Title: Effective Field Theory in Inflation

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Abstract: Though the observed CMB is at very low energy, it encodes ultra high-energy physics in spatial variations of the photon temperature and polarization fluctuations. This effect is believed to be dominated by the initial quantum state of the Universe. I will describe the first theoretical tools by which to construct such a state from fundamental physics. There are three specific observational effects this initial state will produce: a ringing signal in the power spectrum of quantum field fluctuations, an enfolded type of non-Gaussian fluctuations, and a calculable primordial gravitational wave background. We may soon be able to compare these predictions against experiment, allowing one to rule out classes of quantum gravity models. Now is the critical time to undertake such investigations, with a number of ongoing and planned experiments such as WMAP, Planck, and CMBPol poised to collect a wealth of precision data.

Effective Field Theory in Inflation

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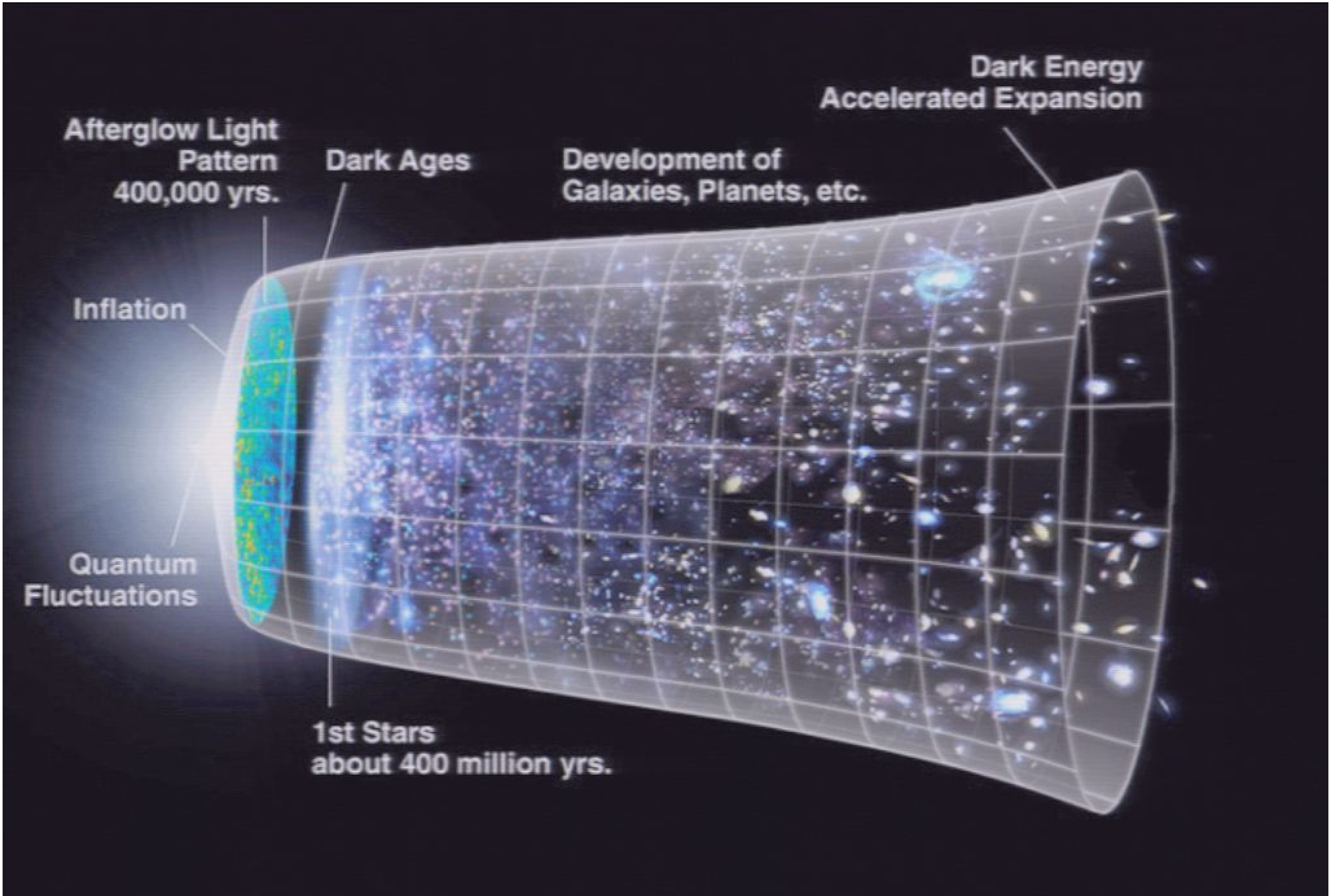
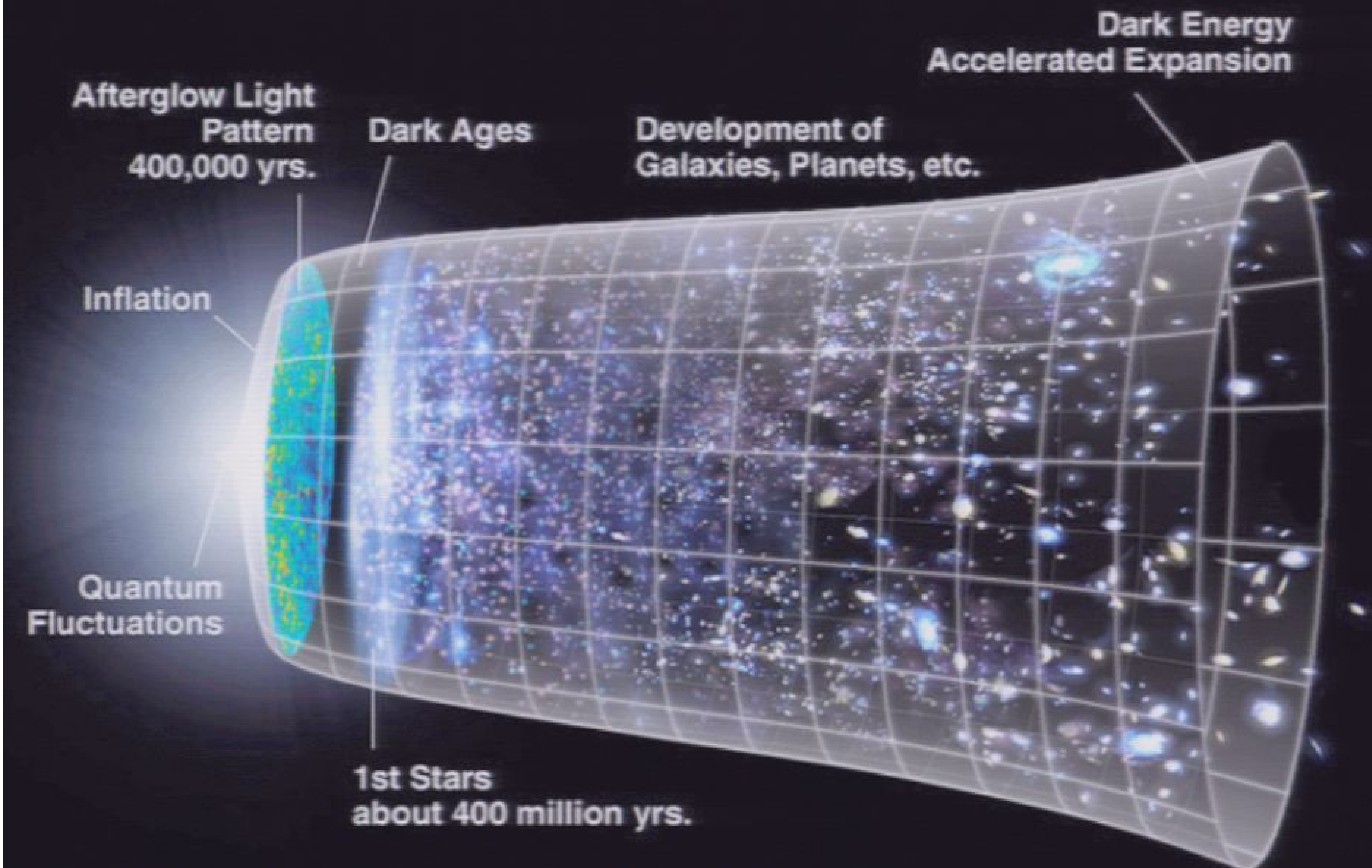
This Fall: APC-Paris

Collaborators: D. Baumann, P. D. Meerburg, J. P. v.d. Schaar, K. Schalm

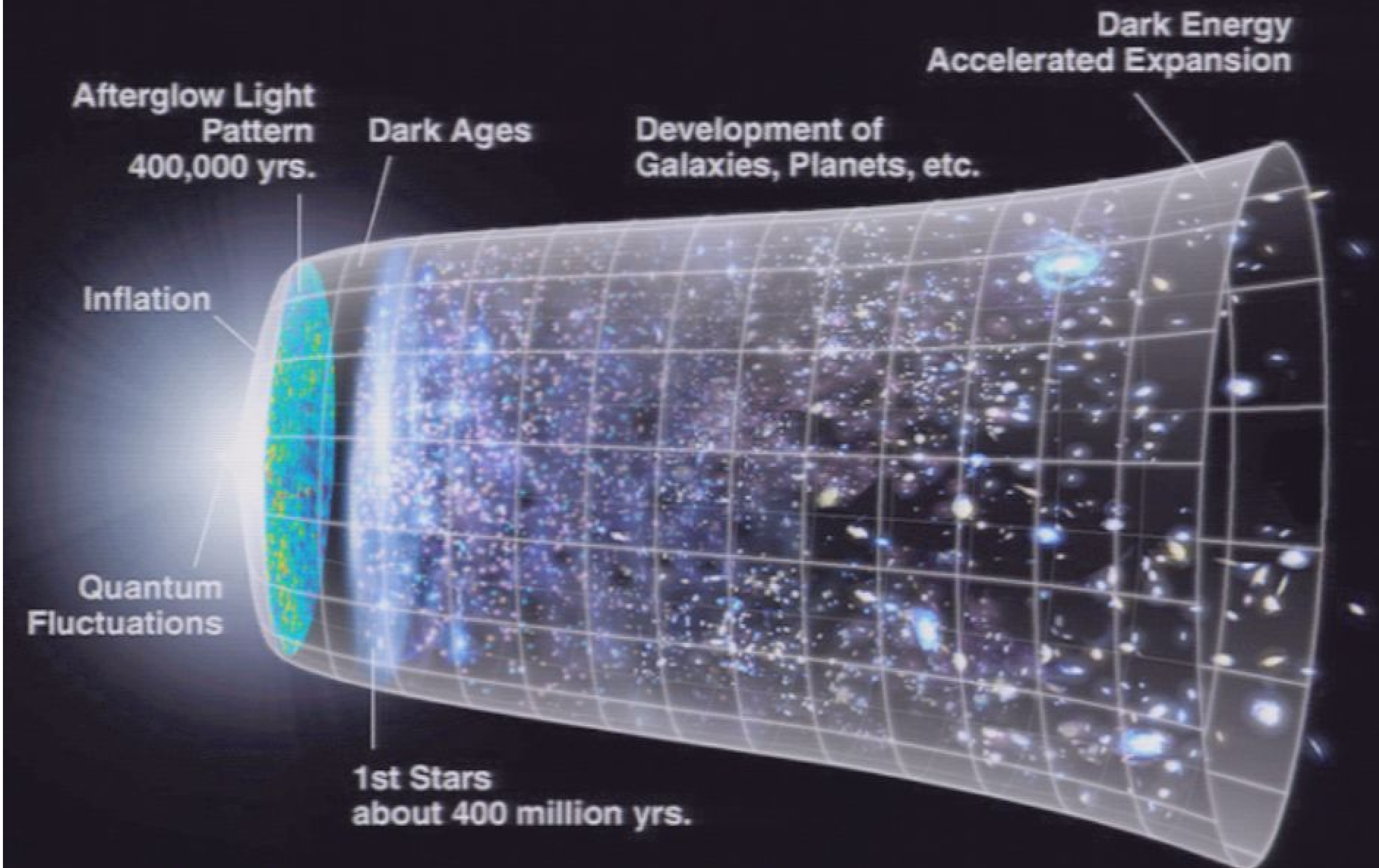
Perimeter Institute

May 31, 2011


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.



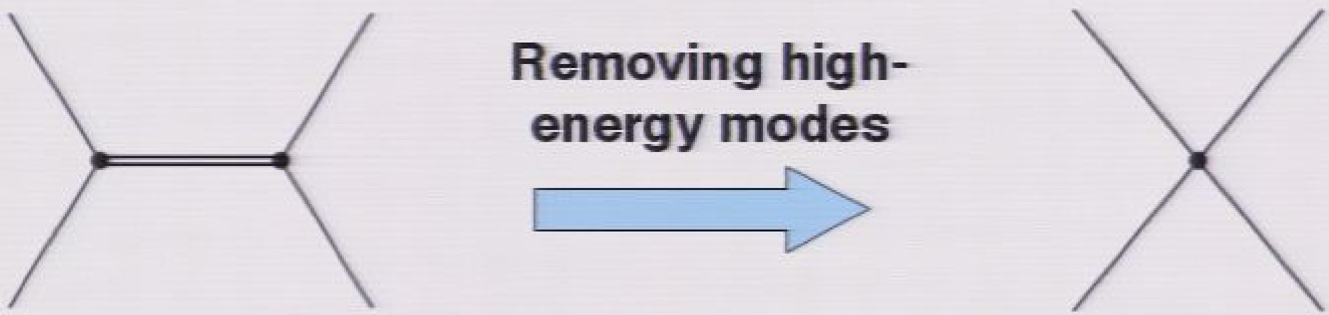
- Inflation is phenomenologically successful, but...



- Inflation is phenomenologically successful, but...
- Ideally it should be embedded in a quantum theory of gravity, such as superstring theory

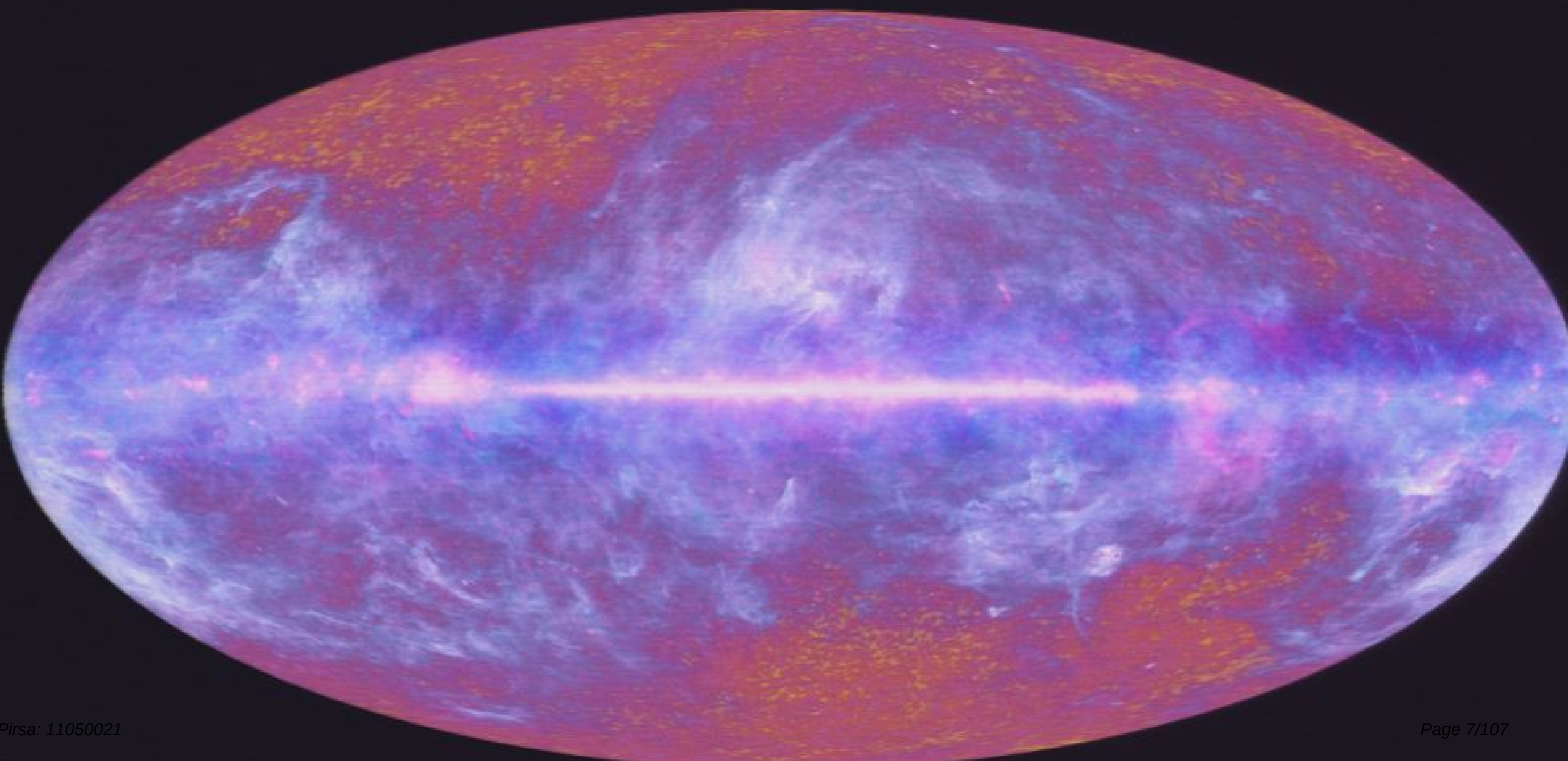
Effective Actions (in Cosmology)

- To test high-energy theories at low energy we rely upon Wilsonian effective actions:



- Unfortunately, standard techniques rely upon energy conservation, which is absent during cosmological expansion
- **To put inflation on a fundamental basis, we need to construct effective actions in a cosmological background.**

A New Hope: The Dawn of Precision Cosmology



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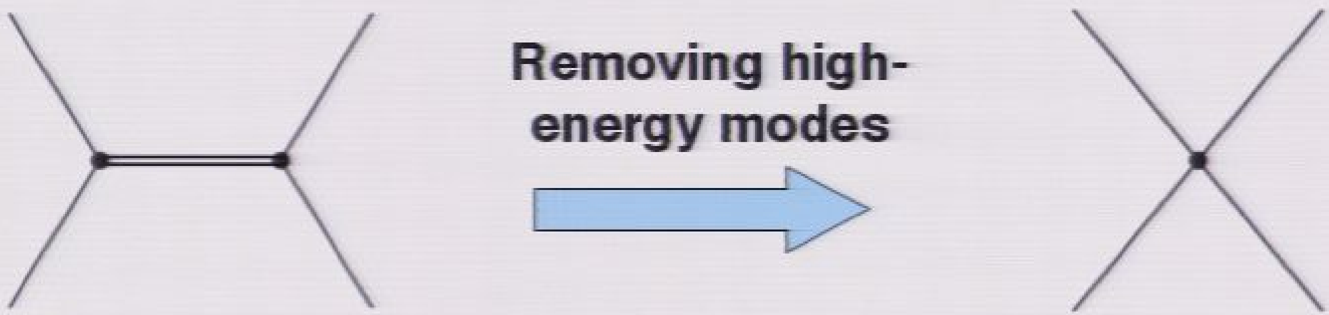

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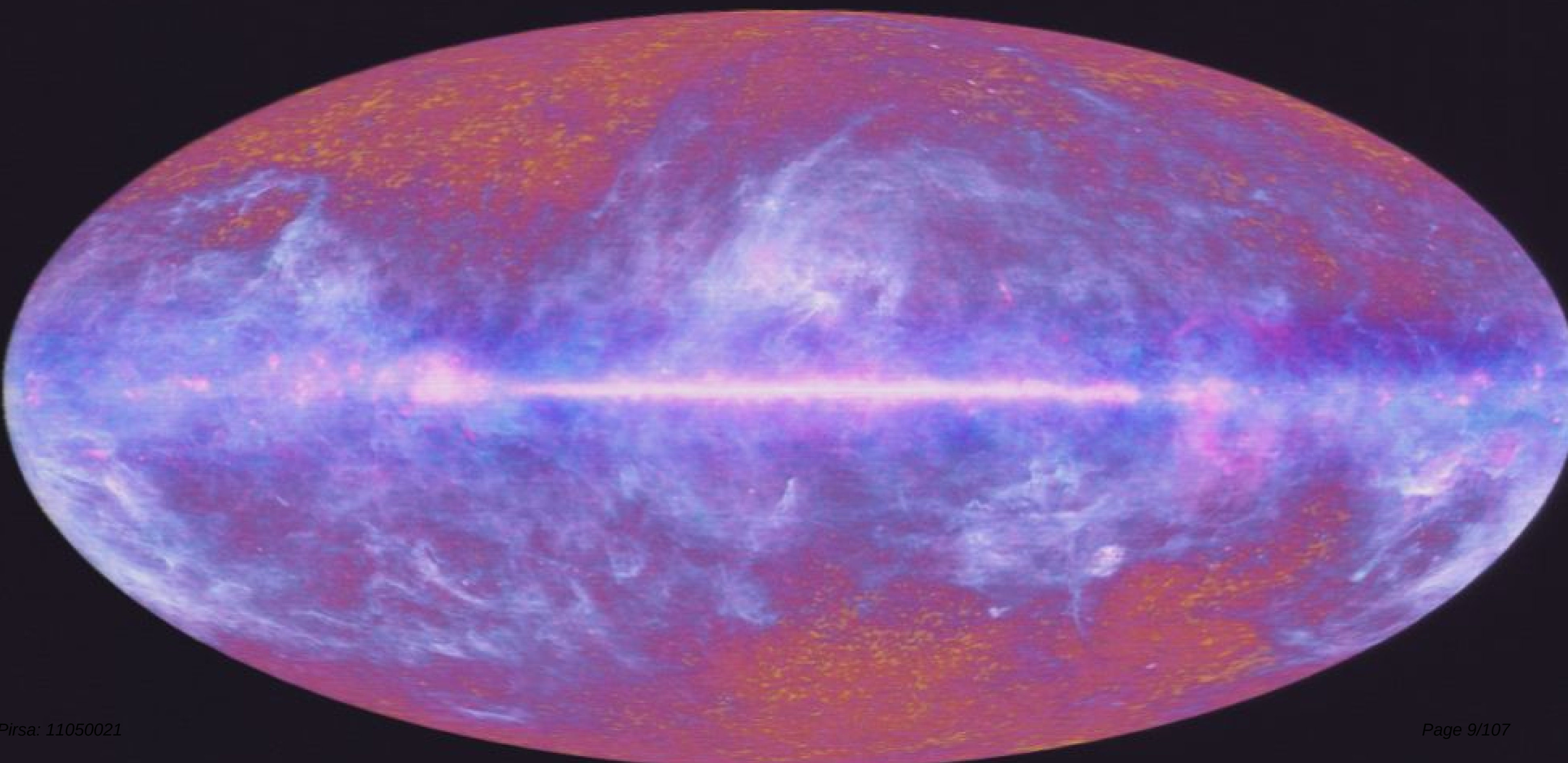


