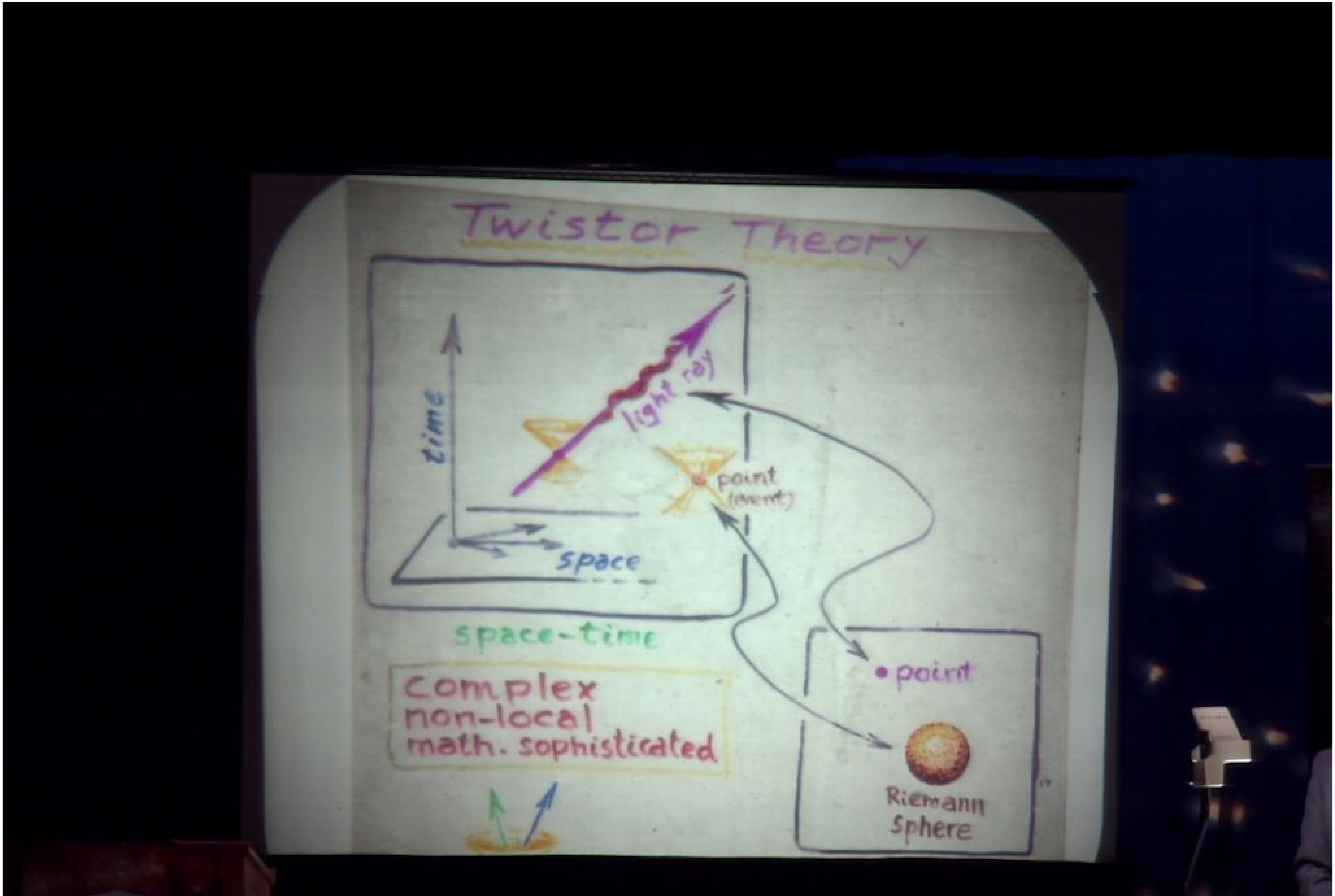
Title: Twistors and Quantum Non-Locality

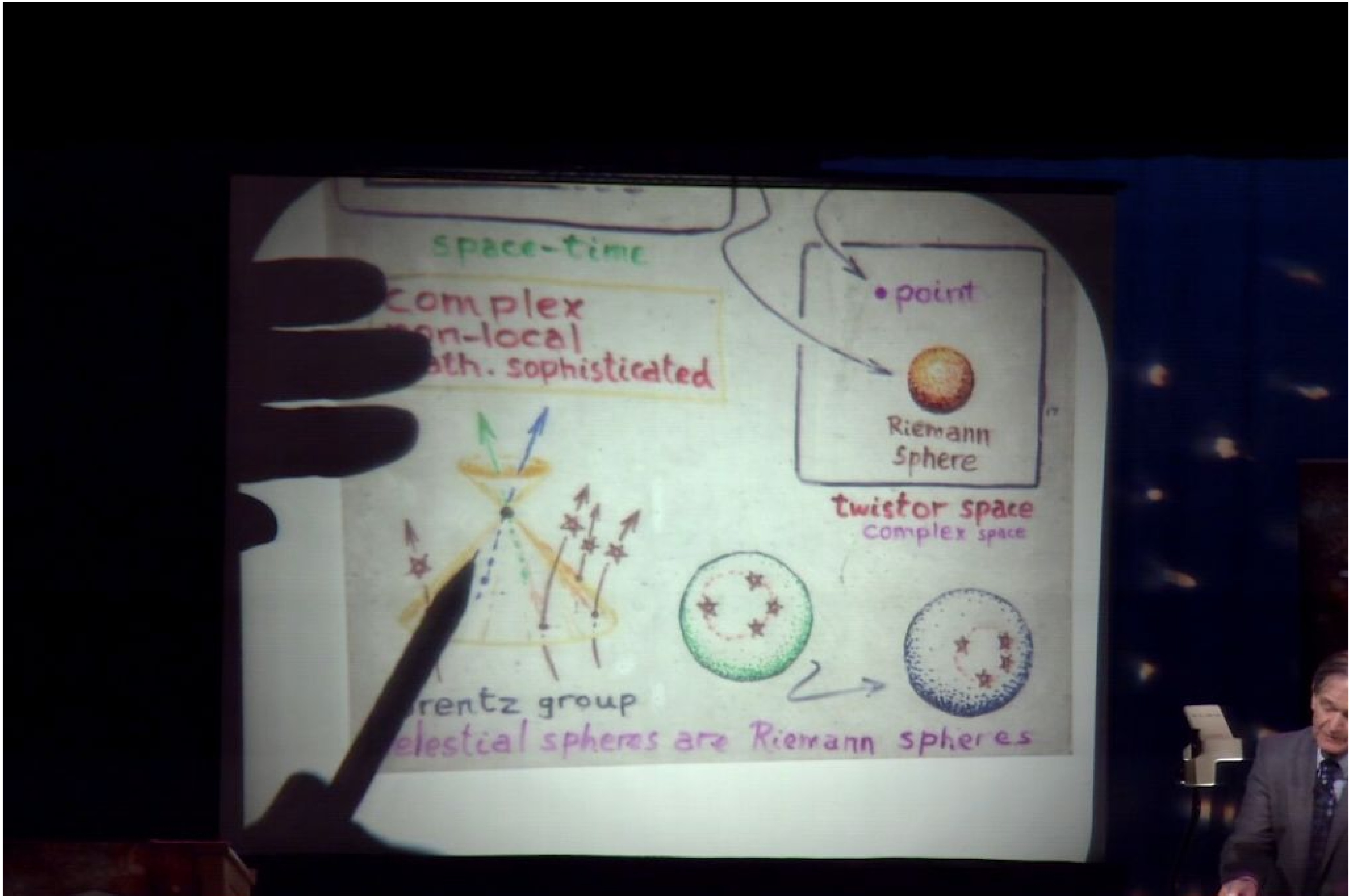
Date: Apr 06, 2011 07:00 PM

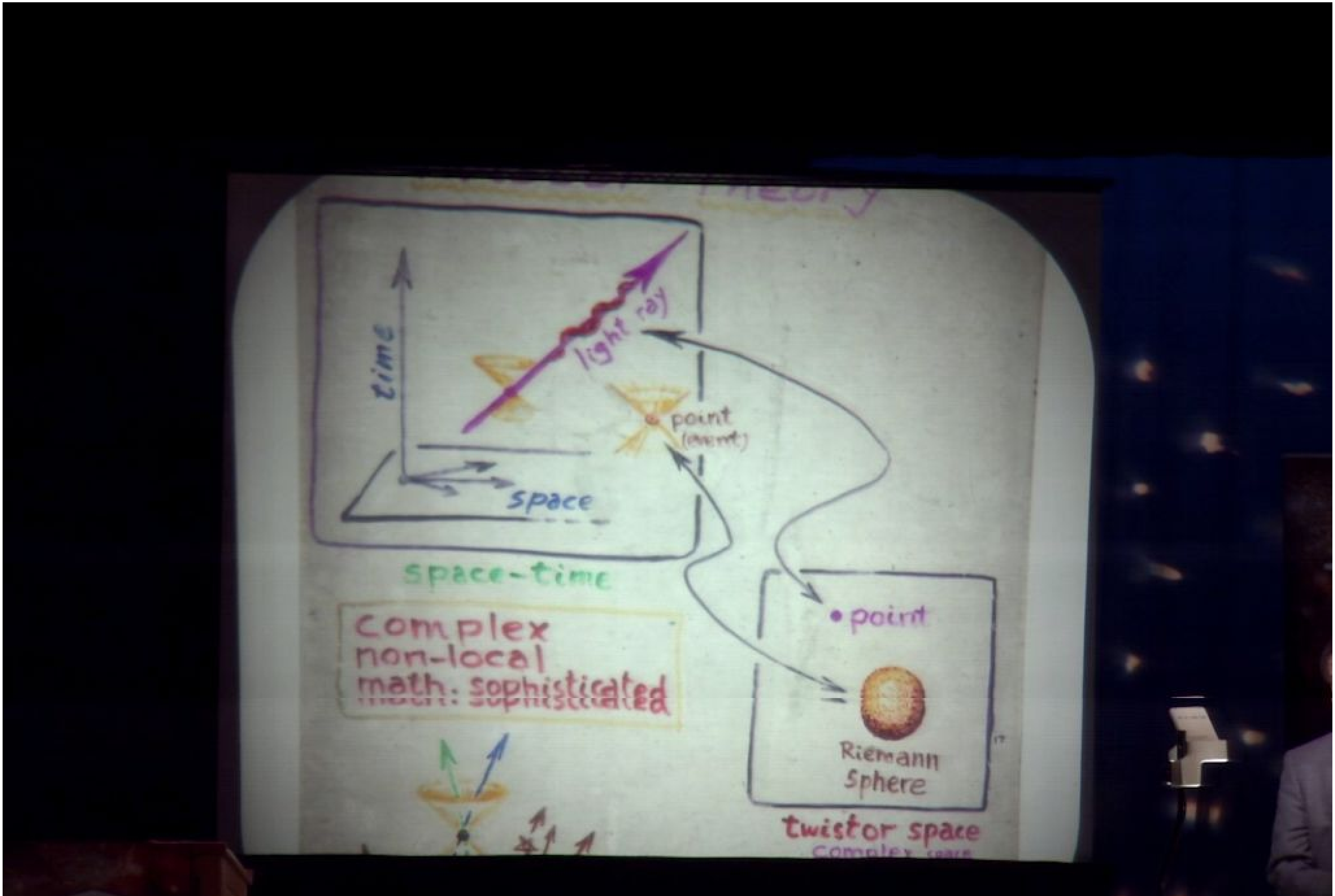
URL: http://pirsa.org/11040119

Abstract: Space and time are two of the universe's most fundamental elements. Relativity combines these two into the unified notion of space-time, but twistor theory goes beyond this replacing both by something entirely different, where the basic elements are the paths taken by particles of light or other particles without mass.
Twistor theory has already found powerful applications in the field of high-energy physics.

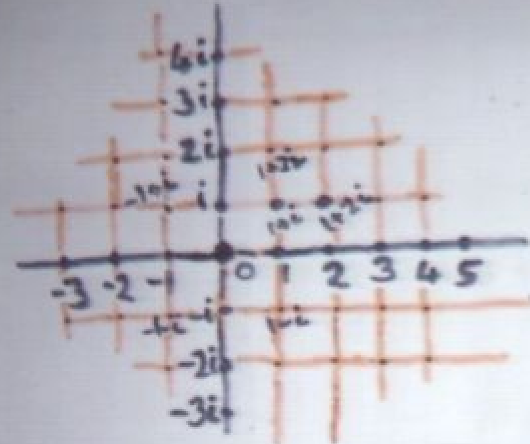
The creation of twistor theory was motivated with the hope that it would shed light on the foundations of quantum physics, a theory that puzzled even Einstein, particularly through the weird effects of quantum non-localityÃƒÂ¢Ã‚Â€Ã‚Â”the phenomenon whereby the behaviour of quantum particles can seem to have instantaneous effects over large distances. In this lecture, Prof. Penrose will describe a deep link between twistor theory and the simplest form of quantum non-locality and how the connection may be generalized in ways that provide a broader understanding of the phenomenon.


Twistor Theory
time
light ray
point
(event)
space
space-time
complex
non-local
math. sophisticated
• point
Riemann
Sphere


space-time
complex
non-local
ath. sophisticated
point
Riemann
Sphere
twistor space
complex space
rentz group
elestial spheres are Riemann spheres


time
light ray
point
(event)
space
space-time
complex
non-local
math: sophisticated
point
Riemann
Sphere
twistor space

$i = \sqrt{-1}$



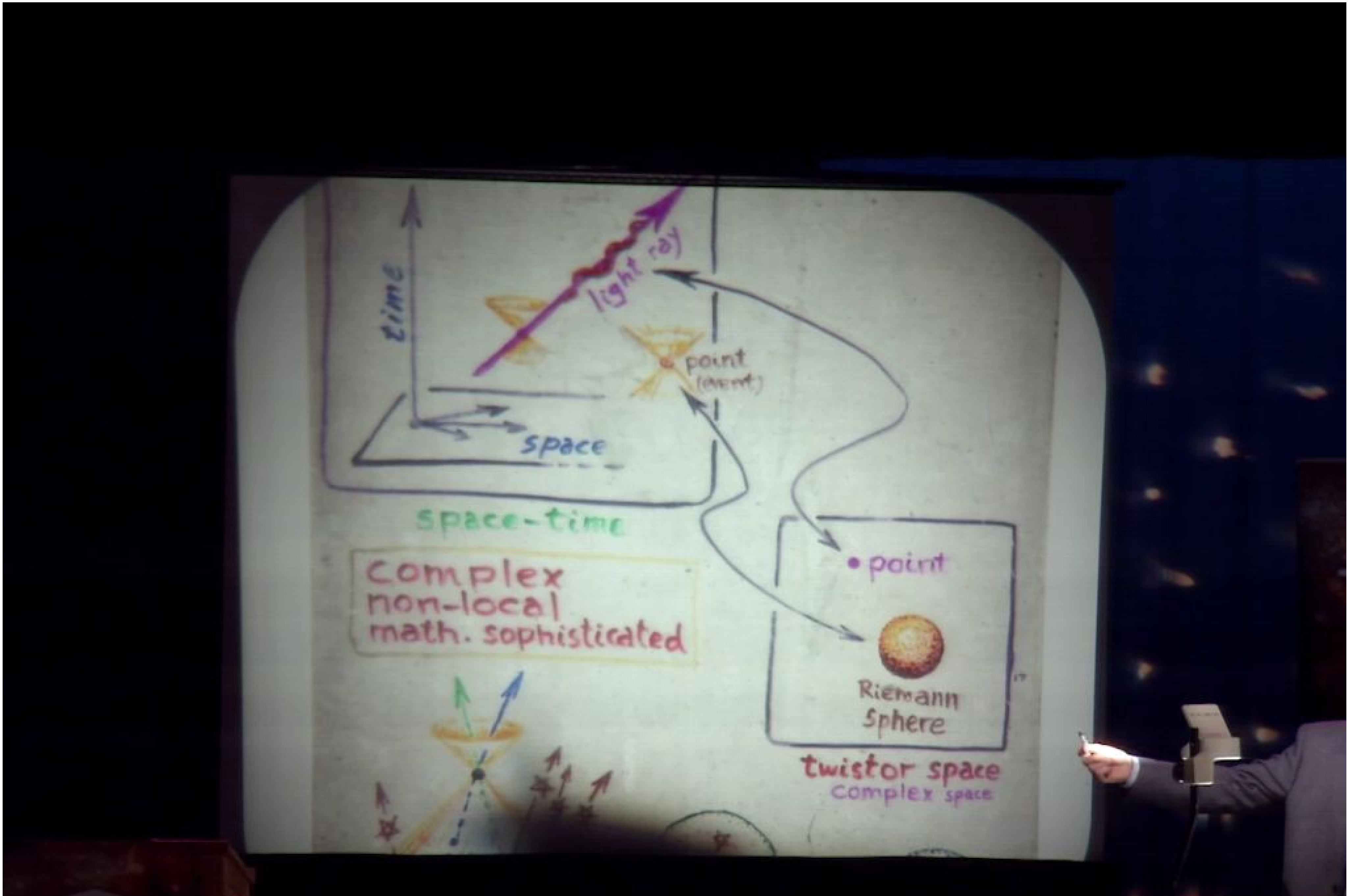
Complex numbers $\mathbb{C}$:

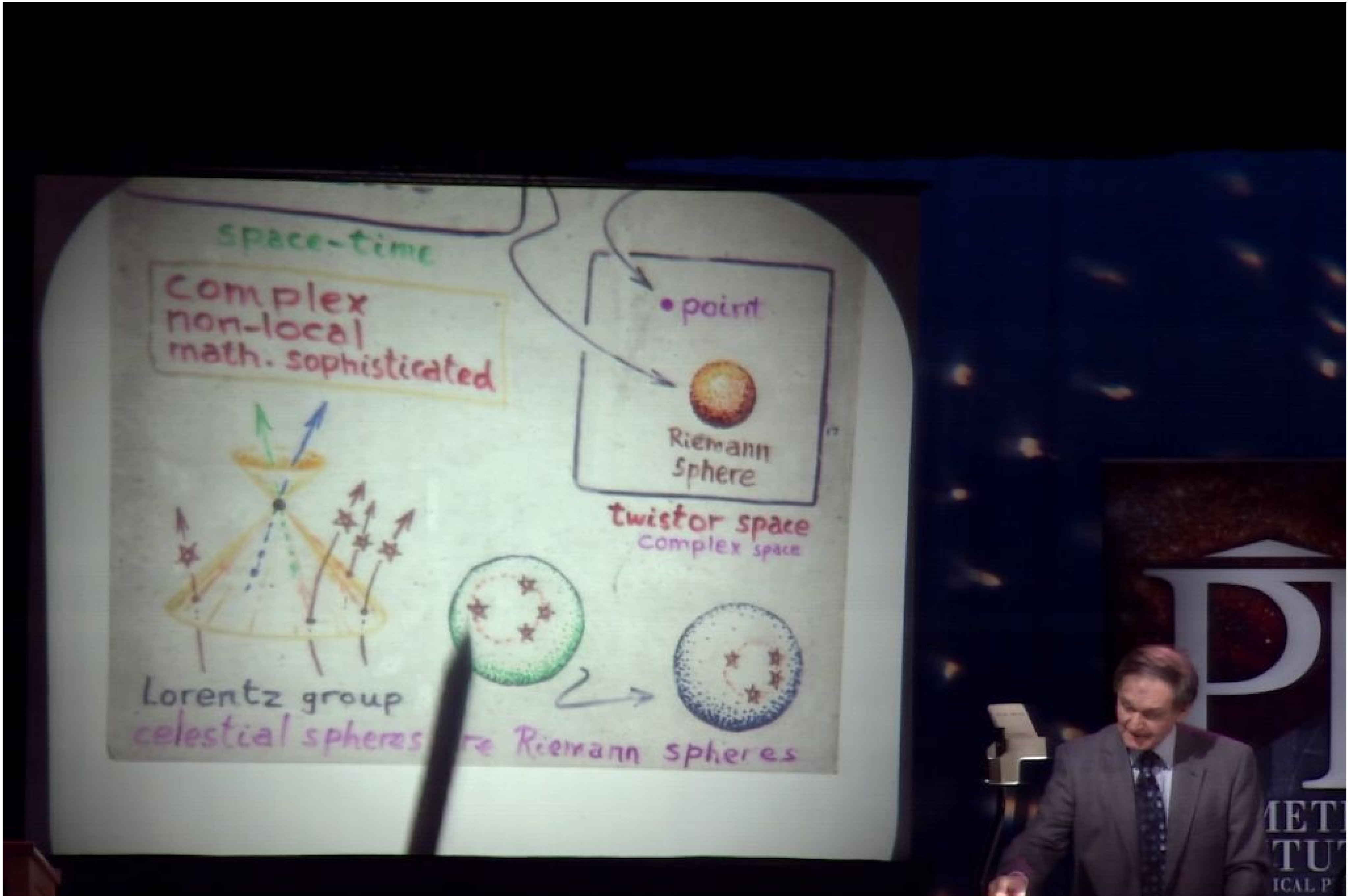
$$\boxed{Z = a + ib}$$

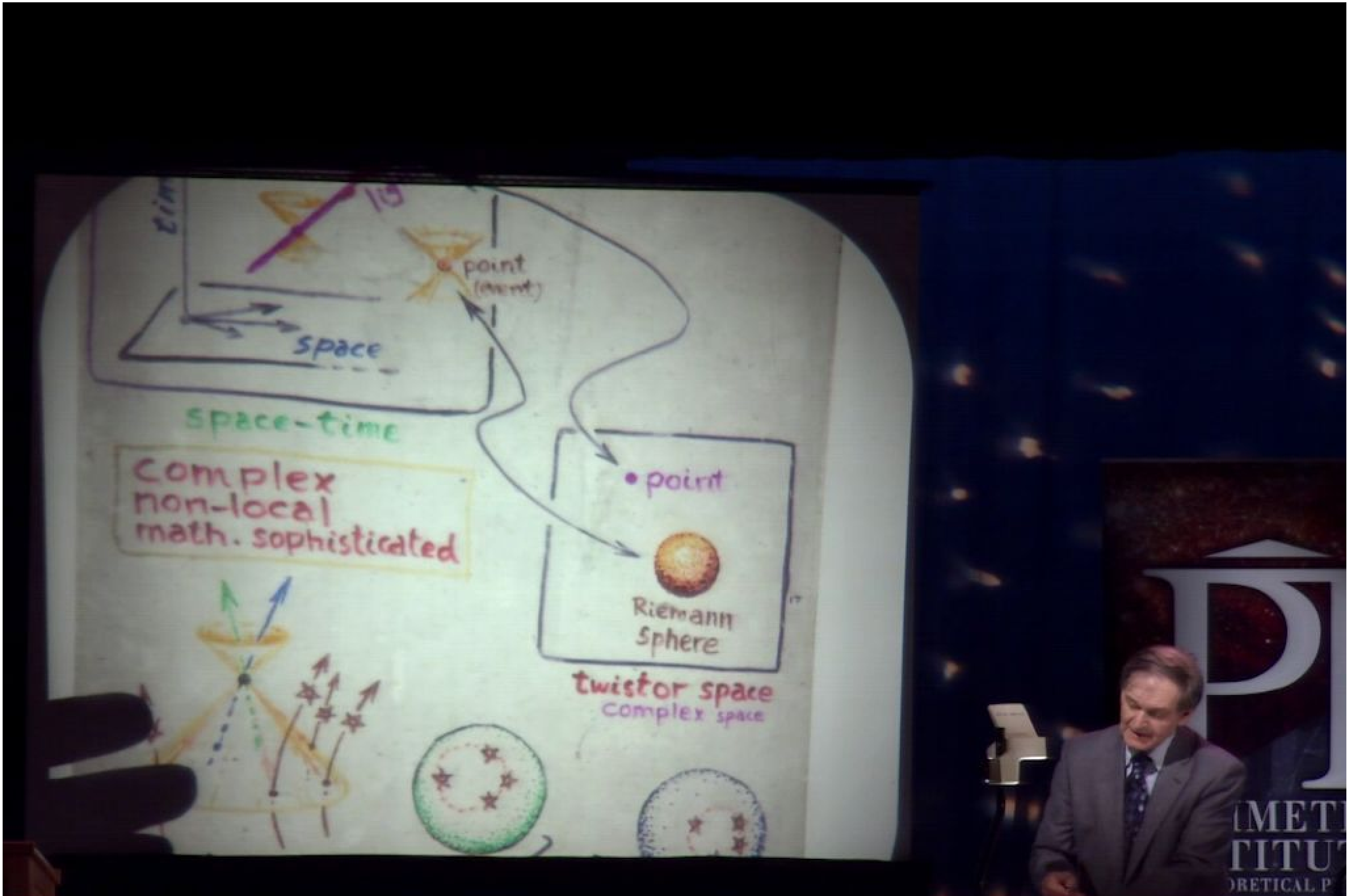
where a and b are ordinary real numbers $\mathbb{R}$

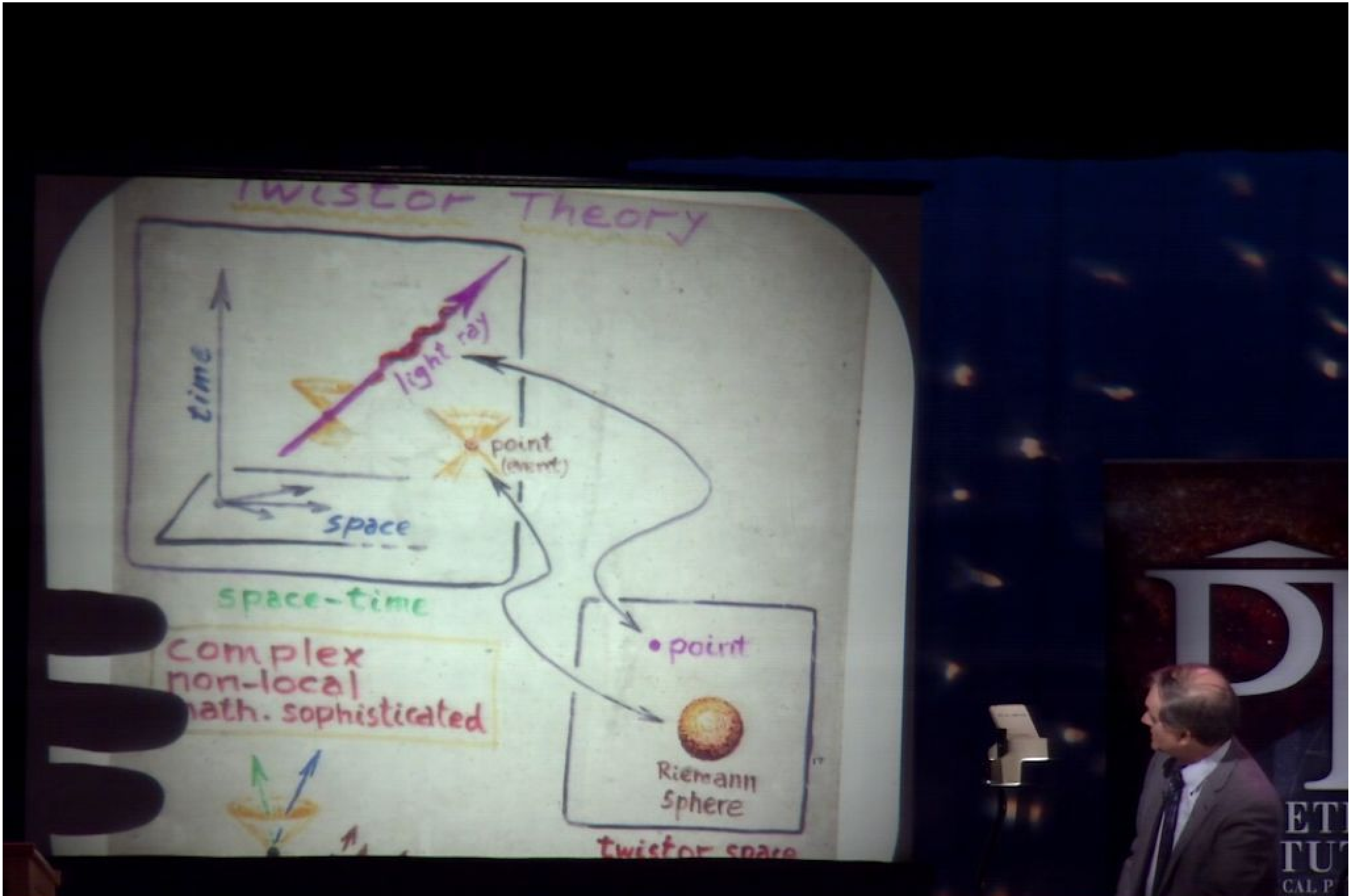
12

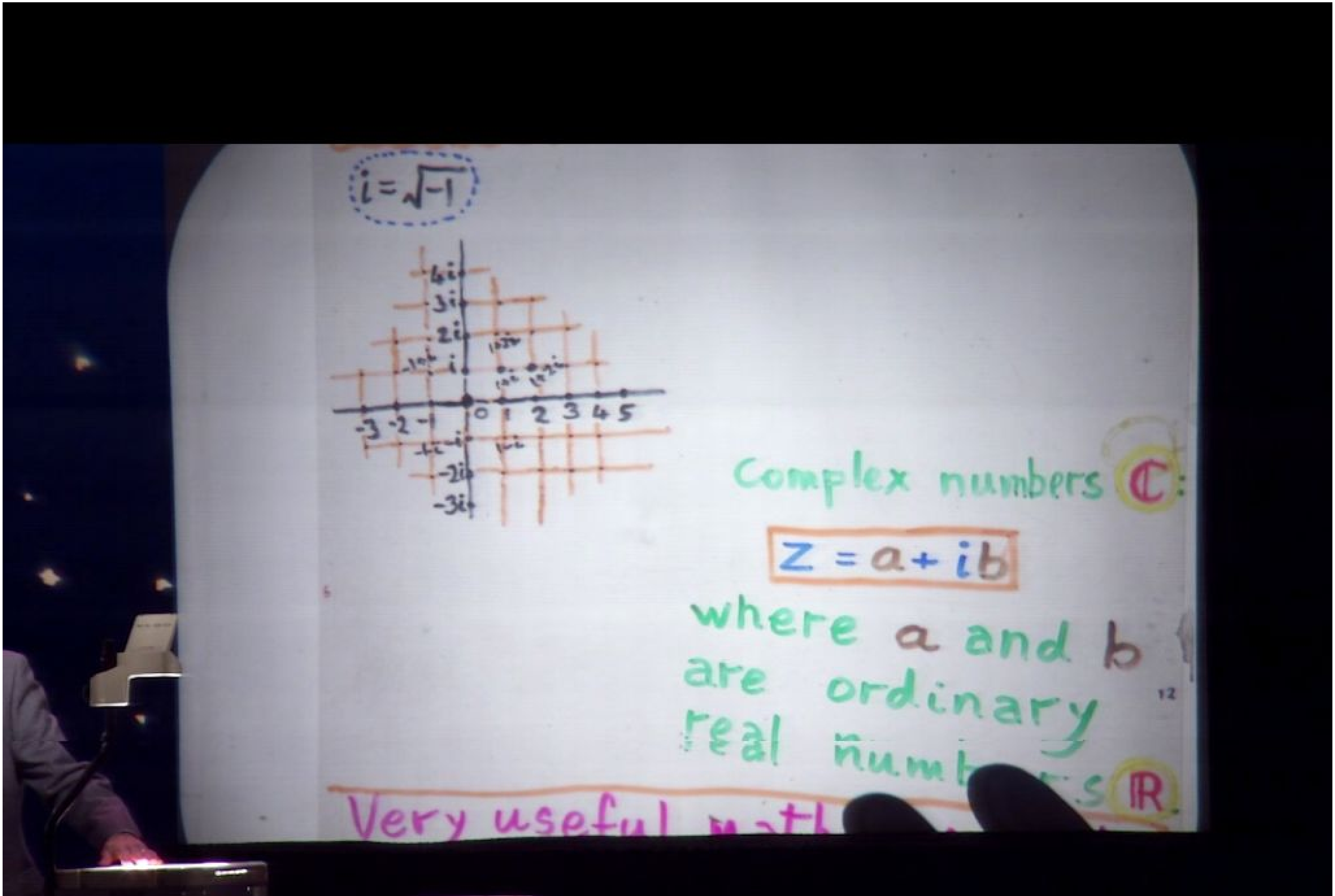
Very useful

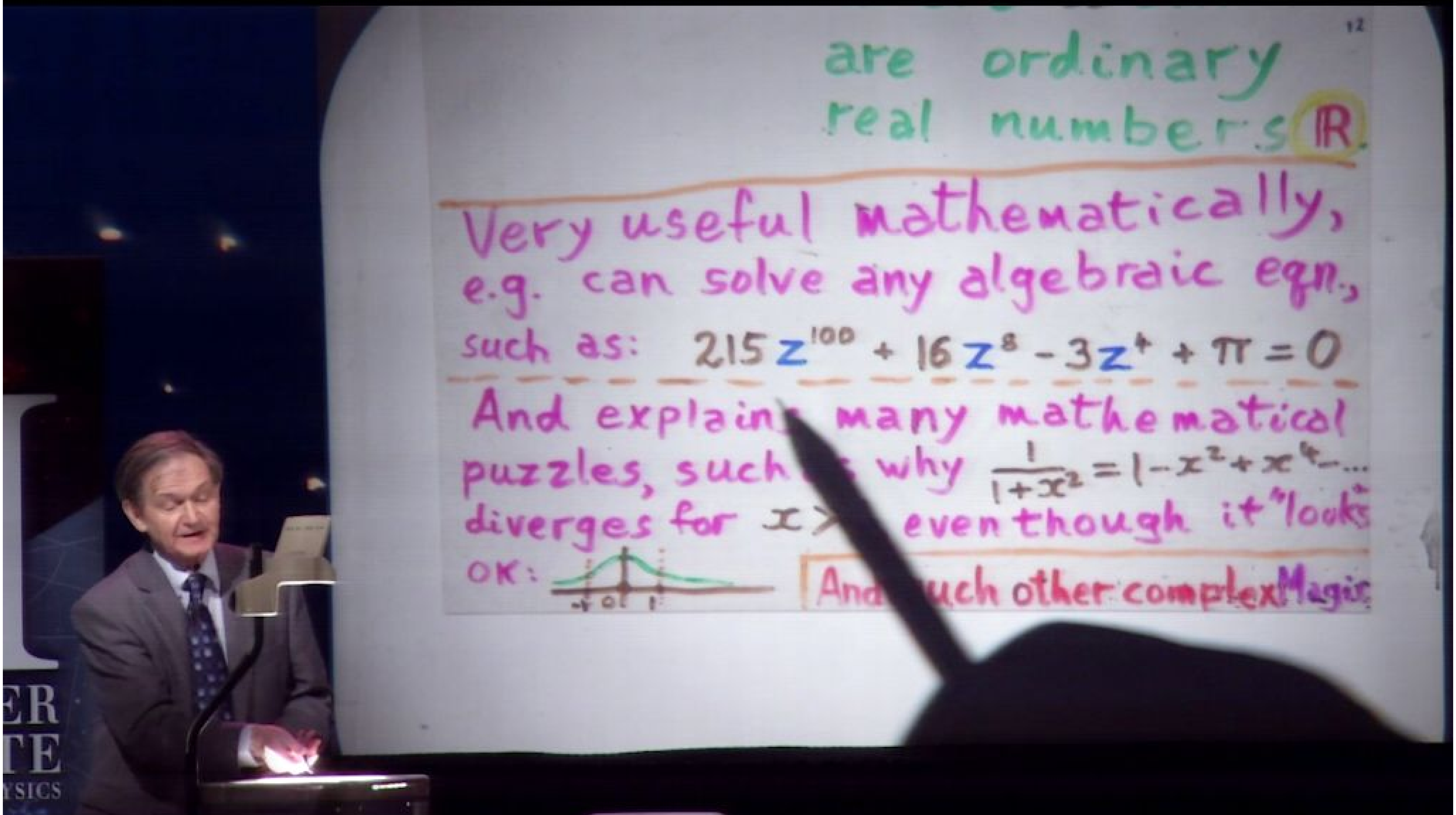

time
light ray
point
(event)
space
space-time
complex
non-local
math. sophisticated
point
Riemann
Sphere
twistor space
complex space

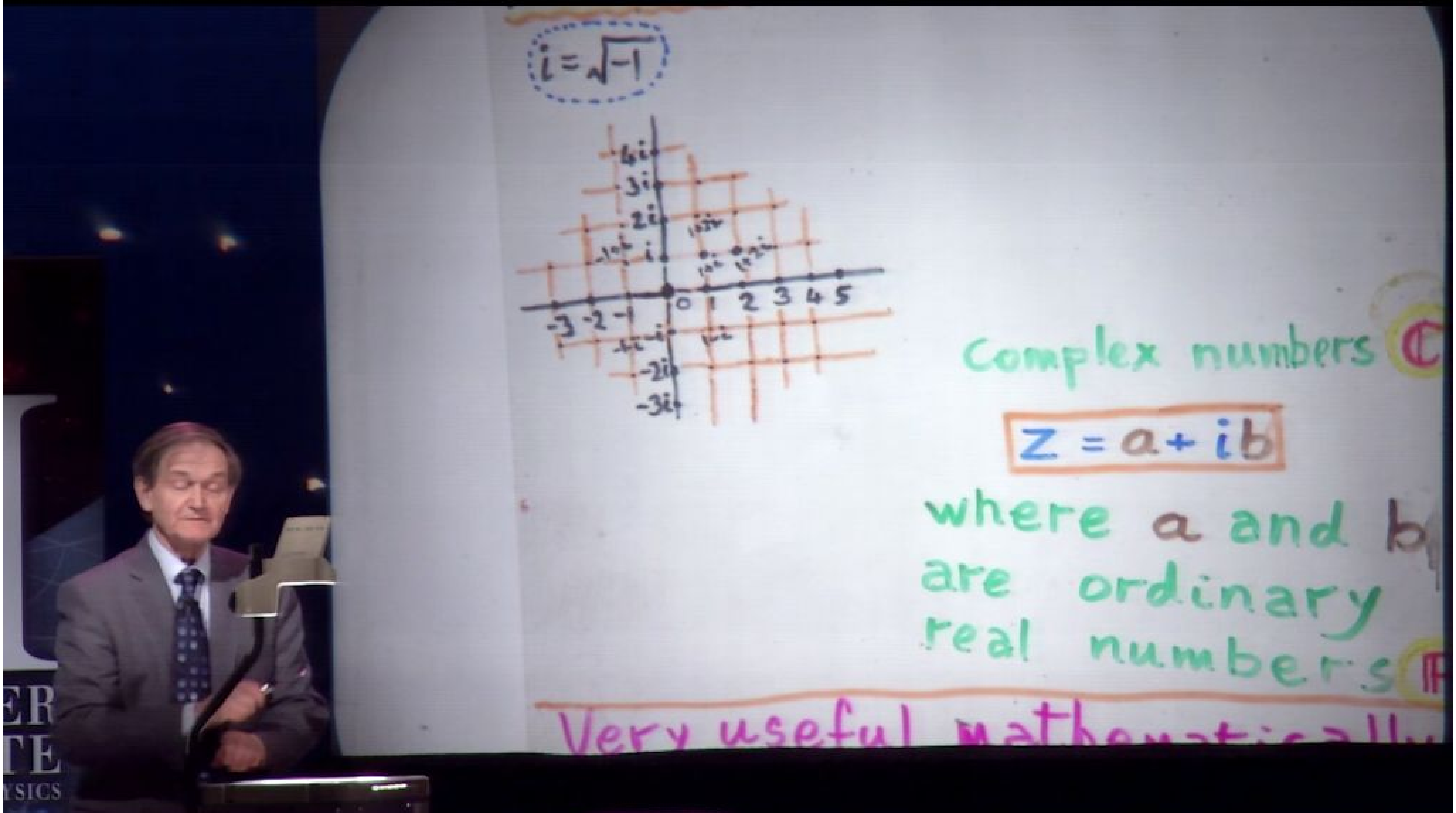

space-time
complex
non-local
math. sophisticated
point
Riemann
sphere
twistor space
complex space
Lorentz group
celestial spheres are Riemann spheres


point
(event)
space
space-time
complex
non-local
math. sophisticated
• point
Riemann
Sphere
twistor space
complex space

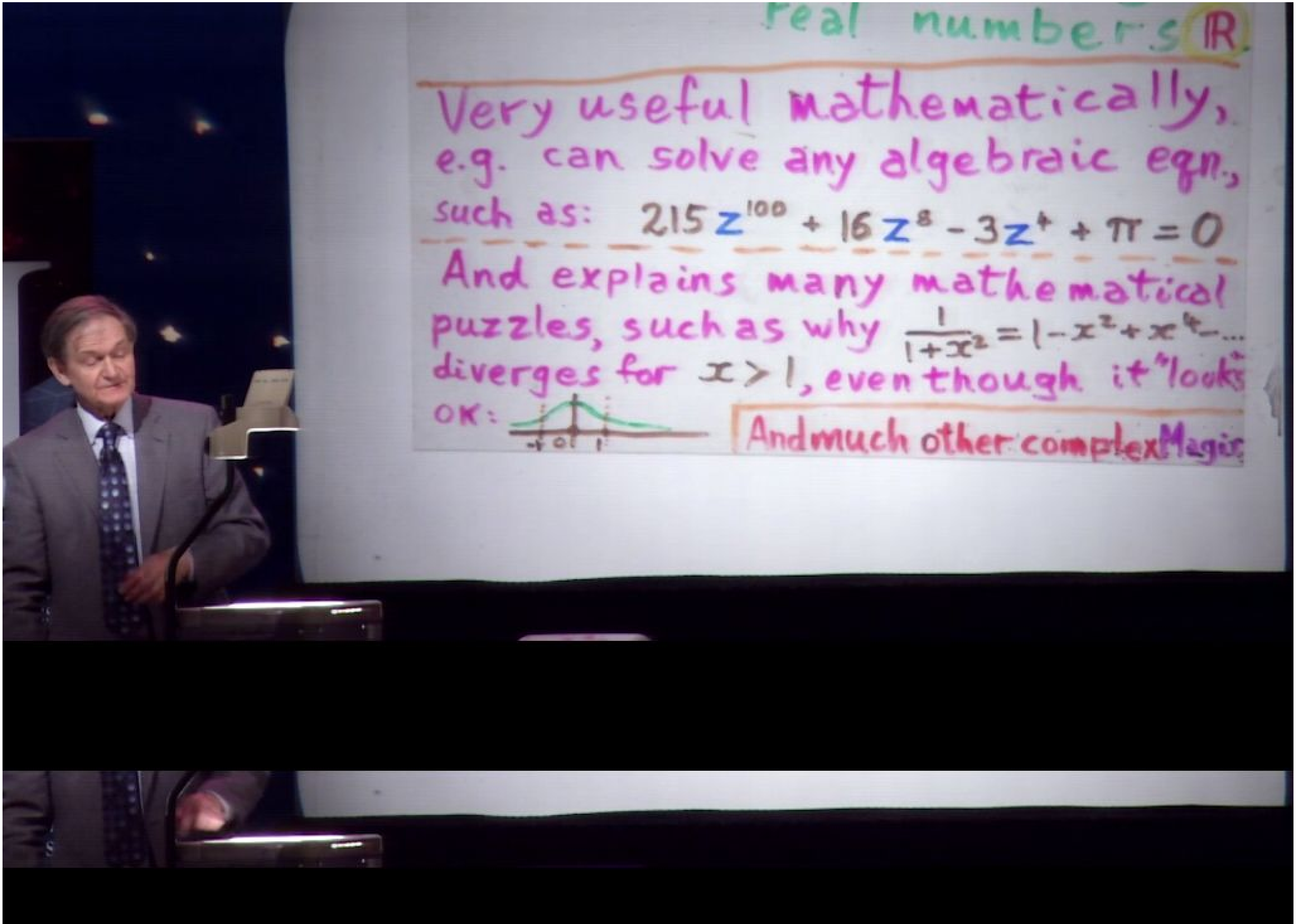

Twistor Theory
time
light ray
point (event)
space
space-time
complex
non-local
math. sophisticated
• point
Riemann Sphere
twistor space

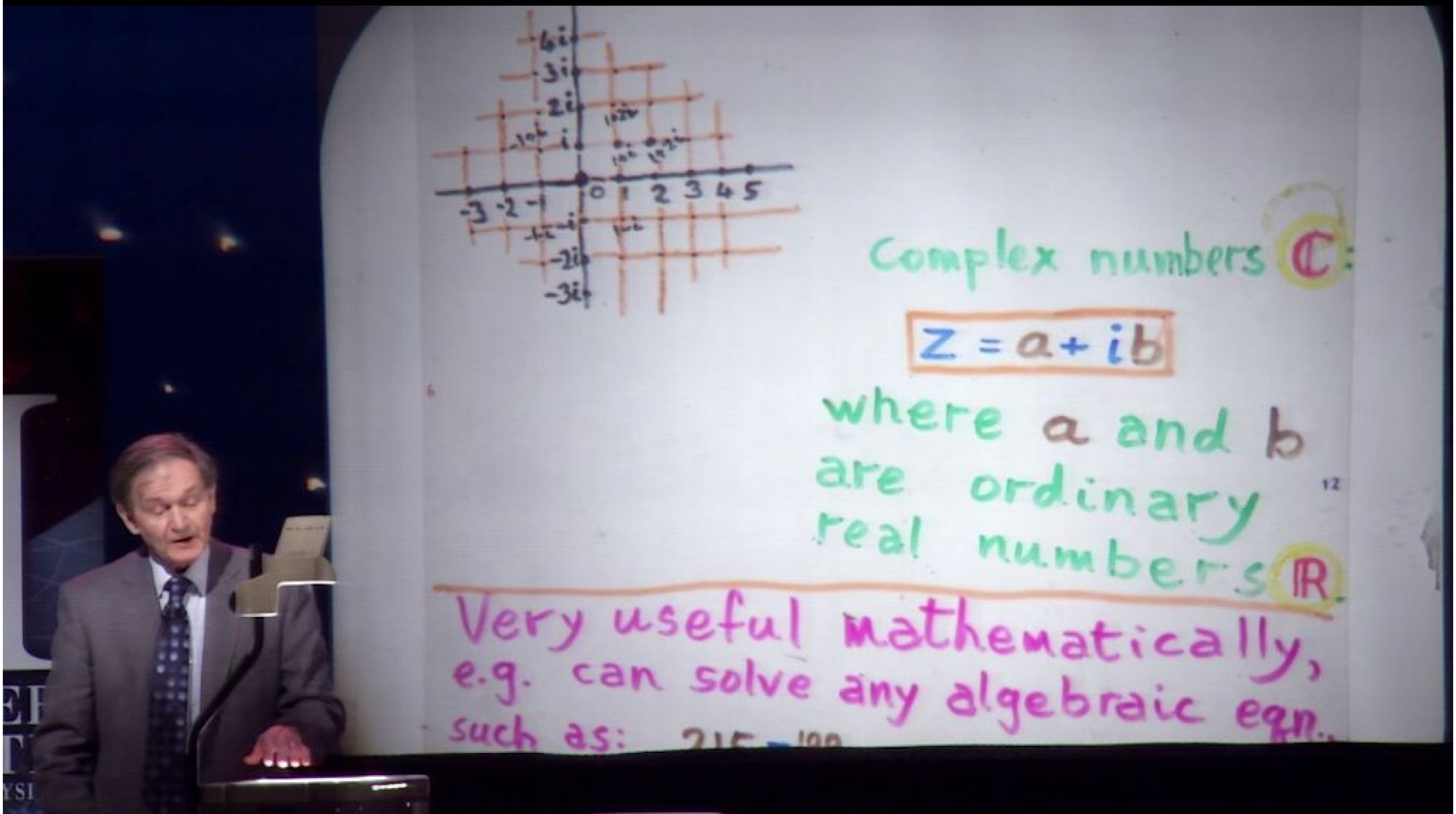

$i = \sqrt{-1}$
4i
3i
2i
i
-i
-2i
-3i
-3 -2 -1 0 1 2 3 4 5
Complex numbers ℂ:
$Z = a + ib$
where a and b
are ordinary
real
ℝ
Very useful

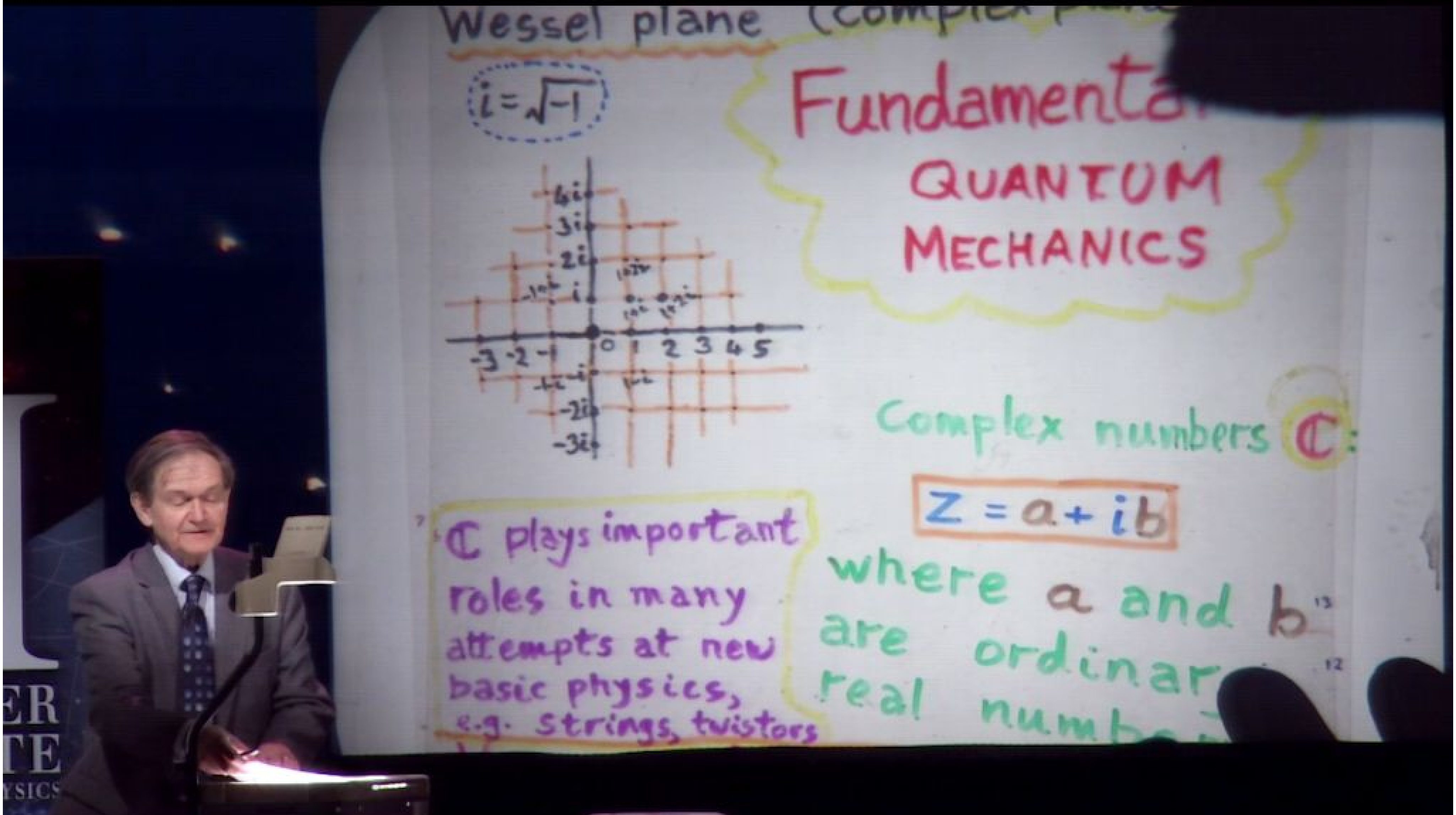

12
are ordinary
real numbers ℝ.
Very useful mathematically,
e.g. can solve any algebraic eqn.,
such as: 215z^100 + 16z^8 - 3z^4 + π = 0
And explain many mathematical
puzzles, such why 1/(1+x^2) = 1 - x^2 + x^4 - ...
diverges for x even though it "looks"
OK:
-1 0 1
uch other complex Magic

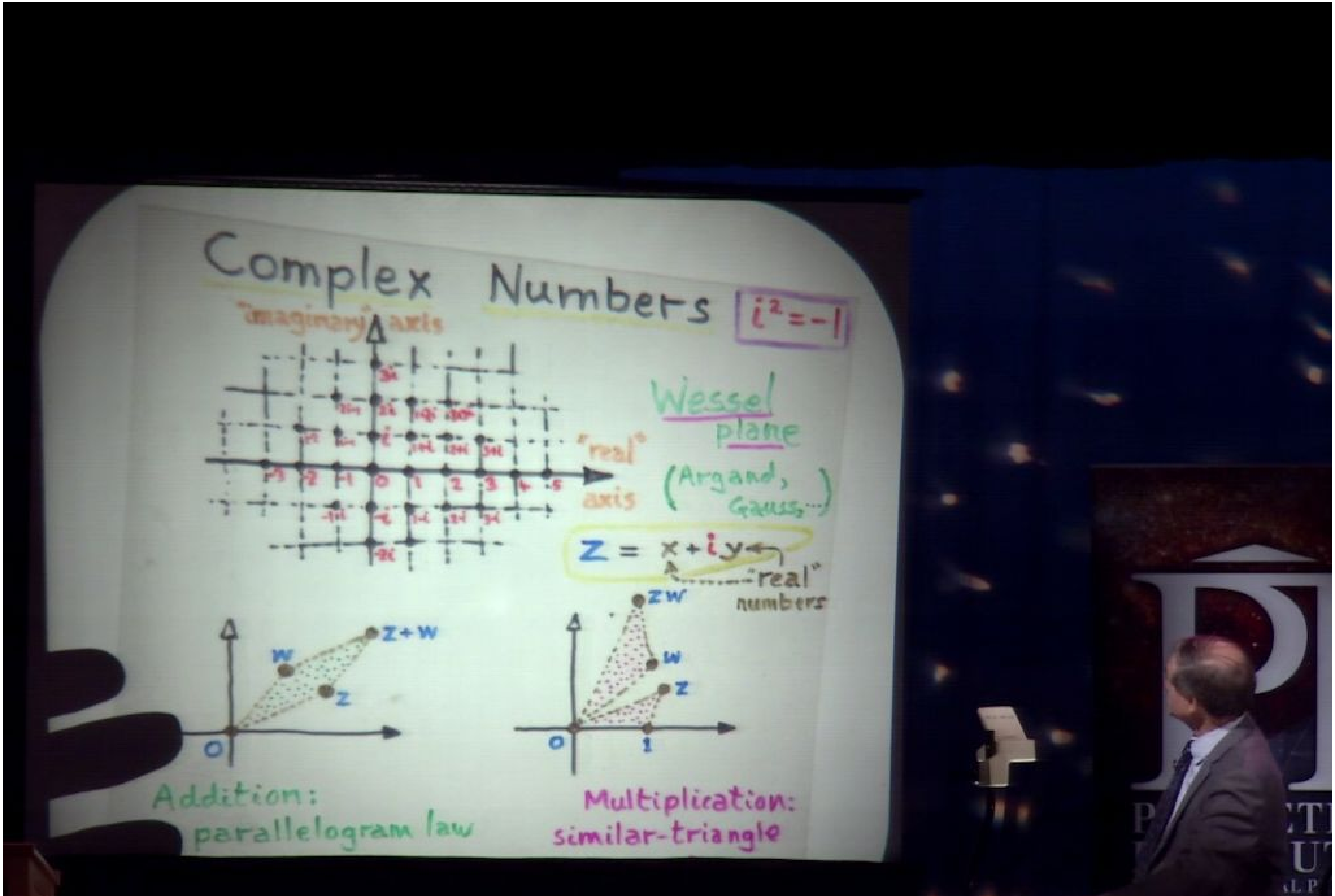


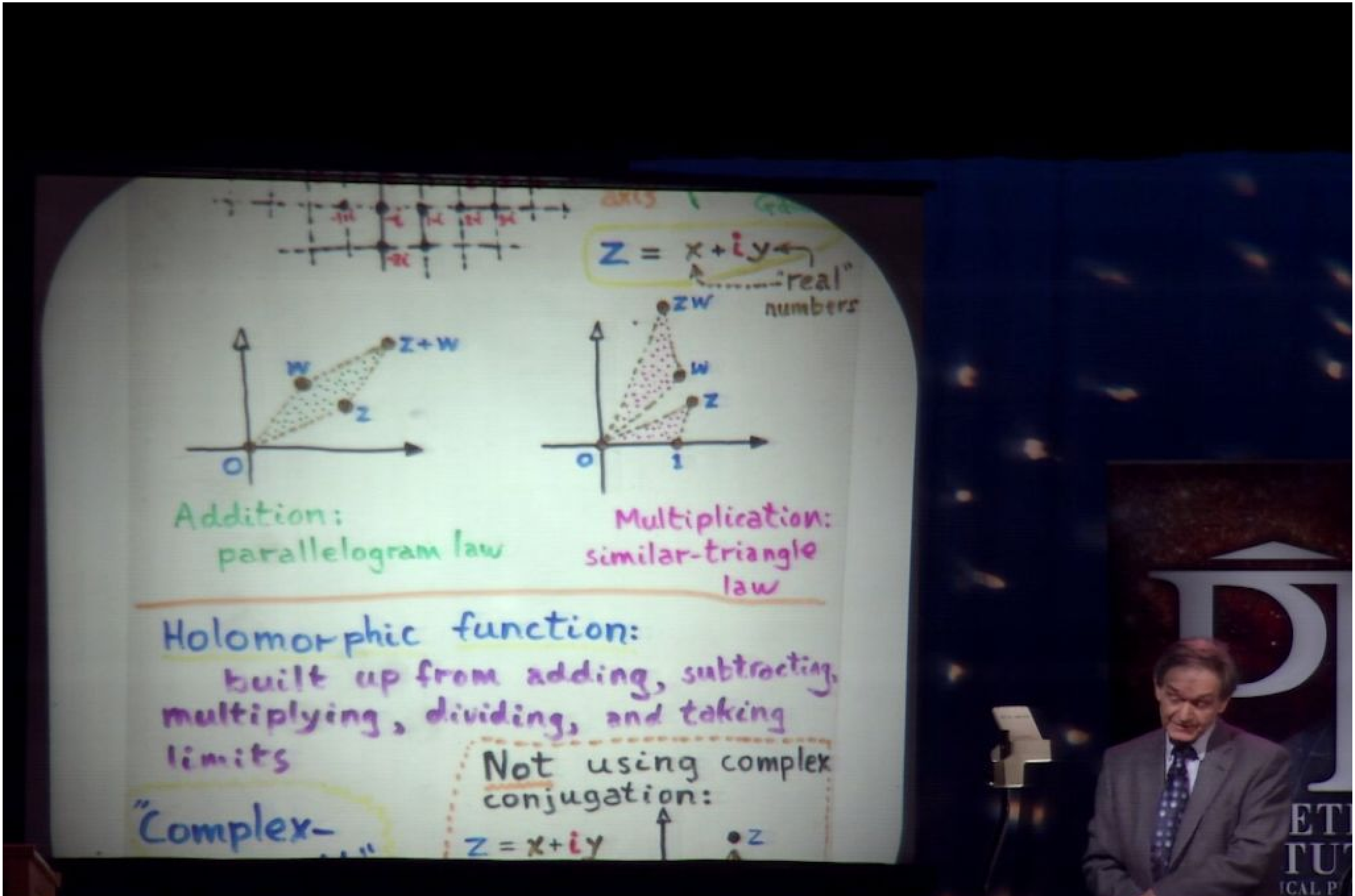
i = √-1
4i
3i
2i
i
-3 -2 -1 0 1 2 3 4 5
-2i
-3i
complex numbers ℂ
z = a + ib
where a and b
are ordinary
real numbers ℝ
Very useful

