Title: Topics in Conformal Field Theory

Date: Apr 07, 2011 02:00 PM

URL: http://pirsa.org/11040110

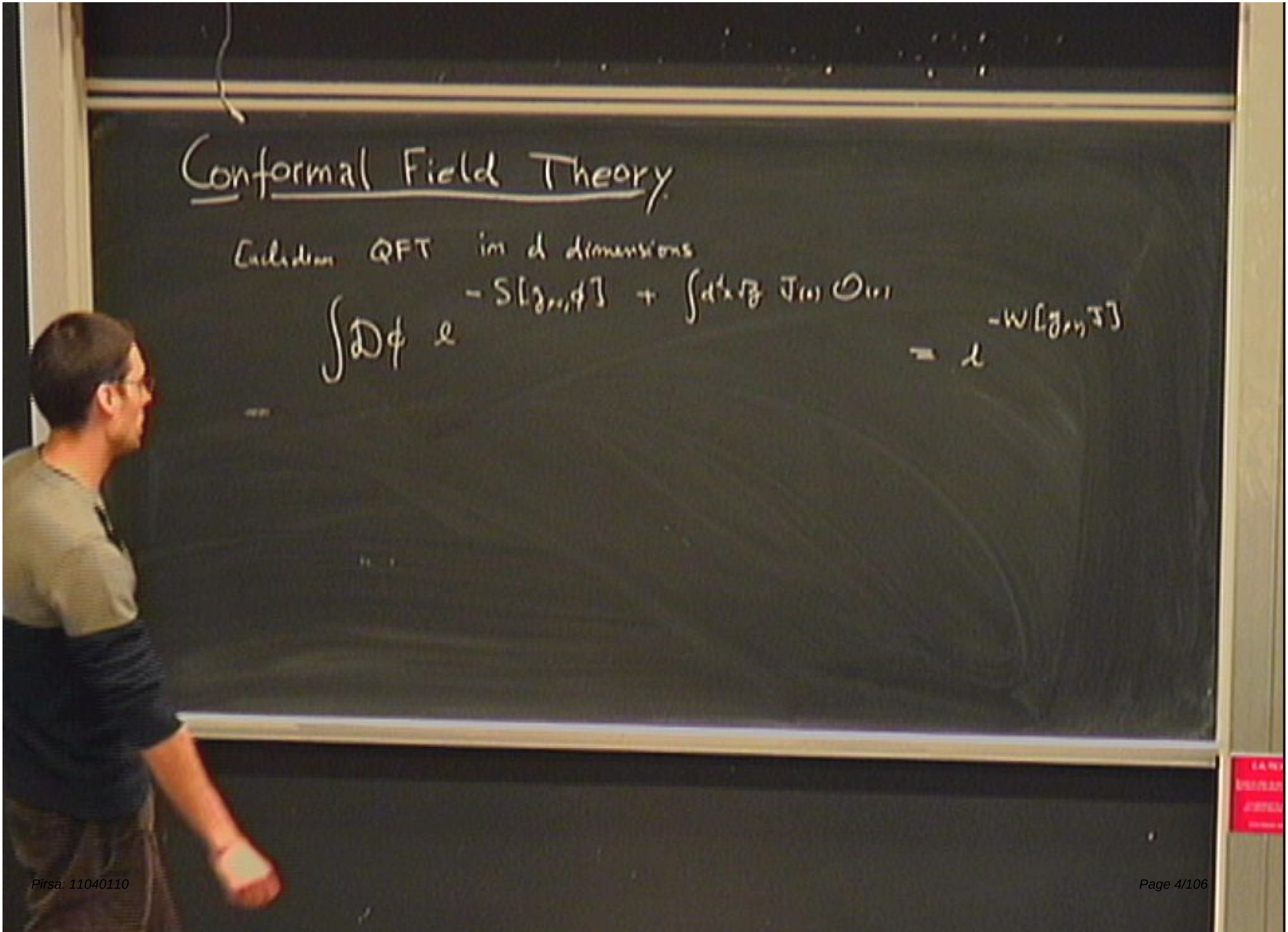
Abstract: I will discuss unitarity bounds on the conformal dimensions of local operators, the operator product expansion, conformal blocks and crossing symmetry

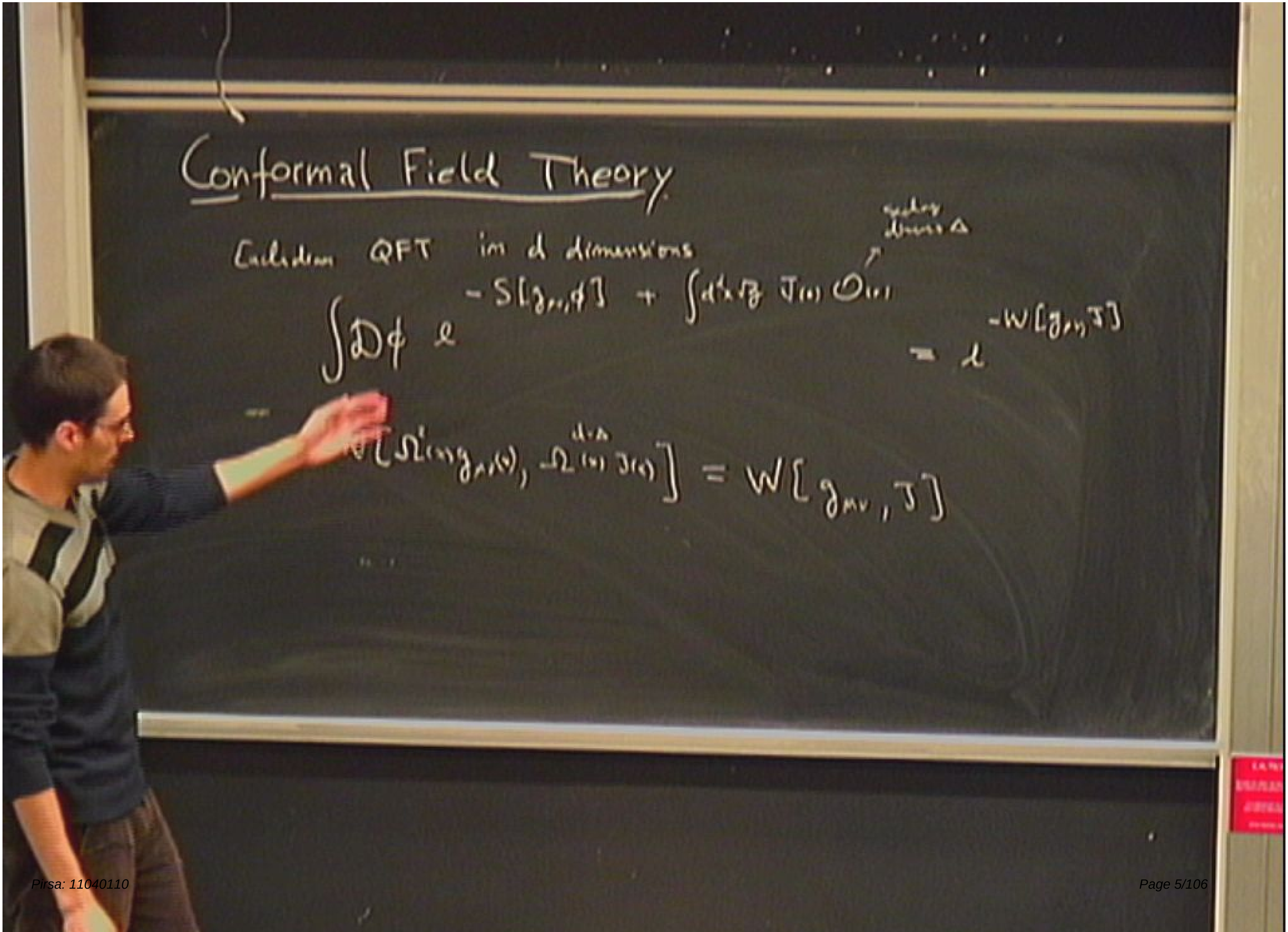

Conformal Field Theory

Conformal Field Theory

Euclidean QFT in d dimensions

$$\int \mathcal{D}\phi \; e^{-S[g_{\mu\nu},\phi] + \int d^dx \sqrt{g}\, J(x)\, \mathcal{O}(x)}$$


Conformal Field Theory
Euclidean QFT in d dimensions
$\int \mathcal{D}\phi \, e^{-S[g_{\mu\nu},\phi] + \int d^dx \sqrt{g}\, J(x)\, \mathcal{O}(x)} = e^{-W[g_{\mu\nu},J]}$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Conformal Field Theory

Euclidean QFT in d dimensions

scaling dimension Δ

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$$\langle \mathcal{O}(x) \rangle = -\frac{1}{\sqrt{g}} \frac{\delta W}{\delta J(x)}\bigg|_{J=0} \qquad \langle T_{\mu\nu} \rangle = -\frac{2}{\sqrt{g}} \frac{\delta W}{\delta g^{\mu\nu}}$$

Conformal Field Theory

Euclidean QFT in d dimensions

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Conformal Field Theory

Euclidean QFT in d dimensions

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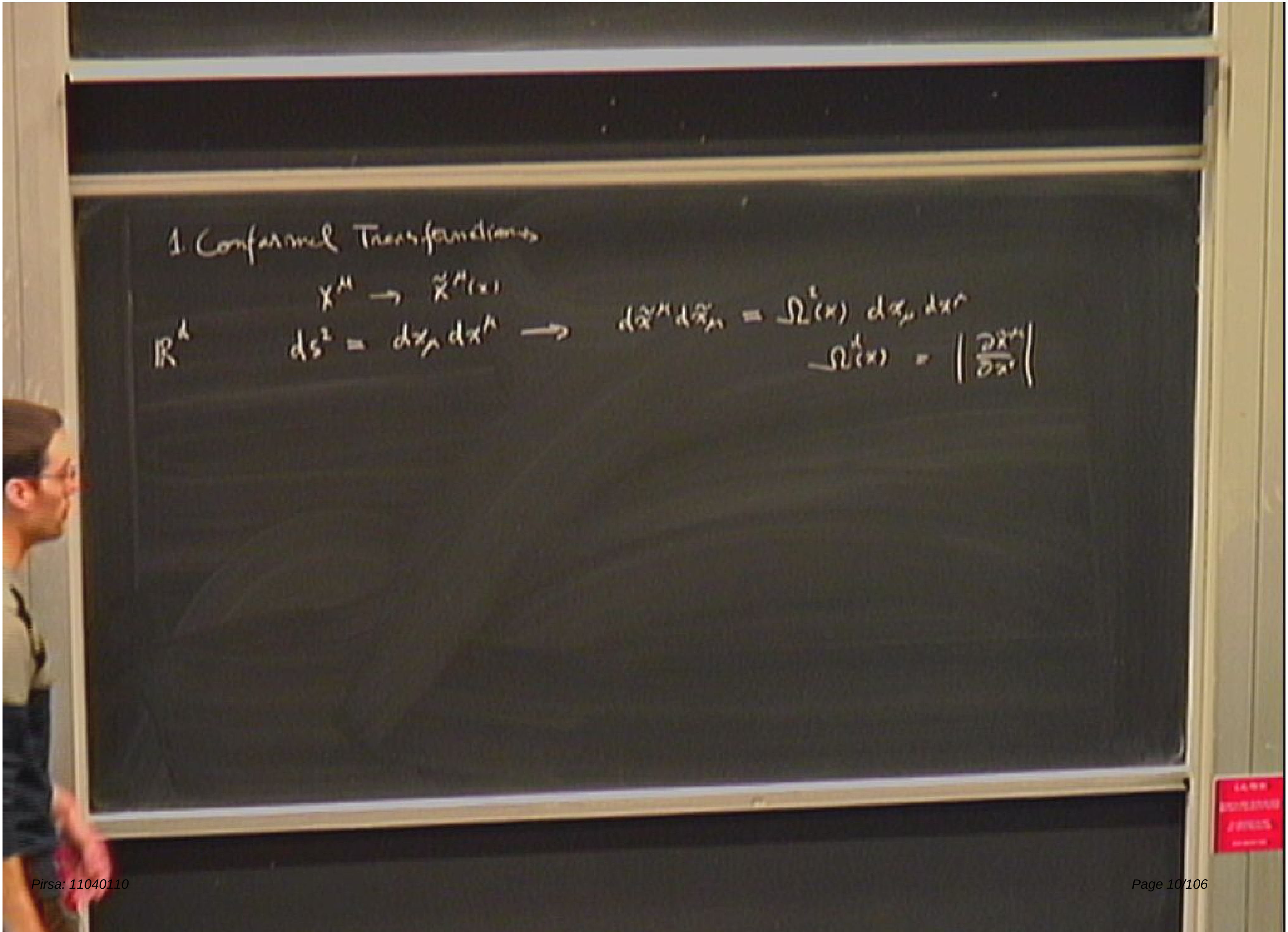
1. Conformal Transformations
$x^\mu \to \tilde{x}^\mu(x)$
$\mathbb{R}^d$ $ds^2 = dx_\mu dx^\mu \longrightarrow d\tilde{x}^\mu d\tilde{x}_\mu = \Omega^2(x)\, dx_\mu dx^\mu$
$\Omega^d(x) = \left| \frac{\partial \tilde{x}^\mu}{\partial x^\nu} \right|$

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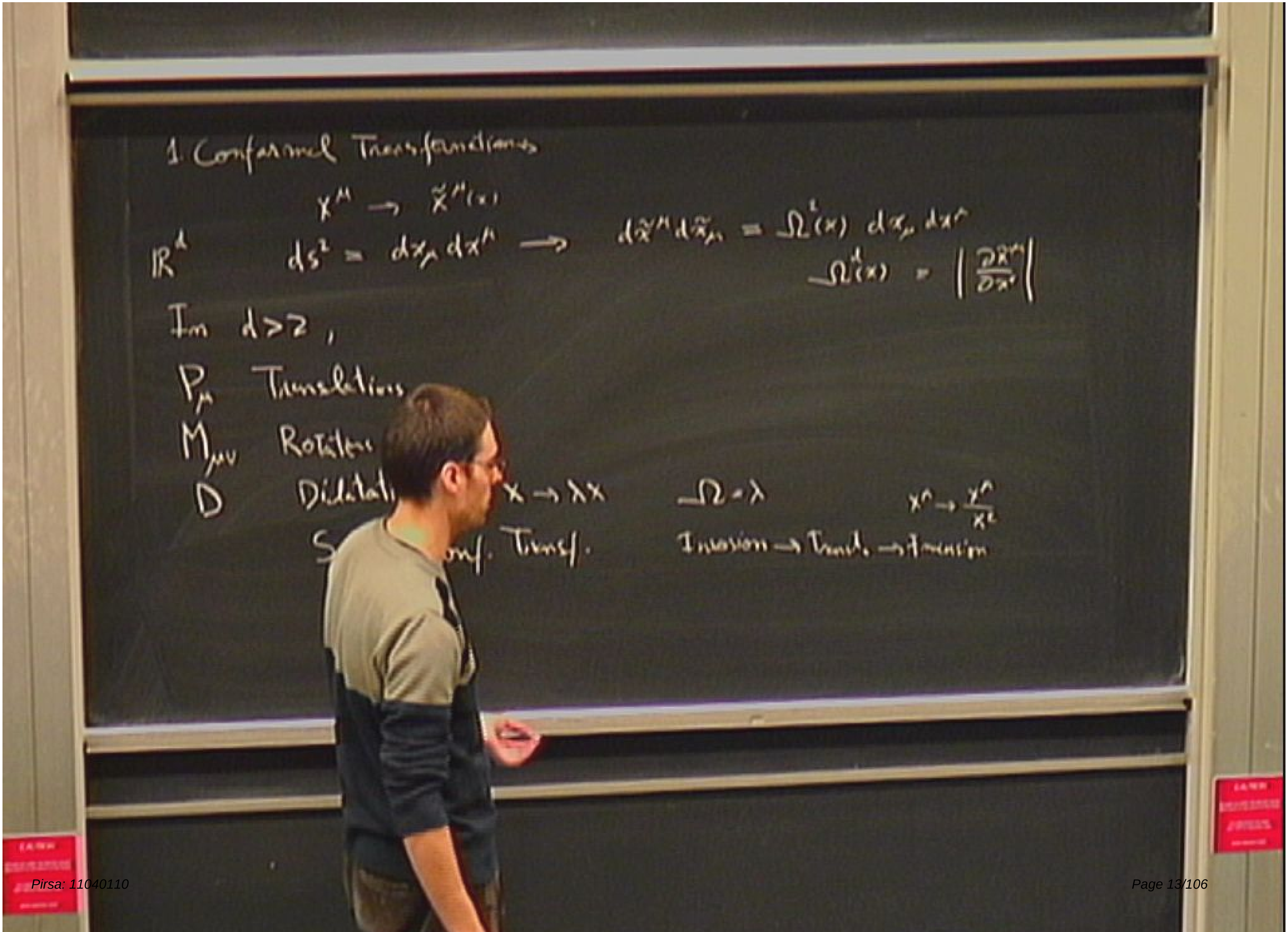
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P_μ Translations
$M_{\mu\nu}$ Rotatio
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1 Conformal Transformations

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In $d>2$,

P_μ Translations

$M_{\mu\nu}$ Rotations $SO(d)$

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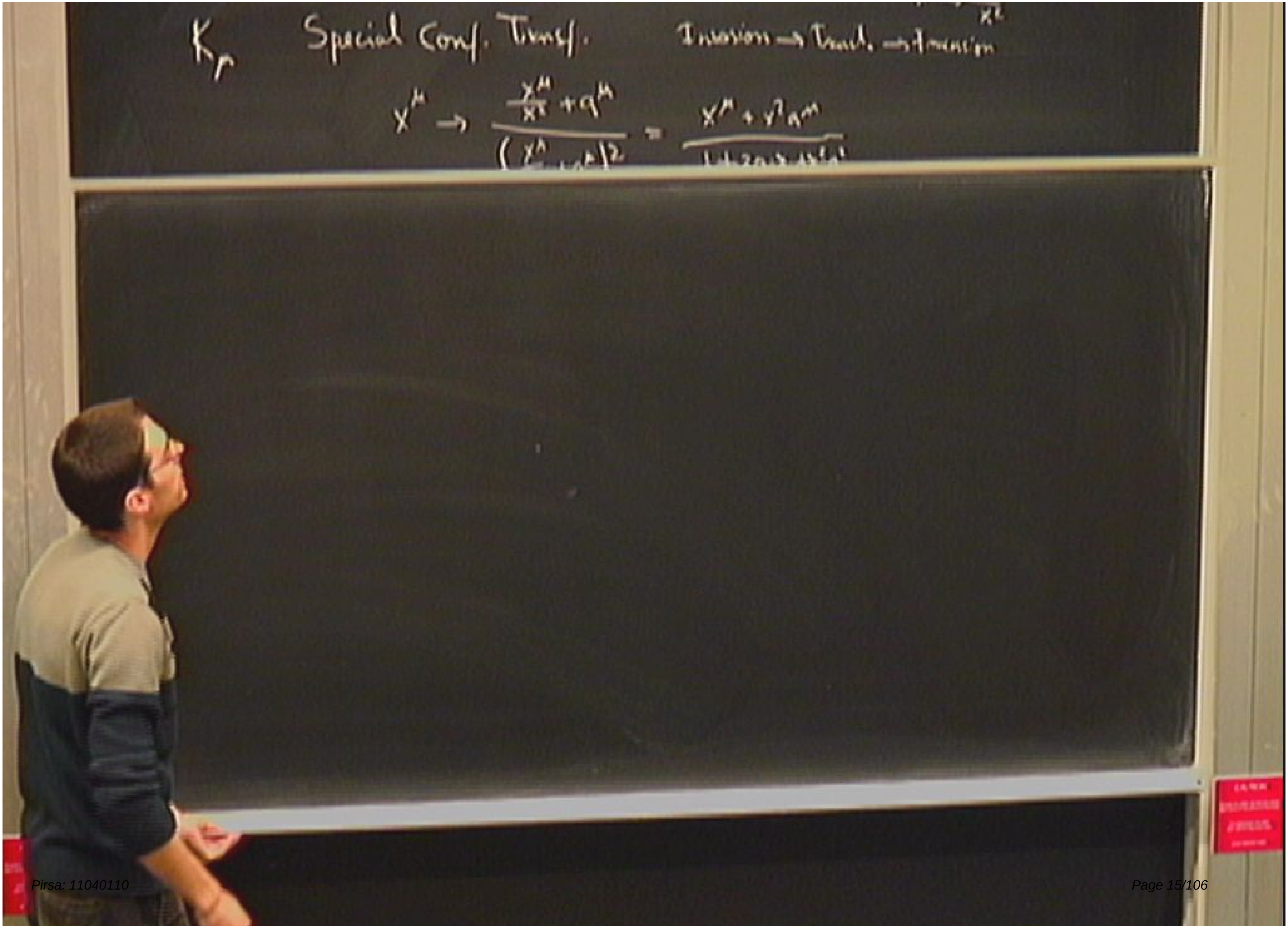
K_μ Special Conf. Transf. $\quad$ Inversion $\to$ Transl. $\to$ Inversion

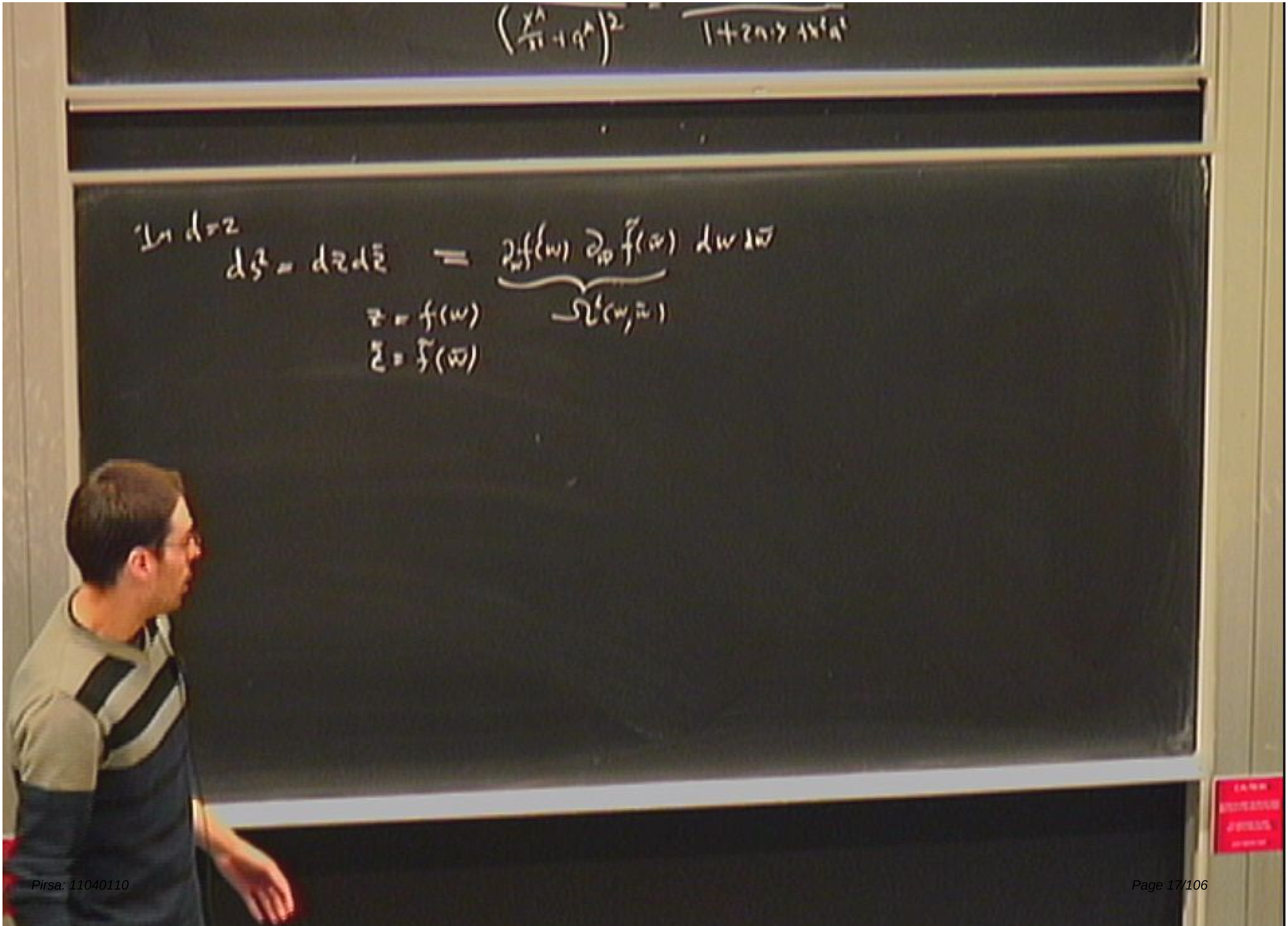
$$x^\mu \to \frac{\frac{x^\mu}{x^2}+a^\mu}{\left(\frac{x^\mu}{x^2}+a^\mu\right)^2} = \frac{x^\mu + x^2 a^\mu}{1+2a\cdot x + x^2 a^2}$$


K_μ
Special Conf. Transf.
Inversion → Transl. → Inversion
$x^\mu \to \frac{\frac{x^\mu}{x^2} + a^\mu}{(\ldots)^2} = \frac{x^\mu + x^2 a^\mu}{\ldots}$

In $d=2$

$$ds^2 = dz\,d\bar{z} = \underbrace{\partial_w f(w)\,\partial_{\bar{w}}\bar{f}(\bar{w})}_{\Omega^2(w,\bar{w})}\,dw\,d\bar{w}$$

$$z = f(w)$$
$$\bar{z} = \bar{f}(\bar{w})$$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Conformal Field Theory

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2. Conformal Algebra

$$[M_{\mu\nu}, P_\rho] = -i(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu) \qquad [M_{\mu\nu}, D] = 0$$

$$[M_{\mu\nu}, K_\rho] = -i(\eta_{\mu\rho}K_\nu - \eta_{\nu\rho}K_\mu)$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = -i(\eta_{\mu\rho}M_{\nu\sigma} + \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\nu\rho}M_{\mu\sigma} - \eta_{\mu\sigma}M_{\nu\rho})$$

$$[D, K_\mu] = i K_\mu \qquad [D, P_\mu] = -i P_\mu$$

$$[K_\mu, P_\nu] = 2i(\eta_{\mu\nu}D + M_{\mu\nu})$$

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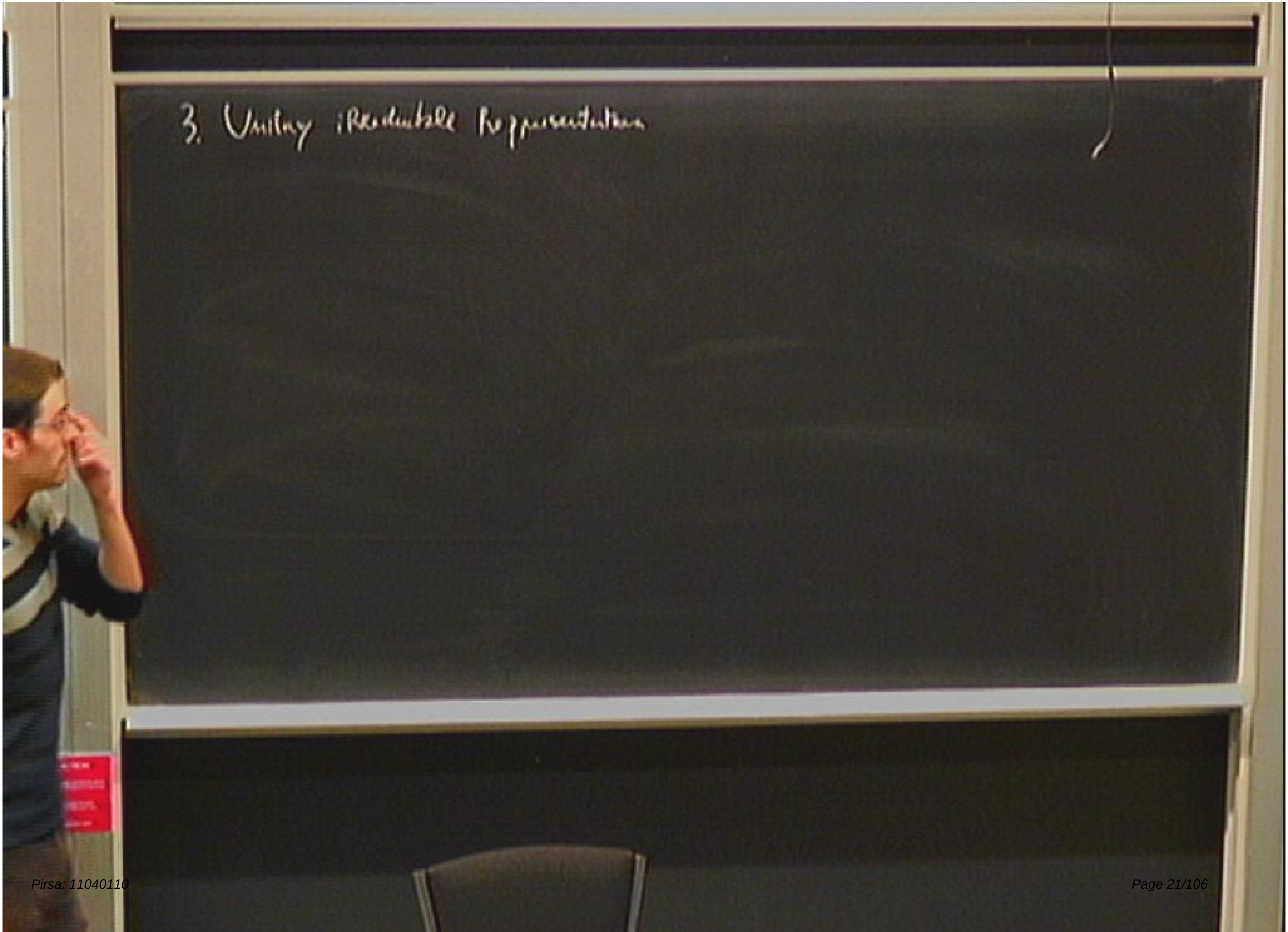
3. Unitary irreducible Representation