Outline

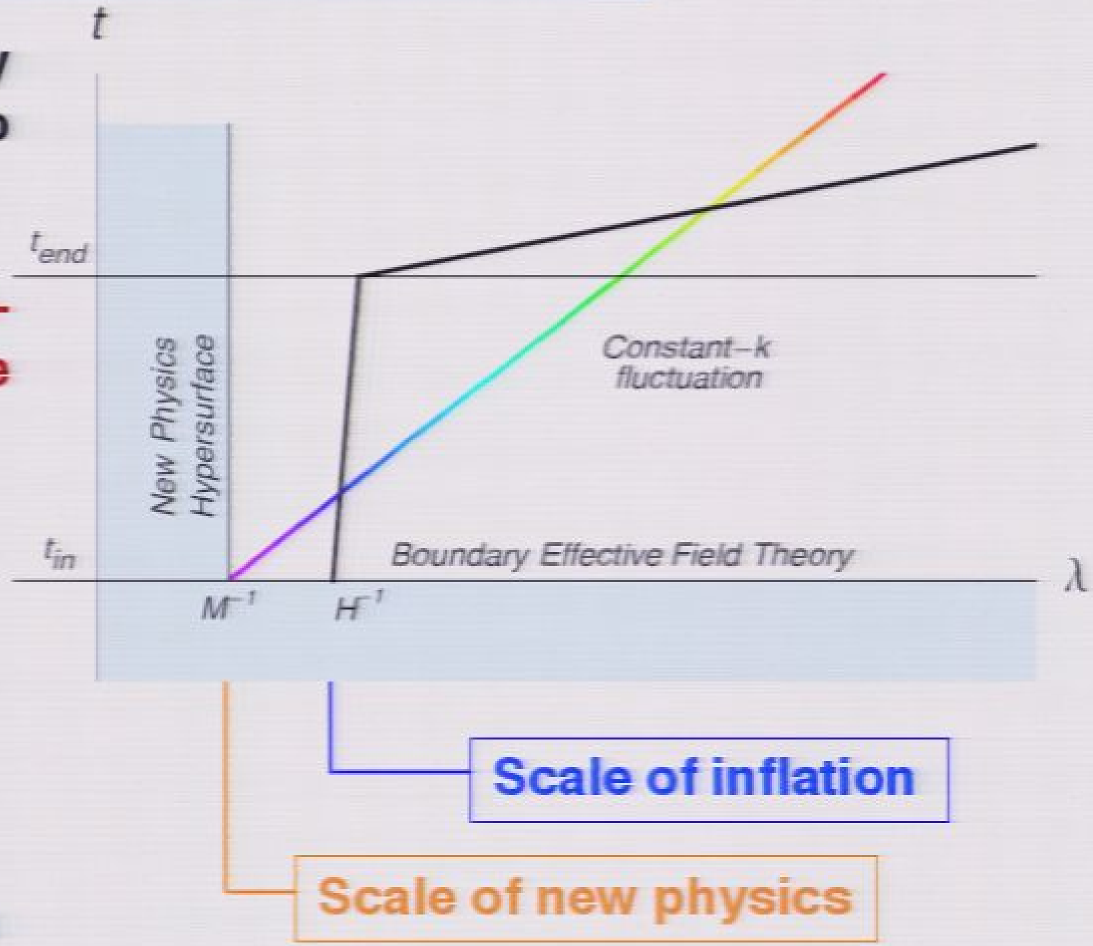
- The Opportunity of Cosmology
- Modified Vacua
- Power Spectrum Oscillations
- Non-Gaussianity
- CMB Polarization and Tensor Modes

The Opportunity of Cosmology: Sensitivity to High Energies

Observed CMB fluctuations today have low energy, but this is due to the cosmological redshifting.

They should be sensitive to high-energy physics, possibly even the Planck scale, as

$$\langle \mathcal{O} \rangle \sim \left(\frac{H}{M} \right)^n$$



Previous effective descriptions,

1. **New Physics Hypersurface,**
2. **Boundary Effective Field Theory,**
3. **Weinberg; Senatore & Zaldarriaga**

either lack a fundamental theory, rely upon energy conservation, or assume a particular vacuum state.

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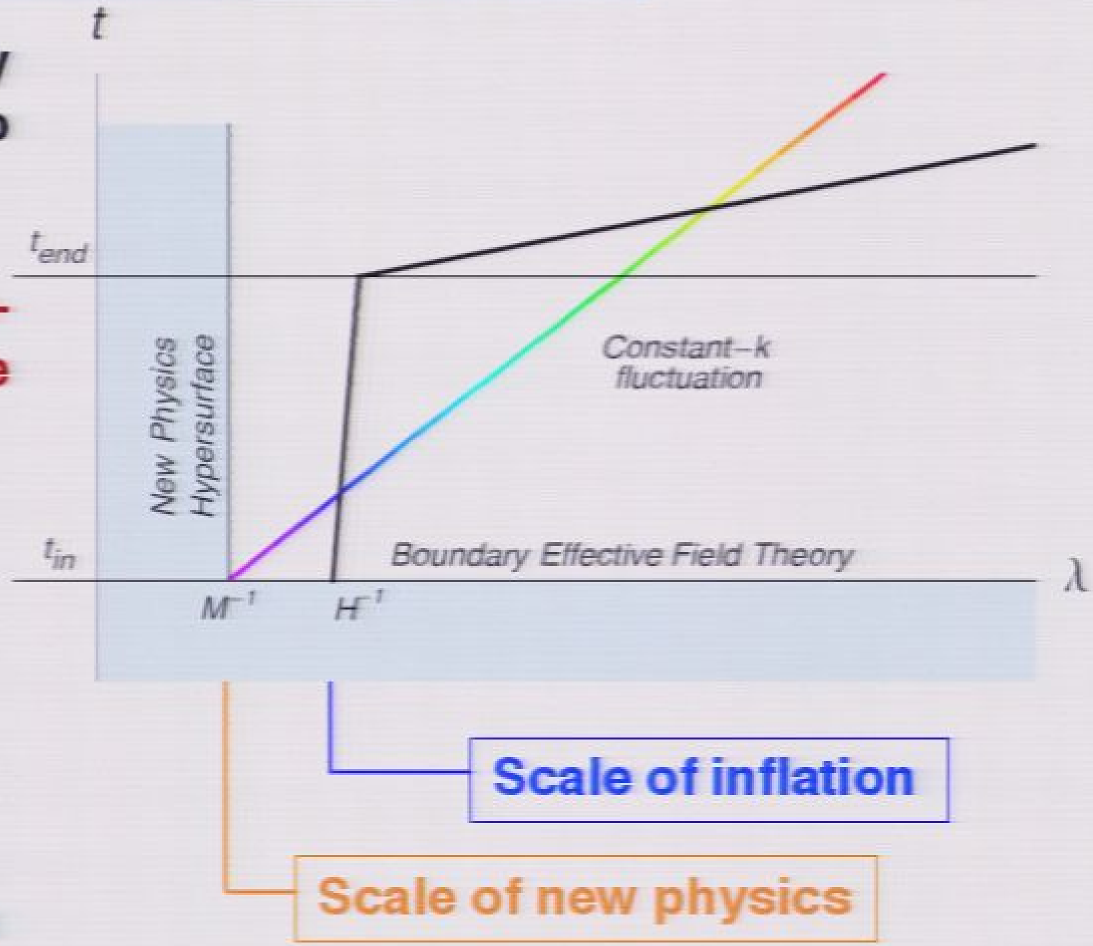

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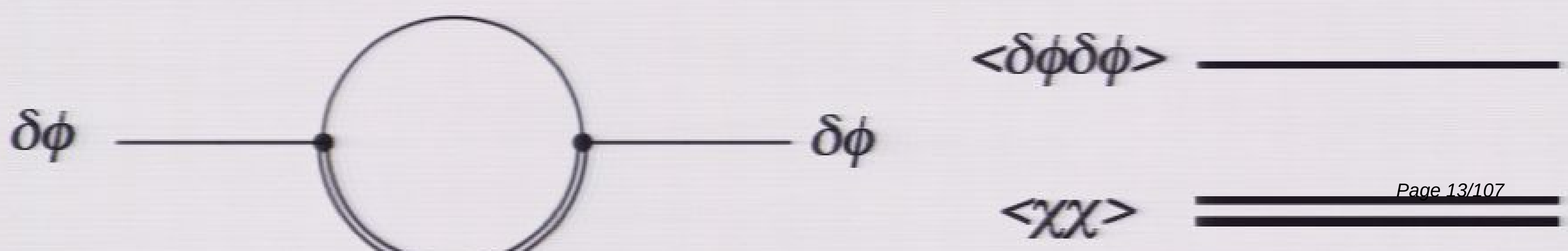
First Observable: The Primordial Power Spectrum

- The power spectrum is simply the 2-pt correlation function of inflaton field fluctuations:

$$P_s(k) = \lim_{t\to\infty} \frac{k^3}{2\pi^2}\langle \delta\phi_{\mathbf{k}}(t)\delta\phi_{-\mathbf{k}}(t)\rangle \quad = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1}$$

$$\text{WMAP7}: \quad A_s = (2.43 \pm 0.11)\times 10^{-9}, \quad n_s = 0.963 \pm 0.012$$

- (Naively) interpreting this as a propagator, we expect that it encodes high-energy physics from e.g. via virtual heavy χ-exchange:



Inflaton Field Effective Action

- **Consider the effective action for ϕ:**

$$S_{eff}[\phi] = \int d^4p\ \phi(p)\phi(-p)\{p^2/2 + H^2/2 + c_0 H^2(H^2/M^2) + c_1 p^2(H^2/M^2)\ + \ldots$$

- **The freezeout scale is $p=H$, thus the 2-pt function is**

$$\langle\phi(p)\phi(-p)\rangle|_{p=H} = H^2 + c_0 H^2(H^2/M^2) + c_1 H^2(H^2/M^2)$$

- **Only even powers of p are allowed in S_{eff}, so we have an expansion in $(H/M)^2$.**

  Which is disastrous, since $H/M \sim 0.01$

  (Brandenberger, Burgess, Cline, Danielsson, Easther, Greene, Lemieux, Kaloper, Kinney, Kleban, Lawrence, Martin, Schalm, Shenker, Shiu, v.d. Schaar, Susskind)

A Possible Solution: Vacuum State Modification

- Fortunately, there appears to be a loophole (Easther, Greene, Kinney, v.d. Schaar, Schalm, Shiu).
- Note that time-localized ('boundary') terms are one energy-dimension lower, and thus would scale only as *H/M*:

$$S_{\text{boundary}} = \int d^4x \sqrt{g}\; m\phi^2 \delta(t - t_c).$$

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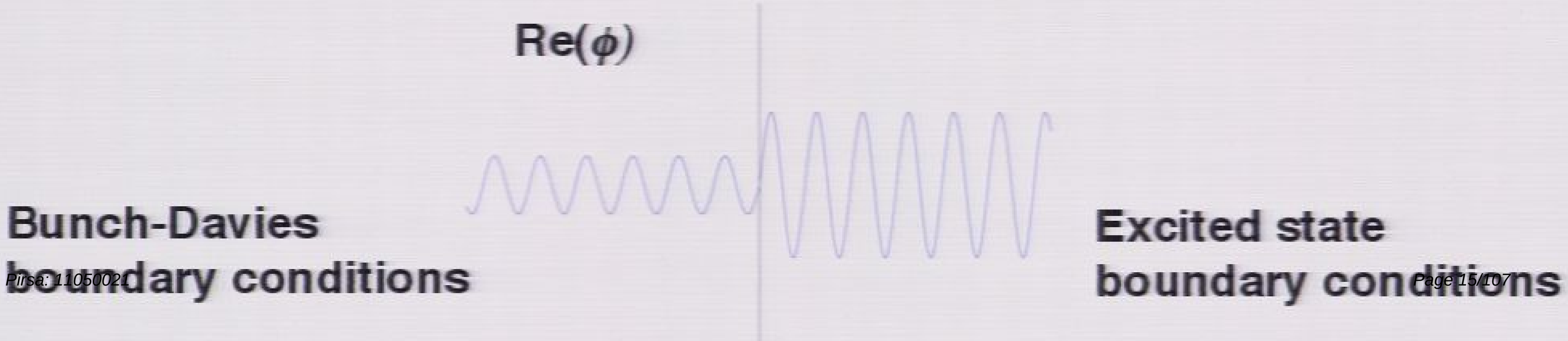

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- In the Hamiltonian description, the Bunch-Davies vacuum is simply the familiar condition

$$a_{\mathbf{k}}|0\rangle = 0,$$

which now becomes generalized to a squeezed coherent state :

$$\left(a_{\mathbf{k}} + \beta_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}}\right)|\beta_{\mathbf{k}}\rangle = 0, \qquad |\beta_{\mathbf{k}}\rangle = \mathcal{N} \exp\left[-\beta_{\mathbf{k}} a^{\dagger}_{-\mathbf{k}} a^{\dagger}_{\mathbf{k}}\right]|0\rangle.$$

- Such a modified state has a very characteristic signature!

Effect of Vacuum Choice on Power Spectrum

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\begin{aligned}
P_\varphi^\beta(k) &= \frac{k^3}{2\pi^2}\langle\beta_{\mathbf{k}}|\varphi_{\mathbf{k}}(0)\varphi_{-\mathbf{k}}(0)|\beta_{\mathbf{k}}\rangle \\
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\end{aligned}
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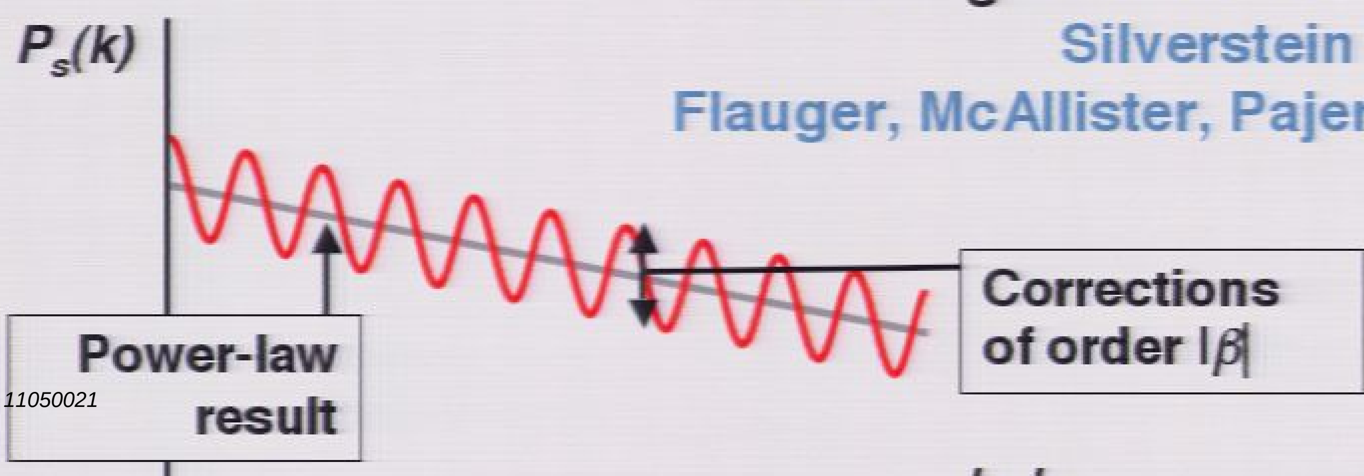
- **These 'wiggles' are a generic, model-independent feature of quantum gravity*, with all new physics encoded in β.**

•And e.g. axion monodromy inflation by
Silverstein and Westphal '08;
Flauger, McAllister, Pajer, Westphal, Xu '09



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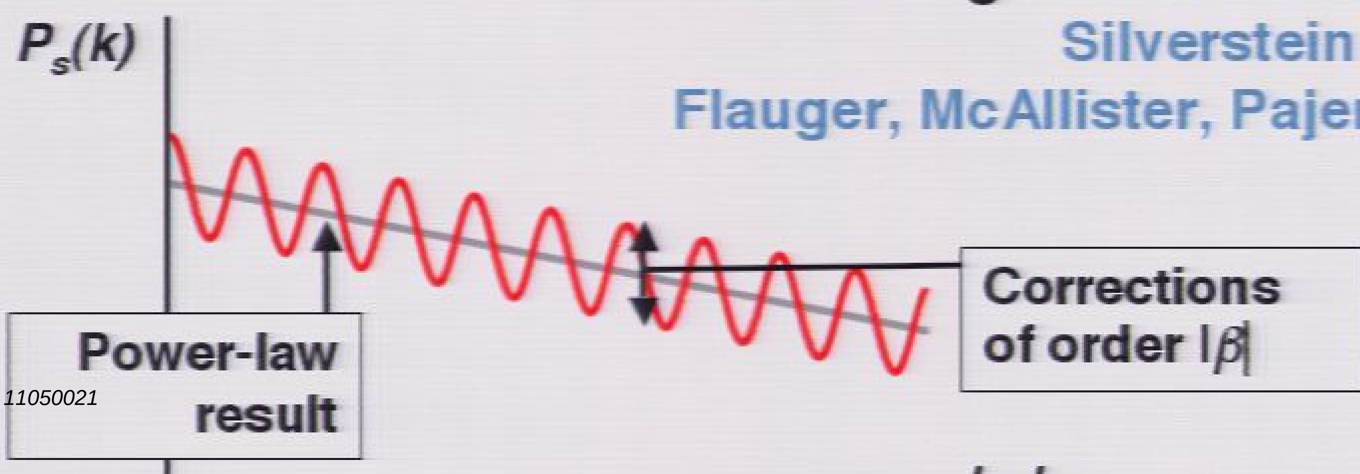

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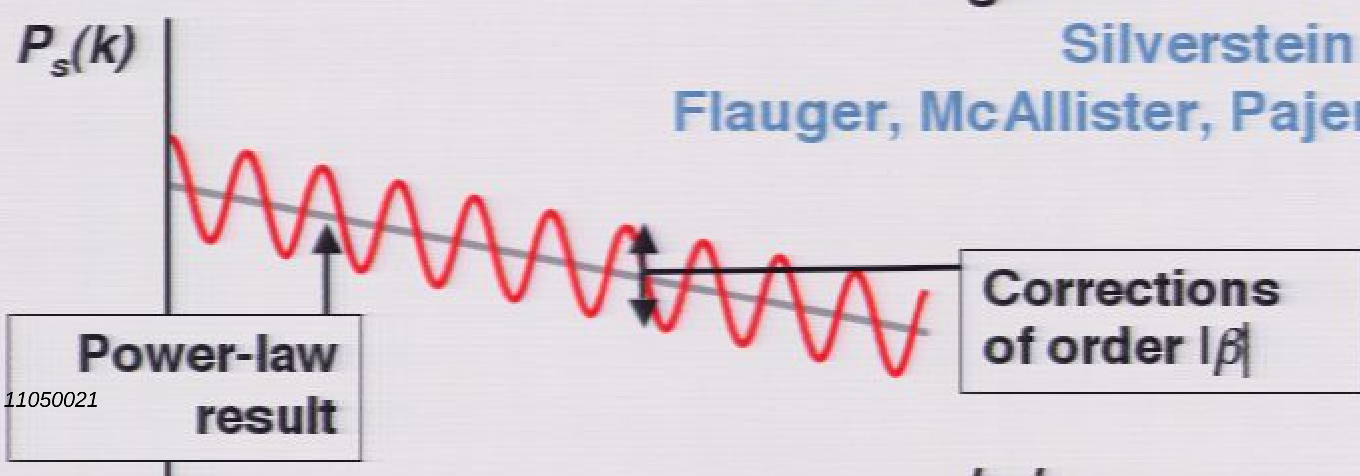

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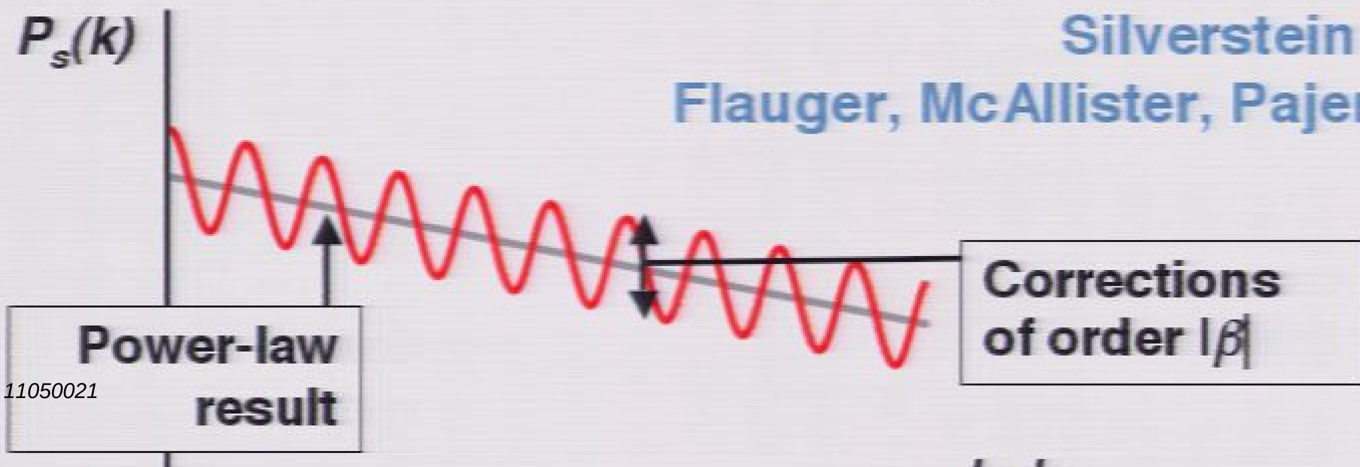

