

Title: Holography of dilatonic black holes

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Abstract: We discuss bulk and holographic features of black hole solutions of 4D anti de Sitter Einstein-Maxwell-Dilaton gravity. At finite temperature the field theory holographically dual to these solutions has a rich and interesting phenomenology reminiscent of electron motion in metals:

phase transitions triggered by nonvanishing VEV of scalar operators, non-monotonic behavior of the electric conductivities etc. Conversely, in the zero temperature limit the transport properties for these models show an universal behavior.

HOLOGRAPHY OF DILATONIC BLACK HOLES



Perimeter Institute for Theoretical Physics May, 13 2011

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Mainly based on M.C, G D'Appollonio, P. Pani, JHEP 1003,100 (2010), [arxiv:0912.3520]

Summary

- Introduction and motivations
- The rudiments of the holographic AdS/CFT correspondence
- Holographic superconductors
- Einstein-Maxwell-dilaton gravity and phase transition triggered by a neutral scalar
- Holography of charged dilatonic black holes: **finite temperature**
- Holography of charged dilatonic black holes: **the zero temperature limit**
- Concluding remarks

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Introduction and motivation

- **Black holes:** Powerful magnifying glass into the realm of quantum gravity . Many recent crucial advances triggered by BH
 - Holographic principle
 - BH entropy from counting microstates
- **Could BH physic be relevant for condensed matter physics?**
 - At the kinematical level, yes . Black holes analogues: formation of horizons seen by perturbations in fluids, dielectric media etc.
 - At dynamical level the answer is also yes mainly because of the **AdS/CFT correspondence** :
 - ✓ In the large N limit one can deal with strongly coupled QFT by investigating classical gravity

- This gives a powerful tool to go beyond the standard paradigm of **weakly coupled quasiparticle** to tackle strongly coupled systems (non Fermi liquids, high temperature superconductors...).
- Other important feature of AdS/CFT : geometric interpretation of **UV-IR flow**. **Asymptotically AdS solutions may become in the near-horizon region domain-wall or Lifshitz solutions**. This allows natural breaking of both spacetime (conformal, Poincare') and internal gauge symmetries in the IR:

□ Meanwhile this holographic approach has become an independent field of research (**AdS/CM correspondence**) and has been used in so many different contexts to give a quantitative description of QFT in terms of **thermodynamics, phase transitions, transport coefficients, spectral functions etc.**

- ❑ A non exhaustive list includes:
 - ✓ Long wavelength (hydrodynamical) limit of strongly coupled QFT
 - ✓ Transport coefficients of relativistic plasmas
 - ✓ Holographic superconductors
 - ✓ Generation of Fermi surfaces
 - ✓ Non Fermi liquids
 - ✓ Quantum phase transitions
 - ✓ Impurities and strange metals
 - ✓
- ❑ In the following I will focus on the holographic description of Einstein-Maxwell gravity non-minimally coupled to scalar field (Einstein-Maxwell-Dilaton gravity (EMDG)). This is important for several reasons

- Generalization of Einstein-Maxwell gravity covariantly coupled to a (charged) scalar, which describes holographic superconductors
- Interesting thermodynamical phase structure (phase transition triggered by a neutral scalar condensate)
- Rich phenomenology concerning the transport properties of the dual QFT
- Non-minimal coupling between scalar and $U(1)$ field very common in SUGRA and Low-energy string theory
- EMDG allow for gravitational backgrounds dual to Lifshitz theories

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Basic dictionary of the AdS/CFT correspondence

A) DUAL THEORIES

- Yang-Mills theory in d -dim \leftrightarrow String theory in AdS_{d+1}
- Relevant string theories live in nontrivial curved background and are not computationally under control
- In the large N limit string theory can be approximated by the effective classical gravity theory but the gauge theory remains strong-coupled, $\lambda = g_{YM} N \gg 1$
- Large N gauge theory in d -dim \leftrightarrow (Semi)classical AdS_{d+1} gravity
- The universal sector of the classical gravitational theory is Einstein-Hilbert AdS_{d+1} gravity. Other relevant bulk fields are $U(1)$ gauge field A_μ and scalar ψ (this is a consistent truncation, mass gap in the spectrum of anomalous dimension)

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} \left(R + \frac{d(d-1)}{L^2} \right)$$

- The maximally symmetric solution of the theory is AdS spacetime

$$ds^2 = \frac{L^2}{x^2} \left(-dt^2 + dy^i dy_i + dx^2 \right)$$

- The isometry group of the ST is the conformal group in d-dim: $SO(d,2)$ acting as a scaling symmetry
- Thus the QFT living in the $x=0$ boundary of the AdS ST is conformally invariant.
- Validity of the semiclassical description requires the AdS radius of curvature to be large in Planck units:

$$c \approx \frac{L^{d-1}}{\kappa^2} \gg 1$$

- c has to be interpreted as the number of DOF in the dual theory (central charge). c will scale as a power of N (e.g. N^2).

B) CORRELATION FUNCTIONS

Gauge inv. operator O in the QFT \leftrightarrow dynamical field ϕ in the bulk; e.g. global current J^μ in the QFT corresponds to Maxwell field A_a in the bulk, scalar operator O_B to scalar field ϕ and so on.

$$Z_{bulk}[\Phi \rightarrow \Phi_0] = \left\langle e^{i \int d^d x \Phi_0 O} \right\rangle_{QFT}$$

- Generating function for the operator O is the partition function of the bulk gravitational theory.
 ϕ_0 is the boundary value of ϕ . n -point functions for O are found by taking functional derivatives of Z_{bulk} with respect to ϕ_0 .

- Scaling dimensions Δ for the QFT operator O is fixed by the masses m of bulk fields. For free bosons and free fermions in the bulk we have

$$\Delta_B(\Delta_B - d) = (mL)^2, \quad \Delta_B = \frac{d}{2} + Lm$$

- For instance for a bulk scalar we have near the boundary $\Phi = \Phi_0 x^{d-\Delta} + \Phi_1 x^{\Delta} + \dots$. And the two-point function for O is

$$\langle O(y_1)O(y_2) \rangle = \frac{2\Delta - d}{L} \frac{\Phi_1}{\Phi_0}$$

- Thus, the holographic correspondence allows computation of correlators in certain **strongly coupled quantum critical theories**. The most universal deformation away from conformal invariance is placing the theory at finite temperature

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C) FINITE TEMPERATURE

- Processes at finite temperature are very difficult to compute also for a weak coupled QFT. This is not so for the holographic correspondence:
- **Boundary QFT at temperature T \leftrightarrow AdS black hole at Hawking temperature T .**
- Thermal states of the boundary QFT are identified with black hole solutions in the bulk (Schwarzschild-AdS black holes)
- The simplest bulk quantities one can calculate at finite temperature are the **free energy $F = -T \log Z$** and the **entropy S**

$$S = \frac{(4\pi)^d L^{d-1}}{2\kappa^2 d^{d-1}} V_{d-1} T^{d-1} \propto c V_{d-1} T^{d-1}; \quad c = \frac{L^{d-1}}{\kappa^2}$$

- The dependence of S from the spatial volume V and the temperature T is fixed by scale invariance but the coefficient c (the central charge) counts the degrees of freedom of the dual QFT. Notice that from the bulk point of view is an area measured in Planck units and that validity of the classical gravity description (large curvature radius of the AdS ST) requires $c \gg 1$ (large N approximation!!)
- The black hole solution describes the theory at equilibrium. Perturbing the system we can compute response functions (correlators) using the same formulas used at $T=0$.

- This basic structure of the holographic correspondence has been used to compute spectral functions of the dual QFT, nonanalyticities at complex frequencies (quasinormal modes), to investigate the long-wavelength dynamics (hydrodynamical limit) of the system etc.. But interesting results come up also when we consider a QFT with finite charge density.

HOLOGRAPHIC SUPERCONDUCTORS

(Horowitz, Hartnoll, Herzog, Gubser....)

- Finite charge density means a VEV for the time-component of a **current** J^0 on the boundary but to build a superconductor we also need a **charged scalar condensate**. In the bulk this corresponds to a U(1) gauge field A_μ and a covariantly coupled complex scalar ψ (from now on $d=3$ and $2\kappa^2=1$)

$$S = \int d^4x \sqrt{-g} \left(R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\nabla\psi - iqA\psi|^2 - m^2\psi^2 \right)$$

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- Now we need to generate a second order phase transition at $T=T_c$ between a phase with unbroken $U(1)$ symmetry, $\langle O \rangle = 0$, for $T > T_c$ and one with broken symmetry $\langle O \rangle \neq 0$ for $T < T_c$.
- In the bulk this can be realised starting from the usual **Electrically charged RN-ADS solution** with a trivial scalar field at $T > T_c$:

$$ds^2 = -g(r)e^{-\chi} dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2); \quad g(r) = \frac{r^2}{L^2} + \frac{Q^2}{4r^2} - \frac{2M}{r};$$

$$A_0 = \frac{Q}{r} - \frac{Q}{r_+}; \quad \chi = 0; \quad \psi = 0$$

if at low temperature the theory allows for a **charged black hole solution with non trivial scalar hair** and the RN solution becomes unstable we have a phase transition generating in the dual QFT a charged scalar condensate and a superconducting phase. This hairy solution has the same asymptotic AdS behaviour and same Q as RN-

- This superconducting instability can be thought of as a polarisation of the spacetime. Above T_c the whole charge is inside the RN black hole. Below T_c the hairy black hole is energetically favourite and the charge is largely carried by the scalar outside the black hole horizon.

- The field equation for the scalar is

$$\psi'' + \left(\frac{g'}{g} - \frac{\chi'}{2} + \frac{2}{r} \right) \psi' - m_{eff}^2 \psi = 0; \quad m_{eff}^2 = m^2 - \frac{q^2 A_0^2 e^\chi}{g}$$

- Stability of the AdS ST requires m^2 to be above the BF bound $-(9/4)L^2$

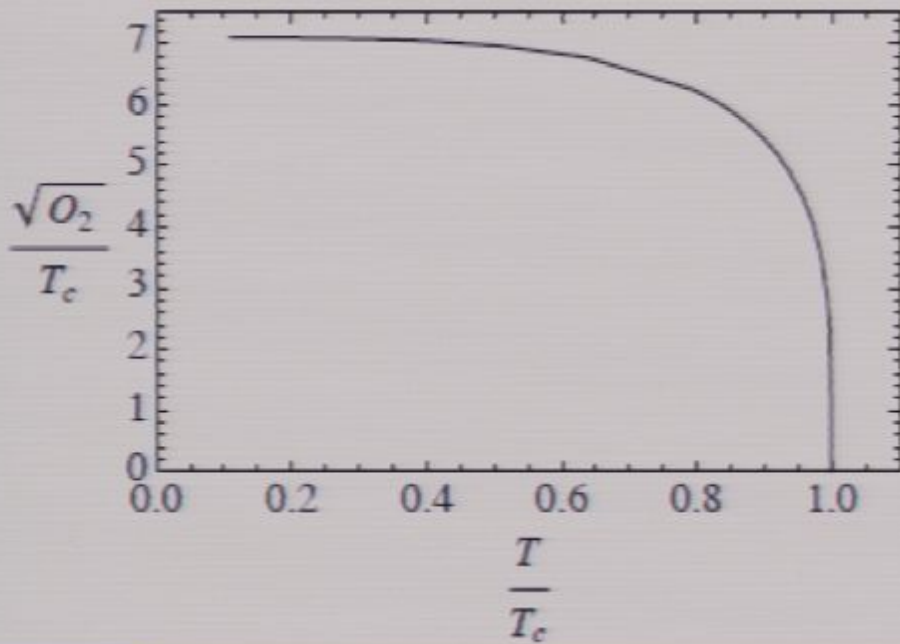
- The instability of the RN solution is due to the negative contribution to the effective mass. At low temperature this will cause the scalar hair to form.

- The scalar and electric potential behaves asymptotically

$$\psi = \frac{\psi_1}{r^{\lambda_+}} + \frac{\psi_2}{r^{\lambda_-}} + \dots; \quad A_0 = \mu - \frac{\rho}{r}; \quad \lambda_{\pm} = \frac{3 \pm \sqrt{9 + 4m^2 L^2}}{2}$$

- The holographic duality implies $\psi_2 = \langle O_2 \rangle \rightarrow$ an hairy black hole will correspond to the formation of a scalar charged condensate on the boundary.

- Using appropriate boundary conditions one can integrate numerically the field equation of the theory and find the dependence of ψ_2 from the temperature

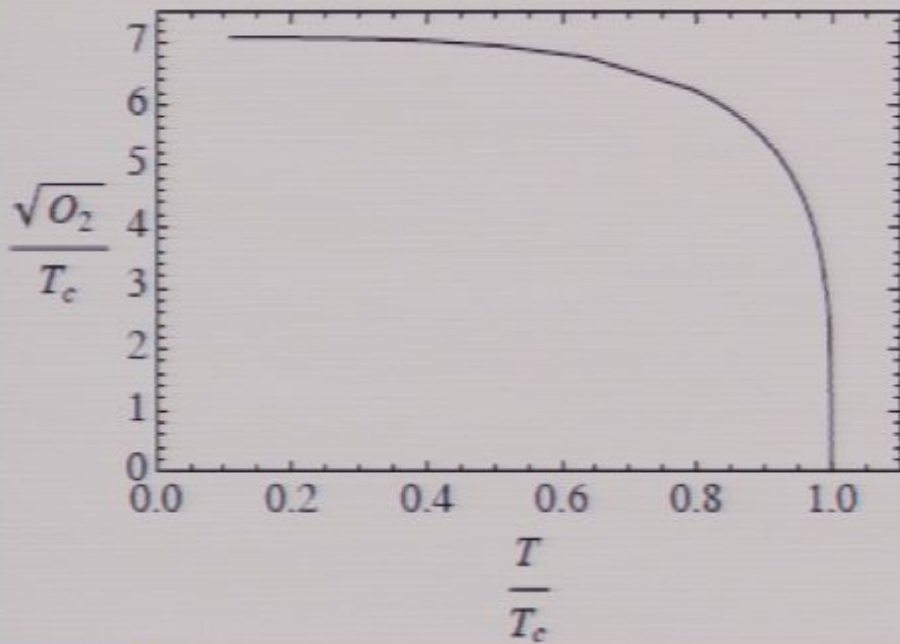


- This curve is qualitatively similar to that obtained in BCS theory and observed in many superconducting materials. The condensate raises quickly when the material is cooled below T_c and goes to a constant as T goes to zero. Near T_c has the square root behavior $O \approx (1 - T/T_c)^{1/2}$ predicted by the Landau-Ginsburg theory

CONDUCTIVITY

To compute the frequency-dependent optical conductivity in the x-direction we perturb A_x in the bulk and consider the equation for perturbations with zero spatial momentum and time dependence $e^{-i\omega t}$

$$A_x'' + \left(\frac{g'}{g} - \frac{\chi'}{2} \right) A_x' + \left[\left(\frac{\omega^2}{g^2} - \frac{A_0'^2}{g} \right) e^{\chi} - \frac{2q^2 \psi^2}{g} \right] A_x = 0$$

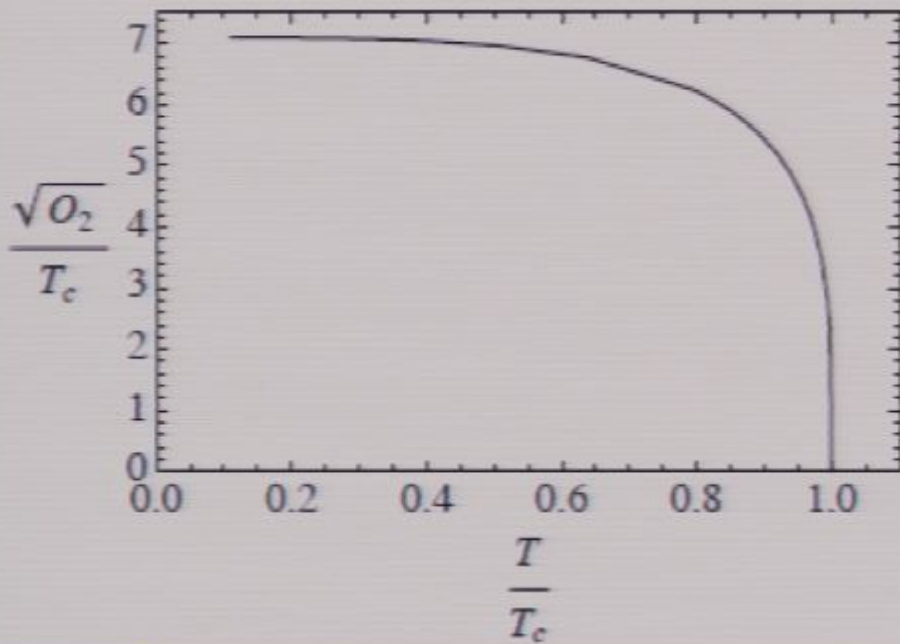


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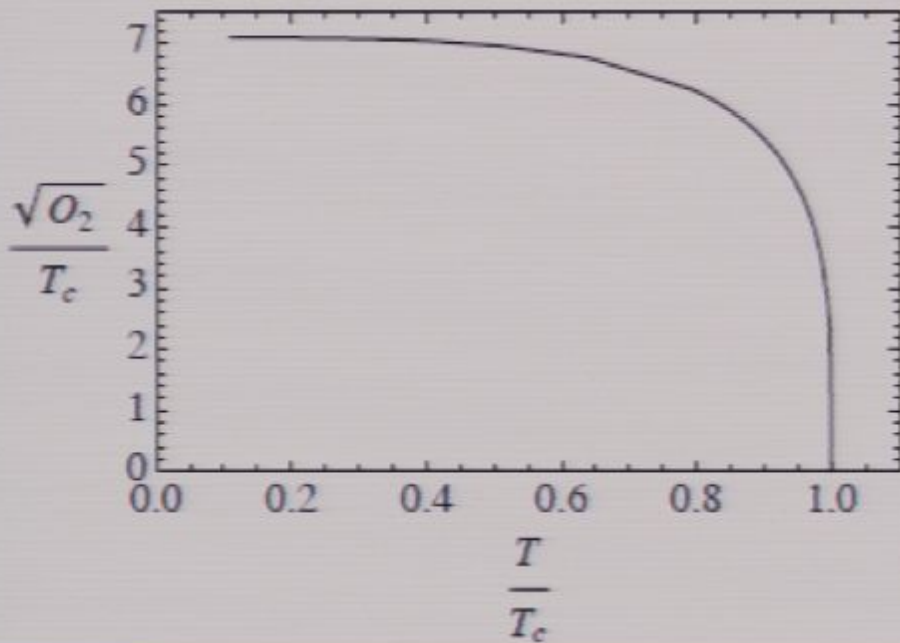
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- The holographic duality implies $\psi_2 = \langle O_2 \rangle \rightarrow$ an hairy black hole will correspond to the formation of a scalar charged condensate on the boundary.

- Using appropriate boundary conditions one can integrate numerically the field equation of the theory and find the dependence of ψ_2 from the temperature

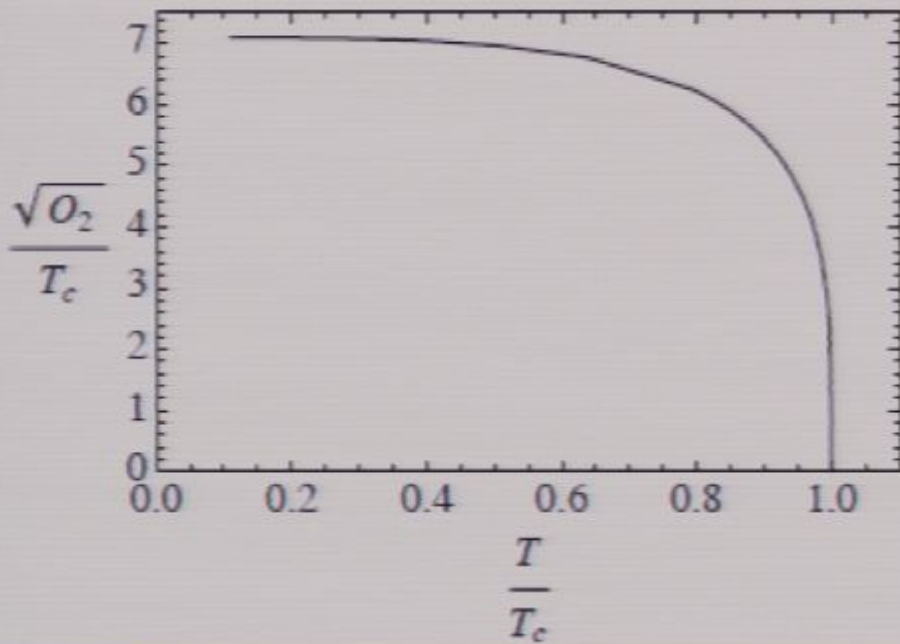


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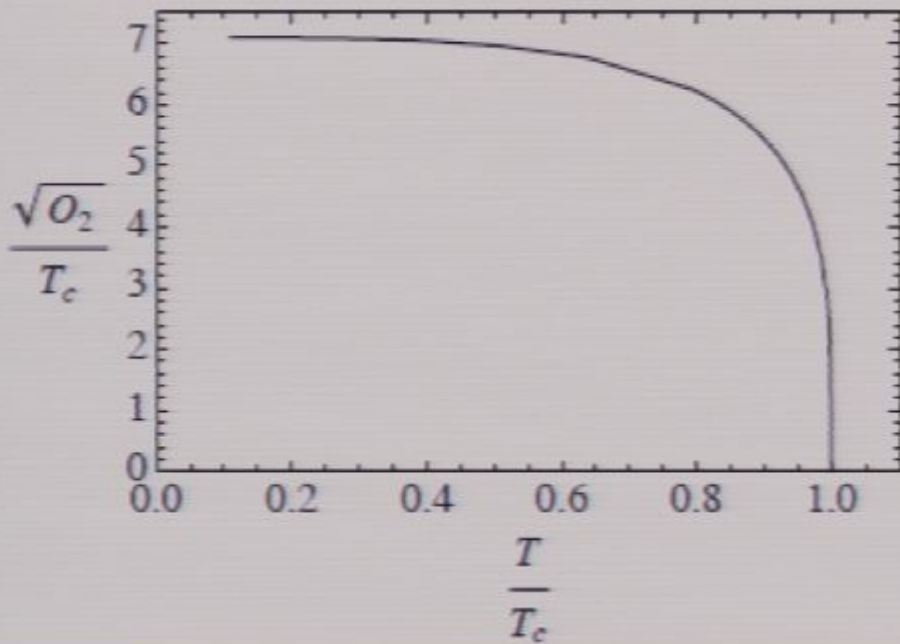