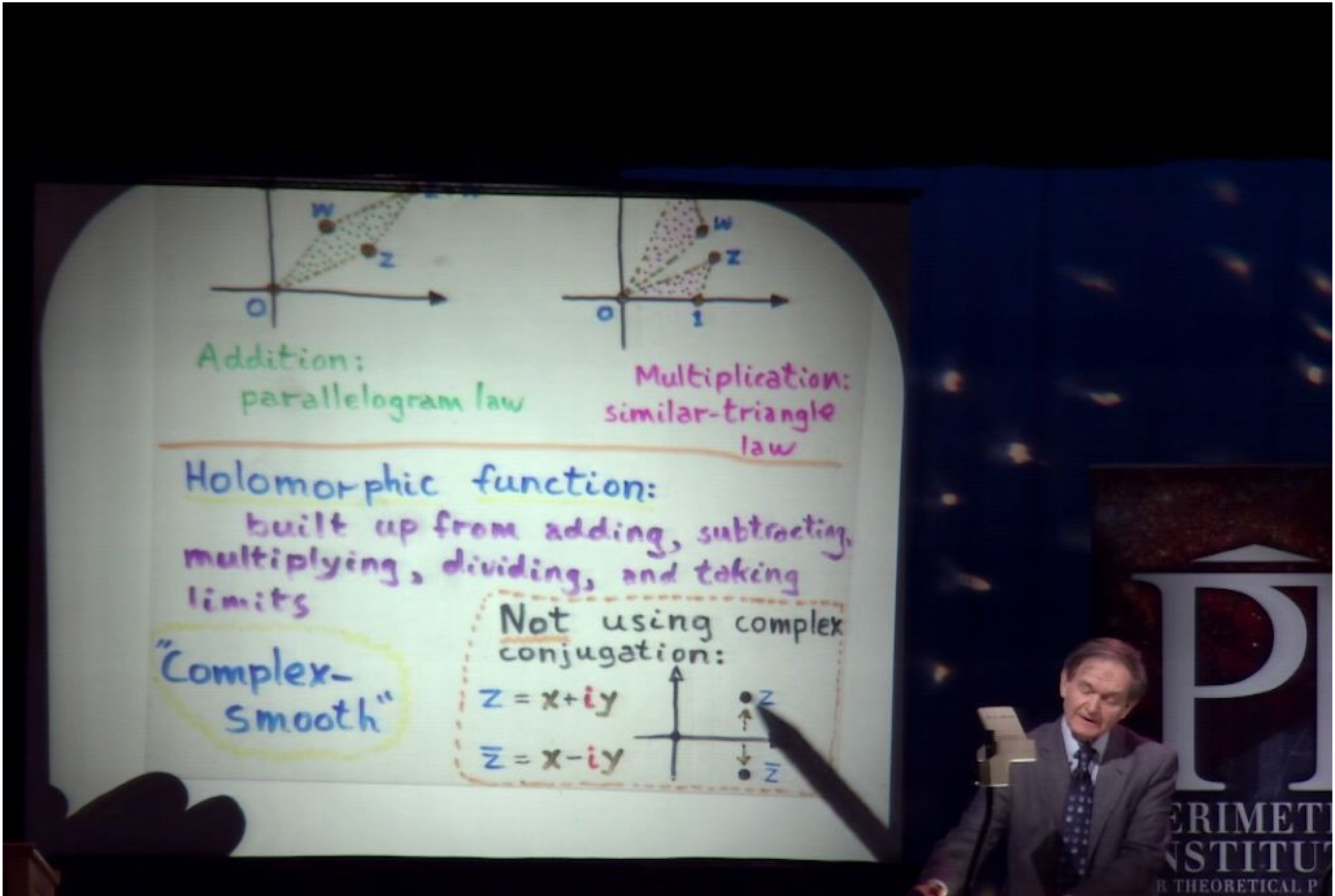

real numbers ℝ
Very useful mathematically,
e.g. can solve any algebraic eqn.,
such as: $215z^{100} + 16z^8 - 3z^4 + \pi = 0$
And explains many mathematical
puzzles, such as why $\frac{1}{1+x^2} = 1 - x^2 + x^4 - \ldots$
diverges for $x > 1$, even though it "looks"
OK:
And much other complex Magic

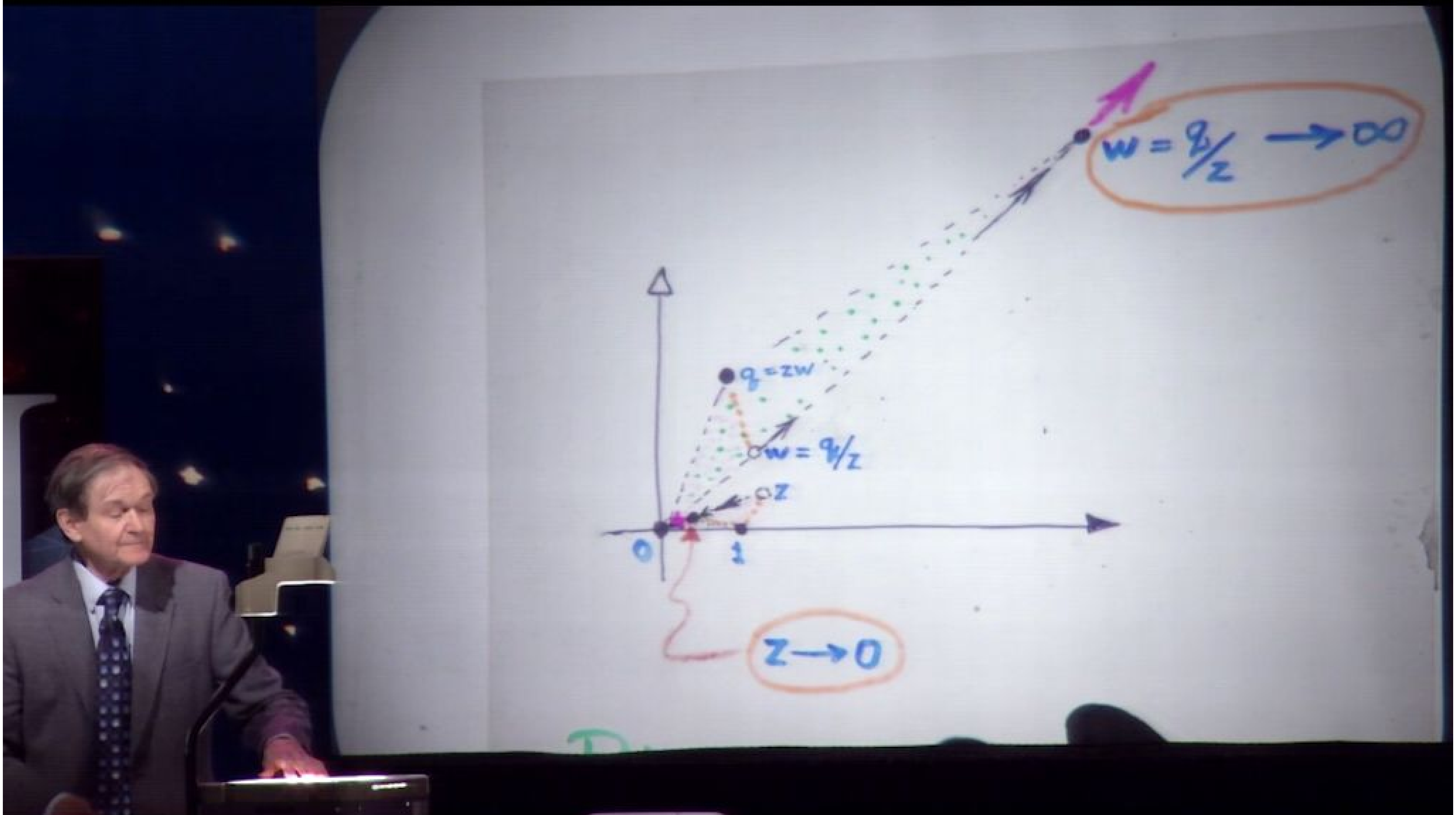

4i
3i
2i
i
-i
-2i
-3i
-3 -2 -1 0 1 2 3 4 5
Complex numbers ℂ:
z = a + ib
where a and b
are ordinary
real numbers ℝ.
Very useful mathematically,
e.g. can solve any algebraic eqn.
such as:


Wessel plane
i = √-1
Fundament
QUANTUM
MECHANICS
Complex numbers ℂ:
z = a + ib
where a and b
are ordinar
real numb
ℂ plays important
roles in many
attempts at new
basic physics,
e.g. strings, twistors

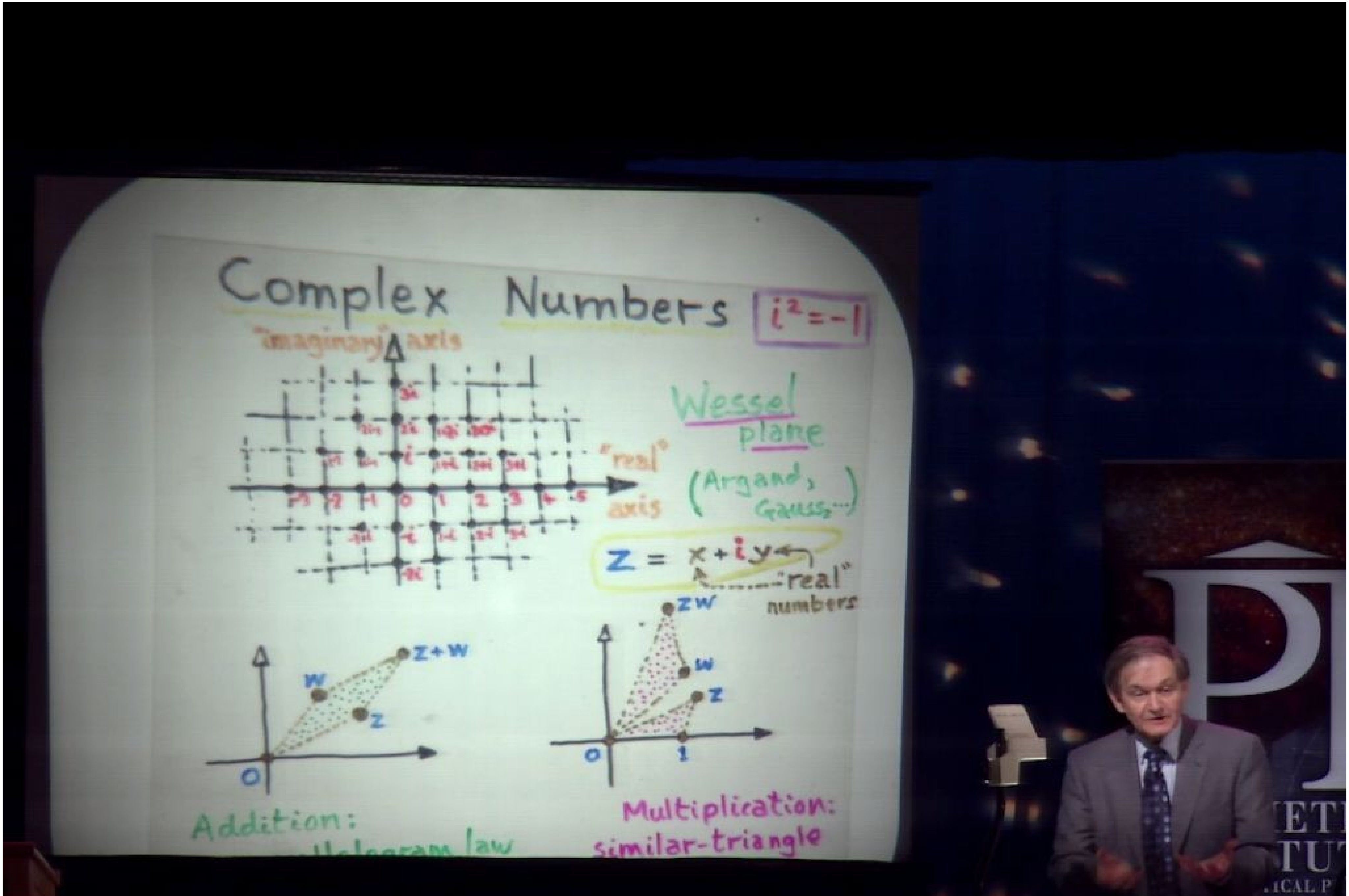

Complex Numbers
i² = -1
"imaginary" axis
"real" axis
Wessel plane
(Argand, Gauss...)
z = x + iy
"real" numbers
w
z
z+w
0
zw
1
Addition: parallelogram law
Multiplication: similar-triangle

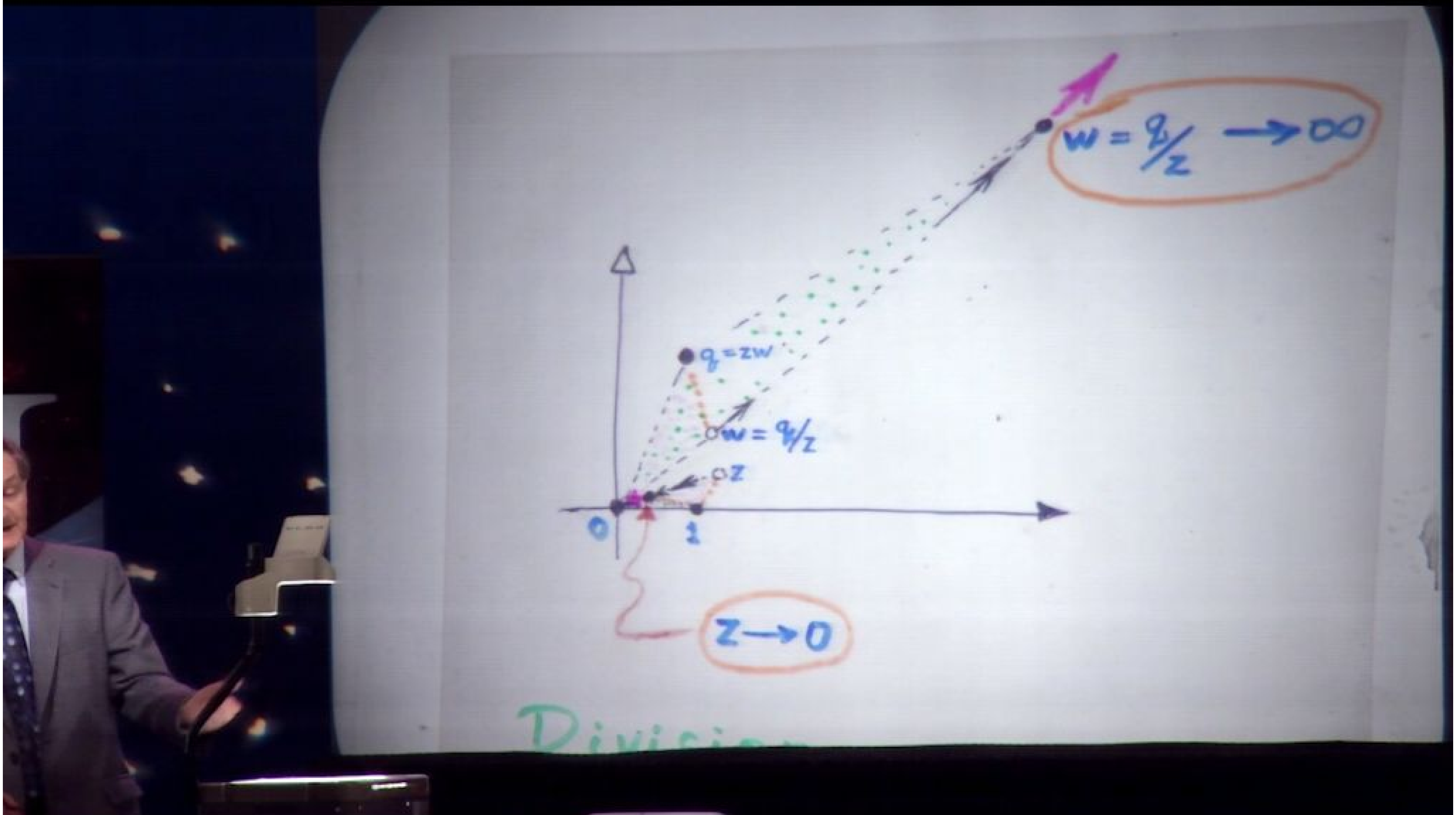

Z = x + iy
"real" numbers
z+w
w
z
0
zw
w
z
0
1
Addition:
parallelogram law
Multiplication:
similar-triangle law
Holomorphic function:
built up from adding, subtracting, multiplying, dividing, and taking limits
Not using complex conjugation:
z = x + iy
z
"Complex-


w
z
0
0
1
w
z
Addition:
parallelogram law
Multiplication:
similar-triangle law
Holomorphic function:
built up from adding, subtracting, multiplying, dividing, and taking limits
"Complex-smooth"
Not using complex conjugation:
z = x+iy
z̄ = x−iy
z
z̄
ERIMET
NSTITUT
R THEORETICAL P

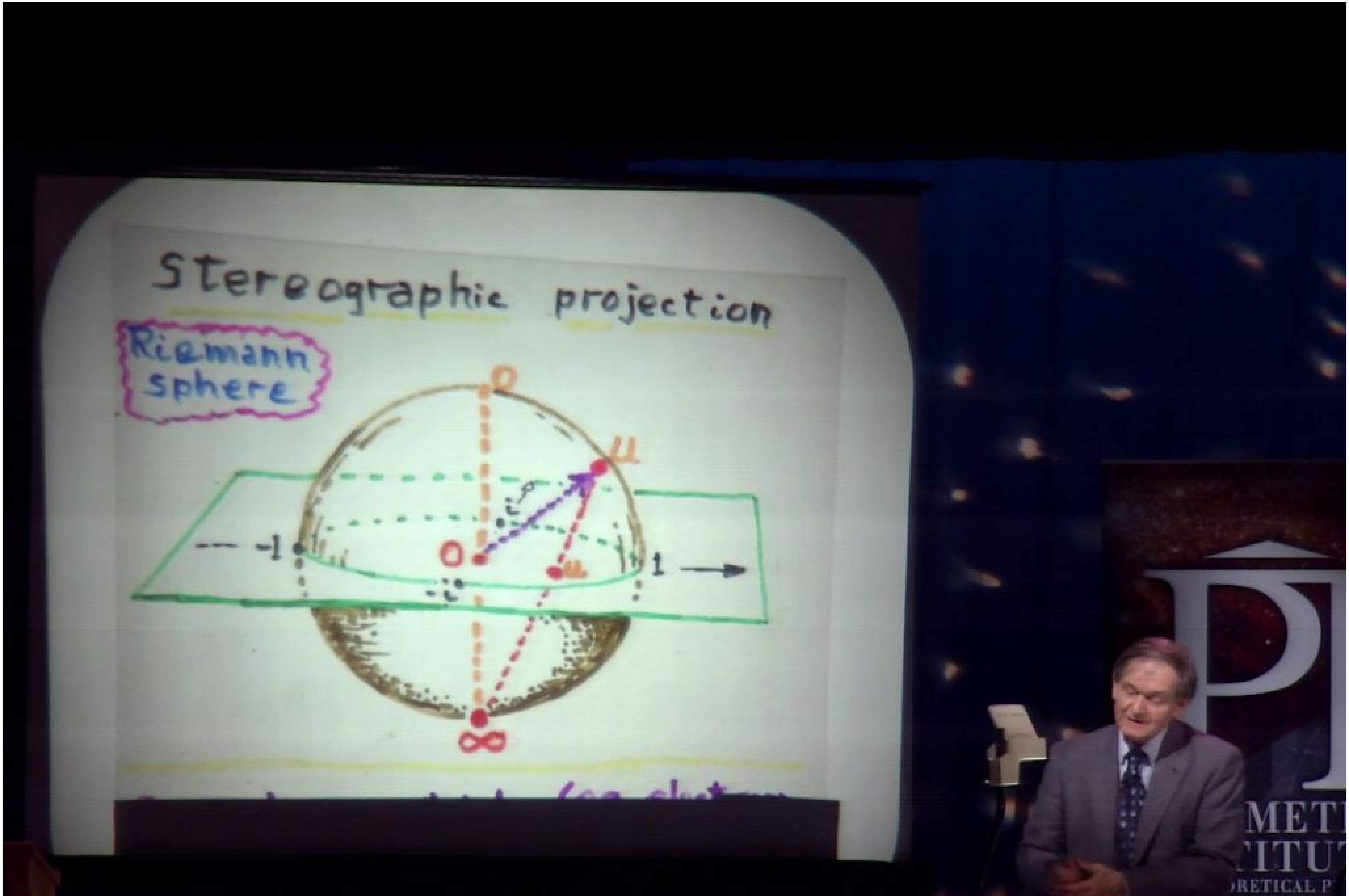


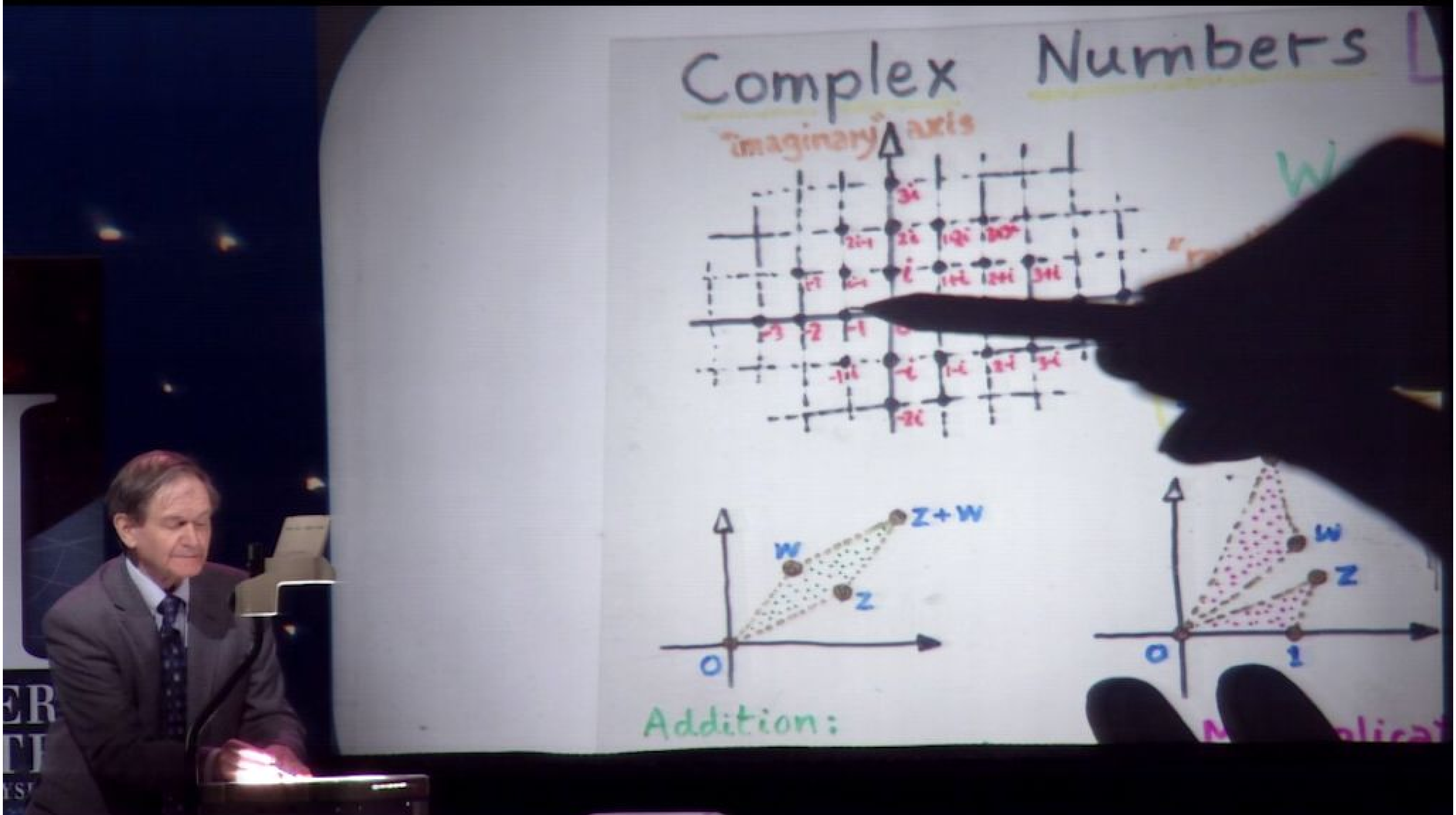
$w = q/z \longrightarrow \infty$
$q = zw$
$w = q/z$
z
0
1
$z \longrightarrow 0$

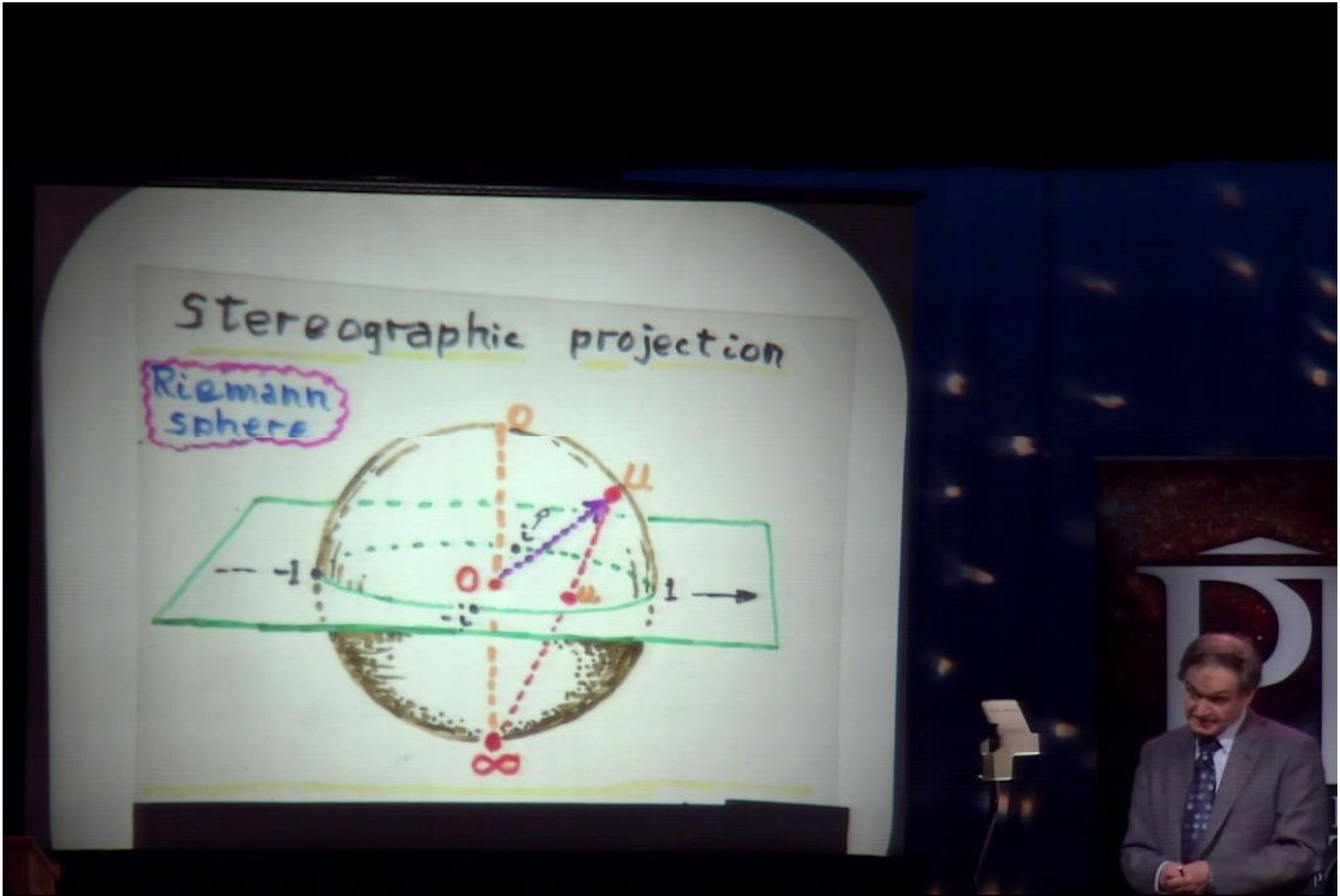

Complex Numbers
$i^2 = -1$
"imaginary" axis
"real" axis
Wessel plane
(Argand, Gauss, ...)
$z = x + iy$
"real" numbers
z+w
w
z
0
zw
1
Addition:
law
Multiplication:
similar-triangle

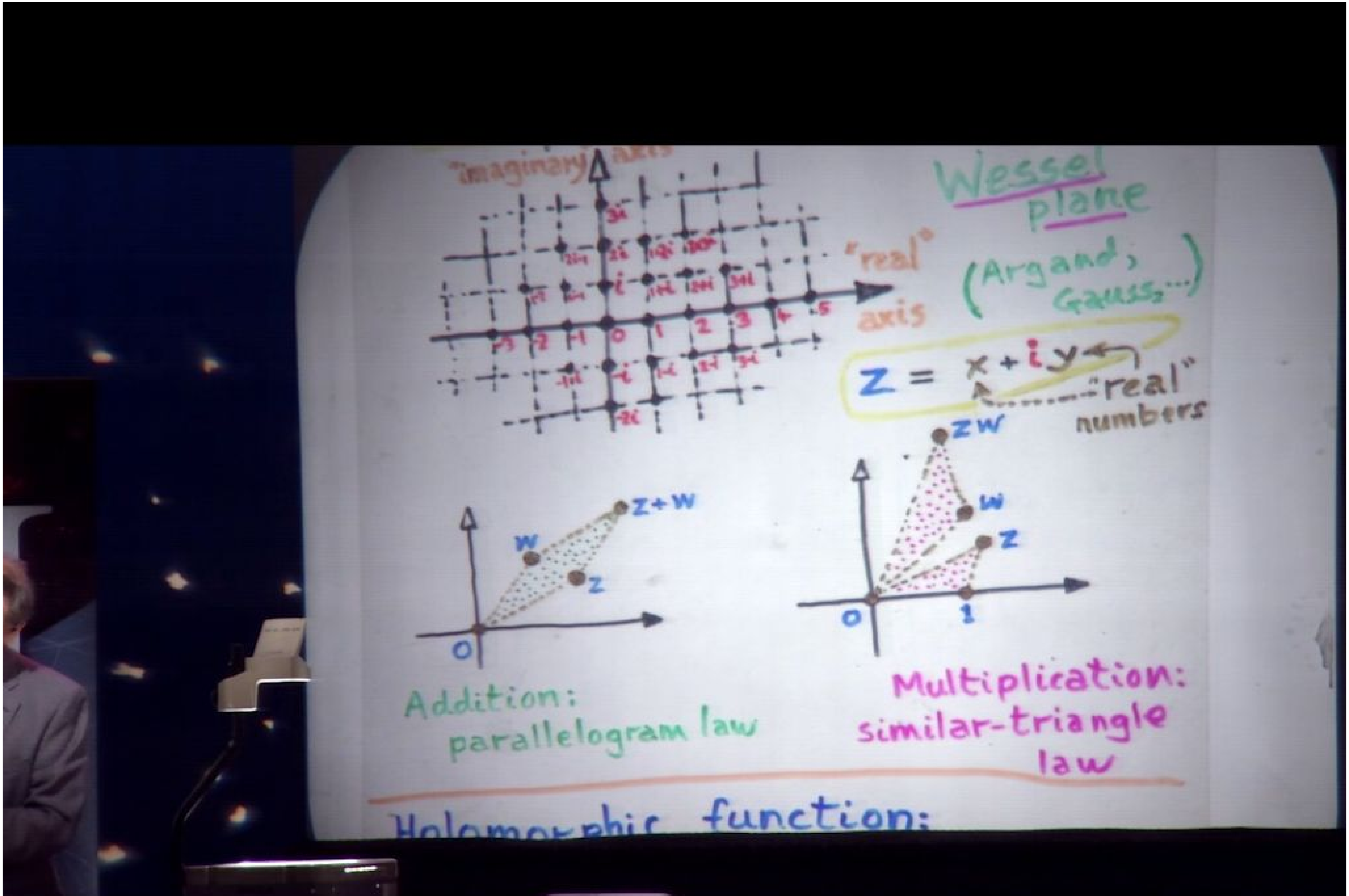


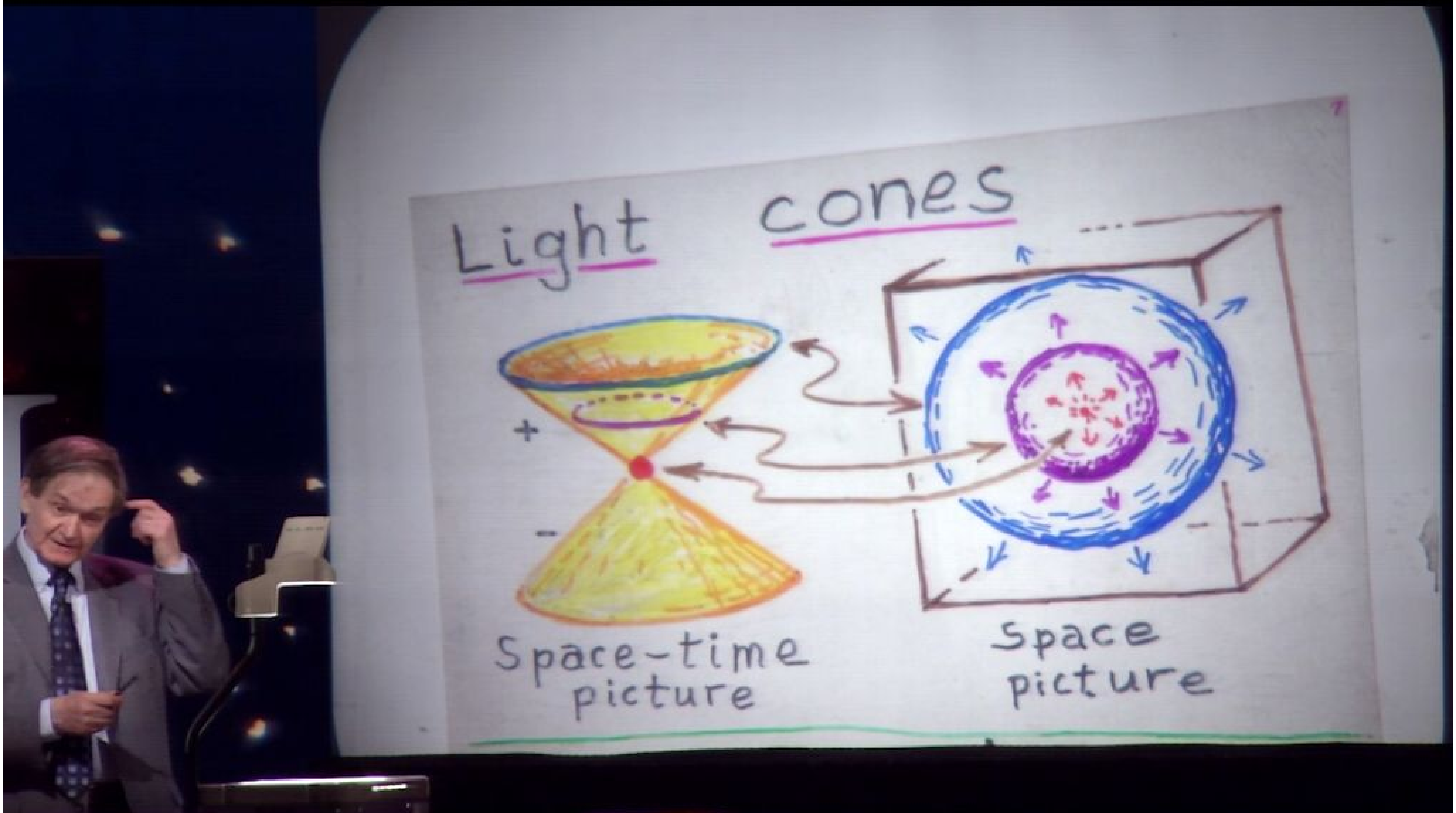
w = q/z ⟶ ∞
q = zw
w = q/z
z
0
1
z ⟶ 0

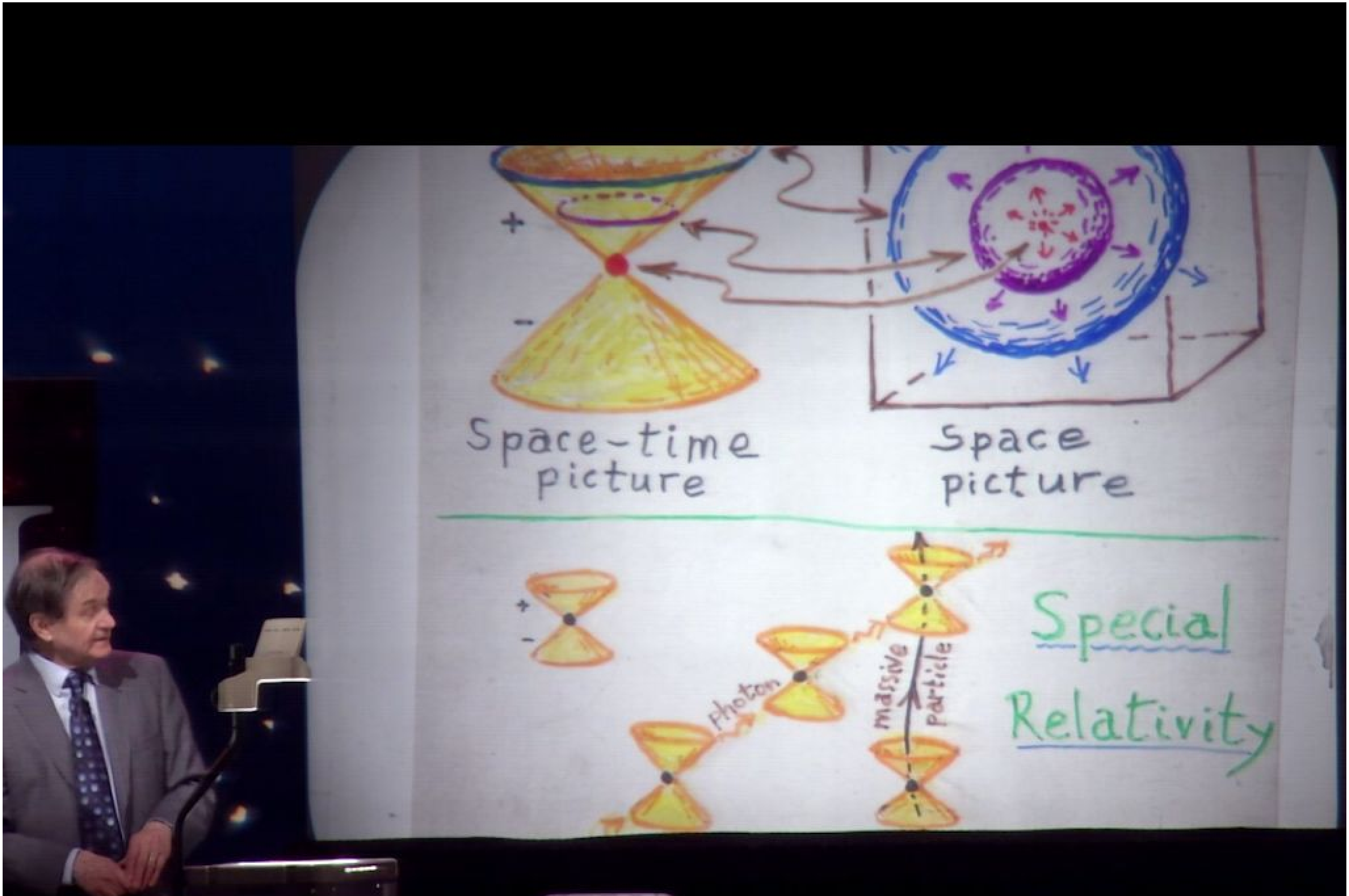

Stereographic projection
Riemann sphere
0
u
ρ
i
-1
O
u
1
-i
∞


Complex Numbers
"imaginary" axis
3i
2i
i
-1
-2
-3
-i
-2i
Z+W
W
Z
0
1
Addition:

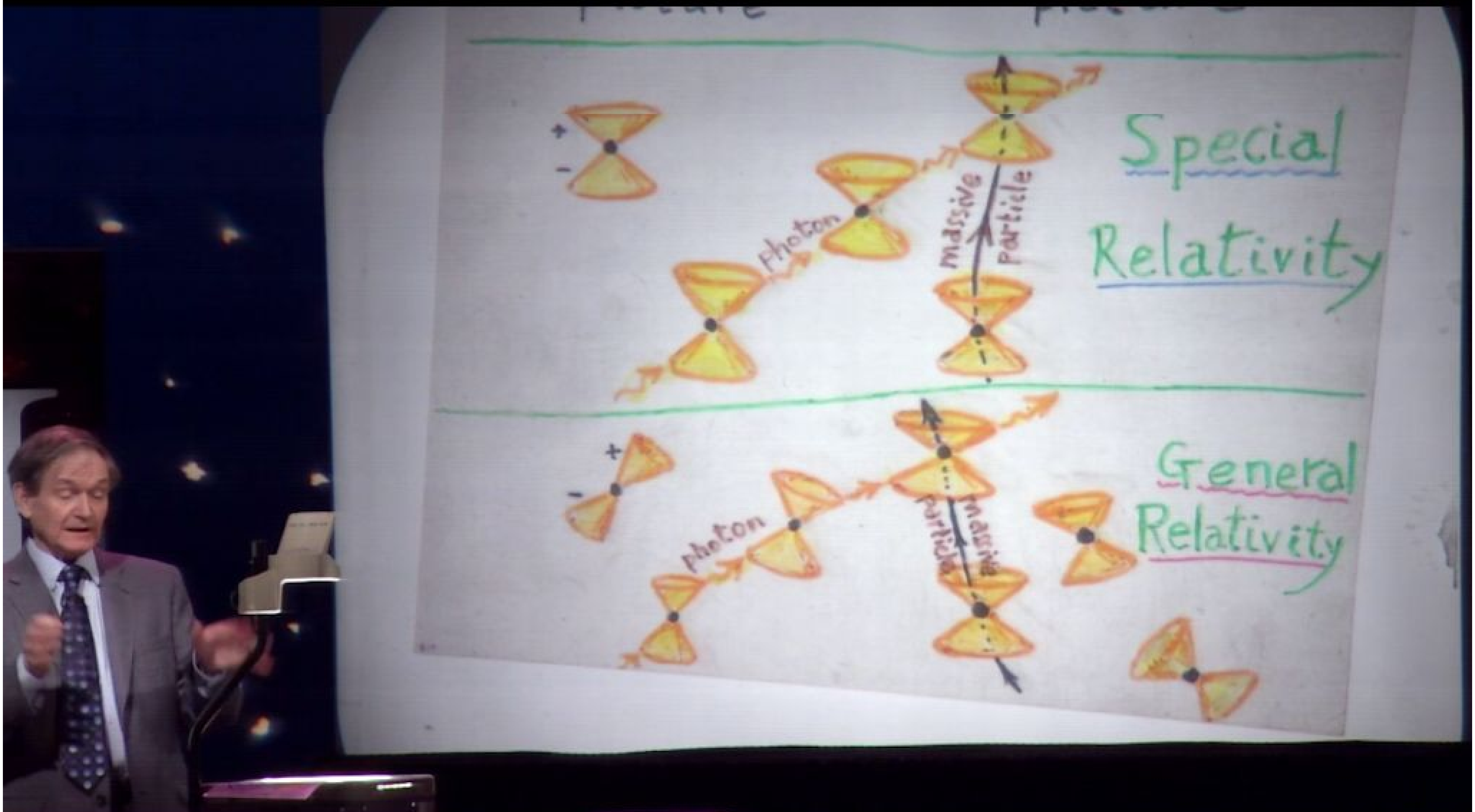

Stereographic projection
Riemann sphere
0
u
u
O
-1
1
∞


"imaginary
Wessel
plane
"real"
axis
(Argand,
Gauss,...)
Z = x + i y
"real"
numbers
z+w
w
z
0
zw
w
z
0
1
Addition:
parallelogram law
Multiplication:
similar-triangle
law
function:

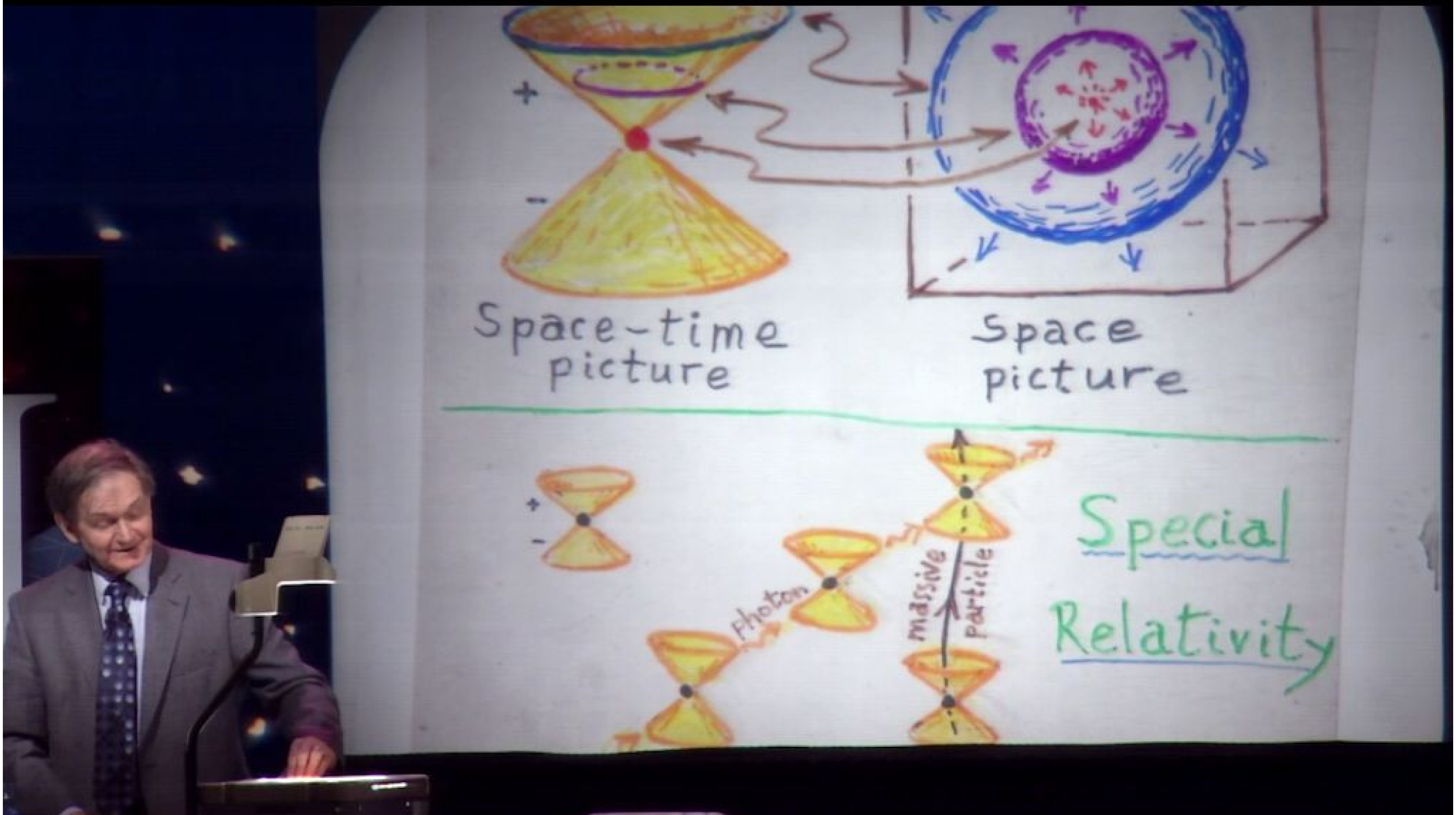

Light cones
+
-
Space-time
picture
Space
picture



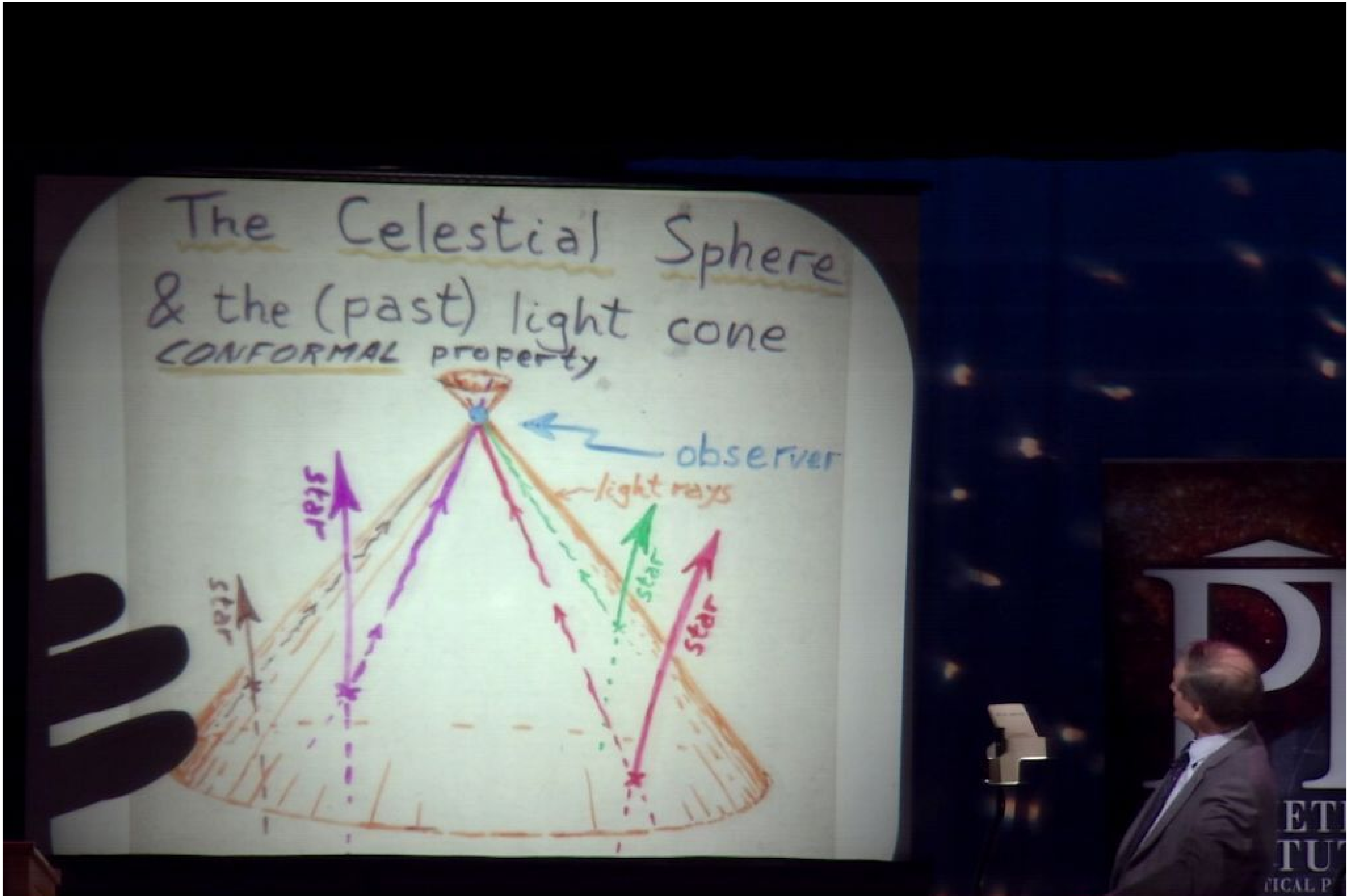
+
−
Space-time
picture
Space
picture
+
−
photon
massive
particle
Special
Relativity



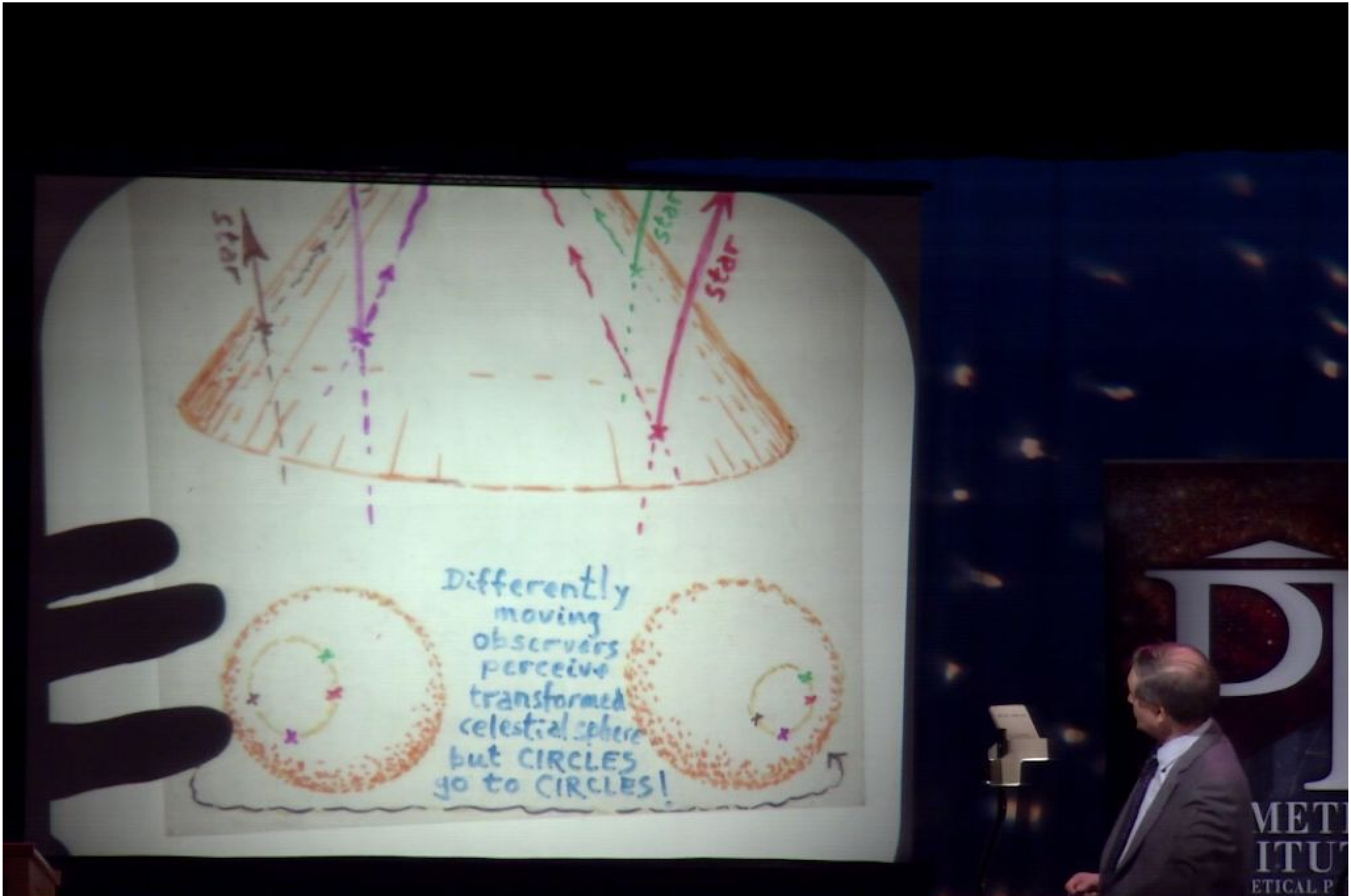
Special
Relativity
photon
massive
particle
+
−
General
Relativity
photon
particle
massive
+
−