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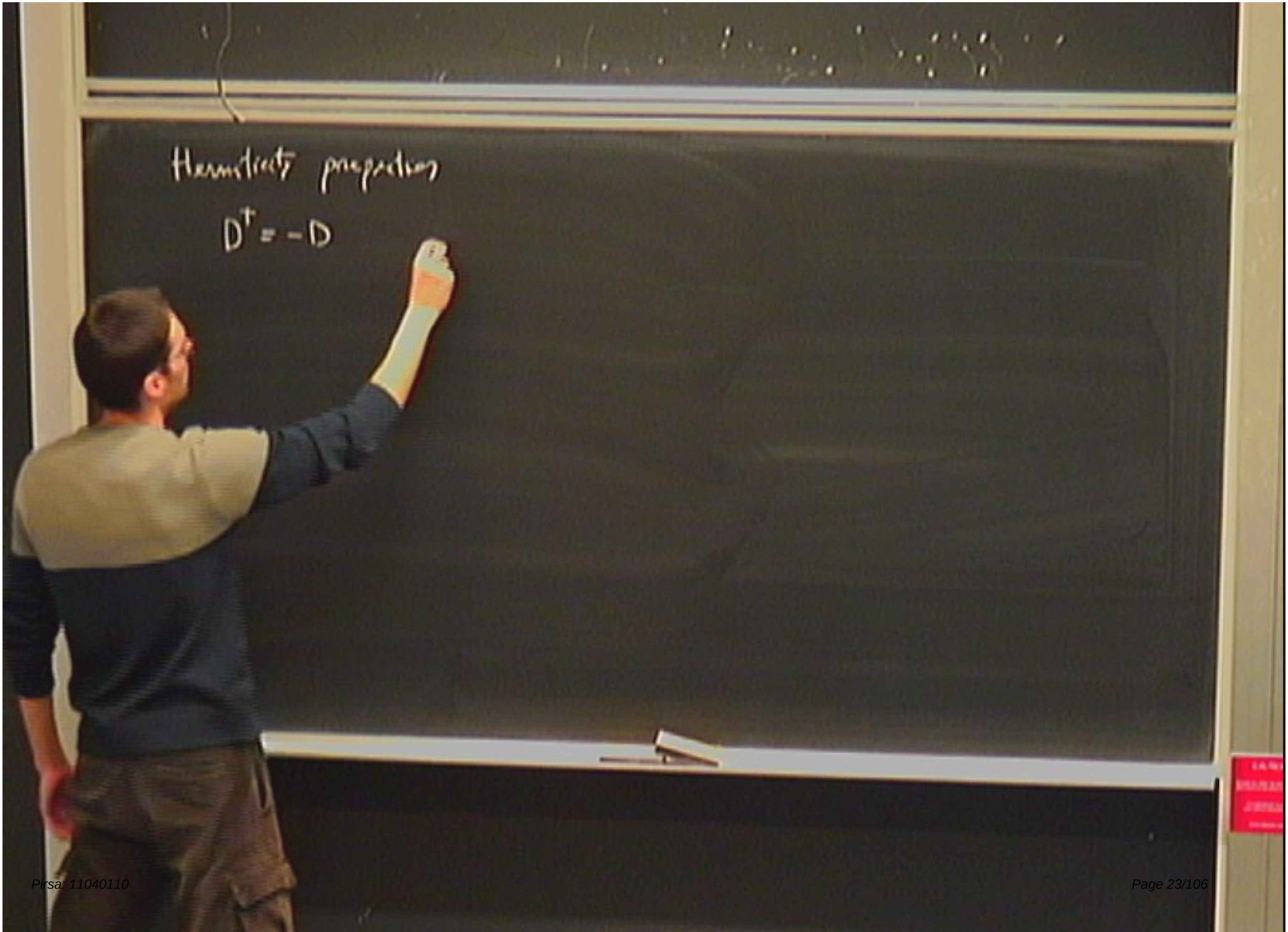
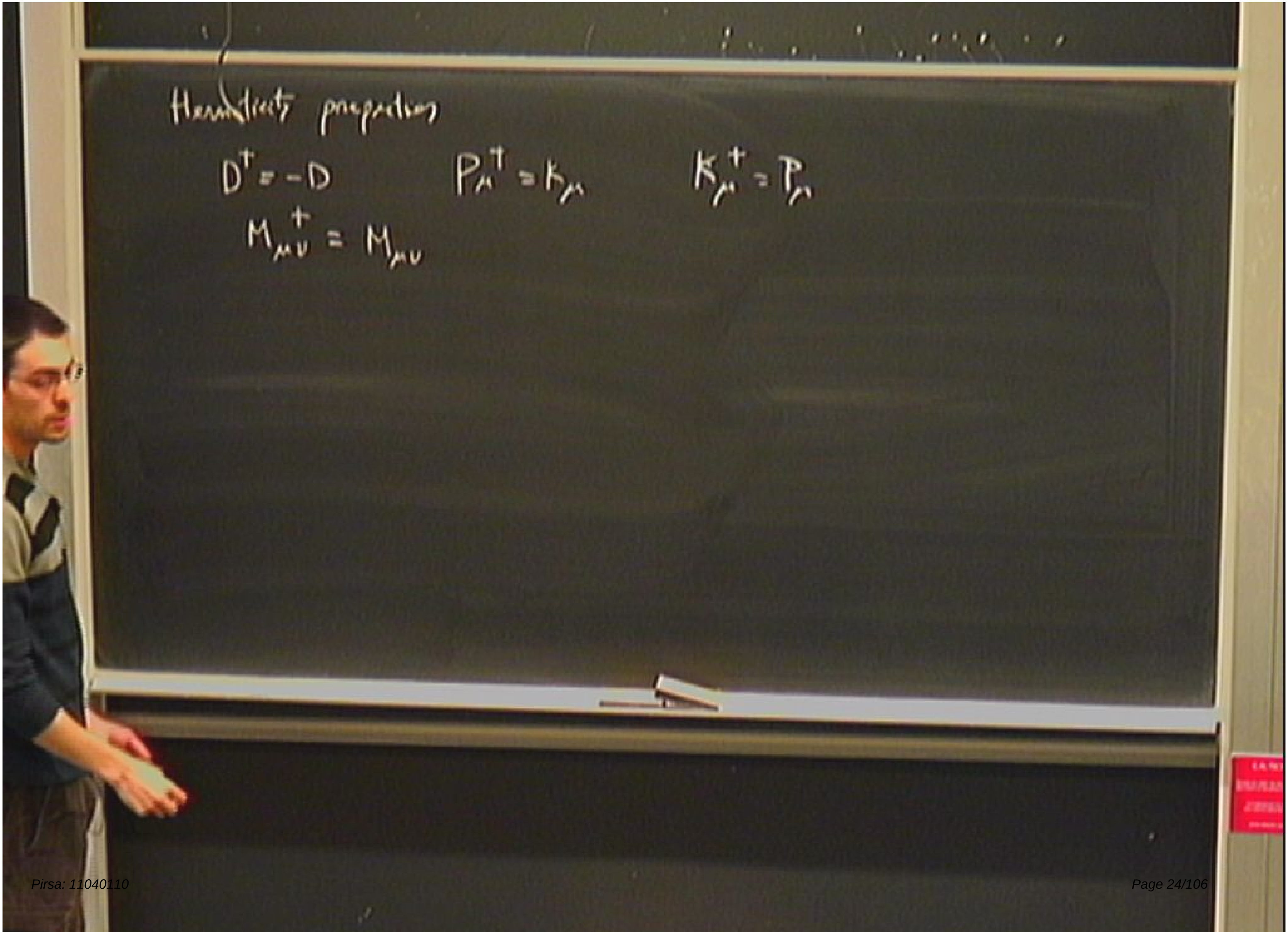
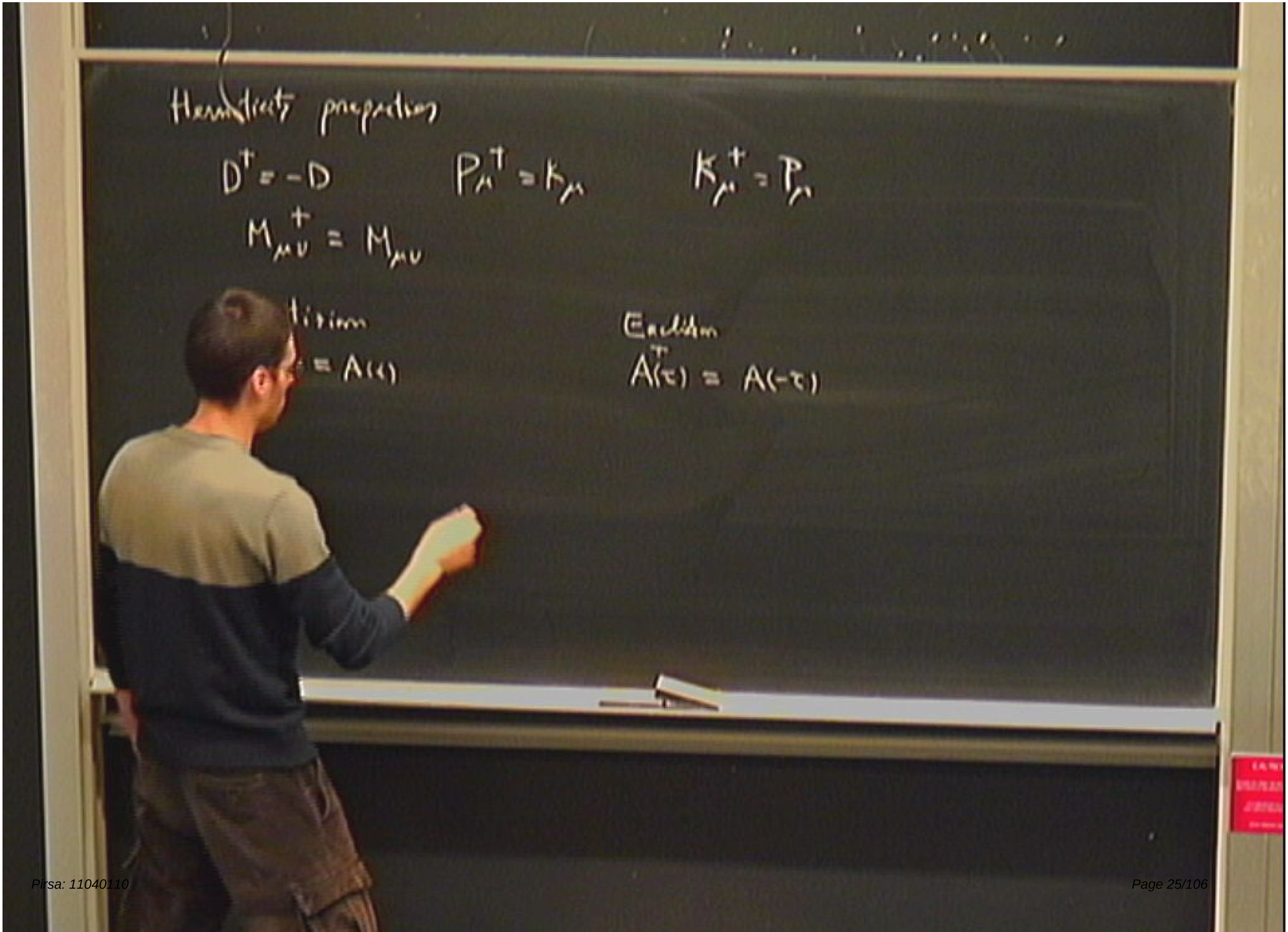
highest weight state – primary

$$D|O\rangle = -i\Delta$$


Hermiticity projection
$D^{\dagger} = -D$


Hermiticity properties
$D^\dagger = -D$
$P_\mu^\dagger = K_\mu$
$K_\mu^\dagger = P_\mu$
$M_{\mu\nu}^\dagger = M_{\mu\nu}$


$D^\dagger = -D$
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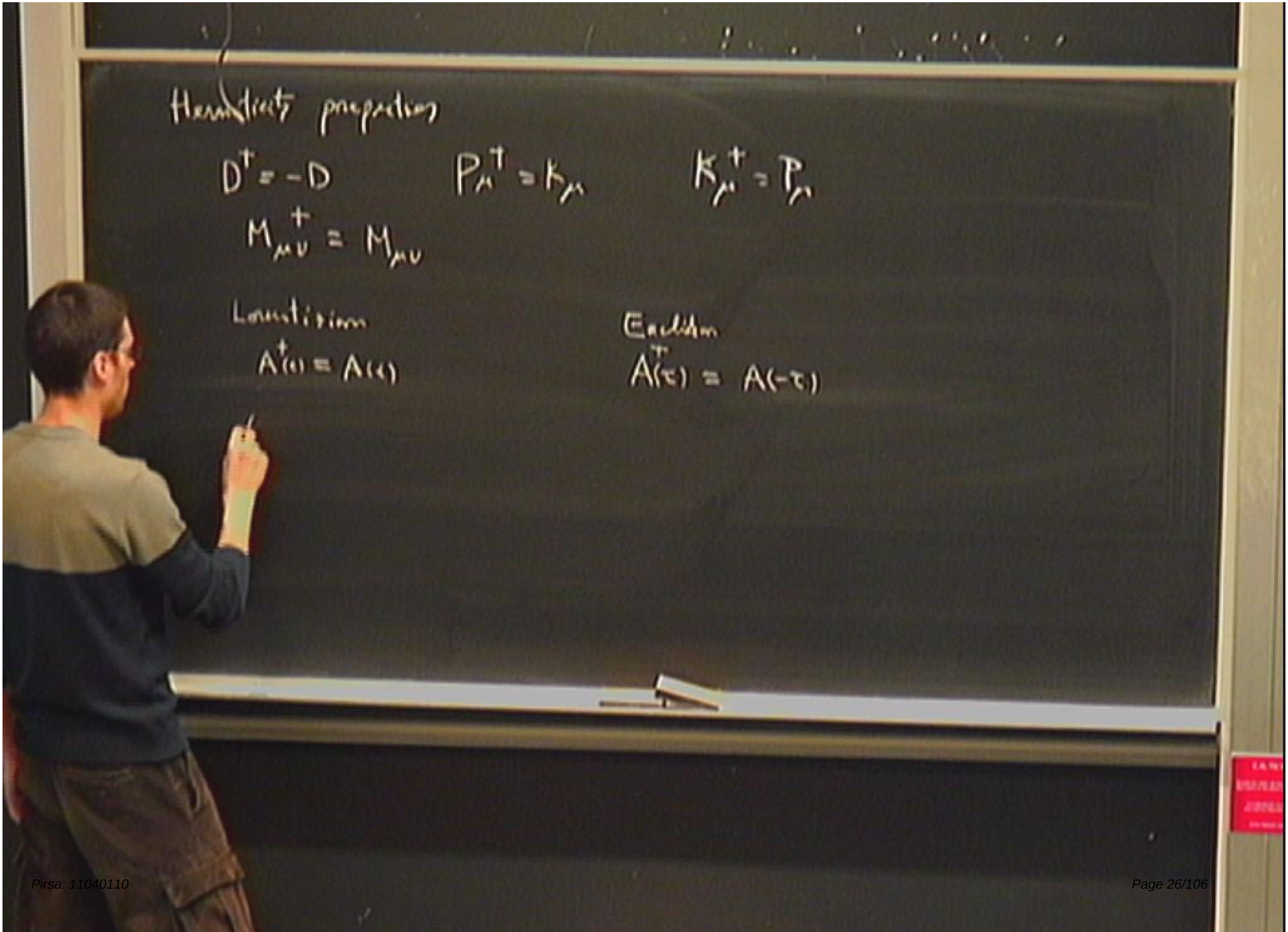
Euclidean
$A^\dagger(\tau) = A(-\tau)$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Lorentzian
$A^\dagger(t) = A(t)$
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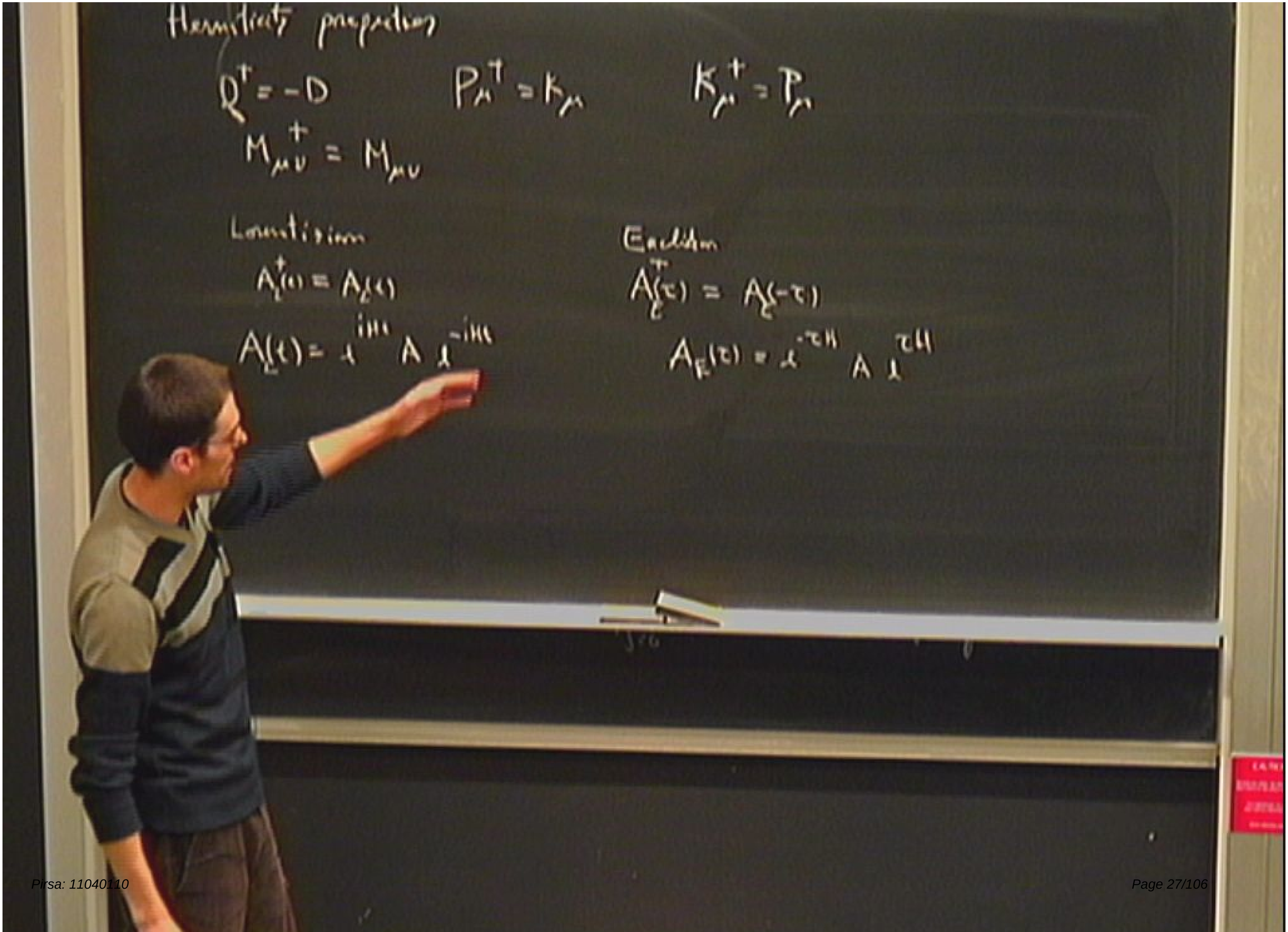
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$A_L(t) = e^{iHt} A e^{-iHt}$
Euclidean
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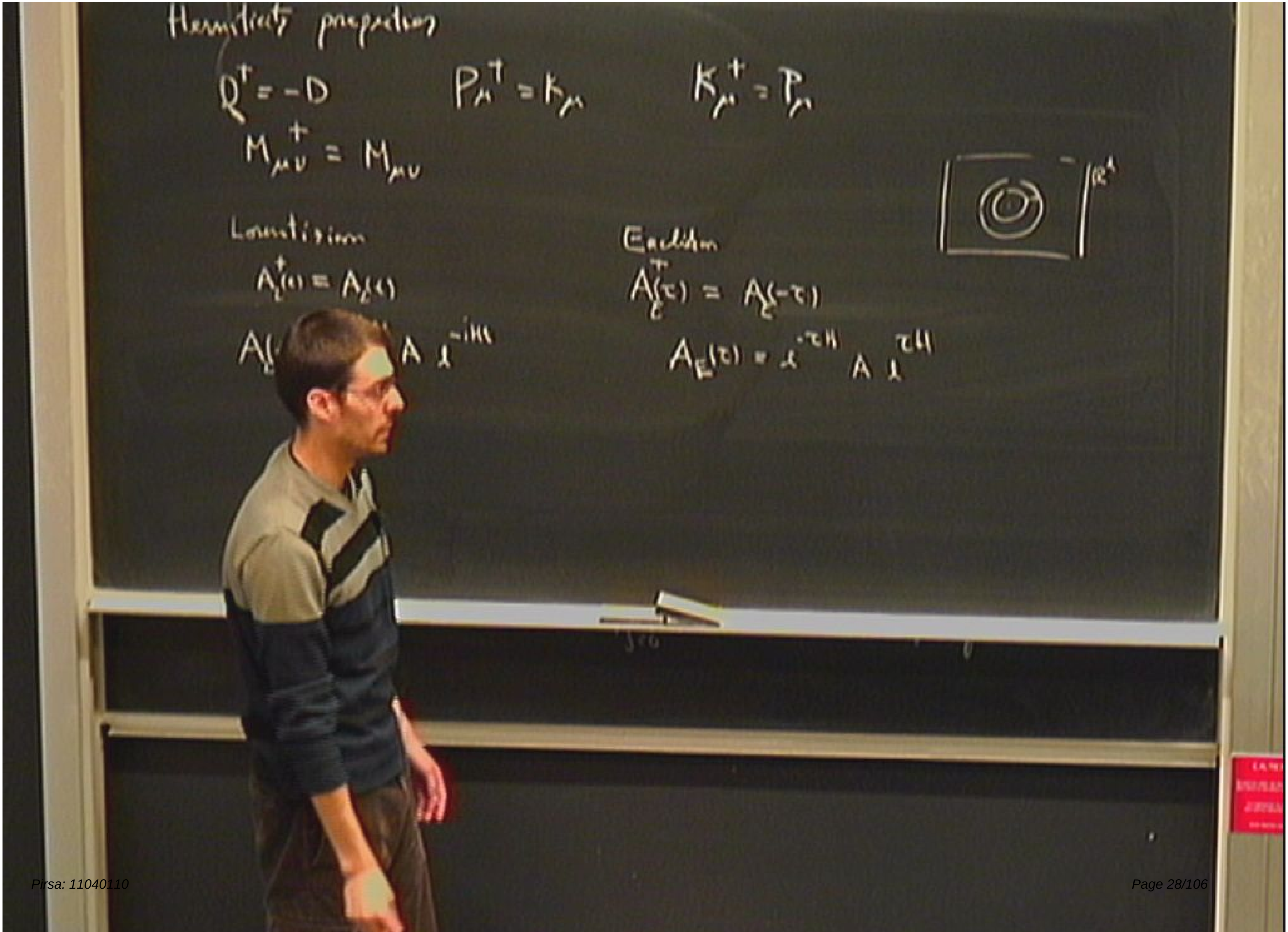
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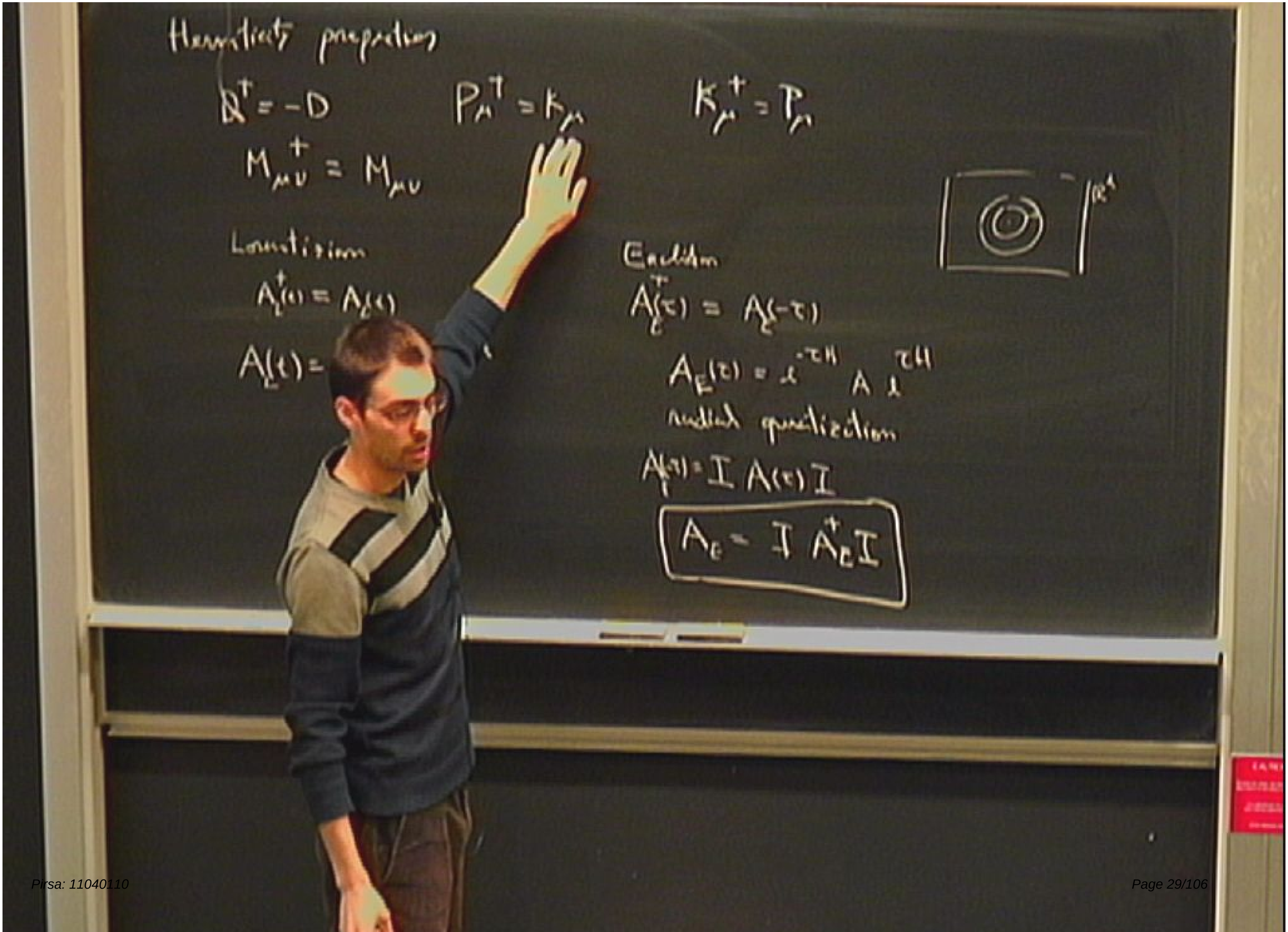
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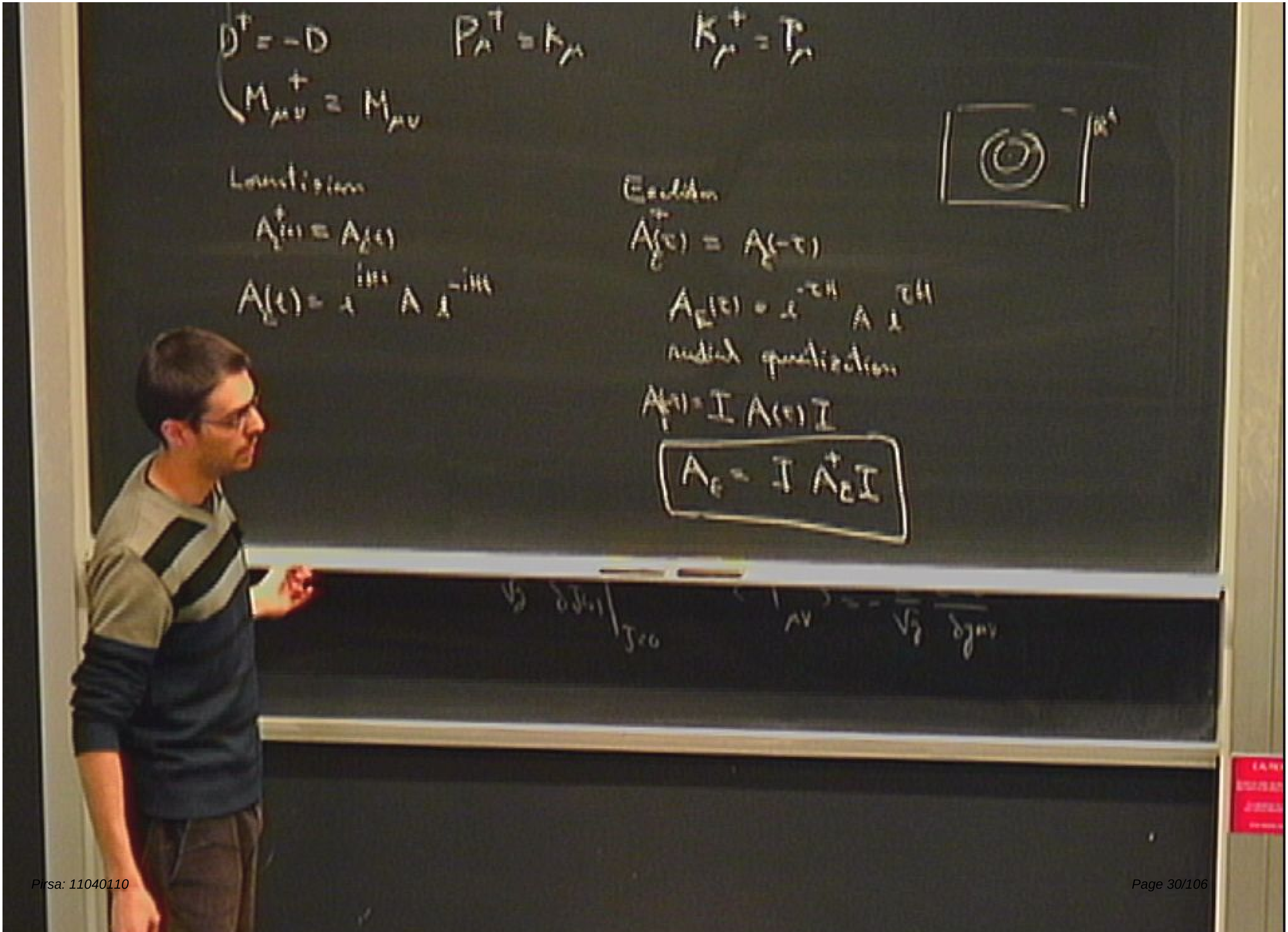
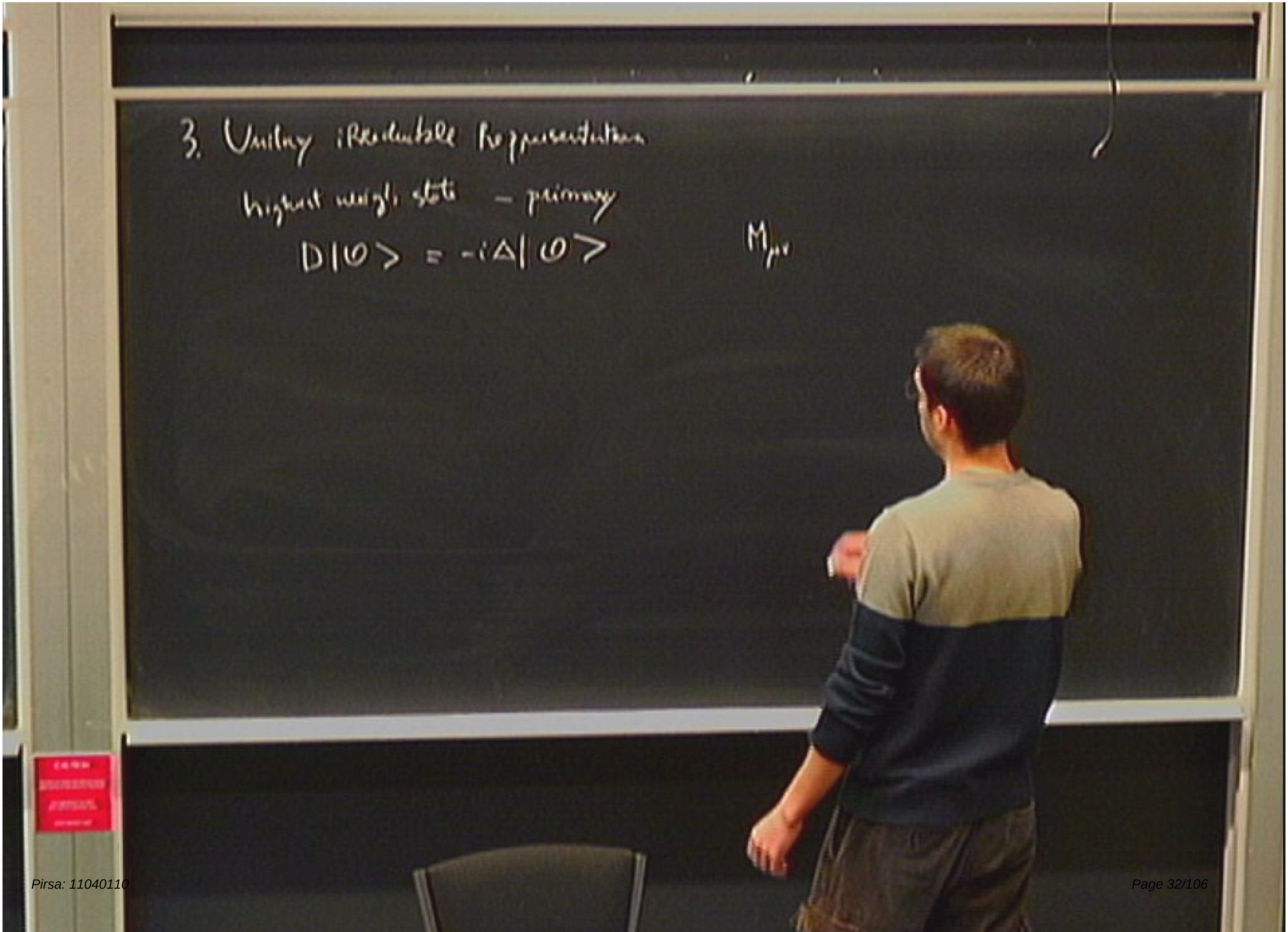
$A_E(\tau) = e^{-\tau H} A e^{\tau H}$
radial quantization
$A(\tau) = I A(\tau) I$
$\boxed{A_E = I A_E^\dagger I}$
$\mathbb{R}^d$


D† = -D
P_μ† = K_μ
K_μ† = P_μ
M_μν† = M_μν
R^d
Lorentzian
A_L†(t) = A_L(t)
A_L(t) = e^{iHt} A e^{-iHt}
Euclidean
A_E†(τ) = A_E(-τ)
A_E(τ) = e^{-τH} A e^{τH}
radial quantization
A_E(τ) = I A(τ) I
A_E = I A_E† I

3. Unitary irreducible Representation

highest weight state — primary

$D|O\rangle = -i\Delta|O\rangle$


3. Unitary irreducible Representation
highest weight state – primary
D|O> = -iΔ|O>
M_μν

3. Unitary irreducible Representation

highest weight state — primary

$D|\mathcal{O}_\Lambda\rangle = -i\Delta|\mathcal{O}_\Lambda\rangle$

$M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}\,|\mathcal{O}_B\rangle$ imp

