Title: Holography for non-relativistic theories

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Abstract: I will discuss the construction of a holographic dictionary for theories with non-relativistic conformal symmetry, relating the field theory to the dual spacetime. I will focus on the case of Lifshitz spacetimes, giving a definition of asymptotically locally Lifshitz spacetimes and discussing the calculation of field theory observables and holographic renormalization.

Holography for non-relativistic theories

SFR & Saremi, 0907.1846 & work in progress

Simon Ross

Centre for Particle Theory, Durham University

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Outline

- Motivation, review of Lifshitz
- Stress tensor complex for non-relativistic theories
- Asymptotically locally Lifshitz spacetimes
- Stress tensor, boundary geometry
- Linearised theory
- Holographic renormalization
- Discussion

ondensed matter physics & holography

pplication of holographic methods to gauge theories long studied.

Vhy condensed matter?

- Rich system:
  - CFTs arise as IR desc near critical points; often strongly coupled.
  - Many different theories; can tune Hamiltonian in some settings.
- AdS/CFT provides a useful new perspective:
  - Few other methods for calculation at strong coupling.
  - Gravity dual calculates observables like transport coefficients directly, no quasiparticle picture.
- Prompts new questions:
  - Charge transport, phase transitions
  - Can have theories with an anisotropic scaling symmetry D: $x^i \to \lambda x^i, t \to \lambda^z t$.

Quantum critical point

Phase boundary at zero temperature.

