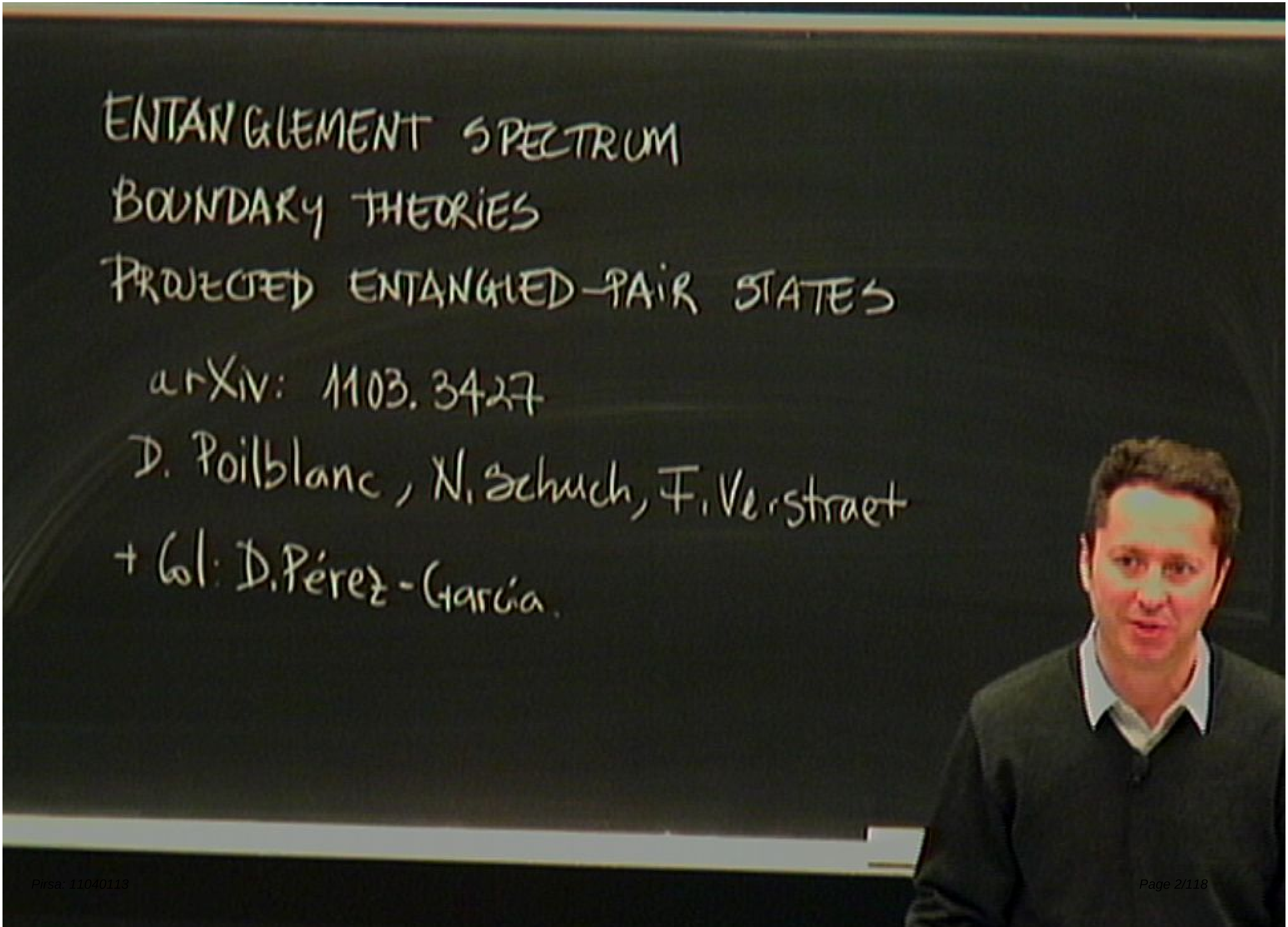
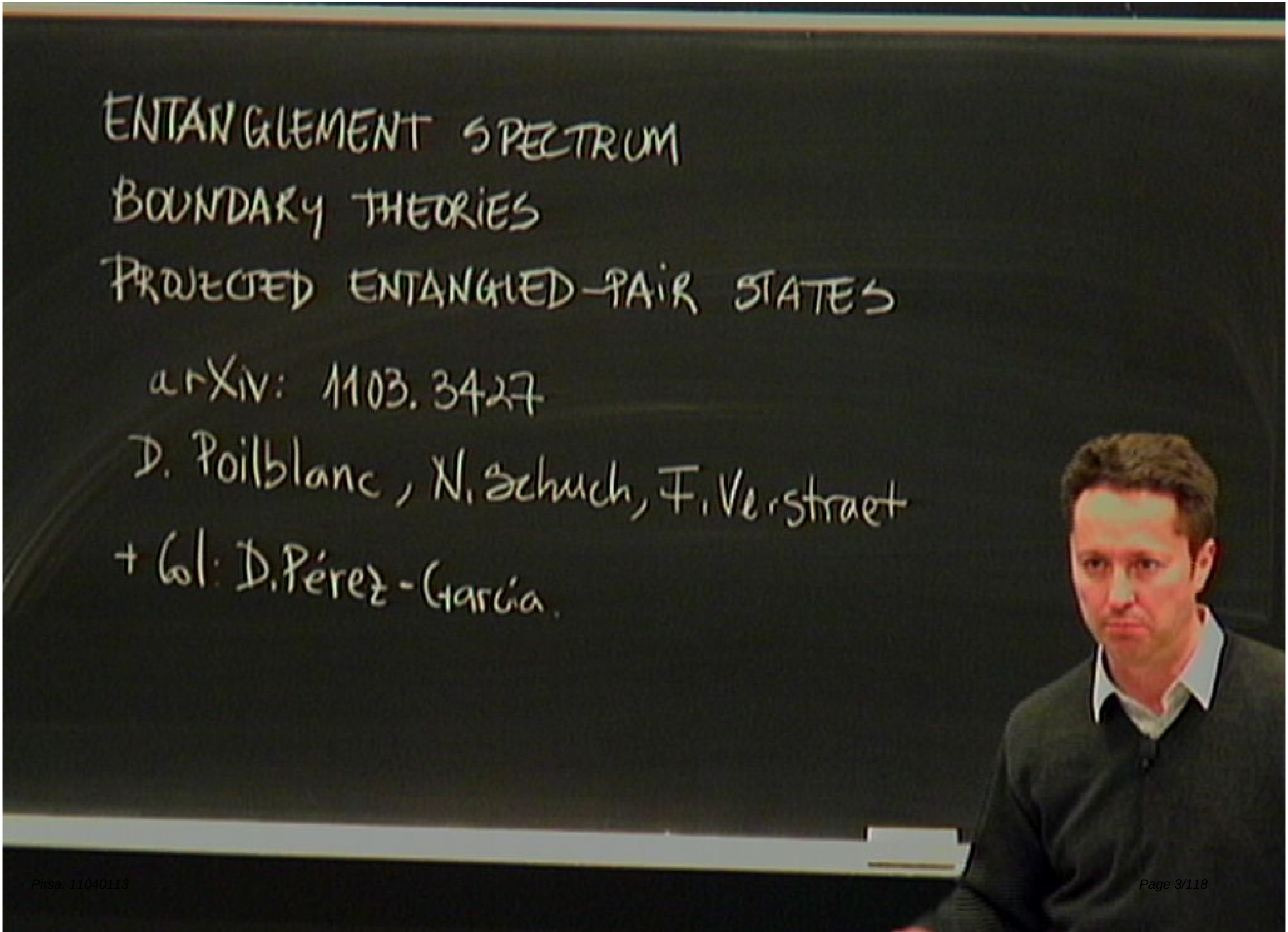
Title: Entanglement spectrum and boundary theories with projected entangled-pair states

Date: Apr 13, 2011 04:00 PM

URL: http://pirsa.org/11040113

Abstract: In many physical scenarios, close relations between the bulk properties of quantum systems and theories associated to their boundaries have been observed. In this work, we provide an exact duality mapping between the bulk of a quantum spin system and its boundary using Projected Entangled Pair States (PEPS). This duality associates to every region a Hamiltonian on its boundary, in such a way that the entanglement spectrum of the bulk corresponds to the excitation spectrum of the boundary Hamiltonian. We study various specific models, like a deformed AKLT , an Ising-type , and Kitaev's toric code, both in finite ladders and infinite square lattices. In the latter case, some of those models display quantum phase transitions. We find that a gapped bulk phase with local order corresponds to a boundary Hamiltonian with local interactions, whereas critical behavior in the bulk is reflected on a diverging interaction length of the boundary Hamiltonian. Furthermore, topologically ordered states yield non-local Hamiltonians. As our duality also associates a boundary operator to any operator in the bulk, it in fact provides a full holographic framework for the study of quantum many-body systems via their boundary. Work done in collaboration with Didier Poilblanc, Norbert Schuch, and Frank Verstraete.


ENTANGLEMENT SPECTRUM
BOUNDARY THEORIES
PROJECTED ENTANGLED-PAIR STATES
arXiv: 1103.3427
D. Poilblanc, N. Schuch, F. Verstraet
+ Col: D. Pérez-García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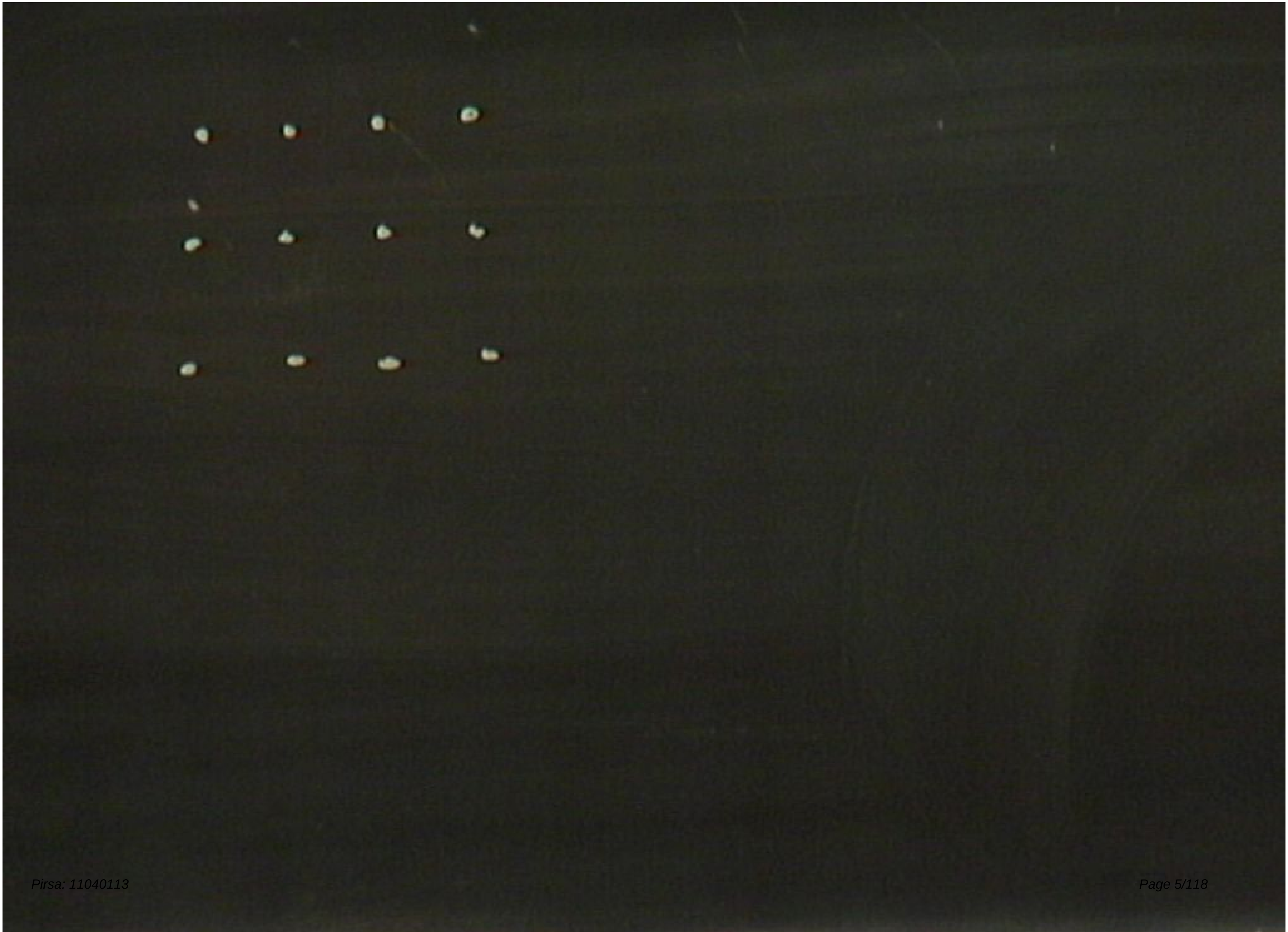
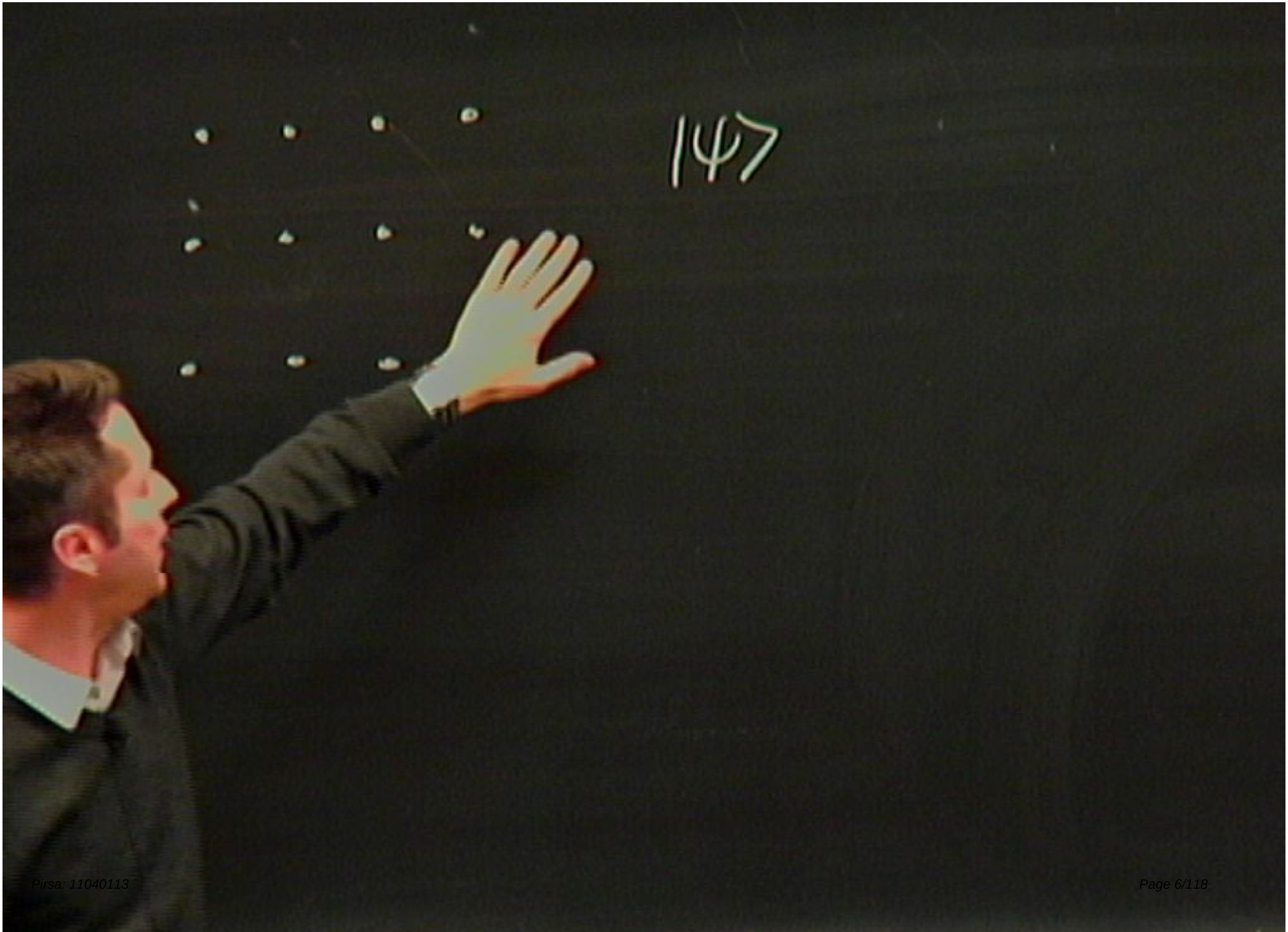
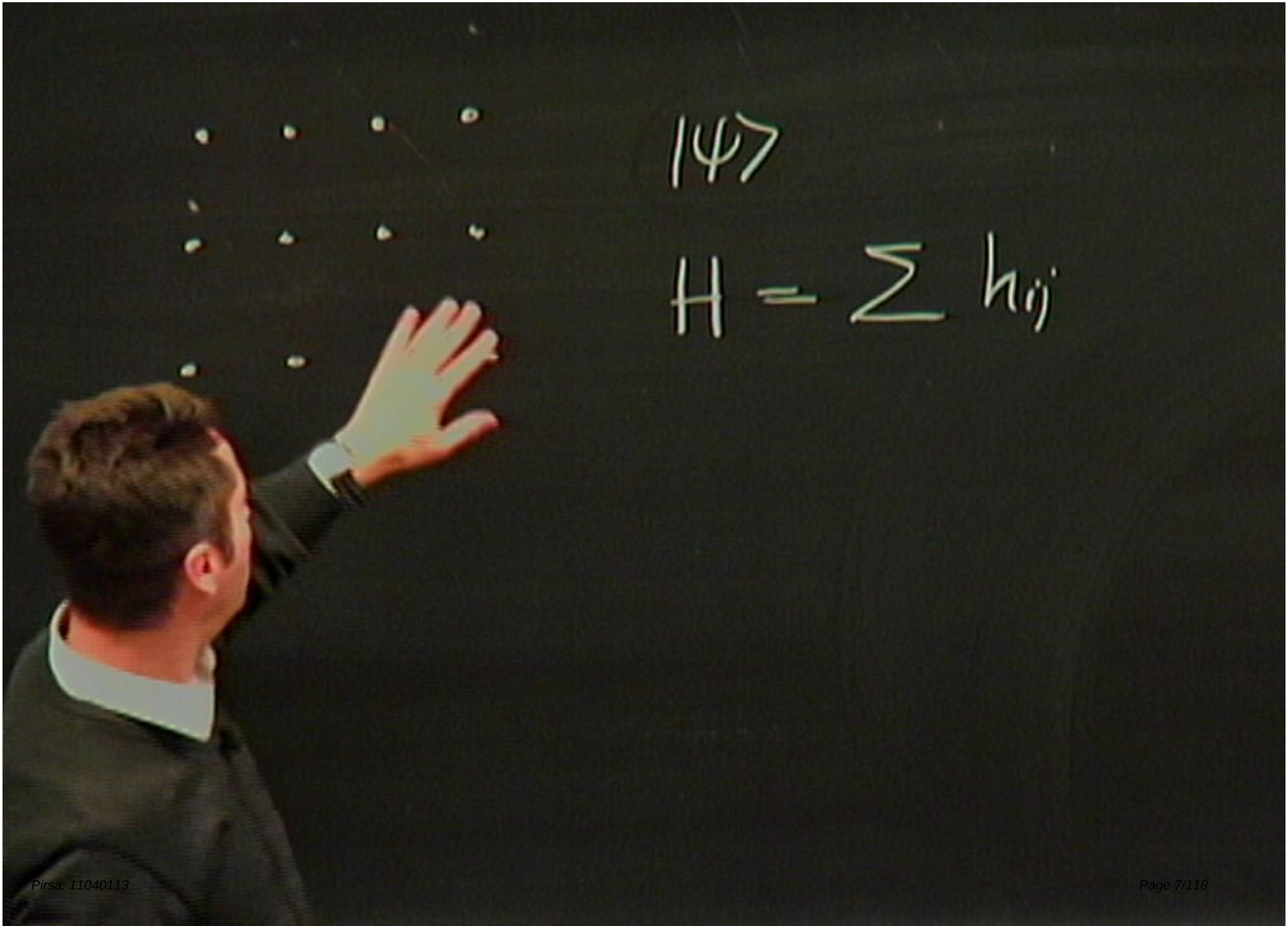
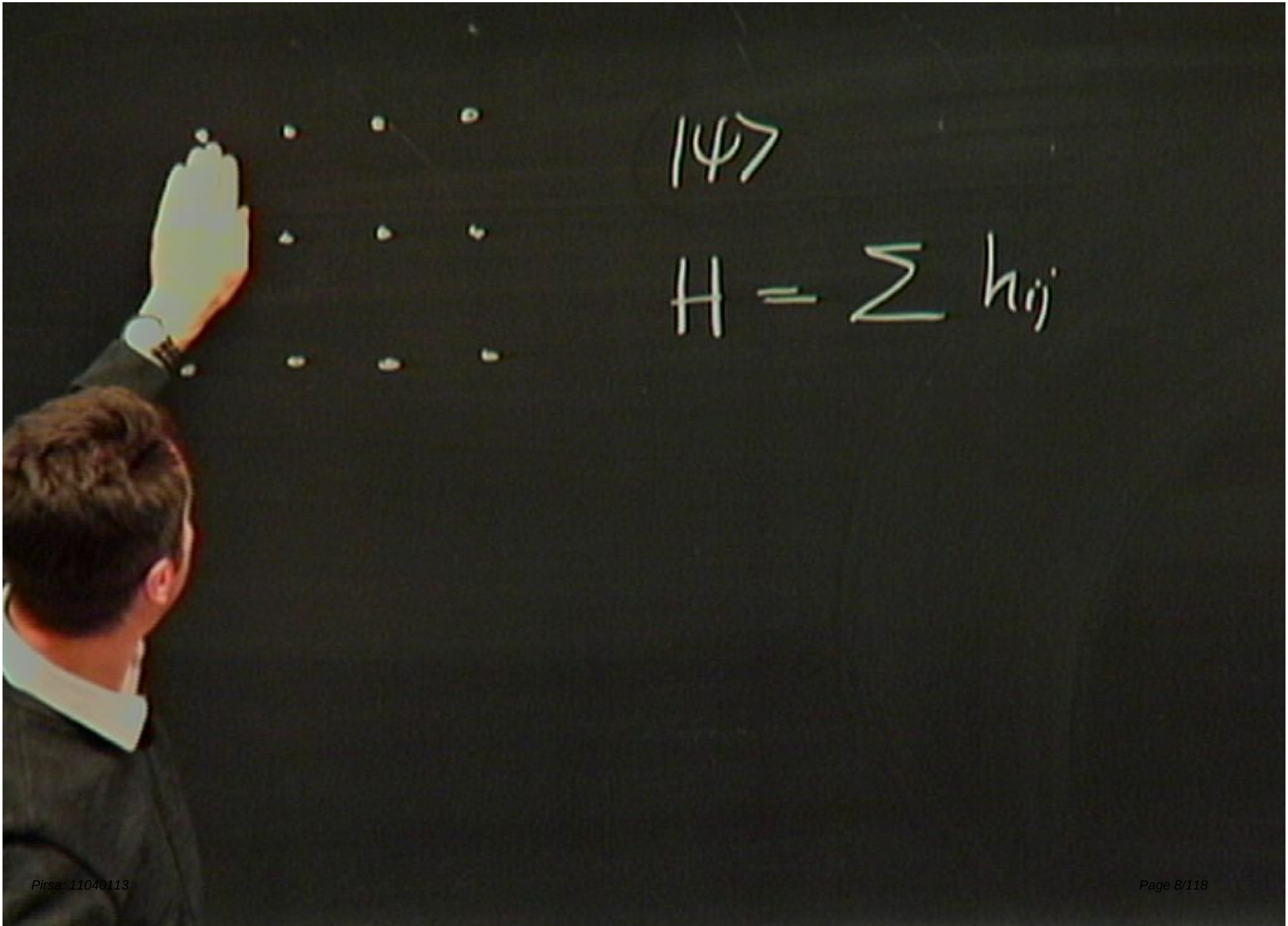
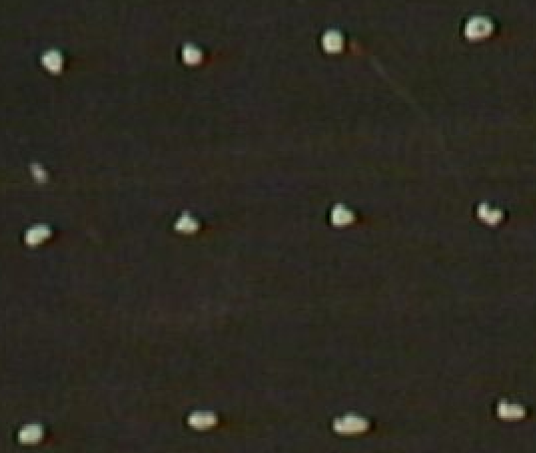



