

Title: Maximizing the scientific return from cosmic non-Gaussianity

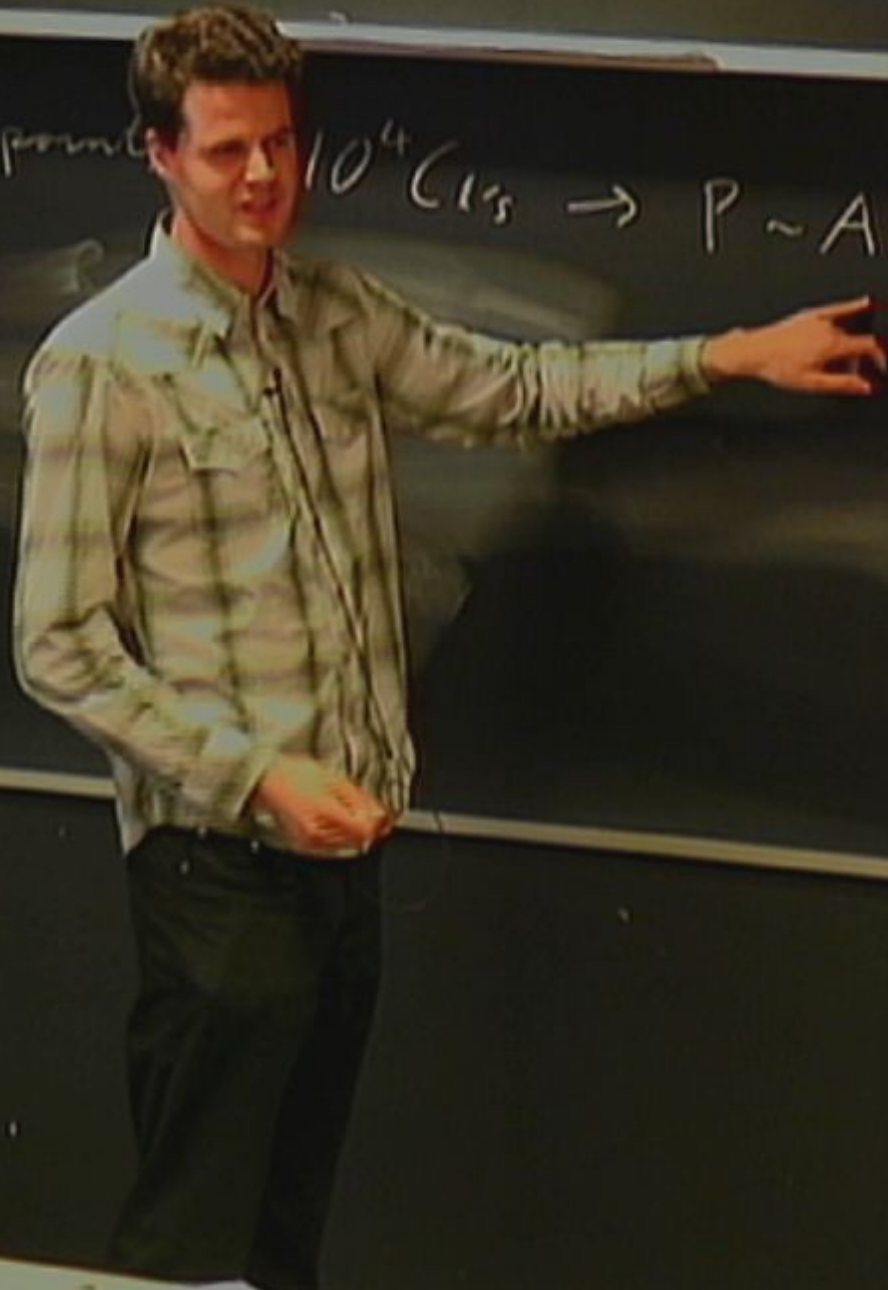
Date: May 17, 2011 02:00 PM

URL: <http://pirsa.org/11050000>

Abstract: The model of local non-Gaussianity, parameterized by the constant non-linearity parameter  $f_{NL}$ , is an extremely popular description of non-Gaussianity. However, a mild scale-dependence of  $f_{NL}$  is natural. This scale dependence is a new observable, potentially detectable with the Planck satellite, which helps to further discriminate between models of inflation. It is sensitive to properties of the early universe which are not probed by the standard observables. In a complementary way, the trispectrum also contains important information about non-Gaussianity which the bispectrum does not capture. We explicitly calculate the scale dependence and trispectrum in several models including one with a very large infrared-loop contribution to the bispectrum and in various realizations of the curvaton scenario.

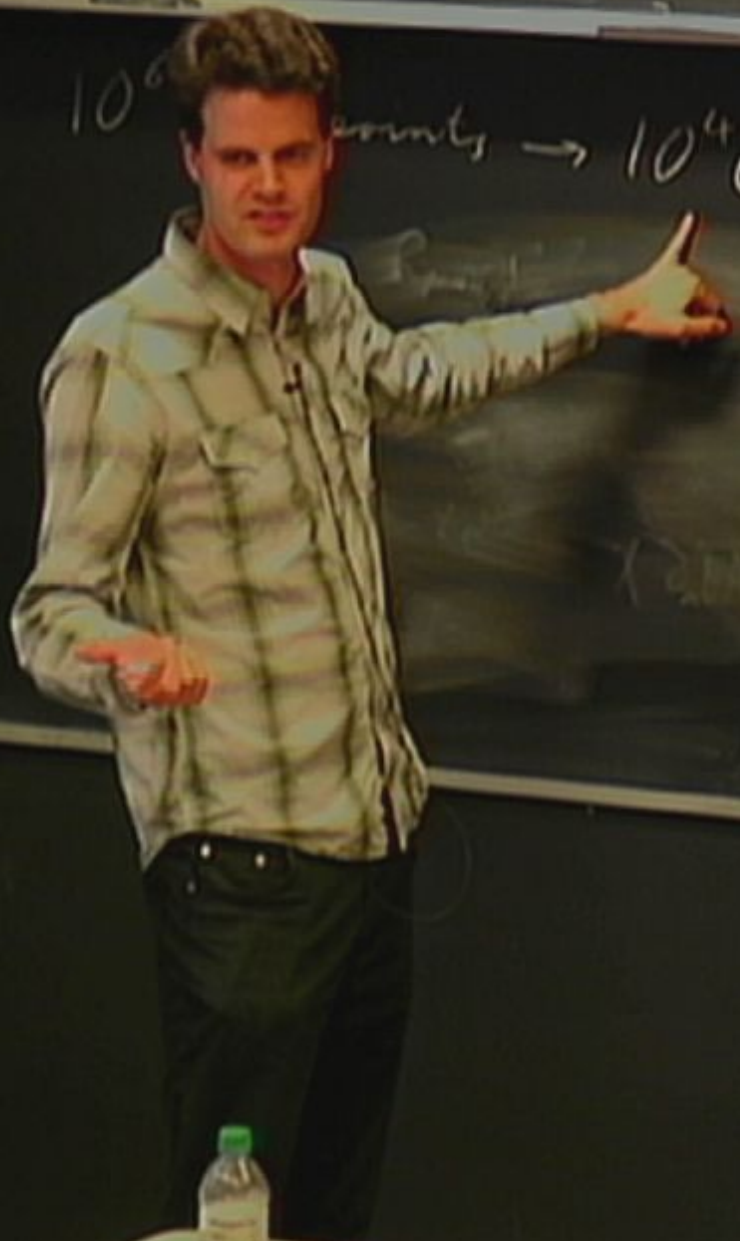
curvaton  
collaborators: C.B. et al 1007.4277  
Engvist, Gerstenlauer, Numma, Takahashi, Tasinato  
Wands 11007.5148 + in preparation

$10^6$  data points  $10^4$  classes  $\rightarrow P \sim A k^{n_s-1}$



CAUTION  
Please do not touch the board  
or the equipment on the board  
if you have any questions  
please ask the lecturer

$10^6$  counts  $\rightarrow 10^4$  classes  $\rightarrow P \sim A k^{n_s-1}$



CAUTION  
Do not touch the board  
Do not touch the board  
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$10^6$  data points  $\rightarrow 10^4$  classes  $\rightarrow P \sim A k^{n-1}$   
i) scale dependence of trispectrum ii) trispectrum

6 data points  $\rightarrow 10^4$  (1's  $\rightarrow P \sim A k^{n_s-1}$   
 scale dependence of trispectrum ii) trispectrum  
 model  $\zeta(x) = \zeta_G(x) + f_{NL} \zeta_G^2(x) + \dots$



ERLEN  
 UNIVERSITÄT  
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 LEHRGEBIET FÜR VERFAHRENSTECHNIK  
 LEHRSTUHL FÜR VERFAHRENSTECHNIK

$10^6$  data points  $\rightarrow 10^{11}$  (1's  $\rightarrow P \sim A k^{n_s-1}$ )

i) scale dependence of spectrum ii) trispectrum

local model

$\{k_1, k_2, k_3\}$

$$\gamma_{\mathbf{k}}(x) + \text{for } \gamma_{\mathbf{k}}^2(x) + \dots$$

$(k_1, k_2)$

$10^6$  points  $\rightarrow 10^4$  (1's  $\rightarrow P \sim A k^{n_s-1}$ )  
 i) second order dependence of bispectrum ii) trispectrum

$$\zeta(x) = \zeta_G(x) + \int_{\text{nl}} \zeta_G^2(x) + \dots$$

$$f_{nl} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$



$10^6$  data points  $\rightarrow 10^4$  (1's  $\rightarrow P \sim A k^{n_s-1}$ )  
i) scale dependence of spectrum ii) trispectrum

local model

$$\zeta(x) = \zeta_G(x) + f_{NL} \zeta_G^2(x) + \dots$$

$$f_{NL} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$

$10^6$  data points  $\rightarrow 10^{11}$  (1's)  $\rightarrow P \sim A k^{n_s-1}$

i) scale dependence of trispectrum ii) trispectrum

local model

$$\zeta(x) = \zeta_G(x) + f_{NL} \zeta_G^2(x) + \dots$$

$$f_{NL} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$

$P(k_1)P(k_2) + \text{cyclic}$



ton scenario

$$\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \dot{\chi}^2 - \frac{1}{2} m^2 \chi^2$$



ton scenario  $\mathcal{L} = \frac{1}{2} \dot{\varphi}^2 - V(\varphi) + \frac{1}{2} \dot{x}^2 - \frac{1}{2} m^2 x^2$   
 $p_x \approx \text{const}$  until  $m$  after