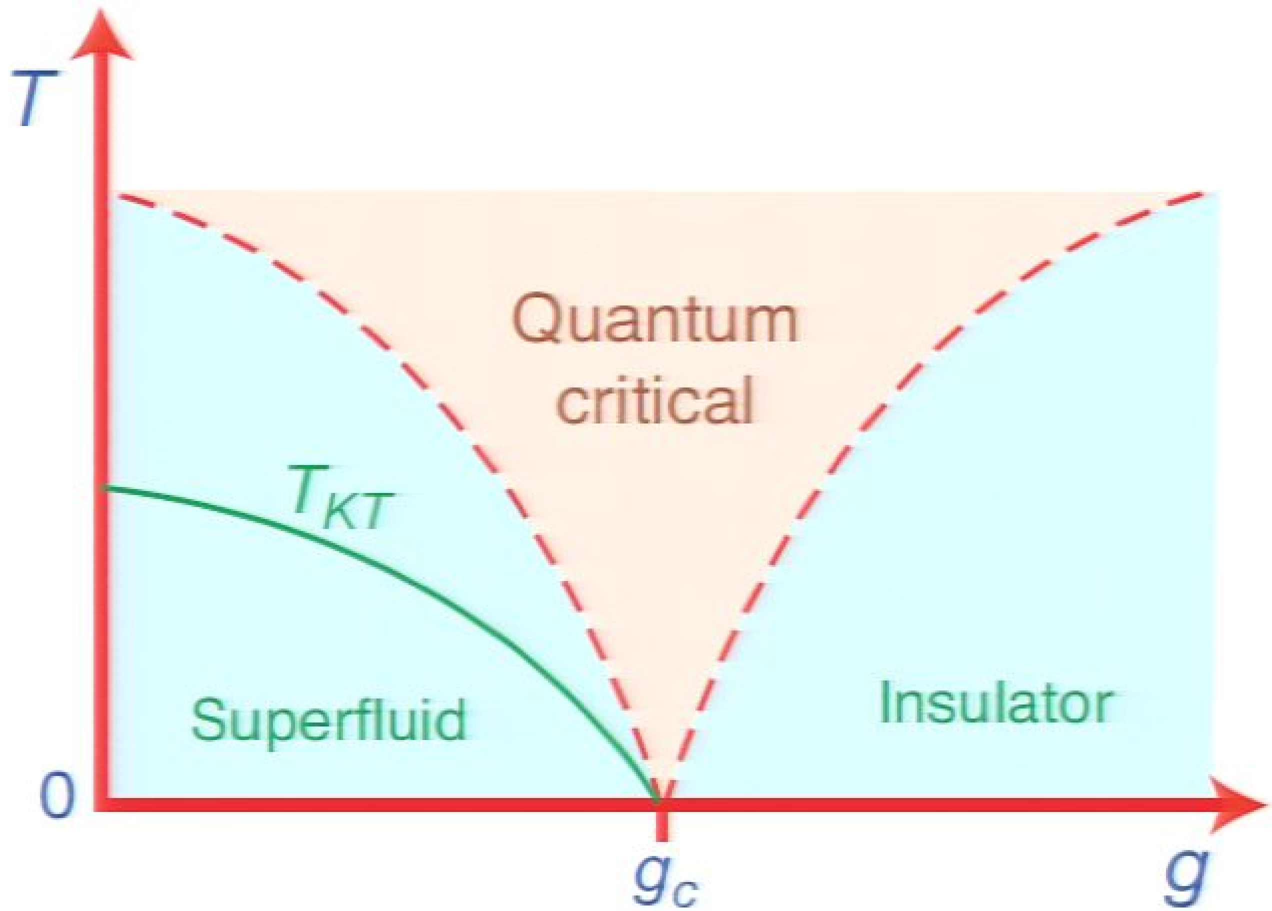

Figure from Sachdev, arXiv:0711.3015

- Critical point described by a CFT;
  finite region described by finite-temperature CFT.

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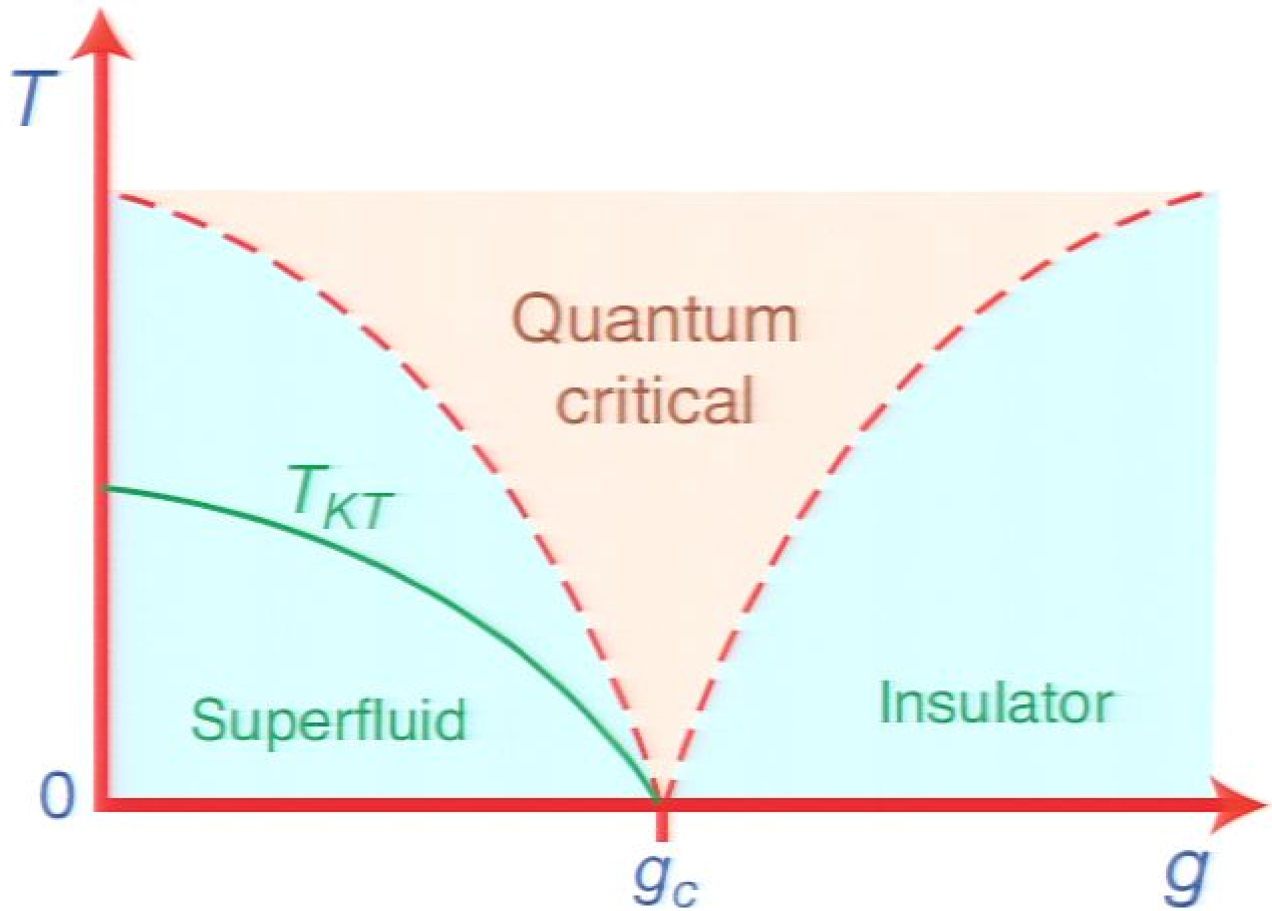