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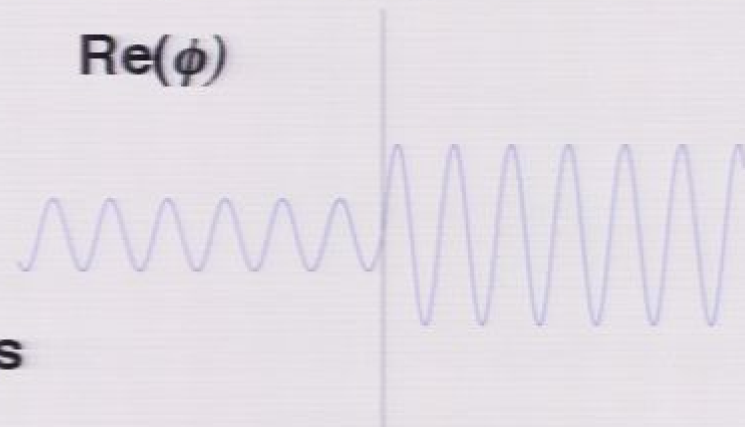

Bunch-Davies boundary conditions

Excited state boundary conditions

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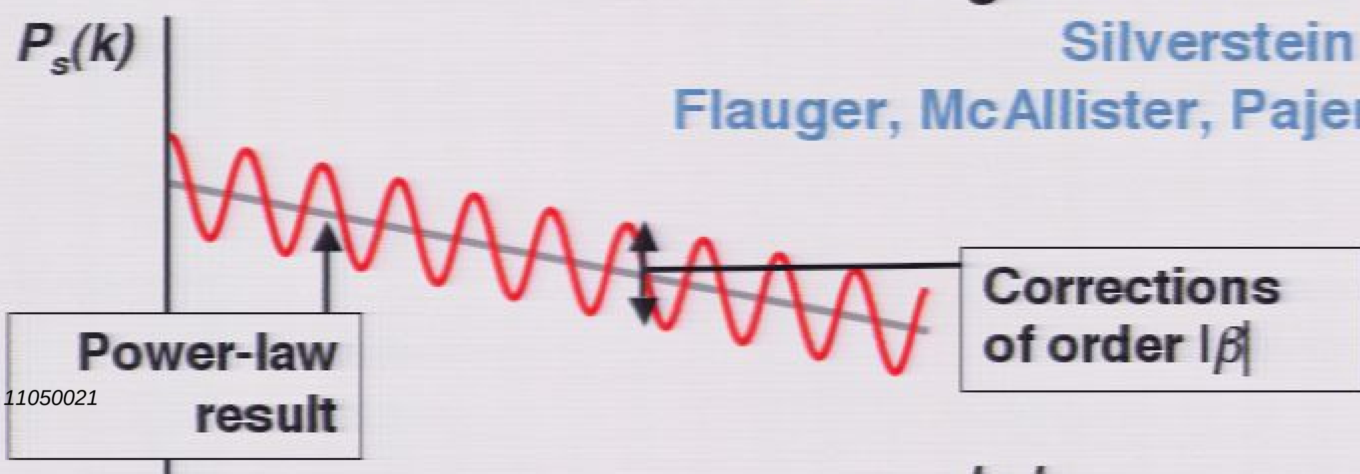

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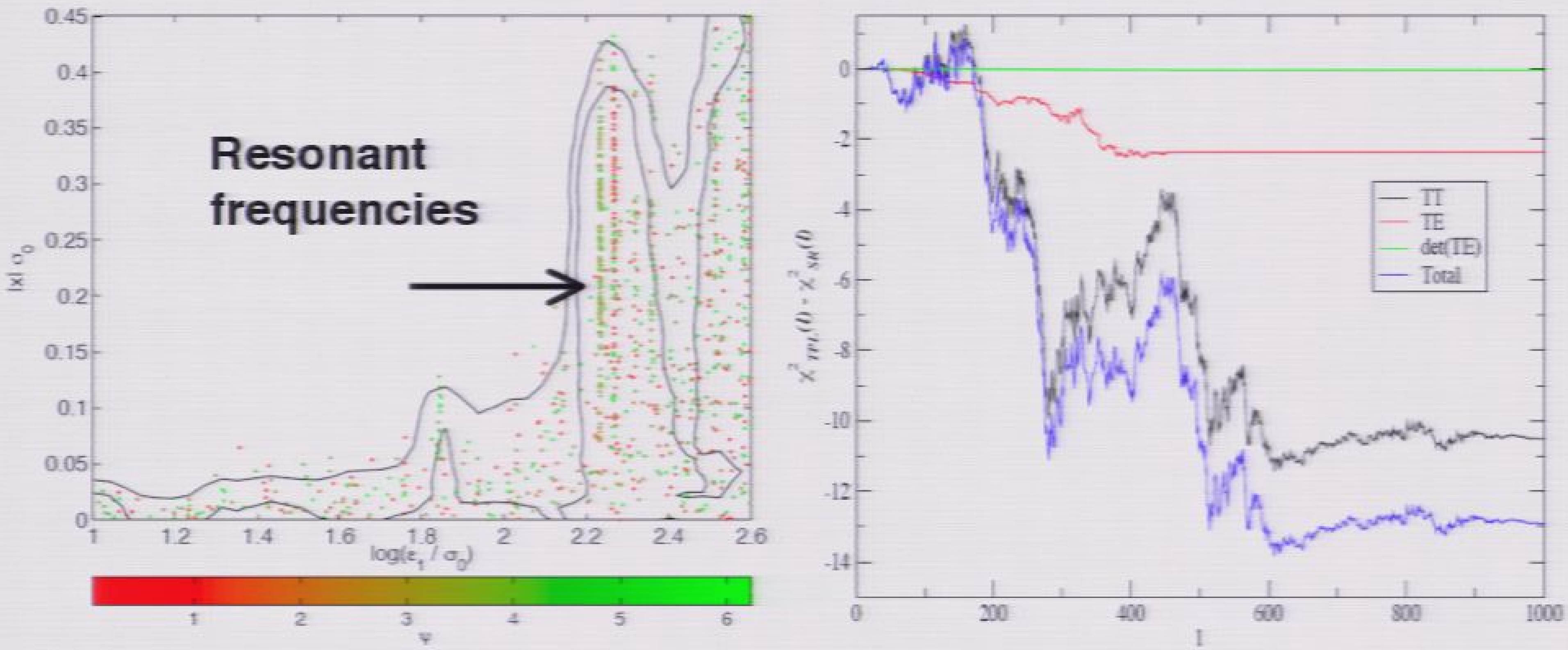
TP physics controlled by these parameters

Parametrizing TP Physics



- They found suggestions of such oscillations
- In the fortunate event of detection, what is the theoretical implication?

Martin and Ringeval '06

Effective Action Construction

- We (MGJ, Schalm '10) recently developed the procedure to construct the effective action representing high-energy physics.
- Begin with inflating system,

$$S_{\rm inf}[\phi] = -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

and add (for example) Yukawa interactions to a heavy field χ :

$$S_{\rm new}[\varphi,\chi] \;=\; -\int d^4x\sqrt{g}\left[\frac{1}{2}(\partial\chi)^2 + \frac{1}{2}M^2\chi^2 + \frac{g}{2}\varphi^2\chi\right]$$

- The power spectrum can then be computed using the in-in formalism:

$$P_\varphi(k) \;=\; \lim_{t\to\infty}\frac{k^3}{2\pi^2}\langle 0(t_0)|e^{i\int_{t_0}^t dt'\mathcal{H}(t')}|\varphi_{\mathbf{k}}(t)|^2 e^{-i\int_{t_0}^t dt''\mathcal{H}(t'')}|0(t_0)\rangle$$

- Note that this can be interpreted as an in-out correlation using

$$\mathcal{S} \equiv S[\varphi_+,\chi_+] - S[\varphi_-,\chi_-]$$

Effective Action Construction

- This suggests we should transform into the new 'Keldysh' field basis given by

$$\bar{\varphi} \equiv (\varphi_+ + \varphi_-)/2, \qquad \Phi \equiv \varphi_+ - \varphi_-,$$
$$\bar{\chi} \equiv (\chi_+ + \chi_-)/2, \qquad \mathrm{X} \equiv \chi_+ - \chi_-$$

- In this basis the action is now

$$\mathcal{S}[\bar{\varphi}, \Phi, \bar{\chi}, \mathrm{X}] = -\int d^4x \sqrt{g}\,\Big[\partial\bar{\varphi}\partial\Phi + \partial\bar{\chi}\partial\mathrm{X} + M^2\bar{\chi}\mathrm{X} + g\bar{\chi}\bar{\varphi}\Phi + \frac{g}{2}\mathrm{X}\left(\bar{\varphi}^2 + \frac{\Phi^2}{4}\right)\Big].$$

- The free field solutions are

$$U_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}(1 - ik\tau)\,e^{-ik\tau}\,,\quad V_{\mathbf{k}}(\tau) \approx -\frac{H\tau \exp\left[-i\int^{\tau} d\tau' \sqrt{k^2 + \frac{M^2}{H^2\tau'^2}}\right]}{\sqrt{2}\left(k^2 + \frac{M^2}{H^2\tau^2}\right)^{1/4}}$$

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Feynman Rules in Keldysh Basis

- The correlations can now be evaluated using these:

$$G^R_{\mathbf{k}}(\tau_1,\tau_2) \equiv i\langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\Phi_{-\mathbf{k}}(\tau_2)\rangle = -2\theta(\tau_1-\tau_2)\,\mathrm{Im}\,[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)]\,,$$

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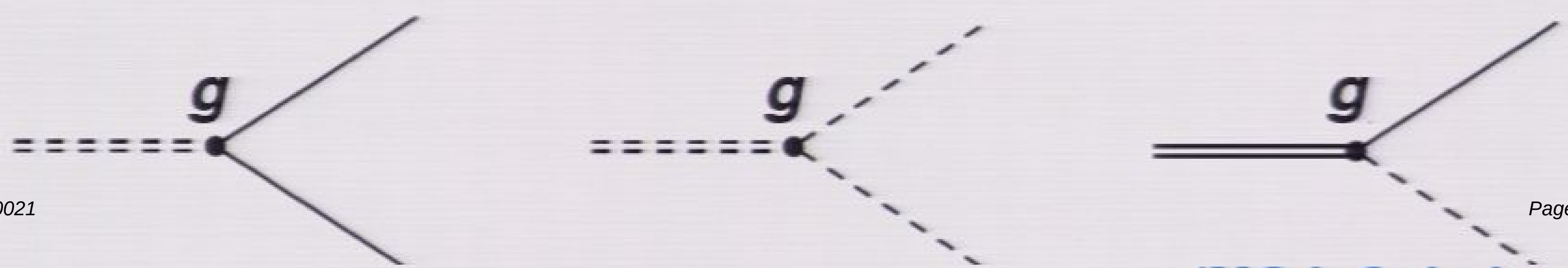

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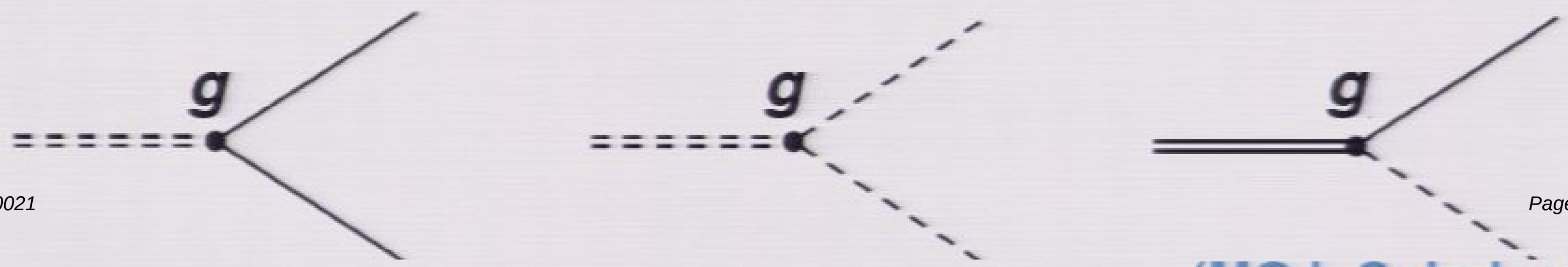
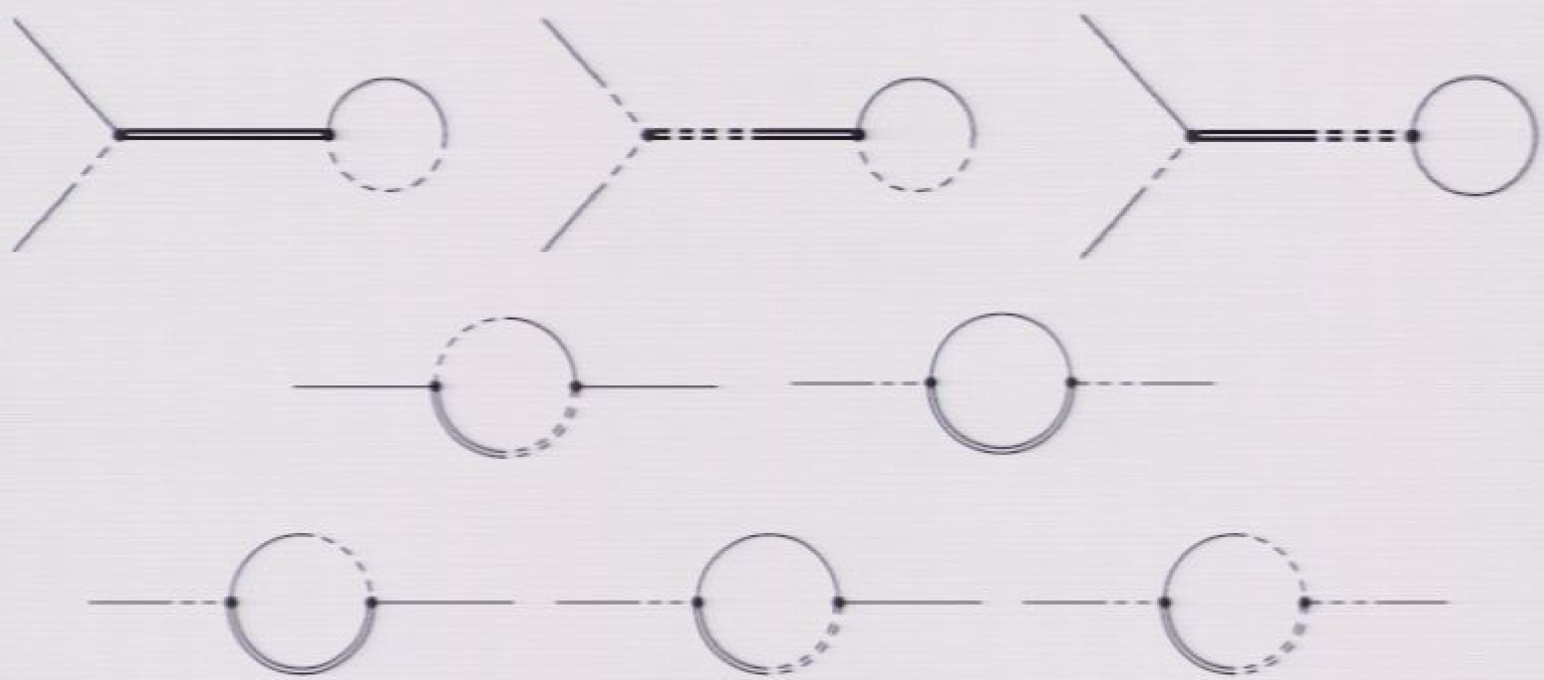


Power Spectrum Corrections

- 2-pt correlation can then be computed using normal methods, producing eight Feynman diagrams:



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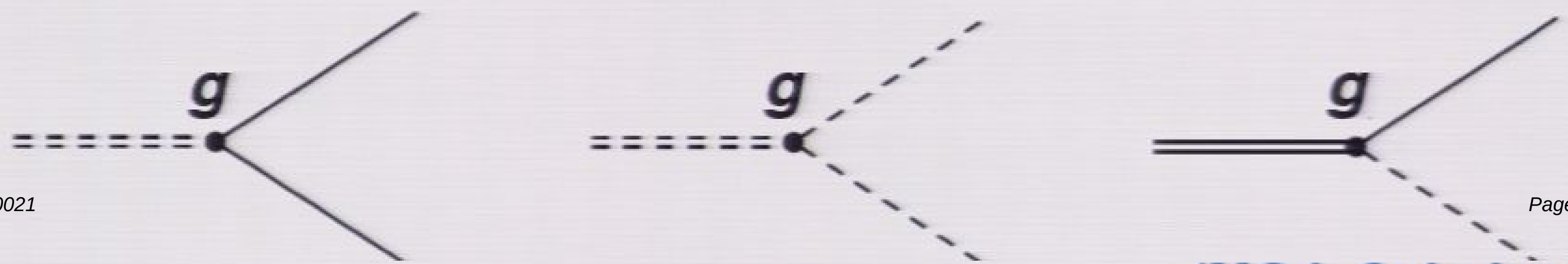

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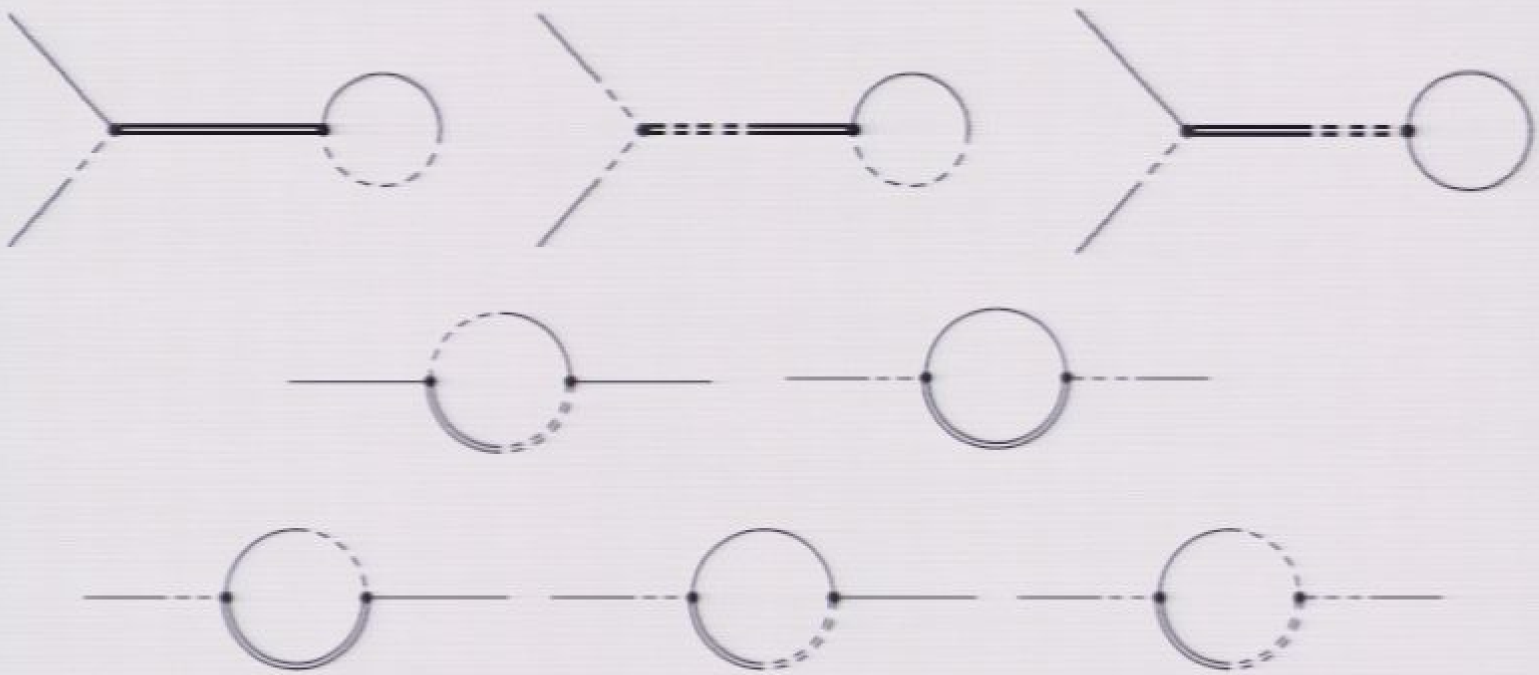

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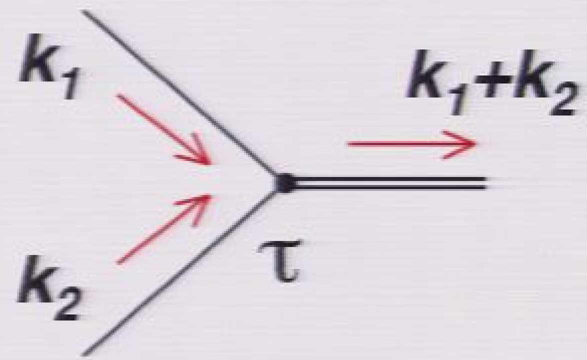