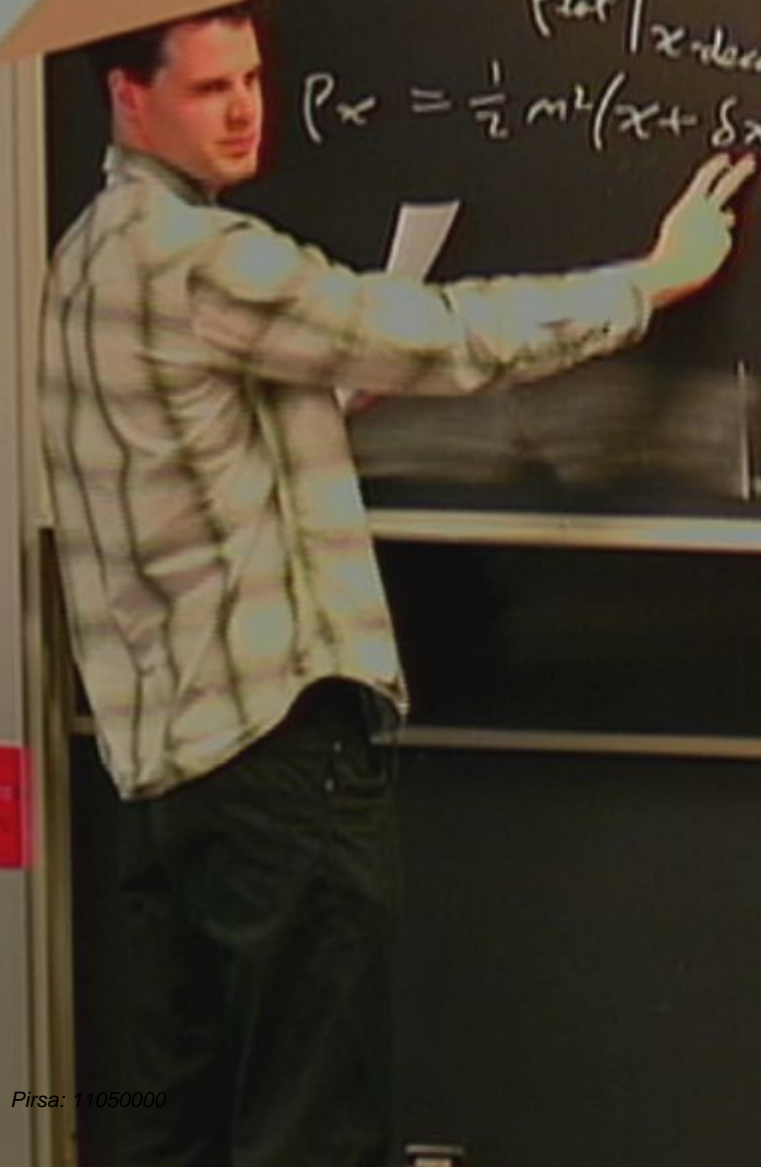
inflation scenario  $\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \dot{\chi}^2 - \frac{1}{2} m^2 \chi^2$   
 $p_\chi \approx \text{const}$  until  $m \sim H$ , after  $p_\chi \sim \frac{1}{a^2}$ ,  $p_\chi \sim \frac{1}{a^3}$

C. scenario  $\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \dot{\chi}^2 - \frac{1}{2} m^2 \chi^2$   
= const until  $m \sim H$ , after  $P_\chi \sim \frac{1}{a^2}$ ,  $P_\chi \sim \frac{1}{a^3}$

$$\Gamma_d = \frac{P_\chi}{P_{\text{tot}}} \Big|_{x\text{-decay}} \ll 1$$

Scenario  $\mathcal{L} = \frac{1}{2} \dot{\phi}^2 - V(\phi) + \frac{1}{2} \dot{x}^2 - \frac{1}{2} m^2 x^2$   
const until  $m \sim H$ , after  $P_\gamma \sim \frac{1}{a^4}$ ,  $P_x \sim \frac{1}{a^3}$

$$\Gamma_x = \frac{P_x}{P_{tot}} \Big|_{x\text{-decay}} \ll 1$$
$$P_x = \frac{1}{2} m^2 (x + \delta x)^2$$



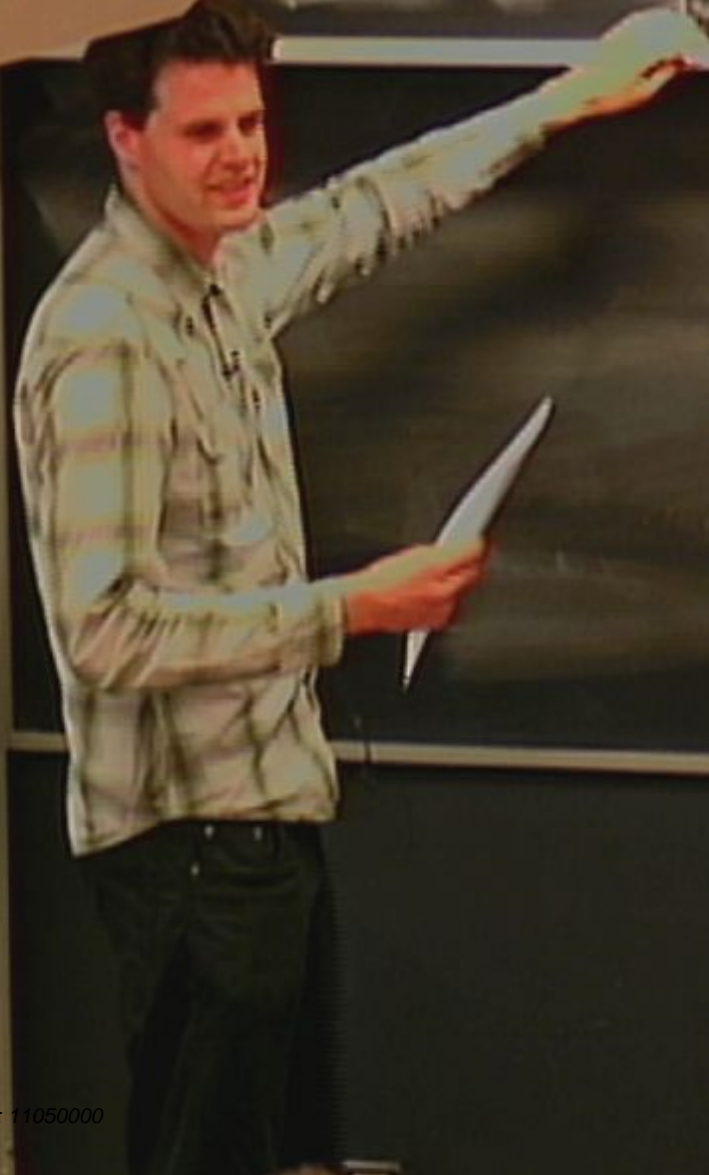
until  $m \sim H$ , after  $P_x \sim \frac{1}{a^2}$ ,  $P_x \sim \frac{1}{a^3}$

$$\frac{P_x}{P_{tot}} \Big|_{x\text{-decay}} \ll 1$$

$$P_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left( \frac{\delta x}{x} \right)^2$$



$$+ 3 H_0 \delta x + m \delta x = 0 \Rightarrow \frac{\delta x}{x} = \text{cst}$$



$$\Gamma_d = \frac{p_x}{p_{tot}} \ll 1$$

$$p_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta p_x}{p_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = -cst$$

$$\delta x_s \sim \delta p_s \sim \frac{H_s}{2\pi}$$

$$p_x = \frac{1}{2} m^2 (\dot{x} + \delta \dot{x})^2 \rightarrow \frac{\delta p_x}{p_x} = \frac{\delta \dot{x}}{\dot{x}} + \left( \frac{\delta \dot{x}}{\dot{x}} \right)^2$$

$$\delta \dot{x} + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta \dot{x}}{\dot{x}} = -c \dot{x}$$

$$\delta x_i \sim \delta \varphi_i \sim \frac{H_i}{2\pi} \quad 3 = \frac{H_i}{\sqrt{\epsilon_x}} \delta \varphi_i + \Gamma_d \left( \frac{\delta \dot{x}}{\dot{x}} + \left( \frac{\delta \dot{x}}{\dot{x}} \right)^2 \right) \Big|_{t_i}$$

$$P_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

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$$\delta x_i \sim \delta \varphi_i \sim \frac{H_i}{2\pi} \quad 3 = \frac{1}{\sqrt{\epsilon_x}} \delta \varphi_i + \Gamma_d \left( \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2 \right) \Big|_{t_i}$$

pure curvature  $\gamma_G = \Gamma_d \frac{\delta x}{x} \Rightarrow$

$$p_x = \frac{1}{2} m^2 (\dot{x} - ax)^2 \rightarrow \frac{\delta p_x}{p_x} = \frac{\delta x}{x} + \left( \frac{\delta x}{x} \right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = -cst$$

$$\delta x_i \sim \delta \varphi_i \sim \frac{H_i}{2\pi} \quad 3 = \frac{1}{\sqrt{\epsilon_x}} \delta \varphi_i + \Gamma_d \left( \frac{\delta x}{x} + \left( \frac{\delta x}{x} \right)^2 \right) \Big|_{t_i}$$

