Title: Exact infinite-time statistics of the Loschmidt echo for a quantum quench

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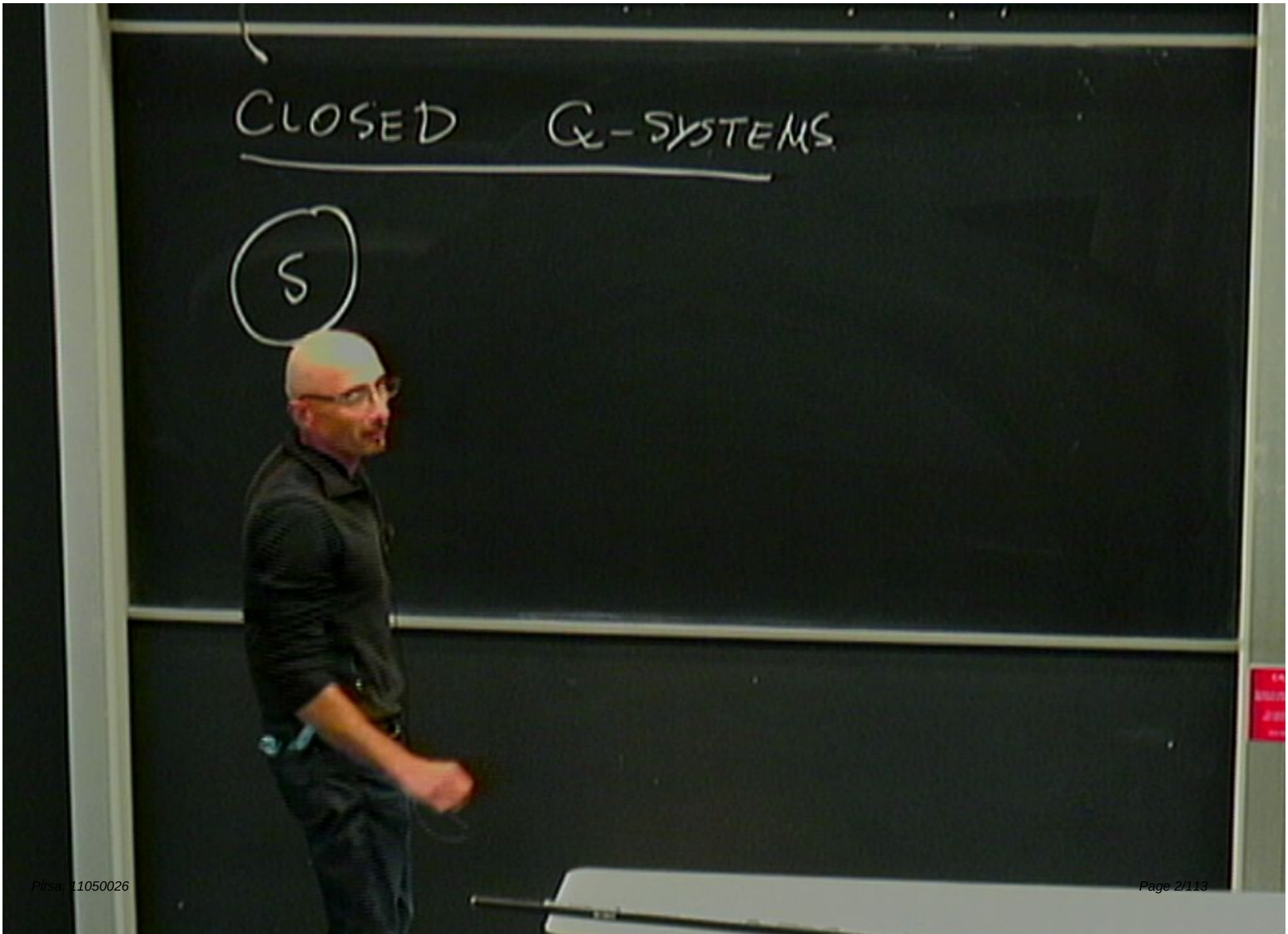
Abstract: The equilibration dynamics of a closed quantum system is encoded in the long-time distribution function of generic observables. In this paper we consider the Loschmidt echo generalized to finite temperature, and show that we can obtain an exact expression for its long-time distribution for a closed system described by a quantum XY chain following a sudden quench. In the thermodynamic limit the logarithm of the Loschmidt echo becomes normally distributed, whereas for small quenches in the opposite, quasi-critical regime, the distribution function acquires a universal double-peaked form indicating poor equilibration. These findings, obtained by a central limit theorem-type result, extend to completely general models in the small-quench regime.


CLOSED Q-SYSTEMS
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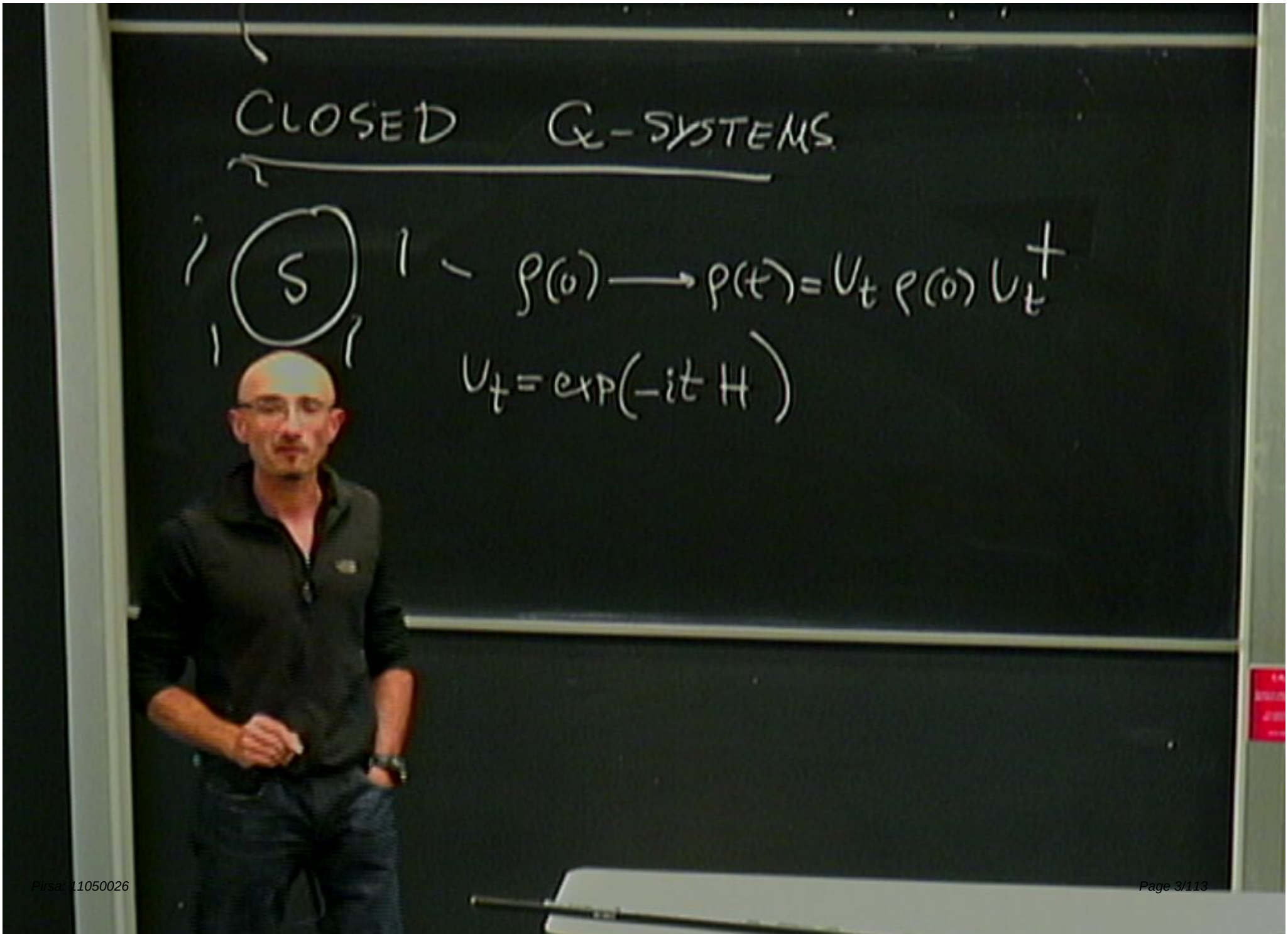
CLOSED Q-SYSTEMS
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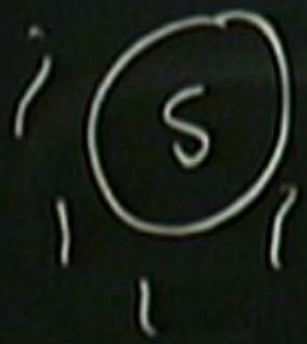
$\rho(0) \longrightarrow \rho(t) = U_t \rho(0) U_t^\dagger$
$U_t = \exp(-it H)$

CLOSED Q-SYSTEMS



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$$H = \sum_m E_m \Pi_m$$

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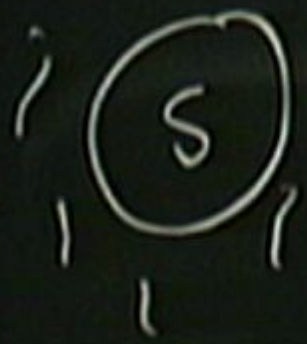

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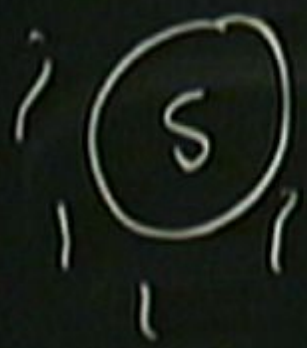

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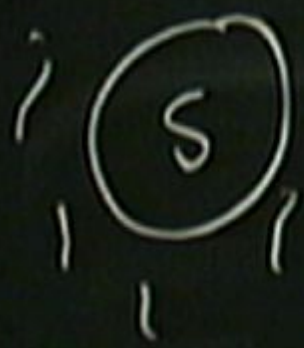
CLOSED Q-SYSTEMS



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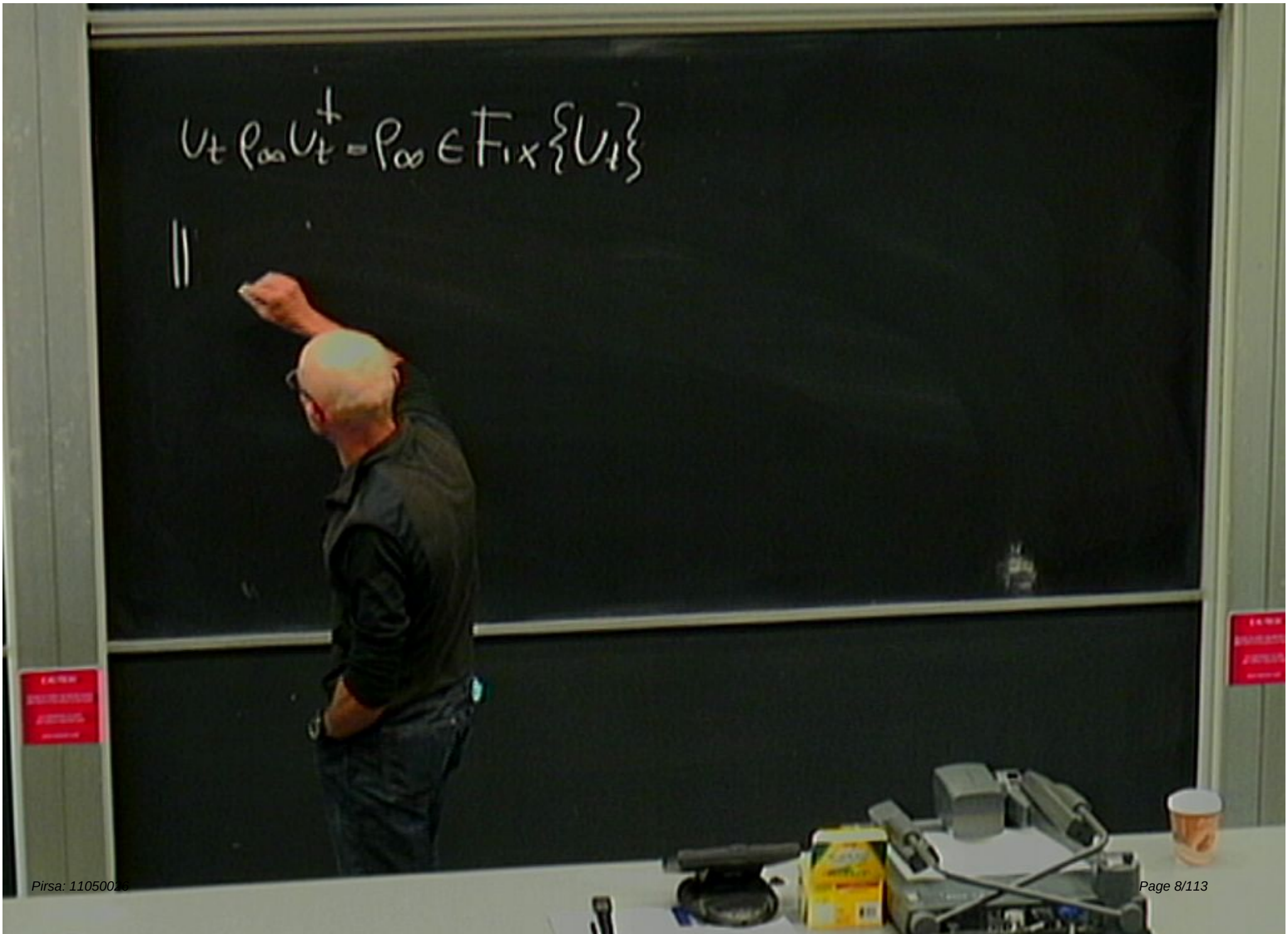
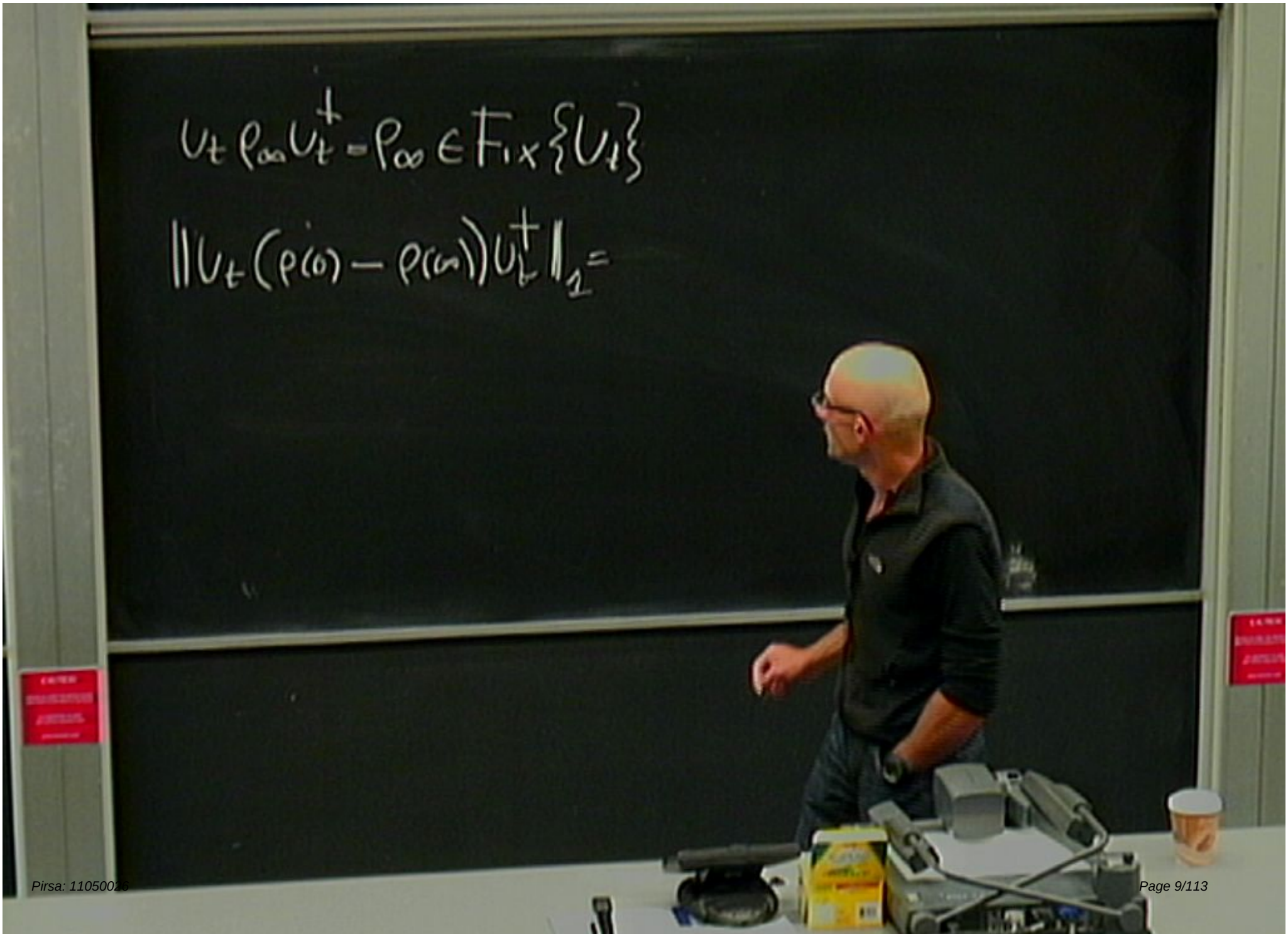

$U_t \rho_\infty U_t^\dagger = \rho_\infty \in \mathrm{Fix}\{U_t\}$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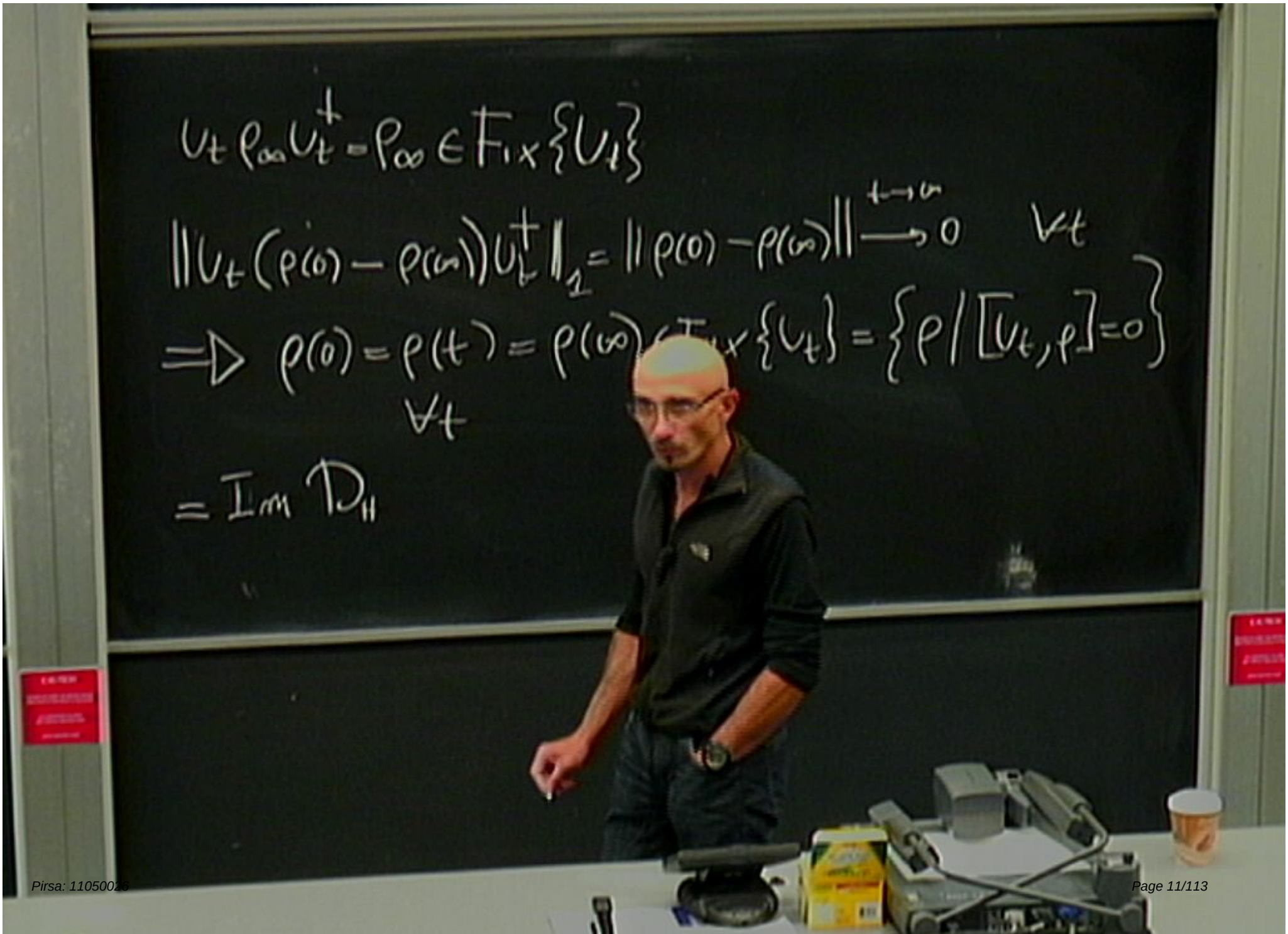
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$\|U_t(\rho(0) - \rho(\infty))U_t^\dagger\|_1 = \|\rho(0) - \rho(\infty)\| \xrightarrow{t\to\infty} 0 \quad \forall t$
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$= \mathrm{Im}\, \mathcal{D}_{\|}$