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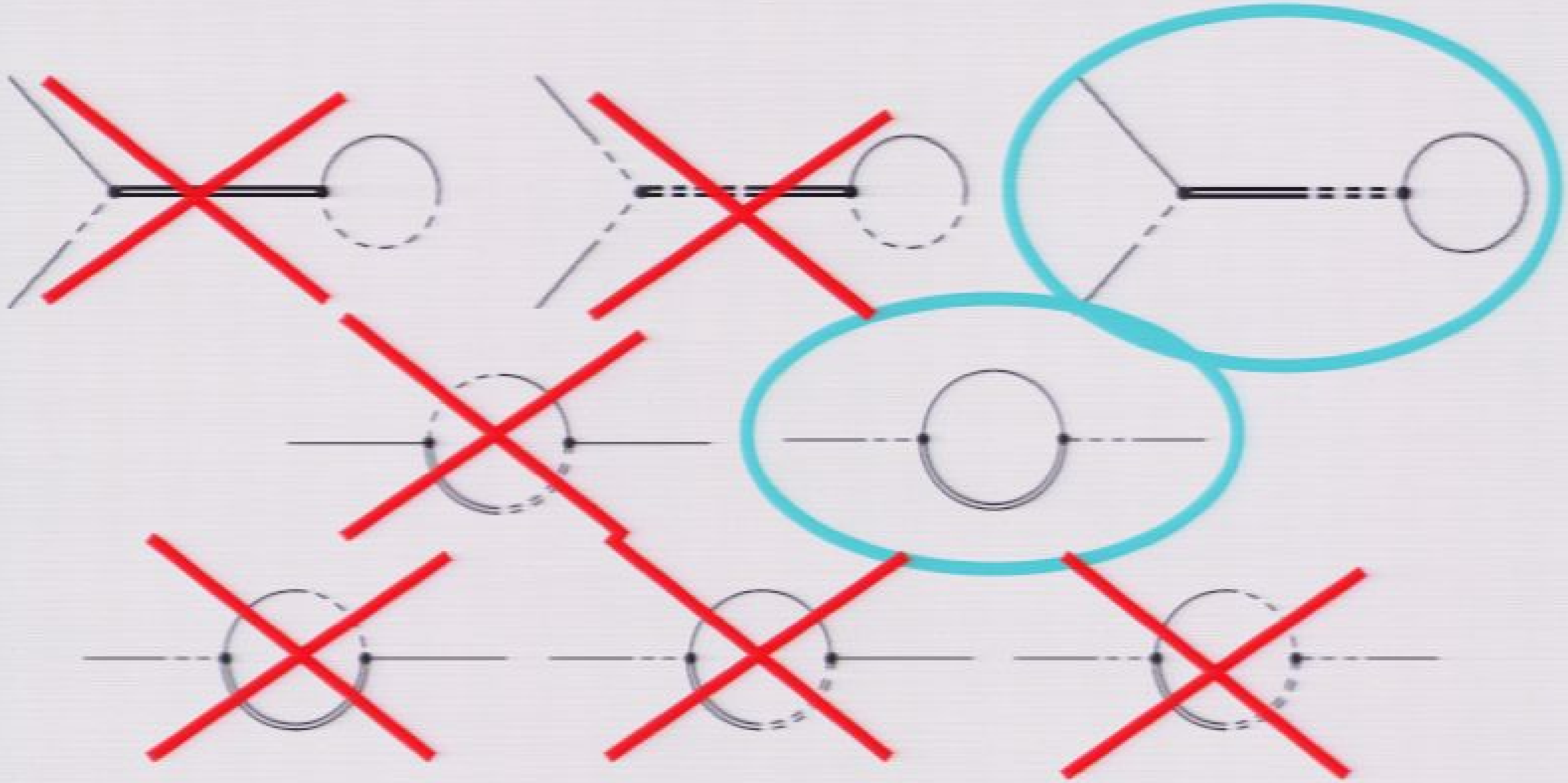

an Rules in Keldysh Basis ction

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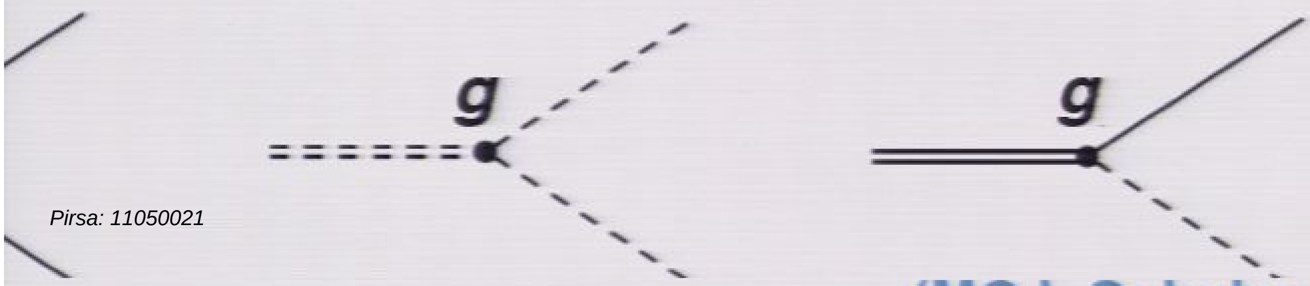

$$\frac{l\tau'\sqrt{k^2+\frac{M^2}{H^2\tau'^2}}\Big]}{\frac{M^2}{H^2\tau^2}\Big)^{1/4}}$$

Effective Action Construction

- This suggests we should transform into the new 'Keldysh' field basis given by

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rrections ectrum Corrections

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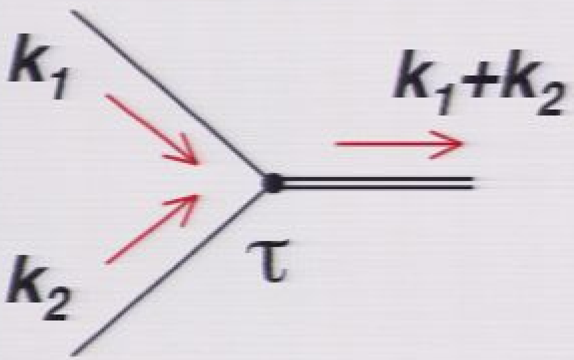

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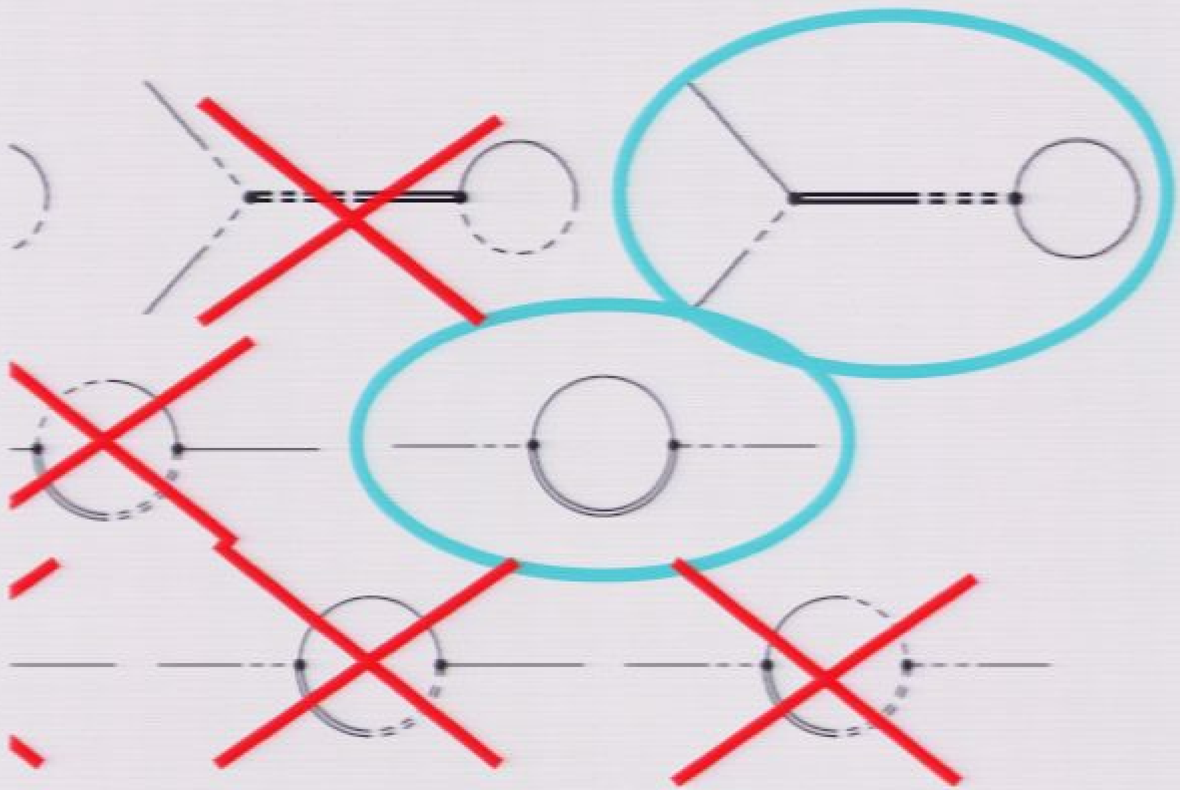

(MG L Schelm (10)

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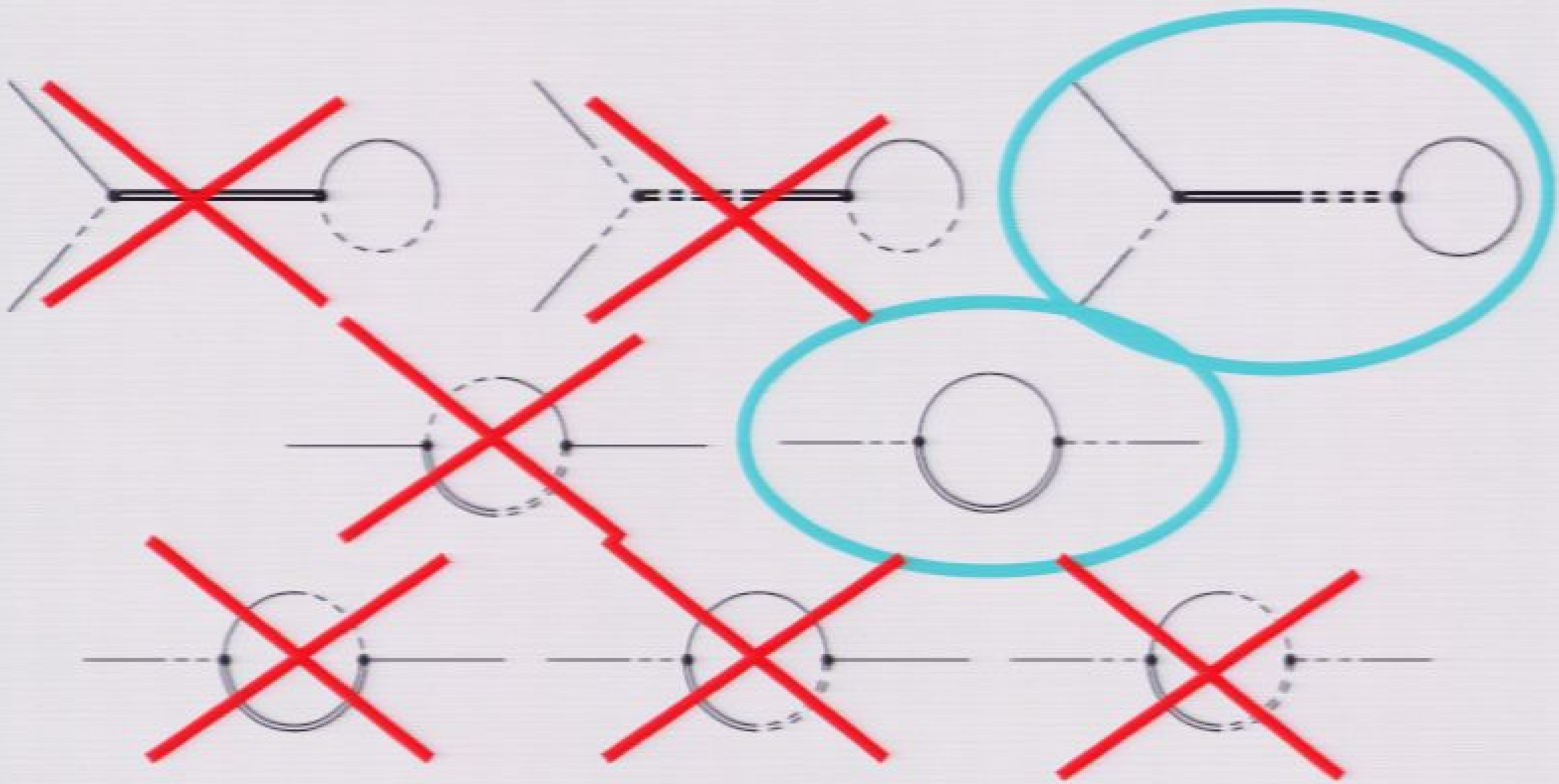

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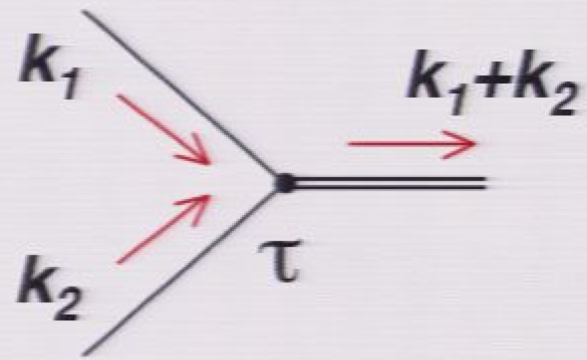

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- Consider this diagram, with loop momentum *q*:

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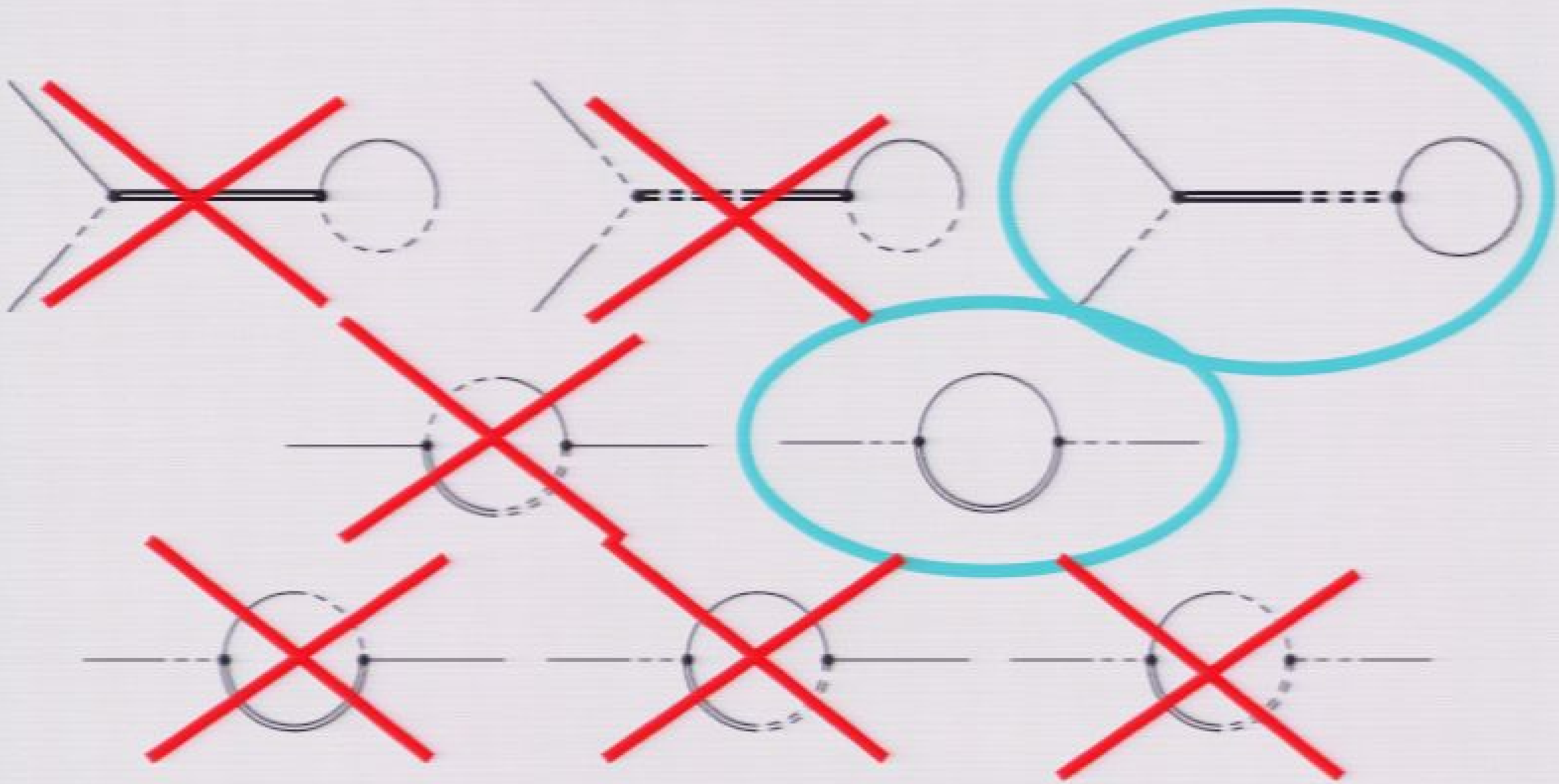

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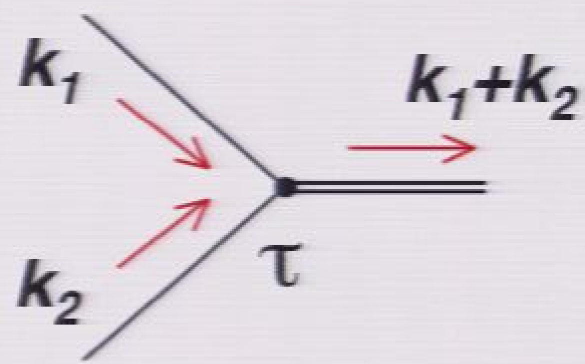

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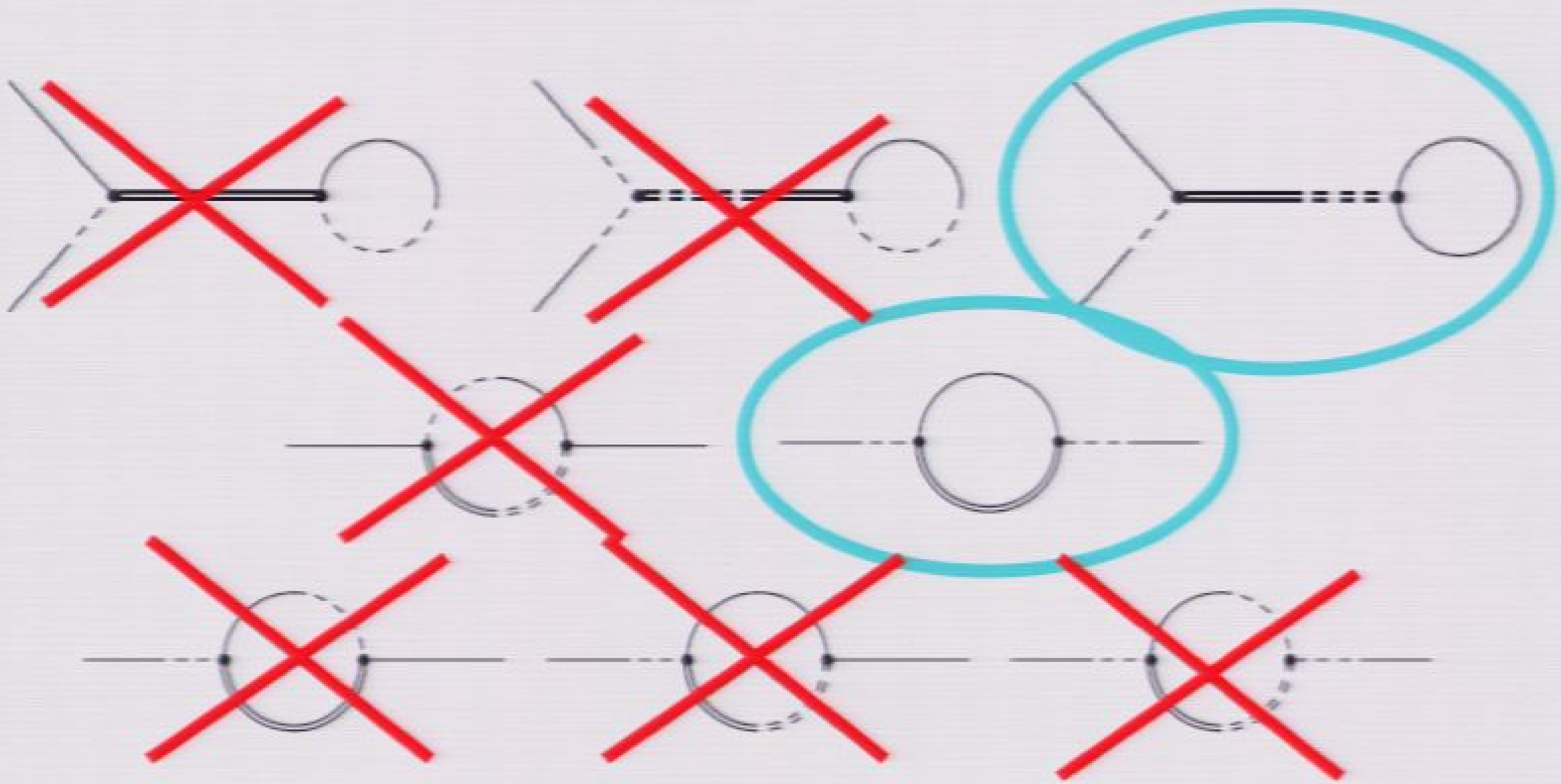