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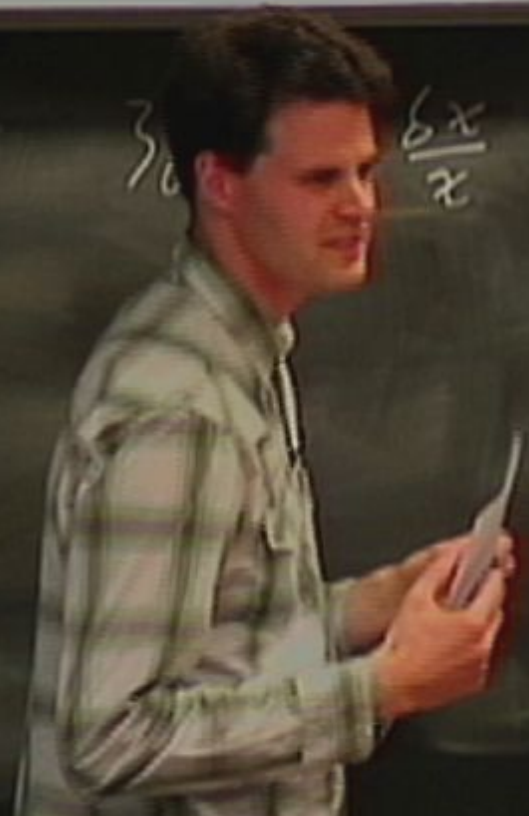
$$P_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = \text{const}$$

$$\delta x_i \sim \delta \varphi_i \sim \frac{H_i}{2\pi} \quad 3 = \frac{1}{\sqrt{\epsilon_x}} \delta \varphi_i + \Gamma_d \left( \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2 \right) \Big|_{t_i}$$

pure curvature

$$3 \approx \frac{\delta x}{x} \Rightarrow f_{NL} = \frac{1}{\Gamma_d}$$



$$P_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = \text{const}$$

$$\delta x \sim \delta \varphi \sim \frac{H_0}{2\pi} \quad 3 = \frac{1}{\sqrt{\epsilon_x}} \delta \varphi + \Gamma_d \left( \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2 \right) \Big|_{t_0}$$

pure curvature

$$\gamma_G = \Gamma_d \frac{\delta x}{x}$$

$$\gamma_{NG} = \frac{1}{\Gamma_d}$$

$$\frac{1}{\sqrt{\epsilon}} \gg 1 \quad \Gamma_d \ll 1 \quad \text{incl}$$

$$P_3 = P_{3q} + P_{3x} = (1+x) P_3$$

$$P_x = \frac{1}{2} m^2 (x + \delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = \text{const}$$

$$\delta x_i \sim \delta \varphi_i \sim \frac{H_i}{2\pi} \quad 3 = \frac{1}{\sqrt{\epsilon_x}} \delta \varphi_i + \Gamma_d \left( \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2 \right) \Big|_{t_i}$$

pure curvature

$$\zeta_G = \Rightarrow f_{NC} = \frac{1}{\Gamma_d}$$

$$\frac{1}{\sqrt{\epsilon}} \gg 1 \quad \Gamma_d \ll 1$$

wide inflaton

$$P_3 = P_{3q} + P_{3x} =$$

$$\lambda = \frac{P_{3q}}{P_{3x}} > 0$$

$$B_3 =$$



$$P_x = \frac{1}{2} m^2 (\delta x)^2 \rightarrow \frac{\delta P_x}{P_x} = \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2$$

$$\delta x + 3 H \delta x + m^2 \delta x = 0 \Rightarrow \frac{\delta x}{x} = \text{const}$$

$$\delta x_s \sim \delta \varphi_s \sim \frac{H_s}{2\pi} \quad \beta = \frac{1}{\sqrt{\epsilon_s}} \delta \varphi_s + \Gamma_d \left( \frac{\delta x}{x} + \left(\frac{\delta x}{x}\right)^2 \right) \Big|_{t_s}$$

pure curvature  $\beta_G = \Gamma_d \frac{\delta x}{x} \Rightarrow f_{NL} = \frac{1}{\Gamma_d}$

$\frac{1}{\sqrt{\epsilon}} \gg 1$   $\Gamma_d \ll 1$  include inflaton

$$P_3 = P_{3q} + P_{3x} = (1+x) P_{3x} \quad \lambda = \frac{P_{3q}}{P_{3x}} > 0$$

$$B_3 = P_{3x} =$$

$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3x}}{(P_{1y} + P_{1x})}$$

$$f_{NL} = \frac{B_3}{P_3^2} = \frac{B_{3x}}{(P_{1y} + P_{2x})^2} = \frac{1}{(\lambda(1+\lambda))^2}$$



$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3\alpha}}{(P_{1\alpha} + P_{2\alpha})^2} = \frac{1}{\Gamma_\lambda (1+\lambda)^2}, \quad \Gamma_\lambda \text{ free parameter, what is } \lambda?$$

$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3\alpha}}{(P_{1\alpha} + P_{2\alpha})^2} = \frac{1}{\Gamma \lambda (1+\lambda)^2}, \quad \Gamma \lambda \text{ free parameter, what is } \lambda?$$

2 ways

$10^6$  data points  $\rightarrow 10^{14}$  (1's)  $\rightarrow P \sim A k^{n_s-1}$

i) scale dependence of bispectrum ii) trispectrum

local model

$$\zeta(x) = \zeta_G(x) + f_{NL} \zeta_G^2(x) + \dots$$

$$f_{NL} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$



$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3\lambda}}{(P_{1\lambda} + P_{2\lambda})^2} = \frac{1}{\Gamma_\lambda (1+\lambda)^2}, \quad \Gamma_\lambda \text{ free parameter, what is } \lambda?$$

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> 2 ways i)





$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3x}}{(P_{1y} + P_{1x})^2} = \frac{1}{\Gamma_2(1+\lambda)^2}, \quad \Gamma_2 \text{ free parameter, what is } \lambda?$$

> 2 ways i)  $\lambda \neq 0$   $f_{NL} \propto \frac{P_{3x}^2}{P_{1y} + P_{1x}}$

$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3\kappa}}{(P_{1\mu} + P_{1\nu})^2} = \frac{1}{\Gamma_2 (1+\lambda)^2}, \quad \Gamma_2 \text{ free parameter, what is } \lambda?$$

> 2 ways

i)  $\lambda \neq 0 \quad f_{NL} \propto \frac{P_{3\kappa}^2}{P_3^2} \propto k^{2(n_s - 1)}$

$$f_{NL} = \frac{B_3}{P_3^2} = \frac{B_{3x}}{(P_{1y} + P_{1x})^2} = \frac{1}{\Gamma_2(1+\lambda)^2}, \quad \Gamma_2 \text{ free parameter, what is } \lambda?$$

> 2 ways

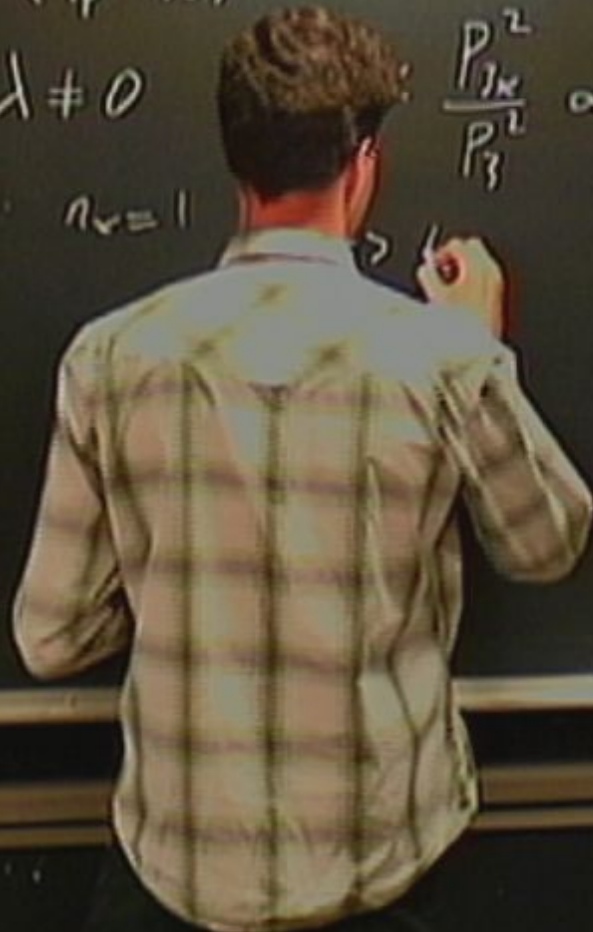
$$i) \lambda \neq 0 \quad f_{NL} \propto \frac{P_{3x}^2}{P_3^2} \propto k^{2(n_x - n_s)} \Rightarrow n_{f_{NL}} = -2(n_x - n_s)$$

$$f_{nu} = \frac{B_3}{P_3} = \frac{B_{3x}}{(P_{1y} + P_{1x})^2} = \frac{1}{r_2((1+\lambda))^2}, \quad r_2 \text{ free parameter, what is } \lambda?$$

> 2 ways

i)  $\lambda \neq 0$   $\frac{P_{3x}^2}{P_3} \propto k^{2(n_x - n_s)} \Rightarrow n_{f_{nu}} = -2(n_x - n_s)$

natural  $n_x = 1$



$$f_{NL} = \frac{B_3}{P_3} = \frac{B_{3\kappa}}{(P_{3\mu} + P_{3\nu})^2} = \frac{1}{r_2(1+\lambda)^2}, \quad r_2 \text{ free parameter, what is } \lambda?$$

