

Title: 3 >> 2

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Abstract: Three-partite quantum systems exhibit interesting features that are absent in bipartite ones. Several instances are classics by now: the GHZ argument, the W state, the UPB bound entangled states, Svetlichny inequalities... In this talk, I shall discuss some on-going research projects that we are pursuing in my group (in collaboration, or in friendly competition, with other groups) and that involve three-partite entanglement or non-locality: * Activation of non-locality in networks. * Device-independent assessment of the entangling power of a measurement. * Can one falsify all models of hidden communication with finite speed? * Information causality in the three-partite scenario. I shall conclude by a blind excursion into uncertainty relations and cryptography, which also shows $3 > 2$ albeit with a different meaning.

Post-doc position open



Commitment to fairness: in case of otherwise equally competent candidates,

$3 > 2$



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$3 > 2$



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The classic examples of $3 > 2$

- Non-locality
 - GHZ argument
 - Svetlichny
- Entanglement theory
 - Unequivalent classes of entanglement (GHZ and W)
 - Bound entanglement for qubits
 - Bound information proved to exist

Here: four topics

1. Activation of non-locality in networks
2. Device-independent tests of entangling measurements
3. Falsify “hidden signaling”
4. Information causality in the 3-partite scenario

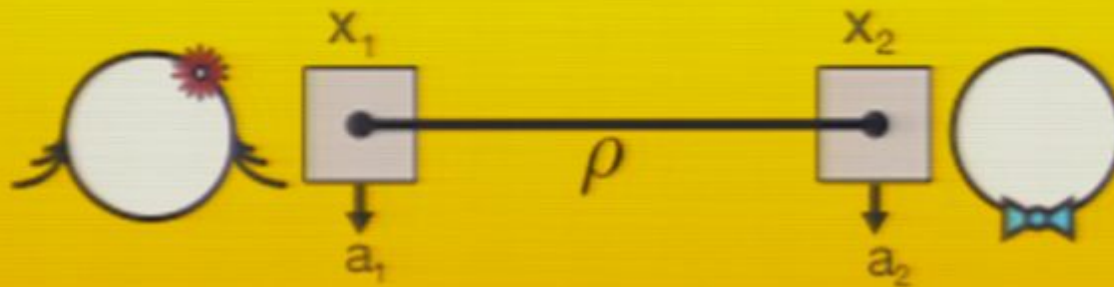
ACTIVATION OF NON-LOCALITY IN NETWORKS

D. Cavalcanti, M. Almeida, V.S., A. Acín, Nature Comm. 2, 184 (2011)

R. Rabelo, D. Cavalcanti, V.S., in preparation

2 parties

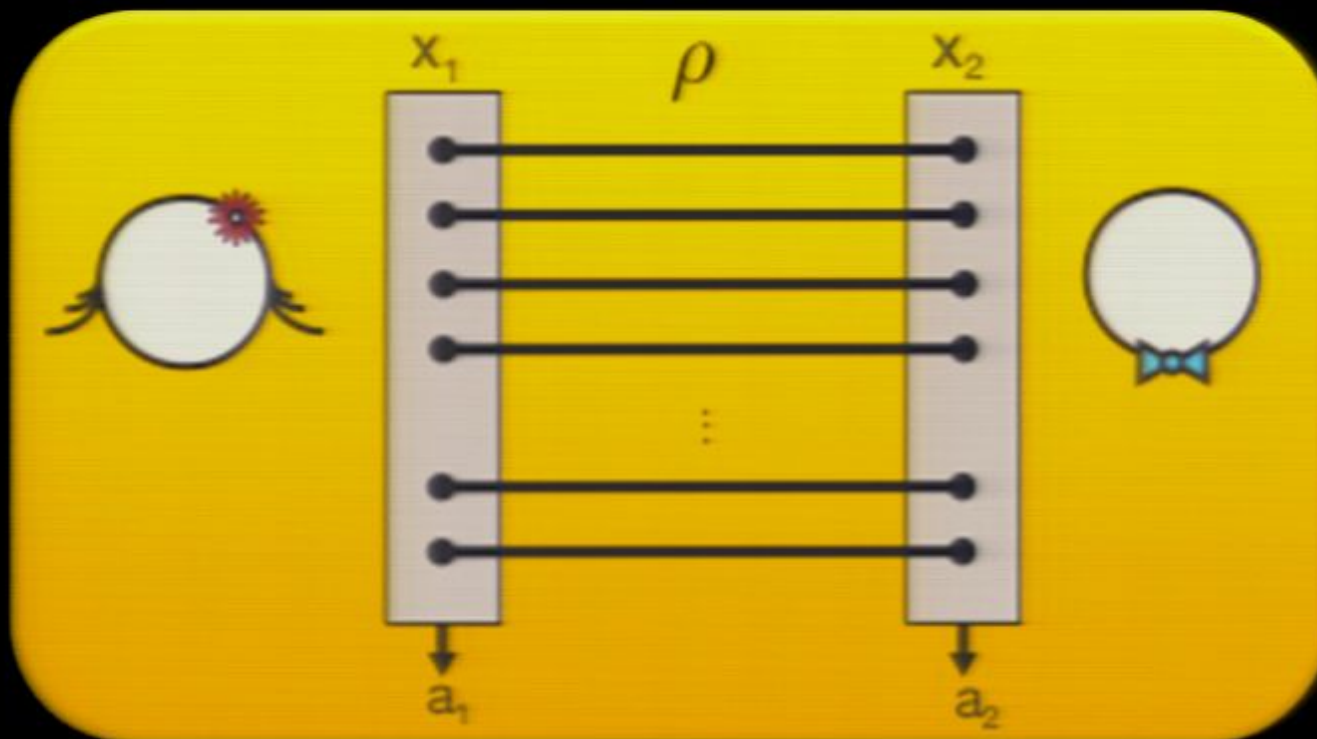
$$P(a_1 a_2 | x_1 x_2) \neq \sum_{\lambda} p(\lambda) P(a_1 | x_1, \lambda) P(a_2 | x_2, \lambda)$$



Can be “local” even if ρ is entangled

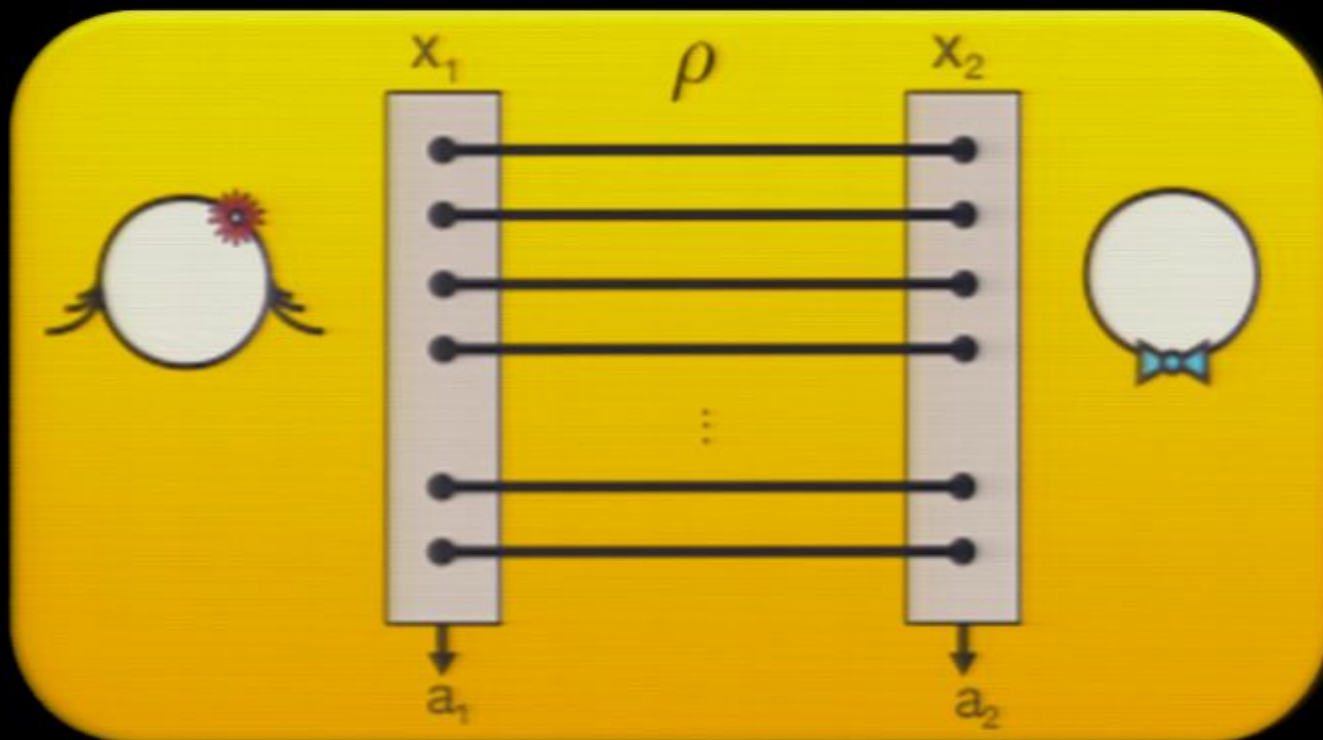
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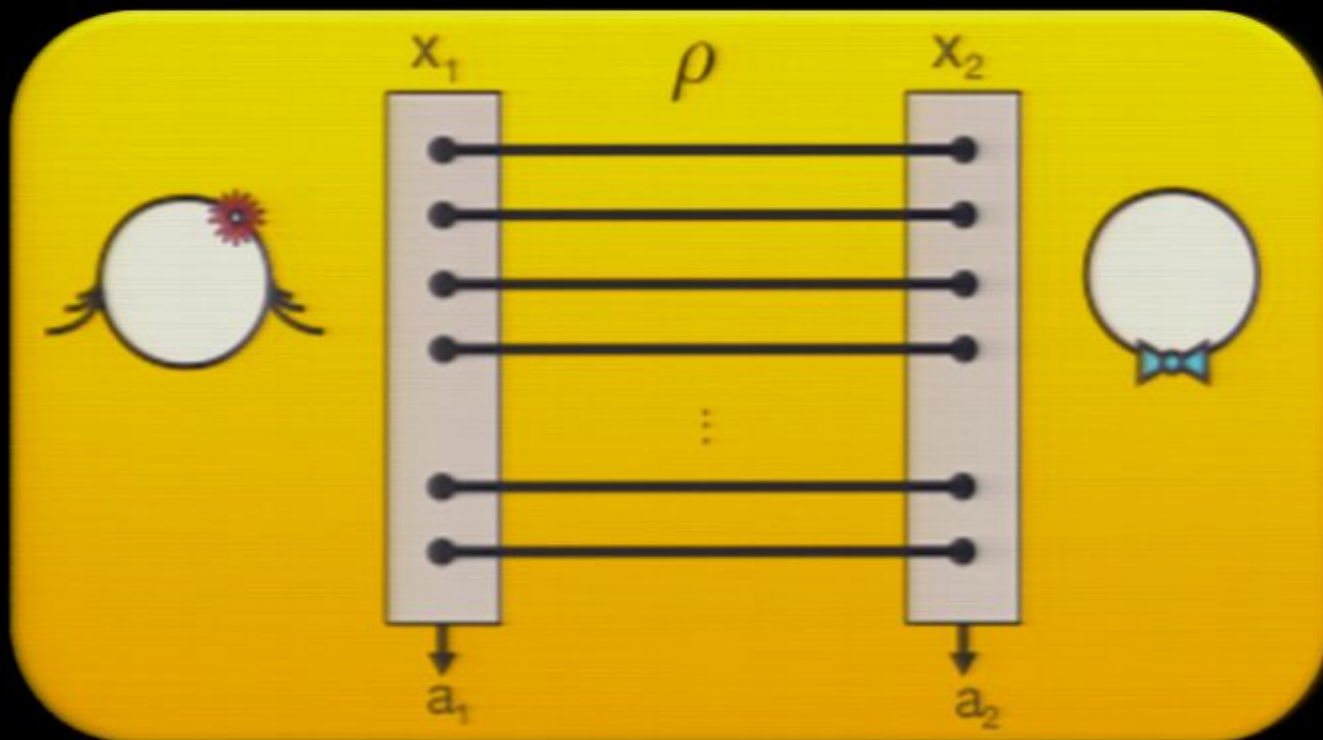
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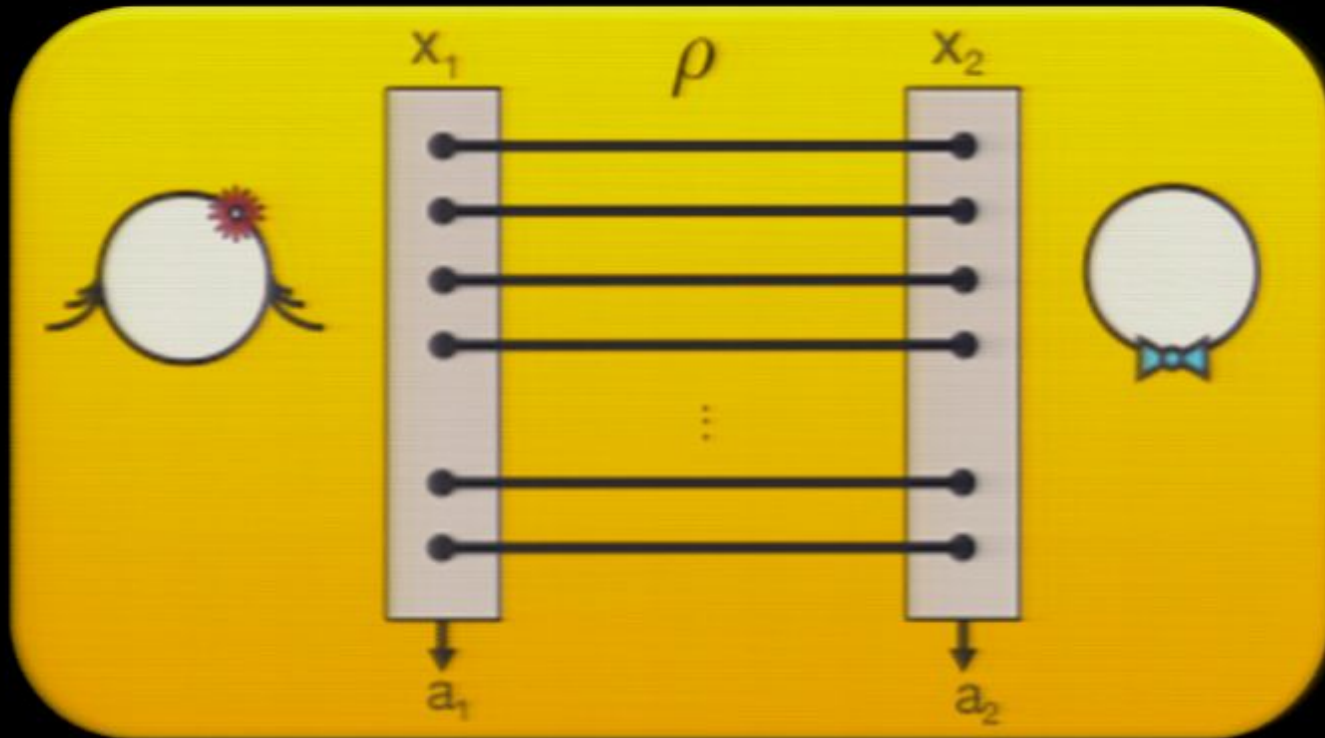
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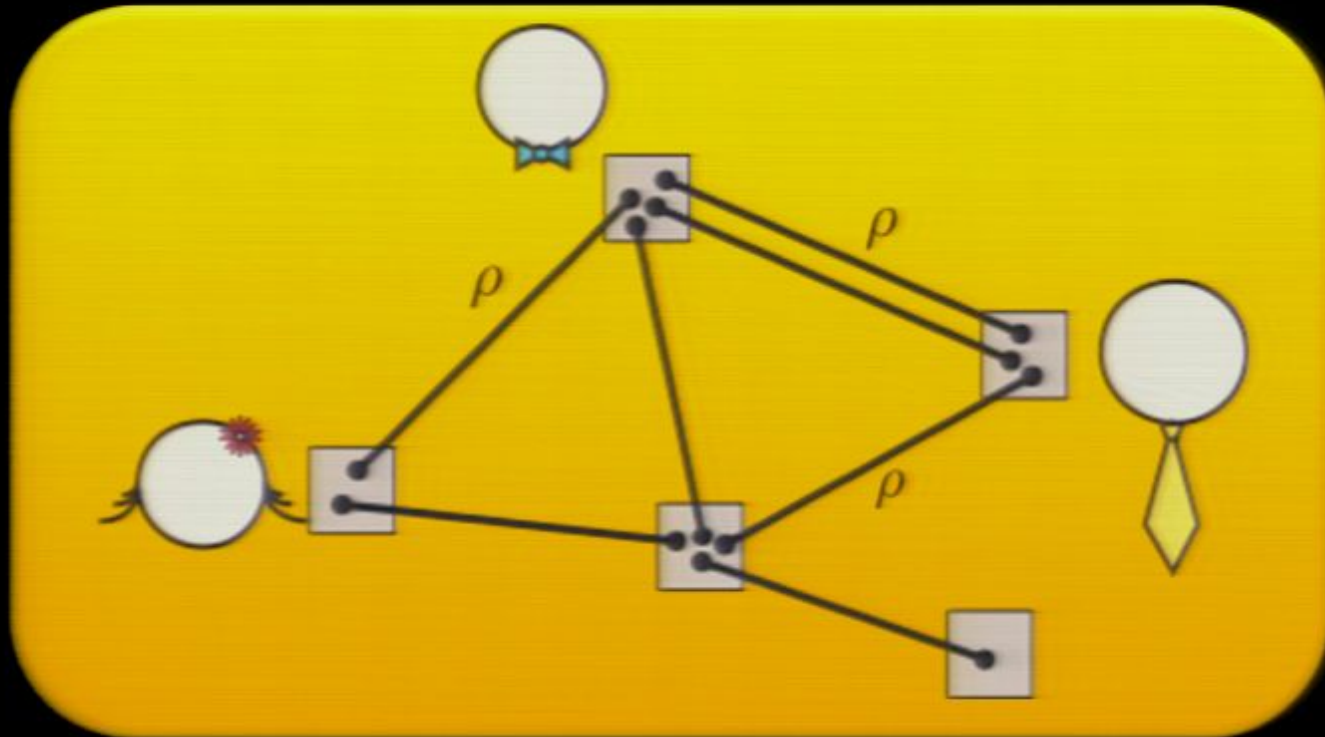
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- Can one see non-locality better with many copies?
- Not equivalent to entg distillation: CC not allowed
- Problem basically open (Navascues-Vertesi: CHSH...)

Network scenario

Network scenario: distribute ρ among more than 2 parties

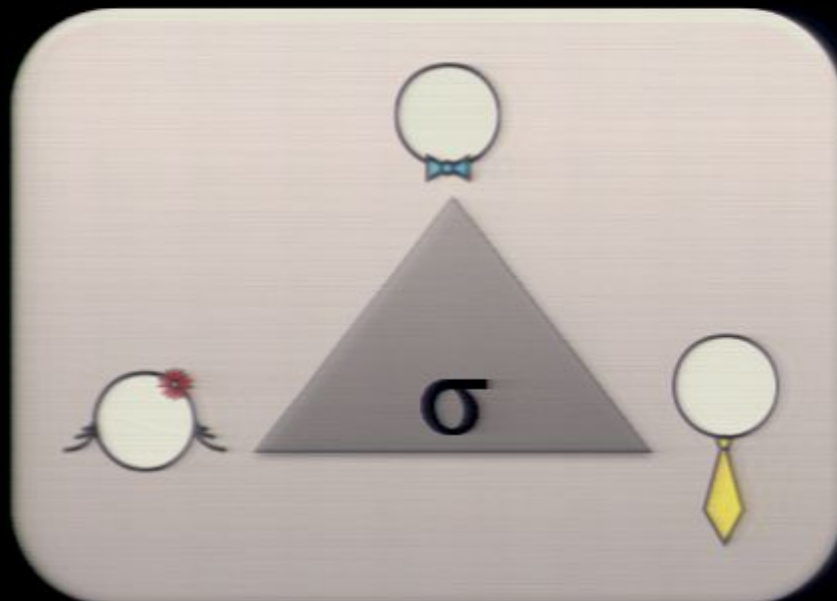


ρ is a “**non-local resource**” if it can provide non-locality for some network

Observation

If there exist local measurements by k parties such that, for one measurement outcome, the resulting state among the remaining $N-k$ parties violates a Bell inequality, then the **initial** N -party state violates a Bell inequality.

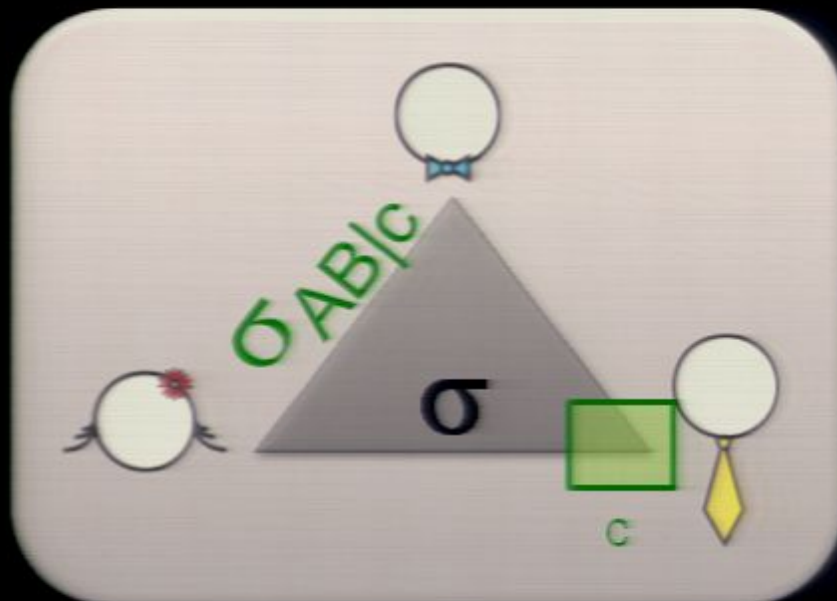
Rk: stronger requirement than hidden-non-locality (Popescu; Masanes-Liang-Doherty)



