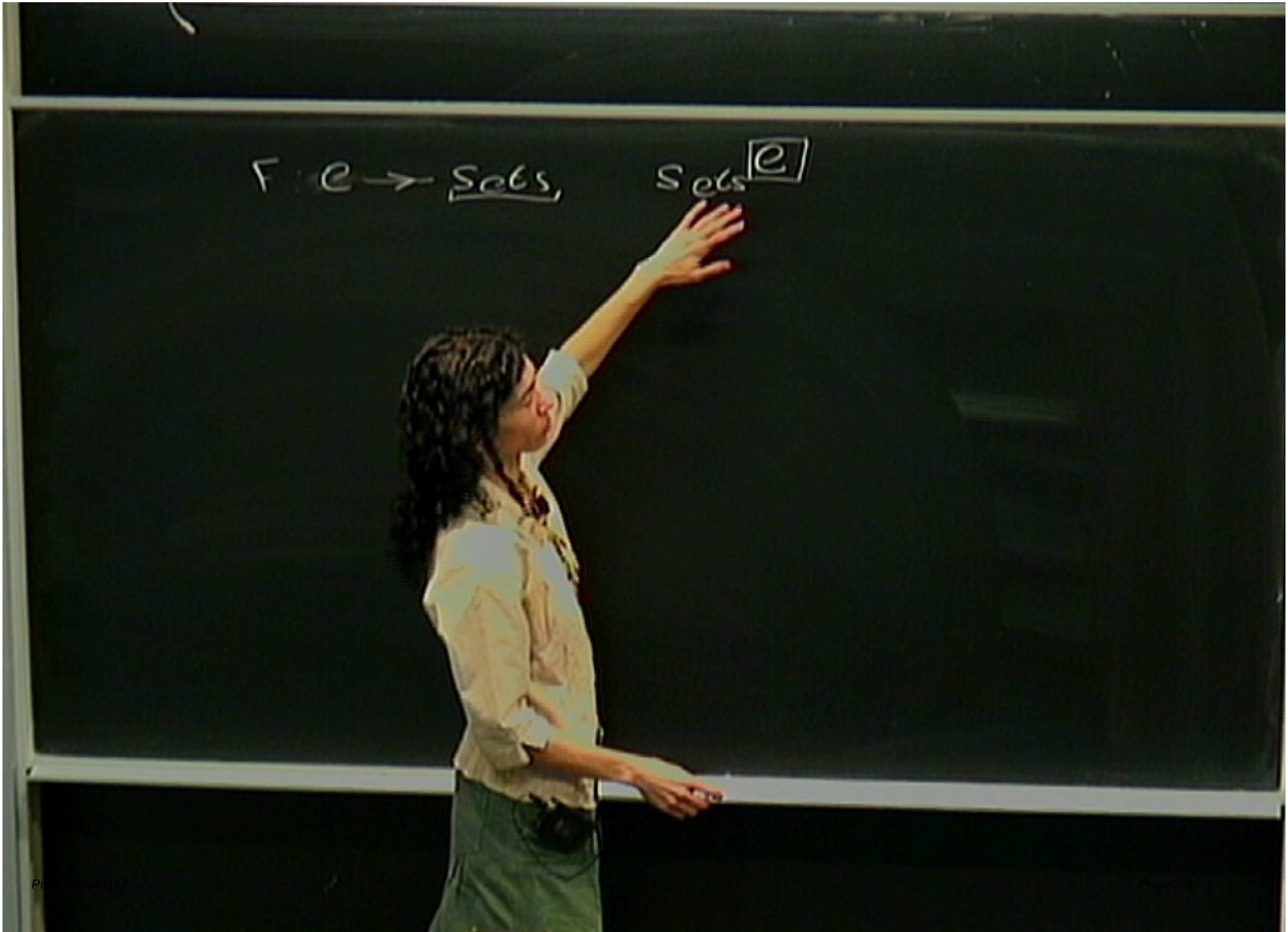
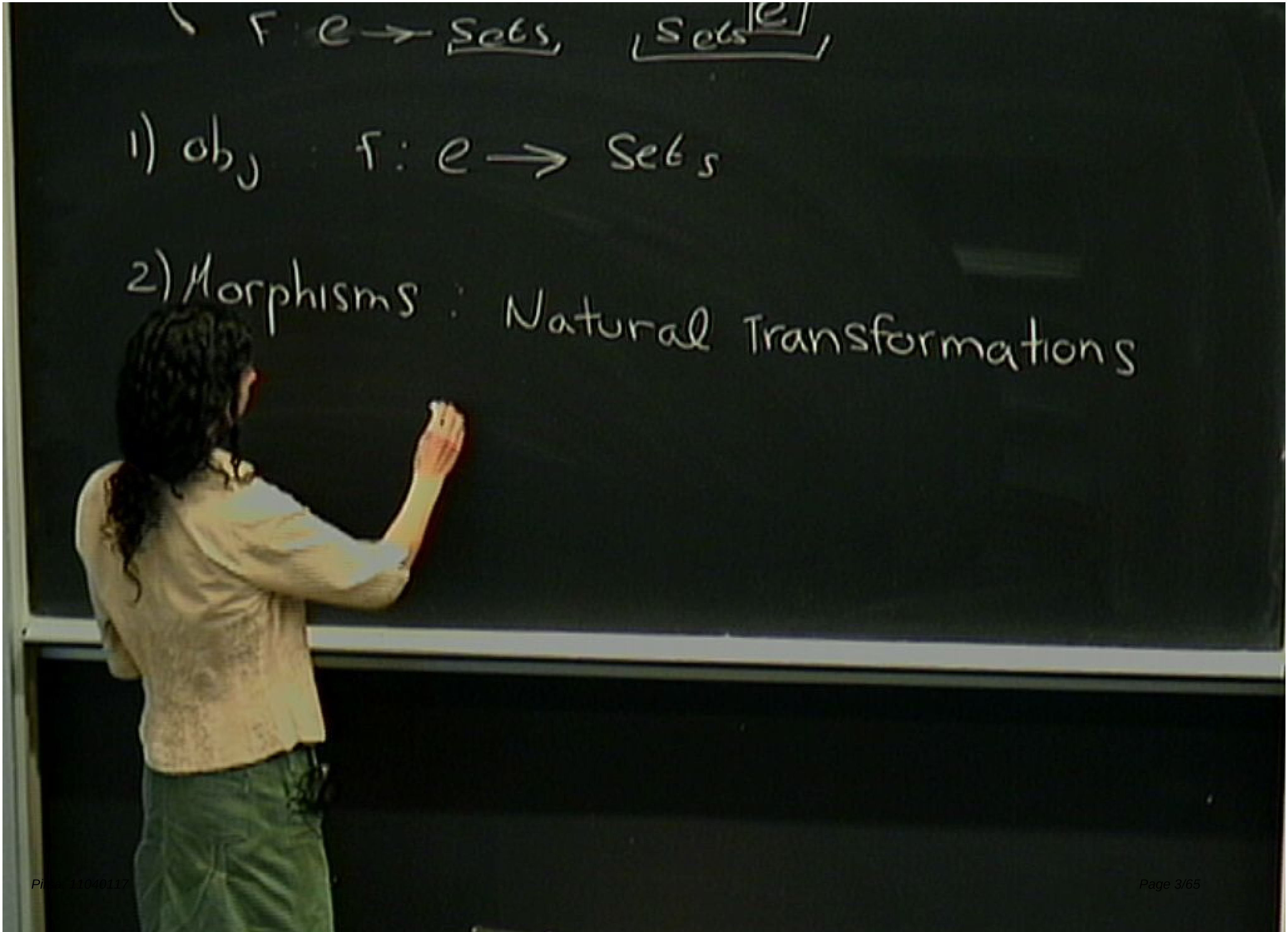
Title: Topos Theory as a Mathematical Universe - Lecture 2

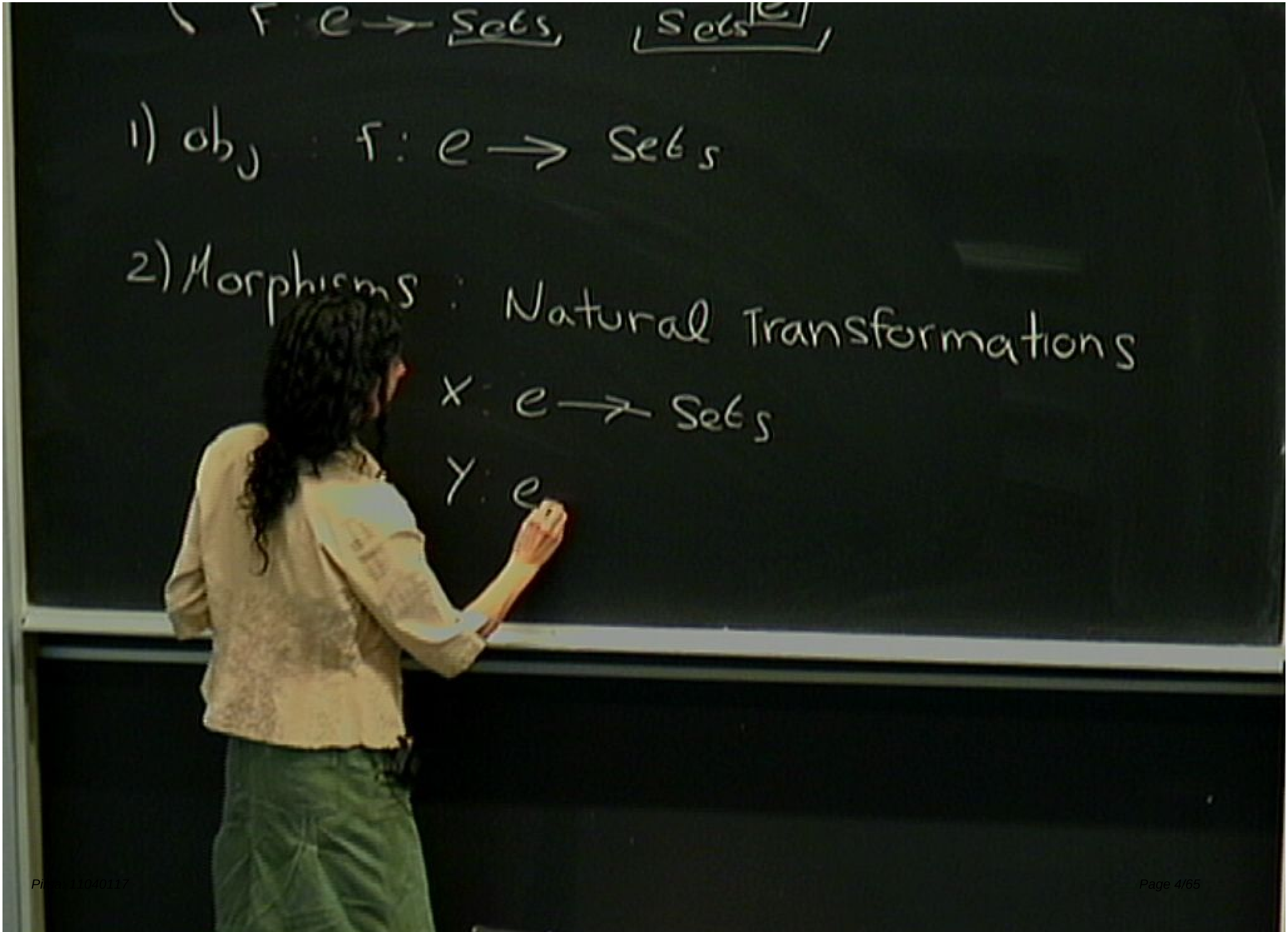
Date: May 05, 2011 11:00 AM

URL: http://pirsa.org/11040117

Abstract:


$F: \mathcal{C} \to \underline{Sets}$
$Sets^{\boxed{\mathcal{C}}}$


F: C → Sets, Sets^C
1) obj F: C → Sets
2) Morphisms : Natural Transformations


F : C → Sets, Sets^C
1) obj   F : C → Sets
2) Morphisms : Natural Transformations
X : C → Sets
Y : C

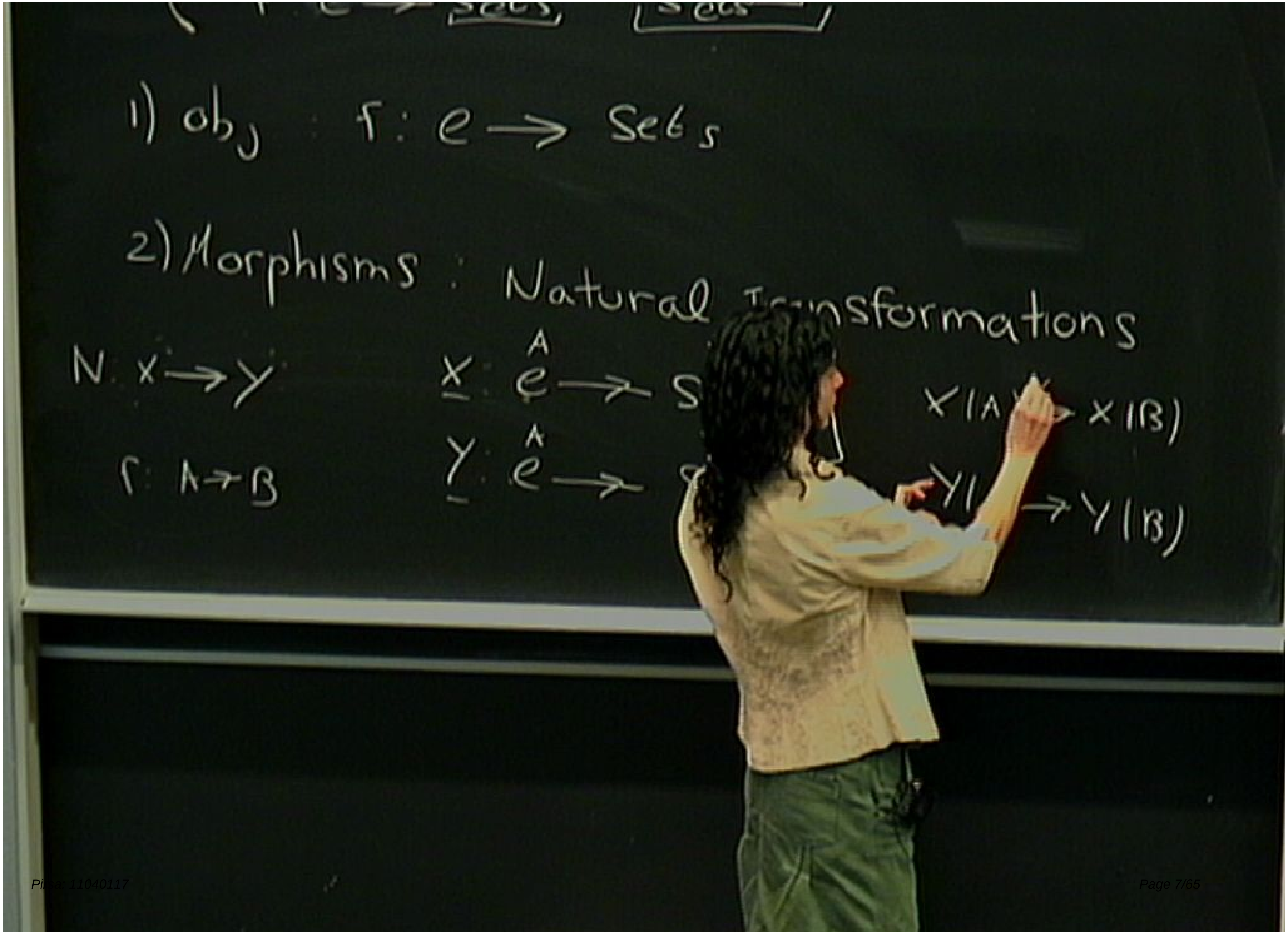

1) obj F : C → Sets
2) Morphisms : Natural Transformations
N : X → Y
X : C → Sets
Y : C → Sets

1) obj $\quad F: \mathcal{C} \to \text{Sets}$

2) Morphisms : Natural Transformations

$N: X \to Y$

$$\begin{array}{ll} \underline{X}: \overset{A}{\mathcal{C}} \longrightarrow \text{Sets} & X(A) \\ & \big\downarrow \\ \underline{Y}: \overset{A}{\mathcal{C}} \longrightarrow \text{Sets} & Y(A) \end{array}$$


1) obj F: C → Sets
2) Morphisms: Natural Transformations
N: X → Y
f: A → B
X: Ĉ →
Y: Ĉ →
X(A) → X(B)
Y(B)

1) obj $F: \mathcal{C} \to Sets$

2) Morphisms : Natural Transformations

$N: X \to Y$

$F: A \to B$

$\underline{X}: \hat{\mathcal{C}} \xrightarrow{} Sets$, $A \mapsto X(A)$

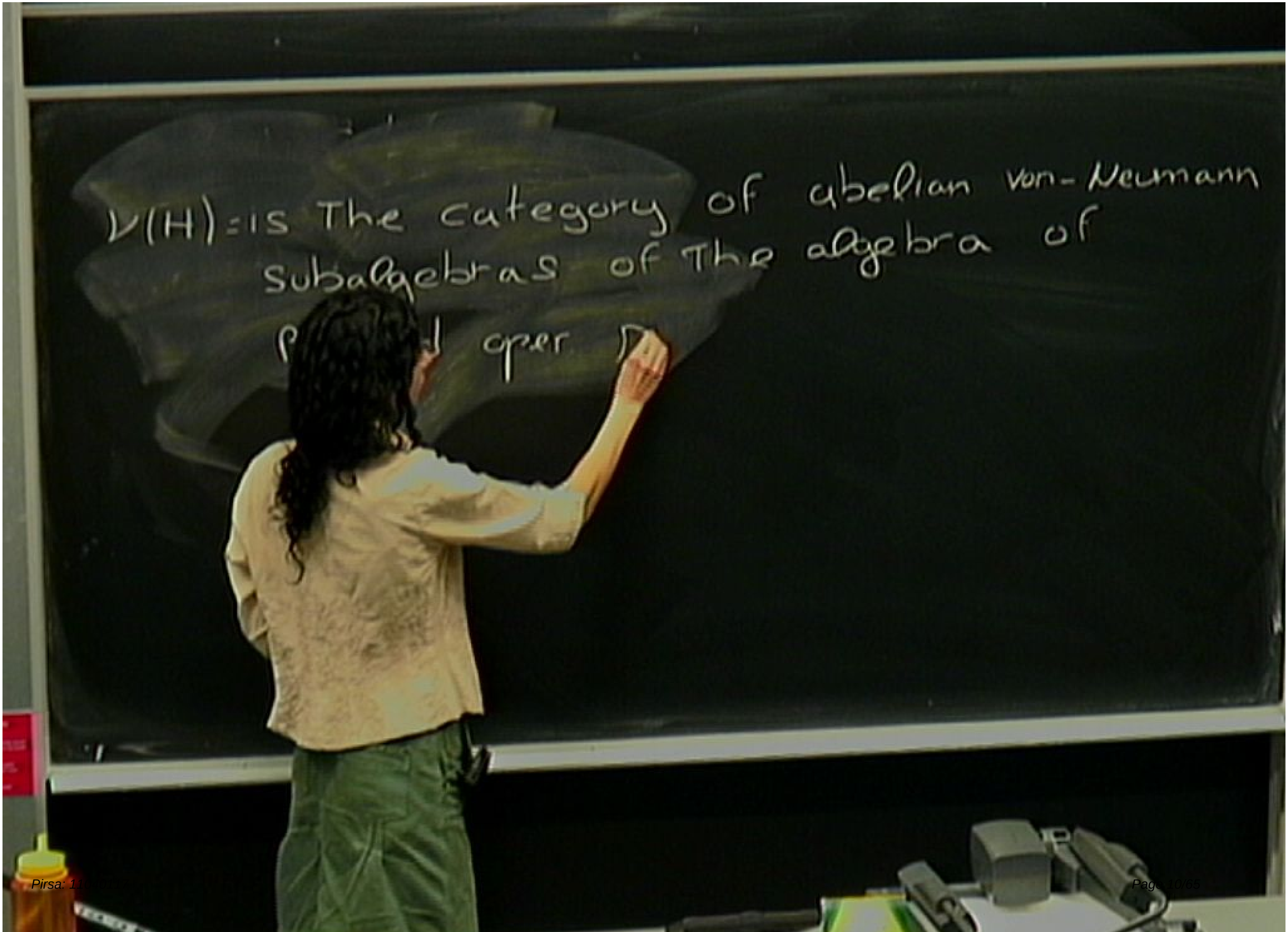
$\underline{Y}: \hat{\mathcal{C}} \xrightarrow{} Sets$, $A \mapsto Y(A)$