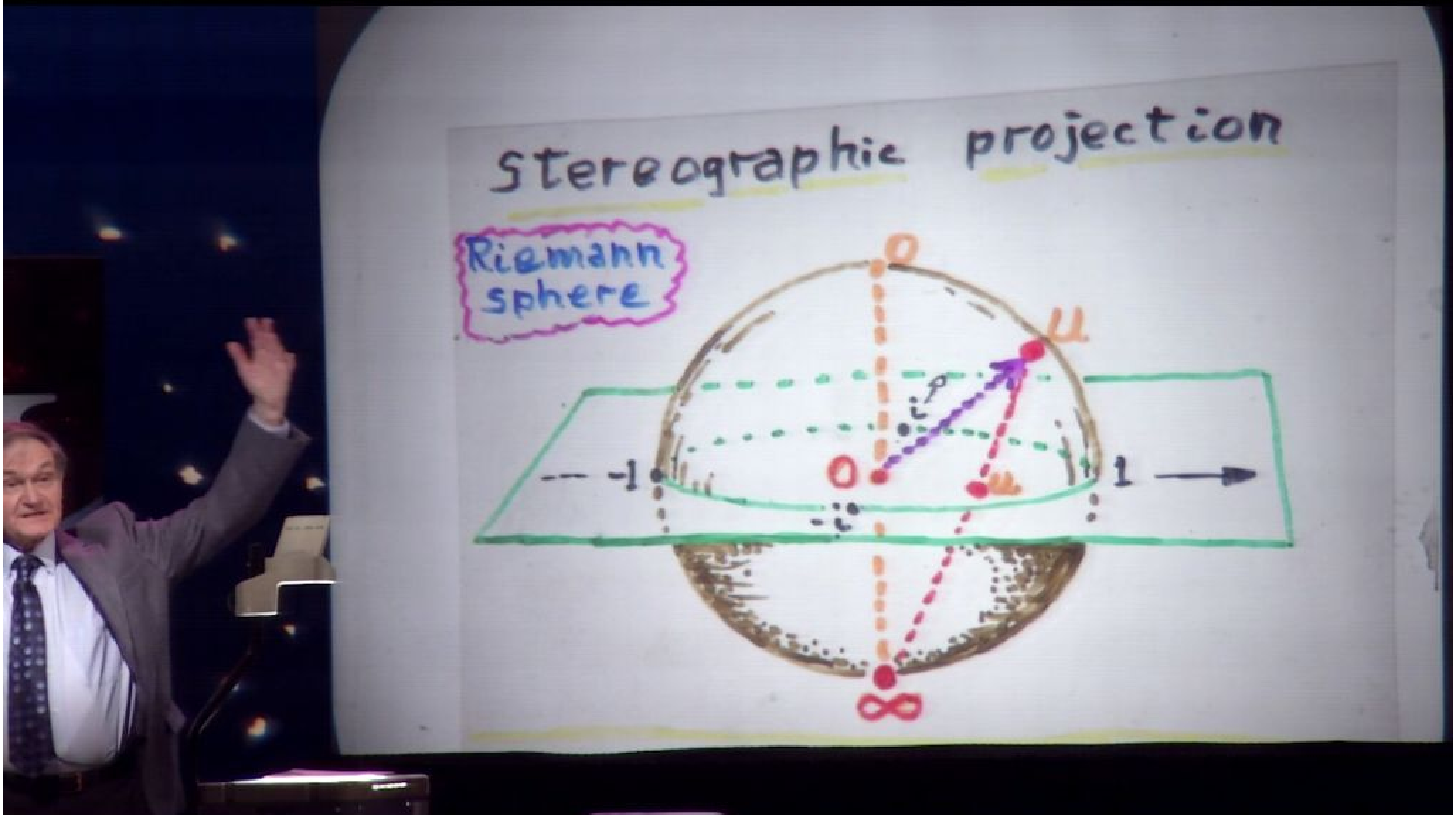


+
−
Space-time picture
Space picture
+
−
photon
massive particle
Special Relativity

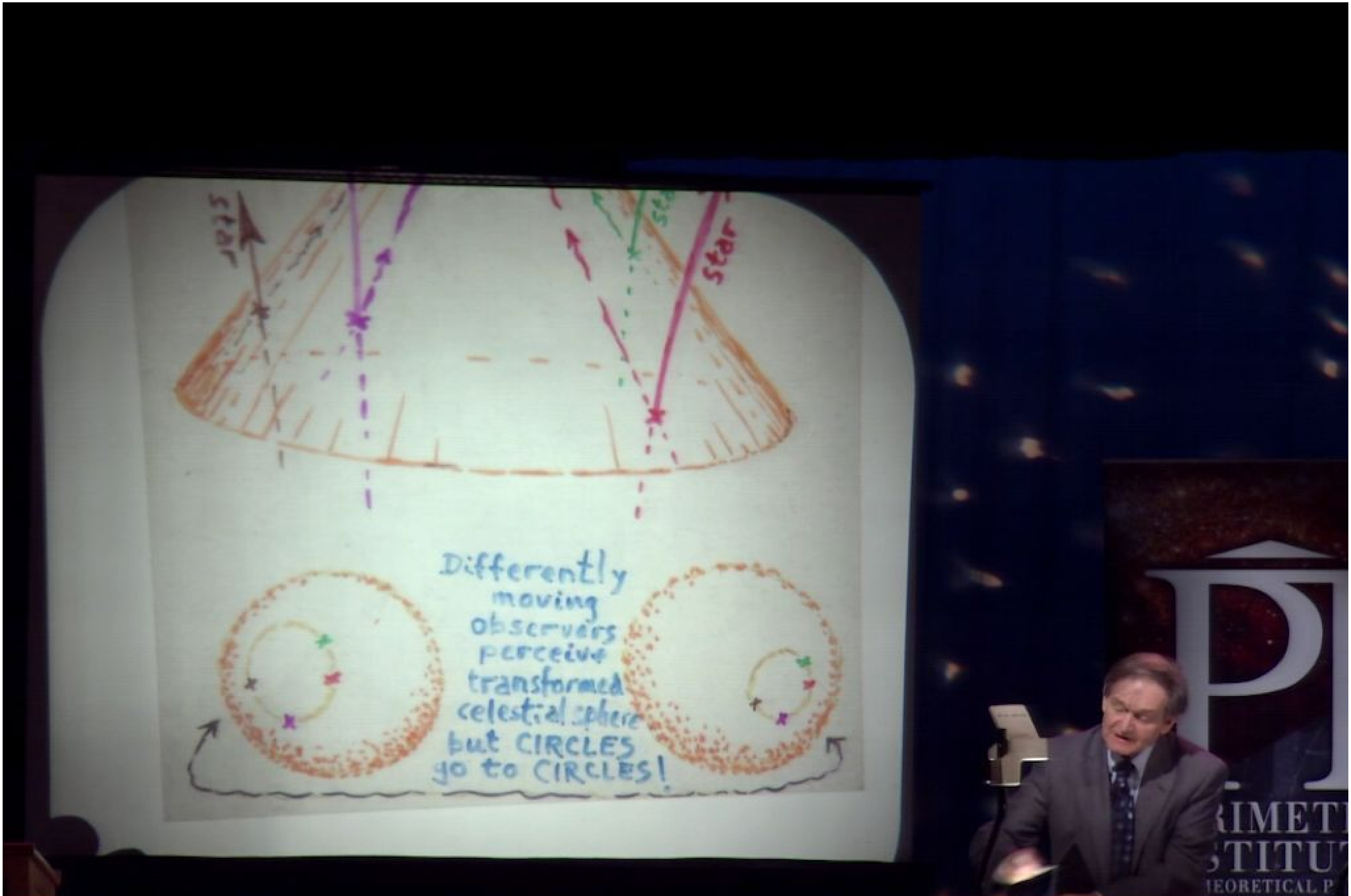


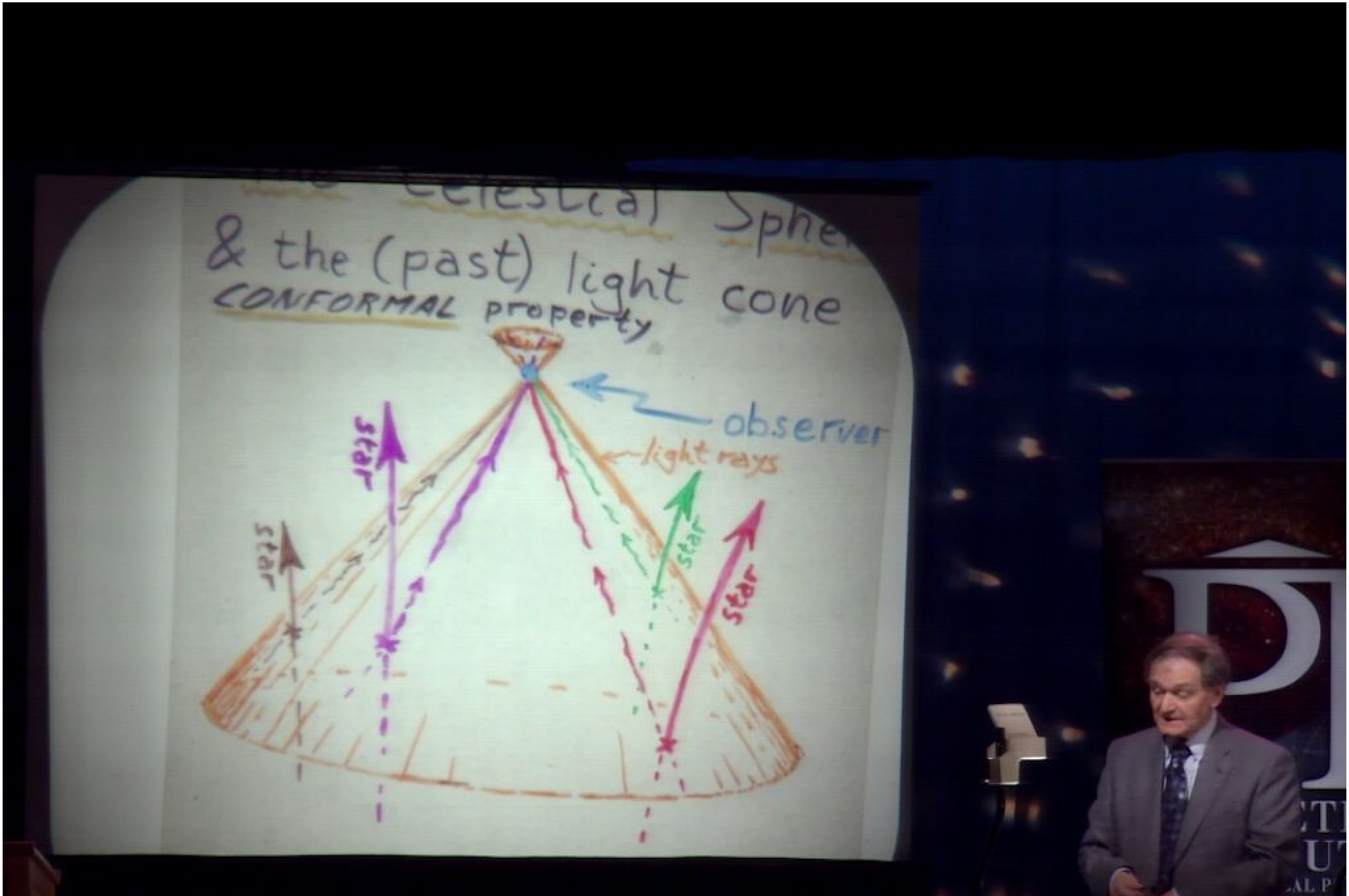
The Celestial Sphere
& the (past) light cone
CONFORMAL property
observer
light rays
star
star
star
star

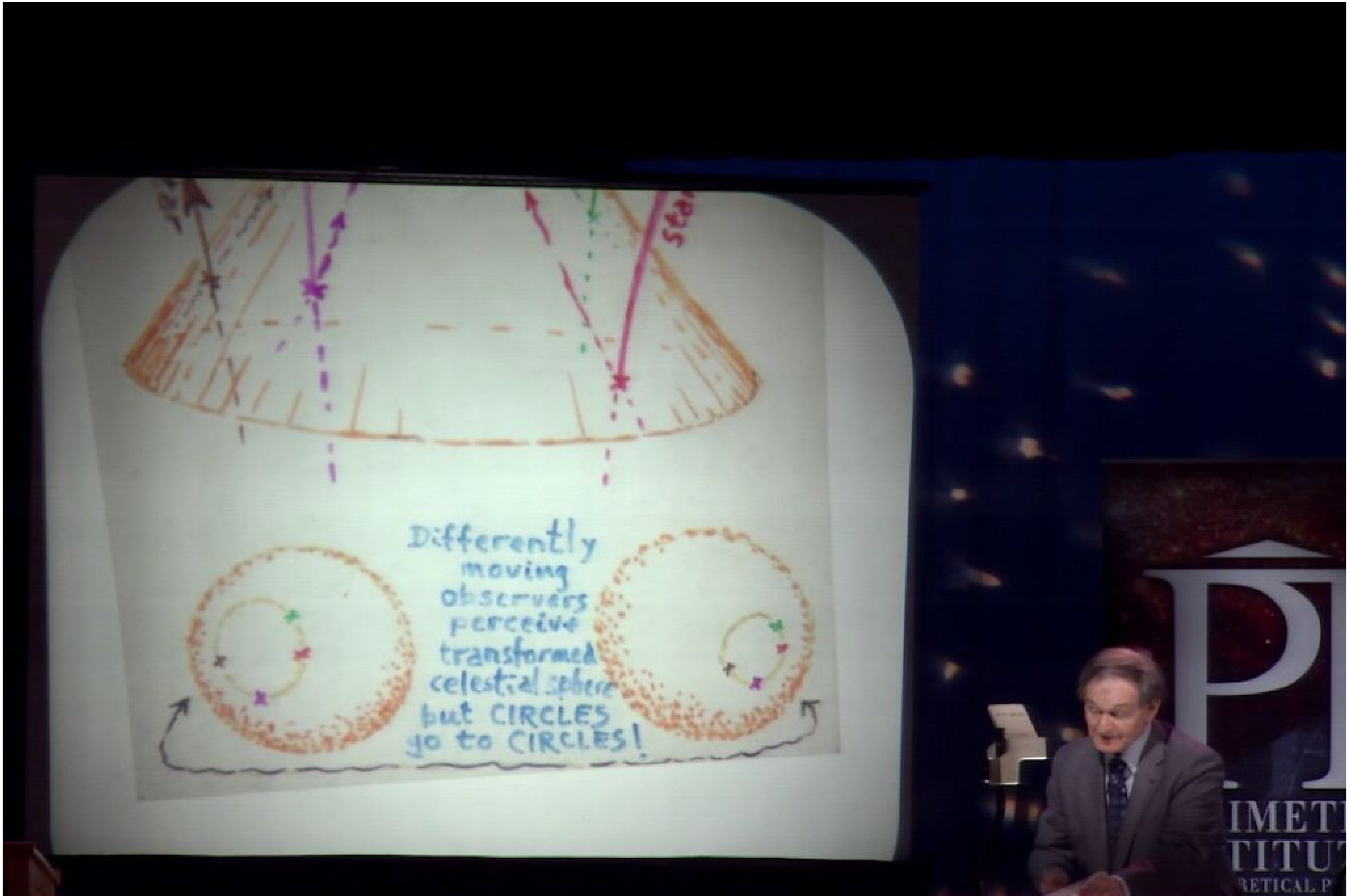

star
star
star
star
Differently
moving
observers
perceive
transformed
celestial sphere
but CIRCLES
go to CIRCLES!

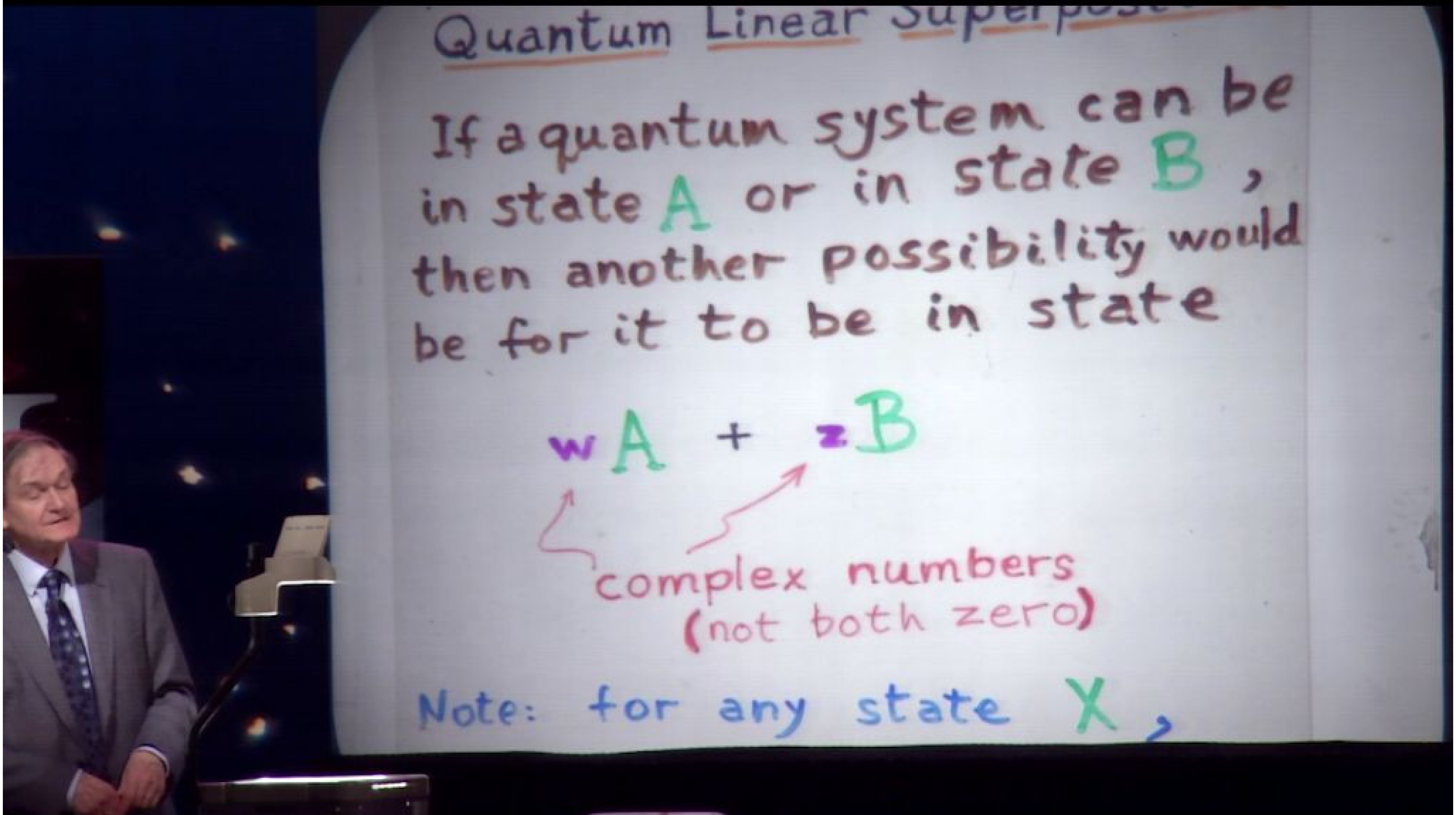


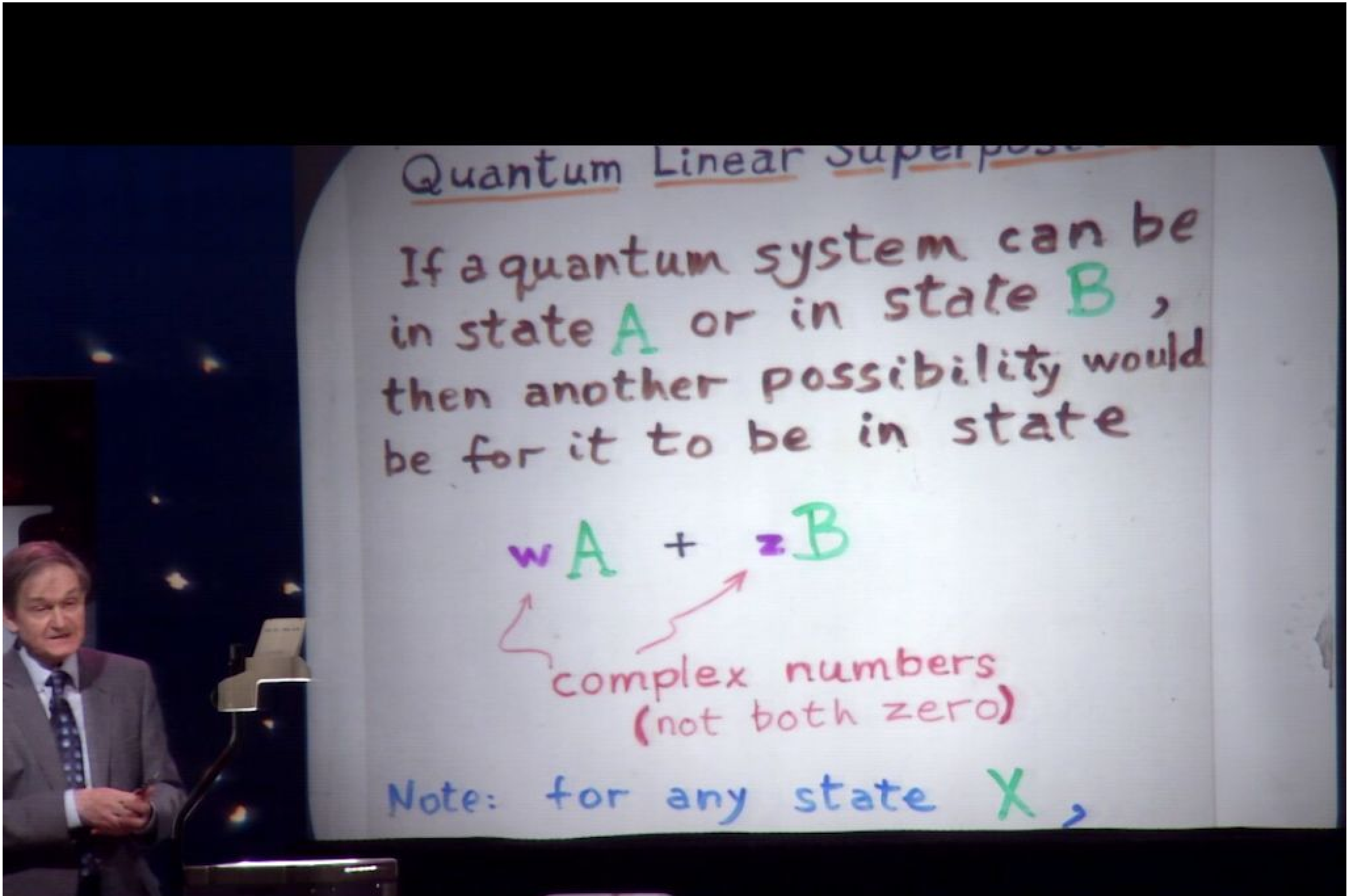
Stereographic projection
Riemann sphere
0
u
ρ
i
-1
0
u
1
-i
∞

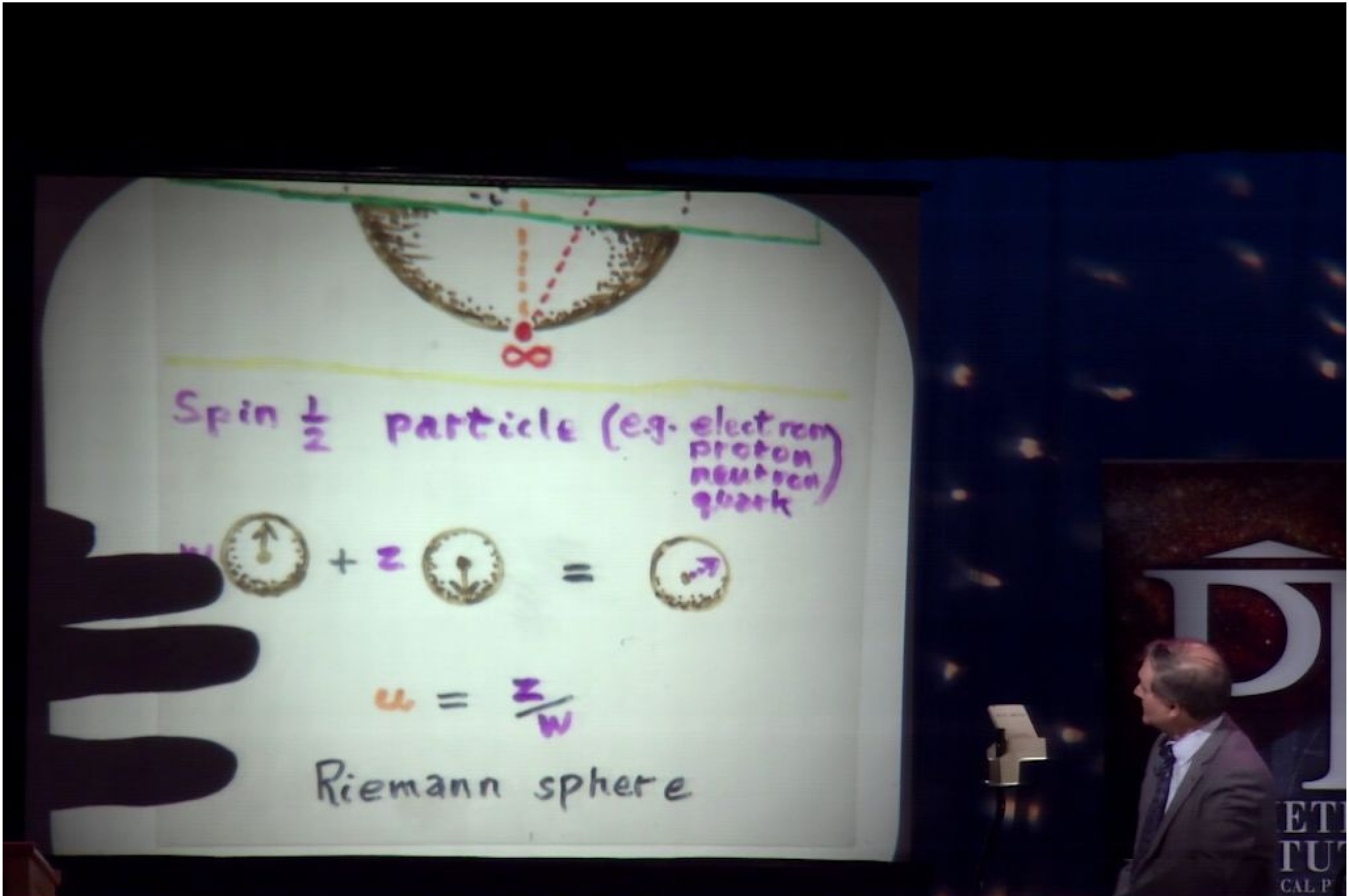

star
star
Differently moving observers perceive transformed celestial sphere but CIRCLES go to CIRCLES!
PI
PERIMETER INSTITUTE
THEORETICAL P

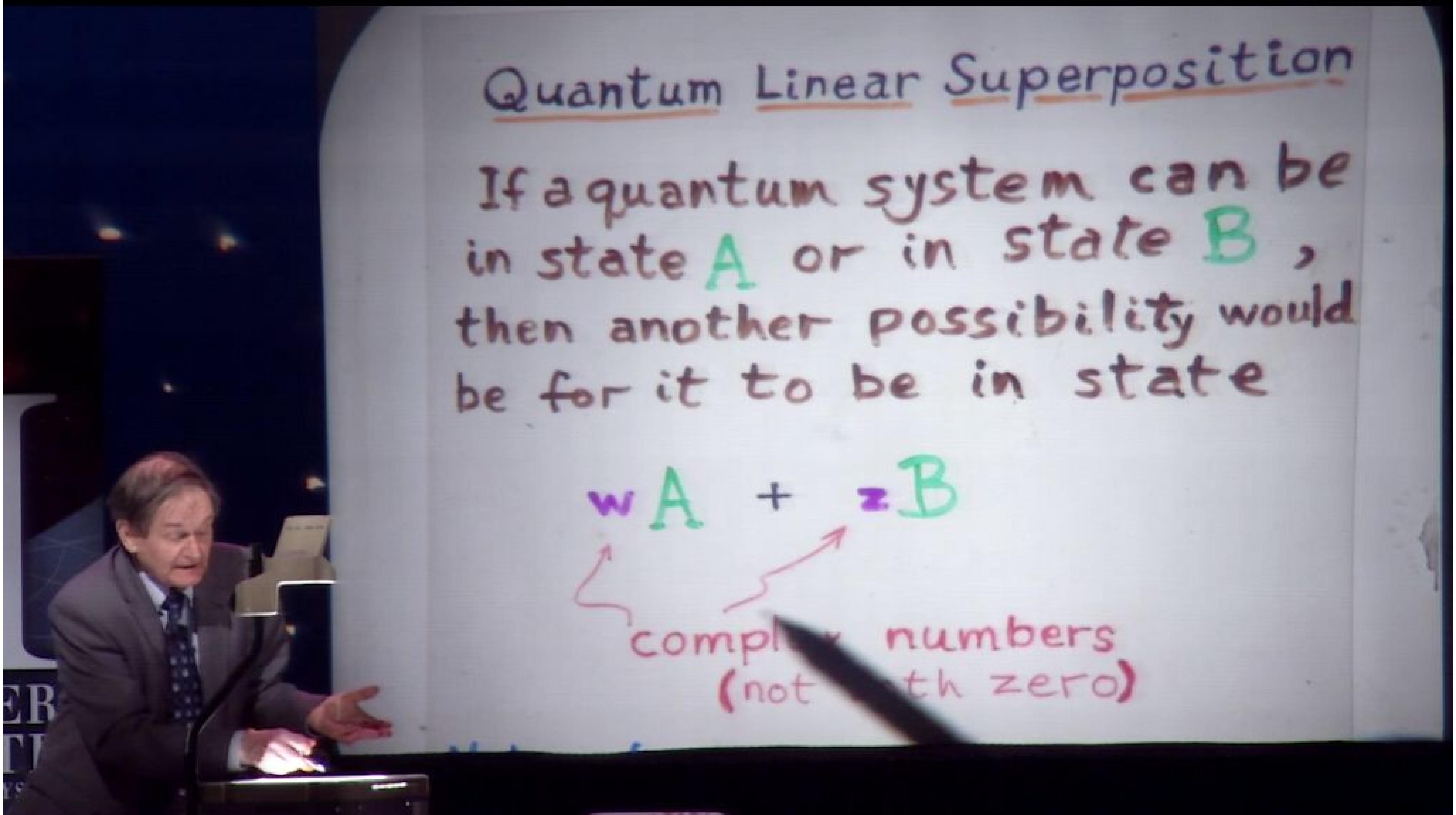

& the (past) light cone
CONFORMAL property
observer
light rays
star
star
star
star

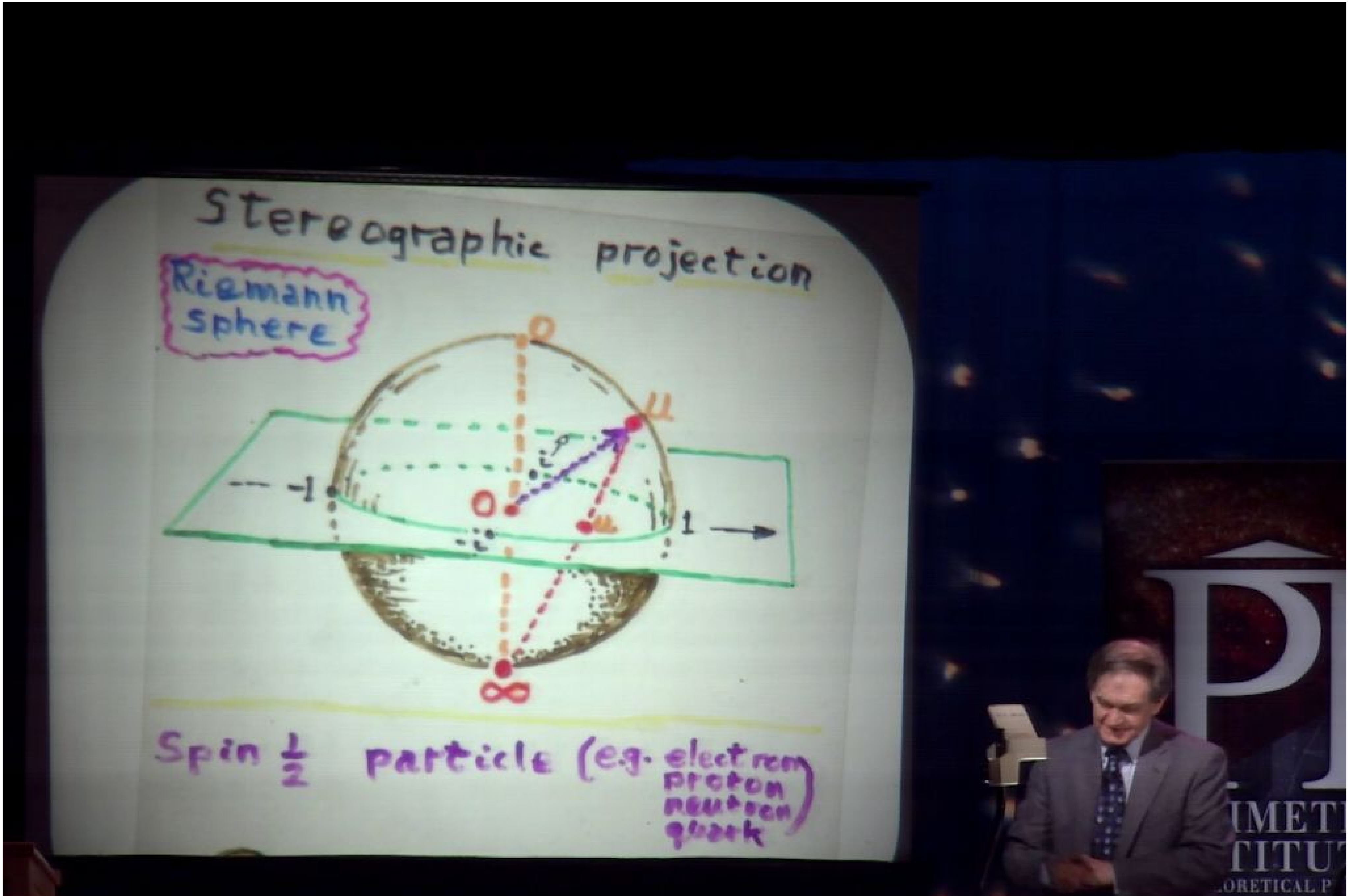

Differently
moving
observers
perceive
transformed
celestial sphere
but CIRCLES
go to CIRCLES!


Quantum Linear
If a quantum system can be
in state A or in state B,
then another possibility would
be for it to be in state
wA + zB
complex numbers
(not both zero)
Note: for any state X,


Quantum Linear
If a quantum system can be in state A or in state B, then another possibility would be for it to be in state
w A + z B
complex numbers
(not both zero)
Note: for any state X,


∞
Spin 1/2 particle (e.g. electron
proton
neutron
quark)
+ z =
u = z/w
Riemann sphere


Quantum Linear Superposition
If a quantum system can be in state A or in state B, then another possibility would be for it to be in state
wA + zB
compl numbers
(not th zero)

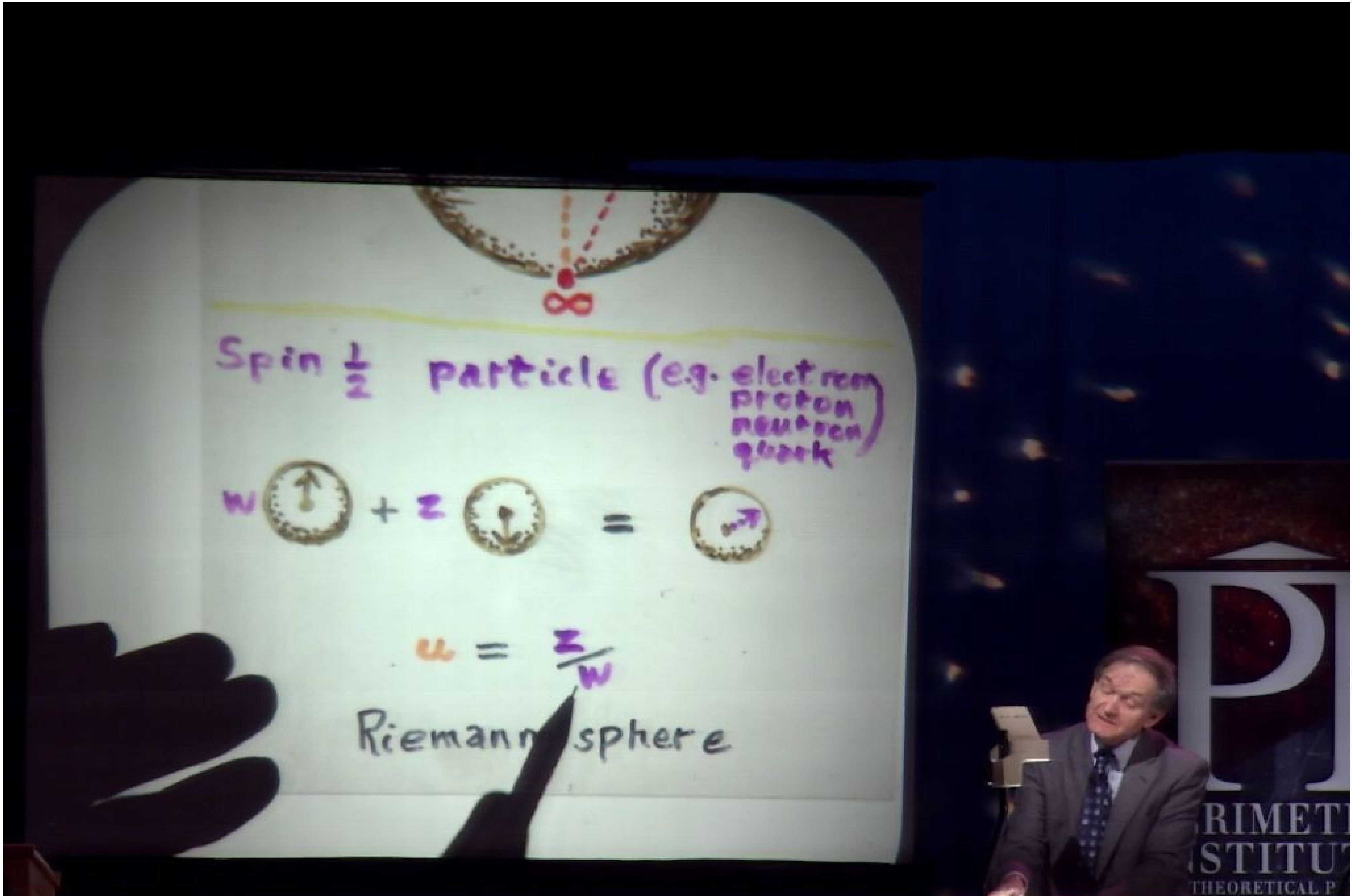

Stereographic projection
Riemann sphere
0
u
ρ
i
-1
O
u
1
-i
∞
Spin ½ particle (e.g. electron
proton
neutron
quark)
PI
IMETE
TITUT
ORETICAL P

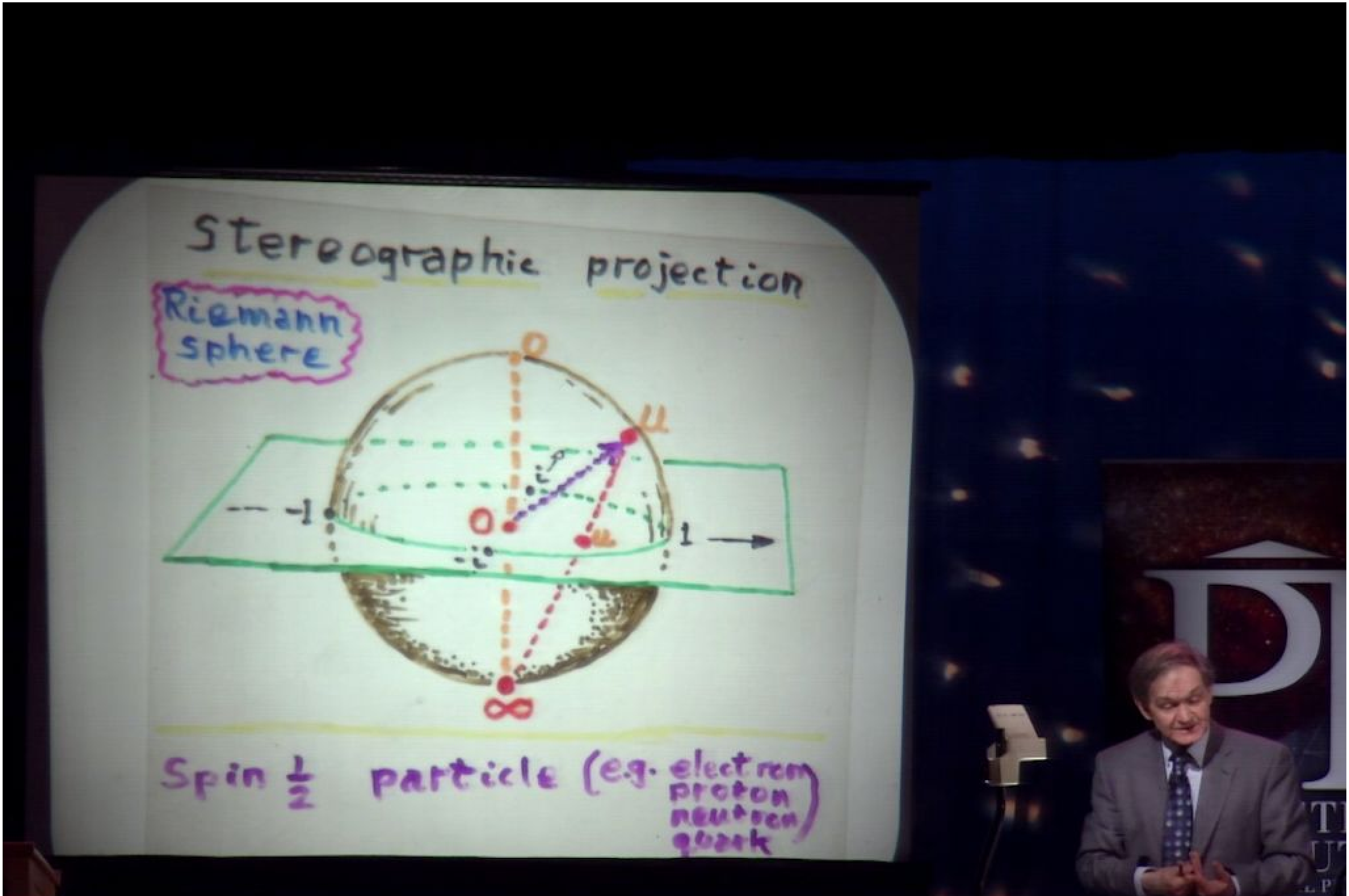
$wA + zB$

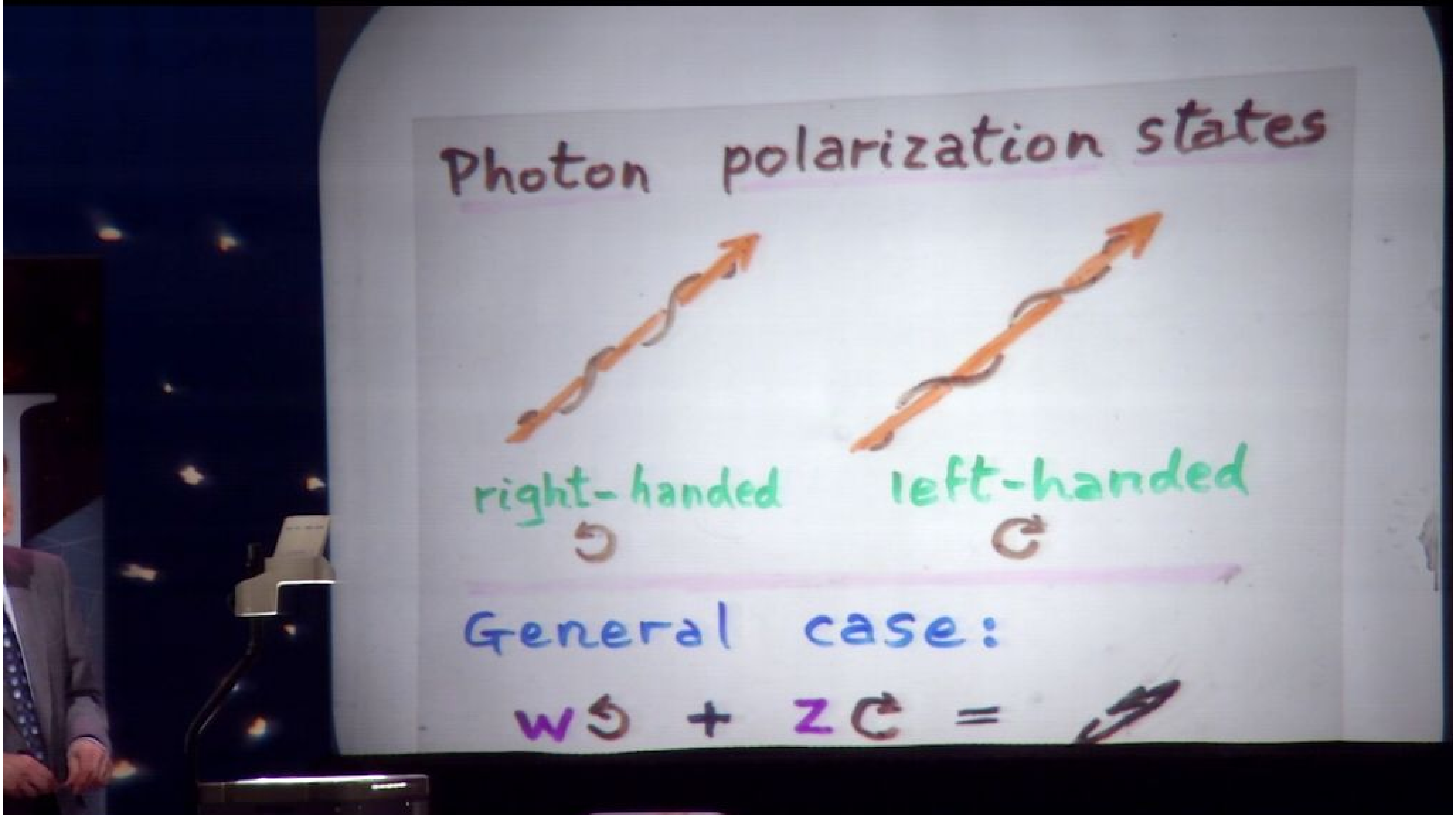
complex numbers
(not both zero)

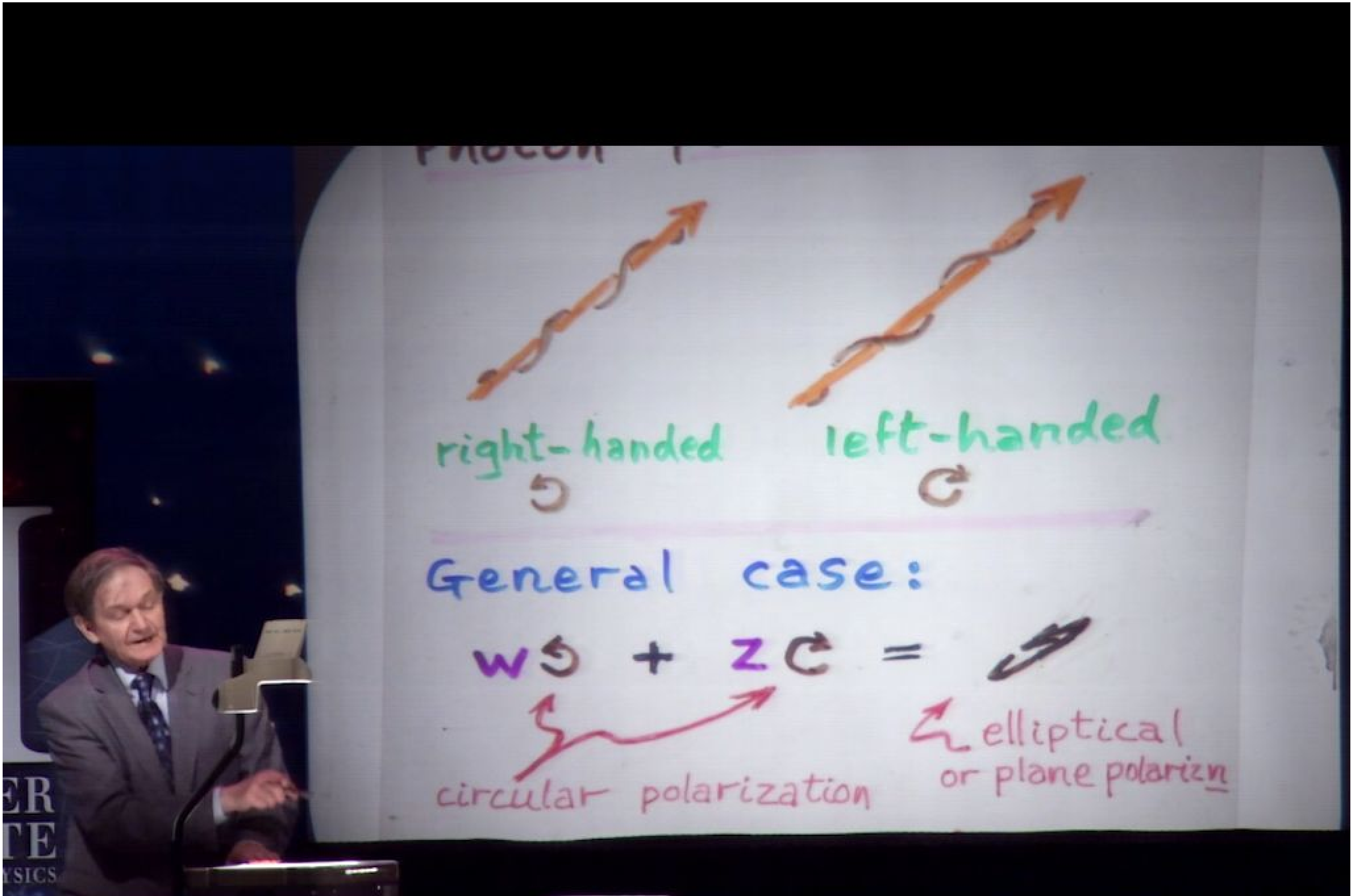
Note: for any state X, multiplying X by any non-zero complex number q: $X \rightsquigarrow qX$, does not change the physical interpretation of the state.

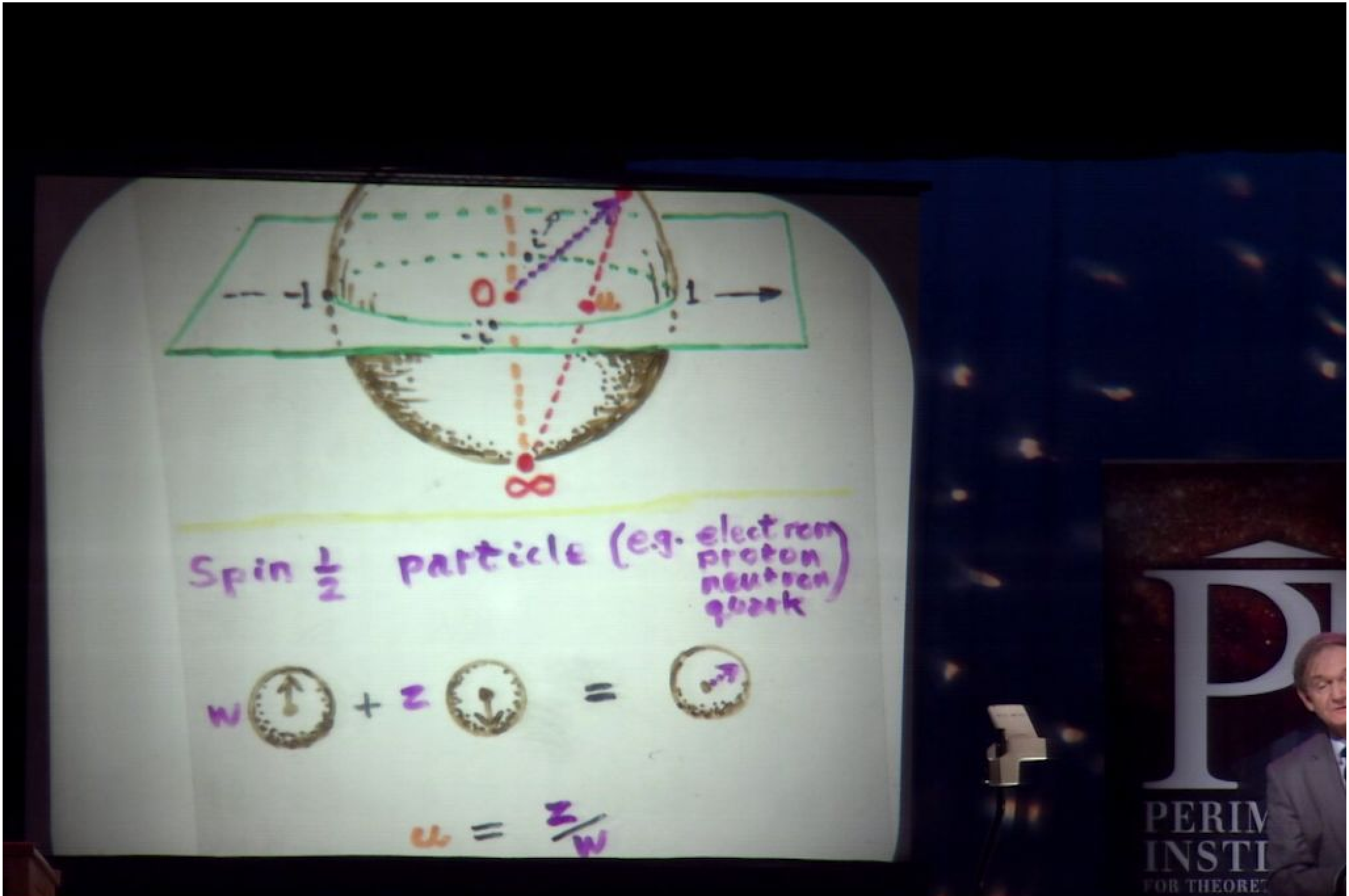
Hence $wA+zB$ is equivalent to $qwA+qzB$ i.e. the physical interpretation of $wA+zB$ depends only on the ratio $\boxed{w:z}$


∞
Spin 1/2 particle (e.g. electron
proton
neutron
quark)
w + z =
u = z/w
Riemann sphere

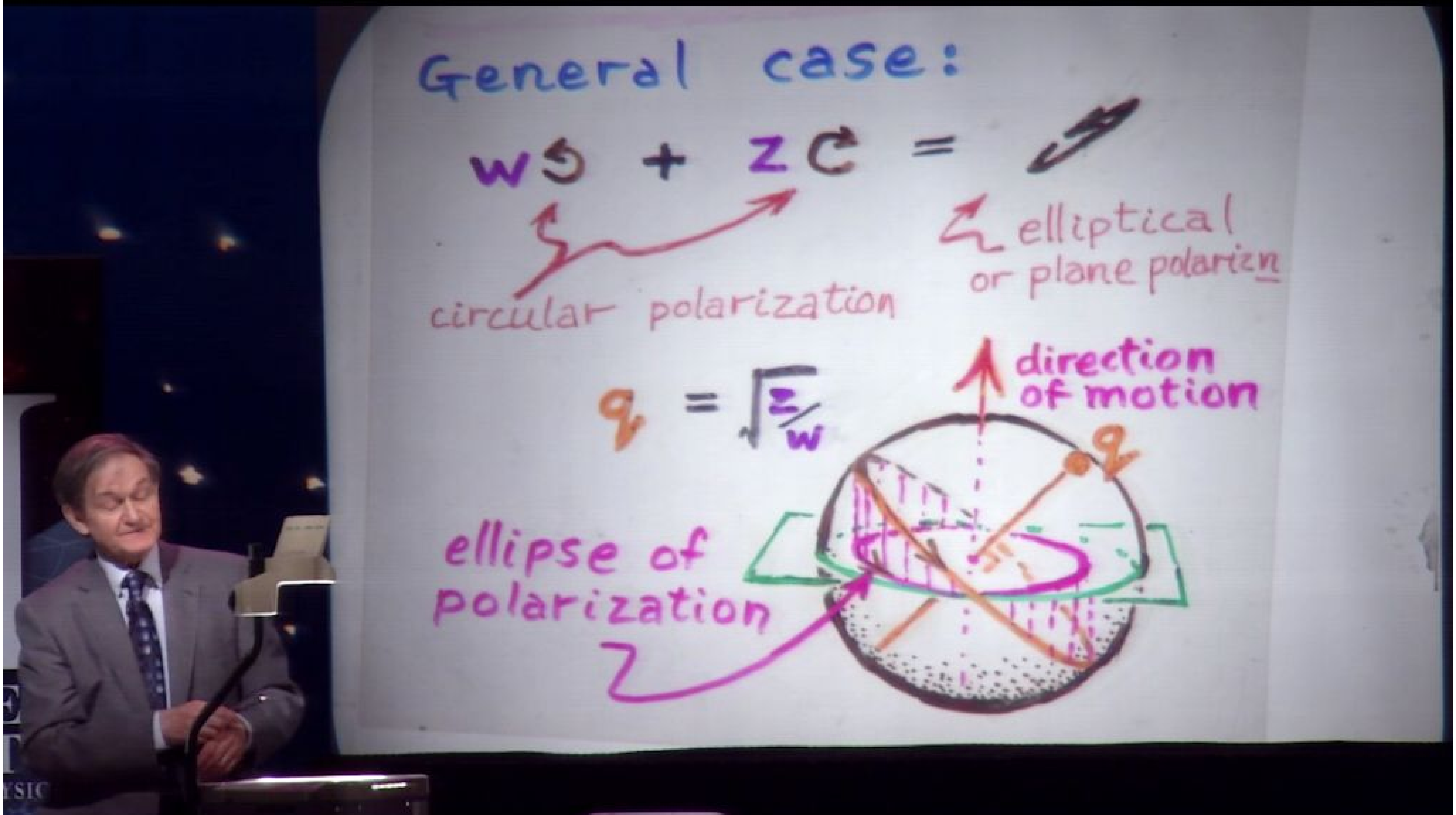

Stereographic projection
Riemann sphere
0
u
ρ
i
O
-1
1
u
-i
∞
Spin ½ particle (e.g. electron proton neutron quark)