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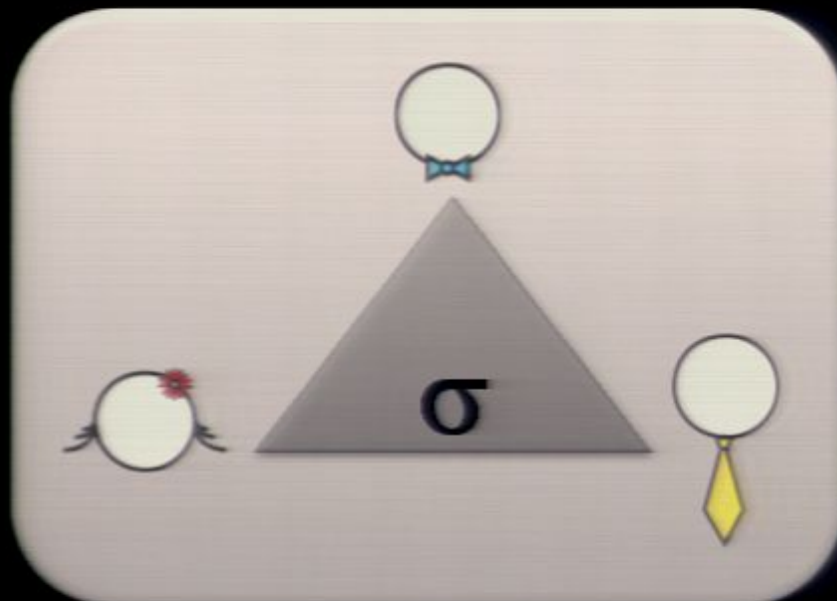
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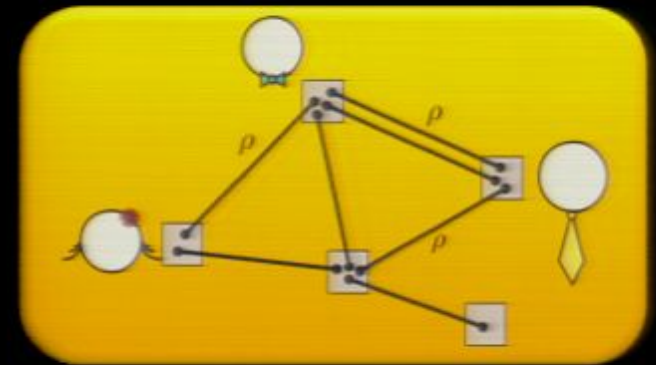
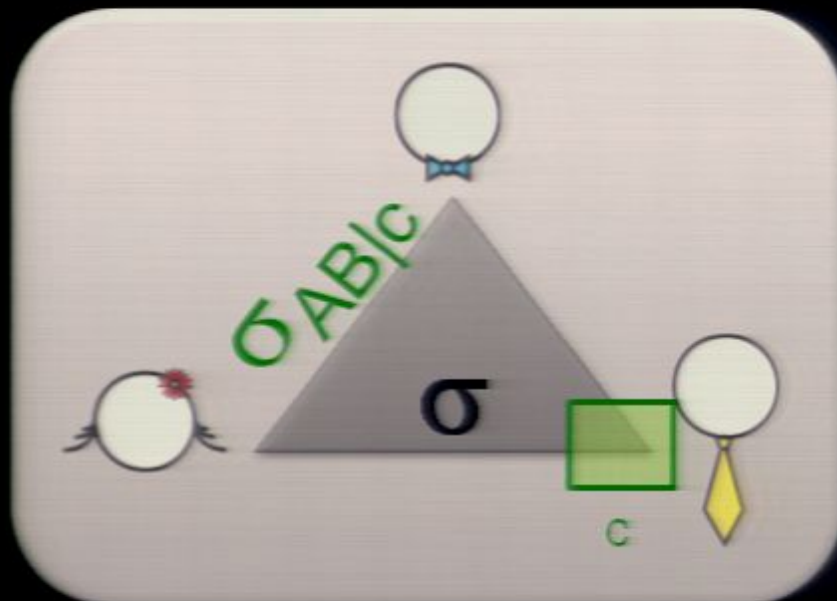
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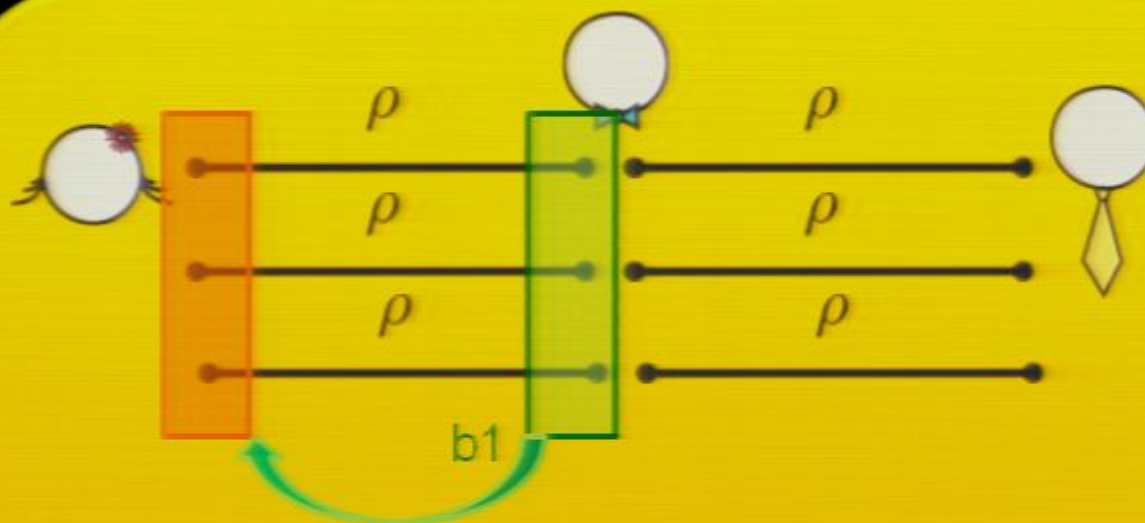


In the case the N -party state is constructed as a network of ρ , this criterion can be used to prove that ρ is a non-local resource

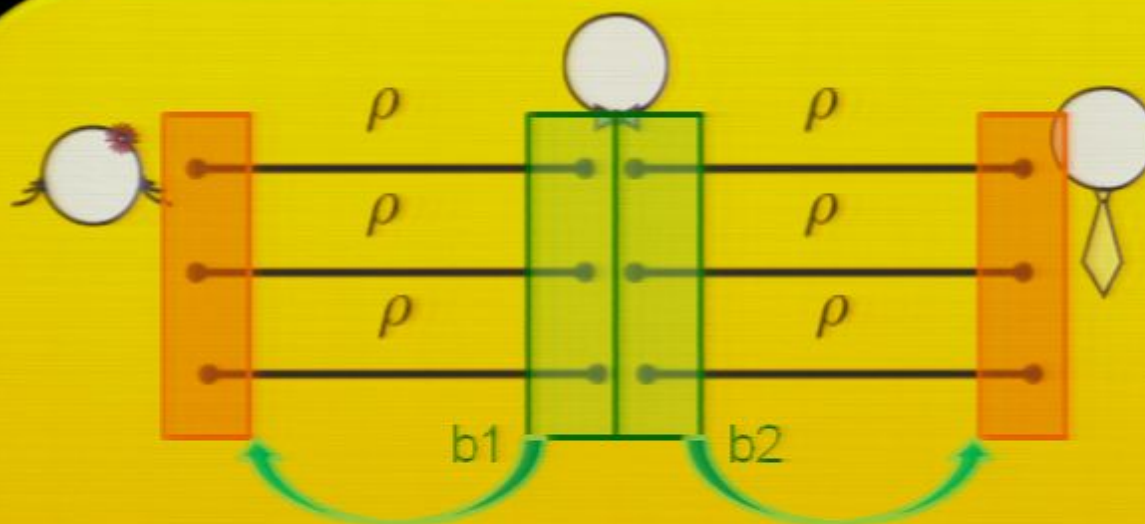
One-way entg dist \Rightarrow NL (1)



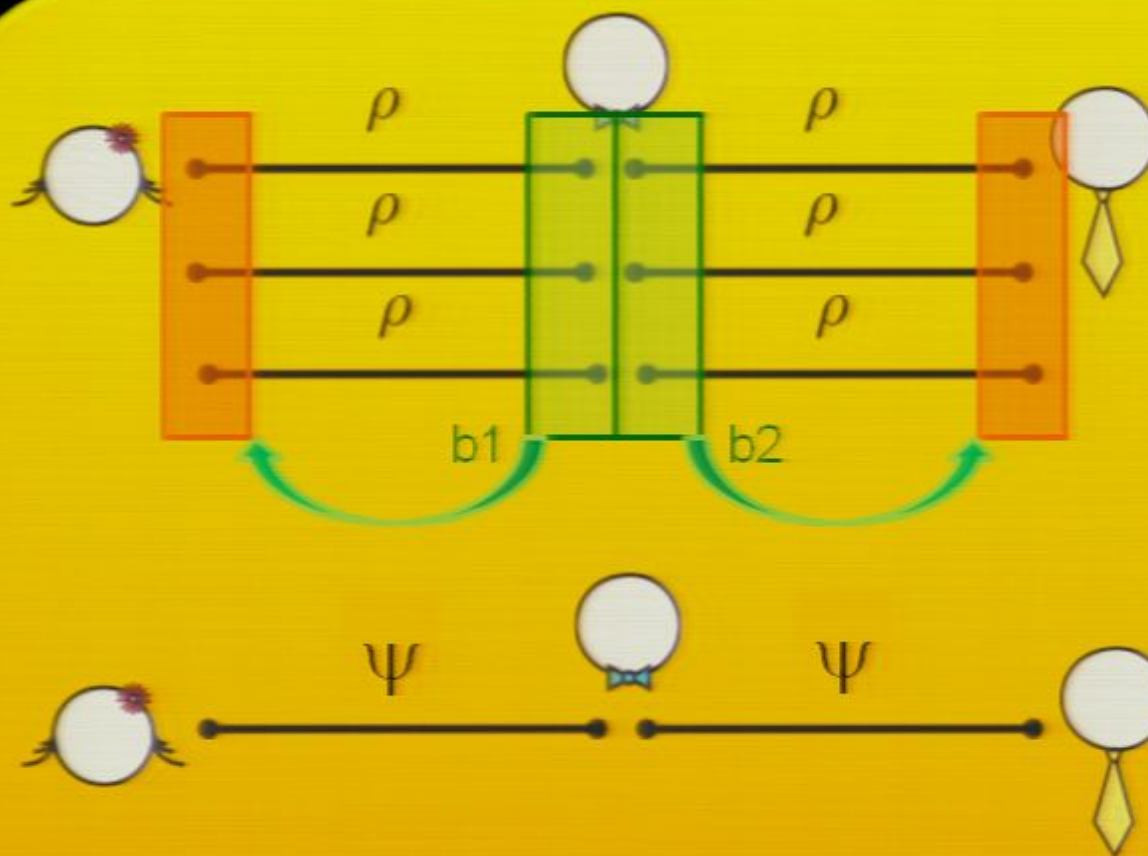
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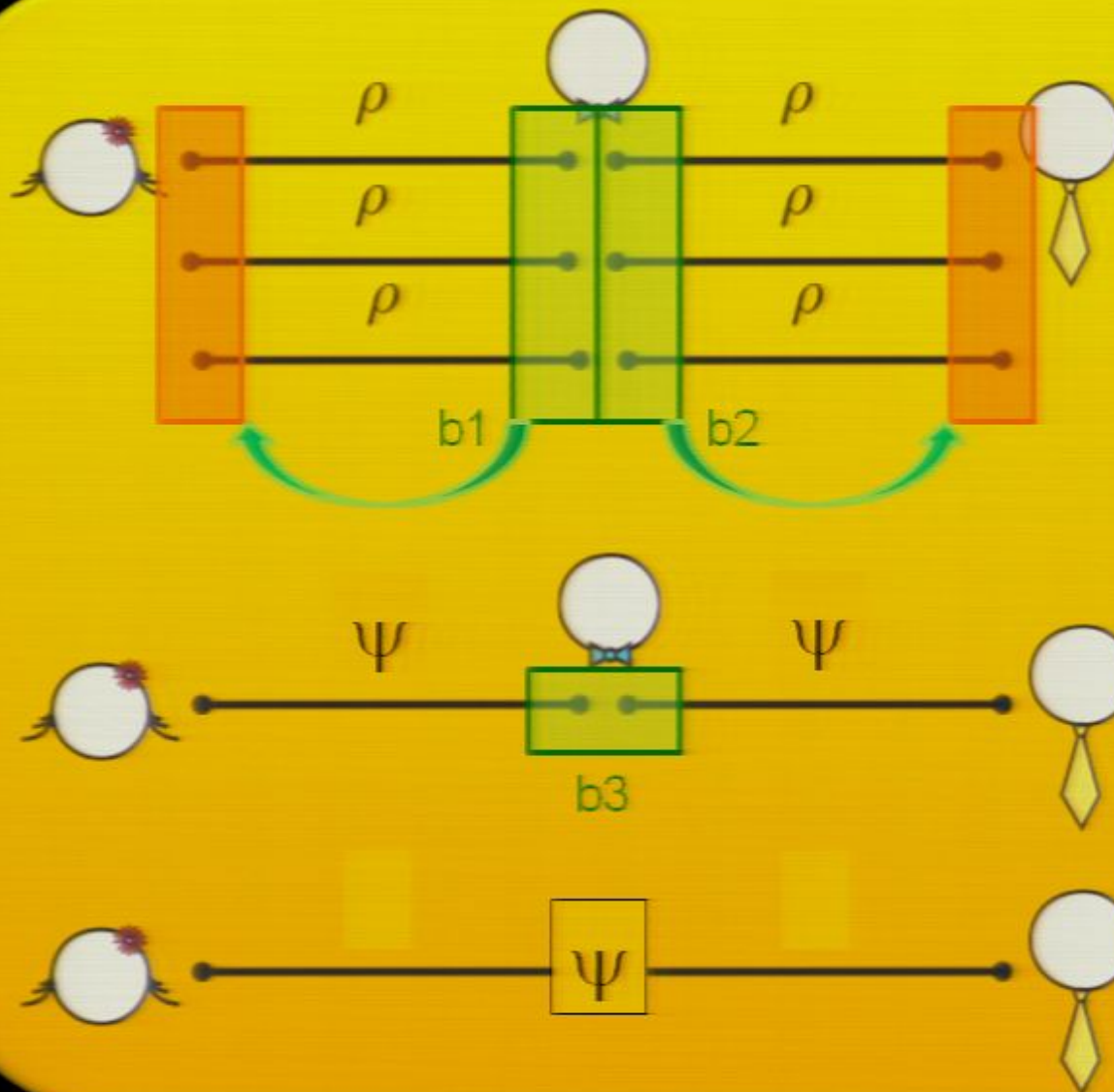
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One-way entg dist \Rightarrow NL (2)

An example of **activation** of non-locality:

Take the state produced by an **erasure channel**:

$$\rho = \frac{1}{k} |\Phi_+\rangle \langle \Phi_+| + \left(1 - \frac{1}{k}\right) \frac{\mathbb{I}}{2} \otimes |2\rangle \langle 2|$$

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- is **k-shareable between Alice and k Bobs** \Rightarrow cannot violate any Bell inequality with k measurements on Bob and arbitrarily many on Alice \checkmark

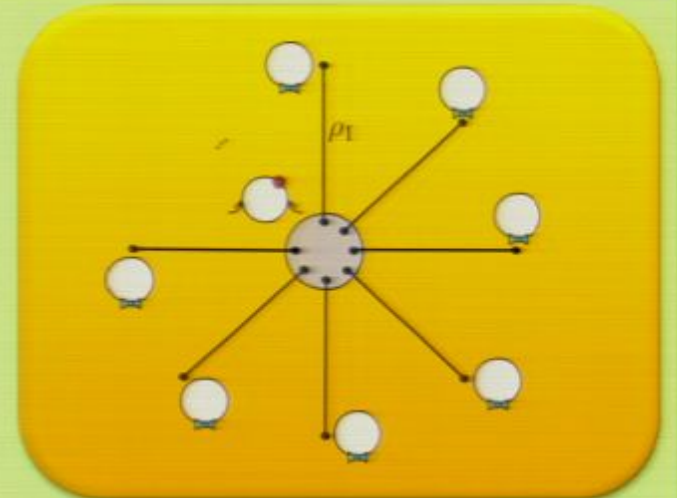
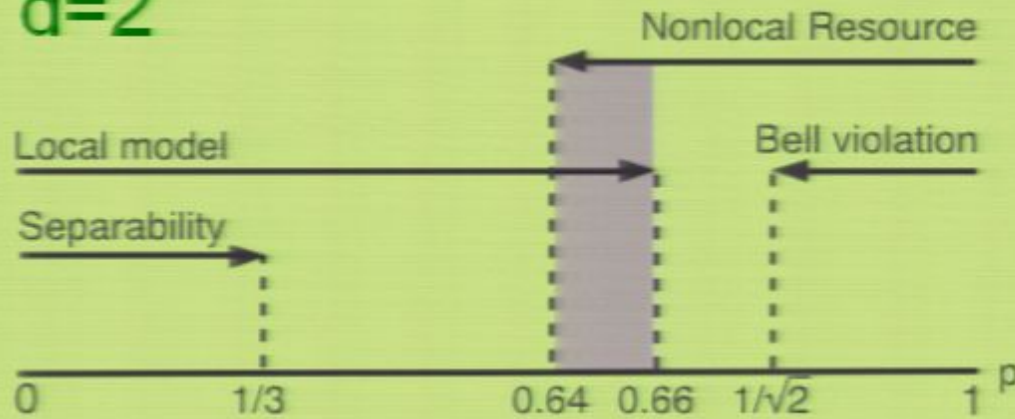
Isotropic states = depolarizing channel

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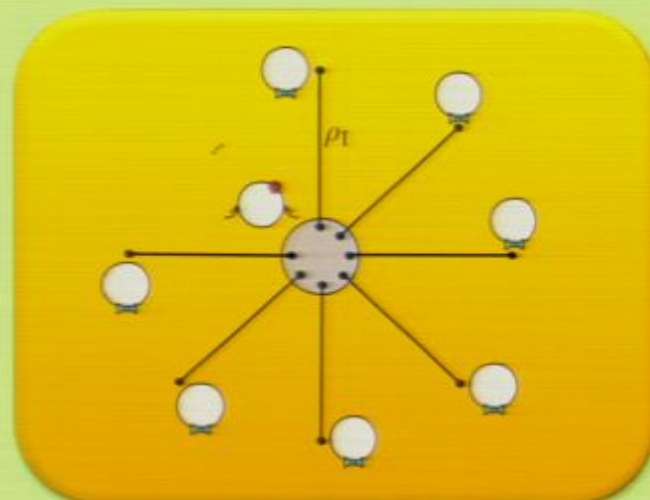
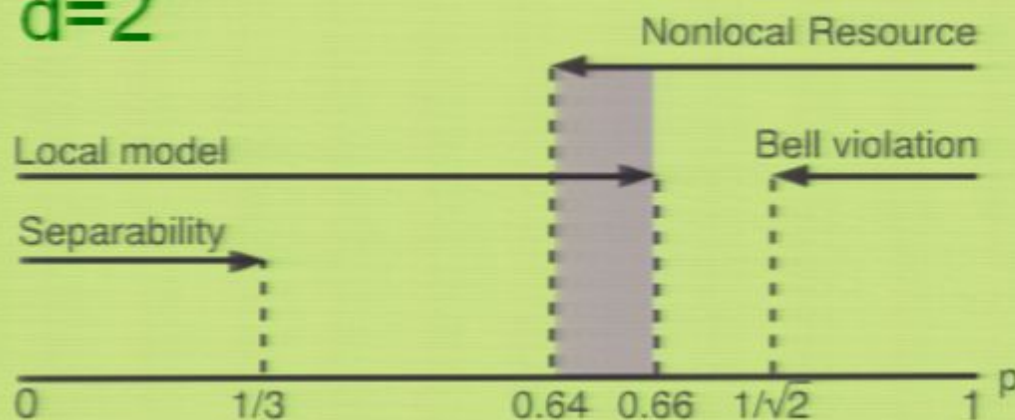


cf. A. Sen et al. PRA 2006

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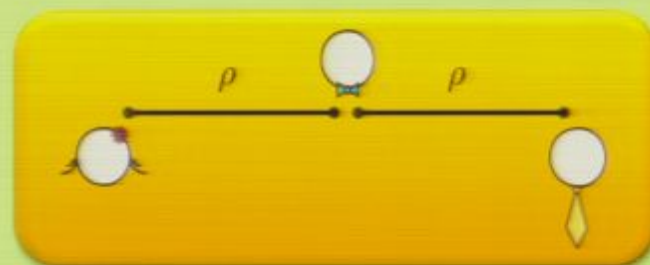
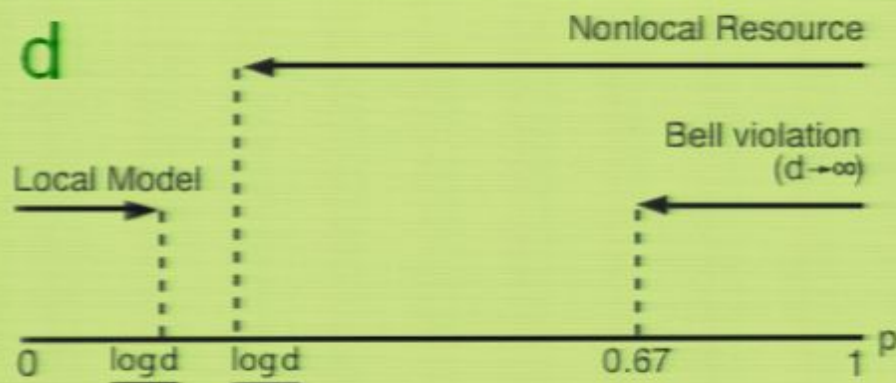
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d



cf. Junge et al., PRL 2010

Violation of Bell's inequalities in a shopping mall (Kurtsiefer et al.)



Activation of non-locality

Local states (like Werner states) can lead to violation of Bell's inequalities if shared among more than two parties.

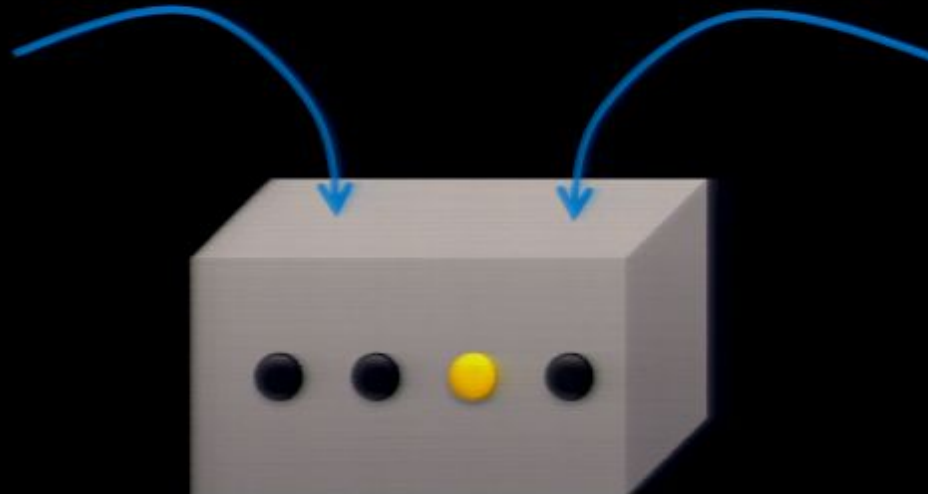
DEVICE-INDEPENDENT TEST OF ENTANGLING MEASUREMENTS

Motivation: circuit testing

A vendor sells allegedly quantum devices: you can buy “sources of entangled pairs”, “local unitaries” ...