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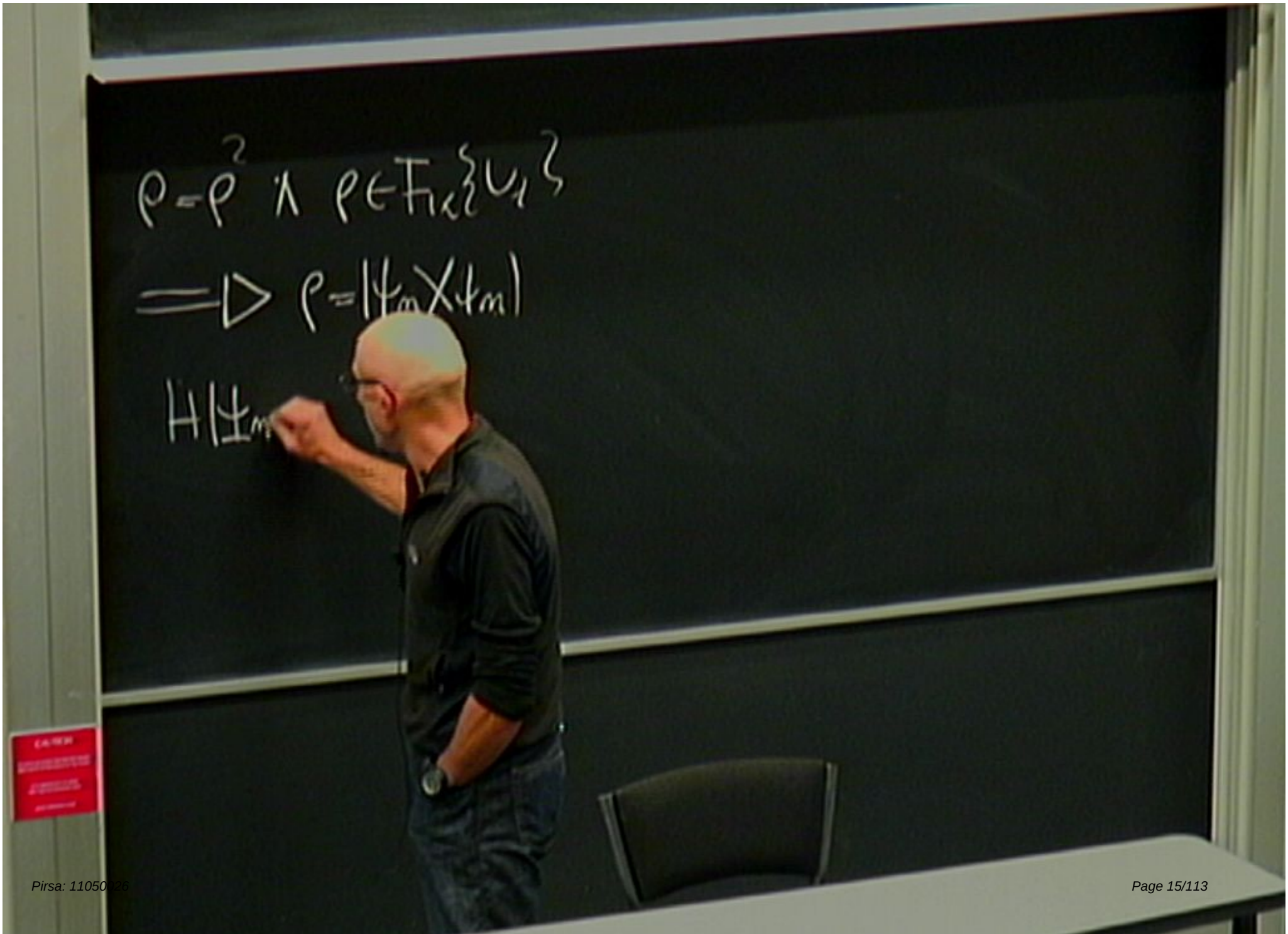
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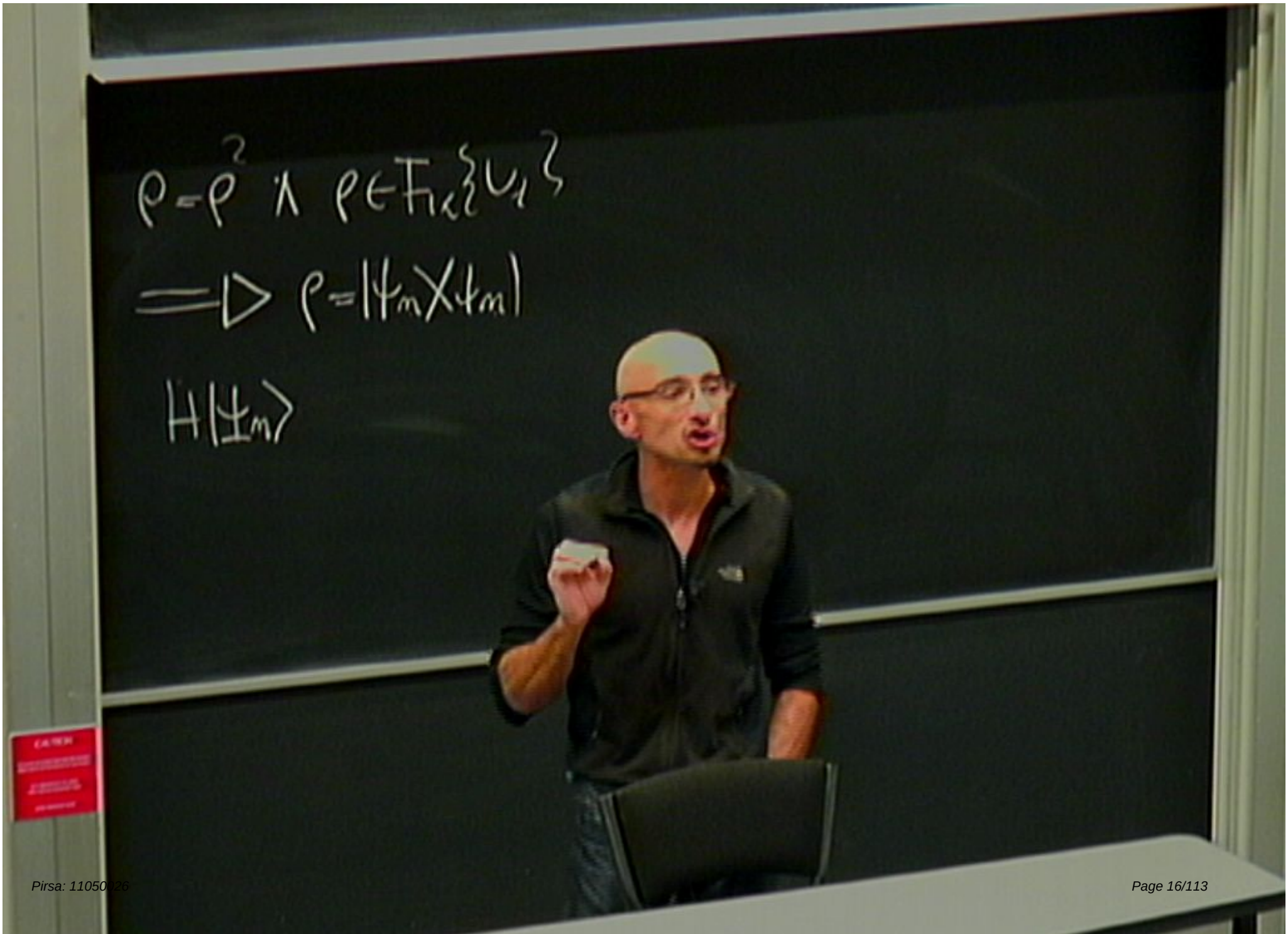
$H|\psi_m$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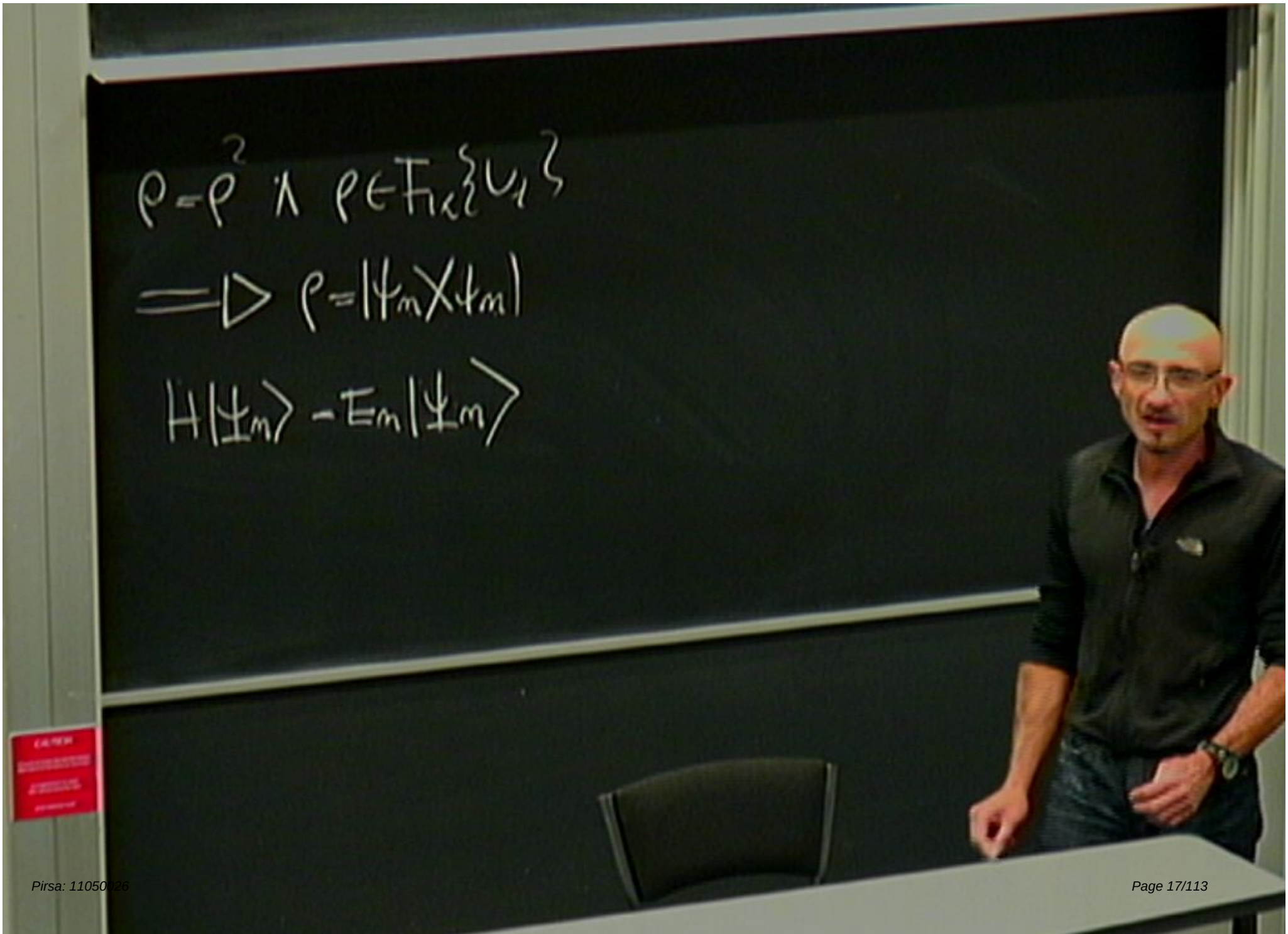
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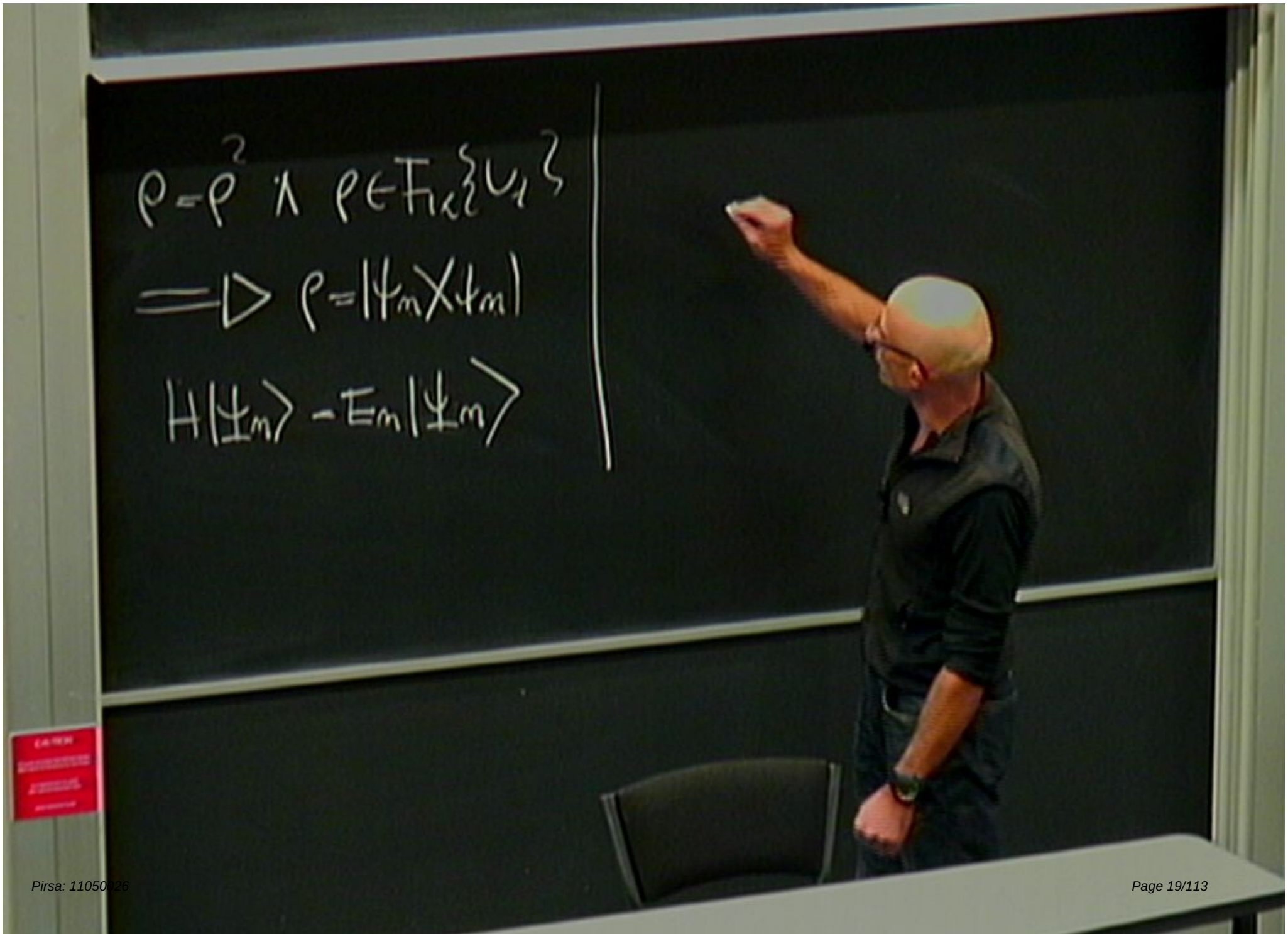
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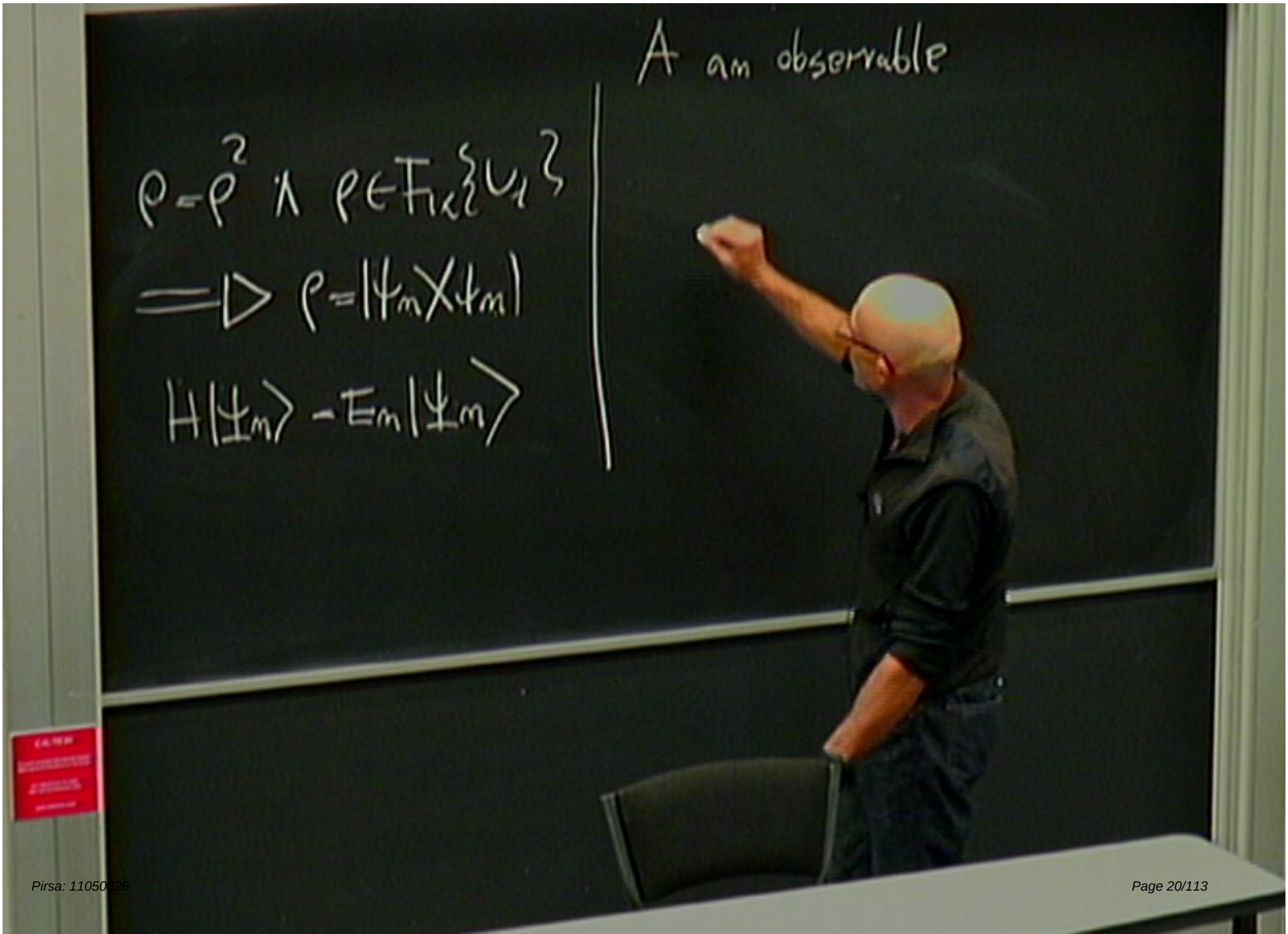
A am observable
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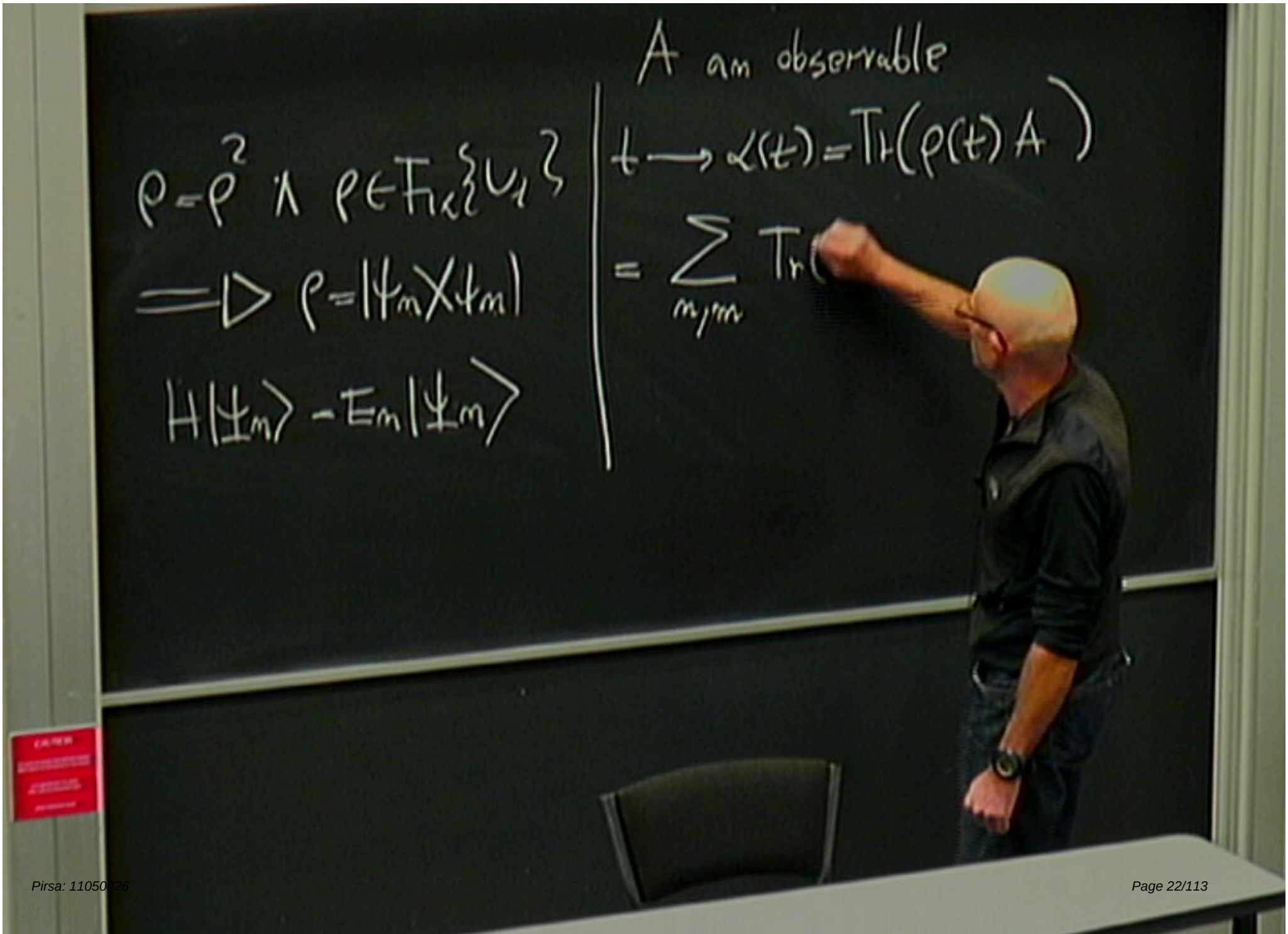
$t \longrightarrow \alpha(t) = \mathrm{Tr}(\rho(t)A)$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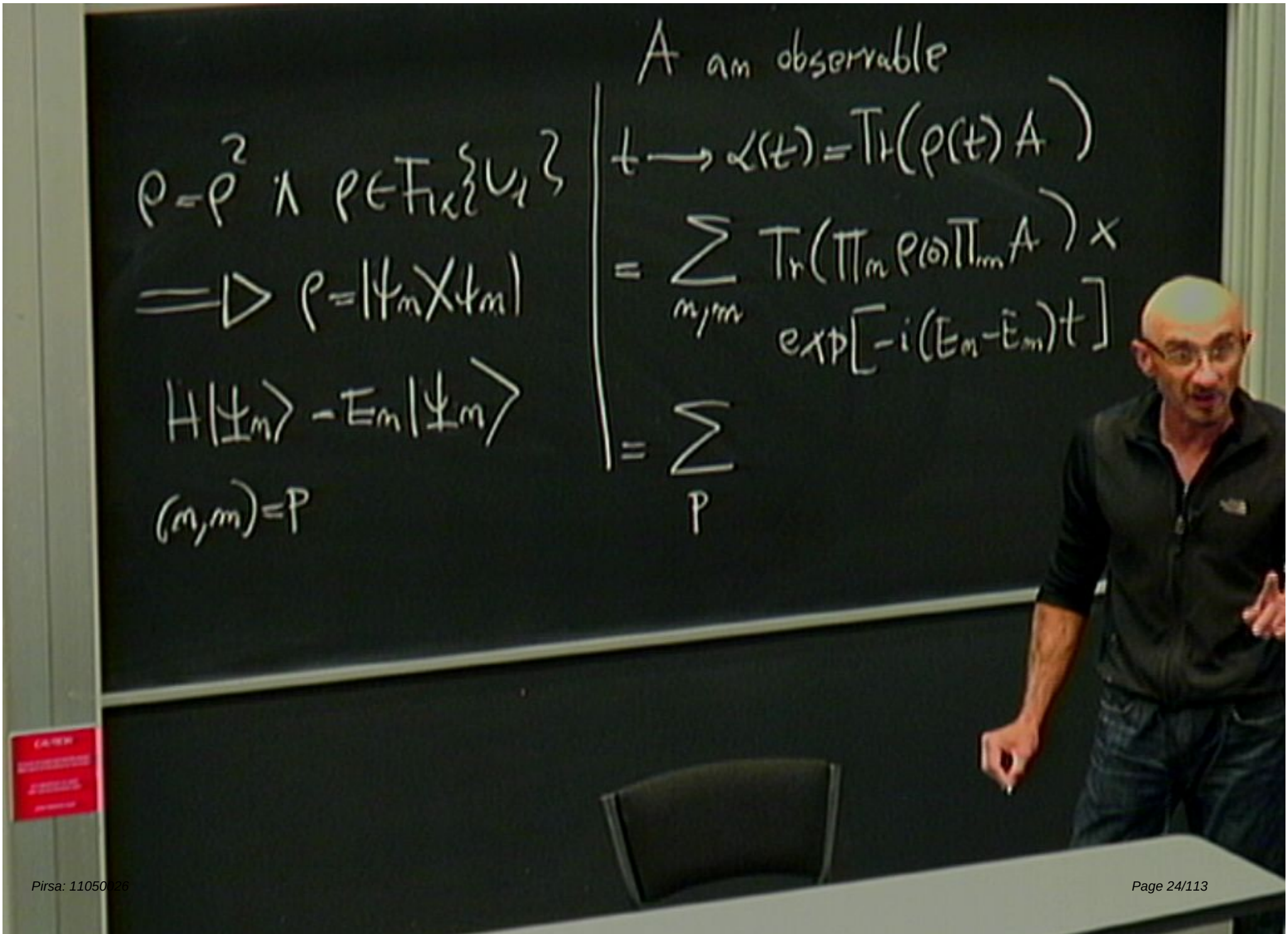
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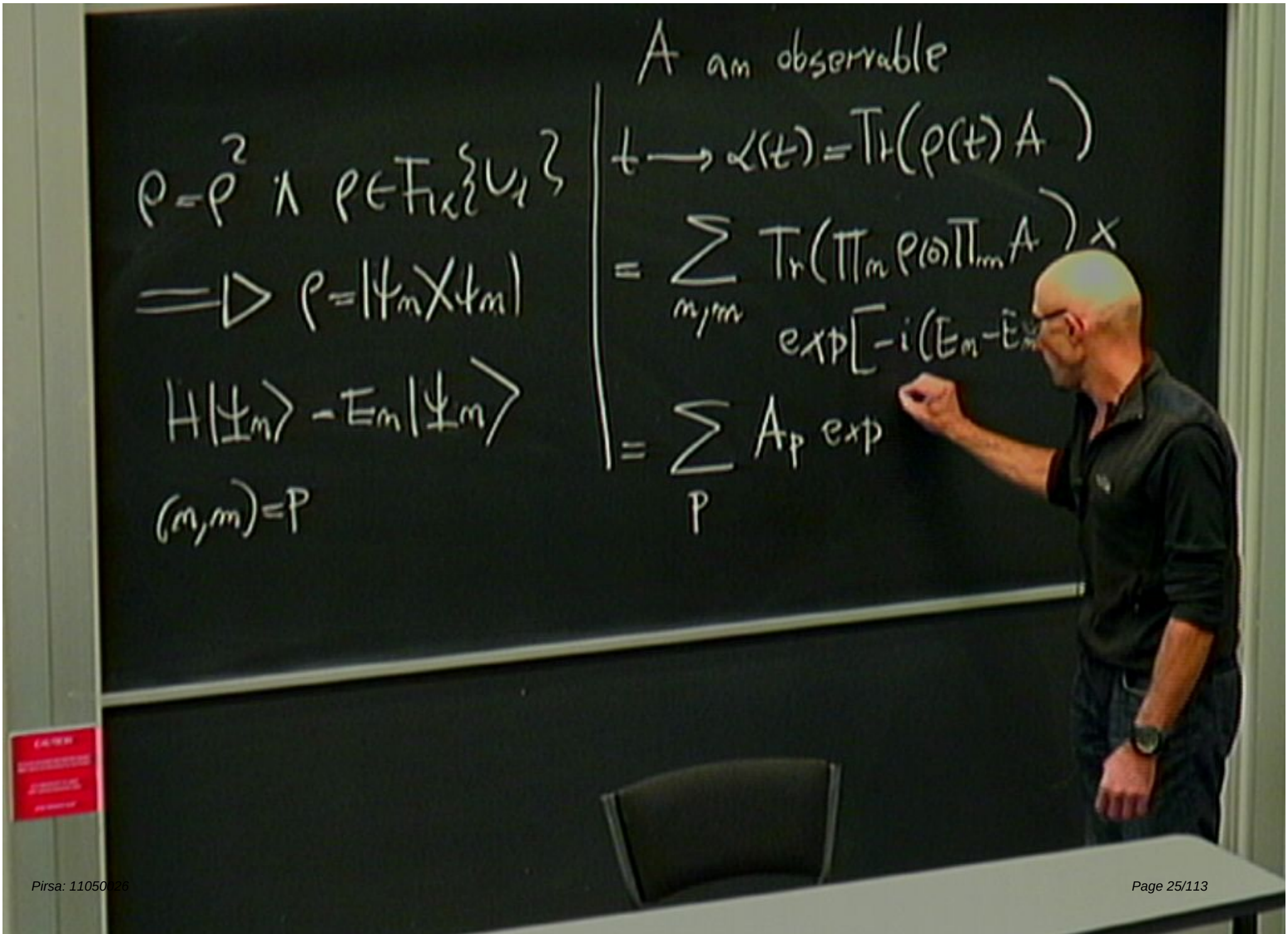
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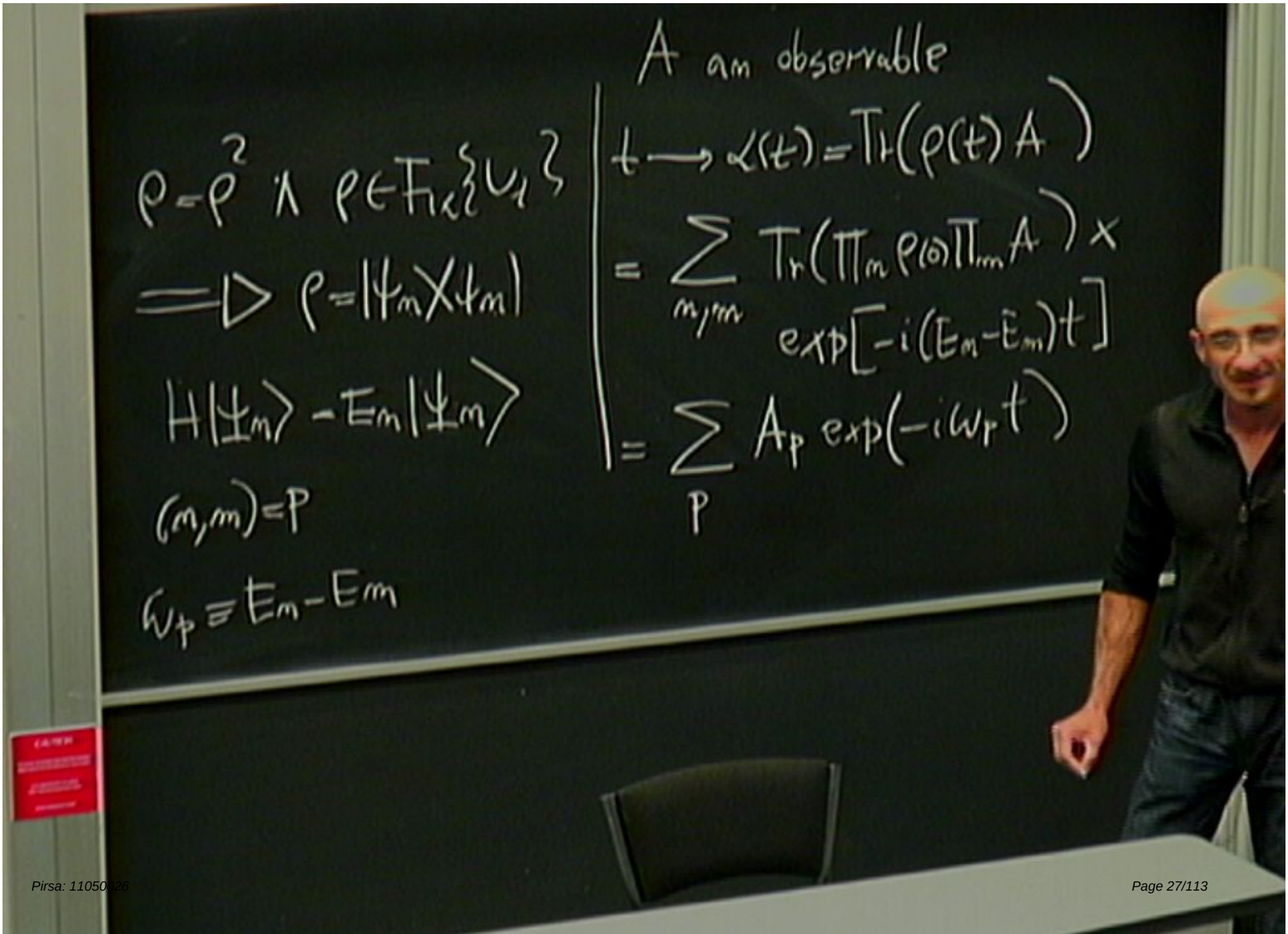
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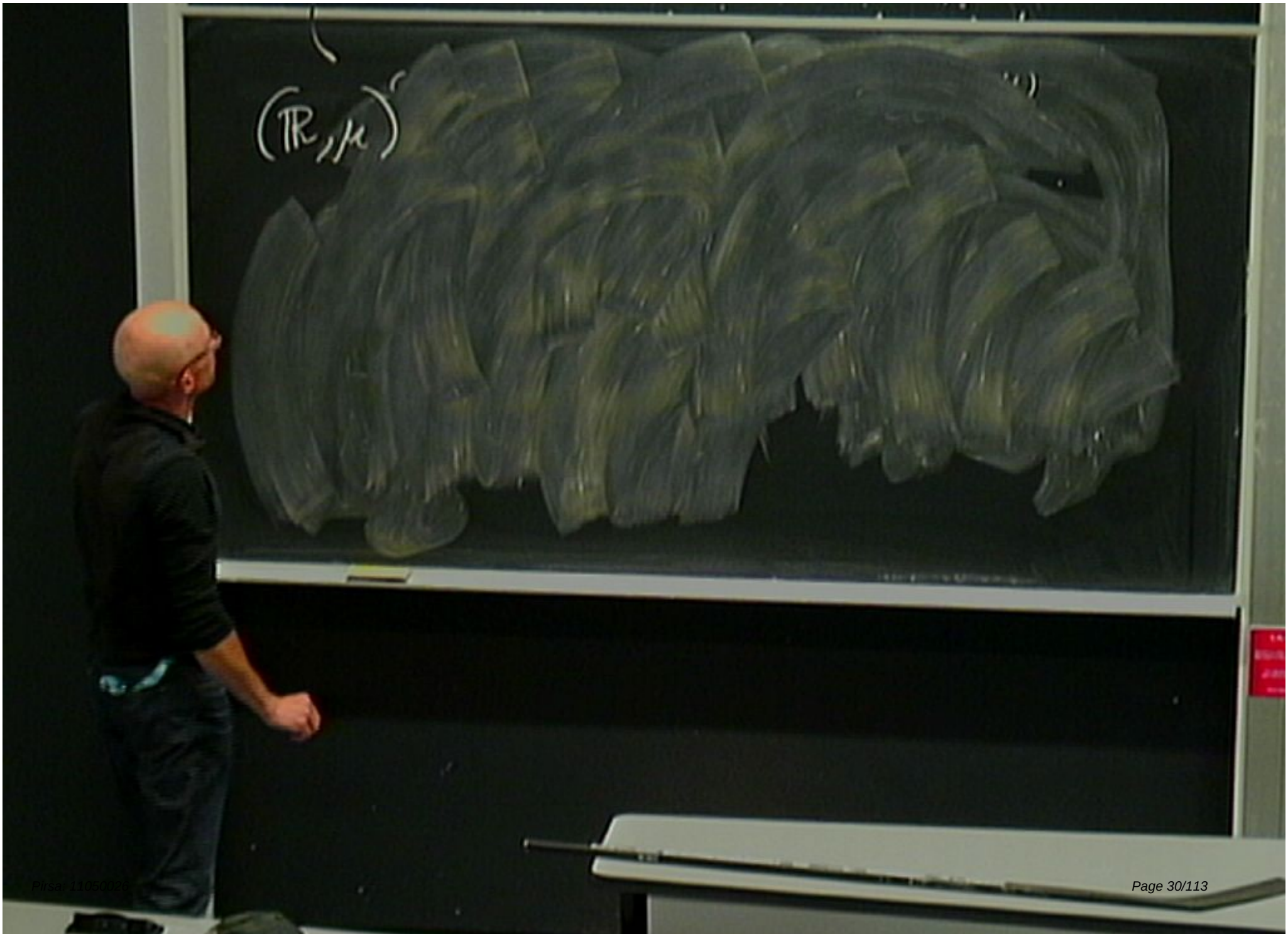
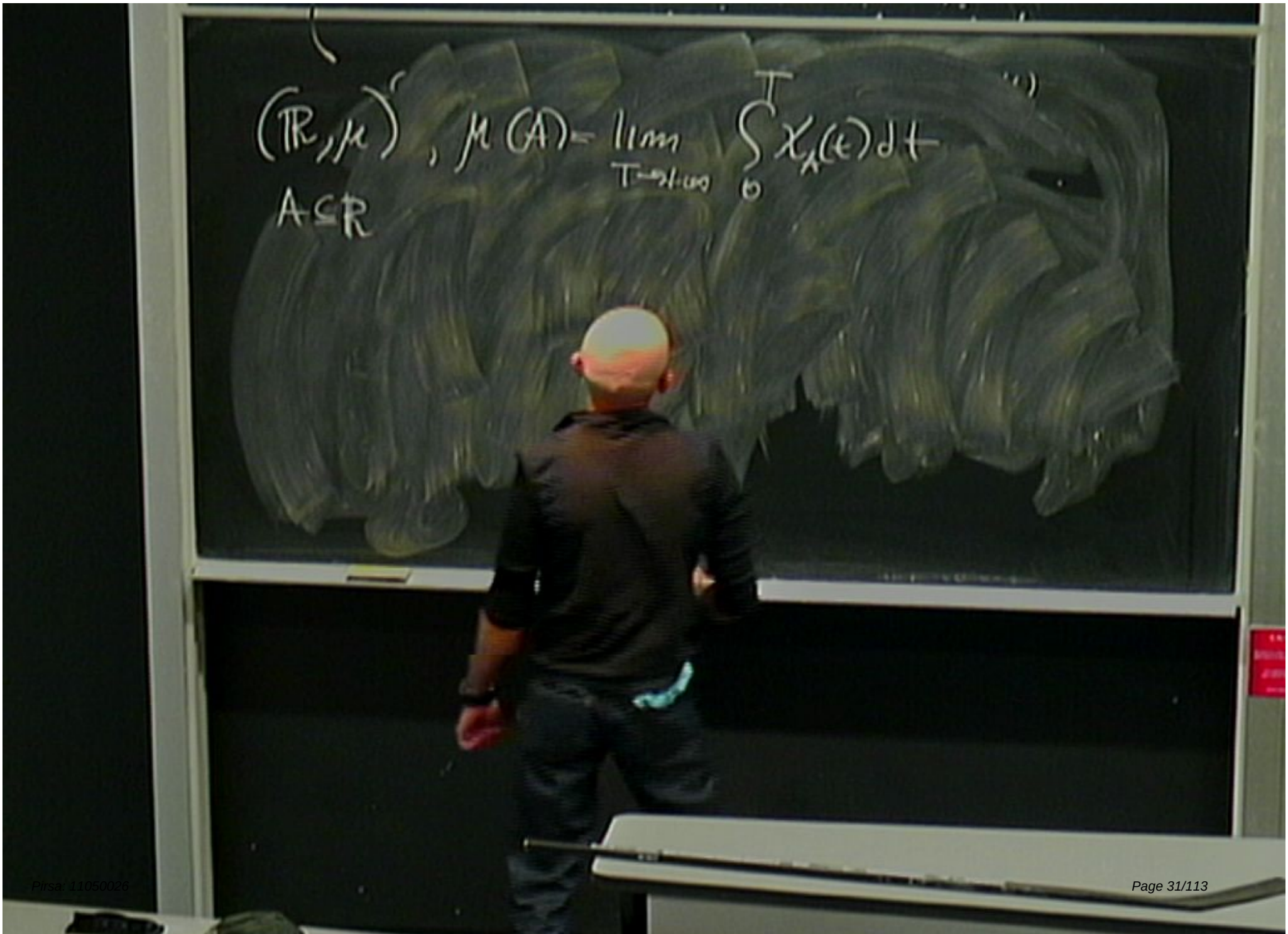
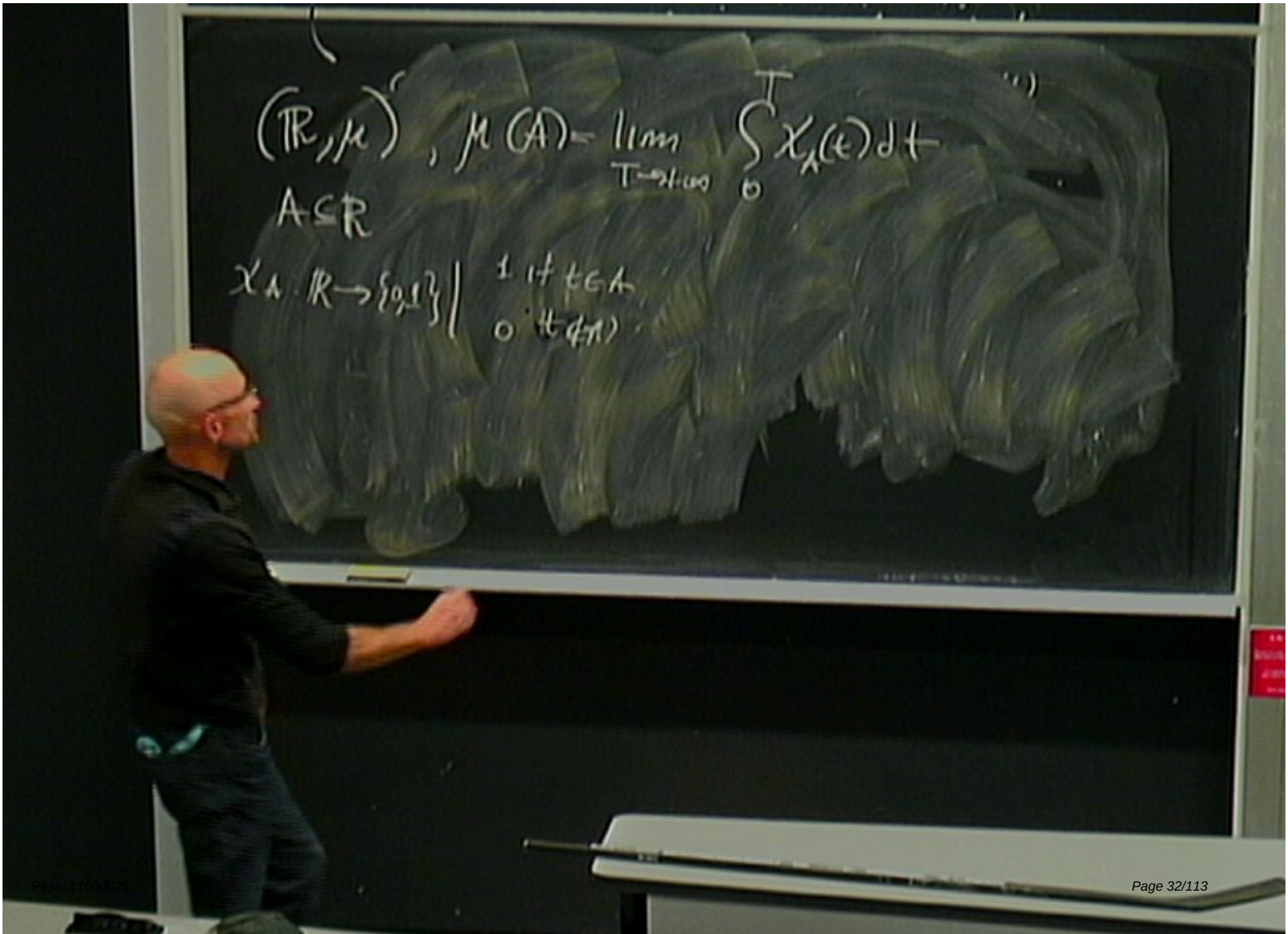