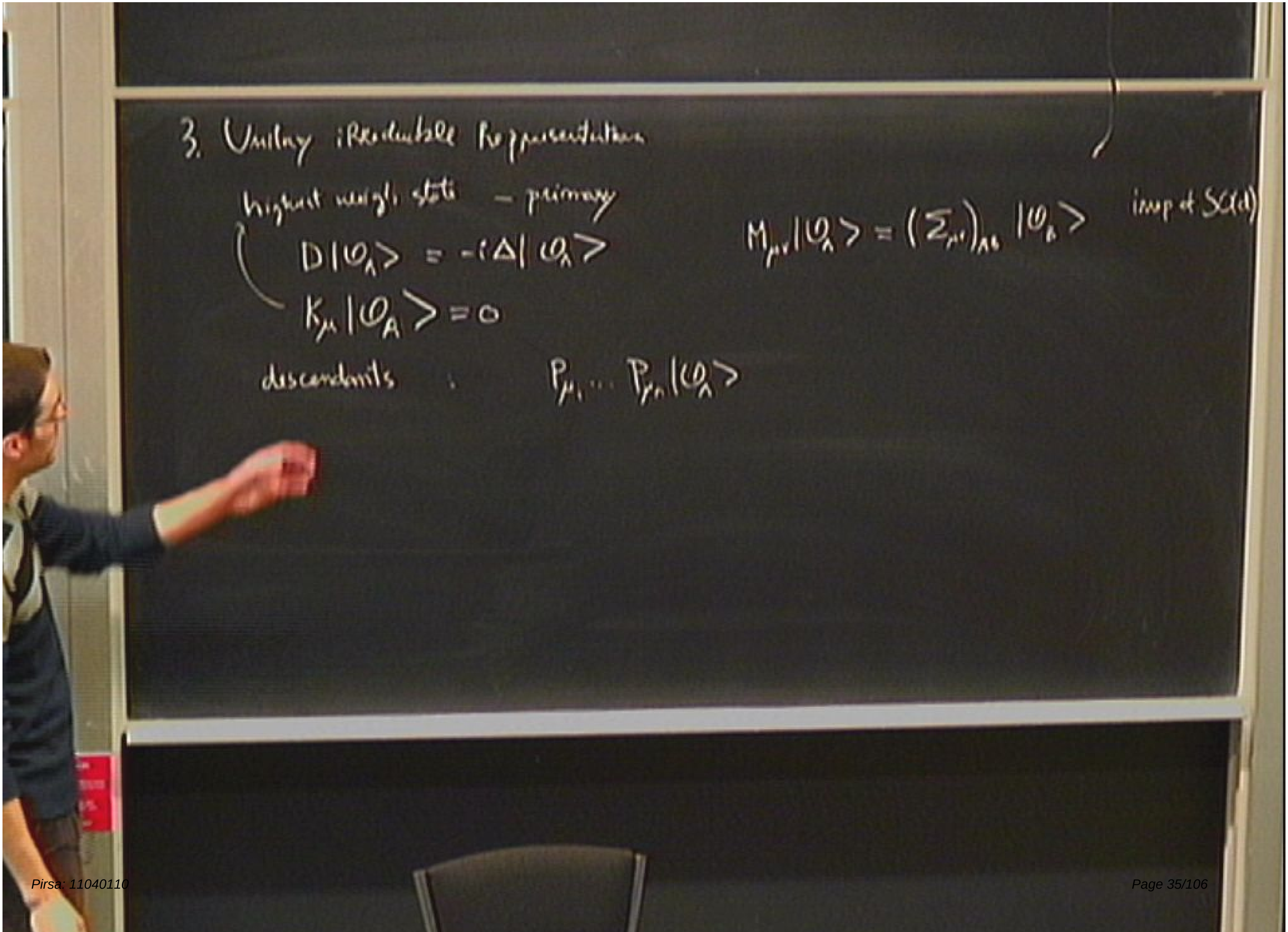
3. Unitary irreducible representation

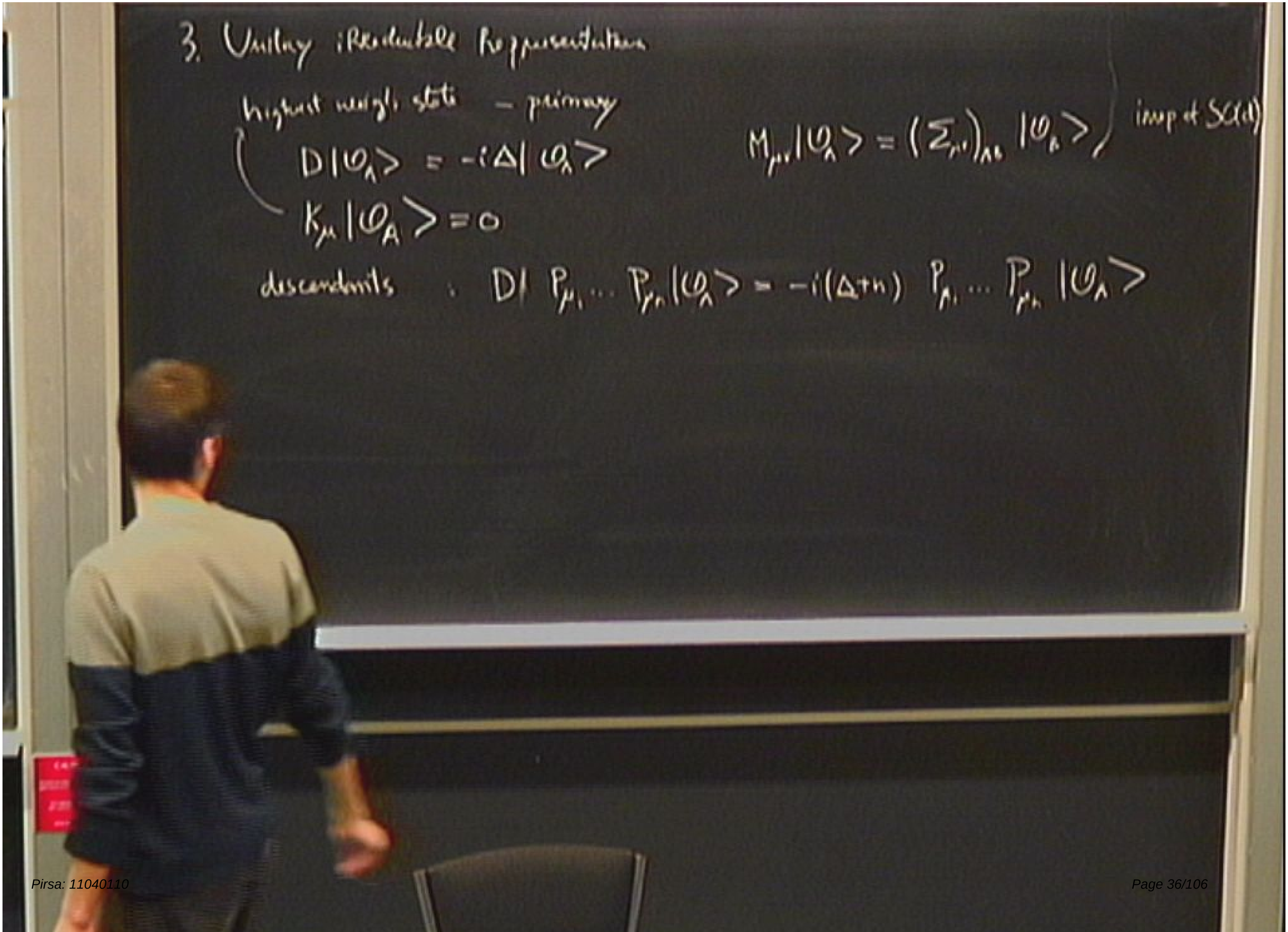
highest weight state – primary

$$D|\mathcal{O}_\Delta\rangle = -i\Delta|\mathcal{O}_\Delta\rangle$$

$$K_\mu|\mathcal{O}_A\rangle = 0$$

$$M_{\mu\nu}|\mathcal{O}_A\rangle = (\Sigma_{\mu\nu})_{AB}|\mathcal{O}_B\rangle$$ irrep of SO(d)


3. Unitary irreducible representation
highest weight state – primary
D|O_Λ> = −iΔ|O_Λ>
K_μ|O_Λ> = 0
descendants : P_μ1 ... P_μn|O_Λ>
M_μν|O_Λ> = (Σ_μν)_ΛB |O_B>
irrep of SO(d)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highest weight state — primary
$D|\mathcal{O}_\Lambda\rangle = -i\Delta|\mathcal{O}_\Lambda\rangle$
$M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|\mathcal{O}_B\rangle$
irrep of SO(d)
$K_\mu|\mathcal{O}_\Lambda\rangle = 0$
descendants : $D|P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle$

3. Unitary irreducible Representation

highest weight state – primary

$$D|\mathcal{O}_\Lambda\rangle = -i\Delta|\mathcal{O}_\Lambda\rangle \qquad M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|\mathcal{O}_B\rangle$$

irrep of $SO(d)$

$$K_\mu|\mathcal{O}_A\rangle = 0$$

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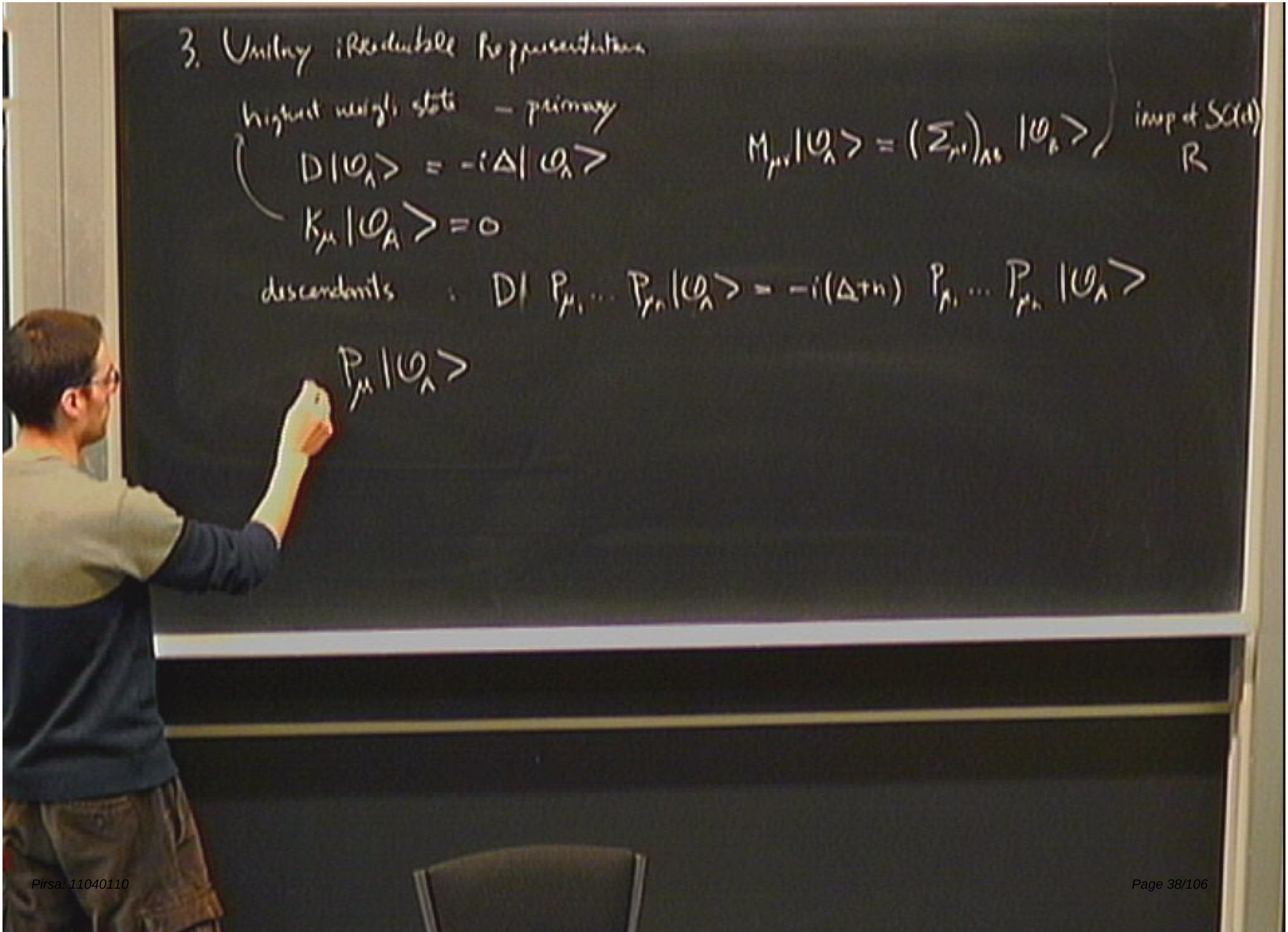
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$M_{\mu\nu}|O_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|O_B\rangle$
irrep of SO(d)
R
descendants : $D| P_{\mu_1}\cdots P_{\mu_n}|O_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|O_\Lambda\rangle$
$P_\mu|O_\Lambda\rangle$

3. Unitary irreducible Representations

highest weight state — primary

$D|\mathcal{O}_\Lambda\rangle = -i\Delta|\mathcal{O}_\Lambda\rangle$ $\qquad$ $M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|\mathcal{O}_B\rangle$ rep of $SO(d)$ R

$K_\mu|\mathcal{O}_\Lambda\rangle = 0$

descendants : $D\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle$

$\| c_{\mu\Lambda} P_\mu |\mathcal{O}_\Lambda\rangle\|^2 \geq 0$

3. Unitary irreducible Representations

highest weight state — primary

$$D|\mathcal{O}_\Lambda\rangle = -i\Delta|\mathcal{O}_\Lambda\rangle \qquad M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|\mathcal{O}_B\rangle, \quad \text{irrep of } SO(d) \ R$$

$$K_\mu|\mathcal{O}_\Lambda\rangle = 0$$

descendants : $D\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle$

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highest weight state — primary

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$$K_\mu|\mathcal{O}_A\rangle = 0$$

$$M_{\mu\nu}|\mathcal{O}_\Lambda\rangle = (\Sigma_{\mu\nu})_{\Lambda B}|\mathcal{O}_B\rangle$$ irrep of SO(d) R

descendants : $D|P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle$

$$\| c_{\mu\Lambda} P_\mu|\mathcal{O}_\Lambda\rangle\|^2 \geq 0$$

$\Leftrightarrow\ T_{A\nu,B\mu} = \langle \mathcal{O}_\Lambda|P_\nu^\dagger P_\mu|\mathcal{O}_B\rangle$ has only non-negative eigenvalues

3. Unitary irreducible Representation

highest weight state — primary

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descendants : $D|P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_\Lambda\rangle$

$$\| c_{\mu\Lambda} P_\mu|\mathcal{O}_\Lambda\rangle\|^2 \geq 0 \quad \forall c_{\mu\Lambda}$$

$\Leftrightarrow$ $T_{A\nu,B\mu} = \langle\mathcal{O}_A|P_\nu^\dagger P_\mu|\mathcal{O}_B\rangle$ has only non-negative eigenvalues

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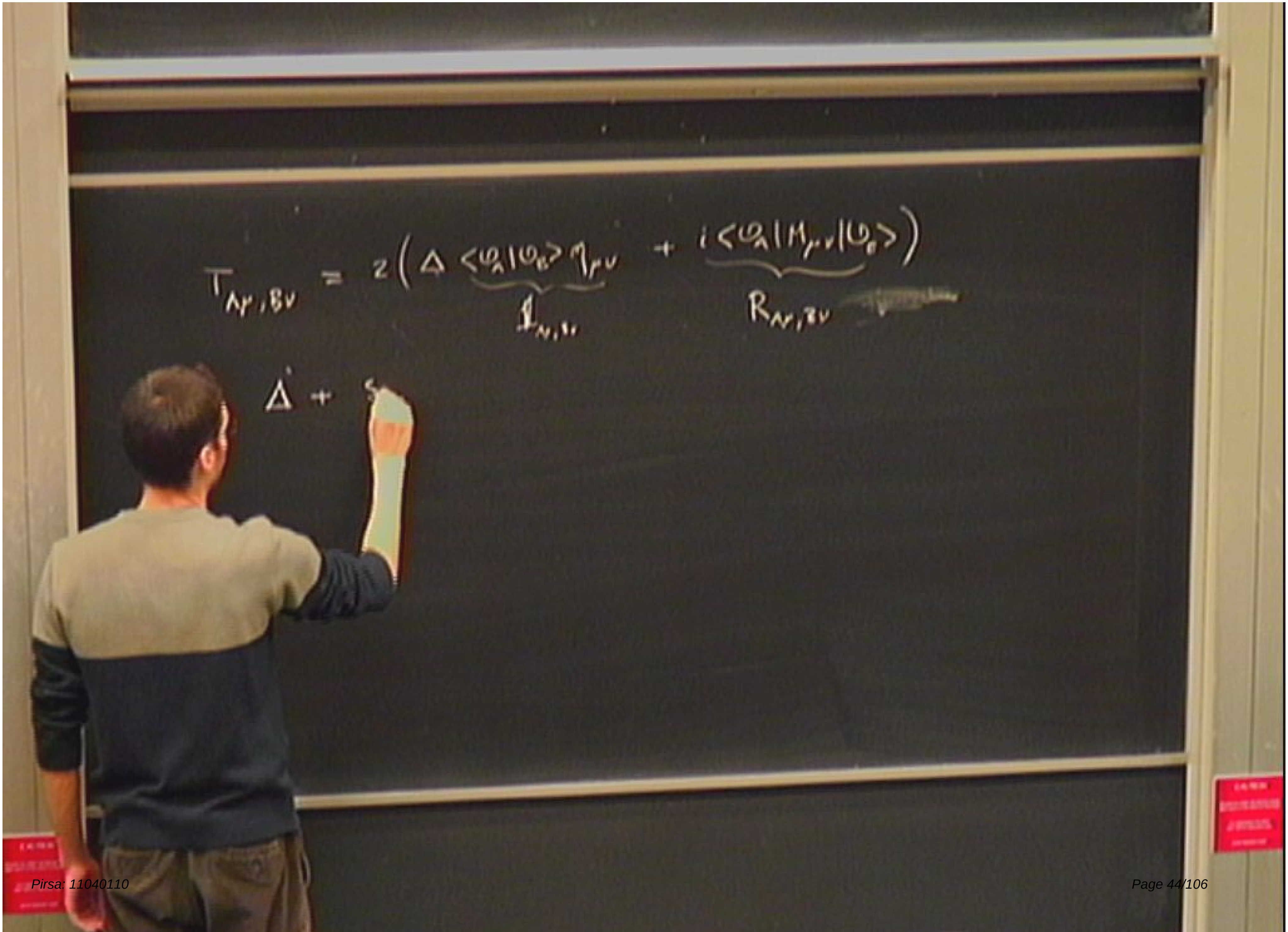
descendants : $D|P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_A\rangle = -i(\Delta+n)\, P_{\mu_1}\cdots P_{\mu_n}|\mathcal{O}_A\rangle$

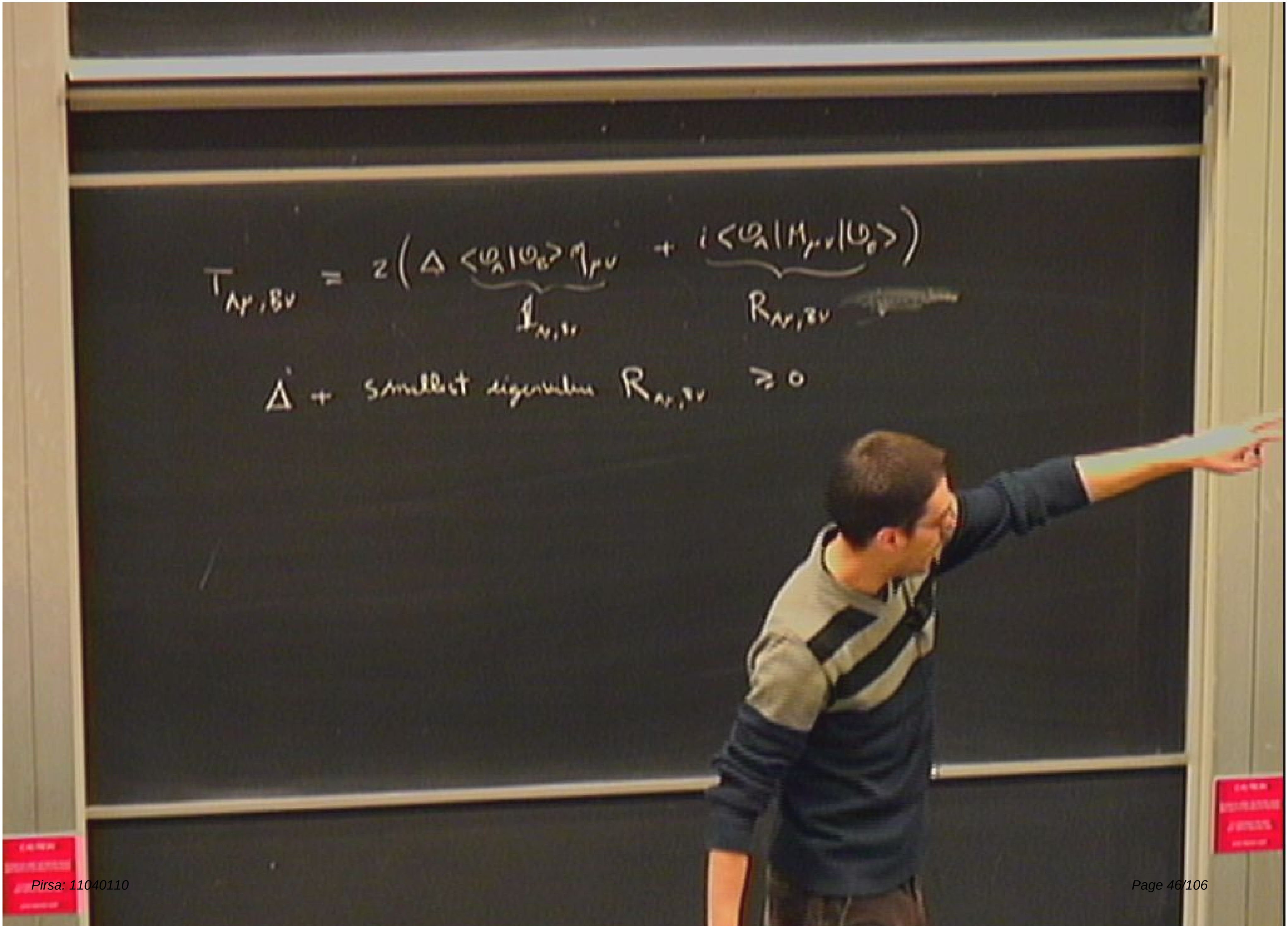
$$\| c_{\mu A} P_\mu|\mathcal{O}_A\rangle\|^2 \geq 0 \quad \forall c_{\mu A}$$

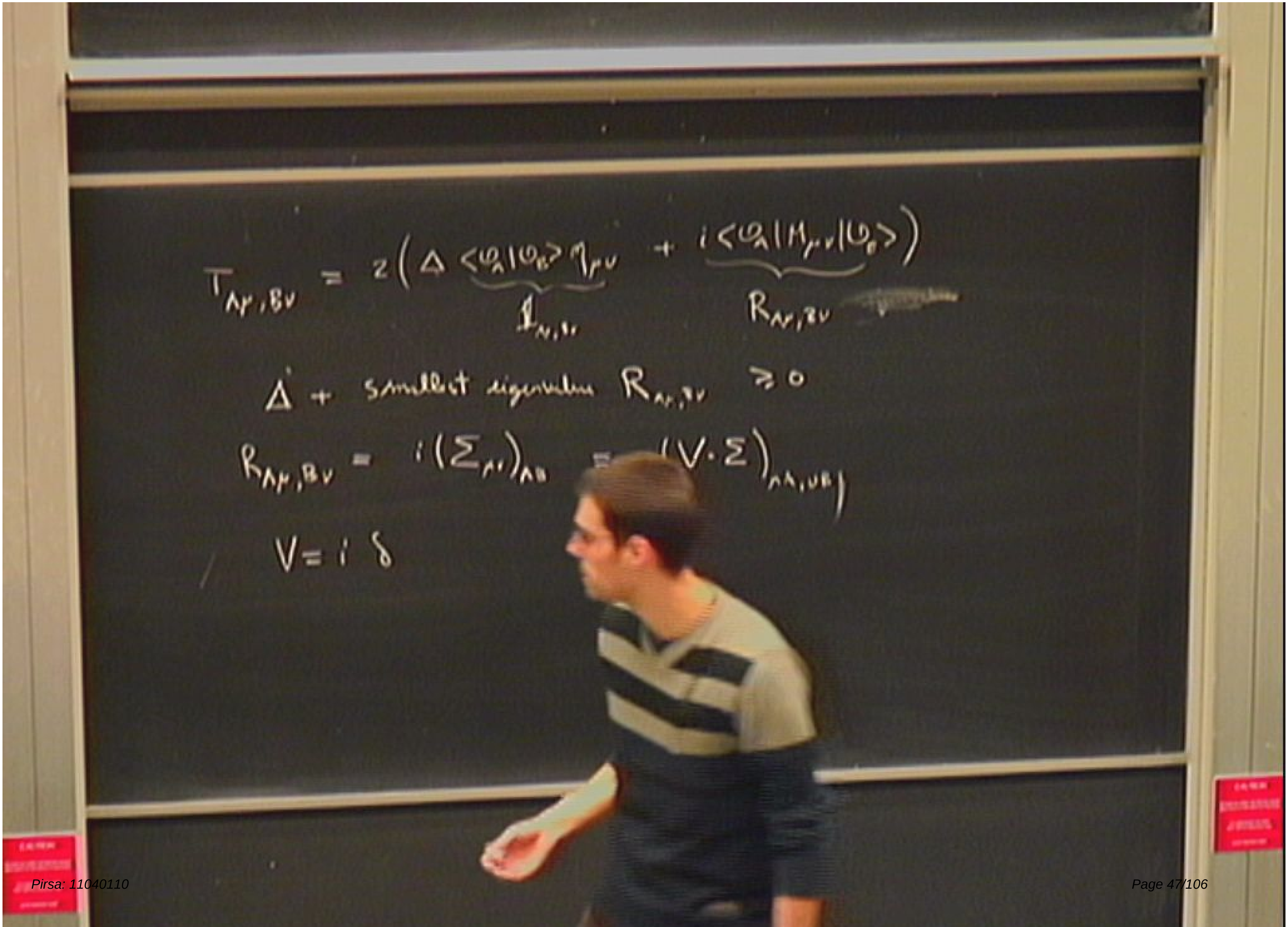
$$\Leftrightarrow \quad T_{A\nu,B\mu} = \langle\mathcal{O}_A|\underset{K_\nu}{P_\nu^\dagger}\, P_\mu|\mathcal{O}_B\rangle$$ has only non-negative eigenvalues

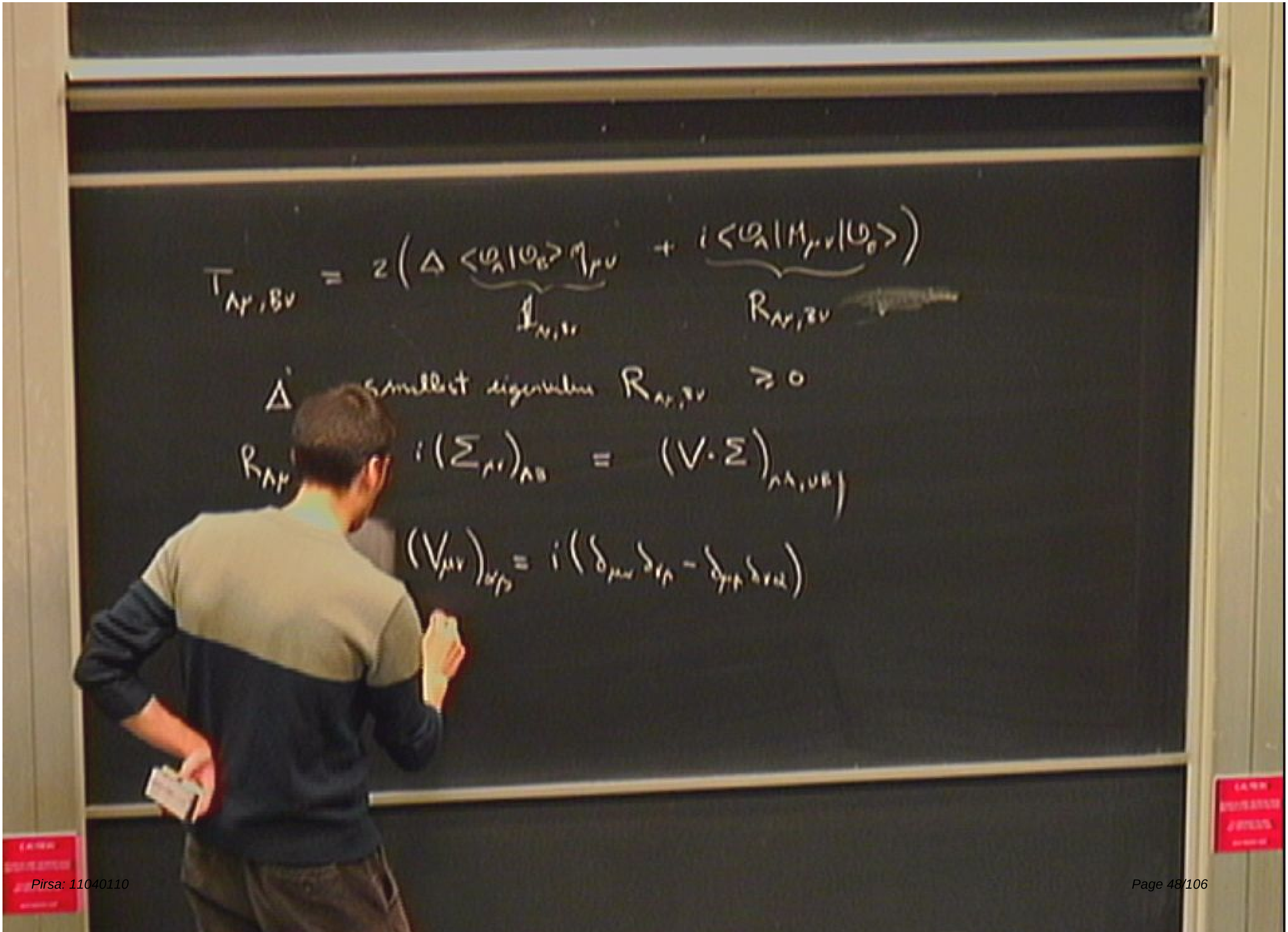

$T_{A\mu,B\nu} = 2\left(\Delta \underbrace{\langle\varphi_A|\varphi_B\rangle \eta_{\mu\nu}} + \underbrace{i\langle\varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\right)$
$\Delta +$

$$T_{A\mu,B\nu} = 2\left(\Delta \underbrace{\langle\varphi_A|\varphi_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle\varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\right)$$

Δ + smallest eigenvalue $R_{A\mu,B\nu} \geq 0$


$T_{A\mu,B\nu} = z\left(\Delta \underbrace{\langle \varphi_A|\varphi_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle \varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\right)$
Δ + smallest eigenvalue $R_{A\mu,B\nu} \geq 0$


$T_{A\mu,B\nu} = 2\Big(\underbrace{\Delta \langle\varphi_A|\varphi_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle\varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\Big)$
Δ + smallest eigenvalue $R_{A\mu,B\nu} \geq 0$
$R_{A\mu,B\nu} = i(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{(\mu A,\nu B)}$
$V = i\,\delta$



$T_{A\mu,B\nu} = 2\left(\Delta \underbrace{\langle\varphi_A|\varphi_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle\varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\right)$
Δ smallest eigenvalue $R_{A\mu,B\nu} \geq 0$
$R_{A\mu}$ $i(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{\mu A,\nu B}$
$(V_{\mu\nu})_{\alpha\beta} = i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$

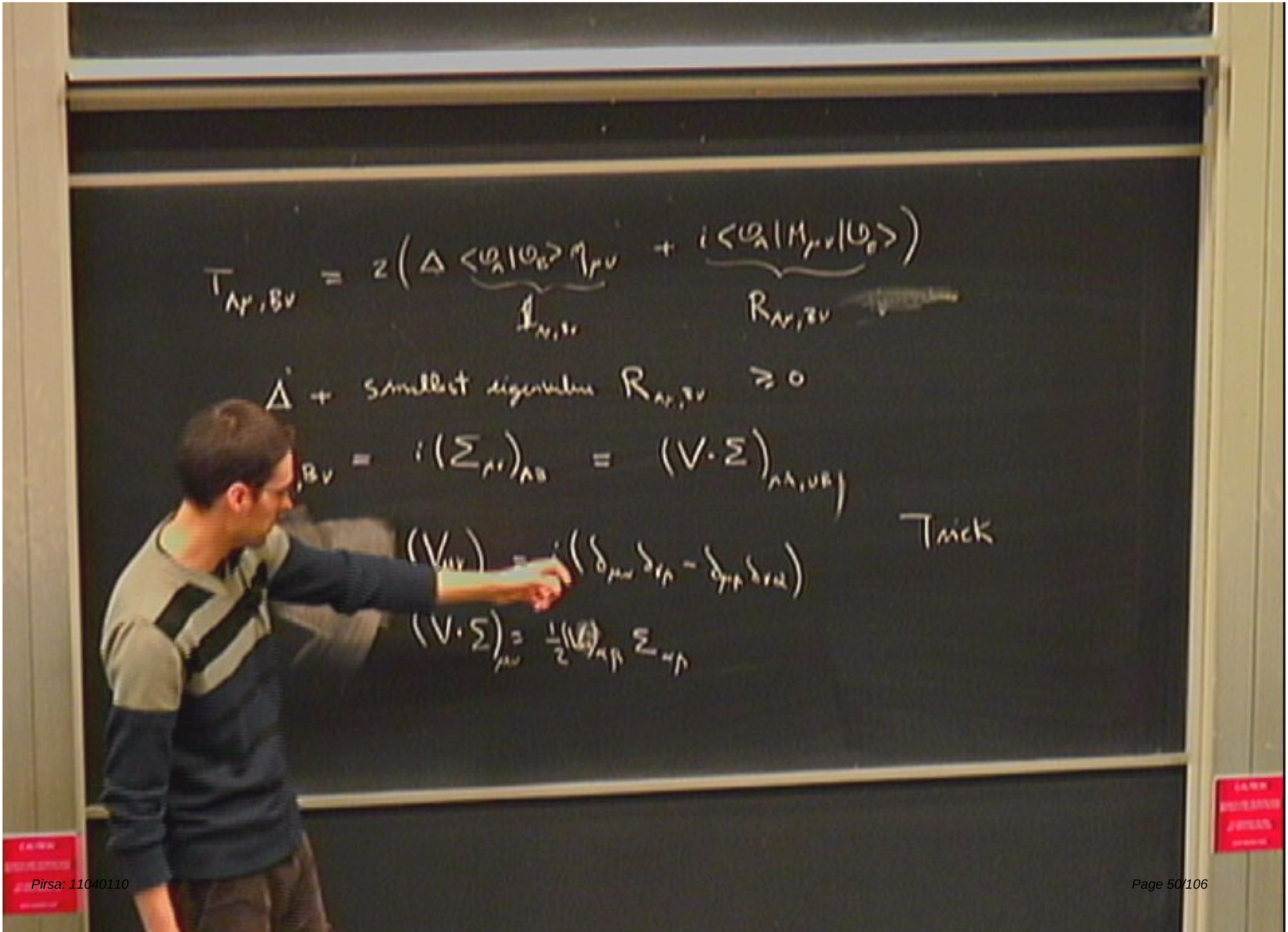
$$T_{A\mu,B\nu} = 2\left(\Delta \underbrace{\langle\omega_A|\omega_B\rangle\, \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle\omega_A|M_{\mu\nu}|\omega_B\rangle}_{R_{A\mu,B\nu}}\right)$$