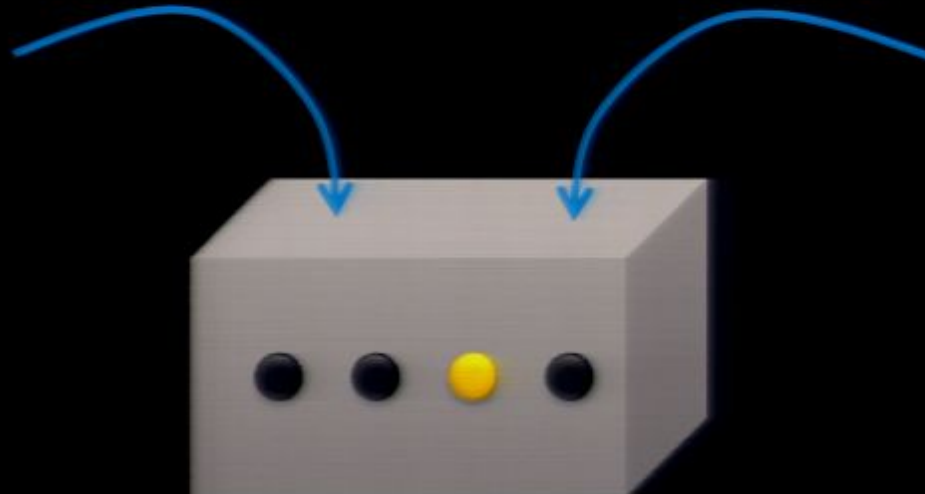
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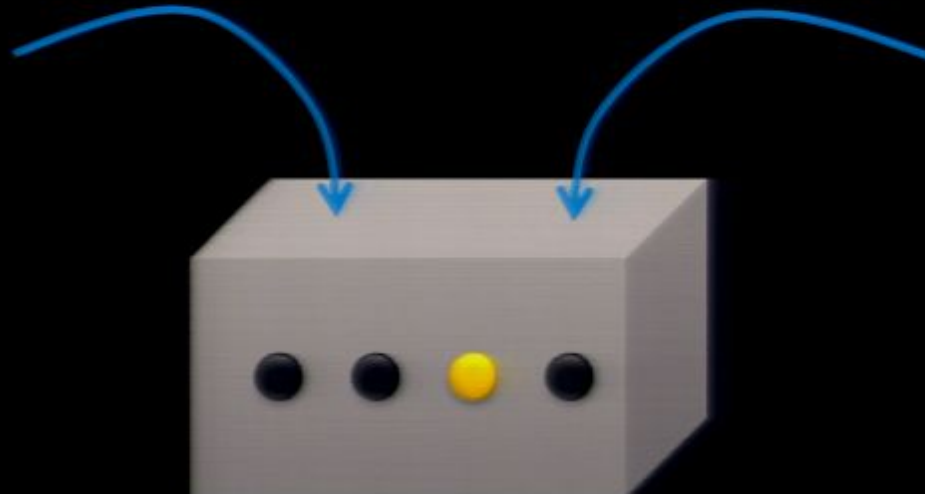
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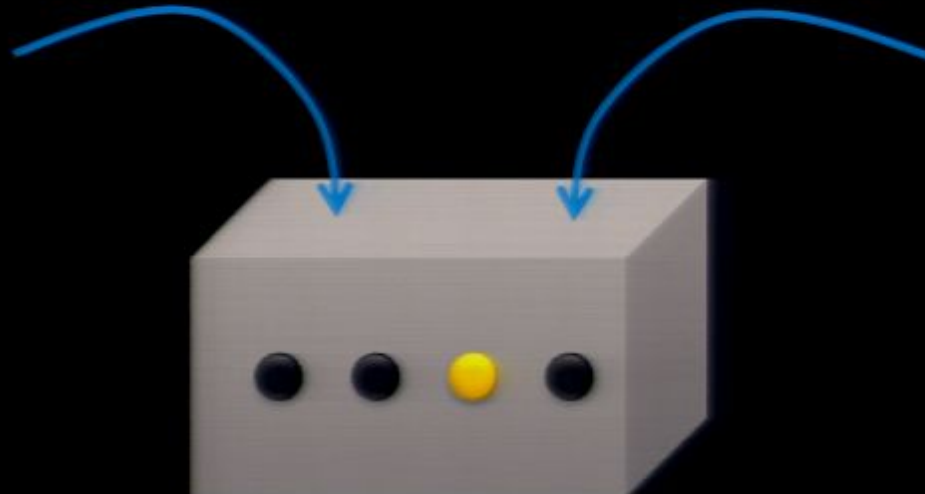


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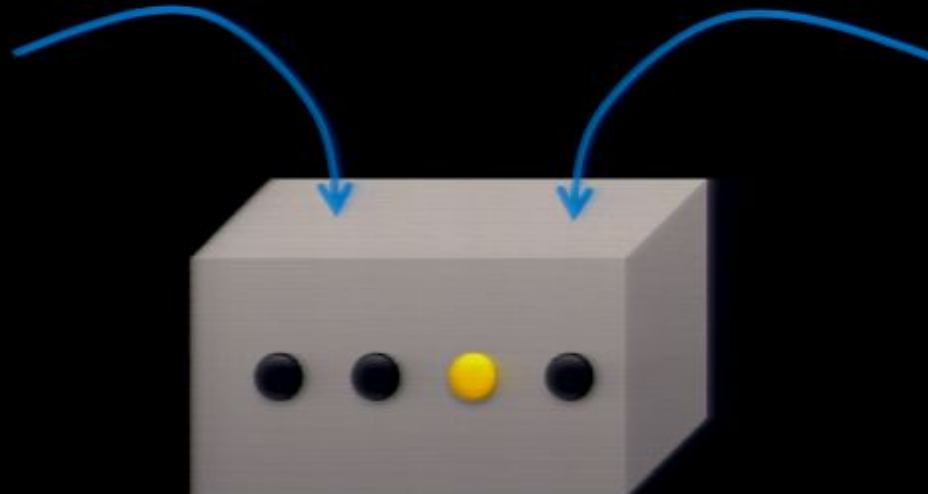


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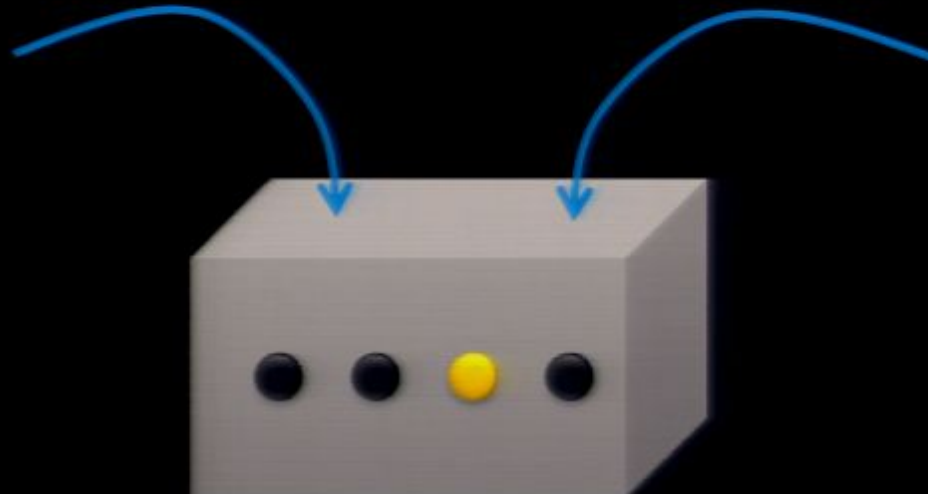
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How close is this box from performing a Bell-state measurement on some 2-dimensional sub-systems in the incoming signals?

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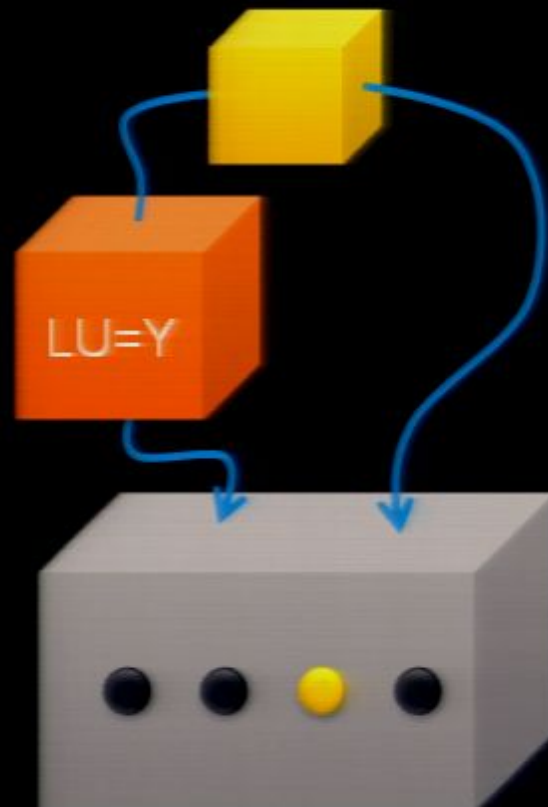
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Of course, everything must be checked with the same vendor's products!

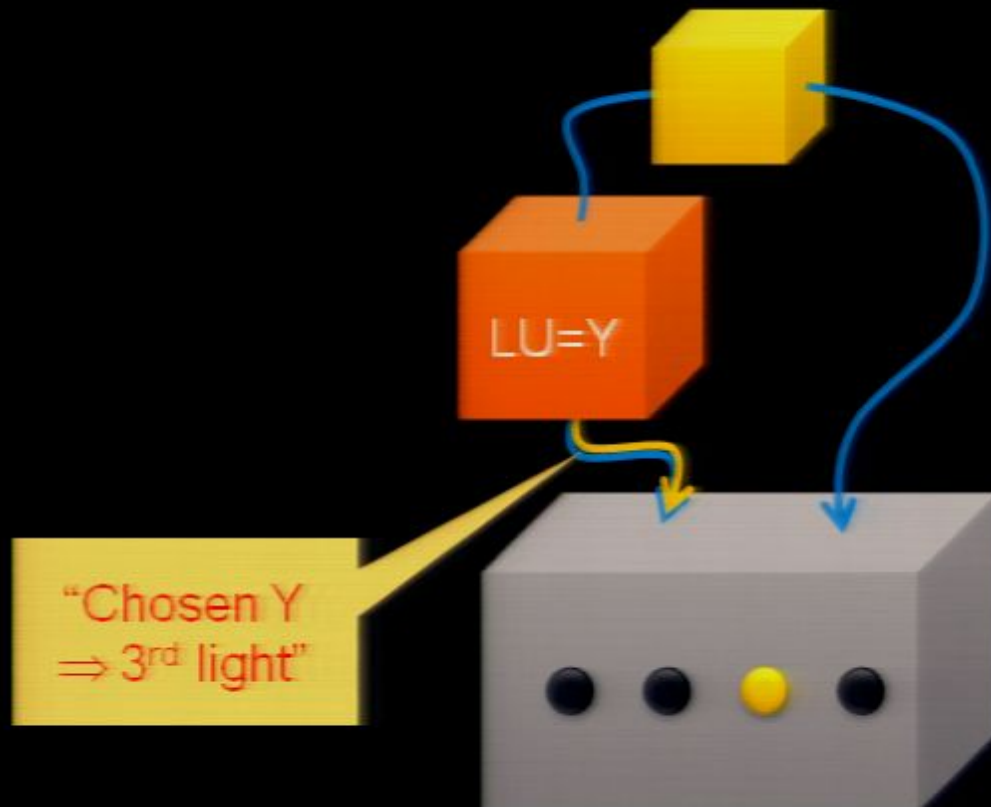
What does not work

0. Buy a “source”, “local measurements”, a “local unitary” and a “BSM”
1. Check the entanglement of the source (e.g. Bardyn et al PRA 2009)
2. Set the local unitary to I , X , Y , Z and check that one light clicks deterministically for each



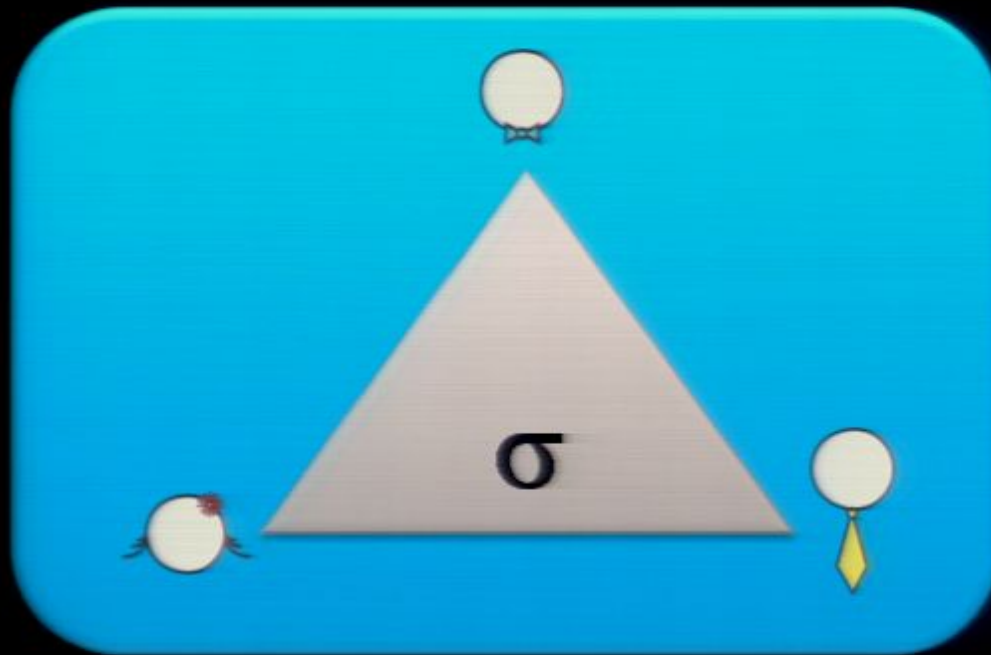
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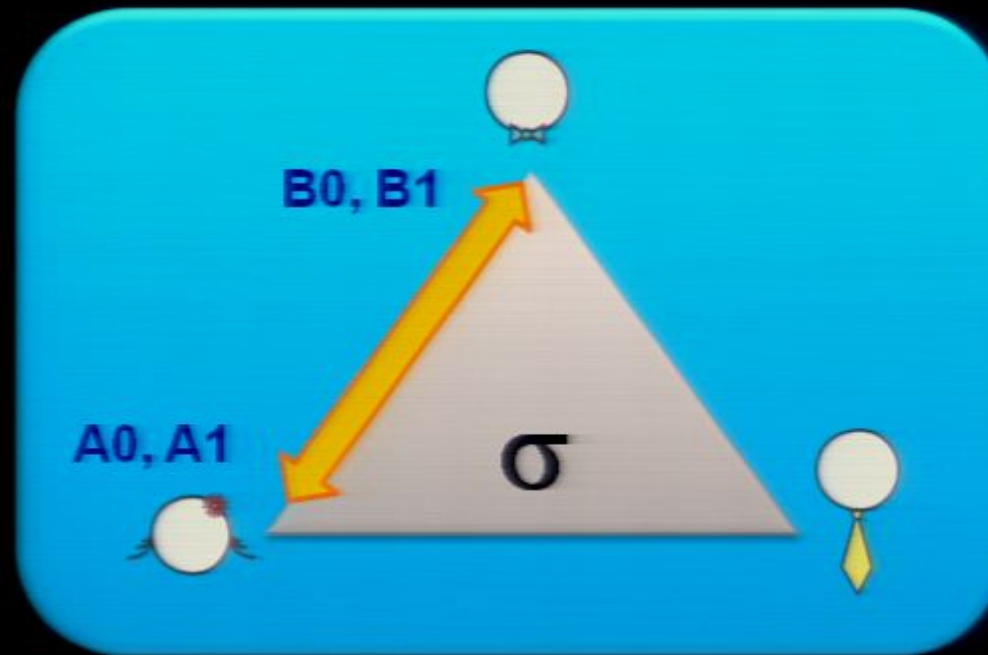
A three-partite strategy

Local measurements: A_0, A_1, C_0, C_1 : 2 outputs; B_0, B_1, B_2 : four outputs



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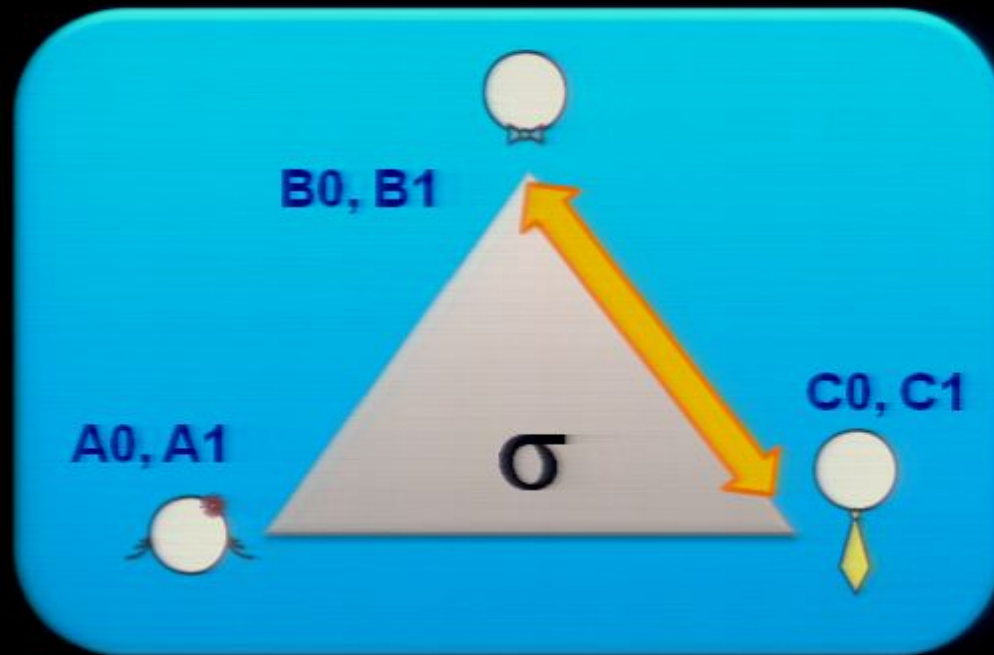
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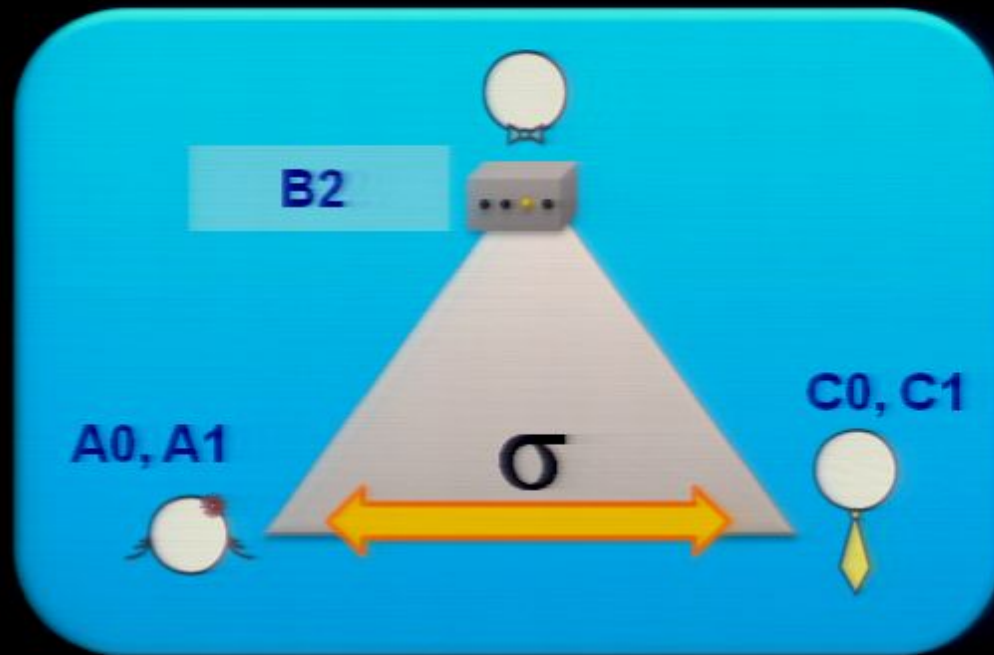


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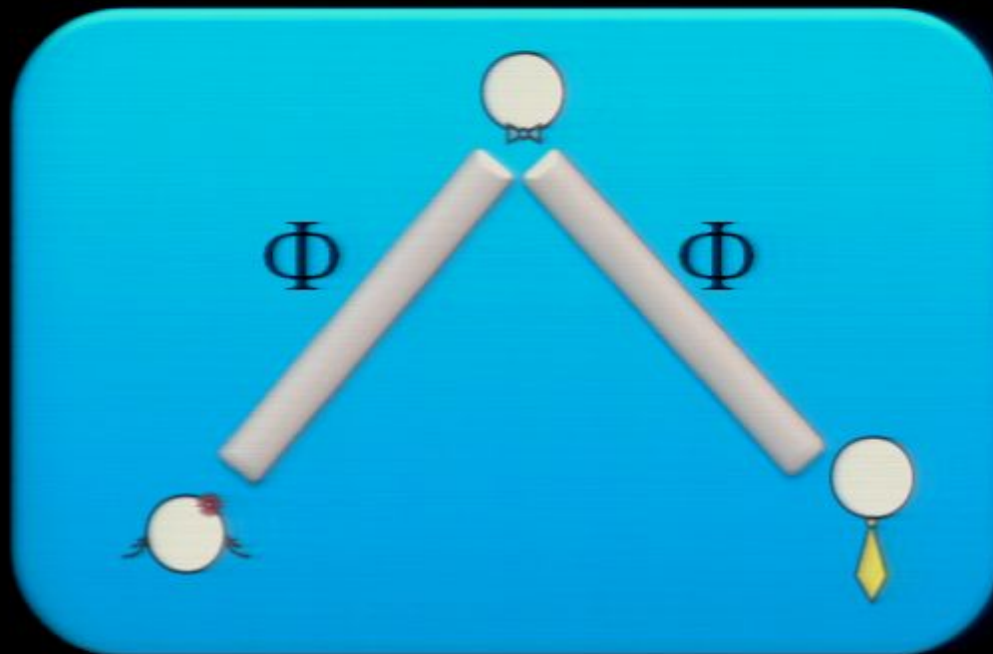
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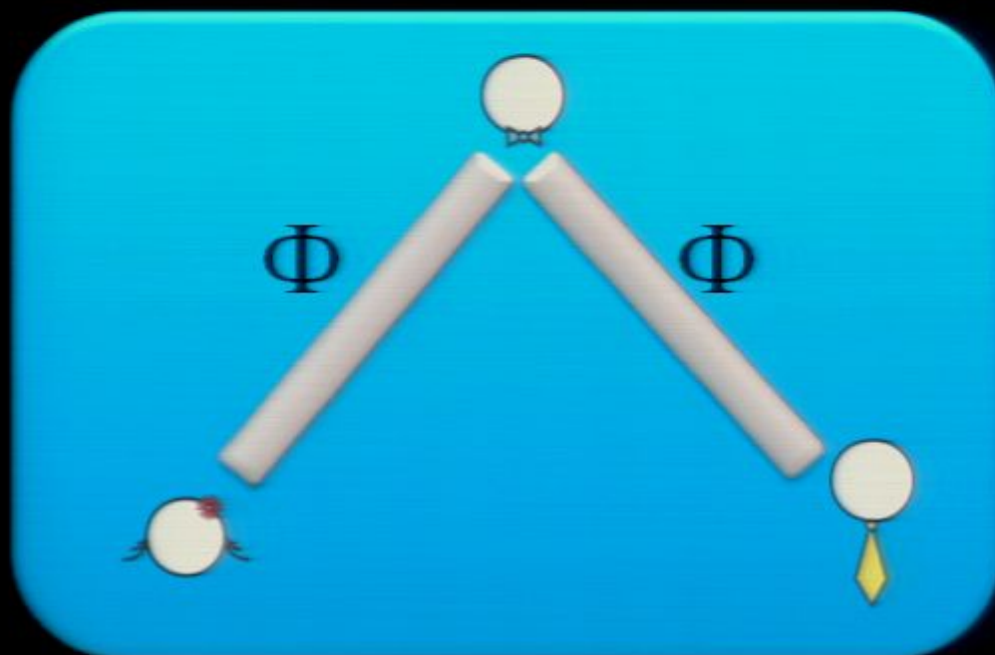
Intuition

What QM can do: entanglement swapping of two singlets



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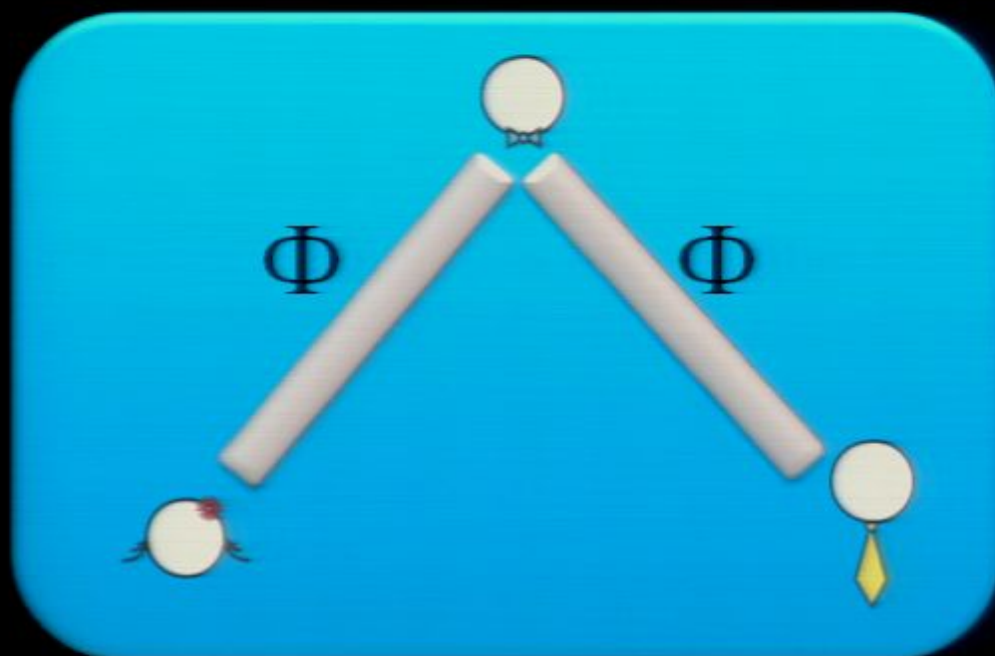


