-dimensionality

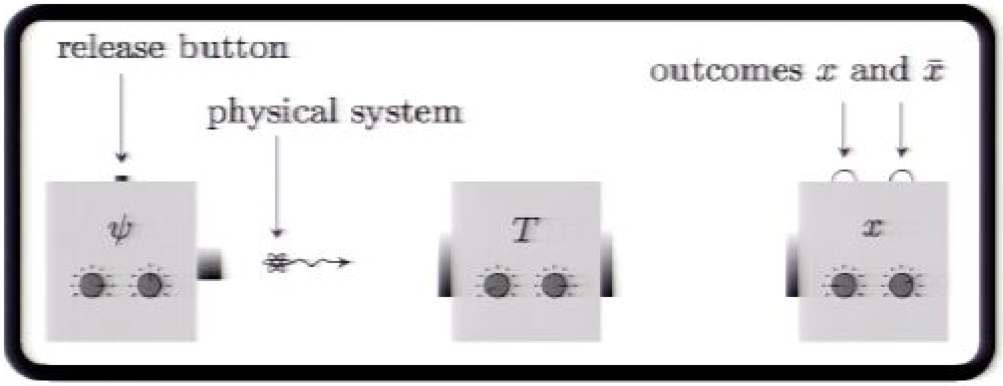
V. All measurements allowed

Determine CPT+QT **uniquely!**

- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories

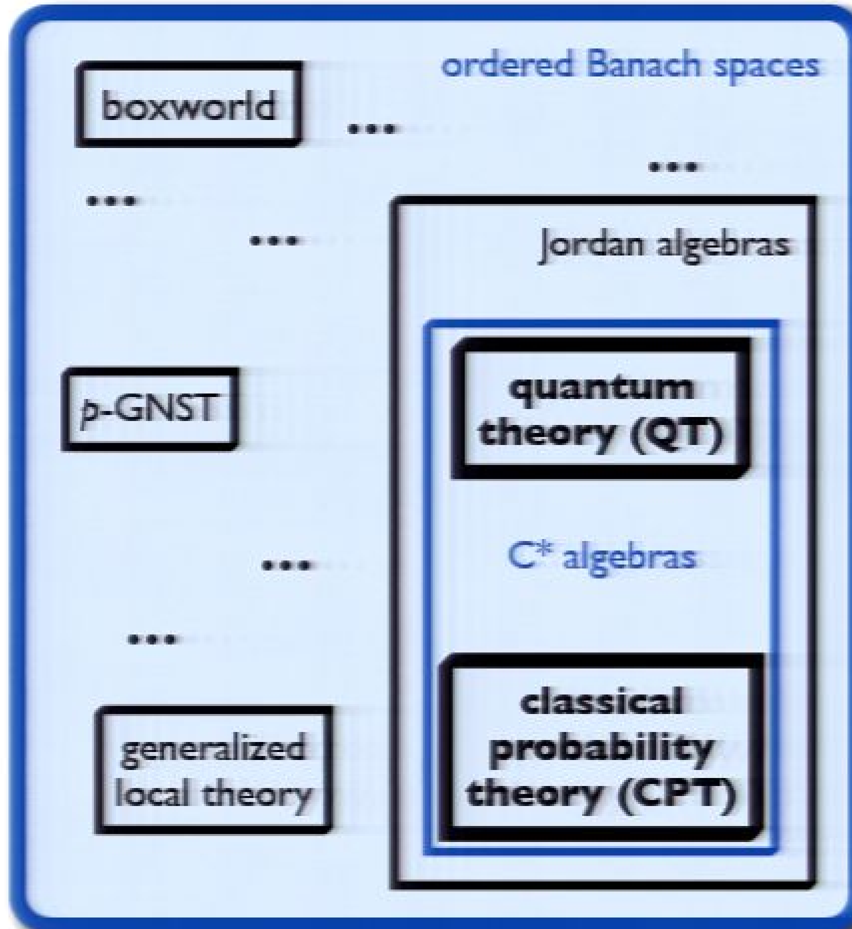




- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

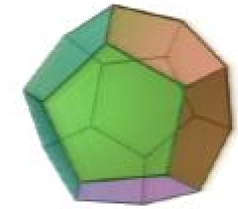
The Axioms:

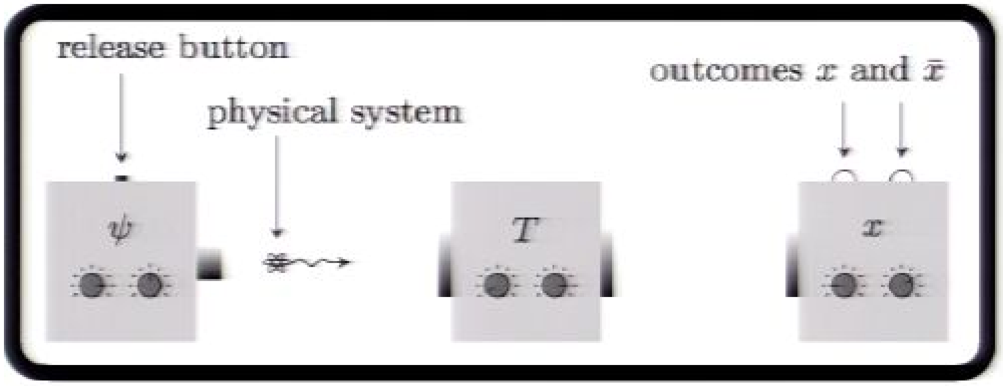
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories



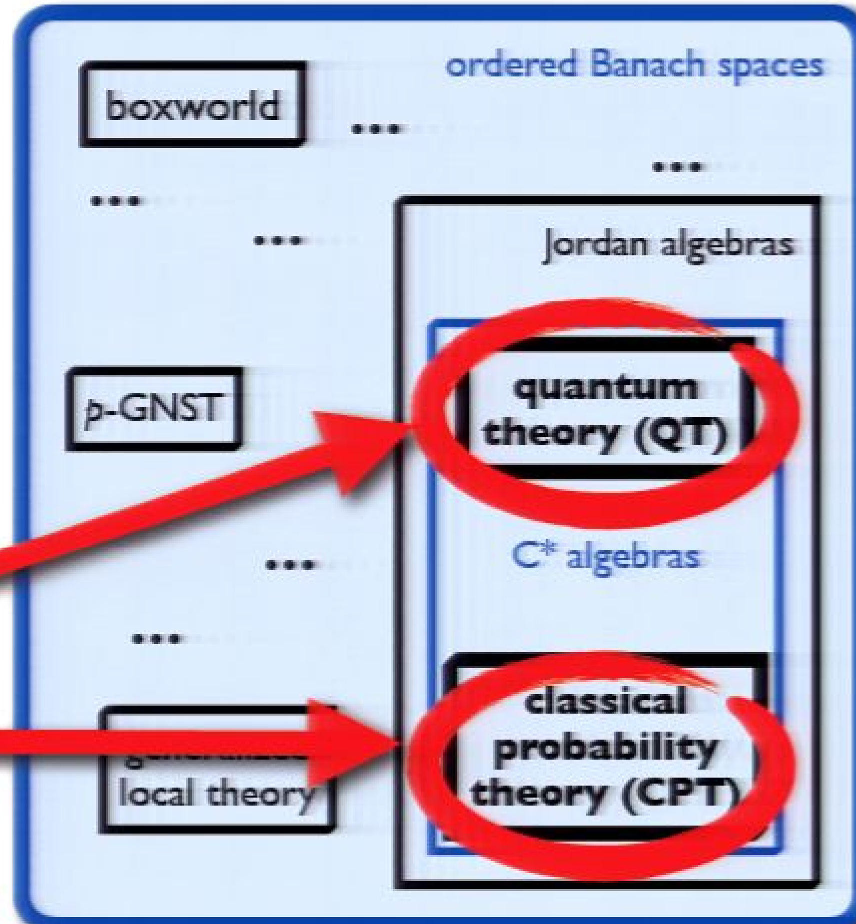


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

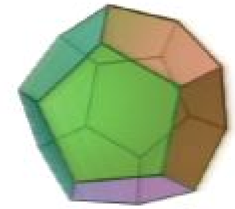
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

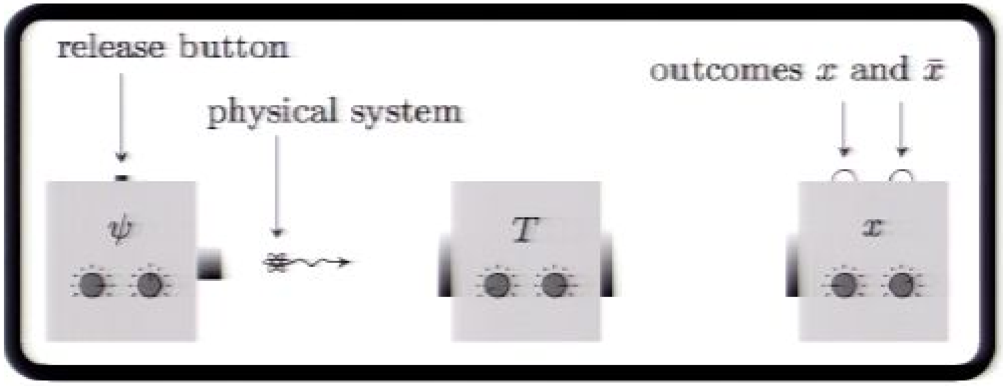
Determine CPT+QT **uniquely!**



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories



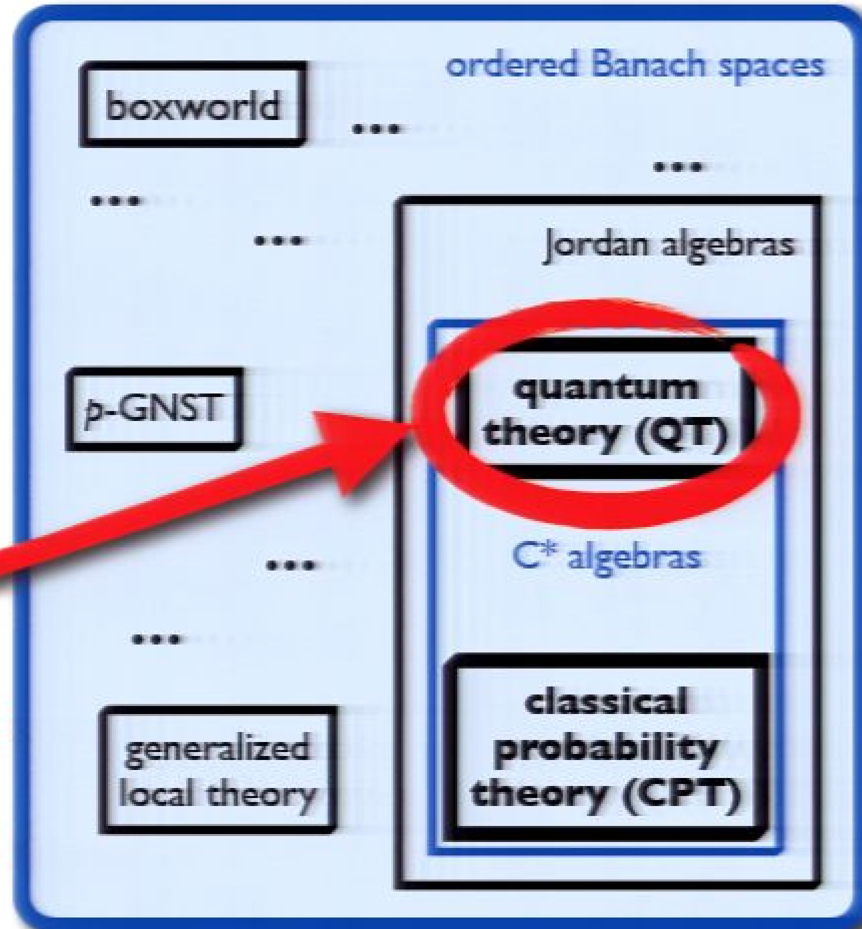


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

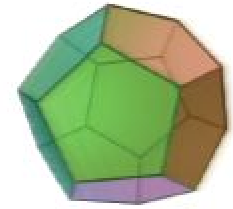
I. Local tomography
II. Continuous Reversib.
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

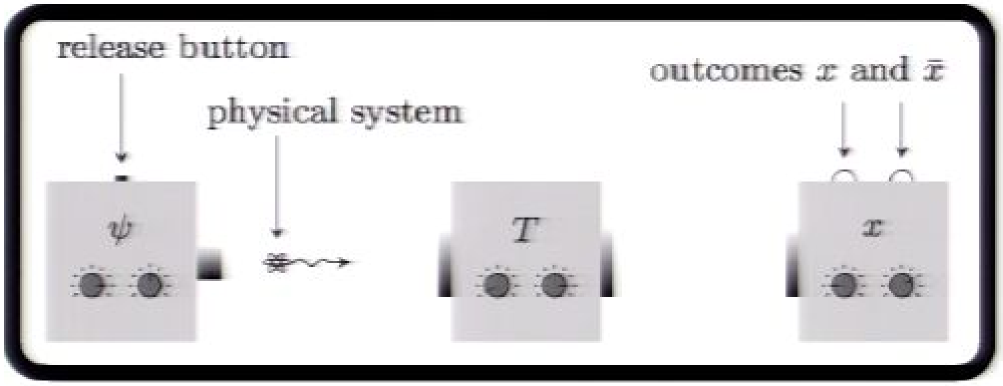
Determines QT **uniquely!**



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories



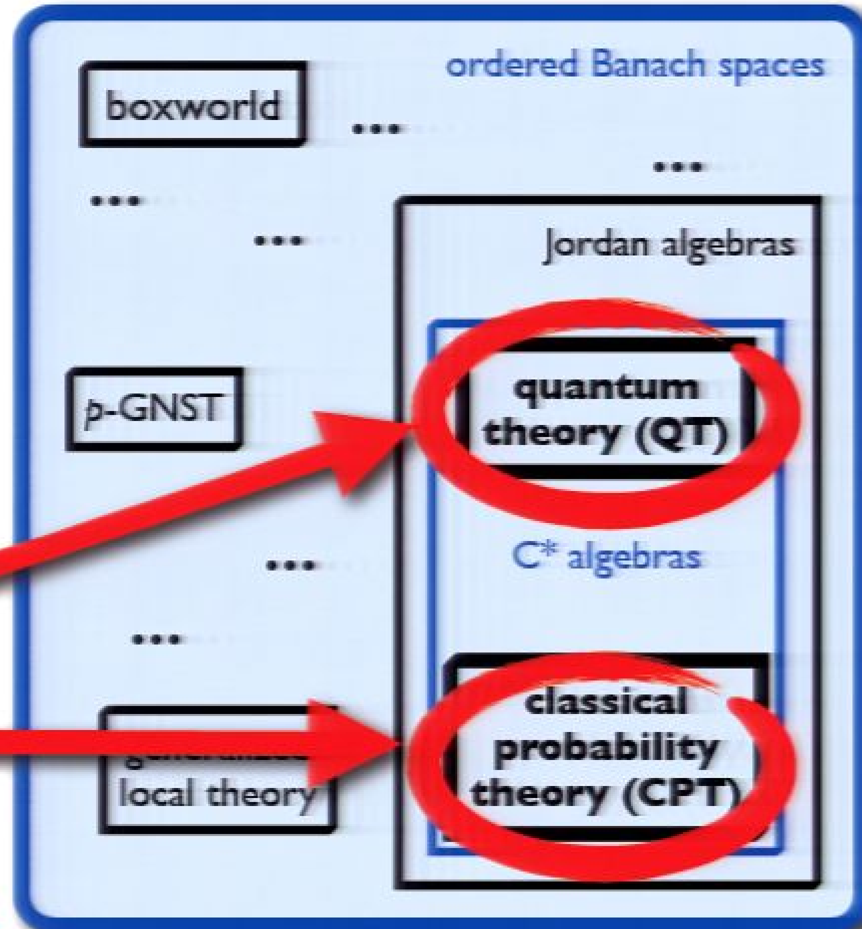


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

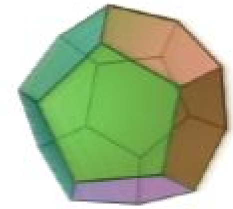
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

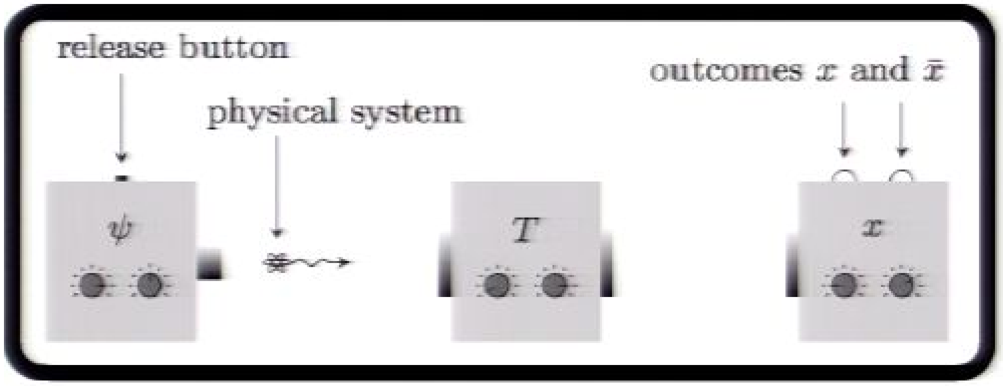
Determine CPT+QT **uniquely!**



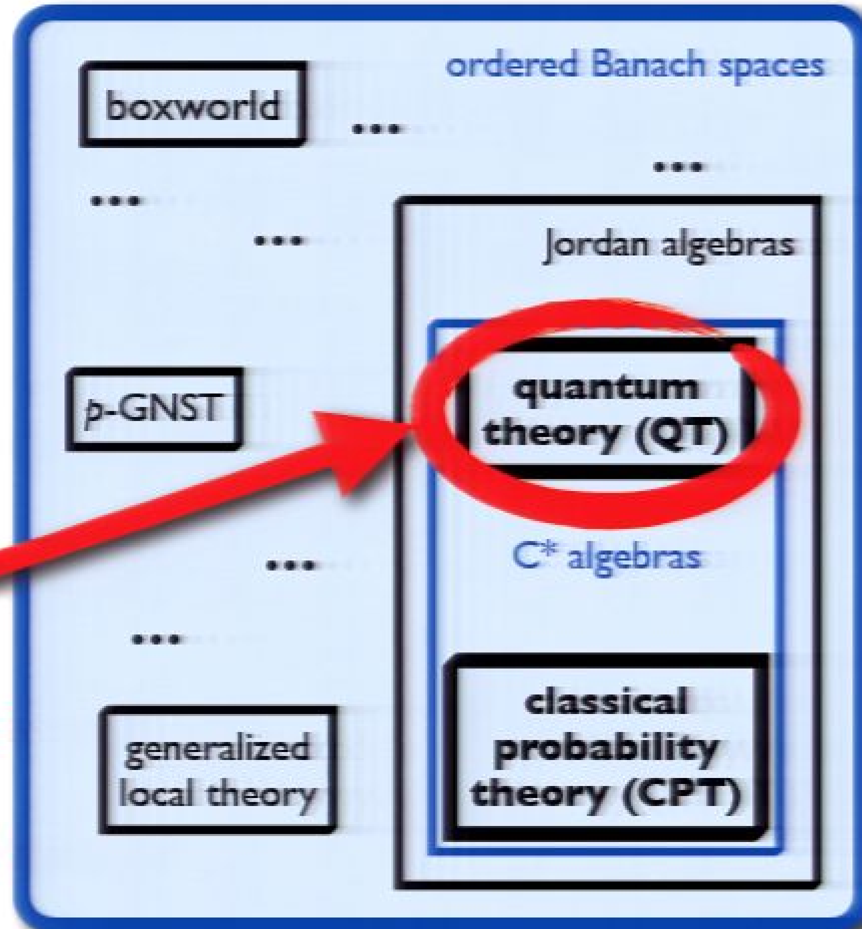
- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories





- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.



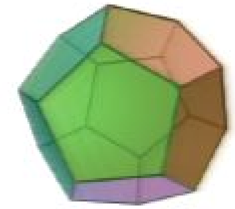
The Axioms:

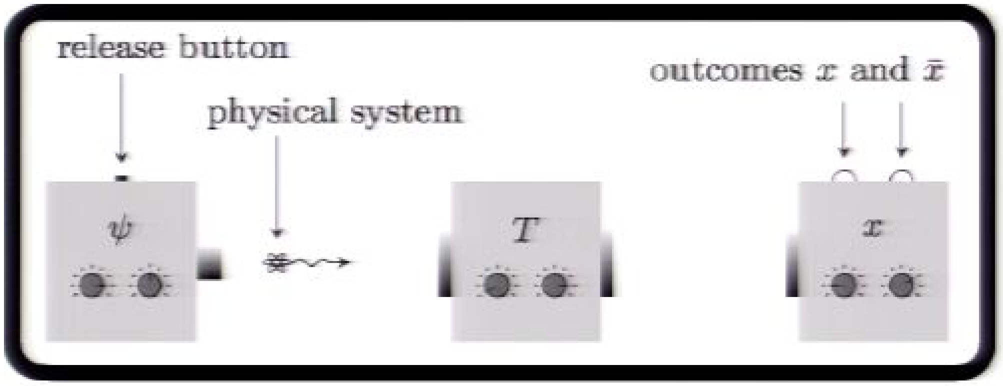
I. Local tomography
II. *Continuous* Reversib.
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

Determines QT **uniquely!**

- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories



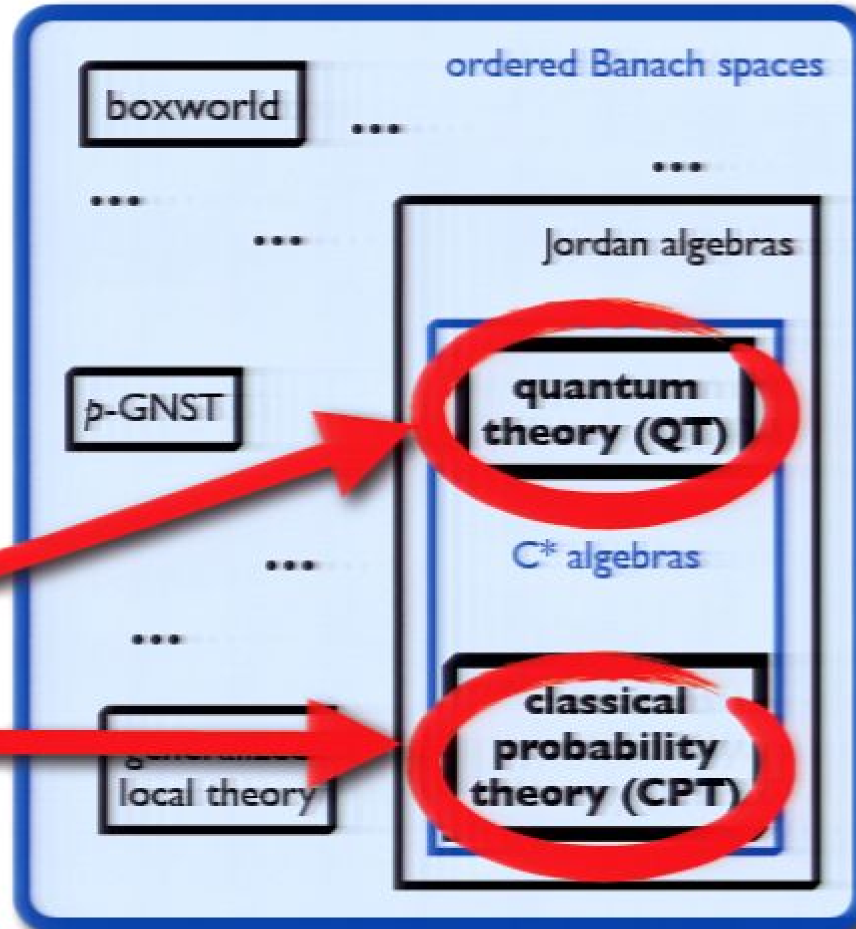


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

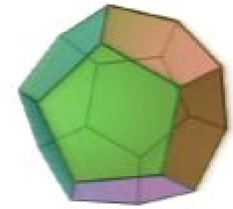
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