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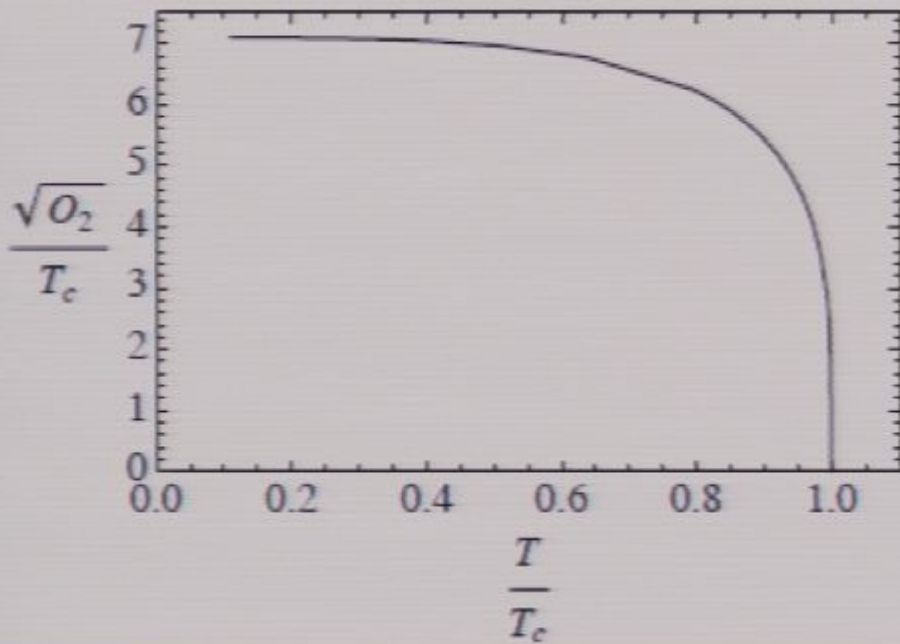


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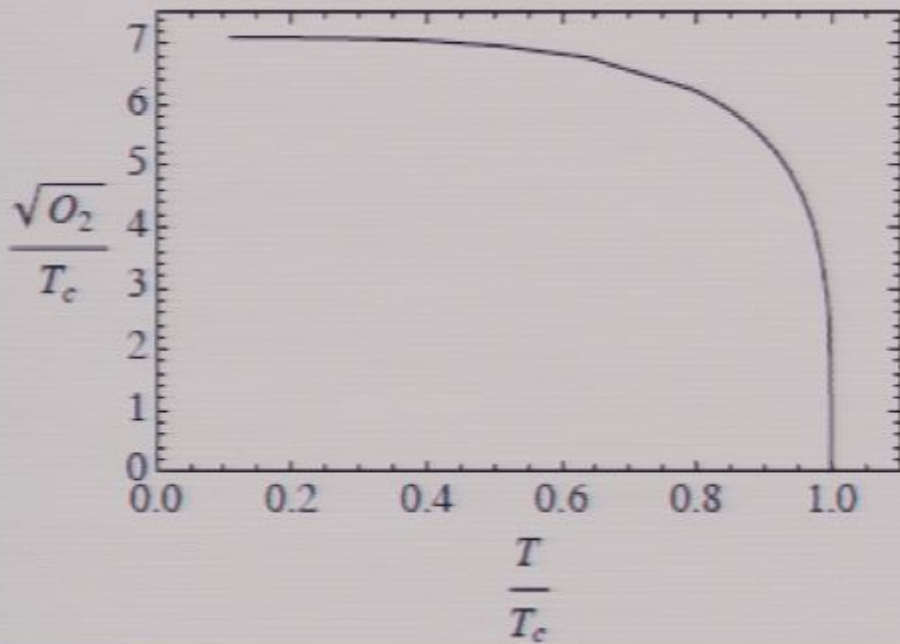


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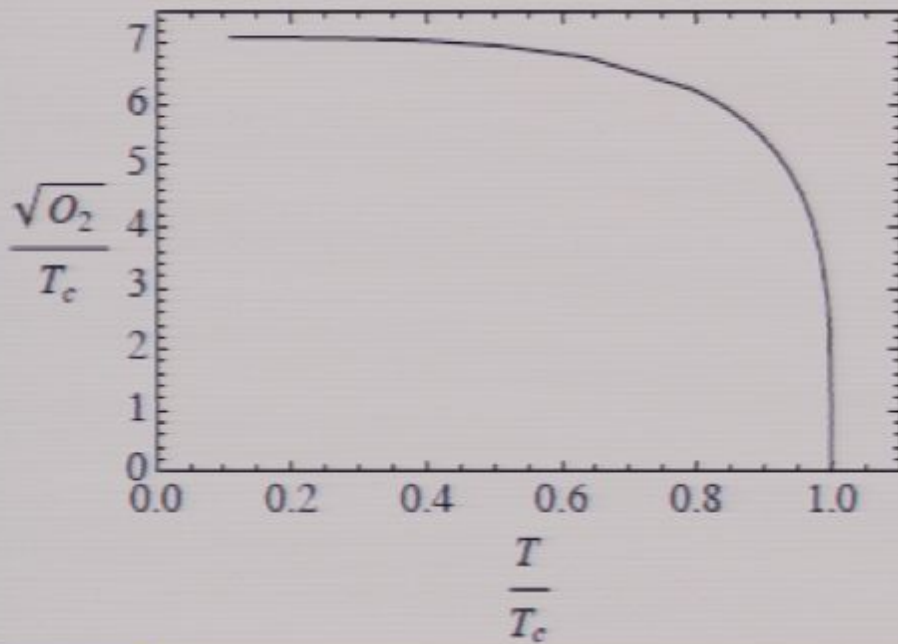


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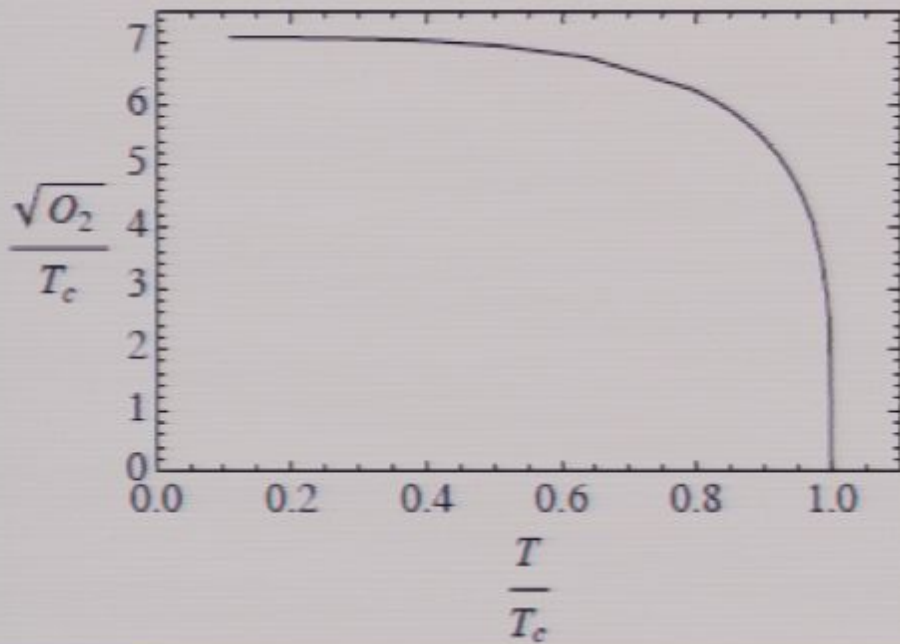


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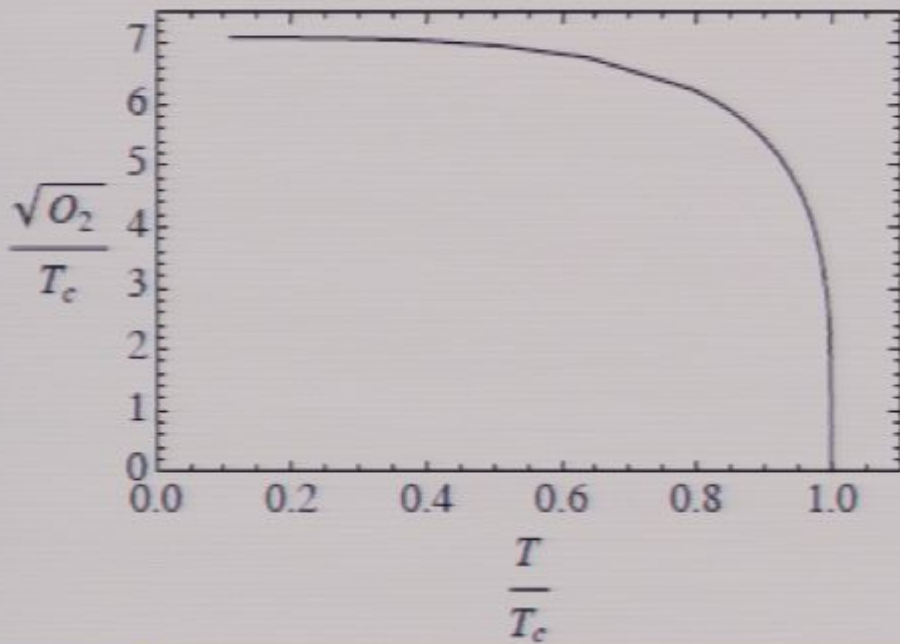


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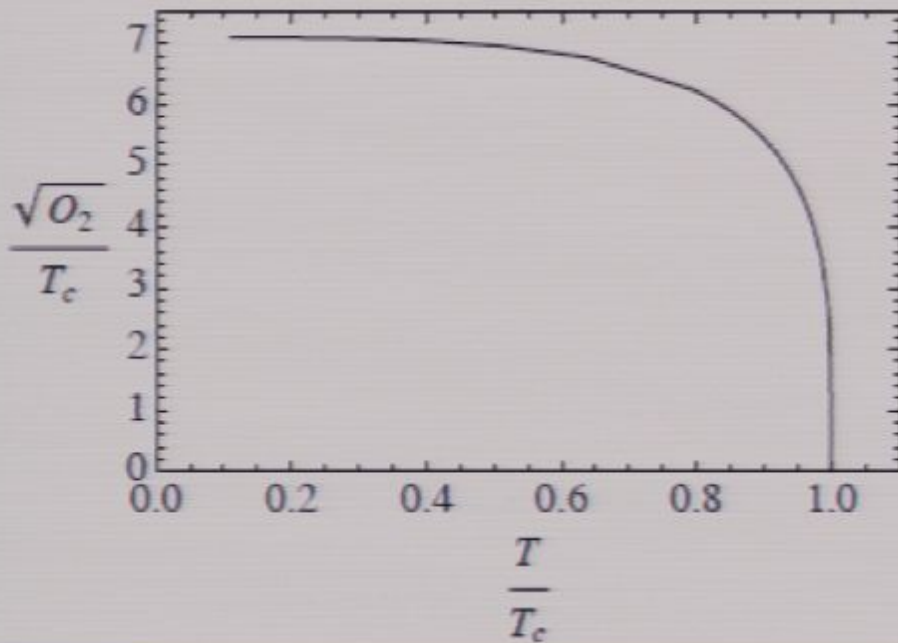


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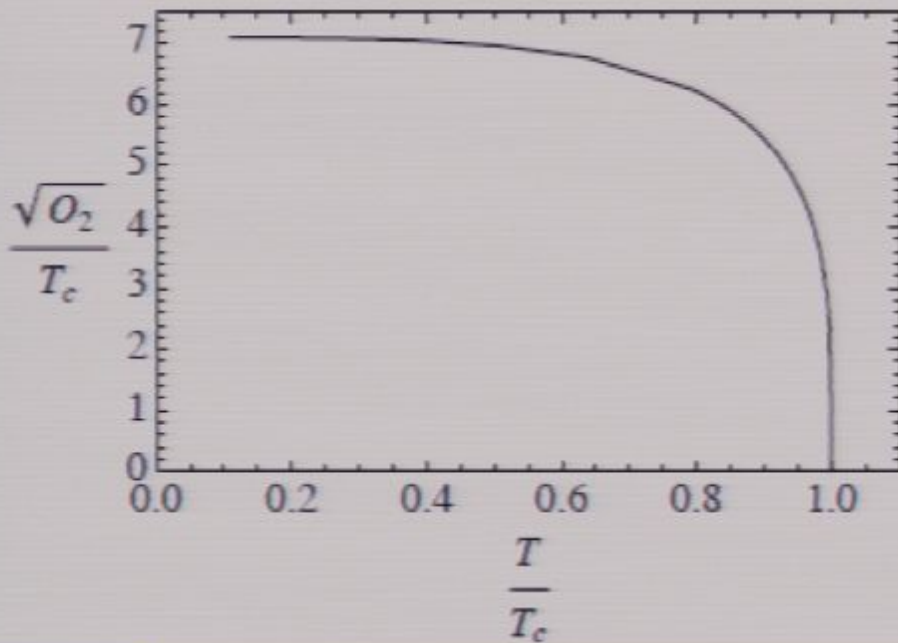


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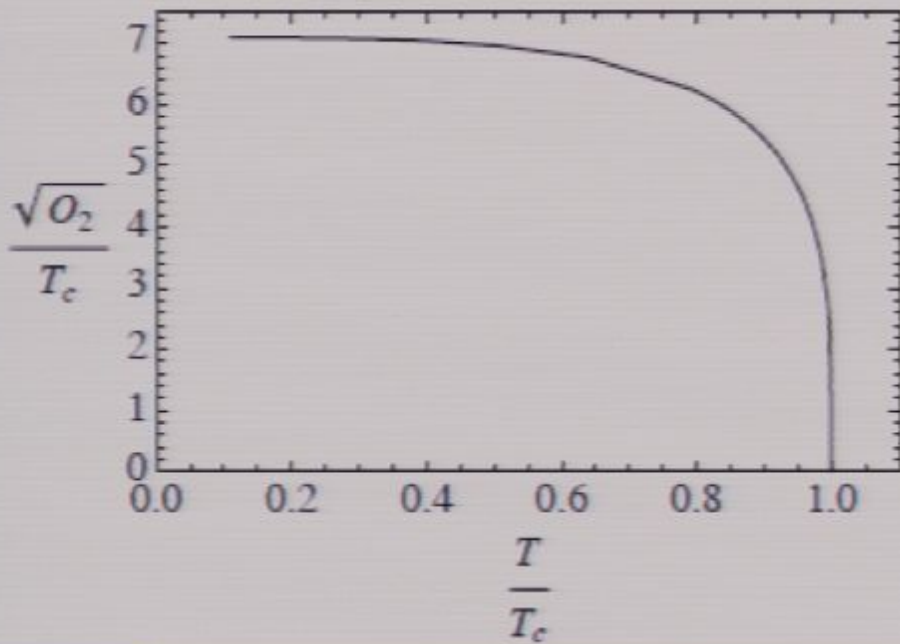


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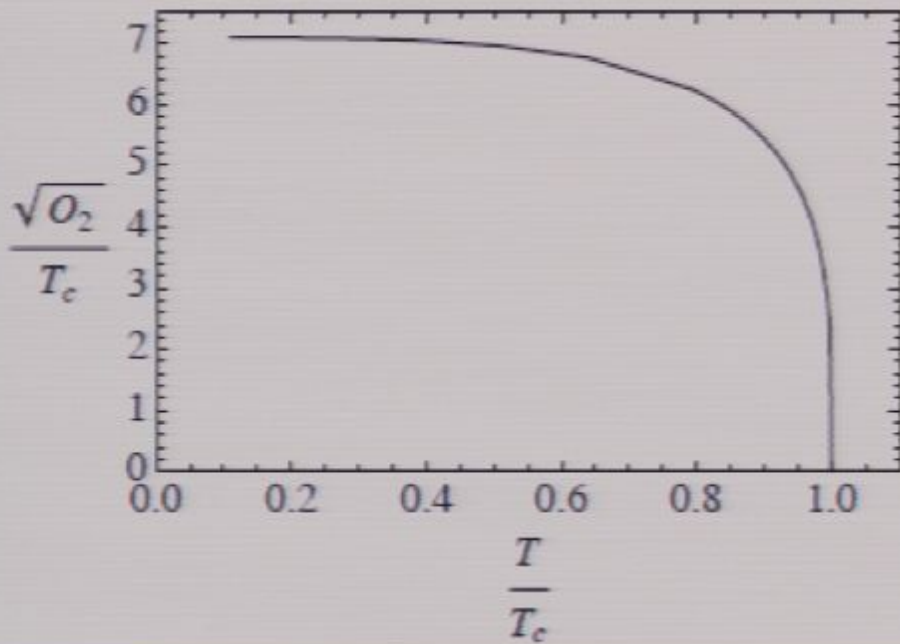


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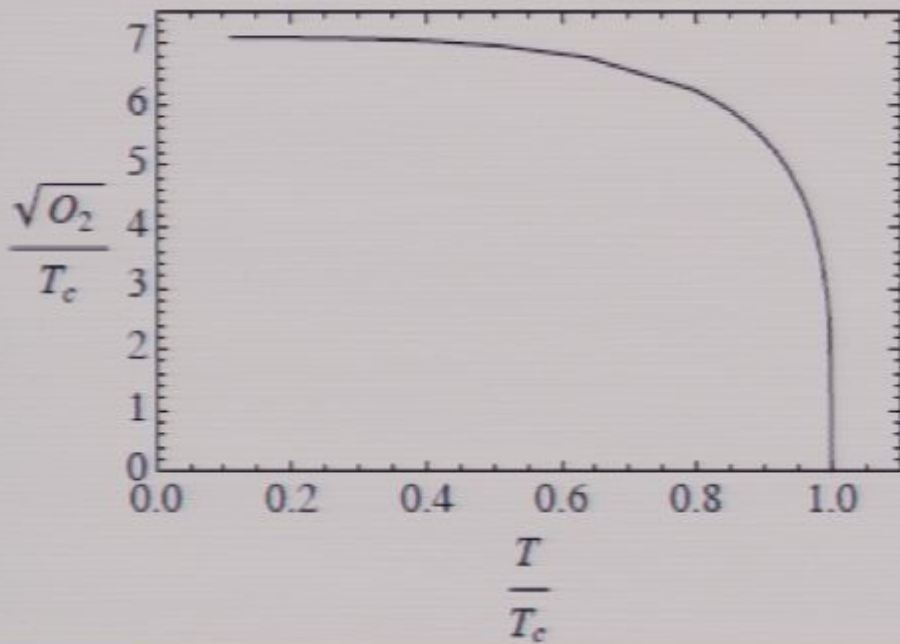


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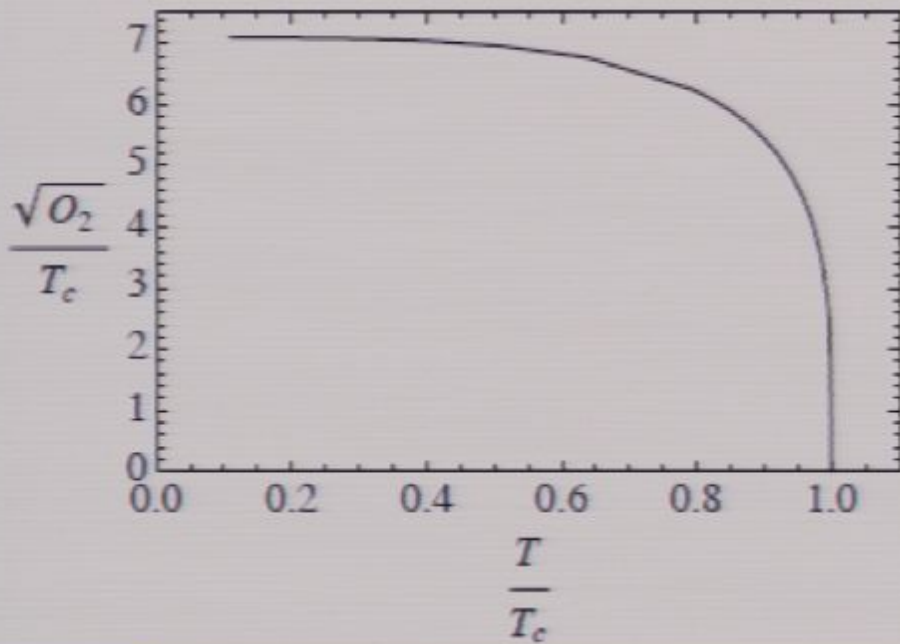


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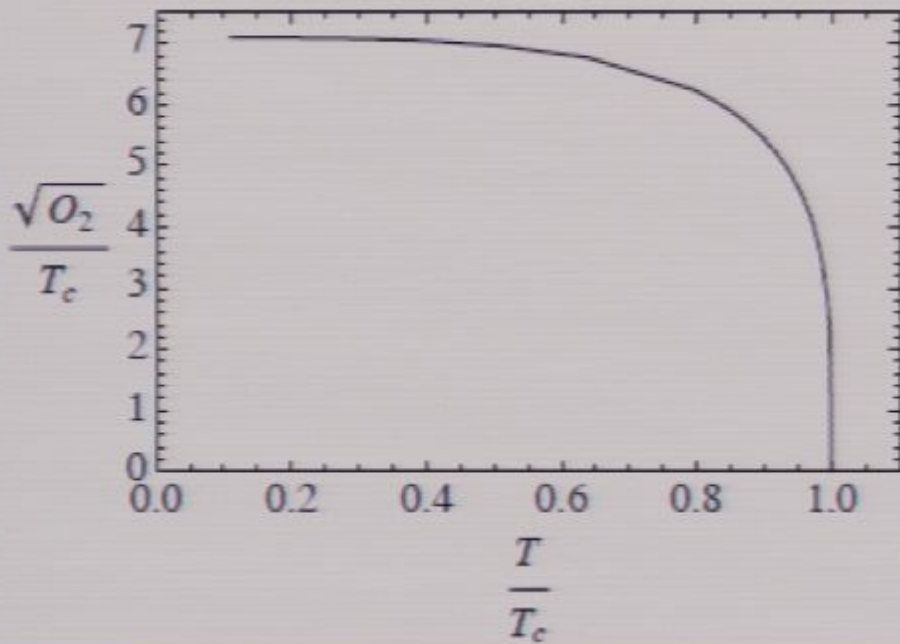


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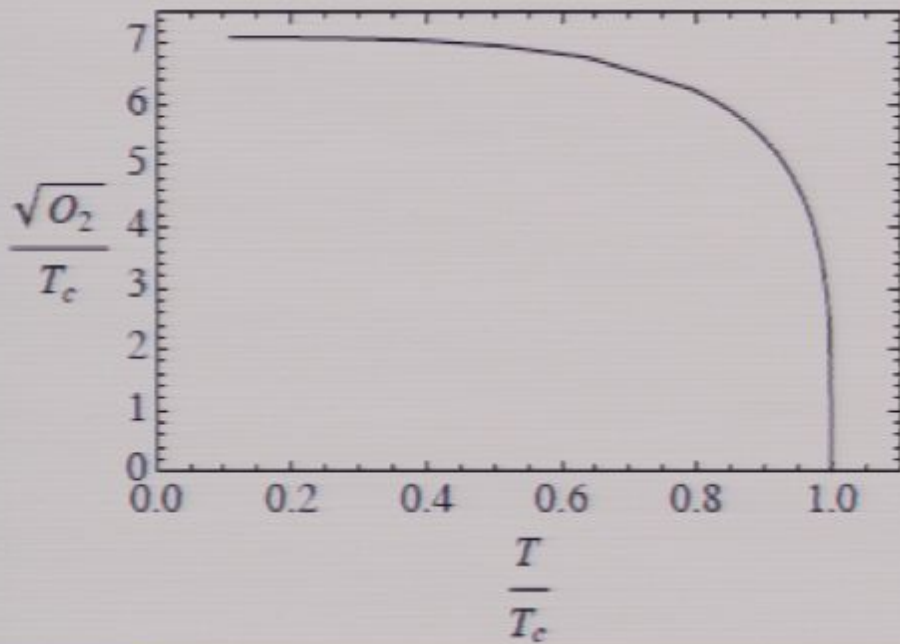