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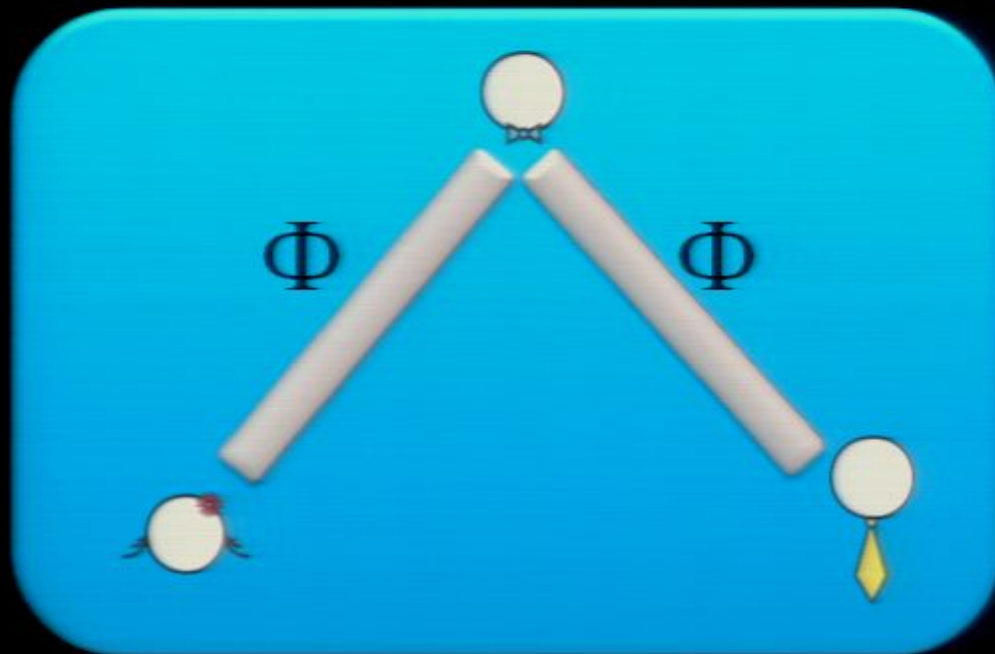
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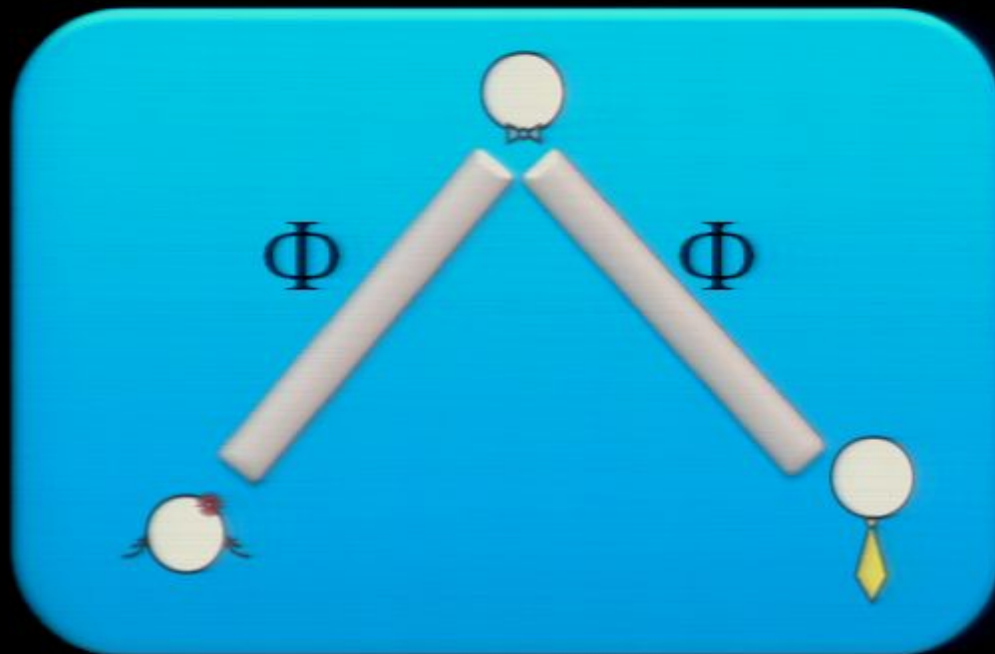
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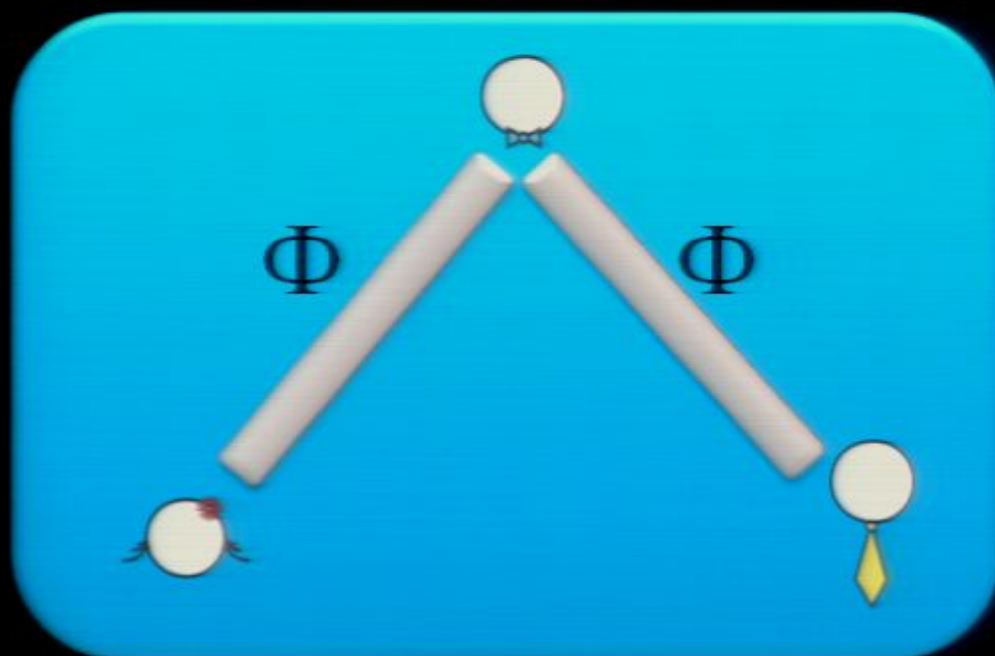
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This last bound is device-independent!

$\text{CHSH}(A,B)=2\sqrt{2} \Rightarrow A$ is measuring on an effective qubit, max entg with something in B (Popescu-Rohrlich, McKague) \Rightarrow not entg with C before B_2 .

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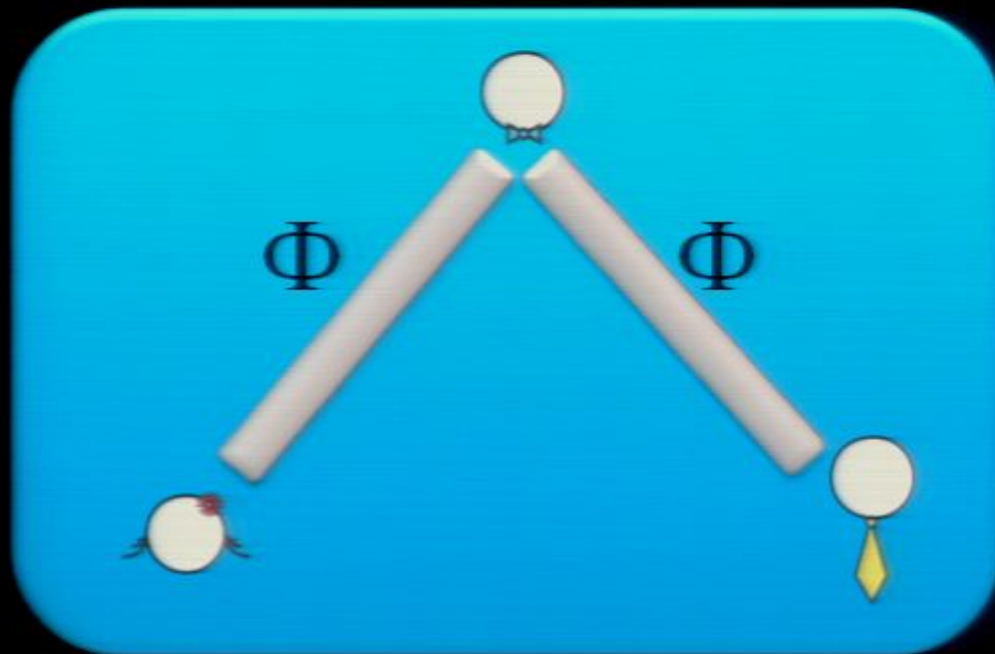
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DI gap between “separable measurement” and “BSM”

\Rightarrow a competent company CAN convince you to buy their BSM

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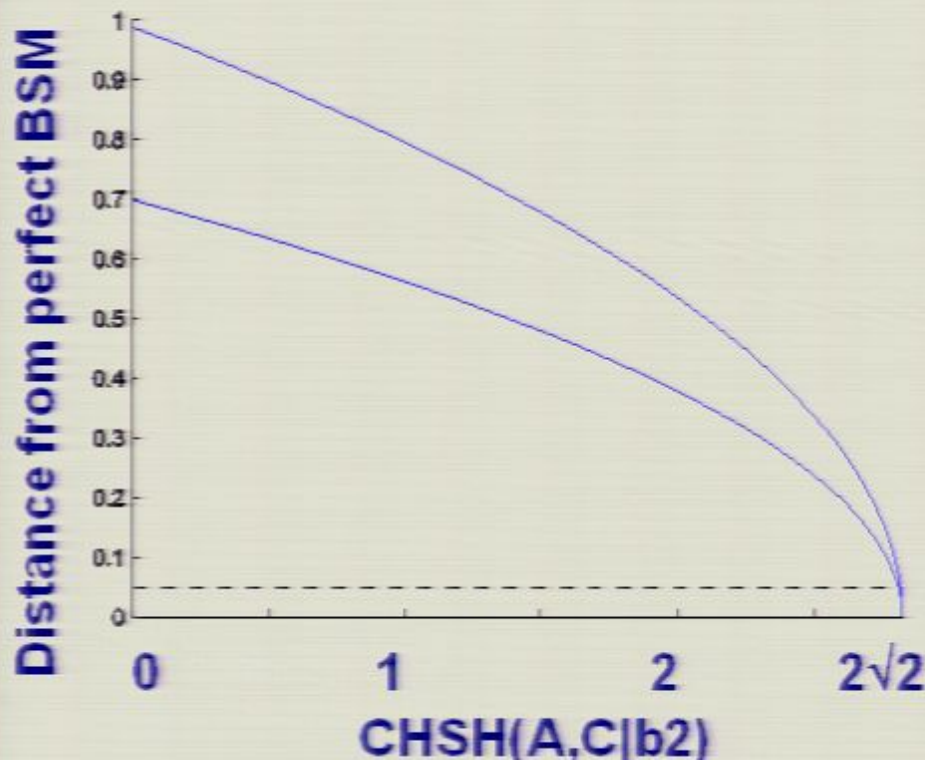
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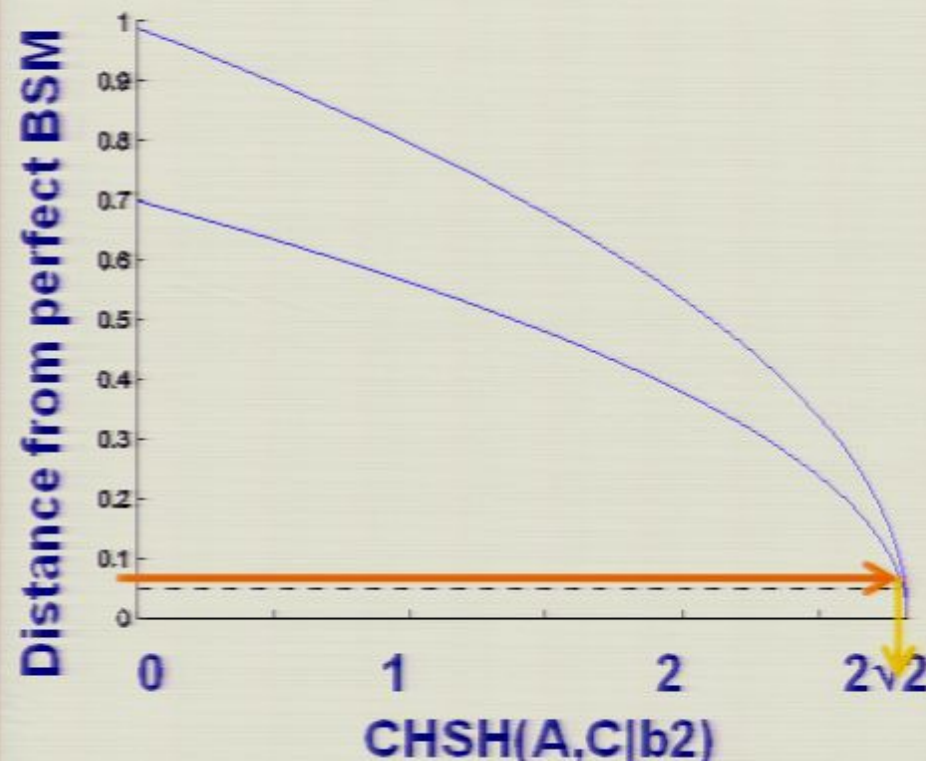
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Certify BSM at 5% of failure probability



$$\text{CHSH} \geq 2\sqrt{2} - 0.5\%$$

Device-dependent group excursion



DI entangling measurement

After QKD, randomness & state estimation, here is another device-independent task.

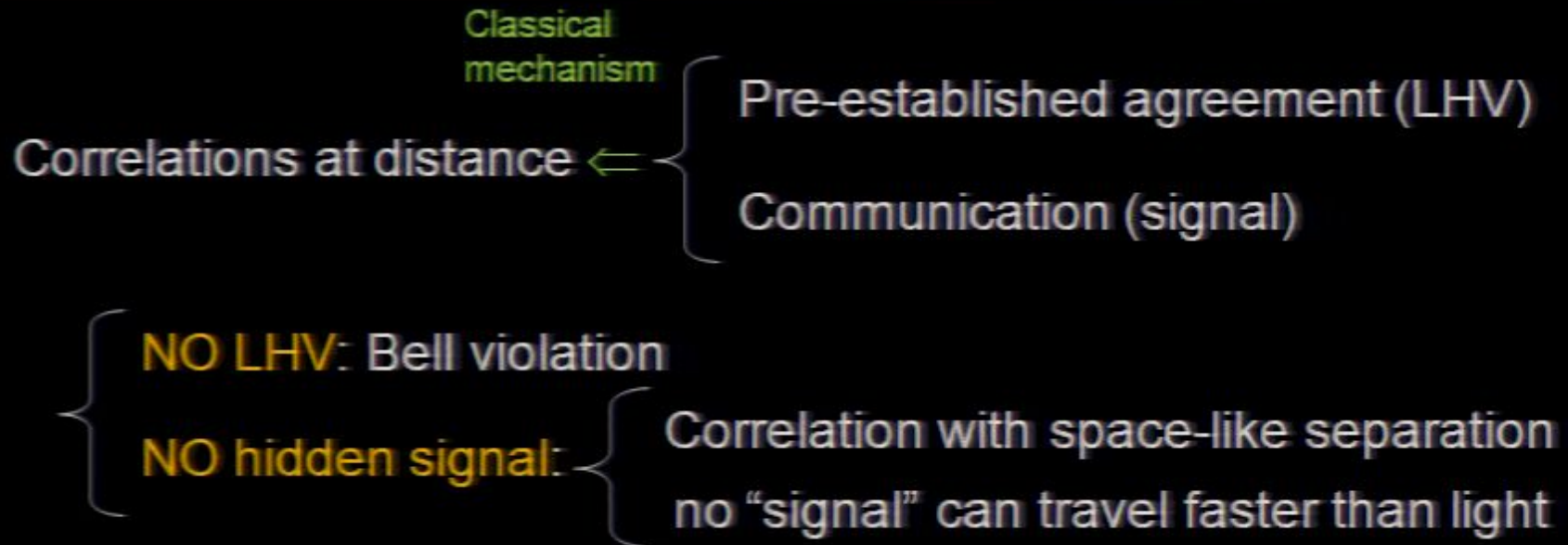
FALSIFY HIDDEN SIGNALING WITH FINITE SPEED

V.S., N. Gisin, Braz. J. Phys. 35, 328 (2005)

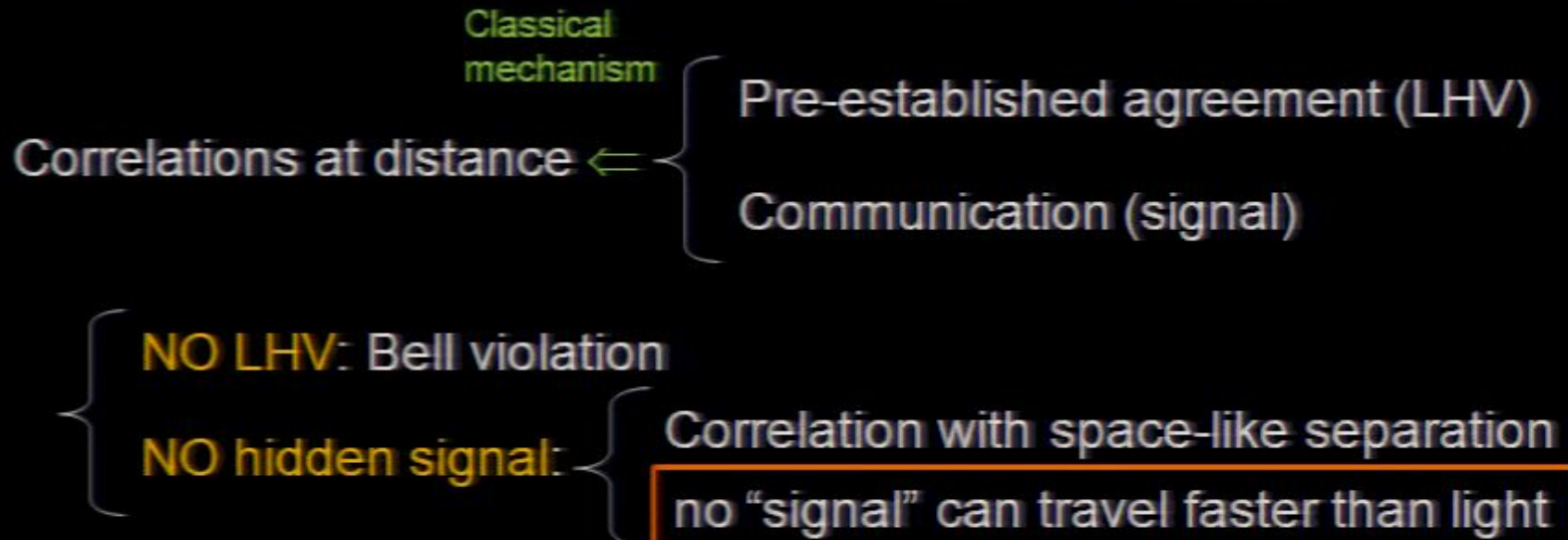
Pirsa: 11050031

S. Coretti, E. Hänggi, S. Wolf, arXiv:1102.5685

A “reverse-Bell” theorem

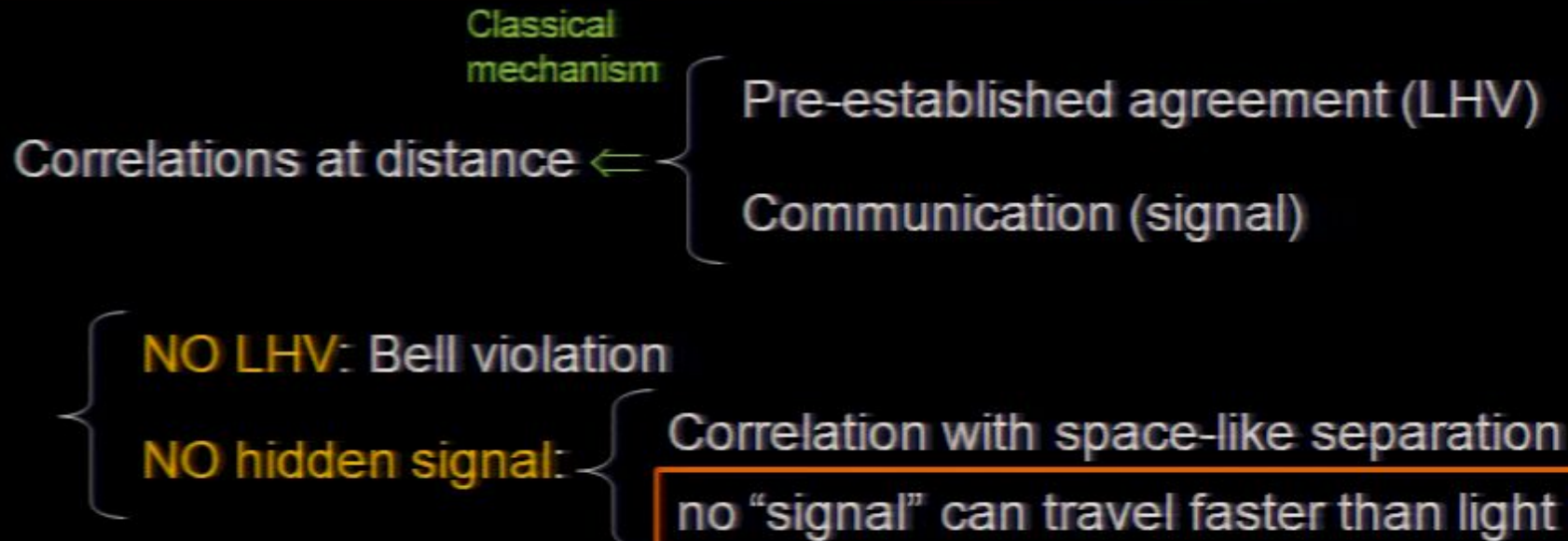


A “reverse-Bell” theorem



Can one assume “hidden signaling models” and falsify them by a direct look at the statistics, without ruling them out by a principle?

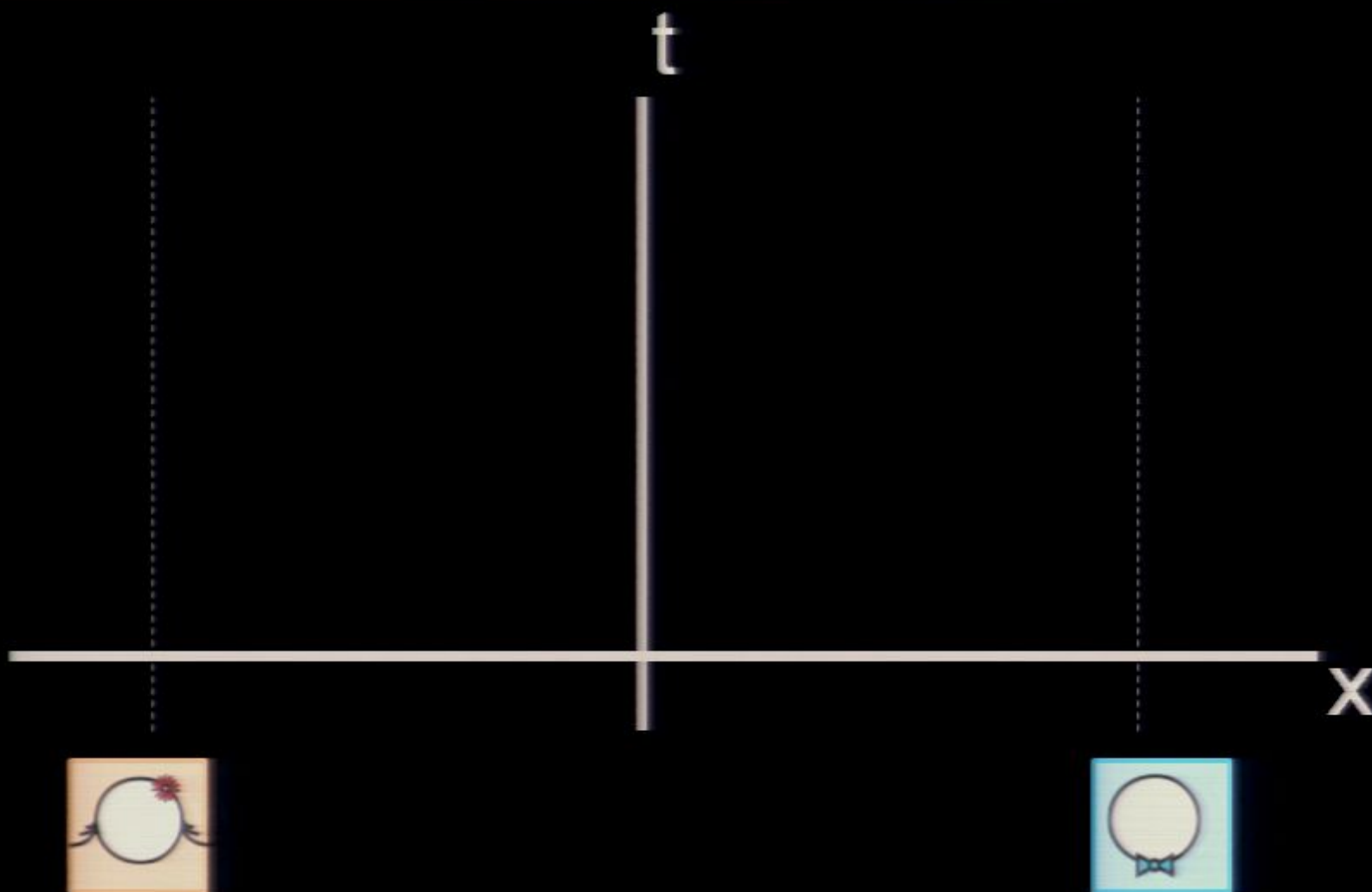
A “reverse-Bell” theorem



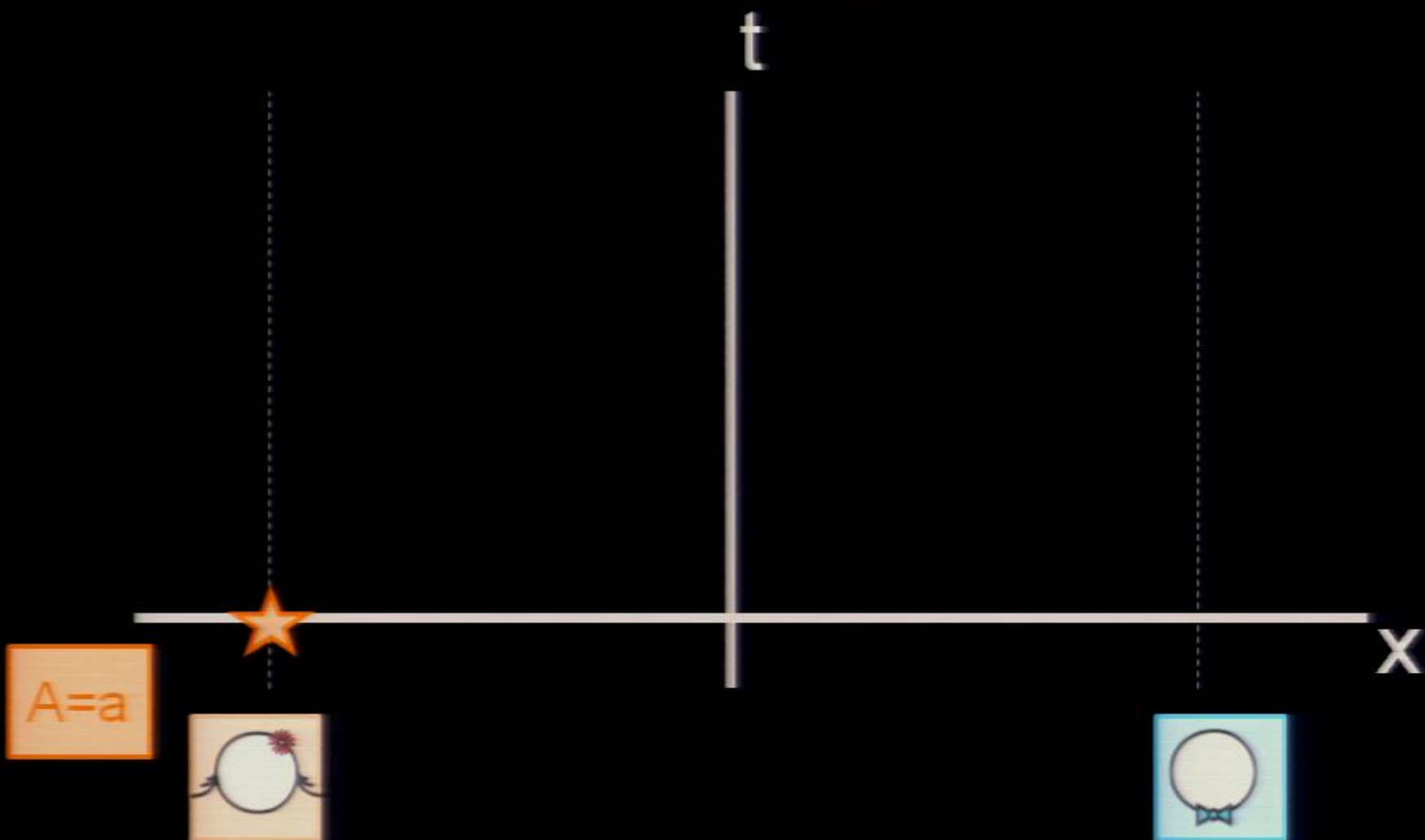
Can one assume “hidden signaling models” and falsify them by a direct look at the statistics, without ruling them out by a principle?