> 2 ways

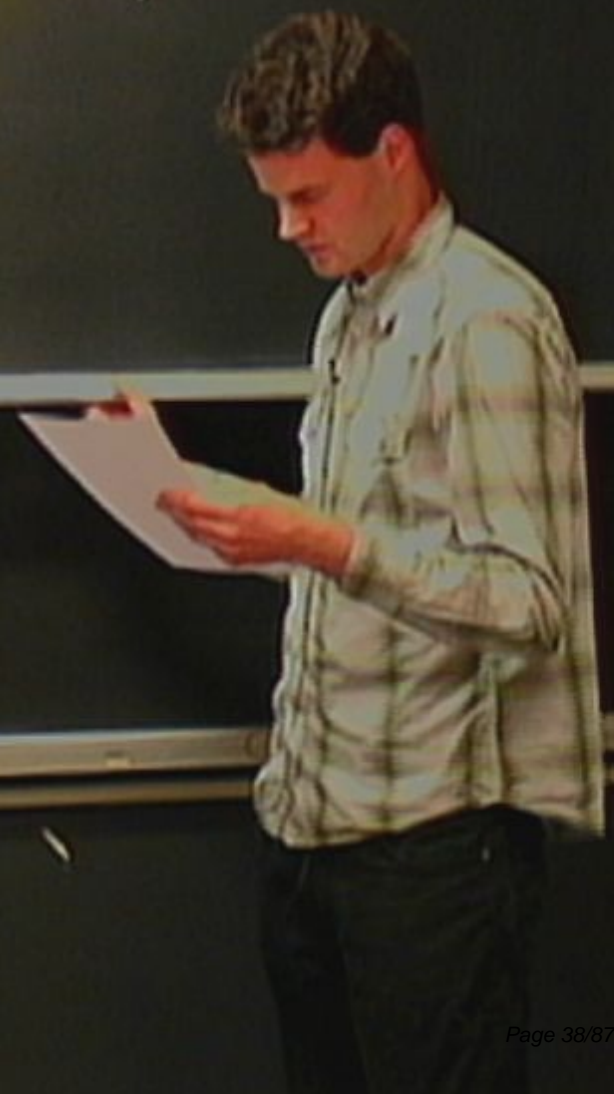
$$i) \lambda \neq 0 \quad f_{NL} \propto \frac{P_{3\kappa}^2}{P_3^2} \propto k^{2(n_\kappa - n_s)} \Rightarrow n_{f_{NL}} = -2(n_\kappa - n_s)$$

natural:  $n_\kappa = 1$

$$\Rightarrow n_{f_{NL}} = -2(n_s - 1) \simeq 0.1$$

$\gamma_3$   $(P_{14} + P_{15})$   
 $\Rightarrow$  2 ways i)  $\lambda \neq 0$   $f_{n_4} \propto \frac{P_{14}^2}{P_3^2} \propto k^{2(n_4 - n_3)} \Rightarrow n_{f_{n_4}} = -2(n_4 - n_3)$   
 natural  $n_4 = 1 \Rightarrow n_{f_{n_4}} = -2(n_3 - 1) \approx 0.1$

$$R = (R, H)$$



$$1) \lambda \neq 0 \quad f_{\text{nu}} \approx \frac{1}{R_3^2} \propto R \quad \Rightarrow n_{f_{\text{nu}}} = -2(n_x - n_s)$$

natural:  $n_x = 1 \quad \Rightarrow n_{f_{\text{nu}}} = -2(n_s - 1) \approx 0.1$

$$R' = (R, \mathcal{H}^1)$$

$$H_{\text{min}}^1(\mathbb{R}, R, |E)$$



> 2 ways

i)  $\lambda \neq 0$   $f_{n_2} \propto \frac{P_{n_2}}{P_{n_1}} \propto k^{2(n_2 - n_1)} \Rightarrow n_{f_{n_2}} = -2(n_2 - n_1)$

natural  $n_2 = 1 \Rightarrow n_{f_{n_2}} \approx -2(n_1 - 1) \approx 0.1$

$n_x$

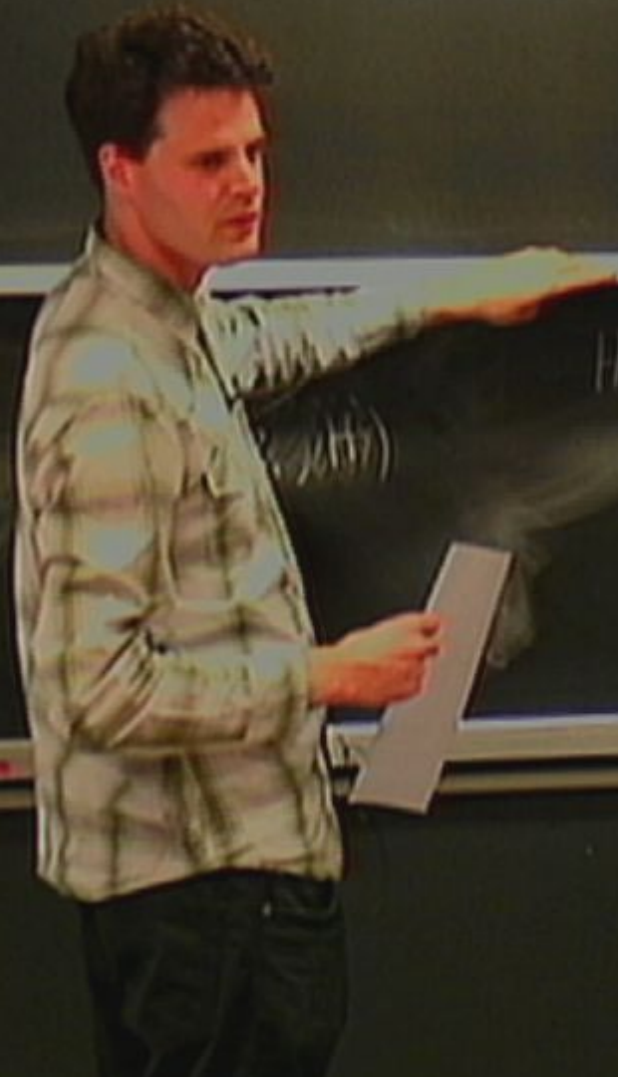
$$H_{\text{min}}(R, R, |E)$$



ways i)  $\lambda \neq 0$   $f_{nl} \propto \frac{f_{nl}}{P_3} \propto k^{2(n_x - n_s)} \Rightarrow n_{f_{nl}} = -2(n_x - n_s)$

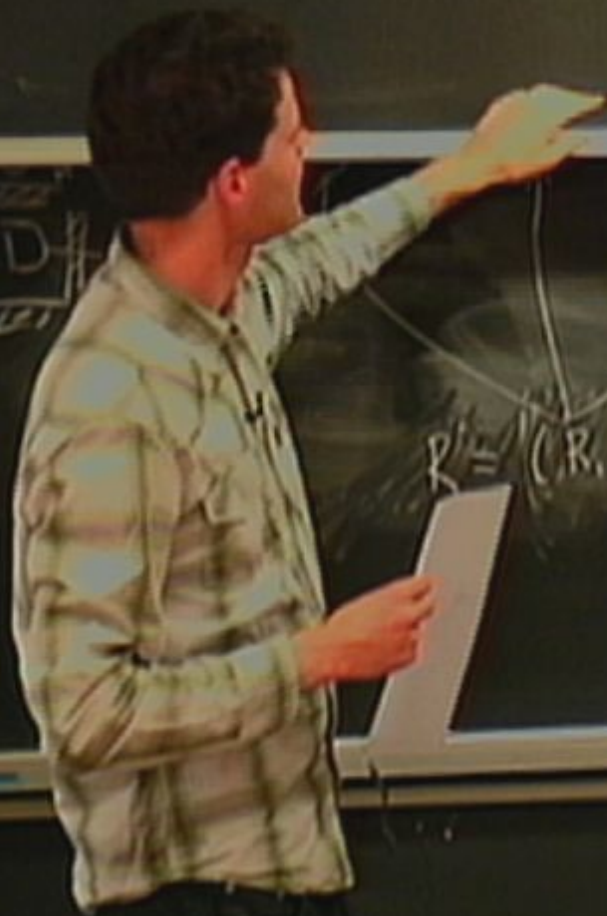
natural  $n_x = 1 \Rightarrow n_{f_{nl}} = -2(n_s - 1) \simeq 0.1$

$n_x = 1 = -2\varepsilon_0 + 2\eta_{x0}$



$H_{min}^1(\mathbb{R}, \mathbb{R}, |E|)$

natural  $n_s = 1 \Rightarrow n_{\text{par}} = -2(n_s - 1) \approx 0.1$   
 $n_{\text{par}} = -2\varepsilon_s + 2\eta_s$



How long  $S$  should be  
 $H_{\text{min}}^e(\tilde{R}, R, |E)$

$$R = (R, \mathcal{H})$$

collaborators: Engquist, Gerstoft, Niu, Takahashi, Tasinato  
Wands

$10^6$  data  $\rightarrow 10^{14}$  (1's  $\rightarrow P \sim A k^{n_s-1}$ )  
i) scale dependence of spectrum ii) trispectrum

local

$$B(k_1, k_2, k_3) + \dots$$
$$\frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$