$|\psi\rangle$


$|\psi\rangle$
$H = \sum h_{ij}$


$|\Psi\rangle$
$H = \sum h_{ij}$



$$|\psi\rangle$$

$$H = \sum h_{ij}$$

$|\psi\rangle$

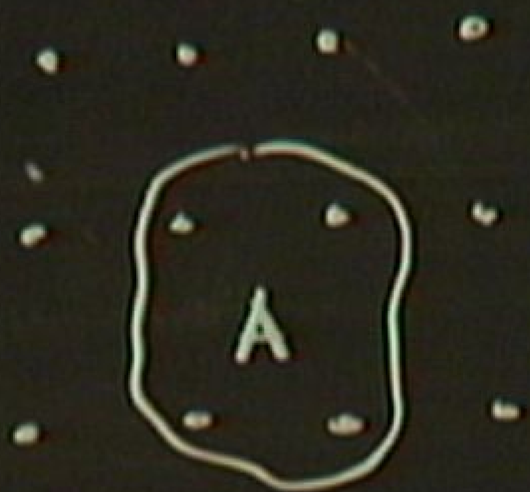
$$H = \sum h_{ij}$$

1

$|\psi\rangle$

$$H = \sum h_{ij}$$

1



$|\psi\rangle$

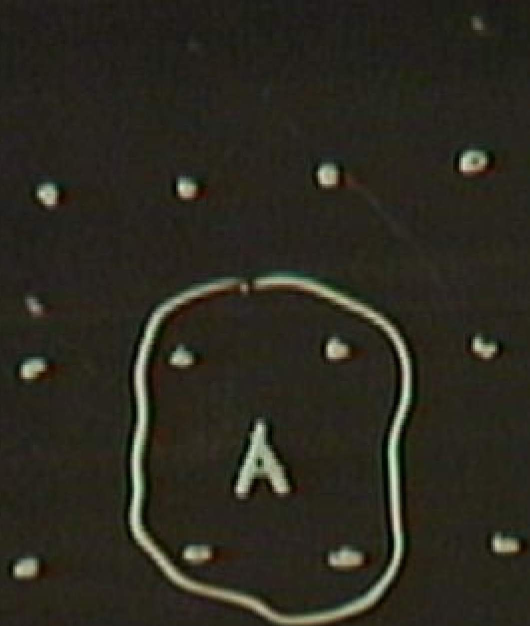
$$H = \sum h_{ij}$$

1 ρ_A

$|\psi\rangle$

$$H = \sum h_{ij}$$

1 ρ_A

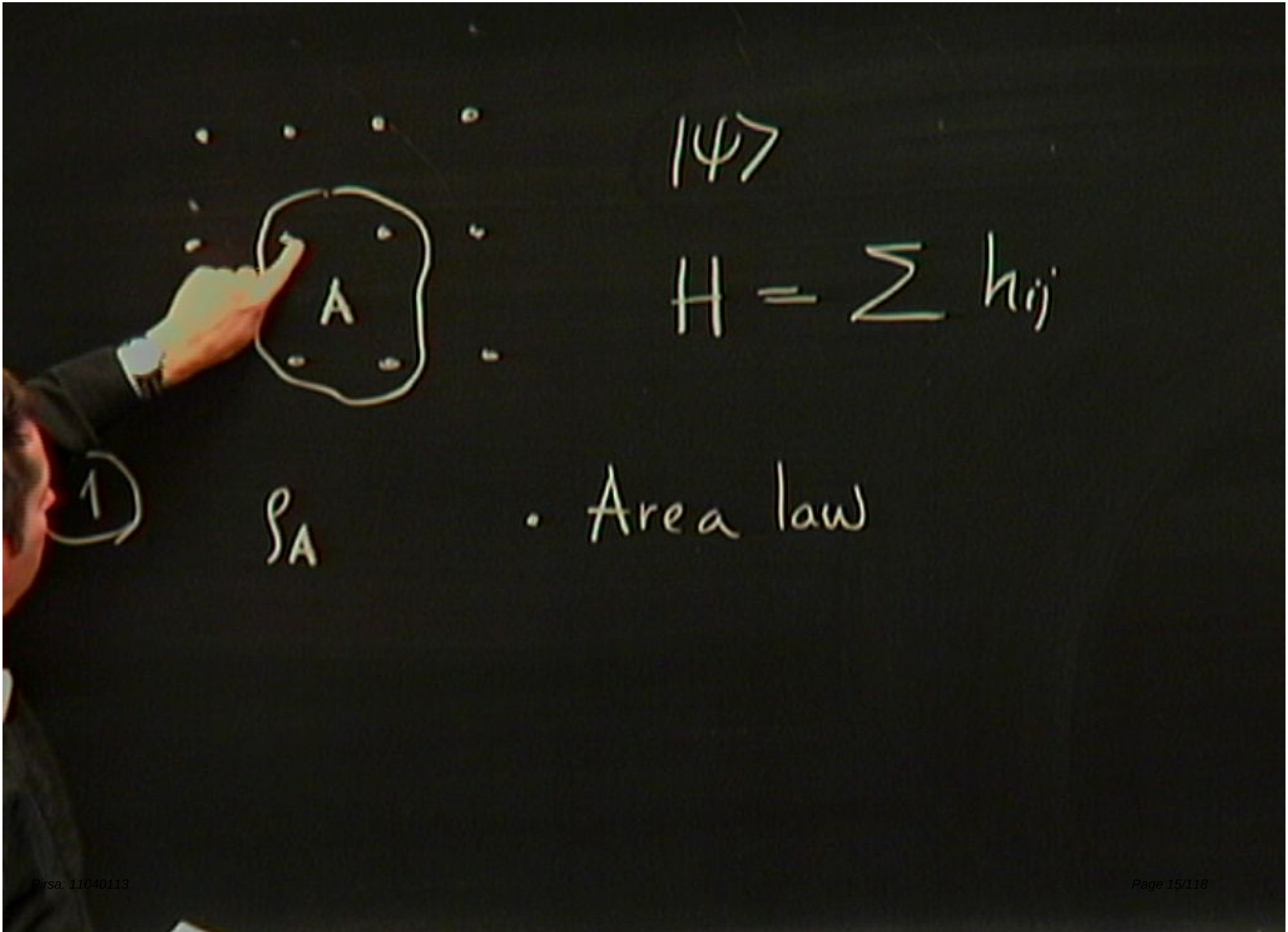


$|\psi\rangle$

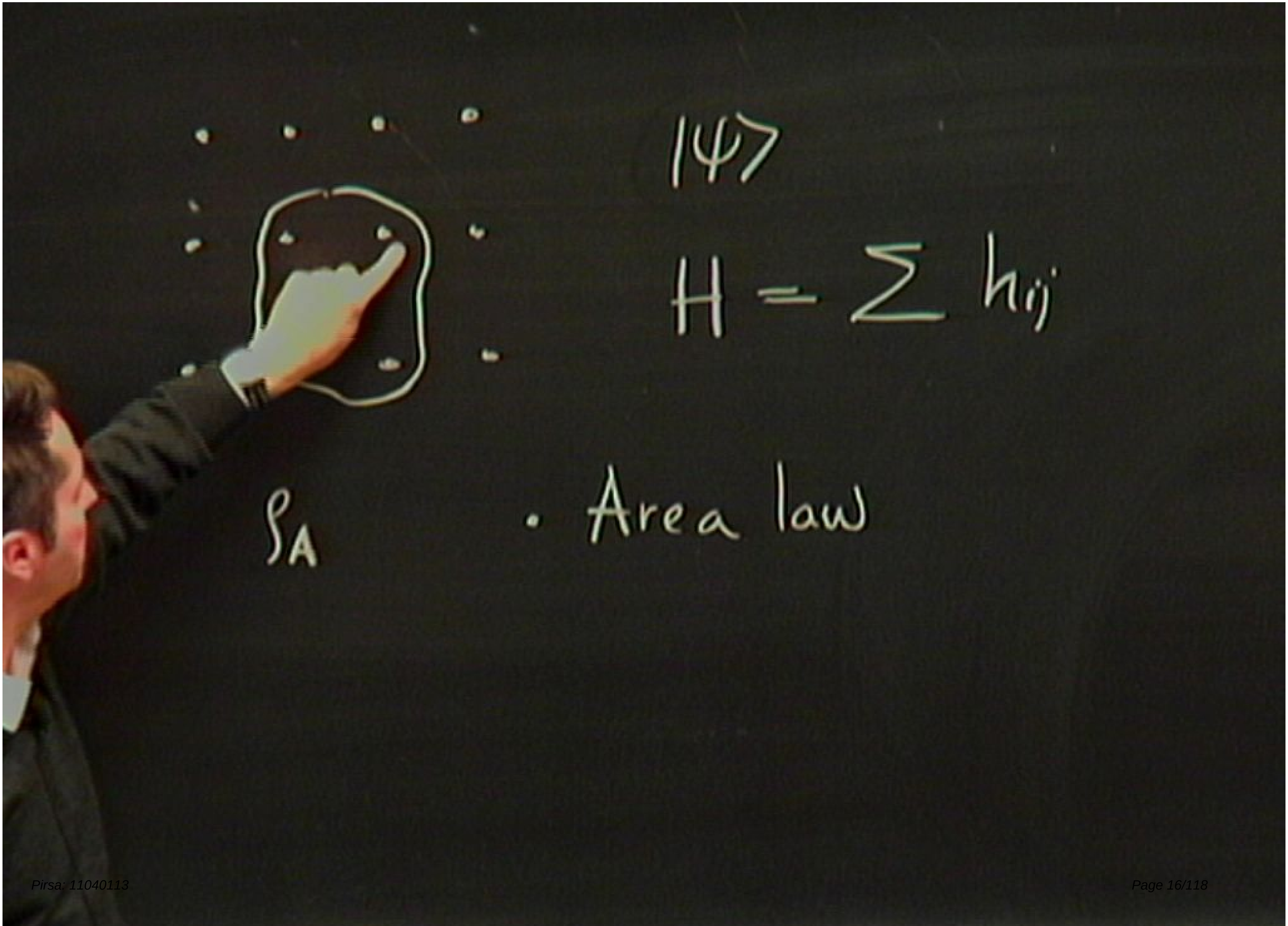
$$H = \sum h_{ij}$$

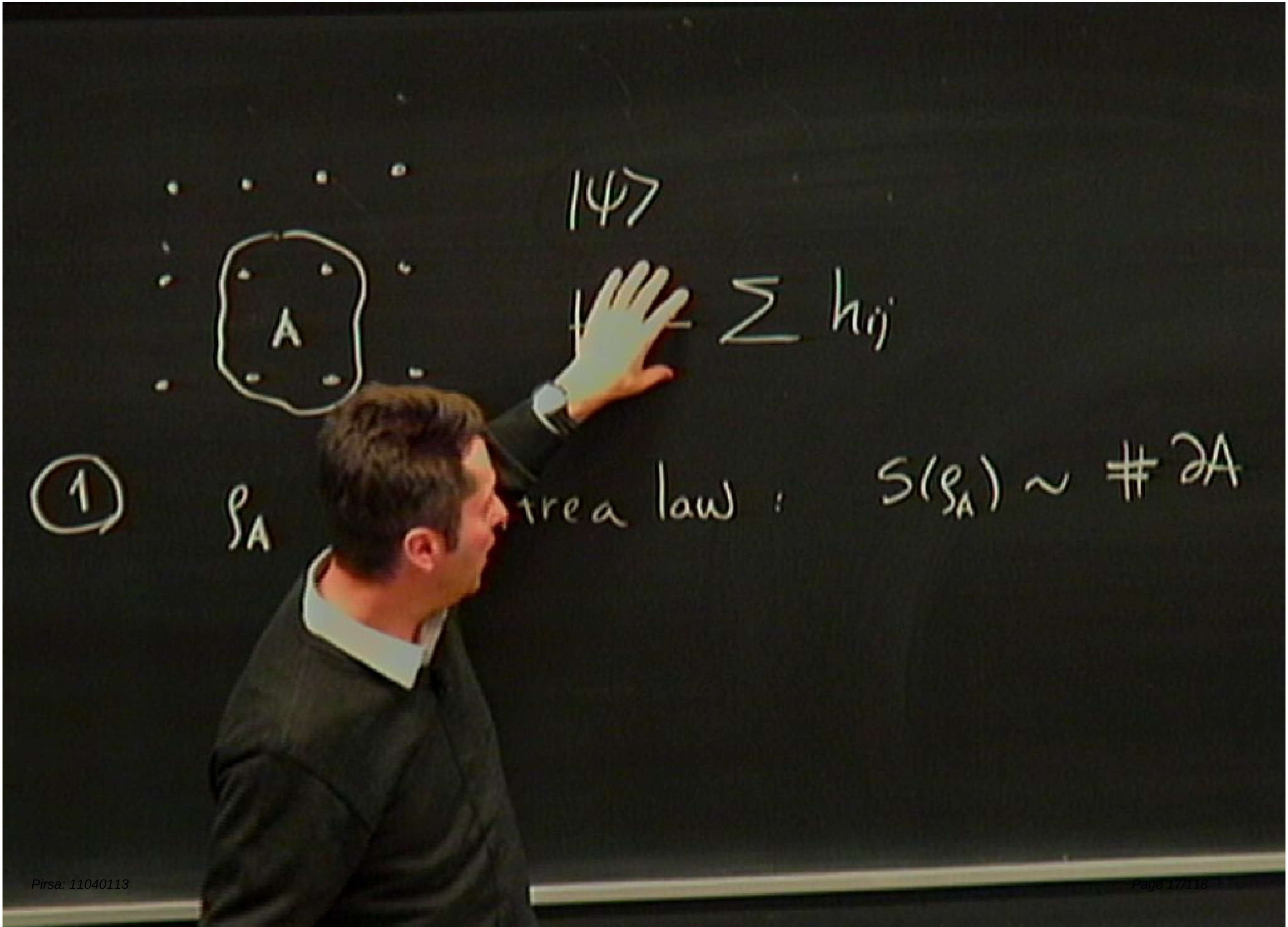
1 ρ_A

- Area law



|ψ⟩
A
H = Σ h_ij
1
ρ_A
• Area law

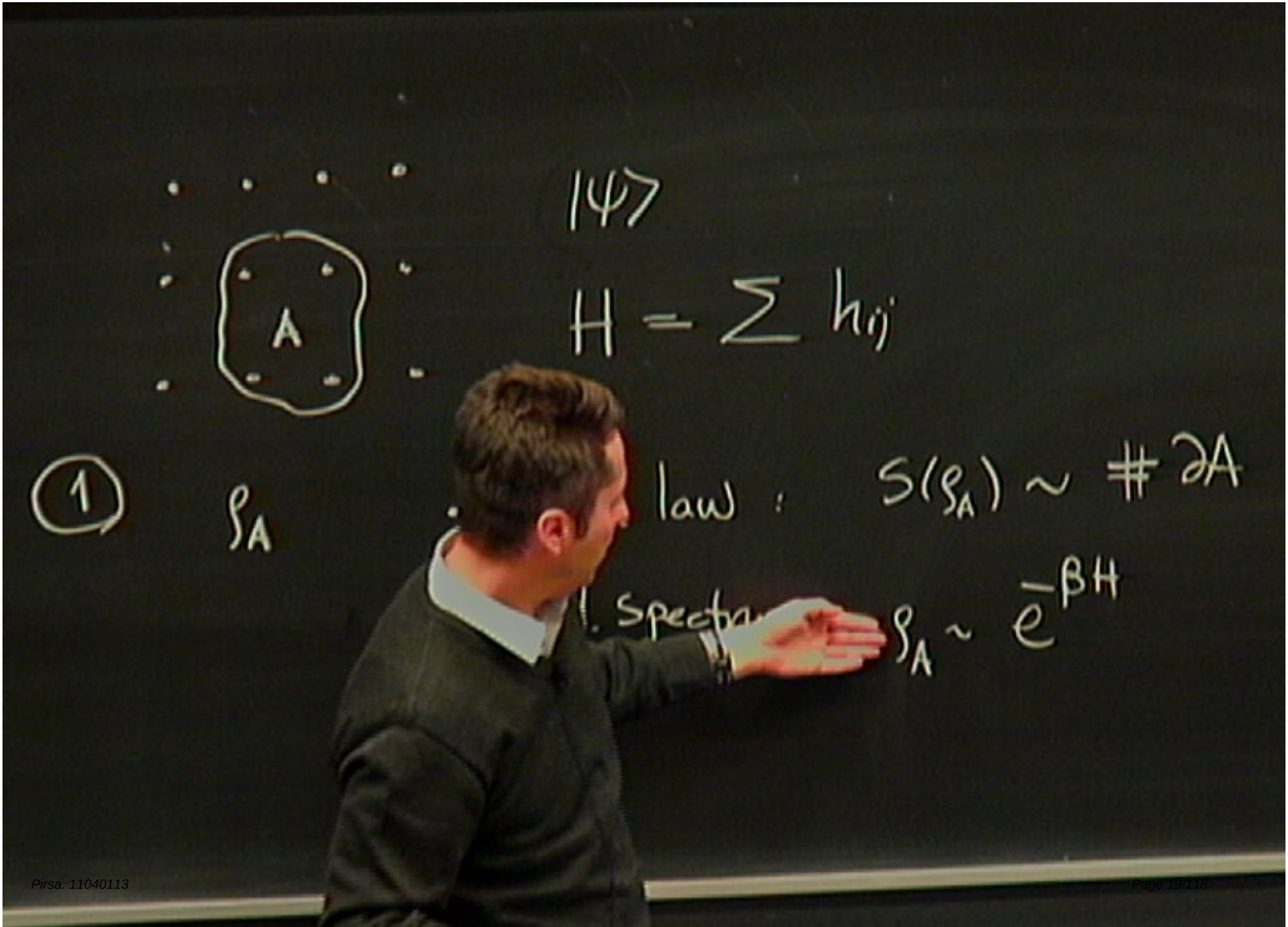

$|\Psi\rangle$
$H = \sum h_{ij}$
ρ_A
• Area law

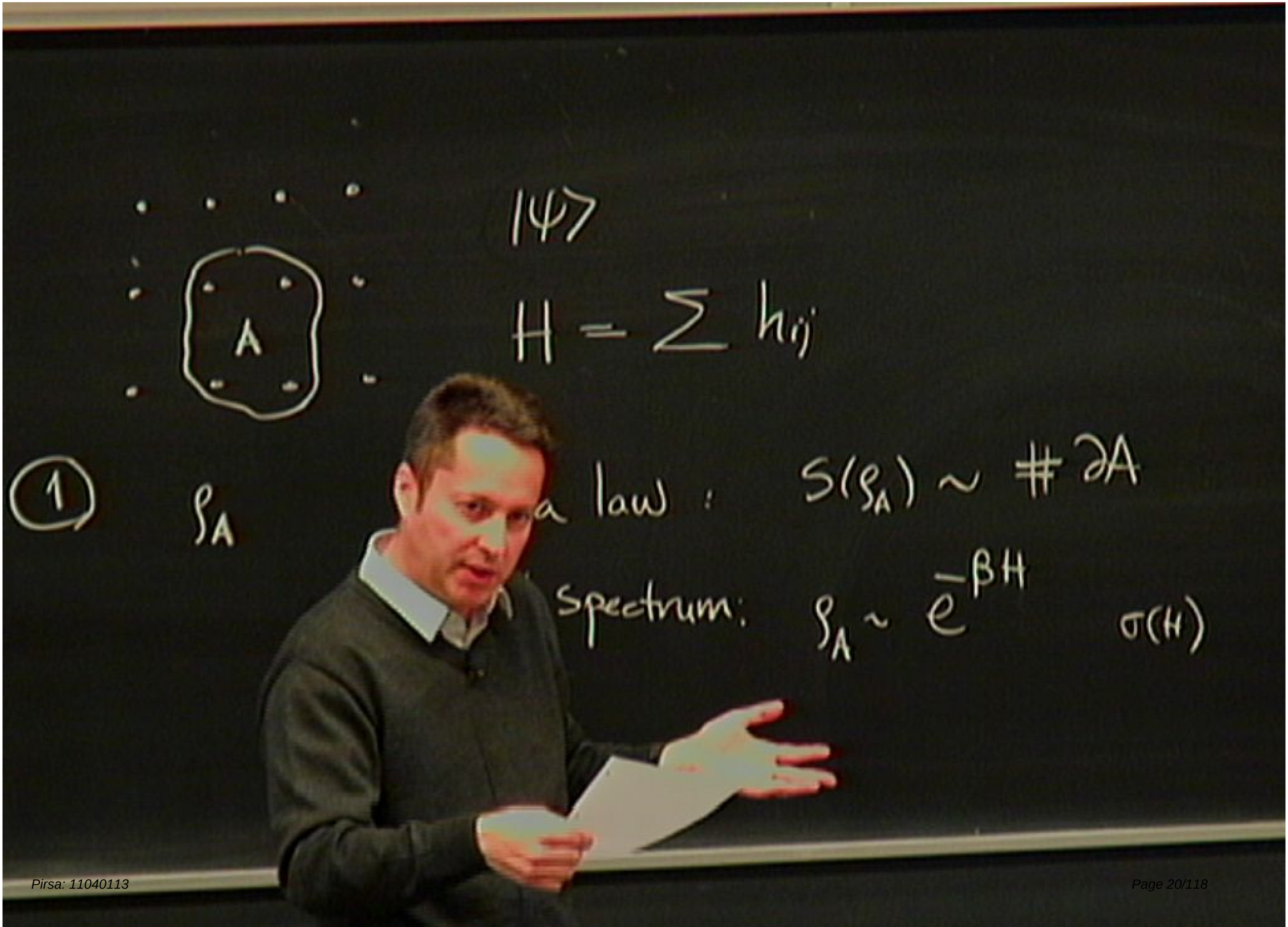

|ψ⟩
A
Σ h_ij
1
ρ_A
rea law :
S(ρ_A) ~ # ∂A

$|\psi\rangle$

$H = \sum h_{ij}$

1 ρ_A • Area law: $S(\rho_A) \sim \# \partial A$


$|\psi\rangle$
$H = \sum h_{ij}$
A
1
ρ_A
law : $S(\rho_A) \sim \# \partial A$
$\rho_A \sim e^{-\beta H}$


$|\psi\rangle$
$H = \sum h_{ij}$
A
1
ρ_A
a law : $S(\rho_A) \sim \# \partial A$
Spectrum: $\rho_A \sim e^{-\beta H}$
$\sigma(H)$

$|\psi\rangle$

$$H = \sum h_{ij}$$

1 ρ_A

- Area law: $S(\rho_A) \sim \# \partial A$
- Ent. spectrum: $\rho_A \sim e^{-\beta H}$ $\quad \sigma(H)$

$|\psi\rangle$

$$H = \sum h_{ij}$$

(1) ρ_A

- Area law: $S(\rho_A) \sim \# \partial A$
- Ent. spectrum: $\rho_A \sim e^{-\beta H}$ $\sigma(H)$

$|\psi\rangle$

$$H = \sum h_{ij}$$

1 ρ_A

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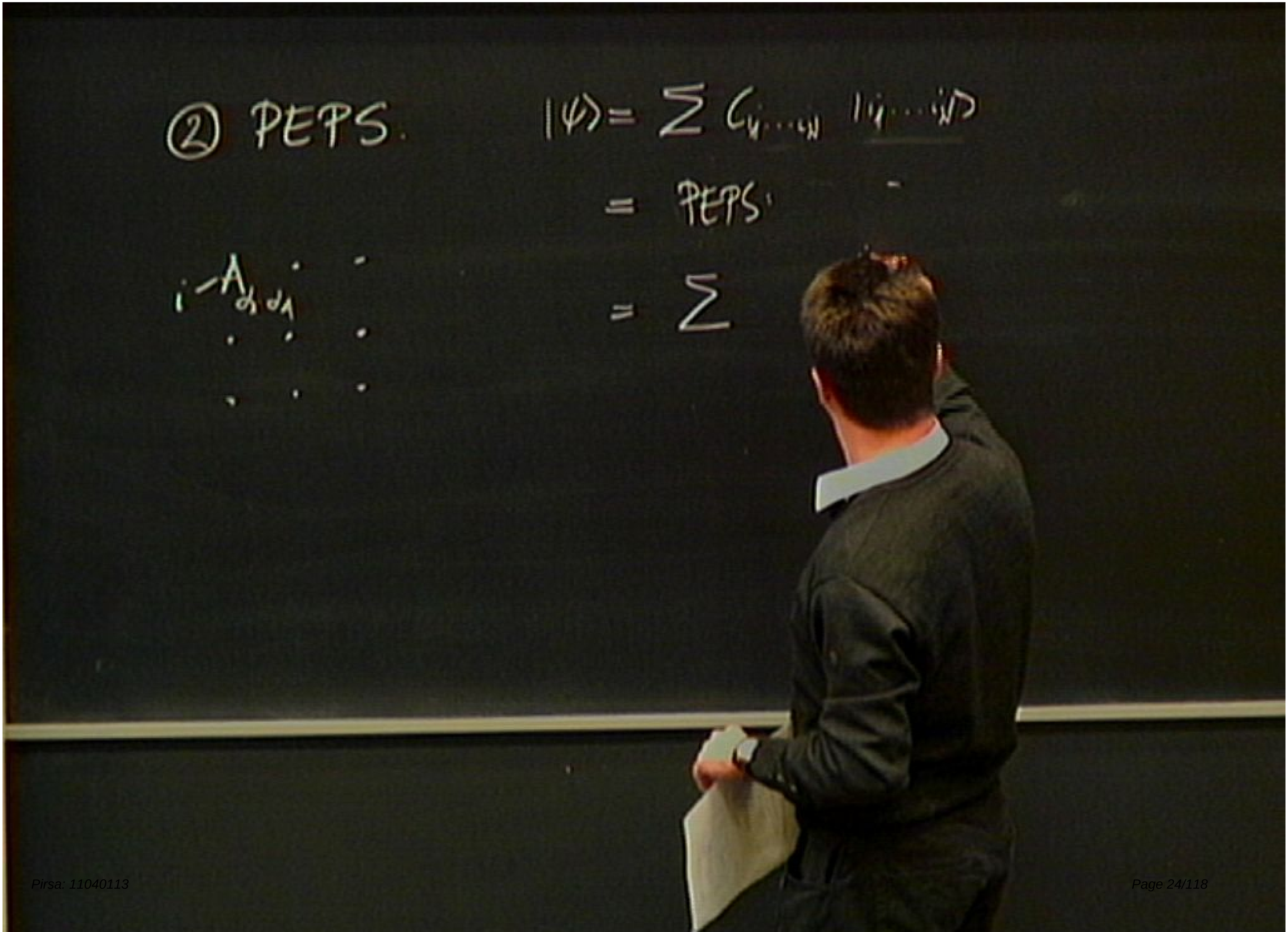
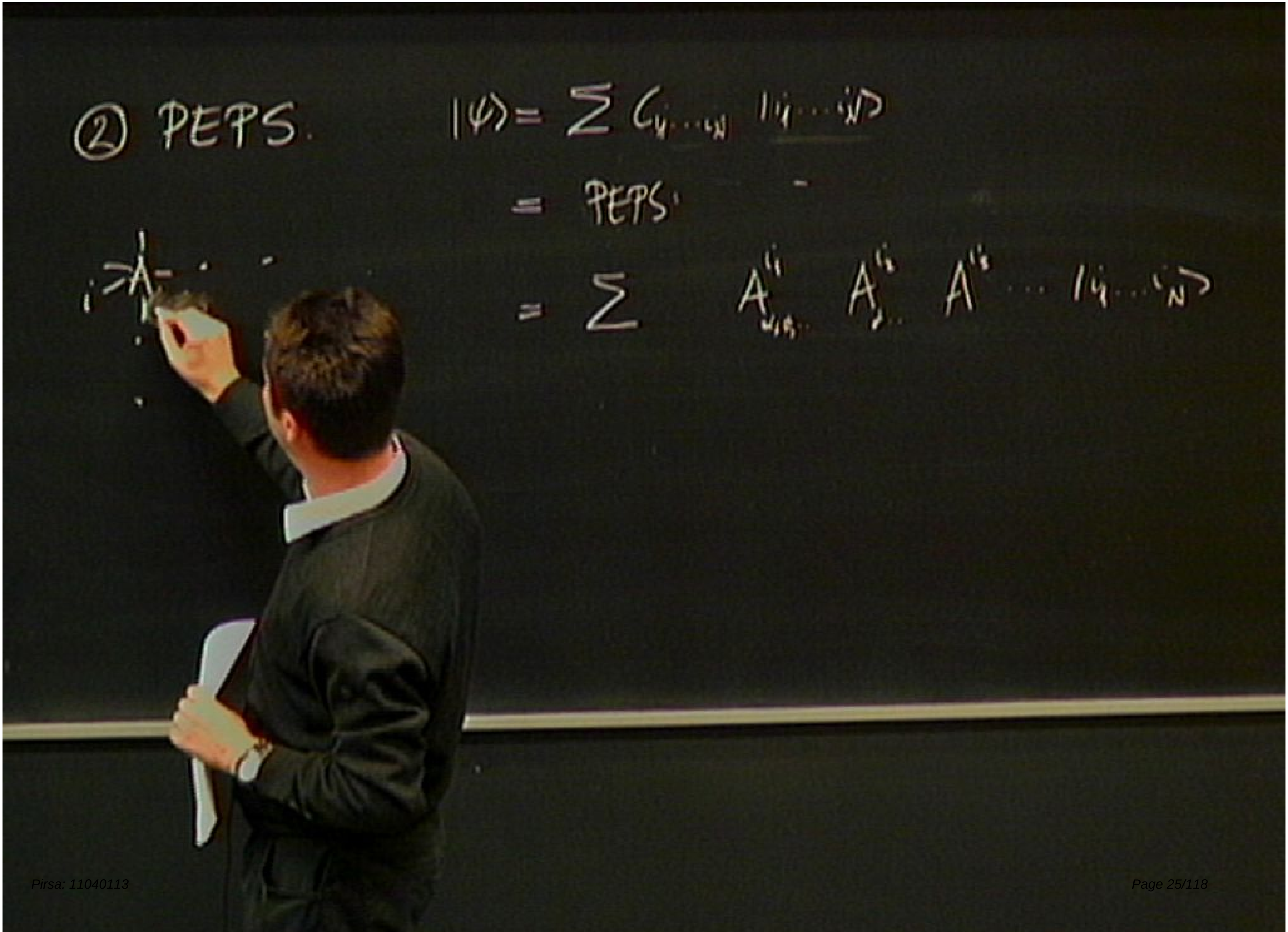
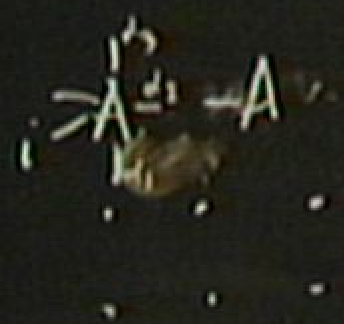
- Ent. spectrum: $\rho_A \sim e^{-\beta H}$ $\sigma(H)$


② PEPS
|ψ⟩ = Σ C_{i1...iN} |i1...iN⟩
= PEPS
= Σ
i — A_{α1 α1}


② PEPS
$|\psi\rangle = \sum C_{i_1 \dots i_N} |i_1 \dots i_N\rangle$
$= \text{PEPS}$
$= \sum A^{i_1} A^{i_2} A^{i_3} \dots |i_1 \dots i_N\rangle$

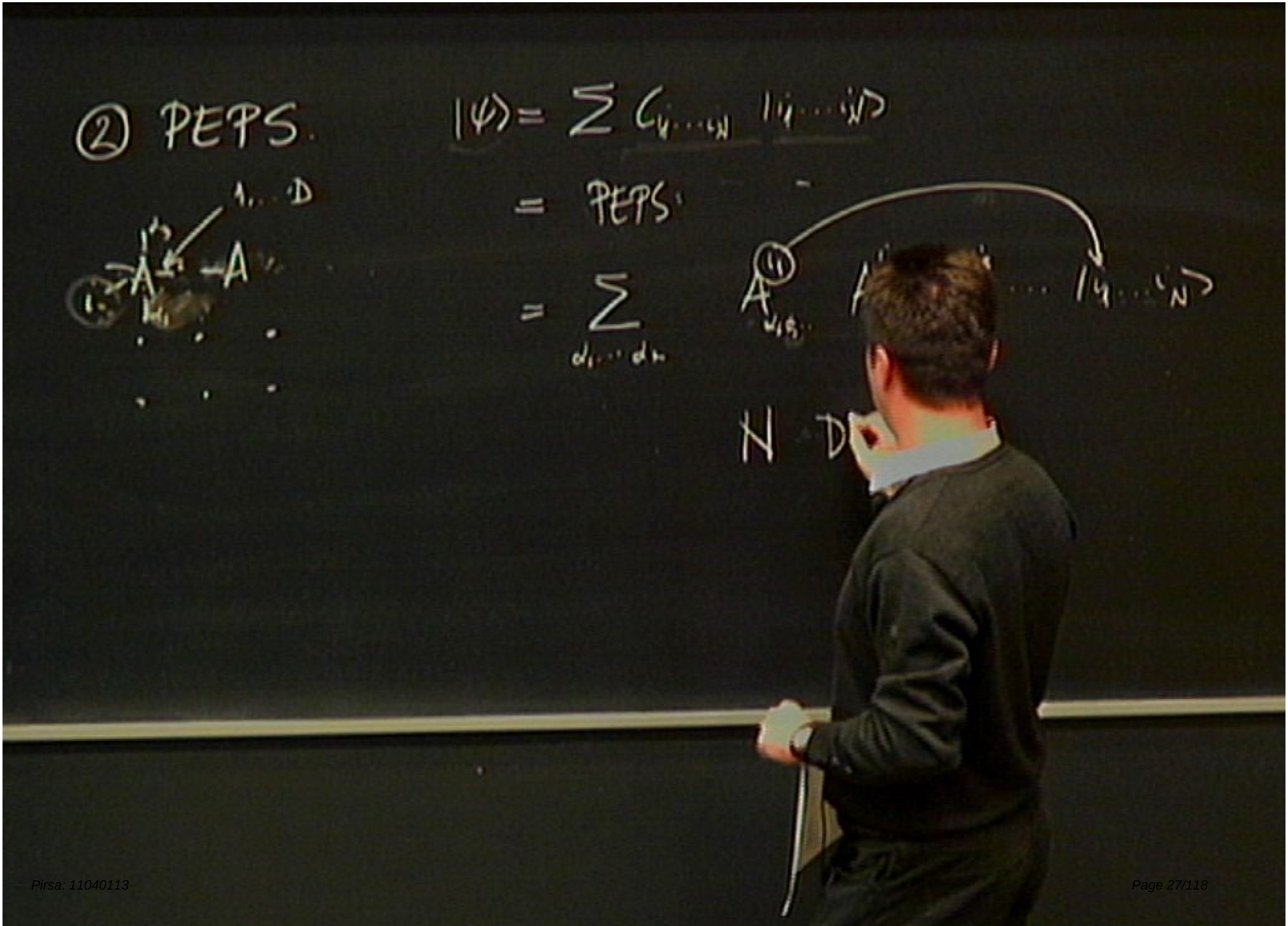
② PEPS.



$$|\psi\rangle = \sum C_{i_1 \cdots i_N} |i_1 \cdots i_N\rangle$$

$$= \text{PEPS}$$

$$= \sum_{d_1 \cdots d_M} A^{i_1}_{d_1 d_2 \cdots} A^{i_2}_{d \cdots} A^{i_3} \cdots |i_1 \cdots i_N\rangle$$



② PEPS
$|\psi\rangle = \sum C_{i_1 \cdots i_N} |i_1 \cdots i_N\rangle$
= PEPS
1, ... D
A A
= $\sum_{d_1 \cdots d_n}$ A
$|i_1 \cdots i_N\rangle$
N D

② PEPS.

$$|\psi\rangle = \sum C_{i_1 \cdots i_N} |i_1 \cdots i_N\rangle$$

$$= \text{PEPS}$$

$$= \sum_{\alpha_1 \cdots \alpha_n} A^{i_1}_{\alpha_1 \alpha_2} A^{i_2}_{\alpha_2 \cdots} A^{i_3} \cdots |i_1 \cdots i_N\rangle$$

$$\boxed{2 N D^z}$$

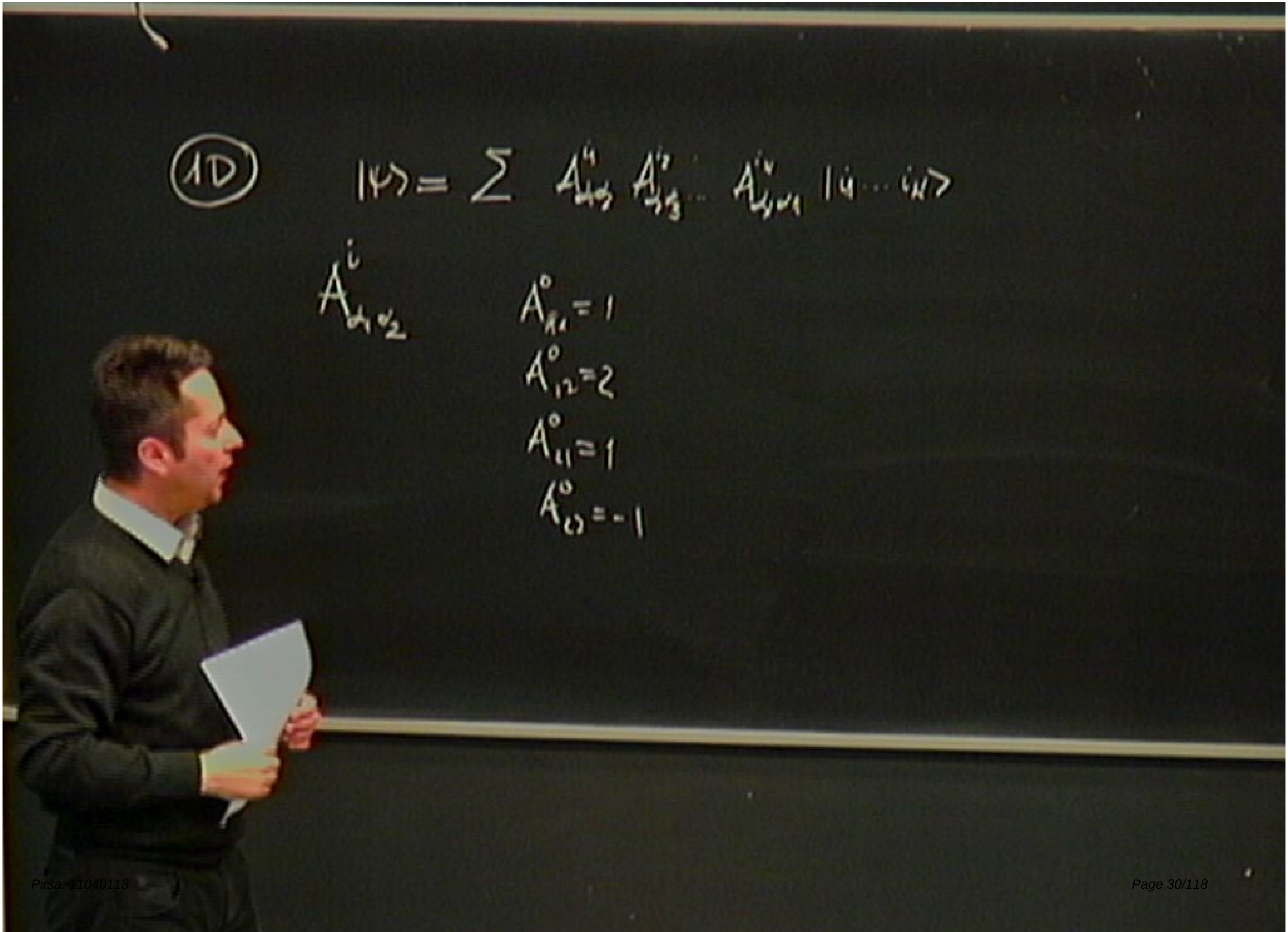
② PEPS

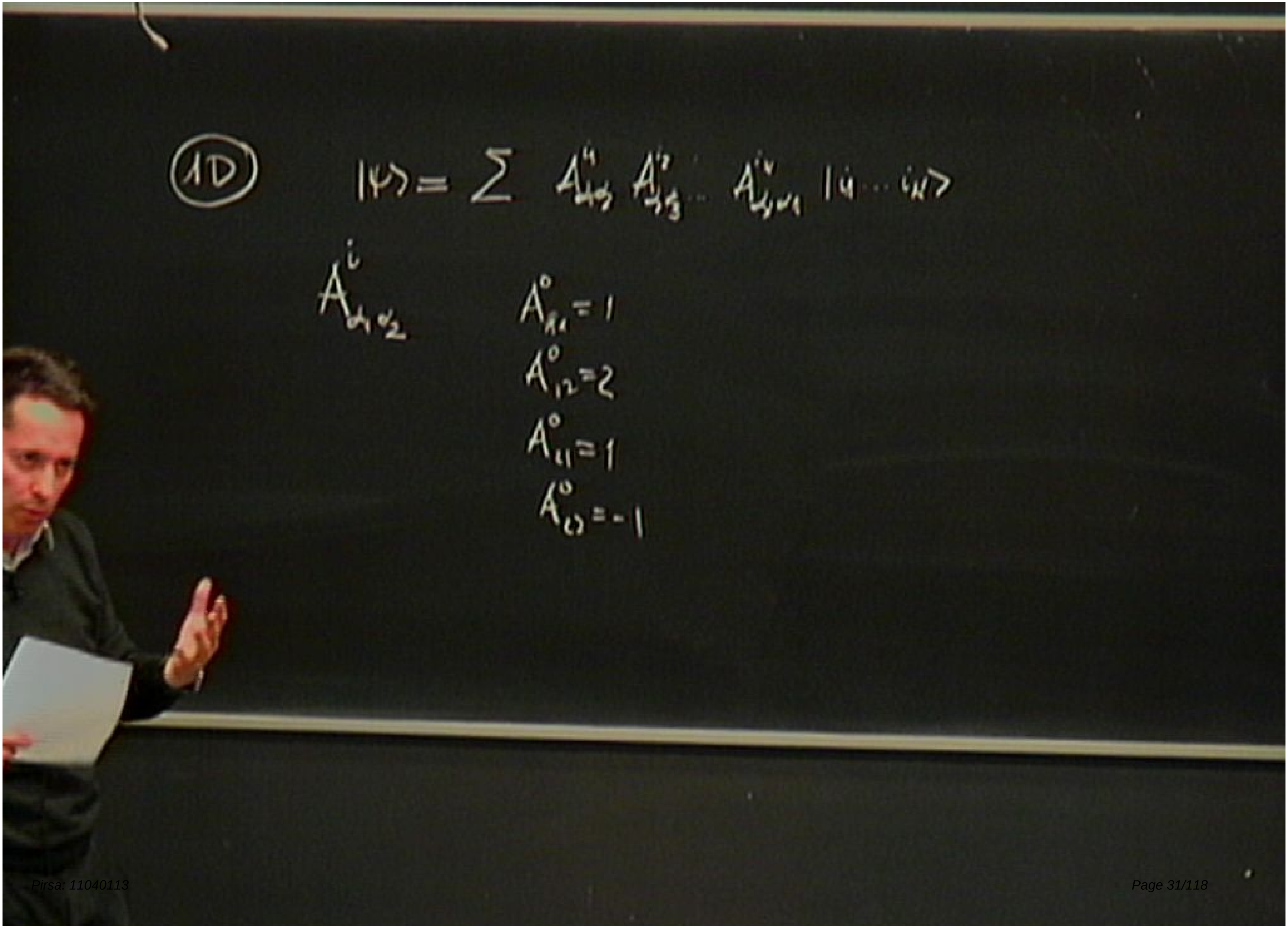
$$|\psi\rangle = \sum C_{i_1 \dots i_N} |i_1 \dots i_N\rangle$$

$$= \text{PEPS}$$

$$= \sum_{\alpha_1 \dots \alpha_M} A^{i_1}_{\alpha_1 \alpha_2} A^{i_2}_{\alpha_1 \dots} A^{i_3} \cdots |i_1 \dots i_N\rangle$$

$$\boxed{2 N D^z}$$


1D
$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$
$A^{i}_{\alpha_1\alpha_2}$
$A^0_{11}=1$
$A^0_{12}=2$
$A^0_{21}=1$
$A^0_{22}=-1$


1D
$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$
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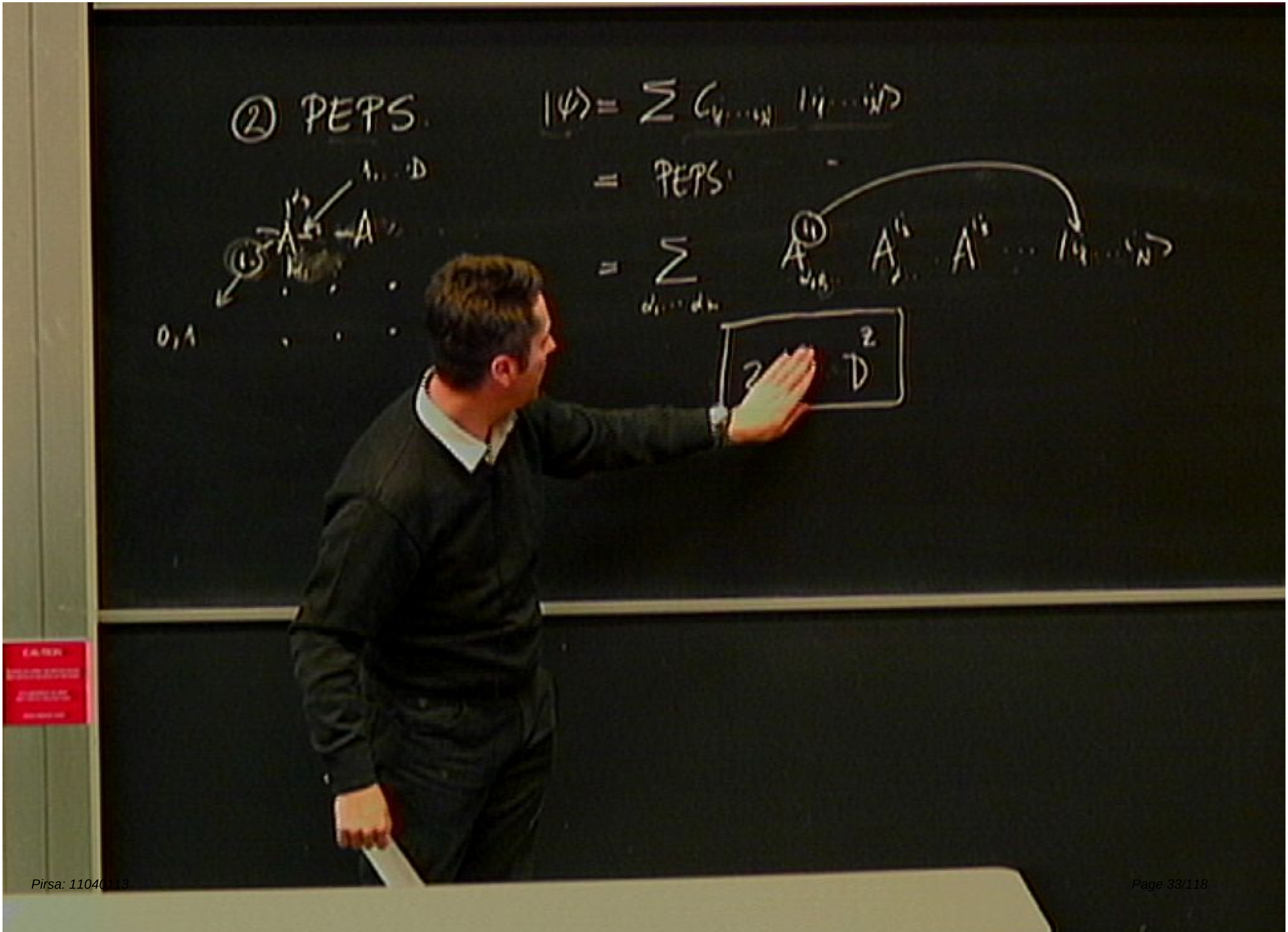
1D

$$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$$

$$A^{i}_{\alpha_1\alpha_2}$$

$$\boxed{\begin{array}{l} A^{0}_{11} = 1 \\ A^{0}_{12} = 2 \\ A^{0}_{21} = 1 \\ A^{0}_{22} = -1 \end{array}}$$


② PEPS
= PEPS
1, ... D
0, 1

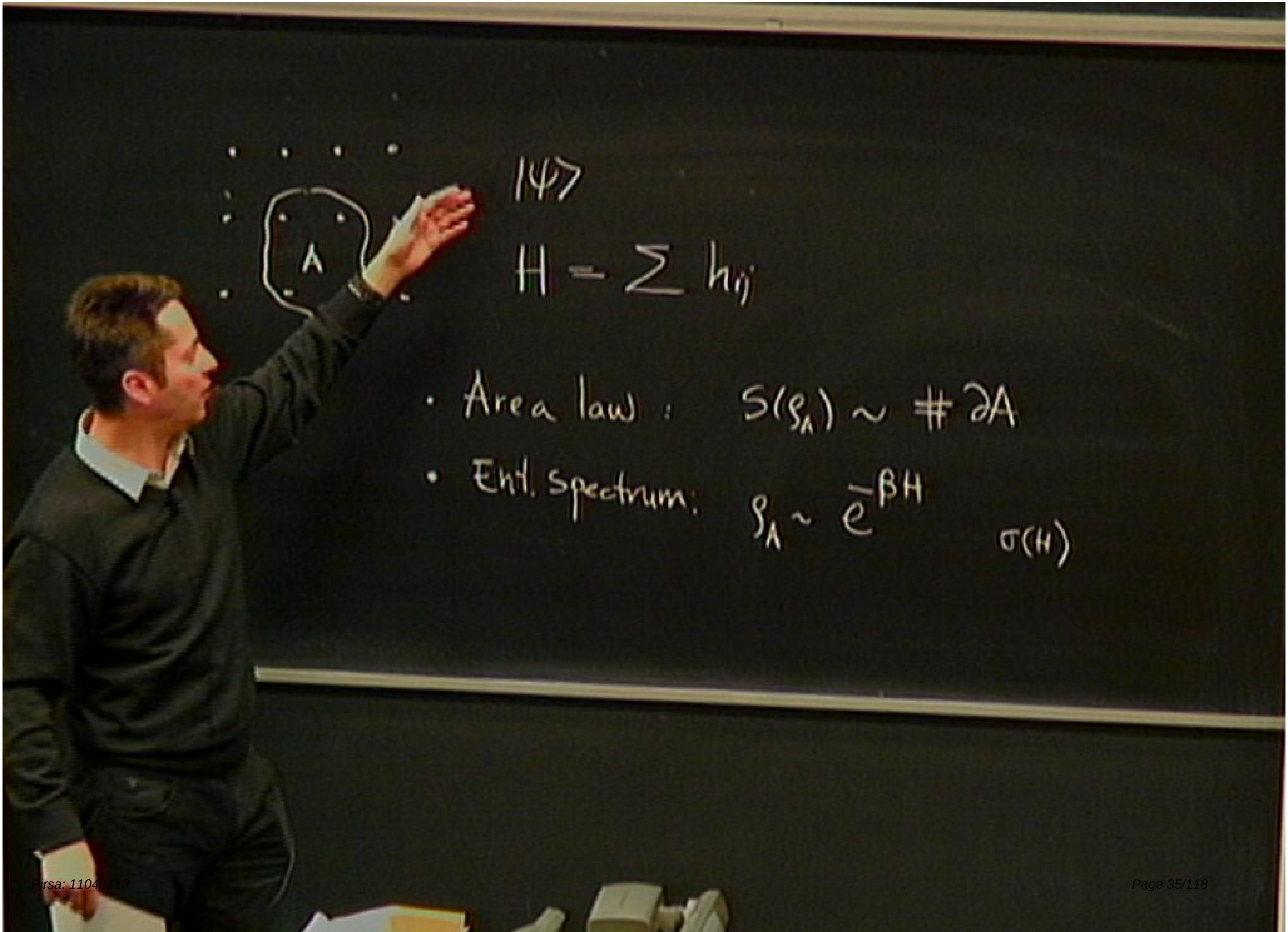
② PEPS

$$|\psi\rangle = \sum C_{i_1\dots i_N} |i_1 \dots i_N\rangle$$

$$= \text{PEPS}$$

$$= \sum_{\alpha_1 \dots \alpha_N} A^{i_1}_{\alpha_1 \alpha_2} A^{i_2}_{\dots} A^{i_3} \cdots |i_1 \dots i_N\rangle$$

$$\boxed{2ND^2}$$


$|\Psi\rangle$
Λ
$H = \sum h_{ij}$
• Area law : $S(\rho_\Lambda) \sim \# \partial A$
• Ent. spectrum: $\rho_\Lambda \sim e^{-\beta H}$
$\sigma(H)$