$(\mathbb{R}, \mu)$


$(\mathbb{R}, \mu)$, $\mu(A) = \lim_{T \to +\infty} \int_0^T \chi_A(t)\,dt$
$A \subseteq \mathbb{R}$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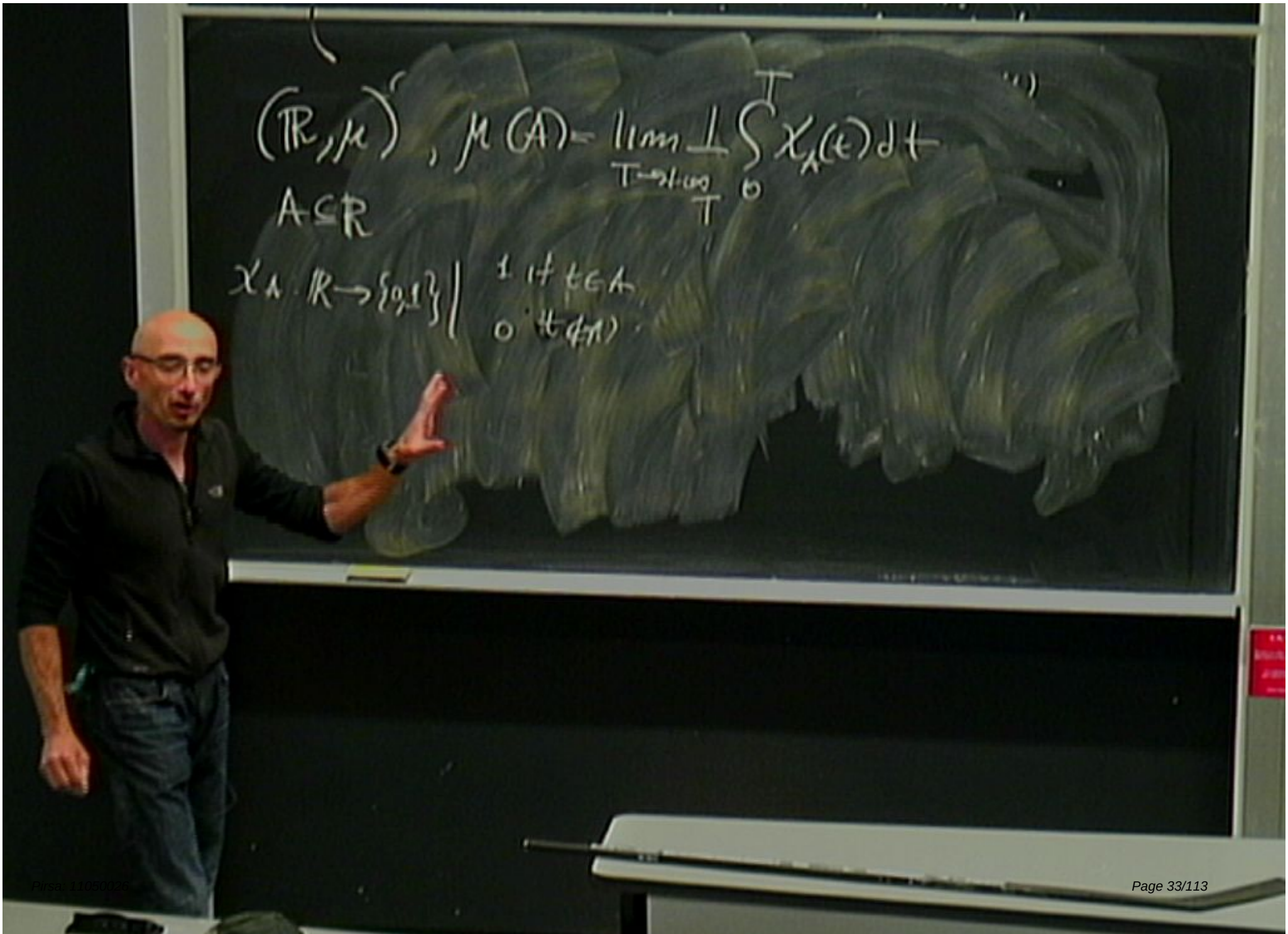
$\chi_A : \mathbb{R} \to \{0,1\}$ | 1 if $t \in A$, 0 if $t \notin A$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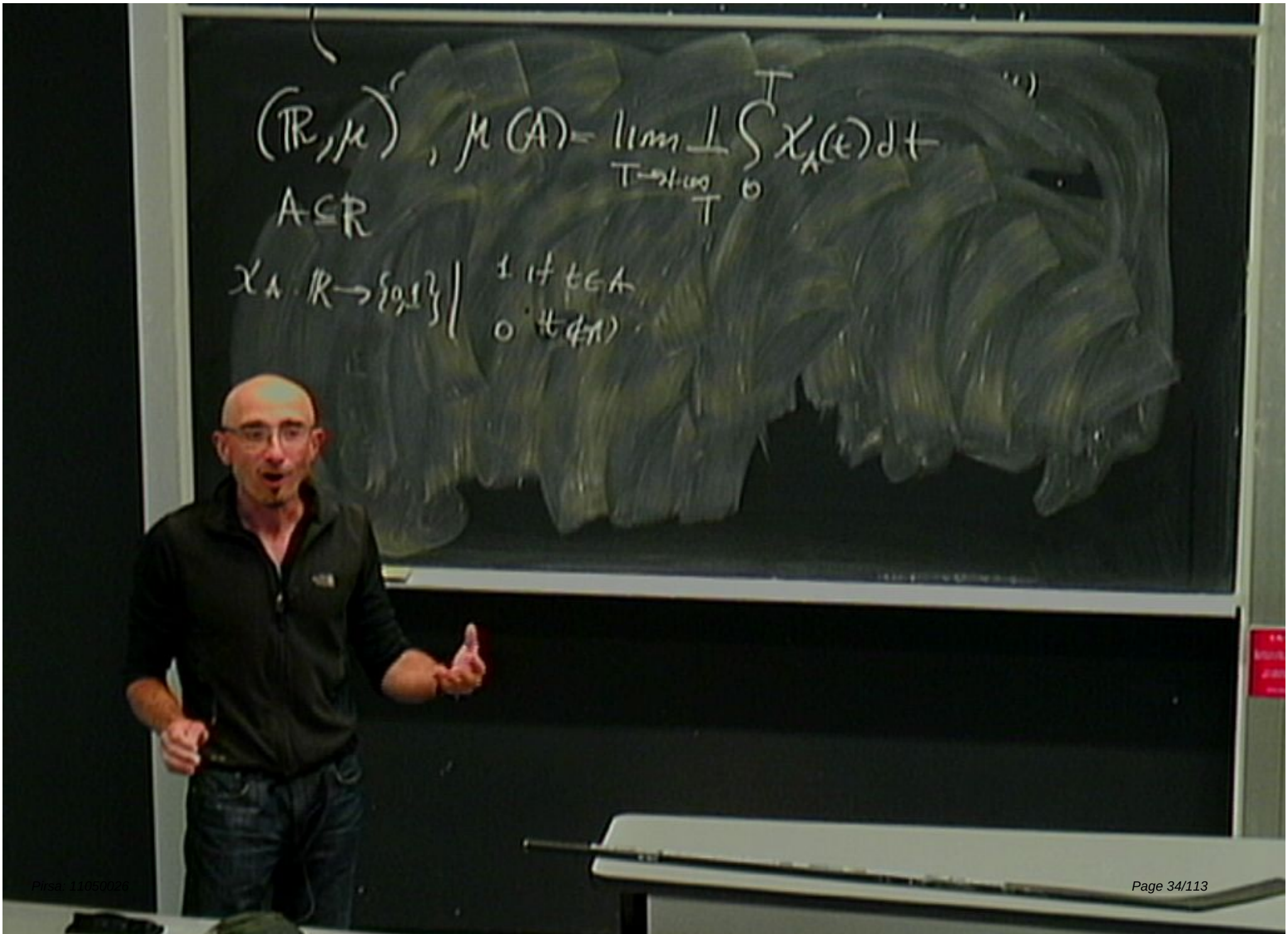
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$$\chi_A : \mathbb{R} \to \{0,1\} \left| \begin{array}{l} 1 \text{ if } t \in A \\ 0 \text{ if } t \notin A \end{array} \right\} \quad P(\alpha) = \overline{\delta(\alpha - \alpha(t))}_t$$

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$$(\mathbb{R}, \mu), \quad \mu(A) = \lim_{T \to +\infty} \frac{1}{T} \int_0^T \chi_A(t)\, dt$$

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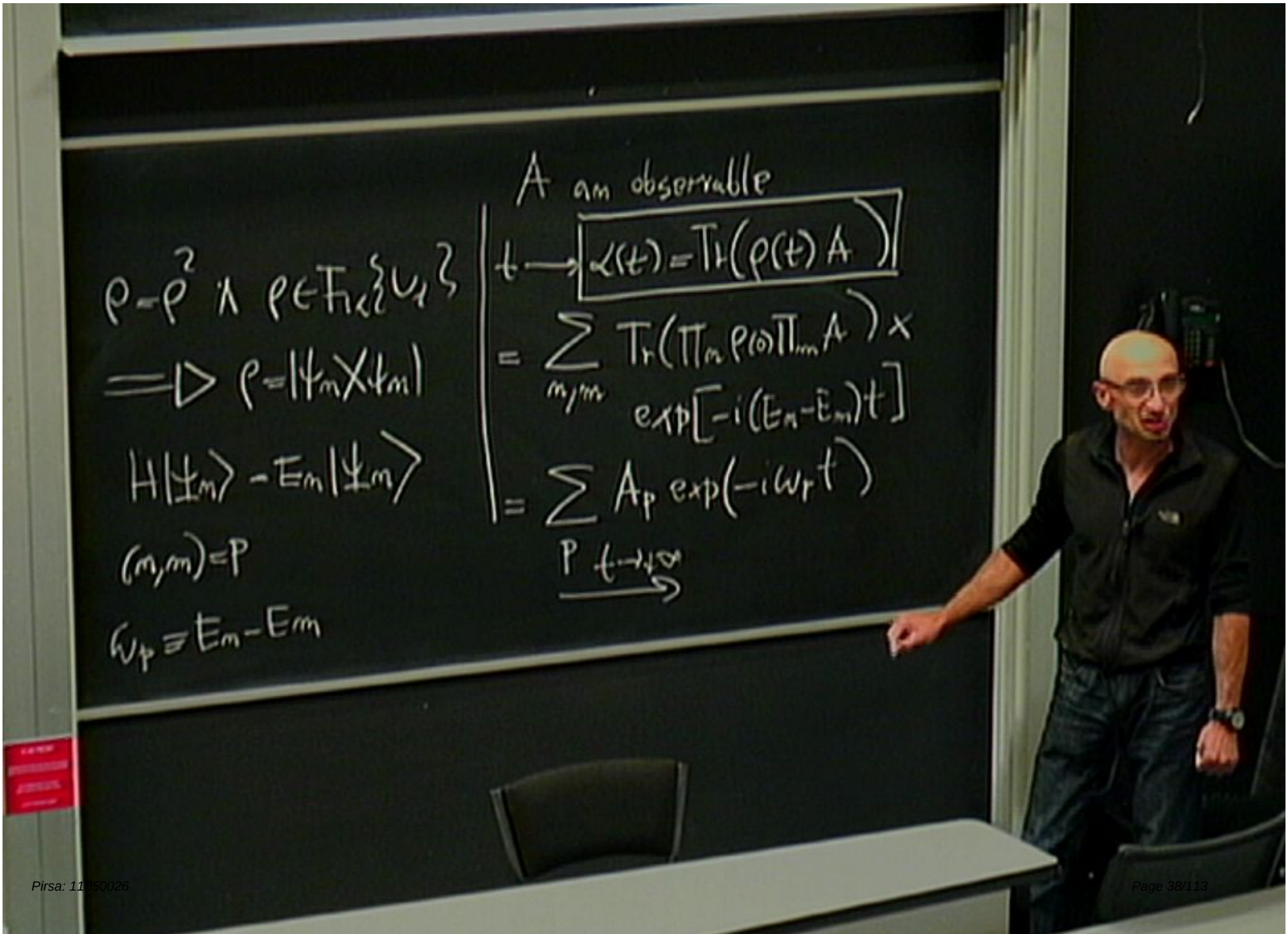
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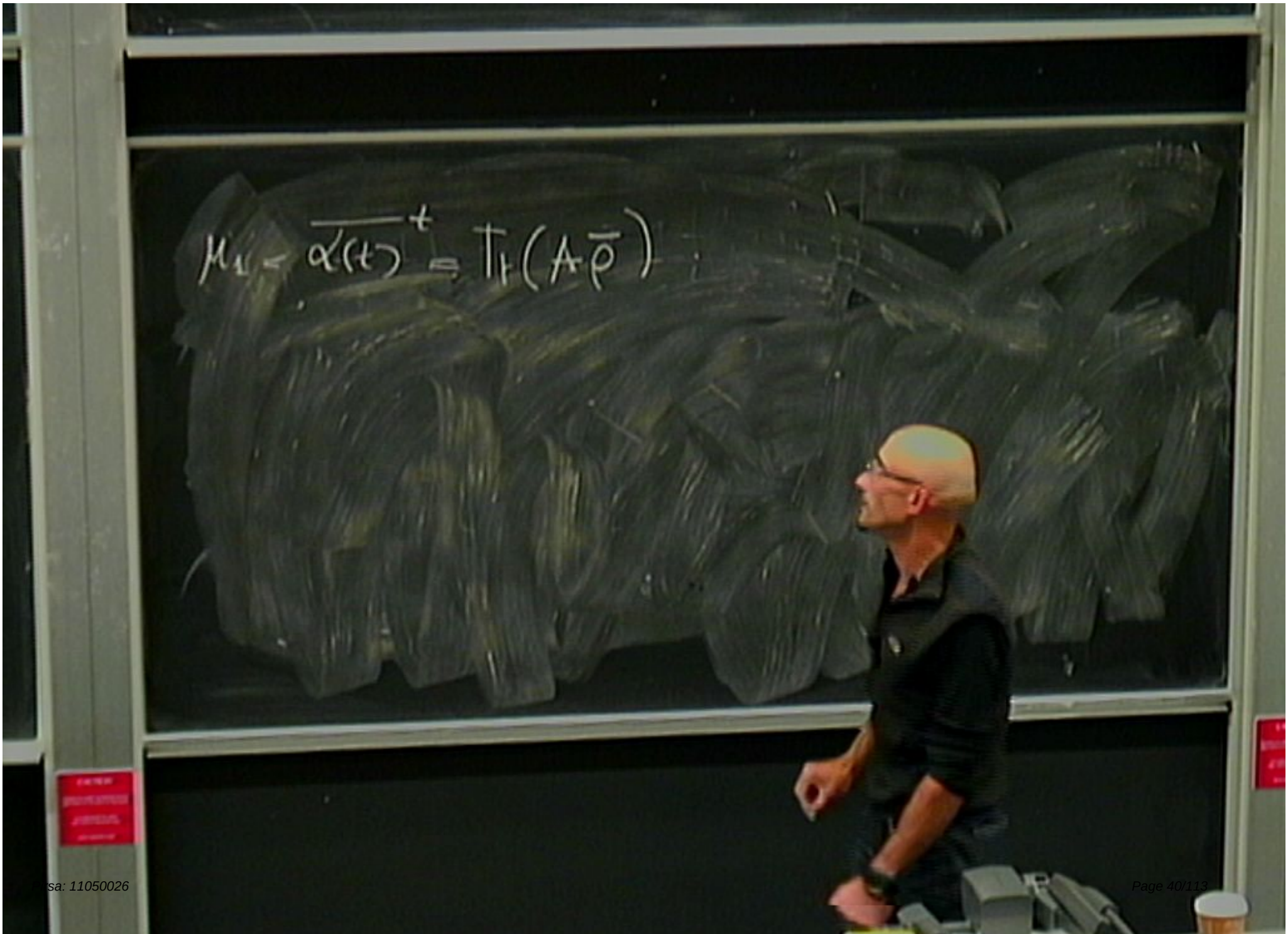
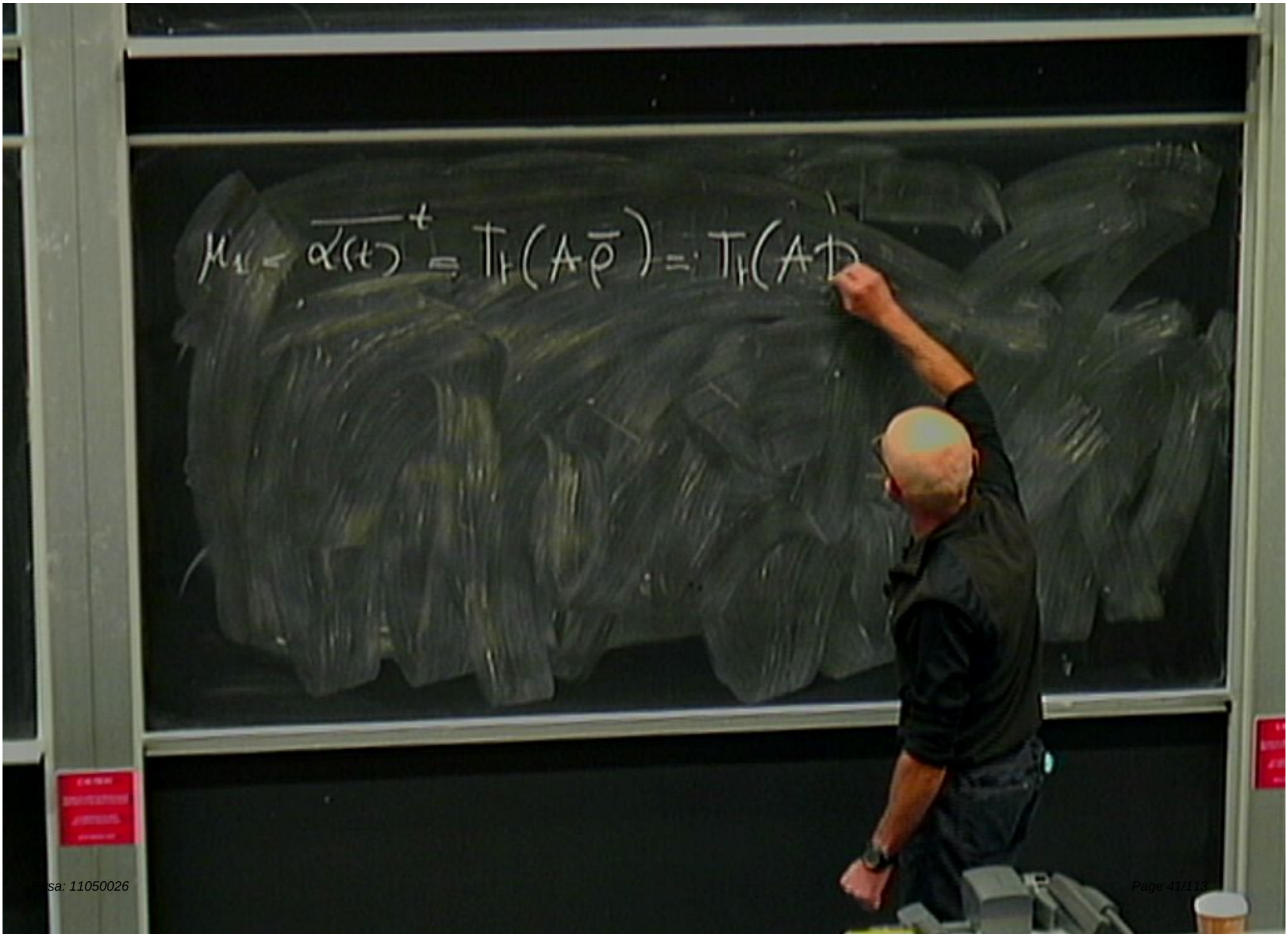
$$= \sum_{n,m} \mathrm{Tr}(\Pi_n \rho(0) \Pi_m A) \times \exp[-i(E_n - E_m)t]$$

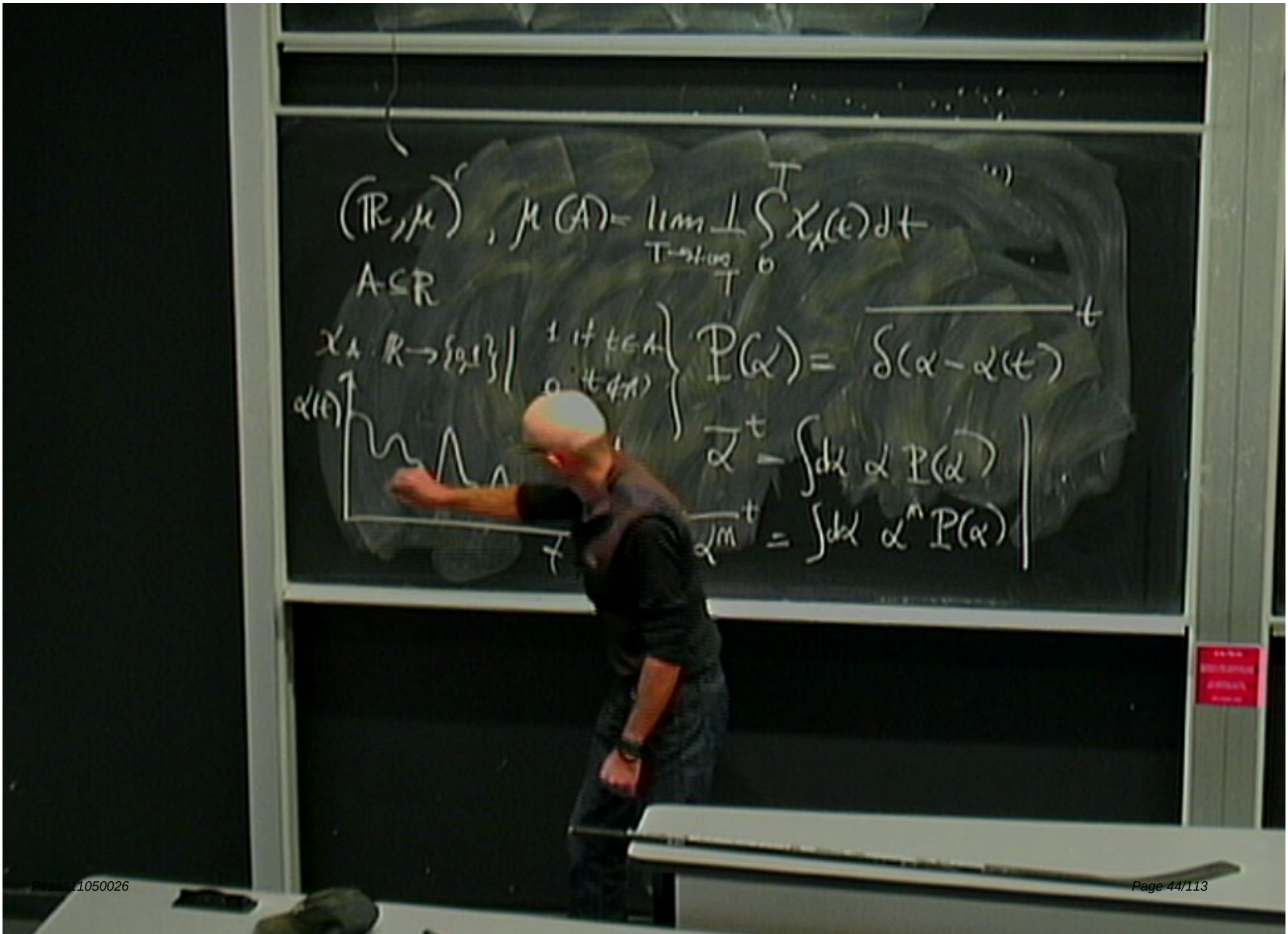
$$= \sum_p A_p \exp(-i\omega_p t) \xrightarrow{t \to +\infty}$$


$\mu_1 = \overline{\alpha(t)}^{\,t} = \mathrm{Tr}(A\bar{\rho})$


$\mu_1 = \overline{\alpha(t)}^{t} = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A$

$$\mu_1 = \overline{\alpha(t)^t} = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A D_H(\rho))$$

$$\mu_1 = \overline{\alpha(t)}^{t} = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A\,D_H(\rho(0)))$$

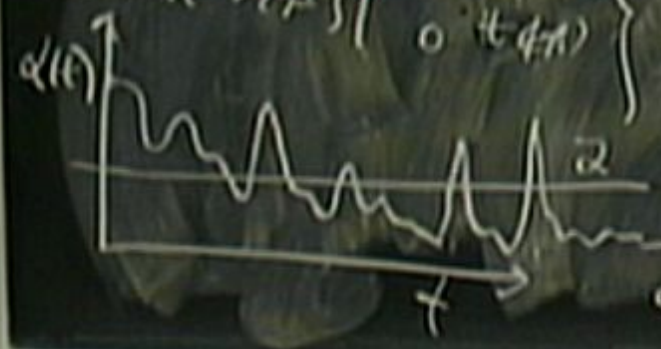

(ℝ, μ), μ(A) = lim_{T→∞} 1/T ∫_0^T χ_A(t) dt
A ⊆ ℝ
χ_A : ℝ → {0,1}
1 if t ∈ A
0 if t ∉ A
P(α) = δ(α − α(t))
α(t)
t
α = ∫ dα α P(α)
α^n = ∫ dα α^n P(α)

$(\mathbb{R}, \mu)$, $\mu(A) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \chi_A(t)\, dt$

$A \subseteq \mathbb{R}$

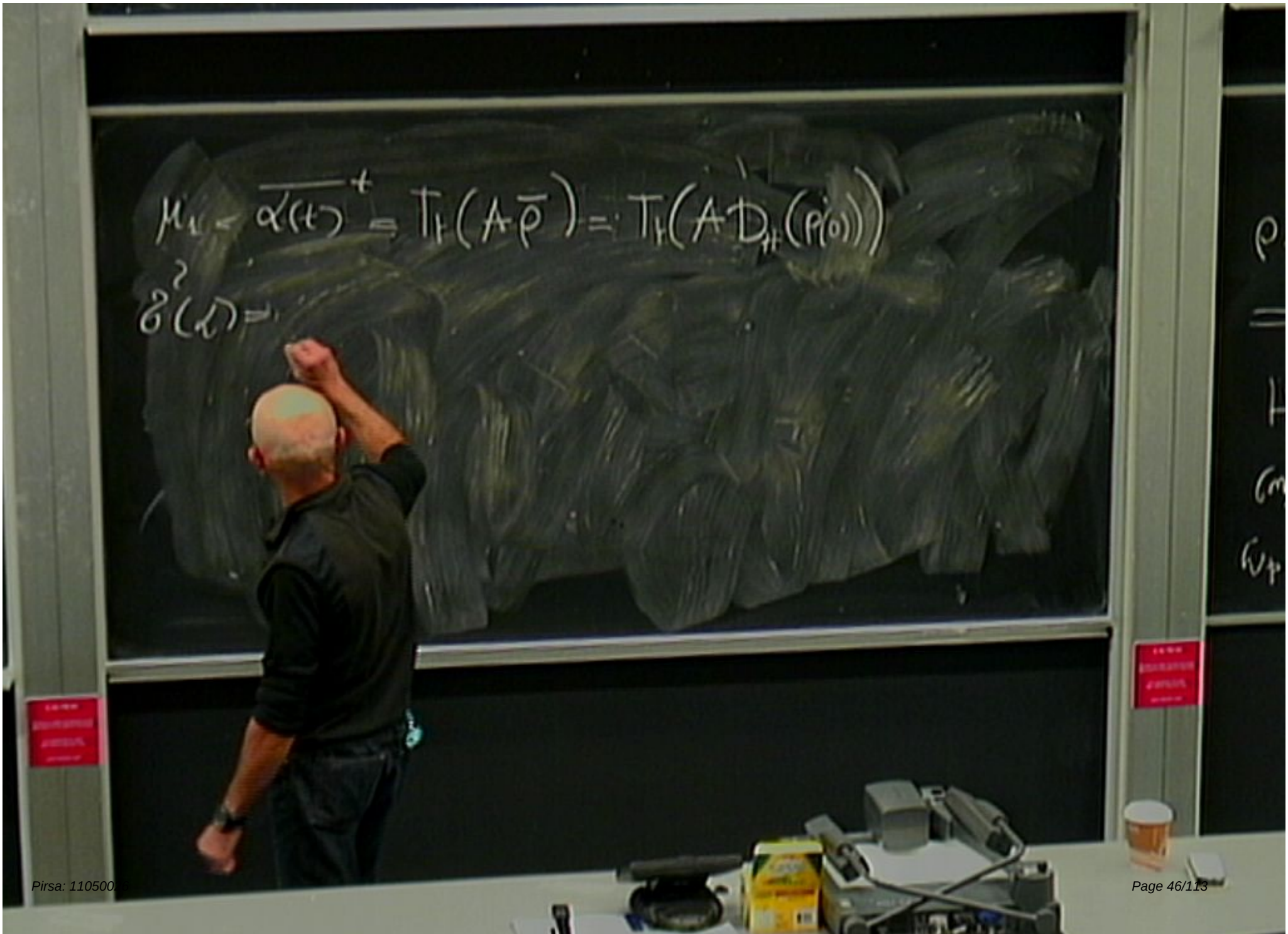
$\chi_A : \mathbb{R} \to \{0,1\} \left| \begin{matrix} 1 & \text{if } t \in A \\ 0 & t \notin A \end{matrix} \right\}$

$P(\alpha) = \overline{\delta(\alpha - \alpha(t))}^{\,t}$



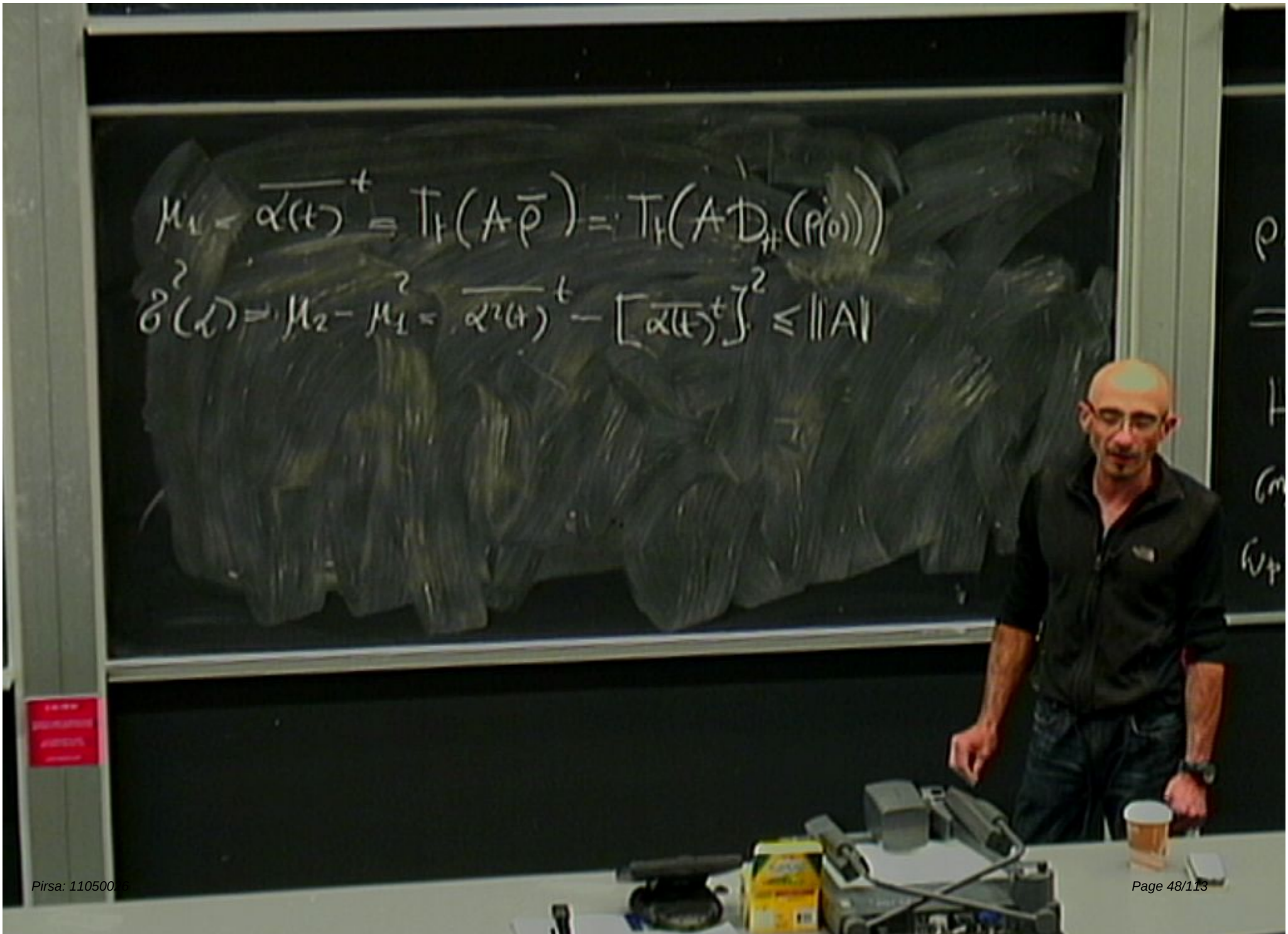
$\bar{\alpha}^t = \int d\alpha\, \alpha\, P(\alpha)$

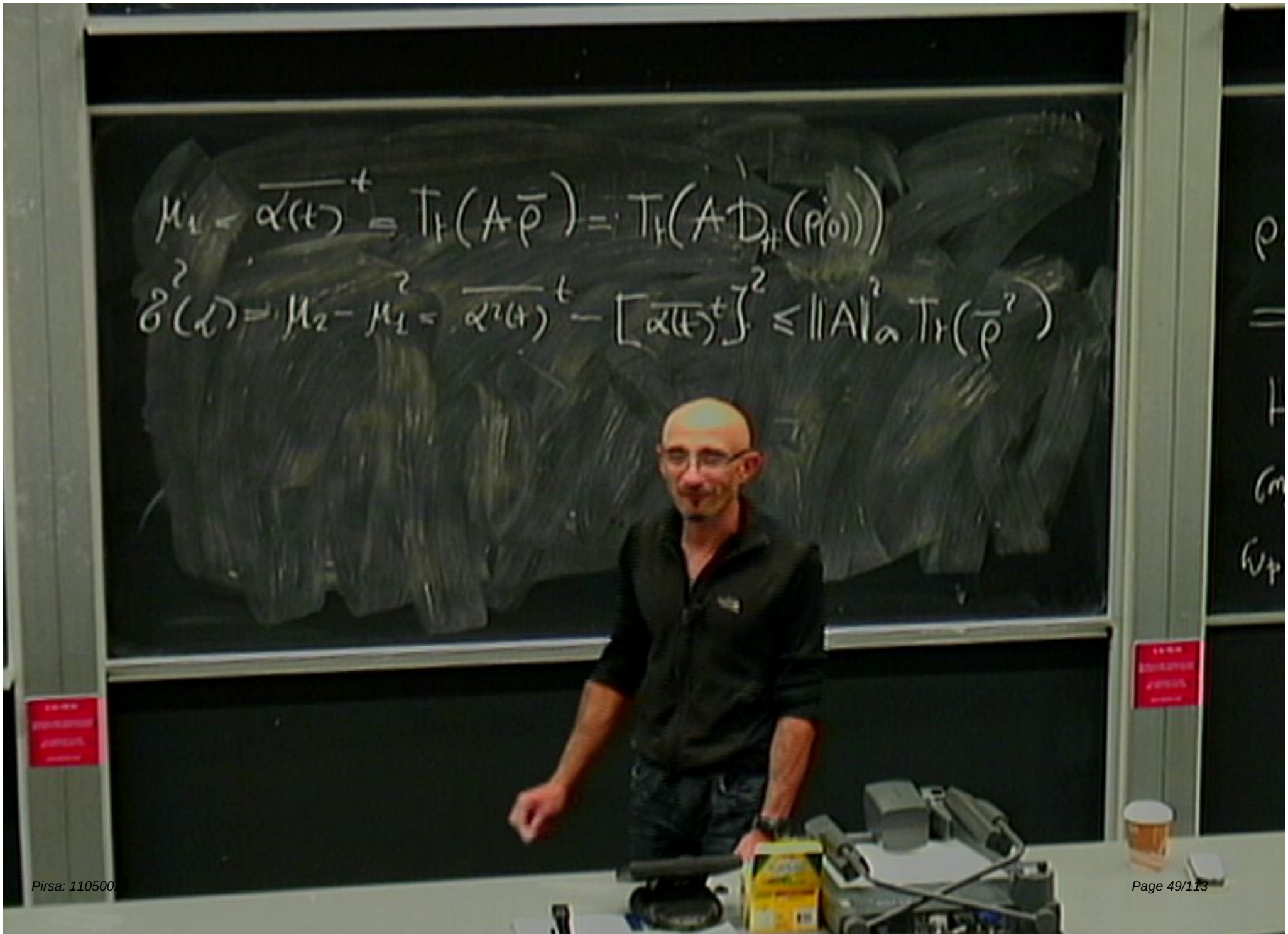
$\overline{\alpha^n}^{\,t} = \int d\alpha\, \alpha^n P(\alpha)$


$\mu_1 = \overline{\alpha(t)}^{+} = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A D_H(\rho(0)))$
$\sigma^2(\alpha) =$

$$\mu_1 = \overline{\alpha(t)}^t = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A\, D_{tt}(\rho(0)))$$

$$\sigma^2(\alpha) = \mu_2 - \mu_1^2 = \overline{\alpha^2(t)}^t - \left[\overline{\alpha(t)}^t\right]^2$$