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- Not to construct a dual to CFTs studied in condensed matter or make direct contact with lattice models. (Known examples of duality are large N gauge theories.)
- Proceed phenomenologically: construct bulk theories with qualitative features of interest.
  - Bottom-up: invent an appropriate classical gravitational Lagrangian
  - Top-down: string theory construction.
- Big question: range of theories/issues for which holography is useful.
  - Matrix large N.
  - Hierarchy in spectrum: $\Delta_{s>2} \gg \Delta_{s\leq 2}$.

dS/CFT review

- $\langle e^{\int \phi_0 \mathcal{O}} \rangle = Z_{string}[\phi_0] \approx e^{-S[\phi_0]}$.
- Calculate e.g. stress tensor as

Henningson Skenderis   Balasubramanian Kraus

$$\langle T_{\mu\nu} \rangle = \frac{1}{\sqrt{\hat{h}}} \frac{\delta S}{\delta \hat{h}^{\mu\nu}},$$

where $ds^2 \approx \frac{dr^2}{r^2} + r^2 \hat{h}_{\mu\nu} dx^\mu dx^\nu + \dots$

- One-point functions for arbitrary sources give full information: obtain correlation functions by varying sources.
  ⋆ Action S for boundary conditions fixing $\hat{h}_{\mu\nu}$
- One-point functions will depend on both sources and states:
  ⋆ Space of 'asymptotically AdS' geometries for a given boundary condition.
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lon-relativistic theory: Stress-energy complex

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- Conservation equations $\partial_t \mathcal{E} + \partial_i \mathcal{E}^i = 0$, $\partial_t \mathcal{P}_i + \partial_j \Pi^j{}_i = 0$.
- Scaling invariance implies $z\mathcal{E} + \Pi^i{}_i = 0$.
- $\mathcal{E}$ dimension $z + d \Rightarrow \mathcal{E}^i$ dimension $2z + d - 1$.
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- Simple deformation of AdS:

Kachru
Liu
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$$ds^2 = -r^{2z} dt^2 + r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2}.$$

- Solution of a theory with a massive vector:

Taylor

$$S = \int d^4x \sqrt{-g} (R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A_\mu A^\mu),$$

with $\Lambda = -\frac{1}{2}(z^2 + z + 4)$, $m^2 = 2z$. Lifshitz solution has

$$A = \alpha r^z dt, \quad \alpha^2 = \frac{2(z-1)}{z}.$$

- Finite temperature black hole solutions obtained numerically.

Danielsson
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Bertoldi
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- Analytic black holes in other theories.