For $v=\infty$, there is a model equivalent to QM: Bohmian mechanics.
What about $c < v < \infty$ in some (unspecified) preferred frame?

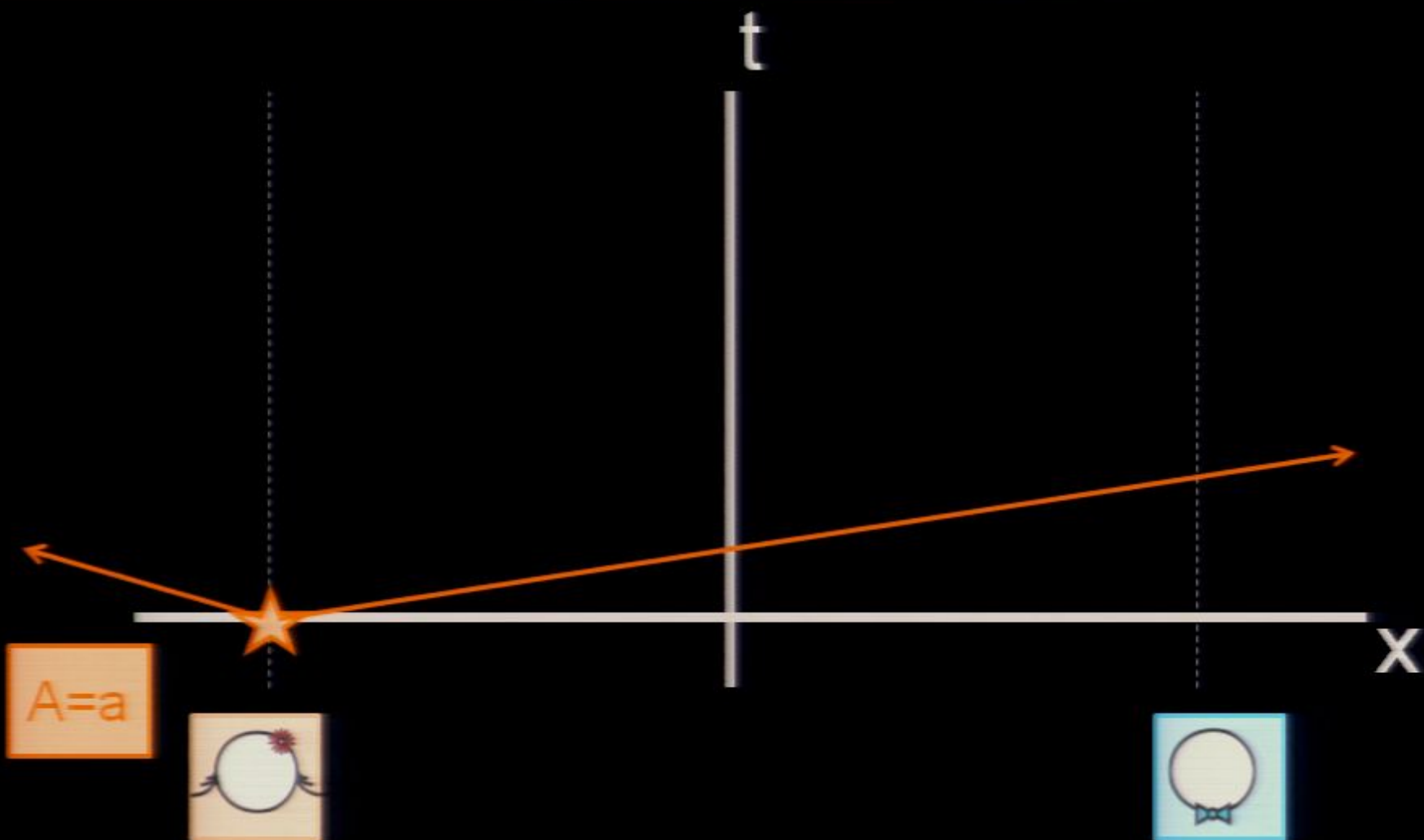
2 parties



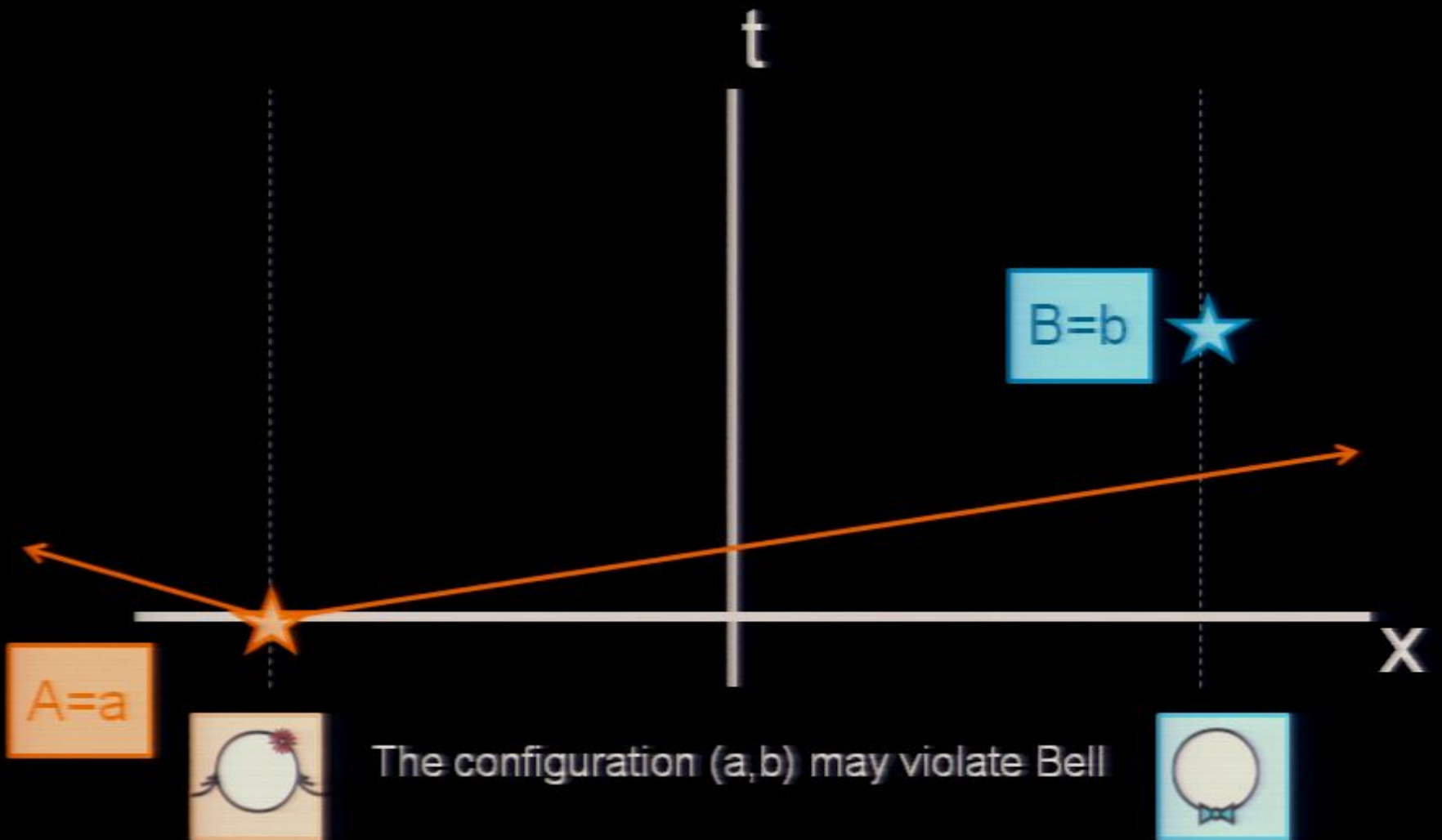
2 parties



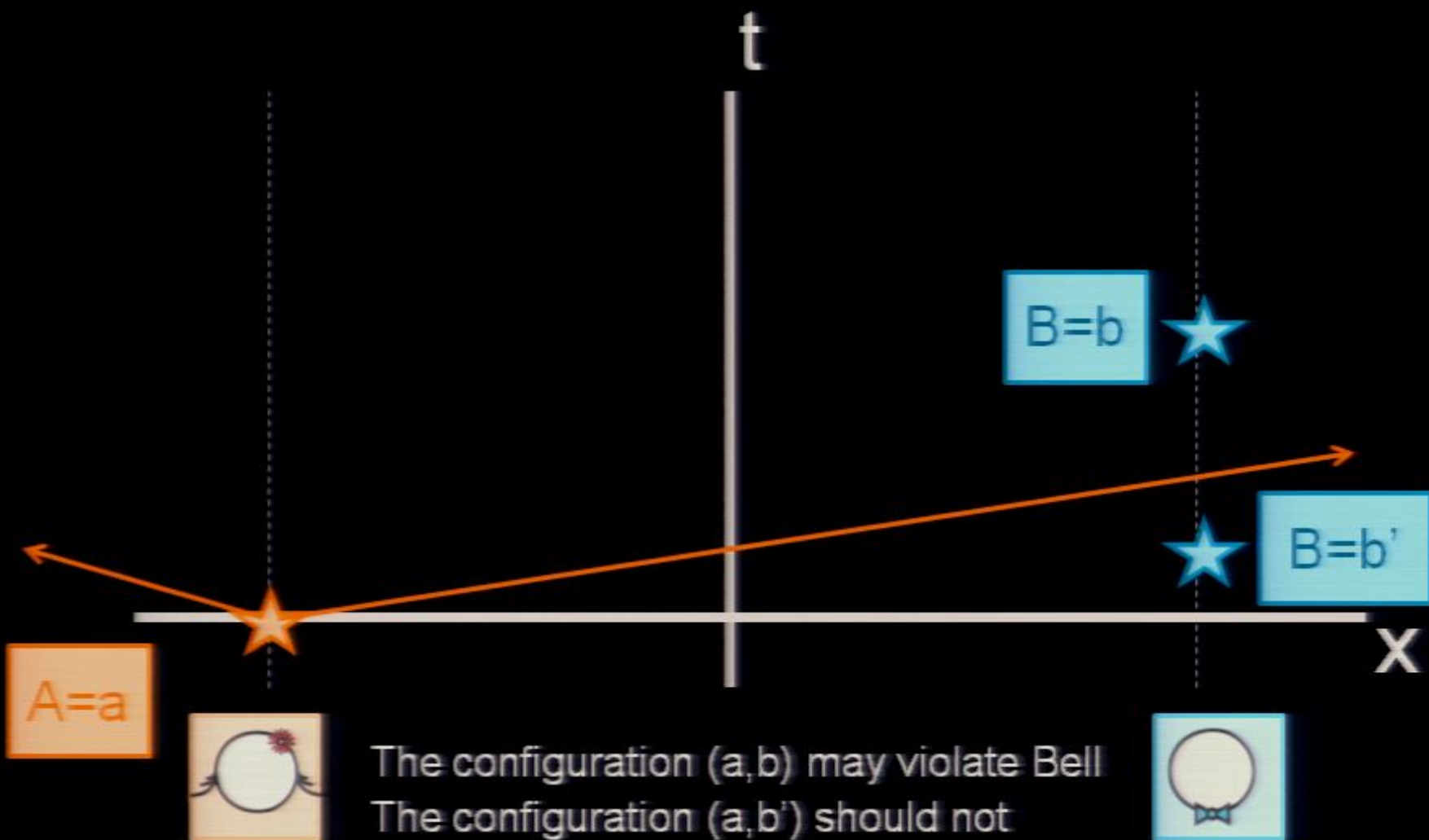
2 parties



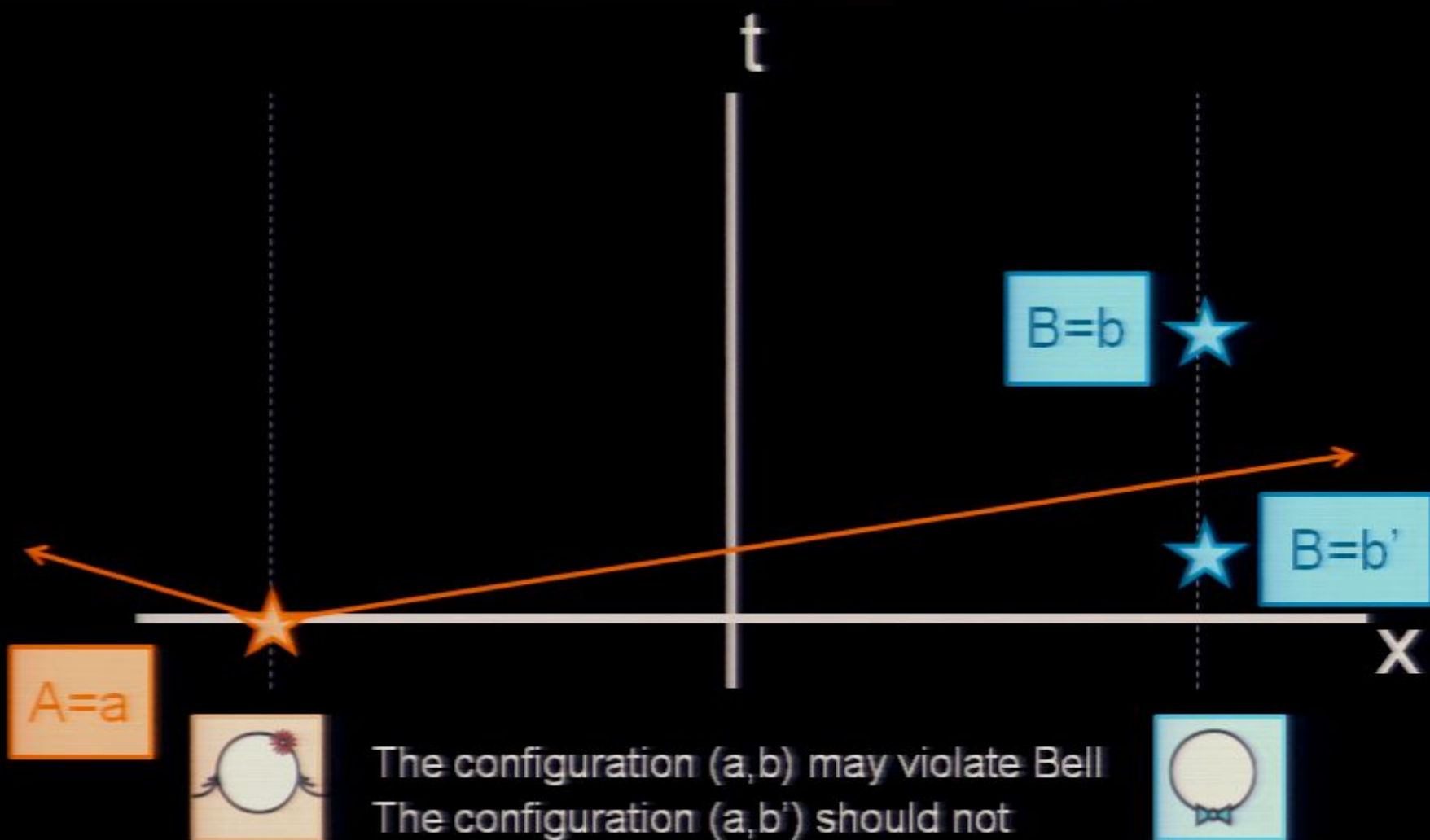
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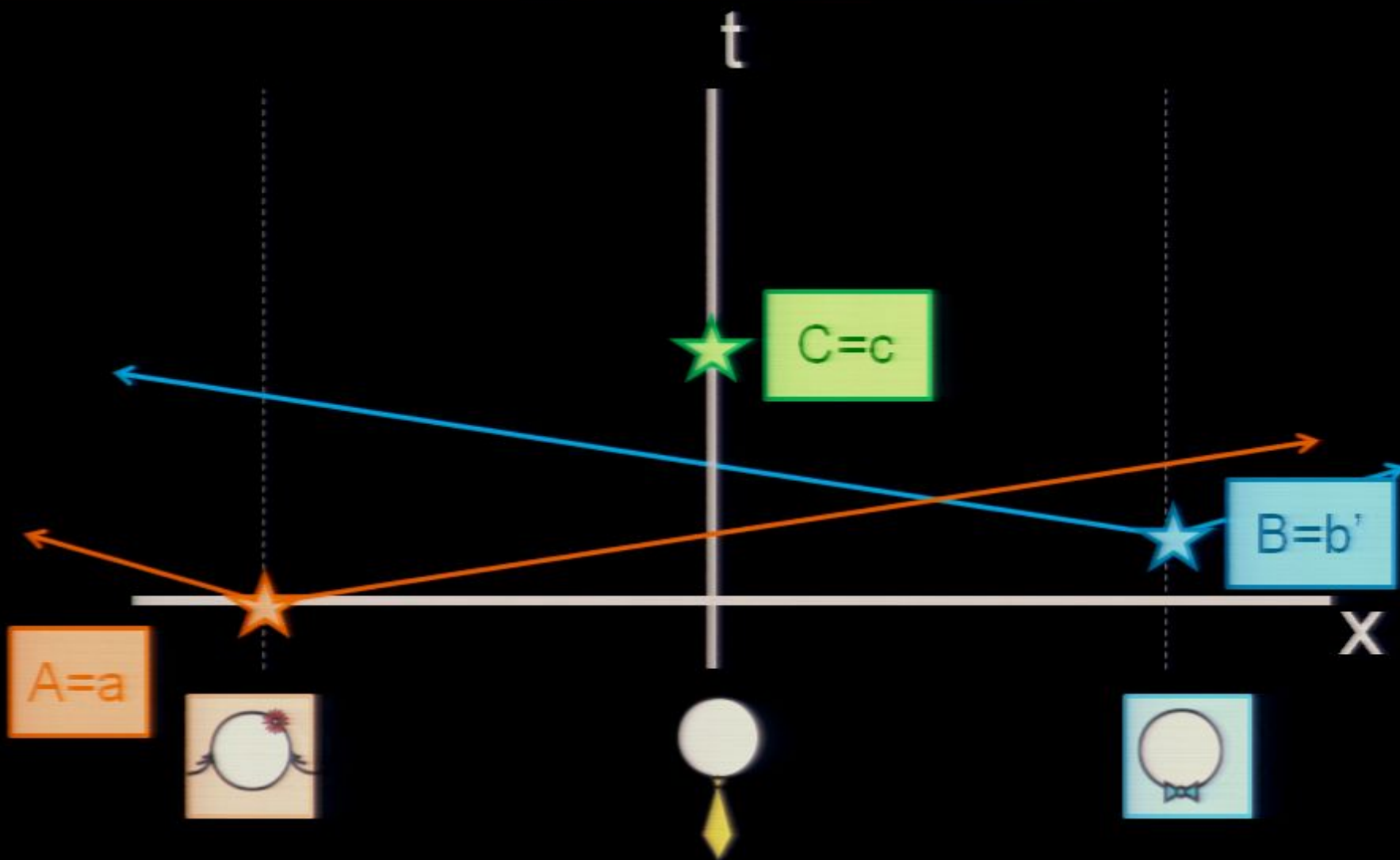


2 parties



Departure from QM if A and B measure "simultaneously enough" in some frame: so far, it could be made consistent...

3 parties



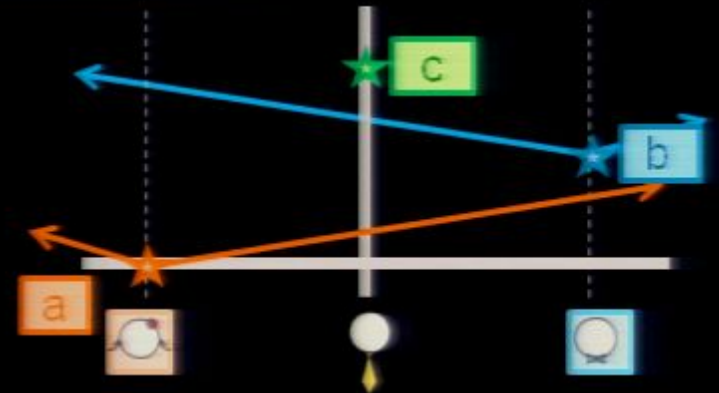
... but a three-party configuration may allow testing the consistency of the alternative theory itself: hidden signaling may not remain hidden!

