

Title: Entanglement and the three-dimensionality of the Bloch sphere

Date: May 09, 2011 02:00 PM

URL: <http://pirsa.org/11050029>

Abstract: We consider theories that satisfy: information causality, reversibility, local discriminability, all tight effects are measurable. A property of these theories is that binary systems (with two perfectly distinguishable states and no more) have state spaces with the shape of a unit ball (the Bloch ball) of arbitrary dimension. It turns out that for dimension different than three these systems cannot be entangled. Hence, the only theory with entanglement which satisfying the above assumptions is quantum theory.

The singularity of entanglement

A generalization of quantum theory

Two qubits

Outline:

A generalization of quantum theory

Two qubits

d -dimensional Bloch sphere

A large family of theories

Relaxing $SO(d)$ -ness

Axioms

Information saturation

Generalizing all this

$$\rho = \frac{1}{4} \left(I \otimes I + \sum_j \mathbf{b}_j I \otimes \sigma_j + \sum_i \mathbf{a}_i \sigma_i \otimes I + \sum_{ij} \mathbf{c}_{ij} \sigma_i \otimes \sigma_j \right)$$

$$\text{state} = \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \quad \mathbf{a}, \mathbf{b} \in \mathbb{R}^3 \quad \mathbf{c} \in \mathbb{R}^3 \otimes \mathbb{R}^3 \quad |\mathbf{a}|, |\mathbf{b}| \leq 1$$

$$\text{constraints for } \mathbf{c} \iff \rho \geq 0$$

Outcome probabilities:

$$\begin{aligned} P(\mathbf{x}, \mathbf{y}) &= \frac{1}{4} \left(1 + \mathbf{y} \cdot \mathbf{b} + \mathbf{x} \cdot \mathbf{a} + (\mathbf{x} \otimes \mathbf{y}) \cdot \mathbf{c} \right) \\ &= \frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \end{aligned}$$

Local reversible transformations $A, B \in SO(3)$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ B\mathbf{b} \\ A\mathbf{a} \\ (A \otimes B)\mathbf{c} \end{bmatrix} = \begin{bmatrix} 1 & \\ & A \end{bmatrix} \otimes \begin{bmatrix} 1 & \\ & B \end{bmatrix} \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

General reversible transformations $G \in \mathcal{G} = \text{adjoint rep of } SU(4)$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow G \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

A generalization of quantum theory

d-dimensional Bloch sphere

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$$\text{state} = \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \quad \mathbf{a}, \mathbf{b} \in \mathbb{R}^d \quad \mathbf{c} \in \mathbb{R}^d \otimes \mathbb{R}^d \quad d = 2, 3, 4, \dots$$

$$|\mathbf{a}|, |\mathbf{b}| \leq 1$$

constraints for \mathbf{c} ?

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$$\text{state} = \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \quad \mathbf{a}, \mathbf{b} \in \mathbb{R}^d \quad \mathbf{c} \in \mathbb{R}^d \otimes \mathbb{R}^d \quad d = 2, 3, 4, \dots$$

$$|\mathbf{a}|, |\mathbf{b}| \leq 1$$

constraints for \mathbf{c} ?

$$P(\mathbf{x}, \mathbf{y}) = \frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \in [0, 1]$$

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d-dimensional Bloch sphere

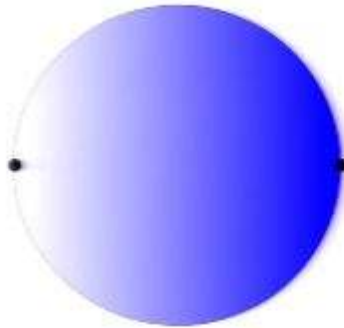
A ball system has 2 distinguishable states



A generalization of quantum theory

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A ball system has 2 distinguishable states



A 2-ball system has 4 or more

A generalization of quantum theory

d-dimensional Bloch sphere

General reversible transformations $G \in \mathcal{G}$

$$\begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix} \longrightarrow G \begin{bmatrix} 1 \\ \mathbf{b} \\ \mathbf{a} \\ \mathbf{c} \end{bmatrix}$$

include local reversible transformations

$$\begin{bmatrix} 1 \\ A \end{bmatrix} \otimes \begin{bmatrix} 1 \\ B \end{bmatrix} \in \mathcal{G}, \quad \text{for all } A, B \in \text{SO}(d)$$