$|\Psi\rangle$

$H = \sum h_{ij}$

1 ρ_A

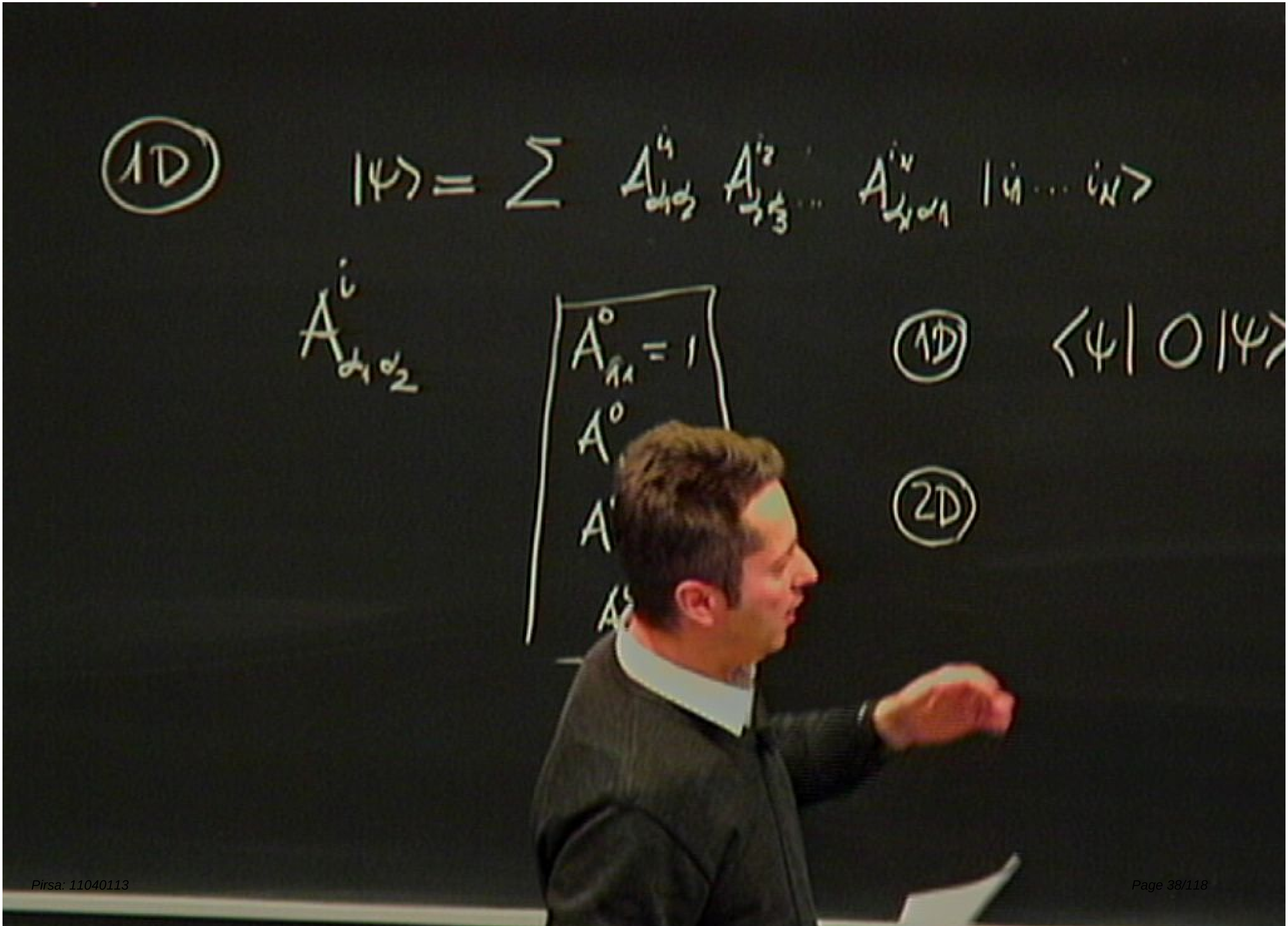
- Area law : $S(\rho_A) \sim \# \partial A$
- Ent. spectrum: $\rho_A \sim e^{-\beta H}$ $\sigma(H)$

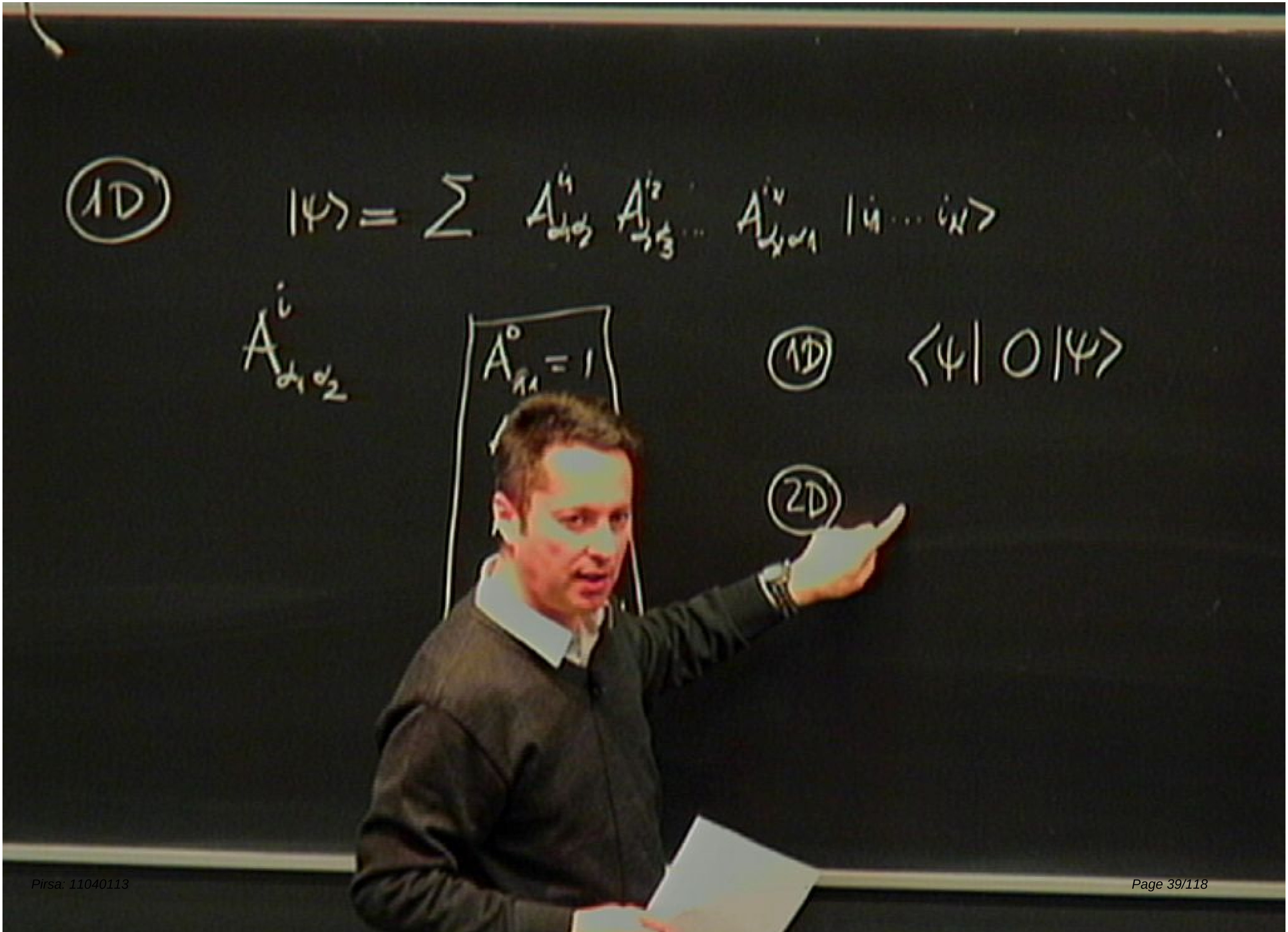
$|\Psi\rangle$

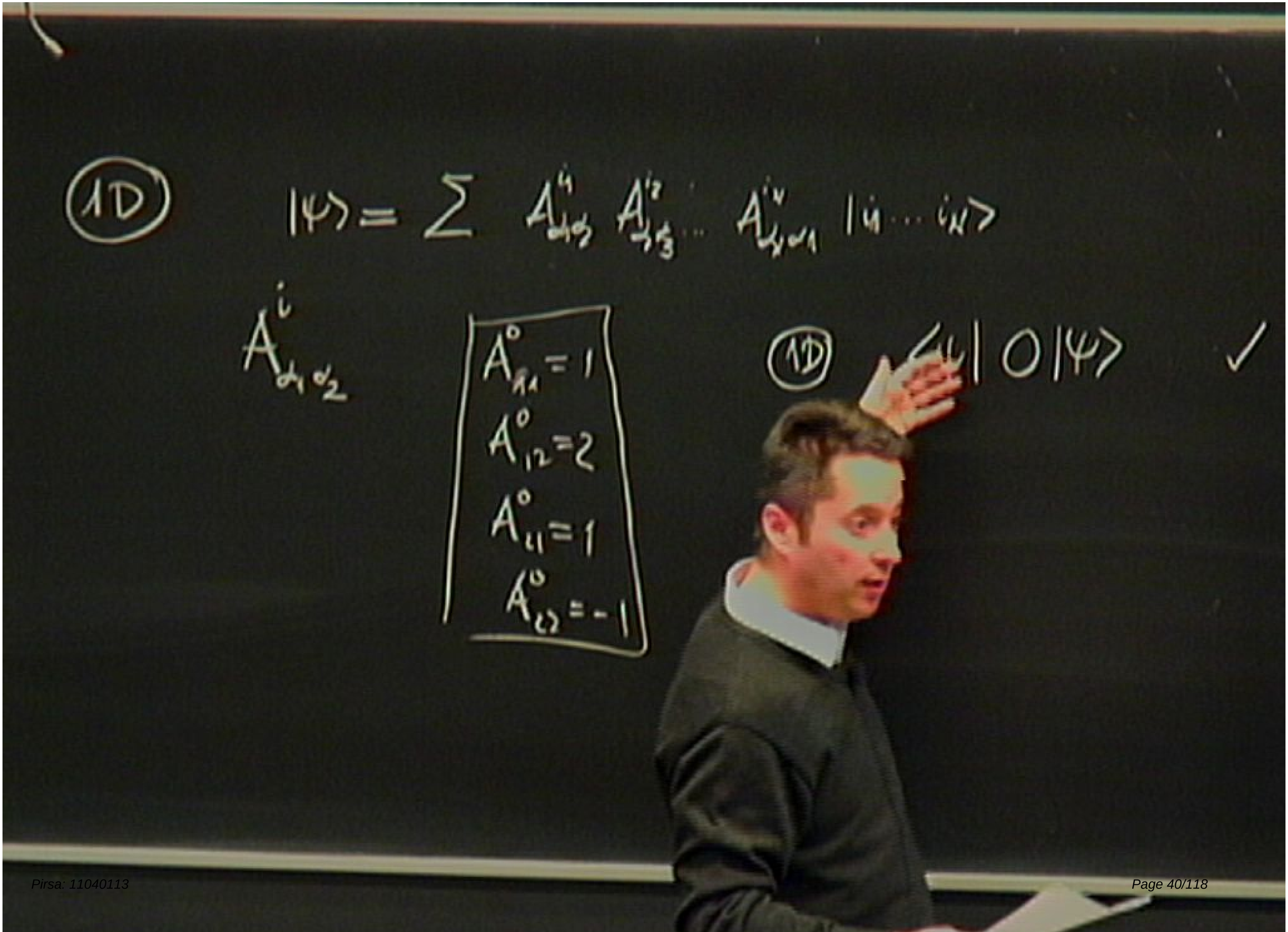
$$H = \sum h_{ij}$$

(1) ρ_A

- Area law: $S(\rho_A) \sim \# \partial A$
- Ent. spectrum: $\rho_A \sim e^{-\beta H}$ $\qquad \sigma(H)$


1D
$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$
$A^{i}_{\alpha_1\alpha_2}$
$A^{0}_{11} = 1$
1D
$\langle\psi| O |\psi\rangle$
2D


1D
$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$
$A^{i}_{\alpha_1\alpha_2}$
$A^{0}_{11} = 1$
1D
$\langle\psi| O |\psi\rangle$
2D



1D
$|\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$
$A^{i}_{\alpha_1\alpha_2}$
$A^0_{11} = 1$
$A^0_{12} = 2$
$A^0_{21} = 1$
$A^0_{22} = -1$
1D
$\langle\psi| O |\psi\rangle$ ✓

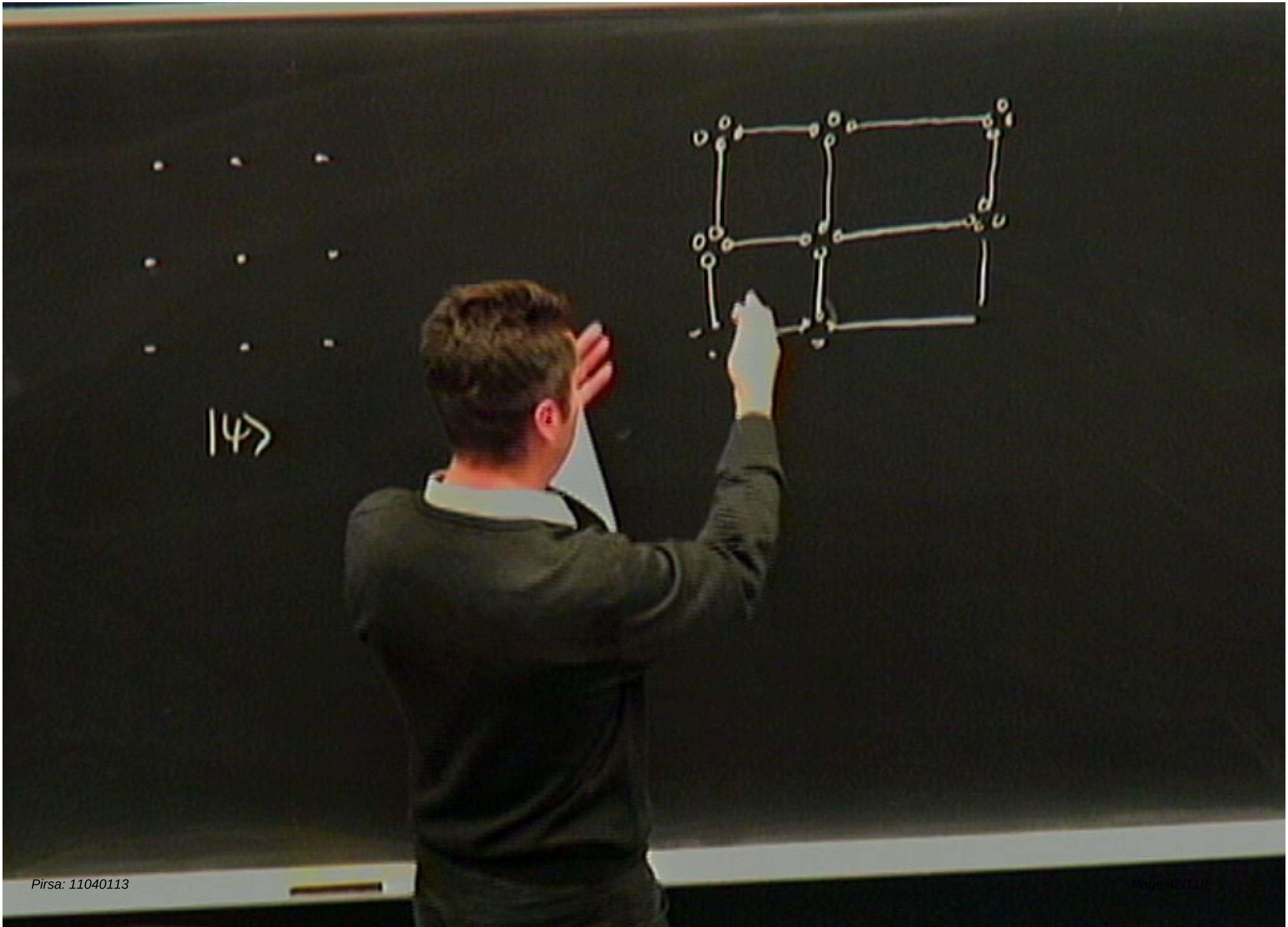
1D $\quad |\psi\rangle = \sum A^{i_1}_{\alpha_1\alpha_2} A^{i_2}_{\alpha_2\alpha_3} \cdots A^{i_N}_{\alpha_N\alpha_1} |i_1 \cdots i_N\rangle$

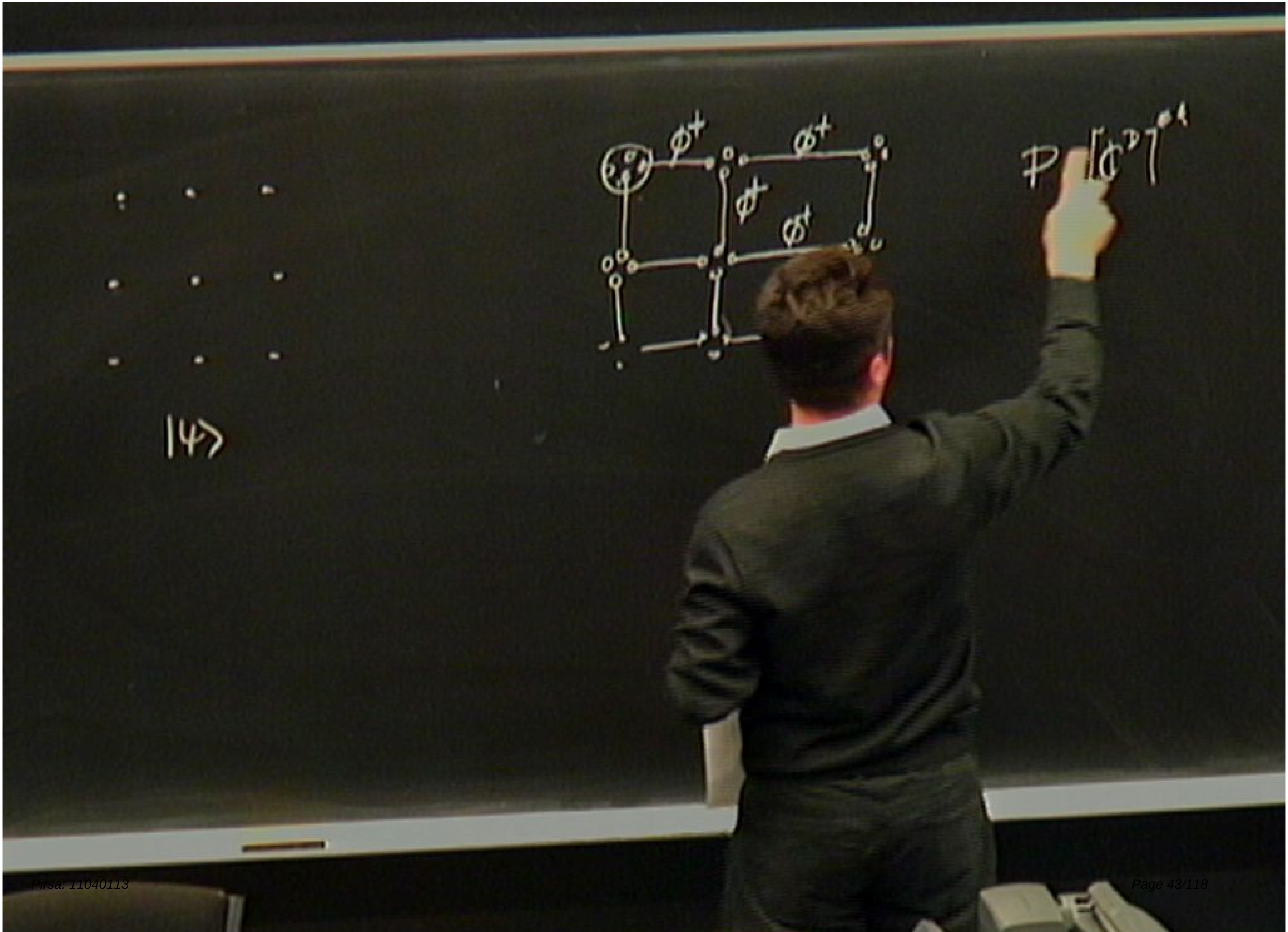
$A^{i}_{\alpha_1\alpha_2}$

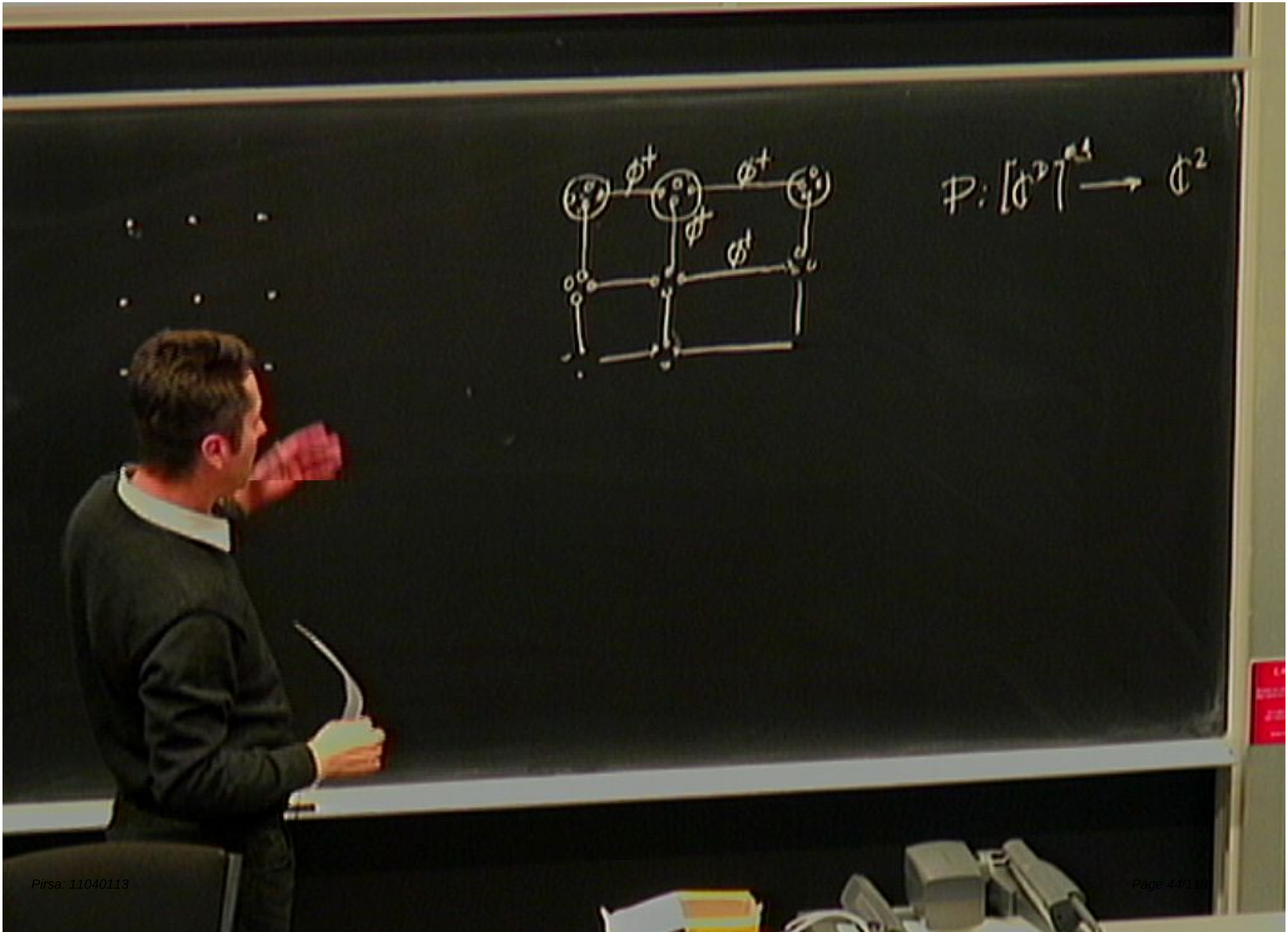
$$\boxed{\begin{array}{l} A^0_{11} = 1 \\ A^0_{12} = 2 \\ A^0_{21} = 1 \\ A^0_{22} = -1 \end{array}}$$

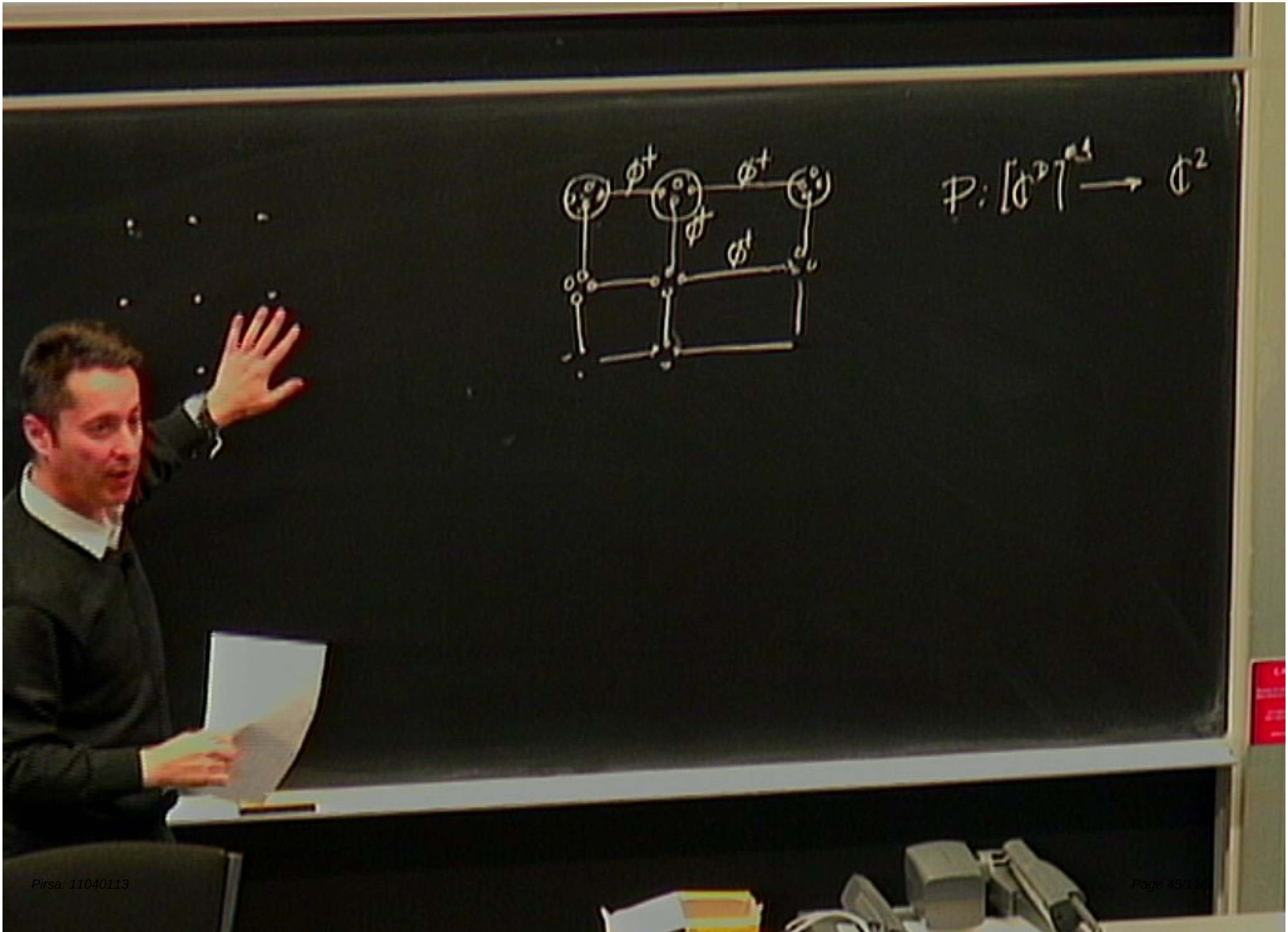
1D $\quad \langle\psi| O |\psi\rangle \quad \checkmark$

