Title: Entanglement and the three-dimensionality of the Bloch sphere

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Abstract: We consider theories that satisfy: information causality, reversibility, local discriminability, all tight effects are measurable. A property of these theories is that binary systems (with two perfectly distinguishable states and no more) have state spaces with the shape of a unit ball (the Bloch ball) of arbitrary dimension. It turns out that for dimension different than three these systems cannot be entangled. Hence, the only theory with entanglement which satisfying the above assumptions is quantum theory.

The singularity of entanglement

Outline:

A generalization of quantum theory
- Two qubits
- d-dimensional Bloch sphere

A large family of theories
- Relaxing $\mathrm{SO}(d)$-ness
- Axioms
- Information saturation

Generalizing all this

$$\rho = \frac{1}{4}\Big(I \otimes I + \textstyle\sum_j \mathbf{b}_j\, I \otimes \sigma_j + \sum_i \mathbf{a}_i\, \sigma_i \otimes I + \sum_{ij} \mathbf{c}_{ij}\, \sigma_i \otimes \sigma_j \Big)$$

$$\text{state } = \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \quad \mathbf{a}, \mathbf{b} \in \mathbb{R}^3 \quad \mathbf{c} \in \mathbb{R}^3 \otimes \mathbb{R}^3 \quad |\mathbf{a}|, |\mathbf{b}| \leq 1$$

constraints for $\mathbf{c} \quad \Longleftrightarrow \quad \rho \geq 0$

Outcome probabilities:

$$P(\mathbf{x}, \mathbf{y}) = \frac{1}{4}\Big(1 + \mathbf{y} \cdot \mathbf{b} + \mathbf{x} \cdot \mathbf{a} + (\mathbf{x} \otimes \mathbf{y}) \cdot \mathbf{c}\Big)$$

$$= \frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

Local reversible transformations $A, B \in \mathrm{SO}(3)$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ B\,\mathbf{b} \\ A\,\mathbf{a} \\ (A \otimes B)\,\mathbf{c} \end{bmatrix} = \begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

General reversible transformations $G \in \mathcal{G} =$ adjoint rep of $\mathrm{SU}(4)$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow G \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

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$|\mathbf{a}|, |\mathbf{b}| \leq 1$

constraints for $\mathbf{c}$?

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$|\mathbf{a}|, |\mathbf{b}| \leq 1$

constraints for $\mathbf{c}$?

$$P(\mathbf{x}, \mathbf{y}) = \frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \in [0, 1]$$

A ball system has 2 distinguishable states

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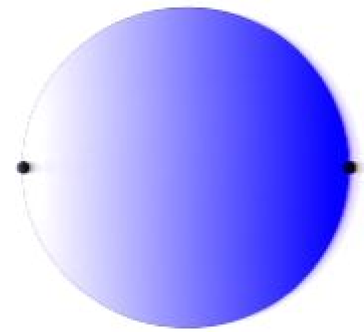

A 2-ball system has 4 or more

General reversible transformations $G \in \mathcal{G}$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow G \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

include local reversible transformations

$$\begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \in \mathcal{G}, \quad \text{for all } A, B \in \mathrm{SO}(d)$$

Reversibility: for every pair of pure states there is a reversible transformation taking one onto the other

$$\text{Set of pure states} = \left\{ G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix}, \quad \text{for all } G \in \mathcal{G} \right\}$$

$\mathcal{G}$ characterizes the set of states

Constraints for $\mathcal{G}$:

$$\frac{1}{4}\begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix} \in [0,1] \quad \text{for all } \mathbf{x}, \mathbf{y}, \mathbf{a}, \mathbf{b}$$

$$\begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \in \mathcal{G} \quad \text{for all } A, B \in \mathrm{SO}(d)$$

$\mathcal{G}$ is connected

Solutions for $d = 3$:

1. $\mathcal{G} = \mathrm{SO}(3) \times \mathrm{SO}(3)$
2. $\mathcal{G} =$ adjoint action of $\mathrm{SU}(4)$
3. its partial transposition

Solutions for $d \neq 3$:

1. $\mathcal{G} = \mathrm{SO}(d) \times \mathrm{SO}(d)$

local state-space $\rightleftarrows$ nonlocality $\rightleftarrows$ reversible transformations

Information processing in generalized probabilistic theories Jonathan Barrett

A generalized no-broadcasting theorem Howard Barnum, Jonathan Barrett, Matthew Leifer, Alexander Wilce

Strong nonlocality: A trade-off between states and measurements Anthony J. Short, Jonathan Barrett

All reversible dynamics in maximally non-local theories are trivial David Gross, Markus Mueller, Roger Colbeck, Oscar C. O. Dahlsten