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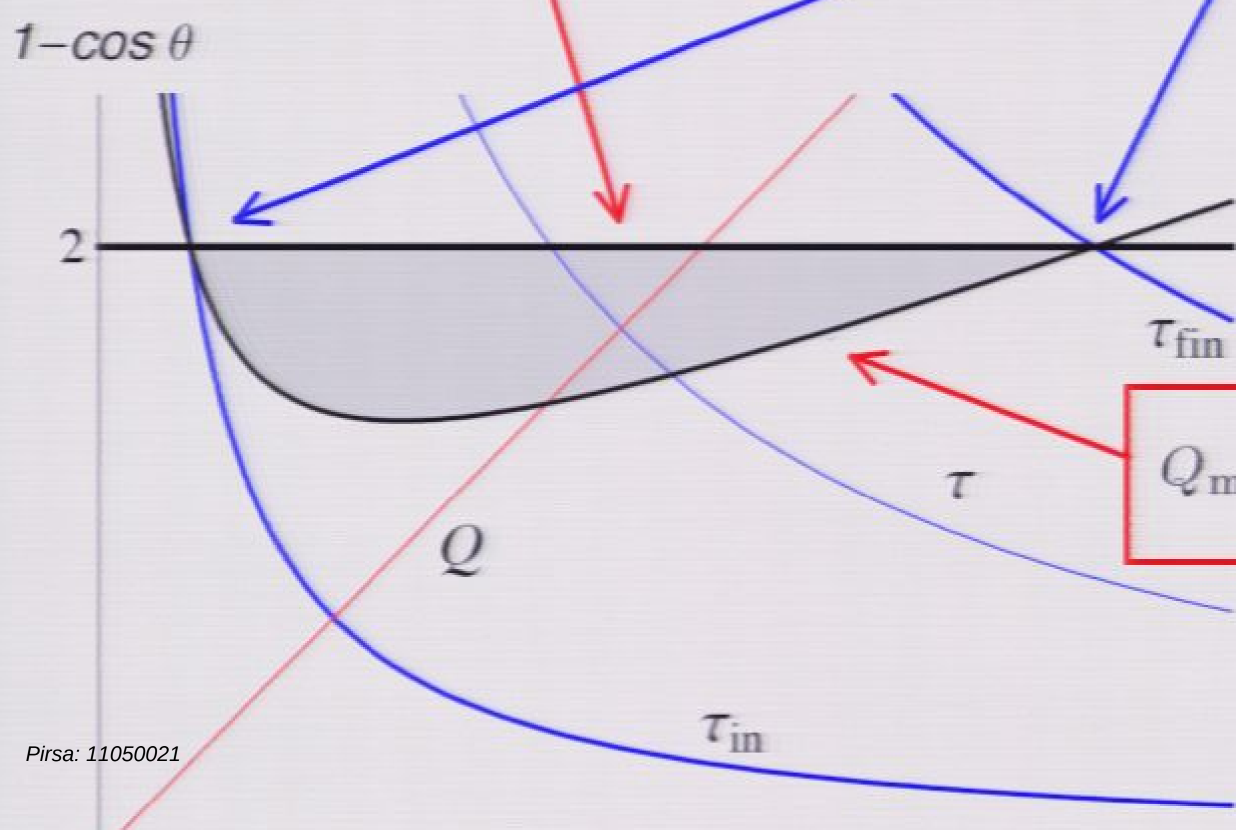
Phase space of energy cutoff

$$Q_{\rm min} = \frac{M^2}{4H(\tau)^2 k|\tau|}.$$

$$\tau_\pm \equiv -\frac{\Lambda \pm \sqrt{\Lambda^2 - M^2}}{2kH_*}.$$



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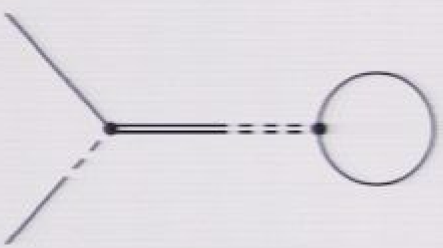
Power Spectrum Corrections

Produces a nearly scale-invariant shift in the power spectrum.

$$\frac{P_\zeta(k)}{P_{\zeta 0}(k_*)} = \frac{g_1^2 \Lambda^3}{24\pi M^4 H_*}\left[1 - \frac{8\epsilon_1}{3} - (2\epsilon_1+\epsilon_2)C - 4\epsilon_1 \ln\left(\frac{4\Lambda H_*}{M^2}\right) + (6\epsilon_1+\epsilon_2)\ln\left(\frac{k}{k_*}\right)\right]$$



The slight deviation from scale-invariance produces an oscillating pattern in the power spectrum.

$$\frac{P_\zeta(k)}{P_{\zeta 0}(k_*)} = \frac{g_1^2\sqrt{\pi\Lambda}}{8(2\pi)^2 M^2\sqrt{H_*}}\left[1 - \frac{\epsilon_1}{2} - (2\epsilon_1+\epsilon_2)C - \left[\epsilon_1\left(\frac{3}{2} + \ln\frac{\Lambda}{H_*}\right) + \epsilon_2\right]\ln\left(\frac{k}{k_*}\right)\right]$$

$$\times \sin\left(\epsilon_1 \frac{M}{H_*}\left(2 - \ln\frac{\Lambda}{M}\right)\ln\frac{k}{k_*}\right)$$

(MGJ, Schalm '11)

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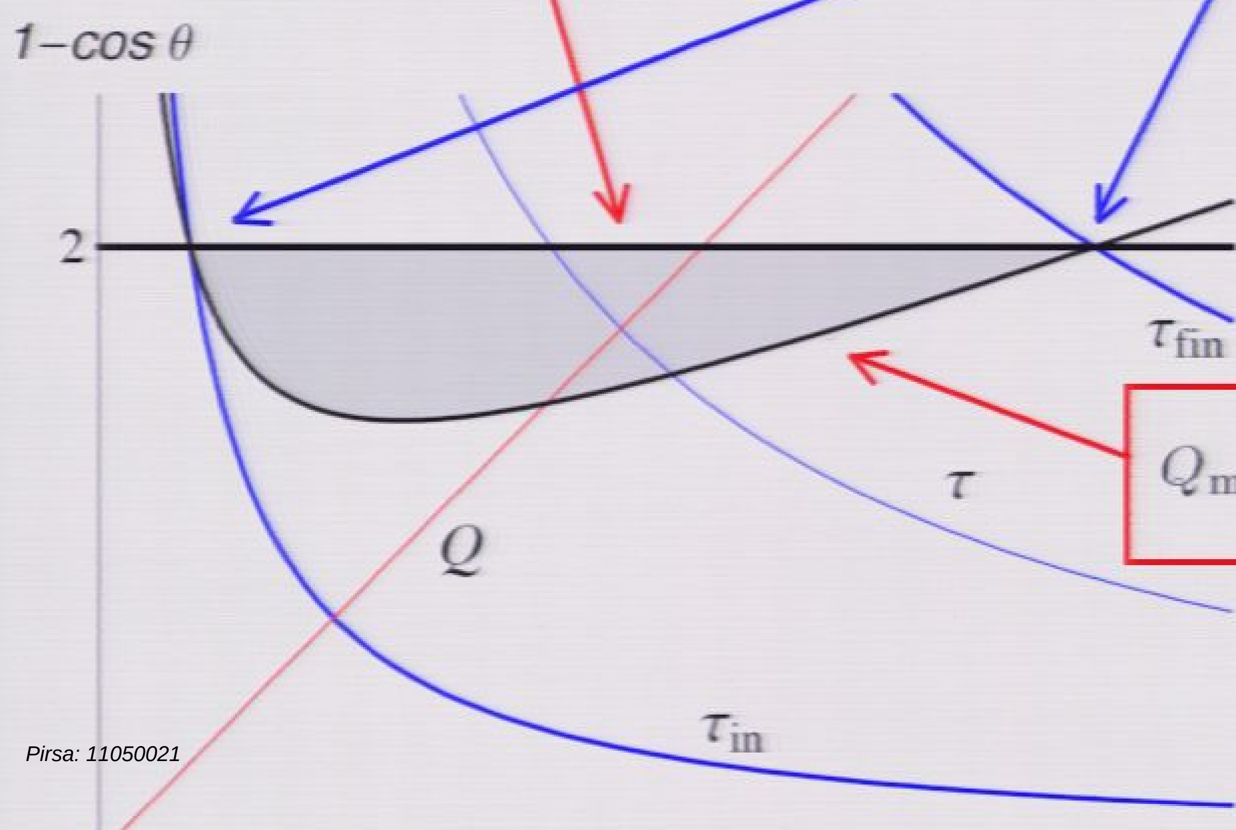

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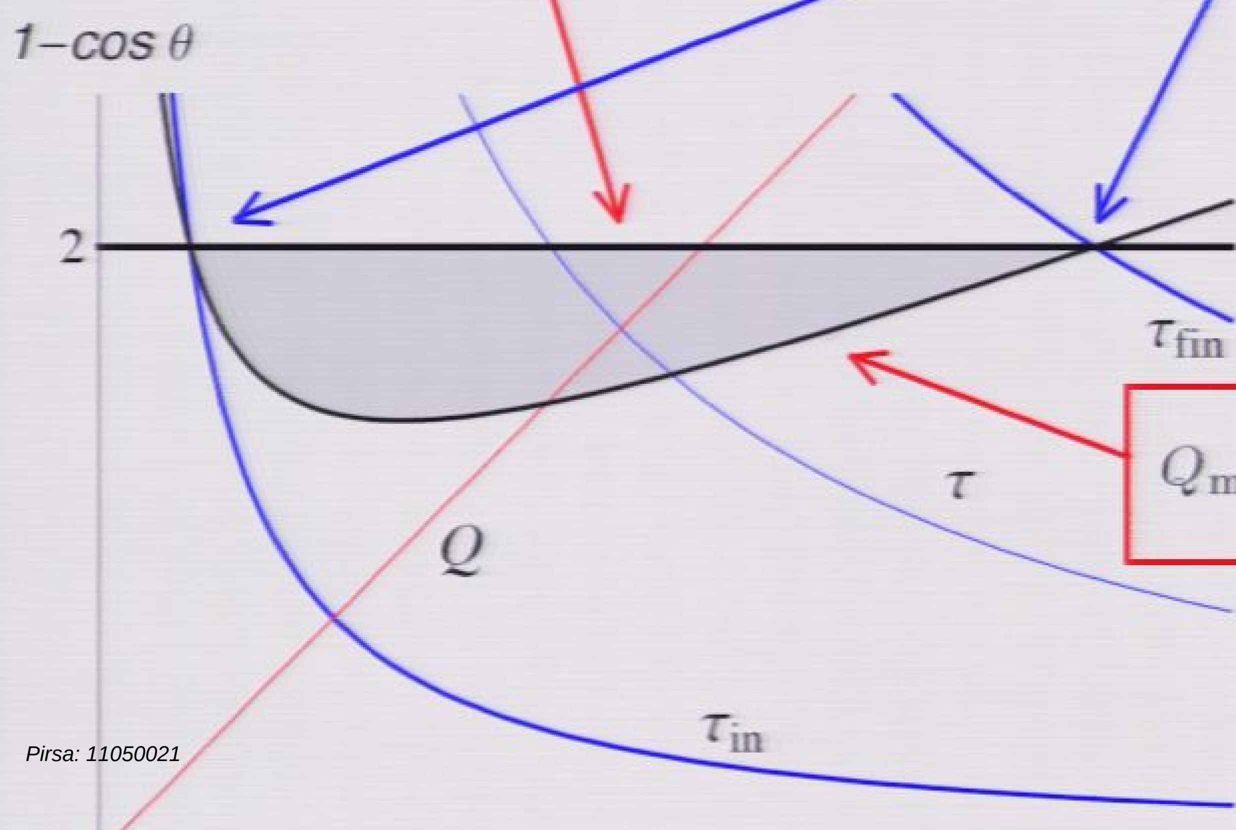

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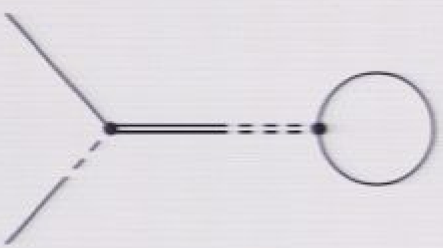

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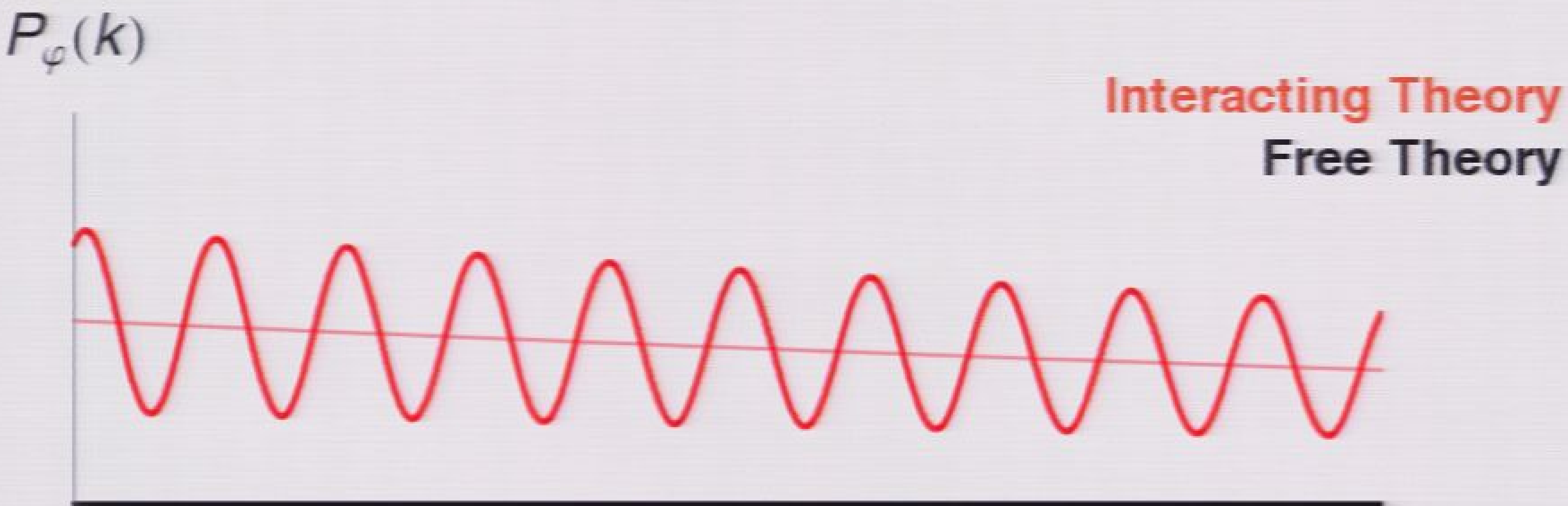

- This is the universal signature of a modified vacuum.
- Currently in collaboration with:
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$\ln k$

(MG-J. Schalm '11)

Power Spectrum Corrections

Produces a nearly scale-invariant shift in the power spectrum.

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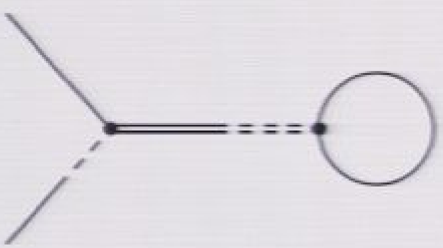

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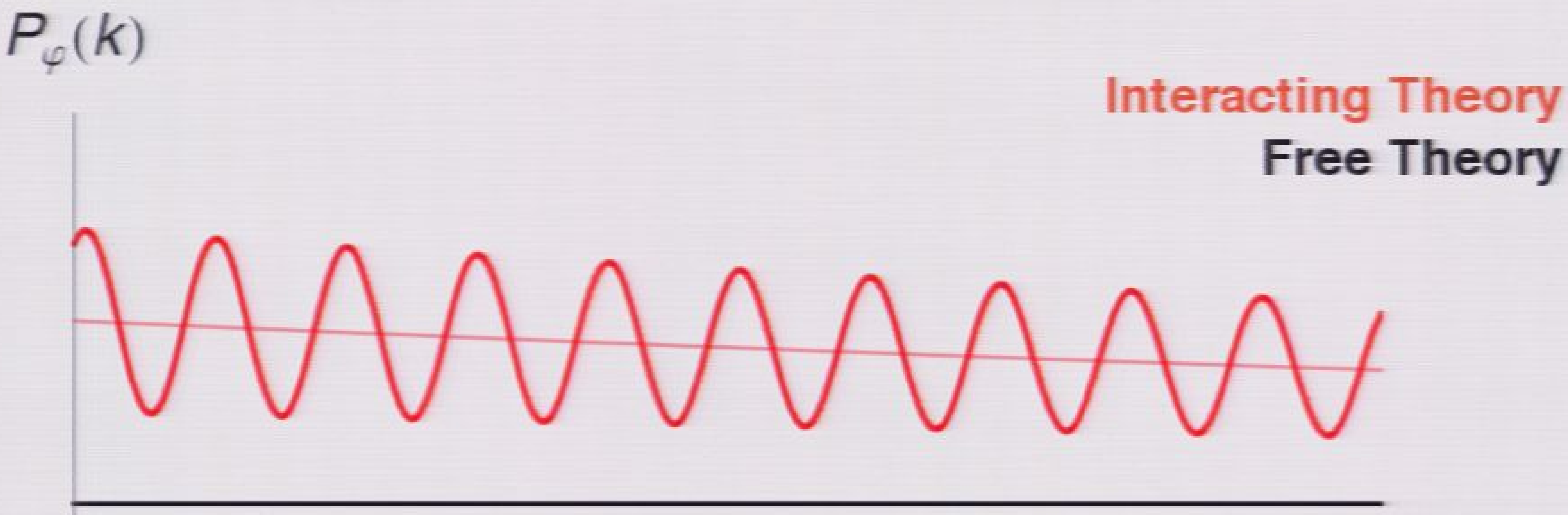

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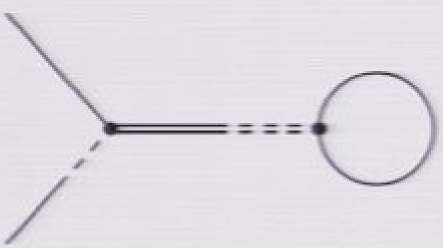

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- Consider this diagram, with loop momentum *q*:

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \to \frac{1}{(2\pi)^2}\int q^2 dq d(1-\cos\theta).$$

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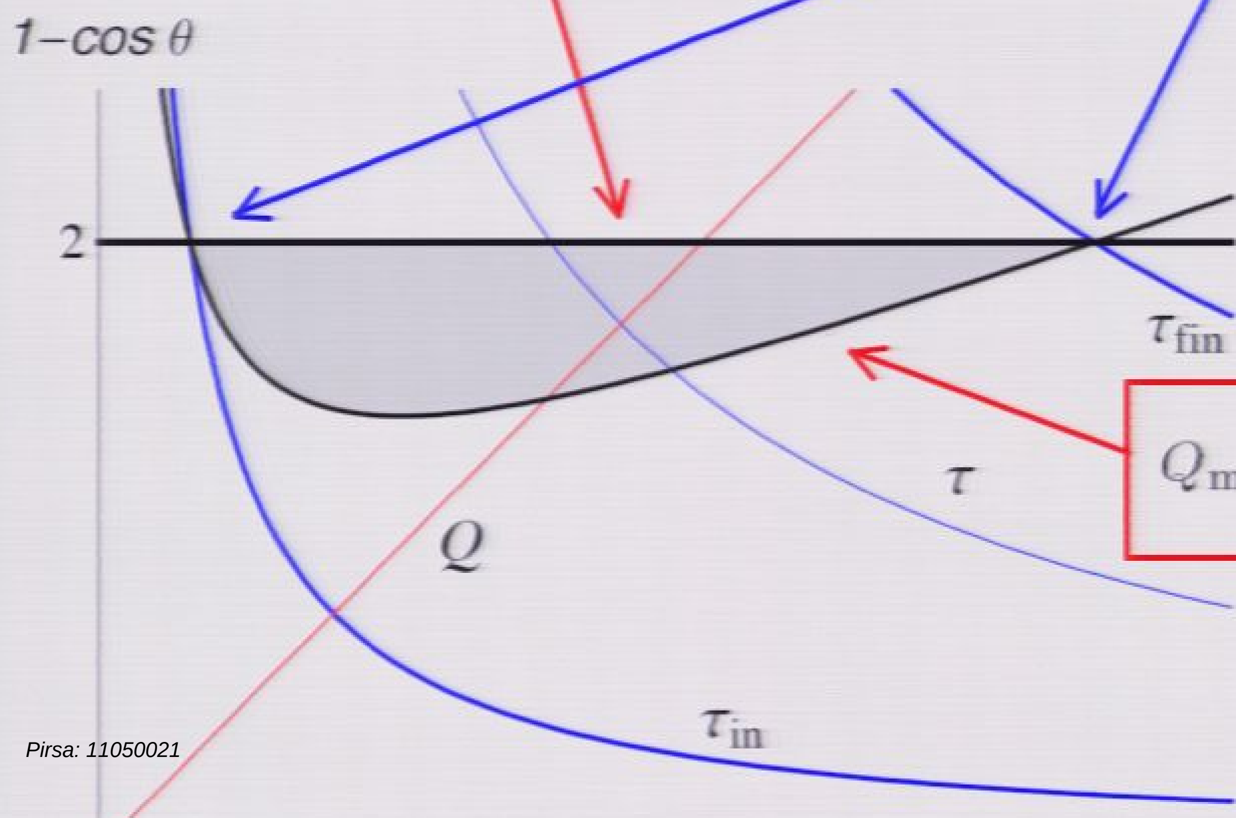

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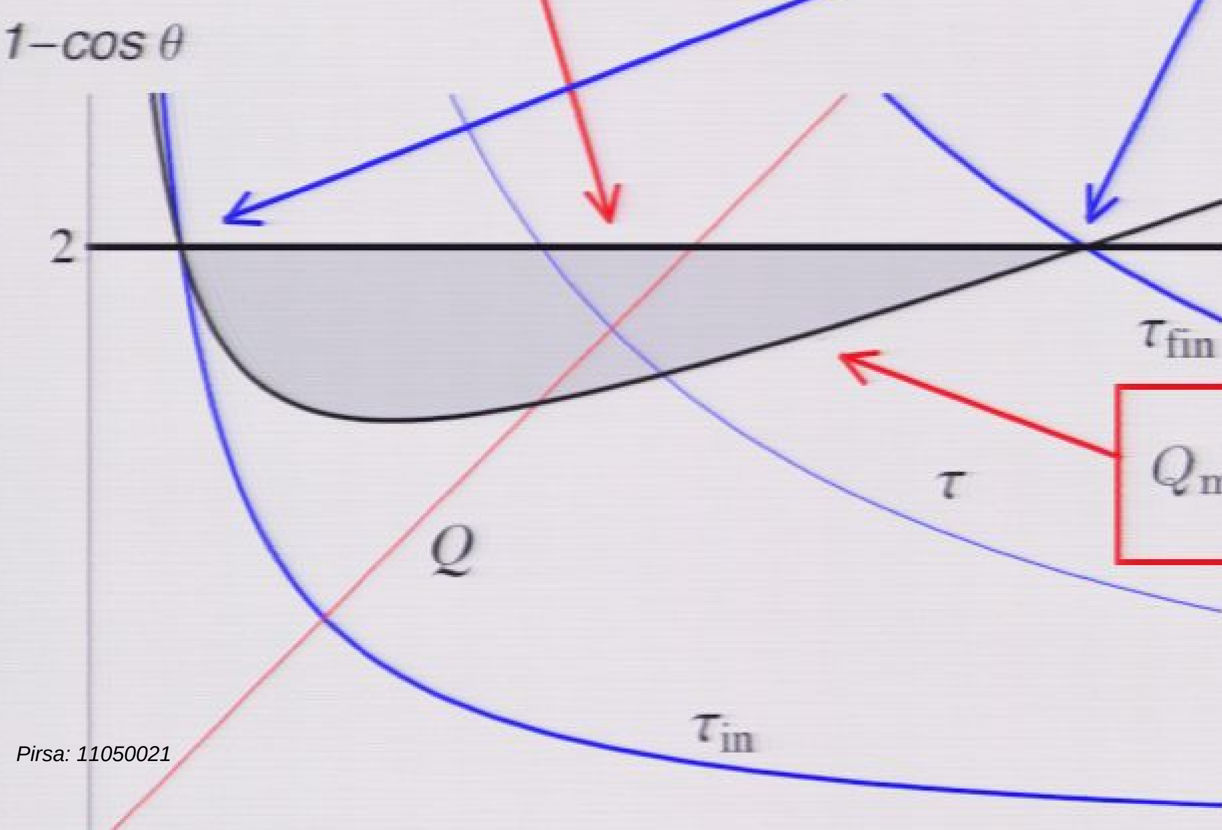