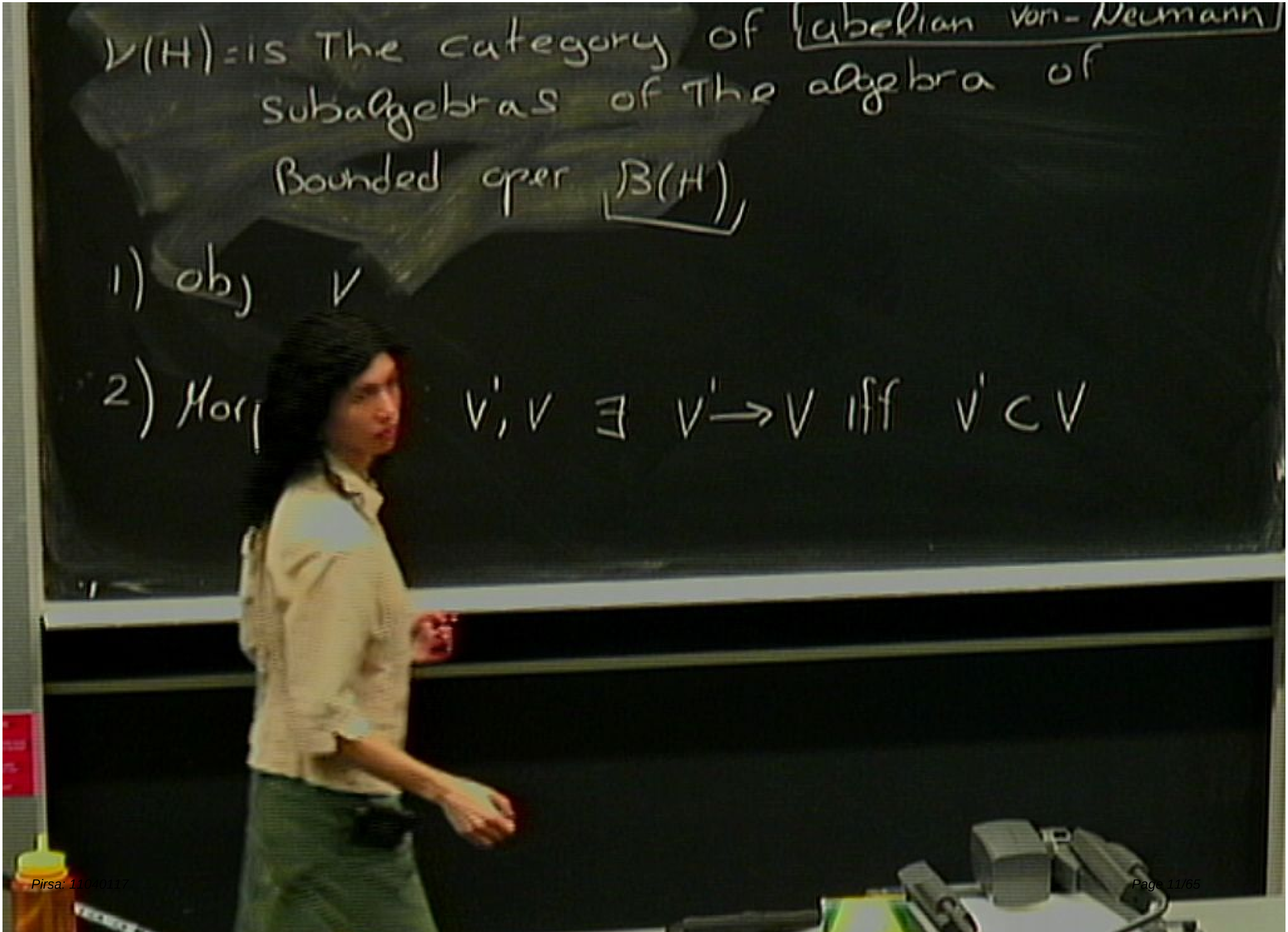
$X(A) \xrightarrow{X(F)} X(B)$

$Y(A) \xrightarrow{Y(F)} Y(B)$

$\forall\ A_i \in ob(\mathcal{C})$

$$\begin{tikzcd} X(A) \arrow[r, "N_A"] \arrow[d, shift left] & Y(A) \arrow[d, shift left] \\ X(B) \arrow[u, shift left, "X(f)"] \arrow[r, "N_B"'] & Y(B) \arrow[u, shift left, "Y(f)"'] \end{tikzcd}$$


V(H) := is The category of abelian von-Neumann
subalgebras of The algebra of
oper


V(H) := is The category of abelian von-Neumann
subalgebras of The algebra of
Bounded oper B(H)
1) obj V
2) Mor
V'; V ∃ V' → V iff V' ⊂ V

$V(H)$:= is The category of $\boxed{\text{abelian von-Neumann}}$ subalgebras of The algebra of Bounded oper $\boxed{B(H)}$

1) obj V

2) Morphisms V', V $\exists$ $V' \to V$ iff $V' \subset V$

$\mathcal{V}(H)$:= is The category of $\boxed{\text{abelian Von-Neumann}}$ subalgebras of The algebra of Bounded oper $\boxed{\mathcal{B}(H)}$

1) obj V

2) Morphisms V', V $\exists$ $V' \to V$ iff $V' \subset V$

$F : \mathcal{C} \to \underline{Sets}$, $\underbrace{Sets^{\boxed{\mathcal{C}}}}$ $\Uparrow$ $\quad Sets^{\mathcal{V}(H)}$

$\{F : \mathcal{V}(H) \to Sets$

1) obj $\quad F : \mathcal{C} \to Sets$

2) Morphisms : Natural Transformations

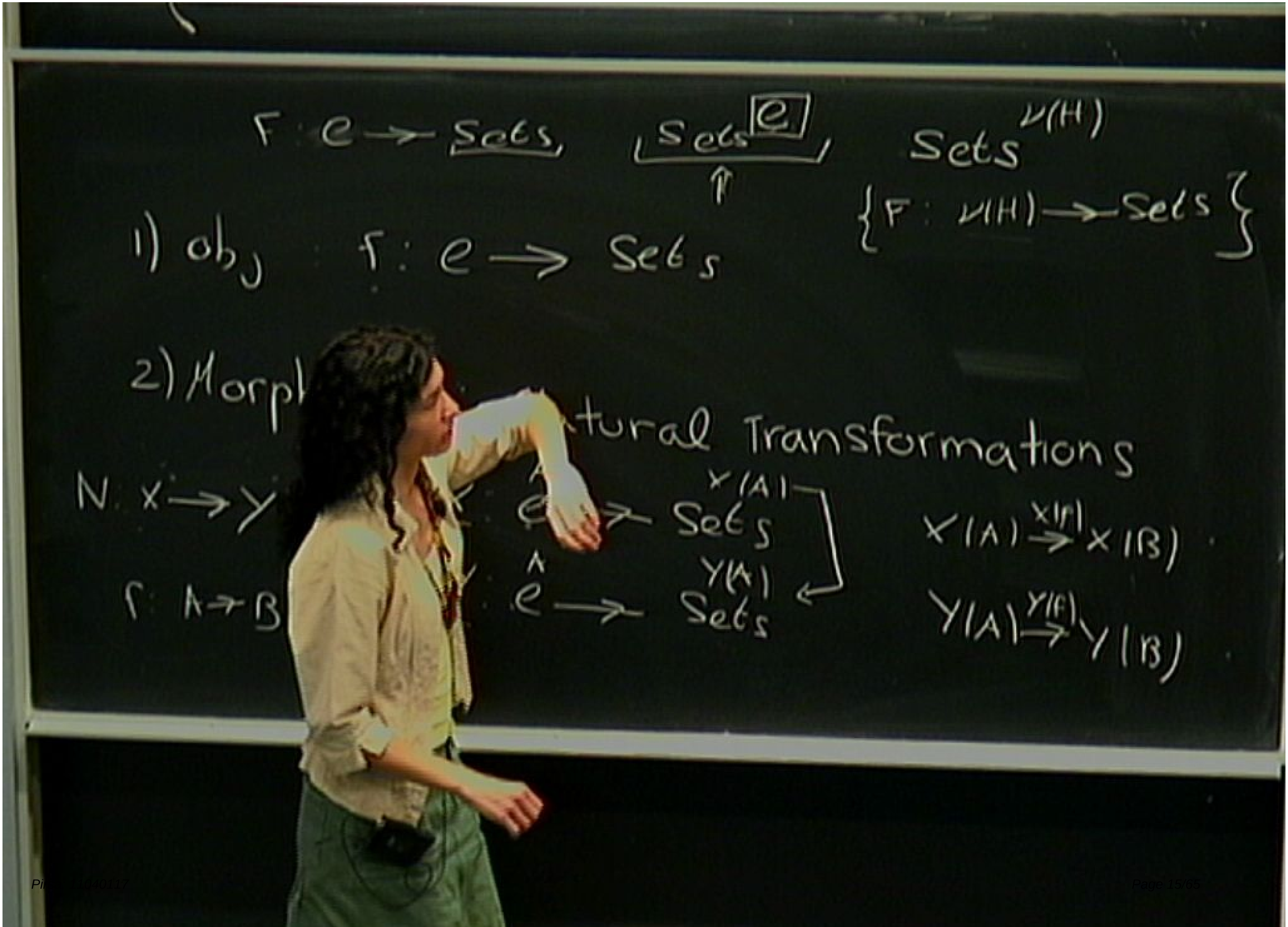
$\to Y$

$\underline{X} : \overset{A}{\mathcal{C}} \to \overset{X(A)}{Sets}$

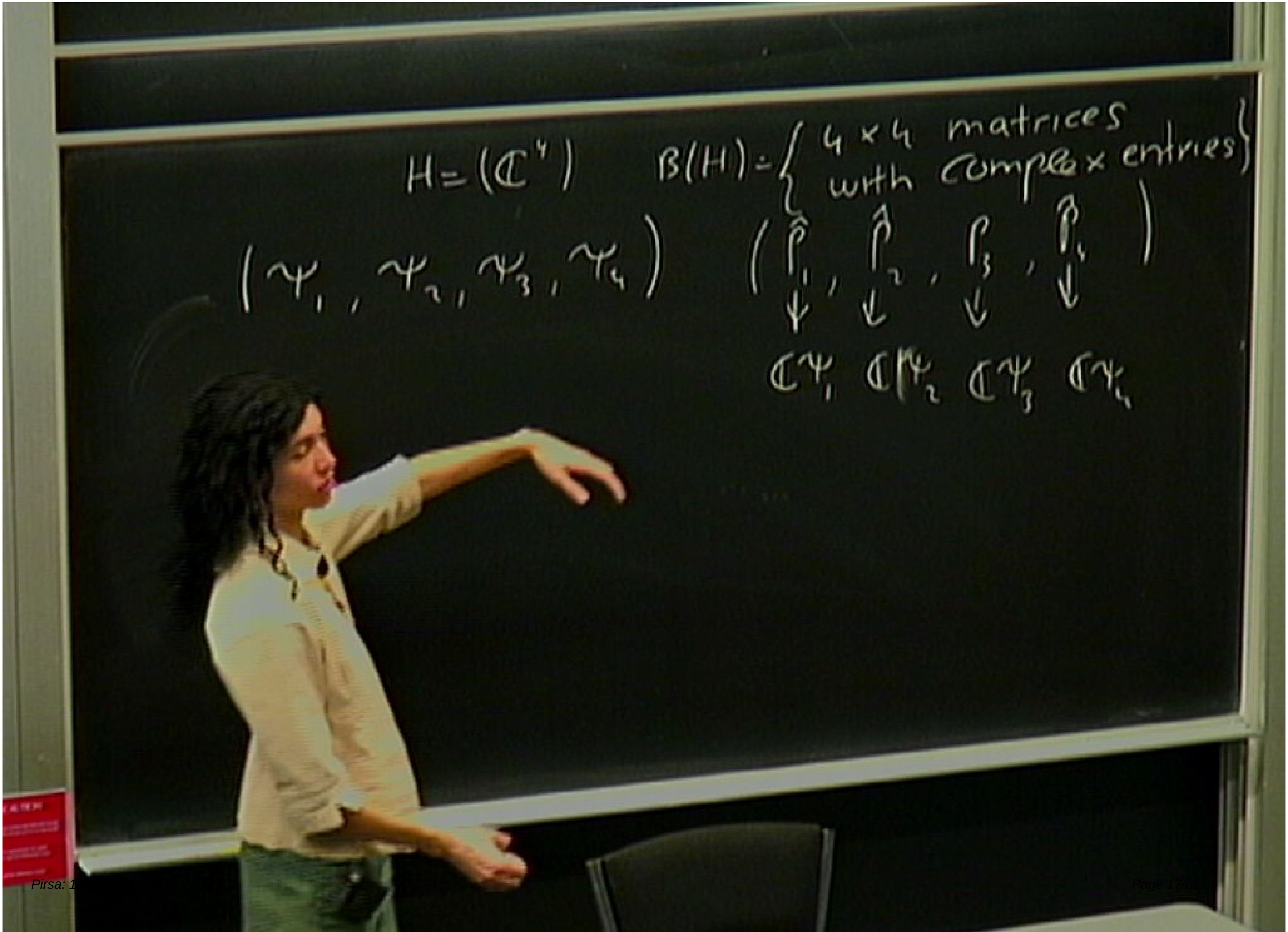
$\underline{Y} : \overset{A}{\mathcal{C}} \to \overset{Y(A)}{Sets}$