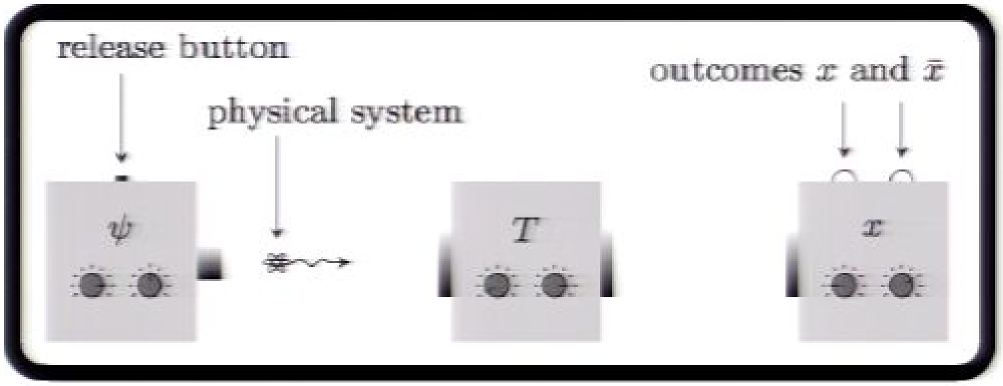
Determine CPT+QT **uniquely!**



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories





- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

I. Local tomography

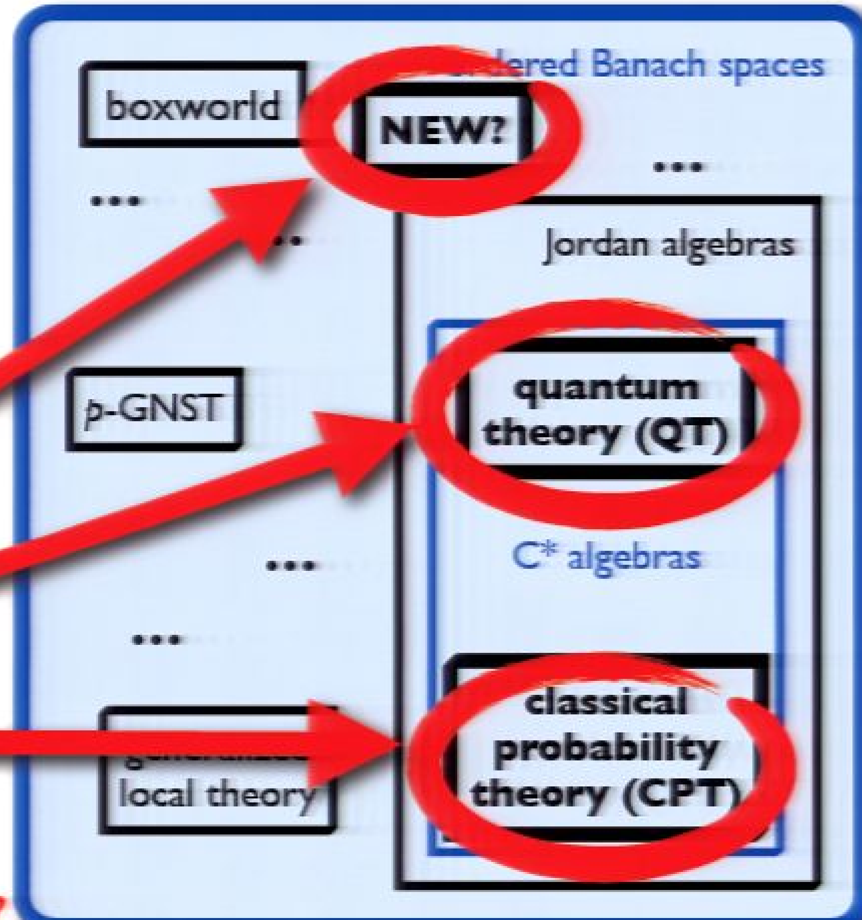
II. Reversibility

~~**III. Subspace axiom**~~

IV. Finite-dimensionality

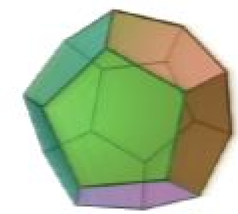
V. All measurements allowed

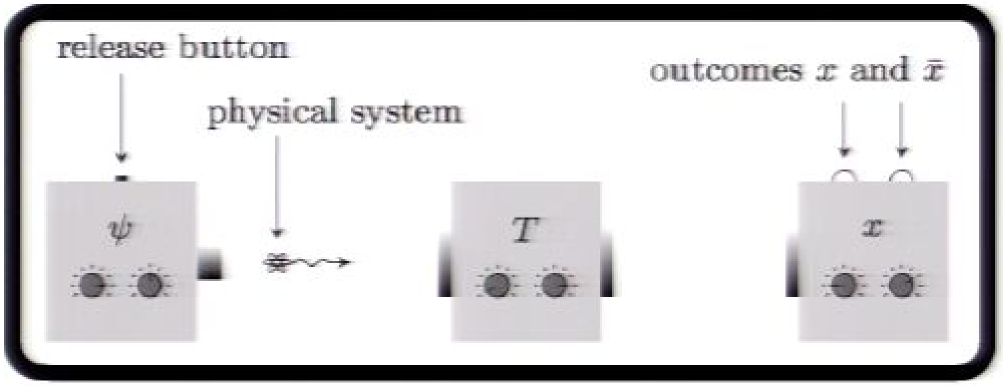
Natural theories **beyond QT?**



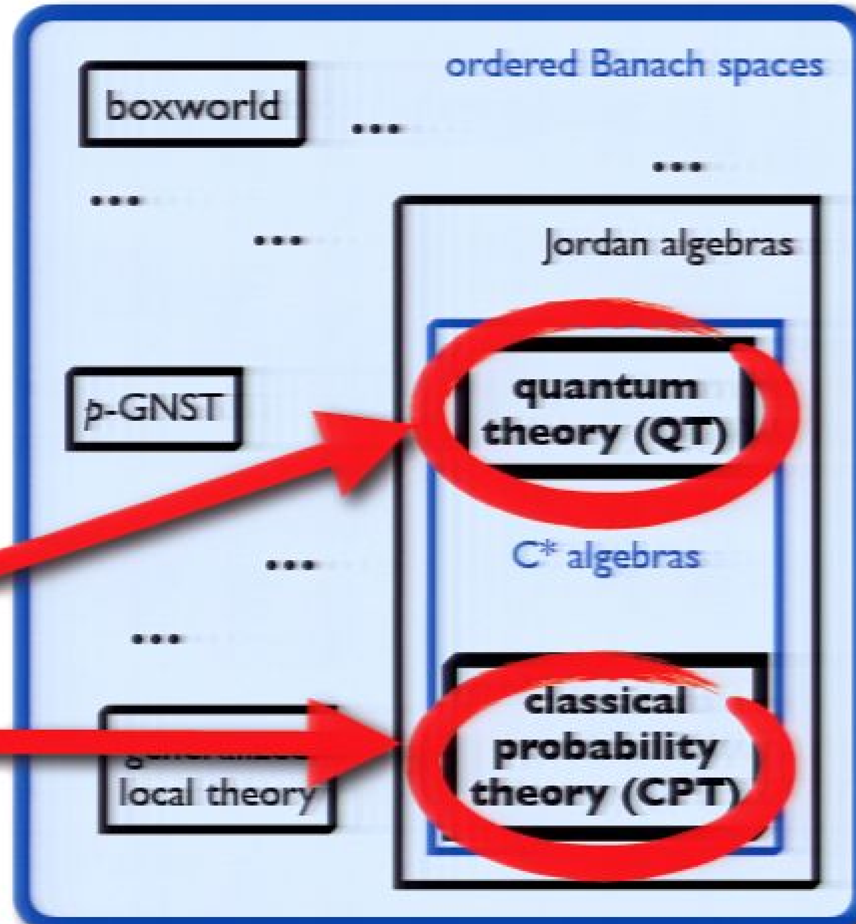
- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories





- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.



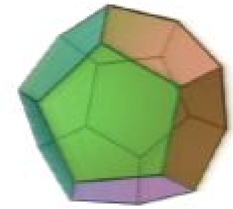
The Axioms:

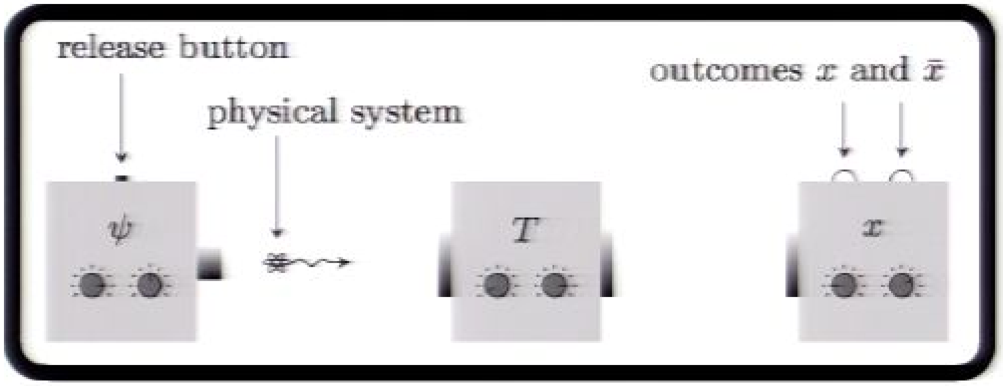
I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed

Determine CPT+QT **uniquely!**

- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories



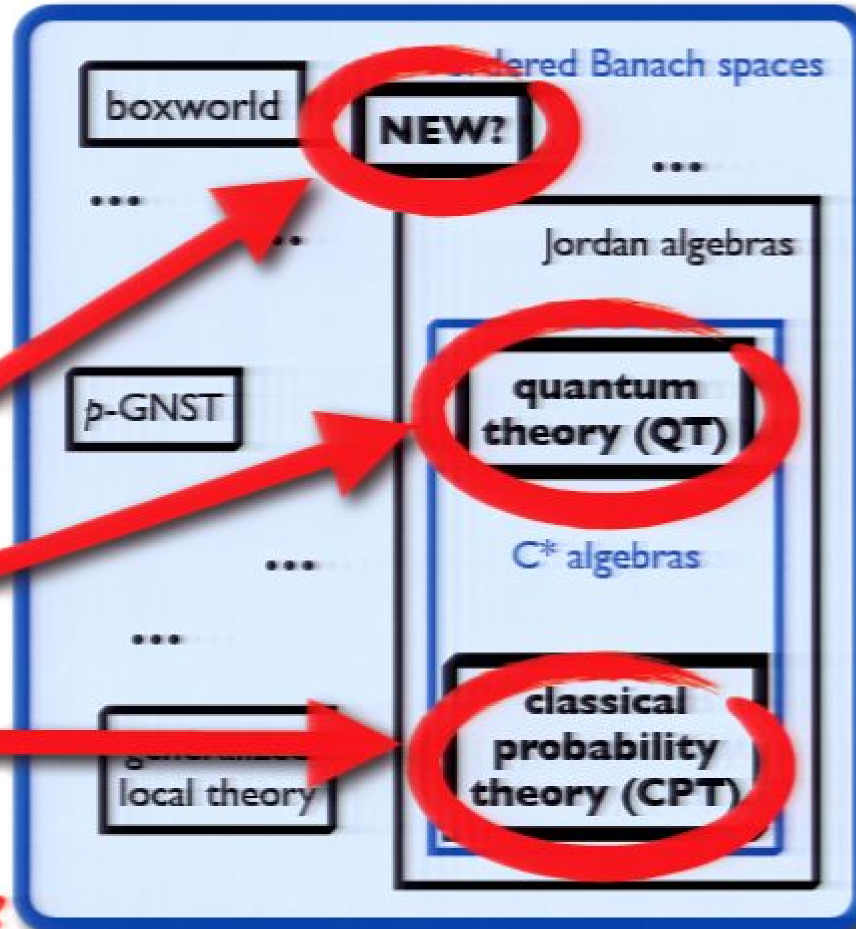


- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

I. Local tomography
II. Reversibility
~~**III. Subspace axiom**~~
IV. Finite-dimensionality
V. All measurements allowed

Natural theories **beyond QT?**



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

What our results are not:

- They offer no resolution of the measurement problem.
- No new interpretation of quantum theory.
- We assume that probabilities exist.
- Only finite-dimensional QT so far.
- Only abstract QT, no mechanics / field theory.

What our results are not:

- They offer no resolution of the measurement problem.
- No new interpretation of quantum theory.
- We assume that probabilities exist.
- Only finite-dimensional QT so far.
- Only abstract QT, no mechanics / field theory.

Statistical Mechanics= Mechanics (Hamiltonian, space, time,...) + Probability Theory

Quantum Mechanics= Mechanics (Hamiltonian, space, time,...) + Abstract Quantum Theory

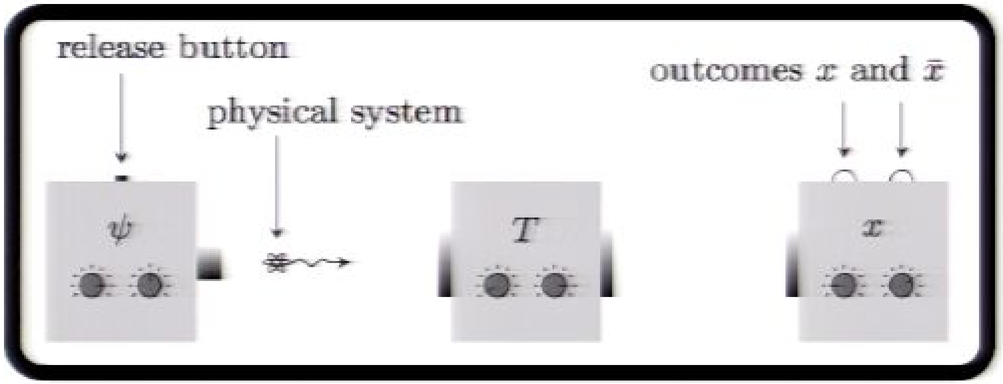
What our results are not:

- They offer no resolution of the measurement problem.
- No new interpretation of quantum theory.
- We assume that probabilities exist.
- Only finite-dimensional QT so far.
- Only abstract QT, no mechanics / field theory.

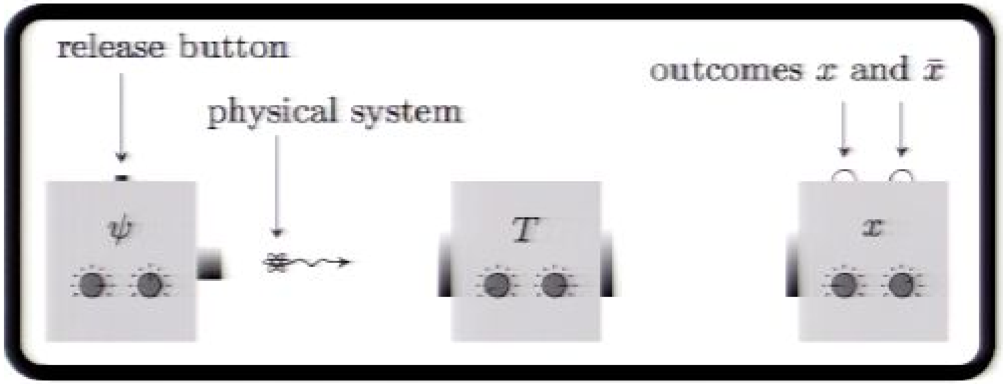
Statistical Mechanics= Mechanics (Hamiltonian, space, time,...) + Probability Theory

Quantum Mechanics= Mechanics (Hamiltonian, space, time,...) + Abstract Quantum Theory

2. The Physical Setup



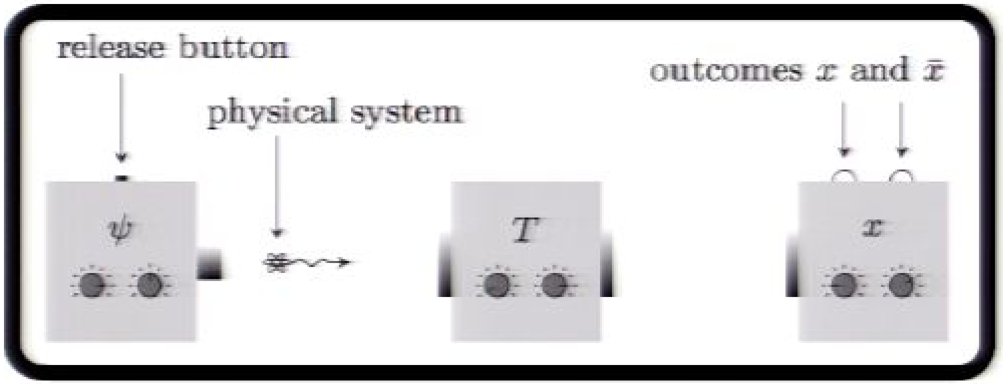
2. The Physical Setup



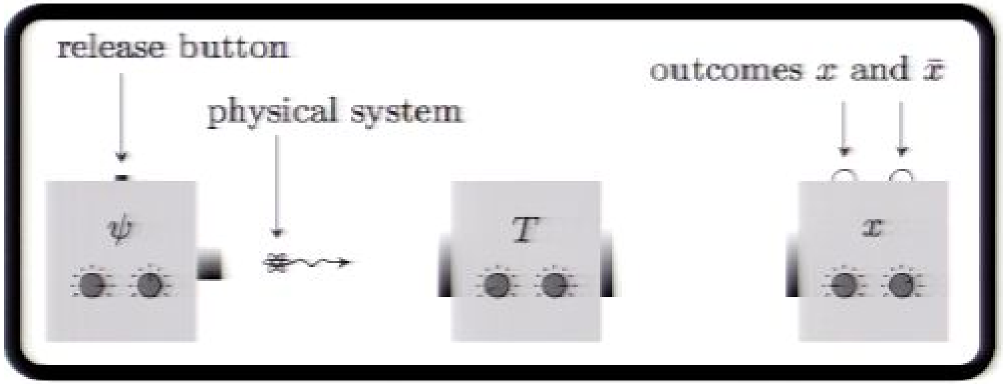
(Unnormalized) state ω =
list of all probabilities of „yes"-outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

2. The Physical Setup



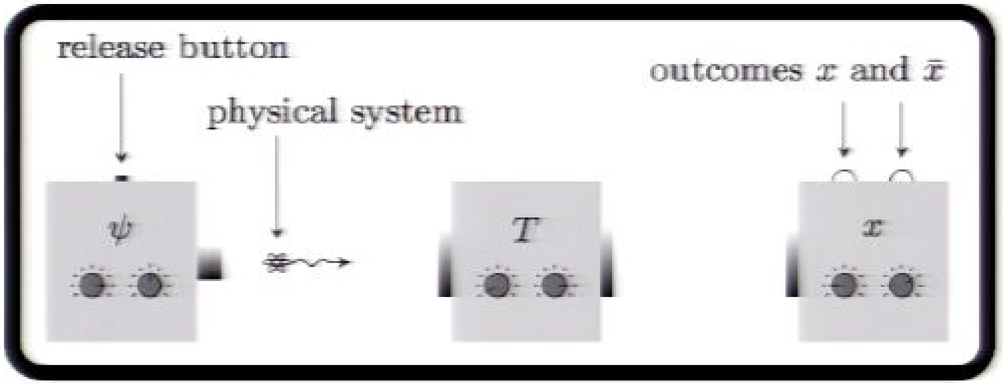
2. The Physical Setup



(Unnormalized) state ω =
list of all probabilities of „yes“-
outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

2. The Physical Setup



(Unnormalized) state ω =
list of all probabilities of „yes"-outcomes of all possible measurements.

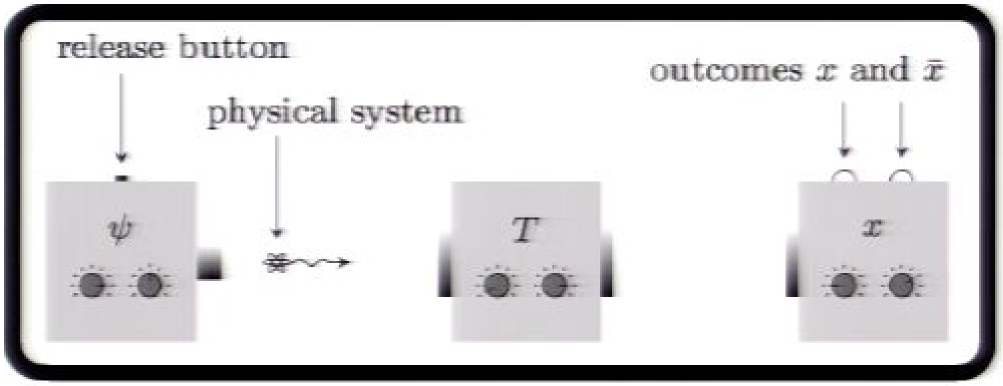
$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

Sometimes, all ω span a finite-dimensional subspace. Ex.: Qubit

- What's the prob. of „spin up" in X-direction?
- What's the prob. of „spin up" in Y-direction?
- What's the prob. of „spin up" in Z-direction?
- Is the particle there at all?

$$\omega = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4$$

2. The Physical Setup



(Unnormalized) state ω = list of all probabilities of „yes“-outcomes of all possible measurements.

$$\omega = (p_1, p_2, p_3, p_4, p_5, p_6, \ldots)$$

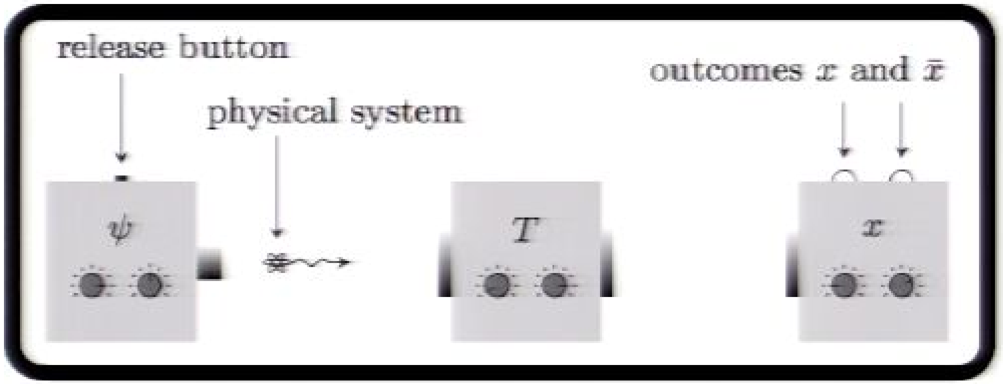
Sometimes, all ω span a finite-dimensional subspace. Ex.: Qubit

- What‘s the prob. of „spin up“ in X-direction?
- What‘s the prob. of „spin up“ in Y-direction?
- What‘s the prob. of „spin up“ in Z-direction?
- Is the particle there at all?

$$\omega = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4$$

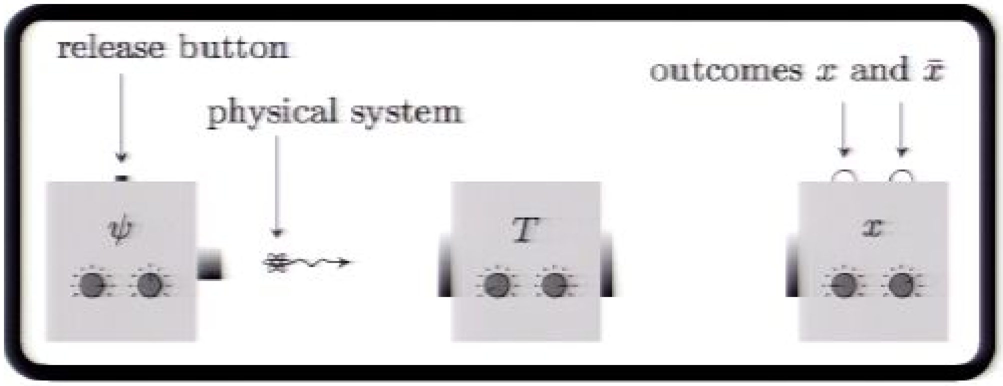
Axiom IV: All state spaces are finite-dimensional.

2. The Physical Setup



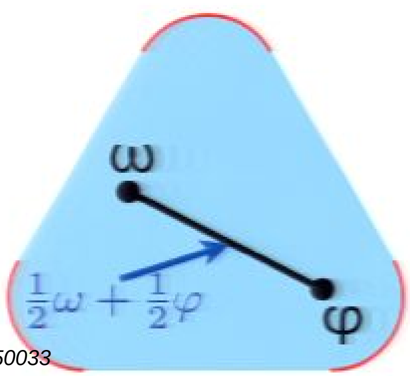
Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

2. The Physical Setup

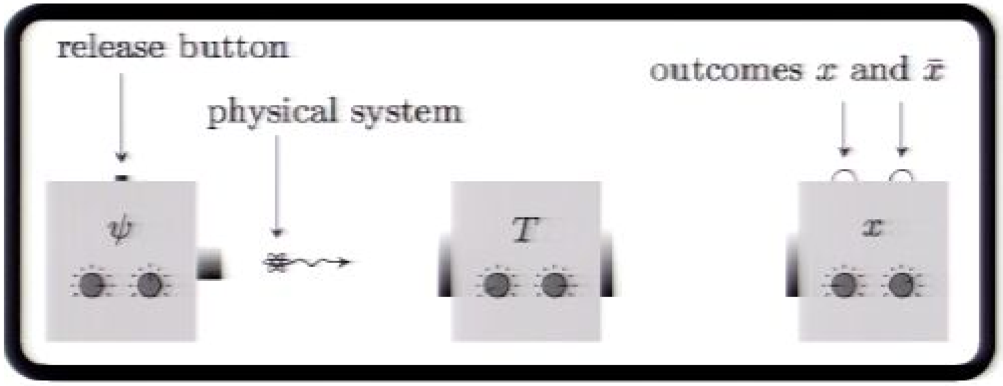


Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

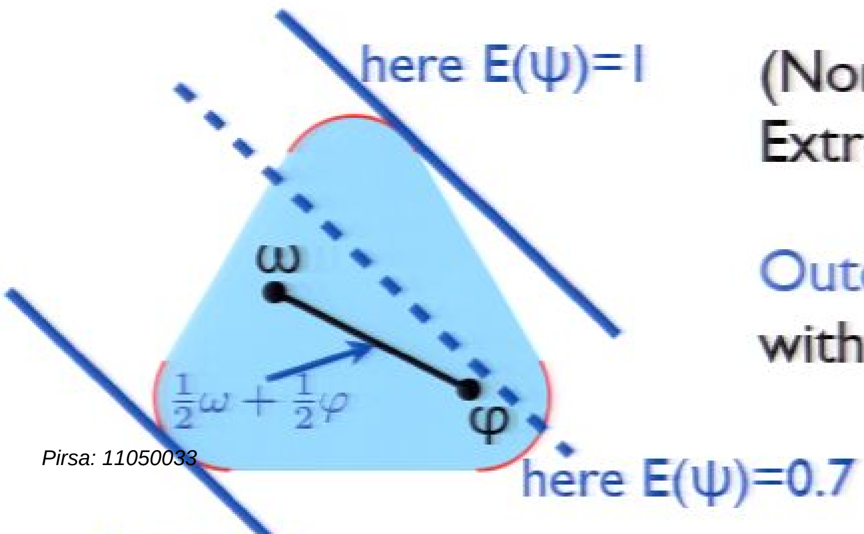
(Normalized) state spaces are convex sets.
Extremal points are pure states, others mixed.