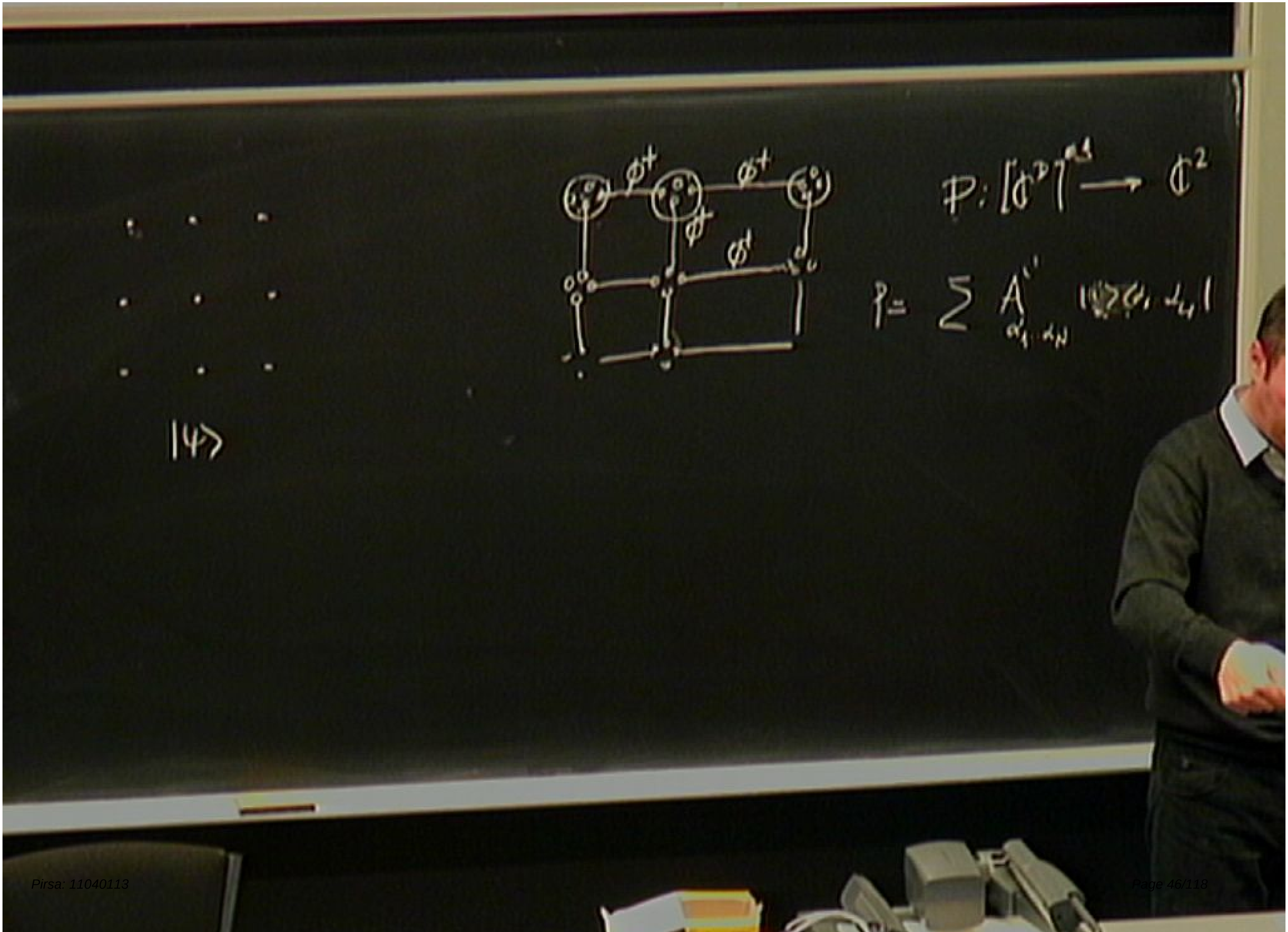
2D


|ψ>


ϕ+
ϕ+
ϕ+
ϕ+
|ψ⟩

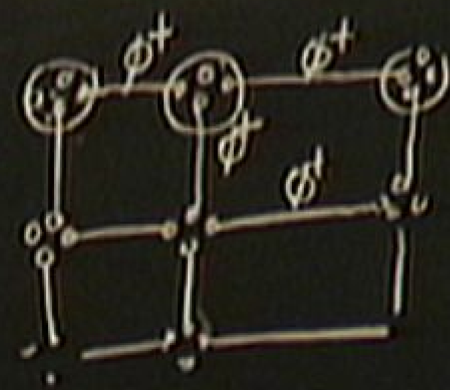

ϕ^+
ϕ^+
ϕ^+
ϕ^+
$P: [\mathbb{C}^2]^{\otimes 4} \to \mathbb{C}^2$


ϕ^+
ϕ^+
ϕ^+
ϕ^+
$P: [\mathbb{C}^D]^{\otimes 4} \to \mathbb{C}^2$



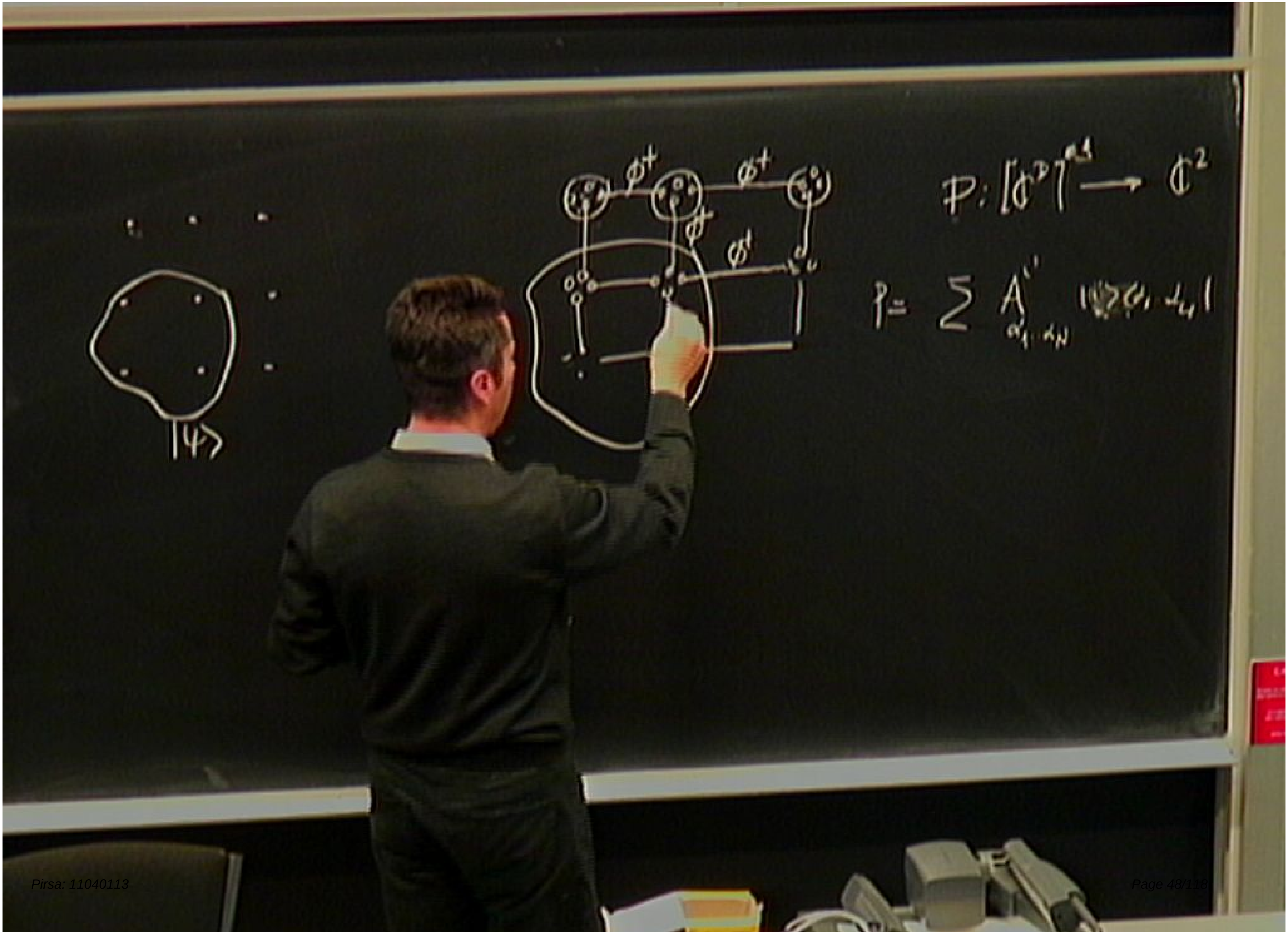
φ+
|ψ⟩
P: [ℂ^D]^{⊗4} → ℂ^2
P = Σ A

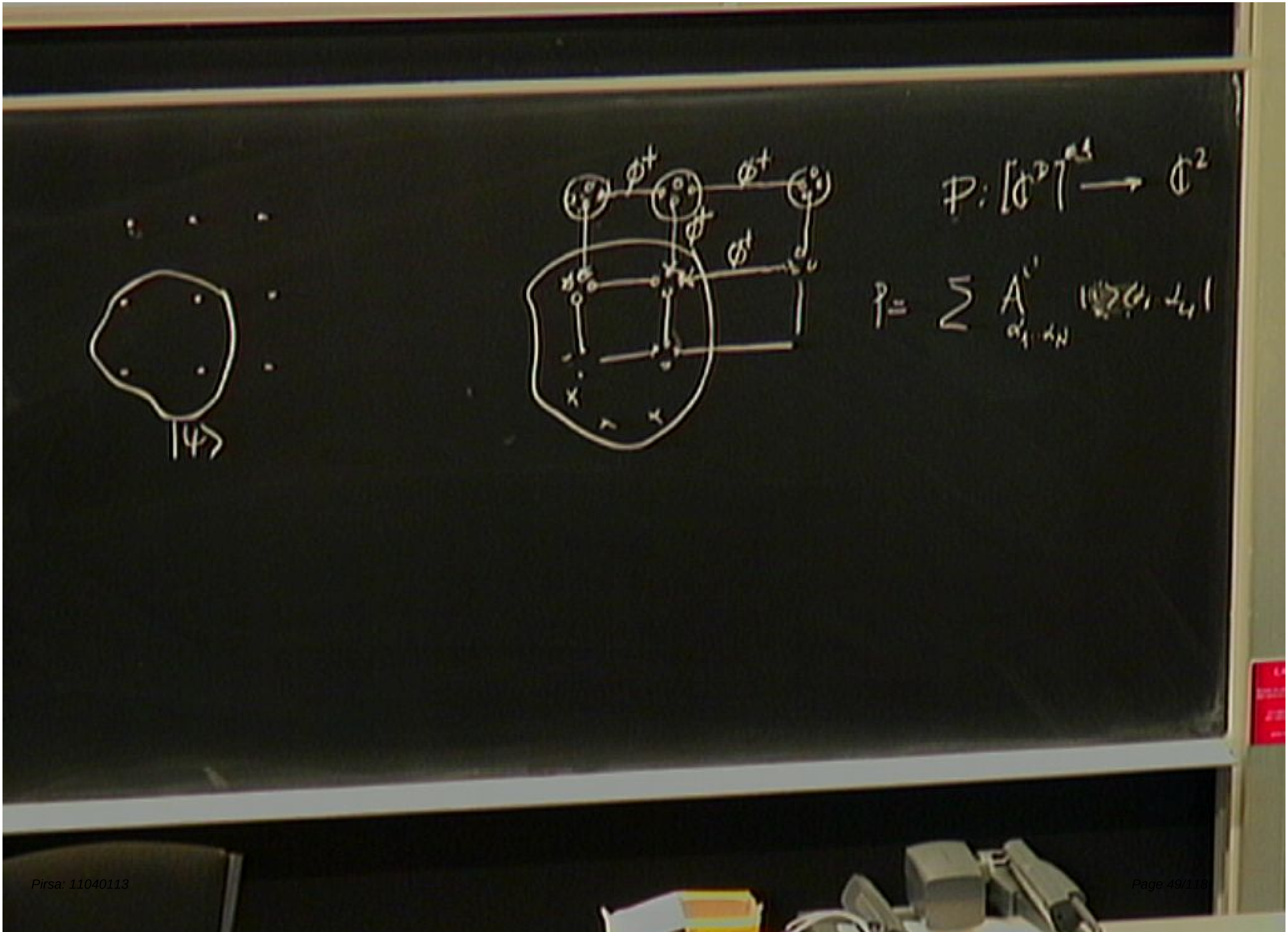
$|\psi\rangle$



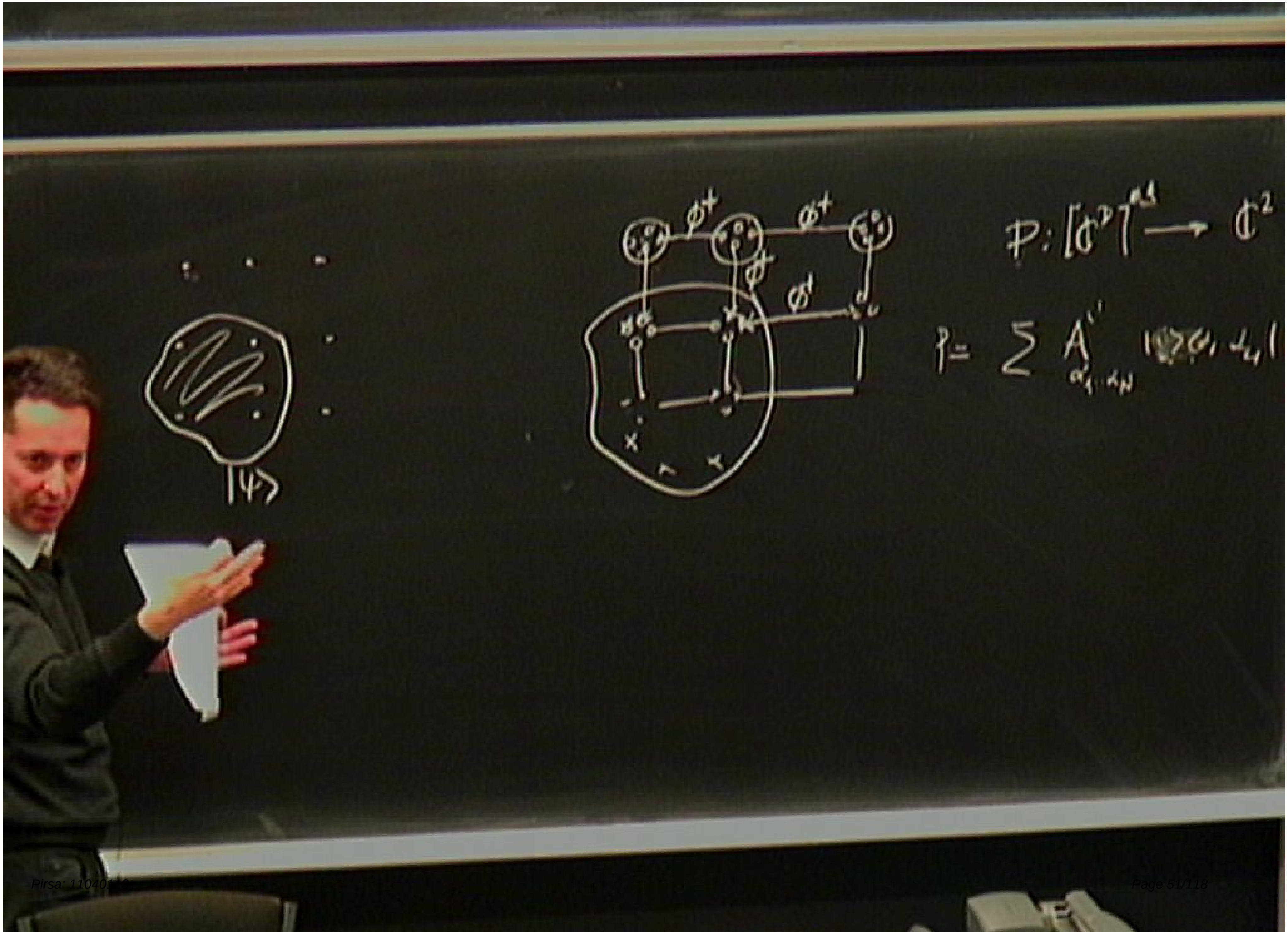
$$P : [\mathbb{C}^D]^{\otimes 3} \longrightarrow \mathbb{C}^2$$

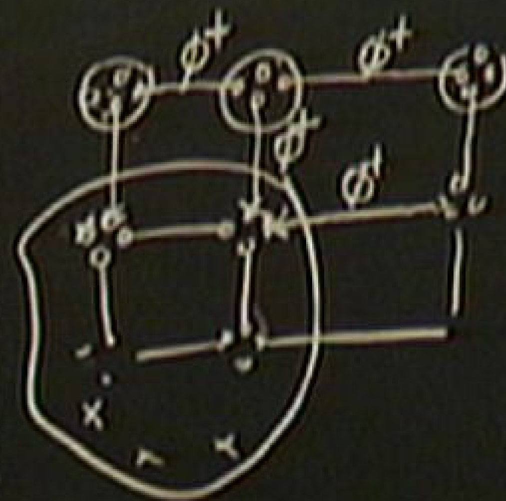
$$P = \sum A^{i}_{\alpha_1 \ldots \alpha_N} |i\rangle\langle\alpha_1 \ldots \alpha_4|$$


|ψ>
φ†


|ψ>
φ+
φ+
φ+
φ+
P : [ℂ²]^⊗4 ⟶ ℂ²

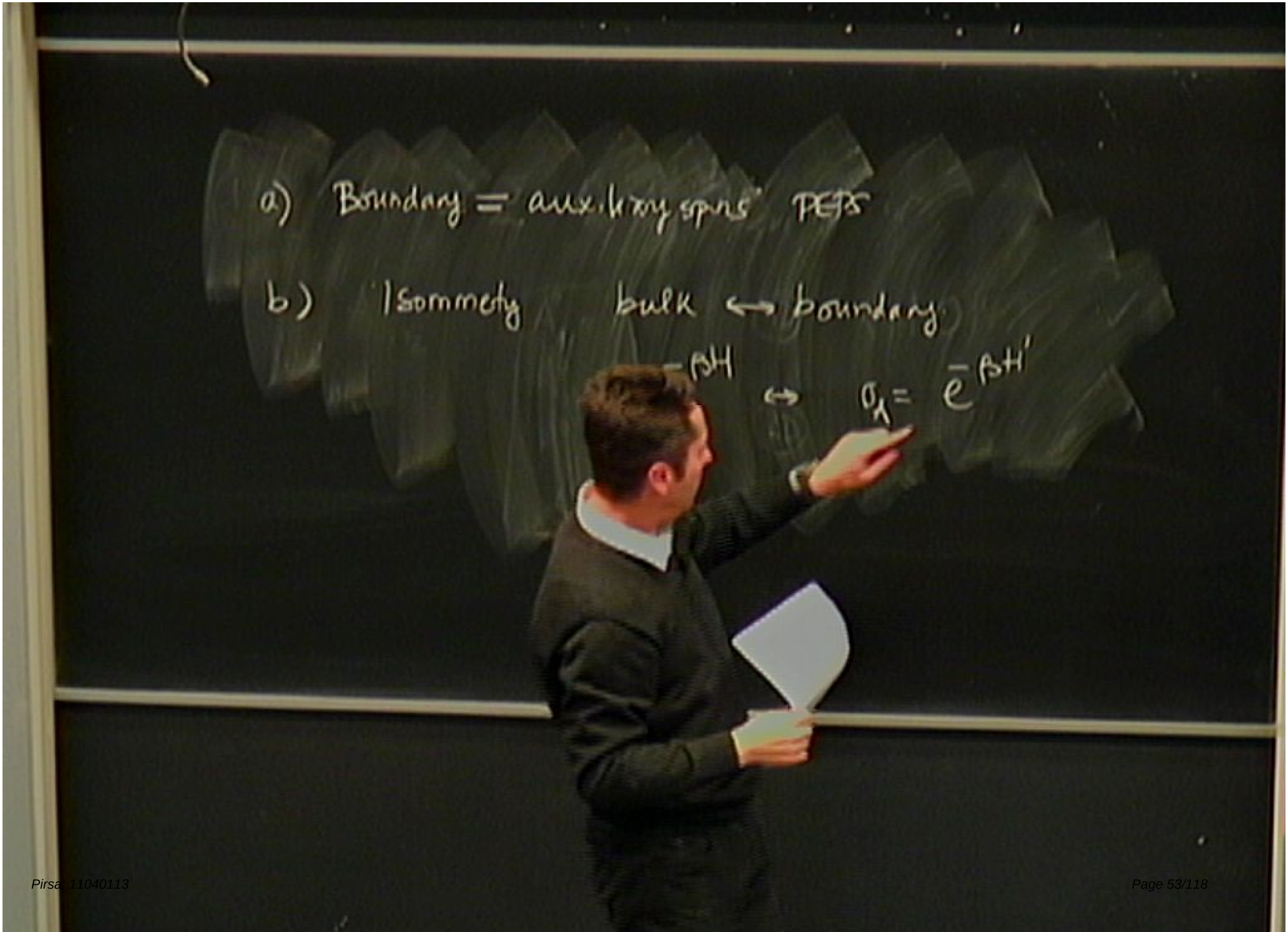
a) Boundary = auxiliary spins PEPS


|ψ>
P: [C^D]^⊗3 → C^2



$$P : [\mathbb{C}^D]^{\otimes 4} \longrightarrow \mathbb{C}^2$$

$$P = \sum A^{i}_{\alpha_1 \ldots \alpha_4} \, |i\rangle\langle \alpha_1 \ldots \alpha_4|$$


a) Boundary = auxiliary spins PEPS
b) Isommetry bulk ↔ boundary
−βH ↔ σA = e−βH'

a) Boundary = auxiliary spins PEPS

b) Isommetry bulk $\leftrightarrow$ boundary

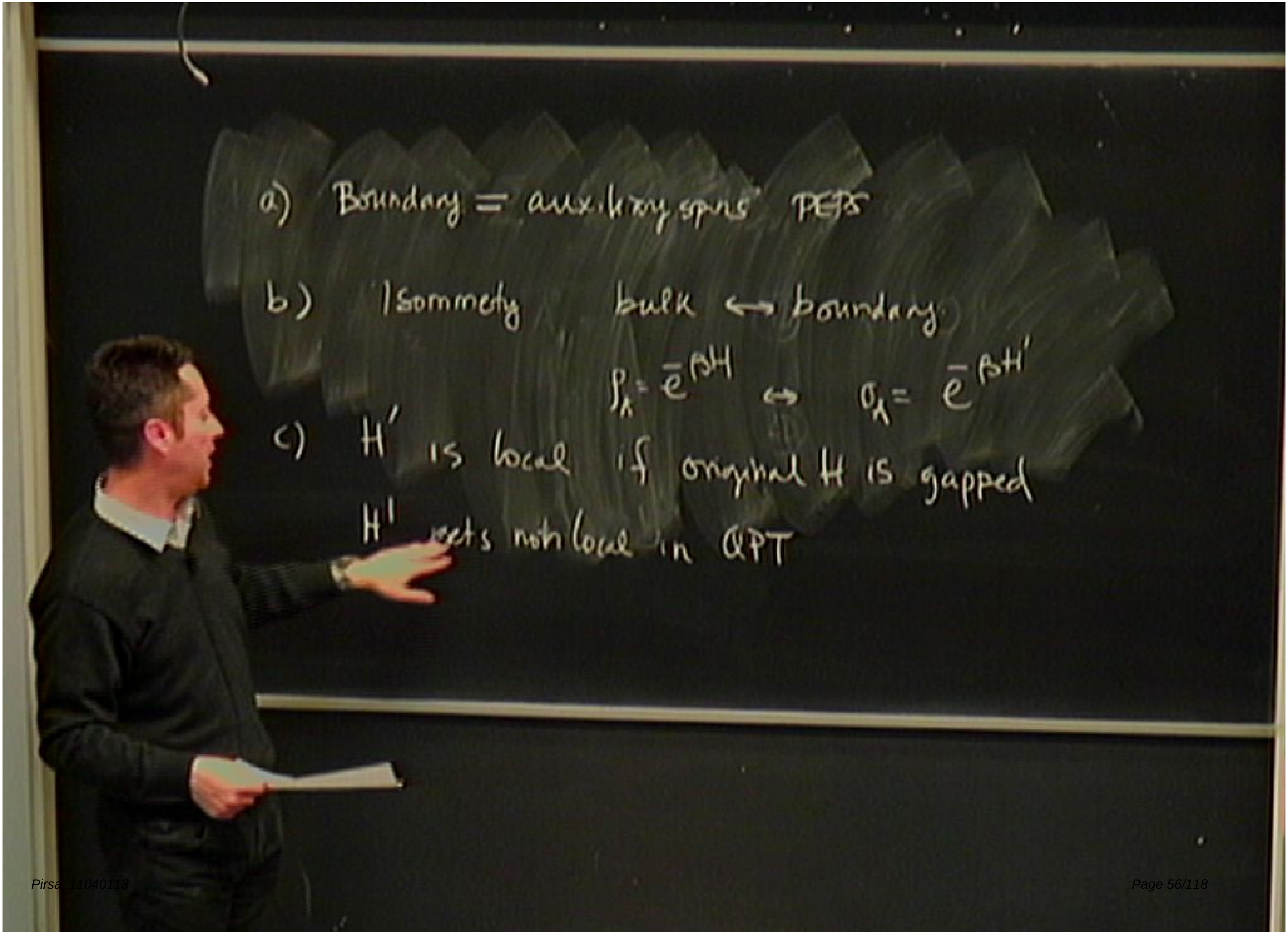
$\rho_A = e^{-\beta H} \leftrightarrow \sigma_A = e^{-\beta H'}$

a) Boundary = auxiliary spins PEPS

b) Isommetry bulk $\leftrightarrow$ boundary

$\rho_A = e^{-\beta H} \leftrightarrow \sigma_A = e^{-\beta H'}$

c) H' is local


a) Boundary = auxiliary spins PEPS
b) Isommetry bulk ↔ boundary
$\rho_A = e^{-\beta H} \leftrightarrow \sigma_A = e^{-\beta H'}$
c) H' is local if original H is gapped
H' gets non local in QPT

a) Boundary = auxiliary spins PEPS

b) Isommetry bulk $\leftrightarrow$ boundary

$\rho_A = e^{-\beta H} \leftrightarrow \sigma_A = e^{-\beta H'}$

c) H' is local if original H is gapped

H' gets non local in QPT

d)

a) Boundary = auxiliary spins PEPS

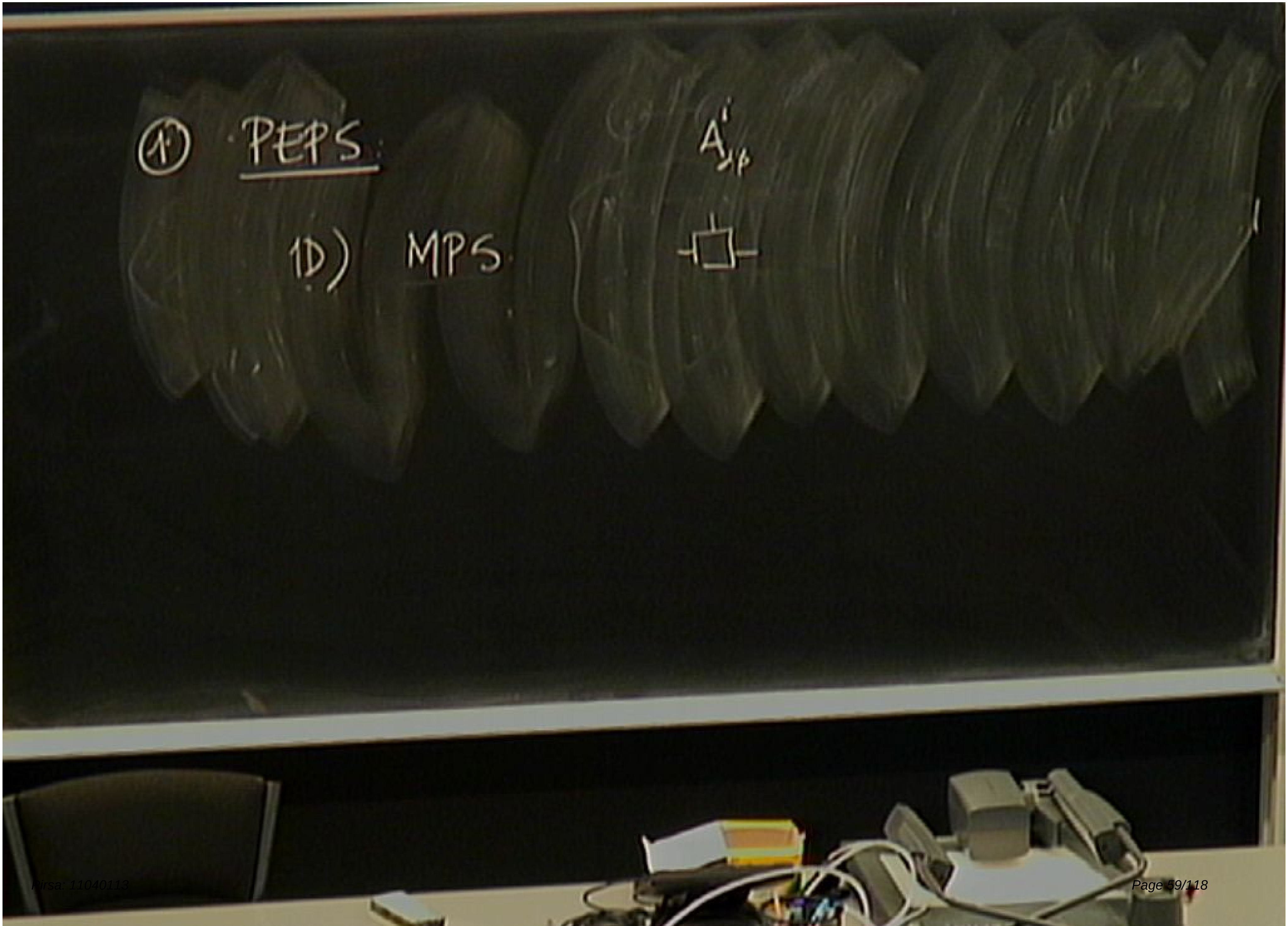
b) Isommetry bulk $\leftrightarrow$ boundary

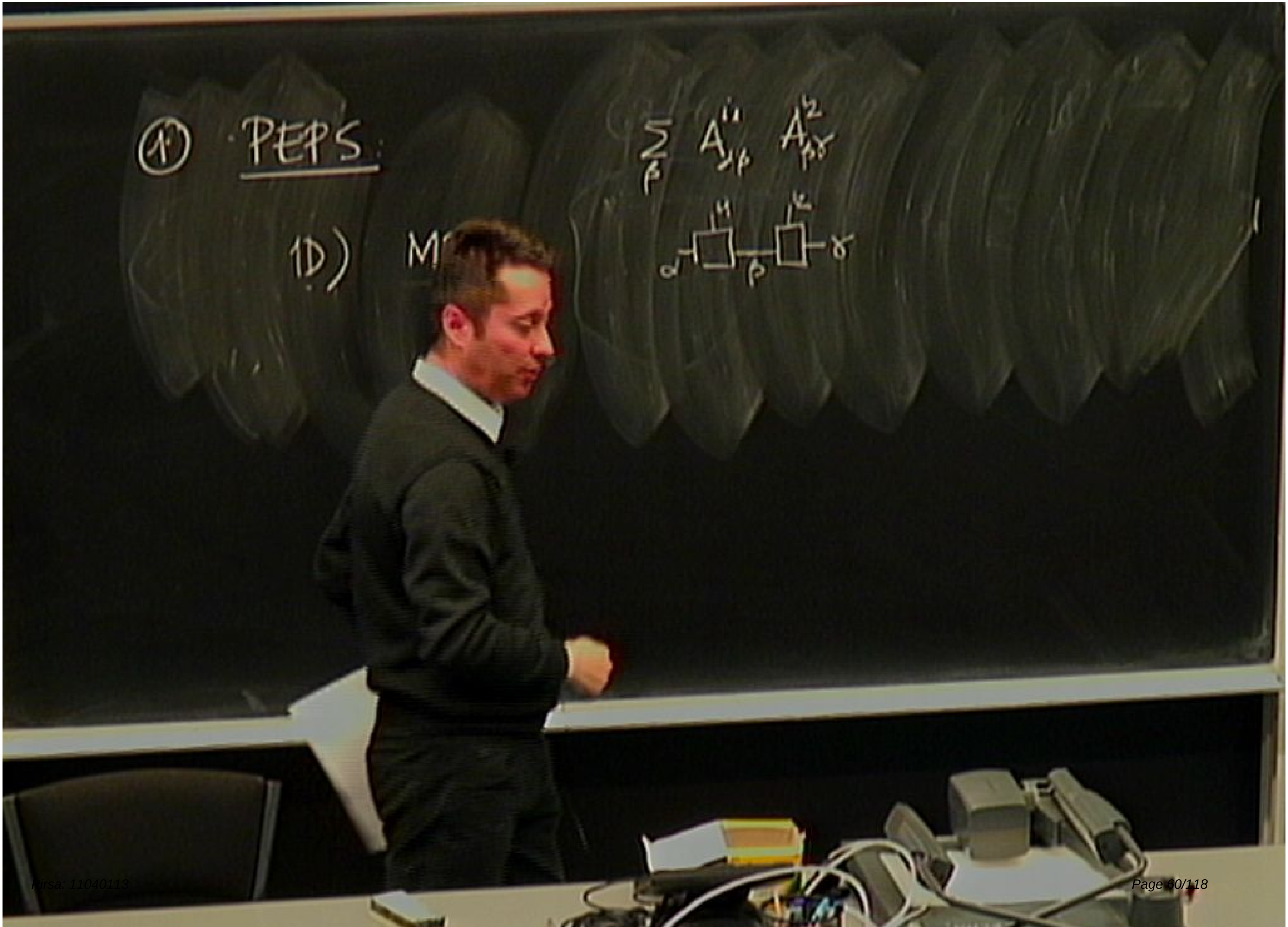
$$\rho_A = e^{-\beta H} \leftrightarrow \sigma_A = e^{-\beta H'}$$

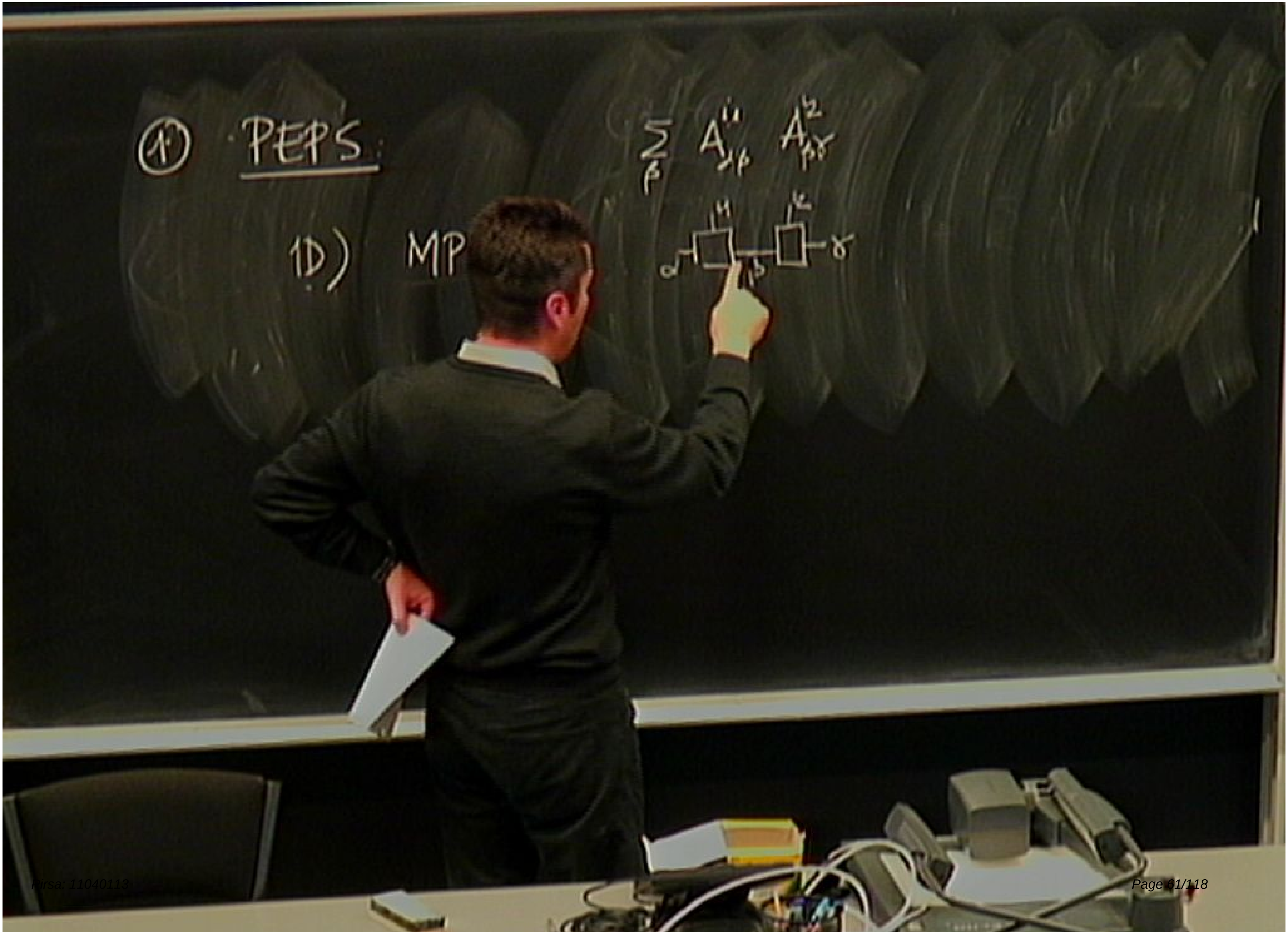
c) H' is local if original H is gapped

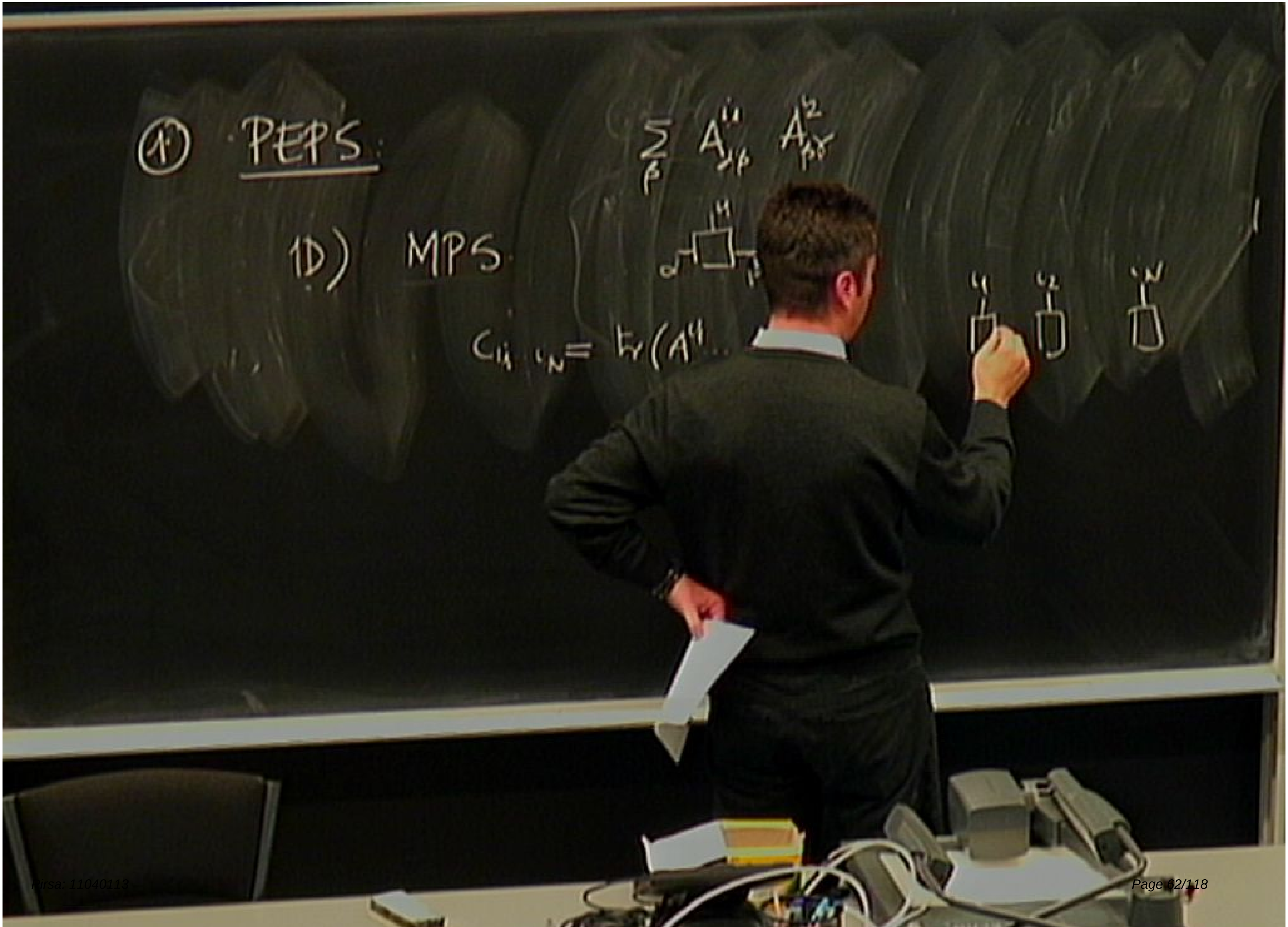
H' gets non local in QPT

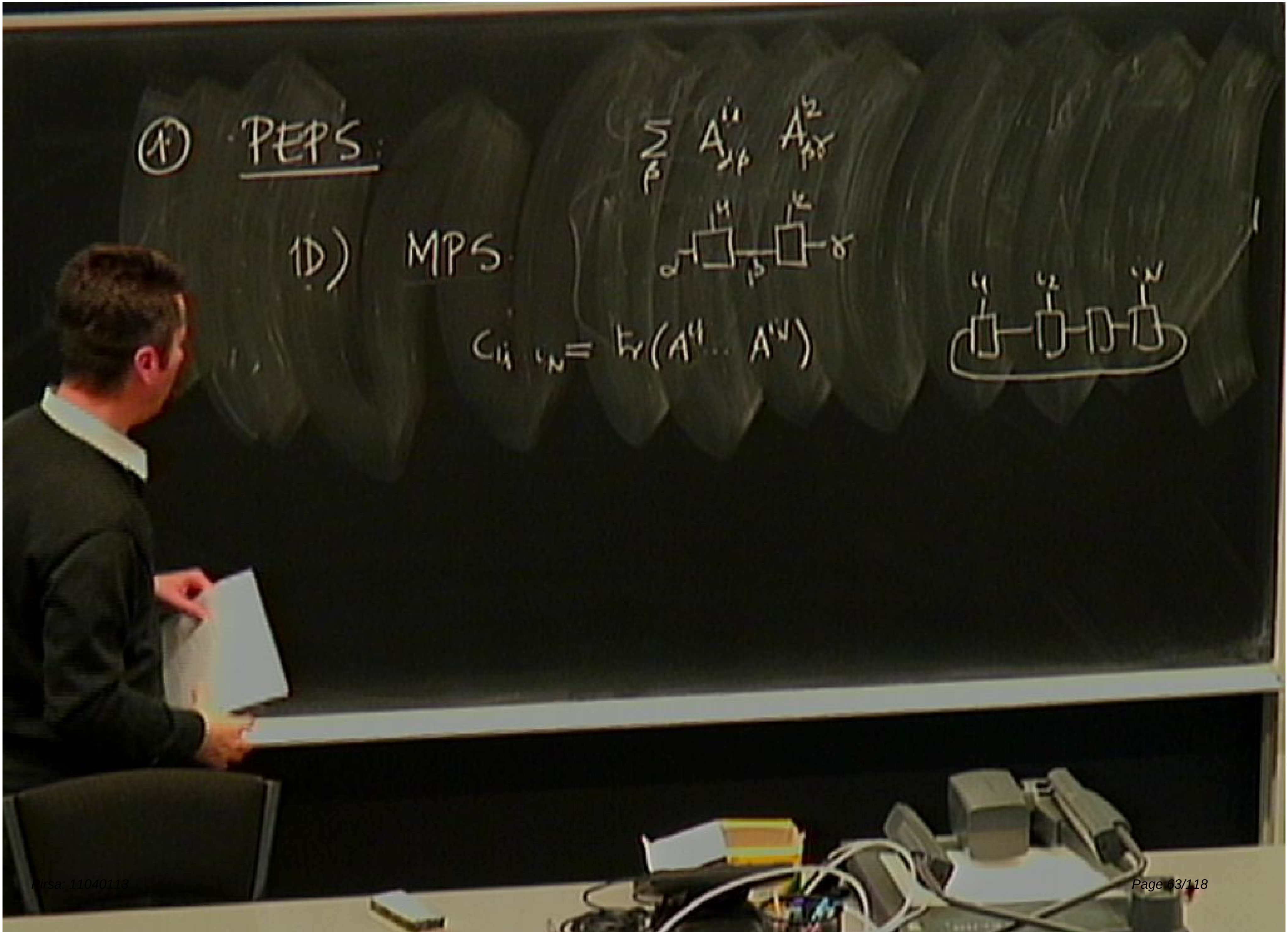
d) PEPS $\longrightarrow$ Efficient $\langle \psi | O | \psi \rangle$

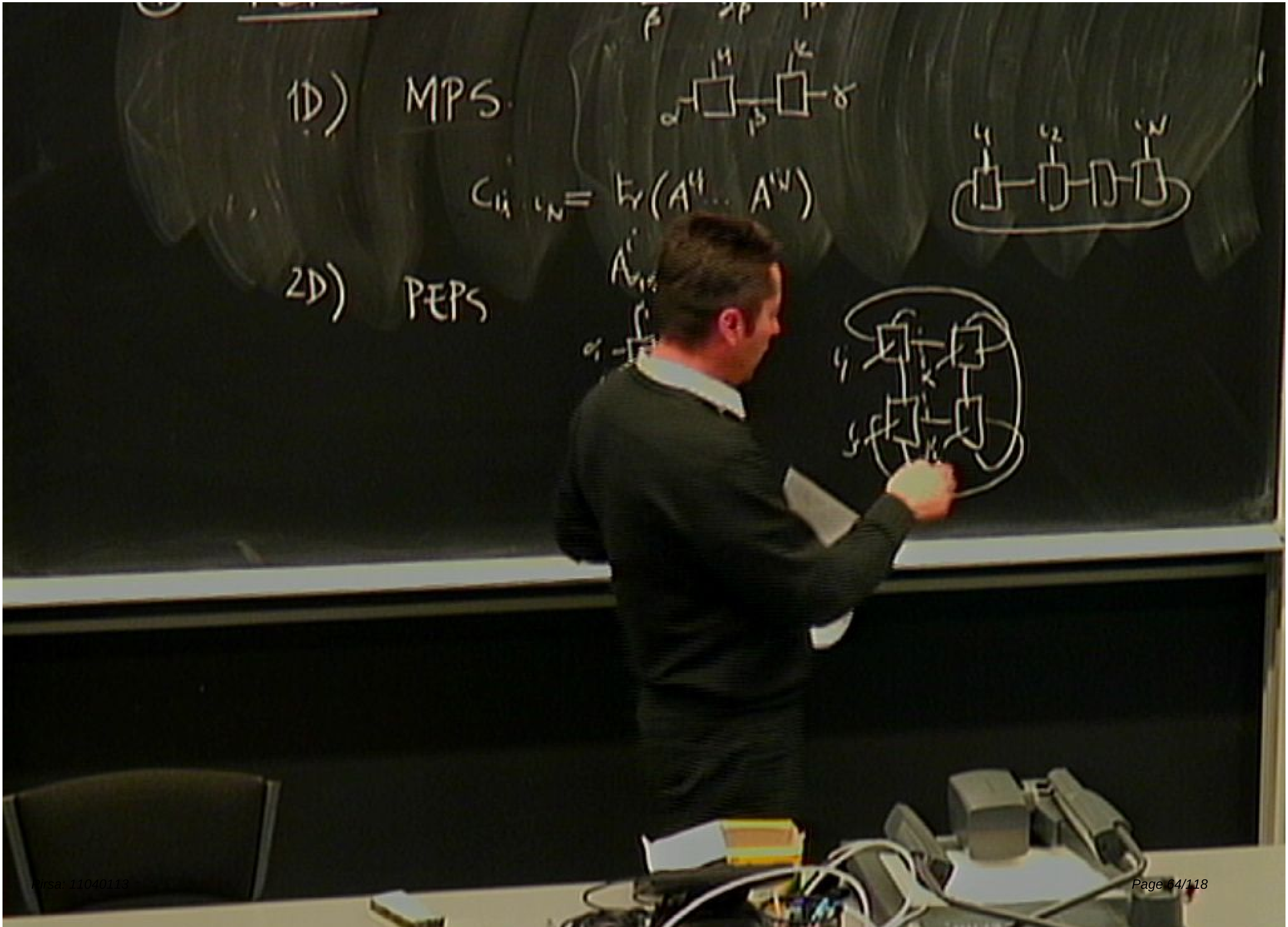

1 PEPS
1D) MPS
$A^{i}_{\alpha\beta}$


1) PEPS
1D)