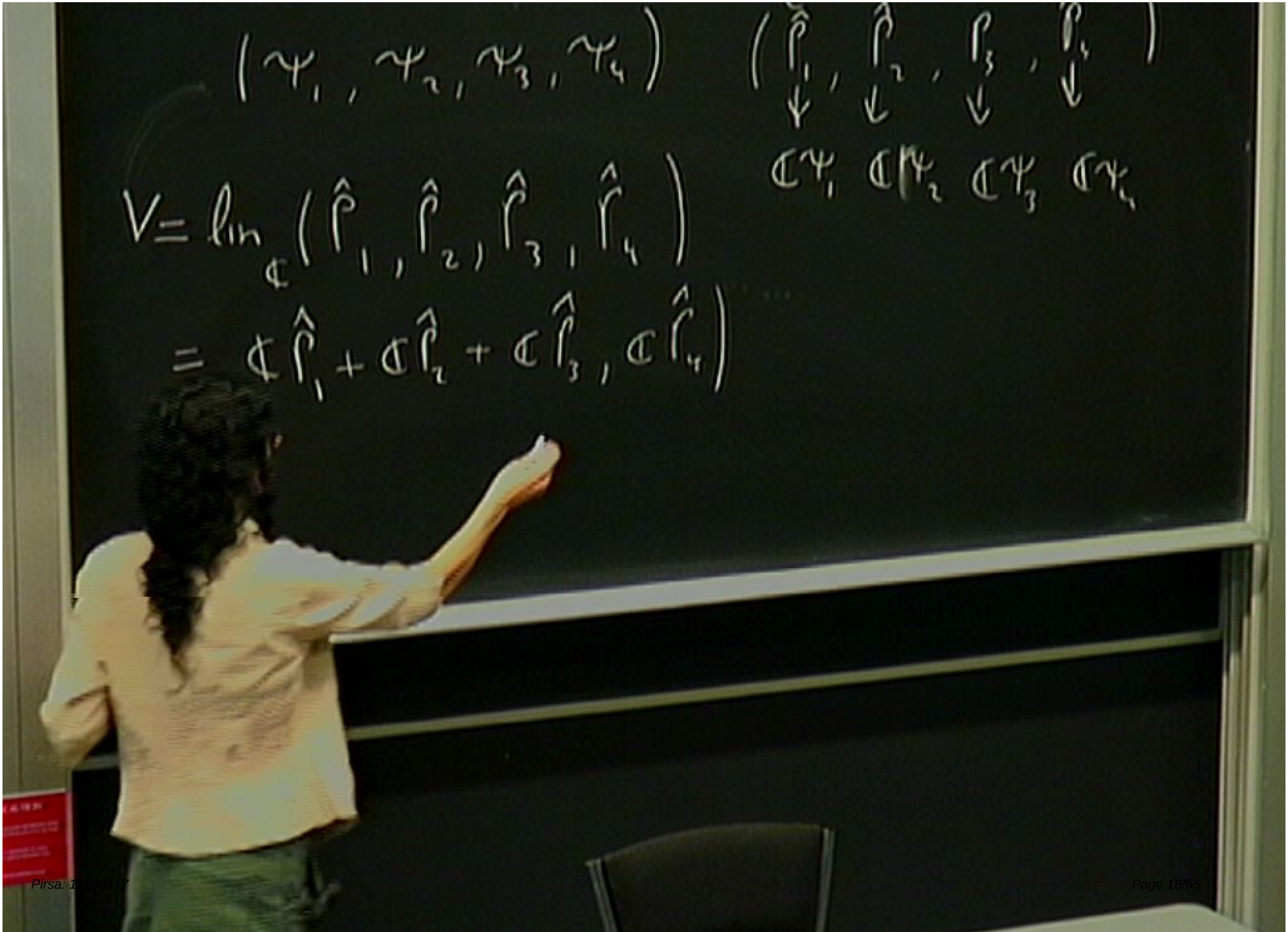
$X(A) \xrightarrow{X(f)} X(B)$

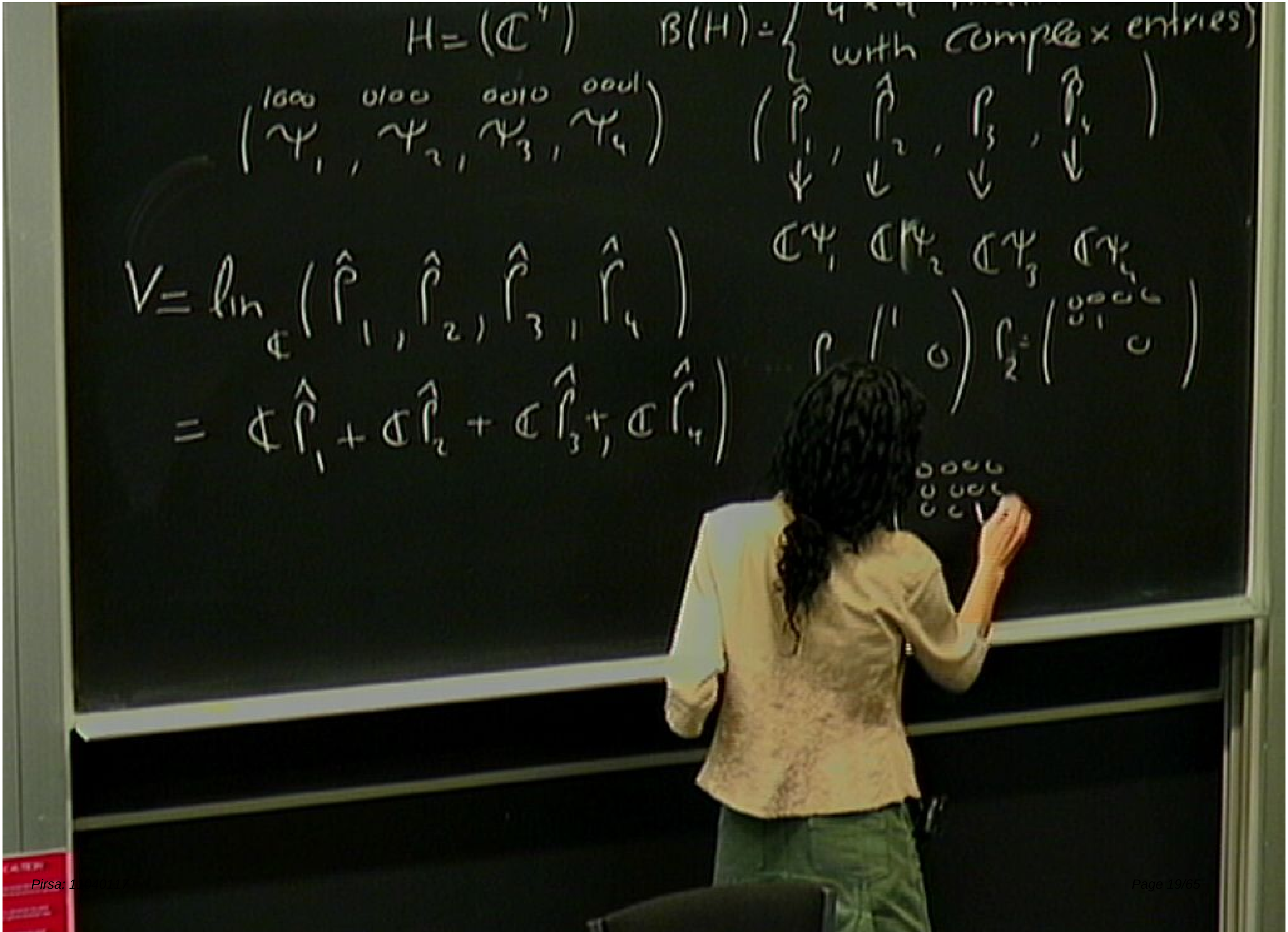
$Y(A) \xrightarrow{Y(f)} Y(B)$

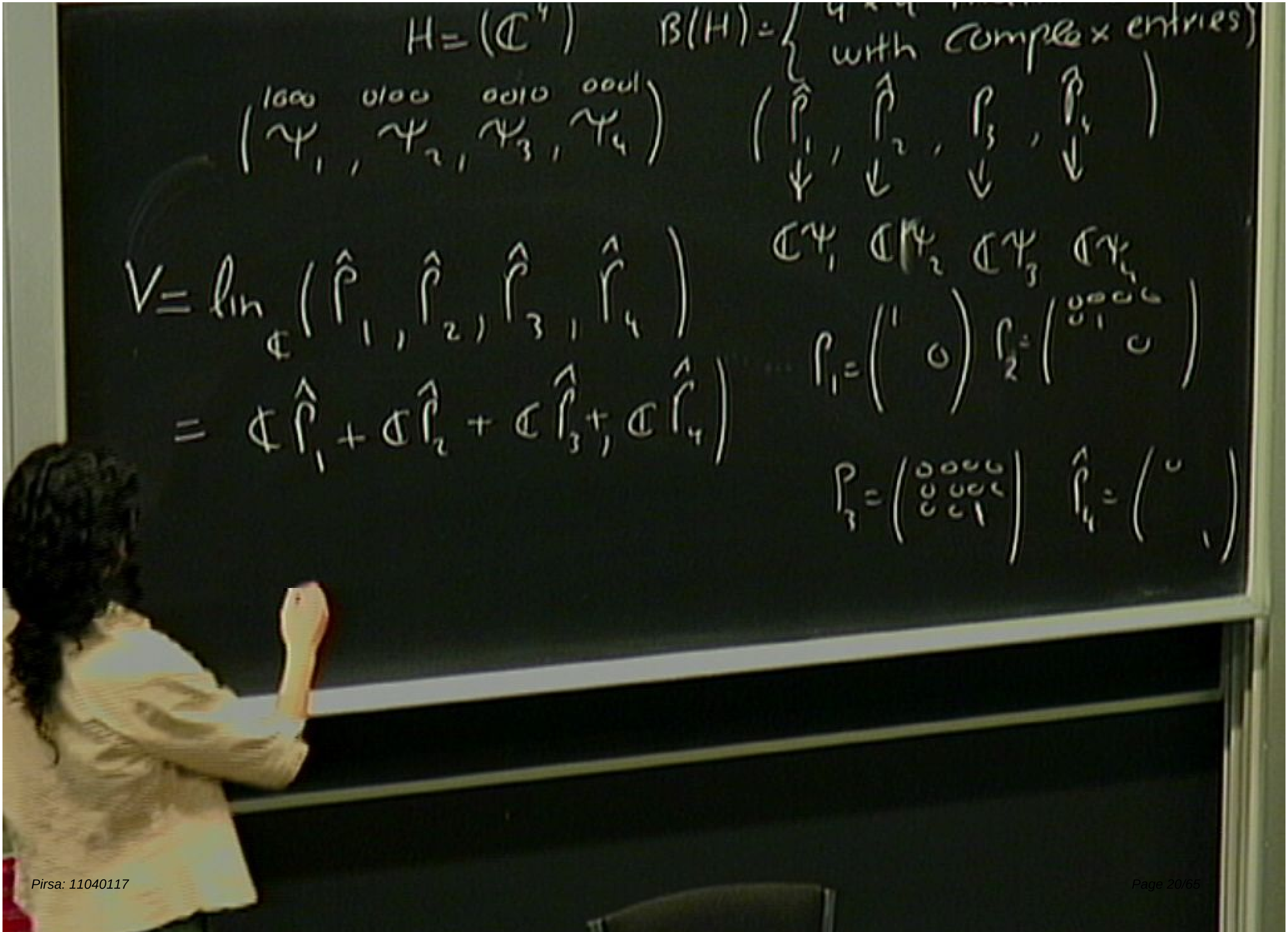

F : C → Sets, Sets^C
Sets^ν(H)
{F : ν(H) → Sets}
1) obj F : C → Sets
2) Morph
tural Transformations
N : X → Y
C → Sets
X(A)
Y(A)
C → Sets
f : A → B
X(A) → X(B)
X(f)
Y(A) → Y(B)
Y(f)

$$H = (\mathbb{C}^4) \qquad B(H) = \left\{ \begin{array}{l} 4 \times 4 \text{ matrices} \\ \text{with complex entries} \end{array} \right\}$$


$H = (\mathbb{C}^4)$
$B(H) = \{$ 4 × 4 matrices with complex entries $\}$
$(\psi_1, \psi_2, \psi_3, \psi_4)$
$(\hat{P}_1, \hat{P}_2, P_3, \hat{P}_4)$
$\mathbb{C}\psi_1$ $\mathbb{C}\psi_2$ $\mathbb{C}\psi_3$ $\mathbb{C}\psi_4$


$(\Psi_1, \Psi_2, \Psi_3, \Psi_4)$
$(\hat{\rho}_1, \hat{\rho}_2, \rho_3, \rho_4)$
$\mathbb{C}\Psi_1 \quad \mathbb{C}\Psi_2 \quad \mathbb{C}\Psi_3 \quad \mathbb{C}\Psi_4$
$V = \mathrm{lin}_{\mathbb{C}}(\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4)$
$= \mathbb{C}\hat{\rho}_1 + \mathbb{C}\hat{\rho}_2 + \mathbb{C}\hat{\rho}_3, \mathbb{C}\hat{\rho}_4)$


H = (ℂ⁴)
B(H) ≐ { 4×4 matrices with complex entries }
(Ψ̃₁, Ψ̃₂, Ψ̃₃, Ψ̃₄)
1000 0100 0010 0001
(ρ̂₁, ρ̂₂, ρ₃, ρ̂₄)
ℂΨ₁ ℂΨ₂ ℂΨ₃ ℂΨ₄
V = lin_ℂ (ρ̂₁, ρ̂₂, ρ̂₃, ρ̂₄)
= ℂρ̂₁ + ℂρ̂₂ + ℂρ̂₃ + ℂρ̂₄


$H = (\mathbb{C}^4)$
$B(H) = \{$ with complex entries $\}$
$(\tilde\psi_1, \tilde\psi_2, \tilde\psi_3, \tilde\psi_4)$
$(\hat\rho_1, \hat\rho_2, \rho_3, \hat\rho_4)$
$\mathbb{C}\psi_1 \quad \mathbb{C}\psi_2 \quad \mathbb{C}\psi_3 \quad \mathbb{C}\psi_4$
$V = \mathrm{lin}_{\mathbb{C}}(\hat\rho_1, \hat\rho_2, \hat\rho_3, \hat\rho_4)$
$= \mathbb{C}\hat\rho_1 + \mathbb{C}\hat\rho_2 + \mathbb{C}\hat\rho_3 + \mathbb{C}\hat\rho_4$

$$\begin{array}{cccc} 1000 & 0100 & 0010 & 0001 \\ \tilde{\Psi}_1, & \tilde{\Psi}_2, & \tilde{\Psi}_3, & \tilde{\Psi}_4 \end{array}$$

$$\begin{pmatrix} \hat{P}_1, & \hat{P}_2, & \hat{P}_3, & \hat{P}_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \mathbb{C}\Psi_1 & \mathbb{C}\Psi_2 & \mathbb{C}\Psi_3 & \mathbb{C}\Psi_4 \end{pmatrix}$$

$$V = \operatorname{lin}_{\mathbb{C}}\left(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4\right)$$

$$= \mathbb{C}\hat{P}_1 + \mathbb{C}\hat{P}_2 + \mathbb{C}\hat{P}_3 + \mathbb{C}\hat{P}_4$$

$$\left\{ \begin{array}{l} 4\times 4 \text{ diagonal matrices} \\ \text{with complex entries} \end{array} \right\}$$

$$P_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ & & & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \end{pmatrix} \quad \hat{P}_4 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$H = (\mathbb{C}^4)$ $\quad B(H) = \left\{ \begin{array}{l} 4 \times 4 \text{ matrices} \\ \text{with complex entries} \end{array} \right\}$

$\left(\overset{1000}{\tilde{\psi}_1}, \overset{0100}{\tilde{\psi}_2}, \overset{0010}{\tilde{\psi}_3}, \overset{0001}{\tilde{\psi}_4} \right)$ $\quad \left(\hat{P}_1, \hat{P}_2, P_3, \hat{P}_4 \right)$

ρ, ρ $\quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\mathbb{C}\psi_1 \quad \mathbb{C}\psi_2 \quad \mathbb{C}\psi_3 \quad \mathbb{C}\psi_4$

$\left(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4 \right)$

$P_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$ $\quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ & & & 0 \end{pmatrix}$

$+ \mathbb{C}\hat{P}_3 + \mathbb{C}\hat{P}_4)$

$\ldots$ diagonal matrices with complex entries $\}$

$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \end{pmatrix}$ $\quad \hat{P}_4 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$

$$H = (\mathbb{C}^4) \qquad B(H) = \left\{ \begin{array}{l} 4\times 4 \text{ matrices} \\ \text{with complex entries} \end{array} \right\}$$

$$\left(\overset{1000}{\psi_1}, \overset{0100}{\psi_2}, \overset{0010}{\psi_3}, \overset{0001}{\psi_4} \right)$$

$$\rho_1, \quad \rho_2, \quad \rho_3, \quad \rho_4$$

$$\left(\hat{P}_1, \hat{P}_2, P_3, \hat{P}_4 \right)$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\mathbb{C}\psi_1 \quad \mathbb{C}\psi_2 \quad \mathbb{C}\psi_3 \quad \mathbb{C}\psi_4$$

$$V = \mathrm{lin}_{\mathbb{C}} \left(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4 \right)$$

$$= \mathbb{C}\hat{P}_1 + \mathbb{C}\hat{P}_2 + \mathbb{C}\hat{P}_3 + \mathbb{C}\hat{P}_4$$

$$\left\{ \begin{array}{l} 4\times 4 \text{ diagonal matrices} \\ \text{with complex entries} \end{array} \right\}$$

$$P_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ & & & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \end{pmatrix} \quad \hat{P}_4 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$H = (\mathbb{C}^4)$ $\quad B(H) = \left\{ \begin{array}{l} 4\times 4 \text{ matrices} \\ \text{with complex entries} \end{array} \right\}$

$$\left(\begin{array}{cccc} 1000 & 0100 & 0010 & 0001 \\ \tilde\psi_1, & \tilde\psi_2, & \tilde\psi_3, & \tilde\psi_4 \\ \rho_1 & \rho_2 & \rho_3 & \rho_4 \end{array} \right) \qquad \left(\begin{array}{cccc} \hat P_1, & \hat P_2, & P_3, & \hat P_4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \mathbb{C}\psi_1 & \mathbb{C}\psi_2 & \mathbb{C}\psi_3 & \mathbb{C}\psi_4 \end{array} \right)$$

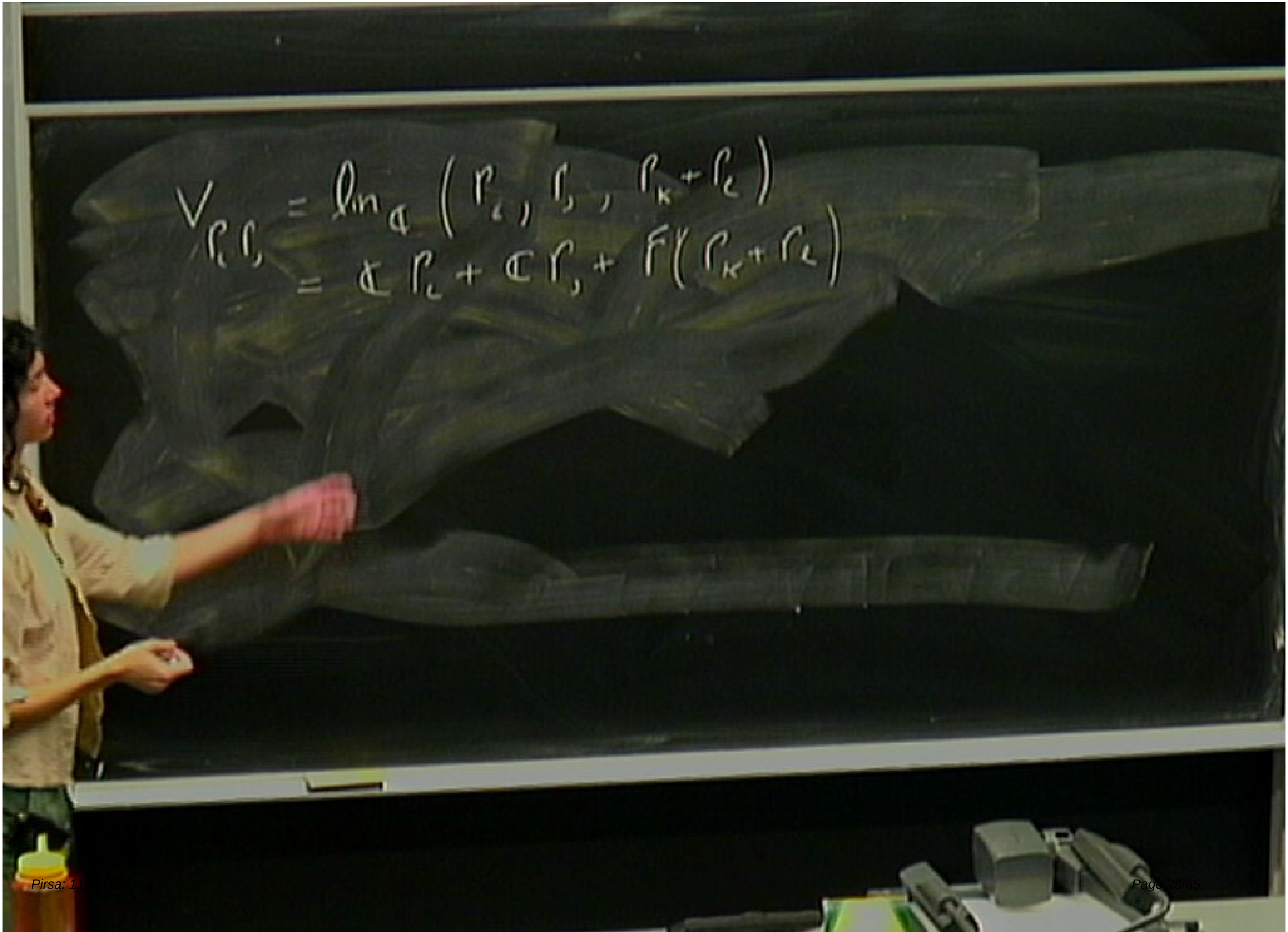
$$V = \mathrm{lin}_{\mathbb{C}} \left(\hat P_1, \hat P_2, \hat P_3, \hat P_4 \right)$$

$$= \mathbb{C}\hat P_1 + \mathbb{C}\hat P_2 + \mathbb{C}\hat P_3 + \mathbb{C}\hat P_4$$

$$P_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ & & 0 & \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \end{pmatrix} \quad \hat P_4 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$$\left\{ \begin{array}{l} 4\times 4 \text{ diagonal matrices} \\ \text{with complex entries} \end{array} \right\}$$



$V_{\rho_i \rho_j} = \mathrm{lin}_{\mathbb{C}}(\rho_i, \rho_j, \rho_k + \rho_l)$
$= \mathbb{C}\rho_i + \mathbb{C}\rho_j + F(\rho_k + \rho_l)$

$$V_{\Gamma_i \Gamma_j} = \mathrm{lin}_{\mathbb{C}} \left(\Gamma_i , \Gamma_j , \Gamma_k + \Gamma_l \right) \qquad \left((\Gamma_1), \hat{Q}_2, \hat{Q}_3, \hat{Q}_4 \right)$$

$$= \mathbb{C}\,\Gamma_i + \mathbb{C}\,\Gamma_j + F\left(\Gamma_k + \Gamma_l\right)$$

$$V_{\Gamma_i} = \mathbb{C}\,\Gamma_i + \mathbb{C}\left(\Gamma_j + \Gamma_k + \Gamma_l\right)$$

$$V_{P_i P_j} = \mathrm{lin}_{\mathbb{C}}\left(P_i,\ P_j,\ P_k + P_l\right) \qquad \left(P_1,\ \hat{Q}_2,\ \hat{Q}_3,\ \hat{Q}_4\right)$$

$$= \mathbb{C} P_i + \mathbb{C} P_j + F\left(P_k + P_l\right)$$

$$V'' = \mathbb{C}\hat{P}_1 + \mathbb{C}\hat{Q}_2 + \mathbb{C}\hat{Q}_3 + \mathbb{C}\hat{Q}_4$$

$$V^{\parallel} = \mathbb{C}\hat{P}_1 + \mathbb{C}\hat{Q}_2 + \mathbb{C}\hat{Q}_3 + \mathbb{C}\hat{Q}_4$$

$$V_{P_1} = \mathbb{C}\hat{P}_1 + \mathbb{C}\left(\hat{P}_2 + \hat{P}_3 + \hat{P}_4\right) =$$

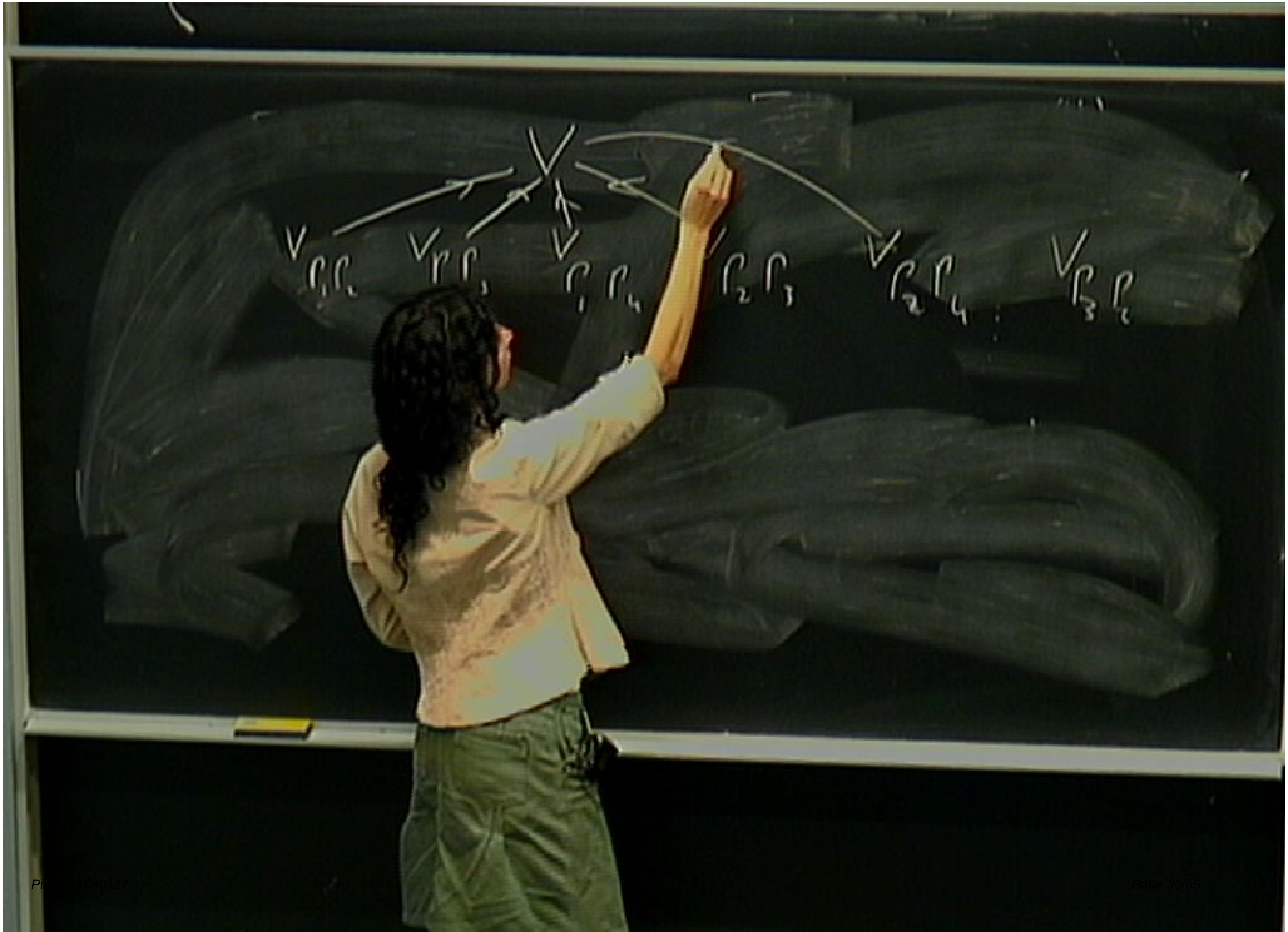
$$= \mathbb{C}\hat{P}_1 + \mathbb{C}\left(\hat{Q}_2 + \hat{Q}_3 + \hat{Q}_4\right)$$