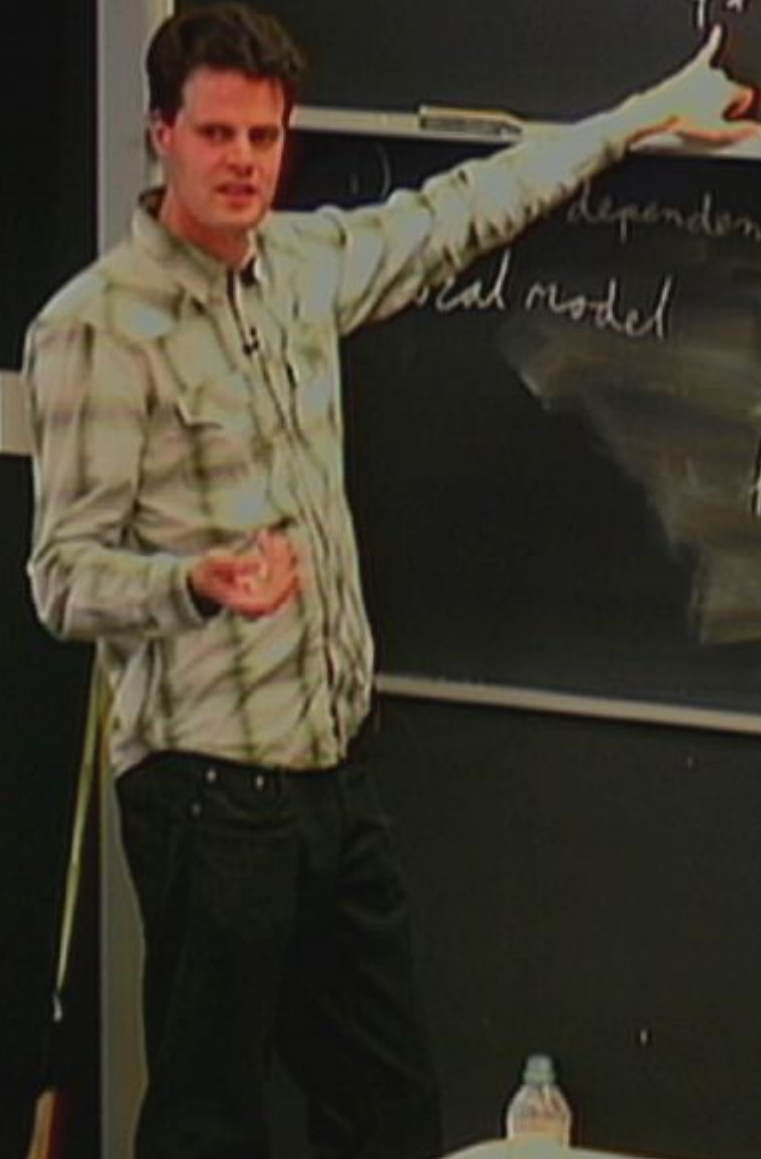
reference (C.B. et al) 1007.4277  
 curvature C.B. et al 1007.5148 + in preparation  
 collaborators: Engvist, Gershtein, Niemi, Takahashi, Tasinato  
 Wands

$$\Phi_x \sim \sqrt{4N} \sim \sqrt{2+0}$$

dependence of spectrum ii) trispectrum  
 local model

$$\zeta(x) = \zeta_G(x) + f_{NL} \zeta_G^2(x) +$$

$$f_{M3} = \frac{B(k_1, k_2, k_3)}{P(k_1)P(k_2) + \text{cyclic}}$$



reference C.B et al 11007.4277  
 curvature C.B et al 11007.5148 + in preparation  
 collaborators: Engvist, Gerstenlauer, Nuomi, Takahashi, Tasinato  
 Wands

$$\varphi_x \sim \sqrt{4N} \sim \sqrt{2+0}$$

$$x_x \approx \delta x_x$$

i) dependence of spectrum ii) spectrum

$$\zeta(x) = \zeta_G(x) + \text{for } \zeta_G^2(x) +$$

$$B(k_1, k_2, k_3)$$

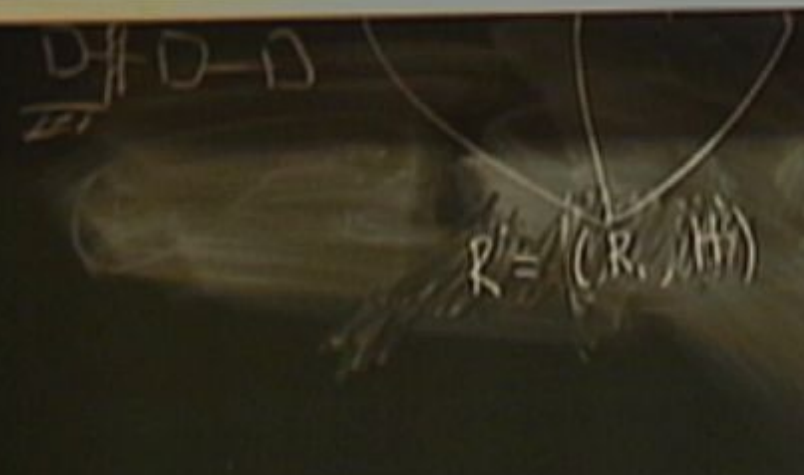

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$$P(k_1)P(k_2) + \text{cyclic}$$



$\lambda \neq 0$   $\frac{1}{R} \propto R$   $\Rightarrow n_{\text{rel}} = -2(n_x - n_s)$   
 natural:  $n_x = 1$   $\Rightarrow n_{\text{rel}} \approx -2(n_s - 1) \approx 0.1$   
 $n_{\text{rel}} = -2\varepsilon_s + 2\eta_s$

ii) trispectrum  6 variables in 2D



How long  $S$  should be  
 $H_{\text{min}}^L(\vec{R}, R, |E)$

natural  $n_s = 1$

$$\Rightarrow n_{eff} \approx -2(n_s - 1) \approx 0.1$$

$$n_{eff} = -2\epsilon_2 + 2\eta_2$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)\alpha} \zeta^{(1)\beta} \zeta^{(1)\gamma} \rangle \rightarrow \tau_{NL}$$

$D \# D \# D$   
 $D \# D \# D$

$$\frac{d}{dt} \ln \left( \frac{\zeta_{NL}}{\zeta_{L}} \right) \approx 0$$

How long S should be

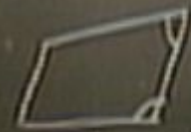
$$H_{min}^L(\mathbb{R}, \mathbb{R}, |E|)$$

natural  $n_v = 1$

$$\Rightarrow n_{\text{free}} = -2(n_s - 1) \approx 0.1$$

$$n_{\text{free}} = -2\varepsilon_v + 2\eta_{\text{free}}$$

ii) trispectrum



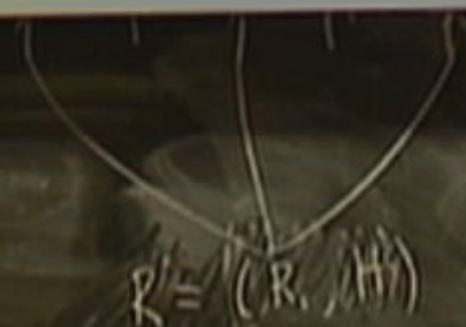
6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3$$