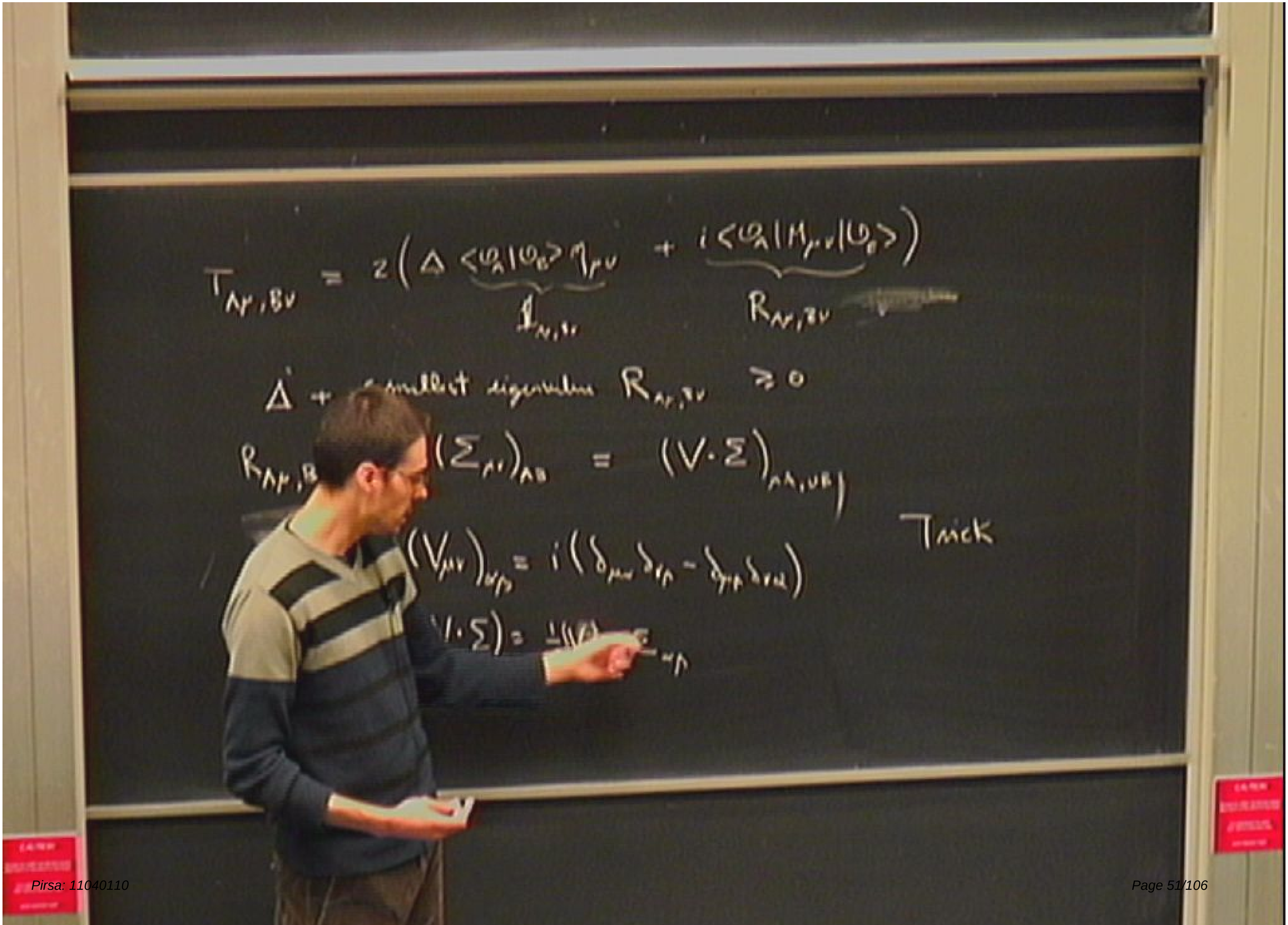
$$\Delta + \text{smallest eigenvalue } R_{A\mu,B\nu} \geq 0$$

$$= i(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{\mu A,\nu B}$$

$$(V_{\mu\nu})_{\alpha\beta} = i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$$

$$V\cdot\Sigma = \frac{1}{2} V_{\alpha\beta}\Sigma_{\alpha\beta}$$


T_{Aμ,Bν} = 2(Δ ⟨φ_A|φ_B⟩ η_{μν} + i⟨φ_A|M_{μν}|φ_B⟩)
R_{Aμ,Bν}
Δ + smallest eigenvalue R_{Aμ,Bν} ≥ 0
= i(Σ_{μν})_{AB} = (V·Σ)
Trick
(V·Σ)_{μν} = ½ (V)_{αβ} Σ_{αβ}



$T_{A\mu,B\nu} = 2\left(\Delta \langle O_A|O_B\rangle \eta_{\mu\nu} + i\langle O_A|M_{\mu\nu}|O_B\rangle\right)$
$R_{A\mu,B\nu}$
Δ + smallest eigenvalue $R_{A\mu,B\nu} \geq 0$
$(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{\mu A,\nu B}$
Trick
$(V_{\mu\nu})_{\alpha\beta} = i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$

$$T_{A\mu,B\nu} = 2\Big(\Delta \underbrace{\langle \varphi_A|\varphi_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A,B}} + \underbrace{i\langle \varphi_A|M_{\mu\nu}|\varphi_B\rangle}_{R_{A\mu,B\nu}}\Big)$$

$$\Delta + \text{smallest eigenvalue } R_{A\mu,B\nu} \geq 0$$

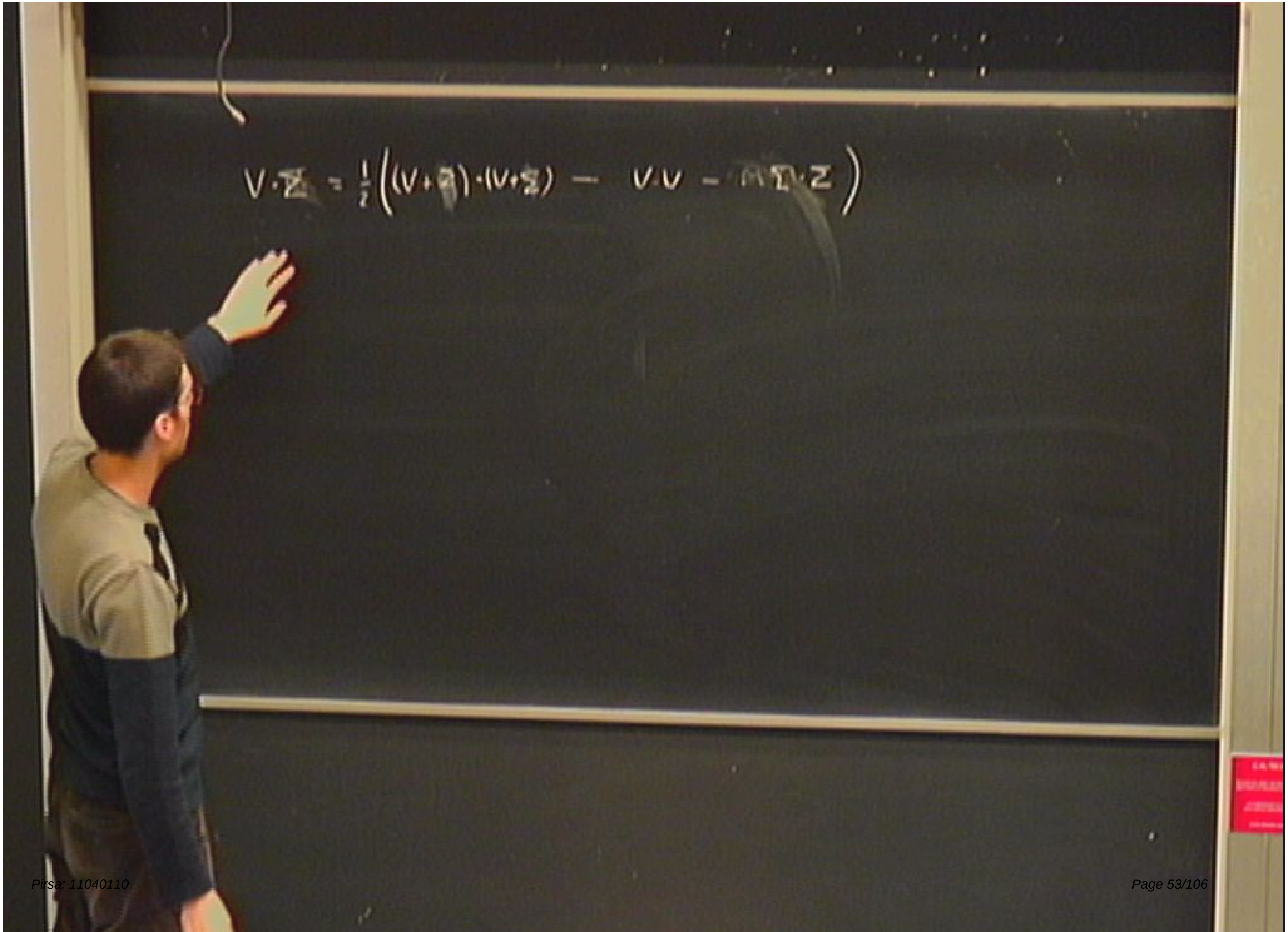
$$R_{A\mu,B\nu} = i(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{A\mu,B\nu}$$

$V \times R$

Trick

$$(V_{\mu\nu})_{\alpha\beta} = i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$$

$$(V\cdot\Sigma)_{\mu\nu} = \tfrac{1}{2}(V)_{\alpha\beta}\Sigma_{\alpha\beta}$$


$V \cdot Z = \frac{1}{2}\left((V+Z)\cdot(V+Z) - V \cdot V - Z \cdot Z\right)$

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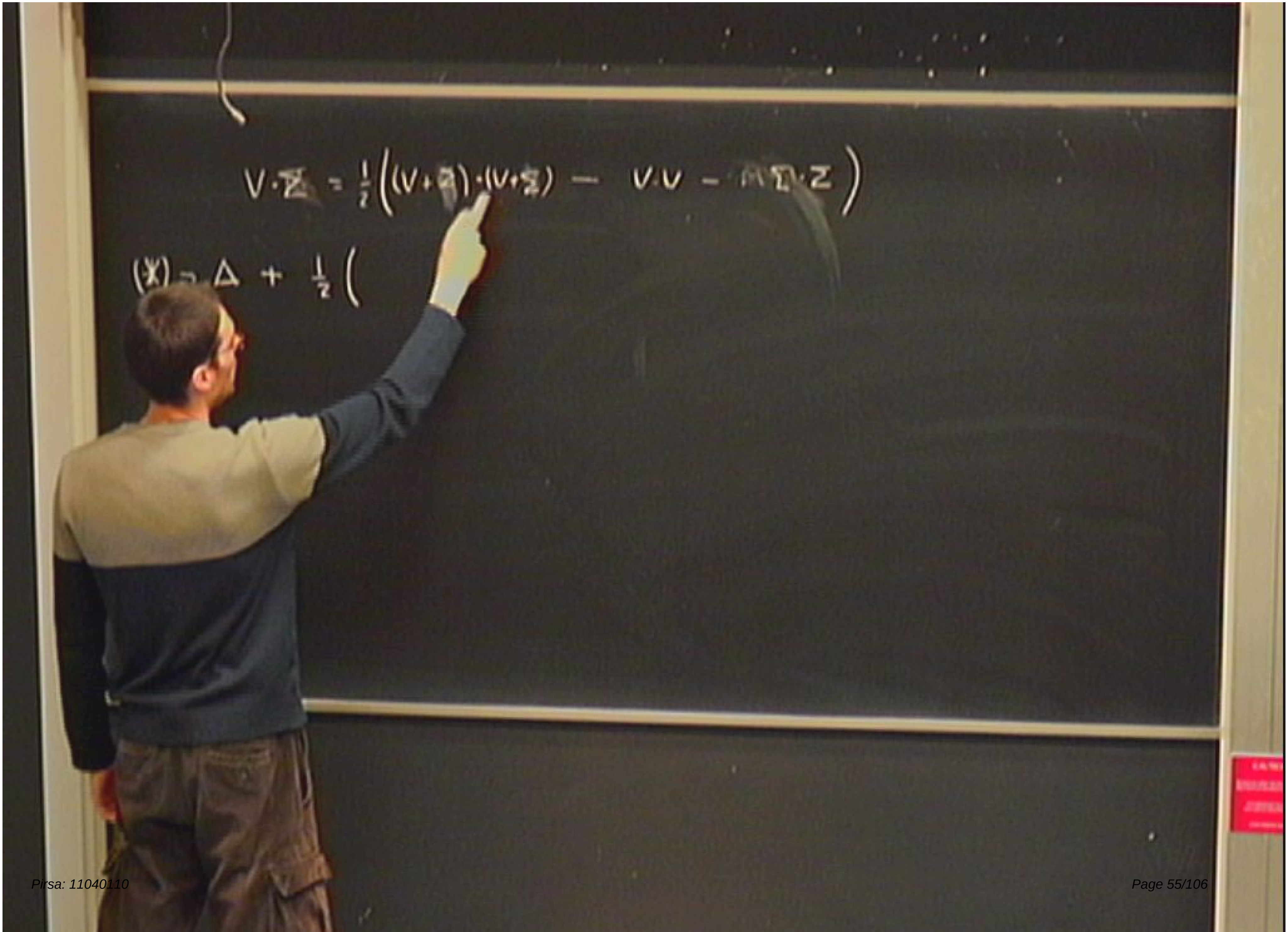
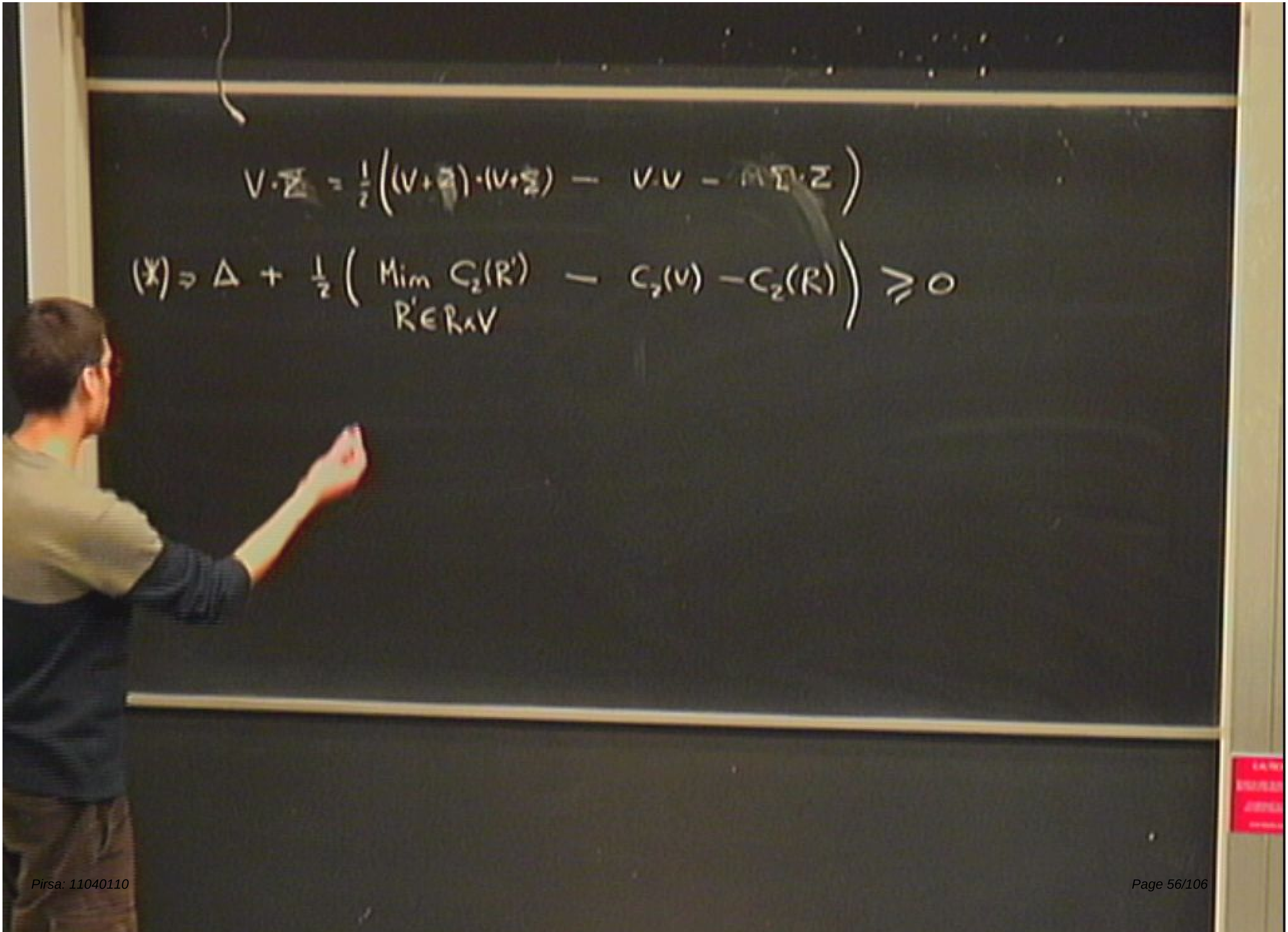

$V \cdot Z = \frac{1}{2}\left((V+Z)\cdot(V+Z) - V \cdot V - Z \cdot Z\right)$
$(*) = \Delta + \frac{1}{2}($


$V\cdot Z = \frac{1}{2}\left((V+Z)\cdot(V+Z) - V\cdot V - Z\cdot Z\right)$
$(*) \Rightarrow \Delta + \frac{1}{2}\left(\min_{R'\in R\otimes V} C_2(R') - C_2(V) - C_2(R)\right) \geq 0$

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<u>$d=3$</u>

$SO(3)$

$$C_2(R) = j(j+1)$$
$$C_2(V) = 2$$
$$C_2(R') = (j-1)\,j \quad , \; j \geq 1$$

$$\Delta \geq j+1$$

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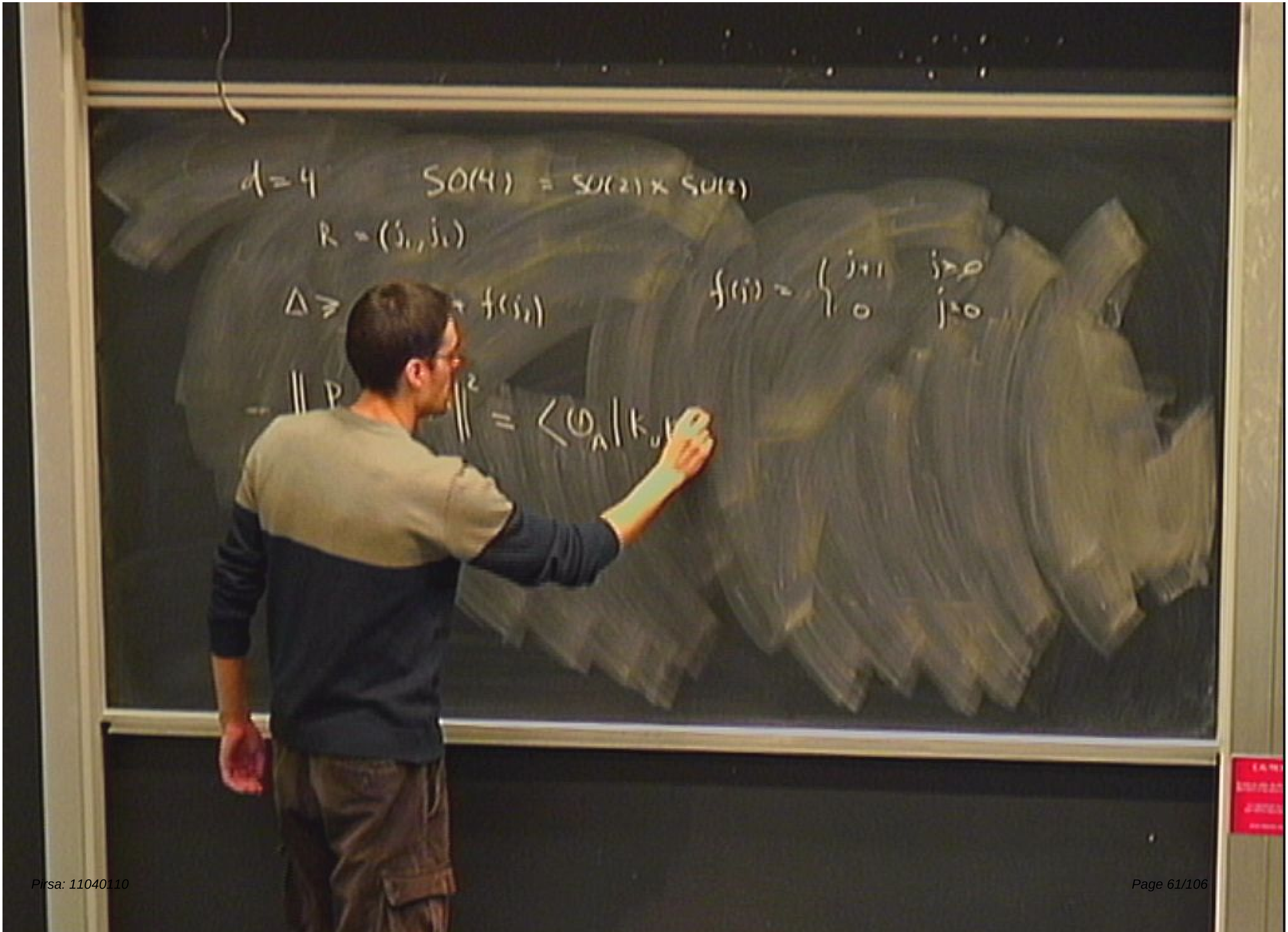
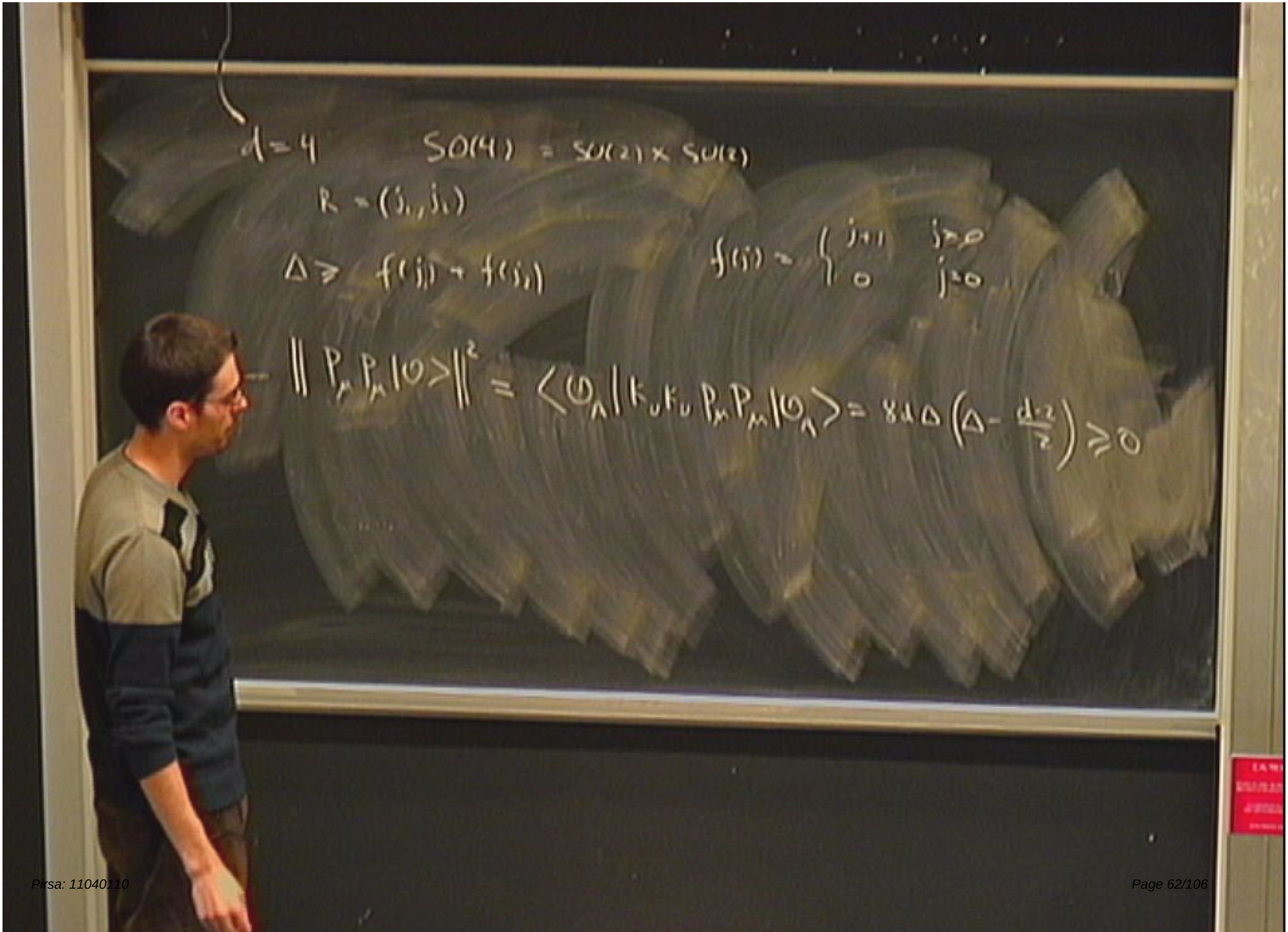
$d=4 \qquad SO(4) = SU(2) \times SU(2)$

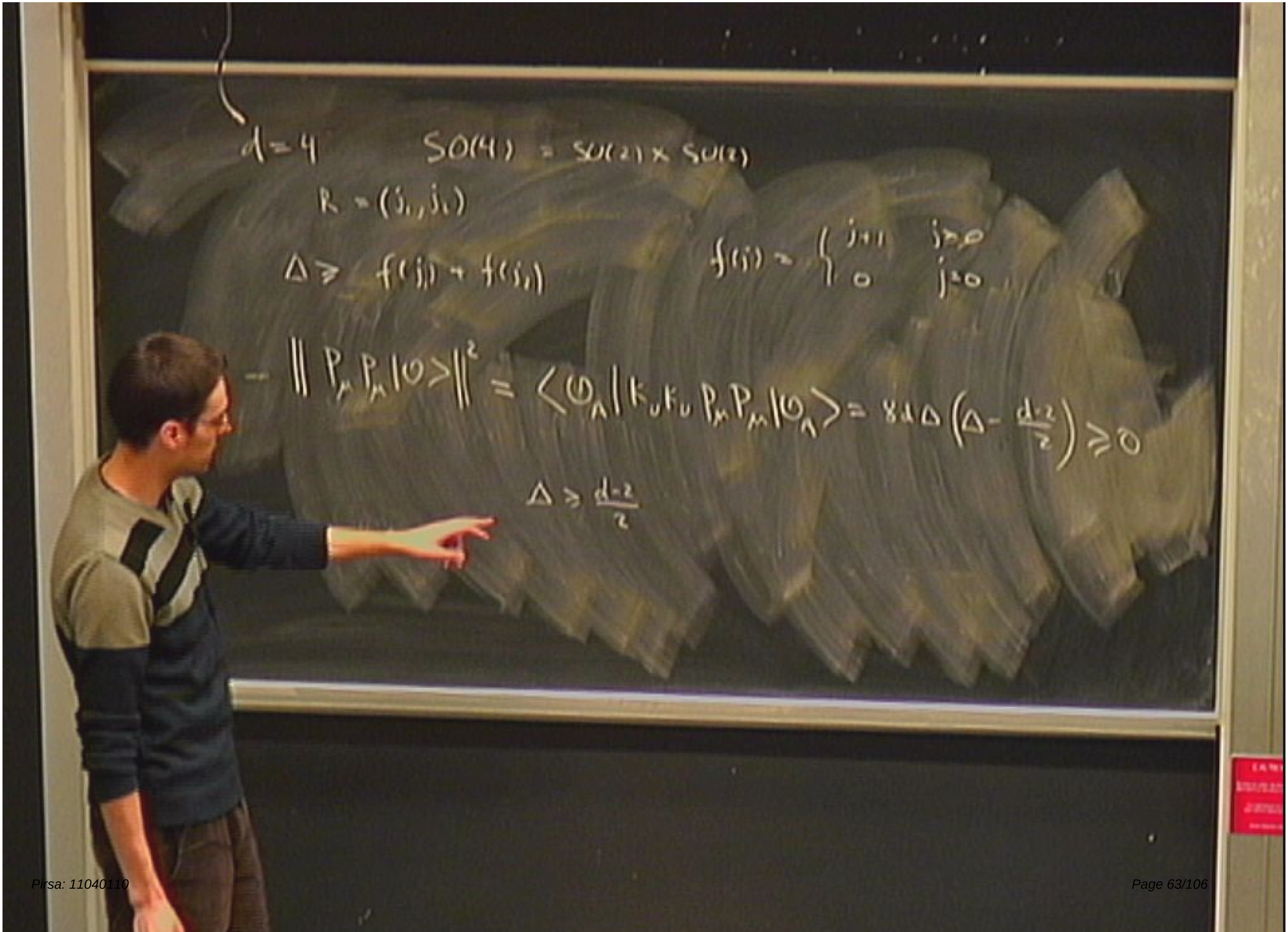
$R = (j_1, j_2)$

$\Delta \geq f(j_1) + f(j_2)$

$$f(j) = \begin{cases} j+1 & j>0 \\ 0 & j=0 \end{cases}$$


d=4
SO(4) = SU(2) x SU(2)
R = (j1, j2)
j=0


d = 4   SO(4) = SU(2) × SU(2)
R = (j₁, j₂)
Δ ≥ f(j₁) + f(j₂)
f(j) = { j+1 j>0 ; 0 j=0
|| P_μ P_μ |O> ||² = <O_Λ| K_ν K_ν P_μ P_μ |O_Λ> = 8dΔ(Δ − (d−2)/2) ≥ 0


$d=4$ $SO(4) = SU(2) \times SU(2)$
$R = (j_1, j_2)$
$\Delta \geq f(j_1) + f(j_2)$
$f(j) = \begin{cases} j+1 & j \neq 0 \\ 0 & j=0 \end{cases}$
$-\| P_\mu P_\mu |O\rangle \|^2 = \langle O_\Delta | K_\nu K_\nu P_\mu P_\mu | O_\Delta \rangle = 8d\Delta\left(\Delta - \frac{d-2}{2}\right) \geq 0$
$\Delta \geq \frac{d-2}{2}$