$\mu_1 = \overline{\alpha(t)}^t = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A\, D_{tt}(\rho(0)))$
$\sigma^2(\alpha) = \mu_2 - \mu_1^2 = \overline{\alpha^2(t)}^t - [\overline{\alpha(t)}^t]^2 \leq \|A\|$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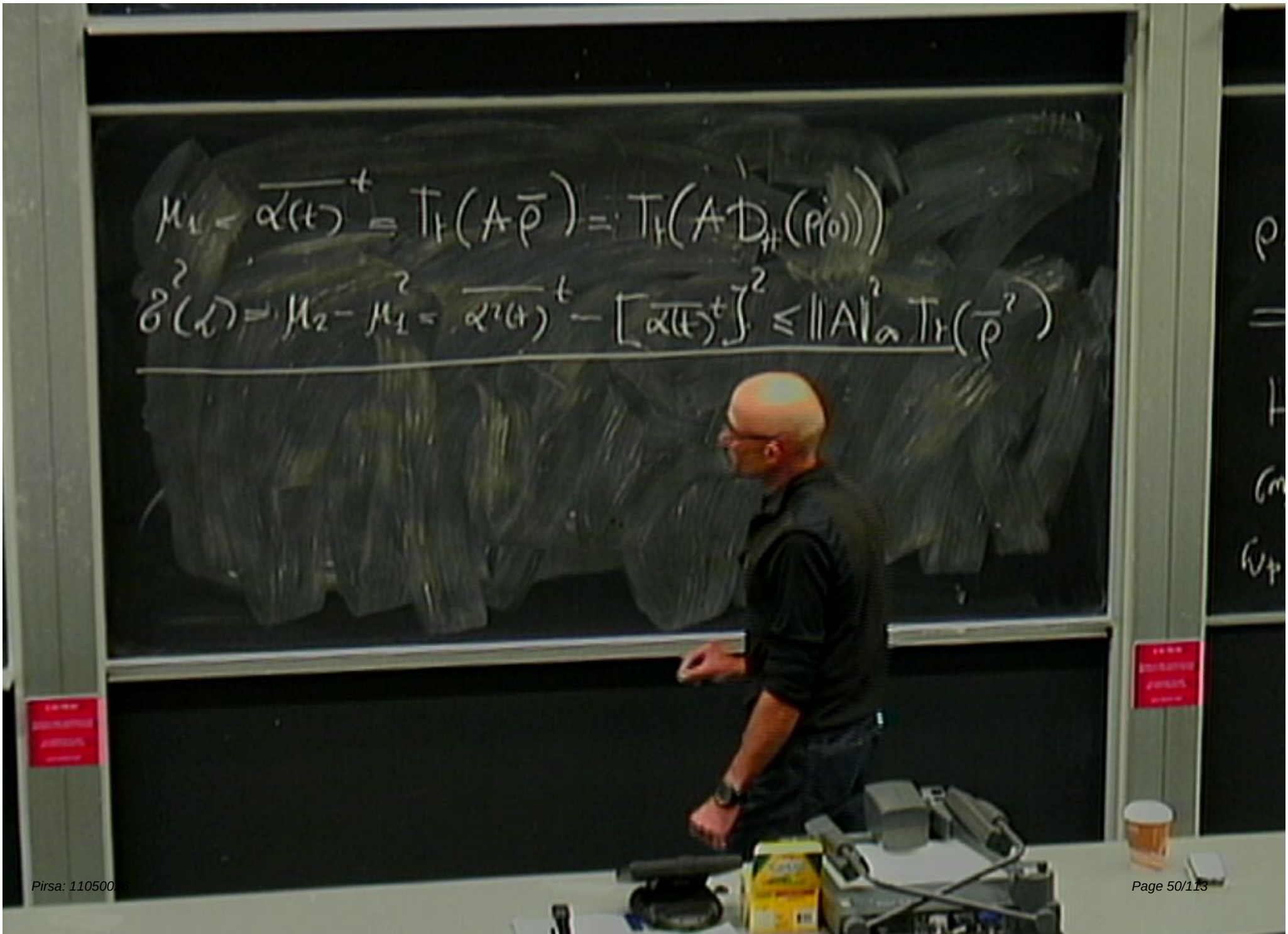
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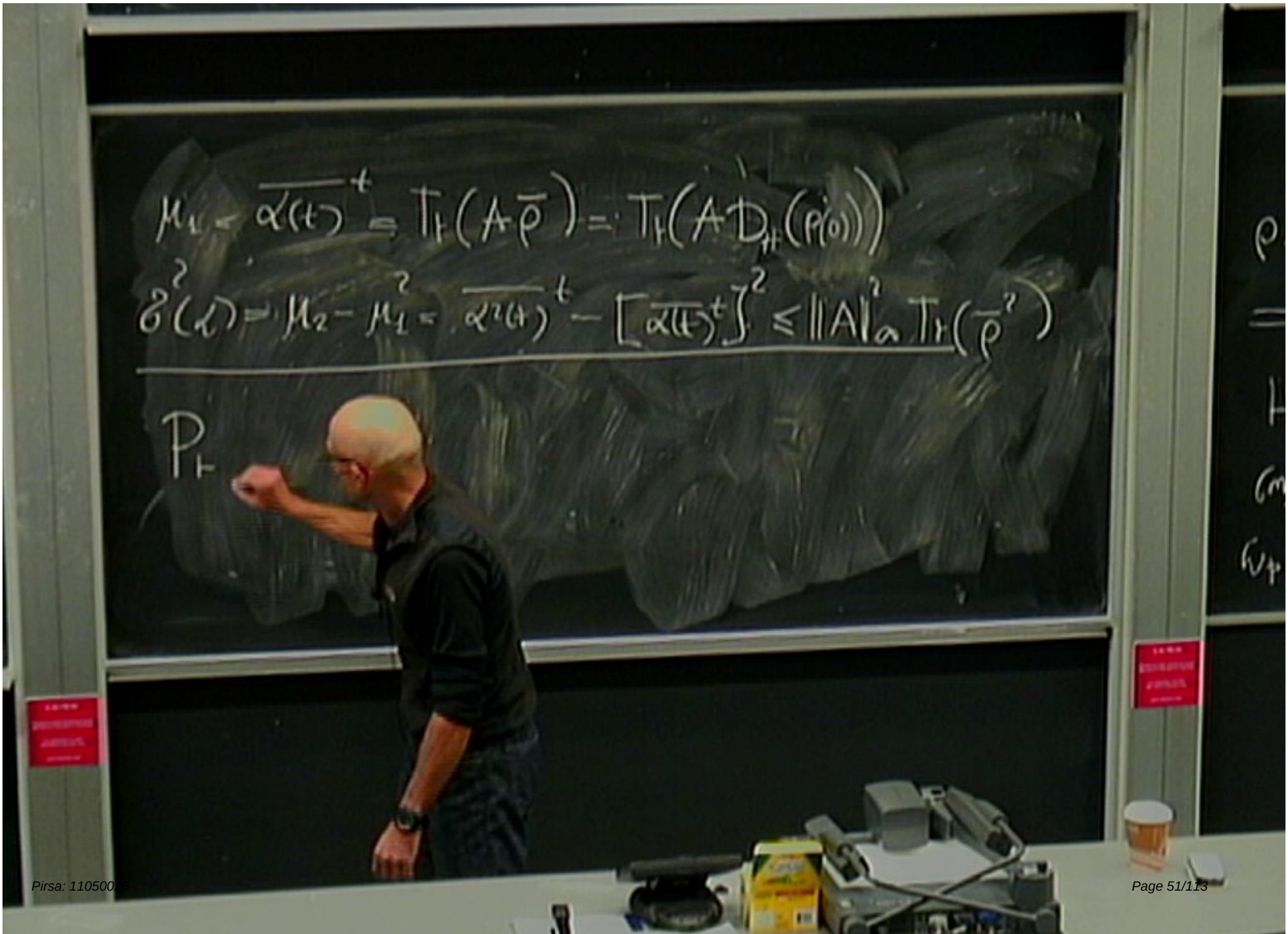
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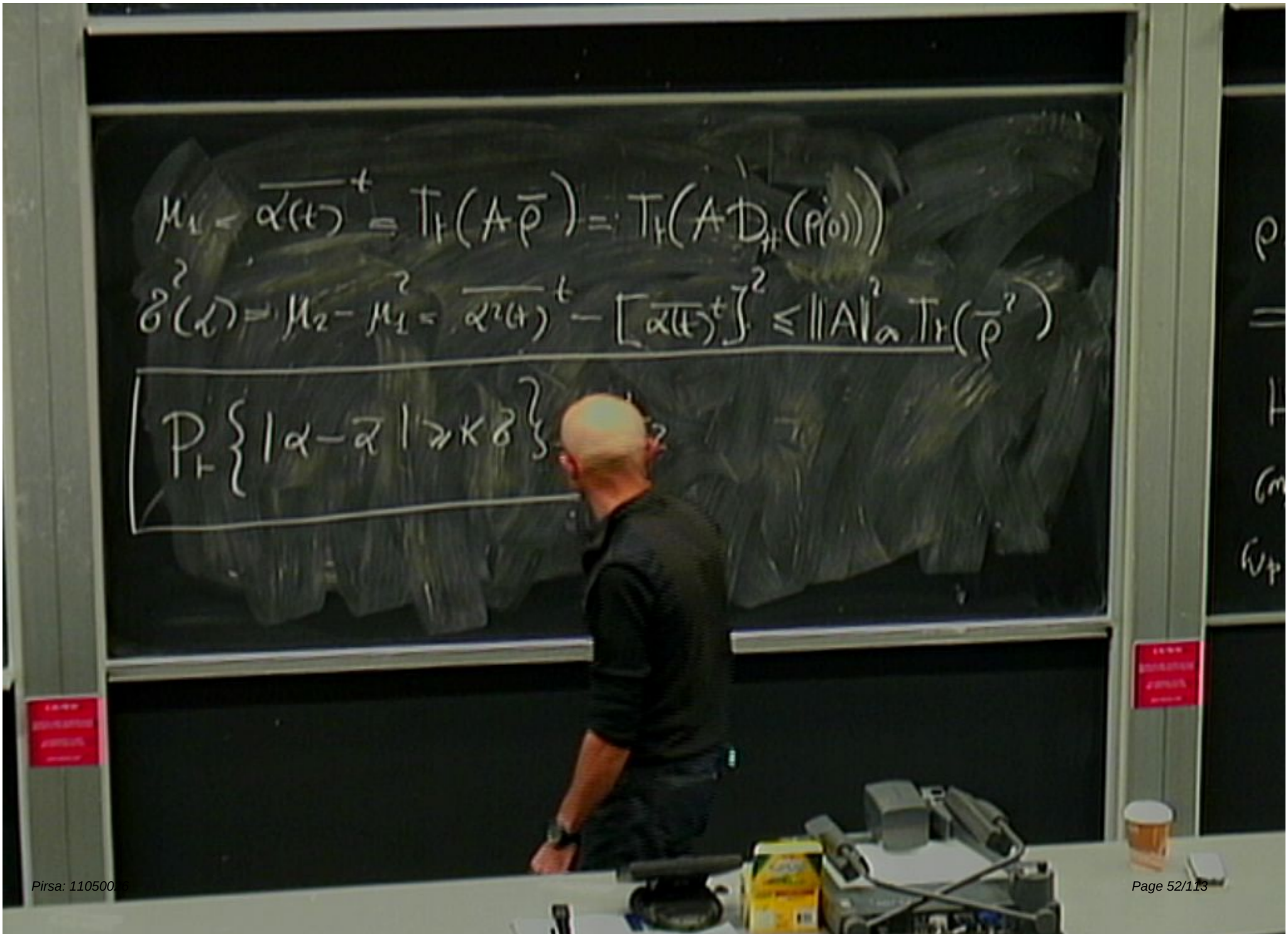
P_t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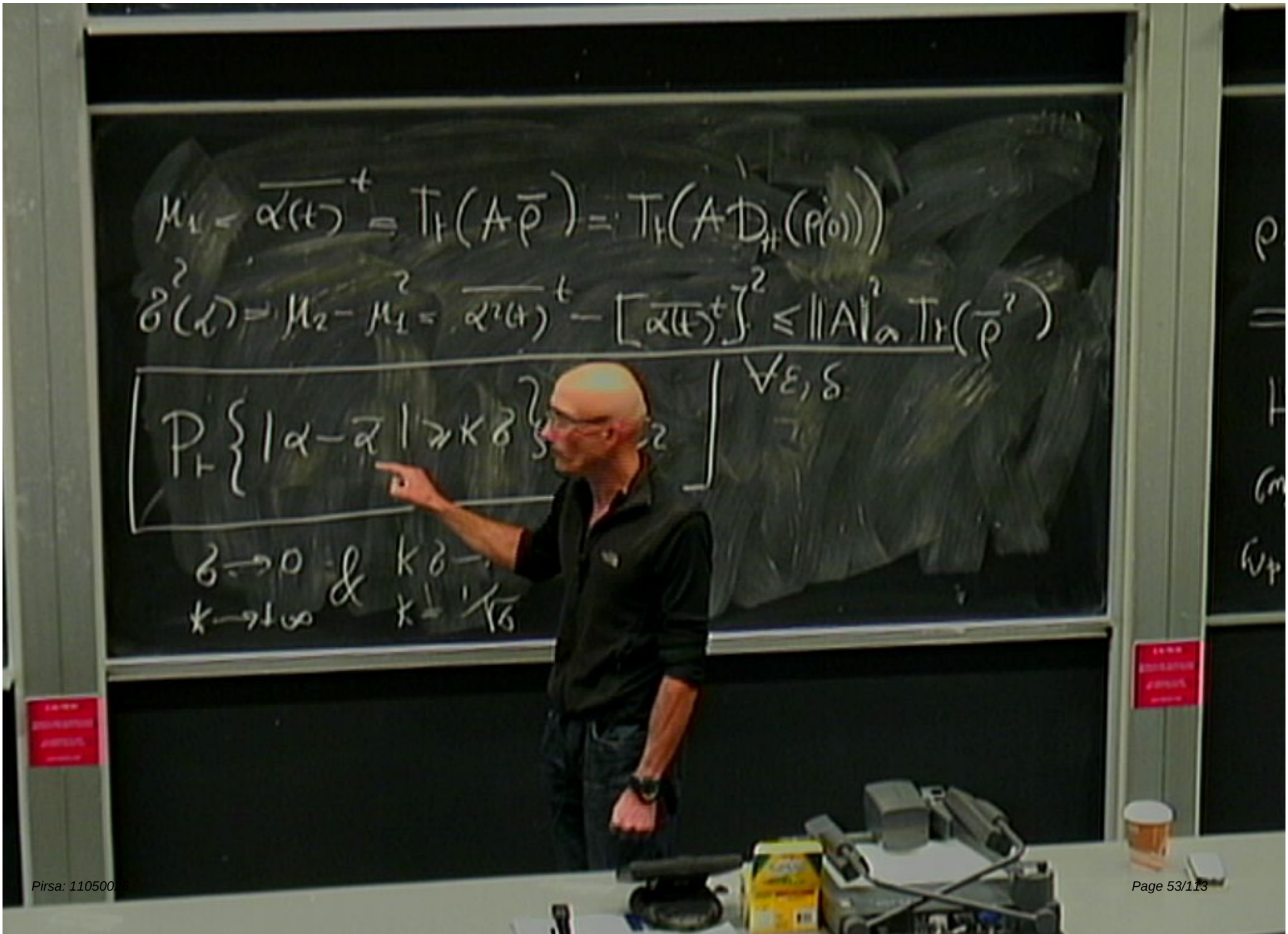
$P_t\{|\alpha - \bar{\alpha}| \geq k\sigma\}$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$\forall \varepsilon, \delta$
$P_t\{|\alpha - \bar{\alpha}| \geq k\sigma\}$
$\sigma \to 0$ & $k\sigma$
$k \to \infty$ & $k = 1/\sqrt{\sigma}$

$$\mu_1 = \overline{\alpha(t)}^t = \mathrm{Tr}(A\bar{\rho}) = \mathrm{Tr}(A\, D_t(\rho(0)))$$

$$\sigma^2(\alpha) = \mu_2 - \mu_1^2 = \overline{\alpha^2(t)}^t - \left[\overline{\alpha(t)}^t\right]^2 \leq \|A\|_\infty^2\, \mathrm{Tr}(\bar{\rho}^2)$$

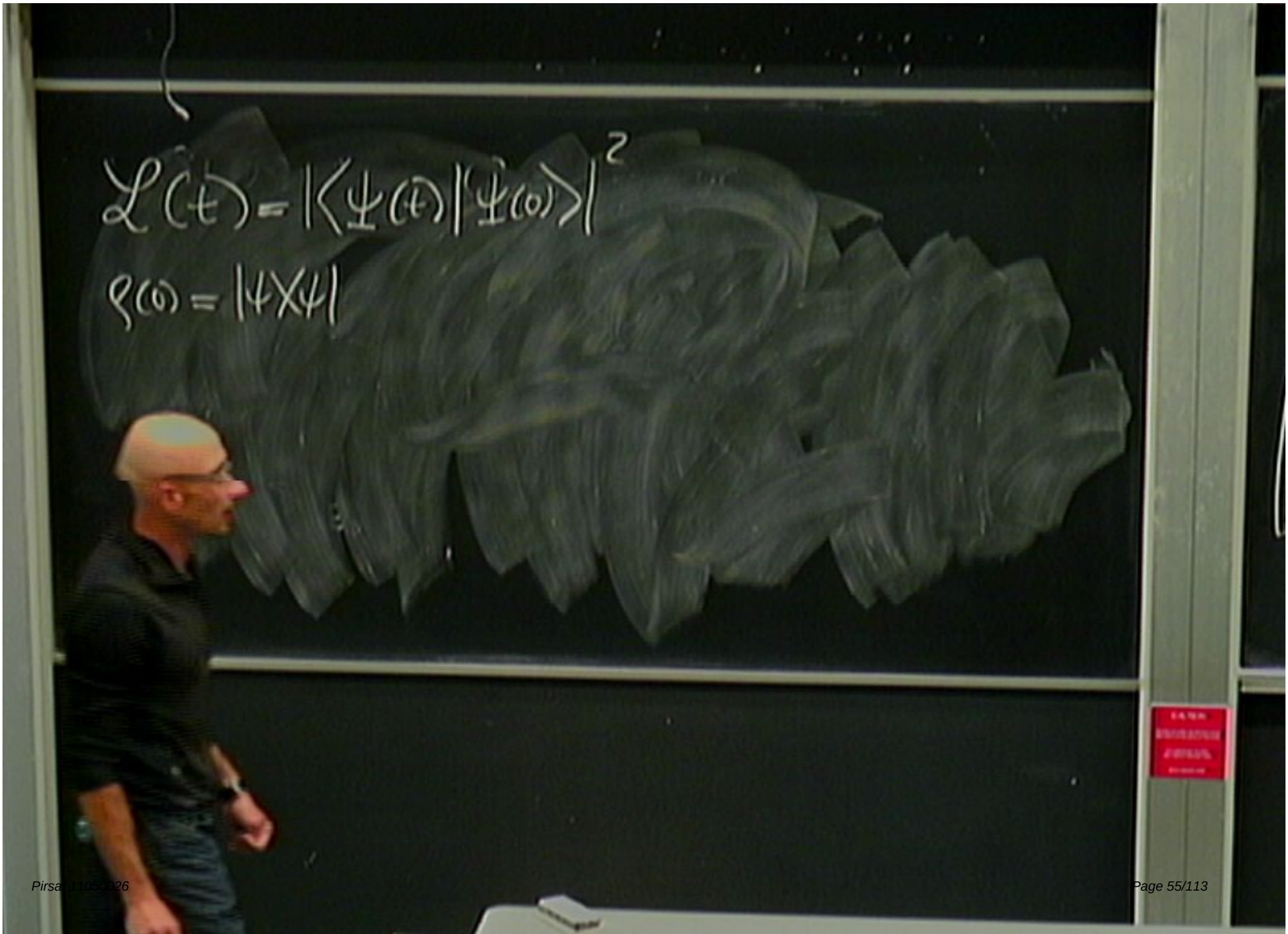
$$\boxed{P_r\{|\alpha - \bar{\alpha}| \geq \varepsilon\} \leq \delta}$$

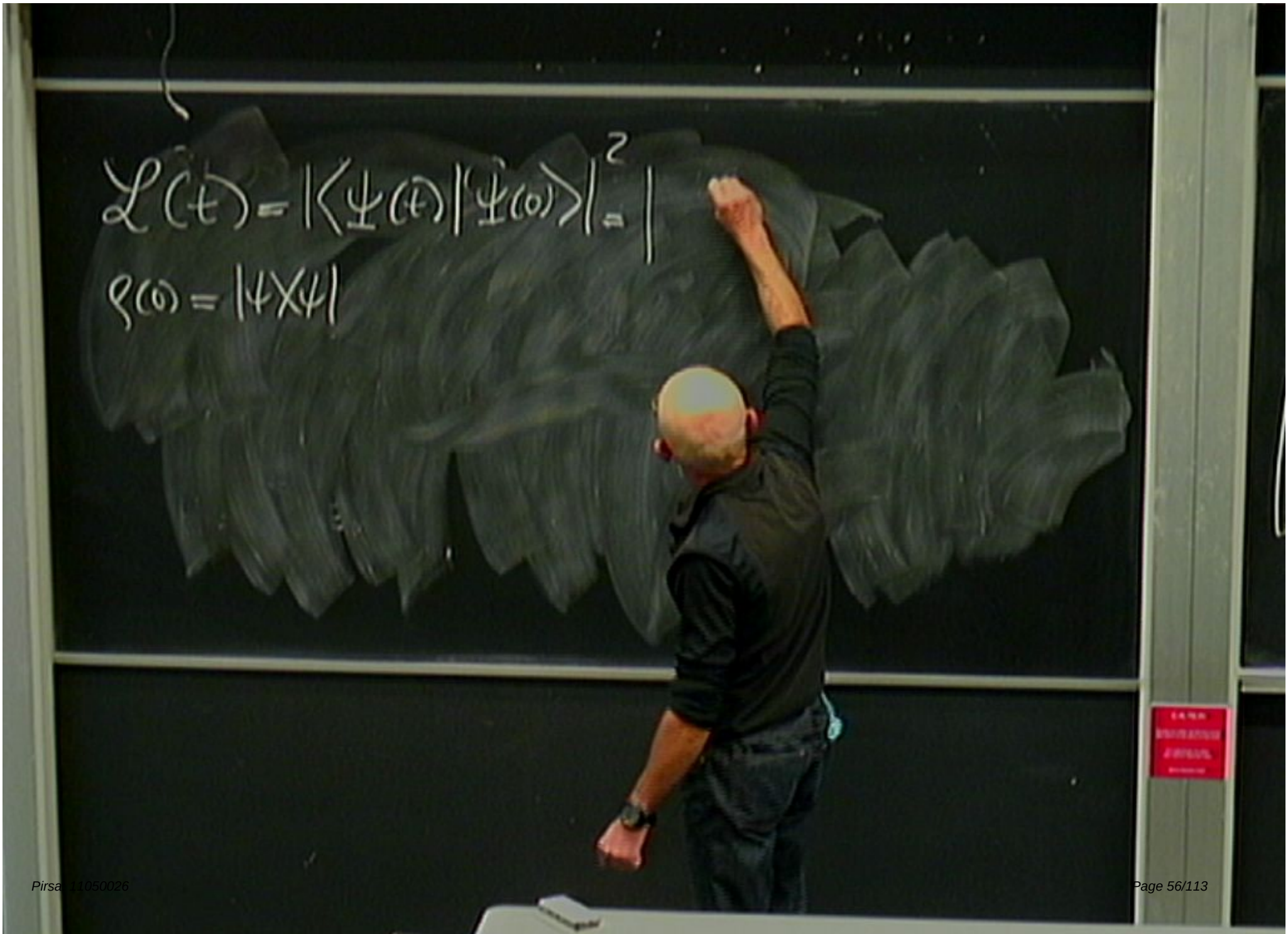
$\forall \varepsilon, \delta$

$$\{\alpha_L(t)\}_{L \in \mathbb{N}} \longrightarrow \bar{\alpha}$$

$$\sigma \to 0 \;\&\; k\sigma \to 0$$

$$k \to +\infty \qquad k = 1/\sqrt{\sigma}$$


$\mathcal{L}(t) = |\langle\Psi(t)|\Psi(0)\rangle|^2$
$\rho(0) = |\psi\rangle\langle\psi|$


$\mathcal{L}(t) = |\langle \Psi(t) | \Psi(0) \rangle|^2 =$
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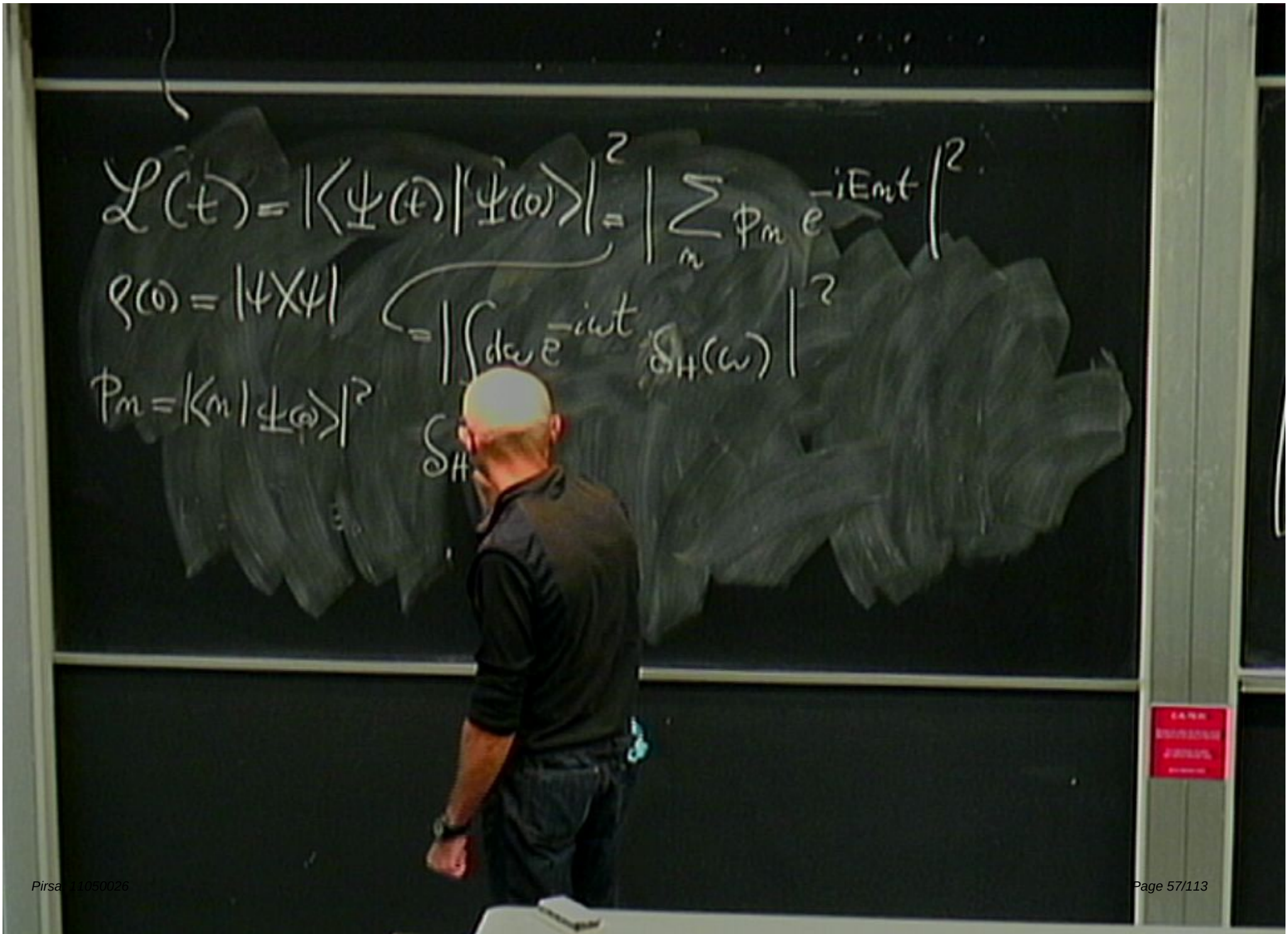

$\mathcal{L}(t) = |\langle\Psi(t)|\Psi(0)\rangle|^2 = \left|\sum_n p_n e^{-iE_n t}\right|^2$
$\rho(0) = |\psi\rangle\langle\psi|$
$= \left|\int d\omega\, e^{-i\omega t}\, S_H(\omega)\right|^2$
$p_n = |\langle n|\Psi(0)\rangle|^2$
S_H

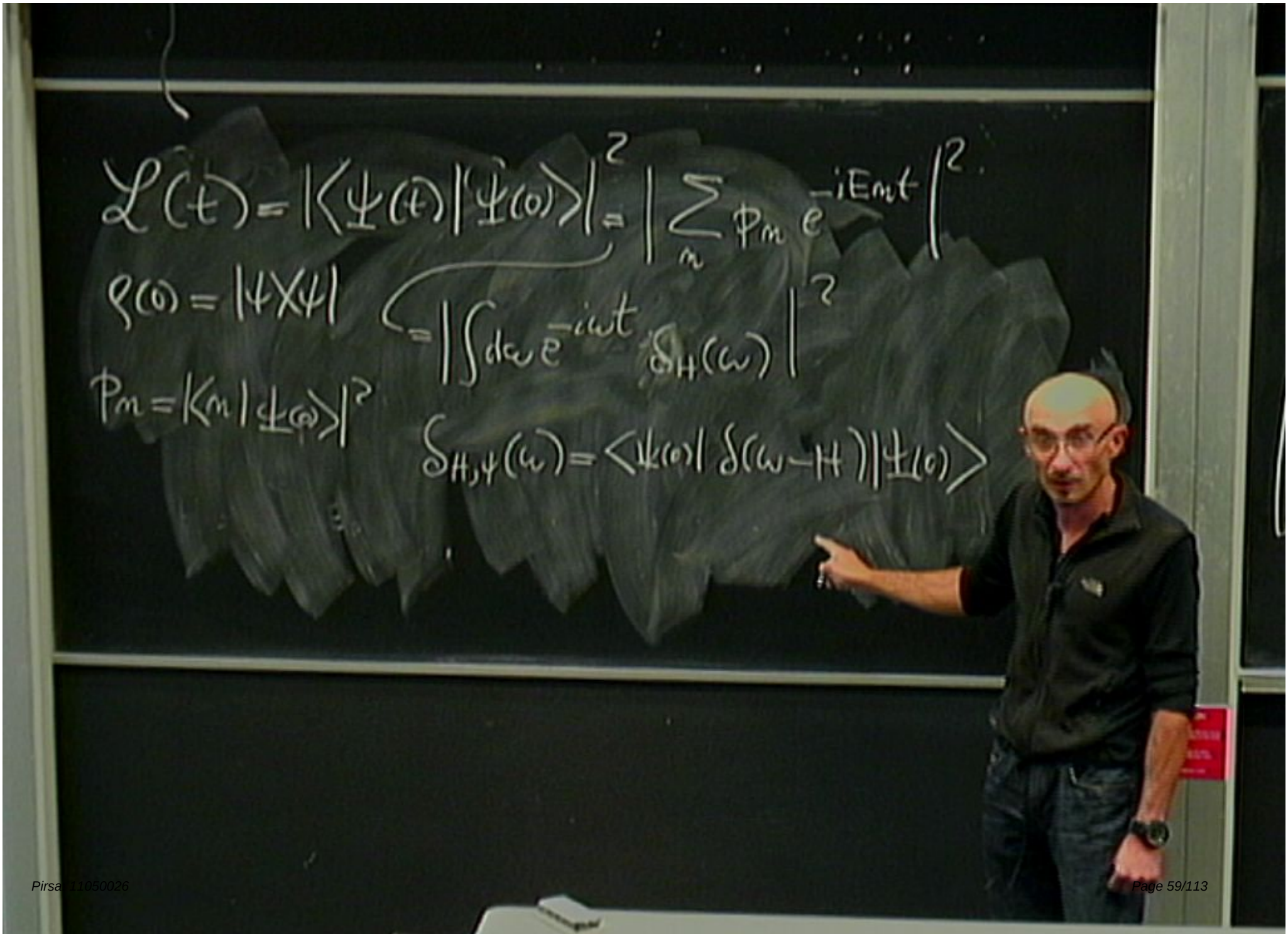
$$\mathcal{L}(t) = |\langle \Psi(t) | \Psi(0) \rangle|^2 = \left| \sum_n p_n e^{-iE_n t} \right|^2$$