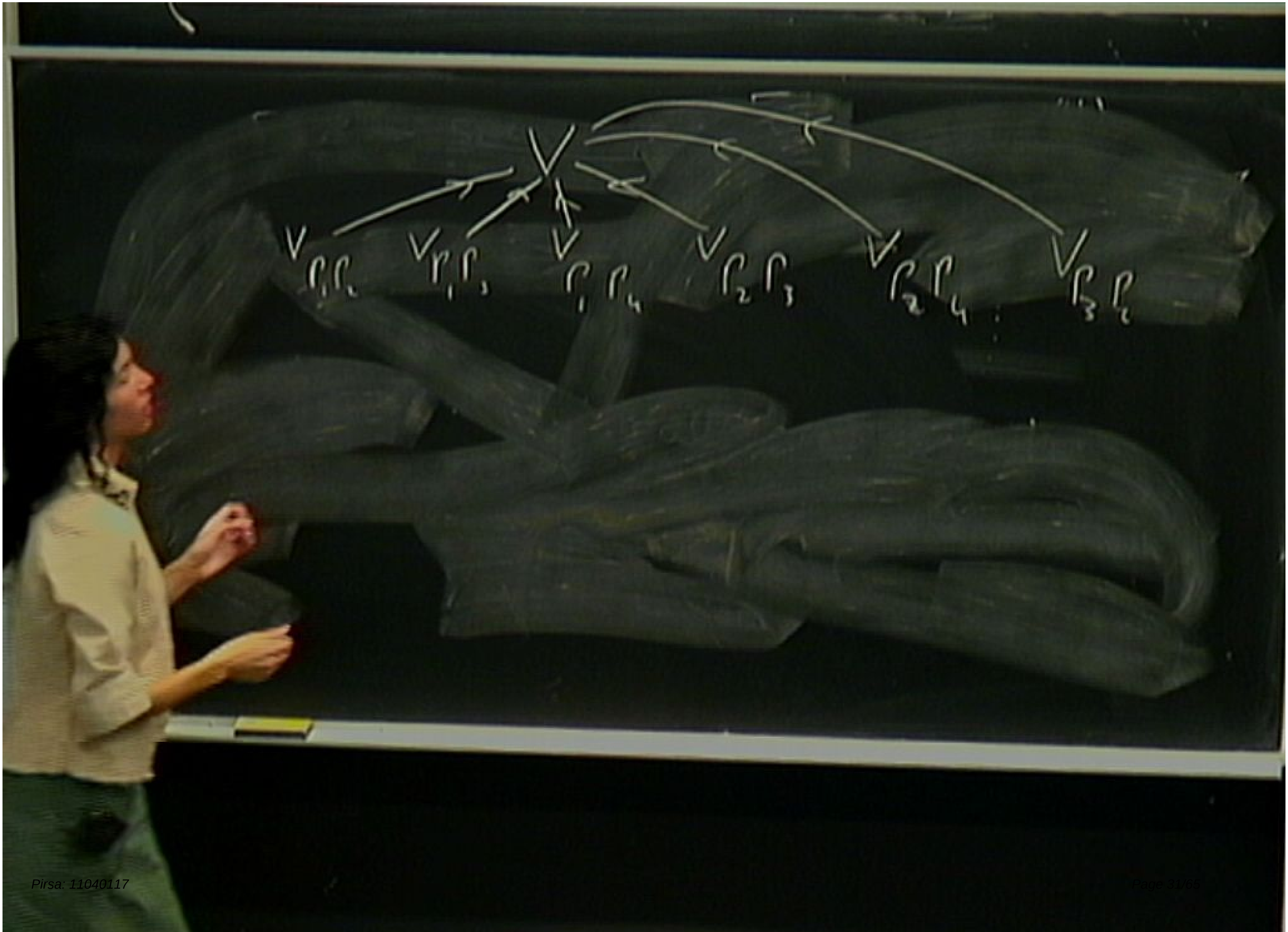
$$V'' = \underbrace{\mathbb{C}\hat{P}_1} + \mathbb{C}\hat{Q}_2 + \mathbb{C}\hat{Q}_3 + \mathbb{C}\hat{Q}_4$$

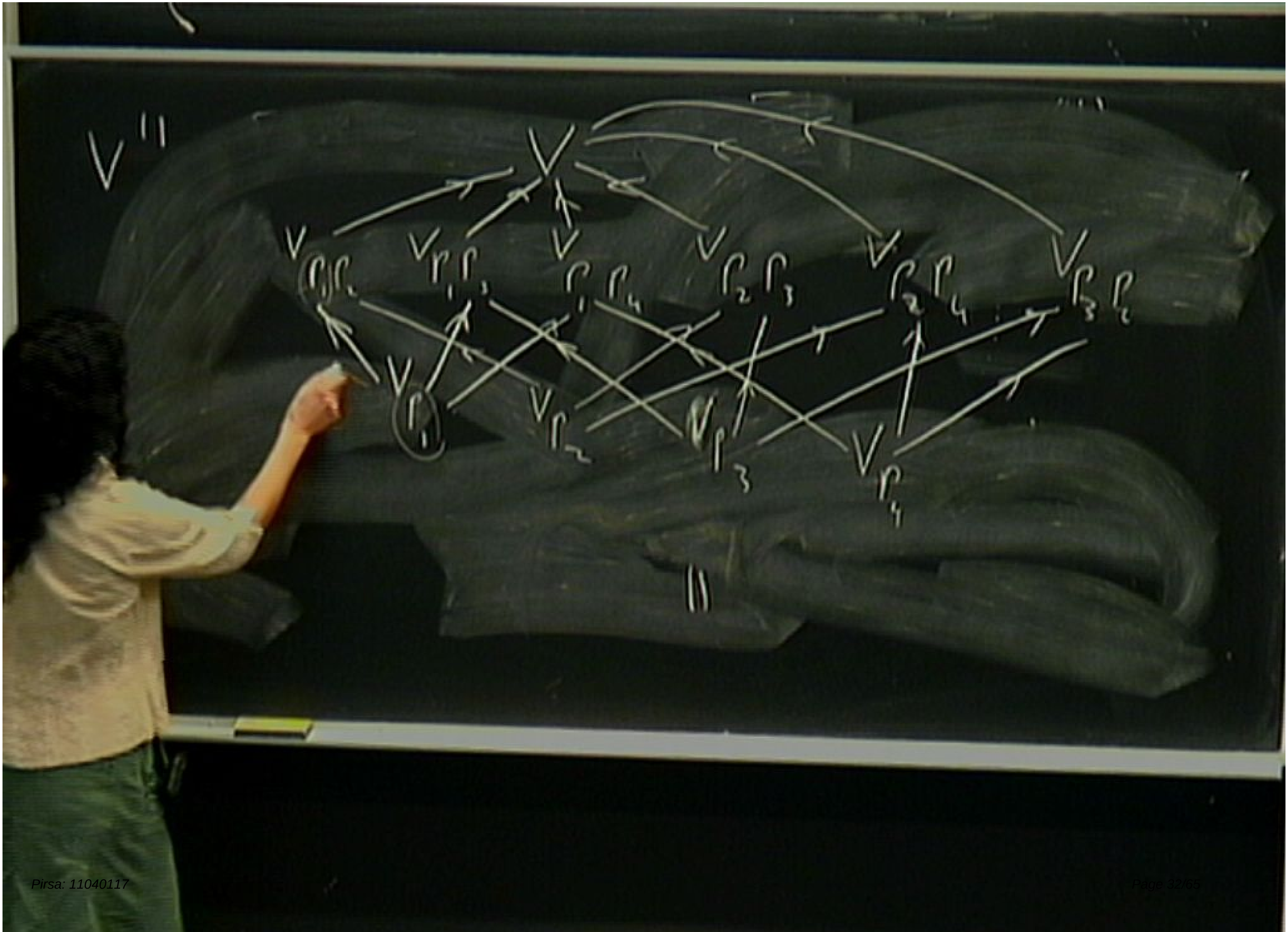
$$\mathbb{1} - \hat{P}_1$$

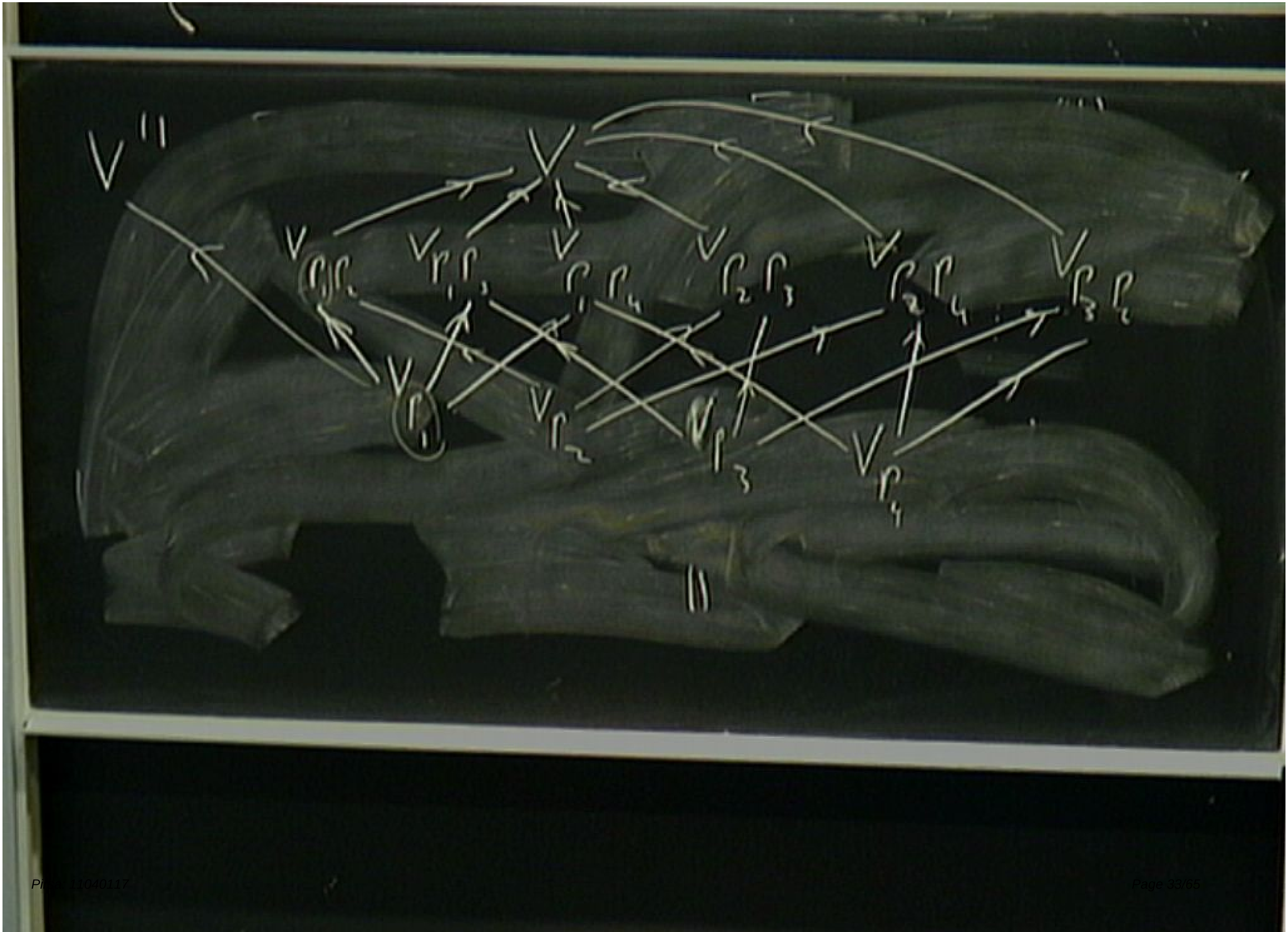
$$V_{P_1} = \mathbb{C}\hat{P}_1 + \mathbb{C}\left(\hat{P}_2 + \hat{P}_3 + \hat{P}_4\right) =$$

$$= \mathbb{C}\hat{P}_1 + \mathbb{C}\left(\underbrace{\hat{Q}_2 + \hat{Q}_3 + \hat{Q}_4}_{\mathbb{1} - \hat{P}_1}\right)$$


V
V Γ1 Γ2
V Γ1 Γ3
V Γ1 Γ4
V Γ2 Γ3
V Γ2 Γ4
V Γ3 Γ4


V
V_{Γ1Γ2}
V_{Γ1Γ3}
V_{Γ1Γ4}
V_{Γ2Γ3}
V_{Γ2Γ4}
V_{Γ3Γ4}




V''
V

$$H = (\mathbb{C}^4) \qquad B(H) = \left\{ \begin{array}{l} 4\times 4 \text{ matrices} \\ \text{with complex entries} \end{array} \right\}$$

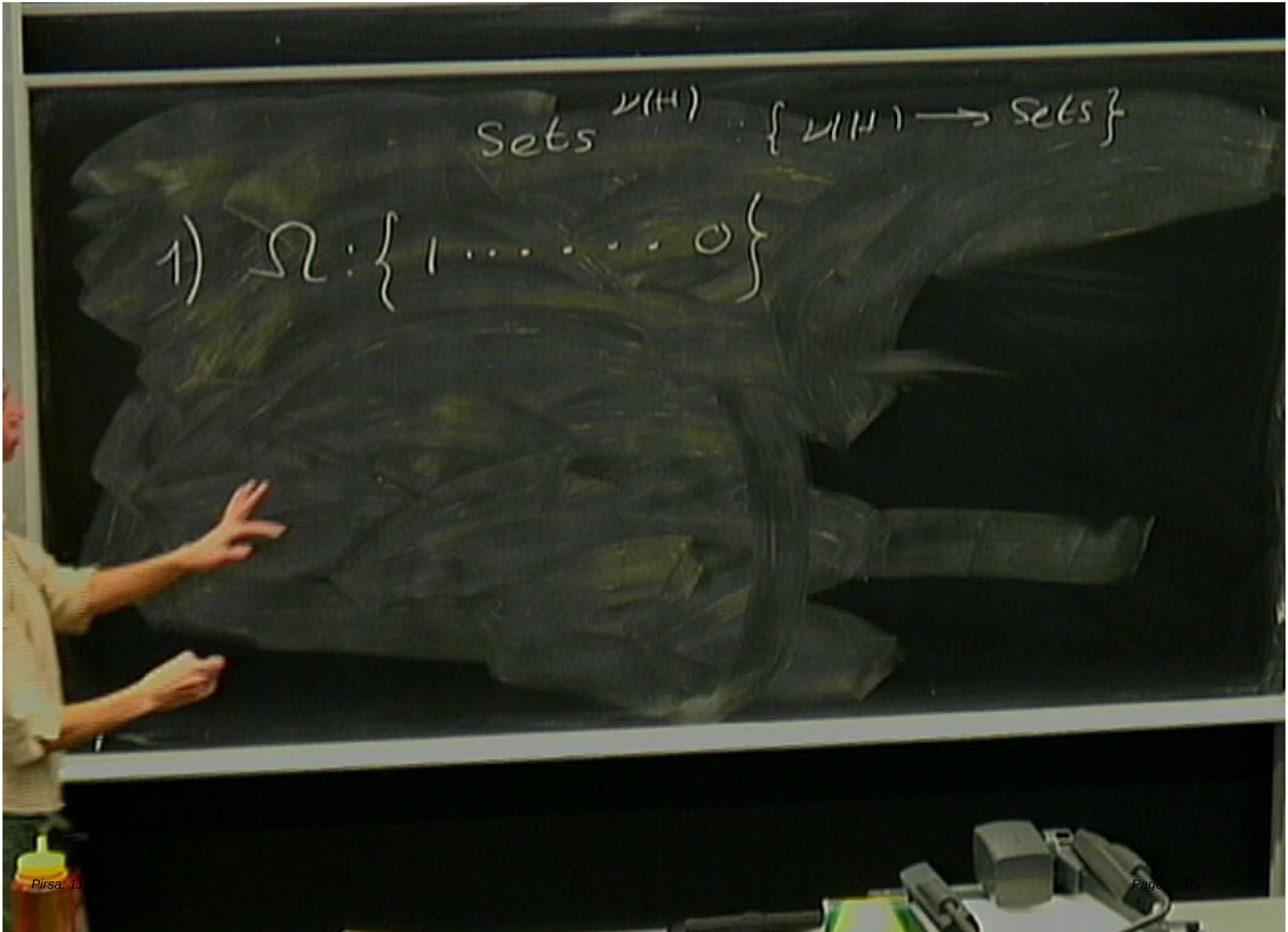
$$\left(\overset{1000}{\tilde{\Psi}_1}, \overset{0100}{\tilde{\Psi}_2}, \overset{0010}{\tilde{\Psi}_3}, \overset{0001}{\tilde{\Psi}_4} \right) \qquad \left(\hat{P}_1, \hat{P}_2, P_3, \hat{P}_4 \right)$$

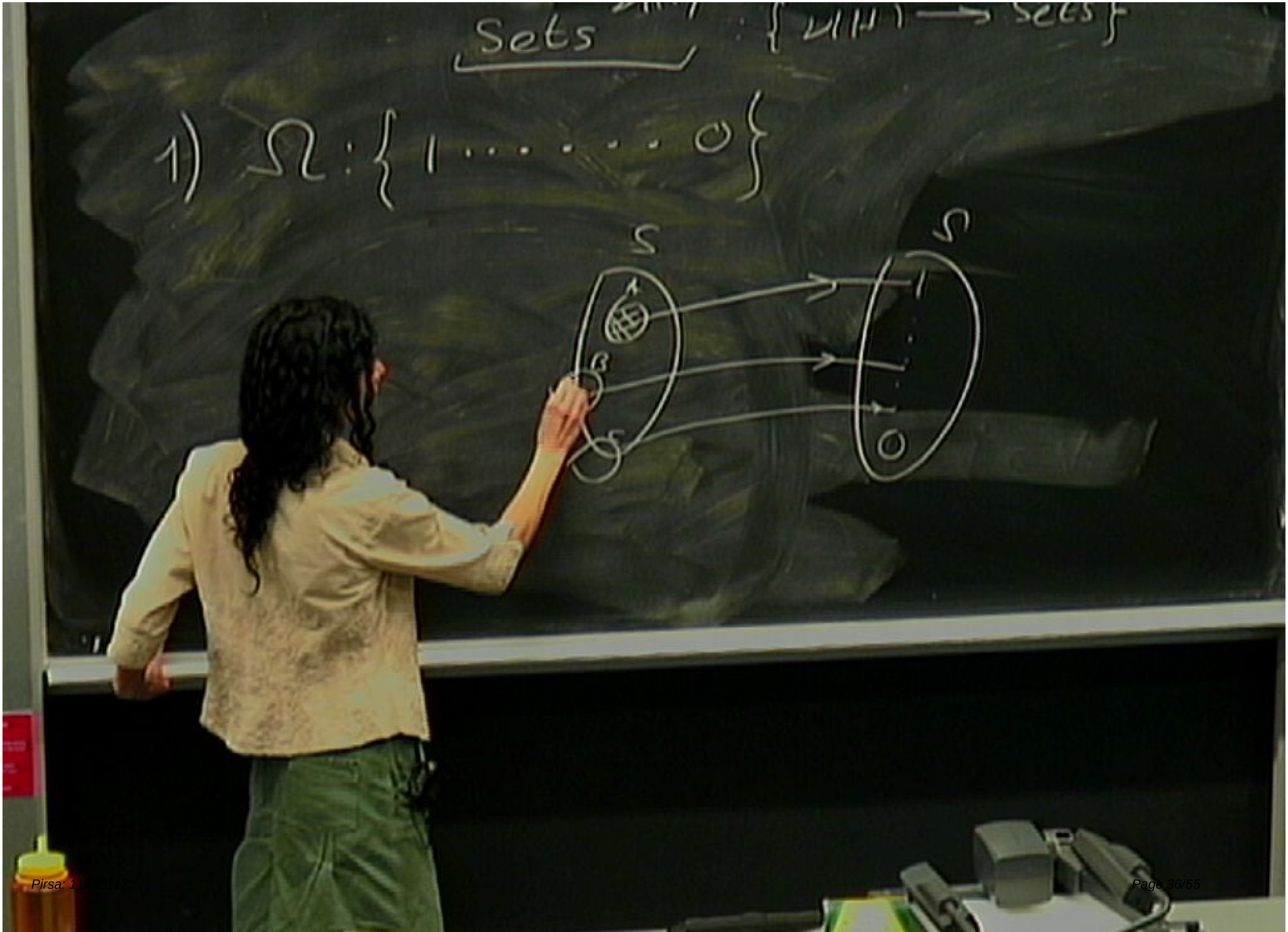
$$\rho_1 \quad \rho_2 \quad \rho_3 \quad \rho_4 \qquad\qquad \mathbb{C}\Psi_1 \quad \mathbb{C}\Psi_2 \quad \mathbb{C}\Psi_3 \quad \mathbb{C}\Psi_4$$