① PEPS
1D) MP

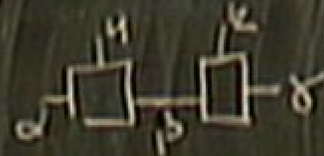

① PEPS
$\sum_{\beta} A^{i_1}_{\alpha\beta} A^{i_2}_{\beta\gamma}$
1D) MPS
$C_{i_1 \dots i_N} = tr(A^{i_1} \dots$


1) PEPS
1D) MPS
$\sum_{\beta} A^{i_1}_{\alpha\beta} A^{i_2}_{\beta\gamma}$
i_1 i_2
α β γ
$c_{i_1 \ldots i_N} = \mathrm{tr}(A^{i_1} \ldots A^{i_N})$
i_1 i_2 i_N


1D) MPS
2D) PEPS

① PEPS

$$\sum_\beta A^{i_1}_{\alpha\beta} A^{i_2}_{\beta\gamma}$$

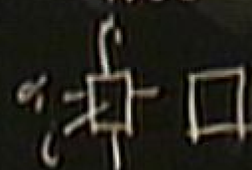


1D) MPS

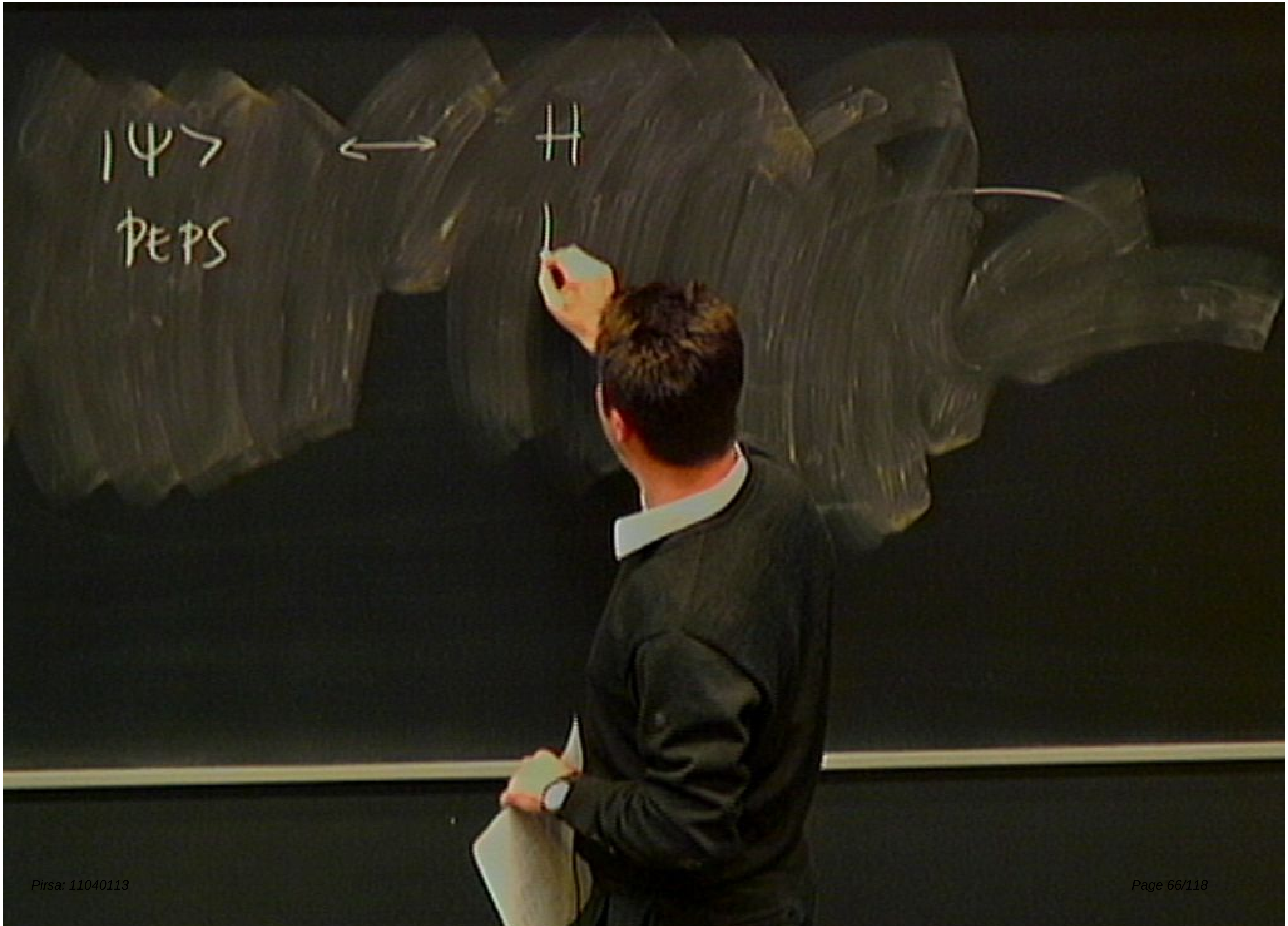
$$C_{i_1 \ldots i_N} = tr(A^{i_1} \cdots A^{i_N})$$

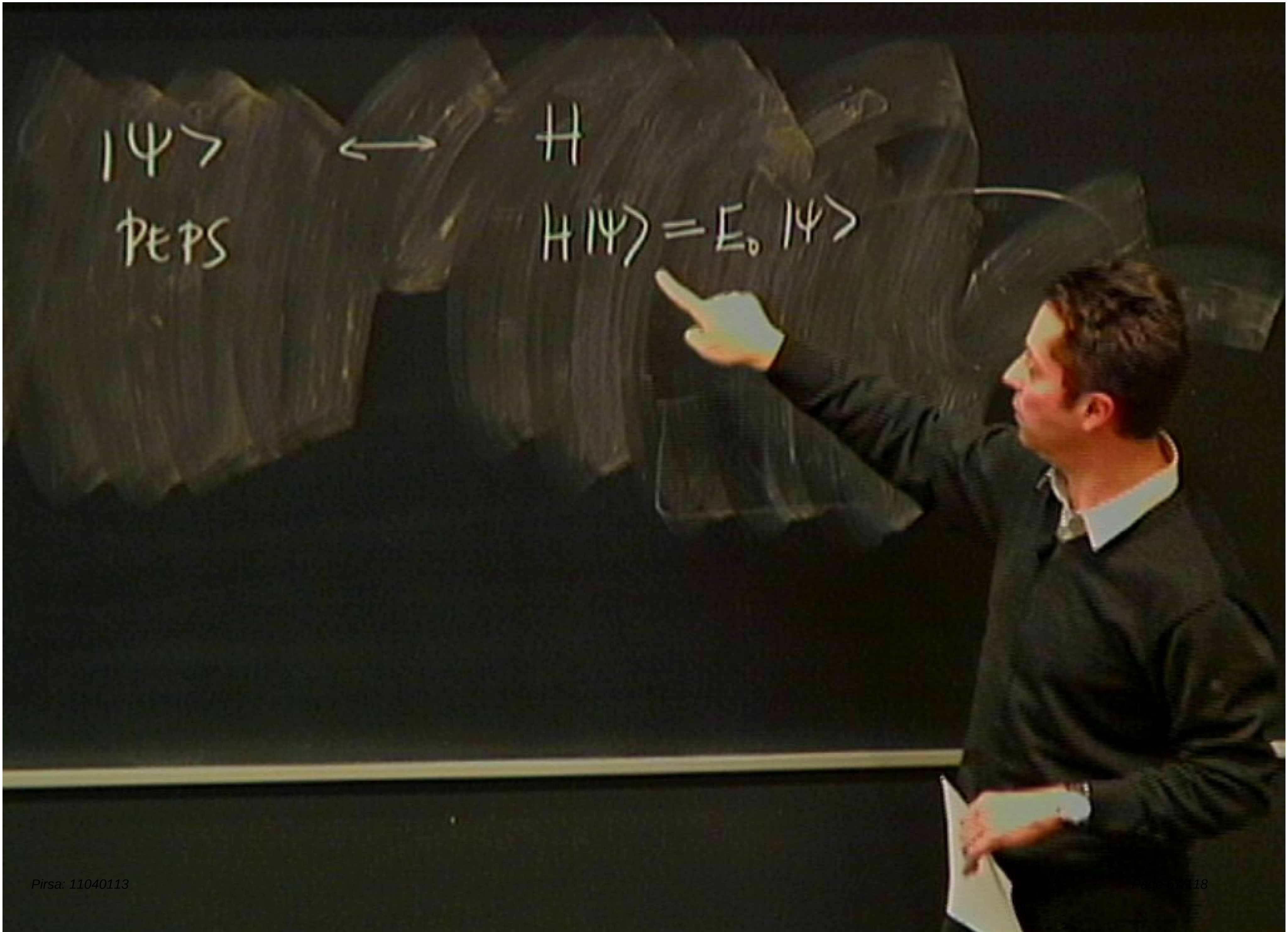
2D) PEPS

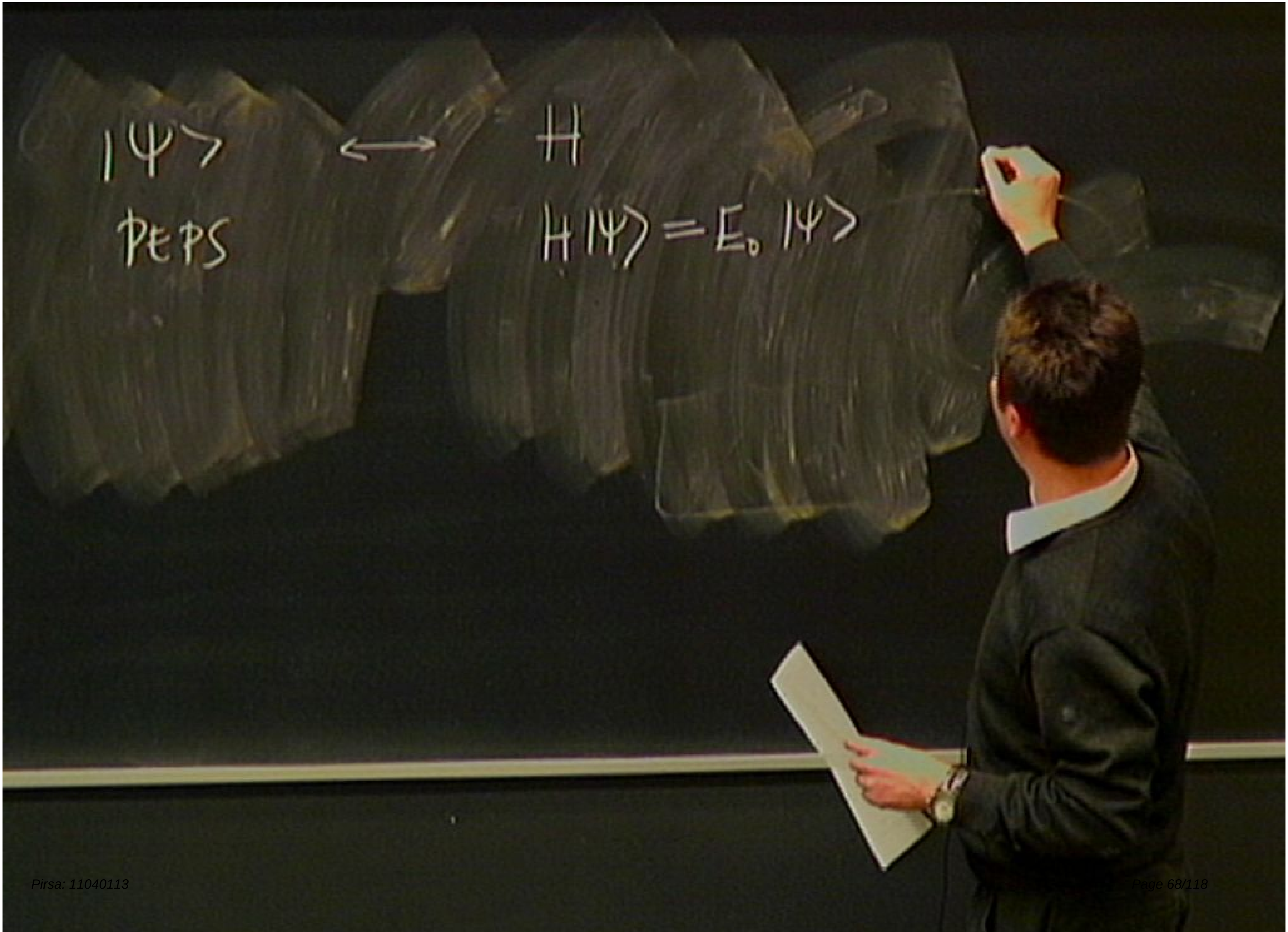
$$A^{i}_{\alpha\beta\gamma\delta}$$



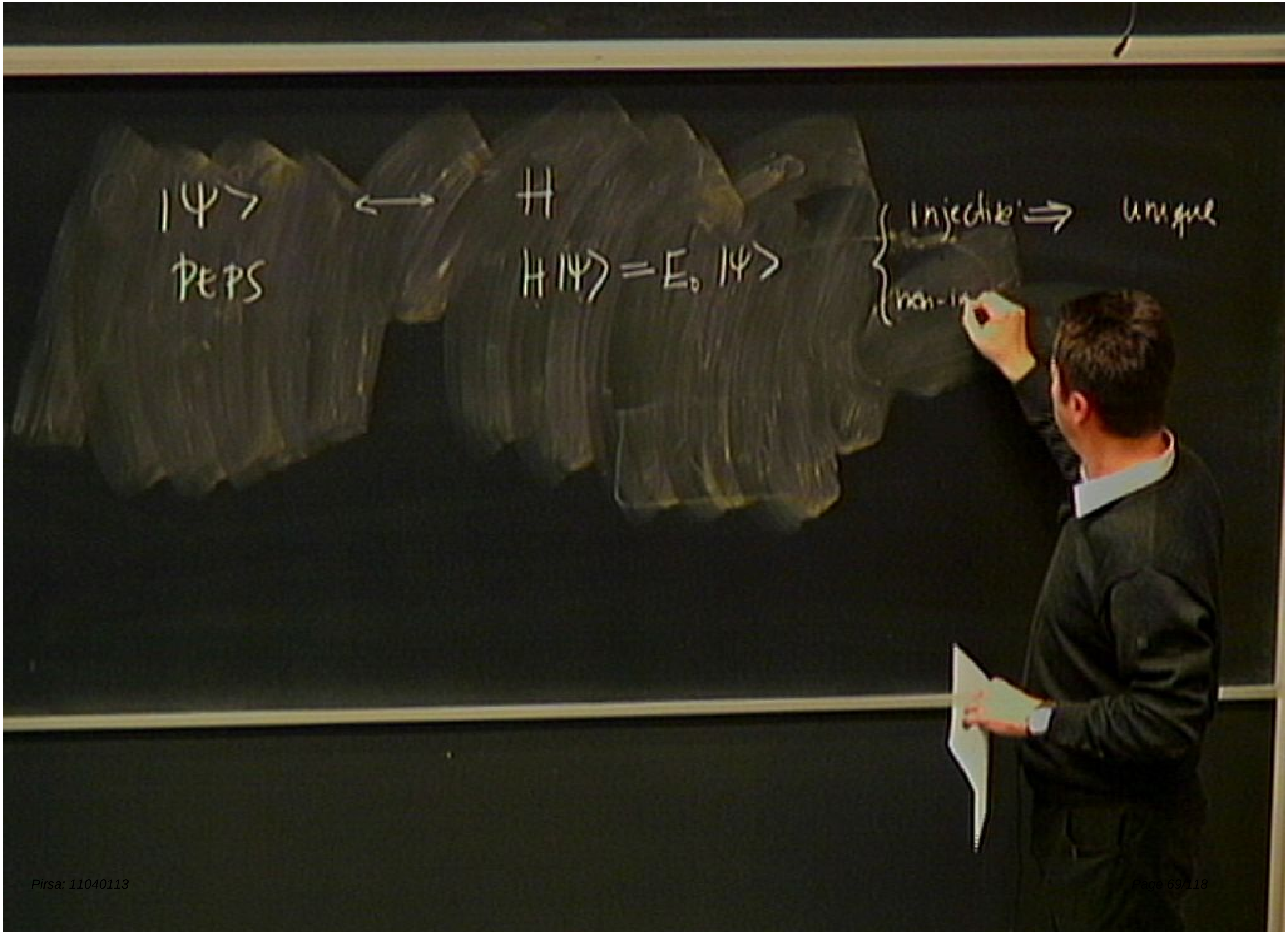
$= C^{i_1 i_2 i_3 i_4}$

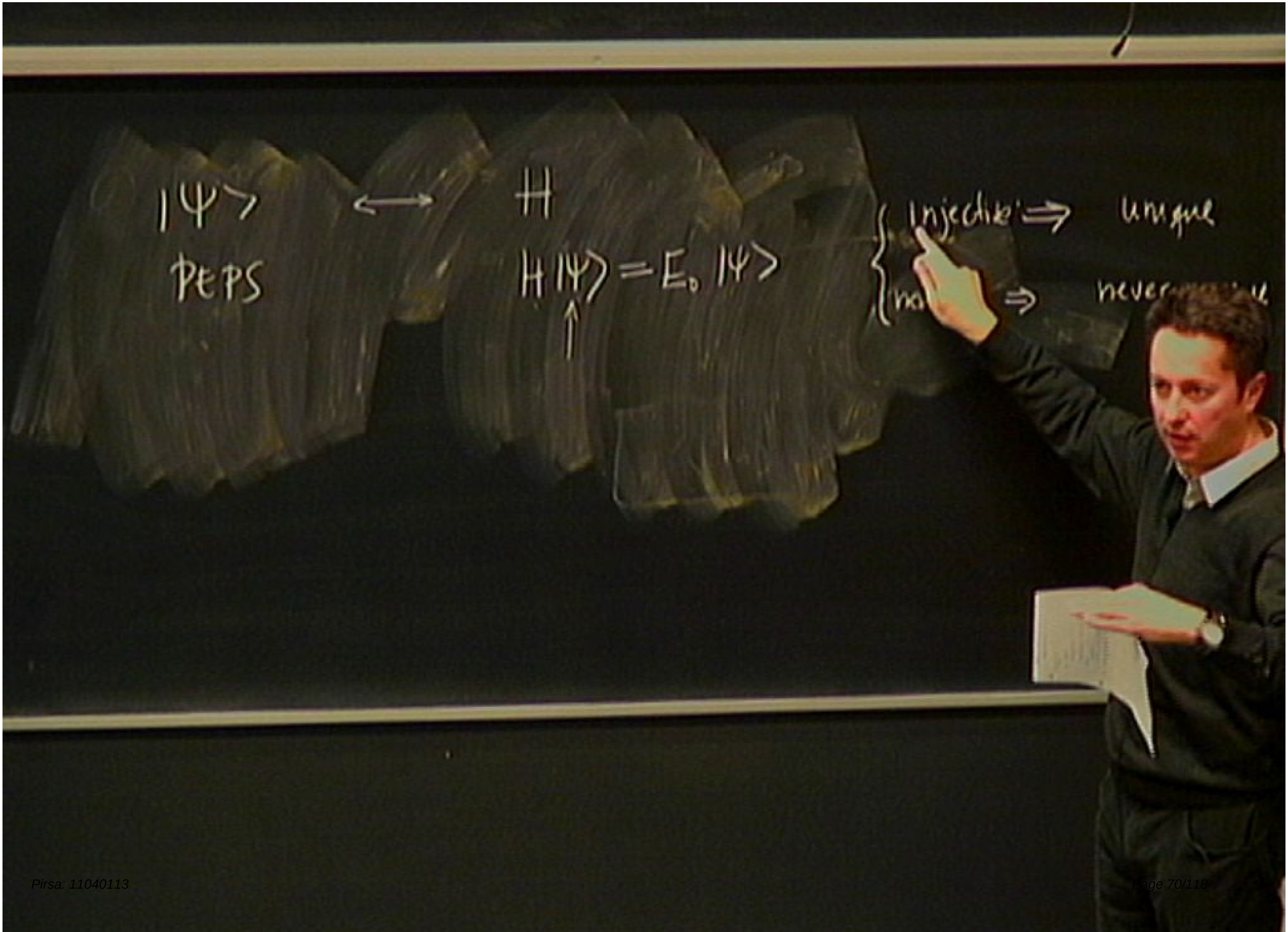

|Ψ⟩ ⟷ H
PEPS


$|\Psi\rangle \longleftrightarrow H$
PEPS
$H|\Psi\rangle = E_0 |\Psi\rangle$



|Ψ⟩ ⟷ H
PEPS
H|Ψ⟩ = E₀|Ψ⟩


|Ψ>
PEPS
⟷
H
H|Ψ> = E0 |Ψ>
injective ⇒ unique
non-in

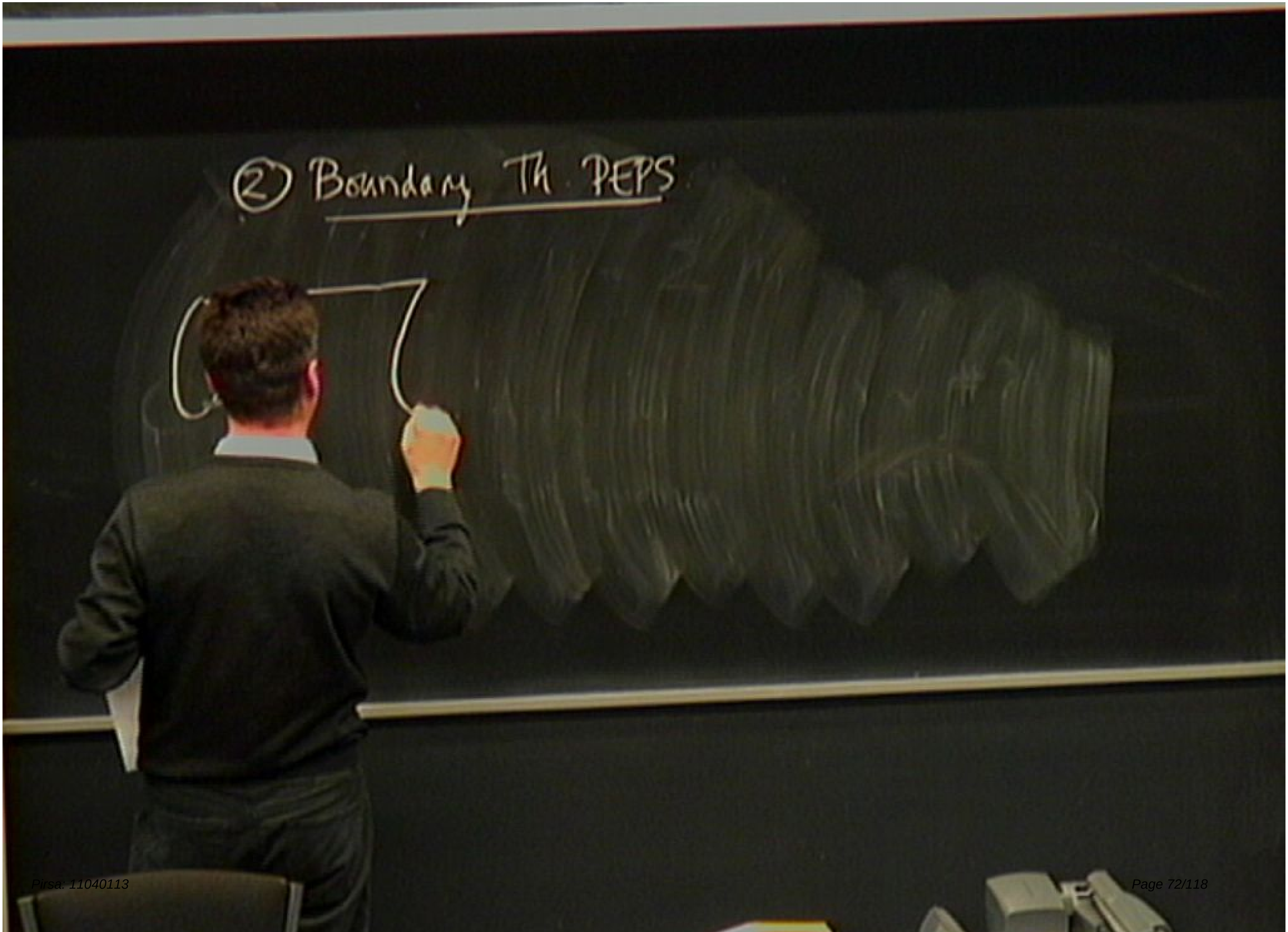


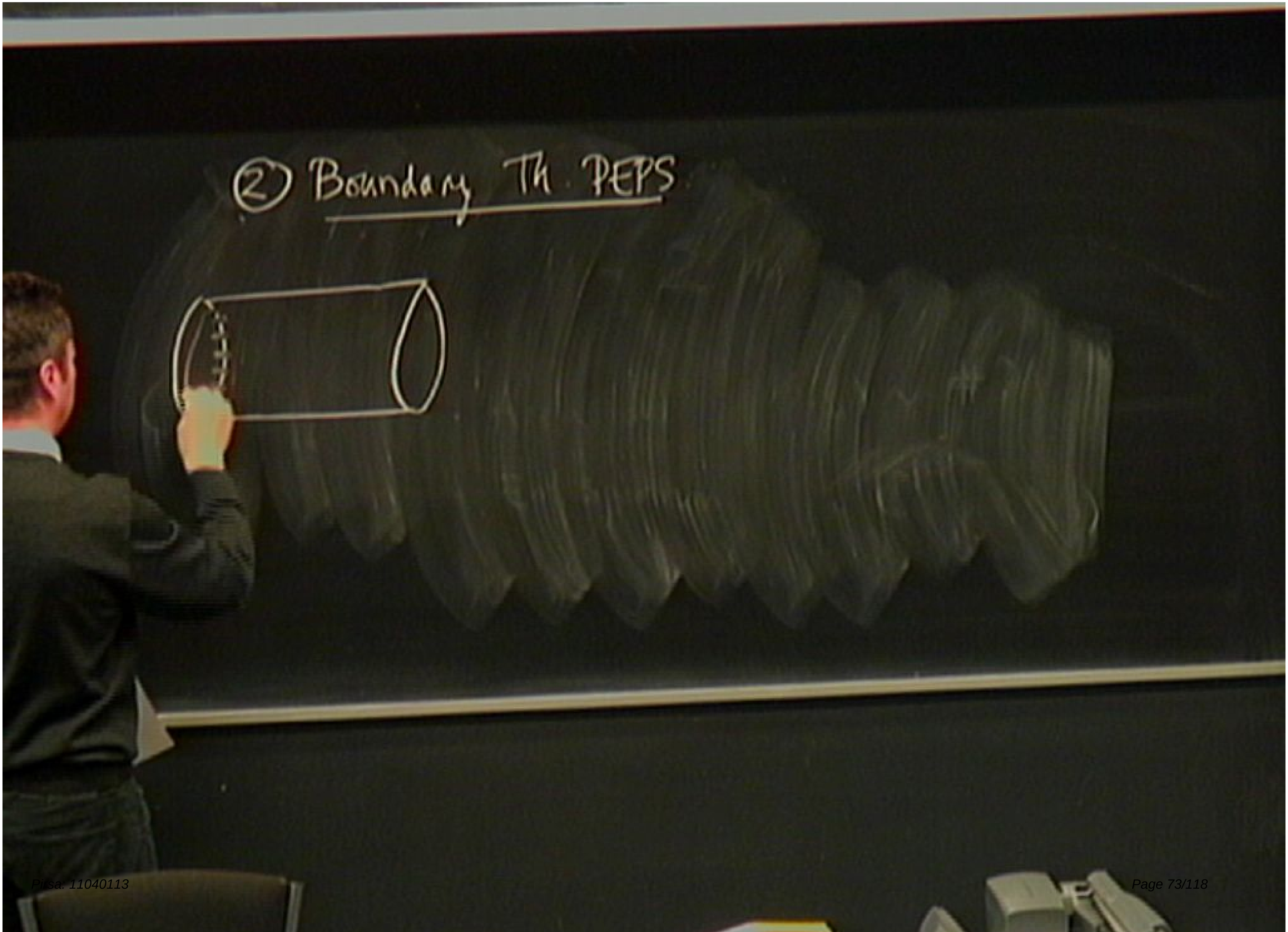
$|\Psi\rangle \longleftrightarrow H$
PEPS
$H|\Psi\rangle = E_0 |\Psi\rangle$
injective $\Rightarrow$ unique

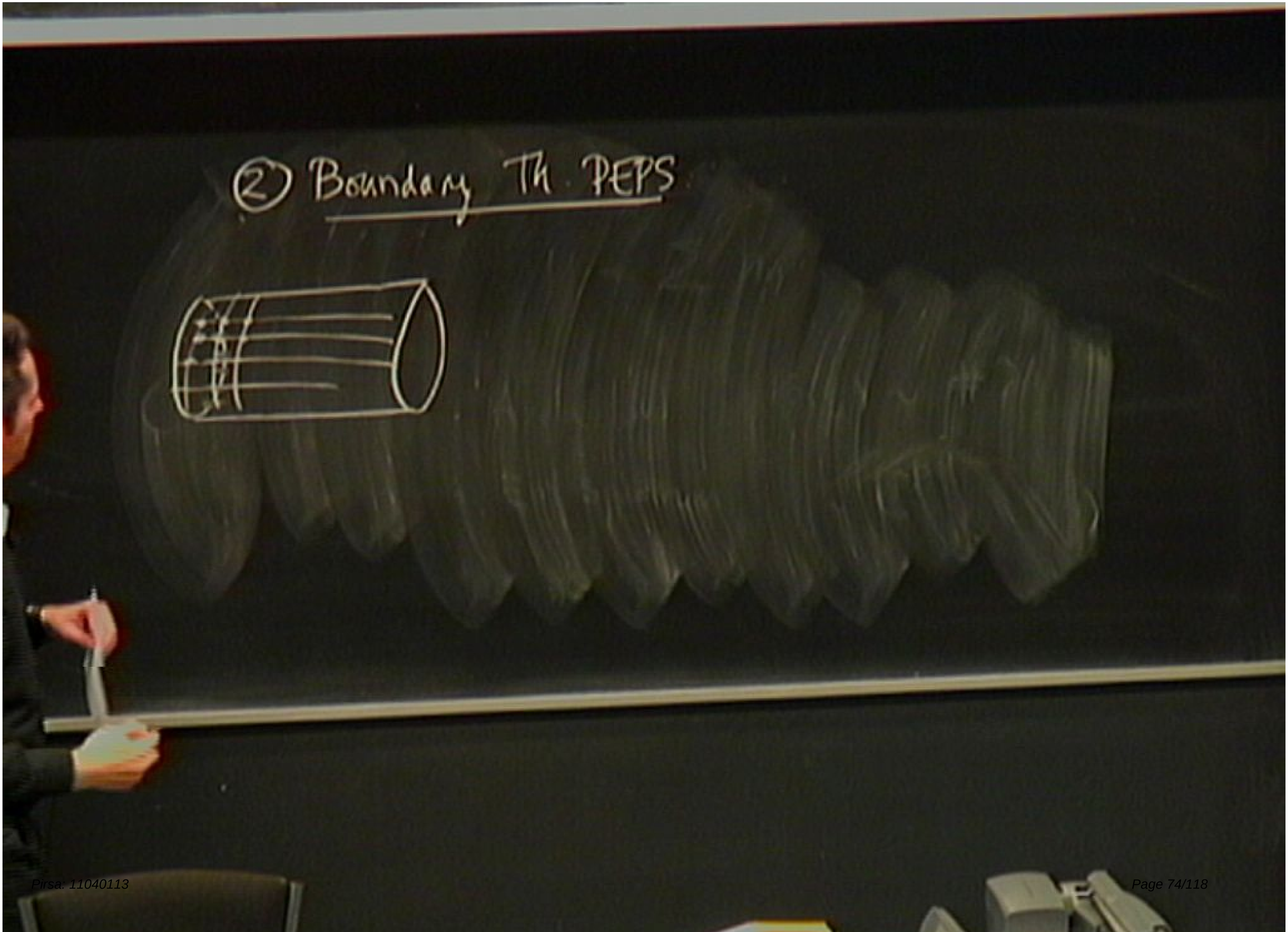
$|\Psi\rangle \longleftrightarrow H$

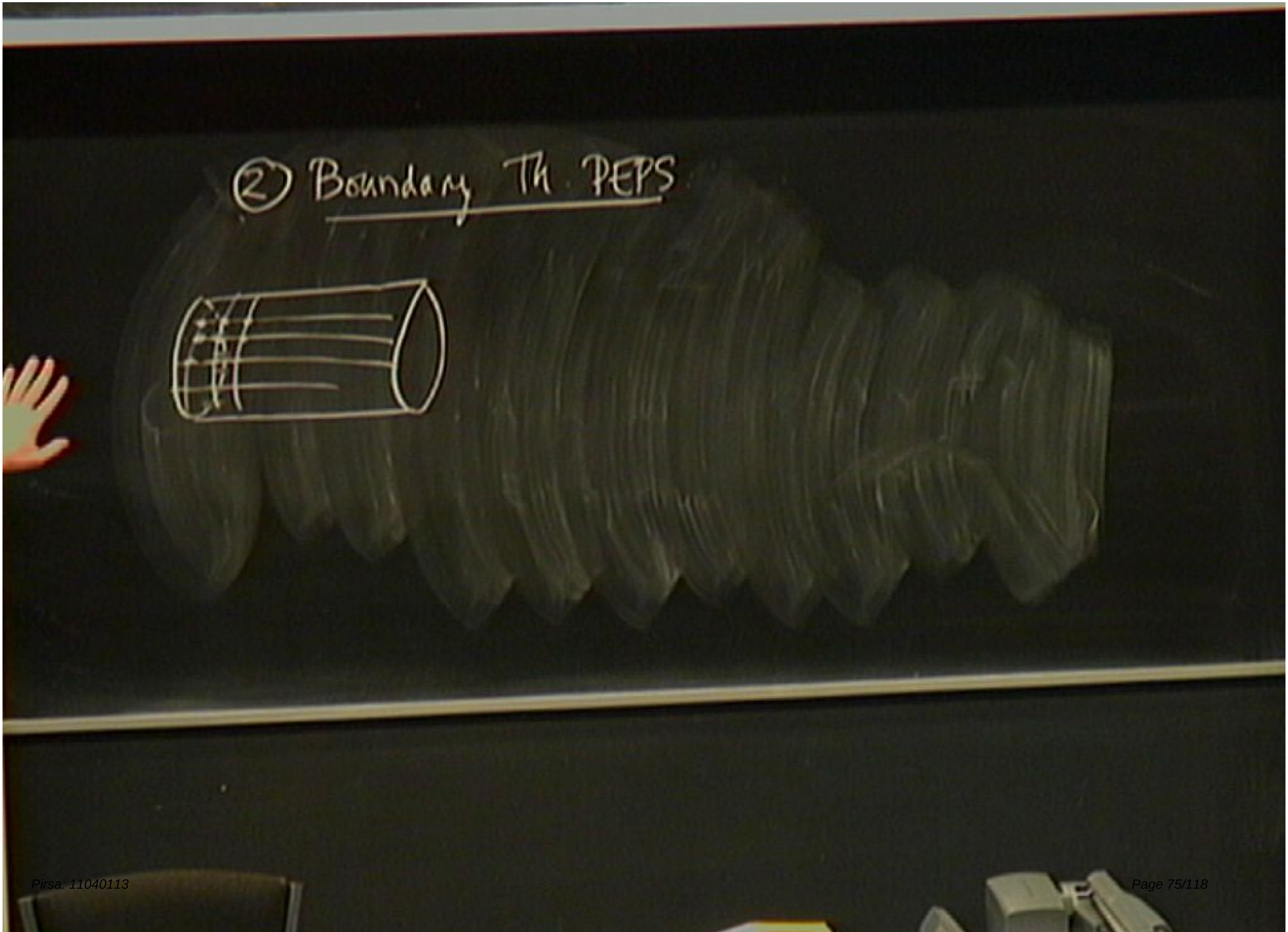
PEPS

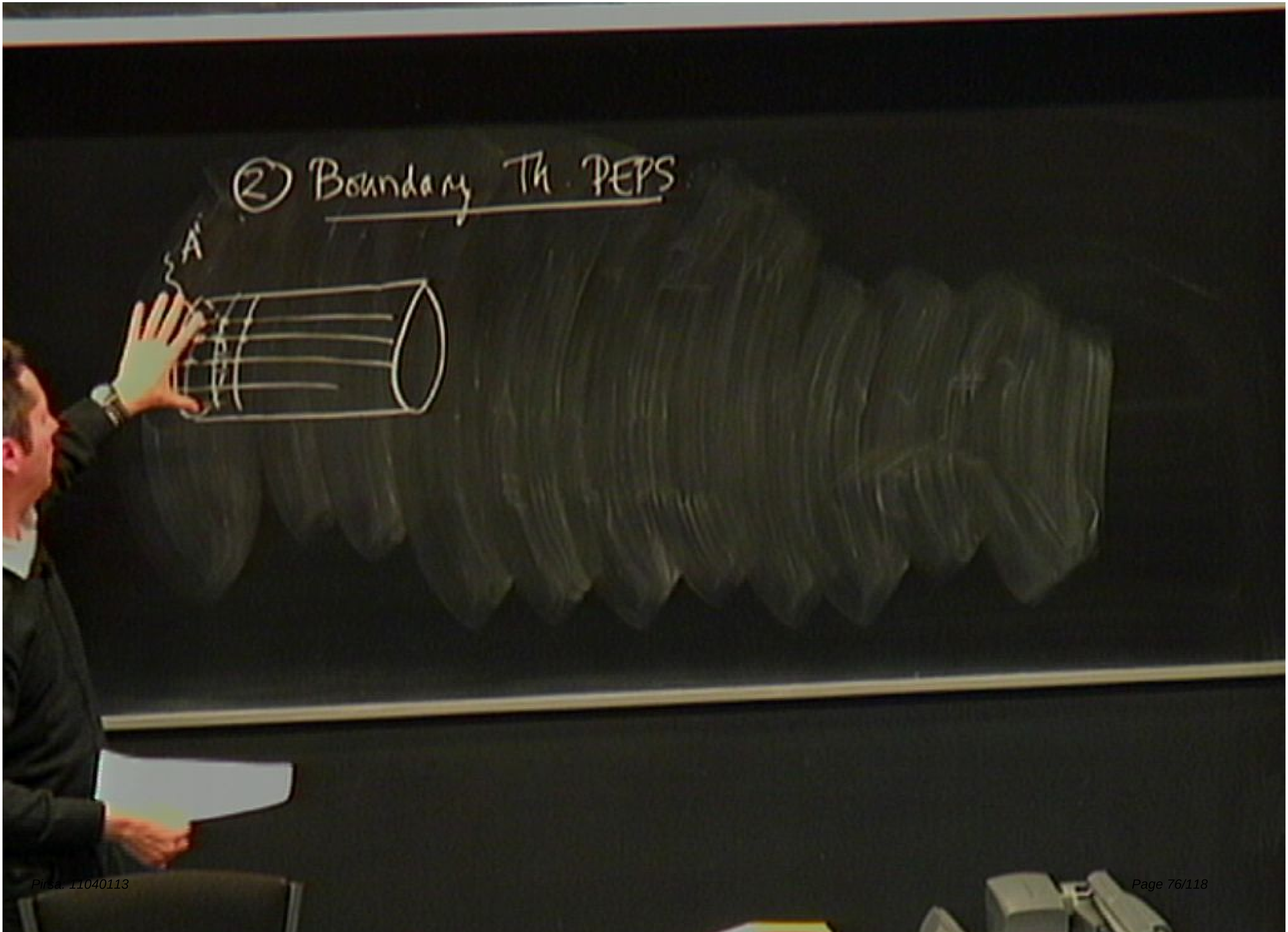
$$H|\Psi\rangle = E_0|\Psi\rangle \quad \begin{cases} \text{injective} \Rightarrow \text{unique} \\ \text{non-inj} \Rightarrow \text{never unique} \end{cases}$$