2. The Physical Setup



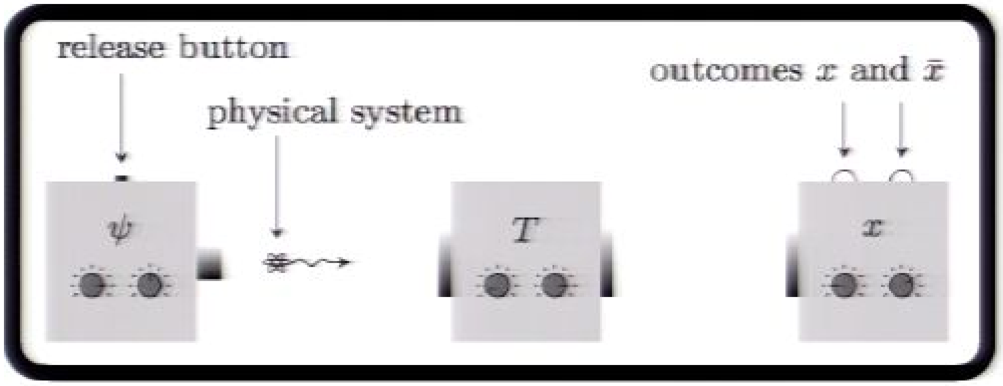
Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$



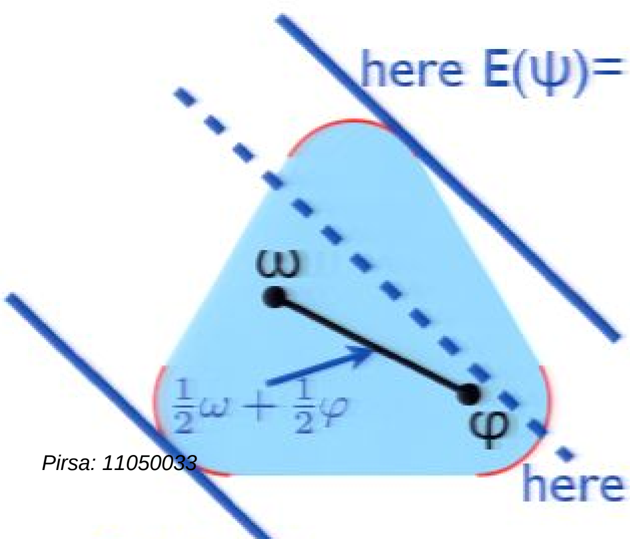
(Normalized) state spaces are convex sets. Extremal points are pure states, others mixed.

Outcome probabilities are linear functionals E with $0 \leq E(\psi) \leq 1$ for all ψ.

2. The Physical Setup



Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

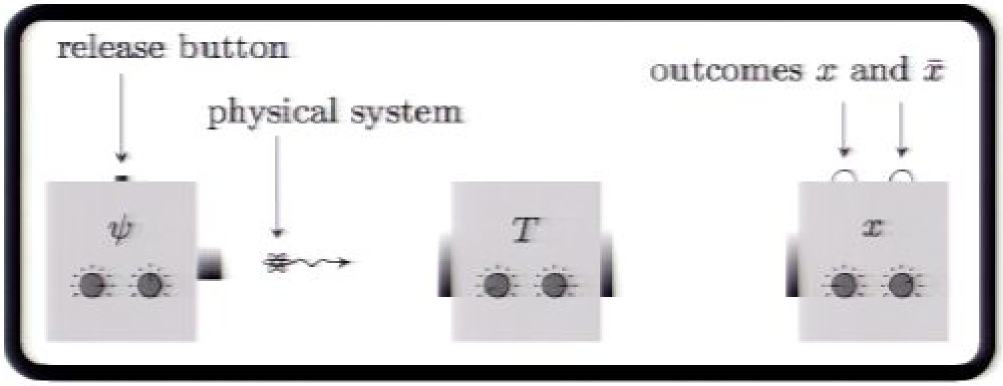


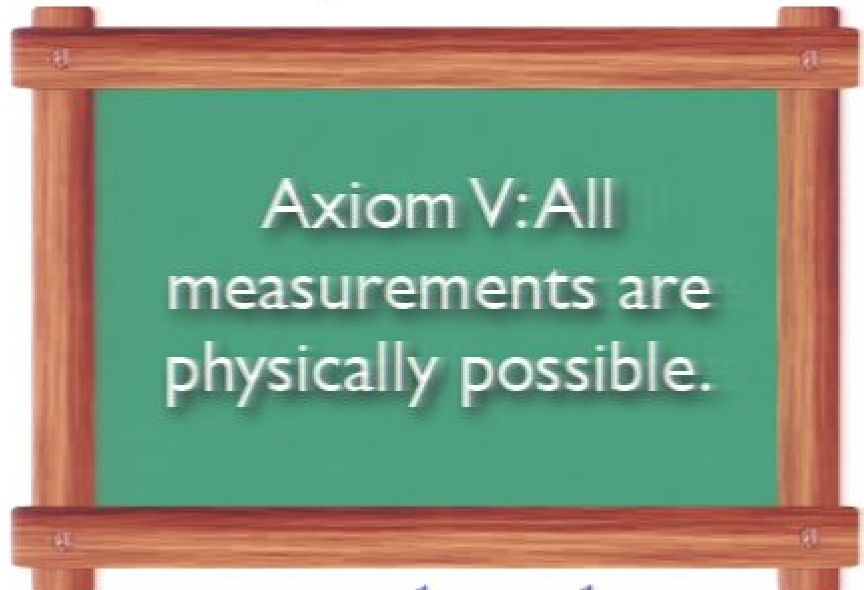
(Normalized) state spaces are convex sets. Extremal points are pure states, others mixed.

Outcome probabilities are linear functionals *E* with $0 \leq E(\psi) \leq 1$ for all ψ.

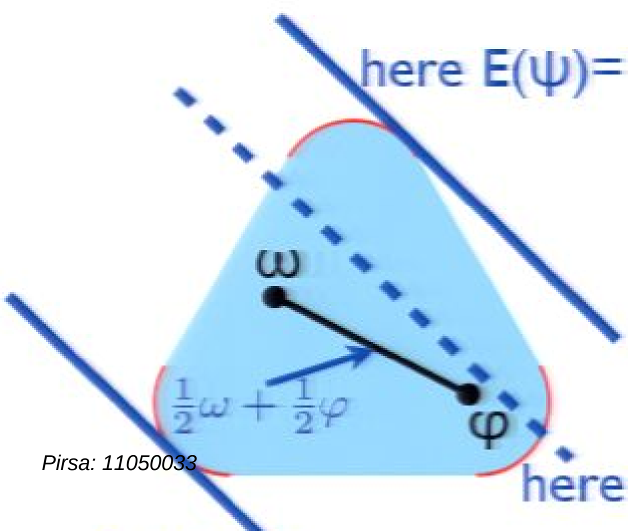
Measurements are $(E_1, E_2, \ldots, E_k)$

2. The Physical Setup





Prepare state ω or φ with prob. ½. Result: $\frac{1}{2}\omega + \frac{1}{2}\varphi$

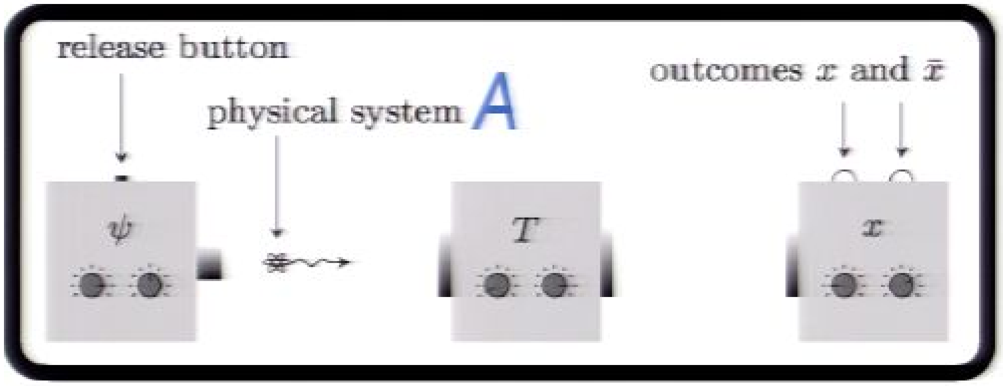


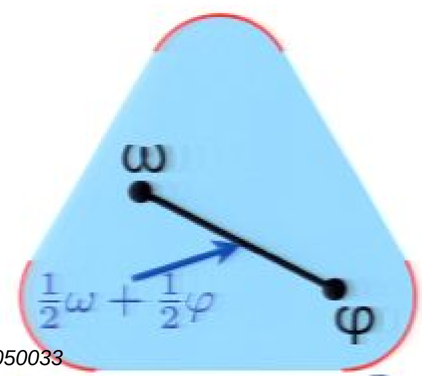
(Normalized) state spaces are convex sets. Extremal points are pure states, others mixed.

Outcome probabilities are linear functionals *E* with $0 \leq E(\psi) \leq 1$ for all ψ.

Measurements are $(E_1, E_2, \ldots, E_k)$

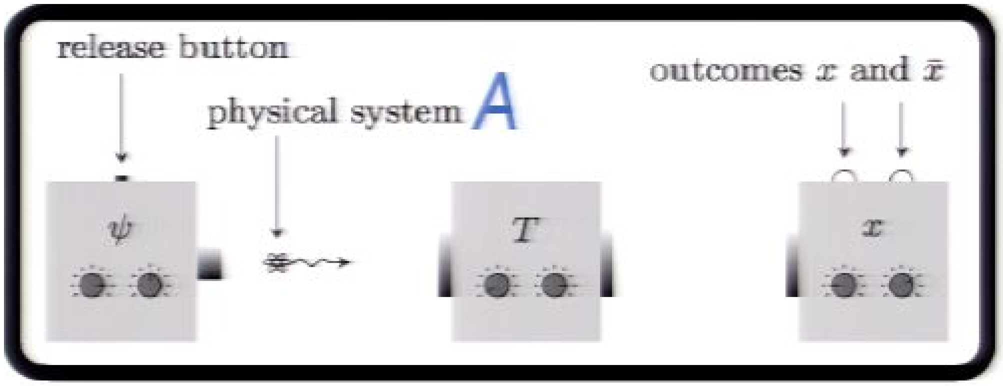
2. The Physical Setup



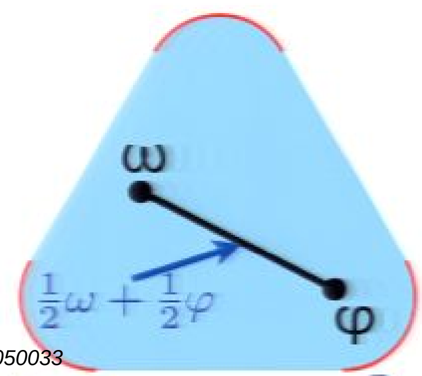


Normalized state space Ω_A

2. The Physical Setup

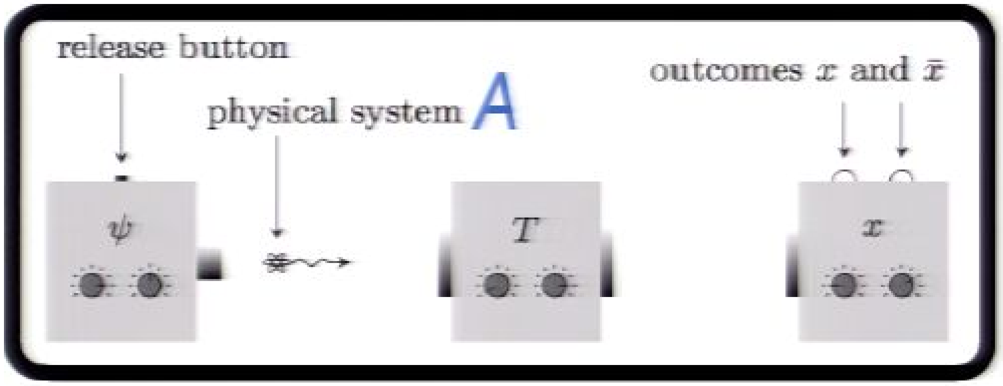


Transformations T map (unnormalized) states to states, and are linear.



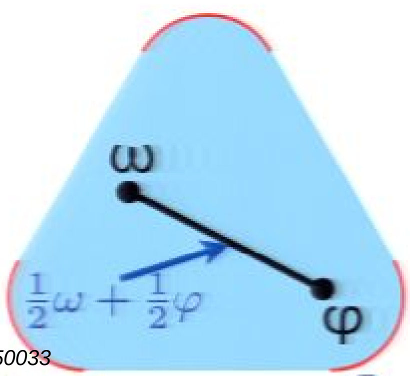
Normalized state space Ω_A

2. The Physical Setup



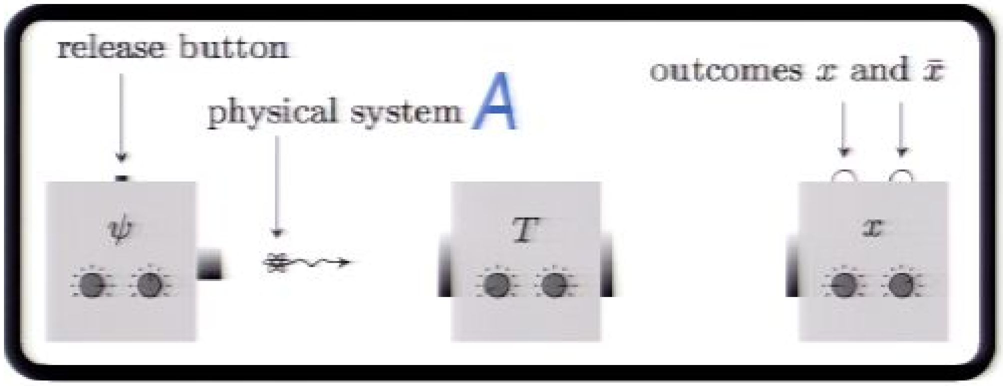
Transformations T map (unnormalized) states to states, and are linear.

Reversible transformations form a group $\mathcal{G}_A$. In quantum theory: $\rho \mapsto U\rho U^\dagger$
They are symmetries of state space: $T(\Omega_A) = \Omega_A$



Normalized state space Ω_A

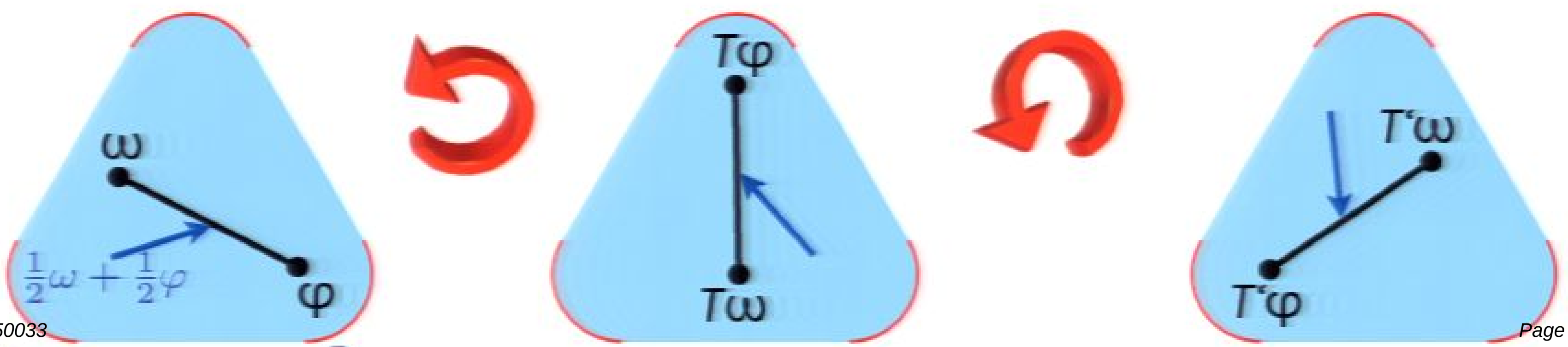
2. The Physical Setup



Transformations T map (unnormalized) states to states, and are linear.

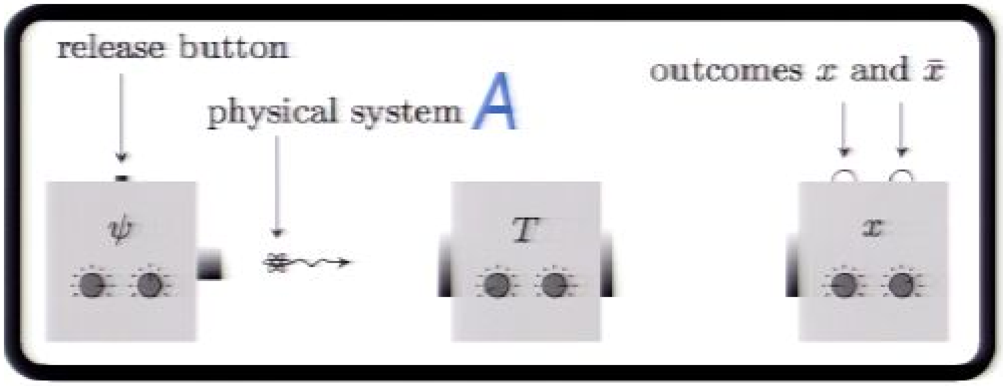
Reversible transformations form a group $\mathcal{G}_A$. In quantum theory: $\rho \mapsto U\rho U^\dagger$

They are symmetries of state space: $T(\Omega_A) = \Omega_A$

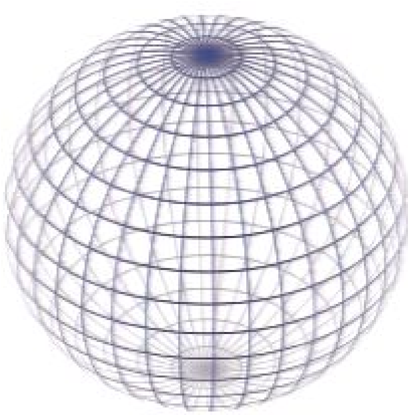


Normalized state space Ω_A

2. The Physical Setup



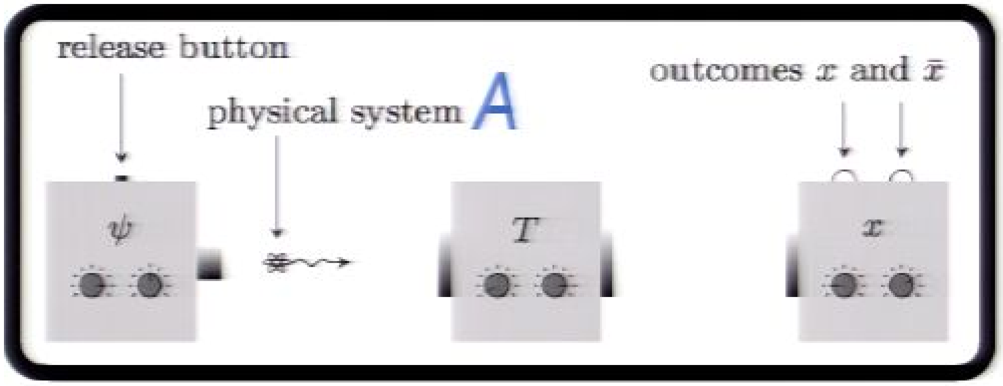
Not all symmetries have to be in $\mathcal{G}_A$.



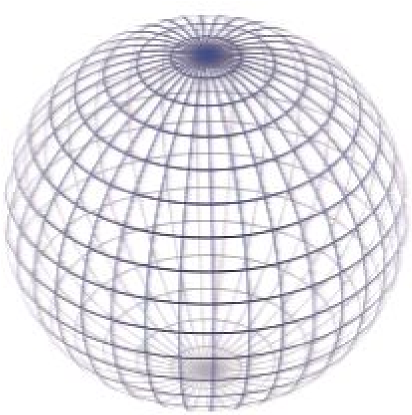
Qubit: Ω_A is the 3D unit ball,

$\mathcal{G}_A = SO(3)$ (no reflections!)

2. The Physical Setup



Not all symmetries have to be in $\mathcal{G}_A$.

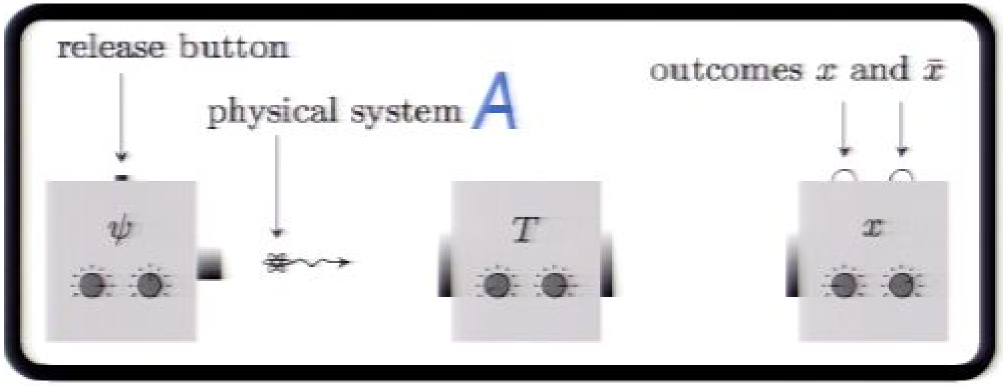


Qubit: Ω_A is the 3D unit ball,

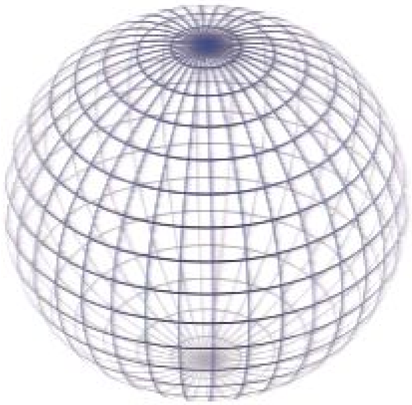
$\mathcal{G}_A = SO(3)$ (no reflections!)

$\Rightarrow$ A system is a pair $(\Omega_A, \mathcal{G}_A)$.

2. The Physical Setup



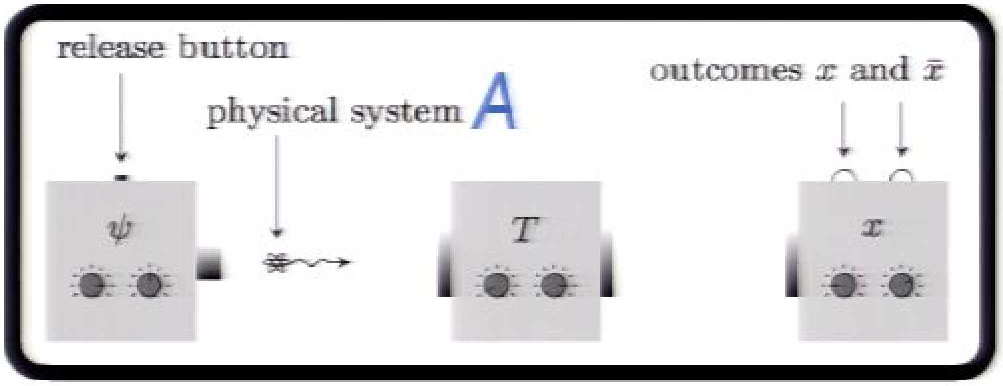
Not all symmetries have to be in $\mathcal{G}_A$.



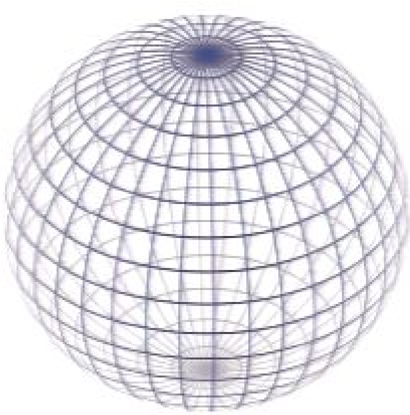
Qubit: Ω_A is the 3D unit ball,

$\mathcal{G}_A = SO(3)$ (no reflections!)