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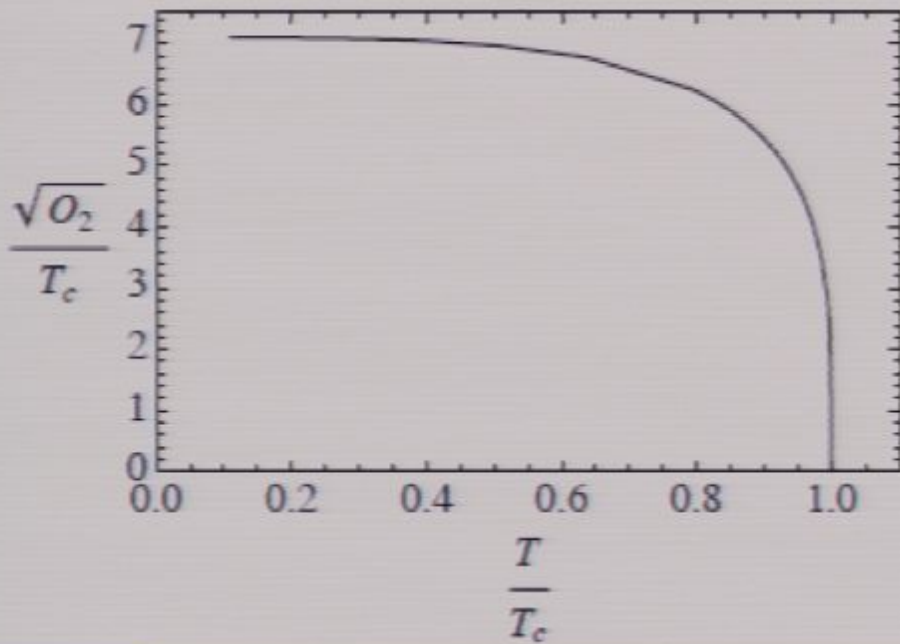


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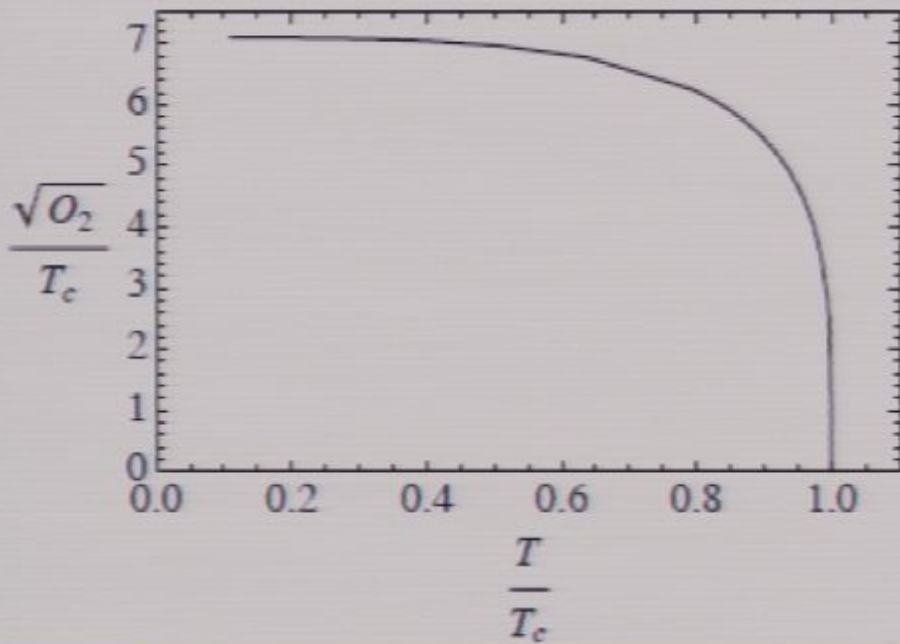


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- This has to be solved with purely ingoing boundary conditions at the horizon and asymptotically ($r=\infty$)

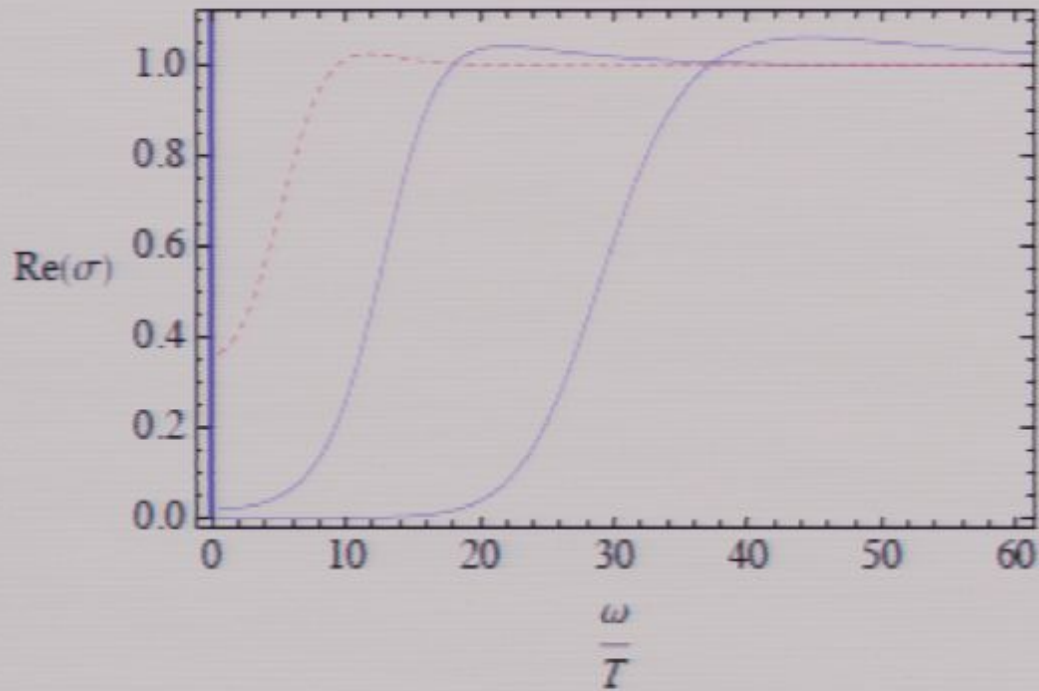
$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \dots$$

From the AdS/CFT dictionary we have

for the electric field E_x , the current J_x and the conductivity σ on the boundary

$$E_x = -\frac{\partial A_x^{(0)}}{\partial t}; \quad J_x = A_x^{(1)}, \quad \sigma = \frac{J_x}{E_x} = -\frac{iA_x^{(1)}}{\omega A_x^{(0)}}$$

- Upon numerical integration one finds the the Real and imaginary part of the conductivity



- As we lower the temperature from T_c a gap opens as expected in BCS theory. There is a delta function at $\omega=0$ (DC conductivity), but this cannot be seen from the numerical solution of the real part. The imaginary part has a pole $\text{Im}(\sigma) \approx 1/\omega$ which from the Kramers-Kronig relations imply $\text{Re}(\sigma) \approx \delta(\omega)$.

EINSTEIN-MAXWELL-DILATON GRAVITY AND PHASE TRANSITION TRIGGERED BY A NEUTRAL SCALAR

- The idea is to consider bulk AdS Einstein-Maxwell dilaton gravity in which a REAL scalar is not covariantly but NONMINIMALLY coupled to the U(1) field

$$S = \int d^4x \sqrt{-g} \left(R + \frac{6}{L^2} - \frac{1}{4} f(\psi) F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial\psi)^2 - V(\psi) \right)$$

- $f(\psi)$ is a coupling function. To allow the RN-AdS black hole solution at $\psi=0$ we must have

$$f(\psi) = 1 + \frac{\alpha}{2} \psi^2 + O(\psi^3); \quad V = -\frac{6}{L^2} + \frac{\beta}{2L^2} \psi^2 + O(\psi^3)$$

- We have shown that in these theories the RN solution may become unstable. Writing the equation for scalar perturbations one finds the effective mass

$$m_{eff}^2 = m^2 - \frac{\alpha}{2} A_0'^2$$

- If the nonminimal coupling α is strong enough it can lower the mass of the excitation below the BF bound and generating a tachyonic mode destabilizing the RN background. Approximate criteria for instability give the critical temperature

$$T_c = \frac{\sqrt{QL}}{16\pi L^2} \left(\frac{12 - \gamma}{\gamma^{1/4}} \right); \quad \gamma = \frac{2}{\alpha} \left(\frac{9}{4} + \beta \right)$$

The presence of this instability is confirmed by explicit numerical solution of the field equations: below T_c appears an hairy solution of the field equations whose free energy $F_{hairy} < F_{RN}$

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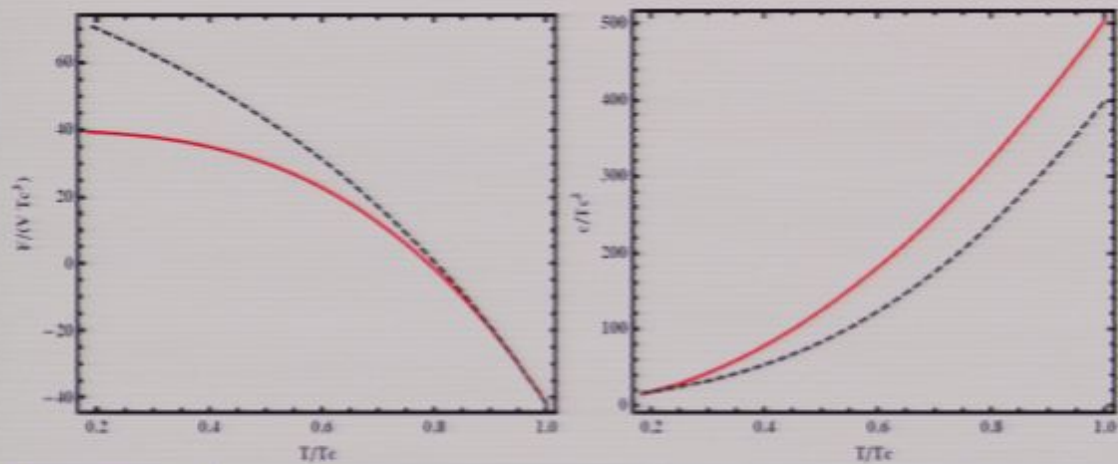
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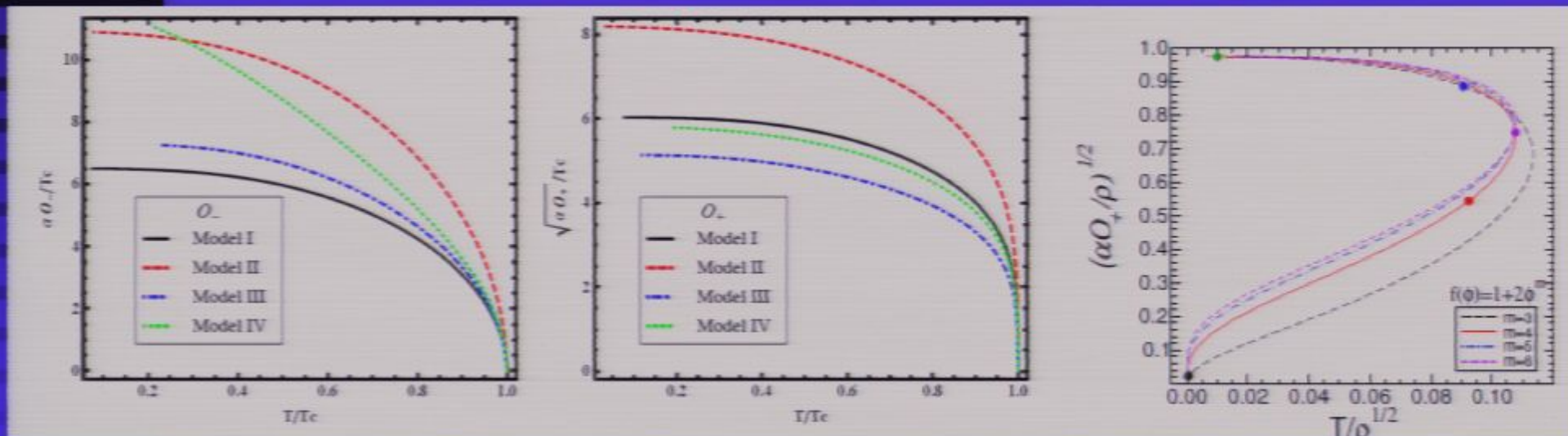
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Free energy (left) and specific heat (right) of the hairy (Red) and RN black hole below T_c

- Below T_c the black hole solution develops a neutral scalar hair \rightarrow in the dual theory we have a second order phase transition and the formation of a neutral condensate.



Near the critical temperature we have the universal Landau-Ginsburg

cooling behaviour $O_+ \sim (1 - T/T_c)^{1/2}$

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$$f(\psi) = 1 + \frac{\alpha}{2} \psi^2 + O(\psi^3); \quad V = -\frac{6}{L^2} + \frac{\beta}{2L^2} \psi^2 + O(\psi^3)$$

- We have shown that in these theories the RN solution may become unstable . Writing the equation for scalar perturbations one finds the effective mass

$$m_{eff}^2 = m^2 - \frac{\alpha}{2} A_0'^2$$

- If the nonminimal coupling α is strong enough it can lower the mass of the excitation below the BF bound and generating a tachyonic mode destabilizing the RN background. Approximate criteria for instability give the critical temperature

$$T_c = \frac{\sqrt{QL}}{16\pi L^2} \left(\frac{12 - \gamma}{\gamma^{1/4}} \right); \quad \gamma = \frac{2}{\alpha} \left(\frac{9}{4} + \beta \right)$$

The presence of this instability is confirmed by explicit numerical solution of the field equations: below T_c appears an hairy solution of the field equations whose free energy $F_{hairy} < F_{RN}$

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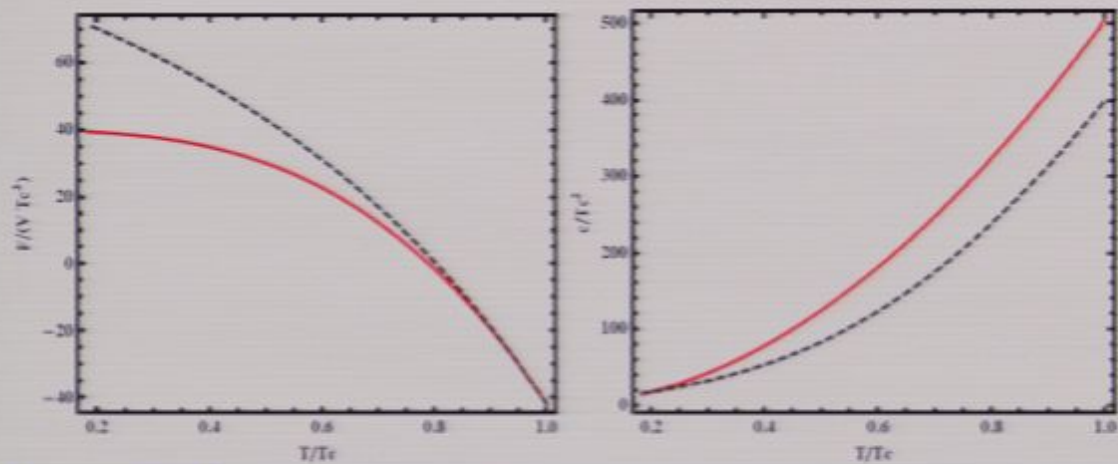
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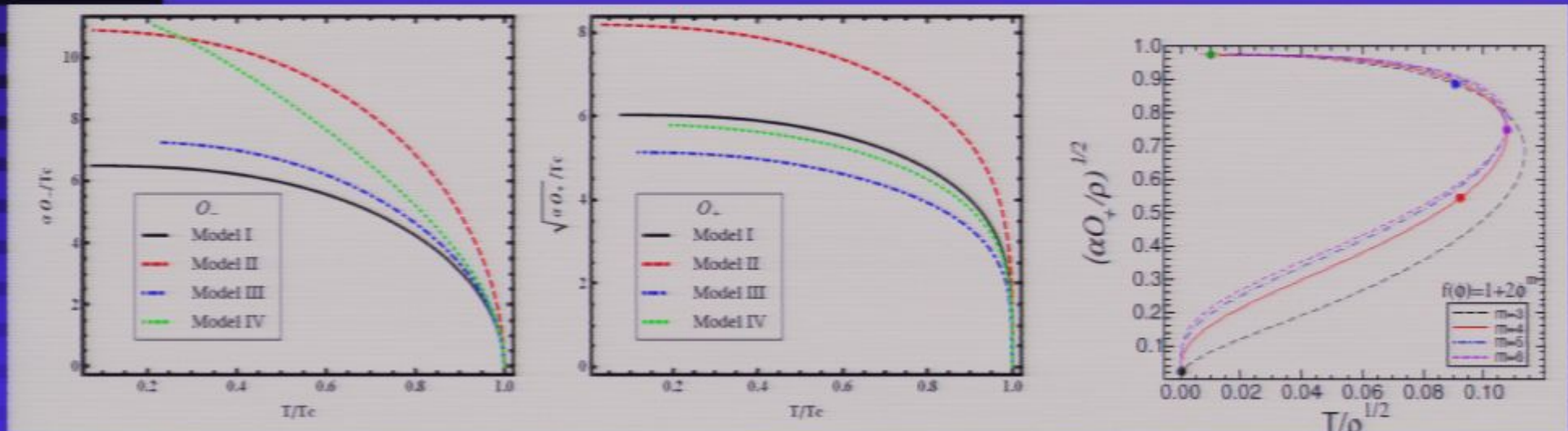
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Free energy (left) and specific heat (right) of the hairy (Red) and RN black hole below T_c

- Below T_c the black hole solution develops a neutral scalar hair \rightarrow in the dual theory we have a second order phase transition and the formation of a neutral condensate.



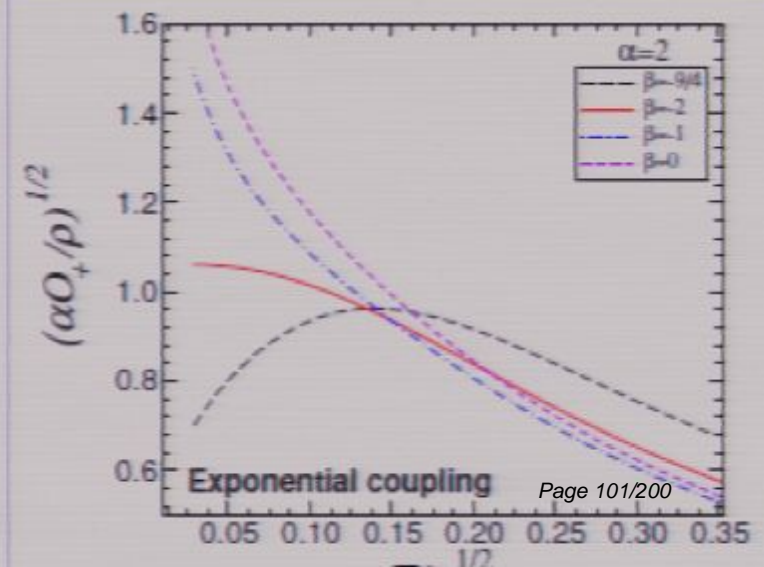
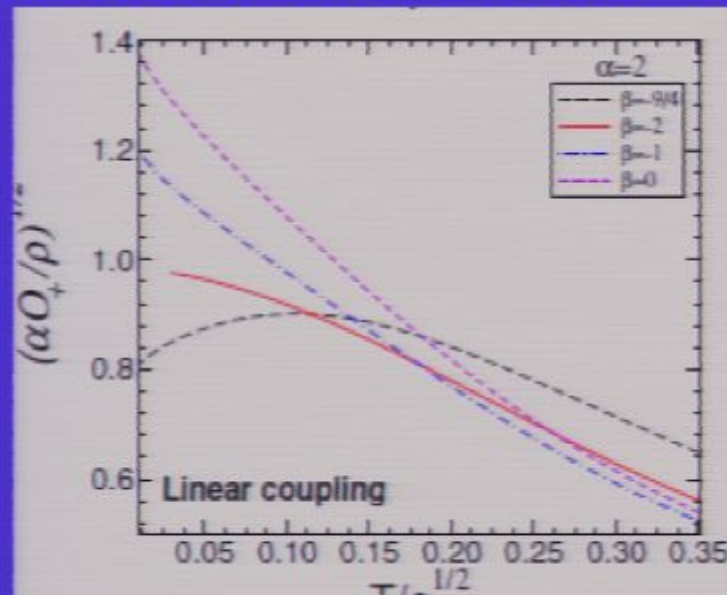
Near the critical temperature we have the universal Landau-Ginsburg

scaling behaviour $O_- \sim (1 - T/T_c)^{1/2}$

- We have investigated and explicitly checked this behaviour for models with the following coupling function and potential (for various values of the parameters)

$$f(\psi) = \cosh(\sqrt{\alpha}\psi), \quad f(\psi) = 1 + \frac{\alpha}{2}\psi^2, \quad f(\psi) = 1 + a\psi^n, \quad V(\psi) = B \cosh(\sqrt{\beta}\psi), \quad V(\psi) = -\frac{6}{L^2} + \frac{\beta\psi^2}{2L^2}$$