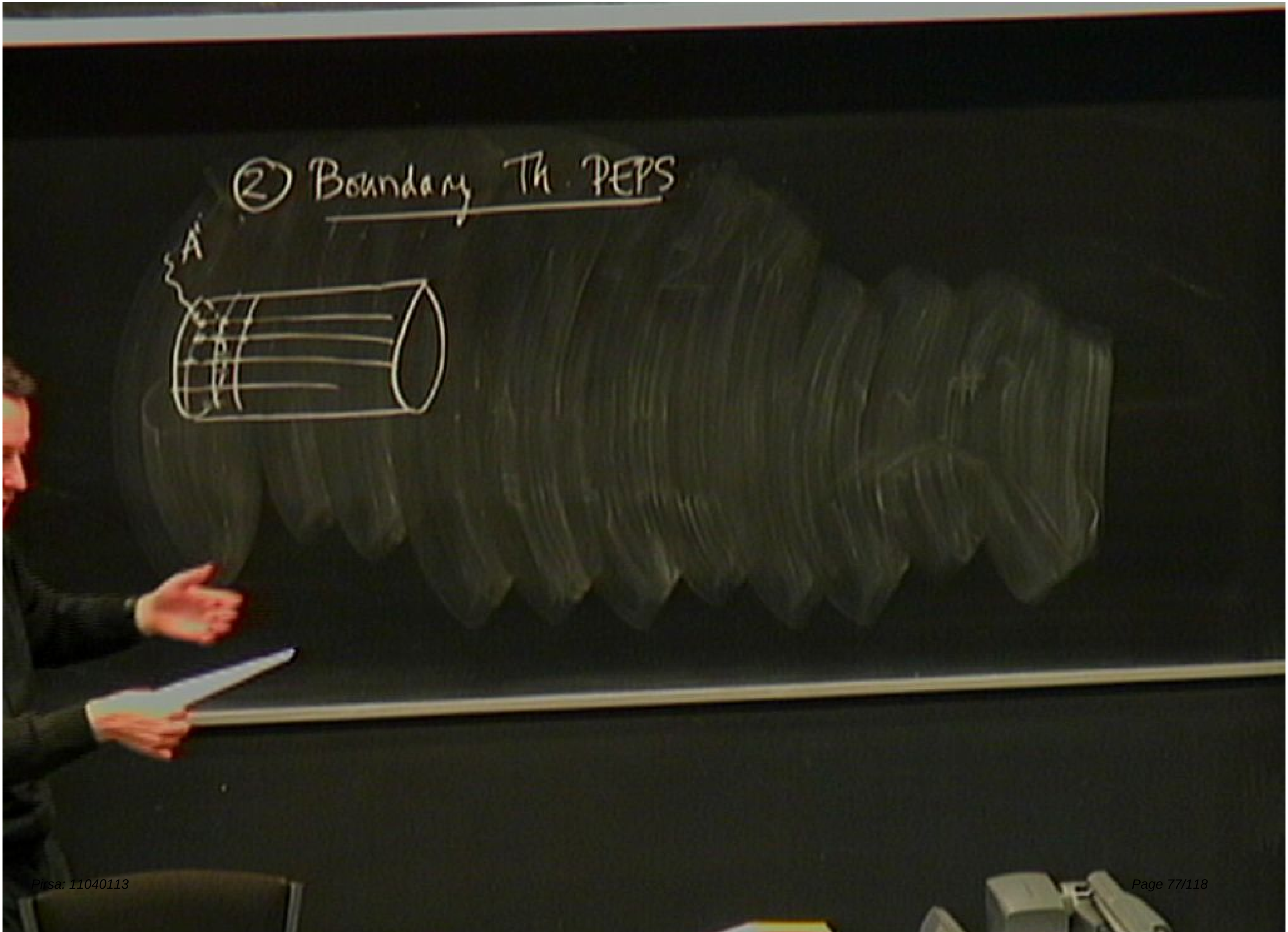

② Boundary Th PEPS

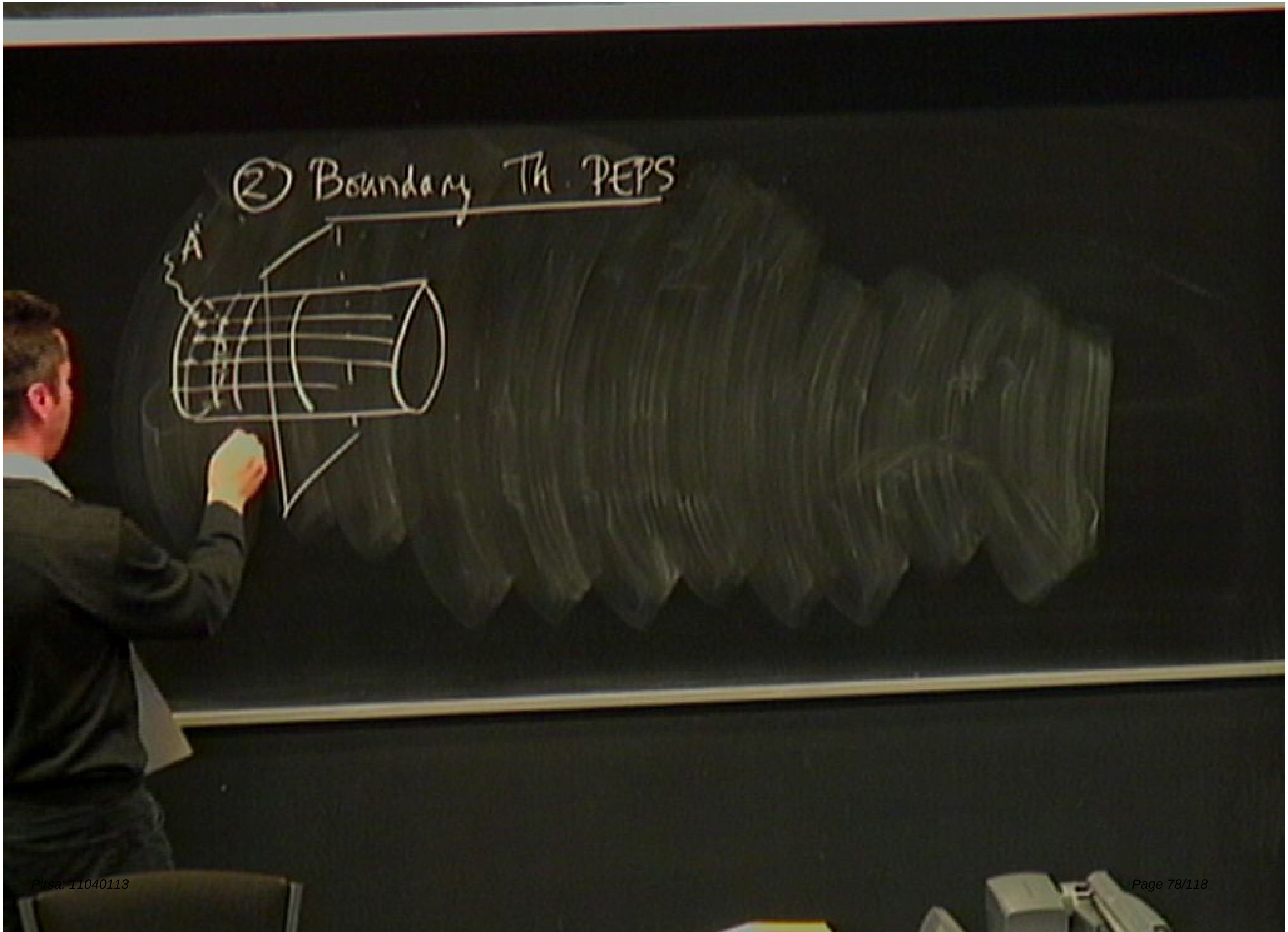

2 Boundary Th PEPS

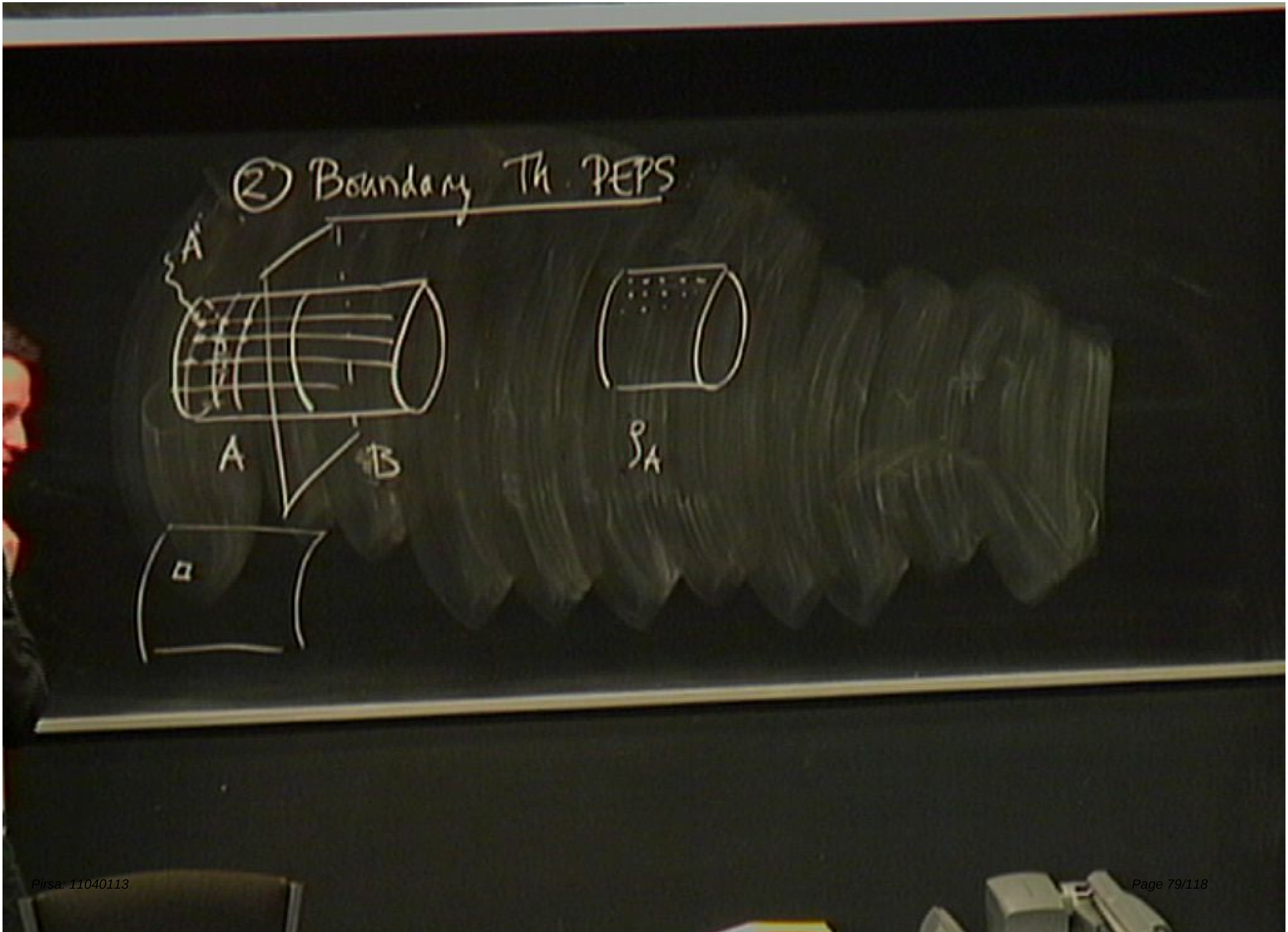

② Boundary Th PEPS


② Boundary Th. PEPS

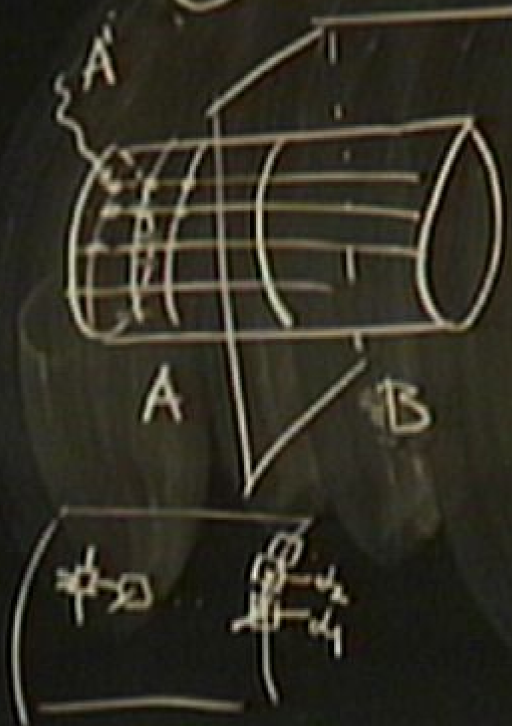

② Boundary Th PEPS
A

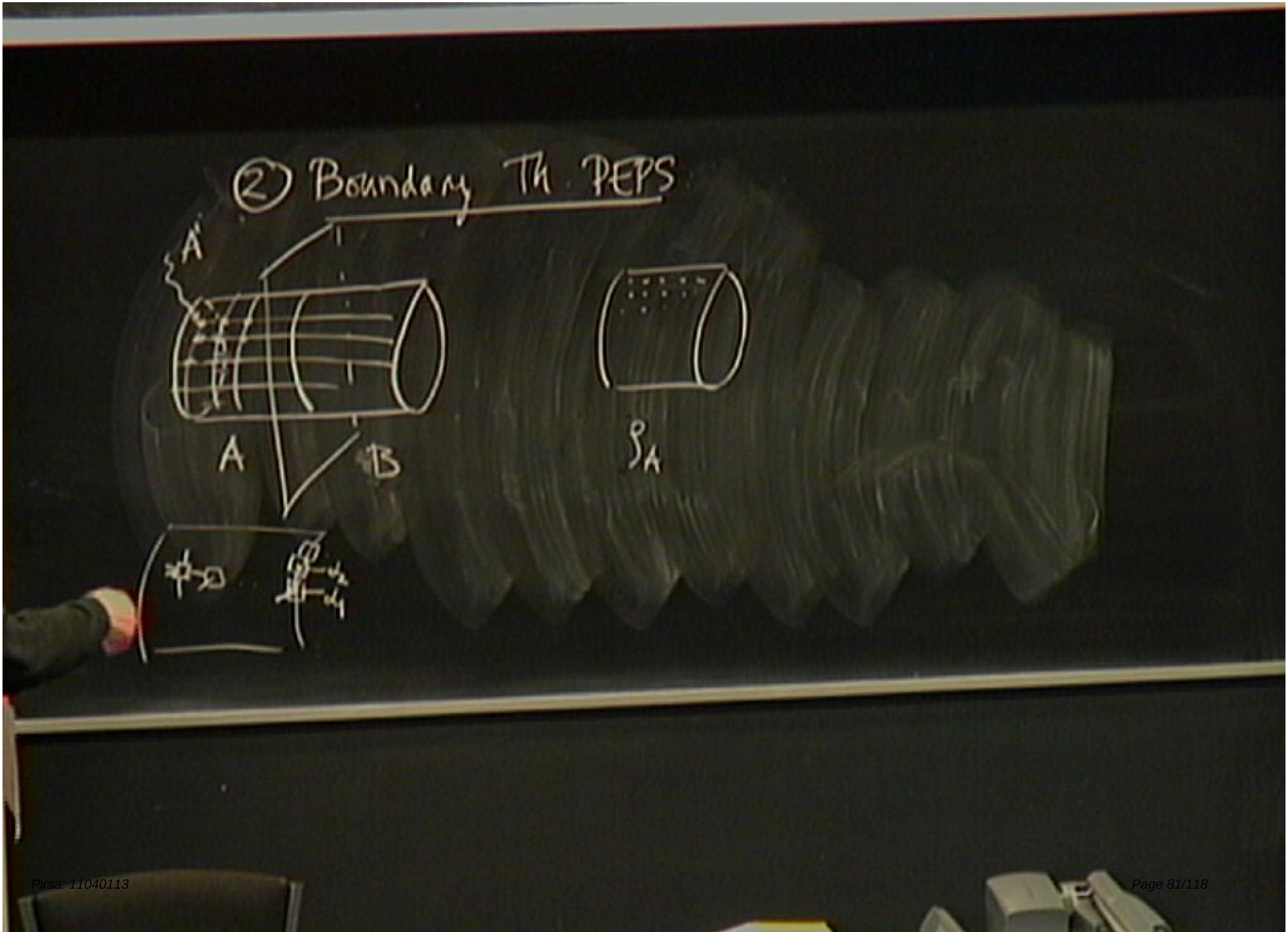

② Boundary Th PEPS
A

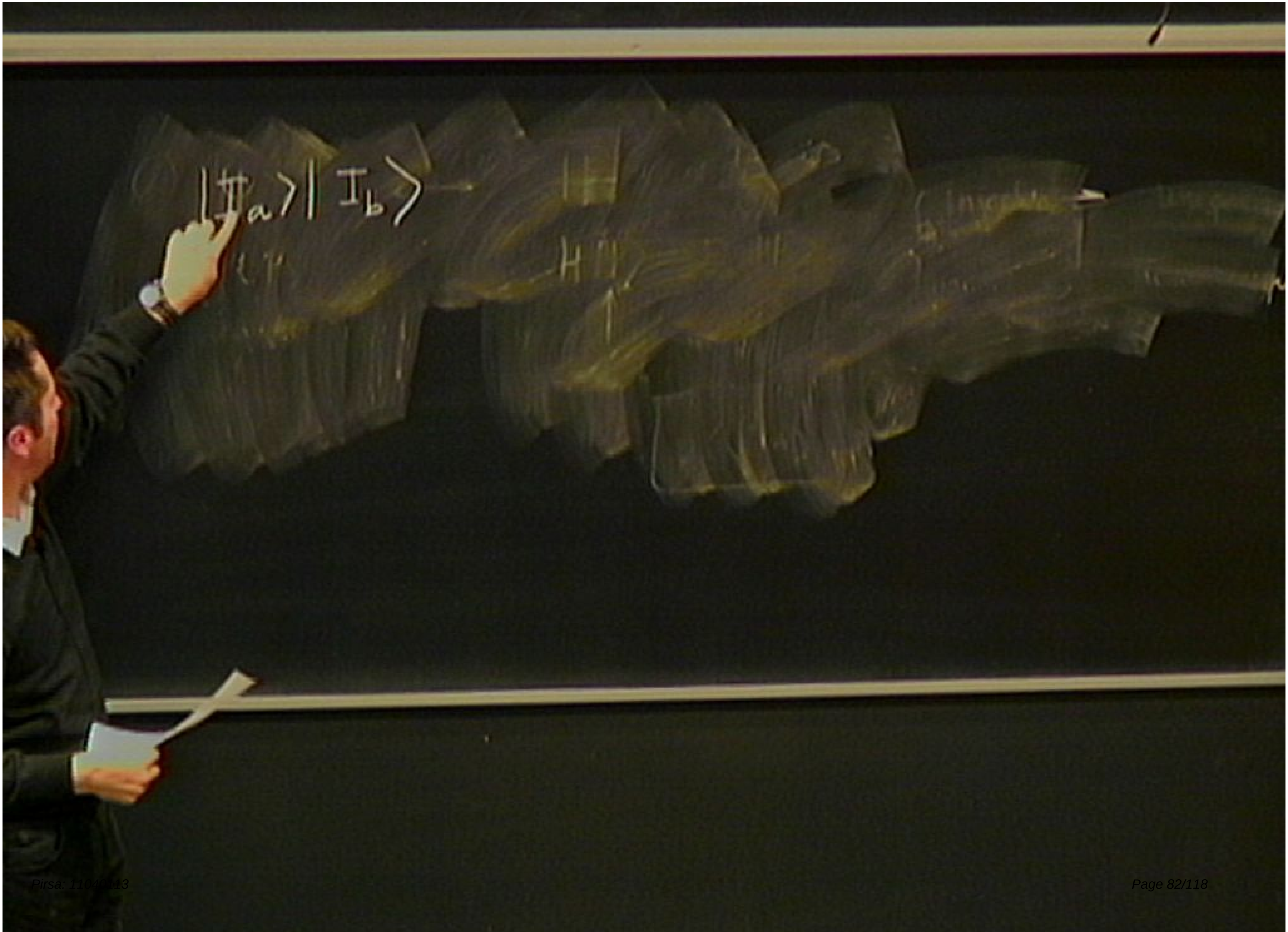

② Boundary Th PEPS
A

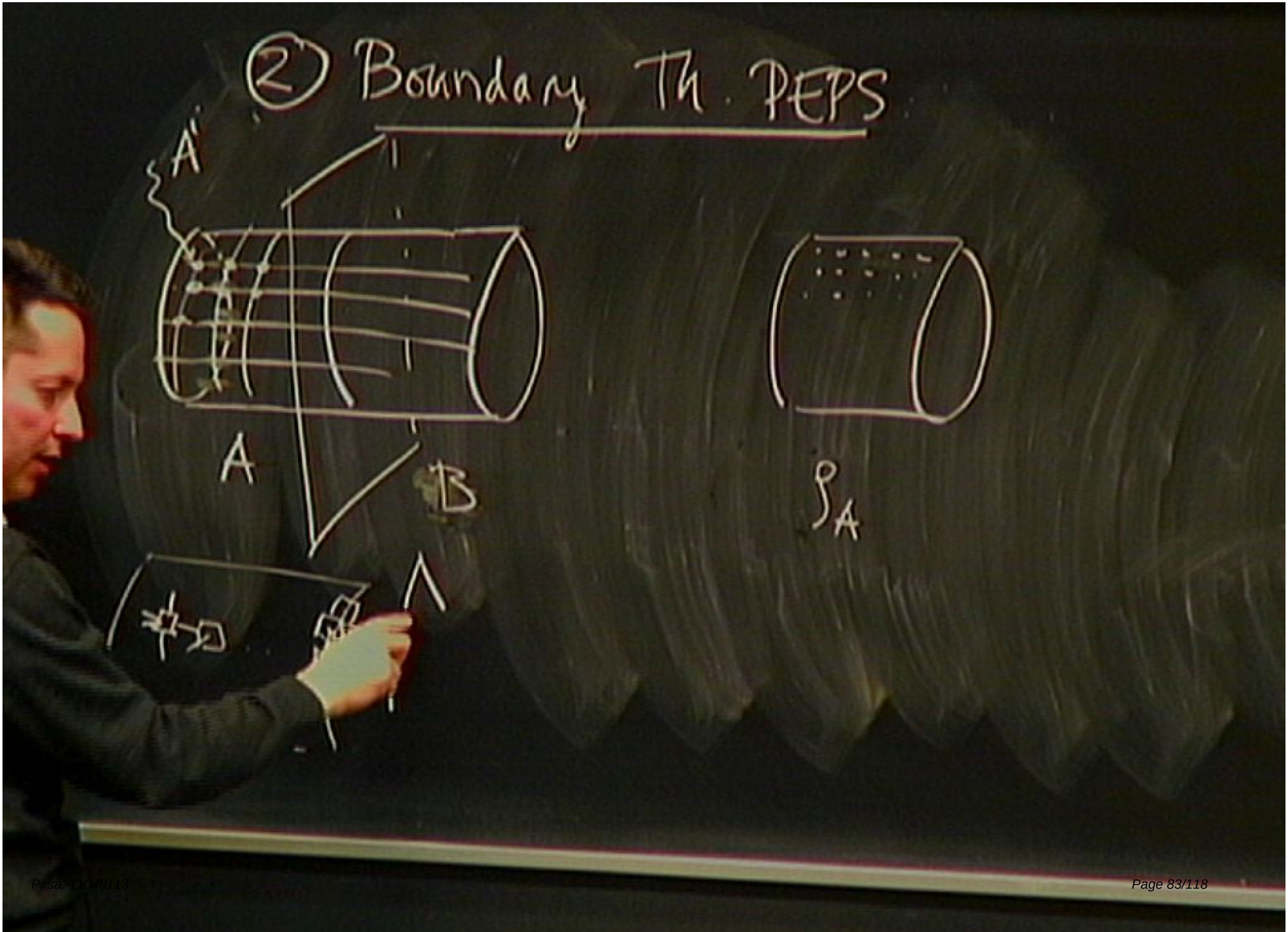

② Boundary Th PEPS
A'
A
B
ρA

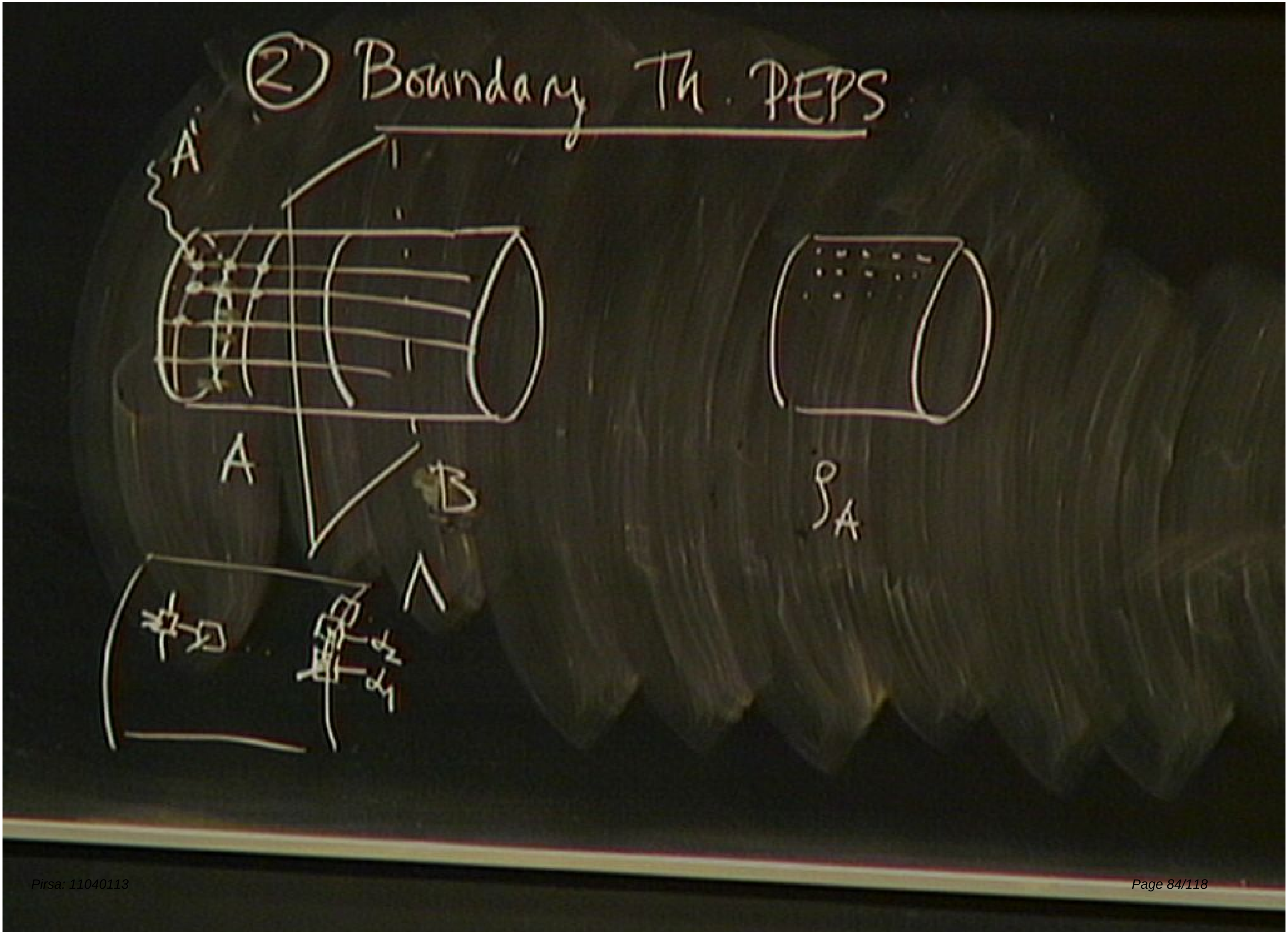
② Boundary Th PEPS




② Boundary Th PEPS
A'
A
B
ρA

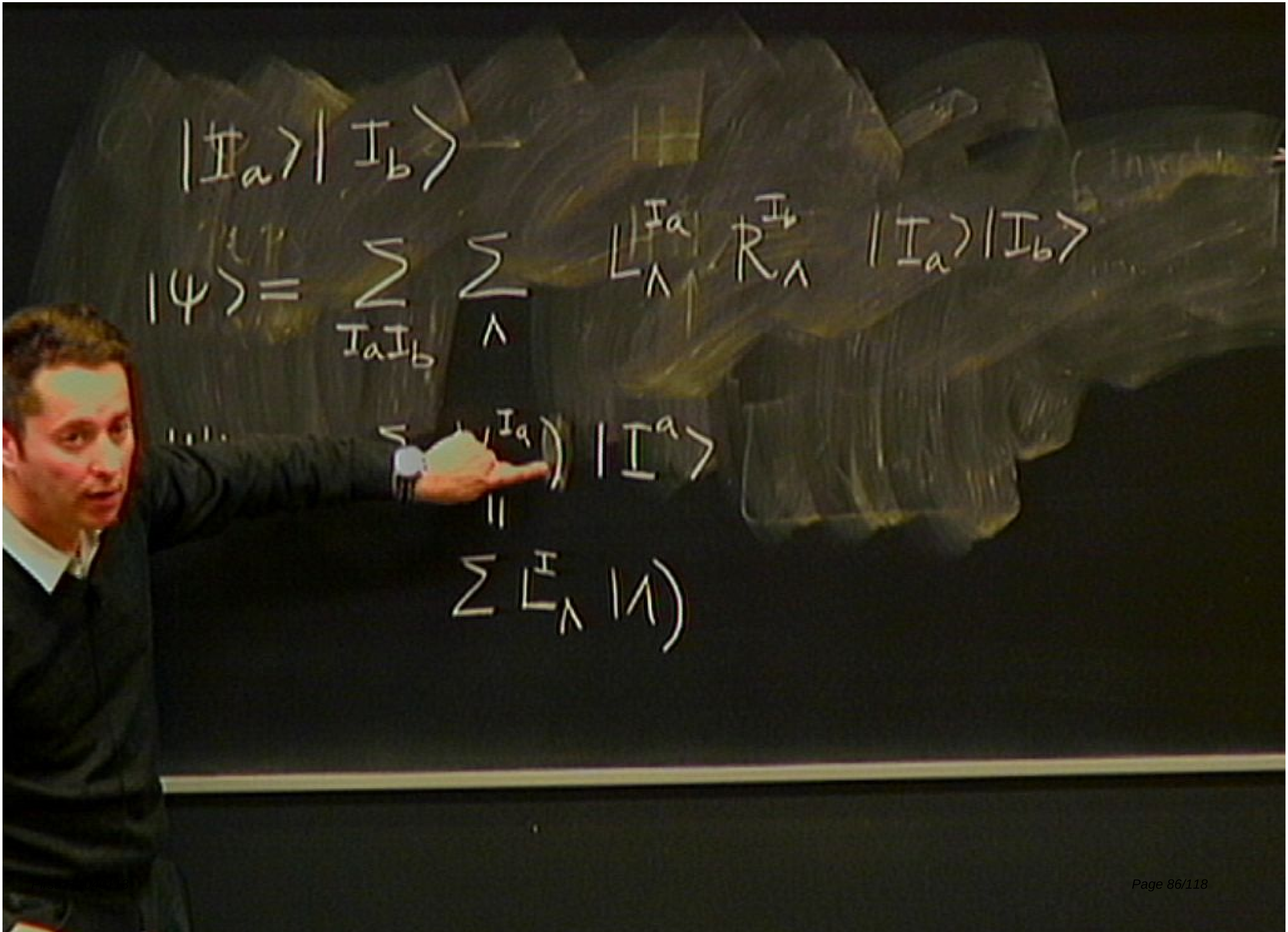

$|II_a\rangle|I_b\rangle$


② Boundary Th. PEPS
A'
A
B
ρ_A


② Boundary Th. PEPS
A'
A
B
ρ_A
α_2
α_1

$$|I_a\rangle |I_b\rangle$$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L^{I_a}_{\Lambda} R^{I_b}_{\Lambda} |I_a\rangle |I_b\rangle$$


|I_a>|I_b>
|ψ> = Σ_{I_a I_b} Σ_Λ L^{I_a}_Λ R^{I_b}_Λ |I_a>|I_b>
|I^a>
Σ L^I_Λ |Λ)

$$|I_a\rangle |I_b\rangle$$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L_{\Lambda}^{I_a} R_{\Lambda}^{I_b} |I_a\rangle |I_b\rangle$$

$$|\psi_L\rangle = \sum |L^{I_a}) \, |I^a\rangle$$

$$\parallel$$

$$\sum L_{\Lambda}^{I} |\Lambda)$$

$$|II_a\rangle|I_b\rangle$$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L_{\Lambda}^{I_a} R_{\Lambda}^{I_b} |I_a\rangle|I_b\rangle$$

$$|\psi_L\rangle = \sum |L^{I_a}) \, |I^a\rangle$$

$$\sum L^{I}_{\Lambda} |\Lambda)$$

$$|I_a\rangle|I_b\rangle \to$$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L^{I_a}_{\Lambda} R^{I_b}_{\Lambda} |I_a\rangle|I_b\rangle$$

$$|\psi_L\rangle = \sum |L^{I_a})|I^a\rangle$$

$$\sigma_L = \sum_{I_a} |L^{I_a})(L^{I_a}|$$

$$|L^{I_a}) = \sum L^{I}_{\Lambda} |\Lambda)$$

$|I_a\rangle |I_b\rangle$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L_{\Lambda}^{I_a} R_{\Lambda}^{I_b} |I_a\rangle |I_b\rangle$$

$$|\psi_{\substack{L\\R}}\rangle = \sum |L^{I_a}) |I^a\rangle \qquad \sigma_{\substack{L\\R}} = \sum_{I_a} |L^{I_a})(L^{I_a}|$$

$$|L^{I_a}) = \sum L_{\Lambda}^{I} |\Lambda)$$

$$|\mathrm{II}_a\rangle |I_b\rangle$$

$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L^{I_a}_{\Lambda} R^{I_b}_{\Lambda} \, |I_a\rangle |I_b\rangle$$

$$|\psi_{\substack{L\\R}}\rangle = \sum |L^{I_a}) \, |I^a\rangle$$

$$\parallel$$

$$\sum L^{I}_{\Lambda} |\Lambda)$$

$$\sigma_{\substack{L\\R}} = \sum_{I_a} |L^{I_a})(L^{I_a}|$$

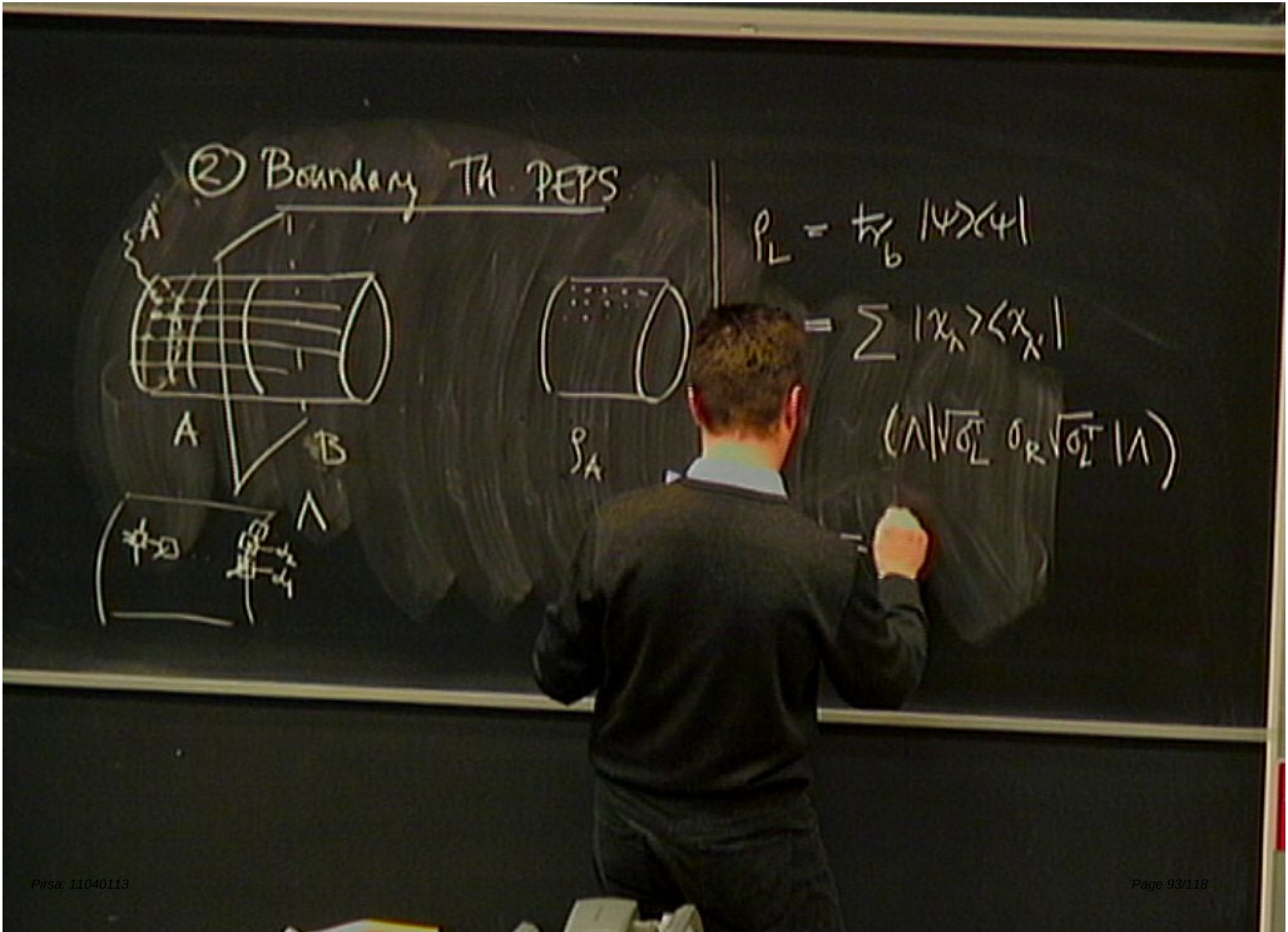
$$|I_a\rangle|I_b\rangle$$

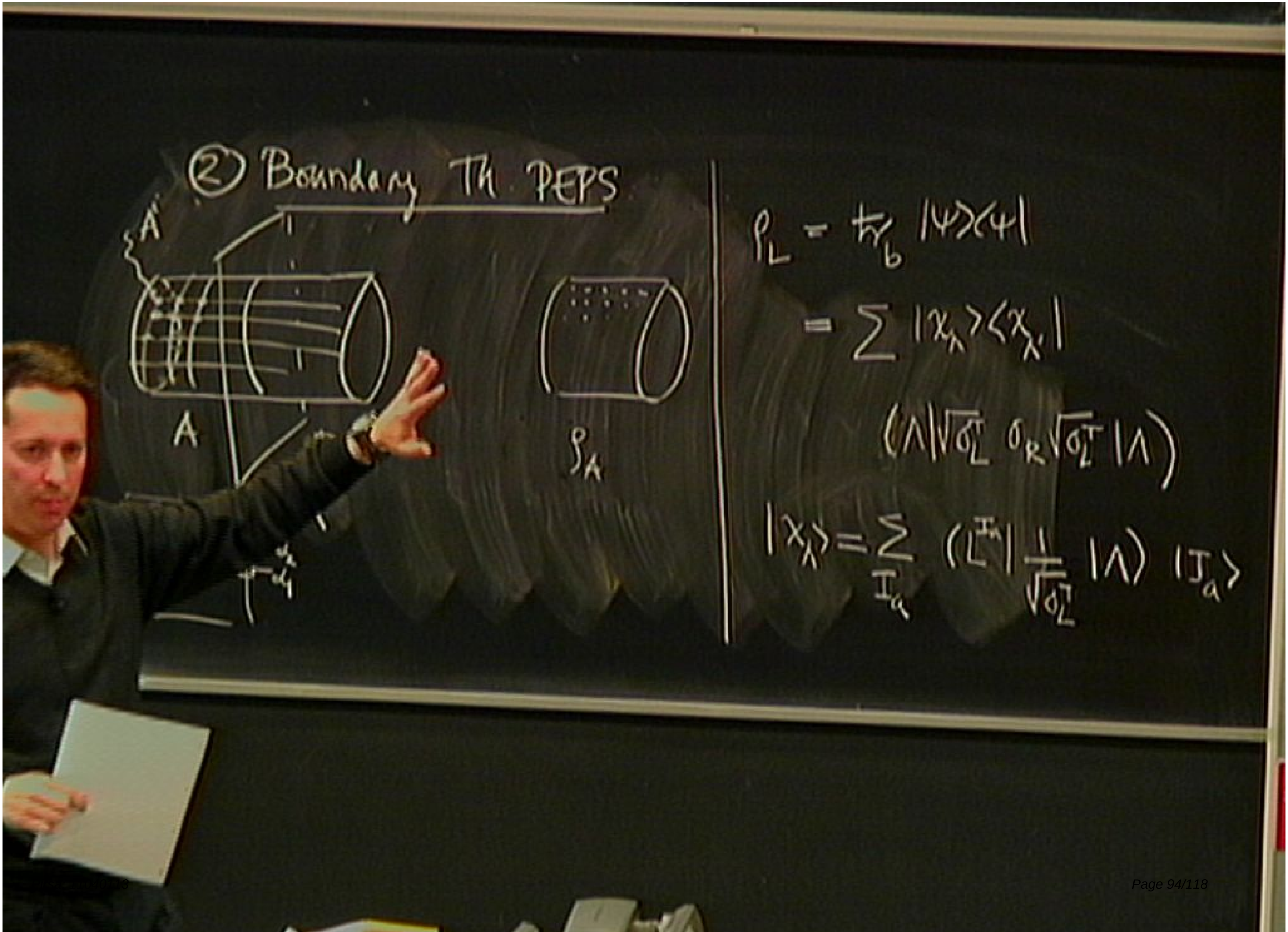
$$|\psi\rangle = \sum_{I_a I_b} \sum_{\Lambda} L^{I_a}_{\Lambda} R^{I_b}_{\Lambda} |I_a\rangle|I_b\rangle$$