Idea of the argument

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Starting point: if all hidden signals arrive in sequence, we have QM:

$$P_Q(a,b,c) = \text{Tr}(\rho A_a B_b C_c)$$

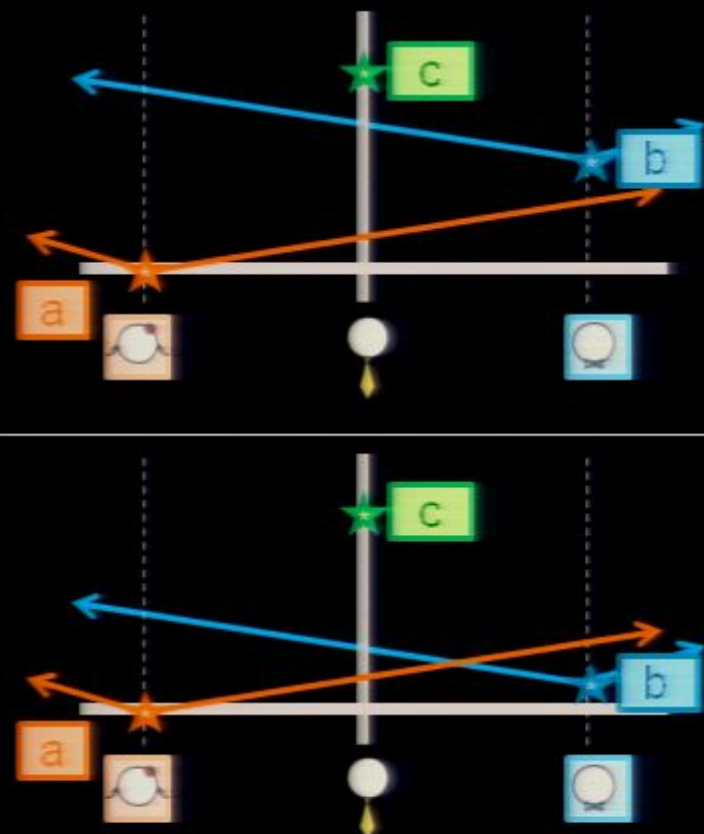


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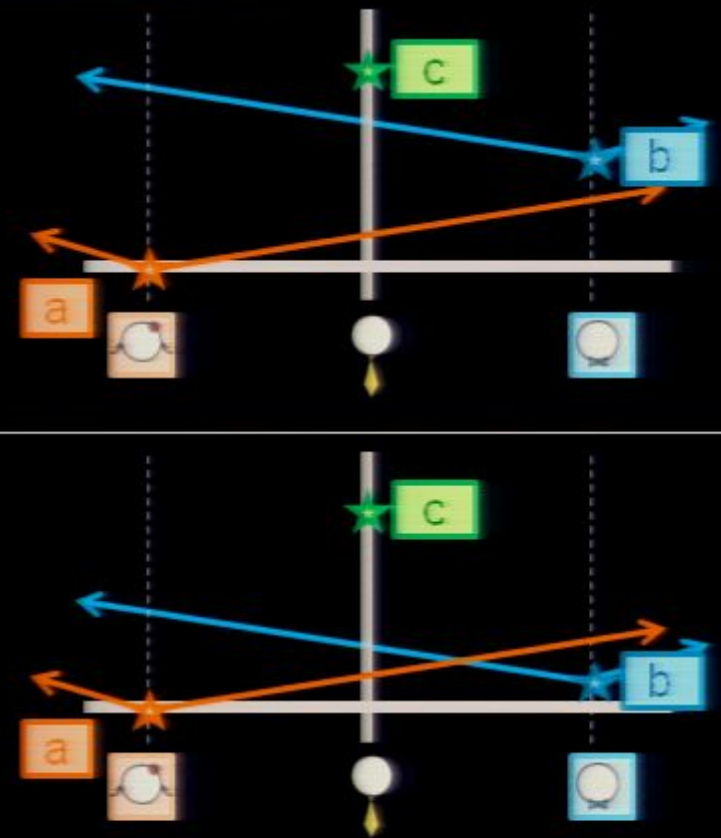


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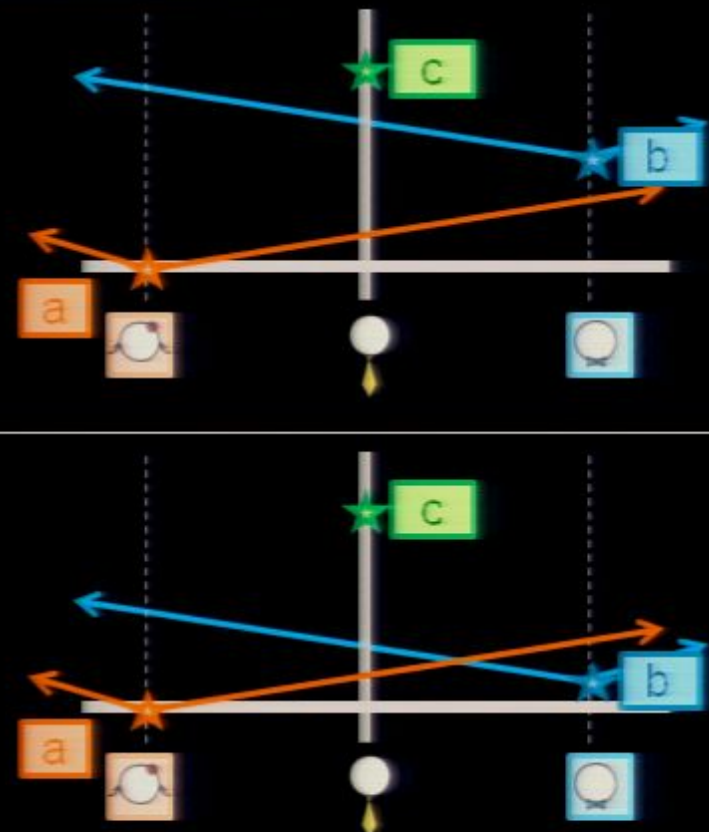
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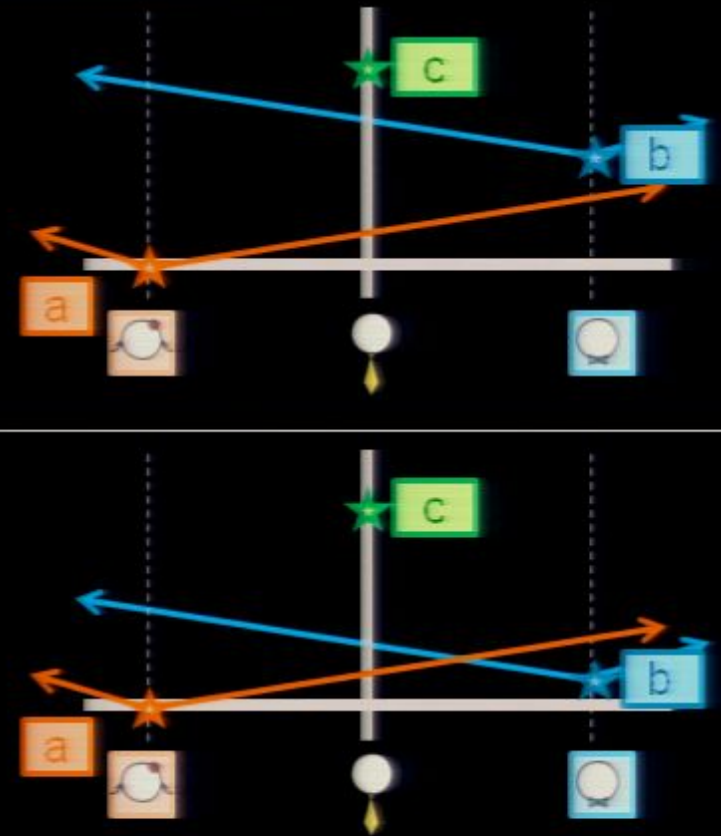
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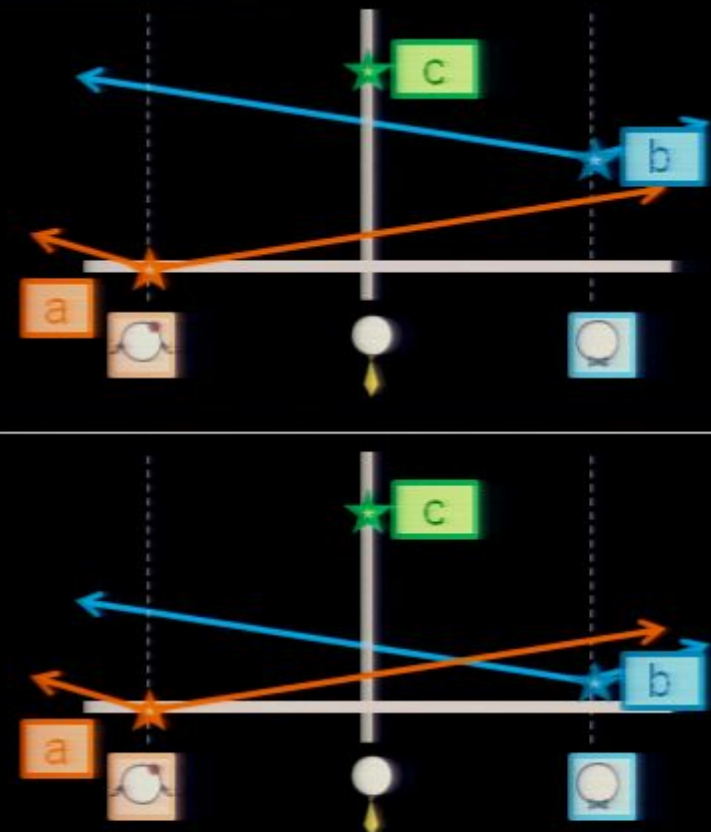
State of the art

- If one restricts the possible P 's to those that can be written as $\text{Tr}(\text{some state} \dots)$, then hidden signaling falsified
 - V.S., Gisin, Braz. J. Phys. 2005
- There exist no-signaling distributions with the desired property, but it is not known if they can be obtained by measuring a quantum system
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- Last step: find a P_Q such that...

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Just graduated (hmmm... maybe with 4 parties it's even better?)



Falsifying hidden communication

It looked impossible, but hopefully YES, we can!

INFORMATION CAUSALITY IN THE THREE-PARTITE SCENARIO

2 parties

$a_0=0,$
 $a_1=1$

Guess a_k



2 parties

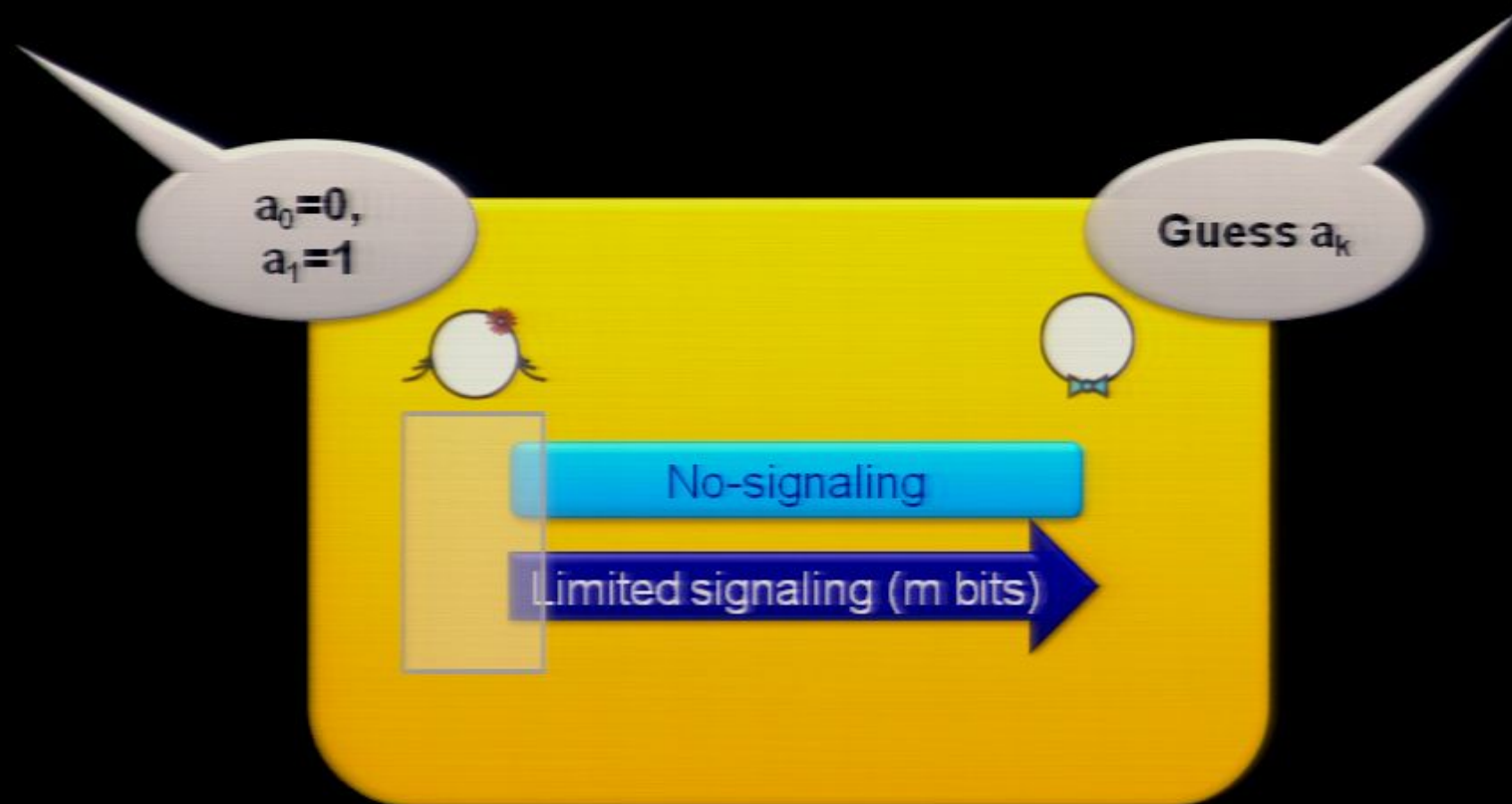
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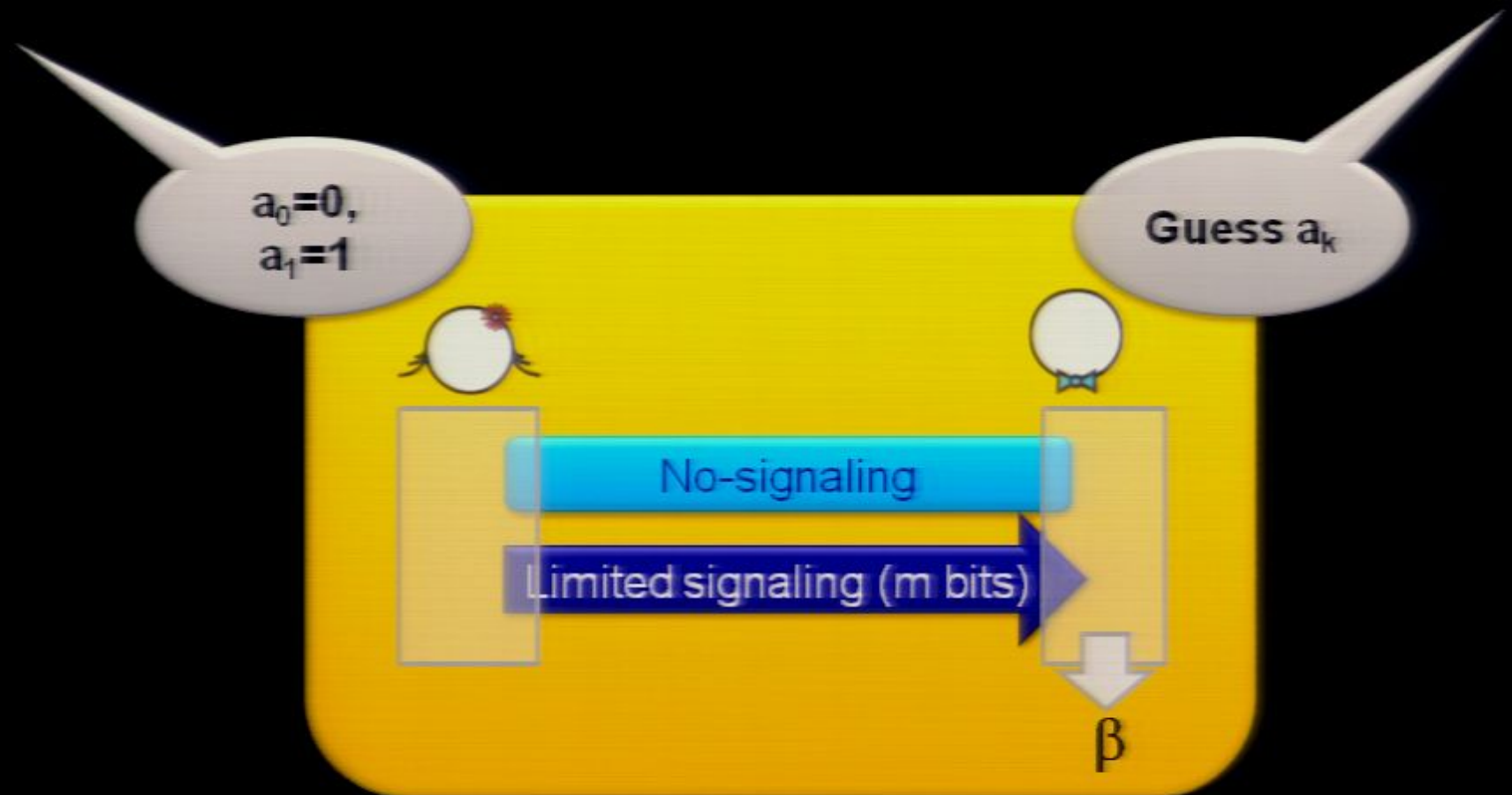
No-signaling

Limited signaling (m bits)

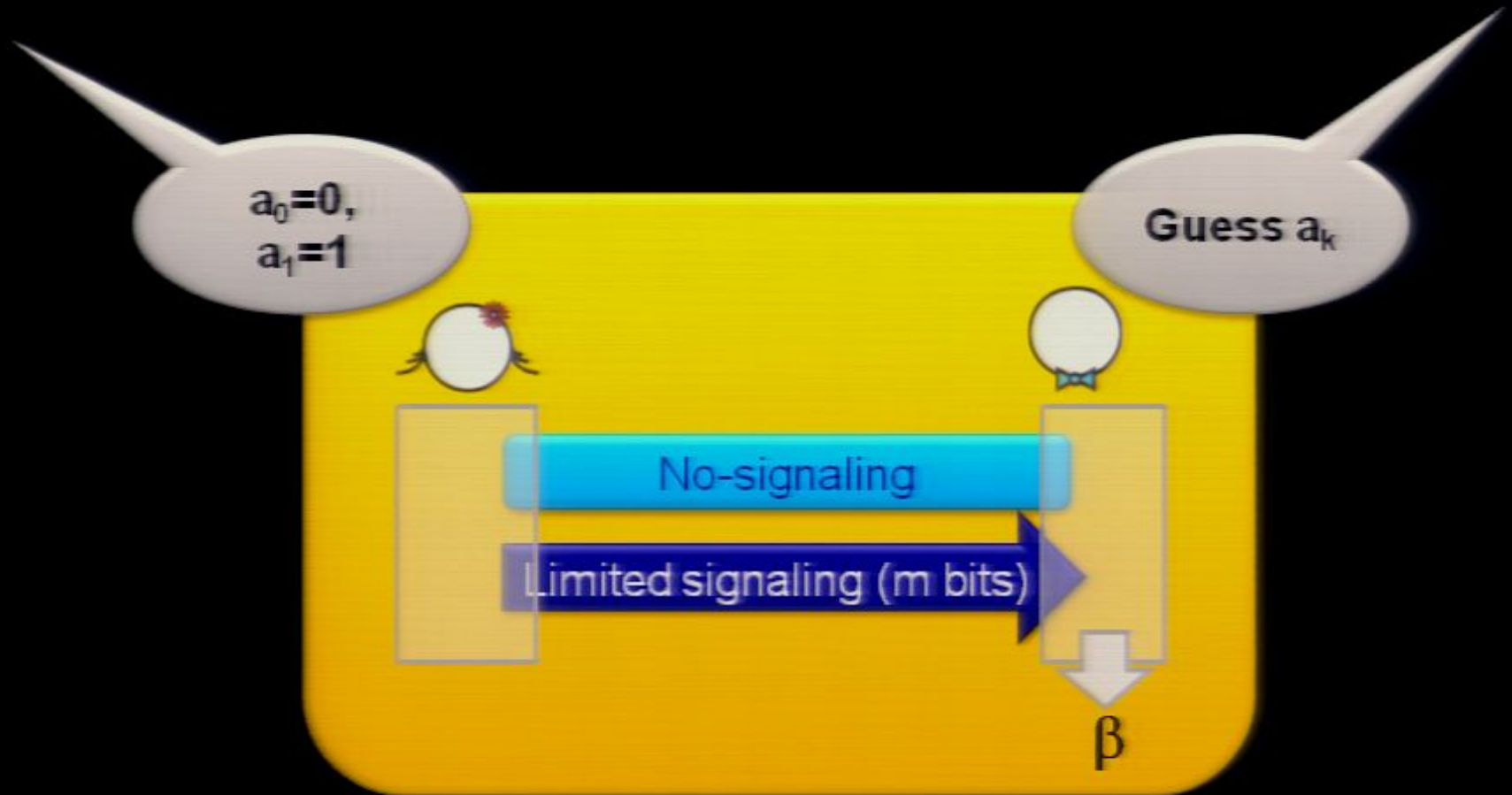
2 parties



2 parties



2 parties



IC respected if
$$\sum_{k=1}^N I(a_k : \beta \mid b = k) \leq m$$

What IC can do for you

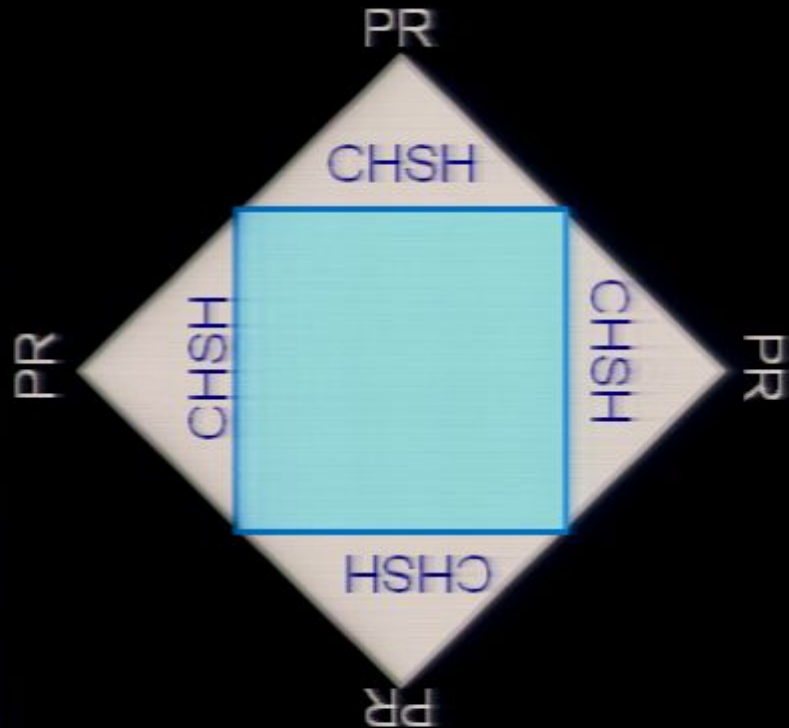
Consider $(2,2;2,2)$



What IC can do for you

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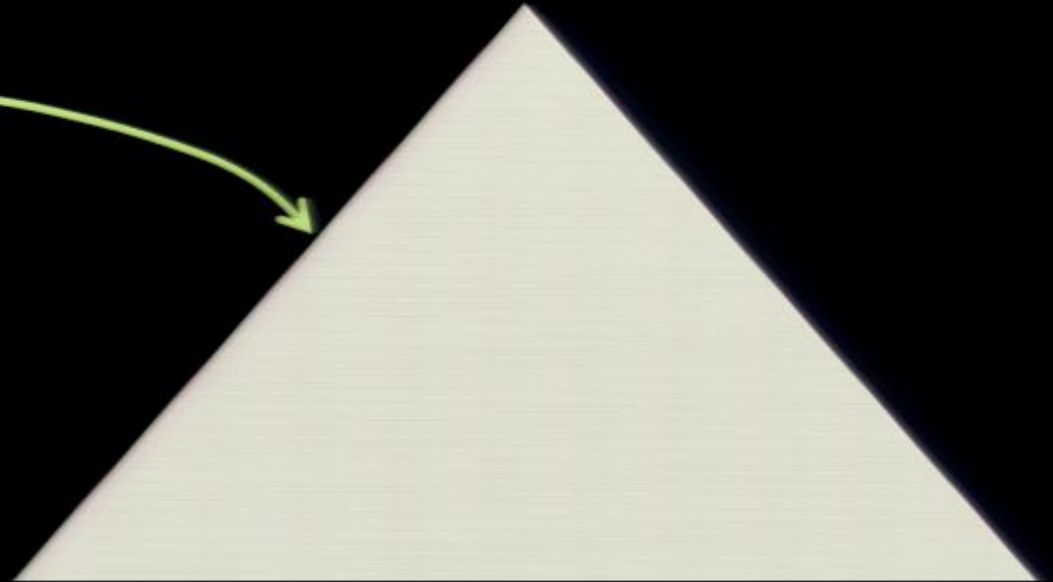
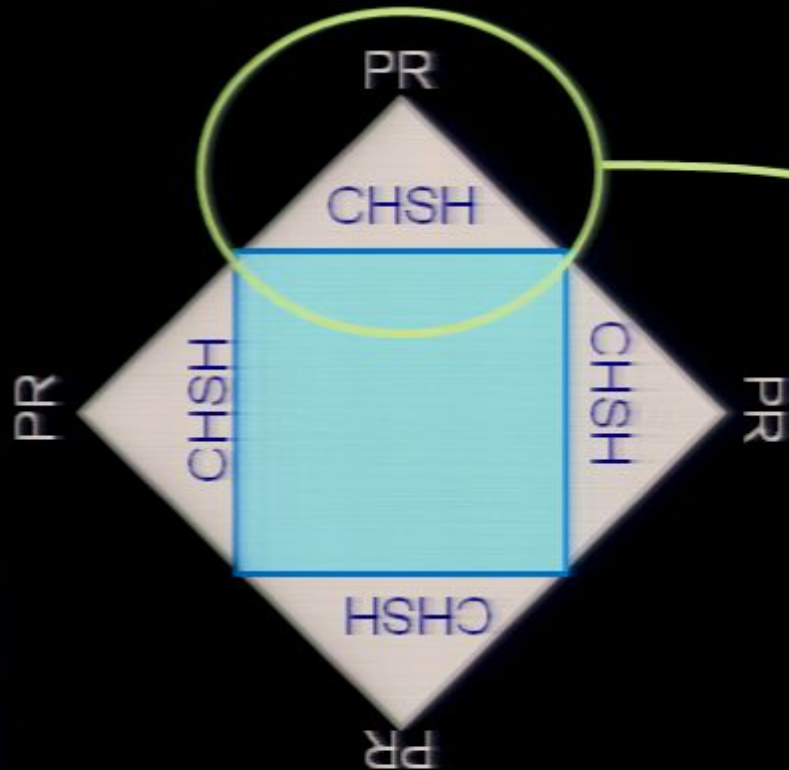
No-signaling polytope