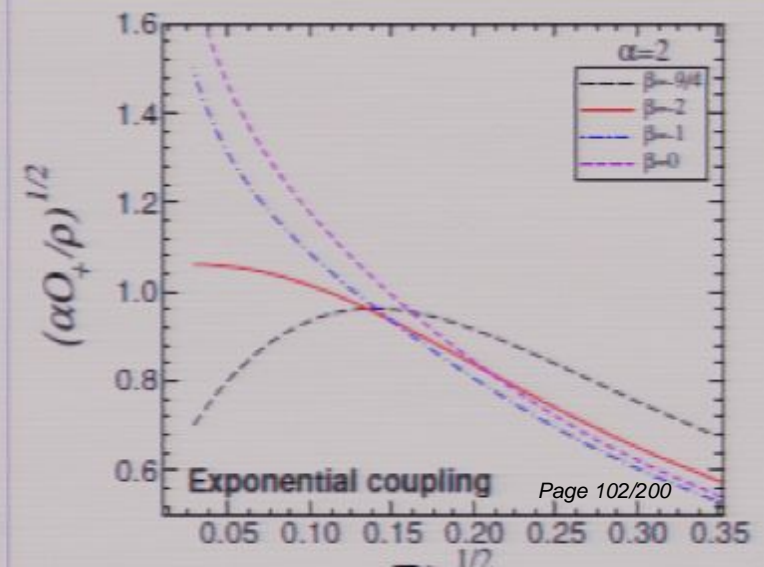
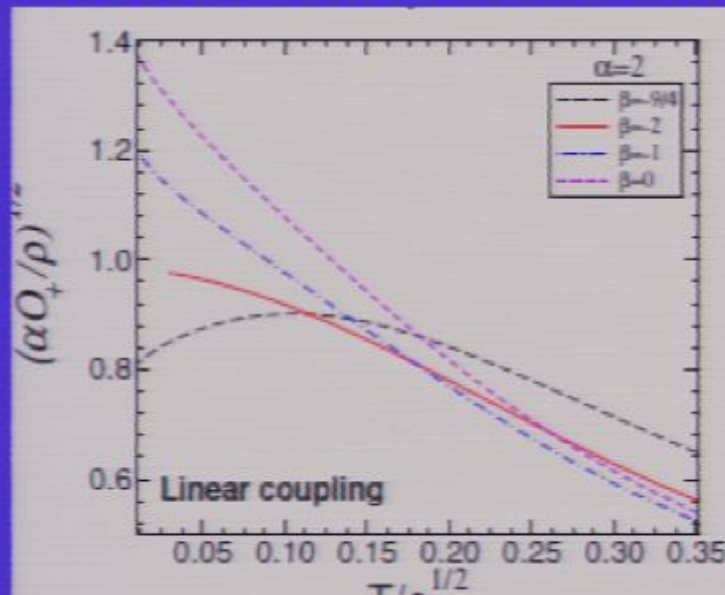
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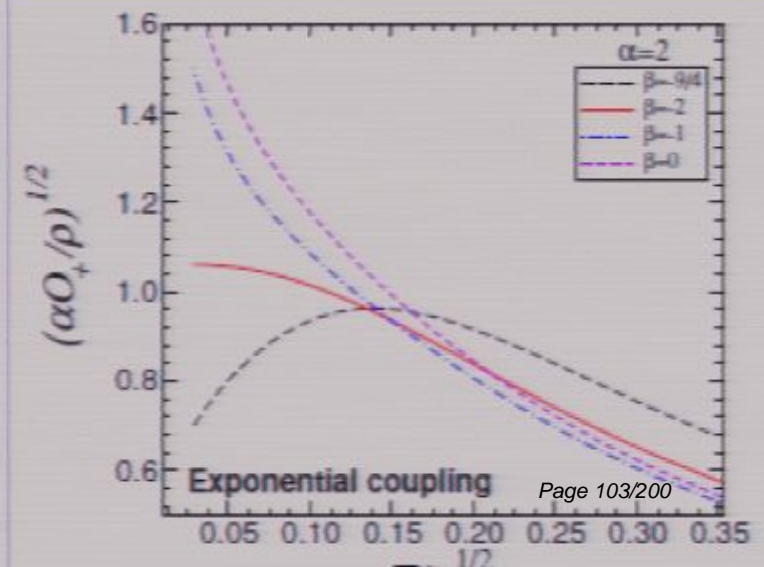
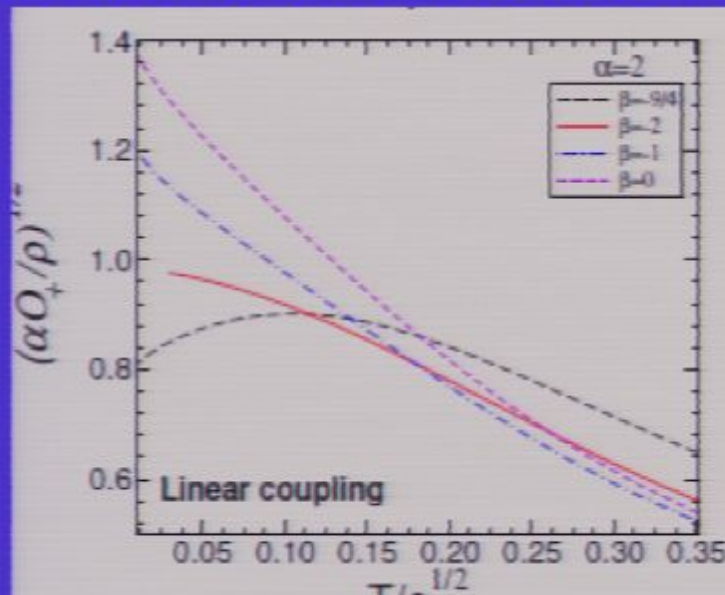
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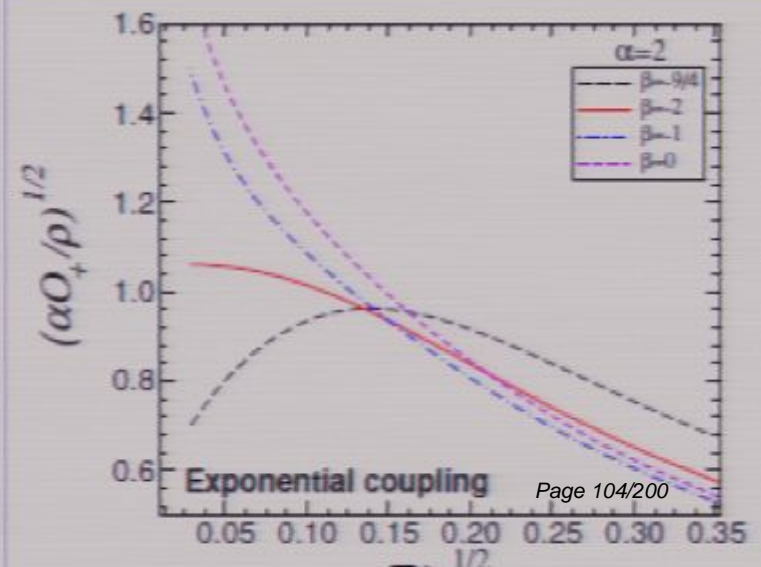
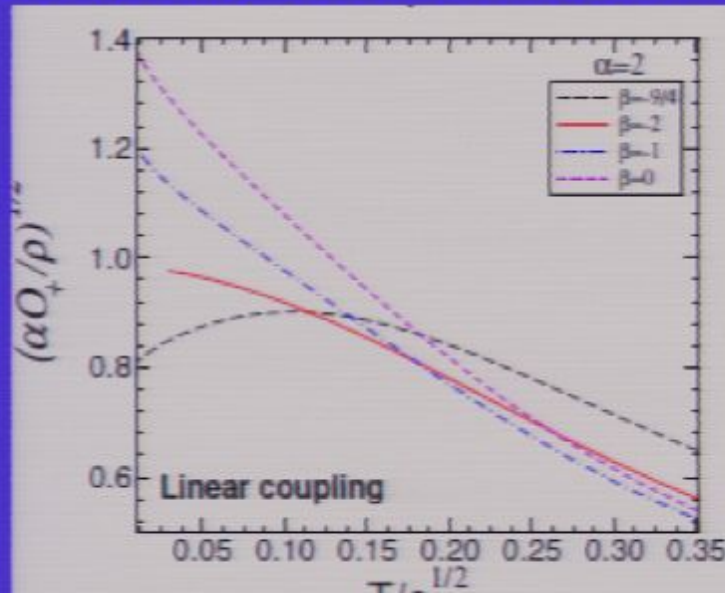
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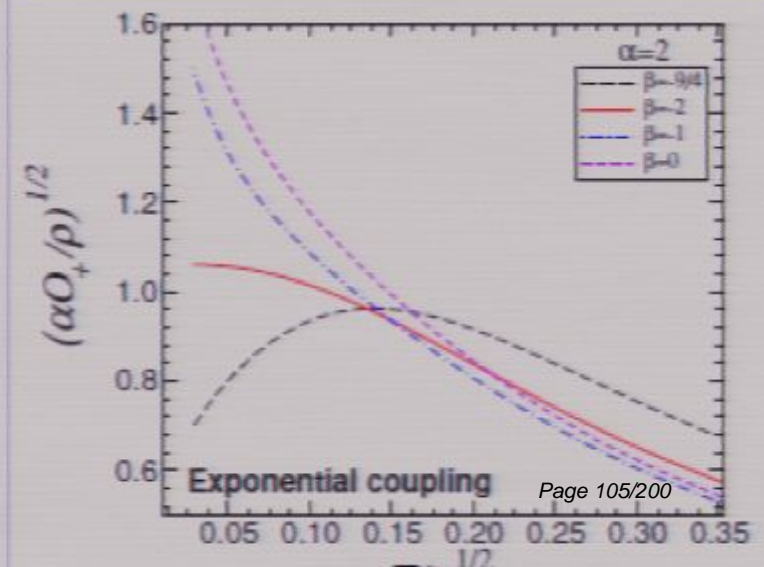
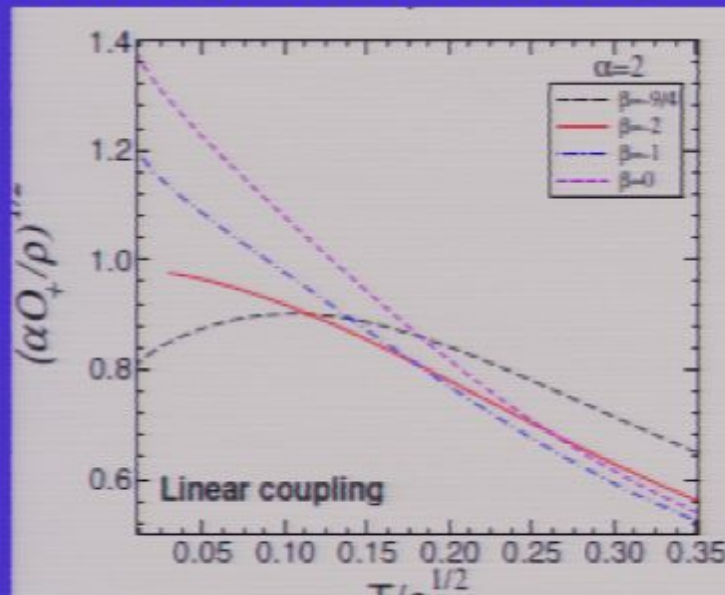
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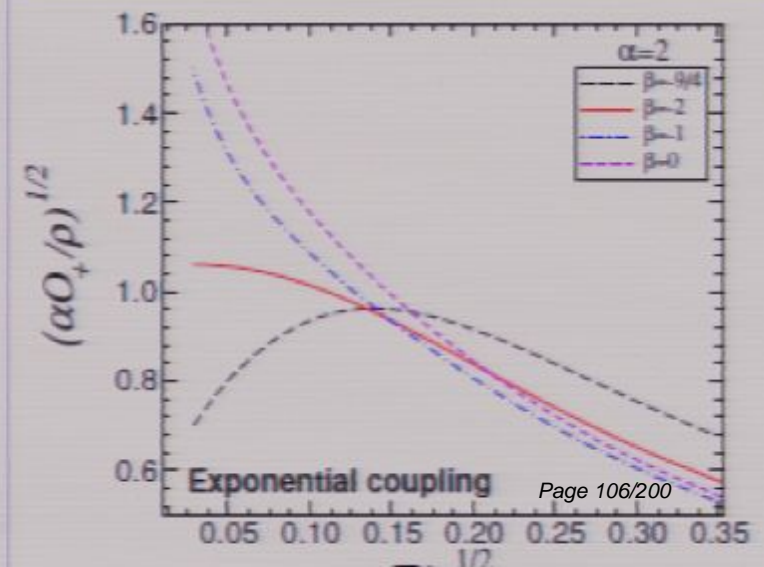
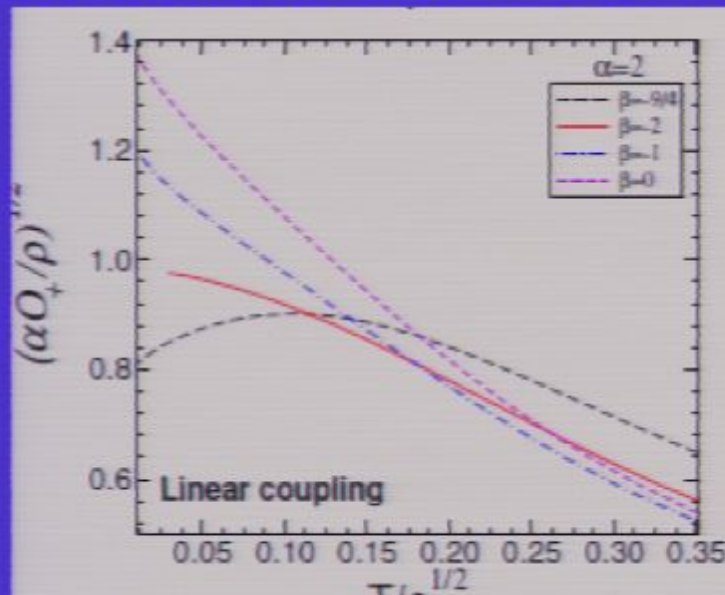
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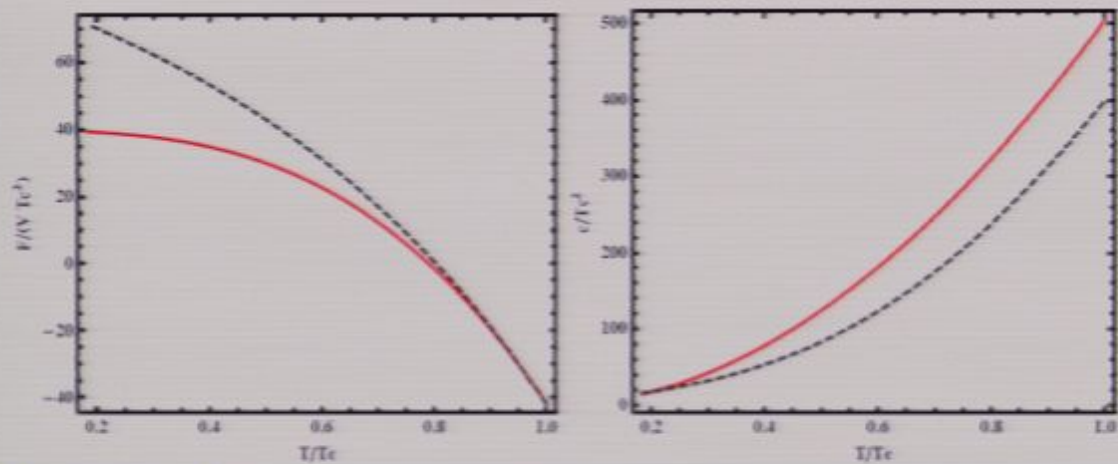


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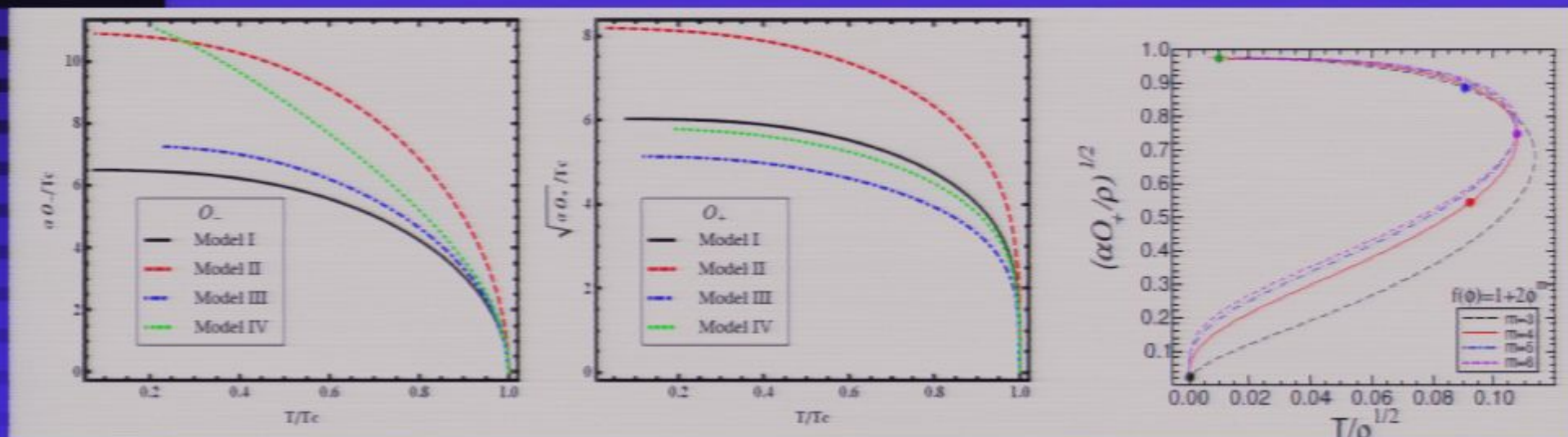
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Free energy (left) and specific heat (right) of the hairy (Red) and RN black hole below T_c

- Below T_c the black hole solution develops a neutral scalar hair \rightarrow in the dual theory we have a second order phase transition and the formation of a neutral condensate.



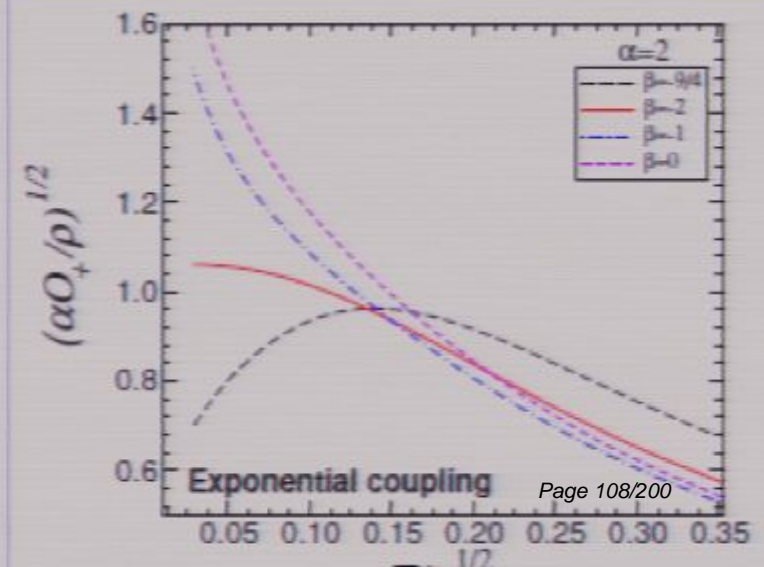
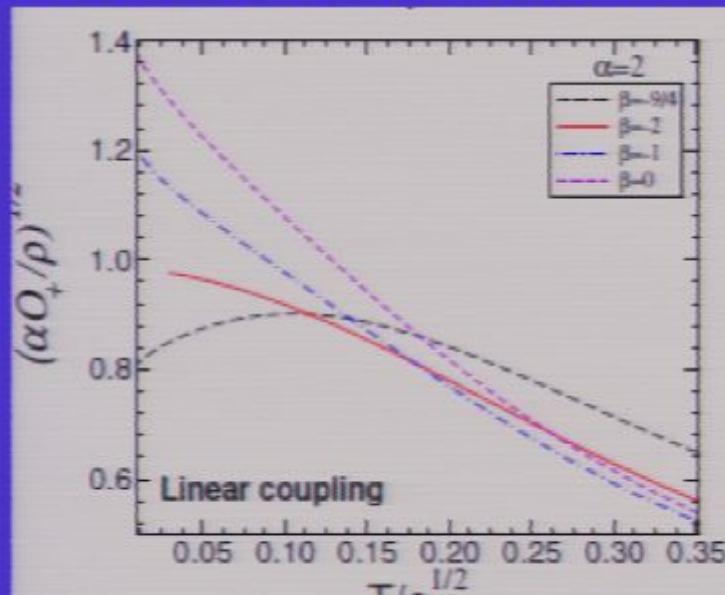
Near the critical temperature we have the universal Landau-Ginsburg