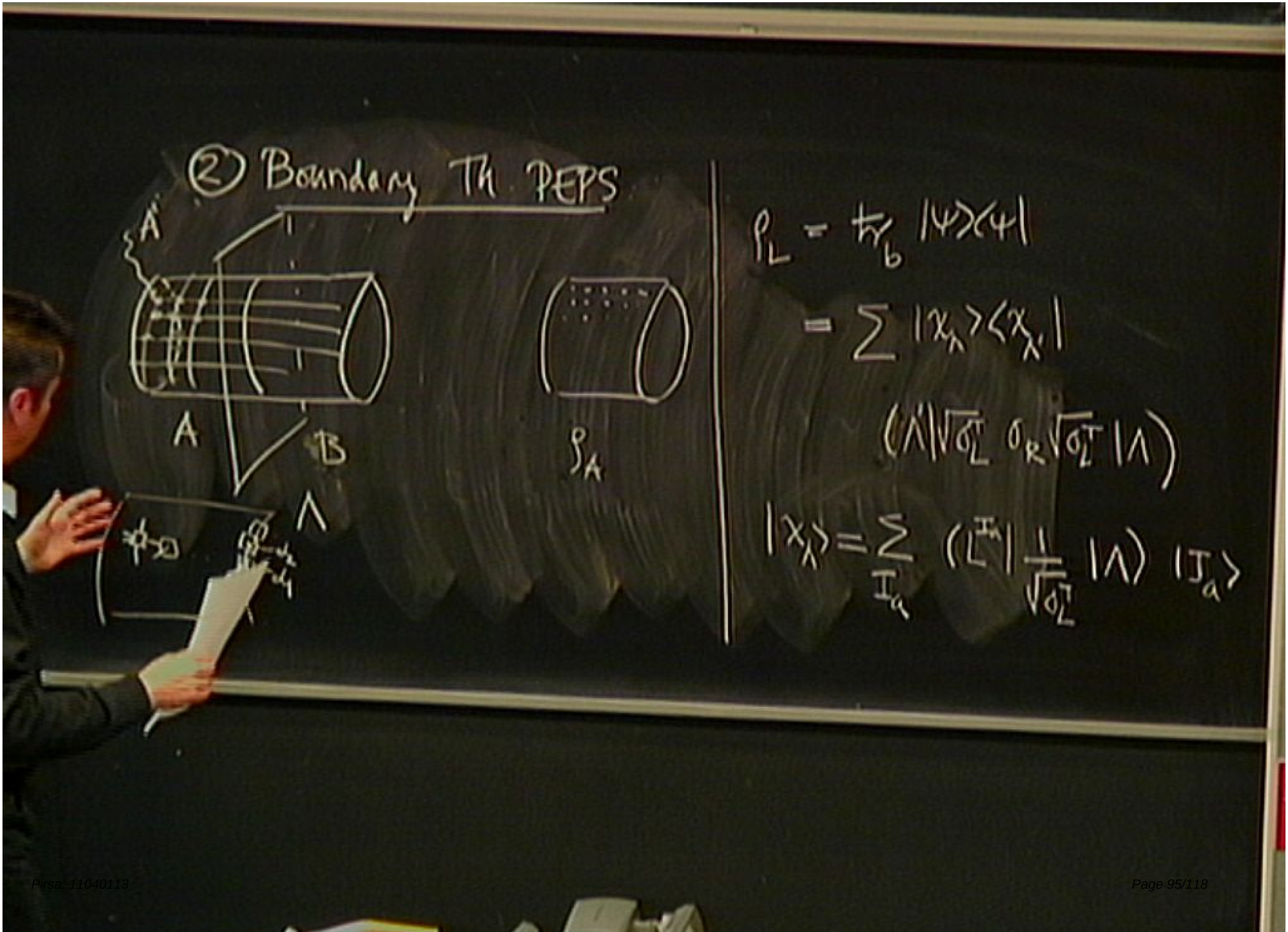
$$|\psi_{\substack{L\\R}}\rangle = \sum |L^{I_a}) \, |I^a\rangle$$

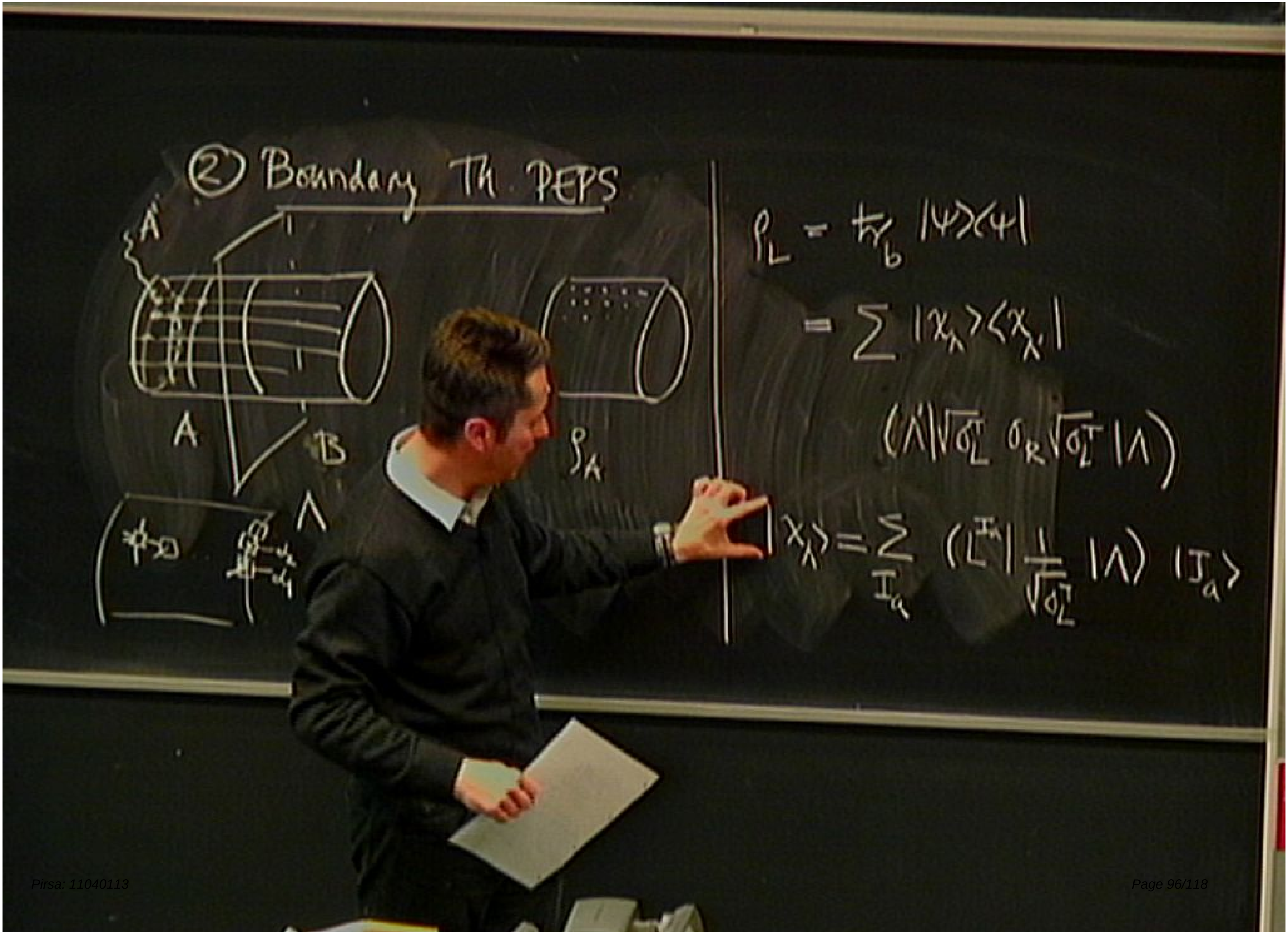
$$|L^{I_a}) = \sum L^{I}_{\Lambda} |\Lambda)$$

$$\sigma_{\substack{L\\R}} = \sum_{I_a} |L^{I_a})(L^{I_a}|$$


② Boundary Th PEPS
A'
A
B
Λ
ρ_A
ρ_L = tr_b |ψ⟩⟨ψ|
= Σ |χ_λ⟩⟨χ_λ|
(Λ|√σ_L^T σ_R √σ_L^T|Λ)


② Boundary Th PEPS
A'
A
ρ_A
ρ_L = tr_b |Ψ⟩⟨Ψ|
= Σ |χ_λ⟩⟨χ_λ|
(Λ|√σ_L σ_R √σ_L^T|Λ)
|χ_λ⟩ = Σ_{I_a} (L^{I_a}| 1/√σ_L |Λ) |I_a⟩


② Boundary Th PEPS
A'
A
B
Λ
ρA
ρL = trb |ψ⟩⟨ψ|
= Σ |χλ⟩⟨χλ|
(Λ'|√σL σR √σL|Λ)
|χλ⟩ = Σ Ia (L^Ia| 1/√σL |Λ) |Ja⟩


② Boundary Th PEPS
A
B
ρ_A
Λ
$\rho_L = \mathrm{tr}_b |\psi\rangle\langle\psi|$
$= \sum |\chi_\lambda\rangle\langle\chi_\lambda|$
$(\Lambda|\sqrt{\sigma_L}\,\sigma_R\sqrt{\sigma_L}|\Lambda)$
$|\chi_\lambda\rangle = \sum_{I_a} (\ldots|\frac{1}{\sqrt{\sigma_L}}|\Lambda)\,|J_a\rangle$

② Boundary Th. PEPS

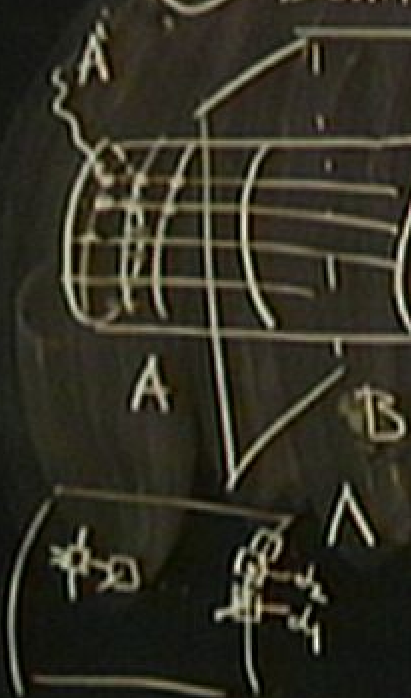
$$\rho_L = \mathrm{tr}_b\, |\psi\rangle\langle\psi|$$

$$= \sum |\chi_\lambda\rangle\langle\chi_\lambda|$$

$$(\Lambda'|\sqrt{\sigma_L}\,\sigma_R\sqrt{\sigma_L^T}|\Lambda)$$

$$|\chi_\lambda\rangle = \sum_{I_a} (L^{I_a}|\frac{1}{\sqrt{\sigma_L^T}}|\Lambda)\,|I_a\rangle$$

② Boundary Th PEPS



$$\rho_L = \mathrm{tr}_b\, |\psi\rangle\langle\psi|$$

$$= \sum |\chi_\lambda\rangle\langle\chi_\lambda|$$

$$(\lambda|\sqrt{\sigma_L^T}\,\sigma_R\sqrt{\sigma_L^T}|\lambda)$$

$$|\chi_\lambda\rangle = \sum_{I_a} (L^{I_a}|\,\frac{1}{\sqrt{\sigma_L^T}}\,|\lambda)\,|I_a\rangle$$

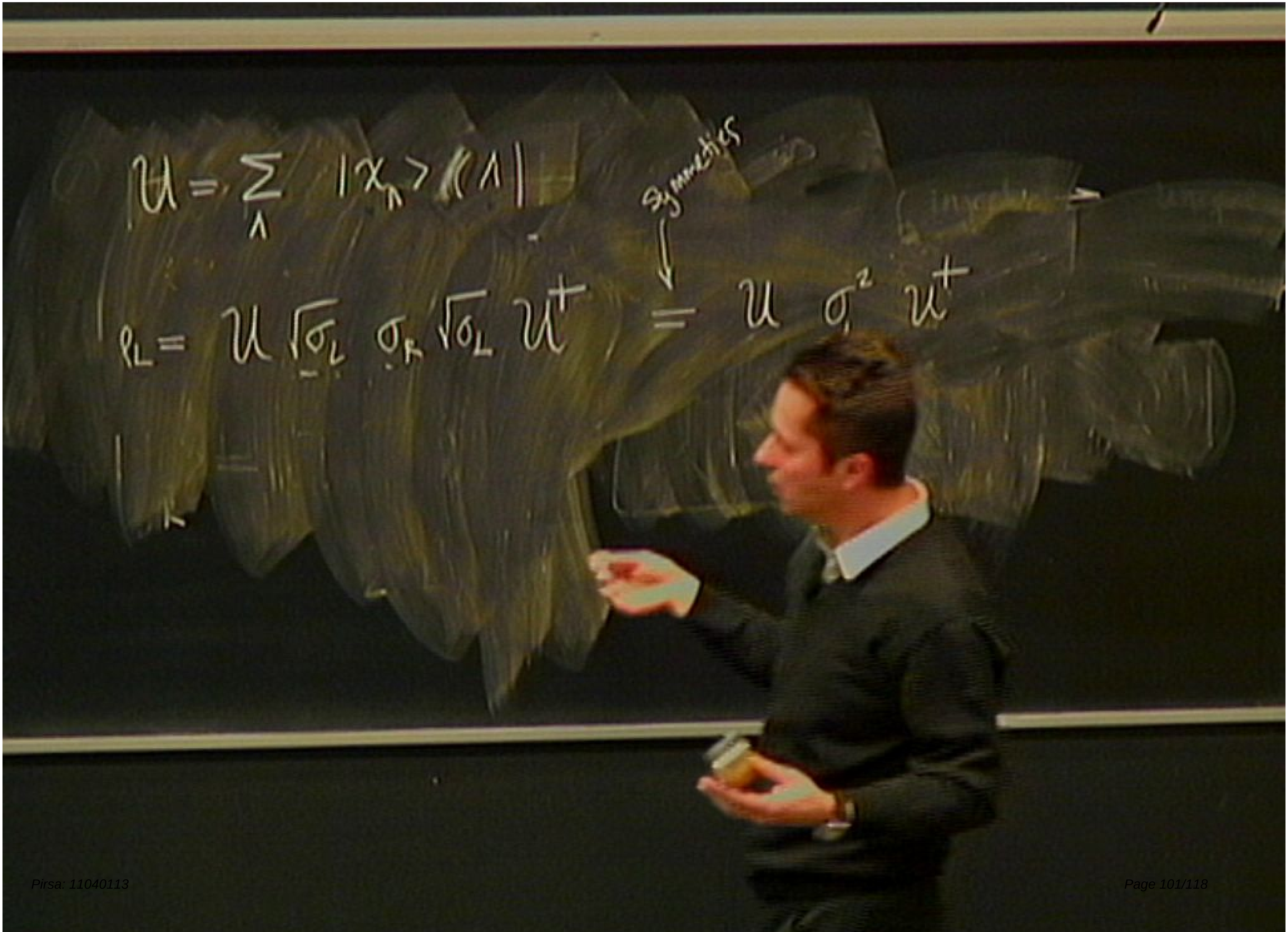
$$\mathcal{U} = \sum_{\Lambda} |x_\Lambda\rangle\langle\Lambda|$$

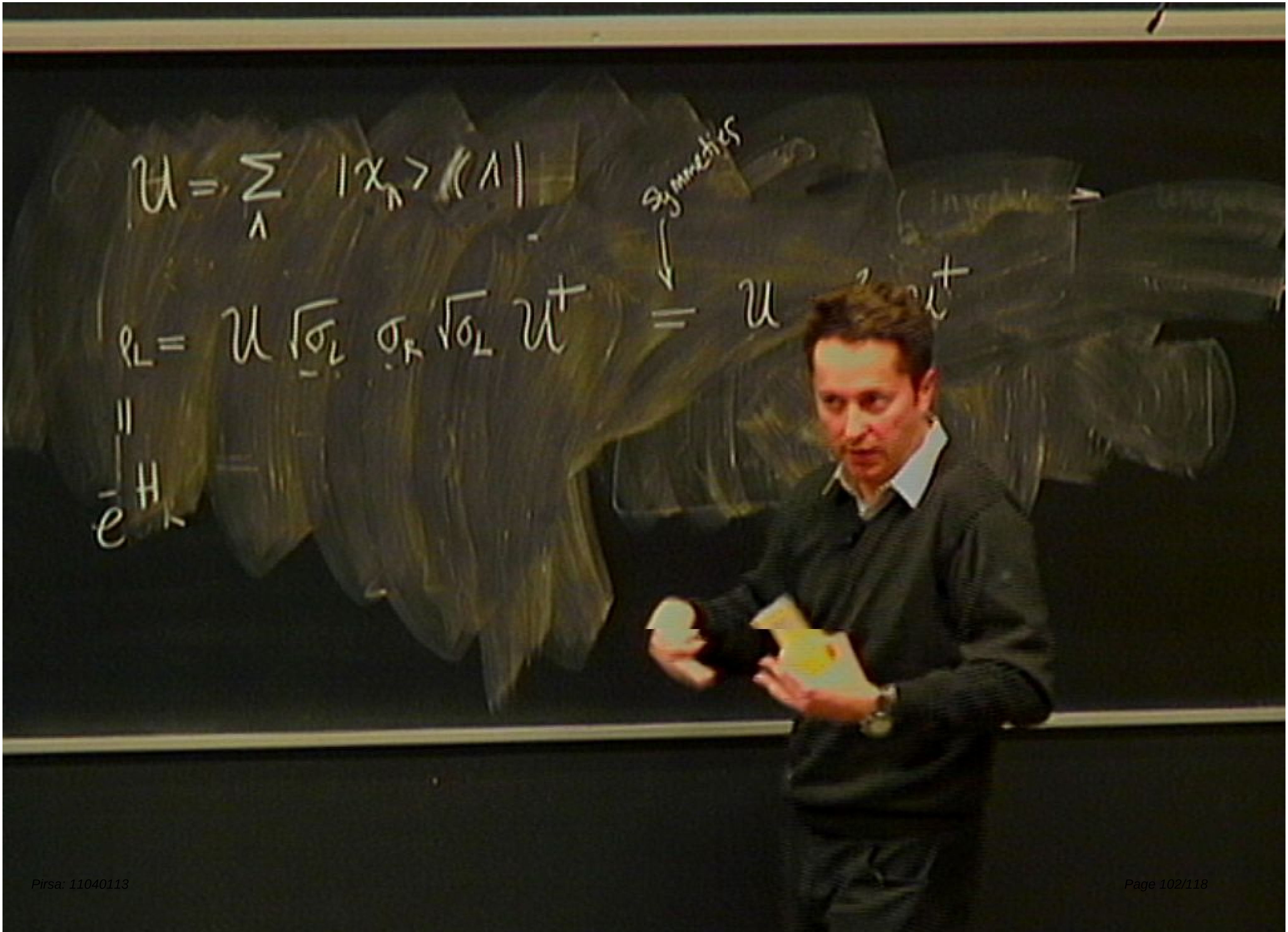
$$\sigma_{\substack{L\\R}} = \sum_{I_A} |L^{I_A})(L^{I_A}|$$

$$\mathcal{U} = \sum_{\Lambda} |x_\Lambda\rangle(\Lambda|$$

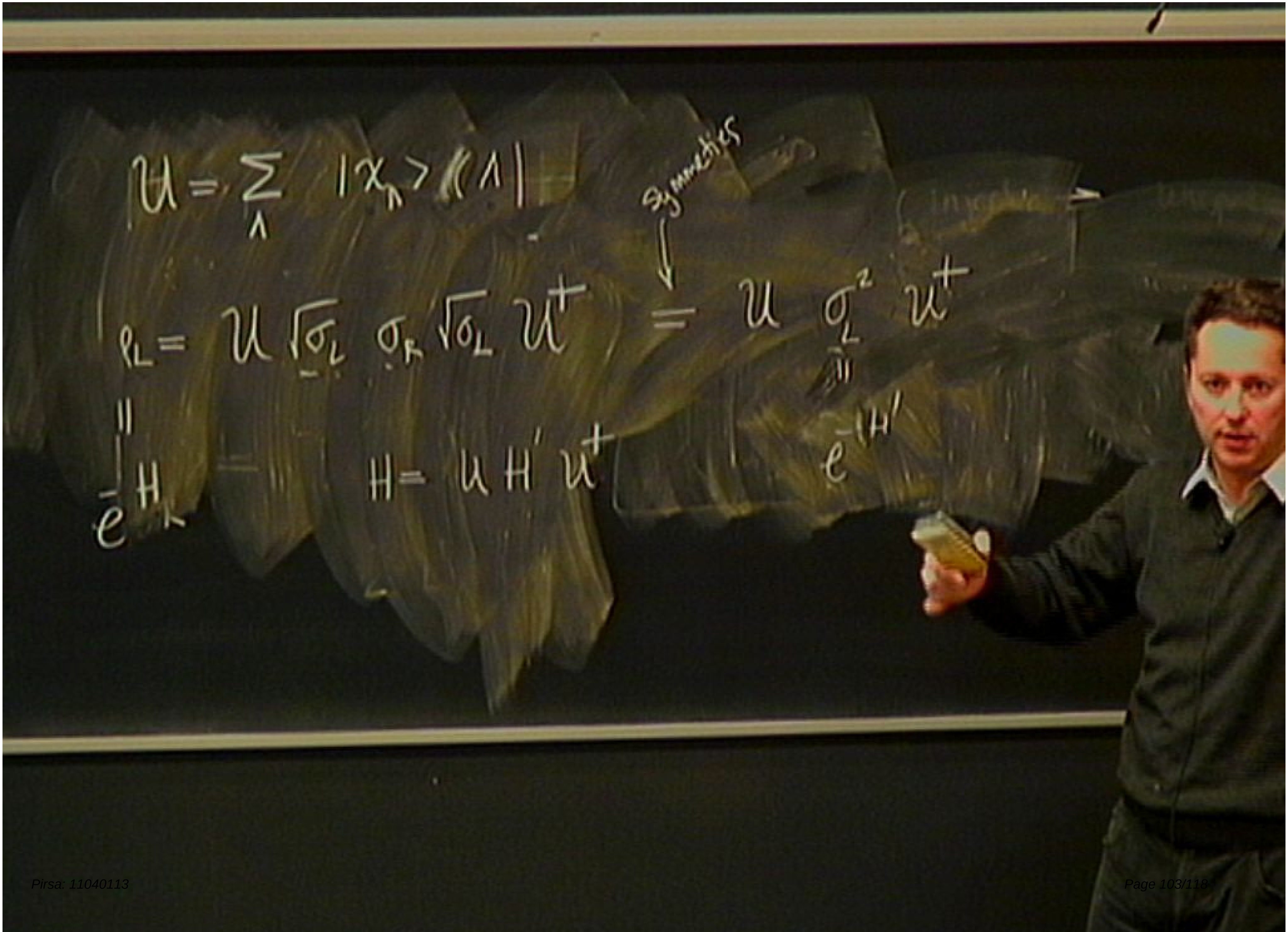
$$\rho_L = \mathcal{U}\sqrt{\sigma_L}\;\sigma_R\sqrt{\sigma_L}\;\mathcal{U}^{\dagger}$$

$$\sigma_{\substack{L\\R}} = \sum_{I_A} |L^{I_A})(L^{I_A}|$$


$U = \sum_{\Lambda} |x_\Lambda\rangle\langle\Lambda|$
symmetries
$\rho_L = U \sqrt{\sigma_L}\, \sigma_R \sqrt{\sigma_L}\, U^\dagger = U \sigma_1^2 U^\dagger$



$\mathcal{U} = \sum_\Lambda |x_\Lambda\rangle\langle\Lambda|$
symmetries
$\rho_L = \mathcal{U}\sqrt{\sigma_L}\,\sigma_R\sqrt{\sigma_L}\,\mathcal{U}^\dagger$


$U = \sum_\Lambda |x_\Lambda\rangle\langle\Lambda|$
symmetries
$\rho_L = U\sqrt{\sigma_L}\,\sigma_R\sqrt{\sigma_L}\,U^\dagger = U\sigma_L^2U^\dagger$
e^{-H_L}
$H = UH'U^\dagger$
$e^{-H'}$

$$\mathcal{U} = \sum_{\Lambda} |\chi_\Lambda\rangle\langle\Lambda|$$

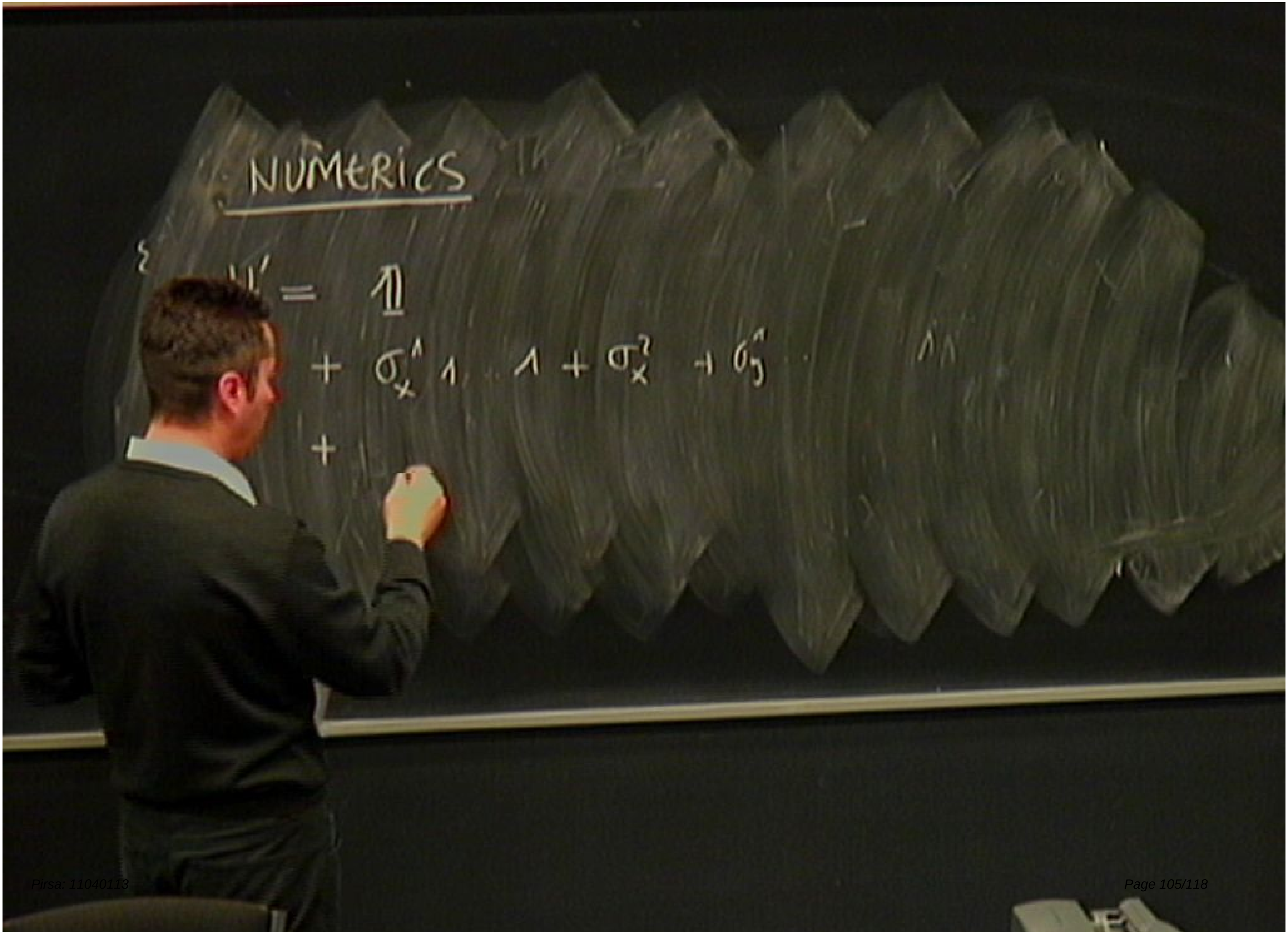
Symmetries

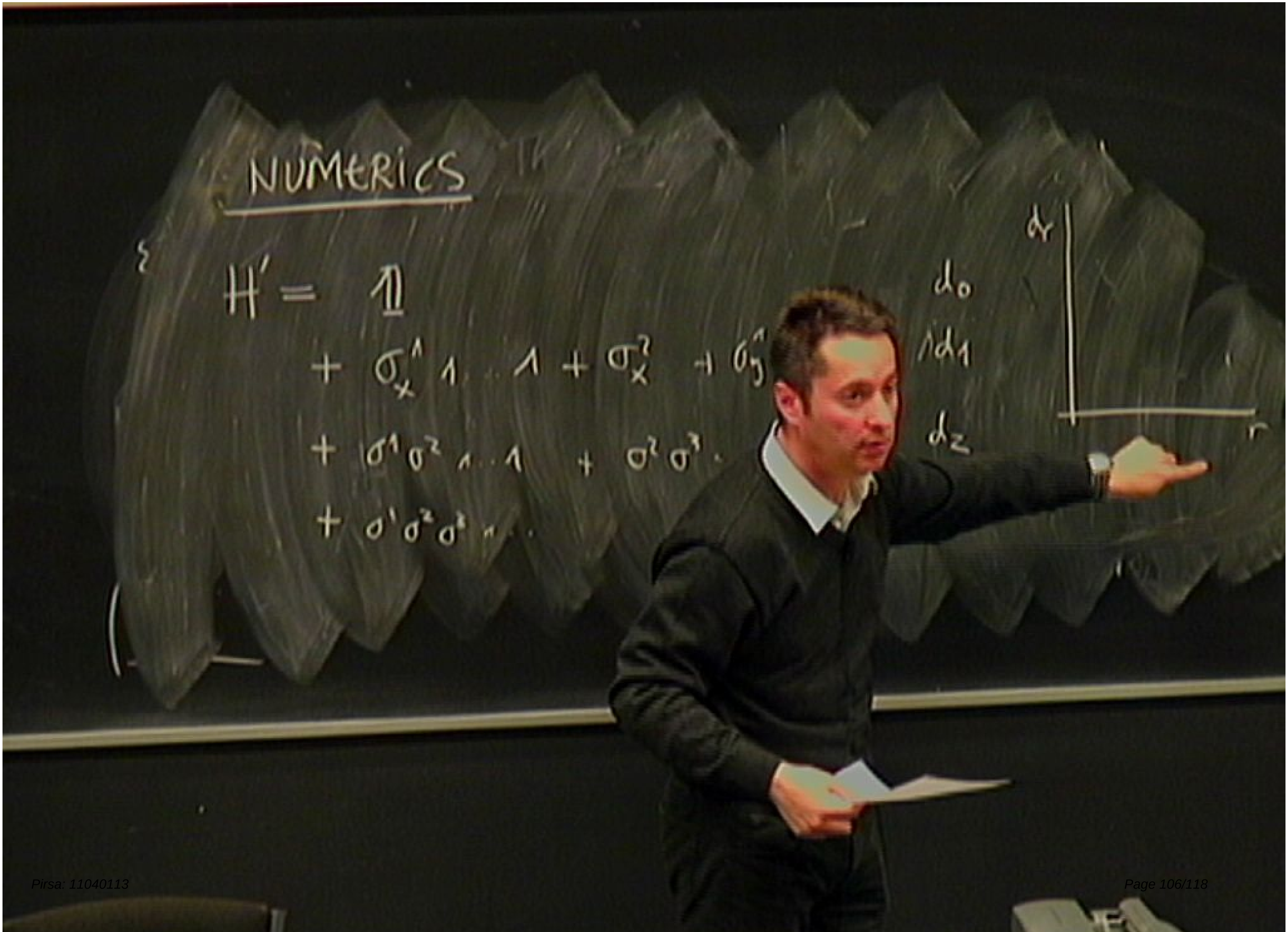
$$\rho_L = \mathcal{U}\sqrt{\sigma_L}\,\sigma_R\sqrt{\sigma_L}\,\mathcal{U}^\dagger = \mathcal{U}\,\sigma_L^2\,\mathcal{U}^\dagger$$