2. The Physical Setup



Not all symmetries have to be in $\mathcal{G}_A$.

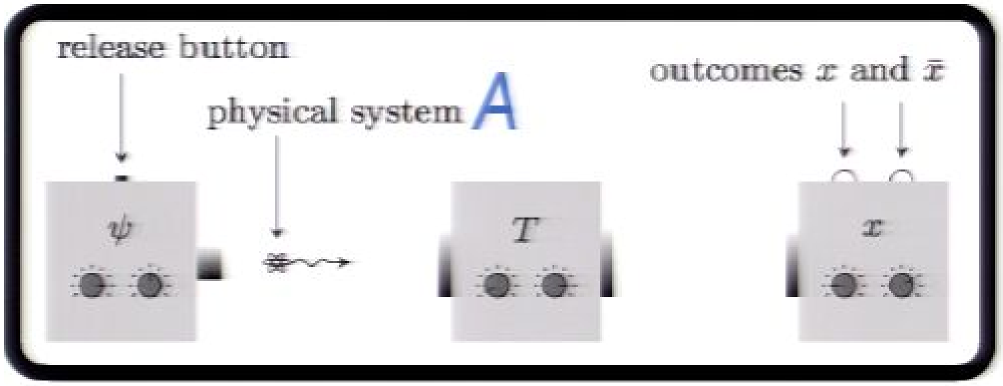


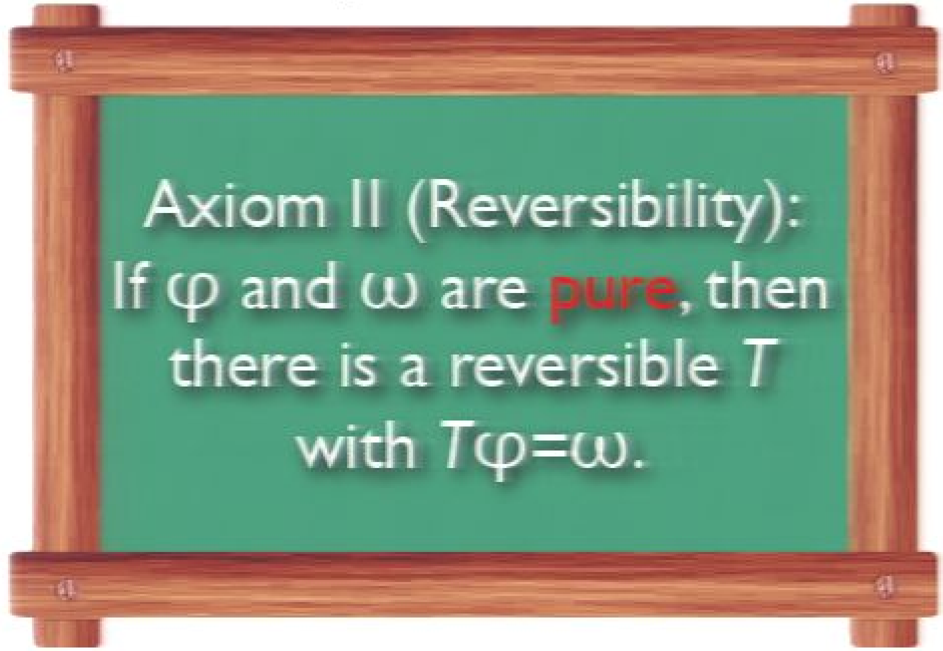
Qubit: Ω_A is the 3D unit ball,

$\mathcal{G}_A = SO(3)$ (no reflections!)

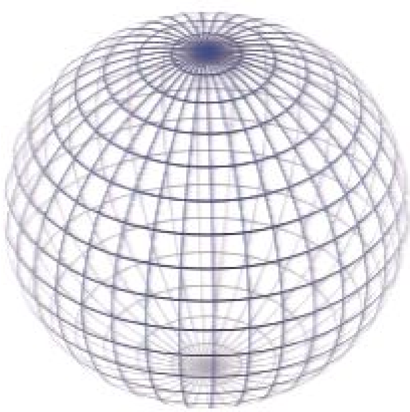
$\Rightarrow$ A system is a pair $(\Omega_A, \mathcal{G}_A)$.

2. The Physical Setup





Not all symmetries have to be in $\mathcal{G}_A$.

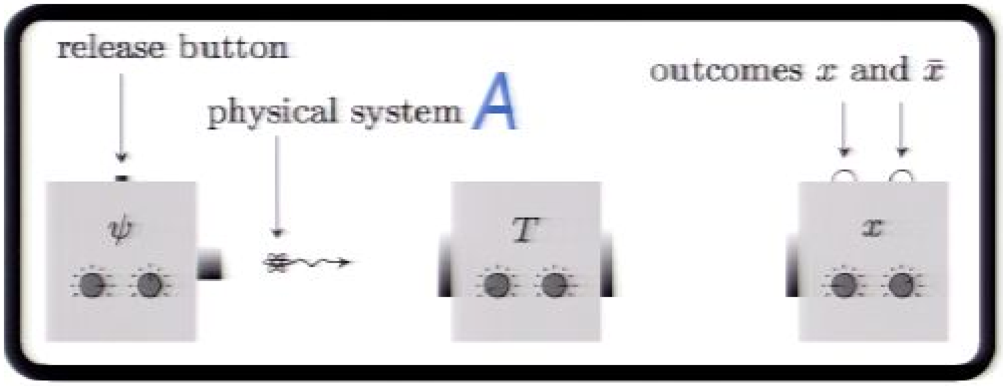


Qubit: Ω_A is the 3D unit ball,

$\mathcal{G}_A = SO(3)$ (no reflections!)

$\Rightarrow$ A system is a pair $(\Omega_A, \mathcal{G}_A)$.

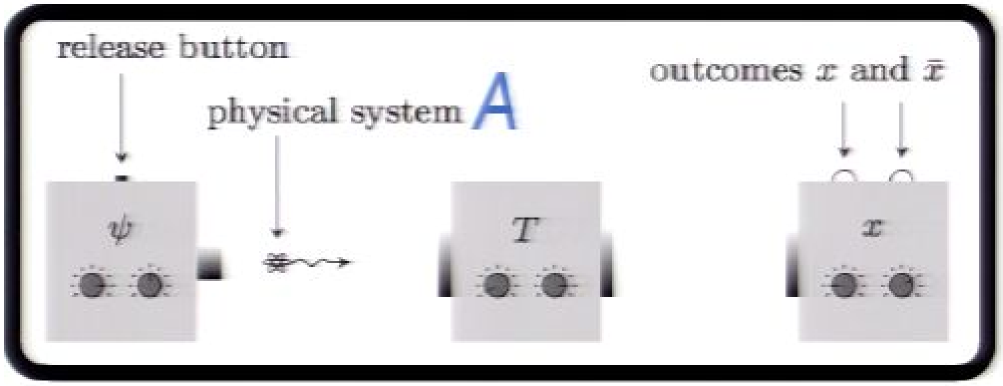
2. The Physical Setup



Axiom II (Reversibility):
If φ and ω are pure, then there is a reversible T with $T\varphi=\omega$.

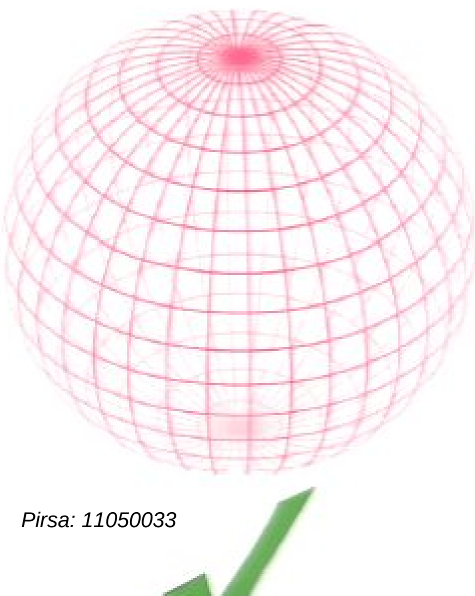
Enforces some symmetry in state space:

2. The Physical Setup

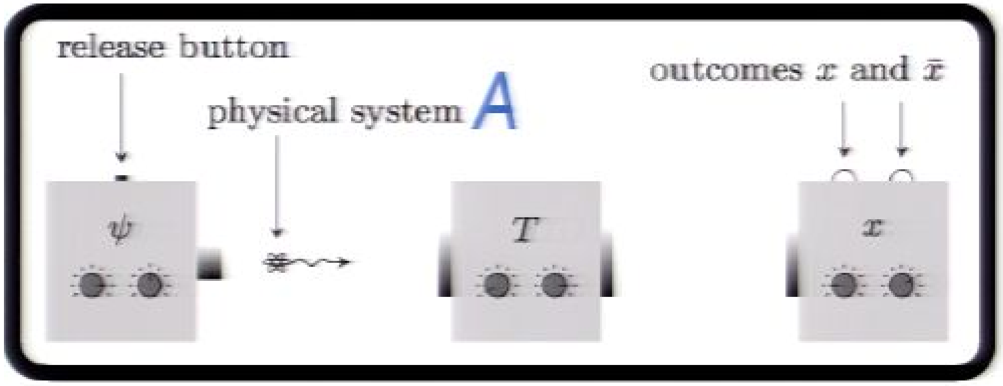


Axiom II (Reversibility):
If φ and ω are pure, then there is a reversible T with Tφ=ω.

Enforces some symmetry in state space:

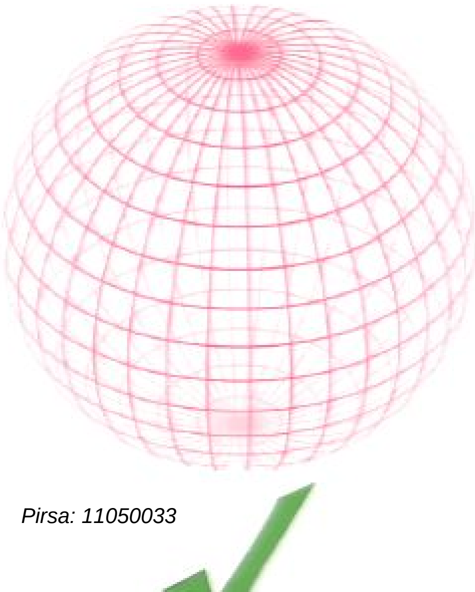


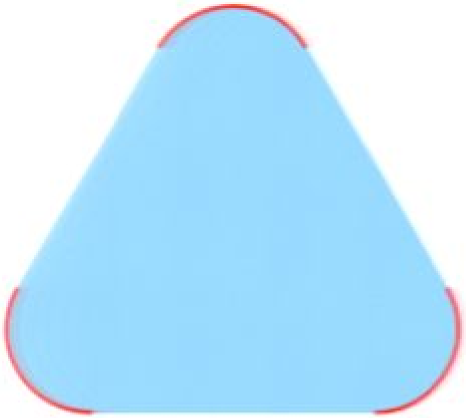
2. The Physical Setup



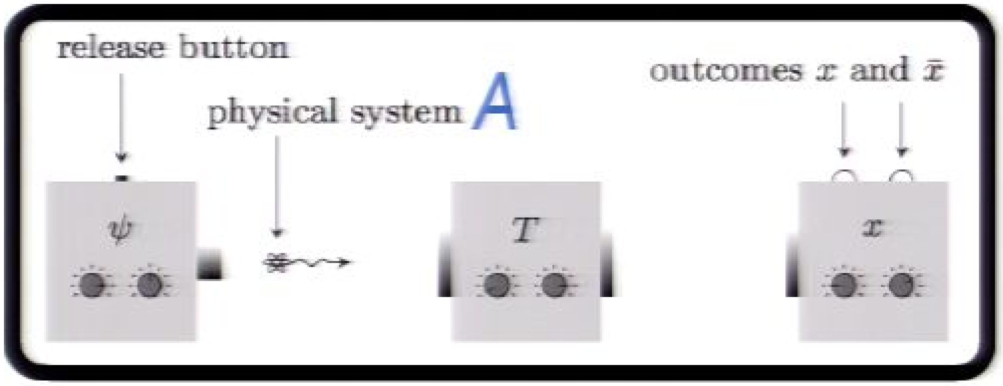
Axiom II (Reversibility):
If φ and ω are pure, then there is a reversible T with $T\varphi=\omega$.

Enforces some symmetry in state space:



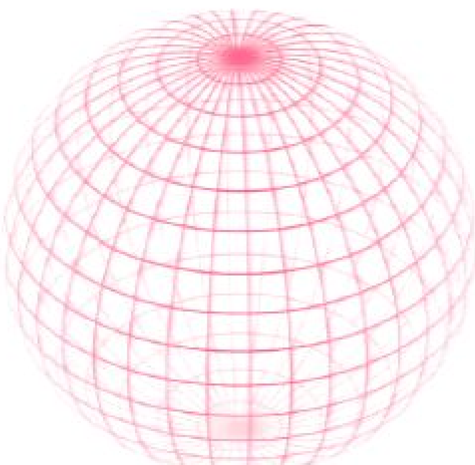


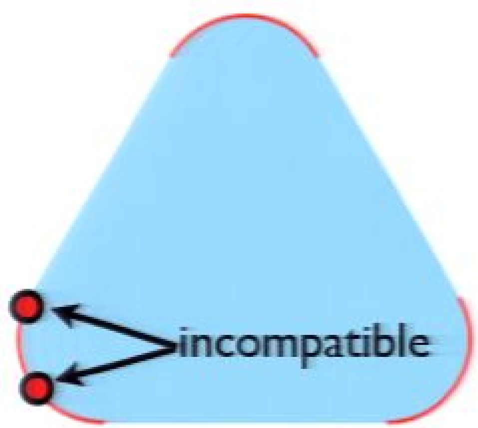
2. The Physical Setup



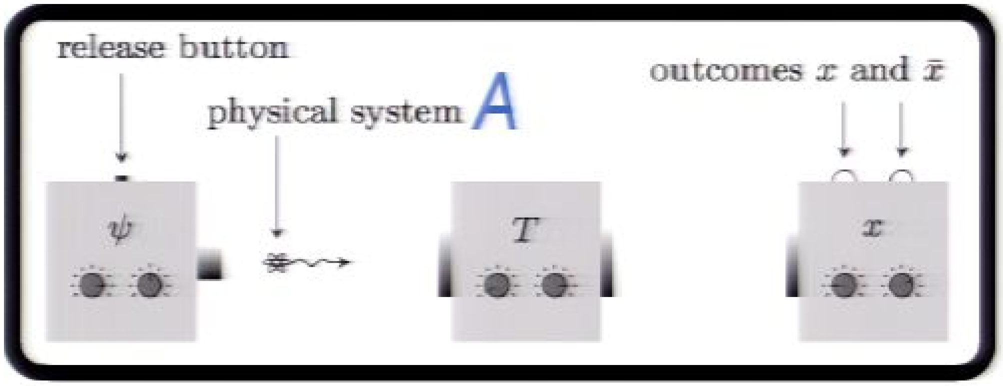
Axiom II (Reversibility):
If φ and ω are pure, then there is a reversible T with $T\varphi=\omega$.

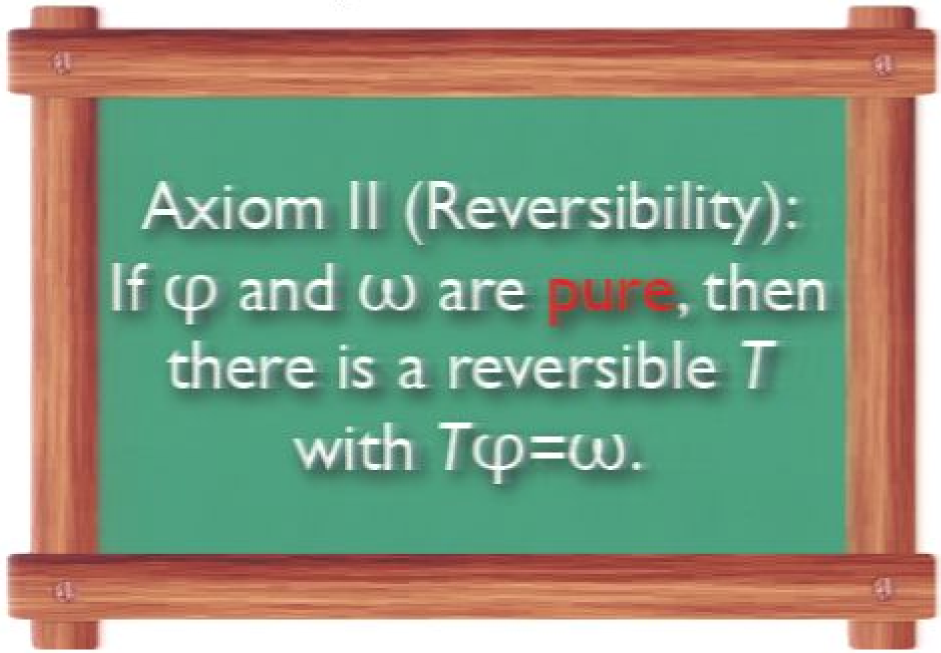
Enforces some symmetry in state space:



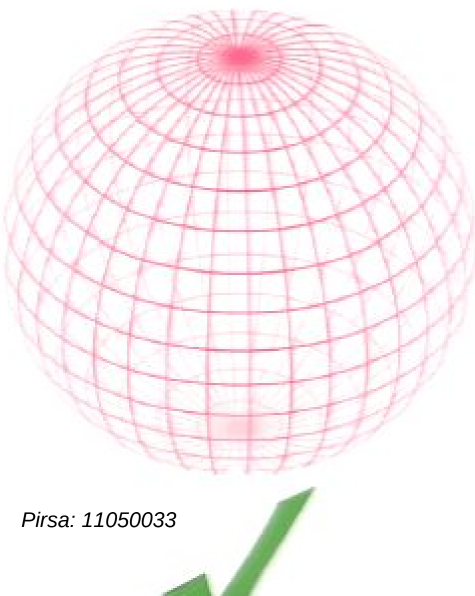


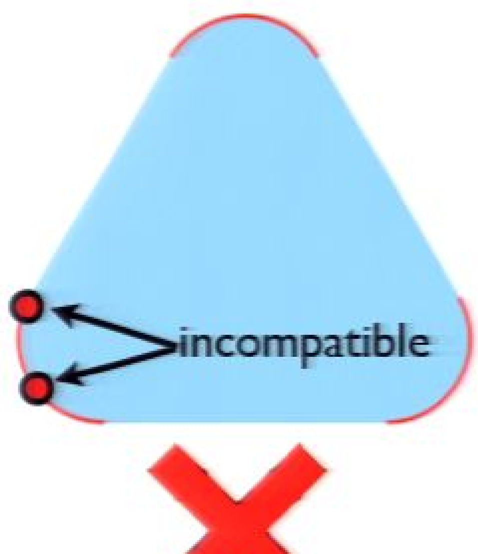
2. The Physical Setup

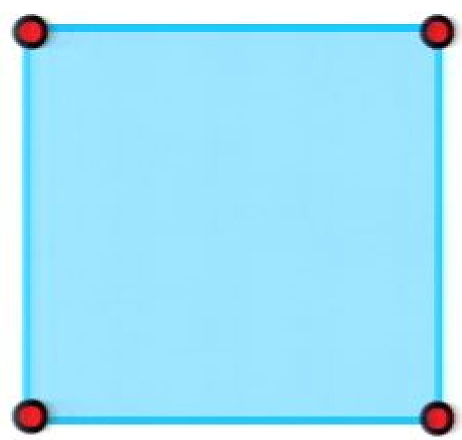




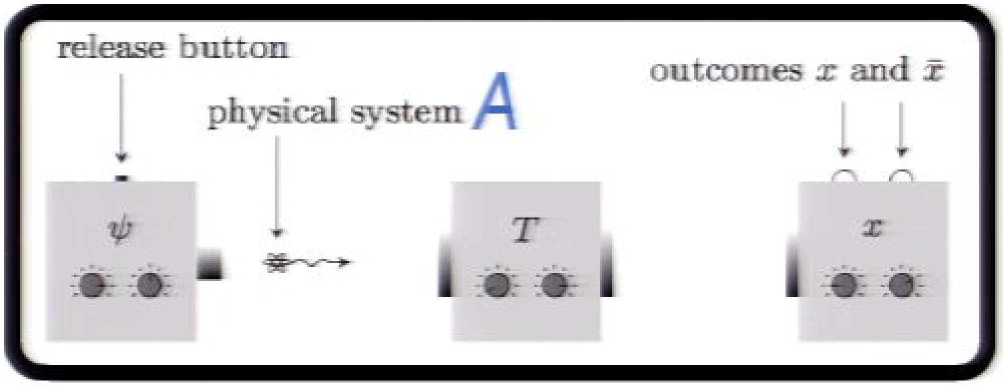
Enforces some symmetry in state space:

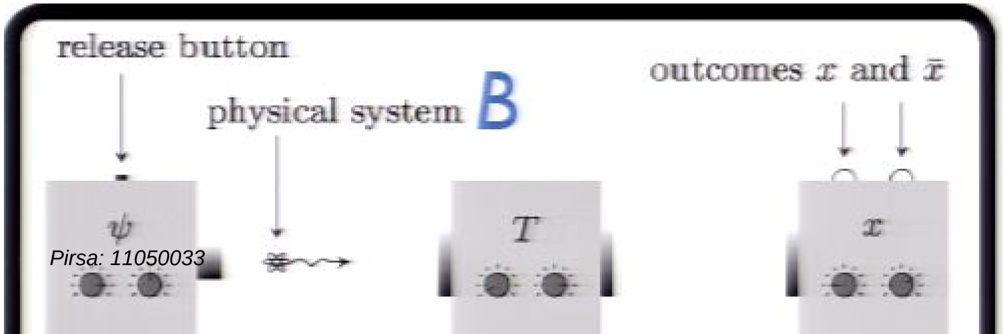




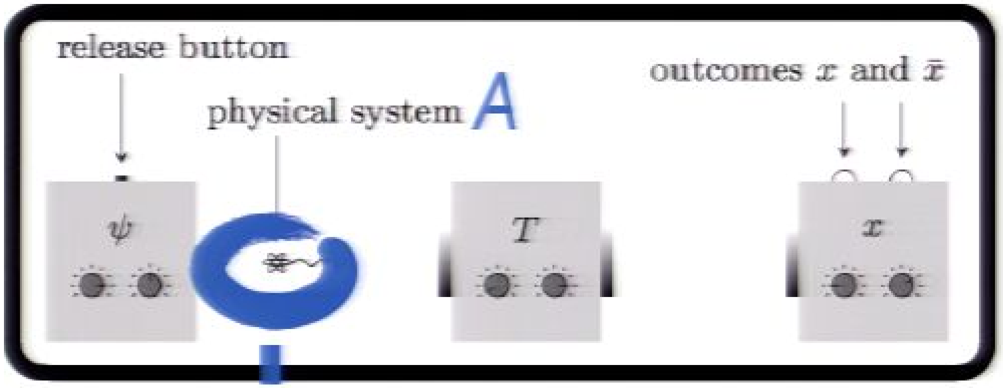


2. The Physical Setup

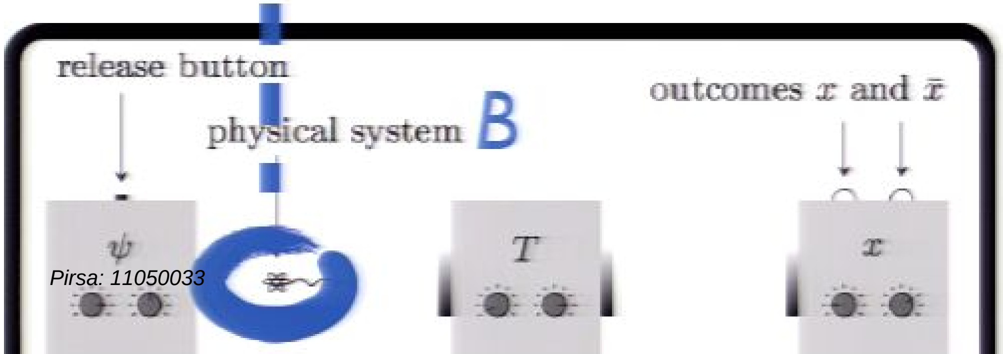




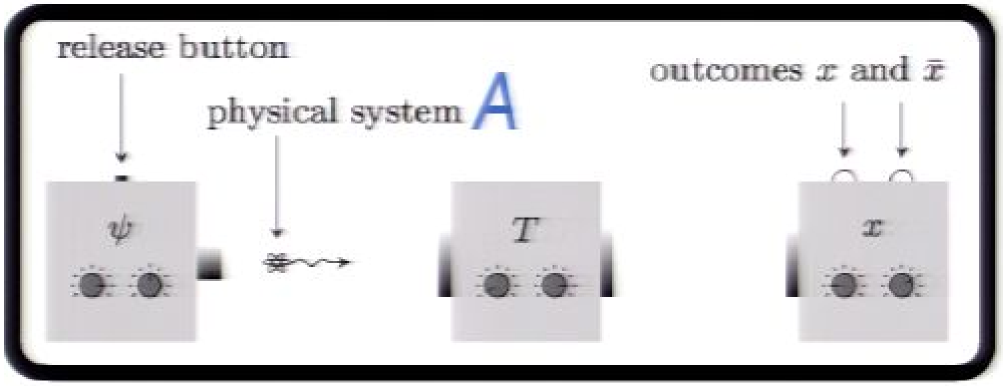
2. The Physical Setup

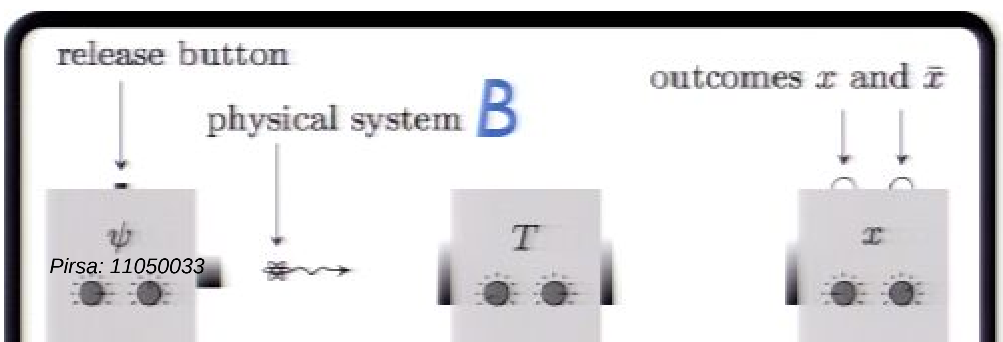


state on AB:
correlations

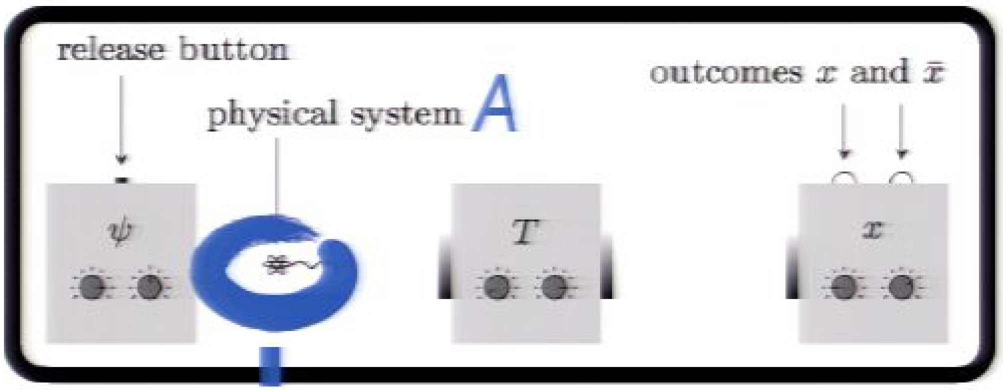


2. The Physical Setup

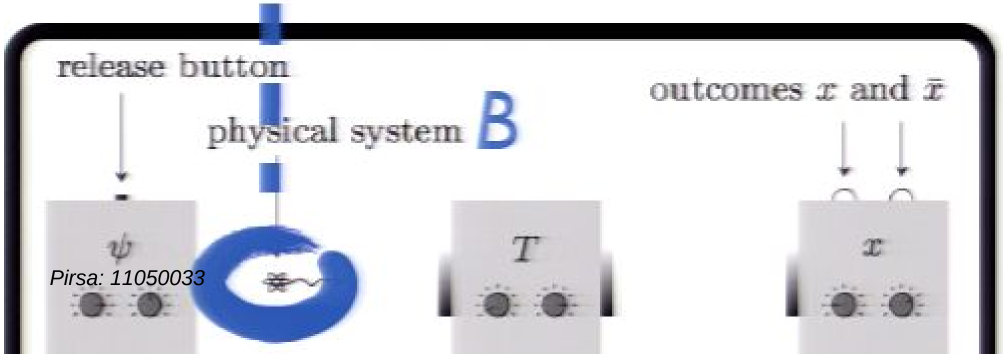




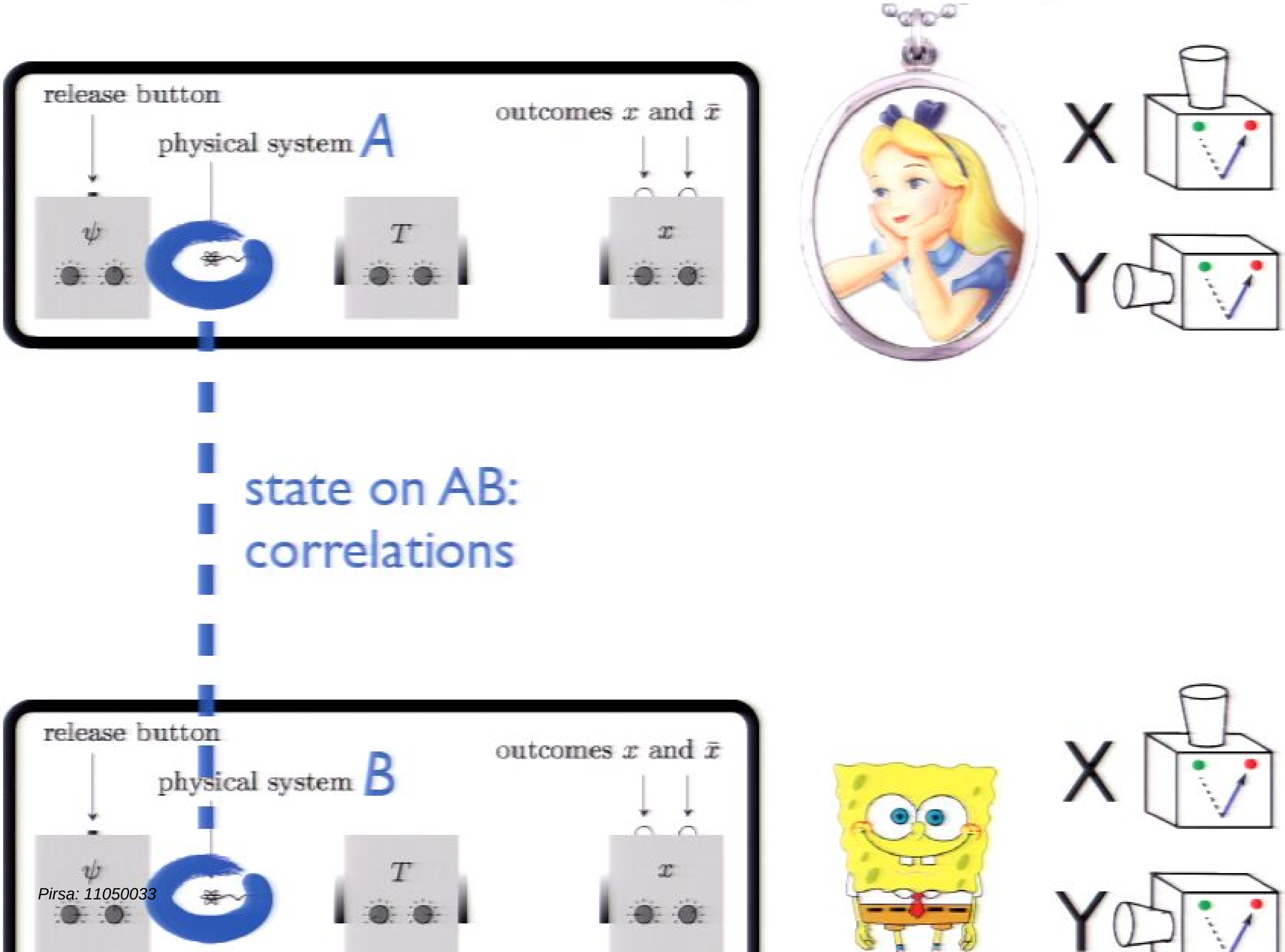
2. The Physical Setup



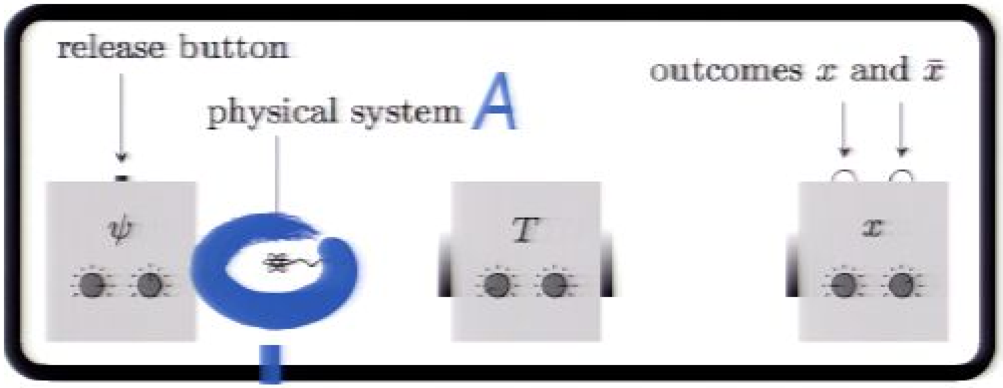
state on AB:
correlations

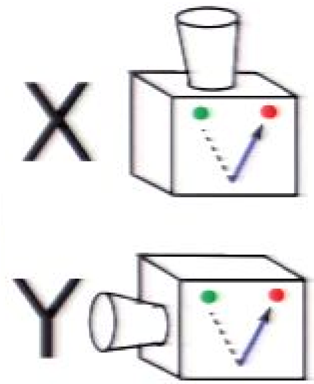


2. The Physical Setup



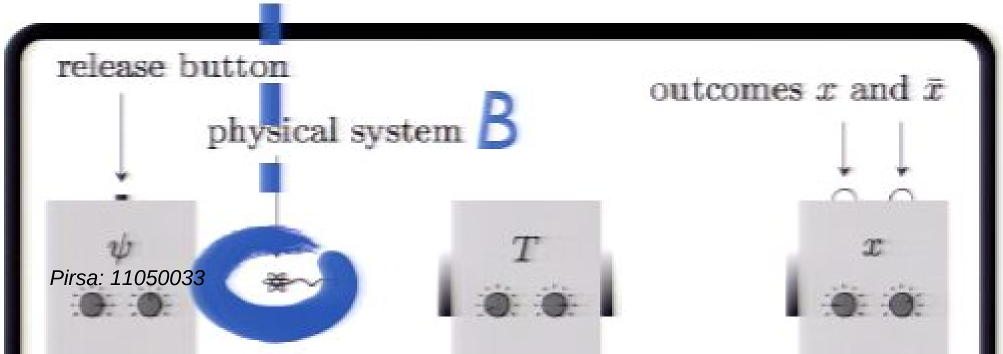
2. The Physical Setup

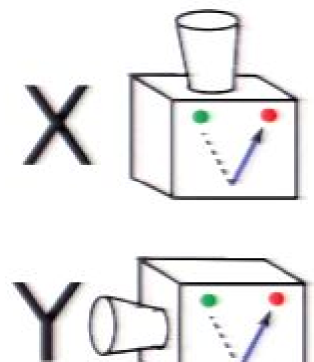




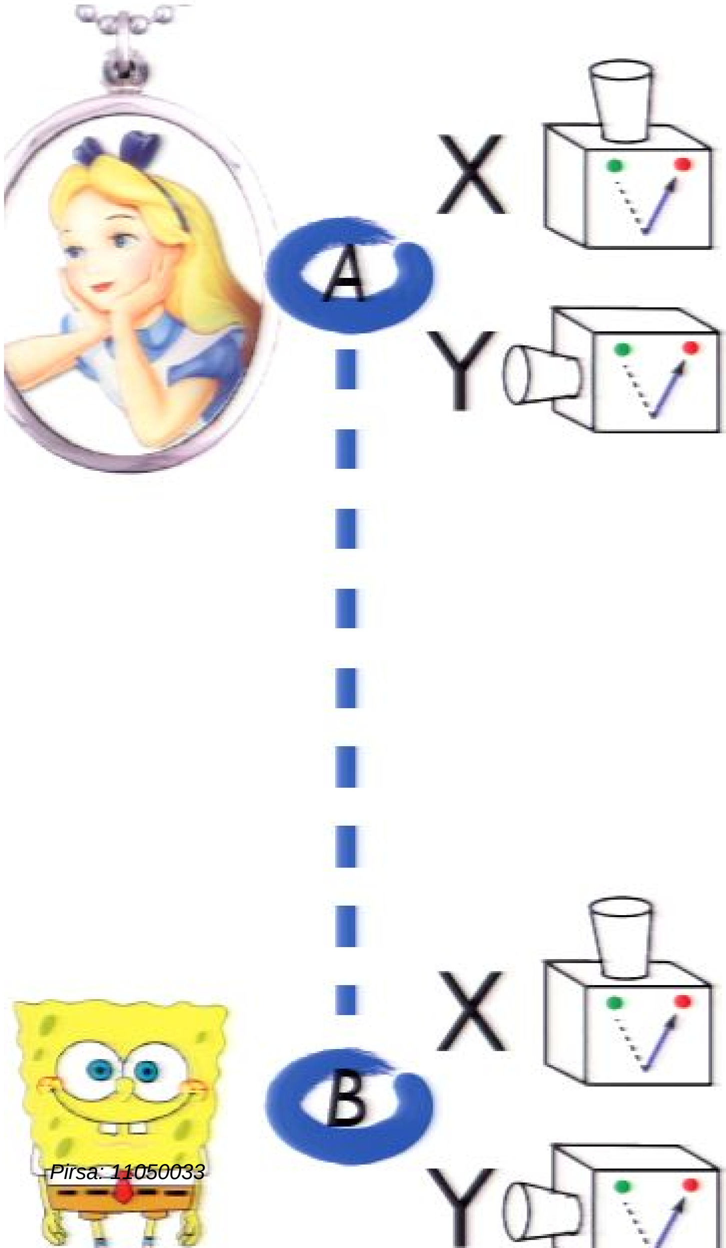
state on AB:
correlations

No-signalling condition:
Alice's probabilities do not depend on Bob's choice of measurement.

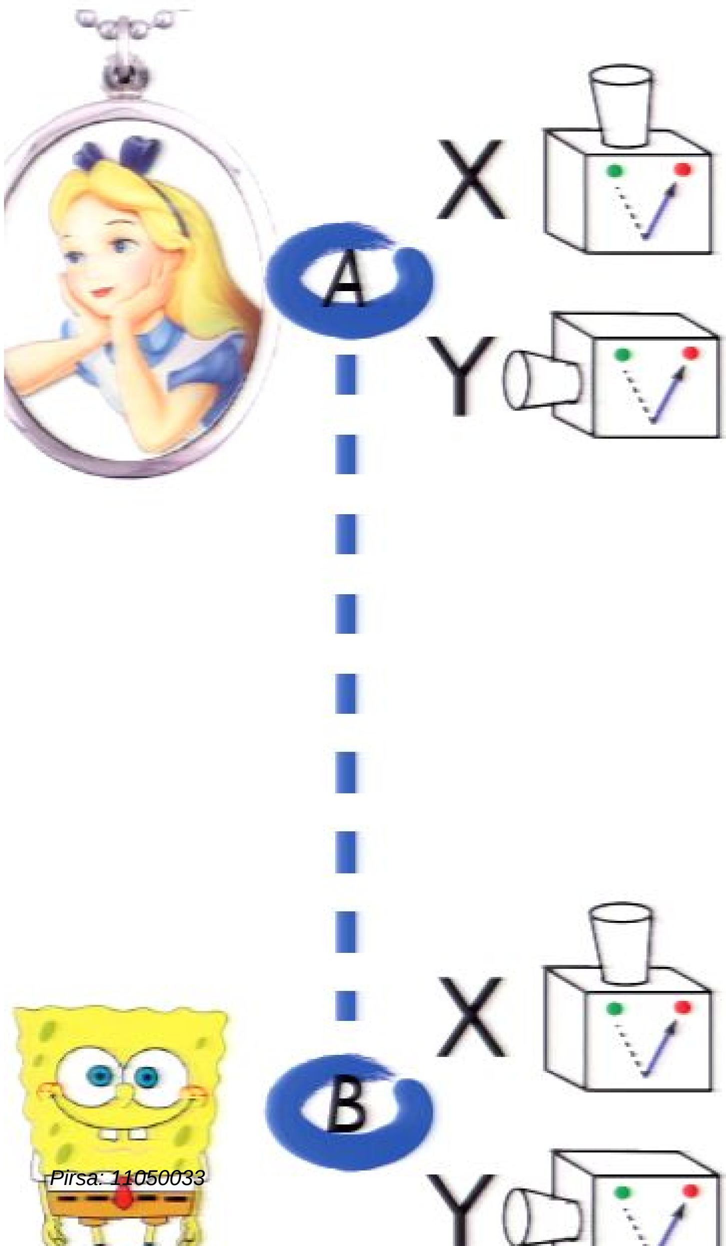




2. The Physical Setup

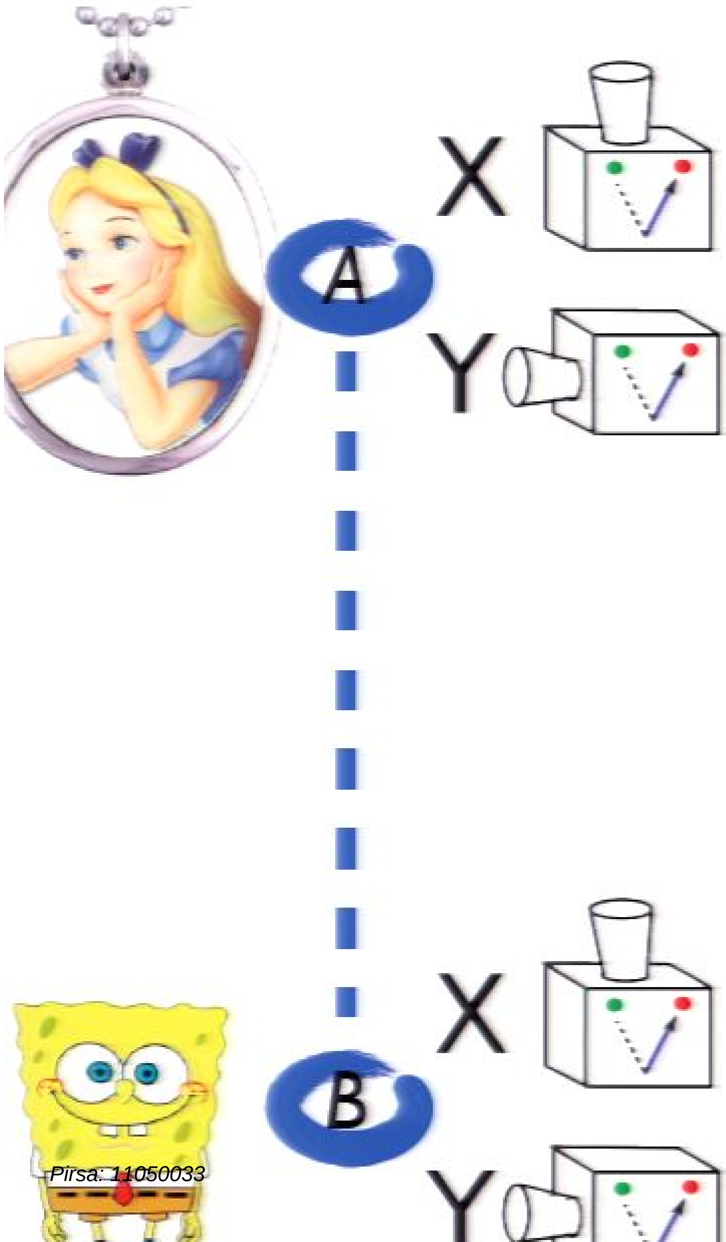


2. The Physical Setup



Axiom I: States on AB are uniquely determined by <u>correlations of local measurements</u> on A,B.

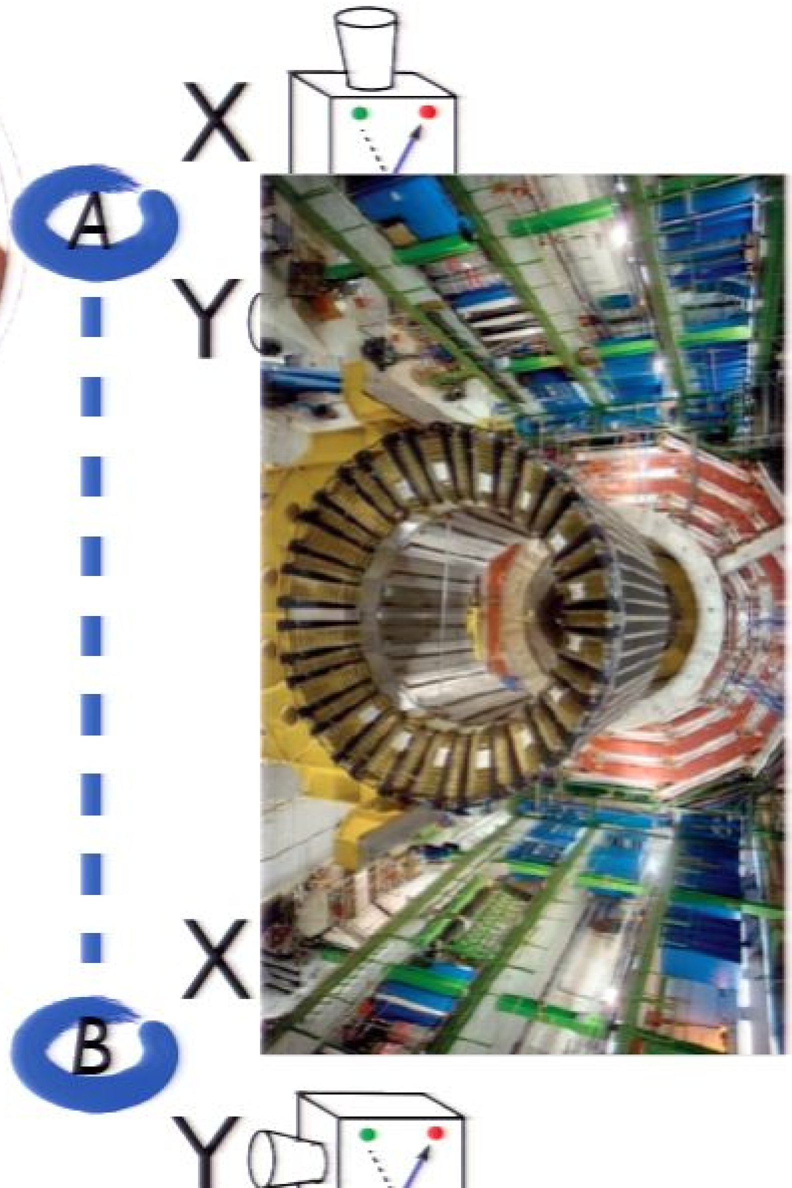
2. The Physical Setup



Axiom I: States on AB are uniquely determined by <u>correlations of local measurements</u> on A,B.

= „Local tomography“:
No non-local measurements necessary.

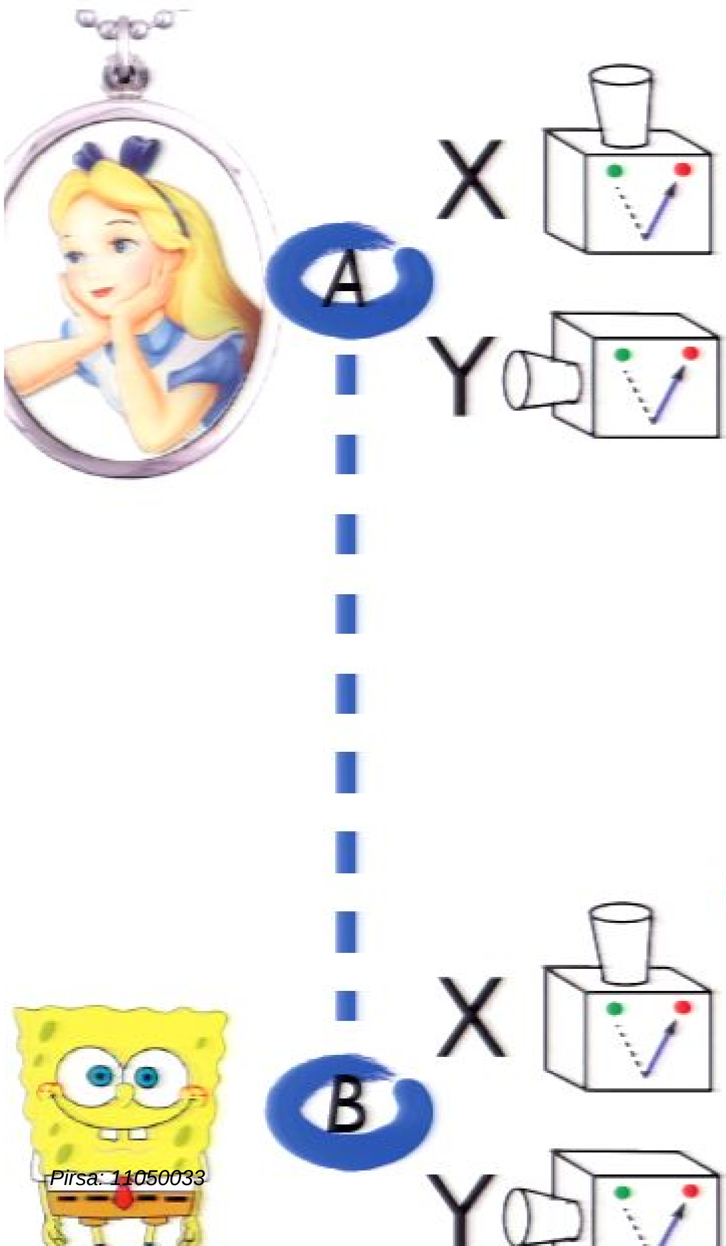
2. The Physical Setup



Axiom I: States on AB are uniquely determined by <u>correlations of local measurements</u> on A,B.