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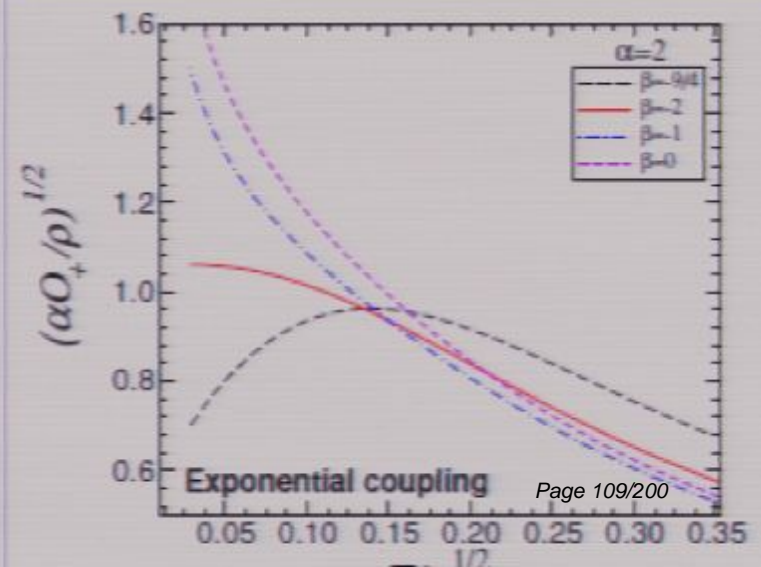
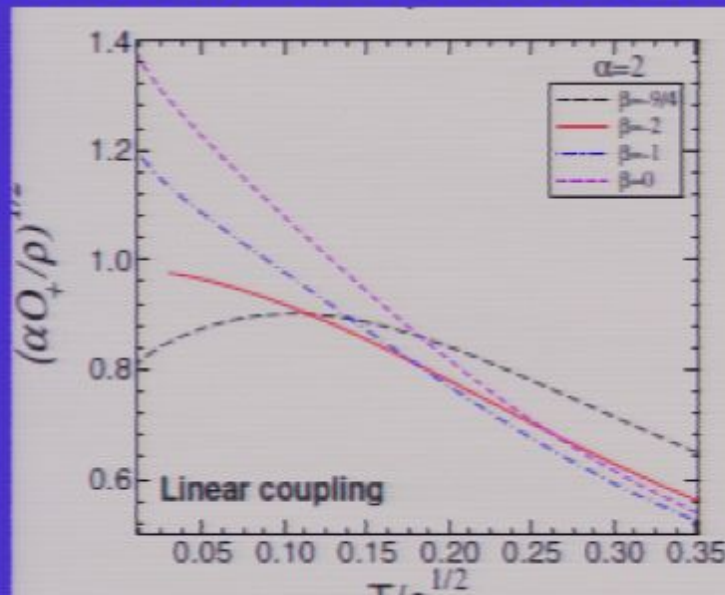
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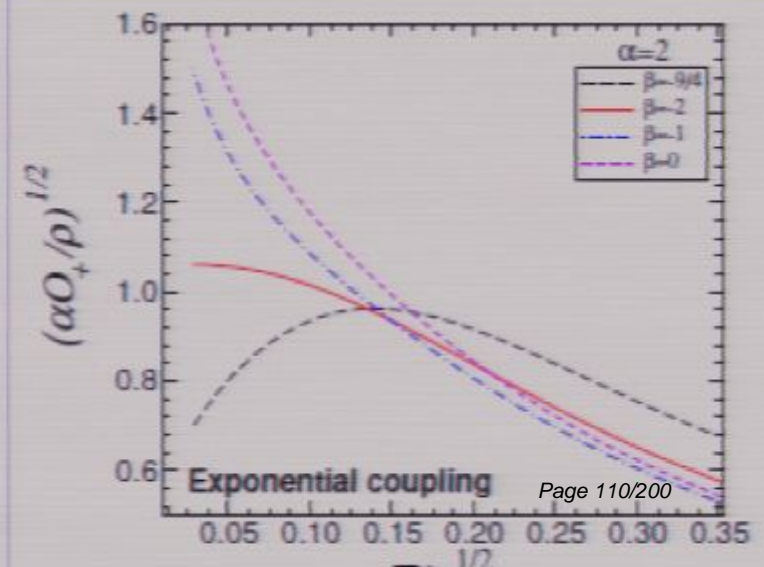
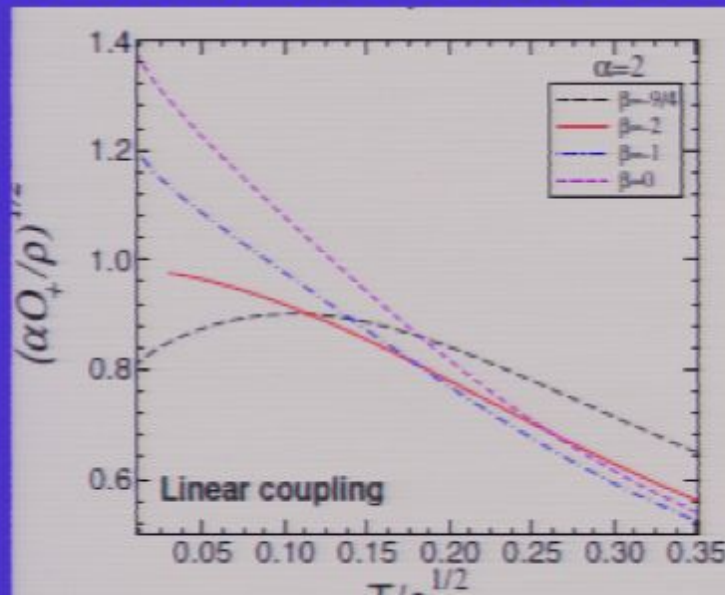
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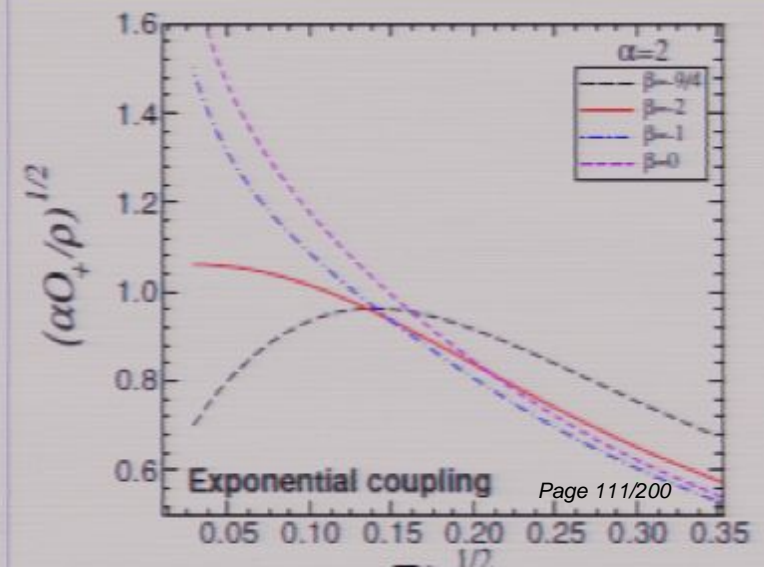
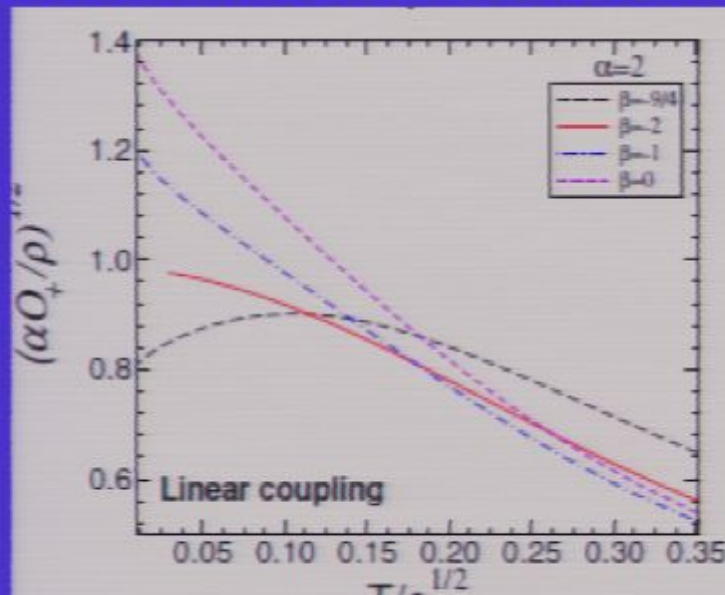
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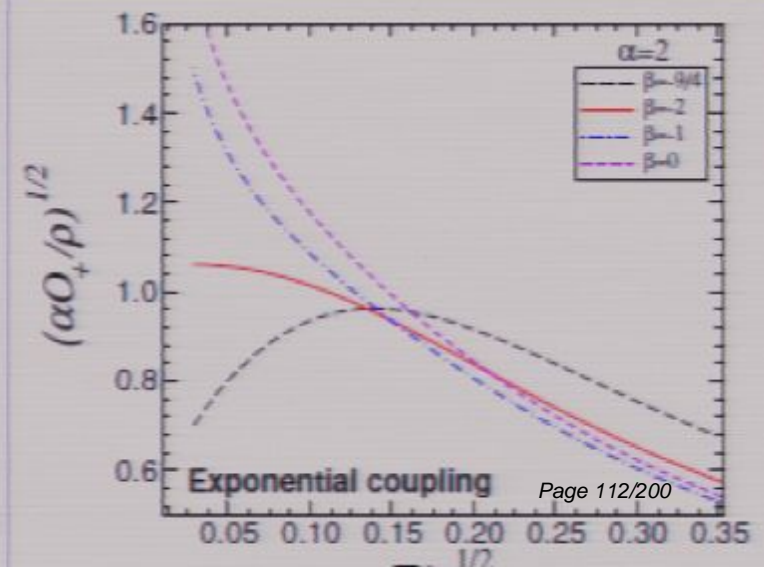
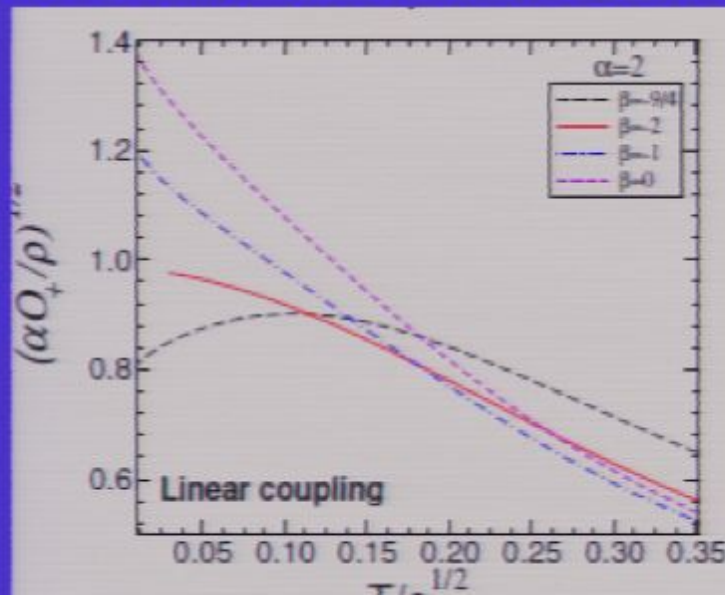
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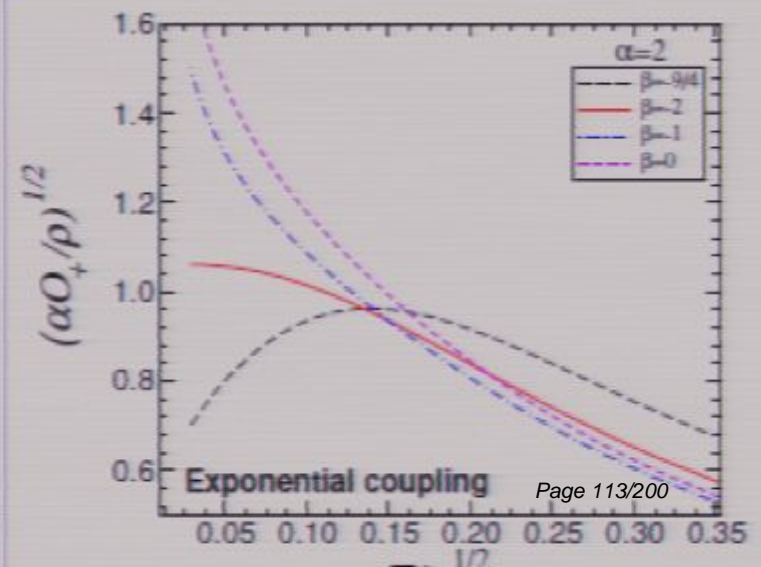
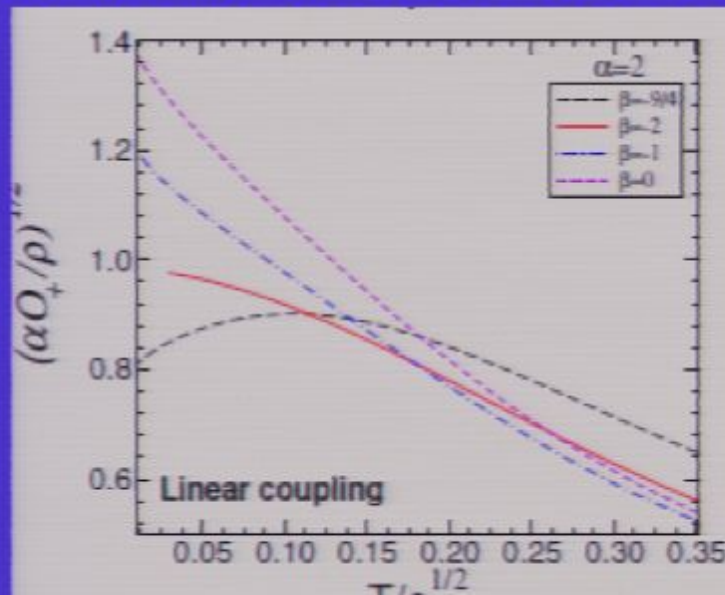
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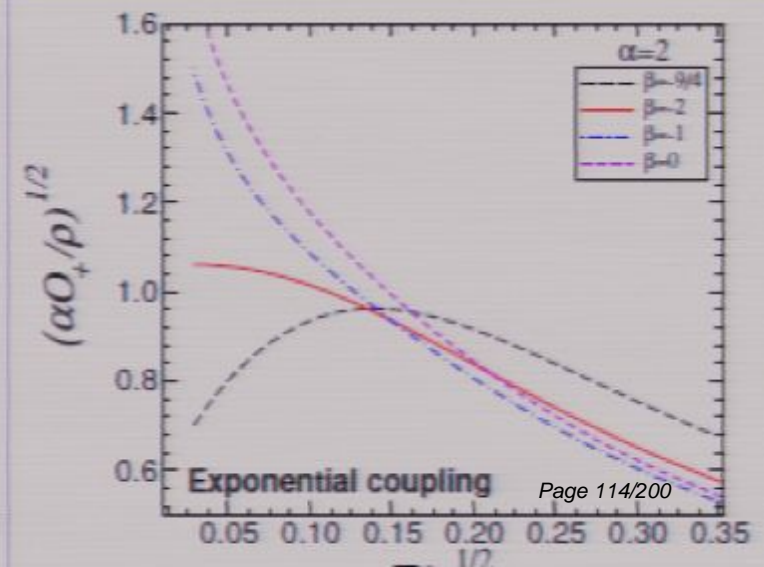
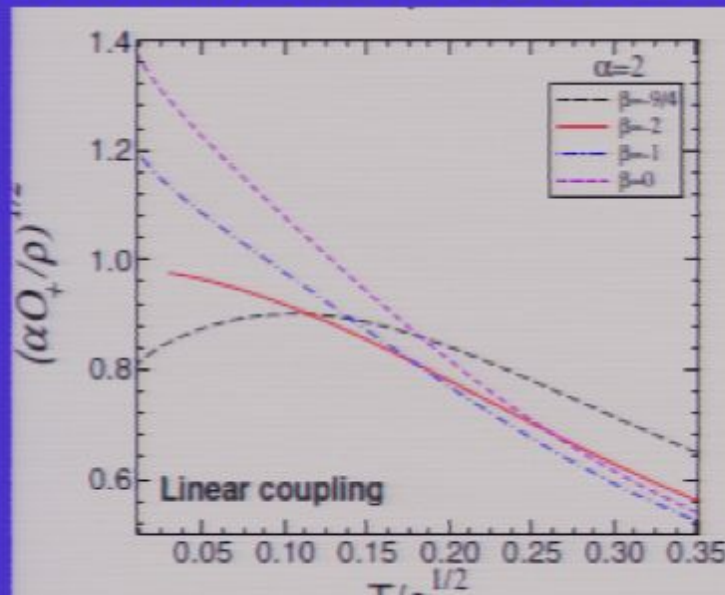
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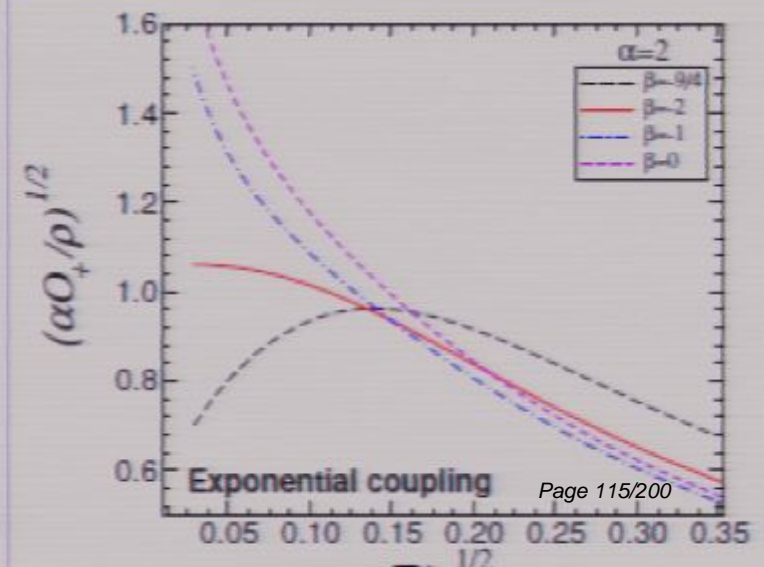
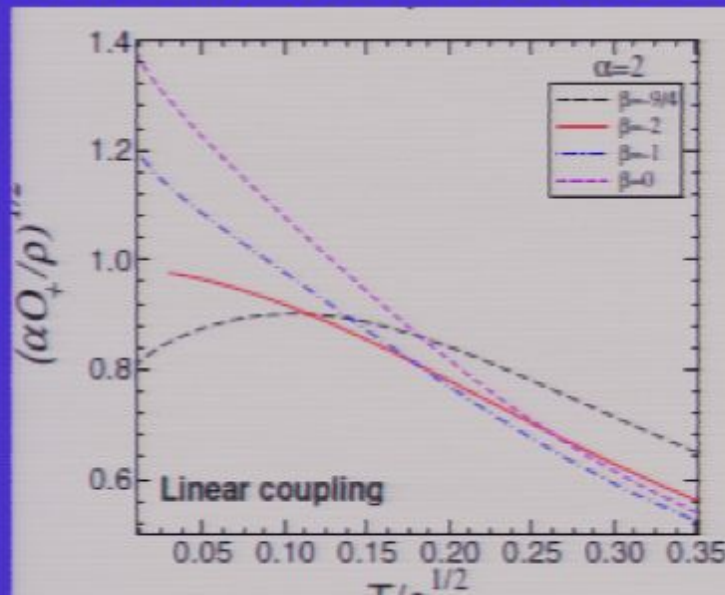
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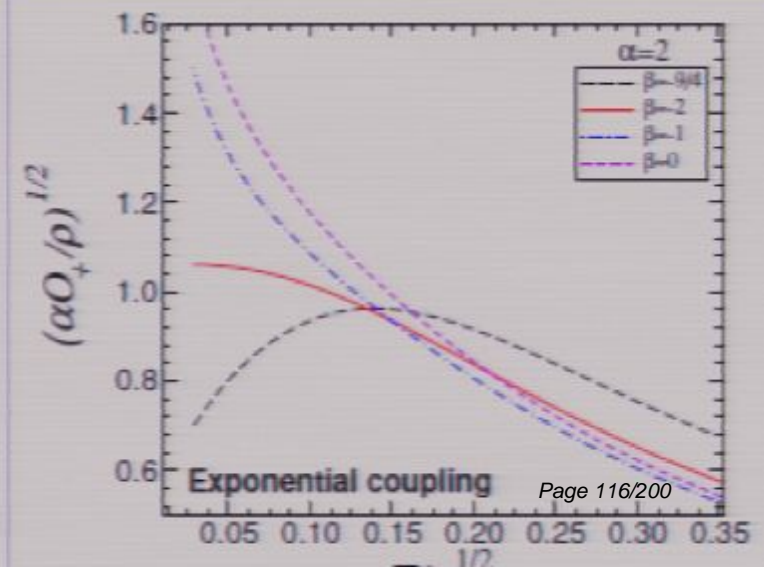
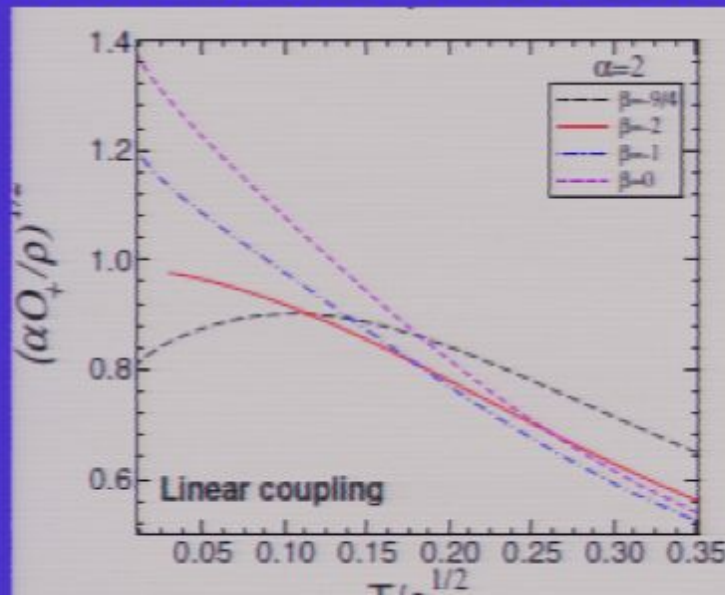
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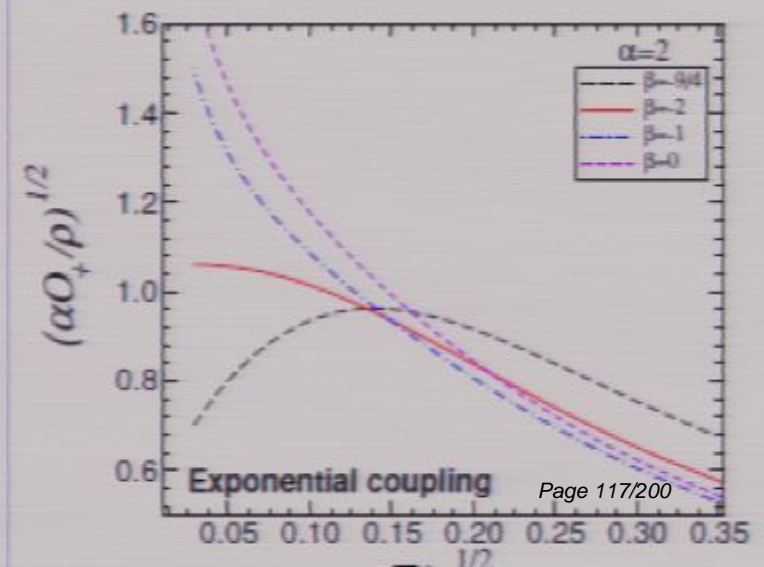
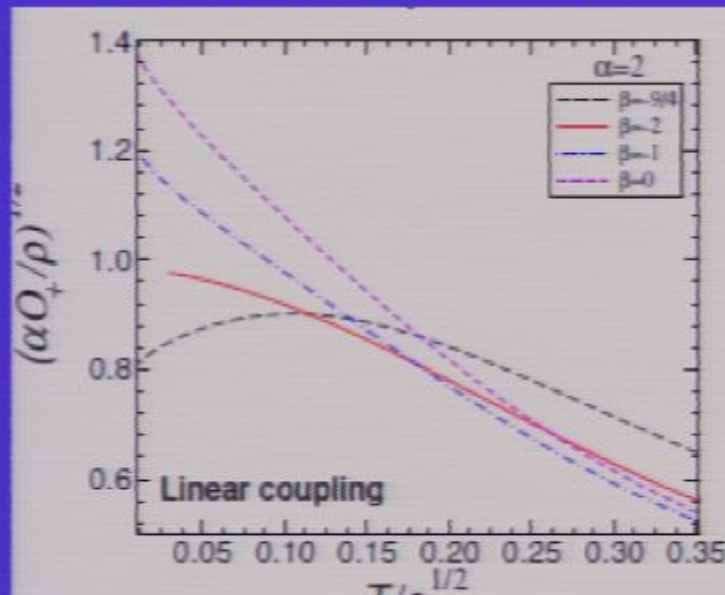
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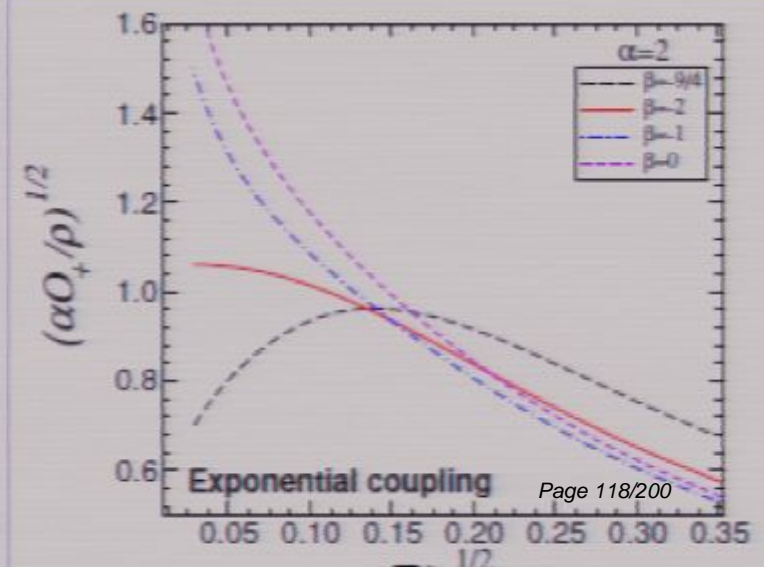
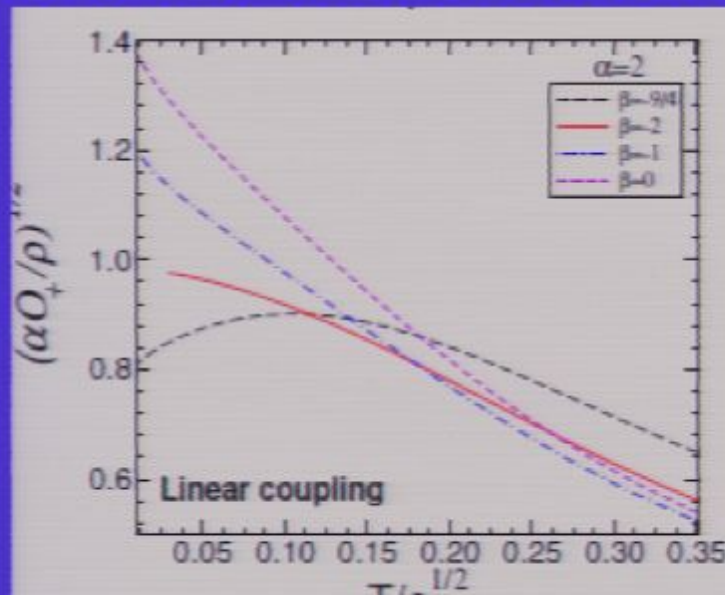
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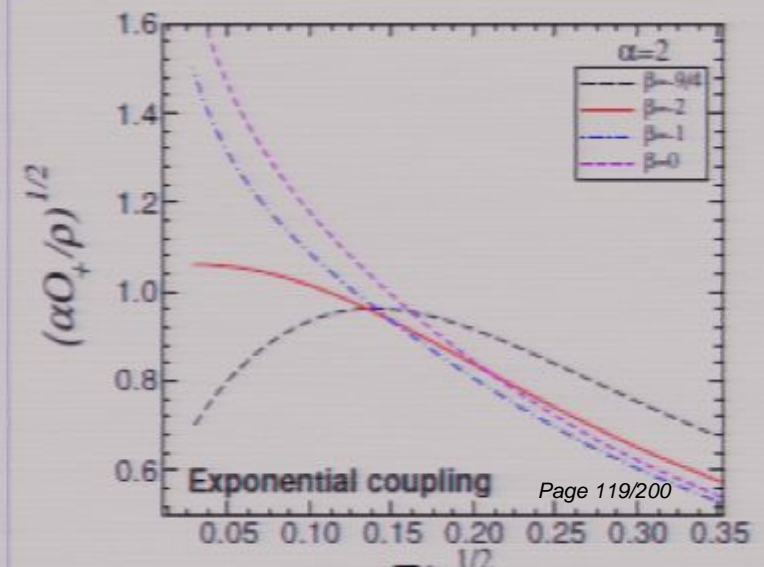
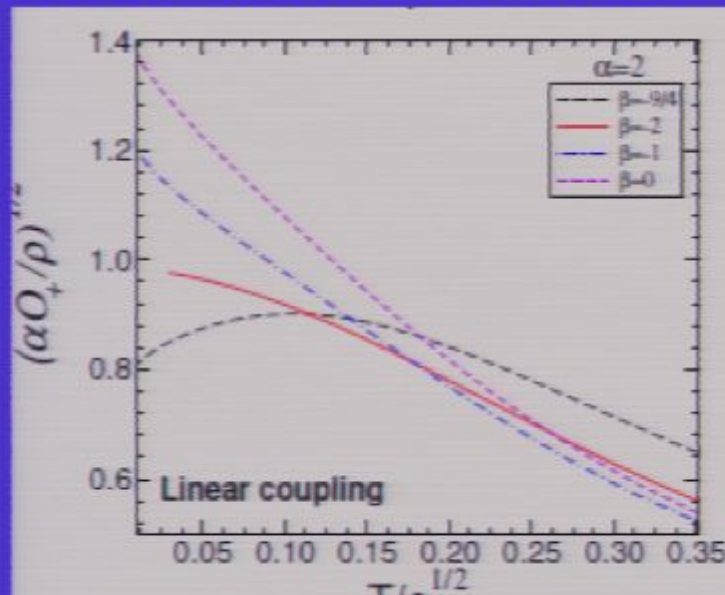
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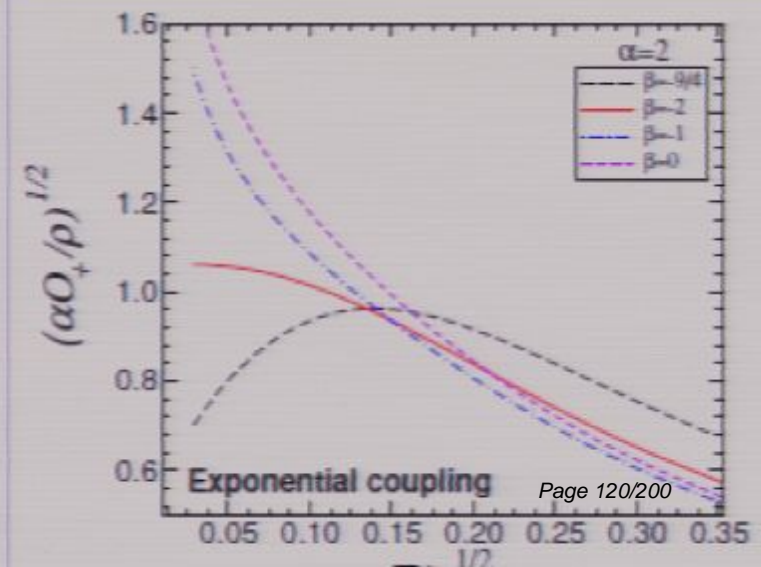
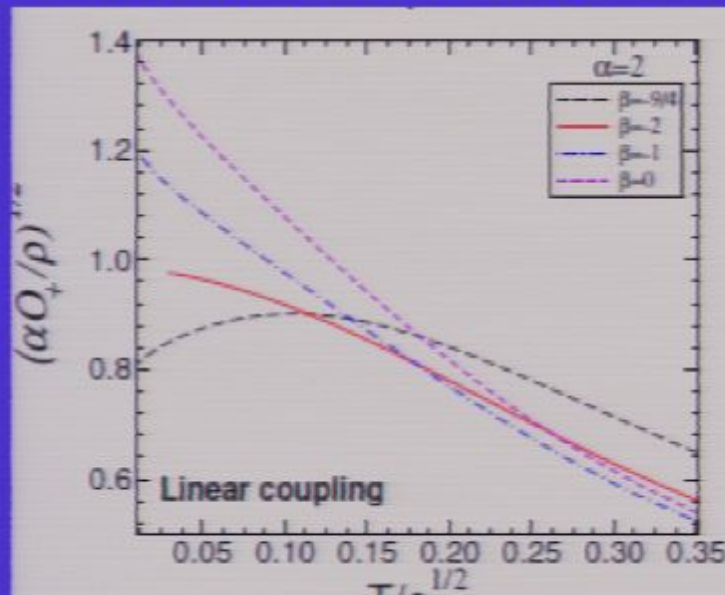
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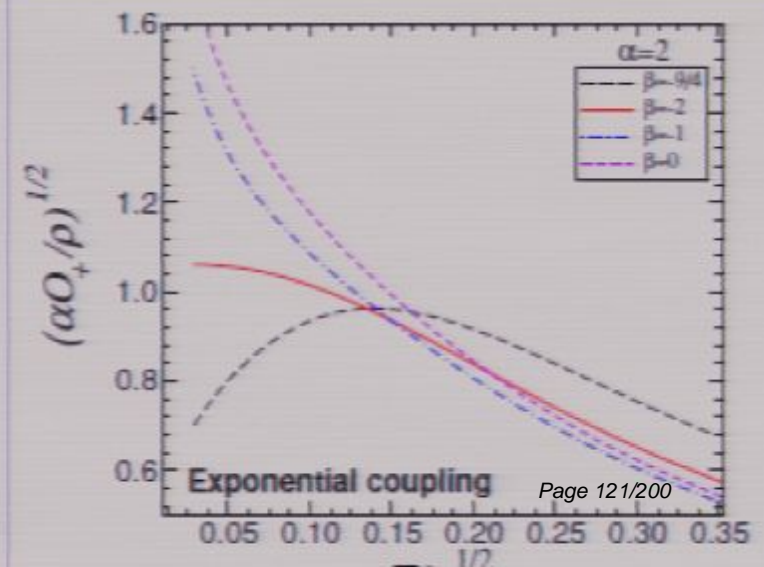
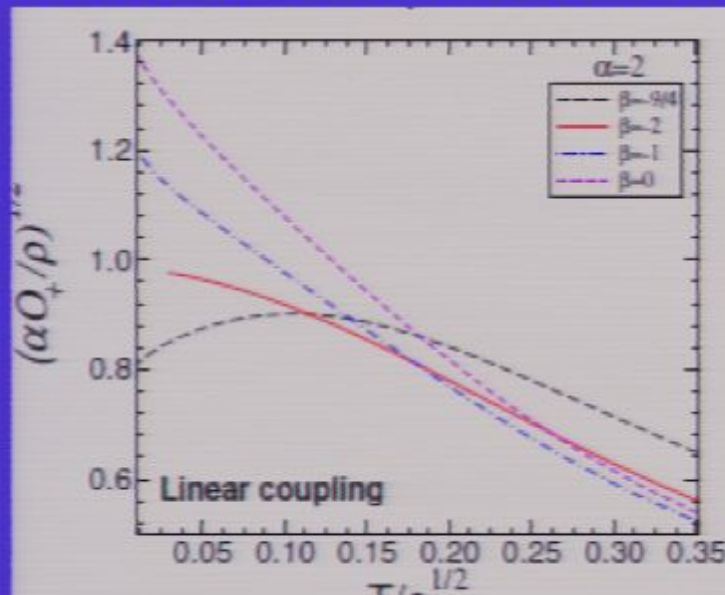