$$\langle \zeta^{(1)3} \zeta^{(1)} \rangle \rightarrow g_{NL}$$

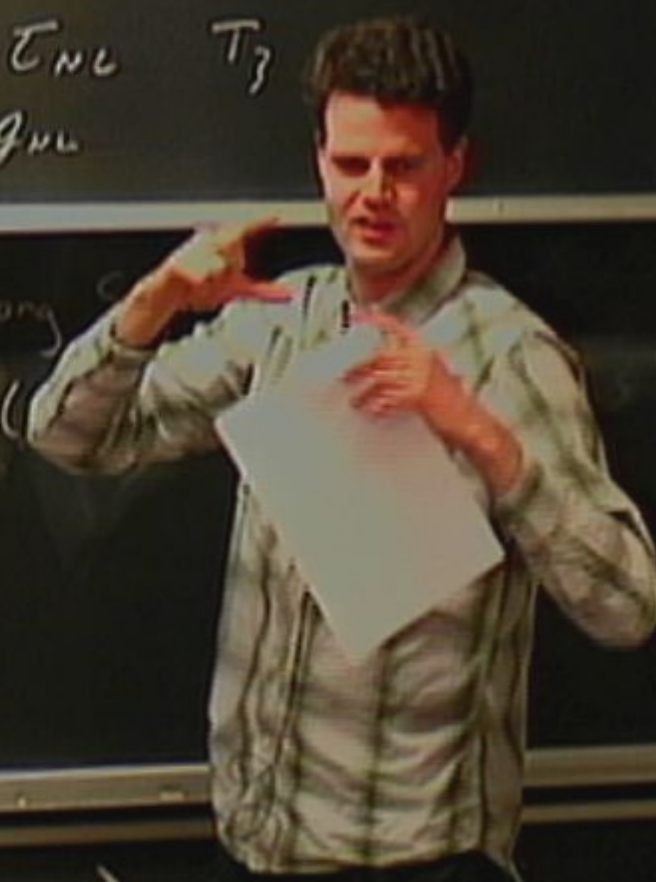
$\frac{D}{dt} D-D-D$



$$R' = (R, H')$$

How long

$$H_{\text{min}}'$$





natural  $n_s = 1$

$$\Rightarrow n_{\text{eff}} \approx -2(n_s - 1) \approx 0.1$$

$$n_{\text{eff}} = -2\epsilon_2 + 2\eta_{\text{eff}}$$

ii) trispectrum

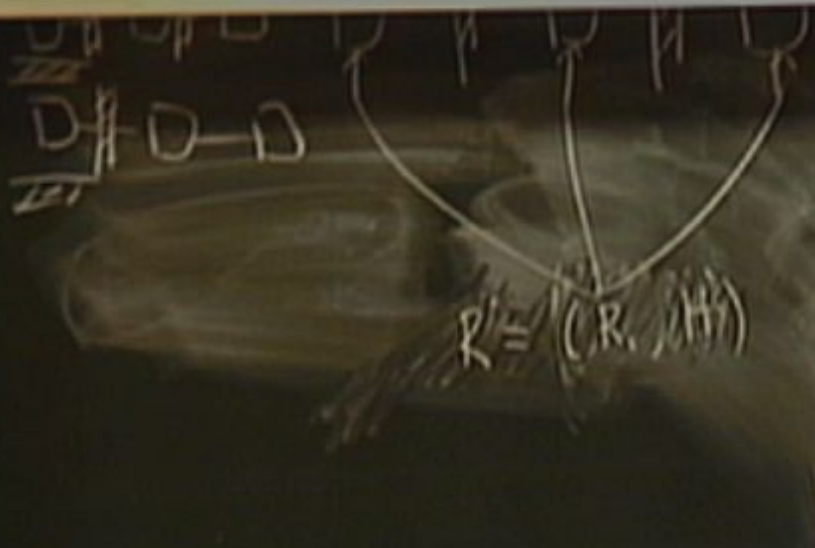


6 variables in 2D

2 terms

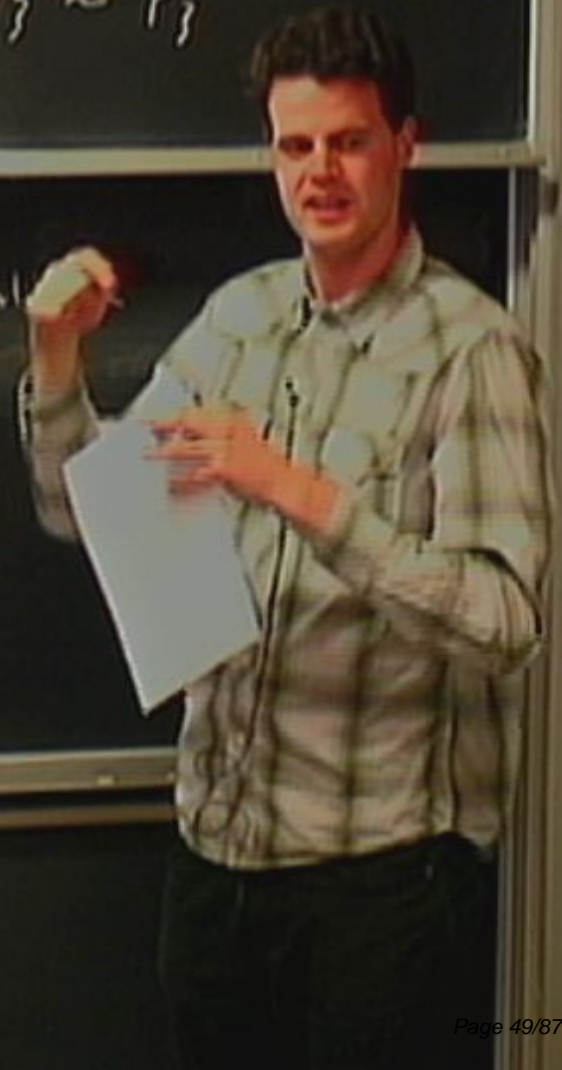
$$\langle \zeta^{(1)\mu} \zeta^{(1)\nu} \zeta^{(1)\lambda} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle \zeta^{(1)\mu} \zeta^{(2)\nu} \rangle \rightarrow g_{NL}$$



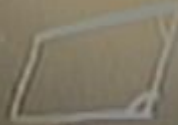
How long S should

$$H_{\text{min}}^L(\mathbb{R}, \mathbb{R}, |E)$$



$$|\psi\rangle = \frac{1}{\sqrt{2}}(\psi_1 + \psi_2)$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle \zeta^{(1)2} \zeta^{(2)} \rangle \rightarrow g_{NL}$$

$\lambda = 0$

$$\tau_{NL} \equiv \frac{T_3}{P_3^3}$$

$$+ 3 H \delta x + m \delta x = 0 \Rightarrow \frac{\delta x}{x} = c \delta t$$

$$3 = \frac{1}{\sqrt{\epsilon_1}} \delta \varphi_1 + \Gamma_d \left( \frac{\delta x}{x} + \left( \frac{\delta x}{x} \right)^2 \right) \Big|_{t_1}$$

pure curvature

$$\gamma_G = \Gamma_d \Rightarrow f_{NL} = \frac{1}{\Gamma_d}$$

$$\frac{1}{\sqrt{\epsilon}} \gg 1 \quad \Gamma_d \ll 1$$

$$P_3 = P_{3q} + P_{3x} = (1 + \dots)$$

$$B_3 = B_{3v} = \dots$$

e inflaton

$$\lambda = \frac{P_{3q}}{P_{1x}} > 0$$

ii) trispectrum



6 variables in 2D

2 terms

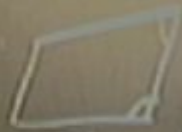
$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$
$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$\lambda=0$

$$\frac{1}{P_3^2} = \frac{1}{P_{NL}^2} = f_{NL}$$

'single source' consistency

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(2)2} \rangle \rightarrow \tau_{NL}$$

$$T_3 \sim P_3^3$$

$$\langle \zeta^{(1)3} \zeta^{(2)} \rangle \rightarrow g_{NL}$$

$\lambda=0$

$$\tau_{NL} \equiv \frac{1}{P_3} = \frac{1}{P_3^2} = f_{NL}$$

single source

$$-4 < f_{NL} < 70$$

2- $\sigma$

$$\tau_{NL} < 10^5$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle \zeta^{(1)3} \zeta^{(1)} \rangle \rightarrow g_{NL}$$

$$\tau_{NL} \equiv \frac{1}{P_3} = \frac{1}{P_{11}^2} = \frac{1}{f_{NL}}$$

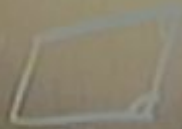
single source

$$-4 < f_{NL} < 70$$

2- $\sigma$

$$\tau_{NL} < 10^5, |g_{NL}| < 10^5 - 10^6$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$$\tau_{NL} \equiv \frac{1}{P_3} = \frac{1}{P_{11}^2} = f_{NL}$$

single source consistency

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, |g_{NL}| < 10^5 - 10^6$$

Planck

ii) trispectrum



6 variables in 2D

2 terms

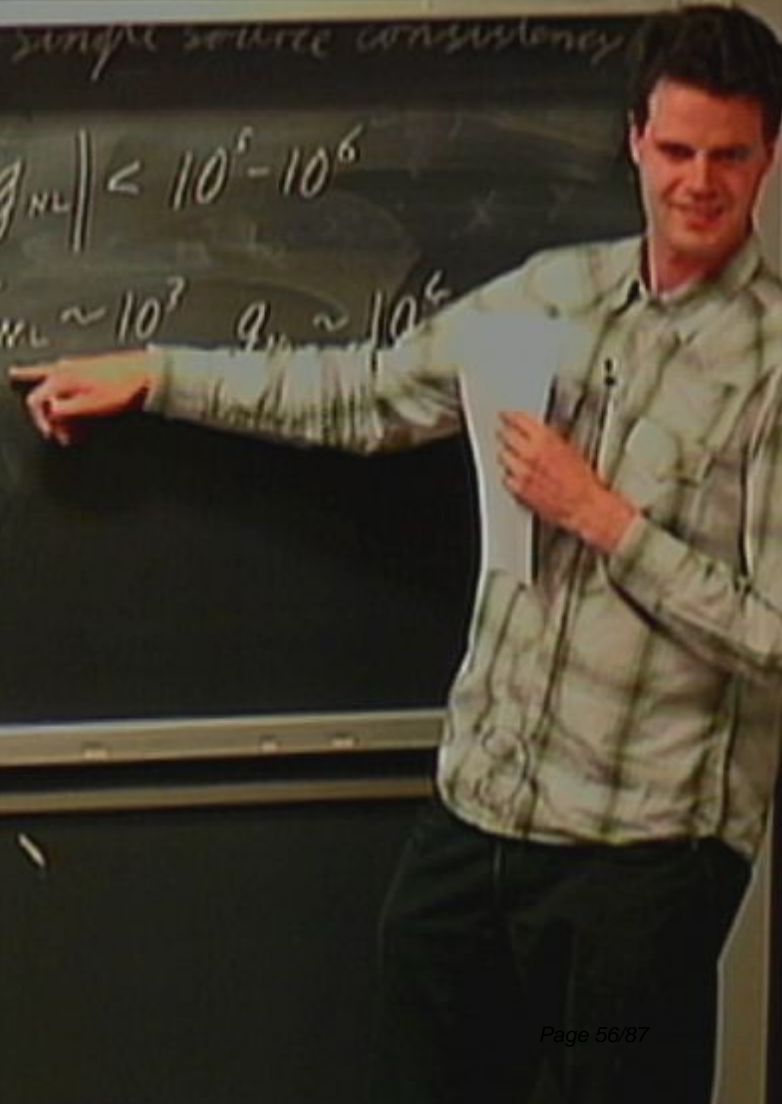
$$\langle \zeta^{(1)2} \zeta^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle \zeta^{(1)3} \zeta^{(1)} \rangle \rightarrow g_{NL}$$

$$\tau_{NL} = \frac{1}{P_3^2} = \Gamma_{11}^2 - f_{NL} \quad \text{single source consistency}$$

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, \quad |g_{NL}| < 10^5 - 10^6$$

Planck forecasts  $f_{NL} \lesssim 10, \tau_{NL} \sim 10^3, g_{NL} \sim 10^5$





ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$$\tau_{NL} = \frac{1}{P_1^2} = \Gamma_{11}^2 - f_{NL}$$

single source consistency

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, |g_{NL}| < 10^5 - 10^6$$

Planck forecasts  $f_{NL} \lesssim 10, \tau_{NL} \sim 10^3, g_{NL} \sim 10^4$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