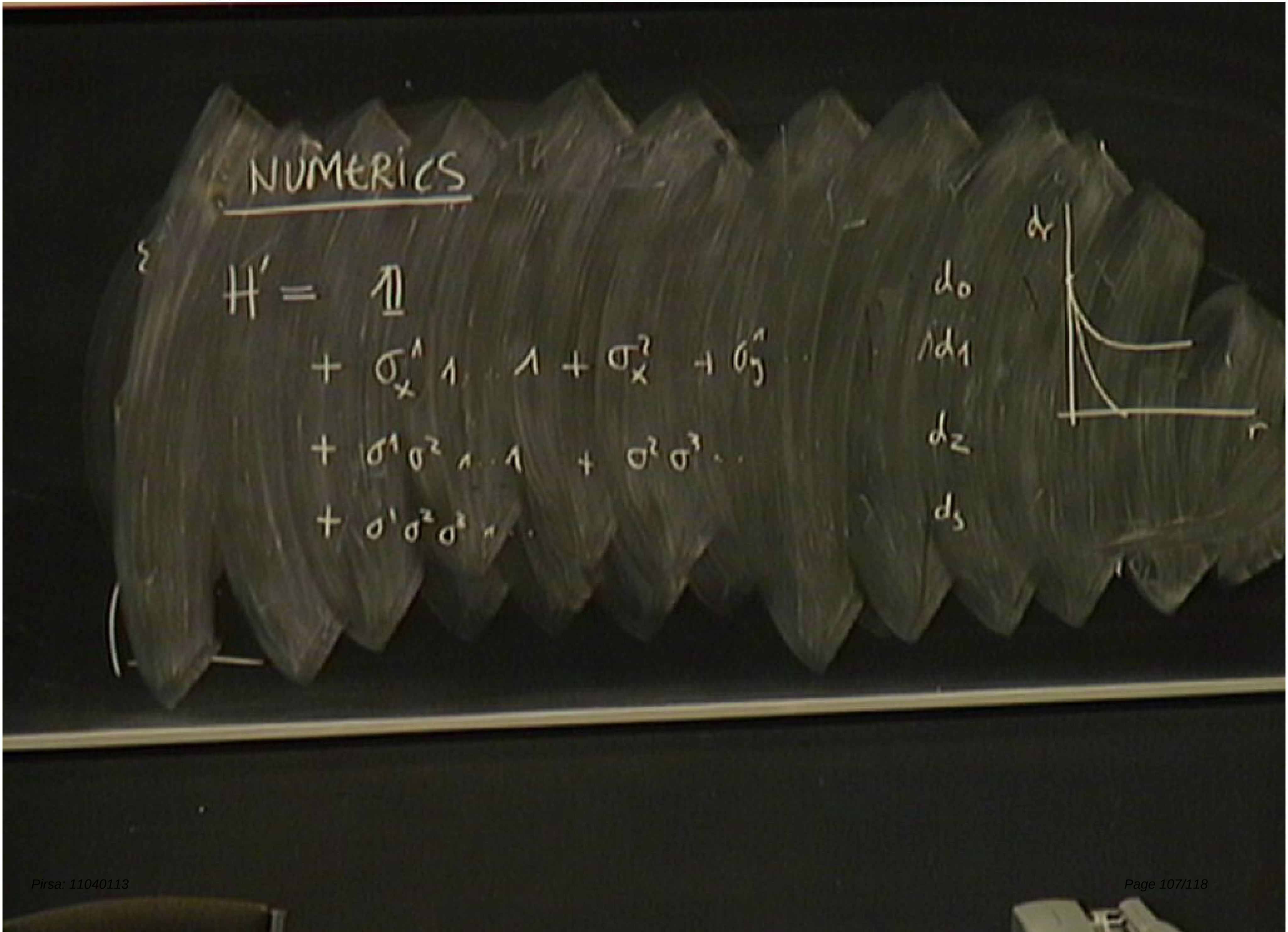
$$\rho_L = e^{-H_L}$$

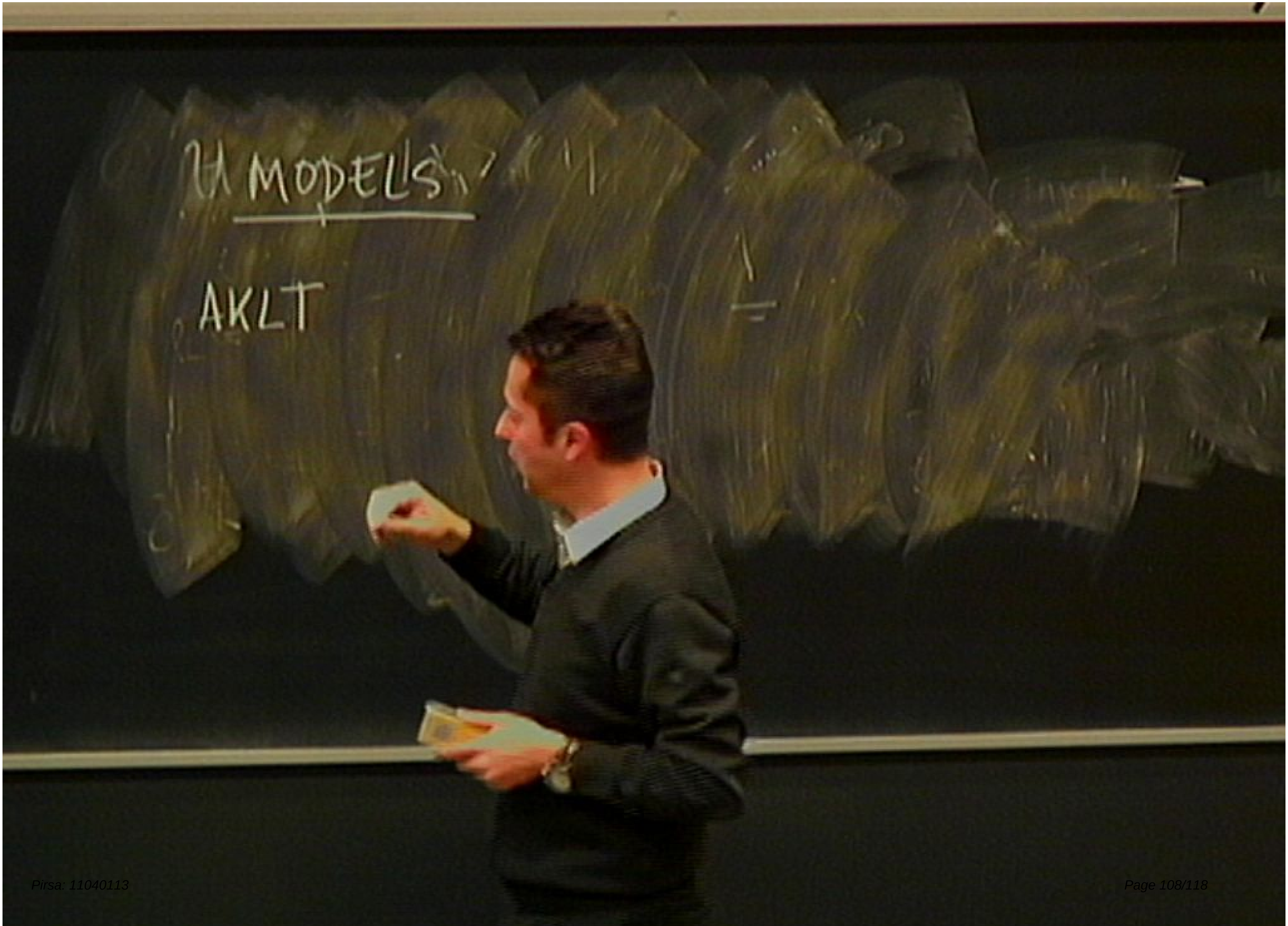
$$\sigma_L^2 = e^{-H'_m}$$

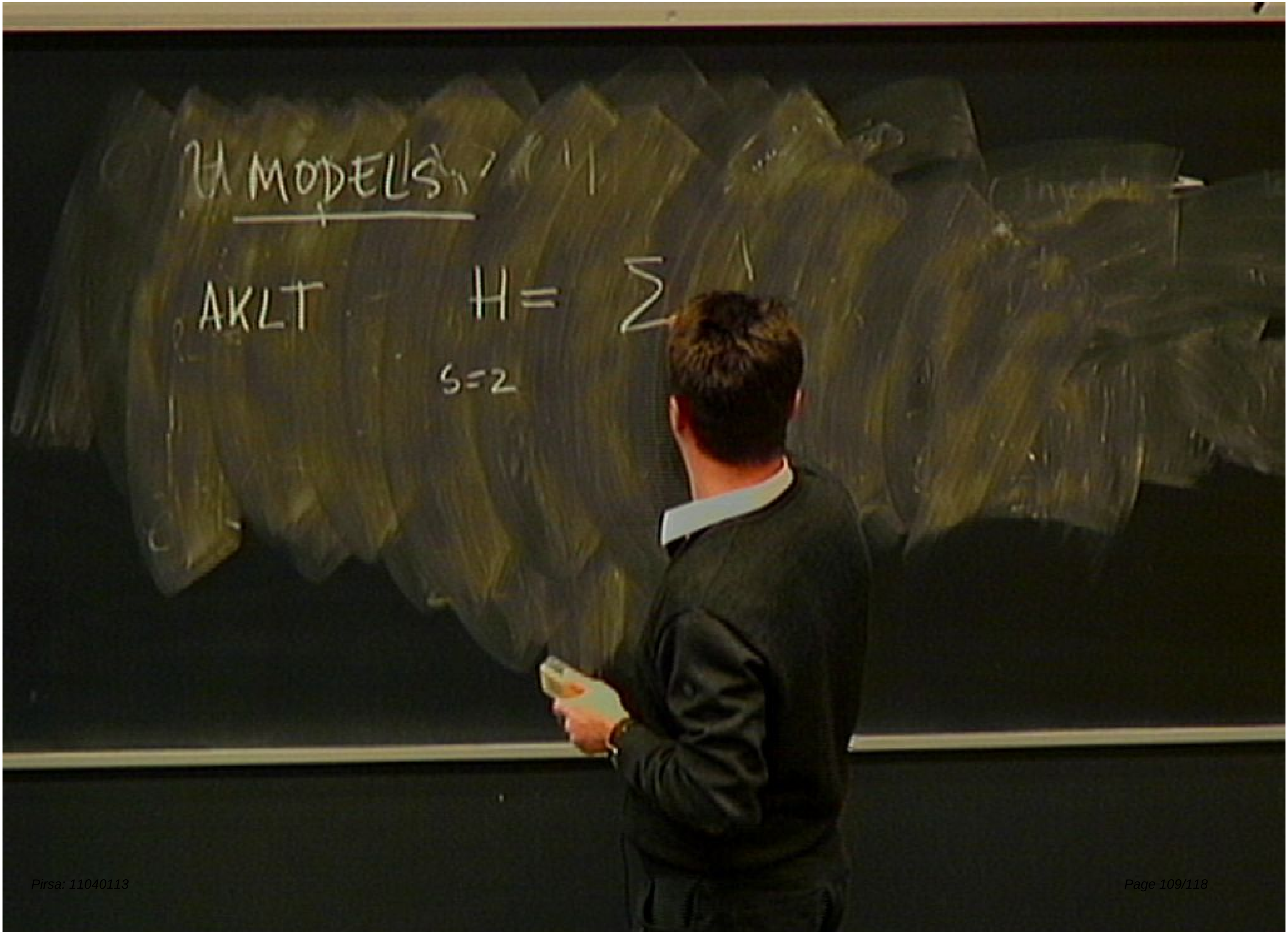
$$H = \mathcal{U} H' \mathcal{U}^\dagger$$

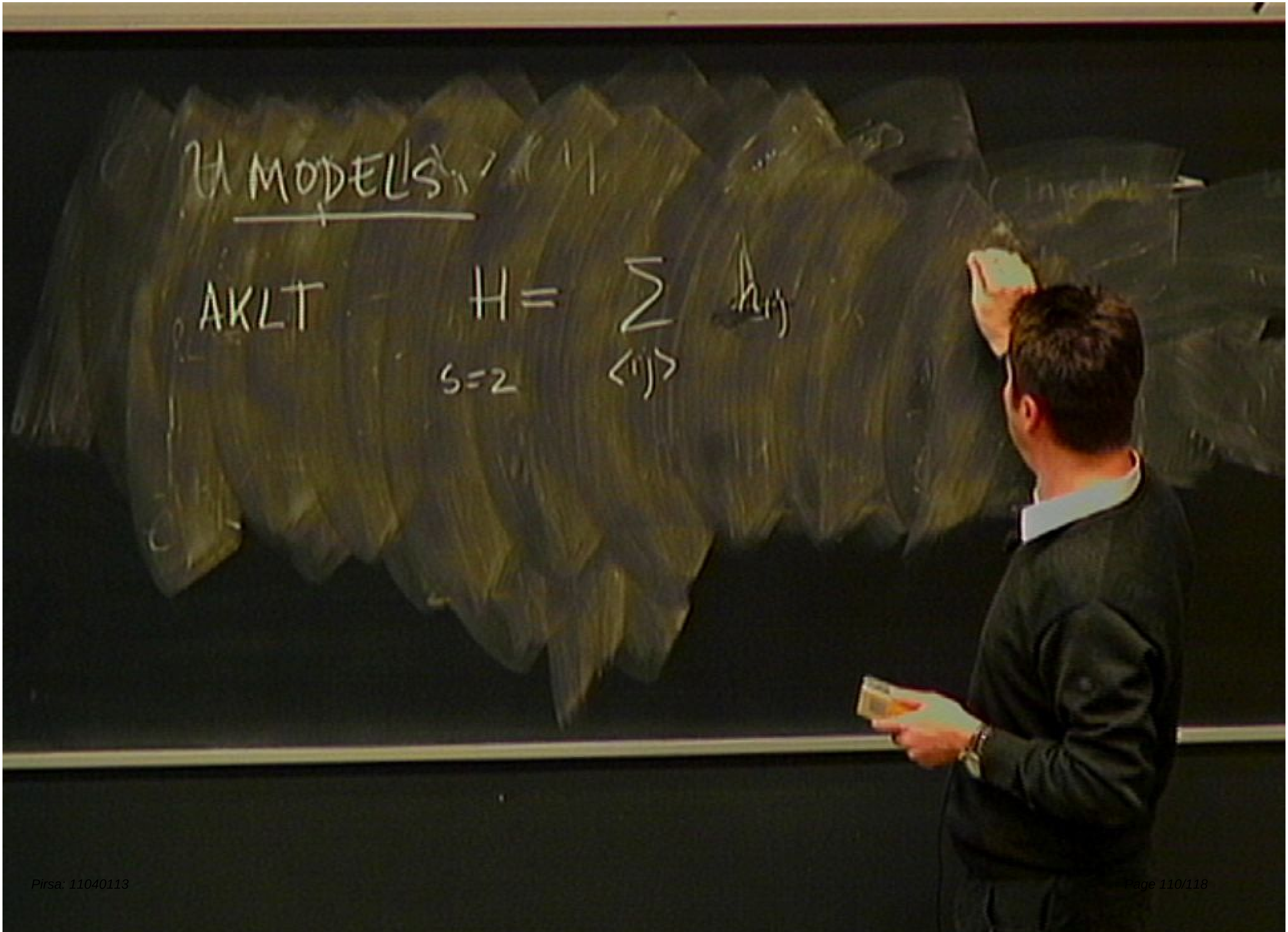

NUMERICS


NUMERICS
H' = 𝟙
d0
d1
d2
r


NUMERICS
H' = 𝟙
+ σ_x^1 1 ... 1 + σ_x^2 + σ_y^1 ...
+ σ^1 σ^2 1 ... 1 + σ^2 σ^3 ...
+ σ^1 σ^2 σ^3 1 ...
d_0
d_1
d_2
d_3
d_r
r


MODELS
AKLT


MODELS
AKLT
H = Σ
S=2


MODELS
AKLT
S=2
H = ∑ h_ij
<ij>

$D = 2$

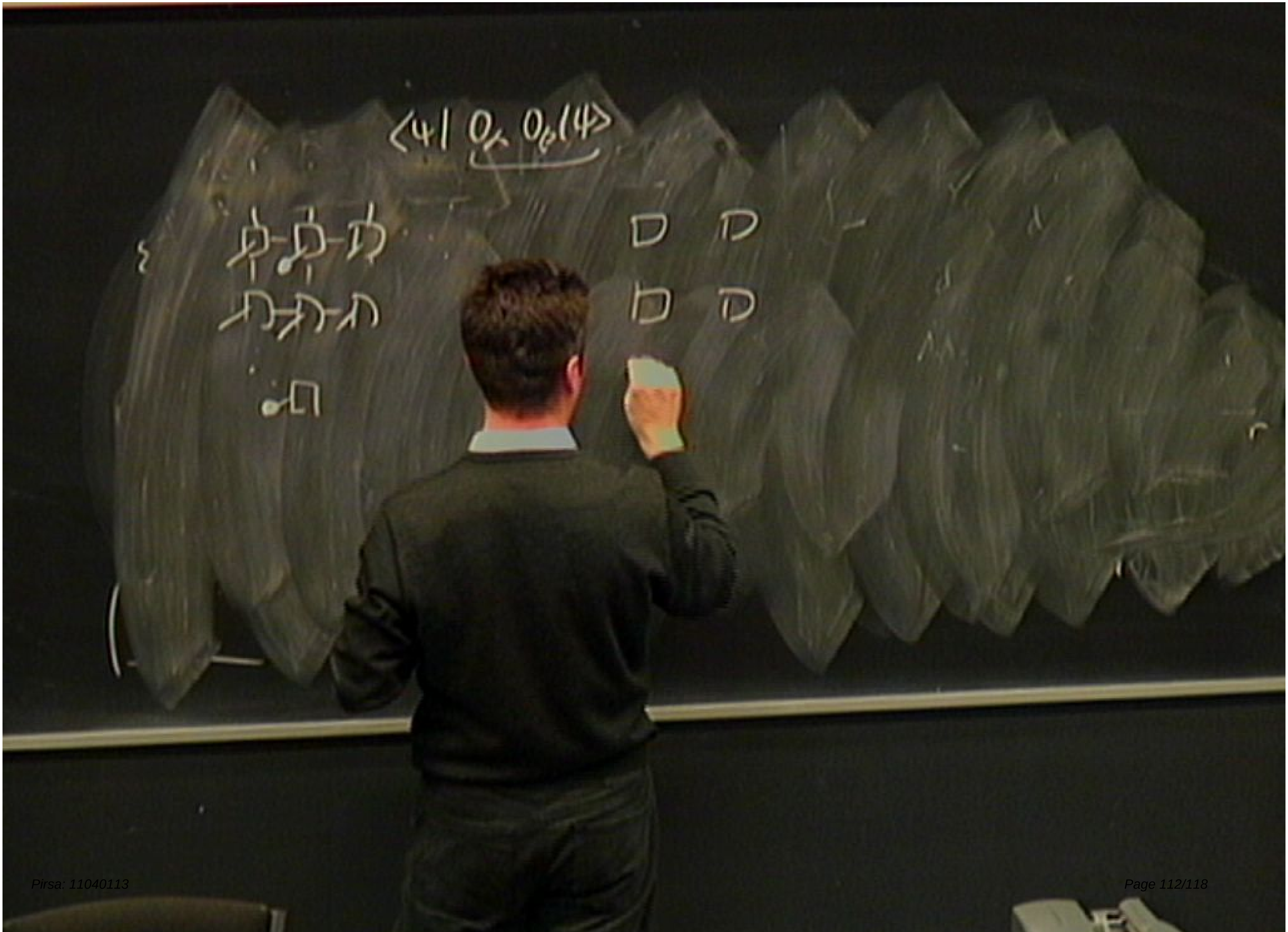
MODELS

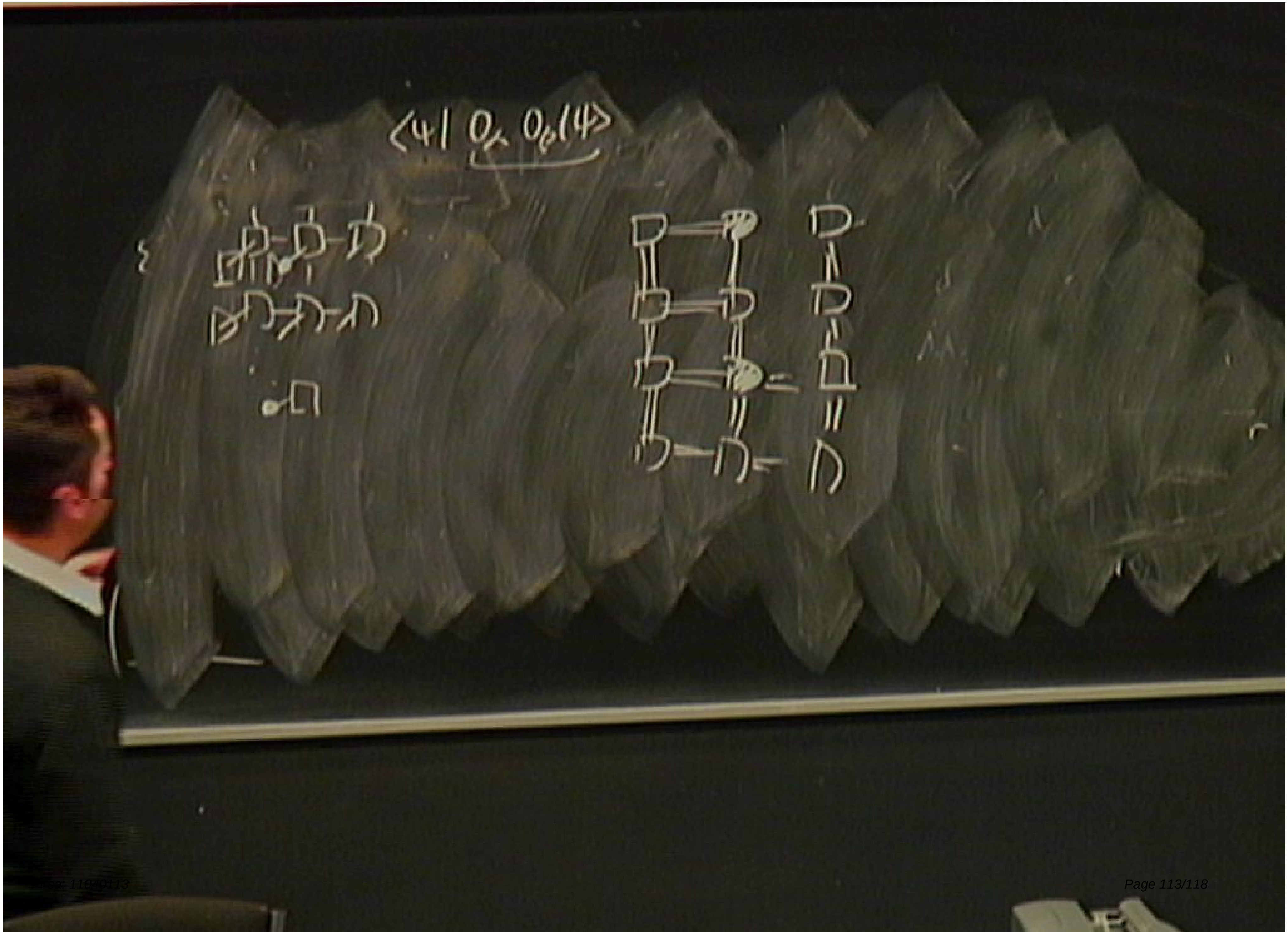
AKLT $\quad H(\theta) = \sum_{\langle ij \rangle} h_{ij}(\theta) \quad SU(2)$

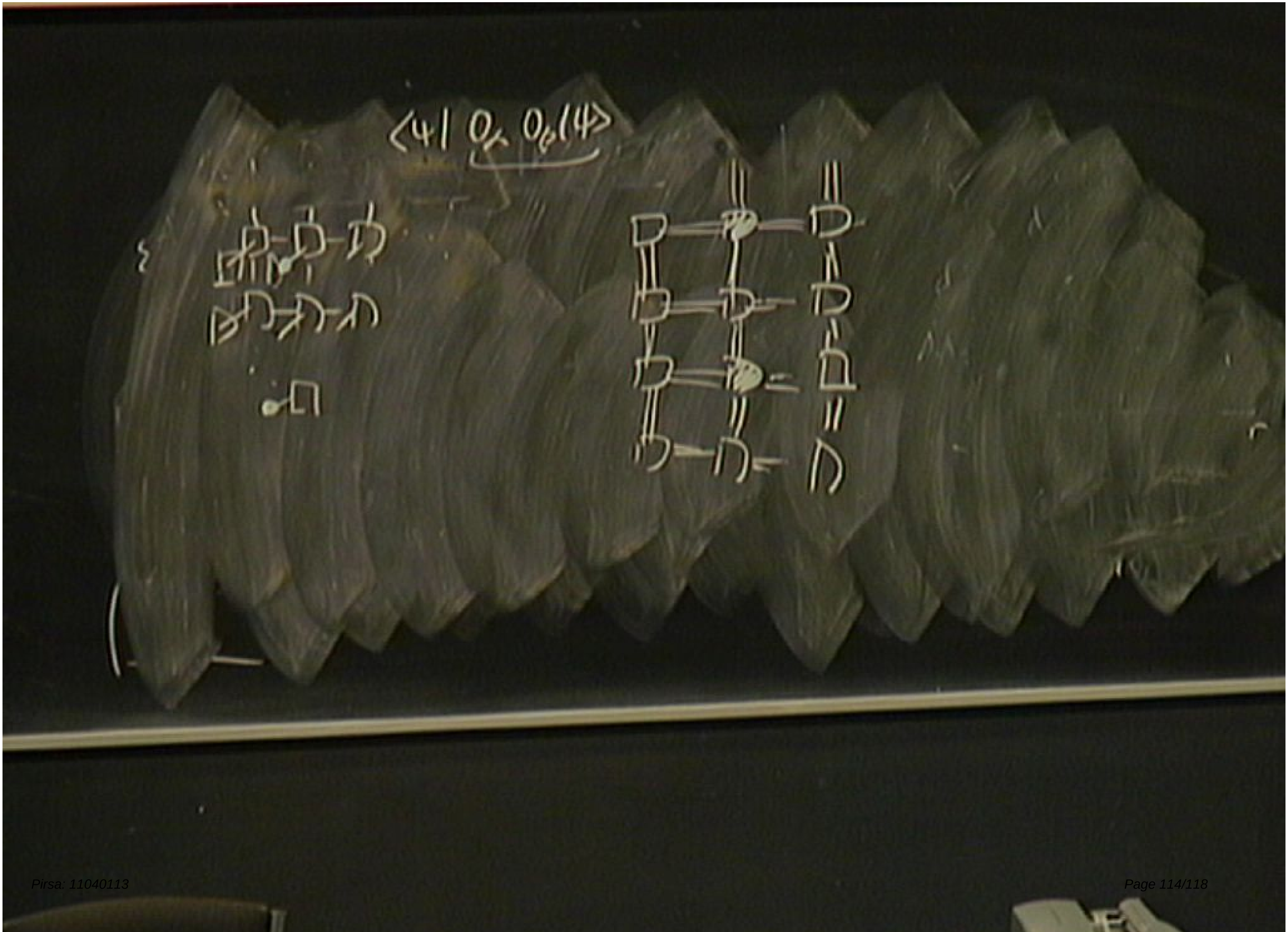
$S = 2$

ISING-LIKE: 5-body $\mathbb{Z}_2$

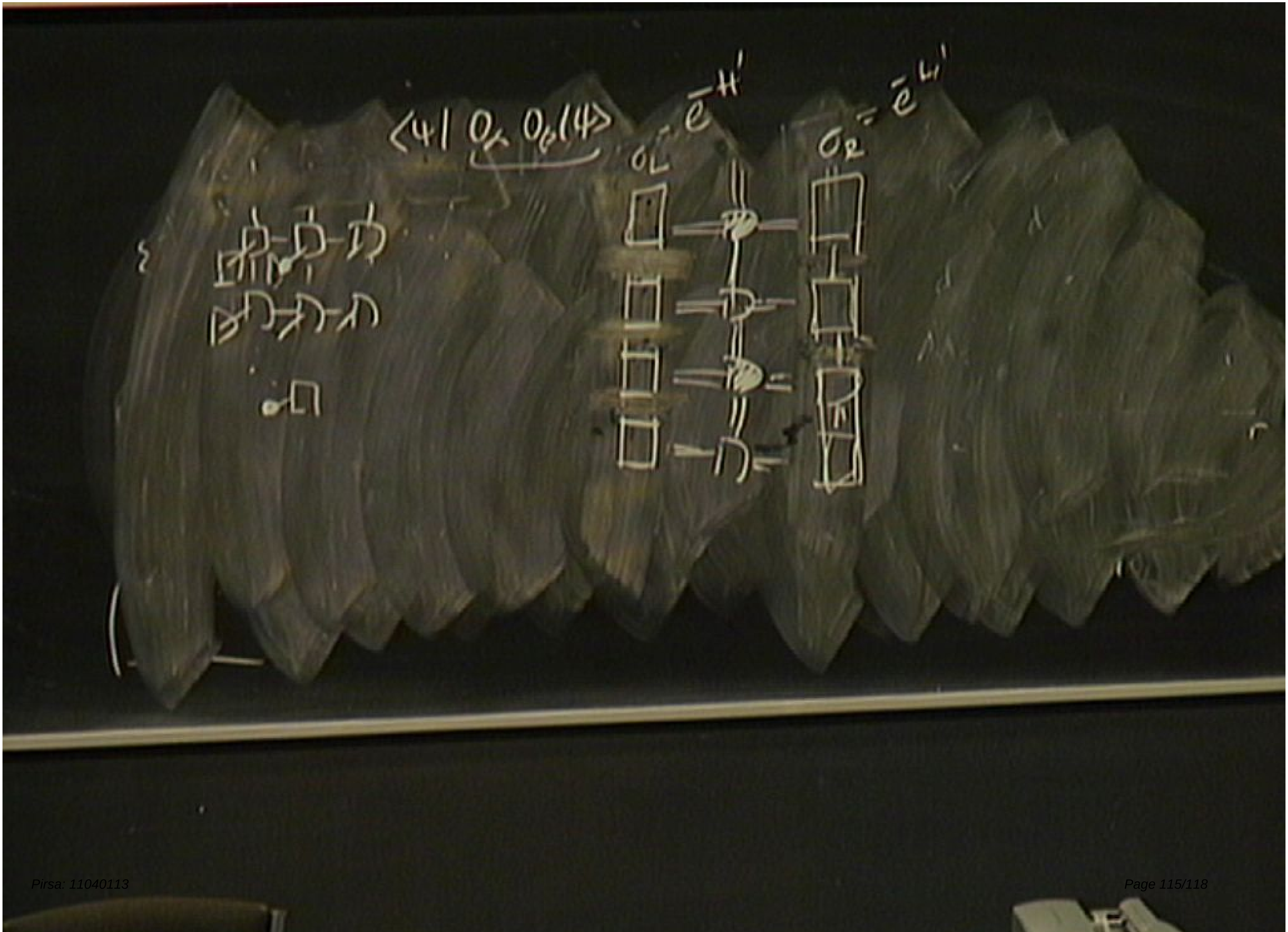
TORIC


⟨ψ| O_r O_p |ψ⟩

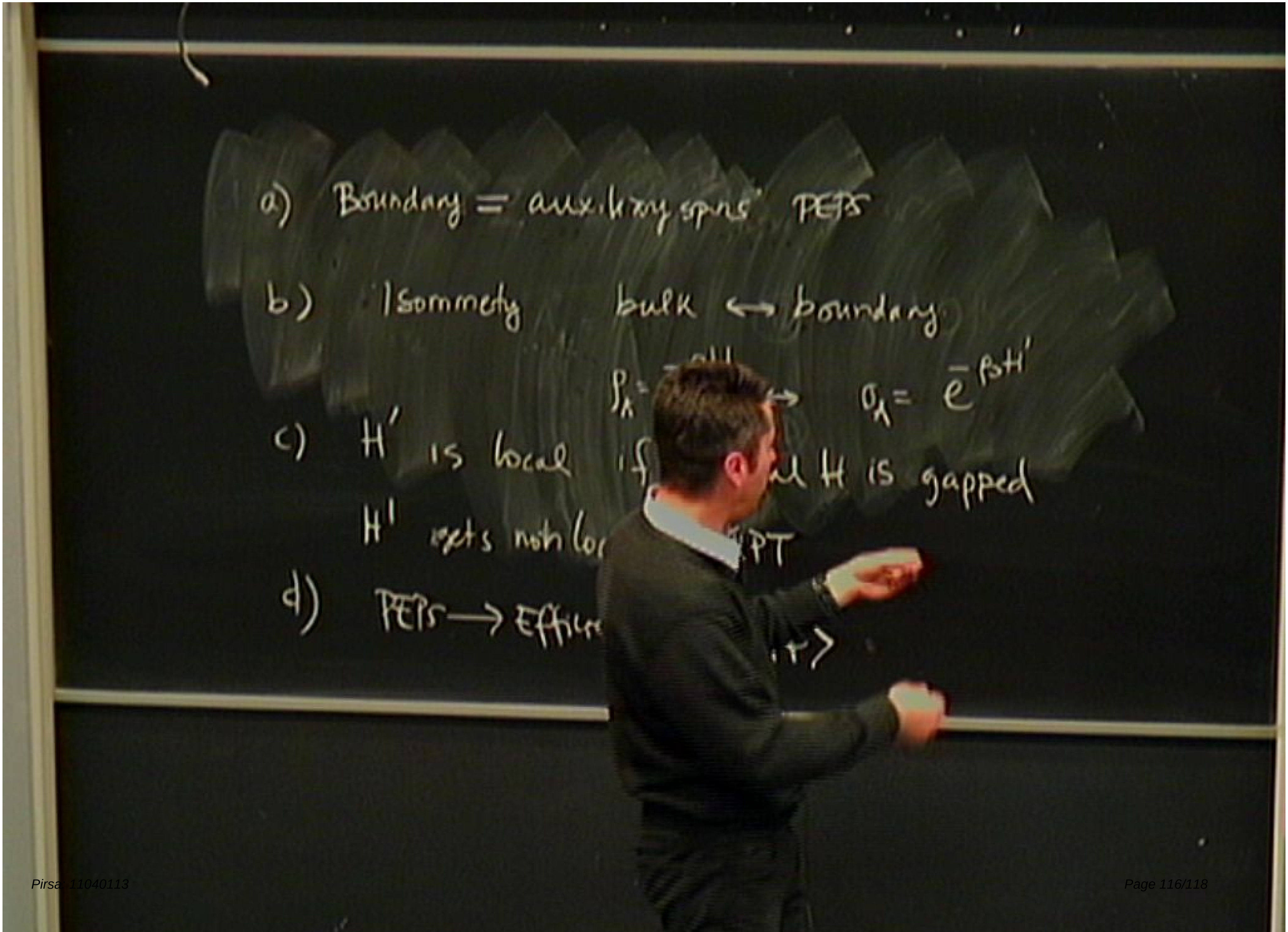

$\langle\psi| O_r \, O_b |\psi\rangle$

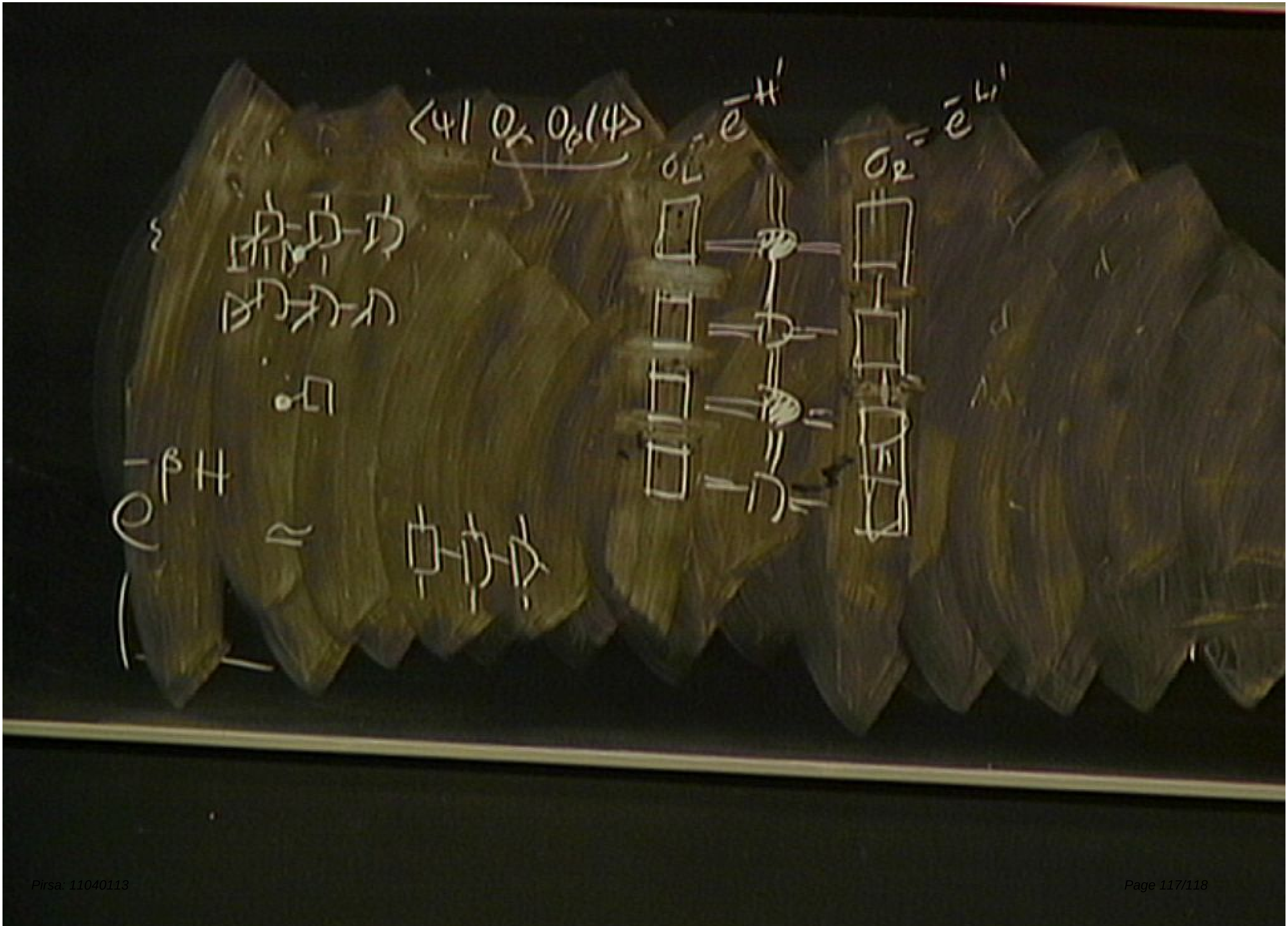


$\langle\psi| O_r O_b |\psi\rangle$



⟨ψ| O_r O_ℓ |ψ⟩
O_L = e^{-H'}
O_R = e^{-H'}


a) Boundary = auxiliary spins PEPS
b) Isommetry bulk ↔ boundary
$\sigma_A = e^{-\beta H'}$
c) H' is local if
H is gapped
H'
d) PEPS →


⟨ψ| O_L O_R |ψ⟩
σ_L = e^{-H'}
σ_R = e^{-H'}
e^{-βH}

MODELS

AKLT $\quad H(\theta) := \sum_{\langle ij \rangle} h_{ij}(\theta) \quad SU(2)$

$S=2$

$D=2$

$H \downarrow A^i \quad D \propto \mathrm{Poly}\, N$

ISING-LIKE: 5-body Z_2

TORIC