$$\rho(0) = |\psi\rangle\langle\psi|$$

$$= \left| \int d\omega\, e^{-i\omega t}\, \delta_H(\omega) \right|^2$$

$$p_n = |\langle n | \Psi(0) \rangle|^2$$

$$\delta_{H,\psi}(\omega) = \langle \Psi(0) | \delta(\omega - H) | \Psi(0) \rangle$$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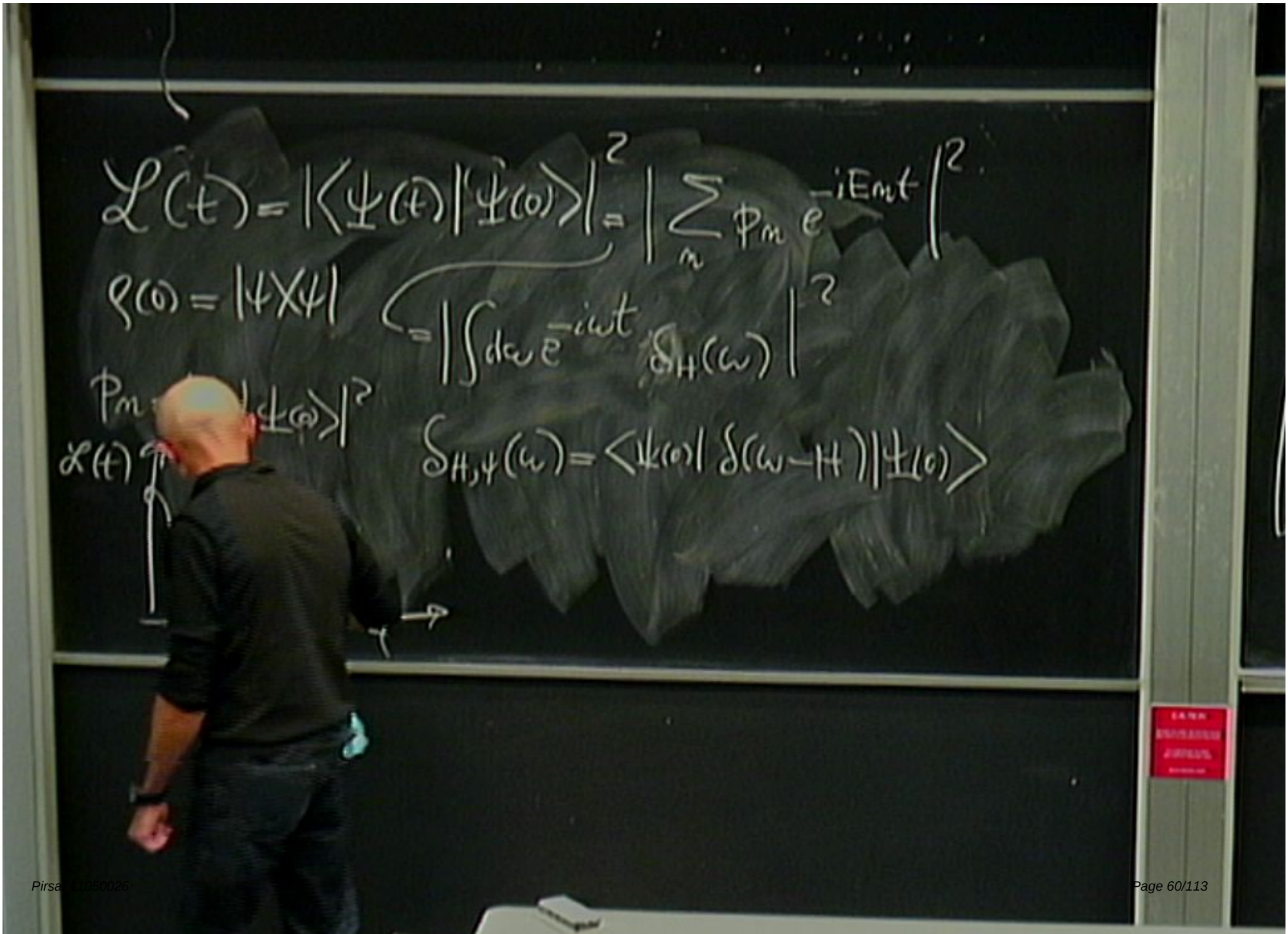
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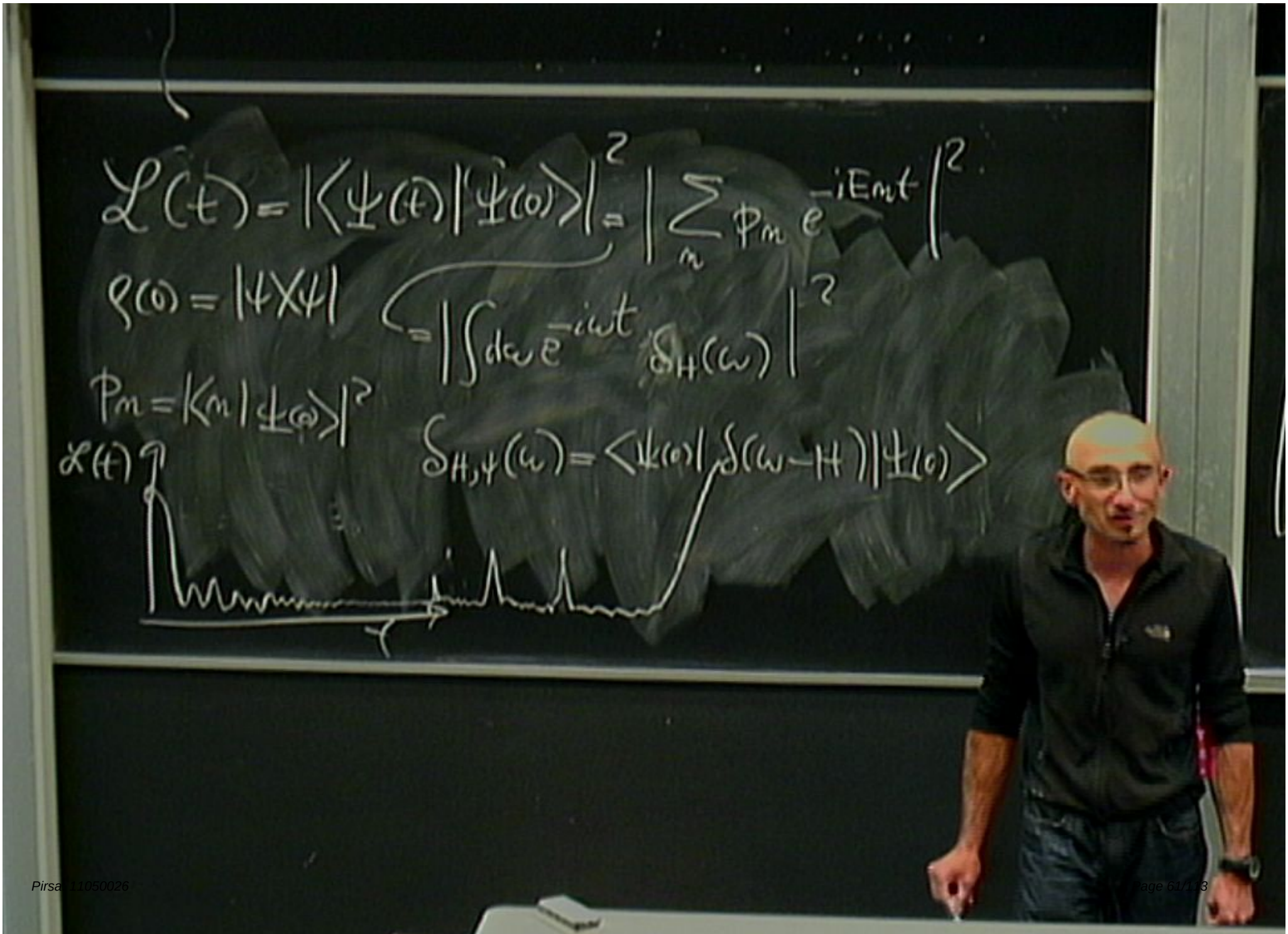
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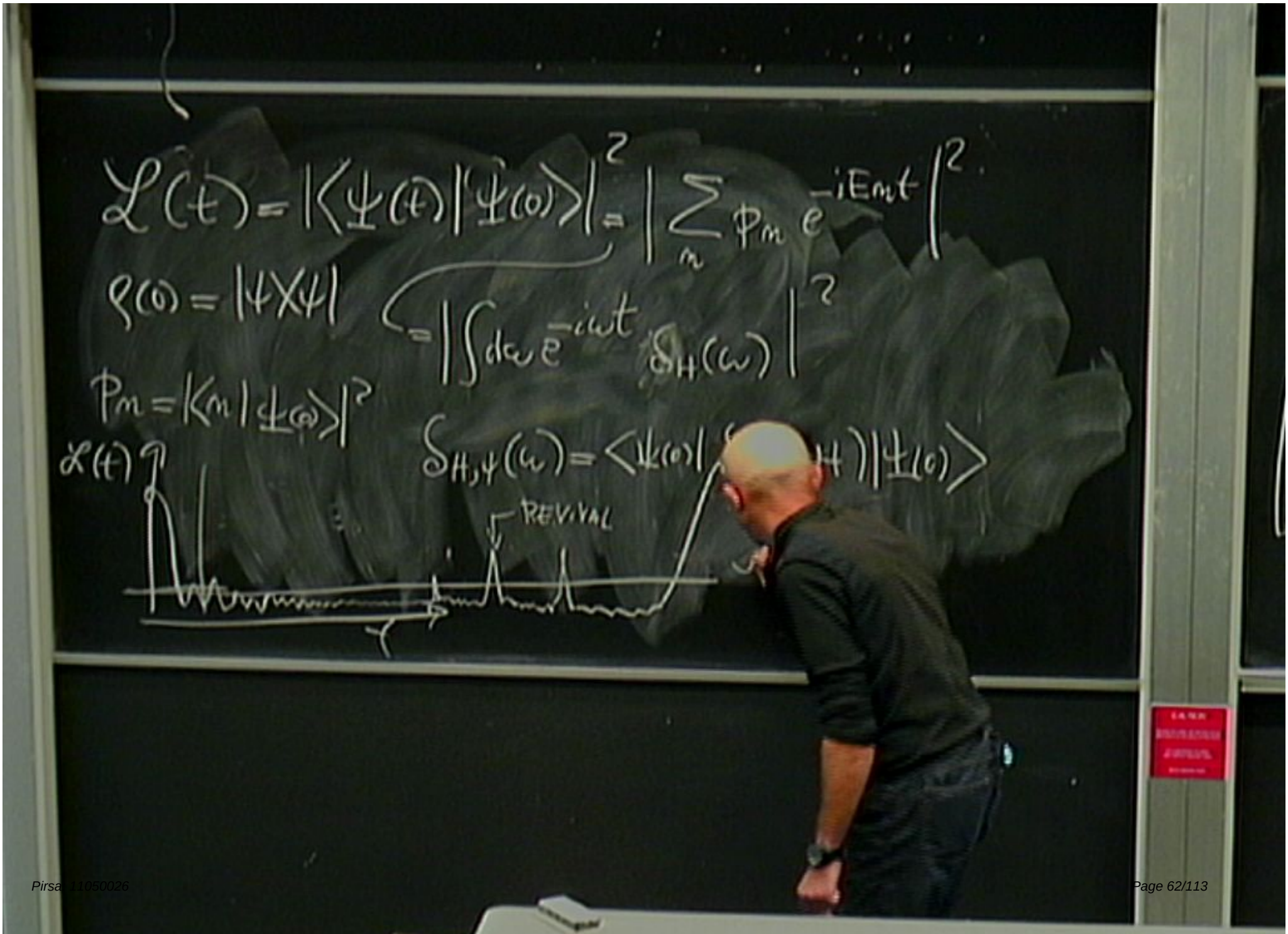
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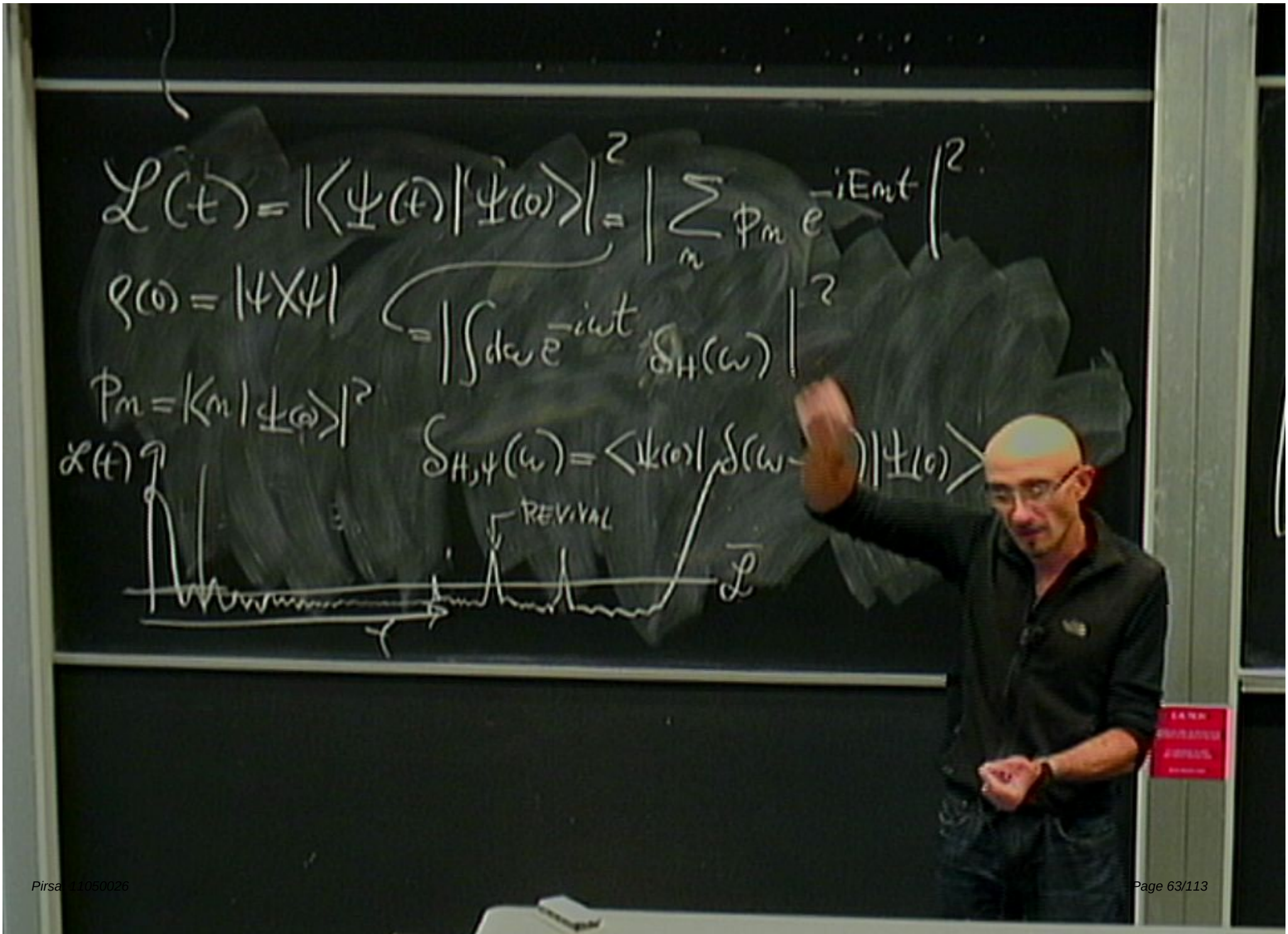
$S_{H,\psi}(\omega) = \langle\psi(0)|\delta(\omega - H)|\Psi(0)\rangle$
$\mathcal{L}(t)$
REVIVAL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REVIVAL
$\bar{\mathcal{L}}$

$$\mathcal{L}(t) = |\langle \Psi(t) | \Psi(0) \rangle|^2 = \left| \sum_n p_n e^{-iE_n t} \right|^2$$

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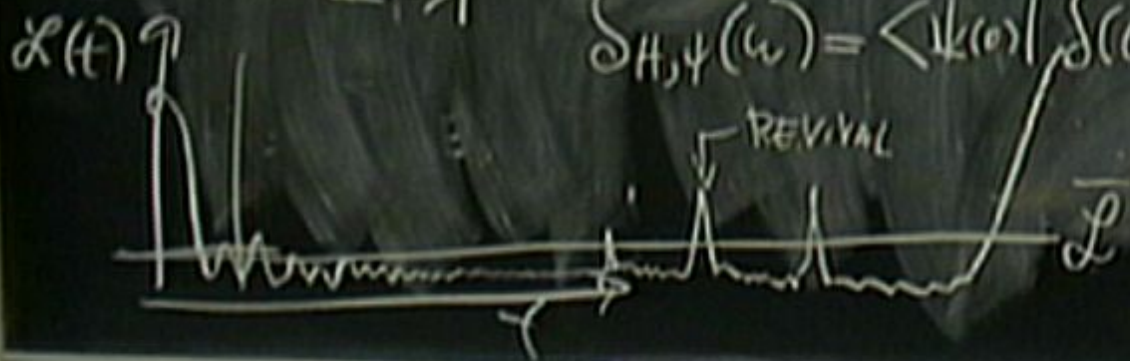
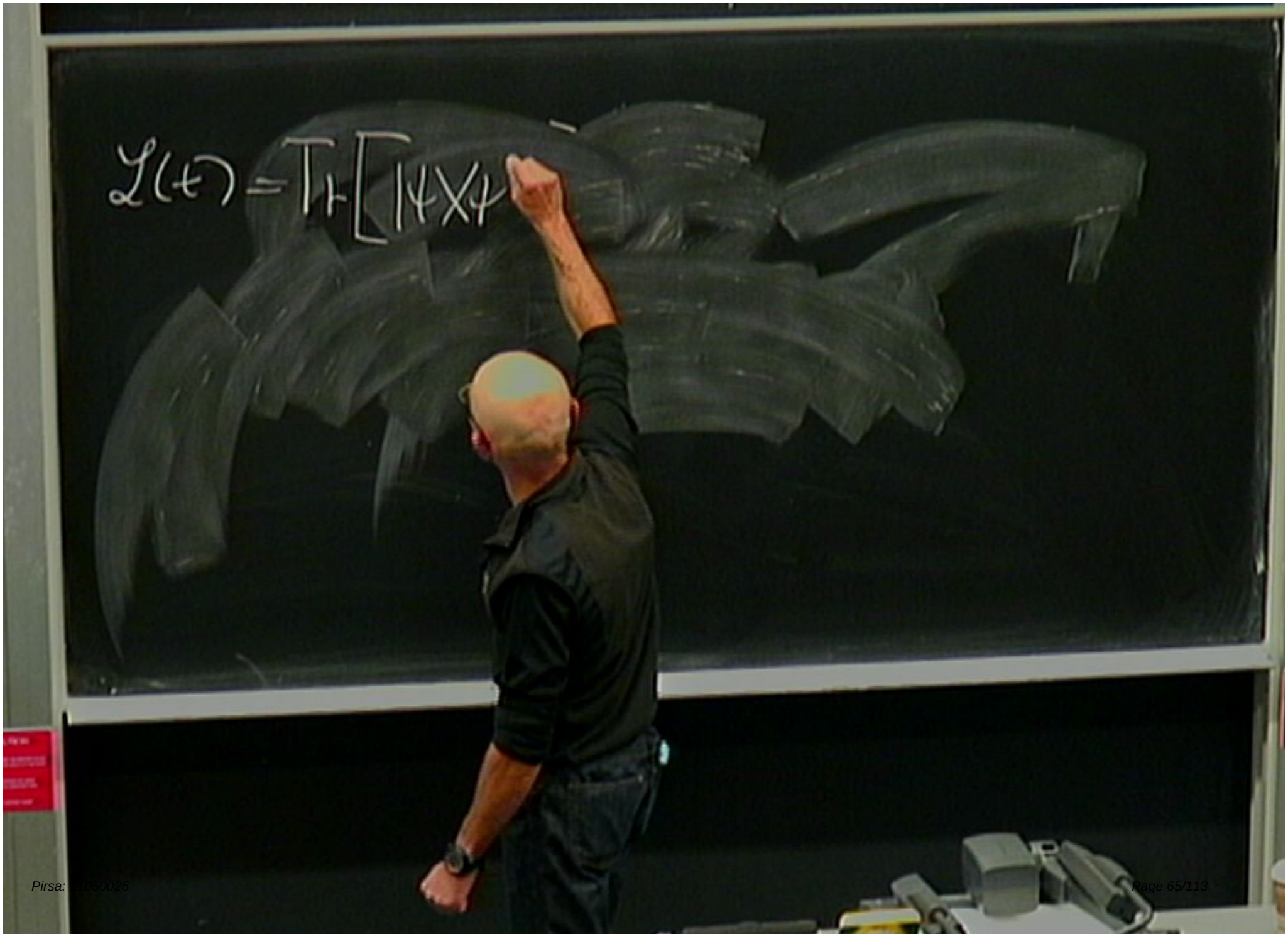
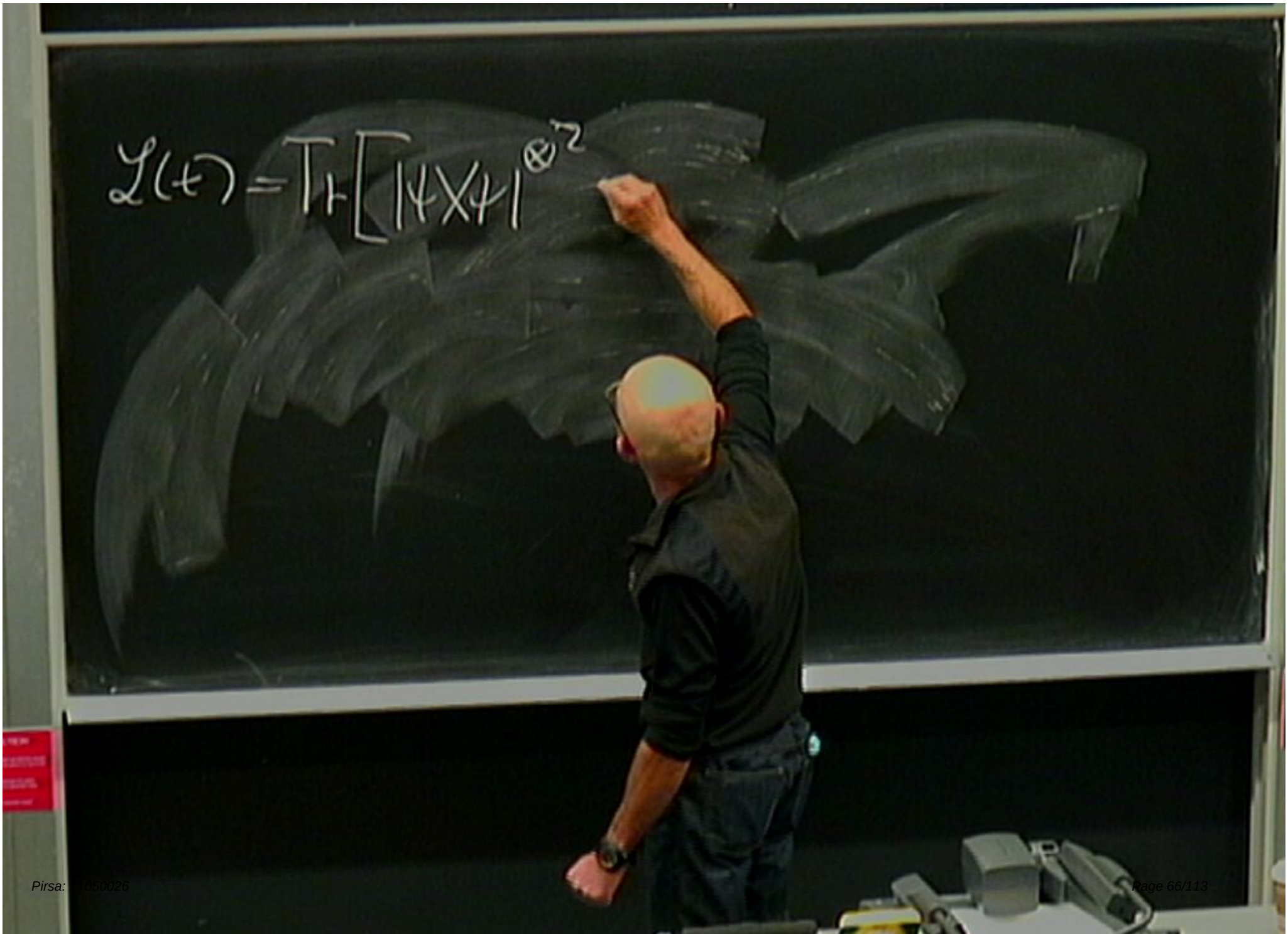
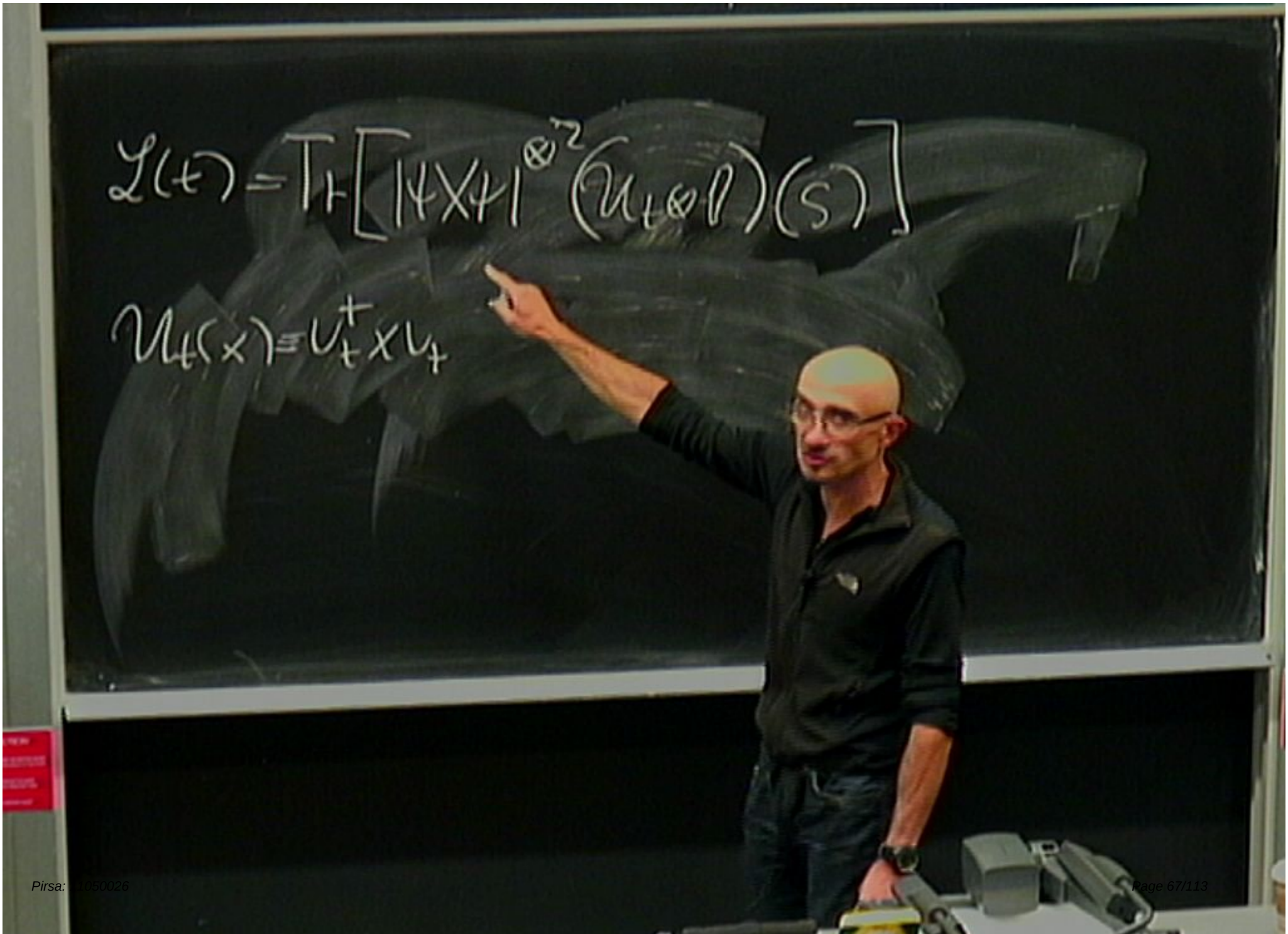
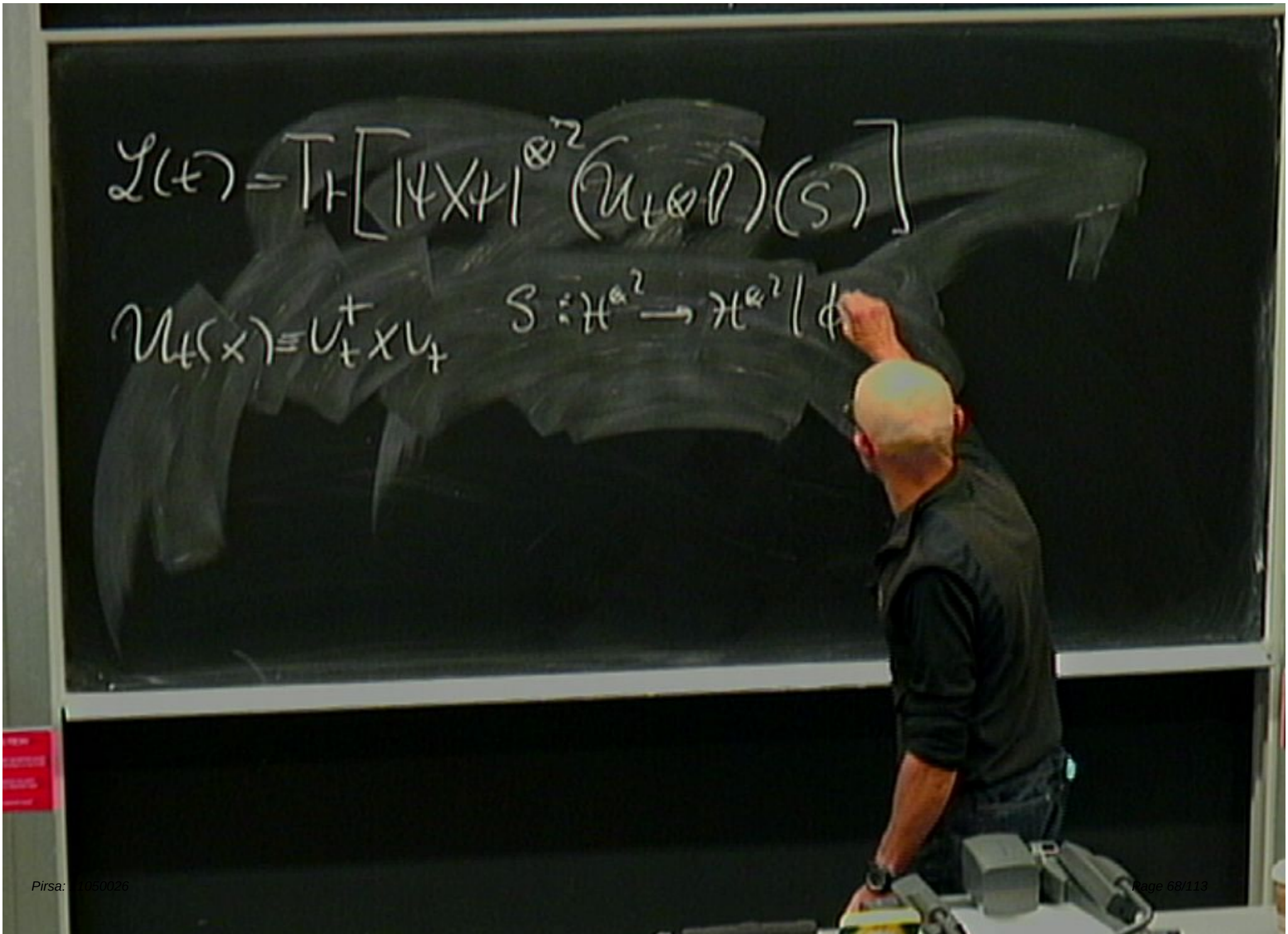