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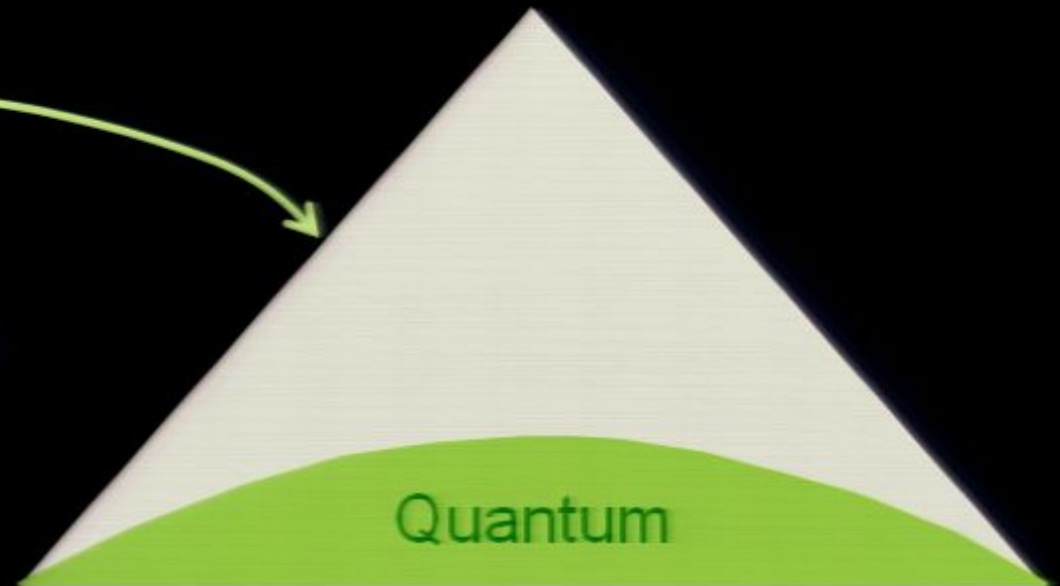
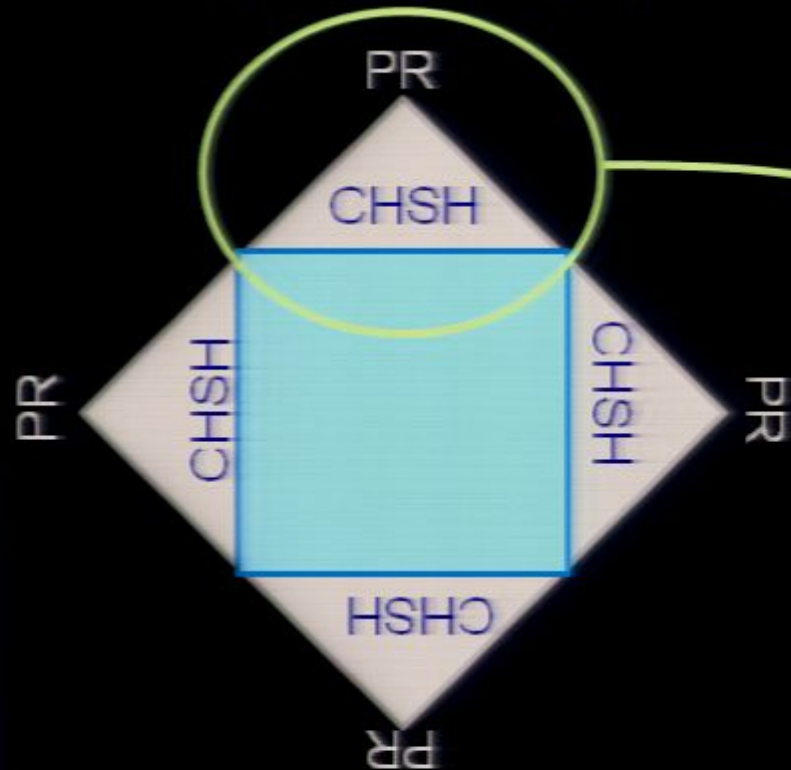
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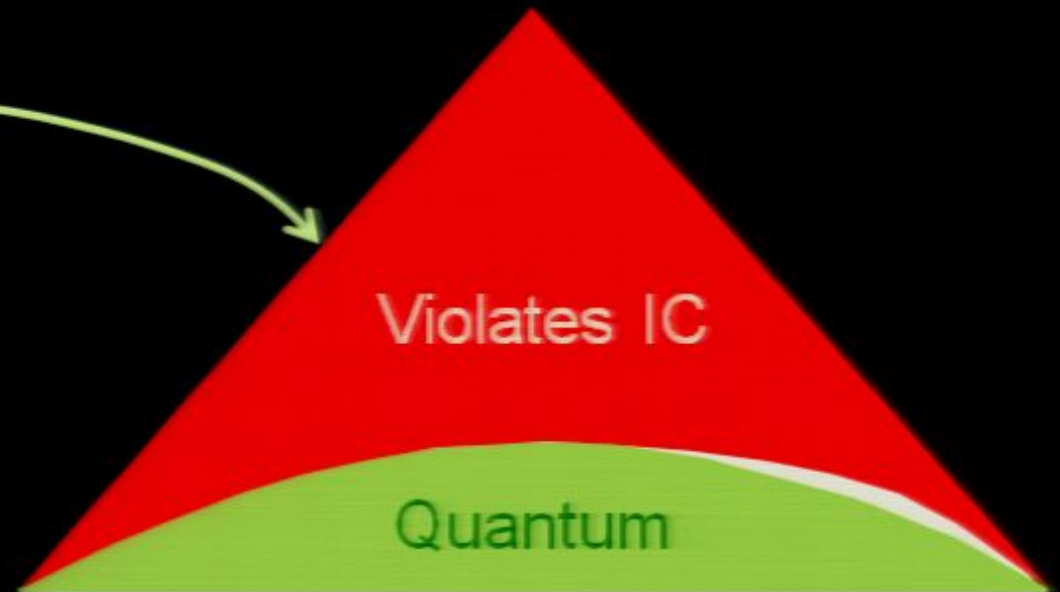
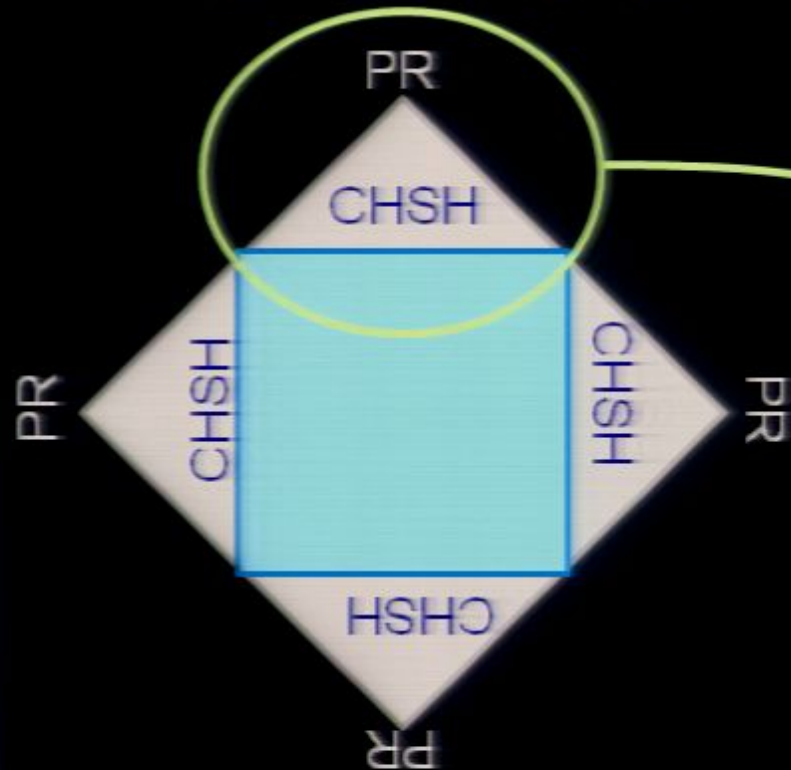
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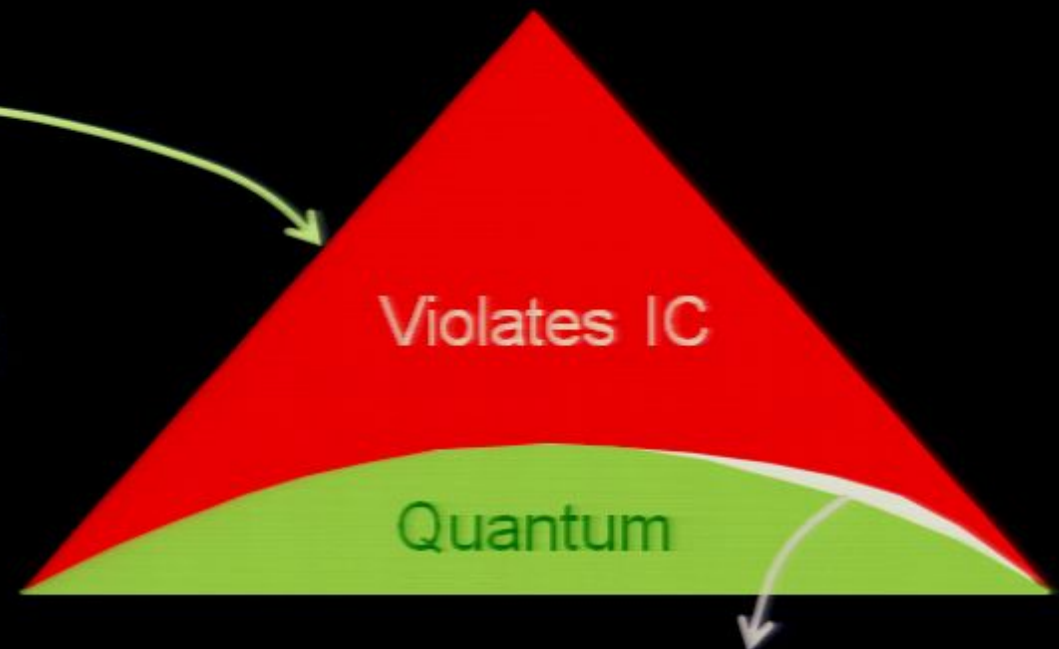
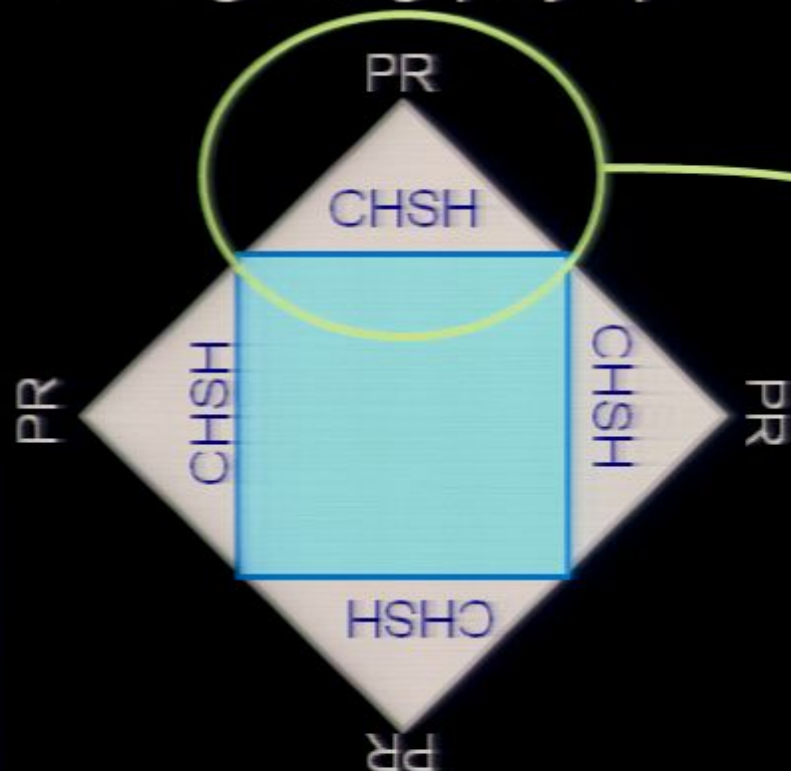
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What IC can do for you

Consider (2,2;2,2)

No-signaling polytope



We don't know...

IC=QM???

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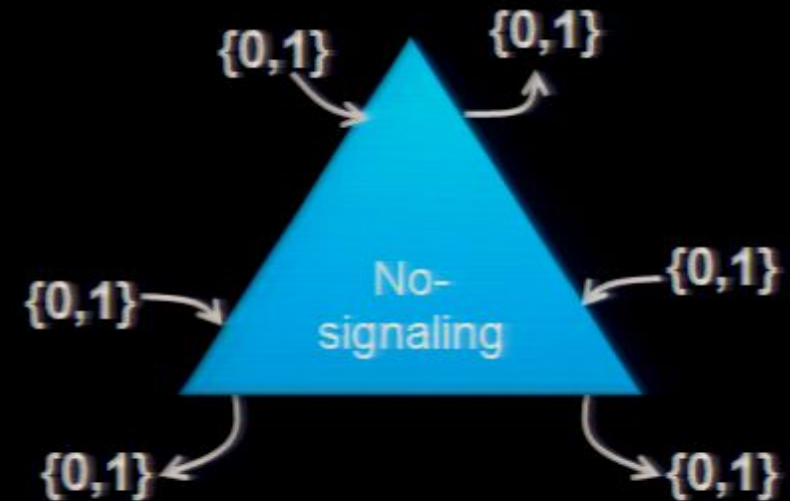
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- Ultimately, one would like to find a solution without having to study each scenario separately...
- And – hey! QM allows more than two parties...

Polytopes: $3 \gg \gg \gg \gg \gg \gg \gg \gg \gg \gg \gg 2$ ☹️

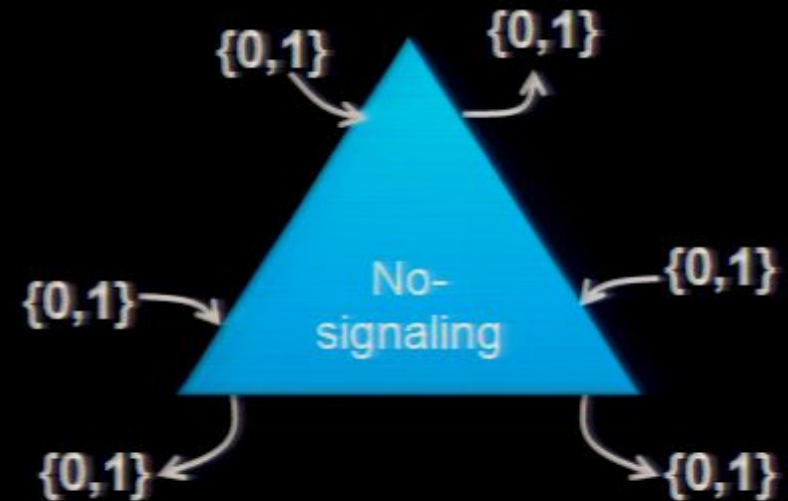
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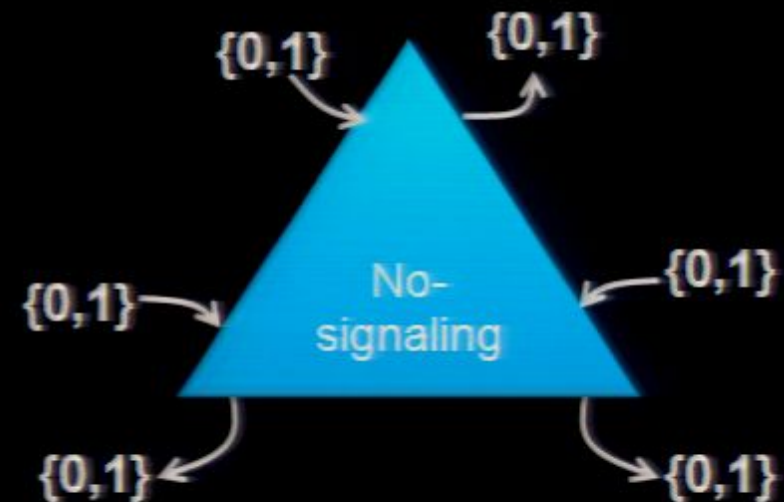


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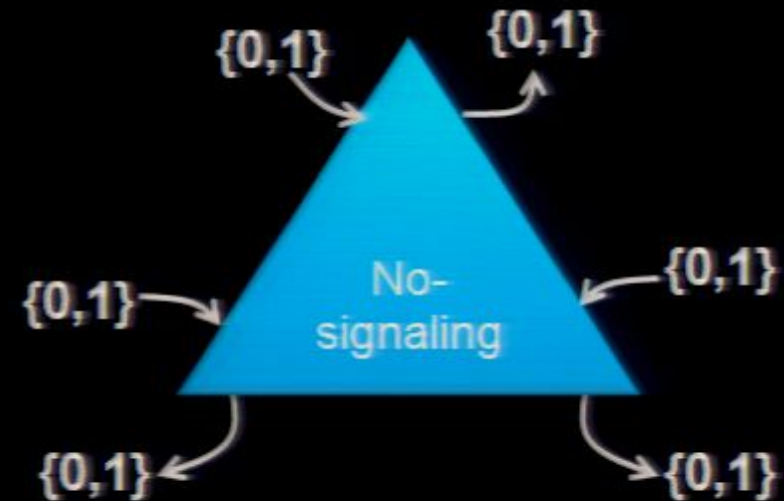
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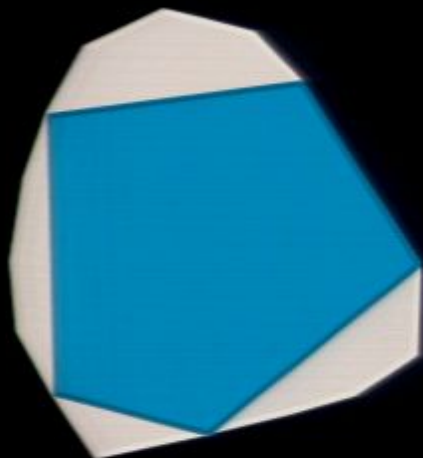
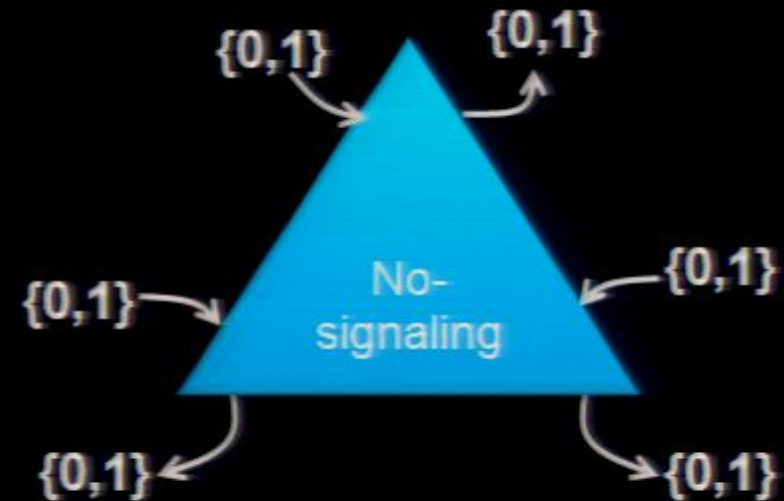
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[Lazy-artist representation]

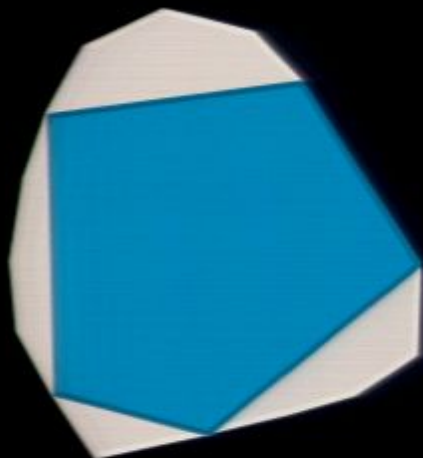
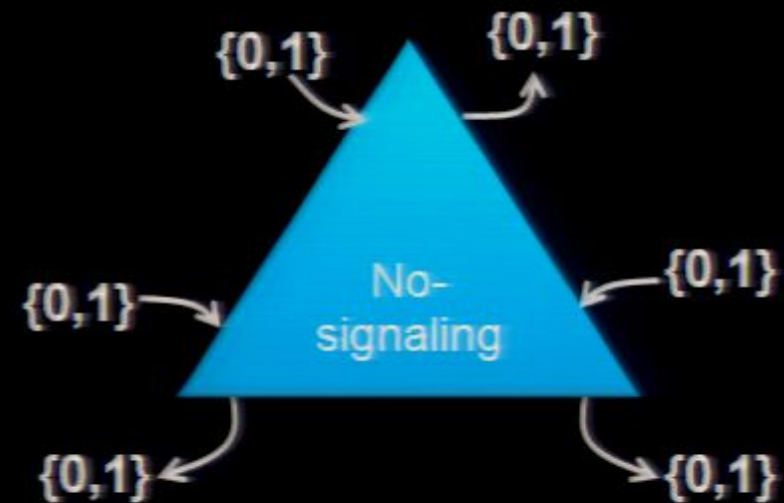
Polytopes: 3>>>>>>>>>>>>2 ☹️

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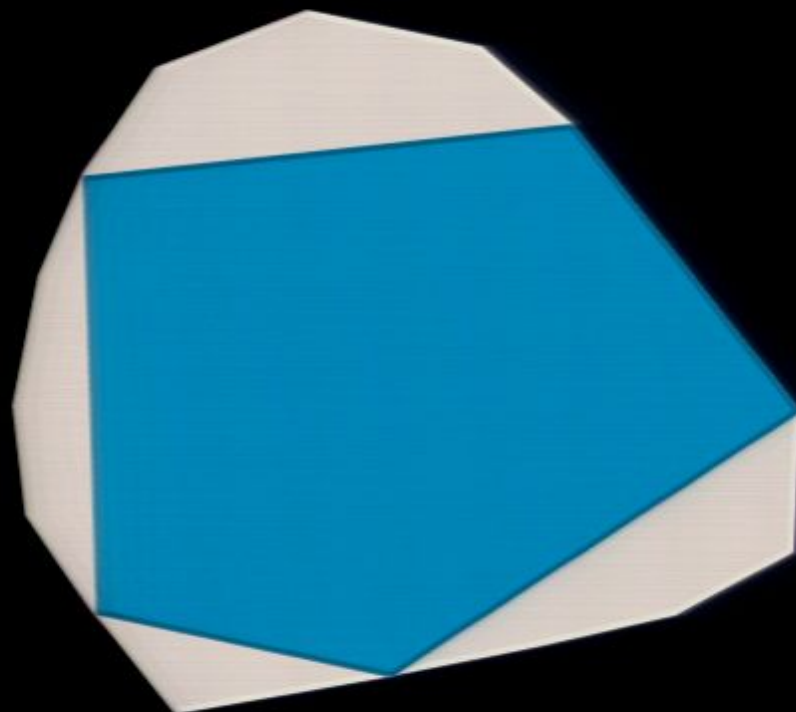
NO obvious correspondence between boxes and facets 😞



IC: even the complete study of this special case seems hopeless... but one has to start somewhere...