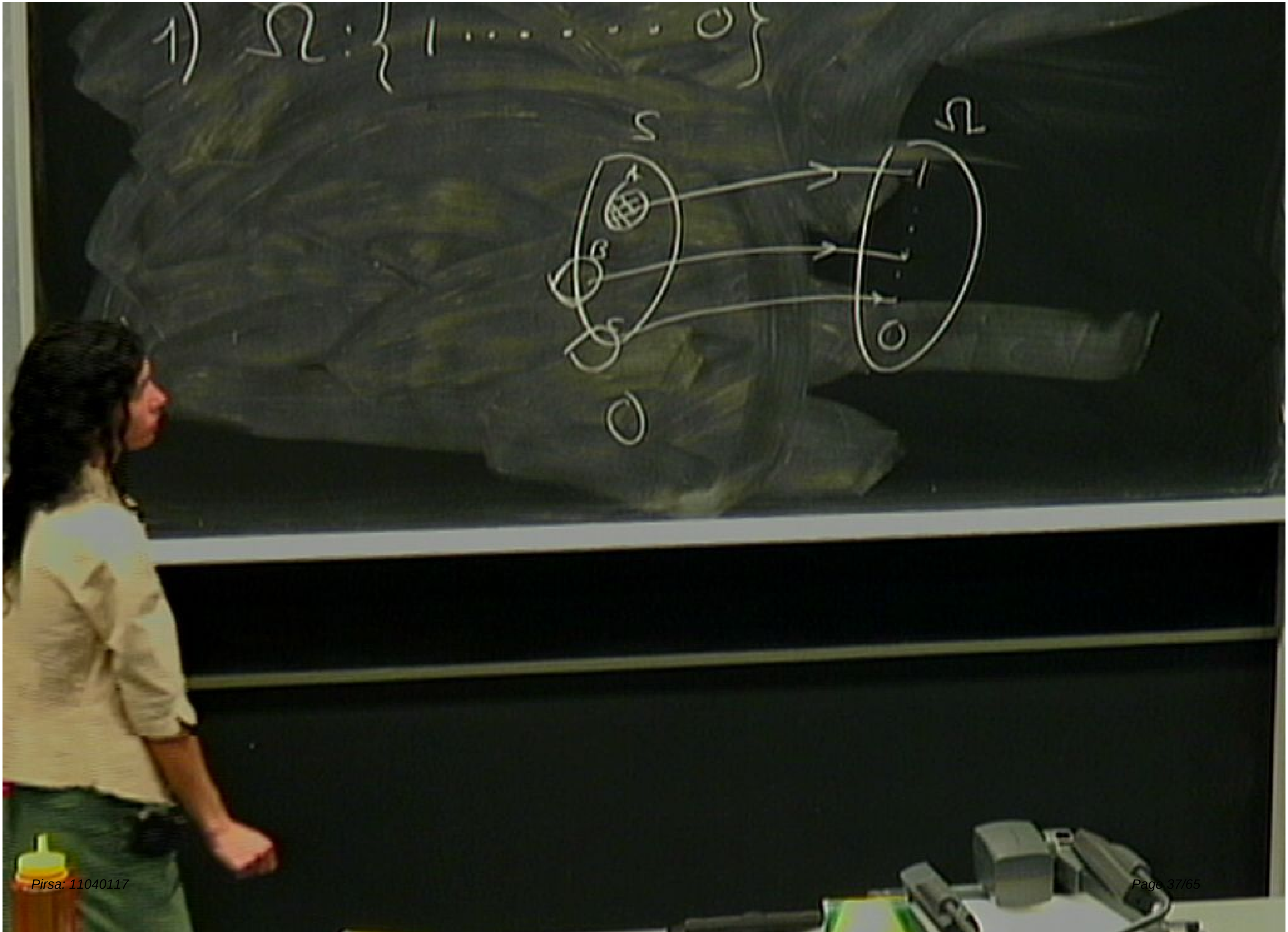
$$V = \mathrm{lin}_{\mathbb{C}} \left(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{P}_4 \right) \qquad P_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & & \\ & & & 0 \end{pmatrix}$$

$$\left(\mathbb{C}\hat{P}_1 + \mathbb{C}\hat{P}_2 + \mathbb{C}\hat{P}_3 + \mathbb{C}\hat{P}_4 \right) \qquad P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \end{pmatrix} \quad \hat{P}_4 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$$

$$\left\{ \begin{array}{l} 4\times 4 \text{ diagonal matrices} \\ \text{with complex entries} \end{array} \right\}$$


$Sets^{V(H)} : \{ V(H) \longrightarrow Sets \}$
1) $\Omega : \{ 1 \ldots \ldots 0 \}$


Sets
1) Ω : { 1 . . . . . . 0 }
S
S
1
0

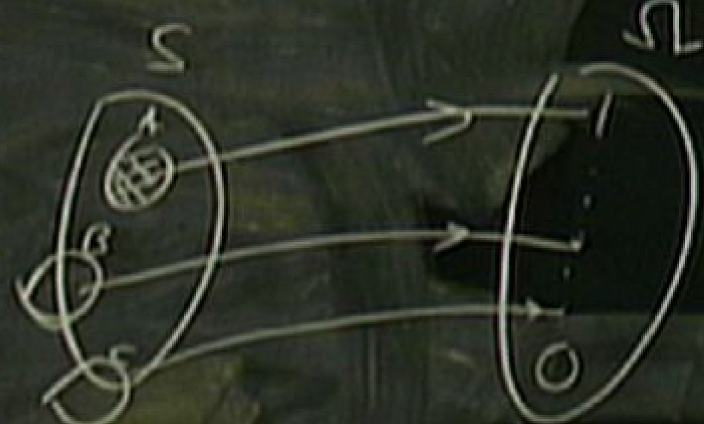

1) Ω: { 1 . . . . . . 0 }
S
Ω
1
0
0

$\underbrace{\mathrm{Sets}^{\mathcal{V}(H)}}$: $\{ \mathcal{V}(H) \longrightarrow \mathrm{Sets} \}$

$S \vee \neg S \leq 1$

$S \cup S^c = 1$

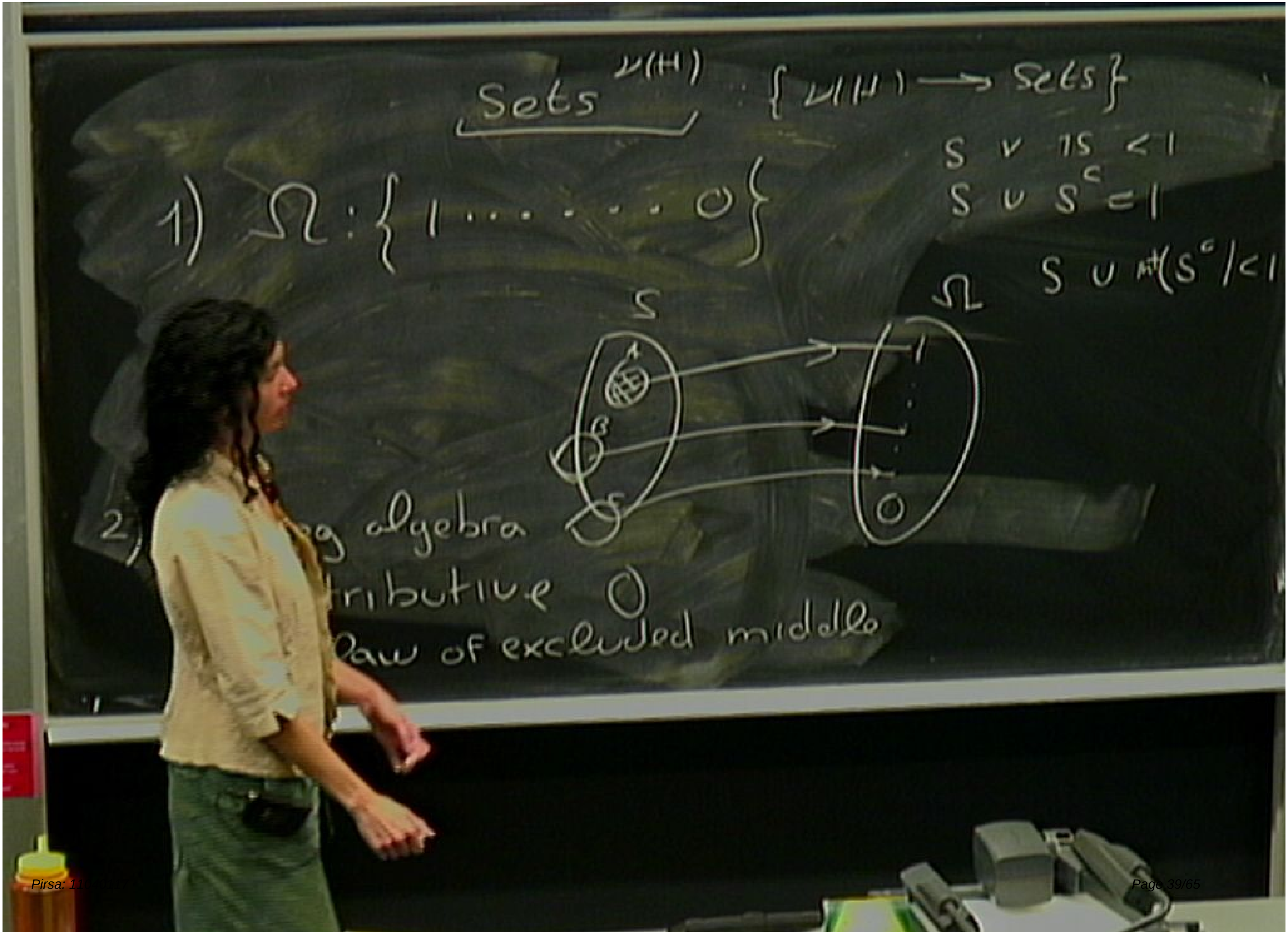
1) $\Omega : \{ 1 \cdots\cdots 0 \}$



2) Heyting algebra

- distributive

- no law of excluded middle

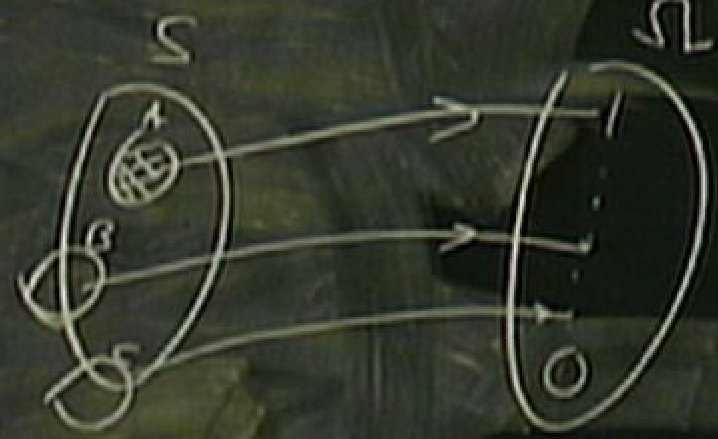


Sets ν(H) : { ν(H) → Sets }
S ∨ ¬S < 1
S ∪ S^c ⊂ 1
1) Ω : { 1 . . . . . . 0 }
S
Ω
1
0
algebra
tributive
law of excluded middle

1) $\Omega : \{ 1 \cdots\cdots 0 \}$

$S \cup S^c = 1$

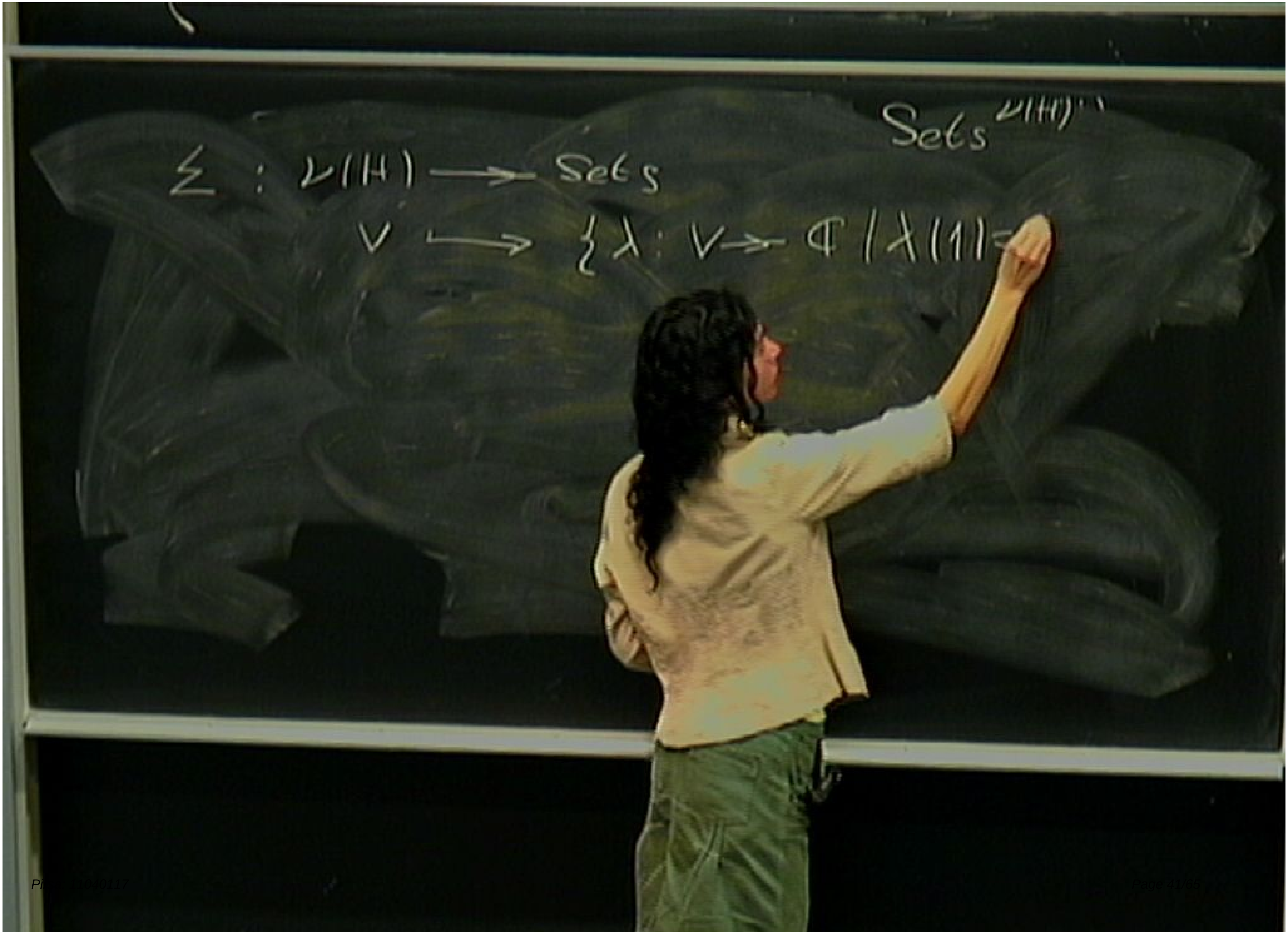
$S \cup \mathrm{int}(S^c) < 1$



2) <u>Heyting algebra</u>

- distributive

- no law of excluded middle

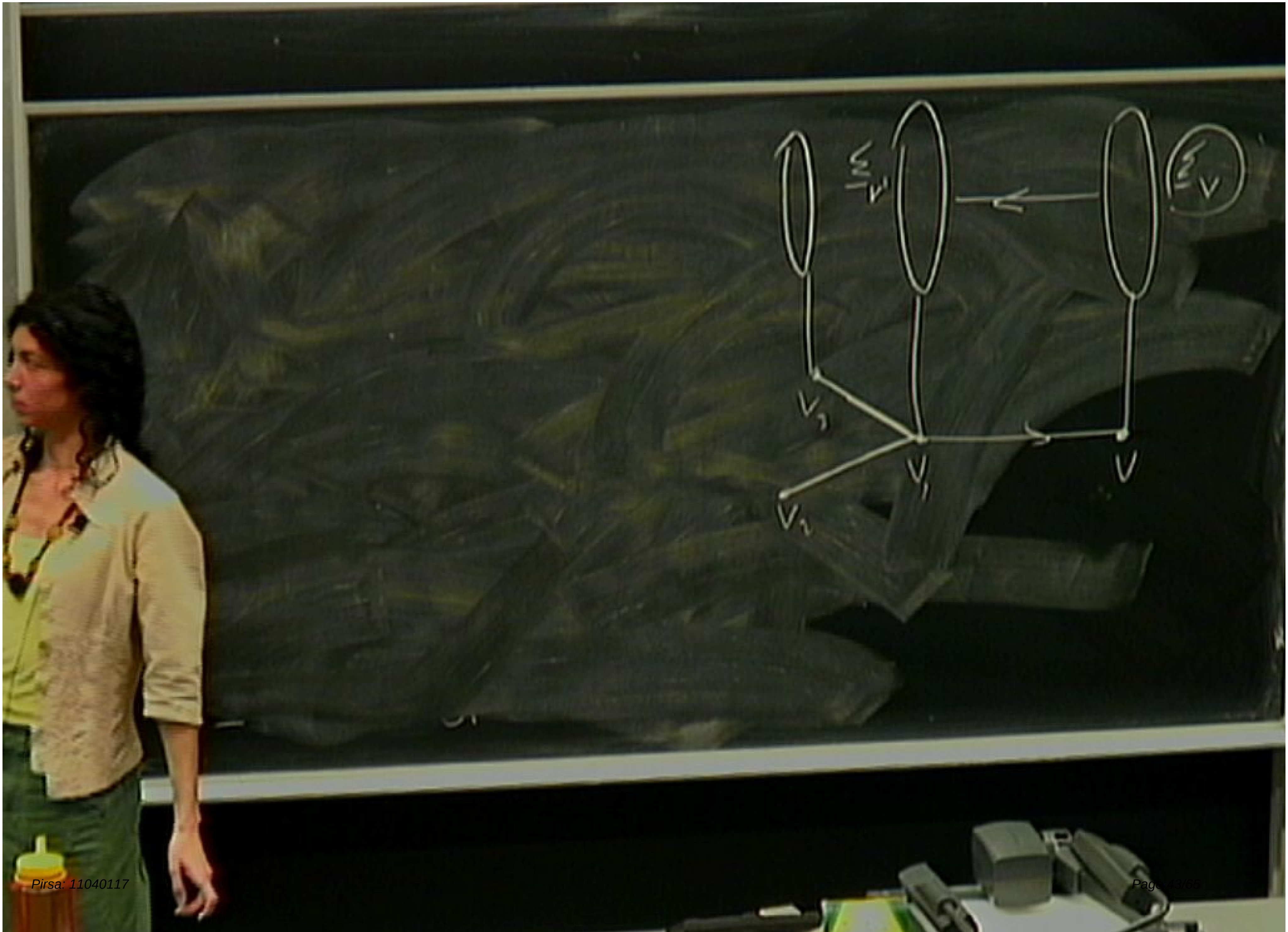

$\Sigma : \mathcal{V}(H) \longrightarrow \mathrm{Sets}$
$V \longmapsto \{\lambda : V \to \mathbb{C} \mid \lambda(1) =$
$\mathrm{Sets}^{\mathcal{V}(H)}$

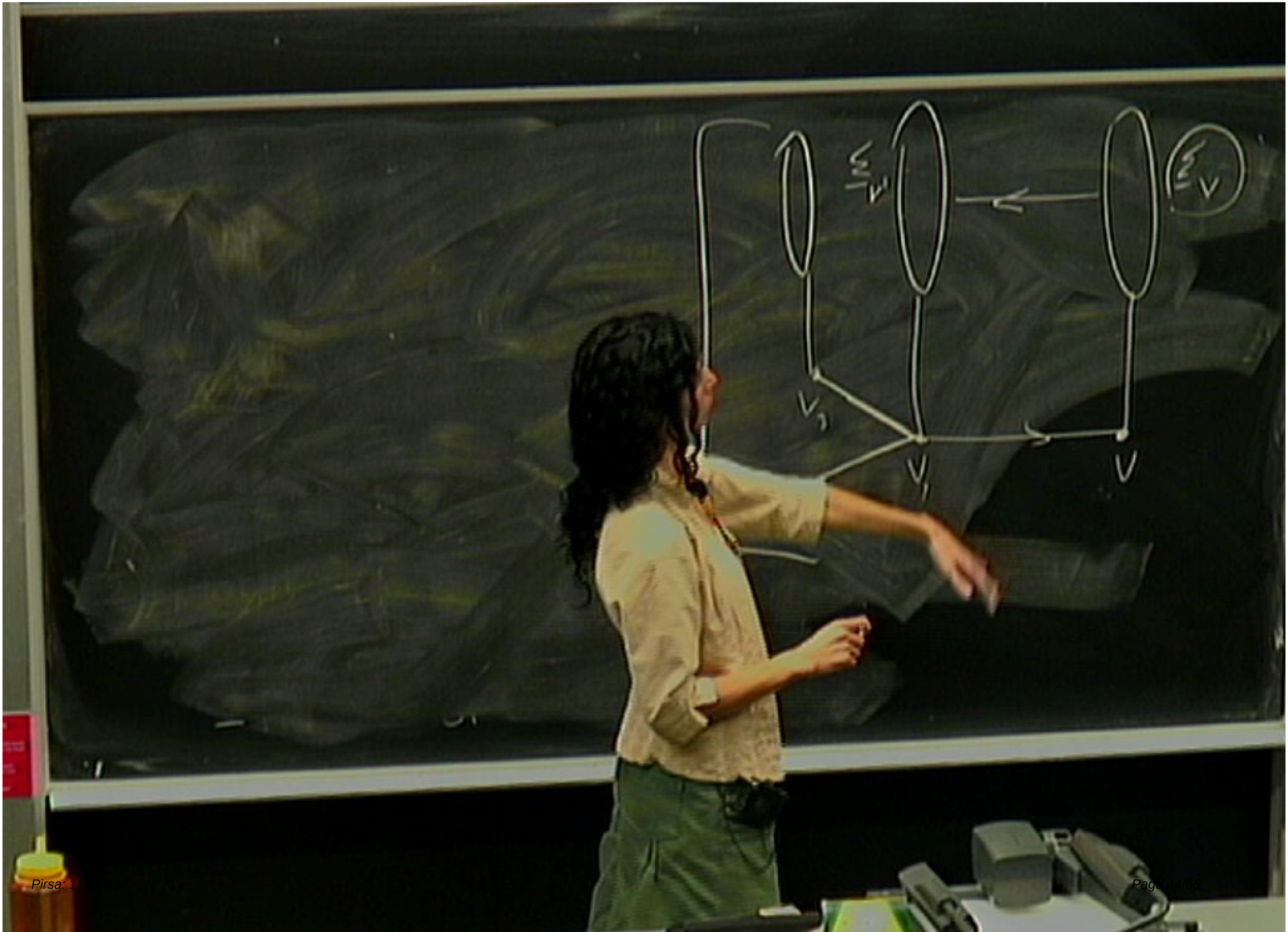
$$\Sigma : \mathcal{V}(H) \longrightarrow \text{Sets}$$