$d = 4 \qquad SO(4) = SU(2) \times SU(2)$

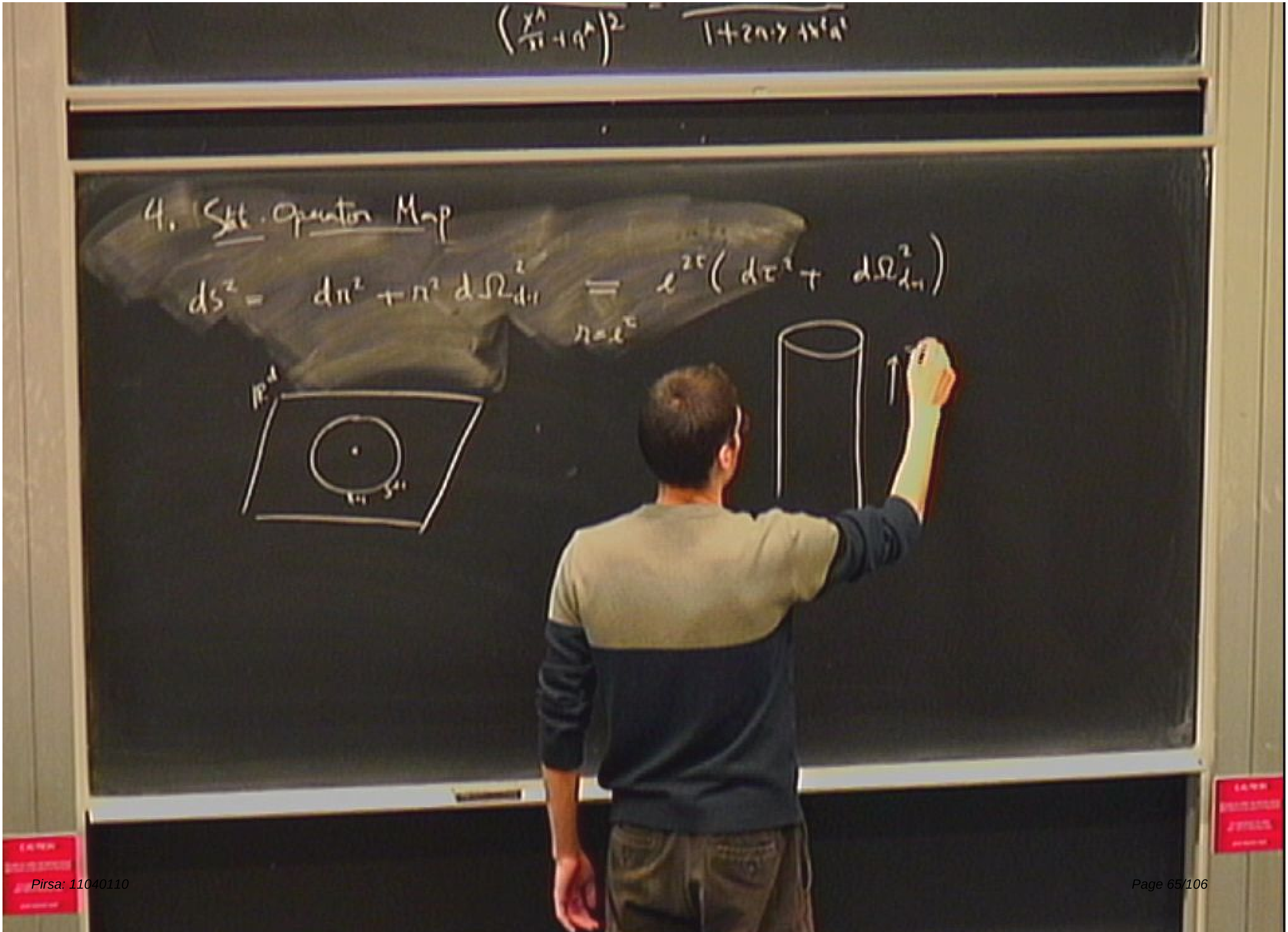
$R = (j_1, j_2)$

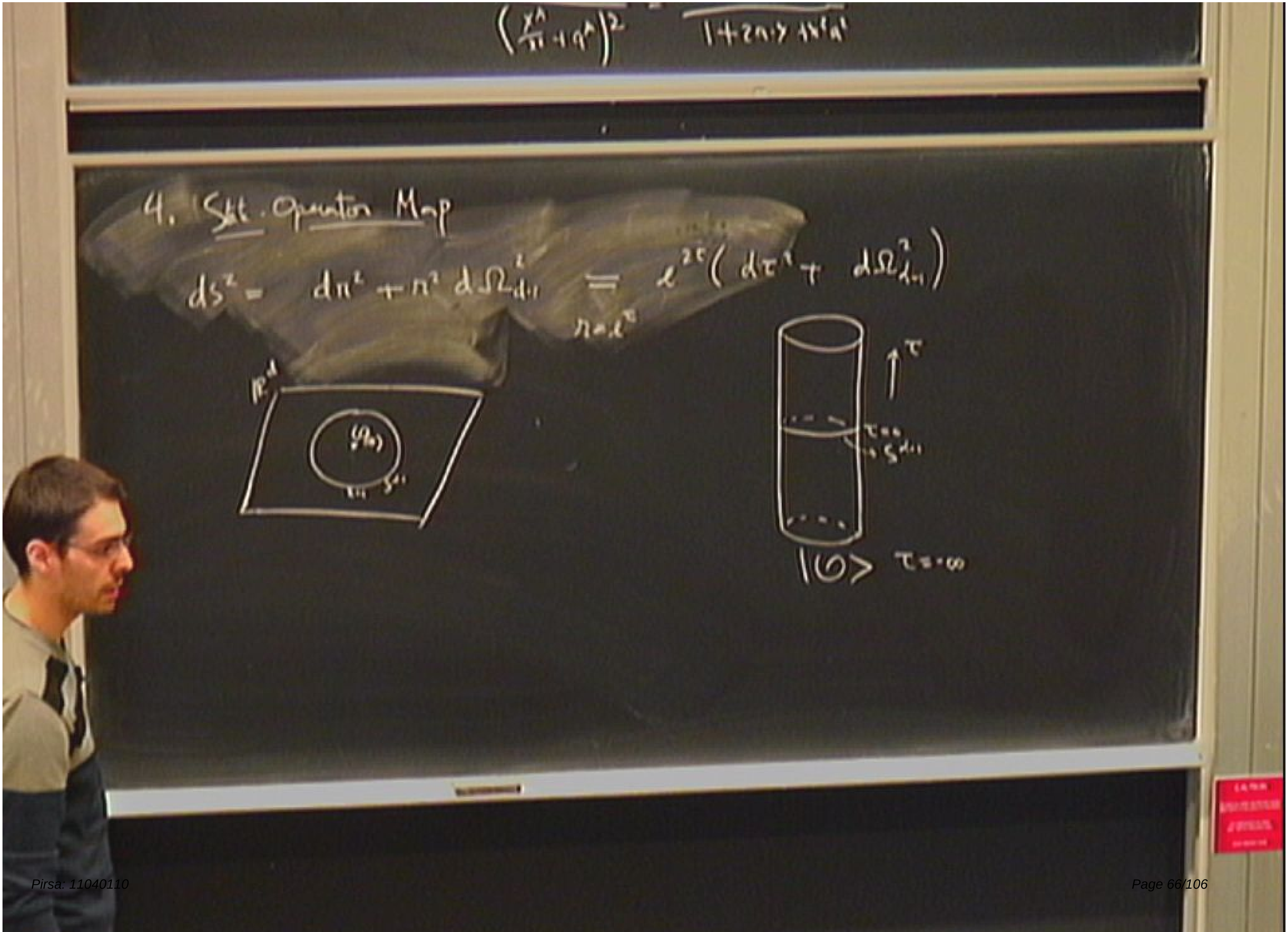
$\Delta \geq f(j_1) + f(j_2) \qquad f(j) = \begin{cases} j+1 & j>0 \\ 0 & j=0 \end{cases}$

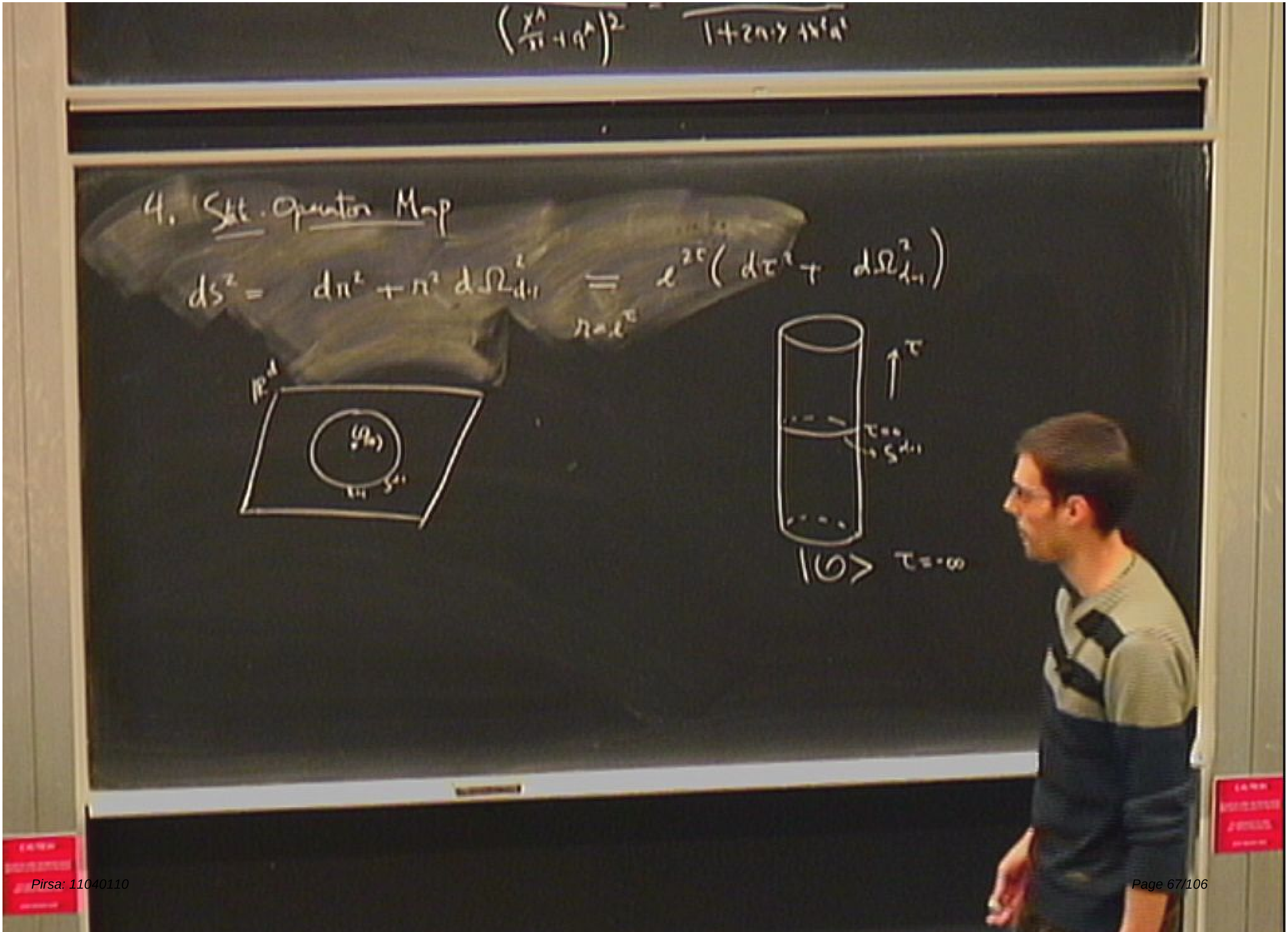
$$\| P_\mu P_\mu |O\rangle \|^2 = \langle O_\Lambda | K_\nu K_\nu P_\mu P_\mu | O_\Lambda \rangle = 8 d \Delta \left(\Delta - \frac{d-2}{2} \right) \geq 0$$

$\Delta \geq \frac{d-2}{2} \qquad \Delta = \frac{d-2}{2} \qquad P_\mu P_\mu |O\rangle = 0$

free massless scalar


4. State - Operator Map
ds² = dr² + r² dΩ²_{d-1} = e^{2τ}(dτ² + dΩ²_{d-1})
r = e^τ
ℝ^d
S^{d-1}


4. State-Operator Map
$ds^2 = dr^2 + r^2 d\Omega_{d-1}^2 \underset{r = e^\tau}{=} e^{2\tau}(d\tau^2 + d\Omega_{d-1}^2)$
$\mathbb{R}^d$
$\mathcal{O}(0)$
$r=1$ S^{d-1}
τ
$\tau=0$
S^{d-1}
$|\mathcal{O}\rangle$ $\tau=-\infty$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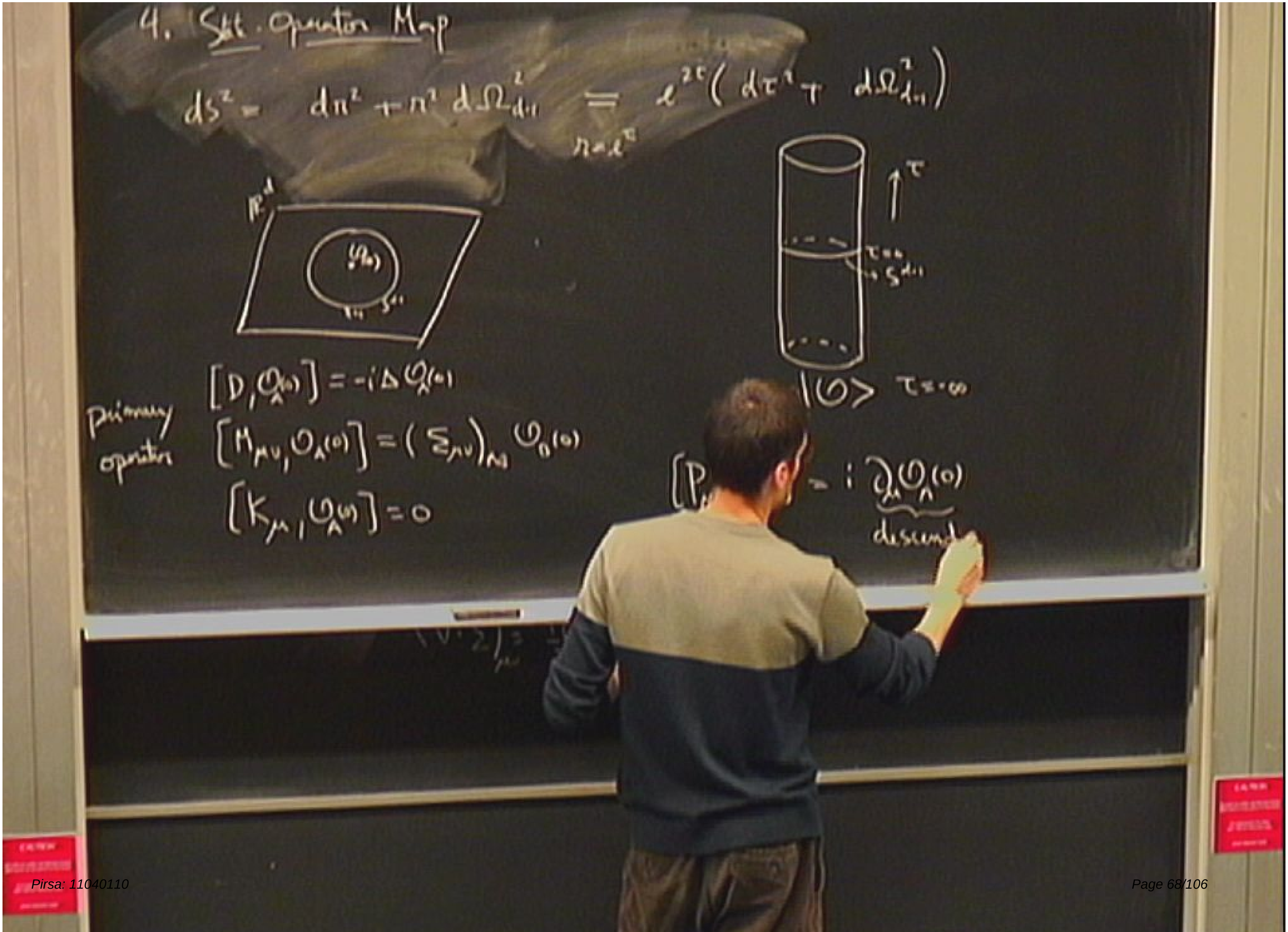
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ds² = dr² + r² dΩ²_{d-1} = e^{2τ}(dτ² + dΩ²_{d-1})
r = e^τ
R^d
O(0)
S^{d-1}
τ
τ = 0
S^{d-1}
|O⟩ τ = -∞
[D, O_Δ(0)] = -iΔ O_Δ(0)
primary operator
[M_{μν}, O_A(0)] = (Σ_{μν})_{AB} O_B(0)
[K_μ, O_A(0)] = 0
= i ∂_μ O_A(0)
descendant

$[K_\mu, \mathcal{O}_A(0)] = 0$

$[P_\mu, \mathcal{O}_A(0)] = i\,\underbrace{\partial_\mu \mathcal{O}_A(0)}_{\text{descendants}}$

$$T_{A\mu,B\nu} = 2\Big(\Delta \underbrace{\langle \mathcal{O}_A|\mathcal{O}_B\rangle \eta_{\mu\nu}}_{\mathbb{1}_{A\mu,B\nu}} + \underbrace{i\langle \mathcal{O}_A|M_{\mu\nu}|\mathcal{O}_B\rangle}_{R_{A\mu,B\nu}}\Big)$$

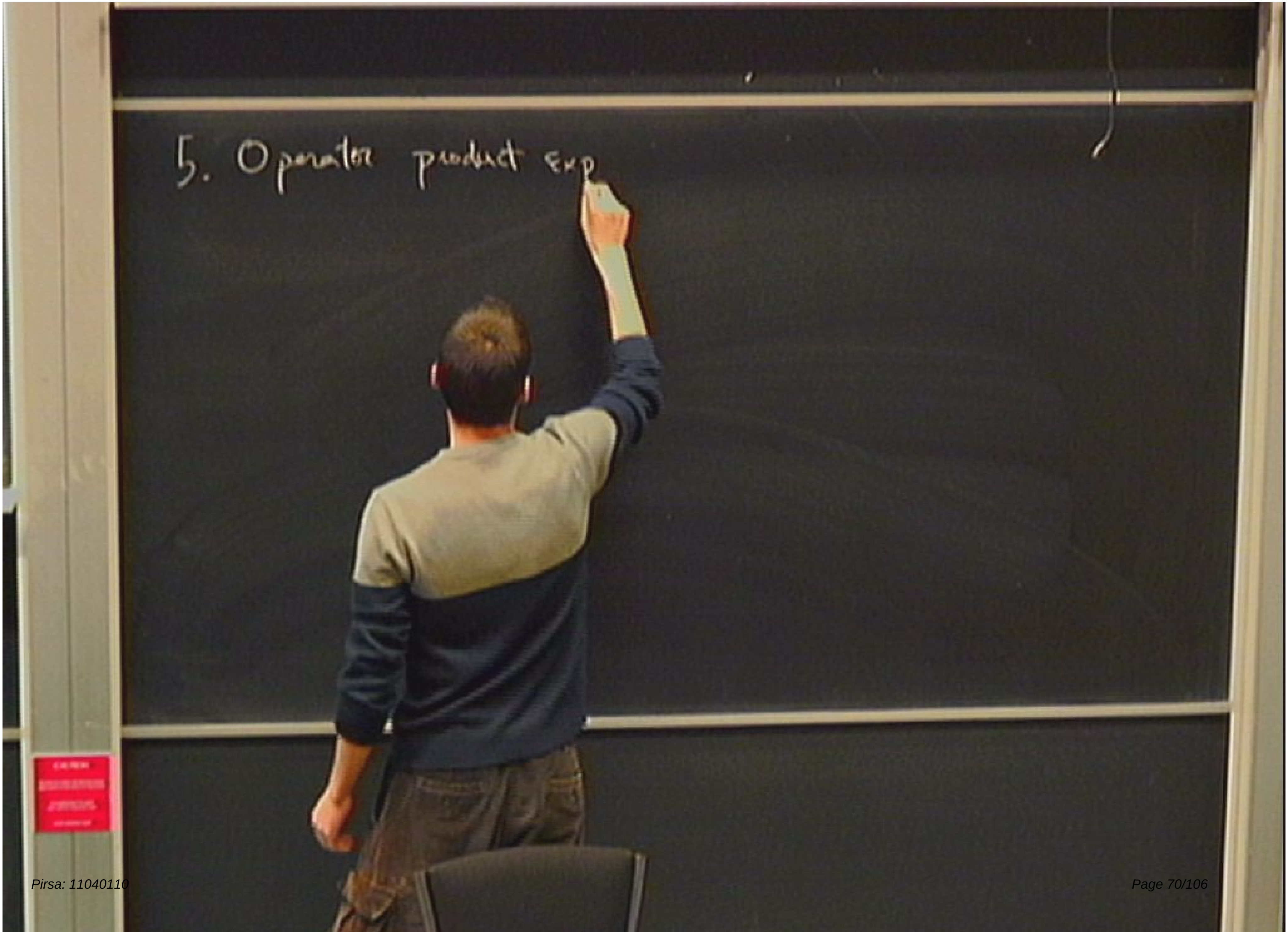
Δ + smallest eigenvalue $R_{A\mu,B\nu} \geq 0$ (*)

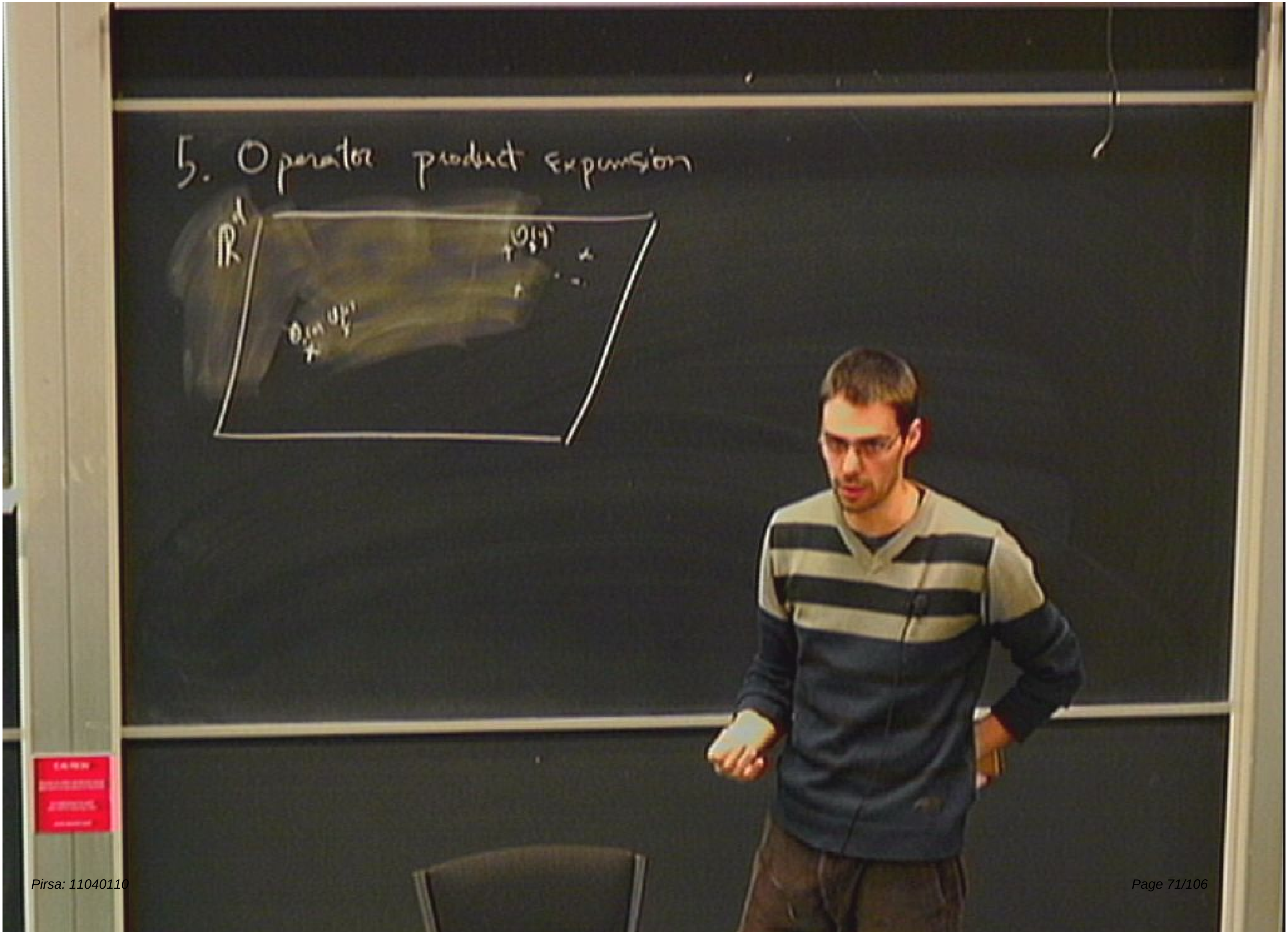
$\ldots = i(\Sigma_{\mu\nu})_{AB} = (V\cdot\Sigma)_{\mu A,\nu B}$ $V \times R$

Trick

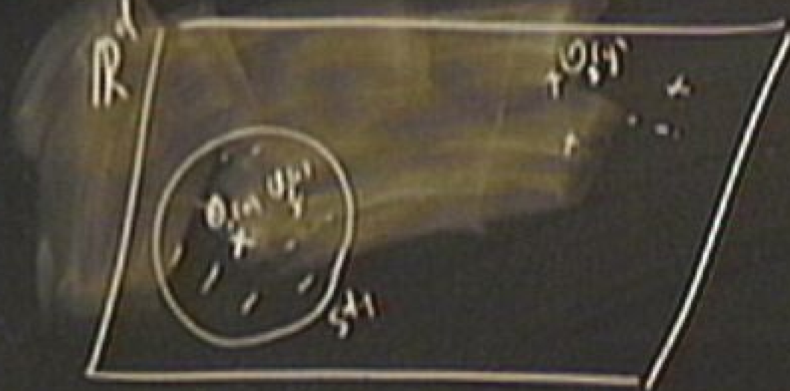
$(V_{\mu\nu})_{\alpha\beta} = i(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha})$

$(V\cdot\Sigma)_{\mu\nu} = \frac{1}{2}(V_{\mu\nu})_{\alpha\beta}\,\Sigma_{\alpha\beta}$


5. Operator product exp

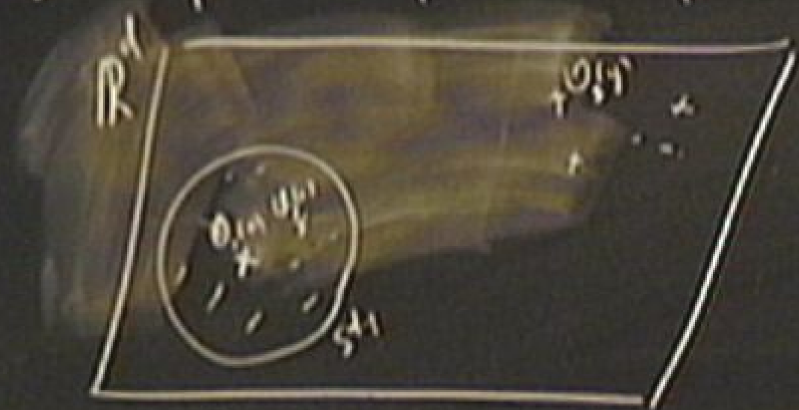

5. Operator product expansion
$\mathbb{R}^d$

5. Operator product expansion



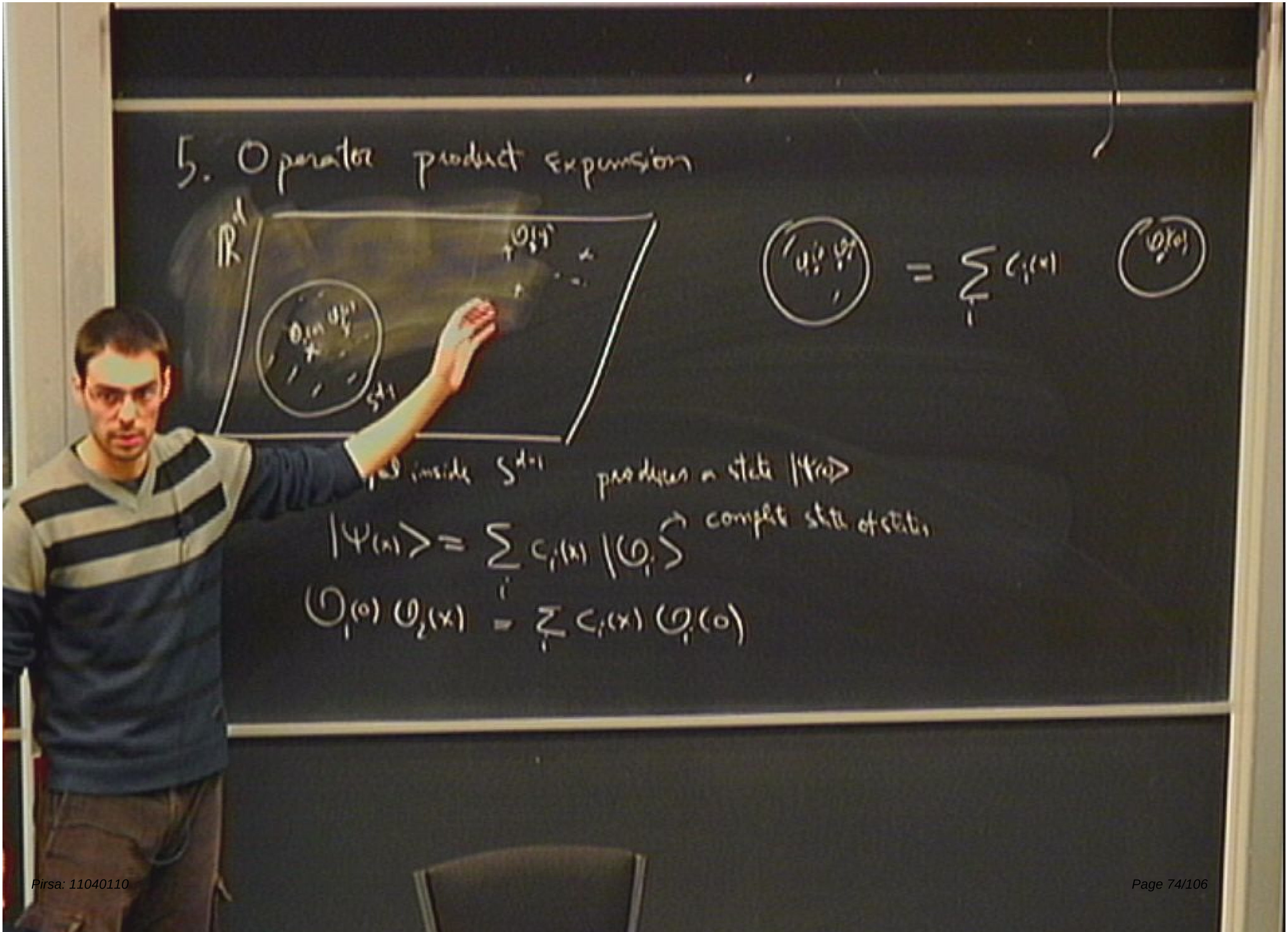
path integral inside S^{d-1}

5. Operator product expansion

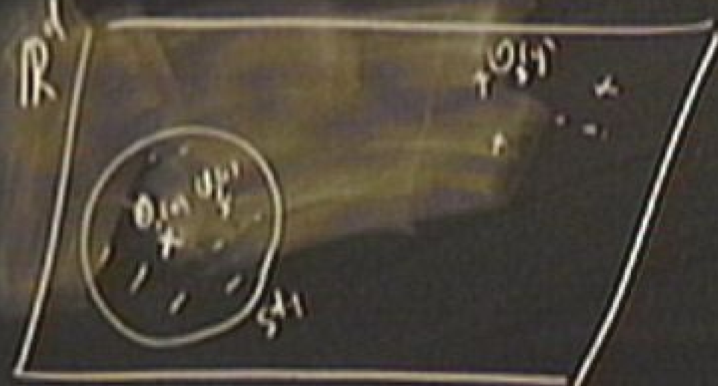


path integral inside S^{d-1} produces a state $|\Psi(x)\rangle$

$$|\Psi(x)\rangle = \sum_i c_i(x)\, |\mathcal{O}_i\rangle$$ complete set of states


5. Operator product expansion
$\mathbb{R}^d$
S^{d-1}
$= \sum_i C_i(x)$
inside S^{d-1} produces a state $|\Psi(x)\rangle$
$|\Psi(x)\rangle = \sum_i C_i(x) |O_i\rangle$ complete set of states
$O_1(0)\, O_2(x) = \sum_i C_i(x)\, O_i(0)$