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- S^1 compactifications of M theory: Balasubramanian Narayan, Donos Gauntlett

$$ds_{11}^2 = ds_4^2 + e^{2T}(d\sigma + A)^2 + e^{2V}D\psi^2 + e^{2U}(ds_1^2 + ds_2^2).$$

$$F_5 = 4e^{T-V-4U}Vol_4 \wedge (d\sigma + A) + 4D\psi \wedge J_1 \wedge J_2,\ H = \sqrt{2}(d\sigma + dk)\wedge(J_1 - J_2).$$

Gives $z = 2$ massive vector theory + scalars.

- Massive IIA on $S^4 \times H^2/\Gamma$: Gregory Parameswaran Tasinato Zavala

Reduction on S^4 gives 6D gauged massive supergravity, $SU(2) \times U(1)$ gauge group. Allowing $F^{(3)}$ flux on H^2/Γ gives 4D Lifshitz with arbitrary z.

- Full 4D theory, asymptotically Lifshitz solutions?

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symptotically locally Lifshitz spacetimes

SFR & Saremi

Vant the leading-order metric at large r to locally take the form

$$ds^2 = -r^{2z} dt^2 + r^2 d\vec{x}^2 + \frac{dr^2}{r^2} + \dots$$

Work with an orthonormal frame $e^{(A)}, e^{(r)}$. $(A = 0, 1, 2;\ I = 1, 2.)$
3y choice of gauge, $e_r^{(A)} = 0$, $e^{(r)} = \frac{dr}{r}$. Require that as $r \to \infty$,

$$e^{(0)} = r^z \hat{e}^{(0)}(r, t, \vec{x}), \quad e^{(I)} = r \hat{e}^I(r, t, \vec{x}),$$

where $\hat{e}^{(0)}(r, t, \vec{x})$, $\hat{e}^I(r, t, \vec{x})$ have finite limits as $r \to \infty$.
3oundary data analogous to conformal metric on boundary.

Horava
Melby-Thompson

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$$\delta S = \int_{\partial M} d^3x \sqrt{-h}\left(T^{\alpha}_{\ B}\delta e^{(B)}_{\alpha} + \pi^{A}\delta A_{A}\right).$$

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    - A_I provides additional vector components; $A_I = 0$ by choice of frame.
    - Remaining scalar dof $\psi = A_0 - \alpha$.
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$$\delta S = \int_{\partial M} d^3x \sqrt{-h}\left(T^{\alpha}_{B}\delta e^{(B)}_{\alpha} + \pi^{A}\delta A_{A}\right).$$

- Variation at fixed A_A implies T_{AB} not a symmetric tensor.
  - A_I provides additional vector components; $A_I = 0$ by choice of frame.
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Hollands
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$$\nabla_{\alpha} T^{\alpha}_{\beta} - \pi^{\alpha}\nabla_{\beta}A_{\alpha} = 0.$$

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symptotically locally Lifshitz spacetimes

SFR & Saremi

Vant the leading-order metric at large r to locally take the form

$$ds^2 = -r^{2z}dt^2 + r^2 d\vec{x}^2 + \frac{dr^2}{r^2} + \dots$$

Vork with an orthonormal frame $e^{(A)}, e^{(r)}$. $(A = 0, 1, 2;\ I = 1, 2.)$
y choice of gauge, $e_r^{(A)} = 0$, $e^{(r)} = \frac{dr}{r}$. Require that as $r \to \infty$,

$$e^{(0)} = r^z \hat{e}^{(0)}(r, t, \vec{x}), \quad e^{(I)} = r\hat{e}^I(r, t, \vec{x}),$$

vhere $\hat{e}^{(0)}(r, t, \vec{x})$, $\hat{e}^I(r, t, \vec{x})$ have finite limits as $r \to \infty$.
oundary data analogous to conformal metric on boundary.

Horava
Melby-Thompson

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SFR
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lolographic renormalization

ixtend beyond linear analysis: want to determine $\langle T^{\alpha}{}_{B}\rangle$, $\langle\mathcal{O}\rangle$ for arbitrary
ources to all orders.

leed to solve eom in asymptotic regime $r \to \infty$.