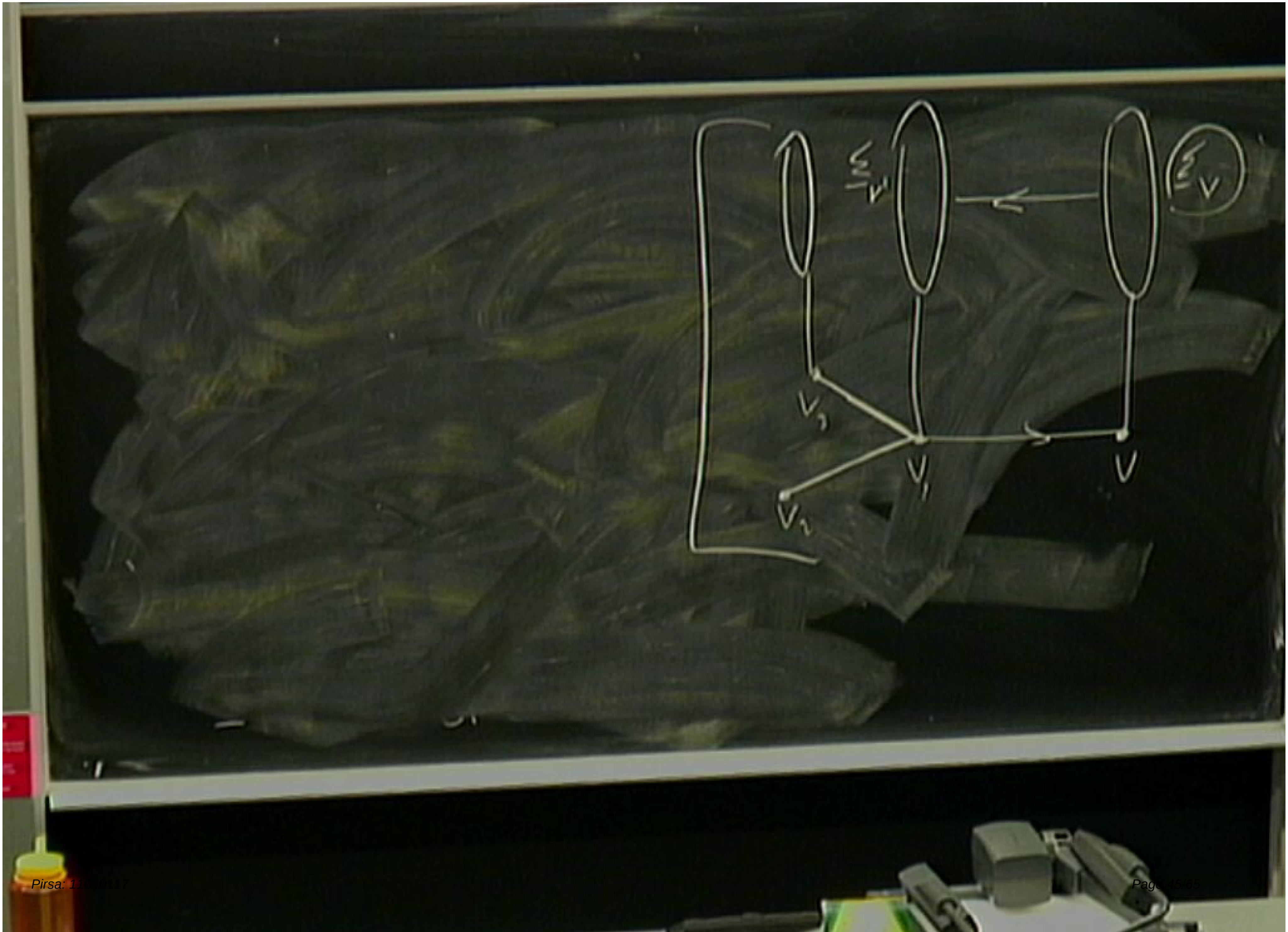
$$V \longmapsto \{\lambda : V \to \mathbb{C} \mid \lambda(\hat{1}) = 1\}$$

$$V' \hookrightarrow V \longmapsto \Sigma_{V'V} : \Sigma_V \longrightarrow \Sigma_{V'}$$

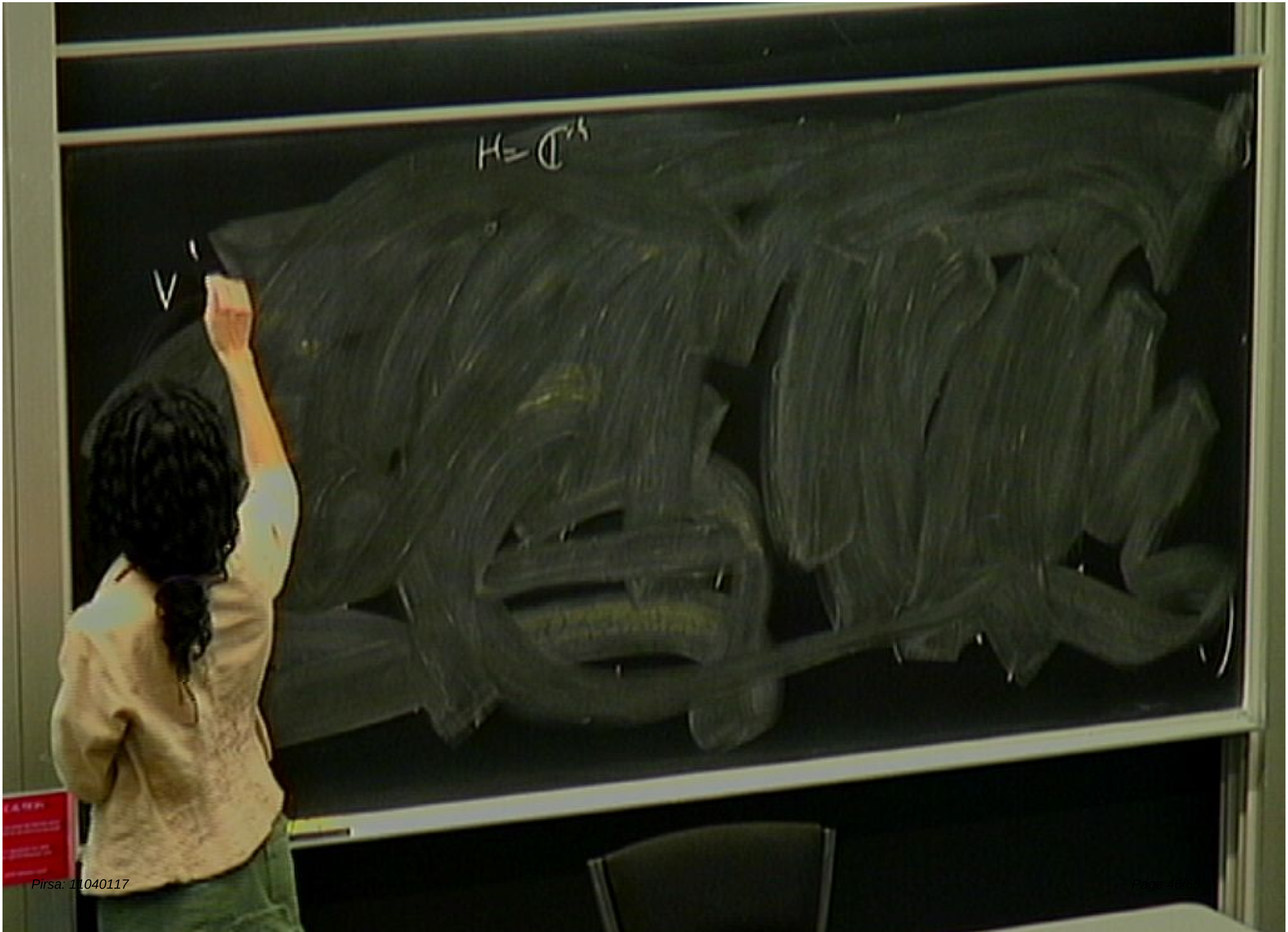
$$\lambda \longmapsto \lambda|_{V'}$$

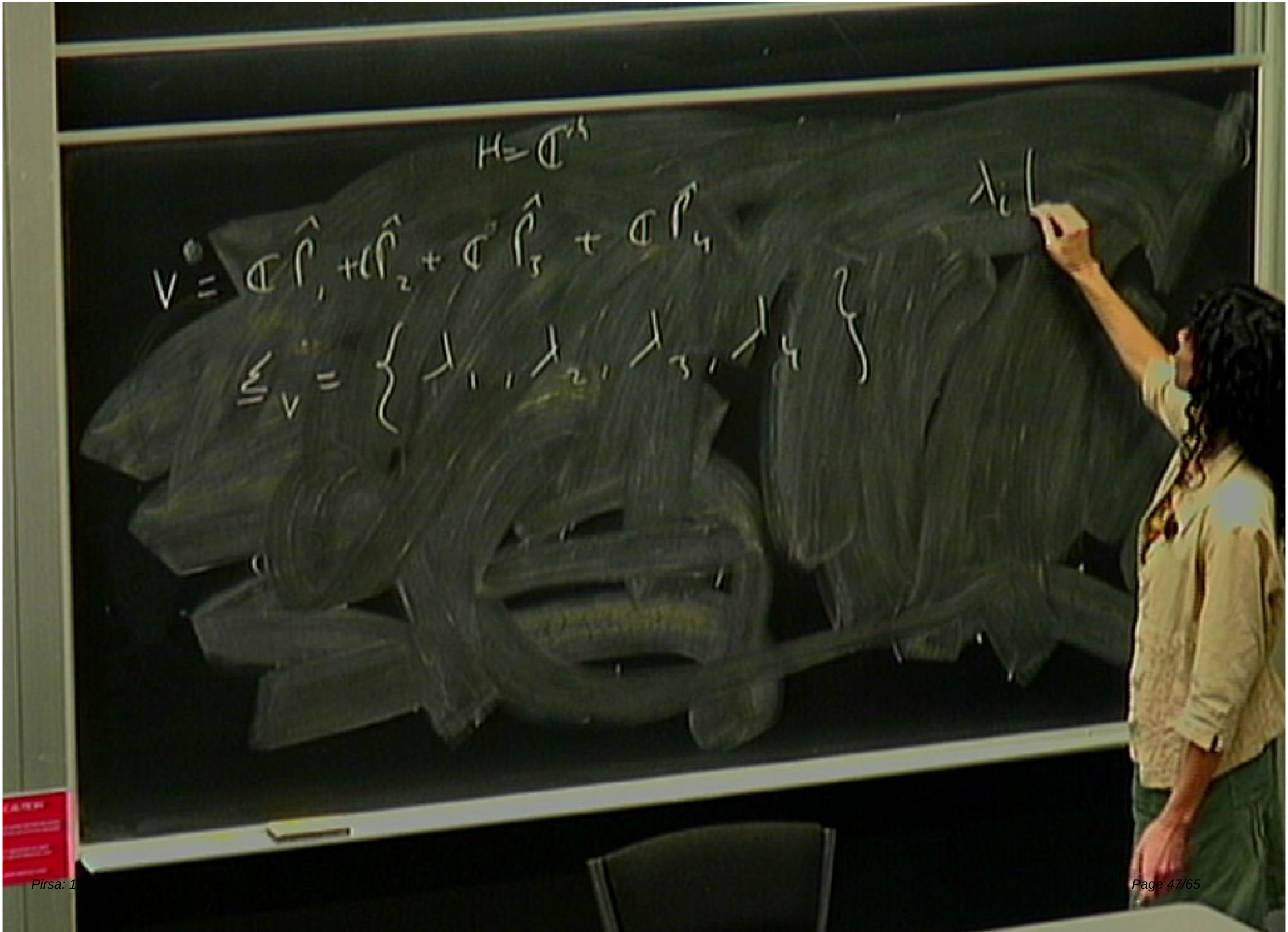

$\Sigma_{v'}$
Σ_v
v_3
v_1
v_2
v


$\mathcal{E}_{V'}$
$\mathcal{E}_V$
V_3
V_1
V



$\xi_{v'}$
ξ_v
v_3
v_1
v_2
v


V



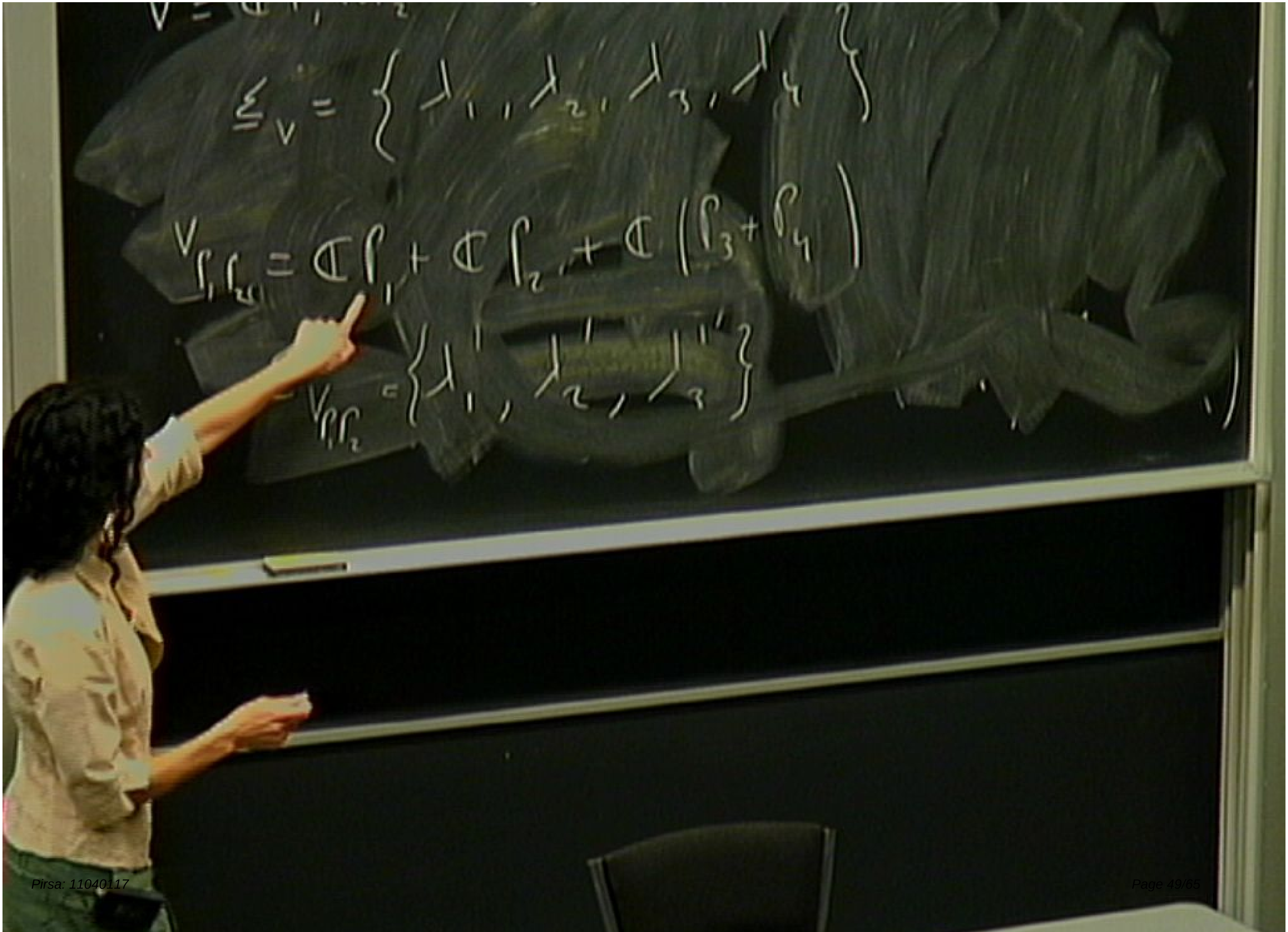
$H = \mathbb{C}^4$
$V = \mathbb{C}\hat{\rho}_1 + \mathbb{C}\hat{\rho}_2 + \mathbb{C}\hat{\rho}_3 + \mathbb{C}\hat{\rho}_4$
$\xi_V = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$
λ_i

$H = \mathbb{C}^4$

$\lambda_i(\rho_j) = \delta_{ij}$

$V = \mathbb{C}\hat{\rho}_1 + \mathbb{C}\hat{\rho}_2 + \mathbb{C}\hat{\rho}_3 + \mathbb{C}\hat{\rho}_4$

$\mathcal{E}_V = \{ \lambda_1, \lambda_2, \lambda_3, \lambda_4 \}$



$\Sigma_V = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$
$V_{\rho_1\rho_2} = \mathbb{C}\rho_1 + \mathbb{C}\rho_2 + \mathbb{C}(\rho_3+\rho_4)$
$\Sigma_{V_{\rho_1\rho_2}} = \{\lambda_1, \lambda_2, \lambda_3\}$

$H = \mathbb{C}^4$

$\lambda_i(\rho_j) = \delta_{ij}$

$V = \mathbb{C}\hat{\rho}_1 + \mathbb{C}\hat{\rho}_2 + \mathbb{C}\hat{\rho}_3 + \mathbb{C}\rho_4$

$\mathcal{E}_V = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$

$\mathcal{E}_{V'V}(\lambda_1') = \lambda_1'$

$V_{\rho_1\rho_2} = \mathbb{C}\rho_1 + \mathbb{C}\rho_2 + \mathbb{C}(\rho_3 + \rho_4)$

$\mathcal{E}_{V_{\rho_1\rho_2}} = \{\lambda_1', \lambda_2', \lambda_3'\}$

$$H = \mathbb{C}^4$$

$$V = \mathbb{C}\hat{\rho}_1 + \mathbb{C}\hat{\rho}_2 + \mathbb{C}\hat{\rho}_3 + \mathbb{C}\hat{\rho}_4$$

$$\Sigma_V = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$$

$$V_{\rho_1\rho_2} = \mathbb{C}\rho_1 + \mathbb{C}\rho_2 + \mathbb{C}(\rho_3 + \rho_4)$$

$$\Sigma_{V_{\rho_1\rho_2}} = \{\lambda_1', \lambda_2', \lambda_3'\}$$

$$\lambda_i(\rho_j) = \delta_{ij}$$

$$\varepsilon_{V'V}(\lambda_1) = \lambda_1'$$

$$\varepsilon_{V'V}(\lambda_2) = \lambda_2'$$

$$\varepsilon_{V'V}(\lambda_3) = \lambda_3'$$

$$H = \mathbb{C}^4$$

$$\lambda_i(\rho_j) = \delta_{ij}$$

$$V_{\rho_1} = \mathbb{C}\hat{\rho}_1 + \mathbb{C}(\rho_2 + \hat{\rho}_3 + \rho_4)$$

$$\mathcal{E}_{V'V}(\lambda_1) = \lambda_1'$$

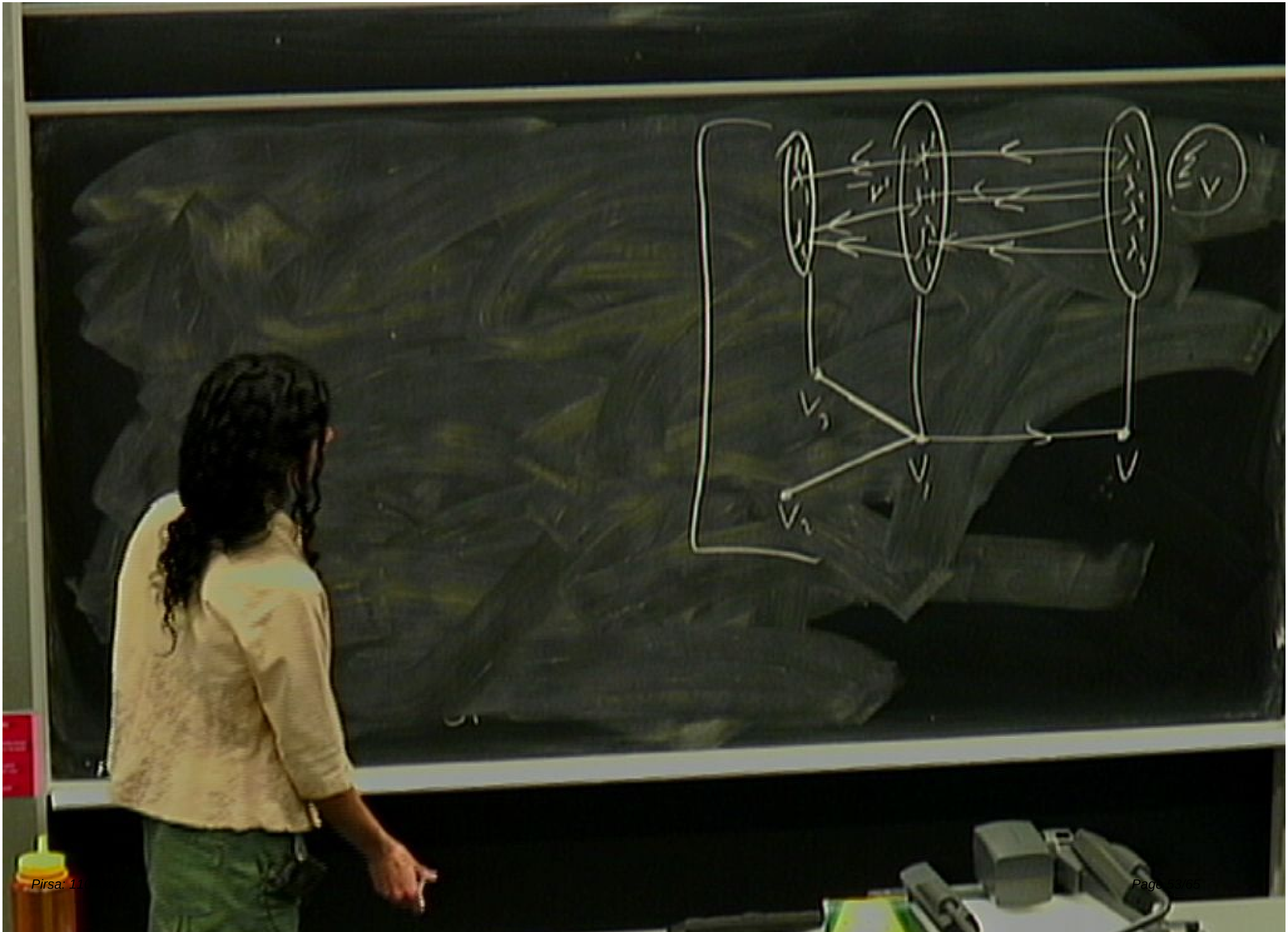
$$\mathcal{E}_V = \{\lambda_1'', \lambda_4\}$$