Photon polarization states
right-handed
left-handed
General case:
w + z =

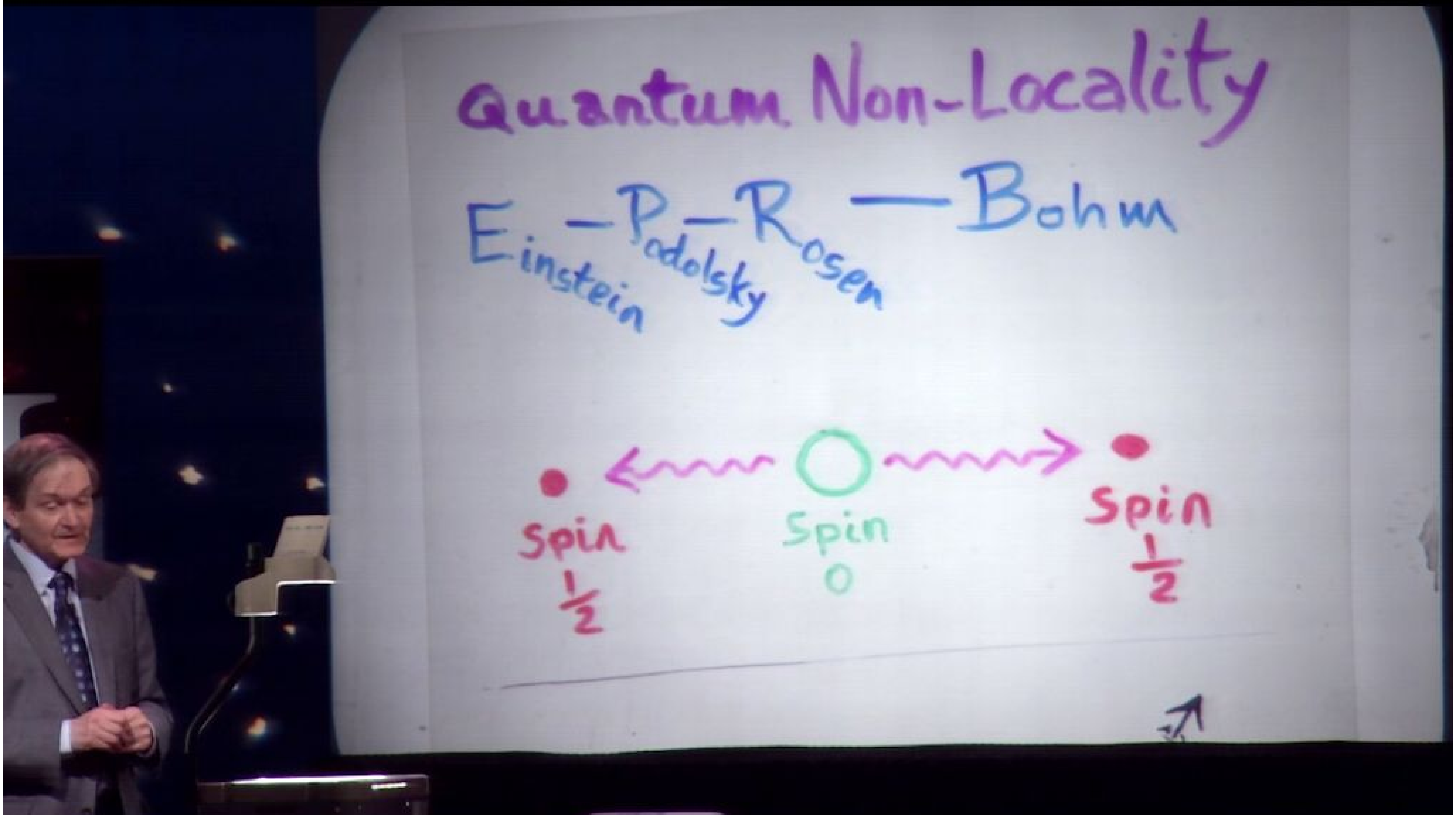

right-handed
left-handed
General case:
w + z =
elliptical
or plane polarizn
circular polarization
ER
TE
YSICS

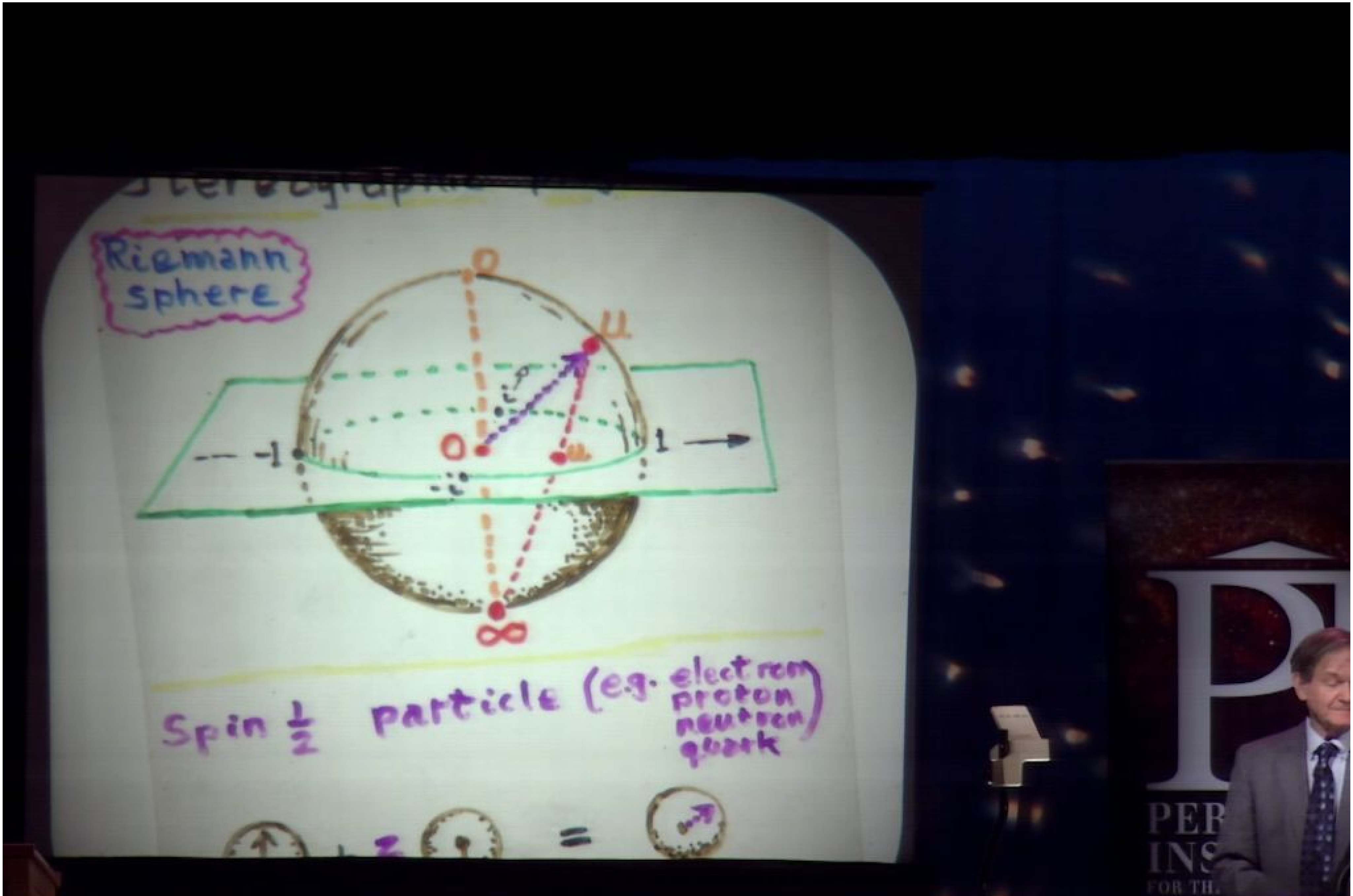


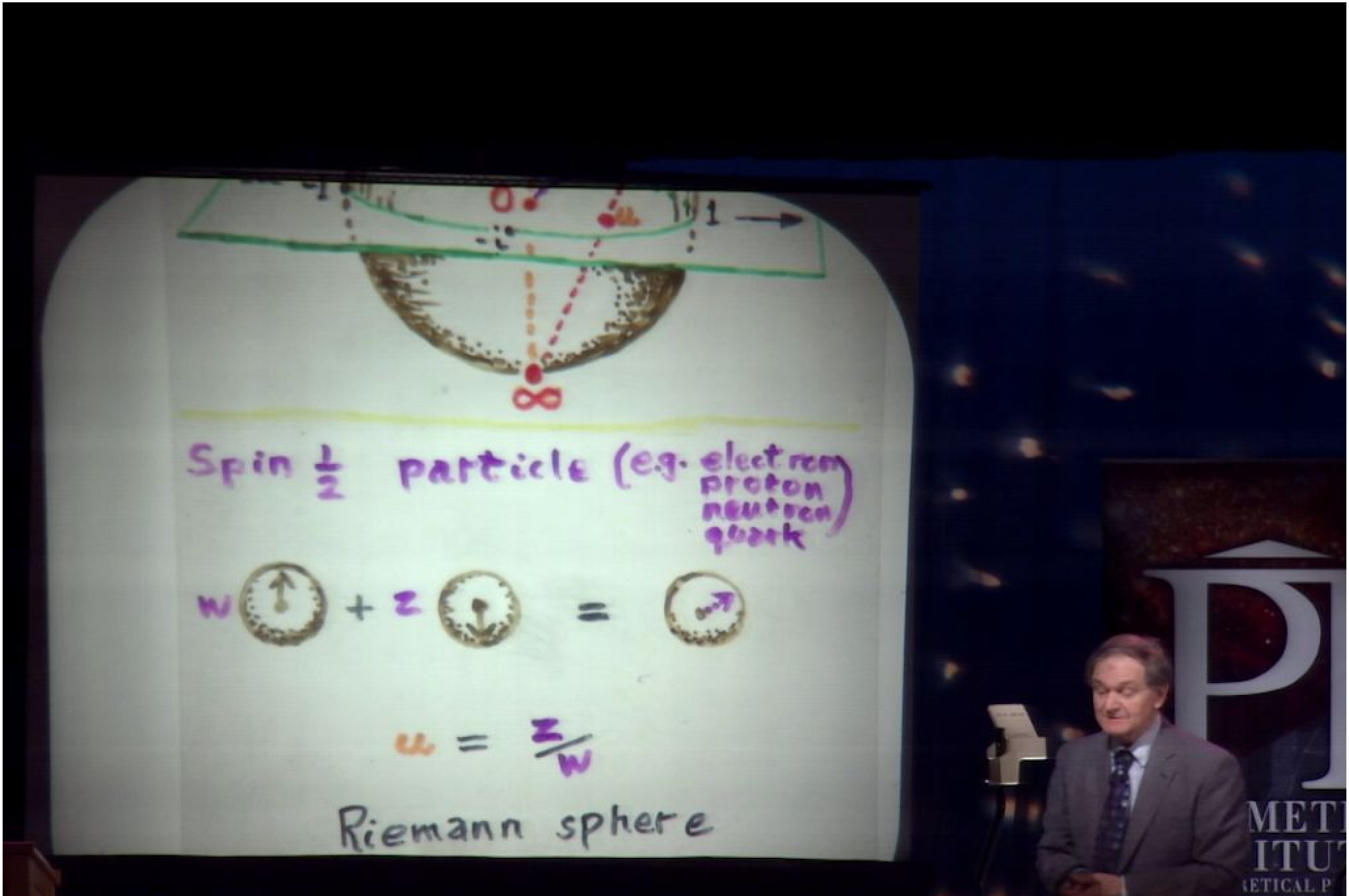
-1
0
u
1
ρ
i
-i
∞
Spin 1/2 particle (e.g. electron proton neutron quark)
w
+ z
=
u = z/w
PERIM
INSTI
FOR THEORET

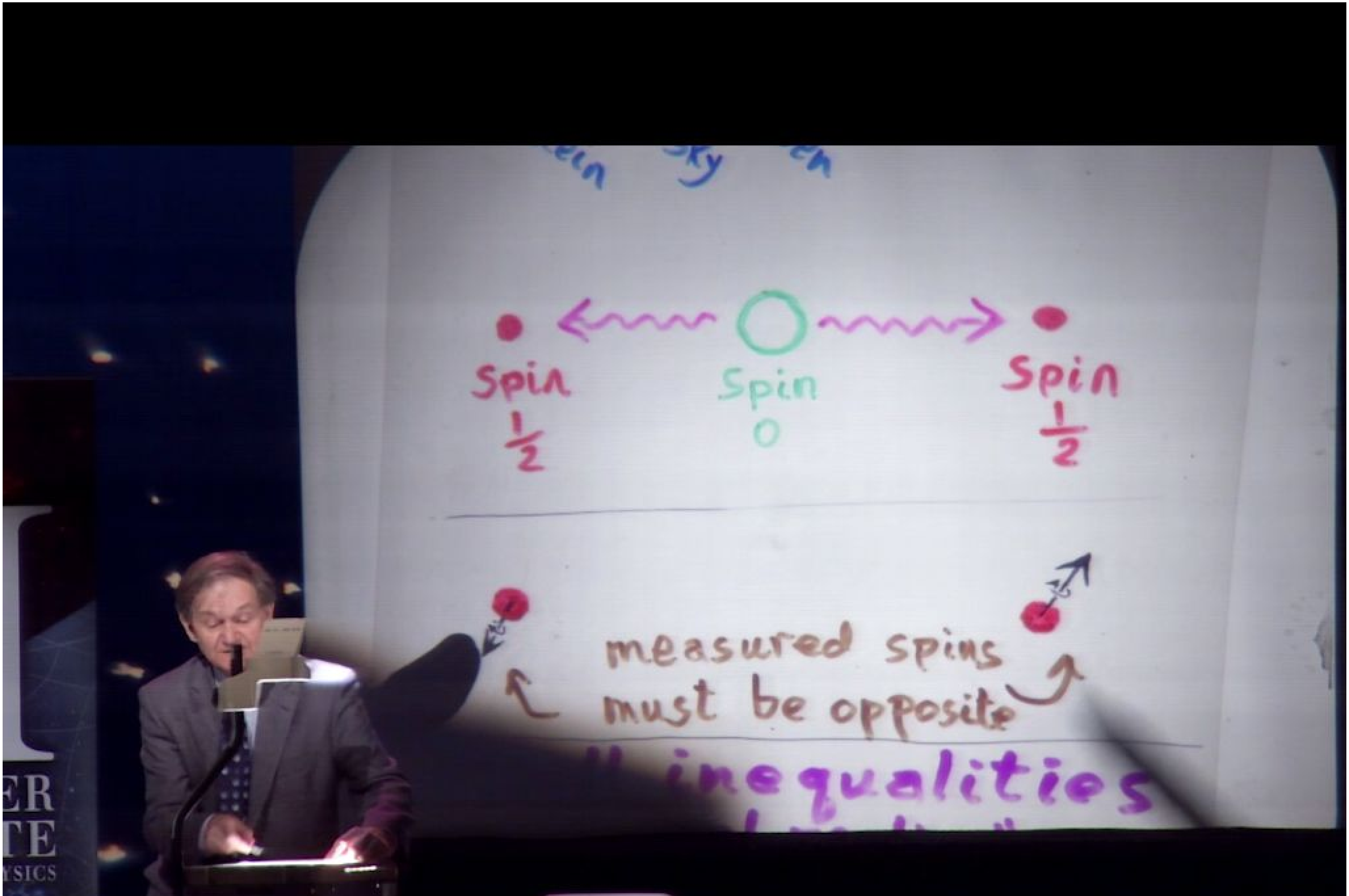


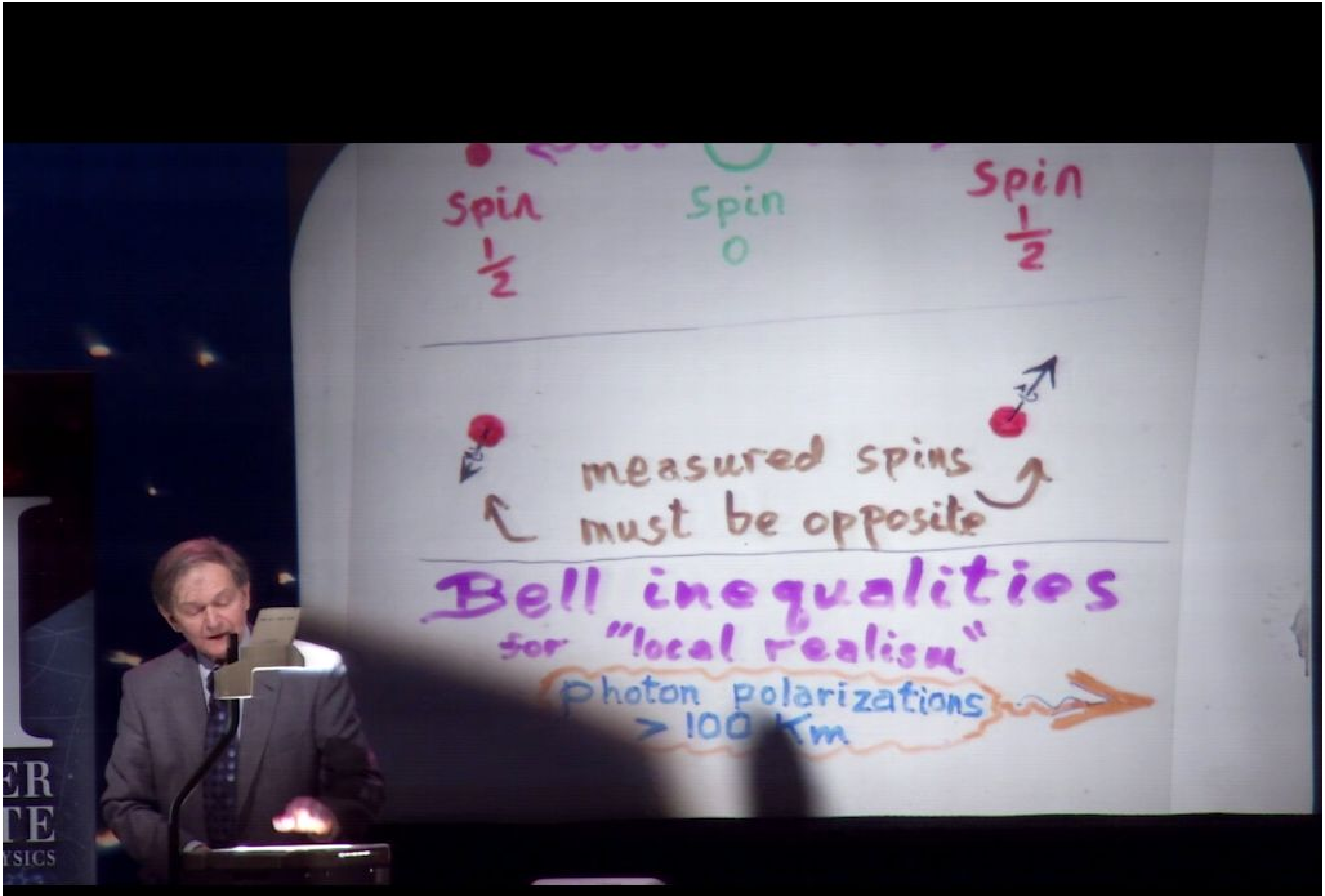
General case:
w ↺ + z ↻ =
circular polarization
elliptical
or plane polarizn
direction of motion
q = √(z/w)
q
ellipse of polarization


Quantum Non-Locality
E—P—R — Bohm
Einstein
Podolsky
Rosen
Spin ½
Spin 0
Spin ½

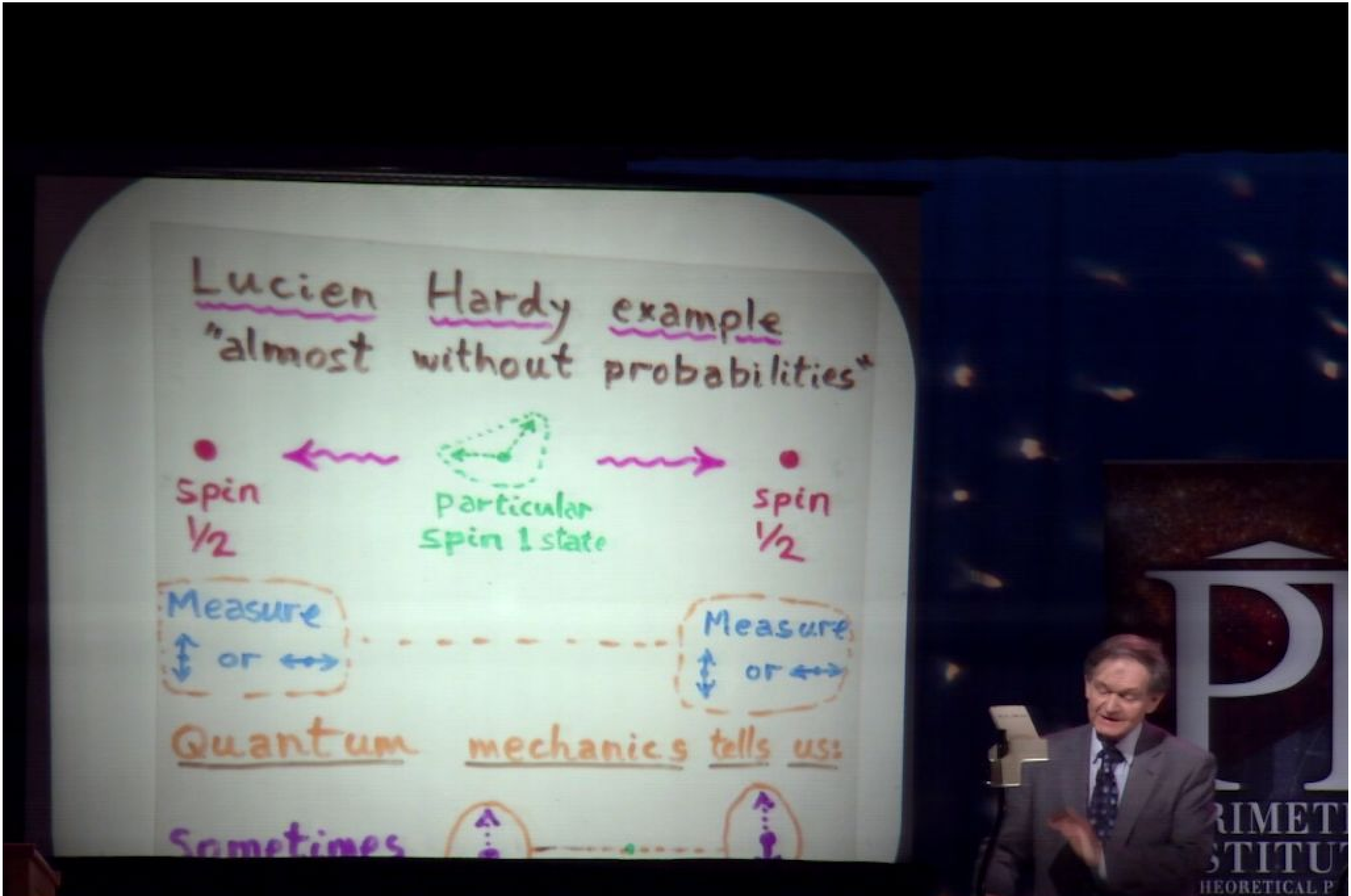

Riemann sphere
u
0
-1
1
∞
Spin ½ particle (e.g. electron proton neutron quark)

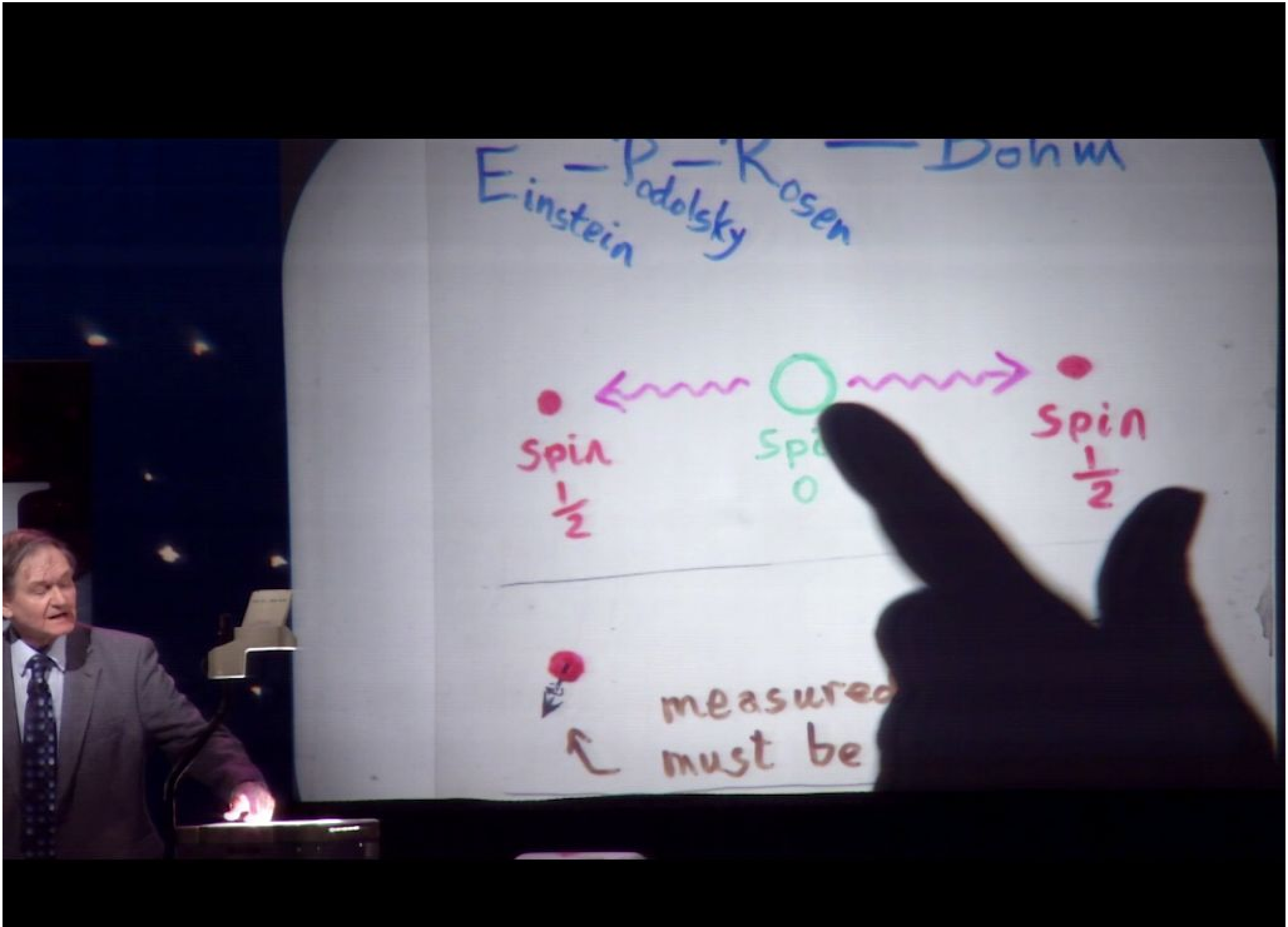

0
1
∞
Spin ½ particle (e.g. electron
proton
neutron
quark)
w + z =
u = z/w
Riemann sphere

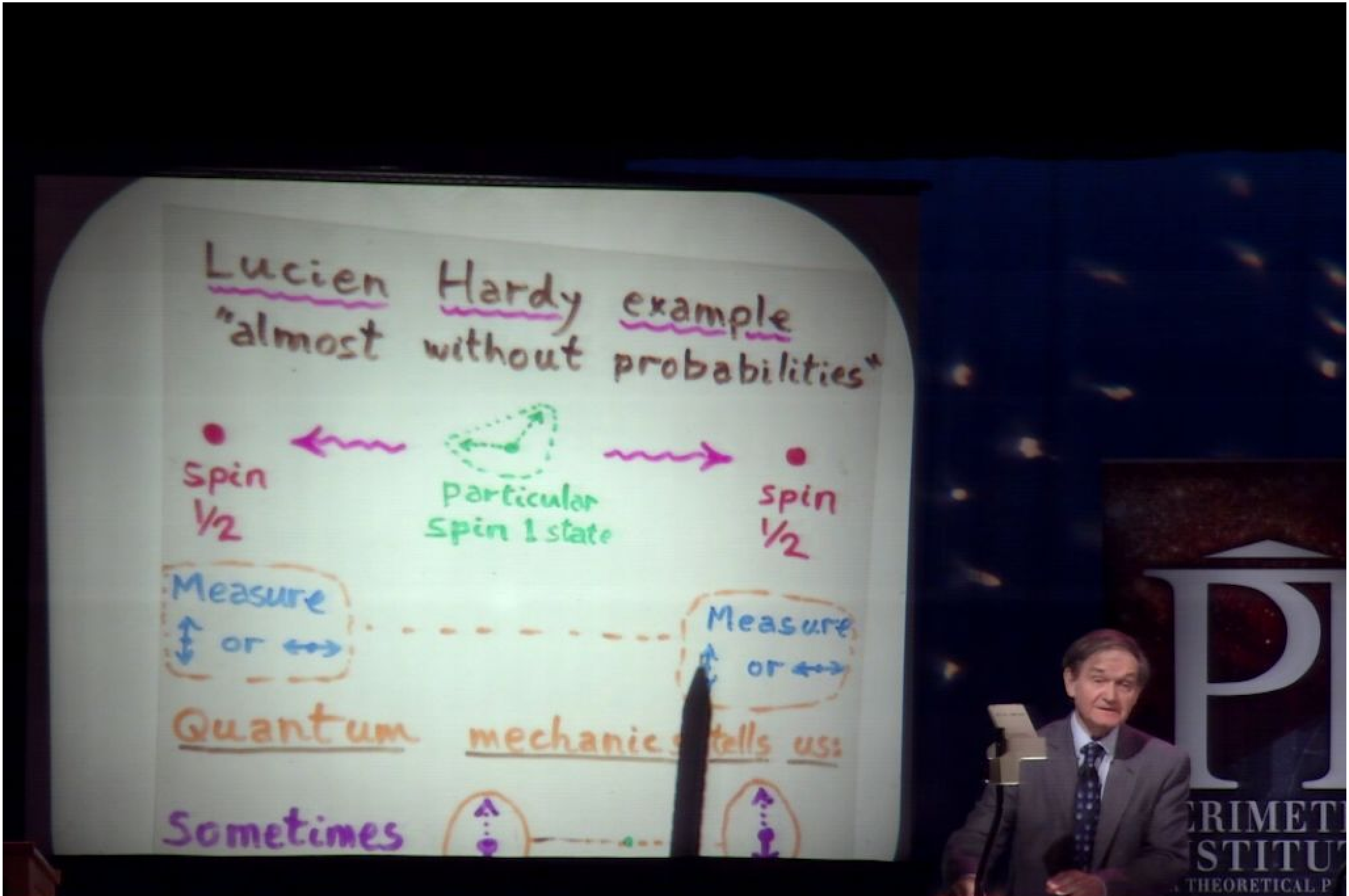

Spin
$\frac{1}{2}$
Spin
0
Spin
$\frac{1}{2}$
measured spins
must be opposite
inequalities

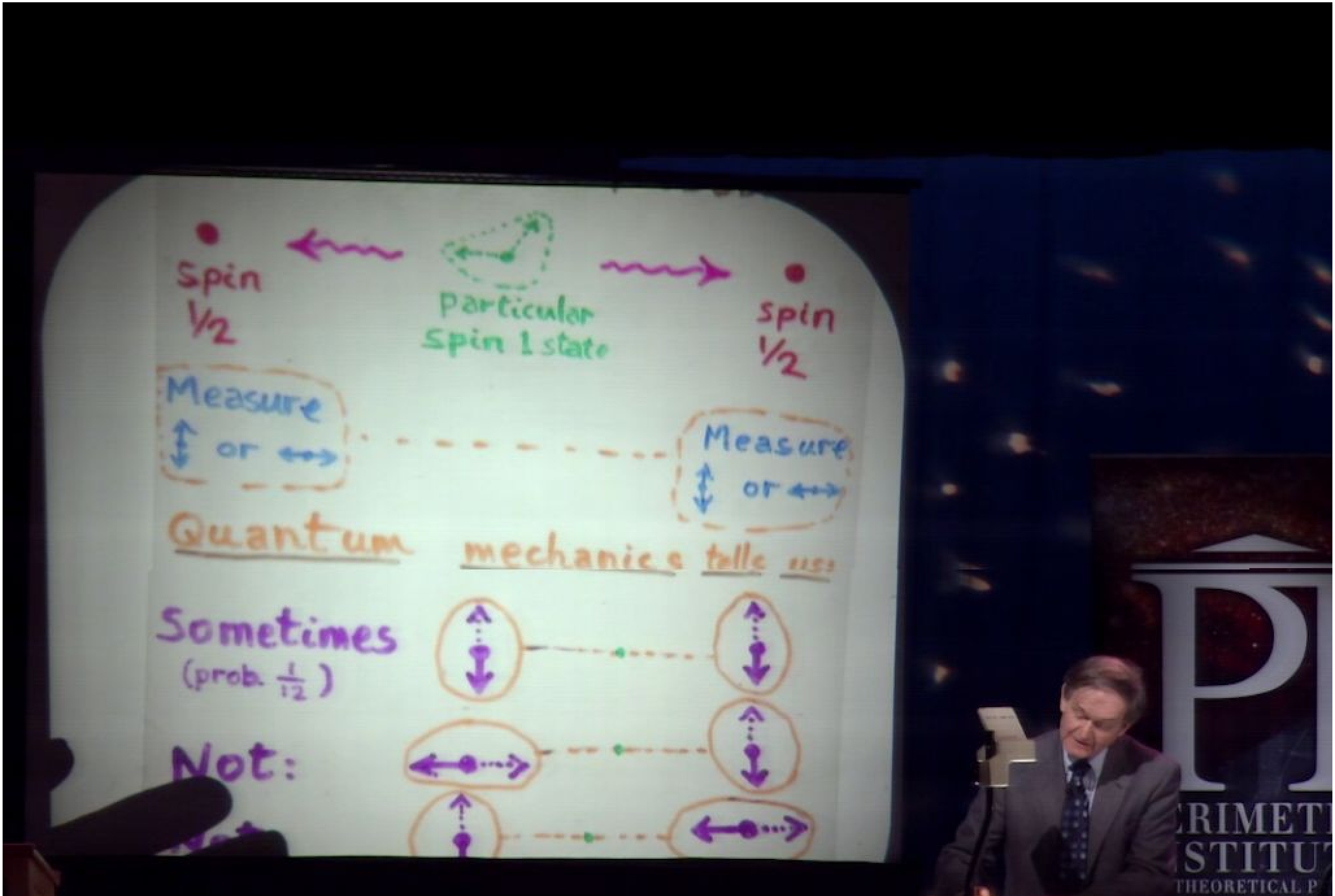


Spin ½
Spin 0
Spin ½
measured spins
must be opposite
Bell inequalities
for "local realism"
photon polarizations
> 100 Km

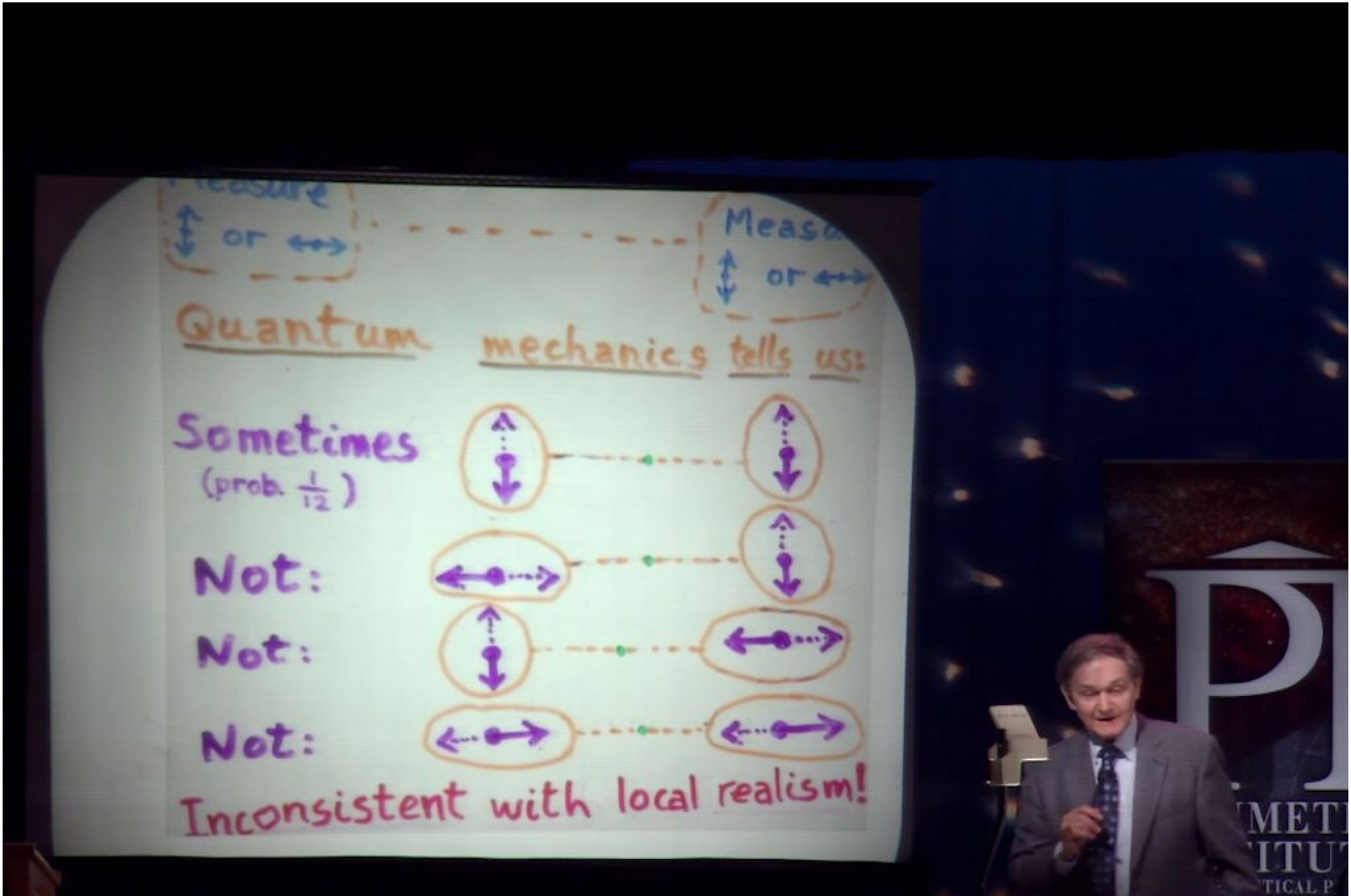

Lucien Hardy example
"almost without probabilities"
spin
1/2
particular
spin 1 state
spin
1/2
Measure
or
Measure
or
Quantum mechanics tells us:
Sometimes
PERIMETER INSTITUTE
THEORETICAL P


E — P — R — Bohm
Einstein
Podolsky
Rosen
spin
1/2
0
spin
1/2
must be

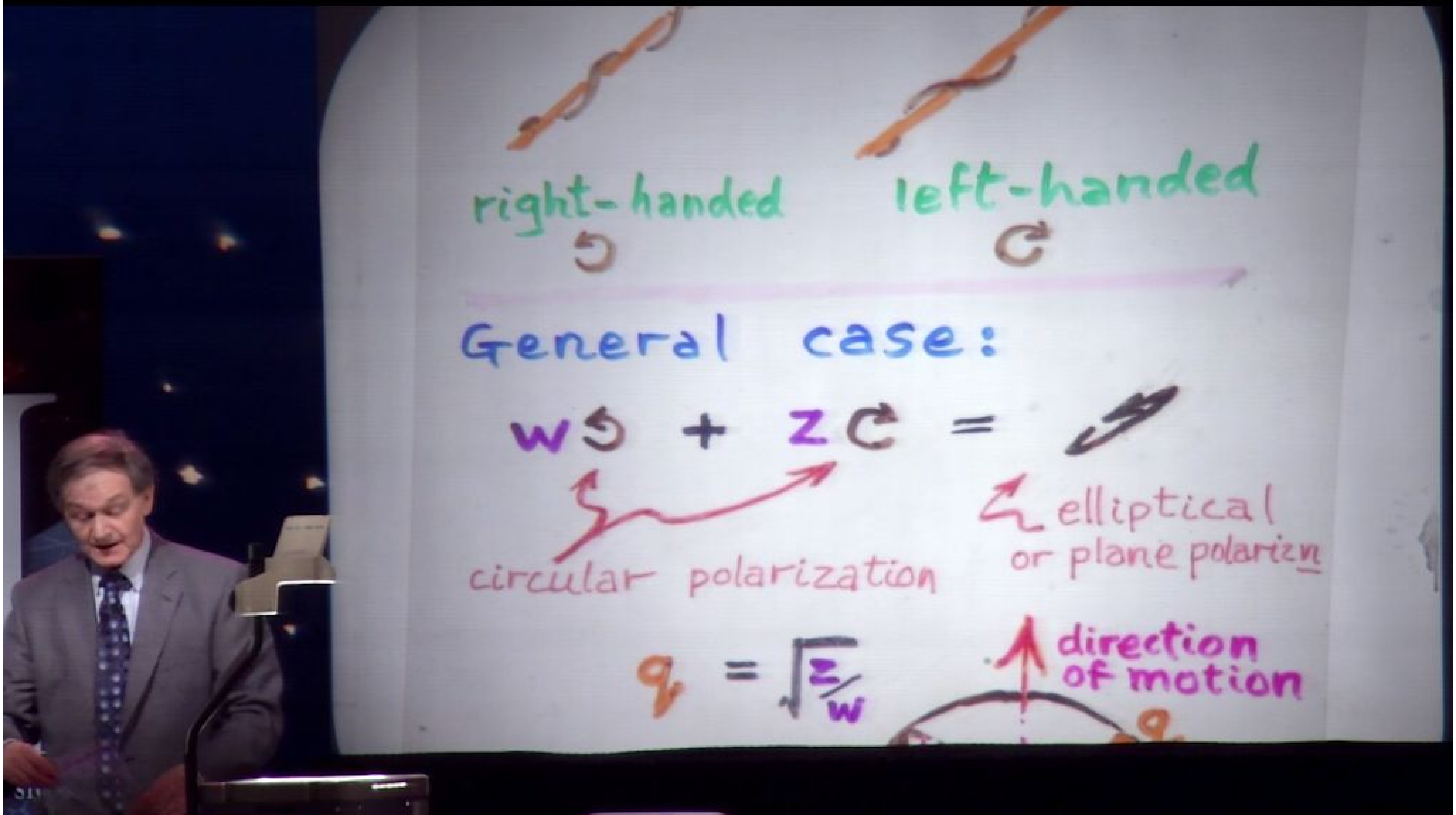

Lucien Hardy example
"almost without probabilities"
spin 1/2
Particular spin 1 state
spin 1/2
Measure
or
Measure
or
Quantum mechanics tells us:
Sometimes

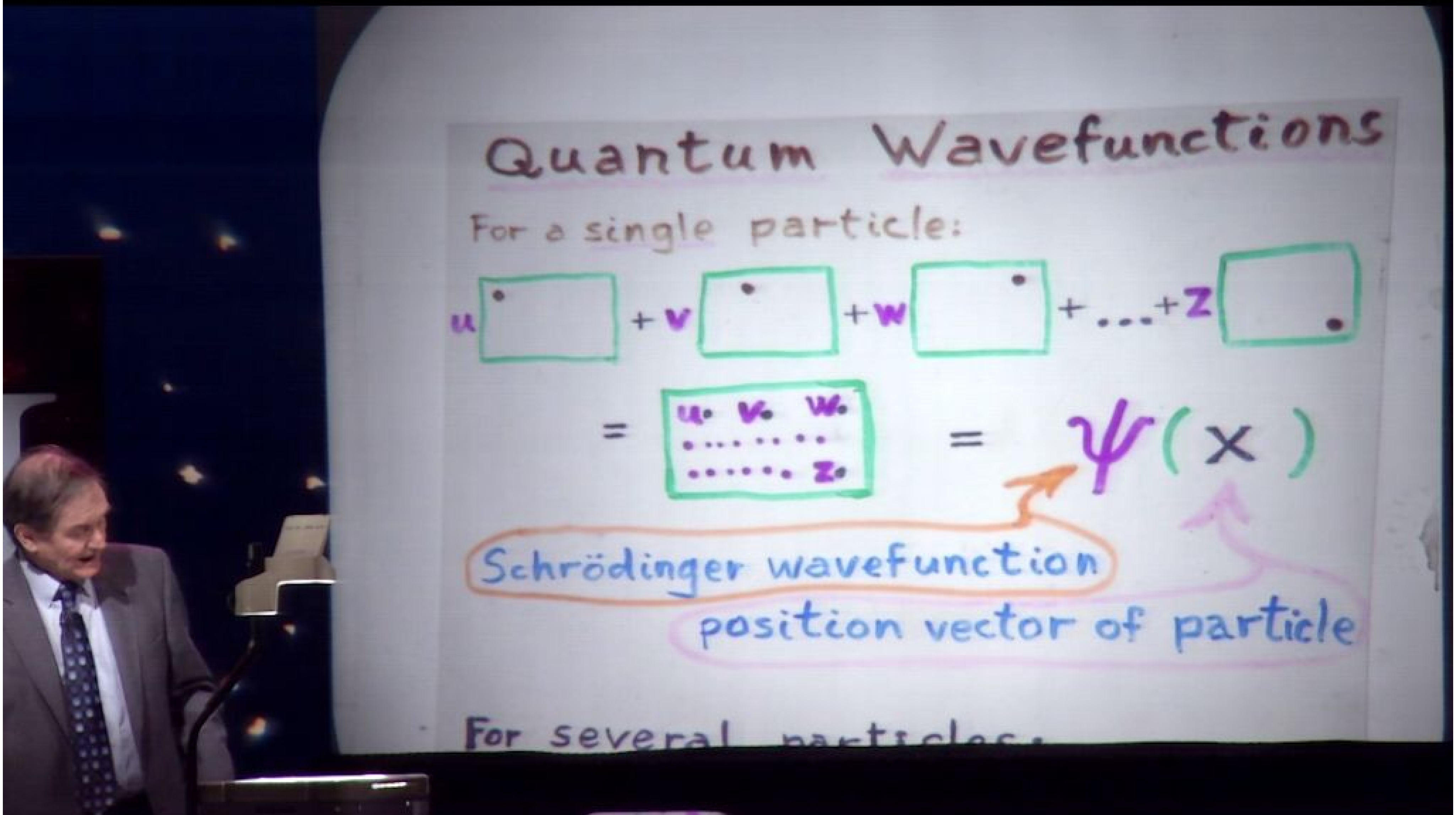


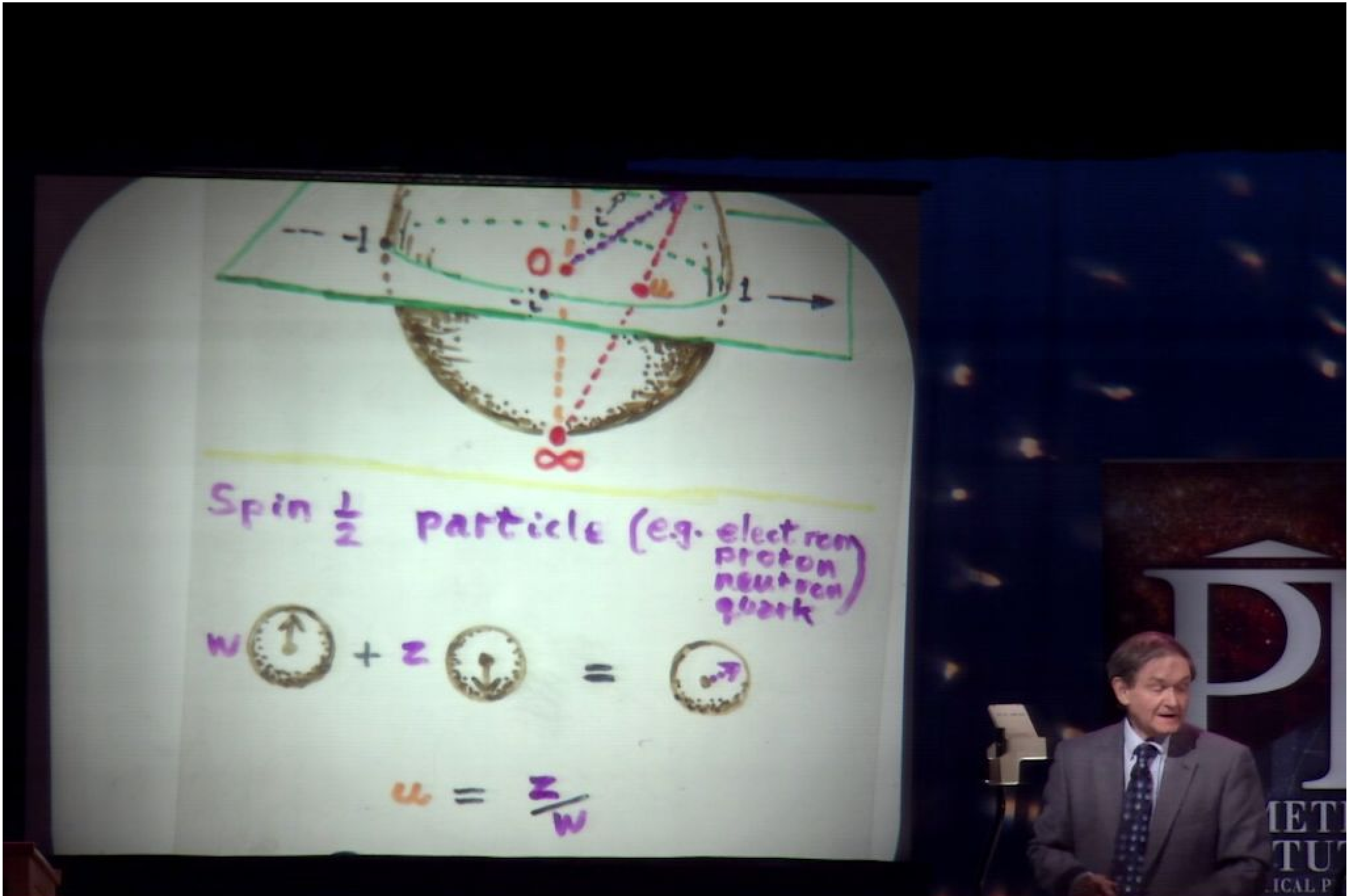
Spin 1/2
Particular
Spin 1 state
spin 1/2
Measure
or
Measure
or
Quantum mechanics tells us:
Sometimes
(prob. 1/12)
Not:

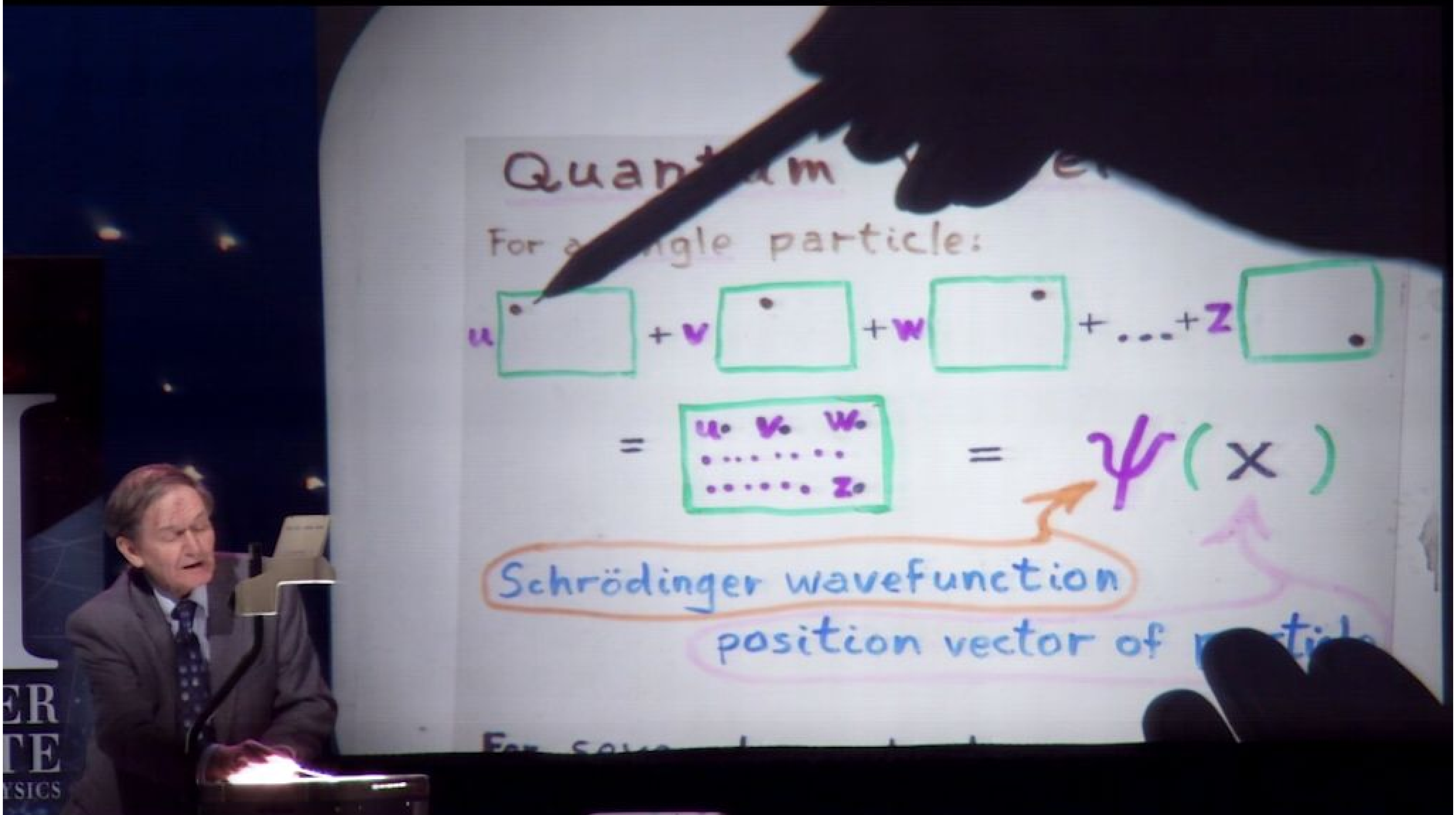


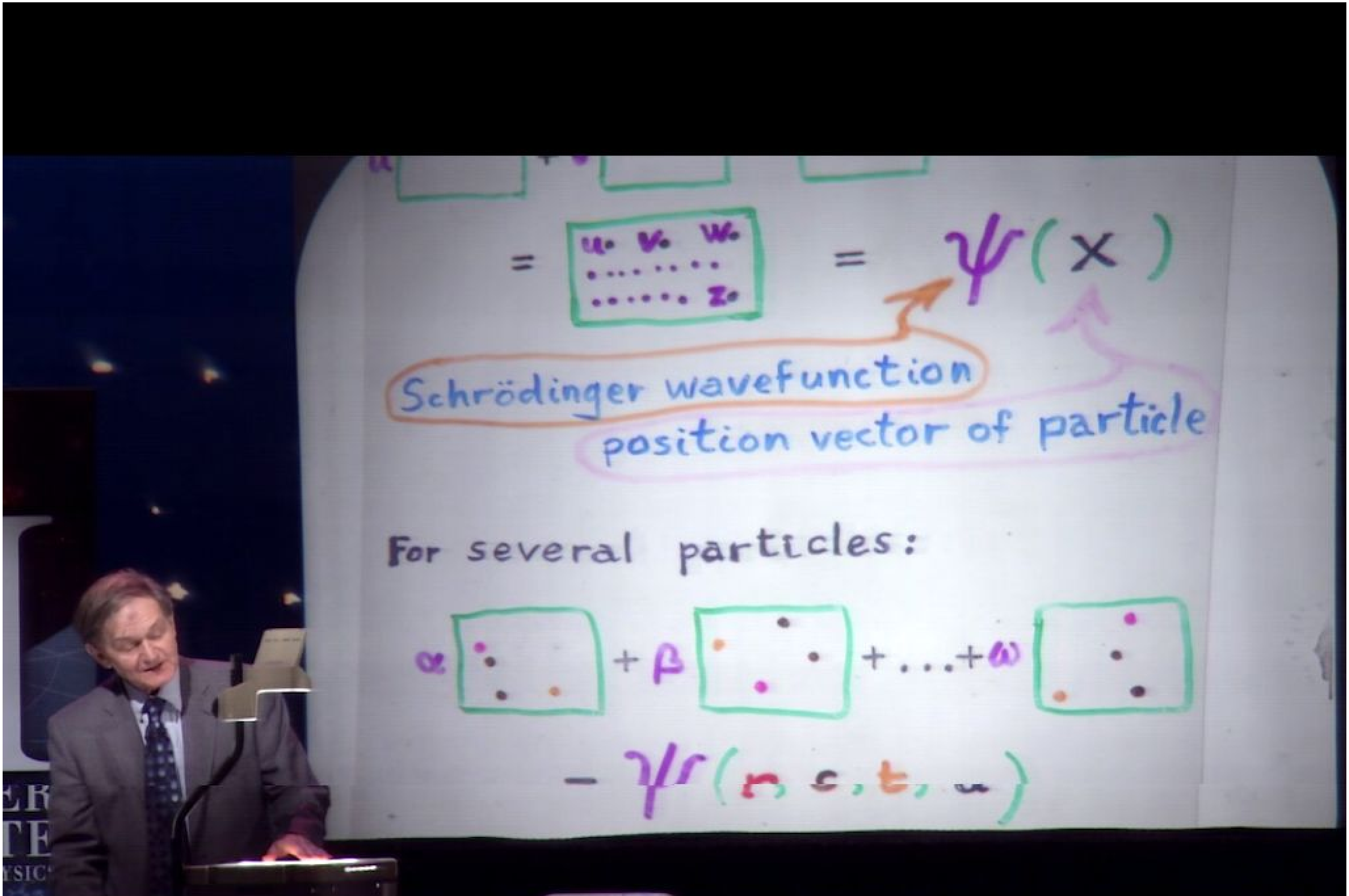
Measure
or
Measure
or
Quantum mechanics tells us:
Sometimes
(prob. $\frac{1}{12}$)
Not:
Not:
Not:
Inconsistent with local realism!


right-handed
left-handed
General case:
w + z =
circular polarization
elliptical
or plane polarizn
q = √(z/w)
direction
of motion