Ise functional differentiation approach:

Papadimitriou
Skenderis

- Introduce "dilatation generator"

$$\delta_D = \int d^3x\sqrt{-h}\left(ze_{\alpha}^{(0)}\frac{\delta}{\delta e_{\alpha}^{(0)}} + e_{\alpha}^{(I)}\frac{\delta}{\delta e_{\alpha}^{(I)}} - (z+2-\Delta)\psi\frac{\delta}{\delta\psi}\right)$$

  expand in eigenvalues of δ_D rather than powers of r.
- Regular expansion exists
  - For arbitrary sources for $z < 2$
  - For zero source for $\mathcal{E}^i$, $\mathcal{O}$ for $z \geq 2$.
- Expansion gives subleading terms in bulk as functions of sources.

lolographic renormalization

)ivergent terms in response functions $\langle T^{\alpha}_{\ B}\rangle$, $\langle \mathcal{O}\rangle$ from dilatation xpansion can be cancelled by local counter-terms in action.

- Write $S_{on-shell} = \int d^3x\sqrt{-h}\lambda$; since $T^A_{\ B} = e^{(A)}_{\alpha}\frac{\delta S}{\delta e^{(B)}_{\alpha}}$,

$$(z+2-\delta_D)\lambda = zT^0_{\ 0} + T^I_{\ I} - (z+2-\Delta)\psi\mathcal{O}.$$

- Using this, $T_{AB} = \pi_{AB} + \pi_A A_B$ and constraint

$$\frac{1}{8}\pi^2 - \frac{1}{4}\pi_{AB}\pi^{AB} - \frac{1}{2}\pi_A\pi^A - \frac{1}{2m^2}(\nabla^A\pi_A)^2 = R - 2\Lambda - \frac{1}{4}F_{AB}F^{AB} - \frac{1}{2}m^2A_AA^A,$$

  determine on-shell action in dilatation expansion: gives divergent terms as functions of sources.

- Term in λ with $\delta_D = z+2$ undetermined; gives finite part of expectation values.

chrödinger geometry

Son Balasubramanian McGreevy

ymmetry Galilean symmetry + anisotropic dilatation D.

mbed Galilean symmetry in $ISO(d+1,1)$ by light-cone quant: $H = \tilde{P}_+$, $_i = \tilde{P}_i$, $K_i = \tilde{M}_{-i}$, $N = \tilde{P}_-$. Extend to embed $Sch(d)$ in $SO(d+2,2)$ by

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ravitational dual: deform AdS_{d+3} to

$$ds^2 = -r^4(dx^+)^2 + r^2(-2dx^+dx^- + d\mathbf{x}^2) + \frac{dr^2}{r^2}.$$

- Solution of a theory with a massive vector, $A_+ = r^2$.
- N discrete implies x^- periodic. Compact null direction?

chrödinger holography

Schrödinger$_{d=2}$ obtained in string theory by TsT from $AdS_5 \times S^5$

pply to Schwarzschild-AdS: obtain asymptotically Schrödinger black hole.

- Two-parameter solutions: r_+, β: temperature, particle number.
- Slow falloffs: $1 + \frac{\beta^2 r_+^4}{r^2}$

pply same prescription for stress tensor:

- For black hole solution, $\mathcal{E} = r_+^4$, $\Pi_{xx} = \Pi_{yy} = r_+^4$, $\rho = 2\beta^2 r_+^4$.
- For solutions obtained by TsT from vacuum AdS solution, agrees with AdS stress tensor.