5. Operator product expansion

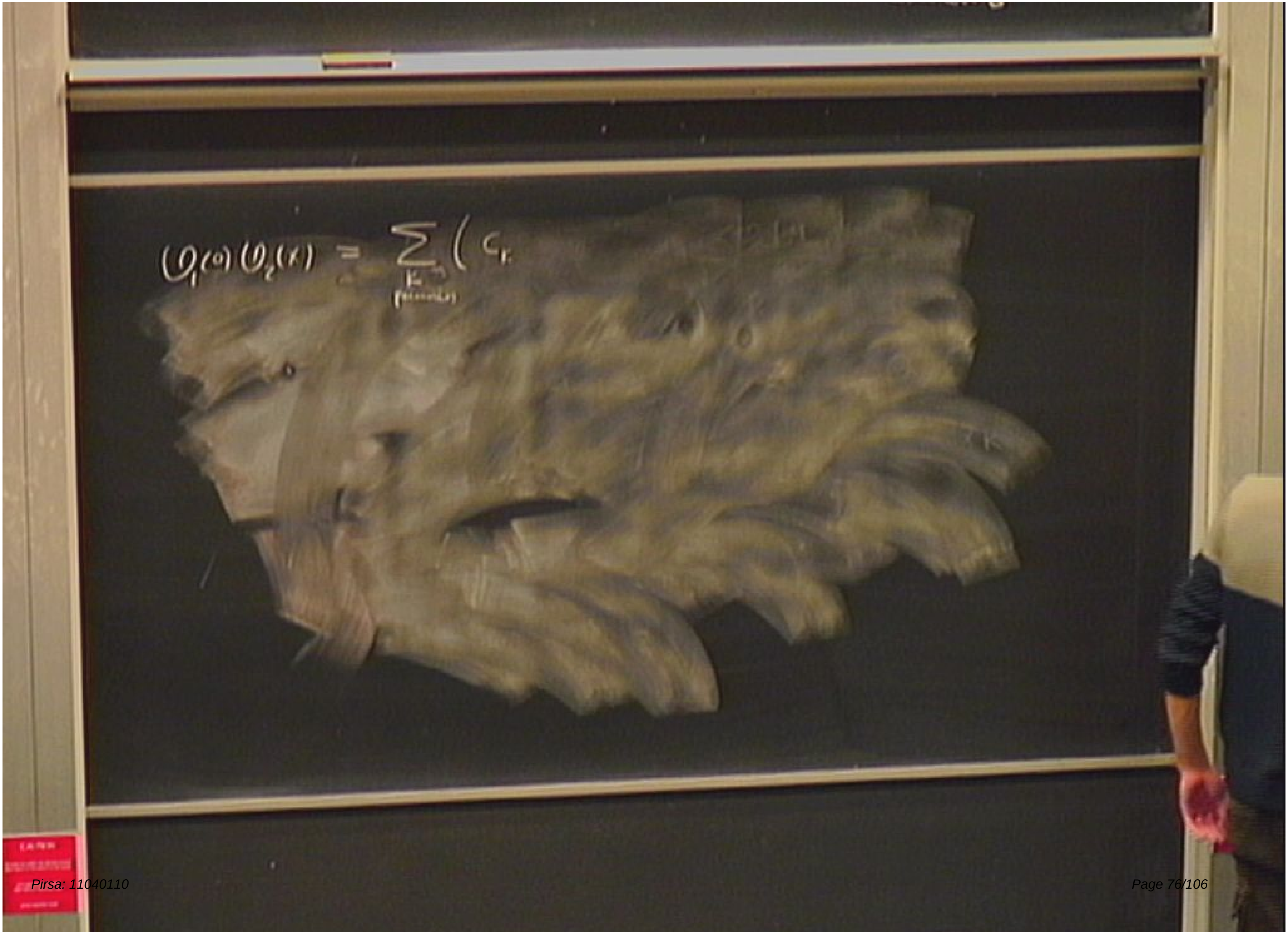


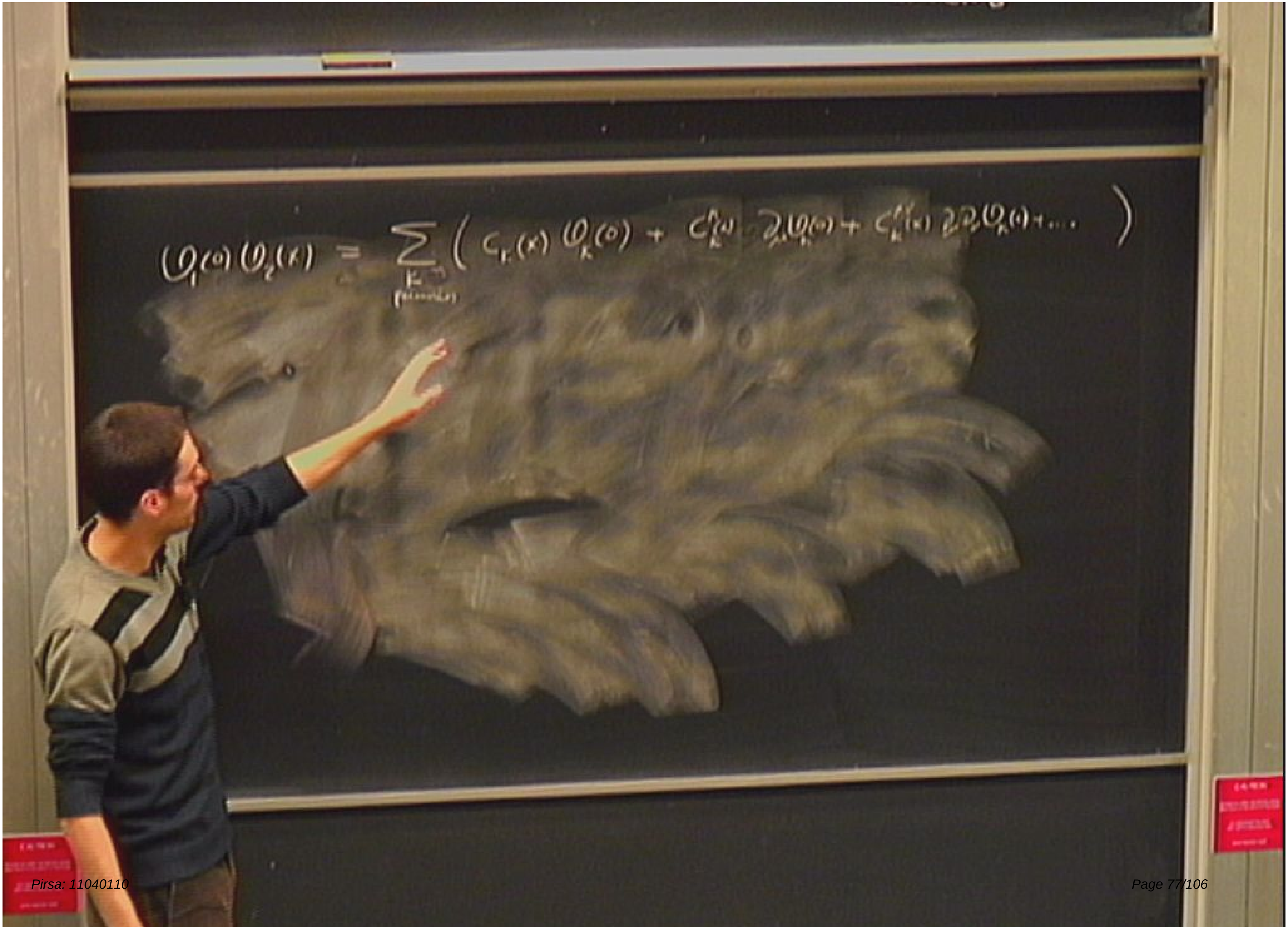
$$\mathcal{O}_i \mathcal{O}_j = \sum_i C_i(x) \; \mathcal{O}_i(0)$$

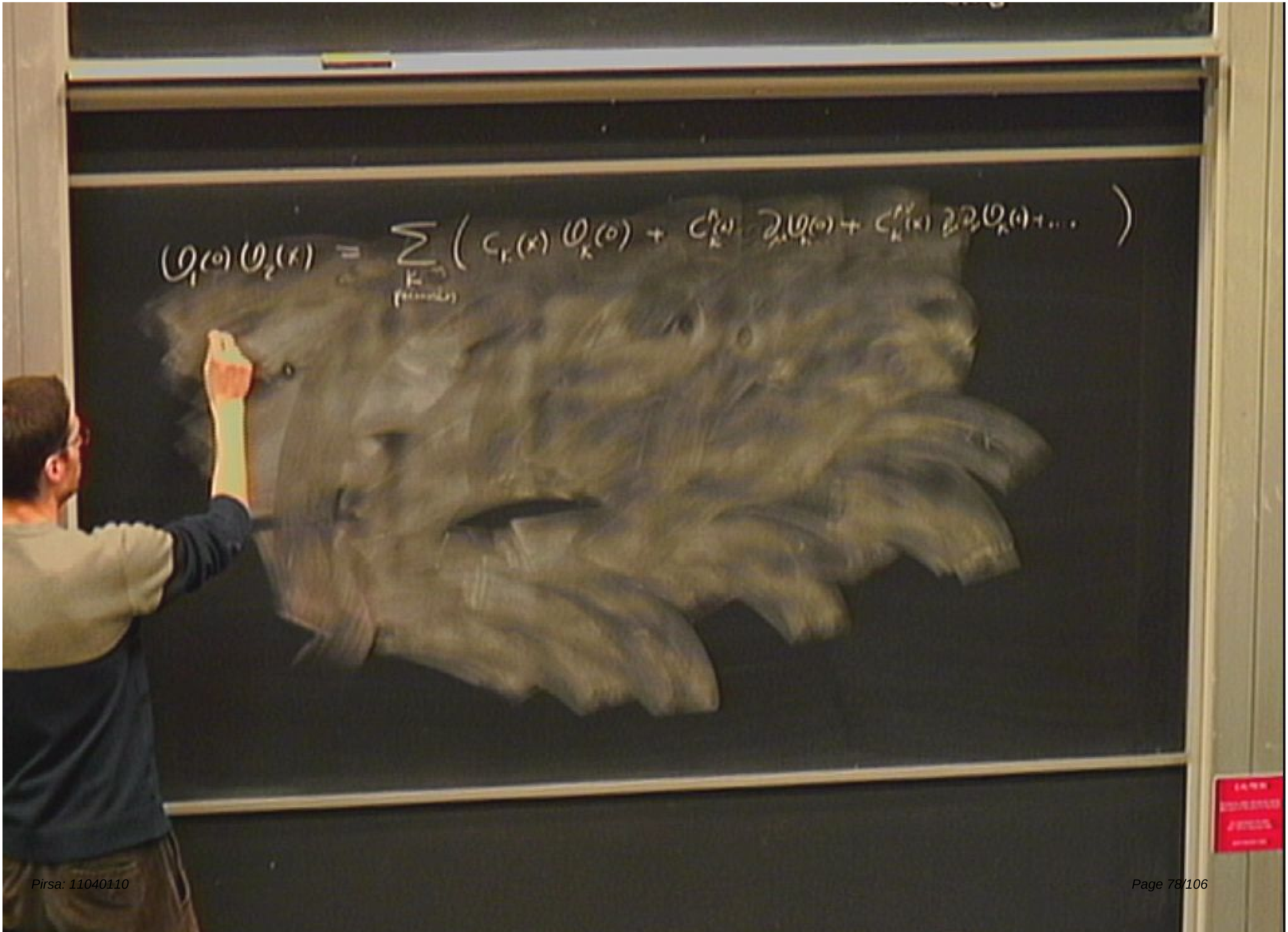
path integral inside S^{d-1} produces a state $|\Psi(x)\rangle$

$$|\Psi(x)\rangle = \sum_i C_i(x) \, |\mathcal{O}_i\rangle$$ complete set of states

$$\mathcal{O}_1(0)\, \mathcal{O}_2(x) = \sum_i C_i(x)\, \mathcal{O}_i(0)$$


$\mathcal{O}_1(0)\,\mathcal{O}_2(x) = \sum_{k \,(\text{primaries})} (c_k$


$\varphi_1(0)\varphi_2(x) = \sum_k \left(c_k(x)\,\varphi_k(0) + c_k^{\mu}(x)\,\partial_\mu\varphi_k(0) + c_k^{\mu\nu}(x)\,\partial_\mu\partial_\nu\varphi_k(0) + \dots \right)$


$\mathcal{O}_1(0)\mathcal{O}_2(x) = \sum_{k} \left(c_k(x)\,\mathcal{O}_k(0) + c_k^{\mu}(x)\,\partial_\mu \mathcal{O}_k(0) + c_k^{\mu\nu}(x)\,\partial_\mu\partial_\nu \mathcal{O}_k(0) + \dots \right)$

$$\mathcal{O}_1(0)\,\mathcal{O}_2(x) = \sum_{\substack{k \\ \text{(primaries)}}} \Big(c_k(x)\,\mathcal{O}_k(0) + c_k^{\mu}(x)\,\partial_\mu \mathcal{O}_k(0) + c_k^{\mu\nu}(x)\,\partial_\mu\partial_\nu \mathcal{O}_k(0) + \dots \Big)$$

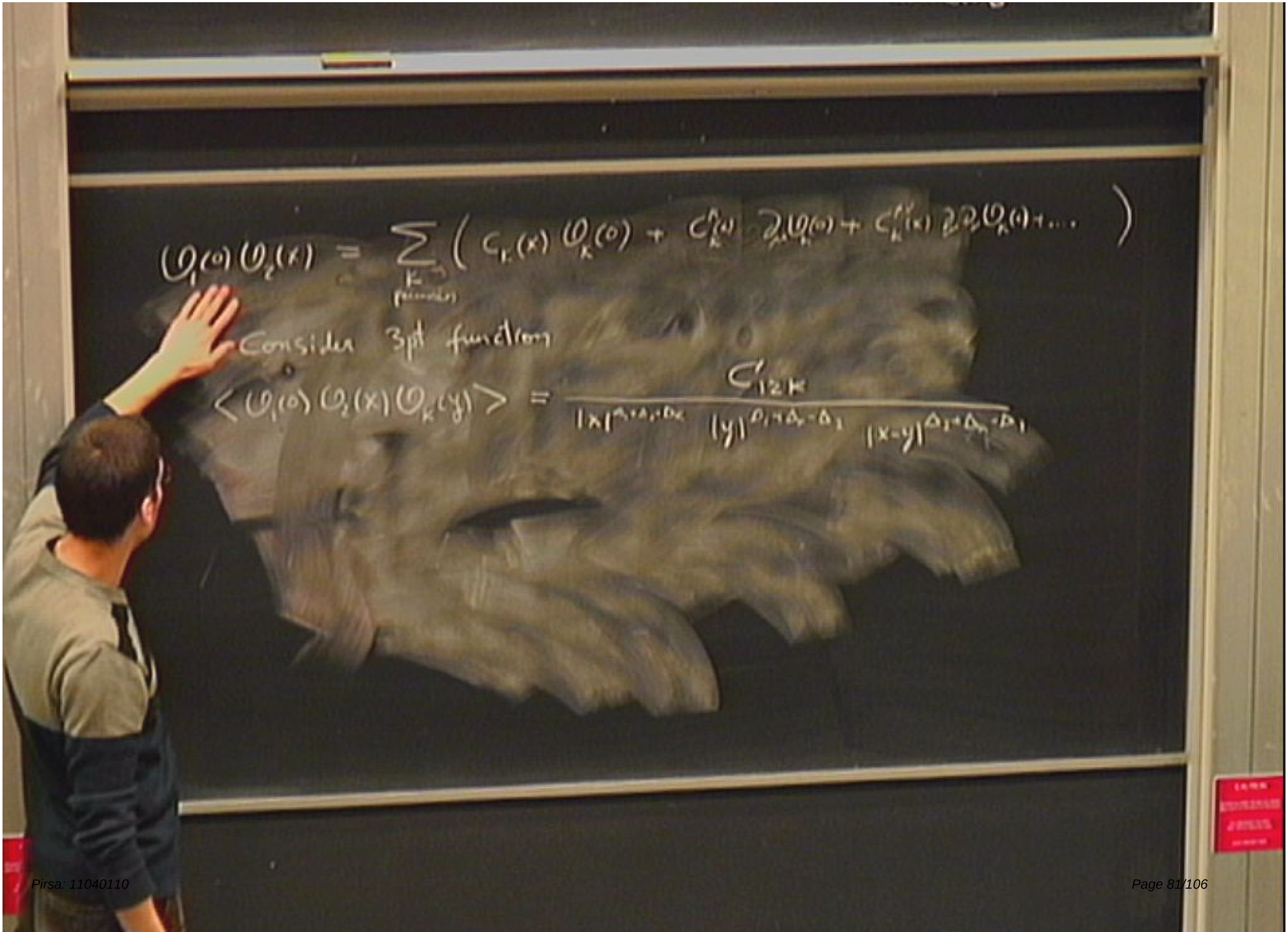
Consider 3pt function

$$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(x)\,\mathcal{O}_k(y) \rangle$$

$$\mathcal{O}_1(0)\,\mathcal{O}_2(x) = \sum_{\substack{k \\ (\text{primaries})}} \left(C_k(x)\,\mathcal{O}_k(0) + C_k^{\mu}(x)\,\partial_\mu \mathcal{O}_k(0) + C_k^{\mu\nu}(x)\,\partial_\mu\partial_\nu \mathcal{O}_k(0) + \dots \right)$$

Consider 3pt function

$$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(x)\,\mathcal{O}_k(y) \rangle = \frac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\;|y|^{\Delta_1+\Delta_k-\Delta_2}\;|x-y|^{\Delta_2+\Delta_k-\Delta_1}}$$


$\mathcal{O}_1(0)\mathcal{O}_2(x) = \sum_{\substack{k \\ (\text{primaries})}} \left(C_k(x)\mathcal{O}_k(0) + C_k^{\mu}(x)\partial_\mu\mathcal{O}_k(0) + C_k^{\mu\nu}(x)\partial_\mu\partial_\nu\mathcal{O}_k(0) + \dots \right)$
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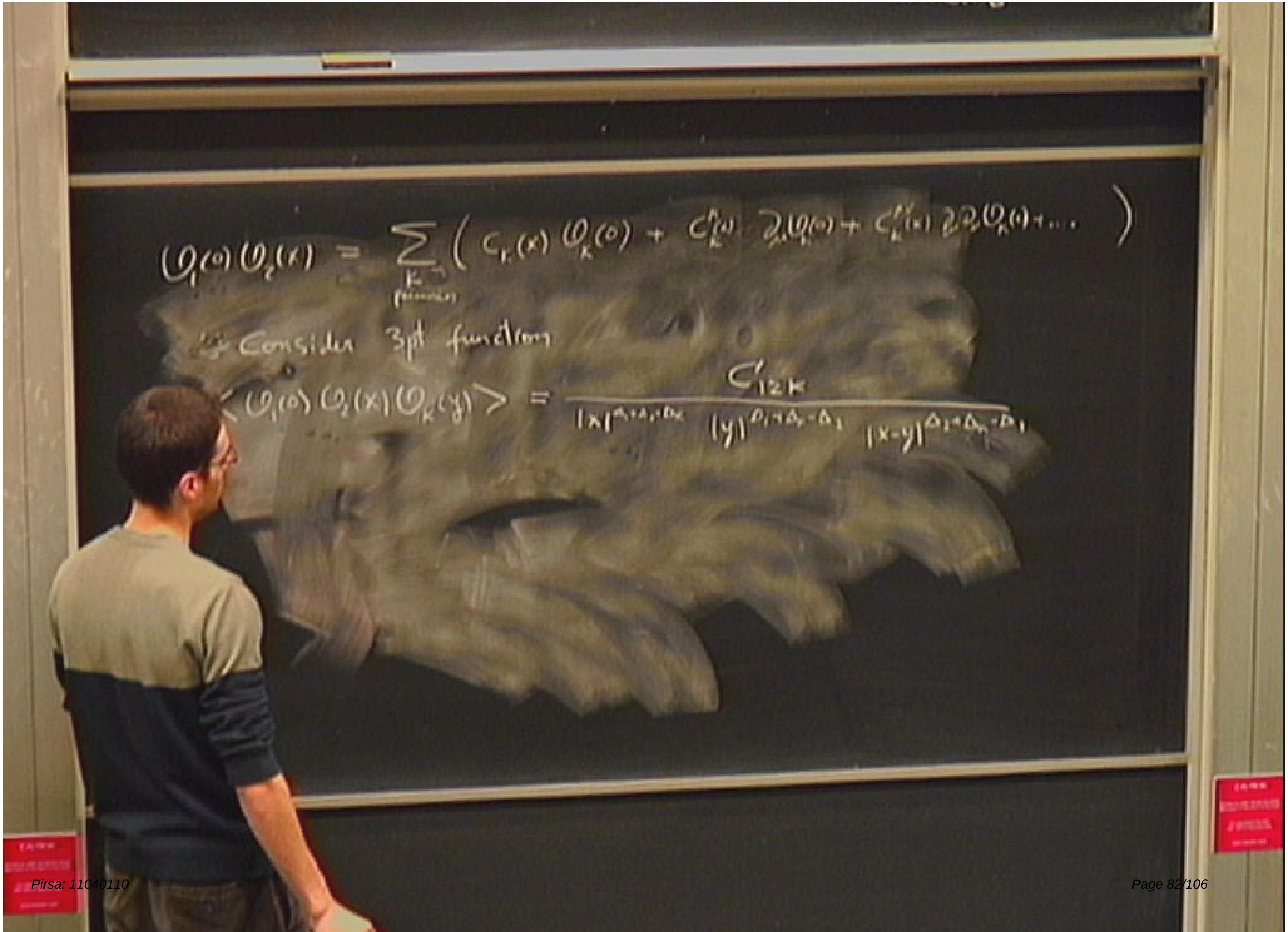
$\mathcal{O}_1(0)\mathcal{O}_2(x) = \sum_{k} \left(C_k(x)\, \mathcal{O}_k(0) + C_k^{\mu}(x)\, \partial_\mu \mathcal{O}_k(0) + C_k^{\mu\nu}(x)\, \partial_\mu\partial_\nu \mathcal{O}_k(0) + \dots \right)$
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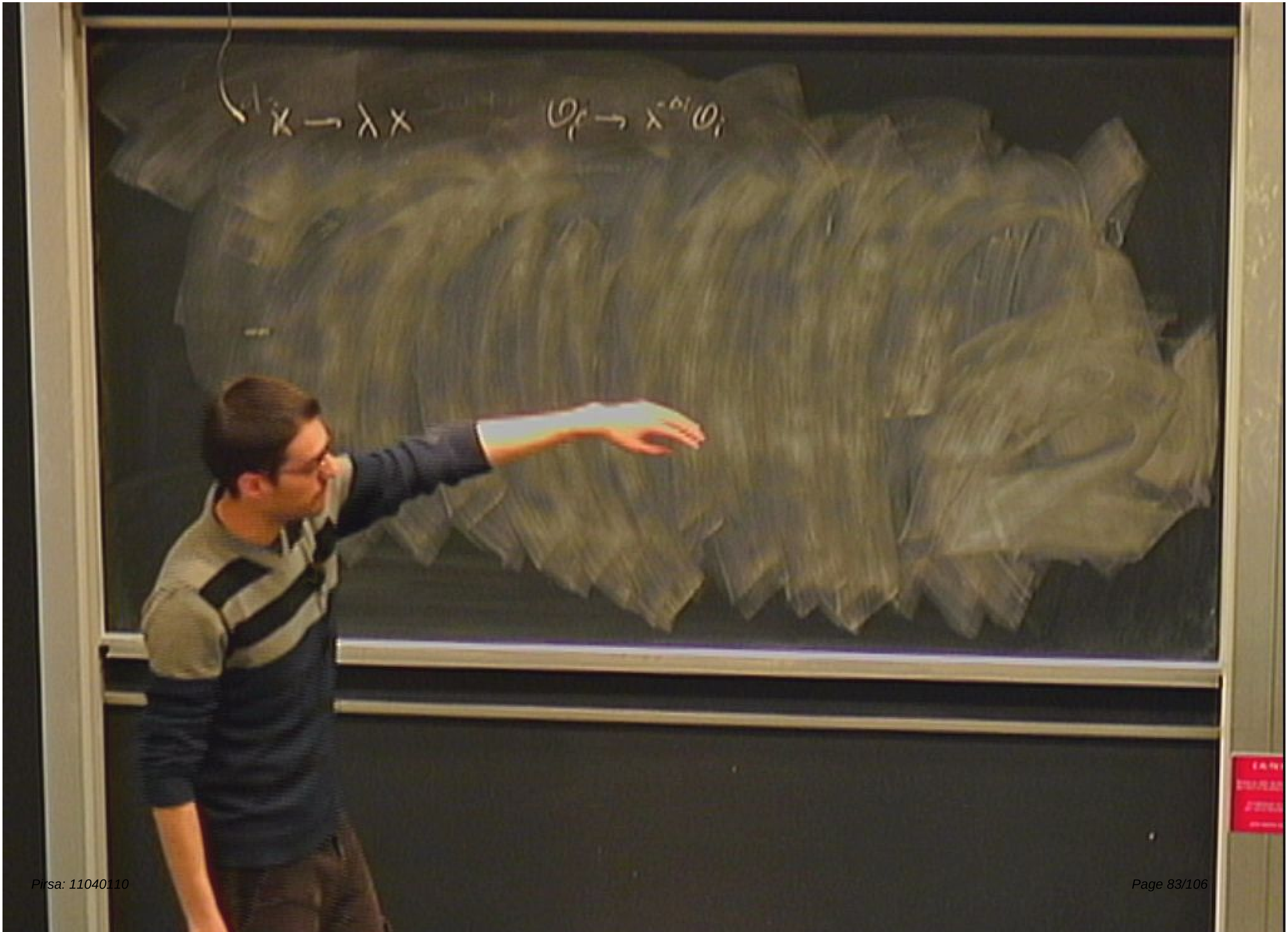
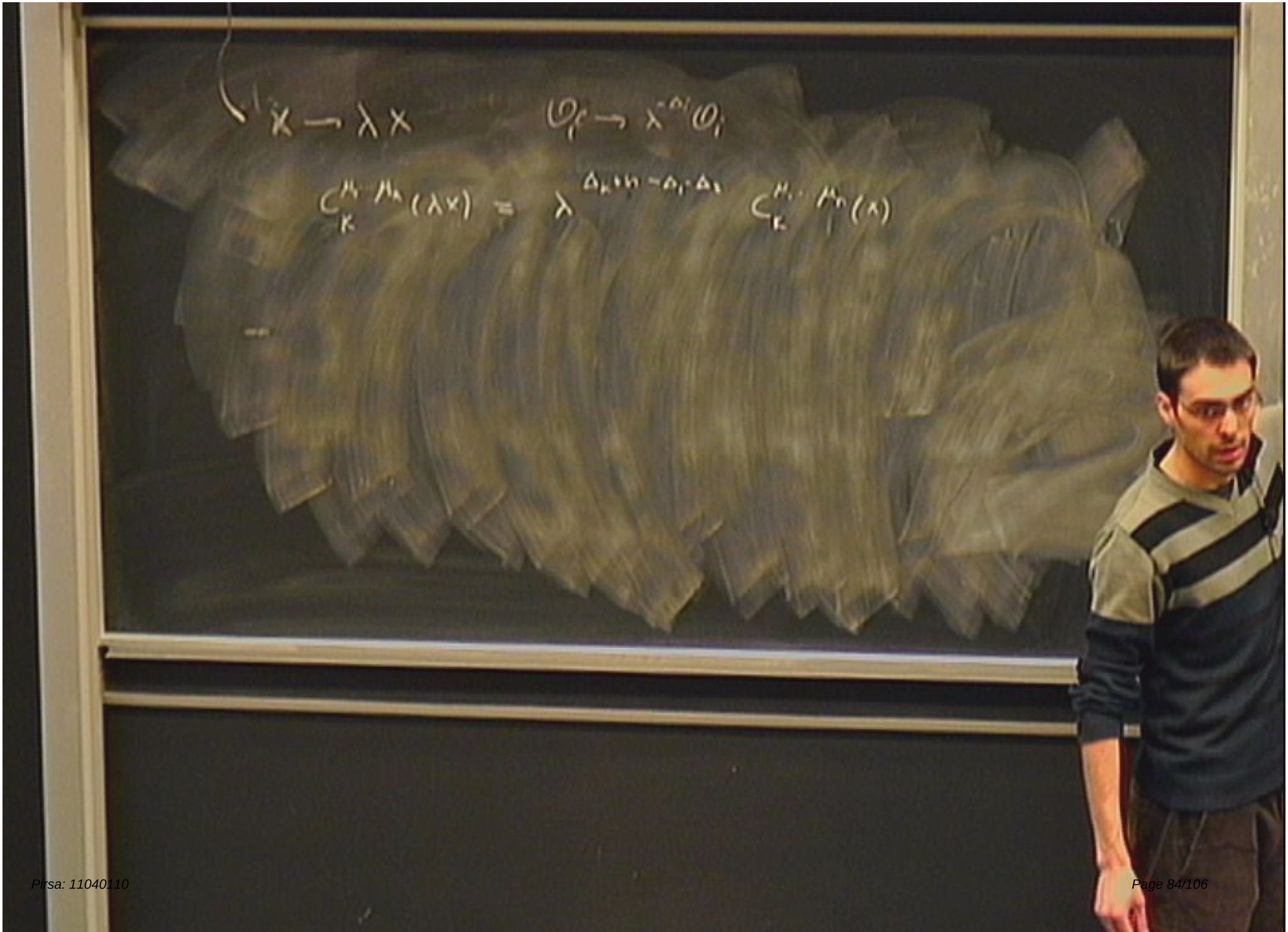
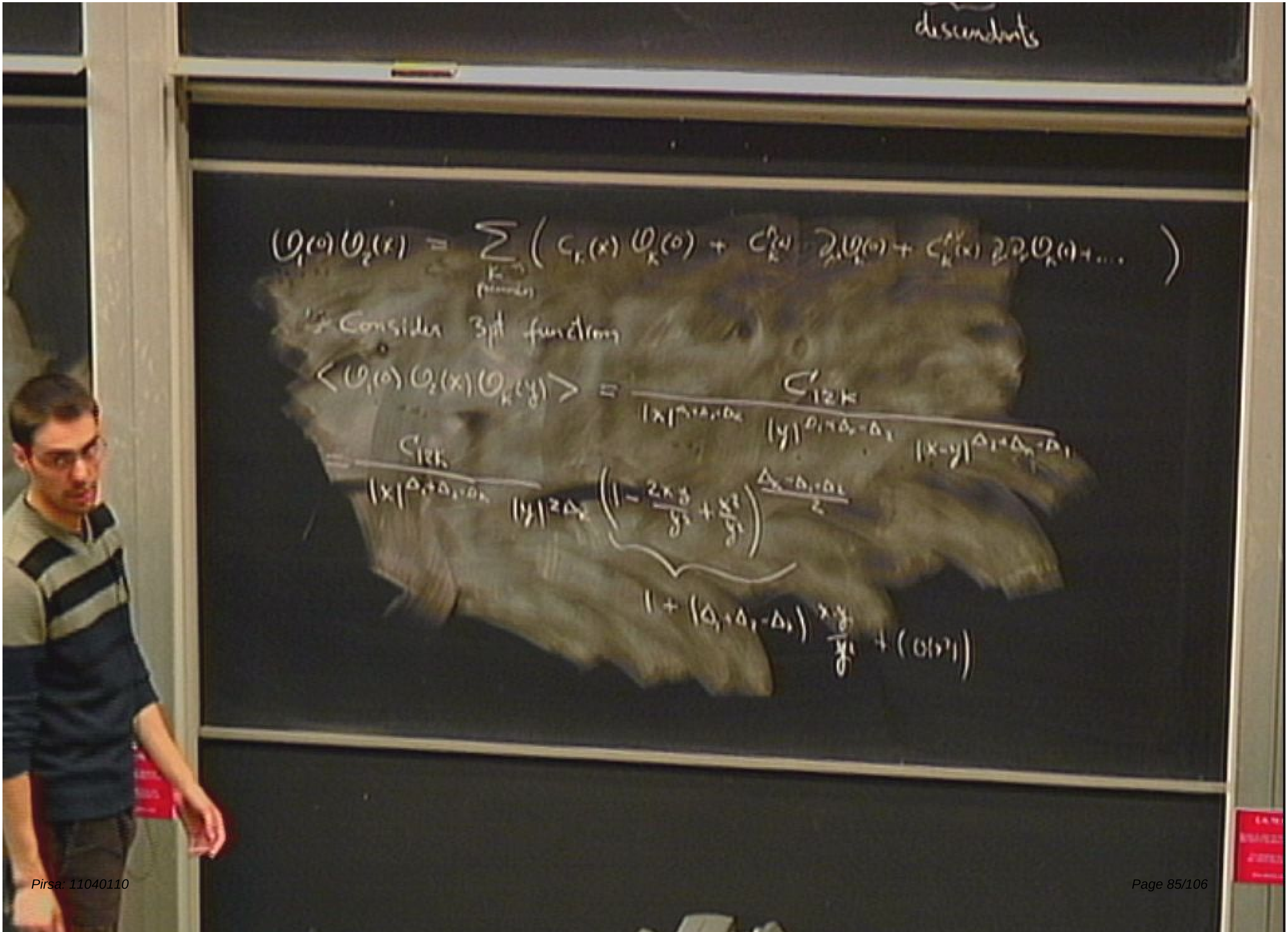
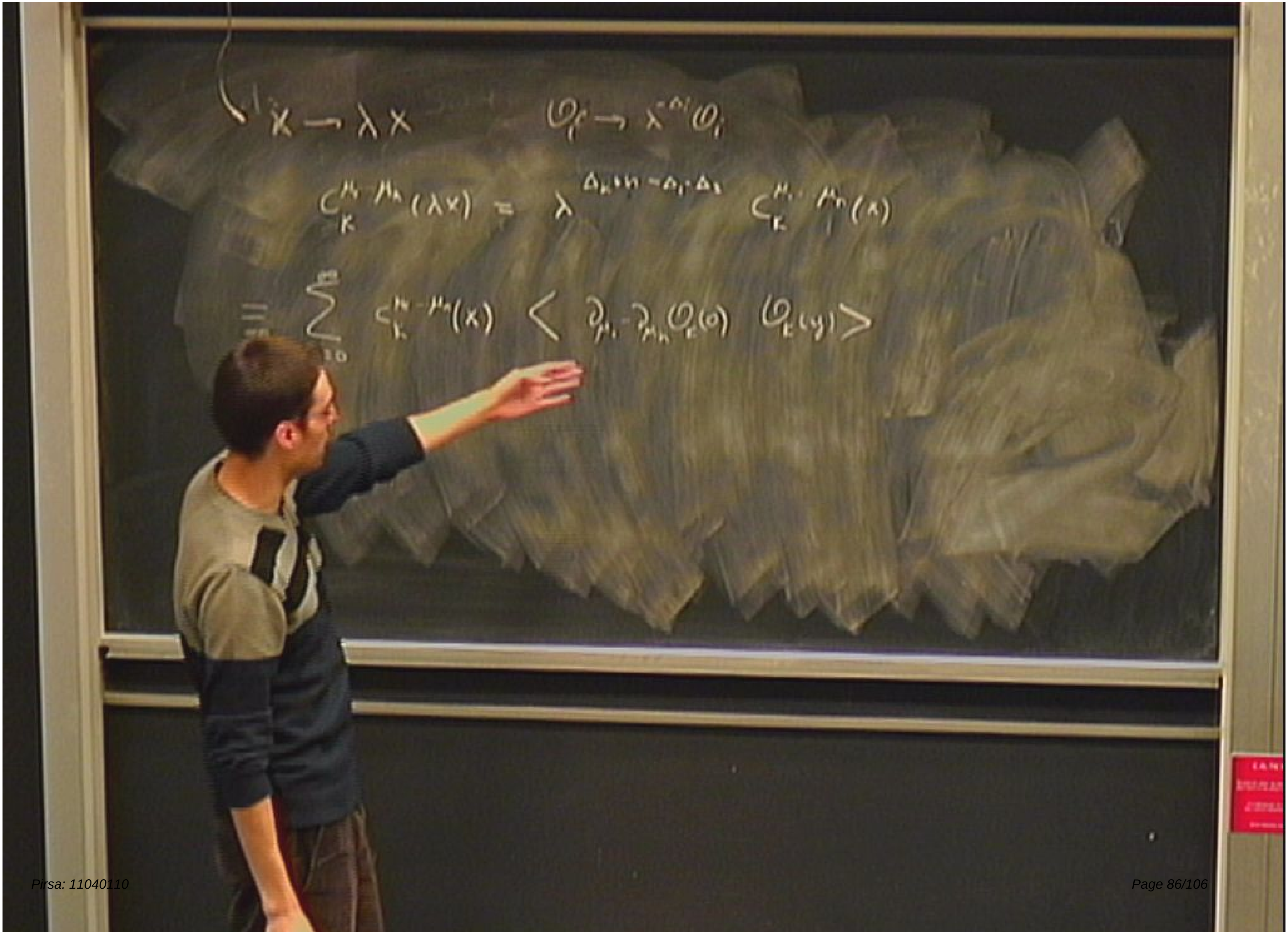
$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(x)\,\mathcal{O}_k(y) \rangle = \dfrac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{\Delta_1+\Delta_k-\Delta_2}\,|x-y|^{\Delta_2+\Delta_k-\Delta_1}}$