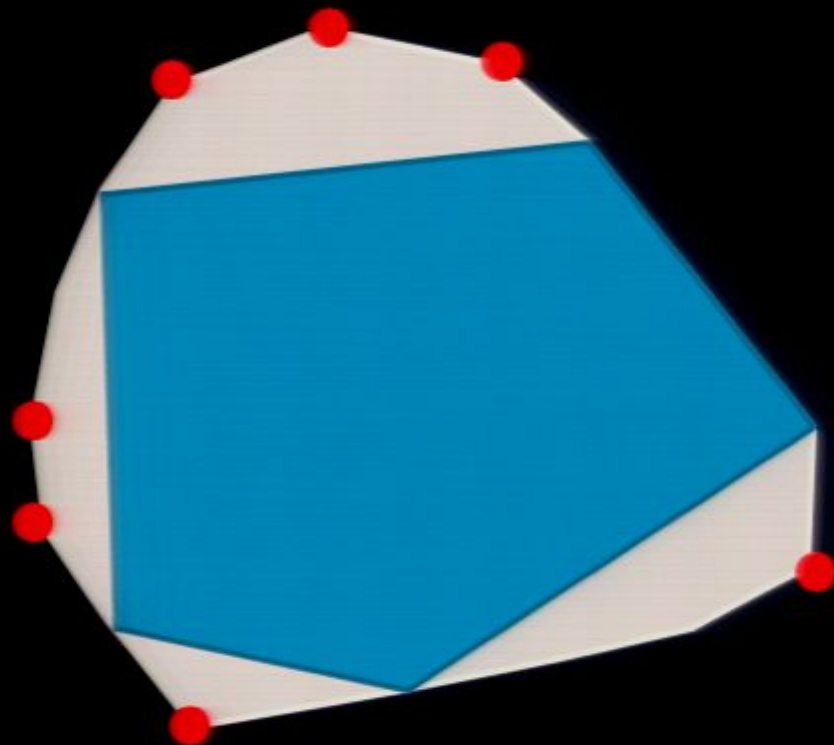
[Lazy-artist representation]

Partial results



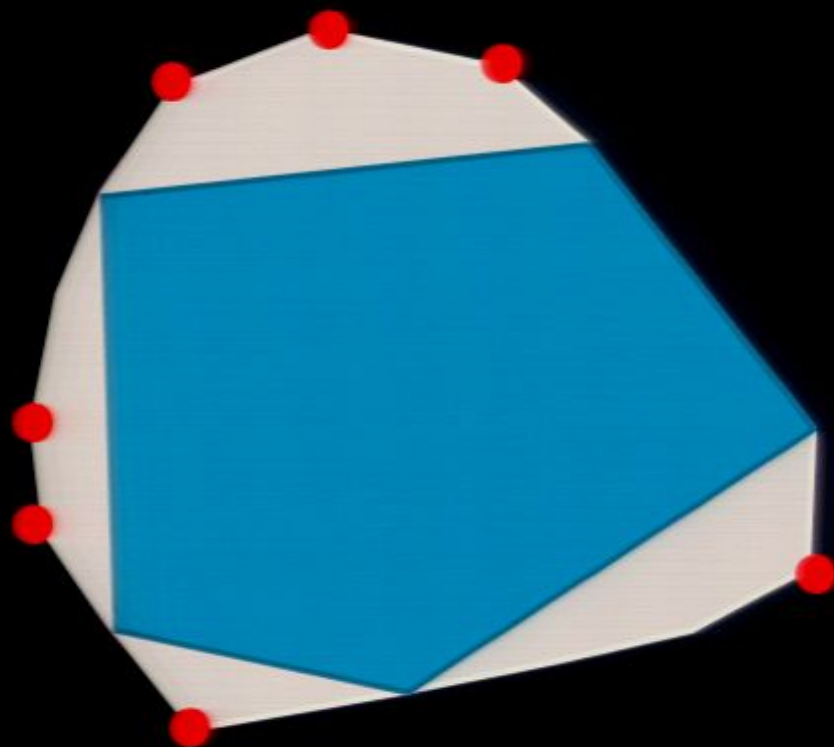
Partial results

1. 43 extremal non-local points violate bipartite IC



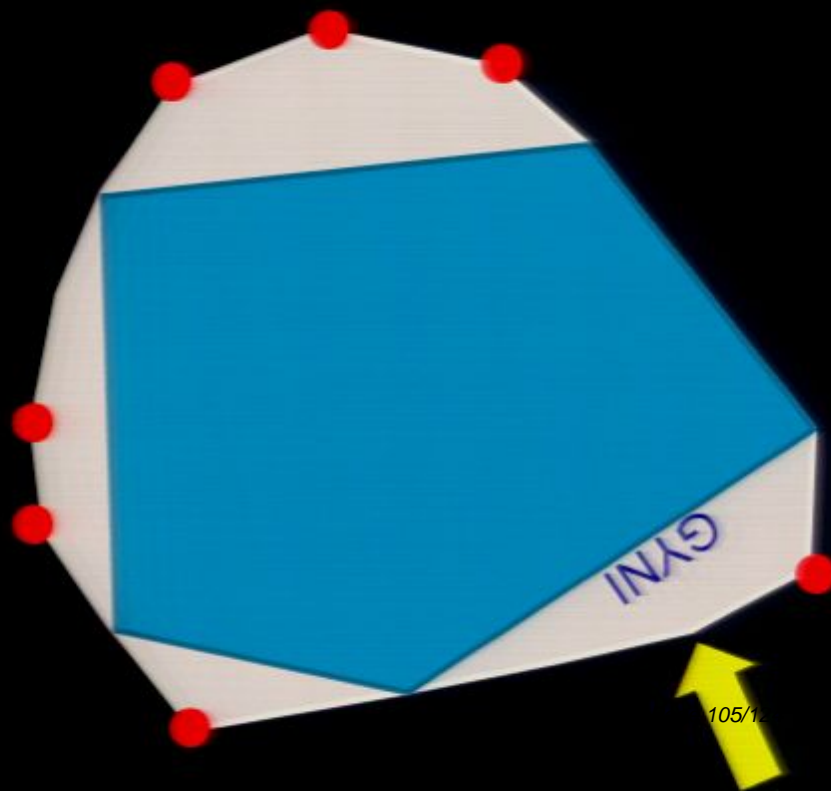
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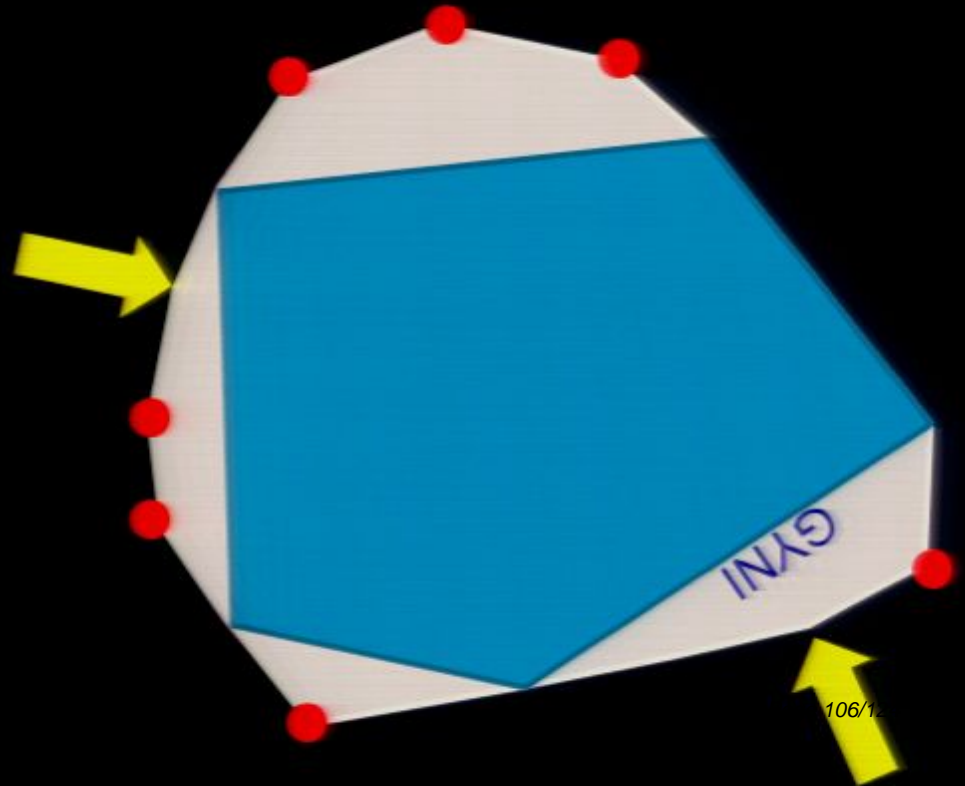
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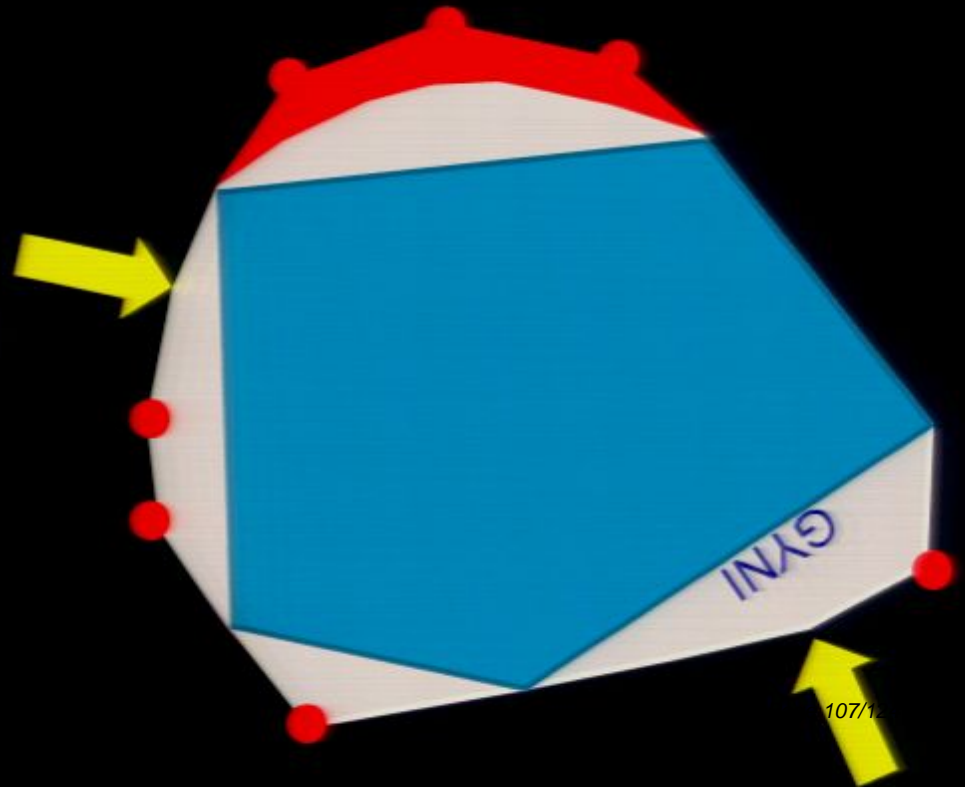
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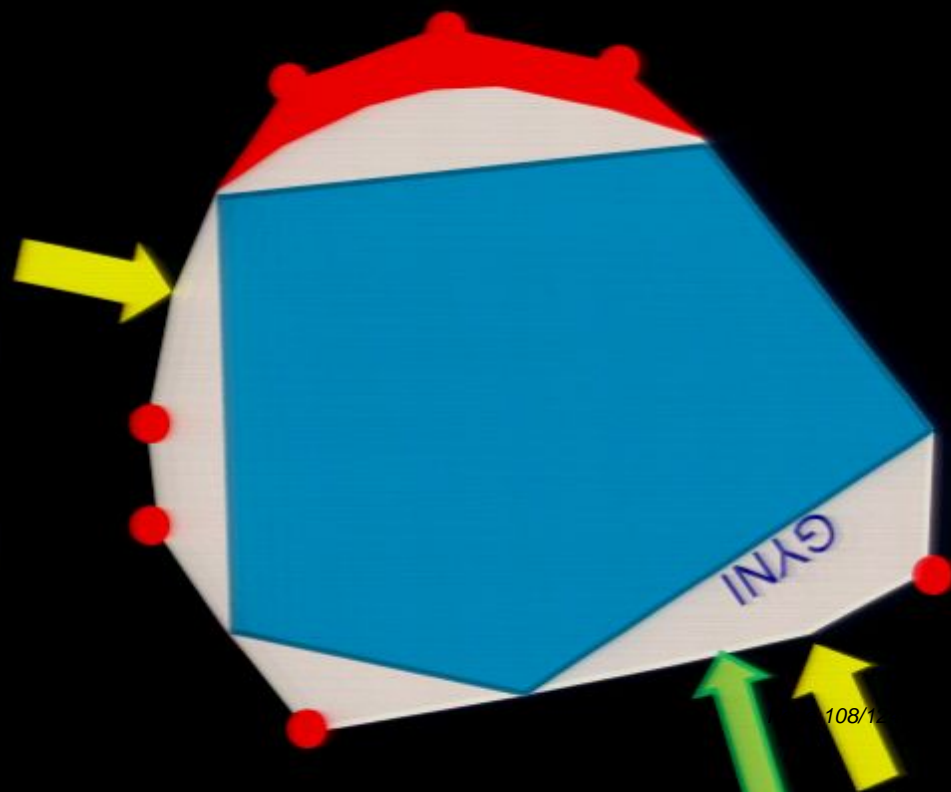
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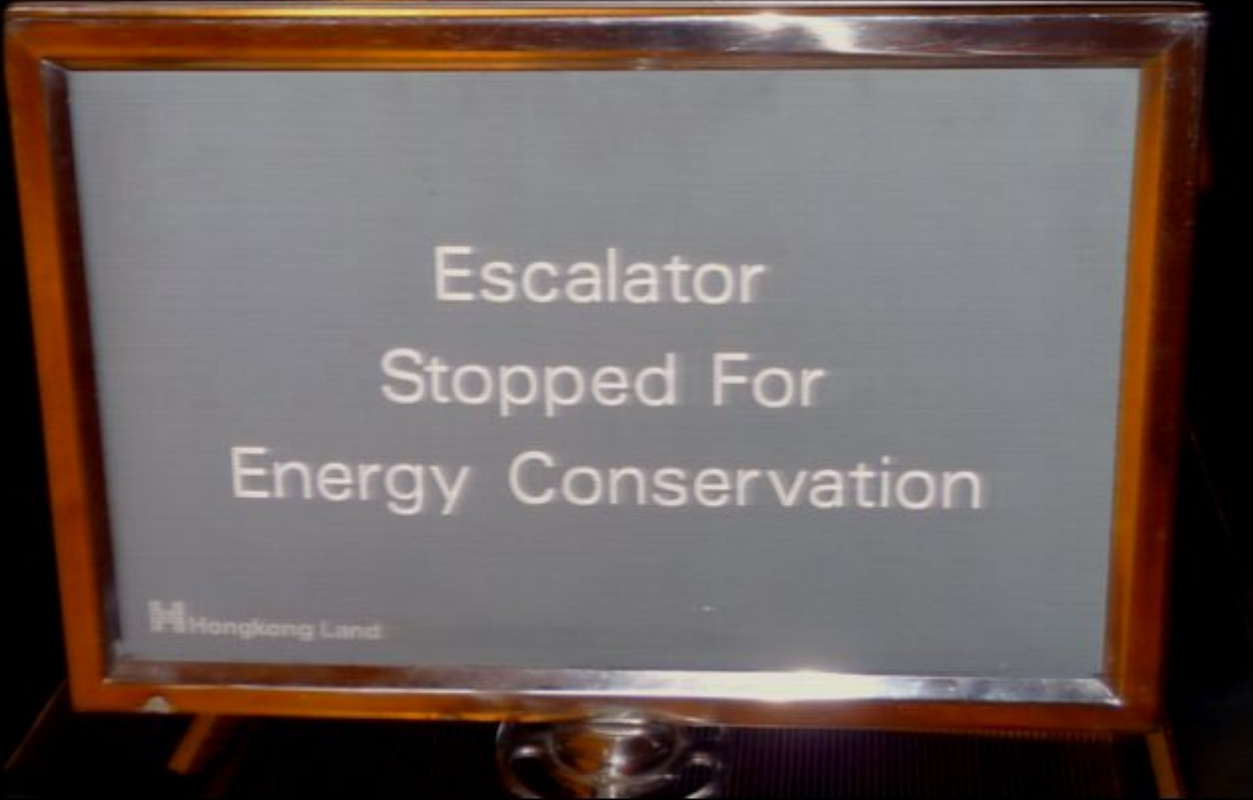


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4. There is a non-extremal point that certainly won't violate bipartite IC (Acin)



In Singapore, we care for the principles of physics!



Escalator
Stopped For
Energy Conservation

Hongkong Land

IC multipartite

If we hope to prove $IC=QM$, we need:

- (i) a multipartite definition of IC
- (ii) Go beyond case-by-case studies

$3 \gg 2$ in a different sense

BB84

$$H_{\min}(Z_A | E) = 1 - h(e_X)$$

3>>2 in a different sense

BB84

$$H_{\min}(Z_A | E) = 1 - h(e_X)$$

inspiration



elegant proof

Unc. rel. $H_{\min}(Z_A | E) = 1 - H_{\max}(X_A | B)$

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inspiration  elegant proof

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Temptation: QKD \Leftrightarrow uncertainty relations

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Temptation: QKD \Leftrightarrow uncertainty relations

Six states

$$H_{\min}(Z_A | E) = 1 - e_Z h\left(\frac{1 - (e_X - e_Y)/e_Z}{2}\right) - (1 - e_Z) h\left(\frac{1 - (e_X + e_Y + e_Z)/2}{1 - e_Z}\right)$$

Good luck to find the corresponding unc. rel. ©

3>>2 in a different sense

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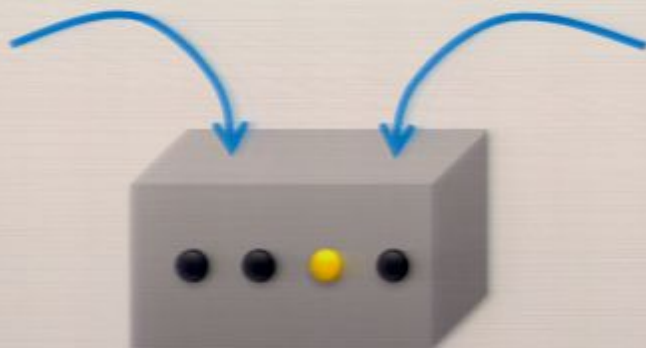
Activation of non-locality in networks



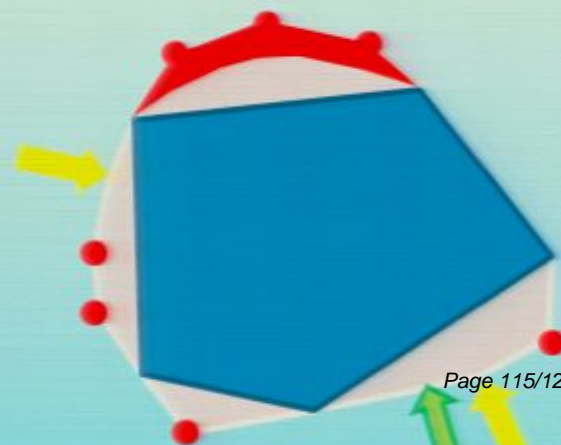
Hope to falsify hidden signaling



Dev-indep entangling measurement



Three-partite information causality











Thank you



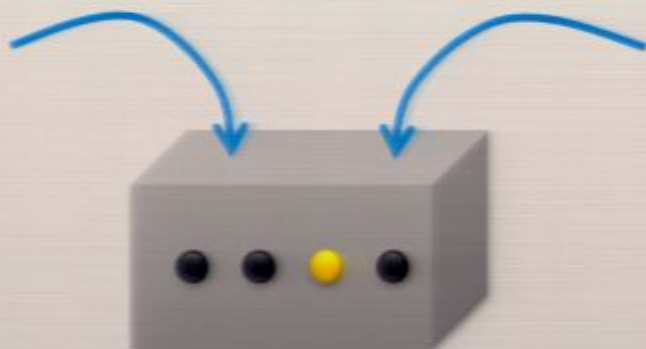
Activation of non-locality in networks



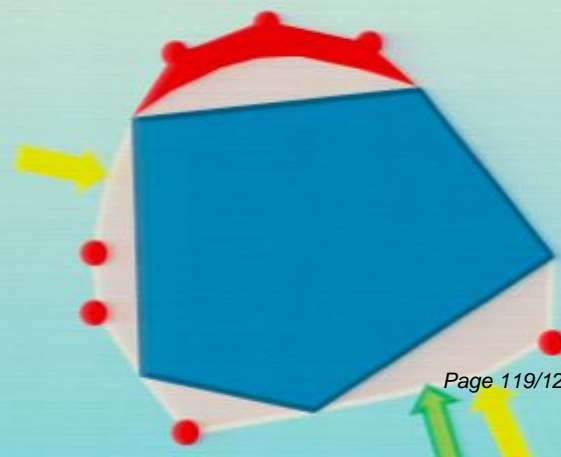
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Three-partite information causality

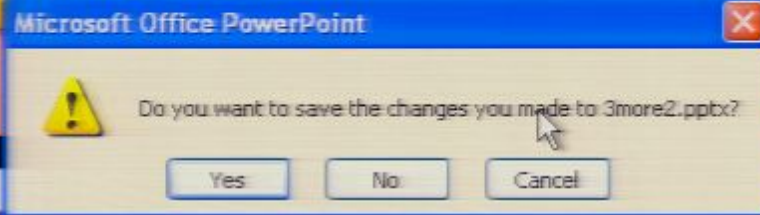
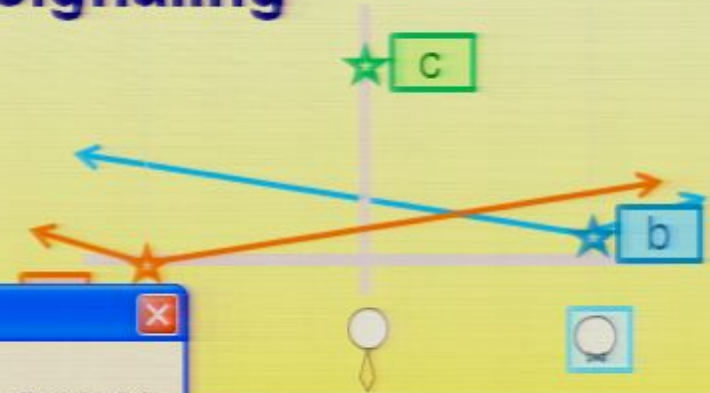


Thank you

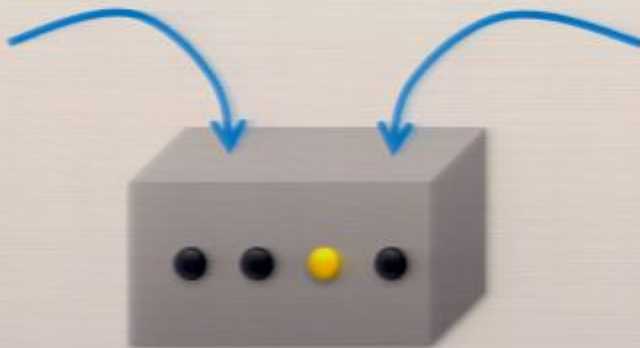
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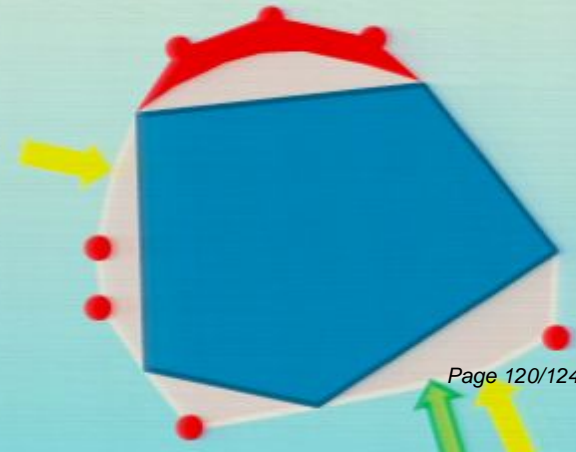
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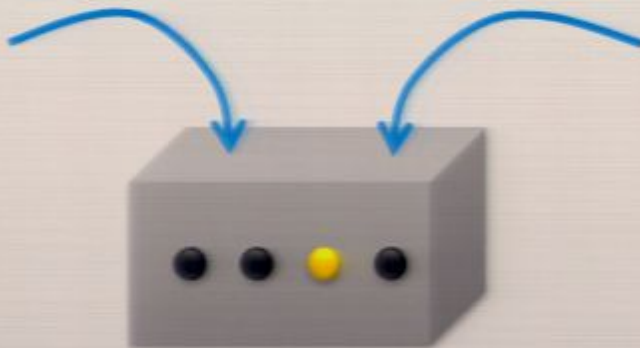
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Partite information causality