$\mathcal{L}(t) = \mathrm{Tr}[\rho(t)X$


$\mathcal{L}(t) = \mathrm{Tr}[|\psi\rangle\langle\psi|^{\otimes 2}$


$\mathcal{L}(t) = \mathrm{Tr}\left[|\psi\rangle\langle\psi|^{\otimes 2} (\mathcal{U}_t \otimes I)(S) \right]$
$\mathcal{U}_t(x) = U_t^\dagger x U_t$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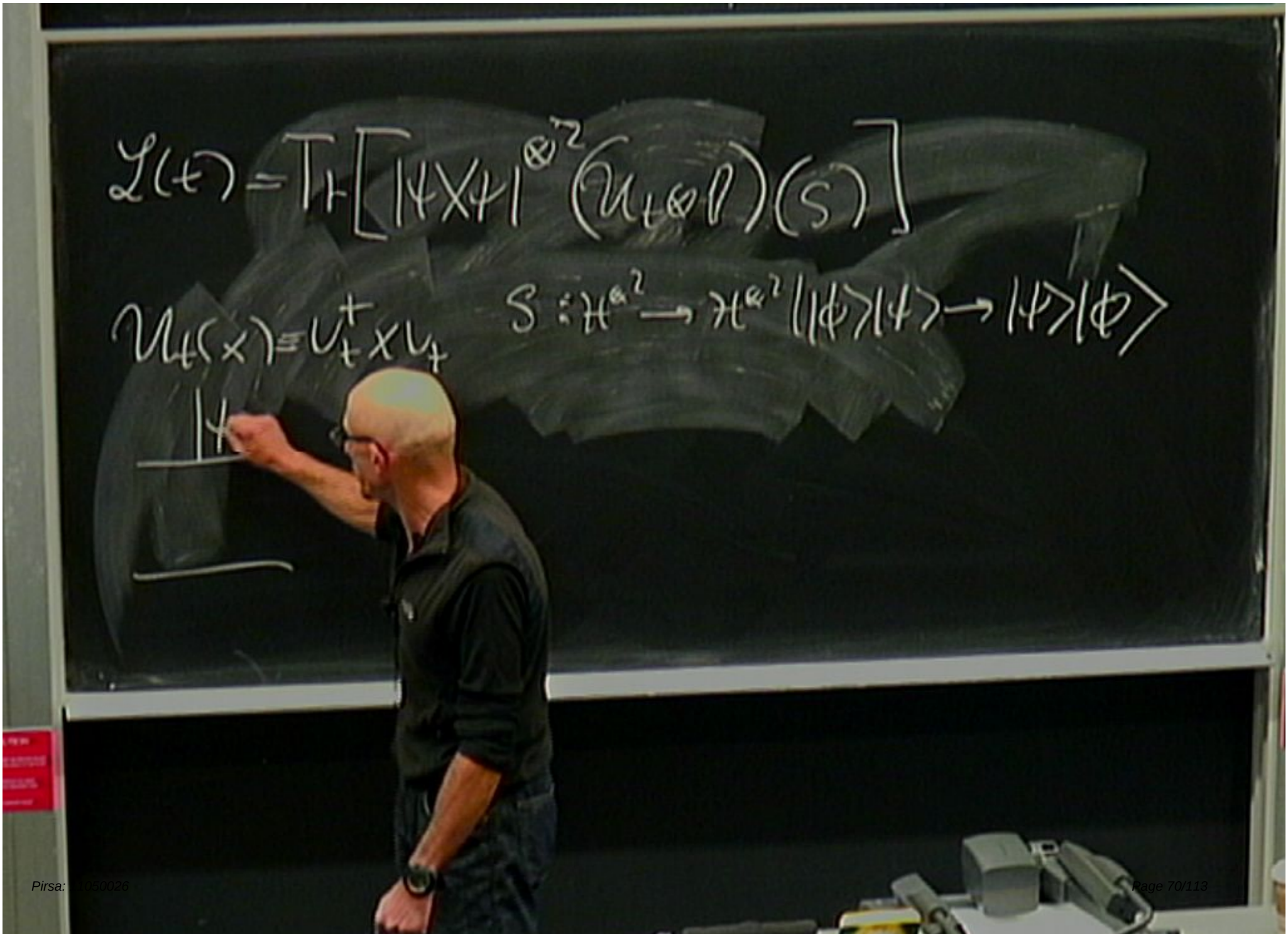
$\mathcal{U}_t(X) = U_t^\dagger X U_t$
$S : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2}$

$$\mathcal{L}(t) = \mathrm{Tr}\left[|\psi\rangle\langle\psi|^{\otimes 2} (\mathcal{U}_t \otimes \rho)(S) \right]$$

$$\mathcal{U}_t(x) = U_t^\dagger x U_t \qquad S : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2} \;\; |\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$$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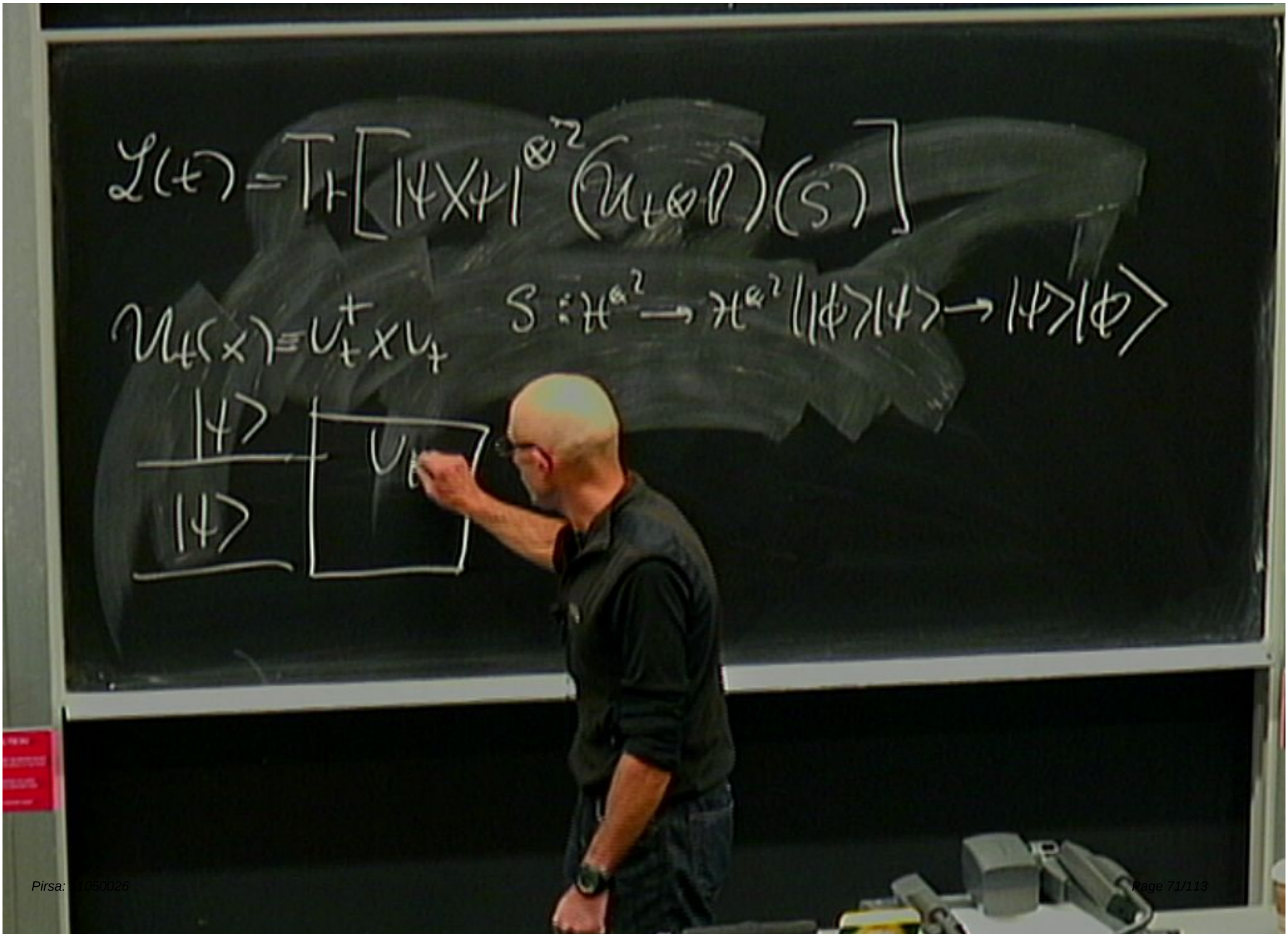
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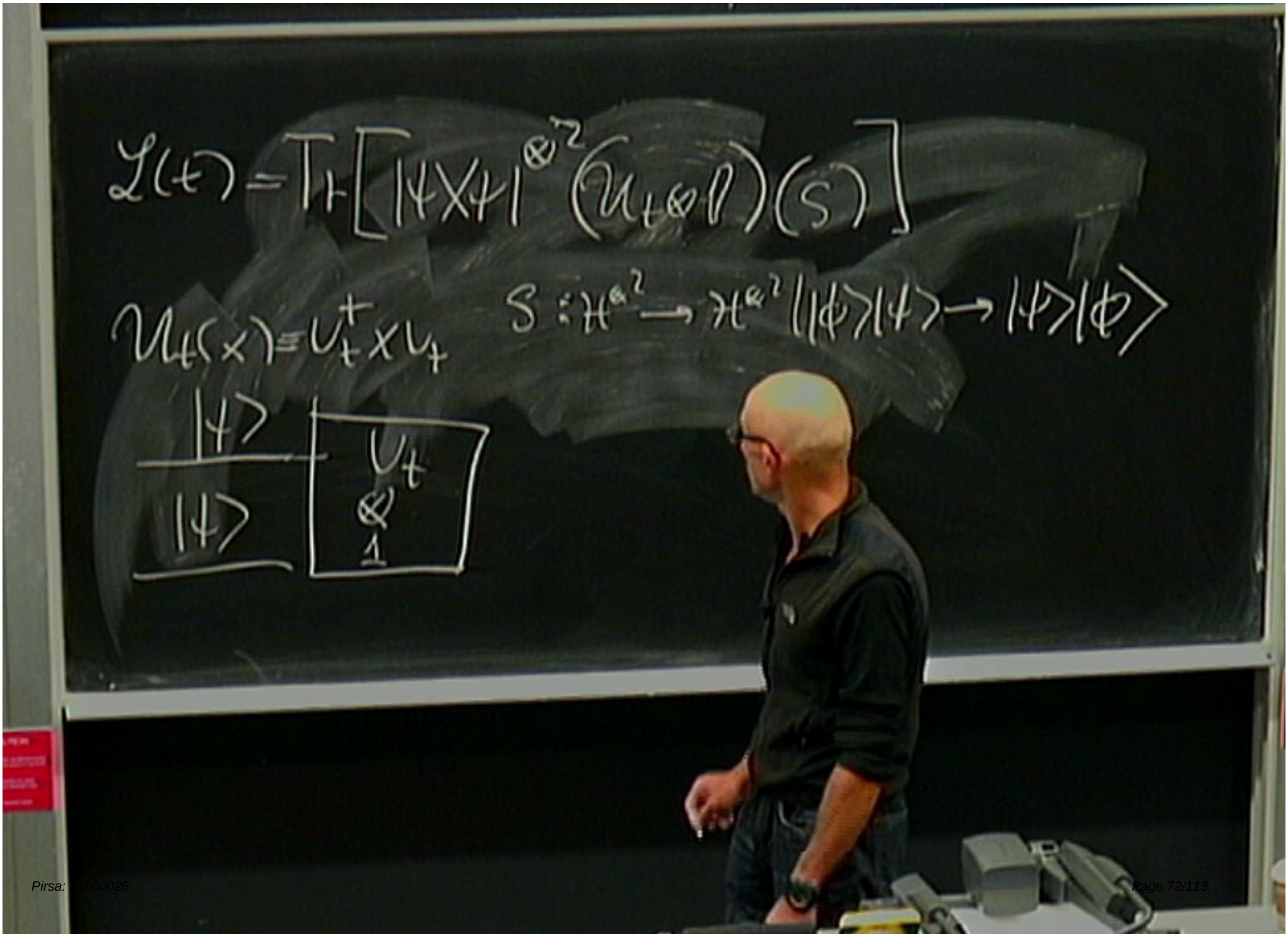
$|\psi\rangle$
$|\psi\rangle$
U_t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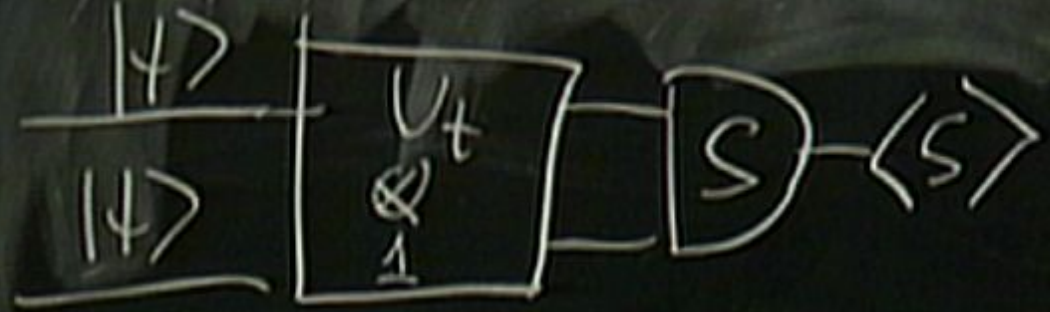
$|\psi\rangle$
$|\psi\rangle$
$U_t \otimes 1$

$$\mathcal{L}(t) = \mathrm{Tr}\left[|\psi\rangle\langle\psi|^{\otimes 2} (\mathcal{U}_t \otimes \mathbb{1})(S) \right]$$

$$\mathcal{U}_t(x) = U_t^\dagger x U_t \qquad S : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2} \;\; |\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$$



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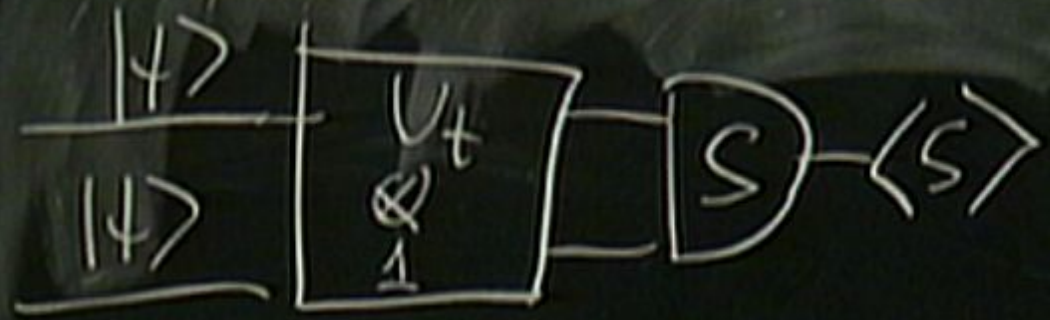
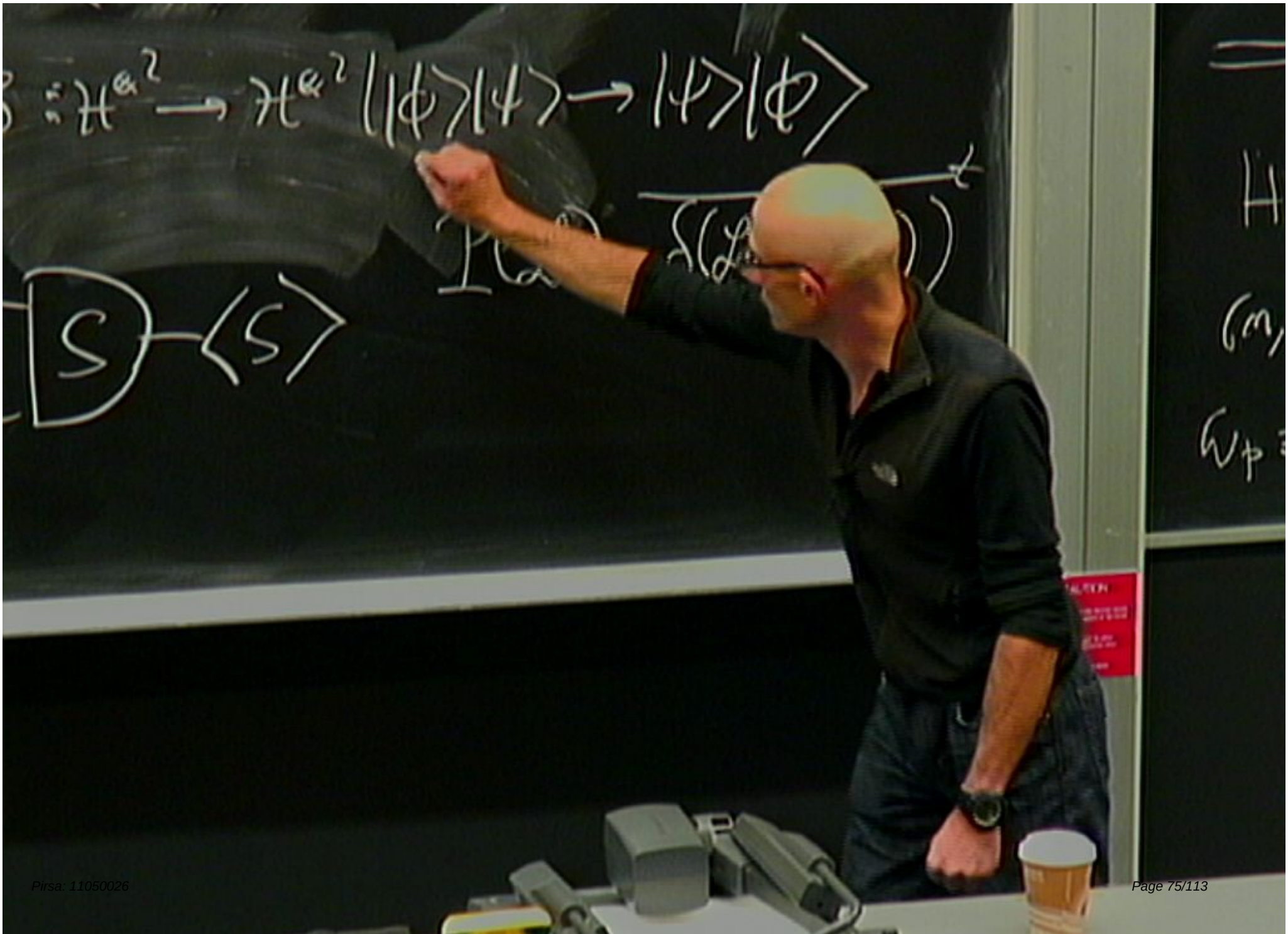
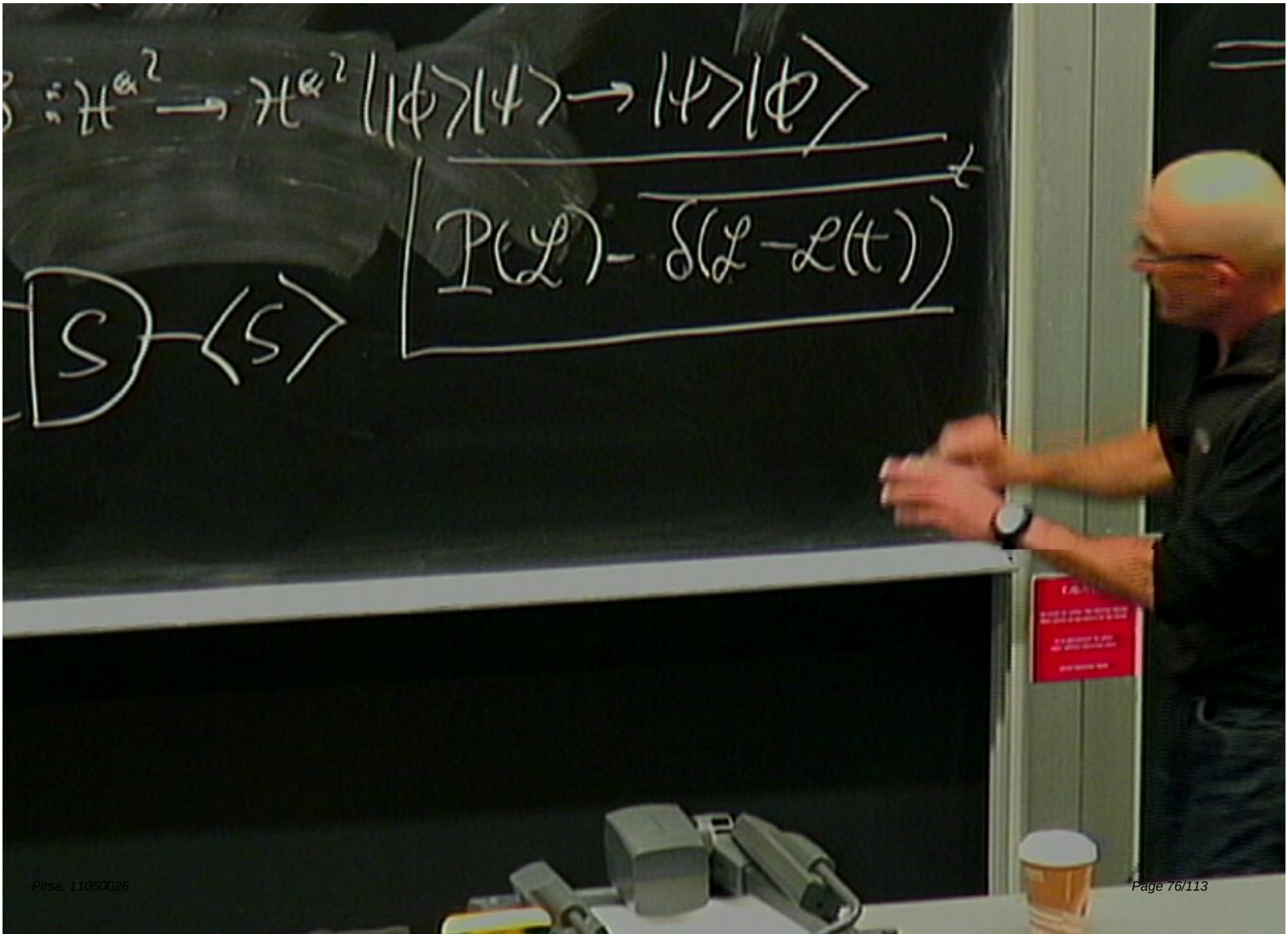
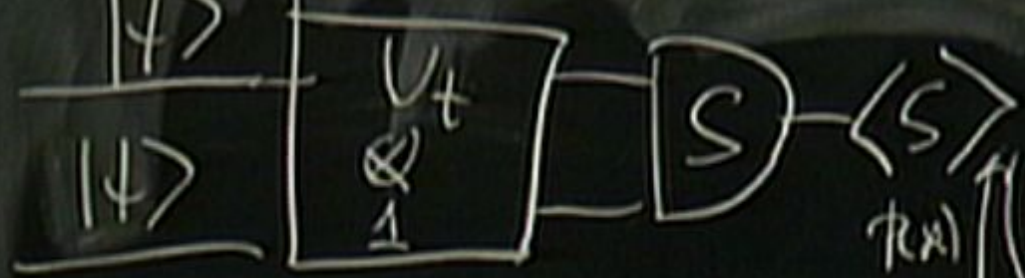



$\mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2}$ $|\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$
S
$\langle S \rangle$


$\mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2}$
$|\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$
$P(\mathcal{L}) - \delta(\mathcal{L} - \mathcal{L}(t))$
S
$\langle S \rangle$

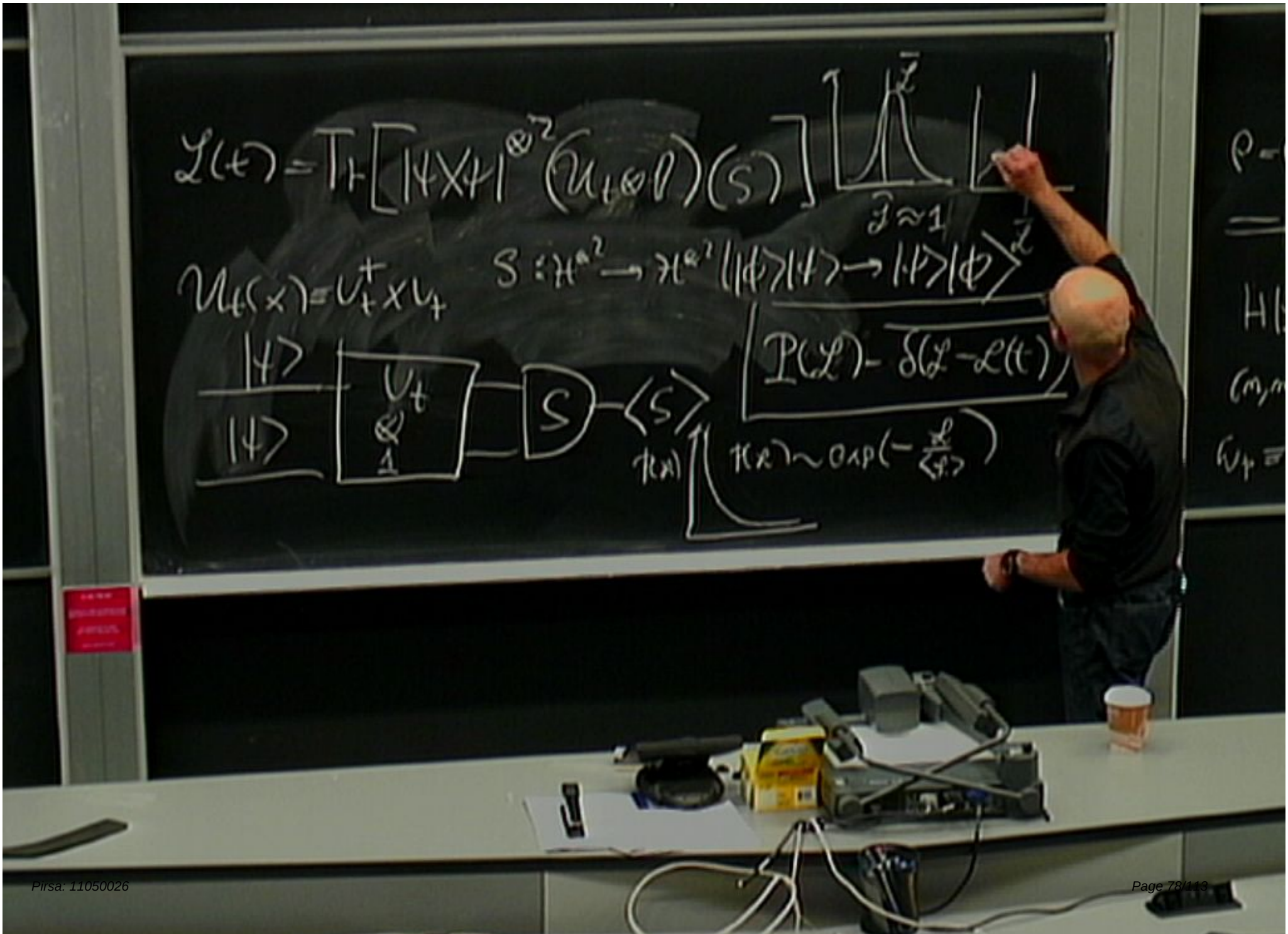
$$\mathcal{L}(t) = \mathrm{Tr}\left[|\psi\rangle\langle\psi|^{\otimes 2} (\mathcal{U}_t \otimes 1)(S) \right]$$

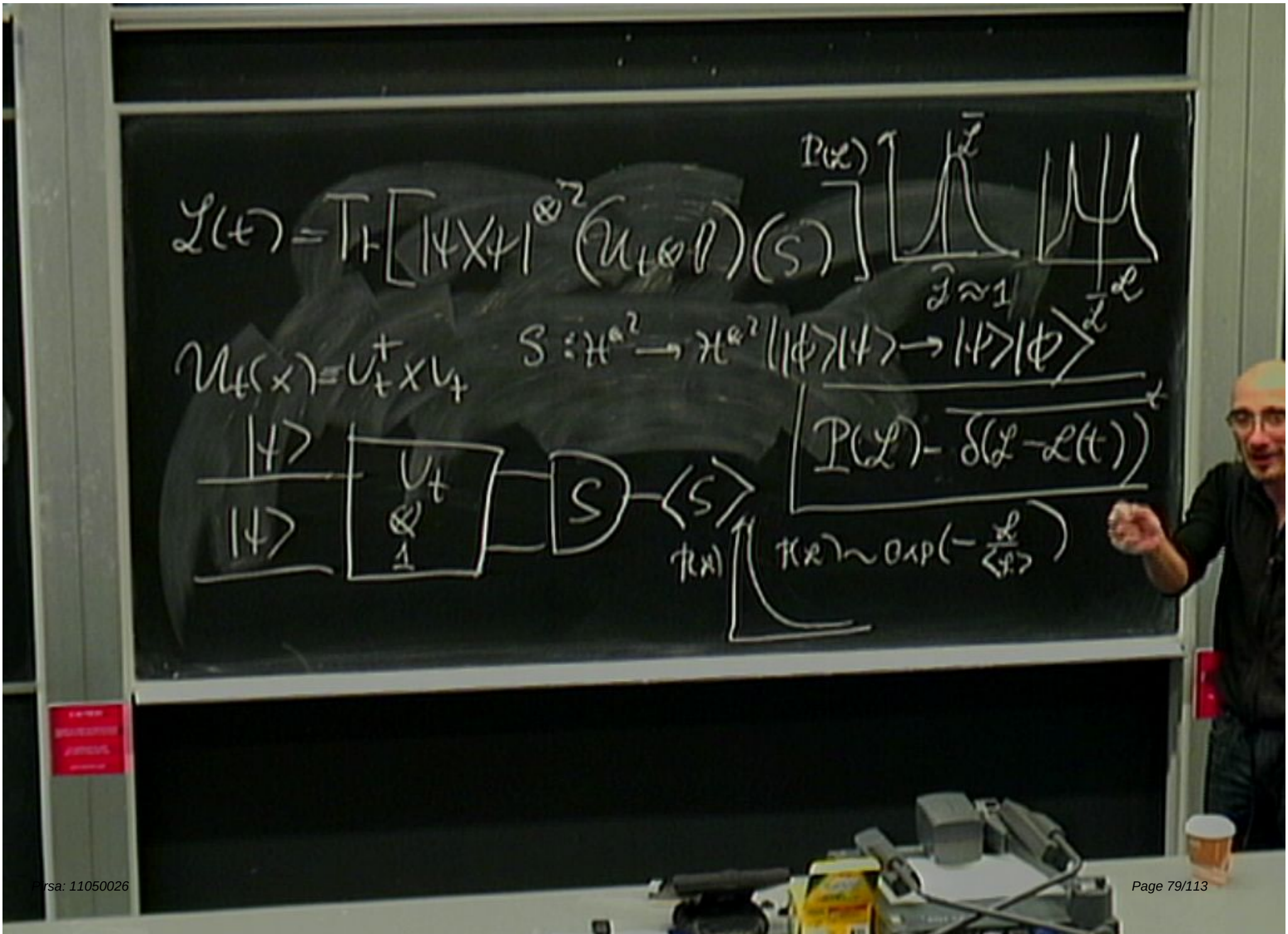
$$\mathcal{U}_t(x) = U_t^\dagger x U_t \qquad S : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2} \quad |\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$$



$$\boxed{P(\mathcal{L}) = \delta(\mathcal{L} - \mathcal{L}(t))}$$

$$\pi(x) \sim \exp\left(-\frac{\mathcal{L}}{\langle\mathcal{L}\rangle}\right)$$


$\mathcal{L}(t) = \mathrm{Tr}[|\psi\rangle\langle\psi|^{\otimes 2}(\mathcal{U}_t \otimes \mathcal{I})(S)]$
$\mathcal{U}_t(x) = U_t^\dagger x U_t$
$S : \mathcal{H}^{\otimes 2} \to \mathcal{H}^{\otimes 2}$
$|\psi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$
$\mathcal{I} \approx 1$
$P(\mathcal{L}) = \delta(\mathcal{L} - \mathcal{L}(t))$
$|\psi\rangle$
$|\psi\rangle$
U_t
$\otimes$
1
S
$\langle S \rangle$
$\pi(x)$
$\pi(x) \sim \exp(-\frac{x}{\langle x \rangle})$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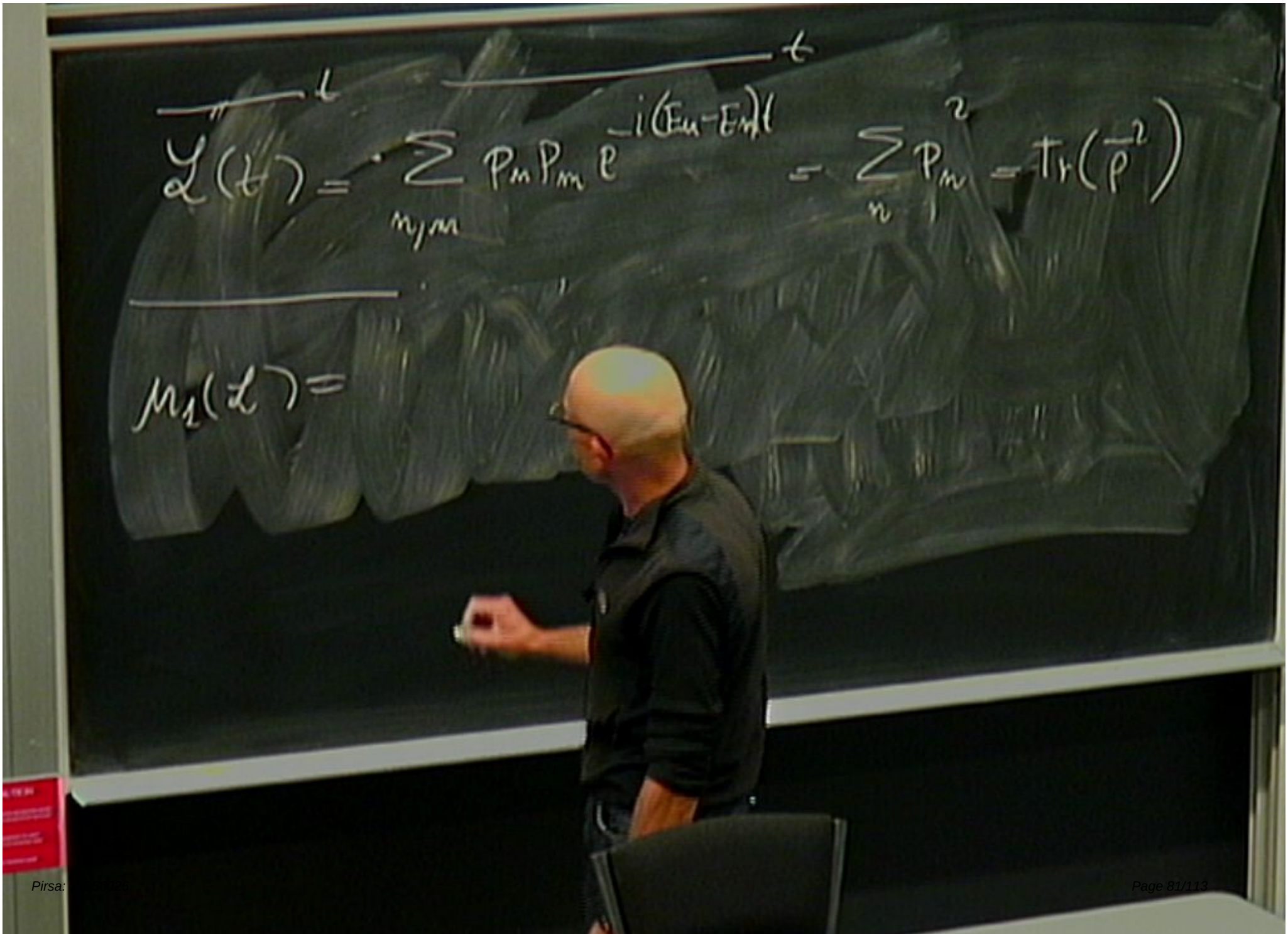
$|\phi\rangle|\psi\rangle \to |\psi\rangle|\phi\rangle$
$P(\mathcal{L}) = \delta(\mathcal{L} - \mathcal{L}(t))$
$|\psi\rangle$
$|\psi\rangle$
$U_t \otimes 1$
S
$\langle S \rangle$
$\pi(x)$
$\pi(x) \sim \exp\left(-\frac{x}{\langle x \rangle}\right)$
$P(\mathcal{L})$
$\bar{\mathcal{L}} \approx 1$
$\bar{\mathcal{L}}$
$\mathcal{L}$

$$\mathcal{L}(t) = \sum_{n,m} p_n p_m e^{-i(E_n - E_m)t} = \sum_n p_n^2 = \mathrm{Tr}(\bar{\rho}^2)$$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$M_1(t) =$

$$\mathcal{L}(t) = \sum_{n,m} p_n p_m e^{-i(E_n - E_m)t} = \sum_n p_n^2 = \mathrm{Tr}(\rho^2)$$

$$\mu_1(t) = \mathrm{Tr}\left[|H\rangle\langle H|^{\otimes 2} D_H^{\otimes 2}(S) \right]$$

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$$M_1(\mathcal{L}) = \mathrm{Tr}\left[|\psi\rangle\langle\psi|^{\otimes 2} D_H^{\otimes 2}(S) \right] \leftarrow \psi \in \mathcal{H}$$

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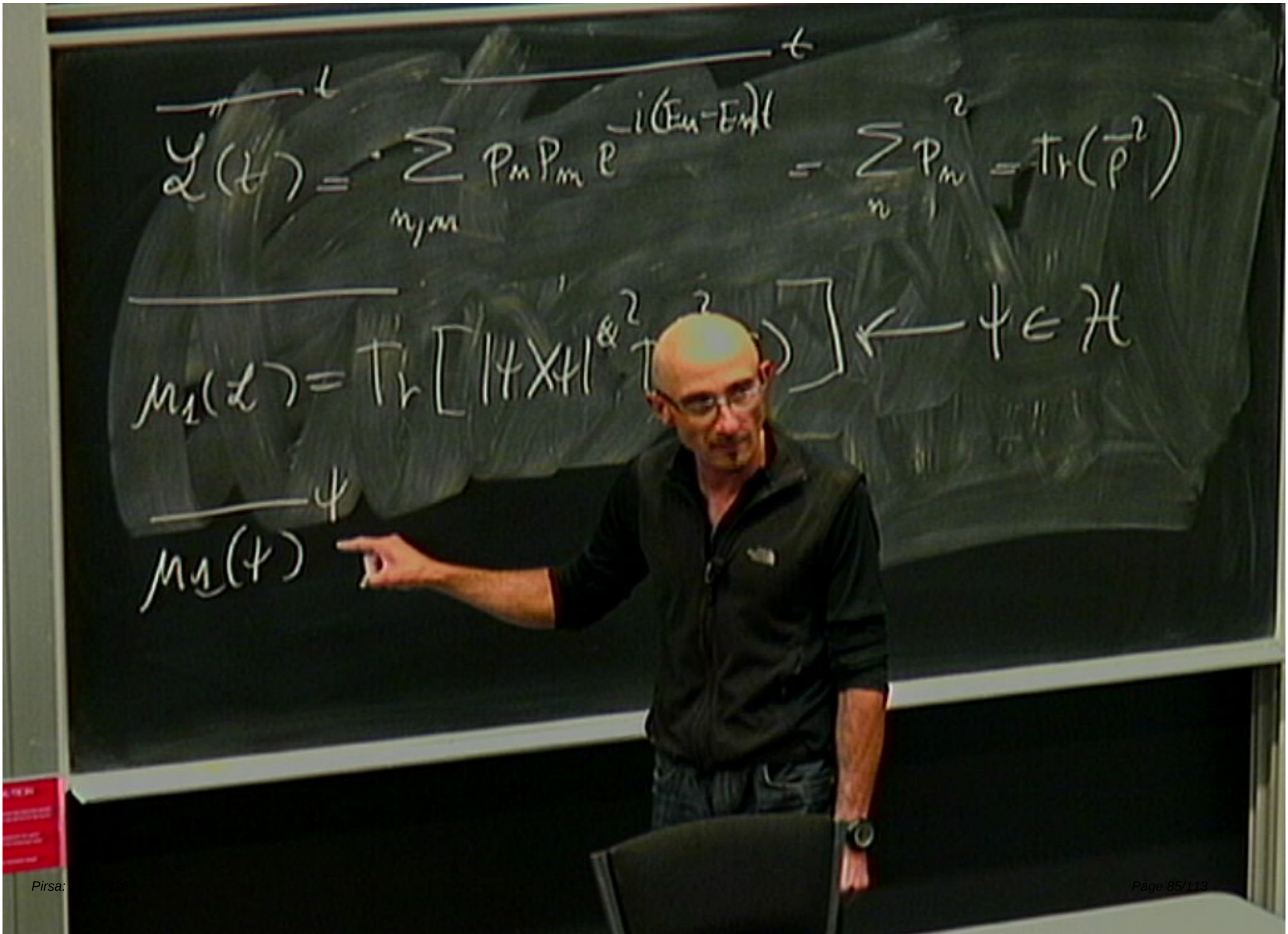
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$\mu_1(\mathcal{L}) = \mathrm{Tr}[|\psi\rangle\langle\psi|^{\otimes 2}]$
$\psi \in \mathcal{H}$
$\mu_1(\psi)$