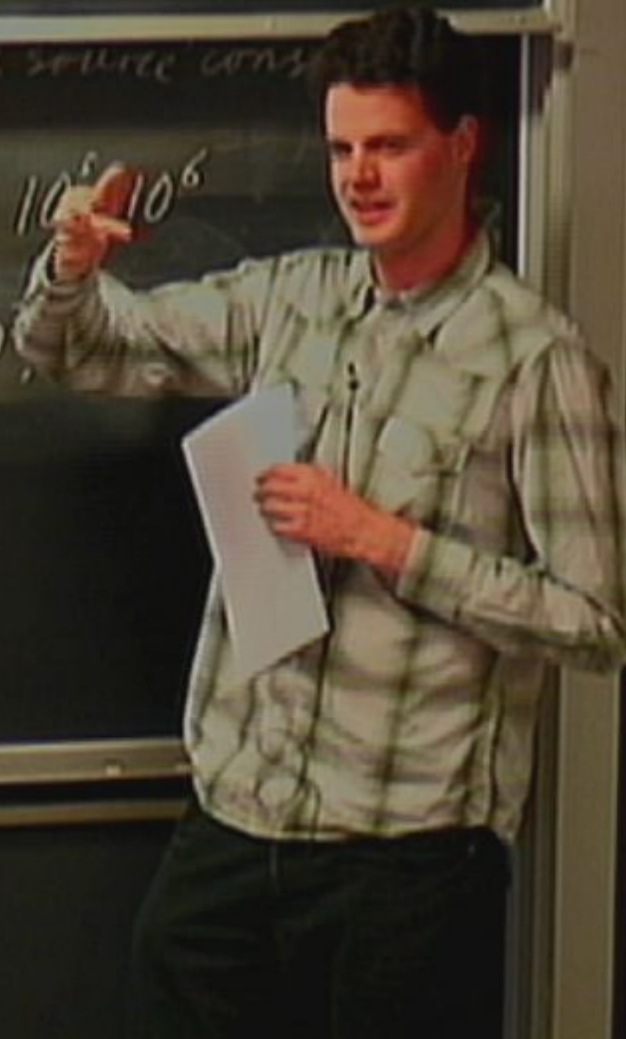
$$\langle \zeta^{(1)3} \zeta^{(2)} \rangle \rightarrow g_{NL}$$

$$\tau_{NL} = \frac{1}{P_1^2} = \tau_{11}^2 - f_{NL}$$

simple source cons

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, |g_{NL}| < 10^5 \cdot 10^6$$

$$\text{Planck forecasts } |f_{NL}| \lesssim 10, |\tau_{NL}| \sim 10^6$$



ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$$-4 < \tau_{NL} < 70 \quad 2-\sigma$$

$$10^3, |g_{NL}| < 10^5 - 10^6$$

Planck forecasts

$$10^3, |\tau_{NL}| \sim 10^3, g_{NL} \sim 10^4$$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{3x}}{(P_{20} + P_{2x})^2} =$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$-4 < r_{NL} < 70$  2- $\sigma$   $\tau_{NL} < 10^3$ ,  $|g_{NL}| < 10^5 - 10^6$

Planck forecasts  $|f_{NL}| \leq 10$ ,  $|\tau_{NL}| \sim 10^3$ ,  $g_{NL} \sim 10^4$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{3\lambda}}{(P_{20} + P_{2\lambda})^2} = \frac{1}{15(1+\lambda)^2}$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle z^{(1)2} z^{(2)} \rangle \rightarrow g_{NL}$$

Planck forecasts

$$|f_{NL}| \lesssim 10, \quad |\tau_{NL}| \sim 10^3, \quad g_{NL} \sim 10^4$$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{3x}}{(P_{20} + P_{2x})^2} = \frac{1}{(1 + \lambda)^2} = f_{NL}^2 (1 + \lambda) \text{ enhanced}$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle \zeta^{(1)2} \zeta^{(1)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$

$$\langle \zeta^{(1)2} \zeta^{(2)} \rangle \rightarrow g_{NL}$$

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, \quad |g_{NL}| < 10^5 - 10^6$$

Planck forecasts  $|f_{NL}| \lesssim 10, \quad |\tau_{NL}| \sim 10^3, \quad g_{NL} \sim 10^4$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{3\lambda}}{(P_{20} + P_{2\lambda})^2} = \frac{1}{16(1+\lambda)^2} = f_{NL}^2 (1+\lambda) \text{ enhanced}$$

$$\tau_{NL} \propto \frac{P_{3\lambda}^3}{P_{2\lambda}^3}$$

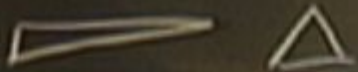
ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$



$$\langle z^{(1)3} z^{(2)3} \rangle \rightarrow g_{NL}$$

Planck forecasts

$$|f_{NL}| \lesssim 10, \quad |\tau_{NL}| \sim 10^3, \quad g_{NL} \sim 10^4$$

$\lambda \neq 0$

$$\tau_{NL} = \frac{T_{3\lambda}}{(P_{2\lambda} + P_{3\lambda})^3} = \frac{1}{(1+\lambda)^3} = f_{NL}^2 (1+\lambda) \text{ ehhan}$$

$$\tau_{NL} \propto \frac{P_{3\lambda}^3}{P_{2\lambda}^3} \propto k^{3(n_+ - n)}$$

$$n_{NL} = \frac{3}{2} n_{2\lambda}$$

ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$



$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

$$\lambda = 0 \quad \tau_{NL} \equiv \frac{T_3}{P_3^3} = \frac{1}{P_{NL}^2} = f_{NL} \quad \text{'single source' consistency}$$

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, \quad |g_{NL}| < 10^5 - 10^6$$

Planck forecasts  $|f_{NL}| \lesssim 10, \quad |\tau_{NL}| \sim 10^3, \quad g_{NL} \sim 10^6$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{3\lambda}}{(P_{3\lambda} + P_{3\lambda}^2)^{3/2}} = \frac{1}{P_{NL}^2 (1+\lambda)^3} = f_{NL}^2 (1+\lambda) \text{ enhanced}$$

$$\tau_{NL} \propto \frac{P_{3\lambda}^3}{P_3^3} \propto k^{3(n_s - n)} \quad n_{NL} = \frac{3}{2} n_{f_{NL}}$$



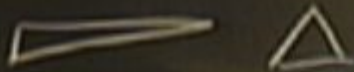
ii) trispectrum



6 variables in 2D

2 terms

$$\langle z^{(1)2} z^{(2)2} \rangle \rightarrow \tau_{NL} \quad T_3 \sim P_3^3$$



$$\langle z^{(1)3} z^{(2)} \rangle \rightarrow g_{NL}$$

1  $\tau_{NL} \equiv \frac{T_3}{P_3^3} = \frac{1}{P_{\delta\delta}^2} = f_{NL}$  'single source' consistency

-4  $< 70$  2- $\sigma$   $\tau_{NL} < 10^5, |g_{NL}| < 10^5 - 10^6$

rank forecasts  $|f_{NL}| \lesssim 10, |\tau_{NL}| \sim 10^3, g_{NL} \sim 10^6$

$$\frac{T_{3x}}{(P_{\delta\delta} + P_{\delta\delta}^2)} = \frac{1}{f_{NL}^2 (1+\lambda)} = f_{NL}^2 (1+\lambda) \text{ enhanced}$$

$$\tau_{NL} \propto \frac{P_{3x}^3}{P_{\delta\delta}^3} \propto k^{3(n_s-1)} \quad \tau_{NL} = \frac{3}{2} \tau_{f_{NL}}$$

$$\lambda = 0 \quad \tau_{NL} \equiv \frac{T_{11}}{P_1^2} = \frac{1}{f_{NL}^2} = f_{NL} \quad \text{'single source' consistency}$$

$$-4 < f_{NL} < 70 \quad 2-\sigma \quad \tau_{NL} < 10^5, |g_{NL}| < 10^5 - 10^6$$

Planck forecasts  $|f_{NL}| \lesssim 10, |\tau_{NL}| \sim 10^3, g_{NL} \sim 10^6$

$$\lambda \neq 0 \quad \tau_{NL} = \frac{T_{32}}{(P_{2n} + P_{2s})^2} = \frac{1}{f_{NL}^2 (1+\lambda)^2} = f_{NL}^2 (1+\lambda) \quad \text{enhanced}$$

$$\tau_{NL} \propto \frac{P_{32}^2}{P_1^3} \propto k^{3(n_s - n)} \quad n_{s,2} = \frac{3}{2} n_{1,2}$$

(Self interacting curvature (drop Sp))

CAUTION  
DO NOT TOUCH THE BOARD  
OR THE CHALK

(Self interacting curvature (drop Sp))

$$U(x) = \frac{1}{2} m^2 x^2 + \frac{m^4}{4} \left(\frac{x}{m}\right)^4$$

(Self interacting curvature (drop  $\delta\rho$ ))

$$U(x) = \frac{1}{2} \kappa x^2 + \lambda m^4 \left(\frac{x}{m}\right)^p, \quad \delta =$$

(Self interacting curvature (drop  $Sp$ ))

$$U(x) = \frac{1}{2} m^2 x^2 + \lambda m^4 \left(\frac{x}{m}\right)^p, \quad S = 2\lambda \left(\frac{x_0}{m}\right)^{p-2}$$

$x_0(x_0)$  - to oscillation

(Self interacting curvature (drop  $Sp$ ))

$$U(\phi) = \frac{1}{2} m^2 \phi^2 + \lambda m^4 \left(\frac{\phi}{m}\right)^p; \quad S = 2\lambda \left(\frac{\phi_0}{m}\right)^{p-2}$$

$\chi_0(\chi)$  oscillation

$$\delta x \propto \chi$$

(Self interacting curvature (drop  $\delta\rho$ ))

$$U(x) = \frac{1}{2} m^2 x^2 + \lambda m^4 \left(\frac{x}{m}\right)^p, \quad \zeta = 2\lambda \left(\frac{x_0}{m}\right)^{p-2}$$

$x_0(x_0)$  - to relation

$\delta x \neq x$

$$f_{NL} = \frac{1}{12} (1 + x)$$



$$P_{ML} = \frac{1}{\sqrt{2}} \left( 1 + \frac{x_1 x_2}{x_0} \right), \quad R_{ML} = \frac{V_{ML}}{H_{ML}} \frac{x_1}{x_0} \frac{1}{\Gamma_{ML}}$$

$$P_3 = P_{3q} + P_{3x} = (1+x) P_{3x} \quad \lambda = P_{3x}$$

$$B_3 = P_{3x} =$$

CAUTION  
 Do not touch the board  
 as it is very hot

$$f_{mc} = \frac{1}{12} \left( 1 + \frac{x_1 x_2''}{x_1''} \right)$$

$$R_{f_{mc}} = \frac{V''''}{H_0^2} \frac{x_1}{x_1} \frac{1}{f_{d/f_{mc}}}$$

indep of  $r_2$

$$P_3 = P_{3q} + P_{3x} = (1+x) P_{3x}$$

$$B_3 = P_{3x} =$$

CAUTION  
 DO NOT TOUCH  
 THE BOARD OR  
 THE CHALK

$$f_{mc} = \frac{1}{\sqrt{2}} \left( 1 + \frac{x_1 x_2''}{x_2''} \right), \quad R_{f_{mc}} = \frac{V''''}{H_0''} \frac{x_1}{x_2} \frac{1}{\Gamma_d f_{mc}}$$

indep. of  $\Gamma_d$

$$= (1+x) P_{3x} \quad \lambda = \frac{P_{3x}}{P_{1x}} > 0$$

CAUTION  
 Do not touch the chalkboard  
 unless you are instructed to do so  
 by the instructor