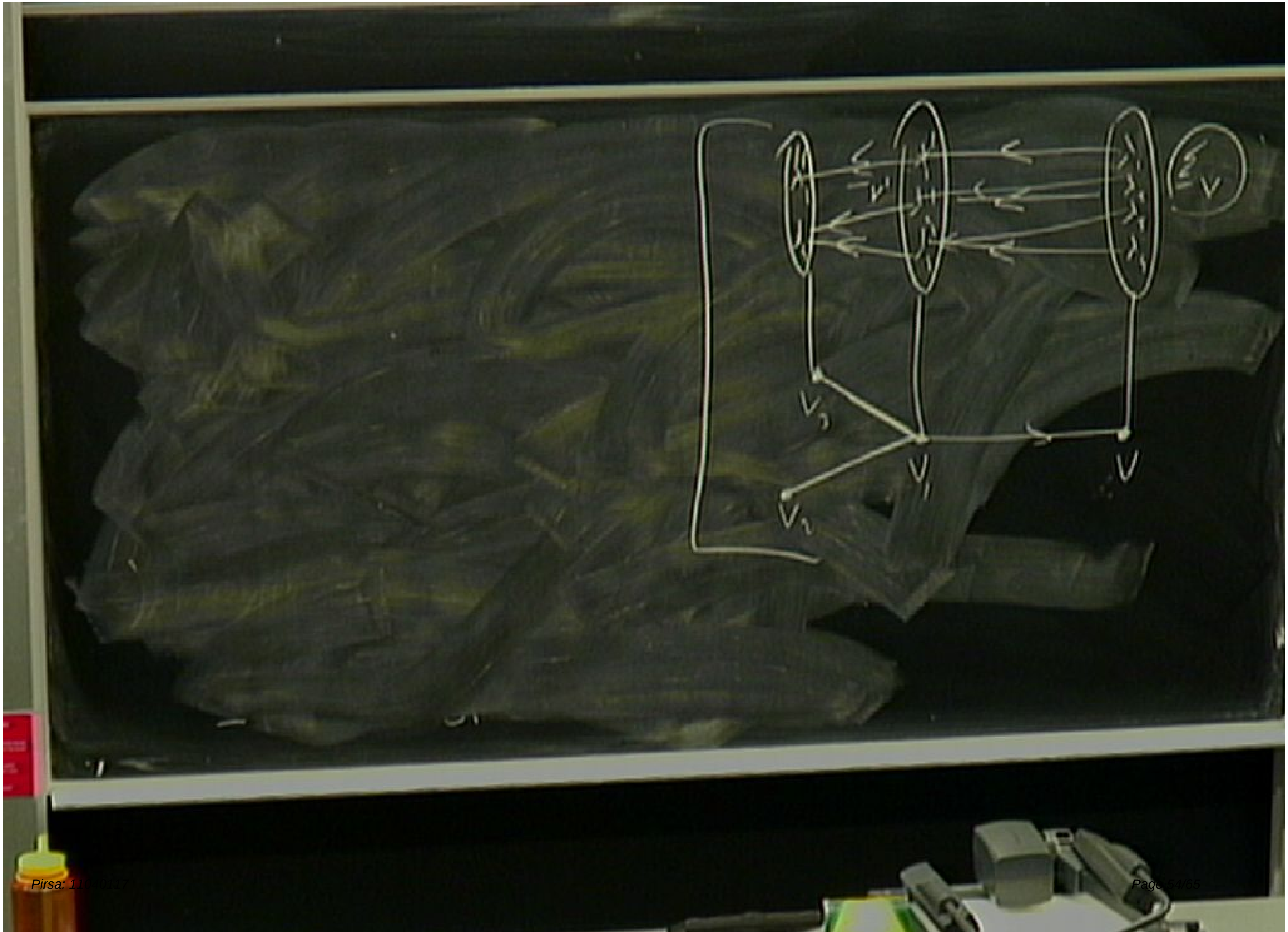
$$\mathcal{E}_{V'V}(\lambda_2) = \lambda_2'$$

$$\mathcal{E}_{V'V}(\lambda_3) = \lambda_3'$$

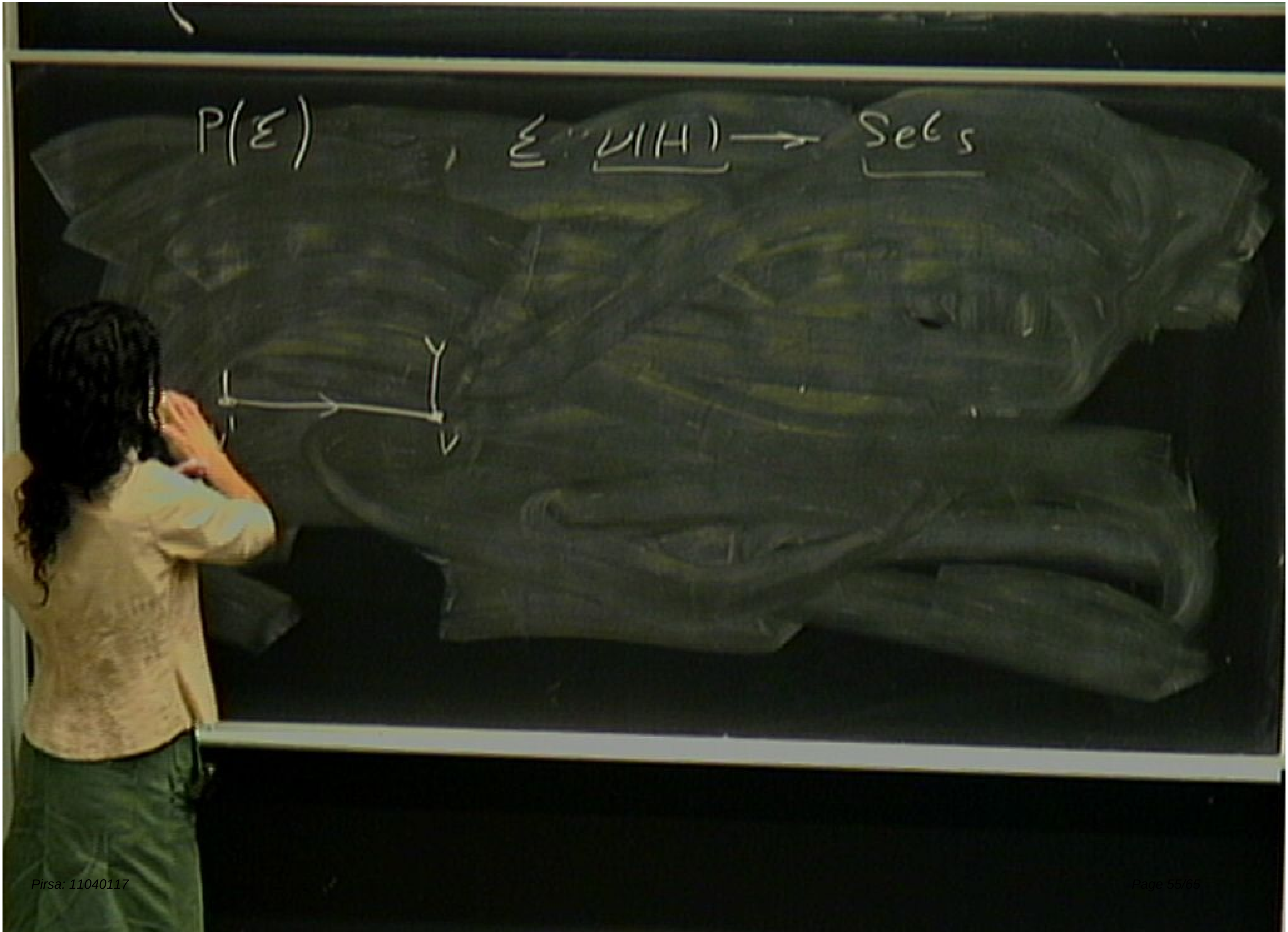
$$V_{\rho_1\rho_2} = \mathbb{C}\rho_1 + \mathbb{C}\rho_2 + \mathbb{C}(\rho_3 + \rho_4)$$

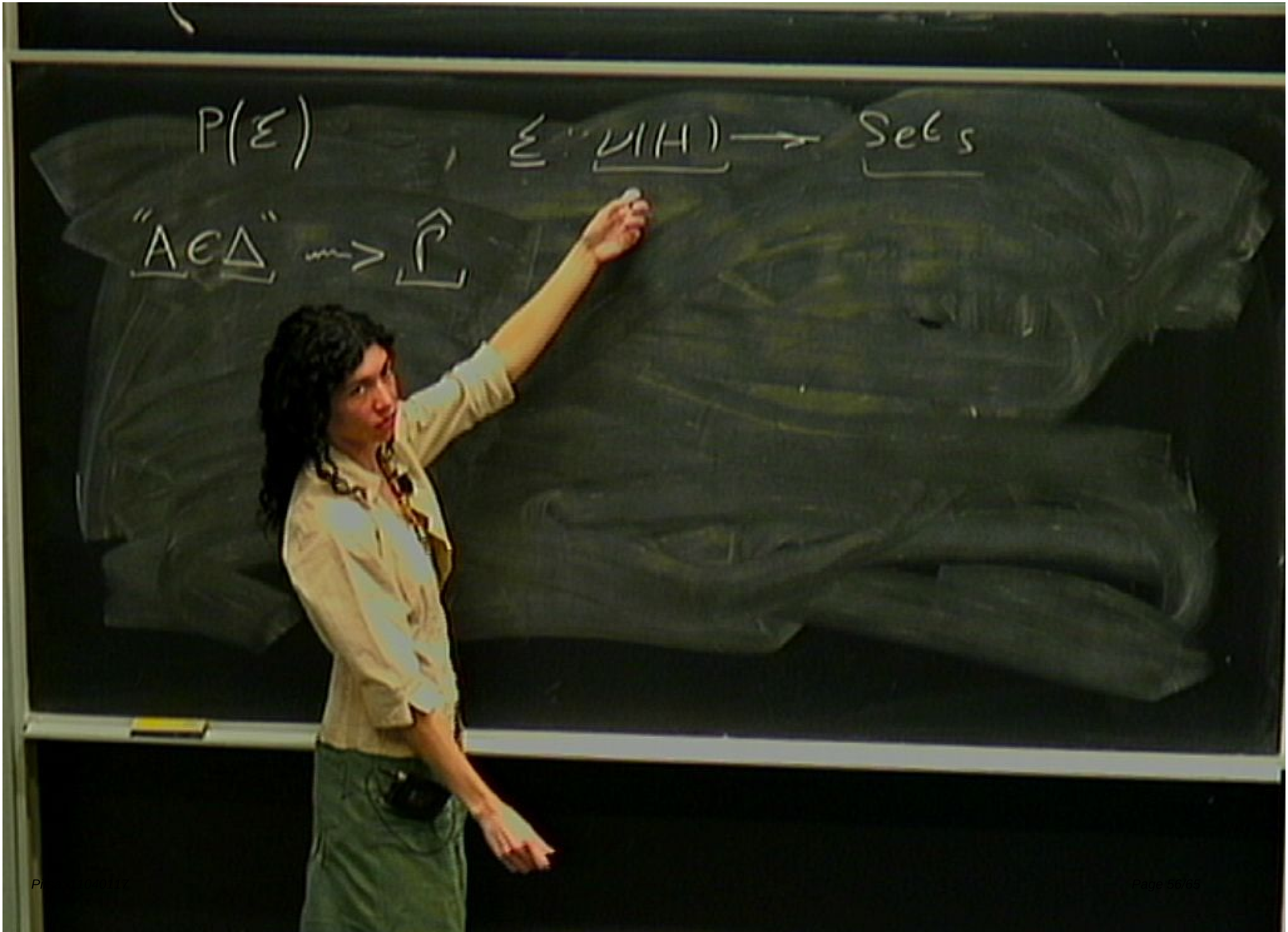
$$\mathcal{E}_{V_{\rho_1\rho_2}} = \{\lambda_1', \lambda_2', \lambda_3'\}$$