A generalization of quantum theory

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Reversibility: for every pair of pure states there is a reversible transformation taking one onto the other

$$\text{Set of pure states} = \left\{ G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix}, \text{ for all } G \in \mathcal{G} \right\}$$

\mathcal{G} characterizes the set of states

A generalization of quantum theory

d-dimensional Bloch sphere

Constraints for \mathcal{G} :

$$\frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix} \in [0, 1] \quad \text{for all } \mathbf{x}, \mathbf{y}, \mathbf{a}, \mathbf{b}$$

$$\begin{bmatrix} 1 \\ A \end{bmatrix} \otimes \begin{bmatrix} 1 \\ B \end{bmatrix} \in \mathcal{G} \quad \text{for all } A, B \in \text{SO}(d)$$

\mathcal{G} is connected

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Solutions for $d = 3$:

1. $\mathcal{G} = \text{SO}(3) \times \text{SO}(3)$
2. $\mathcal{G} =$ adjoint action of $\text{SU}(4)$
3. its partial transposition

Solutions for $d \neq 3$:

1. $\mathcal{G} = \text{SO}(d) \times \text{SO}(d)$

A generalization of quantum theory

d-dimensional Bloch sphere

local state-space \Leftrightarrow nonlocality \Leftrightarrow reversible transformations

Information processing in generalized probabilistic theories Jonathan Barrett

A generalized no-broadcasting theorem Howard Barnum, Jonathan Barrett, Matthew Leifer,

Alexander Wilce

Strong nonlocality: A trade-off between states and measurements Anthony J. Short,

Jonathan Barrett

All reversible dynamics in maximally non-local theories are trivial David Gross,

Markus Mueller, Roger Colbeck, Oscar C. O. Dahlsten

The uncertainty principle determines the non-locality of quantum mechanics

Jonathan Oppenheim, Stephanie Wehner

Limits on non-local correlations from the structure of the local state space

Peter Janotta, Christian Gogolin, Jonathan Barrett, Nicolas Brunner

A large family of theories

Relaxing $SO(d)$ -ness

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Relaxing $SO(d)$ -ness

Group of local transformations: $\mathcal{L} \neq SO(d)$

\mathcal{L} satisfies: reversibility, connectedness

Groups transitive on the $(d - 1)$ -sphere:

| abstract group | d | \mathcal{L} |
|------------------------|----------------|------------------------------------|
| $SO(d)$ | 3, 4, 5 ... | \mathcal{V} |
| $SU(d/2)$ | 4, 6, 8 ... | $\mathcal{V} \oplus \mathcal{V}^*$ |
| $U(d/2)$ | 2, 4, 6, 8 ... | $\mathcal{V} \oplus \mathcal{V}^*$ |
| $Sp(d/4)$ | 8, 12, 16 ... | $\mathcal{V} \oplus \mathcal{V}^*$ |
| $Sp(d/4) \times U(1)$ | 8, 12, 16 ... | $\mathcal{V} \oplus \mathcal{V}^*$ |
| $Sp(d/4) \times SU(2)$ | 4, 8, 12 ... | $\mathcal{V} \oplus \mathcal{V}^*$ |
| G_2 | 7 | \mathcal{V} |
| $Spin(7)$ | 8 | ? |
| $Spin(9)$ | 16 | ? |

Constraints for \mathcal{G} :

$$\frac{1}{4} \begin{bmatrix} 1 \\ \mathbf{x} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{y} \end{bmatrix} \cdot G \begin{bmatrix} 1 \\ \mathbf{a} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ \mathbf{b} \end{bmatrix} \in [0, 1] \quad \text{for all } \mathbf{x}, \mathbf{y}, \mathbf{a}, \mathbf{b}$$

$$\begin{bmatrix} 1 \\ A \end{bmatrix} \otimes \begin{bmatrix} 1 \\ B \end{bmatrix} \in \mathcal{G} \quad \text{for all } A, B \in \mathcal{L}$$

\mathcal{G} is connected

Solutions for $d = 3$:

1. $\mathcal{G} = SO(3) \times SO(3)$
2. $\mathcal{G} =$ adjoint action of $SU(4)$
3. its partial transposition

Solutions for $d \neq 3$:

1. $\mathcal{L} \times \mathcal{L} \leq \mathcal{G} \leq SO(d) \times SO(d)$

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Axioms:

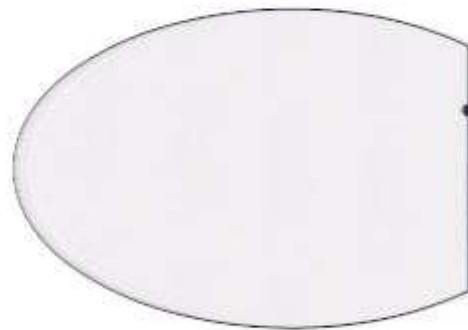
1. Finiteness, causality, convexity, closedness
2. Local tomography
3. Continuous reversibility
4. Ballness

Axioms:

1. Finiteness, causality, convexity, closedness
2. Local tomography
3. Continuous reversibility
4. Perfect distinguishability
5. Information saturation

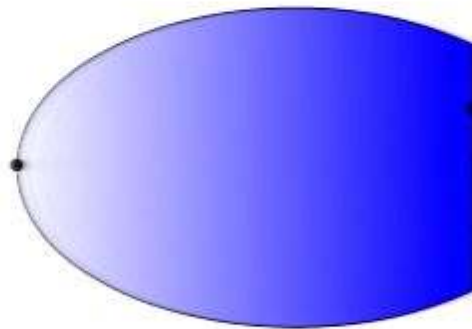
Perfect distinguishability: Every state that is not completely mixed can be perfectly distinguished from some other state.

Informational derivation of Quantum Theory G. Chiribella, G. M. D'Ariano, P. Perinotti



Perfect distinguishability: Every state that is not completely mixed can be perfectly distinguished from some other state.

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All extremal effects correspond to outcome probabilities.

A large family of theories

Information saturation

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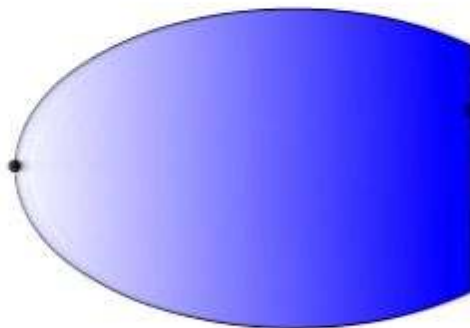
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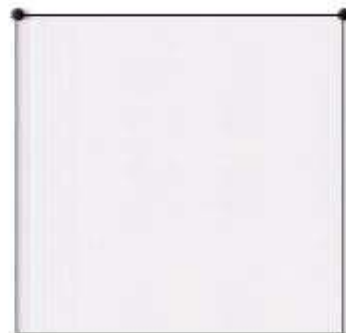
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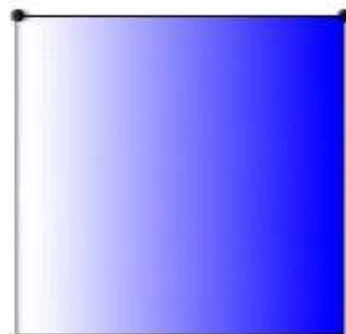
Information saturation: When a system with N distinguishable states is used to perfectly encode a N -valued variable, it cannot additionally encode any other information.