= „Local tomography“:
No non-local measurements necessary.

2. The Physical Setup

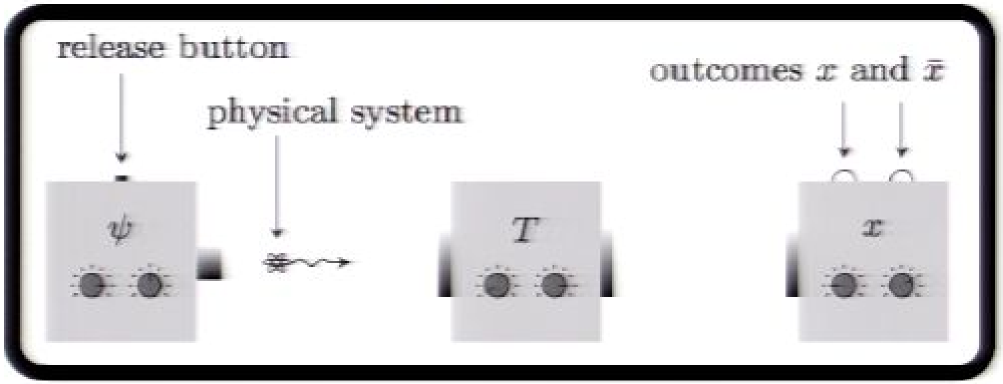


Axiom I: States on AB are uniquely determined by <u>correlations of local measurements</u> on A,B.

= „Local tomography":
No non-local measurements necessary.

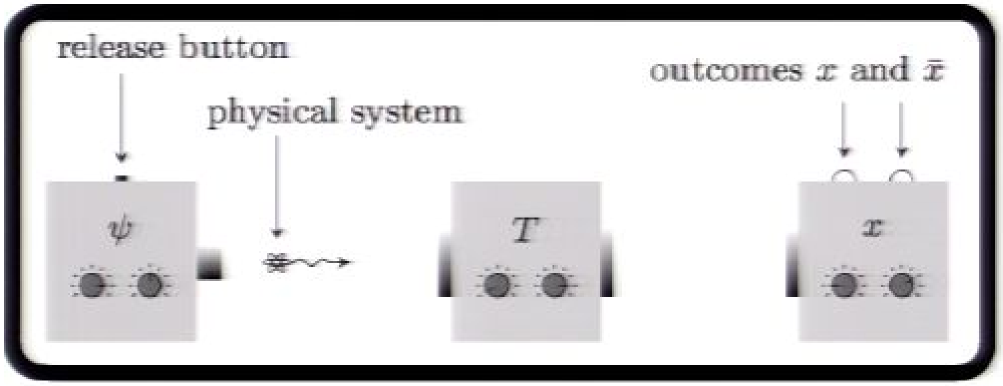
Global state space $\Omega_{AB} \subset A \otimes B$ but not uniquely fixed!

Basic physical / operational assumptions



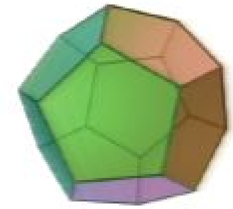
- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

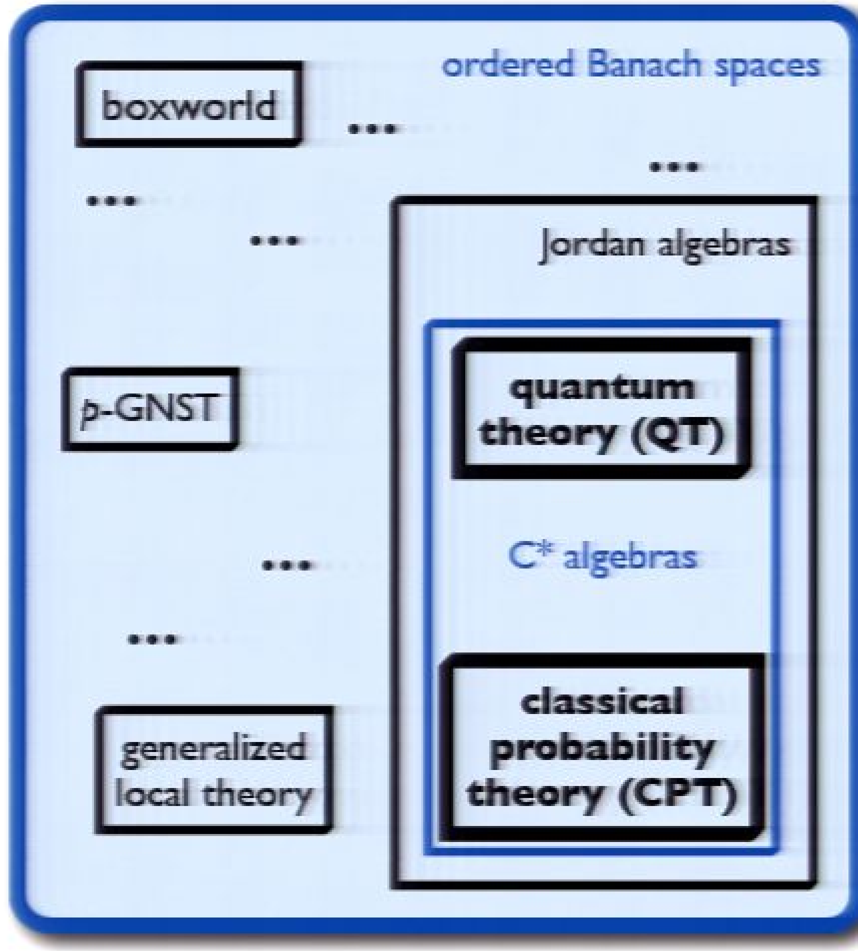
Basic physical / operational assumptions



- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

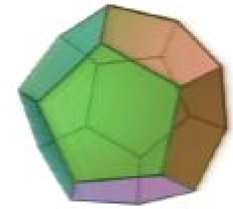
General probabilistic theories

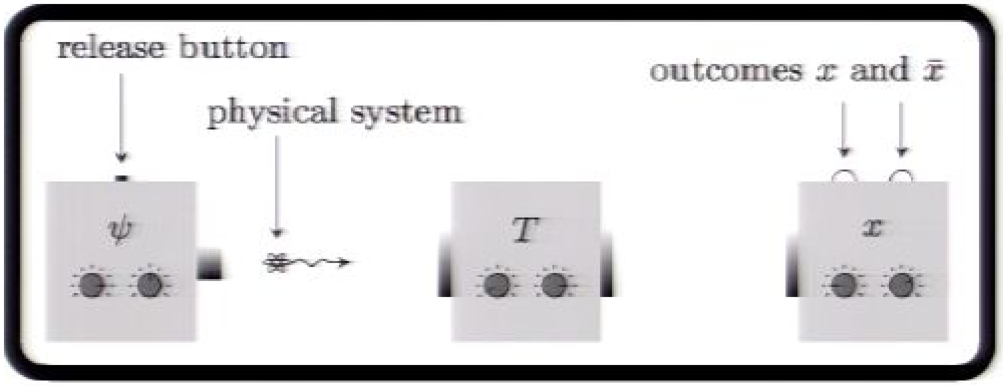




- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions → General probabilistic theories

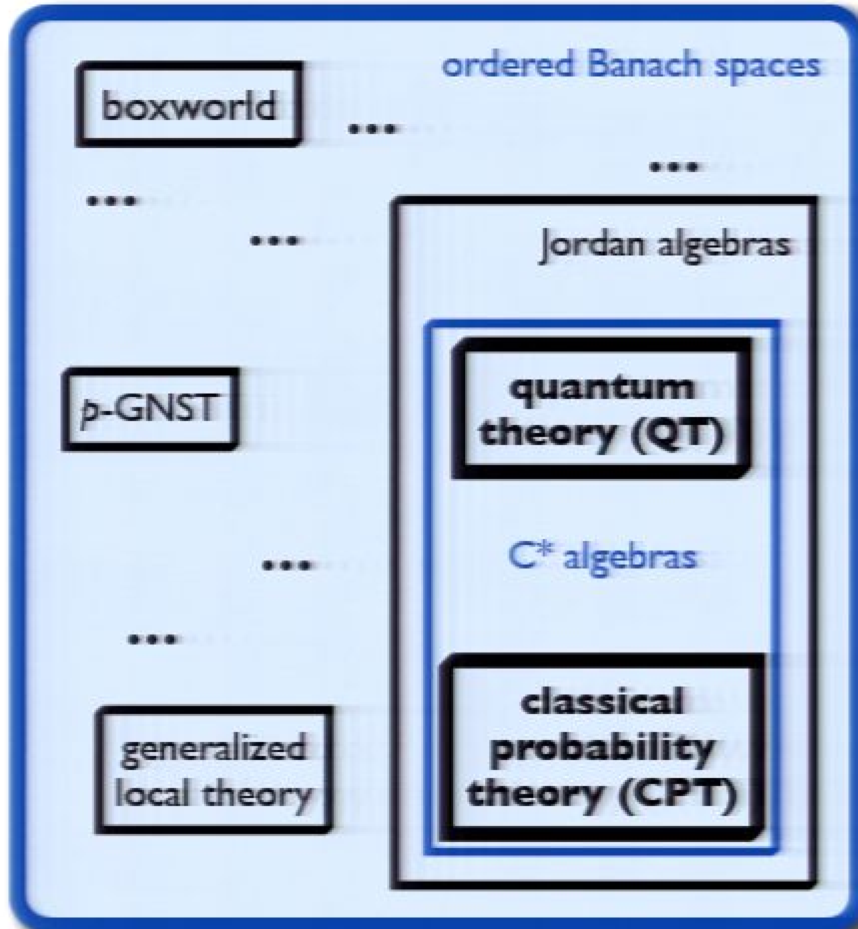




- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

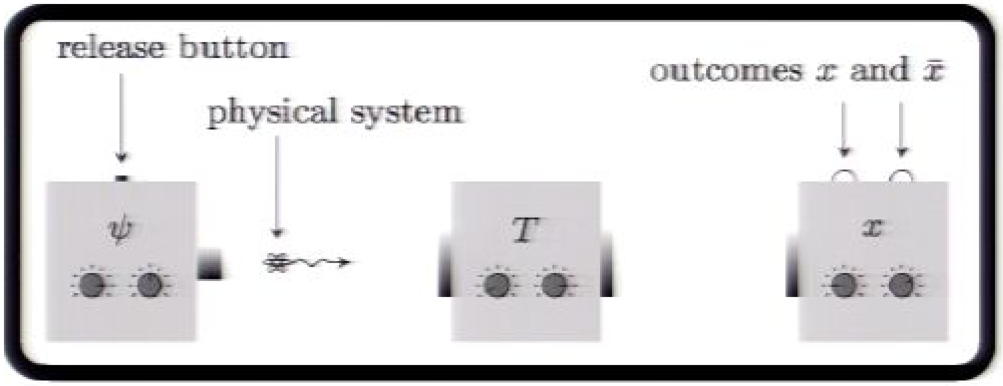
The Axioms:

I. Local tomography
II. Reversibility
III. Subspace axiom
IV. Finite-dimensionality
V. All measurements allowed



- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

Basic physical / operational assumptions

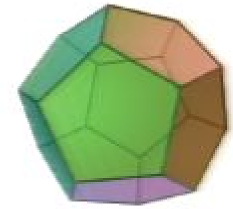


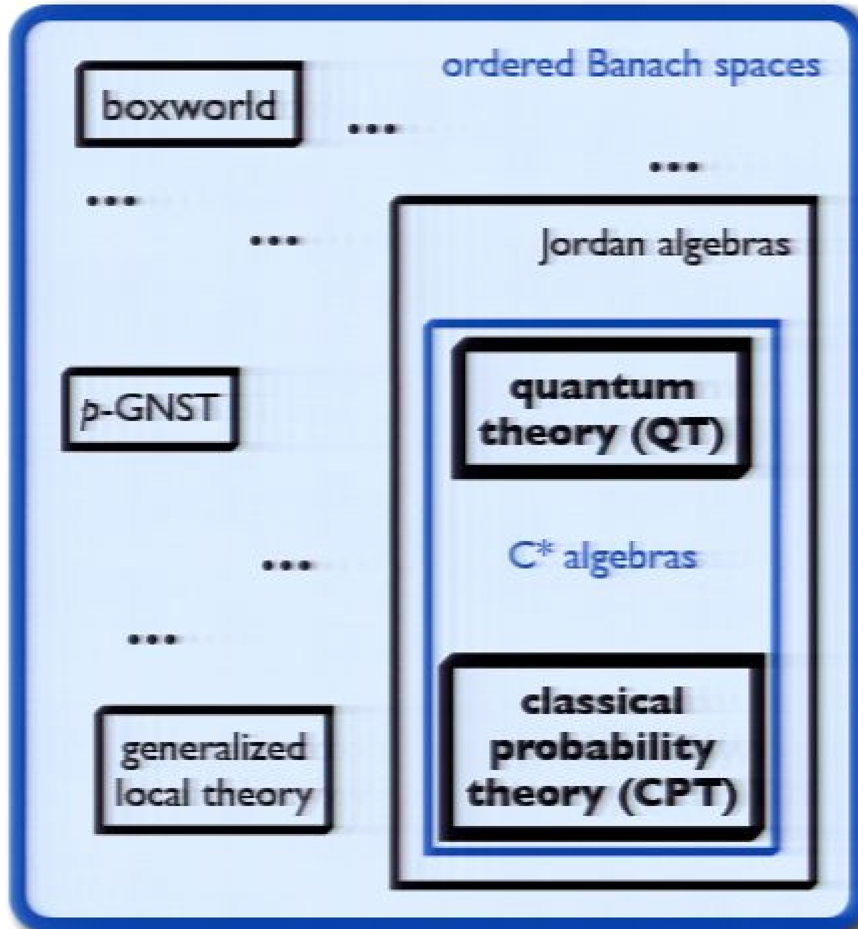
- States, transformations, and measurements with **outcome probabilities**.
- Combined systems: **no-signalling**.

The Axioms:

I. Local tomography ✓
II. Reversibility ✓
III. Subspace axiom
IV. Finite-dimensionality ✓
V. All measurements allowed ✓

General probabilistic theories

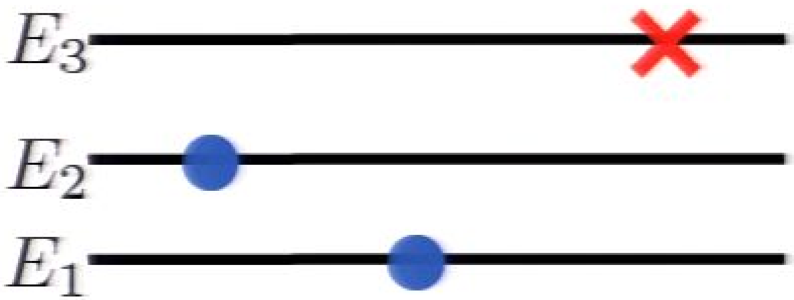




- **No** Hilbert spaces, complex numbers,...
- State spaces: **arbitrary convex sets**.
- Many ways to **combine systems**.

3. The Subspace Axiom

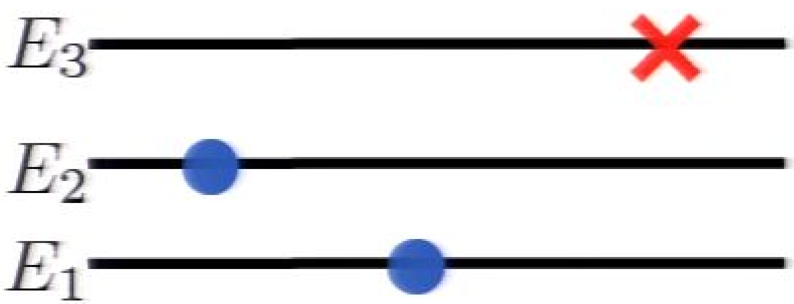
Some 3-level system:



Impossible to put system in 3rd level
⇒ find particle there with probab. 0

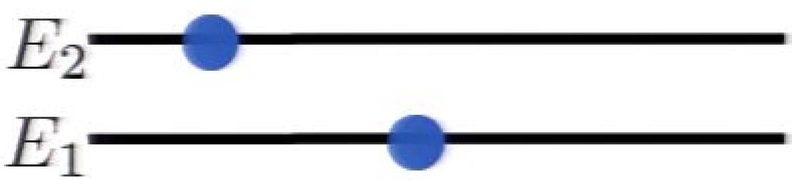
3. The Subspace Axiom

Some 3-level system:



Impossible to put system in 3rd level
⇒ find particle there with probab. 0

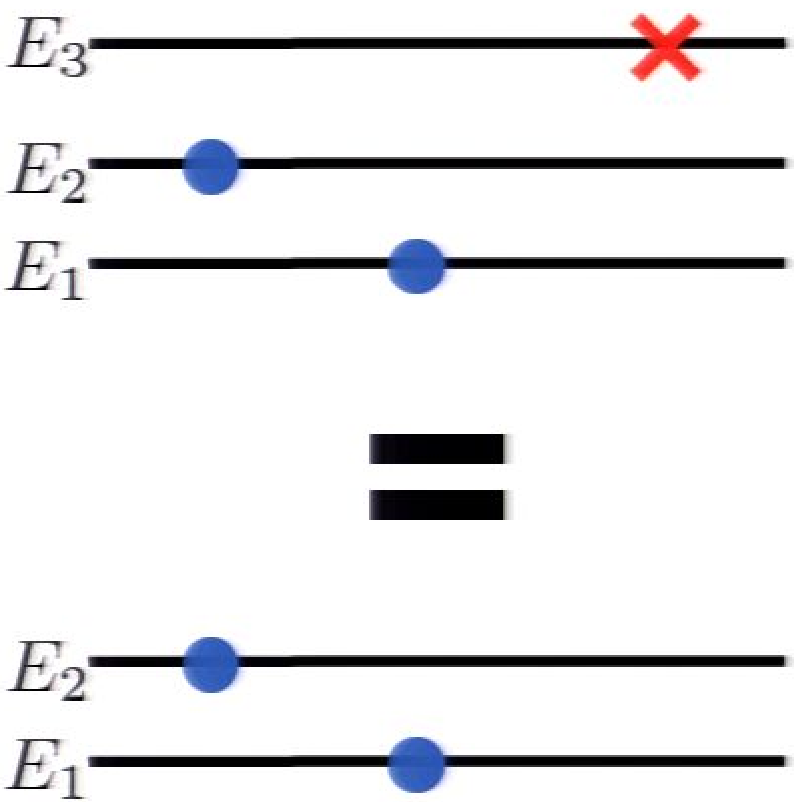
=



2-level system.

3. The Subspace Axiom

Some 3-level system:



2-level system.

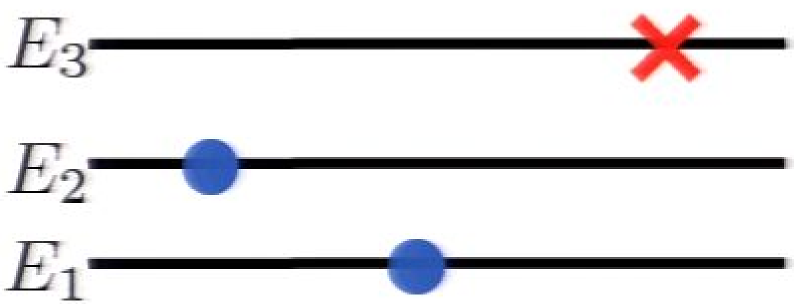
Impossible to put system in 3rd level
$\Rightarrow$ find particle there with probab. 0

QT: $$\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$$

CPT: $$P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$$

3. The Subspace Axiom

Some 3-level system:

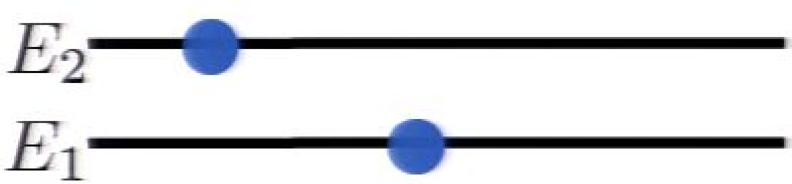


Impossible to put system in 3rd level
$\Rightarrow$ find particle there with probab. 0

$=$

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

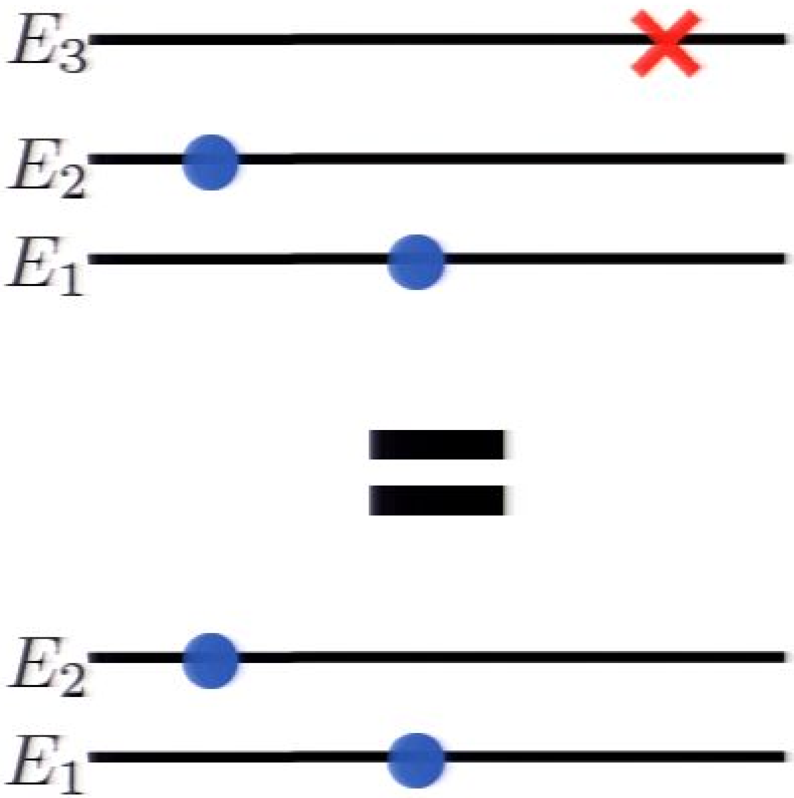


2-level system.

Otherwise, physics would be affected

3. The Subspace Axiom

Some 3-level system:



2-level system.

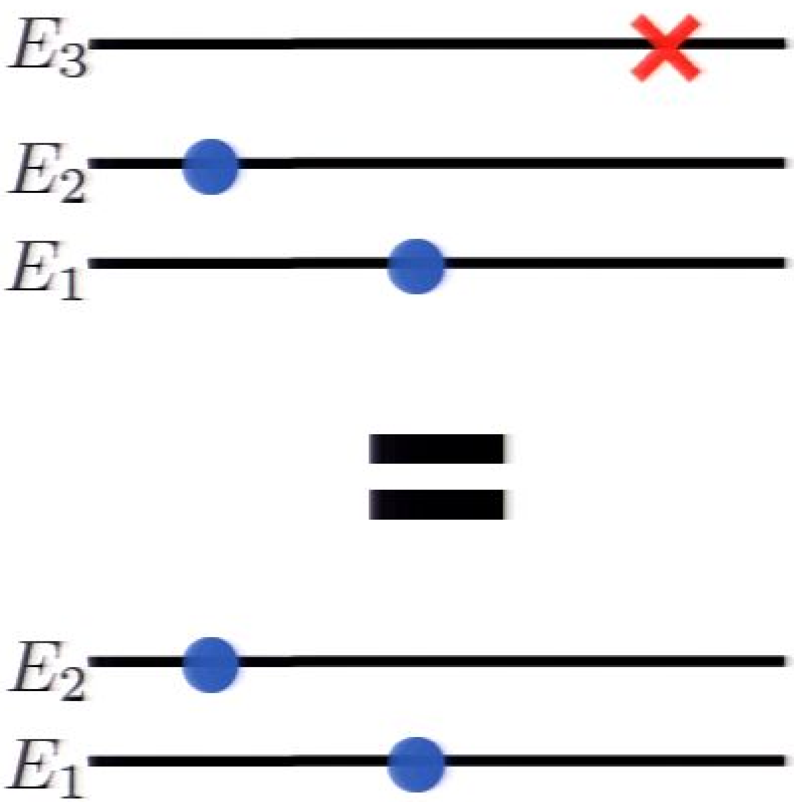
Impossible to put system in 3rd level
$\Rightarrow$ find particle there with probab. 0

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

3. The Subspace Axiom

Some 3-level system:



2-level system.