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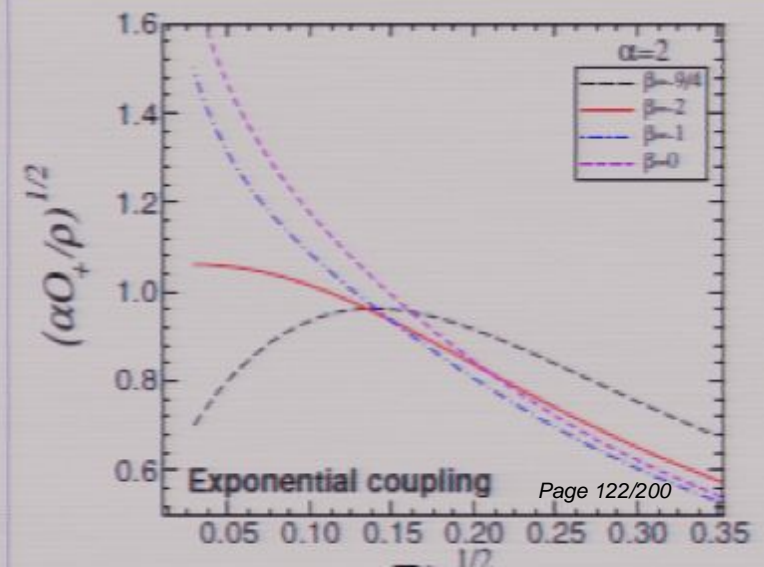
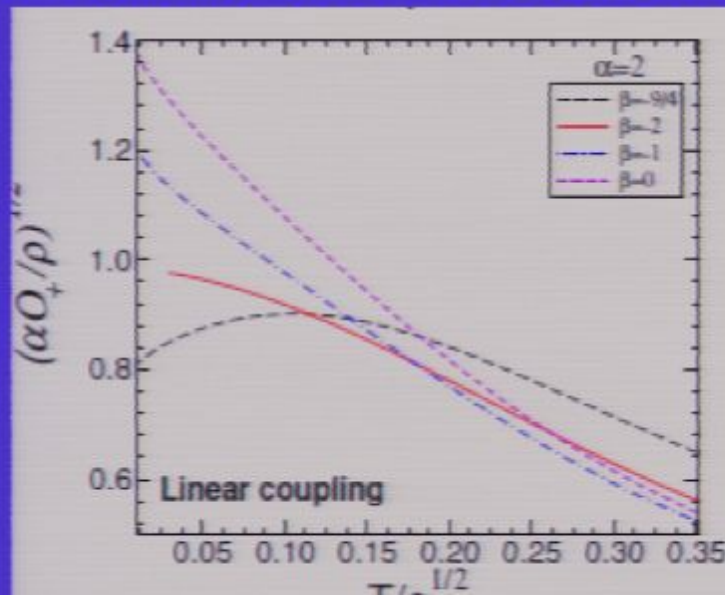


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HOLOGRAPHY OF CHARGED DILATONIC BLACK HOLES: FINITE TEMPERATURE

- As we have seen for the holographic superconductors the AdS/CFT gives a precise prescription for computing the electric conductivity σ of the dual QFT in terms of (bulk) perturbations of A_x .
- Alternatively, σ (in particular its dependence from ω and T) can be calculated by recasting the equation for the perturbations of A_x as a Schroedinger equation. The conductivity can be expressed in terms of the reflection coefficient for a quantum particle incident from the right on a potential barrier generated by an effective potential $V(z)$ (z is a redefinition of the radial coordinate r):

$$\sigma(\omega) = \frac{1 - \mathcal{R}}{1 + \mathcal{R}} - \frac{i}{2\omega} \left[\frac{1}{f} \frac{df}{dz} \right]_{z=0}$$

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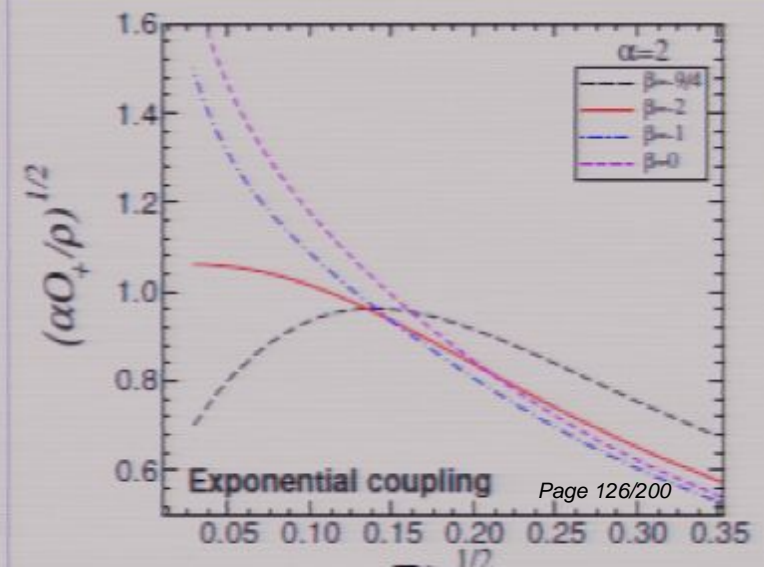
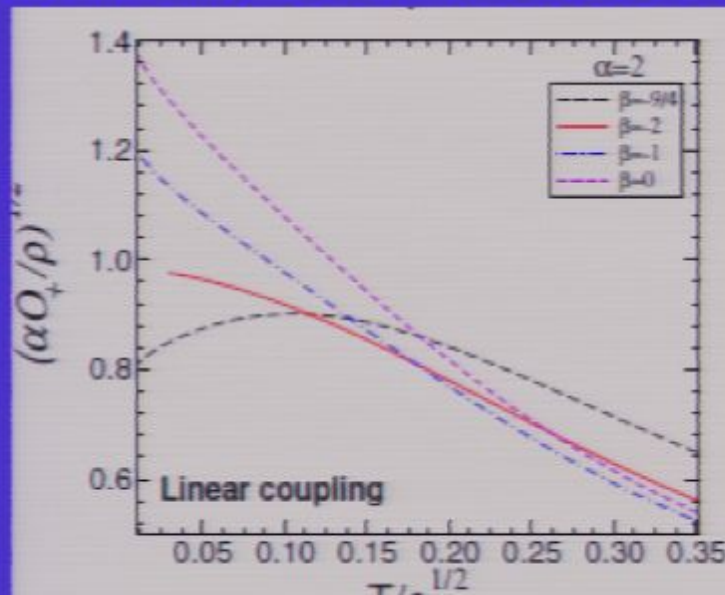
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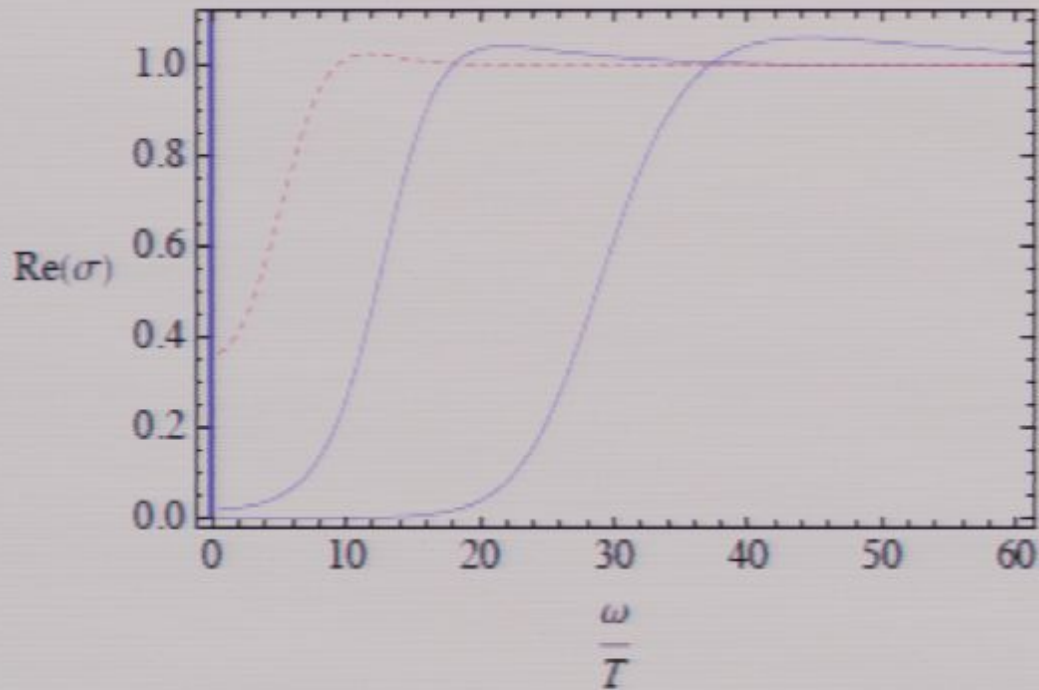
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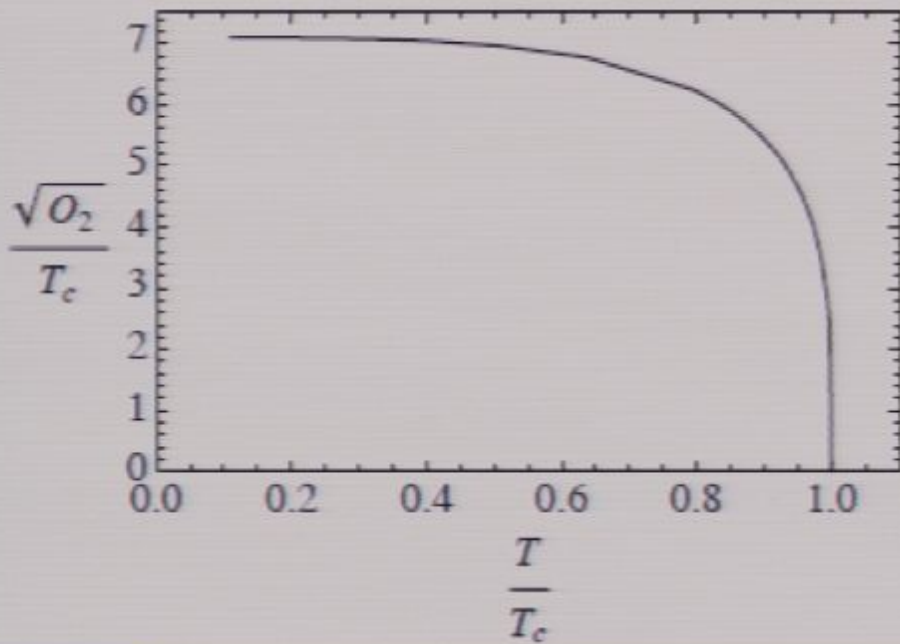
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- As we lower the temperature from T_c a gap opens as expected in BCS theory. There is a delta function at $\omega=0$ (DC conductivity), but this cannot be seen from the numerical solution of the real part. The imaginary part has a pole $\text{Im}(\sigma) \approx 1/\omega$ which from the Kramers-Kronig relations imply $\text{Re}(\sigma) \approx \delta(\omega)$.