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$$d = 2^N$$

$$\mu_1(\psi) = \frac{2}{d+1} \sim 2d^{-1}$$

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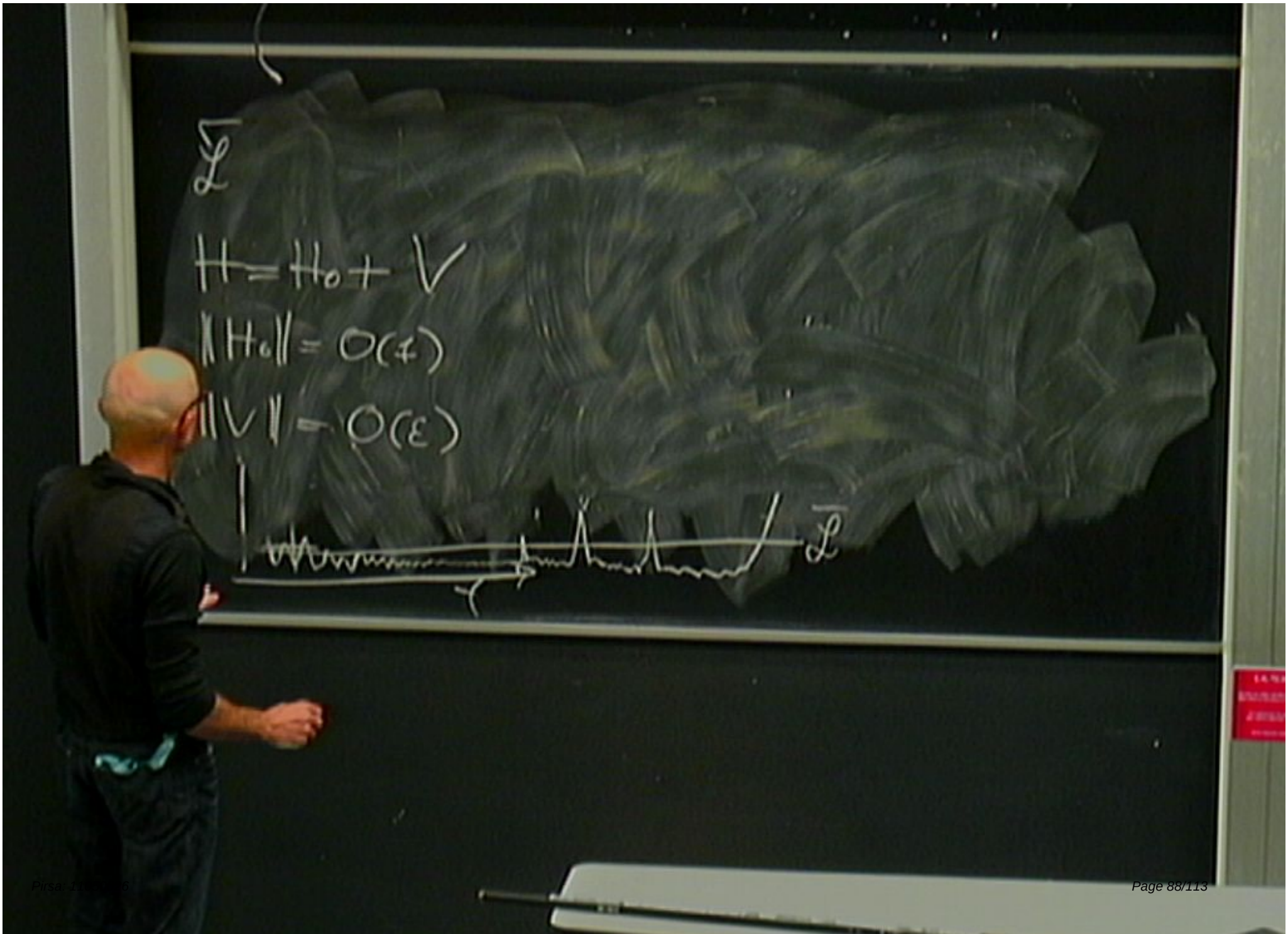
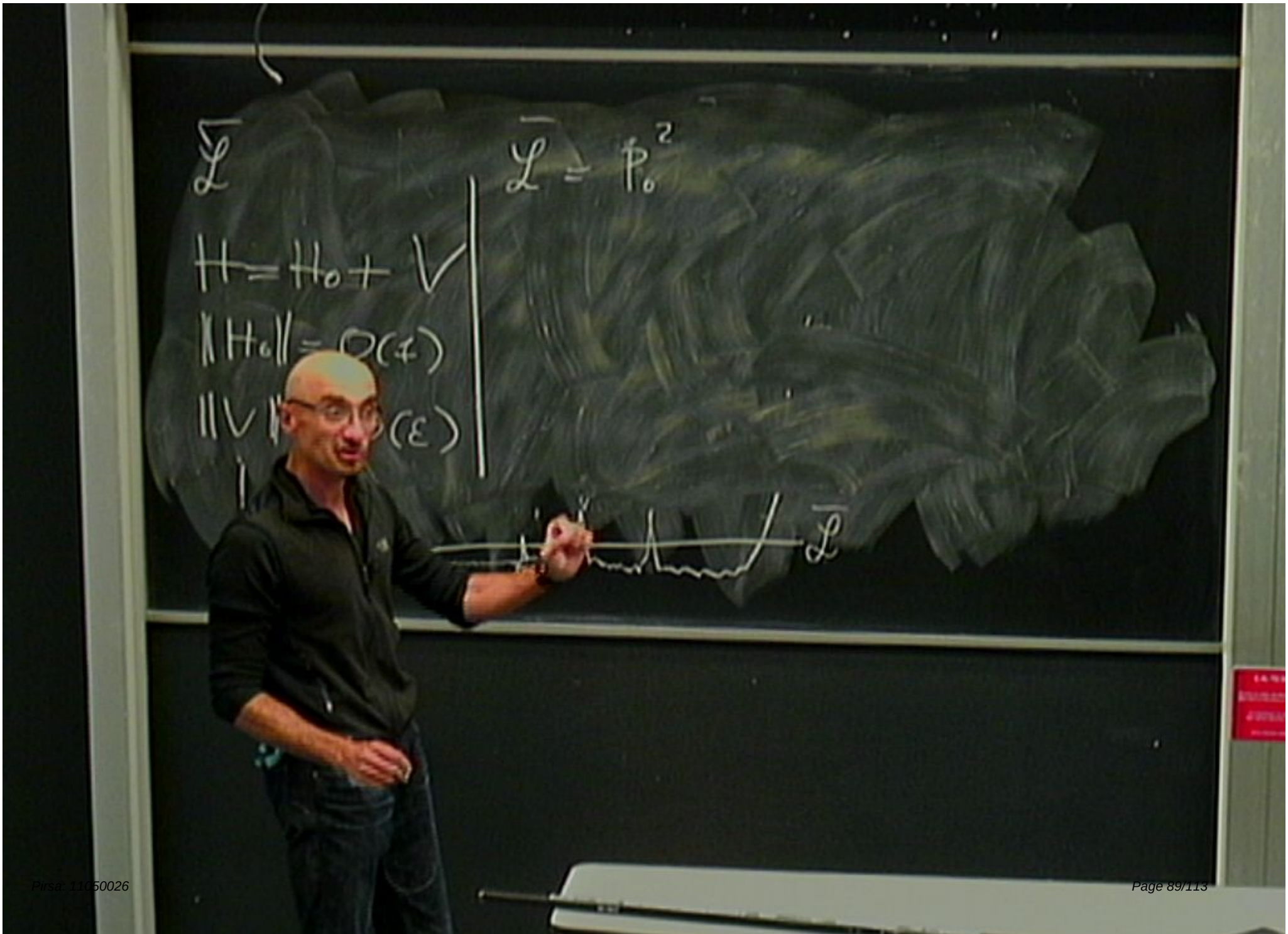
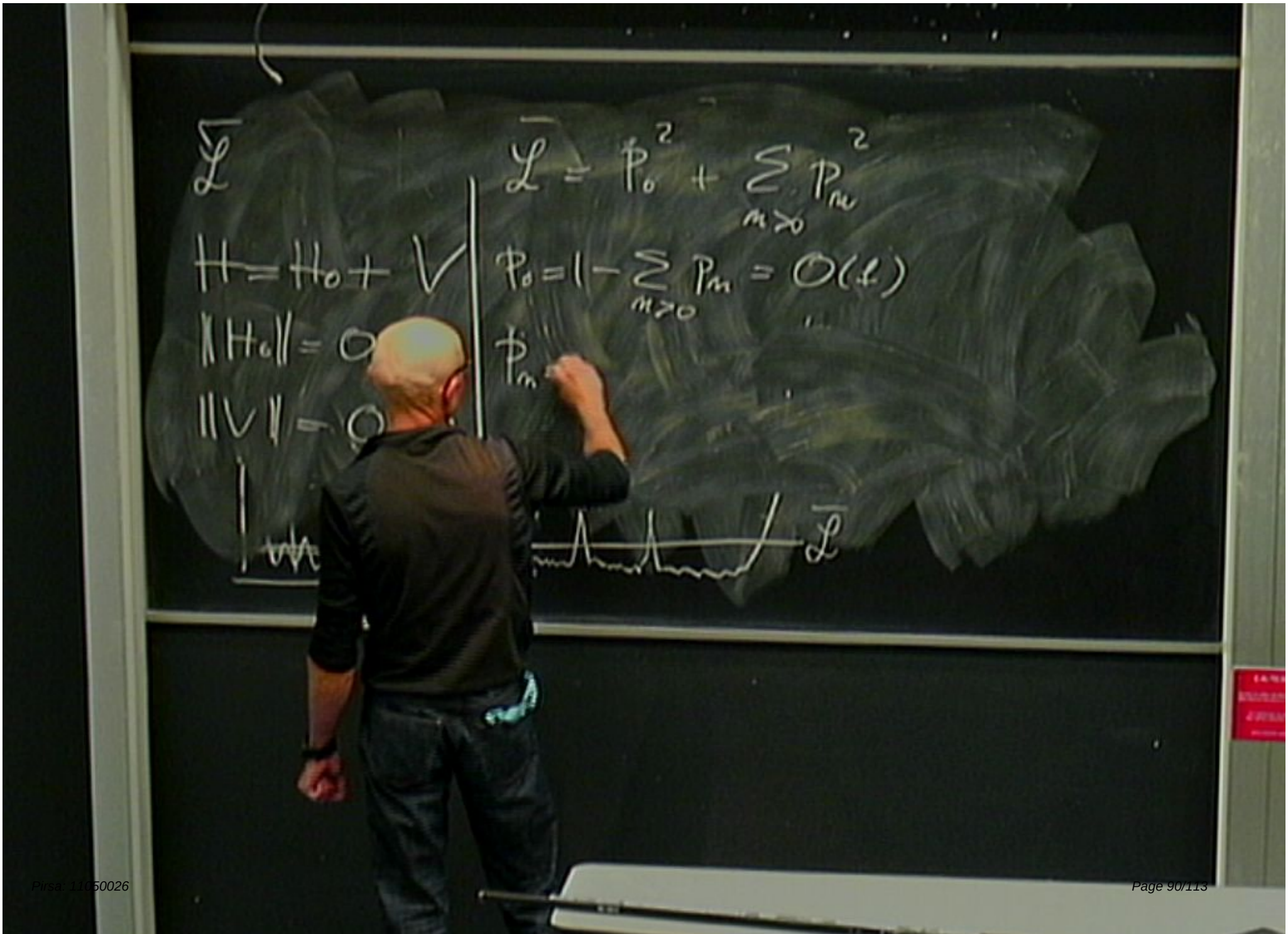

$\mathcal{L}$
$H = H_0 + V$
$\|H_0\| = O(1)$
$\|V\| = O(\varepsilon)$
$\mathcal{L}$


$\bar{\mathcal{L}}$
$\bar{\mathcal{L}} = P_0^2$
$H = H_0 + V$
$\|H_0\| = O(1)$
$\|V\| = O(\varepsilon)$
$\bar{\mathcal{L}}$


$\bar{\mathcal{L}}$
$H = H_0 + V$
$\|H_0\| = O$
$\|V\| = O$
$\bar{\mathcal{L}} = P_0^2 + \sum_{m>0} P_m^2$
$P_0 = 1 - \sum_{m>0} P_m = O(\epsilon)$
P_m
$\bar{\mathcal{L}}$

$\bar{\mathcal{L}}$

$$\bar{\mathcal{L}} = P_0^2 + \sum_{m>0} P_m^2 \approx P_0^2$$

$$H = H_0 + V$$

$$\|H_0\| = O(1)$$

$$\|V\| = O(\varepsilon)$$

$$P_0 = 1 - \sum_{m>0} P_m = O(1)$$

$$P_m = \frac{|\langle m|V|0\rangle|^2}{(E_m - E_0)^2} = O(\varepsilon^2)$$

$\bar{\mathcal{L}}$

$$\bar{\mathcal{L}} = P_0^2 + \sum_{n>0} P_n^2 \approx P_0^2 = \left(1 - \sum_{n>0}^{\infty} P_n\right)^2$$

$$H = H_0 + V$$

$$\|H_0\| = O(1)$$

$$\|V\| = O(\varepsilon)$$

$$P_0 = 1 - \sum_{n>0} P_n = O(\varepsilon) \approx 1 - 2\chi_F$$

$$P_n = \frac{|\langle n|V|0\rangle|^2}{(E_n - E_0)^2} = O(\varepsilon^2)$$

$\bar{\mathcal{L}}$

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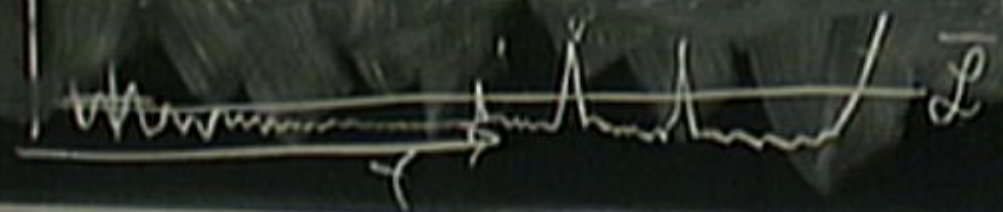
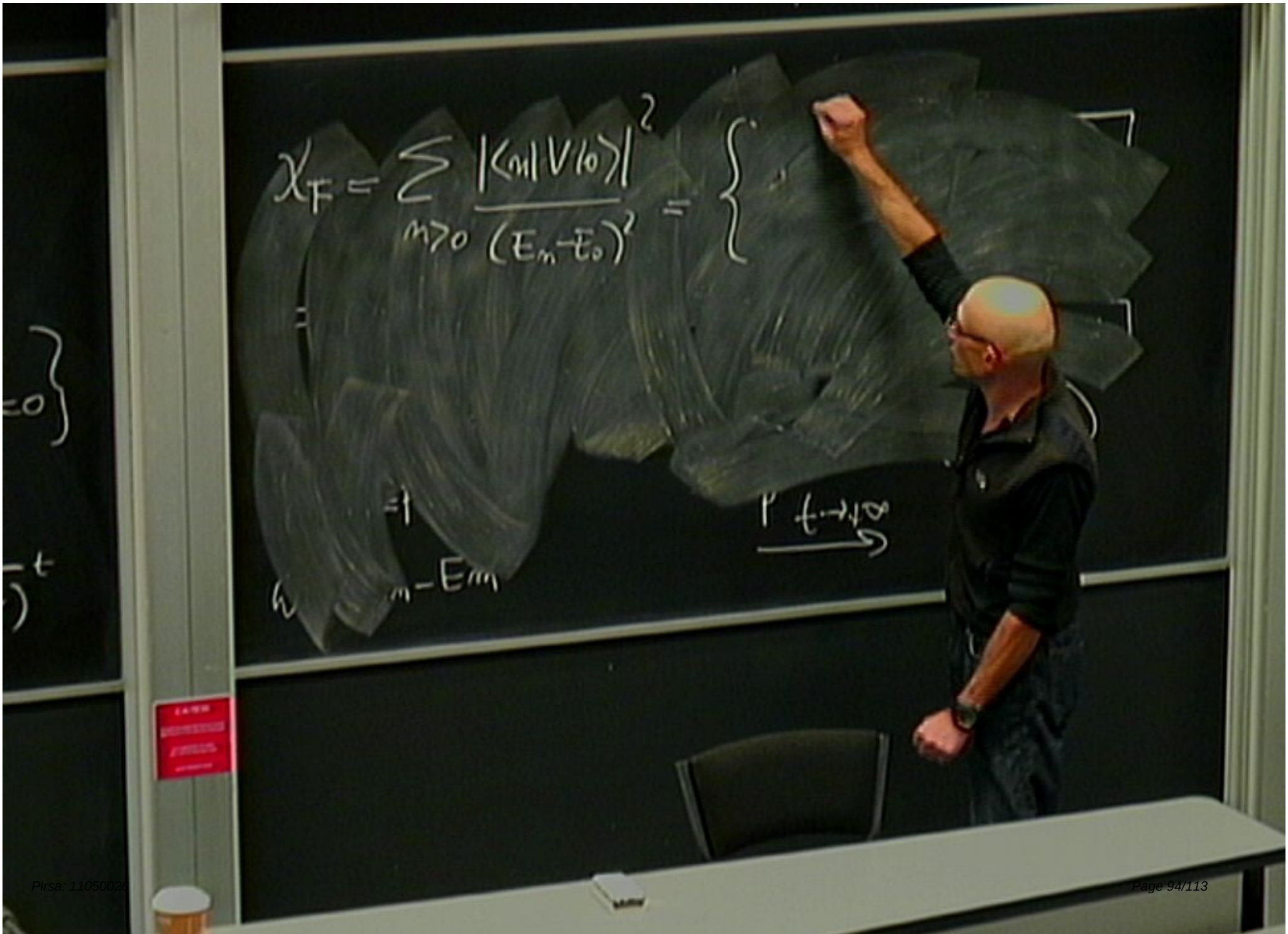
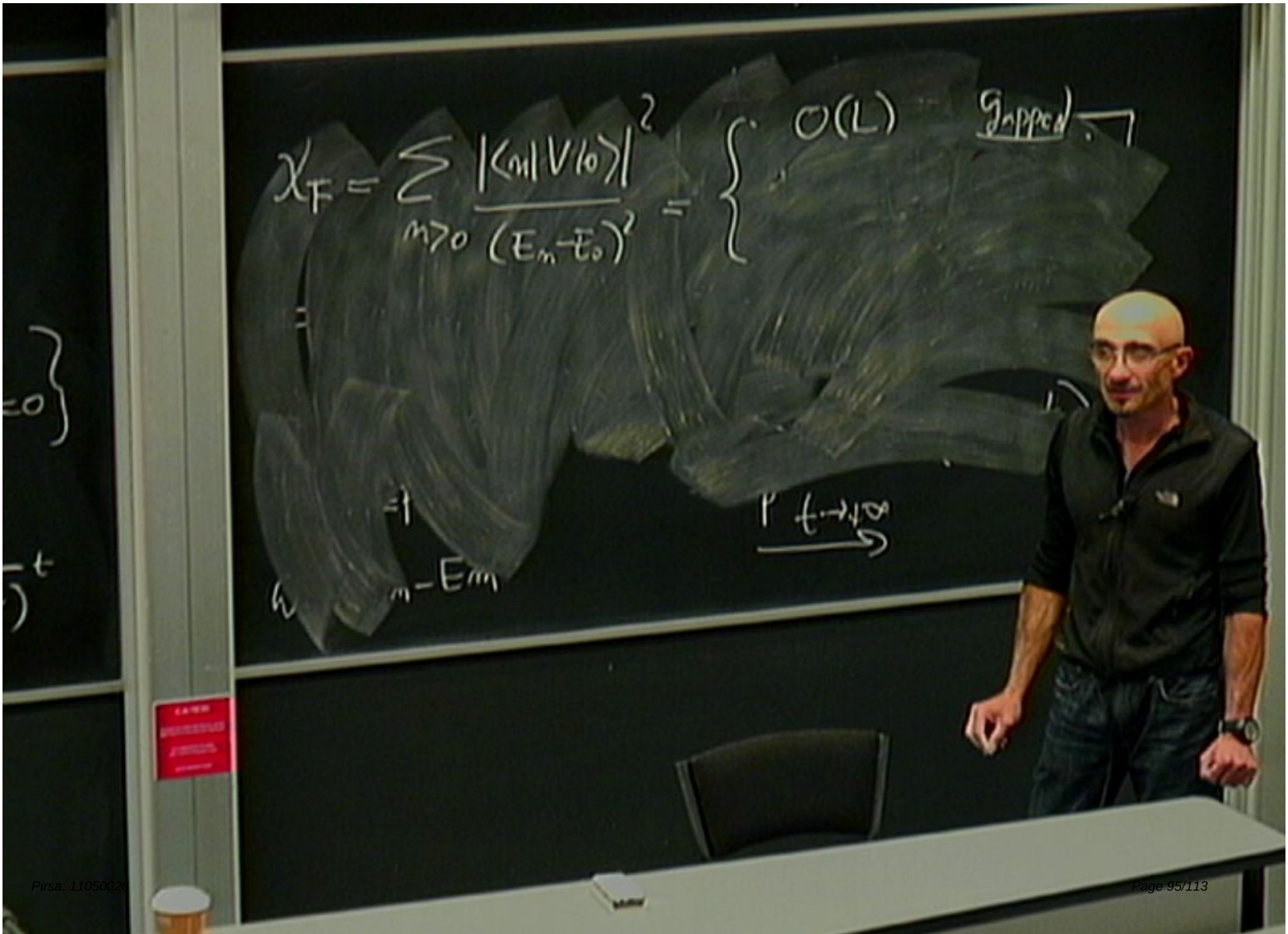



$\chi_F = \sum_{n>0} \frac{|\langle n|V|0\rangle|^2}{(E_n - E_0)^2} = \{$
$t \to +\infty$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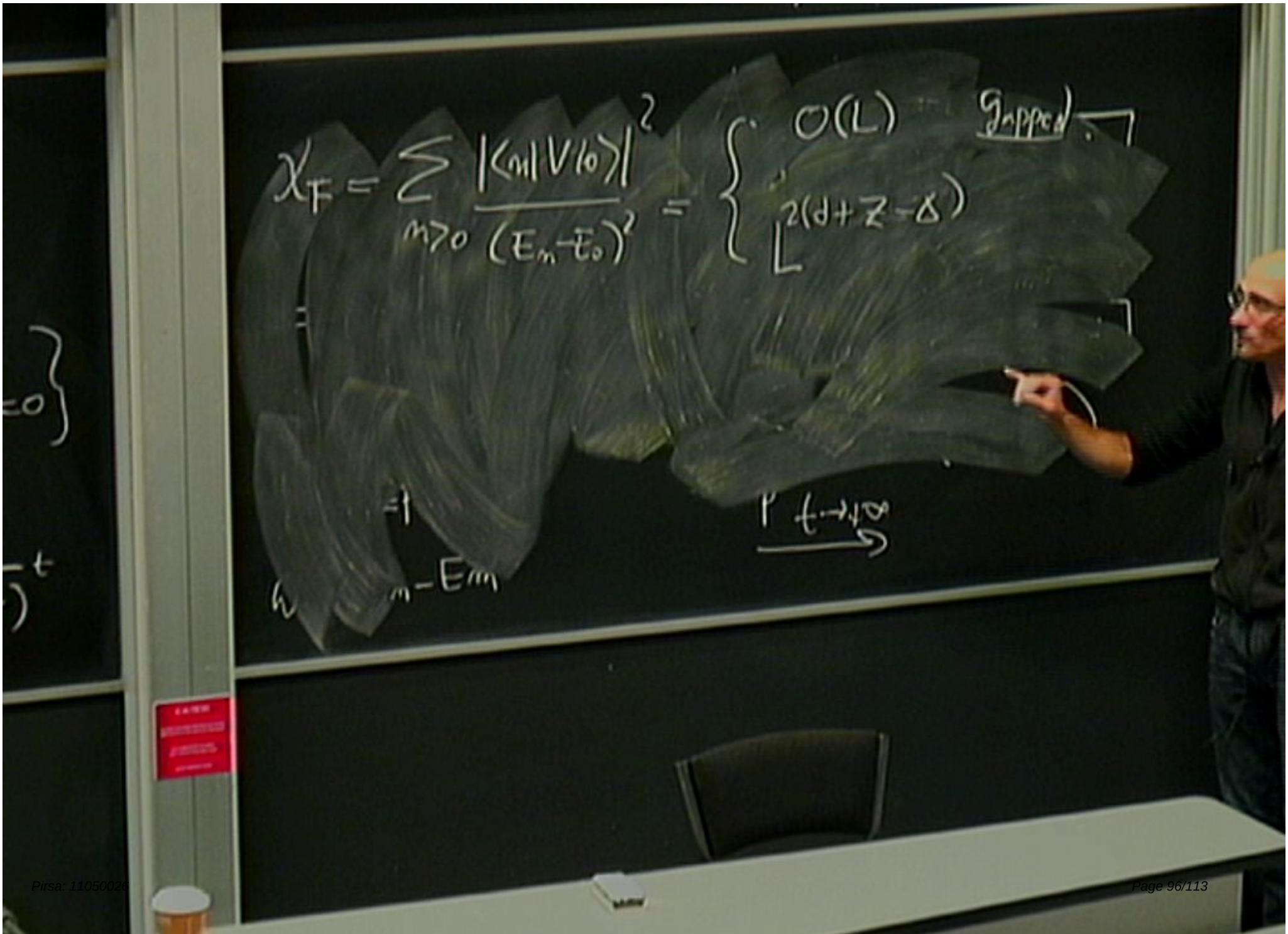
gapped
$t \to +\infty$


$\chi_F = \sum_{n>0} \frac{|\langle n|V|0\rangle|^2}{(E_n - E_0)^2} = \begin{cases} O(L) & \text{gapped} \\ L^{2(d+z-\Delta)} \end{cases}$
$t \to +\infty$

$$\chi_F = \sum_{n>0} \frac{|\langle n|V|0\rangle|^2}{(E_n - E_0)^2} = \begin{cases} O(L) & \text{gapped} \\ L^{2(d+z-\Delta)} & \Delta \leq z + \frac{d}{2} \end{cases}$$

$$\xrightarrow{t \to +\infty}$$

$$\omega \quad {}_n - E_m$$

$$\chi_F = \sum_{m>0} \frac{|\langle m|V|0\rangle|^2}{(E_m - E_0)^2} = \begin{cases} O(L) & \text{gapped} \\ L^{2(d+z-\Delta)} & \Delta \leq d \end{cases}$$

$$|\langle \psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle| \approx \exp\left(-\delta\lambda^2 \chi_F\right)$$

$$\xrightarrow{t \to \infty}$$

$$\omega_{m} = E_m - E_m$$

$$\chi_F = \sum_{n>0} \frac{|\langle n|V|0\rangle|^2}{(E_n - E_0)^2} = \begin{cases} O(L) & \text{gapped} \\ L^{2(d+z-\Delta)} & \Delta \le z + \frac{d}{2} \end{cases}$$

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$$\sum_n - E_m$$

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$$\xrightarrow{t \to +\infty}$$

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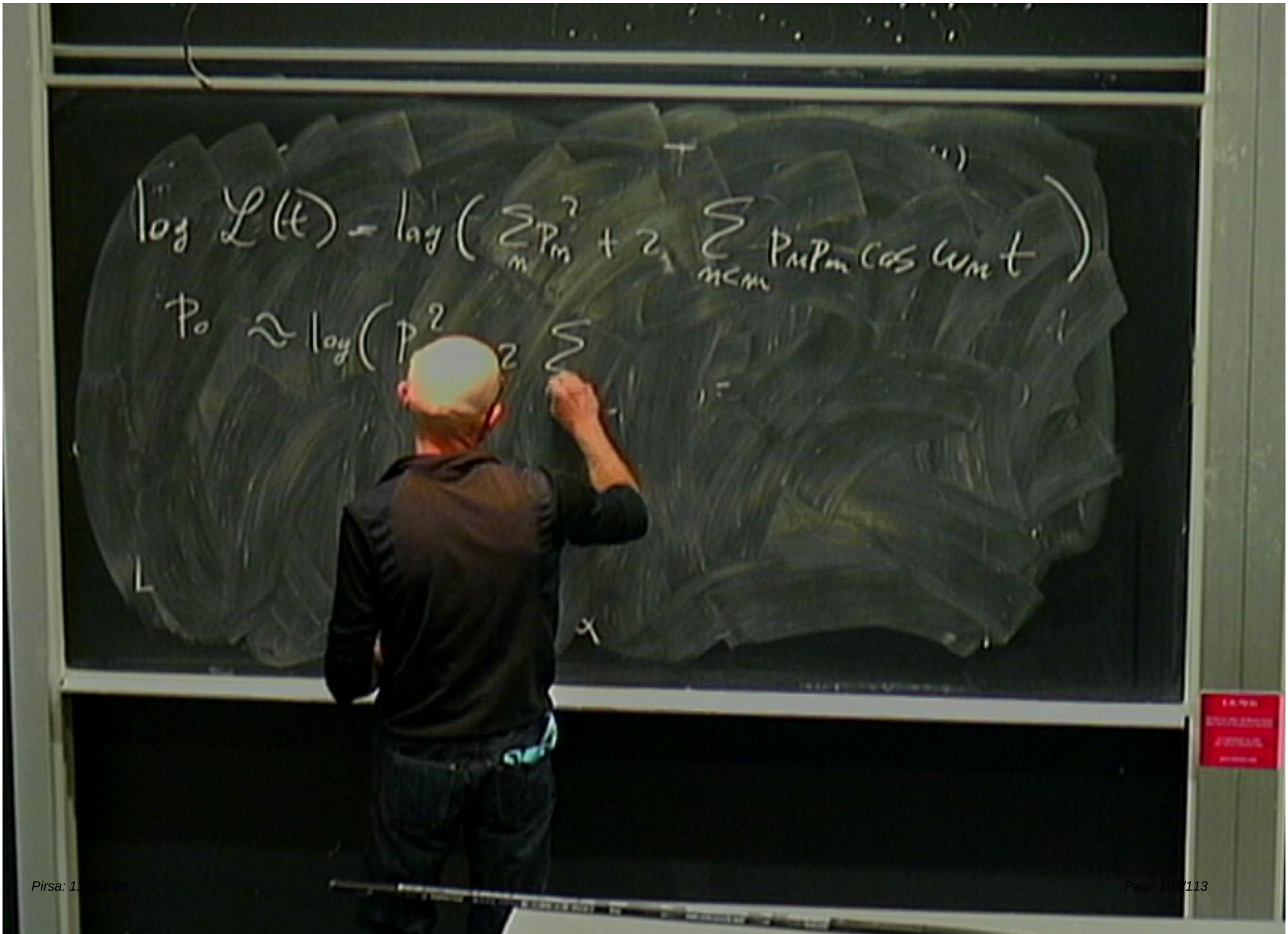
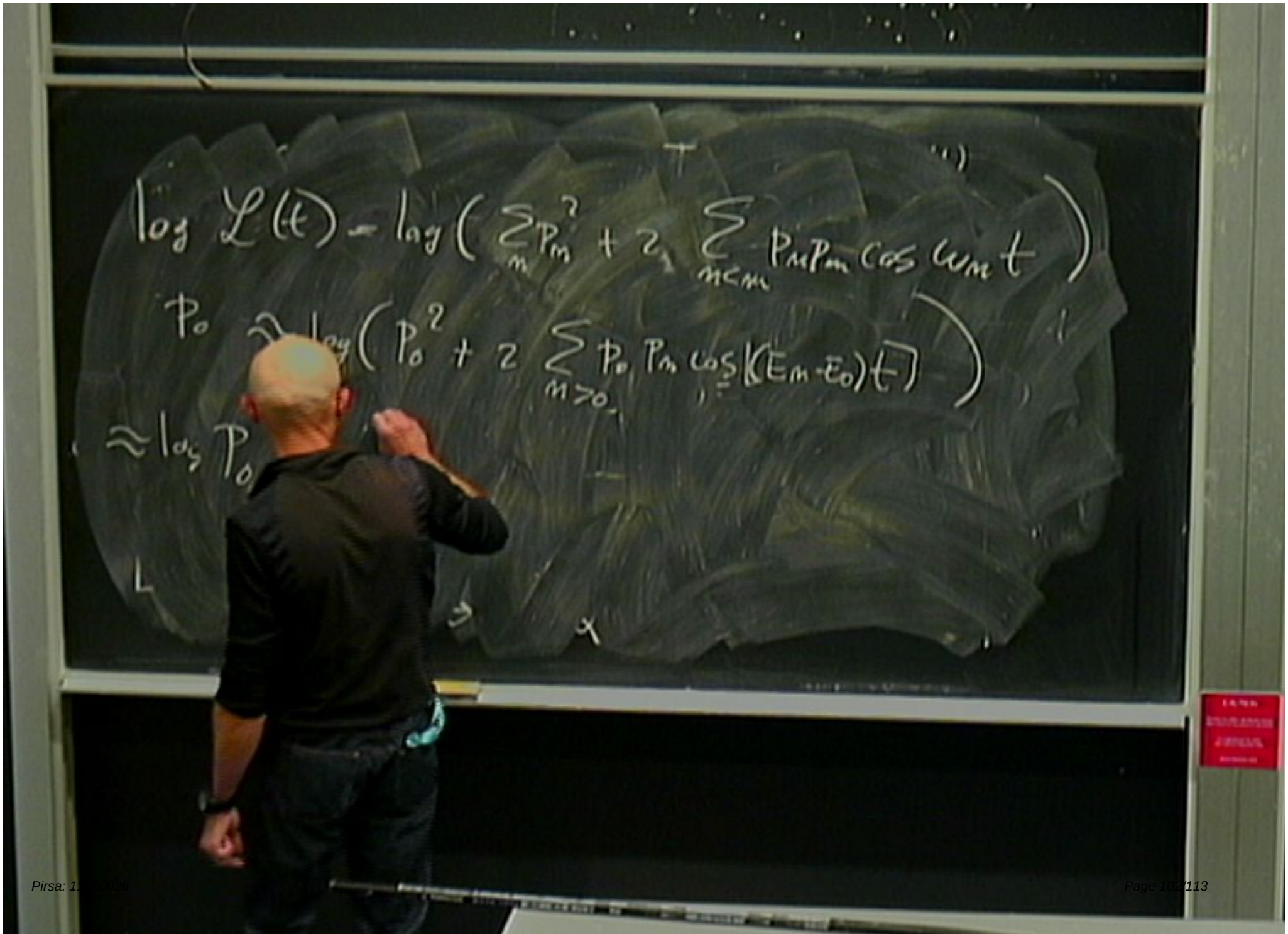

log ℒ(t) = log (Σ_m P_m^2 + 2 Σ_{n<m} P_n P_m cos ω_m t)
P_0 ≈ log (P^2 2 Σ


$\log \mathcal{L}(t) = \log\left(\sum_n P_n^2 + 2\sum_{n<m} P_n P_m \cos \omega_{nm} t\right)$
$P_0 \approx \log\left(P_0^2 + 2\sum_{m>0} P_0 P_m \cos[(E_m - E_0)t]\right)$
$\approx \log P_0$