$$P(a'|a, b) = \delta_{a'}^a \quad \Rightarrow \quad P(b'|a, b) = P(b'|a)$$



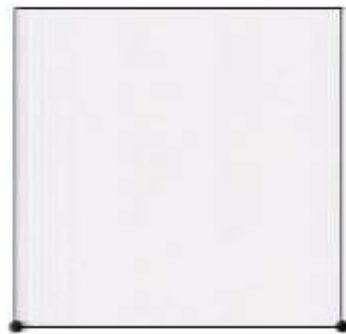
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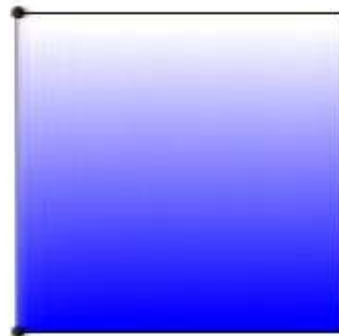
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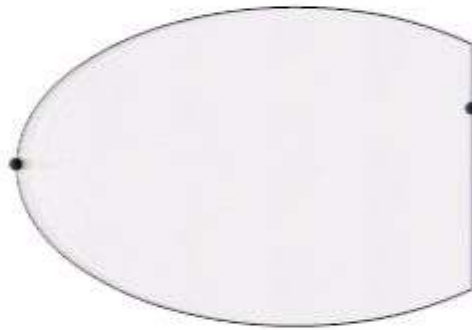
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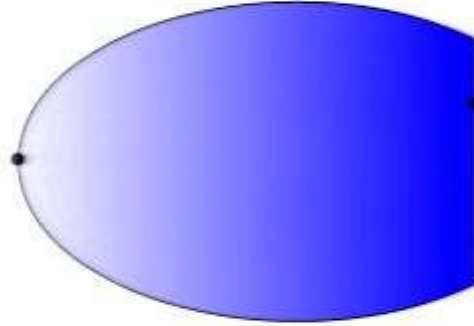
A large family of theories

Information saturation



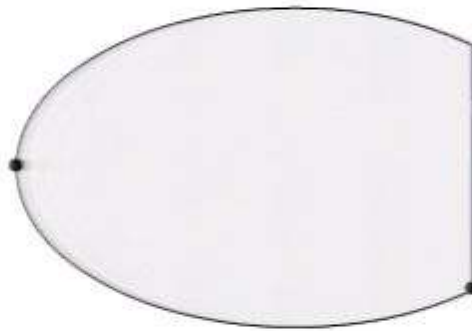
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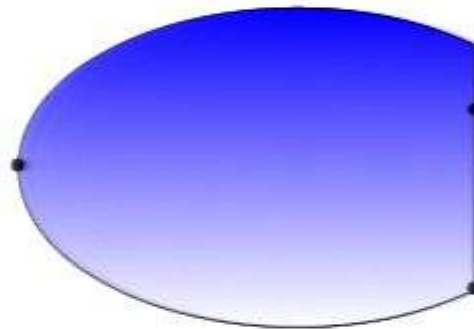
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Information saturation



A large family of theories

Information saturation



Information saturation + Perfect distinguishability \Rightarrow No facets

No facets + Reversibility \Rightarrow Ballness

A large family of theories

Information saturation

QT is the only possibility which allows entanglement and satisfies:

1. Local tomography
2. Continuous reversibility
3. Perfect distinguishability
4. Information saturation

What happens beyond two binary systems?

The team



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Generalizing all this

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Relaxing connectedness:

\mathcal{L} non-connected \Rightarrow nothing changes

\mathcal{G} non-connected $\Rightarrow \mathcal{L} \times \mathcal{L} \leq \mathcal{G}_{\text{conn}} \leq SO(d) \times SO(d)$

Is there a non-connected \mathcal{G} which allows entanglement?

Generalizing all this

Reversible, locally-tomographic theories:

\mathcal{L} is a compact matrix group

$$\text{Set of pure states} = \left\{ \begin{bmatrix} 1 \\ A \mathbf{s} \end{bmatrix}, \text{ for all } A \in \mathcal{L} \right\}$$

$$\text{some effects} = \left\{ \begin{bmatrix} e \\ A^T \mathbf{e} \end{bmatrix}, \text{ for all } A \in \mathcal{L} \right\}$$

$$\begin{bmatrix} 1 \\ A \end{bmatrix} \otimes \begin{bmatrix} 1 \\ B \end{bmatrix} \in \mathcal{G} \text{ for all } A, B \in \mathcal{L}$$

$$(\mathbf{x} \otimes \mathbf{y})G(\mathbf{a} \otimes \mathbf{b}) \in [0, 1] \text{ for all states } \mathbf{a}, \mathbf{b} \text{ and effects } \mathbf{x}, \mathbf{y}$$

Generalizing all this

Is quantum theory the only reversible, locally-tomographic theory
which allows entanglement?