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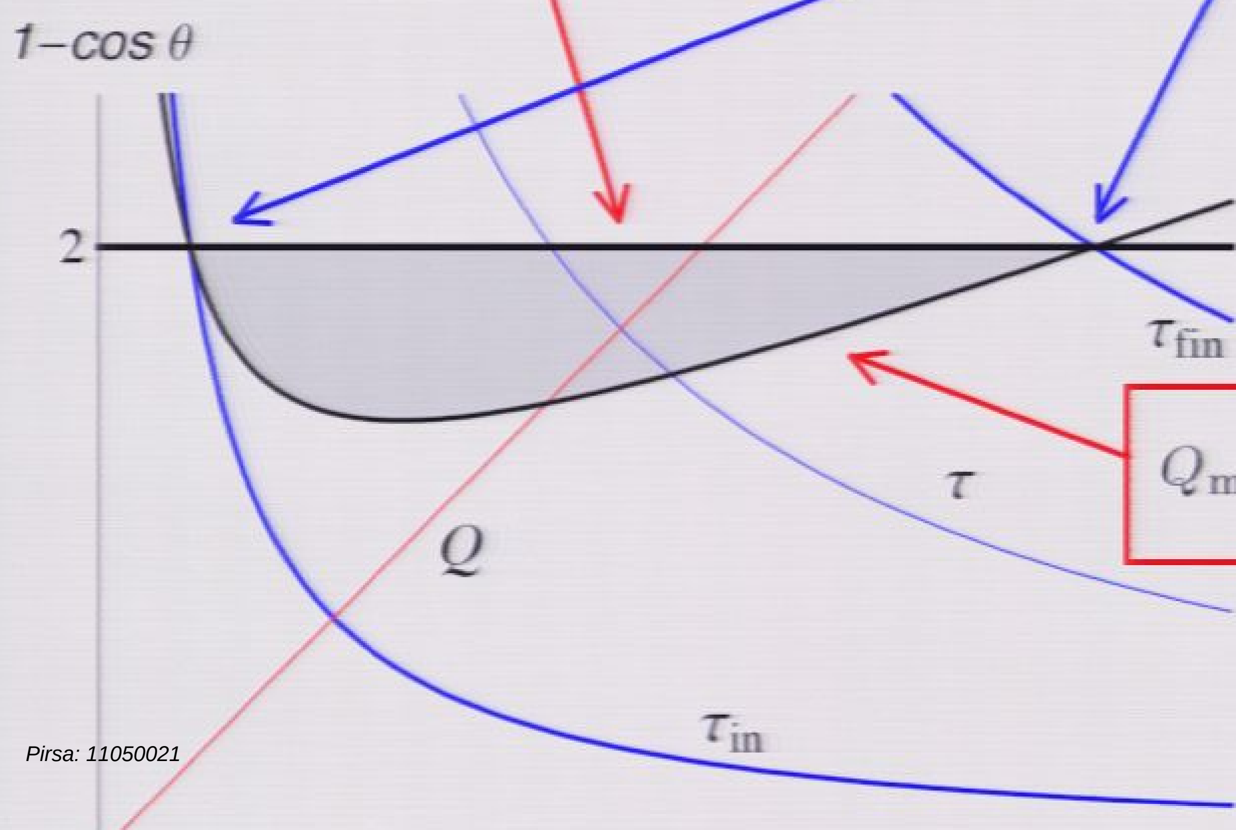

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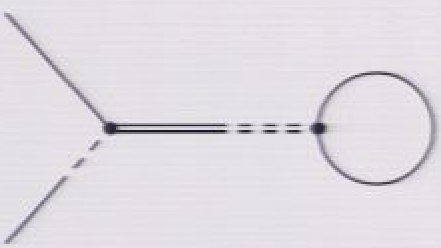

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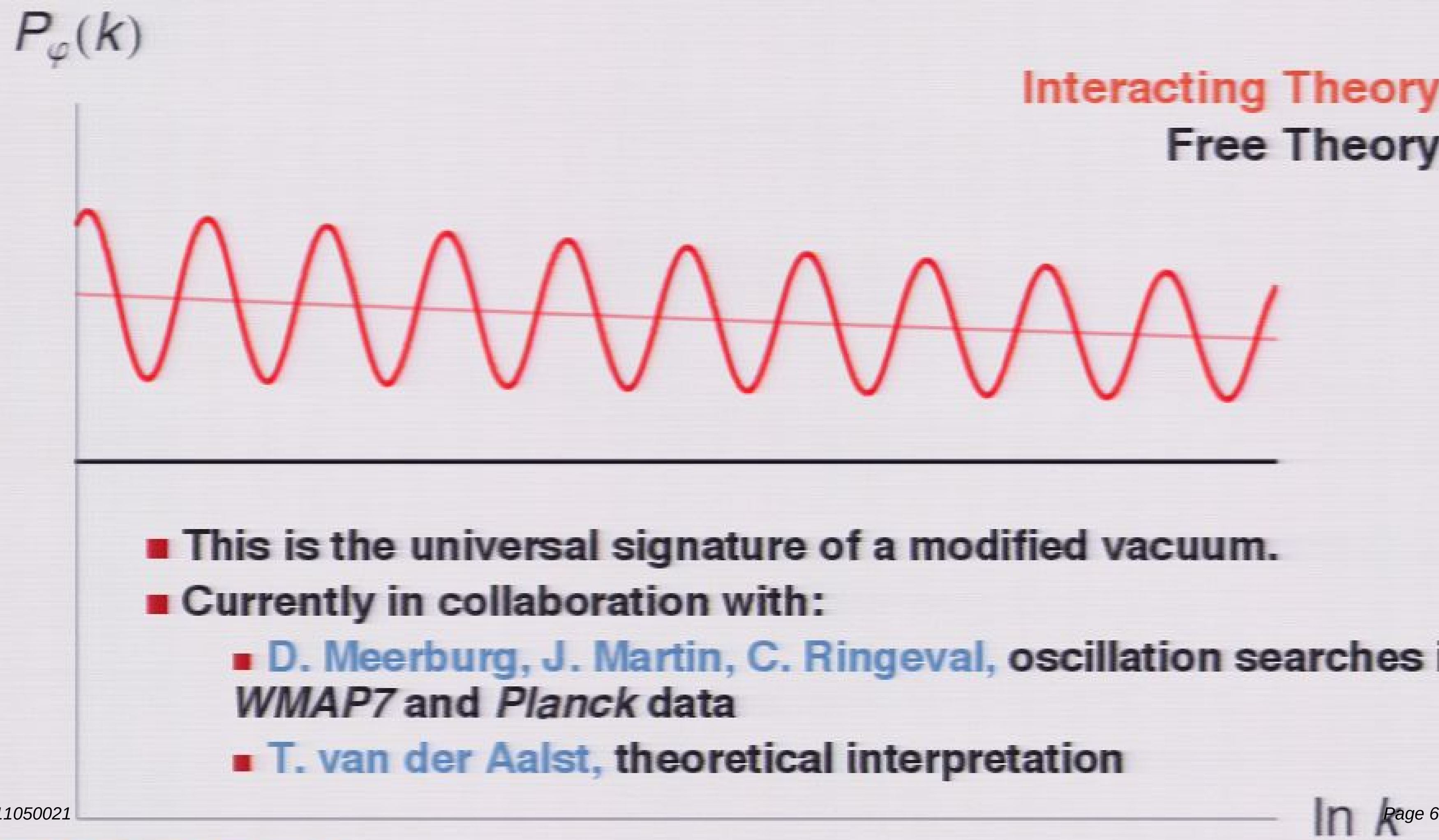

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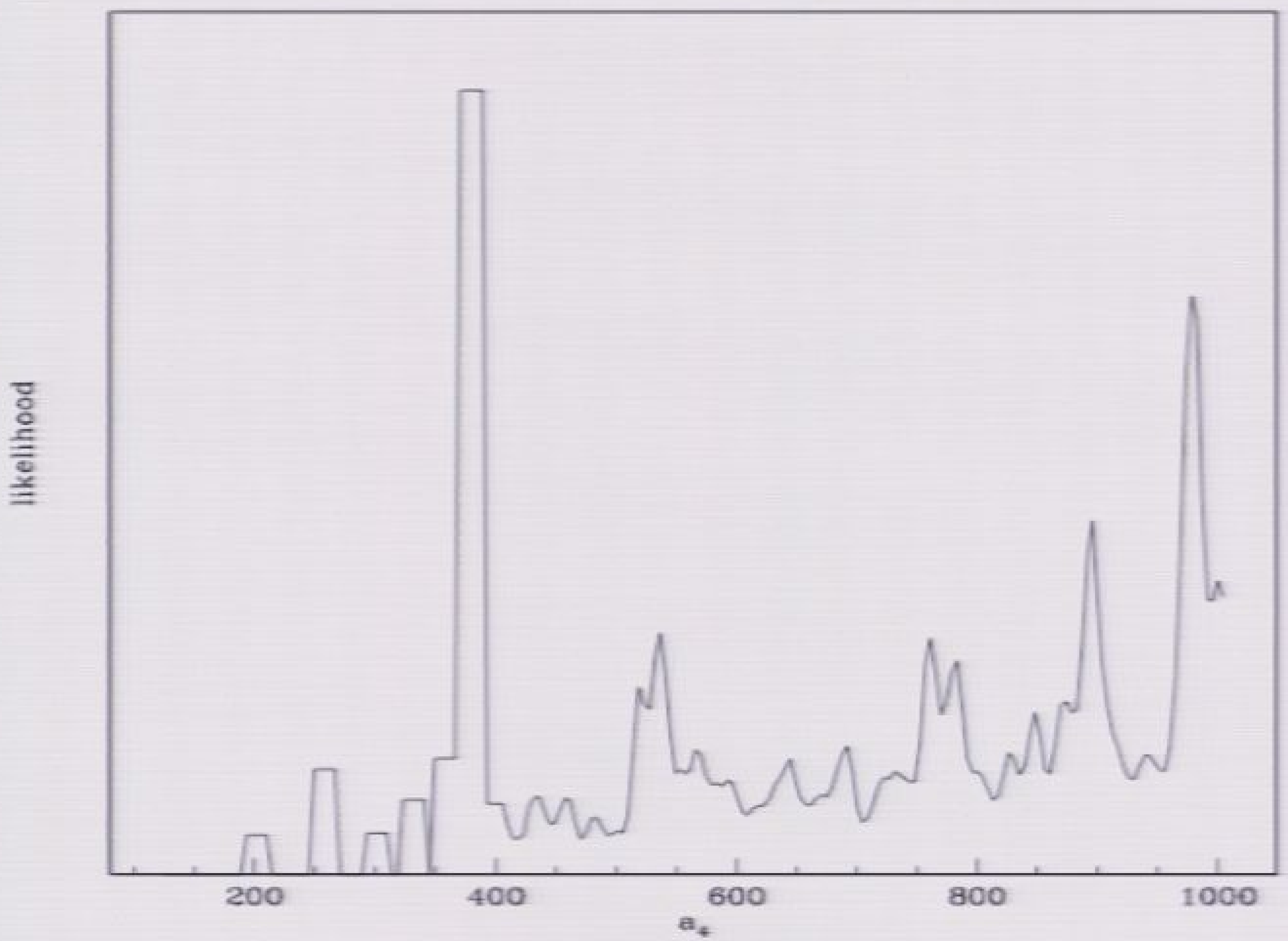
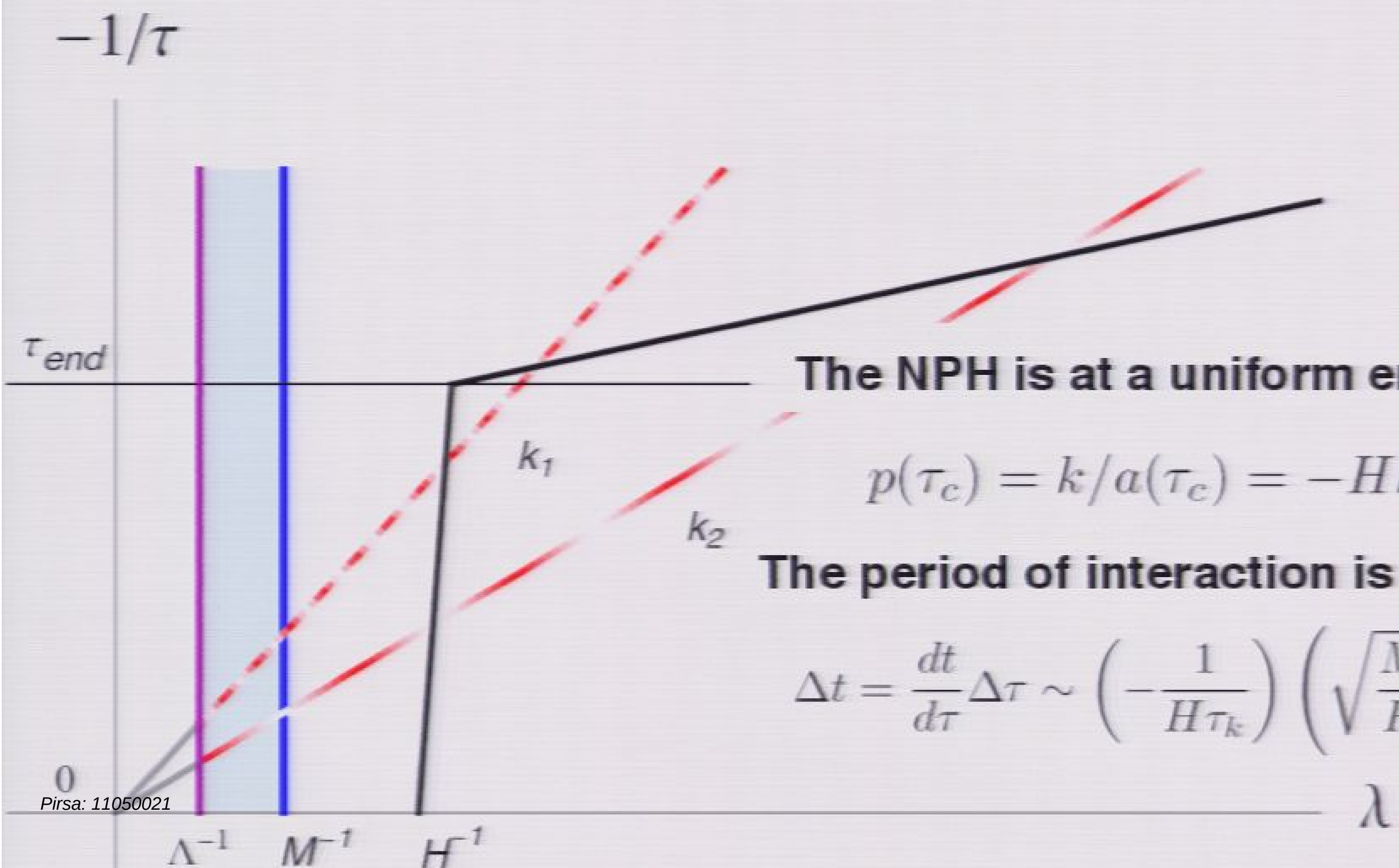
(Very) Preliminary Oscillation Searches in *WMAP7*



MGJ, J. Martin, D. Meerburg, C. Ringeval, K. Schalm '11

Scale-Variance from a Scale-Invariant Theory



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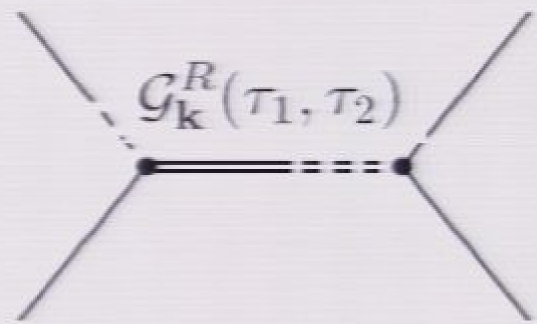

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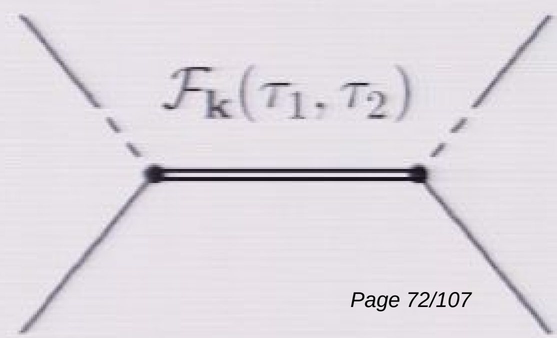

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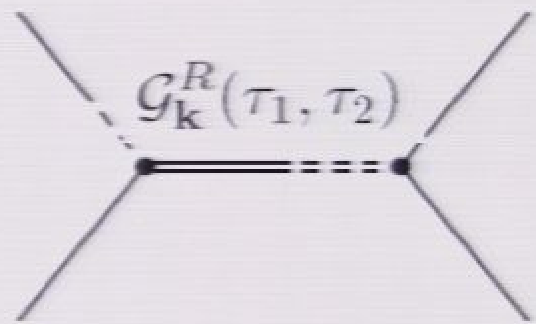

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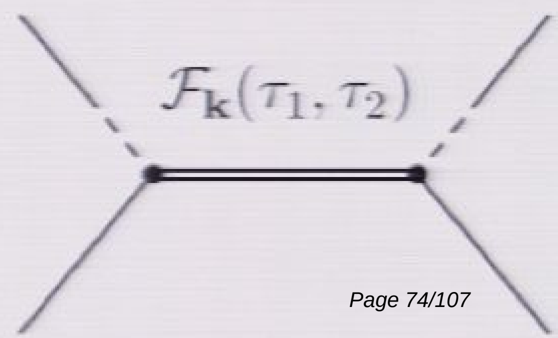

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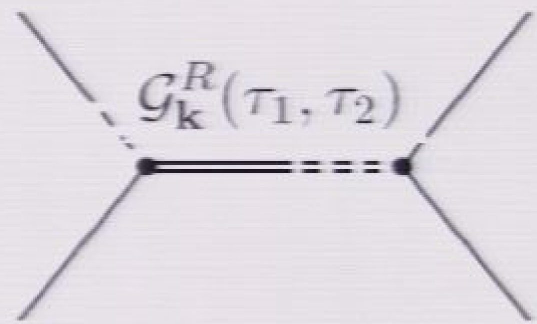

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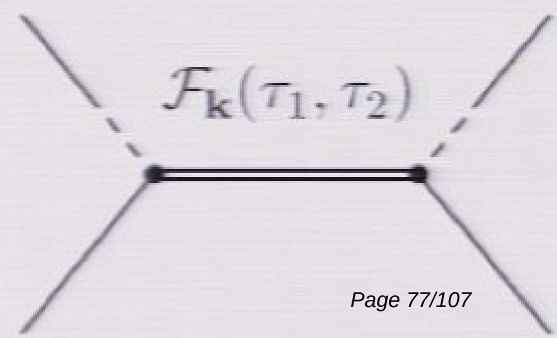

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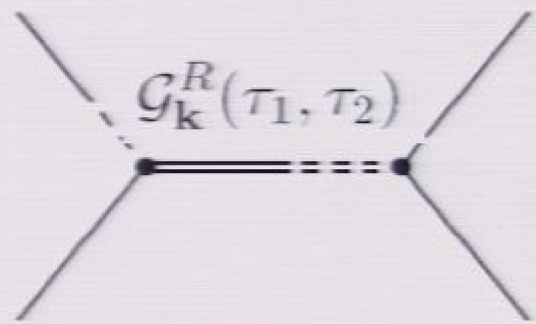

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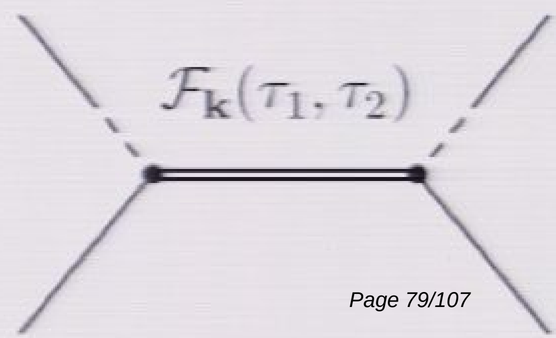