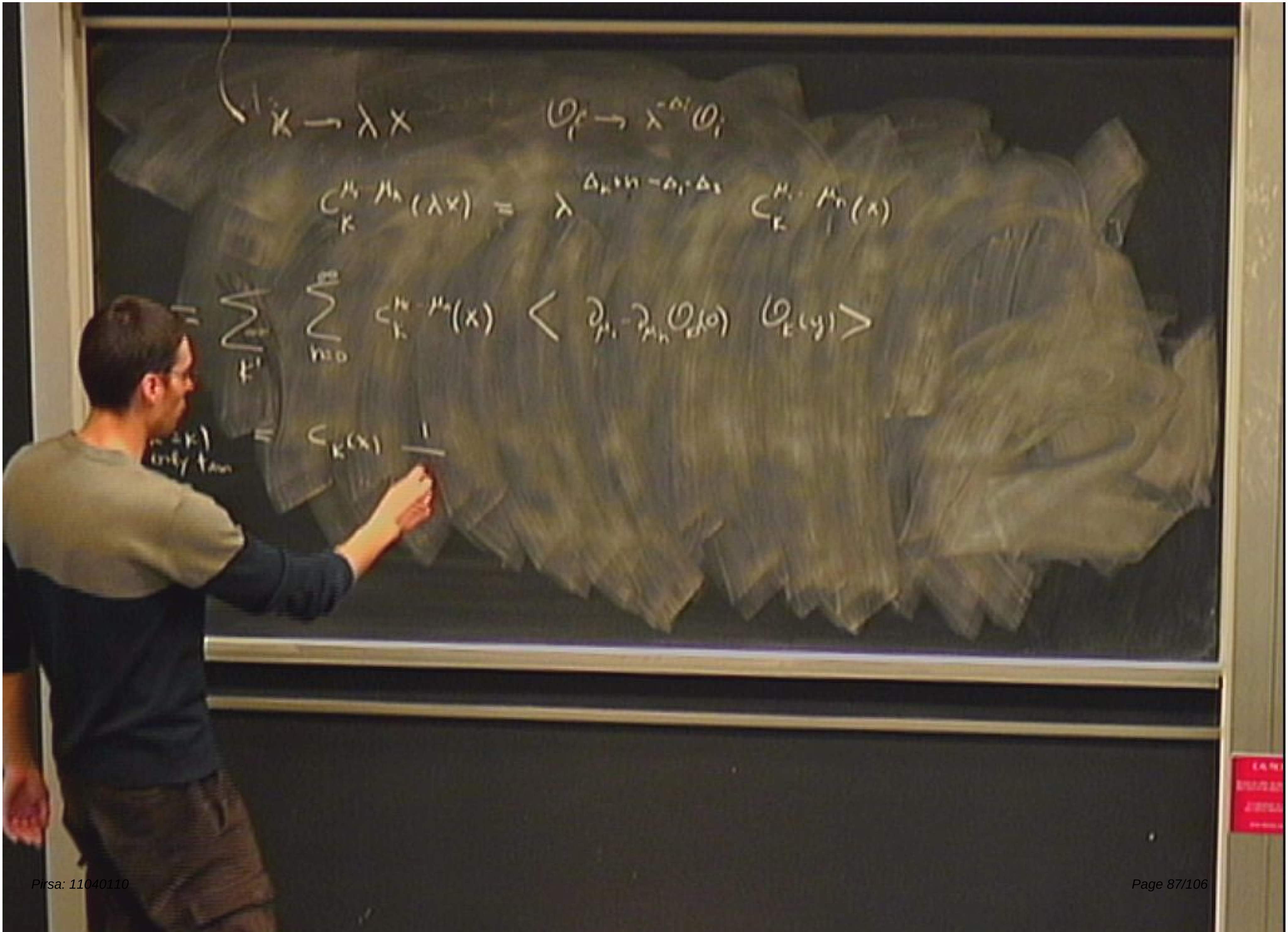

$x \to \lambda x$
$\mathcal{O}_i \to \lambda^{-\Delta_i}\mathcal{O}_i$



$x \to \lambda x$
$\mathcal{O}_i \to \lambda^{-\Delta_i} \mathcal{O}_i$
$C_k^{\mu_1 \cdots \mu_n}(\lambda x) = \lambda^{\Delta_k + n - \Delta_1 - \Delta_2} C_k^{\mu_1 \cdots \mu_n}(x)$


descendants
$\mathcal{O}_1(0)\,\mathcal{O}_2(x) = \sum_{k\,(\text{primaries})} \left(c_k(x)\,\mathcal{O}_k(0) + c_k^{\mu}\,\partial_\mu \mathcal{O}_k(0) + c_k^{\mu\nu}(x)\,\partial_\mu\partial_\nu \mathcal{O}_k(0) + \dots \right)$
Consider 3pt function
$\langle \mathcal{O}_1(0)\,\mathcal{O}_2(x)\,\mathcal{O}_k(y) \rangle = \dfrac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{\Delta_1+\Delta_k-\Delta_2}\,|x-y|^{\Delta_2+\Delta_k-\Delta_1}}$
$= \dfrac{C_{12k}}{|x|^{\Delta_1+\Delta_2-\Delta_k}\,|y|^{2\Delta_k}} \underbrace{\left(1 - \dfrac{2x\cdot y}{y^2} + \dfrac{x^2}{y^2}\right)^{\frac{\Delta_k-\Delta_2+\Delta_1}{2}}}_{1 + (\Delta_k+\Delta_2-\Delta_1)\frac{x\cdot y}{y^2} + (O(x^2))}$


$x \to \lambda x$ $\quad \mathcal{O}_i \to \lambda^{-\Delta_i} \mathcal{O}_i$
$C_k^{\mu_1 \cdots \mu_n}(\lambda x) = \lambda^{\Delta_k + n - \Delta_i - \Delta_j} C_k^{\mu_1 \cdots \mu_n}(x)$
$= \sum_{k=0}^{\infty} C_k^{\mu_1 \cdots \mu_n}(x) \langle \partial_{\mu_1} \cdots \partial_{\mu_n} \mathcal{O}_k(0) \, \mathcal{O}_k(y) \rangle$

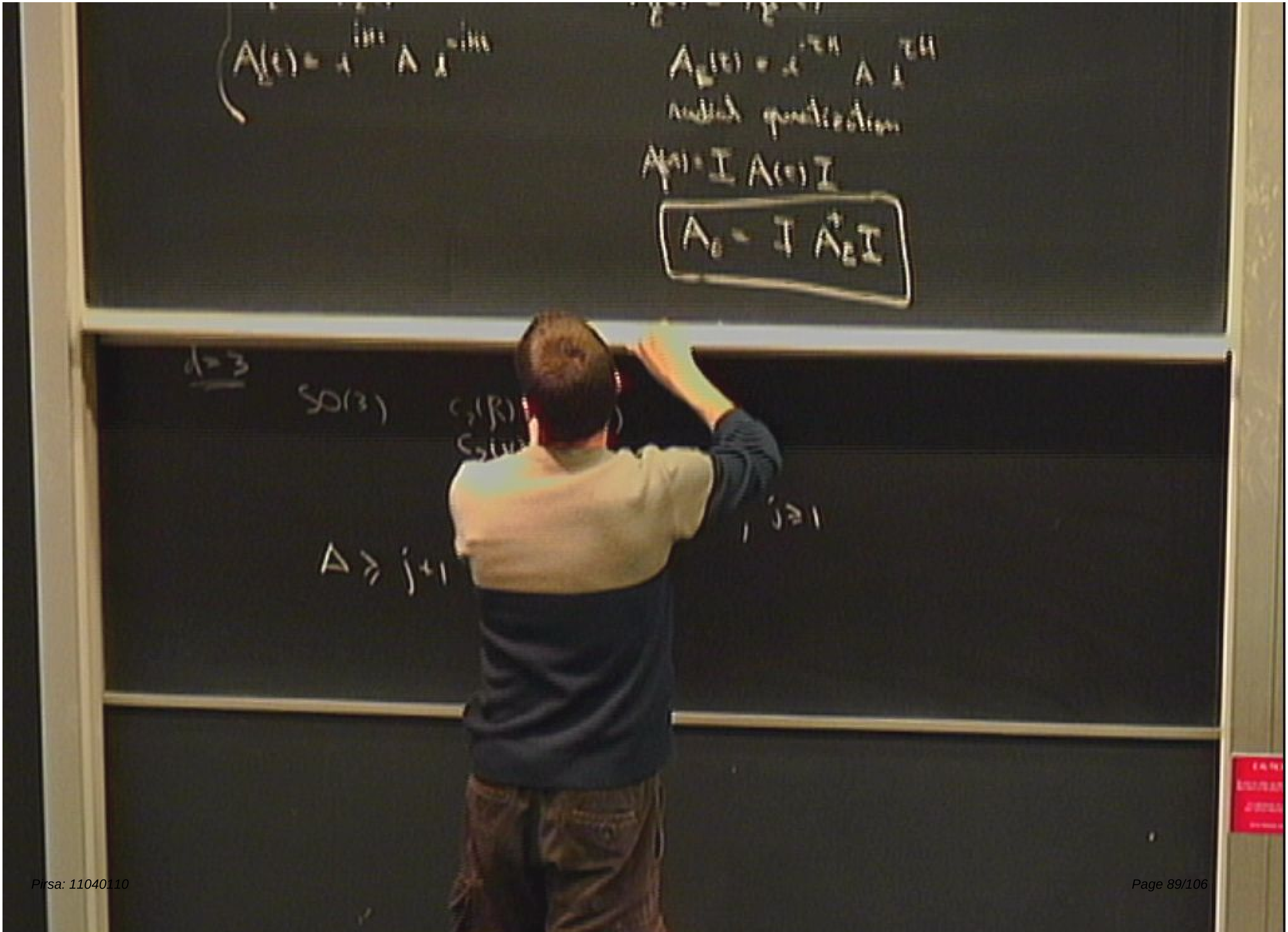

x → λx
O_i → λ^{-Δi} O_i

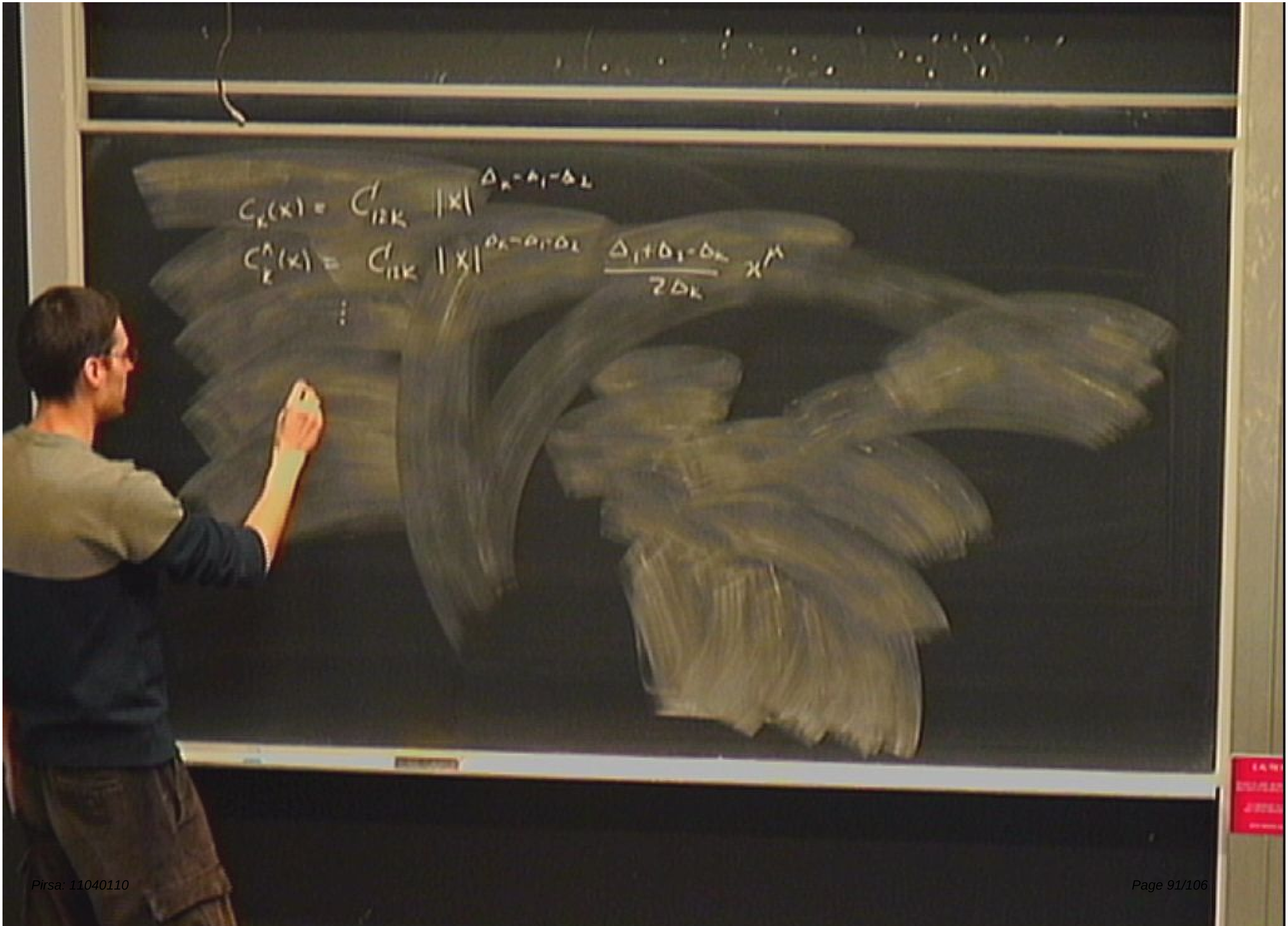
$$x \to \lambda x \qquad \mathcal{O}_i \to \lambda^{-\Delta_i}\mathcal{O}_i$$

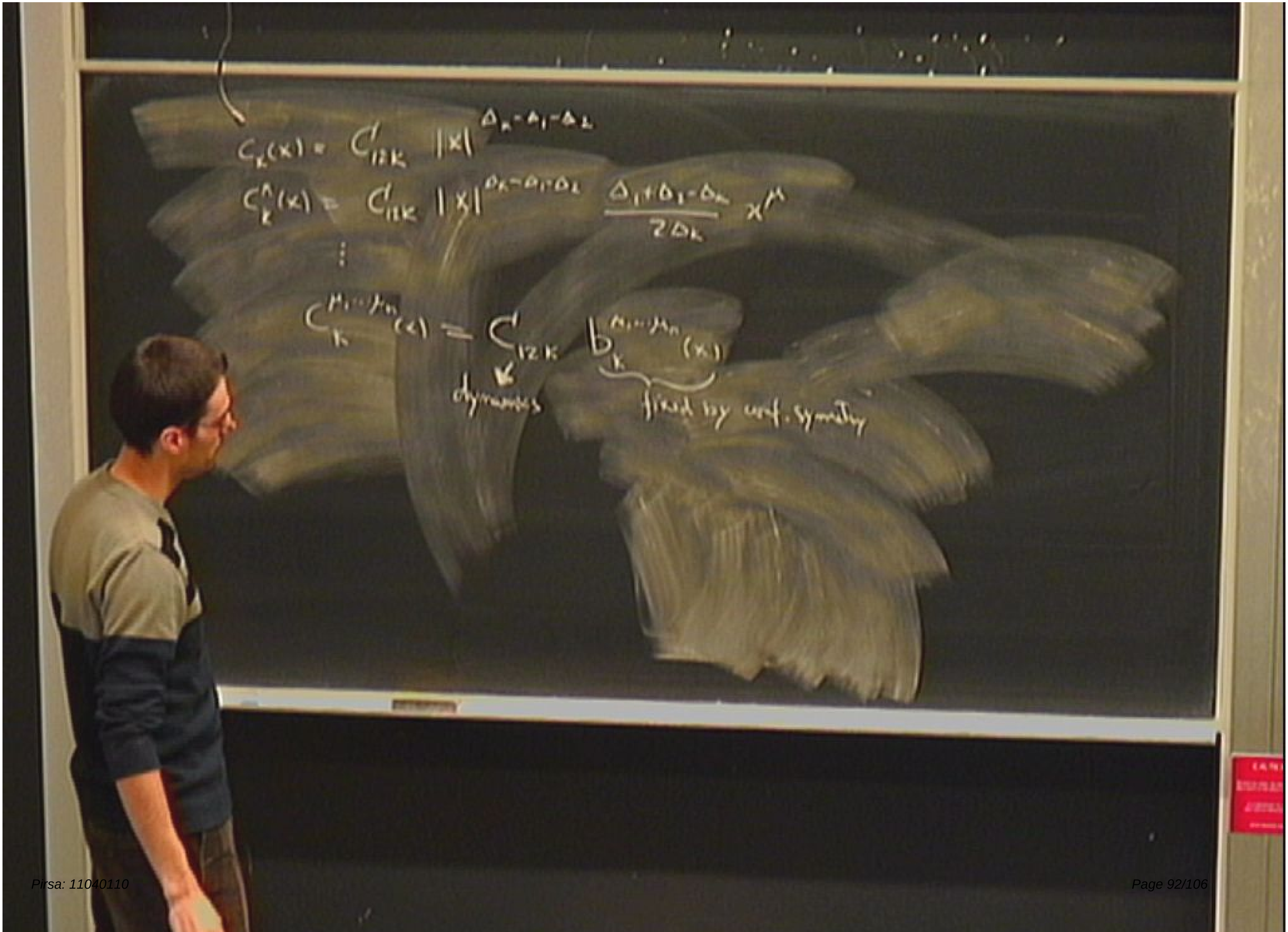
$$C_k^{\mu_1\cdots\mu_n}(\lambda x) = \lambda^{\Delta_k + n - \Delta_1 - \Delta_2} C_k^{\mu_1\cdots\mu_n}(x)$$

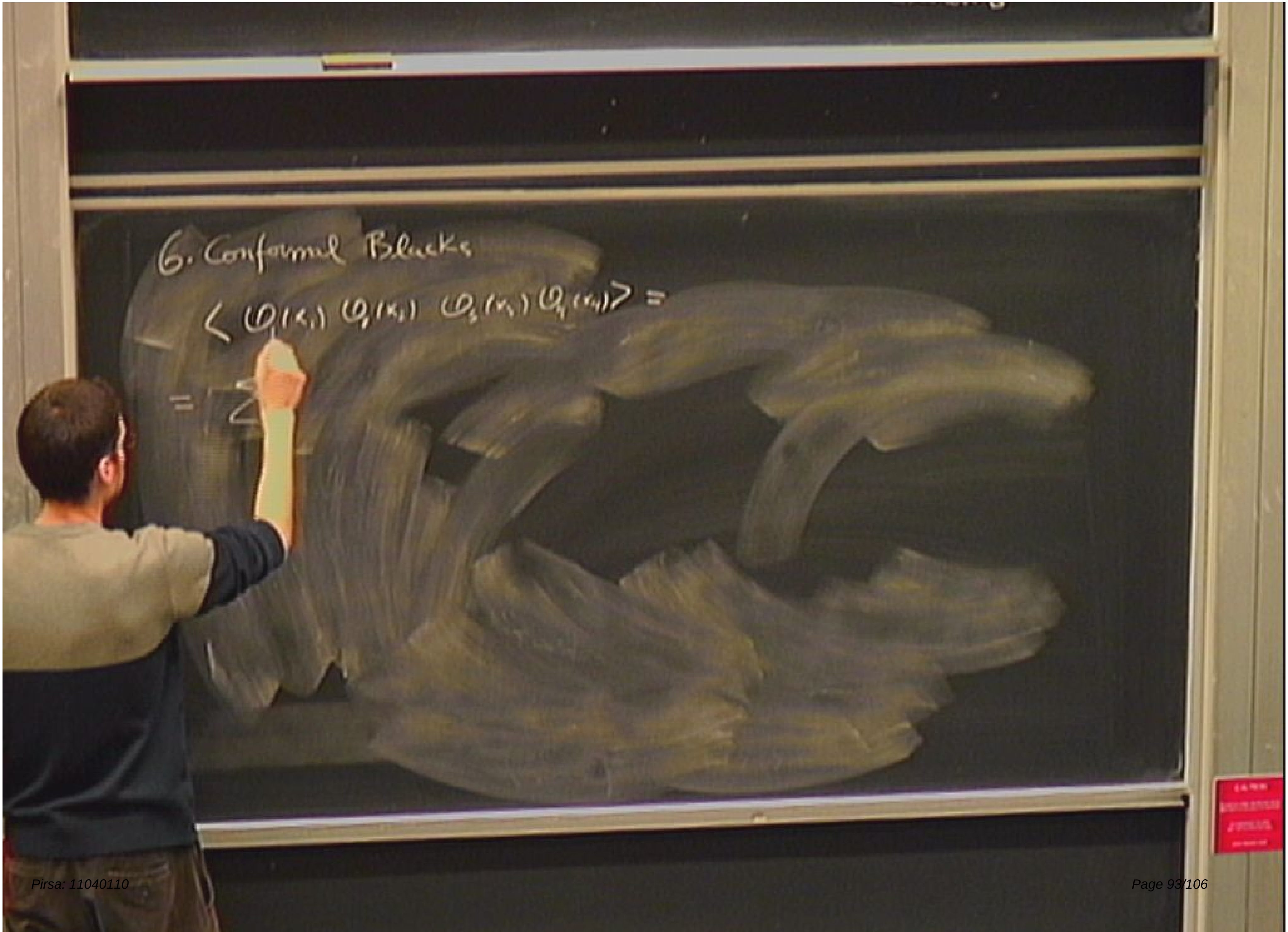
$$= \sum_{k'} \sum_{n=0}^{\infty} C_k^{\mu_1\cdots\mu_n}(x) \left\langle \partial_{\mu_1}\cdots\partial_{\mu_n}\mathcal{O}_{k'}(0)\ \mathcal{O}_k(y)\right\rangle$$

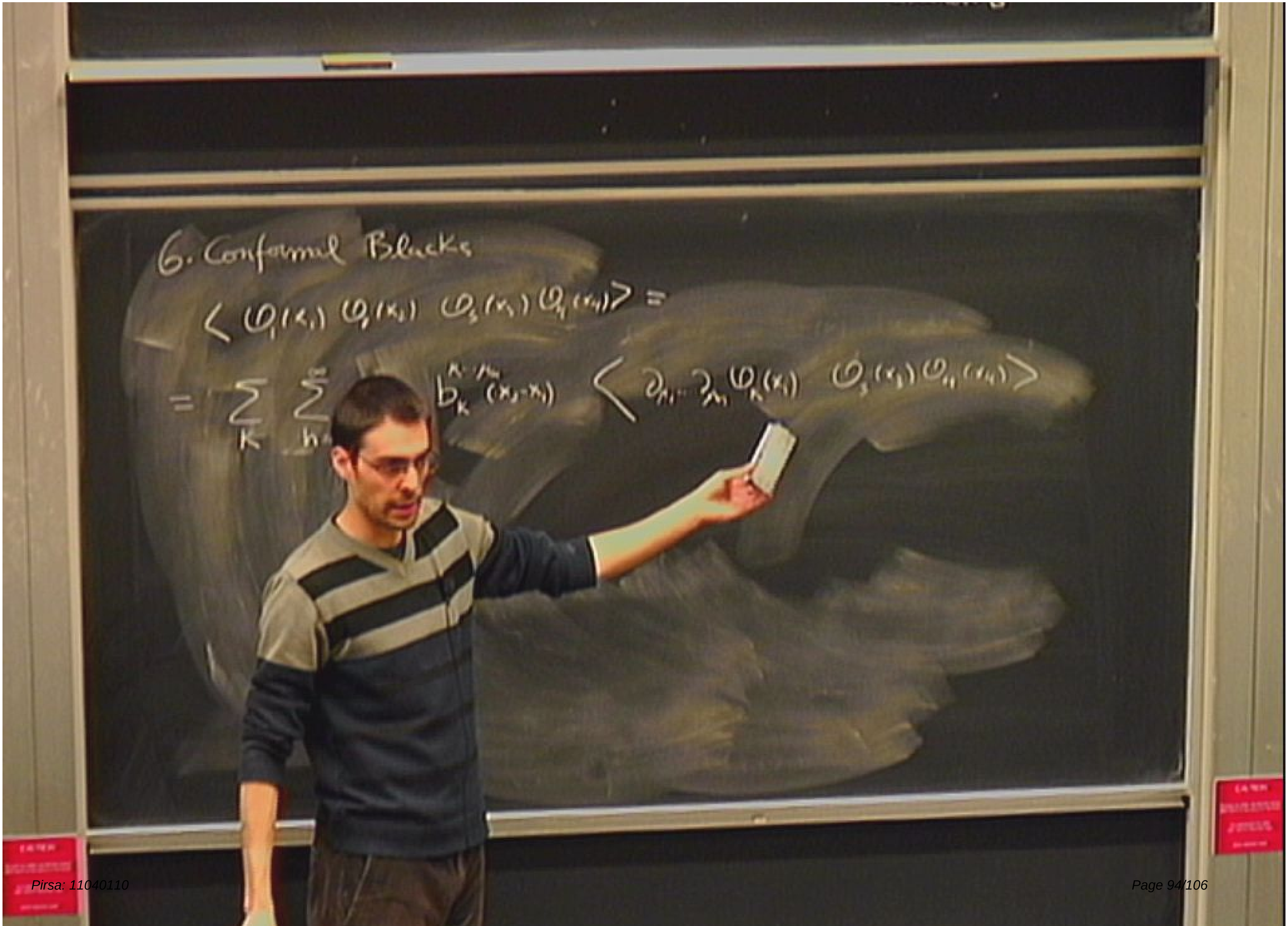
($k'=k$ only term) $$= C_k(x)\,\frac{1}{|y|^{2\Delta_k}}$$

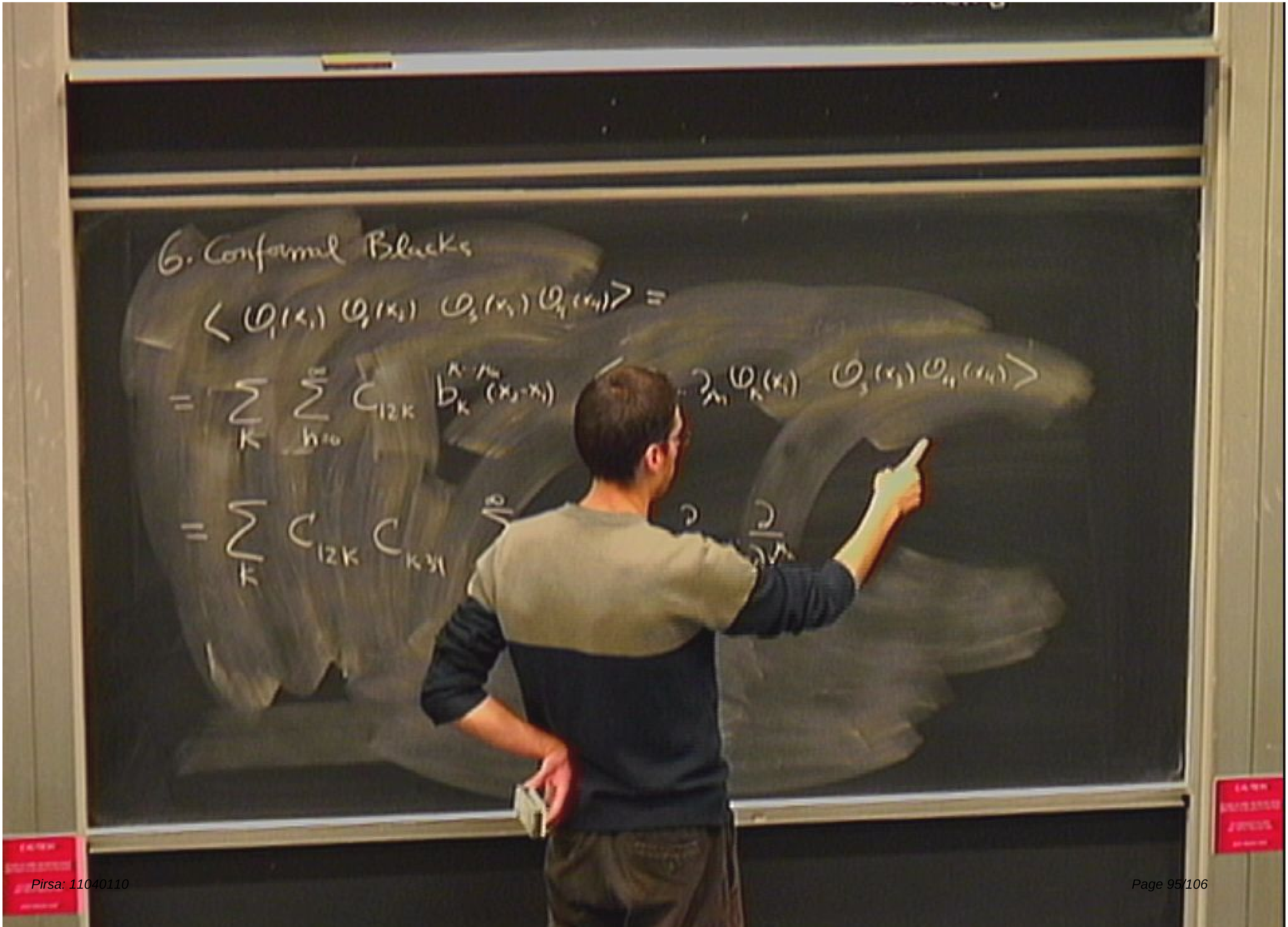

radial quantization
d=3
SO(3)
Δ ≥ j+1
j≥1

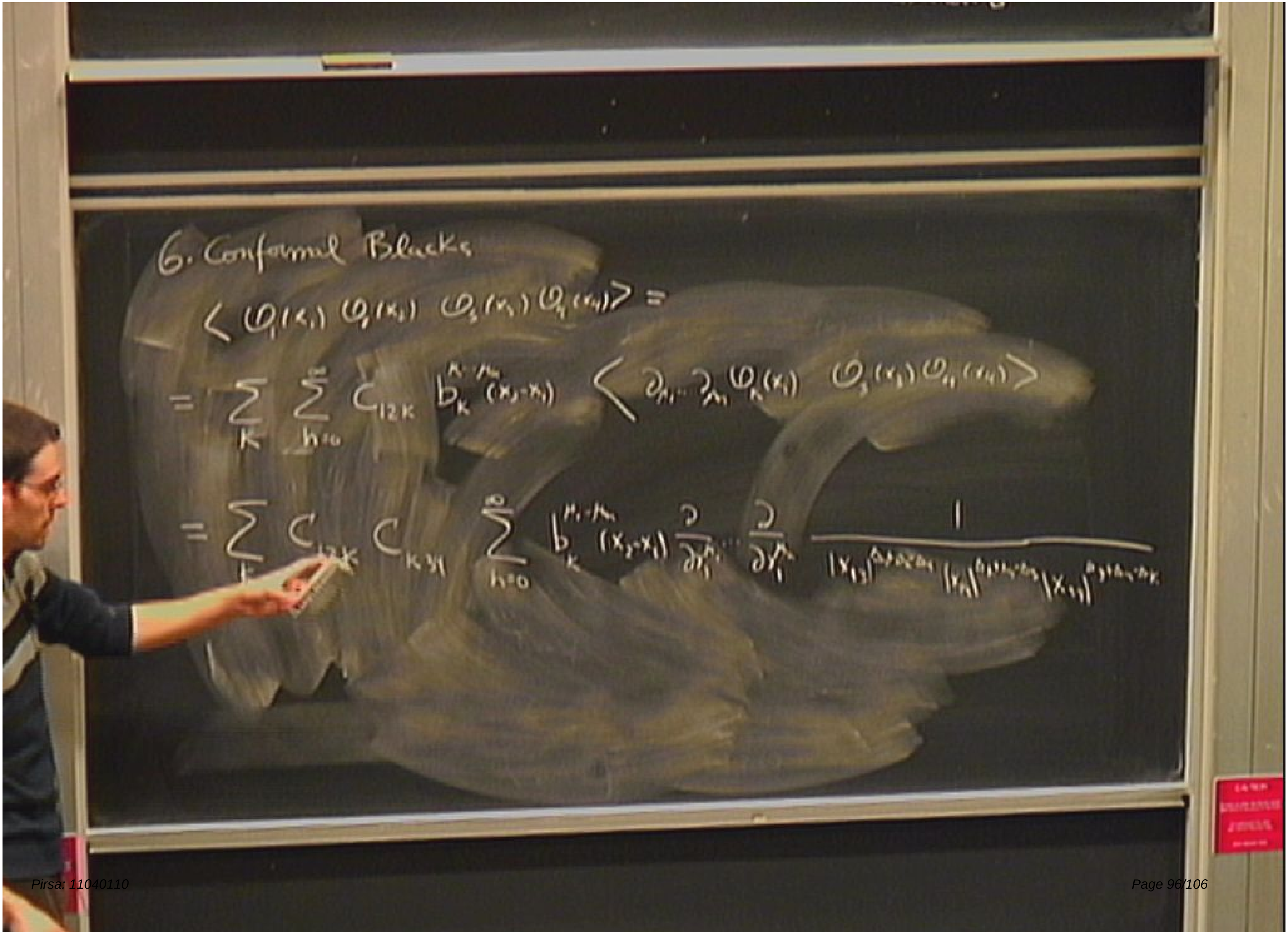

$C_k(x) = C_{12k} |x|^{\Delta_k - \Delta_1 - \Delta_2}$
$C_k^\mu(x) = C_{12k} |x|^{\Delta_k - \Delta_1 - \Delta_2} \frac{\Delta_1 + \Delta_2 - \Delta_k}{2\Delta_k} x^\mu$