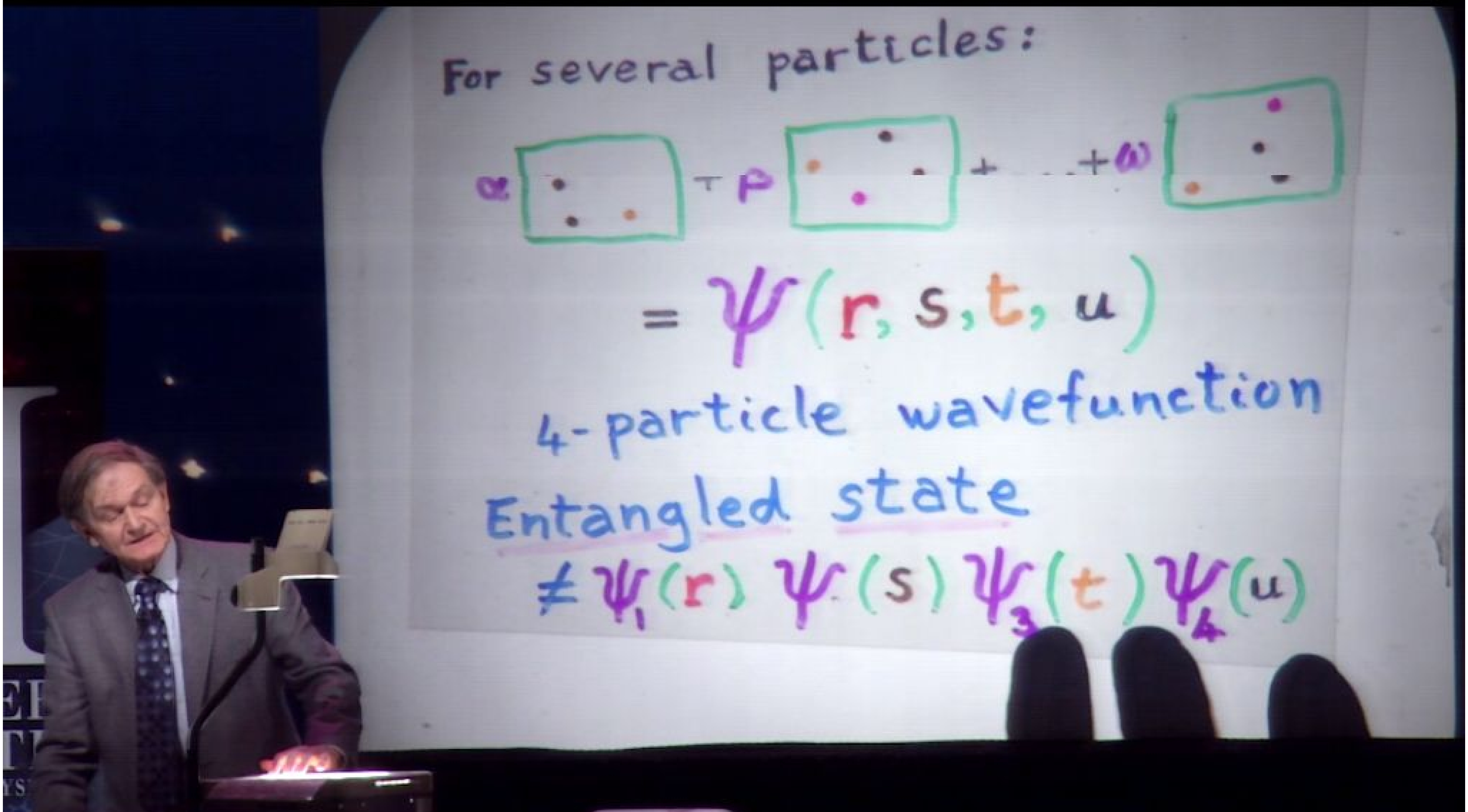

Quantum Wavefunctions
For a single particle:
u
+ v
+ w
+ ... + z
=
u v w
z
=
ψ (x)
Schrödinger wavefunction
position vector of particle
For several particles:

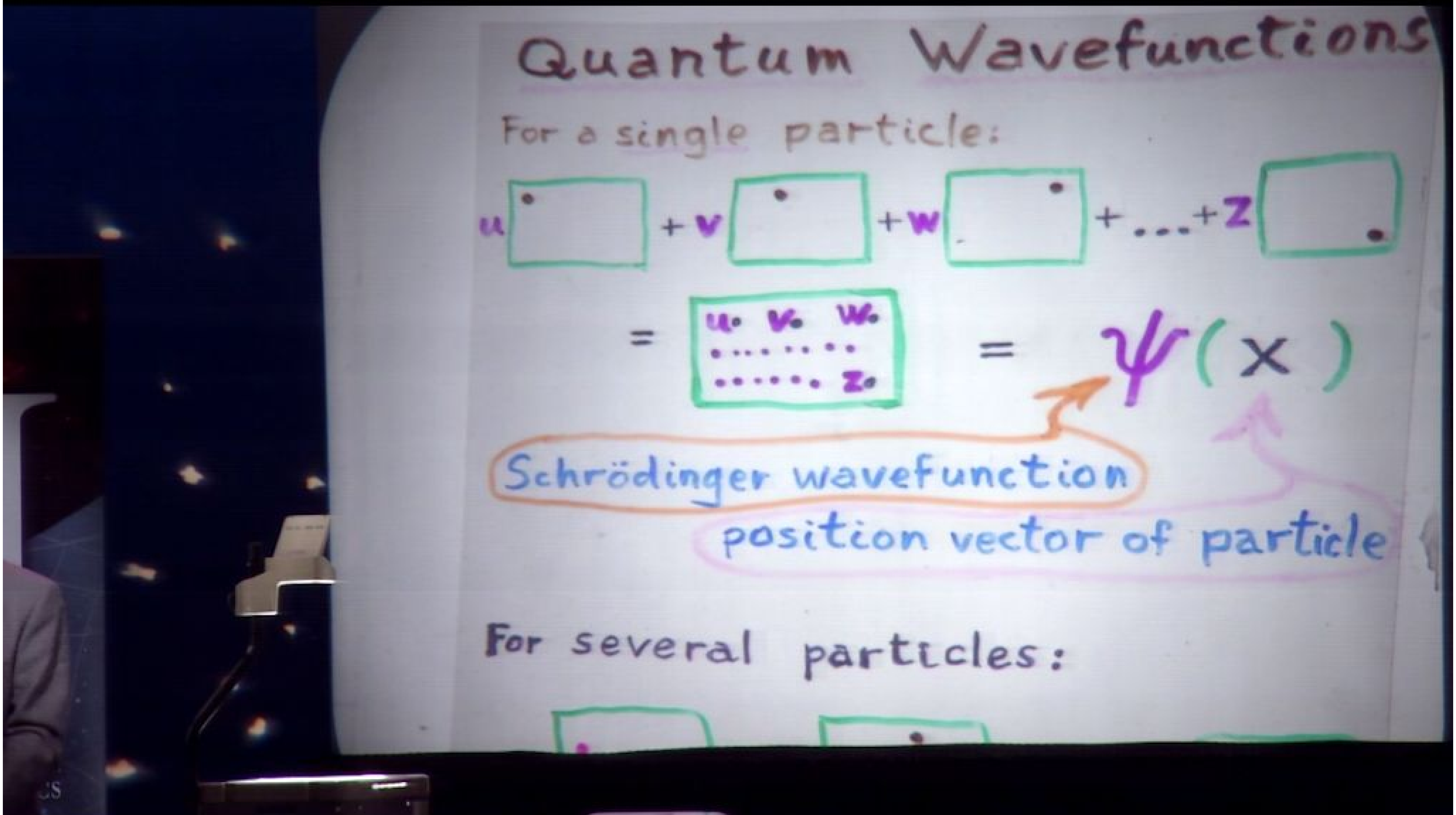

-1
0
i
-i
1
u
∞
Spin 1/2 particle (e.g. electron
proton
neutron
quark)
w + z =
u = z/w

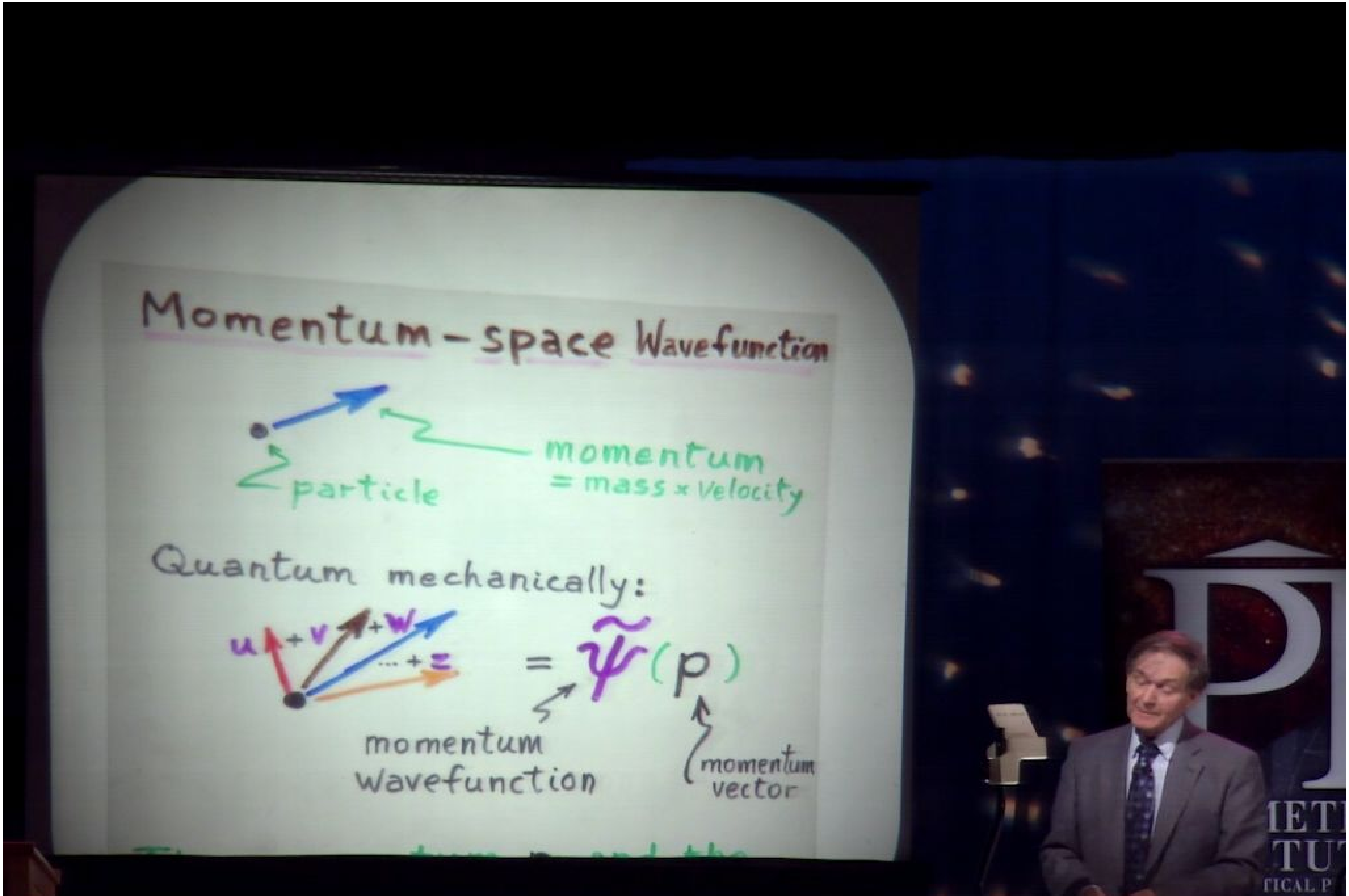

particle:
u + v + w + ... + z
= ψ(x)
Schrödinger wavefunction
position vector of



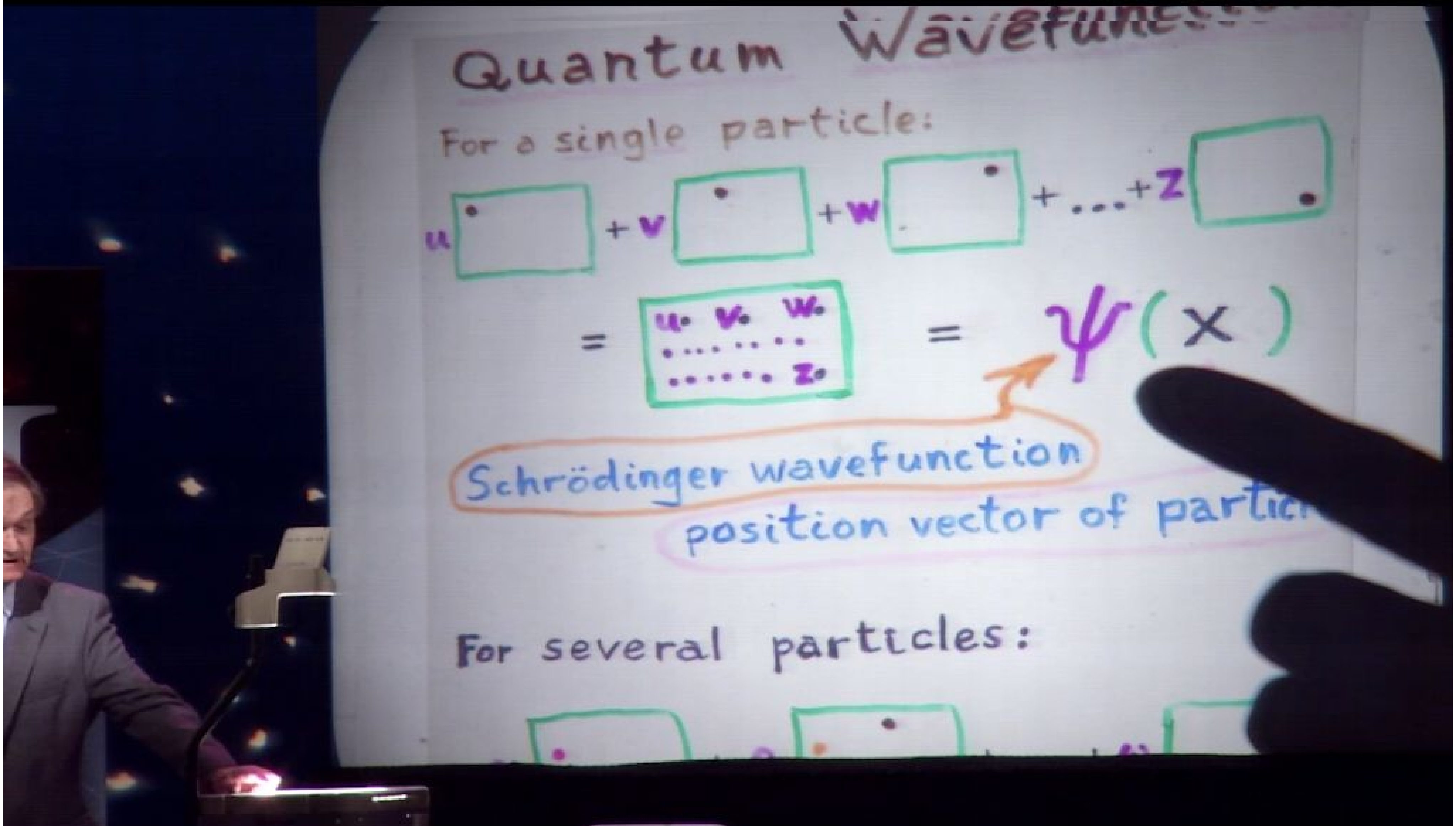
= ψ(x)
Schrödinger wavefunction
position vector of particle
For several particles:
α + β + ... + ω

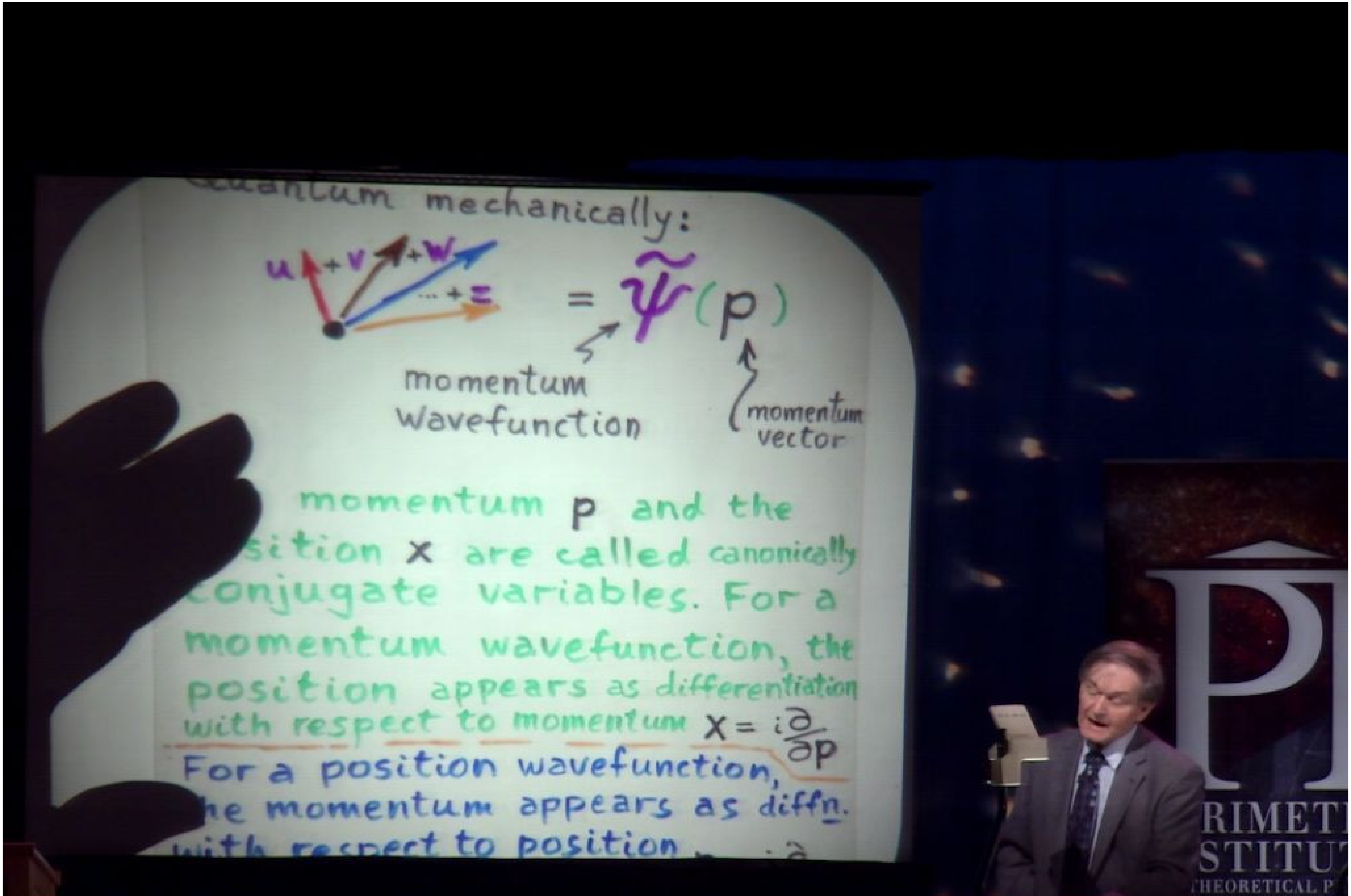

For several particles:
$\alpha \,\boxed{} + \beta \,\boxed{} + \dots + \omega \,\boxed{}$
$= \psi(r, s, t, u)$
4-particle wavefunction
Entangled state
$\neq \psi_1(r)\,\psi(s)\,\psi_3(t)\,\psi_4(u)$

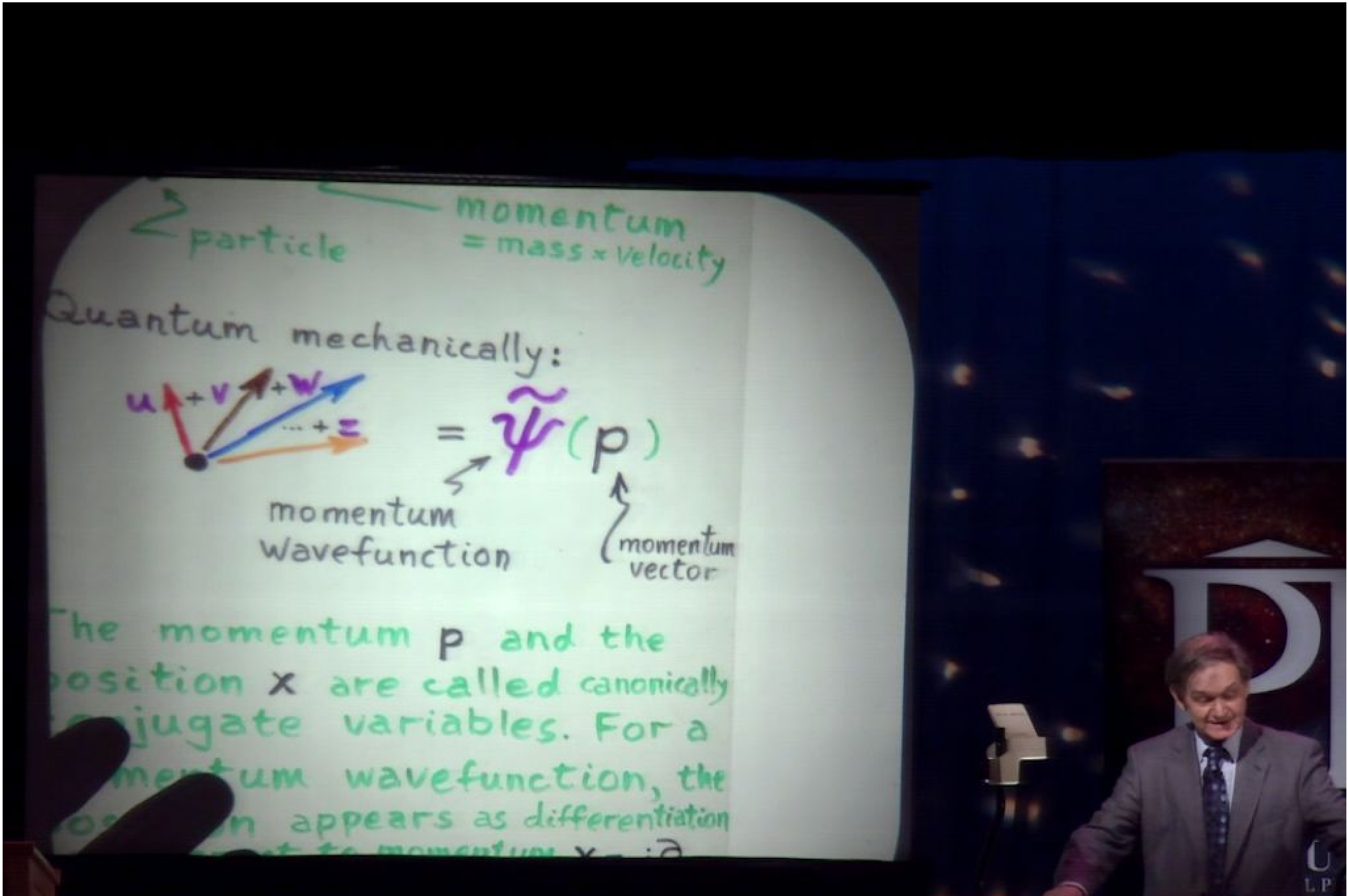

Quantum Wavefunctions
For a single particle:
u + v + w + ... + z
u v w ... z
= = ψ(x)
Schrödinger wavefunction
position vector of particle
For several particles:



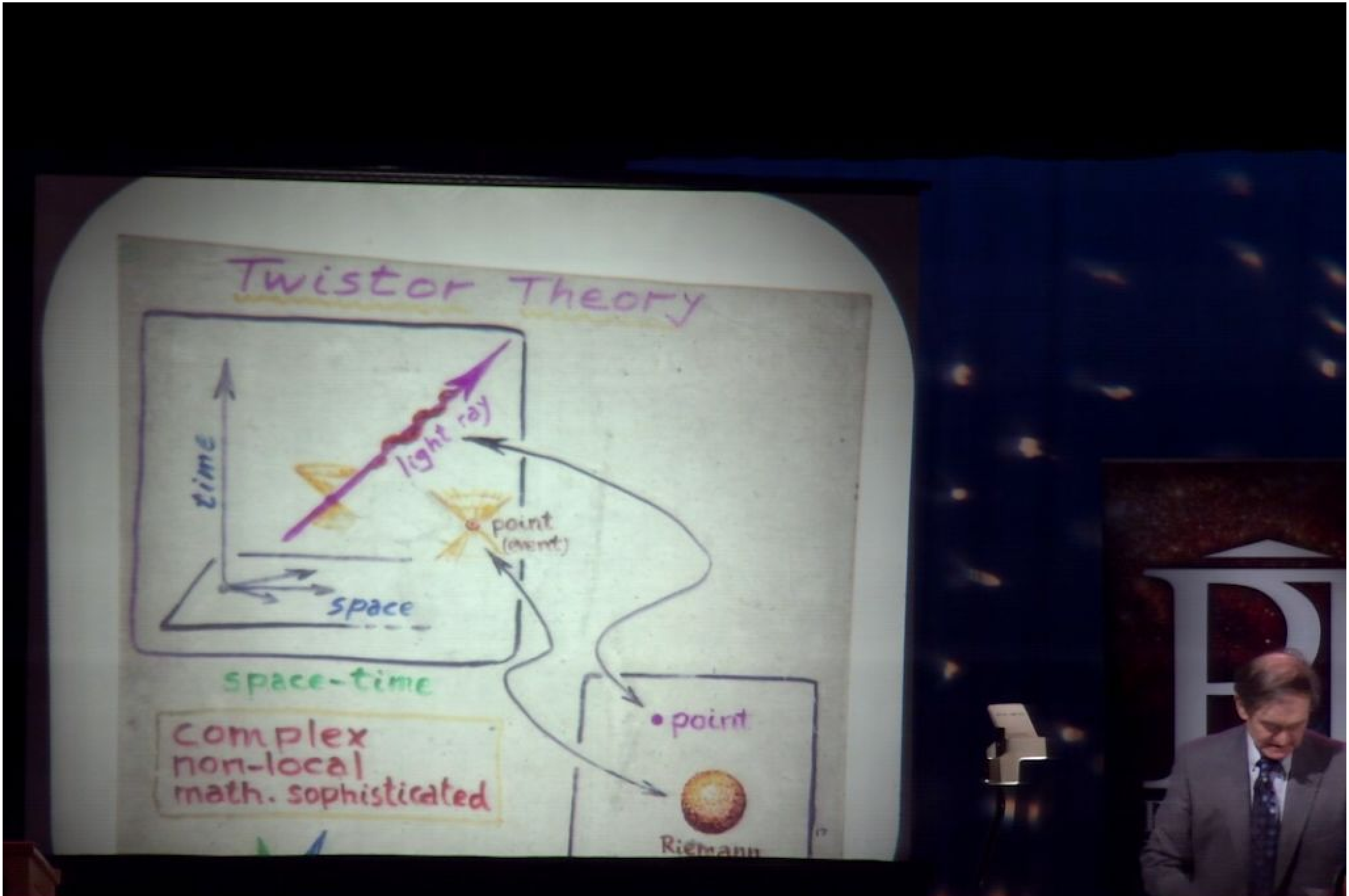
Momentum-space Wavefunction
momentum
= mass × velocity
particle
Quantum mechanically:
u + v + w ... + z
= $\tilde{\Psi}$(p)
momentum
wavefunction
momentum
vector

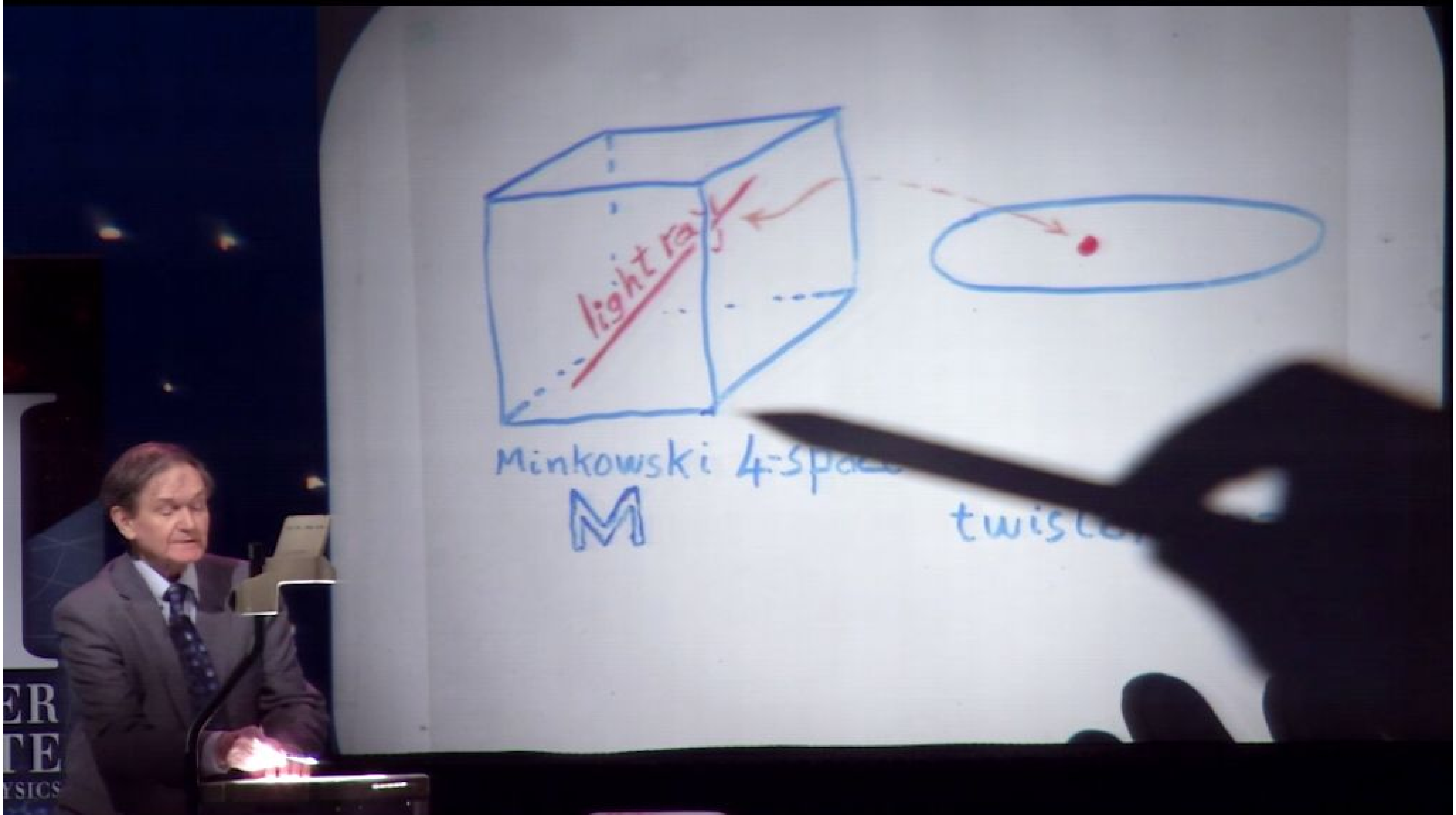

Quantum Wavefunc
For a single particle:
u + v + w + ...+ z
=
u. v. w.
z.
= ψ(x)
Schrödinger wavefunction
position vector of partic
For several particles:

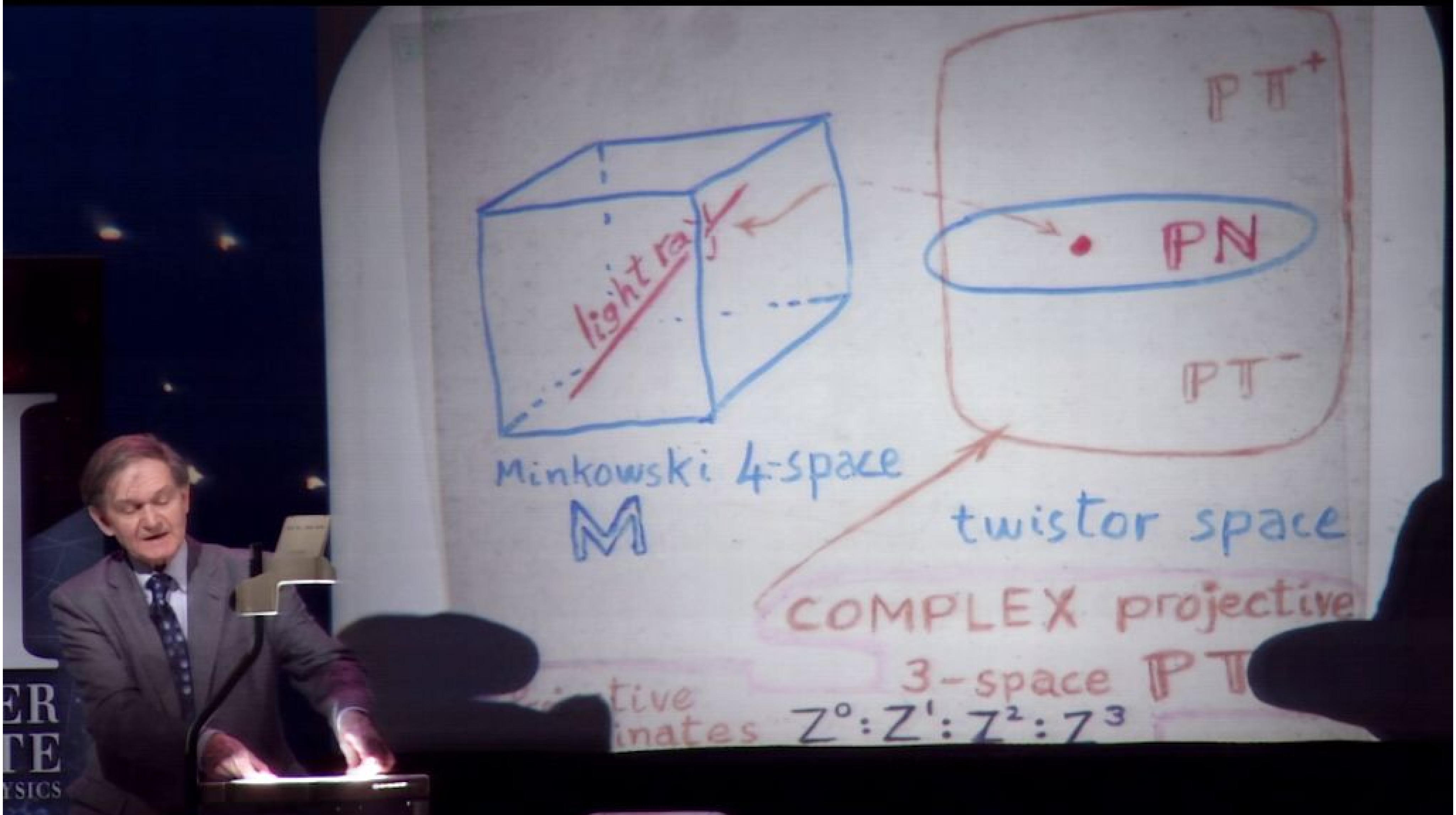

mechanically:
u + v + w ... + z
= $\tilde{\Psi}(p)$
momentum wavefunction
momentum vector
momentum p and the
x are called canonically
variables. For a
momentum wavefunction, the
position appears as differentiation
with respect to momentum $x = i\frac{\partial}{\partial p}$
For a position wavefunction,
momentum appears as diffn.
to position

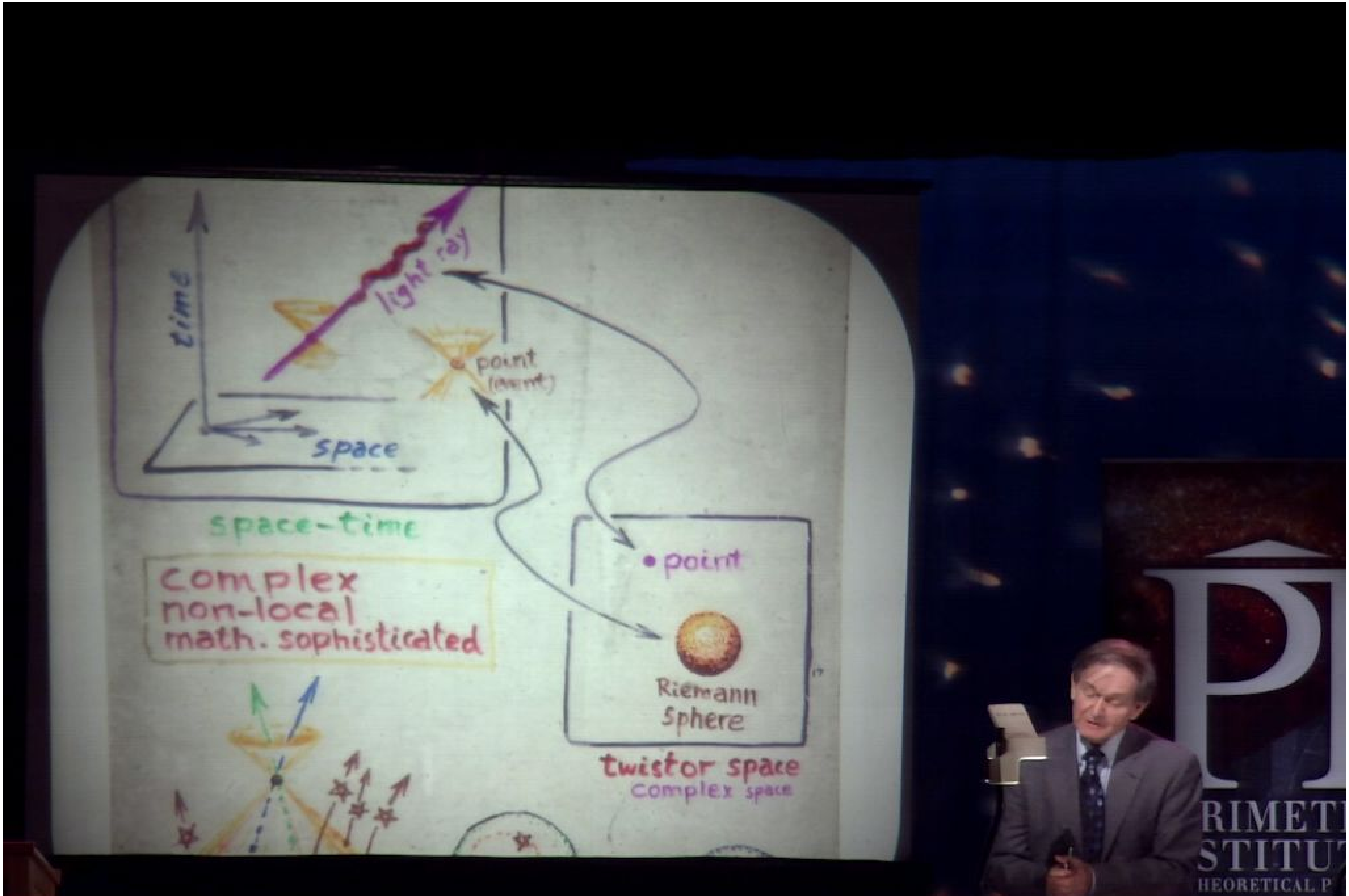


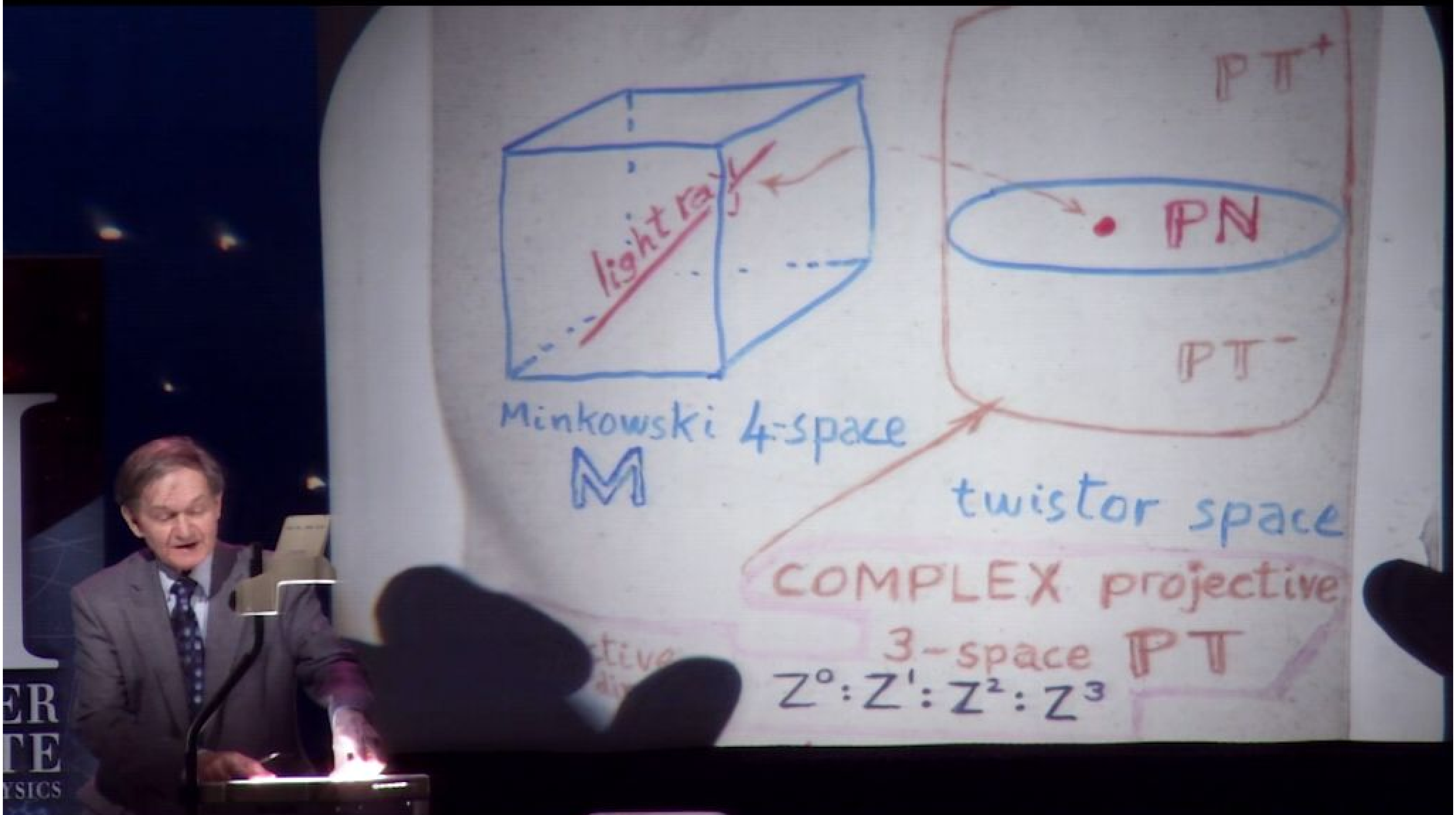
particle
momentum
= mass × velocity
Quantum mechanically:
u + v + w ... + z
$= \tilde{\Psi}(p)$
momentum wavefunction
momentum vector
he momentum p and the
osition x are called canonically
njugate variables. For a
tum wavefunction, the
n appears as differentiation

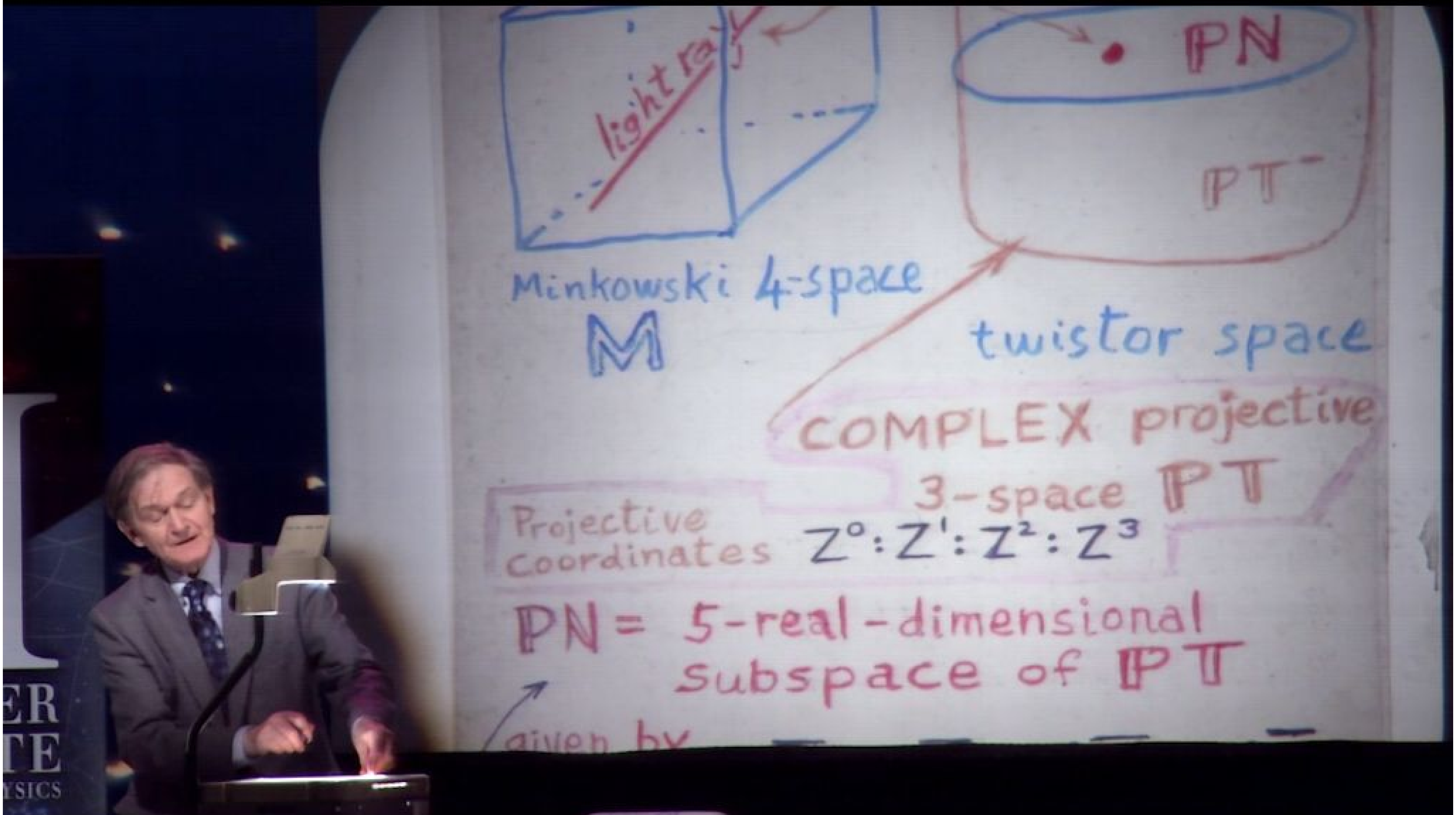

Twistor Theory
time
light ray
point
(event)
space
space-time
complex
non-local
math. sophisticated
• point
Riemann


light ray
Minkowski 4-spa
M
twist

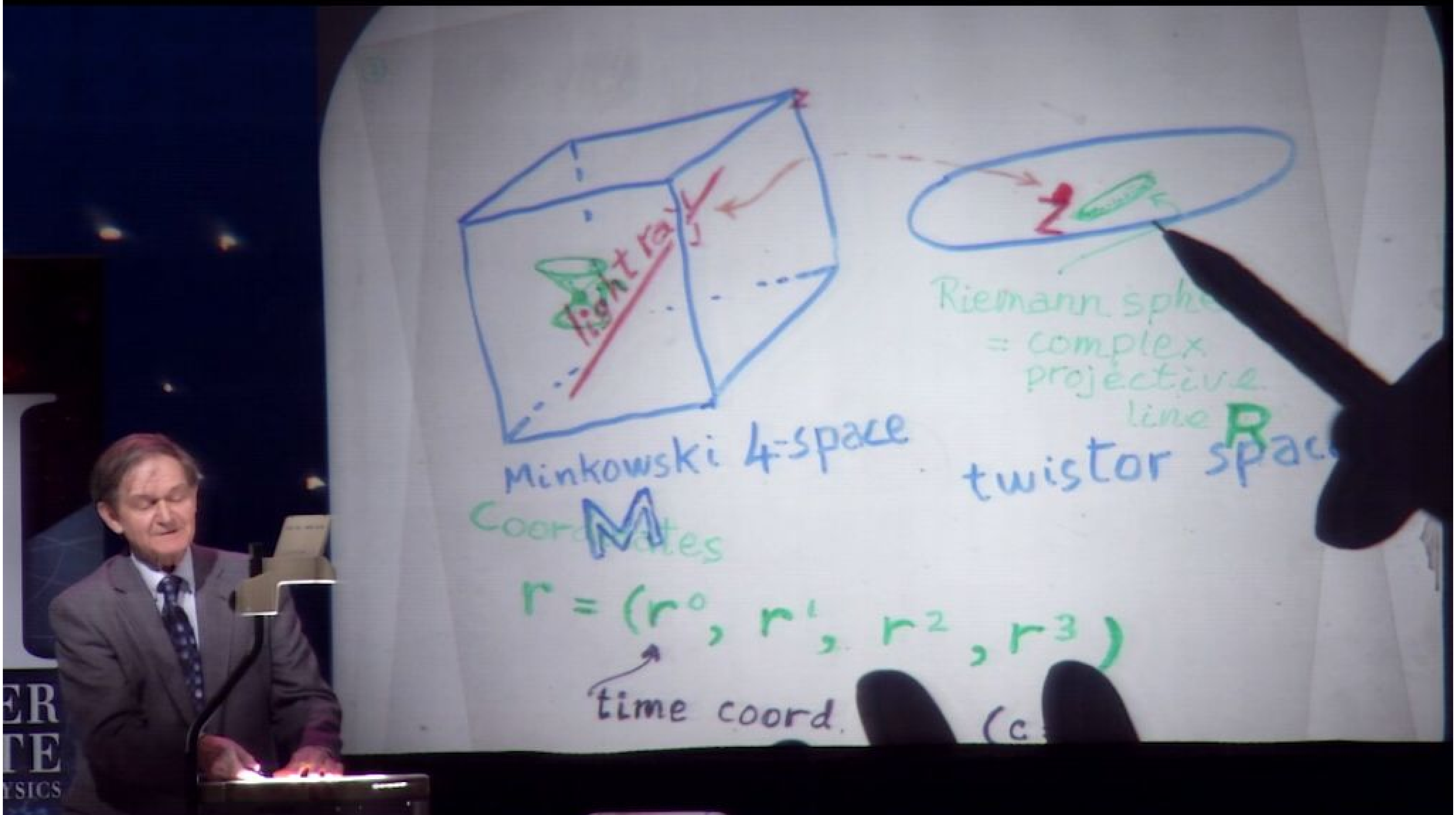

PT+
light ray
PN
PT−
Minkowski 4-space
M
twistor space
COMPLEX projective
3-space PT
tive
inates Z⁰:Z¹:Z²:Z³


time
light ray
space
point
(event)
space-time
complex
non-local
math. sophisticated
• point
Riemann
Sphere
twistor space
complex space

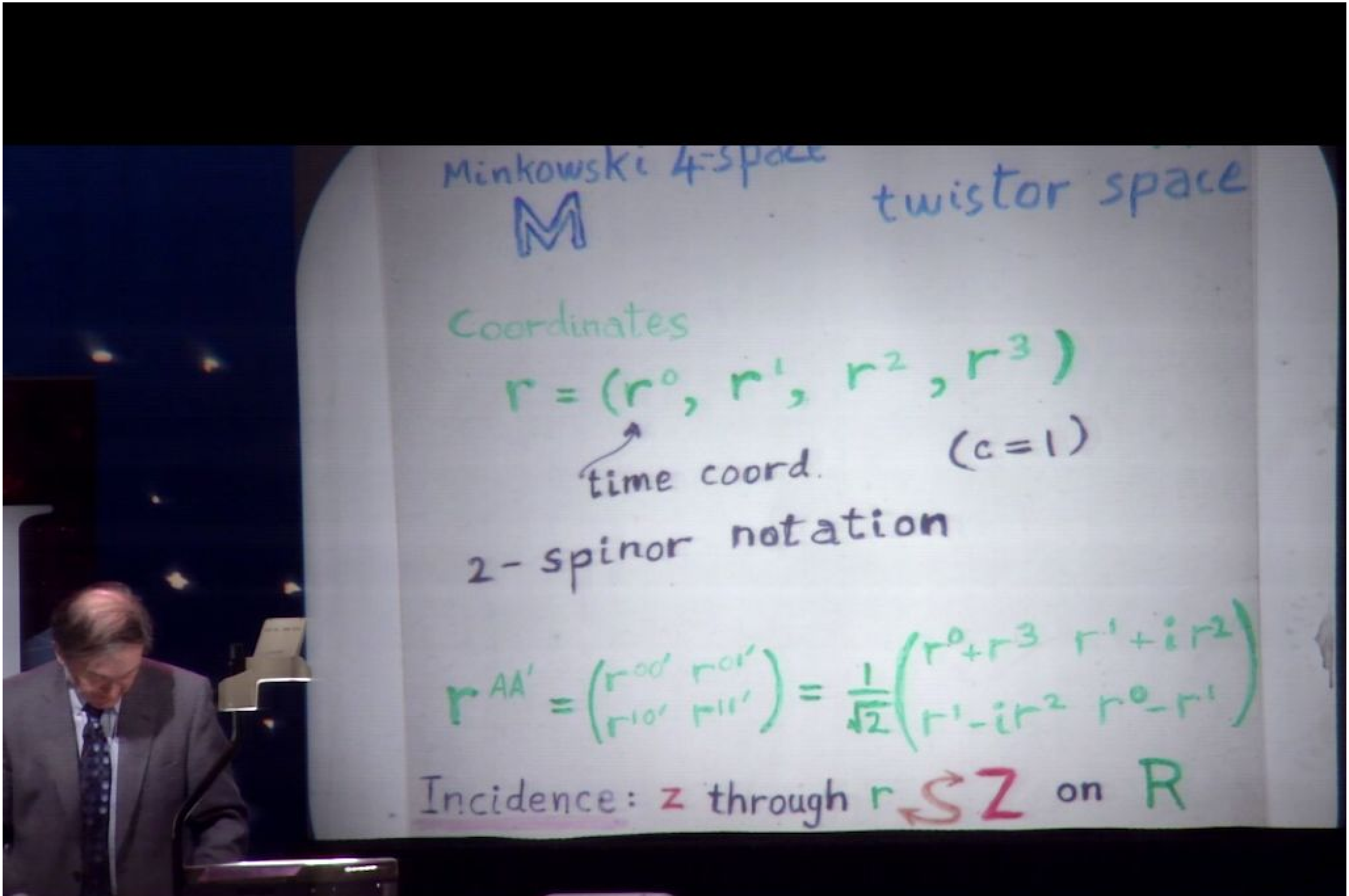

light ray
$\mathbb{P}\mathbb{T}^{+}$
$\mathbb{P}\mathbb{N}$
$\mathbb{P}\mathbb{T}^{-}$
Minkowski 4-space
$\mathbb{M}$
twistor space
COMPLEX projective
3-space $\mathbb{P}\mathbb{T}$
$Z^0 : Z^1 : Z^2 : Z^3$

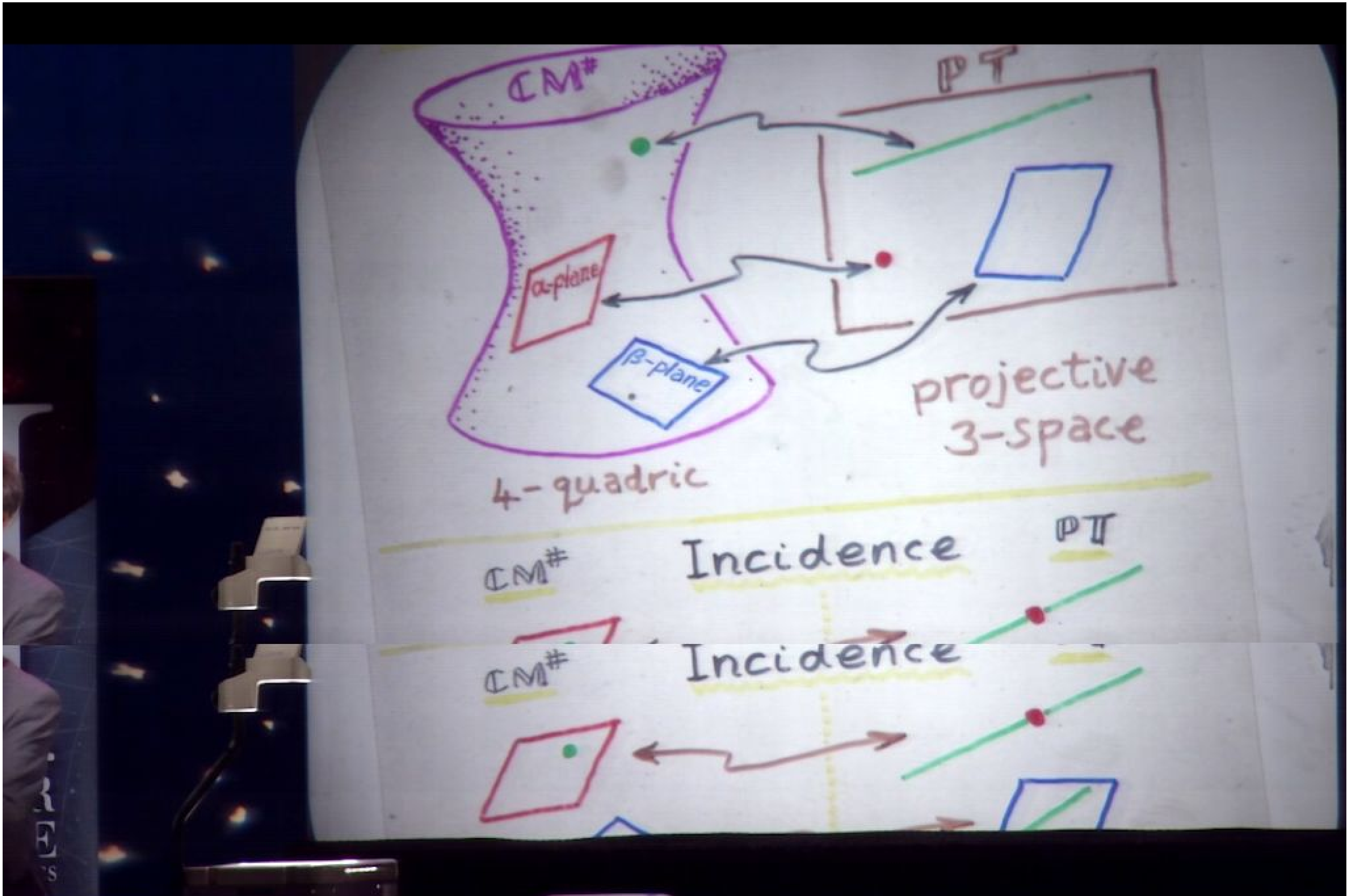


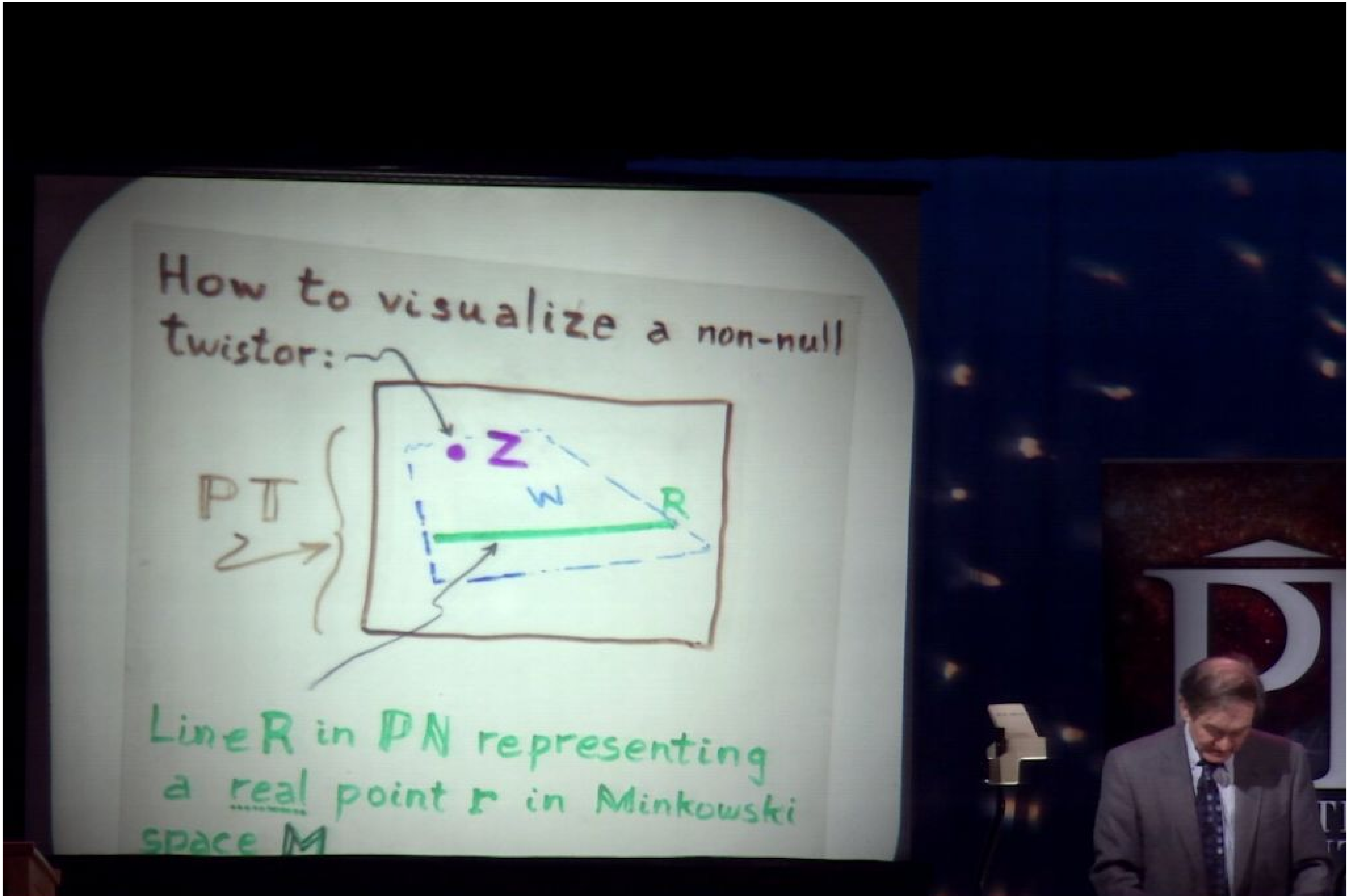
light ray
PN
PT⁻
Minkowski 4-space
M
twistor space
COMPLEX projective
3-space PT
Projective coordinates Z⁰:Z¹:Z²:Z³
PN = 5-real-dimensional
subspace of PT