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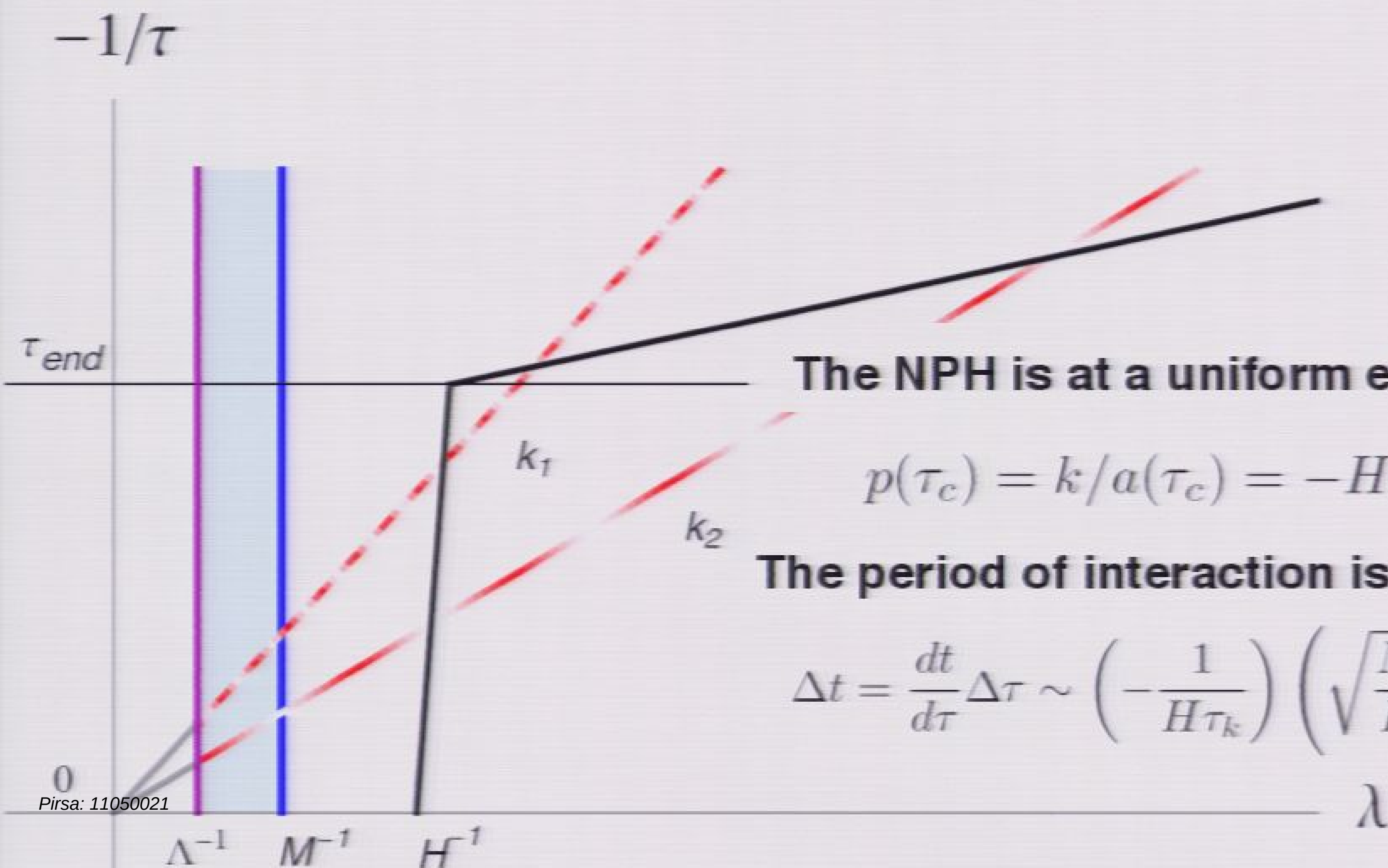

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energy cutoff

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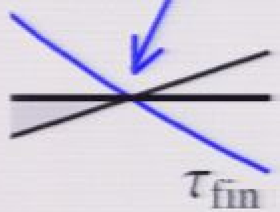


$$Q_{\rm max} = -k|\tau| + \frac{\Lambda}{H(\tau)}$$

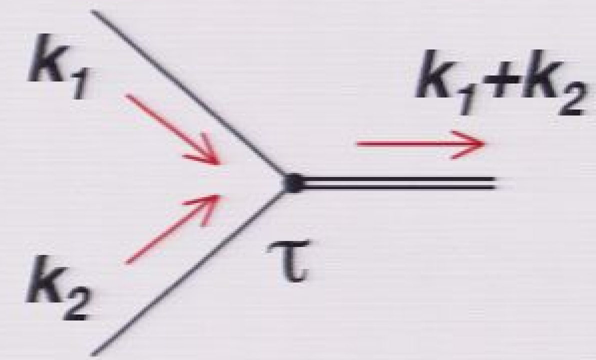
Power Spectrum Corrections

- **Each vertex is an integral over the time of interaction, and has the following form:**

$$\mathcal{A}_1(\mathbf{k}_1,\mathbf{k}_2) \equiv \int_{\tau_0}^{0} d\tau\, a^4(\tau) U_{\mathbf{k}_1}(\tau) U_{\mathbf{k}_2}(\tau) V^*_{-(\mathbf{k}_1+\mathbf{k}_2)}(\tau)$$

$$\approx -\frac{1}{2\sqrt{2k_1^3k_2^3}H}\int_{\tau_0}^{0}\frac{d\tau}{\tau^3}\frac{(1-ik_1\tau)(1-ik_2\tau)}{\left(|\mathbf{k}_1+\mathbf{k}_2|^2+\frac{M^2}{H^2\tau^2}\right)^{1/4}}$$

$$\times \exp\left[-i(k_1+k_2)\tau + i\int^{\tau} d\tau' \sqrt{|\mathbf{k}_1+\mathbf{k}_2|^2+\frac{M^2}{H^2\tau'^2}}\right].$$



- **This admits a stationary phase approximation near the moment of energy-conservation,**

$$\tau_*^{-1} = -\frac{H}{M}\sqrt{2k_1k_2(1-\cos\theta)}, \qquad \cos\theta = \frac{\mathbf{k}_1\cdot\mathbf{k}_2}{k_1k_2}.$$

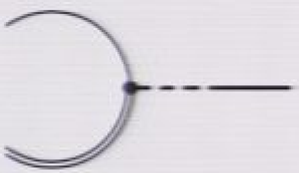
- **The vertex (to leading order in *H/M*) is then simply**

$$\mathcal{A}_1(\mathbf{k}_1,\mathbf{k}_2) \approx -\frac{\sqrt{\pi i}}{\cdots}\sqrt{\frac{H}{M}}\left[\frac{2M}{H}\left(k_1+k_2+\sqrt{2k_1k_2(1-\cos\theta)}\right)\right]^{-i\cdots}$$

f

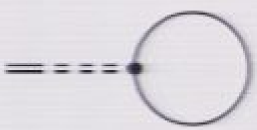
ɔwer Spectrum Corrections

M^2 .



Produces a nearly scale-invariant shift in the power spectrum.

$$= \frac{g_1^2\Lambda^3}{24\pi M^4 H_*}\left[1-\frac{8\epsilon_1}{3}-(2\epsilon_1+\epsilon_2)C-4\epsilon_1\ln\left(\frac{4\Lambda H_*}{M^2}\right)+(6\epsilon_1+\epsilon_2)\ln\left(\frac{k}{k_*}\right)\right]$$



The slight deviation from scale-invariance produces an oscillating pattern in the power spectrum.

$$\frac{g_1^2\sqrt{\pi\Lambda}}{8(2\pi)^2M^2\sqrt{H_*}}\left[1-\frac{\epsilon_1}{2}-(2\epsilon_1+\epsilon_2)C-\left[\epsilon_1\left(\frac{3}{2}+\ln\frac{\Lambda}{H_*}\right)+\epsilon_2\right]\ln\left(\frac{k}{k_*}\right)\right]$$

$$\times \sin\left(\epsilon_1\frac{M}{H_*}\left(2-\ln\frac{\Lambda}{M}\right)\ln\frac{k}{k_*}\right)$$

Schalm '11)

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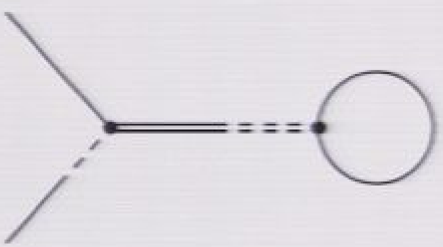

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(MG-J, Schalm '11)

How do you impose an energy cutoff with no energy conservation?

- Consider this diagram, with loop momentum *q*:

$$\int \frac{d^3\mathbf{q}}{(2\pi)^3} \rightarrow \frac{1}{(2\pi)^2}\int q^2 dq d(1-\cos\theta).$$

- *k* and *q* will determine the time of interaction τ via the stationary phase approximation. Let us also define a coordinate *Q ~ E/H* orthogonal to τ,

$$\tau^{-1} \equiv -\frac{H}{M}\sqrt{2kq(1-\cos\theta)}, \qquad Q \equiv q|\tau| = \frac{M}{H}\sqrt{\frac{q}{2k(1-\cos\theta)}}.$$

- Inverting this allows us to transform the *q*-integral into (τ,*Q*):

$$q = -\frac{Q}{\tau}, \qquad 1-\cos\theta = -\frac{M^2}{2H^2 Q\tau k}.$$

$$d(1-\cos\theta)dq \rightarrow \frac{M^2(1+\epsilon_1)}{H^2 Q\tau^3 k} dQ d\tau.$$

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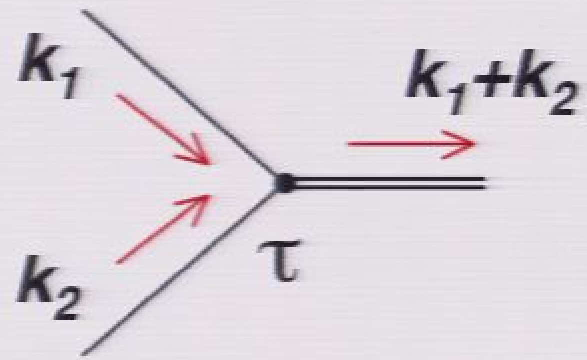

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Feynman Rules in Keldysh Basis

- The correlations can now be evaluated using these:

$$G^R_{\mathbf{k}}(\tau_1,\tau_2) \equiv i\langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\Phi_{-\mathbf{k}}(\tau_2)\rangle$$
$$= -2\theta(\tau_1-\tau_2)\mathrm{Im}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right],$$

$$F_{\mathbf{k}}(\tau_1,\tau_2) \equiv \langle\bar{\varphi}_{\mathbf{k}}(\tau_1)\bar{\varphi}_{-\mathbf{k}}(\tau_2)\rangle$$
$$= \mathrm{Re}\left[U_{\mathbf{k}}(\tau_1)U^*_{\mathbf{k}}(\tau_2)\right],$$

$$\mathcal{G}^R_{\mathbf{k}}(\tau_1,\tau_2) \equiv i\langle\bar{\chi}^{(0)}_{\mathbf{k}}(\tau_1)\mathrm{X}^{(0)}_{-\mathbf{k}}(\tau_2)\rangle$$
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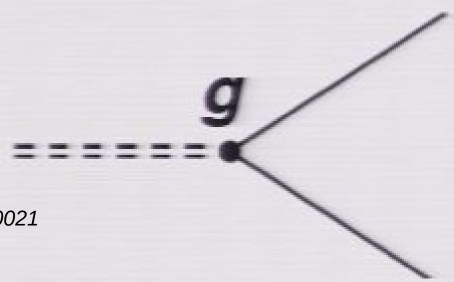
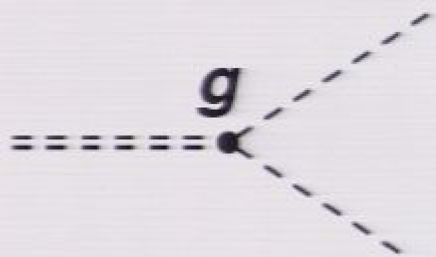
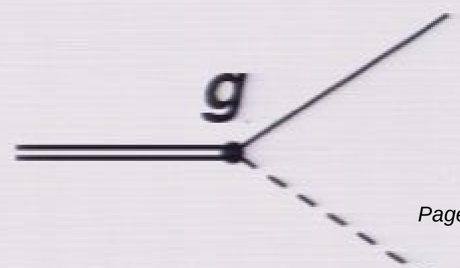
- The interactions are given by:







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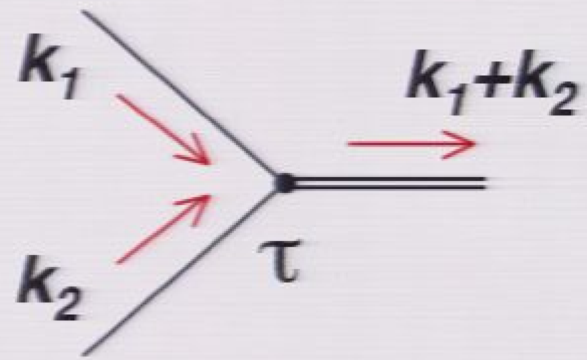

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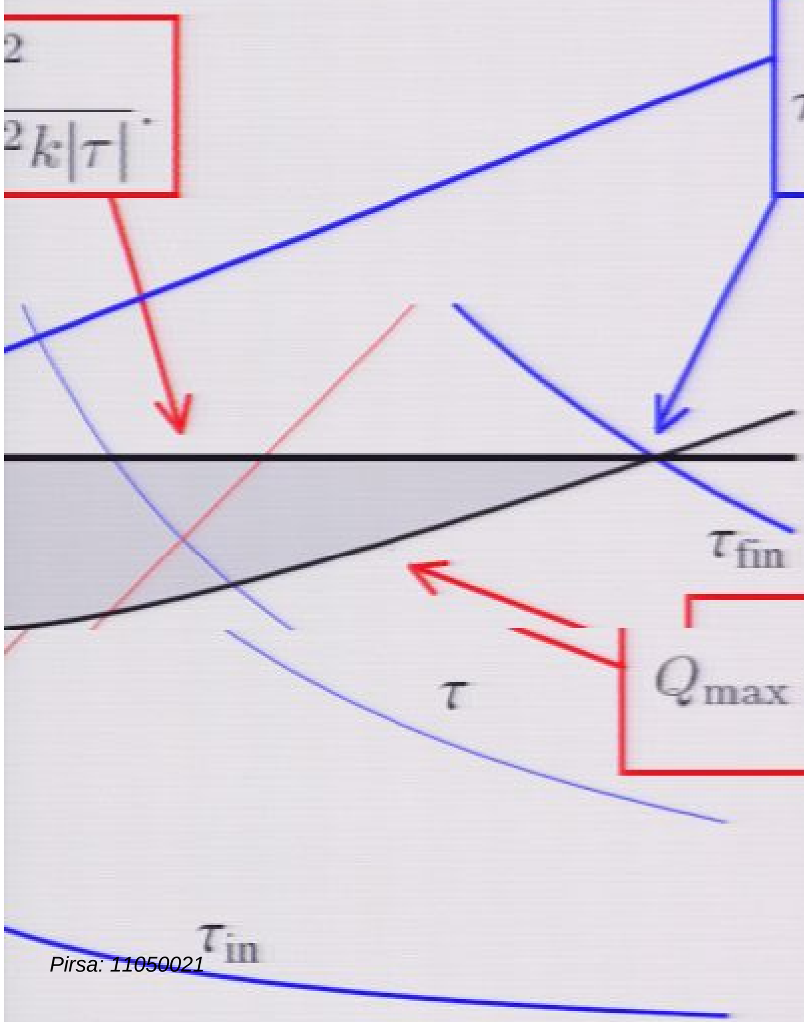
...se space of energy cutoff ...ons



$$\tau_{\pm} \equiv -\frac{\Lambda \pm \sqrt{\Lambda^2 - M^2}}{2kH_*}.$$

$$Q_{\max} = -k|\tau| + \frac{\Lambda}{H(\tau)}$$

...ant shift in the ...wer spectrum.

$$\ldots \epsilon_1 + \epsilon_2) \ln \left(\frac{k}{k_*} \right) \Big]$$

...ance produces ...wer spectrum.

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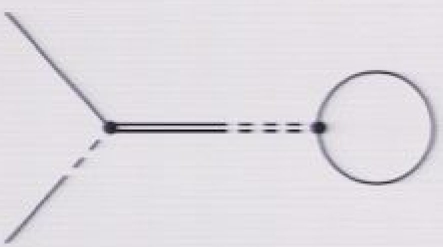

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(MG-L Schalm '11)

What about (H/M)^n?

- The analysis of H/M vs $(H/M)^2$ assumed an expansion in local operators, which works fine in a static background:

$$G(x-y) = -\int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x-y)}}{p^2 - M^2} \approx \frac{1}{M^2}\left[1 + \mathcal{O}\left(\frac{\partial^2}{M^2}\right)\right] \delta^4(x-y).$$

- In an expanding background, the Green's function produces non-local operators, which have very different scaling with energy...

Low-Energy Effective Interactions

- This allows use of the stationary phase approximation,

$$\mathcal{B}(\tau_k;\omega_1,\omega_2,\mathbf{q}) = \int_{\tau_{\rm in}}^{\tau_k} d\tau\ a(\tau)^2 e^{-i(\omega_1+\omega_2)\tau} V_{\mathbf{q}}^*(\tau)$$
$$\approx \frac{\sqrt{i\pi}}{H\tau} \frac{(1+\epsilon_1)e^{-i\theta}}{\left(q^2+\frac{M^2}{H^2\tau^2}\right)^{1/4}} \left(\frac{d^2\theta}{du^2}\right)^{-1/2}\Bigg|_{\tau=\tau_c} .$$

- The effective action, to leading order in *H/M*, is then

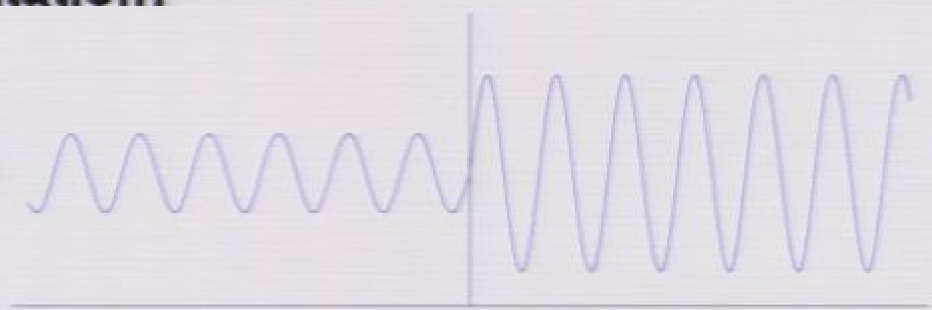
$$\mathcal{S}_{\rm int,4}[\bar\varphi,\Phi] = -\int \prod_i \frac{d\omega_i d^3\mathbf{q}_i}{(2\pi)^4}(2\pi)^3\delta^3(\sum_i \mathbf{q}_i)$$
$$\times \frac{g_1^2}{2!}\Big(2\tilde{\bar\varphi}_1\tilde\Phi_2\theta\left(\tau_{1c}-\tau_{2c}\right)$$
$$\mathrm{Im}\left[\mathcal{B}^*(0;\omega_1,\omega_2,\mathbf{q}_1+\mathbf{q}_2)\mathcal{B}(0;\omega_3,\omega_4,\mathbf{q}_3+\mathbf{q}_4)\right]\tilde{\bar\varphi}_3\tilde{\bar\varphi}_4$$
$$+i\tilde{\bar\varphi}_1\tilde\Phi_2\mathrm{Re}\left[\mathcal{B}^*(0;\omega_1,\omega_2,\mathbf{q}_1+\mathbf{q}_2)\mathcal{B}(0;\omega_3,\omega_4,\mathbf{q}_3+\mathbf{q}_4)\right]\tilde{\bar\varphi}_3\tilde\Phi_4\Big) .$$

Low-Energy Effective Interactions

- The high-energy terms produce a time-localized potential for the inflaton fluctuations:

$$\frac{d}{d\tau}\left[a^2\bar{\varphi}'_{\mathbf{k}}(\tau)\right] = \delta(\tau-\tau_c)a^4 m_{\rm eff}^2\bar{\varphi}_{\mathbf{k}}(\tau)$$

- This produces a change in the fluctuation solution, corresponding to a vacuum excitation:

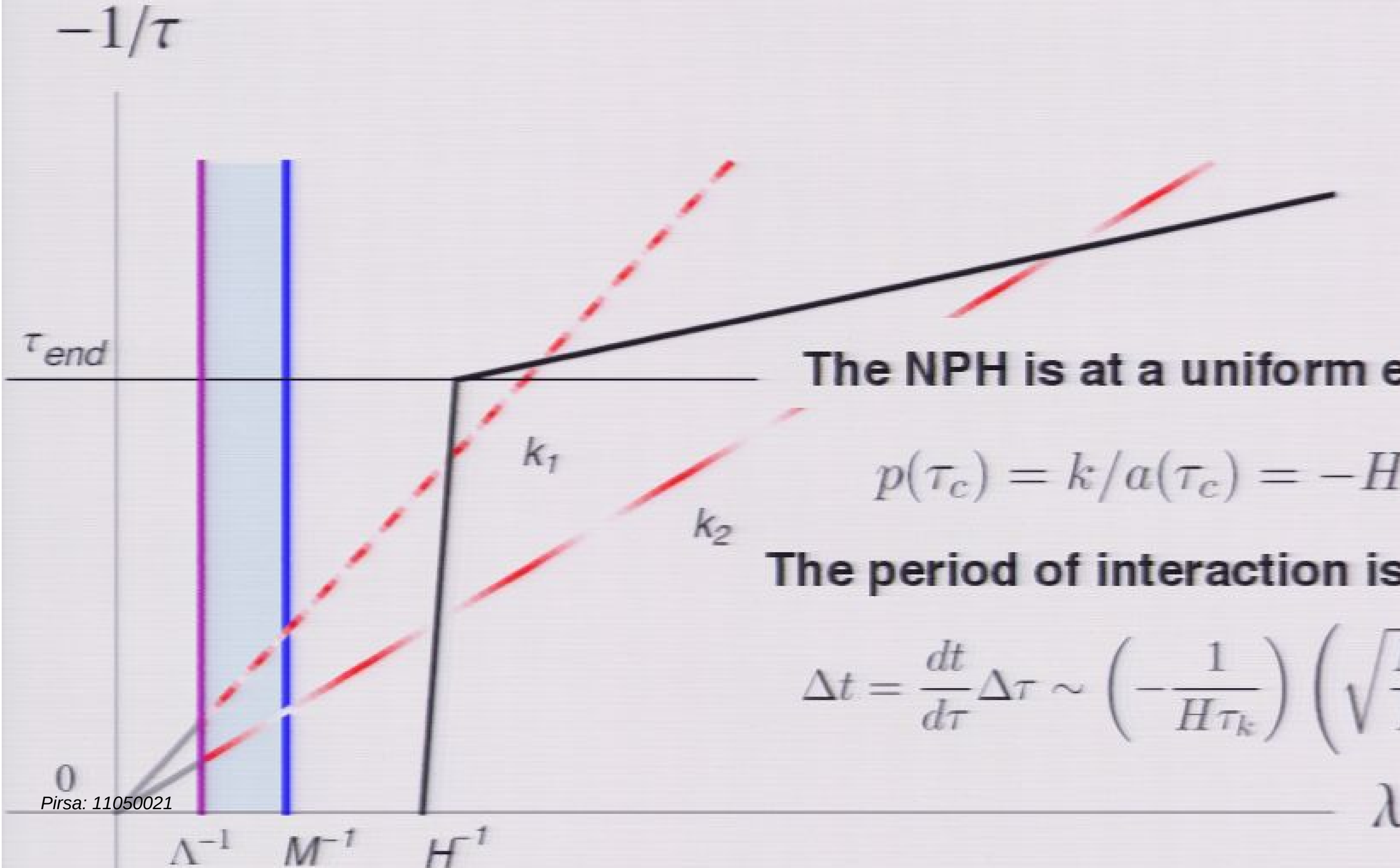
$\bar{\varphi}_{\mathbf{k}}(\tau) = U_{\mathbf{k}}(\tau)$  $\bar{\varphi}_{\mathbf{k}}(\tau) = U_{\mathbf{k}}(\tau) + \beta_{\mathbf{k}}U^*_{\mathbf{k}}(\tau)$

- This is the first time the vacuum rotation has been calculated from a fundamental theory:

$$\beta_{\mathbf{k}} = -\frac{g_1^2\sqrt{\pi\Lambda}}{8\sqrt{2}(2\pi)^2M^2\sqrt{H}}e^{i\frac{M}{H}\left(2-\ln\frac{\Lambda}{M}\right)}$$

(MG-J. Schalm '11)

Scale-Variance from a Scale-Invariant Theory



The NPH is at a uniform energy scale:

$$p(\tau_c) = k/a(\tau_c) = -Hk\tau_c = M/2.$$

The period of interaction is also invariant

$$\Delta t = \frac{dt}{d\tau}\Delta\tau \sim \left(-\frac{1}{H\tau_k}\right)\left(\sqrt{\frac{M}{H}}\frac{1}{k}\right) \sim \frac{1}{\sqrt{HM}}.$$

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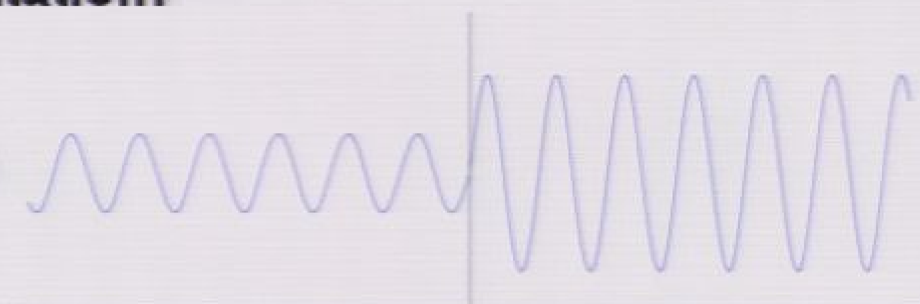

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(MGJ, Schalm '11)

Observability? (!)