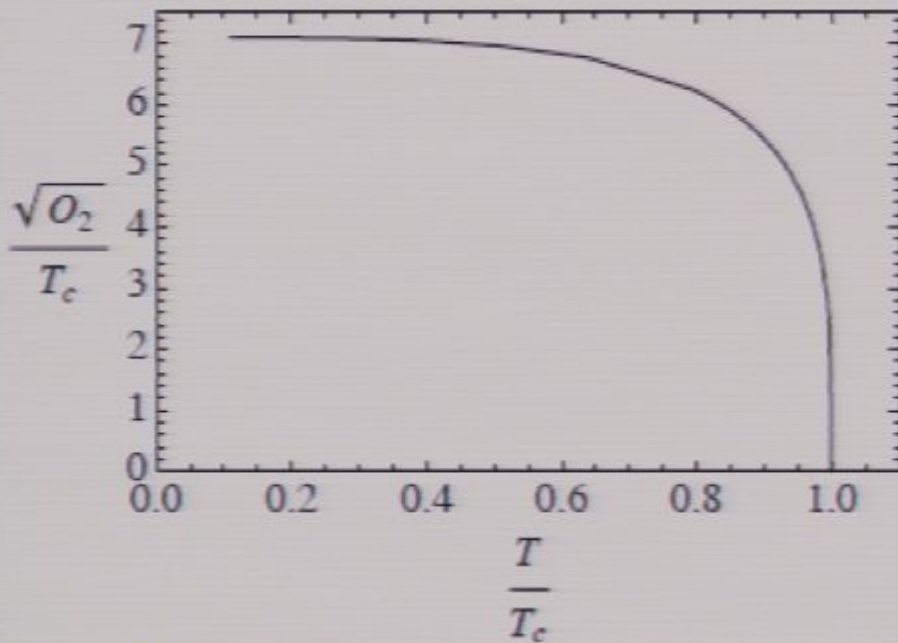


- This curve is qualitatively similar to that obtained in BCS theory and observed in many superconducting materials. The condensate raises quickly when the material is cooled below T_c and goes to a constant as T goes to zero. Near T_c has the square root behavior $O \approx (1 - T/T_c)^{1/2}$ predicted by the Landau-Ginsburg theory

CONDUCTIVITY

To compute the frequency-dependent optical conductivity in the x-direction we perturb A_x in the bulk and consider the equation for perturbations with zero spatial momentum and time dependence $e^{-i\omega t}$

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EINSTEIN-MAXWELL-DILATON GRAVITY AND PHASE TRANSITION TRIGGERED BY A NEUTRAL SCALAR

- The idea is to consider bulk AdS Einstein-Maxwell dilaton gravity in which a REAL scalar is not covariantly but NONMINIMALLY coupled to the U(1) field

$$S = \int d^4x \sqrt{-g} \left(R + \frac{6}{L^2} - \frac{1}{4} f(\psi) F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial\psi)^2 - V(\psi) \right)$$

- $f(\psi)$ is a coupling function. To allow the RN-AdS black hole solution at $\psi=0$ we must have

$$f(\psi) = 1 + \frac{\alpha}{2} \psi^2 + O(\psi^3); \quad V = -\frac{6}{L^2} + \frac{\beta}{2L^2} \psi^2 + O(\psi^3)$$

HOLOGRAPHY OF CHARGED DILATONIC BLACK HOLES: FINITE TEMPERATURE

- As we have seen for the holographic superconductors the AdS/CFT gives a precise prescription for computing the electric conductivity σ of the dual QFT in terms of (bulk) perturbations of A_x .
- Alternatively, σ (in particular its dependence from ω and T) can be calculated by recasting the equation for the perturbations of A_x as a Schroedinger equation. The conductivity can be expressed in terms of the reflection coefficient for a quantum particle incident from the right on a potential barrier generated by an effective potential $V(z)$ (z is a redefinition of the radial coordinate r):

$$\sigma(\omega) = \frac{1 - \mathcal{R}}{1 + \mathcal{R}} - \frac{i}{2\omega} \left[\frac{1}{f} \frac{df}{dz} \right]_{z=0}.$$

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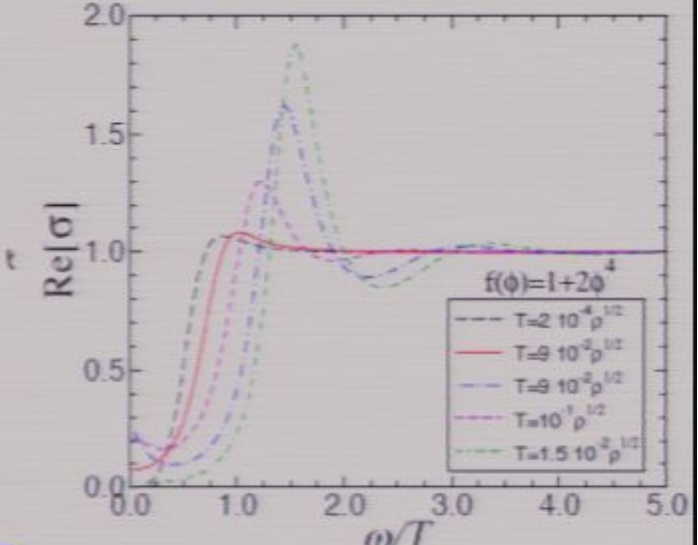
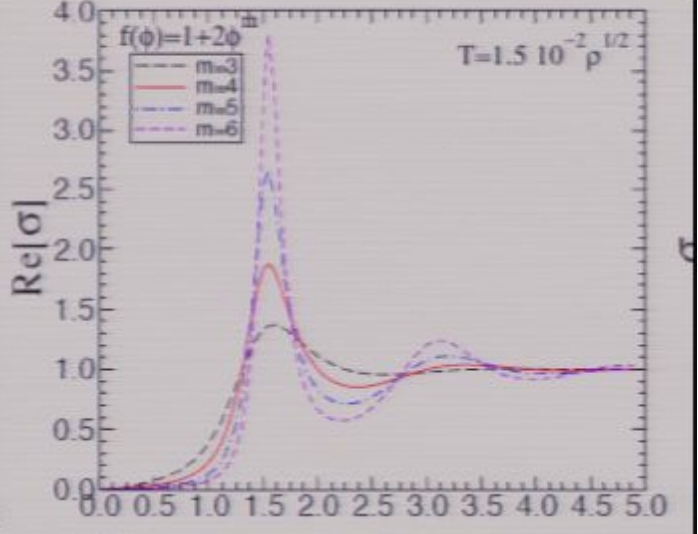
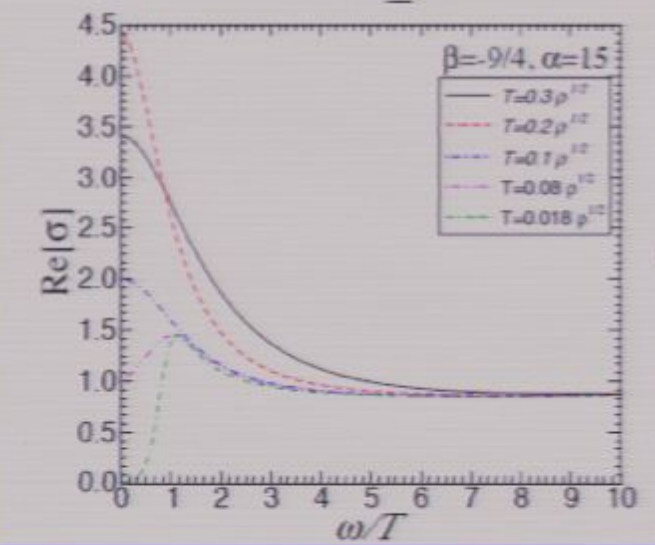
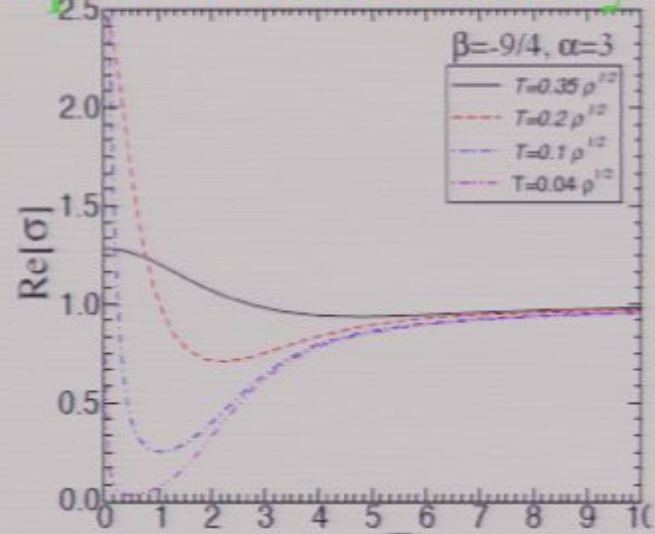
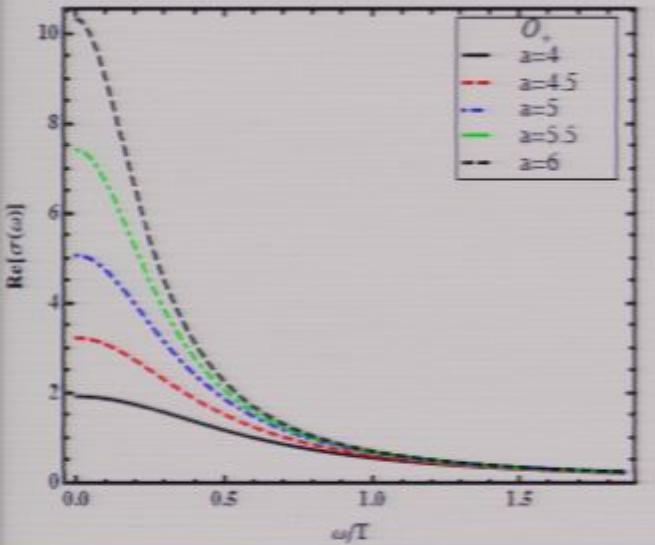
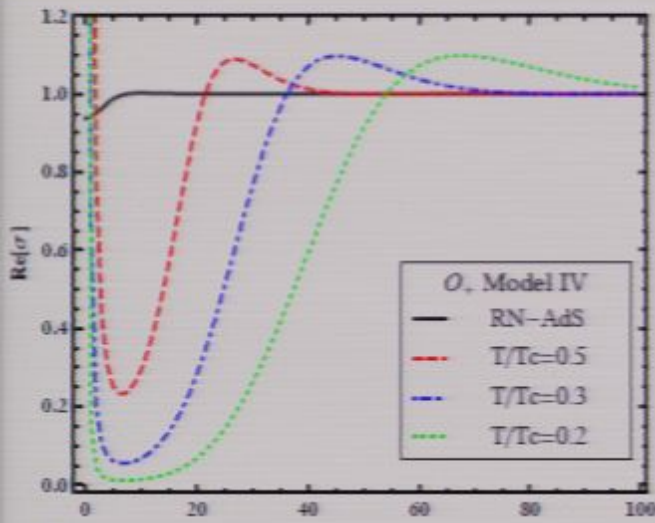
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Optical conductivity



Real part of the conductivity as a function of the frequency for models with $f=\cosh(10\psi)$ and $\beta=-2, \alpha=3^2$

Real part of the conductivity as a function of the frequency for models with exponential (top) and linear

Real part of the conductivity as a function of the frequency for models with power-law coupling function. In the

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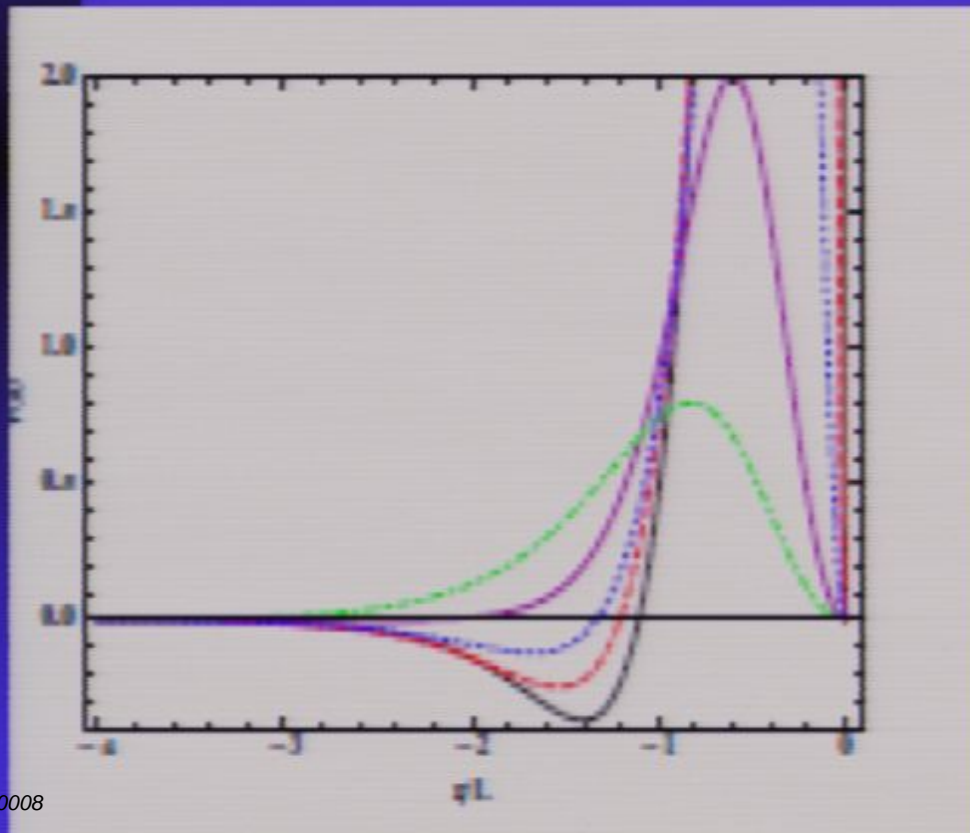
- The AC conductivity has a qualitatively similar behaviour for all the models we have analysed. It has a non monotonic behaviour, approaches a constant value at large ω (determined by a UV relativistic dispersion relation) but reaches constant value at $\omega=0$ which can be considerably larger than the constant value at high ω . This is enhanced for large values of the non-minimal coupling. The effect is reminiscent of a DRUDE PEAK in ordinary metals.

$$\text{Re}[\sigma] = \frac{k\tau}{1 + \omega^2\tau^2}; \quad k = \frac{ne^2}{m}; \quad \tau = \text{relaxation time}$$

- The appearance of a Drude peak can be explained in terms of the effective potential $V(z)$: for large values of α it develops a negative minimum regardless of the form of the coupling function f .

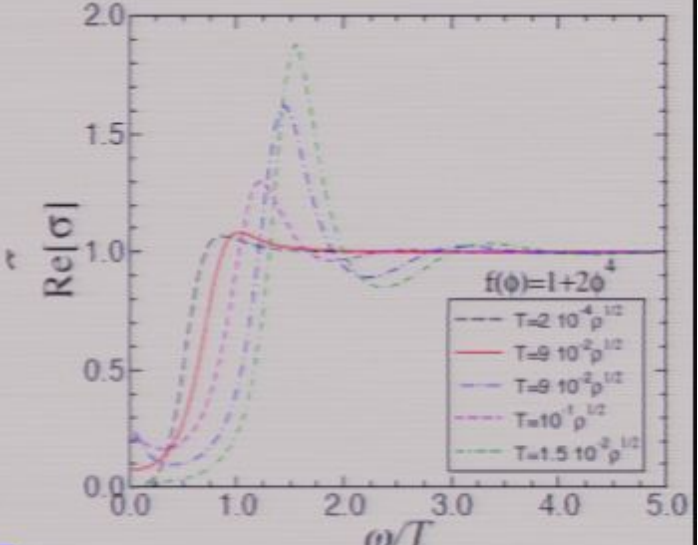
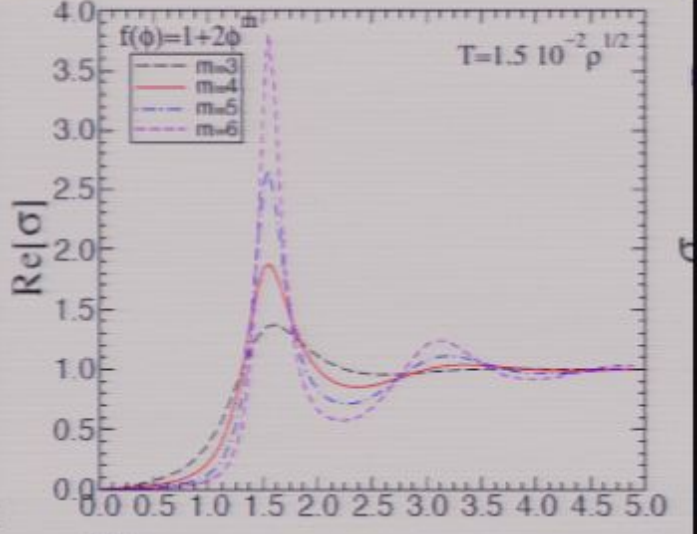
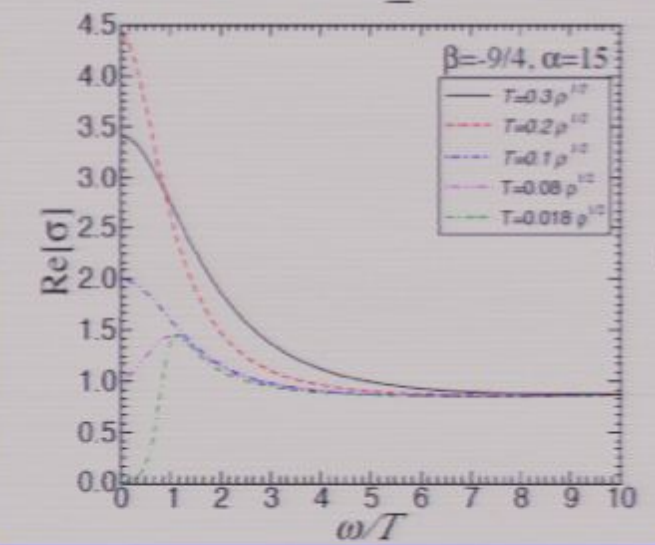
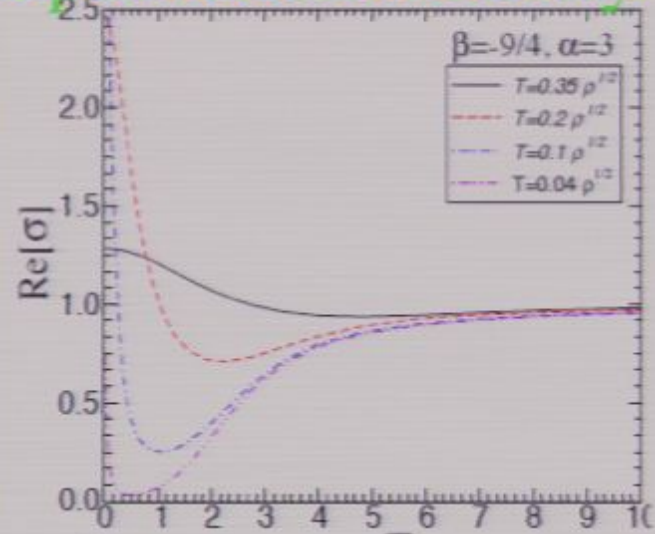
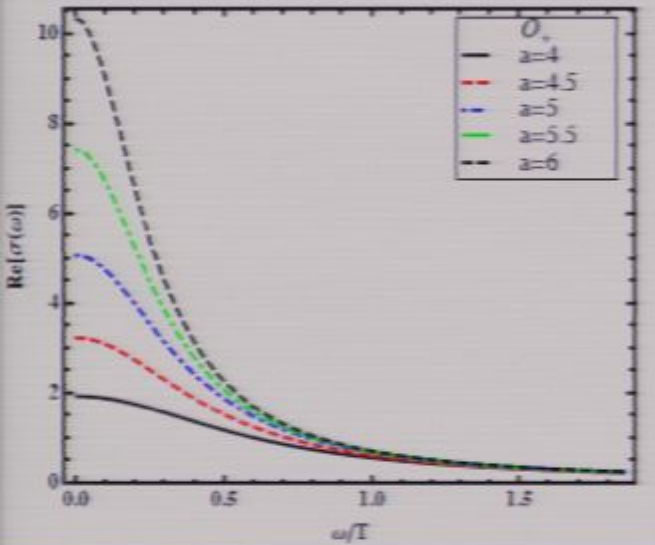
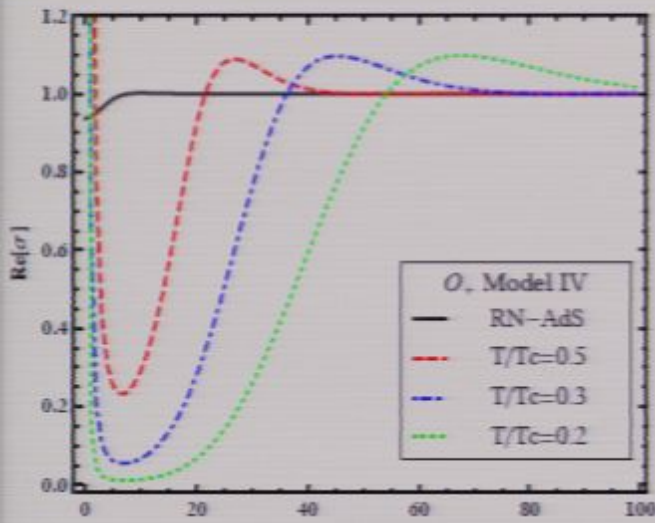
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Effective potential for models with $\alpha=\beta=1/3$, for different temperatures (decreasing from top to below). The lower the temperature, the deeper the minimum.

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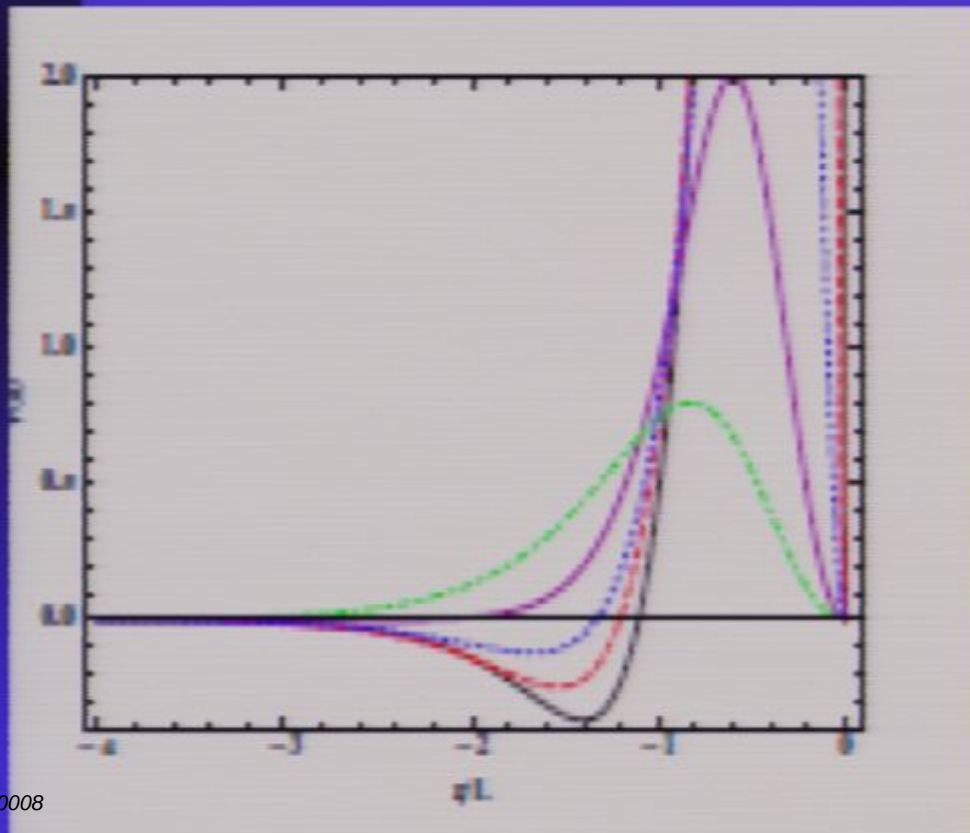
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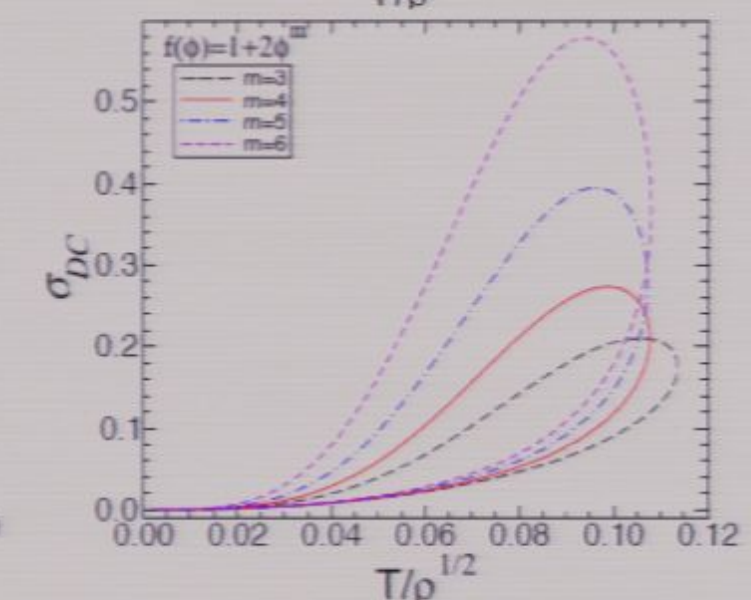
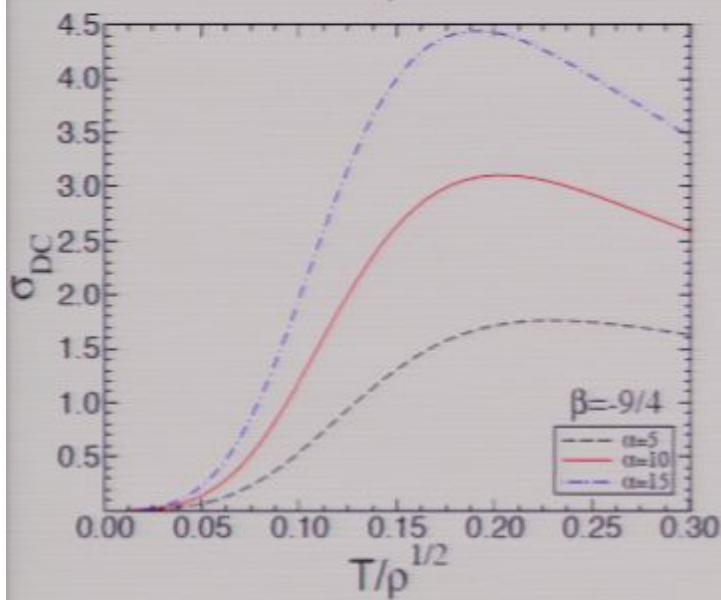
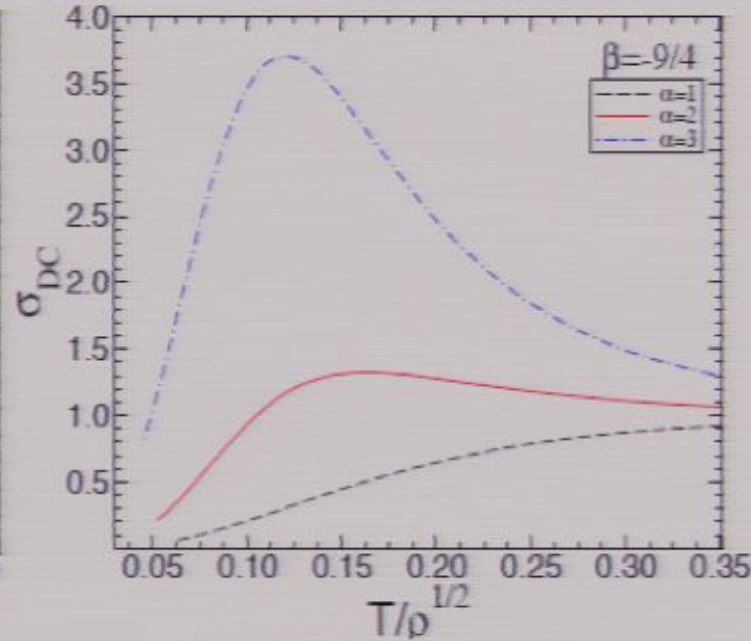
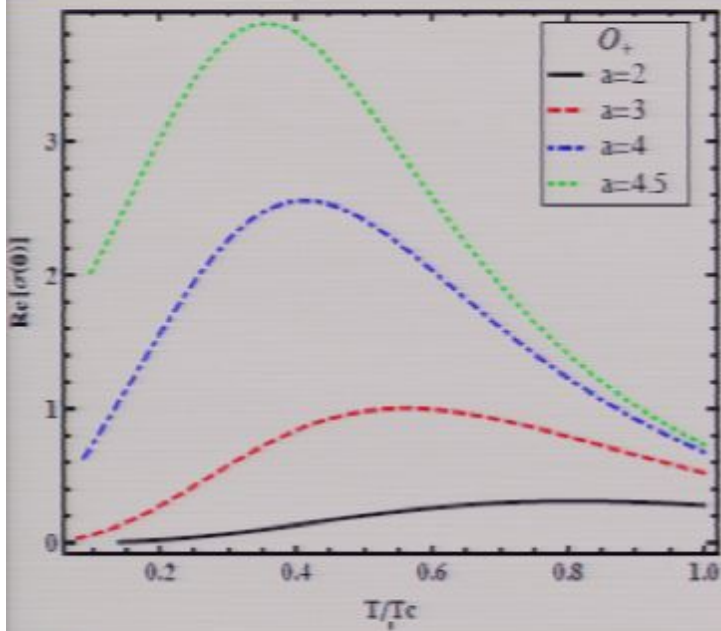
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DC conductivity