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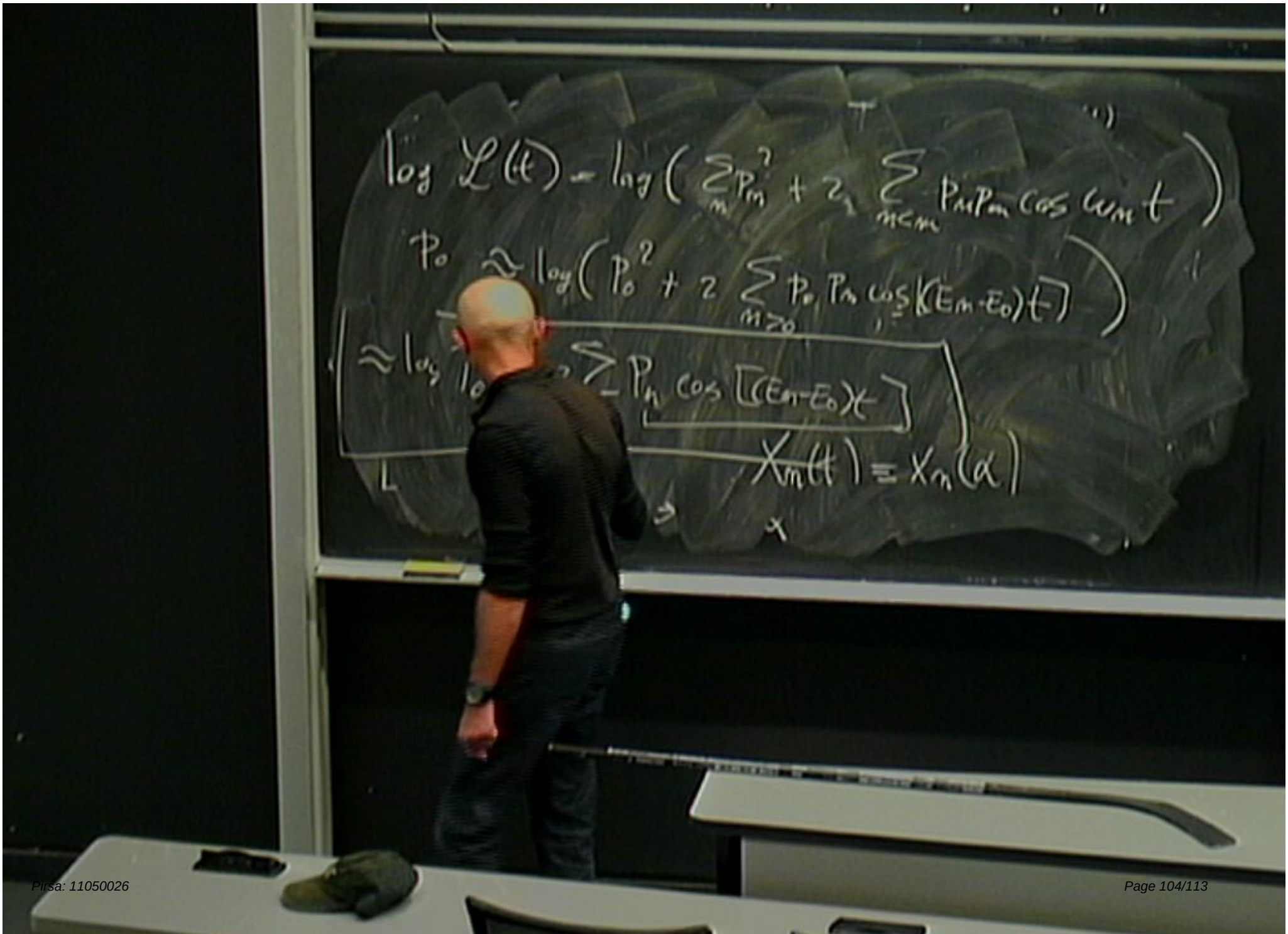
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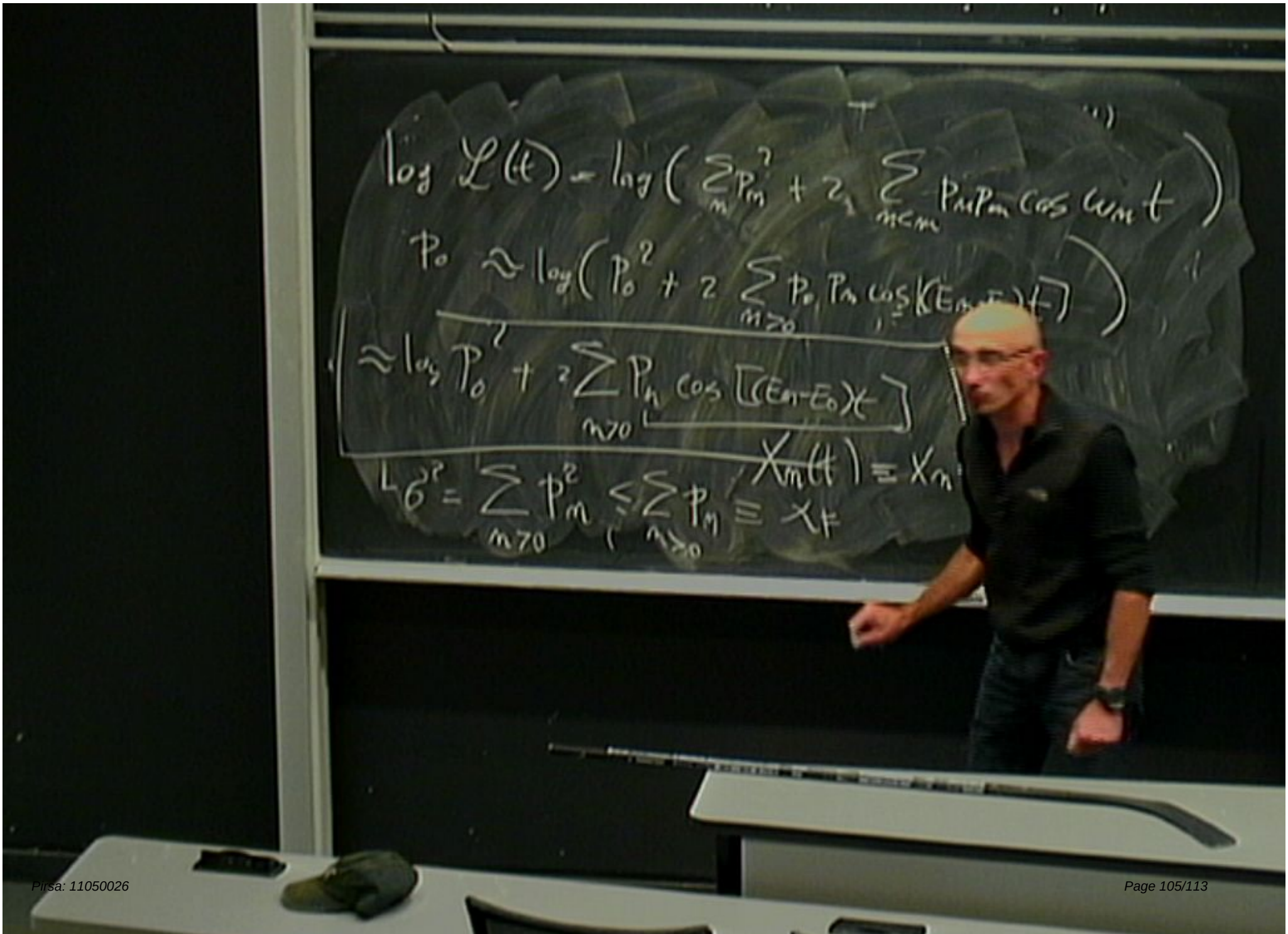
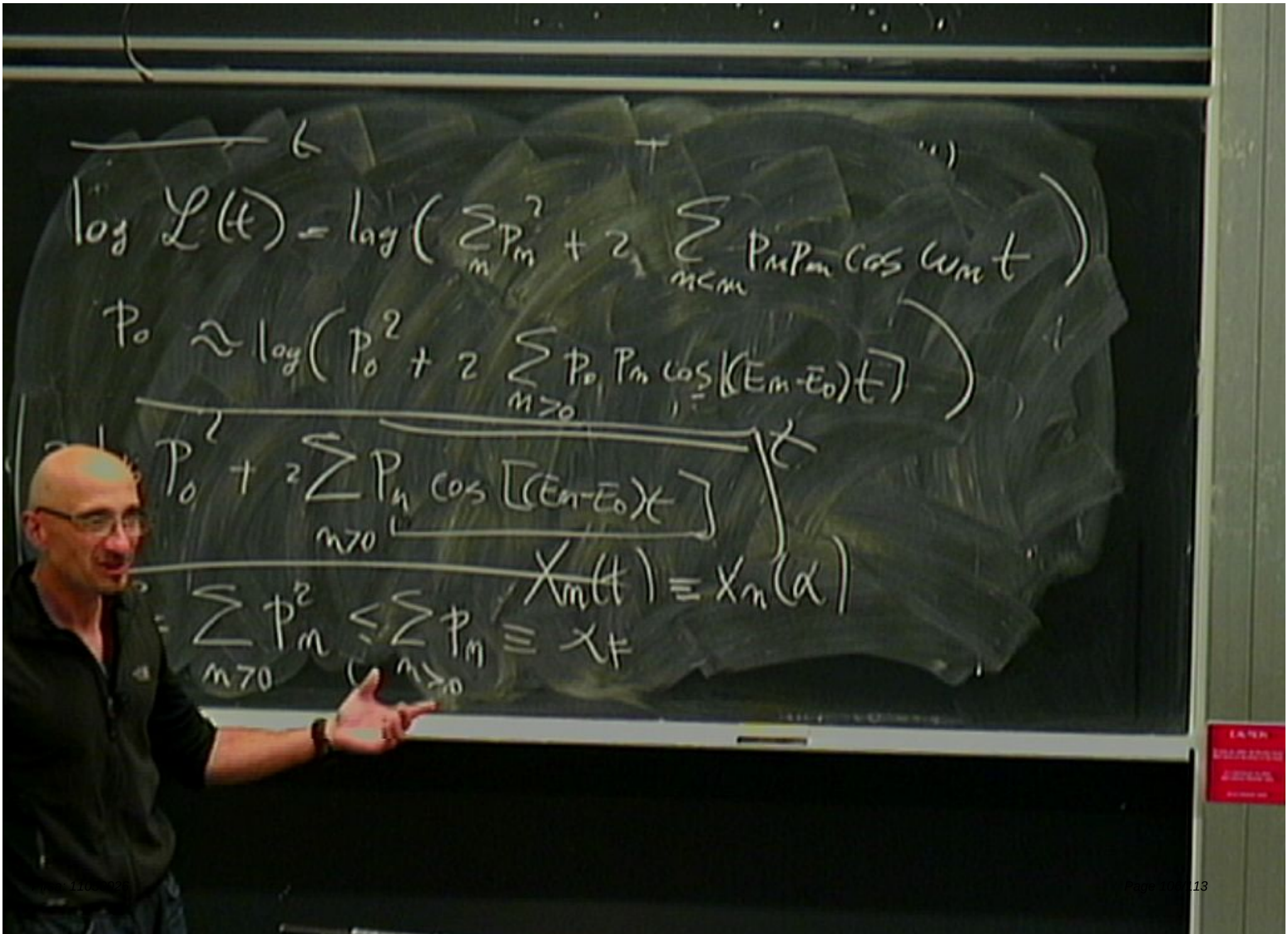
≈ log
Σ P_n cos [(E_n - E_0) t]
X_m(t) ≡ X_n(α)


$\log \mathcal{L}(t) = \log\left(\sum_m p_m^2 + 2\sum_{m<n} p_m p_n \cos \omega_{mn} t\right)$
$\approx \log\left(p_0^2 + 2\sum_{m>0} p_0 p_m \cos[(E_m - E_0)t]\right)$
$\approx \log p_0^2 + 2\sum_{m>0} p_m \cos[(E_m - E_0)t]$
$X_m(t) \equiv X_m$
$\sigma^2 = \sum_{m>0} p_m^2 \leq \sum_{m>0} p_m \equiv X_t$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X_n(t) ≡ X_n(α)
Σ_{n>0} P_n^2 ≤ Σ_{n>0} P_n ≡ X_F

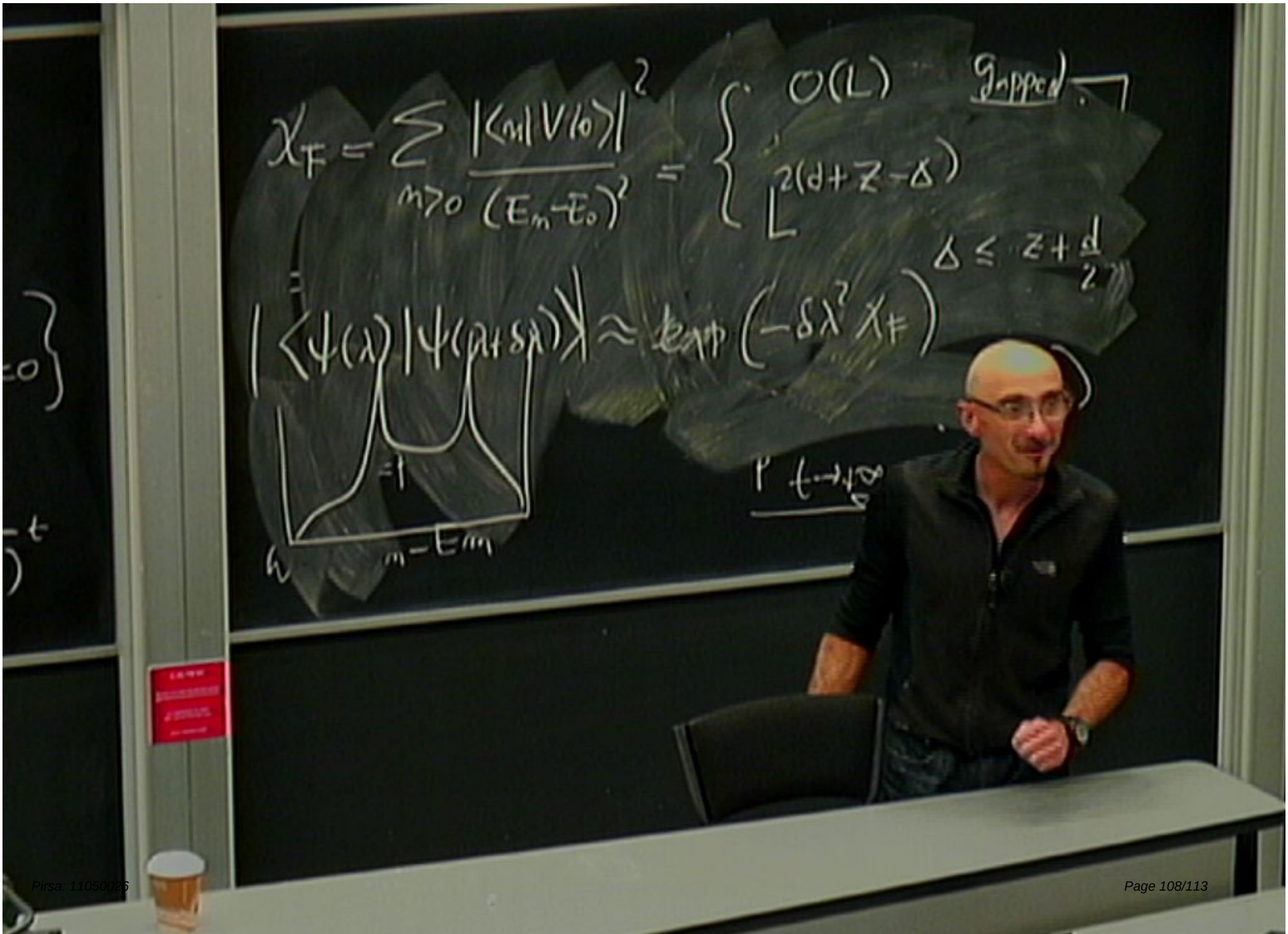
$$\log \mathcal{L}(t) = \log\left(\sum_n P_n^2 + 2\sum_{n<m} P_n P_m \cos \omega_m t\right)$$

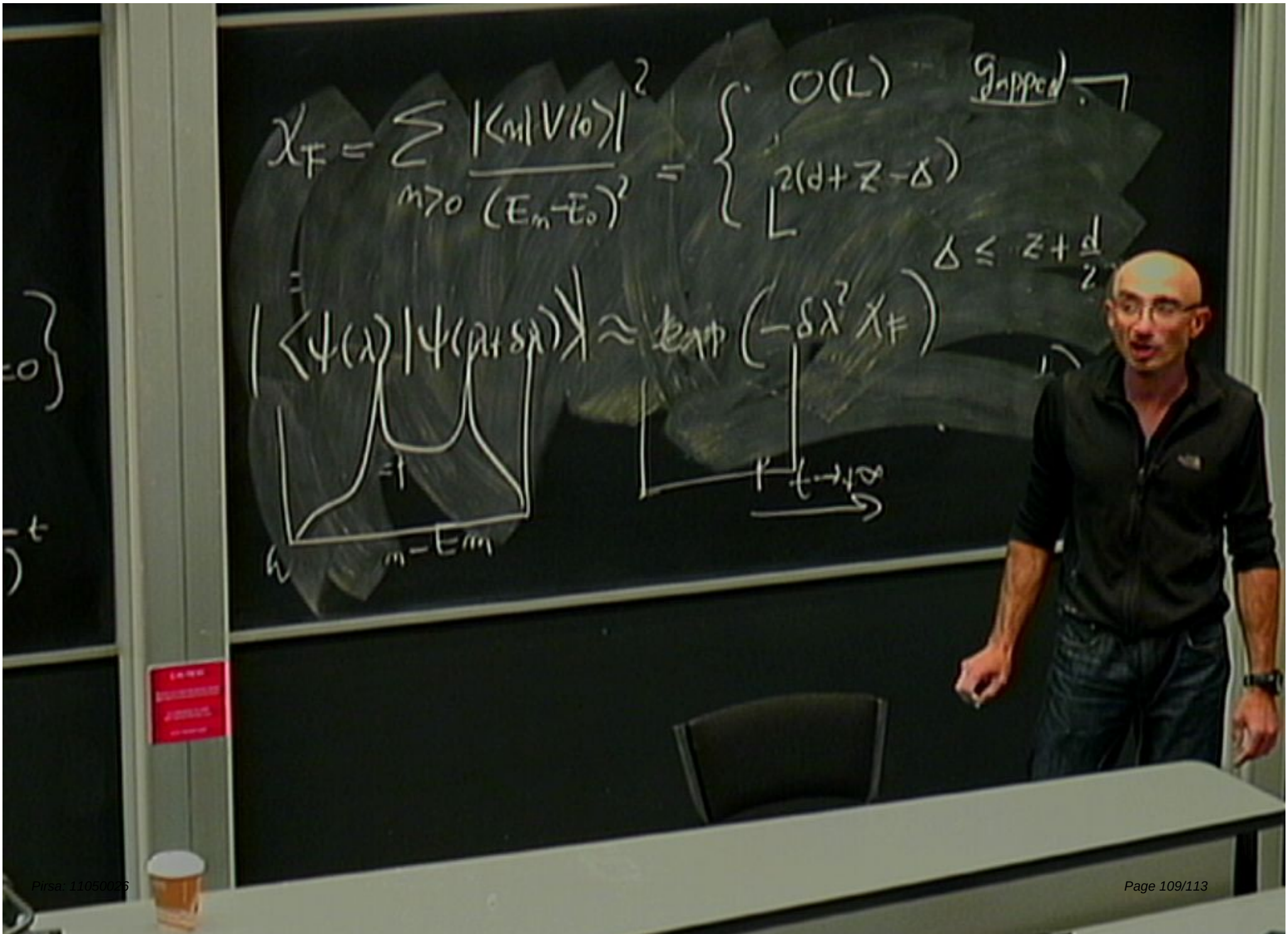
$$P_0 \approx \log\left(P_0^2 + 2\sum_{n>0} P_0 P_n \cos[(E_n - E_0)t]\right)$$

$$\log P_0^2 + 2\sum_{n>0} P_n \cos[(E_n - E_0)t]$$

$$X_n(t) \equiv X_n(\alpha)$$

$$\sum_{n>0} P_n \leq \sum_{n>0} P_n \equiv X_F$$


$\chi_F = \sum_{m>0} \frac{|\langle m|V|0\rangle|^2}{(E_m - E_0)^2} = \begin{cases} O(L) & \text{gapped} \\ L^{2(d+z-\Delta)} & \end{cases}$
$\Delta \leq z + \frac{d}{2}$
$|\langle\psi(\lambda)|\psi(\lambda+\delta\lambda)\rangle| \approx \exp(-\delta\lambda^2 \chi_F)$
$m - E_m$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$m - E_m$
$t \to +\infty$

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$$\Delta \leq z + \frac{d}{2}$$

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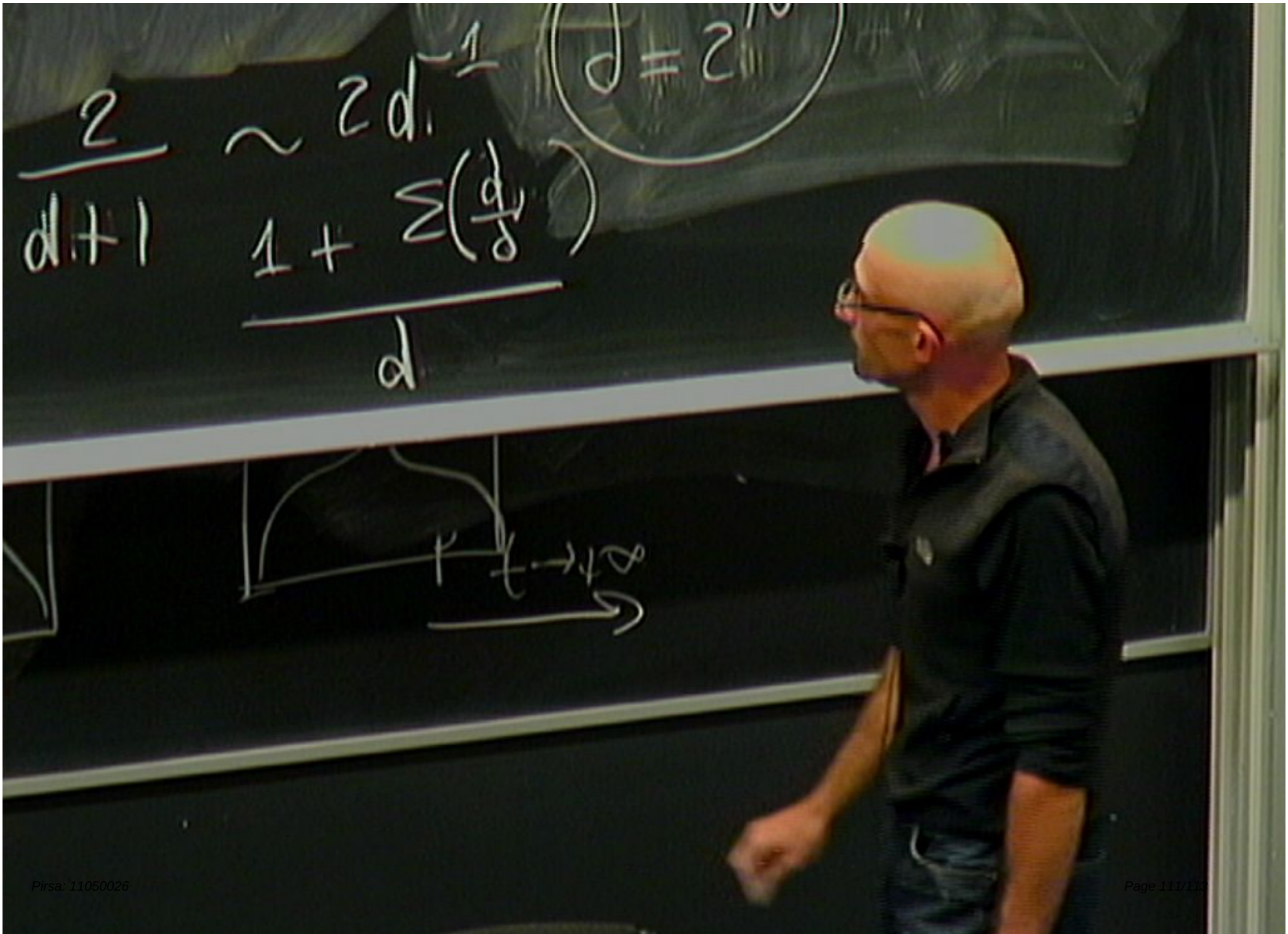
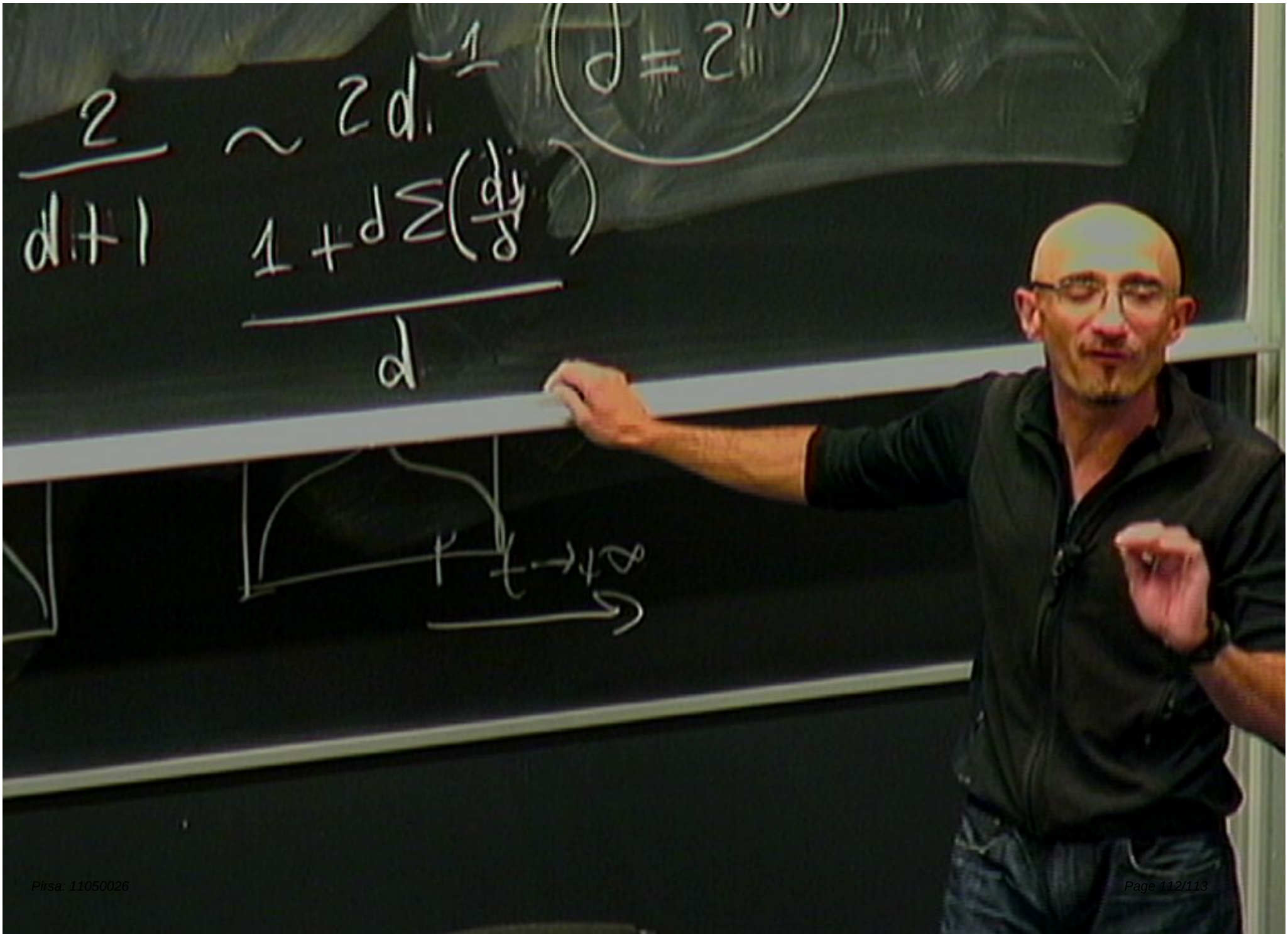

$\frac{2}{d+1} \sim 2d^{-1}$
$\frac{1+\Sigma(\frac{d}{\delta})}{d}$
$\delta = 2^{n}$
$t \to +\infty$


$\frac{2}{d+1} \sim 2d^{-1}$
$\frac{1+d\sum(\frac{d_j}{d})}{d}$
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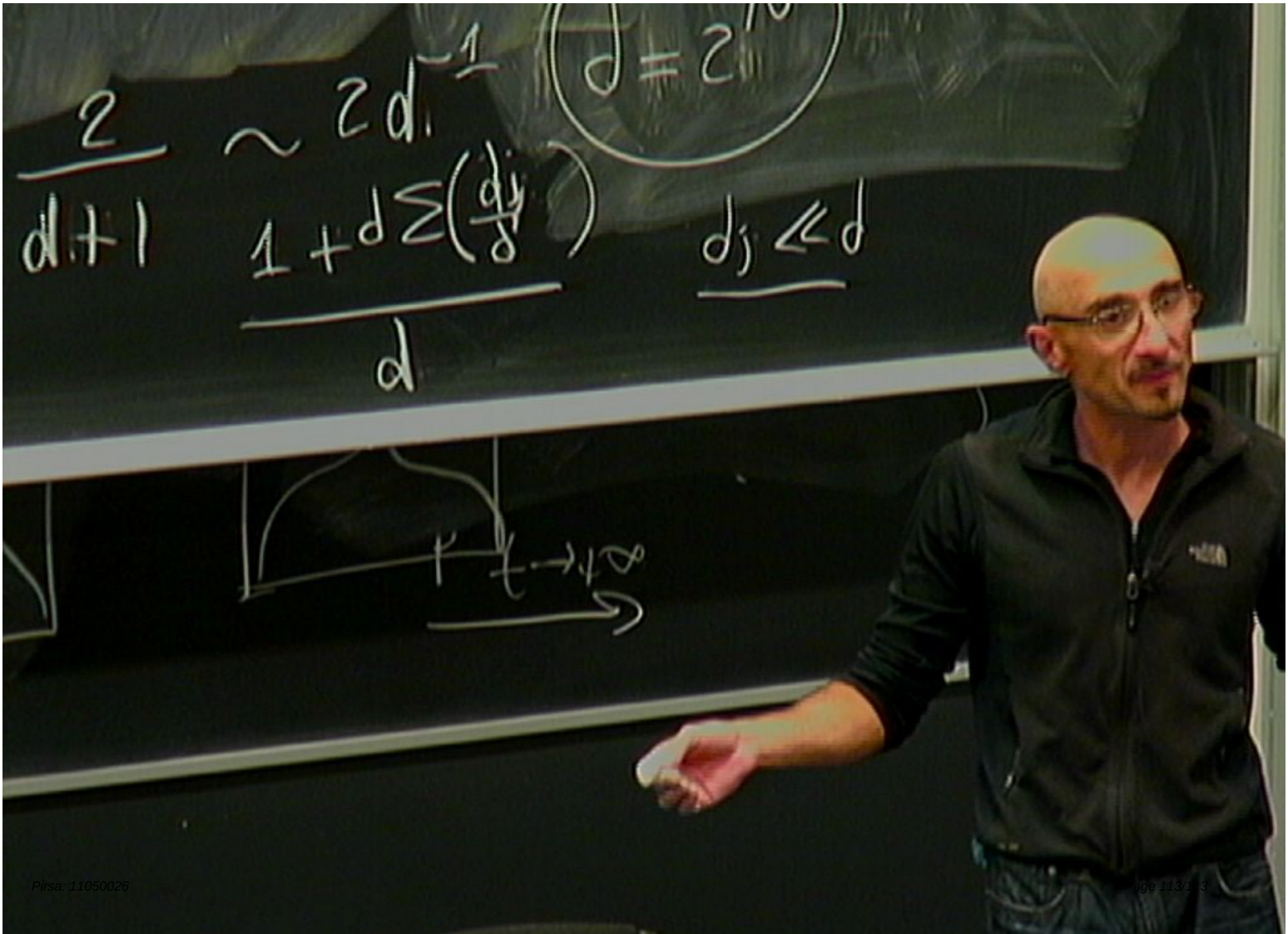
$\frac{2}{d+1} \sim 2d^{-1}$
$d = 2^{10}$
$\frac{1 + d\sum\left(\frac{d_j}{d}\right)}{d}$
$d_j \ll d$
$t \to +\infty$