The uncertainty principle determines the non-locality of quantum mechanics Jonathan Oppenheim, Stephanie Wehner

Limits on non-local correlations from the structure of the local state space Peter Janotta, Christian Gogolin, Jonathan Barrett, Nicolas Brunner

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Group of local transformations: $\mathcal{L} \neq \mathrm{SO}(d)$

$\mathcal{L}$ satisfies: reversibility, connectedness

Groups transitive on the $(d-1)$-sphere:

abstract group	d	$\mathcal{L}$
$\mathrm{SO}(d)$	$3, 4, 5 \dots$	$\mathcal{V}$
$\mathrm{SU}(d/2)$	$4, 6, 8 \dots$	$\mathcal{V} \oplus \mathcal{V}^*$
$\mathrm{U}(d/2)$	$2, 4, 6, 8 \dots$	$\mathcal{V} \oplus \mathcal{V}^*$
$\mathrm{Sp}(d/4)$	$8, 12, 16 \dots$	$\mathcal{V} \oplus \mathcal{V}^*$
$\mathrm{Sp}(d/4) \times \mathrm{U}(1)$	$8, 12, 16 \dots$	$\mathcal{V} \oplus \mathcal{V}^*$
$\mathrm{Sp}(d/4) \times \mathrm{SU}(2)$	$4, 8, 12 \dots$	$\mathcal{V} \oplus \mathcal{V}^*$
G_2	7	$\mathcal{V}$
$\mathrm{Spin}(7)$	8	?
$\mathrm{Spin}(9)$	16	?

Constraints for $\mathcal{G}$:

$$\frac{1}{4}\begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix} \in [0,1] \quad \text{for all } \mathbf{x}, \mathbf{y}, \mathbf{a}, \mathbf{b}$$

$$\begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \in \mathcal{G} \quad \text{for all } A, B \in \mathcal{L}$$

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Solutions for $d \neq 3$:

1. $\mathcal{L} \times \mathcal{L} \ \leq \ \mathcal{G} \ \leq \ \mathrm{SO}(d) \times \mathrm{SO}(d)$

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Axioms:

1. Finiteness, causality, convexity, closedness
2. Local tomography
3. Continuous reversibility
4. Ballness

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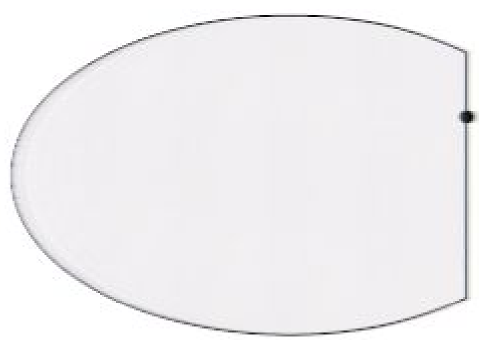
1. Finiteness, causality, convexity, closedness
2. Local tomography
3. Continuous reversibility
4. Perfect distinguishability
5. Information saturation

Perfect distinguishability: Every state that is not completely mixed can be perfectly distinguished from some other state.

Informational derivation of Quantum Theory G. Chiribella, G. M. D'Ariano, P. Perinotti



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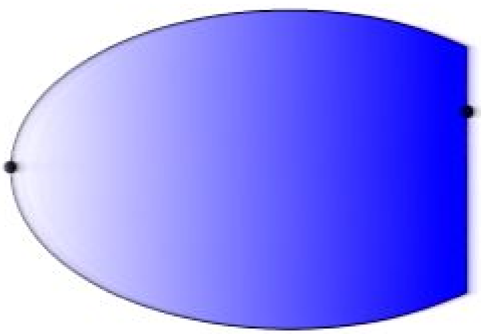

All extremal effects correspond to outcome probabilities.

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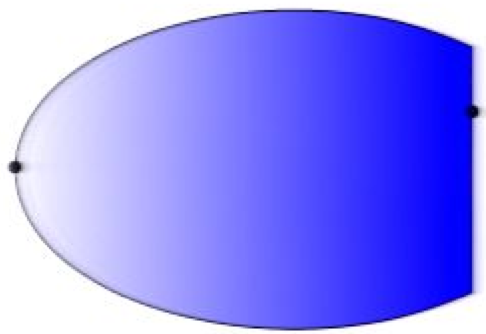

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Information saturation: When a system with N distinguishable states is used to perfectly encode a N-valued variable, it cannot additionally encode any other information.