Impossible to put system in 3rd level
⇒ find particle there with probab. 0

QT: $\rho^{(3)} = \begin{pmatrix} \bullet & \bullet & 0 \\ \bullet & \bullet & 0 \\ 0 & 0 & 0 \end{pmatrix} \longrightarrow \rho^{(2)} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$

CPT: $P^{(3)} = (P_1, P_2, 0) \longrightarrow P^{(2)} = (P_1, P_2)$

Otherwise, physics would be affected

3. The Subspace Axiom

Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

3. The Subspace Axiom

Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities N and N-1. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

Capacity N of Ω = maximal # of perfectly distinguishable states.

$(\omega_1, \ldots, \omega_n)$ perfectly distinguishable, if there is a measurement $(E_1, \ldots, E_n)$ such that $E_i(\omega_j) = \delta_{ij}$.

3. The Subspace Axiom

Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

Capacity N of Ω = maximal # of perfectly distinguishable states.

$(\omega_1, \ldots, \omega_n)$ perfectly distinguishable, if there is a measurement $(E_1, \ldots, E_n)$ such that $E_i(\omega_j) = \delta_{ij}$.

If $n = N$ then $(E_1, \ldots, E_n)$ is complete.

3. The Subspace Axiom

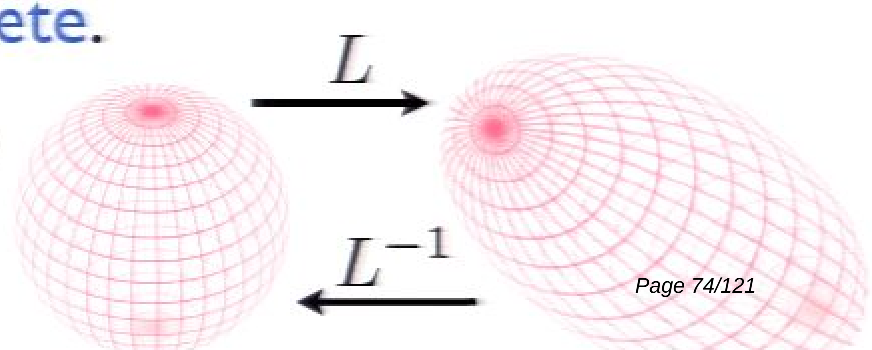
Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

Capacity N of Ω = maximal # of perfectly distinguishable states.

$(\omega_1, \ldots, \omega_n)$ perfectly distinguishable, if there is a measurement $(E_1, \ldots, E_n)$ such that $E_i(\omega_j) = \delta_{ij}$.

If $n = N$ then $(E_1, \ldots, E_n)$ is complete.

Equivalent = same state spaces up to a linear map (incl. transformations!)



4. Derivation of the Hilbert space formalism

Why a bit is described by a ball:

capacity 2 (bit)

3. The Subspace Axiom

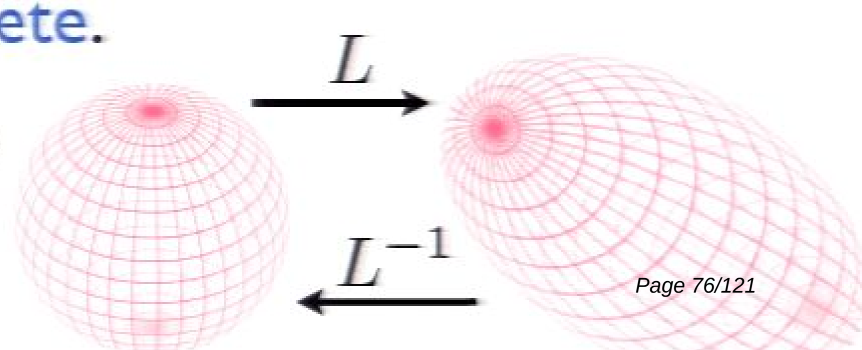
Axiom III: Let Ω_N and Ω_{N-1} be systems with capacities *N* and *N-1*. If $(E_1, \ldots, E_N)$ is a complete measurement on Ω_N, then the set of states ω with $E_N(\omega) = 0$ is equivalent to Ω_{N-1}.

Capacity N of Ω = maximal # of perfectly distinguishable states.

$(\omega_1, \ldots, \omega_n)$ perfectly distinguishable, if there is a measurement $(E_1, \ldots, E_n)$ such that $E_i(\omega_j) = \delta_{ij}$.

If $n = N$ then $(E_1, \ldots, E_n)$ is complete.

Equivalent = same state spaces up to a linear map (incl. transformations!)



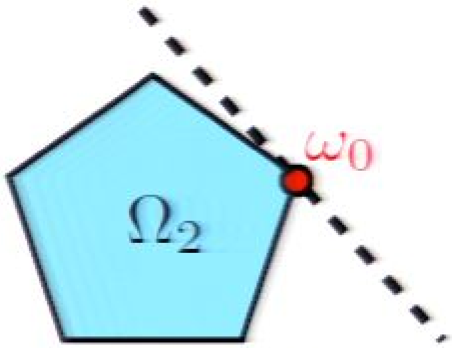
4. Derivation of the Hilbert space formalism

Why a bit is described by a ball:

capacity 2 (bit)

4. Derivation of the Hilbert space formalism

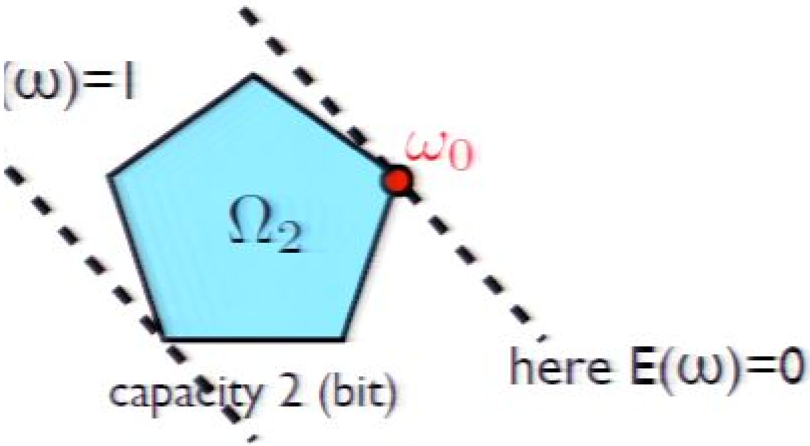
Why a bit is described by a ball:



capacity 2 (bit)

4. Derivation of the Hilbert space formalism

Why a bit is described by a ball:

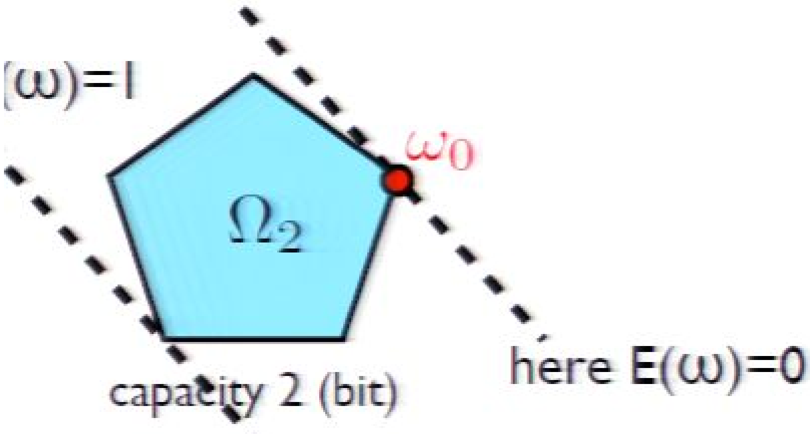


(1-E, E) is complete measurement.

$$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1.$$

4. Derivation of the Hilbert space formalism

Why a bit is described by a ball:



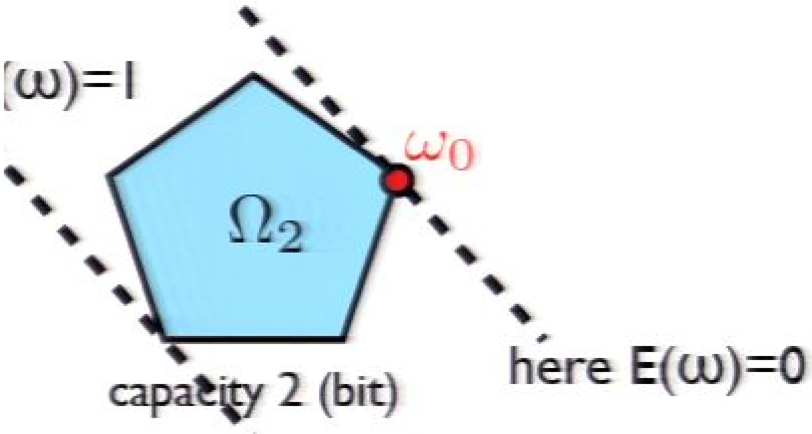
(1-E, E) is complete measurement.

$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1.$

$\Rightarrow \Omega_1$ contains a single state.

4. Derivation of the Hilbert space formalism

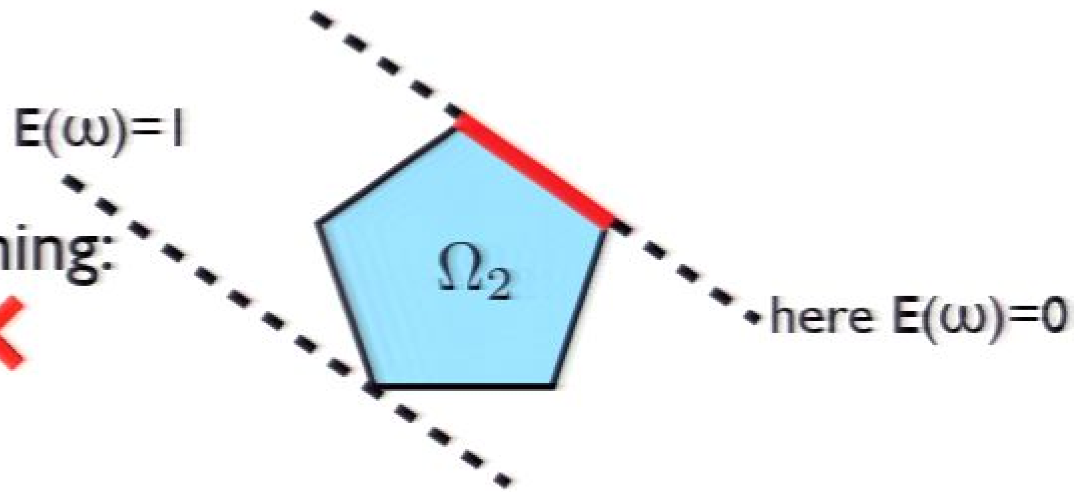
Why a bit is described by a ball:



(1-E, E) is complete measurement.

$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1.$

$\Rightarrow \Omega_1$ contains a single state.

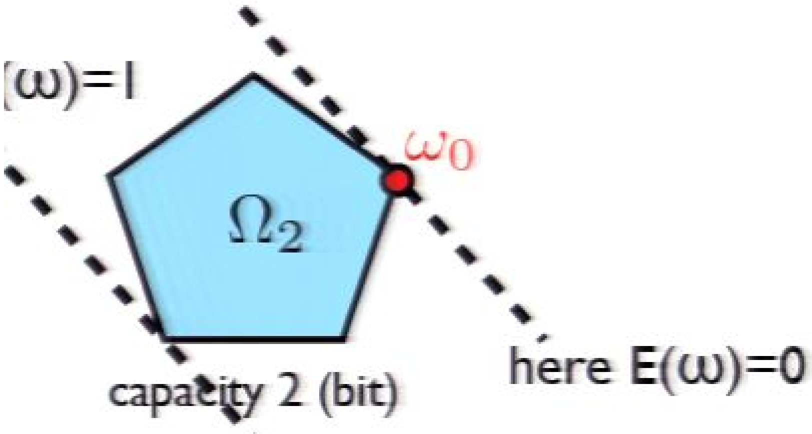


If there is a face, similar reasoning:

Ω_1 contains ∞ many states. ✗

4. Derivation of the Hilbert space formalism

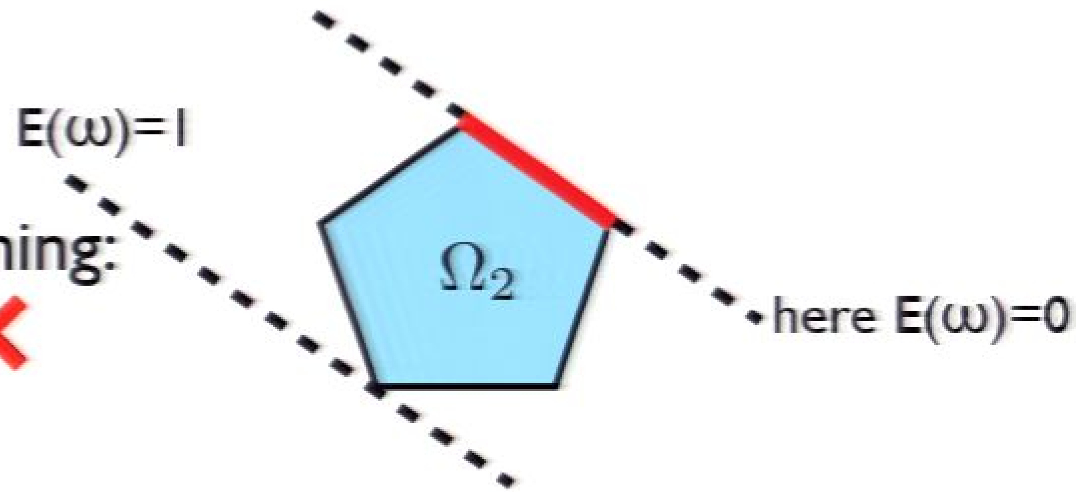
Why a bit is described by a ball:



(1-E, E) is complete measurement.

$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1.$

$\Rightarrow$ Ω_1 contains a single state.



If there is a face, similar reasoning:

Ω_1 contains ∞ many states. ✗

$\Rightarrow$ no faces: 

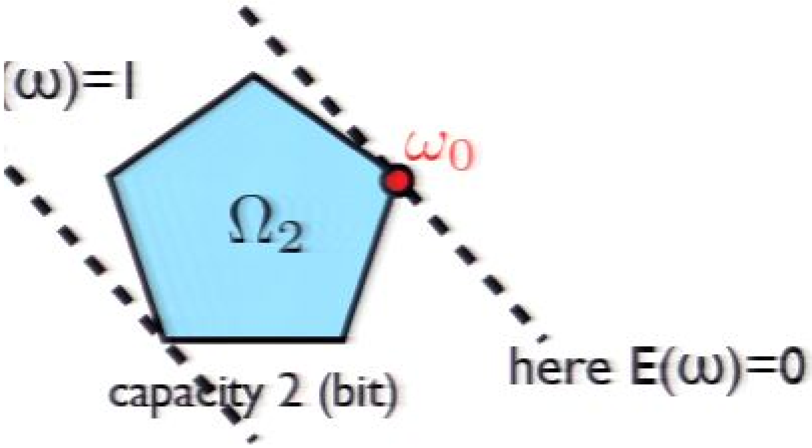
By group rep. theory, Ω_2 eqivalent to a ball.

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

4. Derivation of the Hilbert space formalism

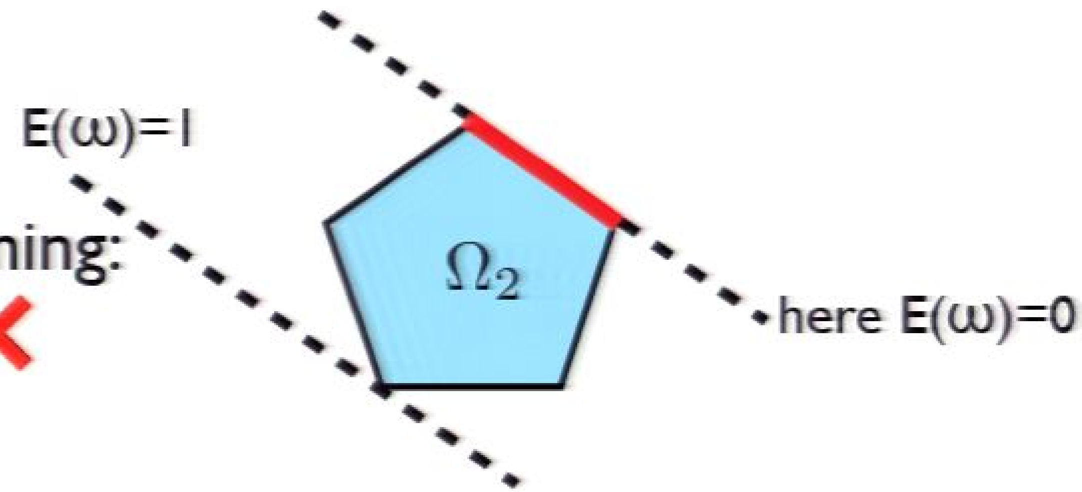
Why a bit is described by a ball:



(1-E, E) is complete measurement.

$\Rightarrow \{\omega : E(\omega) = 0\} = \{\omega_0\} \sim \Omega_1.$

$\Rightarrow \Omega_1$ contains a single state.



If there is a face, similar reasoning:

Ω_1 contains ∞ many states. ✗

$\Rightarrow$ no faces:

By group rep. theory, Ω_2 eqivalent to a ball.

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

- There is maximally mixed state μ with $T\mu = \mu$ for all T.
- $\mu_{AB} = \mu_A \otimes \mu_B$.

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

- There is maximally mixed state μ with $T\mu = \mu$ for all T.
- $\mu_{AB} = \mu_A \otimes \mu_B$.
- There are *N* pure distinguishable states $\omega_1, \ldots, \omega_N$ with

$$\mu = \frac{1}{N} \sum_{i=1}^{N} \omega_i.$$

- capacity $N_{AB} = N_A N_B$ and bit ball dimension

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

- There is maximally mixed state μ with $T\mu = \mu$ for all T,
- $\mu_{AB} = \mu_A \otimes \mu_B$,
- There are *N* pure distinguishable states $\omega_1, \ldots, \omega_N$ with

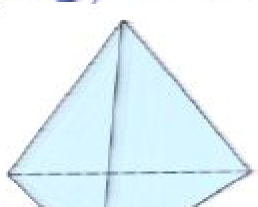
$$\mu = \frac{1}{N} \sum_{i=1}^{N} \omega_i,$$

- capacity $N_{AB} = N_A N_B$ and bit ball dimension

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

If $\dim(\Omega_2) = 1$ then the theory is CPT (easy):

$\Omega_2 =$ 

$\Omega_N =$ 

$\mathcal{G}_N$=permutation group.

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

- There is maximally mixed state μ with $T\mu = \mu$ for all T.
- $\mu_{AB} = \mu_A \otimes \mu_B$.
- There are *N* pure distinguishable states $\omega_1, \ldots, \omega_N$ with

$$\mu = \frac{1}{N} \sum_{i=1}^{N} \omega_i.$$

- capacity $N_{AB} = N_A N_B$ and bit ball dimension

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

4. Derivation of the Hilbert space formalism

Prove step by step (using the axioms):

- There is maximally mixed state μ with $T\mu = \mu$ for all T,
- $\mu_{AB} = \mu_A \otimes \mu_B$,
- There are *N* pure distinguishable states $\omega_1, \ldots, \omega_N$ with

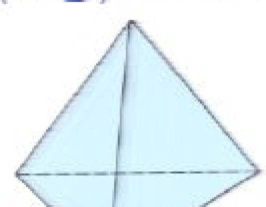
$$\mu = \frac{1}{N} \sum_{i=1}^{N} \omega_i,$$

- capacity $N_{AB} = N_A N_B$ and bit ball dimension

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

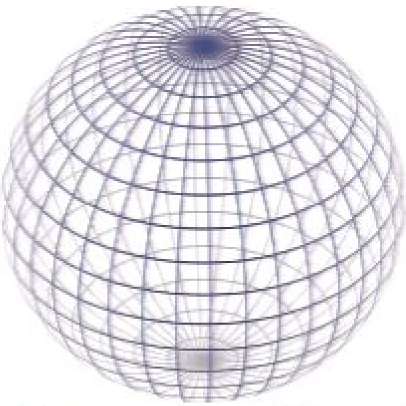
$\Omega_2 =$ 

If $\dim(\Omega_2) = 1$ then the theory is CPT (easy):

$\Omega_N =$ 

$\mathcal{G}_N$=permutation group.

4. Derivation of the Hilbert space formalism



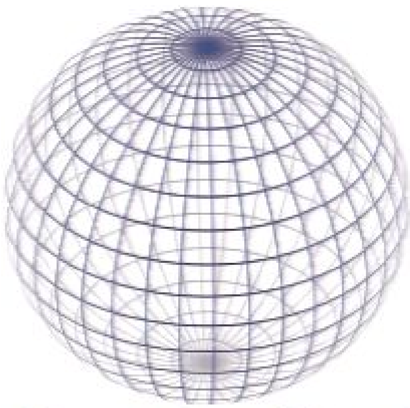
Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

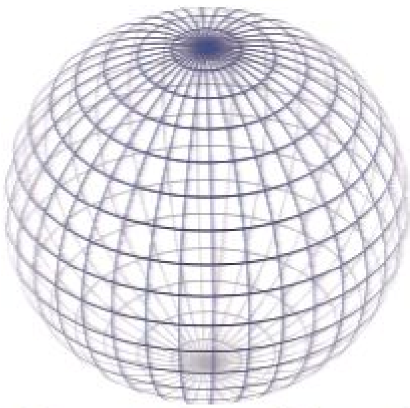
$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$

↗ CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1} :
- if d=even, then many possibilities (like $SU(d/2)$),
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

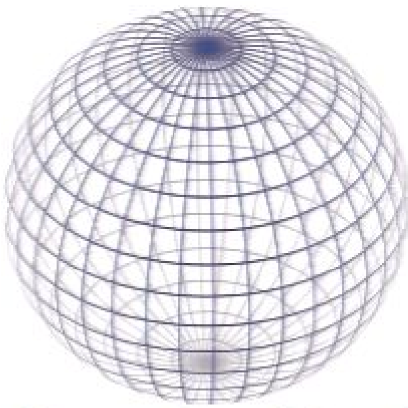
CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1} :

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

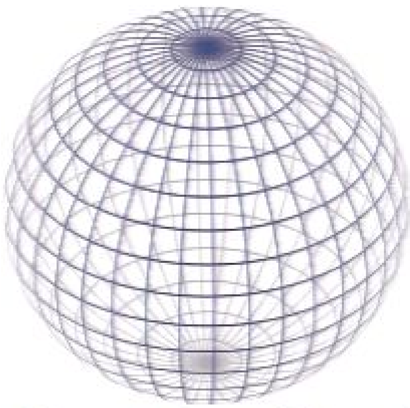
CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1}:

- if *d*=even, then many possibilities (like *SU(d/2)*),
- if *d*=odd and *d*≠7: only *SO(d)*,
- if *d*=7: *SO(7)* and Lie group G_2.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

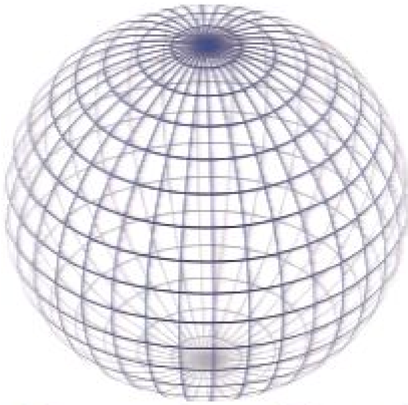
↑ CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1} :

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d\neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

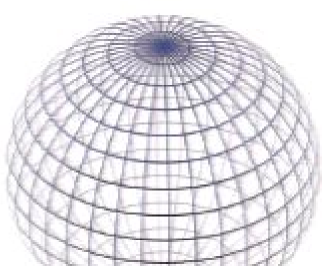
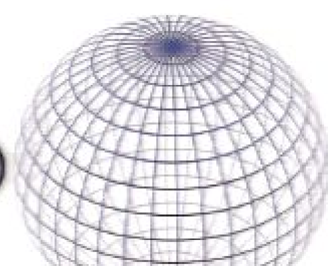
$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$

↗ CPT

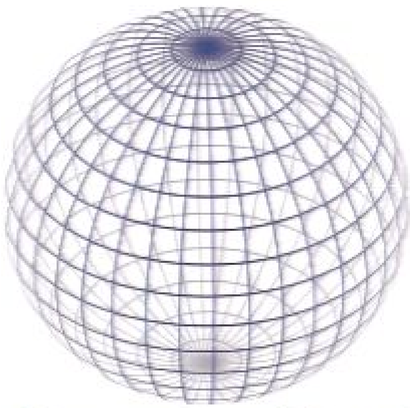
By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1}:

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

Two bits:  $\otimes$ 

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \dots\}.$$

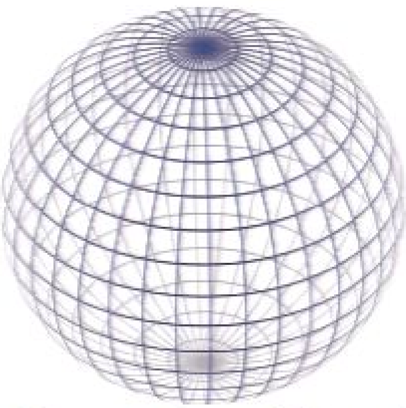
↑ CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

Onishchik `63: Compact connected transitive groups on S^{d-1} :

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

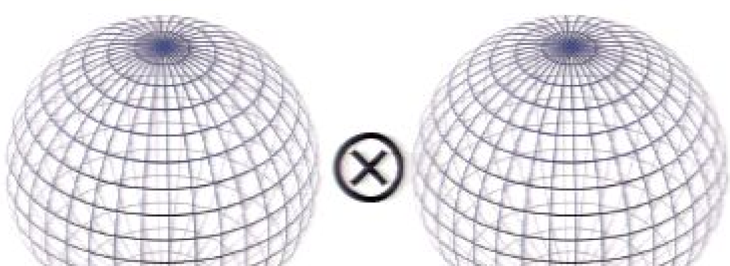
CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

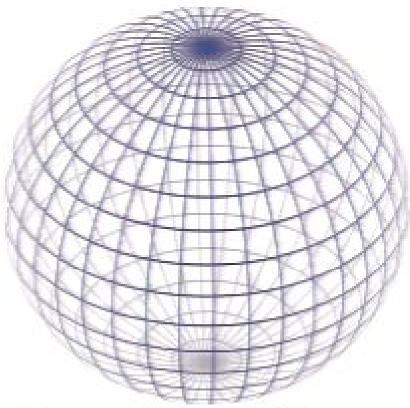
Onishchik `63: Compact connected transitive groups on S^{d-1} :

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

Two bits:



4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

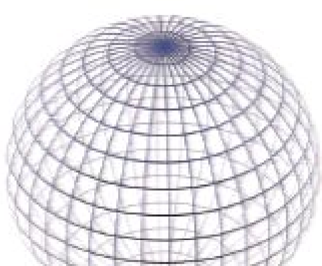
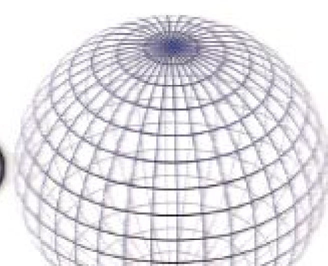
$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

↗ CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

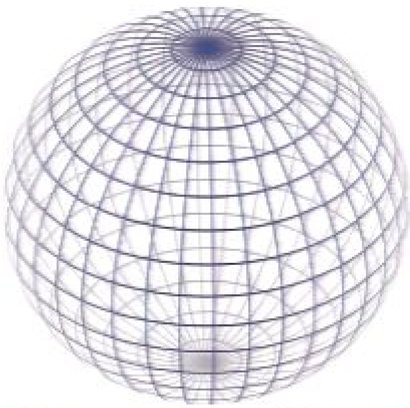
Onishchik `63: Compact connected transitive groups on S^{d-1} :

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

Two bits:  $\otimes$ 

Local transformations contain $\mathcal{G}_2 \otimes \mathcal{G}_2$.