$\mathcal{E}_V$
V_3
V_1
V_2
V

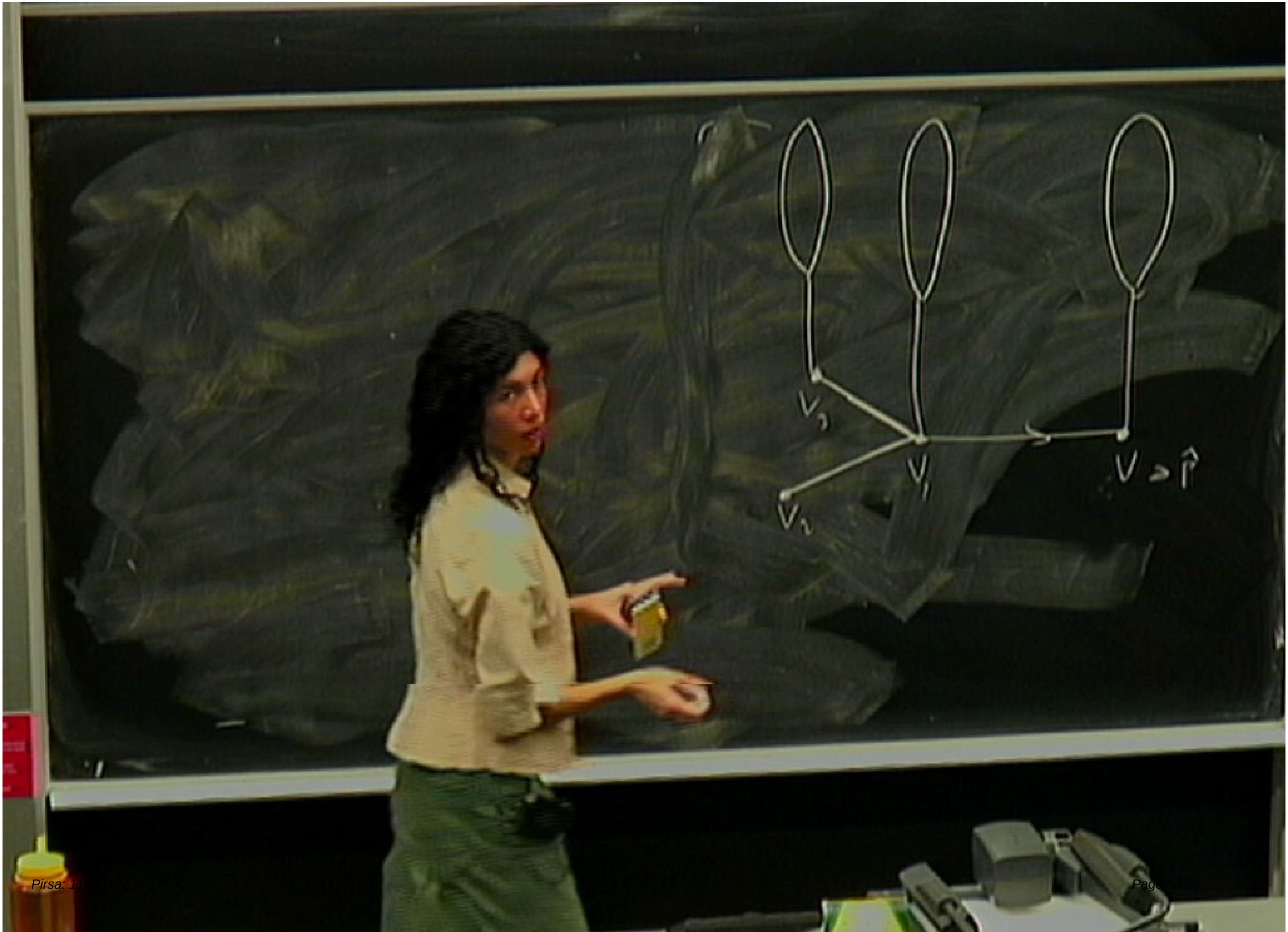

P(ℰ)
ℰ : U(H) → Sets

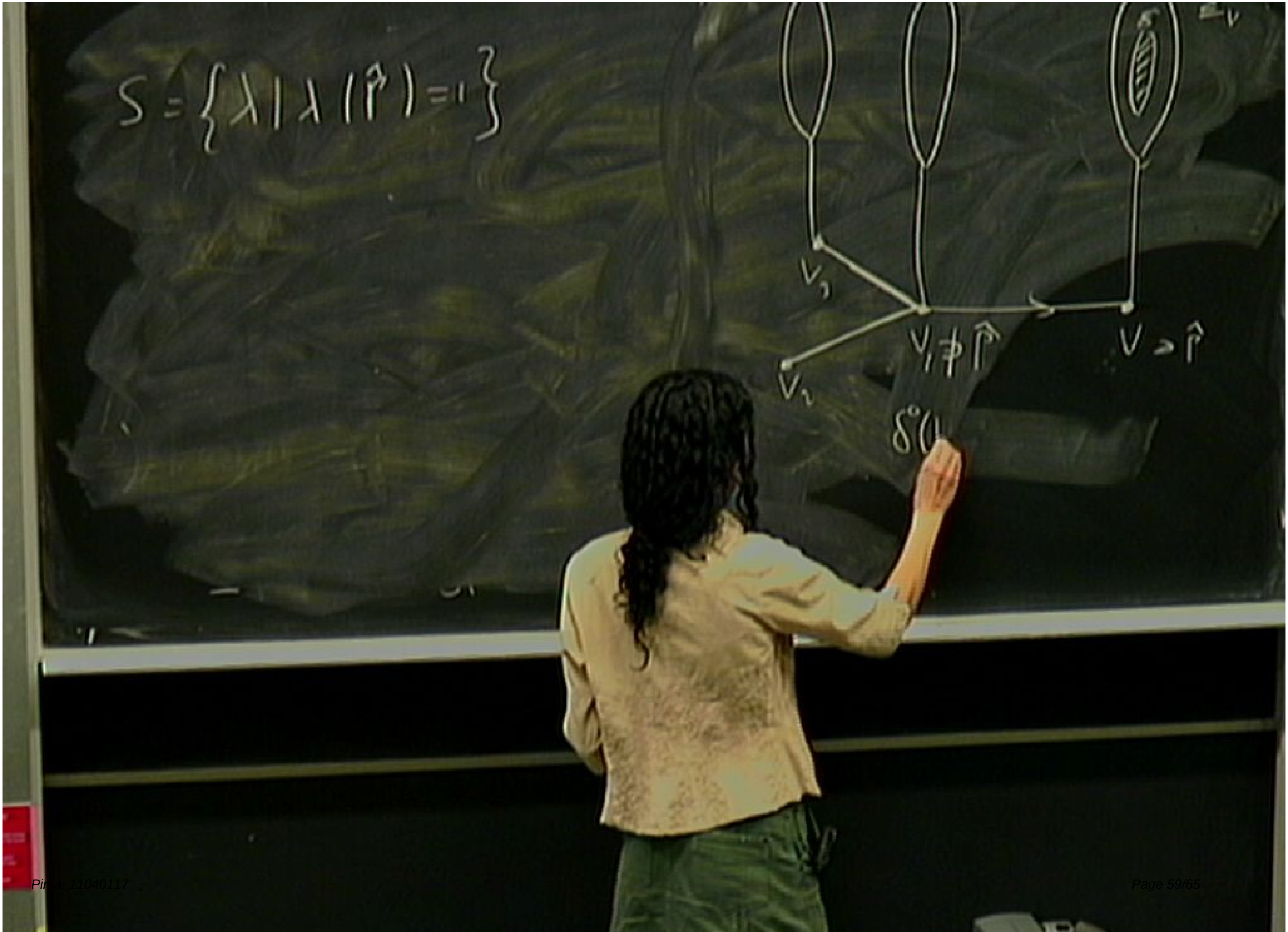


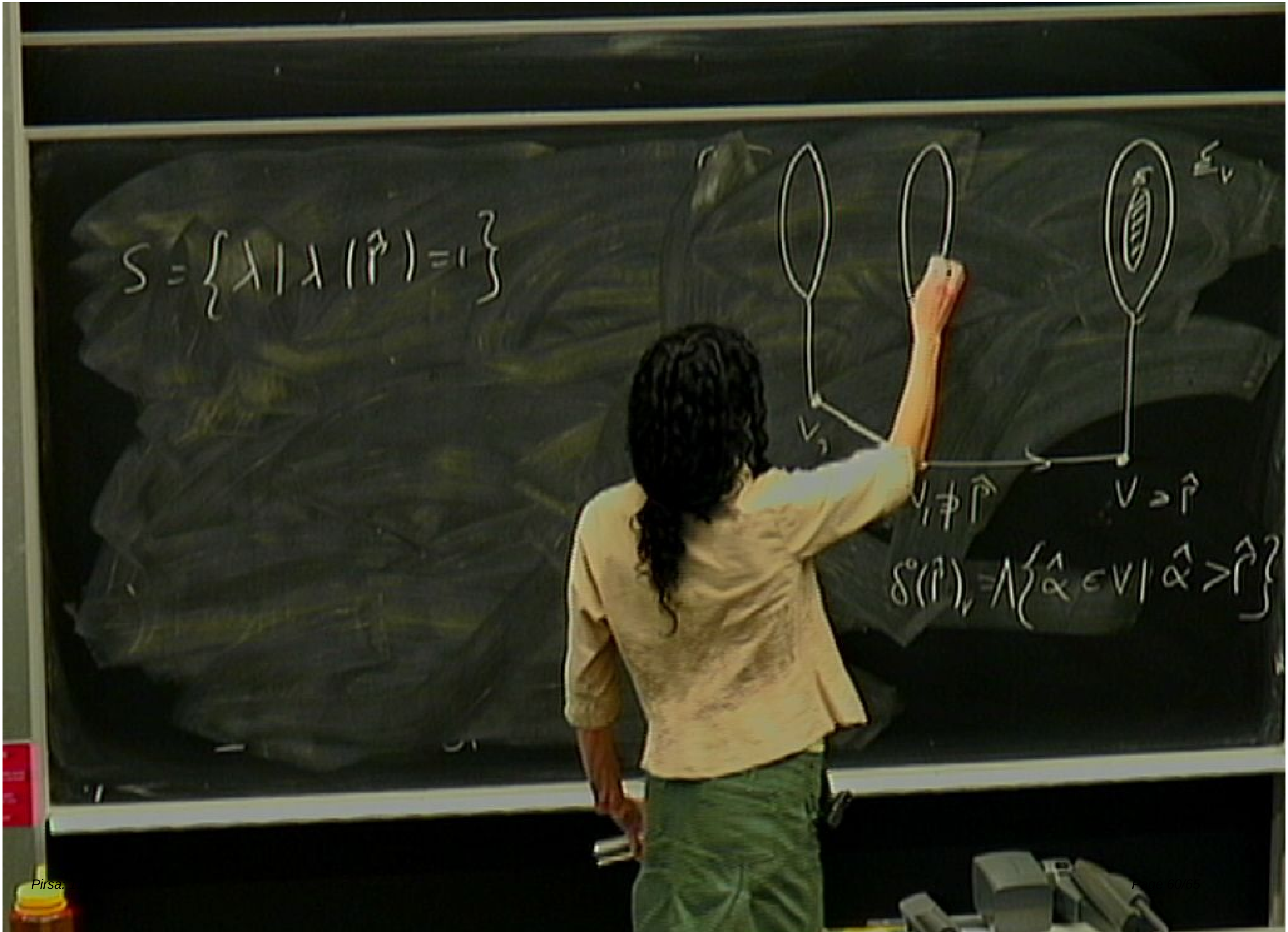
P(Σ) , Σ : U(H) → Sets
"A∈Δ" ⇝ P̂

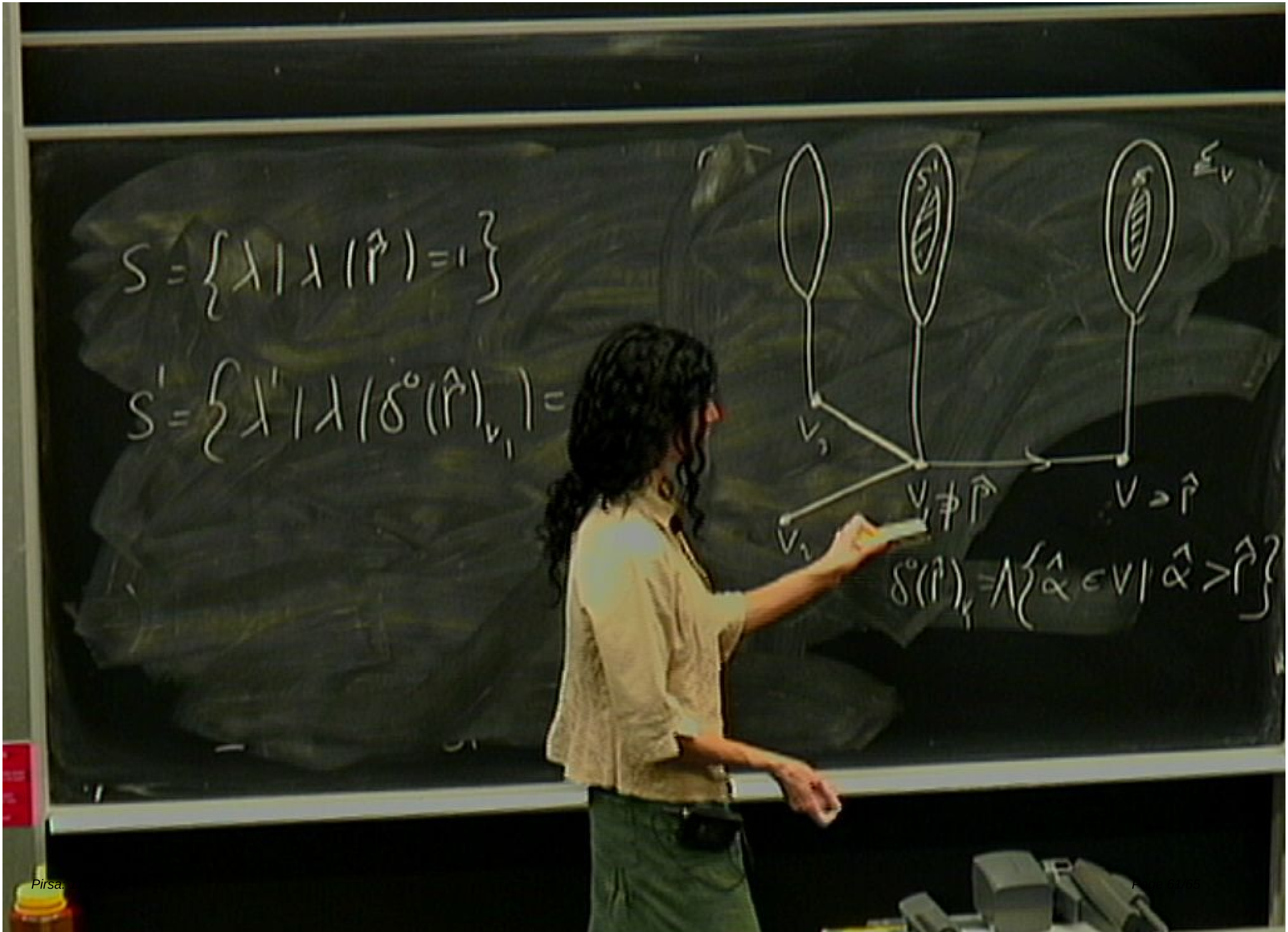
$P(\underline{\Sigma})$, $\underline{\Sigma} : \underline{\mathcal{V}(H)} \longrightarrow \underline{\mathrm{Sets}}$

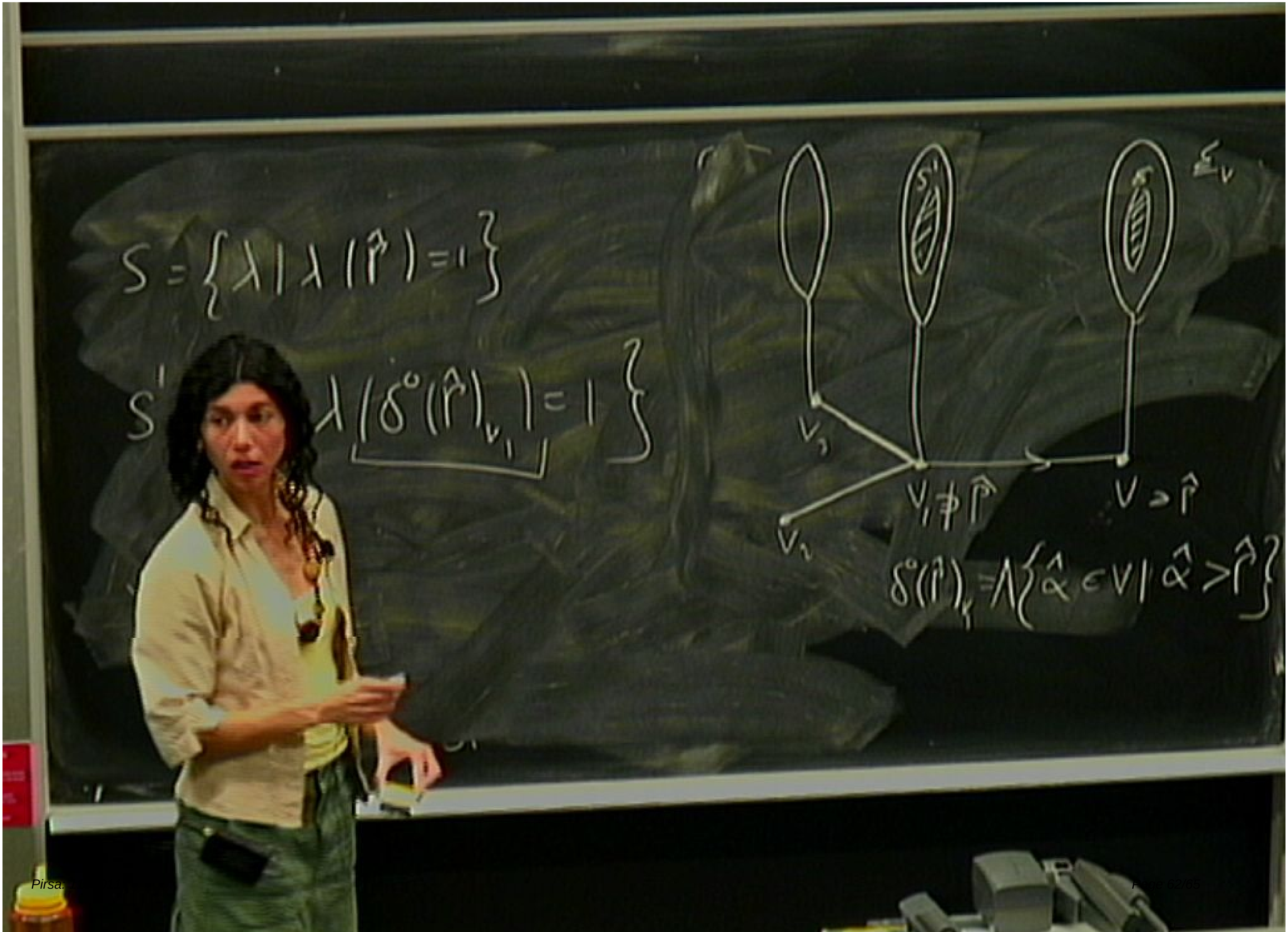
"$\underline{A} \in \underline{\Delta}$" $\rightsquigarrow \underline{\hat{P}}$



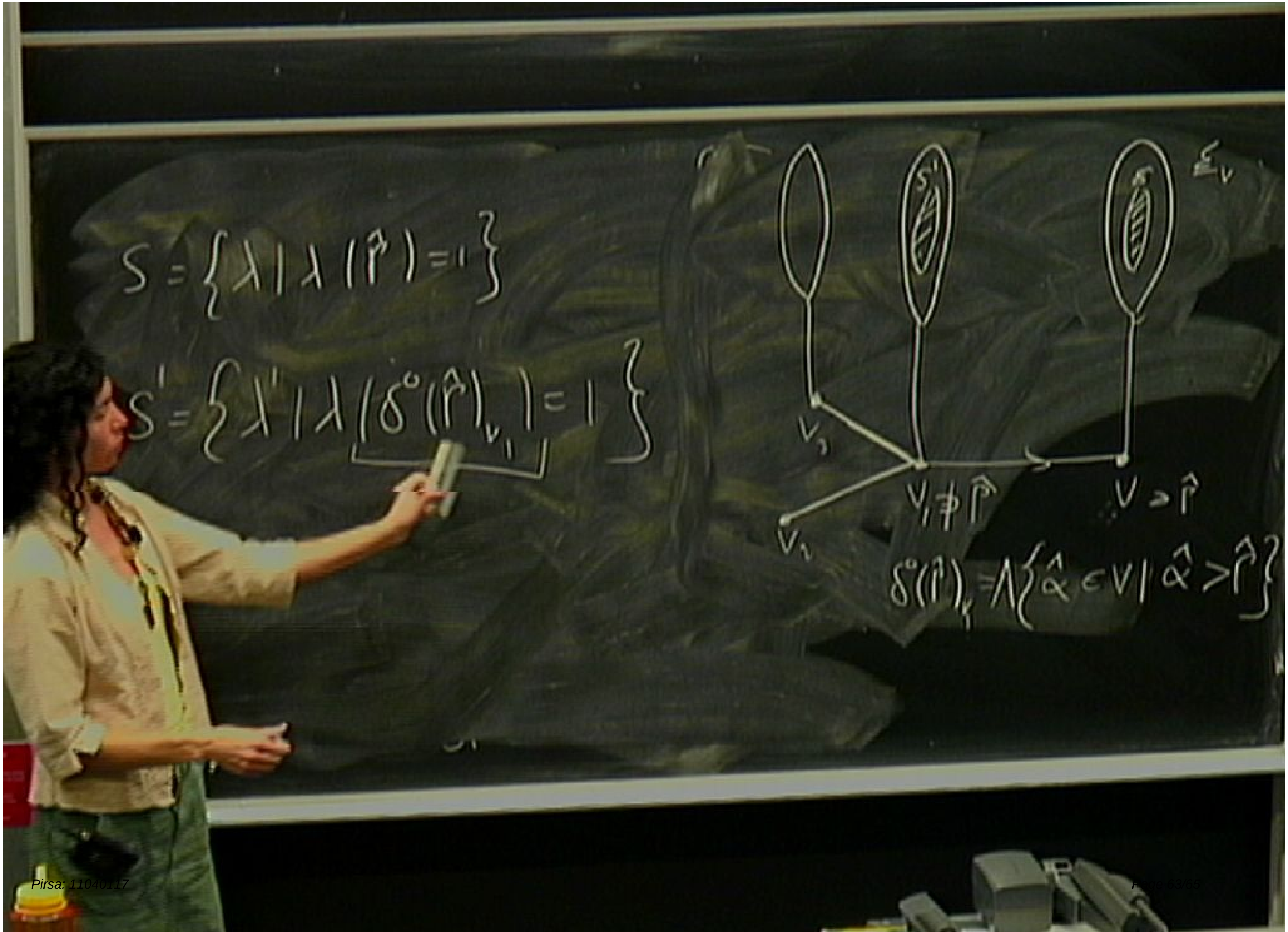

$S = \{\lambda \mid \lambda(\hat{P}) = 1\}$
V_1
V_2
$V_1 \nvDash \hat{P}$
$V \vDash \hat{P}$


S = {λ | λ(P̂) = 1}
S_V
V₁
V ⊇ P̂
δ°(P̂)_V = ∧{α̂ ∈ V | α̂ ≥ P̂}


S = {λ | λ(P̂) = 1}
S' = {λ' | λ(δ°(P̂)_{V_1}) =
V_1
V_2
V ⊉ P̂
V ⊇ P̂
δ°(P̂)_V = ∧{α̂ ∈ V | α̂ ≥ P̂}
S'
≤_V



S = { λ | λ(P̂) = 1 }
S'
λ(δ°(P̂)_{V1}) = 1 }
V1
V2
V1 ⊉ P̂
V ⊇ P̂
δ°(P̂)_V = ∧{ α̂ ∈ V | α̂ ≥ P̂ }

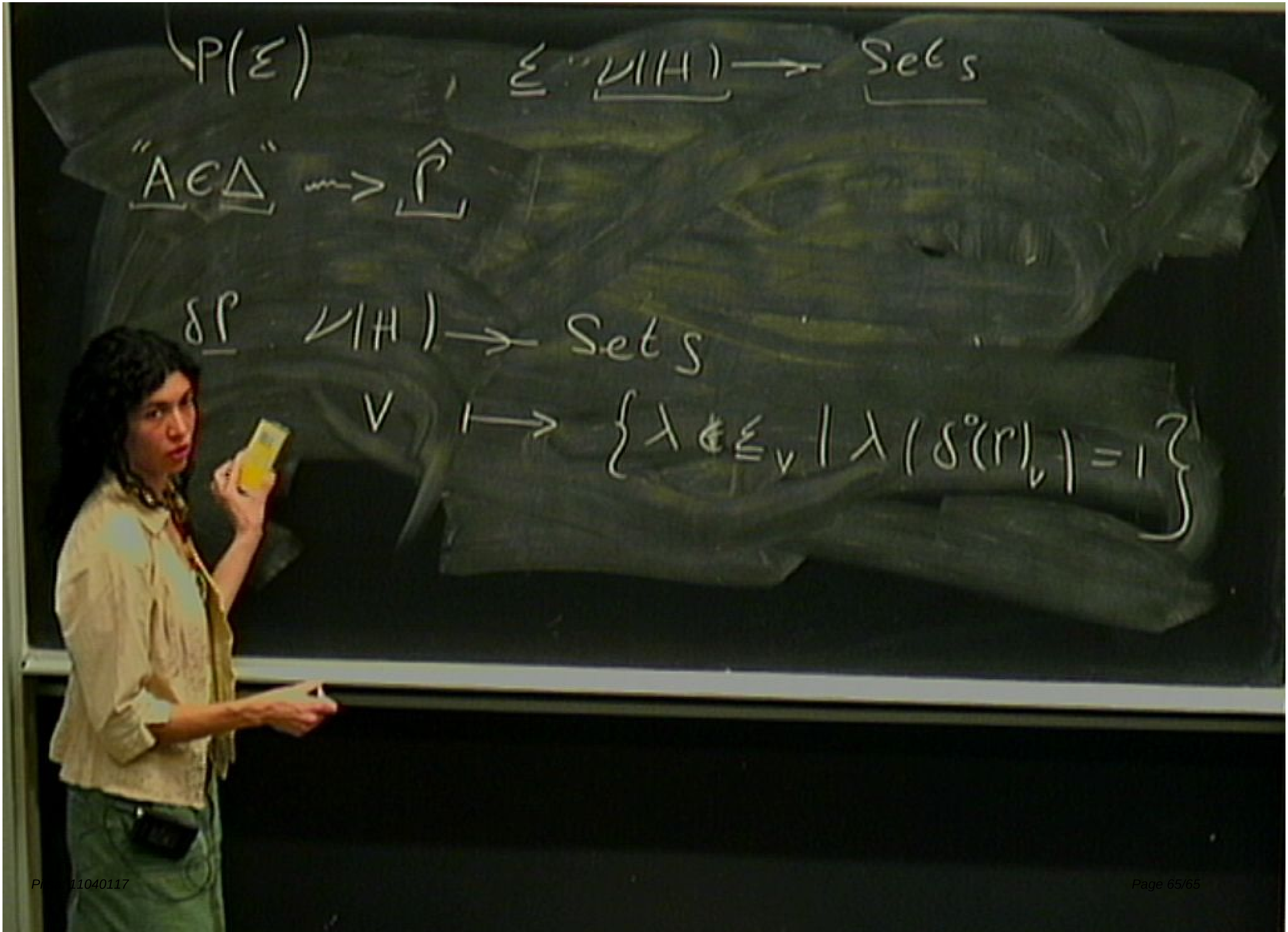

$S = \{ \lambda \mid \lambda(\hat{p}) = 1 \}$
$S' = \{ \lambda' \mid \lambda(\delta^{0}(\hat{p})_{v_1}) = 1 \}$
v_1
v_2
$v_1 \not> \hat{p}$
$v > \hat{p}$
$\delta^{0}(\hat{p})_v = \wedge\{ \hat{\alpha} \in V \mid \hat{\alpha} > \hat{p} \}$
Σ_v

$\varphi(\underline{\mathcal{E}})$, $\underline{\mathcal{E}} : \underline{\mathcal{V}(H)} \longrightarrow \underline{Sets}$

"$\underline{A} \in \underline{\Delta}$" $\rightsquigarrow \underline{\hat{\rho}}$

$\delta\underline{\rho} : \mathcal{V}(H) \longrightarrow Sets$

$$V \longmapsto \{\lambda \in \underline{\mathcal{E}}_V \mid \lambda(\delta^o(\rho)_V) = 1\}$$


P(Σ)
Σ: V(H) → Sets
"A∈Δ" ⇝ P̂
δP V(H) → Sets
V ↦ {λ ∈ Σ_V | λ(δ°(P)_V) = 1}