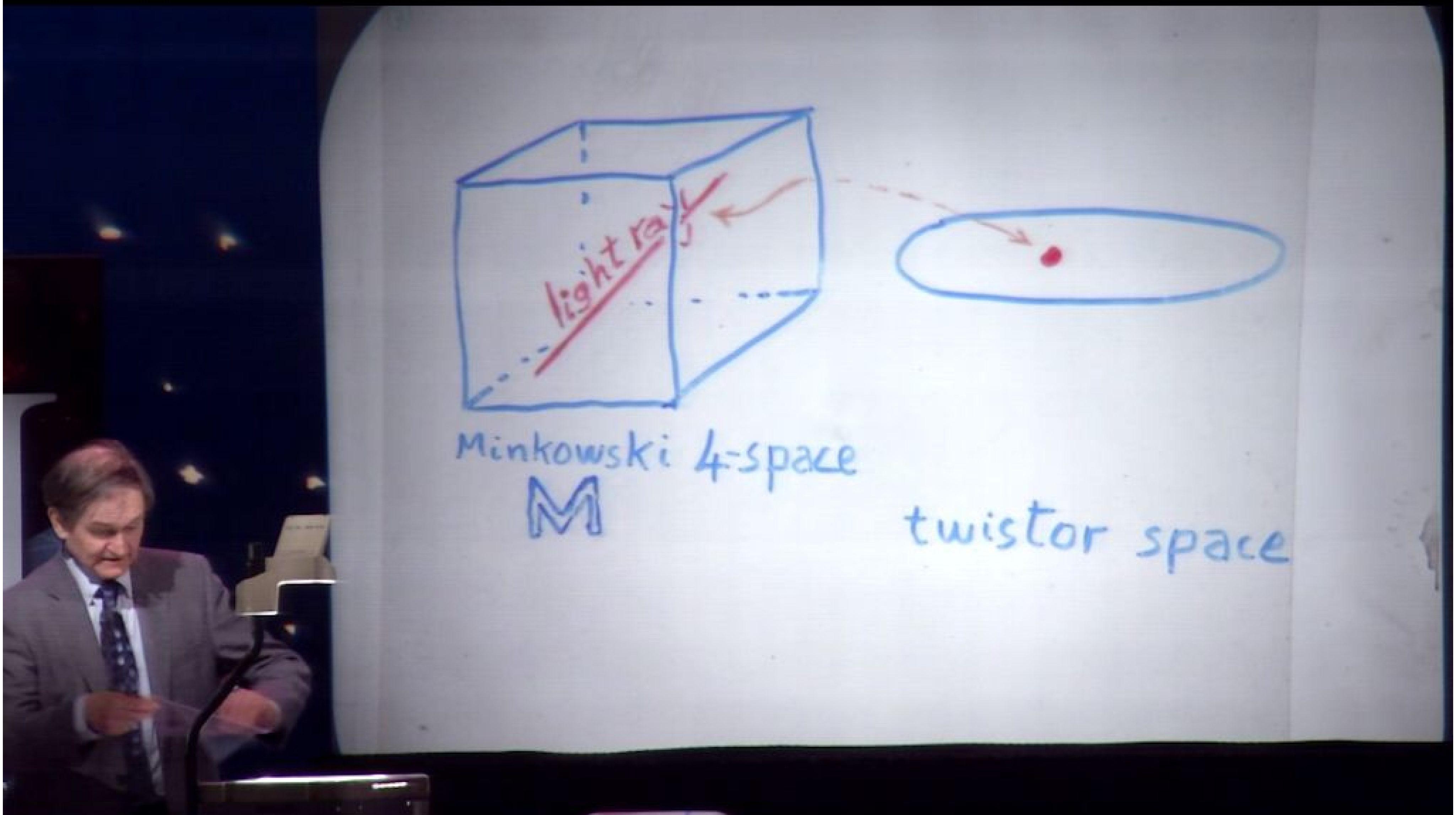


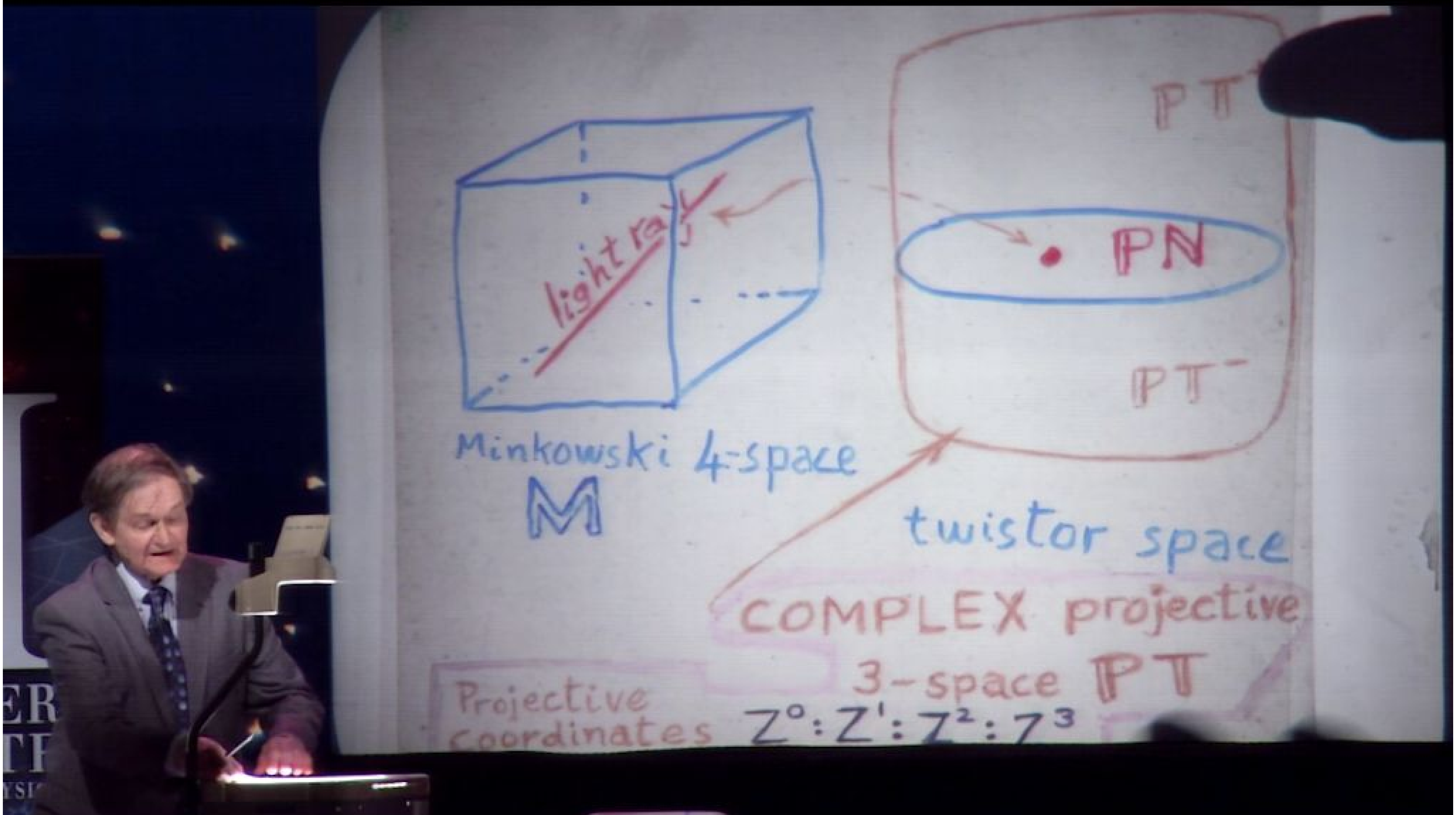
z
light ray
Z
Riemann sphe
= complex
projective
line R
Minkowski 4-space
M
twistor spac
Coordinates
r = (r⁰, r¹, r², r³)
time coord
(c

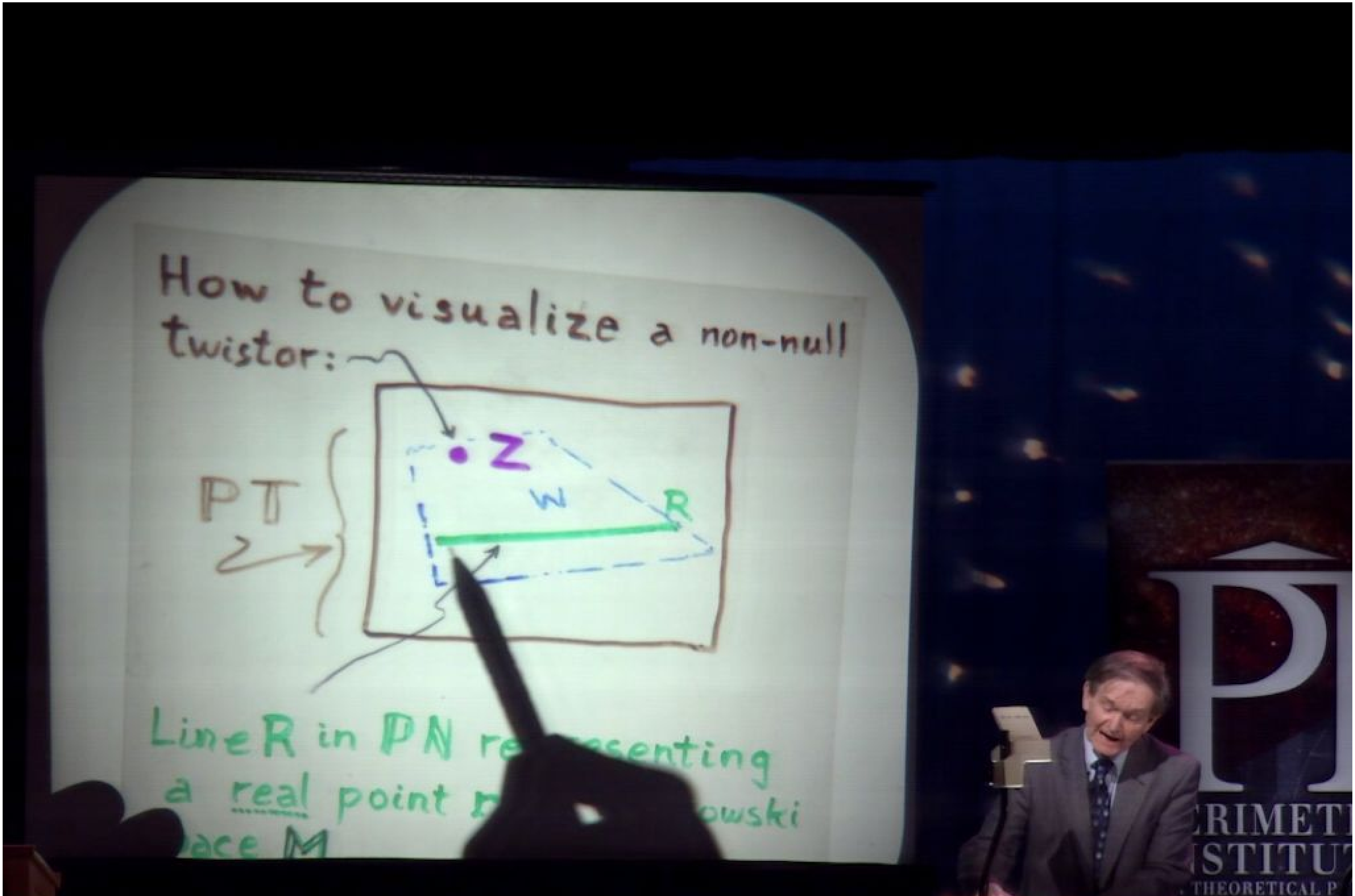

Minkowski 4-space
M
twistor space
Coordinates
r = (r⁰, r¹, r², r³)
time coord.
(c = 1)
2-spinor notation
$r^{AA'} = \begin{pmatrix} r^{00'} & r^{01'} \\ r^{10'} & r^{11'} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} r^0+r^3 & r^1+ir^2 \\ r^1-ir^2 & r^0-r^1 \end{pmatrix}$
Incidence: z through r ⇄ Z on R

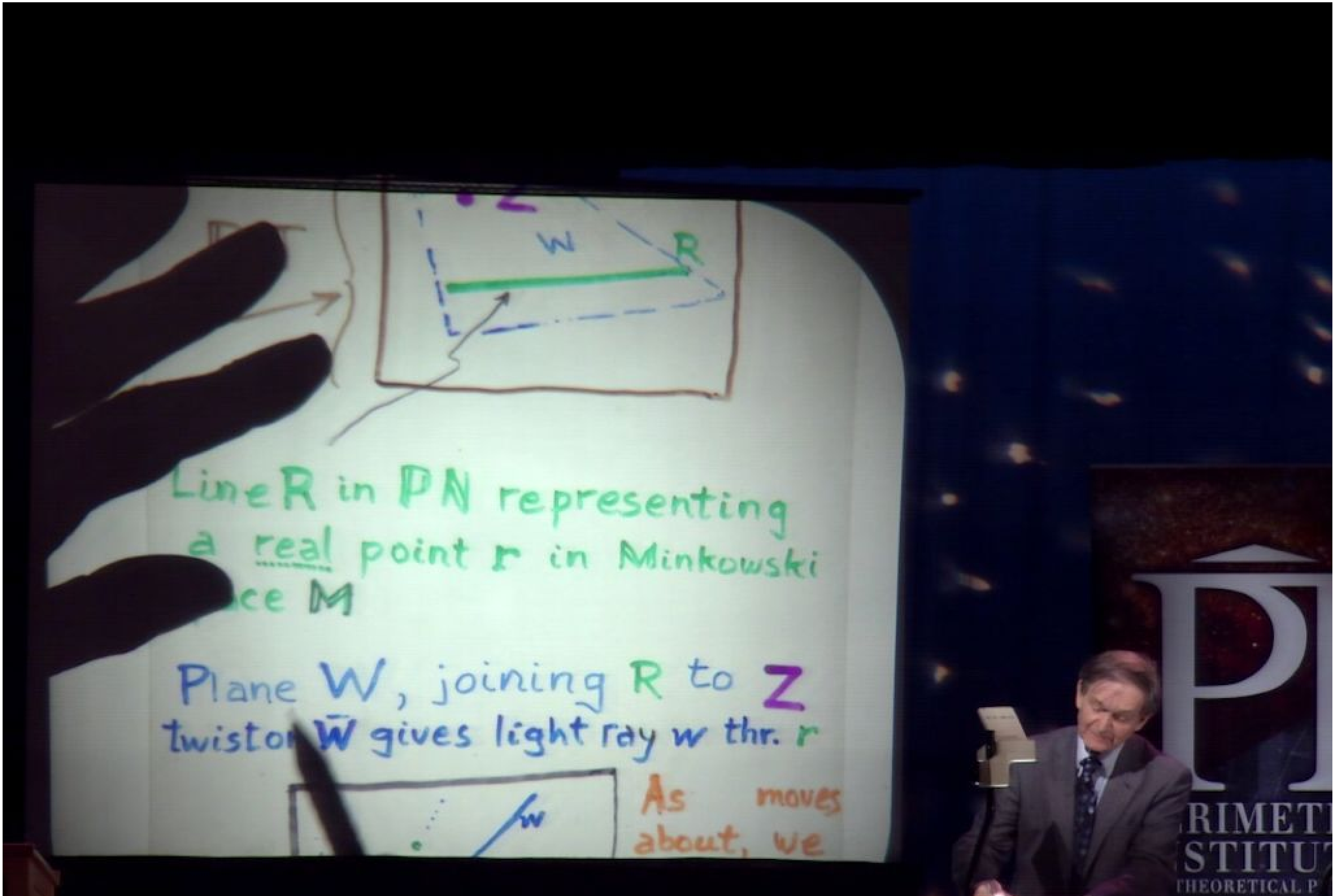

$\mathbb{CM}^\#$
$\mathbb{PT}$
α-plane
β-plane
projective
3-space
4-quadric
$\mathbb{CM}^\#$
Incidence
$\mathbb{PT}$

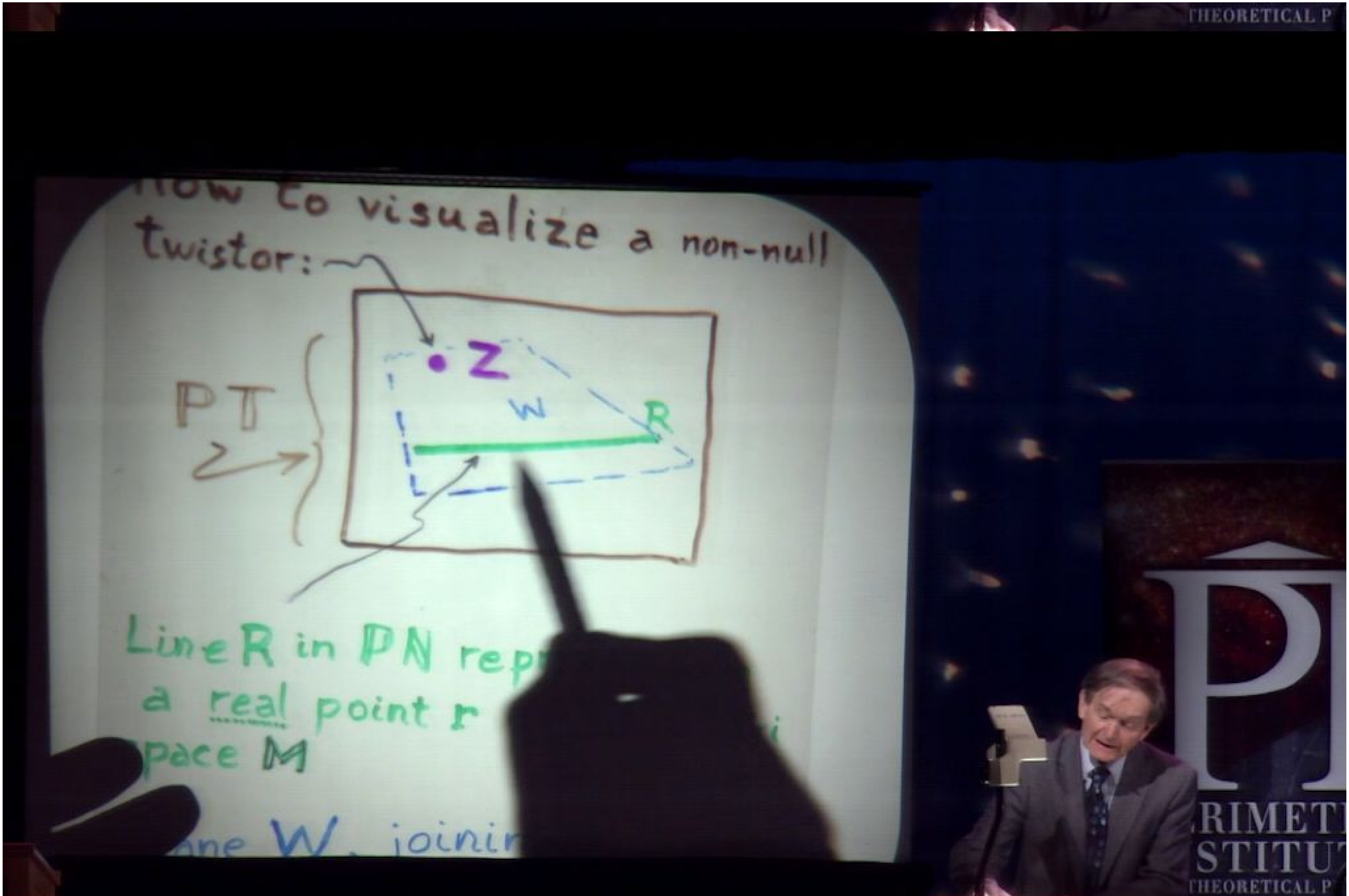

How to visualize a non-null twistor:
Z
PT
W
R
Line R in PN representing a real point r in Minkowski space M

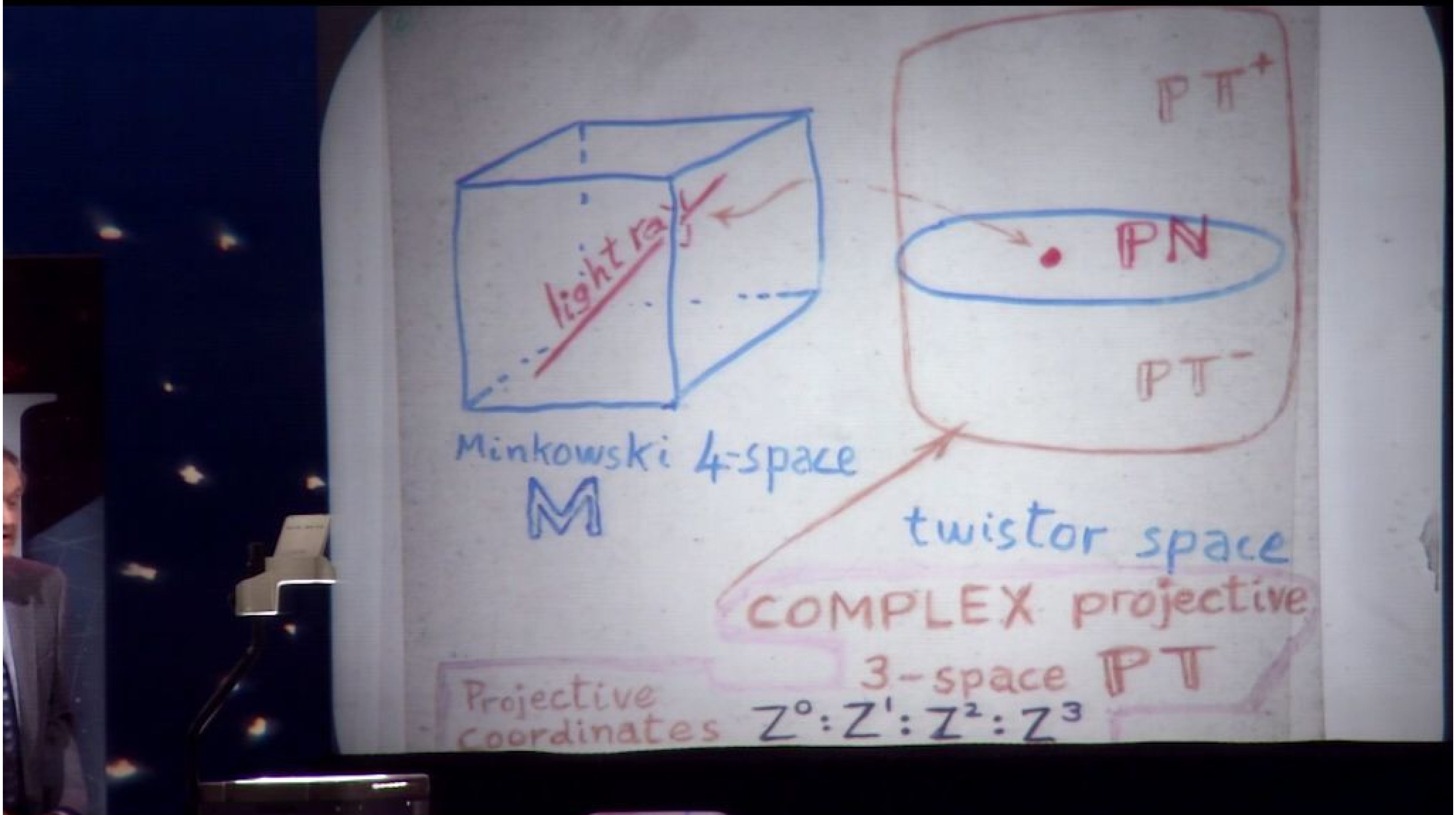

light ray
Minkowski 4-space
M
twistor space


light ray
PT⁺
PN
PT⁻
Minkowski 4-space
M
twistor space
COMPLEX projective
3-space PT
Projective
coordinates Z⁰:Z¹:Z²:Z³

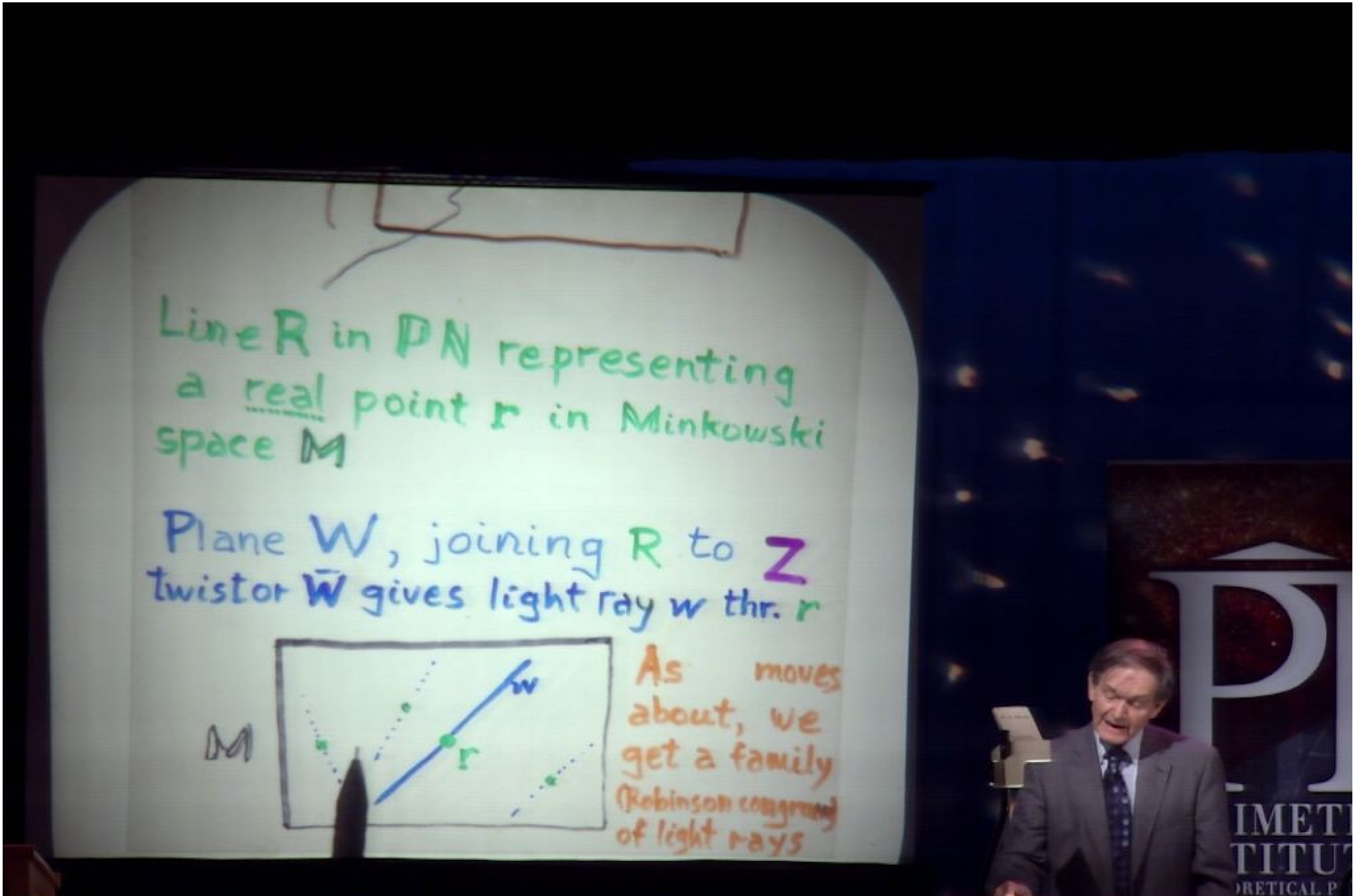

How to visualize a non-null twistor:
Z
PT
W
R
L
Line R in PN re[...]senting
a real point [...]owski
[...]ace M
ERIMET
STITUT
THEORETICAL P

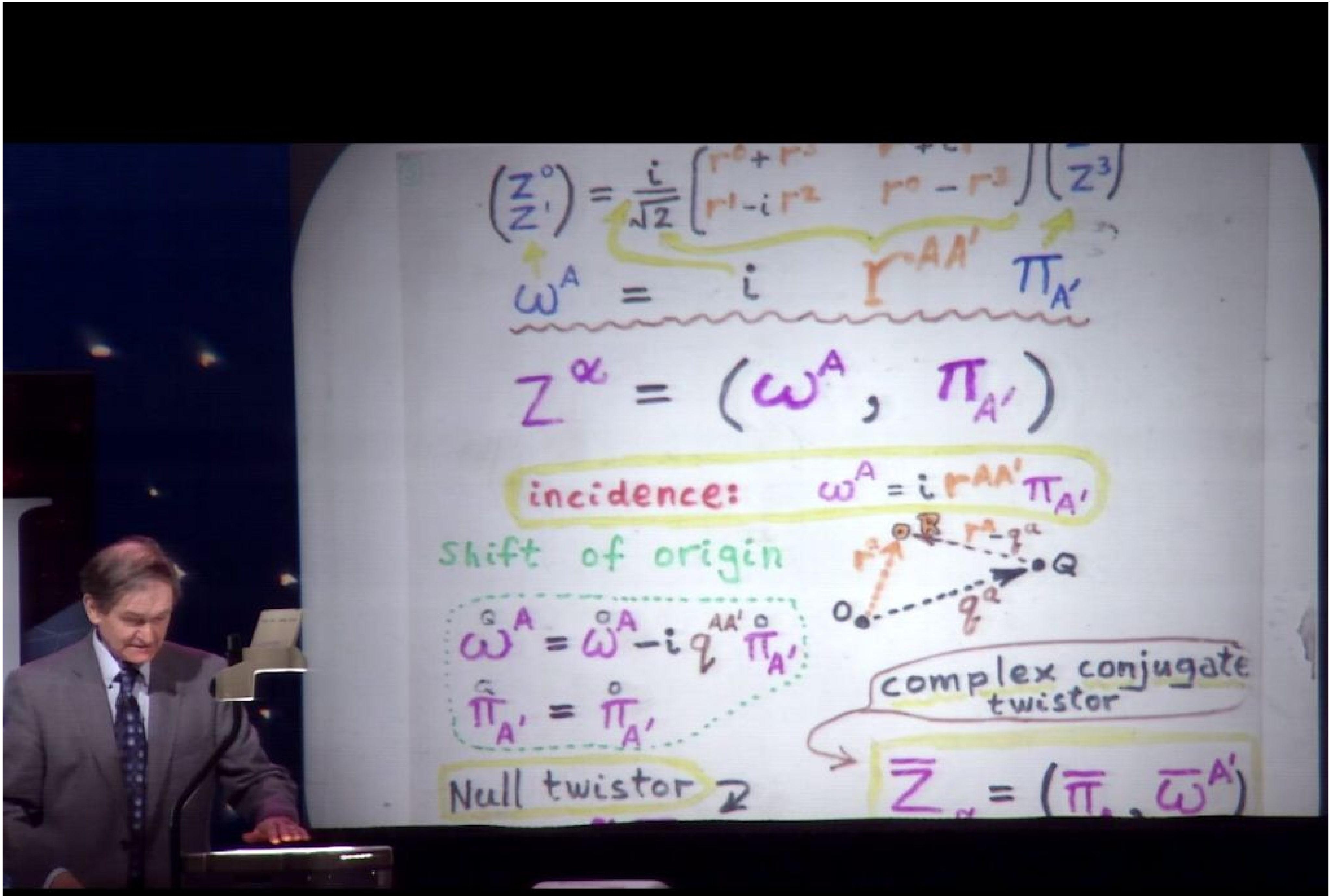

W
R
L
Line R in PN representing
a real point r in Minkowski
ace M
Plane W, joining R to Z
twisto W̄ gives light ray w thr. r
w
As moves
about, we


to visualize a non-null
twistor:
PT
Z
W
R
L
Line R in PN
a real point r
space M
W

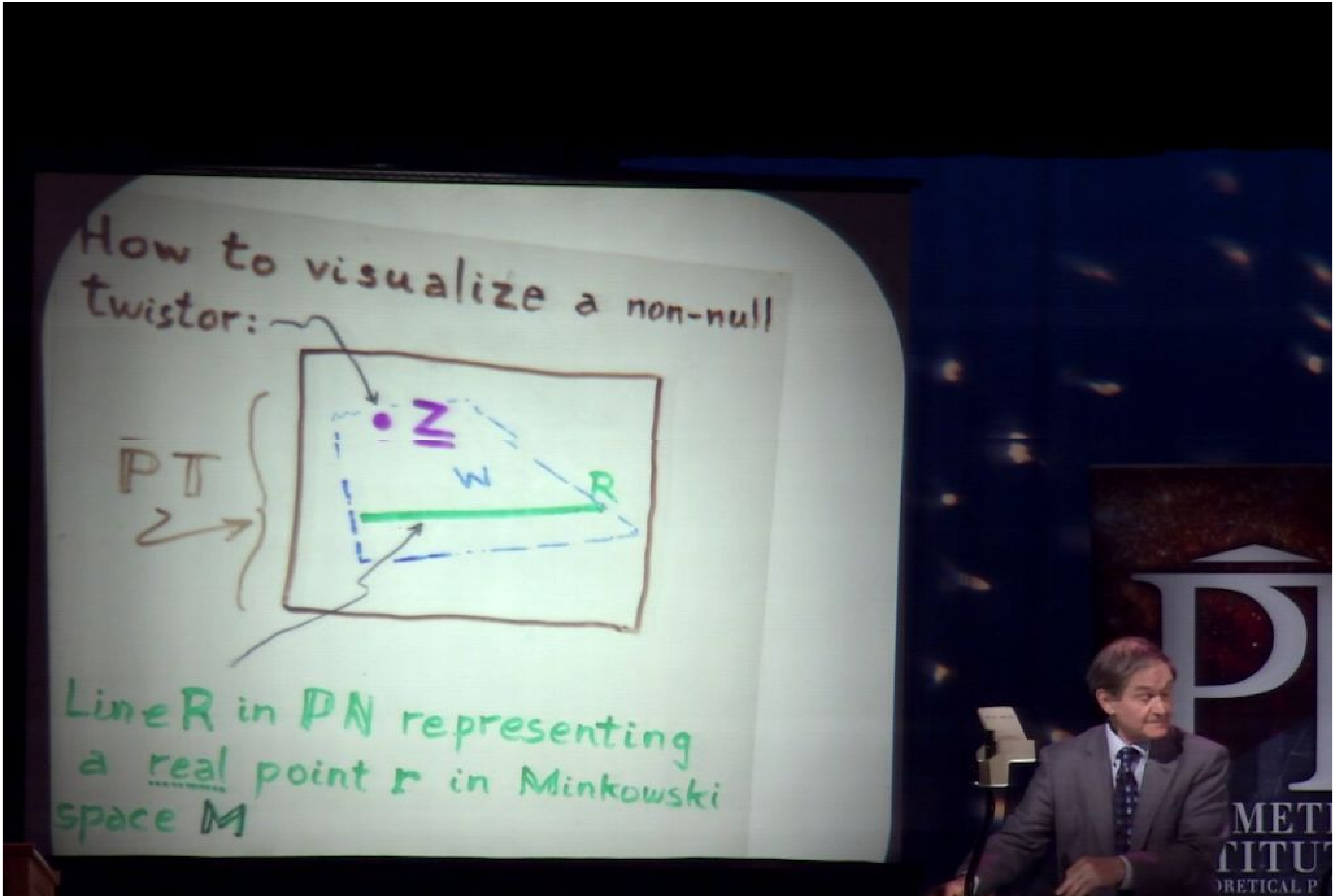


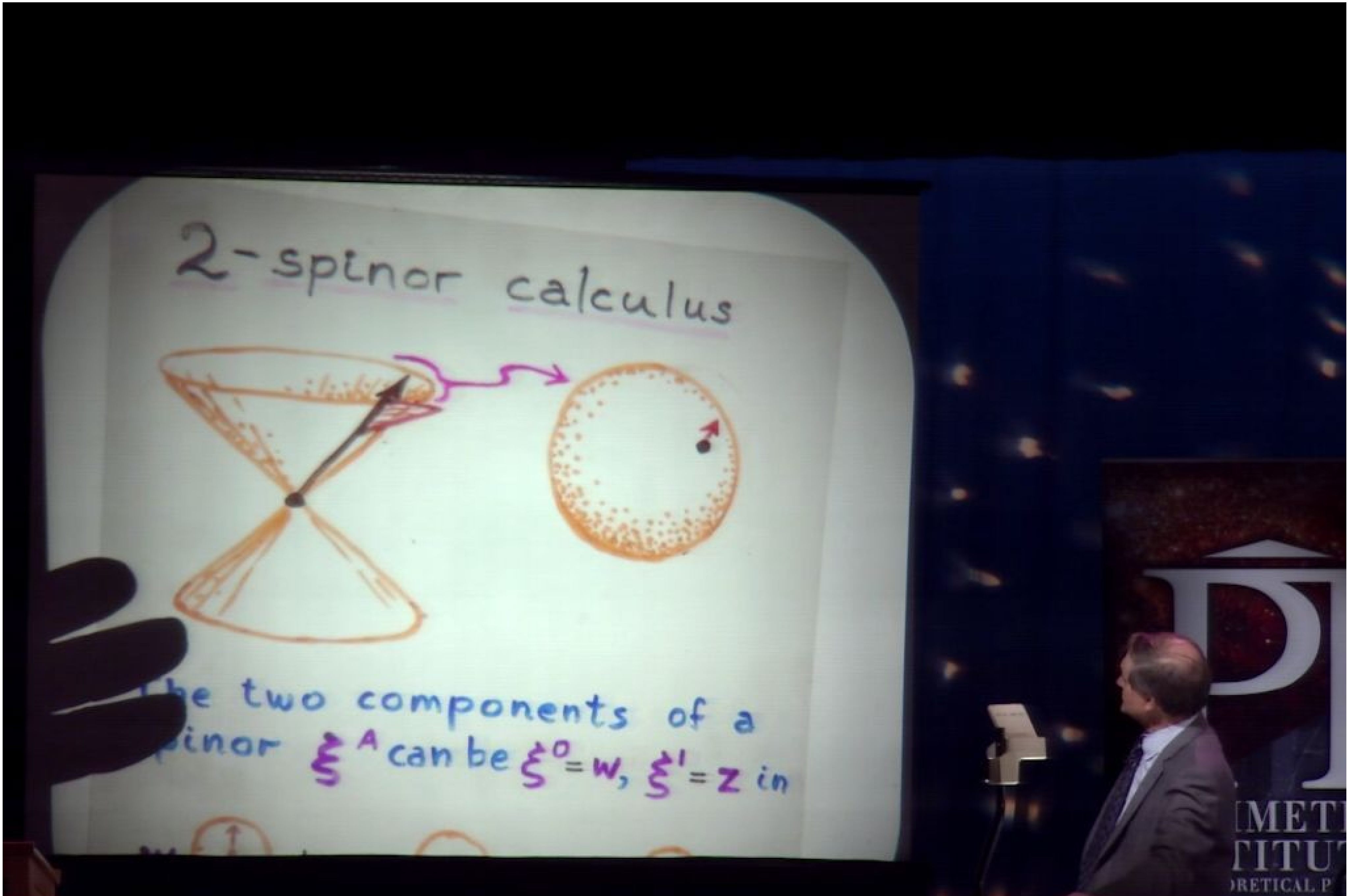
light ray
PT+
PN
PT−
Minkowski 4-space
M
twistor space
COMPLEX projective
3-space PT
Projective
coordinates Z⁰:Z¹:Z²:Z³

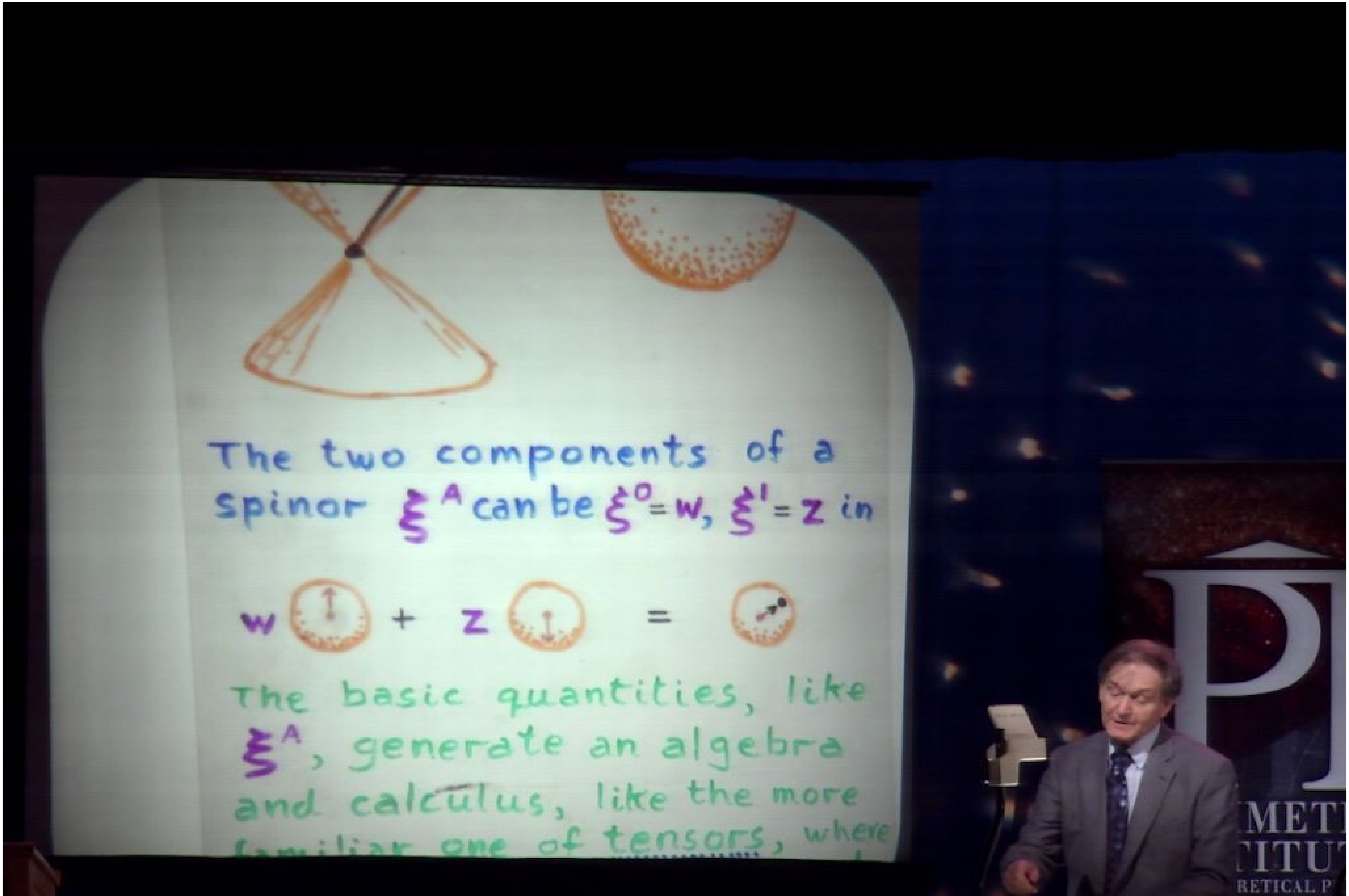

Line R in PN representing
a real point r in Minkowski
space M
Plane W, joining R to Z
twistor W̄ gives light ray w thr. r
M
w
r
As moves
about, we
get a family
(Robinson
of light rays

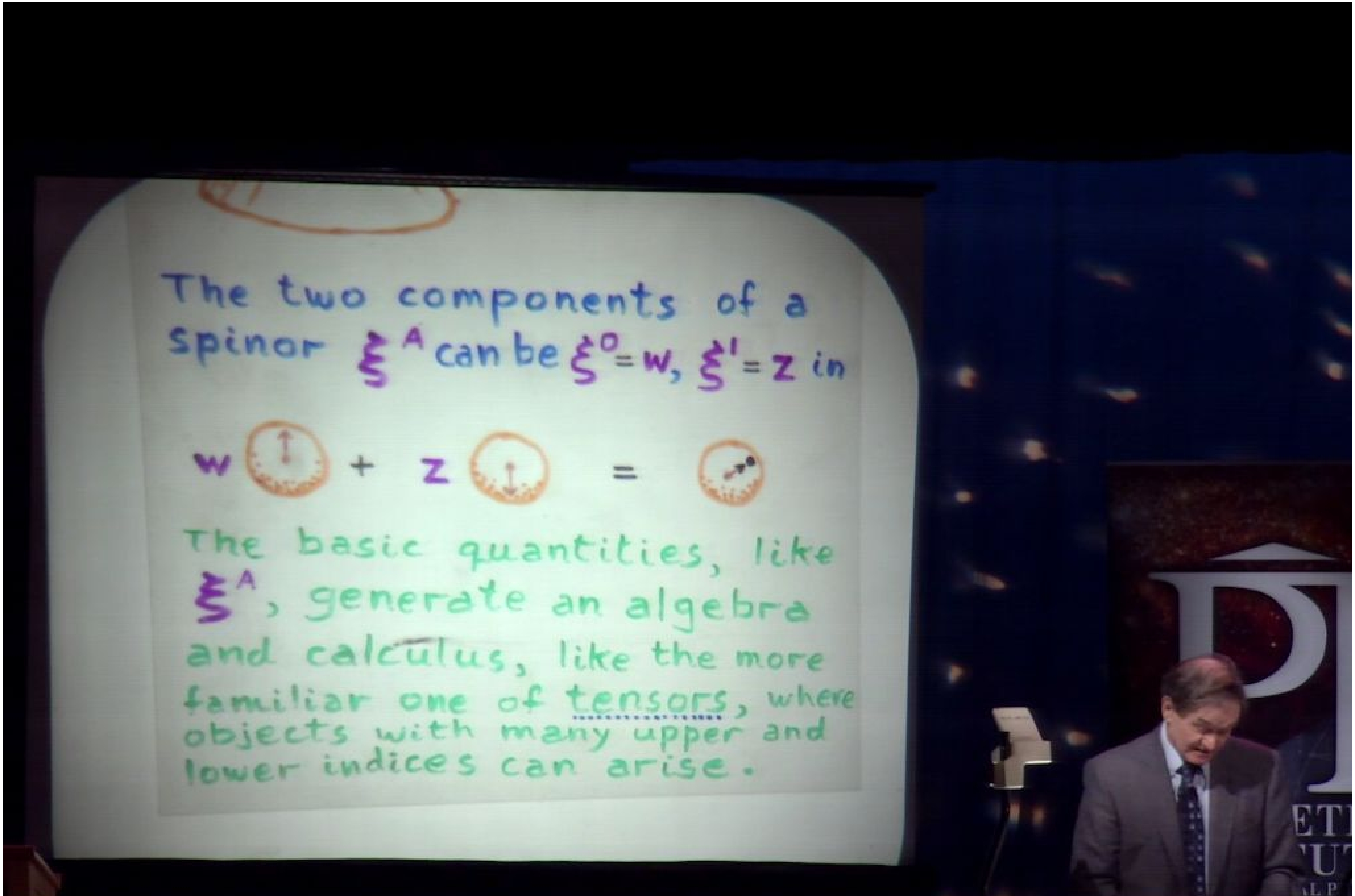


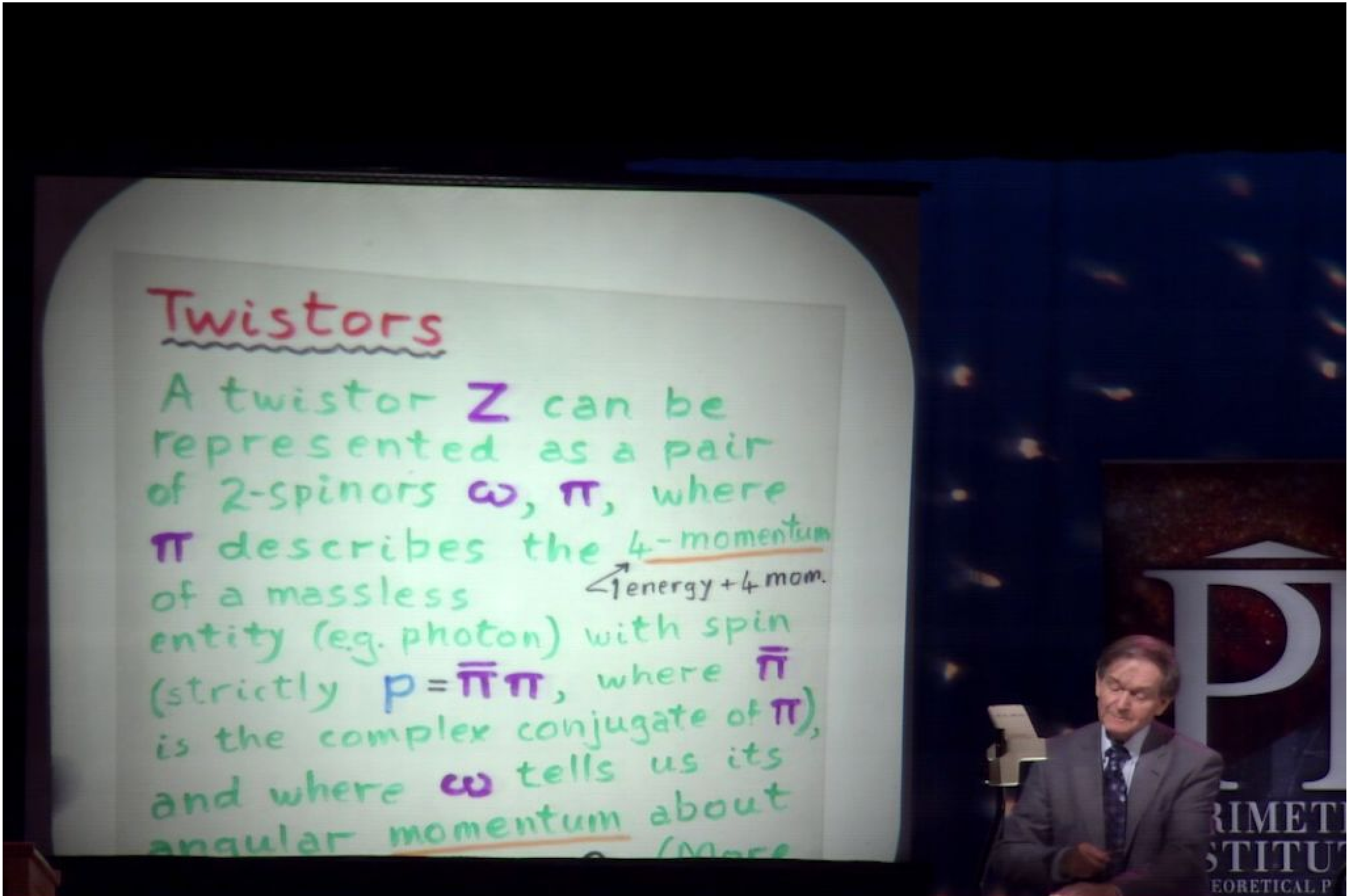
ω^A = i r^AA' π_A'
Z^α = (ω^A, π_A')
incidence: ω^A = i r^AA' π_A'
shift of origin
O
Q
q^a
complex conjugate twistor
Null twistor

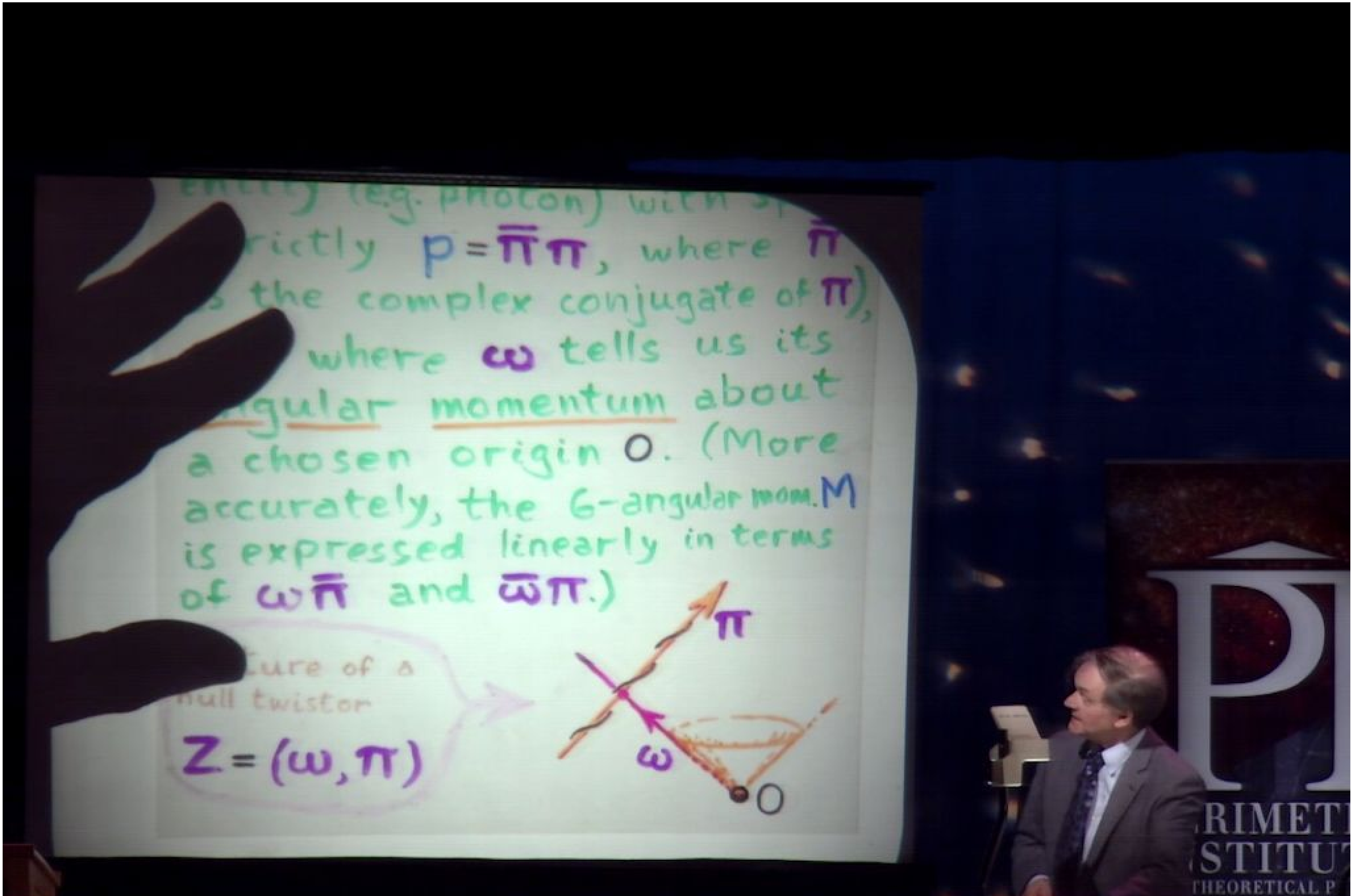

How to visualize a non-null twistor:
Z
PT
W
R
L
Line R in PN representing a real point r in Minkowski space M
METI
TITUT
ORETICAL P