4. Derivation of the Hilbert space formalism



Generalized bit Ω_2

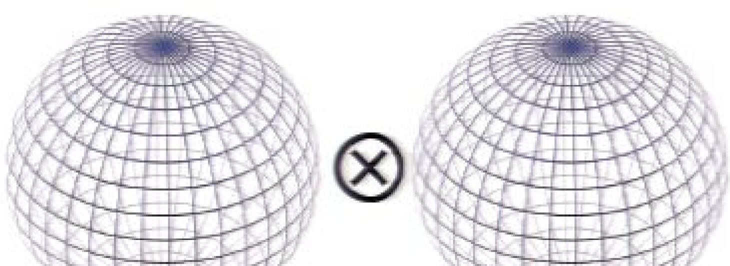
$$\dim(\Omega_2) = 2^r - 1 \in \{1, 3, 7, 15, 31, \ldots\}.$$

CPT

By reversibility axiom, $\mathcal{G}_2$ is transitive on the sphere.

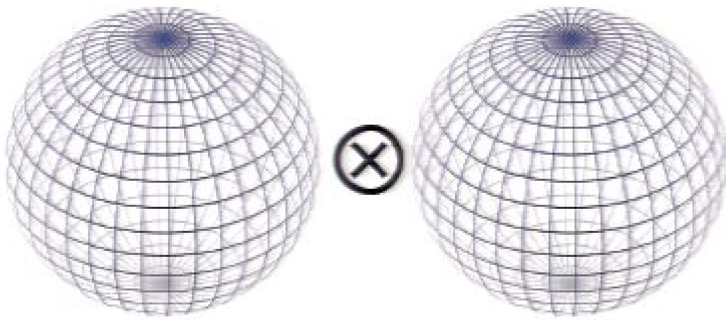
Onishchik `63: Compact connected transitive groups on S^{d-1}:

- ~~if d=even, then many possibilities (like $SU(d/2)$),~~
- if d=odd and $d \neq 7$: only $SO(d)$,
- if $d=7$: $SO(7)$ and Lie group G_2.

Two bits: 

$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

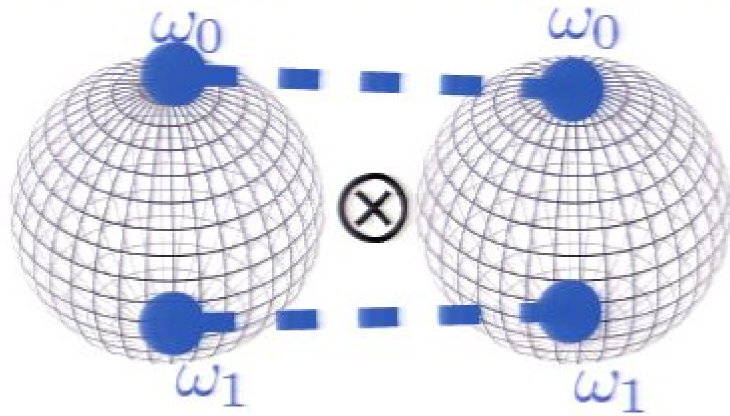
4. Derivation of the Hilbert space formalism

Two bits: 

$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

4. Derivation of the Hilbert space formalism

Two bits:

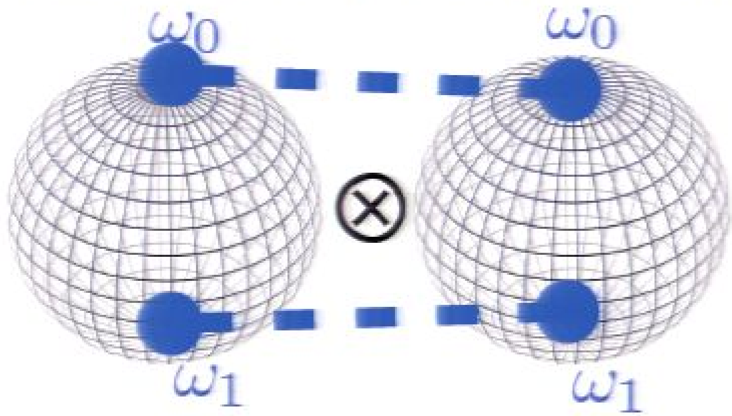


$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace") generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

4. Derivation of the Hilbert space formalism

Two bits:

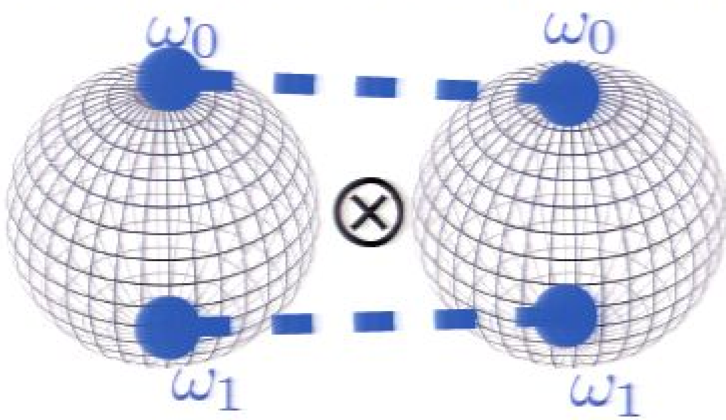


$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace“) generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

- Stabilized by $SO(d-1) \otimes SO(d-1)$.
- Counting dimensions with group rep. theory: if local transformations irreducible then orbit too large.
- But *SO(d-1)* is complex-reducible iff d=3 !

4. Derivation of the Hilbert space formalism

Two bits: 

$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace“) generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

- Stabilized by $SO(d-1) \otimes SO(d-1)$.
- Counting dimensions with group rep. theory: if local transformations irreducible then orbit too large.
- But *SO(d-1)* is complex-reducible iff d=3 !

Take-home message: Bloch ball 3-dimensional because *SO(d-1)* is reducible only for d=3.

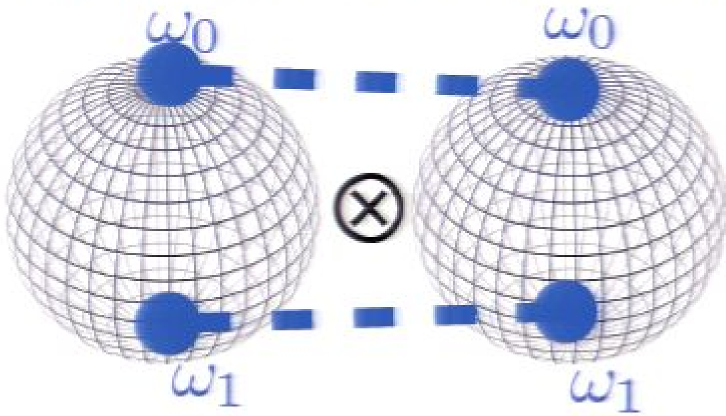
4. Derivation of the Hilbert space formalism

Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

4. Derivation of the Hilbert space formalism

Two bits: 

d≠7: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace“) generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

- Stabilized by $SO(d-1) \otimes SO(d-1)$.
- Counting dimensions with group rep. theory: if local transformations irreducible then orbit too large.
- But *SO(d-1)* is complex-reducible iff d=3 !

Take-home message: Bloch ball 3-dimensional because *SO(d-1)* is reducible only for d=3.

4. Derivation of the Hilbert space formalism

Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

4. Derivation of the Hilbert space formalism

Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

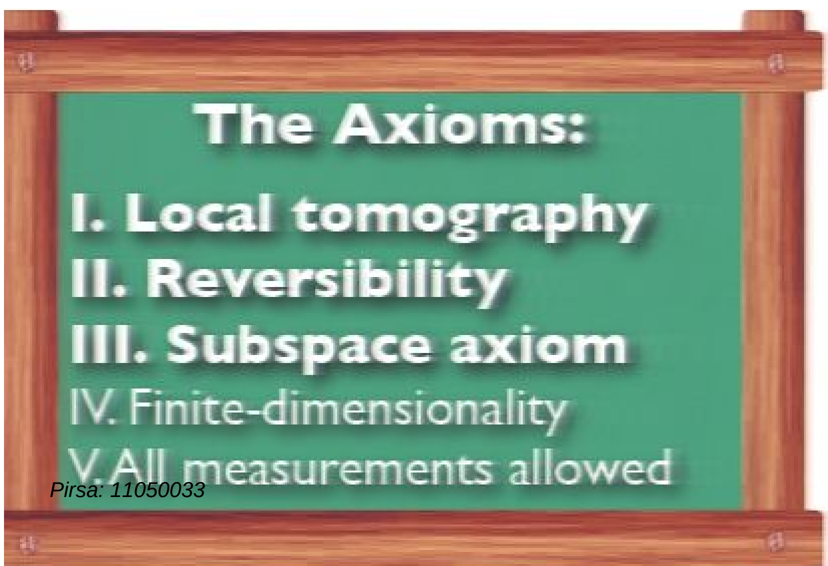
- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

Theorem: Every theory satisfying Axioms I-V (rather than CPT) is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

5. What's beyond quantum theory?



4. Derivation of the Hilbert space formalism

Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

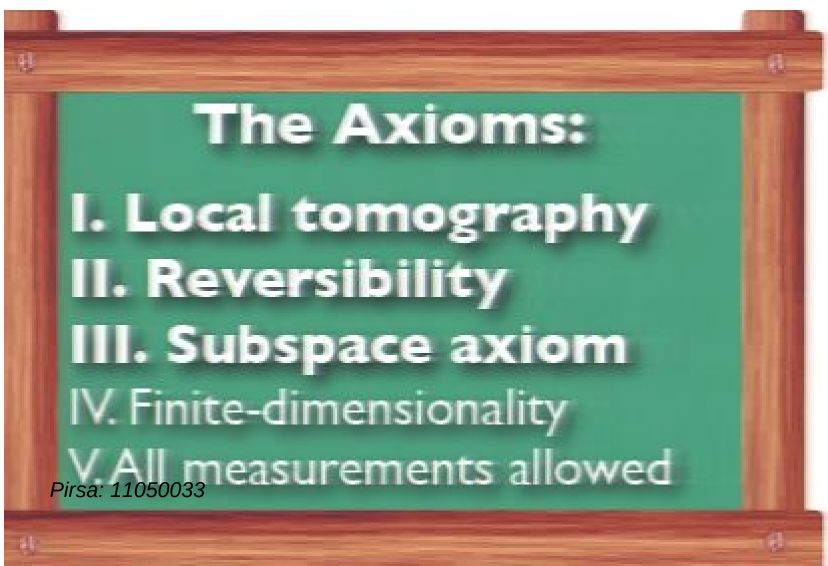
- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

Theorem: Every theory satisfying Axioms I-V (rather than CPT) is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

5. What's beyond quantum theory?



4. Derivation of the Hilbert space formalism

Map 3-vectors to Hermitian matrices: $L(\omega) := \frac{1}{2}\left(\mathbf{1} + \sum_{i=1}^{3} \omega_i \sigma_i\right)$

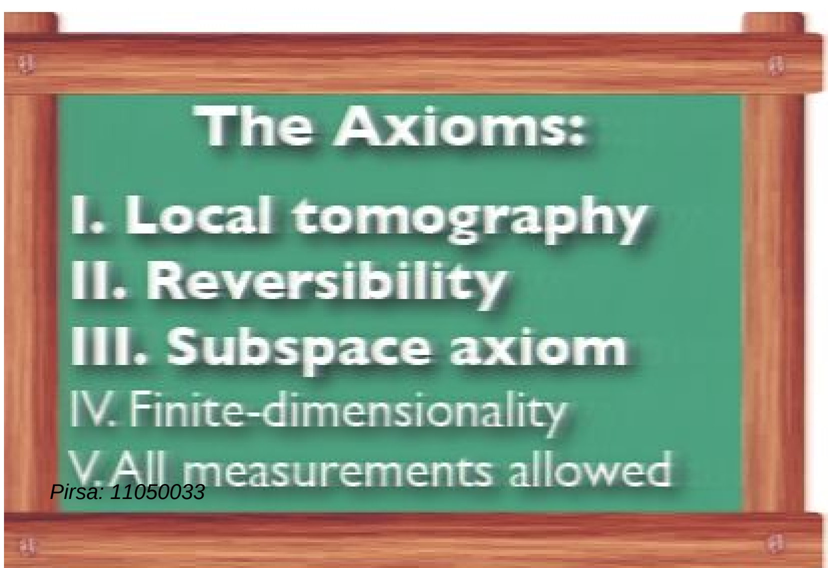
- Facts on universal quantum computation,
- Wigner's theorem
- some other tricks

prove:

Theorem: Every theory satisfying Axioms I-V (rather than CPT) is equivalent to $(\Omega_N, \mathcal{G}_N)$, where

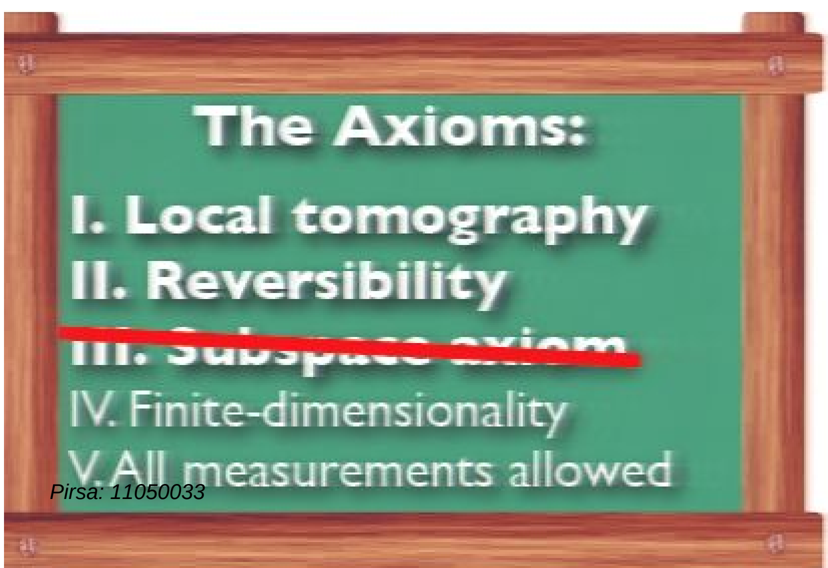
- Ω_N are the density matrices on $\mathbb{C}^N$,
- $\mathcal{G}_N$ is the group of unitaries, acting by conjugation,
- the measurements are exactly the POVMs.

5. What's beyond quantum theory?



5. What's beyond quantum theory?

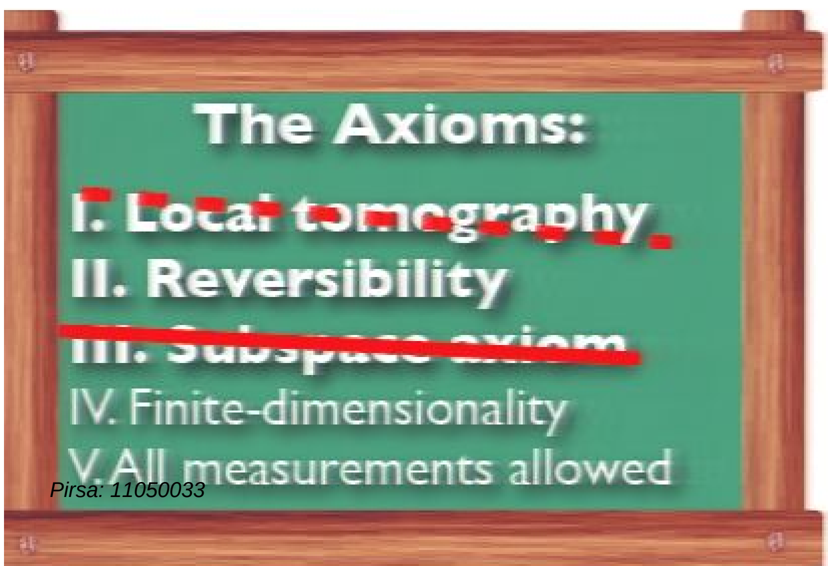
- Ll. Masanes, G. de la Torre (previous talk):
  If local state spaces are balls of dim. d, then entanglement & continuous reversibility for two balls only if $d=3$: only QT!



5. What's beyond quantum theory?

- Ll. Masanes, G. de la Torre (previous talk):
  If local state spaces are balls of dim. d, then entanglement & continuous reversibility for two balls only if $d=3$: only QT!

- MM, O. Dahlsten, V. Vedral, *Subsystem randomization as a universal phenomenon* (in preparation).
  Dynamical state space = state space + transformation group.
  (time evolution / computation)



General probabilistic versions of:

- Purity, Pauli operators,
- Clifford group, H.S. inner product,
- formula for typical entanglement,
- decoupling theorem.

Thank you!

arXiv:1004.1483v2

See also: G. Chiribella et al., arXiv:1011.6451v2
L. Hardy, arXiv:1104.2066v1

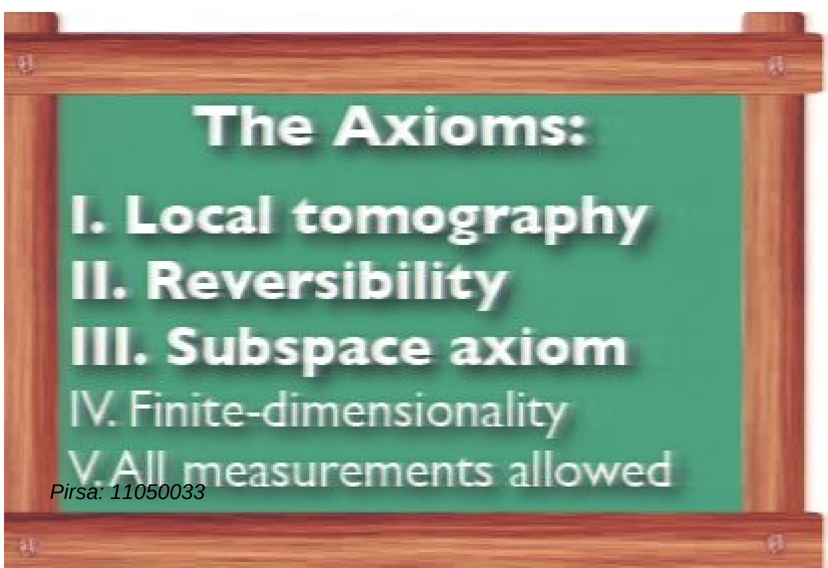
Thank you!

arXiv:1004.1483v2

See also: G. Chiribella et al., arXiv:1011.6451v2
L. Hardy, arXiv:1104.2066v1

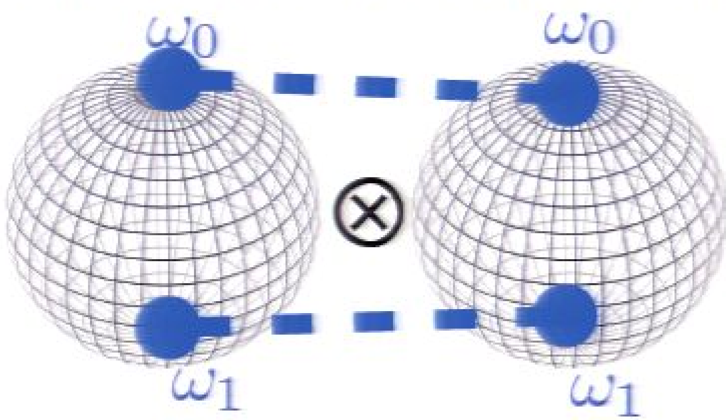
5. What's beyond quantum theory?

- Ll. Masanes, G. de la Torre (previous talk):
  If local state spaces are balls of dim. *d*, then entanglement & continuous reversibility for two balls only if *d*=3: only QT!



4. Derivation of the Hilbert space formalism

Two bits:



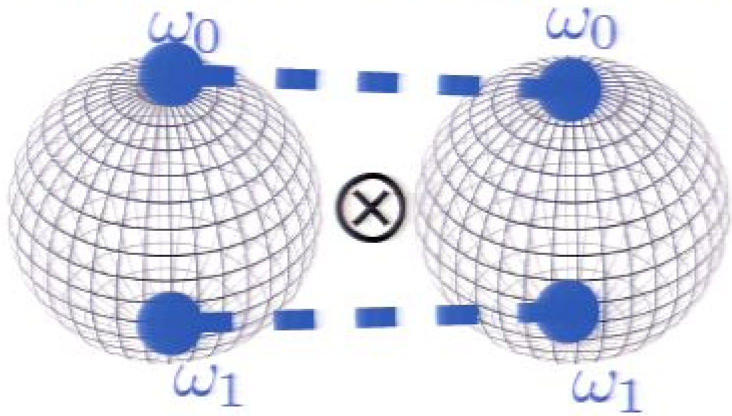
$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace") generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

- Stabilized by $SO(d-1) \otimes SO(d-1)$.
- Counting dimensions with group rep. theory: if local transformations irreducible then orbit too large.
- But *SO(d-1)* is complex-reducible iff d=3 !

4. Derivation of the Hilbert space formalism

Two bits:

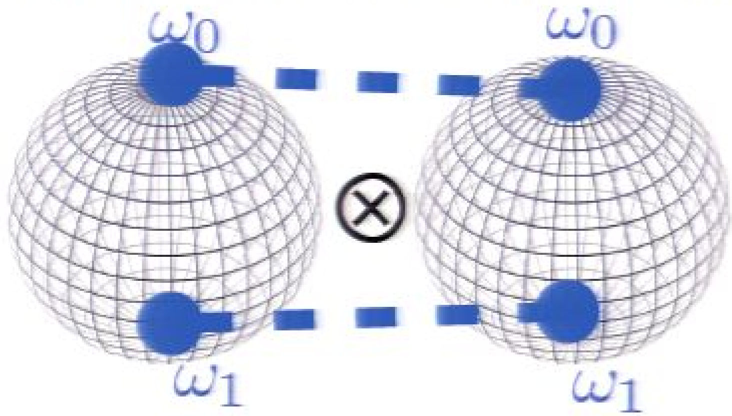


d≠7: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace") generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

4. Derivation of the Hilbert space formalism

Two bits:



$d \neq 7$: Local transformations contain $SO(d) \otimes SO(d)$.

Consider face („subspace") generated by $\omega_0 \otimes \omega_0$ and $\omega_1 \otimes \omega_1$ (again, a bit!)

- Stabilized by $SO(d-1) \otimes SO(d-1)$.
- Counting dimensions with group rep. theory: if local transformations irreducible then orbit too large.
- But *SO(d-1)* is complex-reducible iff d=3 !