$x_0(x_0)$  - to oscillation

$$f_{NL} = \frac{1}{12} \left( 1 + \frac{x_0 x_0''}{x_0'^2} \right)$$

$$g_{NL} \approx \frac{1}{12} H(x_0, x_0', x_0'')$$

$\delta x \neq x$

$$R_{NL} = \frac{V'''}{H_0^2} \frac{x_0}{x_0'} \frac{1}{\Gamma_d f_{NL}} > 0$$

intert of  $\Gamma_d$

$$B_3 = P_{3\omega} =$$



$$J_{NL} \approx \frac{1}{\Gamma_A^2} H(x_0, x_0', x_0'') \sim f_{NL}^2$$

$$R_{NL} = \frac{v'''}{H^2} \frac{x_1}{x_2} \frac{1}{\Gamma_A f_{NL}} > 0$$

indep of  $\Gamma_A$



Include inflation

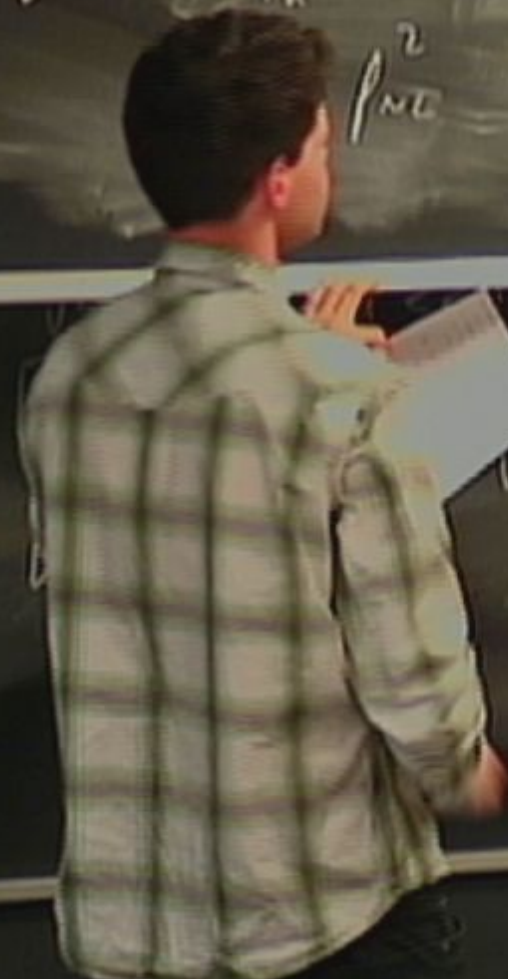
$$= (1+x) P_{3x} \quad \lambda = \frac{P_{3p}}{P_{1x}} > 0$$

CAUTION

$$J_{NL} = \frac{1}{\Gamma_d} \left( 1 + \frac{v''(x_0)}{v'(x_0)} \right), \quad R_{1/d} = \frac{v'''}{v''} \frac{x_0}{v'(x_0)} \frac{1}{\Gamma_d} > 0$$

$$J_{NL} \approx \frac{1}{\Gamma_d^2} H^2(x_0, x_0'', x_0''') \sim f_{NL}^2$$

$\frac{1}{\Gamma_d} \frac{v'''}{v''} \frac{x_0}{v'(x_0)}$   
 intep of  $\Gamma_d$



include inflation

$$(1+x) P_{3x} \quad \lambda = \frac{P_{3p}}{P_{1x}} > 0$$

CAUTION

$$g_{NL} \approx \frac{1}{\Gamma_d^2} H(x_0, x_0', x_0'') \sim \frac{1}{\Gamma_d^2} \frac{1}{\Gamma_d} \text{indep of } \Gamma_d$$

$$g_{NL} \gg \rho_{NL}^2 + \rho_{NL} - 1$$

$$\frac{1}{\sqrt{\epsilon}} \gg 1 \quad \Gamma_d \ll 1 \quad \text{incl}$$

$$P_3 = P_{3q} + P_{3x} = (1+x) P_{3x}$$

$$B_3 = P_{3x} =$$

single source'  $g_{MC} \gg \rho_{MC} \rightarrow \rho_{MC} \sim 1$   
 $\rho_{MC} = \frac{V''(x)}{H^2} \left( \frac{1}{\sqrt{\epsilon}} \right)$

pure curvature  $\gamma_G = \Gamma_d \frac{\delta x}{x}$   $f_{MC} = \frac{1}{\Gamma_d}$

$\frac{1}{\sqrt{\epsilon}} \gg 1$   $\Gamma_d \ll 1$  inflation

$P_3 = P_{3q} + P_{3x} = (1+x) = \frac{P_{3q}}{P}$

$B_3 = P_{3x} =$



$g_{\mu\nu} \gg f_{\mu\nu}^2 \Rightarrow f_{\mu\nu} \sim \sqrt{g_{\mu\nu}}$   
 'single source'  $\Rightarrow \Lambda_{f_{\mu\nu}} = \frac{V''''}{H^4} \left( \frac{\sqrt{g_{\mu\nu}}}{f_{\mu\nu}} \right) \leftarrow \text{only way to probe } V''''$

pure curvature  $\Rightarrow \delta G = \sqrt{-g} \frac{\delta x}{x} \Rightarrow f_{NC} = \frac{1}{\sqrt{x}}$

$\frac{1}{\sqrt{x}} \gg \dots$   
 $\Lambda \ll 1$  include inflation  
 $P_3 = (1, P_{3x})$   
 $B_3$   
 $\lambda = \frac{P_{3p}}{P_{3x}} > 0$

CAUTION  
 Please do not touch the board or the equipment in the room.  
 Thank you for your cooperation.

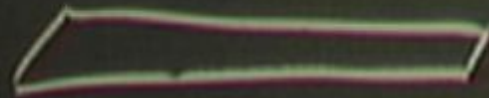
$g_{\text{eff}} \gg f_{\text{pl}}^2 \rightarrow \rho_{\text{pl}} \sim \frac{V''''}{H^2} \left( \frac{\sqrt{f_{\text{pl}}}}{f_{\text{pl}}} \right) \leftarrow \text{only way to probe } V''''$   
 'single source'

adiabatic  $f_{\text{pl}} \sim \frac{1}{n_s - 1} \rightarrow \rho_{\text{pl}} = \frac{-\alpha_s}{n_s - 1} \sim O(\epsilon)$

$g_{\text{eff}} \gg \rho_{\text{eff}}^2 + \rho_{\text{eff}} \sim 1$   
 'single source'  $\rho_{\text{eff}} = \frac{V''''}{H^2} \left( \frac{\sqrt{H^2}}{f_{\text{eff}}} \right) \leftarrow \text{only way to probe } V''''$

kpyrotic  $f_{\text{eff}} \sim \frac{1}{n_s - 1}$   $\rho_{\text{eff}} = \frac{-\alpha_s}{n_s - 1} \sim 0(\epsilon)$

natural  $n_x = 1$



$$n_x^{-1} =$$

) to spectrum



2 terms

$$\left\{ \begin{matrix} \zeta^{(1)2} & \zeta^{(2)2} \\ \zeta^{(1)3} & \zeta^{(3)3} \end{matrix} \right.$$



Planck forecasts

single source'  $\rightarrow$   $\rho_{\text{inc}} + \rho_{\text{sc}} \sim 1$  input of  $\Gamma_d$

$\rho_{\text{sc}} = \frac{V''''}{H^2} \left( \frac{\sqrt{\Gamma_d}}{f_{\text{sc}}} \right)$   $\leftarrow$  only way to probe  $V''''$

chiral

$\rho_{\text{sc}} \sim \frac{1}{n_s - 1}$   $\rho_{\text{sc}} = \frac{-\alpha_s}{n_s - 1} \sim 0(\epsilon)$

$\Gamma = \frac{1}{k_1^3 k_2^3 \|k_1 + k_2\|^3} + \text{|| perms ||}$



$\Gamma_d$  indep of  $\Gamma_d$   
 $g_{NL} \gg f_{NL}^2 + A_{fNL} \sim 1$   
 'single source'  $A_{fNL} = \frac{V''''}{H^2} \left( \frac{\sqrt{F_R}}{f_{NL}} \right)$  ← only way to probe  $V''''$

asymptotic  $f_{NL} \sim \frac{1}{n_s - 1}$   $A_{fNL} = \frac{-\alpha_s}{n_s - 1} \sim 0(\epsilon)$

$T_3 = \tau_{NL} \left( \frac{1}{k_1^3 k_2^3 \|k_1 + k_2\|^3} + \dots \right)$

'single source'  $\mu_{\text{NL}} = \frac{1}{H^2} \frac{1}{f_{\text{NL}}} \text{ to probe } \checkmark$

ekpyrotic  $f_{\text{NL}} \sim \frac{1}{n_s - 1}$   $\mu_{\text{NL}} = \frac{-\alpha_s}{n_s - 1} \sim 0(\epsilon)$

$$T_{\zeta} = \zeta_{\text{NL}} \left( \frac{1}{k_1^3 k_2^3 \|k_1 + k_2\|^3} + \text{all perms} \right)$$

$$\zeta = \zeta_G + f_{\text{NL}} \zeta_G^2 + g_{\text{NL}} \zeta_G^3$$

$\zeta_G$