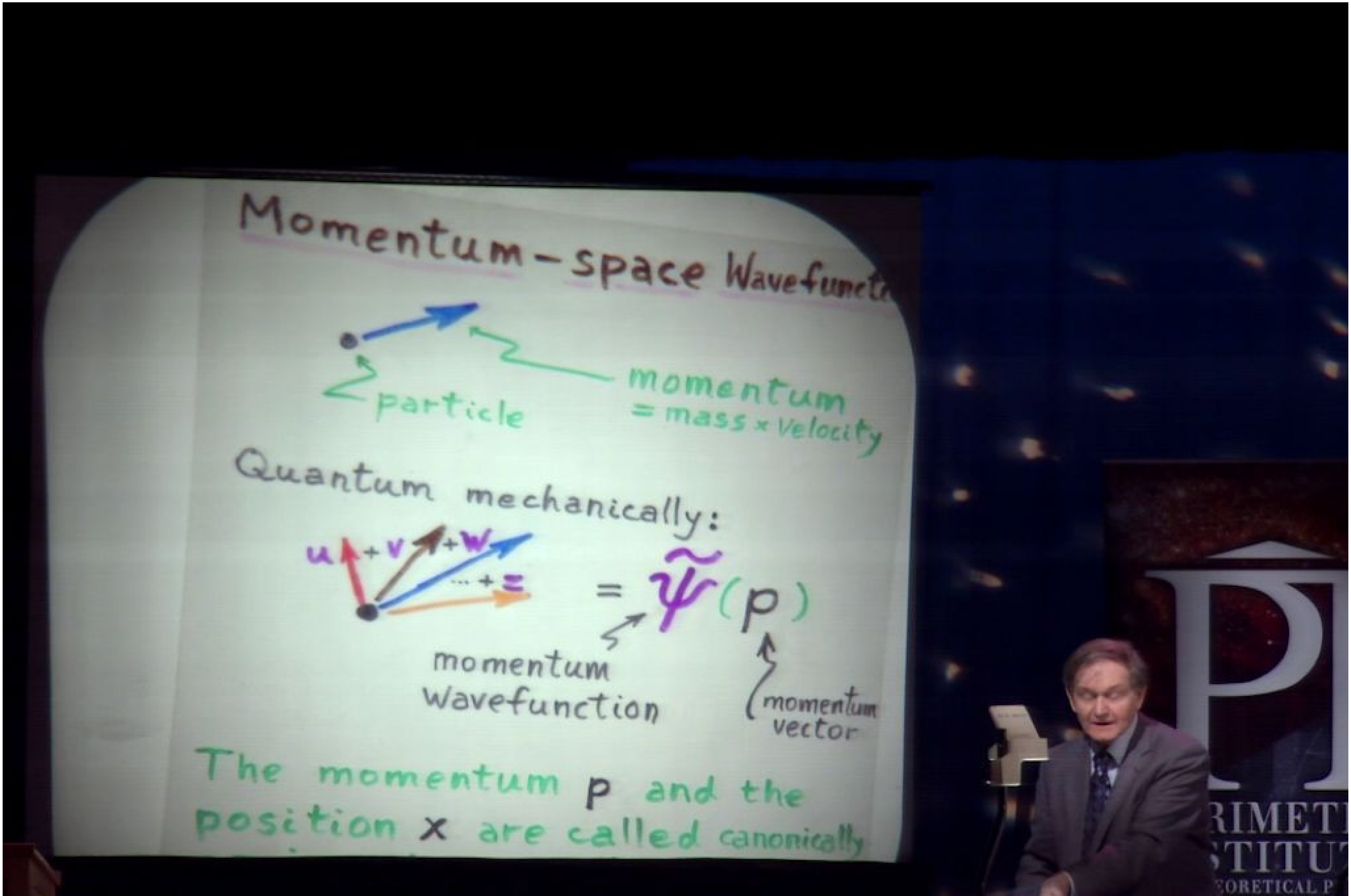

2-spinor calculus
he two components of a
pinor ξ^A can be $\xi^0 = w$, $\xi^1 = z$ in

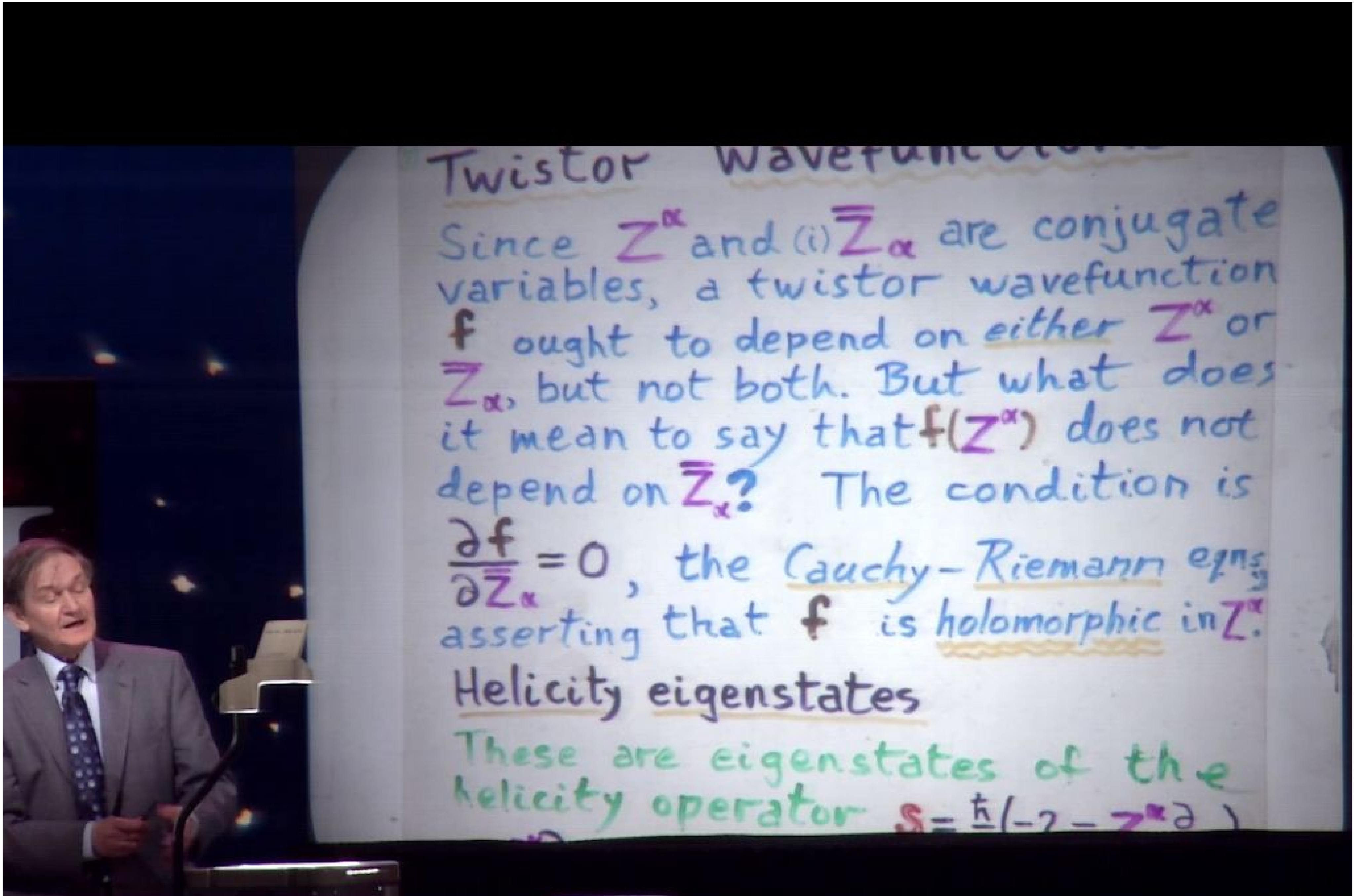

The two components of a
spinor ξ^A can be $\xi^0 = w$, $\xi^1 = z$ in
w + z =
The basic quantities, like
ξ^A, generate an algebra
and calculus, like the more
of tensors, where

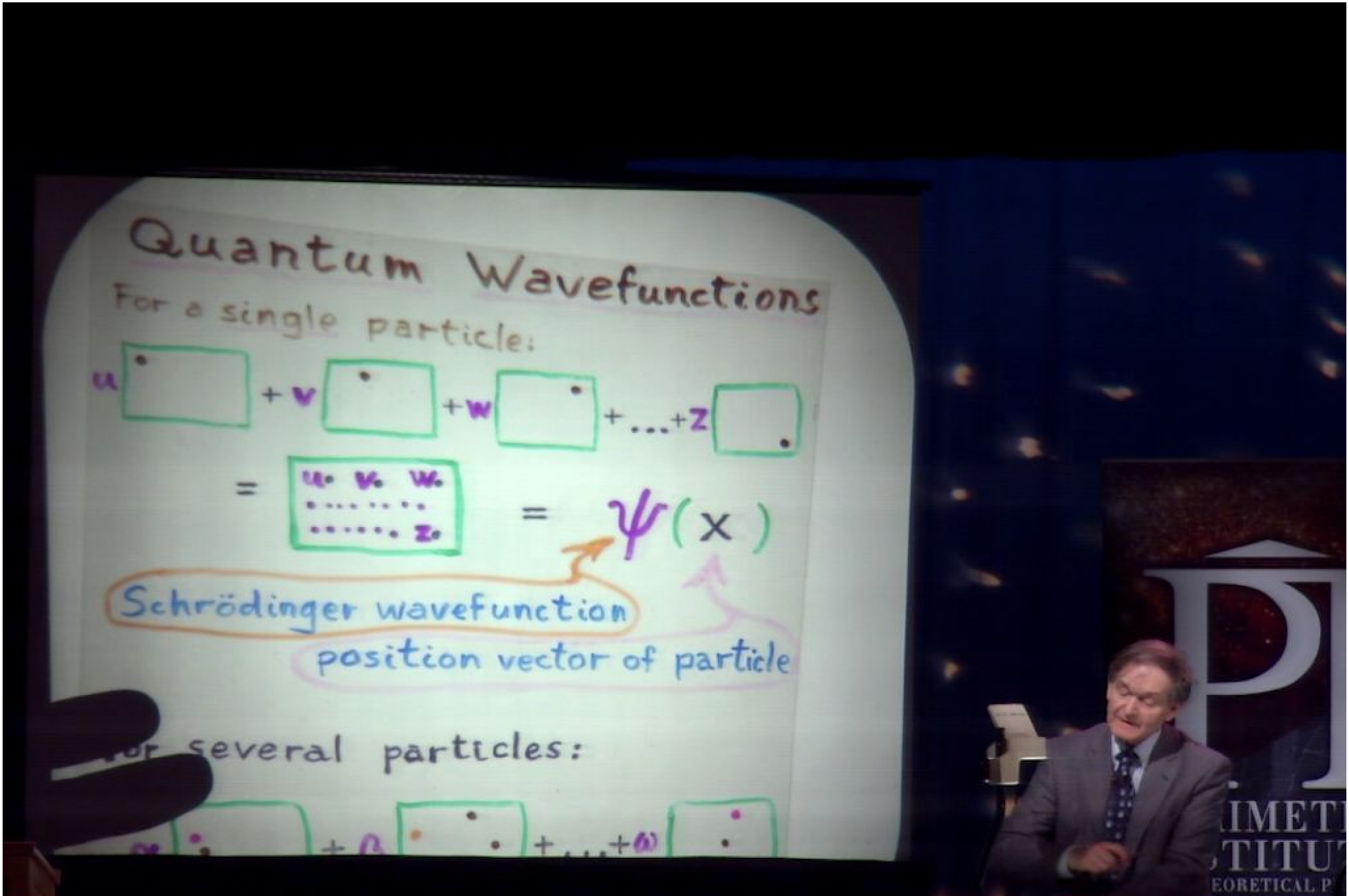

The two components of a spinor ξ^A can be $\xi^0 = w$, $\xi^1 = z$ in
w + z =
The basic quantities, like ξ^A, generate an algebra and calculus, like the more familiar one of tensors, where objects with many upper and lower indices can arise.

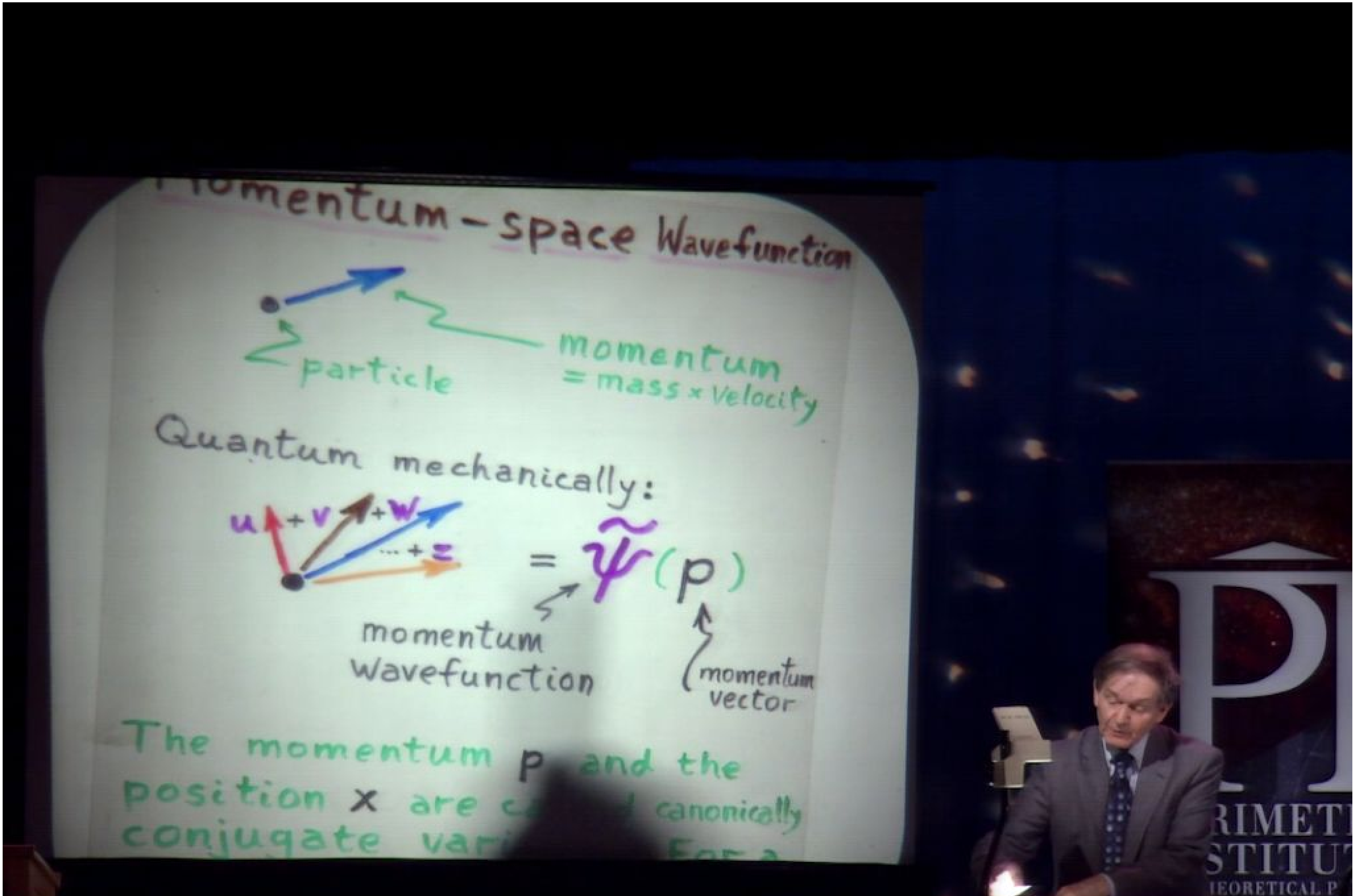

Twistors
A twistor Z can be
represented as a pair
of 2-spinors ω, π, where
π describes the 4-momentum
1 energy + 4 mom.
of a massless
entity (e.g. photon) with spin
(strictly p = π̄π, where π̄
is the complex conjugate of π),
and where ω tells us its
angular momentum about

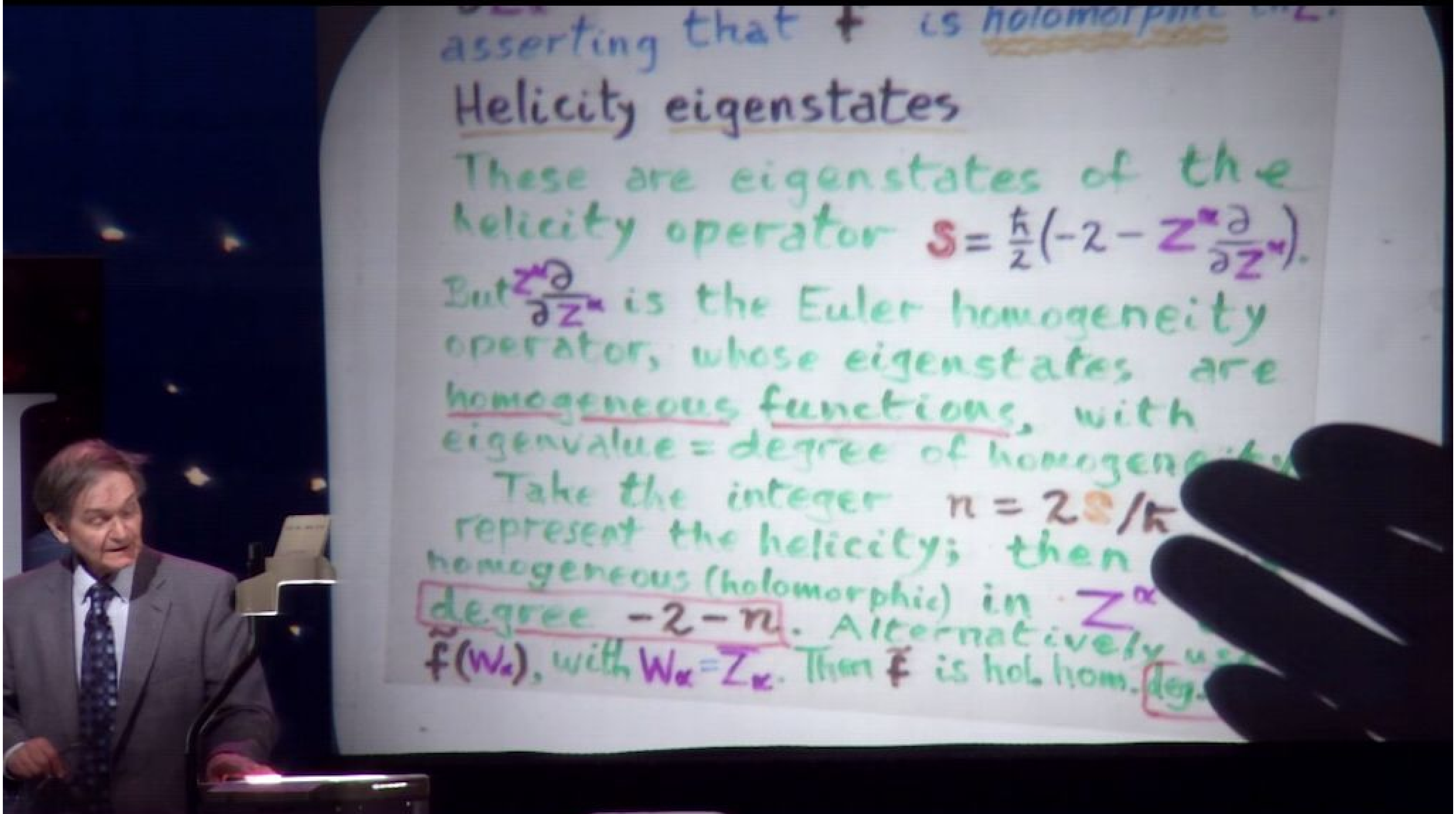

rictly p = π̄π, where π̄
the complex conjugate of π),
where ω tells us its
gular momentum about
a chosen origin O. (More
accurately, the 6-angular mom. M
is expressed linearly in terms
of ωπ̄ and ω̄π.)
π
ture of a
null twistor
Z = (ω, π)
ω
O

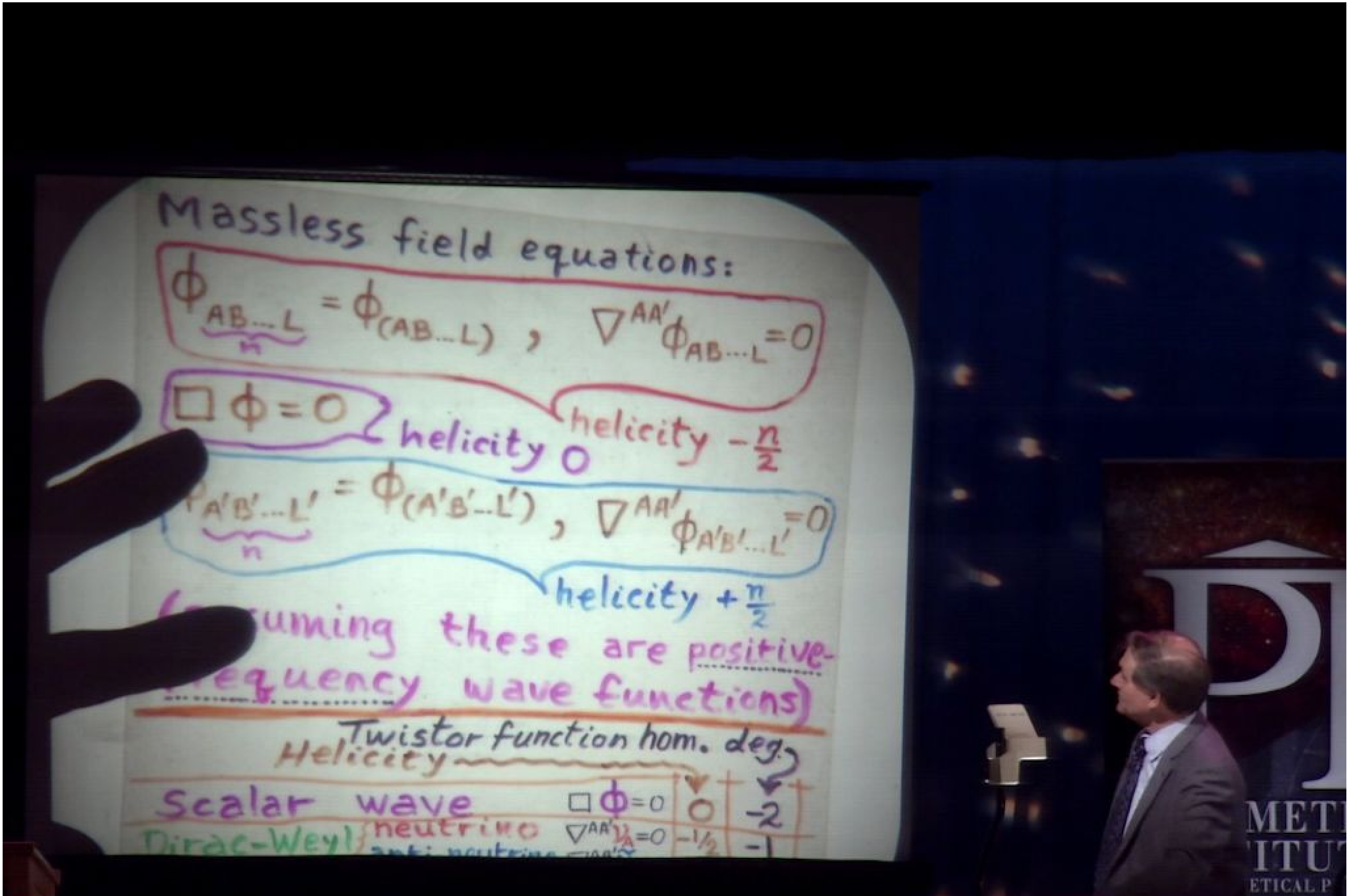

Momentum-space Wavefunct
particle
momentum
= mass × velocity
Quantum mechanically:
u + v + w
... + z
= $\tilde{\psi}(p)$
momentum
wavefunction
momentum
vector
The momentum p and the
position x are called canonically

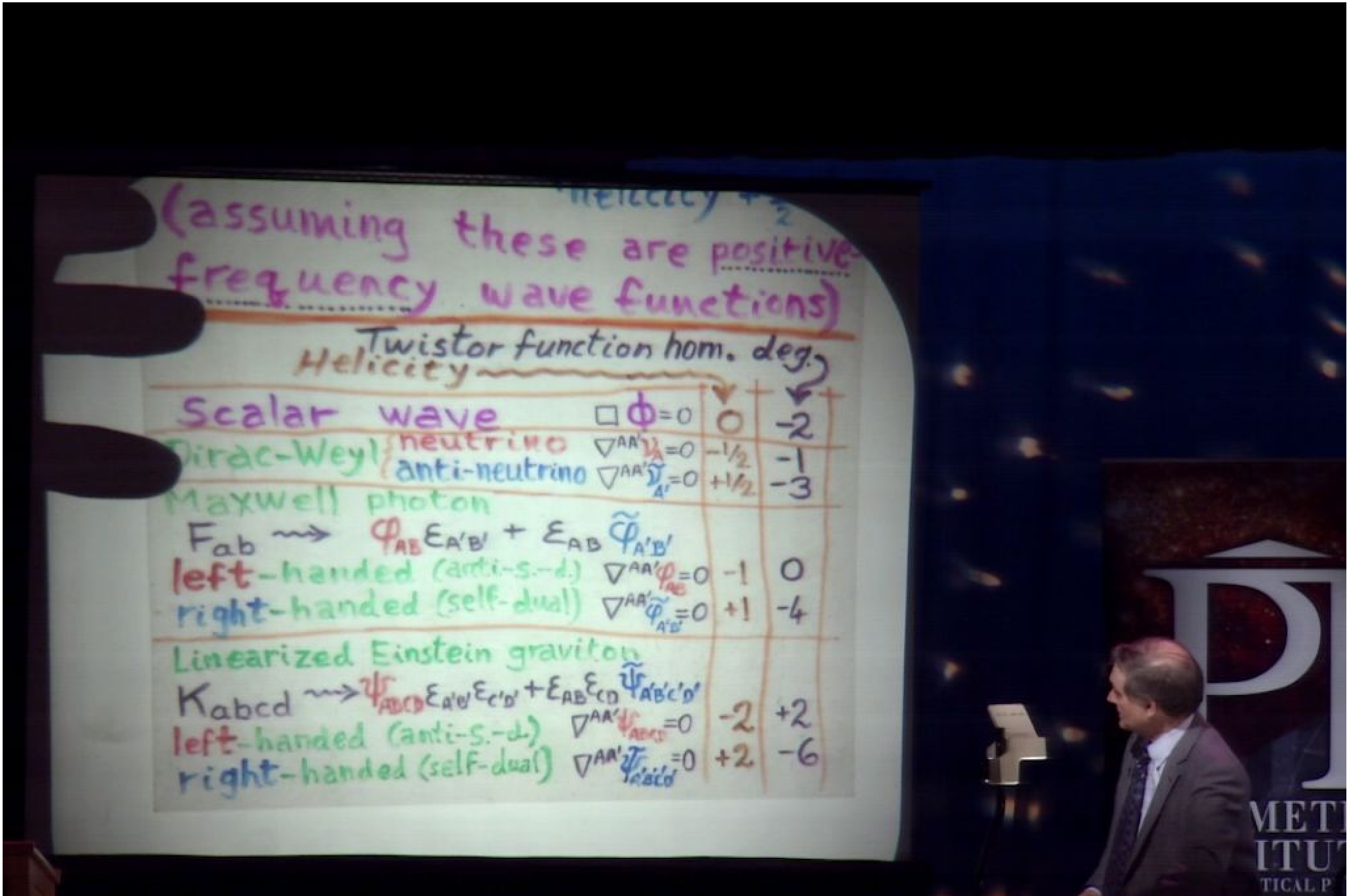

Twistor
Since Z^α and (i)$\bar{Z}_\alpha$ are conjugate variables, a twistor wavefunction f ought to depend on *either* Z^α or $\bar{Z}_\alpha$, but not both. But what does it mean to say that $f(Z^\alpha)$ does not depend on $\bar{Z}_\alpha$? The condition is $\frac{\partial f}{\partial \bar{Z}_\alpha} = 0$, the Cauchy-Riemann eqns, asserting that f is holomorphic in Z^α.
Helicity eigenstates
These are eigenstates of the helicity operator

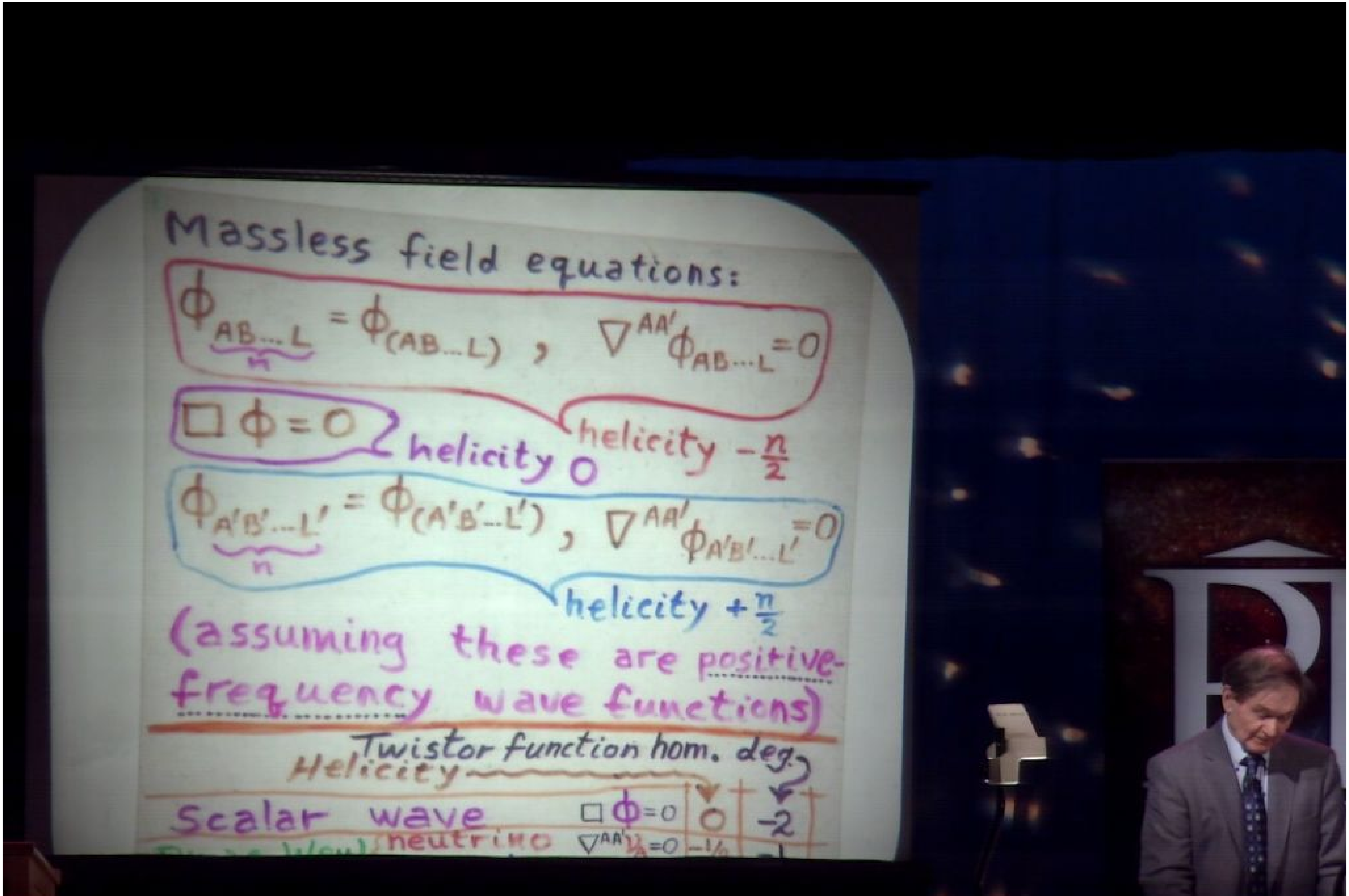

Quantum Wavefunctions
For a single particle:
u + v + w + ... + z
=
= ψ(x)
Schrödinger wavefunction
position vector of particle
several particles:

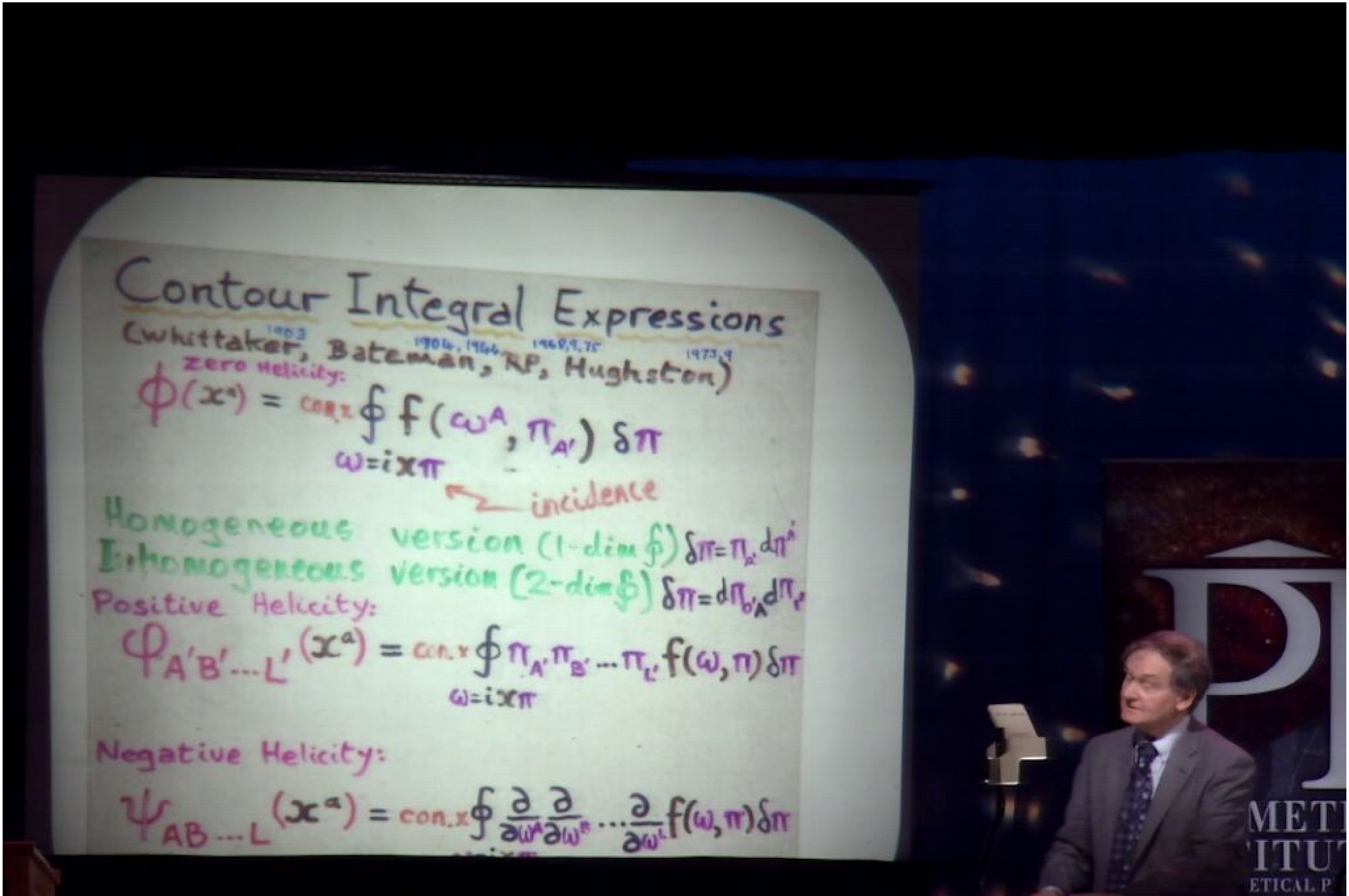

omentum – space Wavefunction
momentum
= mass × velocity
particle
Quantum mechanically:
u + v + w ... + z
= $\tilde{\Psi}(p)$
momentum wavefunction
momentum vector
The momentum p and the
position x are c canonically
conjugate var
RIMET
STITUT
IEORETICAL P

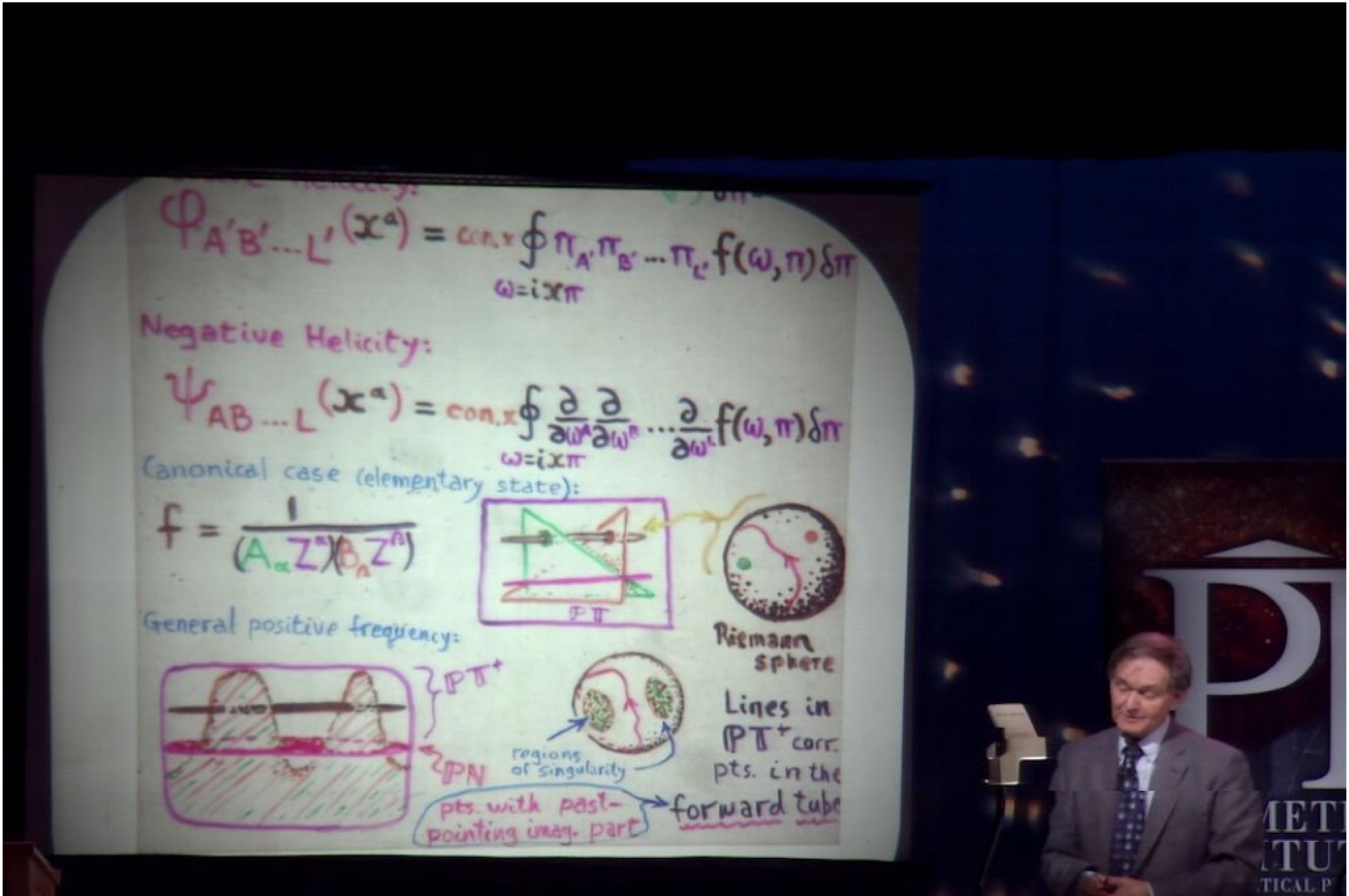

asserting that f is holomorphi
Helicity eigenstates
These are eigenstates of the
helicity operator $S = \frac{\hbar}{2}\left(-2 - Z^\alpha \frac{\partial}{\partial Z^\alpha}\right)$.
But $Z^\alpha \frac{\partial}{\partial Z^\alpha}$ is the Euler homogeneity
operator, whose eigenstates are
homogeneous functions, with
eigenvalue = degree of homogene
Take the integer $n = 2S/\hbar$
represent the helicity; then
homogeneous (holomorphic) in Z^α
degree $-2-n$. Alternatively
$\tilde{f}(W_\alpha)$, with $W_\alpha = \bar{Z}_\alpha$. Then $\tilde{f}$ is hol. hom. deg.


Massless field equations:
$\underbrace{\phi_{AB\ldots L}}_{n} = \phi_{(AB\ldots L)}$, $\nabla^{AA'}\phi_{AB\ldots L} = 0$
$\Box\phi = 0$ helicity 0
helicity $-\frac{n}{2}$
$\underbrace{\phi_{A'B'\ldots L'}}_{n} = \phi_{(A'B'\ldots L')}$, $\nabla^{AA'}\phi_{A'B'\ldots L'} = 0$
helicity $+\frac{n}{2}$
uming these are positive-
equency wave functions)
Twistor function hom. deg.
Helicity
Scalar wave $\Box\phi = 0$ 0 -2
Dirac-Weyl neutrino $-\frac{1}{2}$ -1
METI
ITUT
ETICAL P

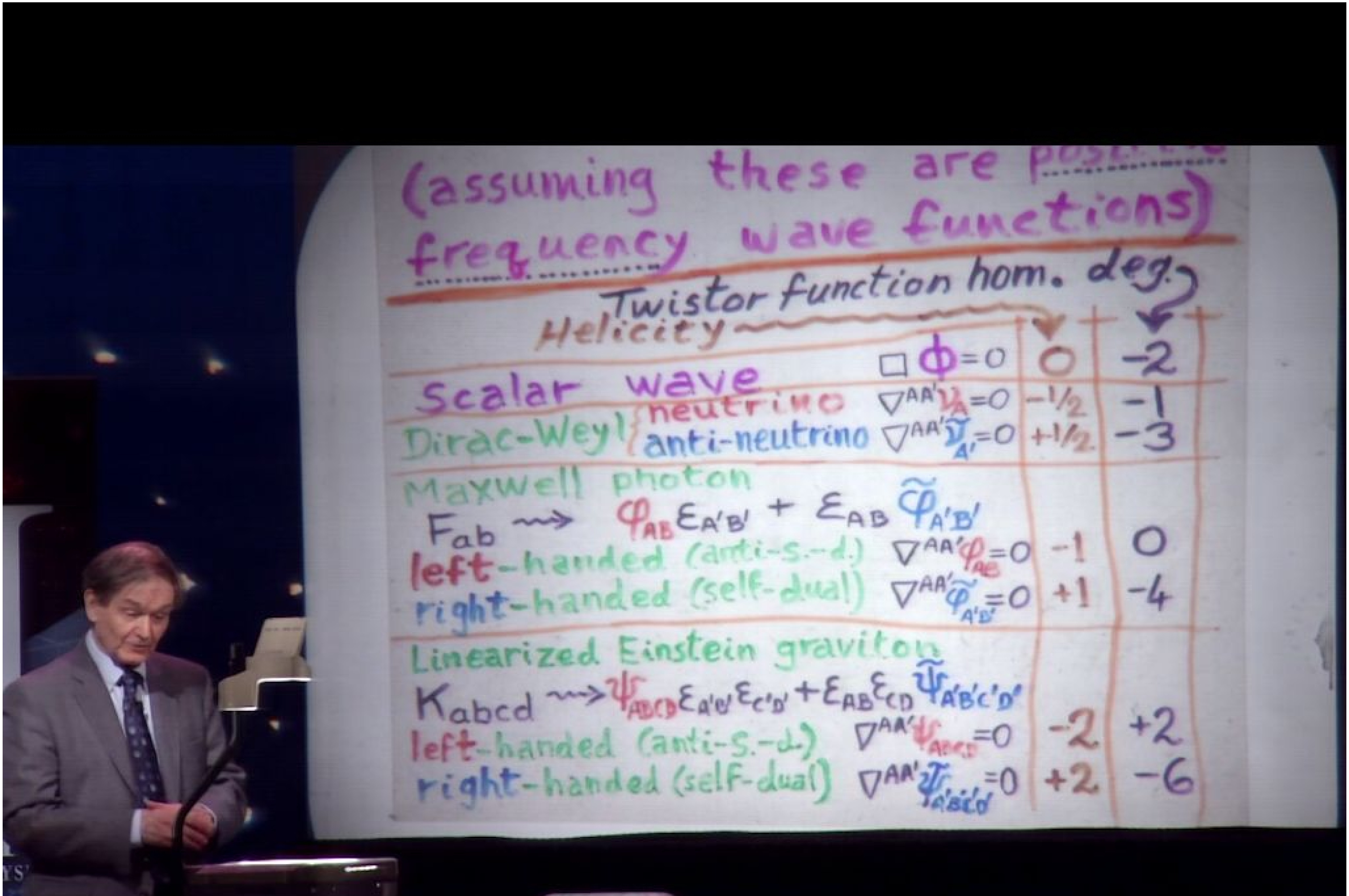

(assuming these are positive-frequency wave functions)
Twistor function hom. deg.
Helicity
Scalar wave □Φ=0 0 −2
Dirac-Weyl neutrino ∇AA'νA=0 −1/2 −1
anti-neutrino ∇AA'ν̄A'=0 +1/2 −3
Maxwell photon
Fab ⇝ φAB εA'B' + εAB φ̃A'B'
left-handed (anti-s.-d.) ∇AA'φAB=0 −1 0
right-handed (self-dual) ∇AA'φ̃A'B'=0 +1 −4
Linearized Einstein graviton
Kabcd ⇝ ΨABCD εA'B' εC'D' + εAB εCD Ψ̃A'B'C'D'
left-handed (anti-s.-d.) ∇AA'ΨABCD=0 −2 +2
right-handed (self-dual) ∇AA'Ψ̃A'B'C'D'=0 +2 −6

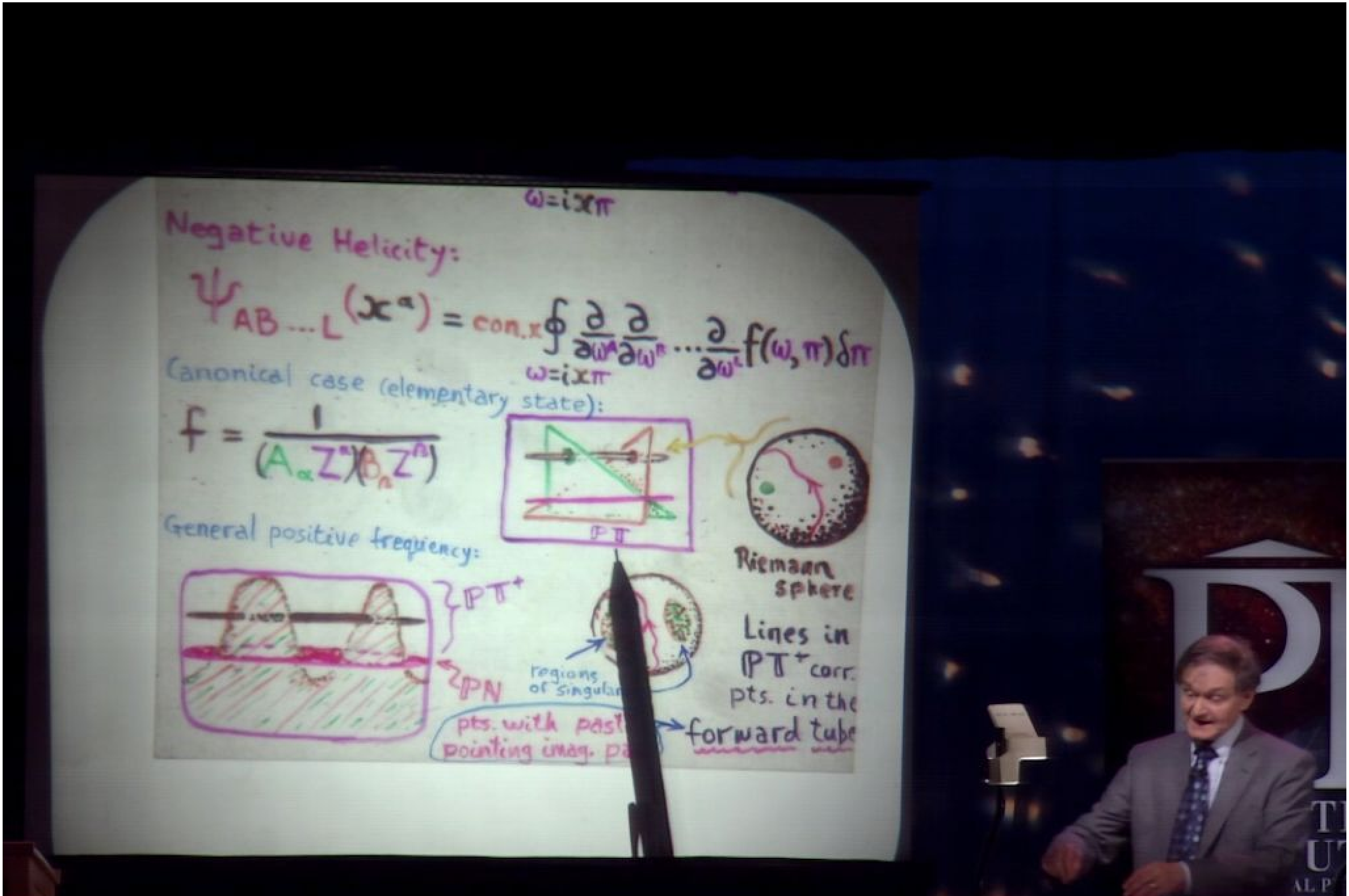

Massless field equations:
$\underbrace{\phi_{AB\ldots L}}_{n} = \phi_{(AB\ldots L)}$, $\nabla^{AA'}\phi_{AB\ldots L}=0$
helicity $-\frac{n}{2}$
$\Box\phi=0$
helicity 0
$\underbrace{\phi_{A'B'\ldots L'}}_{n} = \phi_{(A'B'\ldots L')}$, $\nabla^{AA'}\phi_{A'B'\ldots L'}=0$
helicity $+\frac{n}{2}$
(assuming these are positive-frequency wave functions)
Twistor function hom. deg.
Helicity
Scalar wave $\Box\phi=0$ 0 −2
neutrino

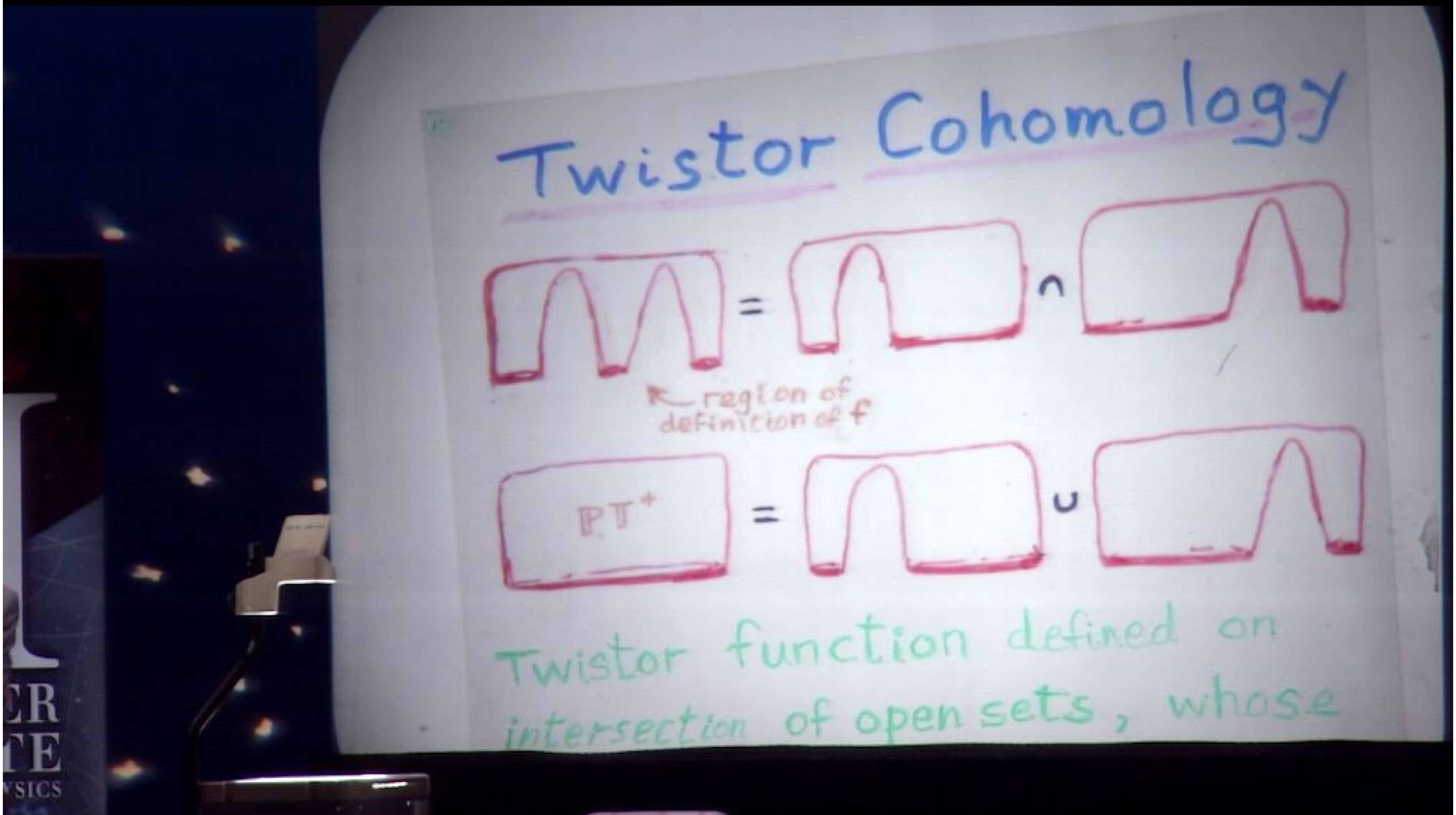

Contour Integral Expressions
(Whittaker, Bateman, RP, Hughston)
1903
1904, 1964
1968,9,75
1973,9
zero Helicity:
$\phi(x^a) = \text{con.}\times \oint f(\omega^A, \pi_{A'})\,\delta\pi$
$\omega = ix\pi$
incidence
Homogeneous version (1-dim $\oint$) $\delta\pi = \pi_{A'}\,d\pi^{A'}$
Inhomogeneous version (2-dim $\oint$)
Positive Helicity:
$\varphi_{A'B'\cdots L'}(x^a) = \text{con.}\times \oint \pi_{A'}\pi_{B'}\cdots\pi_{L'}\, f(\omega,\pi)\,\delta\pi$
$\omega = ix\pi$
Negative Helicity:
$\psi_{AB\cdots L}(x^a) = \text{con.}\times \oint \frac{\partial}{\partial\omega^A}\frac{\partial}{\partial\omega^B}\cdots\frac{\partial}{\partial\omega^L} f(\omega,\pi)\,\delta\pi$

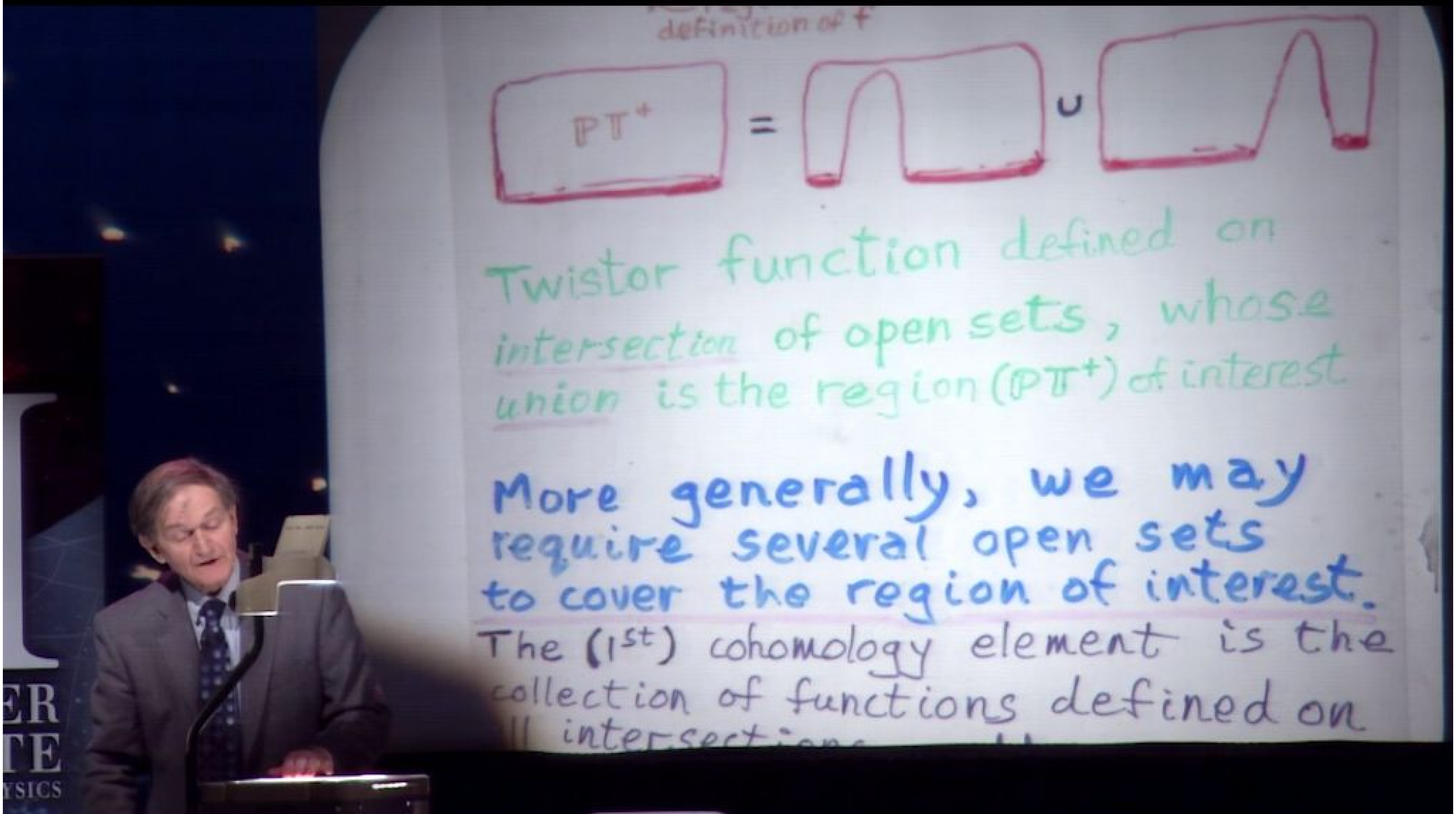


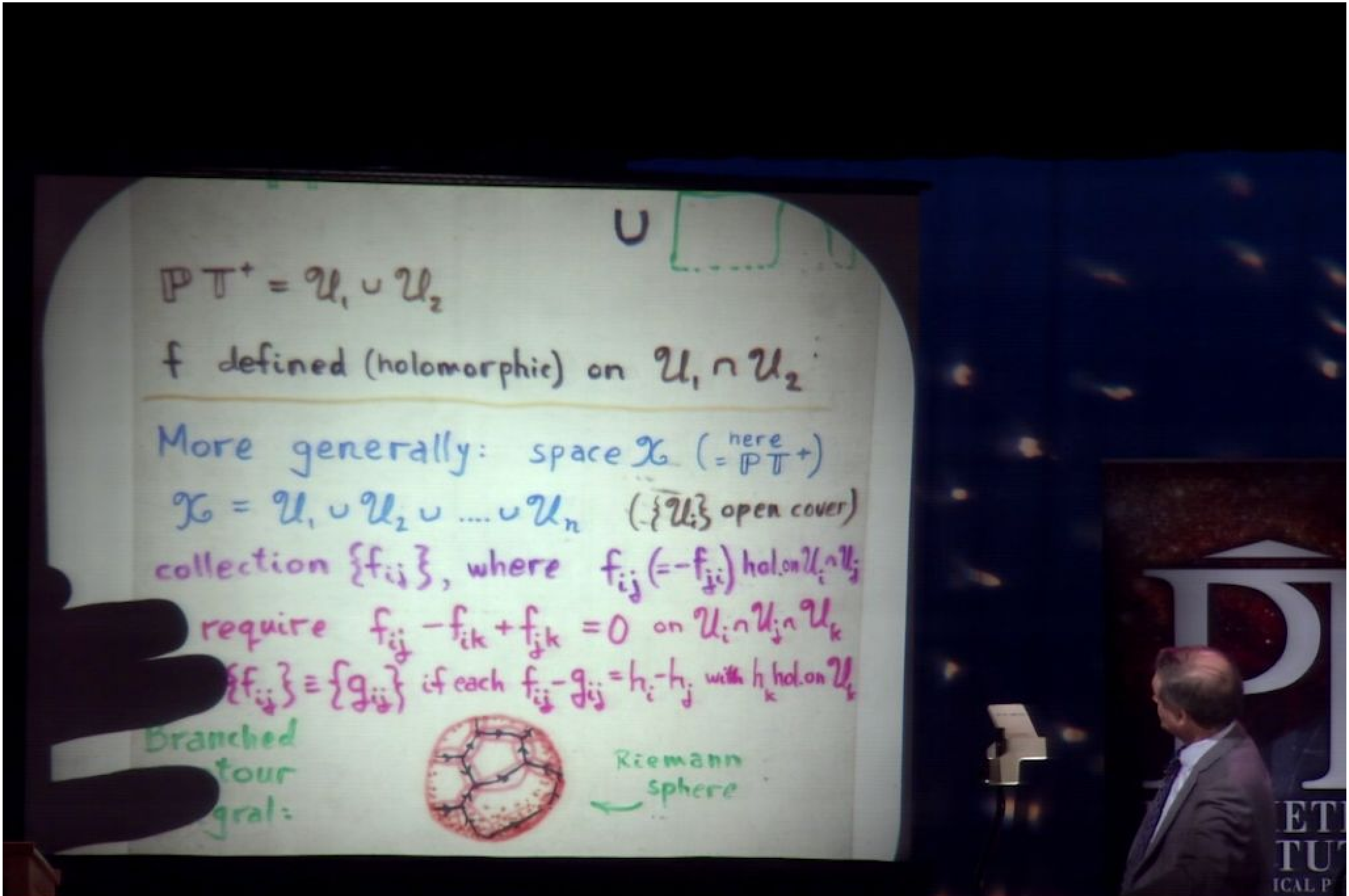
$\varphi_{A'B'\dots L'}(x^a) = \text{con.} \times \oint_{\omega = ix\pi} \pi_{A'}\pi_{B'}\dots\pi_{L'} f(\omega,\pi)\,\delta\pi$
Negative Helicity:
$\psi_{AB\dots L}(x^a) = \text{con.} \times \oint_{\omega = ix\pi} \frac{\partial}{\partial\omega^A}\frac{\partial}{\partial\omega^B}\dots\frac{\partial}{\partial\omega^L} f(\omega,\pi)\,\delta\pi$
Canonical case (elementary state):
$f = \frac{1}{(A_\alpha Z^\alpha)(B_\alpha Z^\alpha)}$
$\mathbb{PT}$
Riemann sphere
General positive frequency:
$\mathbb{PT}^+$
$\mathbb{PN}$
regions of singularity
Lines in $\mathbb{PT}^+$ corr. pts. in the forward tube
pts. with past-pointing imag. part