- We see that integrating out high energy physics produces low energy interactions, but a cosmological background induces boundary terms
- These represent a modified vacuum, appearing in the power spectrum as oscillations
- But is this observable?
- We can see about four decades of comoving *k* in the CMB,

$$k_{\rm min} \leq k_{\rm obs} \leq 10^4 \; k_{\rm min}$$

If $H/M_{\rm string} \sim 10^{-2}$ then we should see about 10^2 oscillations, just at the threshold of *Planck*'s sensitivity.

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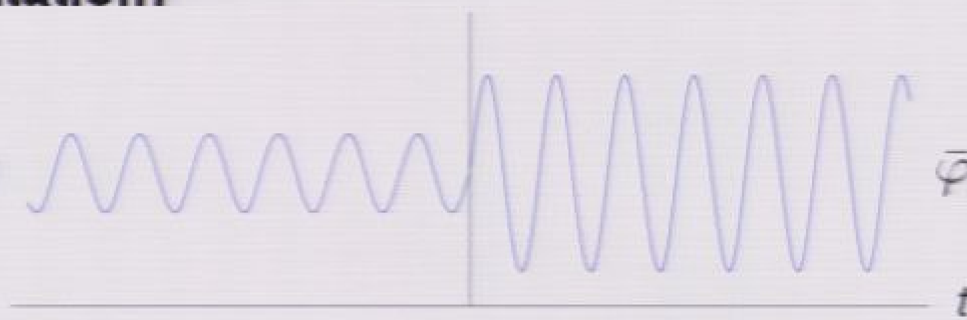

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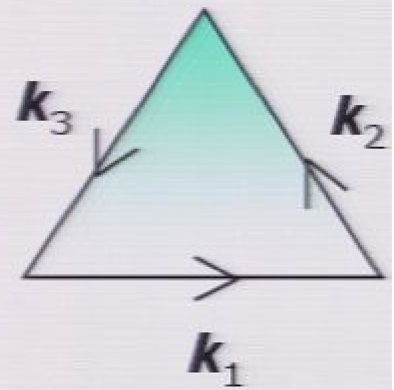
Second Observable: Non-Gaussianity



- We may then consider the 3-pt correlation,

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \, \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \, f_{\rm NL} \, F(k_1, k_2, k_3) \ .$$

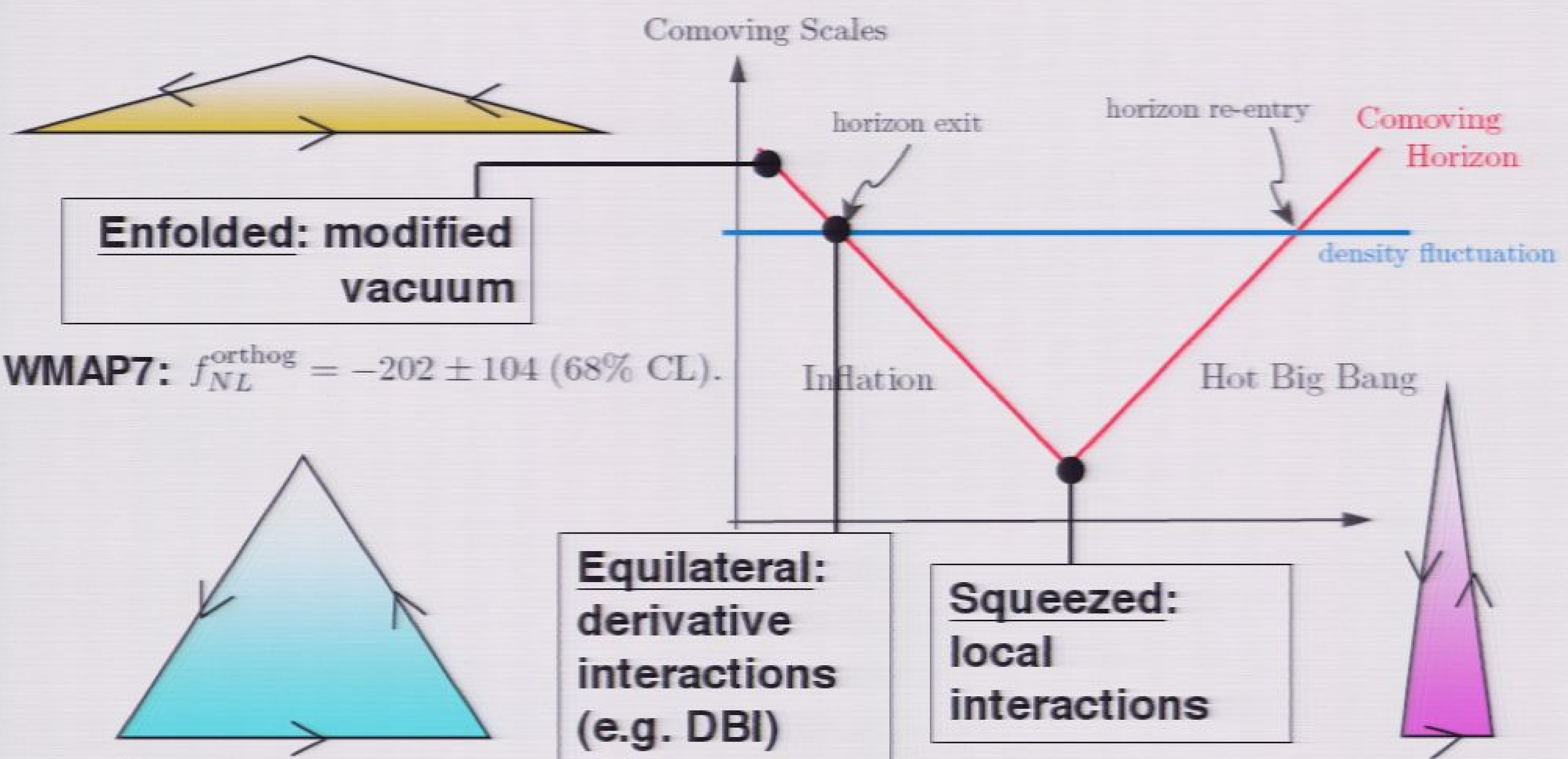
- A free field theory will produce a vanishing 3-pt correlation, and so nG measures interactions: (Creminelli '03)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_P^2 R - \frac{1}{2} (\nabla\phi)^2 - V(\phi) \right] \longrightarrow f_{\rm NL}^{\rm equil.} = \frac{5}{6} (\eta - \frac{23}{6} \epsilon) \ .$$ **Very small**

Adding $\frac{1}{8M^4} (\nabla\phi)^2 (\nabla\phi)^2 \longrightarrow f_{\rm NL}^{\rm equil.} = \frac{35}{108} \frac{\dot\phi^2}{M^4} \ .$ **Potentially very large**

- The shape of the momenta triangle indicates when it was produced (Babich, Creminelli, Zaldarriaga '04) and can serve to easily differentiate models (eg Khoury and Piazza '08)

Types of Non-Gaussianity



WMAP7: $f_{NL}^{\text{orthog}} = -202 \pm 104$ (68% CL).

$f_{NL}^{\text{equil}} = 26 \pm 140$ (68% CL).

$f_{NL}^{\text{local}} = 32 \pm 21$ (68% CL).

anity:

al Small-c_s Models | Vacua

sider a more general class of small-c_s

$$\ldots M_p^2 R - 2P(X,\phi)], \quad \begin{cases} \epsilon = \frac{XP_{,X}}{M_p^2 H^2}, \\ c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} = \frac{M_p^2 H^2 \epsilon}{\Sigma}, \\ \Sigma = XP_{,X} + 2X^2 P_{,XX}, \\ \lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX} = \frac{1}{3}\left(X\frac{\partial \Sigma}{\partial X} - \Sigma\right). \end{cases}$$

a special case,

$$\ldots\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} - V(\phi) \quad \begin{cases} \Sigma = \frac{H^2 M_p^2 \epsilon}{c_s^2}, \\ \lambda = \frac{H^2 M_p^2 \epsilon}{2c_s^4}(1 - c_s^2) \end{cases}$$

equivalent to
rly times:

$$\lambda)\frac{H^6}{\dot\phi_0^6} c_s^3 \times$$

Mixing of positive and negative frequency modes, $\tilde{k}_3 = k_1 + k_2 - k_3$, etc

ar, MG-L '09

Extreme non-Gaussianity: Small c_s with Modified Vacua

- The nG will be multiplied, being equivalent to particles interacting strongly since early times:

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{\rm nBD} = \frac{4}{8}(2\pi)^3\delta^{(3)}(\sum \vec{k}_i)(\Sigma(1-\frac{1}{c_s^2})+2\lambda)\frac{H^6}{\dot{\phi}_0^6}c_s^3 \times$$

$$\mathcal{R}e\left[\sum_j^3 i\beta_{k_j}\int_{\eta_0}^{\eta}\frac{d\eta}{a^2}\frac{1}{k_1k_2k_3}e^{i\tilde{k}_j c_s\eta}\right]$$

Modified initial state

Cutoff time

Mixing of positive and negative frequency modes, $\tilde{k}_3 = k_1 + k_2 - k_3$, etc

Meerburg, v.d. Schaar, MGJ '09

Extreme non-Gaussianity: Small c_s with Modified Vacua

- The nG will be multiplied, being equivalent to particles interacting strongly since early times:

$$\langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle_{\rm nBD} = \frac{4}{8}(2\pi)^3\delta^{(3)}(\sum \vec{k}_i)(\Sigma(1-\frac{1}{c_s^2})+2\lambda)\frac{H^6}{\dot{\phi}_0^6}c_s^3 \times$$

$$\mathcal{R}e\left[\sum_j^3 i\beta_{k_j}\int_{\eta_0}^{\eta}\frac{d\eta}{a^2}\frac{1}{k_1k_2k_3}e^{i\tilde{k}_jc_s\eta}\right]$$

$$= (2\pi)^7\mathcal{R}e[\beta]P_k^2\delta(\sum \vec{k}_i)\frac{1}{k_1^3k_2^3k_3^3}\times$$

$$\left[\sum_j(\frac{1}{c_s^2}-1+\frac{2\lambda}{\Sigma})(k_1c_s\eta_0)^3\mathcal{B}^{(1)}_{k_j}+\ldots\right]$$

Modified initial state

Cutoff time

Mixing of positive and negative frequency modes, $\tilde{k}_3=k_1+k_2-k_3$, etc

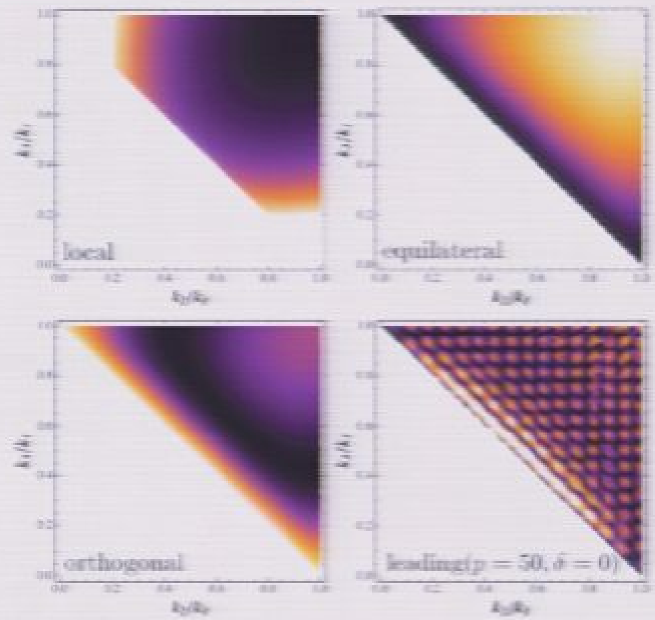
Huge enhancement, $p \sim (10^3)^3$

Oscillating function which peaks near $\tilde{k}_j=0$

Meerburg, v.d. Schaar, MGJ '09

Constraining Small-c_s Models

- **We must project our signal onto the 'equilateral', 'orthogonal', and 'local templates** (Babich, Creminelli, Zaldarriaga '04; Furgesson and Shellard '08; Senatore '10)
- **This lets us use the existing bounds on f_{NL}^{local}, f_{NL}^{equi}, f_{NL}^{ortho} to constrain this type of nG:**



$$f_{\rm NL}^{\rm local} \simeq \frac{1}{176.5}\left(-\frac{5}{4}\cos\delta + \frac{5}{3}p^{-1}\sin\delta\right)\left(\frac{1}{c_s^2} - 1 + \frac{2\lambda}{\Sigma}\right)|\beta|p^2,$$

$$f_{\rm NL}^{\rm eq} \simeq -\frac{5}{3}\left[\left(\frac{1}{c_s^2} - 1\right)\left(\frac{-2}{7.9}|\beta|p\sin\delta + 0.04\right) + \left(\frac{-2}{7.9}|\beta|p\sin\delta - 0.01\right)\frac{2\lambda}{\Sigma}\right],$$

$$f_{\rm NL}^{\rm ort} \simeq \frac{1}{13.8}\left(\frac{5}{6}\cos\delta + 5p^{-1}\sin\delta\right)\left(\frac{1}{c_s^2} - 1 + \frac{2\lambda}{\Sigma}\right)|\beta|p^2.$$

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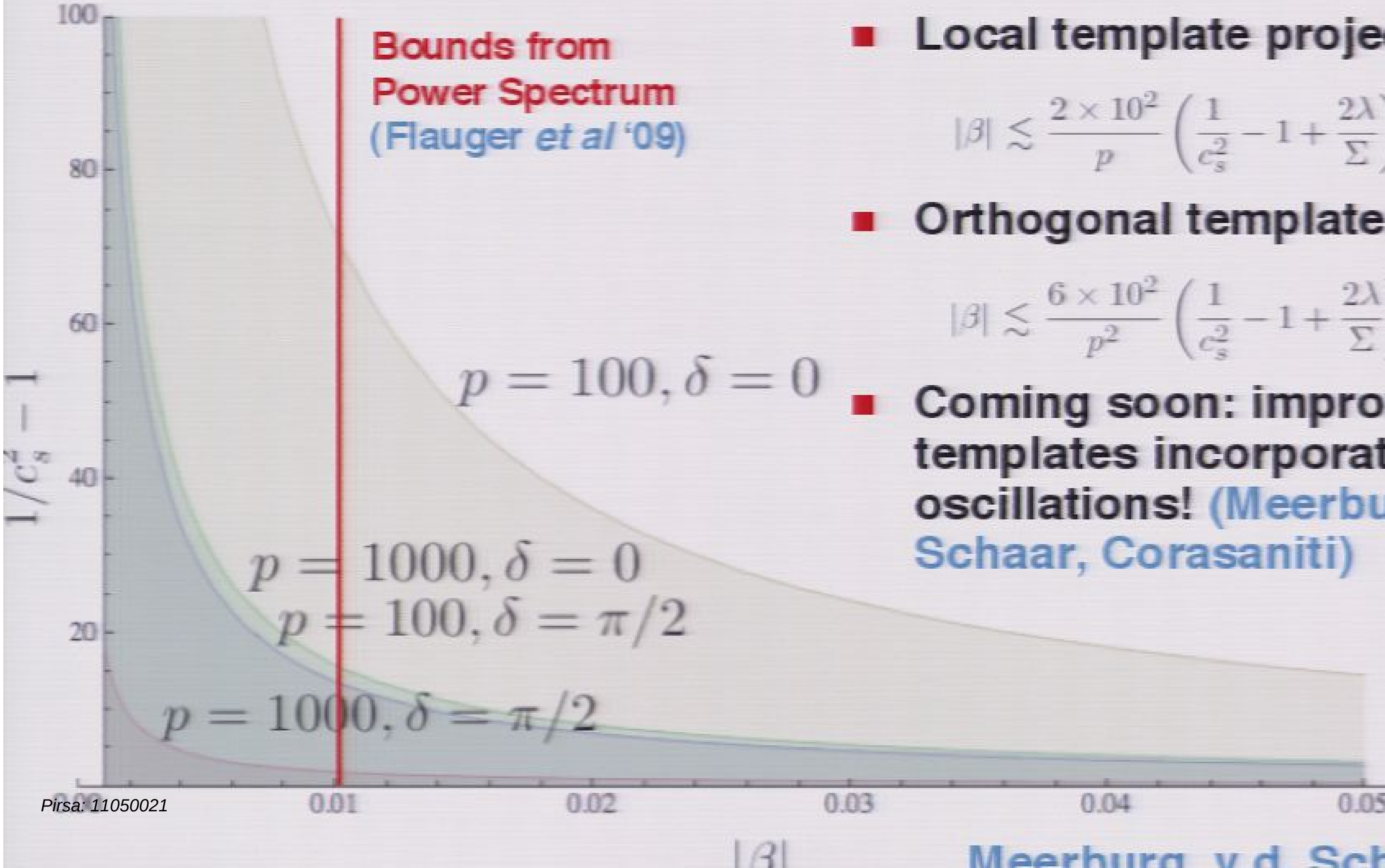
Constraining Vacuum Modification



- **Local template projection:**

$$|\beta| \lesssim \frac{2\times 10^2}{p}\left(\frac{1}{c_s^2} - 1 + \frac{2\lambda}{\Sigma}\right)^{-1}, \quad (\delta = \pi/2).$$

- **Orthogonal template projection:**

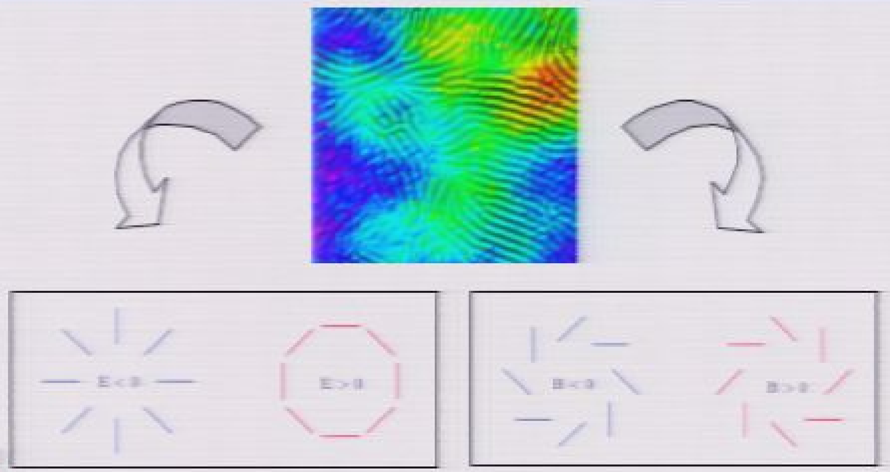
$$|\beta| \lesssim \frac{6\times 10^2}{p^2}\left(\frac{1}{c_s^2} - 1 + \frac{2\lambda}{\Sigma}\right)^{-1}, \quad (\delta = 0).$$

- **Coming soon: improved templates incorporating oscillations!** (Meerburg, van der Schaar, Corasaniti)

Meerburg, v.d. Schaar, MG-J '09

CMB Polarization and High-Energy Signatures

Sourced by scalar and tensor perturbations



Sourced by vector and tensor perturbations

- Utility of ~~*CMBPol*~~ *Inflation Probe*, a polarization-dedicated experiment, studied in NASA/Fermilab Decadal Survey White Paper

  (Baumann, MGJ '08; 57 contributors; 7 countries)

- Conclusion: A detectably large tensor amplitude would demonstrate that inflation occurred at a very high energy scale, comparable to GUTs,

$$P_t \sim (H/M_{\rm pl})^2$$

implying that we should see high-energy effects in upcoming data

Conclusion

- We are anticipating the deluge of upcoming precision cosmological data in several ways:
  1. Effective Field Theory in Inflation can now be performed for fundamental theories
  2. Non-Gaussianity templates including modified vacuum-signatures to study interactions
  3. CMB *B*-Polarization detection implies we will see high-energy effects in e.g. *Planck* data