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pply same prescription for stress tensor:

- For black hole solution, $\mathcal{E} = r_+^4$, $\Pi_{xx} = \Pi_{yy} = r_+^4$, $\rho = 2\beta^2 r_+^4$.
- For solutions obtained by TsT from vacuum AdS solution, agrees with AdS stress tensor.

chrödinger geometry

Son   Balasubramanian McGreevy

ymmetry Galilean symmetry + anisotropic dilatation D.
mbed Galilean symmetry in $ISO(d+1,1)$ by light-cone quant: $H = \tilde{P}_+$,
$_i = \tilde{P}_i$, $K_i = \tilde{M}_{-i}$, $N = \tilde{P}_-$. Extend to embed $Sch(d)$ in $SO(d+2,2)$ by

$$D = \tilde{D} + (z-1)\tilde{M}_{+-}.$$

ravitational dual: deform AdS_{d+3} to

$$ds^2 = -r^4(dx^+)^2 + r^2(-2dx^+dx^- + d\mathbf{x}^2) + \frac{dr^2}{r^2}.$$

- Solution of a theory with a massive vector, $A_+ = r^2$.
- N discrete implies x^- periodic. Compact null direction?

chrödinger holography

Schrödinger$_{d=2}$ obtained in string theory by TsT from $AdS_5 \times S^5$

pply to Schwarzschild-AdS: obtain asymptotically Schrödinger black hole.

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)iscussion

- NRCFT is an interesting and challenging extension of AdS/CFT.
- Lifshitz has a simple spacetime dual, now embedded in string theory.
- Holographic dictionary similar to familiar AdS/CFT case.
- Pretty much under control for $1 < z < 2$
- For $z > 2$, some issues remain:
  - Check counterterms also cancel divergences from fast fall-off modes.
  - Understand divergences in $\langle \mathcal{O} \rangle$ for $z > 2$.
  - Flow between boundary conditions for $z > 4$.

tress tensor

Boundary geometry $\hat{e}^{(0)}(r,t,\vec{x})$, $\hat{e}^{I}(r,t,\vec{x}) \Rightarrow$ sources for stress tensor. Assume we have an action S finite on-shell, $\delta S = 0$ for variations preserving boundary data. Define T^{α}_{B} by $(\alpha = t, x^1, x^2;\ i = x^1, x^2)$

$$\delta S = \int_{\partial M} d^3x \sqrt{-h}\,(T^{\alpha}_{B}\,\delta e^{(B)}_{\alpha} + \pi^{A}\delta A_A).$$

- Variation at fixed A_A implies T_{AB} not a symmetric tensor.
  - A_I provides additional vector components; $A_I = 0$ by choice of frame.
  - Remaining scalar dof $\psi = A_0 - \alpha$.
- Identify with stress tensor complex: $T^{\alpha}_{0} = \mathcal{E}, \mathcal{E}^{i}$; $T^{\alpha}_{J} = \mathcal{P}_j, \Pi^{i}_{j}$.
- Invariance of S under boundary diffeomorphisms $t'(t, x^i), x^{i'}(t, x^i)$ implies conservation equations

Hollands
Ishibashi
Marolf

$$\nabla_{\alpha} T^{\alpha}_{\beta} - \pi^{\alpha}\nabla_{\beta}A_{\alpha} = 0.$$

symptotically locally Lifshitz spacetimes

SFR & Saremi

Vant the leading-order metric at large r to locally take the form

$$ds^2 = -r^{2z}dt^2 + r^2 d\vec{x}^2 + \frac{dr^2}{r^2} + \dots$$

Work with an orthonormal frame $e^{(A)}, e^{(r)}$. $(A = 0, 1, 2;\ I = 1, 2.)$
3y choice of gauge, $e_r^{(A)} = 0$, $e^{(r)} = \frac{dr}{r}$. Require that as $r \to \infty$,

$$e^{(0)} = r^z \hat{e}^{(0)}(r, t, \vec{x}), \quad e^{(I)} = r\hat{e}^I(r, t, \vec{x}),$$

vhere $\hat{e}^{(0)}(r, t, \vec{x})$, $\hat{e}^I(r, t, \vec{x})$ have finite limits as $r \to \infty$.
3oundary data analogous to conformal metric on boundary.

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