$$P(a'|a,b) = \delta^{a}_{a'} \quad \Rightarrow \quad P(b'|a,b) = P(b'|a)$$

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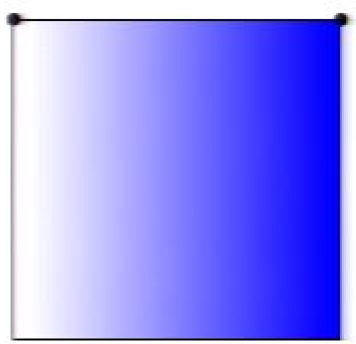

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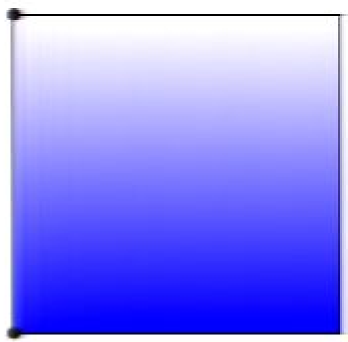

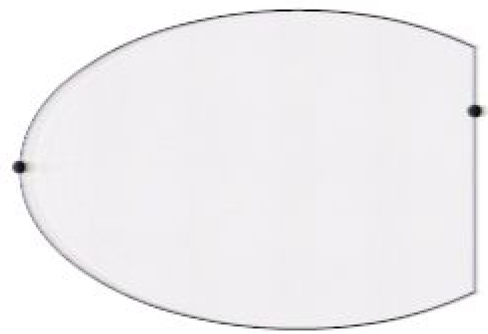

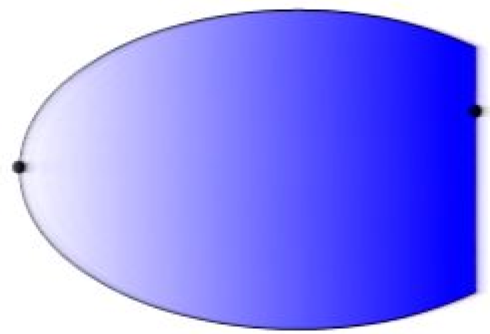

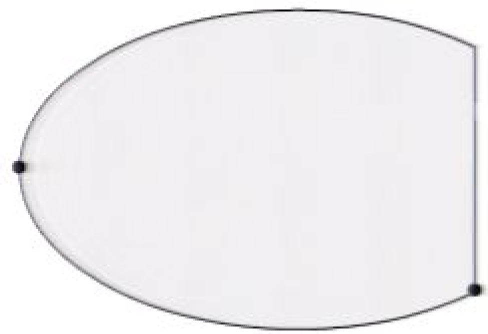

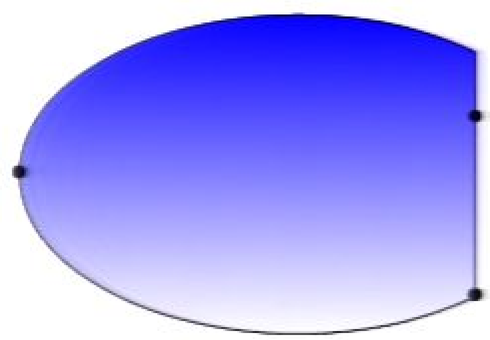

Information saturation + Perfect distinguishability $\Rightarrow$ No facets

No facets + Reversibility $\Rightarrow$ Ballness

QT is the only possibility which allows entanglement and satisfies:

1. Local tomography
2. Continuous reversibility
3. Perfect distinguishability
4. Information saturation

What happens beyond two binary systems?

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Relaxing connectedness:

$\mathcal{L}$ non-connected $\Rightarrow$ nothing changes

$\mathcal{G}$ non-connected $\Rightarrow \mathcal{L} \times \mathcal{L} \leq \mathcal{G}_{\mathrm{conn}} \leq \mathrm{SO}(d) \times \mathrm{SO}(d)$

Is there a non-connected $\mathcal{G}$ which allows entanglement?

Generalizing all this

Reversible, locally-tomographic theories:

$\mathcal{L}$ is a compact matrix group

$$\text{Set of pure states} = \left\{ \begin{bmatrix} 1 \\ A\,\mathbf{s} \end{bmatrix}, \quad \text{for all } A \in \mathcal{L} \right\}$$

$$\text{some effects} = \left\{ \begin{bmatrix} e \\ A^T\,\mathbf{e} \end{bmatrix}, \quad \text{for all } A \in \mathcal{L} \right\}$$

$$\begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \in \mathcal{G} \quad \text{for all } A, B \in \mathcal{L}$$

$(\mathbf{x} \otimes \mathbf{y})G(\mathbf{a} \otimes \mathbf{b}) \in [0, 1]$ for all states $\mathbf{a}, \mathbf{b}$ and effects $\mathbf{x}, \mathbf{y}$

Generalizing all this

Is quantum theory the only reversible, locally-tomographic theory which allows entanglement?