$C_k(x) = C_{12k}\, |x|^{\Delta_k - \Delta_1 - \Delta_2}$
$C_k^{\mu}(x) = C_{12k}\, |x|^{\Delta_k - \Delta_1 - \Delta_2}\, \frac{\Delta_1 + \Delta_2 - \Delta_k}{2\Delta_k}\, x^{\mu}$
$\vdots$
$C_k^{\mu_1 \dots \mu_n}(x) = C_{12k}\, b_k^{\mu_1 \dots \mu_n}(x)$
dynamics
fixed by conf. symmetry


6. Conformal Blocks
< O1(x1) O2(x2) O3(x3) O4(x4) > =
= Σ


6. Conformal Blocks
⟨φ1(x1) φ2(x2) φ3(x3) φ4(x4)⟩ =
= Σ_k Σ b_k^{μ1…μn}(x2−x1) ⟨∂μ1…∂μn φk(x1) φ3(x3) φ4(x4)⟩


6. Conformal Blocks
< O1(x1) O2(x2) O3(x3) O4(x4)> =


6. Conformal Blocks
$\langle \phi_1(x_1)\, \phi_2(x_2)\, \phi_3(x_3)\, \phi_4(x_4) \rangle =$
$= \sum_k \sum_{n=0}^{\infty} C_{12k}\, b_k^{\mu_1 \cdots \mu_n}(x_2-x_1) \langle \partial_{\mu_1} \cdots \partial_{\mu_n} \phi_k(x_1)\, \phi_3(x_3)\, \phi_4(x_4) \rangle$
$= \sum_k C_{12k}\, C_{k34} \sum_{n=0}^{\infty} b_k^{\mu_1 \cdots \mu_n}(x_2-x_1) \frac{\partial}{\partial y_1^{\mu_1}} \cdots \frac{\partial}{\partial y_1^{\mu_n}}$

6. Conformal Blocks

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_2(x_2)\,\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle =$$

$$= \sum_k \sum_{n=0}^{\infty} C_{12k}\, b_k^{\mu_1\cdots\mu_n}(x_2-x_1)\,\langle \partial_{\mu_1}\cdots\partial_{\mu_n}\mathcal{O}_k(x_1)\;\mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\rangle$$

$$= \sum_k C_{12k}\, C_{k34} \underbrace{\sum_{n=0}^{\infty} b_k^{\mu_1\cdots\mu_n}(x_2-x_1)\,\frac{\partial}{\partial x_1^{\mu_1}}\cdots\frac{\partial}{\partial x_1^{\mu_n}}\,\frac{1}{|x_{13}|^{\Delta_k+\Delta_3-\Delta_4}\,|x_{14}|^{\Delta_k+\Delta_4-\Delta_3}\,|x_{34}|^{\Delta_3+\Delta_4-\Delta_k}}}$$

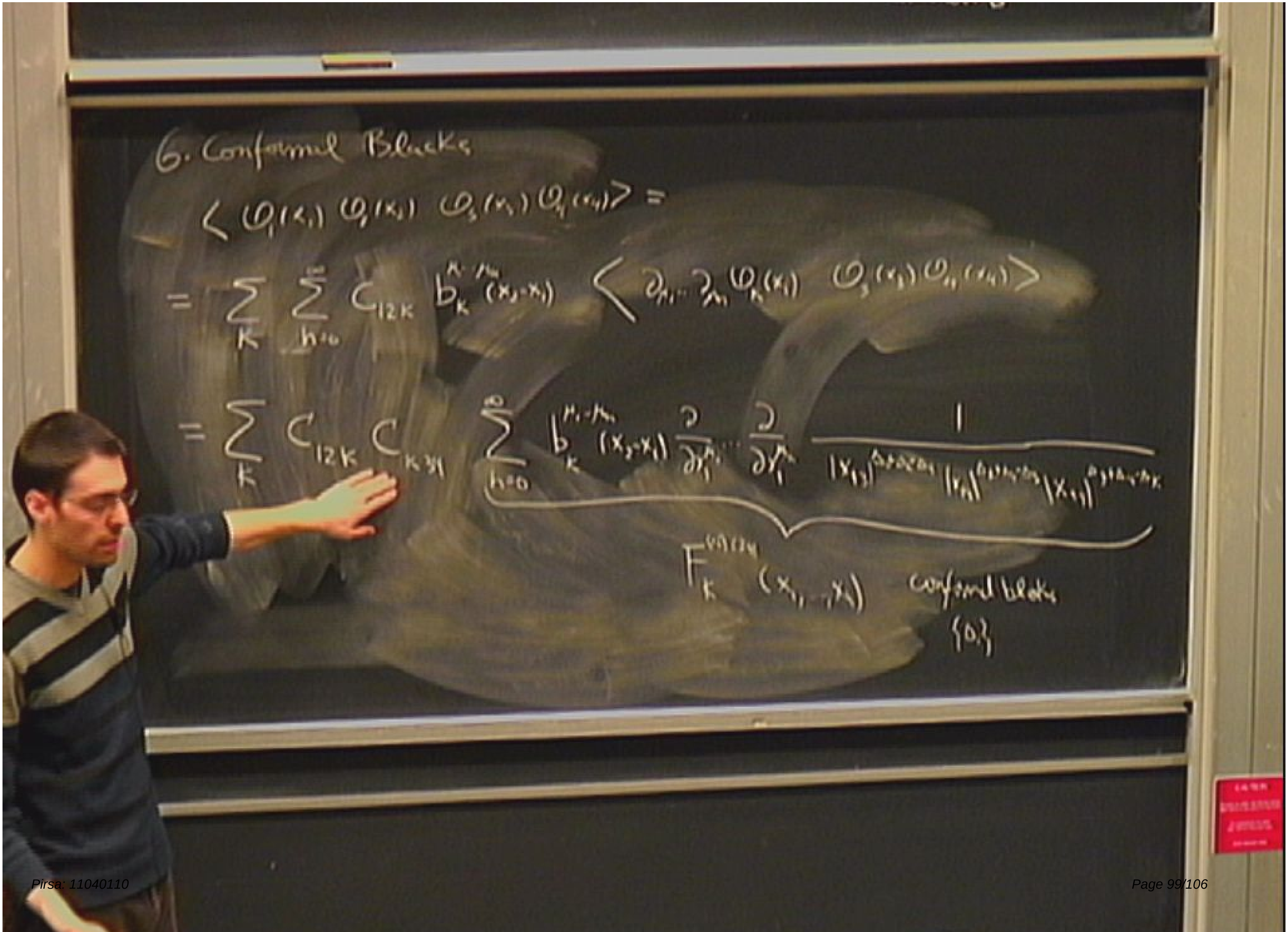
6. Conformal Blocks

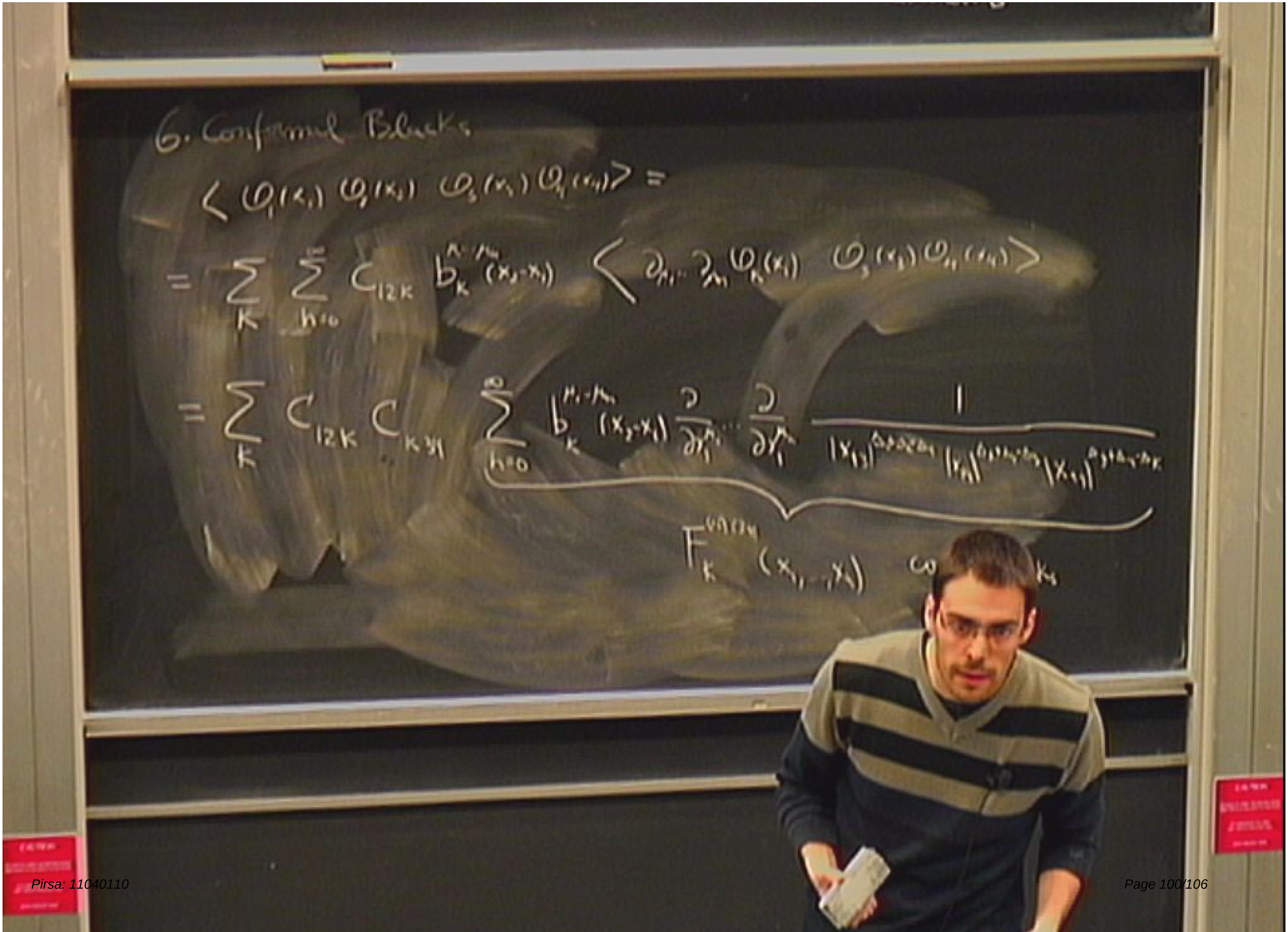
$$\langle \mathcal{O}_1(x_1)\, \mathcal{O}_2(x_2)\, \mathcal{O}_3(x_3)\, \mathcal{O}_4(x_4)\rangle =$$

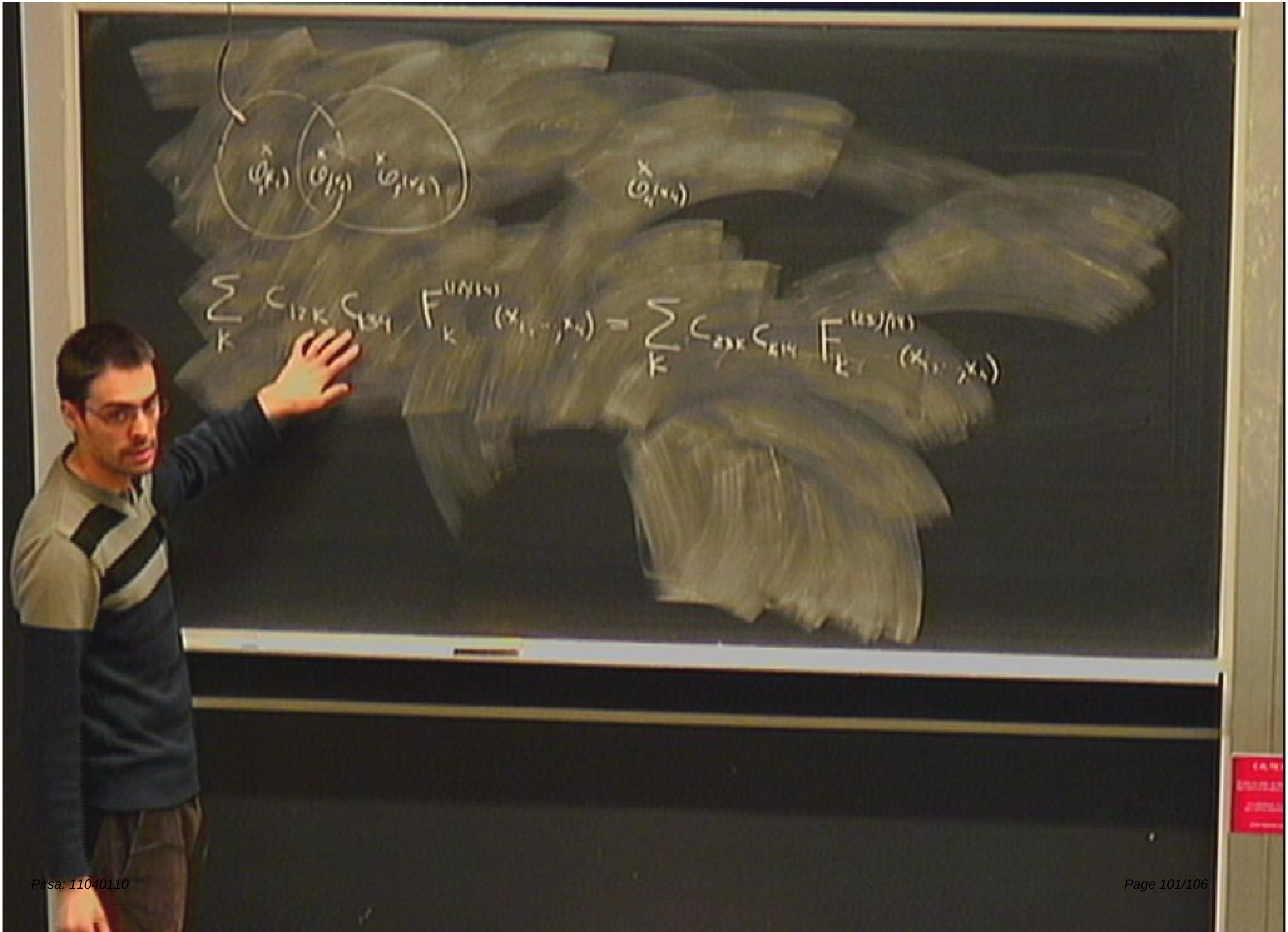
$$= \sum_k \sum_{n=0}^{\infty} C_{12k}\, b_k^{\mu_1\cdots\mu_n}(x_2-x_1) \left\langle \partial_{\mu_1}\cdots\partial_{\mu_n}\mathcal{O}_k(x_1)\; \mathcal{O}_3(x_3)\,\mathcal{O}_4(x_4)\right\rangle$$

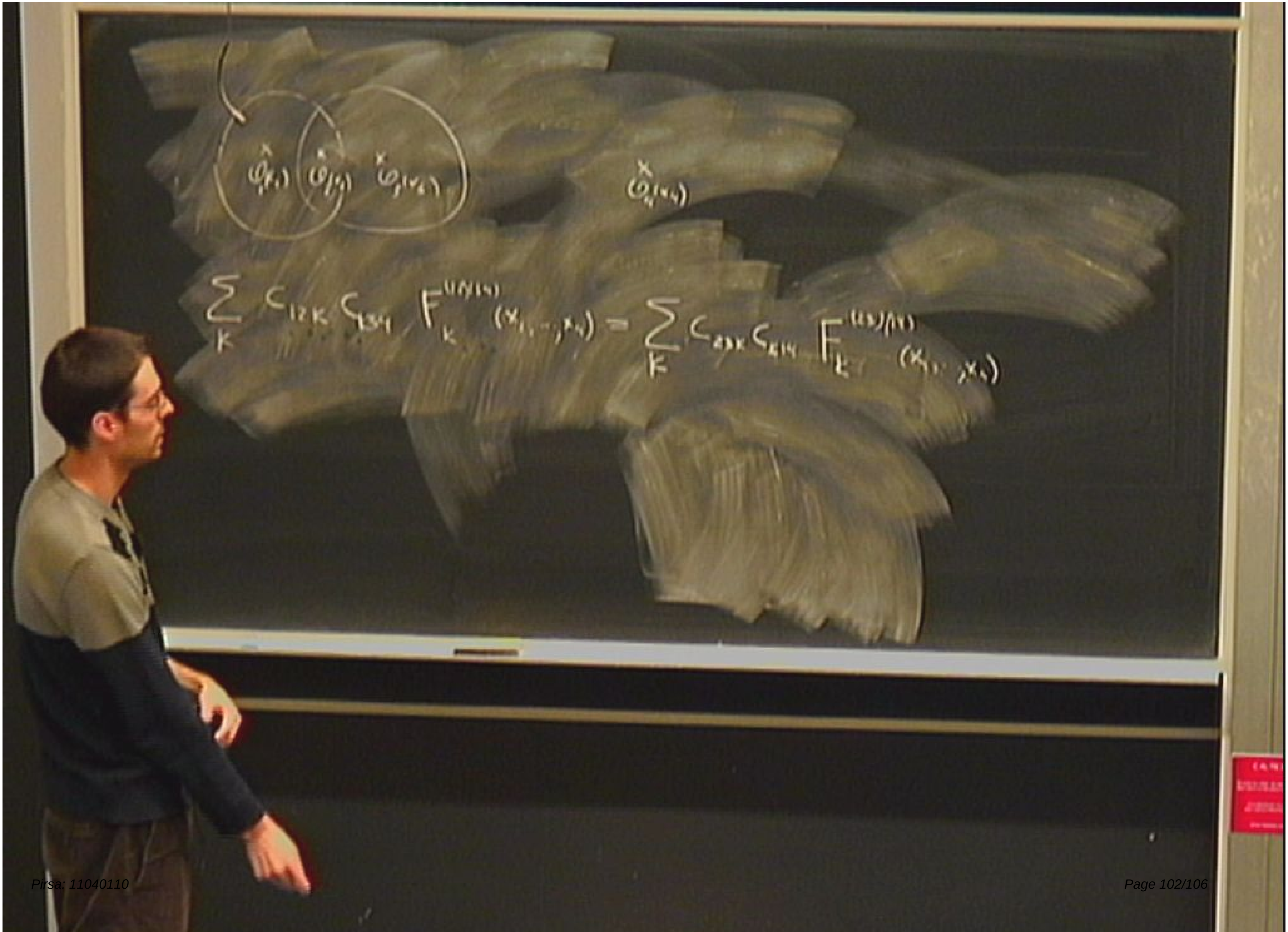
$$= \sum_k C_{12k}\, C_{k34} \underbrace{\sum_{n=0}^{\infty} b_k^{\mu_1\cdots\mu_n}(x_2-x_1)\frac{\partial}{\partial x_1^{\mu_1}}\cdots\frac{\partial}{\partial x_1^{\mu_n}} \frac{1}{|x_{13}|^{\Delta_k+\Delta_3-\Delta_4}\,|x_{14}|^{\Delta_k+\Delta_4-\Delta_3}\,|x_{34}|^{\Delta_3+\Delta_4-\Delta_k}}}_{F_k^{12|34}(x_1,\dots,x_4)}$$

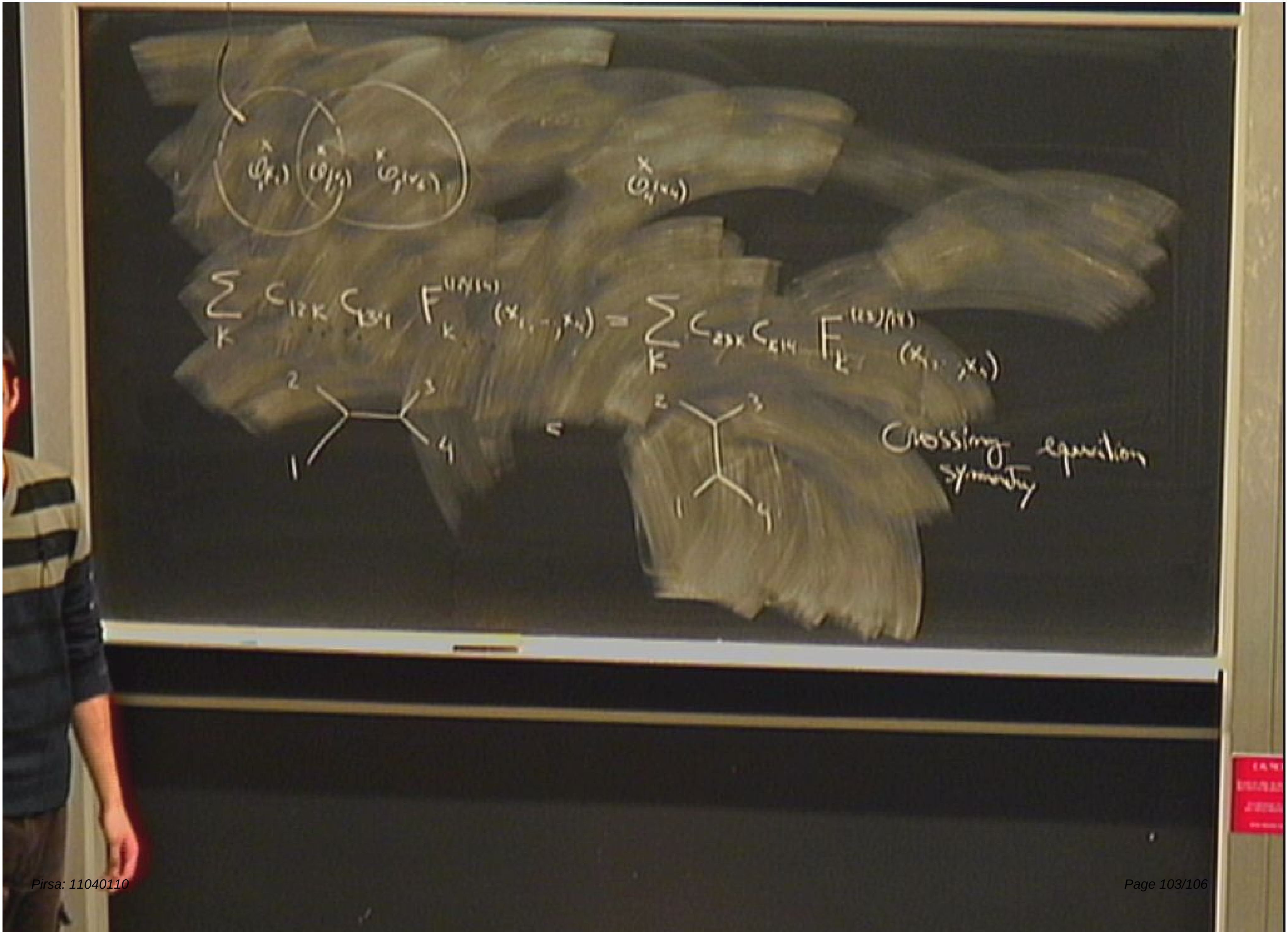
conformal blocks

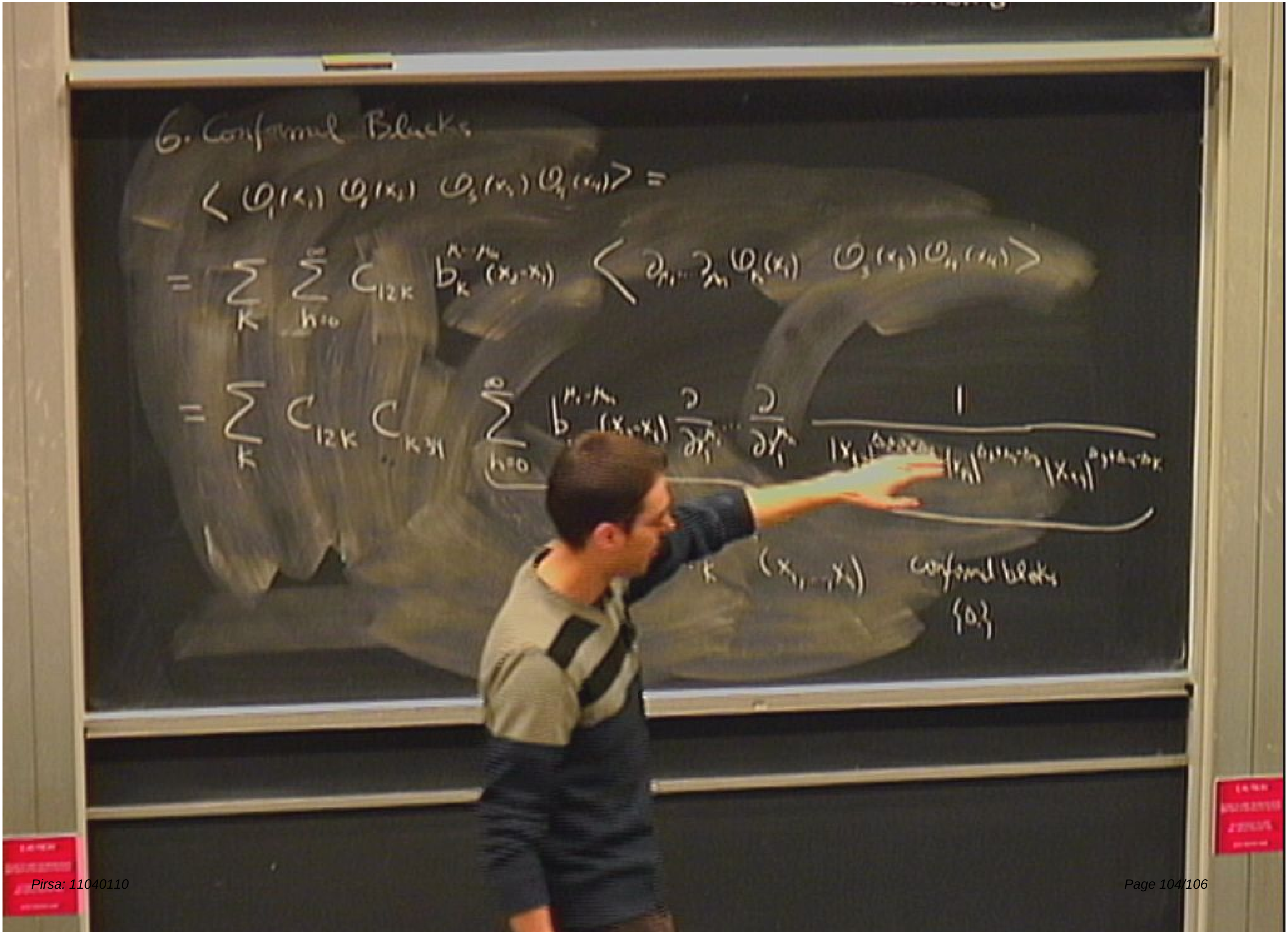

6. Conformal Blocks
$\langle \phi_1(x_1)\, \phi_2(x_2)\, \phi_3(x_3)\, \phi_4(x_4) \rangle =$
$= \sum_k \sum_{n=0}^{\infty} C_{12k}\, b_k^{\mu_1 \cdots \mu_n}(x_2-x_1) \langle \partial_{\mu_1} \cdots \partial_{\mu_n} \phi_k(x_1)\, \phi_3(x_3)\, \phi_4(x_4) \rangle$
$= \sum_k C_{12k}\, C_{k34}\, \underbrace{\sum_{n=0}^{\infty} b_k^{\mu_1 \cdots \mu_n}(x_2-x_1) \frac{\partial}{\partial y_1^{\mu_1}} \cdots \frac{\partial}{\partial y_1^{\mu_n}} \frac{1}{|x_{13}|^{\cdots} |x_{14}|^{\cdots} |x_{34}|^{\cdots}}}_{F_k^{(12)(34)}(x_1, \ldots, x_4)}$
conformal blocks
$\{0_i\}$


6. Conformal Blocks
$\langle \phi_1(x_1)\, \phi_2(x_2)\, \phi_3(x_3)\, \phi_4(x_4) \rangle =$
$= \sum_k \sum_{n=0}^{\infty} C_{12k}\, b_k^{\mu_1 \cdots \mu_n}(x_2 - x_1) \langle \partial_{\mu_1} \cdots \partial_{\mu_n} \phi_k(x_1)\, \phi_3(x_3)\, \phi_4(x_4) \rangle$
$= \sum_k C_{12k}\, C_{k34} \sum_{n=0}^{\infty} b_k^{\mu_1 \cdots \mu_n}(x_2 - x_1) \frac{\partial}{\partial y_1^{\mu_1}} \cdots \frac{\partial}{\partial y_1^{\mu_n}}$
$F_k^{\cdots}(x_1, \ldots, x_4)$

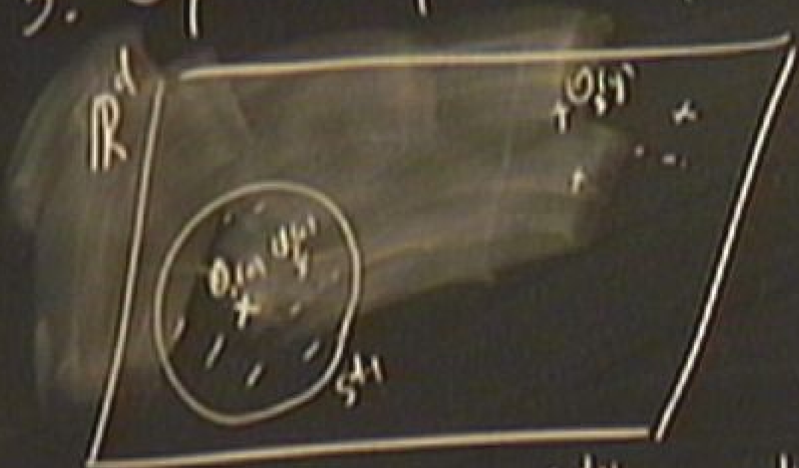



$\sum_k C_{12k} C_{k34} F_k^{(12)(34)}(x_1,\dots,x_4) = \sum_k C_{23k} C_{k14} F_k^{(23)(14)}(x_1,\dots,x_4)$


$\sum_K C_{12K} C_{K34} F_K^{(12)(34)}(x_1, \ldots, x_4) = \sum_K C_{23K} C_{K14} F_K^{(23)(14)}(x_1, \ldots, x_4)$
Crossing symmetry
equation


6. Conformal Blocks
conformal blocks

5. Operator product expansion



$$O_1 \; O_2 = \sum_i C_i(x) \; O_i$$

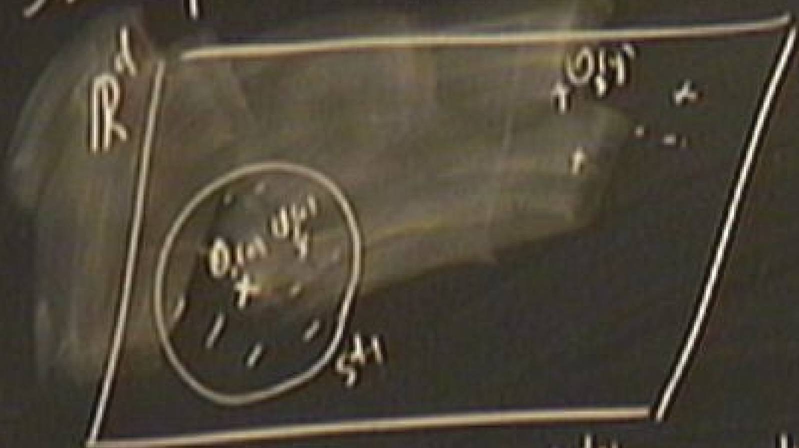
path integral inside S^{d-1} produces a state $|\Psi(x)\rangle$

$$|\Psi(x)\rangle = \sum_i c_i(x) \, |O_i\rangle$$

complete set of states

$$O_1(0) \, O_2(x) = \sum_i \underset{\text{OPE coeff.}}{C_i(x)} \, O_i(0)$$

5. Operator product expansion



$$O_1(x)\, O_2(0) = \sum_i C_i(x)\, O_i(0)$$

path integral inside S^{d-1} produces a state $|\Psi(x)\rangle$

complete set of states

$$|\Psi(x)\rangle = \sum_i C_i(x)\, |O_i\rangle$$

$$O_1(0)\, O_2(x) = \sum_i C_i(x)\, O_i(0)$$

OPE coeff.