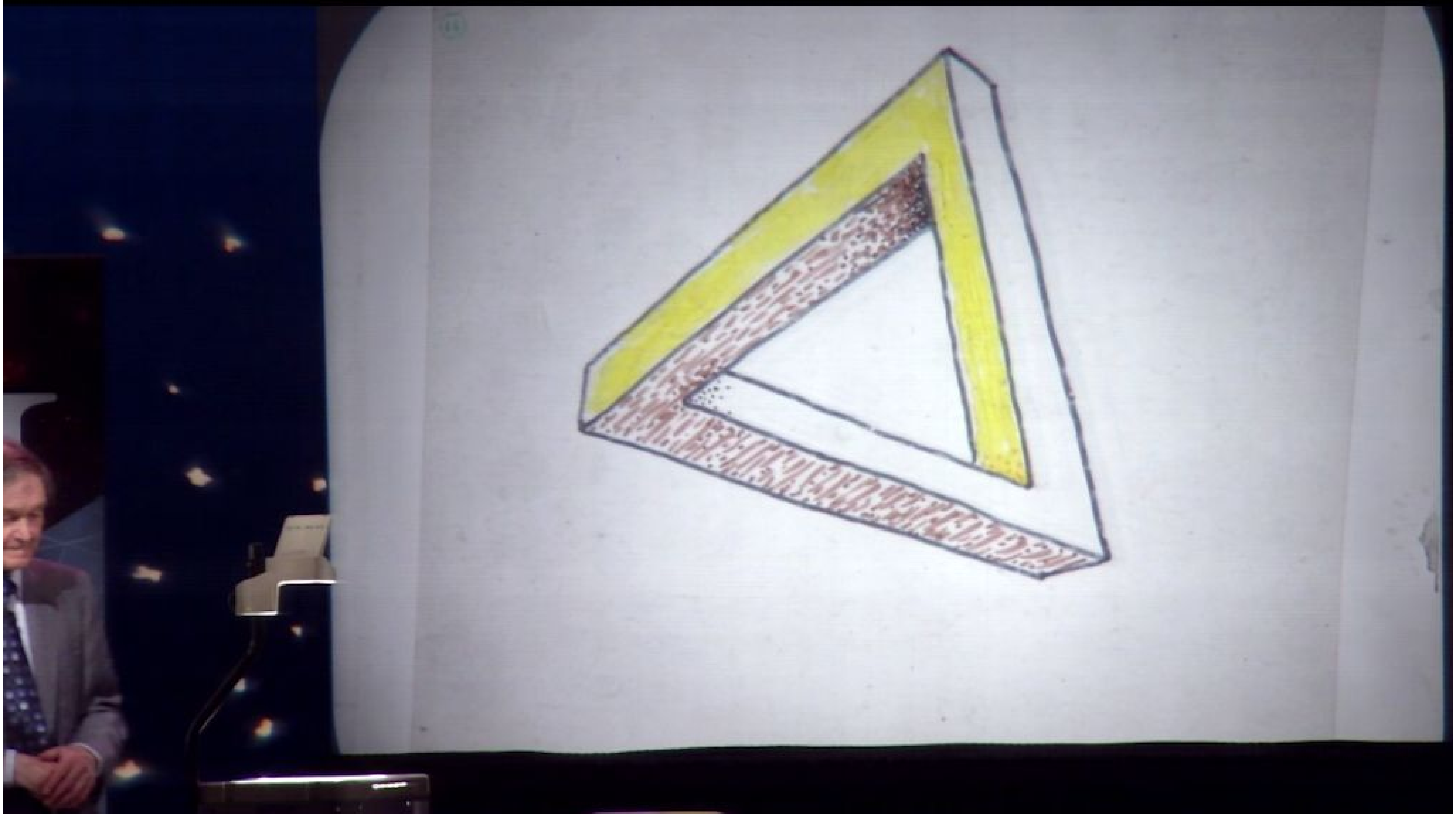

(assuming these are positive
frequency wave functions)
Twistor function hom. deg.
Helicity
Scalar wave $\Box\Phi=0$ 0 −2
Dirac-Weyl { neutrino $\nabla^{AA'}\nu_A=0$ −1/2 −1
anti-neutrino $\nabla^{AA'}\tilde{\nu}_{A'}=0$ +1/2 −3
Maxwell photon
$F_{ab} \rightsquigarrow \varphi_{AB}\varepsilon_{A'B'} + \varepsilon_{AB}\tilde{\varphi}_{A'B'}$
left-handed (anti-s.-d.) $\nabla^{AA'}\varphi_{AB}=0$ −1 0
right-handed (self-dual) $\nabla^{AA'}\tilde{\varphi}_{A'B'}=0$ +1 −4
Linearized Einstein graviton
$K_{abcd} \rightsquigarrow \Psi_{ABCD}\varepsilon_{A'B'}\varepsilon_{C'D'} + \varepsilon_{AB}\varepsilon_{CD}\tilde{\Psi}_{A'B'C'D'}$
left-handed (anti-s.-d.) $\nabla^{AA'}\Psi_{ABCD}=0$ −2 +2
right-handed (self-dual) $\nabla^{AA'}\tilde{\Psi}_{A'B'C'D'}=0$ +2 −6

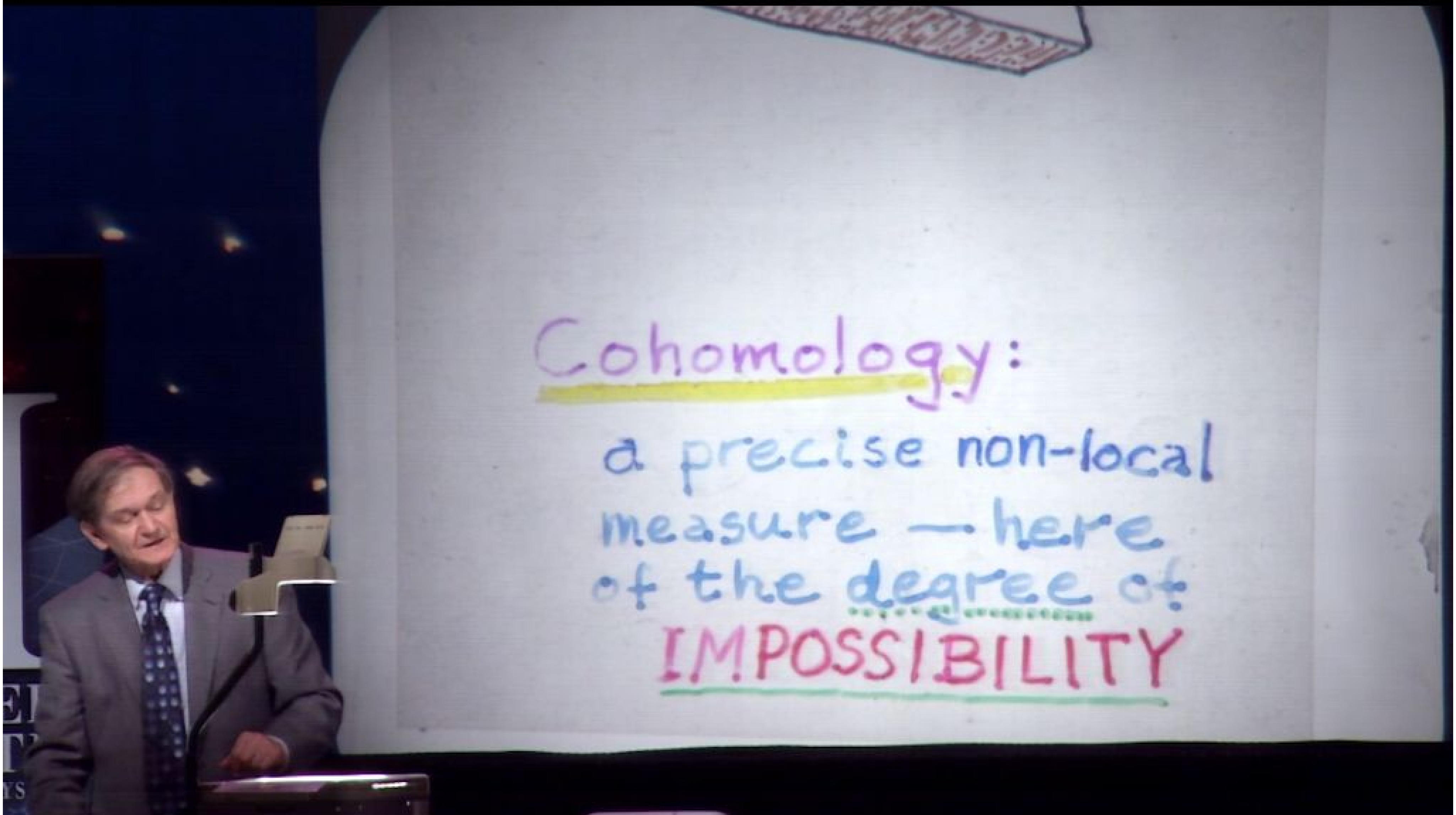

ω=ixπ
Negative Helicity:
$\psi_{AB\ldots L}(x^a) = \text{con.}x \oint \frac{\partial}{\partial\omega^A}\frac{\partial}{\partial\omega^B}\cdots\frac{\partial}{\partial\omega^L} f(\omega,\pi)\,\delta\pi$
ω=ixπ
Canonical case (elementary state):
$f = \frac{1}{(A_\alpha Z^\alpha)(B_\beta Z^\beta)}$
PT
Riemann sphere
General positive frequency:
PT+
PN
Lines in PT+ corr. pts. in the forward tube

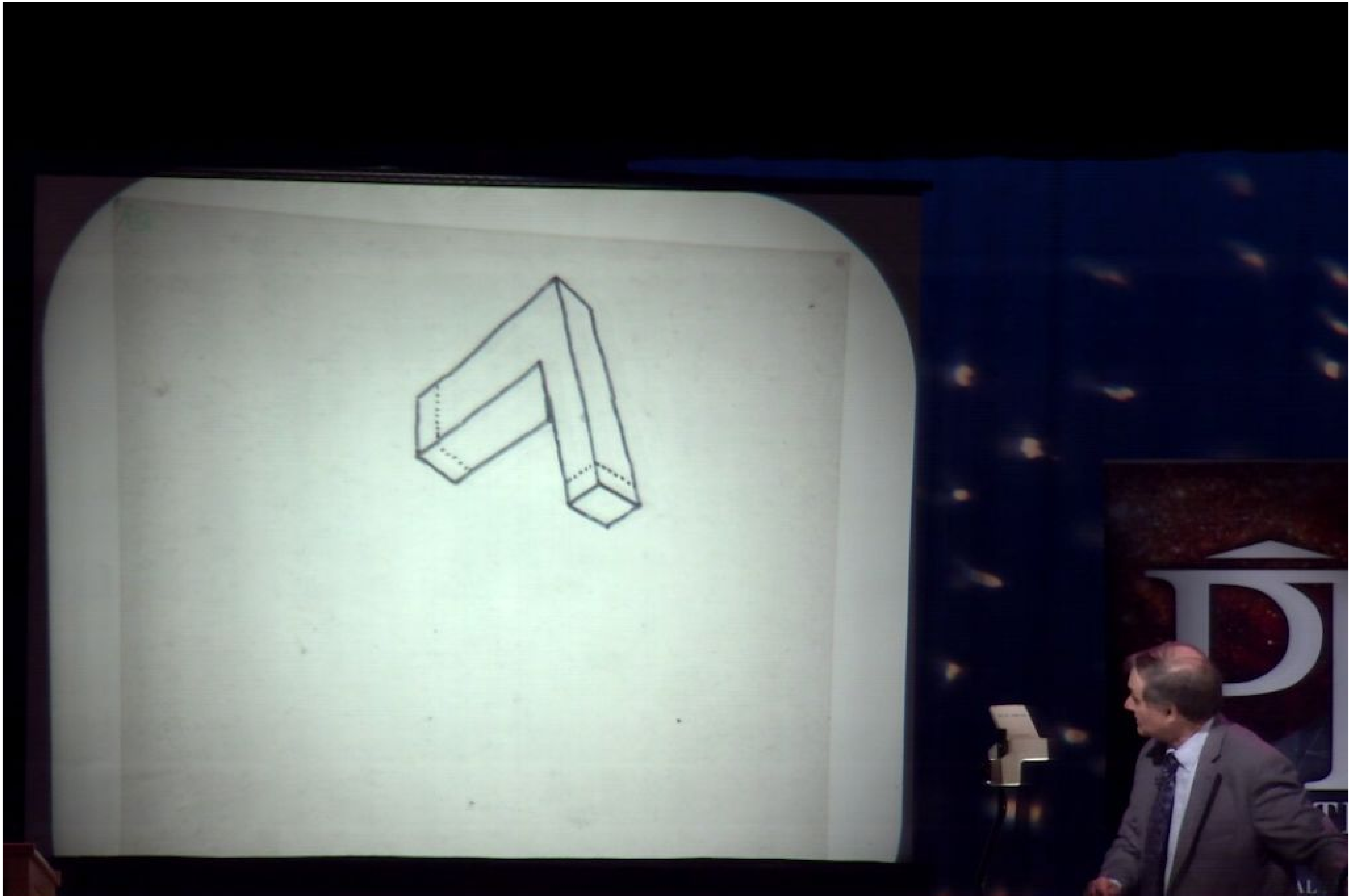

Twistor Cohomology
=
∩
region of definition of f
PT+
=
∪
Twistor function defined on
intersection of open sets, whose

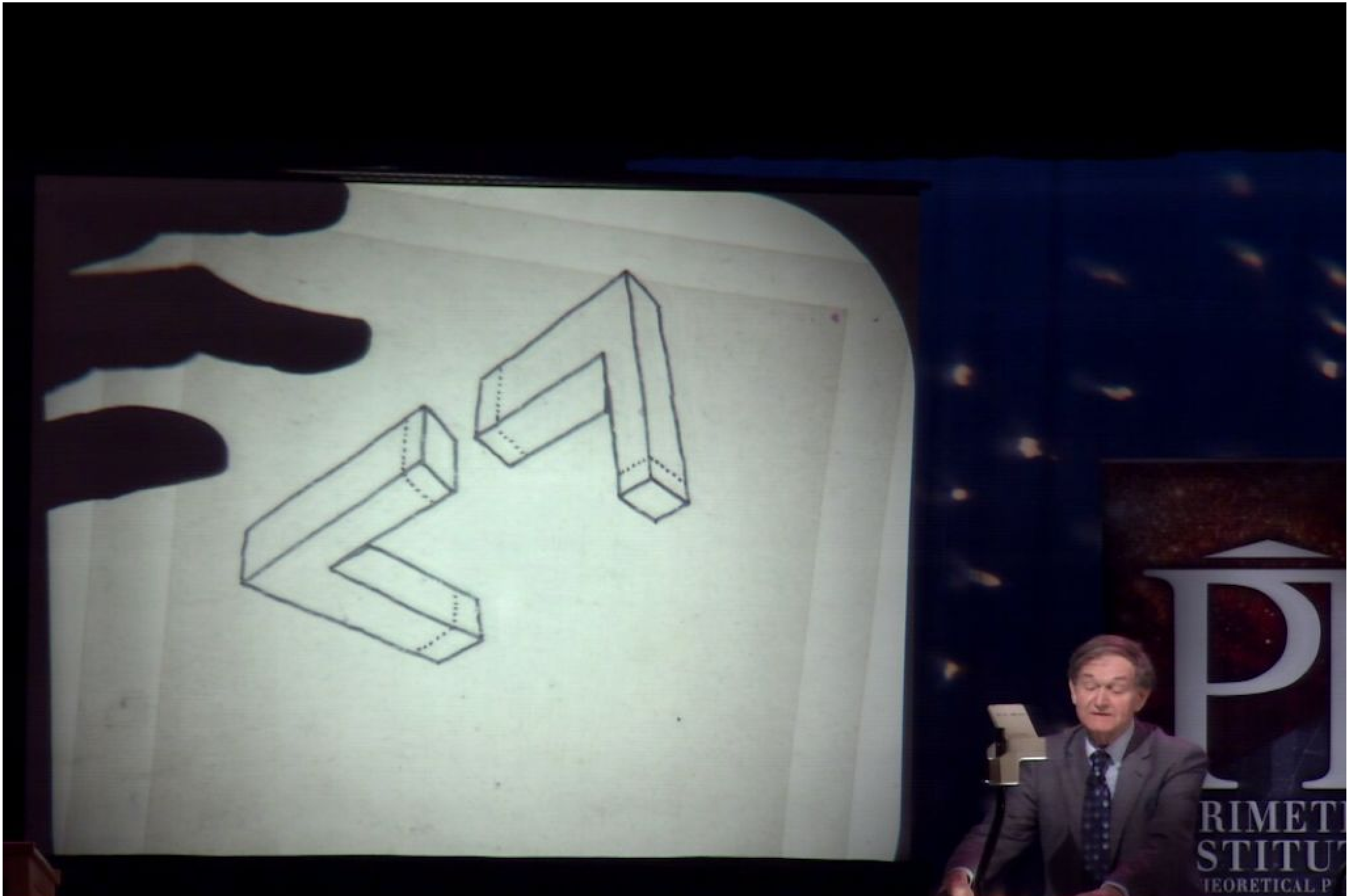

definition of f
PT+
=
∪
Twistor function defined on
intersection of open sets, whose
union is the region (PT+) of interest
More generally, we may
require several open sets
to cover the region of interest.
The (1st) cohomology element is the
collection of functions defined on

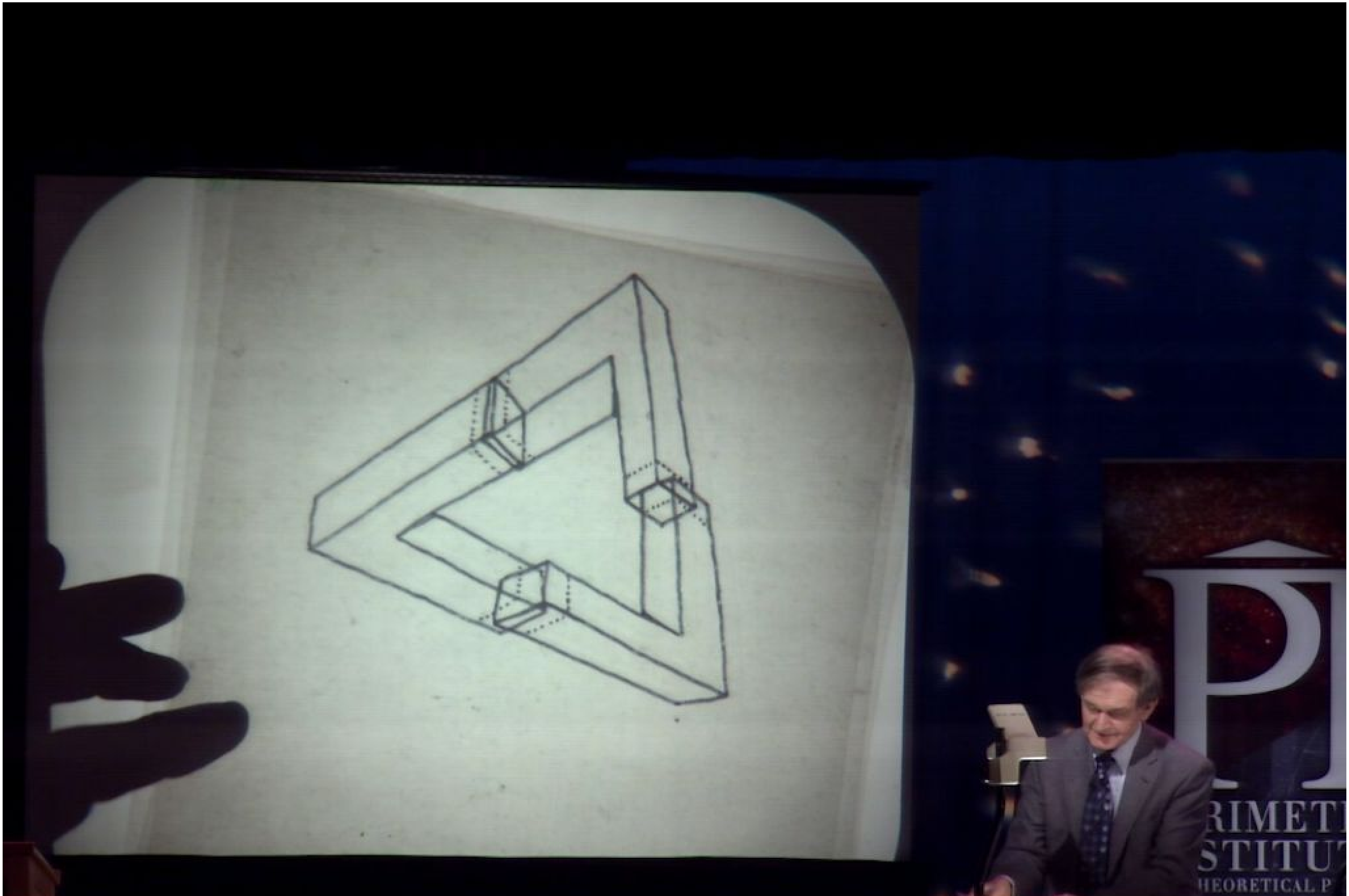

$\cup$
$\mathbb{PT}^+ = \mathcal{U}_1 \cup \mathcal{U}_2$
f defined (holomorphic) on $\mathcal{U}_1 \cap \mathcal{U}_2$
More generally: space $\mathcal{X}$ ($= \overset{\text{here}}{\mathbb{PT}^+}$)
$\mathcal{X} = \mathcal{U}_1 \cup \mathcal{U}_2 \cup \dots \cup \mathcal{U}_n$ ($\{\mathcal{U}_i\}$ open cover)
collection $\{f_{ij}\}$, where f_{ij} $(= -f_{ji})$ hol. on $\mathcal{U}_i \cap \mathcal{U}_j$
require $f_{ij} - f_{ik} + f_{jk} = 0$ on $\mathcal{U}_i \cap \mathcal{U}_j \cap \mathcal{U}_k$
$\{f_{ij}\} \equiv \{g_{ij}\}$ if each $f_{ij} - g_{ij} = h_i - h_j$ with h_k hol. on $\mathcal{U}_k$
Branched
tour
gral:
Riemann sphere

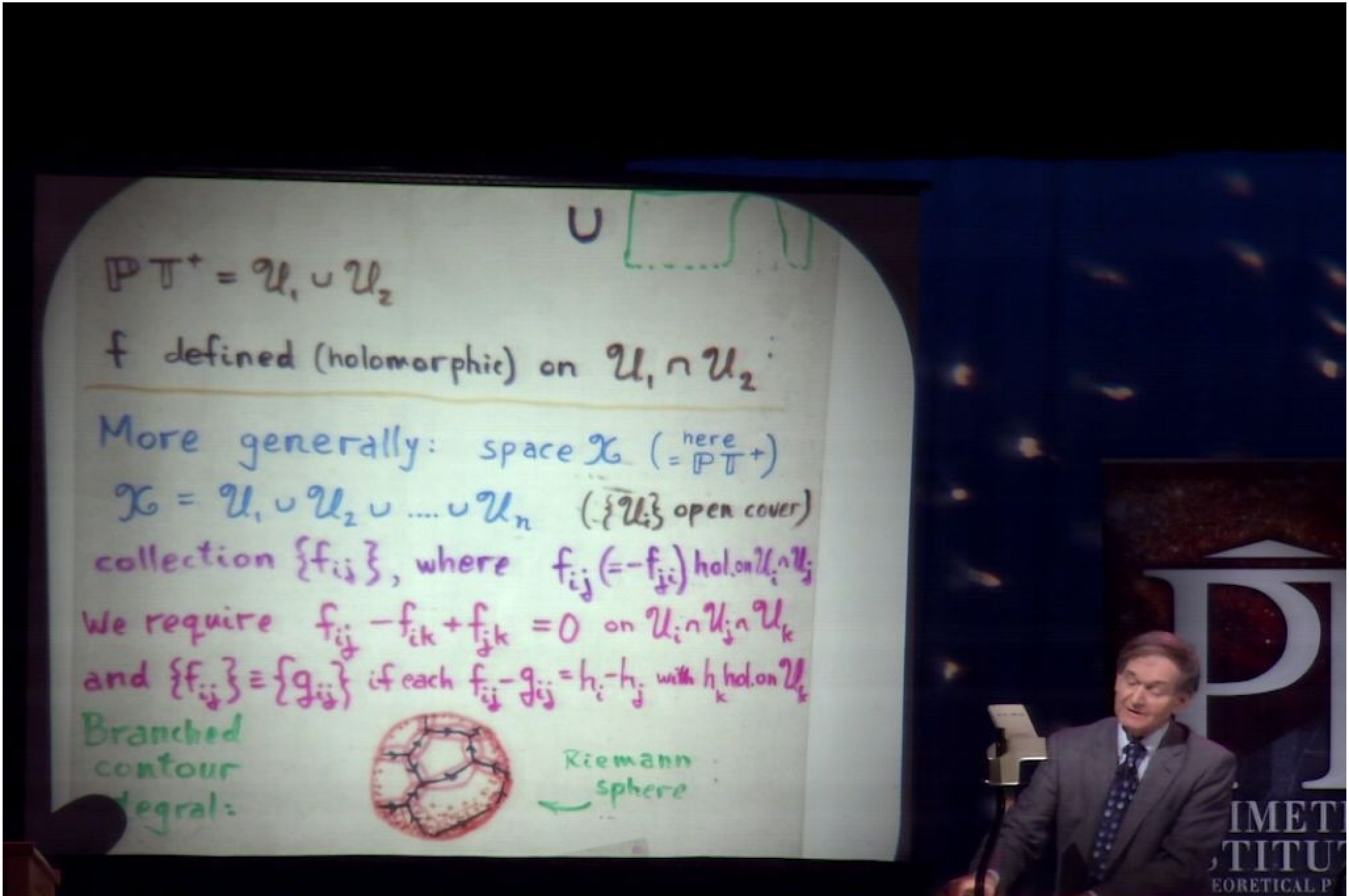



Cohomology:
a precise non-local
measure — here
of the degree of
IMPOSSIBILITY

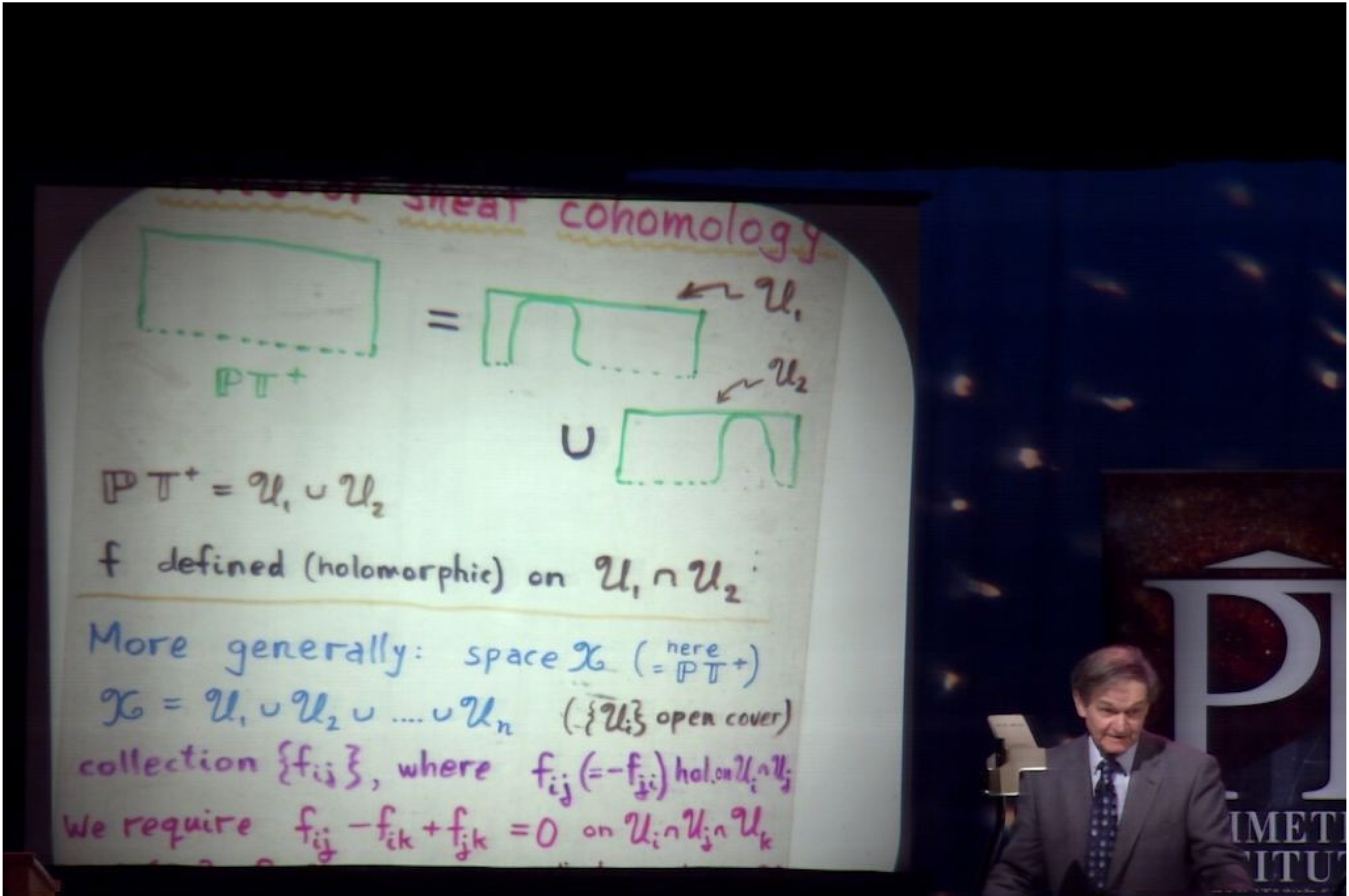


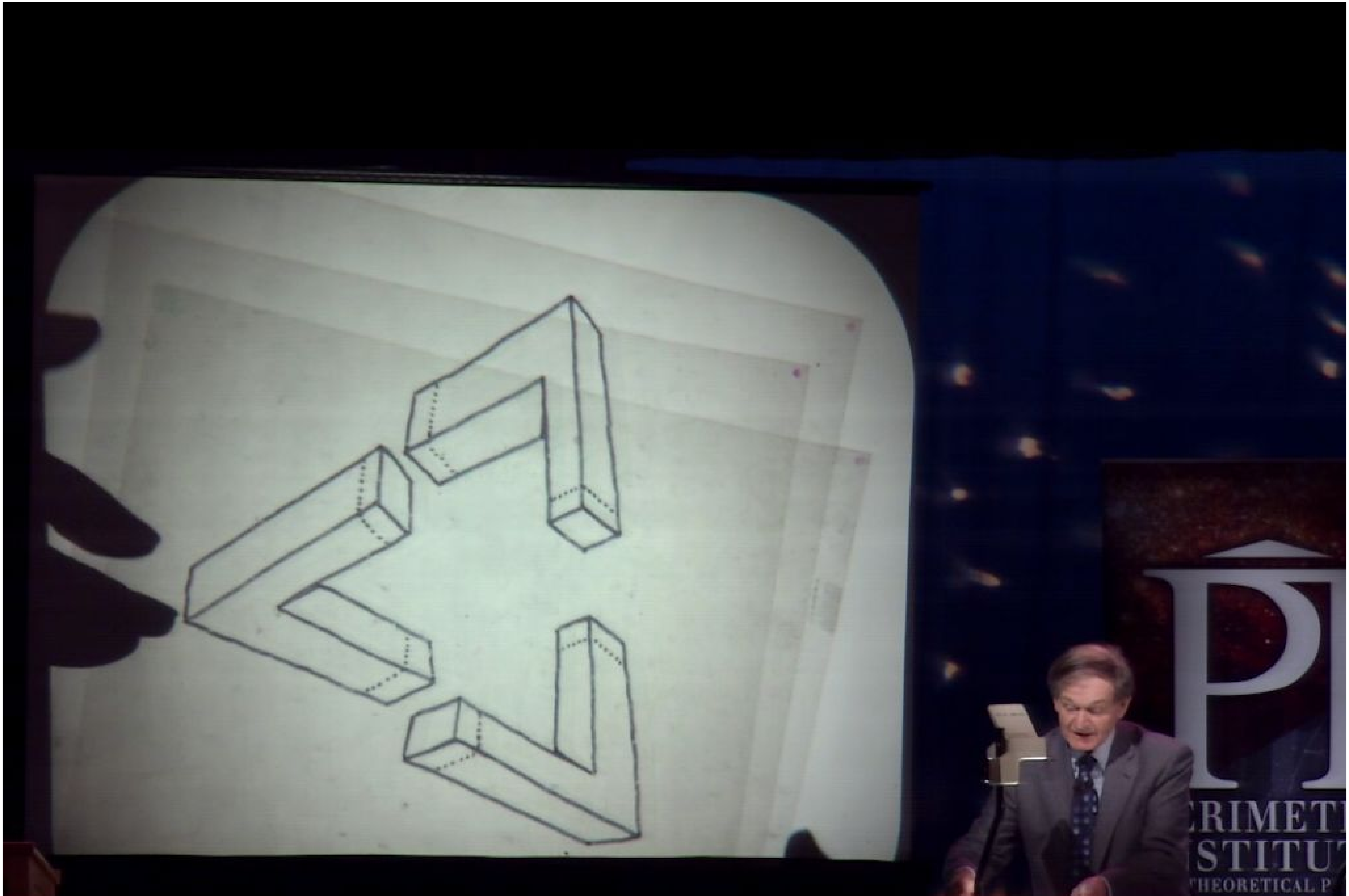


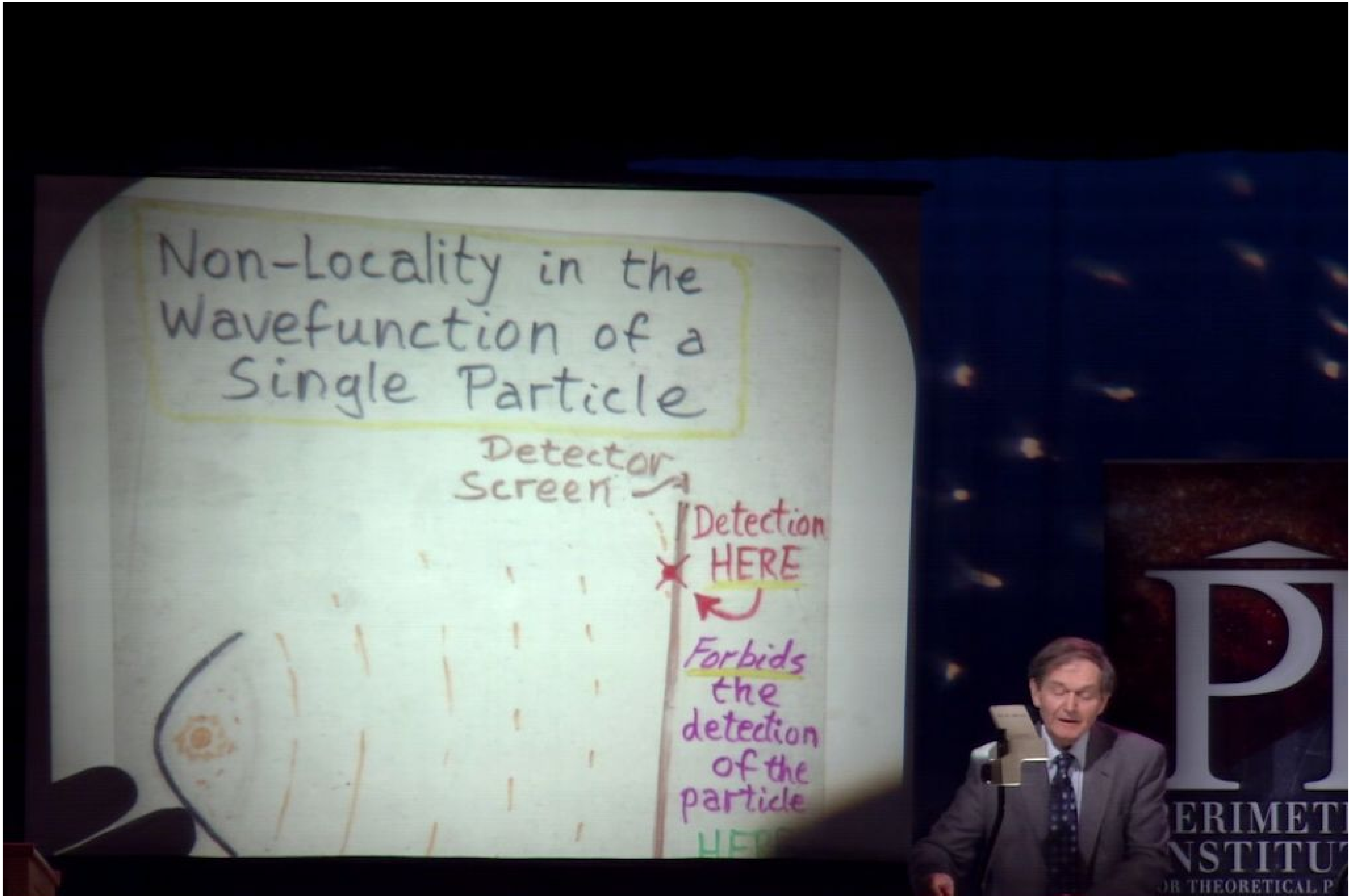


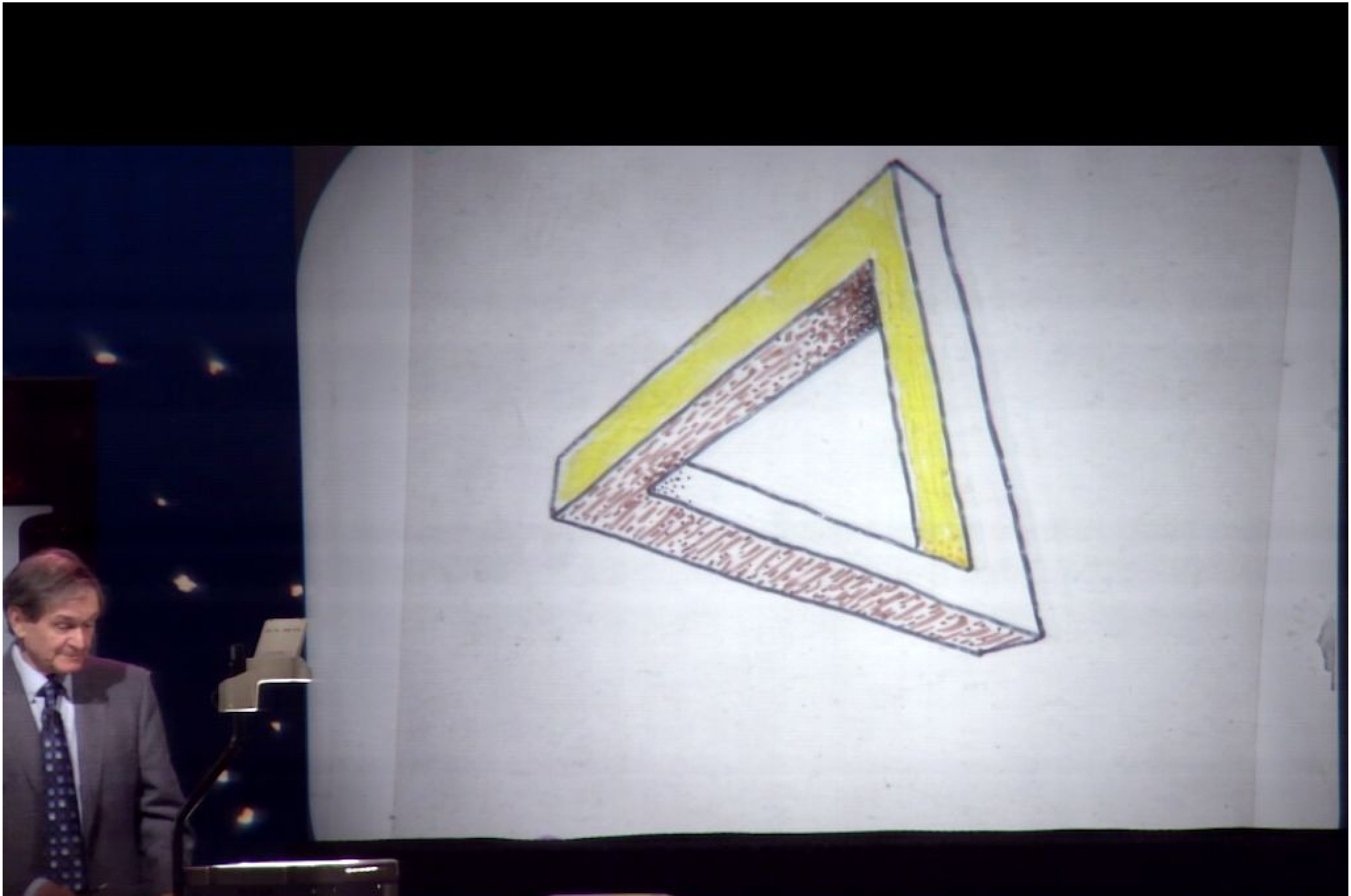


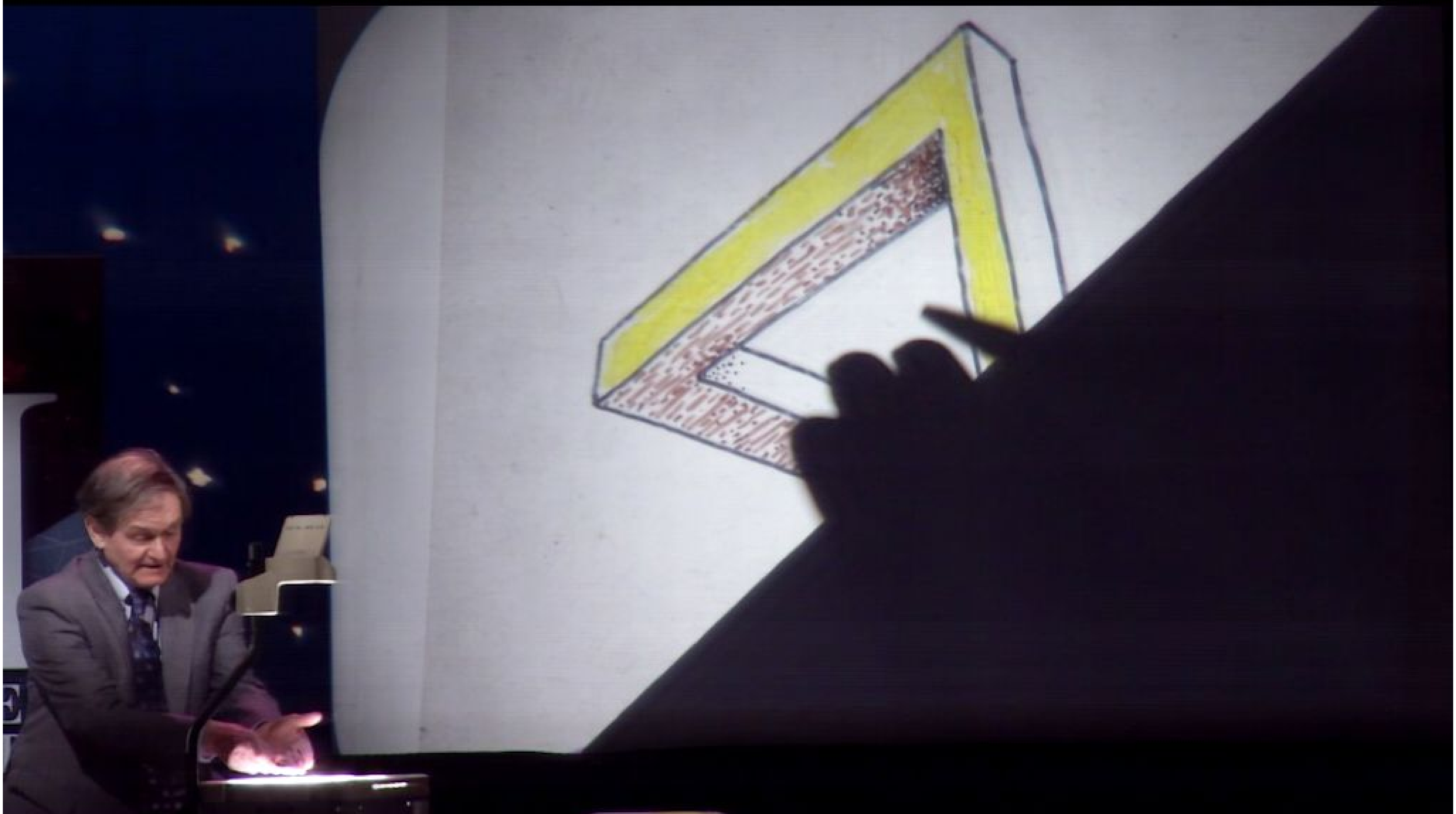
$\mathbb{PT}^+ = \mathcal{U}_1 \cup \mathcal{U}_2$
f defined (holomorphic) on $\mathcal{U}_1 \cap \mathcal{U}_2$
More generally: space $\mathcal{X}$ ($\overset{\text{here}}{=} \mathbb{PT}^+$)
$\mathcal{X} = \mathcal{U}_1 \cup \mathcal{U}_2 \cup \dots \cup \mathcal{U}_n$ ($\{\mathcal{U}_i\}$ open cover)
collection $\{f_{ij}\}$, where f_{ij} $(= -f_{ji})$ hol. on $\mathcal{U}_i \cap \mathcal{U}_j$
We require $f_{ij} - f_{ik} + f_{jk} = 0$ on $\mathcal{U}_i \cap \mathcal{U}_j \cap \mathcal{U}_k$
and $\{f_{ij}\} \equiv \{g_{ij}\}$ if each $f_{ij} - g_{ij} = h_i - h_j$ with h_k hol. on $\mathcal{U}_k$
Branched contour
Riemann sphere

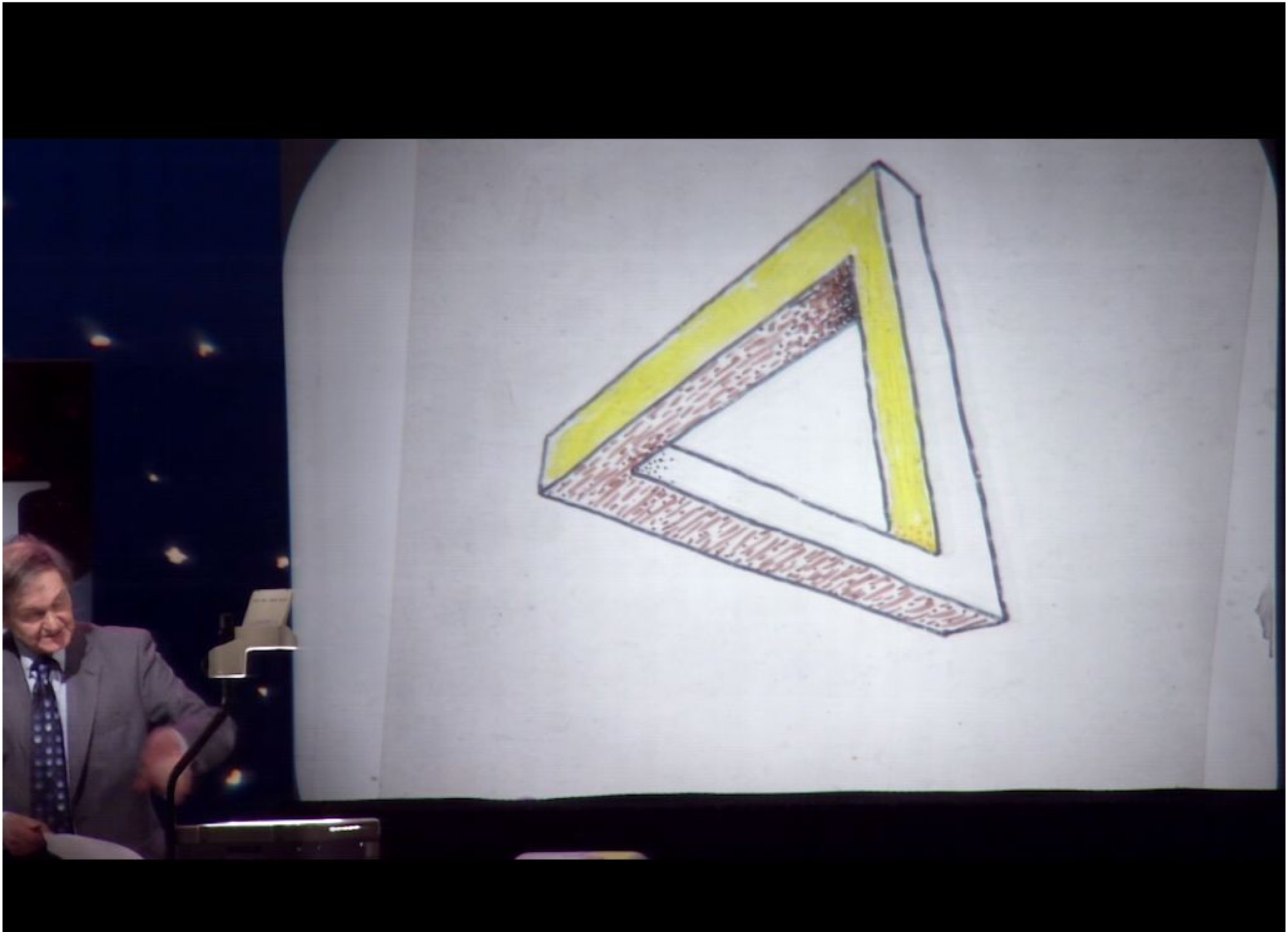

sheaf cohomology
$\mathbb{PT}^+$
$\mathcal{U}_1$
$\mathcal{U}_2$
$\mathbb{PT}^+ = \mathcal{U}_1 \cup \mathcal{U}_2$
f defined (holomorphic) on $\mathcal{U}_1 \cap \mathcal{U}_2$
More generally: space $\mathcal{X}$ (here $= \mathbb{PT}^+$)
$\mathcal{X} = \mathcal{U}_1 \cup \mathcal{U}_2 \cup \dots \cup \mathcal{U}_n$ ($\{\mathcal{U}_i\}$ open cover)
collection $\{f_{ij}\}$, where f_{ij} $(= -f_{ji})$ hol. on $\mathcal{U}_i \cap \mathcal{U}_j$
We require $f_{ij} - f_{ik} + f_{jk} = 0$ on $\mathcal{U}_i \cap \mathcal{U}_j \cap \mathcal{U}_k$

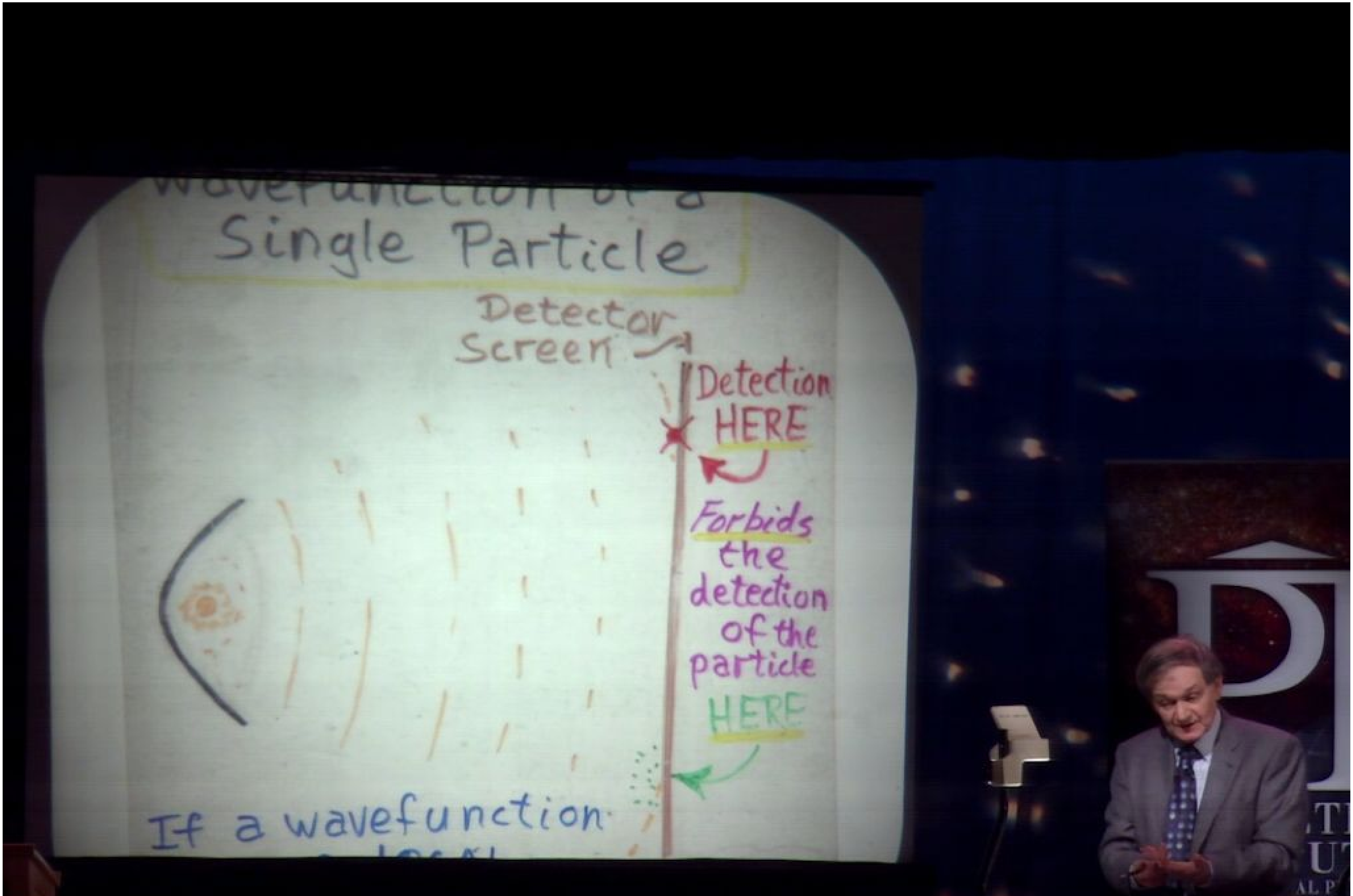



Non-Locality in the
Wavefunction of a
Single Particle
Detector
Screen
Detection
HERE
Forbids
the
detection
of the
particle
PERIMETER
INSTITUTE
FOR THEORETICAL P








Single Particle
Detector Screen
Detection HERE
Forbids the detection of the particle HERE
If a wavefunction