DC conductivity as a function of the temperature for models with $f = \cosh a\psi$ (top left), exponential (top right), linear (bottom left and power-law (bottom right) coupling Function.

Also σ_{DC} has qualitatively similar behaviour for the all models. The conductivity does not decrease monotonically with rising of the temperature as for usual conductors but displays a maximum

- This is reminiscent of the KONDO effect caused in real metals with magnetic impurities by the interaction of the magnetic moment of the conduction electrons with the magnetic moment of the impurity.
- For the case of exponential coupling function, at relatively small values of T the resistivity is very well-fitted by

$$\rho(T) \equiv \frac{1}{\sigma_{\text{DC}}(T)} = a_0 + a_1 T^2 + a_2 \log(T) + a_3 T^{-\gamma},$$

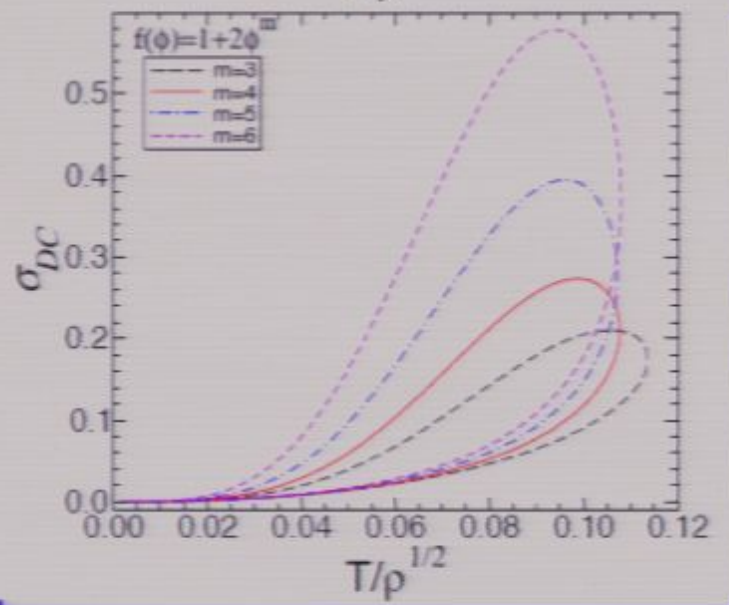
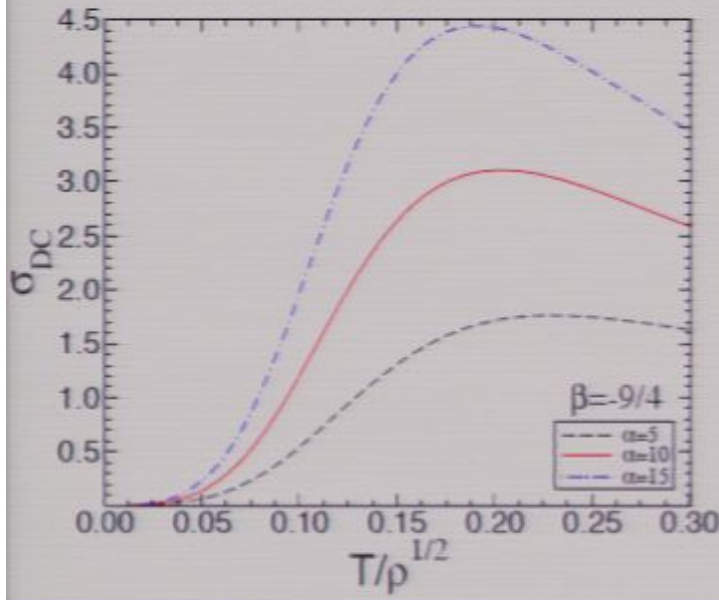
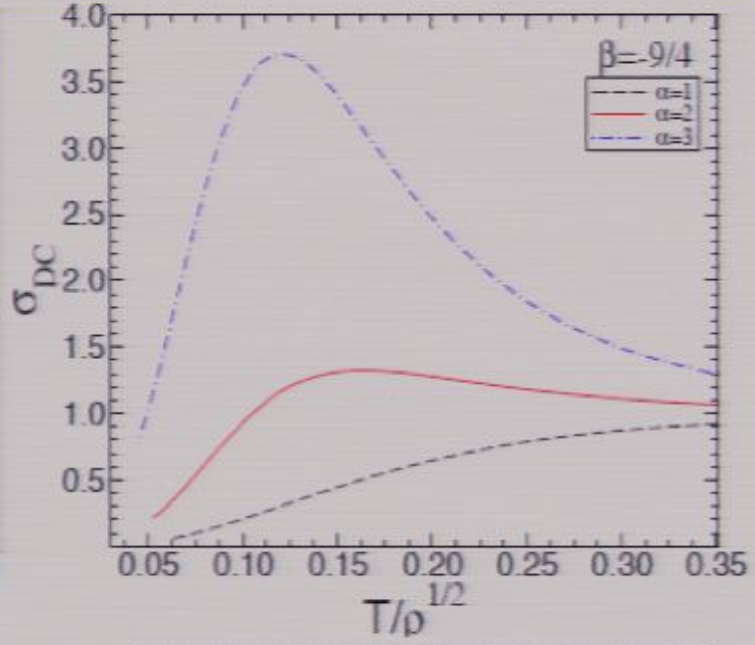
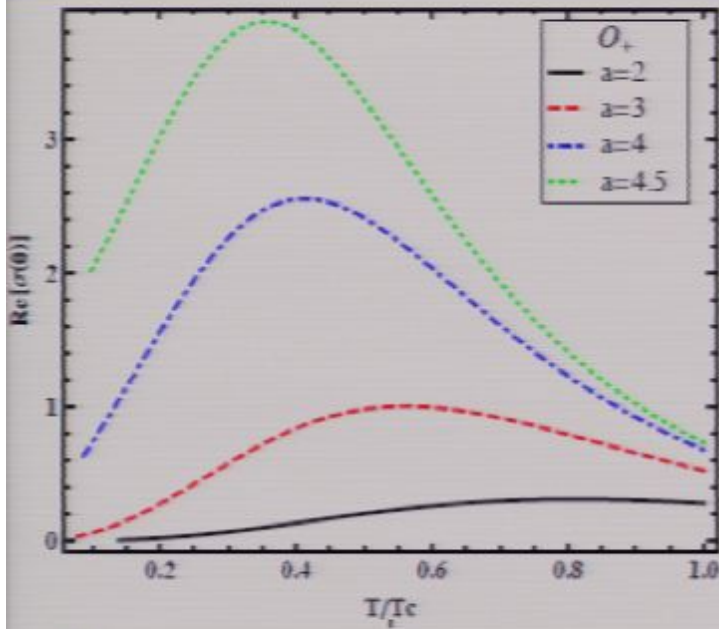
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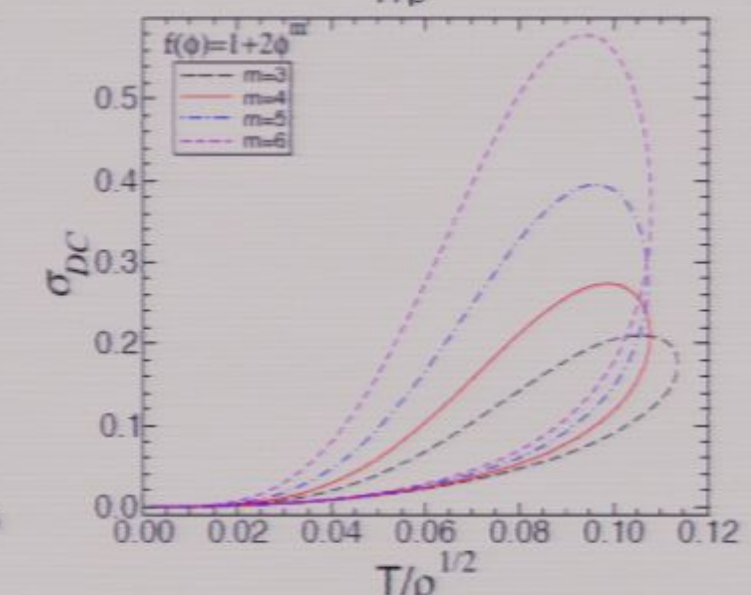
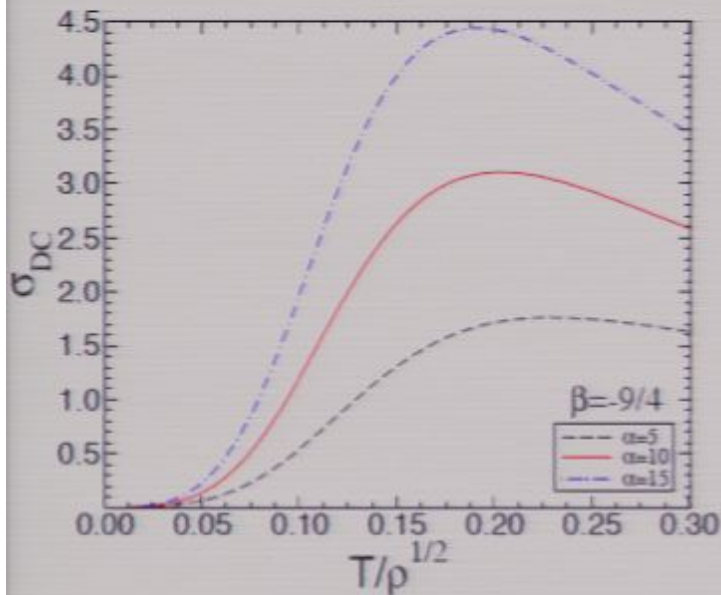
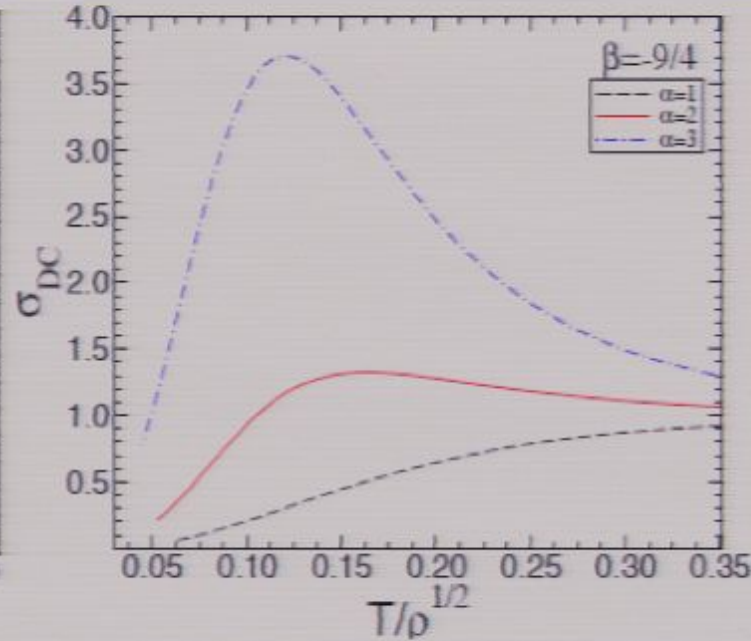
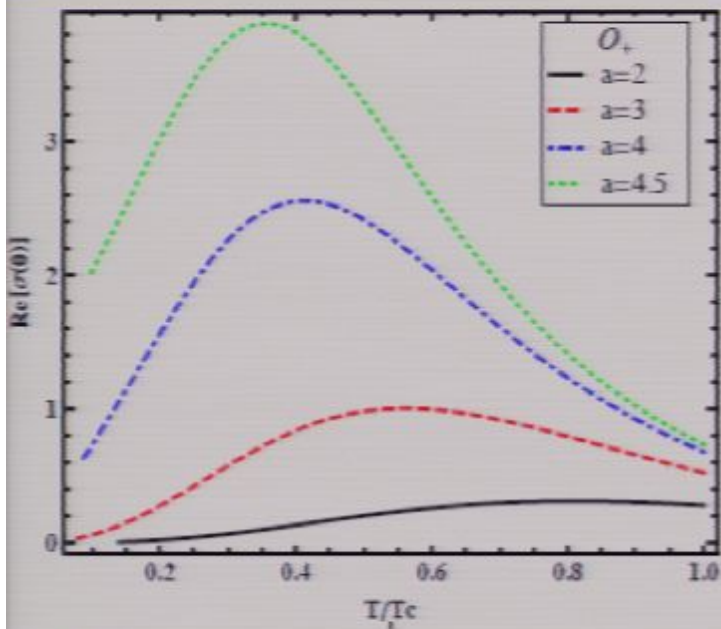
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HOLOGRAPHY OF CHARGED DILATONIC BLACK HOLES: THE ZERO TEMPERATURE LIMIT

- In the near-horizon, near-extremal ($T \approx 0$) regime one can determine analytically the leading term of the black hole solutions. We use now the parametrization of the metric

$$ds^2 = -\lambda(r)dt^2 + \frac{dr^2}{\lambda(r)} + H^2(r)(dx^2 + dy^2) .$$

- One finds that
 - ✓ the metric has a Lifshitz form whenever either the coupling function $f(\psi)$ or the potential $V(\psi)$ behave exponentially in the near-horizon region

$$H = Cr^\gamma, \quad \lambda = -\frac{\alpha^2 a}{4\gamma(1-2\gamma)} r^{2-4\gamma}, \quad \phi = -\frac{4\gamma}{\alpha} \ln r,$$

- ✓ the metric behaves logarithmically for a power-law form of both $f(\psi) \approx \psi^m$ and $V(\psi) \approx \psi^n$

$$H = C(-\ln r)^{-\frac{n+m}{8}}, \quad \lambda = -a\phi_0^n r^2 (-\ln r)^{\frac{n}{2}}, \quad \phi = \phi_0 (-\ln r)^{\frac{1}{2}},$$

- The conductivities in the zero temperature limit are determined using the Schrodinger-like description

$$\frac{d^2\Psi}{dz^2} + (\omega^2 - V_s(z))\Psi = 0, \quad \Psi = \sqrt{f}A_x, \quad \frac{dr}{dz} = \lambda, \quad V_s(z) = \lambda f(\phi)(A'_0)^2 + \frac{1}{\sqrt{f(\phi)}} \frac{d^2\sqrt{f(\phi)}}{dz^2}.$$

- The conductivities are determined by the near-horizon ($z=-\infty$) behavior of the effective potential $V_s(z)$. They show a rather universal, power-law, behaviour as a function of ω and T . We have

✓ For an exponential coupling function f we get

$$V(z) = \frac{C}{z^2},$$

which gives the conductivities

$$\text{Re}[\sigma(T \approx 0)] \approx \omega^s, \quad \sigma_{DC} = \text{Re}[\sigma(\omega \approx 0)] \approx T^s, \quad s \geq 2$$

The exponent s depends on the potential V for the scalar field, $s=2$ when V is a pure exponential. Differently from the $T>0$ case the potential V_z is now positive definite \rightarrow the reflection coefficient is 1 and $\sigma(\omega=0)=0$

✓ For a power-law coupling function f we get

$$V_s(z) \sim \frac{2}{z^2},$$

which gives

$$\text{Re}[\sigma(T \sim 0)] \sim \omega^2,$$

$$\sigma_{\text{DC}} \equiv \text{Re}[\sigma(\omega \sim 0)] \sim r_h^2 \sim T^2 \{-\log[r_h(T)]\}^{-n/2},$$

Notice the puzzling feature: although in this case the Lifshitz metric is deformed by the presence of logarithms and loses the non relativistic scale isometry, the AC conductivity retains the universal quadratic scaling behaviour but the same is not true for the DC conductivity.

GENERALIZATION TO DYONIC BLACK HOLES

- The previous discussion has been generalized to the case of Dyonic black holes, carrying both electric and magnetic charges. Main new features

- The conductivities in the zero temperature limit are determined using the Schrodinger-like description

$$\frac{d^2\Psi}{dz^2} + (\omega^2 - V_s(z))\Psi = 0, \quad \Psi = \sqrt{f}A_x, \quad \frac{dr}{dz} = \lambda, \quad V_s(z) = \lambda f(\phi)(A'_0)^2 + \frac{1}{\sqrt{f(\phi)}} \frac{d^2\sqrt{f(\phi)}}{dz^2}.$$

- The conductivities are determined by the near-horizon ($z=-\infty$) behavior of the effective potential $V_s(z)$. They show a rather universal, power-law, behaviour as a function of ω and T . We have

✓ For an exponential coupling function f we get

$$V(z) = \frac{C}{z^2},$$

which gives the conductivities

$$\text{Re}[\sigma(T \approx 0)] \approx \omega^s, \quad \sigma_{DC} = \text{Re}[\sigma(\omega \approx 0)] \approx T^s, \quad s \geq 2$$

The exponent s depends on the potential V for the scalar field, $s=2$ when V is a pure exponential. Differently from the $T>0$ case the potential V_z is now positive definite \rightarrow the reflection coefficient is 1 and $\sigma(\omega=0)=0$

HOLOGRAPHY OF CHARGED DILATONIC BLACK HOLES: THE ZERO TEMPERATURE LIMIT

- In the near-horizon, near-extremal ($T \approx 0$) regime one can determine analytically the leading term of the black hole solutions. We use now the parametrization of the metric

$$ds^2 = -\lambda(r)dt^2 + \frac{dr^2}{\lambda(r)} + H^2(r)(dx^2 + dy^2) .$$

- One finds that
 - ✓ the metric has a Lifshitz form whenever either the coupling function $f(\psi)$ or the potential $V(\psi)$ behave exponentially in the near-horizon region

$$H = Cr^\gamma, \quad \lambda = -\frac{\alpha^2 a}{4\gamma(1-2\gamma)} r^{2-4\gamma}, \quad \phi = -\frac{4\gamma}{\alpha} \ln r,$$

- ✓ the metric behaves logarithmically for a power-law form of both $f(\psi) \approx \psi^m$ and $V(\psi) \approx \psi^n$

$$H = C(-\ln r)^{-\frac{n+m}{8}}, \quad \lambda = -a\phi_0^n r^2 (-\ln r)^{\frac{n}{2}}, \quad \phi = \phi_0 (-\ln r)^{\frac{1}{2}},$$

✓ For a power-law coupling function f we get

$$V_s(z) \sim \frac{2}{z^2},$$

which gives

$$\text{Re}[\sigma(T \sim 0)] \sim \omega^2,$$

$$\sigma_{\text{DC}} \equiv \text{Re}[\sigma(\omega \sim 0)] \sim r_h^2 \sim T^2 \{-\log[r_h(T)]\}^{-n/2},$$

Notice the puzzling feature: although in this case the Lifshitz metric is deformed by the presence of logarithms and loses the non relativistic scale isometry, the AC conductivity retains the universal quadratic scaling behaviour but the same is not true for the DC conductivity.

GENERALIZATION TO DYONIC BLACK HOLES

- The previous discussion has been generalized to the case of Dyonic black holes, carrying both electric and magnetic charges. Main new features

- ✓ There is a critical value of the MF above which the phase transition does not occur (at fixed electric charge Q)
- ✓ Perturbations of A_x are coupled to those of A_y
- ✓ The conductivity matrix of the dual QFT σ_{ij} has a pole corresponding to a damped cyclotron frequency
- ✓ In the $\omega \rightarrow 0$ limit $\sigma_{xx}=0$, whereas $\sigma_{xy}=Q/B$ (Hall conductivity)
- ✓ Magnetic susceptibility order 1 and positive (boundary QFT strongly diamagnetic)

CONCLUSIONS

- The holographic properties of charged dilatonic black holes seem to be largely determined by the peculiarity of the non-minimal coupling between the scalar and the Maxwell field
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- The transport properties of the dual QFT at finite temperature are characterized by a rich phenomenology typical of electron motion in metals
- The whole holographic picture emerging is that of an interpolation between a repulsive charged plasma at zero temperature and electron motion in real metals at finite temperature
- Intrinsic limitation common to most holographic approaches : almost nothing is known about the actual content of the dual QFT. We start from a well-defined gravitational bulk system and using holographic methods we infer from that some properties of a conjectured dual QFT. But the question is

Who really is this dual QFT?

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